

Dissertation
submitted to the
Combined Faculties of the Natural Sciences and
Mathematics
of the Ruperto-Carola-University of Heidelberg, Germany
for the degree of
Doctor of Natural Sciences

Put forward by
Gerhard Zürn
Born in: Heilbronn
Oral examination: 19.12.2012

Few-fermion systems in one dimension

Referees:

Prof. Dr. Selim Jochim

Prof. Dr. Matthias Weidemüller

Abstract:

This thesis reports on experiments with few-fermion systems in quasi one dimensional confining potentials with tunable interaction. Using ultracold atoms we prepare these systems in a well defined quantum state with fidelities of 90% for up to 8 particles. The interparticle interaction in the 1D environment can be effectively described by a 1D contact interaction where the coupling strength can be tuned with of a confinement induced resonance (CIR).

We investigate a system of two repulsively interacting distinguishable fermions ($|\downarrow\uparrow\rangle$) and compare it to a system of two identical noninteracting fermions ($|\uparrow\uparrow\rangle$). For diverging coupling strength we show the fermionization of two distinguishable fermions, i.e. we observe the energy and the square modulus of the wavefunction of both system to be identical. We also perform radio frequency spectroscopy to measure the energy of a single minority particle interacting repulsively with a defined number of majority particles of different spin ($|\downarrow\uparrow \cdots \uparrow\rangle$). We study the crossover from a few-particle system to a many-particle system by adding majority particles one by one. We observe that already four majority particles are enough to describe the properties of the minority by that of a polaron-like particle, i.e. by a single impurity dressed by a 1D Fermi sea.

Investigating attractively interacting systems we observe that for increasing interaction strength the pair correlations in the system increases. This correlation leads to a strong odd-even effect of the single particle dissociation energy similar to the one observed for nuclei.

Zusammenfassung:

Diese Arbeit beschreibt Experimente an Wenigteilchensystemen aus Fermionen, bei denen wir die Wechselwirkungsstärke einstellen können, in einem quasi eindimensionalen Potential. Mit Hilfe von ultrakalten Atomen präparieren wir solche Wenigteilchensysteme aus bis zu acht Fermionen in wohl definierten Quantenzuständen mit einer Zuverlässigkeit von mehr als 90%. Die Wechselwirkung zwischen den Teilchen kann effektiv durch eine 1D-Kontaktwechselwirkung beschrieben werden, wobei wir die dazugehörige Kopplungskonstante mit Hilfe einer CIR einstellen können.

Wir untersuchen ein System von zwei unterscheidbaren, repulsiv wechselwirkenden Fermionen ($|\downarrow\uparrow\rangle$) und vergleichen es mit einem System von zwei nichtwechselwirkenden identischen Fermionen ($|\uparrow\uparrow\rangle$). Für den Fall von divergierender Kopplungskonstante beobachten wir die Fermionisierung von zwei unterscheidbaren Fermionen, d.h. wir stellen fest, dass die Energie und das Betragsquadrat der Wellenfunktion beider Systeme identisch sind. Mit Hilfe von Radiofrequenzspektroskopie messen wir die Energie eines einzelnen Minoritätsteilchens das repulsiv mit einer festgelegten Anzahl Majoritätsteilchen mit anderem Spin wechselwirkt ($|\downarrow\uparrow \cdots \uparrow\rangle$). Bereits für vier Majoritätsteilchen stellen wir fest, dass das Verhalten des Minoritätsteilchen sich an die polaronischen Eigenschaften eines einzelnen Fremtteilchens in einem 1D Fermi-See annähert.

Im Fall von attraktiv wechselwirkenden Systemen finden wir Hinweise darauf, dass die Paarkorrelation als Funktion der Wechselwirkungsstärke zunimmt. Abhängig von der Teilchenzahl finden wir in der Einteilchen-Dissoziationsenergie einen Ungerade-Gerade Effekt, ähnlich zu dem, der bei Atomkernen gefunden wurde.

Contents

1. Introduction	1
2. Deterministic preparation	5
2.1. Our scheme	5
2.1.1. Alternative schemes	7
2.2. The individual elements of the preparation scheme	9
2.2.1. Preparation of a degenerate Fermi gas	9
2.2.2. Creating a highly degenerate Fermi gas - the dimple trick . .	11
2.2.3. The spilling technique	13
2.2.4. The microtrap	16
2.2.5. High fidelity atom number detection in a MOT	20
2.3. Implementation of the preparation scheme	23
2.3.1. Experimental sequence	23
2.4. Number state preparation, ground state preparation fidelity and lifetime	27
2.4.1. Creating imbalanced systems and spin dependent detection .	30
2.4.2. Effects resulting from finite temperature	32
2.4.3. Summary of the preparation results	33
3. Tuning interaction and few particles with contact interaction in a 1D harmonic trap	35
3.1. Tuning interaction	36
3.1.1. Two-body interaction of ultracold fermions	36
3.1.2. Interaction potential for two distinguishable fermions	39
3.1.3. Tuning the scattering length	40
3.1.4. Tuning the interaction strength via a Feshbach resonance . .	41
3.1.5. Feshbach resonances in ^6Li	42
3.2. Few interacting particles in a harmonic trap	45
3.2.1. Analytic solution for two particles in a 3D cigar shaped trap	45
3.2.2. Two particles in a 1D system	48
3.2.3. Confinement induced resonance and 1D coupling constant .	53
3.2.4. The three-particle and N-particle systems	54

4. Fermionization of two distinguishable fermions	59
4.1. The fermionization experiment	60
4.1.1. Two interacting distinguishable and two noninteracting identical fermions	60
4.1.2. Tunneling measurement and fermionization	63
4.2. Tunneling theory and determination of the potential shape	67
4.2.1. WKB approximation	68
4.2.2. Determination of the potential shape	72
4.3. Energy of the interacting two-particle system	80
4.3.1. Determination of the energy from the tunneling time constants	81
4.3.2. Modulation spectroscopy of two interacting particles	83
4.4. Adding more particles - Fermionization of three fermions	87
4.5. Center-of-mass to relative motion coupling	91
5. Pair correlations in systems with attractive interactions	97
5.1. Tunneling of two attractively interacting fermions	98
5.1.1. Expected tunneling behavior	98
5.1.2. Tunneling model	102
5.1.3. The tunneling experiment	103
5.1.4. Effects of the spin dependent potential	105
5.1.5. Application of the spin dependent model	109
5.1.6. Results for different interaction strength	110
5.2. Observation of the odd even effect	113
5.2.1. Interpretation of the results	117
5.2.2. Comparison to similar systems and prospects for further experiments	117
6. Radio Frequency spectroscopy of few-fermion systems	121
6.1. The Rf-spectroscopy method	122
6.1.1. The concept	122
6.1.2. Illustration of the rf-spectroscopy method	123
6.1.3. Experimental resolution and calibration of the magnetic field	127
6.2. Rf-spectroscopy with trap sideband resolution	128
6.2.1. Decomposition of the interacting wavefunction	129
6.3. RF-spectroscopy of few-fermion systems with two, three and more interacting particles	131
6.3.1. Obtaining the total interaction energy	131
6.4. Precise determination of ^6Li scattering length	135
6.4.1. Rf-spectroscopy of weakly bound molecules with trap sideband resolution	137

6.4.2.	Confinement shift	141
6.4.3.	Binding energy of the weakly bound molecules and position of the Feshbach resonance	142
7.	A single impurity in a finite Fermi system	145
7.1.	Energy of a single impurity	146
7.2.	Theory for a single impurity in a homogeneous system	147
7.3.	How many particles are needed to form a 1D Fermi sea?	148
8.	Conclusion and Outlook	153
8.1.	Conclusion	153
8.2.	Outlook	155
8.2.1.	Ferromagnetic correlations in 1D systems	155
8.2.2.	Antiferromagnetic ordering in a multiple well potential	155
8.2.3.	Dynamical processes in few-fermion systems	156
A.	Appendix	159
A.1.	^6Li level scheme	159
A.2.	Deterministic preparation	160
A.3.	Fermionization experiment	166
A.3.1.	Tuning the trap depth	166
A.3.2.	Ramp speed of the magnetic field	166
A.4.	Modulation spectroscopy	167
A.4.1.	Coherent coupling	167
A.4.2.	Determination of the axial trap parameters	169
A.4.3.	Determination of the radial trap parameters	172
A.5.	COM-REL motion coupling	176
A.6.	Correlations in systems with attractive interaction	178
A.7.	Rf spectroscopy	181
A.8.	Determination of ^6Li Feshbach resonance	183
A.9.	A single impurity	188
	Bibliography	190

1. Introduction

Strongly correlated quantum systems are of high relevance in nature. One prime example are strongly interacting Fermi gases, which play a fundamental role in systems of completely different energy scales. In the upper range of the scale at temperatures of about 10^{12} Kelvin the properties of the Quark Gluon Plasma, a phase of deconfined quarks and gluons are studied. This phase which is postulated to have existed in the early universe is investigated in experiments of heavy ion collisions [ALI04] [RHI05]. On the lower range of the scale, in the ultracold regime of a few nano-Kelvin, the BEC-BCS crossover in the scale-invariant Fermi gas [Ku12] has been studied [Leg80] [Noz85] [Chi04] [Gre05] [Zwi05]. Through a region of strong correlations this crossover connects the bosonic superfluid [Ein25] of tightly bound pairs to the phase of weakly attractively interacting fermions governed by the BCS pairing mechanism [Bar57]. Roughly in the middle of the energy scale between the two extremes in density and temperature are the electron gases in solid state systems whose Fermi temperatures are on the order of a few thousand Kelvin. In these solid state systems the dimensionality of the electron confinement plays an important role. Example are the conductivity in carbon nano-tubes [And00] or in high temperature superconductors [Lee08]. For the occurrence of high- T_c superconductivity the 2D structures of cuprates and pnictides seem to be substantial. One central issue debated in the studies of cuprates is whether these 2D systems behave as a Fermi liquid or not [Mae01]. A restriction to two dimension increases fluctuations [Pet00a] [Kli12a] which modifies the long range order in these system leading to specific superfluid phases described by the Berezinskii–Kosterlitz–Thouless mechanism [Had06] [Fel11] [Frö11] [Des12].

In one-dimensional systems the effect of quantum fluctuations is even stronger. A special feature of these systems is the possibility to exactly solve Hamiltonians of various systems. Using the Bethe ansatz [Bet31] analytic solutions can be found for example for the Heisenberg model [Hei26] and the Kondo model [Kon64]. Due to their integrability 1D systems are an ideal environment to address fundamental open questions of quantum statistical mechanics such as if – or how – an isolated quantum system thermalizes [Kin06] [Rig08] [Gri12] [Ike12]. Another method for solving one dimensional systems is the Bose-Fermi mapping: A 1D bosonic gas in the limit of infinitely strong repulsive interaction, called a Tonks-Girardeau Gas, can be mapped onto a system of identical fermions with a well known solu-

tion [Gir60].

The many-body features of these 1D quantum systems are determined by their microscopic properties. However, for the many-particle case a solution based on the microscopic physics is difficult to obtain. Hence studying few-fermion systems gives us a unique opportunity to understand how the many-body properties of strongly correlated systems emerge from their microscopic physics. Starting from the smallest building block, the two-particle system of two interacting fermions in a 1D harmonic trap [Bus98] [Idz05], one can work its way up to a many-body state by adding particles one by one. In this thesis we have done this by first studying the fermionization of two repulsively interacting distinguishable fermions [Gir10] [Zür12]. By adding more particles of one spin-component we have then studied the convergence to a system of a polaron-like minority particle dressed by a Fermi sea [McG65].

In another line of research we have investigated systems with attractive interactions. Such systems have prominent representatives in nature such as atoms, nucleons or atomic nuclei. In the latter system effective attractive interactions lead to pairing phenomena [Zel03] relevant for various features of nuclei such as the extraordinary stability of some isotopes with even proton and neutron numbers [Bri05], the so-called magic nuclei.

Performing these studies requires a quantum system with control over several degrees of freedom such as the number of particles, the motional degree of freedom of the particles, the interparticle interaction and the confining potential. Systems which have been used for this purpose are quantum dots and atomic clusters, with the recent addition of ultracold atoms [Ser11b] presented in this thesis.

In quantum dots the semiconductor structures serve as a confining potential for electrons. The shape of the confinement can to some extent be chosen at will by microfabrication methods [Kou01] and the number of electrons in the dot can be controlled by changing the applied electric fields which leads to tunneling of electrons onto the dot. As the interaction is dominated by the coulomb repulsion between the electrons these structures can mimic atomic structure and are therefore referred to as 'artificial atoms' [Ash96] [Rei02].

Another approach for realizing artificial atoms is the preparation of atomic clusters. In these systems the number of delocalized electrons which can be regarded to be confined in a mean field potential created by the ionic background can be controlled by selecting the number and the species of the cluster atoms. Some structures exhibit similar properties as elements in the periodic table and thus can be regarded as 'superatoms' which offer the possibility to explore fundamental mechanisms of chemical reactivity. [Cas09].

However, in 'artificial atoms' consisting of quantum dots and clusters the few-particle system under study is coupled to a thermal bath. In comparison, an

advantage of using ultracold atoms for setting up a generic quantum system is their isolation from the environment. This allows to control the full quantum state of the system. Another key feature is the possibility to approximate their interparticle interaction by a contact interaction term described by one parameter which sets the strength of the interaction at these ultracold temperatures [Blo08]. Feshbach resonances [Ino98] [Chi10] allow to tune this interaction parameter without influencing the confining potential. This allows to mimic a large amount of Hamiltonians with various interaction terms. The potential term can be easily changed by different trapping potentials available for ultracold system [Gri00] [Fol02] [Hin08] [Zim11]. The confining potential also determines the dimensionality of the system. If the size of the confinement in two dimensions is as small that its characteristic energy scale – the trap frequency – is much larger than the largest energy scale in the system its dynamic is effectively restricted to one dimension [Ols98] [Stö06] [Osp06] [Hal10]. Also the control on the single particle level in these ultracold systems has become possible as demonstrated in recent experiments [Grü10] [Bak10] [Wei11] [Ser11b] [Kau12].

In this thesis we combine all these features of ultracold atoms to study few fermion system in one dimension. The thesis is structured as follows. In the first part we present the realization of a well-controlled quantum systems using ultracold atoms. In chapter 2 we start with the introduction of our preparation scheme and present deterministic preparation of a few-fermion system. In chapter 3 we introduce the tunability of the inter-particle interaction in our system and discuss theory of few interacting particles in a one dimensional harmonic trap.

In the second part of this thesis we combine the deterministic preparation and the tunability of the interaction strength to study interacting few-fermions systems. In chapter 4 we begin with the investigation of two repulsively interacting distinguishable fermions and show that for infinitely strong repulsive interaction the system can be mapped on a system of two identical fermions. In chapter 5 we study the pair correlations in attractively interacting system with up to six particles. In chapter 6 we introduce the method of rf-spectroscopy with which we determine the energy of few-fermion systems as a function of the particle number and as a function of the interaction strength. Finally, in chapter 7, we investigate a single impurity in a finite Fermi system and address the question of how many identical particles are needed to form a 1D Fermi sea.

2. Deterministic preparation

The achievements of the field of ultracold atoms is owed to the large degree of control and tunability of these systems: A huge variety of different quantum systems could be studied due to the possibility to tune the interparticle interaction and the shape of the external potential. Therefore they are extremely well suited for the investigation of few-fermion systems.

Most of the recent studies of ultracold gases have been focusing on their many-body properties or the underlying few-body physics which have been investigated using samples described in the thermodynamic limit. In these large samples number fluctuations on the order of the shot-noise do not significantly change the thermodynamic quantities such as temperature or pressure. In contrast, few-fermion systems critically depend on the exact number of particles. Hence, to be able to study the crossover from a few-fermion system to a mesoscopic system it is essential to gain exact control of the particle number. To determine the quantities of a distinct few-fermion system we have to repeatably perform measurements on identical copies of this system which requires a deterministic preparation with the control of the particle number and its motional state. To quantify the reproducibility of the preparation of a distinct system we define the preparation fidelity f of a system

$$f = \frac{n_d}{n_d + \overline{n}_d} \quad (2.1)$$

with n_d the number of times of finding the desired system after the preparation procedure and $n_d + \overline{n}_d$ the total number of prepared systems.

The topic of this chapter is the realization of such systems with high fidelity. We first present a scheme which allows a deterministic preparation using ultracold atoms. Then we introduce the individual elements which are required to implement this scheme. Finally we present the results of high fidelity preparation.

2.1. Our scheme

Our approach is to first prepare a quantum gas with temperature T low enough such that the occupation probability of the lowest energy single particle states in the trap approach unity. The removal of excess particles is achieved by controlling

the number of available single particle states in the trap. Finally, the number of particles is detected by a single particle detection method.

Unity occupation probability

The realization of this approach is based on one of the fundamental properties of fermions, the Pauli-principle: Due to the anti-symmetric exchange symmetry of identical fermions, at most one of them can occupy a single particle quantum state in a trapping potential. This is the reason why in a degenerate Fermi gas the occupation probability $P(E)$ of a state with energy $E \ll E_F$ approaches unity for $T \ll T_F$. This can be seen from the Fermi Dirac distribution of a noninteracting Fermi gas

$$P(E) = \frac{1}{e^{(E-\mu)/k_B T} + 1} \quad (2.2)$$

where μ is the chemical potential ¹, E_F the Fermi energy defined by the energy of the particle highest up in the potential at $T = 0$ and $T_F = E_F/k_b$. The Fermi distribution is shown in figure 2.1 a) and clearly shows that for the lowest energy states in the system the occupation probability approaches unity. The fundamental limit of this scheme for the preparation fidelity is determined by the deviation from unity occupation at a finite temperature. Hence, to set this limit as low as possible the sophisticated experimental task is to prepare a highly degenerate Fermi gas where the influence of the temperature on the occupation probability is small.

Control over the number of bound states in the trap

To achieve control over the number of quantum states in the trap we have to control the depth of the trap more precisely than the level spacing as illustrated in figure 2.1 b). With this method we can not only determine the number of states in the trap by choosing an adequate trap depth. If the few-fermion system is automatically in its ground state which fulfills the condition of a well defined motional state. The experimental challenge is to realize such a trap

¹For a highly degenerate Fermi gas with $T \ll T_F$ the chemical potential can be approximated by the Sommerfeld expansion

$$\mu(T, E_F) = E_F \left(1 - \frac{\pi^2}{3} \left(\frac{T}{T_F} \right)^2 \right). \quad (2.3)$$

For simplicity we neglect the second term of the expansion in the following. It has a contribution of $< 3\%$ for $T/T_F < 0.1$ and thus we set $\mu = E_F$.

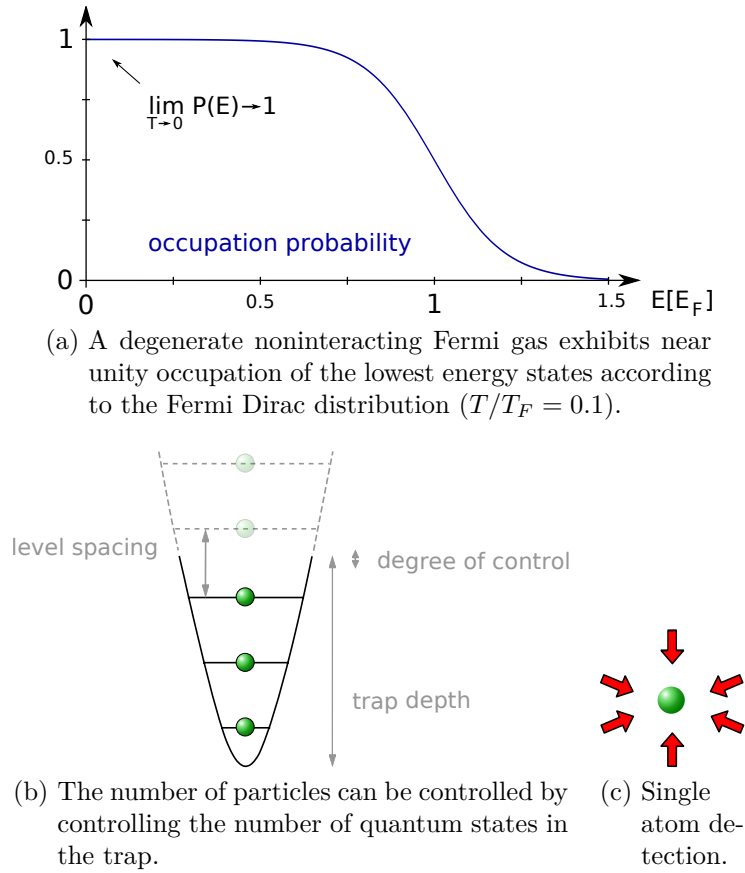


Figure 2.1.: **The preparation scheme.**

where the degree of control on the trap depth is more precise than the separation of single particle levels in this trap.

Single particle detection technique

Finally to test the preparation fidelity of this scheme we need to count the particles of the system on the single atom level (figure 2.1 c). The task is to set up a detection method with a detection fidelity that is larger than the preparation fidelity.

2.1.1. Alternative schemes

Using Bose gases

Using a similar scheme there have been performed experiments with bosonic gases [Chu05] in the Raizen group at UT Austin which realized degenerate samples

of around 60 particles with sub-Poissonian fluctuations by tuning the depth of the trap to control the chemical potential of an interacting Bose gas. For weak interparticle interaction a Bose gas significantly differs from a Fermi gas in the occupation numbers of single particle states. For the energetically lowest states in a noninteracting degenerate gas these numbers exceed 1 in the case of a Bose gas, whereas it converges to 1 in the case of a Fermi gas. Thus, high fidelity preparation in the manner we have sketched could not be achieved with the weakly interacting Bose gas. One could possibly overcome this obstacle by making the bosons behave like fermions which is realized by a strong increase of the repulsive interaction [Dud07]. We will discuss this exciting phenomenon of fermionization in detail for a few-particle system in chapter 4. However, for a many-body system, the lifetime of a repulsive Bose gas at strong repulsive interaction is drastically decreased [Hal09] which again hinders a deterministic preparation. Thus, the Raizen group has also switched over to a fermionic system and are building up a machine for preparing atoms 'on demand' [Rai09].

Single atom cooling approach

Besides our approach there is also an alternative approach which, however, would be more difficult to implement: In a first step one loads a defined number of particles into a trap and in a second step one cools these samples to the ground state of the trapping potential which evidently fulfills the requirement of a well defined motional state. In experiments with ultracold bosonic gases [Sch01a] and Rydberg gases [Grü10] two different loading schemes have been implemented. In these experiments it has been possible to reach probabilities of 50% to 80% for loading a single atom into a trap. Although the number fluctuation are sub-Poissonian, the occurrence of systems with undesirable atom number of 50% to 20% may be still too large to perform further experiments which crucially depend on the exact particle number. In the context of ion-trap experiments the loading and cooling of single atoms to the ground state has been realized [Die89] by using resolved sideband laser cooling [Deh76], which requires the linewidth of the cooling transition to be much smaller than the level spacing of the trap. Unfortunately, this scheme can not be easily transferred to systems of ultracold neutral atoms: The reason is that the typically realizable trap frequencies of optical traps are in the regime of several 10 kHz in contrast to the linewidth of the mostly used elements, the alkali atoms, with linewidths of several MHz. Yet, when writing this thesis, laser cooling of a single neutral atom to the 3D vibrational ground state has been reported [Kau12].

2.2. The individual elements of the preparation scheme

We start with the preparation of a cold Fermi gas and introduce the dimple trick with which we create a highly degenerate Fermi gas to achieve near unity occupation of the lowest trap states. Then we present our microtrap, the experimental realization of a trap with precise control over the trap depth and finally we explain our single atom detection technique. In the subsequent section we then combine these elements and present the results of high fidelity preparation.

2.2.1. Preparation of a degenerate Fermi gas

The preparation of an ultracold Fermi gas of ^6Li atoms near the onset of degeneracy is routinely done in our group to perform various experiments. Here we only briefly sketch its main parts. Details on the experimental apparatus and the different cooling methods are given in [Ser07, Lom08, Koh08, Wen08, Zür09, Ott10, Lom11, Ser11a]. A more general overview of preparing a degenerate Fermi gas can be found in the review [Ket08].

The whole preparation takes place in a vacuum chamber with a background pressure on the order of 10^{-12} mbar in a science chamber (see figure 2.2, No.5) to achieve a long lifetime of the ultracold sample. The preparation process starts with vaporizing ^6Li in an oven (No.3) at 350°C resulting in a mean velocity of the vaporized atoms of about 1500 m/s. Behind a collimator restricting the aperture of the oven the outgoing atomic beam enters a drift tube (No.4) where the atoms are slowed down by a Zeeman slower [Met99]: A counter propagating laser beam resonant to a Doppler shifted optical transition of the ^6Li atoms transfers momentum to the atoms which slows them down. The reduced Doppler shift of the slowed atoms is compensated by a spatially varying external magnetic field which shifts the Zeeman sublevels of the transition back to resonance. After the drift tube the atoms enter the science chamber (No.5) where atoms below a certain velocity are captured by a Magneto Optical trap (MOT) [Met99]. The MOT provides trapping and cooling down to a few hundred μK close to the Doppler temperature [Met99]. As the MOT potential is dissipative further cooling without additional sub-Doppler cooling techniques is not possible in the MOT. Thus, to perform evaporation cooling [Luo06] we transfer the particles into a crossed beam optical dipole trap [Gri00]. Figure 2.3 shows an absorption image [Lom08] of an atomic sample trapped in this potential. Because at low temperatures, a spin polarized gas is noninteracting (see chapter 3), we use a two-component mixture

2.2. The individual elements of the preparation scheme

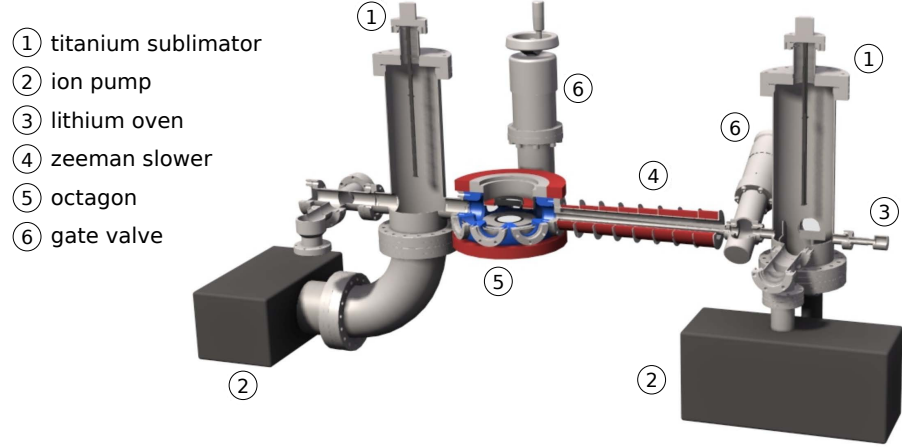


Figure 2.2.: **Vacuum chamber.** The experiments with ultracold atoms take place in the science chamber (No.5) which provides optical access for laser-cooling and -trapping. Several pumping units (No.1,2) and a special surface getter coating guarantee a pressure on the order of 10^{-12} mbar. The low collision rate with the background gas is important to achieve long lifetimes of the ultracold samples.

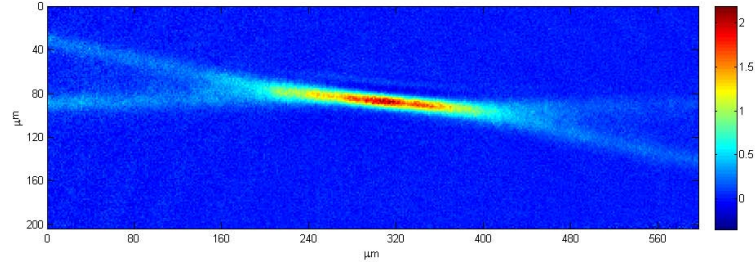


Figure 2.3.: **In-situ absorption image [Lom08] of atoms in the crossed beam optical dipoletrap.** The image is taken of a non-degenerate sample to better visualize the outer regions of the trap.

of the two lowest Zeeman sublevels of the lowest ^6Li hyperfine state $|F = 1/2\rangle$ ². By lowering the trap depth of the dipole trap the particle with largest kinetic energy can leave the trap. Simultaneous thermalization of the sample by scattering between the atoms results in a reduced temperature. Using this technique we are

²We label these states $|1\rangle$ and $|2\rangle$ according to the notation given in the appendix A.1. In this chapter we sketch the state- $|1\rangle$ -atoms by blue circles and the the state- $|2\rangle$ -atoms by green circles. The scattering rate between these two states can be controlled using Feshbach resonances which allow to tune the scattering length a_{3D} by varying a magnetic offset field (see chapter 3).

able to prepare a degenerate Fermi gas of 4×10^4 atoms with a temperature of 250 nK.

2.2.2. Creating a highly degenerate Fermi gas - the dimple trick

The degeneracy of the sample in the crossed beam dipole trap achieved with the evaporative cooling method is $T/T_F \approx 0.5$. A further decrease of degeneracy in the crossed beam dipole trap which we denote as 'reservoir' by further evaporative cooling is challenging due to the Fermi statistics of the gas and the limited collisions needed for thermalization [DeM99]. Sympathetic cooling [Sch01b] as used in different cold atom experiments is not an option for us since we have no bosonic species available in our apparatus. Hence we make use of the dimple trick: By superimposing a small volume trap - the dimple - with the large volume reservoir an arbitrarily large increase in phase-space density in the dimple can be achieved when choosing an extreme ratio of the volumes [Kur98]. The gain in degeneracy for a Fermi gas can be estimated when making the following assumptions:

- the reservoir is large enough to not be perturbed by the presence of the dimple which means that the particle number and the temperature T_{res} remains nearly constant.
- the combined system is in thermal equilibrium.

Under these assumptions one can estimate the degeneracy of the total system: At high filling of the trap the Fermi-temperature $\times k_B$ is approximately given by the energy of the highest level in the trap which is equivalent to the trap depth. In our trap configuration sketched in figure 2.4 the trap depth is almost completely determined by the depth of the dimple U_{dim} and thus the degeneracy of the combined system is given by:

$$T_{\text{res}}/U_{\text{dim}} \quad (250 \text{ nK}/3.3 \mu\text{K} = 0.08)$$

In comparison to the degeneracy of the reservoir

$$T_{\text{res}}/U_{\text{res}} \quad (250 \text{ nK}/0.5 \mu\text{K} = 0.5)$$

where U_{res} is the trap depth of the reservoir, this leads to a gain in degeneracy of a factor of

$$U_{\text{dim}}/U_{\text{res}} \quad (3.3 \mu\text{K}/0.5 \mu\text{K} = 6.6)$$

From the numbers in brackets which are the numbers realized in the experi-

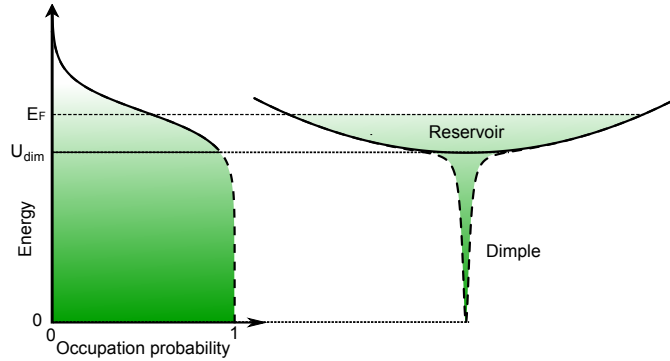


Figure 2.4.: **The dimple trick.** The dimple is so tight that it only provides a few hundred single particle states. Thus, when superimposing the dimple to the reservoir the temperature of the reservoir remains unchanged, whereas the Fermi energy of the combined trap is mainly determined by the trap depth of the dimple. Hence by applying the dimple the degeneracy of the combined system is increased and the occupation probability of the lowest single particle states approaches unity.

ment one sees that we are able to create a highly degenerate Fermi gas of about $T/T_F = 0.08$ in the dimple.

We have not directly proved the assumptions made above, but by estimating an upper bound for the temperature in the dimple we have found evidence that they are sufficiently well fulfilled (see 2.4.2). A qualitative analysis of the transfer efficiency is given in the appendix (figure A.2). Independently we can estimate the effect of the dimple onto the reservoir by finding an upper bound for its temperature increase. Although in the experiment we adiabatically ramp on the dimple trap we assume a sudden switch-on of the dimple which definitely overestimates its effect but serves as an upper bound. From the total number of particles in the reservoir, $N_{\text{res}} = 4 \times 10^4$, we can only transfer $N_{\text{dim}} = 6 \times 10^2$ into the dimple when filling it completely. In this case an upper bound for the energy increase of the reservoir is $\frac{U_{\text{dim}}}{2} \times N_{\text{dim}}/N_{\text{res}} = 25 \text{ nK}$ per particle which is only a 10% effect on the reservoir temperature of $T_{\text{res}} = 250 \text{ nK}$. As the degeneracy in the dimple is directly proportional to the temperature of the reservoir it would maximum decrease the dimple degeneracy from 0.08 to 0.09 [in units of T/T_F].

The remaining question to be answered is how large the occupation probability of the lowest states will be at the realized degeneracy of $T/T_F=0.08$. Inserting this number into the Fermi Dirac distribution we find a occupation probability for the lowest state of 0.9999 which excellently fulfills the requirement of determinism. In a next step we need to control the number of states in the trap. In the following

we will call the dimple 'microtrap' due to its micrometer-size. We will see that the size of the trap will play an important role to get control over the single particle trap states.

2.2.3. The spilling technique

As the occupation probability for the lowest state in the trap in a highly degenerate Fermi gas is close to unity, we just have to control the number of quantum states in the trap to achieve our primary goal. In the following we present a technique with which we have realized this.

One option to reduce the number of trapped states is lowering the depth of microtrap. To perform this step deterministically one needs to control the trap depth of the potential more precise than the level spacing of the highest levels in the trap which is determined by the steepness of the potential slope. Unfortunately, in the case of a Lorentzian shaped optical dipole trap, the slope becomes very flat at the upper boundary of the trap and the density of states diverges which we indicated by the gray shaded area in figure 2.5. By lowering the depth U_0 of the

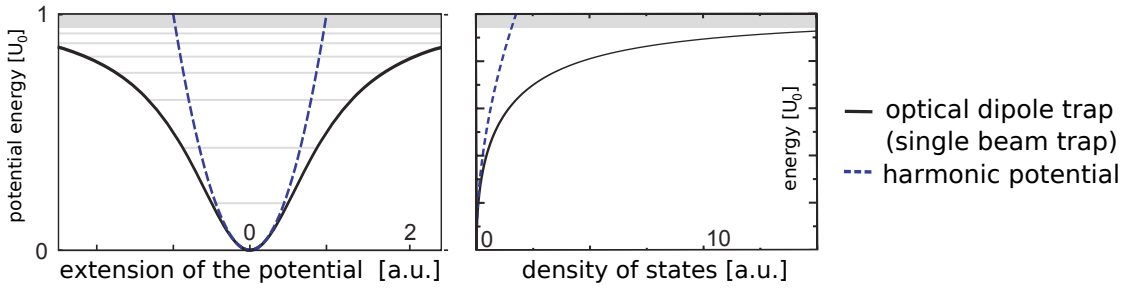


Figure 2.5.: **Density of states in a Lorentzian shaped dipoletrap.** Due to the weak potential slope the density of states in the Lorentzian shaped optical dipole trap diverge close to the continuum which is indicated by the gray region. For this type of trap, control over the individual quantum states is arbitrarily difficult. Plot taken from [Sim10] and adapted.

optical potential the geometric shape of the potential will remain unchanged and thus at the boundary to the continuum the level spacing will always be small. In this regime getting control over the quantum states would be not possible.

We can overcome this problem by modifying the potential by superimposing an additional linear potential as shown in figure 2.6. This results in an asymmetric trap configuration with a local minimum in the center, an open potential above a certain barrier height with unbound states reaching into the continuum on one

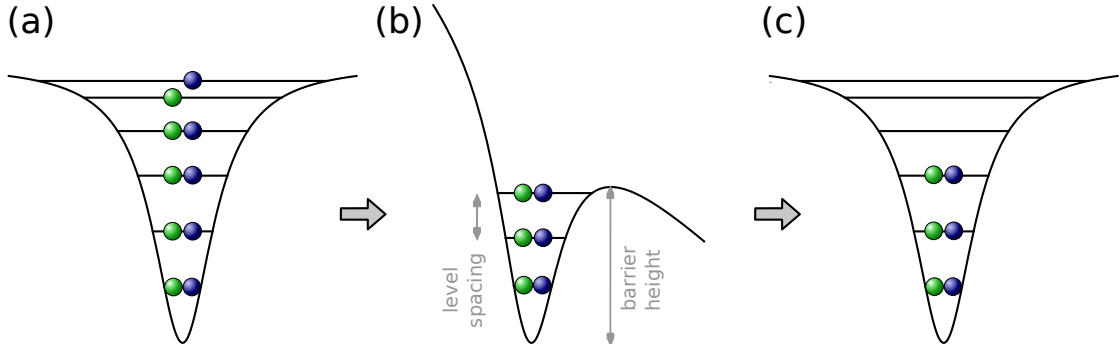


Figure 2.6.: **The spilling technique.** To our optical potential (a) we superimpose an additional linear potential (b). The level spacing of the bound states is sufficiently large to control the number of quantum states. All atoms occupying states above the barrier height become untrapped. After these atoms have escaped from the trap region we switch off the linear potential and end up with a well defined quantum system (c).

side and a very steep potential slope on the opposite side. Due to the steep potential slopes in this configuration the level spacings are large. Now, control over the barrier height more precise than the level spacing is technically possible and a certain barrier height can be reproducibly set up. With adding this linear potential all levels above the barrier height become unbound and one has to wait until the corresponding atoms have escaped from the trap region into the continuum. Finally one can switch off the additional linear potential and end up in a well defined state.

As we use a mixture of two distinguishable spin states, the occupation number per energy level is two as the distinguishable particles do not obey the Pauli-exclusion principle. Hence at the end of this procedure, which we call the spilling process, we expect to have prepared a few-particle system in the ground state with even particle number.

However, the release of atoms from unbound states is not the only process which happens during the spilling process. Just a release of unbound atoms would only be the case in an idealized configuration such as the box potential shown in figure 2.7 a). In the trap configuration which we have chosen, namely the optical dipole trap potential and a linear potential, there is a finite barrier through which the particles can tunnel during the application of the linear potential (see 2.7 b). Although we will later make use of this effect to probe the energy and the pair correlations in the system, for a deterministic preparation this is an unwanted effect which decreases the occupation probability of the bound states inside the potential well. But as long as the timescale τ_{rem} for the tunneling of atoms on the

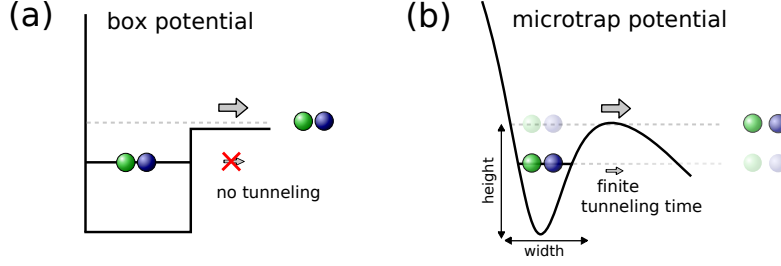


Figure 2.7.: **Comparison between a box potential and our microtrap potential.** In a box potential (a) the lifetime of bound states is infinite. In our microtrap potential (b) atoms on bound states can tunnel through the finite barrier. This sets constraints for the control of the height at a given width of the potential.

uppermost desired state is much longer than the application time t_{app} of the linear potential, the influence on the preparation fidelity is negligible. At the same time, the tunneling timescale τ_{tun} of atoms on the next higher undesirable state has to be much shorter than the application time of the linear potential. Assuming an exponential decay law for the tunneling process this leads to the limiting condition

$$-\ln\left(\frac{1-f}{2}\right) \times \tau_{\text{tun}}^{-1} \geq t_{\text{app}} \geq \tau_{\text{rem}}^{-1} \times -\ln\left(\frac{1+f}{2}\right) \quad (2.4)$$

for which the preparation with a desired target fidelity $f \in [0, 1]$ is possible. We call the parameter region fulfilling this condition the 'window of deterministic preparation'. The tunneling timescales depend on the exact shape of the potential and the calculation of the window needs a quantitative analysis of the tunneling process which will be provided in chapter 4. The results are shown in figure 2.8 and can qualitatively be understood by taking a look at the tunneling properties at a characteristic width of the potential: The smaller the width, the larger the spacing of the relevant levels inside the potential well. Thus a larger difference in tunneling timescales results in a larger region of barrier heights for which the above condition is fulfilled. Hence the smaller the width of the potential the less critical is the precision with which we need to control the barrier height.

Using optical methods for creating the potential, the minimum width that can be achieved is determined by the diffraction limited numerical aperture (NA) of an objective. At maximum NA the minimum resolution that can be achieved with trapping light of wavelength λ is $\frac{\lambda}{2}$. Realistic values for the width that can be achieved with standard objectives using visible or near infrared light are within $1 \mu\text{m}$ and $2 \mu\text{m}$.

For a given experimentally realizable width it is absolutely necessary to achieve

2.2. The individual elements of the preparation scheme

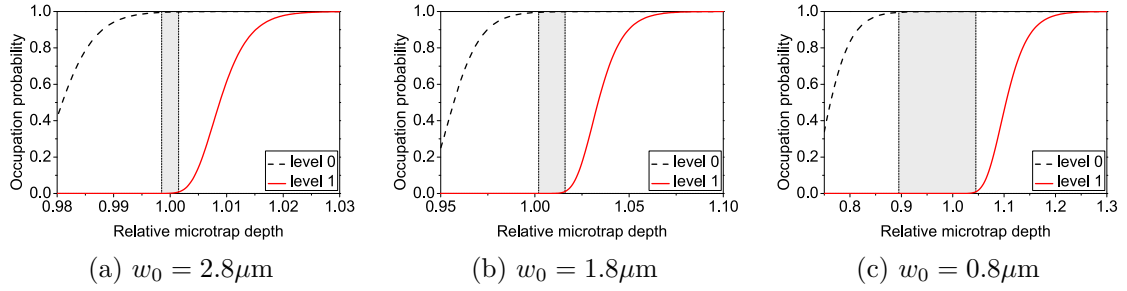


Figure 2.8.: **Window of deterministic preparation.** The individual figures show the occupation probabilities for the lowest trap level (dashed line) and the next higher level (solid line) for a fixed application time of the linear potential. It is given as a function of the microtrap depth (relative scale) for different widths (w_0) of the trap. The gray shaded areas indicate the regimes of trap depths for which the 'survival' probability of atoms on the lowest level is larger than 99.5% and on the next higher level less than 0.5%. For smaller trap widths this window of deterministic preparation gets larger.

a stability of the barrier height of a maximum allowed value to stay within the window of deterministic preparation. Using an elaborate optical design we can achieve a relative stability of the barrier height on the order of 10^{-3} by precisely controlling the intensity of the trapping light (appendix A.3) and the slope of the linear potential (appendix A.5). This is sufficient for a trap with a width of $2\mu\text{m}$ and lower. In the following we introduce the microtrap setup which is designed to realize such a small width.

2.2.4. The microtrap

Setup to create the optical potential

We realize a trap with small width by setting up the simplest realization of an optical trap: a single beam optical dipole trap [Gri00]. To create such a trapping potential we focus a collimated Gaussian beam with a waist of $w = 35\text{ mm}$ to a μm -size spot using a high numerical aperture objective (figure 2.9a). The radial confinement of the trap is given by the focal waist w_0 and the axial confinement is determined by the Rayleigh range $z_r = \pi w_0^2 / \lambda$ of the focused beam. Assuming cylindrical symmetry, the trap frequencies can be deduced by an harmonic

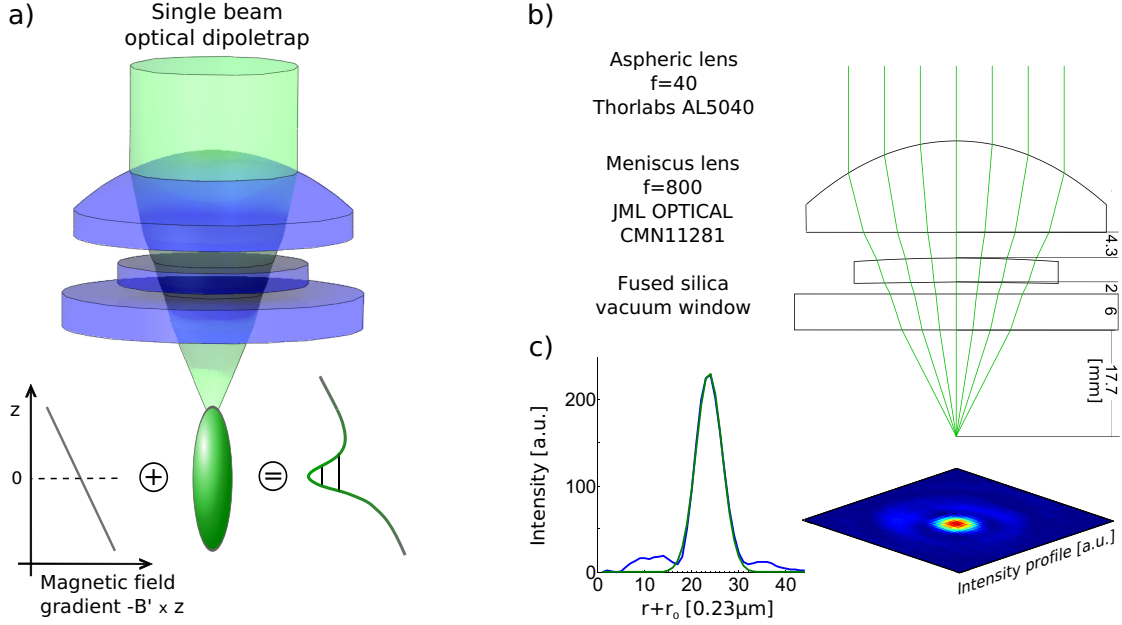


Figure 2.9.: **The microtrap setup.** a) Sketch of the microtrap objective which creates the optical dipole trap. b) Technical drawing of the components of objective. c) Radial intensity profile of the beam in the focal plane with Gaussian fit (red curve).

approximation of the potential [Gri00]:

$$\omega_r = \sqrt{\frac{4U_0}{mw_0^2}} \quad \omega_z = \sqrt{\frac{2U_0}{mz_r^2}} \quad (2.5)$$

where the radial and axial trap frequencies ω_r and ω_z increase with smaller focal waists. Here U_0 is the trap depth and m the ${}^6\text{Li}$ mass. A small focal waist can be achieved by an objective with large diffraction limited NA. One restriction to the objective is the minimum working distance which is limited by the distance of the center of the vacuum chamber to the outer surface of the vacuum window (23.7mm) with a maximum NA of the view port of ≈ 0.8 . Additionally the beam path through the window has to be taken into account when designing a high NA objective.

We have been working with two home built versions ³, the first one set up in 2009

³During writing this thesis a third version of the objective has been tested but not yet implemented into the experimental setup. It is an objective with chromatic correction for trapping and imaging light built by a professional optics manufacturer [JEN11]. The objective has been designed by Friedhelm Serwane [Ser11a] with a targeted diffraction limited NA of 0.6

[Zür09]. The purpose of this first version has been testing the mechanical stability of the objective, the control of the intensity and the transfer of atoms from the reservoir into the microtrap. Although the main result of the test was the necessity of a better light intensity stability which we realized by a completely new optical design (appendix A.3), the determined focal waist of the implemented objective of $\approx 3.7\mu\text{m}$ would have been too large to open the window of deterministic preparation at a relative intensity stability of 10^{-3} .

Hence we developed a second objective with a nominal diffraction limited NA of 0.44 for light of a wavelength of 1064 nm. The optical design was performed by Friedhelm Serwane using the ray tracing software Oslo [Lam10] (for details of the design see [Ser11a]). As the objective should be a temporary solution the design goal was an objective which contains comparatively cheap lenses from stock and a lens mount which could be machined in the institute workshop. The main focusing optic of this objective is a 2 inch diameter, 40 mm effective focal length, plano-convex lens with aspheric curvature compensating spheric aberrations (figure 2.9b). To compensate for the wavefront curvature introduced by the vacuum window we added an additional meniscus lens. As the vacuum window can be considered as a part of the objective, not only the alignment of the aspheric lens to the meniscus lens in the objective mount is critical, also the alignment of the optical axis of the objective mount to the window surface has to be precise on the order of 1 mrad. The mounting and alignment of the objective is realized by a 5-axis translation mount (Newport LP2a) which we modified by adding some external aluminum parts to improve the locking of its degrees of freedom.

The trap parameters of the optical potential

In an external test setup we could achieve a focal waist of $1.3(0.1)\mu\text{m}$ which we have deduced by a Gaussian fit to the radial beam profile (figure 2.9c). After carefully implementing the objective and superimposing the microtrap with the dipole trap we determined the focal waist by exciting atoms to higher trap levels using modulation spectroscopy (chapter 4.3.2). From these measurements we found that the focal waist of about $1.6\mu\text{m}$ is larger than expected. This can be explained by misalignment introduced while superimposing the microtrap with the dipole trap and locking the 5-axis mount. Furthermore we found an anisotropy in the radial symmetry. The most probable reason for that is a slightly anisotropic entrance beam onto the objective. Additionally the Rayleigh range does not correspond to the focal waist. The too large Rayleigh range could be explained by an astigmatism introduced either by a non-perfectly collimated entrance beam or

resulting in a nominal focal waist of $0.72\mu\text{m}$ for the trapping light of a wavelength of 1064 nm and $0.45\mu\text{m}$ for the imaging light of a wavelength of 671 nm.

by misalignment of the objective.

However, the deviation from the profile of a single beam trap, does not affect a deterministic preparation or experiments with few fermions. Knowing the trap parameters we can quantitatively compare our results with theories using these parameters as inputs for the potential. Around its minimum the optical potential can be approximated by

$$V(r, z) = V(x) + V(y) + V(z) \quad (2.6)$$

with Gaussian shape in $r = \sqrt{x^2 + y^2}$ direction

$$V(s) = p V_{0r} \left(1 - e^{-\frac{2s^2}{w_{0s}^2}} \right) \quad [s : x, y] \quad (2.7)$$

and Lorentzian shape in z-direction:

$$V(z) = V_0 \left(1 - \frac{1}{1 + (z/z_r)^2} \right) \quad (2.8)$$

where $w_{0s} \approx 1.6 \mu\text{m}$ and $z_r = \frac{\pi w_0^2}{\lambda}$ with $w_0 = 1.838 \mu\text{m}$. A detailed analysis of the potential shape and the values for the potential depth at the center of the trap, V_{0r} and V_0 , can be found in chapter 4.2.2. According to these parameters the trap has an aspect ratio of $\omega_z/\omega_r \approx 1 : 10$ (cf. equation 2.5). This means that the energetically lowest 10 states contain no excitation in radial direction. Hence, for atoms occupying these states, the tunneling process happens only in one dimension when the linear potential for removing bound states also points into the z-direction.

The magnetic potential for spilling atoms

The linear potential for spilling atoms from the optical potential is created by a magnetic field gradient. The atoms with magnetic moment $\boldsymbol{\mu}$ experience a force $\nabla \boldsymbol{\mu} \mathbf{B}$ in a spatially varying magnetic field $\mathbf{B}$. We first apply a homogeneous magnetic offset field pointing into z-direction which aligns the magnetic moment along this axis. At large magnetic offset fields the magnitude of the magnetic moment μ_m is nearly identical for the three lowest ^6Li hyperfine states and converges to the magnetic field independent value of 1 Bohr magneton μ_B (see figure 2.17). By applying a second non-homogeneous field with a gradient $B' = \frac{\partial B}{\partial z}$ into z-direction the atoms experience a constant force $\mu_B B'$ in a linear magnetic potential.

The combined optical and magnetic potential

Following the previous remarks the total optical and magnetic potential in z -direction is given by

$$V(p, z) = V_{\text{opt}}(p, z) + V_{\text{mag}}(z) = p V_0 \left(1 - \frac{1}{1 + (z/z_r)^2} \right) - \mu_m B' z \quad (2.9)$$

where $V_0 = k_B \times 3.326 \mu\text{K}$ is the initial optical trap depth at an optical beam power of $P_0 = 291.5 \mu\text{W}$, p the trap depth as a fraction of the initial depth, whereas p tunes linear with the optical power P and $z_r = \pi w_0^2 / \lambda$ is the Rayleigh range. The values for the focal waist and the magnetic field gradient are $w_0 = 1.838 \mu\text{m}$ and $B' = 18.92 \text{ G/cm}$ (for details see chapter 4.2.2). When the spilling gradient is switched off then $B' = 0$ and the magnetic part of the potential vanishes⁴. For most of the work presented in this thesis including the deterministic preparation this potential form is the only relevant external potential for the particles.

2.2.5. High fidelity atom number detection in a MOT

To determine the particle number of the few-fermion systems we need to implement a single atom detection method with near unity detection fidelity. In this section we introduce the single atom detection in a magneto-optical trap which we apply after releasing the atomic sample from the microtrap. The detection setup is shown in figure 2.10. The following paragraph describes the detection technique and is taken from the Supporting Online Material of our publication [Ser11b]:

'We detect the number of atoms in the prepared samples by recapturing them in a magneto-optical trap (MOT) and collecting their fluorescence signal[[Hu94](#)]. The magnetic field gradient has a strength of 250 G/cm, the diameter of the MOT beams are $\sim 4 \text{ mm}$ and their frequency is red detuned from the resonance by twice the natural linewidth of the transition. While we cannot determine the recapture efficiency of the MOT directly, it cannot be lower than the highest measured preparation fidelity per atom of 98(1)%. To record the fluorescence from the MOT we image it onto a CCD camera (figure 2.11) with an imaging system with numerical aperture of 0.17, capturing about 1% of the emitted photons. During the 0.5 s exposure time of the CCD one atom scatters about 1.9×10^6 photons. Considering the numerical aperture and the quantum efficiency of the imaging system roughly 1×10^4 photons per atom are detected. The exposure time is much shorter than the $1/e$ -lifetime of 250 s of the atoms in the MOT measured for 8

⁴The atoms experience also a constant gravitational force $-mg$ which is compensated by an additional permanently applied levitation gradient $B'_L = mg/\mu_B$ and therefore neglected in the potential of equation 2.9

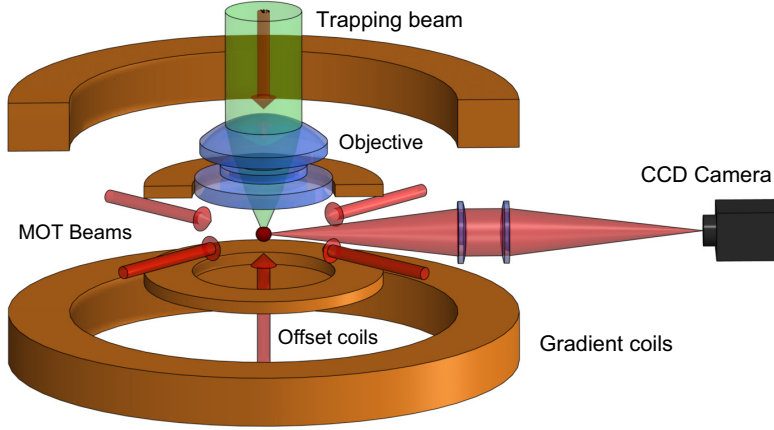


Figure 2.10.: **The MOT setup.** The number of atoms which are prepared in the micrometer-sized optical dipole trap are detected with single-atom resolution by transferring them into a compressed magneto-optical-trap (MOT) and collecting their fluorescence on a CCD camera. Taken from [Ser11b] and adapted.

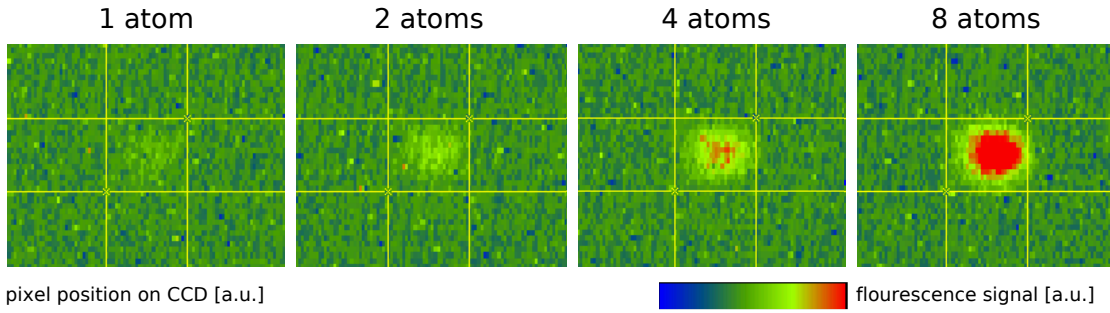


Figure 2.11.: **Fluorescence signal on the CCD camera for different atom numbers.**

atoms. This lifetime is long enough that neither light-induced collisions in the MOT nor collisions with background gas atoms limit our detection fidelity. To deduce the atom number from the fluorescence signal we bin all data from each series of measurements into one histogram. These histograms show distinct peaks, each corresponding to an integer number of atoms in the MOT. From the spacing of the peaks we extract the calibration factor for the mean fluorescence per atom. Because of fluctuations of the intensity of the MOT beams and the detuning of the MOT the fluorescence signal drifts on a few percent level on a timescale of several minutes. To compensate for this drift, we rescale the fluorescence signal of each measurement by a factor which is obtained by taking the average fluorescence per

atom of the ten previous and following measurements. To obtain this average for rescaling we only consider data with fluorescence signals that are close to a peak in the histogram, i.e. maximum 1σ distance. Then, the rescaled atom numbers are binned into a histogram (figure 2.12) and Gaussians are fitted to the peaks.

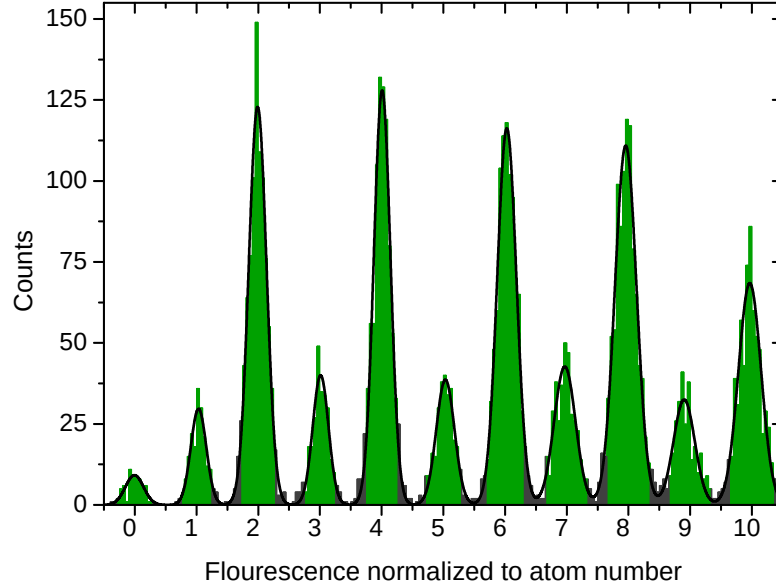


Figure 2.12.: **Histogram of the rescaled fluorescence signals.** The black curves are Gaussian fits to the rescaled atom number. The 2-atom (8-atom) peak center is separated from its adjacent peak centers by 7σ (5.7σ). This is large enough to clearly distinguish the fluorescence signal of different atom numbers. We bin fluorescence data within a 2σ -width of the peaks (green bars) to integer atom numbers; the counts outside the 2σ widths (gray bars) are rejected. Note: the data corresponds to the measurement of figure 2.14 where the occurrence of even-numbered system is enhanced. Taken from [Ser11b] and adapted.

We find a separation of the peak centers of $\sim 6\sigma$. The data points within 2σ of a peak center are binned to integer values which represent the number of atoms in the prepared sample. The 5% of measurements outside of the two-sigma width of the peaks are rejected. This is possible since the measurements are uncorrelated and the atom number detection is independent from the preparation.'

The reader who is interested in further details of the MOT setup can find them in [Ser11a]. What matters for all further experiments presented in this thesis is the ability to measure atom numbers with a fidelity of 98(1)% per particle by

using the described method. Whenever we will quote a particle number less than 1000 we will have used this method for detection. In the work presented in the following we will directly use it to determine if we have deterministically prepared a few-particle systems.

2.3. Implementation of the preparation scheme

To be able to realize our preparation scheme we have to combine the individual elements introduced in the previous section. We use a sketch of a typical experimental sequence to illustrate the combination of these individual elements. In the subsequent section we will then present the results of the high fidelity preparation.

2.3.1. Experimental sequence

To explain the experimental sequence we divide it into five parts labeled by I-V as illustrated in figure 2.13. The upper row of the figure sketches the potential shape in each step of the experimental sequence. The three graphs in the center, which we denote 'timing graph' show the time dependence of the most relevant parameters of the experiment. The first one (black curve) is the optical trap depth p of the microtrap, the second one (green curve) is the magnetic field gradient B' which defines the strength of the linear potential and the third one (blue curve) is the magnetic offset field which determines the scattering length. In the following we discuss the individual parts of this sequence.

I) Ramping on the microtrap

The first elements to be combined are the degenerate Fermi gas in the large volume dipole trap and the microtrap potential. To transfer particles into the microtrap we slowly ramp up the depth of the optical potential of the microtrap at constant reservoir depth. While the depth is ramped up, the scattering length is kept constant at about $-300 a_0$ at an offset field of 300 G to allow thermalization and thus occupation of the lowest states in the microtrap. When increasing the depth of the microtrap potential one has to fulfill the adiabaticity criterion for the lowest states in the trap to not create holes⁵ in the Fermi distribution. The criterion reads

$$\frac{\left| \frac{d}{dt} (e_i - e_j) \right|}{\frac{(2\pi)^2}{\hbar} (e_{ij})^2} \ll 1 \quad (2.10)$$

⁵We denote a hole as an unoccupied state with energy lower than the Fermi energy.

2.3. Implementation of the preparation scheme

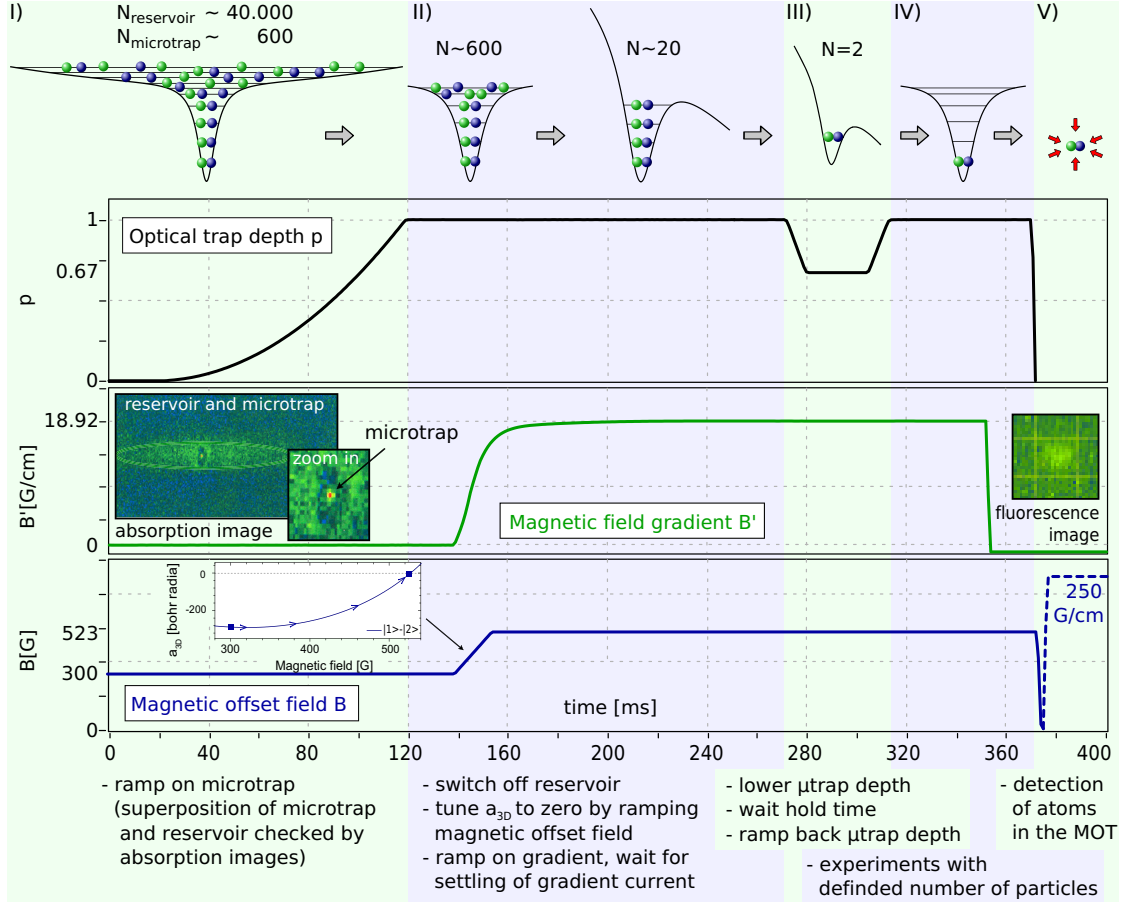


Figure 2.13.: **Experimental sequence for a deterministic preparation.**

and expresses, that the time dependent differential change in energy between two states with energy e_i and e_j must be much smaller than the square of the energy difference e_{ij} between these states [Zen32] to not jump between the different states during the ramp. The right hand side of this differential equation becomes constant when choosing a hyperbolic form $e_n(t) = -\frac{1}{1+t/\tau}$ with time constant τ for the time dependent energy of the states. In a harmonic trap the energy of the eigenstates are $e_n = (n + 1/2) \hbar \omega$ and $\omega \propto \sqrt{p}$. If one further approximates the energy change of two nearby states by the trap frequency $\omega \approx 1$ kHz then the criteria reduces to

$$\tau \gg \frac{1}{\omega} \quad (2.11)$$

and therefore the change of the trap depth p is adiabatic when the time constant of the ramp is much larger than the inverse trap frequency. For the ramp-on of the microtrap we have approximated the hyperbolic form by a parabola which

can be seen by the quadratic increase of the trap depth in the upper part of the timing graph. A quantitative analysis of the adiabaticity condition can be found in [Ser11a].

II) Switching off the reservoir and ramping on the magnetic field gradient

After ramping on the microtrap we keep the scattering length of the sample at about $-300 a_0$ for 20 ms to allow further thermalization. Experimentally we found that for longer periods we do not see an increase of the particle number in the microtrap and therefore we switch off the reservoir after 20 ms by ramping down the optical power of the reservoir. As we want to avoid correlated tunneling of particles from an interacting system we switch off the interaction by tuning the scattering length close to its zero crossing at a magnetic offset field of 523 G⁶. During the ramp of the offset field we switch on the magnetic field gradient for spilling atoms of the trap. Unfortunately the pair of coils which we use to create the quadrupole field to generate the gradient in z-direction has a comparatively long time constant of several ms due to the large inductance of the coils. Additionally to the long time constant, the gain of the digital PID feedback loop which controls the gradient current running through the pair of coils is set to low values to suppress an overshoot of the gradient current above its setpoint. This is the reason why it takes approximately 150 ms until the measured current in the coils reaches its setpoint within a relative precision of 10^{-3} . For the chosen gradient of $B' = 18.92 \text{ G/cm}$ about 20 particles are left in the potential well which corresponds to about 10 ± 1 bound states. As the trap has an aspect ratio of 1:10 all atoms occupying harmonic oscillator levels with radial excitations have left the trap.

III) Tuning the depth of the microtrap

For spilling a well defined fraction of the remaining 20 particles we overcome the long timescale for ramping the gradient by tuning the depth of the optical potential to change the barrier height. This approach has the advantage that the bandwidth of the intensity control of the microtrap beam is about two orders of magnitude faster than the one of the gradient coils and is as fast as the typical tunneling timescales. We ramp the microtrap depth to a certain value p within 8ms. Then

⁶Due to historic reasons we tune the magnetic field to 523 G instead to the zero crossing of the scattering length at $527 \pm 0.5 \text{ G}$ [Bar05]. Yet, this does not effect the spilling scheme since $a_{3D} = -14 a_0$ is still negligibly small.

all levels above a certain energy level become unbound and leave the trap. The timescale is set by the inverse trap frequency $\omega_z \sim 1$ kHz and therefore the escape of particles takes a couple of ms. Thus we wait for a hold time of 25ms. During that time also atoms occupying the highest bound states can tunnel through the barrier. Deterministic preparation is achieved when the tunneling timescale of atoms on the uppermost state is smaller than the holdtime and the tunneling timescale of atoms on the next lower state is much larger than the holdtime. Finally we ramp back to the original optical trap depth of $p_0 = 1$ within the same ramp time of 8ms to switch off all tunneling processes ⁷.

IV) Experiments with a well defined quantum systems

In this part of the sequence we can manipulate the few-particle system. All experiments performed with a few-fermion system presented in this thesis are realized within this part. For now we skip this part of the sequence which will be extensively studied in the next chapters.

IV) Detection of the particle number

In a final step the number of particles in the trap has to be detected. Therefore the introduced single atom detection in a MOT is applied. The atoms in the microtrap are released by instantly switching off the power of the microtrap beam. Consequently the zero-point energy of the atoms in the trap is transferred into kinetic energy and thus the atoms fly away from the trap region with a mean velocity of $\langle v \rangle = \sqrt{\frac{\hbar(2\omega_r + \omega_z)}{m}} \approx 20 \mu\text{m/ms}$. Because for trapping in a MOT the offset field has to be zero it is ramped to zero within 2ms immediately after the microtrap is switched off. As for the single atom detection in the MOT a large magnetic field gradient is required we use the offset coils which can generate a gradient of 250 G/cm when reversing the current flow in one of the coils⁸. The ramp up of this gradient current in the offset coils takes 2ms and thus after 4.3ms time of flight of the atoms the MOT lasers can be switched on and the atoms are recaptured in the MOT. From the fluorescence light of the atoms recorded for 0.5 – 1s the number of particles can be deduced.

⁷As long as the gradient is applied the extension of the barrier is always finite. However, tunneling can be neglected because the tunneling timescales of atoms occupying the energetically lowest states exceed the total experimental time by multiple orders of magnitude.

⁸The gradient generated by the gradient coils is limited to 40 G/cm. To create a quadrupole field with the offset coils the direction of the current flow in one of the offset coil is reversed by an H-bridge consisting of 4 MOSFETS [Koh08].

2.4. Number state preparation, ground state preparation fidelity and lifetime

To realize number state preparation it is necessary to find an adequate barrier height for which atoms from the highest bound state have tunneled within the hold time of 25 ms at a certain trap depth p and atoms on the next energetically lower state have not yet tunneled. To find this adequate barrier height we repeat the experimental sequence several times for one value of the trap depth p and vary p over a certain interval. Figure 2.14 shows the experimental result of this measurement. The plot on the left hand side shows the mean atom number $\langle N \rangle$ as a function of the trap depth p . As expected one finds that the mean particle number decreases with lower trap depth. Additionally, one observes a step-like behavior with plateaus at even atom numbers. This plateaus indicate the region which we denoted as the window of deterministic preparation (see equation 2.4). The reason that the plateaus occur at even atom numbers is attributed to the occupancy of the single particle level by ideally two distinguishable atoms.

The step-like behavior becomes more clearly when investigating the number fluctuations around a certain mean value. The plot on the right hand side in figure 2.14 shows the variance $(\Delta N)^2 = \sum_{i=1}^n (\langle N \rangle - N_i)^2$ of the mean particles number $\langle N \rangle$ at fixed trap depth p , where N_i is the measured particle in a single preparation process. For atom numbers close to even integers the number fluctuations are drastically reduced which is a clear evidence that the window of deterministic preparation exists at the corresponding trap depths.

For a mean number of 8 atoms we find a number fluctuation of

$$\frac{(\Delta N)^2}{\langle N \rangle} = 0.017. \quad (2.12)$$

In contrast in a gas of independent particles the fluctuation follow Poissonian statistics with $(\Delta N)^2 / \langle N \rangle = 1$. The observed drastic reduction of number fluctuations by 18 dB indicate the strong fermionic correlations in our system. To our knowledge this suppression of number fluctuation in an system of eight particles is the largest ever realized in a cold atom experiment.

To quantify the preparation fidelity we exemplarily choose two systems with desired particle number: a two-particle system and an eight-particle system. To deterministically prepare these system we tune the optical trap depth p to the corresponding plateaus where we get the minimum number fluctuation. We repeat the measurement 300 – 600 times and bin the detected fluorescence of each realization into a histogram, which is shown in figure 2.15. We achieve very high preparation fidelities of 96(1)% for a system with a desired particle number of two and

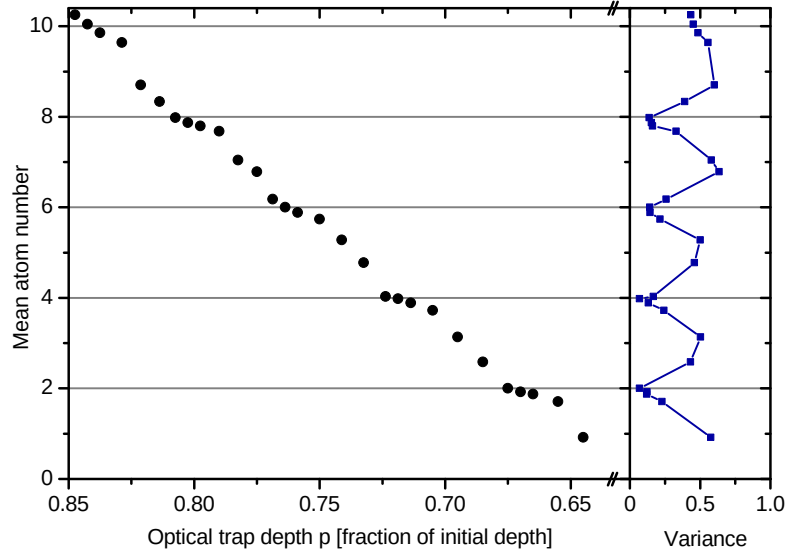


Figure 2.14.: **Controlling the number of quantum states.** When the trap depth is reduced, the mean atom number decreases in steps of two because each energy level in the trap is occupied with one atom per spin state. Each data point is the average of 190 measurements with $(\Delta N)^2$ as the variance (shown on the right). For even atom numbers, the number fluctuations are strongly suppressed. For eight atoms, we achieve a suppression of 18 dB of $(\Delta N)^2/\langle N \rangle$ compared to a system obeying the Poissonian statistics. Taken from [Ser11b] and adapted.

87(1)% for an eight-particle system resulting in an preparation fidelity of 98(1)% per atom. The error is the statistical error calculated by assuming that the occurrence of samples with undesired atom number follows a Poissonian distribution. Although we have an excellent reproducibility, there remains one crucial question: Are the particles in the ground state and thus in a well defined motional state? We test this by applying the spilling process a second time. If the particles where not in the ground state after the first spilling process, for example due to excitations during the ramp of the potential, the particle number would be reduced after the second spilling process (see figure 2.16, second row). We compare the histograms after the first and the second spilling process and find only a small reduction of the occurrence of the desired particle numbers with 92(2)% and 88(1)% for two and eight particles respectively. This is a hint that the probability of exciting the particles during the spilling process can only be on a few percent level. We quantify the excitation probability during the ramp by apply a combinatorial model to estimate the ground state preparation fidelity of the system.

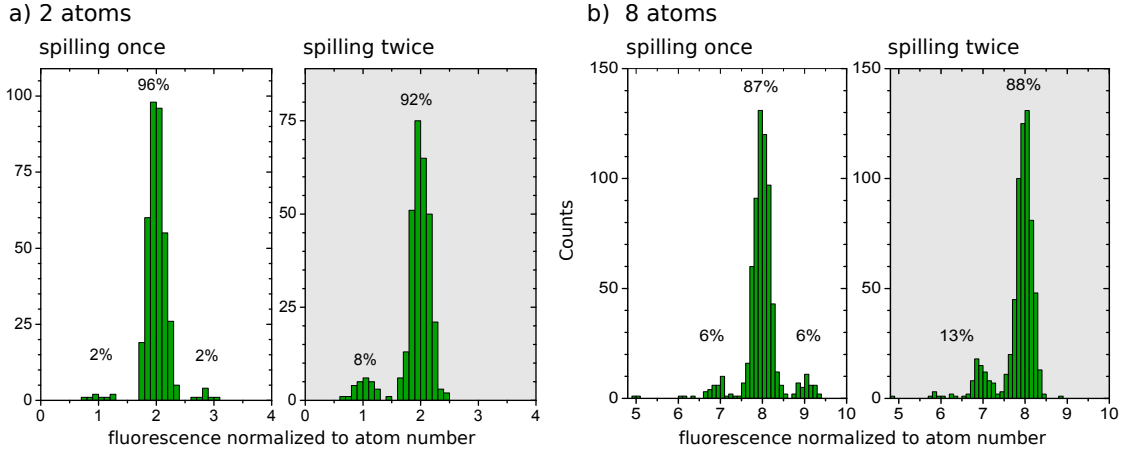


Figure 2.15.: **Histograms after the first and second spilling process for the preparation of two atoms and eight atoms** The numbers above the peaks give the relative occurrences of the counts within the corresponding peaks. The fidelity after the second spilling process (gray shaded histograms) remains almost unchanged, indicating that the ground state is prepared with high fidelity. Taken from [Ser11b].

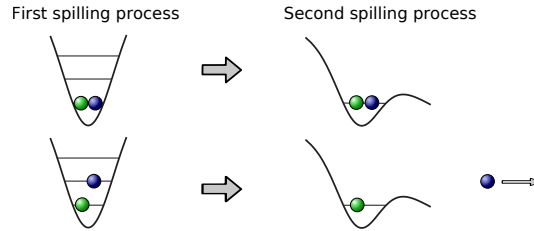


Figure 2.16.: **Fidelity of preparing systems in the ground state.** To determine how many of the prepared few-particle systems are in their ground state, we repeat the spilling process. This removes atoms in higher levels but leaves the ground state unchanged. Taken from [Ser11b].

Ground state preparation fidelity and lifetime, taken from [Ser11b]

To estimate the fidelity for preparing two atoms in the ground state of the trap from the histogram shown in figure 2.15 we only consider the lowest two levels of the trap. This is based on the assumption that atoms in higher levels have only negligible probability to remain trapped after the spilling process.

From the number of prepared samples containing one or three atoms we can deduce upper bounds for the probability to find an atom missing in the lowest level or an

atom remaining in the second level of 2% each. As the atoms are noninteracting during the preparation process we can assume these probabilities to apply to each atom individually. A system containing two atoms which are not in the ground state requires both a hole in the lowest level and an atom on the second level. This is suppressed by a factor of $(0.02)^2 = 4 \cdot 10^{-4}$. From this we conclude that only a negligible fraction of the observed two-atom samples were not prepared in their ground state.

However this does not exclude the possibility of exciting the system when closing the trap at the end of the spilling process. Therefore we perform the spilling process a second time, which removes atoms in higher levels of the trap. We find that the second spilling process reduces the number of samples containing two atoms from 96(1)% to 92(2)%. If we assume that the 2% probability of preparing one atom after the first spilling process is due to states being non-occupied before the spilling process and account for the fact that almost all samples containing three atoms will have two atoms after the second spilling process, we would expect 98(2)% of the samples to contain two and 2(2)% of the samples to contain one atom. This leads us to the conclusion that there is a 6(2)% probability to create an excitation while ramping the barrier up and back down.

Following the same consideration for a system of eight atoms we find the probability that a sample of eight atoms was not in the ground state to be $4 \cdot 10^{-3}$. From spilling twice we also find an upper bound of 6(2)% for the number of excitations during the ramps. If we assume the same excitation probability for ramping up and down, we get an estimated fidelity of 93(2)% to prepare the system in its ground state after ramping the potential back up after the first spilling process. For eight atoms, we find a ground-state preparation probability of 84(2)%. By varying the time between the two spilling processes, we found the 1/e-lifetime of the prepared two-particle system in its ground state to be 60 s, which shows the high degree of isolation from the environment.

2.4.1. Creating imbalanced systems and spin dependent detection

We have demonstrated that we can deterministically prepare a system with an even number of particles. Yet, for the studies presented in this thesis, we also need to prepare spin imbalanced systems, i.e. system where the particle number of one spin component exceeds the number of particles in the other spin component. The component with the larger number is referred to as the majority. To realize such a system we first prepare a balanced system with $N = 2N_{\text{maj}}$. To create the imbalance we take advantage of the magnetic field dependence of the magnetic

moment of the atoms. Around 30 G the magnetic moment of the Zeeman substate $|2\rangle$ vanishes and thus atoms in this state do not experience the spilling potential $V_{\text{mag}} = \mu_z B' z$ generated by the magnetic field gradient B' . The potential shape for this state is indicated by the green line in figure 2.17 b). Yet, the magnetic

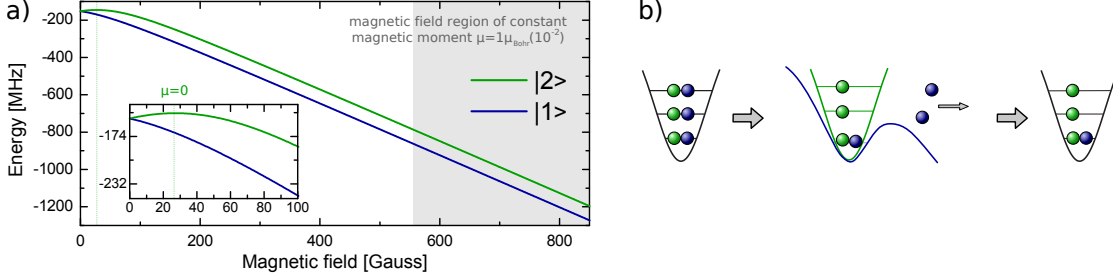


Figure 2.17.: **Preparation of imbalanced systems.** a) Magnetic moment of the two hyperfine states $|1\rangle$ and $|2\rangle$. The inset shows that the magnetic moment of states $|2\rangle$ gets zero at about 30 G. At this field the component $|1\rangle$ can be controlled individually (blue potential shape in b) without influencing the other component (green potential shape).

moment of atoms in state $|1\rangle$ does not vanish and thus the atoms are influenced by the spilling process (blue line in figure 2.17 b). Again, by choosing an adequate value for the optical trap depth parameter p , one can prepare a system with defined number of minority particles. To invert the imbalance to have more particles in state $|1\rangle$ than in $|2\rangle$ we can apply a Landau-Zener passage⁹ which transforms atoms in state $|1\rangle$ to state $|2\rangle$ and vice versa.

This technique can also be used to determine the number of atoms in a certain spin state after a measurement has been performed with some system. To realize this spin dependent detection we remove all atoms which are not in the state of interest by applying the previous spilling technique. In this case the depth of the potential is chosen such that no bound states is left for these atoms. Finally we detect the particle number of atoms left in the potential which corresponds to the number of atoms in the state of interest.

⁹ For a two level system with a coupling Ω between the two state $|i\rangle$ and $|j\rangle$ introduced by a radio-frequency the passage can be understood as follows: When applying a rf-frequency $\omega_{\text{rf}} = \omega_0 - |\delta|$ with large detuning $\delta \gg \Omega$ from the resonance frequency ω_0 to the initial hyperfine state $|i\rangle$, the initial state is equivalent to a dressed state [Dal85] $|d_-\rangle = c_i(\delta, \Omega)|i\rangle - c_j(\delta, \Omega)|j\rangle$ with $c_i = 1$ and $c_j = 0$. When sweeping the detuning slow enough [Zen32] across the resonance the system adiabatically follows the dressed state which evolves to $|d_-\rangle = c_i|i\rangle - c_j|j\rangle$ with $c_i = 0$ and $c_j = 1$ for $\delta \gg \Omega$ and $\omega_{\text{rf}} = \omega_0 + |\delta|$, i.e. after the sweep one ends up in state $|j\rangle$. Using this method we are able to transfer atoms from state $|1\rangle$ to $|2\rangle$ and vice versa. In the microtrap we achieve a transfer efficiency of 95% per atom from one to the other hyperfine state.

2.4.2. Effects resulting from finite temperature

Besides exciting atoms to higher levels during the ramp of the trapping potential also effects from finite temperature might limit the preparation fidelity. We can estimate an upper bound for the temperature of the sample in the microtrap by analyzing the deviation from a perfectly prepared system at $T = 0$. To give this upper bound we assume that all holes are due to thermal fluctuations in the sample. For this consideration we neglect to have two holes at the same time which can be done as long as the single hole probability is small. That this assumption is fulfilled can be seen from the rightmost histogram of figure 2.15 where the single hole probability is 12%. The probability to find such a hole on a certain trap level with energy E_{n_i} is given by the complement of the occupation probability of this level, which is determined by the Fermi Dirac distribution $P(E)$. At finite temperature the total probability to find a single hole on the first 4 levels which are occupied by exactly 8 atoms at $T=0$ is then given by

$$P_h(n_{\max} = 4) = 2 \times \sum_{n_i=1}^{n_{\max}} (1 - P(E_{n_i})) \times \underbrace{\prod_{\substack{n_j=1 \\ n_j \neq n_i}}^{n_{\max}} P(E_{n_j})}_{=1-O(10^{-3})} . \quad (2.13)$$

We insert equation (2.3) for the Fermi Dirac distribution and substitute the chemical potential by the Fermi energy which is a valid approximation for $T \ll T_F$. In our case the Fermi energy is determined by the trap depth of the microtrap and hence $E_F \approx V_0 = k_b \times 3.3 \mu\text{K}$. Then the previous equation reads

$$P_h(n_{\max} = 4) = 2 \times \sum_{n_i=1}^{n_{\max}} \left(1 - \left(e^{\frac{(E_0 + n_i \hbar \omega_z)/E_F - 1}{T/T_F}} + 1 \right)^{-1} \right) \quad (2.14)$$

with $E_0 = \hbar(\omega_r + 1/2 \omega_z) = k_b \times 0.724 \mu\text{K}$ the ground state energy of the trap and $\hbar \omega_z = k_b \times 0.072 \mu\text{K}$ the level spacing. With the experimental result of $P_h = 12\%$ for 8 atoms we can solve equation (2.14) and find

$$T/T_F = 0.19 \quad (2.15)$$

as an upper bound for the temperature in the microtrap. As the temperature in the microtrap is determined by the temperature of the reservoir this value is also an upper bound for the temperature of the reservoir.

Hence the method of measuring the number fluctuations can be used to probe the degeneracy of the lowest energy states of an ultracold Fermi gas as long as the fluctuations do not arise from technical noise. This method is related to work

from different experimental groups where the number fluctuations in a finite sub-volume of a Fermi-gas has been measured providing a sensitive thermometry at low temperatures [San10][Mül10].

Using the upper bound of the temperature we can also estimate the preparation fidelity for a two-particle system if thermal fluctuations were the only limitation. Inserting $T/T_F = 0.19$ in equation (2.14) with $n_{\max} = 1$ would result in a ground state preparation fidelity of 97%. This is consistent with a reference measurement which suppresses the effects of drifts of the barrier height during the spilling process (appendix figure A.6). In this measurement we found an upper bound of 2.4(6)% for the probability of finding a hole in a two-particle system. Comparing these results with the achieved ground state preparation fidelity of 93(2)% we can conclude that one could still gain another 4(2)% in preparation fidelity when further reducing technical noise sources such as excitations and drifts during the spilling process.

2.4.3. Summary of the preparation results

In summary we have implemented a scheme with which we can deterministically prepare a few fermion system. The challenge to built an extremely stable optical setup has been successfully realized. With our reliable setup we achieved preparation fidelities of up to 96% and ground state preparation fidelities of up to 93% with a lifetime of 60 s. Thus, our system is perfectly suited to perform experiments with few interacting fermions. The only missing feature, the tunability of the interparticle interaction, will be presented in the next chapter.

3. Tuning interaction and few particles with contact interaction in a 1D harmonic trap

In the previous chapter we have demonstrated that we are able to deterministically prepare a noninteracting two-component few-fermion system. Yet, the systems becomes most relevant for testing various theoretical models and simulating complex phenomena when the particles are interacting. Ideally, the interaction strength should be tunable to arbitrary values.

To realize this, we make use of one of the most fundamental properties of ultracold gases namely that the interaction potential between two ultracold particles can be described by only one parameter, the s-wave scattering length a_{3D} . Magnetic Feshbach resonances which exist in many ultracold samples of various atomic species allow to tune the scattering length and thus the strength of the interaction.

This tunability enabled researchers to study various kinds of Hamiltonians in which the interaction term is modeled by this parameter. For example in three-component Fermi gases the existence of a universal three-body bound state, the Efimov state [Ef70], could be proven [Kra06] and we were able to measure its binding energies as a function of the scattering length by radio frequency association [Lom10].

In this chapter we will present how the scattering properties of these samples leads to an effective two-body interaction potential which is only dependent on the scattering length, where the latter can be tuned by Feshbach resonances. In the second part of this chapter we discuss the predictions for the properties of few interacting particles which are confined in a potential similar to our microtrap potential. We will first investigate the two-particle system and present regimes in which the interaction properties of the system can be described by a one dimensional model. This leads to the introduction of confinement induced resonances which determine the interaction strength in this quasi-1D regime. Finally we extend our discussion to systems of three particles and present approaches for solving systems with even larger particle numbers.

3.1. Tuning interaction

In the following we introduce the scattering properties of ultracold atoms and present the possibility to tune the interaction strength by means of Feshbach resonances. Further details can be found in [Dal98], [Sch07] and in the reviews [Blo08] [Ket08] [Chi10].

3.1.1. Two-body interaction of ultracold fermions

Although one achieves high phase-space density in systems of ultracold atoms which allow to study degenerate quantum gases these systems are generally dilute meaning that the interparticle distance is much larger than the range of the two-particle interactions. The range of the interaction potential is determined by the van-der-Waals range and is typically less than 5 nm whereas the interparticle spacing is $\sim 1 \mu\text{m}$ determined from the the number density of typically 10^{12}cm^{-3} . This means that the interaction properties can be described by collisional processes in which only two particles are involved at the same time. The two-particle scattering process is analyzed by a transformation into the center-of-mass frame of the system. Then the process is described by a single particle scattered at a scattering center at relative distance $\mathbf{r} = \mathbf{0}$ given by the van-der-Waals interaction potential V_i of the two particles. The corresponding Schrödinger equation reads

$$\left(-\frac{\hbar^2}{2\mu} \nabla^2 + V_i(\mathbf{r}) \right) \Psi_k(\mathbf{r}) = E_k \Psi_k(\mathbf{r}) \quad (3.1)$$

with $E_k = \hbar k^2 / (2\mu)$ the collisional energy and $\mu = m/2$ the reduced mass. In the absence of a confining potential the wavefunction far away from the scattering center can be asymptotically constructed by incoming plane waves and outgoing scattered spherical waves (see figure 3.1)

$$\Psi_k \propto e^{ikz} + f_k(\theta) \frac{e^{ikr}}{r} \quad (3.2)$$

where θ ($0 \leq \theta < \pi$) is the angle between the incident plane wave pointing into z-direction and the direction of observation. Using flux equations one can determine the differential cross section of the scattering process from this ansatz

$$\frac{d\sigma(k)}{d\theta} = |f_k(\theta)|^2 \quad (3.3)$$

with $|f_k(\theta)|$ the scattering amplitude which is defined through the interaction potential. If the interaction potential would be exactly known for all values of $\mathbf{r}$

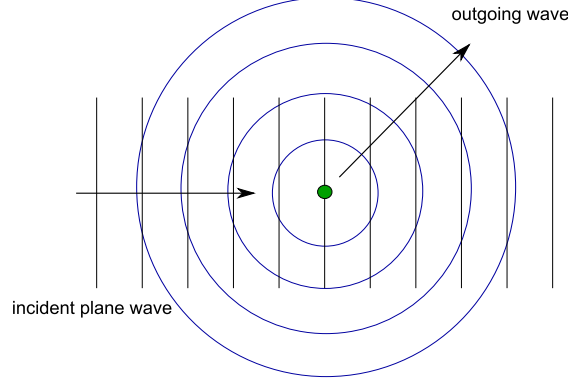


Figure 3.1.: **Scattering of two ultracold atoms.** The scattering potential is given by the van-der-Waals interaction potential of the atoms. Far away from the scattering center the scattering event can be described by incoming plane waves and outgoing spherical waves. At low momenta of the particles only the s-wave of a partial wave expansion contribute to the scattering event. In the case that the de Broglie wavelength is larger than the effective range of the potential the scattering event can be described by one parameter - the s-wave scattering length a_{3D} .

the cross section could be immediately determined. However, we are interested in solving the problem without detailed knowledge of the exact shape of the interaction potential as it is hard to determine it from *ab initio* calculations [Blo08]. Yet, we can make use of the properties of the atoms at ultracold temperatures which are typically on the order of $\sim 1 \mu\text{K}$. Then the wavelength of the incoming waves are on the order of the thermal de Broglie wavelength of atoms of $\sim 1 \mu\text{m}$ which is much larger than the characteristic length scale of the van-der-Waals potential. Hence, the scattered particles far away from the scattering center do not resolve the detailed structure of the potential.

To solve the scattering problem for any potential which extension is smaller than the de Broglie wavelength we apply a partial wave expansion and express the wavefunctions for the incoming and scattered wave in terms of spherical harmonics with angular momentum quantum number l and spherical Bessel functions. When we compare the l -th partial wave of the incoming wave with the scattered one they only differ by a phase shift δ_l which incorporates the effect of the potential on the collision process and determines the scattering amplitude

$$f_k(\theta) = \frac{1}{2ik} \sum_{l=0}^{\infty} (2l+1) P_l(\cos \theta) (e^{2i\delta_l(k)} - 1) \quad (3.4)$$

3.1. Tuning interaction

with $P_l(\cos \theta)$ the Legendre polynomials. The total cross section is then given by the sum of all contributions arising from each partial wave

$$\frac{d\sigma(k)}{d\theta} = \sum_{l=0}^{\infty} \sigma_l(k) \quad (3.5)$$

with the partial cross section

$$\sigma_l(k) = \frac{4\pi}{k^2} (2l+1) \sin^2 \delta_l(k). \quad (3.6)$$

At low kinetic energies as they are present in systems of ultracold atoms the collision energies are too low to overcome the centrifugal barrier in the radial Schrödinger equation of the l -th partial wave which is proportional to $l(l+1)/r^2$. Thus, at sufficiently low temperatures only the partial wave with $l=0$, the s-wave, contribute to the scattering process because higher partial waves are reflected from the centrifugal barrier without probing the interaction potential. This means that only the s-wave scattering contributes to the cross section and it is determined only by the s-wave phase shift δ_0 the particles acquire when they are scattered off the scattering center.

So far we have not considered the symmetry properties of the particles. Yet, in the case of identical fermions we have to consider the antisymmetric exchange symmetry of the total wavefunction. Hence, for identical fermions the spatial wavefunction has to be antisymmetric which can be only realized by partial waves with $l = 2n+1$. Herein, the s-wave is excluded and thus s-wave scattering for identical fermions is forbidden. However, when investigating the properties of spin mixtures of ultracold fermions also scattering events of two distinguishable fermions are present. In this case the antisymmetric exchange symmetry for two distinguishable fermions is not required and thus s-wave interaction occurs.

In the case of only s-wave contribution to the scattering process the scattering amplitude simplifies to

$$f_k(\theta) = \frac{1}{2ik} (e^{2i\delta_0(k)} - 1) \quad (3.7)$$

By an expansion in a power series of k the scattering phase shift can be expressed by

$$k \cot(\delta_0(k)) = -\frac{1}{a_{3D}} + \frac{1}{2} r_{\text{eff}} k^2 + \dots \quad (3.8)$$

which defines the s-wave scattering length a_{3D} and the effective range r_{eff} . For atoms with a de Broglie wavelength much larger than the effective range ($1/k \ll r_{\text{eff}} \approx r_0$) the second term in (3.8) can be neglected and the scattering amplitude is given by

$$f = -\frac{a_{3D}}{1 + ik a_{3D}} \quad (3.9)$$

and depends except for the particles momenta only on one parameter, the s-wave scattering length a_{3D} which is given by

$$a_{3D} = -\lim_{k \rightarrow 0} \frac{\tan \delta_0(k)}{k} . \quad (3.10)$$

The total cross-section can be found by integrating the scattering amplitude (3.9):

$$\sigma_0(k) = \frac{4\pi}{k^2} \sin^2 \delta_0 . \quad (3.11)$$

For small phase shifts in the regime $ka < 1$ we can write the cross section by

$$\sigma_0(k) = 4\pi a_{3D}^2 . \quad (3.12)$$

which only depends on s-wave scattering length a_{3D} . However, as the scattering phase approaches $\pi/2$ the scattering length diverges and so would the cross-section if equation 3.12 applied in this regime. This would result in an unphysical divergence of the outgoing flux of the scattering process. The outgoing flux can maximum become equal to the incoming flux which is defined as the unitary limit [Sak94, Sch07]. In this case where the scattering phase shift becomes $\pi/2$ and the scattering length diverges the scattering cross section becomes independent of the scattering length:

$$\sigma_0(k) = \frac{4\pi}{k^2} \quad (3.13)$$

This is the unitarity regime where the scattering properties are only determined by the particles momenta which allows for example for the investigation of unitary Fermi gases [Ku12].

3.1.2. Interaction potential for two distinguishable fermions

In the presence of a confining potential we are not primarily interested in the cross section or the phase shift which particles acquire far away from the scattering center. We are interested in an general expression for the interaction potential between two particles in the regime of the de Broglie wavelength much larger than the effective range of the interaction potential. Although the exact interacting potential might have a complicated form we do not resolve its fine structure and the physical behavior of the two particles is solely determined by the scattering amplitude (equation 3.7). Any potential whose effective range is smaller than the de Broglie wavelength and which reproduces the scattering amplitude would recover the scattering properties correctly. As described in [Blo08] equation 3.7 is the correct scattering amplitude for the pseudo potential

$$V_i(\mathbf{r}) = \frac{4\pi\hbar^2 a_{3D}}{m} \delta(\mathbf{r})_{\text{reg}} \quad (3.14)$$

with the regularized δ -function¹ $\delta(\mathbf{r})_{\text{reg}} = \delta(\mathbf{r}) \frac{\partial}{\partial r} r$ and $\mathbf{r}$ the relative distance between the two particles. Hence, the interaction potential of two ultracold distinguishable fermions is only determined by the scattering length $a_{3\text{D}}$. As the range of the pseudo potential is zero it represents the properties of a contact interaction. Although we have found a simple parametrization for the interaction potential it is constant as long as the scattering length is fixed. To be able to tune the interaction strength a possibility to tune the scattering length is required.

3.1.3. Tuning the scattering length

To understand how one can tune the scattering length let us assume a box potential of depth U and extension b for the scattering potential. We assume b to be much smaller than the de Broglie wavelength of the scattering particles and thus the scattering process can be described by the above formalism. J. Dalibard [Dal98] has investigated how the scattering length depends on the depth U of this potential. For a box potential with a depth/extend ratio too low to support any bound state near the continuum the resulting scattering length is small and negative. If one has the possibility to tune the depth U of the potential one may increase the depth of the potential. Then the absolute value of the negative scattering length gets larger. When the depth gets as large that it can just support a bound state at the continuum threshold, the phase shift is exactly $\pi/2$ and the scattering length diverges. By further increasing the depth of the potential the phase shifts continuously gets larger whereas the sign of the scattering length has changed due to the dependence of $a_{3\text{D}}$ on $\tan(\delta_0)$. Continuing increasing the depth reduces the absolute value of the positive scattering length until it crosses zero. Then it becomes negative and the initial condition recurs.

With this box-shaped toy potential we have shown that by tuning the depth of the potential we can tune the s-wave scattering length $a_{3\text{D}}$. However, in the case of the ultracold atoms we cannot easily tune the depth of the van-der-Waals interaction potential. Yet, the same effect is achieved when a bound state close to its continuum threshold is tuned with respect to the continuum. When a quasi bound state is close to the continuum, but still bound, the scattering length is large and positive. It becomes resonant when the state is right at the continuum threshold. When it has just become unbound in the potential the scattering length is large and negative. In the following we will describe a possibility with which we can indirectly tune a bound state with respect to the continuum threshold. Due to a coupling to this state the scattering length is tuned.

¹To model the 3D contact interaction potential the regularized δ -function $\delta(\mathbf{r})_{\text{reg}} = \delta(\mathbf{r}) \frac{\partial}{\partial r} r$ has to be used in order to ensure that the Hamiltonian is self-adjoint [Bus98].

3.1.4. Tuning the interaction strength via a Feshbach resonance ²

The difference of a Feshbach resonance to the resonance previously described for the box potential is, that the scattering potential does not have to provide a bound state near the collision energy. Instead a bound state of a different collision channel is resonantly coupled to the scattering channel. For a pictorial explanation of the

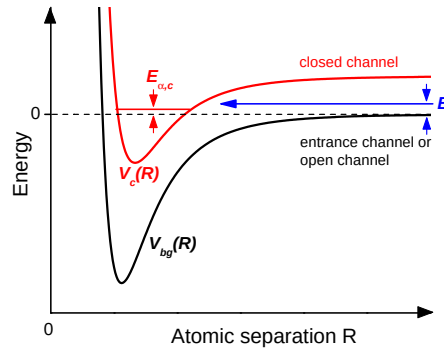


Figure 3.2.: **Illustration of the Feshbach resonance.** The atoms enter the collision region in the open channel with kinetic energy E . If there is a close channel bound state near the entrance energy, the open channel couples to the closed channel. The outgoing scattering wave experiences a phase shift which leads to a divergence of the scattering length at resonance. By controlling the energy difference $E - E_{\alpha,c}$, one can tune the coupling of the states and so the scattering length. Plot taken from [Chi10].

Feshbach resonance we define the channel energy $E_{\alpha,c}$ as the internal energy of the two separated atoms following the more detailed description of [Chi10]. A channel is called an open channel if the total energy $E_{\text{tot}} = E_{\alpha,c} + E$, with E the kinetic energy, is equal or larger then the total energy of the initial collision state; it is called a closed channel if the energy is lower and the atoms are not able to separate to free atoms. A Feshbach resonance occurs when the energy of a bound state crosses the collision energy of the open channel. The situation is illustrated in figure 3.2. By coupling of the entrance channel to the closed channel the atoms can be virtually in the bound state before they separate. By that, they pick up a

²taken from [Zür09] and adapted

phase shift which leads to a resonant scattering length

$$a_{\text{res}} \propto \frac{1}{E_{\text{tot}} - E_{\alpha,c}} \quad (3.15)$$

which is inverse proportional to the energy difference of these states. Again the scattering length can adopt values from $-\infty$ to $+\infty$ dependent on the sign of the energy difference.

In a magnetic Feshbach resonance this difference can be tuned by the magnetic field. If there is a difference in magnetic moment of the closed channel and the open channel,

$$\delta\mu = \mu_{\text{atoms}} - \mu_c \quad (3.16)$$

the energy difference tunes with the magnetic offset field. The resulting effective scattering length a_{3D} depends on the background scattering length a_{bg} and on the width of the resonance Δ_B . It is then given by

$$a_{3D}(B) = a_{\text{bg}} \left(1 - \frac{\Delta_B}{B - B_0} \right). \quad (3.17)$$

with B_0 the position of the resonance.

3.1.5. Feshbach resonances in ^6Li

The ultracold atoms of our choice are ^6Li atoms. ^6Li consist of three electrons with one of them in the valence shell which is the reason for the hydrogen-like structure of its electronic levels. The total spin of the atom is a combination of the nuclear spin $\mathbf{I} = 1$ and the electron spin $\mathbf{s} = 1/2$. Due to its odd-half integer spin ^6Li is a fermionic species. For zero angular momentum ($\mathbf{l} = 0$) the electron spin $\mathbf{s}$ of the valence electron and the nuclear spin $\mathbf{I}$ couple to two hyperfine states, where the lowest states split up in a Zeeman doublet for low magnetic fields (see appendix figure A.1). In the Paschen-Back regime for larger magnetic offset fields, the electron spin tends to decouple from the nuclear spin and is aligned in the external field. In this regime the nuclear spin is aligned separately, with three different possible projections to the quantization axis. We label these states as follows:

state	Paschen Back regime high field	Zeeman regime low field
$ 1\rangle$	$ m_s = -1/2, m_I = 1\rangle$	$ F = 1/2, m_F = 1/2\rangle$
$ 2\rangle$	$ m_s = -1/2, m_I = 0\rangle$	$ F = 1/2, m_F = -1/2\rangle$
$ 3\rangle$	$ m_s = -1/2, m_I = -1\rangle$	$ F = 3/2, m_F = -3/2\rangle$

3. *Tuning interaction and few particles with contact interaction in a 1D harmonic trap*

These states have the advantage that they cannot undergo inelastic spin exchange collisions due to the non existence of energetically reachable final states. Hence, a two-component combination of any of these states is suitable for experiments with interacting few-fermion systems.

In the high magnetic field region the electronic two-particle state has to be a triplet state because of the conservation of the spin projection quantum number ($m_S = -1$). In ^6Li the triplet potential provides a quasi-bound state close above the continuum, from which the large value of the background scattering length of about -2000 Bohr radii (a_0) arises. For lower magnetic fields the interaction potential is a linear combination of triplet and singlet potential where close to zero field the singlet potential is dominant. It has a background scattering length which is close to zero. Between 600 G and 1000 G a Feshbach resonance exist in each of the three different combination of hyperfine states (see figure 3.3). The most commonly used spin combination for our experiments is the $|1\rangle$ - $|2\rangle$ spin mixture which we create by optical pumping into the $|F = 1/2\rangle$ before the transfer of particles from the MOT to the large volume dipole trap. In this combination also the differential magnetic moment with respect to the magnetic field is most similar which is preferred in experiments which involve a magnetic potential. Unfortunately, due to the increase of the background scattering length from almost zero to $-2000 a_0$ for magnetic fields ranging from 0 to 1500 G not all values of the scattering length are accessible. This gap is largest in the $|1\rangle$ - $|2\rangle$ spin combination which is why we occasionally draw on the other spin-combinations with the $|1\rangle$ - $|3\rangle$ combination possessing the smallest gap.

All values of the scattering length are shown in figure 3.3 and can be accessed by tuning the magnetic offset field.

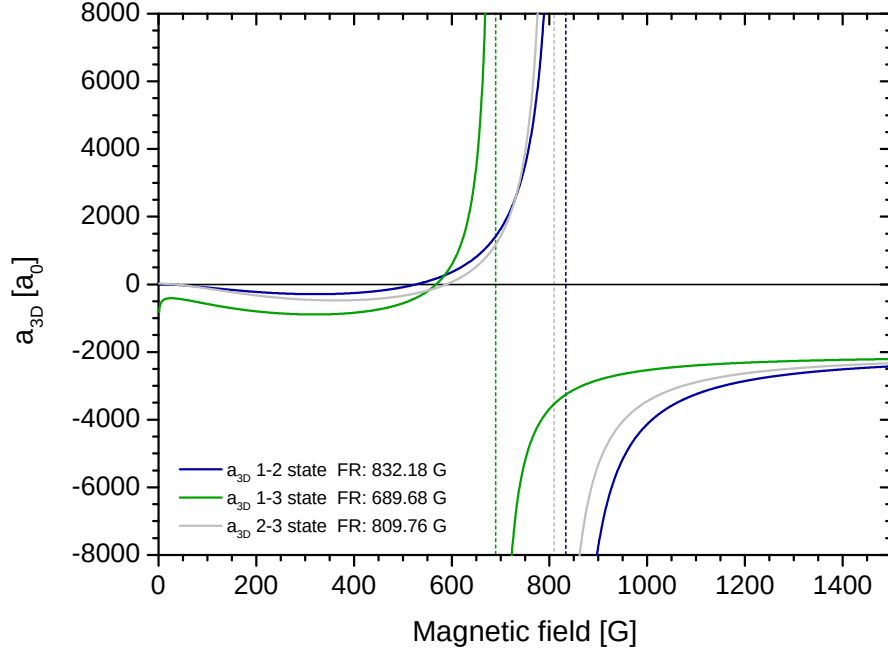


Figure 3.3.: **s-wave scattering length a_{3D} of ${}^6\text{Li}$** . The plot shows a_{3D} as a function of the magnetic offset field for different combinations of hyperfine states. The units of a_{3D} are given in Bohr radii (a_0). For low magnetic field values the scattering length is singlet dominated and thus almost zero. In most of the experiments presented in this thesis we use the $|1\rangle$ - $|2\rangle$ combination of hyperfine states which exhibits a nice broad Feshbach resonance at 832 G with a width of $\Delta_B \approx 300$ G. By tuning the magnetic offset field to the zero-crossing at 527 G we can switch off the interparticle interaction. For large magnetic field values the scattering length converges to the triplet background scattering length of about $-2000 a_0$. The position of the Feshbach resonance has been determined from precise rf-spectroscopy of weakly bound molecules associated with the Feshbach resonance (see chapter 6.4).

3.2. Few interacting particles in a harmonic trap

The presence of a confining potential changes the interaction properties of two particles compared to free space. The trapping potential leads to a discrete energy spectrum of the interacting particles. These energy spectra and the corresponding wavefunctions are of special interest for us to understand and interpret our experiments with interacting few-fermion systems.

In this section we present the solution for the energy of two and three particles in a harmonic trap. For two particles with contact interaction there is an analytic solution for the energy and the wavefunction both in a 1D trap and in a spherical 3D trap [Bus98]. However, in a real experiment a perfect 1D environment does not exist. Nevertheless, it is possible to enter a quasi-1D regime where the confinement in two axis is much tighter than in the other axis. The tight axis is characterized by the harmonic oscillator trap frequency $\omega_{\perp}$ and the weak axis by $\omega_{\parallel}$. In the case $\omega_{\perp} > \omega_{\parallel}$ the properties of the system can be described by a 1D solution with rescaled coupling constant which can be determined from the 3D scattering length [Ols98].

Here, we will first discuss the 3D solution of two particles in a cigar shaped harmonic trap with arbitrary aspect ratio $\eta = \omega_{\perp}/\omega_{\parallel}$ derived by I. Idziaszek et al. [Idz05, Idz06]. Then we consider the case $\eta \rightarrow \infty$ of this solution. We will see that it will recover the 1D solution of two particles in a harmonic trap derived by T. Busch et al. [Bus98]. The used translation from the 3D coupling constant to the 1D coupling constant then reproduces the condition for a confinement induced resonance [Ber03]. In the second part of this section we present the solution for a three-particle system calculated by the group of D. Blume [Gha12]. Finally we briefly discuss some methods to describe systems with even larger particle numbers.

3.2.1. Analytic solution for two particles in a 3D cigar shaped trap

Although we are primarily interested in the properties of a 1D system of two particles in a harmonic trap as sketched in figure 3.4, we first consider a system of two particles in radial symmetric 3D cigar shaped harmonic trap as we use such a trap in our experiment. We present the ansatz and the solution for arbitrary aspect ratio following [Idz06]. Then we set the aspect ratio to that of our experiment and compare the result to the 1D solution. Finally we present the regimes in which the 1D theory is applicable.

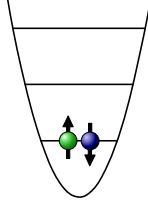


Figure 3.4.: Two-particles in a 1D harmonic trap.

Description of the two-particle system in a 3D cigar shaped trap

The Hamiltonian of the system is given by:

$$H = -\frac{\hbar^2}{2m}\nabla_1^2 - \frac{\hbar^2}{2m}\nabla_2^2 + V_t(\mathbf{r}_1) + V_t(\mathbf{r}_2) + V_i(\mathbf{r}_1 - \mathbf{r}_2) \quad (3.18)$$

with $\mathbf{r}_i$ the position of the two particles, V_t the trapping potential and V_i the interaction potential. The trapping potential is given by the harmonic oscillator potential

$$V_t(\mathbf{r}_i) = \frac{1}{2}m(\omega_\perp^2 \rho_i^2 + \omega_\parallel^2 z_i^2) \quad (3.19)$$

where $\omega_\perp$ and $\omega_\parallel$ are the trap frequencies in radial and axial direction with $\rho_i^2 = x_i^2 + y_i^2$. The interparticle interaction is determined by the s-wave scattering length a_{3D} which describes the interaction properties of ultracold atoms and is given by equation (3.14):

$$V_i(\mathbf{r}_1 - \mathbf{r}_2) = \frac{4\pi\hbar^2 a_{3D}}{m} \delta(\mathbf{r}_1 - \mathbf{r}_2)_{\text{reg}}. \quad (3.20)$$

As the interaction potential only acts on the relative distance between the two particles one can separate the system into a center-of-mass- and a relative-motion term with the corresponding coordinates $\mathbf{R} = \mathbf{r}_1 + \mathbf{r}_2$ and $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$. Then the two commuting Hamiltonians in center-of-mass and relative coordinates read

$$H_{\text{COM}} = -\frac{\hbar^2}{2M}\nabla_{\mathbf{R}}^2 + \frac{M}{m} V_t(\mathbf{R}) \quad (3.21)$$

$$H_{\text{REL}} = -\frac{\hbar^2}{2\mu}\nabla_{\mathbf{r}}^2 + \frac{\mu}{m} V_t(\mathbf{r}) + V_i(\mathbf{r}) \quad (3.22)$$

with $\mu = m/2$ the reduced mass and $M = 2m$ the total mass. The center-of-mass term does not contain any interaction term and the solution is just that of a single particle with mass M in a harmonic oscillator. To simplify the representation of

3. Tuning interaction and few particles with contact interaction in a 1D harmonic trap

the analysis we use dimensionless units by expressing all length scales in units of the harmonic oscillator length of the weak axis $a_{\parallel} = \sqrt{\frac{\hbar}{\mu\omega_{\parallel}}}$ and the energy in units of $\hbar\omega_{\parallel}$. To find the eigenfunctions of the relative-motion Hamiltonian one has to solve the stationary Schrödinger equation

$$\left(-\frac{1}{2}\nabla_r^2 + \frac{1}{2}(\eta^2\rho^2 + z^2) + 2\pi a_{3D}\delta(r)_{\text{reg}}\right)\Psi(\mathbf{r}) = E\Psi(\mathbf{r}) \quad (3.23)$$

where $\eta = \omega_{\perp}/\omega_{\parallel}$ is the aspect ratio. To find the solution one can expand the wavefunctions $\Psi(\mathbf{r})$ into the complete set of the well known eigenfunctions of a harmonic oscillator

$$\Psi(\mathbf{r}) = \sum_{n,k} c_{n,k} \Phi_{n,0}(\rho, \phi) \Theta_k(z) \quad (3.24)$$

with the decomposition coefficients $c_{n,k}$. Due to the cylindrical symmetry the eigenstates are given in polar coordinates (ρ, ϕ) with $\Phi_{n,m}$ the wavefunctions of the two dimensional harmonic oscillator with radial and angular quantum numbers n and m . $\Theta_k(z)$ is the one dimensional h.o. wavefunction with quantum number k . Only states with quantum number $m = 0$ contribute to the sum of the decomposition since all states with higher angular momentum vanish at $r = 0$, the only point where the interaction term is non-zero. By substituting the decomposition into the Schrödinger equation one can derive an equation for the coefficients of the decomposition. Solving for the coefficients is a rather complex analysis which is why we refer to the work of [Idz06] for the algebra. The result of this analysis is an implicit equation for the energy E which relates the scattering length a_{3D} to the energy eigenstates

$$-\frac{1}{a_{3D}} = \frac{1}{\sqrt{\pi}} \mathcal{F}(-\mathcal{E}/2). \quad (3.25)$$

with $\mathcal{E} = E - E_0$, where E is the total energy of the relative motion and E_0 the energy of the zero point motion. The integral representation of $\mathcal{F}(x)$ is given by

$$\mathcal{F}(x) = \int_0^{\infty} dt \left(\frac{\eta e^{-xt}}{\sqrt{1-e^{-t}}(1-e^{-\eta t})} - \frac{1}{t^{3/2}} \right) \quad (3.26)$$

which is valid only for $x > 0$, i.e for interaction energies smaller than 0. Thus, equation (3.26) only describes the energy of the so-called attractive branch³. A general solution, valid for all values of x , is given by the following series representation

$$\mathcal{F}(x) = \frac{\eta}{2\pi} \sum_{n=0}^{\infty} \left(\frac{\Gamma(x+n\eta)}{\Gamma(\frac{1}{2}+x+n\eta)} - \frac{1}{\sqrt{\eta}\sqrt{n+1}} \right) + \frac{\sqrt{\eta}}{2\pi} \zeta\left(\frac{1}{2}\right) \quad (3.27)$$

³for the notation of the branches see figure 3.5

with $\Gamma(x)$ the Euler gamma function and $\zeta\left(\frac{1}{2}\right)$ the Riemann zeta function [Abr72]. Yet, this exact condition for the energy of the system cannot be solved analytically.

Approximated analytic solution for large aspect ratio

For some regimes the solution can be simplified by certain approximations. We are primarily interested in the solution for $\eta \gg 1$. For $x \sim \eta$ and $\eta \gg 1$ the integral in equation (3.26) can be approximated and analytically integrated. Then one obtains the analytic expression of $\mathcal{F}(x)$ for all values of x

$$\mathcal{F}(x) \approx \sqrt{\pi\eta} \zeta_H\left(\frac{1}{2}, 1 + \frac{x}{\eta}\right) + \eta\sqrt{\pi} \frac{\Gamma(x)}{\Gamma(x + \frac{1}{2})} \quad (3.28)$$

with $\zeta_H(s, a) = \sum_{k=0}^{\infty} (k+a)^{-s}$ the Hurwitz zeta function. This expression substituted in equation (3.25) determines the energy eigenstates of the system. I. Idziaszek and T. Calarco have compared the latter approximated analytical solution with a numerical calculation of the the exact result. They have found quite accurate agreement for $x > -\eta$. Thus, as long as we are only interested in the weakly bound attractive state and the first or second repulsive branch we can use expression (3.25) and (3.28) to determine the energy of the system. The blue curve in figure 3.5 shows the calculated energy for an aspect ratio of $\eta = 10$ as it is the case in our experiment. We have labeled the state with $\mathcal{E} < 0$ the 'attractive branch' and the state with $0 < \mathcal{E} < 2$ the 'repulsive branch'. The energy is plotted versus the 1D coupling constant which is introduced in the next section to be able to compare it with the solution of the 1D approach.

3.2.2. Two particles in a 1D system

We want to compare the result for the 3D cigar shaped trap with the analytic solution of a pure 1D system. The relative motion Hamiltonian of this system is given by

$$H = -\frac{\hbar^2}{2\mu} \frac{\partial}{\partial r} + \frac{1}{2} \mu \omega_{\parallel} r^2 + g_{1D} \delta(r) \quad (3.29)$$

with $r = z_1 - z_2$ the relative distance between the two particles. Here g_{1D} is the coupling constant of the contact interaction in 1D⁴. For this 1D case the energy of the states has been derived by T. Busch et al. [Bus98] and is given by the implicit

⁴In 1D the regularization of the δ -potential is not necessary.

3. Tuning interaction and few particles with contact interaction in a 1D harmonic trap

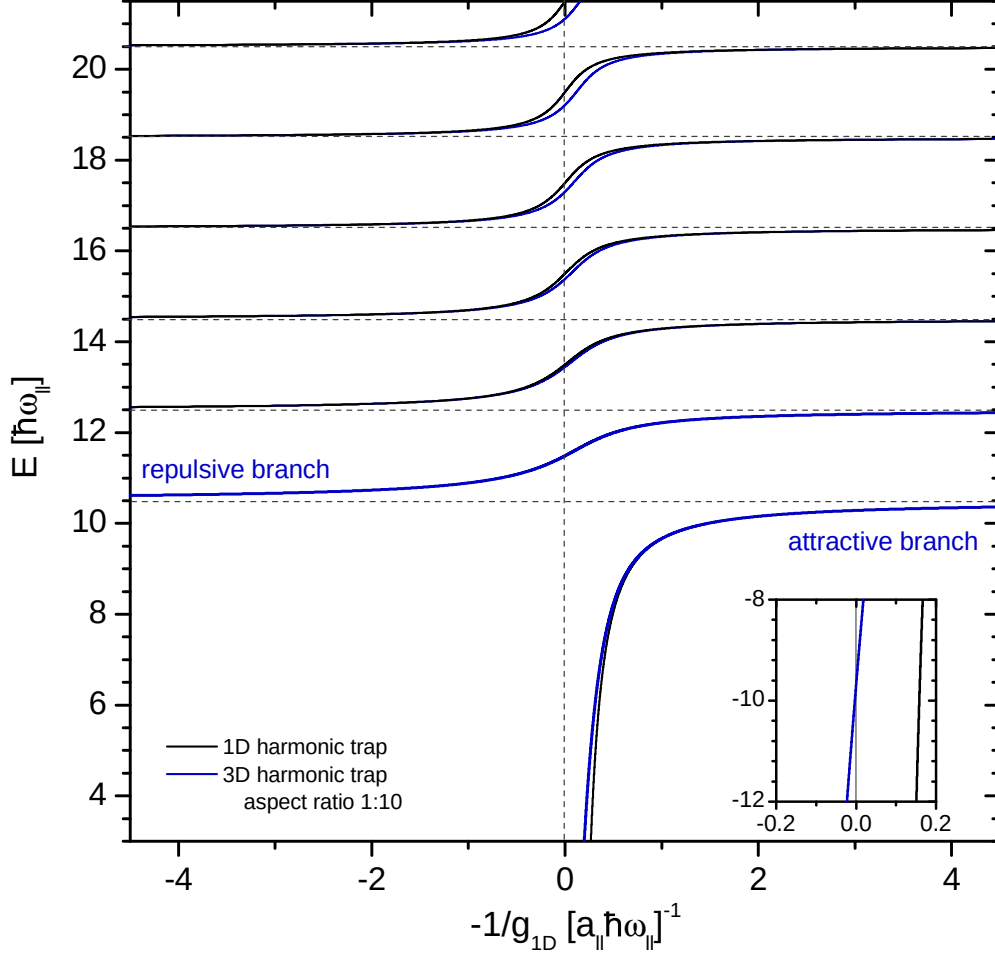


Figure 3.5.: **Energy of two-particles in a harmonic trap.** The energy is given in terms of the negative inverse scattering length $-1/g_{1D}$ where g_{1D} is determined from a_{3D} using equation (3.37). The negative inverse scale is chosen to find the resonance-position of the 1D coupling constant in the center of the plot and the attractive branch of the 1D system on the left hand side of the plot. The solution of the 1D approach is given by the black curve and the solution of the 3D approach with a cigar shaped trap with an aspect ratio of $\eta = \omega_{\perp}/\omega_{\parallel} = 10$ is given by the blue curve. The energy of the ground state is $\frac{1}{2}(2\eta + 1)\hbar\omega_{\parallel} = 10.5\hbar\omega_{\parallel}$. We denote the state with $E < 10.5$ the attractive branch and the state with $10.5 < E < 12.5$ the repulsive branch. We find excellent agreement between the repulsive branches of the 1D and 3D theory. Hence, for experiments involving the repulsive branch we can apply the 1D theory. For experiments involving the attractive branch and large binding energies one has to apply the 3D theory because the energy of the 1D solution approaches $-\infty$ for $g_{1D} \rightarrow -\infty$ whereas the attractive state in the 3D theory crosses the resonance position which is depicted in the inset of this figure.

equation

$$-\frac{1}{g_{1D}} = \frac{\mu}{2\hbar^2} \frac{\Gamma\left(-\frac{\varepsilon}{2}\right)}{\Gamma\left(-\frac{\varepsilon}{2} + \frac{1}{2}\right)} \quad (3.30)$$

The black curve in figure 3.5 represents the energy of the pure 1D system. Generally the energy of the system tunes with $\frac{dE}{dg_{1D}} > 0$.

Interpretation of the results: The repulsive branch.

For the repulsive branch we find very good agreement of the 1D solution compared the 3D solution in a cigar shaped trap with 1:10 aspect ratio. The relative deviation between these two states is less than 1% and is maximal where the coupling constant diverges ⁵. Thus, the physical behavior of two particles on the repulsive branch in a 3D trap with sufficiently large aspect ratio can be described by the 1D theory. The comparison has shown that a 1:10 aspect ratio already fulfills this criterion. Thus, when performing experiments with particles on the repulsive branch we can apply the 1D theory. For the 1D case there is also an analytic solution for the wavefunctions of two particles in a harmonic trap [Gir10]:

$$\Psi(r) = D_{\mathcal{E}}(r) \quad (3.31)$$

with $\mathcal{E}$ the solution of the transcendental equation (3.30) and $D_{\mathcal{E}}$ the parabolic cylinder functions [Abr72]. Figure 3.6 shows the wavefunctions of the relative motion part of the repulsive state depending on the 1D coupling strength. The total wavefunction is the product of the relative motion wavefunction and the center-of-mass wavefunction. The latter is simply given by the well known Hermite polynomials solving for the eigenstates of the harmonic oscillator. For $g_{1D} = 0$ also the relative motion wavefunctions is given by the Gaussian form of the harmonic oscillator ground state. For increasing g_{1D} the particles tend to avoid each other which is expressed by a cusp in the wavefunction appearing at relative distance $r = 0$. For infinitely strong repulsion ⁶ the probability that the particles are at the same point in space vanish. Across the point of diverging g_{1D} the wavefunction continuously changes and acquires probability amplitude at $r = 0$ with two nodal points next to the origin. The two nodal points indicate that the energy of the state has become larger than $1\hbar\omega_{\parallel}$. We call this state the super-repulsive state,

⁵Note: for higher excitations of the repulsive state the deviation between 1D and 3D theory becomes larger.

⁶With infinitely strong repulsion we denote the point where g_{1D} diverges. Except for the divergence of the coupling strength at the resonance position all other physical parameters remain finite and change smoothly across the resonance such as the shape of the wavefunction (figure 3.6.) and the energy of the system (figure 3.5).

3. Tuning interaction and few particles with contact interaction in a 1D harmonic trap

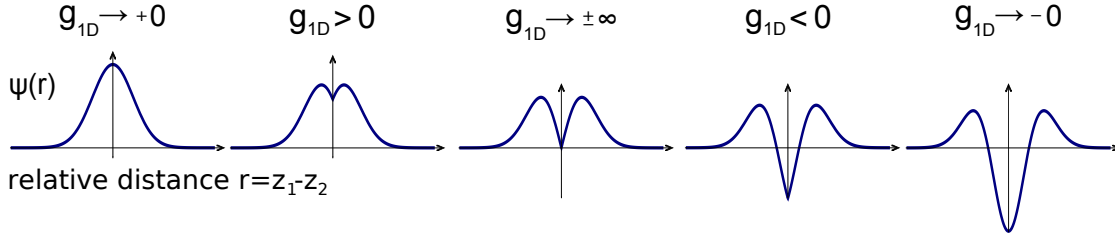


Figure 3.6.: **Wavefunction of the relative motion of two interacting particles in a 1D harmonic trap.** The wavefunctions from the left to the right corresponds to the energy of the repulsive branch in figure 3.5 for different g_{1D} ranging from $g_{1D} = +0$ to $g_{1D} = -0$. Due to the contact interaction the wavefunction develops a point of non-differentiability at the origin. At diverging g_{1D} the probability to find both particles at the same point in space ($r = 0$) vanishes.

because it is the continuation of the repulsive branch across the point of diverging coupling constant with g_{1D} negative ⁷. The region of $g_{1D} < 0$ is often called the attractive side. We do not use this notation because its misleading: It suggests that the particles are attractively interacting in the sense that the particles form a bound state. This is only true for the attractive branch. Above this branch the super-repulsive state exist which is not a bound state. In the context of a large sample of particles this state is referred to as a meta-stable state [Hal09] because it is energetically higher than the ground-state and can relax to the ground state in the presence of a coupling mechanism which is for example introduced by an anharmonicity of the trap (to be discussed in chapter 4.5). However, in a perfect harmonic potential the attractive branch and the repulsive branch are orthogonal since both are eigenstates and thus these states are stable. We will show in our experiments that we are able to access the super-repulsive state even with more than two particles without significant loss.

The attractive branch

Although we find excellent agreement of the 1D solution and the 3D solution with 1:10 aspect ratio for $0 < \mathcal{E} < 2$ the attractive branches in the models deviate

⁷One might think that the negative potential energy contribution from the delta function at $r = 0$ would lower the energy of the $g_{1D} < 0$ solution relative to that of $g_{1D} \rightarrow +\infty$, but that argument is specious, because there is also a positive kinetic energy contribution due to the cusp induced by the delta function; by the Schrödinger equation satisfied by $D_{\mathcal{E}}$, the delta function term $g_{1D}\delta(r)D_{\mathcal{E}}$ and the kinetic energy term sum to $(\mathcal{E} + \frac{1}{2})[\hbar\omega_{\parallel}]D_{\mathcal{E}}$. Taken from [Gir10] and adapted.

3.2. Few interacting particles in a harmonic trap

drastically from each other. In the 3D approach of [Idz06] the energy of the bound state expressed in physical units reads

$$-\frac{a_{\perp}}{a_{3D}} = \zeta_H \left(\frac{1}{2}, \frac{E_0 - E}{2\hbar\omega_{\perp}} \right) \quad (3.32)$$

and is determined solely by the trap frequency of the radial confinement $\omega_{\perp}$ which in the 1D approach is not present. For low binding energies in the case of $-1/g_{1D} > 1$ the description of the 1D approach is still sufficient. Yet, for $-1/g_{1D} < 1$ the deviations increase drastically. At the point of diverging coupling strength both solutions are fundamentally different. In the 1D approach the energy of the bound state diverges to $E \rightarrow -\infty$ for $g_{1D} \rightarrow -\infty$. This means that there exists no bound state in a 1D system for positive g_{1D} . Contrary, the energy of the bound state in the 3D approach crosses the point of diverging coupling strength at $E_B \sim -2\eta$ (see inset figure 3.5). The reason is that for large binding energies, $E_b \gg \hbar\omega_{\perp}$, the extension of the bound state becomes much smaller than the size of the confinement. In this case the effect of the confining trap on the bound state must vanish. Then the energy converges to the energy of a universal bound state which is always associated with this type of interaction potential (equation 3.20). The binding energy of this universal bound state in free space is given by ([Lan87] after [Blo08])

$$E_b = -\frac{\hbar^2}{\mu a_{3D}^2} \quad (3.33)$$

for $a_{3D} > 0$. The corresponding wavefunction exhibits an exponential decay in its asymptotic behavior [Chi05]:

$$\Psi(r) \propto e^{-|r|/a_{3D}}, \quad (3.34)$$

with the extension of the wavefunction, i.e. the size of the molecule solely determined by the scattering length. In free space without a confining potential the universal bound state becomes unbound when the scattering length diverges and there exist no bound state for negative values of the 3D scattering length. The comparison between the 3D approach with confinement and the universal theory of a bound state in free space is shown in figure 6.9 in chapter 6. The effect of the confinement on the bound state for binding energies on the order of $\hbar\omega_{\perp}$ has to be considered for the precise determination of the scattering length from the measurement of the binding energies of a two-particle bound state (to be discussed in chapter 6.4). However, in this regime, the condition of $E_b > -\eta [\hbar\omega_{\parallel}]$ necessary for deriving the analytic approximation of equation (3.28) is not fulfilled. For this case we numerically solve the integral in the exact expression (3.26) for deriving the correct energy of the bound state.

3.2.3. Confinement induced resonance and 1D coupling constant

We have shown that the energy of the repulsive state of the 3D approach for sufficiently large η is very similar to that of the 1D solution of [Bus98]. In the limit $\eta \rightarrow \infty$ the energies of both approaches must match [Ber03] and thus a mapping of the 3D scattering length onto the 1D coupling constant should exist.

In the case $\eta \gg 1$ the implicit equation for deriving the energy is given by substituting $\mathcal{F}(x)$ in equation (3.25) by (3.28):

$$-\frac{1}{a_{3D}} = \sqrt{\eta} \zeta_H \left(\frac{1}{2}, 1 - \frac{\mathcal{E}}{2\eta} \right) + \eta \frac{\Gamma(-\frac{\mathcal{E}}{2})}{\Gamma(-\frac{\mathcal{E}}{2} + \frac{1}{2})}. \quad (3.35)$$

For the energies $E \ll \eta [\hbar\omega_{\parallel}]$ one can neglect the dependence on the energy in the first term on the right hand side. Then $\zeta_H(\frac{1}{2}, 1 - \frac{\mathcal{E}}{2\eta}) \approx \zeta(\frac{1}{2})$ [Idz06]. Comparing this relation to the relation (3.30) solving for the 1D system one finds that

$$\frac{1}{\eta} \left(\frac{1}{a_{3D}/a_{\parallel}} + \sqrt{\eta} \zeta \left(\frac{1}{2} \right) \right) = \frac{2\hbar^2}{\mu} \frac{1}{g_{1D}} / a_{\parallel} \quad (3.36)$$

must be fulfilled when both approaches reproduce the same state. Here we expressed the length scales in physical units by expressing the scattering length and the 1D coupling constant in units of the harmonic oscillator length $a_{\parallel}$. With $a_{\parallel} = \sqrt{\eta} a_{\perp}$ we find the following relation between the 1D coupling constant and the 3D scattering length:

$$g_{1D} = \frac{2\hbar^2 a_{3D}}{\mu a_{\perp}^2} \frac{1}{1 - C a_{3D}/a_{\perp}} \quad (3.37)$$

with $C = -\zeta(\frac{1}{2}) = 1.46\dots$. This expression was derived in a similar way by T. Bergeman, M.G. Moore, and M. Olshanii for finding the condition of a confinement induced resonance (CIR) [Ber03]. When the scattering length becomes of the size of the harmonic oscillator length

$$a_{\perp} = C a_{3D} \quad (3.38)$$

the 1D coupling constant diverges. At this point the relative motion wavefunction of the repulsive branch becomes zero at $r = 0$ and the interaction energy of the system is exactly $\hbar\omega_{\parallel}$. Tuning a_{3D} by means of a Feshbach resonance effects the 1D coupling constant g_{1D} according equation (3.37). We can determine the 1D coupling constant g_{1D} of our system by using the confining harmonic oscillator

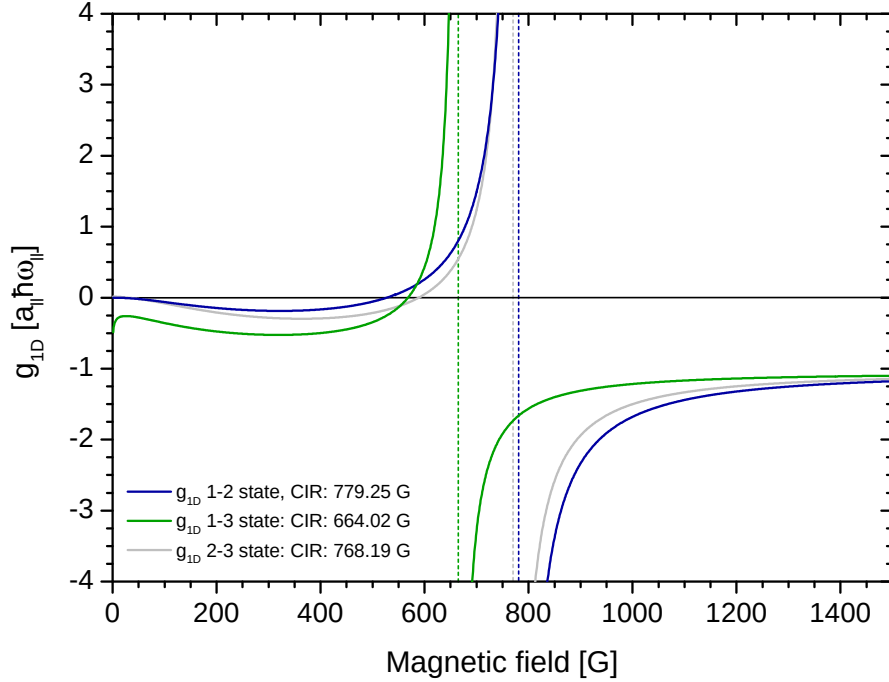


Figure 3.7.: **Confinement induced resonance (CIR) for ^6Li atoms in our microtrap.** In the presence of a cigar shaped trap the interaction properties of this quasi 1D system can be described by a 1D coupling constant g_{1D} which we determine from the scattering length of the ^6Li system using equation (3.37). For the harmonic oscillator length we have used $a_{\perp} = 0.49 \mu\text{m}$ of the microtrap at an optical trap depth of $V_{0r} = 4.12 \mu\text{K}$.

length $a_{\perp} = 0.49 \mu\text{m}$ of the microtrap potential⁸. The functional form and the resonance position are presented in figure 3.7.

In summary with the control of the magnetic offset field we have a tuning knob over the 1D coupling constant. The energy E of any 1D system which we study in this thesis tunes with $\frac{dE}{dB} > 0$.

3.2.4. The three-particle and N-particle systems

The larger the number of particles the more difficult it is to determine the energy and the wavefunction of the system as the dimension of the configuration space

⁸The harmonic oscillator length $a_{\perp}$ tunes with the depth of the microtrap. Here we used the trap parameter $\omega_r = 14.22 \text{ kHz}$ at a depth of $V_{0r} = 4.12 \mu\text{K}$ for the calculation of $a_{\perp}$. For the determination of the trap parameters see chapter 4.2.2.

exponentially increases with the number of particles:

$$H = -\frac{\hbar^2}{2m} \sum_{i=1}^N \frac{\partial^2}{\partial x_i^2} + \frac{1}{2} m \omega_{\parallel}^2 \sum_{i=1}^N x_i^2 + g_{1D} \sum_{i<j}^N \delta(x_i - x_j) \quad (3.39)$$

The three-particle sytems

The group of Dörte Blume was able to numerically calculate the energy of a three-fermion system consisting of two identical fermions and another one which is distinguishable from the others as sketched in figure 3.8 and provided us with their

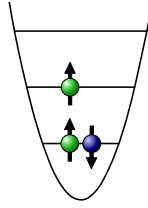


Figure 3.8.: **Three-particles in a 1D harmonic trap - odd parity.**

data. They commented the accuracy of their calculation as follows: *'Our approach builds on the work by Calarco et al. [Idz06]. In particular, we start with the Lippmann-Schwinger equation, and expand the Green's function using what's known about the two-body system for anisotropic traps. For three particles in a spherical symmetric trap the approach was used by [Kes07]. As argued by Peter Drummond and coworkers, the approach can be reinterpreted as a basis set expansion approach [Liu09]. For the anisotropic system, the same argument holds. Errors are introduced by truncating the number of states we use to expand the Green's function and the number of terms we use when evaluating some of the integrals involved by converting integrals to an infinite sums and truncating. For the low-lying states we are looking at, the numerical accuracy should be very good, the error should be much smaller than 1%'* [Gha12].

In their calculation they have distinguished between even and odd parity states of the three particle system. We are primarily interested in systems with minimum energy at $g_{1D} = 0$ due to the way we prepare the few-particle system. In general the parity of our two-component systems with $N_{|1\rangle}$ particles in one spin state and $N_{|2\rangle}$ particles in the other spin state is given by

$$(-1)^{(N_{|1\rangle}-1)+(N_{|2\rangle}-1)} \quad (3.40)$$

This is the reason why in the previous case of a two-particle system we discussed the even-parity states. In the case of a three-particle system we are interested

in the odd-parity system. From the group of Dörte Blume [Gha12] we got data sets of numerical calculations for the odd-parity states. They have performed a calculation for both the three-particle system in an anisotropic 3D cigar shaped trap with an aspect ratio of 1:10 and for a three-particle system in a 1D environment. The energy in this data set has been parametrized in terms of the inverse 3D scattering length in units of the perpendicular harmonic oscillator length $a_{\perp}$. To convert it to the inverse 1D coupling constant in units of $[a_{\parallel} \hbar \omega_{\parallel}]^{-1}$ we have

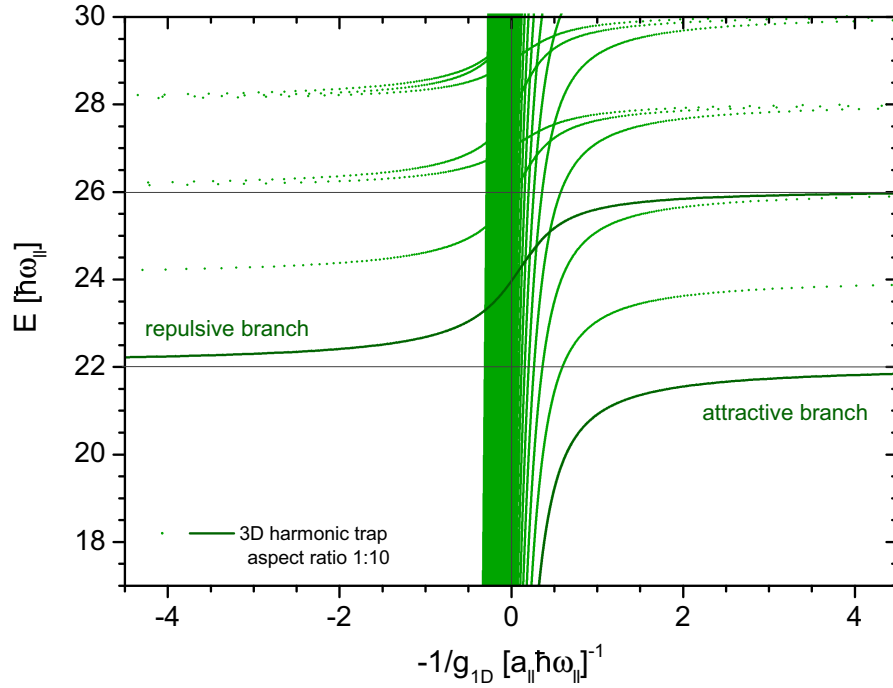


Figure 3.9.: **Energy of three particles in a 3D cigar shaped trap with an aspect ratio of $\omega_{\perp}/\omega_{\parallel} = 10$.** The energy has been calculated in the group of Dörte Blume [Gha12] and has been rescaled using equation (3.41). The energy of the ground state is $(3 \times 1/2 (2\eta + 1) + 1) \hbar \omega_{\parallel} = 22 \hbar \omega_{\parallel}$. The dense cluster of states around $1/g_{1D}=0$ contains states which include molecular states. In the 3D solution these states are present at $g_{1D} > 0$ below the position of the CIR. The upper dark green line indicates the energy of the repulsive branch which does not contain a molecular state.

used the rescaling (3.37) which in this special case takes on the form

$$-\frac{1}{g_{1D}} = - \left(\frac{2\eta}{a_{3D}^{-1} \left(1 - C \frac{\sqrt{\eta}}{a_{3D}^{-1}} \right)} \right)^{-1}. \quad (3.41)$$

3. Tuning interaction and few particles with contact interaction in a 1D harmonic trap

Figure 3.9 shows the energy of the three-particle systems for the 3D case with 1:10 aspect ratio. The spectrum contains a lot of different states which involve a molecular state of two particles. These states can be identified by tracking them towards the resonance position in g_{1D} where they diverge. We again compare the result of the 3D approach to the result of the 1D approach shown in figure 3.10. One finds a quite similar behavior as for the two-particle system. In the 1D case

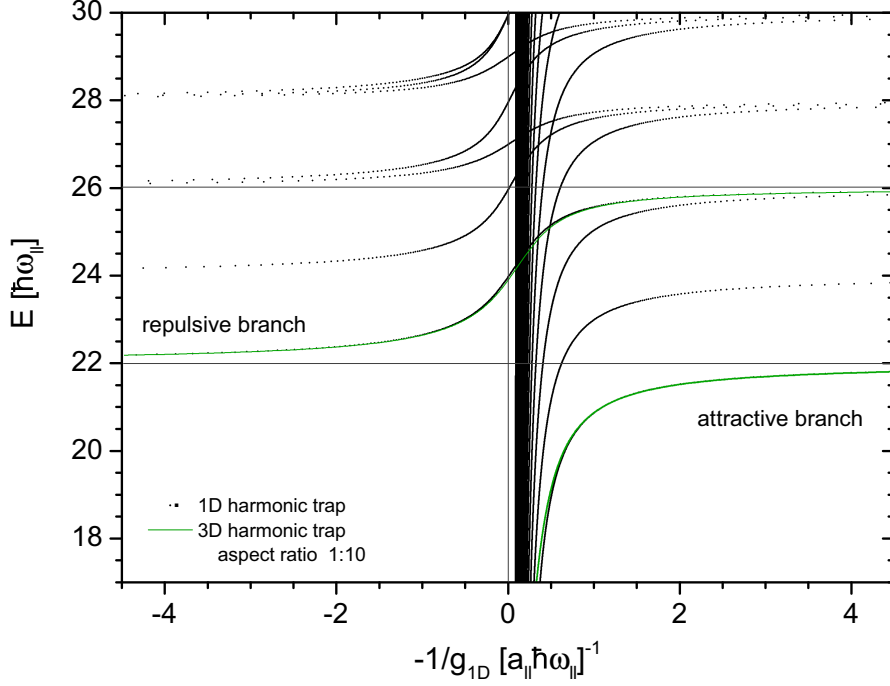


Figure 3.10.: **Energy of three particles in a 1D trap.** The energy has been calculated in the group of Dörte Blume [Gha12] and has been rescaled using equation (3.41). In the 1D solution the states which involve molecules all diverge to $-\infty$ at the CIR. The green line shows the energy of the repulsive state of the 3D approach. For the repulsive branch the 1D solution still represents the real situation in the experiment quite well. However, the deviations for large g_{1D} are already on the order of 2% which has to be taken into account when performing precise rf-spectroscopy measurements of that state.

the attractive state never crosses the g_{1D} resonance position, whereas in the 3D approach the molecular states exist on the side of positive g_{1D} .

Due to the high precision on the order of a few percent of the trap frequency we achieve in rf-spectroscopy measurements we are interested in the energy difference of the repulsive branch in the 3D approach and in the 1D approach. In figure

3.10 one can visually resolve this difference between the black data (1D approach) and the green data (3D approach). Due to the larger interaction energy involved in the three-particle system compared to the two-particle system the effect of the confining trapping potential has become larger. The difference close to the CIR is already in the 2% range. This shift has to be considered when comparing the measured energies of three particles in our microtrap with one dimensional theoretical models.

The N-particle systems

For the four-particle system the previous approach should also work [Gha12]. However, when writing this thesis the calculation has not yet been available. For larger particle numbers ($N = 4, 5, 6$) we draw on the full numerical calculations performed by Ionians Brouzos from the group of Peter Schmelcher using a multiconfigurational approach in terms of Hartree products (MCTDH) [Mey90] in a strict 1D environment.

An alternative approach for larger N developed in their group is the construction of a many-body wavefunction from multiple products of the parabolic cylinder functions (3.31) which solves the two-particle system. The ansatz for this correlated pair wavefunctions reads:

$$\Psi(x_1, \dots, x_N) = C \prod_{i < j}^P D_\mu(\beta |x_i - x_j|) \quad (3.42)$$

where $P = N(N - 1)/2$ is the number of distinct pairs and β and μ parameters which have to be determined [Bro12b]. This analytic approach was first evaluated for bosons and has been recently extended to fermions by enforcing fermionic permutation symmetry of the correlated pair wavefunctions [Bro12c]⁹. For finite g_{1D} their approximated analytic solution deviates from numerical calculations with maximum deviation around $g_{1D} = 1$. Thus, whether this model can be applied depends on the targeted precision of our experiment. Yet, the analytic approach exactly reproduces the many-body state for $g_{1D} \rightarrow 0$ and $g_{1D} \rightarrow +\infty$. The latter case is the fermionization limit [Gir10] for which an exact solution of the many-body Hamiltonian (3.39) exists. This will be the topic of the next chapter.

⁹Unfortunately both methods are constraint to the calculation of the systems ground state and thus cannot describe the super-repulsive state of the N-particle system.

4. Fermionization of two distinguishable fermions

With the possibility to deterministically prepare few fermions in the ground state of a trap (chapter 2) and with the exceptional tunability of the interparticle interaction (chapter 3) we have all ingredients to study interacting few-fermion systems. Here we start with the simplest non-trivial system: The two-particle system of two distinguishable fermions with contact interaction in a 1D harmonic trap.

For particle number larger than two we have already seen that a theoretical description is hard to establish. A powerful tool to reduce the complexity of these systems is to map it onto a system with a simpler solution as done in the work of M.D. Girardeau [Gir60]: He has shown that in a 1D environment there exists a one to one correspondence between a system of interacting bosons and a system of identical fermions. At diverging coupling strength the energy and the square modulus of the spatial wavefunctions $\Psi(x_1, \dots, x_n)$ of such an interacting system becomes equivalent to that of a system of noninteracting identical fermions. This is referred to as fermionization [Pet00b]. At the point of fermionization the local pair correlation ¹ $g^2(0)$ vanishes just like in a gas of noninteracting identical fermions. In experiments with interacting 1D Bose gases the correlations in these so-called Tonks-Girardeau gases have been studied. In the group of David S. Weiss they found strong evidence for fermionization by observing a reduction of the two-particle correlation $g^2(0)$ for increasing but finite coupling strength [Kin04] [Kin05]. The group of Hanns-Christoph Nägerl first demonstrated that the interaction regime across the point of fermionization, which is called the super-Tonks-regime, is accessible [Hal09].

In his recent work, M.D. Girardeau has shown that fermionization occurs regardless of whether the particles are identical bosons or distinguishable fermions [Gir10]. For constructing the correlated many-body wavefunction the main building block is the interacting two-particle system in a 1D harmonic oscillator with contact interaction.

¹In the case of a two particle system, the pair correlation $g^2(r)$ is proportional to the probability that one particle is found at x_1 and the other at x_2 with relative distance $r = x_1 - x_2$ [Fra03]. $g^2(0)$ expresses the probability to find the two particles at the same point in space.

In this chapter we present the experimental demonstration that a system of two distinguishable fermions becomes fermionized at diverging coupling strength. We prove this by directly comparing the energy and square modulus of the wavefunction of two distinguishable fermions at diverging coupling strength with those of two identical fermions [Zür12]. In the second section we determine the energy of the interacting two-particle system which has been studied for large but not diverging coupling strength in the group of Tilman Esslinger [Stö06] and Klaus Sengstock [Osp06]. Finally, we add another particle and find evidence for the fermionization of three particles.

4.1. The fermionization experiment

The goal of the following experiment is the observation of the fermionization of two distinguishable fermions. With our setup we have the exceptional possibility to prepare two distinguishable fermions with tunable interaction in a 1D potential with high fidelity. Their properties can be compared to two identical fermions which are prepared with the same fidelity. By modifying the shape of the potential, the particles can tunnel through a finite potential barrier and we can observe their tunneling dynamics (see figure 4.1). For the comparison of the tunneling properties of the two different systems it is essential that the systems are prepared in exactly the same potential. Then by directly comparing the total energy and the square modulus of the wavefunction of both systems we can give a proof of fermionization.

4.1.1. Two interacting distinguishable and two noninteracting identical fermions

We start with the preparation of two ^6Li atoms in two different Zeeman sublevels of the lowest ^6Li hyperfine state $|F = 1/2, m_F = 1/2\rangle$ and $|F = 1/2, m_F = -1/2\rangle$. Up to this point we used the notation $|1\rangle$ and $|2\rangle$ for these states. For the fermionization experiment presented here these two states are the only relevant hyperfine states. We can relate them to the spin-components of a spin-1/2 system and thus we label them by $|\downarrow\rangle$ and $|\uparrow\rangle$. The distinguishable system and the identical system are then denoted by $|\uparrow\downarrow\rangle$ and $|\uparrow\uparrow\rangle$ respectively.

We prepare the $|\uparrow\downarrow\rangle$ -system with a fidelity of 93(2)% in the ground state of our cigar-shaped microtrap potential. Its shape is determined by the Lorentzian profile in the axial direction of a focused Gaussian beam. A harmonic approximation yields trap frequencies of typically $\omega_{\parallel} = 2\pi \times (1.234 \pm 0.012)$ kHz along the longitudinal direction and $\omega_{\perp} = 2\pi \times (11.79 \pm 0.29)$ kHz along the more confined

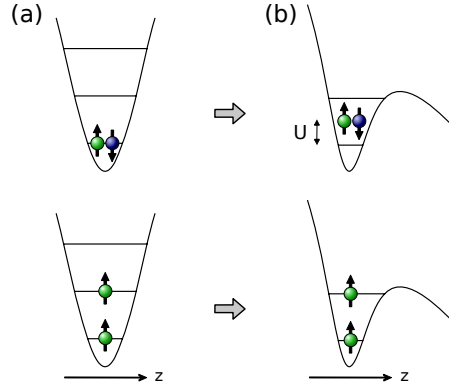


Figure 4.1.: **Sketch of the performed experiment.** a) Deterministic preparation of two fermions in the ground state of a potential well. b) We measure the tunneling dynamics through a potential barrier for a repulsively interacting system of two distinguishable fermions for various interaction energies. The mean interaction energy per particle is indicated by the parameter U . These results are then compared with the tunneling dynamics of two noninteracting identical fermions in the same potential. Taken from [Zür12].

radial direction leading to an aspect ratio of $\omega_{\parallel} : \omega_{\perp} \sim 1:10^2$. Due to the large confinement in 2 dimensions the particles are effectively restricted to 1 dimension. Comparing the 3D-solution of two interacting particles in such a cigar shaped trap to the solution of a 1D calculation for two particles interacting via a 1D-coupling constant g_{1D} , one finds that the lowest non-bound state of the 3D solution is well described by the 1D solution (see chapter 3.2.2). The deviation between both calculations is less than 10^{-2} and therefore our system can be treated in a 1D framework.

The 1D coupling strength $g_{|\uparrow\downarrow\rangle}$ between the two distinguishable fermions can be tuned by a confinement induced resonance (CIR, see chapter 3.2.3). Figure 4.4 b) shows $g_{|\uparrow\downarrow\rangle}$ of the following experiment as a function of the magnetic field with a CIR at 783.1 ± 0.5 G. The coupling constant $g_{|\uparrow\uparrow\rangle} = 0$ for all magnetic field values because s-wave interaction for identical fermions is forbidden.

For two particles in a harmonic trap that interact via contact interaction, the Hamiltonian can be separated into center-of-mass and relative motion, as the interaction term only depends on the relative distance of the two particles. Figure 4.2 shows the relative wavefunction as a function of $g_{|\uparrow\downarrow\rangle}$ and the corresponding kinetic energy of the relative motion (blue and black curves). The derivation of the

²The determination of the trap frequencies is described in section 4.2.2

4.1. The fermionization experiment

solution has originally been performed by Thomas Busch and coworkers [Bus98] and is discussed in chapter 3.2.2.

For the sake of comparison we add the energy of a system of two noninteracting identical fermions plotted in green. Such a system is created by first preparing a

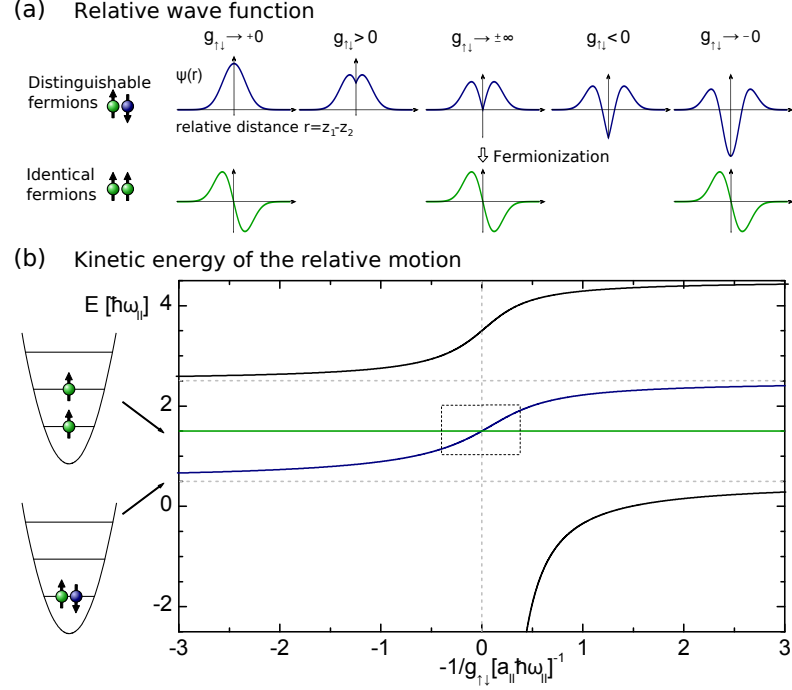


Figure 4.2.: **Two particles in a 1D harmonic potential.** a) Relative wave function of two interacting fermions (blue) and two identical fermions (green) in a 1D harmonic potential. For infinitely strong interaction ($-1/g_{\uparrow\downarrow} \rightarrow 0$) the probability to find the two distinguishable fermions at the same position vanishes. In this case the square modulus of the total wave function of two distinguishable fermions is the same as for two identical fermions. b) Kinetic energy of the relative motion. The blue and black curves show the energy of two interacting fermions in state $|\uparrow\downarrow\rangle$ depending on the coupling strength $g_{\uparrow\downarrow}$ given in units of $a_{\parallel} = \sqrt{\hbar/\mu\omega_{\parallel}}$. Taken from [Zür12]

balanced system with 4 particles and then removing all $|\downarrow\rangle$ -fermions by the method described in chapter 2.4.1. Due to the Pauli exclusion principle the two $|\uparrow\rangle$ -particles remain located at the ground and the first excited single particle levels. This is the ground state of the many-particle system with state $|\uparrow\uparrow\rangle$. We can easily write down the wavefunction of the many-particle ground state in a harmonic oscillator

by considering the antisymmetric particle exchange symmetry

$$\Psi_{|\uparrow\uparrow\rangle}(x_1, x_2) = \frac{1}{\sqrt{2}} (\Phi_0(x_1)\Phi_1(x_2) - \Phi_0(x_2)\Phi_1(x_1)) \quad (4.1)$$

where $\Phi_i(x_j)$ are the single particle eigenstates of the harmonic oscillator. Rewriting the wavefunction in center-of-mass and relative coordinates $R = x_1 + x_2$ and $r = x_1 - x_2$ yields

$$\Psi_{|\uparrow\uparrow\rangle}(R, r) = \Phi_0(R)\Phi_1(r). \quad (4.2)$$

The relative motion part of the wavefunction ³ is thus given by the first excited harmonic oscillator wavefunction (shown in figure 4.2 a) in green) and the energy is given by 3/2 in units of $\hbar\omega_{\parallel}$ (green line in figure 4.2 b).

Comparing both systems one finds that the energy and the square modulus of the wavefunction of the $|\uparrow\downarrow\rangle$ -system at diverging coupling strength are identical to those of a noninteracting $|\uparrow\uparrow\rangle$ -system. This is the point of fermionization.

4.1.2. Tunneling measurement and fermionization ⁴

To perform the fermionization experiment we prepare the initial system as described in chapter 2.3.1. At the end of the preparation process of the two different 2-particle systems we keep the magnetic field gradient at $B' = 18.92$ G/cm as we need the gradient to perform the tunneling measurement. To determine the energy of the two-particle system in state $|\uparrow\downarrow\rangle$ we modify the trapping potential such that there is a potential barrier of fixed height through which the particles can tunnel out of the trap on experimentally accessible time scales. This is done by lowering the depth of the optical trap ⁵. To tune the strength of the interaction we apply magnetic offset fields ranging from 523 G to 900 G ⁶. For starting the measurement of the interaction-induced tunneling we ramp to an optical trap depth of $p = 0.6875$ with a ramp speed of $dp/dt = 0.043$ ms⁻¹ where tunneling occurs on experimentally accessible time scales. In chapter 2.4 we have estimated

³The center-of-mass part of the wavefunction is described by the Gaussian ground state wavefunction of the harmonic oscillator. As for the system of two interacting fermions the center-of-mass term is identical to that of the system of identical fermions, it can be neglected for the further comparison of the two systems.

⁴taken from [Zür12] and adapted.

⁵Before we perform the tunneling measurement, the initial optical trap depth of $p = 1$ has been lowered to $p = 0.795$ ($\pm 1.3 \times 10^{-3}$) with 4 bound states left in the potential with tunneling times much larger than any experimental timescale. The reason for tuning the depth to an already lower value after the preparation process is given in A.3.1 in the appendix.

⁶For the ramp speed of the magnetic field see appendix A.3.2.

4.1. The fermionization experiment

the probability of exciting particles when performing a single ramp of the optical potential at this speed to be consistent with $(3 \pm 1)\%$.

In the presence of repulsive interactions the energy of the system is increased (blue curve in figure 4.2 b). This decreases the effective height of the barrier and the particles tunnel faster. We allow the particles to tunnel out of the trap for different hold times and record the number of particles remaining in the trap. By choosing an adequate barrier height we ensure that the time scale for tunneling is smaller than the lifetime of our samples in the ground state (about 60 s). Additionally obtaining meaningful tunneling time constants requires the timescale of the tunneling to be much larger than the inverse longitudinal trap frequencies of 0.7 ms. By averaging over many experimental realizations we obtain the expectation value of the particle number in the potential for different hold times (see figure 4.3). By performing this measurement for various values of the coupling strength we can determine the dependence of the system's energy on $g_{|\uparrow\downarrow\rangle}$.

We find that for the observed range of interaction energies – which are on the order of $\hbar\omega_{\parallel}$ – only one particle leaves the potential even for long hold times. In a simple picture this can be explained as follows: If one particle tunnels through the barrier the interaction energy is released as kinetic energy, which leaves the other particle in the unperturbed ground state of the potential. This state has a lower energy and thus a tunneling time scale much larger than the duration of the experiment. Thus we can fit⁷ exponentials of the form $N(t) = N_{\text{tunnel}} e^{-\frac{t}{\tau}} + N_{\text{remain}}$ to the mean particle number to deduce the tunneling time constant τ for different magnetic fields. The mean numbers of tunneled (N_{tunnel}) and remaining particles (N_{remain}) are expected to be unity. However, due to the finite preparation fidelity they are slightly lower. In figure 4.4 we show the determined tunneling time constants of a system of two interacting fermions for different interaction energies as a function of the magnetic field. We observe a decrease in the tunneling time constant over two orders of magnitude for increasing magnetic field due to the gain in interaction energy caused by the CIR.

For a direct comparison of the properties of the two interacting distinguishable fermions with those of two identical fermions we perform the same measurement with two fermions in state $|\uparrow\uparrow\rangle$ in the same potential (figure 4.1 c). The results of these reference measurements are shown in figure 4.3 and figure 4.4 (green points). As the identical fermions are noninteracting we find no dependence of the tunneling time constant on the magnetic field in this measurement.

Comparing the results of the two systems we find that the tunneling time con-

⁷The fit is performed with a χ -square minimization algorithm (Levenberg-Marquardt) considering the statistical error of the mean particle number. For all further fits presented in this thesis we use this algorithm.

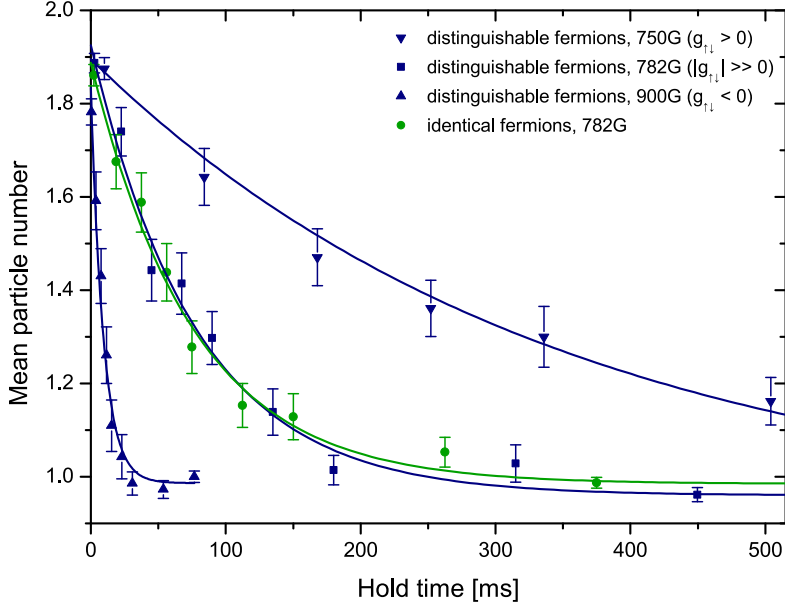


Figure 4.3.: **Mean number of particles remaining in the potential well.**

After modifying the initial potential the particles can tunnel through a barrier of fixed height for a certain hold time. Subsequently, tunneling is switched off and the mean particle number left in the potential is recorded by averaging over many experimental realizations. Exponential fits to the data (solid lines) allow to extract the tunneling time constants of two interacting distinguishable fermions for different interaction strengths (blue) and of two identical fermions (green). Each data point is the average of about 70 measurements except for the first and the last data point in each series (about 230 realizations). The errors are the standard errors of the mean. Taken from [Zür12]

stant for the interacting system decreases monotonically with increasing magnetic field and crosses the magnetic field independent tunneling time constant of the two identical fermions. Thus there is one magnetic field value where the tunneling time constants of both systems are equal. At this point both systems must have the same energy. For a 1D system with given energy there is only one unique solution for the square modulus of the wave function. Therefore, right at the observed crossing point of the tunneling time constants the energy and the square modulus of the wave function $|\psi(z_1, z_2)|^2$ of the two interacting distinguishable fermions and the two noninteracting identical fermions must be equal. Hence, exactly at this crossing point the system of two distinguishable fermions is fermionized. As predicted by theory [Bus98, Gir10] we find the position of the fermionization at

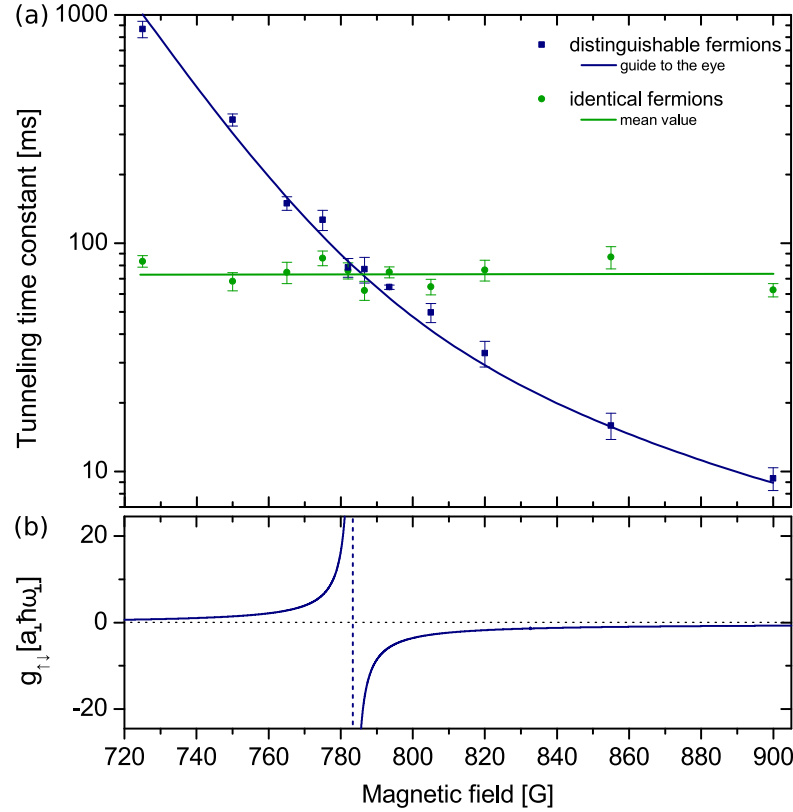


Figure 4.4.: **a) Tunneling time constants for different values of the 1D coupling strength.** The tunneling time constant of two repulsively interacting distinguishable fermions (blue curve) decreases by two orders of magnitude with increasing magnetic field. This is attributed to the gain in interaction energy when ramping across the CIR. The tunneling time constant of two noninteracting identical fermions (green line) remains unaffected by the magnetic field within our experimental accuracy. At the magnetic field value where both curves cross we identify the fermionization of two distinguishable fermions. The errors are the statistical errors of the fits shown in figure 4.3. The blue line is a guide to the eye. **b) One-dimensional coupling constant $g_{|\uparrow\downarrow\rangle}$ with a CIR at $(783.1 \pm 0.5) G$.** For the calculation we used the perpendicular harmonic oscillator length $a_{\perp} = \sqrt{\hbar/\mu\omega_{\perp}}$ of the modified potential. Taken from [Zür12]

the magnetic field value where $g_{|\uparrow\downarrow\rangle}$ diverges due to the confinement-induced resonance.

For magnetic field values below the CIR we have realized the two-particle limit of a Tonks-Girardeau gas [Gir10]. Above the CIR we have created a super-Tonks state consisting of two particles. The super-Tonks state is a strongly correlated metastable state above the attractive ground state branch (see figure 4.2 b). In a system with particle numbers ≥ 3 inelastic three-body collisions lead to a fast decay of the metastable super-Tonks-Girardeau gas [Hal09]. In contrast, our two-particle super-Tonks state is stable against collisional losses since there is no third particle available to undergo an inelastic three-body event.

4.2. Tunneling theory and determination of the potential shape

By mapping the system of two distinguishable fermions onto a system of two identical fermions we could determine its energy at diverging coupling strength at the point of fermionization without any tunneling theory. The only relevant argument has been that a measurement of two identical tunneling time constants for two different systems is equivalent to both systems having the same energy. By knowing the energy of one system, which can be easily found for two identical fermions, we know the energy of the other system, which is the great advantage of Girardeau's mapping technique [Gir10]. Yet, additionally to the point of infinitely strong repulsive interaction we want to determine the energy of the system for finite $g_{|\uparrow\downarrow\rangle}$. Therefore we need to apply some tunneling theory to determine the energy of the system from the tunneling time constants. Whichever theory one applies one needs to know the shape of the potential. This can be obtained by the modulation spectroscopy method and by a WKB approximation which will be introduced in this section.

First we describe the WKB method for the determination of the energy eigenstates of a 1D potential well and for the derivation of the tunneling times through a finite potential barrier. Then we introduce the modulation spectroscopy method which allows to determine the level structure of the potential. By knowing the shape of the potential we can then deduce the energy of an interacting two-particle system for finite coupling strength from the measured tunneling time constants of section 4.1.

4.2.1. WKB approximation

To determine the energy of a single particle with mass m in a 1D potential we have to find the stationary states of the Schrödinger equation

$$-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \Psi(x) = (E - V) \Psi(x). \quad (4.3)$$

If $V(x)$ has a complicated form a solution may be found numerically or by a perturbation theory approach. In the case that $V(x)$ is a slowly varying function of position with respect to the wavelength of the state an approximate analytical solution can be given by the WKB⁸ approximation method. We will introduce this method following the elaborate derivation described in [Mer98, Sch07].

If $V(x)$ was constant the solution would be given by plane waves. As the potential is a slowly varying function this leads to the ansatz

$$\Psi(x) = A(x) e^{iS(x)/\hbar} \quad (4.4)$$

with position depending amplitude $A(x)$ and phase $S(x)$. Inserting this ansatz into the Schrödinger equation (4.3) provides two differential equations for the real and the imaginary part. With the initial condition one can neglect the second derivative of the amplitude with respect to the second derivative of the phase [Sch07]. This leads to the solution

$$S_{\pm}(x) = \pm \int^x dx' \sqrt{2m(E - V(x'))} \quad (4.5)$$

and

$$A_{\pm}(x) = \frac{C_{\pm}}{\frac{dS}{dx}} \quad (4.6)$$

with constants $C_{\pm}$ which will later be defined through further boundary conditions. With this we have found a solution for the wavefunction where we can distinguish two cases:

For the first case, the classically allowed region, where $E > V$ the wavefunction reads

$$\Psi_p(x) = \sum_{\pm} \frac{C_{p,\pm}}{p(x)/\hbar} e^{\pm i \int^x dx' p(x')/\hbar} \quad (4.7)$$

with $p(x)$ defined as

$$p(x) = \sqrt{2m(E - V(x))}. \quad (4.8)$$

The imaginary form of $\Psi_p(x)$ shows the oscillatory behavior of the two counter-propagating wavefunctions in this classical region, with similarities to plane waves

⁸method developed by Wentzel, Kramers and Brillouin

being inherent due to the ansatz. The WKB approximation is valid if the potential changes so slowly with x that as the wave propagates no reflected (scattered) wave is generated [Mer98].

For the case $E < V$ one finds:

$$\Psi_{\kappa}(x) = \sum_{\pm} \frac{C_{\kappa,\pm}}{\kappa(x)/\hbar} e^{\mp \int^x dx' \kappa(x')/\hbar} \quad (4.9)$$

where $\kappa(x)$ is defined as

$$\kappa(x) = \sqrt{2m(V(x) - E)}. \quad (4.10)$$

$\Psi_{\kappa}(x)$ is a real function with an exponential decay or increase. This shows that the wavefunction in this non-classical region decays as the integral in equation (4.9) increases. This part of the wavefunction can be seen as an evanescent wave that penetrates into a classically forbidden region.

Now we can apply this general case of the WKB approximation to our potential. Figure 4.5 sketches the potential that we use in the experiment and consists of a Lorentzian plus a linear potential. For a given energy E one can identify three

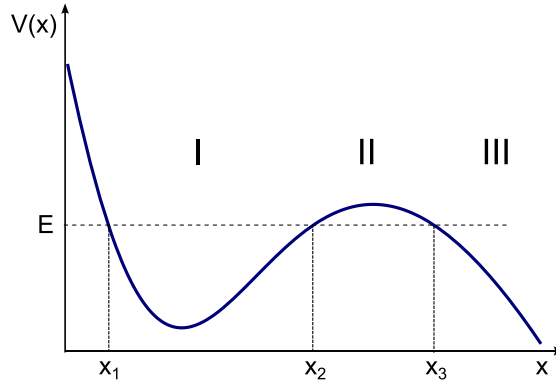


Figure 4.5.: **Classification of our microtrap potential in different regions.**

In region I the kinetic energy E is larger than the potential and extends from x_1 to x_2 which are given by the classical turning points of a classical particle with energy E . Region II is the classically forbidden region through which a particle can tunnel into the continuum in region III.

regions. In region I the kinetic energy E is larger than the potential energy and extends from x_1 to x_2 which are given by the classical turning points of a classical particle with energy E in this potential well. For $x < x_1$ $V(x)$ diverges for

$x \rightarrow -\infty$ and one expects any wavefunction in this non-classical region drop to zero for $x \rightarrow -\infty$. Region II extends from x_2 to x_3 , where x_3 is defined as the maximum of the solution $V(x) = E$. This again is a classically impenetrable region with the difference that the extension of this region is finite. We call this region the potential barrier. For a finite barrier an exponentially decaying wavefunction has not dropped to zero at the point x_3 and the wavefunction continues into region III which is the space $x > x_3$. Here $V(x) \rightarrow -\infty$ for $x \rightarrow \infty$ and therefore plane waves are appropriate solutions in this region resulting in a continuous density of states.

With the WKB method we can determine two parameters in which we are interested: i) we would like to know the energy eigenstates in region I and ii) we would like to know the transmission rate of probability amplitude of these states through the barrier into the continuum of region III .

i) Eigenstates in the potential well

To find a solution in region II we have to determine the coefficients $C_{\pm}^p$ of the wavefunction Ψ_p . To find a meaningful wavefunction over the whole space the wavefunction at the classical turning points which separates the regions has to be continuous,

$$\Psi_p(x_i) = \Psi_{\kappa}(x_i) \quad (4.11)$$

with $x_i = x_1, x_2$. This supplies the additional boundary conditions to determine the coefficients $C_{\pm}^p$ and $C_{\pm}^{\kappa}$. To analytically solve the continuity condition one has to approximate the potential near the classical turning points by a linear form. Then Airy functions [Abr72] serve as a solution for the Schrödinger equation close to the turning points. The wavefunction in region I near both turning points are then given by

$$\Psi^p(x) = \frac{C_1}{p(x)/\hbar} \cos\left(\frac{1}{\hbar} \int_{x_1}^x dx' p(x') - \frac{\pi}{4}\right) \quad (4.12)$$

at turning point x_1 and

$$\Psi^p(x) = \frac{C_2}{p(x)/\hbar} \cos\left(\frac{1}{\hbar} \int_x^{x_2} dx' p(x') - \frac{\pi}{4}\right) \quad (4.13)$$

at turning point x_2 . Both wavefunctions have to coincide which is the case for $C_1 = -C_2$ and

$$\frac{1}{2\pi\hbar} \oint dx' p(x') = n + \frac{1}{2}. \quad (4.14)$$

With this implicit equation for the energy E one can determine the n th bound state in the potential well. This condition is equivalent to the Bohr Sommerfeld

quantization rule which states that the closed integral over the phase of an eigenstate must be a multiple of an integer number n . To calculate the energy of the bound states we numerically solve the equation

$$\frac{1}{\pi\hbar} \int_{x_1}^{x_2} dx' \sqrt{2m(E - V(x'))} = n + \frac{1}{2}. \quad (4.15)$$

ii) Transmission through the barrier and tunneling time constant

As already mentioned the exponentially decaying wavefunction in the barrier does not drop to zero. Thus, a particle penetrating with energy E into the left side of the barrier at x_2 can be transmitted through the barrier into region III. We assume that, when the particle has tunneled into region III, it does not penetrate back into the barrier, because the classically allowed region III extends to infinity and thus the probability to find the particle at x_3 is negligible. To calculate the transmission coefficient through the barrier defined as:

$$T = \frac{|\Psi_{\kappa,\text{trans}}|^2 v_{\text{trans}}}{|\Psi_{p,\text{inc}}|^2 v_{\text{inc}}} = \frac{|\Psi_{\kappa,\text{trans}} \sqrt{\kappa_{\text{trans}}/\hbar}|^2}{|\Psi_{p,\text{inc}} \sqrt{p_{\text{inc}}/\hbar}|^2}, \quad (4.16)$$

one again has to determine the wavefunction $\Psi_{p,\text{inc}}$ and $\Psi_{\kappa,\text{trans}}$ in region I and II with the continuity condition (4.11). For a high and broad barrier the coefficient is given by

$$T = e^{-2 \int_{x_2}^{x_3} dx' \kappa(x')/\hbar}. \quad (4.17)$$

This transmission coefficient is called the Gamow factor referring to the studies of α -decay in nuclei with the transmission of an α -particle through the Coulomb barrier.

One remaining question is how often the particle hits the barrier at the turning point x_2 . In this semi-classical picture we assume that the particle inside the potential well is a classical particle that 'knocks' with frequency

$$\nu = \frac{E}{2\pi\hbar} \quad (4.18)$$

to the barrier. ν is called the knock frequency. The transmission rate γ is then given by the product $\gamma = \nu T$ of the knock frequency ν and the transmission coefficient T . For a system with single loss rate the number of remaining particles in the trap follows a natural decay

$$N(t) = N_0 e^{-\frac{t}{\tau}} \quad (4.19)$$

with

$$\tau^{-1} = \gamma = \frac{E}{2\pi\hbar} \exp \left(-2 \int_{x_2}^{x_3} dx' \sqrt{\frac{2m}{\hbar^2} (V(x') - E)} \right) \quad (4.20)$$

defined as the tunneling time constant. The previous equations describe the tunneling dynamics of a particle inside the potential in the case of a high and broad barrier which is fulfilled for the case $\gamma \ll \nu$. In this thesis we restrict the analysis of the tunneling dynamics to experiments where this condition for the extension of the barrier holds and we can describe the tunneling by the exponential form. In the cases where we perform experiments where γ is on the order of ω we are primarily interested in the fact that the particles has either left or not left the potential with near unity probability. In this case the determination of the details of the tunneling behavior, especially in the case of interacting particles, is non-trivial and requires a more elaborate analysis of the tunneling process [Pon12].

4.2.2. Determination of the potential shape

The knowledge of the potential shape is important to compare our measurements to the theoretical predictions. The axial profile of the potential determines the energy scale of the 1D solution and the radial profile determines the strength of the 1D coupling constant and the position of the confinement induced resonance. Furthermore, to control the motional state of the particles in the potential, we need information about the energy differences between single particle states in the potential.

From the type of trap, a single focused beam optical dipole trap, we know the general overall shape of the trap which is Gaussian in radial direction and Lorentzian in axial direction. Additionally we know the power of the optical beam within an uncertainty of 10%. However, we cannot directly measure the residual parameters - the Rayleigh range and the waist of the focus - using optical measurement methods because the focus is located in the center of the vacuum chamber. One would need a second objective with known point-spread-function on the opposite side of the vacuum chamber to image the focal plane of the microtrap [Zim11].

Hence we make use of the trapped particles to map out the level structure. We motionally excite noninteracting particles in the potential and from the excitation spectrum we can deduce the residual parameters of the potential.

Modulation spectroscopy

To experimentally determine the energy of the eigenstates of a single particle in a harmonic oscillator we couple the particle in the ground state to higher states

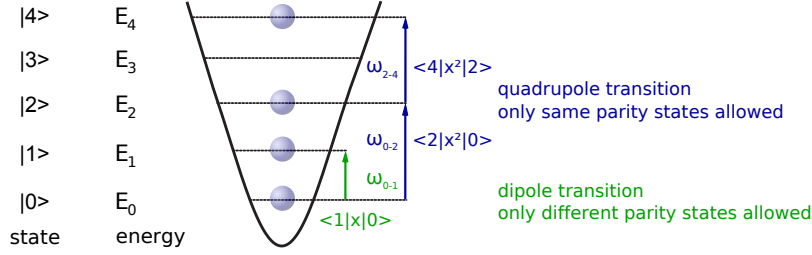


Figure 4.6.: **Mapping out the level structure of the potential.** We motionally excite noninteracting particles in the potential and from the excitation spectrum we can deduce the parameters which determine the confining potential.

by periodically modulating the potential (see figure 4.6). This is actually contrary to setting up an optical dipole trap where one tries to suppress frequency noise in the trap frequency range causing heating [Geh98]. With our setup we have two possibilities to realize modulation of the trap parameters: We can either modulate the position of the focus of the microtrap potential or modulate the intensity of the microtrap beam. In the case of a weak modulation amplitude the coupling process can be described by first order perturbation with the Hamiltonian

$$H = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2 (x + A_d(t))^2 = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2 x^2 + A_d(t)m\omega^2 x + O(A_d^2) \quad (4.21)$$

where A_d is the time dependent position of the focus

$$A_d(t) = a_{h.o} A_{d,0} \cos(\omega_{\text{mod}} t) \quad A_{d,0} \ll 1 \quad (4.22)$$

and $a_{h.o}$ is the harmonic oscillator length. By separating the Hamiltonian into a non-perturbed part and a perturbed part

$$H = H_0 + H_1 \quad (4.23)$$

and by neglecting orders of $A_{d,0}^2$ one can find the transition matrix element which determines the transition probability between two non-perturbed harmonic oscillator states $|m\rangle$ and $|n\rangle$

$$\langle m | H_1 | n \rangle = m\omega^2 \cos(\omega_{\text{mod}} t) \langle m | x | n \rangle. \quad (4.24)$$

The transition matrix element of this dipole transition can only become non-zero, if the two harmonic oscillator states have different parity. Hence for this type of modulation only states with quantum number difference equals $\Delta = 2j + 1$ can be

coupled.

Performing the same analysis for the intensity modulation where the trap frequency $\omega \propto \sqrt{p}$ is modulated by the relative intensity p of the beam we get

$$H = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2 x^2 (1 + A_q(t)) \quad (4.25)$$

with time dependent intensity modulation

$$A_q(t) = A_{q,0} \cos(\omega_{\text{mod}} t) \quad A_{q,0} \ll 1. \quad (4.26)$$

Then the transition matrix element reads:

$$\langle m | H_1 | n \rangle = \frac{1}{2}m\omega^2 A_{q,0} \cos(\omega_{\text{mod}} t) \langle m | x^2 | n \rangle. \quad (4.27)$$

In this case the transition matrix element of the quadrupole transition can only become non-zero, if the two harmonic oscillator states have same parity ($\Delta = 2j$). Further details on the analysis can be found in [Jáu01]

Coherent control of the motional state

Figure 4.7 illustrates the two different coupling processes as they are used in the experiment. After applying the modulation for a certain time we measure the fraction of non-excited particles by removing the particles of the excited states using the spilling technique. Due to the fermionic nature of our two component sample we can only have two particles occupying the same harmonic oscillator state. Thus to obtain the excitation probability with sufficiently low statistical uncertainty we have to repeat the measurement several times. Then the excitation probability is equivalent to $1 - N_{\text{mean}}/N_0$, where N_0 is the number of prepared particles and N_{mean} the mean number of remaining particles in the ground state after the modulation. If we applied the modulation to a harmonic oscillator we would couple all states at the same time when $\omega_{\text{mod}} = \omega$. However, our microtrap potential slightly deviates from the harmonic approximation. Hence the energy differences between three neighboring states are not identical and ω_{mod} can be resonant to one but not the other transitions if the modulation amplitude is weak enough. In this case we can describe the dynamic of a particle which is only weakly coupled to a single other state by that of a two level system. The probability of finding the particle in the excited state after a certain modulation time t is then given by

$$P(e) = \left(\frac{\Omega_0}{\Omega_{\text{eff}}} \right)^2 \sin^2 \left(\frac{\Omega_{\text{eff}}}{2} t \right) \quad (4.28)$$

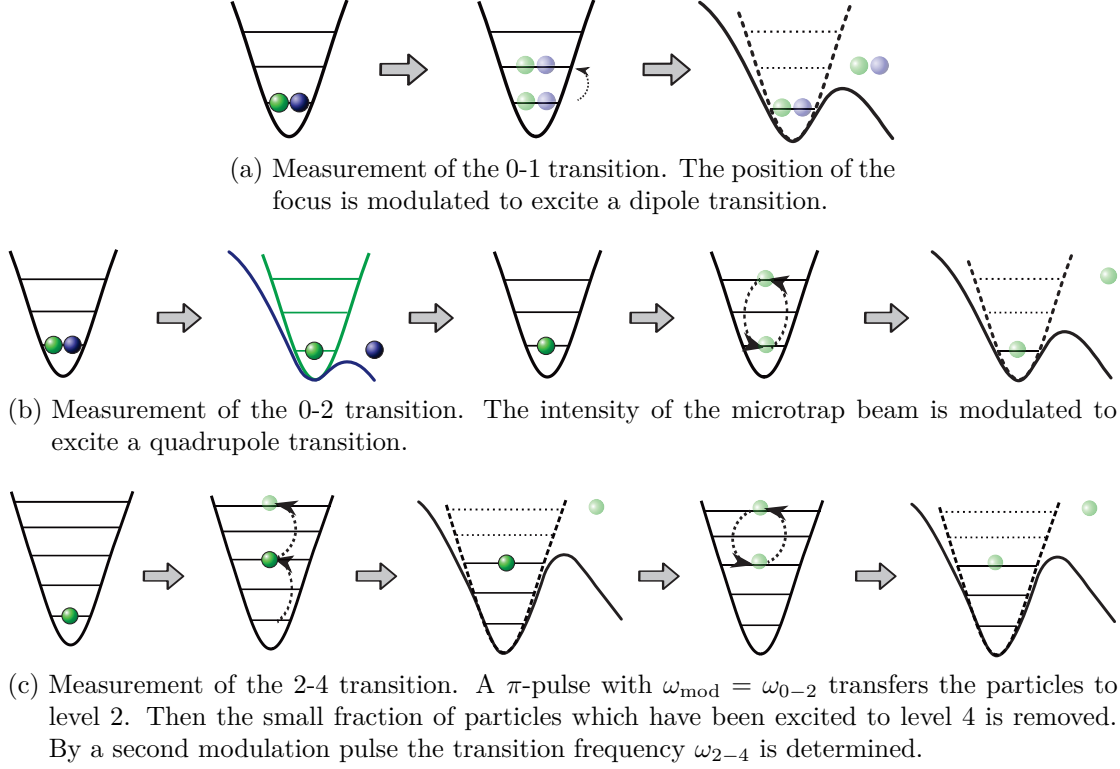


Figure 4.7.: **Measurement of the Transition frequencies.** In the first step we prepare one or two noninteracting particles in the ground state. Then we apply one or several modulation pulses with a certain frequency ω_{mod} . After the modulation pulse the remaining fraction in the ground state is detected using the spilling technique.

with the effective Rabi frequency $\Omega_{\text{eff}} = \sqrt{\Omega_0^2 + \delta^2}$, the detuning $\delta = \omega_{i-j} - \omega_{\text{mod}}$, $\omega_i = E_i/\hbar$ the energy of the i -th unperturbed state and the Rabi frequency at resonance $\Omega_0 \propto A_{q,0} \langle n+1|x|n \rangle$ in the case of a dipole excitation and $\Omega_0 \propto A_{q,0} \langle n+2|x^2|n \rangle$ in the case of a quadrupole excitation.

For the $n = 0$ to $n = 2$ quadrupole transition we have observed more than a full Rabi cycle with some damping showing the coherence of the excitation (see appendix, figure A.7). For the chosen modulation amplitude the Rabi frequency is $2\pi \times 21.3(3)$ Hz which is a fraction of a hundredth of the transition frequency confirming that we are in the weak coupling limit. We are able to apply a π -pulse ($t = \pi/\Omega$, $\delta = 0$) which transfers $\sim 90\%$ of the particles into the excited state. By applying two subsequent $\frac{\pi}{2}$ -pulses with a precession time T between the two pulses we have performed a Ramsey-type experiment (appendix, figure A.8). The fitted

precession frequency coincides with the energy difference of the coupled states. This result impressively show that we have coherent control of the motional state of the particles in the trap.

We have to add that this coherent coupling is only working when the intensity of the microtrap is stable. A drift of 5×10^{-3} of the intensity would cause a drift of ω by 2.5×10^{-3} . At a typical Rabi frequency of $10 \times 10^{-3} \omega$ this would result in a reduction of the contrast by 5%. Although we have a higher stability on a few-day basis such large drifts can still occur within several weeks. Then, a recalibration of the resonance frequency is necessary to be able to perform experiments involving coherent control of the motional states.

For the dipole transition from level 0 to level 1 we have not observed coherent dynamic. This is not unexpected because the way we modulate the position of the microtrap is rather unconventional: We have added a loudspeaker behind the mirror which reflects the optical beam of the microtrap onto the microtrap objective (appendix, figure A.9). By creating sinusoidal sound pulses with the loudspeaker the mirror starts to vibrate and thus modulates the beam angle. Through the objective the variation of the beam angle is then translated to a position modulation of the microtrap focus. Although we have not observed coherent manipulation we could still observe some signal with a maximum of particle transfer into the excited states. In a next generation of the microtrap setup we plan to add an optical device (acousto optical deflector) which is able to deflect the microtrap beam in more controlled way.

Potential shape in axial direction

The level structure and the knowledge of the potential shape in axial direction is important to obtain quantitative results for the energies and tunneling times of the two-particle system. By applying the modulation spectroscopy we determine the structure of the energy levels in the optical potential. The measured transition spectra in axial direction for the pure optical potential are shown in the appendix (figure A.10-A.12). The deduced transition frequencies are listed in table 4.1. Exemplary we present a typical spectrum of the quadrupole transition in figure 4.8. To determine the axial trap parameters we assume that the beam has a Lorentzian shape, neglecting any aberrations that might be introduced by the optical setup. The combined optical and magnetic potential reads

$$V(p, z) = V_{\text{opt}}(p, z) + V_{\text{mag}}(z) = p V_0 \left(1 - \frac{1}{1 + (z/z_r)^2} \right) - \mu_m B' z \quad (4.29)$$

with $z_r = \frac{\pi w_0^2}{\lambda}$ the Rayleigh range, w_0 the waist in the focal plane, λ the wavelength of the trapping light and p the optical trap depth as a fraction of the initial depth.

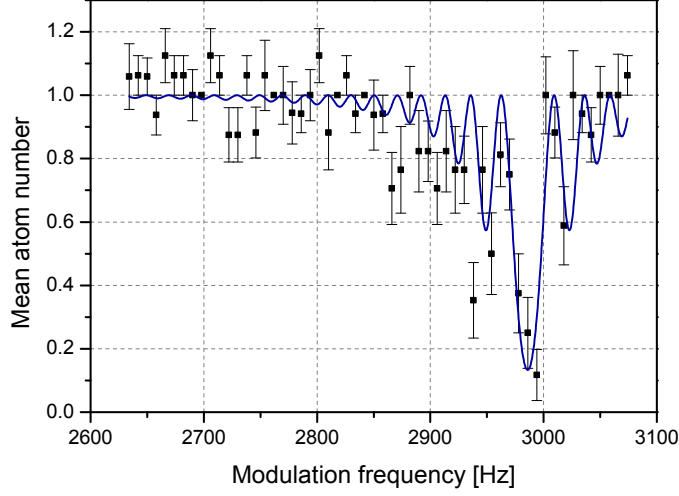


Figure 4.8.: **Quadrupole excitation of the 0-2 transition in axial direction.** We record the number of particles in the ground state after applying a modulation pulse. We determine the transition frequency by fitting equation (4.28) to the spectrum.

transition	transition frequency $\omega_{\parallel}/2\pi$ [kHz]	σ =FWHM (Ω_0) of the peak [Hz]
0-1	1.486	11
0-2	2.985	10 ^(*)
2-4	2.897	20

Table 4.1.: **Transition frequencies in axial direction.** The measurement has been performed at an optical trap depth of $p = 1$. The transition frequencies are determined from fits to the excitation spectra (appendix, figure A.10-A.12). ^(*)Taking the Rabi frequency as the error sets an upper limit for possible deviations. For the 0-2 transition we have measured a full Rabi cycle with 90% contrast at $\omega_{\text{mod}} = 2.98$ kHz. Therefore we can assume a smaller error of $\Omega_0/2$.

For the modulation spectroscopy the gradient has been switched off and thus $V_{\text{mag}} = 0$. The initial trap depth is given by the dipole potential [Gri00]

$$V_0 = -\frac{3\pi c^2}{2\omega_a^3} \left(\frac{\Gamma}{\omega_a - \omega} + \frac{\Gamma}{\omega_a + \omega} \right) \frac{2P_0}{\pi w_0^2}, \quad (4.30)$$

4.2. Tunneling theory and determination of the potential shape

with $\omega_a = 2\pi \times 4.468 \cdot 10^{14}$ Hz the frequency of the atomic transition, $\Gamma = 2\pi \times 5.872$ MHz the linewidth of the transition, $\omega = 2\pi c/\lambda$ and P_0 the power of the microtrap beam. By using the introduced WKB approximation we can calculate the energy of the i -th eigenstates $E_{\text{opt},i}$ in the potential depending on the beam waist w_0 and the power P_0 of the beam. We have to find a value for the waist and the optical power such that the calculated transition frequencies coincide with the measured ones from table 4.1. The description of this routine can be found in A.4.2 in the appendix. The resulting trap parameters for the axial potential are listed in table 4.2.

parameter	value
V_0	3.326 μK
w_0	1.838 μm
$\omega_{\parallel}, p = 1$	$2\pi \times (1.488 \pm 0.014)$ kHz
B'	18.92 G/cm

Table 4.2.: **Parameters of the potential in axial direction.** The parameters are obtained from the axial excitation frequencies in the pure optical potential and from the tunneling time constants of a noninteracting particles. (For the analysis see appendix A.4.2). The parameters V_0 , w_0 and B' modeling the potential according equation (4.29) are used to determine bound states and tunneling times in WKB calculations. As uncertainties in the potential parameters only cause systematic shifts in the latter quantities we do not give their error. The systematic shifts are within a range of 10^{-3} to 10^{-2} .

Potential shape in radial direction

The strength of the perpendicular confinement and thus the position of the confinement induced resonance is defined by the shape of the radial potential. For a perfect Gaussian beam the knowledge of the axial profile would imply the knowledge of the radial profile and vice versa. However, our beam deviates from the ideal form. A main contribution to the deviation might be an astigmatism which effectively increases the Rayleigh range compared to the focal waist in radial direction. Due to this discrepancy we determine the trap parameters in radial direction separately from the parameters in axial direction. Additionally the beam profile is not perfectly radially symmetric which is why we assume an elliptical symmetry in radial direction with Gaussian profile along each main axis. We approximate

transition	quantum number (n_x, n_y, n_z)	transition frequency $\omega_{\perp}/2\pi$ [kHz]	σ =FWHM (Ω_0) of the peak [kHz]
0-1	(1,0,0)	13.96	0.08
0-1	(0,1,0)	14.82	0.09
0-1	(1,0,1)	15.36	0.04
0-2	(2,0,0)	26.43	-
0-2	(0,2,0)	28.26	0.25
0-2	(2,0,2)	29.07	0.14

Table 4.3.: **Transition frequencies in radial direction.** The measurement was performed at an optical trap depth of $p = 1$. The transition frequencies are determined from fits to the excitation spectra. In addition to the two expected transitions in x - and y -direction we observe another peak in the observed frequency range which we attribute to a transition involving an axial excitation. We denote the transitions by their quantum number of a corresponding harmonic oscillator.

the potential by the sum ⁹:

$$V(r, z) = V(x) + V(y) + V(z) \quad (4.31)$$

with $r = \sqrt{x^2 + y^2}$ and

$$V(s) = p V_{0r} \left(1 - e^{-\frac{2s^2}{w_{0s}^2}} \right) \quad [s : x, y] \quad (4.32)$$

with w_{0s} being the different focal waists along the two main axis. We define the anisotropy parameter as the ratio between the two lowest quadrupole transitions in the potential:

$$\eta_{xy} = \frac{\omega_{y\,0-2}}{\omega_{x\,0-2}} \quad (4.33)$$

We again apply the modulation spectroscopy to measure the transition energies in radial direction. The measured frequencies are given in table 4.3. Then we vary the potential parameters of equation (4.32) and apply a WKB calculation to reproduce the measured parameters. The details of this routine are described in A.4.3 in the appendix. The determined parameters of the radial direction are shown in table 4.4. To determine the CIR we only consider one resonance following

⁹In a harmonic approximation the potential separates in a sum of three independent potentials, one per axis. Without this approximation the determination of the eigenstates in the potential

4.3. Energy of the interacting two-particle system

parameter	value
V_{0r}	4.12 μK
w_{0x}	1.637 μm
w_{0y}	1.516 μm
η_{xy}	1.07
$\omega_{\perp}$	$2\pi \times (14.22 \pm 0.35)$ kHz
$\text{CIR}_{ 1\rangle- 2\rangle}, p = 1$	(779.3 ± 0.5) G

Table 4.4.: **Parameters of the potential in radial direction.** The potential parameters are obtained from the radial excitation spectrum using a WKB approximation. The perpendicular trap frequency $\omega_{\perp}$ is determined from the mean of the 0-2 transitions in x - and y -direction. The error is the SEM. The position of the CIR is obtained from equation (3.37).

[Pen11, Sal12] although we have found an anisotropy in our system. We calculate the position of the CIR from the the mean harmonic oscillator length which is determined by the mean trap frequency

$$\omega_{\perp} = \frac{1}{4} (\omega_{x0-2} + \omega_{y0-2}). \quad (4.34)$$

The propagated error of the mean trap frequency serves as the systematic uncertainty of the position of the CIR which is about ± 0.5 G. In the fermionization measurement the optical trap depth is lowered from 1 to $p = 0.6875$ which modifies the confinement by $\sqrt{p}$ resulting in a perpendicular trap frequency of $2\pi \times (11.79 \pm 0.29)$ kHz. The corresponding axial frequency at this depth is $(2\pi \times 1.234 \pm 0.012)$ kHz.

4.3. Energy of the interacting two-particle system

To obtain the energy of an interacting few-particle system we have used three different methods during the course of this thesis. In this section we introduce two of them which we initially applied in our experiment. The first method is the determination of the energy from the tunneling time constants which requires

would require a more complicated numerical treatment of the full 3D potential. The error of the approximation will be determined by the degree of the anharmonicity which is on the order of a few percent.

an elaborate theoretical framework. The second method is the modulation spectroscopy with which we can measure energy differences between the ground and the first excited state. Although this method does not provide the absolute interaction energy, it has the advantage that it can be performed in the pure optical potential without deforming the nearly harmonic potential by applying a magnetic field gradient.

The third and most elegant method which can also be applied in the pure optical potential is the rf-spectroscopy method. This method will be discussed separately in chapter 6.

4.3.1. Determination of the energy from the tunneling time constants ¹⁰

To determine the energy of two interacting fermions from the measured tunneling time constants we use a WKB calculation (see section 4.2.1). To apply this calculation we have determined the shape of the potential¹¹ as described in section 4.2.2.

Within the energy range studied in the fermionization experiment in section 4.1.2 we have observed that only one of the two particles tunnels through the barrier. This particle has an energy identical to the kinetic energy of the relative motion of the two-particle system. Hence we can determine the kinetic energy of the relative motion of the two-particle system by extracting the energy of the tunneled particle from the tunneling time constant τ_{exp} . To map τ_{exp} onto energies we calculate tunneling time constants for a set of energies. By matching these calculated tunneling time constants to the measured ones we determine the kinetic energy of the relative motion of the two-particle systems. A more detailed description of this method is given in [Ser11a]. The energies obtained from the different tunneling time constants of two distinguishable fermions are shown in figure 4.9.

We compare these energies to the analytic theory for a harmonic potential [Bus98] (see figure 4.2). This theory needs two input parameters, the coupling strength and the level spacing. For the coupling strength we use $g_{|\uparrow\downarrow\rangle}$ of our system shown in figure 4.4 b). For the level spacing we use the energy difference $\hbar\omega_{\parallel\text{calc}} = E_0 - E_1 = 2\pi\hbar \times 743 \text{ Hz}$ between the ground and first excited state of the potential which we calculate using the WKB method. With this approximation the energy obtained from the tunneling measurements and the energy obtained from

¹⁰Taken from [Zür12] and adapted.

¹¹The parameters of the optical potential have been determined by precise measurements of the level spacings in the potential. The final parameter to determine the barrier height has been fixed by the measured tunneling time constant of two identical fermions.

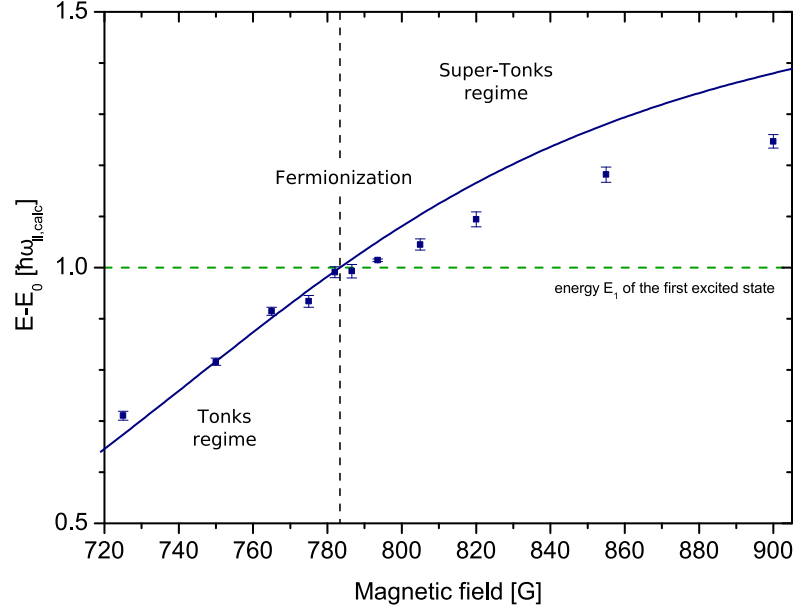


Figure 4.9.: **Interaction energy of two fermions for different interaction regimes.** By using a WKB based calculation we can determine the energy of two distinguishable fermions at different interaction strengths (blue points) from the tunneling time constants presented in figure 4.4 a). The blue curve shows the expected energy shift for a harmonically trapped system (dashed rectangle in figure 4.4). Taken from [Zür12].

the analytic theory [Bus98] are the same at the CIR. For the Tonks regime we find excellent agreement of the experimentally determined energy with the theoretical prediction for a harmonic trap. Above the CIR the harmonic theory is not applicable because the second excited state is not bound in our potential. Additionally, we expect deviations for larger energies due to the limited validity of the WKB approximation for energies close to the continuum threshold. A more precise description could be achieved by adapting the theory described in [Bus98] to our non-harmonic potential using a perturbation theory approach [Ron12b]. By using a more accurate theory for the tunneling process – the quasi-particle wavefunction approach [Bar61, Ron12b] – the results also show good agreement between experiment and theory in the super-Tonks regime (see figure 4.10).

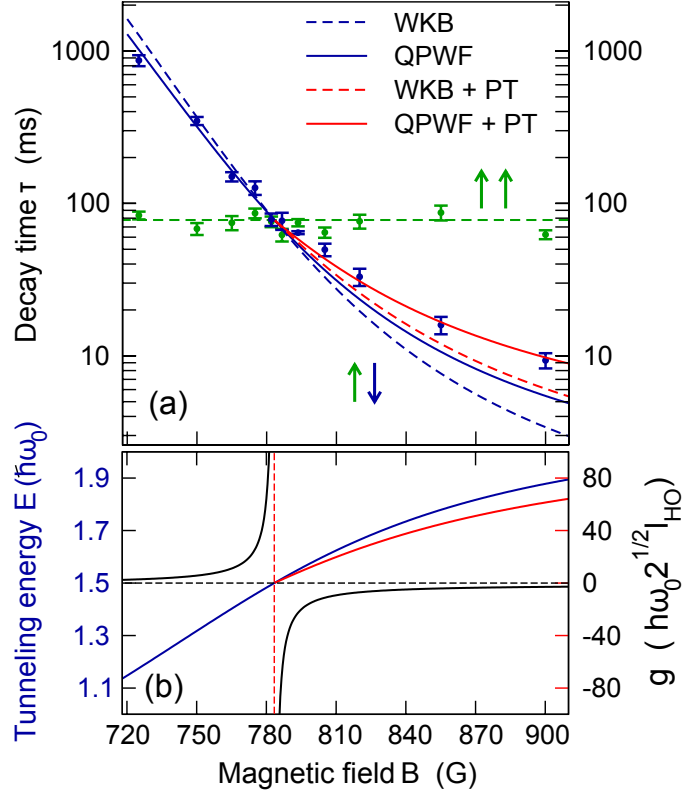


Figure 4.10.: **Tunneling time and interaction energy calculated by the quasi-particle wavefunction approach (QPWF) and perturbation theory approach (PT).** a) Decay time τ depending on the magnetic offset field. The points with error bars represent the experimental data, the dashed and solid lines are the WKB (τ_0) and QPWF predictions. The red lines include the PT correction to the tunneling energy E . b) Interaction strength $g_{|\uparrow\downarrow\rangle}$ (black curve) and E (blue and red curve). $g_{|\uparrow\downarrow\rangle}$ is taken from figure 4.4 and E is computed after [Bus98] (blue curve). The red line includes the PT correction. Taken from [Ron12b] and adapted.

4.3.2. Modulation spectroscopy of two interacting particles

With the modulation spectroscopy method we successfully determined the level structure of noninteracting particles in the confining potential (section 4.2.2). Obviously we were curious whether this method could also be applied to an interacting system. We had been inspired by the measurements of collective modes performed by Elmar Haller et al. [Hal09] who observed a change of the ratio between the

4.3. Energy of the interacting two-particle system

compression modes and the dipole modes of a repulsively interacting 1D Bose gas depending on the interaction strength (figure 4.11). For a many-body state they

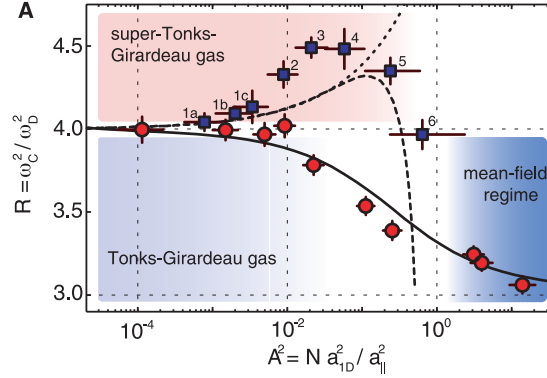


Figure 4.11.: **Excitation of collective modes in a many-body system.** The squared frequency ratio $R = \omega_C^2 / \omega_D^2$ of the lowest compressional mode with frequency ω_C and the dipole mode with frequency ω_D serves as an indicator for the different regimes of interaction. It is plotted as a function of the interaction parameter $A^2 = N a_{1D}^2 / a_{||}^2$ where N is the number of particles, $a_{1D} = -\frac{2\hbar^2}{mg_{1D}}$ is the 1D scattering length, and m the mass of the particles. The squares show the measurements in the super-Tonks regime ($g_{1D} < 0$), providing evidence for the super-Tonks gas. The circles show the transition from the 1D mean-field Thomas-Fermi regime to the Tonks regime ($g_{1D} > 0$). The solid line presents the theoretical data for $g_{1D} > 0$, and the dashed line presents the theoretical data for $g_{1D} < 0$ [Ast05]. Taken from Elmar Haller et al. [Hal09].

observe a reduction of the oscillation frequencies in the Tonks regime and an increase of the modulation frequency in the super-Tonks regime. In the case of our two particle system there is a theoretical prediction for the energy spectrum for all values of g_{1D} [Idz06] (figure 4.12 a). To predict the transition frequencies we subtract the energy of the second repulsive state from the first repulsive state (see figure 4.12 b). The predicted energy difference shows a quite similar behavior with a reduction in the Tonks regime and an increase in the super-Tonks regime. The fact that properties similar to those of the two-particle system are visible in the many-body system confirms that the many-body physics strongly depends on the few-body physics of the system.

We have applied the modulation spectroscopy to interacting two-particle systems by exciting a quadrupole transition. The obtained spectra are shown in figure

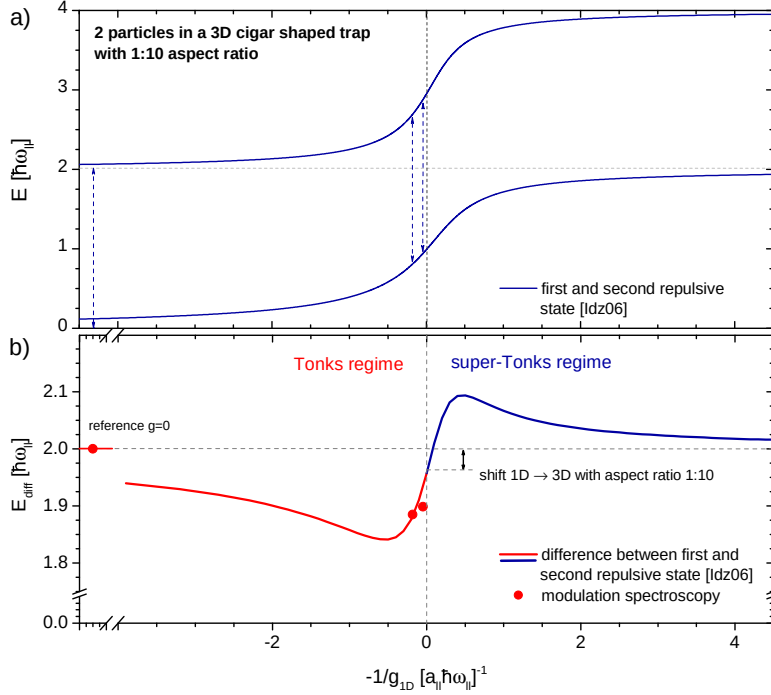


Figure 4.12.: a) Energy and b) energy difference of the first and the second excited repulsive state of a two-particle system [Idz06]. The red points are the results from the modulation spectroscopy (see figure 4.13). In the Tonks regime the frequency of the quadrupole transition is lowered compared to the transition at zero interaction. In the super-Tonks regime the transition frequencies are expected to be increased. The energy at diverging coupling strength in a quasi 1D trap with finite aspect ratio is shifted to a lower value compared to the energy in a pure 1D system with $E_{\text{diff}} = 2\hbar\omega_{\parallel}$.

4.13. The black data shows the spectrum of the noninteracting two-particle system ($g_{1D} = 0$). The oscillatory behavior of the signal is due to the coherence of the coupling expressed by the second term in equation (4.28). For $g_{1D} > 1$ we observe that a second peak appears below the noninteracting transition frequency¹². We interpret the frequency of this peak to be the transition frequency between the two repulsive states of [Idz06]. The oscillatory behavior is reduced in the transition peaks of the interacting systems which might be a signature for the loss of coherence. This is the reason why we fit Gaussians to the envelope of the transition

¹²The reason why the noninteracting peak does not totally disappear in the spectrum of two interacting particles might be due excitations involving center-of-mass states.

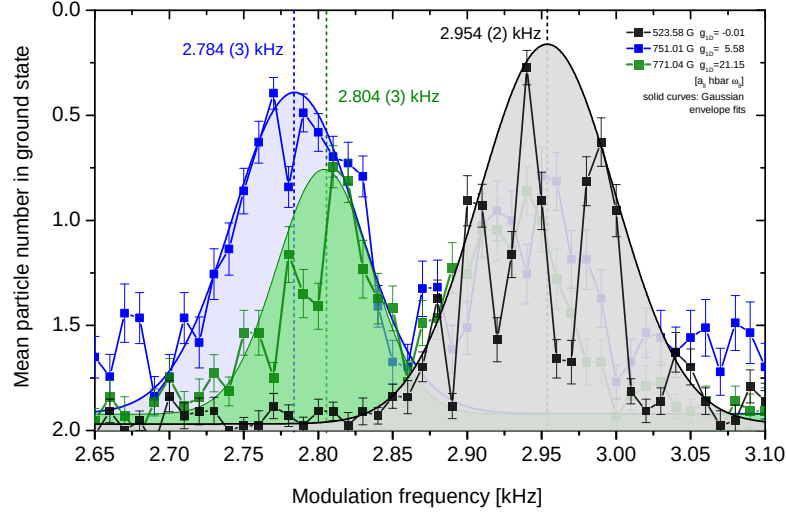


Figure 4.13.: **Modulation spectroscopy of an interacting system.** The amplitude of the modulation depth is as large that one obtains a Rabi-frequency of the noninteracting system of $\Omega_0 = 35$ Hz. The duration of the modulation pulse is $65 \times 2\pi/\omega_{\text{mod}} \approx 20 - 25$ ms. For $g_{1D} > 1$ we observe that a second peak occurs below the noninteracting transition frequency. We fit Gaussians to the envelope of the peaks to determine the transition frequency. The errors given in brackets are the standard errors of the fit.

peaks instead of the functional form for a coherent transition. We find that the determined frequencies agree with the prediction that the transition frequencies are reduced in the Tonks regime (red data points in figure 4.12). For the super-Tonks regime we could not acquire meaningful data because during the time we performed the modulation spectroscopy we were not aware of the center-of-mass to relative motion coupling resonances which lead to loss of particles in system for which we crossed the CIR to slowly (cf. section 4.5). We later performed energy measurements in the super-Tonks regime, even for larger particle numbers. Yet, we then used the more elegant method of rf-spectroscopy. The results are presented in chapter 6.

To summarize, we have measured the energy of two repulsively interacting particle and showed fermionization of two distinguishable fermions. In the following we want to add an additional particle and study the properties of the larger system when tuning it towards strong repulsive interaction.

4.4. Adding more particles - Fermionization of three fermions

With the comparison of the tunneling properties of two distinguishable fermions with two identical ones we have demonstrated the fermionization of two distinguishable fermions. As in a many-body system of interacting distinguishable fermions the correlations of the previously studied two-particle system are dominant these systems are predicted to become fermionized at diverging coupling strength [Gir10]. Thus, to demonstrate fermionization for larger particle numbers, we added another particle to our two particle system. Of course one does not realize a real many-body system by just adding a single particle. However, finding evidence for fermionization in a three-particle system is a next step towards proving fermionization of a many-body Fermi system.

To realize the fermionized three particle system we actually do not have to add a particle in a third spin state which would be distinguishable from each of the other two particles. Adding a third spinstates to the same single well potential would lead to inelastic 3-body recombination at large coupling strength [Ott08]. We can add a particle which is identical to one of the two others because these two identical particles do not interact and thus they are intrinsically fermionized. To prove fermionization we could have repeated the previous tunneling measurement and compared the tunneling properties of the interacting three-particle system to that of three identical fermions. Here, we follow a different approach: By studying the correlations in the system we will illustrate how the fermionic nature of the distinguishable particles is smoothly switched on [Tho12].

We prepare the initial three-particle system as described in the preparation chapter (section 2.4.1). Then we tune the strength of the interaction by tuning the magnetic offset field. At a fixed value of g_{1D} we deform the potential so that particles can tunnel on experimentally accessible timescales. Doing so, we choose the barrier height such that just a single particle leaves during the time we allow for tunneling. Finally we switch off tunneling and detect the spin state of the two remaining particles.

The initial three-particle system of the experiment is sketched on the left side of figure 4.14. When we remove a single particle of the initial noninteracting system by allowing for one particle to tunnel, the result of the state sensitive measurement of the two remaining particles is obvious: We will just remove one of the two identical particles from the second level. The remaining system consists of a single spin up and a single spin down particle and thus the probability of finding a polarized

4.4. Adding more particles - Fermionization of three fermions

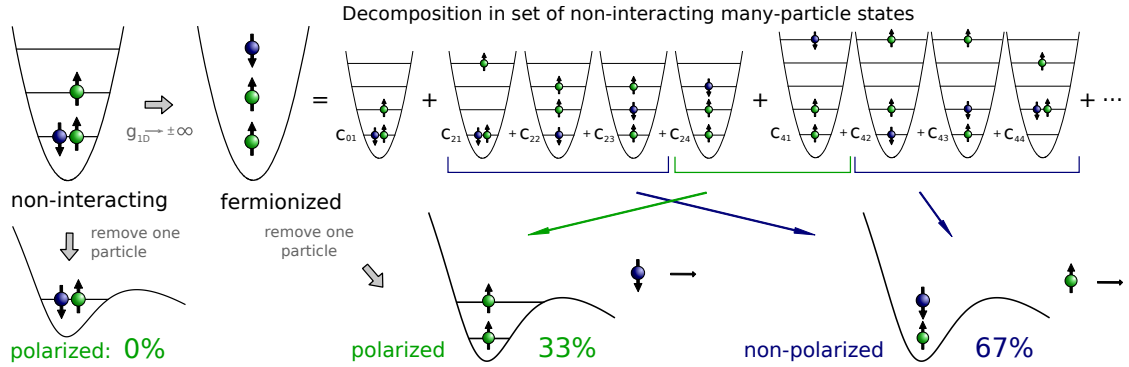


Figure 4.14.: **Initial noninteracting system and decomposition of the fermionized state.** The initial system (left side) is prepared as described in chapter 2.4.1. The probability of finding a polarized state after removing one particle from the noninteracting system is zero. The fermionized state can be decomposed into a set of noninteracting few-body states. Only the states where the distinguishable particle is highest up in the potential contribute to the fraction of finding polarized states. Summing up the coefficients for these states results in a fraction of 33% polarized states for a fermionized system.

system¹³ is zero. To find a polarized system the single minority spin particle has to tunnel. This is highly suppressed as long as the expectation value of the energy of this particle is much smaller than the energy of the identical fermions. However, by making the system more and more repulsively interacting, the minority particle acquires more and more fermionic properties. Thus for strong interaction ($g_{1D} > 1$) we observe that the minority particle starts to tunnel and we observe that the probability of finding a polarized system remaining in the trap increases (see figure 4.15). For infinitely strong coupling the distinguishable particle has acquired maximum fermionic properties: Its probability to tunnel has become equal to the probability of one of the two identical particles to tunnel, i.e. $1/3$ for each particle. We identify this point around the CIR where the fraction of polarized systems crosses $1/3$. This is the point of fermionization where all particles behave like identical fermions.

The result can be visualized by projecting the many-particle wavefunction of the fermionized state for which an analytic solution exists [Gir10] onto a full set of noninteracting many-particle states which is sketched on the left side of figure 4.14. Then one can imagine that the tunneling process acts on each of these basis

¹³We denote a system of two identical fermions as a polarized system due to the alignment of their spins.

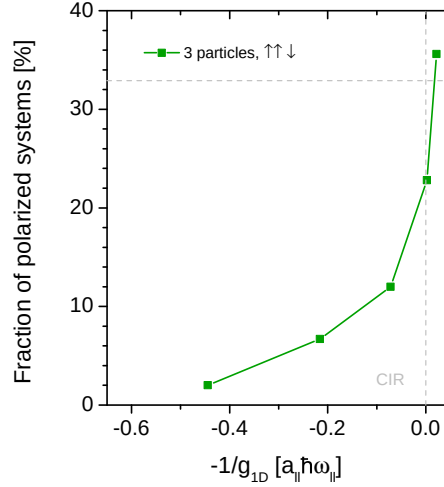


Figure 4.15.: **Fermionization of three fermions.** We prepare a three-particle system of two identical and one distinguishable fermion. After removing one particle from the trap by a tunneling process we record the number of polarized systems. For weak interaction we do not observe polarized systems. For increasing interacting strength the fraction of polarized systems increases and gets $1/3$ close to the CIR which is evidence for fermionization. Note: The crossing at $1/3$ slightly above $1/g_{1D} = 0$ is to some extent due to the systematic uncertainty of the CIR of ± 0.5 G. Additionally, effects from being only in a quasi 1D system with finite perpendicular aspect ratio might lead to a shift. Yet, compared to the whole range of interaction strengths – the plot only shows the strongly interacting regime with $g_{1D} > 1[a_{||} \hbar \omega_{||}]$ – the deviation is small.

state of the decomposition: All states with the distinguishable particle highest up in the potential contribute to the fraction of polarized states (green bracket). All other states where one of the identical is highest up contribute to the non-polarized fraction (blue brackets). To determine the total number of these fractions one has to sum up the corresponding coefficients of the decomposition. Ioannis Brouzos from Peter Schmelchers group in Hamburg [Bro12a] has performed the projection of the fermionized state onto the noninteracting many-particle states and calculated the coefficients $c_{2n,k}$ ¹⁴. Summing up all the coefficients till $c_{4,k}$ results in a fraction of polarized system of 33%. This is exactly what we assumed and also

¹⁴Due to parity conservation the coefficients of the even parity states with energy $2(n+1)\hbar\omega_{||}$ vanish. The contribution of the higher energy states with coefficients $c_{2n>4,k}$ is negligible.

4.4. *Adding more particles - Fermionization of three fermions*

measured for a fermionized system of three particles.

4.5. Center-of-mass to relative motion coupling

In the analysis of two interacting particles in a harmonic trap the relative motion is completely separated from the center-of-mass motion [Bus98] [Idz06]. Yet, due to the anharmonicity of our trapping potential the relative motion state of our interest – the repulsive branch in figure 3.5 (blue curve) – can couple to excited center-of-mass states of molecules. An analysis of this process has been performed by Simon Sala et al. [Sal12]. Here we briefly sum up the most relevant part of this publication and present our experimental results on the COM-REL motion coupling resonances¹⁵. The measurement and its analysis will be content of a joined publication (in preparation). With our measurement of coherent coupling of two particles into a molecular state we can argue that the loss features observed in a 1D system in the Innsbruck experiment [Hal10] is most likely due to this two-body effect followed by a collision of the molecule with a third particle and relaxation into a deeply bound molecule.

In the presence of an anharmonicity in the confining potential there is a potential term in the Hamiltonian which does not separate into center-of-mass and relative motion:

$$W(\mathbf{r}, \mathbf{R}) = V(\mathbf{r}, \mathbf{R}) - V(\mathbf{r}) - V(\mathbf{R}) \quad (4.35)$$

where $V(\mathbf{r})$ and $V(\mathbf{R})$ are the separable parts in terms of the relative coordinate $\mathbf{r} = (x_r, y_r, z_r)$ and the center-of-mass coordinate $\mathbf{R} = (x_R, y_R, z_R)$. This leads to a coupling between the relative motion state $|\Psi_{\text{rep}}\Phi_{(0,0,0)}\rangle$ in the COM ground state and the two molecular states $|\Psi_{\text{att}}\Phi_{(2,0,0)}\rangle$ and $|\Psi_{\text{att}}\Phi_{(0,2,0)}\rangle$ which are in the attractive (molecular) REL ground state and in the second excited COM state. The indices of the states denote the quantum number of excitations in a harmonic oscillator (n_x, n_y, n_z) . The former state corresponds to the repulsive branch in figure 3.5 and the latter to the attractive branch in figure 3.5 with an additional energy of $\hbar\omega_{x,0-2}$ and $\hbar\omega_{y,0-2}$ deposited in the center-of-mass motion. The energy of both states is shown in figure 4.16 a). The corresponding matrix elements of the couplings are

$$\langle \Psi_{\text{rep}}\Phi_{(0,0,0)} | W | \Psi_{\text{att}}\Phi_{(2,0,0)} \rangle \quad (4.36)$$

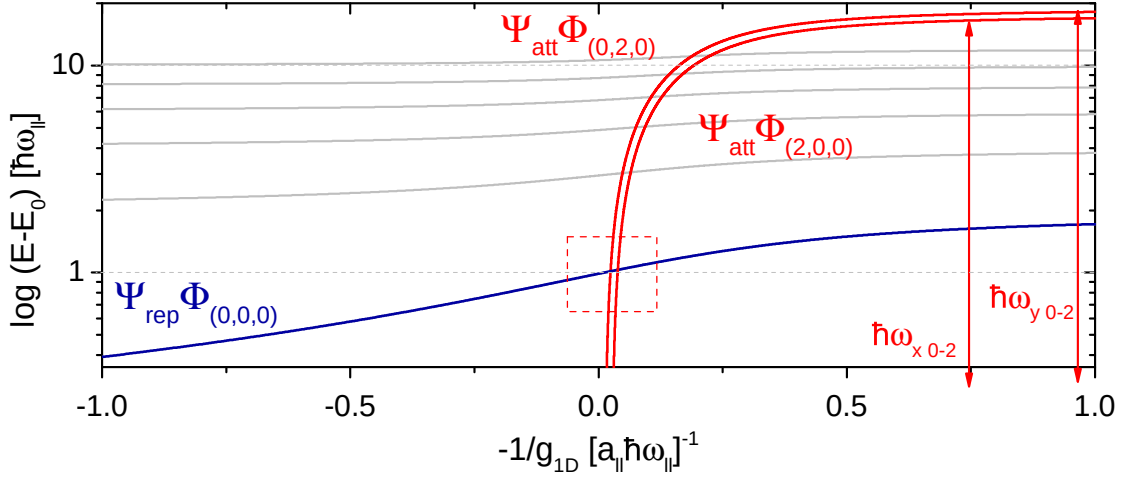
and

$$\langle \Psi_{\text{rep}}\Phi_{(0,0,0)} | W | \Psi_{\text{att}}\Phi_{(0,2,0)} \rangle. \quad (4.37)$$

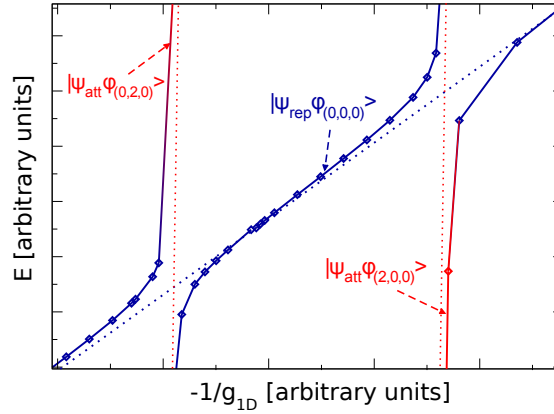
The coupling to a molecular state of the form $(1,0,0)$ is forbidden in a symmetric potential due to parity conservation. All higher radial states with even parity can couple, yet for $n_{x,y} \geq 4$ the coupling is weak and therefore not considered.

¹⁵We use the notation: relative motion: REL, center-of-mass motion: COM.

4.5. Center-of-mass to relative motion coupling



(a) Energy of the repulsive branch (blue) and of the attractive branches (red) [Idz06] with center-of-mass excitation of $\hbar\omega_{x\,2-0}$ and $\hbar\omega_{y\,2-0}$ (logarithmic scale) as a function of the inverse coupling constant. The frequencies $\omega_{x\,2-0}$, $\omega_{y\,2-0}$, $\omega_{||}$ are taken from the tables in section 4.2.2. The estimated anisotropy of the confinement is $\omega_{x\,2-0}/\omega_{y\,2-0} = 1.07$.



(b) Zoom into the dashed window of plot (a). Avoided crossings of the repulsive state and the excited attractive states occur due to the COM-REL motion coupling. Units are in arbitrary scale. The numerical calculation of the energy of the states (points) are performed for the system described in [Sal12]. Plot taken from [Sal12] and adapted.

Figure 4.16.: **Energy spectrum and avoided crossings occurring due to the COM-REL motion coupling.**

The coupling to a bound state along the weakly confined z direction can be neglected due to the anisotropy of the trapping potential[Sal12]. The COM-REL motion coupling leads to avoided crossings of the repulsive state $|\Psi_{\text{rep}}\Phi_{(0,0,0)}\rangle$ and the excited molecular states $|\Psi_{\text{att}}\Phi_{(2,0,0)}\rangle$ and $|\Psi_{\text{att}}\Phi_{(0,2,0)}\rangle$. The minimum energy difference between the adiabatic states are given by the Rabi frequency which is

determined by the matrix element (4.36) and (4.37).

To experimentally resolve these coupling resonances we prepare two noninteracting particles in the ground state of our potential. Then we ramp up the repulsive branch by increasing the magnetic offset field with a speed of 20 G/ms. The ramp speed of the magnetic field is chosen such that g_{1D} tunes fast enough to non-adiabatically jump across the coupling resonances [Zen32]. At distinct magnetic field values we immediately stop the ramp of the magnetic field to be able to create a superposition of the repulsive state and the molecular state. Both states evolve with different phase and therefore we expect an oscillation between the population of the two states of the form $\cos(\Omega_{\text{eff}}t)$ with $\Omega_{\text{eff}} = \sqrt{\Omega_0^2 + \Delta(B - B_0)}$. Here B_0 is the magnetic field value where the energy difference between the adiabatic states is minimal, with minimum Rabi frequency Ω_0 and the width of the resonance Δ . In the experiment we wait for 12.5 ms after the stop of the ramp before we measure the population in the repulsive state. The duration is chosen such that it corresponds to half a cycle of the expected Rabi-Frequency $\Omega_0 = 2\pi \times 80$ Hz which corresponds to a π -pulse. Subsequently, to detect the number of particles in the repulsive state we ramp back the magnetic field value to far below the CIR to 523 G. Any particle which was in the molecular state has become deeply bound and therefore cannot be detected with our detection method. Thus, the number of particles in the molecular state is given by the difference between the mean number of the initial two-particle system N_0 and the number of particles in the repulsive state. To check if there is no other loss channel besides coupling into the molecular state we ramp the magnetic field to a value of 900 G after the stop at the magnetic offset field of interest. At 900 G, above the CIR, the molecules are weakly bound and the individual particles of the molecule can be detected with our detection method. We found $\approx N_0$ particles when measuring the particle number above the CIR. This excludes an additional loss channel besides the coupling to the molecular state.

Figure 4.17 shows the detected number of particles in the repulsive state depending on the magnetic offset field. One clearly observes two peaks which we identify as the COM-REL motion coupling resonances which involve the two molecular states excited in x - and y - direction of the anisotropic confinement with $\omega_{x2-0}/\omega_{y2-0} = 1.07$.

To analyze the dynamics of the coupling we stopped at different values of the magnetic offset field around the observed peaks and varied the duration of the stop. Figure 4.18 a) presents the result of the measurement. The atom number oscillates between the molecular state and the repulsive state showing that we have created a coherent superposition of the molecular and the repulsive state. By a sinusoidal fit we can extract the amplitude and the Rabi frequency Ω_{eff} of the

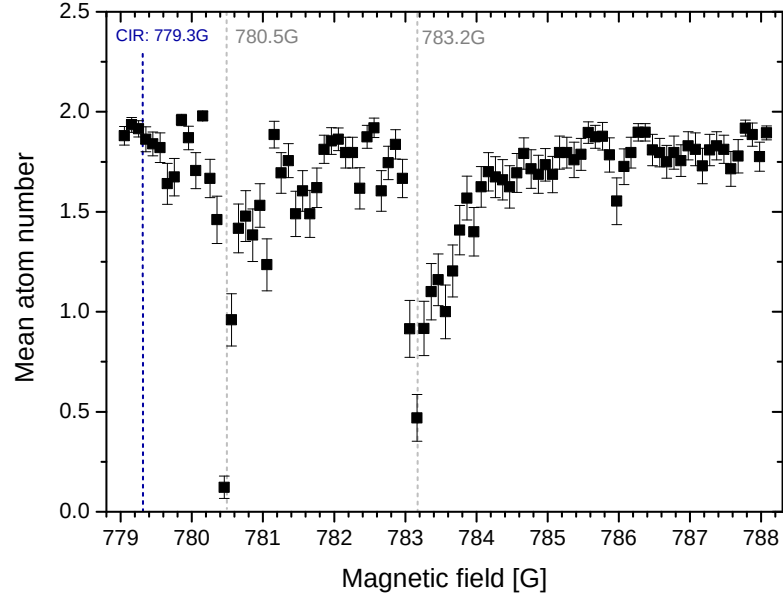


Figure 4.17.: **Disappearance of particles in the repulsive state.** Due to the COM-REL motion coupling the particles in the repulsive state couple into a molecule and disappear when detecting the number of particles in the non-bound state. One observes two peaks indicating COM-REL motion coupling resonances involving two excited molecular states in x - and y -direction of the confinement.

oscillation. The extracted values for different magnetic fields are shown in figure 4.18 b). From a Lorentzian fit to the amplitude we can extract the width (FWHM) of the coupling in terms of the magnetic offset field. Table 4.5 shows the width of the coupling resonances determined from the measurement. We add the position and the coupling strength calculated by Simon Sala who has performed the analysis described in [Sal12] with our trap parameters of section 4.2.2. Compared to the width of the CIR¹⁶ which is about 250 G, we find good agreement between our measurement and the numerical calculation.

Due to the dependence of the energy of the involved states on the magnetic field gradient we observe different positions of COM-REL motion coupling resonances with a gradient of $B' = 18.92$ G/cm applied, see appendix figure A.18. It also shows the dependence of the resonances position on the strength of the confinement which determines the energy of the COM- excitation.

With this measurement we have shown that in a two-particle system coherent

¹⁶The width of the CIR is determined by the width of the Feshbach resonance (see chapter 3.1.5)

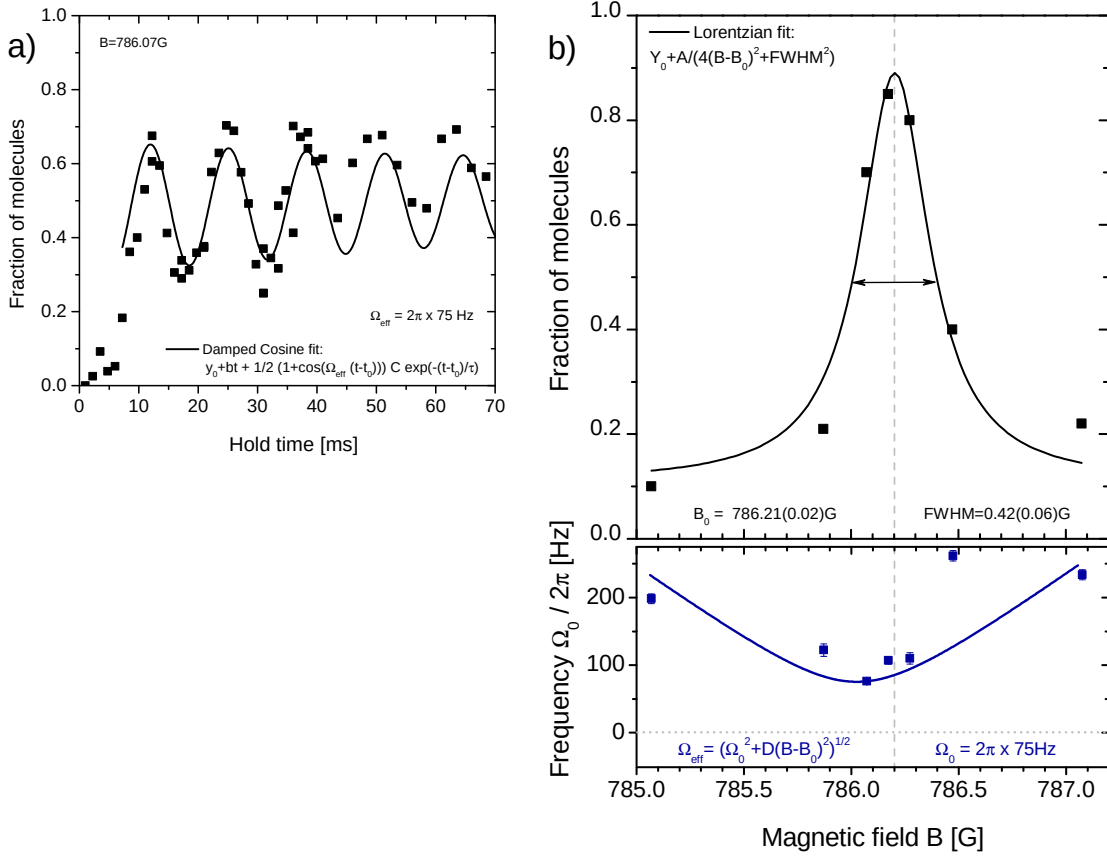


Figure 4.18.: **Dynamic of the COM-REL motion coupling.** a) Oscillation between the non-bound and the molecular state. From a sinusoidal fit we deduce the Rabi-frequency Ω_{eff} and the maximum observed fraction of molecules. b) Maximum amplitude and frequency of the oscillation. The data points are extracted from measurements analog to figure a) at different magnetic offset fields. The measurements are performed with an applied magnetic field gradient $B' = 18.92 \text{ G/cm}$. Similar measurements with the magnetic field gradient switched off are shown in the appendix (figure A.16 and A.17).

coupling into a molecule is possible without having a third particle present. We have contributed to the discussion [Pen11, Mel11, Sal12] which mechanism creates the loss features in the experiment of Elmar Haller et al. [Hal10] (see figure 4.19). These loss features have originally been interpreted to occur due to a maximum loss at CIRs which have been split up from one CIR when introducing an anisotropic confinement. Alejandro Saenz's [Sal12] and our group interpret the loss features

4.5. Center-of-mass to relative motion coupling

COM excitation	Position [G]		FWHM[G]		$\Omega_0[\text{Hz}]/2\pi$	
	exp.	num.	exp.	num.	exp.	num.
$\omega_{y\,2-0}$	780.5	776.01	0.25(0.03)	0.35	83	64
$\omega_{x\,2-0}$	783.2	779.02	0.42(0.06) (*)	0.35	75 (*)	69

Table 4.5.: **Comparison between experiment and numerical calculation**

(*) For the determination of the width and the Rabi Frequency of the coupling to the state in x -direction the magnetic field gradient $B' = 18.92$ G/cm had been applied during the measurement. Yet, the width in the case the gradient is switched off is similar to the one with gradient on which can be seen from figure 4.17.

as COM-REL motion coupling resonances of the type we have observed which is is a two-body effect. We assume that the loss of atoms in their experiment occurs due to collisions of molecules with residual atoms or molecules and relaxation into deeply bound states.

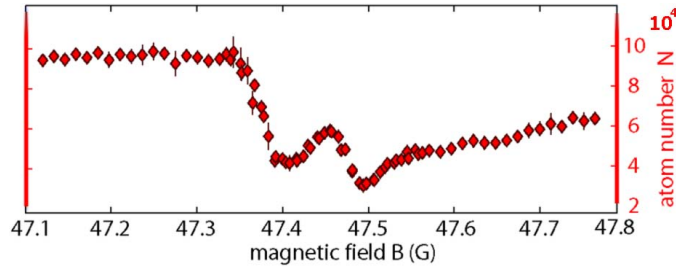


Figure 4.19.: **Two loss features in a 1D system close to the CIR observed in the Innsbruck experiment [Hal10].** The atoms are confined in 1D tubes created by an optical lattice with anisotropic confinement of aspect ratio $\omega_y/\omega_x = 1.10$. For large g_{1D} they observe two loss features which they have interpreted as a result of the splitting of the CIR into two CIR's in the presence of an anisotropic confinement. Plot taken from [Hal10] and adapted.

5. Pair correlations in systems with attractive interactions

Many fundamental properties of strongly correlated quantum systems are determined by the pairing mechanism in these systems. In the previous chapter we have observed that for vanishing pair correlation in an repulsively interacting system of distinguishable fermions the system becomes fermionized at the divergence of the 1D coupling strength [Gir60] [Gir10]. In the case of an effective attractive interaction in a 3D Fermi system macroscopic features such as the conductivity and the heat capacity can be described using the BCS pairing mechanism [Bar57]. The latter theory has originally been developed to explain the occurrence of superconductivity in various metals below a critical temperature and has been extended to describe superfluidity in ^3He [Leg72]. Immediately after the development of the BCS theory the BCS mechanism has also been applied to cover the properties of nuclei [Mig59]. Although the BCS theory can provide exact results in the limit of infinite particle number it is to some extent insufficient to describe mesoscopic Fermi systems such as light nuclei [Zel03]. Yet, pairing mechanism in the nuclei play a fundamental role which can be seen from the odd-even effect in the phenomenological formula of the mass of nuclei [Zel03]. In the context of nuclear physics several models such as the nuclear shell model [May49] [Sue49] and the seniority model [Zel03] have been developed which incorporate pairing effects. In this chapter we present our first steps towards investigating these effects by studying the pair correlations in 1D systems with attractive interparticle interaction for up to six particles. We probe the system using tunneling experiments similar to those introduced in the chapter 4. In the second part of the chapter we present dissociation measurements of a single particle from a finite Fermi-system. We show that the single particle dissociation energy exhibits an odd-even behavior as a function of the particle number similar the one observed for neutron dissociation in nuclei [Bri05].

5.1. Tunneling of two attractively interacting fermions

To begin with, we first discuss the expected degree of pair correlations in a two-particle system predicted by the 1D theory [Bus98] which inspired us to perform the subsequently presented tunneling measurement.

In a system of two attractively interacting fermions the correlations differ from that observed for repulsive interaction. In the latter case we observed that the pair correlation $g^2(0)$ vanishes at the point of Fermionization. Contrary, for two distinguishable fermions with attractive interaction the pair correlation $g^2(0)$ is expected to be increased compared to two noninteracting distinguishable particles; i.e. the probability to find the particles at the same point in space is larger. In the case of a pure state, the pair correlation $g^2(r)$ with r the relative distance between the two particles is proportional to the modulus squared of the two-particle wavefunction [Fra03]. The wavefunction and the energy of two interacting distinguishable fermions is known from the theory of Thomas Busch et al. [Bus98] which we discussed in chapter 3. In figure 5.1 we show the wavefunction of the relative motion depending on the interaction strength. For weak interaction the pair wavefunction resembles the Gaussian trap ground state (right hand side of the figure, $g_{1D} = -0.1$). The pair wavefunction is given by the parabolic cylinder functions with $g_{1D} < 0$ and $E_{\text{int}} < 0$. For stronger attractive interaction the wavefunction gets more and more contracted (left side of the figure). This reveals the increased pair-correlation in the system as the correlation is proportional to the modulus of the pair wavefunction. Simultaneously the binding energy of the system gets larger for increasing absolute values of the negative coupling constant.

5.1.1. Expected tunneling behavior

Before we present the results of the tunneling experiment we will first discuss the expected tunneling properties. We consider two limiting cases, the weakly attractive interacting regime and the strongly attractive interacting regime. We will follow some notes which we thankfully received from Massimo Rontani [Ron12a].

Weakly interacting regime

In the weakly interacting regime ($|g_{1D}| < 1 [a_{\parallel} \hbar \omega_{\parallel}]$) the correlations in the system are not yet developed to a large degree. This can be seen from the wavefunction in figure 5.1 at small g_{1D} . Although there are already correlations in the system the main origin for the localization of $g^2(r)$ is the confinement of the trap. Without

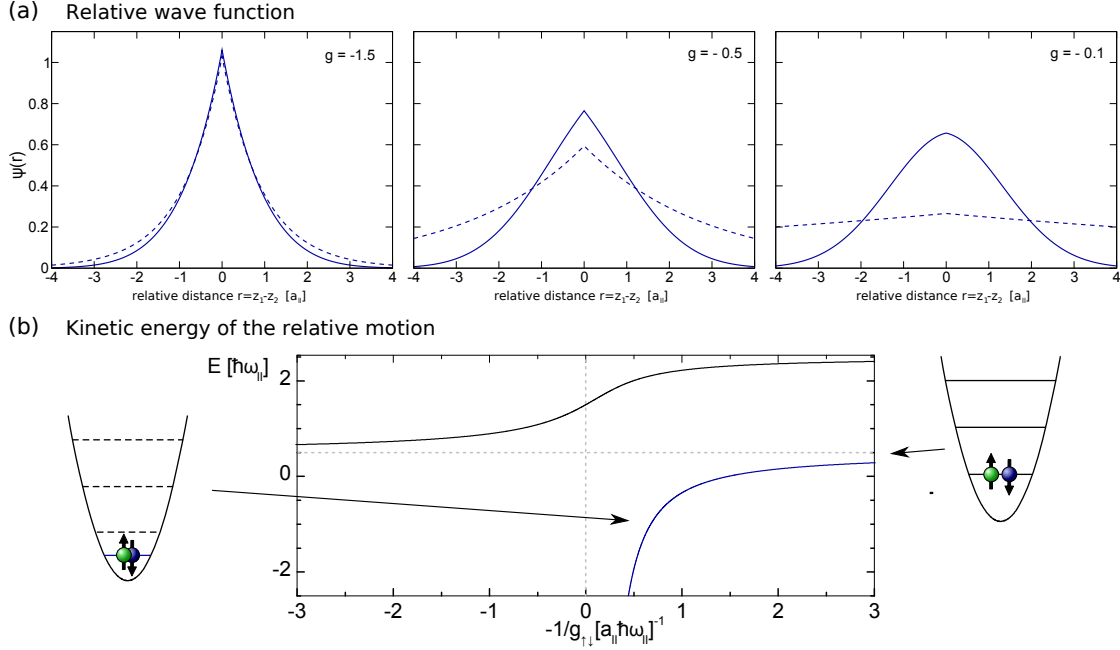


Figure 5.1.: **Pair correlation and pairing energy of two attractively interacting fermions in a 1D harmonic trap** [Bus98]. The pair-correlation is proportional to the modulus squared of the wavefunction [Fra03]. Figure (a) shows the wavefunction of the relative motion depending on the interaction strength. g_{1D} is given in units of $[a_{||}\hbar\omega_{||}]$. The solid curves represent the wavefunctions in a harmonic trapping potential, whereas the dashed curve show the normalized wavefunction in free space. One observes that for weak interacting the wavefunction is modified by the confinement. Yet, for stronger interaction the wavefunction becomes more and more independent of the confinement. For $g_{1D} \rightarrow \infty$ the wavefunction collapses to a single point in space at $r = 0$ with maximized local pair correlation $g^2(0) = 1$ and $g^2(r \neq 0) = 0$. (b) Pairing energy. At the same time as the correlations of the pair increase the interaction energy of the two distinguishable fermions gets larger. For $g_{1D} \rightarrow \infty$ the interaction energy diverges in a pure 1D system. Note: in a realistic trap with finite aspect ratio the quasi-1D approximation breaks down for large pairing energies and converges to the 3D universal bound state. Figure a) taken from [Ron12a] and adapted.

confinement, i.e. in free space, the correlations are much weaker which can be seen from the dashed curve. It represents the normalized pair wavefunction in free

space which is given by ¹

$$\Psi(r) = \frac{\sqrt{-g_{1D}}}{2^{1/4}} e^{\frac{1}{\sqrt{2}}g_{1D}|r|} . \quad (5.1)$$

Due to the weaker correlation one can assume that the particles in a tunneling experiment similar to the one presented in chapter 4, tunnel after each other. In the limit of $g_{1D} \rightarrow 0$ this assumption holds as the particles tunnel completely independent. In the case of weak interaction the tunneling process may be described by quasi-particle tunneling [Ron12b]: One of the two particles tunnels into the continuum leaving the other particle in the unperturbed ground state of the trap. Energy conservation requires the tunneled particle to possess the energy $E = E_0 - E_{\text{int}}(g_{1D})$ in the continuum as well as in the trap before it has tunneled. We have denoted E_0 as the energy of the ground state in the trap and E_{int} as the interaction energy of the pair. When the interaction energy gets larger than E_0 the energy of the quasi-particle has reached the trap bottom and thus the model is no longer applicable. The blue curve in figure 5.2 shows the expected tunneling rate of quasi-particle tunneling in the weakly interacting regime.

Strongly interacting regime

In the case of strong attractive interaction ($|g_{1D}| > 1 [a_{\parallel} \hbar \omega_{\parallel}]$) the size of the pair gets smaller than the width of the barrier and one expects the two particles to tunnel as a pair. When the pair has acquired strong correlations the tunneling rate of the pair should become independent of the binding energy of the pair. In this case the interaction properties can be regarded as an internal degree of freedom of the pair which should not effect the tunneling properties of the pair. The effective confining potential of the pair is shown in figure 5.3 and depends on the interaction strength for intermediate coupling strength. For strong attractive interaction the confining potential becomes more and more independent of g_{1D} . The tunneling barrier of the effective confinement is larger than the single particle potential due to the larger mass of the pair. The calculation of the effective confining potential was derived by Massimo Rontani using time-dependent perturbation theory. For the analysis we refer to future publications which will include this work [Ron12a]. From the effective confinement one can deduce the tunneling rate of the pairs which

¹The exponent in the 1D pair-wavefunction in free space is proportional to the coupling strength g_{1D} whereas in 3D the exponent of the wavefunction is proportional to the inverse scattering length a_{3D}^{-1} (see equation (3.34) in chapter 3). The different dependence of the wavefunction on the coupling parameter is one of the manifestations of the different dimensionality. It is associated with the distinct asymptotic behavior of the interaction energy in the different dimensions with $E_{\text{int},1D} \rightarrow -\infty$ for $g_{1D} \rightarrow -\infty$ and $E_{\text{int},3D} = 0$ in the limit of $a_{3D} \rightarrow \pm\infty$.

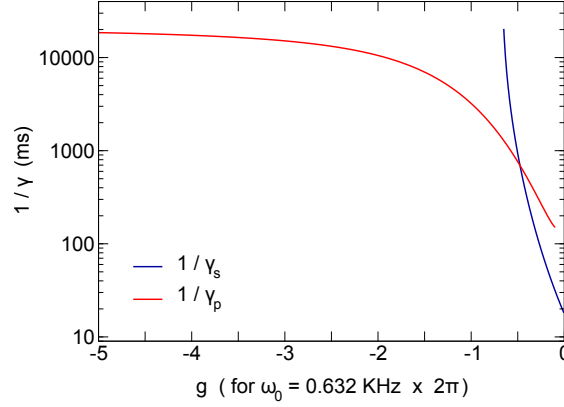


Figure 5.2.: **Single particle and pair tunneling rate.** The blue curve represents the rate for single particle tunneling which is dominant in the weak interacting regime. Single particle tunneling is expected to stop at a certain point when the interaction energy is as large that the quasi-particle has reached the trap bottom. In the strong interacting regime one expects the pair tunneling (red curve) to be dominant. For increasing interaction the tunneling rate become less dependent on the coupling strength. Then the interaction properties can be regarded as an internal degree of the pair and the tunneling rate is nearly not effected by a change of the coupling strength. Plot taken from [Ron12a] and adapted.

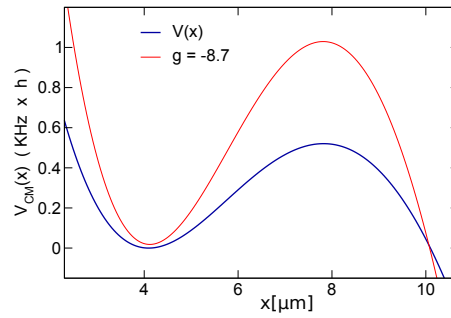


Figure 5.3.: **Single particle potential (blue) and effective potential of the pair (red).** For strong interaction the potential barrier of the pair is larger than the one for a single noninteracting particle. It is expected to converge to the potential of a particle with twice the mass of a single particle. g_{1D} is given in units of $[a_{\text{ron}} \hbar \omega_{\text{ron}}]$ with $\omega_{\text{ron}} = 2\pi \times 250$ Hz. Plot taken from [Ron12a] and adapted.

is shown by the red curve in figure 5.2. For strong interaction only pair tunneling is expected which tunes only weakly with g_{1D} . At a certain point towards weaker interaction the single particle tunneling events are expected to set in and should become the dominant process for small g_{1D} .

5.1.2. Tunneling model

To quantitatively describe the tunneling behavior we implement the two previously discussed processes into a simple tunneling model. Both processes are illustrated in figure 5.4. One possible process is subsequent single particle tunneling. In this

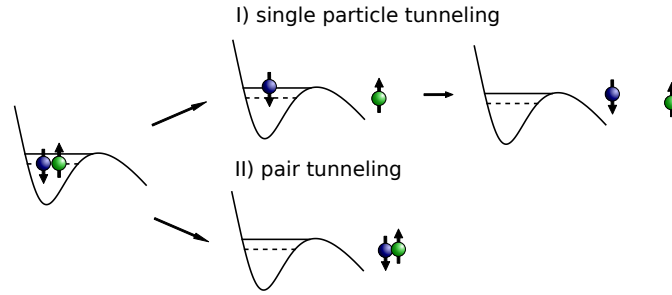


Figure 5.4.: **Tunneling model.** Two processes lead to a decay of the system: I) One particle tunnels first with rate γ_s and leaves the other particle in the unperturbed state of the trap. The second particle tunnels with a rate γ_{s0} of a noninteracting particle. II) Both particles tunnel as a pair with rate γ_p .

case one of the two interacting particles tunnels first with rate γ_s . The other particle is left in the unperturbed ground state of the trap. As the energy of the first particle is reduced by the interaction energy the second noninteracting particle experiences a smaller barrier height than the first one. Thus the second particle leaves the trap faster than the first one with rate γ_{s0} of a noninteracting particle.

The other possible process is pair tunneling. In this case both particles tunnel at the same time with rate γ_p .

To establish the model we introduce the quantities $P_2(t)$, $P_1(t)$ and $P_0(t)$ which are defined as the probability to find two, one or zero particles in the trap after a certain time t . From conservation of probabilities all quantities have to sum up to unity at any time t . The expectation value of the mean particle number in the trap can be calculated from the probabilities and is given by

$$N_{\text{mean}}(t) = 2 P_2(t) + 1 P_1(t) . \quad (5.2)$$

To relate the probabilities to the tunneling rates we consider the effect of the two different tunneling processes on the probabilities. The probability to find two particles in the trap is effected by both – subsequent single particle tunneling and pair tunneling:

$$\frac{dP_2(t)}{dt} = -(\gamma_s + \gamma_p)P_2(t) . \quad (5.3)$$

Here, we have assumed that the second particle of the subsequent tunneling process does not tunnel in the infinitesimal time interval dt . The rate equation can be easily solved and the decay law for the two particle probability reads:

$$P_2(t) = e^{-(\gamma_s + \gamma_p)t} . \quad (5.4)$$

The probability to find one particle in the trap initially increases due to the decay of the two-particle probability by single particle tunneling. Yet, the accumulated population decays by tunneling of the single particle with the rate γ_{s0} of a noninteracting particle. The corresponding rate equation reads:

$$\frac{dP_1(t)}{dt} = \gamma_s P_2(t) - \gamma_{s0} P_1(t) = \gamma_s e^{-(\gamma_s + \gamma_p)t} - \gamma_{s0} P_1(t) . \quad (5.5)$$

We solve the latter equation with the initial conditions $P_2(0) = 1$ and $P_1(0) = P_0(0) = 0$ and obtain the probability of finding one particle in the trap:

$$P_1(t) = \frac{\gamma_s}{\gamma_s + \gamma_p - \gamma_{s0}} \left[e^{-\gamma_{s0}t} - e^{-(\gamma_s + \gamma_p)t} \right] . \quad (5.6)$$

The probability to find no particle left in the trap is given by the identity

$$P_0(t) = 1 - P_2(t) - P_1(t) . \quad (5.7)$$

In the following we want to obtain the quantities $P_2(t)$, $P_1(t)$ and $P_0(t)$ from the actual experiment of two attractively interacting fermions tunneling out of the microtrap potential.

5.1.3. The tunneling experiment

For the tunneling measurement we prepare two particles, one in the hyperfine state $|1\rangle$ and one in state $|3\rangle$, in the ground state of the trapping potential². To probe the system we employ the same method as described in chapter 4.1.2. We deform

²In this section we denote the two distinguishable fermions by their label for the hyperfine state – $|1\rangle$ and $|3\rangle$ – instead of $|\downarrow\rangle$ and $|\uparrow\rangle$. The reason for this is the dependence of the potential on the hyperfine state which makes an indication of their different hyperfine state necessary.

the potential such that there is a potential barrier of defined height through which the particles can tunnel out of the trap. After a certain hold time we ramp the potential back up and measure the number of particles remaining in the trap. By performing many of these measurements at different holdtimes we can map out the time evolution of the particle number in the tilted potential. Figure 5.5 shows the mean particle number in the trap as a function of the hold time for three different values of the interparticle interaction ³. For a system of two noninteracting

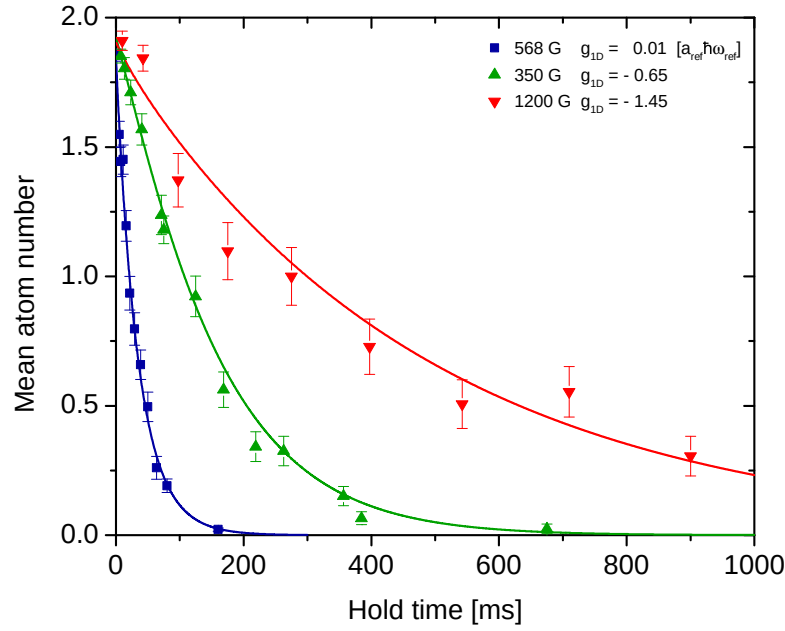


Figure 5.5.: **Tunneling dynamics.** To study the tunneling dynamics of an attractively interacting system we set the barrier to a fixed height and then tune the strength of the attractive interaction. The mean number of remaining particles after a certain hold time increases when tuning the interaction strength from zero (blue data) to larger values (green to red). This demonstrates that the effective barrier height increases as a result of the energy shift due to the increasing attraction.

particles the loss occurs on a timescale with a $1/e$ -lifetime of about 30 ms. In the presence of an attractive interaction with $g_{1D} < 0$ the energy of the system is reduced by the interactions. This leads to an effective increase in the height of the tunneling barrier and therefore to an increased lifetime of the sample. The decreasing decay rate of the total sample already indicates that the system has

³The values of the different interaction parameter g_{1D} are listed in table A.2 in the appendix.

acquired larger pair correlation for increased interaction strength. By recording the number of particles in the trap in each realization of the experiment we can also determine the probability to find two, one or zero particles in the trap.

To characterize the tunneling behavior we want to apply the previously established model to extract the rates γ_{s0} , γ_s and γ_p from the measured probabilities. Unfortunately we cannot simply fit the equations (5.4), (5.6) and (5.7) to the measured probabilities. The reason for that is a spin dependent confining potential. A variation of the coupling strength which is realized by tuning the magnetic offset field also modifies the magnetic potential and thus influences the tunneling rates of interest. Hence, to apply the model we first have to determine the effect of the spin dependent confining potential on the noninteracting tunneling rate γ_{s0} for particles in the two different hyperfine states.

5.1.4. Effects of the spin dependent potential

In the case the potential is the same for all offset fields and all hyperfine states one can deduce the tunneling rate γ_{s0} from the decay of two noninteracting particles at $g_{1D} = 0$ by fitting $N_{\text{mean}} = N_0 e^{\gamma_{s0} t}$ to the mean particle number. This is valid for magnetic field values above 850 G where the magnetic moment of all states are identical within 0.5% (see figure 5.6 a). However, for lower offset fields where we have also performed tunneling measurements, the magnetic moments of state $|1\rangle$ and $|3\rangle$ tune strongly with the magnetic offset field with a deviation between the magnetic moments of the states of more than 1%. Thus, the linear potential V_{mag} of the potential well depends on the offset field and on the hyperfine state of the particle. This means that in a constant optical potential V_{opt} , the tunneling rate $\gamma_{s0|1\rangle}$ and $\gamma_{s0|3\rangle}$ of a single particle in state $|1\rangle$ or $|3\rangle$ are different.

Determination of the spin dependent single particle rates

To determine the spin dependent single particle rates we perform a reference measurement with a single particle at constant magnetic field gradient for two different hyperfine states and two different offset fields. The measured rates are given in table 5.1. In a next step we determine the magnetic moment of each hyperfine state at the different offset fields. To apply the same parametrization of the potential as in chapter 4.2.2, we introduce a new parameter $\mu_{c,B|\text{state}\rangle}$ which depends on the field and the hyperfine state. Then the potential reads:

$$\begin{aligned} V(p, z) &= V_{\text{opt}}(p, z) + V_{\text{mag}}(z) \\ &= p V_0 \left(1 - \frac{1}{1 + (z/z_r)^2} \right) - \mu_m \mu_{c,B|\text{state}\rangle} B' z \end{aligned} \quad (5.8)$$

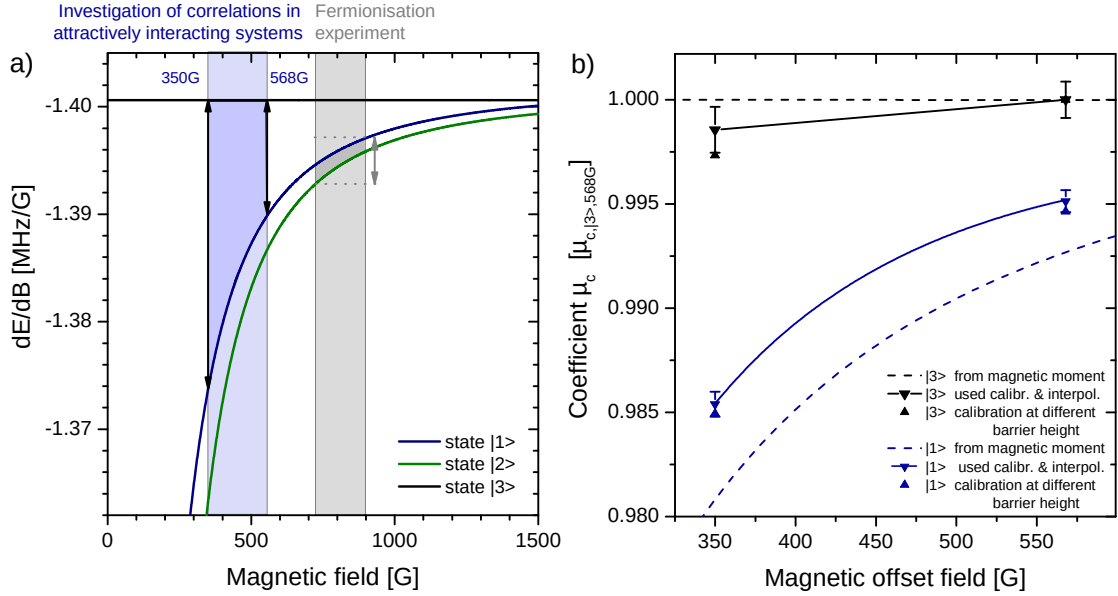


Figure 5.6.: **Origin of the spin dependent potential.** Figure a) shows the magnetic moment of the three lowest hyperfine state of ${}^6\text{Li}$ [Bre31]. In the region of magnetic offset fields where we perform the tunneling measurement the magnetic moments differ by more than 1% (blue shaded area). To incorporate the difference in the magnetic moment into the parametrization of the potential we introduce the coefficient μ_c according to equation (5.8). The solid curves in figure b) show the coefficients for the different hyperfine states calibrated from a reference measurement (data points) .

With the definition $\mu_{c,792G|1\rangle} = 1$ the potential is identical to the one used in chapter 4.2.2 and 4.3. From the functional form of the magnetic moment plotted in figure 5.6 we calculate $\mu_{c,568G|3\rangle}$. Using a WKB approximation (see chapter 4.2.1) we can fix the optical potential parameter p_{ref} of the reference measurement at 568 G by matching the calculated tunneling rate $\gamma_{|3\rangle,\text{WKB}}$ to the experimental one. Performing the same analysis, now with the optical potential fixed at p_{ref} , we can determine $\mu_{c,350G|3\rangle}$, $\mu_{c,568G|1\rangle}$ and $\mu_{c,350G|1\rangle}$ by matching the tunneling times from the WKB calculation with the measured ones at the corresponding magnetic offset field. The results are shown in table 5.1 and figure 5.6 b).

As expected, we find that the coefficient of the linear potential varies within more than 1%. However, the absolute value of the determined coefficient deviates from

magnetic field B G	$ \text{state}\rangle$	$\gamma_{ \text{state}\rangle}$ $\frac{1}{\text{s}}$	$\sigma_{\gamma_{ \text{state}\rangle}}$ $\frac{1}{\text{s}}$	$\mu_{c,B \text{state}\rangle}$	$\sigma_{\mu_{c \text{state}\rangle}}$ 10^{-4}
568	3	35.25	3.57	1.00457	8.7
350	3	30.12	2.81	1.00311	11.0
568	1	21.77	1.12	0.99968	5.3
350	1	8.28	0.49	0.98989	6.0
496	3	interpol.		1.00407	10%
423	3	interpol.		1.00356	10%
496	1	interpol.		0.99806	10%
423	1	interpol.		0.99512	10%
>850	1,3	$\mu_{c,568\text{G} 3\rangle}$		1.00457	8.7

Table 5.1.: **Potential coefficient μ_c .** The table shows the tunneling rates of the reference measurement and the determined coefficient for different hyperfine states and different offset fields. The calibration has been performed using a WKB approximation. For $B > 850$ G the magnetic moment of both states is identical within a relative deviation of 3×10^{-3} . Thus we set $\mu_{c,>850\text{G}|1\rangle,|3\rangle} = \mu_{c,568\text{G}|3\rangle}$

the theoretical prediction⁴. The coefficient of state $|3\rangle$ is not expected to tune with the magnetic offset field. The observed weak increase of $\mu_{c,B|3\rangle}$ for larger fields could be explained by an additional magnetic field gradient created by the offset coils which is proportional to the offset field.

Although the calibrated magnetic moment differs from the prediction we use it to analyze the tunneling data of attractively interacting fermions, because the origin of the deviation should cancel to first order when used for both – for the calibration of the potential and for the analysis of the tunneling dynamics with interaction present.

⁴We have repeated the calibration for a larger optical trap depth which increased the barrier height and thus decreased the tunneling rates by a factor of 5. We have done this to check if the discrepancy between the calibration and the theoretical prediction is a result of the WKB approximation which becomes more inaccurate for smaller barrier heights. However, we do not see any effect from this as the second calibration agrees with the first one within the errors.

Modification of the tunneling model

In the rate equations of equation 5.3 and 5.5 which describe the tunneling dynamics we have to take account for the different magnetic moments of particles in state $|1\rangle$ and $|3\rangle$ which experience different confining potentials (see figure 5.7). Therefore we have to consider different tunneling rates $\gamma_{s0|1}$ and $\gamma_{s0|3}$ for the different states. For the rate with which a two-particle system decays we have to distinguish if a

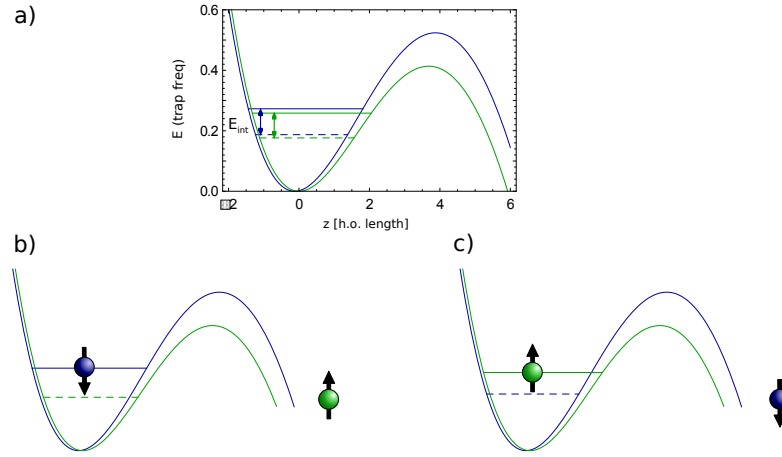


Figure 5.7.: **Spin dependent potential.** Figure a) sketches the potential for the different hyperfine states at a magnetic offset field of 350 G. The blue curve shows the potential for state $|1\rangle$, the green curve the one for state $|3\rangle$. The states shifted by the interaction energy at the corresponding coupling strength of $g_{1D} = -0.65[a_{\text{ref}}\hbar\omega_{\text{ref}}]$ are indicated by the dashed lines. The units are the trap frequency determined by a harmonic approximation to the optical potential and the corresponding harmonic oscillator length. Figure b) and c) depict the two different possibilities of subsequent single particle tunneling with either state $|3\rangle$ tunneling first (b) or state $|1\rangle$ (c).

particle in state $|1\rangle$ or a particle in state $|3\rangle$ leaves the trap first. Then the probability to find two particles in the trap reads:

$$P_2(t) = e^{-(\gamma_{s|1} + \gamma_{s|3} + \gamma_p)t} = e^{-\gamma_2 t} \quad (5.9)$$

If we find one particle in the trap it is either a particle in state $|1\rangle$ or one in state $|3\rangle$. If it is one in state $|1\rangle$ we know that the first particle which has left from the two-particle system must have been a state- $|3\rangle$ -particle and vice versa. This leads to two different probabilities of finding one particle in the corresponding

hyperfine state. The probability of finding one particle in any state is the sum of both probabilities and is given by:

$$P_1(t) = \frac{\gamma_{s|3\rangle}}{\gamma_2 - \gamma_{s0|1\rangle}} \left[e^{-\gamma_{s0|1\rangle}t} - e^{-\gamma_2 t} \right] + \frac{\gamma_{s|1\rangle}}{\gamma_2 - \gamma_{s0|3\rangle}} \left[e^{-\gamma_{s0|3\rangle}t} - e^{-\gamma_2 t} \right] \quad (5.10)$$

Before we can fit the modified probability functions (5.9) and (5.10) to the measured data we first have to determine the relative optical trap depth p to fix the remaining free parameter of the potential of the actual tunneling experiment.

To extract the optical trap depth p we iterative fit a double-exponential decay⁵ of the form

$$N_{\text{mean}} = N_0 \left(e^{-\gamma_{s0,568\text{G}|3\rangle}t} + e^{-r \gamma_{s0,568\text{G}|1\rangle}t} \right) \quad (5.11)$$

to the mean number of two noninteracting particles at 568 G, whereas r is varied in each step. After each fit we apply a WKB calculation in which we tune the parameter p such that $\gamma_{s0,568\text{G}|3\rangle, \text{WKB}}$ becomes equal to $\gamma_{s0,568\text{G}|3\rangle, \text{fit}}$. The parameter r for the next step of the iteration is determined by $\gamma_{s0,568\text{G}|1\rangle, \text{WKB}} / \gamma_{s0,568\text{G}|3\rangle, \text{WKB}}$. We stop the iteration when $\gamma_{s0,568\text{G}|1\rangle}$ and $\gamma_{s0,568\text{G}|3\rangle}$ from the WKB approximation and from the fit coincide within a ratio of 10^{-3} . The derived optical trap depth parameter p is given in table A.3 in the appendix.

5.1.5. Application of the spin dependent model

After the external potential of the particles in the two different states is completely determined we can calculate the rates $\gamma_{s0|1\rangle}$ and $\gamma_{s0|3\rangle}$ of the actual tunneling experiment for any offset field using the WKB approximation. This leaves three free parameters left in the model which we use to describe the tunneling process. However, simultaneously fitting $P_2(t)$ and $P_1(t)$ with $\gamma_{s|1\rangle}$, $\gamma_{s|3\rangle}$ and γ_p as free parameters results in large uncertainties of the derived parameters. Yet, by testing the fit we have observed that the rate for pair-tunneling is comparatively small for an interaction strength of $g_{1\text{D}} = -0.65 [a_{\text{ref}} \hbar \omega_{\text{ref}}]$ at 350 G. Hence, we neglect pair-tunneling in the model and set γ_p to zero for the evaluation of the data. In the case this assumption was wrong, our model would provide a too large probability to find one particle in the trap which we can later check by a comparison to the data.

Although this assumption reduces the number of free parameters to two ($\gamma_{s|1\rangle}$ and $\gamma_{s|3\rangle}$) a fit of P_2 and P_1 might still provide results of large uncertainties. Yet,

⁵Uncorrelated tunneling with two different decay rates is parametrized by a double-exponential decay.

by adding one more condition we can set limits to $\gamma_{s|1}$ and $\gamma_{s|3}$: Due to the assumption of no pair-tunneling only subsequent single particle tunneling leads to the decay of the two particle probability according $P_2 = e^{-\gamma_2 t}$ with $\gamma_2 = \gamma_{s|1} + \gamma_{s|3}$. After one particle has tunneled the other particle with different state is left in the unperturbed ground state. As discussed in section 5.1.1 the kinetic energy of the tunneled particle in the continuum is reduced by the interaction energy. As we know the shape of the potential we can relate the energy $E = E_0 - E_{\text{int},|\text{state}\rangle}$ to $\gamma_{s|\text{state}\rangle}$ using a WKB approximation. With this we obtain a relation for $E_{\text{int},|\text{state}\rangle}$ which monotonically increases with the inverse tunneling rate $\gamma_{s|\text{state}\rangle}^{-1}$. The important additional condition is, that both particles supply the same amount of energy – namely the interaction energy – when they leave the trap first. Thus we set

$$E_{\text{int},|1\rangle} = E_{\text{int},|3\rangle} . \quad (5.12)$$

Then we vary E_{int} and calculate $\gamma_{s|1}$ and $\gamma_{s|3}$ with the WKB approximation until $\gamma_{s|1} + \gamma_{s|3} = \gamma_2$, whereas γ_2 is determined by a fit to the P_2 -data. With this ansatz⁶ we have determined all free parameters of the tunneling model. Finally we have to check if the probability P_1 is consistent with the data of finding one particle in the trap as we have neglected pair-tunneling. We plot P_1 with the determined parameters (green solid line in figure 5.8) to the data (green points) and find good agreement between the model of pure subsequent single particle tunneling and the experiment.

5.1.6. Results for different interaction strength

The good agreement between model and experimental data confirms that pair-tunneling can only play a minor role at an interaction strength of $g_{1D} = -0.65 [a_{\text{ref}}\hbar\omega_{\text{ref}}]$. In the following we present the data and the results of the analysis for different values of the interacting strength⁷.

Evidence for increased correlation

Because of the increasing interaction energy for larger g_{1D} the timescale for tunneling increases significantly. Hence, to compare the probability of finding one particle in the trap at different interacting strength we have to rescale it onto a common value which is the mean particle number $N_{\text{mean}} = 1 \times P_1 + 2 \times P_2$ (see figure 5.9). For zero interaction strength (blue) we find that the probability of finding

⁶The ansatz has included the two assumptions: i) no-pair tunneling ii) whichever particle tunnels first in the subsequent single particle tunneling process, it provides the same interaction energy.

⁷A table of all deduced parameters can be found in the appendix (A.4).

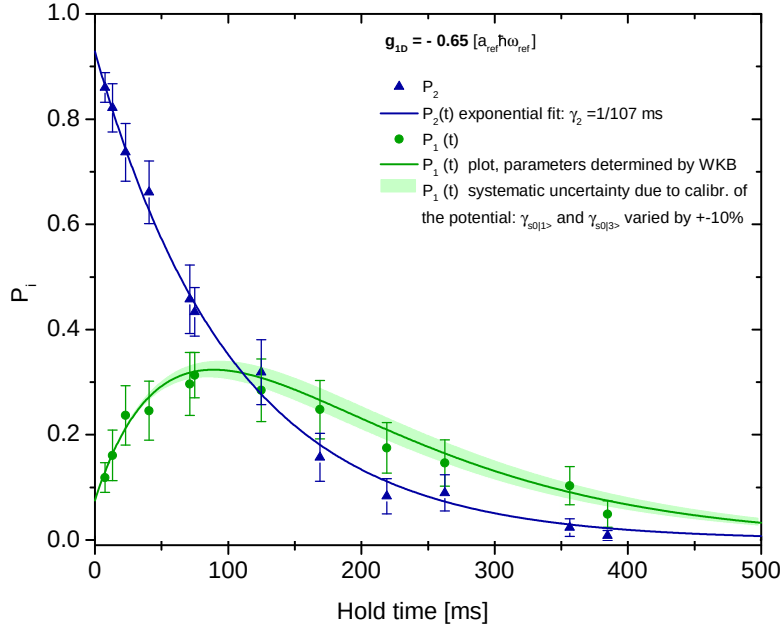


Figure 5.8.: **Probability of finding two or one particle in the trap.** We use the established model to describe the measured probability of finding two particles (blue curve) or just one particle (green curve) in the trap after a certain hold time. In the model we have included the finite preparation fidelity $f < 1$ by setting $P_2(0) = f$ and $P_1(0) = 1 - f$. We find that no pair tunneling is needed to describe the data at that value of the coupling strength ($\gamma_p = 0$). The green shaded band shows P_1 with $\gamma_{s0|1>}$ and $\gamma_{s0|3>}$ varied within its 10% relative error resulting from the systematic uncertainty of the determination of the potential coefficient μ_c .

one atom follows that of completely uncorrelated tunneling which is indicated by the black dashed parabola

$$P_1 = N_{\text{mean}} - \frac{N_{\text{mean}}^2}{2}. \quad (5.13)$$

For increasing attractive interaction we observe that the probability of finding one atom in the trap drastically decreases and deviates from uncorrelated tunneling. Although we just measure the absolute number of particles inside the trap after the tunneling, the observed reduction of P_1 is a strong evidence for increased pair correlation of the two particles in the trap. Up to $g_{1D} = -0.65 [a_{\text{ref}} \hbar \omega_{\text{ref}}]$ the model of subsequent single particle describes the data very well and pair tunneling can

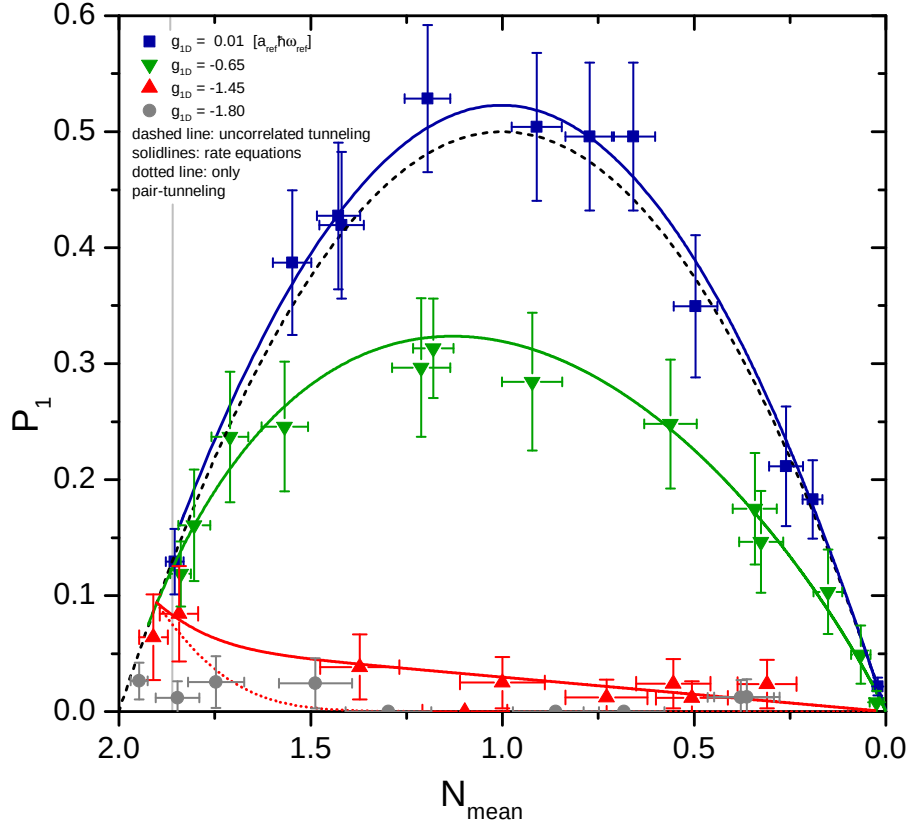


Figure 5.9.: **Probability of finding one particle in the trap for different interaction strength.** The solid curves are determined by fits to the P_2 -data and by application of the model of subsequent single particle tunneling. For zero interaction the probability follows the expectation value of completely uncorrelated tunneling given by the black dashed line which has no fit parameter and is valid for a spin independent potential. For increasing interaction it becomes less likely to find a single particle in the trap. We interpret this effect to be a result of increased pair-correlations at larger interaction strength. The red dotted line represents the result in the case of pure pair-tunneling. The reason to also find single particles in the pair-tunneling model is the finite preparation fidelity taken into account for all presented solid curves.

only play a minor role. For larger g_{1D} this model is also consistent with the data. However, the occurrence of finding one particle in the trap is on the order of a few percent which is as small as the errors. Hence, for the interaction strength of

$g_{1D} = -1.45 [a_{\text{ref}}\hbar\omega_{\text{ref}}]$ we cannot state to which extend the two particles tunnel as a bound object. The data could also be consistent with pure pair tunneling (red dotted line), yet with lower probability. For even larger interaction strength ($g_{1D} = -1.80 [a_{\text{ref}}\hbar\omega_{\text{ref}}]$, gray data points), we do not observe that just one atom is left in the potential at all. For this interaction strength both, subsequent single particle tunneling and pair-tunneling provides the same result, namely zero probability to find one atom in the trap. Thus, for large interaction ($g_{|1\rangle-|3\rangle} > 1$) we cannot make a clear statement which process dominates. Nevertheless with our measurement we have shown that for weak interaction ($g_{1D} < 1$) correlated subsequent tunneling is the dominant process.

Determined interaction energy

While we have extracted the tunneling rates with the model of subsequent single particle tunneling we have also estimated the interaction energy which is shown in figure 5.10. In a comparison with the two-particle theory in a harmonic trap [Bus98] the determined values are approximately a factor of four smaller. We think there are two reasons for this discrepancy. First, the effective g_{1D} in units of twice the energy of the ground state is too large, because the particles do not experience a pure harmonic confinement in the potential well. Second, the overlap between the wavefunction of the two particles is reduced because the particles are trapped in different potentials which are slightly shifted with respect to each other (see figure 5.7). For both effects it is difficult to quantitatively estimate the influence on the measured parameters. Due to this large systematic error of unknown origin it is difficult to compare the determined interaction energy as well as the determined tunneling rates with the theoretical model of Massimo Rontani [Ron12a]. A direct comparison requires further analysis of these effects. Yet, although the results are covered by systemic uncertainties we can still make some quantitative comparison of the interaction energy for different particle number when keeping the interaction strength constant. The dependence of the interaction energy on the particle number will be discussed in the following section.

5.2. Observation of the odd even effect

In the previous section we have set up a model with which we can determine the interaction energy from the tunneling rates of an interaction two-particle system. We have observed that pure subsequent single particle tunneling is sufficient to describe the tunneling behavior in our system for intermediate coupling strength of $g_{1D} = 0.6 [a_{\text{ref}}\hbar\omega_{\text{ref}}]$. In this section we study the effect of the particle number on

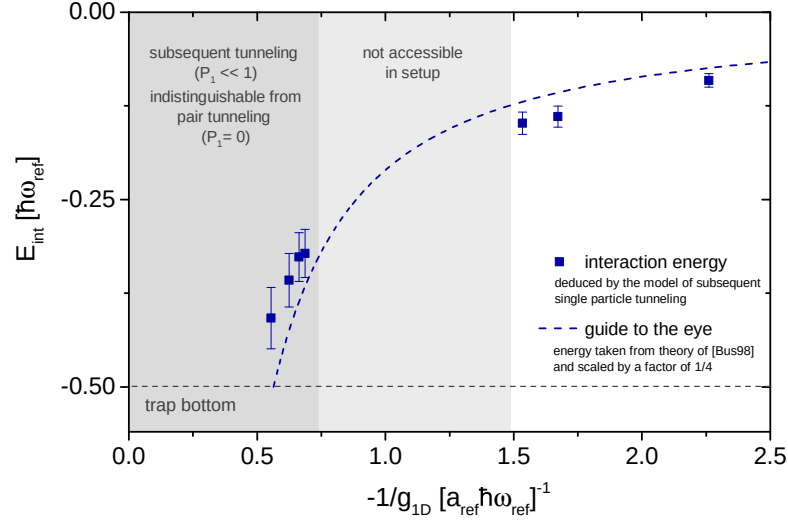


Figure 5.10.: **Interaction energy of two attractively interacting fermions.**

We determined the interaction energy using the model of subsequent single particle tunneling. One observes that for stronger coupling strength the interaction energy increases and follows the trend of [Bus98] (guide to the eye, blue dashed line). Yet, the absolute values of the energy differ by a factor of 4. Values of g_{1D} within the light gray area are not accessible in our system. This is a consequence of non-existent values of the ^6Li scattering length $a_{3D|1\rangle-|3\rangle}$ within a gap from $-900 a_0$ to $-2200 a_0$ for magnetic field values below 1500 G. For values of g_{1D} within the dark gray area subsequent single particle tunneling gets indistinguishable from pair-tunneling. The application of pure subsequent single particle tunneling might not cover the whole physical behavior of the system. An absolute lower boundary for the energy determined by subsequent single particle tunneling is the trap bottom which is indicated by the horizontal dashed line. The error is the 10% relative uncertainty originating from the statistical error of the fit of γ_2 and $\gamma_{s0|1\rangle}$. Not included is the 5 – 10% systematic uncertainty in the shape of the potential due to the errors in the reference measurement.

the energy of the system to gain insight into the correlations of the 1D few-fermion systems.

To perform the tunneling measurements we prepare systems with particle numbers ranging from 2 – 6 particles as illustrated in the upper panel of figure 5.11. Then we deform the potential well such that the particles tunnel on timescales of

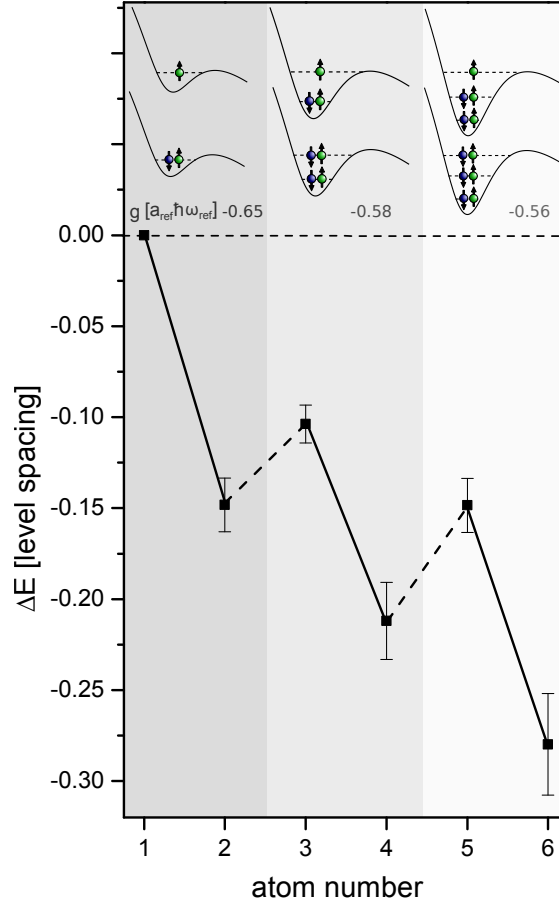


Figure 5.11.: **Odd-even effect.** The single particle dissociation energy depending on the particle number is determined using the model of subsequent single particle tunneling. The energy is given in units of the level spacing of the uppermost two single particle states in the trap. The small difference in g_{1D} for the different particle numbers is due to the dependence of the coupling strength on the confinement which tunes with the depth of the optical potential. Yet, for each of the three configurations (differently gray shaded region) the potential is identical. We find that the system becomes more stable against decay for an even number of particles. The dissociation energies show a strong odd-even effect, similar to the one observed for nuclei. The error is the 10% relative uncertainty originating from the statistical error of the fit of γ_N and $\gamma_{s0|1}$. Not included is the 5 – 10% systematic uncertainty in the shape of the potential due to the errors in the reference measurement.

10 – 1000 ms. After switching of the tunneling barrier we again record the number of particles and obtain the probability of finding N particles left in the potential well after certain hold times. We observe that primarily only the particles highest up in the potential contribute to the tunneling process. For a system with odd-particle number we observe that most of the time one particle leaves the trap during the observation time, whereas for a system with even particle number we observe that two particles leave the trap. Hence, we can apply the model previously established for two particles also to larger systems with even particle number. For systems with odd particle number we simply fit a single-particle decay to the particle highest up in the potential.

We have performed this analysis for systems with $N = 3$ to $N = 6$ particles; the data of the measurement and the determined probabilities of finding the different amount of particles in the trap are given in the appendix (figure A.19 and figure A.20). For the latter analysis we have slightly modified the rate equations to consider the following two issues:

i) After one particle has tunneled from a system with $N \geq 3$ particles the remaining system in the trap is still interacting. Although we then cannot determine the total interaction energy of the initial system we can still extract the energy of the particle which tunnels through the barrier. Besides some offset energy this energy is equivalent to the energy needed to dissociate one particle from the N -particle system. Hence, we denote the energy determined from the model by 'single particle dissociation energy'.

Yet, to apply the model for an even-numbered system, i.e. for the 4- and 6-particle system, we need to know the single particle tunneling rates of the interacting 3- and 5-particle systems. For one hyperfine state we obtain these quantities directly from the tunneling measurements of the corresponding system, for the other spin component we have to apply a WKB calculation to account for the spin dependent potential. The determined rates are given in table A.5 in the appendix.

ii) When the potential is deformed such that the uppermost one or two particles can tunnel on experimental timescales, the particles further down in the trap also exhibit a final lifetime τ_N . Although these rates are much longer than the observation times ($\tau_{N=2} = 40$ s, $\tau_{N=4} = 26$ s) we include these lifetimes as an overall decay in the rate equations.

Considering these modifications we can determine the dissociation energy depending on the particle number in a similar way as in the previous section. The results of the dissociation energies in units of the level spacing are shown in figure 5.11.

5.2.1. Interpretation of the results

We observe a strong odd-even effect in the dissociation energy as a function of the particle number. This reveals the effect of pairing in the 1D system: We can consider the single particle levels in the potential as the shells of the 1D system. In our two-component system each shell can be filled by two distinguishable fermions. With its two-fold degeneracy in the noninteracting case the shell possesses a trivial structure. In the case of weak interaction the energy of the shell is perturbed by the interaction energy of a single particle with a particle on the same shell (intra-shell interaction) and with particles on different shells (inter-shell interaction).

Due to the larger spatial overlap of the wavefunction of two particles occupying the same shell the intra-shell interaction energy is much larger than the inter-shell interaction. In the case of a system with odd particle number the shell at the Fermi edge is filled by just one fermion of a certain spin. The perturbation of the shell is only determined by the inter-shell interaction of this fermion with the other distinguishable fermions occupying lower shells. If we add another distinguishable fermion to the same shell – the system then become an even-particle system – the intra-shell interaction significantly lowers the energy of the shell. Thus, whenever a shell is closed, i.e. occupied by a pair of distinguishable fermions, which is the case for an even-numbered system it becomes more stable against decay. This results in an odd-even effect as we have observed it in the experiment. Additionally to the odd-even effect we observe an overall increase of the mean dissociation energy for larger particle number. We attribute this effect to the increased number of inter-shell interaction 'partners' for larger particle numbers.

5.2.2. Comparison to similar systems and prospects for further experiments

One of the most prominent few-fermion systems for which the single particle dissociation energy depends on the shell structure are light nuclei. The nuclear interaction involving neutral nucleons is governed by meson exchange and results in an effective attractive interaction between neutron and protons [Pov08]. This interaction is incorporated in the seniority model of nuclei [Zel03]. In this model the Hamiltonian including all interaction terms is separated into a mean-field term and a pairing term. The solution of the mean-field Hamiltonian determines the overall spectrum with n -fold degeneracy of single particle levels forming one shell. In this model the pairing term only acts on the n -fold subspace, i.e. on pairs occupying the same shell. If pairing is present the degeneracy of the shell is dissolved. (see figure 5.12). Unpaired particles do not participate in the interaction beyond the mean field level [Ron09] and the energy of the system is minimized

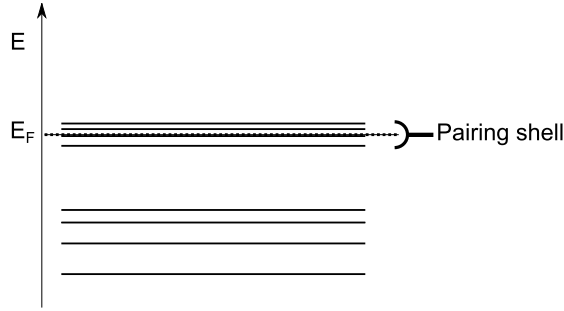


Figure 5.12.: **Energy spectrum of a few-fermion system.** The effects of pairing can be isolated from the rest of the spectrum by assuming that single particle states form one degenerate energy shell near the Fermi surface. Taken from [Ser11a].

in the case of no unpaired particles. In nuclei with odd particle number there is always a particle which is not paired and thus its energy is larger. This results in an odd-even effect of the binding energy depending on the particle number. This effect is similar to our observed structure in the dissociation energy. For nuclei the effect has also been seen from neutron dissociation measurements [Bri05].

In general a completely filled shell in the 3D nuclei structure becomes evident in the extraordinary stability of isotopes with certain nucleon numbers. The particle number for which a shell is closed, i.e. a completely filled shell, are called the magic numbers which in the case of stable nuclei are $N = 2, 8, 20$, etc. [Ots01].

In our 1D system governed by a pure contact interaction the magic numbers are rather trivial with $N = 2, 4, 6$, etc. Yet, the observation of these structure is a first important step towards extending our studies to few-fermion systems in larger dimensions.

Expected shell structure in 2D

In two-dimensional system with contact interaction for instance, a more rich shell structure is expected (figure 5.13 after [Ron09]). For weak interaction (green) the shell structure is dominated by the structure of the 2D harmonic oscillator with completely filled shells for $N = 2$ and $N = 6$. For stronger attractive interaction (blue and black) this structure is gradually washed out and the measured quantity exhibits a pronounced odd-even pattern with pairing effects likely being the underlying mechanism. [Ron09].

The fundamental two-dimensional shell structure has been observed in experiments with quantum dots [Kou01]. Yet, in these experiments the interaction is dominated by Coulomb interaction of the electrons in the quantum dot. The tun-

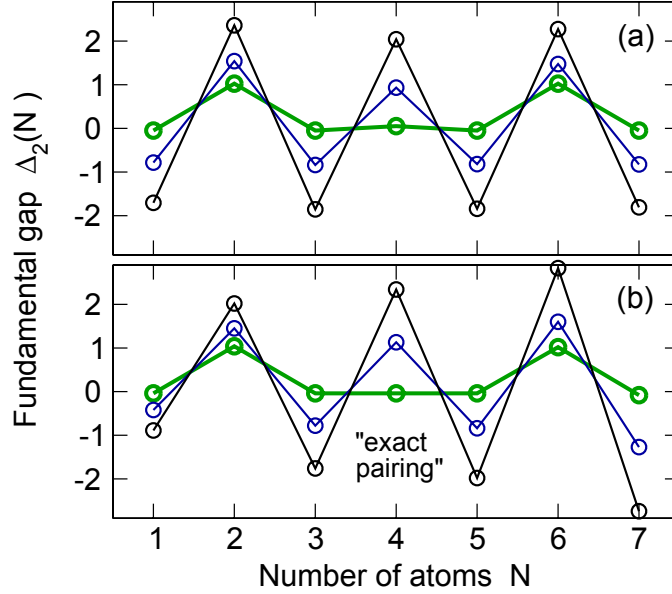


Figure 5.13.: **Fundamental gap in a 2D system.** The fundamental gap is defined as the difference between the modulus of the particle removal and the particle addition energy. For a N -particle system the particle addition (removal) energy is given by the difference of its chemical potential to the system confining $N+1$ ($N-1$) particles. The interaction is modeled by a 2D contact interaction with coupling strength g_{2D} . Plot a) depicts the fundamental gap determined by the full configuration interaction method [Ron08]. In plot b) the seniority model – also called the method of 'exact pairing' – has been applied. Both plots show similar behavior for increasing interaction strength indicated by the different colors. For weak interaction (green curve $g_{2D} = -0.3$ in dimensionless units) the 2D shell structure dominates the fundamental gap with magic numbers for $N = 2$ and $N = 6$. For larger interaction strength (blue curve $g_{2D} = -3$ and black curve $g_{2D} = -5$) the shell structure gets washed out resulting in an odd-even dependence of the fundamental gap. Taken from [Ron09] and adapted.

ability of the coupling strength of ultracold atoms offers the possibility to study a wider range of interaction-regimes. Extending our setup by a more sophisticated confining potential should allow for the investigation of the pairing effects also in 2D or 3D. It may also allow to study the crossover from mesoscopic systems to attractively interacting many-body systems described by BCS-theory [Bar57].

6. Radio Frequency spectroscopy of few-fermion systems

So far we performed tunneling measurements or modulation spectroscopy to measure the energy of our few-fermion systems. For those measurements we either had to deform or to periodically modulate the trapping potential which changes the initial energy of the system. In this chapter we present the rf-spectroscopy method with which we can probe the actual state of the few-fermion system without modifying the potential. The concept of the rf-spectroscopy method is the following: A rf-pulse forces a spin flip into a different Zeeman sublevel of the atom's hyperfine state. Additionally to the energy of the hyperfine-transition the energy difference between the initial and the final state of the system has to be supplied by the rf-photons. Supposing the final state is a noninteracting state the determination of the resonant rf-transition frequency is a direct measurement of the interaction energy in the initial state.

In the field of ultracold atoms this method has been successfully used for various kind of studies. For instance the method allowed the determination of the binding energy of weakly bound molecules [Reg03] [Bar05]. In the BEC-BCS crossover region the pairing energy, the wavefunction, the pair size and the effect of dimensionality could be measured [Chi04] [Sch08a] [Sch08b] [Som12]. Besides experiments with dimers and pairs the method was also applied to measure the energy of Efimov trimers [Lom10] [Nak11]. As we will address in the next chapter, rf-spectroscopy has been used to observe polaronic properties in imbalanced Fermi gases [Sch09] [Koh12].

We use the rf-spectroscopy method to determine the energy of a few-fermion system with up to six particles. In our experiments we measure the transition frequency with a resolution better than 50 Hz due to a stability of the magnetic offset field with only 1 – 5 mG uncertainty at offset fields of around 1000 G. We discuss the possibility of determining the wavefunction of an interacting system by decomposing it into harmonic oscillator states using trap sideband transitions.

Finally we present our precision measurements of the binding energy of weakly bound molecules using the rf-spectroscopy method with trap sideband resolution. We will show that with our preparation scheme and the high rf-resolution we can measure 70 times smaller binding energies compared to previous measurements

[Bar05]. This allows for a more accurate determination of Feshbach resonances in the ${}^6\text{Li}$ system.

6.1. The Rf-spectroscopy method

6.1.1. The concept

To measure the energy difference between two systems which contain atoms in different hyperfine states we flip the spin of one atom by applying a rf-pulse. From the energy of the rf-photon one can determine the energy difference between the two systems.

When applying a resonant rf-pulse with an oscillating magnetic field

$$\mathbf{B}_{\text{rf}}(t) = \mathbf{B}_0 \cos(\omega_{\text{rf}} t) \quad (6.1)$$

the spin $\mathbf{s}$ of the two level system rotates with the rf-Rabi frequency ¹

$$\Omega_{\text{ff}} \propto \mathbf{s} \mathbf{B}_0 = \mu_s B_0 \mathbf{e}_s \mathbf{e}_B \quad (6.2)$$

It is determined by the magnetic moment $\mu_s = |\mathbf{s}|$ of the spin and the orientation of the oscillation rf-field with respect to the quantization axis given by the direction of the magnetic offset field. In the high field region the electronic spin and the nuclear spin are almost decoupled and the hyperfine transition between the lowest three Zeeman sublevels in ${}^6\text{Li}$ can be regarded as a nuclear spin-flip (see hyperfine structure of ${}^6\text{Li}$, appendix, figure A.1). Then one would expect the magnetic moment μ_s to be on the order of the nuclear magnetic moment μ_I which is about a factor of 2000 smaller than the Bohr magneton resulting in a comparatively weak coupling. However, the hyperfine spin still contains an admixture of the electron spin of $\sim 1\%$. Thus the effective magnetic moment of the hyperfine transition is considerably larger than the pure nuclear magnetic moment which results in a higher rf-Rabi-frequency.

Up to now we have only considered the spin degree of freedom of the transition. When coupling two states with a rf-pulse also the external degree of freedom of the atoms, the spatial wavefunction $|\Psi_{\text{sp},i}\rangle$ and $|\Phi_{\text{sp},f}\rangle$ of these systems have to be taken into account. The resonant Rabi frequency of the transition is given by:

$$\Omega_{i \rightarrow f} \propto \langle \Psi_i | \widehat{RF} | \Phi_f \rangle \quad (6.3)$$

¹We use the notation 'free-free', abbreviated 'ff', because the rf-Rabi-frequency of the bare hyperfine transition can be measured by probing the transition of a single atom which is not coupled to any other atom.

with the states

$$|\Psi_j\rangle = |\Psi_{\text{sp},j}\rangle |\text{hfs}_j\rangle \quad (6.4)$$

which incorporate internal and external degree of freedom. As the energy of the rf-photon is in the MHz-regime the corresponding momentum transfer of the photon on the atoms is negligibly small. Thus, the rf-transition operator $\widehat{RF}$ does not act on the spatial wavefunction and the resonant Rabi frequency can be written:

$$\Omega_{i \rightarrow f} \propto \langle \text{hfs}_i | \widehat{RF} | \text{hfs}_f \rangle \langle \Psi_{\text{sp},i} | \Phi_{\text{sp},f} \rangle . \quad (6.5)$$

The resonant coupling strength between the two hyperfine states is given by the rf-Rabi frequency (equation 6.2) and thus

$$\Omega_{i \rightarrow f} = \Omega_{\text{ff}} \langle \Psi_{\text{sp},i} | \Phi_{\text{sp},f} \rangle . \quad (6.6)$$

which depends on the spatial overlap between the initial and the final system. At resonant coupling the energy of single rf-photons E_{rf} match the energy of the hyperfine transition E_{ff} and the energy difference ΔE in the external degree of freedom:

$$E_{\text{rf}} = E_{\text{hfs}} + \Delta E . \quad (6.7)$$

The energy difference in the external degree of freedom can be for instance the interaction energy of a few-fermions system. When a rf-resonance at ω_{rf} is observed one can deduce the energy difference between the initial and final states, knowing the energy of the hyperfine transition $\hbar\omega_{\text{ff}}$:

$$\Delta E = \hbar (\omega_{\text{rf}} - \omega_{\text{ff}}) . \quad (6.8)$$

The energy difference of the three lowest hyperfine states of ^6Li depending on the magnetic offset field is known from the formula of Breit and Rabi [Bre31] and shown in figure 6.1.

6.1.2. Illustration of the rf-spectroscopy method

We illustrate the rather abstract description by using an example system for which we have performed a rf-spectroscopy measurement. Let us assume a one-dimensional system consisting of two atoms in the hyperfine state $|1\rangle$ and $|3\rangle$. Using our preparation technique we can prepare the system in the noninteracting ground state of the microtrap at 568 G. Then we sweep the magnetic offset field to a value of 527 G. According to figure 6.6 which shows the CIR in the system, the 1D coupling strength is tuned to $g_{1\text{D},|1\rangle-|3\rangle} = -0.27 [a_{\parallel} \hbar \omega_{\parallel}]$. Thus, assuming the sweep is slow enough, the system adiabatically follows the attractive branch

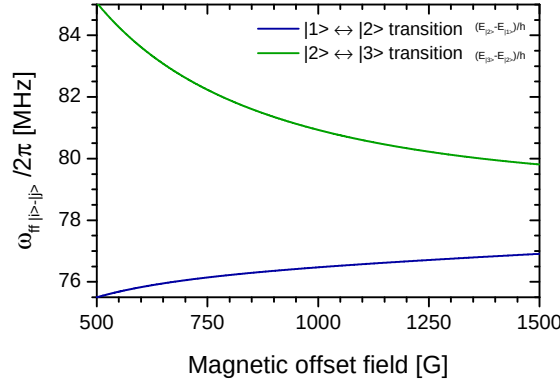


Figure 6.1.: **Energy difference between the three lowest Zeeman sublevels of ^6Li** . To drive a resonant rf-transition the rf-frequency has to match these energies in addition to the energy difference in the external degree of freedom.

(branch shown in figure 5.1, chapter 5). Then we apply a rf-pulse which transfers atoms from state $|3\rangle$ to state $|2\rangle$. The final state of the transition is a two-particle system in state $|1\rangle-|2\rangle$. Since the hyperfine state of the system has changed we have to consider the different coupling strength $g_{1D,|1\rangle-|2\rangle}$, which is zero at 527 G. Hence, by choosing an adequate value of the magnetic field we realized a transition from an interacting to a noninteracting system. The energy difference between the two systems, i.e. the interaction energy of the initial system and the hyperfine energy is extracted by stimulated emission of a rf-photon as illustrated in figure 6.2. In this sketch we have only considered a transition from the initial system in

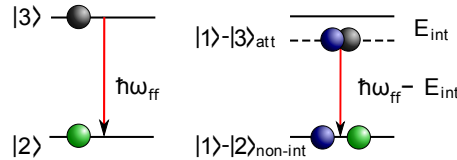


Figure 6.2.: **Determination of the interaction energy.** We deduce the interaction energy from the difference of the rf-transition frequency and the energy difference of the hyperfine states $\hbar\omega_{\text{ff}}$. The free-free transition frequency ω_{ff} is determined by measuring the corresponding rf-transition of a free atom.

the trap ground state to the final system also in the trap ground state. Depending on the energy of the rf-photon other transitions are possible by exciting the final state into higher trap levels. This is referred to as sideband transitions. In the

case of a harmonic trap the transition rates into these states are given by equation (6.6): $\Omega_{i \rightarrow f} = c_j \Omega_{\text{ff}}$ with c_j :

$$\begin{aligned}
 c_0 &= \langle \Psi_{\text{att}}(g_{1D}) | \Phi_0 \rangle & \omega_{\text{rf}} &= \omega_{\text{ff}} & + E_{\text{int}}/\hbar \\
 c_2 &= \langle \Psi_{\text{att}}(g_{1D}) | \Phi_2 \rangle & \omega_{\text{rf}} &= \omega_{\text{ff}} + 2\omega_{\text{h.o.}} & + E_{\text{int}}/\hbar \\
 & \vdots & & & \\
 c_{2n} &= \langle \Psi_{\text{att}}(g_{1D}) | \Phi_{2n} \rangle & \omega_{\text{rf}} &= \omega_{\text{ff}} + 2n\omega_{\text{h.o.}} & + E_{\text{int}}/\hbar
 \end{aligned} \tag{6.9}$$

where $|\Psi_{\text{att}}\rangle$ and $|\Phi_j\rangle$ are wavefunctions of the relative motion; the corresponding resonance frequencies of the rf-transition are listed on the right hand side. For weak attractive interaction ($E_{\text{int}} < \hbar\omega_{\text{trap}}$) the wavefunction of the relative motion $|\Psi_{\text{att}}\rangle$ is given by the parabolic cylinder function $\Psi_{\text{att}}(r) = D_{\varepsilon_{g_{1D}}}(r)$ at the corresponding interaction strength (see chapter 3.2.2). $|\Phi_j\rangle$ are the familiar harmonic oscillator eigenstates. It is not necessary to consider the separated center of mass term of the system: The spatial overlap in the center of mass motion is unity for identical initial and final center of mass states and zero else. This means that it is not possible to change the center of mass motion by a rf-spectroscopy pulse. All possible transitions are sketched in figure 6.3. Transitions which involve states of

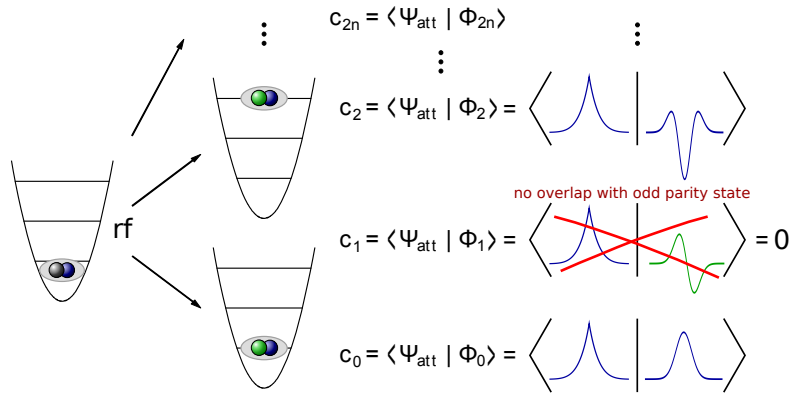


Figure 6.3.: Spatial overlap with trap sidebands. In a transition from an interacting initial state to a noninteracting state the transition rate depends on the overlap between the spatial wavefunction of the interacting state and the wavefunction of the trap sideband. Only sideband transitions with a parity of $\Delta P = 0$ contribute due to parity conservation. Note, the sketch depicts a single particle with reduced mass excited in the harmonic oscillator potential of the relative motion.

different parity do not contribute. As the initial state of the two-particle system

has even parity only sidebands with even symmetry (harmonic oscillator quantum number $2n$) can be addressed by the rf-pulse.

In the experiment we scan the rf-frequency and record the transferred fraction into the final hyperfine state. First we measure the hyperfine transition frequency ω_{ff} of a single atom which is not shifted by any interaction. Instead of a single atom we can use a sample of about 10 identical noninteracting atoms to increase the contrast of the rf-signal. The measured spectrum is given by the red data points in figure 6.4. This peak determines the hyperfine transition energy and we set the peak as the offset frequency and thus the red free-free transition peak appears at zero in the spectrum.

Then we perform the actual measurement with the interacting $|1\rangle$ - $|3\rangle$ system at $g_{1D,|1\rangle-|3\rangle} = -0.27 [a_{\parallel} \hbar \omega_{\parallel}]$. We again apply a rf-pulses and scan the frequency. We

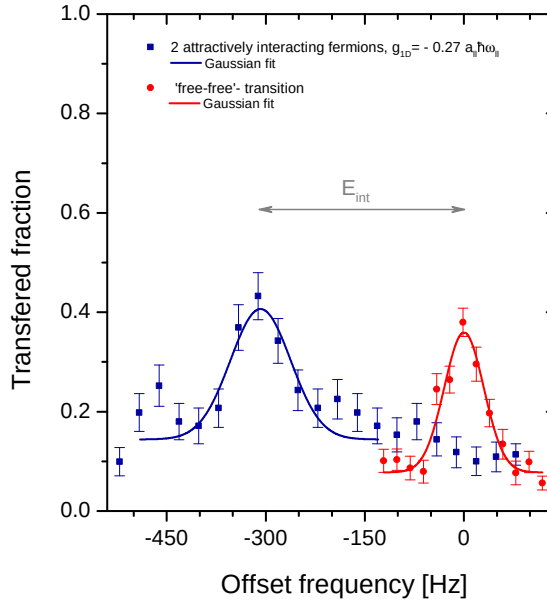


Figure 6.4.: **RF spectrum of two interacting fermions - lowest trap sideband.** The red circle show the spectrum of the free-free transition. The center-frequency of the free-free peak is set as the zero-point of the offset frequency. The first trap sideband peak (blue squares) appears next to the free free-peak shifted by the interaction energy E_{int} .

observe the first transition peak shifted from the free-free transition (blue data points). The energy shift exactly corresponds to the interaction energy of the $|1\rangle$ - $|3\rangle$ system as the final state is noninteracting. Subtracting both peak positions directly provides the interacting energy of the two-particle system.

In the following we discuss the experimental resolution of the spectra and the

calibration of the magnetic field. The discussion of transitions to higher trap sidebands which allow to decompose the wavefunction of an interacting system is given in the subsequent section. We can also use the rf-spectroscopy to determine the energies of few-fermion systems with larger particle numbers which we then present in section 6.3 for up to six particles.

6.1.3. Experimental resolution and calibration of the magnetic field

In this section technical details of the rf-spectroscopy measurement are described and we discuss the origin of the errors of the determined rf-transition frequencies. The natural linewidth of the rf-transition is very narrow as an electric dipole transition between two hyperfine states is forbidden. The experimental resolution of the transition peak is determined by the stability of the magnetic offset field and by the duration of the rf-pulse. According to figure 6.1 the energies of the two transitions tune with the magnetic offset field. This means that the resonance condition of the rf-transition depends on the offset field². Thus, fluctuations in the magnetic field broaden the measured transition spectrum. With our setup we achieve a magnetic field stability of 1 – 5 mG using a high precision digital feedback loop for stabilizing the current of the magnetic field coils. The current transducer, Danfysik Ultrastab 866 ($I_{\text{max}} = 600 \text{ A}$), for measuring the current in the coils has a precision of 10^{-6} , the ADC of the corresponding digital PID controller has a resolution of 17 bit over a dynamical range of 1500 G which corresponds to a precision of 8×10^{-6} . The ADC's sampling rate is about 100 times faster than the time constant of the coils and thus the generated current is determined from an average of many ADC samples. For this reason we can achieve an even higher precision than the nominal resolution of the ADC.

For the rf-pulse we choose a pulse length of 12.5 ms which leads to a Fourier limit of the frequency on the order of the frequency width introduced by the magnetic field instability. In total this leads to a typical width of 60 – 80 Hz FWHM of the transition peak. Due to the statistical origin of the magnetic field fluctuations we fit Gaussians to the peaks to determine the center frequency of the transition. The standard error of the fit, which is typically 3 – 15 Hz, serves as the statistical error of the center frequency.

To avoid saturation of the transition the rf-power is chosen small enough that about 40% of the atoms are transferred into the final state. After the rf-pulse

²In the high field region around 500 – 1000 G where we perform most of our experiments the $|1\rangle$ - $|2\rangle$ transition tunes with $\sim 1 \text{ kHz/G}$ and the $|2\rangle$ - $|3\rangle$ transition tunes with $\sim -10 \text{ kHz/G}$.

the N-particle system gets either projected to the final state or back to the initial state. Hence we average over about 50 repetitions of the same measurement to obtain the transferred fraction to the final state as shown in the spectrum of figure 6.4.

During the measurement of a single transition spectrum of the few-fermion system which typically takes 8 hours the magnetic field may have drifted. Reasons for that could be drifts of the temperature and of the pressure of the water-cooling for the magnetic field coils. This especially happens if the water cooling of other devices is switched on or off during the measurement. Room temperature drifts may lead to small drifts of the high precision measurement resistors necessary to convert the secondary current of the current transducer into a measurable voltage. To incorporate the drifts of the magnetic field during the actual measurement into the error budget we measure the free-free hyperfine transition before *and* after the actual rf-spectroscopy measurement. The weighted mean of both frequencies determines the free-free transition frequency ω_{ff} . Its error is given by the weighted error of weighted mean (see appendix table A.6). The interaction energy is then determined according equation (6.18).

To calibrate the magnetic field which we need to determine g_{1D} we calculate the field from the free-free transition frequency ω_{ff} using the formula of Breit and Rabi [Bre31]. The error in the magnetic field calibration does not significantly effect g_{1D} as it tunes only weakly with the magnetic field.

6.2. Rf-spectroscopy with trap sideband resolution

We have discussed that the rf-transition depends on the spatial overlap between the initial and the final state of the few-particle system. In the case of an interacting initial system the overlap to higher trap sidebands is non-zero. If the experimental width of the rf-transition is smaller than the level-spacing one can resolve the transitions to individual trap sidebands. In our system the FWHM of the rf-transition peak is 60 – 80 Hz which is significantly smaller than the level spacing of 1.5 kHz in axial direction of the microtrap. Hence, by scanning the rf-frequency to lower values in the previously described rf-measurement a sideband peak should occur with a distance of twice the level spacing. Yet, the transition rates into the higher sidebands with even parity has not necessarily to be of the same order as for the ground state transition and depends on the overlap between the wavefunctions (equation 6.9). In the case of the two-particle system with $g_{1D,|1\rangle-|3\rangle} = -0.27 [a_{\parallel} \hbar \omega_{\parallel}]$ we had to increase the power of the rf-pulse by 16 dB and the length of the pulse by a factor of 10. Then we indeed resolved the trap sideband which can be seen from figure 6.5. The measured distance between the

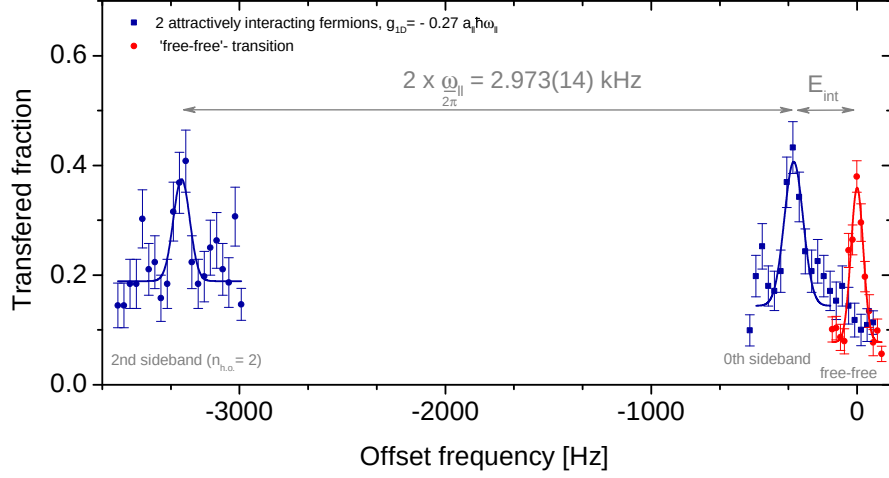


Figure 6.5.: **RF spectrum of two interacting fermions - second trap sideband.** Next to the lowest sideband transition we observe the peak of the second sideband transition. The shifted frequency matches well to twice the trap frequency. Due to a smaller spatial overlap we had to increase the power and the length of the rf-pulse to observe the transition at similar rates.

zeroth sideband and the second sideband is $2.973(14)$ kHz. Within the error this fits perfectly to the axial 0-2 transition in the microtrap of $\omega_{||0-2} = 2.985(10)$ kHz which we have determined from the modulation spectroscopy measurement (see chapter 4, table 4.2). Hence, with the measurement of the trap sideband transitions we have implemented a complementary method to determine the level spacing of the confining potential. Even more important is the possibility to determine the wavefunction of an interacting few-fermion system which we discuss next.

6.2.1. Decomposition of the interacting wavefunction

As already described in chapter 4.4 the wavefunction of an interacting few-fermion system in a harmonic trap can be decomposed into a set of noninteracting harmonic oscillator wavefunctions. Using the completeness relation the general decomposition of an even parity state $|\Psi_{\text{att}}(g)\rangle$ in harmonic oscillator states Φ_n reads

$$|\Psi_{\text{att}}(g)\rangle = \sum_{n=1}^{\infty} |\Phi_{2n}\rangle \langle \Phi_{2n} | \Psi_{\text{att}}(g) \rangle \quad (6.10)$$

with coefficients

$$c_{2n} = \langle \Phi_{2n} | \Psi_{\text{att}}(g) \rangle \quad (6.11)$$

which fulfill the identity

$$\sum_{i=1}^{\infty} |c_i|^2 = 1 . \quad (6.12)$$

If one compares the coefficients of this decomposition with the transition probabilities of the rf-transition one finds that the coefficients are given by the ratio between the Rabi frequency to higher trap sidebands and the free-free Rabi frequency:

$$c_i = \frac{\Omega_{i \rightarrow f}}{\Omega_{\text{ff}}} . \quad (6.13)$$

This means by measuring the resonant Rabi frequency to the trap sidebands the rf-spectroscopy allows to determine the wavefunction of an interacting system. From the previous measurement of the weakly attractive interacting two-particle system we can infer the coefficients from the spectrum of figure 6.5. As the system is only weakly interacting we can truncate the Hilbert space at low n of the decomposition 6.10. Here we only consider 2 states, the zeroth and the second trap sideband. Then the identity (6.12) simplifies to

$$|c_0|^2 + |c_2|^2 = 1 . \quad (6.14)$$

From the increase of the rf-power by $\Delta P_{\text{rf}} = 16$ dB and the increase of the pulse-length by a factor of $\Delta T = 10$ which has been needed to obtain similar peak heights in the spectrum we infer that the ratio between the Rabi frequencies are

$$\frac{c_0}{c_2} = \Delta T \times \sqrt{\Delta P_{\text{rf}}} \cong 60 . \quad (6.15)$$

Using the identity (6.14) of the sub-system we find the coefficients of the decomposed wavefunction:

$$c_0 \cong 0.98 \quad (6.16)$$

$$c_2 \cong 0.02 . \quad (6.17)$$

Note, the determination of the Rabi frequencies by comparing the two sideband peaks for different power and pulse-length is not accurate as decoherence effects might influence the height of the peaks for longer pulses. Yet, when measuring the transition rates precisely, the rf-measurement should provide a quantitative method to determine the wavefunction of an interacting system.

6.3. RF-spectroscopy of few-fermion systems with two, three and more interacting particles

We can apply the rf-spectroscopy also to systems where the initial and final state consists of more than two particles. As we have illustrated we can directly determine the interaction energy of the initial system if the final state is noninteracting. Unfortunately, for the 3 lowest states in ${}^6\text{Li}$, there are not many configurations for which the final state is noninteracting³. A view on the 1D coupling strength in figure 6.6 tells us, that there are only three distinct magnetic field values, where the interacting strength is zero: Namely at the three zero-crossings of g_{1D} which are equivalent to the zero-crossings of a_{3D} . For the different combinations of hyperfine states these positions are as follows: $|1\rangle\text{-}|2\rangle$: $B = 527\text{ G}$, $|1\rangle\text{-}|3\rangle$: $B = 568\text{ G}$ and $|2\rangle\text{-}|3\rangle$: $B = 589\text{ G}$. We have performed rf-spectroscopy measurements at these values of the magnetic field. The results for different particle numbers is shown in figure 6.7.

Nevertheless, we want to determine the interaction energy also for different values of g_{1D} . This is possible since the rf-spectroscopy can also be applied to configurations in which the final states are interacting. Then the obtained energy is the difference in the interaction energy of the initial state and the final state. Exemplary this is illustrated for a three-particle system by the green arrows in figure 6.7. Although in these cases we can not directly measure the absolute value of the interacting strength we can still extract information from these measurements. For example we can identify the branches on which we ramp up the few-fermion system starting from a noninteracting sample.

6.3.1. Obtaining the total interaction energy

We stated that the total interaction energies can only be directly obtained when the final state is noninteracting. However, by using a little trick we can also deduce the interaction energy for systems where both initial and final states are interacting. The idea is to measure the interaction energy in two or more steps as shown in figure 6.8. We start with a hyperfine combination (I-i) at a magnetic offset field where $g_{1D} = g_{I-i}$ and drive a rf-transition to a final intermediate state (I-f) which is also interacting with coupling constant g_{I-f} . Then we change the magnetic field (horizontal line in figure 6.6) and choose a different hyperfine-combination (II-i) with $g_{I-i} = g_{II-f}$. By matching the values of g_{1D} we ensure that both systems, (II-i) and (I-f), have the same energy. Then we apply a rf-pulse which transfers

³A species offering a hyperfine configurations with a wide range of final noninteracting states such as ${}^{40}\text{K}$ would be advantageous to investigate interacting systems.

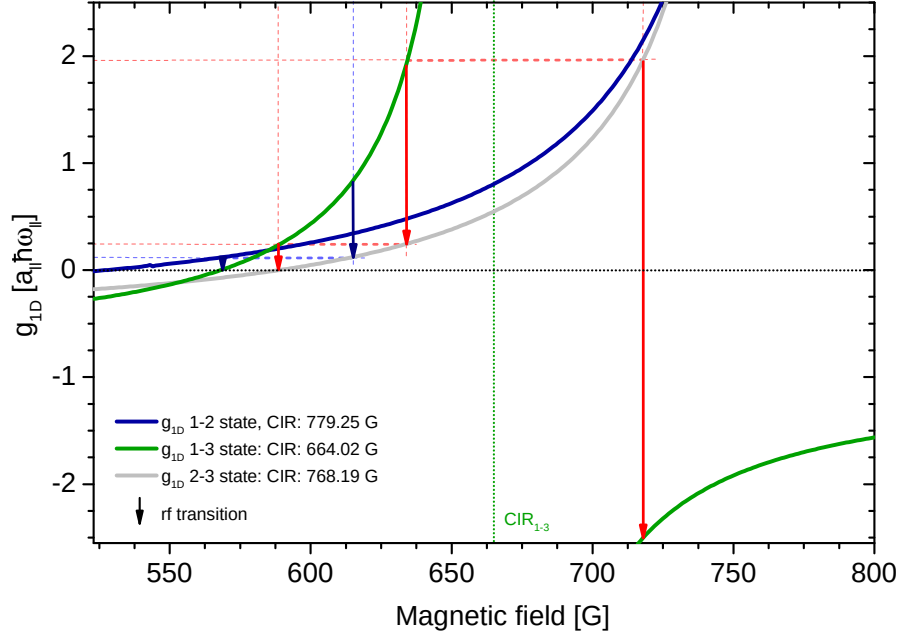


Figure 6.6.: **Combination of rf-transitions with noninteracting final states.**

For the three lowest hyperfine states of ${}^6\text{Li}$ there are only three configurations of two different spin combinations where the interaction strength of the final state is noninteracting. These configurations can be accessed at the zero-crossings of the coupling constant. Yet, by performing several subsequent measurements for which we match the coupling constant of intermediate states (dashed lines) we can also determine the interaction energy at different values of the magnetic offset field (blue and red arrows).

state (II-i) to the final noninteracting state (II-f). When realizing this trick in the experiment one has to choose $g_{\text{I-i}}$ such that the previously explained condition is fulfilled. The arrows in figure 6.6 indicate such possible combinations of g_{ID} . The blue arrow indicates a combination with one intermediate state, the combination illustrated by the red arrow includes two intermediate states.

Using this method the total interaction energy of the initial interacting system can be deduced by adding up the energies of the rf spectroscopy measurement:

$$E_{\text{int}} = \Delta E_{\text{I}} + \Delta E_{\text{II}} . \quad (6.18)$$

The error of E_{int} is determined by a quadratic addition of $\sigma_{E_{\text{I}}}$ and $\sigma_{E_{\text{II}}}$. There is also an error in the interaction strength g_{ID} due to a non-perfect matching of the intermediate coupling strength as a result of non-accurate magnetic field cali-

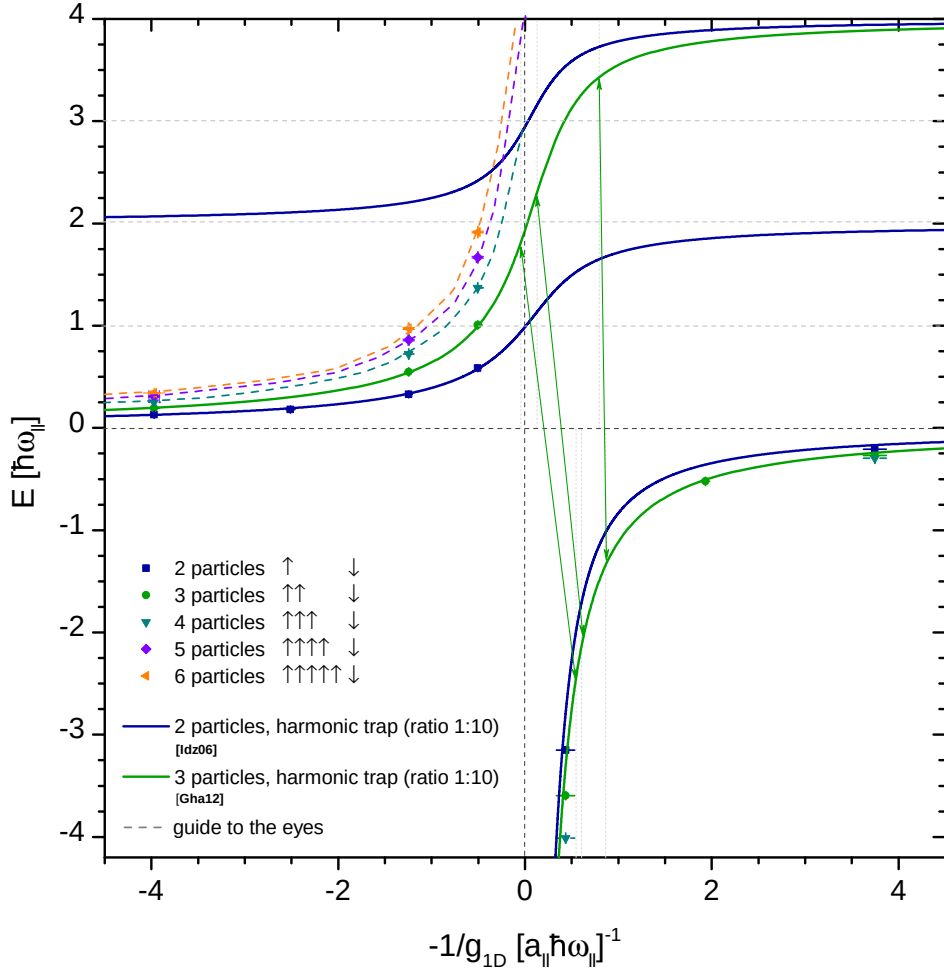


Figure 6.7.: **Results of the rf-spectroscopy for two, three and more particles.** For distinct values of the coupling constant (data points) we have determined the absolute value of the interaction energy of few-fermion systems. The results for two and three particles (points in blue and green) fit excellently to the two and three particle theory (blue and green solid lines [Idz06] [Gha12]). For larger systems with particle number $N > 3$ we only sketched the course of the interaction energy (dashed lines). For all of the sketched curves we know that the energy at $1/g_{1D} \rightarrow 0$ has to match $(N - 1) \times \hbar\omega_{||}$ due to the fermionization occurring in this quasi-1D regime. For other values of the interaction energy we can only measure the energy difference between these branches. We exemplarily have chosen some transitions for three particles indicated by the green arrows. All residual measurement data can be found in tables in the appendix (A.21-A.24). Transitions between states with same initial and final g_{1D} which would appear as straight vertical arrows in the energy spectrum cannot be measured, as these states are orthogonal and thus do not exhibit a spatial overlap.

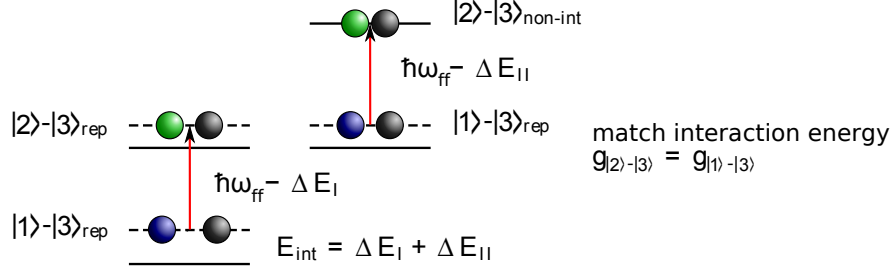


Figure 6.8.: **Trick to determine the interaction energy.** For some systems we can measure the total interaction energy E_{int} in a two step process: First we measure the energy difference ΔE_I to an intermediate state which is also interacting. By changing the magnetic field (see figure 6.6) we can find a different configuration of hyperfine states with the same interaction energy. Its energy ΔE_{II} can then be determine in a second rf-measurement.

bration during the measurement. The error of g_{1D} is determined by the difference $g_{II-i} - g_{I-f}$. All determined interaction energies are shown in figure 6.7.

We have performed measurements for systems with up to $N = 6$ particles. They consist of a single minority atom in one hyperfine state and $N - 1$ majority particles in a different hyperfine state. To measure the energy of the system we have always changed the hyperfine state of the minority atom. The interpretation of the results for a single minority particle with a large number of identical majority particles is topic of the next chapter.

For a two- and a three-particle system we can compare the experimental results with theories of two and three particles derived by [Idz06] and [Gha12] (blue and green curve in figure 6.7). We find excellent agreement between our experiment and the few-particle theory. Besides one data set where we assume that a systematic error had occurred during the measurement, theory and experiment match within less than 5% (see appendix table A.21 - A.24). The origin of the error is expected to be partially due to systematic uncertainties such as the exact position of the CIR and the anharmonicity of the trap. By decreasing this uncertainties we are convinced that one could obtain agreement within 1 – 2%. The high resolution and the good agreement with the two- and three-particle theory demonstrates that the rf-spectroscopy is ideally suited to probe few-fermions.

6.4. Precise determination of ^6Li scattering length

In previous work rf-spectroscopy of weakly bound dimers has been used to determine the position of Feshbach resonances [Reg03] [Bar05]. As we have noted in chapter 3, for an interaction potential parametrized by a delta-potential there is an universal bound state with binding energy

$$E_b = \frac{\hbar^2}{\mu a_{3D}^2} \quad (6.19)$$

where μ the reduced mass and a_{3D} the scattering length. The energy of the bound state in free-space as a function of the inverse scattering length is shown in figure 6.9. At the point where the scattering length diverges ($1/a_{3D} = 0$) this state becomes unbound. Hence by measuring the binding energy as a function of the magnetic field ⁴ one can determine the position of the resonance position - in our case the magnetic Feshbach resonance for the hyperfine states $|1\rangle$ - $|2\rangle$ around 830 G.

So far the most precise determination was performed by Bartenstein et al. with an uncertainty of 1.5 G of the resonance position [Bar05]. They measured the binding energy of the associated bound state – the so-called Feshbach molecules – at binding energies of around 100 kHz at various intermediate scattering length of about $2000 a_0$. They were not able to perform measurements for smaller binding energies closer to the Feshbach resonance which would decrease the uncertainty of the determined position because the density in their two-component Fermi gas was too large and thus many-body effects of the gas started to play a role. The latter results in a shift of the rf-transition with respect to the pure two-body transition which would effect the determination of the resonance position. To avoid this effect they had to perform experiments comparatively far away from the position of the Feshbach resonance where the binding energy of the Feshbach molecules is about 100 kHz.

With our system we are able to prepare and detect few-particle samples with high fidelity. This allows us to prepare a very dilute sample in which density effects do not play a major role even for large scattering lengths. We will show that we are able to determine the binding energy close to the Feshbach resonance with binding energies of around 2 kHz. Here shifts in the binding energy due to the confining trap are relevant and they are therefore considered in the analysis.

In this section we present the experimental results for the binding energies from

⁴In the non-universal regime when the scattering length becomes on the size of the effective range the dependence of a_{3D} on B is not as simple as equation (6.19). It depends on the detailed shape of the van-der-Waals singlet and triplet potential. To determine the position of the Feshbach resonance and to determine the scattering length coupled channel calculations are used [Bar05] [Joc12a].

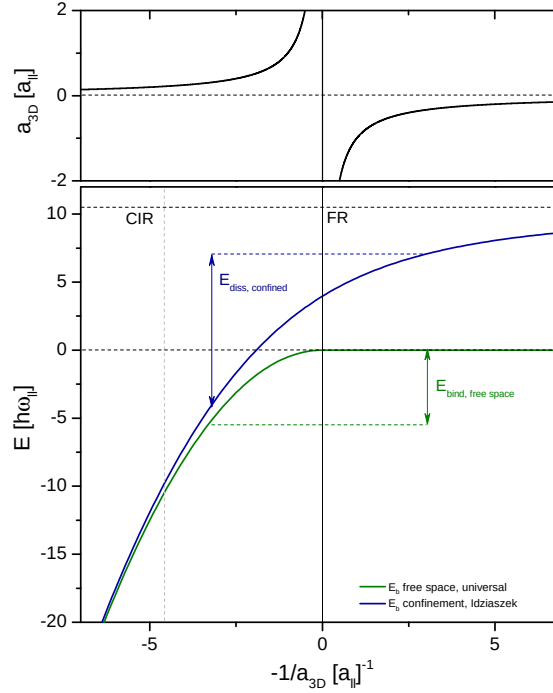


Figure 6.9.: **Universal bound state.** The green curve shows the universal bound state associated with the contact interaction potential [Lan87]. At the point where the scattering length diverges at the Feshbach resonance (FR, upper panel) the universal bound state reaches the continuum. By measuring its binding energy in free space (green arrow) as a function of the magnetic field one can determine the position of the Feshbach resonance. The blue curve shows the universal bound state in the presence of the confinement ([Idz06], $\eta = 10$) and the blue arrow indicates a corresponding rf-transition. Due to the confinement the dissociation frequency is shifted with respect to the one in free space.

which one can determine the Feshbach resonance more precisely. Using our data, the position of the Feshbach resonance could be determined with an accuracy better than 0.1 G. This section gives an overview over the measurement and the corresponding analysis. Further details can be found in [Joc12a].

6.4.1. Rf-spectroscopy of weakly bound molecules with trap sideband resolution

To perform the rf-spectroscopy we start with a BEC of 10^5 $|1\rangle$ - $|2\rangle$ molecules in our large volume optical dipole trap at a magnetic offset field of 760 G. For details on the preparation of a molecular BEC we refer to [Lom11]. For minimizing density dependent shifts we reduce the particle number and increase the temperature of the sample. To achieve this we first superimpose the microtrap and load about 10^3 molecules into it. By applying the spilling technique we further decrease the particle number and end up with a sample of about 30 molecules with sub-Poissonian number fluctuations (± 2 molecules). The total particle number may differ by $\pm 50\%$ due to systematic uncertainties in the calibration of the single atom detection in the MOT at the time we performed the measurement. To reduce the density and to create a non-degenerate sample, we transfer the molecules back into the large volume dipole trap by switching off the microtrap. This non-adiabatic release transforms the potential energy of the molecules in the microtrap into kinetic energy resulting in a mean kinetic energy per particle of $\geq k_B \times 0.4 \mu\text{K}$. The radial trap frequency of the dipole trap is $\omega_r = 2\pi \times 349(3)\text{Hz}$ which we determine from the sideband peak separation in the rf-spectra (see appendix A.27). From the known trap geometry [Lom11] we calculate the axial trap frequency $\omega_a = 2\pi \times 34(1)\text{Hz}$ and the trap depth ($k_B \times 2 \mu\text{K}$). To determine the dissociation frequencies for different magnetic offset fields we ramp to the corresponding field and wait 5 ms. This time is long enough for the decay of eddy currents in the steel vacuum chamber which guarantees a stable offset field, but short enough to avoid dissociation of a large fraction of molecules at low binding energies due to collisions. Then we apply a 10 ms rf-pulse with frequency ν_{bf} to fractionally dissociate the $|1\rangle$ - $|2\rangle$ molecules into free atoms in state $|1\rangle$ and $|3\rangle$. By measuring the fraction of unbound atoms we obtain spectra as shown in figure 6.10 (blue data points). To limit saturation effects we choose the pulse power such that the maximum transferred fraction is only 30 – 40%. In order to estimate the density dependent systematic shift we redo the experiment with a sample of 200 molecules (before 30). From the comparison of the two measurements we can infer that the density dependent shift of our 30 molecules is smaller than 8 Hz (see appendix A.28).

Determination of the free-free-transition and magnetic field calibration

To calculate the dissociation frequencies and to calibrate the magnetic field we additionally measure the 'bare' transition frequencies of single atoms from state $|2\rangle$ to state $|3\rangle$ (red data points in figure 6.10). This calibration is done once before

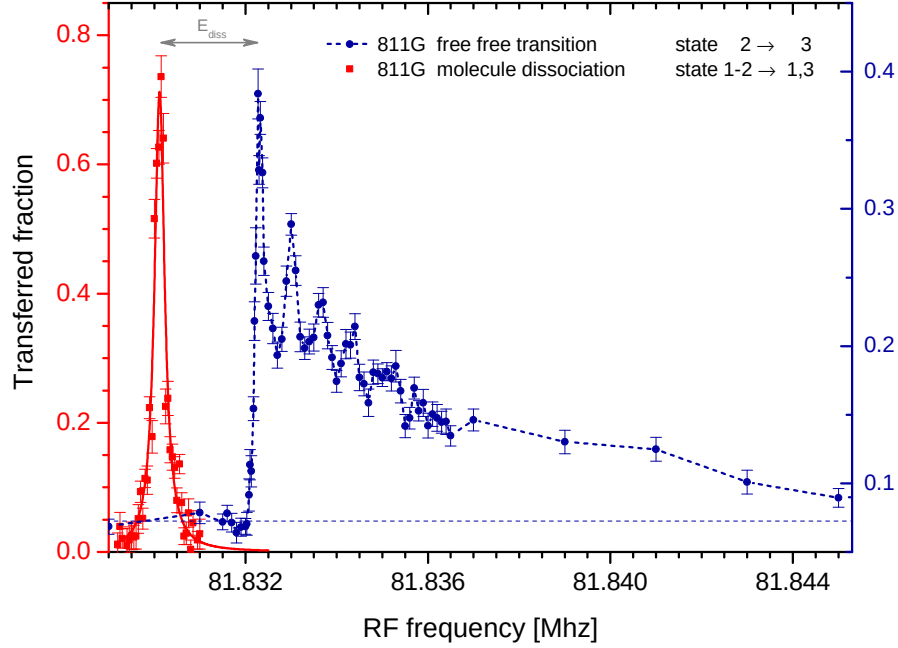


Figure 6.10.: **Dissociation spectrum at low binding energy.** The red squares show the free-free transition. The blue circles show the dissociation spectrum with a characteristic peak structure for smaller dissociation energies and a decaying tail for larger energies. The peak structure results from transitions to different radial trap sidebands. For larger transition energies we do not resolve the sideband structure. The shape of the decaying tail is determined by the wavefunction overlap between the molecule ($\Psi \propto e^{\frac{-r}{a_{3D}}}$) and higher trap sidebands ($\Phi_{2n}(r)$). The distance between the free-free peak and the first trap sideband determines the dissociation frequency.

(ν_{ff1}) and once after (ν_{ff2}) the actual molecule dissociation measurement to minimize the uncertainty in the magnetic field and to be able to estimate the magnetic field stability over the course of the measurements. We fit the resulting spectra with the coherent lineshape for a pulse of a certain length and obtain the free-free transition frequencies ν_{ff1} and ν_{ff2} before and after the molecule dissociation measurement (see appendix figure A.25). For the final magnetic field calibration we use the mean of these two measurements. The results of the free-free transition measurements and the magnetic field calibration can be found in table A.6 in the appendix.

Interpretation of the bound-free spectra

To interpret the obtained dissociation spectrum (figure 6.10) we have to discuss the trap sideband transition. Since we have loaded the particles from the microtrap into the large volume dipole trap we have to consider different trap parameters as in the previous section. As mentioned, the trap frequencies of the large volume cigar shaped dipole trap are $\omega_{\perp} = 2\pi \times 349(3)\text{Hz}$ in radial direction and $\omega_{\parallel} = 2\pi \times 34(1)\text{Hz}$ in axial direction. Our frequency resolution in terms of the FWHM of the transition peak is 120 Hz. Hence, we cannot resolve the axial trap sidebands of this trap. Yet, the radial sidebands can be resolved which is illustrated in figure 6.11. The envelope of all resolved and not resolved peaks determines the

Radial sidebands resolved (finite experimental resolution ~ 60 Hz)

Trap geometry

cigar shaped large volume dipoletrap

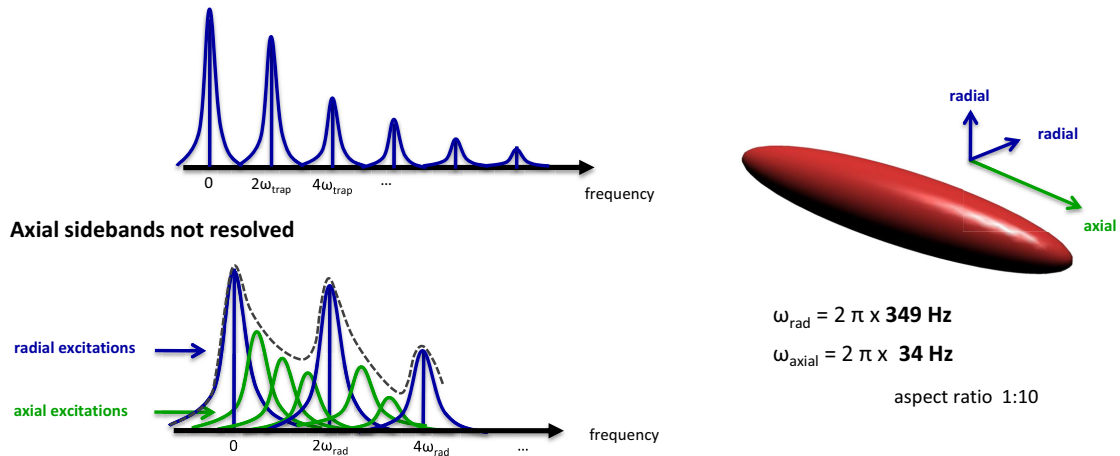


Figure 6.11.: **Trap sideband resolution in the large volume dipole trap.**

With our experimental resolution we can only resolve the radial trap sidebands of the dipole trap (right hand side). The axial trap sidebands are not resolved. The total dissociation spectrum (dashed line) is given by the sum of the radial transition peaks (blue) and the axial transition peak (green).

shape of the dissociation spectra. To extract the binding energy we have to fit the sum of all sideband transitions with the corresponding experimental resolution to the measured spectra. Therefore we need to know the overlap coefficients of the molecular wavefunction to the individual sidebands. To obtain these coefficients we make an estimate for the binding energy which we deduce from the difference of the free-free transition peak to the zeroth radial sideband peak. From the binding energy we can then determine the scattering length. Since for some offset fields

6.4. Precise determination of ${}^6\text{Li}$ scattering length

the scattering length is of the size of the effective range of the ${}^6\text{Li}$ interaction potential we consider effective range corrections to first order. In this case the binding energy reads [Gri93]:

$$E_b = \frac{\hbar^2}{\mu(a_{3D} - \bar{a})^2} \quad (6.20)$$

with μ the reduced mass and with the so-called mean scattering length

$$\bar{a} \approx 0.487 r_{\text{vdw}} \quad (6.21)$$

with r_{vdw} being the range of the van-der-Waals potential [Lom11]. This expression allows to make an estimate for the scattering length. Then we can approximately calculate the molecular wavefunction which is expressed in the exponentially decaying form [Chi05]

$$\Psi \propto e^{\frac{-r}{a_{3D}}}. \quad (6.22)$$

Using the wavefunction we calculate the overlap $c_{2n_x, 2n_y, 2n_z}$ between the bound state and the even parity trap excitations in relative motion. The energy of the final trap states, $\hbar((2n_x + 2n_y)\omega_{\perp} + 2n_z\omega_{\parallel})$, determines the position of the sideband transition peaks in the spectrum. In our model the width of the peak has been infinitesimally small with a finite integral given by the overlap coefficients. To include the experimental resolution we convolute the discrete transition peaks with normalized Lorentzians⁵. After summing up all sideband contributions up to $n_{x,y} = 3$ and $n_z \approx 45$ we have completed a model for the spectrum. (see figure 6.12, dashed line).

The overall shape of the model fits quite well to the measured spectra at different magnetic offset fields. There are some minor deviations for larger sidebands which are probably due to the anharmonicity of the trap and due to saturation effects in the rf transition. Thus, we only consider the lowest 10 axial trap sidebands and the zeroth radial sideband for the fit. The fit has two free parameters, the overall height of the model (plus offset) and the absolute frequency position ν_{bf} of the zeroth trap sideband. The lines in figure 6.12 show the result of the fit to the spectra. Although our fitting routine precisely determines the absolute position of the peaks it does not provide its standard error. Hence we fit a single Lorentzian to the rising slope of the zeroth trap sideband peak. Its standard error serves as the error of ν_{bf} . All values of the bound-free transition ν_{bf} for the different offset fields are recorded in table A.6 in the appendix.

⁵We originally used Gaussians for the shape of the transition as the magnetic field uncertainty has a statistical origin. Yet, we have found that Lorentzians better represent the overall lineshape. We have not exactly analyzed the lineshape and therefore treat the difference of the fit results between the Gaussian and the Lorentzian model as a systematic error.

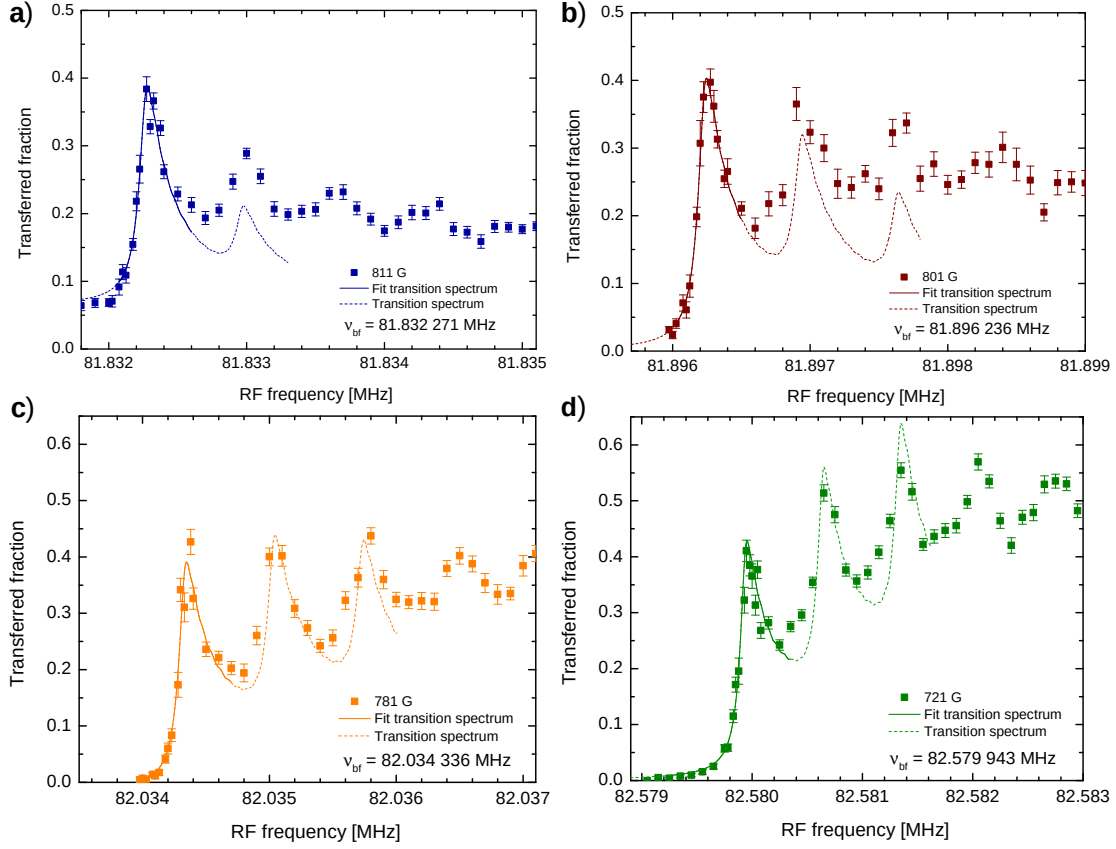


Figure 6.12.: **Dissociation spectra for different magnetic offset fields.** To determine the binding energy we measure the dissociation spectra at different magnetic fields. The lines show the theoretical model, where the solid line indicates the fitting region. For increasing interaction energy and smaller scattering length (blue $\rightarrow$ brown $\rightarrow$ orange $\rightarrow$ green) the particles get more localized according to $\Psi \propto e^{\frac{-r}{a_{3D}}}$ and the overlap to higher sidebands with respect to the lower sidebands increase. This is the reason for the change from an initially-decaying overall shape (blue) to an initially-decreasing shape (green) for the lowest trap sidebands.

6.4.2. Confinement shift

As we initially mentioned we are interested in the binding energy of the Feshbach molecules in free-space. Yet, although we have performed the spectroscopy in a very shallow trap, we still have to consider the effect of the confining potential on the binding energy. In figure 6.9 we show the comparison between the universal

bound state in free space and the bound state in a cigar shaped trap with 1:10 aspect ratio. The difference between the two curves is plotted in figure 6.13. As

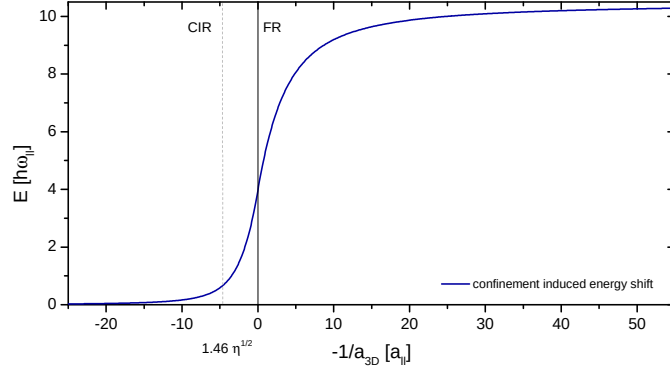


Figure 6.13.: **Confinement induced shift.** The shift is determined from the binding energy of the free-free transition frequency and the binding energy in the presence of the confinement (difference between green and blue curve in figure 6.9).

discussed in chapter 3.2.2 both curves must coincide at large binding energies and therefore the confinement shift vanishes for $-1/a_{3D} \rightarrow -\infty$. At $1/a_{3D} = 0$ the universal bound state in free space has reached the continuum whereas in the trap the bound state still exists also for negative a_{3D} . For $-1/a_{3D} \rightarrow \infty$ the binding energy of the confinement induced molecule becomes zero and approaches the zero-point energy of the trap of $\frac{1}{2}(2\eta + 1)\hbar\omega_{\parallel}$. The confinement shift in the initial and final state of the rf-transition at the relevant magnetic field is given in the following table:

magnetic field B [G]	$-1/a_{3D 1\rangle- 2\rangle}$ [$a_{\parallel}^{-1}$]	$-1/a_{3D 1\rangle- 3\rangle}$ [$a_{\parallel}^{-1}$]	shift initial [Hz]	shift final [Hz]
811	-10.11	52.37	5.6	358.5
801	-15.72	50.21	2.4	358.2
781	-28.35	45.26	0.7	357.2
721	-84.21	21.29	0.0	345.8

6.4.3. Binding energy of the weakly bound molecules and position of the Feshbach resonance

We can calculate the free-space dissociation energy of the Feshbach molecules by subtracting the free-free transition energy from the bound-free transition energy

while including the confinement shift:

$$E_b = 2\pi\hbar \times (\nu_{\text{bf}} - \nu_{\text{ff}} + \nu_{\text{initial}} - \nu_{\text{final}}) \quad (6.23)$$

The results for the binding energy depending on the four different positions of the magnetic offset field are listed in the following table:

magnetic field B [G]	stat. error σ_B [mG]	binding energy E_b [kHz]	stat. error σ_{E_b} [Hz]	syst. error Δ_{E_b} [Hz]
811.139	1	1.803	8	17
801.115	5	4.341	33	17
781.057	1	14.157	7	17
720.965	1	127.115	14	17

The error budget is discussed in the appendix (A.7).

We conclude that in our experimental setup we have realized a high magnetic field stability of only 1 – 5 mG uncertainty. For the measurement of the binding energy of weakly bound ^6Li molecules we have achieved a high precision with 24 – 50 Hz uncertainty including statistical and systematic errors. This allows for an accurate determination of the Feshbach resonance position and the scattering length as a function of the magnetic field. The analysis has been performed by Jeremy Hutson and Paul Julienne using coupled channel calculations [Bar05]. Further details can be found in a joined publication [Joc12a]. The resulting new resonance positions are:

state	resonance position [G]	σ [mG]
$ 1\rangle - 2\rangle$	832.18	80
$ 1\rangle - 3\rangle$	689.68	80
$ 2\rangle - 3\rangle$	809.76	50

7. A single impurity in a finite Fermi system

The interaction of a single impurity with a surrounding bath of other particles is of high interest in physics. In solid state systems for example, single conduction electrons can interact with the ions in a dielectric crystal which is polarized by the electrons [Dev00]. The single impurity can be regarded as a fermionic quasi-particle – the polaron – exhibiting interesting dynamics with an effective mass depending on the strength of the interaction with the bath. In the case the bath is a Fermi-sea the polaron is referred to as a Fermi-Polaron. Within the formalism of particle-hole excitations the polaron’s wavefunction can be described by the following variational ansatz [Che06] [Mas11]:

$$|\psi_{\mathbf{p}}\rangle = \Phi_0 a_{\mathbf{p}\downarrow}^\dagger |\text{FS}\rangle + \sum_{q < k_F < k} \phi_{\mathbf{k},\mathbf{q}} a_{\mathbf{p}+\mathbf{q}-\mathbf{k}\downarrow}^\dagger a_{\mathbf{k}\uparrow}^\dagger a_{\mathbf{q}\uparrow} |\text{FS}\rangle \quad (7.1)$$

where $|\text{FS}\rangle$ is a noninteracting Fermi sea of $\uparrow$ -majority particles plus a $\downarrow$ -minority particle with momentum $\mathbf{p}$. The Fermi sea is perturbed by annihilation of particles with momentum $q < k_F$ within the Fermi sea ($a_{\mathbf{q}\uparrow}$) and by creation of particles outside the sea ($a_{\mathbf{k}\uparrow}^\dagger$). To satisfy momentum conservation, the minority particle acquires a momentum $\mathbf{q} - \mathbf{k}$ ($a_{\mathbf{p}+\mathbf{q}-\mathbf{k}\downarrow}^\dagger$) [Che06]. This ansatz describes the impurity-dressing by particle-hole pairs [Mas11]. A corresponding measurable quantity is the quasi-particle residue which quantifies how much of the wavefunction of the noninteracting particle is contained in the polaron’s wavefunction [Koh12].

In recent experiments the quasi-particle residue and the effective masses have been measured in 3D Fermi systems [Sch09] [Koh12] as well as in a 2D environment [Kos12]. In all these experiments they do not observe a difference in the polaronic features for large impurity concentration up to one fifth of the particle number of the Fermi sea. This raised the question, how many majority particles are needed to describe the behavior of the impurity in the limit of a single impurity immersed in an infinite number of majority particles.

With our system we are able to built up a Fermi sea from the bottom up by successively adding more majority particles to the trap containing the single impurity. Thus we can measure the interaction energy of the system as a function of the

number of majority particles for different values of the interaction strength. In this chapter we summarize the results of these measurements ¹ and compare them to an exact theory for a single impurity immersed in a homogenous Fermi sea. We find that it takes only a few majority particles to converge to the limit of a 1D Fermi-sea consisting of an infinite number of particles.

7.1. Energy of a single impurity

To determine the influence of the number of majority particles N_{maj} on the impurity we measure the interaction energy of systems which contain a single impurity and an increasing number of majority particles confined in a 1D harmonic trap. For the noninteracting case these systems consist of the impurity particle in the ground state and a number of majority particles which fill the trap up to the Fermi energy E_F . These system are shown in figure 7.1. Like for all few-fermion systems

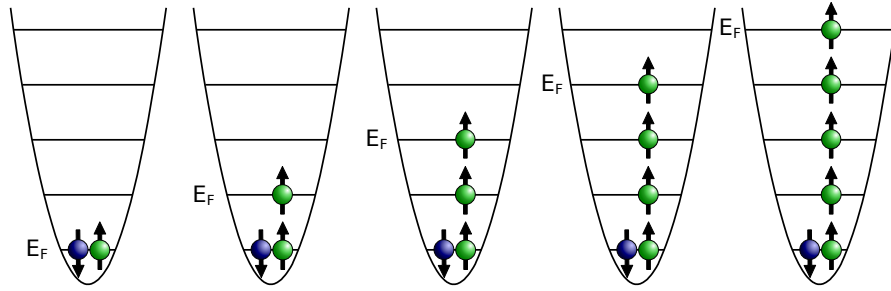


Figure 7.1.: **A single impurity in systems with different number of majority particles.** To determine the influence of the majority number on the impurity we successively add majority particles and measure the interaction energy of the system.

presented in this thesis the coupling constant g_{1D} between the $\uparrow$ -particles and the $\downarrow$ -particles can be tuned by a confinement induced resonance (CIR, see chapter 3.2.3) while the majority fermions are indistinguishable and therefore do not interact (see chapter 3.1.1). To measure the energy of the repulsively interacting few-fermion systems we have performed rf-spectroscopy on the impurity particle. Figure 7.2 a) shows the obtained spectra for different number of majority particles at a coupling strength of $g_{1D} = 2.0 [a_{\parallel} \hbar \omega_{\parallel}]$.

One observes that the interaction energy increases with the number of majority particles as can be see from figure 7.2 b). This increase in interaction energy can

¹The measurement has just been completed while writing this thesis. Further details on the analysis can be found in future publications [[Joc12b](#)]

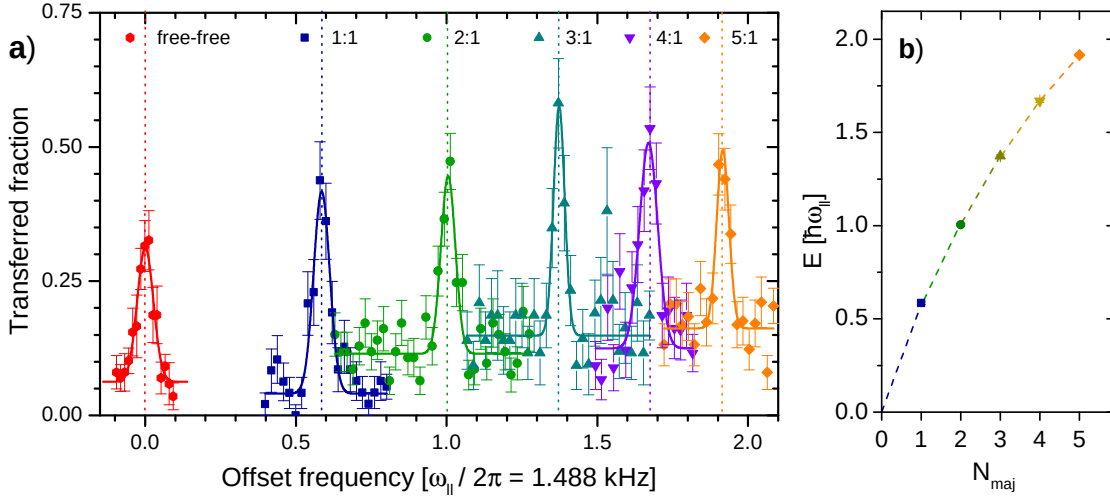


Figure 7.2.: **Rf-spectra and interaction energy.** a) Rf-spectra of a system with a single impurity and different majority number for $g_{1D} = 2.0 [a_{||}\hbar\omega_{||}]$. b) energy obtained from the rf-spectra. For increasing impurity number, the energy increases due to the repulsive interaction.

be explained by the larger amount of interaction partners which all contribute to the repulsive interaction. Yet, the increase of the interaction energy per majority particle is less or at most equal to the interaction energy of two distinguishable particles in a harmonic trap for $-1/g_{1D} < 0$. Note that adding more majority particles also increases the density in the 1D harmonic trap. This has to be taken into account when comparing our results to the theoretical model which we introduce in the next section.

7.2. Theory for a single impurity in a homogeneous system

For the case $g_{1D} \rightarrow +\infty$ we have already introduced a powerful theoretical model with exact solution which we experimentally confirmed for 2 and 3 distinguishable fermions in chapter 4: at diverging coupling strength a 1D system of i identical spin- $\downarrow$ -fermions and j identical spin- $\uparrow$ -fermions can be mapped onto a system of $(i + j)$ noninteracting identical fermions [Gir60] [Gir10]. Hence, the interaction energy in a harmonically confined system with one spin impurity in a Fermi sea of

7.3. How many particles are needed to form a 1D Fermi sea?

N_{maj} particles is exactly $N_{\text{maj}}\hbar\omega_{\parallel} = E_F$. In a very simplified picture ² the minority particle has to get all the way up to the Fermi surface to get fermionized for which it has to gain an energy of E_F .

For a single impurity in 1D there is an exact solution of the spin-impurity Hamiltonian in a homogeneous system, i.e. without a confining potential, for all values of the coupling strength $g_{\text{1D-McGuire}}$ of the 1D contact interaction potential [McG65]. In the limit of $N_{\text{maj}} \rightarrow \infty$ with $\gamma = g_{\text{1D-McGuire}}/k_F$ constant this approach provides the following formula for the interaction energy

$$E = \frac{k_F^2}{\pi} \left(\gamma/2 + \tan^{-1}(\gamma/2) - (\gamma/2)^2 \left(\frac{\pi}{2} - \tan^{-1}(\gamma/2) \right) \right) \quad (7.2)$$

where k_F denotes the Fermi momentum of the Fermi sea of majority particles. In the homogeneous case $k_F = (N_{\text{maj}} - 1)\pi/L$ with L being the size of the system [Bro12a]. An approach using the ansatz of particle-hole excitations as introduced in the beginning of this chapter provides similar results [Gir09].

7.3. How many particles are needed to form a 1D Fermi sea?

To answer the question of how many majority particles are needed to describe them by an infinitely large Fermi sea we compare our measured energies to the above mentioned theory by McGuire [McG65]. In this theory the energy of a single impurity in a homogeneous system is given as a function of the interaction parameter γ which depends on $1/k_F$ and thus is proportional to the inverse line density $(N/L)^{-1}$. However, in our case the situation is somewhat different as the particles are confined in a harmonic trap. In a 1D harmonic trap the Fermi energy is $E_F = N_{\text{maj}}\hbar\omega_{\parallel}$ and thus $k_F = \sqrt{\frac{2m}{\hbar}}\omega_{\parallel}N$. ³ Hence, to obtain γ we have to scale g_{1D} by $\frac{1}{\sqrt{N}}$. ⁴ Although the theory of McGuire is developed for a homogeneous system we can still use it for comparison: In a local density approximation the trapped impurity experiences a homogeneous Fermi sea. If the impurity was located in the trap minimum the density would be the same as in the theory of McGuire. For any other spatial distribution of the single impurity in the trap the

²When projecting the wavefunction of the interacting system onto the noninteracting single particle basis one of the component of the decomposition corresponds to the simplified picture.

³In all expressions for the energy of the system we neglect the zero-point energy $\frac{1}{2}\hbar\omega_{\parallel}$ of the trap.

⁴ $\gamma = g_{\text{1D}}/k_F$ is given in dimensionless units. $g_{\text{1D}} = \sqrt{2}g_{\text{1D-McGuire}}$ as the theory of McGuire addresses the problem in terms of the particle mass m whereas we consider the reduced mass $\mu = m/2$.

local k_F is smaller than the peak k_F and thus the theory of McGuire serves as an upper boundary for the energy in our trapped system.

To compare the interaction energy E for different densities and particle numbers we scale the interaction energy by the Fermi energy. Hence, to obtain the normalized energy E/E_F we divide the measured interaction energies by $N\hbar\omega_{\parallel}$.⁵

Applying this scaling we can plot the theory of the two limiting cases – one impurity with one majority particle (1:1) and one impurity with an infinite number of majority particles (1: ∞) – into one graph (figure 7.3). The blue curve represents the already frequently cited solution for two particles in a harmonic trap [Bus98], i.e. a system of a single ‘impurity’ with one ‘majority’ particle [Bus98]. The other limiting case is McGuire’s theory of one impurity with an infinite number of majority particles shown by the yellow curve. Both theories approach unity for $k_F/g_{1D} \rightarrow 0$ which is the point of Fermionization. One can observe that for increasing number of majority particles (from blue to orange) our measurement of the normalized interaction E/E_F energy approaches the theory of McGuire. This can be seen in detail in figure 7.4 where we have plotted enlarged views for different interaction strengths. To these graphs we have also added the results for a 1:2 system derived by the group of Doerte Blume [Gha12] and the MCDTH calculation of Ioannis Brouzos for finite systems of larger imbalance [Bro12c]. One can already guess from these theories that the energy rapidly converges to the limit at $N \rightarrow \infty$. Our data confirms this behavior for increasing particle number. We find that already for a system of 3 majority particles the energy is very close to the theory of McGuire. For a 1:4 and a 1:5 system we find surprisingly good agreement of our measurement with McGuire’s theory although it has been established for a homogeneous system which serves as an upper boundary for our trapped system. The deviation to the MCDTH method could be due to systematic effects such as anharmonicity and finite aspect ratio of the microtrap potential which we might have over-estimated in our correction.

Yet, these systematic effects cancel to first order if we regard only the change of the interaction energy per added majority particle. Therefore we consider the quantity

$$s = \frac{\Delta_N \left(\frac{E}{E_F} \right)}{\Delta_N \left(\frac{k_F}{g_{1D}} \right)} \quad (7.3)$$

⁵For this scaling we take the anharmonicity of the trap into account. E_F is replaced by the energy of the n -th single particle level in the anharmonic trap derived by a WKB calculation (see appendix figure A.29). We also considered the shift discussed in chapter 3 between the 1D solution and the solution for the 3D trap with 1:10 aspect ratio (see appendix figure A.30). The shift caused by these effects is shown in figure A.31 in the appendix.

7.3. How many particles are needed to form a 1D Fermi sea?

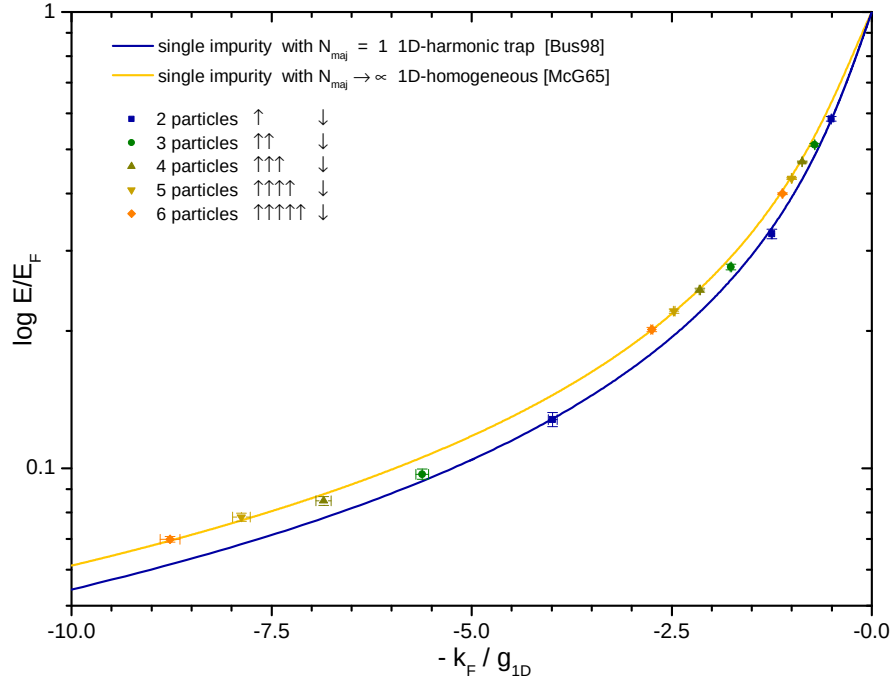


Figure 7.3.: **Normalized interaction energy as a function of the interaction parameter k_F/g_{1D} and depending on the particle number.** The data points (blue $\rightarrow$ green $\rightarrow$ orange) show the interaction energy for systems with increasing number of majority particles. For a 1:1 system we find excellent agreement with the theory of T. Busch et al. [Bus98]. For larger values of N_{maj} the energy gets closer to the theory of McGuire for a single impurity in a homogeneous system of infinite N_{maj} [McG65].

with the function $\Delta_N(x) = x_{N_{\text{maj}}} - x_{N_{\text{maj}}+1}$ at constant g_{1D} . s_{exp} represents the slope of the connecting line between two neighboring data points in figure 7.4. From the similar slopes of the different theory curves for $N_{\text{maj}} = 1, \dots, 5$ and $N_{\text{maj}} = \infty$ in a homogeneous system at constant k_F/g_{1D} we assume that the exact theory for a single impurity immersed in a large Fermi sea in a harmonic trap⁶ has approximately the same derivative

$$\frac{d\left(\frac{E}{E_F}\right)}{d\left(\frac{k_F}{g}\right)}_{\text{h.o.}} = \frac{d\left(\frac{E}{E_F}\right)}{d\left(\frac{k_F}{g}\right)}_{\text{hom.}} \quad (7.4)$$

⁶To our knowledge when writing this thesis a theory for a single impurity immersed in a large Fermi sea in a harmonic trap has not been available.

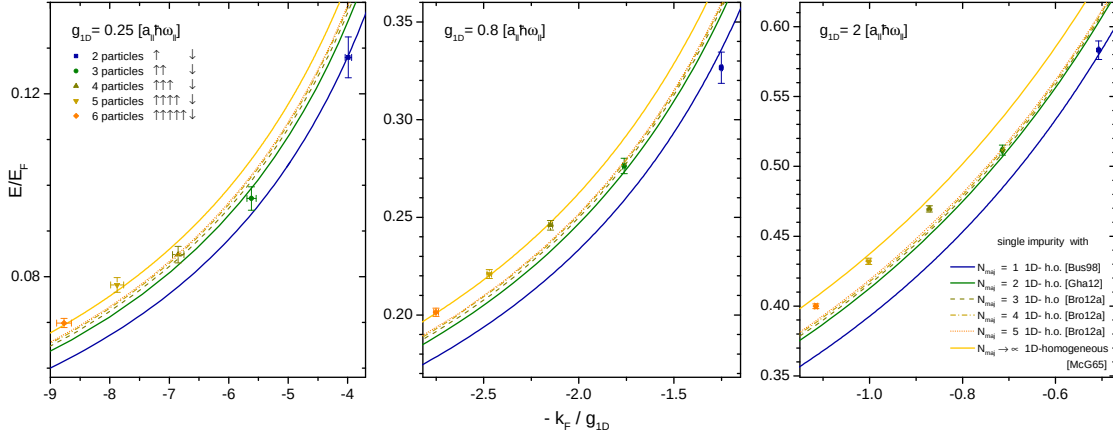


Figure 7.4.: **Enlarged views of the normalized energy for three different values of g_{1D} .** The notation for the measured data points is the same as the one in figure 7.3. The blue curve shows the 1:1 theory [Bus98] and the yellow curve shows the upper limit represented by a single impurity in a homogeneous Fermi sea of infinite number of majority particles [McG65]. Additionally we have added the 1:2 theory of [Gha12] (solid green curve) and the MCDTH calculation results for a harmonically trapped system of up to 1:5 particles [Bro12c] (dashed curves).

as the homogeneous theory of McGuire⁷. Hence, to determine the change of energy with respect to the coupling parameter for the harmonically trapped system we derive the slope of McGuire's theory s_{theo} in the same way as done for the experimental data. Now, we can relate the change of the energy with respect to the majority number to the derivative of the theoretical prediction. The result is shown in figure 7.5 for two values of the 1D coupling constant g_{1D} .⁸ We find that for small majority number the ratio is considerably larger than 1 which means that the measured change of the energy with respect to the majority number differs from the change predicted by the theory. Yet, the change from a 1:3 to a 1:4 system is almost equivalent to that of the theory and the transition from a 1:4 to a 1:5 system follows the same slope. This means that by adding further majority particles we do not change the properties of the dressed impurity except for changing the density of the Fermi sea.

⁷We expect that the theory curve for the energy of the harmonically trapped case lies between the dotted line (MCDTH 1:5) and the solid line (McGuire 1: ∞ homogeneous system)

⁸As the relative errors for weak interaction energies are larger we have not considered the slope for $g_{1D} = 0.25 [a_{\parallel} \hbar \omega_{\parallel}]$

7.3. How many particles are needed to form a 1D Fermi sea?

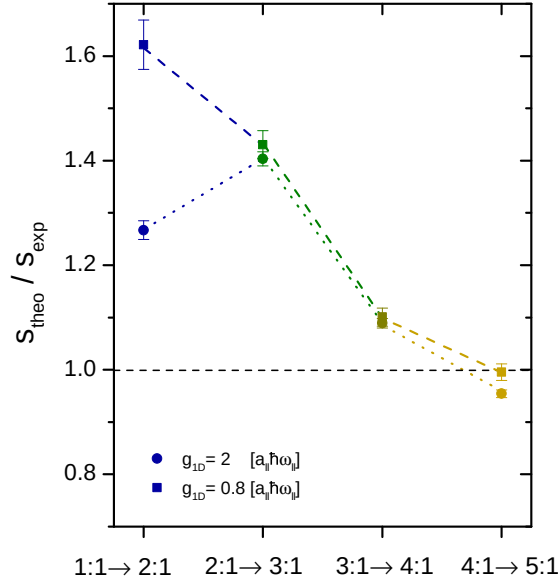


Figure 7.5.: **Relating experiment and theory.** The graph shows the change of the energy as a function of the majority particle number (s_{exp} , equation 7.3) related to the theoretical prediction for a system of an infinite majority number (s_{theo} , equation 7.4). We find that the change in the experimental data approaches the change in the theory already for a system of four majority particles. By further adding majority particles the property of the impurity is not anymore modified and follows that of a single impurity immersed in an infinitely large Fermi sea.

This observation allows us to answer the initially posed question: It seems like already four majority particles are sufficient to describe the system by a single impurity immersed in an infinite number of majority particles. Or to put it in other words: Four particles are enough to form a 1D Fermi sea.

8. Conclusion and Outlook

8.1. Conclusion

Using the advantages of ultracold atomic systems we have successfully set up a generic quantum system which allowed us to perform experiments with one-dimensional few fermion systems. To achieve this we have designed and built a reliable setup with precise control over the key experimental parameters such as the depth of the confining potential and the strength of the magnetic offset field. This gives us outstanding control over several degrees of freedom of the quantum system.

A generic quantum system

- With our scheme we can deterministically prepare a few-fermion system with a fidelity of 98% per particle. We have achieved control over the motional state of the particles in the trap. Our preparation scheme has a large ground state preparation fidelity of up to 93% [Ser11b]. By coherent coupling we can transfer a two-particle system from the ground state to an excited motional state with $\sim 90\%$ fidelity. In these quasi-1D systems we are able to tune the interaction strength using confinement induced resonances.
- To probe the few-fermion system we have implemented several diagnostic tools: By periodically modulating the trap depth we can determine the shape of the confining potential with a high precision.
- By measuring the tunneling dynamic of fermions which tunnel through a well-defined potential barrier we can deduce the energy and the correlations of the few-fermion system.
- Using rf-spectroscopy with a resolution much higher than the characteristic energy scale of the system allows us to precisely measure its interaction energy. As we can resolve single trap sidebands the rf-spectroscopy offers a possibility to map out the wavefunction of an interacting system. We demonstrated the capability of our high resolution rf-spectroscopy by measuring the binding energy of weakly bound dimers which allowed for a

precise characterization of ^6Li Feshbach resonances. We lowered the uncertainty of the resonance position by almost two orders of magnitude compared to previous experiments.

Using this system with the versatile diagnostic toolbox we studied several few-fermion systems during the course of this thesis.

Investigated few-fermion systems

- By comparing systems of two distinguishable fermions with two identical fermions we directly observed the mechanism of fermionization [Zür12] which gives rise to the physics of strong repulsively interacting systems such as the Tonks and Super-Tonks gas .
- In two-particle systems with increasing attractive interaction we found evidence for increased-pair correlations. For larger systems we observed a strong odd-even effect in the single particle dissociation energies as a function of the particle number. This pairing phenomena exhibits similarities to nuclei which suggests that our system is suited to study further pairing mechanisms related to nuclei.
- Using rf-spectroscopy we measured the energy of few-fermion systems with up to six particles. For two and three particles we found excellent agreement to the theoretical models. By tuning the systems across the CIR and measuring rf-transitions to different energy states we have demonstrated that we can create metastable systems in the so-called super-Tonks regime with up to four particles without major loss.
- We have investigated the properties of a single impurity which repulsively interacts with an increasing number of majority particles. We have started with the smallest possible few-particle system, a single impurity and one 'majority' particle and measured the interaction energy while adding more and more majority. We have observed that already for majority numbers of $N \geq 4$ the interaction properties can be described by that of a polaron-like particle immersed in a Fermi sea. Thus, for this particular system we have shown the crossover from a few-particle system to a system well described in the limit of $N \rightarrow \infty$.

With the presented measurements we have investigated various fundamental few-fermion systems in a 1D environment. Yet, our setup has the capability to study even more complex phenomena in future experiments such as spin-correlations or dynamical processes in higher dimensional mesoscopic Fermi-systems.

8.2. Outlook

8.2.1. Ferromagnetic correlations in 1D systems

An extensively debated issue is the question if a two-component mixture of interacting spin-1/2 fermions in a homogeneous system can form ferromagnetic domains [Jo09] [San12]. A model which states a possible condition for ferromagnetic order is the Stoner model [Sto39]: In the case that the effect of the repulsive interaction between two distinguishable fermions is larger than the Pauli repulsion of two identical fermions, it is more favorable to form magnetic domains. For our 1D system we have found a regime where the Stoner criterion is fulfilled: We accessed it by tuning the coupling strength of a few-fermion system across the CIR to the super-Tonks regime where the energy of the system is larger than the energy of a corresponding spin-aligned system. Measurements which we have performed with 3 and 4 particles on this meta-stable state have yielded promising results: We have observed that in the super-Tonks regime a polarized system remains after one particle has tunneled from the correlated N-particle super-Tonks state (cf. chapter 4.4). This is a strong indication that the system which we have created, actually exhibits ferromagnetic correlations. The exact analysis of the observed properties is current work in progress. An interesting issue is the question if the investigation of ferromagnetic correlations can be extended to systems in higher dimensions?

Studying few-fermion systems in higher dimensions is one of the major goals for future experiments. For instance in systems with attractive interaction we want to investigate how the initial 2D shell structure alters when increasing the strength of the interaction (cf. chapter 5.2.2).

To be able to create different confining potentials we have designed a new optical setup [Kli12b] containing a high NA objective [Ser11a] with the goal to also create multiple well potentials.

8.2.2. Antiferromagnetic ordering in a multiple well potential

The possibility to create multiple adjacent potential wells will allow us to study spin-1/2 systems in finite periodic potentials. In the case of an infinite chain or lattice of wells this system can be described by the Fermi-Hubbard model [Lie93]. It incorporates the kinetic energy by a tunneling term which describes hopping from one site to the other. The interaction in this model is parametrized by an on-site interaction term. For repulsive interaction the many-particle ground state is predicted to be an ordered system with alternating spin orientation in neighboring sites which is referred to as antiferromagnetic ordering. This is the opposite

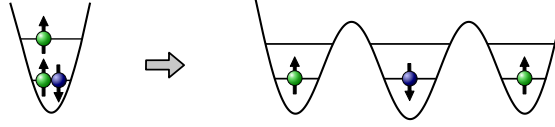


Figure 8.1.: **Antiferromagnetic ordering in a multiple well potential.** In the case we can adiabatically change a single well potential containing a repulsively interacting few-fermion system into a multiple well configuration we expect to observe antiferromagnetic ordering of the particles for a certain ratio of the tunneling rate and the strength of the repulsive on-site interaction.

regime of the previously discussed ferromagnetic order with the Pauli repulsion dominating over the interaction energy and a localization of the individual particles.

We want to investigate these systems by starting from a N -particle ground state in a single well, where N is the number of wells which we want to realize (see figure 8.1). By smoothly splitting up the single well into a finite size periodic potential while the system should stay in the ground state, we expect to observe the appearance of anti-ferromagnetic order.

8.2.3. Dynamical processes in few-fermion systems

The precise control over our system should also allow to study dynamical processes in few-fermion systems. Partially we have already started to investigate dynamical phenomena by observing tunneling of particles through a static potential barrier. We want to excite even more interesting dynamics by periodically modulating the shape of the potential barrier which we can achieve by sinusoidally change the orientation of the linear potential relevant for creating the tunneling barrier.

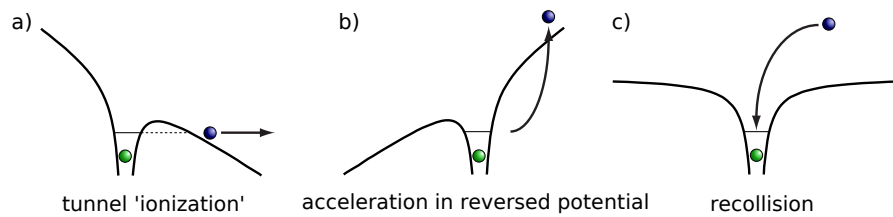


Figure 8.2.: **Dynamical process similar to ionization of atoms in strong laser fields.** The process can be described in a three-step model: a) One particle is 'ionized' by applying a time-dependent magnetic field gradient. b) The particle is accelerated in the oscillating potential. c) The particle comes back to the trap region where it interacts with the residual particles. Taken from [Gal12] and adapted.

Then the finite size Fermi-system in our microtrap system would be similar to an atom which electrons are excited by a strong laser pulse [Cor93]: In the dipole approximation the potential created by the electrical field of the laser is identical to our linear potential (see figure 8.2). Although the characteristic timescale of this ultra-fast physics is orders of magnitudes away from that of our system, we expect to be able to study similar physics.

Besides non-adiabatically changing the potential we want to excite a finite Fermi system into a non-equilibrium state by performing an interaction 'quench', a non-adiabatic change of the coupling strength. By probing the time evolution of the system in different dimensions we can study how or if the isolated quantum system thermalizes which is still an open question [Rig12].

A. Appendix

A.1. ${}^6\text{Li}$ level scheme

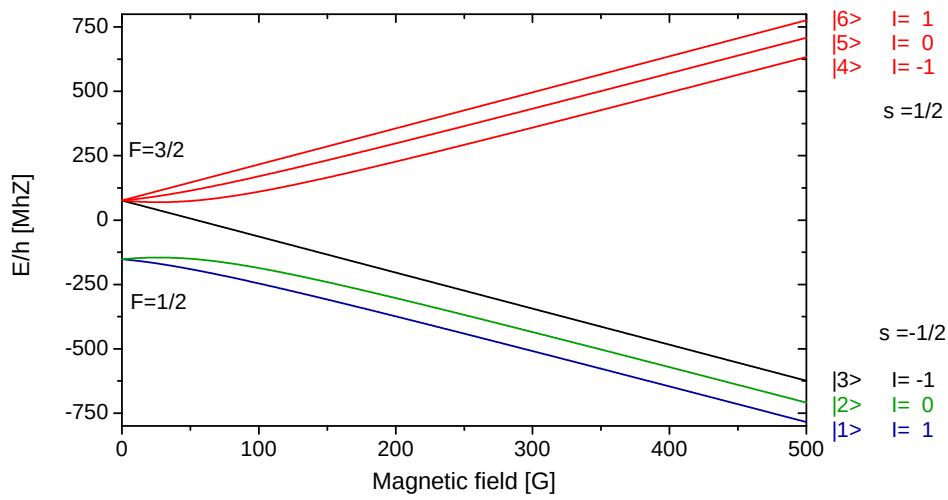


Figure A.1.: **Level scheme of ${}^6\text{Li}$.** The energy of the hyperfine levels has been calculated using the corresponding formula of Breit and Rabi [Bre31].

A.2. Deterministic preparation

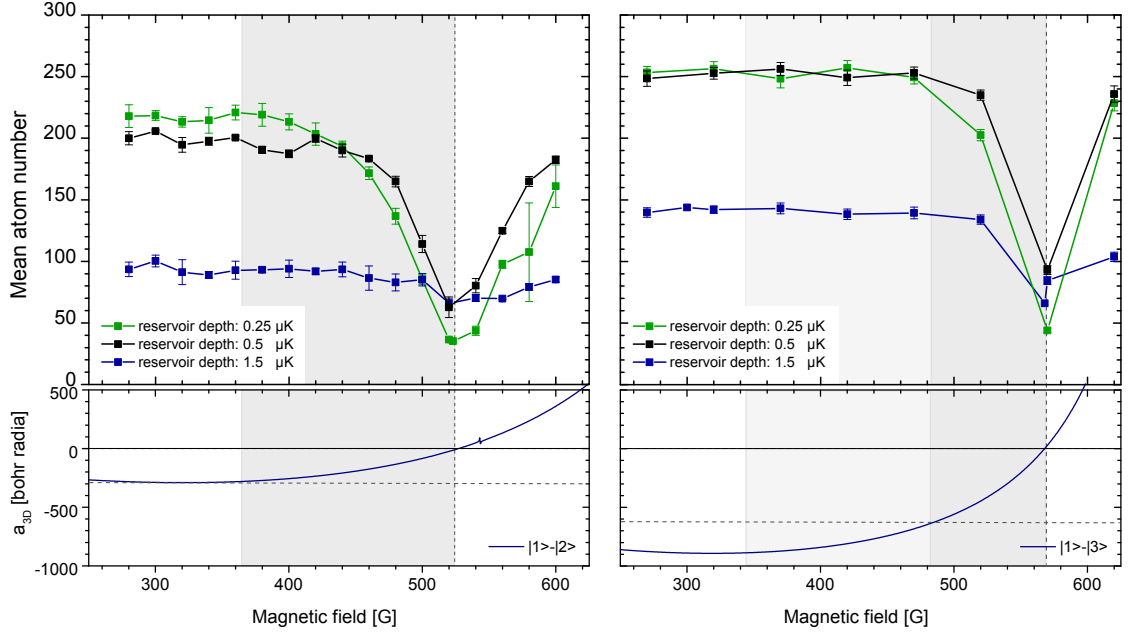


Figure A.2.: **Qualitative analysis of the transfer into the dimple.** The plots show the number of transferred particles in dependence of the reservoir's trap depth as a function of the magnetic offset field which determines the scattering length a_{3D} (lower panel). At lower reservoir depth, T_{res} is decreased as well as the density n_{res} of the reservoir. The efficiency of thermalization is determined by the scattering rate which is proportional to the scattering length. In the gray shaded area of the left plot, the number of transferred particles depends on the scattering length over the whole range which indicates the system has not equilibrated yet. For this mixture of $|1\rangle$ - $|2\rangle$ atoms the value of the scattering length cannot be increased further (white region, lhs.). This limitation can be overcome by choosing a mixture of $|1\rangle$ - $|3\rangle$ atoms where the scattering length can be tuned to larger values (right plot). Then, by further increasing $|a_{3D}|$ (light gray shaded region) the number of transferred particles saturates, which is a signature of thermalization. We also find no measurable change in the number of transferred particles at lower temperature (right plot, green data compared to black data) which indicates that we have entered the highly degenerate regime with near unity filling of the states in the microtrap. Note: the maximum number of transferred particle is lower than the 600 particles stated in chapter 2.2.2 which is due to the lower depth of the microtrap used for this analysis.

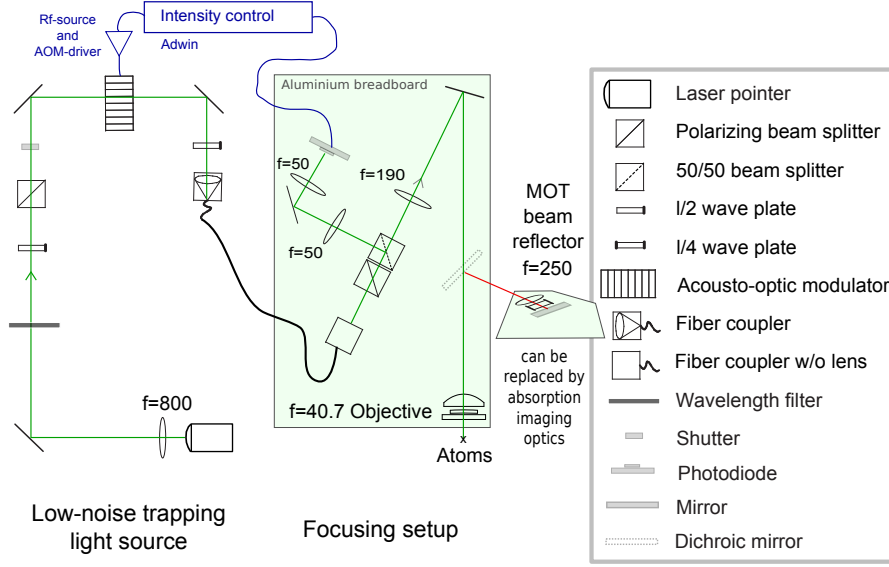


Figure A.3.: **Light source and focusing setup for generating a stable optical potential.** For the light source we use a laser with a wavelength of 1064 nm. A low noise laser source is important as frequency noise on the order of the trap frequency leads to excitation of atoms to higher trap levels with a rate depending on the amplitude of the modulation [Geh98]. Table A.1 shows the relative intensity noise [Ser11a] for three different types of laser which we have used in the experiment. For the demonstration of high fidelity preparation we have used a modified laser-pointer (figure A.4), later we changed to the commercial laser source Mephisto S (Innolight). The intensity of the beam is varied using an AOM which is controlled by a digital PID controller [Zür09]. The light is transferred via an optical fiber to the focusing setup. In this setup a portion of the light is outcoupled from the beam and focused onto a photodiode which measures the intensity of the beam. This signal is fed into the PID controller to stabilize the intensity of the microtrap potential. It is extremely crucial that the intensity ratio between the out-coupled light used for stabilization and the light focused into the vacuum chamber by the microtrap objective is constant within 10^{-3} to achieve a sufficiently large window of deterministic preparation (see chapter 2.2.3). We ensure this by suppressing polarization drifts which could be translated into intensity drifts by polarization dependent optical devices. This is done using a polarizing beam splitter cube (suppression of undesired polarization of 10^{-4}) in order to clean the polarization and a subsequent non-polarizing beam splitter cube to couple out the light. To furthermore suppress intensity drifts caused by interference of the coherent light reflected from different surfaces, we use anti-reflection coated optics. Additionally, we have removed the cover glass of the photodiode (Hamamatsu G8370-81) which caused intensity drifts as it had been acting as an etalon. In a test measurement we confirmed the desired intensity stabilization of 10^{-3} [Ser11a].

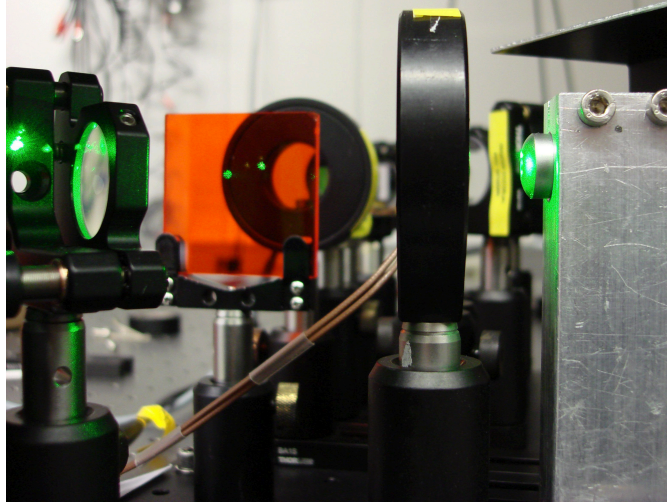


Figure A.4.: **Modified laser pointer as a source for the trapping light.** To demonstrate the deterministic preparation of a few fermion system [Ser11b] we used a modified laser pointer designed for emitting light at a wavelength of 532 nm. This green light is generated by intracavity frequency doubling of light with 1064 nm wavelength which corresponds to the actual laser transition of the laser-crystal which is pumped by a 800 nm laser diode. Due to the large power of the pumping diode and the poor 1064 nm anti-reflection coating of the cavity surface, about 6 mW of the 1064 nm-light is leaking out of the green laser pointer. Using a filter-plate (orange plate in the picture) we filtered out the residual 100 mW green light and used the 1064 nm-light as the laser source for the microtrap.

Laser source	RIN [dB/Hz]
IPG YLM-5-LP	-96
modified laser pointer	-116
Mephisto S (Innolight)	-113
Mephisto S with noise eater function	-117

Table A.1.: **Relative intensity noise (RIN) of different laser sources.** The right column shows the relative intensity noise below 100 kHz, the relevant range in which frequency noise can cause undesired excitations of atoms in our system with typical trap frequencies of 30 kHz. For all three lasers the RIN below 100 KHz is independent of the frequency. Due to the uncertainty in the calibration of the detector the absolute values might differ by 10 dB. The first laser which we have used – the IPG fiber laser – has been replaced by a modified laser pointer which noise level is 20 dB lower (see figure A.4). It is as good as the commercial Mephisto S with noise eater option. Yet, for the modified laser pointer it has been more difficult to operate it in a reliable mode because the current of its pumping diode has not been actively stabilized. This is the reason why we changed to the Mephisto S which operates in its low-noise mode without any readjustment within months. We have not quantitatively analyzed the effect of the RIN onto the heating rate in the microtrap. Qualitatively however, we have found an improvement of the preparation fidelity after replacing the IPG fiber laser by the modified laser pointer. Yet, as we have also changed other components of the setup, we cannot clearly state if the 20 dB improvement of the noise level by replacing the light source is responsible for the increase in preparation fidelity.

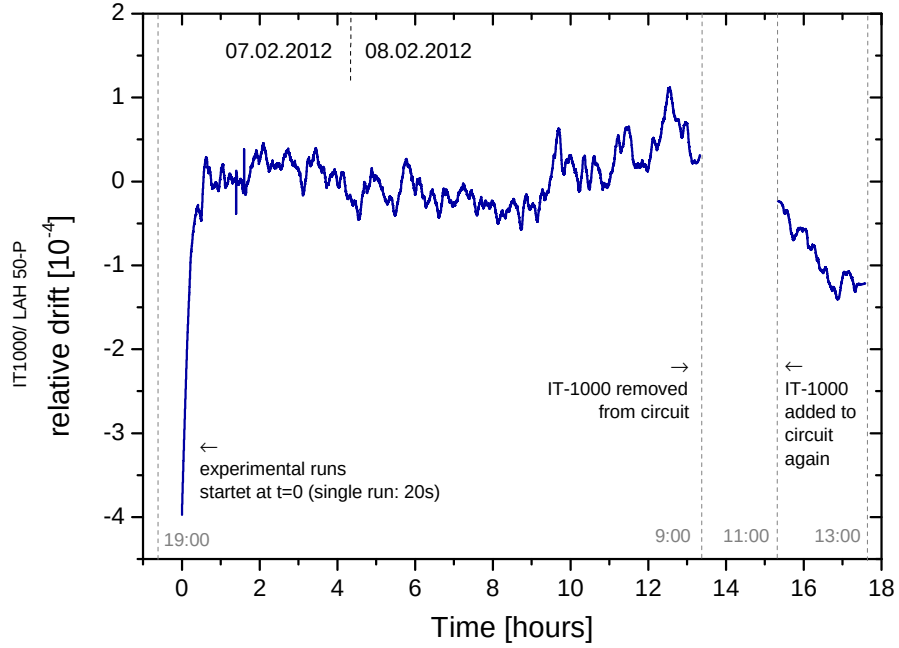


Figure A.5.: **Stability of the gradient current.** The current which generates the magnetic field gradient is stabilized using the current transducer LEM LAH 50-P in combination with a precision resistor (Vishay S102J, 2ppm/K) which transfers the secondary current of the transducer into a measurable voltage. The LAH 50-P has a specified accuracy of 2.5×10^{-3} . If drifts were occurring in this range they would disturb a deterministic preparation. To check the stability of the LAH 50-P we added a second transducer to the same circuit. The second transducer is a LEM IT-1000 with a accuracy of $< 50 \times 10^{-6}$ (offset current + self magnetization + effect of earth magnetic field) used in combination with eight in parallel connected Vishay RCK-HR02 resistors (2ppm/K). To measure drifts of the gradient current we recorded the ratio between both transducers for 18 hours. Within the first half an hour we observe a fast increase between the ratio of both sensors. After this increase of 4×10^{-4} the current stayed within $\pm 1 \times 10^{-4}$. Hence, after switching on the experiment one should wait at least one hour until the current transducer has saturated to its 1×10^{-4} temporary accuracy which is much better than the specified absolute accuracy. To check which one of transducers actually drifts we removed the high precision sensor (IT-1000) from the circuit. Afterwords we waited 2 hours to allow the IT-1000 to relax to its steady state in offline mode. After that we reconnected the transducer with the circuit. We observed no fast change in contrast to our initial observation. From this we deduced that the large drift at $t = 0$ stems from the LAH 50-P.

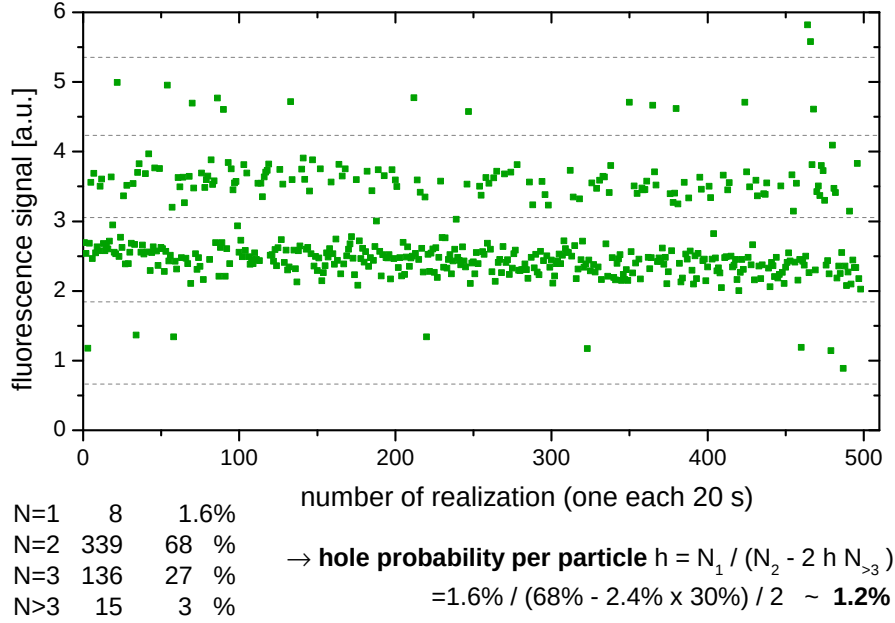


Figure A.6.: **Hole probability per particle of the lowest trap state.** A hole is defined as an unoccupied single particle state in the system. We adjust the tunneling barrier such that the probability to find three particles left in the potential is approximately one third. In this case the tunneling rate of the lowest trap level is decreased compared to the case when we try to remove all atoms from the second level ($n_{\text{h.o.}} = 1$). This procedure should reduce the effect of drifts which lead to tunneling of particles from the ground state. We estimate that the hole probability per particle for the lowest state in the trap is 1.2(4)%.

A.3. Fermionization experiment

A.3.1. Tuning the trap depth

Changing the trap depth parameter p , which is done by modifying the power of the microtrap beam, also changes the harmonic oscillator length of the confinement $a_{\perp} = \sqrt[4]{p} a_{\perp,0}$. This results in a slight change of the coupling constant $g_{|\uparrow\downarrow\rangle}$ and the position of the CIR (see equation (3.37) chapter 3). We have observed that there are two loss channels with a width of 0.4 G (FWHM) close to the CIR which cause loss of unbound-particles depending on the position of the CIR: Due to the anharmonicity of the trapping potential relative motion states can couple to excited center-of-mass states of molecules which is discussed in chapter 4.5. The positions of these loss channels depending on the magnetic field and the harmonic oscillator length and are shown in figure A.18. Hence, after ramping to a constant magnetic field for setting up a certain interaction strength, we do not want to modify the harmonic oscillator length $a_{\perp}$ to a large extent in order not to enter one of the two loss channels. Therefore we set p already to a value lower than the initial depth of $p = 1$ right after the preparation process. Then $a_{\perp}$ tunes only by a small amount when we perform experiments at constant magnetic field values next to the position of the loss channels. However, we cannot tune p to arbitrarily low values before the measurement starts because all single particle states of the potential contribute to the wavefunction of the interacting $|\uparrow\downarrow\rangle$ -system with decreasing fraction for larger energy states. To make sure that the system does not tunnel before the actual tunneling measurement starts, we have to leave all the states which considerably contribute to the wavefunction bound in the trap. Experimentally we found that 4 bound states are sufficient to ensure that less than a fraction of a few percent has tunneled before the actual tunneling measurement starts.

A.3.2. Ramp speed of the magnetic field

We avoid magnetic field values close to the two loss channels mentioned previously in A.3.1. To not be effected by these channels we non-adiabatically ramp across the resonances with sufficiently high speed of 20 G/ms.

A.4. Modulation spectroscopy

A.4.1. Coherent coupling

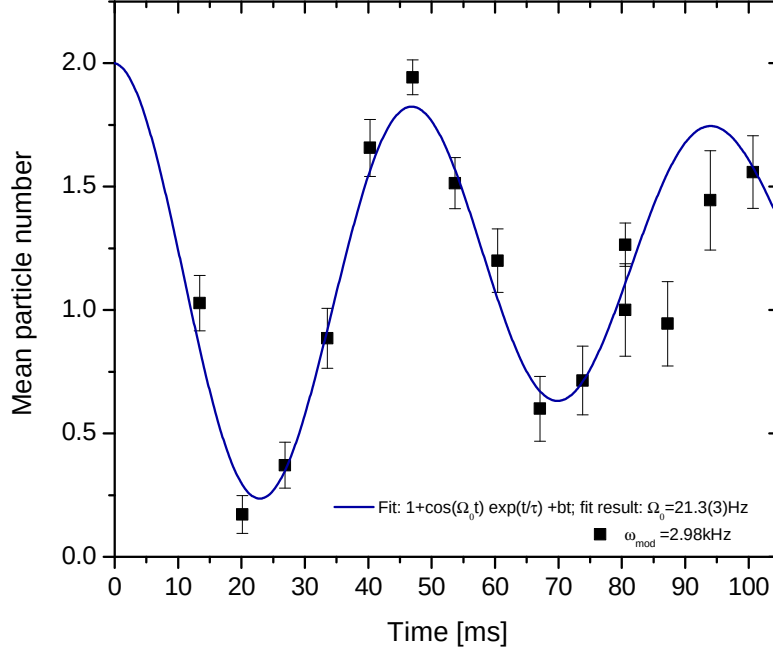


Figure A.7.: **Coherent excitation.** The modulation pulse coherently couples atoms in the ground state to the second axial trap level (quadrupole transition). After half of a Rabi-cycle about 90% of the population is transferred to the excited state. Due to the coherent coupling almost the whole population comes back to the ground state after a full Rabi cycle. The error is the standard error of the fit.

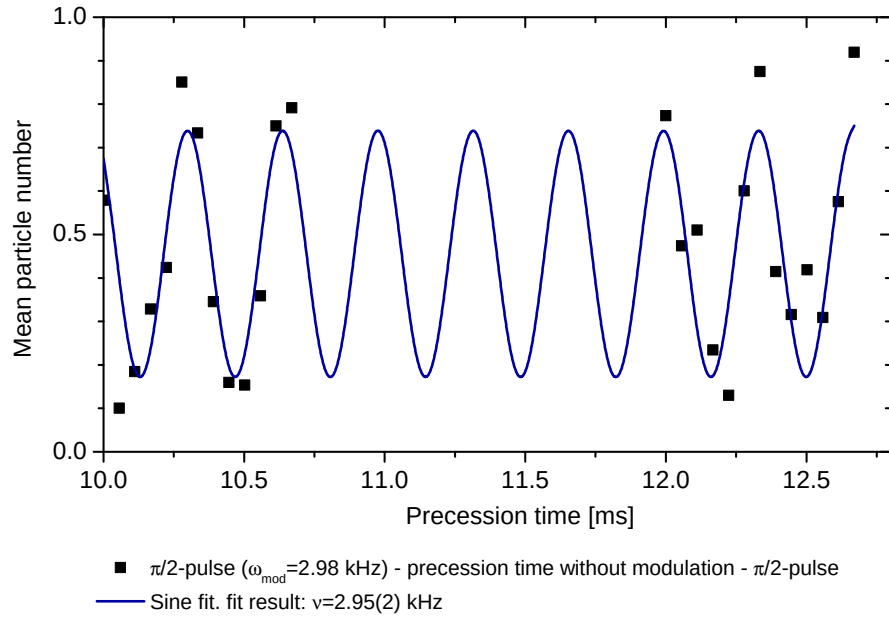


Figure A.8.: Ramsey type experiment for the 0-2 quadrupole transition in axial direction.

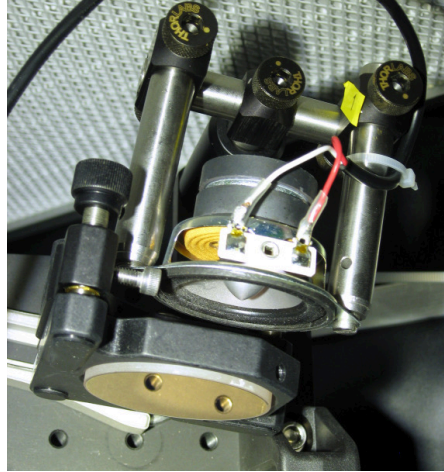


Figure A.9.: Loudspeaker for exciting the dipole transition.

A.4.2. Determination of the axial trap parameters

The following description of the routine for deriving the axial trap parameters is taken from [Zür12].

To obtain the waist we vary w_0 and P_0 within its error σ_{P_0} to minimize

$$\sum_{(i,j)=(0,1)(0,2)(2,4)} \frac{1}{\sigma_{\omega_{\parallel i-j}}} [(E_{opt j} - E_{opt i}) - \hbar\omega_{\parallel i-j}]^2 \quad (\text{A.1})$$

with $E_{opt i,j}$ being the energy of the calculated bound states of the varied optical potential. We find a minimum deviation for $w_0 = 1.838 \mu\text{m}$ and $P_0 = 291.5 \mu\text{W}$ resulting in an initial depth at the center of the optical trap of $V_0 = k_B \times 3.326 \mu\text{K}$. For these parameters two of the three frequencies match the calculated bound states within their errors and all three match within 2σ . After fixing the parameters for the optical potential we have to determine the strength of the linear magnetic potential which is created by a magnetic field gradient B' . From a levitation measurement we obtain $B' = (18.9 \pm 0.2) \text{ G/cm}$. To obtain a more accurate parametrization of the potential barrier we use the tunneling measurement of two identical fermions at a trap depth of $p = 0.6875$ as a calibration. We perform a WKB calculation of the tunneling rate and modify the value of B' in the calculation such that the resulting tunneling time constant agrees with the experimentally observed tunneling time constant of $(74.1 \pm 2.7) \text{ ms}$. From this we obtain $B' = 18.92 \text{ G/cm}$. Using this magnetic field gradient the calculation is consistent with the deterministic preparation of (2, 4, 6, 8, 10) particles (see figure 2.14) at $p=(0.6575, 0.7025, 0.7475, 0.7863, 0.8200)$ in the experiment. The parameters are summarized in table 4.2. At the optical trap depth of $p = 0.6875$, where we perform the tunneling measurements for the fermionization experiment, $\omega_{\parallel}$ has to be scaled by $\sqrt{p}$ and is given by $(2\pi \times 1.234 \pm 0.012) \text{ kHz}$ calculated from the excitation frequencies $\omega_{\parallel 0-1}$ and $\omega_{\parallel 0-2}$.

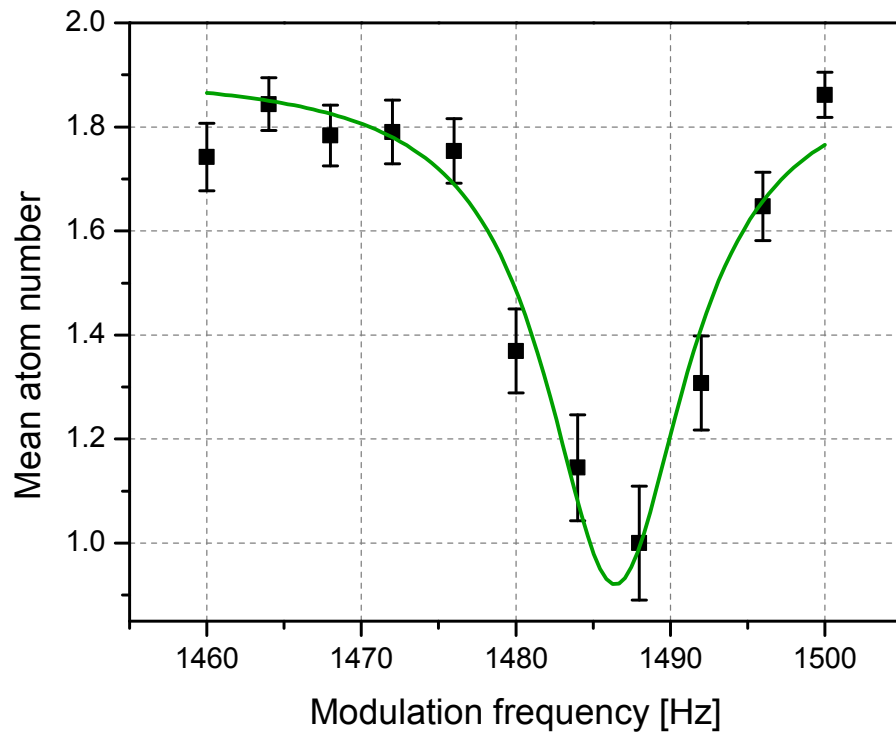


Figure A.10.: **Axial excitation determined by modulation spectroscopy.**
Dipole transition level 0-1.

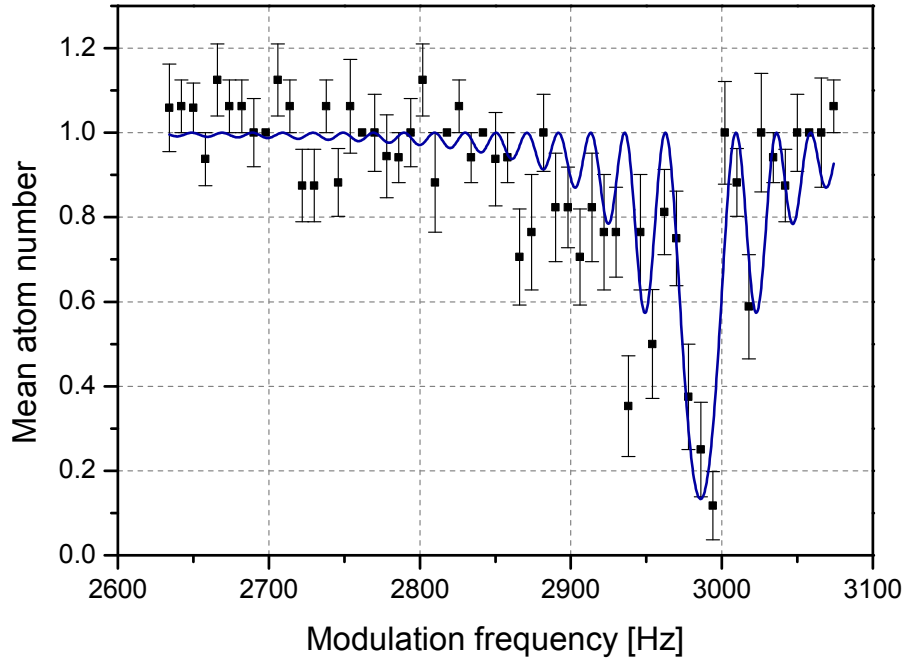


Figure A.11.: **Axial excitation.** Quadrupole transition level 0-2, identical to figure 4.8 in chapter 4.

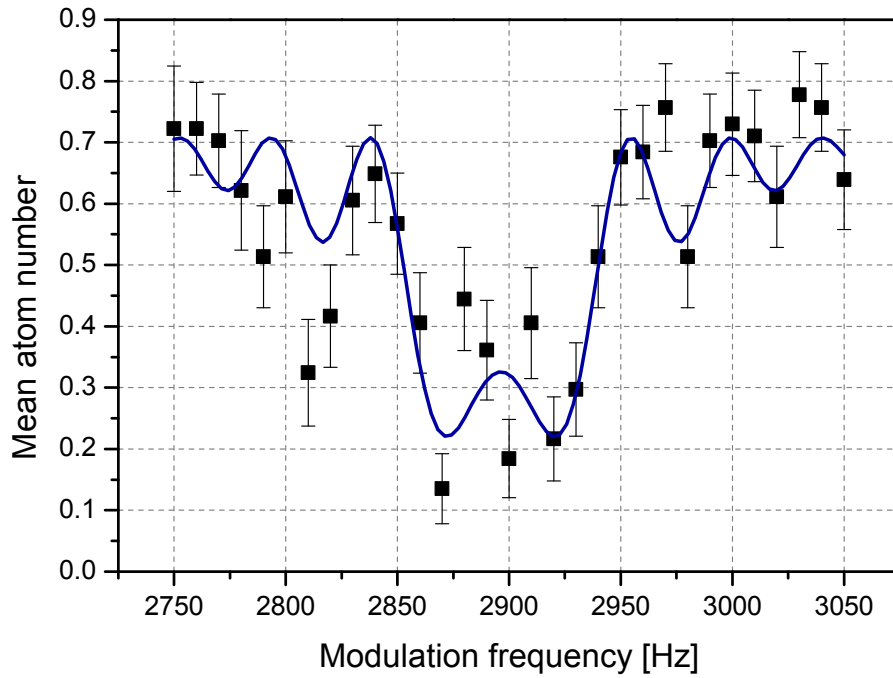


Figure A.12.: **Axial excitation.** Quadrupole transition level 2-4.

A.4.3. Determination of the radial trap parameters

Figure A.13 shows the modulation spectroscopy measurement for the radial transition frequencies from which we determine the trap parameters. In a first evaluation of the quadrupole spectrum (figure A.13 b) we have identified two peaks (blue and red curve) from which we calculated $\eta = 1.03$ [Ser11a] [Zür12]. Yet, a WKB approximation which reproduces these peaks is not consistent with the determined resonances of the dipole spectrum (figure A.13 a). Thus we needed to evaluate the shape and the position of the individual peaks more precisely: We found that the upper peak of the quadrupole-measurement is narrower than the lower peak. Also the upper peak in the dipole-measurement is narrower than the two other peaks. Furthermore, the distance between the lower peak and the upper peak in the dipole-measurement is ~ 1.4 kHz which corresponds to the axial trap frequency. Due to this distance and the weaker coupling we assign this peak to a transition with energy $\hbar(\omega_{x0-2} + \omega_{||0-1})$ with h.o. quantum numbers (1,0,1). The allocation of the x-axis to one of the two main axis of the elliptic confinement is arbitrary. The center peak is assigned to the (0,1,0)-transition with energy $\hbar\omega_{y0-1}$. A WKB calculation based on the dipole spectrum suggests that also in the quadrupole transition spectrum a peak should appear at $\sim (13.96/14.82) \times 28.26 \text{ kHz} = 26.6 \text{ kHz}$. Unfortunately we had not extended our range to such low frequencies when we performed this measurement.

Thus we had to repeat the experiment one year later, when we reconsidered the analysis of the level structure in radial direction. During that time the overall power calibration may have changed by $\sim 1\%$ due to a readjustment of the reference beam onto the photodiode (see setup in figure A.3). A readjustment of the microtrap beam has not been performed which is why the geometric shape of the potential should have remained unchanged. Hence, we observed a shift of the absolute position of the (0,2,0)-peak and the (0,2,2)-peak. As expected we observed a peak at lower frequencies which we assign to the (2,0,0)-transition (figure A.14). From the ratio between the (2,0,0)- and (0,2,0)-transition frequencies of the new measurement we calculate the aspect ratio $\eta_{xy} = 1,0695$ from which we determined $\omega_x = 26.43 \text{ kHz}$. All relevant frequencies are listed in table 4.3 in chapter 4.

Additionally to the (0,2,0)- and the (2,0,0)-peak we observed a fourth peak in the new measurement which we could not assign to any transition. The origin of this peak could be a transition of the form (1,1,0) which actually does not couple in a perfect harmonic potential. However, due to the anharmonicity and the asymmetry of the potential this transition could be allowed. To check if the peak height tunes as a function of anharmonicity we increased the potential depth by a factor of two which reduces the anharmonicity. (figure A.15). We observed that the

height of the center peak became smaller which is a hint that it indeed corresponds to the (1,1,0) transition.

However, also the aspect ratio of the two relevant peaks changed when increasing the trap depth by a factor of two ($p = 2$). Although this is expected in an anisotropic trap with Gaussian shape it exceeded the normal level. Hence we assumed this to be a result of the non-perfect Gaussian beam profile. This is in accordance with the findings in an external test setup where we mapped out the profile in radial direction and saw small deviations from the Gaussian profile (see figure 2.9 chapter 2).

Nevertheless, the approximation of the shape by a Gaussian should be sufficient for our purpose. To extract the shape of the potential we have to determine two parameters - the depth and the width - of each Gaussian in x - and y -direction. The idea is to vary the parameters in order to get the right transition frequencies of the two measurements at $p = 1$ and $p = 2$. Leaving both parameters free we can uniquely reproduce the two frequencies. However, we also have to consider another constraint: The integral of the whole profile must reproduce the optical power P_0 . Furthermore we assume that the trap depths in both directions are identical due to the nearly radial symmetry.

With this constraint we only have the parameters w_{0x} and w_{0y} left as free parameters which we vary to reproduce the transition frequencies (2,0,0) and (0,2,0) from table 4.3. The resulting trap parameters are listed in table 4.4.

The blue dashed lines in the excitation spectra indicate the transition frequencies calculated from the determined radial profile. Although the simulation slightly deviates from the dipole transition peaks it qualitatively reproduces the measurement. The gray dashed line in the dipole spectrum indicates the mean trap frequency calculated by $\omega_{\perp} = \frac{1}{4}(\omega_{x0-2} + \omega_{y0-2})$ which is necessary to determine the position of the CIR.

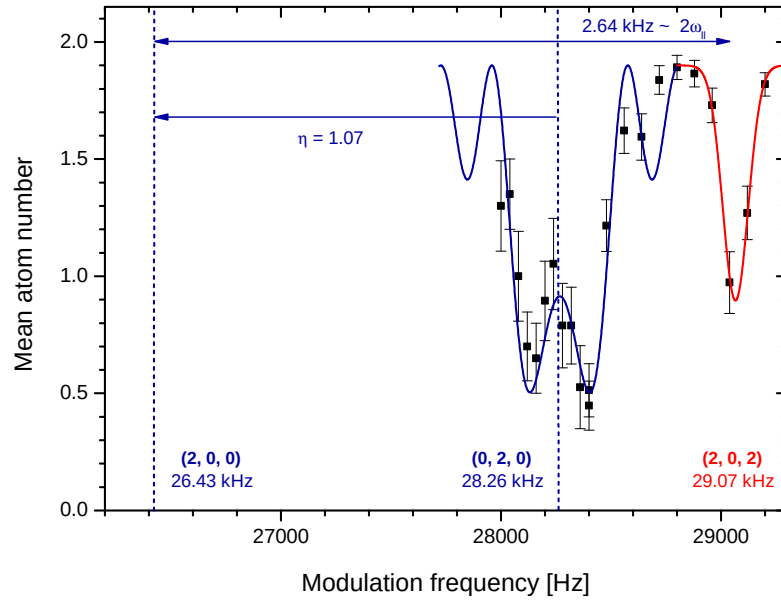
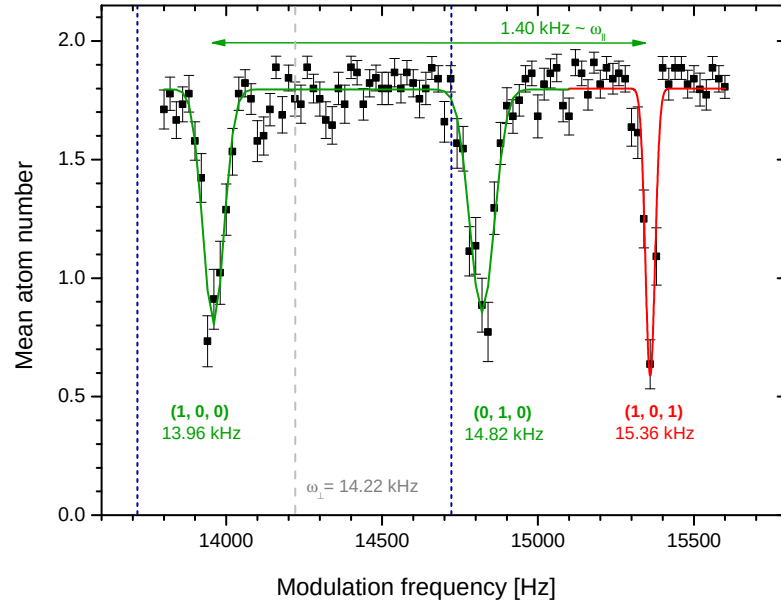


Figure A.13.: **Radial excitation spectra determined by modulation spectroscopy.** The blue dashed lines show the transition frequencies of the WKB simulation of the potential shape. The gray dashed line indicates $\omega_{\perp}$ from which we calculate the position of the CIR.

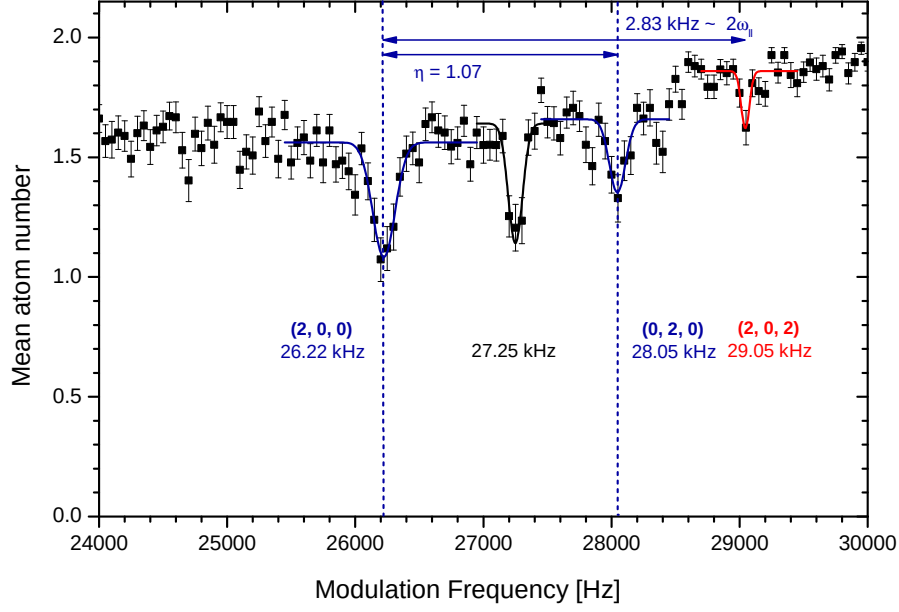


Figure A.14.: **Radial excitation.** Quadrupole transition, repeated measurement with trap depth $p = 0.986$.

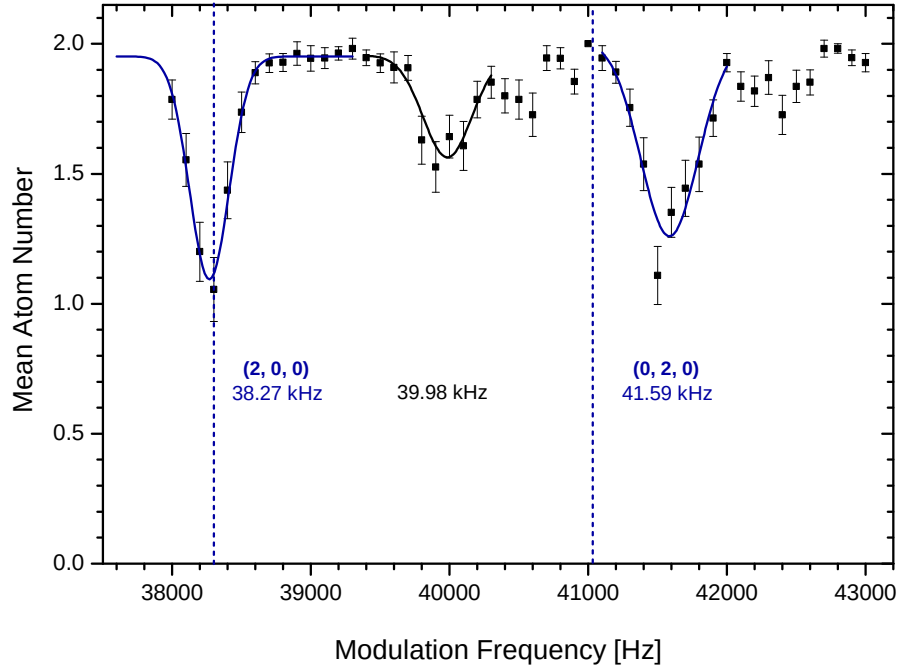


Figure A.15.: **Radial excitation.** Quadrupole transition, repeated measurement with microtrap depth doubled: $p = 2 \times 0.986$.

A.5. COM-REL motion coupling

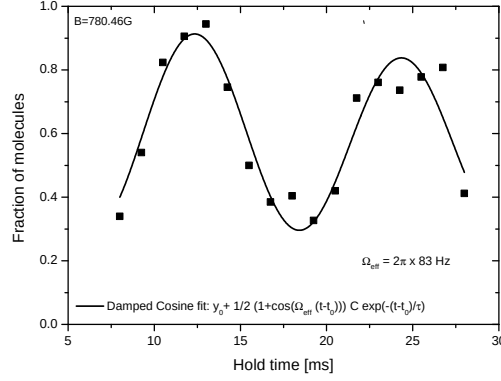


Figure A.16.: **Oscillation between the non-bound and the molecular state.** From a sinusoidal fit we deduce the Rabi-frequency Ω_{eff} and the maximum observed fraction of molecules. The measurement was performed with the magnetic field gradient switched off.

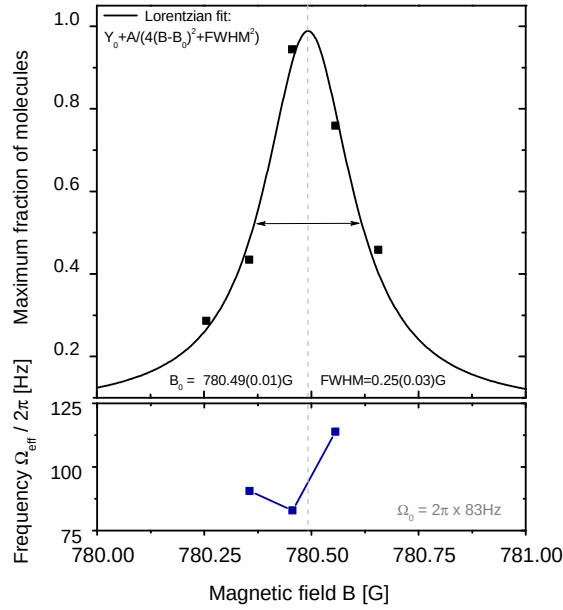


Figure A.17.: **Maximum amplitude and frequency of the oscillation between the non-bound and the molecular state.** The data points are extracted from measurements analog to figure A.16 at different magnetic offset fields. The gradient has been switched off.

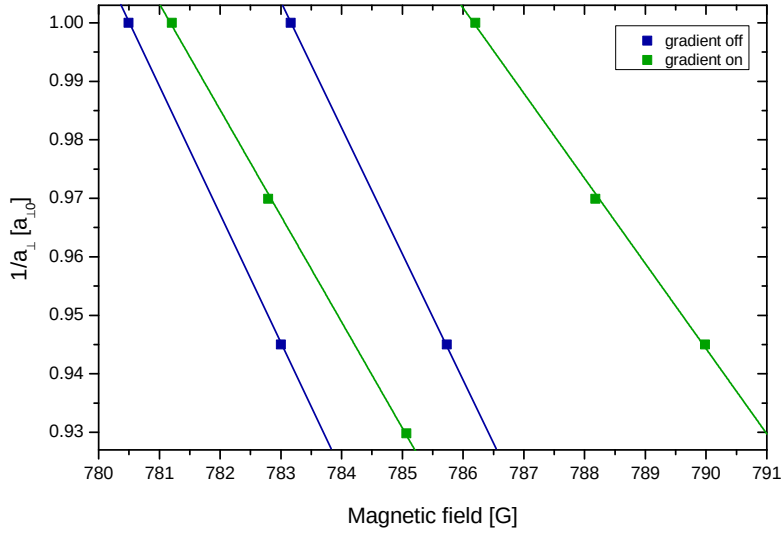


Figure A.18.: **Position of the COM-REL motion resonances depending on the strength of the confinement.** Due to the dependence of the energy of the involved states on the magnetic field gradient we observe different positions of COM-REL motion coupling resonances with the gradient of $B' = 18.92$ G/cm applied (green) and switched off (blue).

A.6. Correlations in systems with attractive interaction

magnetic field G calibrated	$g_{1D, \text{closed trap}}$ $a_{\parallel} \hbar \omega_{\parallel}$ $\omega_{\parallel}/2\pi=1488$ Hz	g_{1D} $a_{\text{ref}} \hbar \omega_{\text{ref}}$ $\omega_{\text{ref}}/2\pi=634$ Hz	$g_{1D, \text{ron}}$ [Ron12a] $a_{\text{ron}} \hbar \omega_{\text{ron}}$ $\omega_{\text{ron}}/2\pi=250$ Hz
350	-0.43	-0.65	-1.04
423	-0.39	-0.60	-0.95
496	-0.29	-0.44	-0.70
569	0.01	0.01	0.02
851	-1.18	-1.80	-2.87
958	-1.04	-1.60	-2.55
1074	-0.98	-1.51	-2.40
1202	-0.95	-1.45	-2.32

Table A.2.: Interaction parameter g_{1D} for different reference potentials.

prepared particle number N	potential parameter p [fraction of initial depth]
2	0.63496
3	0.69232
4	0.69232
5	0.73227
6	0.73136

Table A.3.: Optical trap depth parameter p for the different N-particle systems.

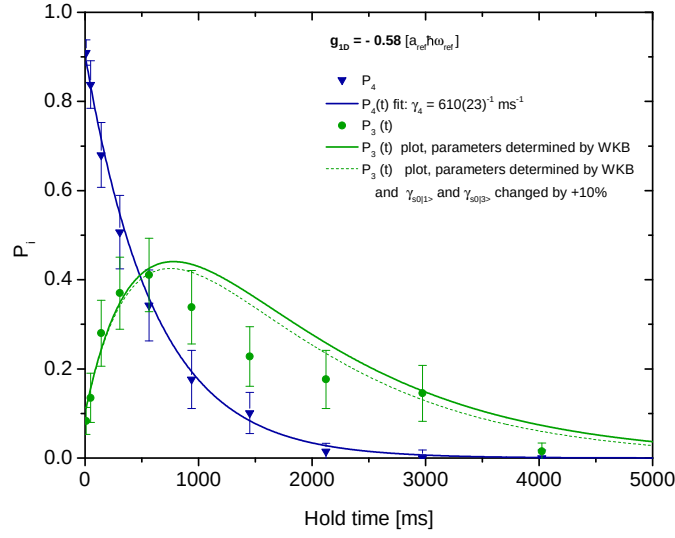


Figure A.19.: **Tunneling from a 4-particle system.** Probability of finding 4 and 3 particles in the trap. The green curve is determined by the model of subsequent single particle tunneling.

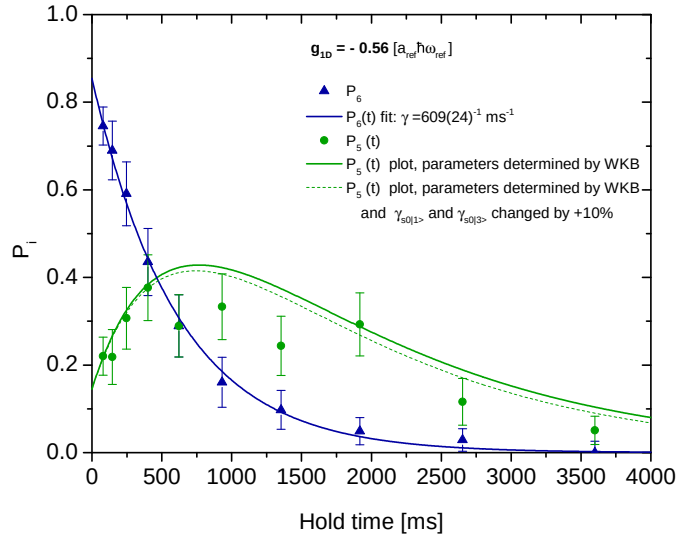


Figure A.20.: **Tunneling from a 6-particle system.** Probability of finding 6 and 5 particles in the trap.

A.6. Correlations in systems with attractive interaction

g_{1D} [$a_{\text{ref}}\hbar\omega_{\text{ref}}$]	$\gamma_{2,\text{fit}}$ [1/s]	$\gamma_{s 1},\text{WKB}$ [1/s]	$\gamma_{s 3},\text{WKB}$ [1/s]	$E_{\text{int},\text{WKB}}$ [$\hbar\omega_{\text{ref}}$]
-0.44	23.16 ± 1.55	8.42	14.75	-0.091
-0.60	12.28 ± 1.14	3.73	8.55	-0.139
-0.65	9.32 ± 0.33	1.98	7.46	-0.148
-1.45	2.08 ± 0.17	$\gamma_2/2$		-0.322
-1.51	1.949 ± 0.110	$\gamma_2/2$		-0.327
-1.60	1.224 ± 0.053	$\gamma_2/2$		-0.358
-1.80	0.505 ± 0.023	$\gamma_2/2$		-0.408

Table A.4.: **Tunneling rates of a two-particle system for different interaction strength.**

N	$\gamma_{s0 1},\text{fit}$ [1/s]	$\gamma_{s0 3},\text{fit}$ [1/s]	g_{1D} [$a_{\text{ref}}\hbar\omega_{\text{ref}}$]	$\gamma_{N,\text{fit}}$ [1/s]	$\gamma_{s 1},\text{WKB}$ [1/s]	$\gamma_{s 3},\text{WKB}$ [1/s]	$E_{\text{int},\text{WKB}}$ [$\hbar\omega_{\text{ref}}$]
2	37.7 ± 2.1	24.8 ± 1.4	-0.65	9.32 ± 0.33	1.98	7.46	-0.148
3	-	9.83 ± 1.03	-0.58	3.43 ± 0.19	-	γ_3	-0.104
4 $g=0$ $g=-0.58$	5.51 ± 0.45 0.67	9.78 ± 0.79 γ_3		1.59 ± 0.06	0.25	1.34	-0.212
5	-	12.15 ± 1.85	-0.56	3.09 ± 0.11	-	γ_5	-0.148
6 $g=0$ $g=-0.56$	7.79 ± 0.29 0.54	14.32 ± 0.53 γ_5		1.49 ± 0.06	0.22	1.28	-0.280

Table A.5.: **Tunneling rates for system with 2 to 6 particles.**

A.7. Rf spectroscopy

g1D abs all hbar wll		1:1 trans-freq wll/2pi		1:1 theory hbar wll	deviation %	2:1 trans-freq wll/2pi		2:1 theory hbar wll	deviation %	3:1 trans-freq wll/2pi	
g1D abs	error <--	2 particles experiment	error <--	2 particles theory [ldz06]	2 particles relative deviation theory-exp	3 particles experiment	error <--	3 particles theory [Gha12]	3 particle relative deviation theory-exp	4 particles experiment	error <--
-0.517	0.010	--	--	--	--	-0.522	0.018	-0.506	3.1	--	--
-0.267	0.009	-0.207	0.006	-0.167	23.7	-0.267	0.009	-0.244	9.7	-0.294	0.022
0.252	0.004	0.129	0.005	0.129	0	0.194	0.005	0.198	2.2	0.252	0.005
0.398	0.008	0.179	0.010	0.193	6.9	--	--	--	--	--	--
1.979	0.006	0.585	0.007	0.578	1.2	1.005	0.007	0.996	0.9	1.373	0.006
-2.298	0.518	-3.152	0.008	-2.787	13.1	-3.595	0.009	-3.308	8.7	-4.010	0.008

Figure A.21.: **Interaction energy of few-fermion systems I.** For the determination of the data points in the last row ($-1/g_{1D} = 0.5$) there are three different rf-spectroscopy measurements involved. Due to error propagation this leads to a large error in g_{1D} (marked in red). The data points at $-1/g_{1D} = 3.7$ deviate from theory by more than 10% which we attribute to a systematic error which probably occurred during the measurement at this interaction strength (probably due to a drift of the magnetic field between the measurement of the interaction energy and the free-free reference transition).

g1D ini all hbar wll		4:1 trans-freq wll/2pi		5:1 trans-freq wll/2pi	
g1D abs	error <--	5 particles experiment	error <--	6 particles experiment	error <--
0.252	0.004	0.458	0.009	0.507	0.007
1.979	0.006	2.485	0.013	2.851	0.011

Figure A.22.: **Interaction energy of few-fermion systems II.**

g1D ini all hbar wll	g1D fin all hbar wll	1:1 trans-freq wll/2pi		1:1 initial hbar wll	1:1 final hbar wll	1:1 diff. hbar wll	deviation %
initial state	final state	2 particles experiment	error <--	2 particles theory [ldz06]	2 particles theory [ldz06]	2 particles theory [ldz06]	2 particles relative deviation theory-exp
2.492	-2.298	3.608	0.004	0.638	-2.787	3.425	5.3
21.736	-1.800	3.007	0.004	0.938	-1.940	2.878	4.5
-7.574	-1.616	2.872	0.006	1.139	-1.653	2.792	2.9
-1.248	-1.139	2.649	0.003	1.652	-0.997	2.649	0

Figure A.23.: **Energy difference between states with non-zero interaction strength I** We find good agreement between experiment and theory (bold numbers). The relative deviation ranges from 1% to 5% only. The residual small deviations probably result from systematic errors such as anharmonicity, position of the CIR, magnetic offset field drifts during the rf-spectroscopy measurement, etc.

g1D ini all hbar wll	g1D fin all hbar wll	2:1 trans-freq wll/2pi		2:1 initial hbar wll	2:1 final hbar wll	2:1 diff. hbar wll	deviation %	3:1 trans-freq wll/2pi	
initial state	final state	3 particles experiment	error <--	3 particles theory [Gha12]	3 particles theory [Gha12]	2 particles theory [Gha12]	3 particles relative deviation theory-exp	4 particles experiment	error <--
2.492	-2.298	4.601	0.006	1.121	-3.308	4.429	3.9	--	--
21.736	-1.800	4.307	0.004	1.806	-2.393	4.199	2.6	5.443	0.007
-7.574	-1.616	4.462	0.008	2.310	-2.073	4.383	1.8	--	--
-1.248	-1.139	4.692	0.004	3.436	-1.313	4.749	1.2		

Figure A.24.: **Energy difference between states with non-zero interaction strength II**

A.8. Determination of ^6Li Feshbach resonance

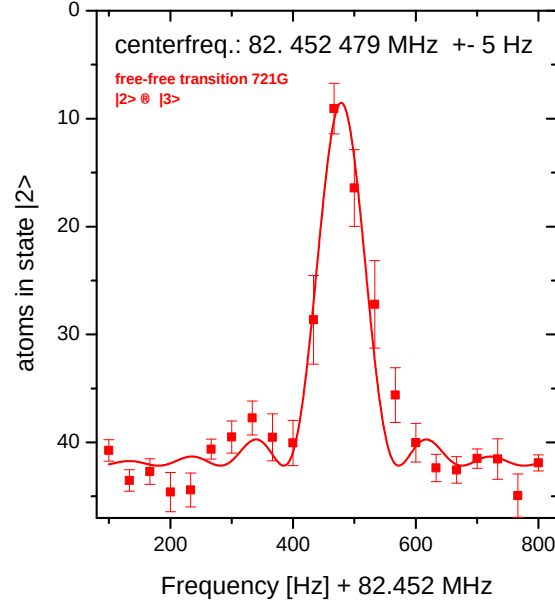


Figure A.25.: **Free-free transition.** The pulselength is 10 ms and thus shorter than the decoherence time of 23 ms determined from figure A.26. Hence, the free free spectrum is fitted using the coherent lineshape given by equation (4.28).

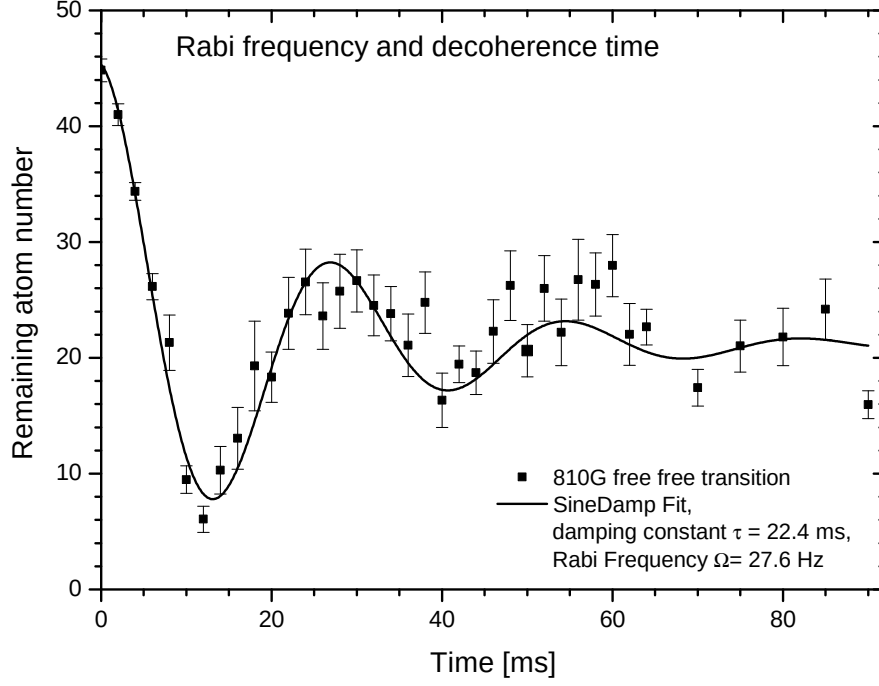


Figure A.26.: **Rabi frequency and decoherence time.** To determine the decoherence time we start with about 50 atoms in state $|2\rangle$ and apply a resonant rf-frequency ν_{ff} . Thus state $|2\rangle$ is coupled to state $|3\rangle$ and the occupation probability oscillates between those states. The contrast decays with a time constant of 23 ms. The reason for that is most likely due to dephasing in the trap. The atoms with different kinetic energy in the trap experience a different oscillation frequency due to the anharmonicity.

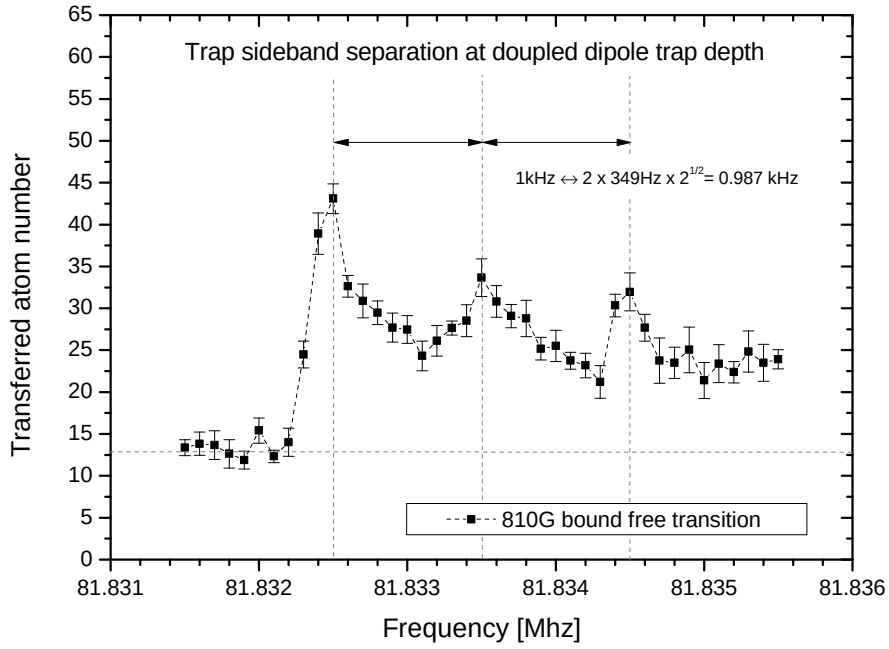


Figure A.27.: **Trap sideband separation.** To check whether the sideband-peaks tune with the trap frequency we double the depth of the trap. We observe the expected scaling of $\sqrt{2}$ of the trap sideband separation. The trap frequency in the actual dissociation measurement has been 349(3) kHz. The error is the SEM deduced from the determination of the peak separation in the four different rf-spectra of figure 6.12.

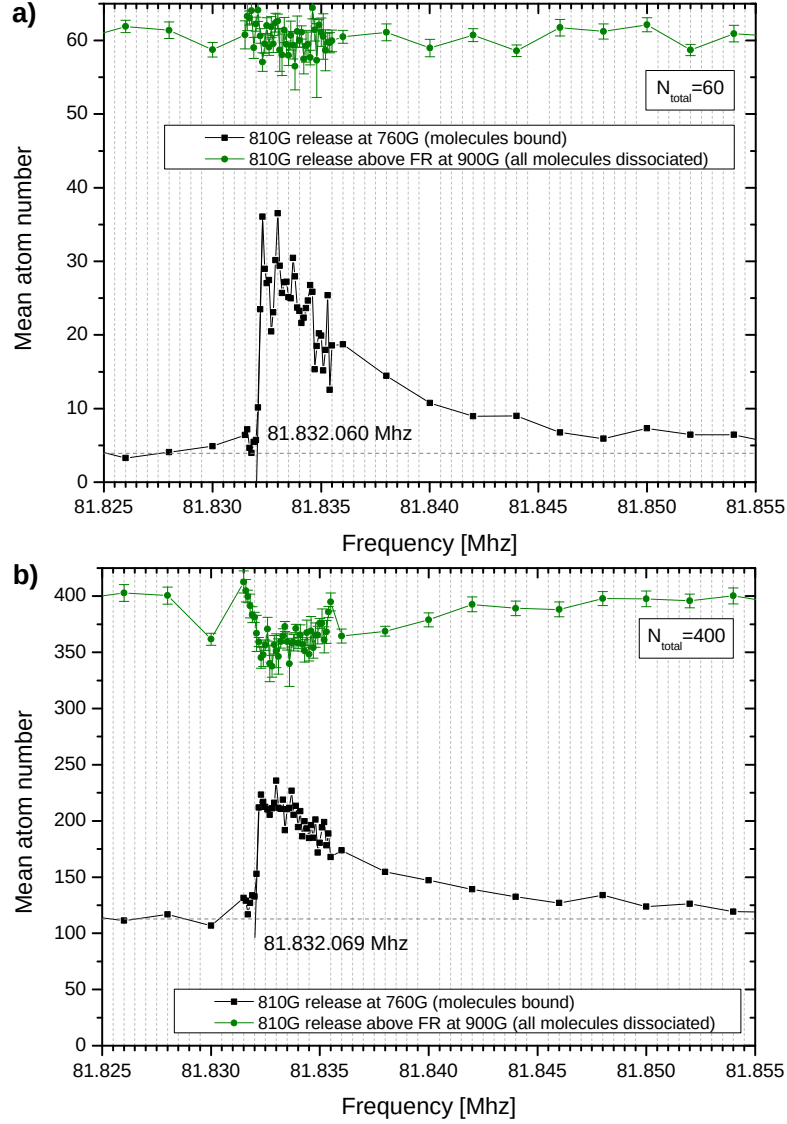


Figure A.28.: **Density dependent shift.** To determine the density dependent shift we increase the particle number by a factor of 7. From both spectra we deduce that the shift must be less than 50 Hz. Assuming monotonic behavior of the dissociation energy with the particle number we estimate the shift to be 0.125 Hz per particle in a linear approximation. This results in a density dependent systematic uncertainty for the dissociation of 30 molecules (60 atoms) of 8 Hz.

magn. field B [G]	free-free 1st ν_{ff1} [MHz]	free-free 2nd ν_{ff2} [MHz]	ff weighted mean ν_{ffm} [MHz]	bound-free transition ν_{bf} [MHz]	dissociation freq. $\delta\nu_{\text{ffm-bf}}$ [kHz]
811.139 (1)	81.830 120 (8)	81.830 113 (5)	81.830 115 (3)	81.832 271 (7)(8)	2.156 (8)(16)
801.115 (5)	81.891 515 (3)	81.891 583 (4)	81.891 539 (33)	81.896 236 (3)(8)	4.697 (33)(16)
781.057 (1)	82.019 822 (2)	82.019 824 (3)	82.019 823 (1)	82.034 336 (6)(8)	14.513 (6)(16)
720.965 (1)	82.452 484 (4)	82.452 479 (5)	82.452 482 (2)	82.579 943 (13)(8)	127.461 (13)(16)

Table A.6.: **Transition and dissociation frequencies.** For the magnetic field calibration we use the Breit-Rabi formula [Bre31]. The origin of the individual errors are as follows: $\sigma_{\nu_{\text{ff1}}}$ and $\sigma_{\nu_{\text{ff2}}}$ are the statistical errors of the fit to the free-free transitions; $\nu_{\text{ffm}} = \frac{\sum_i \frac{1}{\sigma_{\nu_{\text{ff}i}}^2} \nu_{\text{ff}i}}{\sum_i \frac{1}{\sigma_{\nu_{\text{ff}i}}^2}}$ (weighted mean);

$\sigma_{\nu_{\text{ffm}}} = \sqrt{\frac{\sum_i \frac{1}{\sigma_{\nu_{\text{ff}i}}^2} (\nu_{\text{ff}i} - \nu_{\text{ffm}})^2}{\sum_i \frac{1}{\sigma_{\nu_{\text{ff}i}}^2}}}$ (weighted error of weighted mean); σ_B is the statistical error resulting from $\sigma_{\nu_{\text{ffm}}}$ (Breit-Rabi formula); 1st of $\text{err}_{\nu_{\text{bf}}}$: statistical error $\sigma_{\nu_{\text{bf}}}$: fit error of single Lorentzian fit to first slope of dissociation spectrum, 2nd of $\text{err}_{\nu_{\text{bf}}}$: systematic error $\Delta_{\nu_{\text{model}}}$: max. of model dependent shift (Lorentz vs. Gauss); 1st of $\text{err}_{\nu_{\text{ffm-bf}}}$: statistical error $\sigma_{\delta\nu_{\text{ffm-bf}}}$: quadratic addition of $\sigma_{\nu_{\text{ffm}}}$ and $\sigma_{\nu_{\text{bf}}}$, 2nd of $\text{err}_{\nu_{\text{ffm-bf}}}$: $\Delta_{\delta\nu_{\text{ffm-bf}}}$ systematic error: $\Delta_{\nu_{\text{model}}} + \Delta_{\nu_{\text{density}}}$.

dissociation freq. $\delta\nu_{\text{ffm-bf}}$ [kHz]	confinement shift ν_{cs} [kHz]	binding energy ν_{E_b} [kHz]	scattering lenght $a_{3D 1\rangle- 2\rangle}$ [a_0]	scattering lenght $a_{3D 1\rangle- 3\rangle}$ [a_0]
2.156 (8)(16)	0.353 (3)(1)	1.803 (8)(17) (25)	18,340	-3,541
4.697 (33)(16)	0.356 (3)(1)	4.341 (33)(17) (50)	11,800	-3,693
14.513 (6)(16)	0.356 (3)(1)	14.157 (7)(17) (24)	6,542	-4,097
127.461 (13)(16)	0.346 (3)(1)	127.115 (14)(17) (31)	2,202	-8,709

Table A.7.: **Dissociation frequencies and binding energies.** The origin of the individual errors are as follows: 1st of $\text{err}_{\nu_{\text{cs}}}$: $\sigma_{\nu_{\text{cs}}}$ SEM of trap frequency determination, 2nd of $\text{err}_{\nu_{\text{cs}}}$: $\Delta_{\nu_{\text{cs}}}$ systematic error: position of 1-3 resonance; 1st of $\text{err}_{\nu_{E_b}}$: statistical error $\sigma_{\delta\nu_{E_b}}$: quadratic addition of $\sigma_{\nu_{\text{ffm}}}$, $\sigma_{\nu_{\text{bf}}}$ and $\sigma_{\nu_{E_0}}$, 2nd of $\text{err}_{\nu_{E_b}}$: $\Delta_{\delta\nu_{E_b}}$ systematic error: $\Delta_{\nu_{\text{model}}} + \Delta_{\nu_{\text{density}}} + \Delta_{\nu_{\text{cs}}}$, 3rd: sum of statistical and systematic error.

A.9. A single impurity

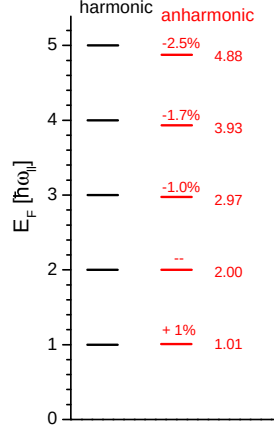


Figure A.29.: **Fermi energy in our slightly anharmonic microtrap (red).** We use the half of the measured 0-2 quadrupole-transition-frequency as the frequency reference $\omega_{||} = 2\pi \times 1.488 \text{ kHz}$ as this has been the most precise trap frequency measurement we have performed (see figure A.8) .

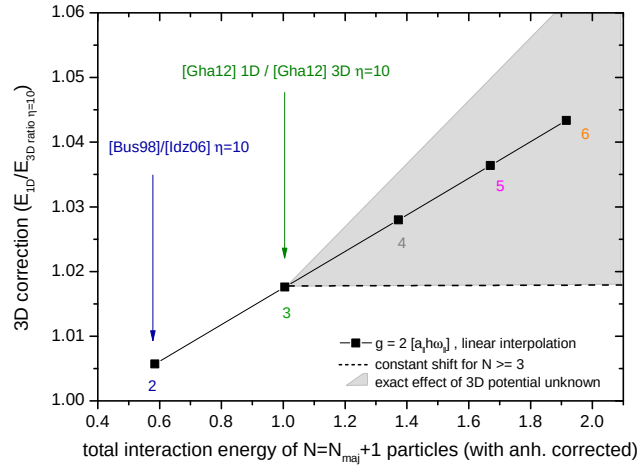
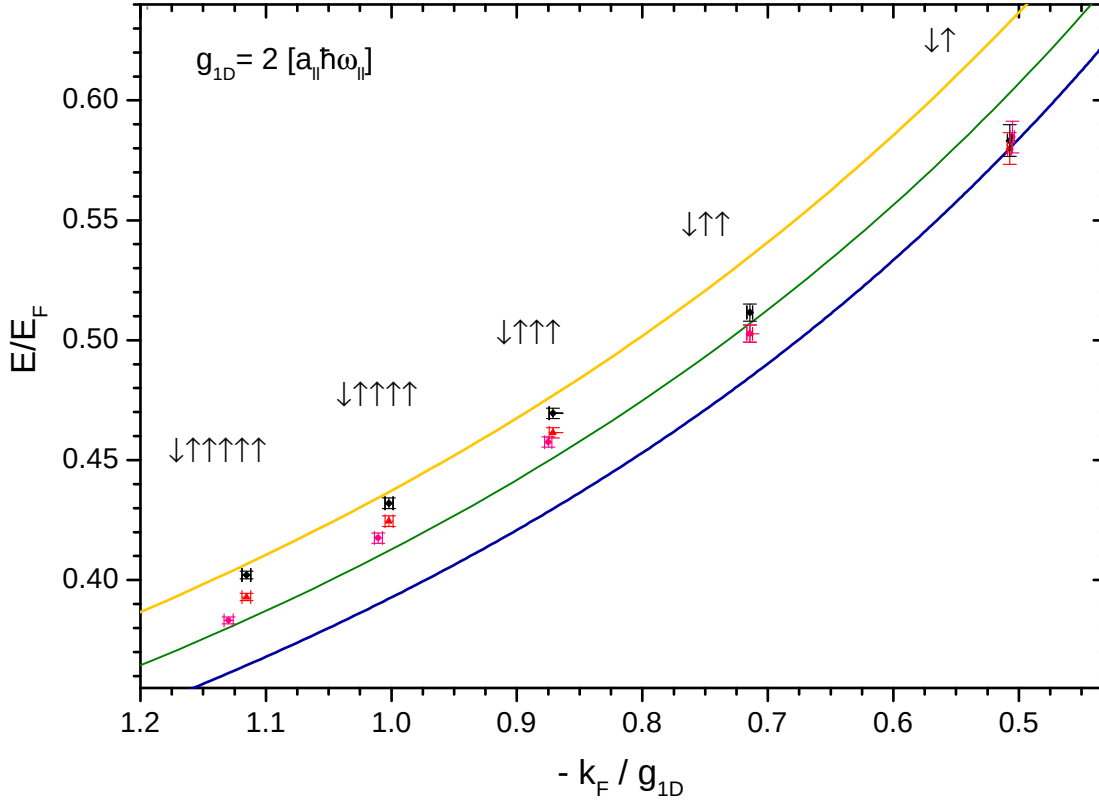


Figure A.30.: **Finite-aspect-ratio correction.** The energy of a few-fermion system in a 3D cigar-shaped trap with finite aspect ratio is smaller than in a pure 1D trap. This has to be taken into account when comparing our measurement with 1D theories. For a two and three particle system we have deduced these shifts from the comparison of the different theories. For large particle numbers we do not know the effect of these shifts (sketched by the gray shaded area). To still correct for it to some extent we use the shift of a three particle system for all other system with $N > 3$ (dashed line).



- A single impurity in a homogeneous 1D system of infinite N_{Maj} , **McGuire JMP 6, 432 (1965)**
 - 2 particles 1D, **T. Busch et al., Found Phys Vol.28, No.4 549-559 (1998)**
 - 3 particles 1D, **S. E. Gharashi and D. Blume, private communication (2012)**
 - (pink) Experimental data $g_{1D} = 0.25; 0.8; 2$, raw data
 - (red) Experimental data $g_{1D} = 0.25; 0.8; 2$, anharmonicity corrected (WKB)
 - (black) Experimental data $g_{1D} = 0.25; 0.8; 2$, anharmonicity + 3D-ness corrected
- 3D $\rightarrow$ 1D correction: $N=2$: shift from [ldz06] $\rightarrow$ [Bus98] ($g=2$: 0.6%, $g=0.2$: 0.2%)
for $N>2$: constant shift, taken from 3 particles [Gha12] 3D $\rightarrow$ [Gha12] 1D ($g=2$: 1.8%, $g=0.8$: 1.0%)
($g=0.25$: 3D-ness correction neglected (1.8×10^{-3} for 3 particles))

Figure A.31.: **Anharmonicity and finite-aspect-ratio correction.** The anharmonicity shift (figure A.29) reduces E_F and k_F whereas the finite aspect ratio (figure A.30) shifts the energy to larger values.

Bibliography

- [Abr72] M. Abramowitz, I. A. Stegun (Ed.), *Handbook of Mathematical Functions* (Dover Publications, 1972).
- [ALI04] ALICE Collaboration, *ALICE: Physics Performance Report, Volume I*, J. Phys. G: Nucl. Part. Phys. **30**, 1517 (2004).
- [And00] T. Ando, *Theory of transport in carbon nanotubes*, Semiconductor Science and Technology **15**, R13 (2000).
- [Ash96] R. C. Ashoori, *Electrons in artificial atoms*, Nature **379**, 413 – 419 (1996).
- [Ast05] G. E. Astrakharchik, J. Boronat, J. Casulleras, S. Giorgini, *Beyond the Tonks-Girardeau Gas: Strongly Correlated Regime in Quasi-One-Dimensional Bose Gases*, Phys. Rev. Lett. **95**, 190407 (2005).
- [Bak10] W. S. Bakr, A. Peng, M. E. Tai, R. Ma, J. Simon, J. I. Gillen, S. Fölling, L. Pollet, M. Greiner, *Probing the Superfluid-to-Mott Insulator Transition at the Single-Atom Level*, Science **329**, 547–550 (2010).
- [Bar57] J. Bardeen, L. N. Cooper, J. R. Schrieffer, *Theory of Superconductivity*, Phys. Rev. **108**, 1175–1204 (1957).
- [Bar61] J. Bardeen, *Tunnelling from a Many-Particle Point of View*, Phys. Rev. Lett. **6**, 57–59 (1961).
- [Bar05] M. Bartenstein, A. Altmeyer, S. Riedl, R. Geursen, S. Jochim, C. Chin, J. H. Denschlag, R. Grimm, A. Simoni, E. Tiesinga, C. J. Williams, P. S. Julienne, *Precise Determination of ^6Li Cold Collision Parameters by Radio-Frequency Spectroscopy on Weakly Bound Molecules*, Phys. Rev. Lett. **94**, 103201 (2005).
- [Ber03] T. Bergeman, M. G. Moore, M. Olshanii, *Atom-Atom Scattering under Cylindrical Harmonic Confinement: Numerical and Analytic Studies of the Confinement Induced Resonance*, Phys. Rev. Lett. **91**, 163201 (2003).

- [Bet31] H. Bethe, *Zur Theorie der Metalle I. Eigenwerte und Eigenfunktionen der linearen Atomkette* **71**, 205–226 (1931).
- [Blo08] I. Bloch, J. Dalibard, W. Zwerger, *Many-body physics with ultracold gases*, *Rev. Mod. Phys.* **80**, 885 (2008).
- [Bre31] G. Breit, I. I. Rabi, *Measurement of Nuclear Spin*, *Phys. Rev.* **38**, 2082–2083 (1931).
- [Bri05] D. Brink, R. Broglia, *Nuclear Superfluidity* (Cambridge University Press, 2005).
- [Bro12a] I. Brouzos, *A repulsive impurity in one dimension*, private communication (2012).
- [Bro12b] I. Brouzos, P. Schmelcher, *Construction of Analytical Many-Body Wave Functions for Correlated Bosons in a Harmonic Trap*, *Phys. Rev. Lett.* **108**, 045301 (2012).
- [Bro12c] I. Brouzos, P. Schmelcher, *Two-component few-fermion mixtures in a one-dimensional trap: numerical versus analytical approach*, *ArXiv e-prints* **1209.2891** (2012).
- [Bus98] Thomas Busch, B.-G. Englert, K. Rzazewski, M. Wilkens, *Two Cold Atoms in a Harmonic Trap*, *Foundations of Physics* **28**, 549–559 (1998).
- [Cas09] A. W. Castleman, S. N. Khanna, *Clusters, Superatoms, and Building Blocks of New Materials*, *The Journal of Physical Chemistry C* **113**, 2664–2675 (2009).
- [Che06] F. Chevy, *Universal phase diagram of a strongly interacting Fermi gas with unbalanced spin populations*, *Phys. Rev. A* **74**, 063628 (2006).
- [Chi04] C. Chin, M. Bartenstein, A. Altmeyer, S. Riedl, S. Jochim, J. H. Denschlag, R. Grimm, *Observation of the Pairing Gap in a Strongly Interacting Fermi Gas*, *Science* **305**, 1128–1130 (2004).
- [Chi05] C. Chin, P. S. Julienne, *Radio-frequency transitions on weakly bound ultracold molecules*, *Phys. Rev. A* **71**, 012713 (2005).
- [Chi10] C. Chin, R. Grimm, P. Julienne, E. Tiesinga, *Feshbach resonances in ultracold gases*, *Rev. Mod. Phys.* **82**, 1225 (2010).

-
- [Chu05] C.-S. Chuu, F. Schreck, T. P. Meyrath, J. L. Hanssen, G. N. Price, M. G. Raizen, *Direct Observation of Sub-Poissonian Number Statistics in a Degenerate Bose Gas*, Phys. Rev. Lett. **95**, 260403 (2005).
- [Cor93] P. B. Corkum, *Plasma perspective on strong field multiphoton ionization*, Phys. Rev. Lett. **71**, 1994–1997 (1993).
- [Dal85] J. Dalibard, C. Cohen Tannoudji, *Dressed-atom approach to atomic motion in laser light: the dipole force revisited*, J. Opt. Soc. Am. B **2**, 1707–1720 (1985).
- [Dal98] J. Dalibard, *Collisional dynamics of ultracold atomic gases*, in M. Inguscio, S. Stringari, C. Wieman (Ed.), *Bose-Einstein Condensation in Atomic Gases*, Vol. Course CXL of *Proceedings of the International School of Physics Enrico Fermi* (IOS Press, Varenna, 1998).
- [Deh76] H. G. Dehmelt, *Entropy reduction by motional sideband excitation*, Nature **262**, 777–777 (1976).
- [DeM99] B. DeMarco, D. S. Jin, *Onset of Fermi Degeneracy in a Trapped Atomic Gas*, Science **285**, 1703–1706 (1999).
- [Des12] R. Desbuquois, L. Chomaz, T. Yefsah, J. Leonard, J. Beugnon, C. Weitenberg, J. Dalibard, *Superfluid behaviour of a two-dimensional Bose gas*, ArXiv e-prints **1205.4536** (2012).
- [Dev00] J. Devreese, *Polarons*, eprint arXiv:cond-mat/0004497 (2000).
- [Die89] F. Diedrich, J. C. Bergquist, W. M. Itano, D. J. Wineland, *Laser Cooling to the Zero-Point Energy of Motion*, Phys. Rev. Lett. **62**, 403–406 (1989).
- [Dud07] A. M. Dudarev, M. G. Raizen, Q. Niu, *Quantum Many-Body Culling: Production of a Definite Number of Ground-State Atoms in a Bose-Einstein Condensate*, Phys. Rev. Lett. **98**, 063001 (2007).
- [Efi70] V. Efimov, *Energy levels arising from resonant two-body forces in a three-body system*, Physics Letters B **33**, 563 – 564 (1970).
- [Ein25] A. Einstein, *Quantentheorie des einatomigen idealen Gases. Zweite Abhandlung.*, Sitzungsberichte der Preussischen Akademie der Wissenschaften **1**, 3–14 (1925).

- [Fel11] M. Feld, B. Fröhlich, E. Vogt, M. Koschorreck, M. Köhl, *Observation of a pairing pseudogap in a two-dimensional Fermi gas*, Nature **480**, 7578 (2011).
- [Fol02] R. Folman, P. Krüger, J. Schmiedmayer, J. Denschlag, C. Henkel, *Microscopic atom optics: from wires to an atom chip*, Advances in Atomic, Molecular, and Optical Physics **48**, 263–356 (2002).
- [Fra03] S. Franke-Arnold, S. Barnett, G. Huyet, C. Sailliot, *Coherence properties of two trapped particles*, Eur. Phys. J. D **22**, 373 – 383 (2003).
- [Frö11] B. Fröhlich, M. Feld, E. Vogt, M. Koschorreck, W. Zwerger, M. Köhl, *Radio-Frequency Spectroscopy of a Strongly Interacting Two-Dimensional Fermi Gas*, Phys. Rev. Lett. **106**, 105301 (2011).
- [Gal12] L. Gallmann, C. Cirelli, U. Keller, *Attosecond Science: Recent Highlights and Future Trends*, Annual Review of Physical Chemistry **63**, 447–469 (2012).
- [Geh98] M. E. Gehm, K. M. O’Hara, T. A. Savard, J. E. Thomas, *Dynamics of noise-induced heating in atom traps*, Phys. Rev. A **58**, 3914–3921 (1998).
- [Gha12] S. E. Gharashi, K. M. Daily, D. Blume, *Three s-wave interacting fermions under anisotropic harmonic confinement.*, private communication (2012).
- [Gir60] M. Girardeau, *Relationship between Systems of Impenetrable Bosons and Fermions in One Dimension*, J. Math. Phys. **1**, 516–523 (1960).
- [Gir09] S. Giraud, R. Combescot, *Highly polarized Fermi gases: One-dimensional case*, Phys. Rev. A **79**, 043615 (2009).
- [Gir10] M. D. Girardeau, *Two super-Tonks-Girardeau states of a trapped one-dimensional spinor Fermi gas*, Phys. Rev. A **82**, 011607 (2010).
- [Gre05] M. Greiner, C. A. Regal, D. S. Jin, *Probing the Excitation Spectrum of a Fermi Gas in the BCS-BEC Crossover Regime*, Phys. Rev. Lett. **94**, 070403 (2005).
- [Gri93] G. F. Gribakin, V. V. Flambaum, *Calculation of the scattering length in atomic collisions using the semiclassical approximation*, Phys. Rev. A **48**, 546–553 (1993).

-
- [Gri00] Rudolf Grimm, Matthias Weidemüller, Yurii B. Ovchinnikov, *Optical dipole traps for neutral atoms*, Adv. At. Mol. Opt. Phys. **Vol. 42**, 95–170 (2000).
- [Gri12] M. Gring, M. Kuhnert, T. Langen, T. Kitagawa, B. Rauer, M. Schreitl, I. Mazets, D. A. Smith, E. Demler, J. Schmiedmayer, *Relaxation and Prethermalization in an Isolated Quantum System*, Science **337**, 1318–1322 (2012).
- [Grü10] T. Grünzweig, A. Hilliard, M. McGovern, M. F. Andersen, *Near-deterministic preparation of a single atom in an optical microtrap*, Nature Physics **6**, 951954 (2010).
- [Had06] Z. Hadzibabic, P. Krüger, M. Cheneau, B. Battelier, J. Dalibard, *Berezinskii-Kosterlitz-Thouless crossover in a trapped atomic gas*, Nature **441**, 1118 (2006).
- [Hal09] E. Haller, M. Gustavsson, M. J. Mark, J. G. Danzl, R. Hart, G. Pupillo, H.-C. Nägerl, *Realization of an Excited, Strongly Correlated Quantum Gas Phase*, Science **325**, 1224–1227 (2009).
- [Hal10] E. Haller, M. J. Mark, R. Hart, J. G. Danzl, R. Lukas, V. Melezhik, P. Schmelcher, H.-C. Nägerl, *Confinement-Induced Resonances in Low-Dimensional Quantum Systems*, Phys. Rev. Lett. **104**, 153203 (2010).
- [Hei26] W. Heisenberg, Z. Phys. **38**, 441 (1926).
- [Hin08] E. A. Hinds, M. Trupke, B. Darqui. Goldwin, G. Dutier, *Progress in atom chips and the integration of optical microcavities*, Proceedings of the 18th International Conference On Laser Spectroscopy (ICOLS), 272-282 and ArXiv e-prints **0802.0987** (2008).
- [Hu94] Z. Hu, H. J. Kimble, *Observation of a Single-Atom in a Magneto-optical Trap*, Opt. Lett. **19**, 1888–1890 (1994).
- [Idz05] Z. Idziaszek, T. Calarco, *Two atoms in an anisotropic harmonic trap*, Phys. Rev. A **71**, 050701 (2005).
- [Idz06] Z. Idziaszek, T. Calarco, *Analytical solutions for the dynamics of two trapped interacting ultracold atoms*, Phys. Rev. A **74**, 022712 (2006).
- [Ike12] T. N. Ikeda, Y. Watanabe, M. Ueda, *Finite-Size Scaling Analysis of the Eigenstate Thermalization Hypothesis in a One-Dimensional Interacting Bose gas*, ArXiv e-prints **1202.1965** (2012).

- [Ino98] S. Inouye, M. R. Andrews, J. Stenger, H.-J. Miesner, D. M. Stamper Kurn, W. Ketterle, *Observation of Feshbach resonances in a Bose-Einstein condensate*, Nature **392**, 151–154 (1998).
- [Jáu01] R. Jáuregui, *Nonperturbative and perturbative treatments of parametric heating in atom traps*, Phys. Rev. A **64**, 053408 (2001).
- [JEN11] JENOPTIK Optical Systems GmbH (2011), 07745 Jena, Germany.
- [Jo09] G.-B. Jo, Y.-R. Lee, J.-H. Choi, C. A. Christensen, T. H. Kim, J. H. Thywissen, D. E. Pritchard, W. Ketterle, *Itinerant Ferromagnetism in a Fermi Gas of Ultracold Atoms*, Science **325**, 1521–1524 (2009).
- [Joc12a] S. Jochim research group, *Precise characterization of ^6Li Feshbach resonances using trap-sideband resolved RF-spectroscopy of weakly bound molecules*, in preparation (2012).
- [Joc12b] S. Jochim research group, *A single impurity in a finite Fermi system*, in preparation (2012).
- [Kau12] A. M. Kaufman, B. J. Lester, C. A. Regal, *Laser cooling a neutral atom to the three-dimensional vibrational ground state of an optical tweezer*, ArXiv e-prints **1209.2087** (2012).
- [Kes07] J. P. Kestner, L.-M. Duan, *Level crossing in the three-body problem for strongly interacting fermions in a harmonic trap*, Phys. Rev. A **76**, 033611 (2007).
- [Ket08] W. Ketterle, M. W. Zwierlein, *Proc. of the International School of Physics 'Enrico Fermi', Course CLXIV* (2008).
- [Kin04] T. Kinoshita, T. Wenger, D. S. Weiss, *Observation of a one-dimensional Tonks-Girardeau gas*, Science **305**, 1125–1128 (2004).
- [Kin05] T. Kinoshita, T. Wenger, D. S. Weiss, *Local Pair Correlations in One-Dimensional Bose Gases*, Phys. Rev. Lett. **95**, 190406 (2005).
- [Kin06] T. Kinoshita, T. Wenger, D. S. Weiss, *A quantum Newton's cradle*, Nature **440**, 900–903 (2006).
- [Kli12a] S. Klimin, J. Tempere, J. Devreese, *Pseudogap and preformed pairs in the imbalanced Fermi gas in two dimensions*, ArXiv e-prints **1207.0706** (2012).

-
- [Kli12b] V. Klinkhamer, *An apparatus for few-fermion systems in multiple well potentials* (2012), master thesis, Ruprecht-Karls-Universität Heidelberg.
- [Koh08] M. Kohnen, *Ultracold Fermi Mixtures in an Optical Dipole Trap*, <http://www.mpi-hd.mpg.de/biblio/preprints/2008-009.pdf> (2008), diploma thesis, Ruprecht-Karls-Universität Heidelberg.
- [Koh12] C. Kohstall, M. Zaccanti, M. Jag, A. Trenkwalder, P. Massignan, G. M. Bruun, F. Schreck, R. Grimm, *Metastability and coherence of repulsive polarons in a strongly interacting Fermi mixture*, *Nature* **485**, 615618 (2012).
- [Kon64] J. Kondo, *Resistance Minimum in Dilute Magnetic Alloys*, *Prog. Theor. Phys.* **32 No. 1**, 37–49 (1964).
- [Kos12] Marco Koschorreck, Daniel Pertot, Enrico Vogt, Bernd Fröhlich, Michael Feld, Michael Köhl, *Attractive and repulsive Fermi polarons in two dimensions*, *Nature* **485**, 619622 (2012).
- [Kou01] L. P. Kouwenhoven, D. G. Austing, S. Tarucha, *Few-electron quantum dots*, *Reports on Progress in Physics* **64**, 701 (2001).
- [Kra06] T. Kraemer, M. Mark, P. Waldburger, J. G. Danzl, C. Chin, B. Engeser, A. D. Lange, K. Pilch, A. Jaakkola, H.-C. Nägerl, R. Grimm, *Evidence for Efimov quantum states in an ultracold gas of caesium atoms*, *Nature* **440**, 315–318 (2006).
- [Ku12] M. J. H. Ku, A. T. Sommer, L. W. Cheuk, M. W. Zwierlein, *Revealing the Superfluid Lambda Transition in the Universal Thermodynamics of a Unitary Fermi Gas*, *Science* **335**, 563–567 (2012).
- [Kur98] D. M. Stamper Kurn, H.-J. Miesner, A. P. Chikkatur, S. Inouye, J. Stenger, W. Ketterle, *Reversible Formation of a Bose-Einstein Condensate*, *Phys. Rev. Lett.* **81**, 2194–2197 (1998).
- [Lam10] Lambda Research Corporation (2010), Littleton, MA 01460, USA.
- [Lan87] L. D. Landau, E. M. Lifshitz, *Quantum Mechanics* (Pergamon, New York, 1987).
- [Lee08] P. A. Lee, *From high temperature superconductivity to quantum spin liquid: progress in strong correlation physics*, *Rep. Prog. Phys.* **71**, 012501 (2008).

- [Leg72] A. J. Leggett, *Interpretation of Recent Results on He^3 below 3 mK: A New Liquid Phase?*, Phys. Rev. Lett. **29**, 1227–1230 (1972).
- [Leg80] A. J. Leggett, *Cooper pairing in spin-polarized Fermi systems*, J. Phys. (Paris) Coll. **41**, C7–19 (1980).
- [Lie93] E. Lieb, *The Hubbard Model: Some Rigorous Results and Open Problems*, eprint arXiv:cond-mat/9311033 (1993).
- [Liu09] X.-J. Liu, H. Hu, P. D. Drummond, *Virial Expansion for a Strongly Correlated Fermi Gas*, Phys. Rev. Lett. **102**, 160401 (2009).
- [Lom08] T. Lompe, *An apparatus for the production of molecular Bose-Einstein condensates*, <http://www.mpi-hd.mpg.de/biblio/preprints/2008-021.pdf> (2008), diploma thesis, Ruprecht-Karls-Universität Heidelberg.
- [Lom10] T. Lompe, T. B. Ottenstein, F. Serwane, A. N. Wenz, G. Zürn, S. Jochim, *Radio-Frequency Association of Efimov Trimers*, Science **330**, 940–944 (2010).
- [Lom11] Thomas Lompe, *Efimov Physics in a three-component Fermi gas*, Dissertation, Ruprecht-Karls-Universität Heidelberg, <http://www.mpi-hd.mpg.de/biblio/preprints/2011-023.pdf> (2011).
- [Luo06] L. Luo, B. Clancy, J. Joseph, J. Kinast, A. Turlapov, J. E. Thomas, *Evaporative cooling of unitary Fermi gas mixtures in optical traps*, New Journal of Physics **8**, 213 (2006).
- [Mae01] S. Maekawa, T. Tohyama, *Charge and spin in low-dimensional cuprates*, Rep. Prog. Phys. **64**, 383428 (2001).
- [Mas11] P. Massignan, G. Bruun, *Repulsive polarons and itinerant ferromagnetism in strongly polarized Fermi gases*, Eur. Phys. J. D **65**, 83–89 (2011).
- [May49] M. G. Mayer, *On Closed Shells in Nuclei. II*, Phys. Rev. **75**, 19691970 (1949).
- [McG65] J. B. McGuire, *Interacting Fermions in One Dimension. I. Repulsive Potential*, J. Math. Phys. **6**, 432 (1965).
- [Mel11] V. S. Melezhik, P. Schmelcher, *Multichannel effects near confinement-induced resonances in harmonic waveguides*, Phys. Rev. A **84**, 042712 (2011).

-
- [Mer98] E. Merzbacher, *Quantum Mechanics* (John Wiley, 1998), 3rd Edn.
- [Met99] H. J. Metcalf, P. van der Straten, *Laser Cooling and Trapping* (Springer-Verlag, New York, 1999).
- [Mey90] H.-D. Meyer, U. Manthe, L. Cederbaum., *The multi-configurational time-dependent Hartree approach.*, Chem.Phys.Lett. **165**, 73 (1990).
- [Mig59] A. Migdal, *Superfluidity and the moments of inertia of nuclei*, Nuclear Physics **13**, 655–674 (1959).
- [Mül10] T. Müller, B. Zimmermann, J. Meineke, J.-P. Brantut, T. Esslinger, H. Moritz, *Local Observation of Antibunching in a Trapped Fermi Gas*, Phys. Rev. Lett. **105**, 040401 (2010).
- [Nak11] S. Nakajima, M. Horikoshi, T. Mukaiyama, P. Naidon, M. Ueda, *Measurement of an Efimov Trimer Binding Energy in a Three-Component Mixture of ^6Li* , Phys. Rev. Lett. **106**, 143201 (2011).
- [Noz85] P. Nozières, S. Schmitt Rink, *Bose Condensation in an Attractive Fermion Gas: From Weak to Strong Coupling Superconductivity*, J. Low Temp. Phys. **59**, 195–211 (1985).
- [Ols98] M. Olshanii, *Atomic Scattering in the Presence of an External Confinement and a Gas of Impenetrable Bosons*, Phys. Rev. Lett. **81**, 938 (1998).
- [Osp06] C. Ospelkaus, S. Ospelkaus, L. Humbert, P. Ernst, K. Sengstock, K. Bongs, *Ultracold Heteronuclear Molecules in a 3D Optical Lattice*, Phys. Rev. Lett. **97**, 120402 (2006).
- [Ots01] T. Otsuka, R. Fujimoto, Y. Utsuno, B. A. Brown, M. Honma, T. Mizusaki, *Magic Numbers in Exotic Nuclei and Spin-Isospin Properties of the NN Interaction*, Phys. Rev. Lett. **87**, 082502 (2001).
- [Ott08] T. B. Ottenstein, T. Lompe, M. Kohnen, A. N. Wenz, S. Jochim, *Collisional Stability of a Three-Component Degenerate Fermi Gas*, Phys. Rev. Lett. **101**, 203202 (2008).
- [Ott10] T. B. Ottenstein, *Few-body physics in ultracold Fermi gases*, Dissertation, Ruprecht-Karls-Universität Heidelberg, <http://www.mpi-hd.mpg.de/biblio/preprints/2010-001.pdf> (2010).
- [Pen11] S.-G. Peng, H. Hu, X.-J. Liu, P. D. Drummond, *Confinement induced resonances in anharmonic waveguides*, ArXiv e-prints **1107.2725** (2011).

- [Pet00a] D. S. Petrov, M. Holzmann, G. V. Shlyapnikov, *Bose-Einstein Condensation in Quasi-2D Trapped Gases*, Phys. Rev. Lett. **84**, 2551–2555 (2000).
- [Pet00b] D. S. Petrov, G. V. Shlyapnikov, J. T. M. Walraven, *Regimes of Quantum Degeneracy in Trapped 1D Gases*, Phys. Rev. Lett. **85**, 3745–3749 (2000).
- [Pon12] M. Pons, D. Sokolovski, A. del Campo, *Fidelity of fermionic-atom number states subjected to tunneling decay*, Phys. Rev. A **85**, 022107 (2012).
- [Pov08] B. Povh, K. Rith, C. Schulz, F. Zetsche, *Particles and Nuclei* (Springer-Verlag Berlin Heidelberg, 2008), 6th Edn.
- [Rai09] M. G. Raizen, S.-P. Wan, C. Zhang, Q. Niu, *Ultrahigh-fidelity qubits for quantum computing*, Phys. Rev. A **80**, 030302 (2009).
- [Reg03] C. A. Regal, C. Ticknor, J. L. Bohn, D. S. Jin, *Creation of ultracold molecules from a Fermi gas of atoms*, Nature **424**, 47–50 (2003).
- [Rei02] S. M. Reimann, M. Manninen, *Electronic structure of quantum dots*, Rev. Mod. Phys. **74**, 1283 (2002).
- [RHI05] RHIC Collaboration, *First Three Years of Operation of RHIC*, Nuclear Physics A **757**, 1–284 (2005).
- [Rig08] M. Rigol, V. Dunjko, M. Olshanii, *Thermalization and its mechanism for generic isolated quantum systems*, Nature **452**, 854–858 (2008).
- [Rig12] M. Rigol, M. Srednicki, *Alternatives to Eigenstate Thermalization*, Phys. Rev. Lett. **108**, 110601 (2012).
- [Ron08] M. Rontani, S. Aberg, S. M. Reimann, *Configuration interaction approach to the few-body problem in a two-dimensional harmonic trap with contact interaction* (2008).
- [Ron09] M. Rontani, J. R. Armstrong, Y. Yu, S. Aberg, S. M. Reimann, *Cold Fermionic Atoms in Two-Dimensional Traps: Pairing versus Hund’s Rule*, Phys. Rev. Lett. **102**, 060401 (2009).
- [Ron12a] M. Rontani, *Notes on pair tunneling of two atoms out of a trap*, private communication (2012).
- [Ron12b] M. Rontani, *Tunneling Theory of Two Interacting Atoms in a Trap*, Phys. Rev. Lett. **108**, 115302 (2012).

-
- [Sak94] J. J. Sakurai, *Modern Quantum Mechanics* (Addison-Wesley Publishing Company, Inc., 1994).
- [Sal12] S. Sala, P.-I. Schneider, A. Saenz, *Inelastic Confinement-Induced Resonances in Low-Dimensional Quantum Systems*, Phys. Rev. Lett. **109**, 073201 (2012).
- [San10] C. Sanner, E. J. Su, A. Keshet, R. Gommers, Y.-i. Shin, W. Huang, W. Ketterle, *Suppression of Density Fluctuations in a Quantum Degenerate Fermi Gas*, Phys. Rev. Lett. **105**, 040402 (2010).
- [San12] C. Sanner, E. J. Su, W. Huang, A. Keshet, J. Gillen, W. Ketterle, *Correlations and Pair Formation in a Repulsively Interacting Fermi Gas*, Phys. Rev. Lett. **108**, 240404 (2012).
- [Sch01a] N. Schlosser, G. Reymond, I. Protsenko, P. Grangier, *Sub-poissonian loading of single atoms in a microscopic dipole trap*, Nature **411**, 1024–1027 (2001).
- [Sch01b] F. Schreck, G. Ferrari, K. L. Corwin, J. Cubizolles, L. Khaykovich, M.-O. Mewes, C. Salomon, *Sympathetic cooling of bosonic and fermionic lithium gases towards quantum degeneracy*, Phys. Rev. A **64**, 011402 (2001).
- [Sch07] F. Schwabl, *Quantenmechanik* (Springer, 2007), 7th Edn.
- [Sch08a] A. Schirotzek, Y.-i. Shin, C. H. Schunck, W. Ketterle, *Determination of the Superfluid Gap in Atomic Fermi Gases by Quasiparticle Spectroscopy*, Phys. Rev. Lett. **101**, 140403 (2008).
- [Sch08b] C. H. Schunck, Y. Shin, A. Schirotzek, W. Ketterle, *Determination of the fermion pair size in a resonantly interacting superfluid*, Nature **454**, 739–743 (2008).
- [Sch09] A. Schirotzek, C.-H. Wu, A. Sommer, M. W. Zwierlein, *Observation of Fermi Polarons in a Tunable Fermi Liquid of Ultracold Atoms*, Phys. Rev. Lett. **102**, 230402 (2009).
- [Ser07] F. Serwane, *The setup of a Magneto Optical Trap for the preparation of a mesoscopic degenerate Fermi gas*, <http://www.mpi-hd.mpg.de/biblio/preprints/2007-031.pdf> (2007), diploma thesis, Ruprecht-Karls-Universität Heidelberg.

- [Ser11a] F. Serwane, *Deterministic preparation of a tunable few-fermion system*, Dissertation, Ruprecht-Karls-Universität Heidelberg, <http://www.mpi-hd.mpg.de/biblio/preprints/2011-048.pdf> (2011).
- [Ser11b] F. Serwane, G. Zürn, T. Lompe, T. B. Ottenstein, A. N. Wenz, S. Jochim, *Deterministic Preparation of a Tunable Few-Fermion System*, *Science* **332**, 336–338 (2011).
- [Sim10] L. Simon, W. T. Strunz, *Bose gas in a single-beam optical dipole trap*, *Phys. Rev. A* **81**, 063620 (2010).
- [Som12] A. T. Sommer, L. W. Cheuk, M. J. H. Ku, W. S. Bakr, M. W. Zwierlein, *Evolution of Fermion Pairing from Three to Two Dimensions*, *Phys. Rev. Lett.* **108**, 045302 (2012).
- [Sto39] E. C. Stoner, *Collective Electron Ferromagnetism. II. Energy and Specific Heat Proc.*, *R. Soc. Lond. A* **169**, 339–371 (1939).
- [Stö06] T. Stöferle, H. Moritz, K. Günter, M. Köhl, T. Esslinger, *Molecules of Fermionic Atoms in an Optical Lattice*, *Phys. Rev. Lett.* **96**, 030401 (2006).
- [Sue49] H. E. Suess, O. Haxel, J. H. D. Jensen, *Modellmässige Deutung der ausgezeichneten Nucleonenzahlen Im Kernbau*, *Naturwissenschaften* **36**, 155 (1949).
- [Tho12] J. Thomas, *Synopsis: Fermionic Switch*, American Physical Society (February 16 2012), <http://physics.aps.org/synopsis-for/10.1103/PhysRevLett.108.075303>.
- [Wei11] C. Weitenberg, M. Endres, J. F. Sherson, M. Cheneau, P. Schausz, T. Fukuhara, I. Bloch, S. Kuhr, *Single-spin addressing in an atomic Mott insulator*, *Nature* **471**, 319–324 (2011).
- [Wen08] A. Wenz, *Few-Body Physics in a Three-Component Fermi Gas*, <http://www.mpi-hd.mpg.de/biblio/preprints/2008-023.pdf> (2008), diploma thesis, Ruprecht-Karls-Universität Heidelberg.
- [Zel03] V. Zelevinsky, A. Volya, *Nuclear pairing: New perspectives*, *Physics of Atomic Nuclei* **66**, 1781–1801 (2003).
- [Zen32] C. Zener, *Non-Adiabatic Crossing of Energy Levels*, *Proc. R. Soc. Lond. A* **137**, 696–702 (1932).

- [Zim11] B. Zimmermann, T. Müller, J. Meineke, T. Esslinger, H. Moritz, *High-resolution imaging of ultracold fermions in microscopically tailored optical potentials*, New Journal of Physics **13**, 043007 (2011).
- [Zür09] G. Zürn, *Realization of an Optical Microtrap for a Highly Degenerate Fermi Gas*, <http://www.mpi-hd.mpg.de/biblio/preprints/2009-013.pdf> (2009), diploma thesis, Ruprecht-Karls-Universität Heidelberg.
- [Zür12] G. Zürn, F. Serwane, T. Lompe, A. N. Wenz, M. G. Ries, J. E. Bohn, S. Jochim, *Fermionization of Two Distinguishable Fermions*, Phys. Rev. Lett. **108**, 075303 (2012).
- [Zwi05] M. W. Zwierlein, J. R. Abo Shaeer, A. Schirotzek, C. H. Schunck, W. Ketterle, *Vortices and superfluidity in a strongly interacting Fermi gas*, Nature **435**, 1047–1051 (2005).

Danksagung

Ich möchte all denen danken, die es ermöglicht haben, dass diese Arbeit entstehen konnte.

Selim für sein Wissen, seine freundliche und exzellente Betreuung und für seine Begeisterung, die uns immer wieder mitgerissen hat.

Timo für seine Optikexpertise und seine fast unbegrenzte Geduld und Gelassenheit.

Friedhelm für das hervorragend funktionierende Optik Design und für die vielen angenehmen Stunden im Labor.

Thomas für die guten Ideen und die schnellen Lösungen, die fast immer zum erwünschten Ziel führen.

Andre für sein fundiertes Theoriewissen, seine guten Vorschläge und die hilfreichen Diskussionen, die wir oft mit erhobener Stimme ausgefochten haben.

Martin für seine Vakuum Kenntnisse, mit denen wir das Experiment nach längerem Defekt wieder in Betrieb nehmen konnten.

Andrea, Vincent, Simon, Mathias, Kirsten, Philipp, Johanna und der ganzen Gruppe für ihr Hilfe und die wirklich schöne gemeinsame Zeit.

Prof. Matthias Weidemüller für das Lesen dieser Arbeit.

Ioannis Brouzos, Simon Sala, Massimo Rontani, Prof. Dörte Blume und Prof. Thomas Busch für die theoretischen Ausarbeitungen und die sehr aufschlussreichen Diskussionen.

Der Gruppe von Prof. Joachim Ulrich und der Gruppe von Prof. Klaus Blaum für die gute Atmosphäre und die zur Verfügung gestellten Räumlichkeiten und das geliehene Laborequipment.

Florian Säubert und der Lehrwerkstatt, für ihre technische Unterstützung und die außerordentlich gut funktionierende Feinmechanik.

Herrn Schuhmann, Herrn Geerds und Herbert Flindt, meinen Physiklehrern, die ursprünglich mein Interesse geweckt haben.

Allen, die mich im Studium begleitet haben, vor allem *Bernhard und Valentin* für die vielen Diskussionen über unsere Arbeit spät abends beim Stammtisch. Und natürlich *Kathrin*, die geduldig die Gespräche mitverfolgt.

Meinen Geschwistern *Claudia, Regina und Markus* und meinen Eltern *Elisabeth und Wolfgang*, denen ich am Meisten zu verdanken habe und die mir so vieles mit auf den Weg geben.

Meiner lieben *Lisa* für all ihre Unterstützung und für die vielen Stunden und Tage an denen sie mich aufmuntert mit ihrer Freude, die sie ausstrahlt.

