New considerations on construction methods of the Ancient Egyptian pyramids

The pyramids of the Old Kingdom have always fascinated visitors of Egypt by their astonishing size and the vast proportions of the stone blocks. Not only do they leave a lasting impression, but they also prompt questions such as why did the ancient Egyptian kings build such gigantic burials? In searching for answers, we have grown accustomed to viewing the construction of the pyramids in the context of the religious mindset and the society of the time, and also in relation to the technical facilities and logistic proficiency of the ancient architects.

Throughout the past centuries, Egyptologists, archaeologists, engineers, and amateurs have attempted to explain the techniques and processes involved in the construction of the pyramids. However, as Egyptologists and archaeologists are not usually specialized to deal with the technical aspects of the construction process, many assumptions widely acknowledged in the academic world are flawed in this respect. On the other hand, authors without conventional training in Egyptology have made interesting contributions to the discussion of construction issues, adding remarkable explanations and calculations, but frequently they have overlooked important archaeological evidence. Consequently, the suggestions of 'non-archaeologists' are, as it were, less than well accepted among academic Egyptologists.

With reference to issues such as construction process, timing, technical devices, and workforce employed in the construction of the pyramids of the Old Kingdom, no conclusive explanation has been proposed so far, despite the wealth of books and articles published on the subject.

In the present study I shall suggest a new solution to the construction of the Step Pyramids[1] of the Old Kingdom, starting from current research in Egyptology and taking into account archaeological evidence, taking particular care to avoid internal contradictions.[2] I base my analysis on the following assumptions:

- Only tools, transport devices and construction techniques proven by archaeological evidence to have been available in the Old Kingdom can sensibly be assumed to have been employed.
- The same applies to the architectural characteristics and construction techniques of the Old Kingdom pyramids.
- Not only technical aspects should be taken into account, but also safety issues concerning the casing and its subsequent smoothing, as well as secure accessibility of the respective construction site.
- Backsighting must have been possible on a regular basis throughout the building process.
- Building time calculations based the construction hypothesis developed in the following discussion are presented for several pyramids.

[1] With the term 'Step Pyramids', I refer to buildings with a stepped core, resembling a series of platforms built one on top of the other, each smaller than the preceding one. The outer walls of the steps consist of large, finely hewn blocks with a batter of about 80°. To fill the core, stones of varying format were used, and also rubble, gravel, tafla, and mortar.

[2] Müller-Römer, F. 2011. *Der Bau der Pyramiden im Alten Ägypten*. Utz, Munich.

Construction techniques in the Old Kingdom

The laws of statics and mechanics were neither explored in theory nor by scientific experiments throughout the Old Kingdom. There is no evidence for any sort of static reckoning similar to that which we are accustomed to today. Proficiency in construction, as in other crafts, was developed through observation of nature and by experience, and thus brought to perfection. The techniques employed in the production of columns, architraves, obelisks, and so forth, as well as the relevant means for building and transport, continued to evolve, culminating in the Late Kingdom. For example, alterations in work processes or construction techniques would emerge on the grounds of experiences made with different types of material like local bedrock. Indeed, if one may say so, solutions to construction issues tend to be as 'simple' as the available techniques.

On the other hand, the construction of the pyramids is unthinkable without extensive preparation and detailed plans. Exact schedules for each task and a highly sophisticated infrastructure were indispensable. Choosing quarries yielding adequate material and finding a site suitable as a harbor for the shipping of material, surveying the pyramid's base area and orienting it, calculating and providing, marking and storing of building material were tasks that required an experienced and exceedingly well-organized team. We have evidence that in the Middle Kingdom stone blocks were inscribed with precise descriptions comprising the date of production, the mason in charge, the transport pathway, the storage area, and so forth.[3] It seems reasonable to assume that a comparable level of organization also existed in the Old Kingdom.

Preparations for work in the entire construction area required painstaking organization. Similarly, regulations were needed for transfer of material from the quarries and the harbor, and also for the recruiting and maintenance of the workforce. Just looking at the number of blocks and the building time as such, we must inevitably conclude that some sort of 'just in time' principle was employed to coordinate quarries, harbor, storage, and building site. Unfortunately, no construction reports are available for the Third and Fourth Dynasties. However, a pictorial representation from the tomb of the royal architect Senedjem-jb Inti, dated to the late Fifth Dynasty, can be regarded as proof that the owner of the tomb was in charge of a plan for the pyramid complex of King Djedkare Isesi.[4]

Considering the life expectancy of the ancient Egyptian rulers, the building schedule for the pyramid complexes was extremely tight. Works at several stages of construction must have been carried out in various areas of the site at the same time.

Much information pertaining to ancient Egyptian craftsmanship can be found in pictorial representations, notably reliefs in tombs of the kings' relatives and officials. Such representations can be compared to other archaeological evidence, and conclusions can be drawn with reference to the reconstruction of long-forgotten techniques. Below, I shall discuss some construction techniques that have emerged from such research with reference to the relevant archaeological findings and sources.

Notably, the sarcophagus of Khufu, the first known sarcophagus of the Old Kingdom to be made from granite, has been extensively studied, and it has been proven that copper saws in combination with sanding were used to accomplish its remarkable exterior design.[5] Further to

[3] Arnold, F. 1990. *The control notes and team marks*. New York, 14.
[4] Stadelmann, R. 1990. *Die großen Pyramiden von Giza*. Graz, 248.
[5] Stocks, D.A. 1999. "Stone sarcophagus manufacture in Ancient Egypt," *Antiquity* 73, 918-922.

this, Stocks has published an in-depth study of the usage of saws, including documentation of experimental approaches and other aspects of stone working.[6]

Drills are shown on various representations from the Old Kingdom period. Borchardt, for example, refers to a picture of a drill for carving hollow vessels from the Fourth Dynasty. The drill has a forked shaft, and pieces of flint could be inserted as drill bits depending on the required diameter. At its upper end, an arched crank handle is attached, together with two stones on ropes that serve as weight and flywheel at the same time. If pressure is kept constant, the stone weights assure an accurate centering of the drill's working end. In the tomb of Ti in Saqqara, a workman is depicted with a drill of this type (fig. 1).

The simplest lifting device attested from the Old Kingdom onwards is a wooden lever beam. It enables the user to lift heavy weights attached to the shorter arm with relatively little force by moving the longer arm of the lever. This method was used to loosen stone blocks from the bedrock in the quarries of Giza, and also for laying and adjusting stones for the pyramid's core and casing.

The usage of drills and lever beams bears witness to the fact that in the Old Kingdom, ***the functioning principle of cranks and rollers*** was already understood and put to practical use; the principle of ***force intensification*** was therefore known implicitly.

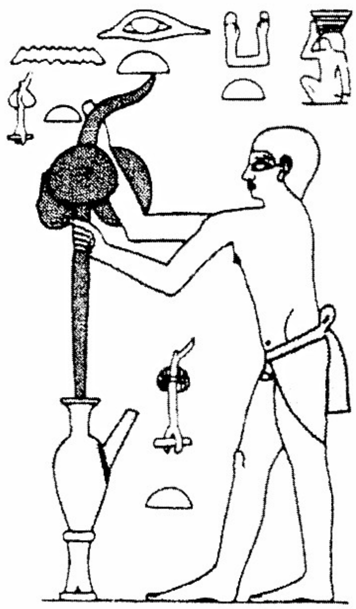

Fig. 1 The workman from the tomb of Ti.[7]

There is ample evidence for devices needed for ***lowering*** and ***lifting*** of heavy weights in Ancient Egypt. The earliest evidence is a bearing stone used to erect building elements by means of rope deflection. It was found on the site of the Valley Temple (possible harbour area) of Menkaure and is dated to the Old Kingdom (fig. 2).[8]

[6] Stocks, D.A. 2003. *Experiments in Egyptian Archaeology. Stone working technology in Ancient Egypt*. London.
[7] Steindorff, G. 1913. *Das Grab des Ti*. Leipzig, pl. 134.
[8] Reisner, G.A. 1931, *Mycerinos. The temples of the Third Pyramid at Giza*. Cambridge MA: Harvard University Press, 276 pl. A.

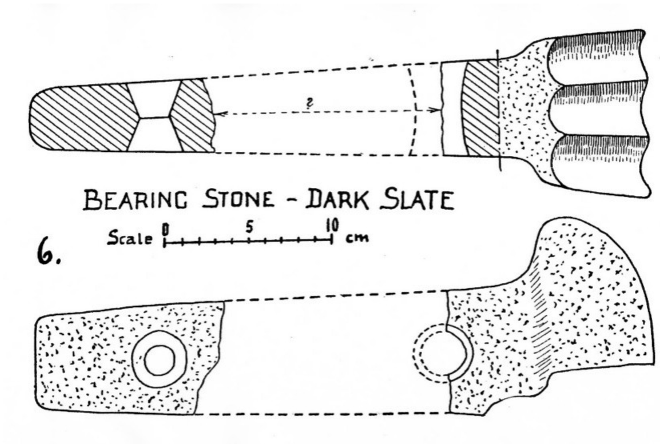

Fig. 2 Bearing stone.

Arnold suggests that it was used in a wood structure.[9]

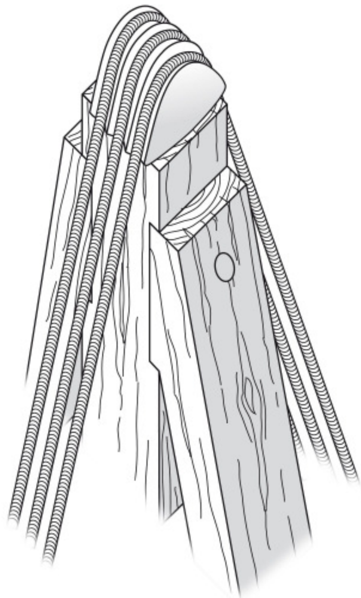

Fig. 3 Bearing stone in use, as reconstructed by Arnold.

In the upper part of the antechamber of the pyramid of Khufu, semicircular grooves can be seen in the West wall, which served as rests for wooden rollers used for lowering the plugging blocks (fig. 4).

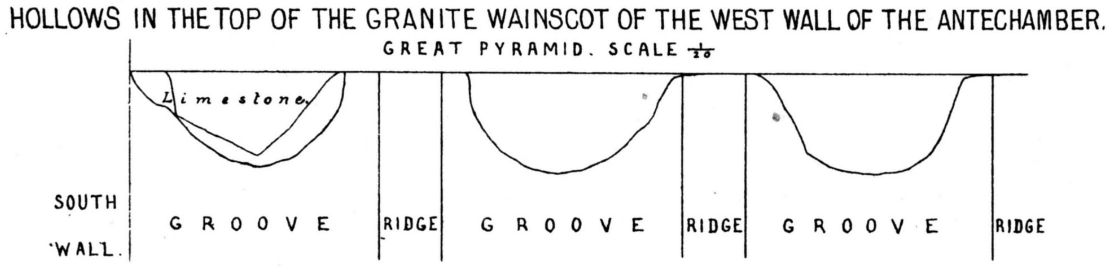

Fig. 4 Grooves in the wainscot of the antechamber in the pyramid of Khufu.[10]

[9] Arnold, D. 1991. *Building in Egypt*. Oxford: Oxford University Press, 283.
[10] Petrie, W.M. Flinders 1883. *Pyramids and Temples of Gize.* (London, 1883), pl. 12.

Thus, on the basis of archaeological finds (rollers embedded in semicircular grooves (fig. 4) together with fixed rope redirection devices, (fig. 2 and fig. 3), it can be argued that the principle of force redirection was known the **Old Kingdom**. Furthermore, mobile disk wheels (sheaves) are attested for the Fifth Dynasty. Therefore, mobile wooden rope rolls, functioning according to the winch principle, may already have existed in the Old Kingdom. Consequently, while pulleys as such are not documented for the Old Kingdom, they cannot be discounted altogether.[11]

In a tomb at Lisht North, three wooden rope rolls were found, which were obviously used for guiding two ropes each (unpublished information, fig. 5). Arnold initially dated them to the late Twelfth Dynasty, that is, to the **Middle Kingdom**.[12] Later, they were used for lowering a limestone slab which displays two drill holes for attaching the ropes. This is the earliest evidence for rope redirection by means of a roll revolving on a roller. However, one of these rope rolls kept at the Metropolitan Museum in New York is now dated to the Nineteenth or Twentieth Dynasty.[13]

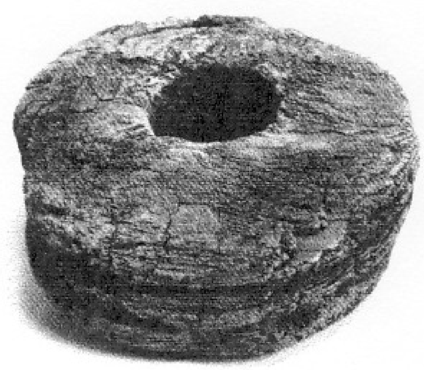

Fig. 5 Wooden roll with two guide grooves for ropes (dated either to Twelfth or Nineteenth to Twentieth Dynasty).

Rolls such as these can be used to construct a pulley. Further similar rope rolls, including some larger ones, are documented for the **New Kingdom** (fig. 6).[14] At Deir el-Medina, in tomb no. 1353 which is dated to the Eighteenth Dynasty, a revolving roll on a roller, or respectively a winch, with a raffia rope was found.

Similarly, Lauer describes a wooden rope roll discovered at the site of the pyramid of Djoser at Saqqara, which was apparently intended to revolve on a roller, and thus used for rope redirection (fig. 7).[15] However, Clarke and Engelbach argue for a Roman dating of this redirection roller,[16] while Arnold dates it to the Late Period.[17]

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[11] Stadelmann, R. 1990. *Die großen Pyramiden von Giza*. Graz, 135.

[12] Arnold, D. 1991. *Building in Egypt*. Oxford: Oxford University Press, 71 and 103 with n. 56 (text pertaining to note 55).

[13]http://www.metmuseum.org/works_of_art/collection_database/egyptian_art/block/objectview.aspx?collID=10&OID=100027095, accessed 24 October 2010, 11:30 am.

[14] Bruyère, B. 1937. *Les Fouilles Deir el Médineh 1933-1934*. Cairo: Institut Francais, 122 fig. 54.

[15] Lauer, J.-P. 1936. *La Pyramide à Degrés*. Vol. 1: *L'Architecture: Texte.* Cairo: Institute Français d'Archéologie orientale, 52.

[16] Clarke, S. and Engelbach, R. 1930. *Ancient Egyptian Masonry.* London: Oxford University Press, 44 n. 2.

[17] Arnold, D. 1991. *Building in Egypt*. Oxford: Oxford University Press, 71.

Fig. 6 Rope roll (Eighteenth Dynasty).

Arranged on a round beam with a diameter of approx. 8cm, it served as a kind of sheave in a bearing device. The round beam seems to have rested on supports at either end. From this model, a clearance of approx. 60cm can be deduced as a basis for the following calculations.

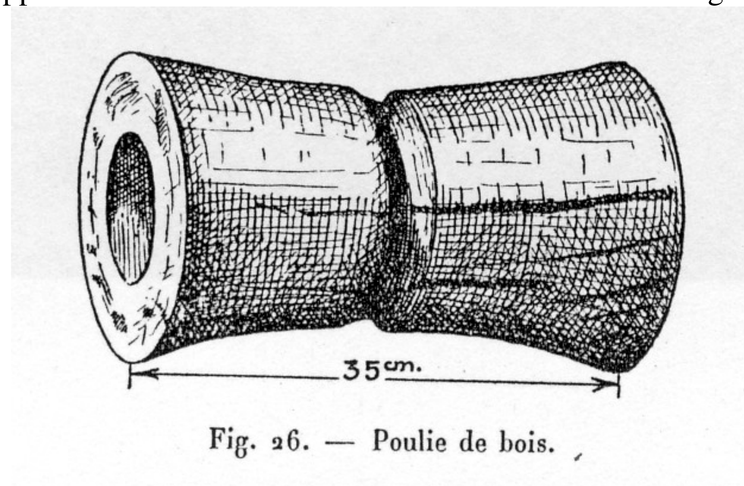

Fig. 7 Rope roll from the site of the pyramid of Djoser.

In order to estimate the force which the rope roll is intended to redirect, I base my calculcations on a maximum resistance to bending stress of 1000 kg/cm² for hardwood[18] and 750 kg/cm² for nile acacia.[19] A total diameter of approx. 10cm can be deduced from the rope roller's width and from the fact that it would have needed to rest safely on the round beam. Resistance in the rope roll's middle must therefore be at least 125cm³ ([radius 5cm]³), so that the resulting bending moment amounts to 125 000 (hardwood) or respectively 93 750 kgcm (nile acacia). [20] Thus, the maximum force to be redirected amounts to 4 167 or 3 125kg respectively.[21] For a redirection roll with an integrated roller and an estimated diameter of 15cm, the maximum force to be deflected amounts to 14 067 or respectively 10 547kg.

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[18] dos Santos, A. 1998. "Theorien zur Bautechnik der Großen Pyramide" In *Kemet* 7 (3) 27 sqq. – According to Croon, African Bulletrie wood resists strains of up to 2000 kg/ cm^2 (Croon, L. 1925. *Lastentransport beim Bau der Pyramiden. Dissertation*. Hannover: Buchdruckerei des Stephanstifts)

[19] Croon 1925, 42 sqq.

[20] The bending moment is defined as the resistance moment multiplied by bending strength.

[21] The maximum force to be deflected results from dividing bending strength (125 000 bzw. 93 750 kgcm) by half the clearance (30 cm).

Rope rolls of this type in various designs and measurements were used in Ancient Egypt for force redirection, including heavy loads and transport on sloping ground. The pyramid of Khufu is the first building in which rollers were employed for controlled lowering of plugging blocks. Later, in the antechamber of the pyramid of Menkaure, the same principle was employed.

Therefore, the use of redirection rollers should be taken into account when discussing lifting and lowering of loads and transport across inclined planes. Such connections between the various groups of archaeological evidence have not previously been discussed to satisfaction, but it emerges that a well-functioning system of lifting devices must have been available in the Old Kingdom from the Fourth Dynasty onwards at the very least. In fact, the combination of lifting devices and an inclined plane is virtually the only way of explaining how the gabled roofs in the pyramid of Khufu, or the burial chambers built in open cut construction as used in the pyramids of the Fifth and Sixth Dynasty, or even the gabled roof in the burial chamber of Djedefre in Abu Roash were built. Constructions such as these require lifting devices combined with slopes or ramps (inclined plane).

Building material and similar objects were conveyed on a ***level plane*** by means of sledges. Several pictorial representations from tombs of officials of the Old Kingdom show transports of statues and other goods on sledges. In a relief from the tomb of Hetepherakhti, a high-ranking official from Memphis (Fifth Dynasty),[22] two bulls are pulling a device possibly made from wood, upon which the statue is secured against falling or shifting (fig. 8).

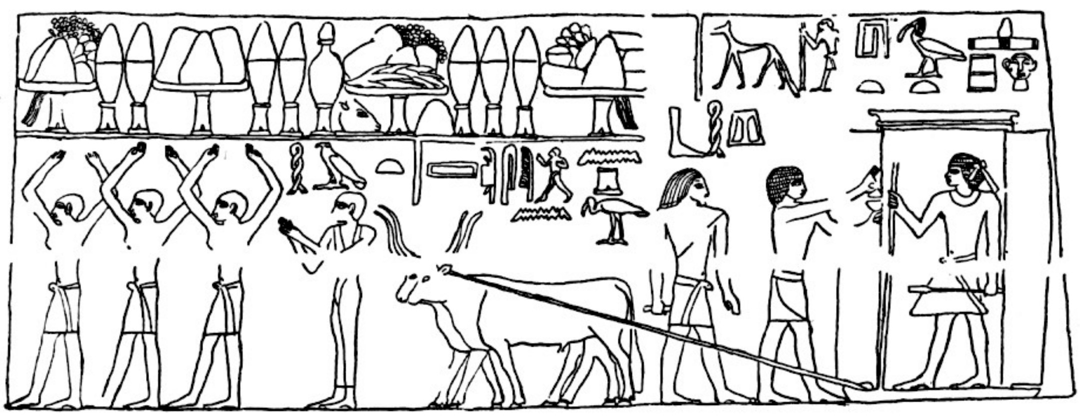

Fig. 3

Fig. 8 Transport of a statue – two bulls pulling a carriage, from the tomb of Hetepherakhti.

One of the ***principles of Egyptian construction technique*** was to pull, rather than lift, large or heavy stone blocks. This was the case for transports to the building site, and for bringing the blocks into their definite position. An inclined plane, or ramp, was commonly used, as is attested by a remarkable number of Old Kingdom ramps connecting quarries and pyramid sites, and by the causeways.

When towing crews or draught animals are employed to pull loads on ramps, it is a necessary precondition that frictional force R

[22] Mohr, H.T. 1943. *The Mastaba of Hetep-her-Akhti*. Leiden: Brill, 36 fig. 3.

$$R = (\mu \text{ [coefficient of friction]}^{23} \cdot Q \text{ [load]} \cdot \cos \alpha \text{ [Angle of elevation of the plane]})$$

exceeds the gravity forces on batter, for only then backwards sliding of the sledges can be prevented, as they stop automatically whenever the towing crew pauses or a rope tears.

Striction of each transport object is determined by the contact surfaces and the normal force exerted on its support by the object. In the present case, it has to exceed kinetic friction by about 20%. However, it is easily overcome when additional thrust is applied, or by means of leverage.

Thus, a maximum slope of approx. 15° results for our ramp when we take into account the coefficient of friction applicable when stone blocks are conveyed on ramps covered with sand or gravel.[24] Transport ramps with a lower coefficient of friction (smooth stone surface, wet surface, facilitation of bull carriage towing by round beams positioned on the track horizontally[25]) may not exceed a slope of 8° with a view to prevention of backwards sliding, as explained above. Accordingly, the slope of the ramp at the Mortuary Temple of Menkaure is approx. 7°, the ascent from the Valley Temple of Khafre and his pyramid is 6°[26] and the angle of the ramp running from the quarry in the South toward the pyramid of Khufu is 6° (with a slope of 10%).[27] Under such circumstances, it is easily explained how bull carriages could haul stone blocks weighing up to 200tons to the Mortuary Temple of Menkaure, or respectively blocks of up to 425tons to the Mortuary Temple of the pyramid of Khafre.[28] Stops were possible whenever necessary, as stiction would prevent slippage.

Only the small Layer Pyramids of Lisht, Meidum and Sinki have perpendicular ramps which were used in building the pyramid itself. Other than that, only ramps that delivered material to the area surrounding the immediate construction site are attested. No sufficient amount of ramp material has been found in the necropolis areas, not even in the case of pyramids that were abandoned in mid-construction.[29] Disposal sites attested in the pyramid area of Giza consist largely of a mixture of limestone, gypsum and tafla without any obvious remnants of Nile mud. They are usually interpreted as waste that was disposed of during the building of the pyramids.[30]

This being the state of archaeological research, one cannot but draw the conclusion that a building technique was used during the Old Kingdom which did not depend on perpendicular ramps that required enormous amounts of building material as their height had to be gradually increased throughout the construction process.

Steep ramps displaying a gradient ratio of 2:1 (26°33’54’’) with only minor exceptions to this rule are used in the pyramids of the Fourth Dynasty for downward and upward bound

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[23] The friction coefficient for a smooth surface (wood or stone) is 0.6 (wood on stone) or 0.3 (stone on sand or gravel).
[24] Dörnenburg, F. 2008. *Pyramidengeheimnisse? Enträtselte Mysterien*. Munich: Brose, 148.
[25] These round beams can be compared to railroad sleepers in that they are lodged firmly in a ballast bed (limestone chips, small stones). They are able to carry considerable weight, because the ballast functions as solid underground which precludes sideways slippage by canting of the stones.
[26] Stocks, D.A. 2007. “Das Bewegen schwerer Steinobjekte im Alten Ägypten. ” In *Sokar* 15 (2), 74 sqq.
[27] Lehner, M: 1985. “The Development of the Gisa Necropolis: The Khufu Project.” In *MDAIK* 1985, 127.
[28] Maragioglio, V. and Rinaldi, C.A. 1966. *L’Archittetura delle Piramidi Menfite*, vol. 5. Torino: Officine Grafiche Canessa, 64 sqq.
[29] Stadelmann, R. 1997. *Die ägyptischen Pyramiden*. Mainz: von Zabern, 226.
[30] Lehner, M: 1985. “The Development of the Gisa Necropolis: The Khufu Project.” In *MDAIK* 1985, 124 and 132.

corridors. Hence, steep ramps are attested in the Old Kingdom.[31]. However, in these cases it is unlikely that the loads were pulled by towing crews and bulls.

In the Debeheni Mastaba at Giza, dating to the early Fifth Dynasty, a depiction of a ramp with a gradient of 2 : 1 from the Old Kingdom is preserved (Fig. 9).[32]

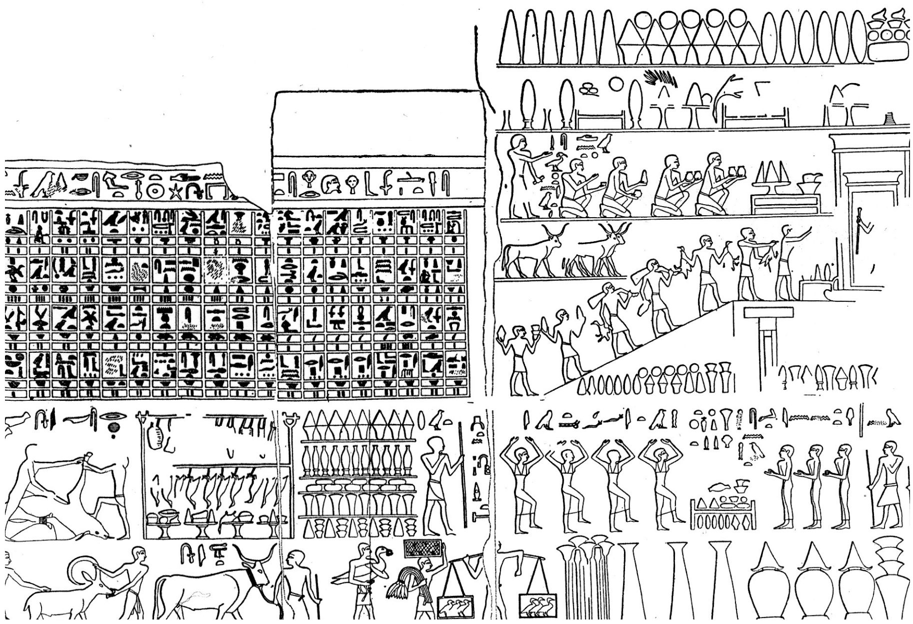

Fig. 9 Ramp from the tomb of Debeheni as published by Hassan.

It is also interesting to consider the picture of a ramp from the tomb of Rekhmire (TT 100 Thebes West) although it is dated to the Eighteenth Dynasty (fig. 10). To the right, three walls or columns can be seen, the intervals between which are filled with brick walls. To the left, a ramp runs towards a building under construction, apparently enabling upward transport of stone blocks to the next layer. The gradient ratio of the ramp is 10:5, or 2:1 (26.5°).

Fig. 10 Brick ramp, from the tomb of Rekhmire.[33]

[31] Becker, J. 2003. "Die Funktion der Pyramidenkorridore als vermessungstechnische Einrichtungen. " In *Sokar* 6 (1), 14-21.
[32] Hassan, S. 1943. *Excavations at Giza*, 1932–1933, vol. 4. Cairo, Fig. 122.
[33] Davies, N. de Garis 1943. *The Tomb of Rekh-mi-Re at Thebes*, vol. 2. *Egyptian Expedition* vol. 11. New York: Publications of the Metropolitan Museum of Art, pl. LX.

Archaeological features of the Old Kingdom pyramids

Initially started as a mastaba, the ***Pyramid of Djoser*** was built in layers (two to each step), and is therefore rightly referred to as Layer Pyramid. The basic techniques for the construction of such Layer Pyramids were developed at that time. Blocks were arranged around the core, with inward-leaning layers. The pyramids of king Sekhemkhet and Khaba (?), of which only the stumps are preserved, and the small pyramids (possibly of Huni?) were built according to this principle. There is practically no evidence for a smoothed casing here.[34]

The first ***Pyramid of Seneferu*** at Meidum was built as a Layer Pyramid in several stages. In stage E1 and E2, limestone blocks were used for the visible surface of the steps, and dressed after application. After the finishing of the Layer Pyramid in stage E2, the steps were filled with another layer (E3), so that the façade now formed a constant slope, rather than a stepped outline. Afterwards, it was cased with finely grained limestone. In this monument, we are able to witness the transition from a visibly stepped Layer Pyramid to a pyramid with steps filled to form a facade with a constant slope (‘true pyramid’). This development took place parallel to the construction of the Bent Pyramid. The cult pyramid of Meidum was also built as a Layer Pyramid.

It is virtually impossible to offer an archaeologically sound explanation for the construction process of the ***Bent Pyramid***, especially its inward structures. The start of its construction is usually dated to twelve to fifteen years after the start of the construction of the Meidum pyramid, and at this time, no severe difficulties had occurred with the Layer Pyramid style. It is therefore likely that the Bent Pyramid was initially designed as a Layer Pyramid as well. When the first stage of construction was completed, backing and casing blocks were added in the next stage to form a constant slope in its lower half. When the additional casing had been finished, the pyramid’s construction was continued from a height of 90 cubits (47m) onwards with a less steep slope. Moreover, the construction technique was modified: The stone courses underneath the casing were then set horizontally, that is to say the degree of inward leaning was lessened as the building gained height. Also, the stone blocks used were smaller than those used in the casing below.

The design of the attached Satellite Pyramid then followed the ‘new’, modified construction principle with horizontal courses of stone blocks. This might have served as a test for the construction technique later adopted in other pyramids.

To sum up, the transition from the principle of inward-leaning layers, as seen in the Layer Pyramids, to construction by means of horizontal courses, the latter being characteristic for the Fourth Dynasty, probably took place during the reign of Seneferu. Having been pinned down to the building process of the upper half of the Bent Pyramid, the attached cult pyramid, and the Red Pyramid, it can be explained from the difficulties that occurred during construction of the Bent Pyramid.

The ***Red Pyramid*** is the first to display fully horizontal courses of casing blocks. Due to its good state of preservation, archaeologists have not been able to obtain any definite information as to the structure of its core. However, it is hard to imagine that the core would consist of inward-leaning layers after the disturbing experiences made during the building of the Bent Pyramid.

[34] The casing of the lowest step of the small pyramid at Saujet el-Meitin is an exception to this rule.

Another significant change took place during the construction of the Red Pyramid, because the large blocks familiar from the Giza pyramids are used here for the first time. Consequently, new transport techniques must have been developed for these blocks. Considering the chronology of Seneferu's reign, the pyramid's construction started relatively late, and it is possible that the builders were pressed for time, so that changes in the construction technique were at least partly dictated by tight schedules.

It is not a far-fetched thought that the first and foremost way of improving construction techniques to such a degree as to enable the building of the Great Pyramids was to collect solutions developed from difficulties encountered previously in this epoch characterized by intense pyramid building activity. Therefore, we may conclude that the Red Pyramid consists of steps with horizontal courses, marking the beginning of the Step Pyramid style.

Scholars generally speak of a transition from the so-called Step Pyramids (as the Layer Pyramids are often mistakenly called) towards the 'true' pyramid during the reign of Seneferu. However, the actual circumstances of this transition are hardly ever considered. Within eighty years after the erection of the first pyramid by Djoser, the 'classic' pyramid was developed as the type of royal burial that should dominate the ancient Egyptian necropolis throughout the upcoming centuries. Seen from an evolutionary perspective, the development of Egyptian burial structures first started from the mud brick mastaba, moving on to the Layer Pyramid and the Step Pyramid and reaching a climax in the 'classic' pyramid with a constant slope.

Observations made on the breach in the South face of the pyramid of Khufu and in the tomb robbers' tunnel on the North side, and also on the breach in the North face of the pyramid of Menkaure (fig. 11) suggest that the horizontal courses of the casing do not match those of the core. In both pyramids the filling within the core walls that form the steps consists of roughly hewn stones of various sizes and shapes piled on top of each other irregularly, and joined by mortar. Some of these are clearly unworked quarry stones.

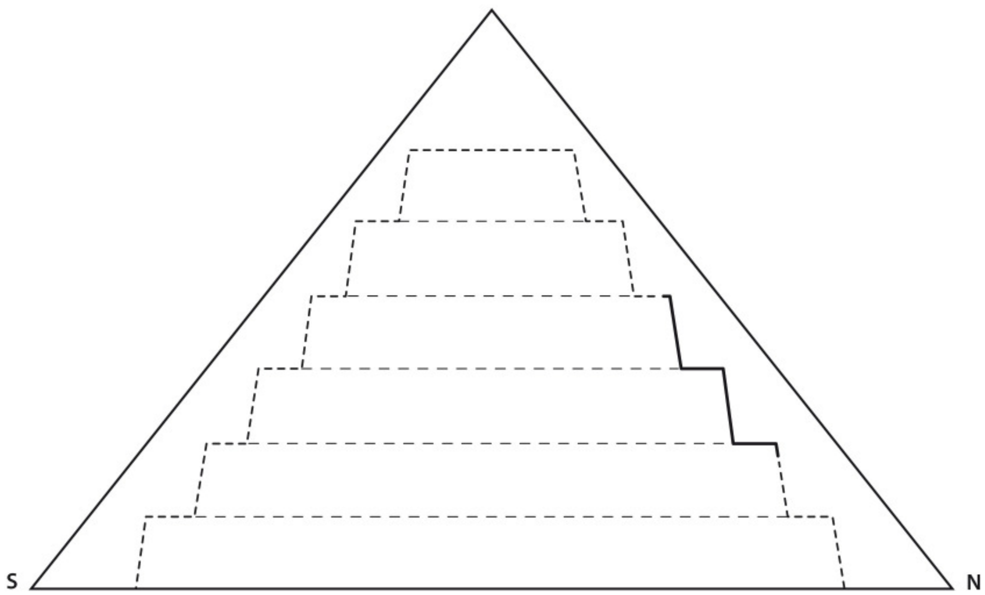

Fig. 11 Cross-section (S-N) of the pyramid of Menkaure with steps archaeologically attested in the breach on the North face (drawn lines) and the other steps of the core reconstructed by analogy (dotted lines).

Production, transport and laying of the stones for the core would have required less effort than for the casing. The stepped core served to improve the inner stability of the building, while also increasing its resilience against external damage. However, the actual structure of the core of the Khephren pyramid remains unknown up until today.

Exact positioning and measuring of the casing's base was essential. This was the only way to ensure that the sides would actually converge at the top. After each measurement, any necessary fine tuning could be executed in the following layer. The remarkable degree of exactness that was reached in the measurement of the base of the pyramid of Khufu – even while the rock spur in its middle would have precluded diagonal measuring – shows that measuring of this sort would not have posed insurmountable difficulties.

Any distortion of the pyramid was prevented by measuring the length of the sides of a finished course of casing blocks at the pyramid stump from one corner stone to the other ("length measurement") and control of the horizontal course ("horizontal leveling"),. Furthermore, straight lines were established between the corner stones which had been prefabricated to have a rectangular base and a defined slope of the outer ridge, indicated by marks on the upper surface of the blocks.[35]

There is no sufficient evidence for orientation on the points of the compass for this purpose during construction of the pyramids, and with a view to other well documented measurement procedures, it does not seem probable either.

The so-called distortion observed by Petrie at the top of the pyramid of Khephren which is quoted by Unterberger as support for his hypothesis that the points of the compass were used during construction does not seem cogent upon closer examination.

While working at a considerable distance, Petrie measured the deviation of the lower preserved layers of casing in the upper part of the pyramid of Khephren from the directions of the pyramid's base as follows: NE corner + 1.7 inch, SE corner + 0.6 inch, SW corner +0.3 inch, NW corner + 0.3 inch. Consequently, he arrived at an average of + 00°01'40" of distortion inbetween the four sides. However, this argument is not effective, because such differences are more likely to suggest that minimal inaccuracies in the production of the corner stones. Rather, it is a remarkable achievement that there are only such minuscule irregularities from the ideal measurements. This is yet another proof for the astonishing degree of advancement of construction techniques in the Old Kingdom.

The foundations of the casing were leveled out and measured permanently. The same is true for the other courses of the casing and their respective offset. It was essential to maintain the same height of the steps in order to achieve a constant batter. Depending on the quarrying, the stone layers may have had different heights. Under such circumstances, exact monitoring of the batter was the only viable way to check the angle of inclination, on which the height of the pyramid depended. The fact that the pyramids' exact slopes vary is most likely a result of different designs, or else just slightly inexact measuring. Besides, it has to be admitted that today, the angle of inclination can often be only roughly determined due to the pyramid's state of preservation. It seems likely, however, that the actual height of the pyramid was not included in the initial design, rather, in effect, it resulted from the added offset of all steps.

[35] Müller-Römer, F. 2011. *Der Bau der Pyramiden im Alten Ägypten*. Utz, Munich, 112.

The Step Pyramid style was kept in use throughout the Fifth and Sixth Dynasties, as is attested by archaeological findings on all pyramids of this period, including the cult pyramids and Queens' Pyramids. The pyramids of Menkaure and Userkaf mark the beginning of a period of pyramids with less monumental dimensions than those of the Fourth Dynasty. The development continued in this direction until the end of the Sixth Dynasty (Pepi II). However, the technique used in constructing the steps changed during the Fifth and Sixth Dynasty. The stepped core walls of the steps consist of an outer wall executed in well-hewn stones with slight batter, while towards the inside, we find a filling of blocks of various shapes and sizes that are only roughly hewn and mixed with mortar and rubble. Thus, by using smaller blocks, it was possible to lower the expenses for production and transport in comparison to the pyramids of the Fourth Dynasty.

The courses of large well-hewn stones casing blocks characteristic of the Fourth Dynasty pyramids disappear, making way for smaller backing stones of irregular shape filling the spaces between the steps and the outer limestone casing. As a result, when the casing was stripped, the mortar was exposed to the elements so that parts of the core masonry collapsed. Stadelmann, Lehner and others speak of slack workmanship here; similarly, Maragioglio and Rinaldi attribute the state of the building to the inferior quality of the masonry. However, I should like to argue that this construction technique was the result of experiences made in pyramid building to the extent that in smaller pyramids, a stable outer casing would have sufficed to protect the core from weathering or erosion. Surely, experience in pyramid building would have become increasingly profound over the years. Moreover, in order to keep within tight building schedules while also minimizing the costs, the small pyramids did not receive such a closely-fitted casing as the pyramids of the Fourth Dynasty, whose inner casing masonry even resists the weather and most of the erosion when much of the outer casing is stripped.

To sum up, all pyramids of the Old Kingdom, or at least from the pyramid of Khufu onwards, have stepped cores (layered or stepped construction).[36]

In the immediate surroundings of the pyramids, no ramps have been attested by archaeological evidence that could have been used for building the pyramid itself rather than for transport of stone blocks to the construction area. The pyramid of Seneferu in Meidum is the one notable exception to this rule.

The pyramids of the Old Kingdom were planned and built over a period of roughly four hundred years, and a broad spectrum of construction techniques must have been invented and applied, while the techniques themselves were improved continually.

Some previous explanations of pyramid building

Nearly all relevant explanations of pyramid construction have been formulated with respect to the pyramid of Khufu. As stated above, with a view to archaeological evidence, the techniques did not change much throughout the period of four hundred years (Third to Sixth Dynasty), during which roughly twenty-five pyramids were built, with the one notable exception of the transition from Layer to Step Pyramid during the reign of Seneferu. Other improvements were introduced in a less dramatic way and can hardly be traced nowadays.

[36] In the case of the Bent Pyramid, the Red Pyramid, and the Pyramid of Khephren, this has not previously been proven.

Therefore, we have good reason to assume that any theories on the building of the pyramid of Khufu should also serve to explain the building of other pyramids of the Old Kingdom.

An analysis of the suggestions published so far with reference to the building of the ancient pyramids in general, and the pyramid of Khufu in particular, yields the following three categories:

- solutions involving perpendicular ramps
- solutions involving ramps built along the pyramid's sides
- solutions making use of lifting or towing devices

Any discussion of the subject should fulfil the prerequisites listed above, for otherwise it does not measure up to the requirements of the situation, thus failing to meet the standards of academic discussion.

Perpendicular ramps as suggested by Arnold,[37] Stadelmann,[38] Lauer,[39] Lattermann,[40] and others ***can be excluded as a means of construction*** on the grounds of the following observations:

- The ramps require continuous increase in height and accordingly, width. This leads to regular disruption of the construction works, or at least considerable delays.
- The ramps have a maximum slope higher than 7° respective 15°.
- The volume of the ramps would have been enormous, easily reaching or even exceeding the volume of the pyramid itself, depending on the design of the ramp.
- No archaeological evidence has been offered that cogently proves the existence of ramps or appropriately large rubble deposits.
- Free-standing ramps built from clay or mud bricks sustain their own weight only up to a height of 120m.
- Archaeological evidence from the Step Pyramids, notably the pyramid of Menkaure, contradicts the hypothesis that the core was erected layer-wise, casting doubts on an idea that is crucial to the ramp models.
- Due to safety concerns and aspects of construction technique, the application of the casing and dressing required scaffolding or other attachments to the pyramid's facade.

Spiral ramps, as proposed by Goyon,[41] Lehner,[42] Klemm and Klemm,[43] Houdin,[44] and others ***can also be excluded as a method of construction*** on the grounds of the following observations:

- At the corners of the pyramid, spirals ramps would have necessitated transport of building material at an angle of 90°, even if the corners were chamfered. This was hardly possible, and even if it was, it would have been an excessively time-consuming

[37] Arnold, D. 1981. “Überlegungen zum Problem des Pyramidenbaus.” In *MDAIK* 37, 15-28.
[38] Stadelmann 1990, 266-275.
[39] Lauer, J.-P. 1989. “Le Problème de la Construction de la Grande Pyramide.” In *RdE* 40, 91-111.
[40] Lattermann, W. 2002. *Der Bau der Cheopspyramide*. München.
[41] Goyon, G. 1990. *Die Cheopspyramide*. Augsburg.
[42] Lehner, M: 1985. “The Development of the Gisa Necropolis: The Khufu Project.” In *MDAIK* 1985, 109-143.
[43] Klemm, R. and D. Klemm 1998. “Die Integralrampe als Konstruktionselement großer Pyramiden.” In H. Guksch, ed., *Stationen. Beiträge zur Kulturgeschichte Ägyptens. R. Stadelmann gewidmet*. Mainz: von Zabern, 87-94.
[44] Houdin, J.-P. 2007. *Cheops. Die Geheimnisse um den Bauprozess der Großen Pyramide*. Mainz: von Zabern.

task. Also, no feasible solutions have been offered as to the redirection of the traction force.
- The idea that the core masonry was laid layer-wise is not corroborated by archaeological evidence.
- The limited transport capacity of the spiral ramp leads to a schedule for the pyramids' construction that by far exceeds the attested time frame.
- As stated above with reference to other ramp models, due to safety concerns and aspects of construction technique, the application of the casing and dressing required scaffolding or other attachments to the pyramid's facade.
- Exact measuring throughout the building process would have been difficult as spiral ramps are thought to have enclosed the pyramid, partly obscuring its outer face and corners. It would have been very difficult indeed to ensure a constant batter.

Furthermore, the authors suggesting ramps often omit calculations of transport capacities, thus failing to set their solution in relation to the time frame suggested by archaeological evidence, which in the case of the pyramid of Khufu is a maximum of twenty-three years total.

The solutions offered by Arnold,[45] Isler,[46] and Graefe,[47] who propose ***constructions involving steps*** leaning against the pyramid's faces at right angles, are no more convincing. These models pose static problems because the steep slope would have required upward leverage of the stone blocks, which was practically impossible.

Several hypotheses about pyramid building (Graefe, Landt,[48] Hölscher) share the idea that from the stepped core of the pyramids, which is archaeologically attested, it can be deduced that ***ramps*** were used which ran ***in line with the steps' lateral surfaces***, and whose slope was adjusted to the transport method chosen for the respective material. On the broad lower steps, several ramps could be installed on all sides of the pyramid, so that even the vast amount of material needed for the lower area of the pyramid could be transported and laid within an economical frame of time and effort. Keyssner suggests that stepped construction platforms were attached to the sides of the pyramid, with winches positioned on the steps to enable hauling up of the stone blocks.[49] Again, we must hold against this that the concept of layer-wise building of the core employed in this model is not corroborated by archaeological evidence which rather suggests a stepped core.

Most hypotheses start from the assumption that traction force was employed, which is to say, bull carriages or workmen were used for hauling. If the loads were heavier than usual, or the ramp slopes steeper, larger carriages or more workmen would have been required, and also increasingly long ramps.

Some suggestions concerning the transport of building material for the pyramids are based on rather complex construction processes, so that one may wonder whether the ancient Egyptian architects would have indeed employed such sophisticated methods. What is more, the issues of constructing the top of the pyramid, setting the pyramidion, casing, and dressing often suffer neglect. The same applies to safety concerns. From all this, we cannot but draw the

[45] Arnold, D. 1981. "Überlegungen zum Problem des Pyramidenbaus." In *MDAIK* 37, 15-28.
[46] Isler, M. 1985. "On Pyramid Building," *JARCE* 22, 129-142, and Isler, M. 1987. "On Pyramid Building II," *JARCE* 24, 95-112.
[47] Graefe, E. *Über die Determinanten des Pyramidenbaus bzw. Wie haben die Alten Ägypter die Pyramiden erbaut?* at http://www.uni-muenster.de/IAEK/org/WMA/graefe/pyr/index.html.
[48] Landt, E. 1923. *Ein neuer Kampf um die Cheopspyramide*. Berlin: Weidmann.
[49] Keyssner, H.K. 2007. *Baustelle Giza. Kritische Untersuchung zum Bau der Cheopspyramide.* Karlsruhe: Institut für Baugeschichte der Universität Karlsruhe.

conclusion that yet another construction method was used which neither involved perpendicular, nor spiral ramps, nor quite so large quantities of material, nor a step construction.

My suggestion, which I shall outline in the following chapter, combines the idea of ramps set out parallel to the pyramid's sides and that of hauling devices with winches in connection with ramps with a steeper slope. Again, I should like to emphasize that I have incorporated only such techniques and procedures which are either supported directly by archaeological evidence or conclusions drawn from archaeological data.

Ramps and redirection rollers in the construction of the pyramids[50]

Starting from archaeological evidence, I shall propose an explanation for the building of the pyramids and an estimate of the necessary time frame, using the pyramid of Menkaure as an example. I have chosen this pyramid because its stepped construction, the laying technique of inner and outer casing, and its measures are easily accessible, and therefore subject to archaeological documentation and discussion.

My hypothesis involves the assumption that construction took place ***in several phases***, which can be summarized as follows:

- First, the ***stepped core*** is built up to the sixth step. Transport of stone blocks takes place via ramps with a gradient ratio of 2:1 positioned on all four sides of the pyramid on the step platforms of the core, in line with the sides of the pyramid. These ramps are removed as soon as the core is completed.
- Next, from the lowest course of casing upwards, a stepped structure is erected around the pyramid which serves as a ***working platform***.[51] The platform enables the workers to perform in one continuous process the laying of backing and casing stones. Transport of stone blocks for this purpose runs via steep ramps with a gradient ratio of 2:1 positioned on the working platform which, as we have said, surrounds the pyramid on all sides.
- After the setting of the pyramidion, the stepped surrounding structure (working platform) is ***removed***, and at the same time dressing takes place starting from the top of the pyramids, proceeding downwards.

The ***point of my argument*** is that on the one hand, the stepped core was built using steep ramps and redirection rollers, while ***on the other hand***, a working platform was erected around the pyramid's circumference to enable the positioning of the pyramidion and the laying, working and dressing of the casing, which was again made possible by steep ramps. In both cases, the building process would continue simultaneously on the four sides of the pyramid. Thus the complete building process including the setting of the pyramidion can be understood as one continuous process.

Redirection rollers allow even heavy loads to be hauled up over steep ramps without large towing crews. What is more, because this method combines two construction techniques that

[50] Müller-Römer, F. 2011. *Der Bau der Pyramiden im Alten Ägypten*. Utz, Munich, 335.
[51] A working platform attached to the pyramid on all sides enables best possible access to the construction site, and safety issues are thus taken care of satisfactorily. Both are necessary concerns during the laying of the casing, notably the on-site fitting of backing and casing blocks' sides to match their finely-hewn horizontal surfaces.

are archaeologically attested (ramps and rollers), it certainly could have been available in the Old Kingdom and is therefore a likely candidate.
At first glance, it may seem contradictory to install two independent ramp systems and to remove both in the course of the building process. However, the archaeological evidence does suggest that the core was built in a distinct phase preceding the application of the casing. To quote another example, the Queens' Pyramids near the pyramid of Menkaure also consist of the core only, as the casing was not finished. Furthermore, in the breach at the North of the pyramid of Menkaure, we can observe that the backing stones were fitted into the outer masonry of the core. This evidence plainly excludes layer-wise simultaneous laying of core, backing and casing with the aid of one single ramp. It is probable that static concerns prompted the separate building of the core: The intention was to avoid damage to the building through earthquakes or ground shifts by combining stable outer core walls with a filling.

If the solution I have offered holds true for the pyramid of Menkaure, it is applicable to all pyramids of the Old Kingdom, with individual modifications.

In the Queens' Pyramids of Menkaure, it can be observed that the heights of their single steps are roughly equal, apart from the lowest step, which is not quite as high as the others. Therefore, we may assume that in the pyramid of Menkaure, two more steps exist below the breach. The total height of these two steps is about 16.39m, and thus not quite as high as that of the uppermost steps. This characteristic is comparable to the construction principles of the Queens' Pyramids. The reconstruction shown below (fig. 12) suggests that the core consists of seven steps. The heights of the fifth to seventh step and their batter are inferred from the measure attested for the third and fourth steps (height 8.5m, width 4.2m). It is also possible that the seventh step consists not of a core masonry wall with filling, but rather from well-hewn blocks that were installed in alternate courses, as can be seen at the top of the pyramid of Khufu.

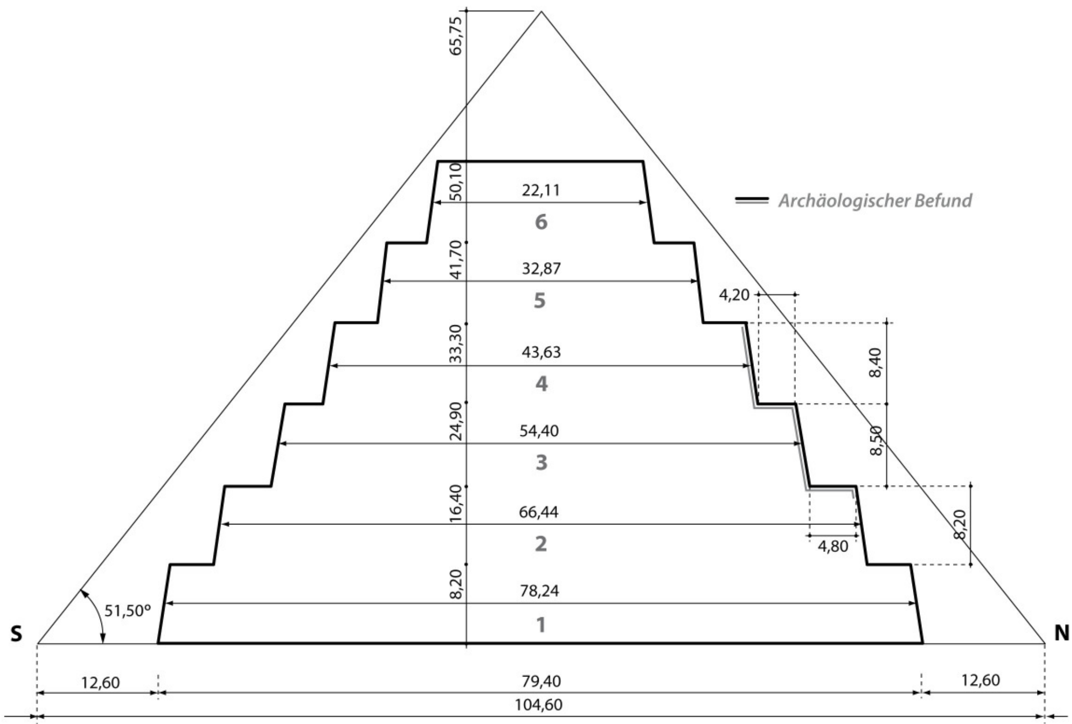

Fig. 12 Stepped core of the pyramid of Menkaure.

The building phases

Once a decision had been reached on the pyramid's construction, planning was completed and the site chosen. Then the building site was leveled and areas presenting loose rubble were supported with stone slabs. The transport ways connecting the quarries and the newly-built harbor were determined and equipped with the necessary facilities. Furthermore, the pyramid's base was fixed and aligned towards the North. The length of the base line on each side is about 80m according to Maragioglio and Rinaldi (150 cubits?). Also, the base line of the lowest layer of the casing was fixed at 196 x 200 cubits (102.2 x 104.6m). At the same time, the construction of the passages and chambers was planned, and building started.[52] As soon as these preliminary works were finished, that is, approximately a year later, the building of the actual pyramid began.

Building of the core

Archaeological evidence suggests that the second to fourth steps of the core masonry are slightly divergent in their height and width. There is no unified alignment of the steps' edges, at the best about 54°30', but this measure does not cover all of the edges in question, for example the edge of the second step is situated further inwards. The same divergence in term of step height and width can be found in the Queens' Pyramids G III c and G III b. Thus it becomes clear that a constant slope could only be achieved by exact laying of backing and casing blocks. Starting from the blocks in the lowest layer of the casing, a continuous batter had to be kept on the side faces as well as at the corners. In fact, this seems to be the reason for the outer angle's always being slightly smaller (51°30') than that formed by the edges of the steps of the core masonry (54°30'). Consequently, the outer angle of inclination could be determined separate from the core, and it was not necessary for the upper edges of the steps to be positioned as precisely as the corner stones of the backing and casing.

Right from the start of the building works, the lowest courses of blocks of the core walls were laid simultaneously at all sides of the pyramid, together with the respective filling material (stones of various sizes and shapes, gravel, tafla, sand, mortar). The stones were conveyed from one layer of blocks to the next by means of ramps which grew higher as the building increased in height. As one layer of the core walls after the other was finished, and as the respective filling was completed, the ramps were increased in height, and thus in length. However, the part of the building that had already been completed, did not require changes on the ramps. The winches however had to be moved upwards.

The transport ramps probably had a slope of 26.5°, at a ratio of base to height of 2:1, with a width to match that of the respective steps of the core (4.8m on the first two steps, and 4.2m on the other steps). As mentioned above, a gradient ratio of 2:1 (26°33'54) can be observed in all passage systems of the pyramids of the Fourth Dynasty. The ramps consist of the actual transport way with a set of steps running alongside it, adding up to a total width of 3m, which corresponds to the total width of the respective step of the core. Fig. 13 illustrates a ramp combined with a roller.

[52] Due to the constraints of this paper, I will not further discuss the question whether the access to the antechamber which was later built over constituted an expansion or change to the original plan, or whether it could have served as a transport way for the granite slabs for the burial chamber's wainscot.

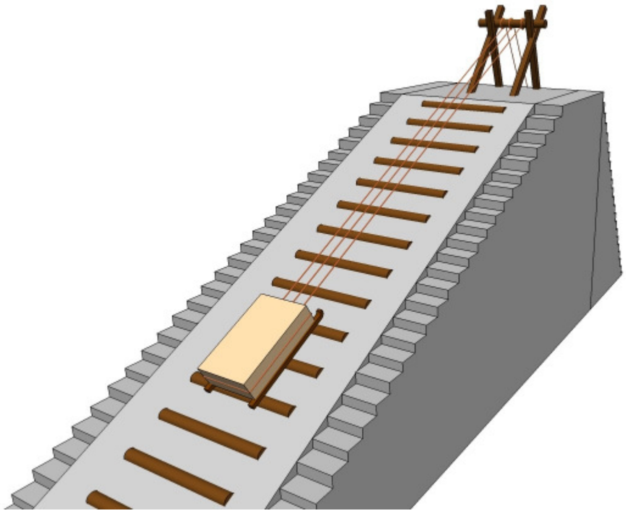

Fig. 13 Transport of blocks by a revolving redirection roller.

Several possibilities exist for the design of the transport way on the ramp:

- The surface was made from smoothed limestone.
- Ruts were engraved along the ramp surface, lined with mortar and filled with loose dolerite pellets. The ruts were slightly wider than the sledge runners.
- The surface consisted of dolerite pellets embedded in mortar.
- The surface was engraved with horizontal grooves into which wood beams were fitted, which would revolve under the block which was hauled, and return to their initial position as soon as the load had moved on.

One possible arrangement of ramps on the core steps is shown in fig. 14. On the first to sixth step, ramps could be positioned on each side of the building.

The outer faces of the ramps consist of finely hewn blocks, forming a stable wall with a slight inward slope comparable to the outer shell of the core masonry. The interior of the ramps was filled with roughly hewn stones and mud bricks.

Considering the measures of the transport ramps (fig. 14), ramps can be positioned on each side of the pyramid as follows:

step 1: 2 ramps
step 2: 1 ramp
step 3: 1 ramp
step 4: 1 ramp
step 5: 1 ramp
step 6: 1 ramp

This serves to illustrate once more that construction works on the steps of the core may have well been carried out simultaneously on each side of the pyramid, using one or two ramps. Thus, a transport capacity emerges which is significantly higher than that of perpendicular or spiral ramps.

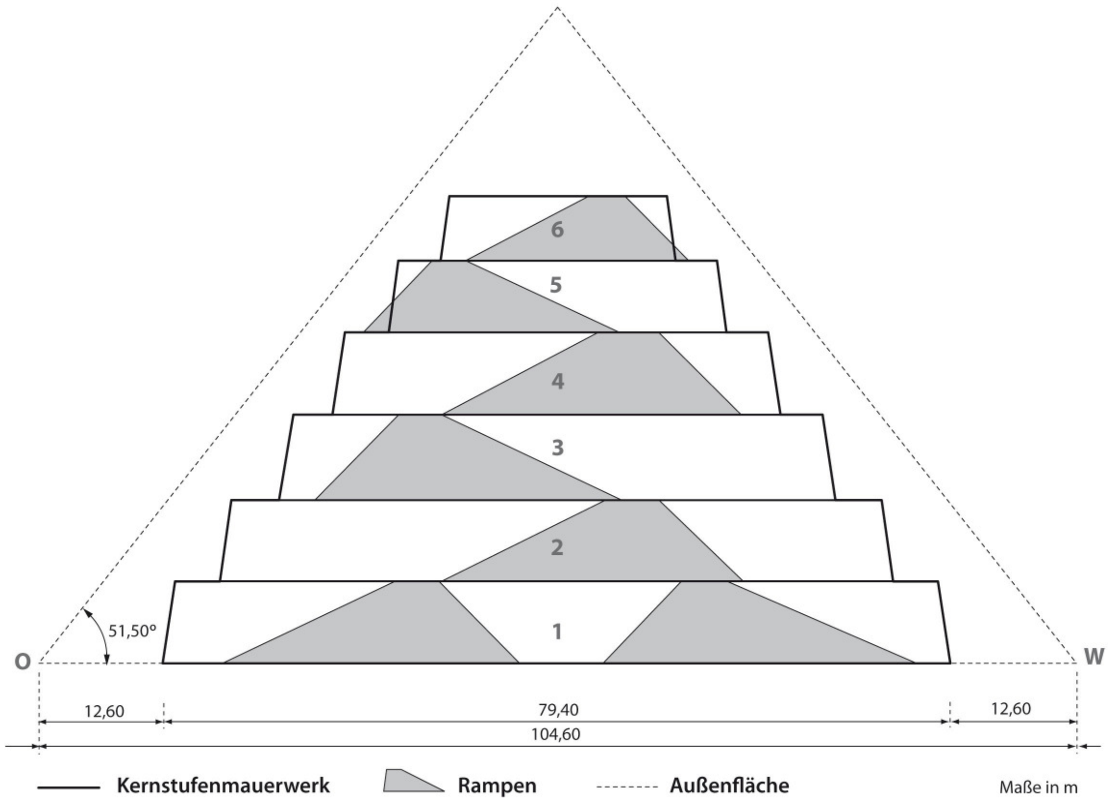

Fig. 14 Position of the ramps for redirection rollers on steps 1 to 6 of the core.

The following remarks are intended to clarify what I refer to as the principle of transport by means of a steep ramp. I have had to adopt some common sense parameters, as there was no opportunity to confirm them by practical experiments. On the uppermost platform of each ramp (length: 5m), a redirection roller is positioned at the side opposite the sloped surface fig. 15.

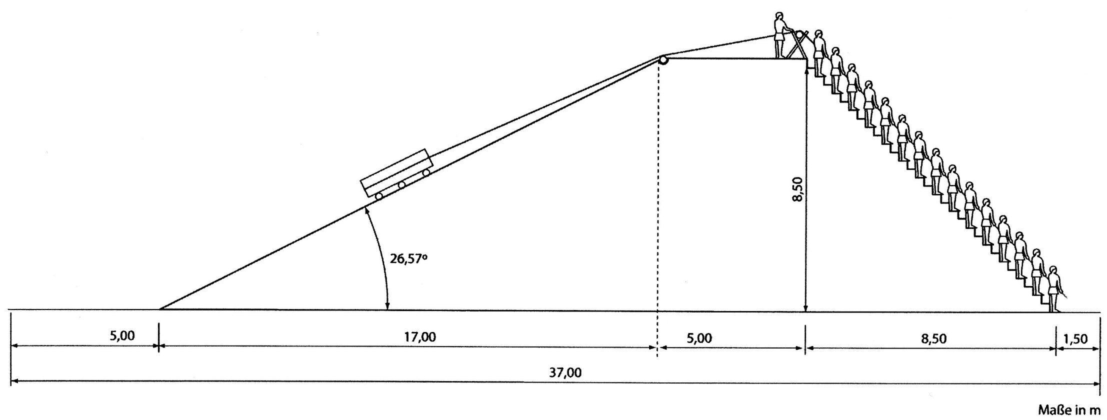

Fig. 15 Ramp with redirection roller

The redirection roller consists of wooden beams that are crossed and bound together, then firmly planted into the ground, while the roller is resting on top of them. As the largest blocks have sizes of approx. 2.3m x 1.4m at a height of 0.6m (weight 4.5tons),[53] the rollers' supports have to be positioned with enough space in-between to allow the sledge to be drawn right up onto the platform together with its load. However, we have not taken into account the weight of the roller itself. Through use of hardwood and greasing of the roller's rests, the frictional force can be kept at a minimum, so that we safely may discount it.

The hauling vehicles, presumably sledges or similar devices made from wood, are likely to have measured approx. 2.5m (length) to 1.5m (width). On the other hand, it is quite possible that the stones themselves were pulled up on the ramp without further supporting devices. For that purpose however, a sufficient traction force would have been needed. The advantage of this method would have been that the time consuming tasks of loading and unloading of the stone blocks, and also the transport of the sledges back down, were not needed. In any case, the hauling devices must be designed in such a way as to ensure that they can be pulled in both directions (change of direction at the top of one ramp to be pulled towards the base of another).

3100kp is the traction force necessary for transport of a stone block over a smoothed limestone ramp with a gradient ratio of 2:1 or with a slope of 26.5°, and a friction coefficient of 0.25, that is to say the force needed in the ***least favorable circumstances***. 45 workers are needed. Supposing that sledges or other wood devices with less friction were used, or horizontally embedded wood beams,[54] fine sand strewn over the sloped ground,[55] or another kind of support covered with dolerite pellets, a traction load of a mere 2200kp has to be achieved, [56] which may be further lowered when smaller blocks are moved.

Then again, friction is necessary to ensure that the roller does not spin freely under the coiled rope. This is guaranteed by the use of a roughened wood surface, the natural roughness of the ropes, and a traction force (workers) along the ramp down to its base.

Horizontal transport of a block on either end of the ramp towards its definite position in core or casing, or towards the starting point of its transport to the next level, would be carried out by means of levers or stone pellets. As in the pyramid G III c (Menkaure), layers are also visible in the core filling. It is therefore quite possible that the blocks were moved across a surface covered with loose stone pellets. Furthermore, we have reason to assume that the uppermost horizontal layer of the ramps was covered with dolerite pellets on the first few meters after the ending of the slope, in order to allow the sledges or the blocks to be rotated if necessary, and to await further transport as they were sitting on a bed of stone pellets.

The building phases of the core construction by means of tangentially positioned ramps and redirection rollers are illustrated by the following drawings.

[53] Blocks in the steps of the outer wall of the core.
[54] This technique is archaeologically attested only from the Middle Kingdom onwards, but then again, the principle of the roller is known already from the Old Kingdom.
[55] The favourable effect of fine sand is comparable to the lowering of rolling friction by means of stone pellets.
[56] As I am only aiming at a rough outline at present, I have not taken into account the minimal friction caused by the roller's rests.

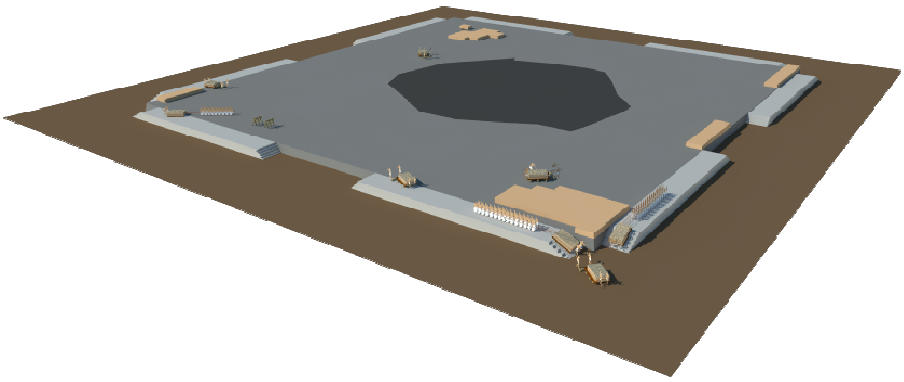

Fig. 16 Laying of the second layer of the first step of the core. Note the rock spur in the middle which is enclosed by this step.

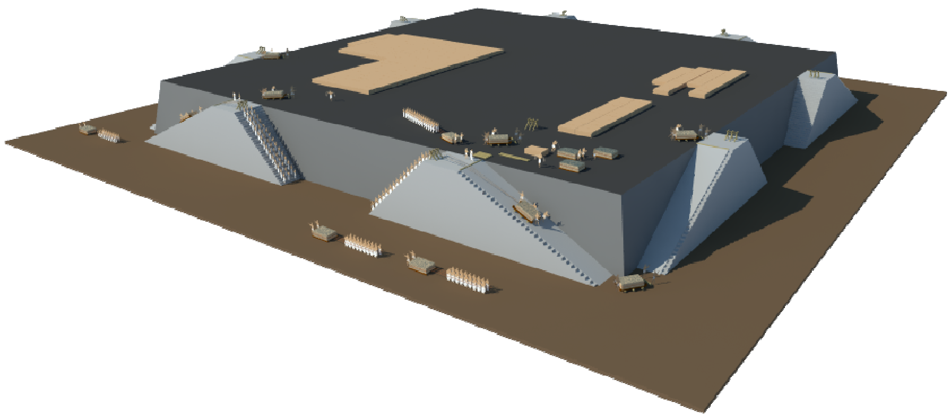

Fig. 17 The first step of the core is complete.

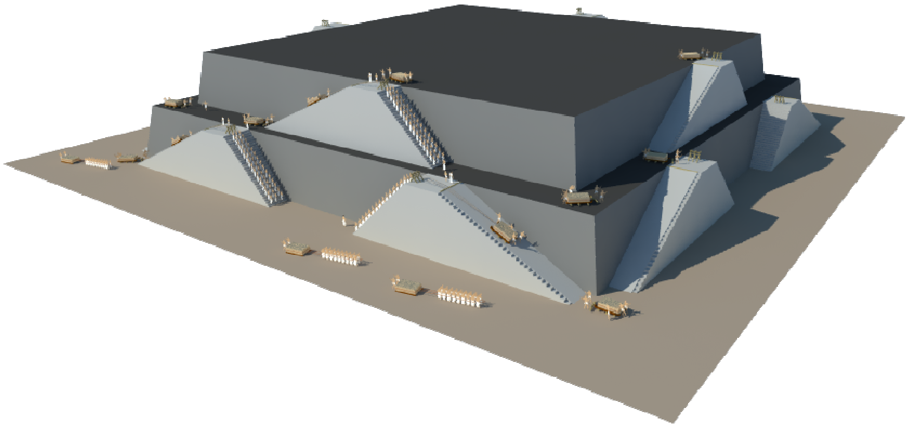

Fig. 18 The second step of the core is complete.

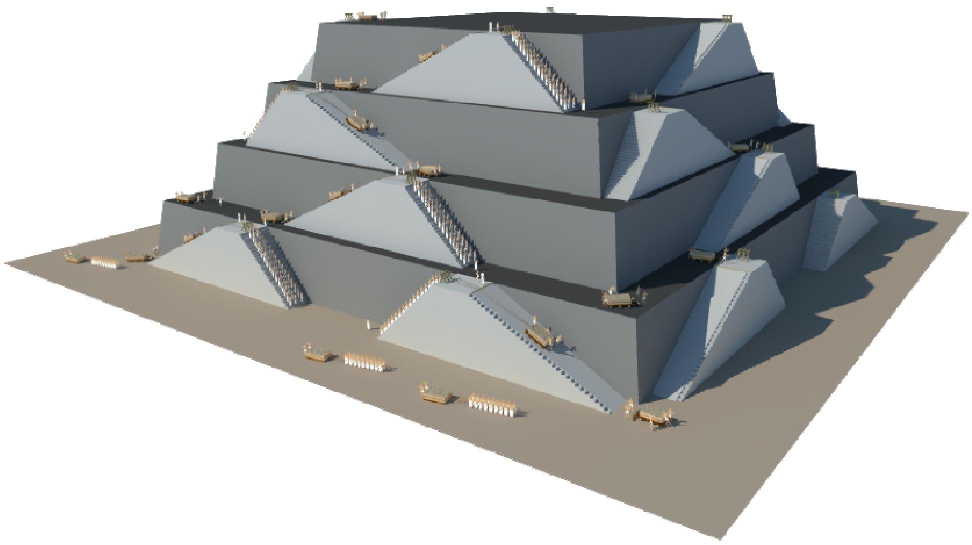

Fig. 19 The fourth step of the core is complete.

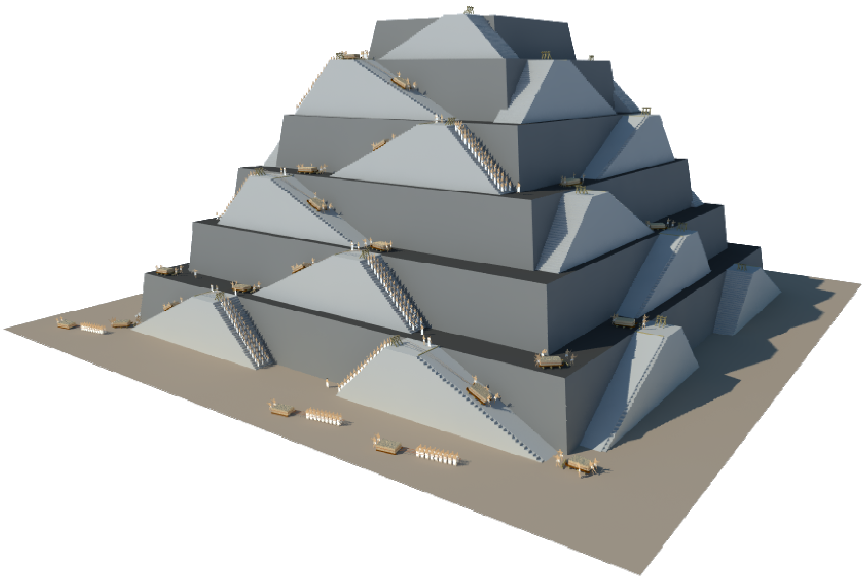

Fig. 20 The core's six steps are complete.

As soon as the sixth step of the core is finished, the ramps are removed (fig. 21).

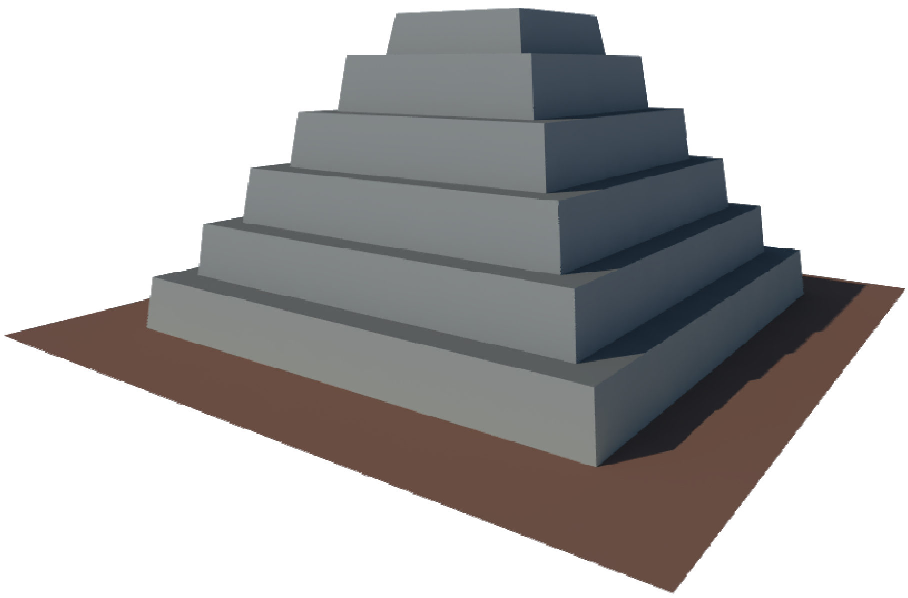

Fig. 21 Finished core after removal of the ramps.

Application of the backing blocks

In many pyramids it is still obvious today that measuring and laying of the base course of the casing was carried out most meticulously. This supports the assumption that the backing blocks[57] and the casing[58] were completed starting from the base of the pyramid as a rule.[59]

In some pyramids, like the Queens' Pyramids of Menkaure G III b and G III c, only the core was finished.[60] I should like to put particular emphasis on this fact at is rules out virtually any other model than that involving separate construction phases for core and casing.

Exact laying and working of backing and casing blocks requires a platform surrounding the pyramid, for no other solution ensures the safety of the workplaces. Ramps would have been erected on these platforms to match those suggested with reference to the core construction phase (see above).

As can be seen in the breach on the North side, the core steps were filled out with backing stones to match the respective batter. This process is technically impossible without an attached working platform. The same is true for laying and working the stones of the casing. Only an attached working platform enables the workmen to fit the blocks quite so neatly into the given spaces. Also, this model explains how the pyramid's surface could be dressed top down during removal of the attached working platforms. Up until then, these blocks would not only have been as yet unsmoothed, but also parts of them would have protruded from the pyramid's walls. Their rough surfaces would have ensured safe lodging of the ramps leaning onto the pyramid with blocks resting on the surface of the pyramid's outermost masonry. As the ramps are built layer by layer, the ramp blocks facing the pyramid are safely positioned in this manner, lending sufficient security to the ramps walls.

For the transport of blocks on the respective steps of the platform, I would suggest ramps with a width of approx. 5m, similar to the core ramps, made from mud bricks or smaller stones, with outer flanks made up from stone walls. These brick ramps display a gradient ratio of 2:1, and they grow with the platform from layer to layer. Within one layer, first the backing is completed and then the casing is laid, each new course of casing blocks on top of the previous one. The working platform may well have served for the construction of the seventh step of the core also (if indeed there was one), and of course for the setting of the pyramidion.

The blocks of the casing were worked with remarkable precision even before transport on their horizontal faces. Their fronts, however, were left undressed. The sides of these blocks were often cut very finely, partly during laying. As can be seen in several instances, some of the cutting was not executed vertically, but rather to fit the situation found on-site. This kind of work also requires a platform attached to the pyramid's face, because only then, the required sawing on both sides would have been possible in safe circumstances. Only very closely fitted casing blocks could prevent leakage of water and sand into the masonry, thus offering protection from damage through erosion. Before a casing block was fitted, the slope

[57] By backing, or backing stones, I mean the horizontal layers of stone blocks between the steps of the core and the casing, which consists of Tura limestone and Assuan granite.

[58] The blocks of the casing from Tura limestone or Assuan granite were left in boss at the stage of fitting the casing, to be dressed later.

[59] Stadelmann has noted that in the Bent Pyramid, the casing was laid right at the beginning, together with the core. In that case it was laid starting from the base, moving towards the top (Stadelmann 1997. *Die ägyptischen Pyramiden*. Mainz, 226).

[60] Jánosi, P. 1996. *Die Pyramidenanlagen der Königinnen*. Wien: Verlag der Österreichischen Akademie der Wissenschaften, 85.

of the pyramid was sketched on the previous block so that later, dressing could start from this mark in keeping with the exact slope. Dressing of the outer faces of the casing blocks previously left in boss, starting at the top of the pyramid and proceeding downwards as the platform was removed, was feasible because the brick ramps were dismantled when work at the top had been completed.[61]

The building phases and subsequent removal of auxiliary structures (working platforms, ramps) as seen in the construction model that utilizes redirection rollers are illustrated in fig. 22 to 24.

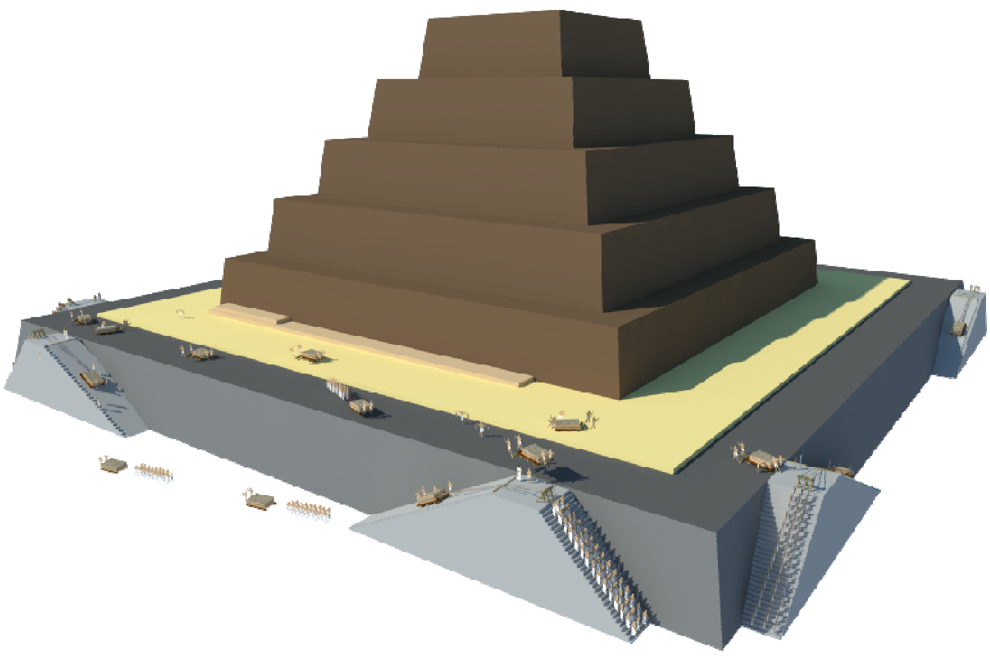

Fig. 22 Finished working platform 1 with ramps.

[61] Herodotus reports that the pyramid was finished top down (Herodotus, *Historiae* II.125. *Herodot Historien. Deutsche Gesamtausgabe* 1961. Translated by A. Horneffer, Stuttgart: Kröner). According to Diodorus, the construction of the pyramids involved earth walls (mud bricks) along the outer face of the pyramids (von Bissing, W. 1901. *Der Bericht des Diodor über die Pyramiden.* Berlin: Duncker). Pliny mentions a terrace system (Goyon, G. 1990. *Die Cheopspyramide*. Augsburg, 117).

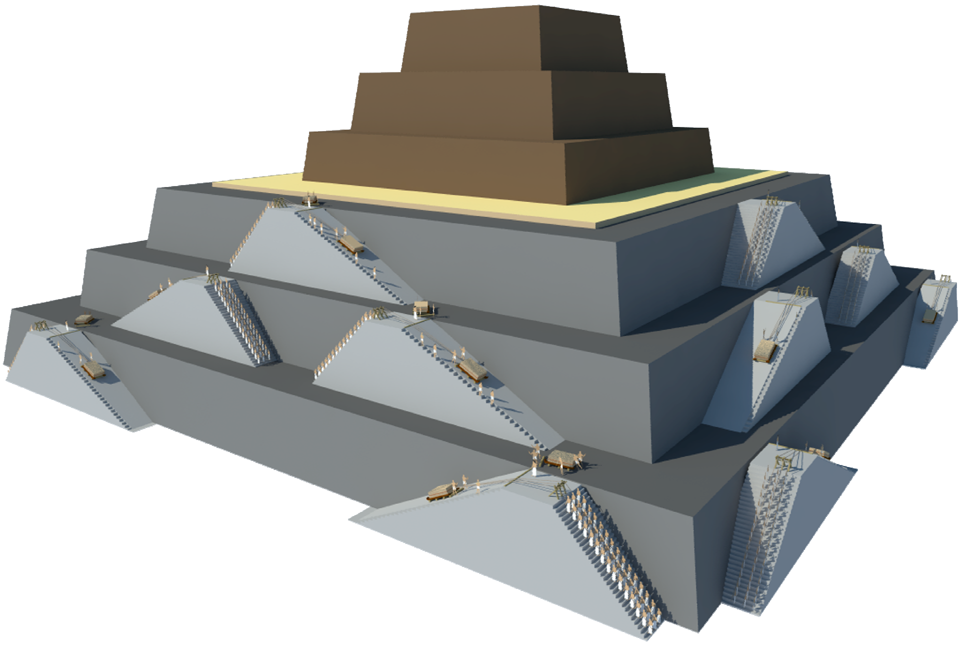

Fig. 23 Finished working platform 3 with ramps.

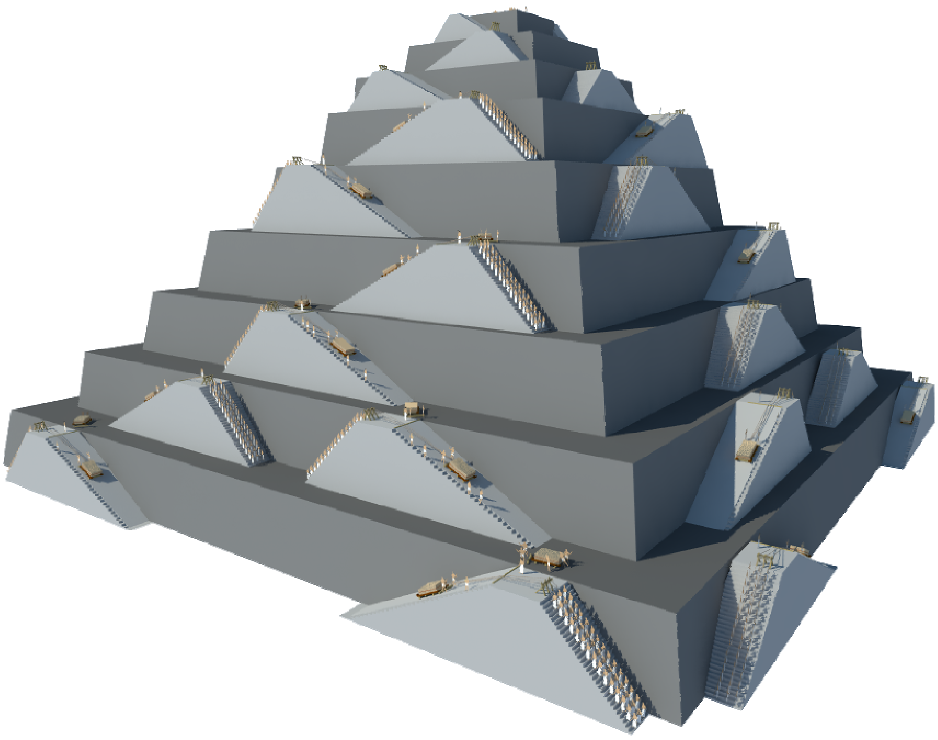

Fig. 24 Positioning of the pyramidion following completion of the uppermost working platform.

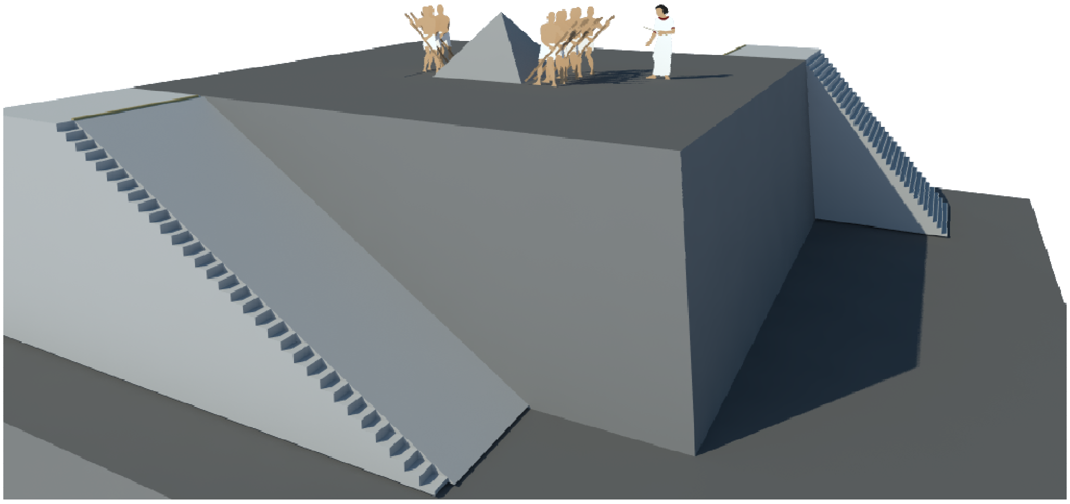

Fig.25 Positioning of the pyramidion

As soon as the pyramidion has been positioned, the working platform is used for top down dressing the outer casing. At the same time, it is removed step by step (fig. 26).

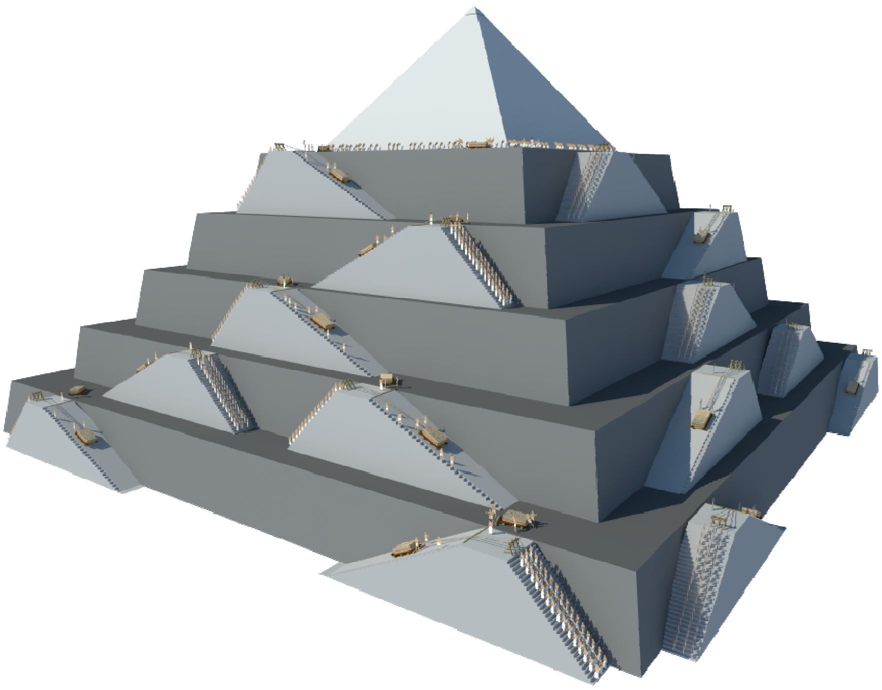

Fig. 26 Removal of the working platform and the outer ramps while dressing is carried out.

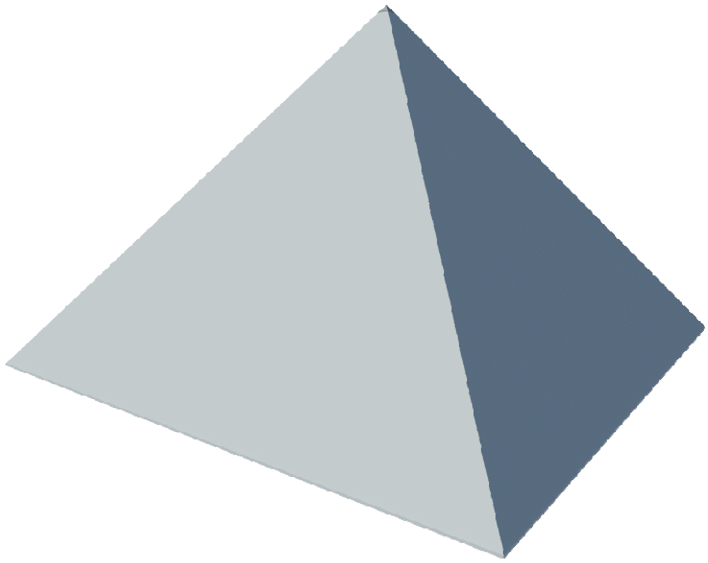

Fig. 27 Finished pyramid following dressing.

Calculating the building time of the pyramid of Menkaure

In calculating the efforts made in building the pyramid for Menkaure, and the resulting building time, we must use rough estimates in some points, especially as far as the block sizes are concerned, which are different in each layer. However, as the burial chambers of the pyramid of Menkaure are located below the core steps, estimating these numbers is comparatively straightforward.

- The measures of core and backing blocks are approx. 1.3m x 1.3m x 0.7m (1.2m^3), thus their weight amounts to approx. 3tons, as the specific weight of limestone is 2.4tons per m^3.[62] This calculation accounts for the fact that the core stones have slightly smaller dimensions than those of the outer walls of the core. These numbers serve the purpose of transparent reckoning, as the average block size will be used in the calculations of sledge loads and towing below.
- For the stones of the casing, the mean measure is 0.7m x 0.7m x 1.1m (0.54m^3, approx. 1.5tons). Two of these blocks would form a sledge or hauling load.
- Transport efforts as calculated here account for the building itself with core and casing, as well as the attached working platform. Moving of the winches would only become necessary occasionally, depending on the number of transport units. Therefore, I have excluded this task from my calculations.
- Also, the flat rate stone volume assumed for spaces filled with mortar, sand and so forth is negligible.

Throughout the construction process, a broad range of transport and building tasks was employed. After upward transport, the stones had to be moved horizontally into their definite position. Due to the limited number of ramps, upward transport required far more time than horizontal transport within one layer, as a greater number of workmen could be set to the

[62] Specific weight of limestone 2.4tons per m^3: Arnold, D. 1997. "Kalkstein" in *Lexikon der ägyptischen Baukunst*. Düsseldorf: Albatros, 119.

latter task simultaneously. Notably, horizontal transport may therefore be regarded as comparatively irrelevant to the building schedule, compared to ramp-dependent transport. Consequently, I have only taken into account ramp-dependent transport.

My model of transport of a sledge load, or a hauling process, up to a height of one step, is based on the following practice-based assumptions: The duration of transport up to the next step amounts to approx. 5min, that is a towing speed of approx. 4m per minute. For fastening and releasing of the ropes from the sledge or the block, the shifting of the sledge or respectively the load, and the back transport of the empty vehicle, rope and so forth, 5 minutes should suffice. Therefore, a unit duration of 10 minutes is realistic.

Further to the schedule calculation, we need to keep in mind that the blocks for the upper steps had to be moved over several ramps, which was a more time-consuming procedure. On the other hand, if we start from the assumption that a block was conveyed continuously from one ramp to the next to its final destination, it follows that a new block would reach the construction level every 10min, be it via sledge or hauling. The additional effort necessitated by the greater height that had to be overcome could be compensated by an increased workforce. The assumed average values for block weight and unit duration directly influence the estimated building time. If these sizes are altered, a new schedule will result. Obviously, calculations such as these are always more or less hypothetical, depending on the numbers inserted above.

The respective volumes of the six core steps of the pyramid of Menkaure add up to a volume of 140 736m^3, that is 117 280 stone blocks at a volume of 1.2m^3 each and an equivalent number of sledge loads. Taking into account the number of ramps, the total work units amount to 24 093 with redirection rollers. The top of the pyramid with its volume of 900m^3 was built at a later stage when the platform had been installed (see above). Further to a calculation of the building time of the core, we propose that a sufficient workforce was available on three hundred days of the year, ten hours per day (shift work), which means that under the circumstances outlined above (10 minutes per load), a total of 18 000 units were conveyed in a year. Therefore, the building of the core took 1.3 years. We still have to add construction and removal of the ramps on the first to sixth step, that is 28 ramps at a total volume of approx. 15 400m^3, requiring a total of 0.3 years if work took place on all sides of the pyramid. Therefore the entire ***core*** could have been readily completed in just **1.6 *years***.

My calculation of transport volume and building schedule for the ***working platform*** attached to the outside, plus the pyramids's top, backing and casing, covers a volume of 141 358m^3 or 22 458 work units. If again we base our calculations on the availability of the workforce as outlined above, a total ***1.5 years*** results. Next, the ramps and the platform need to be removed, and the stones left in boss need to be dressed. The ***removal*** of said structures requires ***0.6 years*** in keeping with the framework set out above.

Only a rough estimate can be offered for the ***dressing works on the facade***. A time frame of about ***0.2 years*** seems realistic in the light of the following considerations. Firstly, the lowest sixteen layers were left partly unfinished (granite wainscot). Secondly, we may assume that on one day (10 hours of work in shifts) one team of workmen succeeded in dressing a facade area of 1.0m x 1.0m (Tura limestone from the seventeenth layer onwards). In the case of the granite wainscot, the area would have only been 0.3m x 0.3m. By employing a large number of teams, the same work process can be undertaken in the lower part of the pyramid, on all four sides at the same time. At a height of 65m, excluding the eight lowest layers, an approximate duration of 434 days or 1,5 years results.

The dressing works would take slightly longer than the time needed for removal of the working platform (0.6 years). However, we must still keep in mind that the removal of the working platform and the ramps must be coordinated with the dressing and disruptions due to mutual impairment must be dealt with. To account for such unforeseeable circumstances, another ***0,2 years*** are added.

The debris accumulated during removal of the ramps and the working platform, largely mud bricks, amounted to roughly 60 000m^3, that is approx. 25% of the pyramid's volume. While mud bricks could be distributed for use on the surrounding agricultural domains, the rest of the debris was stored on the North Eastern slopes of the desert plateau and in the area towards the South of the Giza plateau.

The ***building of the pyramid of Menkaure***, not counting the time spent on pre-construction preparations, therefore took ***approx. 4.8 years***. All in all, an average volume of 202m^3 of stone or 186 blocks were moved per day.

Towards a comparison with the building times of the Red Pyramid and the Pyramid of Khufu

How does the schedule proposed for the pyramid of Menkaure compare to that of the other pyramids? In the following chapter, I propose for comparison some calculations for two other pyramids, for which a good enough time frame is widely acknowledged. Again, my calculations are based on estimates in some instances. I base my arguments on the points made above with reference to the core steps, ramps, and unit duration. Similarly, the construction of backing, casing, and working platform is assumed to follow the same principle as illustrated above.

Building time calculation for the Red Pyramid

In order to estimate the building time for the Red Pyramid, we need to first establish a framework of data that can only partly be verified by archaeological evidence due to the pyramid's state of preservation.

- The height of the respective steps is assumed to be 10m. Therefore, the pyramid has a total of nine steps.
- The average size of the backing and core blocks is approx. 1.1m x 1.0m x 0.7m,[63] which is a volume of 0.77m^3, or a weight of 1.9tons.
- Although the ramps are slightly longer, the duration of a single unit can be fixed at 10min, because the average weight is lower in the Red Pyramid than in the one previously discussed.
- The number of working days per year and hours per day is equal to that discussed above.

[63] Perring has measured the height of twenty-one steps of backing blocks starting from the bottom step (Perring, J.F. 1839. *Pyramids* vol. 1. London: Fraser). In total, their height is 21.4m, which leads to an average step height of 1.02m. The width is said to be 0.9m. Maragioglio and Rinaldi suggest the heights of the stones are 0.5 to 0.7m and their depth 0.9 to 1.2m (Maragioglio, V. and Rinaldi, C.A. 1964. *L'Architettura delle Pyramidi Menfite* vol. 3. Torino: Officine Grafiche Canessa, 126, and also *Addenda*, pl. 19 fig. 6). Stadelmann writes that the height of the corner blocks of the backing, up to a point at two thirds of the height of the pyramid, was 1 to 1.3m (Stadelmann, R. 1982. "Die Pyramiden des Snofru in Dahschur. Erster Bericht über die Ausgrabungen an der nördlichen Steinpyramide". In *MDAIK* 38, 380-381). Lepsius describes the blocks of the core masonry to be altogether '… perhaps not quite as large as in Giza …' (Lepsius, R. 1897. *LD* vol. 1: Texts. Berlin, 206).

- The building of the two burial chambers is not taken into account, as these are subterraneous structures.

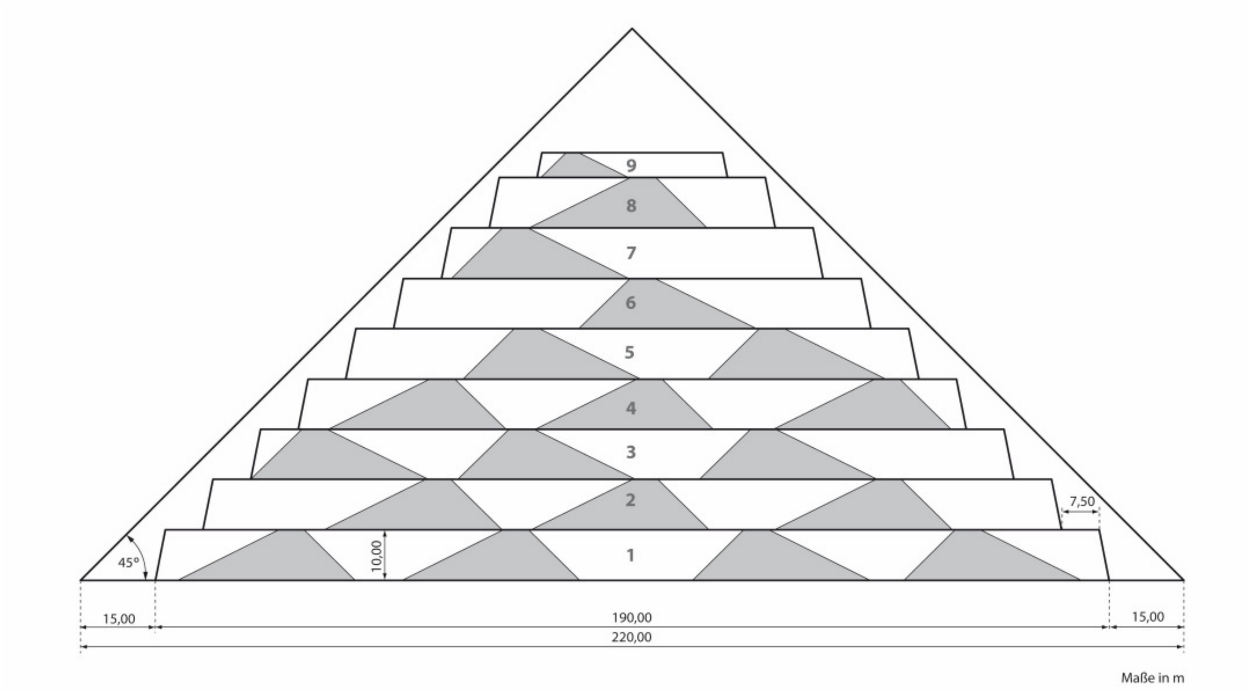

Fig.28 Red Pyramid: Proposl for Rampsystem

Our estimation of the building schedule of the Red Pyramid, not counting the preliminary preparations, will again start from the assumption that construction works took place on three hundred days of the year, on ten hours of each day, in shift work. It follows that a total of 18 000 transport units of 10 minutes each could be carried out per year. Thus, ***the construction of the core*** of the Red Pyramid took **11.6 *years***. The time needed for the construction of the working platform, backing and casing, adds up to ***4.9 years***. For subsequent removal of ramps and platform, and dressing of the blocks left in boss, ***3 years*** should suffice.
The time frame for dressing comprises approx. 223 work days or 0.8 years, to which we must add half a year during which the removal of the working platform and dressing would need to be coordinated. If we assume that this time must be added to that needed for the removal of the working platform and so forth, the actual ***building time of the Red Pyramid*** results at ***18.7 years**,* during which in total an average of 443 m^3 of stone, or 576 blocks, were moved each day.

Building time calculation for the Pyramid of Khufu

Again, as in the case of the Red Pyramid, some assumptions have to be made with reference to the measurements and the number of the core steps that are partly impossible to corroborate by archaeological evidence as yet. However, the following assumptions can be made:

- My starting point are my own observations and studies. In the tomb-robbers' tunnel the structure of the core masonry changes abruptly in layer seven (lower edge at 7.06m over foundation level), about 15m behind the entrance. Considering the horizontal measures of the outer wall of the first core step (1m) and of the casing which is not preserved (approx. 3m), and also the inclination of the outer wall of the core steps

(80°), there results a base length of the first step of the core of approx. 197m (375 cubits).[64]

– The height of the steps is likely to be 11m and the number of steps twelve.[65]
– The width of the steps accordingly is 5.75m, if we take into account the alignment of 54°30’ along the steps’ edges on the lateral surface.
– The unit frequency differs the pyramid of Menkaure, because the ramps are significantly longer, thus approx. 12min.
– The volume of the bed rock core in the base of the pyramid is assumed to amount to 7.9% by Haase,[66] and 7.7% in more recent studies, i. e. 200 000m^3.[67] Goyon supposes a volume of 127 000 to 160 000m^3.[68] I shall use the volume of the rock core given by Haase.
– Further construction data as the number of working days per year and the size and average weight of the stone blocks and so forth are the same as for the pyramid of Menkaure.[69]

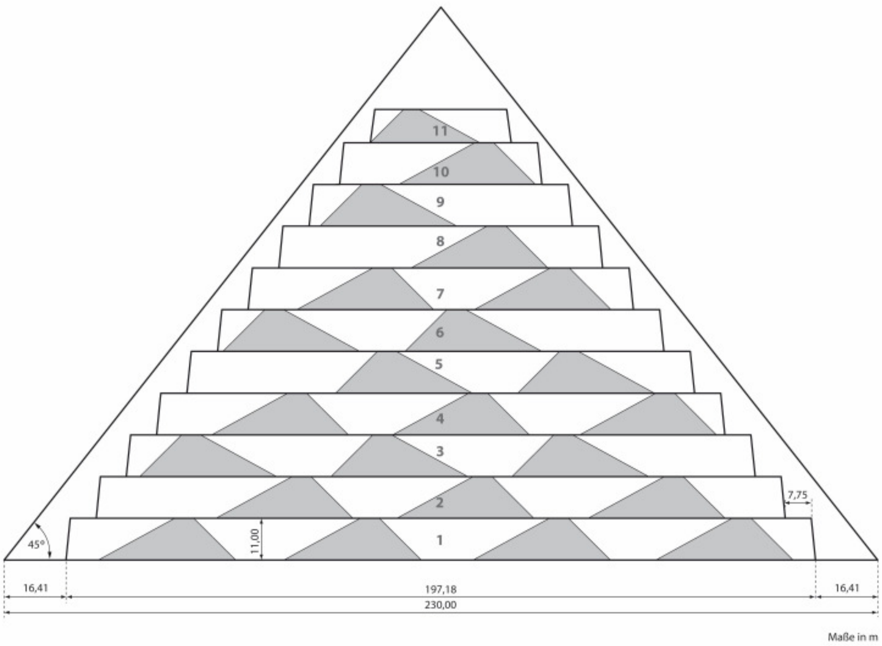

Fig.29 Pyramid of Khufu: Proposl for Rampsystem

In order to estimate the building time of the pyramid of Khufu, not counting the preliminary preparations, we again assume that a sufficient workforce was available on three hundred days of the year, ten hours a day, in shift work, so that at a duration of each transport unit of 12 minutes a total of 15000 rounds per year could be achieved. Accordingly, for the ***core*** of

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[64] This is an assumption based on calculation, which does not include the streaks of solid rock that reach the backing. The same is true for the suggested ramp arrangement on the first step of the core.

[65] Graefe proposes eleven steps at a height of 13m each for the core steps, at a width of 7.35m (Graefe, E. *Über die Determinanten des Pyramidenbaus bzw. Wie haben die Alten Ägypter die Pyramiden erbaut?* at http://www.uni-muenster.de/IAEK/org/WMA/graefe/pyr/index.html). Thus, the line of the steps’ edges on the lateral surface also reaches 54.5.

[66] Haase, M. 1993. “Der Felskern der Cheopspyramide.” In *Zeitschrift für Archäologie und archäologische Grenzwissenschaften* 1/1993, 5-13.

[67] Haase, M. 2004. *Eine Stätte für die Ewigkeit. Der Pyramidenkomplex des Cheops*. Mainz: von Zabern, 17.

[68] Goyon, G. 1990. *Die Cheopspyramide*. Augsburg, 117.

[69] In a number of publications, an average volume of 1m^3 is assumed for each stone block, from which an hypothetical volume of the pyramid of 2.3 x 10^6 m^3 results. The solid rock core, cavities, and the spaces filled with mortar and rubble are neglected.

the pyramid of Khufu, the time needed for transport of blocks amounts to 13.4 years, or just ***12.8 years*** if we subtract the volume of the bedrock core. The building time for the working platform, backing and casing amounts to a total of ***6.9 years***. Finally, the ramps and the platforms need to be removed, and the blocks left in boss require dressing, which all in all takes ***2.8 years***. The time spent on dressing amounts to approx. 286 work days, or ***one year***, to which we may add another year as a time reserve for any necessary coordination procedures. If we assume that this must be added to the removal of the working platform and so forth, the actual ***building time of the pyramid of Khufu*** results in ***22.5 years years***, during which an average amount of $393m^3$, or 327 blocks per day were moved.

I should like to emphasize again that these observations are based on rough estimates for the block sizes and weights, and to be precise, they ought to be modified by adding some more detailed explanations, for example as to how the large limestone blocks over the entrance on the North side of the pyramid, or the granite blocks of various shapes (Grand Gallery, burial chambers, stress-relieving chambers, plugging blocks) were moved. It may well be the case that in the lower area at pyramids' East some tangential ramps with a less steep inclination were erected in order to convey the larger blocks with a weight of up to 60tons, which were delivered along the causeway.

On the other hand, it is quite possible that the causeway was continued at an even inclination (10 : 1) towards the base of the pyramid or into the actual building site, reaching a height of 46 to 53m (fig.30) above base level, at which height the ceiling slabs could be conveyed further upwards. These considerations, however, must be continued elsewhere.

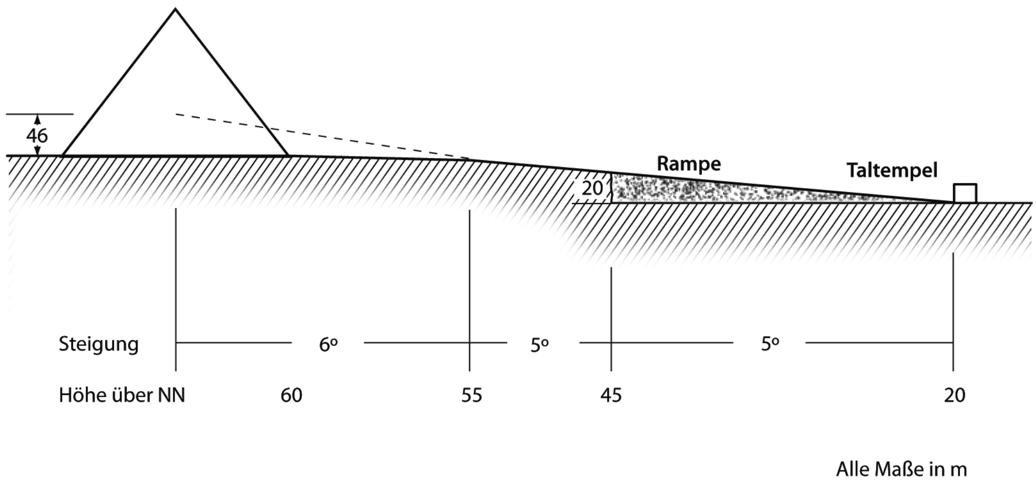

Fig.30

Starting from his own observations on the topographical situation at the SW corner, Lehner shows that the ramp approaching the pyramid from the Southern quarry, would have met the pyramid at about 30m above base level. [70] Also, we should not go without mentioning a solution for heavy load transports during the building of the pyramid of Khufu which Haase has published recently.[71]

[70] Lehner, M. 1985. "The Development of the Gisa Necropolis: The Khufu Project." In *MDAIK* 1985, 109-143.
[71] Haase, M. 2007. "Eine Rampe für Schwertransporte beim Bau der Cheops-Pyramide," In *Sokar* 15 (2), 48-49 and Haase, M. 2010. "Die megalithische Mauer westlich der Cheopspyramide," In *Sokar* 20 (1), 22 sqq and Haase, M. 2010. „Eine Transportrampe am Gisa-Plateau?, In *Sokar 21*, 22 sqq.

Summary of results

Several solutions that have so far been offered for issues of the construction of the Ancient Egyptian pyramids by Stadelmann, Arnold, Lehner, Goyon, G., Klemm and Klemm, Lattermann and Houdin, are based on the assumption that perpendicular or integrated ramps were used. Other explanations involve steep ramps running parallel to the core steps (Landt, Graefe). Still other authors suggest the use of winches, for example Riedl, Pitlik, and Keyssner. However, most of the authors assume that the core was built layer-wise; moreover, many do not offer solutions as to how the pyramidion was set, and how safety was ensured when the casing was applied and dressed. Building time calculations are also only offered occasionally.

The observations which I have presented here and elsewhere, lead to the conclusion that difficulties encountered in building the Bent Pyramid prompted the architects of the Red Pyramid and later pyramids to revise their construction methods in order to control the threat to the pyramid through ground erosion and earth quakes. At the same time, it was crucial to keep the building time as short as possible to meet the king's request for swift completion. As a result, a stepped core was erected, to offer more stability on the one hand, and on the other hand to enable ramps to be positioned on all four sides of the pyramid parallel to the core steps, in order to allow for material transports in a shorter time frame. Also, larger stone blocks were used. Subsequently, during the building of the pyramid of Khufu, the same methods were employed, and the size of the blocks was increased still further, in order to finish the vast building as soon as possible.

My hypothesis for the building of the Step Pyramids of the Old Kingdom involves combined use of ramps and redirectional rollers, which are both archeologically attested. Compared to previous suggestions involving transport of material by towing crews, the ramps were positioned in line with the steps of the core and to the working platform and were thus built with a steeper slope. As we have seen in the case of the pyramid of Menkaure, there is no longer a need for perpendicular ramps, or integrated ramps with less slope. Under these circumstances, we may dispense with any more complex solutions for the constructions. The use of several ramps with a steeper gradient ratio (26.5°, 2:1) on the core steps and the platform, on each side of the pyramid, leads to a steady and frequent influx of material, thus emerging as the best time-saving device that has been proposed so far.

Following the construction of the core and the removal of the respective ramps, backing and casing were applied layer-wise, after stepped working platforms with ramps had been installed around the pyramid, all of which were removed as the dressing of the casing proceded top down. This same platform also offers a convenient solution for works at the pyramid's top including the pyramidion.

The calculations of the transport effort and the resulting building time are based on the least favorable circumstances in terms of block size and weight, the ramp surface's friction, and transport unit duration. If any of these conditions are found to be more favorable, either the number of ramps could be reduced, or the building could be finished sooner. I have occasionally mentioned alternative solutions to some details, such as the ramp surfaces, but these do not have a significant effect on the overall construction method.

A calculation for the building times of the pyramids of Menkaure, Seneferu (Red Pyramid), and Khufu would realistically render the following results:

***Pyramid of Menkaure*:**	planning and preparation – 1 year building time, including dressing – approx. 4.8[72] total ***5.8 years*.**
***Red Pyramid*:**	planning and preparation – 2 years building time, including dressing – approx. 18.7 years total ***20.7 years*.**
***Pyramid of Khufu*:**	planning and preparation – 2 years building time, including dressing – approx. 22.5 years total ***24.5 years*.**

These building times fit well within the accepted dating of the reign of Seneferu (thirty-five years),[73] Khufu (twenty-three years),[74] and Menkaure (twenty-eight,[75] or six,[76] years).

The present article offers a solution that has been developed from archaeological evidence. It is centred around the idea of 'pyramid building with ramps and redirection rollers'. The solution involves steep ramps positioned in line with the steps of the pyramid's core, equipped with rollers in the form of redirection rollers for transport, and superseded by a working platform attached to the pyramid's sides to enable safe setting of the pyramidion and dressing of the casing.

[72] The dressing of the casing is not fully finished.
[73] According to new results, thirty-three years (Krauss, R. and D.A. Warburton, 2006. *Ancient Egyptian Chronology*. Leiden: Brill, 490). The building of the Red Pyramid was begun in year fifteen of the reign of Seneferu.
[74] According to new results, twenty-six years (Krauss, R. and D.A. Warburton, 2006. *Ancient Egyptian Chronology*. Leiden: Brill, 491).
[75] von Beckerath, J. 1997. *Chronologie des pharaonischen Ägypten*. Mainz: von Zabern, 188.
[76] According to new results, only six years (Krauss, R. and D.A. Warburton, 2006. *Ancient Egyptian Chronology*. Leiden: Brill, 485).