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Sebastian Kraus

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New Inflationary Scenarios from D7-Brane Dynamics

Referees: Prof. Dr. rer. nat. Arthur Hebecker
Prof. Dr. rer. nat. Jörg Jäckel

Neue Inflationsszenarien mit dynamischen D7-Branes

Die vorliegende Arbeit beschäftigt sich mit der Frage, ob der Positionsmodulus einer D7-Brane in Typ IIB Stringkompaktifizierungen ein geeigneter Inflatonkandidat sein kann. Es werden zwei sehr verschiedene Szenarien analysiert: Das erste der beiden identifiziert den Abstand zweier D7-Branes mit dem Inflaton. Ein Branefluss induziert eine D-Term-Energie, welche zu Inflation führt. Die Branes nähern sich an, bis eine Tachyonkondensation die Inflationsphase beendet. Dies ist also eine stringtheoretische Variante von D-Term Hybrid Inflation. Die Rolle des Inflatons im zweiten Modell spielt der Positionsmodulus einer einzelnen D7-Brane. Während der Inflationsphase durchschreitet dieser eine große Distanz in Planck-Einheiten. Eine Monodromie im Feldraum macht dies möglich. Die Branebewegung bedingt eine sich verändernde F-Term-Energie, welche zu (chaotischer) Inflation führt. Die vorliegende Arbeit analysiert jeweils das Zusammenspiel der beiden Szenarien mit Kählermodulistabilisierung. Da das Fixieren dieser Moduli auf Termen höherer Ordnung im skalaren Potential beruht, sind die Modulimassen typischerweise relativ klein. In den vorliegenden Modellen kann dennoch gezeigt werden, dass das Inflatonpotential mit der Kählermodulistabilisierung kompatibel ist. Zuletzt erläutert die Arbeit, dass beide Modelle mit den aktuellen kosmologischen Beobachtungen in Übereinstimmung gebracht werden können, während die Implikationen für das Verhältnis der tensoriellen zu den skalaren Perturbationen in der kosmischen Mikrowellenhintergrundstrahlung sehr verschieden sind.

New Inflationary Scenarios from D7-Brane Dynamics

We analyze whether a D7-brane position modulus in Type IIB string compactifications can be a suitable inflaton candidate. To this end, we study two rather different scenarios: In the first, the inflaton is associated with the distance of two D7-branes. Inflation is driven by a brane-flux-induced D-term energy and proceeds as the branes approach each other. It ends in a tachyonic instability. This model thus represents a string-theoretic version of D-term hybrid inflation. In the second model, the inflaton is the position modulus of a single D7-brane. During inflation this modulus traverses a large distance in Planck units. This is possible due to a monodromy in field space. The brane displacement leads to a continuously changing F-term energy which drives (chaotic) inflation. We explicitly analyze the intricate interplay of each scenario with moduli stabilization. In particular, since Kähler moduli are fixed by higher-order terms in the scalar potential, their masses are typically relatively small. We demonstrate that, nonetheless, in our models the inflaton potential does not upset Kähler moduli stabilization. Finally, we show that both models can be in agreement with the most recent cosmological observations, while their implications for the size of the tensor-to-scalar ratio are very different.

Contents

1. Introduction

Understanding the laws of nature at ever increasing energies is one of the main objectives in theoretical and experimental investigations of fundamental physics. It is a path alongside which we expect to find crucial hints towards the underlying principles of matter and its interactions. Walking that path, we have learned about four fundamental forces in nature. Three of those have a common description in the framework of renormalizable quantum field theories: The standard model of particle physics very successfully describes the strong, weak, and electromagnetic interactions of particles. Including gravity in this picture leads to an effective theory which, at least naively, becomes non-perturbative at a certain high scale, the Planck scale M_p. However, the correct description of physical processes at Planck-scale energies is yet to be discovered. It seems that this theory either involves new degrees of freedom such that it is rendered perturbative, or it is of a truly non-perturbative nature, in which case new techniques need to be developed to treat this theory. In our investigations we will explore the first, in some sense more conservative, of these two possibilities.

A promising and widely discussed candidate for a fundamental theory which contains gravity and which remains perturbative even at and beyond the Planck scale is string theory [1–4]. It posits that the fundamental objects in nature are one-dimensional, with a characteristic length ℓ_s and associated mass scale $M_s = 1/\ell_s$, above which the extended nature of these objects becomes relevant. The idea is that this 'string scale' is somewhat below M_p, such that the string captures the degrees of freedom for processes which involve Planck-scale energies. Below M_s the theory (ideally) gives rise to the standard model coupled to gravity, supplemented by (non-renormalizable) interaction terms which are suppressed by some power of the energy scale involved in a given process, divided by the string scale. The latter terms quickly become irrelevant at low energies.

Given that modern collider experiments operate at energies very far below M_p, one might wonder how phenomenological a discussion about processes at the Planck scale can be. It turns out that the theory of 'primordial inflation' is in fact sensitive to Planck-scale physics: Primordial inflation [5–9] describes a period of accelerated expansion in the early universe which nicely explains some puzzles of the standard Big Bang theory, such as the horizon and flatness problems. In its simplest field-theoretic realization, the accelerated expansion is driven by the potential energy density of a scalar field, the 'inflaton' φ, whose vacuum expectation value slowly evolves in time. As gravity is only an effective theory, valid below (at most) M_p, we expect terms e.g. of the form $\sim \mathcal{O}_6[\varphi]/\Lambda^2$ to appear in the effective Lagrangian. Here, $\mathcal{O}_6[\varphi]$ is some dimension-six operator which is introduced by integrating out modes with masses at some scale Λ, which is at most M_p. Operators of this kind are thus *determined* by 'UV completions' of quantum gravity, such as string theory. Inflationary phenomenology is affected (or sometimes even governed) by such non-renormalizable interaction terms. Therefore, inflation provides a window to physics at the string and even the Planck scale. In this thesis we will assume that string theory offers the correct UV completion of quantum gravity and explore this window and its consequences for consistent string-derived models of inflation.

Even irrespective of primordial inflation, higher-dimensional operators play a crucial role in the phenomenological discussion of effective theories derived from string theory. This very fact makes it highly non-trivial to construct string-theoretic models of inflation. In the following let us give some details of how this comes about. String theory is the quantum theory of spatially one-dimensional objects which move in spacetime. It is defined in terms of a two-dimensional quantum field theory on the string 'worldsheet', i.e. on the surface which is swept out by the string as it evolves in time. The theory is highly symmetric. In fact, it is invariant under the group of superconformal transformations in two dimensions, which is infinite-dimensional. This leads to a very constrained setting. In particular, a consistent perturbative definition of the quantum theory demands ten spacetime dimensions. The spectrum of string theory contains massless states which transform in various representations of the ten-dimensional Lorentz algebra. One finds a graviton, several gauge field quanta, and scalars.

There are two expansion parameters in string theory. First, one can expand the action describing the string motion in powers of the curvature of the ten-dimensional spacetime. Second, in analogy to ordinary quantum field theory, the diagrams for string scattering can be organized in terms of a loop expansion, i.e. in powers of the 'string coupling' g_s. At leading order in these two expansions one can now compute scattering amplitudes for the lowest string excitations and match them to scattering amplitudes of (massless) states in a ten-dimensional *supergravity* theory.[1] The latter thus gives the correct *effective* ten-dimensional field theory description of string theory in the limit of low energies, i.e. neglecting the extended nature of strings.

To make contact with observations, six of the ten dimensions need to be 'compactified'. This means finding solutions to the equations of motion of the ten-dimensional theory which are such that, of the ten dimensions of the string theory target spacetime only four are large. The remaining six describe an internal compact space with a small characteristic length scale, compatible with the non-observation of extra dimensions. Having found such a solution, one can 'dimensionally reduce' the ten-dimensional theory to obtain an effective four-dimensional description. While the choice of a solution adds a huge amount of ambiguity and complication to the subject, the requirement of being left with a *supersymmetric* four-dimensional theory gives a lot of structure to the compact six-dimensional space. In the simplest cases, this space is a Calabi-Yau manifold [10].

These compactification manifolds are by no means rigid: The metric of the ten-dimensional theory is dynamical and, upon dimensional reduction, gives rise e.g. to scalar fields in the four-dimensional theory which parametrize deformations of the compact space. Deriving these four-dimensional effective (massless) degrees of freedom from the ten-dimensional theory is particularly well-studied for Calabi-Yau manifolds [11].

A common feature of these deformation modes is the fact that, naively, their potential in the four-dimensional theory is exactly flat (they are 'moduli'). The presence of such fields is observationally excluded. The flatness of the moduli potential can be partially lifted by considering so-called fluxes, i.e. non-trivial field configurations of p-form gauge potentials along the compact dimensions [12]. This is particularly well understood in four-dimensional theories arising from a specific 10d theory called 'Type IIB string theory', which is why

[1] In fact, there exist several consistent string theories in ten dimensions which thus give rise to different effective supergravity theories. They are connected by a web of dualities.

we will mainly be concerned with this class of 4d theories. However, turning on fluxes is not enough to stabilize all moduli in the tree-level effective action. In particular, so-called 'Kähler moduli' still correspond to flat directions in the effective potential. They are associated with metric fluctuations which describe, for example, deformations of the overall volume of the internal manifold. In order to stabilize these Kähler moduli one needs to consider higher-order effects (e.g. loop corrections or instantons) in the four-dimensional effective action [13–16]. The latter arise, for example, from integrating out massive string modes. As the masses of these states depend on the details of the compactification, these higher-order effects will typically involve many of the moduli, not only on the Kähler moduli [17, 18].

In string inflation scenarios one of these moduli fields will be associated with the inflaton. Consequently, the inflaton is likely to enter the higher-order terms which stabilize Kähler moduli. We have argued that precisely such inflaton-dependent higher-order terms play a pivotal role in inflationary phenomenology. As an important result, one cannot in general disentangle the quest of finding viable inflation scenarios in string theory from moduli stabilization. Rather, in any successful inflation scenario one needs a very precise idea of how the Kähler moduli obtain a mass. Otherwise, an a priori good-looking scenario might be ruined completely if one takes into account the moduli dynamics. This is the intimate link between string compactifications with moduli stabilization and inflation, on which this thesis is based.

Since there are quite a number of different scalar fields which arise in the 4d effective action of a string compactification (the deformation modes of the compact space being only a subset), many different string inflation models have been discussed in the past. For a recent review see [9]. They vary in the extent to which the issue of moduli stabilization is addressed, as well as in their predictions regarding inflationary observables. Our interest will be in string inflation scenarios in which the inflaton is associated with a deformation modulus of a so-called D7-brane [19–29]. These proposals have certain advantages which we will discuss in the further course of this thesis.

Dp-branes are hypersurfaces with p spatial dimensions which fill up the external space and wrap some $(p-3)$-dimensional submanifold in the compact space [30, 31]. They are associated with stable solutions to the equations of motion of Type II string theories. In particular, moduli stabilization in the Type IIB theory generally *requires* D-branes to be present [12]. In perturbative string theory D-branes arise as hypersurfaces to which the endpoints of open strings are confined (i.e. open strings have Dirichlet boundary conditions in the directions transverse to the D-branes). The quantum theory of open strings contains fluctuations both along the brane and perpendicular to it. While the fluctuations along the brane give rise to a gauge theory on the D-brane 'worldvolume', the perpendicular fluctuations correspond to fluctuations of the brane position. Thus, a D-brane is a dynamical object in its own right. In particular, the D-brane can move in the transverse space, its location being described by the vacuum expectation values of scalar fields in the four-dimensional theory.

A scalar field which describes the (relative) position of (two) D7-branes will be associated with the inflaton in the following (see [32] for the first proposal of 'brane inflation'). We will thus analyze the (quantum-corrected) moduli space of D7-brane positions in the context of inflation and moduli stabilization. In particular, since any four-dimensional supergravity theory is characterized by a Kähler potential and a superpotential (and a gauge kinetic function), we will study those quantities as they arise from compactifying Type IIB string theory with D7-branes to four dimensions.

One interesting aspect of our investigations is the observation that a D7-brane position modulus can realize two vastly different classes of single-field slow-roll inflation, namely large-field and small-field inflation. In the first class, the inflaton traverses a super-planckian distance during inflation, $\Delta\varphi > M_p$, while in the second class the field excursion is sub-planckian, $\Delta\varphi < M_p$. As we will detail in the course of this thesis, these two classes are very distinct regarding model building and predictions of some inflationary observables. In particular, while our realization of small-field inflation can appear rather naturally in a string compactification, the large-field version is highly 'tuned', i.e. various terms in the effective theory need to cancel each other with high precision, which is non-generic.[2] Despite the fact that such a fine-tuning is not desirable from a model building point of view, the results of the recent BICEP2 experiment seem to force us to go in this direction [33].

In order to convey the gist of our two inflation models, let us summarize the basic mechanisms which underlie the two scenarios. For a suitable choice of fluxes (to stabilize some of the moduli) and in the absence of any source of supersymmetry breaking, D7-branes don't feel any force and can be displaced arbitrarily in the compact space. A potential for the D7-brane position moduli (i.e. the inflaton potential) can be introduced in different ways: One option is to let two D7-branes attract each other. This is achieved by considering supersymmetry-breaking flux on a pair of D7-branes which is, however, chosen such that it does not individually stabilize the D7-brane positions at tree-level.[3] In this scenario the *distance* between the two branes is associated with the inflaton. The flux induces a supersymmetry-breaking energy density on the branes which drives inflation. Furthermore, it leads to a mass difference between bosonic and fermionic excitations of the open strings whose endpoints lie on the two branes, such that the typical SUSY cancellation of bosonic and fermionic loops in the computation of the effective potential no longer takes place. Thus, being non-vanishing, these loop corrections induce an attractive potential for the D7-brane pair, i.e. an inflaton potential. As soon as the branes approach a certain critical distance, a tachyon appears in the spectrum of the open strings, which triggers a phase transition towards the true vacuum, taking the potential to zero very quickly. This scenario of 'fluxbrane inflation' [21] is therefore a stringy version of supersymmetric 'hybrid inflation' [34,35] and is of the small-field type. The size of the loop-induced inflaton-dependent potential term relative to the constant energy density which drives inflation is controlled by the supersymmetry-breaking parameter. For a pair of D7-branes the potential can be flat enough to maintain an extended period of slow evolution of the inflaton.[4]

Most notably, the inflaton potential in fluxbrane inflation is obtained via integrating out massive string excitations. The inflaton-dependent term which arises in this manner will also depend on Kähler moduli which are not stabilized at tree-level. Thus, one might fear that moduli and inflaton masses are always of the same magnitude in our model, which

[2]To the best of our understanding, the latter is true for all realizations of large-field inflation in string theory which have been proposed so far.

[3]This means, in particular, that an appearance of the D7-brane modulus in the superpotential is avoided.

[4]The fact that this does not generically work for branes of all dimensions was one of the reasons for proposing the fluxbrane inflation scenario in [21]. In particular, in popular scenarios where D3-brane positions play the role of the inflaton [36–38], the maximum field displacement which can be attained in a compact space is not sufficient to obtain a potential suitable for slow-roll inflation.

would, at best, imply a departure from the single-field slow-roll regime. This is, however, not the case. We will show that it is possible to parametrically separate the scale of moduli and inflaton masses in fluxbrane inflation [23, 29]. This works due to the existence of an operator which manifestly does not contain the inflaton and which, under certain conditions, is parametrically dominant amongst the various higher-order terms.

In our second model, the inflaton is associated with a position modulus of a single D7-brane. The inflaton potential is induced at tree-level via a suitable choice of fluxes, which results in a small (tuned) dependence of the superpotential on the D7-brane modulus. This leads to a monodromy (i.e. a multi-covering) in the moduli space of the D7-brane position, which enlarges the a priori sub-planckian field range such that large-field inflation can be accommodated. Our scenario is along the lines of earlier axion monodromy inflation proposals [39, 40], however, with the crucial advantage of being formulated within the context of spontaneously-broken supergravity. This allows us to explicitly address certain control issues. The resulting potential in our 'D7-brane chaotic inflation' model [28] is of the type known from the chaotic inflation scenario [41], entailing the corresponding phenomenological implications such as a large tensor-to-scalar ratio.

Also in this second case we have to worry about the interplay between inflation and moduli stabilization. At first sight, the situation is even worse in D7-brane chaotic inflation, as the inflaton potential is a tree-level (rather than a loop) effect in this model. However, this is precisely why the aforementioned tuning is needed: It suppresses the inflaton mass scale below the scale at which moduli are stabilized. Consequently, also this large-field realization of inflation, using a single D7-brane, works in a parametrically controlled way.

In the rest of this introduction we will provide some more details of the theory of inflation and string theory, focusing on the material which is useful to follow the thesis. In section 1.3 we will give a summary of our most important findings.

1.1. Field-Theoretic Models of Inflation – A Window to UV Physics

Primordial inflation [5–9] has become an important cornerstone of the standard theory for the cosmological evolution. It resolves several puzzles of Big Bang cosmology. In its simplest field-theoretic realization the accelerated expansion of our universe, associated with this period of primordial inflation, is driven by the potential energy density of a slowly rolling scalar field, the inflaton φ. This vacuum energy mimics, to a first approximation, a cosmological constant, which is known to lead to an exponential expansion. The slow motion of the inflaton field in its non-trivial potential leads to a slow time evolution of the energy density. Eventually, as soon as the time variation becomes too big, the energy density of the scalar field looses its interpretation as a cosmological constant and, consequently, the period of accelerated expansion ends. Quantum fluctuations of the metric and the inflaton field during inflation [42–46] are a source of temperature anisotropies in the cosmic microwave background (CMB) radiation and provide the seeds for structure formation, in astonishing agreement with measurements [47, 48].

In this section we highlight two important facts: First, since we are able to observe anisotropies of potentially primordial origin, e.g. the CMB temperature fluctuations, on a range of different scales, we can significantly constrain the time evolution of the energy density during some part of the inflationary epoch. This means that we can constrain some part of the potential for the inflaton. Having string theory in mind, we thus constrain the potential for the string modulus which we want to associate to the inflaton. Second, the fact that the evolution of the energy density is very slow means that the inflaton needs to have a very flat potential in the effective theory. Since scalar fields typically obtain large masses via quantum corrections, the latter is unnatural, unless some symmetries are involved. We will discuss two classes of models, large-field and small-field inflation, and detail how symmetries can be used to protect the inflaton from large mass terms in the effective action. In particular, we discuss the importance of higher-dimensional operators in the two classes.

1.1.1. Slow-Roll Inflation

The need for an epoch of primordial inflation is most easily motivated via the horizon problem, which we will therefore very briefly review, following mostly [9, 49–51]. Recall that the universe on large scales is to a good approximation spatially homogeneous and isotropic, and can thus be described by the Friedmann-Robertson-Walker metric[5]

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2 \left(\frac{\mathrm{d}r^2}{1-kr^2} + r^2 \mathrm{d}\Omega^2 \right) , \tag{1.1}$$

where $a(t)$ is the 'scale factor' and $k = 0, \pm 1$ is the curvature signature. The energy-momentum tensor describing the content of the universe is assumed to be well-approximated by the energy-momentum tensor of a perfect fluid. The latter is fully characterized by the energy density ρ and the pressure p and takes the form $T^{\mu}_{\nu} = \mathrm{diag}(-\rho, p, p, p)$ in the fluid's rest frame. Energy density and pressure are related to the scale factor by the first law of thermodynamics

$$\mathrm{d}(\rho a^3) = -p\mathrm{d}(a^3). \tag{1.2}$$

During matter domination ($p = 0$) this implies $\rho \sim a^{-3}$, while radiation domination (tracelessness of the energy-momentum tensor, i.e. $p = \rho/3$) gives $\rho \sim a^{-4}$. The time evolution of $a(t)$ is governed by Friedmann's equations which are obtained from Einstein's field equations using the ansatz (1.1)

$$H^2 := \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{3} - \frac{k}{a^2}, \tag{1.3}$$

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3p). \tag{1.4}$$

Here we have set the reduced Planck mass to one, $M_p := (8\pi G)^{-1/2} = 1$. For radiation and matter domination Friedmann's equations imply $a(t) \sim t^n$ for some $0 < n < 1$. Thus, the universe expands and was dominated by radiation at some early epoch. Importantly, $a \to 0$ at early times, which corresponds to a singularity in spacetime.

The 'particle horizon' is the comoving distance a photon has traveled since the singularity at $t = t_0$ (we choose $t_0 = 0$ by convention). It is given by

$$\Delta\tau = \int_0^t \frac{\mathrm{d}t'}{a(t')} = \int_0^a \frac{\mathrm{d}\ln a}{aH}. \tag{1.5}$$

[5] We will use units in which $\hbar = c = 1$.

From the above it is clear that aH decreases during radiation and matter domination, such that the 'Hubble radius' $(aH)^{-1}$ increases. In particular, one can evaluate the comoving distance a photon has traveled between the initial singularity at $t = 0$ and the time of recombination, where the universe became transparent and the cosmic microwave background (CMB) was formed. This quantity can be compared to the distance a photon has travelled between CMB formation and today. The latter turns out to be much bigger, leading to the implication that, if the above picture was correct, CMB photons coming from different directions in the sky and being observed by us today would never have been in causal contact in the history of the universe. But how can we then explain correlations in the photon abundance (i.e. correlations of temperature fluctuations in the CMB) across the whole sky? This puzzle is the horizon problem.

It is resolved by postulating an epoch of accelerated expansion, i.e. an epoch of decreasing Hubble radius, in the early universe. This changes the above conclusions. In particular, $\Delta\tau$ evaluated between the onset of inflation and recombination can be bigger than $\Delta\tau$ evaluated between recombination and today. This solves the horizon problem. Also note that, due to the fact that the horizon decreased during inflation, a fixed comoving distance can be larger than the Hubble radius at recombination, but has been 'inside the horizon' (i.e. smaller than aH) at some earlier stage of the cosmological evolution. This will become important later.

An accelerated expansion is most easily realized in a universe whose energy density is dominated by a cosmological constant, in which case the energy-momentum tensor is $T_{\mu\nu} \sim \Lambda g_{\mu\nu}$. This leads to a constant Hubble rate, $H = \text{const.}$, and an exponentially growing scale factor, $a(t) \sim e^{Ht}$.

However, inflation has to end at some point. Thus, a cosmological constant cannot be the final answer. Rather, in single-field slow-roll inflation one postulates that a scalar field, the inflaton, provides the energy density which drives inflation. The equation of motion for a scalar field φ with potential $V(\varphi)$ reads

$$\ddot{\varphi} + 3H\dot{\varphi} = -V'. \tag{1.6}$$

Here we have neglected spatial variations of φ. Although generically present, they are inflated away quickly. In order to see under which conditions the scalar field can drive an accelerated expansion we rewrite $\ddot{a} > 0$ in terms of H

$$\epsilon := -\frac{\dot{H}}{H^2} < 1. \tag{1.7}$$

Combining Friedmann's equations with the equation of motion for the inflaton, using $\rho = \dot{\varphi}^2/2 + V(\varphi)$, yields $\dot{H} = -\dot{\varphi}^2/2$. Here we have neglected the curvature contribution, which quickly becomes irrelevant during inflation. We thus find

$$\epsilon = \frac{1}{2}\frac{\dot{\varphi}^2}{H^2} < 1. \tag{1.8}$$

Inflation proceeds as long as (1.8) is satisfied. Combining (1.8) with (1.3) gives

$$3H^2 \simeq V(\varphi). \tag{1.9}$$

In order to express $\dot{\varphi}$ in (1.8) one can use the equation of motion for the inflaton, neglecting $\ddot{\varphi}$. Smallness of $\ddot{\varphi}$ is usually assumed in slow-roll inflation, though strictly speaking it is not necessary for driving an accelerated expansion (see e.g. [52,53]). However, for non-negligible $\ddot{\varphi}$ inflation typically does not last long enough to explain e.g. the observed correlation of temperature fluctuations in the CMB. Using (1.6) we find

$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 \ll 1, \tag{1.10}$$

while smallness of $\ddot{\varphi}$ translates to

$$|\eta| := \left|\frac{V''}{V}\right| \ll 1. \tag{1.11}$$

These two conditions are often called 'slow-roll conditions'.

Inflation now proceeds as follows: Suppose the inflaton potential has a flat region, such that (1.8) is satisfied. As long as the inflaton happens to be in that region the energy density is approximately constant, leading to an approximately constant Hubble rate H and to an approximately exponential increase of the scale factor. Subsequently, the inflaton enters a region where its kinetic term becomes sizable and thus, inflation ends. To solve the horizon problem the required number of 'e-foldings' is given by [48]

$$N := \ln\left(\frac{a_{\mathrm{end}}}{a_{\mathrm{beginning}}}\right) \simeq 50 \text{ to } 60. \tag{1.12}$$

In fact, it logarithmically depends on the scale of inflation and on the details of reheating, but the numbers quoted above correspond to the commonly used values and are valid for a broad range of inflation models. In this thesis we will use $N = 60$ throughout. Using $\mathrm{d}a \simeq -\frac{a}{\sqrt{2\epsilon}}\mathrm{d}\varphi$, which is true for approximately constant H, we find

$$N \simeq \int_{\varphi_0}^{\varphi_N} \frac{\mathrm{d}\varphi}{\sqrt{2\epsilon}}, \tag{1.13}$$

where φ_0 denotes the field value at which inflation ends and φ_N denotes the field value N e-folds before the end of inflation.

1.1.2. Quantum Fluctuations during Slow-Roll Inflation

Amazingly, besides providing a way to resolve the horizon problem and other issues related to the Big Bang scenario, inflation can extremely accurately describe the spectrum of temperature fluctuations in the CMB [42–46] as measured e.g. by the Planck collaboration [48]. In fact, in the inflationary scenario these temperature variations can be traced back to quantum fluctuations of the metric and the inflaton itself, which are stretched to cosmological scales during inflation.

The treatment of quantum fluctuations of metric and inflaton is somewhat involved. One reason for this is the link between the two via Einstein's field equations. Let us nevertheless try to develop some intuition for the underlying physics.

Fluctuations of the metric divide into scalar $\delta g_{ij} \sim \delta_{ij}$ and tensor fluctuations $\delta g_{ij} \sim h_{ij},\ h^i_i = 0$, where i, j label the spatial components of the metric. Focusing on the scalar perturbations, one may use diffeomorphism invariance to remove the inflaton fluctuations, $\delta\varphi = 0$, and write down the action for the 'comoving curvature perturbation' $\mathcal{R}$, where

$\delta g_{ij} \sim a(t)^2 \mathcal{R} \delta_{ij}$. This is done by plugging the ansatz for δg_{ij} into the action containing the Einstein-Hilbert term and the inflaton. Canonically normalizing the fluctuations, $v = a\sqrt{2\epsilon}\mathcal{R}$, and going to Fourier space, one recovers the Mukhanov-Sasaki equations for the Fourier modes v_k, associated with the comoving wavenumber k. For pure de Sitter[6] they read

$$v_k'' + \left(k^2 - \frac{2}{\tau^2}\right) v_k = 0, \quad \tau = -\frac{1}{aH} = \text{conformal time}. \tag{1.14}$$

The primes denote derivatives with respect to conformal time τ. In approximate de Sitter the situation is more complicated. However, our discussion of the de Sitter case will suffice to illustrate the most important features. Deviations from this idealized situation are expected to be suppressed by the slow-roll parameters.

On 'subhorizon' scales, $k \gg aH$, equation (1.14) describes a simple harmonic oscillator. This is intuitive: small-wavelength modes cannot detect whether the space is Minkowski or de Sitter. The properly normalized 'positive frequency' modes are given, as usual, by

$$v_k = \frac{1}{\sqrt{2k}} e^{-ik\tau}. \tag{1.15}$$

On the other hand, on 'superhorizon' scales, $k \ll aH$, we find a growing solution $v_k = C(k)/\tau$. Note that for the Fourier modes of $\mathcal{R}$ this implies that they are 'frozen' (i.e. constant) on superhorizon scales.

Upon quantizing the fluctuations, an ambiguity concerning the choice of a vacuum arises due to the fact that the 'frequency' $\left(k^2 - \frac{2}{\tau^2}\right)$ in (1.14) is time-dependent. This ambiguity is resolved by observing that, at sufficiently early times, all relevant scales were inside the horizon and the corresponding frequencies were approximately constant. The mode functions which approach (1.15) in the far past, $\tau \to -\infty$, are defined to be positive frequency modes (i.e. multiplying annihilation operators). This corresponds to the *choice* of a Minkowski vacuum in the far past, where all relevant scales were inside the horizon (these are the 'Bunch-Davies' initial conditions).

As long as a given scale is inside the horizon it will fluctuate according to (1.15). When it gets stretched outside the horizon it will follow the growing solution. To determine the normalization $C(k)$ we can, to a first approximation, match the two solutions at horizon crossing, $k = aH$. This gives $C(k) = 1/\sqrt{2k^3}$ and thus $|v_k|^2 = (aH)^2/(2k^3)$ on superhorizon scales. For the Fourier modes of the field $\mathcal{R}$ this implies that

$$P_{\mathcal{R}}(k) := |\mathcal{R}_k|^2 = \frac{1}{4k^3} \frac{H^4}{\dot{H}} \tag{1.16}$$

on superhorizon scales. The above quantifies the power of zero-point fluctuations of a mode with wavenumber k. The 'dimensionless power spectrum' is defined via

$$\Delta_{\mathcal{R}}^2 := \frac{k^3}{2\pi^2} P_{\mathcal{R}}(k) = \frac{1}{2\epsilon} \left(\frac{H}{2\pi}\right)^2. \tag{1.17}$$

In these expressions it is understood that the right-hand side is to be evaluated at horizon crossing, i.e. $k = aH$ (this is where we matched the sub- and superhorizon solutions

[6] Strictly speaking, the relation $v = a\sqrt{2\epsilon}\mathcal{R}$ is ill-defined in pure de Sitter space, as $\epsilon = 0$. For performing the following calculations one usually assumes $\epsilon = \text{const}$. During slow-roll inflation, ϵ will be non-vanishing and approximately constant, with deviations suppressed by the slow-roll parameters.

of the Mukhanov-Sasaki equation). Importantly, (1.17) is independent of k in pure de Sitter space. By contrast, for primordial inflation (which represents a small deviation from de Sitter) (1.17) will depend on k, i.e. the power spectrum is 'scale-dependent'. Since the departure from de Sitter is small, suppressed by the slow-roll parameters, inflation produces a *nearly* scale-invariant spectrum.

A similar computation can be performed for tensor fluctuations of the metric. One again finds the Mukhanov-Sasaki equation for the canonically normalized field, only this time the process of canonically normalizing does not involve ϵ. Therefore, the power spectrum of tensor perturbations does not involve ϵ and is given by

$$\Delta_{\mathcal{T}}^2 = \frac{2}{\pi^2} H^2. \tag{1.18}$$

Thus, this quantity is directly related to the scale of inflation.

Upon reheating the above fluctuations become density variations in the primordial plasma. Driven by gravitational pull and radiation repulsion they start oscillating coherently as soon as the scale associated with a given fluctuation re-enters the horizon. At recombination a snapshot of these fluctuations is imprinted in the CMB radiation, which can be measured today. In particular, modern experiments like the Planck satellite [48] are able to resolve a range of different scales in the CMB, i.e. we can probe a range of different wavenumbers k. This constrains the shape of the inflaton potential. For example, measurements of the amplitude of scalar perturbations determine [48]

$$\sqrt{12\pi^2}\Delta_{\mathcal{R}} = \sqrt{\frac{V}{2\epsilon}} = 5.1 \cdot 10^{-4} \tag{1.19}$$

at the pivot scale $k_* = 0.002$ Mpc^{-1}.

To first order, the deviation from scale invariance of the scalar power spectrum is conveniently parametrized by the spectral tilt

$$n_s - 1 := \frac{\mathrm{d}\ln \Delta_{\mathcal{R}}^2}{\mathrm{d}\ln k}. \tag{1.20}$$

In the limit of small ϵ we find $\mathrm{d}\ln k = H\mathrm{d}t$. Using furthermore (1.9) and (1.6) (with $\ddot{\varphi} \approx 0$) we obtain $\mathrm{d}\ln k = -\frac{1}{\sqrt{2\epsilon}}\mathrm{d}\varphi$, and thus

$$n_s - 1 = -6\epsilon + 2\eta. \tag{1.21}$$

As expected, the departure from scale invariance is quantified in terms of the slow-roll parameters. The spectral tilt is measured to be $n_s = 0.9603 \pm 0.0073$ at k_* [48].

Another important quantity is the tensor-to-scalar ratio

$$r := \frac{\Delta_{\mathcal{T}}^2}{\Delta_{\mathcal{R}}^2} = 16\epsilon. \tag{1.22}$$

Until recently, for this quantity only upper bounds existed, the latest one being $r < 0.11$ [48]. This situation has changed: In [33] it is claimed that $r = 0.2^{+0.07}_{-0.05}$ has been detected. If confirmed, this would determine the scale of inflation to be roughly the GUT scale, $V^{1/4} \simeq 2 \cdot 10^{16}$ GeV. At the moment, the validity of this result is still debated.

Interestingly, in view of (1.13) there is a close relationship between the field range $\Delta\varphi$, which is traversed by the inflaton during ΔN e-folds, and the tensor-to-scalar ratio. Assuming approximately constant r it reads

$$|\Delta\varphi| \simeq \sqrt{\frac{r}{8}}|\Delta N|. \tag{1.23}$$

In particular, assuming $r \simeq 0.2$ to be roughly constant during the phenomenologically required 50 to 60 e-folds, this implies $\Delta\varphi \gg 1$ [54, 55]. On the other hand, the scales that we can presently resolve in the CMB correspond to less than about 10 e-folds. Therefore, the BICEP2 result [33] does not immediately imply that the field excursion of the inflaton was super-planckian.

1.1.3. Models of Slow-Roll Inflation in Effective Field Theory

Models of slow-roll inflation can be classified according to the distance in field space which is traversed by the inflaton during the last $N = 60$ e-folds of inflation. By obvious use of nomenclature they are called 'large-field' ($\Delta\varphi > 1$) and 'small-field' ($\Delta\varphi < 1$) models. Let us highlight some of the important features of these classes in turn.

In terms of bottom-up model building it is very easy to write down a potential for a large-field model of inflation. A simple

$$V(\varphi) = m^2\varphi^2 \tag{1.24}$$

is in reasonable agreement with the observational constraints for a properly chosen m [41].[7] More precisely, the measured value of n_s fixes $\varphi_N^2 \simeq 200$ via (1.21).[8] Furthermore, (1.19) determines $m \simeq 0.5\cdot 10^{-5}$. Crucially, the tensor-to-scalar ratio is large, $r \simeq 0.16$, consistent with the BICEP2 result.

So far this potential looks very promising. One general challenge in the bottom-up construction of inflationary scenarios is, however, that it is non-generic to have light scalars in an effective theory. Rather, starting in the UV and integrating out heavy degrees of freedom, scalars will typically obtain large masses. For example, if the UV theory contains the marginal coupling $\lambda\varphi^2\chi^2$ to some heavy scalar χ with mass M, the inflaton mass m^2 will receive quantum corrections proportional to λM^2. Thus, retaining $m \simeq 0.5 \cdot 10^{-5}$ under the inclusion of quantum corrections implies a tuning.[9] What is more, generically all higher-dimensional operators $\simeq \mathcal{O}_d[\varphi]/M^{d-4}$ will be introduced in the effective theory. Since $M \lesssim M_p$, they will dominate the potential for $\varphi \gg 1$ and generically ruin the large-field inflation model. As an example, consider the marginal operator $\kappa\varphi^4$ which, even if not present at tree-level, will be introduced via a χ-loop with a coupling strength $\kappa \sim \lambda^2$. For λ not too small this operator will dominate over (1.24) in the regime of large field values, $\varphi \gg 1$.

One way to proceed from here is to assume that the UV theory features a symmetry which forbids (or limits the size of) the dangerous higher-dimensional operators. Possible

[7] There is some ongoing discussion how to reconcile the BICEP2 measurement with the Planck bound $r < 0.11$, one option being to allow for a sizable running of the spectral index, which weakens the Planck bound [48]. However, such a large running cannot be provided by the simple quadratic potential (1.24).

[8] By choice, the origin $\varphi = 0$ corresponds to the minimum of the potential.

[9] Clearly, if one has reasons to assume $\lambda \ll 1$ (or even $\lambda = 0$) for *all* couplings of this type, tuning might not be necessary.

candidates for such symmetries are supersymmetry [56] or a shift symmetry. Supersymmetry is well-known to control the size of loop corrections to scalar masses, e.g. in the context of the standard model Higgs particle. However, supersymmetry will be broken by the positive vacuum energy density during inflation, leading to a cutoff which is generically $\Lambda \sim H$ and therefore to a Hubble-scale inflaton mass, inconsistent with (1.11). A shift symmetry, i.e. invariance of the UV theory under $\varphi \to \varphi + \text{const.}$, is more promising in this respect. It forbids all dangerous higher-dimensional operators and leads to an exactly flat potential for the inflaton. If the shift symmetry is broken only by a small effect, then all quantum corrections breaking the shift symmetry will be suppressed by the parameter controlling the size of this small effect. For instance, if the breaking is due to a mass term (1.24), quantum corrections to the inflaton potential will be suppressed by m divided by some high scale (e.g. the Planck scale if one considers a graviton loop). In this sense, the above model of chaotic inflation can be 'technically natural' [57].

An important caveat to this is a folk theorem which states that global symmetries are not consistent in a quantum theory of gravity (for a recent discussion see [58]). While there is no rigorous proof of this theorem, it is certainly true that no continuous global symmetries are encountered in presently known string compactifications. By this folk theorem, any global shift symmetry is expected to be broken by Planck-suppressed operators, which threatens the discussed large-field inflation model. We thus learn that bottom-up model building clearly has limitations. In this thesis we therefore follow a different strategy: We start with string theory as a candidate theory for a UV completion of gravity and *derive* the effective theory for the inflaton. This approach automatically warrants the UV compatibility of the inflation model.

Examples of shift symmetries do arise in string theory, the most prominent ones being axionic shift symmetries which descend from higher-dimensional gauge invariance. These shift symmetries are typically gauged and always broken by non-perturbative effects. For example, the axionic coupling $\frac{a}{f} F\tilde{F}$ induces a periodic potential $\sim \cos(a/f)$ for the axion a via gauge instanton effects. This potential can lead to natural inflation [59] if the suppression scale f of the higher-dimensional operator (commonly called the 'axion decay constant') is much larger than the Planck scale, $f \gg 1$. Note that in the limit $f \to \infty$ the axion decouples from the gauge sector and the shift symmetry becomes global, which is forbidden by the folk theorem. Accordingly, this limit should not be available in string theory. Indeed, all attempts to realize $f > 1$ in weakly-coupled string theory failed so far [60], which is why there is no successful embedding of natural inflation, using one axion, in string theory.[10] These arguments do not rule out large-field inflation in string theory. For example, models where the inflaton dynamics is mainly due to a tree-level breaking term, as in (1.24), can still work.

In conclusion, we have seen that consistency with the UV completion severely constrains inflationary models. In particular, an inflaton potential which is written down naively might seem promising at first sight and can even be technically natural, but may be incompatible with a UV theory of quantum gravity. In string theory, the question of UV compatibility of inflationary models can in principle be addressed explicitly, which motivates the investigations performed in this thesis.

Let us now discuss 'small-field inflation', where $\Delta\varphi < 1$ during the whole $N = 60$ e-folds of inflation. We will see that, by contrast to the large-field case, in small-field inflation

[10] Generating an *effectively* large axion decay constant using multiple axions has been discussed in [61,62].

only a finite number of higher-dimensional operators needs to be controlled. An inflaton potential which is suitable for inflation with sub-planckian field excursion is given by (see e.g. [55])

$$V(\varphi) = V_0(1 - \alpha\varphi^2 + \ldots), \tag{1.25}$$

where the ellipses denote terms of higher order in φ. Here, the origin $\varphi = 0$ is chosen such that it corresponds to a local maximum of the potential. Inflation works for $\alpha \ll 1$. In this limit, $\epsilon \ll |\eta|$ and thus (1.21) determines $\alpha \simeq \frac{1-n_s}{4} \simeq 0.01$. Furthermore, $N = \frac{1}{2\alpha}\ln\left(\frac{\varphi_0}{\varphi_N}\right)$ and

$$r = \frac{2(1-n_s)^2}{\left(e^{\frac{(1-n_s)}{2}N} - 1\right)^2}(\varphi_0 - \varphi_N)^2 \simeq 6 \cdot (\varphi_0 - \varphi_N)^2 \cdot 10^{-4} \tag{1.26}$$

for $N = 60$. Here we have assumed that at φ_0 higher-order terms in (1.25) become important and slow-roll ends. The only free model parameter is φ_0. The value φ_N is then determined in terms of φ_0 and N. Since $\alpha \simeq 0.01$, we have $N \simeq 50 \cdot \ln\left(\frac{\varphi_0}{\varphi_N}\right)$. Thus, $N = 60$ can be achieved for sub-planckian field excursions. The overall scale V_0 is set by (1.19).

From this simple but sufficiently generic example we can learn a couple of lessons: First, since in the above model the inflaton rolls a sub-planckian distance in field space, $|\varphi_0 - \varphi_N| < 1$, we can safely neglect almost all higher-dimensional operators in (1.25), something which was not possible in the large-field model discussed before. Second, as expected from (1.23), the tensor-to-scalar ratio is tiny. The situation is actually particularly extreme in the above model, since r is monotonically increasing during inflation. A naive Lyth-bound estimate, using $\Delta N = 60$, would give $r \simeq 2 \cdot (\varphi_0 - \varphi_N)^2 \cdot 10^{-3}$, which is larger than the actual value. A string-inspired scenario where the Lyth bound in this strong form is evaded by a non-monotonic evolution of r will be discussed in chapter 2 (see also [24]). Third, in order for the model to be viable we needed a small dimensionless coefficient in front of the inflaton-dependent term in the potential, something which we have no reason to believe it appears generically. Instead, similar to the large-field case, one would naively expect mass corrections of Planck-scale size, which would be disastrous for the inflation model. Supersymmetry helps, but again we expect $\Delta m^2 \sim H^2$ because supersymmetry is broken spontaneously during inflation. This mass is too large to be consistent with (1.11) by a factor of a hundred. One can, nevertheless, obtain a viable model of inflation using supersymmetry, by explicitly computing all operators which contribute inflaton mass corrections and subsequently invoking explicit tuning. The most prominent string inflation scenario where this strategy has been employed is the KKLMMT scenario [38]. Crucially, such a strategy can only be successful with knowledge about the UV completion, which again motivates the discussion of inflation in string theory. The other option is to resort to shift symmetries in order to protect the inflaton mass. As we have already mentioned, such shift symmetries are broken, e.g. by non-perturbative effects, and the size of the breaking terms determines whether or not an inflationary model is viable. In chapter 2 we will discuss the example of hybrid natural inflation, a scenario in which a shift symmetry and supersymmetry are employed to built a model in which the inflaton naturally remains light.

1.2. String Theory and its Low-Energy Description

In this section we provide some relevant string theory background which is needed to follow the discussion in the later chapters. A more detailed treatment can be found in [1–4].

1.2.1. Ten-Dimensional Effective Actions from String Theory

String theory is the presently best understood candidate theory for describing the UV behavior of the standard model, including gravity. It posits that the fundamental degrees of freedom at and above the string scale M_s are the ones of vibrating closed (and sometimes open) strings, rather than point particles. The string scale M_s is set by the tension of the strings.

The classical dynamics of a bosonic string moving in a d-dimensional spacetime is governed by the Polyakov action [63, 64]

$$S = -\frac{1}{4\pi\alpha'}\int_W \mathrm{d}^2\sigma\sqrt{-g}g^{\alpha\beta}\partial_\alpha X^M\partial_\beta X^N\eta_{MN}, \tag{1.27}$$

where the string tension T is defined in terms of the 'Regge slope' α' as $T := 1/(2\pi\alpha')$. The string sweeps out a 'worldsheet' W as it moves in time. Coordinates on the worldsheet are denoted by $\sigma \equiv (\tau, \sigma)$, and the $X^M(\sigma)$, $M = 0, \ldots, d-1$, describe the embedding of the worldsheet in the d-dimensional 'target spacetime'. In (1.27) the latter is a Minkowski space with metric η_{MN}. In the above Polyakov formulation the metric on the worldsheet $g_{\alpha\beta}$ is an independent quantity which is introduced in order to simplify the treatment of the action. Equation (1.27) thus describes a field theory coupled to gravity on the two-dimensional worldsheet. The extra redundancies associated with the worldsheet metric $g_{\alpha\beta}$, namely the freedom to choose coordinates on the worldsheet and invariance under Weyl rescalings, $g_{\alpha\beta} \to e^{2\omega(\sigma)}g_{\alpha\beta}$, can be partially removed by the gauge choice $g_{\alpha\beta} = \eta_{\alpha\beta}$. What is left are diffeomorphisms on the worldsheet which can be undone by a Weyl rescaling. These are conformal transformations. Crucially, preserving conformal invariance at the quantum level requires $d = 26$ [65].

The action (1.27) has d-dimensional Poincaré invariance as an internal symmetry. Upon quantization the massless states of the theory described by (1.27) can be organized into representations of this internal symmetry group. In particular, one finds a 'graviton' G_{MN} (i.e. a rank-two symmetric tensor), an antisymmetric tensor B_{MN}, and the 'dilaton' Φ.

One can consider non-trivial backgrounds for these massless excitations, in which case the action of the string moving in such a background becomes a non-linear sigma-model

$$S = -\frac{1}{4\pi\alpha'}\int_W \mathrm{d}^2\sigma\sqrt{-g}\left(\left\{g^{\alpha\beta}G_{MN}(X) + \epsilon^{\alpha\beta}B_{MN}(X)\right\}\partial_\alpha X^M\partial_\beta X^N + \alpha'\Phi(X)R(g)\right), \tag{1.28}$$

where $R(g)$ is the two-dimensional Ricci scalar. The above describes an interacting theory of scalars which generally cannot be solved exactly. One can, however, expand the background fields appearing in (1.28) in fluctuations about a classical solution X_0^M, which describes a string sitting at a fixed point. The coupling constants multiplying the interaction terms in this expansion are then given by derivatives of the background fields. They can be organized in powers of $\frac{\sqrt{\alpha'}}{R}$, where R is some typical length scale of the target space.

Besides requiring $d = 26$, preserving conformal invariance at the quantum level imposes additional constraints on the background fields in terms of equations of motion in the target spacetime, which can be analyzed order by order in $\frac{\sqrt{\alpha'}}{R}$. At leading order, these equations of motion can be shown to arise from an action

$$S = \frac{1}{2\kappa_d^2}\int_{\mathcal{M}_d} \mathrm{d}^d X\sqrt{-G}e^{-2\Phi}\left(R + 4(\partial\Phi)^2 - \frac{1}{2}|H_3|^2 - \frac{2(d-26)}{3\alpha'} + \mathcal{O}(\alpha')\right), \qquad (1.29)$$

where $H_3 := \mathrm{d}B_2$, $|H_3|^2\sqrt{-G}\mathrm{d}^d X \equiv H_3 \wedge *H_3$, and $d = 26$ for the bosonic string. The constant κ_d^2 (which is proportional to Newton's constant in d-dimensions) can be expressed in terms of α', but its precise form is irrelevant for us. Crucially, (1.29) involves the d-dimensional Einstein-Hilbert action in a non-canonical frame, the 'string frame'. Thus, at leading order in α', bosonic string theory gives a particular 26-dimensional effective field theory, containing gravity.

Bosonic string theory has several drawbacks. For instance, it contains tachyons and has no fermionic excitations. Both problems are absent in superstring theory, i.e. a theory which is based on a two-dimensional *supersymmetric* worldsheet action and which thus contains fields which transform in the spinor representation in two dimensions. In contrast to the bosonic string, superstring theory is consistently defined in $d = 10$ target spacetime dimensions. Furthermore, one finds that there exists not only one consistent theory. Rather, one is free to make several choices, e.g. choose different periodicity conditions for the worldsheet fermions, which give rise to *five* different string theories in ten dimensions, with associated ten-dimensional effective *supergravity* actions.

For simplicity, in the following we consider only one of these five theories, namely 'Type IIB' string theory. From the perspective of moduli stabilization, to be discussed later, it is the one which is best understood. The bosonic field content in Type IIB string theory comprises, in addition to the fields G_{MN}, B_{MN}, and Φ, the p-form potentials C_p, $p = 0, 2, 4$. Denoting their field strengths by $F_{p+1} := \mathrm{d}C_p$, it is convenient to define the axio-dilaton $\tau := C_0 + ie^{-\Phi}$, and $G_3 := F_3 - \tau H_3$. Furthermore, $\tilde{F}_5 := F_5 - \frac{1}{2}C_2 \wedge H_3 + \frac{1}{2}B_2 \wedge F_3$. The effective action for the bosonic fields then reads

$$\begin{aligned} S_{\mathrm{IIB}} =& \frac{1}{2\kappa_{10}^2}\int_{\mathcal{M}_{10}}\left(\mathrm{d}^{10}X\sqrt{-G_E}R_E - \frac{\mathrm{d}\tau\wedge *\mathrm{d}\overline{\tau}}{2\,\mathrm{Im}(\tau)^2} - \frac{G_3\wedge *\overline{G}_3}{2\,\mathrm{Im}(\tau)} - \frac{\tilde{F}_5\wedge *\tilde{F}_5}{4}\right) \\ &+ \frac{1}{8\kappa_{10}^2}\int \frac{C_4\wedge G_3\wedge \overline{G}_3}{i\,\mathrm{Im}(\tau)}, \end{aligned} \qquad (1.30)$$

where $2\kappa_{10}^2 = (2\pi)^7(\alpha')^4$. Note that the self-duality constraint $\tilde{F}_5 = *\tilde{F}_5$ has to be imposed at the level of the equations of motion. The action (1.30) has been written in a frame where the Einstein-Hilbert term is canonical (i.e. in the ten-dimensional 'Einstein frame'), which is achieved via a Weyl rescaling $G_{MN,\mathrm{E}} := e^{-\Phi/2}G_{MN}$.

Just as the one-form potential in electromagnetism is sourced by the electron, there exist sources for the C_p gauge potentials appearing in (1.30), so-called 'D-branes'. In perturbative string theory they arise as subspaces on which open strings can end (i.e. the open strings have Dirichlet boundary conditions in the directions transverse to the D-brane). Fluctuations of the open strings transverse to the D-brane can be interpreted as fluctuations of the D-brane itself, whereas fluctuations along the D-brane directions correspond, in the effective theory, to gauge fields living on the D-brane worldvolume. The effective action for the massless excitations associated with a Dp-brane with p spatial dimensions is given by [66]

$$S_{\mathrm{D}p} = -T_p \int_{\mathrm{D}p} \mathrm{d}^{p+1}\xi e^{-\Phi} \sqrt{-\det\left(G_{ab} + B_{ab} + 2\pi\alpha' F_{ab}\right)} + \mu_p \int_{\mathrm{D}p} \sum_q C_q \wedge e^{B_2 + 2\pi\alpha' F_2}, \tag{1.31}$$

where F_2 is the field strength two-form of the D-brane gauge theory, and the D-brane charge μ_p equals its tension T_p (a D-brane is a BPS object [67, 68]) which is given by $\mu_p = T_p := (2\pi)^{-p}(\alpha')^{-(p+1)/2}$. The second term in (1.31) captures the coupling of the D-brane to the C_q gauge potentials, in analogy to the coupling of the electron to a background one-form gauge potential. The 'induced metric' on the D-brane worldvolume is given by the pullback

$$G_{ab} = \frac{\partial X^M}{\partial \xi^a}\frac{\partial X^N}{\partial \xi^b} G_{MN}, \quad a, b = 0, \ldots, p, \tag{1.32}$$

where G_{MN} is the ten-dimensional string frame metric and the $X^M(\xi)$ describe the embedding of the D-brane in the target spacetime. The latter fields thus encode the *position* of the D-brane. They will be associated with the inflaton in the string inflation scenarios discussed in this thesis. Finally, B_{ab} is the pullback of B_{MN} to the worldvolume of the Dp-brane. In Type IIB string theory, only Dp-branes with odd p correspond to stable solutions of the equations of motion.

1.2.2. Four-Dimensional Effective Actions from String Compactifications

The action (1.30) (possibly supplemented by (1.31)) will be the starting point of our further investigations. The first task is to find classical solutions to the equations of motion derived from (1.30). Following our reasoning from the previous subsection, these solutions are then backgrounds for a worldsheet theory without quantum anomalies, at least at leading order in α'. In order to make contact with phenomenology one typically looks for solutions of the type

$$\mathcal{M}_{10} = \mathcal{M}_4 \times X_3, \tag{1.33}$$

where $\mathcal{M}_4$ describes the four-dimensional spacetime we observe and X_3 is some compact six-dimensional internal space, small enough such that its existence has escaped experiments so far.[11] If one has obtained a solution to the equations of motion, obeying (1.33), the theory is then said to be 'compactified' on X_3.

Having found a solution one can perform a 'dimensional (Kaluza-Klein) reduction' [69, 70], i.e. one expands the ten-dimensional fields in terms of eigenfunctions on the internal space with respect to the differential operator appearing in the equations of motion. The coefficients in this expansion depend on the non-compact dimensions and thus correspond to fields in a four-dimensional effective theory. At leading order one takes into account only the massless fields in four dimensions, neglecting all higher modes with masses at the KK scale.

The ten-dimensional effective theory, whose action for the bosonic fields is given by (1.30), enjoys $\mathcal{N} = 2$ supersymmetry. Generic solutions to the equations of motion, obeying (1.33), will lead to four-dimensional effective theories with no supersymmetry. However, for

[11] Often, a direct product of the form (1.33) is not general enough for phenomenological purposes. In 'flux compactifications', to be discussed momentarily, one typically considers warped products of a maximally symmetric four-dimensional spacetime with some internal six-dimensional space [12].

many purposes it is adequate to focus on compactifications which preserve some amount of supersymmetry in four dimensions.[12] In the case of trivial vacuum expectation values for the fields C_p and B_2, and for constant Φ, those solutions are 'Calabi-Yau' spaces [10], i.e. complex three-dimensional Kähler manifolds with vanishing first Chern class. This implies in particular that the Ricci tensor vanishes (for suitably chosen coordinates), which means that these spaces are solutions to the *vacuum* equations of motion.

Moduli in String Compactifications

A common feature of such supersymmetric compactifications is that they come with many 'moduli', i.e. four-dimensional scalar fields with exactly flat potential. They arise upon dimensionally reducing the massless ten-dimensional fields to four dimensions. The equations of motion for the massless fields in ten dimensions imply that the eigenfunctions multiplying the massless four-dimensional fields in the KK expansion are *harmonic* forms of the Calabi-Yau space. The number of different harmonic (p, q)-forms on a compact complex manifold X_3 is given by the dimension $h^{p,q}(X_3)$ of the corresponding Dolbeault cohomology group $H^{p,q}(X_3)$. Thus, the massless spectrum of the four-dimensional effective theory is determined by purely topological data of the Calabi-Yau. For instance, the ten-dimensional field C_4 can be expanded as $C_4 = c_I(x)\tilde{\omega}^I + C_2^{\tilde{I}}(x) \wedge \omega_{\tilde{I}} + \ldots$, where $\tilde{\omega}^I$, $I = 1, \ldots, h^{2,2}(X_3)$, is a basis of $H^{2,2}(X_3)$. Furthermore, $\omega_{\tilde{I}}$, $\tilde{I} = 1, \ldots, h^{1,1}(X_3)$, is a basis of $H^{1,1}(X_3)$, and x denotes the coordinates along the non-compact directions. Importantly, on a Calabi-Yau manifold one has

$$\begin{array}{ccccccc} & & & h^{0,0}=1 & & & \\ & & h^{1,0}=0 & & h^{0,1}=0 & & \\ & h^{2,0}=0 & & h^{1,1}=h^{2,2} & & h^{0,2}=0 & \\ h^{3,0}=1 & & h^{2,1}=h^{1,2} & & h^{1,2}=h^{2,1} & & h^{0,3}=1 \\ & h^{3,1}=0 & & h^{2,2}=h^{1,1} & & h^{1,3}=0 & \\ & & h^{3,2}=0 & & h^{2,3}=0 & & \\ & & & h^{3,3}=1 & & & \end{array} \tag{1.34}$$

That is, only $h^{1,1}(X_3) = h^{2,2}(X_3)$ and $h^{2,1}(X_3) = h^{1,2}(X_3)$ are not determined.

The fact that the Calabi-Yau vacuum solutions do not provide a potential for the four-dimensional moduli fields represents a serious drawback. We will discuss how this is resolved in more detail in the following subsection 1.2.3. Here we just note that, according to [12], at least some of the moduli can be 'stabilized' (i.e. they become massive) if one turns on a non-trivial field configuration ('fluxes') for F_3 and H_3. Consistency (that is, charge cancellation in the internal compact space) then typically requires to include localized sources with negative tension. One is thus led to consider Type IIB 'orientifold' compactifications [71–73]. These are Type IIB theories where states which are not invariant under a certain geometric involution, combined with worldsheet parity inversion, are projected out. They contain the desired localized sources in terms of D-branes and 'O-planes'. As described before, D-branes are dynamical (positive-tension) objects and therefore come with their own moduli fields. On the other hand, O-planes are by definition the fixed-point

[12] For example, supersymmetry makes the treatment of the four-dimensional effective actions tractable. Furthermore, we will see in chapter 2 that supersymmetry controls the size of shift-symmetry-breaking quantum corrections in hybrid natural inflation, whose stringy embedding (fluxbrane inflation) will be discussed in a large part of this thesis.

loci of the geometric involution and merely a tool to describe the effect of the orientifold quotient. They can be assigned a charge and tension (which turns out to be negative), but they are not associated with dynamical physical objects. We will only consider orientifold compactifications in the following.

Crucially, under the orientifold action the cohomology groups decompose into orientifold-odd and -even parts, $H^{p,q}(X_3) = H^{p,q}_+(X_3) \oplus H^{p,q}_-(X_3)$. The transformation property of a given field then determines whether its expansion is along elements of $H^{p,q}_+(X_3)$ or $H^{p,q}_-(X_3)$.

Geometric Moduli in Orientifold Compactifications

Recall that in the original Kaluza-Klein proposal, i.e. the reduction of a five-dimensional gravity theory on a circle to four dimensions, there appeared a massless scalar field in the four-dimensional action which was associated with the component of the metric along the circle, i.e. to its 'volume'. A similar situation is encountered in the more complicated case discussed here: Deformations of the metric on the internal space which preserve the vanishing of the Ricci scalar (i.e. which preserve supersymmetry of the four-dimensional theory) will correspond to massless fields (moduli) in the four-dimensional effective action. Identifying them is the question of identifying the moduli space of Calabi-Yau manifolds [11]. In order to achieve this, recall that on a Calabi-Yau manifold there exist two specific (p, q)-forms of paramount importance: The Kähler form J is a real $(1, 1)$-form and measures sizes of $p = 2, 4, 6$-dimensional subspaces (p-cycles). For instance, the overall volume of the internal space is given by $\mathcal{V} = \int_{X_3} J \wedge J \wedge J$.[13] On the other hand, the holomorphic $(3, 0)$-form Ω specifies the 'complex structure' (i.e. a choice of complex coordinates) on the manifold. The geometric moduli in the effective four-dimensional theory now correspond to deformations of Kähler form and complex structure.

The Kähler form, being invariant under the orientifold involution, is expanded in a basis of $H^{1,1}_+(X_3)$,

$$J = t^I(x)\omega_I, \quad I = 1, \dots, h^{1,1}_+(X_3), \tag{1.35}$$

which is normalized such that the (real) t^I measure the sizes of a basis of integral two-cycles. Deformations of the Kähler form correspond to a 'motion' of J in $H^2_+(X_3)$, i.e. to a variation of the four-dimensional moduli t^I. Clearly, there are $h^{1,1}_+(X_3)$ independent such deformations.

It turns out that there are two different possible choices for the orientifold involution, both of which lead to consistent theories. The latter involve either O7- and O3-planes (and the corresponding D7- and D3-branes), or O5- and O9-planes (and the corresponding D5- and D9-branes). Specifying to orientifold involutions of the first type, the holomorphic three-form Ω_3 is odd under the involution [71].[14] Variations of the complex structure thus correspond to variations of the $(3, 0)$-form Ω_3 in $H^3_-(X_3)$. Infinitesimally, these variations are elements of $H^{2,1}_-(X_3)$. In analogy to the t^I, the four-dimensional (complex) fields z_i, which parametrize the complex structure (and its deformations), arise by integrating Ω_3 over a basis of three-cycles Σ_i,

$$z_i \sim \int_{\Sigma_i} \Omega_3, \quad i = 1, \dots, h^{2,1}_-(X_3). \tag{1.36}$$

[13] Our conventions for measuring lengths are summarized in appendix A.

[14] The Kähler form J is invariant under the involution in both cases.

Much more can be said, e.g. concerning the choice of the basis Σ_i. However, the above suffices to give some intuition, which is all we need in the following. For more details we refer the interested reader to the literature [11].

In summary, the four-dimensional effective theory will contain the moduli fields which arise from reducing the 2- and 4-form potentials appearing in (1.30) along corresponding 2- and 4-cycles. It will further contain the axio-dilaton τ and the geometric moduli t^I and z_i.

Kähler and Superpotential for the Effective Theory

As the effective theory is supersymmetric, we will be able to phrase it in terms of properly chosen *complex* (super-)fields ϕ_α. Furthermore, its Lagrangian can be specified in terms of a Kähler potential K (which is a real function of the complex fields) and the holomorphic superpotential W [56, 74]. In the absence of gauge interactions it is given by

$$\mathcal{L} = -K_{\alpha\overline{\beta}}\partial^\mu\phi^\alpha\overline{\partial_\mu\phi^\beta} - V_F, \tag{1.37}$$

where $K_{\alpha\overline{\beta}} := \partial_\alpha\partial_{\overline{\beta}}K$ is the Kähler metric and

$$V_F = e^K\left(K^{\alpha\overline{\beta}}D_\alpha W\overline{D_\beta W} - 3|W|^2\right), \quad D_\alpha W := (\partial_\alpha + K_\alpha)W \tag{1.38}$$

is the scalar F-term potential.

The task is now to derive K and W from (1.30). As an example, consider the axio-dilaton τ. If we assume a constant profile for that field along the internal space, the integration over X_3 can be performed, giving only a factor of the overall volume which will be absorbed in the definition of the four-dimensional Planck mass. The kinetic term for τ is thus easily seen to descend from a Kähler potential

$$K \supset -\ln(-i(\tau - \overline{\tau})). \tag{1.39}$$

For the other fields appearing in the four-dimensional action the derivation of the Kähler potential is somewhat harder. We only quote the results [11, 71]:

$$K = -2\ln\mathcal{V} - \ln(-i(\tau - \overline{\tau})) - \ln\left(-i\int_{X_3}\Omega_3 \wedge \overline{\Omega}_3\right). \tag{1.40}$$

Here, the volume $\mathcal{V} = \frac{1}{3!}\kappa_{IJK}t^It^Jt^K$ is to be read as a function of the 'complexified four-cycle volumes'

$$T_I := \frac{1}{2}\kappa_{IJK}t^Jt^K + ic_I + \ldots, \tag{1.41}$$

where κ_{IJK} is the triple self-intersection matrix of the Calabi-Yau four-cycles and the ellipses denote terms which depend on the fluxes and which are not of relevance for us. In particular, the latter are absent for $h^{1,1}_-(X_3) = 0$. As the name suggests, $\tau_I := \frac{1}{2}\kappa_{IJK}t^Jt^K$ measures the volume of the four-cycle which is Poincaré-dual to ω_I. Without additional ingredients, the superpotential for these fields vanishes identically.

In the above we have completely neglected D-brane moduli. For our purposes, the most relevant of those are D7-brane position moduli. They will be identified with the inflaton in our string inflation scenarios. The low-energy effective theory for these moduli has been discussed in [72, 73]. In their presence, the Kähler variable τ needs to be redefined (and is then commonly called S), and the D7-brane position moduli enter the Kähler potential together with the redefined axio-dilaton in the schematic form

$$K \supset -\ln\left(-i(S-\overline{S}) - k_{\mathrm{D7}}(z,\overline{z};c,\overline{c})\right). \tag{1.42}$$

Here, k_{D7} is the Kähler potential for the D7-brane position moduli c. It depends on the complex structure moduli.

1.2.3. Moduli Stabilization

The presence of moduli in the four-dimensional effective theory has profound phenomenological consequences. For example, after inflation some significant amount of energy may be contained in the moduli sector, which can then decay into 'dark radiation' candidates and as such be observed in principle. Therefore, understanding how and at which scale moduli obtain masses is crucial for providing a description of the cosmological history.

In the context of Type IIB string theory it was found [12] that turning on F_3 and H_3 'fluxes', i.e. allowing those field strengths to have components along non-trivial three-forms on the internal space, will generically lead to a potential for the complex structure moduli and the axio-dilaton. The Dirac quantization condition [75] requires these fluxes to be quantized according to $\int F_3 = (2\pi)^2\alpha' m$, $\int H_3 = (2\pi)^2\alpha' n$, where $m, n \in \mathbb{Z}$. Without extra ingredients, only AdS vacua are possible [76], which is undesirable.[15] One way of directly obtaining Minkowski (or de Sitter) vacua is to include localized sources e.g. in the form of D7- and D3-branes and their orientifold planes. The compact spaces which arise as solutions to the equations of motion in these theories can be of the 'conformal Calabi-Yau' type, which means that they describe warped products of maximally symmetric external spaces with internal Calabi-Yau manifolds. The warping effect is small in the limit of large volume (in units of $\sqrt{\alpha'}$) of the internal manifold [12]. This allows us, in the large-volume (dilute-flux) limit, to use all the concepts discussed so far, which is the reason why moduli stabilization is best studied in Type IIB compactifications.

In the four-dimensional supergravity action the flux effect is captured by a non-trivial superpotential [79]

$$W = \int_{X_3} G_3 \wedge \Omega_3, \tag{1.43}$$

which is independent of the Kähler moduli to all orders in perturbation theory [80, 81]. To analyze the effect of (1.43) let us write the scalar F-term potential in the following way

$$\begin{aligned} V_F = e^K \Big(& K^{T_I \overline{T}_J} D_{T_I} W \overline{D_{T_J} W} - 3|W|^2 \\ & + K^{\tau\overline{\tau}} D_\tau W \overline{D_\tau W} + K^{i\overline{j}} D_i W \overline{D_j W} \Big), \end{aligned} \tag{1.44}$$

where i, j label the complex structure moduli. Crucially, as $\mathcal{V}^2$ is a homogeneous function of degree 3 in the T_I, the first line in (1.44) identically vanishes. This is famously called 'no-scale structure' [82, 83]. It leaves us with the potential

$$V_F = e^K \left(K^{\tau\overline{\tau}} D_\tau W \overline{D_\tau W} + K^{i\overline{j}} D_i W \overline{D_j W} \right). \tag{1.45}$$

[15] In such 'pure-flux compactifications' (see e.g. [77] and the nice discussion in [78]) the AdS scale is generically at the KK scale, which makes it hard to 'uplift' to the phenomenologically required quasi-Minkowski vacuum in a controlled fashion, using quantum corrections.

It can be shown that if there exists a point in moduli space where

$$D_i W = D_\tau W = 0, \quad \forall i, \tag{1.46}$$

this point will be a minimum of the potential (1.45). Thus, (1.45) can stabilize the complex structure moduli and the axio-dilaton supersymmetrically, i.e. at vanishing F-terms.

Since the complex structure moduli and the axio-dilaton are now massive by the above mechanism, they can be integrated out. After having integrated out these fields, the superpotential is a constant, W_0. Below the stabilization scale we are then left with an effective theory which only contains the Kähler moduli T_I. Crucially, as reviewed above, they don't have a potential at tree-level. The Kähler moduli receive their masses only through higher-order corrections to the scalar potential [13–16] which are suppressed by the string coupling g_s or some inverse power of $\mathrm{Re}(T_I)$ with respect to the natural size of the tree-level terms (1.45) away from the minimum. Therefore, these corrections are parametrically small in the limit of large volumes and small string coupling. Accordingly, their effect on the stabilization of the complex structure moduli and the axio-dilaton is small. This justifies the procedure of integrating out the latter.

The form of the higher-order corrections to the potential which stabilize the T_I will be given in chapter 4, which also contains a detailed description of a particular Kähler moduli stabilization mechanism. One characteristic feature of these higher-order corrections is that they generally mix moduli. For example, loop corrections to the Kähler potential are known to depend on complex structure moduli, the axio-dilaton, Kähler moduli, and open-string moduli [17, 18], and therefore involve potential inflaton candidates.[16] They precisely represent the shift-symmetry-breaking quantum corrections discussed at the end of section 1.1, which are relevant for inflation phenomenology. What is more, since the tree-level vacuum energy density vanishes due to (1.46), the higher-order corrections set the scale of the true vacuum energy density, i.e. the Hubble scale (e.g. V_0 in (1.25)). We have thus learned two lessons: First, a study of inflationary scenarios in string theory must, at the same time, be a study of (Kähler) moduli stabilization. Second, it is clear that potentials like (1.25) (with *constant* V_0) do not arise trivially in in this context.

1.3. D7-Brane Inflation: Summary of Results

In this section we introduce the two inflation models which we analyze in this thesis and summarize the main results. Since moduli stabilization is such a crucial issue in string inflation, we work in the context of Type IIB string compactifications where this matter is best understood. It turns out that, amongst the various moduli fields which arise in these compactifications, D7-brane position moduli are promising inflaton candidates which can realize small-field as well as large-field inflation in string theory.

1.3.1. Hybrid Inflation and String-Theoretic Constraints

String-derived inflation models are typically of the small-field type. This can easily be seen in the case of D-brane inflation [32, 36], i.e. in models where the inflaton is associated with the position modulus of a D-brane: Schematically, the four-dimensional Lagrangian which

[16] Recall that we are going to identify an open-string modulus, more precisely the position modulus of a D7-brane, with the inflaton.

contains the Einstein-Hilbert term and the kinetic term for a D-brane position modulus takes the form

$$\mathcal{L} \sim \sqrt{-\tilde{g}} \left(g_s^{-2} \hat{\mathcal{V}} \tilde{R} + g_s^{-1} \hat{\mathcal{V}}_{||} \tilde{g}^{\mu\nu} \partial_\mu \hat{r} \partial_\nu \hat{r} \right). \tag{1.47}$$

This form arises by assuming a constant dilaton profile and performing the integration over the internal dimensions in (1.30) and (1.31) (starting in the ten-dimensional string frame). The quantity $\hat{\mathcal{V}}$ is the volume of the compact space, measured in the ten-dimensional string frame in units of the ten-dimensional string length $\ell_s = 2\pi\sqrt{\alpha'}$. Furthermore, $\hat{\mathcal{V}}_{||}$ measures the volume of the subspace of the internal manifold which is wrapped by the D-brane. Finally, $\hat{r}$ denotes the position of the D-brane in one (arbitrary) transverse direction. The kinetic term for $\hat{r}$ in (1.47) arises from the pullback of the metric to the worldvolume of the brane (1.32). In order to obtain a canonical Einstein-Hilbert term in four dimensions we rescale $\tilde{g}_{\mu\nu} = \frac{g_s^2}{\hat{\mathcal{V}}} g_{\mu\nu}$. This gives a kinetic term $\sim g_s \frac{\hat{\mathcal{V}}_{||}}{\hat{\mathcal{V}}} (\partial \hat{r})^2$ for the brane position modulus and thus the corresponding canonically normalized field is defined as

$$\phi \sim \hat{r} \sqrt{g_s \frac{\hat{\mathcal{V}}_{||}}{\hat{\mathcal{V}}}}. \tag{1.48}$$

If we now assume that the internal manifold has only one typical length scale $\hat{R}$, this entails $\hat{r}_{\text{max.}} \sim \hat{R}$ for the maximal value of $\hat{r}$ and hence

$$\phi_{\text{max.}} \sim \sqrt{g_s \hat{R}^{p-7}}. \tag{1.49}$$

In the weak-coupling and large-volume limit this is smaller than unity for $p \leq 7$, implying a sub-planckian field range.[17]

Such sub-planckian field spaces occur more generally in the case of axions,[18] Wilson lines, and also in the case of most Kähler moduli.[19] We will make this more explicit in chapter 3, where we also take care of numerical factors. In view of these numerous examples of small field spaces, we will focus on small-field inflation in this section.

As we have motivated in section 1.1.3, realizing small-field inflation requires either some non-generic structure in the UV theory, such as a shift symmetry, or some sort of tuning (see e.g. [38]). While the latter is certainly a viable possibility to consider, we attempt to derive non-fine-tuned inflation models with shift symmetries from string theory.

The presence of a shift symmetry guarantees the leading-order flatness of the potential. The shift symmetry is broken (cf. section 1.1.3) by non-perturbative effects and, possibly, by explicit breaking terms in the theory, such as a mass term or interactions with other fields. A breaking only by non-perturbative effects does not lead to a viable inflation model due to the smallness of the axion decay constant in string theory (cf. the discussion in section 1.1.3 and the computation of axion decay constants in section 2.3). On the other hand, a breaking via a coupling of the inflaton φ to another field χ, e.g. of the form

[17] D8-branes can naively realize a super-planckian field range. However, they do not occur in Type IIB string compactifications.

[18] This is precisely the statement that no super-planckian axion decay constants are obtained in weakly-coupled string theory [60].

[19] An exception occurs for some bulk moduli such as the breathing mode of the compact space which, however, does not enjoy a shift symmetry. Nevertheless, owing to the non-generic structure of the Kähler moduli Kähler potential, there exist proposals for large-field and small-field inflation using Kähler moduli as the inflaton [84, 85].

$\lambda\varphi^2\chi^2$, can lead to a viable inflation model in the context of 'hybrid inflation' [34,35]. The leading-order potential for this class of models is given by

$$V(\varphi, \chi) = \lambda\varphi^2\chi^2 + \kappa(\chi^2 - \chi_0^2)^2, \tag{1.50}$$

which includes a tachyonic mass term for the field χ, plus an effective χ-mass whose size depends on the vev of φ. During inflation φ is non-zero, such that $\chi = 0$, and inflation is driven by the vacuum energy $\kappa\chi_0^4$. A loop-induced potential for φ, generated by its coupling to χ, leads to a slow decrease in φ, such that, at some point, χ obtains a tachyonic mass and inflation ends in a waterfall instability.

In chapter 2 we review from [86–88] how this hybrid inflation scenario is technically natural when put into a supersymmetric context. Furthermore, we elaborate on a particular virtue of this model: Since the evolution of the slow-roll parameter ϵ is generically non-monotonic in hybrid natural inflation, the tensor-to-scalar ratio can be considerably larger than the naive Lyth-bound expectation (1.23) suggests. For this to happen, ϵ has to be relatively large during the initial few observable e-foldings of inflation, implying a relatively large r. Subsequent to this initial stage of inflation, the bulk of the $N = 60$ e-folds is accumulated in a phase of small ϵ. We quantify the maximal possible size of the tensor-to-scalar ratio as $r \lesssim 7.6 \cdot 10^{-4}$ in such a scenario. The main obstructions are the limited size of the axion decay constant as well as the observational bound on the running of the spectral index, $\mathrm{d}n_s/\mathrm{d}\ln k$. As an additional result, we report that a curvaton-dominated regime is not possible in hybrid natural inflation.

1.3.2. Introduction to Fluxbrane Inflation

In chapter 3 we attempt to realize the appealing field theory model of hybrid inflation in string theory. To this end, we identify the relative position modulus of a pair of non-supersymmetric D7-branes (i.e. D7-branes which break *relative* supersymmetry) with the inflaton. The waterfall field χ corresponds to a recombination mode of this D7-brane pair.

To intuitively understand the relative supersymmetry breaking, recall that the supersymmetry algebra includes the generators of translations. A D-brane breaks translational invariance in its transverse directions. Consequently, in ten-dimensional flat space some amount (more precisely: half) of the previously present supersymmetry will be broken by a D-brane.[20] In case two (or more) D-branes are present, depending on their dimensionality and relative angles, supersymmetry is generically broken completely. If this is true, stability of the vacuum (which is usually ensured by supersymmetry) is not granted anymore and there can be unstable directions along which a tachyon condensation process takes the system to its true minimum. For two non-supersymmetric D-branes a tachyon arises from the zero mode of the open string which stretches between the branes. The mass-squared of this zero mode depends on the brane distance, i.e. on the inflaton in our model, and can be either positive or negative. This is precisely the structure encountered in (1.50).

'Relative fluxes' on the D-branes (i.e. fluxes for a certain linear combination of the U(1) gauge theories living on the two branes) have the same effect as relative angles: Just like a magnetic field obstructs the motion of an electron, fluxes break translational invariance

[20] This is also true in compactifications on Calabi-Yau orientifolds with D-branes and O-planes which give rise to $\mathcal{N} = 1$ theories in four dimensions, in contrast to $\mathcal{N} = 2$ for the Calabi-Yau case without branes [89].

along the D-brane directions due to the coupling of the open-string endpoints to the gauge field

$$\int_{\partial W} \mathrm{d}X^M A_M, \tag{1.51}$$

where ∂W is the boundary of the worldsheet. Consequently, brane fluxes generally also break supersymmetry. In fact, fluxes (rather than the relative orientation of D-branes) are the appropriate language in Type IIB compactifications [89].

Turning on a flux $\mathcal{F} := B_2 + 2\pi\alpha' F_2$ for the 'relative' U(1) theory of two D7-branes (cf. chapter 3) leads, amongst others, to the coupling

$$\frac{\mu_7}{2} \int_{\mathrm{D7}} C_4 \wedge \mathcal{F} \wedge \mathcal{F} \tag{1.52}$$

from the Chern-Simons term in (1.30). Expanding $C_4 = C_2^\alpha \wedge \omega_\alpha + c_\alpha \tilde{\omega}^\alpha + \dots$ (cf. section 1.2.2) and $\mathcal{F} = \mathcal{F}_{\text{int.}} + \mathcal{F}_{\text{ext.}}$ we find the four-dimensional coupling

$$\sim q_\alpha \int_{\mathcal{M}_4} C_2^\alpha \wedge \mathcal{F}_{\text{ext.}}, \tag{1.53}$$

with the charge $q_\alpha \sim \int_\Sigma \omega_\alpha \wedge \mathcal{F}_{\text{int.}}$, where Σ is the internal four-cycle wrapped by the D7-brane. The presence of such a term implies the gauging of the shift symmetry of the axion dual to C_2^α which, by the self-duality constraint $\tilde{F}_5 = *\tilde{F}_5$, is $c_\alpha = \mathrm{Im}(T_\alpha)$ (see e.g. the nice account in [90, 91]). Thus, under a gauge transformation the superfield T_α shifts according to $T_\alpha \to T_\alpha + i q_\alpha \epsilon$, where ϵ is the gauge parameter [91]. This *gauged isometry* in the Kähler moduli space leads to the appearance of a D-term potential

$$V_D = \frac{g_{\mathrm{YM}}^2}{2} \left(q_\alpha \partial_{T_\alpha} K + q_I |\chi_I|^2 \right)^2 \tag{1.54}$$

in the effective action [56], where g_{YM} is the gauge coupling.

In the above expression the χ_I are scalar fields with charges q_I under the gauge field. Microscopically, they arise from the zero modes of open strings stretched between the branes. A field χ_J with $q_J < 0$ corresponds to the waterfall field in the fluxbrane inflation model. It couples supersymmetrically to the modulus measuring the brane separation, i.e. the inflaton, via a superpotential term $W \sim \sqrt{\lambda}\varphi\chi_I\chi_J$ [92]. Owing to the resulting effective supersymmetric mass term, the vev of χ_J vanishes during inflation, which means that the D-term potential contributes a positive energy density

$$V_D = \frac{g_{\mathrm{YM}}^2 \xi^2}{2}, \quad \xi := q_\alpha \partial_{T_\alpha} K = \frac{1}{4\pi} \frac{\int_\Sigma J \wedge \mathcal{F}}{\mathcal{V}}, \tag{1.55}$$

which drives inflation. Here, J is the Kähler form and $\mathcal{V}$ is the volume of the Calabi-Yau, both given in the ten-dimensional Einstein frame. Upon tachyon condensation, the field χ_J obtains a non-trivial vev and the potential is taken to zero quickly. This is the idea of D-term hybrid inflation [93, 94], put in a stringy context.

Relation to other Brane Inflation Proposals

Compared to other brane inflation scenarios, inflation with fluxed D7-branes ('fluxbrane inflation') has some crucial advantages: The first explicit brane inflation proposal [36] used a brane-antibrane pair whose distance played the role of the inflaton. The potential for such a pair of branes follows from a Coulomb law in the transverse $9-p$ dimensions and thus reads $\sim -\sqrt{-\tilde{g}}g_s^{-1}\hat{\mathcal{V}}_{||}\left(A - Bg_s/\hat{r}^{7-p}\right)$. Again, the factor $\hat{\mathcal{V}}_{||}$ comes from the integration over the brane-parallel internal directions, and A and B are some positive $\mathcal{O}(1)$ numbers. Canonically normalizing $\hat{r}$ according to (1.48) gives

$$-\eta \sim \frac{B}{A}\left(\frac{\hat{R}}{\hat{r}}\right)^{9-p}. \tag{1.56}$$

Thus, to obtain $|\eta| \ll 1$ one has to realize $\hat{r} \gg \hat{R}$, which is impossible physically. What is more, the presence of the antibrane generally breaks supersymmetry at the string scale, which hinders putting the model in a supersymmetric language.[21]

The latter issue is avoided if one considers $p = 3$ and replaces the antibrane by a D7-brane with flux [17, 37, 95–109]. Like in our fluxbrane inflation proposal, this flux leads to a D-term energy density which drives inflation. Thus, much in the spirit of the above discussion, the D3/D7 inflation model in principle allows for a description in terms of D-term hybrid inflation. However, since the flux on the D7-brane merely corresponds to dissolved D3-branes (and, most importantly, is quantized), the same negative conclusion regarding the field range (equation (1.56)) applies to this model.

In [38] the issue of the small field range in brane-antibrane inflation was evaded by considering warped geometries. Opposed to the spirit of using shift symmetries, in this model a sufficiently flat potential is obtained by explicit tuning of parameters of the warped geometry. This significantly departs from the original hybrid inflation idea.

Fluxbrane inflation is proposed to save the idea of hybrid inflation in string theory.[22] The scenario is conceptually similar to the D3/D7 model, featuring a potential

$$V(\varphi) = \frac{g_{\mathrm{YM}}^2\xi^2}{2}\left(1 + \alpha\ln\left(\frac{\varphi}{\varphi_0}\right)\right). \tag{1.57}$$

The second term arises as a Coleman-Weinberg loop correction to the effective potential [125]. Consequently, α is essentially given by the gauge coupling, $\alpha \sim g_{\mathrm{YM}}^2/16\pi^2$. For the gauge theory living on the D7-branes, $g_{\mathrm{YM}}^2 = 2\pi/\mathcal{V}_{\mathrm{D7}}$ (cf. (1.31)), where $\mathcal{V}_{\mathrm{D7}}$ is the volume of the internal four-cycle Σ wrapped by the D7-brane. It is thus immediately clear, using (1.48), that $|\eta| \ll 1$ cannot be achieved generically for a D3-brane moving in the background of a D7-brane, while it is easily realized for a pair of D7-branes.[23]

As we will detail in chapter 3, a further considerable virtue of the fluxbrane inflation model concerns the tension of cosmic strings, produced in the phase transition at the end of the inflationary epoch. In the standard D-term inflation model [93, 94], α is given by

[21] It is argued that the use of a supergravity description may still be possible if the antibranes are put in a warped region of the internal manifold, such that their contribution to the energy density is highly redshifted [13].

[22] A closely related idea is that of Wilson line inflation [110, 111] and inflation from branes at angles [112–122] (see [123, 124] for related earlier proposals). These two scenarios can be viewed as T-dual Type IIA versions of the fluxbrane inflation model. We believe that investigating inflation models in the IIB context can be more fruitful, since moduli stabilization is better understood in these models.

[23] In fact, for D7-branes the moment of 60 e-folds before reheating corresponds to a brane distance which is below the string length. One might thus fear that the Coleman-Weinberg intuition fails in this regime. However, we showed in [21] that this is, in fact, not the case and the potential (1.57) remains valid.

$\alpha = g_{\rm YM}^2/16\pi^2$. Thus, using the phenomenological constraints (1.13) and (1.19), the D-term ξ is generically fixed at a value corresponding to a cosmic string tension which is too large to be consistent with observations.[24] This conclusion equally applies to the stringy embedding in terms of D3/D7 inflation. On the other hand, in fluxbrane inflation

$$\alpha = \frac{g_{\rm YM}^2}{16\pi^2}\left(-2\int_\Sigma \mathcal{F}\wedge\mathcal{F} + \frac{g_{\rm YM}^2}{2\pi}\left(\int_\Sigma J\wedge\mathcal{F}\right)^2\right). \tag{1.58}$$

The first term measures the induced (quantized) D3 charge on the D7-brane. Its presence would lead to the same negative conclusion regarding the size of the cosmic string tension. However, in fluxbrane inflation we can choose a flux satisfying $\int_\Sigma \mathcal{F}\wedge\mathcal{F} = 0$ and suppress the term $\int_\Sigma J\wedge\mathcal{F}$ below its natural value. This solves the cosmic string problem of D3/D7 inflation.

Requiring the fluxbrane inflation model to reproduce the correct amplitude of curvature perturbations fixes the overall volume at $\mathcal{V} \simeq 1.7\cdot 10^6$. The cosmic string bound is satisfied owing to a mild hierarchy in the compact space, such that $\int_\Sigma J\wedge\mathcal{F}/\sqrt{\mathcal{V}_{\rm D7}}$ is slightly smaller than unity.[25] For $\alpha \ll 1$, potentials of the form (1.57) fix $n_s = 1 - 1/N$, which is slightly too large to be consistent with the most recent measurements [48]. However, since we expect alterations of the model as soon as moduli stabilization is taken into account, we do not worry too much about this slight discrepancy.

1.3.3. Moduli Stabilization in Fluxbrane Inflation

All of the above assumes that all moduli, in particular the Kähler moduli entering ξ, $g_{\rm YM}$ and α, are stabilized. This clearly needs some justification. In fact, while the leading-order F-term potential for the Kähler moduli vanishes (cf. (1.44)), the D-term potential (1.55) generically causes a runaway of the overall volume to infinity. Thus, taken on its own, the leading-order analysis of chapter 3 is too naive. Rather, the fluxbrane inflation model needs to be combined with a viable Kähler moduli stabilization scenario and suitability of the potential for supporting inflation should be checked *after* moduli stabilization. This is done in chapter 4, using the established Large Volume Scenario [15]. In particular, we demonstrate consistency with the phenomenological requirements derived in chapter 3.

The Large Volume Scenario uses a combination of α'-corrections to the Kähler potential[26] [127]

$$K = -2\ln\left(\mathcal{V} + \frac{\xi}{2g_s^{3/2}}\right), \quad \xi = -\frac{\zeta(3)\chi(X_3)}{2(2\pi)^2}, \tag{1.59}$$

and non-perturbative D3-instanton corrections to the superpotential

$$W = W_0 + \sum_p A_p e^{-2\pi T_p}, \tag{1.60}$$

[24] This can be avoided if the trilinear superpotential coupling λ is extremely small, such that the inflaton rolls a small distance during the last $N = 60$ e-folds of inflation, i.e. $\varphi_N \approx \varphi_0$ [126]. However, in D3/D7 inflation as well as fluxbrane inflation an underlying $\mathcal{N} = 2$ supersymmetry relates λ to $g_{\rm YM}$, excluding this particular parameter regime.

[25] An alternative strategy is to appeal to a partial cancellation of terms appearing in $\int_\Sigma J\wedge\mathcal{F}$. We will discuss this possibility in chapter 5.

[26] Unfortunately, the symbol ξ is used for the D-term as well as the quantity defined in (1.59). In order not to cause confusion by differing from the notation used in the papers on which this thesis is based, we continue using ξ for both quantities. It will be clear from the context what is meant in each case.

to stabilize Kähler moduli. Here, $\chi(X_3)$ is the Euler characteristic of the Calabi-Yau, $\zeta(3) \simeq 1.20$ is the Riemann zeta-function, and A_p are some prefactors which depend on the complex structure moduli and which are assumed to be constant. In its simplest version the Large Volume Scenario is implemented on a Calabi-Yau manifold which has two Kähler moduli, one corresponding to a large four-cycle which sets the size of the manifold, and one corresponding to a 'diagonal' small blow-up cycle. The volume is then given by

$$\mathcal{V} = b\tau_b^{3/2} - c\tau_s^{3/2}, \quad \text{where} \quad \tau_i := \mathrm{Re}(T_i), \quad \tau_s \ll \tau_b. \tag{1.61}$$

Taking into account only the non-perturbative effects associated with the small cycle and plugging the two expressions (1.59) and (1.60) into the standard formula for the F-term potential (1.38) yields (after integrating out the axion $\mathrm{Im}(T_s)$)

$$V_F \sim \alpha \frac{\sqrt{\tau_s} e^{-4\pi\tau_s}}{\mathcal{V}} - \beta \frac{|W_0|\tau_s e^{-2\pi\tau_s}}{\mathcal{V}^2} + \gamma \frac{\xi |W_0|^2}{\mathcal{V}^3}. \tag{1.62}$$

This potential stabilizes the field τ_s at the mass scale $\frac{|W_0|}{\mathcal{V}}$, while τ_b is stabilized at $\frac{|W_0|}{\mathcal{V}^{3/2}}$ (after canonical normalization). In the resulting AdS minimum all three terms in (1.62) are, to a good approximation, of equal size, which means that the scale of the vacuum energy density is $\sim -|W_0|^2/\mathcal{V}^3$. The volume is exponentially large, $\mathcal{V} \sim |W_0| e^{2\pi\tau_s}$. A more detailed account of the Large Volume Scenario can be found in appendix B.

There are several issues which prohibit combining this simplest scenario with fluxbrane inflation. First, it is obvious that, at large volume and $\mathcal{O}(1)$ values for the superpotential $|W_0|$, the D-term potential $V_D \sim \mathcal{V}^{-2}$ is completely dominant, leading to a runaway behavior of the potential to infinite volume. This can be avoided only by making $|W_0|$ parametrically large, such that at least $|W_0| \sim \sqrt{\mathcal{V}}$. Such a scaling, however, implies that at large volume the gravitino mass $m_{3/2}$ becomes parametrically larger than the Kaluza-Klein scale m_{KK}, meaning that there is no regime in which the leading-order supergravity description, taking into account only the zero modes of the string excitations, is valid. Second, using only one large Kähler modulus we cannot suppress α in (1.58) below its natural size: The compact space is characterized by only one typical length and all quantities scale in the expected way with this length.

These issues can be resolved in models with more than two Kähler moduli. Consequently, in chapter 4 we consider a model with four Kähler moduli of hierarchically different sizes.[27] Since the corrections (1.59) and (1.60) involve only the overall volume and the volume of the small blow-up cycle, further higher-order effects need to be taken into account in order to stabilize all Kähler moduli. We consider string loop corrections to the Kähler potential [18, 128–131]

$$\delta K_{(g_s)} \sim g_s \frac{C_i^{\mathrm{KK}} t^i}{\mathcal{V}} + \frac{C_i^{\mathrm{W}}}{t^i \mathcal{V}}, \tag{1.63}$$

with some constants $C_i^{\mathrm{KK,W}}$,[28] in addition to (1.59) and (1.60). They arise from Kaluza-Klein and winding excitations of strings propagating on two-cycles. Consequently, these corrections involve not only the overall volume, but also some different combinations of

[27] It turns out that the minimal extension to a model with three Kähler moduli does not work.

[28] In general the $C_i^{\mathrm{KK,W}}$ will depend on open-string moduli such as the D7-brane positions. We will ignore this for the moment, assuming that the dependence is very weak or even absent, but come back to this important issue in the next subsection.

Kähler moduli and are thus suitable for stabilization of all moduli. Importantly, while the first term in (1.63) dominates over the corrections in (1.59), its effect in the F-term potential is $\sim |W_0|^2/\mathcal{V}^{10/3}$ and thus subleading with respect to the terms in (1.62). This is the 'extended no-scale structure' [128, 130, 131]. We will have more to say about this structure in the following subsection.

Our model now works as follows: The potential (1.62) stabilizes the overall volume and τ_s in an AdS minimum. This minimum then needs to be uplifted to a Minkowski minimum, which can in principle be achieved via some anti-D3-branes in a warped throat [13], or via another D-term.[29] The latter possibility turns out not to work trivially in the concrete model we consider, which is why we resort to an uplift through antibranes. The inflationary uplift to de Sitter is provided by the D-term (1.55), which depends not only on the overall volume, but also on other combinations of Kähler moduli (e.g. through $\int_\Sigma J \wedge \mathcal{F}$). Assuming a stabilized overall volume, this will drive these other combinations of Kähler moduli towards a minimum in which the D-term potential vanishes. At some point, however, the string loop corrections (1.63) will become relevant and balance the D-term. This will stabilize all Kähler moduli in a minimum with non-vanishing D-term. Neglecting the W_0 dependence, the balancing happens to occur at the natural size of the LVS F-terms (1.62), i.e. at $\sim \mathcal{V}^{-3}$.

In our model we choose $\mathcal{F}$ to be Poincaré-dual to an actual (physical) two-cycle of the internal manifold. It turns out that for $|W_0| = 1$ the balancing of the D-term against loop corrections would imply a shrinking of this two-cycle below the string scale, which potentially spoils the supergravity approximation. While this issue can be avoided for a more general flux choice (see the discussion below), in chapter 4 we choose to increase the value of the tree-level superpotential to $|W_0| \simeq 2 \cdot 10^3$ in order to maintain perturbative control. This entails $\mathcal{V} \simeq 3.5 \cdot 10^7$, while the string coupling remains small, $g_s \simeq 3 \cdot 10^{-2}$.

1.3.4. Towards a Consistent Model of Fluxbrane Inflation

In the above analysis we completely neglected a possible open-string dependence in the F-term potential. In particular, we assumed the factors $C_i^{\mathrm{KK,W}}$ as well as the tree-level superpotential W_0 to be independent of the D7-brane moduli. Furthermore, in the F-term potential we did not take into account the tree-level open-string Kähler potential. This is clearly an oversimplification: In all three places the inflaton generically shows up, threatening the nice properties of our inflation scenario. In chapter 5 we approach these issues, aiming towards a consistent overall picture of fluxbrane inflation.

For a generic leading-order Kähler potential $K \sim c\bar{c}$ for the canonically normalized D7-brane position modulus c the F-term potential is too steep to support inflation (see the discussion in chapter 2). This is the 'supergravity η-problem' [139]. As briefly mentioned before, in fluxbrane inflation we appeal to a shift symmetry to ensure the flatness of the inflaton potential. That is, we assume that the Kähler potential (1.42) in fact reads

$$K \supset -\ln\left(-i(S-\overline{S}) - k_{\mathrm{D7}}(z,\overline{z};c-\overline{c})\right). \tag{1.64}$$

This would imply invariance of the Kähler potential under $c \to c+\delta$, $\delta \in \mathbb{R}$, and the inflaton would be identified as $\varphi \sim \mathrm{Re}(c)$.

Such a structure of the Kähler potential is indeed expected to arise in the vicinity of the 'large complex structure' point of the Calabi-Yau manifold. To get some intuition for

[29] For some proposals of D-term uplifting see [91, 132–138] and also the discussion in [23].

this statement, note that there exist various dualities which map different string theories onto another. For example, Type IIA string theory compactified on a circle with length $\hat{L}$ (measured in units of the string length in the ten-dimensional string frame) is the same as Type IIB string theory compactified on a circle with length $1/\hat{L}$. This is called 'T-duality'. A T-duality transformation exchanges Dirichlet and Neumann boundary conditions and therefore changes the dimensionality of a D-brane. Starting with Type IIB with D7-branes and performing three T-dualities, two on circles along the D7-brane directions and one on a circle transverse to the D7-branes, the D7-brane distance will be mapped to a Wilson line on D6-branes in Type IIA string theory. This Wilson line descends from a gauge field component along the internal dimensions of the D6-branes and enjoys a perturbative shift symmetry to all orders in α' as a consequence of the higher-dimensional gauge invariance. This shift symmetry thus holds as long as the α'-expansion is valid, which is the case at large volume in the Type IIA theory. Under T-duality complex structure moduli are identified with Kähler moduli.[30] Thus, the large-volume limit in the IIA description corresponds to a 'large complex structure' limit in the IIB description, which is why we expect the structure (1.64) to be present in precisely this region of moduli space. We will give further evidence for the shift symmetry in the D7-brane position moduli sector in section 5.1.1.[31]

Let us now consider the superpotential. In the presence of fluxed D7-branes the leading-order superpotential (1.43) is extended to

$$W = W_{\text{bulk}} + W_{\text{brane}} = \int_{X_3} G_3 \wedge \Omega_3 + \int_{\Gamma_5} \mathcal{F} \wedge \Omega_3 + \ldots , \tag{1.65}$$

where Γ_5 is a five-chain which is swept out as the D7-branes are deformed into each other. The second term in this expression as well as further contributions, which were discussed in [29], will generically introduce a dangerous brane-position dependence in the superpotential. The brane-dependent term shown in (1.65) can in fact be avoided if the flux $\mathcal{F}$ is chosen to be of type $(1,1)$, which is consistent with the fluxbrane inflation proposal. On the other hand, terms hidden in the ellipses in (1.65) are generally non-vanishing even in this case. To avoid an appearance of those terms a non-generic (bulk) flux choice is necessary. The analysis is model-dependent and was performed for a compactification of F-theory on $K3 \times K3$ in [29]. We will not review this here, but give some more general statements about the structure of the superpotential in section 5.1.2.

Finally, let us discuss the quantities $C_i^{\text{KK,W}}$. Couplings of the inflaton φ e.g. to the waterfall fields, as in (1.50), clearly break the shift symmetry. They induce an inflaton dependence of the potential at one-loop. These higher-order terms arise from Kaluza-Klein and winding-mode exchange between the branes and correspond precisely to the Kähler potential corrections (1.63), used to stabilize Kähler moduli in chapter 4. Thus, the $C_i^{\text{KK,W}}$ are generically functions of φ. Consequently, one should ask two questions. First, the extended no-scale structure, mentioned below (1.63), is known to hold for *constant* $C_i^{\text{KK,W}}$. This structure is essential to ensure the subleading nature of loop corrections with

[30] This can be understood intuitively on a rectangular torus with volume $\hat{L}_1\hat{L}_2$ and complex structure $i\hat{L}_2/\hat{L}_1$. T-duality along the circle with length $\hat{L}_1$ gives the dual theory on a torus with volume $\hat{L}_2/\hat{L}_1$, which was the imaginary part of the complex structure in the original theory.

[31] In inflation models with D3-branes no shift symmetry will be present generically. This is because the moduli space of D3-brane positions is just the compactification Calabi-Yau which, if it has full SU(3) holonomy, has no isometries. Only in special examples of compactifications on manifolds with reduced holonomy there may arise shift-symmetric Kähler potentials for D3-brane moduli [98, 102, 105].

respect to (1.59). Will this structure persist, even if the $C_i^{\mathrm{KK,W}}$ depend on an additional light degree of freedom, namely the brane position modulus? Second, even if the extended no-scale structure continues to hold, we saw that the scale of the vacuum energy in the scenario discussed in chapter 4 is set by the size of the loop corrections, naively implying $|\eta| \sim 1$. Can this negative conclusion be avoided?[32]

The answer to the first question is "yes". In section 5.2 we will demonstrate explicitly that the extended no-scale structure continues to hold, even if the brane position modulus is present as an additional light field in the effective theory. The answer to the second question is also "yes", but there are two sides to it: One might hope that in particular models the functions $C_i^{\mathrm{KK,W}}$ are such that they feature a brane-independent part which is enhanced with respect to the brane-dependent part e.g. by some power of the complex structure moduli. This seems to be the case in toroidal models and is currently investigated [140]. However, expecting such a structure to arise more generally is likely to be too optimistic. Rather, in chapter 5 we propose a slightly different variant of Kähler moduli stabilization in the Large Volume Scenario, which leads to a consistent fluxbrane inflation scenario without making non-generic assumptions about the structure of the $C_i^{\mathrm{KK,W}}$. The idea is to use additional fluxes on D-branes wrapping different four-cycles to stabilize ratios of four-cycle volumes at leading order in a minimum of vanishing D-terms, as in [141]. The overall volume is then stabilized in the usual Large Volume Scenario, leading to an AdS minimum $\sim -|W_0|^2/\mathcal{V}^3$. This is then uplifted via non-vanishing D-terms first to Minkowski and then to de Sitter. These uplift D-terms have to be tuned to small values in order not to destabilize the LVS minimum. This is an explicit tuning in the Kähler moduli sector. As explained in chapter 5, this tuning is part of the tuning of the cosmological constant in the D-term uplifting proposal.[33] Therefore, the fluxbrane inflation scenario arises quite naturally in string compactifications with D-term uplifting.

It turns out that in most of the parameter space the dominant contribution to the inflaton potential arises from (periodic) corrections to the F-term potential induced by loop corrections to the Kähler potential. While they dominate over the previously analyzed corrections to the D-term potential, they are parametrically subleading with respect to the higher-order terms which stabilize the volume moduli. Consequently, Kähler moduli stabilization is not upset during inflation.

We will demonstrate that the resulting inflaton potential is in agreement with the most recent measurements of the spectral index n_s and the amplitude of curvature perturbations. It gives rise to $N = 60$ e-folds of inflation, the running of the spectral index is moderately small $n_s' \lesssim 10^{-2}$, and the tensor-to-scalar ratio is tiny $r \lesssim 2.6 \cdot 10^{-5}$.

[32] From a different perspective, one might naively wonder why this extended no-scale structure was no issue in the field-theoretic analysis of hybrid natural inflation [86–88]. The reason is that the vacuum energy density in these references arises from leading-order F-terms, such that loop corrections indeed only add subleading terms. By contrast, in conventional Type IIB moduli stabilization the F-term vacuum energy density vanishes at tree-level, due to the no-scale structure of the leading-order Kähler potential. Thus, even neglecting the problem of a potential destabilization in the Kähler moduli space, the extended no-scale structure is needed to ensure a suitable hierarchy between the various higher-order corrections, such that the vacuum energy density and the inflaton-dependent term are parametrically separated and inflation is possible.

[33] Actually, implementing fluxbrane inflation in a model with D-term uplifting slightly worsens the tuning needed to get the correct present-day cosmological constant. The reason is a small hierarchy between the inflationary de Sitter energy density and the AdS energy density before uplifting. This small hierarchy is needed to ensure stability of the uplift.

1.3.5. Introduction to D7-Brane Chaotic Inflation

As explained in the beginning of this section, inflation in string theory is often of the small-field type, simply because the moduli spaces of many inflaton candidates only allow for sub-planckian field excursions. An exception occurs in the case of some bulk moduli: The breathing mode of a Calabi-Yau can realize large-field inflation [85], however, with a tiny tensor-to-scalar ratio r.

Nevertheless, even from moduli with an a priori sub-planckian field range one can sometimes construct large-field inflation models.[34] This can work e.g. by considering $N \gg 1$ axions, all with sub-planckian decay constants f, and associating the inflaton to a *diagonal* direction in the combined axion moduli space. The resulting effective axion decay constant $\sqrt{N}f$ can be super-planckian [62, 156, 157]. Embedding this in string theory seems to require N as large as 10^5 [158], see however, [159].

Instead of considering multiple fields one can also enlarge the field range by breaking the periodicity of an axion to obtain a multi-covering of the field space [28, 29, 39, 40, 160–165], a mechanism also analyzed in field theory [166–172]. The model we proposed in [28] and will discuss in chapter 6 is of this type. The idea is to again use the position modulus c of a D7-brane to embed inflation in string theory. Consequently, many of the investigations performed so far have a direct use also in this realization of large-field inflation. The central insights are the following: In analogy to the fluxbrane inflation scenario, the shift-symmetric Kähler potential for the D7-brane position modulus, depending only on $(c - \overline{c})$, will stabilize $\mathrm{Im}(c)$ via leading-order F-terms, but leave $\mathrm{Re}(c)$ as a flat (periodic) direction. Turning on fluxes will generically break the shift symmetry and stabilize the full complex field c. In fluxbrane inflation we therefore chose non-generic flux, such that the superpotential was independent of c. By contrast, in 'D7-brane chaotic inflation' we want this tree-level breaking of the shift symmetry to obtain a monodromy along $\mathrm{Re}(c)$, which we identify with the inflaton. Thus, the superpotential will involve c. We parametrize this dependence as

$$W = W_0 + \alpha c + \frac{\beta}{2}c^2 + A_s e^{-2\pi T_s}. \tag{1.66}$$

Due to the fact that the leading-order Kähler potential does not mix c with Kähler moduli, only the term

$$V_F \supset e^K K^{c\overline{c}} |D_c W|^2 \tag{1.67}$$

needs to be taken into account in addition to the generically subleading corrections (1.62). In the minimum of the scalar potential, (1.67) vanishes and the Kähler moduli are stabilized in the Large Volume Scenario AdS minimum. This minimum is then uplifted to Minkowski either via anti-D3-branes [13], D-terms [23, 91, 132–138], or some alternative positive contribution to the energy density.

During inflation c is displaced from its minimum, such that (1.67) is non-vanishing. Clearly, if non-vanishing, (1.67) dominates over (1.62) for a generic superpotential (the terms in (1.67) scale as $\sim \mathcal{V}^{-2}$, while (1.62) scales as $\mathcal{V}^{-3}$). Since (1.67) is positive definite, this would lead to a runaway behavior towards infinite volume. However, a viable inflation model can be obtained if the superpotential depends only very weakly on c (i.e. $|\alpha|$ and $|\beta|$ are small), which can be achieved via flux tuning. Owing to this tuning and the shift

[34] Sparked by the recently claimed detection of primordial tensor perturbations, a plethora of different string embeddings of large-field inflation have been proposed [142–152], some extending older ideas [61, 153–155].

symmetry in the Kähler potential of the D7-brane position modulus, the scalar potential is much steeper in the direction $\mathrm{Im}(c)$ than in $\mathrm{Re}(c)$. Thus, $\mathrm{Im}(c)$ is essentially fixed by (1.67) and the inflaton trajectory is mostly along the $\mathrm{Re}(c)$ direction. Due to the tuning of α and β the energy density during inflation associated with (1.67) remains smaller than the F-term contributions (1.62), such that the Large Volume Scenario is not upset. In terms of explicit numbers, we show that the model works e.g. for $|W_0| = 10$, an overall volume $\mathcal{V} = 10^3$, and $|\beta| = 0.5 \cdot 10^{-2}$.

The resulting potential in our model turns out to be quadratic, reminiscent of the chaotic inflation example discussed in section 1.1.3. It thus entails a large tensor-to-scalar ratio, $r \simeq 0.16$.

Let us emphasize again that the three pillars on which the fluxbrane inflation scenario is built are crucial ingredients also in the D7-brane chaotic inflation model: We have discussed the relevance of the shift-symmetric Kähler potential and the non-generic flux choice already. The validity of the extended no-scale structure also in the presence of light D7-brane moduli is crucial for the viability of the LVS in general, and therefore also in the D7-brane inflation scenario.

Our model represents a very explicit proposal for how to realize large-field inflation in spontaneously-broken supergravity, with constraints from moduli stabilization taken into account. Previous to this proposal, no such model was available. For example, the original scenarios [39, 40] need the presence of antibranes in order to obtain a non-flat potential for the axion. Accordingly, supersymmetry is broken at the string scale and the validity of the description in terms of an effective four-dimensional supersymmetric action is questionable [173]. Consequently, the discussion of moduli stabilization is not easily possible in these models. By contrast, compatibility with moduli stabilization can be checked explicitly in D7-brane chaotic inflation and the amount of fine tuning needed to ensure viability of the model can be quantified. Importantly, the required flux tuning can in principle be discussed in explicit models, though this had to be left to future work.

1.3.6. D7-Brane Inflation in Light of the BICEP2 Results[35]

Recently, the BICEP2 collaboration has reported the measurement of B-mode polarization [33]. They claim that the measurement is well fit by a B-mode spectrum sourced by primordial gravitational waves which are produced during an epoch of slow-roll inflation. The corresponding amplitude of primordial tensor perturbations relative to the amplitude of scalar perturbations is given by $r = 0.2^{+0.07}_{-0.05}$.

B-modes are sourced by various effects (see e.g. [174–178]). For example, it was shown in [179] that the conclusion of the BICEP2 team that $r = 0$ is ruled out with high significance is altered if one includes cosmic strings in the model (see, however, [180]).[36] We believe that, while the attribution of the B-mode signal to primordial tensor modes is tempting, it will take additional time and effort to prove this claim and reliably exclude other possible sources.

The predicted value of the tensor-to-scalar ratio r in the two D7-brane inflation scenarios outlined above is certainly one important feature which phenomenologically distinguishes the models from each other. In particular, if the measurement in [33] and its attribution to

[35] This subsection is taken from the introduction of [29], which was written by the author of this thesis.

[36] Another interesting issue has been raised in [181], where it was stated that 'radio loops' may dominate over the primordial B-mode signal in some regions of the sky.

primordial gravitational waves is correct, this would imply that models of small-field inflation, such as fluxbrane inflation [21, 23, 29], are ruled out [54, 55] (see, however, [182–188] and references therein). On the other hand, in such a situation D7-brane chaotic inflation [28] looks very promising: The leading-order inflaton potential in this model takes a quadratic form, well-known since the early proposal of chaotic inflation [41]. Correspondingly, the tensor-to-scalar ratio is large, $r \simeq 0.16$, in reasonable agreement with the BICEP2 results. Confirmation or rejection of the gravitational wave signal is thus crucial to be able to tell whether D7-brane chaotic inflation in the form proposed in [28] is phenomenologically viable.

1.3.7. Outline of this Thesis and Specification of Sources

The structure of this introduction also represents the order in which the various topics are covered in this thesis. That is, we start in chapter 2 with an introduction to hybrid natural inflation. We review that this model can be technically natural in field theory and discuss its phenomenology with an eye on the stringy embedding in terms of fluxbrane inflation. Furthermore, we analyze stringy constraints on the size of the axion decay constant in specific examples. A particular focus will be on the maximal possible size of the tensor-to-scalar ratio in such models. Section 2.1.1 until section 2.4 is mainly copied from [24], which is work done in collaboration with Arthur Hebecker and Alexander Westphal. The first draft of section 2.2 until but not including section 2.2.1 was written by A. Westphal, but heavily edited by myself afterwards. The remaining sections, starting from section 2.2.1, were written by myself.

In chapter 3 we then introduce in detail the fluxbrane inflation model as a stringy implementation of hybrid natural inflation. Much of sections 3.1 and 3.2 and a small part of the outlook is copied from [21]. This is work done in collaboration with Arthur Hebecker, Dieter Lüst, Stephan Steinfurt, and Timo Weigand, during the time when S. Steinfurt and myself were diploma students [19, 20]. Most of the borrowed content was written by myself, however, with some parts by T. Weigand.

Moduli stabilization in the phenomenologically required regime is then discussed in chapter 4. Most of this chapter is copied from [23], which is work done in collaboration with Arthur Hebecker, Moritz Küntzler, Dieter Lüst, and Timo Weigand. A first discussion of Kähler moduli stabilization in the fluxbrane inflation scenario is contained in the diploma thesis of M. Küntzler [22]. However, many relevant constraints were neglected in that analysis, such as the issue of a too large ratio $m_{3/2}/m_{\mathrm{KK}}$ and the tadpole constraint on the size of W_0. Therefore, the discussion in chapter 4 completely differs from that diploma thesis. The corresponding sections in [23] were written by myself, with some parts of section 4.3 edited by A. Hebecker. The appendices A, B, and C are slightly modified versions of the respective appendices in [23]. Appendix B grew from an appendix contained in [22], but was adapted to the discussion in [23] by myself.

The shift-symmetric structure of the Kähler potential, the brane-(in)dependence of the superpotential, the extended no-scale structure in the presence of light D7-brane position moduli, as well as the phenomenology of the fluxbrane inflation model in the presence of F-term corrections is analyzed in chapter 5. The structure of superpotential and Kähler potential was discussed in the diploma theses of Max Arends [25] and Konrad Heimpel [26]. Furthermore, the extended no-scale structure in fluxbrane inflation was analyzed in the diploma thesis of Christoph Schick [27]. These analyses were extended by myself, in collaboration with Arthur Hebecker, Dieter Lüst, Christoph Mayrhofer, and Timo Weigand, and published in [29]. A concise version of the discussion regarding the shift symmetry and the superpotential was part of [28], which is work done in collaboration with Arthur Hebecker and Lukas Witkowski. Section 5.1 is an extended version of that part of [28], which was written by myself. Sections 5.2 and 5.3 are mainly copied from [29]. Sections 5.2.1 and 5.2.2 are based on [27], however, with significant changes starting below (5.18) until the end of section 5.2.1, altering some conclusions of [27]. The parts in [29] which correspond to the remaining sections in chapter 5 (starting from section 5.2.3) were written by myself.

Finally, the D7-brane chaotic inflation model is analyzed in chapter 6. Most of this chapter is copied from [28]. All copied sections are written by myself, except for section 6.3.2 which was written by L. Witkowski. The illustrations of the D7-brane field space were also done by L. Witkowski.

2. Case Study: Hybrid Natural Inflation and Tensor Modes

Before we enter the discussion of how to derive inflation scenarios from string theory, we start in this chapter by taking a closer look at inflation in field theory. Clearly, our analysis will not be completely disentangled from the rest of this thesis. In particular, we take seriously the most severe constraints which arise in typical string inflation scenarios. Namely, we work in the context of supergravity, where the structure of the scalar F-term potential generically introduces a 'supergravity η-problem'. Furthermore, we face the fact that (as we discuss in more detail in section 2.3) the field range of typical inflaton candidates in string theory is bound to be sub-planckian, i.e. during the phenomenologically required $N = 60$ e-foldings of inflation we have $\Delta\varphi < 1$.[1] We will show that, under these circumstances, hybrid natural inflation [86, 87, 189–192] appears as an appealing inflation scenario.

Interestingly, in hybrid natural inflation the tensor-to-scalar ratio r can be larger than expected from a naive use of the Lyth bound [54, 55]. Thus, in the present chapter we study hybrid natural inflation with a special focus on tensor modes.

Section 2.1.1 until section 2.4 is mainly copied from [24], which is work done in collaboration with Arthur Hebecker and Alexander Westphal. The first draft of section 2.2 until but not including section 2.2.1 was written by A. Westphal, but heavily edited by myself afterwards. The remaining sections, starting from section 2.2.1, were written by myself.

2.1. Introducing Hybrid Natural Inflation

We start by making a case for hybrid natural inflation. Recall from the introduction the standard form of the scalar F-term potential in supergravity,

$$V_F = e^K \left(K^{i\bar{j}} D_i W \overline{D_j W} - 3|W|^2 \right), \quad D_i W = (\partial_i + K_i) W, \tag{2.1}$$

where W is the superpotential and K the Kähler potential of the 4d effective $\mathcal{N} = 1$ supergravity theory. During inflation supersymmetry is broken by F-terms,[2] i.e. the first term in (2.1) is non-vanishing. This term sets (an upper limit on) the size of the constant V_0 in the small-field inflation potential $V(\varphi) = V_0(1 + \ldots)$ (cf. (1.25)). A minimal Kähler potential $K \sim \Phi\overline{\Phi}$ (where Φ denotes the chiral superfield which contains the (real) inflaton φ) therefore entails a supergravity η-problem, i.e. $\eta = \mathcal{O}(1)$ [139], unless a non-generic cancellation takes place: For example, if the inflationary energy density is provided by the

[1] This means, in particular, that we leave aside the possibility to enlarge the field range using e.g. a monodromy. The latter is discussed in chapter 6.

[2] There exist proposals of inflation driven by *constant* D-terms ('FI-terms') at vanishing F-term potential, e.g. [93, 94]. However, constant D-terms seem to be inconsistent in supergravity [58, 81, 193–200]. In particular, in models derived from string theory the D-term potential is always moduli-dependent and contributions of similar size (or larger) to the F-term potential are required to stabilize these moduli.

F-term potential, the exponential in (2.1) leads to $V(\varphi) = V_0(1+c_1\varphi^2+\ldots)$, with $\eta \simeq 2c_1$ some $\mathcal{O}(1)$ constant. Via the general link between D-terms and F-terms (see footnote 2) the same negative conclusion applies if it is a D-term energy density which drives inflation.

Thus, if one does not want to give up the idea of non-fine-tuned (small-field) inflation, these arguments lead us to consider *non-minimal* structures of the Kähler potential. Such structures are encountered, for example, in the axion moduli space, where a shift symmetry protects the axion field from appearing in the effective potential to all orders in perturbation theory. In this case the Kähler potential contains only the combination $\Phi - \overline{\Phi}$, where the axion is now associated with the real part of the scalar component of Φ and thus drops out. Non-perturbative effects will, nevertheless, introduce a (periodic) potential for the axion which can be parametrized as

$$V(\varphi) = V_0 \left(1 - \cos\frac{\varphi}{f}\right), \tag{2.2}$$

and which is suitable for inflation [59], given a super-planckian axion decay constant, $f \gg 1$. The resulting inflation model is, however, of the large-field type. Note that, besides the fact that such large axion decay constants do not arise in weakly coupled string theory (see again the discussion in section 2.3), it is questionable if they can be attained in quantum gravity [201].

One can, nevertheless, use the shift symmetry for constructing small-field models of inflation by introducing additional fields. Here, we focus on hybrid inflation [34, 35]. In this class of models the inflaton φ is coupled to an additional scalar χ (the 'waterfall field'), whose mass depends on the vev of the inflaton. The tree-level potential in this model is given by

$$\begin{aligned} V(\varphi,\chi) &= \lambda\varphi^2\chi^2 + \kappa(\chi^2 - \chi_0^2)^2 \\ &= \kappa\chi_0^4 + (\lambda\varphi^2 - 2\kappa\chi_0^2)\chi^2 + \kappa\chi^4. \end{aligned} \tag{2.3}$$

Inflation proceeds at

$$\varphi > \varphi_c, \quad \varphi_c^2 = \frac{2\kappa\chi_0^2}{\lambda}, \tag{2.4}$$

driven by the vacuum energy density of (2.3) at $\chi = 0$, i.e.

$$V_0 = 3H^2 = \kappa\chi_0^4, \tag{2.5}$$

where H is the Hubble parameter. Once φ falls below the critical value φ_c, the mass of χ turns tachyonic and inflation ends via a waterfall instability.

Clearly, the coupling of the inflaton to the waterfall fields in (2.3) violates the shift symmetry. Since during inflation $\chi = 0$, this coupling will not introduce a tree-level mass for the inflaton. However, loop corrections will certainly lift the flat direction and introduce a non-trivial potential for φ. Being a scalar, the corrections will depend quadratically on the scale of new physics Λ, i.e.

$$\Delta m_\varphi^2 \simeq \frac{\lambda}{16\pi^2}\Lambda^2. \tag{2.6}$$

Using (2.4) and (2.5) we easily rewrite

$$\Delta m_\varphi^2 > \frac{H^2\Lambda^2}{16\pi^2\varphi_N^2\chi_0^2}, \tag{2.7}$$

where φ_N is the value of the inflaton at the beginning of the last N e-foldings. Successful (small-field) inflation requires $\Delta m_\varphi^2 \simeq \eta \cdot V_0 \simeq 0.02 \cdot V_0$ and thus [86–88]

$$\frac{\Lambda}{\varphi_N \chi_0} < 1. \tag{2.8}$$

For $\varphi_N, \chi_0 < 1$ (which was our assumption in the present chapter) this is only possible for new physics at $\Lambda \ll 1$, cutting off the quadratic divergence. Luckily, a supersymmetric theory above the scale Λ is a sort of new physics which is tailor-made for this job.

Let us therefore return to (2.1) and suppose that inflation is driven by an F-term energy density. I.e. we assume that some $F^i \neq 0$, where $F^i = e^{K/2} K^{i\bar{j}} \overline{D_j W}$. (In Type IIB string embeddings the label i typically denotes closed-string moduli such as Kähler moduli.) Unless a non-generic cancellation in (2.1) takes place we find

$$V_0 \simeq |F|^2. \tag{2.9}$$

The constant V_0 directly enters the formula for the soft masses [202] of the matter fields which run in the loop, correcting the inflaton mass. Above the scale of these soft masses the theory is supersymmetric, i.e. boson and fermion loops cancel each other, which is why we can identify $\Lambda^2 \simeq V_0$. It is now obvious that, for $\lambda \lesssim 1$, we automatically have

$$\Delta m_\varphi^2 \simeq \frac{\lambda}{16\pi^2} \Lambda^2 \ll V_0. \tag{2.10}$$

This looks very promising and shows that hybrid natural inflation, i.e. inflation using a shift-symmetric Kähler potential and supersymmetry to protect the inflaton from a large loop-induced mass, appears very naturally in field theory.

In summary, we explained that for small-field inflation models in the supergravity framework a non-minimal Kähler potential is needed in order to avoid the well-known η-problem. A shift symmetry in the Kähler potential is suitable for that purpose. For accommodating small-field inflation additional degrees of freedom need to be considered, e.g. in the form of a waterfall field which terminates inflation as soon as the vev of the inflaton falls below a critical value. Shift-symmetry-breaking interactions of the inflaton with the waterfall field then lead to loop corrections to the inflaton mass, which can be naturally small in the context of spontaneously broken supergravity. In particular, the loop-induced inflaton mass is suppressed with respect to the constant of the inflaton potential by a loop factor, as in (2.10).

All of the above analysis was performed with an eye on string theory, where the field space of most inflaton candidates is bound to be sub-planckian. Furthermore, a supergravity description arises naturally in Calabi-Yau compactifications and shift symmetries are certainly present for some fields in such constructions.

An important caveat is implied in the innocent-looking *unless a non-generic cancellation in* (2.1) *takes place* above (2.9): String theory features non-generic Kähler potentials. As a result, in Type IIB string compactifications a non-generic cancellation often *does* take place, the prime example being the well-known 'no-scale' cancellation in the Kähler moduli sector. The above arguments thus do not trivially apply to that situation. In fact, a sizable part of chapter 5 is devoted to discussing this more specific case. What saves the stringy hybrid natural inflation model is the so-called 'extended no-scale structure' which suppresses loop corrections beyond their expected size (2.10).

2.1.1. Tensor Modes in Hybrid Natural Inflation

If the field excursion of the inflaton during inflation is smaller than the Planck mass, which is the case in the hybrid natural inflation model discussed here, the amplitude of tensor perturbations in the cosmic microwave background is tiny compared to the amplitude of scalar perturbations. This conclusion is easily drawn by considering the Lyth bound [54,55]: In slow-roll inflation the tensor-to-scalar ratio (i.e. the ratio of the gravitational-wave power spectrum $\Delta^2_{\mathcal{T}} \sim H^2$ and the scalar power spectrum $\Delta^2_{\mathcal{R}} \sim H^2/\epsilon$) is proportional to the first slow-roll parameter, $r = \Delta^2_{\mathcal{T}}/\Delta^2_{\mathcal{R}} = 16\epsilon$. This can be rewritten in terms of the field variation per e-fold as

$$\sqrt{\frac{r}{8}} \simeq \left|\frac{\mathrm{d}\varphi}{\mathrm{d}N}\right|. \tag{2.11}$$

Thus, if r is roughly constant or monotonically increasing during inflation, the size of tensor modes produced within the initial, observable 10 e-folds of the cosmologically needed 60 e-folds of inflation is bounded by the total field excursion during these 60 e-folds.[3] In particular, for small-field inflation r is typically negligibly small.

In many small-field models, inflation ends because the evolution of the inflaton smoothly changes from slow-roll to fast-roll. In this sense, a monotonically increasing ϵ is a common feature. However, there are certainly exceptions: For example, if inflation ends in a 'waterfall' classical instability of a second scalar field, as in hybrid inflation[4] [34, 35], ϵ can decrease monotonically, allowing for a large tensor signal during the observable ~ 10 e-folds. This conclusion applies in particular to the hybrid natural inflation model[5] analyzed in the present chapter. In this model, loop corrections and non-perturbative effects introduce a non-trivial inflaton dependence in the leading order flat potential (2.5). As the inflaton typically parametrizes a periodic direction in field space (in fluxbrane inflation it parametrizes a displacement of D7-branes in a compact space), the contributions to the potential are typically periodic and are thus given, at leading order, by a cosine-shaped potential. Consequently, a non-monotonic evolution of ϵ occurs quite generically in hybrid natural inflation. If, in such a model, the waterfall sets in close to the minimum of the potential, ϵ can be sizable during the observable e-folds of inflation, while the bulk of the required 60 e-folds is accumulated at later stages, when ϵ is very small. This quite naturally provides us with a potentially detectable tensor signal, in spite of the small field range.

For avoiding the Lyth bound, it is essential that the waterfall sets in very close to the minimum of the potential. From a purely field-theoretic perspective, this requires some

[3] The bound arising from a situation of monotonically increasing ϵ during the last 60 e-folds of inflation is considered in [55]. The original paper [54] only takes into account the field excursion during the observable ~ 10 e-folds.

[4] The production of gravitational waves in the context of supersymmetric hybrid inflation, incorporating various corrections, was investigated in the minimal SUSY hybrid inflation program [203, 204] where it was found that, in some regions of parameter space, this model can produce sizable gravity waves [205]. More generally, large tensor signals in small-field inflation can be obtained whenever a sufficiently complicated potential is tuned in order to achieve a non-monotonic evolution of ϵ (see for example [206, 207]). Other suggestions for avoiding the Lyth bound and related constraints are discussed, e.g., in [174, 175, 177, 178, 208, 209].

[5] The inflaton can be a pseudo-Nambu-Goldstone boson [86, 189–192] or a Wilson line [86, 87]. The proposed models go by various names, such as 'little inflatons' or 'pseudonatural inflation'. Wilson line inflation was put into a stringy context in [110]. For other ideas of combining axions with hybrid inflation see e.g. [210, 211].

tuning of Lagrangian parameters or an appropriate model building effort.[6] From the point of view of the stringy implementation we have in mind, namely fluxbrane inflation (to be discussed in the subsequent chapters, see also [21, 29]), things look different: The inflaton in this model is associated with the distance of two D7-branes. The energy of this system is minimized in a situation where the branes come very close to each other. It is precisely in this regime of small distances where the tachyon appears. Namely, the critical field value φ_c, at which the waterfall stage sets in, is determined by the square root of the FI-parameter $\sqrt{\xi}$, which is bound to be small, $\sqrt{\xi} \lesssim 10^{-3}$. Finally, there is generically a hierarchy between the maximal and the critical brane-to-brane distance (see in particular section 3.2). Thus, the necessary prerequisites for avoiding the Lyth bound in hybrid natural inflation (i.e. large variation of φ/f during inflation and tachyon condensation close to $\varphi/f = 0$) are naturally satisfied in the fluxbrane inflation model.

In the further course of this chapter we discuss the phenomenology of a general hybrid inflation model with a periodic potential. In this model a sizable tensor-to-scalar ratio can be obtained for a planckian axion decay constant, $f = 1$. However, stringy consistency conditions dictate bounds on those decay constants. The examples of Kähler and complex structure axions are examined. The latter is of particular interest for us as, from an F-theory perspective, the inflaton, being a D7-brane deformation modulus, is part of the complex structure moduli space of the fourfold. We argue that for generic values of the complex structure the tensor-to-scalar ratio can be as large as $r \sim 10^{-3}$.

2.2. Phenomenology of Hybrid Natural Inflation

The effective loop-corrected scalar potential of hybrid natural inflation can be parametrized as (cf. (2.3))

$$V(\varphi, \chi) = \kappa(\chi^2 - \chi_0^2)^2 \cdot \left[1 - \alpha \cos\left(\frac{\varphi}{f}\right)\right] + \lambda\varphi^2\chi^2. \tag{2.12}$$

As we will show in the following, in the limit $f \ll 1$ this entails $\alpha \ll 1$. (We choose $\alpha > 0$ by convention.) Inflation ends via a waterfall instability in χ once $\varphi < \varphi_c$. Keeping the vacuum energy $\kappa\chi_0^4$ during inflation fixed, in view of (2.4) we can adjust $\varphi_c \ll 1$ as small as we like by choosing appropriately $\kappa \ll \lambda < 1$.

During inflation $\varphi > \varphi_c$, and the dynamics is governed by the effective potential

$$V(\varphi) = \kappa\chi_0^4 \cdot \left[1 - \alpha \cos\left(\frac{\varphi}{f}\right)\right]. \tag{2.13}$$

[6]For example, the authors of [191, 192] find that inflation most naturally starts and ends above the inflection point of the potential. In this regime ϵ increases monotonically and thus the Lyth bound applies, leading to a small tensor-to-scalar ratio. By contrast, the authors of [190] achieve waterfall near the minimum through field theory model building, but they do not consider tensor modes.

The slow-roll parameters are [86, 191, 192]

$$
\begin{aligned}
\epsilon &= \frac{1}{2}\left(\frac{V'}{V}\right)^2 = \frac{\alpha^2}{2f^2}\,\frac{\sin^2(\varphi/f)}{\left(1-\alpha\cos(\varphi/f)\right)^2}, \\
\eta &= \frac{V''}{V} = \frac{\alpha}{f^2}\,\frac{\cos(\varphi/f)}{1-\alpha\cos(\varphi/f)}, \\
\tilde{\xi}^2 &= -\frac{V'V'''}{V^2} = \frac{\alpha^2}{f^4}\,\frac{\sin^2(\varphi/f)}{\left(1-\alpha\cos(\varphi/f)\right)^2},
\end{aligned}
\tag{2.14}
$$

from which the two-point function observables can be computed as

$$
\begin{aligned}
n_s &= 1 - 6\epsilon + 2\eta, \\
r &= 16\epsilon, \\
\frac{\mathrm{d}n_s}{\mathrm{d}\ln k} &= 16\epsilon\eta - 24\epsilon^2 + 2\tilde{\xi}^2.
\end{aligned}
\tag{2.15}
$$

We see from (2.14) that, for a given potential, maximizing r means taking $\eta \to 0$. Hence, in a first step, we choose $\varphi_N = \varphi_0 := \pi f/2$ to be the fixed starting point, because there we have V' maximal, while $V'' = 0$.

We can now compute the number of e-folds N accumulated between the initial field value $\varphi_N = \varphi_0$ and the final value φ_c in the limit $\varphi_c/f \ll 1$:

$$
N = \int_{t(\varphi_c)}^{t(\varphi_0)} \mathrm{d}t\, H = \int_{\varphi_c}^{\varphi_0} \frac{\mathrm{d}\varphi}{\sqrt{2\epsilon}} \simeq \frac{f^2}{\alpha}\ln\left(\frac{4\varphi_0}{\pi\varphi_c}\right).
\tag{2.16}
$$

This can be dialed by choosing the waterfall exit φ_c appropriately, so that $N = 60$, i.e.

$$
\varphi_c \simeq \frac{4}{\pi}\varphi_0 \exp\left(-\frac{60\alpha}{f^2}\right).
\tag{2.17}
$$

As φ_0 is now the point at 60 e-folds before the end of inflation, the observables at CMB scales are evaluated at $\varphi = \varphi_0$. This gives the two-point function observables

$$
\begin{aligned}
n_s &\simeq 1 - 6\epsilon = 1 - \frac{3}{8}r \simeq 0.962 + 0.038\,(1 - r/0.1), \\
\frac{\mathrm{d}n_s}{\mathrm{d}\ln k} &= 2\tilde{\xi}^2 + \mathcal{O}(\alpha^4) \simeq 2\frac{\alpha^2}{f^4}, \\
r &= 16\epsilon = 8\frac{\alpha^2}{f^2} \simeq 4f^2\,\frac{\mathrm{d}n_s}{\mathrm{d}\ln k}.
\end{aligned}
\tag{2.18}
$$

We now see that a choice of a planckian axion decay constant, $f = 1$, and $\alpha = 0.1$ produces a red tilt $n_s \simeq 0.97$ and a sizable tensor-mode fraction $r \simeq 0.08$, while keeping the running of the spectral index $\mathrm{d}n_s/\mathrm{d}\ln k \simeq 0.02$ moderately small.

However, the embedding of this effective description into a string theory model dictates additional constraints on the parameters. For our purposes, the most relevant restrictions are on axion decay constants, which are subject of section 2.3. Guided by this discussion we choose to work with the fiducial bound $f \lesssim \sqrt{3}/4\pi$. Furthermore, we have to implement the observational constraints on n_s and its running. These are $n_s = 0.9603 \pm 0.0073$ and $\mathrm{d}n_s/\mathrm{d}\ln k = -0.0134 \pm 0.0090$ [48]. Using the constraints $f \lesssim \sqrt{3}/4\pi$ and $\mathrm{d}n_s/\mathrm{d}\ln k \lesssim 0.01$, equation (2.18) dictates the bound $r \lesssim 7.6 \cdot 10^{-4}$, which in turn forces $n_s \simeq 1$, a value excluded by Planck at the level of 5σ.

However, $n_s < 1$ is easily achieved by letting inflation start slightly above $\varphi_0 = \pi f/2$. In this region of field space η takes the form

$$\eta \simeq -\frac{\alpha}{f^3}\left(\varphi_N - \varphi_0\right). \tag{2.19}$$

The measured value $n_s = 0.9603$ dictates $\varphi_N/\varphi_0 \simeq 1.18$.

Thus, we have consistently realized $r \simeq 7.6 \cdot 10^{-4}$ in a string-motivated setting. By contrast, the Lyth-bound estimate of (2.11), assuming constant r and $\Delta N = 60$, would give $r \simeq 1.4 \cdot 10^{-4}$. Hence, we gain a factor of $\simeq 5$ as compared to the Lyth approximation.

We can now compare this with the estimated precision of future cosmological probes. While the B-mode polarization search in the CMB is expected to yield sensitivity of $r = (\text{a few}) \cdot 10^{-2}$ for Planck [212], the dedicated CMB polarization probe candidates CMBpol/EPIC [212] and PIXIE [213] can detect a tensor-to-scalar ratio down to $r \simeq 10^{-3}$. Even more promising is the analysis of the angular power spectra and weak lensing contribution to the 21 cm radiation, which can yield a B-mode detection down to $r \simeq 10^{-9}$ [214,215]. The recent results reported by the BICEP2-collaboration [33], which point towards $r = 0.2$, would certainly rule out the hybrid natural inflation model as presented above. However, these results are not confirmed yet and it is fair to say that, at the moment, it is unclear whether or not the signal stays.

2.2.1. Production of Primordial Black Holes

The potential of our hybrid axion inflation model flattens towards the end of the slow-roll regime. This means that the amplitude of curvature perturbations grows, implying the threat of primordial black hole overproduction [216–218]. To evaluate the situation, we compare the curvature perturbations $\Delta^2_{\mathcal{R}} \propto H^2/\epsilon$ at the beginning and the end of inflation, $\Delta_{\mathcal{R},N}/\Delta_{\mathcal{R},c} \simeq \sqrt{\epsilon_c/\epsilon_N}$. Using (2.14) and (2.17) we find

$$\frac{\Delta_{\mathcal{R},N}}{\Delta_{\mathcal{R},c}} \simeq 2\exp\left(-\frac{60\alpha}{f^2}\right) \sim 10^{-2}. \tag{2.20}$$

Now, the most recent value [48] for the power spectrum is $\Delta^2_{\mathcal{R}} \equiv A_s \simeq 2.2 \cdot 10^{-9}$ at the fiducial scale $k = 0.05\,\mathrm{Mpc}^{-1}$. This can be identified with our $\Delta^2_{\mathcal{R},N}$. It can be compared to the most conservative primordial black hole production bound, which is $\Delta^2_{\mathcal{R},c} < 10^{-3}$ (see [219, 220] and references therein). One finds $\Delta_{\mathcal{R},N}/\Delta_{\mathcal{R},c} \gtrsim 10^{-3}$. Thus, in view of (2.20), our model is completely safe.

2.2.2. Curvaton in Hybrid Natural Inflation

Before ending this section we pause to analyze whether the curvature perturbations can be generated by an additional light scalar, i.e. a curvaton σ [221–223] (for the specific formulae used below see [224] and references therein). Due to the high friction and the small mass, the field value of the curvaton is constant during inflation $\sigma \equiv \sigma^*$. The time-evolution of the scalar power spectrum $\Delta^2_\sigma \sim H^2/(\sigma^*)^2$ produced by the curvaton is thus governed by the time-evolution of the Hubble parameter H during inflation. Consequently, one finds that in the curvaton dominated regime ($\epsilon \gg (\sigma^*)^2$) the spectral index is given by

$$n_s - 1 = -2\epsilon. \tag{2.21}$$

Thus, the first slow-roll parameter is bound to be $\epsilon \simeq 0.02$. Furthermore, it is clear from (2.14) together with the bound $f \lesssim 0.1$ that slow-roll inflation (i.e. $\epsilon, \eta \ll 1$) can only be achieved for $\alpha \ll 1$. Hence, in the first line of (2.14), we can replace $1 - \alpha \cos(\varphi/f)$ by 1. Together with $\epsilon \simeq 0.02$ this equation then implies $\alpha \gtrsim 0.2f$ and therefore

$$\eta \gtrsim \frac{0.2}{f} \cos\left(\frac{\varphi}{f}\right). \tag{2.22}$$

For the fiducial value $f = \sqrt{3}/4\pi$ this gives $\eta \gtrsim 1.4 \cdot \cos(\varphi/f)$. Now consider the running of the spectral index which, in the curvaton dominated regime, is given by

$$\frac{\mathrm{d}n_s}{\mathrm{d}\ln k} = 4\epsilon\left(\eta - 2\epsilon\right). \tag{2.23}$$

This value can become incompatible with the data, $|\mathrm{d}n_s/\mathrm{d}\ln k| \lesssim 0.01$, if η becomes large too quickly. Recall that η is small close to the inflection point φ_0. We thus have to assume $\varphi_N \approx \varphi_0$ in order not to get into conflict with the bound on the running from the very beginning. In analogy to (2.16), we can then derive a bound on the number ΔN of e-foldings which are generated while the inflaton rolls from φ_0 to some smaller value φ: $\Delta N \lesssim 0.7 \cdot \ln\left(\frac{4\varphi_0}{\pi\varphi}\right)$. (Here we have used again the fiducial value $f = \sqrt{3}/4\pi$.) It is thus clear that the inflaton leaves the region where $\cos(\varphi/f) \ll 1$ already during the first e-fold, giving $\eta = \mathcal{O}(1)$. In view of (2.23) and with $\epsilon = 0.02$, the running of the spectral index predicted by the curvaton model in hybrid natural inflation then violates the bound after one e-folding. Thus, taking the stringy bound on f seriously, it is impossible to realize a curvaton-dominated power spectrum in hybrid natural inflation.

2.3. Stringy Constraints

String theory dictates additional constraints on the parameters, in particular on the axion decay constant [60]. Our focus will be on two types of axionic scalars: the imaginary parts of Kähler moduli and the real parts of complex structure moduli. Kähler axions descend from p-form potentials of the 10d theory upon dimensional reduction to 4d. On the other hand, to understand that complex structure moduli have anything to do with axions, recall that under mirror symmetry the complex structure moduli space of Type IIB string theory is mapped to the Kähler moduli space of Type IIA. Thus, at large complex structure (which corresponds to large volume on the Type IIA side), we expect an axionic shift symmetry to act on the complex structure moduli as well.

As a simple example, consider the axio-dilaton $S = i/g_s + C_0$, where g_s is the string coupling and C_0 is the Ramond-Ramond (RR) zero-form potential. The Kähler potential for this modulus is $K = -\ln\left(-i\left(S - \overline{S}\right)\right)$, giving rise to a kinetic term

$$\mathcal{L} \supset K_{S\overline{S}}\,|\partial S|^2 \supset \left(\frac{g_s}{2}\right)^2 (\partial C_0)^2. \tag{2.24}$$

The canonically normalized 'would-be' inflaton is $\varphi = \frac{g_s}{\sqrt{2}} C_0$. The periodicity $C_0 \to C_0 + 1$ of the RR zero-form field [78] implies a periodicity $\varphi \to \varphi + g_s/\sqrt{2}$, and hence $f = \frac{g_s}{\sqrt{2}\, 2\pi}$. Therefore, already at the self-dual point $g_s = 1$, the axion decay constant is much smaller than one. It decreases even further at weak coupling, $g_s \ll 1$.

From the F-theory perspective, S is part of the complex structure moduli space of the fourfold. Therefore, we expect similar considerations to apply in the case of complex structure axions. The same is true for deformation moduli of D7-branes, as they are part of the complex structure of the F-theory fourfold as well. The analogs of $g_s \sim 1$ and $g_s \ll 1$ are generic and large imaginary parts of the complex structure moduli, respectively. We thus expect that the axion decay constant f in (2.13) can take values as large as $f \sim 1/4\pi$ at generic complex structure and $g_s \sim 1$.

In the example of Kähler axions [137, 225–227] one obtains, via dimensional reduction, the term

$$\mathcal{L} \supset \frac{1}{4\pi} r^{i\alpha} c_\alpha \operatorname{tr}(F_i \wedge F_i)\,, \tag{2.25}$$

which displays the coupling of the axions c_α to the field strength of the gauge field living on a D7-brane (wrapping a four-cycle labeled by the index i). The c_α are the coefficients of an expansion of the RR four-form in terms of a basis of four-forms (labeled by the index α) of the threefold. The integers $r^{i\alpha}$ arise from integrating the four-form labeled by α over the four-cycle labeled by i. Quantization of $\int \operatorname{tr}(F_i \wedge F_i)$ implies that the term (2.25) is trivial for integer values of c_α [227].

In order to read off the axion decay constant one has to canonically normalize the axion, using the Kähler potential $K = -2 \ln \mathcal{V}$. From here it is apparent that the axion decay constant typically scales with the inverse of some four-cycle volume, the precise value depending on the volume form. For example, for a scenario with one large four-cycle with volume[7] τ and corresponding Kähler modulus $T = \tau + i\, c$, such that $\mathcal{V} \sim \left(T + \overline{T}\right)^{3/2}$, one finds [227]

$$f = \frac{\sqrt{3}}{4\pi\tau}. \tag{2.26}$$

We take $f \lesssim \sqrt{3}/4\pi$ as our fiducial value, corresponding to an Einstein frame four-cycle volume of unity. In the example of a compactification on a square $(T^2)^3$ and for $g_s = 1$, this is the T-self-dual point. If one instead evaluates f at the point where instanton corrections $\sim e^{-2\pi\tau}$ become important, the bound on f is generally weakened by a factor of 2π.

2.4. Outlook

In this chapter we have analyzed scenarios which are a cross between axionic (= natural) and hybrid inflation. They represent a particularly appealing class of models since, as we have reviewed, they rely on a combination of a shift symmetry and supersymmetry which can naturally stabilize the inflaton mass at a small value, without the need to fine-tune. A sufficient amount of e-foldings is produced within a sub-planckian field range. The typical periodic higher-order corrections to the flat tree-level potential imply a non-monotonic evolution of ϵ. This generates a significant tensor-mode contribution (up to $r \sim 10^{-3}$) in

[7] The (dimensionless) Einstein frame four-cycle volume τ is related to the string frame four-cycle volume as $\tau = g_s^{-1} \ell_s^{-4} \tau^s$, where $\ell_s = 2\pi\sqrt{\alpha'}$ is the string length (see also appendix A).

early inflation, where ϵ is sizable. The required number of e-foldings is accumulated later on, when ϵ approaches zero.

We have also demonstrated that, within this class of inflationary models, it is impossible to generate the curvature perturbations by a curvaton field. The crucial obstacle is the string-theoretic bound on the field range of the axion.

In the following chapters we set out to implement this field-theoretically attractive class of inflationary models in string theory. More precisely, in chapter 3 we introduce fluxbrane inflation as a stringy version of D-term hybrid inflation. In this first approach towards a string inflation model, we completely neglect all F-term contributions to the potential and study the (loop-corrected) D-term potential and its phenomenological applications. Moduli stabilization in this setup is discussed in chapter 4. We return to a detailed analysis of F-term contributions and their phenomenology in chapter 5.

3. Fluxbrane Inflation as a Stringy Implementation of Hybrid Natural Inflation

In this chapter we introduce our idea of how to embed the appealing model of hybrid natural inflation, analyzed in the preceding discussion, in string theory. We propose a scenario where the inflaton is associated with the relative position of two D7-branes wrapped on homologous four-cycles. Non-supersymmetric gauge flux induces an inflaton-independent tree-level D-term energy density and an attractive higher-order inter-brane potential. The latter arises from a Coleman-Weinberg-type loop correction due to strings stretched between the branes and is sufficiently flat in the supergravity regime of large volume moduli. This 'inflaton potential' can be calculated explicitly via an open-string one-loop computation on toroidal backgrounds [20, 21]. A generalization of this result to genuine Calabi-Yau manifolds is given in [21].

The end of inflation is marked by the condensation of tachyonic recombination fields, arising from strings stretched between the D7-branes, triggering a phase transition towards the Minkowski vacuum. Hence our model fits in the framework of D-term hybrid inflation [93, 94].

In the present chapter we consider in detail the setup for fluxbrane inflation. In particular, we specify how inherently stringy quantities (like the brane distance) map to 4d quantities (like the canonically normalized inflaton). These relations will be useful in the further course of this thesis. Regarding the string-derived inter-brane potential and its proposed generalization to Calabi-Yau manifolds, we only quote the results from [19–21]. We then work out the main phenomenological properties of our D-term inflation potential. The latter analysis underlies the discussion of moduli stabilization in the following chapter 4. As part of our findings we report that, contrary to previously proposed stringy embeddings of D-term hybrid inflation, in fluxbrane inflation the field range of the inflaton during the last 60 e-foldings corresponds to a brane displacement which is easily accommodated within the compact space. Furthermore, our inflationary scenario provides a mechanism to avoid the familiar clash of D-term inflation with observational constraints on the cosmic string tension (cf. also section 1.3.2).

Throughout this chapter we only compute the D-term potential and completely neglect the effects induced by non-vanishing F-terms. These F-terms, however, generically appear as soon as moduli stabilization is consistently taken into account. They are dealt with in chapters 4 and 5. In this context let us mention that, while the initial motivation to consider fluxbrane inflation mainly concerned the field range of the inflaton and the suppression of the cosmic string tension, D7-brane position moduli are appealing inflaton candidates also from the F-term point of view. In fact, these moduli enjoy a shift-symmetric Kähler potential in some region of the moduli space, to be discussed in more detail in chapter 5, which allows to avoid the well-known supergravity η-problem, along the lines of chapter 2.

Much of sections 3.1 and 3.2 and a small part of the outlook is copied from [21]. This is work done in collaboration with Arthur Hebecker, Dieter Lüst, Stephan Steinfurt, and Timo Weigand, during the time when S. Steinfurt and myself were diploma students [19,20]. Most of the borrowed content was written by myself, however, with some parts by T. Weigand.

3.1. The Geometric Calabi-Yau Setup for D7-Brane Hybrid Inflation

In this section we describe the geometric configuration underlying our fluxbrane inflation scenario. We consider a Type IIB orientifold compactification on a Calabi-Yau threefold X_3 modded out by the orientifold action $\Omega(-1)^{F_L}\sigma$. The holomorphic involution σ is chosen such that it gives rise to O3- and O7-planes compatible with the addition of D3- and D7-branes. The four-dimensional effective action of such Type IIB orientifolds has been studied in detail in [71–73, 91].

The key players of our inflationary scenario are spacetime-filling D7-branes wrapping holomorphic four-cycles of X_3. The inflaton is related to the position modulus for a particular D7-brane as follows: In flat space, two parallel D7-branes can be separated from each other in such a way that there is a non-zero distance r in the perpendicular complex plane at every point of the branes. This means that there exists a modulus y associated with the brane separation, such that $|y| = r$. The canonically normalized field related to this modulus will be the inflaton in our model. As discussed in some detail in [21], this simple picture receives interesting modifications for curved branes on general manifolds. Nonetheless, even in this more general situation one can maintain the concept of a relative D7-brane deformation and of an associated modulus – the inflaton. Here, we will ignore all complications occurring on curved spaces as compared to flat backgrounds.

Concretely, let us denote by $\Sigma \in H_4(X_3, \mathbb{Z})$ a divisor class with a geometric deformation modulus; i.e. a D7-brane wrapped along a representative in the class Σ can move in X_3. We assume for simplicity that this brane does not intersect the orientifold plane or its orientifold image in the class $\sigma^*\Sigma$. A pair of D7-branes $\mathcal{D}_a$ and $\mathcal{D}_b$ along two representatives Σ_a, Σ_b of the divisor class Σ can then be deformed with respect to one another.

Our second ingredient is non-supersymmetric relative gauge flux along the two branes and the resulting attractive D-term potential. If the two branes are separated from each other, the four-dimensional gauge group is $\mathrm{U}(1)_a \times \mathrm{U}(1)_b$. On the D7-branes we switch on non-trivial U(1) gauge bundles L_a and L_b with first Chern class

$$c_1(L_a) = \frac{1}{2\pi}(\ell_s^2 F_a) + \iota^* B_+ \in H^2(\Sigma_a, \mathbb{Z}/2), \tag{3.1}$$

and analogously for L_b. Here we distinguish between the (dimensionful) expectation value of the curvature $F_i = \mathrm{d}A_i$ and the pullback[1] with respect to the embedding $\iota : \Sigma \to X_3$ of a discrete B-field described by elements of $H^{1,1}_+(X_3)$ that are even under the orientifold involution σ. By contrast the quantity

$$\mathcal{F}_a = \frac{1}{2\pi}(\ell_s^2 F_a) + B \tag{3.2}$$

[1] In the following we will omit writing ι^* explicitly all the time. It will be clear from the context whenever we need to pull back a form to Σ.

refers to the full B-field including its non-integer piece along orientifold odd elements of $H^{1,1}_{-}(X_3)$. In our conventions the string length ℓ_s is related to the Regge slope α' as $\ell_s = 2\pi\sqrt{\alpha'}$ (cf. appendix A).

The dynamics of the relative brane motion during inflation involves only the relative gauge group $\mathrm{U}(1)_-$ with Abelian generator $Q_- = \frac{1}{\sqrt{2}}(Q_a - Q_b)$. In general there will be open strings stretched between $\mathcal{D}_a$ and $\mathcal{D}_b$ charged under $\mathrm{U}(1)_-$. In their ground state sector they give rise to chiral multiplets Φ^i_{ab} with charge $(-1_a, 1_b)$ and $\tilde{\Phi}^j_{ab}$ with charge $(1_a, -1_b)$. In flat space, as soon as the parallel branes are separated, these strings would necessarily acquire a supersymmetric mass term proportional to the brane separation with modifications on curved spaces discussed in [21]. Apart from appearing with a supersymmetric mass the bosonic components of Φ^i_{ab} and $\tilde{\Phi}^i_{ab}$ enter the four-dimensional $\mathcal{N} = 1$ supergravity D-term potential for $\mathrm{U}(1)_-$ of the standard form[2]

$$V_D = \frac{1}{2}\,\mathrm{Re}(f)^{-1}\left(-\sqrt{2}\sum_i |\Phi^i_{ab}|^2 + \sqrt{2}\sum_j |\tilde{\Phi}^j_{ab}|^2 + \xi_{ab}\right)^2 . \tag{3.3}$$

Here f represents the gauge kinetic function associated with the four-dimensional gauge group $\mathrm{U}(1)_-$. To first order its real part is given by

$$\mathrm{Re}(f) = \frac{1}{2\pi}\left(\frac{1}{2}\int_\Sigma J\wedge J - e^{-\phi}\int_\Sigma \frac{1}{2}\mathcal{F}_{ab}\wedge\mathcal{F}_{ab}\right). \tag{3.4}$$

Here $J \in H^2(X_3)$ is the Kähler form on X_3 as appearing in the ten-dimensional Einstein frame. It is related to the Kähler form $\hat{J}$ in the ten-dimensional string frame via

$$J = e^{-\phi/2}\hat{J} \tag{3.5}$$

and is normalized such that $\hat{\mathcal{V}}(\Sigma) = \frac{1}{2}\int_\Sigma \hat{J}\wedge\hat{J}$ is dimensionless and measures the string frame volume of the divisor Σ in units of the string length ℓ_s.[3] Furthermore, we have defined the relative flux as $\mathcal{F}_{ab} = \frac{1}{\sqrt{2}}(\mathcal{F}_a - \mathcal{F}_b)$. Finally, ϕ denotes the axio-dilaton.

The quantity ξ_{ab} is known, by slight abuse of nomenclature, as the field-dependent Fayet-Iliopoulos term and serves as an order parameter for the amount of relative supersymmetry breaking. In the above conventions we have [91]

$$\xi_{ab} = \frac{M_p^2}{4\pi}\frac{\int_\Sigma J\wedge\mathcal{F}_{ab}}{\mathcal{V}(X_3)}, \quad \mathcal{V}(X_3) = \frac{1}{6}\int_{X_3} J\wedge J\wedge J, \quad M_p^2 = \frac{4\pi}{\ell_s^2}, \tag{3.6}$$

where M_p denotes the four-dimensional reduced Planck mass.

If the two branes are separated and $\xi_{ab} \neq 0$, an attractive potential between the branes arises. We will discuss the precise form of this potential in section 3.2. Clearly, the amount of supersymmetry breaking responsible for this potential depends dynamically on the Kähler moduli appearing in (3.6). Thus, stabilization of the Kähler moduli in a non-supersymmetric manner is key to a successful realization of inflation. However, we

[2] We use the same symbol Φ to denote the scalar component of a chiral superfield Φ. It will always be clear from the context to which of the two we are referring.

[3] Our conventions are summarized in appendix A. Hatted quantities are measured in the ten-dimensional string frame, while quantities without a hat are measured in the ten-dimensional Einstein frame.

postpone the question of moduli stabilization to chapter 4 and treat ξ_{ab} as a parameter in the present analysis.

The end of inflation is marked by the critical distance $\hat{r}_{\text{crit.}}$ at which one of the fields Φ^i_{ab} or $\tilde{\Phi}^j_{ab}$ becomes tachyonic. To determine when this happens we must take into account, in addition to the supersymmetric mass term for the string modes proportional to the brane distance, the non-supersymmetric mass from the D-term potential. To arrive at an expression for $\hat{r}_{\text{crit.}}$ we consider the case of a compactification on a factorisable torus $\prod_{I=1}^3 T_I^2$. Modifications of these results for curved backgrounds are discussed in [21]. Separating the two branes $\mathcal{D}_a$ and $\mathcal{D}_b$ by a distance $\hat{r}$, measured in units of ℓ_s, yields a supersymmetric mass square $(2\pi/\ell_s)^2\hat{r}^2$ of the open-string states Φ^i_{ab} [3]. To quantify the non-supersymmetric mass, suppose that the relative flux density is non-vanishing on one torus only (say the one corresponding to the internal directions X^a, $a = 4,5$) and parametrized by

$$\mathcal{F}^{ab}_{45} = \frac{1}{\sqrt{2}}\left(\mathcal{F}^a_{45} - \mathcal{F}^b_{45}\right) = \frac{1}{\sqrt{2}}\left(\tan\theta_a - \tan\theta_b\right). \tag{3.7}$$

Then, by T-duality, one can use the familiar result of the branes-at-angles picture for the mass of the lightest state [4, 228]

$$m^2 = \frac{(2\pi)^2}{\ell_s^2}\hat{r}^2 - \frac{2\pi\theta_{ab}}{\ell_s^2}, \tag{3.8}$$

where we assumed (without loss of generality) that moduli stabilization has resulted in $\theta_{ab} := \theta_a - \theta_b > 0$. From this expression one can read off that the lightest state becomes tachyonic at the critical distance

$$\hat{r}^2_{\text{crit.}} = \frac{\theta_{ab}}{2\pi}. \tag{3.9}$$

To obtain the corresponding expression in terms of a canonically normalized field $\varphi \equiv |\Phi|$ (the inflaton) in four dimensions we use the relation to the eight-dimensional modulus y [36][4]

$$\frac{\varphi}{M_p} = \hat{r}\sqrt{\frac{g_s}{4}\frac{\hat{\mathcal{V}}(\Sigma)}{\hat{\mathcal{V}}(X_3)}}, \quad \hat{r} \equiv |y|. \tag{3.10}$$

With the help of (3.6) it follows that

$$\varphi^2_{\text{crit.}} \simeq \frac{\xi_{ab}}{\sqrt{2}} \tag{3.11}$$

for small θ_{ab}. This is precisely the result one would obtain by embedding hybrid inflation from D-terms in $\mathcal{N} = 2$ supersymmetry, where there is a relation between the trilinear coupling λ in the superpotential and the gauge coupling g_{YM} of the form $\lambda^2 = 2g^2_{\text{YM}}$ [229, 230]. The four-dimensional mass squared of the tachyon in this model is given by

$$m^2_{4D} = 2\,\text{Re}(f)^{-1}\left(\varphi^2 - \frac{\xi_{ab}}{\sqrt{2}}\right). \tag{3.12}$$

Tachyon condensation leads to the formation of a bound state between $\mathcal{D}_a$ and $\mathcal{D}_b$ and breaks the gauge group $\text{U}(1)_a \times \text{U}(1)_b$ to $\text{U}(1)_+$ with generator $\frac{1}{\sqrt{2}}(Q_a + Q_b)$. This is

[4] Note that the authors of [36] use different conventions for the rescaling of the metric in order to go from ten-dimensional string frame to four-dimensional Einstein frame.

typical of D-term hybrid inflation, where the condensing tachyon Φ^i_{ab} plays the role of the waterfall field.

In more general setups on curved backgrounds, discussed in [21], there is no unambiguous definition of a distance $\hat{r}$ between the D-branes. Instead, one can use the relation (3.10) as a definition of $\hat{r}$ in terms of the four-dimensional inflaton φ.

3.2. Inflaton Potential and Phenomenology

In this section we quote from [21] the inflaton potential for the setup outlined in section 3.1 and perform a phenomenological analysis. The outcome of this analysis is the basis of chapter 4.

String-theoretically speaking, the attractive inter-brane potential arises as a Coleman-Weinberg-type loop correction from open strings which stretch between the two fluxbranes. We have computed these corrections for a toroidal compactification in [20,21]. The result is non-vanishing in case the brane flux breaks supersymmetry. The purpose of this string-loop calculation was twofold: First, we derived the potential at large brane-separation, which contains a constant plus a logarithmic dependence on the brane separation, as expected by analogy to the Coleman-Weinberg result [125]. We highlighted the possibility to suppress the logarithmic term by more than just the conventional loop factor. As we will detail below, this feature enables us to circumvent the problem of cosmic string overproduction in D-term hybrid inflation. Secondly, we also showed that, surprisingly, the form of the potential remains valid for $\hat{r} < 1$, as long as $\hat{r}$ is not too small. In particular, this is true as long as $\hat{r}$ is larger than a certain lower bound which is parametrically given by the distance at which the lowest-lying state in the open-string spectrum becomes tachyonic (equation (3.9)). This is a crucial result because, as we will argue in the following, D7/D7 inflation takes place precisely in the regime where $\hat{r} < 1$.

We will not review the form of the fluxbrane potential in the toroidal example, but directly move to its generalization on a genuine Calabi-Yau orientifold X_3, which was derived in [19, 21]. It reads

$$V_D = \frac{1}{2} g_{\mathrm{YM}}^2 \xi^2 \left[1 + \frac{g_{\mathrm{YM}}^2}{16\pi^2} \left\{ \frac{\left(\int_\Sigma J \wedge \mathcal{F}\right)^2}{\left(\frac{1}{2}\int_\Sigma J \wedge J\right)} - 4 \cdot \left(\frac{1}{2} \int_\Sigma \mathcal{F} \wedge \mathcal{F} \right) \right\} \log\left(\frac{\varphi}{\varphi_0}\right) \right], \tag{3.13}$$

where the gauge coupling is given by

$$g_{\mathrm{YM}}^{-2} = \frac{1}{2\pi} \left(\frac{1}{2} \int_\Sigma J \wedge J \right), \tag{3.14}$$

and ξ is defined as in (3.6). $\mathcal{F}$ is the integrally-quantized brane flux and Σ denotes the divisor wrapped by the D7-branes. Furthermore, φ is the canonically normalized inflaton and φ_0 is a normalization which, at our level of precision, is arbitrary.

The idea for how to obtain (3.13) is to analyze the action of a fluxed probe D7-brane in the warped background generated by a fluxless brane. The potential appearing in the action is then expanded for small flux density, leading to (3.13). On the factorisable torus, the above expression reduces precisely to the potential obtained in the string calculation. The similarity of (3.13) with corresponding expressions in D-term hybrid inflation [93, 94] is manifest and allows to interpret our result as the Coleman-Weinberg potential of a 4d gauge theory, with the massive waterfall-fields running in the loop. Their masses are split after SUSY-breaking due to the presence of the FI-term ξ, giving rise to the non-vanishing loop contribution.

Regarding the suppression of the logarithmic correction, note that the first term in the big round brackets in (3.13), proportional to $\int_\Sigma J \wedge \mathcal{F}$, can be made parametrically small by adjusting the 'angle' between the Kähler form J and the flux $\mathcal{F}$. At the same time, $\mathcal{F}$ can in principle be chosen in such a way that the induced D3 charge $\int_\Sigma \mathcal{F} \wedge \mathcal{F}$ on the D7-branes vanishes. In such situations one arrives at a highly suppressed logarithmic term which specifically arises in our D7-brane context.

At this stage it is instructive to compare our inflationary brane potential to the setup in D3/D7 inflation. From [105] we recall that the D3/D7 potential takes the generic form

$$V = \frac{1}{2} g_{\mathrm{YM}}^2 \xi^2 \left(1 + \frac{g_{\mathrm{YM}}^2}{16\pi^2} \log \frac{\varphi}{\varphi_0} \right) . \tag{3.15}$$

In particular, there is no analogue of the term proportional to $(\int_\Sigma J \wedge \mathcal{F})^2 / \frac{1}{2} \int_\Sigma J \wedge J$, which arises from the non-alignment of relative D5-brane charge. Rather, the expression only involves the relative D3-brane charge of the fluxed D7 and the mobile D3-brane. To match this with the D7/D7 potential (3.13) we note the general result (see e.g. [231] for details) that for gauge flux that can be made supersymmetric inside the Kähler cone the expression $-\int_\Sigma \mathcal{F} \wedge \mathcal{F}$ is positive and thus measures D3 (as opposed to anti-D3) charge.

Phenomenological Analysis

We now collect the basic phenomenological properties of the inflationary D-term potential (3.13). As one of our main results we will show how, in the D7/D7 inflationary scenario, one is able to overcome the clash with observational bounds due to cosmic string production at the end of inflation. In fact these bounds have turned out to be a notorious problem in D-term inflation models with an underlying $\mathcal{N} = 2$ structure [229].[5]

Let us parametrize the potential (3.13) as

$$V(\varphi) = V_0 \left(1 + \alpha \log \frac{\varphi}{\varphi_0} \right) , \tag{3.16}$$

where

$$V_0 = \frac{1}{2} g_{\mathrm{YM}}^2 \xi^2 , \qquad \alpha = \frac{g_{\mathrm{YM}}^2}{16\pi^2} \left(-2 \int_\Sigma \mathcal{F}^2 + \frac{g_{\mathrm{YM}}^2}{2\pi} \left(\int_\Sigma J \wedge \mathcal{F} \right)^2 \right) . \tag{3.17}$$

The choice of φ_0 corresponds to some choice of normalization for the potential. Its value is irrelevant at our level of precision. For convenience we will choose φ_0 such that it corresponds to the bifurcation point $\varphi_0 \equiv \varphi_{\mathrm{crit.}}$ of our potential, defined in (3.11), which is where the tachyon appears and inflation ends. Close to this point the simple functional form (3.16) is no longer valid.

Let us first analyze the field range required to obtain $N = 60$ e-foldings in the course of inflation. To this end we recall that, in terms of the inflaton potential $V(\varphi)$, N is

[5] Recall the discussion below (3.11).

determined as [232]

$$N = \int_{t_N}^{t_0} \mathrm{d}t\, H = \int_{\varphi_0}^{\varphi_N} \mathrm{d}\varphi\, \frac{V}{V'}, \tag{3.18}$$

where t_N denotes the time associated with the onset of the last N e-foldings and t_0 marks the end of inflation; the corresponding values of the inflaton are $\varphi_N \equiv \varphi(t_N)$ and $\varphi_0 \equiv \varphi(t_0)$. In our model inflation starts out far from the bifurcation point of the potential (i.e. $\varphi_N \gg \varphi_0$). A simple parametric analysis shows that in a regime where the supergravity approximation is valid (i.e. the typical length scales of the compactification manifold are large in units of the string length), the constant of the potential dominates over the distance-dependent term throughout inflation, i.e. $\alpha \log(\varphi/\varphi_0) \ll 1$. This allows us to evaluate (3.18) as

$$N = \frac{1}{2\alpha}\left(\varphi_N^2 - \varphi_0^2\right). \tag{3.19}$$

Using $\varphi_N \gg \varphi_0$ we thus find that the field value of the inflaton at the beginning of the last 60 e-foldings is given by

$$\varphi_N \simeq \sqrt{2\alpha N}. \tag{3.20}$$

The slow-roll parameters are readily evaluated, in the approximation (3.20), as

$$\epsilon := \frac{1}{2}\left(\frac{V'}{V}\right)^2\Bigg|_{t=t_N} = \frac{1}{2}\frac{\alpha^2}{\varphi_N^2} = \frac{\alpha}{4N}, \tag{3.21}$$

$$\eta := \frac{V''}{V}\Bigg|_{t=t_N} = -\frac{\alpha}{\varphi_N^2} = -\frac{1}{2N}. \tag{3.22}$$

Since $\alpha \ll 1$ it follows that $\epsilon \ll |\eta|$ and thus, for $N = 60$, the slow-roll condition $\epsilon \ll 1$, $|\eta| \ll 1$ is easily satisfied. From the above we extract a prediction for the spectral index n_s via

$$n_s = 1 - 6\epsilon + 2\eta \simeq 1 + 2\eta = 1 - \frac{1}{N} = 0.983. \tag{3.23}$$

This value lies marginally outside the 1σ value $n_s = 0.968 \pm 0.012$ according to WMAP7 [233].[6]

The inflationary potential is further constrained by measurements of the amplitude of adiabatic curvature perturbations. They set a value for the ratio $V^{3/2}/V'$ at time t_N as [233]

$$\tilde{\zeta} := \frac{V^{3/2}}{V'}\Bigg|_{t=t_N} = 5.4 \cdot 10^{-4}. \tag{3.24}$$

[6] At the time of publication of [21] this was the most up-to-date measurement of n_s. The more recent value $n_s = 0.9603 \pm 0.0073$ [48] is inconsistent with the inflaton potential analyzed in this section at more than 3σ. On the other hand, F-term corrections are expected to change the phenomenological implications of our model. In fact, we will show in chapter 5 that fluxbrane inflation can consistently describe $n_s = 0.9603$.

Using the smallness of the distance-dependent term relative to the constant of the potential (3.16), as discussed above, we can evaluate this constraint in the approximation (3.20) and for $N = 60$ as

$$\frac{V_0}{\alpha} = \frac{\tilde{\zeta}^2}{2N} = 2.4 \cdot 10^{-9}. \tag{3.25}$$

To analyze the implications on the parameters of our potential it is more convenient to consider the inverse combination

$$\frac{\alpha}{V_0} = \frac{1}{(2\pi)^2\xi^2}\left(\int_\Sigma -\mathcal{F}^2\right) + 2\frac{\mathcal{V}^2(X_3)}{\mathcal{V}(\Sigma)} = 4.2 \cdot 10^8. \tag{3.26}$$

Crucially, the first summand involves the FI-term ξ. For positive and integral $\int_\Sigma -\mathcal{F}^2$ this therefore sets a lower bound on ξ, which turns out to lie above the observational bound from cosmic strings. In particular, this is the situation encountered in D3/D7 inflation, where $\int_\Sigma -\mathcal{F}^2$ is replaced by a positive order one number.

The cosmic string bound constrains the energy density (i.e. tension μ) of these topological defects as

$$G\mu \lesssim 6.4 \cdot 10^{-7}, \tag{3.27}$$

where G is Newton's constant. This was found in [234] using the Abelian-Higgs model to simulate the evolution of the string network.[7] This value implies a contribution of $\lesssim 9.3\%$ from cosmic strings to the total power in the CMB at multipole moment $l = 10$. The string tension is related to the FI-term ξ as $G\mu = \xi/4$ [239]. We note that for D3/D7 inflation the value of ξ required by the measured value for the amplitude of curvature perturbations (and determined via the D3/D7 analog of equation (3.26)) lies above the cosmic string bound (3.27). By contrast, our D7/D7 inflation model is in a fundamentally different position. Namely, by a suitable choice of gauge flux it is possible to achieve $\int_\Sigma \mathcal{F}^2 = 0$ so that the FI-term completely drops out from (3.26). In this situation, what is constrained by (3.26) is the ratio of the volume of Σ and of the Calabi-Yau X_3,

$$\frac{2N}{\tilde{\zeta}^2} = \frac{\alpha}{V_0} = \frac{2\mathcal{V}^2}{\frac{1}{2}\int_\Sigma J \wedge J} \simeq 4.2 \cdot 10^8 \quad \text{for} \quad N = 60. \tag{3.28}$$

For simplicity let us assume that the internal manifold can be characterized by a single length scale R. Equation (3.28) then implies

$$R \simeq 11, \quad \mathcal{V} \simeq 1.7 \cdot 10^6. \tag{3.29}$$

Due to our choice of a flux vector $\mathcal{F} \in H^2(\Sigma)$ which satisfies $\int_\Sigma \mathcal{F}^2 = 0$, the FI-term is in principle unconstrained by (3.26) and thus the measured value for the amplitude of curvature perturbations is not in conflict with bounds on ξ from cosmic strings: Let $\xi_{\text{crit.}}$ be the 'critical' value of the FI-term for which the cosmic string bound is saturated, i.e.

[7] There are alternative approaches to look for signatures of cosmic strings, some of which include [235–237]. However, they seem to find upper bounds on $G\mu$ which are comparable to the one cited above. Over the years the comic string bound has become tighter (see e.g. [238], which is the value used in our most recent analysis [29]). This does not qualitatively change our analysis. We will stick with (3.27) until the end of chapter 4.

$\xi_{\text{crit.}}/4 \simeq 6.4 \cdot 10^{-7}$. With the help of (3.28) one can re-express the cosmic string bound $\xi \lesssim \xi_{\text{crit.}}$ as

$$\frac{\left(\int_\Sigma J \wedge \mathcal{F}\right)^2}{\frac{1}{2}\int_\Sigma J \wedge J} \lesssim 8\pi^2 \xi_{\text{crit.}}^2 \cdot 4.2 \cdot 10^8 \simeq 0.2. \tag{3.30}$$

This constraint can easily be satisfied by appealing to a mild hierarchy of four-cycle volumes in the compact space or by a partial cancellation in the numerator of the above expression, consistent with the normalization of the amplitude of curvature perturbations (3.26). We will discuss the solution in terms of a mild hierarchy in chapter 4, while the possibility of a partial cancellation is addressed in chapter 5. Both solutions will naively not significantly alter the prediction $R \simeq 11$, which is why we continue to work with this value. A more detailed investigation will be performed in chapter 4.

We proceed by briefly discussing the implications of the above analysis for the field range during inflation. Recall that in all of the above we assumed that inflation starts far away from the bifurcation point, which is the point where inflation ends. This means that we have to require $\varphi_0 \ll \varphi_N$ and thus, in view of (3.20),

$$\varphi_0^2 \ll 2\alpha N. \tag{3.31}$$

According to the discussion in section 3.1 the bifurcation point is just $\varphi_0^2 \simeq \xi/\sqrt{2}$. Assuming that $\int_\Sigma \mathcal{F}^2 = 0$ and that the cosmic string bound (3.27) is saturated (i.e. $\xi \simeq \xi_{\text{crit.}} = 2.6 \cdot 10^{-6}$) the requirement (3.31) can be rewritten as

$$3.7 \cdot 10^2 \ll \frac{\mathcal{V}^2(X_3)}{\mathcal{V}^2(\Sigma)}. \tag{3.32}$$

This condition is in agreement with the prediction $R \simeq 11$ for a typical length scale of our (isotropic) compactification manifold.

As a final step we deduce from the above analysis the brane separation $\hat{r}_N$ of the two D7-branes at the beginning of the last 60 e-foldings of inflation. The field value of the inflaton at this time is given by (3.20). Considering for simplicity the case of a toroidal compactification, we may use (3.10) to calculate $\hat{r}_N$

$$\hat{r}_N^2 = 16\pi N g_s^{-\frac{1}{2}} \xi^2 \frac{\mathcal{V}^3(X_3)}{\mathcal{V}^3(\Sigma)}, \tag{3.33}$$

where $\hat{r}_N$ is measured in units of ℓ_s and ξ is measured in units of M_p. Assuming again that the cosmic string bound is saturated (i.e. $\xi \simeq \xi_{\text{crit.}}$) it is obvious that a roughly isotropic compactification manifold with typical length $R \simeq 11$ leads to inflation in the regime where $\hat{r}_N < 1$, more precisely it leads to $\hat{r}_N^2 \approx 10^{-2}/\sqrt{g_s}$. This crucial conclusion makes it necessary to perform the full string computation as outlined in section 3.2 in order to derive the inter-brane potential because, generically, the supergravity approximation can be trusted only at distances larger than the string length.

3.3. Outlook

In this chapter we have outlined our basic idea of how to embed the hybrid natural inflation scenario in string theory. That is, we have introduced the fluxbrane inflation proposal, in which the distance between two D7-branes with non-supersymmetric flux plays the role of the inflaton. Inflation is driven by a D-term energy and the end of inflation occurs via a tachyonic instability, as in D-term hybrid inflation. Our analysis has focused on the form of the attractive D-term potential between separated D7-branes and has shown that this potential as it stands is of a type suitable for inflation. In particular, it easily gives rise to $N = 60$ e-foldings of slow-roll inflation, even in the regime of sub-stringy brane distances.

Crucially, the fluxbrane inflation scenario provides a mechanism to overcome the familiar clash of standard D-term hybrid inflation with the observational bound on the cosmic string tension. The spectral index is slightly too red, but additional inflaton-dependent terms are likely to alter this prediction. In fact, the appearance of such additional terms is intimately linked with moduli stabilization, which needs to be incorporated in a successful inflationary scenario: Our analysis so far has treated the flux-induced D-term as a given order parameter for supersymmetry breaking. In view of (3.6), however, it is apparent that this D-term depends dynamically on the Kähler moduli. Their stabilization in a *supersymmetry-breaking* regime is thus of pivotal importance. Note that this requirement is equally relevant for all variants of D-term inflation, including the scenario of D3/D7 inflation or the T-dual inflation with branes at angles. In particular, it is crucial not only to stabilize the overall Calabi-Yau volume, but also the particular combination of Kähler moduli entering the D-term ξ; this would be the prime candidate for a runaway direction that could spoil inflation. Stabilization of the Kähler moduli in the fluxbrane inflation scenario will be discussed in chapter 4. Our strategy will be to approach this in the context of the Large Volume Scenario [15, 135], where the overall volume modulus can be stabilized by a combination of α'-corrections in the Kähler potential and non-perturbative corrections in the superpotential. We will demonstrate explicitly how ξ can obtain its value dynamically and show that this can be achieved in a way such that no runaway direction is introduced.

A related challenge concerns the inevitable appearance of F-term contributions to the D7-brane modulus potential that may compete with the D-term potential. In fact, there are three qualitatively different sources for such a contribution. These are a leading-order appearance of the inflaton in the Kähler potential, a direct appearance of the brane moduli in the flux-induced superpotential, or brane-modulus-dependent loop corrections to the Kähler potential. We will discuss these effects in detail in chapter 5. It turns out that, while the leading-order terms from Kähler and superpotential can be avoided by a suitable (bulk) flux choice and a shift-symmetric Kähler potential for the D7-brane modulus, the loop-induced F-terms will dominate the potential in a large part of the parameter space and will significantly alter the phenomenological analysis of the fluxbrane inflation model.

4. Fluxbranes: Moduli Stabilization and Inflation

In the analysis of fluxbrane inflation in chapter 3 (see also [21]) moduli stabilization was taken for granted. This is a strong assumption for two reasons. On the one hand, our scenario requires specific values for certain parameters of the compactification (e.g. for the overall volume $\mathcal{V}$). It has therefore to be checked that these values can indeed be attained. On the other hand, the physical effects invoked to stabilize moduli tend to destroy the flatness of the inflationary potential, an effect well familiar also in other brane inflation scenarios [17, 38, 102, 240, 241]. Hence, the flatness of the potential has to be checked *after* moduli stabilization.

In the present chapter we discuss a new explicit moduli stabilization procedure, combining the F- and D-term scalar potentials. It is based on fluxed D7-branes in a geometry with three large four-cycles of hierarchically different volumes. In this scenario, the D-term is dynamically stabilized at a parametrically small value, such that a D-term-induced runaway direction towards infinite volume is avoided. Subsequently, we combine this moduli stabilization with the fluxbrane inflation idea, demonstrating in particular that CMB data (including cosmic string constraints) can be described within our setup of 'hierarchical' large volume CY compactifications, while maintaining parametric control over all four-cycle volumes. We also explain why recently raised concerns about constant FI-terms do not affect the consistent, string-derived variant of D-term inflation discussed in this thesis.

The size of curvature perturbations in fluxbrane inflation is governed by the inverse volume, forcing us into a regime where the volume $\mathcal{V}$ is very large (cf. equation (3.29)). Kähler moduli stabilization is then naturally realized in the Large Volume Scenario [15, 242]. The latter is based on the interplay between α'- and non-perturbative corrections to Kähler and superpotential, giving rise to a non-supersymmetric AdS minimum at exponentially large volume. The potential is uplifted by some additional positive contribution to the vacuum energy density, such that the minimum becomes Minkowski. This can be done via fluxes on D7-branes [91, 132–138] or via $\overline{\text{D3}}$-branes in a warped throat [13, 38, 243]. As will be worked out in the example of a simple two-Kähler-moduli model in section 4.1, using the first of these two possibilities in the context of fluxbrane inflation requires two independent flux effects: One of them annihilates at the end of inflation, when the two relevant D7-branes meet. The other flux can not annihilate given that certain topological requirements are fulfilled. This second flux is responsible for the uplift to a Minkowski vacuum, which has to remain intact after reheating.

Appropriately suppressing the cosmic string contribution to CMB fluctuations is a crucial issue in brane or D-term inflation. In our scenario, the stability of cosmic strings is not completely trivial (cf. the discussion in [193]). To be on the safe side, we consider the worst-case scenario of topologically stable (local) cosmic strings, rather than their semilocal cousins [100, 193, 244–246]. Cosmic string suppression then requires a hierarchy of four-cycle volumes in the internal manifold (see the discussion in the previous chapter). This forces us to go beyond the simple 'warm-up' example of section 4.1. Thus, in section 4.2, we embed our model of inflation in a 'hierarchical' Large Volume Scenario, along the lines of [131, 247]. This requires to consider string loop corrections to the Kähler potential, in addition to the α'- and non-perturbative corrections to Kähler and superpotential, in order to stabilize all Kähler moduli. These string loop corrections generically involve our inflaton candidate and thus give rise to additional (potentially dangerous) terms in the inflaton potential. We will neglect this inflaton-dependence in the present chapter, but discuss it in detail in chapter 5.

Combining the idea of 'fluxbrane inflation' (with all its phenomenological constraints) with moduli stabilization within hierarchical Large Volume Scenarios turns out to lead to a rather restrictive setting: For example, it was not possible to find a model with only three Kähler moduli in which one has parametric control over the size of the ratio of the gravitino mass and the Kaluza-Klein scale, $m_{3/2}/m_{\rm KK}$, and, at the same time, has all intermediate-size two-cycles parametrically large. While we were able to overcome this issue in a model with four Kähler moduli, we had to give up the idea of realizing the uplift of the AdS minimum to Minkowski via a D-term. Thus, in section 4.2 we perform this uplift by means of a different contribution to the vacuum energy density which, for concreteness, we chose to be $\overline{\rm D3}$-branes. It would be interesting to investigate in more detail under which circumstances the idea of an uplift induced by fluxes on D7-branes can be realized. However, this is beyond the scope of this thesis.

Most of this chapter is copied from [23], which is work done in collaboration with Arthur Hebecker, Moritz Küntzler, Dieter Lüst, and Timo Weigand. A first discussion of Kähler moduli stabilization in the fluxbrane inflation scenario is contained in the diploma thesis of M. Küntzler [22]. However, many relevant constraints were neglected in that analysis, such as the issue of a too large ratio $m_{3/2}/m_{\rm KK}$ and the tadpole constraint on the size of W_0. Therefore, the discussion in chapter 4 completely differs from that diploma thesis. The corresponding sections in [23] were written by myself, with some parts of section 4.3 edited by A. Hebecker. The appendices A, B, and C are slightly modified versions of the respective appendices in [23]. Appendix B grew from an appendix contained in [22], but was adapted to the discussion in [23] by myself.

4.1. Moduli Stabilization – Basic Setup

4.1.1. Moduli Stabilization in the Large Volume Scenario

It was found in [15,242] that under certain topological conditions there exists a non-supersymmetric AdS minimum of the scalar potential of Type IIB string theory compactified on a Calabi-Yau orientifold. This minimum appears at an exponentially large volume of the internal manifold and is therefore suitable for our purposes. To find this minimum one applies a two-step procedure: First, the complex structure moduli and the axio-dilaton are stabilized via bulk fluxes [12] and integrated out at a high scale, giving rise to a constant tree-level superpotential W_0. Due to the 'no-scale structure' of the Kähler moduli Kähler potential the resulting leading-order F-term potential is identically zero. A non-zero scalar potential arises at subleading order through α'-corrections in the Kähler potential and non-perturbative corrections in the superpotential. In a second step one then minimizes the effective potential for the Kähler moduli resulting from these higher-order effects.

D3-instantons can wrap internal four-cycles of the Calabi-Yau manifold. The corrected superpotential is given by

$$W = W_0 + \sum_p A_p e^{-a_p T_p}, \tag{4.1}$$

where the $T_p = \tau_p + ic_p$ denote the complexified Kähler moduli of the instanton four-cycles.[1] The constants a_p are given by $a_p = 2\pi$, while the Pfaffian prefactors A_p depend on the complex structure moduli and the axio-dilaton (which are assumed to be fixed) as well as the open-string moduli. The latter dependence could well be an issue for the viability of our brane inflation model. For example, it is known that in the presence of D3-branes the one-loop Pfaffian A_p involves the D3-brane position [241]. A similar effect was argued to occur for D7-branes which carry flux with non-vanishing induced D3-brane charge [248]. Recall from the discussion in chapter 3 that our flux[2] $\mathcal{F}_-$ is chosen such that the induced D3-brane charge vanishes (and the same can also be imposed on $\mathcal{F}_+$). The effect of [241, 248] is therefore not expected to occur in our setup. It remains an open question whether, for D7-branes, there is a possible open-string dependence of the non-perturbative superpotential beyond these effects. In particular, one must sum over all flux configurations on the D3-instantons [249], which may introduce such a dependence via the flux-induced D(-1) charge. This could be avoided in geometries for which $H^{(1,1)}$ of the instanton divisor only contains elements which are even under the orientifold involution, such that the instanton cannot carry flux [249].

The second ingredient apart from the superpotential (4.1) is the Kähler potential, including a certain α'^3-correction [127] (which can be shown to be the leading correction in inverse powers of the volume [131])

$$K = -2\log\left(\mathcal{V} + \frac{\xi}{2g_s^{3/2}}\right) - \log(-i(S-\overline{S})) + K_{\text{cs}}. \tag{4.2}$$

Here, $\xi = -\frac{\zeta(3)\chi(X_3)}{2(2\pi)^3}$ and $\chi(X_3)$ is the Euler characteristic of the Calabi-Yau threefold X_3. The Kähler variable S is related to the axio-dilaton $\tau = C_0 + ie^{-\Phi}$ via a non-trivial field redefinition [72, 73], the details of which are, however, not important in this chapter. The α'-contribution to the scalar potential, resulting from (4.2), is $\sim 1/\mathcal{V}^3$. On the other hand, the non-perturbative corrections in the superpotential are exponentially small, leading to a contribution $\sim e^{-a_p\tau_p}/\mathcal{V}^2$. For both contributions to be equally relevant for creating a large-volume minimum of the scalar potential, one has to require some of the four-cycles on which instantons are wrapped to be exponentially smaller than the overall volume of the threefold. Suppose that there is one such small four-cycle whose modulus we will call τ_s and whose intersection form is 'diagonal' with respect to all other four-cycles, in the sense that the only non-vanishing triple intersection number involving τ_s is its triple self-intersection[3] κ_{sss}. Then the volume can be written in terms of the four-cycle moduli

[1] Our conventions for how lengths are measured are summarized in appendix A.

[2] As explained in section 3.1 the two U(1) gauge theories on the two D7-branes are most conveniently described in terms of their 'overall' ($\mathrm{U}(1)_+$) and 'relative' ($\mathrm{U}(1)_-$) piece. Accordingly, in the present chapter we will distinguish between the gauge flux $\mathcal{F}_+$ for $\mathrm{U}(1)_+$ and the gauge flux $\mathcal{F}_-$ for $\mathrm{U}(1)_-$. We will have more to say about $\mathcal{F}_+$ later on. What is important for the inter-brane potential is the relative gauge flux $\mathcal{F}_-$ (which was called $\mathcal{F}$ in chapter 3) in terms of which the effective potential for the canonically normalized inflaton φ is given by (3.16).

[3] Note that in our conventions $\kappa_{sss} > 0$. As the small four-cycle with modulus τ_s is contractible to a point, this means that in the expansion of the Kähler form J the coefficient t^s of the $(1,1)$-form ω_s is negative, $t^s < 0$. Here, ω_s is Poincaré-dual to the small four-cycle.

as

$$\mathcal{V} = \tilde{\mathcal{V}}(\tau_{q,q\neq s}) - c\tau_s^{3/2}, \tag{4.3}$$

where c is related to κ_{sss} as $c = \frac{2^{3/2}}{3!\sqrt{\kappa_{sss}}}$. Furthermore, we require $\tau_{q\neq s} \gg \tau_s$ such that the overall volume $\mathcal{V}$ is large, measured in units of ℓ_s in the 10d Einstein frame. In this limit the scalar potential is given by [15] (see also appendix B)

$$V_F(\mathcal{V}, \tau_s) = V_{0,F}\left(\frac{\alpha\sqrt{\tau_s}e^{-2a_s\tau_s}}{c\mathcal{V}} - \frac{\beta|W_0|\tau_s e^{-a_s\tau_s}}{\mathcal{V}^2} + \frac{\xi\gamma|W_0|^2}{g_s^{3/2}\mathcal{V}^3}\right), \tag{4.4}$$

with α, β and γ some positive constants which depend only on $|A_s|$, and $V_{0,F}$ some overall g_s-dependent prefactor. Their precise form is given in (B.7). This potential is already minimized in the axionic (i.e. c_s) direction. Since it is only the absolute value of W_0 and A_s which enter (4.4), in the following we will write W_0 instead of $|W_0|$ etc.

Extremization with respect to τ_s in the limit $a_s\tau_s \gg 1$ gives

$$a_s\tau_s = \log\left(\frac{2\alpha\mathcal{V}}{\beta c W_0}\right) - \frac{1}{2}\log\tau_s = \log\left(\frac{4a_sA_s}{3c}\frac{\mathcal{V}}{W_0}\right) - \frac{1}{2}\log\tau_s \tag{4.5}$$

and thus

$$V_F(\mathcal{V}) \simeq V_{0,F}\left(\frac{\xi\gamma W_0^2}{g_s^{3/2}\mathcal{V}^3} - \frac{c\beta^2W_0^2}{4\alpha\mathcal{V}^3a_s^{3/2}}\log^{3/2}\left(\frac{2\alpha\mathcal{V}}{c\beta W_0}\right)\right), \tag{4.6}$$

where we have neglected terms $\sim \log\tau_s$. Both terms in the above expression roughly scale like $\sim W_0^2\mathcal{V}^{-3}$. Generically, the same will be true for the value of the F-term potential at its minimum. As this minimum is AdS, we need some extra positive contribution to the energy density which lifts the potential at least to Minkowski. In our setup it seems most natural to do this via a D-term. As we saw in chapter 3, such a D-term scales like $\mathcal{V}^{-2}$ and will thus, in general, give rise to a runaway potential for $\mathcal{V}$, unless the size of the above F-terms is enhanced. The latter enhancement can be achieved via a large W_0. Considering only $\mathcal{V}$ and W_0, we expect from these arguments that roughly $W_0^2 \sim \mathcal{V}$.

Before addressing in detail this question of a dynamical runaway in the closed-string moduli space, we first explain why potential instabilities in the open-string moduli can generally be avoided geometrically: During inflation the uplifting D-term is due to both the relative gauge flux $\mathcal{F}_-$ and the overall flux $\mathcal{F}_+$ on the two D7-branes. As detailed in the previous chapter (see also [21]), the end of inflation is marked by a generalized recombination process between the two D7-branes: $\mathcal{F}_-$ is responsible for a tachyonic mode in the spectrum between both branes. The resulting condensation leads to a bound state between the two branes in which the relative $\mathrm{U}(1)_-$ is higgsed. The remaining bound state continues to carry gauge flux $\mathcal{F}_+$, whose D-term ξ_+ is responsible for the uplift to Minkowski/de Sitter after reheating. To guarantee stability of this D-term, apart from the potential runaway in the Kähler moduli discussed below, it must be ensured that no further condensation process occurs. The only such process would be a generalized recombination between the brane bound state and its orientifold image along their common locus, or possibly a recombination between the bound state and a different brane stack in the model. The appearance of a tachyon depends on the pullback of $\mathcal{F}_+$ to the respective intersection loci and can thus be controlled by a suitable choice of flux, see [21] for details. In particular, this requires an explicit choice of orientifold projection from which the brane-image brane intersection can be deduced. While we do not present such a concrete geometry in this work, these arguments are sufficient to show that a runaway in the open-string sector is in general not a problem.

4.1.2. A Two-Modulus Fluxbrane Inflation Model

As an illustrative example consider a two-modulus swiss-cheese model similar to the one discussed e.g. in the original LVS publication [15]. In such a model the overall volume can be expressed in terms of the two four-cycle volumes τ_b and τ_s as

$$\mathcal{V} = b\tau_b^{3/2} - c\tau_s^{3/2}, \tag{4.7}$$

where $b = \frac{2^{3/2}}{3!\sqrt{\kappa_{bbb}}}$, $c = \frac{2^{3/2}}{3!\sqrt{\kappa_{sss}}}$, and $\tau_b \gg \tau_s$. Wrapping the fluxed D7-branes around the large four-cycle called D_b and choosing flux $\mathcal{F}_\pm = n_\pm[D_b]$ for the overall/relative U(1) theory U(1)$_\pm$ of the brane pair induces a D-term potential [21, 72, 73][4]

$$V_D^\pm(\mathcal{V}) = \frac{1}{16\pi\mathcal{V}^2}\frac{\left(\int_{D_b} J \wedge \mathcal{F}_\pm\right)^2}{\frac{1}{2}\int_{D_b} J \wedge J} = \frac{1}{16\pi\mathcal{V}^2}2n_\pm^2\kappa_{bbb}. \tag{4.8}$$

The full scalar potential thus reads

$$V(\mathcal{V}) \simeq \frac{V_{0,F}}{\mathcal{V}^3}\left(\frac{\xi\gamma W_0^2}{g_s^{3/2}} - \frac{c\beta^2 W_0^2}{4\alpha a_s^{3/2}}\log^{3/2}\left(\frac{2\alpha\mathcal{V}}{c\beta W_0}\right) + \frac{2n^2\kappa_{bbb}}{g_s}\mathcal{V}\right), \tag{4.9}$$

where $n^2 = n_+^2 + n_-^2$. Note that from now on we work in a gauge where $e^{K_{\rm cs}} = 1$. Let $f(\mathcal{V})$ denote the term in the brackets on the right-hand side of (4.9). Then, in the Minkowski minimum after annihilation of $\mathcal{F}_-$ (i.e. $V_D(\mathcal{V}) = V_D^+(\mathcal{V})$) we have $f(\mathcal{V}_{\rm min.}) = f'(\mathcal{V}_{\rm min.}) = 0$. Vanishing of $f(\mathcal{V})$ in the minimum yields (to leading order in $a_s\tau_s \simeq \log\left(\frac{4a_sA_s}{3c}\frac{\mathcal{V}_{\rm min.}}{W_0}\right)$, using also $f'(\mathcal{V}_{\rm min.}) = 0$)

$$\frac{a_s}{g_s}\left(\frac{\xi}{2c}\right)^{2/3} \simeq \log\left(\frac{4a_sA_s}{3c}\frac{\mathcal{V}_{\rm min.}}{W_0}\right), \tag{4.10}$$

which can be used to rewrite $f'(\mathcal{V}_{\rm min.}) = 0$ as

$$W_0 \simeq \frac{2}{3}\frac{n_+^2\kappa_{bbb}}{A_s g_s}\frac{e^{a_s\tau_s}}{\tau_s^{1/2}}. \tag{4.11}$$

Plugging this back into (4.10) gives

$$\mathcal{V}_{\rm min.} \simeq \frac{c\kappa_{bbb}}{2a_sA_s^2}\frac{n_+^2}{g_s}\frac{e^{2a_s\tau_s}}{\tau_s^{1/2}}. \tag{4.12}$$

[4]Since the flux of the relative U(1)$_-$ theory will annihilate upon brane recombination, we cannot use it for uplifting the minimum value of the potential to zero. Instead, we use V_D^+ for the Minkowski uplift, while V_D^- is some additional energy density which is present during inflation and which decays into standard model d.o.f. upon reheating.

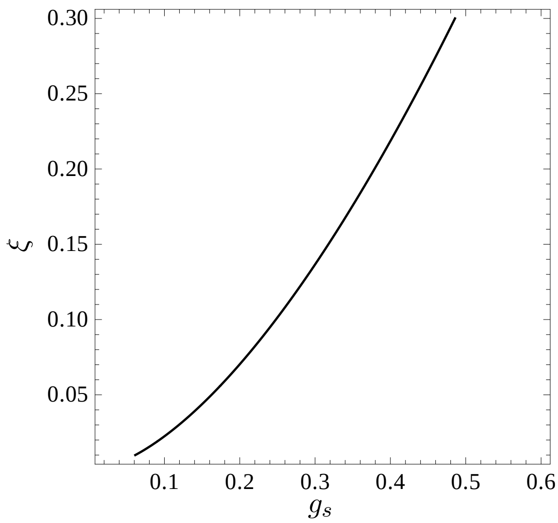

Figure 4.1.: Allowed values for g_s and ξ in the simple two-Kähler-moduli model.

Setting $n_+ = 5$,[5] $\kappa_{bbb} = 5$, $\kappa_{sss} = 1$ (such that $c = \sqrt{2}/3$),[6] and $A_s = 1$ we find that for $\mathcal{V}_{\text{min.}} = 1.7 \cdot 10^6$ (cf. (3.29)) the parameters can be chosen to lie in the phenomenologically viable regime (see figure 4.1): A value $\xi = 0.1$ implies $g_s \simeq 0.25$. In view of equation (4.11) this means

$$W_0 \simeq 1 \cdot 10^5. \tag{4.13}$$

It turns out that there is a tension between such a large W_0 and the requirement to cancel the D3 tadpole: The authors of [253] were able to reformulate the tadpole cancellation condition in a way which makes it obvious that, as long as all F-terms for the complex structure moduli vanish in the minimum, W_0 or rather $\sqrt{g_s/2}\, W_0$ is bounded by $\sqrt{\chi(X_4)/24}$, where $\chi(X_4)$ is the Euler characteristic of the associated F-theory fourfold X_4. In our example, $\sqrt{g_s/2}\ W_0 \simeq 3.6 \cdot 10^4$, which would require $\chi(X_4) \simeq 3.2 \cdot 10^{10}$. To the best of our knowledge, no fourfold with such a large Euler characteristic is known. Therefore, even before considering the production of cosmic strings, the simple two-Kähler-moduli model turns out not to work quite generically. One can go ahead and try to choose manifolds with different intersection numbers and tune A_s etc. However, we will not go down this road because, to be on the safe side concerning the cosmic string bound, we have to consider models beyond this simple one anyhow (see the discussion in sections 3.2 and 4.2.1). Instead, we will show that in a slightly more complicated situation W_0 can actually be much smaller, such that the tension described above is absent.

5 It turns out that there is a lower bound on n_+, which is easy to understand: As n_- is integrally quantized, for a given n_+ the uplift to de Sitter cannot be arbitrarily small. However, a large extra D-term from the relative $\mathrm{U}(1)_-$ on top of the uplift to Minkowski may potentially wash out the de Sitter minimum for the volume modulus. Therefore, the relative change in the size of the D-term before and after inflation cannot be too large or, in other words, n_+ has a lower bound. An analytical estimate of this lower bound, performed in appendix C, gives $n_+ \geq 4$. For what follows we use a slightly more conservative $n_+ = 5$. It is then possible to show numerically that one can indeed obtain a Minkowski minimum for $n_- = 0$ which is uplifted to a stable de Sitter minimum for $n_- = 1$.

6 Large intersection numbers tend to exacerbate the problems discussed below. Here and in the next section we take $\kappa_{bbb} = 5$ which is, for example, the triple self-intersection number of the quintic [250] and appropriate blow-ups thereof [251, 252].

Some further comments are in order, which also apply to the more general setup discussed in section 4.2:

- As τ_s is given by equation (4.5) we find that, in view of (4.10),

$$\tau_s \simeq \frac{1}{g_s}\left(\frac{\xi}{2c}\right)^{2/3} . \tag{4.14}$$

  This can also be found using a different method (cf. appendix B).

- Uplifting an AdS vacuum through magnetized D7-branes has been discussed in different variants in [91, 132–138]. Unless one appeals to a partial cancellation using charged fields [135–138], the D-term potential scales as $1/\mathcal{V}^2$. Since the F-term potential scales as $W_0^2/\mathcal{V}^3$, a successful uplift generically requires $W_0^2 \sim \mathcal{V}$. This is problematic for the following reason:[7] Estimating the Kaluza-Klein scale on the basis of T^6 with equal radii, we have $m_{\text{KK}} = \sqrt{\pi}/\mathcal{V}^{2/3} \sim 1/\mathcal{V}^{2/3}$. At the same time $m_{3/2} \sim W_0/\mathcal{V}$, which should be parametrically smaller to justify the use of a 4d supergravity analysis. However, one finds (with W_0 normalized as in [12])

$$\frac{m_{3/2}}{m_{\text{KK}}} = \frac{\sqrt{g_s}}{4\pi} \cdot \frac{W_0}{\mathcal{V}^{1/3}} \sim \mathcal{V}^{1/6} . \tag{4.15}$$

  This 'goes the wrong way' at large $\mathcal{V}$ (though it does so very weakly). In [135] it was argued that due to the appearance of a large numerical factor $16\pi^4$ in the denominator of V_D it is possible, for $\mathcal{V} \sim 10^3$, to uplift the AdS minimum to a stable de Sitter vacuum with $W_0 = \mathcal{O}(1)$. In view of (4.9) we believe that the situation is not quite as simple: The only relative factors of 2π between F- and D-term contributions come from the definition of ξ. They suppress the F-term, making the situation naively worse, but can be easily compensated by a large $\chi(X_3)$. However, using the explicit formulae (4.11), (4.12) and (4.14), we can make equation (4.15) more precise:

$$\frac{m_{3/2}}{m_{\text{KK}}} = \frac{n_+}{3\sqrt{\pi}}\sqrt{\frac{\kappa_{bbb}}{c\sqrt{\tau_s}}} \cdot \mathcal{V}^{1/6} . \tag{4.16}$$

  Assuming $n_+ \sim c \sim \kappa_{bbb} \sim \tau_s \sim 1$ this suggests that, at least in rough numerical agreement with [135], a fairly large $\mathcal{V}$ can indeed be tolerated in spite of the 'parametrical' clash between $m_{3/2}$ and m_{KK}. However, it is not clear that a manifold of swiss-cheese-type with such intersection numbers exists. Furthermore, as elucidated above, $n_+ = 1$ does not allow for a stable de Sitter uplift. Larger intersection numbers and a larger value of n_+ both deteriorate the situation, reducing the maximal size of the overall volume consistent with the requirement $m_{3/2} < m_{\text{KK}}$. On the other hand, the four-cycle volume τ_s, which could in principle suppress the size of the ratio (4.16), is essentially fixed at a value ~ 1 by (4.12) and the requirement $\mathcal{V}_{\text{min.}} = 1.7 \cdot 10^6$. In particular, with the numbers used and computed in this section we find $m_{3/2}/m_{\text{KK}} \simeq 34$ which means that there is no regime in which the supergravity approximation is valid.

  The authors of [135, 254] furthermore propose to use warping to suppress the D-term even further. While this is certainly a very appealing possibility, we are hesitant

[7] We thank Joseph Conlon and Fernando Quevedo for pointing this out.

to use it for the D-term driving inflation: We fear that it might clash with the shift symmetry that we need to keep our inflaton potential flat. On the other hand, including a further sector of D7-branes with flux in a warped region is certainly an option: It might be used for the uplift from AdS to Minkowski.

Concerning the 'inflationary D-term', our suggested solution to the 'D-term-suppression problem' is a hierarchy between large four-cycles. Given that we have several of those with significantly different volumes, we can arrange for the D-term to be parametrically smaller than $1/\mathcal{V}^2$.

In other proposals [135–138] the stabilization mechanism crucially depends on the presence of non-trivial vevs for some of the charged matter fields which appear in the D-term. These would arise from the intersection of the mobile D7-brane with other branes in the compactification. A suitable choice of gauge fluxes can in general ensure the absence of such matter fields. Indeed this conforms with our assumptions described at the end of section 4.1.1 concerning absence of extra instabilities in the open-string sector.

Finally, in [134] the authors consider only one Kähler modulus which is charged under the anomalous U(1) and which also appears in the non-perturbative superpotential.

Other proposals for uplifting mechanisms, put forward in the recent literature, include [254, 255].

- We will find that in the hierarchical case discussed in section 4.2 the D-term is not suitable for uplifting the AdS minimum to Minkowski. Therefore, we will need to do the uplift by means of a different mechanism. For concreteness we will consider $\overline{\text{D3}}$-branes.

  One might expect that such an $\overline{\text{D3}}$ uplift would also help in the isotropic setup discussed here, since the large n_+^2 in (4.11) would be absent. However, this turns out not to be the case: The lack of parametric control over the ratio $m_{3/2}/m_{\text{KK}}$ is still an issue. The size of W_0 is reduced only by a factor of $\sqrt{3/5}$ which appears because of the different volume-scaling of the $\overline{\text{D3}}$-brane energy density as compared to the D-term. This factor is not nearly sufficient for solving the gravitino mass and the D3 tadpole problems.

- From $f'(\mathcal{V}) = 0$ we find that the D-term contribution to the scalar potential (i.e. the third term in (4.9)) is suppressed by a factor of $a_s\tau_s = \frac{a_s}{g_s}\left(\frac{\xi}{2c}\right)^{2/3}$ relative to the first and second term in that expression. This means that the required uplift (i.e. the value of the F-term potential at its AdS minimum) is smaller than the naive parametric expectation, in agreement with the alternative derivation in appendix B. While this tends to exacerbate the 'D-term-suppression problem' discussed earlier, the effect is already included in equation (4.16) and does not change the moderately optimistic conclusion drawn above.

- It should be clear from the above that in our scenario SUSY is broken at a high scale, $m_{3/2} \sim 10^{-3}$, avoiding the Kallosh-Linde problem [256] in a 'trivial' way. While it is interesting to investigate the possibility that, after reheating, a different moduli stabilization mechanism takes over and low-scale SUSY is recovered [257, 258], we do not pursue this idea in the present paper.

4.2. Moduli Stabilization – Hierarchical Setup

While we saw in the previous section that, within the Large Volume Scenario, it is possible to stabilize the Kähler moduli in an AdS minimum at exponentially large overall volume, we ran into trouble trying to uplift the minimum to inflationary dS via a D-term potential: For $\mathcal{V} \simeq 1.7 \cdot 10^6$ the required size of W_0 is in tension with the D3 tadpole constraint and makes $m_{3/2}$ unacceptably large. On the other hand, this clash is not expected to be a generic feature because, in situations with more than two Kähler moduli, there are further potentially small or large numbers to be considered. These are, in particular, the relative sizes of four-cycles or two-cycles, respectively, and they may well improve the situation, depending on the precise intersection structure. In fact, considering these more involved models has turned out to be essential for appropriately suppressing the cosmic string tension: One of the promising outcomes of chapter 3 (see also [21]) was that, in fluxbrane inflation, this can be achieved via a mild hierarchy of four-cycle volumes.

In this section we thus consider moduli stabilization in a toy model with four Kähler moduli and discuss how, in this case, moduli stabilization can be achieved in a way which is consistent with the fluxbrane inflation proposal.

4.2.1. Cosmic Strings and the Need for a Hierarchy

As explained in section 3.2, the non-observation of cosmic string signatures (e.g. in the CMB) constrains their tension, the latter being directly related to the FI-term [239]. Due to the complicated nature of the bound state formed at the end of inflation, it is actually not immediately clear whether the produced cosmic strings are topologically stable (local) in our fluxbrane scenario. Since a detailed investigation of this interesting question is beyond the scope of this thesis, we assume a worst case scenario of local cosmic strings.[8] We use the results from [260],[9] which constrain the product $G\mu$ of the cosmic string tension μ and Newton's constant G as $(G\mu)_{\text{crit.}} = \frac{1}{4}\xi_{\text{crit.}} \simeq 0.42 \cdot 10^{-6}$, i.e.

$$\frac{\left(\int_\Sigma J \wedge \mathcal{F}_-\right)^2}{\frac{1}{2}\int_\Sigma J \wedge J} \lesssim 9.4 \cdot 10^{-2}, \tag{4.17}$$

with Σ the D7-brane divisor. It is thus clear that our compactification manifold needs to have at least two 'large' four-cycles with hierarchically different volumes in order for the cosmic string bound to be satisfied. This leads us to consider hierarchical compactification proposals similar to those discussed for example in [247, 261]. As it turns out the minimal modification of our previous setup with just one additional Kähler modulus is not sufficient for satisfying all phenomenological constraints at the same time. Hence, we will focus on a situation with four Kähler moduli. In this chapter we are primarily interested in investigating a general mechanism for stabilizing the moduli in a manner suitable for fluxbrane inflation rather than in constructing explicit compactification manifolds that furnish concrete realizations of this mechanism. We therefore content ourselves with making reasonable assumptions about the topology of the compactification space. With this

[8] In the semilocal case [100, 193, 244–246] the constraints are weakened [259].

[9] This was the most recent constraint on the cosmic string tension at the time when [23] was published. The analysis of [238] has further tightened this bound. However, as we will find in section 4.2.5, the cosmic string bound is actually not a constraining factor in fluxbrane inflation with hierarchical Kähler moduli stabilization. Even the tighter bound of [238] is easily satisfied in the scenario discussed in this chapter.

understanding, let us assume, for definiteness, that the volume form is of the type

$$\mathcal{V} = \frac{1}{2}\kappa_{112}(t^1)^2 t^2 + \frac{1}{2}\kappa_{133} t^1 (t^3)^2 + \frac{1}{2}\kappa_{223}(t^2)^2 t^3 + \frac{1}{6}\kappa_{sss}(t^s)^3. \tag{4.18}$$

For an overview of our conventions see appendix A. As the $(1,1)$-form ω_s is dual to a four-cycle which is contractible to a point, t^s is negative.

We choose the following brane and flux setup: The pair of D7-branes is wrapped around the four-cycle D_2 dual to the $(1,1)$-form ω_2, while the brane flux is given by $\mathcal{F}_\pm = n_\pm \omega_2$. Thus, the induced D3-brane charge ($\sim \int_{D_2} \mathcal{F}_\pm \wedge \mathcal{F}_\pm$) vanishes due to $\kappa_{222} = 0$. We now consider the limit $|t^1| \gg |t^2|, |t^3|$.[10] It turns out that for the Kähler metric $K_{T_i \overline{T_j}}$ to be positive definite in this limit we need $\kappa_{133} < 0$.[11]

It will be convenient to express all quantities in terms of τ_s, $\mathcal{V}$ and the quantities

$$x := \frac{t^3}{t^1}, \quad y := \frac{t^2}{t^1}. \tag{4.19}$$

For example, the constraints (3.28) and (4.17) can now be rewritten as

$$\mathcal{V}^{4/3} y^{2/3} = \frac{\kappa_{112}^{1/3}}{2^{4/3}} \cdot 4.2 \cdot 10^8, \tag{4.20}$$

$$x^2 \lesssim \frac{\kappa_{112}}{2 n_-^2 \kappa_{223}^2} \cdot 9.4 \cdot 10^{-2}. \tag{4.21}$$

As we will learn presently, the regime in which the model works is at small x and y.

4.2.2. String Loop Corrections

As we saw in section 4.1.1 the interplay between α'-corrections to the Kähler potential and non-perturbative corrections to the superpotential allows for a minimum of the scalar potential with the overall volume $\mathcal{V}$ stabilized at an exponentially large value and the small instanton four-cycle stabilized at $a_s \tau_s \sim \log(\mathcal{V}/|W_0|)$. However, for a model with more than two Kähler moduli there will be directions transverse to $\mathcal{V}$ which remain flat. As was shown in [131, 247] these transverse directions may be stabilized by string loop corrections to the Kähler potential. In toroidal compactifications those corrections are well known [18, 128, 129]. Based on this work the authors of [130] conjectured that on a general Calabi-Yau manifold string loop corrections to the Kähler potential take the form

$$\begin{aligned}\delta K_{(g_s)} &= \delta K_{(g_s)}^{\mathrm{KK}} + \delta K_{(g_s)}^{\mathrm{W}} \\ &= \sum_{i=1}^{h_{1,1}} g_s \frac{C_i^{\mathrm{KK}}(U, \overline{U})(a_{ij} t^j)}{\mathcal{V}} + \sum_{i=1}^{h_{1,1}} \frac{C_i^{\mathrm{W}}(U, \overline{U})}{(b_{ij} t^j)\mathcal{V}}.\end{aligned} \tag{4.22}$$

These corrections originate from the exchange of Kaluza-Klein (KK) modes (with respect to a two-cycle $a_{ij} t^j$) between D7-branes and O7-planes, and of winding (W) modes of strings (along a two-cycle $b_{ij} t^j$ on which the D7-branes intersect). In the example of a

[10] In section 4.2.5 we will show that it is indeed possible to stabilize the Kähler moduli in this regime.

[11] As we only specify the intersection numbers (4.18) rather than a concrete geometry, it is not possible to actually compute the Mori cone. In the general spirit of our approach we take $t^i > 0$, $i = 1, 2, 3$, $\kappa_{112}, \kappa_{223} > 0$ as part of the assumptions on our toy model.

toroidal compactification with $\mathcal{O}(1)$ values of the complex structure U the functions $\mathcal{C}^{\mathrm{KK,W}}$ were calculated to be of the order 10^{-2} (see e.g. [129]). Importantly, these functions will, generically, depend on the open-string moduli. However, as stated in the introduction to this chapter, we will neglect this dependence in the present discussion and come back to this issue in chapter 5.

Although the g_s-corrections coming from KK modes are the leading corrections in the Kähler potential in terms of the scaling with the Kähler moduli, it was found [131] that in the F-term potential actually the α'-corrections are dominant. This feature is called 'extended no-scale structure' and is crucial to ensure the overall consistency of the approach. Furthermore, as the g_s-corrections depend not only on the overall volume $\mathcal{V}$ but also on the two-cycle moduli t^i, it is intuitively clear that these corrections potentially stabilize the flat directions.

Following [130] we will assume that in our scenario the g_s-corrections take the form

$$\delta K_{(g_s)} = \frac{g_s}{\mathcal{V}} \sum_{i=1}^{3} C_i^{\mathrm{KK}} t^i + \frac{1}{\mathcal{V}} \sum_{i=1}^{3} \frac{C_i^{\mathrm{W}}}{t^i}. \tag{4.23}$$

From these terms one can compute the corresponding leading-order corrections to the scalar F-term potential as (cf. [131])

$$\begin{aligned} \delta V_{(g_s)} &= V_{0,F} \frac{W_0^2}{\mathcal{V}^{10/9}} \left\{ \frac{g_s^2 \left(C_1^{\mathrm{KK}}\right)^2}{2^{1/3} \kappa_{112}^{2/3}} \frac{1}{y^{2/3}} - (4\kappa_{112})^{1/3} \left(C_2^{\mathrm{W}} \frac{1}{y^{2/3}} + C_3^{\mathrm{W}} \frac{y^{1/3}}{x} \right) + \dots \right\} \\ &= V_{0,F} \frac{W_0^2}{\mathcal{V}^{10/3}} \left\{ \mathcal{A} g_s^2 \frac{1}{y^{2/3}} + \mathcal{B} \frac{1}{y^{2/3}} + \mathcal{C} \frac{y^{1/3}}{x} + \dots \right\}, \end{aligned} \tag{4.24}$$

with

$$\mathcal{A} = \frac{\left(C_1^{\mathrm{KK}}\right)^2}{2^{1/3} \kappa_{112}^{2/3}} > 0, \quad \mathcal{B} = -\left(4\kappa_{112}\right)^{1/3} C_2^{\mathrm{W}}, \quad \mathcal{C} = -\left(4\kappa_{112}\right)^{1/3} C_3^{\mathrm{W}}. \tag{4.25}$$

The ellipses in (4.24) denote terms which are suppressed by further powers of x and y in the limit of small x, y as compared to the leading-order contributions. Note that the C_i^{W} can have either sign.

4.2.3. Stabilizing Ratios of Two-Cycles

The string loop corrections discussed in the previous section, together with the D-term potential, will stabilize the ratios x and y. We now analyze the way in which this happens. In view of our ignorance concerning the prefactors $\mathcal{A}$, $\mathcal{B}$, and $\mathcal{C}$ in (4.24), we will *assume* in the sequel that $\mathcal{B}$ and $\mathcal{C}$ are positive and $g_s^2 \mathcal{A} \ll \mathcal{B}, \mathcal{C}$, such that the dominant terms of the g_s-corrections at small x, y are

$$\delta V_{(g_s)} = V_{0,F} \frac{W_0^2}{\mathcal{V}^{10/3}} \left\{ \mathcal{B} \frac{1}{y^{2/3}} + \mathcal{C} \frac{y^{1/3}}{x} \right\}. \tag{4.26}$$

Thus, y will be stabilized at

$$y_{\mathrm{min.}} = \frac{2\mathcal{B}}{\mathcal{C}} x. \tag{4.27}$$

Plugging this back into (4.26) yields

$$\delta V_{(g_s)} = V_{0,F} \frac{W_0^2}{\mathcal{V}^{10/3}} \frac{\mathcal{D}}{x^{2/3}}, \tag{4.28}$$

where $\mathcal{D} = \left(3\mathcal{B}^{1/3}\mathcal{C}^{2/3}\right)/2^{2/3}$. The runaway of x to infinity will be stopped by the D-term potential which, in the hierarchical model specified in section 4.2.1, is given by [21, 72, 73]

$$V_D = \frac{1}{16\pi\mathcal{V}^2} \frac{\left(\int_{D_2} J \wedge \mathcal{F}\right)^2}{\frac{1}{2}\int_{D_2} J \wedge J} = \frac{\alpha n^2 x^2}{16\pi\mathcal{V}^2}\left(1 + \ldots\right). \tag{4.29}$$

The ellipses denote terms which are higher-order in x and y. Furthermore, we write $\alpha = \left(2\kappa_{223}^2\right)/\kappa_{112}$ and[12] $n = \sqrt{n_+^2 + n_-^2}$. The above D-term potential is enhanced by powers of the overall volume $\mathcal{V}$ as compared to (4.24). For some fixed $\mathcal{V}$ it will drive x to small values, thereby lowering the relative size of the D-term potential with respect to the F-terms. Regarding the required suppression of the cosmic string energy density, this is precisely the regime where we want to be. Furthermore, it is this feature which allows us to choose W_0 much smaller than in the 'warm-up' model of section 4.1.

Minimizing in the x-direction gives

$$x_{\text{min.}} = \left(\frac{1}{3}\frac{g_s W_0^2 \mathcal{D}}{\alpha n^2 \mathcal{V}^{4/3}}\right)^{3/8}. \tag{4.30}$$

We thus find a flux-dependent energy density given by

$$V_{\text{flux}}(\mathcal{V}) = \frac{g_s W_0^2}{16\pi\mathcal{V}^3}\left(\frac{4}{3^{3/4}}\left(\frac{\alpha n^2}{g_s W_0^2}\right)^{1/4}\mathcal{D}^{3/4}\right). \tag{4.31}$$

Comparing this expression to (4.6) it is apparent that the flux-induced energy density is not suitable for an uplift to dS. The reason is that the volume scaling of (4.31) is exactly the same as in the first term of (4.6). Thus, the same argument which shows that (4.6) gives rise to an AdS minimum applies here. Therefore, we need some additional contribution to the vacuum energy density which is suitable for uplifting the minimum to dS. $\overline{\text{D3}}$-branes in a warped throat are a prime candidate for this purpose [13, 38, 243].[13] Including their contribution, the full scalar potential reads

$$V(\mathcal{V}) = \frac{g_s W_0^2}{16\pi\mathcal{V}^3}\left(\frac{3}{4}\frac{\xi}{g_s^{3/2}} - \frac{3}{2}\frac{c}{a_s^{3/2}}\log^{3/2}\left(\frac{4a_s A_s}{3c}\frac{\mathcal{V}}{W_0}\right) + \mathcal{E}\mathcal{V}^{5/3} + \frac{4}{3^{3/4}}\left(\frac{\alpha n^2}{g_s W_0^2}\right)^{1/4}\mathcal{D}^{3/4}\right). \tag{4.32}$$

The quantity $\mathcal{E}$ scales with the fourth power of the warp factor at the position of the $\overline{\text{D3}}$-brane inside the warped throat. We will assume that it can be tuned arbitrarily by tuning the fluxes which determine the strength of the warping.

[12] Note that we will choose to have $n_+ \neq 0$, although, as we will find out presently, the uplift to dS cannot be done via a D-term. By keeping $n_+ \neq 0$ the stabilization of x before and after reheating relies on the same mechanism, which simplifies the discussion.

[13] We are aware of the recent discussion [262–264] (see also [265]) of potential problems with the supergravity solution corresponding to an $\overline{\text{D3}}$-brane in a warped throat. While these investigations have to be taken very seriously, we think it is fair to say that, until now, no definite conclusion disproving the viability of such an uplift has been established. As already stated at the end of section 4.1, D7-branes with flux in a warped region might be a good alternative.

4.2.4. Parametric Analysis

In this section we would like to give an argument why, parametrically, the situation in the hierarchical setup is improved as compared to the 'warm-up' model.

Recall that in the model discussed in section 4.1 the tree-level superpotential had to be tuned large, $W_0 \sim \sqrt{\mathcal{V}}$, such that the F-terms and the D-terms had about the same size. The necessity to go in this parameter regime can be seen most easily from (4.9) (all three terms should be of the same order). Furthermore, in section 3.2 the overall volume $\mathcal{V}$ was fixed by the requirement that the right amount of curvature perturbations is produced. The so determined $\mathcal{V}$ led to a value of W_0 which was incompatible with the D3 tadpole cancellation constraint.

On the other hand, the situation in the hierarchical model looks quite different: The D-term features a suppression proportional to x^2 and thus the tuning $W_0 \sim \sqrt{\mathcal{V}}$ is not necessary anymore. The gravitino mass, measured in units of the Kaluza-Klein scale, is given by[14] $m_{3/2}/m_{\mathrm{KK}} \sim W_0/(\mathcal{V}^{1/3}y^{1/6})$. Furthermore, we have $t^2 \sim yt^1 \sim y^{2/3}\mathcal{V}^{1/3}$ and $t^3 \sim xt^1 \sim W_0^{3/4}\mathcal{D}^{3/8}/(\mathcal{V}^{1/6}y^{1/3})$. Now we use (4.27) together with the constraint $\mathcal{V}^{2/3}y^{1/3} = \zeta^{-1}$ (cf. (4.20)) to find

$$\begin{aligned} \frac{m_{3/2}}{m_{\mathrm{KK}}} &\sim W_0\sqrt{\zeta}, \\ t^2 &\sim \left(\frac{\mathcal{B}^{3/2}}{\mathcal{C}}W_0\right)^{1/2}, \\ t^3 &\sim \left(\frac{\mathcal{C}}{\mathcal{B}^{1/2}}W_0\right)^{1/2}. \end{aligned} \tag{4.33}$$

Here, ζ is related to $\tilde{\zeta}$ defined in section 3.2 via $\zeta = \left((2\kappa_{112})^{1/3}N\right)^{-1/2}\tilde{\zeta}$, where N is the number of e-foldings which we took to be 60. We see immediately that, for $\mathcal{B}, \mathcal{C} = \mathcal{O}(1)$, we have to choose W_0 somewhat large in order to have $t^2, t^3 \gg 1$. Considering in addition the required smallness of $m_{3/2}/m_{\mathrm{KK}}$ we should choose $1 \ll W_0 \ll \zeta^{-1/2}$. For example, setting $W_0 \sim \zeta^{-1/3}$ we find

$$t^2 \sim t^3 \sim \zeta^{-1/6} \gg 1, \quad \frac{m_{3/2}}{m_{\mathrm{KK}}} \sim \zeta^{1/6} \ll 1. \tag{4.34}$$

Therefore, in the hierarchical setup it is indeed possible to make $m_{3/2}/m_{\mathrm{KK}}$ parametrically small and, at the same time, have the two-cycle volumes t^2 and t^3 parametrically large. In the following section we will demonstrate, using explicit numbers, that the tuning $W_0 \sim \zeta^{-1/3}$ can be done in a way in which the D3 tadpole constraint is not violated.

4.2.5. Quantitative Results in the Hierarchical Setup

We conclude this chapter by showing that it is possible to consistently choose or compute explicit numbers for all the quantities which are involved in the setup under discussion. This can be done in a way such that all phenomenological constraints are satisfied.

[14] This estimate is based on the approximation $L_{\mathrm{max.}} \sim \sqrt{t^1}$, where $L_{\mathrm{max.}}$ is the volume of the largest cycle on which KK states propagate.

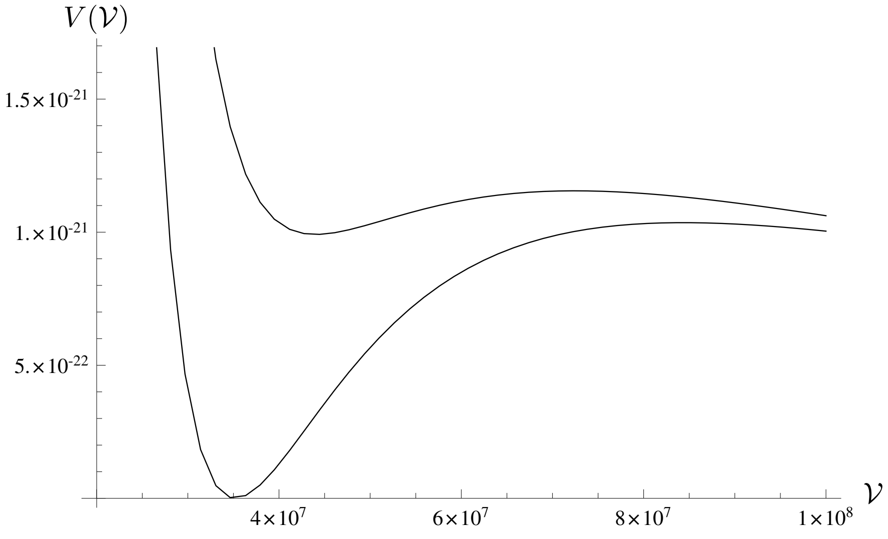

Figure 4.2.: Plot of (4.32) for $n_+ = 1$, $n_- \in \{0, 1\}$.

Starting point is the expression (4.32) with $n_- = 0$. Assuming that the threefold has an Euler characteristic[15] $\chi(X_3) = 5$ we get $\xi \simeq 1.2 \cdot 10^{-2}$. We furthermore choose $W_0 = 2 \cdot 10^3$, $g_s = 3 \cdot 10^{-2}$, $\mathcal{D} = 7 \cdot 10^{-1}$, $\kappa_{112} = \kappa_{223} = 5$, $\kappa_{sss} = 1$ and thus $c = \sqrt{2}/3$, $A_s = 1$, and $n_+ = 1$. Then, via $V(\mathcal{V}_{\text{min.}}) = V'(\mathcal{V}_{\text{min.}}) = 0$, we can determine $\tau_s \simeq 2.01$. This gives an overall volume $\mathcal{V} \simeq 3.5 \cdot 10^7$ and, furthermore, $y \simeq 3.9 \cdot 10^{-3}$, $t^2 \simeq 6.0$. On the other hand, the ratio x is now determined to be $x \simeq 3.3 \cdot 10^{-3}$ and thus $t^3 \simeq 5.1$. Note that this value of x is easily compatible with the cosmic string bound (4.21). Furthermore, the requirement $V'(\mathcal{V}_{\text{min.}}) = 0$ can be used to compute $\mathcal{E} \simeq 3.8 \cdot 10^{-14}$.

A plot of the potential (4.32) for $n_- \in \{0, 1\}$ is shown in figure 4.2. Note that the value $W_0 = 2 \cdot 10^3$ requires $\chi(X_4) \gtrsim 1.4 \cdot 10^6$ (cf. the discussion at the end of section 4.1). Fourfolds with such Euler characteristics are known in the literature (see e.g. [266]). Furthermore, in view of (4.15) we find $m_{3/2}/m_{\text{KK}} \simeq 0.4$. Clearly, $m_{3/2}/m_{\text{KK}}$ is only marginally smaller than one and some of our cycle volumes are only marginally larger than the string length. However, as we have demonstrated in section 4.2.4, these crucial inequalities hold parametrically, where the small parameter can be chosen, for example, as the quantity ζ. Then, we of course have to plug in actual numbers, hoping that our parametrically controlled approximation continues to hold in the physical regime. This works sufficiently well to expect that models with the desired type of stabilization exist.

4.3. Consistency of the Effective Theory

While the question of a potential inconsistency of (constant) FI-terms in supergravity is not a novel issue (see, e.g. [81, 193]), it has attracted an increased amount of interest more recently [58, 194–199]. Given that D-term inflation in its original form [93, 94] relies on the presence of a (constant) FI-term and that the existence of consistent gravity models with this feature is doubtful, we find it necessary to devote a section of the present chapter to this issue. For example, the arguments above have led the authors of [267] to conclude that

[15] If, alternatively, we start from a more typical value of e.g. $\chi(X_3) = 100$, the value of g_s increases to $g_s = 0.2$ with all other quantities changed only marginally.

D3/D7 inflation as well as fluxbrane inflation are subject to rather stringent constraints. As we will explain, we believe that our construction can not come into conflict even with the most stringent no-go theorems concerning FI-terms that are being debated.

The viability of D-term inflation in view of supergravity constraints on FI-terms has also been discussed in [193]. However, since our perspective and (part of) our conclusions are different, we believe that it is worthwhile to revisit this issue.

4.3.1. Issues in String D-Terms

Recall that the D-term in supergravity is given in general by [56]

$$\xi = iK_i X^i(z)\,, \tag{4.35}$$

where K is the Kähler potential and $X(z)$ is the holomorphic Killing vector generating the (gauged) isometry of the moduli space. We denote the coordinates z^i on that moduli space collectively by z. The D-term potential then reads

$$V_D = \frac{g_{\rm YM}^2}{2}\xi^2\,. \tag{4.36}$$

The consistency question alluded to above is, roughly speaking, under which circumstances one may write

$$\xi = iK_i X^i(z) + \xi_0 \tag{4.37}$$

for some constant $\xi_0 \neq 0$. For our purposes, the precise answer to this question is, in fact, irrelevant. We are only interested in string-derived models and hence for us it is sufficient to know that no such constant arises in the low-energy limit of string compactifications [268–271] (at least there are no such examples). Moreover, as we will work out in more detail momentarily, our D-term potential is described by the (undebated) part $iK_iX^i(z)$. Since this has given us a viable model of inflation, one might think that the 'FI-term-issue' in fluxbrane inflation is thus closed.

Things are not quite as simple, though. Given that the D-term potential drives inflation, the moduli in $iK_iX^i(z)$ must be stabilized. In fact, this was the main theme of the present investigation. Hence one might expect to encounter, somewhere between the moduli-stabilization scale and the SUSY-breaking scale, an effective theory with constant FI-term. This would not only be potentially inconsistent, it turns out to be technically impossible in models where no FI-term is originally present [193, 194, 196]. How can any stringy version of D-term inflation then exist? The answer suggested in [193] was to have a *small* F-term potential giving a *large* mass to the relevant moduli, which might be possible with a special choice of Kähler potential. Jumping ahead, our answer is different: In our scenario the SUSY-breaking scale is enhanced as compared to the scale at which the Kähler moduli are stabilized. This fact is easy to understand: As we will confirm momentarily, the Kähler moduli naturally have masses $m_\tau^2 \sim V_D \sim \mathcal{V}^{-2}$. On the other hand the gravitino mass is given by $m_{3/2}^2 \sim W_0^2/\mathcal{V}^2$. Recall that, as a result of the approximate no-scale structure in the F-term potential and the requirement $V_D \sim V_F$, we work at parametrically large W_0. Therefore, the gravitino mass is parametrically larger than m_τ. Due to this particular hierarchy of scales our model avoids the above constraints 'trivially'. We will come back to this fact at the end of this section.

4.3.2. Moduli Masses in Fluxbrane Moduli Stabilization

We first put our D-term potential in the standard $\mathcal{N}=1$ supergravity form, following [91]: Let D_j be a divisor with dual two-form $[D_j]$ and let the fluxed D7-brane be wrapped on a divisor $D_{\mathcal{F}}$. The four-cycle modulus T_j, whose real part τ_j parametrizes the size of D_j, gets charged under the U(1) on the D7-brane if the flux living on the intersection $D_j \cap D_{\mathcal{F}}$ is non-vanishing. Since the symmetry (i.e. the isometry of the moduli space) which is gauged is an axionic shift symmetry, the corresponding Killing vector is just $X_j = iq_j$ with q_j the charge of T_j. Thus, in the low-energy effective action a D-term

$$\xi = -q_j K_{T_j} \tag{4.38}$$

appears. The charge q_j depends on the flux [91]: $q_j \sim \int_{D_{\mathcal{F}}}[D_j] \wedge \mathcal{F}$. In particular, with this input one can show that (4.38) is equivalent to [72, 73]

$$\xi = \frac{1}{4\pi}\frac{\int_{D_{\mathcal{F}}} J \wedge \mathcal{F}}{\mathcal{V}}, \tag{4.39}$$

which was used in section 4.1 and section 4.2.

In the simple two-Kähler-moduli example at the end of section 4.1, we wrapped the D-brane on D_b and chose a flux $\mathcal{F} = n[D_b]$. As κ_{bbb} is non-zero, T_b is charged under the U(1), generating a D-term of the form (4.39). The potential terms relevant for the mass of τ_b are (4.6) and (4.8). The corresponding Kähler potential is

$$K = -3\log\tau_b + \dots. \tag{4.40}$$

For simplicity we compute the mass in the Minkowski minimum (the result changes only by an $\mathcal{O}(1)$ factor when going to de Sitter). Working in addition to leading order in $\log\left(\frac{2\alpha\mathcal{V}}{c\beta W_0}\right)$, we find

$$m^2_{\tau_b} = \frac{1}{K_{\tau_b\tau_b}}\partial^2_{\tau_b}V\bigg|_{\tau_b^{\text{min.}}} = \frac{3}{4}V_D\bigg|_{\tau_b^{\text{min.}}}, \tag{4.41}$$

where V_D is given in (4.8).

This can be compared to the mass of the vector boson which gauges the axionic shift-symmetry. The relevant terms in the Lagrangian are

$$\mathcal{L} \supset -\frac{1}{4g^2_{\text{YM}}}F^2 + K_{i\bar{j}}D_\mu z^i \overline{D^\mu z^j}, \tag{4.42}$$

with

$$D_\mu z^i = \partial_\mu z^i - A_\mu X^i(z). \tag{4.43}$$

Thus, using also (4.35) and (4.36) the gauge boson mass is

$$m_V^2 = 2g^2_{\text{YM}}K_{T_b\overline{T_b}}\left|X^{T_b}\right|^2 = \frac{4}{3}V_D\bigg|_{\tau_b^{\text{min.}}}. \tag{4.44}$$

We conclude that the masses of vector boson and corresponding Kähler modulus are of the same order of magnitude. Moreover, since $V_D \sim H^2$, both masses are related to the Hubble scale. While the purist might object both to calling this D-term inflation (since

$V_F \sim V_D$) and to calling it single-field inflation (since $m_{\tau_b} \sim H$), we are, for the time being, satisfied with this outcome.

Finally, the analysis in appendix C gives

$$\frac{\delta V_D}{V_D} \lesssim \frac{2}{3^3}, \tag{4.45}$$

where δV_D represents the additional energy density due to $\mathcal{F}_- \neq 0$ during inflation. This implies that the Hubble scale during inflation is given by $H_{\text{infl.}} \lesssim 0.07 \cdot V_D$. Therefore, τ_b is actually somewhat heavier than suggested by the parametric analysis above ($m_{\tau_b}/H_{\text{infl.}} \gg 1$) and its dynamics can be disregarded during inflation.

We note that our result can be understood more generally (see e.g. [193,194]): In unbroken SUSY the mass of the vector and the mass of the volume modulus are the same because the U(1) is higgsed by the axionic scalar from the volume superfield (in our case T_b). This equality can only be lifted by SUSY breaking. Thus, if the mass of the volume modulus is stabilized at a scale much above the vector mass, supersymmetry must be broken at this high scale. As a result, there can be no energy domain where τ_b is consistently integrated out while the gauge boson is kept as a dynamical degree of freedom in a supersymmetric theory. In other words, as mentioned earlier, even an *effectively* constant FI-term can not arise.

In our specific setting (at least in the toy model version of section 4.1), we have $H^2 \sim m_V^2 \sim m_{\tau_b}^2 \sim V_D \sim V_F$, as demonstrated above. By contrast, $m_{3/2}^2$ is much larger. This is due to the (approximate) no-scale cancellation which makes V_F smaller than its naive parametric size $|e^K W_0^2|$. Hence, we are indeed more than safe from any regime with unbroken SUSY and an effectively constant FI-term. Of course, the analysis in the present section dealt just with the toy model of section 4.1. An analogous discussion of the hierarchical model of section 4.2 is qualitatively similar but much more involved. While the various 'low-lying' mass scales from H to m_V are now somewhat different, the much larger size of $m_{3/2}$ is a generic feature. It will continue to ensure that SUSY is broken before the moduli are frozen.

4.4. Outlook

The main objective of this chapter was the study of moduli stabilization in the fluxbrane inflation scenario, introduced in chapter 3 (see also [21]). We briefly outlined the basics of Kähler moduli stabilization in the Large Volume Scenario and then discussed an example with four Kähler moduli and some specific intersection structure which was motivated by the phenomenological requirements. While, in our moduli stabilization program, the overall volume and some small blow-up four-cycle are fixed along the lines of the standard Large Volume Scenario, relative sizes of Kähler moduli are stabilized by an interplay of loop corrections and the D-term potential, thereby leading to a dynamical suppression of the size of the D-term which drives inflation. This was phenomenologically required in order not to run into conflict with a too large ratio of $m_{3/2}/m_{\text{KK}}$ or the D3 tadpole constraint. We demonstrated that moduli stabilization in this manner can work parametrically and also given the specific values for the parameters of the compact space which are required for a successful inflation phenomenology. Finally, we discussed our model of stringy D-term inflation in view of the fact that constant D-terms seem not to be consistent in supergravity, and explained why this issue does not affect fluxbrane inflation.

In the 'hierarchical model' discussed in section 4.2 it was not possible to achieve the uplift of the AdS vacuum to a Minkowski vacuum via a flux-induced D-term: The phenomenological requirements, which need to be imposed on the model, were too restrictive for that purpose. While, for the present analysis, we contented ourselves with the idea of realizing the uplift with $\overline{\text{D3}}$-branes instead, it would be interesting to further investigate the D-term uplifting proposal in more general setups, using the dynamical suppression mechanism as outlined in the present chapter. This is, however, beyond the scope of this thesis.

In the analysis contained in the present chapter we have neglected corrections to the inflaton potential due to moduli stabilization. As briefly mentioned already in several instances, there are various sources for such corrections. For example, D7-brane position moduli usually appear in the superpotential and Kähler potential. Furthermore, the loop corrections used to stabilize the relative sizes of Kähler moduli are also known to generically depend on the open-string scalars. All these quantities show up explicitly in the Large Volume Scenario, introducing inflaton-dependent terms which potentially spoil (or at least alter) the investigated scenario. A discussion of these effects is contained in the following chapter 5.

5. Fluxbrane Inflation with F-Term Corrections

As we have emphasized several times in the course of this thesis, any successful model of slow-roll inflation in string theory has to address the crucial issue of moduli stabilization and associated corrections to the inflaton potential. In the specific example of our fluxbrane inflation model the entanglement between inflation and (Kähler) moduli stabilization is most apparent when considering the D-term (equation (3.6)) which drives inflation. This D-term involves Kähler moduli (e.g. the overall volume $\mathcal{V}$ of the Calabi-Yau, but also some other linear combinations of cycle volumes) which are unstabilized at leading order and correspond, in fact, to runaway directions in moduli space (e.g. towards $\mathcal{V} \to \infty$). In the previous chapter we have stabilized these Kähler moduli by an interplay of loop corrections and the D-term. It has become clear that, in this case, the vacuum energy density during inflation can actually be at most as large as the size of the loop corrections in the minimum of the F-term potential. This is clearly a serious problem for inflation as soon as the loop corrections depend on the D7-brane modulus, i.e. the inflaton. Although generically present [18], we have neglected this dependence in the previous chapter.

In the present chapter we address the inflaton-dependent F-term corrections and analyze, under which circumstances we can parametrically suppress those with respect to the leading-order constant in the scalar potential. We start in section 5.1 by identifying possible F-term contributions to the inflaton potential. They can be attributed to explicit appearances of the brane moduli in the superpotential and the Kähler potential. Probably the most prominent such contribution is the one which arises from a generic Kähler potential, introducing the well-known η-problem in supergravity, mentioned already in section 2.1. As discussed in that section, the η-problem can be evaded e.g. if a shift symmetry protects the inflaton from appearing in the Kähler potential. Here we will analyze how such a shift symmetry comes about in the D7-brane context. Regarding an inflaton dependence in the superpotential, our strategy for avoiding these terms will be via a suitable flux choice.

Even if, owing to the shift symmetry, the leading-order Kähler potential does not depend on the inflaton, loop corrections will induce a dependence due to couplings in the open-string sector (i.e. inflaton – waterfall field couplings), in analogy to section 2.1. This entails two problems for the fluxbrane inflation scenario:

First, in previous analyses of Kähler moduli stabilization [129–131] it was assumed that all moduli appearing in loop-induced Kähler potential terms (including the open-string moduli) are stabilized supersymmetrically by their respective F-terms, the only exception being the Kähler moduli. If this is the case, it is well-known that loop corrections feature the 'extended no-scale structure', cf. section 4.2.2, which makes them subleading with respect to the α'^3-corrections [127]. This is an important prerequisite for successful Kähler moduli stabilization in the Large Volume Scenario. However, in fluxbrane inflation we need to avoid stabilization of at least one open-string modulus via leading-order F-terms. One might thus worry that the above analysis fails and the Large Volume Scenario is not compatible with the fluxbrane inflation proposal. In section 5.2 we will explicitly demonstrate that the extended no-scale structure is not spoiled if one includes an additional light degree of freedom, the inflaton, in the effective theory which is valid below the scale where complex structure moduli and the axio-dilaton are stabilized.

Second, as mentioned above, if loop corrections are used to stabilize Kähler moduli, as in the previous chapter 4, one can not generically separate the scale of the vacuum energy density driving inflation and the inflaton mass, implying an $\mathcal{O}(1)$ η-parameter. As explained in section 5.3.3 this negative conclusion can be avoided by an alternative stabilization proposal for the 'transverse' Kähler moduli directions: We will assume that relative sizes of four-cycles are stabilized by the condition of vanishing D-terms. The overall volume is then fixed in terms of the standard Large Volume Scenario. The uplift of the AdS vacuum obtained in this way to a Minkowski vacuum is achieved by a further (non-vanishing) D-term. This D-term has to be tuned to a small value (since it is generically dominant in the scalar potential of the Large Volume Scenario), which can be realized by tuning the position in Kähler moduli space. The D-term tuning is part of our version of the D-term uplifting proposal and can be viewed as the tuning of the cosmological constant. This tuning is only slightly worsened by insisting on an inflationary model which relies on a D-term for a different U(1) theory, but involving the same combination of Kähler moduli.[1] Importantly, in fluxbrane inflation no additional fine-tuning is needed in order to have a small η-parameter.

The goal of the phenomenological section 5.3 is to establish a parametrically controlled realization of fluxbrane inflation. The latter is achieved in section 5.3.6, where the phenomenological implications of the fluxbrane inflation model are thoroughly discussed. We find that in most of the parameter space the F-term loop corrections dominate over the D-term corrections, thus altering the predictions derived in the previous two chapters. Our scenario is able to reproduce the correct value for the spectral index, the number of e-foldings, and the amplitude of curvature perturbations. It satisfies the cosmic string bound and the running of the spectral index is moderately small, $n_s' \lesssim 10^{-2}$. The tensor-to-scalar ratio is tiny, $r \lesssim 2.6 \cdot 10^{-5}$.

This chapter is based on the publications [28, 29]. The structure of superpotential and Kähler potential was discussed in the diploma theses of Max Arends [25] and Konrad Heimpel [26]. Furthermore, the extended no-scale structure in fluxbrane inflation was analyzed in the diploma thesis of Christoph Schick [27]. These analyses were extended by myself, in collaboration with Arthur Hebecker, Dieter Lüst, Christoph Mayrhofer, and Timo Weigand, and published in [29]. A concise version of the discussion regarding the shift symmetry and the superpotential was part of [28], which is work done in collaboration with Arthur Hebecker and Lukas Witkowski. Section 5.1 is an extended version of that part of [28], which was written by myself. Sections 5.2 and 5.3 are mainly copied from [29]. Sections 5.2.1 and 5.2.2 are based on [27], however, with significant changes starting below (5.18) until the end of section 5.2.1, altering some conclusions of [27]. The parts in [29] which correspond to the remaining sections in chapter 5 (starting from section 5.2.3) were written by myself.

[1] Realizing inflation with a D-term involving a *different* combination of Kähler moduli does not help since it requires a further tuning.

5.1. Inflaton Dependence of Kähler and Superpotential

The scalar F-term potential in the supergravity Lagrangian is specified in terms of a Kähler and superpotential. We will discuss the latter two quantities in the fluxbrane inflation model and analyze under which circumstances a leading-order dependence of these quantities on the D7-brane position modulus can be avoided.

5.1.1. Shift-Symmetric Kähler Potential

In the low-energy effective theory arising from compactifying Type IIB string theory to four dimensions, let c denote a complex scalar which describes a D7-brane deformation modulus (the case where c describes the *relative* deformation of two D7-brane will be discussed at the end of this section). Such deformation modes are known to enter the Kähler potential in the form $K \supset -\ln\left(-i(S-\overline{S}) - k_{\mathrm{D7}}(z,\overline{z};c,\overline{c})\right)$ [72,73]. Here, $S = C_0 + i/g_s$ is the axio-dilaton and z collectively denotes complex structure moduli of the Calabi-Yau threefold. This Kähler potential arises in the weak-coupling limit from the F-theory Kähler potential for the fourfold complex structure moduli, given by

$$K = -\ln\left(\int \Omega_4 \wedge \overline{\Omega}_4\right), \tag{5.1}$$

where Ω_4 is the holomorphic (4,0)-form of the fourfold. Taking the weak-coupling limit one finds [78]

$$K_{g_s\to 0} = -\log\left((S-\overline{S})\pi_A(z)Q^{AB}\overline{\pi}_B(\overline{z}) + f(z,\overline{z};c,\overline{c})\right) + \dots, \tag{5.2}$$

where $\pi_A(z)$ are the periods of the threefold, i.e. integrals of the holomorphic (3,0)-form over a symplectic basis of three-cycles with intersection matrix Q^{AB}. Furthermore, $f(z,\overline{z};c,\overline{c})$ is some function involving brane and complex structure moduli, but not the axio-dilaton. The above expression holds up to corrections which are suppressed at large $\mathrm{Im}(S) = 1/g_s$.

In case $f(z,\overline{z};c,\overline{c})$ is a generic function of the brane moduli we expect a large inflaton mass,

$$m_c \simeq m_{3/2}, \tag{5.3}$$

leading to an η-parameter of $\mathcal{O}(1)$. This is the supergravity η-problem (cf. section 2.1). It is avoided if the D7-brane position moduli space possesses a shift symmetry, such that $f(z,\overline{z};c,\overline{c}) \equiv f(z,\overline{z};c-\overline{c})$. Such a shift symmetry comes about as follows: Under mirror symmetry [272] the Kähler potential (5.1) is identified with the Kähler moduli Kähler potential of the mirror fourfold which is known, at large volume, to involve the volume moduli of the fourfold, but not the corresponding axions. I.e. the Kähler moduli Kähler potential is shift-symmetric at large volume. Thus, via mirror symmetry we expect that (5.1) takes the shift-symmetric form

$$K^{\mathrm{LCS}} = -\ln\left(\frac{\kappa_{ijkl}}{4!}(u^i-\overline{u}^i)(u^j-\overline{u}^j)(u^k-\overline{u}^k)(u^l-\overline{u}^l) + \dots\right), \tag{5.4}$$

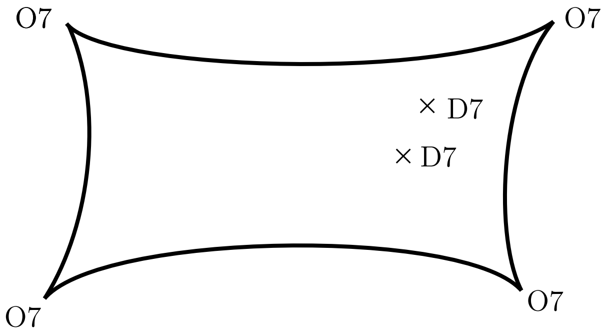

Figure 5.1.: Illustration of the D7-brane position modulus parameter space in the example of F-theory on $K3 \times K3$, which reduces to Type IIB string theory on $K3 \times T^2/\mathbb{Z}_2$ in the weak-coupling limit.

in the large complex structure limit [273, 274],[2] which is indeed explicitly visible in the expressions derived in [275, 276]. Here, κ_{ijkl} is the self-intersection matrix of the mirror fourfold divisors, u^i are the complex structure moduli of the fourfold, and the ellipses denote corrections to this shift-symmetric structure.

In the weak-coupling limit one of the u^i is identified with the axio-dilaton S, others are identified with D7-brane position moduli c^p, and the rest are complex structure moduli z^a of the threefold. Writing down the brane moduli dependence explicitly, we expect the structure

$$\begin{aligned} K^{\mathrm{LCS}}_{g_s\to 0} = -\ln\Bigg(&\frac{\kappa^{(1)}_{abc}}{3!}(S-\overline{S})(z^a-\overline{z}^a)(z^b-\overline{z}^b)(z^c-\overline{z}^c) \\ &+\frac{\kappa^{(2)}_{abpq}}{4!}(z^a-\overline{z}^a)(z^b-\overline{z}^b)(c^p-\overline{c}^p)(c^q-\overline{c}^q)+\dots\Bigg)\,. \end{aligned} \tag{5.5}$$

Focusing on one D7-brane deformation modulus c, we thus conjecture the following general form for the Kähler potential at large complex structure

$$K^{\mathrm{LCS}}_{g_s\to 0} = -\ln\left(-i(S-\overline{S}) - k_{\mathrm{D7}}(z,\overline{z};c-\overline{c})\right) - \ln\left(i\pi_A(z)Q^{AB}\overline{\pi}_B(\overline{z})\right). \tag{5.6}$$

The Kähler potential is then invariant under

$$c \to c+\delta,\ \ \delta\in\mathbb{R}, \tag{5.7}$$

and the inflaton will be associated with the real part of c, i.e. $\varphi \sim \mathrm{Re}(c)$.

An instructive example is F-theory on $K3\times K3$, where the Kähler potential is known explicitly and the shift symmetry is manifest [277, 278] (cf. also the discussion in [29]). In the orientifold limit the model is described by Type IIB string theory compactified on

[2]The point of 'large complex structure' (LCS) [273, 274] is defined as follows: It is a singular point in the complex structure moduli space, where the divergence structure of the periods is characterized by certain monodromies. Let w^i be a suitable set of local coordinates on the complex structure moduli space in which the LCS point is at $w^i = 0$, $\forall i$. Then, for a certain set of three-cycles Σ_A there is one invariant period. This period is scaled to one in the end. For the periods associated with the special coordinates u^i one finds $u^i \sim \log w^i$ in the vicinity of the LCS point. Furthermore, at leading order the remaining periods then have a very simple structure, leading to the Kähler potential (5.4).

$K3 \times T^2/\mathbb{Z}_2$ with D7-branes and O7-planes wrapping $K3$. The parameter space of c is thus $T^2/\mathbb{Z}_2$, which is depicted in figure 5.1. The brane-position-dependent part of the Kähler potential in this case reads

$$K^{K3} = -\log\left[-\left((S-\overline{S})(U-\overline{U}) - \sum_a (c^a - \overline{c}^a)^2\right)\right] + \ldots, \tag{5.8}$$

where U is the complex structure of $T^2/\mathbb{Z}_2$, and the c^a are 16 brane positions.[3]

What changes if one considers a pair of D7-branes (as in fluxbrane inflation) instead of one isolated brane? As indicated in (5.2) the S-dependence of the Kähler potential in the weak-coupling limit is very simple. In particular, S does not show up in $f(z,\overline{z};c,\overline{c})$. Corrections of the Kähler potential in the weak-coupling limit due to brane-brane interactions (which are higher order in $\mathrm{Im}(S)^{-1}$) are thus exponentially suppressed in S [78] and therefore not part of $f(z,\overline{z};c,\overline{c})$ (the branes don't 'see' each other at this order). Consequently, in analogy to the $K3$-example, we expect the function $f(z,\overline{z};c,\overline{c})$ to be additive in a suitable parametrization of the brane positions. Terms which break this structure are suppressed as $\sim e^{-\mathrm{Im}(S)}$.[4]

An alternative view on the shift symmetry for the D7-brane position moduli is provided via T-duality to Type IIA string theory. Upon T-duality the D7-brane position modulus is mapped to a Wilson line in Type IIA. The Wilson line arises from a 10d gauge potential. As a consequence of the 10d gauge invariance it couples only derivatively in the effective action and thus enjoys a shift symmetry [280–283]. This shift symmetry is protected to all orders in α'. It is broken by gauge theory loops, to be discussed in the following sections of the present chapter, and worldsheet instanton effects. The latter arise from open strings wrapping a disk with topologically non-trivial boundary. Such disks become large in the limit of large volume on the Type IIA side, which corresponds to the large complex structure limit on the Type IIB side. We thus expect that the shift symmetry in the D7-brane moduli space is present at large complex structure of the Calabi-Yau orientifold.

Note that the conjecture motivated by the Type IIA Wilson line picture is actually stronger than the one motivated from the fourfold perspective: In the latter case the shift symmetry was present in the limit of large complex structure of the F-theory *fourfold*. In the weak-coupling limit this would correspond to a situation where also the $\mathrm{Im}(c^p)$ are large. Motivated by the Wilson line picture we do, however, believe that the conjecture in its strong form is valid, i.e. the shift symmetry indeed exists at weak coupling and large complex structure of the Calabi-Yau orientifold. Moreover, as we will detail in the following chapter 6, minimizing the $\mathrm{Im}(z^a)$ at large values will lead to a minimum at large $\mathrm{Im}(c^p)$ as long as there is no (strong) dependence of the superpotential on the c^p.

5.1.2. Superpotential

For a generic flux choice the D7-brane deformation modulus will enter the scalar potential through an appearance in the superpotential. In Type IIB string theory the superpotential

[3]This Kähler potential is exact, i.e. there are no instanton-type corrections (this is due to certain integrability conditions of the Picard-Fuchs equations, see e.g. [279]).

[4]In addition, there are of course the usual gauge theory loops which add corrections to the scalar potential. But this is a different issue and will be discussed in section 5.2.

can be split into a bulk and a brane part, $W_{\rm IIB} = W_{\rm bulk} + W_{\rm brane}$, where the bulk part takes the well-known form

$$W_{\rm bulk} = \int_{X_3} G_3 \wedge \Omega_3, \tag{5.9}$$

independent of the brane moduli. The brane part contains the term [73, 284, 285]

$$W_{\rm brane} \supset \int_{\Gamma_5} \tilde{\mathcal{F}} \wedge \Omega_3, \tag{5.10}$$

where Γ_5 is the five-chain swept out by a pair of D7-branes, wrapped on Σ, as they are pulled off the O7-plane. Furthermore, $\tilde{\mathcal{F}}$ is the brane flux $\mathcal{F} := F_2 - B_2$ continued to that five-chain, such that its pullback to Σ gives $\mathcal{F}$.

Having in mind our inflation proposal, a simple strategy for avoiding the presence of the term displayed in (5.10) is to choose the flux $\mathcal{F}$ such that it is expanded along two-forms on Σ which are inherited from the bulk. For a Calabi-Yau the latter are automatically $(1,1)$-forms. Consequently, $\mathcal{F} \in H^{(1,1)}(\Sigma)$ and the term in (5.10) vanishes. Note that, since J is also of type $(1,1)$, it is precisely such flux which generates a D-term (cf. (3.6)).

On the other hand, it turns out that the superpotential $W_{\rm IIB}$ generically has a brane dependence beyond the term shown explicitly in (5.10). This was emphasized in [26, 29], where we computed $W_{\rm IIB} = W_{\rm bulk} + W_{\rm brane}$ from a decomposition of the F-theory superpotential

$$W = N^i \Pi_i(u) \tag{5.11}$$

in the weak-coupling limit. Here, the N^i are flux quantum numbers and $\Pi_i(u)$ is the period vector of the fourfold. The latter arises (in analogy to (1.36)) by integrating the holomorphic $(4,0)$-form of the fourfold over some basis of four-cycles and schematically reads (see e.g. [275, 276, 286])

$$\Pi(z) \sim (1, u^i, \kappa_{ijkl} u^i u^j, \kappa_{ijkl} u^i u^j u^k, \kappa_{ijkl} u^i u^j u^k u^l), \tag{5.12}$$

up to corrections which are subleading in the limit of large complex structure. In the weak-coupling limit this reduces to (5.9)+(5.10) [78], plus some additional contribution which is non-zero even if the flux $\mathcal{F}$ is of type $(1,1)$. As a result, even if no brane flux is present (or if the brane flux is of type $(1,1)$) the D7-brane coordinates may appear in the superpotential, leading to a stabilization of the branes via leading-order F-terms. This is well-known in explicit examples, e.g. the compactification of F-theory on $K3 \times K3$ [278] and its corresponding Type IIB limit. It is due to the fact that even those fourfold periods which, in the orientifold limit, reduce to bulk complex structure moduli have a brane moduli dependence. Accordingly, simply choosing $\mathcal{F}$ of type $(1,1)$ is not enough for avoiding a brane dependence of the superpotential. Rather, to obtain a brane-independent superpotential also the Type IIB bulk fluxes have to be chosen properly.

In reference [29] we analyzed the explicit example of a compactification of F-theory on $K3 \times K3$ and specified a flux which does not stabilize the D7-branes. We concluded that this is certainly possible, though we did not manage to find an explicit flux in the $K3 \times K3$-example which, at the same time, stabilizes both the axio-dilaton and all complex structure moduli. We will not discuss this any further in this thesis, but rather continue under the well-motivated assumption that an appropriate flux can be chosen also in more general compactifications. In the following we concentrate on the phenomenologically interesting subleading effects which induce the inflaton potential.

For completeness we furthermore recall from the discussion after equation (4.1) that non-perturbative effects from fluxed D3-brane instantons can introduce an explicit inflation dependence in the superpotential, which can be avoided by suitable constraints on the geometry of the instanton divisors.

5.2. Loop Corrections to the Kähler Potential

Having discussed the shift symmetry in the D7-brane position moduli space and the possibility to choose flux, such that the resulting leading-order scalar potential is independent of the inflaton, we now analyze loop effects which break the shift symmetry. They are induced by the coupling of the brane position moduli to the charged zero modes of strings stretching between the two branes and correct the Kähler potential of the effective theory [18].[5]

Our focus will be on the size of the corrections to the scalar potential which are induced by these Kähler potential corrections. Regarding string loop corrections in Type IIB orientifolds without any dependence on open-string or complex structure moduli (i.e. those moduli are assumed to be fixed at some higher scale and the only light fields are Kähler moduli), it was shown [128, 130, 131] that the leading-order contributions to the scalar potential induced by those corrections cancel due to the 'extended no-scale structure'. This structure renders g_s-corrections less important in the limit of large volume than, for example, α'-corrections [127]. As the main result of this section, we will demonstrate that the extended no-scale structure holds even when including partially unstabilized branes, at least in an exemplifying toy model.

5.2.1. Tree-Level Masses in D7-Brane Inflation

To set the stage we calculate some tree-level masses for open-string moduli in fluxbrane inflation. More precisely, we consider the open-string sector of a pair of D7-branes and compute masses for the components of the 4d scalar SU(2) multiplet which contains the relative deformation modulus of the two branes. In the fluxbrane inflation model this multiplet describes, amongst others, the inflaton field. As we will see, the structure of the Kähler and superpotential which is derived from string theory is crucial for the fluxbrane inflation model to be viable: Assuming a shift-symmetric Kähler potential for the brane deformation modulus, the only term which violates the shift symmetry in the F-term potential corresponds to the SUSY mass term for the zero modes which couple to the deformation modulus (the waterfall fields in fluxbrane inflation). By this mass term, these modes are stabilized at zero vev during inflation.

To obtain this result, recall that the open-string sector of a pair of D7-branes can be described in terms of a higher-dimensional SU(2) multiplet, more precisely a $\mathcal{N}=1$ vector multiplet in 8d, consisting of a vector, a complex scalar, and fermions. These components can be thought of as arising from the reduction of a 10d vector multiplet to 8d. From the 4d perspective we can construct the 8d multiplet in terms of several 4d $\mathcal{N}=1$ multiplets, namely a vector multiplet and three chiral multiplets (see e.g. [92]).

Upon dimensional reduction the 10d vector will give rise to three complex scalars which

[5] The superpotential is not corrected perturbatively (cf. [81]).

are the lowest components of chiral superfields ϕ_i in 4d,

$$\phi_{j\,|\theta=\bar{\theta}=0} = \frac{1}{\sqrt{2}}\left(A_{4+2j} + iA_{3+2j}\right), \quad j \in \{1,2,3\}, \tag{5.13}$$

plus a 4d gauge field which is contained in a vector multiplet V. The action for the dimensionally reduced gauge theory is given by [92]

$$\begin{aligned} S_{10} = &\int d^{10}x \int d^2\theta \mathrm{Tr}\left(\frac{1}{4kg^2}W^\alpha W_\alpha + \frac{1}{2kg^2}\epsilon^{ijk}\phi_i\left(\partial_j\phi_k + \frac{1}{\sqrt{2}}[\phi_j,\phi_k]\right)\right) \\ &+ \int d^{10}x \int d^4\theta \frac{1}{kg^2}\mathrm{Tr}\left((\sqrt{2}\overline{\partial}^i + \overline{\phi}^i)e^{-V}(-\sqrt{2}\partial_i + \phi_i)e^V + \overline{\partial}^i e^{-V}\partial_i e^V\right) \\ &+ \text{WZW term.} \end{aligned} \tag{5.14}$$

Here, $\phi \equiv \phi^a T^a$, where T^a are the SU(2) generators, normalized according to $\mathrm{Tr}\, T^a T^b = k\delta^{ab}$. Furthermore, we denote $\phi^a =: \{c, \chi^1, \chi^2\}$. The non-Abelian field strength superfield is given by $W_\alpha = -\frac{1}{4}\overline{D}\,\overline{D}e^{-V}D_\alpha e^V$ [56].

After compactification to 4d (and using the same symbol for the 10d fields and their zero modes in 4d) one can read off the superpotential

$$W \sim \mathrm{Tr}\left(\epsilon^{ijk}\phi_i[\phi_j,\phi_k]\right) \tag{5.15}$$

and the Kähler potential

$$K \sim \mathrm{Tr}\left(\overline{\phi}^i\phi_i\right). \tag{5.16}$$

Using the structure constants of the SU(2) algebra, together with $\mathrm{Tr}\, T^a T^b = k\delta^{ab}$, one finds that the only non-vanishing terms in the superpotential are the ones $\sim \phi_i^a\phi_j^b\phi_k^c$, where $a \neq b \neq c$ and $i \neq j \neq k$. We now recall the parametrization of ϕ in terms of c, χ^1, χ^2 to obtain the superpotential

$$W \sim \lambda c_i \chi_j^1 \chi_k^2, \quad i \neq j \neq k. \tag{5.17}$$

Furthermore, the Kähler potential is given by

$$K \sim \overline{\chi}_i^1\chi_i^1 + \overline{\chi}_i^2\chi_i^2 + \overline{c}_i c_i. \tag{5.18}$$

The inflaton in fluxbrane inflation will be associated with the diagonal (neutral) component of the 8d complex SU(2) scalar. Let's call this component c_3 for definiteness. Due to the completely antisymmetric structure of the superpotential, it is multiplied by components of the 4d chiral SU(2) multiplets which arise from dimensionally reducing the 8d vector. Those components are χ_1^1, χ_2^2 as well as χ_1^2, χ_2^1. For simplicity we focus only on $\chi_1 := \chi_1^1$ and $\chi_2 := \chi_2^2$. The origin of these fields is important for extracting the Kähler moduli dependence of their *supergravity* Kähler potential. While the Kähler potential for the component transverse to the brane is given by $K \sim c\overline{c}$, the one for the components parallel to the brane reads $K \sim \chi_i\overline{\chi}_i/(T+\overline{T})$ [287],[6] where T is a Kähler modulus whose

[6] Note that this expression is related to (5.18) via a field redefinition.

real part $\mathrm{Re}(T) = \tau$ measures a four-cycle volume. Additionally, from our previous considerations we expect the Kähler potential to have a shift-symmetric structure, such that it is independent of $\mathrm{Re}(c)$. We therefore work with

$$K = -3\ln\left(T+\overline{T}\right) - \frac{(c-\overline{c})^2}{2} + \frac{1}{T+\overline{T}}\left(\chi_1\overline{\chi}_1 + \chi_2\overline{\chi}_2\right) \tag{5.19}$$

and

$$W = W_0 + \lambda c \chi_1 \chi_2. \tag{5.20}$$

The scalar F-term potential computed from these quantities reads

$$\begin{aligned} V_F =& e^{-\frac{1}{2}(c-\overline{c})^2} \Bigg\{ \frac{1}{(T+\overline{T})^2}|\lambda|^2 c\overline{c}(\chi_1\overline{\chi}_1 + \chi_2\overline{\chi}_2) \\ &- \frac{1}{(T+\overline{T})^3}\left[(c-\overline{c})^2|W_0|^2 + \left\{(-(c-\overline{c}) + c(c-\overline{c})^2)\overline{W}_0\lambda\chi_1\chi_2 + \text{h.c.}\right\}\right] \\ &- \frac{1}{(T+\overline{T})^4}(c-\overline{c})^2(\chi_1\overline{\chi}_1 + \chi_2\overline{\chi}_2)|W_0|^2 \Bigg\} \\ &+ \text{terms higher order in } \chi_i \text{ and } \frac{1}{(T+\overline{T})}. \end{aligned} \tag{5.21}$$

There are several observations to be made:

- After canonically normalizing $\chi_i \to \chi_i\sqrt{T+\overline{T}}$, the first term (being the SUSY mass for the fields which couple to the inflaton in the superpotential) scales with $g_{\mathrm{YM}} \sim (T+\overline{T})^{-1}$, exactly as expected from the analysis of [21].
- The second term is the only one without χ_i-dependence. It fixes the imaginary part of c at the origin.
- All terms except for the first one are proportional to $(c-\overline{c})$. Since this difference is stabilized at zero, no SUSY-breaking mass term for the charged fields χ_i is obtained at tree-level. This is similar in spirit to the fact that for a Kähler potential of the form $K = -3\ln\left(T+\overline{T}-\chi\overline{\chi}\right)$, known from no-scale supergravity [82, 83] (see also [202]), the potential is exactly flat, i.e. no SUSY-breaking χ-mass is induced by non-vanishing F-terms of the modulus T.
- A SUSY-breaking mass for the waterfall fields is thus introduced only by subleading effects, such as loop corrections.

5.2.2. Extended No-Scale Structure with Dynamical Branes

We now consider loop corrections to the above setting. The conjectured form of string loop corrections [18] to the Kähler potential on general Calabi-Yau manifolds is given by (4.22) (cf. [130]). The functions C_i^{KK} and C_i^{W} will generically depend on complex structure and open-string moduli. The latter have been neglected in chapter 4. Regarding the dependence of these corrections on the Kähler moduli, they dominate over the α'-corrections (4.2) used to stabilize the overall volume in the Large Volume Scenario. However, due to the 'extended no-scale structure' [128, 130, 131] the leading-order terms in the scalar F-term potential induced by these loop corrections cancel, which makes the Large Volume Scenario viable. In order to arrive at this conclusion the complex structure and open-string moduli are usually assumed to be stabilized at some higher scale.

In fluxbrane inflation this assumption is violated: We explicitly choose flux such that the inflaton, being associated with one particular open-string modulus, obtains no mass at tree-level. Nevertheless, even in this more general situation the extended no-scale structure continues to hold, as we will demonstrate in the following.

Consider a toy model of Type IIB string theory compactified to 4d whose low-energy spectrum contains only one Kähler modulus $T = \tau + ic_4$ (cf. (1.41)) and dynamical branes, whose position moduli are denoted by c. The fluxes are chosen such that all complex structure moduli and the axio-dilaton are heavy and can be integrated out. Additionally, we will not consider the χ_i to be dynamical fields, as they are stabilized supersymmetrically by the leading term in the potential (5.21) (i.e. we take the superpotential to be $W \equiv W_0$). The Kähler potential for the dynamical fields is given, at leading order, by

$$K_0 = -3 \ln \left(T + \overline{T} \right) + K_{\mathrm{D7}}(c, \overline{c}). \tag{5.22}$$

Here, $K_{\mathrm{D7}}(c, \overline{c})$ denotes the Kähler potential for the open-string moduli c. It takes the shift-symmetric form $K_{\mathrm{D7}} \sim (c - \overline{c})^2$ (cf. (5.19)). Let us now consider string loop corrections to the Kähler potential (4.22) which are, at leading order in $\mathrm{Re}(T)$, of the type

$$\delta K = \frac{\beta(c, \overline{c})}{T + \overline{T}}, \tag{5.23}$$

and compute the effect of such a term in the scalar potential

$$V = e^K \left(K^{i\overline{\jmath}} D_i W_0 D_{\overline{\jmath}} \overline{W}_0 - 3|W_0|^2 \right). \tag{5.24}$$

In performing this calculation we follow the methods used for similar purposes in [131]: Suppose one would like to calculate the inverse of a matrix $K^0_{\overline{\imath}j} + \delta K_{\overline{\imath}j}$, where $\delta K_{\overline{\imath}j}$ is thought of as a correction to the leading-order expression $K^0_{\overline{\imath}j}$. Then we rewrite $K^0_{\overline{\imath}j} + \delta K_{\overline{\imath}j} = K^0_{\overline{\imath}k}(\delta^k{}_j + K_0^{k\overline{l}} \delta K_{\overline{l}j})$ and thus, using the Neumann series $(1 - B)^{-1} = \sum_{i=0}^{\infty} B^i$, we find

$$K^{i\overline{\jmath}} := K_0^{i\overline{\jmath}} + \delta K^{i\overline{\jmath}} + \ldots = K_0^{i\overline{\jmath}} - K_0^{i\overline{l}} \delta K_{\overline{l}k} K_0^{k\overline{\jmath}} + \ldots. \tag{5.25}$$

Using the explicit expressions (5.22) and (5.23) we obtain, for $i, j \in \{T, c\}$,

$$K^0_{\overline{\imath}j} = \begin{pmatrix} \frac{3}{(T+\overline{T})^2} & 0 \\ 0 & K^{\mathrm{D7}}_{\overline{c}c} \end{pmatrix}, \tag{5.26}$$

$$K_0^{i\overline{\jmath}} = \begin{pmatrix} \frac{(T+\overline{T})^2}{3} & 0 \\ 0 & \left(K^{\mathrm{D7}}_{\overline{c}c}\right)^{-1} \end{pmatrix}, \tag{5.27}$$

$$\delta K_{\overline{\imath}j} = \begin{pmatrix} \frac{2\beta}{(T+\overline{T})^3} & -\frac{\beta_c}{(T+\overline{T})^2} \\ -\frac{\beta_{\overline{c}}}{(T+\overline{T})^2} & \frac{\beta_{\overline{c}c}}{(T+\overline{T})} \end{pmatrix}, \tag{5.28}$$

$$\delta K^{i\overline{\jmath}} = \begin{pmatrix} -\frac{2\beta}{9(T+\overline{T})^{-1}} & \frac{\beta_c \left(K^{\mathrm{D7}}_{\overline{c}c}\right)^{-1}}{3} \\ \frac{\beta_{\overline{c}} \left(K^{\mathrm{D7}}_{\overline{c}c}\right)^{-1}}{3} & -\frac{\left(K^{\mathrm{D7}}_{\overline{c}c}\right)^{-2} \beta_{\overline{c}c}}{(T+\overline{T})} \end{pmatrix}, \tag{5.29}$$

in terms of which the correction to the F-term potential at linear order in δK reads

$$\begin{aligned}\delta V_1 = e^{K_0}\Big(& K_0^{T\overline{T}}K_T^0\delta K_{\overline{T}} + K_0^{T\overline{T}}\delta K_T K_{\overline{T}}^0 + \delta K^{T\overline{T}}K_T^0 K_{\overline{T}}^0 \\ &+ K_0^{c\overline{c}}K_c^0\delta K_{\overline{c}} + K_0^{c\overline{c}}\delta K_c K_{\overline{c}}^0 + \delta K^{c\overline{c}}K_c^0 K_{\overline{c}}^0 \\ &+ \delta K^{T\overline{c}}K_T^0 K_{\overline{c}}^0 + \delta K^{c\overline{T}}K_c^0 K_{\overline{T}}^0 \\ &+ \delta K K_0^{c\overline{c}}K_c^0 K_{\overline{c}}^0\Big)|W_0|^2.\end{aligned} \tag{5.30}$$

The first line contains only derivatives with respect to T. It thus vanishes as a result of the extended no-scale structure. Using (5.26)-(5.29), only the last term of the second line and the term in the last line survive. All others cancel against each other. This leaves us with

$$\delta V_1 = e^{K_0}\left(\beta - \left(K_{\overline{c}c}^{\mathrm{D7}}\right)^{-1}\beta_{\overline{c}c}\right)\frac{\left(K_{\overline{c}c}^{\mathrm{D7}}\right)^{-1}K_c^{\mathrm{D7}}K_{\overline{c}}^{\mathrm{D7}}}{(T+\overline{T})}|W_0|^2. \tag{5.31}$$

This term is $\sim |W_0|^2/(T+\overline{T})^4$ and thus parametrically dominant in the scalar potential. However, as already discussed in section 5.2.1, the leading-order term

$$V_0 = e^{K_0}\left(\left(K_{\overline{c}c}^{\mathrm{D7}}\right)^{-1}K_c^{\mathrm{D7}}K_{\overline{c}}^{\mathrm{D7}}\right)|W_0|^2 \tag{5.32}$$

fixes $\mathrm{Im}(c)$ at

$$K_c^{\mathrm{D7}} = K_{\overline{c}}^{\mathrm{D7}} = 0, \tag{5.33}$$

such that (5.31) in fact vanishes, $\delta V_1 = 0$. We have thus demonstrated that extended no-scale structure continues to hold in the fluxbrane inflation inflation scenario, where at least one open-string modulus is not stabilized at leading order.

5.2.3. Relevance of Loop Corrections

From the results of the previous subsection we expect the inflaton-dependent loop corrections to appear at $\mathcal{O}\left(|W_0|^2\mathcal{V}^{-10/3}\right)$ in the scalar potential. On one hand, this is good news as the inflaton, being an additional light degree of freedom entering the loop corrections, does not spoil the extended no-scale structure. On the other hand, these loop corrections have previously been used to stabilize Kähler moduli which are ‘transverse’ to the overall volume. This presents a problem for the fluxbrane inflation model: The stabilization mechanism used in chapter 4 balances loop corrections against the D-term, thereby stabilizing the latter at a small value which is phenomenologically required. But since the loop corrections are generically inflaton-dependent, this leads to an η-problem, i.e. the D-term vacuum energy density will be of the same size as the loop-induced mass term for the inflaton.

One can ask whether the loop corrections involving the inflaton are suppressed by additional small numbers. After all, we found in [21] (cf. also chapter 3) that the loop-induced D-term potential for the inflaton features an additional suppression by the quantity $g_{\mathrm{YM}}^2\left(\int_\Sigma J\wedge\mathcal{F}\right)^2$ which needs to be small in order to satisfy the cosmic string bound. However, expecting this would indeed be too optimistic: Let us consider the D-term potential, including the Coleman-Weinberg-type loop corrections, which was calculated in [21]:

$$V_D = V_0 \left(1 + \alpha_{\ln} \ln\left(\frac{\varphi}{\varphi_0}\right)\right), \tag{5.34}$$

$$\alpha_{\ln} = \frac{g_{\rm YM}^2}{16\pi^2}\left(-2\int_\Sigma \mathcal{F}\wedge\mathcal{F} + \frac{g_{\rm YM}^2}{2\pi}\left(\int_\Sigma J\wedge\mathcal{F}\right)^2\right). \tag{5.35}$$

In chapters 3 and 4 (see also [21, 23]) the first term in the expression for $\alpha_{\ln}$ was turned off (by an appropriate flux choice) for phenomenological purposes. However, assume for a moment that this term is there and, instead, neglect the (potentially small) second term $\sim \left(\int_\Sigma J\wedge\mathcal{F}\right)^2$. In this case, $\alpha_{\ln}$ is proportional to the number of chiral multiplets running in the loop (or, equivalently, the induced D3-brane charge) and the potential is the usual one of D-term hybrid inflation, found in field-theoretic approaches [93, 94] as well as, for example, in D3/D7 inflation [37, 105]. Let us now try to rephrase the loop correction in (5.34) in terms of a correction to the Kähler potential. To this end we make the following educated guess[7]

$$K \supset -3\ln(T+\overline{T}) - \frac{(c-\overline{c})^2}{2} + \frac{3g_{\rm YM}^2}{16\pi^2}\ln\sqrt{c\overline{c}}, \quad g_{\rm YM}^2 = \frac{4\pi}{T+\overline{T}}, \quad \varphi = {\rm Re}(c). \tag{5.36}$$

Using the general form of the D-term potential in supergravity (following e.g. the conventions of [91])

$$V_D = \frac{g_{\rm YM}^2}{2}\left(\frac{Q}{(2\pi)^2}\partial_T K\right)^2, \tag{5.37}$$

we now precisely reproduce (5.34) up to the factor $2\int_\Sigma \mathcal{F}\wedge\mathcal{F}$ which, as stated above, only counts the number of fields running in the loop and could easily be included in the above ansatz. Here, Q is the charge of the superfield T which shifts under the U(1) symmetry and $g_{\rm YM}^{-2}$ is the real part of the gauge kinetic function.

Now the naive hope might be that turning off $\int_\Sigma \mathcal{F}\wedge\mathcal{F}$ could also turn off the above correction. What remains would be suppressed by a further power of $g_{\rm YM}^2 \left(\int_\Sigma J\wedge\mathcal{F}\right)^2$ (*in addition* to the $g_{\rm YM}^2$-suppression) which can be stabilized at some very small value as discussed in chapter 4. This is, however, indeed too naive generically: While the supergravity calculation performed in section 4 of [21] actually admits the cycle which is wrapped by the D7-brane to be compact (and therefore captures the corrections $\sim g_{\rm YM}^2\int_\Sigma \mathcal{F}\wedge\mathcal{F}$), curvature-induced D3-brane charge [288–296] was neglected. As long as this charge is not canceled *locally* there is no reason to assume the BHK-type corrections [18, 129–131] to be absent.

Here, we therefore follow a different strategy and stabilize the 'transverse' Kähler moduli by leading-order D-terms, rather than loop corrections. More details on this stabilization are contained in the following sections.

Still, gaining a better understanding of the structure of loop corrections to the Kähler potential (in particular concerning their behavior at large complex structure) is desirable. This includes an analysis of known corrections on toroidal orbifolds [18] and on $K3\times T^2/\mathbb{Z}_2$. Besides being compactifications in which the loop corrections are actually computable, in these examples the D7-branes can be parallel-displaced, i.e. there is no self-intersection curve of the D7-brane divisor. Other such examples may be provided by $K3$-fibrations. Having in mind our fluxbrane inflation model, such settings are rather attractive.

[7] A similar type of correction was considered already in the Wilson line inflation papers [86, 87].

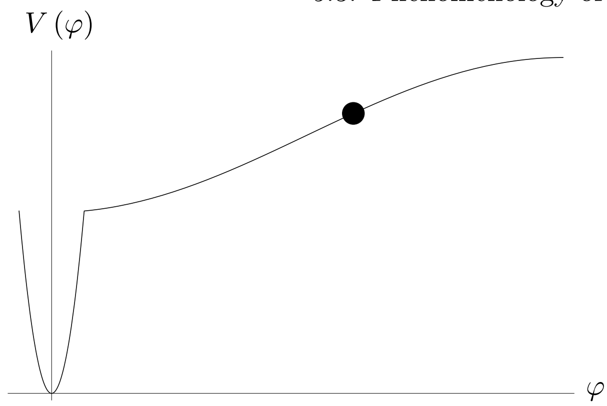

Figure 5.2.: Plot of the potential (5.38). For illustrative purposes the relative size of the variation with respect to the constant is exaggerated.

5.3. Phenomenology of Fluxbrane Inflation

Having determined the parametric size of the leading-order inflaton-dependent quantum corrections to the F-term potential, we now turn to the phenomenology of the fluxbrane inflation model. Recall from chapters 2 and 3 that fluxbrane inflation is a stringy version of supersymmetric hybrid natural inflation [191, 192]. As such, the inflaton potential in this model is parametrized as

$$V(\varphi) = V_0 \left(1 - \alpha \cos\left(\frac{\varphi}{f}\right)\right) \tag{5.38}$$

(cf. (2.13)), where $\varphi \sim \mathrm{Re}(c)$ is the canonically normalized inflaton. The periodicity in (5.38) captures the periodicity in the field space of the D7-brane modulus.

5.3.1. Phenomenological Constraints

The potential (5.38) has to satisfy a number of phenomenological constraints: One easily computes the slow-roll parameters at the beginning of the last N e-folds in the limit $|\alpha| \ll 1$ as (see also (2.14))

$$\begin{aligned} \epsilon &:= \frac{1}{2}\left(\frac{V'}{V}\right)^2 \simeq \frac{1}{2}\left(\frac{\alpha}{f}\right)^2 \sin^2\left(\frac{\varphi_N}{f}\right), \\ \eta &:= \frac{V''}{V} \simeq \frac{\alpha}{f^2}\cos\left(\frac{\varphi_N}{f}\right), \\ \tilde{\xi}^2 &:= -\frac{V'V'''}{V^2} \simeq \frac{2\epsilon}{f^2}. \end{aligned} \tag{5.39}$$

While the inflaton rolls from φ_N to φ_0 the universe undergoes an accelerated expansion with the number of e-folds given by

$$N := \int_{\varphi_0}^{\varphi_N} \frac{\mathrm{d}\varphi}{\sqrt{2\epsilon}} \simeq \frac{f^2}{\alpha} \ln\left(\frac{\tan\left(\frac{\varphi_N}{2f}\right)}{\tan\left(\frac{\varphi_0}{2f}\right)}\right). \tag{5.40}$$

Being a variant of hybrid inflation, fluxbrane inflation ends when the mass-square of the waterfall field becomes tachyonic. Figure 5.2 shows a schematic plot of the potential, including the waterfall transition. The tachyon exists due to the presence of supersymmetry-breaking brane flux which leads to a D-term in the effective theory. During inflation, when the waterfall fields are stabilized at zero vev, the D-term is given by (cf. equation (3.3))

$$V_D = \frac{g_{\mathrm{YM}}^2 \xi^2}{2} \equiv V_0, \tag{5.41}$$

where g_{YM} is the coupling of the gauge theory living on the branes and ξ is the Fayet-Iliopoulos parameter. In the present subsection we will assume that the system enters the waterfall regime at φ_0, i.e., at this point in field space the tachyon appears in the spectrum. This can be achieved by adjusting the coupling of the inflaton to the waterfall fields appropriately. However, as noted below (3.11), in our stringy realization of the hybrid natural inflation model there is a relation between this superpotential coupling and the gauge coupling constant. This relation is a remnant of an underlying $\mathcal{N} = 2$ supersymmetry [21, 229, 230]. As a consequence, φ_0 will eventually be set by the FI-parameter ξ, with no further model building freedom. We will further discuss the consequences of this relation in the next subsection.

The model as described above can be characterized by the parameters α, f, V_0, φ_0, and g_{YM}^2. The quantity φ_N is then adjusted in order to satisfy phenomenological requirements. The model parameters are constrained by experiment [48] as

$$\begin{aligned}
&N \simeq 60, \\
&n_s \simeq 1 - 6\epsilon + 2\eta \simeq 0.9603 \pm 0.0073, \\
&\tilde{\zeta} := \frac{V^{3/2}}{V'} \simeq \sqrt{\frac{V_0}{2\epsilon}} \simeq 5.10 \cdot 10^{-4}, \\
&n_s' := \frac{\mathrm{d}n_s}{\mathrm{d}\ln k} = 16\epsilon\eta - 24\epsilon^2 + 2\tilde{\xi}^2 = -0.0134 \pm 0.0090.
\end{aligned} \tag{5.42}$$

Generically, during tachyon condensation cosmic strings will form. If existent, those topological defects will leave an imprint on the CMB spectrum which can, in principle, be measured. The fact that no such signal has been observed yet constrains the (dimensionless) cosmic string tension $G\mu = \xi/4$ as

$$\frac{\xi}{4} \lesssim 1.3 \cdot 10^{-7}, \quad \text{(see [238])}. \tag{5.43}$$

Note that this bound depends on various things, such as the way in which the cosmic string network is modeled as well as the dataset used for constraining the string tension. In (5.43) we quote the most stringent bound from [238].

5.3.2. Embedding Hybrid Natural Inflation in String Theory

Given the model parameters in the field theory description, how do they relate to the parameters of the underlying string embedding? In order to answer this question, let us give the following intuitive general picture of how we think fluxbrane inflation works: In the simplest setup two D7-branes, whose positions are encoded by the vevs of two fields

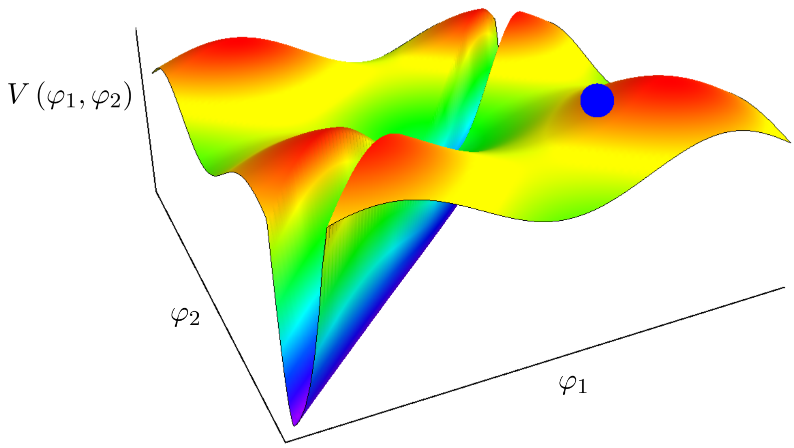

Figure 5.3.: Plot of the combined potential $\delta V = \sum_i \tilde{\alpha}_i \sin(\varphi_i/f)$.

c_i, $i = 1, 2$, will move in the transverse space along a S^1 with circumference R.[8] This S^1 corresponds to the directions $\mathrm{Re}(c_i)$ in field space, along which the leading-order potential is flat. At subleading order, this potential receives periodic corrections which we assume to be, at lowest order, $\delta V_i = \tilde{\alpha}_i \sin(\varphi_i/f)$. Here, the $\varphi_i \sim \mathrm{Re}(c_i)$ are canonically normalized fields and the field displacement $\Delta\varphi_i = 2\pi f$ corresponds to shifting one of the D7-branes once around the S^1. The total potential, which is displayed in figure 5.3, is then a function of both φ_i, $i = 1, 2$. It is thus clear that a 'generic' trajectory of the canonically normalized inflaton φ, corresponding to the distance of the two D7-branes, can be parametrized, at leading order, by (5.38) (see figure 5.2).

The D7-branes will wrap a four-cycle whose volume we denote by $\mathcal{V}_{\mathrm{D7}}$. Furthermore, $\mathcal{V}$ will be the volume of the whole Calabi-Yau. The circumference R of the transverse periodic direction, along which the D7-branes are separated, can be translated into the size of the field space for the canonically normalized inflaton (cf. chapter 3, see also [21]):

$$2\pi f \simeq R\sqrt{\frac{g_s}{4}\frac{\mathcal{V}_{\mathrm{D7}}}{\mathcal{V}}} =: \frac{1}{2}\sqrt{\frac{g_s}{z}}. \tag{5.44}$$

This equation defines the 'complex structure modulus' $z = \mathcal{V}/(\mathcal{V}_{\mathrm{D7}} R^2)$.

Recall from chapter 3 that in fluxbrane inflation a supersymmetry-breaking flux configuration on the D7-branes leads to the appearance of a D-term in the effective action. This D-term gives rise to a tachyonic mass term for the waterfall field. It reads

$$V_D = \frac{g_{\mathrm{YM}}^2 \xi^2}{2}, \quad \text{where} \quad g_{\mathrm{YM}}^2 = \frac{2\pi}{\mathcal{V}_{\mathrm{D7}}}, \quad \xi = \frac{1}{4\pi}\frac{\int_\Sigma J \wedge \mathcal{F}}{\mathcal{V}} =: \frac{x}{4\pi}\frac{\sqrt{\mathcal{V}_{\mathrm{D7}}}}{\mathcal{V}}. \tag{5.45}$$

The last equation defines $x = \int_\Sigma J \wedge \mathcal{F}/\sqrt{\mathcal{V}_{\mathrm{D7}}}$. Finally, the point where the tachyon condensation sets in is given by

$$\varphi_0 = \frac{\sqrt{\xi}}{2^{1/4}} = \frac{1}{2^{1/4}}\sqrt{\frac{x}{4\pi}}\sqrt{\frac{\sqrt{\mathcal{V}_{\mathrm{D7}}}}{\mathcal{V}}}. \tag{5.46}$$

[8] We choose the convention $\ell_s = 2\pi\sqrt{\alpha'}$ and measure all lengths in the ten-dimensional Einstein frame, i.e. after rescaling the metric $g^s_{MN} = e^{\phi/2} g^E_{MN}$, where $g_s = \langle e^\phi \rangle$ (cf. appendix A).

We have thus identified f, V_0, φ_0, and g^2_{YM} in terms of quantities which generically parametrize the fluxbrane inflation scenario. We now turn to the crucial and much more involved issue of deriving an expression for α in terms of stringy model parameters.

5.3.3. Moduli Stabilization

From the analysis in section 5.2.2 we know that the generic size of the loop corrections calculated by [130] is[9]

$$\delta V_{\mathrm{loop}}(\varphi) \sim \frac{g_s|W_0|^2}{\mathcal{V}^{10/3}}\beta(\varphi), \tag{5.47}$$

where $\beta(\varphi)$ is some function which involves the brane deformation modulus, i.e. the inflaton, and which we assume to have no specific structure except for its periodicity.[10] What matters now is the relative size of (5.47) with respect to the constant energy density during inflation. To quantify this we have to specify the vacuum of our theory, i.e. we have to discuss moduli stabilization.

We start with the assumption that the axio-dilaton as well as all complex structure moduli are stabilized by fluxes at some high scale, such that we are left with an effective theory of the Kähler and D7-brane moduli, in accordance with section 5.2.2. Recall from chapters 3 and 4 that one needs more than two Kähler moduli in order to implement the fluxbrane inflation scenario. This comes about as follows: In our model the constant energy density during inflation is due to supersymmetry-breaking flux on the D7-branes which annihilates at reheating. In the effective theory this flux gives rise to a non-vanishing contribution to the D-term potential (5.45). The cosmic string bound (5.43) imposes

$$\xi = \frac{1}{4\pi}\frac{\int_\Sigma J \wedge \mathcal{F}}{\mathcal{V}} \lesssim 4 \cdot 1.3 \cdot 10^{-7}, \tag{5.48}$$

which forces us to stabilize $x = \int_\Sigma J \wedge \mathcal{F}/\sqrt{\mathcal{V}_{\mathrm{D7}}}$ at a small value (cf. the discussion in chapter 3). This can only be achieved in models with more than two Kähler moduli.

Chapter 4 discusses a situation in which the flux $\mathcal{F}$ is dual[11] to an effective curve (i.e. a curve inside the Mori cone) on the brane. Therefore, given a certain fixed overall volume, there was a minimal value of x below which the volume of this curve becomes sub-stringy. If this is the case, one cannot trust the supergravity approximation anymore. Consequently, in the discussion of chapter 4 there was a lower bound on the size of x. In more generic situations, however, this lower bound will not be present due to the fact that the dual two-cycle need not be an effective curve of the brane (in particular it can be a linear combination of effective two-cycles with coefficients of either sign). In this case, nothing prevents x from being extremely small during inflation.

Stabilization of the directions in Kähler moduli space which are 'transverse' to the overall volume can be achieved in different ways: The strategy pursued in chapter 4 was to stabilize those moduli via loop corrections of the form discussed in section 5.2. This thus forces a balance of those loop corrections and the D-term, which is undesirable because one would then have $V_0 \sim \delta V_{\mathrm{loop}}(\varphi)$, implying $\eta \sim 1$.

One can, however, also follow a different strategy and stabilize those 'transverse' directions via D-terms as in [141]. That means one turns on worldvolume fluxes on D7-branes

[9] Regarding our normalization conventions, we follow chapter 4.
[10] Note that $\beta(\varphi)$ may contain additional factors of g_s. We analyze this in the course of this section.
[11] 'Dual' in this case refers to Poincaré duality on the worldvolume of the 7-brane.

other than the branes which are responsible for inflation. These fluxes induce parametrically dominant D-terms in the scalar potential which are strictly positive and stabilized at zero value, thereby fixing relative sizes of two-cycle volumes. The remaining two flat directions are then stabilized within the standard Large Volume Scenario, giving rise to a vacuum energy density $V_{\rm AdS} \sim -|W_0|^2/\mathcal{V}^3$. This vacuum is uplifted first to a Minkowski minimum and then, subsequently, to dS via two different D-terms.[12] In order not to introduce a runaway potential for the overall volume $\mathcal{V}$, these D-terms have to be parametrically smaller than their generic value $\sim 1/\mathcal{V}^2$, which can be achieved by fine-tuning the relative sizes of two-cycle volumes. The maximum value of the energy density during inflation is thus roughly given by $|V_{\rm AdS}|$, and we will parametrize

$$V_0 = \gamma |V_{\rm AdS}| \tag{5.49}$$

with $\gamma \lesssim 1$ in the following.[13] From a model building point of view we consider the tuning in the D-term superior to a tuning of loop coefficients, as the ability to compute D-terms exceeds by far the ability to compute loop corrections in general Calabi-Yau compactifications. It would be interesting and instructive to construct an example for this uplifting proposal.

5.3.4. The Relative Size of Loop Corrections from a Microscopic Viewpoint

What is the generic size of V_0 and $\delta V_{\rm loop}$, including the relevant parameters and factors of π? The AdS minimum in the Large Volume Scenario is at (see appendix B)

$$V_{\rm AdS} = -\frac{3}{8}\frac{(2\gamma_s)^{1/3}}{(4\pi)^3\sqrt{2\pi}} \frac{\hat{\xi}^{2/3}}{\sqrt{\ln\left(\frac{8\pi A_s}{3\gamma_s}\frac{\mathcal{V}}{|W_0|}\right)}} \frac{|W_0|^2}{\mathcal{V}^3}, \tag{5.50}$$

where $\hat{\xi} = -\chi(X_3)\zeta(3)/2$,[14] A_s is the prefactor of the instanton correction to the superpotential involving the small four-cycle of the Large Volume Scenario, $\delta W = A_s e^{-2\pi\tau_s}$, and $\gamma_s = \frac{2^{3/2}}{3!\sqrt{\kappa_{sss}}}$, where κ_{sss} is the triple self-intersection number of the small four-cycle. In this minimum, the small four-cycle is stabilized such that

$$2\pi\tau_s = \ln\left(\frac{8\pi A_s}{3\gamma_s}\frac{\mathcal{V}}{|W_0|}\right) = \frac{1}{2\pi g_s}\left(\frac{\hat{\xi}}{2\gamma_s}\right)^{2/3}. \tag{5.51}$$

The loop corrections are known explicitly only for certain orbifolds/orientifolds of the factorisable torus $T_1^2 \times T_2^2 \times T_3^2$, where they take the form [18]

$$\begin{aligned} \delta K_{\rm BHK} &= \delta K_{\rm BHK}^{\rm KK} + \delta K_{\rm BHK}^{\rm W} \\ &= -\frac{2 g_s}{(4\pi)^4}\sum_{I=1}^{3}\frac{\mathcal{E}_I^{\rm KK}(\varphi, U^I)}{\tau_{T_I^4}} - \frac{2}{(4\pi)^4}\sum_{I=1}^{3}\frac{\mathcal{E}_I^{\rm W}(\varphi, U^I)}{\tau_{T_J^4}\tau_{T_K^4}}\Bigg|_{I\neq J\neq K}. \end{aligned} \tag{5.52}$$

[12] Think of the corresponding two U(1) theories as linear combinations of the gauge theories living on the two fluxbranes, as in section 4.1.2.

[13] Values for γ in the range $\gamma = 10^{-1}\ldots 10^{-2}$ are required quite generally if the flux responsible for the uplift to Minkowski and the flux responsible for the inflationary de Sitter uplift live on the same two-cycle (see appendix C). In this case, stability of the uplifted vacuum requires the flux quantum number for the annihilating flux to be a fraction of the total flux quantum number, leading to a hierarchy between $|V_{\rm AdS}|$ and V_0.

[14] Note the absence of the factor $(2\pi)^3$ in this definition, as compared to the definition of ξ in chapter 4.

Here, U^I is the complex structure of the two-torus T_I^2 and $\tau_{T_I^4}$ is the volume of the four-torus $T_J^2 \times T_K^2$ (where $I \neq J \neq K$). The superscripts KK and W indicate whether the corrections are due to Kaluza-Klein or winding modes of the strings. The KK corrections arise from the exchange of Kaluza-Klein modes between D7-branes (and, potentially, D3-branes and the respective O-planes). The W corrections are due to the exchange of strings which wind around one-cycles along the intersection locus of two D7-branes [130]. Therefore, the presence of those winding corrections in a given model depends on the topology of the compact space and, in particular, of the intersection locus of the two D7-branes. For example, in the $K3 \times K3$-model discussed in [29] the D7-branes do not intersect at all. Thus, we expect the corrections associated with winding modes to be absent in this model [140].

The KK corrections are precisely of the form (5.23) and are thus subject to the extended no-scale structure, investigated in section 5.2.2. On the other hand, the W corrections enter the Kähler potential as a homogeneous function of degree -2 in the four-cycle volumina and are therefore expected to appear in the scalar potential at linear order in $\delta K^{\mathrm{W}}_{\mathrm{BHK}}$. Counting π-factors in the toroidal computation [18] and factors of g_s, the loop corrections due to winding modes are generically the largest in the scalar F-term potential.

The functions $\mathcal{E}_I^{\mathrm{KK,W}}(0, U^I)$ in (5.52) are proportional to a particular non-holomorphic Eisenstein series,

$$E_2(U) = \sum_{(n,m)\neq(0,0)} \frac{\mathrm{Im}(U)^2}{|n+mU|^4}. \tag{5.53}$$

For a square torus one has $E_2(i) \simeq 6$. The factors of proportionality (which we will call N^{KK}, N^{W}) depend on the particular orbifold/orientifold model and its brane content. They are essentially traces over matrices which specify the action of the orbifold and orientifold group on the CP labels of an open-string state. We view them as integral, topological data. While they can be of the order of thousands (see e.g. [129]), we do not expect such large factors to be a generic feature. The general form of the correction to the scalar potential, induced by (5.52), is thus

$$\delta V_{\mathrm{loop}} \simeq \frac{g_s}{16\pi} \frac{|W_0|^2}{\mathcal{V}^2} \frac{1}{(4\pi)^4 \mathcal{V}^{4/3}} \left\{ \frac{g_s^2 \left(N^{\mathrm{KK}} C^{\mathrm{KK}}\right)^2}{(4\pi)^4} \beta^{\mathrm{KK}}(\varphi) + N^{\mathrm{W}} C^{\mathrm{W}} \beta^{\mathrm{W}}(\varphi) \right\}, \tag{5.54}$$

expecting that this also applies in the more general Calabi-Yau context, as long as there are no big hierarchies of four-cycle volumes (except for potential blow-up modes). The quantities $C^{\mathrm{KK,W}}$ account for the complex structure dependence of this expression and are expected to be $\mathcal{O}(1)$ generically. As anticipated, the KK corrections are suppressed with respect to the W corrections, as the corresponding corrections to the Kähler potential feature a common $(4\pi)^{-4}$-factor, but the KK corrections are subject to the extended no-scale structure. Therefore, for the KK corrections the $(4\pi)^{-4}$-factor is squared in the scalar potential.

Now let us compute α, defined in (5.38), microscopically. Recall that α quantifies the relative size of the loop corrections (5.54) with respect to the constant (5.49) of the potential. From (5.54) it is clear that, generically, α_{micro} will be dominated by the loop corrections due to winding modes. However, as noted above, those are only present as long as there are non-trivial one-cycles along the intersection locus of D7-branes. This need not be the case. Therefore, we will distinguish $\alpha^{\mathrm{KK}}_{\mathrm{micro}}$ and $\alpha^{\mathrm{W}}_{\mathrm{micro}}$. Regarding the inflaton dependence of the loop corrections, we assume that the $\beta^{\mathrm{KK,W}}(\varphi)$ in (5.54) vary by $\mathcal{O}(1)$ as φ moves the maximal distance in field space. Thus, assuming $N^{\mathrm{KK,W}} = C^{\mathrm{KK,W}} = 2\gamma_s = 1$, $\alpha_{\mathrm{micro}}^{\mathrm{KK,W}}$ is directly computed as

$$\alpha_{\mathrm{micro}}^{\mathrm{KK}} = \frac{2}{3}\frac{g_s^{5/2}}{\gamma(4\pi)^6\hat{\xi}^{1/3}\mathcal{V}^{1/3}}, \tag{5.55}$$

$$\alpha_{\mathrm{micro}}^{\mathrm{W}} = \frac{2}{3}\frac{\sqrt{g_s}}{\gamma(4\pi)^2\hat{\xi}^{1/3}\mathcal{V}^{1/3}}. \tag{5.56}$$

5.3.5. Consequences of Experimental Constraints

In order to analyze the experimental constraints for the relative size of V_0 and $\delta V_{\mathrm{loop}}(\varphi)$ (i.e. the magnitude of α in (5.38)), let us now return to the phenomenological analysis of section 5.3.2. For exploring the parameter space of the fluxbrane inflation model we parametrize it in terms of the quantity φ_0/f, which describes the point of tachyon condensation in units of the total field space (up to a factor of 2π). Using the expression for η (5.39) and N (5.40) together with the experimental constraints (5.42) and assuming $\epsilon \ll |\eta|$ we find

$$N = \frac{2\cos\left(\frac{\varphi_N}{f}\right)}{n_s - 1}\ln\left(\frac{\tan\left(\frac{\varphi_N}{2f}\right)}{\tan\left(\frac{\varphi_0}{2f}\right)}\right). \tag{5.57}$$

For $N = 60$ this implicitly defines φ_N/f in terms of φ_0/f. Furthermore, the running of the spectral index is easily computed as

$$n_s' = \frac{(n_s - 1)^2}{2}\tan^2\left(\frac{\varphi_N}{f}\right). \tag{5.58}$$

Combining (5.57) and (5.58) we obtain a relation between φ_0/f and φ_N/f which is monotonic. The requirement $n_s' \lesssim 0.01$ thus puts a lower bound on φ_0/f which is given by

$$\frac{\varphi_0}{f} \gtrsim 0.032. \tag{5.59}$$

Figure 5.4 shows φ_N/f and φ_0/f for several different values of n_s'.

From (5.41) and (5.42) we now compute

$$\epsilon = \frac{1}{4}\left(\frac{g_{\mathrm{YM}}\xi}{\tilde{\zeta}}\right)^2. \tag{5.60}$$

We thus have introduced the additional parameter $(g_{\mathrm{YM}}\xi)^2$ which, together with φ_0/f, parametrizes the model. It is bounded from above due to $\xi \lesssim \xi_{\mathrm{max.}} := 5.2 \cdot 10^{-7}$ and $g_{\mathrm{YM}}^2 \lesssim 2\pi$.[15]

The quantity α is now expressed as

$$\alpha = \frac{2\epsilon}{\sqrt{n_s'}}\sqrt{2 + \frac{(1 - n_s)^2}{n_s'}}, \tag{5.61}$$

[15] In order to trust the supergravity approximation we require $\mathcal{V}_{\mathrm{D7}} \gtrsim 1$ and thus, in view of (5.45), we find $g_{\mathrm{YM}}^2 = 2\pi/\mathcal{V}_{\mathrm{D7}} \lesssim 2\pi$.

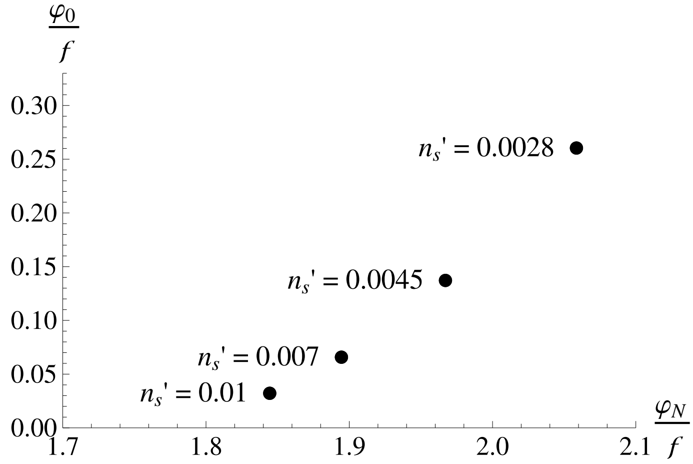

Figure 5.4.: Field values of the inflaton at the beginning of the last 60 e-foldings and the end of inflation for different values of n_s'.

and is thus completely determined by φ_0/f and $(g_{\rm YM}\xi)^2$. For $g_{\rm YM}^2 = 2\pi$, $\xi = \xi_{\rm max}$ and $\varphi_0/f = 0.032$, such that $n_s' = 0.01$, one finds $\alpha = 4.8 \cdot 10^{-5}$. This corresponds to the situation where ϵ and therefore the tensor-to-scalar ratio $r = 16\epsilon$ is maximal and given by

$$r = 4\left(\frac{g_{\rm YM}\xi}{\tilde{\zeta}}\right)^2 \simeq 2.6 \cdot 10^{-5}. \tag{5.62}$$

This is smaller than the value $r \simeq 7.6 \cdot 10^{-4}$ found in chapter 2. The reason for this is the cosmic string bound which was not taken into account in this chapter. Without the cosmic string bound, the limiting factors are the bounds on n_s' and f, from which one then determines a maximal value for ϵ which lies above the value computed via (5.60) with $g_{\rm YM}^2 = 2\pi$, $\xi = \xi_{\rm max}$. This ϵ then leads to the larger value for r.

One can ask what the maximal possible value for α in our model is. In order to determine that value we observe that the axion decay constant is given in terms of ϵ and n_s' as

$$n_s' = \frac{4\epsilon}{f^2}. \tag{5.63}$$

In view of (5.61) it is clear that α is maximized for large ϵ and small n_s', i.e. large f. However, recall that, in a regime where one controls the effective theory of a string theory compactification, the size of the field space of the axion is constrained as $f \lesssim 1/4\pi$ [60] (see also [137, 225–227] for an explicit discussion in the case of Kähler axions).[16] Also in field theory there are arguments that the quantity f should take only sub-planckian values [201]. Using the fiducial value $f = 1/4\pi$ one finds

$$\alpha^{(4\pi)} = \frac{\sqrt{\epsilon}}{4\pi}\sqrt{2 + \frac{(1-n_s)^2}{(4\pi)^2 4\epsilon}}. \tag{5.64}$$

This is a monotonically increasing function and maximized for large ϵ. Therefore, setting again $g_{\rm YM}^2 = 2\pi$ and $\xi = \xi_{\rm max}$ we find $\alpha_{\rm max}^{(4\pi)} = 1.9 \cdot 10^{-4}$. Values larger than this one

[16] Proposals of realizing inflation in string theory with larger f include [142, 143].

are not compatible with the data. This is a rather stringent constraint which needs to be satisfied by the string theory model.

5.3.6. Translation to Parameters of the String Theory Model

We saw that the field theory model of hybrid natural inflation can be parametrized by φ_0/f (or, equivalently, n_s') and $g_{\mathrm{YM}}\xi$. We now analyze how these two parameters map to corresponding quantities constructed from $\mathcal{V}$, $\mathcal{V}_{\mathrm{D7}}$, g_s, x and z of the stringy embedding. Let us start by combining (5.44) with (5.60) and (5.63) to obtain

$$\frac{g_s}{z} = \frac{(4\pi)^2}{\tilde{\zeta}^2} \frac{(g_{\mathrm{YM}}\xi)^2}{n_s'}. \tag{5.65}$$

Furthermore, from (5.45) it follows that

$$\frac{x}{\mathcal{V}} = \sqrt{8\pi}\, g_{\mathrm{YM}}\xi. \tag{5.66}$$

On the other hand, we can combine (5.45) with (5.46) to get

$$\sqrt{\mathcal{V}_{\mathrm{D7}}} = \frac{\sqrt{4\pi}}{\tilde{\zeta}^2} \frac{g_{\mathrm{YM}}\xi}{n_s'} \left(\frac{\varphi_0}{f}\right)^2. \tag{5.67}$$

We have thus expressed $\mathcal{V}_{\mathrm{D7}}$, $x/\mathcal{V}$ and g_s/z in terms of φ_0/f and $g_{\mathrm{YM}}\xi$. The quantity α is then calculated via (5.61).

In order to determine the absolute values of z, g_s, $\mathcal{V}$ and x we have to implement the constraints which come from moduli stabilization. As we have discussed in section 5.3.3 the maximum uplift in our model, and thus the maximum energy density during inflation, is given by

$$V_0 = \gamma \, |V_{\mathrm{AdS}}| \,, \quad \gamma \lesssim 1. \tag{5.68}$$

Furthermore, since the positive energy density is provided by a D-term (5.41) which annihilates at the end of inflation, we find

$$\frac{(g_{\mathrm{YM}}\xi)^2}{2} = \frac{3}{8} \frac{1}{(4\pi)^3\sqrt{2\pi}} \frac{\gamma \hat{\xi}^{2/3}}{\sqrt{\ln\left(\frac{16\pi}{3}\frac{\mathcal{V}}{W_0}\right)}} \frac{W_0^2}{\mathcal{V}^3}. \tag{5.69}$$

Here we have assumed that $A_s = 1$. Furthermore, from now on W_0 will denote the absolute value $|W_0|$. Given some W_0, $\hat{\xi}$ and γ, this equation determines $\mathcal{V}$. Finally, the string coupling g_s is given by

$$g_s = \frac{\hat{\xi}^{2/3}}{2\pi \ln\left(\frac{16\pi}{3}\frac{\mathcal{V}}{W_0}\right)}. \tag{5.70}$$

The following table shows the model parameters for a couple of different values of n_s' and ξ. The constant value of the tree-level superpotential is chosen to be $W_0 = 1$ and, furthermore, $\hat{\xi} = \gamma = 1$. We obtain

n_s'	ξ	$\mathcal{V}$	$\mathcal{V}_{\mathrm{D7}}$	g_s	z	x	R^2	τ_s	α
0.01	$5.2 \cdot 10^{-7}$	380.7	1.84	0.018	0.324	$1.8 \cdot 10^{-3}$	639	1.39	$2.6 \cdot 10^{-5}$
0.007	$5.2 \cdot 10^{-7}$	683.0	11.0	0.017	1.27	$1.3 \cdot 10^{-3}$	49.1	1.49	$5.3 \cdot 10^{-6}$
0.007	10^{-7}	1172	2.11	0.016	6.23	$1.0 \cdot 10^{-3}$	89.1	1.57	$1.0 \cdot 10^{-6}$

Here, R^2 is the length-squared of the S^1 in the fiber which is transverse to the D7-brane, cf. (5.44). Decreasing n_s' decreases α. Lowering ξ lowers the energy scale of inflation and therefore also the values of ϵ and α. Note that for $n_s' = 0.01$ and $\xi = 5.2 \cdot 10^{-7}$ the value of z is rather low, such that one might worry about the relevance of non-perturbative corrections to the Kähler potential. However, slightly decreasing n_s' and ξ increases z, which thus helps in this respect. Furthermore, $n_s' = 0.01$ and $\xi = 10^{-7}$ would have given $\mathcal{V}_{\mathrm{D7}} < 1$, which might pose a problem for the control of the effective theory. This is why we chose $n_s' = 0.007$ and $\xi = 10^{-7}$ in the last row of the above table.

Let us now compute α_{micro}. One finds that it varies only very weakly with n_s' and ξ. In particular

n_s'	ξ	$\alpha_{\text{micro}}^{\text{KK}}$	$\alpha_{\text{micro}}^{\text{W}}$
0.01	$5.2 \cdot 10^{-7}$	$1.0 \cdot 10^{-12}$	$7.9 \cdot 10^{-5}$
0.007	$5.2 \cdot 10^{-7}$	$7.3 \cdot 10^{-13}$	$6.3 \cdot 10^{-5}$
0.007	10^{-7}	$5.3 \cdot 10^{-13}$	$5.1 \cdot 10^{-5}$

The quantity $\alpha_{\text{micro}}^{\text{W}}$ is thus too big by a factor of at least 3 as compared to α computed above. We will come back to this issue after a brief comment on the KK scale.

Masses of Kaluza-Klein States

Having a compact space which contains hierarchically different length scales, one faces the danger of light KK modes in the spectrum which might spoil the validity of the supergravity approximation. In particular, the ratio $m_{3/2}/m_{\text{KK}}$ might become of order one. Let us briefly estimate the size of this ratio. For that purpose we picture the space transverse to the D7-branes, whose size is given by $\mathcal{V}/\mathcal{V}_{\mathrm{D7}}$, as a product of two circles, one of length R (along which the branes are separated), and the other one of length L. Schematically we then find

$$z = \frac{\mathcal{V}}{\mathcal{V}_{\mathrm{D7}}} \frac{1}{R^2} = LR \frac{1}{R^2} = \frac{L}{R}. \tag{5.71}$$

The 'complex structure modulus' z thus becomes large in the limit of large L. Since we want z to be large, KK modes propagating along the circle with length L might be worrisome. Their mass is given by $m_{\text{KK}} = \frac{2\pi}{\ell_s L^s}$, where the superscript s denotes that L^s is measured in units of ℓ_s in the ten-dimensional string frame. It is related to L via $L g_s^{1/4} = L^s$. The gravitino mass is $m_{3/2} = \frac{\sqrt{g_s}}{\sqrt{16\pi}} \frac{W_0}{\mathcal{V}}$. Furthermore, recall that $\ell_s^{-1} = \frac{g_s}{\sqrt{4\pi \mathcal{V}^s}} M_p$. Putting everything together we have, for $n_s' = 0.01$, $\xi = 5.2 \cdot 10^{-7}$, $W_0 = 1$, and $\hat{\xi} = \gamma = 1$,

$$\frac{m_{3/2}}{m_{\text{KK}}} = \frac{\sqrt{g_s}}{4\pi} \frac{W_0}{\sqrt{\mathcal{V}}} zR = \frac{\sqrt{g_s} W_0}{4\pi} \sqrt{\frac{z}{\mathcal{V}_{\mathrm{D7}}}} \simeq 4.5 \cdot 10^{-3}. \tag{5.72}$$

Therefore, the supergravity approximation is under control in this example.

Parametric Analysis in Fluxbrane Inflation

In order to see whether adjusting the values of W_0, $\hat{\xi}$, and γ helps decreasing $\alpha_{\text{micro}}^{\text{W}}$, let us analyze the scaling of the above results with these quantities. Treating the logarithm in (5.69) as constant we find

$$\mathcal{V} \sim W_0^{2/3} \hat{\xi}^{2/9} \gamma^{1/3}, \quad g_s \sim \hat{\xi}^{2/3}, \quad z \sim \hat{\xi}^{2/3}, \quad x \sim W_0^{2/3} \hat{\xi}^{2/9} \gamma^{1/3}, \quad R^2 \sim W_0^{2/3} \hat{\xi}^{-4/9} \gamma^{1/3}. \tag{5.73}$$

Most importantly,

$$\alpha \sim \text{const.}, \quad \alpha_{\text{micro}}^{\text{KK}} \sim \frac{\hat{\xi}^{34/27}}{W_0^{2/9}\gamma^{10/9}}, \quad \alpha_{\text{micro}}^{\text{W}} \sim \frac{1}{W_0^{2/9}\hat{\xi}^{2/27}\gamma^{10/9}}. \tag{5.74}$$

It thus seems that we should increase $\hat{\xi}$ and W_0 as much as possible. However, a natural upper bound on $\hat{\xi}$ is given by the requirement $g_s \lesssim 1$. Additionally, in view of (5.75) we find that

$$\frac{m_{3/2}}{m_{\text{KK}}} \sim W_0 \hat{\xi}^{2/3}. \tag{5.75}$$

As $\hat{\xi} \gtrsim 1$, for fixed n_s' and ξ this scaling puts an upper bound on W_0. A large $\hat{\xi}$ in (5.75) can in principle be compensated by a small W_0. On the other hand, since in the above example both $m_{3/2}/m_{\text{KK}} \simeq 10^{-2}$ and $g_s \simeq 10^{-2}$ and both scale with the same power of $\hat{\xi}$, this 'compensation' by a small W_0 is actually not constructive. Still, as z scales with a positive power of $\hat{\xi}$, increasing the latter helps suppressing non-perturbative corrections to the Kähler potential. Therefore, if one insists on having $z > 1$ it is most efficient to increase both, W_0 and $\hat{\xi}$ at the same time, up to a point where $m_{3/2}/m_{\text{KK}} \ll 1$ is still valid. Let us thus choose $n_s' = 0.01$, $\xi = 5.2 \cdot 10^{-7}$, $\gamma = 1$, and $W_0 = \hat{\xi} = 10$. We then find

$\mathcal{V}$	$\mathcal{V}_{\text{D7}}$	g_s	z	x	R^2	τ_s
2962	1.84	0.09	1.55	$1.4 \cdot 10^{-2}$	1040	1.35

α	$\alpha_{\text{micro}}^{\text{KK}}$	$\alpha_{\text{micro}}^{\text{W}}$	$m_{3/2}/m_{\text{KK}}$
$2.6 \cdot 10^{-5}$	$1.2 \cdot 10^{-11}$	$4.0 \cdot 10^{-5}$	0.22

Clearly, the quantity $\alpha_{\text{micro}}^{\text{W}}$ is still slightly larger than α. In particular, as discussed at the end of section 5.3.3, it is generally not possible to realize $\gamma = 1$, which further deteriorates the situation. However, if we assume the presence of winding-mode corrections, insist on $z > 1$, and ignore that typically $\gamma < 1$, the above table summarizes the best we can do.

D-Term Corrections

In the analysis presented so far we have neglected the Coleman-Weinberg-type loop corrections which were computed and analyzed for the fluxbrane model in [21, 23] (see also chapters 3 and 4). Recall that they were found to correct the tree-level D-term potential as

$$V(\varphi) = V_0 \left(1 + \alpha_{\ln} \ln\left(\frac{\varphi}{\varphi_0}\right) + \dots\right), \quad \alpha_{\ln} = \frac{g_{\text{YM}}^2}{(4\pi)^2} x^2 = \frac{1}{8\pi \mathcal{V}_{\text{D7}}} x^2. \tag{5.76}$$

This gives, for $n_s' = 0.01$, $\xi = 5.2 \cdot 10^{-7}$, and $\gamma = 1$,

W_0	$\hat{\xi}$	$\alpha_{\ln}$
1	1	$7.3 \cdot 10^{-8}$
10	10	$4.4 \cdot 10^{-6}$

The second value of $\alpha_{\ln}$ in this table is rather large, due to the relatively large value of x. On the other hand, it is still almost one order of magnitude smaller than the corresponding α, which means that the corrections to the F-term potential remain dominant. Nevertheless, the constraint $\alpha, \alpha_{\text{micro}}^{\text{KK,W}} \gg \alpha_{\ln}$, needed for being able to consistently neglect the Coleman-Weinberg-type loop term, is generically non-trivial and should be taken into account carefully.

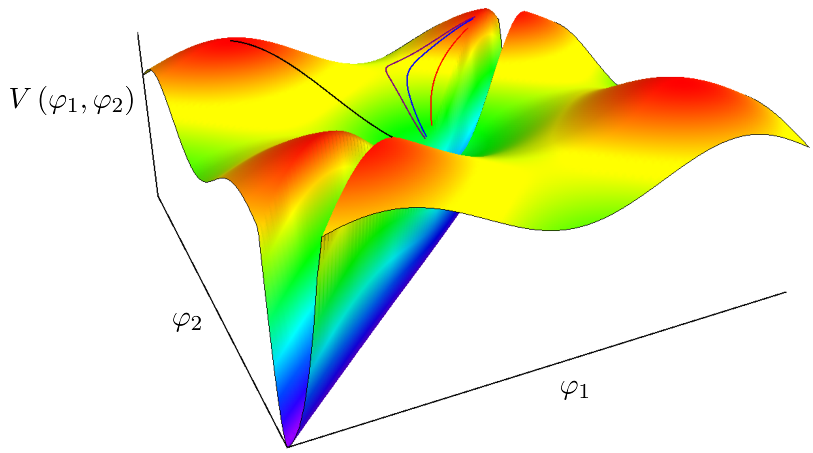

Figure 5.5.: Various trajectories in the field space of the two brane coordinates.

Switching off Winding-Mode Corrections

Up to now we assumed the worst case scenario, i.e. we assumed that the winding-mode corrections, which are the largest (regarding the accompanying π-factors), exist and depend on the inflaton. However, as detailed in section 5.3.4, these corrections are absent in models in which there are no one-cycles along the intersection loci of two D7-branes [130]. In particular, for F-theory on $K3 \times K3$ in the orientifold limit these corrections are expected to be absent. Let us therefore concentrate on models in which the intersection curves of D7-branes (at least of the ones on which we realize our fluxbrane inflation model) do not have non-trivial one-cycles. Then, the phenomenological quantity α is determined microscopically by $\alpha_{\text{micro}}^{\text{KK}}$ which, for the above values for n_s' and ξ, tends to be too small. For $n_s' = 0.007$, $\xi = 10^{-7}$, $\hat{\xi} = 100$, $\gamma = 10^{-2}$ and $W_0 = 1$ we find

$\mathcal{V}$	$\mathcal{V}_{\text{D7}}$	g_s	z	x	R^2	τ_s
708.9	2.11	0.37	142	$6.1 \cdot 10^{-4}$	2.37	1.49

α	$\alpha_{\text{micro}}^{\text{KK}}$	$\alpha_{\ln}$	$m_{3/2}/m_{\text{KK}}$
$1.0 \cdot 10^{-6}$	$3.3 \cdot 10^{-8}$	$7.1 \cdot 10^{-9}$	0.39

Larger values for n_s' and ξ increase the ratio $\alpha/\alpha_{\text{micro}}^{\text{KK}}$. A lower value for n_s' leads to $R^2 < 1$, whereas a lower ξ leads to an increase in $m_{3/2}/m_{\text{KK}}$, both of which is undesirable. This leaves us with a considerable difference in size between α and $\alpha_{\text{micro}}^{\text{KK}}$ in the above table. However, this discrepancy can easily be resolved by having a larger number of N^{KK} or C^{KK} in (5.54). For example, the function $E_2(U)$ gives $E_2(i) \simeq 6$ and grows for larger $\text{Im}(U)$.

Additional Remarks

Before we end this subsection we would like to make some closing remarks:

- As detailed in section 5.3.3, x has to be tuned to a small value in order to have F- and D-term of comparable size. The sum of D-term and (negative) F-term energy density in the minimum of the scalar potential after inflation determines the cosmological constant in our D-term uplifting scenario. Thus, the tuning of x in our

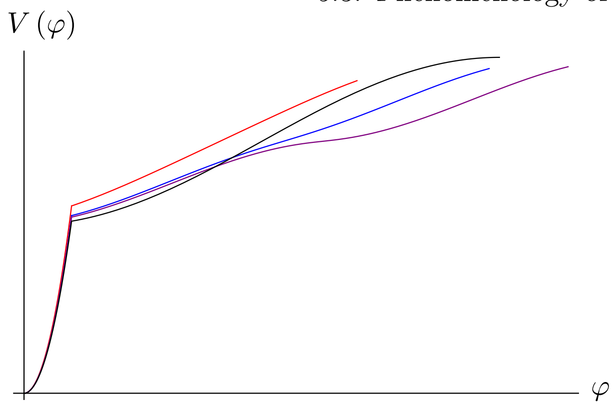

Figure 5.6.: One-dimensional plot of the potential along the trajectories drawn in figure 5.5.

model constitutes part of the tuning of the cosmological constant to the famous value 10^{-120}. As mentioned already at the end of section 5.3.3, D-terms can be computed very easily in a given model and thus the tuning of x can in principle be analyzed very explicitly. If one furthermore requires $\gamma \ll 1$ for stability reasons (see footnote 13 and appendix C), this amounts to additional fine tuning. In the context of a D-term inflation model with an unwarped D-term uplift and moduli stabilization in terms of the Large Volume Scenario we do not see any way to circumvent this tuning.

- If one could relax the cosmic string bound one would have more freedom. Using $\xi = 10^{-6}$, $n_s' = 0.01$, $\hat{\xi} = 30$, $W_0 = 10$, and $\gamma = 1$, one finds

$\mathcal{V}$	$\mathcal{V}_{\text{D7}}$	g_s	z	x	R^2	τ_s
3039	3.54	0.18	1.67	$2.0 \cdot 10^{-2}$	514	1.36

α	$\alpha^{\text{W}}_{\text{micro}}$	$\alpha_{\ln}$	$m_{3/2}/m_{\text{KK}}$
$5.0 \cdot 10^{-5}$	$4.0 \cdot 10^{-5}$	$4.6 \cdot 10^{-6}$	$2.3 \cdot 10^{-1}$

- Alternatively, if one drops the assumption that the inflaton is responsible for the generation of CMB perturbations, for $\tilde{\zeta} = 5.1 \cdot 10^{-5}$, $n_s' = 0.014$, $W_0 = \gamma = 1$, $\hat{\xi} = 100$ and $\xi = 10^{-7}$ one finds that

$\mathcal{V}$	$\mathcal{V}_{\text{D7}}$	g_s	z	x	R^2	τ_s
4304	5.16	0.31	5.80	$2.4 \cdot 10^{-3}$	144	1.78

α	$\alpha^{\text{W}}_{\text{micro}}$	$\alpha_{\ln}$	$m_{3/2}/m_{\text{KK}}$
$2.9 \cdot 10^{-5}$	$3.1 \cdot 10^{-5}$	$4.4 \cdot 10^{-8}$	$4.7 \cdot 10^{-2}$

Here, we have assumed for simplicity that still $n_s = 0.9603$.

5.3.7. Alternative Trajectories

We would like to emphasize that the phenomenological analysis performed in the previous subsections was assuming a cosine-shaped potential. In view of figure 5.3, which displays the two-dimensional field space, parametrizing the positions of the two D7-branes along a circle (cf. section 5.3.2), it is obvious that this assumption corresponds to the 'generic case'.

However, potentials which significantly depart from the cosine shape are also possible. Figure 5.5 shows various one-dimensional inflaton trajectories in the two-dimensional field space. The corresponding potentials along the trajectories are shown in figure 5.6. Some of them look very different from a cosine and are thus likely to lead to a rather different phenomenological discussion. We will not pursue these ideas any further in this thesis.

Furthermore, we note that, in view of figure 5.3, fluxbrane inflation is really a multifield inflation model. Accordingly, isocurvature modes can be important [297]. An analysis of those modes is beyond the scope of this thesis.

5.4. Summary

In this chapter we have arrived at a consistent overall picture of fluxbrane inflation, in particular taking into account inflaton-dependent corrections to the scalar F-term potential which have been neglected so far and which are, in fact, the dominant contribution to the inflaton potential in most of the parameter space.

We started by analyzing under which circumstances dangerous leading-order contributions to the inflaton potential are absent. This requires the presence of a shift symmetry and a suitable choice of bulk and brane fluxes. Regarding the shift symmetry we argued that the moduli space of D7-brane position moduli possesses such a symmetry in the vicinity of the point of large complex structure of the compactification space. This can be motivated via T-duality to Type IIA string theory, where the brane separation becomes a Wilson line whose flat potential is protected against perturbative α'-corrections. Non-perturbative effects are exponentially small in the limit of large Type IIA volume, which corresponds to large complex structure on the Type IIB side.

An appropriate flux choice which does not stabilize the D7-brane position moduli was analyzed for a compactification of F-theory on $K3 \times K3$ in the orientifold limit in [29]. As opposed to the presence of the shift symmetry, there is no general strategy for ensuring the absence of brane positions in the superpotential. Rather, the situation has to be analyzed on a case-by-case basis for each model.

Beyond these leading-order effects, an inflaton dependence will generically be induced by loop corrections to the Kähler potential. Neglecting complex structure and open-string moduli (i.e. integrating out these moduli), those loop corrections feature the so-called extended no-scale structure, which is essential for the viability of the Large Volume Scenario, in terms of which Kähler moduli are stabilized in the fluxbrane inflation model. In the present chapter we demonstrated that this extended no-scale structure continues to hold in the presence of a further light degree of freedom (namely the inflaton) in the effective theory. As a result, the Large Volume Scenario is in principle compatible with the fluxbrane inflation model.

We discussed how moduli can be stabilized in a way which allows for a parametric suppression of the inflaton-dependent loop corrections with respect to the constant energy density of the potential. This enables us to make the slow-roll parameter η parametrically small in the fluxbrane inflation model, such that we do not have to appeal to fine tuning.

Finally, we performed a detailed phenomenological analysis of the potential arising in fluxbrane inflation. We found that, by a suitable choice of parameters which describe the string compactification we can fit the phenomenologically required values of the quantities which parametrize the inflaton potential. The suppression of loop corrections associated with the exchange of Kaluza-Klein modes is sufficient for reproducing the required relative size α between the constant and the inflaton-dependent term in the potential. On the other hand, loop corrections due to winding modes of the string around potential one-cycles of D7-brane intersections are on the verge of being too large. The absence of those corrections can be achieved in cases where the self-intersection of the D7-brane divisor is either empty or contains no non-contractible one-cycles. Still, even in the presence of corrections due to winding modes the discrepancy between the phenomenologically computed α and the one obtained microscopically is rather small. Thus, a given model which features such winding-mode corrections may well reproduce the correct size of α due to the appearance of $\mathcal{O}(1)$-factors which were neglected in our investigation. We were able to fit the correct value of the spectral index, the amplitude of curvature perturbations, and the number of e-foldings. Furthermore, the fluxbrane inflation model satisfies the current cosmic string bound and the running of the spectral index is small, $n_s' \lesssim 10^{-2}$. However, being a small-field inflation model, the tensor-to-scalar ratio is tiny, typically $r \lesssim 2.6 \cdot 10^{-5}$.

6. D7-Brane Chaotic Inflation

Models of slow-roll inflation can be classified according to the distance the inflaton rolls during inflation and are either of the large-field type, $\Delta\varphi > M_p$, or of the small-field type, $\Delta\varphi < M_p$. As reviewed in chapter 2, typical inflaton candidates in string theory, like D-brane positions [32, 36], Wilson lines [110], and axions, generically have a field range which is limited to sub-planckian values. This applies in particular to the inflaton field in the fluxbrane inflation scenario, which we have discussed so far in this thesis. The same limitation of the field range occurs for Kähler moduli [84], except for models in which the inflaton is identified with a breathing mode of the compact space [85]. Thus, while there has been much progress in constructing small-field models in string theory (for a review see [9, 298]), realizing large-field models is notoriously difficult.

Clearly, there are several possible ways how one can, despite the limited field range, construct scenarios in string theory which are *effectively* of the large-field type. A brief account of these models is contained in section 1.3.5. Here we propose a novel scenario which realizes large-field inflaton in string theory, using the position modulus of a single D7-brane as the inflaton. Our model features the appealing mechanisms of a shift symmetry and a monodromy. Therefore, it is similar in spirit to the proposals of [39, 40, 160, 161], however, with one major advantage: Our model allows for a description in terms of an effective supergravity Lagrangian and thus does not suffer from the control issues associated with the need to include antibranes. Furthermore, a rather explicit discussion of moduli stabilization, e.g. in the Large Volume Scenario [15], is possible.

The basic ingredients for our proposal of large-field inflation with a D7-brane are the following: First, we recall from the previous chapter that the Kähler potential for a D7-brane position modulus features a shift symmetry in the vicinity of the large complex structure point. Second, in the absence of fluxes the D7-brane modulus parametrizes a Riemann surface which generically has one-cycles, such that the field space of the modulus is periodic.[1] In fact, all we need in our model is a closed trajectory along the shift-symmetric direction in the D7-brane position moduli space. Fluxes will lead to an appearance of the brane modulus in the superpotential, such that the periodicity will be broken and a monodromy arises.[2] Inflation in this 'D7-brane chaotic inflation' model occurs along the shift-symmetric direction in the D7 moduli space. The situation is illustrated in figure 6.1.

As a result of working in Type IIB string theory, Kähler moduli stabilization can be analyzed very explicitly in our model, e.g. in the Large Volume Scenario, and gives non-trivial constraints on the size of the overall volume of the compact space and the coefficients of the brane-moduli-dependent terms in the superpotential. For example, displacing the D7-brane from its minimum leads to F-terms in the effective action which generically

[1] Immediately after [28] appeared (on which the present chapter is based) the possibility of realizing inflation on Riemann surfaces was proposed in [299].

[2] Inflation using a monodromy in the field space of a D3-brane was analyzed in [300]. However, it is acknowledged in that paper that, since the proposal relies on the existence of non-trivial one-cycles in the compact space, much of the recent progress regarding moduli stabilization is not applicable in that model.

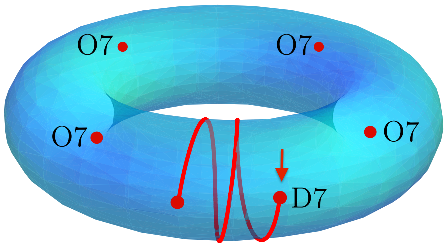

Figure 6.1.: Illustration of the D7-brane position modulus parameter space. Inflation occurs when the D7-brane moves along a one-cycle in the parameter space, which need not necessarily be non-trivial in homology.

destabilize the potential, i.e. they lead to a runaway direction in the Kähler moduli space. Therefore, in order to ensure stability of the system during inflation, we have to tune the coefficients of the brane moduli in the superpotential to small values. This can be viewed as a tuning of complex structure moduli by a suitable choice of fluxes. We assume that the landscape will provide a model with this feature and will not discuss this tuning in any detail in this chapter. Rather, given the very limited understanding of large-field inflation in string theory, we think it is important to demonstrate that such models can be realized in principle in a controlled string-derived supergravity framework.

As mentioned in the introduction, the recent measurement of B-mode polarization [33] by the BICEP2 collaboration serves as an additional motivation for studying large-field inflation in string theory. In fact, the reported value $r = 0.2^{+0.07}_{-0.05}$ for the tensor-to-scalar ratio is in reasonable agreement with the value $r = 0.16$ which we find in our model.

Most of this chapter is copied from [28]. All copied sections are written by myself, except for section 6.3.2 which was written by L. Witkowski. The illustrations of the D7-brane field space were also done by L. Witkowski.

6.1. Ingredients

The low-energy effective description of our model is in terms of a supergravity Lagrangian which is built from a Kähler and superpotential. Let us discuss these two quantities in more detail for our model.

6.1.1. Shift-Symmetric Kähler Potential in D7-Brane Chaotic Inflation

The form of the Kähler potential for a D7-brane deformation modulus was discussed in section 5.1.1. It was argued that at weak coupling and in the vicinity of the large complex

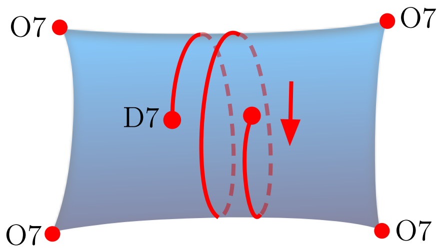

Figure 6.2.: Illustration of the D7-brane position modulus parameter space in the example of F-theory on $K3 \times K3$, which reduces to Type IIB string theory on $K3 \times T^2/\mathbb{Z}_2$ in the weak-coupling limit.

structure point it takes the form[3]

$$K^{\mathrm{LCS}}_{g_s\to 0} = -\ln\Bigg(\frac{\kappa^{(1)}_{abc}}{3!}(S-\overline{S})(u^a-\overline{u}^a)(u^b-\overline{u}^b)(u^c-\overline{u}^c) \\ +\frac{\kappa^{(2)}_{abpq}}{4!}(u^a-\overline{u}^a)(u^b-\overline{u}^b)(c^p-\overline{c}^p)(c^q-\overline{c}^q)+\ \ldots\Bigg)\,, \tag{6.1}$$

where S is the axio-dilaton, the u^a are complex structure moduli of the threefold, and the c^p describe D7-brane positions. All these moduli arise from complex structure moduli z^i of the F-theory fourfold.

Identifying one of the c^p with the deformation modulus c of the D7-brane with which we would like to realize inflation and integrating out all other moduli, we conjecture the following general structure for the Kähler potential

$$K = -\ln\left(A + iB(c-\overline{c}) - \frac{D}{2}(c-\overline{c})^2\right)\,, \tag{6.2}$$

where $A, B, D \in \mathbb{R}$. Recall the instructive example of F-theory on $K3 \times K3$ in the orientifold limit (equation (5.8)), where the parameter space of c is just $T^2/\mathbb{Z}_2$, which is depicted in figure 6.2. A linear term $\sim (c-\overline{c})$ is not present in this example, but we think that this is a special feature of the $K3$-manifold. Indeed, if we start from a generic quadratic expression in the brane moduli c^p the situation looks different: Assuming that all but one (which we call c) are stabilized at a high scale, both a quadratic and a linear term in $(c-\overline{c})$ (the latter coming from mixed terms of type $(c^p-\overline{c}^p)(c-\overline{c})$) arise. Furthermore, it is not clear to us whether terms of higher than quadratic order in $(c-\overline{c})$ appear generically in the above expression. However, they would just slightly complicate the computations, but not alter our conclusions qualitatively.

Assuming that we make all the z^i of the fourfold homogeneously large in the large complex structure limit, we expect the scalings $A \sim \mathrm{Im}(z)^4$, $B \sim \mathrm{Im}(z)^3$, $D \sim \mathrm{Im}(z)^2$. Here we have treated the axio-dilaton, the complex structure moduli of the threefold, and all brane coordinates except for c on similar grounds. This is, of course, a very coarse approximation. As a first estimate, however, it is certainly a valid assumption.

[3]Note the relabeling $z^i \leftrightarrow u^i$ with respect to (5.5), in order to be consistent with the notation in [28].

6.1.2. Superpotential in D7-Brane Chaotic Inflation

Applying similar arguments to the F-theory superpotential (5.11), (5.12), focusing on its dependence on one brane modulus c, we expect the general structure

$$W = W_0 + \alpha c + \frac{\beta}{2} c^2 + \dots . \tag{6.3}$$

In the example of F-theory on $K3 \times K3$, precisely this structure arises. It is again not clear if cubic and quartic terms will arise in the generic case. Such terms would certainly alter the phenomenology of our model. However, for the time being we assume that it is possible to either restrict to models in which such terms are absent (such as $K3 \times K3$), or to choose fluxes such that the superpotential contains terms only up to quadratic order. In this case, (6.3) captures the structure of the superpotential in our model.

As outlined in the introduction to this chapter, we need to tune $|\alpha|$ and $|\beta|$ to small values in order for the induced F-terms to be small enough not to interfere with moduli stabilization during inflation. The merit of our model is that it indeed admits a rather explicit discussion of moduli stabilization and therefore, non-trivial constraints on α and β are obtained and reported in the subsequent sections.

6.2. The Model

Given the Kähler potential (6.2), supplemented by the Kähler moduli part, i.e.

$$K = -2 \ln \mathcal{V} - \ln \left(A + iB(c - \bar{c}) - \frac{D}{2}(c - \bar{c})^2 \right), \tag{6.4}$$

and the superpotential (6.3), supplemented by instanton corrections on small blow-up cycles

$$W = W_0 + \alpha c + \frac{\beta}{2} c^2 + e^{-2\pi T_s}, \tag{6.5}$$

we can now write down the F-term potential:

$$V_F = e^K \left(K^{T_\gamma \overline{T}_\delta} D_{T_\gamma} W \overline{D_{T_\delta} W} - 3|W|^2 + K^{c\bar{c}} |D_c W|^2 \right). \tag{6.6}$$

Here, the T_γ are complexified Kähler moduli whose real part measures the size of a four-cycle of the threefold in units of the string length. Furthermore, $\mathcal{V}$ is the volume of the threefold. As usual, the complex structure moduli, the axio-dilaton and almost all brane moduli are assumed to be stabilized by their respective F-terms, with the exception of c whose F-term we included explicitly in (6.6). The reason for doing so is the very weak dependence of W on c which, due to the shift symmetry in the Kähler potential, leaves $\mathrm{Re}(c)$ unstabilized in a first approximation.

Owing to the fact that the Kähler metric is block-diagonal in the Kähler and complex structure moduli, no terms with mixed derivatives in c and T_γ appear in (6.6). Therefore, in the first two terms we can formally substitute $\tilde{W}_0 = W_0 + \alpha c + \frac{\beta}{2} c^2$ and the no-scale structure leads to a cancellation of the leading-order terms in T_γ. Thus, the third term in (6.6) is dominant and stabilizes c in a supersymmetric minimum, i.e. at $D_c W = 0$.

Now, Kähler moduli stabilization proceeds as in the plain-vanilla Large Volume Scenario [15], giving rise to an AdS minimum which scales as $\sim -|\tilde{W}_0|^2/\mathcal{V}^3$. This minimum is then uplifted to a Minkowski minimum via one of the various proposed uplifting mechanisms. We are now interested in the c-dependence of the resulting terms, as inflation occurs along $\mathrm{Re}(c)$. It is clear that the terms from $-|\tilde{W}_0|^2/\mathcal{V}^3$ are subleading in the inverse overall volume with respect to the terms from the third term in (6.6). Therefore, the leading-order mass term for the inflaton in our model is contained in $e^K K^{c\bar{c}}|D_cW|^2$. In order for this mass term not to interfere with the volume stabilization we tune $|\alpha|$ and $|\beta|$ to small values. This ensures stability in the Kähler moduli directions along the whole inflaton trajectory.

One crucial fact for the viability of the Large Volume Scenario is the existence of the 'extended no-scale structure' [128, 130, 131] which ensures that loop corrections are subleading with respect to the α'^3-corrections [127] used to stabilize the overall volume. In the above references it is generally assumed that complex structure moduli, the axio-dilaton and all brane moduli are integrated out at a higher scale. However, it turns out that the extended no-scale structure persists even if the low-energy theory includes a complex scalar which does not appear in the superpotential and which enters the Kähler potential only shift-symmetrically, such that one real scalar remains light. We have demonstrated this explicitly in section 5.2.2 (see also [29]). Clearly, in our setting this structure will be broken by the explicit dependence of the superpotential on c. However, since the extended no-scale structure is restored in the limit of vanishing α and β, the breaking will be small in the limit of small $|\alpha|$ and $|\beta|$ and the overall picture remains consistent.

6.2.1. Minimizing the Potential

Let us analyze the stabilization of c in more detail. We will work in the limit of small $|\alpha|$ and $|\beta|$ throughout. From $D_cW = 0$ we obtain the equation

$$\frac{\alpha + \beta c}{W_0 + \alpha c + \frac{\beta}{2}c^2} = \frac{iB - D(c-\bar{c})}{A + iB(c-\bar{c}) - \frac{D}{2}(c-\bar{c})^2}. \tag{6.7}$$

In the following we will write $c = x + iy$ with $x, y \in \mathbb{R}$. At 0^{th} order in α and β, the left-hand side of this equation vanishes and y is stabilized at

$$y_0 = \frac{B}{2D}. \tag{6.8}$$

Furthermore, we observe that the RHS of (6.7) is purely imaginary. Requiring the real part of the LHS to vanish leads, at 1^{st} order in α and β, to

$$x_0 = \frac{\mathrm{Im}(\beta\overline{W}_0)y_0 - \mathrm{Re}(\alpha\overline{W}_0)}{\mathrm{Re}(\beta\overline{W}_0)}. \tag{6.9}$$

Thus, recalling the scaling of A, B, and D with $\mathrm{Im}(z)$ we find $y_0 \sim x_0 \sim \mathrm{Im}(z)$. These expressions will be corrected at higher order in α and β. However, since these coefficients have to be tuned to small values anyhow, for our purposes the above analysis is sufficient.

6.2.2. Computing the Mass

We now compute the mass for the inflaton. As motivated above, the mass term will arise from $|D_cW|^2$. Furthermore, since $D_cW = 0$ in the minimum, it suffices to expand this term in leading order in the variation of the real part of c, i.e. in δx. Furthermore, since stabilization enforces $K_c = W_c/W$ (cf. (6.7)) and the latter scales linearly with α and β, displacing x from its minimum simply gives

$$\delta D_c W = \delta\left(K_c W + W_c\right) \simeq \delta W_c = \beta\delta x \tag{6.10}$$

in linear order in α and β, leading to

$$e^K K^{c\bar{c}}|\beta|^2\delta x^2 + \text{higher order in } \alpha, \beta, \delta x. \tag{6.11}$$

Now, δx is related to the inflaton via canonical normalization. The kinetic term for δx is contained in $K_{c\bar{c}}|\partial c|^2$. Recalling the scaling $K_{c\bar{c}} \sim \mathrm{Im}(z)^{-2}$ and $e^K \sim \mathrm{Im}(z)^{-4}$, we find

$$m_\varphi^2 \sim \frac{1}{\mathcal{V}^2}\frac{1}{\mathrm{Im}(z)^4}\,\mathrm{Im}(z)^2\,\mathrm{Im}(z)^2|\beta|^2 = \frac{|\beta|^2}{\mathcal{V}^2}, \tag{6.12}$$

where the two factors of $\mathrm{Im}(z)^2$ come from canonically normalizing the inflaton and from the $K^{c\bar{c}}$ factor in the F-term potential, respectively. Interestingly, $\mathrm{Im}(z)$ does not show up in m_φ^2.

6.3. Phenomenology

The phenomenology of quadratic inflation is, of course, well known [41]. Let us briefly recall the basic results. For a potential $V = m^2\varphi^2$ the slow-roll parameters are determined as

$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 = \frac{2}{\varphi^2}, \tag{6.13}$$

$$\eta = \frac{V''}{V} = \frac{2}{\varphi^2}. \tag{6.14}$$

The spectral index can be expressed in terms of these two quantities as

$$n_s - 1 = -6\epsilon + 2\eta = -\frac{8}{\varphi^2}. \tag{6.15}$$

Since this quantity is measured to be $\simeq -0.04$ [48], the field displacement at the beginning of the last ~ 60 e-folds of inflation is determined to be $\varphi^2 \simeq 200$. The tensor-to-scalar ratio is thus fixed as

$$r = 16\epsilon \simeq 0.16. \tag{6.16}$$

On the other hand, the measured value for the amplitude of curvature perturbations determines [48]

$$\sqrt{\frac{V}{2\epsilon}} = 5.1 \cdot 10^{-4}, \tag{6.17}$$

which leads to $m \simeq 0.5 \cdot 10^{-5}$.

This can be translated into requirements on our stringy model of large-field inflation. In particular,

$$m_\varphi = \frac{|\beta|}{\mathcal{V}} \overset{!}{=} 0.5 \cdot 10^{-5}. \tag{6.18}$$

This is, however, not the only constraint which m_φ has to satisfy. As mentioned before, in order not to interfere with Kähler moduli stabilization we need to require

$$m_\varphi^2 \varphi^2 \simeq 0.5 \cdot 10^{-8} \ll \frac{|W_0|^2}{\mathcal{V}^3} \tag{6.19}$$

along the whole inflationary trajectory.

To give a few specific numbers, let us choose $\mathcal{V} = 10^3$. This determines, via (6.18), $|\beta| = 0.5 \cdot 10^{-2}$. Then, (6.19) is satisfied for $|W_0| = 10$. But this is by no means the only possible realization: Also a choice $\mathcal{V} = 10^2$, leading to $|\beta| = 0.5 \cdot 10^{-3}$, works fine, even for $|W_0| = 1$.

6.3.1. Stability during Inflation

During inflation, the real part of c traverses a large distance in field space. We should thus make sure that the stabilization of the imaginary part is not significantly affected by this field displacement. Recall that the kinetic term for $x = \mathrm{Re}(c)$ reads

$$K_{c\bar{c}}(\partial \delta x)^2 \sim \frac{(\partial \delta x)^2}{\mathrm{Im}(z)^2}. \tag{6.20}$$

At the beginning of the last $N \simeq 60$ e-folds of inflation the canonically normalized inflaton φ takes the value $\varphi_N \simeq 14$, giving

$$\delta x_N \sim 14 \cdot \mathrm{Im}(z). \tag{6.21}$$

Now consider the stabilization equation (6.7). One can easily convince oneself that, writing $y = y_0 + \delta y$, the consistency requirement $|\delta y| \ll y_0$ is satisfied as long as

$$\frac{14|\beta|\,\mathrm{Im}(z)^2}{|W_0|} \ll 1. \tag{6.22}$$

Choosing $\beta = 0.5 \cdot 10^{-3}$ and $|W_0| = 1$, $\mathrm{Im}(z)$ is constrained as

$$\mathrm{Im}(z) < 12. \tag{6.23}$$

This potentially presents a conflict with the large complex structure limit. However, since the suppression of the correction terms is of exponential nature, even in view of (6.23) one can choose z large enough in order to suppress these corrections.

6.3.2. Cubic and Quartic Terms

Beyond the mass term (6.12) the potential will also exhibit cubic and quartic terms in δx. Expanding the potential (6.6) in δx about the minimum one finds

$$\begin{aligned} V \sim \frac{|\beta|^2}{\mathcal{V}^2} \frac{\delta x^2}{\mathrm{Im}(z)^2} \Bigg\{ 1 &+ \mathcal{O}\left(\frac{\alpha^3}{\beta W_0^2}, \frac{\alpha^2 c_0}{W_0^2}, \frac{\alpha \beta c_0^2}{W_0^2}, \frac{\beta^2 c_0^3}{W_0^2} \right) \delta x \\ &+ \mathcal{O}\left(\frac{\alpha^2}{W_0^2}, \frac{\alpha \beta c_0}{W_0^2}, \frac{\beta^2 c_0^2}{W_0^2} \right) \delta x^2 \Bigg\}. \end{aligned} \tag{6.24}$$

The above is derived by first expanding (6.6) in both δx and δy about $c_0 = x_0 + iy_0$. Further, we solve $D_c W = 0$ for $y = y_0 + y_1$ to first order in α, β. Equation (6.24) is finally obtained by substituting $\delta y = y_1$.

Here we examine the relevance of the cubic and quartic terms at the onset of the last 60 e-folds of inflation at $\delta x_N \sim 14 \cdot \mathrm{Im}(z)$. As $|c_0| \sim \mathrm{Im}(z)$ we find that the parametrically most important term is

$$V \supset \frac{|\beta|^2}{\mathcal{V}^2\ \mathrm{Im}(z)^2}\ \mathcal{O}\left(\frac{\beta^2 c_0^2}{W_0^2}\right) \delta x^4. \tag{6.25}$$

Terms involving α are not dangerous as we can always tune α independently of the phenomenological discussion above. For $|\beta| = 0.5 \cdot 10^{-3}$ and $|W_0| = 1$ we find that, if we set $\mathrm{Im}(z) \sim 10$, the term (6.25) becomes comparable to the mass term at the onset of the last 60 e-folds:

$$\frac{|\beta|^2 |c_0|^2}{|W_0|^2} \delta x_N^2 \sim 0.25 \cdot 10^{-6} \cdot 10^2 \cdot 200 \cdot 10^2 \sim 1\ . \tag{6.26}$$

For larger δx we then transition to a regime where the potential is dominated by the quartic term. However, such a large $\mathrm{Im}(z)$ is close to the upper bound (6.23). For $\mathrm{Im}(z) < 10$, the quartic term is still subleading compared to the mass term at δx_N. In this case, the quartic term in (6.24) can be made comparable again by choosing an appropriate value for α. Such corrections to the inflaton potential have been discussed recently in [172].

6.3.3. Non-Perturbative and Loop Corrections

So far we have completely neglected the mirror-dual version of the Type IIA worldsheet instanton corrections to the Kähler potential. These are expected to give oscillatory contributions at the order

$$\sim e^{-2\pi y_0} \frac{|W_0|^2}{\mathcal{V}^2} \tag{6.27}$$

to the F-term potential. Thus, in view of (6.8), they are exponentially suppressed in the limit of large complex structure. Furthermore, loop corrections due to the exchange of Kaluza-Klein modes between branes [18, 129–131] will also lead to periodic corrections, roughly at the order

$$\sim \{\alpha, \beta\} \cdot \frac{|W_0|^2}{\mathcal{V}^{8/3}}. \tag{6.28}$$

The induced corrections can be parametrized at leading order as

$$V = m^2\varphi^2 + \gamma \cos\left(\frac{\varphi}{f} + \delta\right). \tag{6.29}$$

The phenomenology of such a periodic modulation of a monomial inflaton potential, in particular its effect on the power spectrum and the bispectrum, was investigated for axion monodromy inflation (i.e. with a linear rather than a quadratic potential) in [301] and more generally in [167, 302]. Since the axion decay constant is small, roughly bounded by $f \lesssim 1/4\pi$ (see chapter 2 and also [24] and references therein), during the initial observable e-folds the inflaton typically crosses more than one period of the oscillatory piece in (6.29). Thus, if present and sufficiently large, the oscillatory features leave their imprint in the observable CMB modes. An explicit computation of the oscillatory terms in our model and a detailed analysis of the observational implications along the lines of [167] would be interesting, but is beyond the scope of this thesis. In any case, the periodic modulations become small in the limit of large $\mathrm{Im}(z)$ and small $|\alpha|$ and $|\beta|$.

6.4. Summary

In this chapter we have introduced a large-field variant of string inflation using a D7-brane position modulus as the inflaton. Our D7-brane chaotic inflation scenario is related to earlier proposals of axion monodromy inflation, where the periodicity of the a priori sub-planckian field space of an axion is broken by introducing certain branes. This leads to a multi-covering of the field space, such that field variations larger than the Planck mass can be attained. In our scenario the periodicity is of geometric nature, corresponding to the motion of a D7-brane around a closed trajectory (of sub-planckian circumference) in the D7-brane position moduli space. The Kähler potential is shift-symmetric along that trajectory. If, in addition, the superpotential is independent of the brane moduli this corresponds to a flat periodic direction in the scalar potential. The periodicity is broken by turning on fluxes, such that the desired monodromy in the D7-brane position moduli space is introduced.

As one of the main virtues of this new approach, the above scenario can be formulated purely in terms of spontaneously-broken supergravity, without the necessity to include explicitly supersymmetry-breaking terms. Moreover, having its home in Type IIB string theory, the model can be combined with the currently best-developed moduli stabilization scenarios. In this chapter we employed the Large Volume Scenario and analyzed the constraints imposed on the model by requiring a viable moduli stabilization. These are, most prominently, a very weak (tuned) dependence of the superpotential on the D7-brane moduli. This is needed to obtain a hierarchy of the terms in the scalar potential which stabilize the non-shift-symmetric direction in the D7-brane moduli space, the Kähler moduli, and the inflaton, respectively. In particular, while the non-shift-symmetric direction is stabilized by leading-order F-terms and the Kähler moduli are stabilized in the conventional Large Volume Scenario, the potential terms for the inflaton are suppressed by their tuned coefficients in the superpotential. Owing to this hierarchy, a displacement of the inflaton from the minimum of its potential will lead to an energy density which is small compared to the potential terms which stabilize Kähler moduli. Consequently, the Large Volume Scenario is not upset. The tuning corresponds to a non-generic flux choice which should be available given the large number of string vacua.

Several aspects of the analysis performed previously for the fluxbrane inflation model can be applied also to this large-field scenario. Most notably, the shift-symmetric Kähler potential, present at large complex structure, plays a pivotal role in D7-brane chaotic inflation. Furthermore, the validity of the extended no-scale structure in the presence of an additional light field is essential for the model to work.

Our parametric analysis demonstrates that the above-mentioned flux-tuning allows us to prevent moduli destabilization during inflation: We investigated the example of Kähler moduli stabilization in the Large Volume Scenario, where the most dangerous runaway direction is that of the overall volume. Destabilization is avoided e.g. for a tree-level superpotential $|W_0| = 1$, a volume $\mathcal{V} = 10^3$, and a coefficient $|\beta| = 0.5 \cdot 10^{-2}$ of the quadratic term in the superpotential.

Regarding the inflation phenomenology, the inflaton potential is quadratic at leading order, entailing in particular $r \simeq 0.16$. The potential receives cubic and quartic corrections which can be sizable at the onset of the last 60 e-folds of inflation. Whether or not they invalidate the quadratic approximation of the potential depends on the coefficients in the superpotential as well as the complex structure. Importantly, the parameters can be chosen such that the quadratic term is dominant throughout.

7. Summary and Outlook

Cosmological observations may offer crucial hints towards the fundamental constituents of nature. In particular, the theory of primordial inflation in its best understood field-theoretic realization makes assumptions about the structure of the UV theory which replaces the standard model of particle physics and gravity at high energies. For any candidate UV theory it is thus crucial to check whether it is compatible with these assumptions. This also applies to string theory, being the currently best studied candidate for a UV completion of the standard model coupled to gravity.

In this thesis we have analyzed inflation models with D7-branes in Type IIB string theory. These D7-branes are solitonic objects which extend in eight of the ten dimensions of the superstring theory target spacetime and which can be deformed in their transverse directions. The associated deformation modes correspond to scalar fields in the effective four-dimensional theory, one of which is identified with the inflaton in our models.

Most of the thesis analyzes fluxbrane inflation, which is a string-theoretic variant of hybrid natural inflation. Accordingly, we started our discussion by reviewing the appealing scenario of hybrid natural inflation in field theory. In this model, the leading-order flatness of the inflaton potential is ensured by a shift symmetry. This shift symmetry is broken by couplings of the inflaton to a waterfall sector, i.e. to fields which develop tachyonic masses as soon as the inflaton vacuum expectation value falls below some critical value. These couplings induce quantum corrections which lead to an inflaton potential. We reiterated that this model, when put into a supersymmetric context, can be technically natural. Subsequently, we performed a phenomenological analysis with a particular focus on the tensor perturbations produced during inflation. The limited size of the axion decay constant and the observational bounds on the running of the spectral index constrained the tensor-to-scalar ratio r to a value $r \lesssim 7.6 \cdot 10^{-4}$. Prominently, this is larger than the expectation derived from the Lyth bound. We furthermore showed that the power spectrum in hybrid natural inflation cannot be curvaton-dominated.

This discussion was followed by a more thorough introduction of fluxbrane inflation as a string-theoretic version of hybrid natural inflation. In this string embedding, the inflaton is associated with the distance between two D7-branes. The energy density which drives inflation is due to a D-term which is induced by supersymmetry-breaking brane flux. The fluxbrane inflation model allows to avoid some drawbacks encountered in previous proposals of brane inflation and D-term hybrid inflation. Most importantly, it is possible to accommodate the phenomenologically required field range (microscopically speaking, the brane separation) within the compact space, something which was not easily achieved in related brane inflation scenarios. Moreover, the problem of cosmic string overproduction in D-term hybrid inflation is overcome in fluxbrane inflation due to the specific form of the inflaton potential, which allows for a suppression of the inflaton-dependent part below the size expected from the field theory model. Our scenario can fit the amplitude of curvature perturbations, while the spectral index is slightly too large to be compatible with the most recent measurements.

In this first analysis of fluxbrane inflation we did not take into account the issue of stabilizing all moduli (i.e. scalar fields with leading-order flat potentials) which appear in the four-dimensional effective theory. The interplay of inflation with moduli stabilization is, however, a highly non-trivial issue in models of string inflation. The reason is that both, stabilization of volume moduli (i.e. 'Kähler moduli') in Type IIB string theory as well as inflation are sensitive to higher-order corrections in the effective potential and can therefore not be considered separately. Consequently, subsequent to our introduction to fluxbrane inflation we approached the issue of moduli stabilization.

In our first attempt we chose to neglect a possible inflaton dependence of the higher-order corrections which stabilize the Kähler moduli. Moreover, we assumed the superpotential to be independent of the inflaton and neglected the supergravity η-problem which generically appears for a canonical Kähler potential. It was then possible to demonstrate that Kähler moduli stabilization can be achieved in the phenomenologically required regime. More precisely, we employed the Large Volume Scenario to stabilize the overall volume (and some blow-up four-cycle) and used an interplay of loop corrections to the Kähler potential and the D-term to stabilize the remaining Kähler moduli. In the resulting minimum the D-term was parametrically suppressed with respect to its natural size. This was required to ensure stability of the system and satisfy the cosmic string bound. Naively, another possibility to grant stability was to enhance the size of the F-terms by increasing the value of the tree-level superpotential W_0. However, on its own this strategy was not successful due to the need of having the gravitino mass smaller than the Kaluza-Klein scale. In the end, a combination of a mildly large W_0 and a hierarchy of four-cycle volumes in the internal space allowed us to realize fluxbrane inflation with moduli stabilization in a parametrically controlled fashion.

Neglecting the inflaton dependence of Kähler and superpotential in the computation of the F-term scalar potential was certainly too naive. In fact, there are three distinct issues which one has to address in order for the inflation model to be viable. First, a canonical Kähler potential for the superfield containing the inflaton will lead to a supergravity η-problem, i.e. the supergravity F-term potential will be too steep in the inflaton direction. Second, a generic appearance of the inflaton in the superpotential will equally not be compatible with inflation, due to a too large inflaton mass. Third, the coefficients of the higher-order corrections (loop corrections to the Kähler potential), which were previously used to stabilize Kähler moduli, will involve the inflaton, something which we did not take into account so far.

To resolve these issues we first realized that the D7-brane moduli space enjoys a shift symmetry in the vicinity of the large complex structure point. Such a shift symmetry forbids the appearance of the inflaton in the leading-order Kähler potential, thereby solving the supergravity η-problem. Regarding the superpotential, we reiterated that in the F-theory formalism it is given by a product of the period vector with a flux vector. Some of the periods will involve the inflaton. However, by a suitable flux choice one can avoid the appearance of the corresponding components of the vector in the superpotential. This analysis is model-dependent, with the example of a compactification of F-theory on $K3 \times K3$ contained in [29].

Regarding the loop corrections to the Kähler potential, there is no general mechanism to achieve the absence of the inflaton in these terms. This entails two problems. First, one might fear that the 'extended no-scale structure' is jeopardized. However, this structure is essential for the validity of our model. It ensures that the naively dominant loop corrections are in fact subleading in the Kähler potential, which is crucial for the Large Volume Scenario to work. Originally, the presence of the extended no-scale structure was proven only in a situation where almost all moduli are stabilized at a high scale and the only light degrees of freedom entering the loop corrections are the Kähler moduli. This is, however, clearly not the case of interest for us, as we also want the inflaton to remain as a light degree of freedom in the effective theory. We managed to show that the extended no-scale structure persists, even with a D7-brane position unstabilized, and were therefore able to resolve this issue. Second, loop corrections were previously used to stabilize some Kähler moduli, in which case the size of the vacuum energy which drives inflation was bounded by the size of these loop corrections. As soon as there is an inflaton dependence in the loop corrections, one must thus fear to encounter an inflaton potential which is too steep to maintain slow-roll inflation. To resolve this issue we proposed to stabilize linear combinations of Kähler moduli by leading-order D-terms, while the overall volume is stabilized in the Large Volume Scenario, implying a parametric separation of the vacuum energy density and the loop corrections. In conclusion, all the above obstacles can be overcome in our model.

We argued that the fluxbrane inflation model appears rather naturally in string compactifications in which the uplift of the AdS vacuum, obtained in the Large Volume Scenario, to Minkowski is achieved via a D-term. In such a scenario, the correct present-day cosmological constant is obtained via a tuning in Kähler moduli space. Realizing fluxbrane inflation in such a setting makes this tuning only marginally worse.

The scenario of fluxbrane inflation gives rise to a viable realization of hybrid natural inflation in string theory. For a suitable choice of parameters the model is able to reproduce the correct amplitude of curvature perturbations, the spectral index n_s, and the number of e-foldings. It satisfies the cosmic string bound, while the running of the spectral index is small. Furthermore, the tensor-to-scalar ratio is tiny.

Finally, we explored the possibility of realizing large-field inflation with the position modulus of a single D7-brane. Despite the a priori sub-plankian (periodic) field range of this modulus, turning on fluxes can lead to a monodromy, breaking the periodicity and 'unfolding' the field space to accommodate a super-planckian field excursion. This proposal is along the lines of previously considered axion monodromy inflation scenarios, however, with the crucial advantage of being formulated in terms of spontaneously-broken supergravity. We explicitly addressed moduli stabilization and derived constraints on the size of the coefficients of the brane modulus in the superpotential. Our model is of the chaotic inflation type with a quadratic inflaton potential, entailing a large tensor-to-scalar ratio. This large-field realization is of great interest nowadays, as the recent claim of detection of gravitational waves might be confirmed soon.

Clearly, while we discussed how fluxbrane inflation and D7-brane chaotic inflation can be realized under certain (reasonable) assumptions about the Type IIB string compactification, we did not implement the models on explicit Calabi-Yau spaces. To achieve this one would need to understand not only the volume form of a given manifold, but also the structure of the periods. The latter is crucial to analyze whether the non-generic flux choices which are required in both inflation models (e.g. the fine-tuned weak dependence of the superpotential on the D7-brane moduli in the chaotic inflation model) can indeed be attained. It has become clear in [29] that this is highly entangled with the task of stabilizing complex structure moduli and the axio-dilaton. A further complication arises as this stabilization needs to be in the vicinity of the large complex structure point. Such minima are statistically disfavored [303], nevertheless, a large enough number is expected to exist [29] (see also the explicit computation in [304]). Finally, it would be reassuring to observe explicitly the exponential suppression of corrections to the leading-order shift-symmetric Kähler potential.

At least naively, a somewhat easier task is to analyze the Kähler moduli space in concrete examples. It would be very interesting to explicitly implement the Kähler moduli stabilization program outlined in chapter 5. This includes, in particular, the tuning of relative sizes of four-cycle volumes to realize a controlled D-term uplift. In this context it seems to be easily within reach to propose a viable version of the D-term uplifting proposal, either by explicit tuning as in chapter 5, or by a dynamical stabilization of the D-term at a small value via a balancing against loop corrections (whose exact form is, of course, generally unknown). We attempted to achieve the latter in chapter 4. However, the constraints imposed on the intersection structure by requiring a viable fluxbrane inflation scenario did not allow for a D-term uplift to de Sitter.[1] In the present situation, where no undebated uplifting proposal exists, a viable D-term uplifting scenario would certainly be a valuable contribution, even independently of a concrete inflation model.

On more general grounds, a better understanding of loop corrections to the Kähler potential would be desirable. They are known on some toroidal orientifolds, but beyond these examples their structure remains unclear. Even their Kähler moduli dependence is only conjectured. Furthermore, while we argued that, in suitable geometries, the prefactor of the instanton correction in the superpotential, associated with the small blow-up four-cycle, is independent of the D7-brane position moduli, explicitly demonstrating this would be desirable.

Also on the phenomenological side there are some interesting aspects which deserve further investigation. For example, in the fluxbrane inflation model alternative inflaton trajectories in the two-dimensional field space, depicted in figure 5.5, may lead to rather different model parameters, constraints, and potentially even observational signatures. Furthermore, the analysis of subleading corrections in the D7-brane chaotic inflation model was certainly not exhaustive. In particular, since the inflaton potential is quadratic, oscillatory contributions can become important during the later stages of inflation, even if they are completely subdominant during the initial few e-foldings. Finally, the backreaction of the inflaton on the stabilization of the complex structure moduli seems to be an issue in the D7-brane chaotic inflation model which needs further attention. Investigating these interesting effects is left for future work.

We hope that we have made a valuable contribution to the ongoing quest of implementing inflation in a string-theoretic context, with all other moduli of the string compactification stabilized. Our work contains many interesting results in this direction and indicates several possible avenues for further research.

[1] It might be that this negative conclusion was caused by our choice of a brane flux along an effective curve, and thus can be avoided in a more general setup.

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A. Definitions and Conventions

In this appendix we collect some definitions and conventions used in this thesis.

The string length ℓ_s is defined in terms of the Regge slope α' as $\ell_s = 2\pi\sqrt{\alpha'}$. Furthermore, the metrics in the ten-dimensional string and Einstein frames are related by

$$g^{\mathrm{S}}_{MN} = e^{\frac{\Phi}{2}} g^{\mathrm{E10}}_{MN}, \tag{A.1}$$

where Φ is the dilaton. Its vev sets the string coupling, $g_s = \langle e^{\Phi} \rangle$.

The Einstein frame volume $\mathcal{V}$ of the compactification space is given by the integral

$$\mathcal{V} = \frac{1}{\ell_s^6} \int \mathrm{d}^6 x \sqrt{g^{\mathrm{E10}}_6}. \tag{A.2}$$

After dimensional reduction, the canonical 4d Einstein-Hilbert term is recovered in the 4d Einstein frame defined via

$$g^{\mathrm{E10}}_{\mu\nu} = \frac{1}{\mathcal{V}} g^{\mathrm{E4}}_{\mu\nu}. \tag{A.3}$$

The relation of the four-dimensional Planck mass to the ten-dimensional string frame string length is then given by

$$M_p^2 = \frac{4\pi}{\ell_s^2}. \tag{A.4}$$

It is set to one in all four-dimensional field-theory formulae.

Note that the quantity ℓ_s is used differently in different contexts. As defined here, it is the string length in the ten-dimensional string frame. Quite obviously, its inverse does not define the string scale in four dimensions. To compute the mass of a string excitation which, in the ten-dimensional string frame, is for example given by ℓ_s^{-1}, we have to rescale this mass according to (A.1) and (A.3). This gives the mass

$$M_s = \frac{g_s^{1/4}}{\sqrt{\mathcal{V}}\ell_s} \tag{A.5}$$

in four dimensions. Sometimes, one defines a four-dimensional string length which, confusingly, is also called ℓ_s, via $\ell_s := 1/M_s$. Then, $M_p^2 = \frac{4\pi\mathcal{V}}{\sqrt{g_s}\ell_s^2}$ as opposed to (A.4).[1] In this thesis we use both definitions of ℓ_s. However, it will always be clear from the context which one is is meant.

[1]Note that this relation is often given in terms of the string-frame volume $\mathcal{V}_s$. In this case the g_s-factor changes to g_s^{-2}.

B. *F*-Term Scalar Potential

The main purpose of this appendix is to analyze the F-term potential discussed in section 4.1.1. Such a potential arises in the original Large Volume Scenario as proposed in [15] as well as in more elaborate versions thereof [247], which is the case of interest for us.

Starting point is the expression[1]

$$\begin{aligned} V_F = & \frac{e^K}{8\pi} \Big[K^{s\bar{s}} a_s^2 |A_s|^2 e^{-2a_s\tau_s} \\ & - a_s K^{s\bar{p}} \partial_{\bar{p}} K e^{-a_s\tau_s} \left\{ W\overline{A}_s e^{ia_sc_s} + \overline{W} A_s e^{-ia_sc_s} \right\} \\ & + \frac{3\xi|W_0|^2}{4g_s^{3/2}\mathcal{V}} \Big] \end{aligned} \tag{B.1}$$

which is obtained after plugging (4.1) and (4.2) into the standard supergravity formula for the F-term potential

$$V = \frac{e^K}{8\pi} \left(K^{a\bar{b}} D_a W D_{\bar{b}} \overline{W} - 3|W|^2 \right), \tag{B.2}$$

expanding in leading order in $1/\mathcal{V}$, and neglecting all terms $\propto e^{-a_p\tau_p}$, $p \neq s$ (cf. [14]).

Consider the second line of equation (B.1). We can rewrite the term in the brackets as

$$2|W_0||A_s| \cos\left(\arg(W_0) - \arg(A_s) + a_s c_s\right). \tag{B.3}$$

Furthermore, using the identity

$$K^{s\bar{p}} \partial_{\bar{p}} K = -2\tau_s + \text{higher orders in } 1/\mathcal{V} \tag{B.4}$$

(cf. e.g. [131]) it is clear that minimizing V_F with respect to the axion c_s will give $\cos(\ldots) \to -1$ in (B.3) and thus the second term in (B.1) becomes $-4a_s\tau_s e^{-a_s\tau_s}|W_0||A_s|$.

Now we turn to the first line in (B.1): Using $\mathcal{V}(\tau_p) = \tilde{\mathcal{V}}(\tau_{p\neq s}) - c\tau_s^{3/2}$ we find

$$K_{s\bar{s}} \simeq \frac{3}{8} \frac{c}{\mathcal{V}\tau_s^{1/2}} \ , \quad K_{p\bar{s}} \simeq -\frac{3}{4} \frac{c(\partial_p \mathcal{V})\tau_s^{1/2}}{\mathcal{V}^2}, \tag{B.5}$$

i.e. $K_{p\bar{q}}$ is block-diagonal in leading order in $1/\mathcal{V}$. Therefore, $K^{s\bar{s}} \simeq \frac{8}{3}\frac{\mathcal{V}\tau_s^{1/2}}{c}$ in leading order. Combining all the results we find

$$V_F = V_{0,F} \left(\frac{\alpha\sqrt{\tau_s} e^{-2a_s\tau_s}}{c\mathcal{V}} - \frac{\beta|W_0|\tau_s e^{-a_s\tau_s}}{\mathcal{V}^2} + \frac{\gamma\xi|W_0|^2}{g_s^{3/2}\mathcal{V}^3} \right) \tag{B.6}$$

[1] There seems to be a disagreement in the literature concerning the overall prefactor of the supergravity potential (see [12] and [242]). However, this factor is irrelevant for our purposes as we can simply choose to work with a differently normalized W_0.

with

$$V_{0,F} = \frac{g_s e^{K_{\rm cs}}}{16\pi}, \quad \alpha = \frac{8a_s^2|A_s|^2}{3}, \quad \beta = 4a_s|A_s|, \quad \gamma = \frac{3}{4}. \tag{B.7}$$

We now compute the large-volume minimum of (B.6). To this end, we evaluate

$$\frac{\partial V}{\partial \mathcal{V}} = 0, \quad \frac{\partial V}{\partial \tau_s} = 0. \tag{B.8}$$

The first equation gives

$$\mathcal{V} = \frac{\beta|W_0|c\sqrt{\tau_s}e^{a_s\tau_s}}{\alpha}\left(1 - \sqrt{1 - \frac{3\alpha\gamma\xi}{g_s^{3/2}c\beta^2}\frac{1}{\tau_s^{3/2}}}\right) \tag{B.9}$$

while the second one reads

$$\frac{\mathcal{V}\alpha e^{-a_s\tau_s}}{\beta c|W_0|\sqrt{\tau_s}} = \frac{1 - a_s\tau_s}{\frac{1}{2} - 2a_s\tau_s}. \tag{B.10}$$

One can rewrite these two expressions, using (B.9), to obtain

$$1 - \sqrt{1 - \frac{3\alpha\gamma\xi}{g_s^{3/2}c\beta^2}\frac{1}{\tau_s^{3/2}}} = \frac{1 - a_s\tau_s}{\frac{1}{2} - 2a_s\tau_s}. \tag{B.11}$$

For $a_s\tau_s \gg 1$ the right-hand side of the above equation becomes a constant, such that, at leading order,

$$\tau_s = \frac{1}{g_s}\left(\frac{4\gamma\alpha\xi}{c\beta^2}\right)^{2/3} = \frac{1}{g_s}\left(\frac{\xi}{2c}\right)^{2/3}, \tag{B.12}$$

$$\mathcal{V} = \frac{\beta|W_0|c\sqrt{\tau_s}e^{a_s\tau_s}}{2\alpha} = \frac{\beta|W_0|c}{2\alpha g_s^{1/2}}\left(\frac{\xi}{2c}\right)^{1/3} e^{\frac{a_s}{g_s}\left(\frac{\xi}{2c}\right)^{2/3}}. \tag{B.13}$$

From the above analysis and the definition of ξ below equation (4.2) it is clear that the requirement $a_s\tau_s \gg 1$ is satisfied as long as $-\chi(X_3) \gg \frac{4c}{\zeta(3)}(2\pi g_s)^{3/2}$. By definition, $\chi(X_3) = 2(h^{(1,1)} - h^{(2,1)})$. Furthermore, in the models considered in this thesis $h^{(1,1)}$ need not be big. Thus, the implied lower bound on the number of complex structure moduli is easily satisfied for g_s in the perturbative regime.

A quantity which is crucial for our phenomenological discussion is the value of the scalar F-term potential at its minimum. This value is obtained by replacing $\mathcal{V}$ in (B.6), using (B.10):

$$V_F = V_{0,F}\frac{\alpha^2}{\beta c^2|W_0|}e^{-3a_s\tau_s}\left(X - X^2 + \frac{\alpha\gamma\xi}{\beta^2 c g_s^{3/2}}\tau_s^{-3/2}X^3\right), \tag{B.14}$$

where we have defined

$$X = \left(\frac{\frac{1}{2} - 2a_s\tau_s}{1 - a_s\tau_s}\right). \tag{B.15}$$

On the other hand we can use this definition of X with (B.11) to obtain

$$\frac{\alpha\gamma\xi}{\beta^2 c g_s^{3/2}}\tau_s^{-3/2} = \frac{1}{3}\left(2X^{-1} - X^{-2}\right). \tag{B.16}$$

Combining the two results finally gives

$$V_F = V_{0,F}\frac{\alpha^2}{\beta c^2|W_0|}e^{-3a_s\tau_s}\left(\frac{2}{3}X - \frac{1}{3}X^2\right) \approx V_{0,F}\frac{\alpha^2}{\beta c^2|W_0|}e^{-3a_s\tau_s}\left(-\frac{1}{a_s\tau_s}\right) \tag{B.17}$$

for $a_s\tau_s \gg 1$. Interestingly, the minimum value of the F-term potential is smaller than the three individual terms in (B.6) by a factor of $(a_s\tau_s)^{-1}$. Using (B.12) and (B.13) we can express the above result as

$$V_F = -\frac{3M_p^4\sqrt{g_s}e^{\mathcal{K}_{cs}}}{128\pi^2}\frac{c|W_0|^2}{\mathcal{V}^3}\left(\frac{\xi}{2c}\right)^{1/3} \tag{B.18}$$

where we chose to explicitly write $\mathcal{V}$ and $|W_0|$ in order to get a feeling for the size of the F-term potential in its minimum. It is clear, that via (B.12) and (B.13) one can express the minimum value solely in terms of ξ, g_s, c etc.

C. Lower Bound on the Brane Flux Quanta

In this appendix we give a rough estimate for the lower bound on n_+, as motivated in section 4.1.2. To this end we write (4.9) as

$$\frac{V(\mathcal{V})}{V_{0,F}} = \frac{1}{\mathcal{V}^3} f(\mathcal{V}), \tag{C.1}$$

where $f(\mathcal{V}) = A - B \log^{3/2}(C\mathcal{V}) + D\mathcal{V}$. Furthermore, we choose to expand the potential at the minimum $\mathcal{V}_{\text{min.}}$ as

$$\frac{V(\mathcal{V})}{V_{0,F}} = \frac{f''(\mathcal{V}_{\text{min.}})}{2\mathcal{V}^3} (\mathcal{V} - \mathcal{V}_{\text{min.}})^2 + \dots . \tag{C.2}$$

Maximizing (C.2) gives[1] $\mathcal{V}_{\text{max.}} = 3\mathcal{V}_{\text{min.}}$ and

$$\frac{V(\mathcal{V}_{\text{max.}})}{V_{0,F}} \simeq \frac{2}{3^3} \frac{f''(\mathcal{V}_{\text{min.}})}{\mathcal{V}_{\text{min.}}} . \tag{C.3}$$

Computing $f''(\mathcal{V}_{\text{min.}})$, keeping only the leading term in $\log(C\mathcal{V}_{\text{min.}})$, and using $f'(\mathcal{V}_{\text{min.}}) = 0$ one finds

$$\frac{V(\mathcal{V}_{\text{max.}})}{V_{0,F}} \simeq \frac{2}{3^3} \frac{D}{\mathcal{V}_{\text{min.}}^2} . \tag{C.4}$$

An estimate of how big the uplift can be such that it does not destroy the local minimum of the potential is given by requiring

$$\frac{V(\mathcal{V}_{\text{max.}})}{V_{0,F}} \gtrsim \frac{\delta V(\mathcal{V}_{\text{min.}})}{V_{0,F}} = \frac{\delta D}{\mathcal{V}_{\text{min.}}^2} . \tag{C.5}$$

This then implies

$$\frac{\delta D}{D} = \frac{n_-^2}{n_+^2} \lesssim \frac{2}{3^3} . \tag{C.6}$$

Thus, $n_+ \geq 4$, $n_- = 1$ is ok. Obviously, this is only a very coarse analysis. However, the result can be confirmed very easily by a straightforward numerical analysis.

[1] Actually, as $(\mathcal{V}_{\text{max.}} - \mathcal{V}_{\text{min.}}) > \mathcal{V}_{\text{min.}}$ the expansion (C.2) breaks down. However, the calculation still gives a first idea for the required size of n_+ which can be confirmed numerically.

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