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Cultural Analysis Situs [°]

"**Space** *is the order of the coexisting …*
Situs *is the mode of coexistence."*
Gottfried Wilhelm Leibniz

"This is where the book is going (into n-dimensional space)."
Gilbert Strang

The disciplines of complex network science, of art and cultural history, and of computation have a common ancestor in the *analysis situs* of Gottfried Wilhelm Leibniz. Unfortunately, this shared conceptual origin remains hidden so far within a history of science that is tragically bifurcated, due to the branching evolution of disciplinary focus, due to changes in language, and due to sometimes forced scholarly migration. This chapter breaks the mutual tear lines of citation between disciplines to enable a common future. What lies at stake is the surprisingly deep-rooted and shared foundation of the emerging enterprise of *a systematic science of art and culture*.* This enterprise currently flourishes mainly in departments of multidisciplinary information science, network and complexity science, and applications in industry. It promises nothing less than an integration of humanistic inquiry and a physics of cultures and cultural production.

TALES OF THE ANCESTORS

Every academic discipline has its own creation myth. Leonard Euler, for example, functions as a holy saint or hero of *network science* as introductory textbooks trace the root of the field to mathematical *graph theory* and further in particular to Euler's *solution* of the so-called Königsberg bridge problem.* The popular and highly worthwhile network science introductions of Barabási 2016, Estrada 2015, and Caldarelli & Catanzaro 2012, for exam-

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* Notes and references see pages 21 to 36.

ple, mention Euler's contribution explicitly as the "birth", the "start", and the "foundational moment" of network science, while Newman 2010 characterizes the Königsberg solution, more cautious, as "one of the oldest proofs in graph theory". Newman 2018 curiously cuts out the anecdote.*

Euler gained his position in network science by solving the riddle, if it was possible to walk all seven bridges in the city of Königsberg without crossing a single bridge twice. His solution brought proof that there is no such viable *path*. To some, Euler's insight even verges on the sacred. The complexity scientist Stuart Kauffman, for example, mentions the episode as an instance of quasi-divine inspiration, that cannot be the product of "algorithmic thinking".* Consequently, neither Kauffman nor the mainstream network science textbooks discuss the fact that the Königsberg solution is in fact explicitly rooted in Leibniz.

Similarly, Aby Warburg has acquired quasi-mythical status as a quasi-messianic hero of *art history* and *cultural studies*. He is most frequently cited as the father of *Iconology* and *Mnemosyne*.* Both are famous methods in art history and cultural research, the first still in use, the second a much discussed yet enigmatic product of 1920s scholarship. Mnemosyne is of broader interest as it currently experiences a silent Renaissance in literally millions of *Pinterest boards*, and more explicitly in combination with methods of *computer vision*, particularly so-called *deep learning*, a branch of *machine learning* or *artificial intelligence*.* Warburg's status in art research is underscored by an abundance of research about himself, seemingly eclipsing applications of the more radical approaches for which he and his circle are famous.*

Other similar foundational figures of disciplinary creation would include Charles Darwin for evolutionary biology, Plato and Aristotle for Western philosophy, Newton and Einstein for physics, Turing and Neumann for computation, Raphael and Winckelmann for classical archaeology, Max Weber for the social sciences, Ramón y Cajal for neuroscience, or Roberto Busa for the so-called digital humanities. Not by conincidence, the embellished, often purely male stories of the associated disciplinary initiation mostly follow the narrative structure of other "*imagined communities*"*, from illuminating the life of heroes at the root of princely states in the Hellenistic era to the prophets, messiahs, and saints at the root of many religious congregations. Much like the accounts of Romulus and Remus, the wulf-fed founders of Rome, or Telephos, the panther-fed founder of Pergamon, son of Hercules, Newton's proverbial "*giants*" on whose "shoulders we stand" are often presented in their respective narrative as examples of virtue, whom it is worthwhile to follow within our respective discipline or social group.*

The goal here is not to validate such heros or establish new messiah figures to letigimate yet another discipline, as limited as any other.* Instead we strive for an integration of multiple disciplines and perspectives, complementary and overlapping as needed. Our goal is similar to the necessary realization that *the meaning of human existence** is characterized by living in an ecosystem that spans the whole planet, all species, and where we simply cannot ignore the existing structure, interactions, and dynamics beyond and across the self-chosen or inculturated *filter bubbles* of our own limited minds, our disciplines, and our own tribes.* Opposed to creating new absolute heroes, all this chapter intends to do, is to break the mythical tear lines of *network science*, *art history*, and by coincidence *computation* towards one of their most recent common ancestors. The story told here consciously breaks with some of the *TV Tropes* of first-semester lectures and keynote slide sequences in related fields.* And while the presented story is just as true as the established ones, it again is but one of many tracing back a path in the river delta of all possible versions of the *Ancestor's Tale*.* The novel or, one should say, renovated "*genius*" figures of our story are much closer to the original Roman meaning of the word, intermediary "*spirits of becoming*", rather than absolute, exclusive, or princely targets of divine inspiration or adoration.*

EULER'S PATH

The first tear line that we are about to break is to see Leonard Euler's Solution of the Königsberg Bridge problem as the root of graph theory and network science. While Euler's solution, first published in 1736*, is widely reported and prominently used as a textbook example, it is obviously not widely read or studied. The biggest inhibitor for direct study seems to be that the article is written in Latin, back then the most prominent so-called *lingua franca* or common language of scholarship, competing with French, then German, and eventually English. Yes, a translation into English is available, but still the situation is a first source of disciplinary bifurcation obviously, especially preceding the now increasing abundance of *visualization* and *scientific figures* as the prime mode of global multidisciplinary communication.* If more scholars of modern network science would have read Euler's article, its status as a *prime object* of network science would be less likely.* Indeed, Euler himself would probably strongly disagree. Euler's title contains the first hint, namely that the article provides a solution regarding an older established issue, what Euler calls "geometriam situs", literally translated to English and German – and this is important for later – as the *geometry of position* or *Geometrie der Lage*. Furthermore, literally in the first sentence, Euler points to his own source, namely the writings of Gottfried Wilhelm Leibniz. Euler does not provide a proper scholarly citation that would satisfy the conventional formatting standards of a graduate studies committee in a modern English literature department, yet he characterizes the issue well enough, so we can identify the source with surgical precision. In fact, there can be no doubt that Euler indeed refers to what Leibniz himself calls *analysis situs*, the *analysis of position* or *Analyse der Lage*.⋆

Euler, continuing in paragraph one, first differentiates Leibniz's "almost unknown" geometry of position from regular geometry: While regular geometry deals with measurement, magnitude, and distance, the Königsberg bridge problem, Euler points out, can be solved without distance, focusing only on postions and properties of positions. Put in simple words, the problem can be solved without knowing the length of the bridges or the angle between consecutive bridges. Case in point, in paragraph two, Euler furthermore generalizes the problem by replacing the central Kneiphof island in Königsberg with the *symbol* "A", and the seven bridges with the symbols "a, b, c, d, e, f and g". Consequently the Königsberg bridge problem reduces to a combination of symbols for positions and symbols for properties of positions. In the words of more recent *graph theory* we are looking at a problem of *vertices* with connected *edges*, or according to modern *network science* a problem of *nodes* with adjacent *links*.⋆ Yet, let's not blind ourselves by Euler's approach prefiguring our own concept of network science, as the implications are even more profound. Indeed, taken together, it seems crystal clear that Euler's approach in this specific case is firmly rooted, arguably yet more broadly, in Leibniz's foundational concept of mathematics.

LEIBNIZ'S ANALYSIS SITUS

Leibniz's foundational notion of mathematics, which Euler picks up via one source or another, becomes clear most succinctly in a sidenote of Leibniz's *mathesis universalis*. This work in turn, as Ernst Cassirer outlines, seems to develop in reaction and opposition to René Descartes' concept of geometry. As such, Leibniz's *mathesis universalis* provides both, and this is curious, a base for *calculus* (Infinitesimalrechnung), which is all about numbers and magnitudes, and second for *analysis situs*, which is the new form of geometry Euler is referring to, without distance or magnitude. According to the 1910 interpretation of Ernst Cassirer, here translated from German ...

> "... mathematics is hereby not the general science of *magnitude*, but of *form*, not of *quantity*, but of *quality*. *Combinatorics* hereby becomes the proper foundational science: If one does not understand it as the doctrine of the *number* of connections of given elements, but as the universal representation of possible kinds of relation in general and their mutual dependence."⋆

I consciously quote Leibniz here via the interpretation of Cassirer, as he will emerge as one of our most central spirits of becoming. His translation notably also includes the emphasis of the terms *quality* versus *quantity*, *relation* versus *magnitude*, and *kinds* versus *number*, which includes a consequential abbreviation in relation to Leibniz original text. But let's stick with Leibniz for a moment. In yet another fragment, titled "*Initia Rerum Mathematicarum Metaphysica*", which initially carried the more simple title "*De calculo situm*", Leibniz further lays out the relevant foundations:

> "If a plurality of things is the given status of existence,
> nothing opposing involved, existence is said to be **Simul**."
> "**Tempus** is the order of existence of those that are not simul."
> "**Duratio** is the temporal magnitude."
> "**Spatium** *is the order of coexistence* or
> the order of existence between those that are simul."
> "**Extensio** is the spatial magnitude."
> "**Situs** *is the mode of coexistence*.
> Therefore not involving quantity, but quality".
> "**Quantitas** or magnitude is, what can only be known
> in things compresent (or in simultaneous perception)."
> "**Qualitas** however is, what can be known
> in things with one observation."*

What is crucial here is that Leibniz's definiton *from first principles* allows for qualtitative coexistence, including a definition of space without or at least preceding a notion of extension or magnitude. We may also add, at least in Cassirer's interpretation, that not only space but also *form* is defined without or even in opposition to magnitude, implying a *situational configuration*, yet not necessarily a shape with an associated *extension*. This notion is of great consequence for a systematic science of art and culture, as Cassirer's interpretation may be considered in some sense more radical than Leibniz's himself, at least if we follow Vincenzo de Risi's recent interpretation of Leibniz's analysis situs in the 2018 "Oxford Handbook of Leibniz", which takes into account some so-far unpublished and unexplored manuscripts.*

Combining all the definitions as given above, one could say, maybe naively, that Euler's Königsberg *symbols* and symbolic *relations* sum up to a *symbolic form* constituting what we now call a topological *space* of symbols that does not necessitate a measured extension. As such, the situational configuration precedes a *projection* of the situation onto a screen or a piece of paper, much like the configuration of a complex network precedes network visualization in a node-link diagram. Put in other words, Euler's solution, and successively mathematical *graph theory* and modern *network science*, can be seen as specific instances, even if creative misinterpretations, of Leibniz's core definition of *analysis situs*. Yet, this is not all. *Nodes*, *links*, and *networks* are but one possible instance of such *symbolic positions*, *symbolic relations*, and *symbolic form*.

POINCARÉ'S ANALYSIS SITUS

Network science, since the early 2000s, has focused mostly on networks of *nodes* and *binary links*, which can be imagined as *points* and *line segments* in a network diagram. Currently, in 2019, this notion is increasingly extended towards a more general *topological data analysis*, that includes higher-order aspects of *network geometry*, including situations of points, edges, triangles, tetrahedra, etc., i.e. so-called *simplices*, the plural of *simplex*, and so-called *simplicial complexes*, i.e. combinations of said simplices.∗ While recent work in this area already revolutionizes data-driven network science, neural science, biology, and machine learning, the new practioners of topological data analysis are very explicit about their own roots in the history of mathematical *topology*. Maybe curiously, via a single hop, the field in essence roots itself in yet another *analysis situs*, as developed by the French mathematician Henri Poincaré, starting with a first publication in 1895 including a definition of higher-order simplices called *generalized tetrahedra*.∗ In Poincaré's *analysis situs* there is no explicit reference to Leibniz, yet like in Euler's case, here too, we can break the tear line. Indeed reading the following lines from Poincaré's 1910 article on "The Future of Mathematics" there can be no doubt:

> "There is a science called Analysis Situs and which has for its object the study of the positional relations of the different elements of a figure, apart from their sizes. This geometry is purely qualitative; its theorems would remain true if the figures, instead of being exact, were roughly imitated by a child. We may also make an Analysis Situs of more than three dimensions. The importance of Analysis Situs is enormous and cannot be too much emphasized; the advantage obtained from it by Riemann, one of its chief creators, would suffice to prove this. We must achieve its complete construction in the higher spaces; then we shall have an instrument which will enable us really to see in hyperspace and supplement our senses."∗

Given this passage, it won't surprise the reader, that Poincaré was commissioned to edit the mathematical writings of Leibniz for publication a couple of years before he began developing his own *analysis situs*.∗ True, Poincaré's work, extending over Klein, Riemann and others, provides a rigorous foundation of modern mathematical *topology*, which very much transcends the larger system of Leibniz, yet it is also obvious that Poincaré's *analysis situs* is inspired by Leibniz's core definitions, as given above. In sum, we should not blame Poincaré for the ommission of an explicit citation or his apparent lack of engagment as an editor. It would take yet another century to fully edit the prolific and fragmented mathematical manuscripts of Leibniz. Yet still, it is interesting and should be noted that the basic core concepts of Leibniz's *analysis situs* are a common ancestor of both network science and mathematical topology in a broader sense. Yet again, this is not all. The *afterlife* of Leibniz's *analysis situs* that we are still about to explore is nothing short of epic.

CASSIRER AS THE MISSING LINK

To break the tear line towards art and cultural history, it makes sense to take a closer look at the works of Ernst Cassirer, who already put some essential nuts into the chocolate above. Cassirer is best known to a broader audience as a Kantian philospher due to a famous 1929 debate with Martin Heidegger on "The Magic Mountain" in Davos, a future Jewish emigrant versus a future Nazi party member.* In academia, Cassirer is best known for his "*Philosophy of Symbolic Forms*", which was first published in three volumes in German from 1923 to 1929.* This happened at a time, when Cassirer's favorite scholarly "hang out" was the famously weird yet excellent library of Aby Warburg at the University of Hamburg.* Cassirer later summarized and extended the three volumes of his *philosophy of symbolic forms* in "An Essay on Man", published in 1944 shortly preceding his death.* The Essay on Man contains chapters on myth and religion, language, art, history, and science. Unfortunately, Cassirer's work is rarely considered looking at all multidisciplinary aspects in their entirety and mutual co-dependence. Highly specialized scholars, which are the vast majority across all fields, are much more likely to read the respective book or chapter closest to their own core interest, which explains, why there is another tear line to break in the first place. What makes the situation even more difficult, particularly from the perspective of art and cultural history, is that the Essay's chapter on *science* "does not of course claim to give an outline of a *philosophy of science*", as according to Cassirer's first footnote in the chapter, he has discussed this issue in previous works, including two books published in 1910 and 1921, the first titled "*Substance and Function*" ("Substanzbegriff und Funktionsbegriff"),* while the second is on "Einstein's Theory of Relativity considered from the epistomological standpoint".* To provide a taste for the level of thought, just consider that the manuscript for the latter was proofread by Albert Einstein himself. Both books, together outlining a philosophy of science are now more obscure, typically to be ordered as a hardcover rather than readily available as a paperback in your local university book shop. Yet, as we will see, both are of key relevance for the mission of a systematic science of art and culture.

To provide an impression how acquainted Cassirer was with matters of natural science and mathematics, before he joined the Warburg circle in 1920, when he embarked on his *philosophy of symbolic forms*, let's consider the following: Cassirer's very first book of 1902 summarizes "Leibniz' system", meaning Leibniz's body of work considering a broad range of multi-disciplinary aspects. This means, like Poincaré a decade earlier, Cassirer was deeply familiar with the work of Leibniz. Already in his first book, Cassirer discusses "the geometric problem of space and the analysis of position" as an issue of central importance.* In "Substance and Function", Cassirer then outlines an up to date *philosophy of science* in which Leibniz's sidenote, as quoted above, again takes an even more central *position*.* As a whole, the book on "Substance and Function" emphasizes the modern shift in science from *things* to *relations*, or as the title suggests from *substance* to *function*. This not only foreshadows the inclusion of network science into ever more disciplines by a larger part of a century on average. It is also aligned with contemporary mathematics, including the geometry of David Hilbert, and natural science including the cognitive psychology of Helmholtz.* Indeed Cassirer adopted Helmholtz's notion of cognition, which itself foreshadows ideas that only became mainstream in US resesarch much later with the so-called *cognitive revolution* after World War II.* It is this *situation*, with Cassirer at the cutting edge of scientific understanding, that silently sets the base for Cassirer's own *philosophy of symbolic forms*, as later applied to the domains of myth, religion, language, and art. To understand the consequences better, we need to further discuss the notions of *space*, of *symbols*, and *relations*.

CASSIRER'S SPACE OF SPACES

The relevant notions of *space* become clear via a brief excursion into art history. Artworks are often situated regarding the three dimensions of *Euclidean space*. This obviously makes great sense in museum inventories where artworks of the traditional genres of painting, sculpture, and architecture are characterized and identified according to the dimensions of width, height, and depth. It becomes more questionable, when more conservative art historians try to convince you that these three dimensions of Euclidean space are still the foundation of art research, as outlined in a 1893 lecture by August Schmarsow, who claims that "the three directional axes are defined even in the smallest cellular nucleus of every concept of space".* Some more recent specialist practitioners in *geographical information science*, *digital art history*, or *computer applications in archaeology* may want to convince you of the same, depending how exclusive their respective imagination and database systems center on the (re)construction of three-dimensional objects.*

Since the 19th century, art historians have of course extended over such a simple Euclidean notion of space.* In 2003, art historian David Summers, for example, has published a massive book titled "*Real Spaces*", which deals with a variety of notions, including *virtual space*, *social space*, and *metaoptical space*, the latter according to Summers is "the universal metric space of modern Western physics and technology" or the "modern Western co-ordinate space", which allows us to refer to objects and places that are beyond our immediate field of view. By now, we know that such measured "meta-opticality" is neither "Western" nor "modern".* Indeed, neural scienctists have discovered *grid-cells* in the brains of rats, which allow them to represent their environment beyond the immediatly visible in a hexagonal grid.* So intrinsic grid-based metaopticality, while admittedly not infinite, seems to be around for quite some time. Other than that, Summers is on the right track, when he contrasts the metric notion of *cartesian* "co-ordinate space" against "more primordial" notions of space that are "qualitative". Unfortunately, this is where we find yet another tear line: The crucial starting point for Summers is one of the most central texts of 20th century art history: Erwin Panofsky's "*Perspective as Symbolic Form*", which was first presented as a lecture in 1924/25 at the Warburg library.* The title of this text and lecture is obviously a direct reference to *symbolic form*, meaning the central concept of Ernst Cassirer's philosophy.* Summers follows Panofsky and indirectly Cassirer in differentiating the notion of metric coordinate space from what he quotes as *psychophysiological space*. And here, it is worthwhile to follow the lead, to find out what exactly Summers has in mind.

Panofsky himself explains, speaking of perspectival drawings, that "perspective transforms psychophysiological space into *mathematical space*." With "psychophysiological space", Panofsky explicitly refers to a combination of "*visual space*" ("Gesichtsraum", literally *facial space*) and "*tactical space*" ("Tastraum"). Both of these Panofsky considers "anisotropic and unhomogenous in contrast to the *metric space of Euclidean geometry*". In short, both Summers and Panofsky set up a dualism between the space of immediate perception and the space of 3D orthogonal measurement and construction.

From here we can break the tear line again. Panofsky's lecture on "Perspective as Symbolic Form" curiously coincides with the completion of the second volume of Cassirer's "Philosophy of Symbolic Forms". One does not need computational *paraphrase detection* or *sequence alignment* to see how Panofsky's contribution behaves to Cassirer's original text.⁎ As Max Friedländer once put it in relation to art: "The original is like an organism, the copy is like a machine."⁎ It is clear that Panofsky either read Cassirer's manuscript, or both had a substantial chat at the proverbial whiteboard. Cassirer indeed not only uses the same concepts in a strikingly similar but different and much more profound argument. Cassirer not only differentiates *psychophysiological space* from the *metric space* of Euclidean geometry in a meaningful dualism. In not more than a fulminant three pages, indeed, Cassirer concentrates all his previous thought on the notion of space, while extending it to the mythical domain. As the relevant definitions of space transcend the limits of language, Cassirer brilliantly plays a *naming game*,⁎ using a variety of characterizations for particular notions of space, which eventually may have confused Panofsky, but are crystal clear given the Leibnizian foundation.

The three pages in question initiate chapter two of Cassirers volume presenting a "foundational outline of a doctrine regarding the form of myth" , with which Cassirer means the *symbolic form* of myth. In the three pages Cassirer differentiates four different forms of space.⁎ First, Cassirer introduces the concept of *mythical space*, which, in his words, takes "a curious *intermediate position* (Mittelstellung) between the sensual space of perception and the space of pure knowledge". Second, we find a variety of characterizations for the *sensual space of perception* (sinnlicher Wahrnehmungsraum), including *seeing and tactical space* (Seh- und Tastraum), also *facial space* and *tactical space* (Gesichtsraum und Tastraum), together referred to as the two *physiological spaces*. Third, Cassirer introduces the *metric space* (metrischer Raum) that "Euclidean geometry underlies its constructions", to in essence outline the very same argument on which Panofsky builds his lecture on perspective. What's curious is that Panofsky's *psychophysiological space* seems to conflate Cassirer's *physiological space* with a fourth *space of pure knowledge*. This is a smart move on behalf of Panofsky, who was talking to an audience of art historians with little or no background, or interest, in pure mathematics. The fourth notion of space according to Cassirer deserves another paragraph.

CASSIRER'S DENKRAUM

Cassirer's *space of pure knowledge* (Raum der reinen Erkenntnis), is of central importance for our mission. The *alternative names* for this notion of space in Cassirer's three pages include the *space of geometric intuition* (Raum der geometrischen Anschauung), the *thought space of pure mathematics* (»Gedankenraum« der reinen Mathematik), the *thinking space of geometry* (Denkraum der Geometrie), the *pure space of geometry* (»reiner« Raum der Geometrie), the *geometric conceptual space* (geometrischer Begriffsraum), and the *abstract* *space of pure knowledge* (»abstrakter« Raum der reinen Erkenntnis). What Cassirer means with this *litany* becomes crystal clear, when he explains that the "elements" or "points" that "connect" to "constitute" "geometric space" "are nothing else but *positional situations*" (Lagebestimmungen), which "have no content" "outside these relations". "Their being is taken up in their mutual situation: It is a purely *functional*, not a *substantial* being." So, as Cassirer writes fifteen years earlier in "Substance and Function", "again it is the Leibnizian foundational conception of mathematics, to which we are seeing ourselves guided back to."*

Importantly, Cassirer does not conflate the *space of pure knowledge* with the *metric space* of three-dimensional Euclidean geometry. He clarifies this through the qualifier "*pure*", which again fits with "Substance and Function", where he observes that "modern mathematics has converged ever more precisely and consciously to the ideal that Leibniz set up for it", and where he also explains that "in *pure geometry* this shows itself most clear-cut in the *general concept of space*", which "even before it is defined as *Quantum*, has to be understood in its *qualitative* character as *order in togetherness* (ordre des coexistences possibles)".

We can even go further back to Cassirer's first book on "Leibniz' System", publised 23 years earlier in 1902, to clarify the enigmatic concept of "*Denkraum*", defined by Cassirer as a *pure space of geometry*.* Here Cassirer points out, that the *principle of situation* (Prinzip der Lage), in the sense of analysis situs, emerges as a *pure principle of thinking* (Prinzip des Denkens), in which the point, meaning position, appears as the basis of *space*. This means, the *space of pure knowledge* Cassirer is referring to as a *pure space of geometry*, is obviosuly not necessarily a *metric space*, and also not necessarily restricted to three dimensions.

A bit later, in "Substance and Function" Cassirer makes the potential multitude of dimensions more clear, when he says that "the method of *pure* mathematics" reaches its sharpest expression in the geometry of David Hilbert, which he refers to as a *pure doctrine of relationship* (reine Beziehungslehre).* This clearly includes the notion of multiple dimensions, as Hilbert's doctrine famously helped to revolutionize our concept of physics around the same time. In fact Hilbert's mathematics helped change our model of the *real outer world* (Reale Aussenwelt in the sense of Max Planck*), from Newtonian mechanics in three dimensions to *quantum mechanics* in multidimensional state space. As Susskind and Friedman (2014) neatly summarize "the vector spaces we use to define quantum mechanical states are called *Hilbert spaces*. We won't give the mathematical definition here, but you may as well add this term to your vocabulary. [...] A Hilbert space may have either a finite or an inifinite number of dimensions."* One may add that the Hilbert spaces of quantum mechanics are metric, so they are less general than Cassirer's pre-quantified notion of "*pure*". On the other hand, it is also important to note that the associated quantum mechanical states include continous wave functions, which implies significantly more mathematical sophistication than the more basic, purely *discrete* applications of network analysis across many disciplines, including current truely "digital art history".*

Without going into further detail, and pending further scholarship, we may simply note that Cassirer's *pure* notion of *relational space* is obviously rooted in Leibniz's core definition of *analysis situs*, and as such contains the analysis of nodes, links, and networks as one possible application. At the same time, we may also come up with an extended systematic science of art and culture in the future, which may transcend the core concepts of basic network science, and maybe even the sophistication of quantum mechanics, while still staying within the symbolic framework of Cassirer.

Inspired by Leibniz's "Monadology", we could for example imagine, in a cartoon-like manner, a Hilbert space of Hilbert spaces, one each for every human individual, each representing the state space of the outer world, plus another additional Hilbert space representing the state space of the outer world itself. Leibniz indeed imagined that every *monad* in such a the system, would contain a cognitive mirror-image of the whole world within it.* Poincaré, again likely in silent *allusion* to Leibniz, would later interject that "the head of the scientist, which is only a corner of the universe, could never contain the universe entire", which obviously puts a bound on the problem.* And indeed recent advances in neural science point to cognitive limits of more than three yet practically a limited amount of dimensions, at least at the local level of neurons, apparent for example within the *imagination of faces* in primates.* Be that as it may, even a more complete model of 7 billion monads mutually perceiving the world, equivalent to the human world population, all including each other, would still stay within the notion of *space*, *symbols*, *symbolic relations*, and *symbolic forms* as defined by Cassirer inspired by Leibniz.

A much more concrete and currently practical instance of a *geometrical thought space* in Cassirer's sense would be the metric *vector spaces* of machine learning, such as the spaces resulting from in the popular *word2vec* method, where words in a large corpus of text are embedded as vectors so one can for example subtract "female" from "queen" to obtain "king".*

A maybe less sophisticated, yet still powerful example, would be to constitute a *feature space* via the straightforward measurement of a variety of visual features in digital images, as applied in art history by Lev Manovich, and abundant within more classic approaches of computer vision and machine learning.* It is unfortunate that such vector and metric feature spaces are often seen as subject to the so-called *curse of dimensionality*. In particular, it is a pity that we usually react with methods of *dimensionality reduction*, without further discussion, while we may still dream of "*seeing in hyperspace*", as Poincaré put it above.*

Probably the simplest way to constitute a *Denkraum*, in the sense of Cassirer, would be via a so-called *bipartite classification*, where objects or images are related to properties or features, much like publications are related to subject headings in a library catalog. As detailed in later chapters, such classifications are simple to define and collect, yet surprisingly complex in their structure.* While typically still encompassing a variety of data types, such as *nominal*, yet also *ordinal*, or *quantitative*.* The difference to a fully fledged *feature space* would be that bi-partite classifications are typically constituted via a single link type in the sense of "has-classification" or "is-related-to", as opposed to a multitude of specific *kinds of relations* such as "has-name", "has-location", "has-inventory-number", "has-shape-count", "has-length", etc. We will return to this issue.

CASSIRER'S SYSTEMATIC FRAMEWORK

Now that we have broken a whole number of tear lines, we can pull out all the stops. What follows is not meant to be a review or a final word, but a wild yet hopefully inspiring improvisation over the established motifs. Art Historians with access to a good library may by now have "Warburg's Denkraum" in mind or on the table, an excellent 2014 book by a group of excellent art historians.* Flipping back and forth, they may wonder, how Aby Warburg's *thinking space* ("*Denkraum*"), *thinking space of enlightenment* ("Denkraum der *Besonnenheit*"), and *devotional space* ("*Andachtsraum*") are related to Cassirer and Leibniz. Mario Wimmer, in a 2017 article, provides two interesting leads, when he quotes Warburg himself assuming these spaces are created via "*vergeistigte Verknüpfung*", i.e. an ambiguous *spiritual relationship* or *relationship put in mind*, and when he quotes Warburg's librarian Gertrud Bing using the word "*Denklage*" or *situation of thinking*.* This word *Lage* of course does not mean "*layer*". It refers to Leibniz's *analysis situs* (Analyse der Lage).

Gertrud Bing was not a conventional librarian. Bing joined the Warburg library, after writing her PhD with Ernst Cassirer on the relation of Leibniz and Lessing, so she was very likely very familiar with both the writings of Cassirer and Leibniz. Bing, according to Wimmer, uses the term *Denklage* to characterize the mutual position of books within the Warburg library. So not only did the central oval space of the Warburg library reflect the central space in the library of Wolfenbüttel, where Leibniz himself developed the cataloging system. The curious order of books in the Warburg library, which famously followed "*the law of the good neighbor*" to maximize serendipity, can in fact be seen, as Bing obviously did, as a manifestation of a *geometric conceptual space* in the sense of Leibniz's *analysis situs*.*

More doubtful art historians and humanists may now say, but Cassirers *philosophy of symbolic forms* cannot be conflated with mathematics as it is more specific with regards to religion, language, myth, and art, while the art historical conceptions of the Warburg circle are again different and more specific in their own ways. Yes, and no. Cassirer himself tackled the issue in a German lecture on *the problem of symbol and its position in the system of philosophy*.* Published in 1927, this most crucial article includes a telling debate at the end, where the now almost forgotten art historian Alois Schardt,* who later emigrated to New Mexico, confronts the naysayers. Schardt emphasizes that Cassirer's contribution lies in aligning the manifold notions of the *symbol* with the modern worldview, which is driven by modern mathematics and science, explicitly including the example of Hilbert. In the final answer within this debate, Cassirer then explicitly confirms the summary of Schardt, making clear that his intention is to do both, differentiate the issue of symbolic form in all its discipilinary specifics and permutations, while also putting the issue into a meaningful multidisciplinary relational framework.

Obviously, Cassirer was not able to convince the majority of specialists, who, as Warburg put it earlier, were stuck within their own disciplines, not only living in "two worlds", but exhibiting a kind of *border police mentality* ("grenzpolizeiliche Befangenheit").* Yet for a while, Cassirer's notion of *symbols in the widest sense* certainly did convince a close circle of colleagues and successors. In 1955, Rudolf Wittkower, for example, explicitly deals with "visual symbols in their widest conceptual meaning".* Earlier, in his 1939 refined outline of *Iconology*, Erwin Panofsky probably sticks most closely to the meme by striving for "*a history of cultural symptoms – or symbols in Ernst Cassirer's sense – in general*".* In fact, improving over the first German version of 1932, Panofsky aligns the three stages of Iconology very closely with the system of Cassirer, literally shifting the *function of meaning* to another "*level*". Panofsky, as every student of art history knows, famously defines his method along the three stages of interpretation, including first "*natural* subject matter" that is practically experienced, second *conventional* subject matter known from written sources, and third "*intrinsic meaning* or *content*, constituting the world of 'symbolical' values".

What is hitherto unknown, or at least lesser known, is that Panofsky's 1939 definition of Iconology perfectly coincides with the above 1927 article, where Cassirer not only argues for a notion of *the symbolic in general*.* Indeed, Cassirer suggests nothing less than a *most general systematic reference framework of thought* ("allgmeinstes gedankliches Bezugssystem"), notably including three *dimensions*:

Cassirer's first dimension is the *expressive function* (Ausdrucksfunktion), where "a sensual-outer posesses the force to express an inner being". This is obviously in line with and preceding Panofsky's first stage of Iconology, i.e. the natural subject matter.

Cassirer's second dimension is the *representational function* (Darstellungsfunktion), including a *conventional setting of sentences* (bestimmte Satzung) as found in language, but explicitly including more general scientific theory – Wittgenstein's *tractatus* comes to mind, and also the fact that the *second law of thermodynamics*, for example, is called *Zweiter Hauptsatz der Thermodynamik* in German.⁎ This is obbviously in line with Panofky's second stage of *iconography* ("from Greek" "graphein" that is to "write").

Cassirer's third dimension finally is the *meaning function* (Bedeutungsfunktion), which again he exemplifies by pointing to contemporary math and physics. The parallel to Panofky's third stage of *iconology* or *iconography in a deeper sense* is again obvious.

The essential difference between the two is that Panofsky speaks of *layers*, which again may be rooted in a misunderstanding of the word *Lage*, which for Cassirer clearly means *position* in the sense of *analysis situs*.⁎ Cassirer's threefold system is not *layered*. Instead it differentiates into three *functional dimensions*, which is obviously more general, as one can ask how independent (or orthogonal) these dimensions actually are, as opposed to considering them separate *a priori*. Finally, Gertud Bing's dissertation⁎ comes to mind, when Cassirer refers to Lessing, before eventually ending up with a model of the arts, where the subject of inquiry is not the *substantial form* but the *functional form*, including *laws of formation*.⁎

At this point it is probably less mind-blowing than it should be, at least for art historians, that Panofsky's *Iconology* of 1939 is not only directly rooted in Cassirer's *most general systematic reference framework of thought* of 1927, which itself is explicitly tied to the 1910 *Substance and Function*, but in turn both are firmly rooted in the system of Leibniz.⁎ Clarity, in this respect, comes from a 1994 article by George Gale that analyzes the substantial influence of Leibniz on the *cybernetics* of Norbert Wiener – that is the same Wiener, who not only made crucial contributions to algorithmic computation, but also famously exclaimed "Back to Leibniz!" in 1932 in relation to modern physics.⁎ Feeding into our own issue, Gale presents a diagram that summarizes *Leibniz's complete system*, based on the same body of work that inspired Poincaré and Cassirer before. According to this diagram Leibniz's system includes the three strata of the *observable*, of the *explanatory*, and the *metaphysical*. The latter implies *analysis situs*, as we know from the title of Leibniz's late fragment that includes the core definitions of *space* and *situs*, as cited above.

So now we have clearly delineated a bridge from art history to Euler's trick of replacing parts of Königsberg with symbols, such as the letter "A". Indeed, one could say, *art and cultural history* following Cassirer and *network science* following Euler are in some sense a *ring species in time*, much like humans and other apes. The difference is of course that *horizontal meme transfer* between disciplines is possible.*

SHANNON'S IDEOGRAPHY

Adding computation to our story of disciplinary bifurcation enables an even broader perspective. And again the common ancestor reappears. Leibniz's development of *calculus*, in parallel with Isaac Newton, is partially driven by his innovations in symbolic mathematical notation. Indeed, Leibniz comes up with a symbolic framework called *calculus ratiocinator*, which directly precedes modern *algorithmic computation*.* In this case, there is no need to break a tear line as the references are explicit. Yet it seems valuable to raise broader awareness, particularly as the episode contains a crucial argument regarding the primacy of text in the so-called *ideographic* humanities: It is well acknowledged that Claude Shannon's ground-breaking master thesis, published in 1938, pioneers the Boolean logic functions of modern digital computers in "a *symbolic analysis of relay and switching circuits*", that is what later became the "*network analysis*" of *electrical engineering*.* Shannon's thesis is cited over 1400 times according to Google Scholar while its diagrammatic conventions and functional principles found application in a myriad of devices. What is less acknowledged is that Shannon cites "*the algebra of logic*" by Louis Couturat (cited by 40) as one of two key foundations – the 1914 English translation to be precise. Not only is Couturat the most eminent scholar of Leibniz's manuscripts besides Cassirer and Bertrand Russell.* This particular English translation of "*the algebra of logic*" also contains a preface by Philip Jourdain that emphasizes the conribution of Leibniz and makes a number of essential clarifications: One is that the "algebra of logic" should not be confused with the "logic of mathematics", which for us means there is or may be a difference between our algorithmic computations and the pure *meaning function* in the sense of Cassirer and Panofsky, that is between our digital computations and whatever our computations intend to model. Yet second, again firmly rooted in Leibniz, Jourdain also raises a major caveat for so-called traditional humanists that prefer sticking to text:

"The ambiguities of ordinary language are too well known [...]. The object of a complete logical symbolism are: firstly, to avoid this disadvantage by providing an *ideography*, in which the signs represent ideas and the relations between them *directly* (without the intermediary of words)."⁎ The consequences of these insights have obviously been profound. And much like they have propelled electrical enginering, computation, and natural science, they can also be further harnessed to make headway in our understanding of the *meaning function* in art and cultural history, much like Cassirer envisioned. This is why we see more and more applications of computation, mathematical models, and diagrammatic visualization in the social sciences and the humanities as well.

Today X can be a symbol, an equation can be a symbol, a matrix of equations can be a symbol, a relation can be a symbol, an image can be a symbol, a symbolic form can be a symbol, indeed everthing can be a symbol, and subject to *analysis situs*. Some scientists reading this may by now chuckle and think of Stephen Wolfram's intonation in his "Introduction to the Wolfram Language", in which "everything is symbolic".⁎ And yes indeed, the point of a systematic science of art and culture is, maybe a bit more cautious, that *everything that is symbolic* can be subject to symbolic thought, symbolic computation, and also symbolic analysis situs in the sense of Leibniz.

This does not mean that math will take over the humanities, or that we will end up with a general systematic science in the sense of an *egg-laying-wool-producing-milk-giving-pig* (Eierlegendewollmilchsau), where one method or perspective solves all issues. Instead, the *general framework of a systematic science of art and culture* will likely contain several, certainly overlapping approaches, much like the current situation in physics, where, for now, a multitude of models seems to resist further unification.⁎ A tangible example for such a framework necessitating a variety of approaches would be the less advertised components of the not-so-general state-of-the-art in *artificial intelligence* or *cognitive computing*, where methods such as *convolutional neural networks* processing unstructured data, more standard *data science* dealing with texts, and a more "pure" network science of *knowledge graphs* are combined to deal with *unstructured*, *semi-structured*, and *structured* data, all together, yet different.⁎

KINDS OF RELATION AS SUBSTANCE

As I said at the beginning, it is not my intention here to create new absolute heroes. Both Leibniz and Cassirer, like many others, are brilliant geniuses, meaning nothing more than *intermediary spirits of becoming*, whose work is *inspiring* and helpful to clarify the *situation* of art and cultural history in relation to network science and computation. Like Poincaré has developed his own *analysis situs* to substantially advance what we now call *topology*, we have to develop our own *systematic science of art and culture*, driven by novel *kinds* and *amounts* of data, and made possible by novel methods, including *network science*, *topological data analysis*, *high-performance computing*, general *machine learning*, more specific *computer vision*, yet also plain but enhanced *qualitative inquiry* using our own eyes, hands, and brains.

What I present throughout the upcoming Cultural Interaction book, by summarizing two decades of my own work, is a hopefully critical mass of sometimes fragmentary proofs of concept, feeding into such a novel systematic framework. As Fritz Saxl, the head of the Warburg library, pointed out in obvious allusion to Ernst Cassirer in a 1947 lecture: "*I am not a philosopher*, nor I am able to talk about the philosophy of history. It is the concrete historical material that has always attracted me..."* That said, let me use the opportunity to connect my own work back to Leibniz touching all three critical paths, one via network science and graph theory, a second via knowledge graphs and computation, and a third via the theoretical ideas of Cassirer and the followers of Aby Warburg. Adding more detail in later chapters, I can keep it short:

While I was studying *art history*, *classical archaeology*, and *psychology* at the University of Munich in the mid-1990s, I started to earn a living through gradually becoming a "*database pathologist*", consulting large projects in art research while dealing with what we now call *graph databases* or *knowledge graphs*. In such *graph databases* records are directly linked to each other in a granular fashion while carrying any kind of property. This is different from then more standard *relational databases*, where tables are linked, containing conventional records that all share the same properties. *Graph databases* and *knowledge graphs*, then more esoteric, have by now achieved a major breakthrough with abundant web-scale applications. Of course, compared to the current scale, the graph database system, which I had the pleasure to work with in the 1990s used much more modest hardware, yet an ideosyncratic software concept that prefigured or was in line with many of the functionalities later to be found in *Freebase.com* (as published in 2007), the now standard ways we process *Linked Data* (since 2006), or recently more widely applied graph database systems, such as *Neo4J* (since 2007).* It was this relatively early experience with the graph database paradigm (since 1996) that enabled me to pioneer *complex network science* in the realm of art history during my PhD work from 2002 to 2005. Within this work, I was able to reveal some (then still) surprising properties in the relations of *visual similarity* and *implicit visual citation*.*

It was the same experience with "large" knowledge graphs in art research, which later allowed me, again relatively early around 2008 to 2009, to argue for a network science of *multiple kinds of relations*. The resulting 2010 book chapter "*Revealing Matrices*", and another 2013 book chapter in German translating to "*Networks of Complex Networks in Art Research/Science*" both remained more obscure.* Yet the proposed mapping of multiple node and link types did enable the 2014 "*Network Framework of Cultural History*" in Science Magazine,* which is not only is the first such paper by an art historian as a first author, but also the first such paper to analyze the content of Freebase.com. Beyond that, the validity of the approach to focus on multiple *kinds* of nodes and links concurrently, has been resoundedly confirmed in its relevance through the recent explosion of mathematically rigorous multidisciplinary research on *multiplex* and *multilayer networks* with mutliple *node types* and *link types*.*

As such, my own work too is not the product of divine inspiration, but makes sense rooted within a broad heterogeneity of intermediary spirits on whose shoulders I stand. The modern notions of *multilayer networks* are prefigured in the *kinds of relations* that Cassirer saw in Leibniz's definition of analysis situs. And it was Cassirer who likely brought the issue to the forefront of attention in Aby Warburg. He in turn opens the *Mnemosyne Atlas* – which is more famous for its focus on the *similarity relation* of the *pathos formula* – on plate A with "*different systems of relations*", including visual diagrams to exemplify *cosmic situation*, *geographical exchange* rooted in birth places, and *social position*.* It will probably amuse young network scientists focusing on *multi-relational networks* when thinking about their PhD advisors, that Warburg goes on to say on the same page that in *magical thinking* all these relations are unified, as the separation of link types "already presupposes performance of thinking". Personally, I wasn't any better in thinking than other network scientists preceding the current rise of multilayer network research. Instead, through my work as a "*database pathologist*", I was lucky to get acquainted with data that was built on the insights around Cassirer and Warburg in the 1920s.

In 1910 Cassirer observes that "there is no objectivity outside the framework of *number* and *magnitude*, *continuity* and *variation*, *causality* and *interaction*."* "*Continuity and variation in the meaning of images*" are the silent core issues of Warburg's Mnemosyne, and the explicit title of a 1947 lecture by Fritz Saxl, i.e. Warburg's assistant, deputy, and successor.* *Continuity and variation* also resonate closely with the core relations of *antique reception* and *replicas* in a database project called the "Census of Antique works of Art and Architecture Known in the Renaissance",* which too was co-founded by

Saxl, in 1946-47. This database was continously funded and fed until 2018. It was translated from a *paper card index* into a *relational database* under the leadership of Arnold Nesselrath in the 1980s and into a *graph database* by Ralf Biering and Vinzenz Brinkmann, with my own involvement in 1997, before it became a subject of my PhD from 2002 to 2005. Most of the multiple node types within the *Census*, which I went on to visualize in the 2010 “Revealing Matrices” book chapter, are already enumerated explicitly in Saxl's 1947 lecture on *Continuity and Variation*, if one reads between the lines.* So all in all, it turns out, I was allowed to pioneer complex network science in art research, due to a tradition of scholarship in art and cultural history that is as “Leibnizian” as network science itself, even though most art historians in the wake of Cassirer forgot or never followed the lead.

SO WHY CARE ABOUT ART AND CULTURE?

Finally, scientists may ask why they should care about the domain of art and culture beyond computational social science? – As Ferreiros and Grey point out in 2005, “for Cassirer there are no facts”,* which is in line with Danny Hillis' more recent observation that we cannot record facts, only “*assertions about relations*”.* This, in turn, is in line with Max Planck when he states in 1930: “We live in a weird world” within “a time of severe crises”, where “*mathematics in its pure form* alone can't help us”, as “all it can do is to *connect the dots*”. “Healthy reason”, Max Planck goes on, “requires a step into the metaphyiscal, namely that our experiences are not the physical world itself, but that they merely give notice from another world.” The metaphysical step, according to Planck, is to accept that a shared *real outer world* (reale Aussenwelt) does indeed exist. This step is metaphysical, as the *pure geometric spaces of knowledge* that are subject to a generalized *analysis situs*, as expressed in our minds, in our external products of collective cognition, and in our computing machines, can only register a *multiplicity of opinion*, based on imperfect tools of measurement and cultural technologies, based on our own imperfect senses, our biology, and our inculturations. As such, all science is intertwined with the question of symbolic form in cultures and cultural production. This is why *a systematic science of art and culture* is necessary to disambiguate the omnipresent multiplicity of opinion, where everthing goes, but *everything goes with a probability*.

Notes

This text caters to a multidisciplinary audience, including scientists and humanists. The notes provide links and supplement the ***list of references at the end***, to facilitate mutual understanding, to indicate essential sources, and to provide further perspective. Each note is referenced by an asterisk in the main text.*

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The first opening quote is taken from a Latin manuscript fragment by Leibniz, with uncertain date, obviously preceding his death in 1716: "**Spatium** *est ordo coexistendi* [...] **Situs** *est coexistendi modus*." The emphasis in the main text is Leibniz's own; see Leibniz 1863 pp. 17-29.

The second opening quote refering to ***n-dimensional space*** is taken from Strang 2016 p.1. For those who want to get serious about *a systematic science of art and culture*, beyond basic network analysis, this branch of mathematics is foundational; The highly approachable "*Matrixology*" course by Peter Dodds at the University of Vermont is based on Strang's book (cf. http://www.uvm.edu/pdodds/teaching/courses/matrixology/).

A perspective paper regarding "***a systematic science of art and culture***" see Schich 2016.

The original article, ***Euler 1736*** pp. 128-140, is based on a 1735 lecture. English translation with comments see Leibniz et al. 1986 pp. 3-11. A succinct summary see Estrada & Knight 2015 pp. 5-7.

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The following textbooks provide a variety of perspectives regarding the foundation of modern ***complex network science***, while mentioning Euler's solution: Barabási 2016, Estrada & Knight 2015, Caldarelli & Catanzaro 2012, and Newman 2010. The following textbooks are equally foundational yet do not mention Euler's solution: Newman 2018, and Easley & Kleinberg 2010. My own introduction into the field of network science precedes the emergence of comprehensive textbooks. As for many others starting in 2002, my first sources were journal papers such as Barabási & Albert 1999, Broder et al. 2000, and Watts & Strogatz 1998. The first comprehensive accounts of modern network science came out in 2002/3, in form of a popular book and arguably the first comprehensive review paper within the emerging field: Barabási 2002 & Newman 2003. The latter also forms the nucleus for Newman 2010 & 2018.

Stuart Kauffman is best known for a brilliant book, Kauffman 1993, which Barabási 2016 also cites as one of the most inspiring influences. For the more controversial notion of Euler's "invention" in an otherwise excellent and literally "inspiring" account see Kauffman 2008 pp. 184-186.

Literature on ***Aby Warburg*** and the associated Hamburg school of art and cultural history is abundant in art history. Essential works include Warburg 1998-2019, including the *Mnemosyne Atlas*, Warburg 2008. An alternative in a single volume without figures is Warburg 2010.

Explicitly connecting Warburg's Mnemosyne with computer vision and ***machine learning***, and more broadly ***cultural analytics*** and ***data science*** see Impett & Moretti 2017, Impett & Süsstrunk 2016, and Hristova 2016. Rigorous applications of ***machine learning and data science in art history*** see also Elgammal et al. 2018, Sigaki et al. 2018, Seguin et al. 2017, De La Rosa & Suárez 2015, Crowley & Zisserman 2014, Kim et al. 2014, and Johnson et al. 2008. An attempt of my own from the dark ages using "human computation" see Schich & Ebert-Schifferer 2008. For ***deep learning in general*** see the literature following Goodfellow et al. 2016 and LeCun et al. 2015. More general textbooks on ***machine learning*** include Russell & Norvig 2016 and Bishop 2011. For ***data science*** see Leskovec et al. 2014 and one of my personal favorites, the brief and brilliant Janssens 2014.

Some ***recent accounts on Warburg***, Cassirer, and their circle include Johnson 2012, Treml & Flach 2014, Vos 2016, and Didi-Huberman 2017.

A relevant book on ***Imagined communities*** with almost 100,000 citations see Anderson 2006.

Newton's original letter featuring the famous adage of ***standing on the shoulders of giants*** see Newton et al. 1675. For the pre-Newtonian roots see Merton 1993. A likely inspiration for the adage becoming the motto for the Google Scholar citation database is Garfield 1998.

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The mission of not creating new ***heroes*** is in line with the Cassirer 1946. Indeed, Cassirer's arguments made Hitler's architect Albert Speer realize later in jail how his devotion for the dictator made himself a monster. cf. Speer 1969 pp. 62 & 532.

The meaning of human existence see Wilson 2014. This worldview is in line with Alexander von Humboldt's statement within the *cosmos* lectures that *everything is interaction* ("Alles ist Wechselwirkung"); see Humboldt 1803. *Cosmos* literally means *order*, which again resonates well with Leibniz's *analysis situs*.

Beyond the popular notion of ***filter bubbles***, as introduced by Pariser 2011, more scientific accounts are relevant, such as Lorenz et al. 2011.

TV Tropes 2004-2019 is a large, somewhat messy, yet scholarly underrated crowd-sourced collection of *tropes*, *memes*, or *archetypes* across a large variety of *media genres*.

The ancestor's tale see Dawkins & Wong 2005, itself inspired by Chaucer & Ellis 2014.

For the Roman meaning of ***genius*** as "Werdegeist" see Roscher 1894-1937 s.v. Genius vol. 1,2 cols. 1613-1625.

Euler 1736 (Latin original) and Euler 1986 (English translation).

For ***scholarly figures***, ***information design*** and ***data visualization*** as a ***lingua franca*** see, literally, Viégas 2018. Excellent general introductions to the area of information visualization include Munzner 2014, Meirelles 2013, and Cairo 2012. A classic, since 1967, is Bertin 2010. The latter includes a brilliant 2004 "brief presentation of graphics" on pp. 418-434, which I prefer to assign as the single most required "reading" or better "staring" in courses on data visualization.

The ***prime object*** as a denominator of first occurrence in art and culture is a concept introduced and diagnosed as exceedingly rare within Kubler 1962 pp. 39-53.

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Leibniz's analysis situs is also mentioned in the history of graph theory, Biggs et al. 1986 p. 21, quoting Gauss, who acknowledged both Leibniz as the root and Euler's solution, while also pointing to the intially very slow progress. The relevance of Leibniz's analysis situs within the history of the humanities is typically less explicit; cf. the otherwise stunningly comprehensive Bod 2013.

The terminologies of ***graphs***, ***vertex***/vertices, and ***edges*** in graph theory, and ***networks***, ***nodes***, and ***links*** in network science have many more specific equivalents across disciplines, not always identical yet still related in meaning. Concrete examples for nodes and links would be *actors* and *ties* in *social network analysis*, *entities* and *relationships* in *database ontology*, *atoms* and *bonds* in a *molecule*, or the *components* and *wires* in an *electric circuit*. A comprehensive dictionary of symbolic terminology that would translate the respective variant names and variant meanings of symbols, symbolic relations, symbolic forms, and symbolic systems remains an open desiderate.

Cassirer's interpretation of Leibniz' Analysis Situs in Cassirer 1910 p.98 is accompanied with the following ***original quote*** from Leibniz: "Hinc etiam prodit ignorata hactens vel neglecta sub-ordinatio Algebrae ad partem Combinatoriam, seu Algebrae Specioae ad Speciosam generalem, seu scientiae de formulis quantitatem significantibus ad doctrinam de formulis, seu ordinis, similitudinis, relationis, etc. expressionibus in universum, vel scientiae generalis de quantitatae, ad scientiam generalem de qualitatae, ut adeo speciosa nostra Mathematica nihil aliud sit quam specimen illustre Artis Combinatoriae seu speciosae generalis." taken from "***Mathesis Universalis.***" Leibniz 1863 p. 61. Cassirer obviously abreviates "ordinis, similitudinis, relationis, etc. expressionibus in universum" to "kinds of relation" ("Weisen der Verknüpfung"). The diligent reader may surmise that Cassirer's rejection of "number of connections" is at odds with Euler's solution of counting links: Here Leibniz's Latin original is important, which does not imply an exclusion of counting qualitative relations, but generalizes over the measurement of quantitative extension (cf. next note).

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Leibniz's own definition in Latin: *"Si plures ponantur existere rerum status, nihil oppositum involventes, dicentur existere* **simul**. *[...]* **Tempus** *est ordo existendi eorum quae non sunt simul. [...]* **Duratio** *est temporis magnitudo. [...]* **Spatium** *est ordo coexistendi* seu ordo existendi inter ea quae sunt simul. *[...]* **Extensio** *est spatii magnitudo. [...]* **Situs** *est coexistentiae modus. [...]* **Quantitas** *seu Magnitudo est, quod in rebus sola compraesentia (seu perceptione simultanea) cognosci potest. [...]* **Qualitas** *autem est, quod in rebus cognosci potest cum singulatim observantur.*" taken from "***Initia rerum mathematicarum metaphysica.***" Leibniz 1863 p. 18.

De Risi's recent summary and interpretation of Leibni'z Analysis Situs see De Risi 2018 pp. 247-558. De Risi dates Leibniz's work on analysis situs from the 1672 to 1676 Paris phase until the end of his life 1714 to 1716, with the quoted fragment above among the latest. De Risi also points to yet deeper roots in Aristotle, Pascal, and early modern perspective books.

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Regarding higher-order ***topological data analysis (TDA)*** and ***network geometry***, which is of increasing interest in the communities of network and complexity science, see Mulder & Bianconi 2018, and Patania et al. 2017, who explicitly refer to Aleksandrov 1972. For the unprecedented potential to connect hitherto separate streams of research cf. Patania et al. 2018. In addition, higher order topology is used to discretize and study Riemannian manifolds in natural computation, which opens an avenue to study topological data spaces metric-independent, meaning without reduction to a metric or vector space. cf. Merelli et al. 2015.

Sur l'analysis situs. Poincaré 1892 was extended five times in 1899, 1900, twice in 1902, and in 1904. English translation see Poincaré 2010.

The Future of Mathematics. Poincaré 1910.

For a direct trace of ***Poincaré***'s study of Leibniz see his contribution in Leibniz et al. 1881. For more detail on the giant project of ***editing Leibniz*** including Poincaré see De Risi 2007: "No monograph has been written on [Leibniz's] analysis situs. No one has as yet devoted more pages to the subject than Couturat and Cassirer had - whose contributions, in spite of the excellence [...] are now a hundred years old."

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On ***Cassirer and Heidegger in Davos*** see Gordon 2010 and Rheinberger 2015.

For ***Philosophie der symbolischen Formen***, Erster Teil, ***Die Sprache***; Zweiter Teil, ***Das mythische Denken***; Dritter Teil, ***Phänomenologie der Erkenntnis***, see Cassirer 1923/1927/1929.

For an anecdotal account of ***Cassirer in the circle of Warburg*** see Levine 2013. For Warburg's own perspective on Cassirer see Warburg 2010 pp. 680-703.

An Essay on Man. Cassirer 1944 p. 223 (207) note 1.

Substanzbegriff und Funktionsbegriff. Cassirer 1910. English version Cassirer 1923b.

Zur Einsteinschen Relativitätstheorie. Cassirer 1921. English version Cassirer 1923b. Cassirer acknowledges input from Einstein in the preface.

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Leibniz' System in seinen wissenschaftlichen Grundlagen. Cassirer 1902, including "the geometric problem of space and the analysis of position" pp. 142-143.

Cassirer 1910 p. 98 on ***Leibniz's sidenote*** and p. 99 regarding the ***geometry of Hilbert***. See also Ferreiros and Grey 2005 p. 388: "It emerges that Cassirer too had moved towards Leibniz: not only does logic embrace the study of relations, but Hilbert's formulation of geometry is a 'pure doctrine of relations'" – This seems important in relation to Cassirer's image as a Neo-Kantian.

Cassirer 1910 p. 315-316 & 353-374 introduces a modern concept of what we now call ***cognition***, based on ***Helmholtz*** 1896 pp. 947f., before making a case for a "psychology of relations". This is relevant regarding the relation of cognition and art history: cf. Schich 2009 pp. 150-155, including alternative pointers to Leibniz and the relation of Helmholtz to a more recent cognitive science of vision. On the ***cognitive revolution*** see Miller 2003 and Murray 1995 in relation to older approaches.

Schmarsow 1894 provides a figleaf within the so-called state-of-the-art in German art history: Jantzen 1938 for example mentions "space as symbol", "thought systems", "symbolic form", and "knowledge-theoretical space", clearly alluding and actually making an argument for the theory of Cassirer. At the same time Jantzen avoids explicit citation in favor of concocting a clearly bullshit argument claiming foundation in Schmarsow and primacy of the "nordic" within a prime example of aligned Nazi rhetoric. At roughly the same time within the Warburg circle, ***the actual state-of-the-art in art history is curious***: Gombrich. E. 1937 p. 109, for example, attributes "the projection of inner images" to Panofsky, which in essence signifies a double tear line for Cassirer. Neither "pure Germans", nor "pure art historians" would explicitly and directly acknowledge his concept of Denkraum as "purely mathematical". Curiously, the disconnect to the "unspoken Kantian" still exists: cf. Freedberg 2018.

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For a nutshell intro to ***geographical information science*** see the still classic Zeiler 1999. Space in ***digital art history*** conventionally means Euclidean space to a majority of practioners: cf. the contributions in Klinke et al. 2018. Similarly, a large fraction of ***computer applications in archaeology*** focus on 3D aspects, implicitly assuming geographical space as the foundation: cf. the Proceedings of the CAA 1973-2019 (i.e. the archaeology CAA, not the art history CAA).

Regarding the fortune of ***more advanced theories of space***, including Leibniz, Euler, Cassirer, and in the last case Warburg see Günzel 2015, Dünne & Günzel 2006, and Hofmann 2015.

Summers 2003, in particular p. 21.

On ***grid-cells*** see Moser et al. 2014. Regarding a "metaoptical" represenation of space in rats see Carpenter et al. 2015.

Die Perspektive als 'symbolische Form'. Panofsky 1927. English: Panofsky 1991.

My discussion is based on the original German version of ***Cassirer's breakdown of space in mythical awareness*** or "Gliederung des Raums im mythischen Bewusstsein" opening the chapter "Grundzüge einer Formenlehre des Mythos". See Cassirer 1927 pp. 98-100.

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For the state-of-the-art in ***sequence alignment***, ***meme-tracking***, and ***paraphrase detection*** see the literature citing the following: Searls 1997, Leskovec et al. 2009, Socher et al. 2011.

Friedländer 1946 p. 146: "Das ***Original*** gleicht einem Organismus, die ***Kopie*** einer Maschine." – Put more positively, Panofsky "has taken Cassirer at his word" and developed from there; see Müller 2017.

Several scholars have previously analyzed the relation of Panofsky's perspective to Cassirer's symbolic form; they all differ from my interpretation, as none of them takes into account the link to Leibniz's analysis situs; as they make excellent points, the situation is pending further debate and integration: Damisch & Goodman 1994, Neher 2005, Hub 2010, Alloa 2015.

Regarding the definition of ***naming games*** in ***evolutionary game theory*** see Young 2015 and Centola & Baroncchelli 2015. The issue is related to Wittgenstein's notion of ***language game*** (Sprachspiel) in Wittgenstein & Schulte 2001 or PU §48/64, which is closely related to the concept of *family resemblance* (Familienähnlichkeit) in PU §66. Wittgenstein's example in PU §2 of a builder and an assistant using different words to talk about shared concepts, indeed, could be seen as a translation game between language and a more abstract "geometric meaning", in the sense of Cassirer's "meaning dimension", or Panofsky's level of iconology, as defined following Cassirer, in 1939 (see below).

Cassirer's breakdown of space see Cassirer 1927 pp. 98-100. Cassirer's ***essential definition of geometric thought space*** is explained regarding its homogeneity: "Die Homogeneität des geometrischen Raumes beruht letztendlich darauf, daß die Punkte, die sich in ihm zusammenschließen, nichts als einfache Lagebestimmungen sind, die aber außerhalb dieser Relation, dieser Lage, in welcher sie sich zueinander befinden, nicht noch einen eigenen Inhalt besitzen. Ihr Sein geht in ihrem wechselseitigen Verhältnis auf: Es ist ein rein funktionales Sein, kein substantielles sein."

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Regarding Leibniz's ***foundational concept of mathematics***, Cassirer 1910 pp. 97/98 implies multiple possible "kinds of relations" or "Weisen der Verknüpfung", based on "ordinis, similitudinis, relationis etc. expressionibus in universum" as given in Leibniz 1863 p. 61. This means Cassirer assumes a possibly broader definition of Leibniz's analysis situs than De Risi 2018 implies for Leibniz himself. In this broader interpretation the ***situs function*** could indeed consist of simple Euclidean distances to constitute a 3D metric space (following de Risi's interpretation), but also, "in expressionibus universum", in line with modern network science, a purely topological yes/no binary kind of relation, a more general weighted kind of relation, or any much more complex or complicated kind of relation.

Cassirer's Denkraum already appears in Cassirer 1902 p. 148.

Cassirer 1910 pp. 99 & 108, discusses modern ***pure mathematics*** or "reine Mathematik" exemplified in the geometry of Hilbert, while refering to the geometry of Veronese with respect to the potential of multiple dimensions. This has to do with the fact that Cassirer writes at a time when "the choir has not yet sung their last notes".

Reale Aussenwelt see Planck 1931.

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On ***Hilbert spaces*** succinctly see Susskind & Friedman 2014 p. 25, and thoroughly Debnath & Mikusinski 2005 and Young 1988. Hilbert spaces are by no means the most general or only way to define a multidimensional space.

Network analysis in ***digital art history*** see the contributions in Kienle 2017 and several contributions in Schich et al. 2010-2017.

The ***cartoon model*** is ***in line with Leibniz's Monadology*** §56, where "each simple substance has relations that express all the others, and is in consequence a perpetual living mirror of the universe." See Leibniz 1714 and Rescher 2014.

For "the head of ***the scientist***, which ***is only a corner of the universe***" see Poincaré 1910 p. 77.

Recent evidence points to a distribution of ***optimal dimensionality of individual neurons***, often responding to more than three stimulus dimensions associated with a multidimensional feature space, for example in face recognition; cf. Freiwald et al. 2009 and more recent related Finkelstein et al. 2018.

The issue is not only interesting regarding a toy model in line with Poincaré's notion of each scientist, or one could say, cognitive agent being "only a corner of the universe". It is also of great interest regarding the ***thermodynamics of computation***, meaning "the astonishing difference in efficiency" between current machines, including those developed by Google Brain, and biologically evolved systems that compute, such as our own brain; cf. Kempes et al. 2017.

Foundational on vector spaces see Strang 2016 p.124-134. For ***vector spaces as geometric spaces of meaning*** see the recent rise of ***Word2Vec***, where 9000 citations have replaced peer review: Mikolov et al. 2013. An application to embbed more *pure* knowledge graphs in a *metric* vector space see Ristoski & Paulheim 2016.

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In ***machine learning (ML)***, a ***feature space*** conventionally connotates the ***vector space*** associated with a set of n-dimensional ***feature vectors***. For a succinct introduction see Leskovec et al. 2014. For a straight-forward application in digital art history see Manovich 2015.

The so-called ***curse of dimensionality*** in machine learning is usually overcome via methods of ***dimensionality reduction***, such as ***umap***, ***t-sne***, ***PCA***, and others: Becht et al. 2019, Maaten & Hinton 2008, Jolliffe 2011, and Van Der Maaten et al. 2009. Notwithstanding the abundant use of such dimensionality reduction methods, it seems worthwhile to explore alternatives, for example following Sanderson 2017, who puts it poignantly: "Just because you can't visualize something doesn't mean you can't still think about it visually".

My own work dealing with ***bipartite classifications in art history and archaeology*** see for example Schich et al. 2008, Schich 2009, Schich et al. 2009, Schich & Coscia 2011, Park et al. 2015.

Super-succinctly on ***nominal***, ***ordinal***, and ***quantitative*** data types see Meirelles 2013 pp. 204/205.

On ***Warburg's Denkraum*** see Treml & Flach 2014, in particular Wedepohl 2014 and Weigel 2014 ibid. pp. 17-50/143-180. Both clarify the awareness of Warburg, Wind, and Cassirer for cutting edge results in psychology and physics, explcitly resulting in a desire to establish a "Physik des Geistigen" or ***physics of the mind***, which we should probably take more seriously. Indeed, according to Wedepohl, Warburg enthusiastically notes down a model of a "Symbolfunktion" or *symbolic function,* as proposed by Edgar Wind in a conversation close to Warburg's death in 1929. This model makes very much sense as a "Prolegomena for a physics of mental distance", that is the distance of an observer's mental situation from an observed situation: The "*± energetic transformations"* term would be necessary to translate between Cassirer's four notions of space, much like Lorentz transformations translate between spatial reference frames within Einstein's special relativity; the *"magnetizing Zeitgeist"* term would approximate the unknown environment, much like a mean-field approximation of magnetism averages over a large number of individual components; the more difficult to translate term "÷Hingabe += Behauptung", loosely speaking, would negotiate between information given to and put forward by the mind in the process of cognition. This notion is obviously closely in line with Helmholtz and more recent cognitive science. In sum, what we should call the 1929 ***Wind-Warburg model*** is a surprisingly fancy yet precise characterization of Cassirer's ***animal symbolicum*** as a cognitive *agent* that could operate in an *agent based model* of a *complex adaptive system*. cf. Helbing & Balietti 2011 and Miller & Page 2009.

Gertrud Bing's ***Denklage*** see Wimmer 2017.

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Leibniz role in Wolfenbüttel, the ***oval space*** in the Warburg library, and the ***law of the good Neighbor*** are mentioned in Johnson 2012 p. 69; on Leibniz as head librarian and the oval space in Wolfenbüttel in more detail see Bredekamp 2004 pp. 137-143.

Cassirer's systematic framework of *expressive function*, *representational function*, and *meaning function* see Cassirer 1927b pp. 295-322 including Schardt's *Aussprache*. ibid. pp. 319-320 and Cassirer's *Schlußwort*. ibid. pp. 320-322. The framework itself is defined on pp. 301-322.

Regarding ***Alois Schardt*** see Heftrig et al. 2013, including a contribution by Heinrich Dilly (pp. 1-18), who we will also encounter as a pioneer of quantitative art history in chapter four.

Warburg 1912 p. 396 contrasts his ***iconologic analysis*** to the "grenzpolizeiliche Befangenheit" or ***border police mentality*** as exhibited by art historians that split the discipline by period or prefer to separate free from applied art. It is in this sense, that Warburg 1924 p. 681. frames the Warburg library as "a college of small bridge builders", while Cassirer "helps to put up a novel wide bridging arc across stream of Lethe", that is the river of forgetting.

Wittkower 1955 pp. 109-124, here quoted from the German translation in Kaemmerling 1994 p. 227 (author's translation).

Building on Warburg's concept of iconologic analysis, there are three differing versions of ***Panofsky's three-layer scheme of iconology***: The "preliminary" Panofsky 1932, the second, "canonical" Panofsky 1939, which aligns with Cassirer 1927b, and the third "out of favor" Panofsky 1955. Regarding Panofsky's development yet without reference to Cassirer 1927b see Kaemmerling 1994 pp. 487-501. The explicit reference to "a history of *cultural symptoms* – or *'symbols'* in Ernst Cassirer's sense – in general" first appears in Panofsky 1939 p. 16.

Cassirer 1927b (see *Cassirer's systematic framework* above).

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Wittgenstein 1922.

Further ***differences between the threefold frameworks of Panofsky and Cassirer*** imply the clearest signature of an active disciplinary bifurcation: While Cassirer aims for an *analysis* of the *symbolic* in general including the *sciences*, Panofsky emphasizes iconology as an approach of *synthesis* bringing together *all the humanities*, which only sounds inclusive.

Leibniz and Lessing see Bing 1923.

Regarding ***laws of formation***, Krakauer & Rockmore 2015 pp. 591-597 seem applicable.

According to the summary figure in Gale 1997 pp. 247-261, ***Leibniz's complete system*** includes the *observable*, the *explanatory* and the *metaphysical*, which obviously resonates with *Cassirer's systematic framework* and *Panofsky's iconology*. From here, one could open yet another can of worms, looking at the design principles of human computer interaction, where my dear colleague Mihai Nadin has delineated an influential ***semiotic paradigm***, that includes a three part system of *iconic*, *indexical*, and *symbolic* dimensions, all present and optimized in the little desktop icons that we got so used to on our screens; see Nadin 1988. He consulted both Steve Jobs and Danny Hillis regarding the matter. For explicit references to Leibniz see Nadin 1997.

On ***Leibniz preceding modern physics*** see Wiener 1932. This and ***Leibniz preceding modern algorithmic computation*** including the role of Gödel, Wiener, and von Neumann see Dyson 2012 pp. 101-107/335.

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On ***ring species in time*** see Dawkins & Wong 2005 pp. 308-320; on ***horizontal gene transfer*** see Soucy et al. 2015; and ***memes*** see Dawkins 2016.

For another connection of ***Leibniz's calculus ratiocinator and modern computation*** see also Dyson 2012, in particular pp. 104-105.

Shannon 1938 slightly misquotes ***Couturat 1914*** as Cauturat and omits the year, but includes the publisher, which makes the source identifyable.

Couturat also studied the manuscripts of Leibniz intensively. Regarding the relation of ***Russell*** 1900, ***Couturat*** 1901, and ***Cassirer*** 1902 to Leibniz *analysis situs* in particular see De Risi 2007 pp. 114-115.

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Jourdin's preface see Couturat 1914 pp. i-x, including the quote on p. vii.

Wolfram 2014 ***is all symbolic***, yet still "just a corner of the universe" in the sense of Poincaré 1910.

For the current ***multiplicity of models in physics*** cf. Fuhrmann & Wolchover 2015.

A good example for the interplay of ***unstructured, semi-structured,*** and ***structured data*** in so-called *artificial intelligence* or *cognitive computing* lies under the hood of *question answering systems*, such as the instance of IBM Watson that won *Jeopardy*. Far from the neat public avatar, such systems typically contain a complicated architecture, including a structured ontology (or knowledge graph), a semi-structured corpus (of annotated text), and unstructured data (in the form of machine learning models); cf. figure 1 in Clark et al. 2017.

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"I am not a philosopher..." see Saxl 1947.

A recent approach to ***graph databases*** (exemplified in ***Neo4J***) see Robinson et al. 2013. For the previously more popular ***relational database model*** see the literature following Codd 1970. For ***knowledge graphs***, including in the *semantic web*, *linked data*, *Freebbase.com* and *Wikidata* see the literature following Paulheim 2017, Vrandečić & Krötzsch 2014, Bollacker et al. 2008, and Bizer et al. 2011.

My PhD monograph, Schich 2009, deserves some background regarding priority in pioneering network science in art history: I submitted my PhD manuscript shortly after finalizing it in November 2005; After more than a year of "administrative processing time" I defended the "magna cum laude" text in a "summa cum laude" defense in May 2007, and eventually published the book unchanged in 2009.

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My early book chapters dissecting a network multiplicity see Schich 2010 and Schich 2013 pp. 161-178.

My breakthrough paper introducing ***A network framework of cultural history*** in Science Magazine, Schich et al. 2014, was accompanied by the Nature video ***Charting Culture,*** Schich & Martino 2014. Both resulted in more than 120 press items in 28 languages. As the first author I have invested approximately twice the amount of labor in the project than I did in my upcoming second book, or as Aby Warburg would say: I ask you to weigh it on the scales of gold, not the scales used for meat.

More recent on ***multilayer networks*** see Bianconi 2018, Kivelä et al. 2014, and Mucha et al. 2010.

Der Bilderatlas Mnemosyne, Warburg 2008 plate A, presents "Different systems of relations, in which the human is put into, cosmic, terrestrial, genealogic. Unification of all these relations in magical thinking, as separation of ancestry, birth place, and cosmic situation already presuppuses performance of thinking" (translation by the author). Regarding the notion of ***kinds of relation*** see the note on *Cassirer's interpretation of Leibniz' Analysis Situs* above.

Cassirer 1910. p. 334: "Es gibt keine Objektivität, die außerhalb des Rahmens der Zahl und Größe, der Beharrung und Veränderlichkeit, der Kausalität und Wechselwirkung stünde".

Continuity and Variation in the Meaning of Images see Saxl 1947.

For the ***Census project (1946-2018)***, including the current database version see Nesselrath 1946-2018. The database as analyzed in my PhD see Nesselrath 1946-2006. On ***early project history*** see Trapp 1999. On the 1980s conversion from a ***paper card index*** to a ***relational database*** see Nesselrath 1993. In 1997, the Census was converted into a ***Dyabola graph database,*** which I have co-developed, preserving the original datamodel, while allowing for graph queries, and Freebase-style association of property-types. In 2006, HU-Berlin hired another company to reconvert the Census into a more conventional ***relational database*** system, opting against my suggestion to dump the dyabola graph into a then state-of-the-art ***semantic web graph***. A decade later, the Brandenburg Academy of the Sciences (BBAW) finally decided that it was time to convert the Census into ***Linked Open Data*** driven again by a graph database: see http://www.bbaw.de/telota/projekte/census-lod (2017).

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Saxl 1947 in ***Continuity and Variation in the Meaning of Images*** relates *images* to *meaning*, *time*, *place*, *creators*, and other *ideas*, (including a dynamic relation of *magnetic attraction between images and meaning*). The title of my PhD, Schich 2009, "Rezeption und Tradierung", roughly translates to *similarity and visual citation*, i.e. again *continuity and variation*. Back then, I was not aware of the roots in Cassirer, while throwing another valid tentacle from art history to graph theory and network science via Kubler 1962 and Ore 1962.

Ferreirós & Gray 2006 pp. 388/389 point out that "***for Cassirer there are no facts***". The insight of Danny Hillis that we can only record "***assertions about relations***" dates to the mid-1980s; see Hillis 2017. The profoundness of this insight cannot be overestimated, as it not only led to *Freebase.com*, but revolutionized both web search and artificial intelligence through John Giannandrea, the other co-founder of Metaweb/Freebase. Personally, I had a similar insight in 1996, rooted in the fragmented yet overlapping inventory card index of the Glypothek Museum in Munich (see chapter two).

Reale Aussenwelt see Planck 1931.

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