THE GEOMETRY IN ART & NATURE

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Abstract: This study elaborates the premise that underpinning great works of art – invisible, beneath the surface, as it were – lies a geometrical infrastructure adhering to basic geometric principles. It’s the infrastructure that, to the beholder’s eye, gives the work in question its profoundly experienced sense of coherence and integrity, and that, once understood instead of just being intuited, more clearly accounts for its aesthetic appeal. Similar geometric arrangements are also often to be seen in natural treescapes, with leaves and limbs obeying relevant laws of physics and biology. As examples of these intrinsic patterns, some works of the fifteenth century Northern European artist Albrecht Dürer are examined in detail – in particular, his set of woodcuts accompanying an early edition of the biblical Book of Revelations.

1. Introduction

A painting is a two-dimensional expression creating a visual sense of three-dimensions. Leonardo wrote: “The art of painting is to take a flat surface and by embracing the forms with chiaroscuro make it rise to another plane without increasing its physical volume.” A great painting meets the viewer as an entire statement. Its two-dimensional reality encourages a design system that controls the visual surface - this is the geometry of
Art. All the elements are connected to the external edges of the work by sympathetic lines or colors.

Geometrical patterns tie a painting's subject matter to the picture plane, making a united environment that is embodied in the ratios and proportions that are expressed. These patterns are not so much an artistic solution as a tool that the artist can employ well or badly. Using this device without proper feedback can give poor results. Using it creatively, through talent or plain hard work, can furnish the maker results bound only by his or her imagination. In many other art forms, this sort of control and order is considered to be more or less obvious.

In literature, in theater, in musical composition, artists liberally and deliberately employ plot thickeners, tension builds, stage directions, programmed rhythm changes, and silence itself.

These devices create a bond between artist and audience, a suspension of disbelief that is at the heart of every aesthetic transaction. Time and space themselves, when properly acknowledged, can be considered the shared concepts that allow the reader/viewer/listener to participate in the artistic partnership. In the two-dimensional visual arts, though, the viewer is thought to see the entire field instantly. Because of this, extreme prejudice exists against the idea that the artist might have had a preconceived geometric infrastructure in mind that can only be expressed through the dimensions of space and time. When this structural idea, namely, the two-dimensional geometric subplot, is successfully integrated into the three-dimensional illusion, the formalism involved remains, as it were, invisible, and it undoubtedly contributes to achieving the desired aesthetic effect.

The secret ingredient, this geometric structure, is related to the methods that the Renaissance artists used to conquer perspective. Perspective is a wonderful tool to express an ordered spatial
relationship. However, when employed at the front of the picture plane, it often results in a "fish eye" lens distorting everything, since everything inside the frontal picture plane gets smaller as it moves back. In like manner the surface geometry should not encompass the entire form. Rather, to use a rough rule of thumb, 40 to 60 % of the enclosure can be delineated. This rule applies mainly to circles occurring on the visual surface. It is obvious that a 100 percent circle would look like an NRA target.

The viewer's attention reconstructs, as it were, an image. The surface image occupies the picture plane and is held in tension by the substructure. In the best examples of the art, the harmonious marriage of surface image and substructure unfolds slowly in a finite space of time, and new relationships and subtleties are caught by the eye in a sequence of evolving responses.

2. Historical Background

The idea that geometry is the authentic force, much like an undertow, uniting various aspects of the visual image, though seeming perhaps somewhat radical in this day and age, actually appears in the literature as early as Plato.

In *Timaeus*, Socrates argues, "But it is impossible to combine satisfactorily two things without a third one: we must have between them a correlating link.... Such is the nature of proportion...." In fact, in their quest for ideal beauty and harmony, it seemed natural for the Greeks to turn to mathematics. Plato, who developed Pythagoras's Aesthetics of Number into the Aesthetics of Proportion, wrote in *Epinomis*, "Numbers are the highest degree of knowledge," and, in the same work, "Number is knowledge itself."

It's worthwhile to stay with the Greeks a little longer. Their preoccupation with both algebra and geometry – in other words, number and space – is relevant to the argument. For
Pythagoras and the secret society he founded, numbers constituted different shapes, different ineffable entities. The connections were deep and mysterious. The members of the society were sworn to secrecy on pain of death. Their seriousness, even fanaticism, of purpose found a victim in the hapless Hippasus of Metapontum, who talked to outsiders about the group's discovery of the existence of irrational numbers. The discovery threatened the Pythagorean ratio-universe ideology, and the unfortunate gossip was put to death by drowning, on the direct orders of Pythagoras himself. The Pythagorean symbol was a pentagram, a regular polygon in which was inscribed a five-pointed star.

It's easy to show that the lines emanating from every vertex of the figure, as well as any intersections of those lines, are in the ratio of the golden mean, about which more a bit later. *Ratio* in Greek means *logos*, which also means *word*. In addition it can be seen that the five-pointed star holds an inverted pentagram, and algorithmic inscriptions of pentagrams can continue to infinity. The concept of infinity was anathema to the Greeks. The Pythagoreans rejected the infinite, along with their rejection of zero, or the void. Aristotle as well denied existence to both the infinite and the void in his theory of God and the cosmos. The insistence on this principle by the Greeks, Pythagoreans, and others, overwhelmed any opposition. The Pythagoreans as a secret society committed nothing to writing. Some two hundred years after the death of Pythagoras, Euclid of Alexandria wrote the *Elements* on a parchment roll and circulated his theorems and his ratios.

To return to the golden mean, it describes the ratio of the sides of the so-called perfect rectangle, "the golden section." The golden section is the only rectangle the gnomon of which is a square. That is, draw a rectangle such that, when inscribed
with a square on the smaller side, the resulting rectangle is in the same ratio as the original.

This repetition of a rectangle of the same ratio within the picture plane has fascinated artists throughout history – Dürer so much that he applied the concept four times in “Riders of the Apocalypse.” A gnomon is the remainder of a parallelogram after the removal of a similar parallelogram containing one of its corners. The Pythagorean theorem itself is demonstrated in the diagonal of the figure as hypotenuse (see illustration). The formula for this is the shorter side times the irrational number 1.618....

Let's now move a thousand years forward to another numerical luminary. Leonardo of Pisa, the first great mathematician of the 13th century, was instrumental in introducing Arabic notation into European mathematical practice. He was educated in North Africa, and favored the anti-ecclesiastical, anti-
Aristotelian concept of the void – that is, a zero, which allows for the digital description of numbers. It’s to be noted that the digit zero as already being used by the Hindu, Arab, and Mayan cultures, would be rejected by the universities of Italy and Europe for another two hundred years.

Not surprisingly, with its more pragmatic approach, the business community took to the cipher immediately. Zero freed calculations from dependence on the abacus, tally sticks, and other mechanical encumbrances. A double entry balance book dating from 1340 used the number zero to demonstrate equal balance. The two systems, the Arabic algorists and the mechanical abacists, coexisted for a long time, the latter having the advantage of tradition. In speed contests, though, a skilled algorist always won.

Leonardo of Pisa, known as Fibonacci, is best remembered for the number sequence that bears his name. The Fibonacci series, in which each subsequent number after 0, 1, comprises the sum of the two previous numbers, proceeds as follows: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, etc. The number progression possesses characteristics that replicate growth patterns in nature that are called phyllotaxis, meaning leaf arrangement. Many artists intuitively recognized this formation of tree branching and accordingly repositioned their landscape vistas to conform with this naturally occurring phenomenon.

Close observation reveals that nature chooses the geometry by which the leaves and flowers get the proper amount of light and water. An ancillary feature is the joy of seeing the partaking elements emerging in the observer’s eye in regular geometric shapes: circles, squares and equilateral triangles. These forms can also easily be seen in the growth structures of pineapples and sunflower seed arrangements. Another interesting feature of the number series is that if you divide any term of the Fibonacci sequence by the previous, adjacent term, the series of quotients
approaches 1.618 as the terms move higher and higher. The result is of great, perhaps not coincidental interest in that the sides of the Euclidean Pythagorean golden rectangle comply with the same ratio, 1.618!

Thousands of words have been written about the golden section, for and against, as the perfect container for the artistic statement. Aside from its historical interest, the matter is not really important one way or the other. What is significant is that the ratio represents the beginning of the division of the space in the picture plane. Everything inside the plane is of a piece, unified to the very edges of the plane in a manner germane to the proportion of the plane itself. The order is by no means a sacrosanct one guaranteeing the success of the resulting artwork. Many examples exist that show the concept used in a clumsy fashion. But with thought, hard work, and creative diligence, the division can be an effective way of organizing an artistic work.
3. Albrecht Dürer
The ideas and concepts described above find powerful, hermeneutical expression in the works and writings of the fifteenth century artist Albrecht Dürer (1471-1528). A harbinger of the great artist he was to become is evident in a self-portrait the youthful Dürer did when he was 13 years old. The silverpoint drawing is extremely delicate. The paper is coated with a solution that is sensitive to the silver rod. A very thin rod leaves a thin line just barely visible. it cannot be erased. If a thicker line is needed, a thicker rod is used, or many thin lines are combined. The drawing young Dürer produced is charming and innocent. It has a script on the top that states, “I did this portrait of myself in front of a mirror in 1484 when I was still a child.” In the drawing Dürer is pointing to the left. At the end on the index finger is a very slight line that continues a short distance from the finger, a direct graphic clue toward the artist’s statement in the script.
The directed finger represents a singular introduction to the manner in which artists take control of the forms of geometry and how they exquisitely use these forms to their advantage in organizing the work in question. The geometric forms exist in all forms of life, but artists capture and shape them so that they act unseen, beneath the surface.

The drawing by the youthful Dürer only touched the surface, as it were, with his basic geometry. A few years later, he became the consummate geometric analyst of his time. In addition, he traveled back and forth to and from Italy and so was well versed in all the discoveries of the Renaissance masters to the south. They shared many ideas in this unusually creative age.

1498, Dürer published his first large collection of prints that accompany an edition of the Apocalypse section of the biblical Book of Revelations – the Apocalypse being a subject he was very concerned about, both spiritually and artistically.

The completed publication differed markedly from other illustrated bibles of the time. To begin with, the work was entirely Durer’s. He served as both artist and publisher. Most other bibles had outside sponsors. The format also represented a significant departure. Previous bibles were composed of text with either illustrations scattered throughout the work or small illustrations sunk into the text. When, as in a few cases, the illustrations were more prominently featured, the accompanying text consisted of a minimal caption. By contrast, Dürer clearly intended the illustrations to virtually bestride the text (sacred though the biblical words are), inasmuch as the text be significantly more important than the text only appeared on the back side of the print.

Another feature, in the prints related to the Apocalypse, was his mixing of the order of events in the illustrations, whereas the text itself is presented in the traditional order.

Thus the reader experiences two distinct narratives – the chronological progress of the text and the spatio-temporal impact of
Although he did not invent the illustrated Bible, Dürer absorbed and abstracted the contents of the preceding editions such as the Quentell Bible of 1479, the Koberger Bible of 1483, and the Gruninger Bible of 1485. Whereas these works suggested much of the general subject matter and some indication of the structure, they were visually very primitive. Dürer had been to Italy where he’d met Mantegna and Leonardo. He was a master geometrician conversant with the best mathematicians in Europe. He was part of the Renaissance but in relation to the German renaissance, Dürer’s influence was singularly pervasive. The artist was totally committed to the graphic arts and to the vast communications possibilities that the print medium offered. This is not surprising since as an artist, he worked on the cutting edge of this new means of expression, just some forty years after the invention of the printing press by Gutenberg. The publication of Dürer’s intense and energetic prints exercised an enormous impact on German art in particular and on all of European art in general.

The very nature of preparing a sketch for printing was that it required many drafts and studied changes. The finished selection was glued on a pear tree plank, so that the carver could cut away the parts that were to be left white, leaving the raised surface to be inked black. Any area that was to be a large white section was carved deeper to limit the possible inking of the part by a faulty rub of the burin.

This attention to the smallest detail was necessary since the number of prints could be endless, and Dürer’s self-imposed standards were demanding. His control of chiaroscuro was elegant, inasmuch as it gave a deeper spatial effect to the two dimensional media. Although he completely understood perspective as a three-dimensional measure, he never used it in the Apocalypse series but instead chose Geometry as the creator of space. The choice is interesting given that St John’s Book of Revelations abounds with
numerical classifications of life and death with unrelenting repartitions. There was an expectation throughout the continent of Europe that the grand Apocalypse would be 0ccur in the upcoming centennial year 1600.

A study of Dürer's graphic productions, his woodcuts in particular, demonstrates a consistent and profound investigation of how to totally control the picture plane. The work comprises Dürer's brilliant integration of three-dimensional illusion comprehended in a two-dimensional substructure. In the Apocalypse series, this geometry is laid out in precise harmony with the numerology contained in the text printed, as noted, on the reverse side of the page. The reader was thus obliged to study the Dürer illustration before reading the text, which Dürer had reordered and simplified. The original text without illustrations ran on and on with redundancies and repetitions in order to make its various points. To encapsulate the excesses of the text, Dürer used two principles: concentration and, as continually supplied by the text, a powerful visualization of the heavenly fierce wrath raining down upon the sinners on Earth. The concentration was necessary since space on the back side of the 15 prints was condensed to 8 pages of 6 point type in the Bible. Generally in his geometric configurations, Dürer employed basic shapes that could not be compromised, e.g., equilateral triangles, right angle triangles, and rectangles in the same ratio as the original picture plane. Also evident are squares and circles. This was basis of the unseen organizing elements.
The numerology contained in the original text refers to images not included in the St John’s original. There are references such as “St John and the Twenty Four Elders in Heaven”, “St John’s Vision of Christ and the Seven
Candlesticks”, “Four Angels Staying the Winds,” etc. In each case, if the title didn’t contain a number, the number was to be found within the text itself. Only the first print, “The Martyrdom of St. John,” doesn’t possess a number that can be assigned, this because the Roman Emperor prosecuted all Christians. Being submerged in boiling oil, however, did not kill St John – he was the only apostle to die of natural causes. For the woodcuts, Dürer took numbers from the text and translated them into geometrical statements that are only apparent after careful analysis. For instance, in the fourth illustration, ”The Four Riders,” he interlocks four rectangles of the same ratio as the original picture plane:
Further, in the ninth print, "St. John Devours the Book," the text mentions "seven thunders." Dürer starts diagonal lines from the two lower corners of the picture plane, and these delineate two squares, which deviate by exactly seven degrees.
Lest this be taken as a coincidence, one can discern in the picture exactly seven instances of this event! In the twelfth plate, "The Sea Monster and the Beast with Lamb's Horns," the
text reveals that the Beast shall inhabit the Earth and blaspheme and defile at will for 42 months – the rectangles in the picture tilt at precisely 42 degrees! The triangle is 42 X 48 x 90 degrees.

These results are but appetizers in the grand cookbook of Geometry and Art. In his writings, Dürer remarks, “And since geometry is the right foundation of all painting, I have decided to teach its rudiments and principles to all youngsters eager for art... I hope that this my undertaking will not be criticized by any reasonable man, for it may benefit not only painters, but also goldsmiths, sculptors, stonemasons, carpenters, and all those who have to rely on measurment.”

Thus the enterprise continues – moving on from Dürer to Veermer, Chardin, Ingres, Cézanne, and de Kooning, as well as to many others. Some artists cling tightly to the geometric tradition. Others, like De Kooning, like to find order in the chaos they create. In retrospect, Dürer himself represents the epitome of geometric design – likely since The Book of Revelations itself overflows in numerical references. As we’ve seen, both Dürer and Pythagoras loved numbers and the metaphorical shapes they represented with an all consuming passion.
A GUIDE TO FINDING GEOMETRIC STRUCTURES THAT ARE USED IN ORGANIZING THE VISUAL ARTS

THIS IMAGE OF DURER’S FOUR RIDERS SHOWS THE APPROXIMATE PERCENTAGE OF THE CIRCLE THAT IS USED IN THE SURFACE IMAGE OF THE ILLUSTRATION. IN THIS CASE IT IS ABOUT 60 PERCENT. THIS ALLOWS THE VISUAL IMAGE TO WORK WITH THE HIDDEN ORGANIZATION WITHOUT LOOKING LIKE A TARGET. USING THE TEMPLATE LOCATOR
WITH PHOTOSHOP SIMPLIFIES THE TASK BUT A COMPASS WILL ALSO DO THE JOB. WITH PRACTICE, THE GEOMETRY OF NATURAL CONFIGURATIONS SUCH AS PHYLLOTAXIS (LEAF ARRANGEMENT) CAN BE FOUND IN THE WOODS. IT CAN BE TRYING, BUT IT IS STILL REWARDING.
St. John was exiled to the penal colony of Patmos by Emperor Domitian because he was a fiery Christian. He wrote *The Book of Revelations* at age 92. Christ, the alpha and omega figure, appeared and dictated the text. John was warned by an angel not to change a word, since it was an apocalyptic message of Christ’s Second Coming and, as such, the final judgement upon mankind’s miscreants. As noted, Dürer severely edited the text.
As also noted, his illustration was first seen, the text itself being on the other side of the printed woodcut.

This second woodcut did have a numerical clue and it was Seven. Dürer did not use perspective, since it wasted space and was more of a problem than a solution. His geometric order was a super control system without being visible or hindering the illustration. As for the seven mentioned in the *Revelations*,
there were seven churches in what is now Turkey, seven spirits of God, and seven stars in the right hand. The stars, Christ said are the angels of the seven churches, and the gold candlesticks symbolize the seven churches.

There are seven twenty-four degree triangles and two rectangles of the same ratio as the original rectangle. The illustration has two-thirds of the original rectangle busy in heaven, with the remainder in the rural countryside of Earth.
The geometric graphic interlaces the art work like a hand in a glove without having to be aware of the connections. There are also six circles and four squares, all working in a catholic manner that embraces the heavens floating over the pastoral scene.
THE FOURTH WOODCUT['S HOMAGE TO FOURNESS

THE FOURTH RECTANGLE COMPLETE THE TIGHT INTEGRATION OF GESTURE AND GEOMETRY PRODUCE PURE BEAUTY
The fifth seal was to give white robes to protect the chosen of the twelve tribes of the sons of Israel. The sixth seal brought to Earth a violent earthquake where the sun was black as sackcloth and the moon was blood as the stars fell on Earth. The force moved mountains and islands where kings, princes, the rich and the strong, and bondmen and freemen were ensconced, hiding from the fury of the heavenly action.
6. The Angels Stay the Winds

The angels, who were prepared to hurt the Earth and seas, were commanded to cease their mission until the tribes of Israel – namely, Judah, Reuben, Gad, Asher, Naph’tali, Mana’she, Simeon, Levi, Issachar, Zeb’ulun, Joseph, and Benjamin: all 12 thousand of each. “Amen! Blessing and glory and wisdom and thanksgiving and honor and power and might be to our God for ever and ever! Amen.” And thus the chosen were signed in the calm and still as the Angels held the wind.
7. The Seven Trumpets

An angel with a censer filled it with fire and threw it onto the Earth, which drew forth peals of thunder, voices, flashes of lightning, and an earthquake. A third of the Earth was burnt up. The next angel blew his trumpet and a great mountain burning with fire was tossed into the sea, and a third of the sea turned to blood, and a third of the creatures in the sea died. The star Wormwood was tossed into the sea, making it bitter and killing of the men who drank it. An eagle screamed “woe, woe, woe to all who dwell on Earth. What follows is worse!”
8. The Battle of the Angels

The battles of the angels were the remaining woes. The cavalry horses hurled fire and sulphur, saturating the air that mankind inhaled. The other power of the horses lay in their tails. These were like serpents or scorpions, flailing around and wounding all creatures in their way. It may be true that the unrepentant members of mankind thought that choosing the final judgement might be easier to take than the wrath of the angels and the calvary, since they could still enjoy the pleasures of their sins during the wait.
9. St John Eats the Scroll

An Angel came down and had one foot in the sea and the other on land. He’d brought with him a small scroll from heaven. His face was like the sun and his voice was like a lion roaring, a vast thunder. The angel said, “Seal up what the seven thunders said and do not write it down. Take the scroll from the angel.” The angel continued, “Take it and eat, it will be bitter in your stomach but sweet as honey in your mouth.” And St. John found this to be so. The angel said, “You must warn many people and nations and tongues and kings to be aware of the consequences.”
The woman clothed in the sun was preparing to give birth very soon and needed a place of shelter and calm, since the seven headed serpent lusted after her child and wanted too eat it. God gave the woman the wings of a great eagle, and she flew off to a protected place where she and her male child were nurtured for 1,260 days or 42 months (a month at that time was 30 days). The serpent vomited forth a river hoping to carry the woman away from safety, but, the earth swallowed up the river denying the serpent’s wish.
11. The Archangels Drive Satan Down to Earth

There was a great battle in heaven between Michael and his angels and Lucifer and his angels. It being a battle among angels, none of them could be injured, yet God wanted Lucifer to be cast down to Earth, because he was challenging authority and was therefore unwelcome. So Lucifer became Satan, or the devil, and was driven down to Earth. And it was heard, “Woe to you of Earth and sea, for the Devil has come down to you in great wrath, because he knows that his time is short.”
In addition to the beast with seven heads, another rose from the sea. It looked like a leopard, with the feet of a bear and the mouth of a lion, and all the powers of the first beast. It was the power to blaspheme the earth for 42 months. The beast opened his mouth and, like a dam bursting, a constant curse against God, his tabernacle, and them who dwelt there, poured forth. In the war against the saints, the aim was to overcome them and gain power over the tribes of Israel.
There was great multitude of all nations, tribes, peoples, and tongues before the lamb, clothed in white robes, with palm branches in their hands. They chanted and sang, “Amen, Benediction and glory, wisdom, honor, and power to God and the lamb for ever and ever, Amen.” The lamb stood on Mt. Sion and withstood a hundred and forty-four thousand with God’s name written on their foreheads. And they sang the words that only they knew, with love and devotion.
14. The Harlot of Babylon

FOR HER SINS AND CALAMITOUS WAYS AN ANGEL THREW A HUGE MILLSTONE INTO THE SEA CAUSING A MIGHTY WAVE WHICH SWEPT AWAY THE GREAT CITY OF BABYLON AND ALL ITS EVILNESS

amen

She sits astraddle of the seven-headed serpent holding up her chalice of wine inviting all to join her in corporeal pleasures. The merchants of Earth have grown rich by the power of her delicacies. She shares her lust for the finest encapsulations of Earthly delights that can be discovered again and again. The flavor of her wine has nose, a perfume that enhances boldness and courage in a life of repititous beginnings such as daybreak and sunset. A meeting of the senses offered in the great city of Babylon brings the repititous into ordered habit.
15. New Jerusalem

NUMBER 15, THE FINAL APOCALYPSE WOODCUT AN ANGEL SHOWS ST JOHN THE NEW JERUSALEM. ANOTHER ANGEL HAS BOUND SATAN FOR 1,000 YEARS AND THRUSTS HIM INTO THE BOTTOMLESS PIT. AT THE END OF THIS ERA, SATAN IS RELEASED FOR A WHILE NEW JERUSALEM LOOKS LIKE NUREMBERG DURER’S HOMETOWN

There came down from heaven an Angel with the key to the bottomless pit and a great chain. The Angel bound the evil Satan for a thousand years. This began at the end of Babylon or pagan Rome until the efforts of Gog and Magog against the church, toward the end of the world. The second resurrection will be that of the body and the day of the general judgement. The bottomless pit was sealed shut so that Satan should no more seduce the nations till the thousand years be finished. And after that he must be loosed again, for a little time, into the world.