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Abstract

We develop a parsimonious model of bubbles based on the assumption of imprecisely known market depth. In a speculative bubble, traders drive the price above its fundamental value in a dynamic way, driven by rational expectations about future price developments. At a previously unknown date, the bubble will endogenously burst. We provide a general condition for the possibility of bubbles depending on the risk-free rate, uncertainty about market depth, and traders' degree of leverage. This allows us to discuss several policy measures. Bubbles always reduce aggregate welfare. Among others, certain monetary policy rules, minimum leverage ratios, and a correctly implemented Tobin tax can prevent their occurrence. Implemented incorrectly, however, some of these measures backfire and facilitate bubbles.

Keywords: Bubbles, Rational Expectations, Market Depth, Liquidity, Financial Crises, Leveraged Investment, Bonuses, Capital Structure.

JEL-Codes: G01, G12, E44,

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1 Introduction

Under what conditions are asset price bubbles possible? Which policies can prevent them? In the light of recent economic experience, these questions seem important and topical. Although the phenomenon of bubbles has long been recognized, economic policy has been unable to prevent their repeated occurrence in different settings and circumstances. In this paper, we propose a fundamental mechanism that fosters the emergence of bubbles, based on a minimal set of assumptions. In fact, we believe that this mechanism belongs to the simplest and most general ones that can give rise to bubbles. Developing a broadly applicable understanding should eventually help to guide policymakers. To this end, we build a theoretical dynamic model and derive analytical conditions under which bubbles can occur. We show that such bubbles reduce welfare and discuss various policy measures that can prevent them.

Our workhorse model is based on the fundamental assumption of finite but imprecisely known *market depth*, as one important aspect of market liquidity. Market depth is defined here as the maximum amount of money that a market can attract. No further assumption about asymmetric or incomplete information, the relationship between the growth rate of the economy and the real interest rate, irrationality of agents, changes in fundamentals or similar complications have to be taken, which constitutes the first main contribution of this paper. Uncertainty about market depth allows a bubble to continue growing each period with an endogenous probability, as rational and otherwise perfectly informed traders are only willing to invest in a bubble if they believe that there can be yet another future market participant to whom they can sell at an even higher price. We thereby combine two polar cases. Blanchard (1979) and Blanchard and Watson (1982) show in a nutshell that price bubbles are possible if an economy has potentially infinite market depth. The expectation of prices that rise further and further can then be rationalized. Tirole (1982) takes the opposite position and shows that in a simple finite economy, bubbles are impossible.[1] Our model takes an intermediate position, in which the possible existence of bubbles depends on a number of parameters. "What makes bubbles possible?" is a question that can thus be asked and answered, giving rise to important policy implications regarding their prevention. These manifold implications are the second main contribution of the paper.

[1]Also Santos and Woodford (1997) show that the conditions for the existence of bubbles are very restrictive if one assumes a fixed number of households that participate in the asset market and that own finite aggregate endowments. Note that a stochastic growth rate of the economy would induce an uncertain future market size, as is assumed here. The model of Zeira (1999) is similar in spirit to our model as he also assumes an unknown market size after, e. g., a financial liberalization. This uncertainty, however, creates booms and crashes of an asset's price via its influence on its fundamental value, above which the price cannot rise. Similarly, Allen and Gale (2000) show in a two-period model that expected expansions in credit can generate uncertainty about the steady-state price, which influences prices in previous periods. Prices can then also fall depending on the realized expansion of credit. In Froot and Obstfeld (1991), asset prices can also overreact to movements in fundamentals but are nevertheless linked to them. In our model, bubbles drive asset prices dynamically above their steady-state prices that are constant and known to all agents. Furthermore, no expected or realized changes in exogenous variables are needed to generate bubbles.

For a low degree of uncertainty about market depth, the highest possible price of an asset can be computed sufficiently accurately. By backward induction, no bubble can then exist in the first place. For a higher degree of uncertainty, bubbles can emerge but will endogenously burst at an unknown date. Our assumption of finite but imprecisely known market depth therefore endogenously determines a time-varying probability of bursting, which is assumed exogenously in Blanchard (1979) and Blanchard and Watson (1982). Assuming imprecise information about market depth seems natural in light of increasingly complex and opaque financial markets. The assumption of a exactly known future market depth, on the other hand, strikes us as highly unrealistic. The recent financial crisis in particular has shown that domestic and international capital flows can be very intransparent. The size and end point of these capital flows are not precisely predictable, such that the maximum amount of resources that can flow into a particular market can only be estimated.

In our model, a steady-state price always exists. A *bubble* is defined as a price path that deviates from the steady state. Therefore, bubbles exist if and only if there are multiple equilibrium price paths. In such a bubble, traders are aware that they are in a bubble.[2] A high price reduces the probability that current asset holders will find future buyers at an even higher price. Given this increased risk, buyers demand a higher expected return from the asset. In a bubble, this accelerator mechanism increases expected prices over time until the bubble collapses because the previously unknown market depth is reached (i. e., the current owners of the asset do not find buyers), the underlying fundamental breaks down (e. g., due to bankruptcy of the issuing firm), or until a sudden change in agents' expectations about future prices occurs.

Importantly, the model allows for bubbles in some parameter ranges but not in others: the above-explained feedback between higher prices and increased fragility of the bubble may not have a fixed point. If no fixed point exists, even at some future date, there is no possible price path besides the fundamental path; bubbles are then unfeasible. Hence, depending on the interaction of leverage (discussed below), uncertainty about the market depth, riskiness of the asset, and the risk-free interest rate, the prerequisites for bubbles may be fulfilled or not.[3] We can thus analyze several policies to prevent bubbles. One of the widely discussed possible policy measures is a financial transaction (Tobin) tax. We find that such a measure can actually enlarge the parameter range in which bubbles are possible if the tax levied on *all* financial assets. If imposed only on potential bubble-assets, the tax prevents the emergence of bubbles. Similarly, also a system that reduces bonus payments can backfire

[2] Conlon (2004) argues that in many bubble periods, the overvaluation of assets was widely discussed. Referring to the dot-com bubble, Brunnermeier and Nagel (2004) provide evidence that hedge funds were riding the bubble, a result similar to a previous finding by Wermers (1999). The authors relate this to a short-term horizon of the managers, among other elements. This notion is consistent with our model.

[3] Using the latest US housing bubble as an example, we find that all conditions that are favorable for the emergence of bubbles were fulfilled. Low interest rates prevailed for a long period, whereas increasingly international financial flows and more complex financial instruments obscured potential market depth. Furthermore, the Securities and Exchange Commission's 2004 decision to allow large investment banks to assume more debt raised their leverage and further increased uncertainty about market depth. Kaminsky and Reinhart (1999), among others, also indicate an empirical connection between financial liberalization, credit expansion, and bubble emergence.

and increase the parameter range in which bubbles are feasible. Only a sufficiently low cap on bonuses effectively prevents the emergence of bubbles. Additionally, mandatory long-term compensation and/or capital requirements for traders can render bubbles impossible. Finally, a monetary policy rule that considers asset price inflation fulfills the same purpose. Galí (2014) shows in a different setup that higher interest rates set by the central bank in response to a bubble let it actually grow faster instead of preventing its existence. The reason is that in equilibrium the bubble component of an asset price needs to rise with a rate equal to the interest rate. This effect is also present in our model. If the interest rate reacts sufficiently strongly to asset price movements, however, the implied increasing increasing speed of the bubble build-up lets the risk of an exhausting market depth outweigh the benefits of a higher return, causing the bubble to burst. This creates a channel through which monetary policy that "leans against the wind" can prevent the emergence of bubbles, generalizing the result of Galí.

While imprecise information about market depth alone can already lead to the emergence of bubbles, additionally assuming that traders are leveraged increases the parameter space in which bubbles are possible significantly. In this setting, traders borrow from rational, risk-neutral households in addition to investing their own funds. These households know whether a bubble exists. They can anticipate but not monitor the behavior of traders, constituting the only friction. Leveraged traders behave as if they were risk-loving, investing in situations that they would deem too risky if they were only able to invest own funds.[4] They profit strongly from advancing stock prices, but their losses are limited to their own invested funds. The same results are obtained if one assumes that an investor delegates investment to a fund manager, whose compensation includes bonus payments that cannot become negative.[5] Hence, the model directly applies to any type of intermediated finance with limited liability, such as through banks, investment banks, insurance companies, and private equity firms as well as to non-intermediated, debt-financed investments. We show that depending on traders' investment opportunities compared with those of households, the latter may still be willing to lend their money to traders with limited liability even if the existence of a bubble is common knowledge. Under certain parameter constellations, however, bubbles can be ruled out if households' participation constraints will be violated at a future date. We provide a necessary and sufficient condition. Interestingly, it turns out that households are rationally lending funds to the traders under the same parameter constellations that support the development of bubbles, reducing their expected payoff relative to a world without bubbles. Policy interventions that can prevent bubbles are thus justified, as households cannot credibly commit to stop lending at a later date.

[4]Examining the effects of leverage alone, we find that the induced risk appetite of risk-neutral traders pushes asset prices above their fundamental values (as already shown by Allen and Gale, 2000). Because of limited liability in case of a low or zero return, a trader can increase her expected payoff by engaging in riskier assets. Equilibrium asset prices are therefore driven above fundamentals but in a static way. These price deviations are not induced by expectations about higher future prices (speculation) and are not subject to sudden corrections (bursts).

[5]According to the OECD database on institutional investors' assets, in 2007, institutional investors in the U. S. managed assets worth 211.2% of GDP, showing the investors' prominent role in investment decisions. Furthermore, the assets' size has grown steadily over the last decade with a yearly average growth rate of 6.6% from 1995-2005 within the OECD(17) area (see Gonnard, Kim, and Ynesta, 2008).

Because of the common but imperfect information about the market depth, bubbles can exist although the existence of a bubble is known to all agents in the model, just like the fact that the initial sellers of a bubbly asset gain and some later traders will loose. Furthermore, while households and traders are distinct, agents in each group are perfectly symmetric and hold symmetric information and priors. The existence of bubbles under these circumstances contrasts a series of papers that assume asymmetric traders, asymmetric information or priors among them.[6] An early contribution is Harrison and Kreps (1978), where the option to sell an asset to more optimistic investors in the future raises its price. In Allen, Morris, and Postlewaite (1993), private information of traders can drive a price above its fundamental value as they hope to sell the asset to a 'greater fool'. Similarly, Scheinkman and Xiong (2003) and Bolton, Scheinkman, and Xiong (2006) assume that buyers of an asset hope to sell it to overoptimistic agents in the next period. This process is only possible in the context of heterogeneous beliefs. The presence of overoptimistic agents and short-sale constraints creates bubbles also in Hong, Scheinkman, and Xiong (2006) in which prices drop after an increase in asset float. In Conlon (2004), the state of the world is known to the nth degree to asymmetrically informed traders, assigning higher-order beliefs a potentially important role. Allen, Morris, and Shin (2006) also highlight the role of higher-order expectations if traders have asymmetric information. Allen and Gorton (1993) and Barlevi (2008) show that asymmetry of information between investors and heterogeneous managers can lead to deviations in prices from fundamentals if liquidity needs are stochastic. The model of Brunnermeier and Abreu (2003) relies on dispersed opinions that, with the resulting coordination failure, enable bubbles to occur. Froot, Scharfstein, and Stein (1992) analyze which information can influence trading and potentially lead to herding equilibria, while Scharfstein and Stein (1990) relies on unknown abilities of heterogenous managers to generate herding. In DeMarzo, Kaniel, and Kremer (2008) herding can occur because of relative wealth concerns, which are not present in our setup. Firms' liquidity requirements that are not fully met by uninformed investors can lead to bubbles in Farhi and Tirole (2012). In DeLong, Shleifer, Summers, and Waldmann (1990), rational traders' behavior is influenced by noise traders, who follow positive-feedback strategies. In Plantin and Shin (2011), not all traders have market access at a given date. Alternatively, Kocherlakota (2008) shows that bubbles are possible if agents face a solvency constraint.[7]

Another large strand of literature goes back to Samuelson (1958) and Tirole (1985), who analyzed bubbles in overlapping-generations (OLG) models. In this setup, bubbles can arise if the real interest rate is smaller than the growth rate of the economy (see also Weil 1987). Hirano and Yanagawa (2014) study the effects of a bailout scheme in this

[6]Brunnermeier (2001) provides a more extensive survey of bubble models based on asymmetric information. Our overview of the literature on bubbles is by no means meant to be complete; this would require far more space.

[7]A different strand of literature relies on learning. Agents in Adam and Marcet (2010) use Bayesian updating about asset price movements, leading to bubbles. Similarly, in Branch and Evans (2011), bubbles can arise because of risk-averse agents who use least-squares learning. In Pástor and Veronesi (2006, 2009) there is initial noisy information and learning over time as well, leading to stock-price behavior that can be confused with a bubble.

context. A recent contribution by Martin and Ventura (2012) overcomes this restriction by assuming heterogeneous investors who differ in their productivity. Productive investors demand liquidity, whereas unproductive investors supply it in the form of bubbly assets. While the economy can be on average dynamically efficient, bubbles require that at least some investments in the economy are dynamically inefficient. As common in the OLG-growth setup, bubbles can theoretically grow forever but not faster than the growth rate of the economy. Our model differs in these respects: since the probability of a collapse of the bubble can rapidly reach high levels over time, prices can steeply rise far above the fundamental value. Furthermore, bubbles cannot grow forever but will burst at some point in time. Neither heterogenous investors, dynamic inefficiency, nor a positive growth rate of the economy are necessary for bubble creation. Contrary to our setup, bubbles are mostly welfare-enhancing in the mentioned types of OLG models.[8]

The remainder of this paper is organized as follows. Section 2 introduces the model and then develops a steady-state (rational-expectations) equilibrium price process. Section 3 provides a necessary and sufficient condition for the existence of bubbles. The section begins with the construction of a special type of bubble, which then serves as a limiting case for the general case. The condition lends itself to basic policy analysis, which is performed in Section 4 by discussing several policy measures, beginning with a welfare analysis. Interpreting the trader's payoff scheme as a compensation package that pays bonuses but does not allow traders to participate in losses allows us to evaluate the policy measures of a cap on bonuses and long-term compensation. Section 5 concludes. Appendix A provides a necessary and sufficient condition for the households' participation constraint. All proofs are in Appendix B.

2 The Model

2.1 Setup

Consider an infinite horizon economy with a series of cohorts of risk-neutral households and traders. In period t, a continuum of measure N households and a continuum of measure N traders are born. Each household has an initial endowment of D dollars, and each trader owns E dollars. N is fixed over time but unknown. The perceived probability of N has the distribution $F(N)$; the density $f(N)$ exists and has unbounded support.[9] Households and traders invest in period t, earn their returns, withdraw from the market,

[8] An exception is Caballero and Krishnamurthy (2006). In their model, however, the bubble growth rate is again limited by the growth rate of the economy and is predictable in the case of a continuation. As our model allows for multiple price paths, the growth rate of bubbles is unpredictable even conditional on a continuation.

[9] The unlimited support of the function implies that in a bubble, agents can never be completely sure that market depth will be exhausted in the next period.

and consume in the next period $t+1$, i. e., they both have a life span of two periods. The duration of a period stands for the investment horizon of a trader.[10]

There are two types of assets, safe assets (short: storage) of unlimited supply and a single risky asset (short: the asset) of volume 1. Storage assets bear a risk-free return of $Y > 1$ to a trader. The return of a safe asset to a household is λY, with $0 \leq \lambda < 1$. The inverse of λ thus measures the investment advantage of traders, which may arise because of better abilities or better information. Traders have access to the risky asset, which can be interpreted as shares of a firm. Shares cannot be sold short. The firm pays dividends of d in each period. There is a probability $1-q$ in each period that the firm goes bankrupt and ceases to pay dividends forever; the firm's shares are then no longer traded.[11] Hence, the time of bankruptcy follows a geometric distribution. The risky asset is traded in a competitive market in each period, and its price follows an endogenous discrete-time stochastic process $\{\tilde{p}_t\}_{t\geq 0}$.

To benefit from better access to investment, households can provide loans to traders, causing them to act as financial intermediaries who can invest $D+E$ rather than only E. Households cannot observe the investment choices of traders. The endogenous gross loan rate is called $r > 1$, including principal. It is endogenized in Appendix A. If R is a trader's gross return rate to investment, then her income from investment is $R\,(D+E)$, and the profit is $R\,(D+E) - r\,D$ if that is positive; otherwise, the profit is zero due to limited liability. The trader's target function is thus

$$\max\{\alpha\,(R-\beta);\,0\} \tag{1}$$

with $\alpha := (D+E)$ and $\beta := r\,D/(D+E)$. Note that higher leverage $D/(D+E)$ increases β, and limited liability becomes more severe as traders are liable only with their own funds E. Traders maximize the expected target function. Note that $\beta = 0$ corresponds to the case without leveraged traders ($D = 0$), thus if households prefer not to lend to traders.

We solve for stochastic rational expectations equilibria. We say that a bubble exists if there are multiple stochastic processes satisfying the rational expectations equilibrium condition. The model never predicts which price process is chosen. We now begin by discussing the steady-state price path.

2.2 The Steady-State Price

Consider the following simple stochastic process $\{\tilde{p}_t\}_{t\geq 0}$. The price of the asset is a constant, $\tilde{p}_t = \overline{p}$. The price drops to zero only if the underlying firm goes bankrupt (with probability $1-q$), and cash ceases to flow. Hence, the price follows a simple binomial

[10] Traders enter and exit the market in an OLG fashion to generate trade each period. We do not see this OLG structure as representing actual generations, but as a shortcut for non-modeled market imperfections, such as heterogeneous liquidity preferences of traders.

[11] One may also interpret the asset as real estate. If a house is used as rental property, d denotes the rent per period, whereas $1-q$ is the probability that the house becomes uninhabitable.

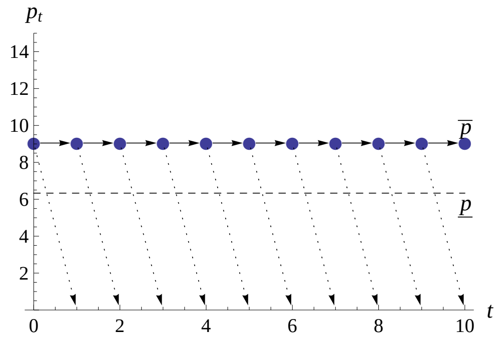

Figure 1: The Price Process in Steady State.

Parameters are $\beta = 0.9$, $q = 95\%$, $d = 1$, and $Y = 1.1$.

process with $\Pr_t\{\tilde{p}_{t+1} = \overline{p} | p_t = \overline{p}\} = q$. Zero is an absorbing state. Let us derive the price $\overline{p}$ for which this process is a rational-expectations equilibrium.

In a market equilibrium, prices must be such that the traders' expected return is the same for storage and for the risky asset. If a trader opts for storage, her compensation is $\max\{\alpha\,(Y-\beta);\,0\} = \alpha\,(Y-\beta)$ with $\beta \leq Y$ because otherwise, traders would not want to borrow. If the trader buys shares of the firm at a price $p_t = \overline{p}$, she benefits from the dividend with probability q. She thus earns d/p_t with probability q. In the absence of a bankruptcy, the price remains at $\tilde{p}_{t+1} = \overline{p}$, and the trader additionally receives $p_{t+1}/p_t = \overline{p}/\overline{p} = 1$ from selling the asset. This stochastic process is depicted in Figure 1.

In the steady state, an intermediary's expected compensation from holding the risky asset on date t is

$$E_t \max\left\{0;\, \alpha\left(\frac{\tilde{p}_{t+1}}{p_t} + \frac{d}{p_t} - \beta\right)\right\} = q\,\alpha\left(\frac{\overline{p}+d}{\overline{p}} - \beta\right) \tag{2}$$

For the market to clear, traders must be indifferent between storage and the risky investment:

$$\alpha\,(Y-\beta) = q\,\alpha\left(\frac{\overline{p}+d}{\overline{p}} - \beta\right), \tag{3}$$

$$\Rightarrow \quad \overline{p} = \frac{d\,q}{Y - q - \beta\,(1-q)}. \tag{4}$$

Therefore, the *steady-state price* $\overline{p}$ depends on the compensation scheme (β). The *fundamental value* is obtained in the absence of leverage:

$$\underline{p} := \frac{d\,q}{Y-q}. \tag{5}$$

Hence, only if $\beta = 0$ (no leverage) or if $q = 1$ (no risk), the fundamental value and steady-state price are equal, $\underline{p} = \overline{p}$. The effect of leveraged traders pushing prices of risky assets above their fundamental levels has been analyzed previously by Allen and Gale (2000), but this systematic deviation from fundamentals is not driven by expectations. The following comparative statics follow immediately.

Proposition 1 *The ratio between the steady-state price $\overline{p}$ and the fundamental value $\underline{p}$ is high for a low yield Y on safe assets, for high fundamental risk (low q), and for substantial leverage (high β).*

A large ratio $\overline{p}/\underline{p}$ amplifies the effect of a change in fundamentals on the steady-state price. Let us clarify three aspects. *First*, the price $\overline{p}$ can only be achieved if the market is deep enough, i. e., if enough money is in the market, $N\,(D+E) \geq \overline{p}$. Otherwise, there is cash-in-the-market pricing, and $\overline{p} = N\,(D+E)$. *Second*, households may not wish to provide loans to traders at all. If the households invest themselves, they receive λYD from the safe asset. If the households provide a loan to a trader, the trader invests in the risky asset with probability $\frac{\overline{p}}{N\,(D+E)}$ because this fraction of overall investment is taken by the risky asset. In this case, the loan can be repaid only partially with probability q. Consequently, if the asset is too risky, and/or the trader's return rate is only slightly above that of the households, traders receive no outside financing. We derive the households' participation constraint in Appendix A.

Third, in the numerical example for Figure 1, the fundamental value is $\underline{p} = 6.33$, but the steady-state price is $\overline{p} = 9.05$. This price deviation is due to leverage. However, the deviation is *static* and driven by fundamentals (risk q, dividends d, and investment opportunity Y) and the traders' financial contracts (β) but *not* by traders' expectations about future price developments. The deviation is constant over time and cannot burst, so its existence is less interesting from a financial stability perspective. Nevertheless, this deviation can magnify price movements resulting from, e. g., changing dividend payments. In contrast, the bubble described in the following section is dynamic by nature and can only be sustained if the price is expected to further increase in the future. Large price deviations are fueled by the *expectation* that future traders will buy at an even higher price. A bubble grows dynamically, and it can burst at any time.

3 Bubbles

Consider a situation in which the price p_t is above the steady-state price $\overline{p}$ at a certain date t. The only conceivable reason to buy is that traders expect the price to rise even further, at least with a certain probability. Otherwise, traders would prefer to store rather than to invest in the asset. We define this expectations-driven price deviation as a *bubble*.

Definition 1 (Bubble) *A bubble is a price process in a rational-expectations equilibrium in which the price deviates from the steady-state price.*

In the model, because the steady-state price process always exists, the existence of a bubble is equivalent to the existence of multiple equilibria. Hence, the search for bubbles is equivalent to the search for multiple equilibria. We now state our main result.

Theorem 1a *Define*

$$\gamma := \lim_{N\to\infty} N \frac{f(N)}{1-F(N)} \tag{6}$$

if the term converges in $\mathbb{R}_+ \cup \{\infty\}$*. If* $\beta \leq (\gamma-1)/\gamma$*, bubbles cannot exist in a rational expectations equilibrium. If* $\beta > (\gamma-1)/\gamma$*, bubbles are possible if and only if*

$$\gamma^{\gamma}\left(\frac{\beta}{\gamma-1}\right)^{\gamma-1} \leq \frac{q}{Y-\beta}. \tag{7}$$

If $\gamma < 1$*, bubbles are always possible. In the limiting case* $\gamma = \infty$*, bubbles never exist.*

If the term in (6) does not converge, we can derive a more general version of Theorem 1a. Instead of the limit, we take the supremum and infimum of the term (6), which always exists in $\mathbb{R}_+ \cup \{\infty\}$. The supremum enters into a sufficient condition for the existence of bubbles, and the infimum enters into a necessary condition. However, for common distribution functions, the limit does exist; the distinction between supremum and infimum is then unnecessary.

Theorem 1b *If the limit in equation* (6) *does not exist, define*

$$\underline{\gamma} := \inf_{N\to\infty} N \frac{f(N)}{1-F(N)}, \qquad \textit{and} \qquad \overline{\gamma} := \sup_{N\to\infty} N \frac{f(N)}{1-F(N)}, \tag{8}$$

with $0 \leq \underline{\gamma} < \overline{\gamma} \leq \infty$*. If* $\beta \geq (\overline{\gamma}-1)/\overline{\gamma}$ *and*

$$\overline{\gamma}^{\overline{\gamma}}\left(\frac{\beta}{\overline{\gamma}-1}\right)^{\overline{\gamma}-1} < \frac{q}{Y-\beta}, \tag{9}$$

bubbles are possible; the price process has multiple rational expectations equilibria. If $\beta < (\underline{\gamma}-1)/\underline{\gamma}$ *or*

$$\underline{\gamma}^{\underline{\gamma}}\left(\frac{\beta}{\underline{\gamma}-1}\right)^{\underline{\gamma}-1} > \frac{q}{Y-\beta}, \tag{10}$$

bubbles are impossible; the price process is unique. If $\overline{\gamma} < 1$*, bubbles are always possible. In the limiting case* $\underline{\gamma} = \infty$*, bubbles never exist.*

Before deriving Theorems 1a and 1b in detail, let us provide a brief explanation. In the first part of the theorem, we analyze the tail behavior of the distribution of N by calculating γ. The definition of γ is the limit of a relative version of a hazard rate, which is intuitive as

	Static deviation (Prop. 1)	Bubble condition (Prop. 2)	HH participation (Prop. 6, App. A)
γ	—	$\searrow$	$\searrow$
Y	$\searrow$	$\searrow$	$\searrow$
β	$\nearrow$	$\nearrow$	$\nearrow$
q	$\searrow$	$\nearrow$	$\nearrow$
d	— (but multiplier)	—	—

Table 1: Effects of Variations of Parameters

An upward arrow in the left column indicates that the deviation increases. An upward arrow in the middle and the right columns signifies that the necessary condition for a bubble to emerge is fulfilled for a larger range of all other parameters.

we are interested in the probability of a bursting bubble given a price increase in per cent. Condition (7) verifies whether the tail of the distribution is thick enough to constrain the risk of a burst such that investing is still attractive (a low γ denotes a thicker tail). A higher β denotes higher leverage of the traders, which makes it more attractive to assume risk. In case of a very thick tail ($\gamma < 1$), traders are willing to invest their own funds into a publicly known bubble even without limited liability ($\beta = 0$). A high return Y of the risk-free asset and a high intrinsic risk of the risky asset (low q) reduce traders' willingness to invest in the risky asset.

For $\gamma < 1$, we get the result of Blanchard (1979) and Blanchard and Watson (1982): bubbles are always possible. For $\gamma \to \infty$, we obtain Tirole (1982)'s impossibility result. The following proposition and Table 3 (middle column) summarize the comparative statics. An upward arrow signifies that the necessary condition for a bubble to emerge is fulfilled for a larger range of all other parameters. Figure 2 visualizes the parameter constellations that allow for bubbles. In the figure, bubbles exist for parameter constellations *below* the plane. The range of β starts at $(\gamma - 1)/\gamma$ as bubbles cannot exist below.

Proposition 2 *Bubbles tend to be possible for a low risk-free yield Y, low fundamental risk (large q), large uncertainty about market depth (low γ), and high leverage (high β).*

In Appendix A, we prove a second pair of theorems with the same structure. Whereas Theorems 1a and 1b address the incentives of traders to invest in the potentially bubbly asset, Theorems 2a and 2b will address households' incentives to lend to a trader. If the parameter γ exists, we can derive a sufficient and necessary condition. If the parameter does not exist, we will again use the supremum for a sufficient condition and the infimum for a necessary condition. To summarize the results, parameter constellations that tend to make bubbles possible (see Proposition 2) also make household's participation constraint less restrictive (see the right column of Table 3, or Proposition 6 on page 29).

Again, note the difference between a *dynamic* price deviation from the steady-state price and the *static* deviation of the steady-state price from the fundamental value. There is

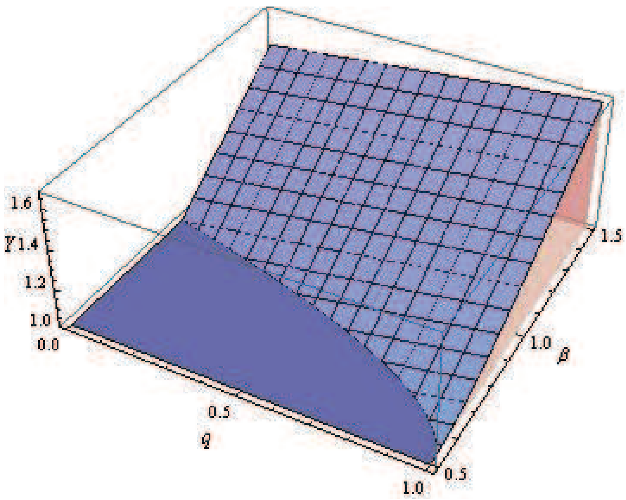

Figure 2: Parameter Range where Bubbles are Possible

Here, $\gamma = 2$. For parameter constellations *below* the surface, bubbles are possible.

one major difference between these two cases in the left and the middle columns. A static deviation is larger for inherently risky assets, but bubbles tend to emerge for inherently safe assets. Note one subtle but important difference between the inherent and the financial risks of an asset. To provide an example, building a house may be inherently safer than buying stock in a firm. However, considering financial risk, a house may be a riskier investment, especially if it is built during a bubble. Our model distinguishes between these notions of risk. Inherent risk is captured by $1-q$, the risk of failure of the underlying asset. Additional financial risk can occur if movements in fundamentals are strongly magnified because the asset price deviates from the fundamental value in a static way or because condition (7) holds, and a bubble can form. Interestingly, the two sources of financial risk react similarly to most parameter changes, but their reactions with respect to the underlying risk $1-q$ are directly opposed.

This fine differentiation suggests different explanations for the two most prominent bubbles in recent decades. Real estate and mortgages are inherently safe; thus, according to Lemma 1, a speculative bubble on these assets should be feasible. In this sense, the theory matches the recent real estate bubble.[12] However, dot-com firms are inherently risky. A speculative bubble might be thus impossible, but the above static deviation that potentially changes fundamentals will be particularly large. Consequently, according to our theory, the bursting of the dot-com "bubble" might have been a correction of

[12] As argued in footnote 3, the development of the housing bubble was also promoted by the constellation of the remaining parameters, i. e., low interest rates, opaque financial markets, and the 2004 decision of the Securities and Exchange Commission to allow the large investment banks to assume more debt, which corresponds to a higher β (see Section 4.3).

expectations bloated by a large multiplier.[13]

The proof of Theorem 1a and 1b proceeds in three steps. We first concentrate on simple (trinomial) bubble paths and a subset of distributions $F(N)$. The ensuing lemma serves both to illustrate the mechanics of a bubble and as a stepping stone for the general case. Section 3.2 then drops the assumption of a trinomial price path, yielding another lemma, and Section 3.3 drops the assumption of the distribution of N, yielding the above Theorem 1a and 1b. The intuition for the proofs is in the main text, and the formalism is in Appendix B.

3.1 "Trinomial" Bubbles

First, concentrate on a *trinomial* process with

$$\tilde{p}_{t+1} = \begin{cases} 0, & \text{with probability } 1-q \\ \overline{p}, & \text{with probability } q - Q_t \\ p_{t+1}, & \text{with probability } Q_t \end{cases} \tag{11}$$

with $Q_t \leq q$.[14] All variables $\{p_t, Q_t\}_t$ are determined endogenously. For our purposes, trinomial processes are the simplest ones, allowing for a fundamental default of the firm (case one), a bursting of the bubble (case two), and the continuation of the bubble (case three).

Second, let us concentrate on a parameterized version of the number of households and traders $F(N)$. To be concrete, we assume that $\log N$ is exponentially distributed; thus, $F(N) = 1 - e^{\gamma(\log N - \log N_0)} = 1 - (N/N_0)^{-\gamma}$ for some $N_0 > 0$ and $\gamma > 1$. Here, N_0 is a lower bound on the number of households, and γ measures the thinness of the tail. Because the number of traders is also N, and they can acquire additional funds from households, the aggregate amount of intermediated investment L is $(D+E)\,N$. L is the maximum amount of investment in the asset, i. e., the market depth. This value has the distribution function

$$\begin{aligned} \bar{F}(L) &= \Pr\{(D+E)\,N \leq L\} = F\big(L/(D+E)\big) \\ &= 1 - \Big(\frac{L}{(D+E)\,N_0}\Big)^{-\gamma} = 1 - \Big(\frac{L}{L_0}\Big)^{-\gamma}. \end{aligned} \tag{12}$$

with $L_0 := (D+E)\,N_0$. γ measures the precision of the information on market depth; for $\gamma \to \infty$, the market depth is $L = L_0$ with certainty. Because each intermediary can invest $D + E$ dollars, an asset's price p can never exceed $L = N\,(D+E)$.

[13] Pástor and Veronesi (2009) model the learning process about the productivity of new technologies and apply it to the introduction of railroad and internet technologies. The mentioned multiplier is then an amplification of movements generated by learning.

[14] Note the notational difference between $\tilde{p}_{t+1}$ and p_{t+1}. $\tilde{p}_{t+1}$ is the stochastic price at date $t+1$ that can assume three different values. $p_{t+1} > p_t$ is the largest of these realizations.

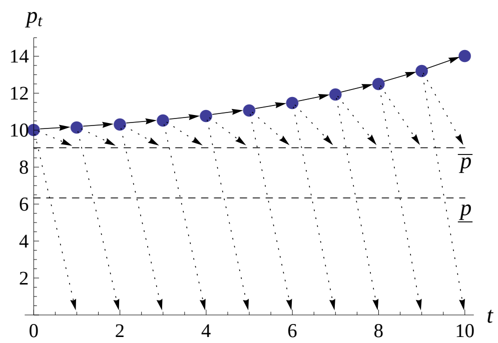

Figure 3: A Trinomial Price Process with a Bubble

As in Figure 1, parameters are $\gamma = 2$, $\beta = 0.9$, $q = 95\%$, $d = 1$, and $Y = 1.1$.

Construction of a Bubble. Next, we proceed to the discussion of bubble paths. A possible price process is depicted in Figure 3. The process begins at a certain price $p_0 > \overline{p}$; the resulting bubble can then grow further and further, $p_0 < p_1 < p_2 < \ldots$ For a given price increase from p_t to p_{t+1}, more money will be absorbed by the market in $t+1$ than in t. As a consequence, p_{t+1} may exceed the market depth L at a certain date. In this case, the price reaches a ceiling, and no further price increases are possible, i. e., traders cannot expect to sell the asset at a higher price in the future. Hence, the bubble must collapse back to the steady-state price, $\tilde{p}_{t+1} = \overline{p}$. This ceiling L is not pictured in the figure as it is unknown. The date at which the bubble bursts is (and must be) unknown, but with certainty, the ceiling will be reached at some date. Initially, the amount of money absorbed by the risky asset is relatively small. However, as the bubble grows, more traders are attracted by the risky asset, and it absorbs more money. This money is diverted from investment into the safe asset (storage).

Alternatively, if the underlying firm goes bankrupt, the price drops to $\tilde{p}_{t+1} = 0$. The conditional probability of a continuation (*non*-collapse) of the bubble is then

$$Q_t = q \ \Pr\{p_{t+1} \leq L | p_t \leq L\} = q \, \frac{1 - \bar{F}(p_{t+1})}{1 - \bar{F}(p_t)}$$
$$= q \, \frac{L_0^\gamma / p_{t+1}^\gamma}{L_0^\gamma / p_t^\gamma} = q \, p_t^\gamma / p_{t+1}^\gamma, \tag{13}$$

where q is the probability that a firm continues to operate, and Q_t is the probability that the firm's asset price continues to rise. The probability that the bubble bursts although the firm is still solvent is thus $1 - Q_t - (1-q) = q - Q_t = q\,(1 - p_t^\gamma / p_{t+1}^\gamma)$.

Because no dividends are paid, and the shares are no longer traded if the firm is insolvent, the profit of the trader is $\alpha \max\{0/p_t + 0/p_t - \beta; 0\} = 0$ in this case. Alternatively, if the share price falls because a bubble bursts, the price drops to $\overline{p}$. For now, let us simply assume that there is no profit if a bubble bursts. We provide a condition and analyze the alternative in the proof of Lemma 1 in Appendix B. The asset market can only be in equilibrium if a modified version of (3) holds, considering the probability of a burst and the increased profit if the bubble does not burst:

$$\begin{aligned}\alpha\,(Y-\beta) &= Q_t\,\alpha\left(\frac{p_{t+1}+d}{p_t}-\beta\right),\\ &= q\left(\frac{p_t}{p_{t+1}}\right)^{\gamma}\alpha\left(\frac{p_{t+1}+d}{p_t}-\beta\right),\\ \Rightarrow\qquad \frac{Y-\beta}{q} &= \left(\frac{p_t}{p_{t+1}}\right)^{\gamma}\left(\frac{p_{t+1}}{p_t}+\frac{d}{p_t}-\beta\right).\end{aligned} \tag{14}$$

Equation (14) implicitly determines a price process in a rational expectations equilibrium. For any given $p_0 > \overline{p}$, (14) implicitly defines p_1, and (13) provides the corresponding Q_0; thus, all variables for $\tilde{p}_1$ in (11) are defined. Then, if we begin from p_1 in a second step, (14) and (13) define p_2 and Q_1; thus, $\tilde{p}_2$ is obtained. Following this procedure provides the complete process recursively. One such process is shown in Figure 3.

The Existence of Bubble Processes. Equation (14) does not necessarily yield a solution for any set of parameters. As discussed above, the higher the potential future price increase, the more likely it is that the market depth L is reached and the bubble bursts. However, the more likely the bubble is to burst, the larger the expected price increase must be to compensate traders for the risk they face. This multiplier effect does not necessarily reach an equilibrium price p_{t+1} for all p_t. In this case, there is a $\hat{p}$ above which potential future price increases cannot compensate for the accompanying higher risk of a burst. Because all market participants can calculate the date t at which this $\hat{p}$ is reached, if it exists, a bubble will burst with certainty at a certain date $t+1$, i. e., $Q_t = 0$. If the bubble cannot be sustained at a date of $t+1$, traders anticipate this. By backward induction, the bubble is not sustainable from the beginning.

We are interested in conditions under which a bubble can or cannot be sustained. To be sustainable, the implicit equation (14) must have a solution at any date t, or equivalently, for any initial price p_t. Rewriting (14) by defining the auxiliary variable $\phi_t = p_{t+1}/p_t$ as the relative price increase yields

$$Y-\beta = q\,\phi_t^{-\gamma}\left(\phi_t+\frac{d}{p_t}-\beta\right). \tag{15}$$

The left side of the equation is independent of p_t, but the right side depends on the starting point p_t.

Figure 4 shows the left side (thin solid line) and the right side for two starting prices, $p_t = \overline{p}$ (thick dashed curve) and $\lim p_t \to \infty$ (thick solid curve). If the curve intersects with the line, a bubble is possible. For $\gamma < 1$, they always intersect, for $\gamma \to \infty$, they never intersect. Consider first the case $p_t = \overline{p}$. The intersection with the thin line is at $\phi_t = 1$, which implies that $p_{t+1} = \phi_t\,p_t = p_t$, so there is no price increase. Starting with $p_t = \overline{p}$, we are in the steady state; the price does not change over time.

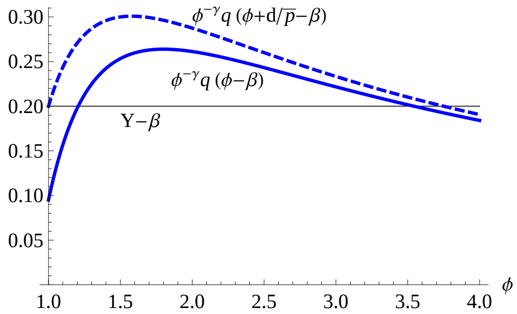

Figure 4: Possibility of a Bubble

As in Figure 3, parameters are $\gamma = 2$, $\beta = 0.9$, $q = 95\%$, $d = 1$, and $Y = 1.1$.

If the initial price is slightly above $\overline{p}$ due to higher expectations, the curve shifts downwards (and slightly changes its shape), implying that it intersects with the line at a certain $\phi_t > 1$. In the next period, the price will be higher still, so the intersection ϕ_{t+1} will be even higher. A bubble emerges, and the growth rate $\phi_t = p_{t+1}/p_t$ increases with time. For $\lim p_t \to \infty$, the limiting line $q\,(\phi - \beta)$ is reached (solid curve). Because the intersection point moves right as p_t increases, the bubble becomes less stable; the probability of a burst, $1 - Q_t = 1 - q/\phi_t$, increases.

When the asset price increases and the curve moves downward, an intersection between the line and the curve can cease to exist (if $\gamma > 1$), depending on the parameters. In this case, it is common knowledge that the asset price cannot rise without bounds. Because of this upper bound, a backward induction argument applies. No rational price deviation can exist in the first place.

Condition for Existence. To show that a bubble can be sustained in a market, it is sufficient to consider large prices p_t. In the limit of $p_t \to \infty$, (15) simplifies to

$$Y - \beta = q\,\phi^{-\gamma}\,(\phi - \beta). \tag{16}$$

The equation does not depend on time; thus, we have dropped the index t. If (16) has a solution for $\phi > 1$, the corresponding market can sustain a bubble.[15] This formula implies that for arbitrarily high prices p_t, there is always a price p_{t+1} that is high enough to induce traders to buy at date t. If (16) does not have a solution for ϕ, then there exists a price

[15]There are at most two solutions to (16). In the following explanation, we concentrate on the lower solution. As we are mainly interested in the existence of bubbles, the magnitude of the solution is irrelevant. Moreover, as households become less willing to invest money with increasing ϕ, they might be willing to invest in the lower solution (see Appendix A) but not the higher solution. Combinations of the two equilibria are implicitly addressed in Lemma 2. Note also that the higher solution is unstable. Furthermore, a solution must satisfy $\phi \geq 1$. Values of $\phi < 1$ would stand for bubbles with falling prices and, formally, negative probabilities of a burst. A sufficient condition is $\beta \geq (1-\gamma)/\gamma$; see the proof of Lemma 1.

$\hat{p}$ beyond which no further increase is impossible. Nobody buys, and the bubble bursts. Hence, according to backward induction, the bubble cannot begin to form at date $t = 0$. The only possible price is then $p_0 = \overline{p}$.

Although one cannot provide a closed-form solution of (16), one can provide a necessary and sufficient condition for existence, which leads to the following lemma.

Lemma 1 *If* $\log N$ *is exponentially distributed, and* $\beta > (\gamma - 1)/\gamma$*, a price process can exhibit a trinomial bubble in a rational expectations equilibrium if and only if* (7) *holds, hence if*

$$\gamma^{\gamma} \left(\frac{\beta}{\gamma - 1} \right)^{\gamma - 1} \leq \frac{q}{Y - \beta}. \tag{17}$$

If $\gamma < 1$*, it can always exhibit a trinomial bubble. If* $\beta \leq (\gamma - 1)/\gamma$ *or* $\gamma = \infty$*, trinomial bubbles cannot exist.*

As for the steady-state price above, we discuss the participation constraint of the households in Appendix A. In sum, households' relative return to investing λ must be low enough to maintain intermediated investment attractive for them despite the risk of a bursting bubble.

Recall that higher risk of a bursting bubble implies a larger potential price increase to compensate traders and that a larger potential price increase in turn increases the likelihood of bursting. If this problem has a fixed point at all times, a bubble can emerge. If risk-free rates are higher, storage becomes more attractive to traders, and the traders must be compensated by a larger potential price increase of the risky asset to hold it. However, this process further increases the likelihood of a burst, which impedes the convergence to a new fixed point. Hence, for a larger risk-free yield Y, bubbles might cease to be possible. This finding is consistent with the idea that central banks can puncture bubbles by increasing interest rates and that bubbles are particularly likely if interest rates are low. Furthermore, bubbles can exist particularly if q is high, that is, if the underlying asset is rather safe, which decreases the likelihood of a burst. The parameter γ captures the uncertainty in the market. The smaller the value of γ, the larger are the mean and the variance of the distribution, and the more uncertain is the potential market size. The parameter N_0 does not appear in the analysis, which shows that for the existence of a bubble, only the shape of the upper tail matters; bubbles tend to exist for smaller values of γ. For $\gamma < 1$, the expected market size becomes infinite. Bubbles can then always exist. On the other hand, if $\gamma \to \infty$, the market depth is almost surely L_0, and a bubble can never be sustained independent of the values of other parameters. Finally, the parameter β describes the degree of leverage and thus the importance of limited liability. The larger the value of β, the more traders rely on external financing, and the more prominent the effect of limited liability becomes. Hence, we obtain the result that the emergence of bubbles may become possible in the context of a high degree of leverage.

3.2 General Bubble Price Path

We have argued that a special type of bubble process, the trinomial bubble, exists if and only if (7) holds. We now generalize this result by showing that if (7) fails to hold, the *only* rational expectations equilibrium process is the non-bubble process with the steady-state price $\overline{p}$. Then, no conceivable bubble paths exist. We provide an intuitive explanation here; the proof is in Appendix B. Trinomial bubbles do not exist if the thick curve and the thin straight line in Figure 4 do not intersect, i. e., if there is no solution for ϕ, and hence prices in a bubble eventually increase too quickly to be sustainable. The point on the graph at which the line and the curve are closest corresponds to the price increase with the lowest ratio of risk (of a bubble burst related to ϕ^γ) to potential gains (of a price increase related to ϕ). Concentrating all probability mass on this point maximizes the attractiveness of an investment in the risky asset, thereby creating favorable conditions for the emergence of bubbles. Distributing probability mass to other price increases, i. e., deviating from the assumption of a trinomial price process, lowers the willingness of traders to invest in the risky asset and therefore reduces the parameter space in which bubbles are possible. Thus, for a given set of parameters, if not even a trinomial bubble path exists, no bubble can exist at all. However, if a bubble does exist, its path depends on the evolution of price expectations; it cannot be unique.

Lemma 2 *If* $\log N$ *is exponentially distributed, and* $\beta > (\gamma - 1)/\gamma$*, a price process can exhibit a bubble in a rational expectations equilibrium iff* (7) *holds, hence if*

$$\gamma^\gamma \left(\frac{\beta}{\gamma - 1}\right)^{\gamma - 1} \leq \frac{q}{Y - \beta}.$$

If $\gamma < 1$*, it can always exhibit a bubble. If* $\beta \leq (\gamma - 1)/\gamma$ *or* $\gamma = \infty$*, bubbles cannot exist.*

3.3 General Distribution of Market Depth

Thus far, we have assumed that $\log N$ and therefore also $\log L$ are exponentially distributed. We now derive a bubble condition for general distributions of the market depth L. However, the bubble condition (7) used the distribution-specific parameter γ. For general distributions, we must find the property that is captured by γ. In fact, γ describes the fatness of the distribution's tail, thereby measuring the uncertainty regarding the market depth. For example, if $\log N$ is exponentially distributed, the hazard rate is

$$\frac{f(N)}{1 - F(N)} = \frac{\gamma}{N}. \tag{18}$$

A relative version of the hazard rate, $\frac{\mathrm{d}}{\mathrm{d}\phi} F(\phi N)\big|_{\phi=1} \big/ \big(1 - F(\phi N)\big)\big|_{\phi=1} = N \frac{f(N)}{1-F(N)}$, is constant and equals γ for the example distribution. Hence, the possibility of a bubble will depend on the relative hazard rate of $F(N)$ for large N. Following this approach yields Theorem 1a in which this relative hazard rate is provided by the parameter γ. However, the parameter need not converge, and if it does not, one must use the supremum and the infimum instead. This yields Theorem 1b.

Let us provide some intuition. We already know that the possibility of a bubble depends on the shape of the distribution $f(L)$ for large L, thus for large prices. The relevant measure for the thickness of the tail is the relative hazard rate as it denotes the probability that today's price realization plus one percent is still below N (which is the applicable question for calculating returns). Therefore, for a general distribution of N, we must calculate the relative hazard rate in the upper tail of the distribution. This rate need not converge, but the supremum (infimum) provides an upper (lower) limit, hence a lower (upper) boundary for the thickness of the tail. Theorem 1b states that if a bubble were possible already for a constant relative hazard rate at $\overline{\gamma}$, then bubbles are even more possible for even thicker tails; we obtain a sufficient condition. If the tail is thinner as described by $\underline{\gamma}$, we know from Theorem 1b that bubbles are not possible, resulting in a necessary condition.

4 Policy Measures

In this section, we first show that bubbles reduce welfare in our setting. We then examine whether certain policy measures that have been suggested in public debates can prevent the creation of bubbles. Specifically, we examine an asset-price augmented Taylor rule, capital requirements, and a financial transaction (Tobin) tax. We then interpret α and β as parts of a compensation package and can therefore discuss caps on bonuses and mandatory long-term compensation as possible policy measures.

4.1 Welfare

To justify any policy measure for the prevention of bubbles, the welfare effect of bubbles must be analyzed.[16] In our model, agents consume in their second period. To aggregate the utilities of different cohorts of traders, we introduce a discount factor $\rho < 1$, which is also the discount factor between cohorts. Then

$$E_0[W] = \sum_{t=0}^{\infty} \rho^t E_0[C_t], \tag{19}$$

where C_t is the aggregate expected consumption at date t. Payments between traders and households in the same cohort are mere transfers and do not directly enter the welfare function. In the absence of a bubble, the price of the asset is $\overline{p}$ until the asset defaults. Hence, the cohort that consumes on date 0 earns $C_0 = \overline{p}$ from selling the asset. Cohort 1 pays $\overline{p}$ for the asset. Because there are N households and N traders owning D and E dollars, respectively, the aggregate endowment of cohort 1 is $N\,(D+E)$. A part $\overline{p}$ is spent

[16] We restrict the analysis to the case of $\gamma \geq 1$. Otherwise, bubbles cannot be prevented by economic policy.

on the risky asset, so the investment in the risk-free asset is $N(D+E)-\overline{p}$ with a gross yield of Y. With probability q, cohort 1 also obtains $\overline{p}$ from selling the asset plus the dividend d. Hence, the aggregate expected consumption of cohort 1 is

$$E_0[C_1] = q\,(d+\overline{p}) + \big(N\,(D+E)-\overline{p}\big)\,Y. \tag{20}$$

Cohort 2 buys the asset only with probability q; with probability $1-q$, the firm is bankrupt and there is nothing to buy. Hence,

$$E_0[C_2] = q^2\,(d+\overline{p}) + \big(N\,(D+E)-q\,\overline{p}\big)\,Y. \tag{21}$$

The equations for the following cohorts are similar. Discounting, aggregating over cohorts, and taking the derivative with respect to $\overline{p}$ yields

$$\frac{\mathrm{d}}{\mathrm{d}\overline{p}}\Big(E_0[C_0] + \sum_{t=1}^{\infty} \rho^t E_0[C_t]\Big) = -\frac{\rho\,Y-1}{1-\rho\,q}. \tag{22}$$

This derivative is negative if $\rho\,Y > 1$. A high price $\overline{p}$ causes consumption to be shifted from the future to the present, increasing welfare due to less discounting. However, a high $\overline{p}$ also leaves less for real investment in the Y-technology. As a result, if $Y > 1/\rho$, a lower market price $\overline{p}$ increases welfare.

The condition $Y > 1/\rho$ indicates that risk-free investment increases welfare initially; otherwise, no investment would occur at all and no bubble could emerge. For the rest of the paper, we assume $Y > 1/\rho$. A social planner would then set the price of the risky asset to zero if she could. However, this solution is not feasible in a decentralized equilibrium. Consequently, the welfare-optimal price path is the one with the smallest possible prices – the steady state.

Proposition 3 *The welfare in a bubble is smaller than in the steady state.*

Note the difference from models of rational bubbles in overlapping-generations models, which build upon the mechanism developed in Tirole (1985). In these models, bubbles are possible if the economy features a certain form of dynamic inefficiency. Investing less in capital increases available resources currently and in the future such that bubbles can be purchased by future generations and increase welfare. Our model does not require dynamic inefficiency for bubbles to exist. Furthermore, because we assume that the Y-technology exhibits constant returns, investment is always welfare enhancing as long as $\rho\,Y > 1$. Investment reduces welfare if this condition does not hold, but then bubbles are not possible.

The analysis of aggregate welfare conceals the fact that interests can be different from an ex-interim perspective. The first cohort always profits from the existence of a bubble. From an ex-ante perspective, the second cohort suffers in expectations. However, once a cohort has purchased the overpriced asset, that cohort no longer wants the bubble to burst. Hence, although a welfare-maximizing regulator should avoid bubbles, her interests might be different if she is influenced mostly by the current cohort.

4.2 Monetary Policy

We assume that by setting interest rates, the central bank has at least an indirect impact on the return to safe assets over short time horizons. As discussed above, higher returns to safe assets can puncture a bubble. Hence, discretionary monetary policy can prevent bubbles.[17] Following a Taylor rule that considers asset price inflation, let us now analyze the impact of an automatic, pre-announced interest-rate increase in the case of a bubble. Specifically, assume a version of the rule used in Bernanke and Gertler (1999, 2001),

$$i_t = \bar{i} + \psi_\pi (\pi_t - \bar{\pi}) + \psi (p_t/p_{t-1} - \bar{\pi}), \tag{23}$$

where i_t is the nominal interest rate, π_t is the gross consumer price index (CPI) inflation, and p_t/p_{t-1} is the asset price inflation of the only risky asset in the economy as defined above. For simplicity, we neglect the influence of asset-price inflation on CPI inflation by setting CPI inflation equal to its target rate $\bar{\pi}$, which is itself set to unity.[18] Hence, in steady state the nominal interest rate $\bar{i}$ equals the real rate, which is given by the return $\bar{Y}$ of the safe asset. This assumption does not influence our conclusions below. For simplicity, we furthermore assume that returns from storage depend on the interest rate in a linear way, $Y_t = \iota\, i_t$ with $\iota > 0$. As in the above analysis, in a bubble, p_{t+1}/p_t converges towards a constant ϕ. Inserting (23) into (16) yields

$$\iota \left(\bar{Y} + \psi (\phi - 1)\right) - \beta = q\, \phi^{-\gamma} (\phi - \beta). \tag{24}$$

As for (7), we can derive a condition for parameters $\bar{Y}$, ψ, β, γ and q to determine whether (24) has a solution for $\phi > 1$. Intuitively, a positive ψ transforms the flat line $Y - \beta$ of Figure 3 into an upward sloping line; see the left side of equation (24). As shown in the figure, for a sufficiently steep slope, the intercept disappears, implying that the risky asset becomes too unattractive for investments.

Figure 5 shows the values of ψ dependent on $\bar{i}$ for which bubbles can exist, fixing $\gamma = 2$, $\beta = 0.9$, $\iota = 1$, and $r = 10\%$. The figure shows that to prevent the emergence of bubbles, the central bank can threaten to raise interest rates in the future if a bubble should occur by committing to a Taylor rule with a positive ψ. For lower $\bar{i}$, a higher ψ must be implemented. In equilibrium, the central bank never actually needs to raise interest rates: interest rate increases occur only as a consequence of asset price movements, but because of the credible announcement of this policy (with a sufficiently large ψ), asset prices do not rise, and bubbles are prevented.[19]

[17] Conlon (2014) also finds that welfare can be increased by central-bank actions. His argument, however, is very different from the present model. In his setup, bubbles burst because the central bank reveals information to asymmetrically informed traders.

[18] Hence, i_t corresponds to the real interest rate. As said, we thus assume that the central bank has at least a certain impact on short-term real rates, which is standard in monetary economic theory. If inflation reacts negatively to interest rates, the value of ψ must be correspondingly higher to prevent bubbles.

[19] In this respect, the model differs from that of Bernanke and Gertler (1999, 2001), who show that monetary policy should not react to asset prices based on the assumption of *exogenous* bubbles.

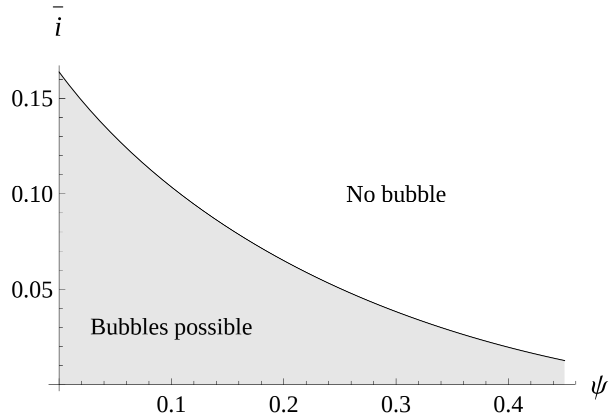

Figure 5: Effects of the Taylor Rule

Proposition 4 *Monetary policy that systematically reacts to asset price increases ($\psi > 0$) reduces the range of parameters under which bubbles are possible.*

This argument shows that compared with discretionary interest-rate policies, a Taylor-type rule could cause fewer distortions. However, if the central bank cannot differentiate between price movements due to bubbles and changes in the underlying fundamentals (such as the probability of bankruptcy, $1 - q$), or if the bank is uncertain of which assets to monitor, it faces a tradeoff between preventing bubbles and the risk of unnecessarily changing the interest rate in periods without bubbles. A thorough examination of this trade-off would require a fully specified dynamic stochastic general equilibrium model, which is beyond the scope of this paper.

4.3 Capital Requirements

Analyzing capital requirements is straightforward. For E dollars of equity, a trader can borrow D dollars. The balance sheet total is thus $D+E$, and the equity ratio is $E/(D+E)$. If the regulator stipulates stricter capital requirements, the equity ratio must increase; hence, D must decline. Because $\beta = r\,D/(D+E)$, a smaller D leads to a smaller β. However, based on Proposition 2, we know that a smaller β tends to entail a unique rational expectations equilibrium and thus no bubble. Hence, stricter capital requirements reduce the range of parameters in which bubbles are possible.

Recall that if the price path is too steep, bubbles do not exist because the potential bubble is highly likely to burst. Therefore, traders cannot be compensated for an investment in an overpriced asset by further phantasies regarding price increases. However, if traders are highly leveraged, they do not shun investment in overpriced assets and can easily be compensated. Because capital requirements decrease leverage, they can eliminate potential bubbles. Furthermore, note that a policy that reduces β also reduces the steady-state price. Hence, the policy enhances welfare in the steady state in addition to potentially eliminating bubbles.

4.4 Financial Transaction Tax

There are several possible ways to implement a so-called Tobin tax. In the following, we assume that the tax must be paid by the buyer of an asset. We call τ the tax rate on transactions of the safe asset and τ' the (potentially different) tax rate on the risky asset. Under such a tax regime, the market-clearing condition (16) changes to

$$Y - \beta - \tau = q\,\phi^{-\gamma}\,(\phi - \beta - \tau'). \tag{25}$$

The modified condition for the existence of bubbles is then

$$\gamma^{\gamma}\left(\frac{\beta + \tau'}{\gamma - 1}\right)^{\gamma-1} \leq \frac{q}{Y - \beta - \tau}, \tag{26}$$

with the additional restriction that $\beta + \tau' \geq (\gamma - 1)/\gamma$. The derivative of the left side of inequality (26) with respect to τ' is positive, i. e., increasing the tax on transactions of the risky asset tends to make bubbles impossible. The latter inequality, however, goes in the opposite direction. Hence, a situation could emerge in which small increases in τ' make bubbles possible, while further increases prevent them. Additionally, the way in which the tax is implemented is crucial. If the tax is levied on all financial assets, including the safe one, τ equals τ', and the derivative of the right side of (26) with respect to the common tax rate is larger than the derivative of the left side. Hence, in such a case, the possibility of bubbles can actually be created by the Tobin tax. Furthermore, modifying equation (3) similarly to equation (25) shows that increasing τ raises the steady-state price, whereas a higher τ' reduces it. For identical $\tau = \tau'$, a higher tax rate also increases the steady-state price. Hence, the impact of bubble-eliminating measures are reinforced. The revenues from the Tobin tax are welfare neutral if they are redistributed to the households.

Proposition 5 *If a large enough financial transaction tax is levied on the risky asset only, the range of parameters in which bubbles are possible is reduced.*

4.5 Policy Measures for Intermediated Investment

In the following explanation, we evaluate two more frequently proposed policy measures: caps on bonuses and long-term compensation. For this purpose, we interpret the above payoff scheme as a contract in an intermediated-investment setup. Assume that households do not have direct access to investment but must employ institutional traders. These traders are compensated according to (1).[20] The parameter β is then the hurdle rate the trader must outperform to obtain a bonus.

[20] There are several ways to endogenize such a contract. For example, a model with costly state verification, moral hazard or risk aversion on the part of households would lead to this type of compensation scheme. In the latter case, the parameters α and β and the shape of the contract are the solution to an optimal risk-sharing problem.

Caps on Bonuses. After the financial crisis, the amount of bonus payments has been heavily debated. In this context, we wish to analyze whether bubbles can be prevented by decreasing bonus payments. The bonus payment to a trader is $B_t = \alpha\left(\phi_t + d/p_t - \beta\right)$ if the underlying asset continues to pay off (probability q) and if the bubble does not burst (probability $1 - Q$). If a bubble is absent, this bonus payment is a constant. Let us first ask whether a potential cap on this bonus would bind early or late in the life of a bubble. In terms of the bonus payment, ϕ_t increases over time, but d/p_t decreases. In sum, due to (15), we have

$$B_t = \alpha\left(\phi_t + d/p_t - \beta\right) = \alpha\, \phi_t^{\gamma}\, (Y - \beta)/q.$$

Hence, bonuses increase over time in a bubble, and caps on bonus payments become binding in later stages of a bubble. As a consequence, we can concentrate on large prices p_t so that ϕ_t approaches a constant and the maximum bonus is

$$B = \alpha\left(\phi - \beta\right) = \alpha\, \phi^{\gamma}\, (Y - \beta)/q.$$

Now, assume that the regulator places restrictions on the bonus. There are several ways in which the regulation can be implemented. First, the compensation scheme could be adjusted such that bonuses are uniformly lower, for example by reducing α or increasing β. However, according to Proposition 1, α does not have an effect on the existence of bubbles, and an increase in β would favor the emergence of bubbles. Hence, this policy could backfire and render bubbles possible.

Second, one could place a cap $\bar{B}$ on bonuses. The compensation scheme accordingly adjusts to $\min\{\max\{\alpha\left((p_{t+1} + d)/p_t - \beta\right);\, 0\};\, \bar{B}\}$. The bubble then bursts with certainty at a specific point if $\alpha\left(\phi - \beta\right) > \bar{B}$ and thus if $\phi > \bar{B}/\alpha + \beta$. Economically, from a certain point on, traders' bonuses cannot rise further to compensate them for the continually increasing risk of a bursting bubble. A backward induction argument then shows that the bubble cannot exist in the first place. Consequently, for a given compensation scheme with parameters α and β, a cap on bonus payments $\bar{B}$ renders a bubble unfeasible if $\bar{B}/\alpha + \beta < \phi$, with ϕ implicitly defined by (16).

Summing up, to prohibit the emergence of a bubble by decreasing bonuses, raising β is counterproductive, reducing α is irrelevant, and placing a cap $\bar{B}$ on bonuses is effective if the cap is low enough.

The model can indicate which types of assets might require a cap on bonus payments and for which assets the cap must be lower. First, relatively safe assets (high q) tend to develop bubbles, and the price increase ϕ is especially low for safe assets, which implies that traders in markets with relatively safe assets (e. g., mortgages, bond markets) should have a ceiling in their bonus contracts that should be relatively low. Second, bubbles emerge especially for high hurdle rates β, and the limit price increase ϕ is lower for large β. Because $\bar{B} < \alpha\,(\phi - \beta)$, the minimal effective cap $\bar{B}$ depends negatively on β. With a higher benchmark, the cap must be stricter.

Long-term Compensation. In recent political discussions, it has often been argued that traders' incentives should concentrate more on long-term goals and should avoid short-termism. The same argument might apply to the traders in our model. To analyze this question, let us assume that the trader receives $\max\{0;\, \alpha\,(R-\beta)\}$ as before but that she is liable with her compensation for potential future losses. Hence, she receives nothing if the accumulated yield is negative next period. In a steady state, the market price is

$$\alpha\,(Y-\beta) = q^2\,\alpha\,\big((p_t+d)/p_t-\beta\big),$$
$$p_t = \overline{p} := \frac{d\,q^2}{(1-\beta)\,(1-q^2)+Y-1}, \tag{27}$$

i. e., smaller than without long-term liability. If a bubble exists, the probability that the bubble does not burst after two periods is

$$Q = q^2\,p_t^\gamma/p_{t+2}^\gamma = q^2/\phi^{2\,\gamma}.$$

As a consequence, the one-period price increase ϕ is determined by

$$\alpha\,(Y-\beta) = Q\,\alpha\,\big(\phi-\beta\big) = q^2/\phi^{2\,\gamma}\,\alpha\,\big(\phi-\beta\big),$$
$$\phi^{2\,\gamma}\,(Y-\beta) = q^2\,\big(\phi-\beta\big).$$

The equation is similar to (16), but γ is replaced by $2\,\gamma$, and q is substituted by q^2. Because bubbles exist especially for small γ and large q, according to Proposition 2, we find that long-term liability prevents the formation of bubbles. For a longer liability period, the effect would be even larger. This argument concentrates on trinomial bubbles but can be generalized to bubbles in general. If traders are liable for future developments with their bonuses, the range of parameters in which bubbles are possible is reduced. In short, there are fewer bubbles.

This measure includes a side effect. Replacing q by q^2 in equation (4) shows that the steady-state price in (27) rises, which is also visible in Table 3. As this condition implies that the risky asset's overvaluation increases, and less resources are devoted to the risk-free asset, welfare is reduced in the steady state. Intuitively, the long-term compensation magnifies the negative effects of limited liability. The occurrence of bubbles depends on expectations for which probabilities cannot be calculated. Thus, we cannot determine whether the overall welfare effect of preventing bubbles and simultaneously raising the steady-state price is positive or negative.

5 Conclusion

Our model endogenizes a specific reason why the price of an asset may deviate from its fundamental value. If the market depth is unknown, a fund manager may be willing to spend more than the fundamental value on an asset because she expects to earn even more when she sells the asset. This price deviation can occur with unchanged fundamentals, is completely driven by expectations and is dynamic, typically involving large, unpredictable abnormal returns until the bubble bursts. Furthermore, leveraged fund managers may increase the price of risky assets. The price deviation is not driven by expectations and is constant over time; the deviation involves no dynamics if all exogenous variables are constant. Combined with imprecisely known market depth, however, leverage significantly enlarges the parameter space in which bubbles are possible.

These two mechanisms are consistent with anecdotal evidence. During the dot-com bubble (1998–2001), fantasies about the potential of internet firms were exuberant. The asset prices of these firms might have been even more exaggerated due to the traders' leverage, even absent a dynamic bubble. Hence, leverage allows the exuberance to appear magnified. When expectations became more realistic, asset prices collapsed because the correction of expectations was again magnified. This complete argument follows from the static price deviation due to leverage. According to our model, the story applies especially to risky assets, such as the stock of dot-com firms.

Following the "as long as the music is playing, you've got to get up and dance" explanation for the recent U. S. housing bubble, managers bought securities because they assumed they could sell them at a higher price later, which increased prices. This argument follows from the unknown market depth. According to our model, the story applies especially to fundamentally safe assets, such as real estate.

Our model suggests several ways to avoid such bubbles. By virtue of its relative simplicity, the model lends itself to further discussion. For example, one could consider multiple assets and discuss whether the collapse of a bubble in one market can be contagious for other markets. One could also apply bubbles to macro models and investigate business cycle and growth effects. Especially after the recent burst of the housing bubble, applications seem both numerous and relevant.

A Appendix: The Participation Constraint of Households

In this appendix, we investigate the circumstances in which households are willing to lend to traders.[21] The answer depends on the households' opportunities. We have assumed that the risk-free return is Y for traders but only $\lambda Y < Y$ for households. If there were no risky assets, households would always lend. However, in the presence of risky assets, there are several reasons why households might not lend to traders.

First, even in the absence of a bubble, households anticipate that traders might invest in an overvalued asset. The probability depends on the price and thus on the market capitalization of the risky asset. Willingness to lend also depends on households' expectations regarding market depth. For a given lower bound of N, more uncertainty about N implies a higher expected N. The market capitalization is then smaller compared to total wealth,

[21] For $\gamma < 1$, bubbles can emerge even with unleveraged traders. Household lending is thus crucial only for $\gamma \geq 1$, we therefore concentrate on that case in this section.

and the probability that a trader invests in the asset is lower. We derive a condition under which households lend to traders.

Second, within a bubble, households are even more reluctant to lend because they suffer from a bursting bubble. This condition implies that households might be willing to lend in the absence of a bubble but not in the presence of a bubble. As the bubble evolves, the probability of a burst increases; hence, a fortiori, households become more reluctant to invest. If households stop lending completely at some future date, then the bubble bursts because no limited liability is present in this case. By backward induction, the bubble then cannot emerge in the first place. We can show that given all other parameters, a $\bar{\lambda} > 0$ exists such that for any $\lambda \leq \bar{\lambda}$, the participation constraint of households does not constrain any bubble equilibrium. Furthermore, we derive $\bar{\lambda}$ for trinomial bubbles.

To maintain the problem's tractability, we impose two additional assumptions. First, traders and households know that there is enough money in the market to support the current price, but they do not know more. Hence, at a current price p, the minimum N_0 satisfies $p = N_0\,(D+E)$. Second, there is only debt finance, and households possess complete bargaining power. Consequently, traders must be indifferent between borrowing D and investing $D+E$ and simply investing E. Hence,

$$Y\,E = Y\,(D+E) - r\,D \quad\Longrightarrow\quad r = Y \quad\Longrightarrow\quad \beta = Y\,\frac{D}{D+E}, \tag{28}$$

as in (1). Because $D/(D+E)$ is a trader's leverage, β is simply the risk-free gross return times leverage. We begin by discussing trinomial bubbles, always assuming that $\log F(N)$ is exponentially distributed.

The Steady State. The asset price in a steady state is provided by (4), $\overline{p} = d\,q/\big(Y - q - \beta\,(1-q)\big)$. A household investing D can always obtain $\lambda\,Y D$ from storage. When the household delegates investment to a trader, it does not know whether the trader uses the money to buy a risky asset. An aggregate amount of $\overline{p}$ is invested into the risky asset, so the remaining $N\,(D+E) - \overline{p}$ is invested safely. The probability of a risky investment is thus $\overline{p}/(N\,(D+E))$. In this case, the investor is repaid $r\,D = YD$ only with probability q. If the trader stores the capital and earns a gross return Y, the household is repaid with certainty. The expected return to a household in steady state is thus

$$\int_{\overline{p}/(D+E)}^{\infty} \Big(\frac{\overline{p}}{N\,(D+E)}\,q\,YD + \frac{N\,(D+E)-\overline{p}}{N\,(D+E)}\,YD\Big)\,f(N)\,\mathrm{d}N.$$

This value must at least match $\lambda\,YD$. The density $f(N)$ depends on the available information about N. Because $N \geq N_0 = \overline{p}/(D+E)$, the conditional distribution is $F(N) = 1 - (N\,(D+E)/\overline{p})^{-\gamma}$. The expected return then becomes

$$\frac{1+q\,\gamma}{1+\gamma}\,YD \geq \lambda\,YD \qquad\Longrightarrow\qquad \lambda \leq \frac{1+q\,\gamma}{1+\gamma}. \tag{29}$$

Otherwise, the market breaks down. From the households' point of view, the risk of traders investing their money in the overpriced risky asset is too high in comparison to storage. Condition (29) is depicted as a gray grid in Figure 6 (page 30).

For λ below the gray grid, households will invest. For λ above, households will not invest. The condition does not depend on traders' equity ratio $E/(D+E)$ (upper picture). The riskier the asset (low q), the more careful are households. The higher Y, the more willing are households to participate (lower picture). The impact of γ is barely perceptible. For higher γ, households are less willing to participate because for a given N_0 a high γ implies an expected smaller market depth, which makes investment in the risky asset rather likely.

The Example Bubble. Consider the case of a bubble with a trinomial price path and $\log N$ exponentially distributed. The current price is p_t, and the next period's price will be $p_{t+1} = \phi_t \, p_t$. At this point, the household faces a twofold risk. The trader might invest into the risky asset, and if she does, the bubble might burst. Actually, the bubble bursts if $N\,(D+E) < \phi_t \, p_t$. In this case, the household receives back $\overline{p}/p_t$ times its investment D. With $r = Y$, we can thus calculate the household's expected return as

$$\begin{aligned} g(\phi_t) &:= \int_{p_t/(D+E)}^{\phi_t\,p_t/(D+E)} \Big(\frac{p_t}{N\,(D+E)}\, q \frac{\overline{p}}{p_t}\, D + \frac{N\,(D+E) - p_t}{N\,(D+E)}\, Y D \Big)\, f(N)\, \mathrm{d}N \\ &\quad + \int_{\phi_t\,p_t/(D+E)}^{\infty} \Big(\frac{p_t}{N\,(D+E)}\, q\, Y D + \frac{N\,(D+E) - p_t}{N\,(D+E)}\, Y D \Big)\, f(N)\, \mathrm{d}N \qquad (30) \\ &= \frac{D}{\gamma+1} \left(\gamma\, q\, \frac{\overline{p}}{p_t} \left(1 - \frac{1}{\phi_t^{\gamma+1}} \right) + Y \left(1 + \frac{\gamma\, q}{\phi_t^{\gamma+1}} \right) \right). \qquad (31) \end{aligned}$$

This term must exceed $\lambda Y D$; hence, we can calculate a critical $\bar{\lambda}$. If λ is above this critical point, then households are better off with the opportunity to buy the safe asset themselves, and they do not participate. The critical $\bar{\lambda}$ depends on p_t and ϕ_t. As the bubble evolves, both p_t and ϕ_t increase. Both effects reduce households' expected return (31). Consequently, as the bubble evolves, $\bar{\lambda}$ decreases, and households might become unwilling to lend at a certain stage. The expected negative value of potential investment into the bubbly asset can then no longer be compensated with possible interest from investment in storage. The bubble then bursts with certainty. However, this event can be anticipated. Hence, the bubble cannot emerge in the first place; the price path is then unique. To derive a condition, we must verify whether households are willing to invest even at arbitrarily high p_t. The first addend in the brackets in (31) vanishes, and we can set ϕ_t to the limit ϕ. The expected return (31) becomes

$$\frac{D}{\gamma+1} \left(1 + \frac{\gamma\, q}{\phi^{1+\gamma}} \right) Y. \qquad (32)$$

This value must exceed $\lambda Y D$. Solving for λ, a bubble can thus evolve only if

$$\lambda \leq \frac{1 + \gamma\, q/\phi^{\gamma+1}}{1+\gamma}, \qquad (33)$$

with ϕ defined by (16), $Y - \beta = q\,\phi^{-\gamma}\,(\phi - \beta)$. This condition is depicted as the colored surface in Figure 6. First, note that if (7) fails to hold, then $\phi = 1$, and bubbles are impossible. In the figure, this effect occurs for high E because traders wish to invest in a bubble only when they are highly leveraged and for low q because traders are deterred by high fundamental risk. When (7) holds and a $\phi > 1$ is defined, households still participate only if their opportunity investment is not profitable enough. In the figure, λ must be below the colored surface. Hence, for parameters *below* the gray grid, households are willing to participate in the absence of a bubble. For parameters below the colored surface, households are willing to participate even if there is a bubble. Between the gray grid and the above colored surface, households would be willing to participate in the initial stage of a bubble. However, as the bubble becomes livelier over time, households' participation constraint is violated. This condition is anticipated right away, so bubbles cannot emerge. In regions without any colored surface, bubbles never emerge due to the arguments in Section 3.1, independent of the households' participation constraint.

We now have two conditions. If (7) fails to hold, traders do not invest in bubbles. If (33) fails to hold, households do not lend to traders. How does this new condition (33) depend on exogenous parameters? The two pictures in Figure 6 show one general property. Towards the parameter constellation in which bubbles do not exist at all, the colored surface slopes down because households dislike lending when a bubble is extremely risky, close to overheating. However, overheating (with certainty) also explains why no bubble might exist in the first place. Hence, an exogenous parameter that tends to invalidate condition (7) also tends to invalidate (33). In particular, households are less willing to lend in a bubble when the underlying asset is risky (low q), when the trader's capital ratio $E/(D+E)$ is high, when uncertainty about the market depth N is precise (high γ) or if the opportunity yield Y is high. Two of these statements deserve additional explanation. *First*, why is a household reluctant to lend to a trader with high capital ratio? If the capital ratio is sufficiently high, bubbles cannot exist because traders themselves refrain from investing in a bubbly asset. However, if the capital ratio is slightly below this parameter range, traders are already reluctant to invest; hence, if a bubble exists nevertheless, its expected price path must be steep. This condition compensates traders for investing in the bubbly asset but worsens the condition of households. *Second*, why is the effect of Y negative? The effect of a higher Y is detrimental for households because with a more profitable safe investment, the bubble must grow faster to achieve market clearing. It is thus more likely to burst.

Bubbles in General. We can show analogously to the main part of the paper that condition (33), stemming from the households' participation constraint, holds not only for our example bubbles with a trinomial price process and $\log N$ exponentially distributed but also for bubbles in general.

Theorem 2a *Define γ as in equation (6) of Theorem 1a. If $\gamma < 1$ and/or*

$$\lambda < \frac{1 + \gamma\, q/\phi^{\gamma+1}}{1+\gamma}, \tag{34}$$

with ϕ defined implicitly by $Y - \beta = q\,\phi^{-\gamma}\,(\phi - \beta)$, the households' participation constraint does not restrict the existence of bubbles. If this condition does not hold, bubbles are impossible because households do not lend to traders.

As in Theorem 1a, the statistic γ need not exist. Again, using the supremum for a sufficient condition and the infimum for a necessary condition, we can prove a more general version of Theorem 2a.

Theorem 2b *Define $\overline{\gamma}$ and $\underline{\gamma}$ as in equation (8) of Theorem 1b. If $\overline{\gamma} < 1$ and/or*

$$\lambda < \frac{1 + \overline{\gamma}\,q/\overline{\phi}^{\overline{\gamma}+1}}{1 + \overline{\gamma}}, \tag{35}$$

where $\overline{\phi}$ is implicitly defined by $Y - \beta = q\,\overline{\phi}^{-\gamma}\,(\overline{\phi} - \beta)$, the households' participation constraint does not restrict the existence of bubbles. If $\underline{\gamma} \geq 1$ and

$$\lambda > \frac{1 + \underline{\gamma}\,q/\underline{\phi}^{\underline{\gamma}+1}}{1 + \underline{\gamma}}, \tag{36}$$

with $\underline{\phi}$ implicitly defined by $Y - \beta = q\,\underline{\phi}^{-\gamma}\,(\underline{\phi} - \beta)$, bubbles are impossible because households do not lend to traders.

In sum, the households' participation constraint does limit the possible existence of bubbles. If $\gamma < 1$, bubbles can also exist without limited liability. In this case, traders would invest their own money, and the households' participation constraint is not even needed. For $\gamma \geq 1$, there is a critical value $\bar{\lambda}$ such that for smaller λ households participate even in fully-grown bubbles. For larger λ, households would not participate; hence, bubbles cannot emerge. For trinomial bubbles, a necessary and sufficient condition is (33). This condition is pictured in Figure 6; the comparative statics are provided in the following proposition and summarized in the right column of Table 3 (page 10).

Proposition 6 *In a bubble, households tend to employ traders for high investment advantages of traders (low λ), low risk-free returns of traders Y, low fundamental risk (large q), large uncertainty about market depth (low γ), and high leverage (high β).*

Note that in addition to the added effect of λ, the effects of variations in the parameters of the model provide the same conclusions as in Proposition 2. We can therefore conclude the following.

Proposition 7 *Parameter changes that increase households' incentives to lend to traders also make bubbles more likely. In particular, policy measures that tend to make bubbles impossible according to Section 4 also tend to discourage households from lending to traders.*

Because the same parameter constellations that induce households to lend to traders make bubbles possible, maximizing individual household payoffs by lending to traders increases the probability of bubbles via increased leverage. In turn, bubbles reduce the expected payoff of households in equilibrium relative to the no-bubble case.

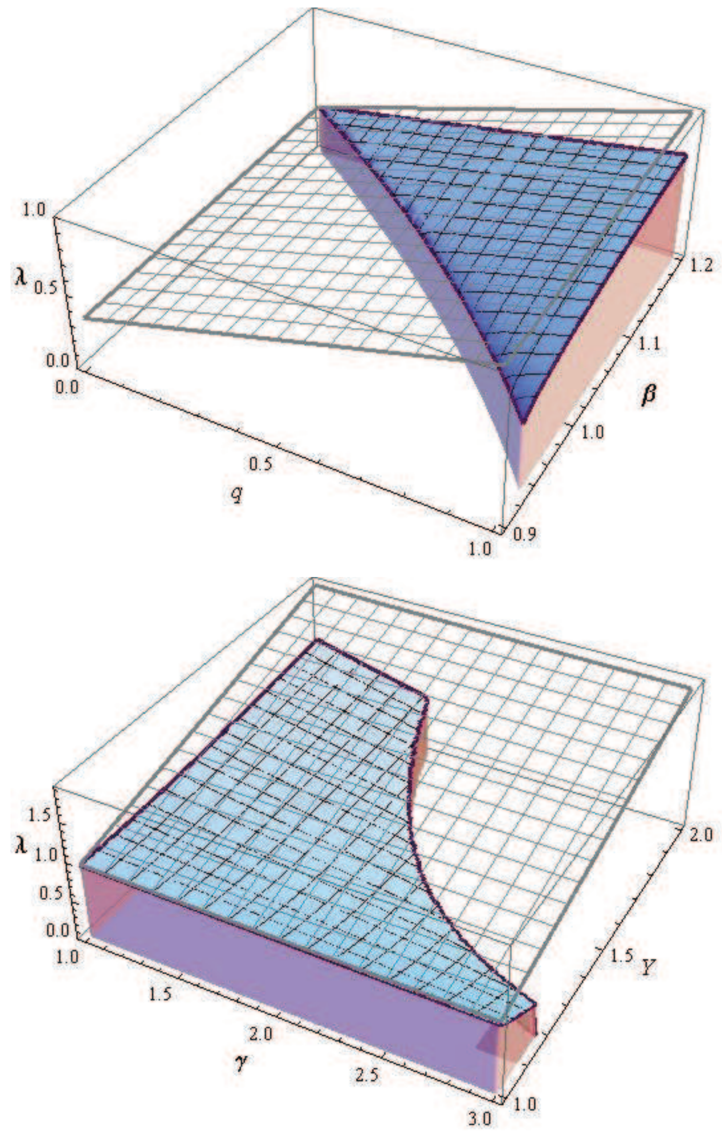

Figure 6: Parameter Range in which Bubbles are Possible

In the first picture, $\gamma = 2$, and $Y = 1.2$, whereas q and β vary. In the second picture, $q = 0.9$, and $E/(D + E) = 0.1$, and γ and Y vary. For parameters *below* the gray grid, households are willing to participate in the absence of a bubble. For parameters below the colored surface, households are willing to participate even if there is a bubble. The first picture is comparable to Figure 2. Figure 2, read at $Y = 1.2$, shows the range of q and β in which traders would invest in a bubble. The surfaces in Figure 6 exists only in this region. The surfaces then show how small λ must be such that households will invest in the presence of a bubble.

B Proofs

Proof of Proposition 1. The proof consists of taking derivatives. ■

Proof of Theorems 1a and 1b. The proof of Theorem 1a consists of the following Lemma 1 and 2. The proof of Theorem 1b follows.

Proof of Lemma 1. In the exposition in the main text, we have treated only the case in which traders are not paid if a bubble bursts. Hence, we begin the proof of the lemma by providing a condition for this case and analyzing the alternative. If a bubble bursts without the firm going bankrupt, the firm still pays the dividend. The payment to the trader is then

$$\alpha \max\left\{\frac{d}{p_t} + \frac{\overline{p}}{p_t} - \beta; 0\right\} = \alpha \max\left\{\frac{d + \frac{d\,q}{Y - q - \beta\,(1-q)}}{p_t} - \beta; 0\right\}. \tag{37}$$

This equation implies that if the price is only slightly above the steady-state price $\overline{p}$ (i. e., the bubble is small), the trader receives a payment even when the bubble bursts. The corresponding condition is

$$p_t < \check{p} := \Big(d + \frac{d\,q}{Y - q - \beta\,(1-q)}\Big)/\beta. \tag{38}$$

Now, if p_t is less than $\check{p}$ such that (38) is satisfied, a modified version of (14) applies. In market equilibrium,

$$\begin{aligned} \alpha\,(Y-\beta) &= Q_t\,\alpha\,\big((p_{t+1}+d)/p_t - \beta\big) + (q - Q_t)\,\alpha\,\big((\overline{p}+d)/p_t - \beta\big) \\ \iff \quad \frac{Y-\beta}{q} &= \Big(\frac{p_t}{p_{t+1}}\Big)^{\gamma}\frac{p_{t+1}}{p_t} + \Big(1 - \Big(\frac{p_t}{p_{t+1}}\Big)^{\gamma}\Big)\frac{\overline{p}}{p_t} + \frac{d}{p_t} - \beta. \end{aligned} \tag{39}$$

Again, beginning from p_t, we have an implicit equation for p_{t+1} in a rational expectations equilibrium. Substituting $p_{t+1} = \phi_t\,p_t$, we obtain

$$\phi_t^{\gamma}\,\frac{Y-\beta}{q} = \phi_t + (\phi_t^{\gamma} - 1)\,\frac{\overline{p}}{p_t} + \phi_t^{\gamma}\Big(\frac{d}{p_t} - \beta\Big). \tag{40}$$

However, in a bubble, the price p_t increases over time and eventually exceeds the threshold $\check{p}$. Therefore, to determine whether bubbles are feasible, it suffices to consider the case $p_t > \check{p}$ as done in the main text.

We have already argued that the probability that a bubble bursts increases with p_t. Because p_t is an increasing function of t, a bubble is sustainable if and only if it is sustainable for $p_t \to \infty$. Hence, if (16) has a solution for ϕ, the bubble is sustainable. Consider the limiting case in which the curve $q\,\phi^{-\gamma}\,(\phi - \beta)$ and the line $Y - \beta$ only touch. At the point of contact, the slopes must be equal,

$$\phi^{-\gamma-1}\,q\,(\gamma\,\beta + (1-\gamma)\,\phi) = 0, \tag{41}$$

which implies that the point of contact is $\phi = \beta\,\gamma/(\gamma-1)$. Substituting this solution in (16), we find that the limiting case is obtained at

$$Y - \beta = q\left(\frac{\beta\,\gamma}{\gamma-1}\right)^{-\gamma}\left(\frac{\beta\,\gamma}{\gamma-1} - \beta\right). \tag{42}$$

Some algebra yields (7), the condition under which (16) has a solution. In a bubble, prices must increase; otherwise, the algebraic term for the probability of a burst would become negative. Formally, ϕ must exceed 1 at the curve's highest point. Given that $\lim_{\phi\to\infty} q\,\phi^{-\gamma}\,(\phi-\beta) = 0$ and $q\,(1-\beta) < Y - \beta$, it is sufficient to show that $\phi > 1$ at this point. This is the case for $\beta > \frac{\gamma-1}{\gamma}$. Note that because $\beta \geq 0$, this condition is always fulfilled for $\gamma < 1$. Furthermore, inequality (7) is not a necessary condition in this case, as the curve $q\,\phi^{-\gamma}\,(\phi-\beta)$ increases monotonically, starting from a value lower than $Y-\beta$. Hence, there always exists an intersection, which implies that bubbles are always possible for $\gamma < 1$. Finally, for $\gamma = \infty$, the condition $\beta > (\gamma-1)/\gamma$ becomes $\beta > 1$. However, in (7), the term $\gamma^{\gamma}\left(\frac{\beta}{\gamma-1}\right)^{\gamma-1}$ converges to ∞ for $\beta > 1$, which implies that (7) cannot be met; bubbles are impossible for $\gamma = \infty$. ■

Proof of Lemma 2. Assume that a price process exhibits a bubble, i.e., $p_t > \overline{p}$ at a date t, and that $\tilde{p}_{t+1}$ is distributed with distribution $\bar{F}(\tilde{p}_{t+1})$. Then, in a rational expectations equilibrium,

$$\begin{aligned}
\alpha\,(Y-\beta) &= \int_0^\infty q\frac{\tilde{p}_t^{\gamma}}{\tilde{p}_{t+1}^{\gamma}}\,\alpha\,\max\left\{\frac{\tilde{p}_{t+1}+d}{p_t} - \beta; 0\right\}\mathrm{d}\bar{F}(\tilde{p}_{t+1}),\\
\frac{Y-\beta}{q} &= \int_0^\infty h(\tilde{p}_{t+1})\,\mathrm{d}\bar{F}(\tilde{p}_{t+1}), \text{ where}\\
h(\tilde{p}_{t+1}) &:= \max\left\{\frac{p_t^{\gamma}}{\tilde{p}_{t+1}^{\gamma}}\left(\frac{\tilde{p}_{t+1}+d}{p_t} - \beta\right); 0\right\}
\end{aligned} \tag{43}$$

is an auxiliary function. The p_{t+1} implicitly defined by (14) solves this equation for a distribution that has probability mass only at one point p_{t+1} (and zero and $\overline{p}$). From this three-point distribution, can we shift probability mass to other prices such that the above (43) still holds? The answer depends on the shape of $h(\tilde{p}_{t+1})$. Some straightforward analysis shows that $h(\tilde{p}_{t+1})$ is zero up to $\tilde{p}_{t+1} = \beta\,p_t - d$, then increases and decreases again. For $\tilde{p}_{t+1} \to \infty$, the value approaches zero. The maximum of the integral is reached if all probability mass is located at

$$\tilde{p}^*_{t+1} = \gamma\,\frac{\beta\,p_t - d}{\gamma - 1} > \beta\,p_t - d.$$

Hence, a trinomial process with the possible events p^*_{t+1}, $\overline{p}$, and 0 maximizes the right side of (43). Shifting probability mass to other parts of $h(\tilde{p}_{t+1})$ reduces the value of the integral. Note that no bubble can emerge if the right side of (43) is lower than the left side for any price path. We can therefore conclude that if no trinomial bubble process exists, no other bubble process can exist either. However, if a trinomial bubble process exists, it exemplifies a general bubble process. As a consequence, (7) is the general condition for the existence of bubble processes in a rational expectations equilibrium. ■

Proof of Theorem 1b. Based on the definition of $\underline{\gamma}$, for all $\varepsilon > 0$, there exists an $\underline{N}$ such that for all $N > \underline{N}$,

$$N \frac{f(N)}{1 - F(N)} < \underline{\gamma} + \varepsilon.$$

Now, choose ε sufficiently small such that (10) holds even after substituting $\underline{\gamma}$ by $\underline{\gamma}+\varepsilon$. This is possible because the inequality is strict and continuous in $\underline{\gamma}$. Now, if the distribution of $\log N$ were exponential with parameter $\underline{\gamma} + \varepsilon$, there would be multiple price processes (bubbles). However, for the actual distribution $F(N)$, the probability of a bubble burst is even smaller. Consequently, if (10) holds, bubbles are possible. The argument for the second statement is analogous. ■

Proof of Proposition 2. There are two conditions that might become increasingly strict or lax. First, condition (7) is satisfied if and only if

$$(Y - \beta)\, \gamma^{\gamma} \left(\frac{\beta}{\gamma - 1}\right)^{\gamma - 1} - q \leq 0. \tag{44}$$

The derivative of this term with respect to q is negative, so the condition is more likely to be satisfied for large q. The derivative with respect to Y is positive; hence, (7) holds for small Y. The derivative with respect to γ is

$$(Y - \beta)\, \gamma^{\gamma} \left(\frac{\beta}{\gamma - 1}\right)^{\gamma - 1} \log \frac{\beta\, \gamma}{\gamma - 1}.$$

Recall that the point at which the curves touch is $\phi = \beta\, \gamma/(\gamma - 1) > 1$. The above logarithm is therefore positive, and the complete derivative with respect to γ is positive. A larger γ makes bubbles less likely. Finally, we wish to show that an increase in β makes bubbles more likely, i. e., an increase in β reduces the left side of (44) whenever this value is negative. We first show that the left side of (44) is concave in β. It is continuous, and the derivative with respect to β is

$$\gamma^{\gamma} \left(\frac{\beta}{\gamma - 1}\right)^{\gamma - 1} \frac{Y\,(\gamma - 1) - \beta\, \gamma}{\beta},$$

which is positive for $\beta < Y\,(\gamma - 1)/\gamma$ and negative for $\beta > Y\,(\gamma - 1)/\gamma$. It thus creates concavity. We must consider the region in which (44) holds. In the decreasing region (large β), the proof is complete. In the increasing region (small β), remember that $\phi > 1$ must hold; hence, $\beta > (\gamma - 1)/\gamma$. At $\beta = (\gamma - 1)/\gamma$, the left side of (44) becomes $1 - q + \gamma\,(Y - 1) > 0$. Hence, an increase in β always reduces the left side of (44) if bubbles are possible, which completes the argument. The second condition, $\beta > \frac{\gamma - 1}{\gamma}$, has the same comparative statics. ■

Proof of Proposition 3: We need to examine the expected consumption in a bubble. Cohort 0 obtains $p_0 > \overline{p}$ from selling the asset. Cohort 1 buys the asset at price p_0 but expects the price to rise to p_1 with probability Q_0, to fall to $\overline{p}$ with probability $q - Q_0$, and to fall to 0 with probability $1 - q$. Hence,

$$E_0[C_1'] = Q_0\,(d + p_1) + (q - Q_0)\,\overline{p} + (N\,(D + E) - p_0)\,Y, \tag{45}$$

and so forth. Now consider the welfare differences

$$\begin{aligned} C_0' - C_0 &= p_0 - \overline{p}, \\ E_0[C_1' - C_1] &= Q_0\,(p_1 - \overline{p}) - Y\,(p_0 - \overline{p}), \\ E_0[C_2' - C_2] &= Q_1\,Q_0\,(p_2 - \overline{p}) - Q_0\,Y\,(p_1 - \overline{p}), \end{aligned} \tag{46}$$

and so forth. Hence, the aggregate welfare difference amounts to

$$\begin{aligned} E_0[\Delta W] &= (p_0 - \overline{p}) + \sum_{t=1}^{\infty} \rho^t \prod_{t'=0}^{t-2} Q_{t'} \Big(Q_{t-1}\,(p_t - \overline{p}) - Y\,\big(p_{t-1} - \overline{p}\big) \Big) \\ &= \sum_{t=0}^{\infty} \rho^t\,(p_t - \overline{p})\,\big(1 - \rho\,Y\big) \prod_{t'=0}^{t-1} Q_{t'}, \end{aligned} \tag{47}$$

which is negative if $\rho\,Y \geq 1$. ■

Proof of Proposition 4. The proposition becomes obvious by a geometric argument. Bubbles are possible if (24) has a solution in ϕ. The right side of (24) resembles the thick curve in Figure 4. For $\psi = 0$, the left side is identical to the horizontal line. An increase in ψ leaves the axis intercept constant but increases the slope and is equivalent to a counterclockwise rotation. Clearly, for a certain ψ, the line will barely touch the curve. For even larger ψ, bubbles cease to exist. The critical ψ can be calculated algebraically, but the equation yields no further insights. ■

Proof of Proposition 5. Consider a very general tax system, and denote a tax on transactions of the safe asset by τ, a tax on selling the risky asset by τ'', and a tax on buying the risky asset by τ'. Hence, $\tau'' = 0$ means that the buyer of an asset must pay the tax (the case analyzed in the main text), whereas τ' equals zero if the seller pays the tax. The equilibrium condition (16) is then

$$Y - \beta - \tau = q\,\phi^{-\gamma}\,\big(\phi\,(1 - \tau'') - \beta - \tau'\big).$$

Following the steps taken in the proof of Lemma 1 yields $\phi = \gamma(\beta + \tau')/[(\gamma - 1)(1 - \tau'')]$ as the potential touching point of both sides of the above equation. The condition for $\phi > 1$ at this point becomes

$$\frac{\beta + \tau'}{1 - \tau''} > \frac{\gamma - 1}{\gamma}. \tag{48}$$

The modified condition for the existence of bubbles is then

$$\left(\frac{\gamma}{1 - \tau''}\right)^{\gamma} \left(\frac{\beta + \tau'}{\gamma - 1}\right)^{\gamma - 1} \leq \frac{q}{Y - \beta - \tau}.$$

The right side of this equation increases in τ, whereas the left side increases in τ'' and τ'. Considering equal tax rates on the risky and the safe assets, one can use equation (48) to show that the derivative of the right side with respect to the common tax rate is larger than the derivative of the left side for both cases, $\tau = \tau''$ and $\tau' = 0$, or alternatively $\tau = \tau'$ and $\tau'' = 0$. ■

Proof of Theorems 2a and 2b. For trinomial bubbles, we have already derived a necessary and sufficient participation constraint, equation (33). Let us first show that the condition remains valid for non-trinomial bubbles. It is clear that if (33) holds, the households' participation constraint does not curb the existence of multiple equilibria because the trinomial bubble is one example of an alternative price path. It remains to be demonstrated that if (33) fails to hold, households' participation constraint is violated also for any other type of bubble. Hence, we need to show that of all possible bubble paths, the trinomial bubble is the most preferred. Then, if for a certain parameter constellation, trinomial bubbles do not exist because of households' participation constraint, households are even more reluctant to invest in a non-trinomial bubble.

As in the proof of Lemma 2, the function $h(p_{t+1})$ provides the value of the future price for the trader, considering that the bubble might burst. As defined in (30), $g(p_{t+1})$ provides the return to households. The more evolved a bubble is, the more reluctant households are to invest; hence, we can concentrate on large prices p_{t+1} and consider the relative return ϕ rather than the absolute price p_{t+1}. The functions then become

$$\begin{aligned} h(\phi) &= q\,\phi^{-\gamma}\,(\phi - \beta), \qquad \text{and} \\ g(\phi) &= D/(\gamma+1)\left(1 + \gamma\, q\, \phi^{-\gamma-1}\right). \end{aligned} \tag{49}$$

These functions are plotted in the left panel of Figure 7. The lower blue curve defines the market clearing condition. In a trinomial bubble, the market clears when the blue curve intersects with $Y - \beta$ (dashed line), and traders are indifferent with respect to investing in the bubbly asset. In a general bubble, the return can assume several values with different probabilities. This equation defines a probability distribution for ϕ. For the market to clear, $\mathrm{E}_\phi[h(\phi)] = Y - \beta$ must hold. The households' expected return is then $\mathrm{E}_\phi[g(\phi)]$. We need to show that for all distributions of ϕ with strictly positive variance, the households' expected return falls short of that in the trinomial bubble in which ϕ assumes only one value. We hence need to solve

$$\max \mathrm{E}_\phi[g(\phi)] \quad \text{s.t.} \quad \mathrm{E}_\phi[h(\phi)] = Y - \beta, \tag{50}$$

where the max operator is taken over all probability distributions of ϕ.

We rescale the problem by distorting the ϕ-axis. We substitute $h(\phi) \mapsto x$; thus, $\phi \mapsto h^{-1}(x)$. This equation leads to the right panel of Figure 7. The function $h(\phi)$ has become $h(h^{-1}(x))$, the identity, and $g(\phi)$ has become $g(h^{-1}(x))$, a concave function. Problem (50) becomes

$$\max \mathrm{E}_x[g(h^{-1}(x))] \quad \text{s.t.} \quad \mathrm{E}_x[x] = Y - \beta, \tag{51}$$

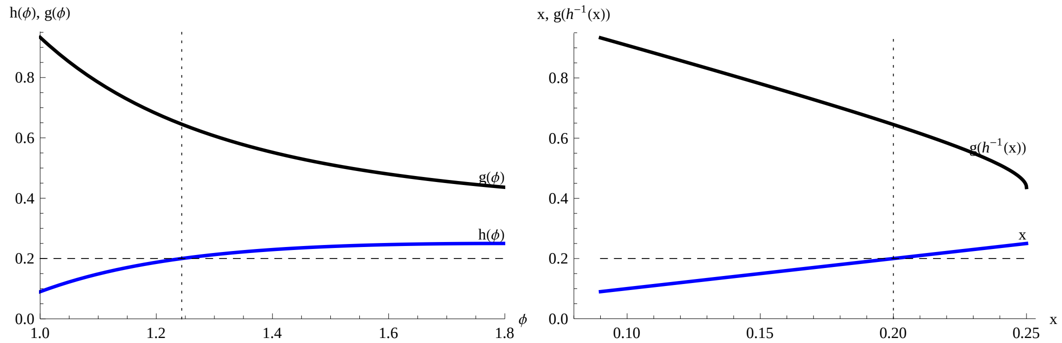

Figure 7: Market clearing condition h and households participation constraint g

Parameters are $\gamma = 2$, $\beta = 0.9$, $q = 95\%$, $d = 1$, and $Y = 1.1$.

where the max operator is taken over all probability distributions of x. In the figure, because $g(h^{-1}(x))$ is concave, a mean preserving spread deteriorates $\mathrm{E}_x[g(h^{-1}(x))]$. Problem (51) is thus solved by the degenerate one-point distribution. Hence, if we show that $g(h^{-1}(x))$ is always concave, then households prefer trinomial bubbles in which only one ϕ is possible. The implicit function theorem yields

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2} g(h^{-1}(x)) = \frac{h'(\phi)\, g''(\phi) - h''(\phi)\, g'(\phi)}{h'(\phi)^3}. \tag{52}$$

Any ϕ with positive probability mass must be in the increasing part of $h(\phi)$; hence, $h'(\phi) > 0$, and the denominator $h'(\phi)^3$ is positive. The numerator is

$$D\,\gamma\, q^2\, \phi^{-2\,(\gamma+2)} \big(\beta\,\gamma - 2\,\phi\,(\gamma-1)\big). \tag{53}$$

If the variance of N is finite, we must have $\gamma \geq 2$. This condition implies $\beta\,\gamma \leq 2\,\phi\,(\gamma - 1)$; thus, the numerator is negative. Consequently, $g(h^{-1}(x))$ is always concave, and households prefer trinomial bubbles above all other types. If households' participation constraint is violated within the class of trinomial bubbles, it is violated for any bubble.

The remainder of the proof is analogous to that of Theorem 1b. For all $\varepsilon > 0$, there is an $\underline{N} > 0$ such that for all larger N, we find functions with decay parameter $\underline{\gamma} + \varepsilon$ (or $\overline{\gamma} - \varepsilon$) that can be used as a lower (or upper) bound. In sum, if traders are willing to invest in bubbly assets according to Theorems 1a and 1b, then if (35) holds, the price path is not unique. If (36) fails to hold, the price path is unique. ■

Proof of Proposition 6. Consider condition (34). The effect of changes in λ is straightforward, but when evaluating the effects of changes in the other parameters, one must consider their effect on ϕ via the market clearing condition (16), $q\,\phi^{-\gamma}(\phi-\beta) - (Y-\beta) = 0$. Implicitly differentiating this equality shows that ϕ depends positively on Y and γ and negatively on β and q. Recall that the right side of (16) depends positively on ϕ in the relevant region (see footnote 15). The derivative of $q\,\phi^{-\gamma}(\phi-\beta) - Y + \beta$ with respect to

ϕ is hence positive. Furthermore, the right side of condition (34) depends negatively on ϕ and γ and positively on q, which completes the proof.[22] ■

Proof of Proposition 7. The proof of Proposition 6 shows that changing the parameters β, γ, q, and Y in a way that makes bubbles less likely according to Proposition 2 also tends to discourage households from lending to traders. As the policy measures in Section 4 aim directly or indirectly to change these parameters, the measures tend to make bubbles impossible because there is no bubble equilibrium (Proposition 2), and/or households stop lending to traders (Proposition 6). The only policy measure that functions differently is the financial transaction tax. The new participation constraint of the household under a tax system as discussed in the proof of Proposition 5 is $\lambda < \frac{1+\gamma q/\phi^{\gamma+1}}{1+\gamma}$, with ϕ implicitly defined by

$$\phi^{\gamma}\left(Y-\beta-\tau\right)=q\left(\phi\left(1-\tau''\right)-\beta-\tau'\right).$$

Note that we assume that the tail statistic γ exists. Otherwise, the statistics $\overline{\gamma}$ and $\underline{\gamma}$ must be used (see the proof of Theorems 2a and 2b). Implicit differentiation of the transformed market-clearing condition $q\left(\phi\left(1-\tau''\right)-\beta-\tau'\right)-\phi^{\gamma}\left(Y-\beta-\tau\right)=0$ shows that ϕ depends positively on τ' and τ'' but negatively on τ. Furthermore, setting $\tau=\tau'$ and $\tau''=0$ or alternatively $\tau=\tau''$ and $\tau'=0$ provides a negative dependence of ϕ on τ. Combining this result with the discussion in the proof of Proposition 6 shows that *a)* levying a financial transaction tax only on the risky asset ($\tau'>0$ and/or $\tau''>0$) can discourage households from lending to traders, and *b)* levying a tax on the risky and the safe asset ($\tau=\tau'$ and $\tau''=0$ or $\tau=\tau''$ and $\tau'=0$) raises incentives for households to lend to traders. ■

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[22]When calculating the effect of γ, it is helpful to insert the transformed equality $\phi^{-\gamma}=q^{-1}(Y-\beta)/(\phi-\beta)$ into (34) and to calculate the direct effect of γ and the indirect effect via ϕ on the resulting inequality. Observe that $\phi>Y$ because of the market-clearing condition.

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