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Patrick Mangat

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Axions, Wormholes and Inflation

in the String Landscape

Referees: Prof. Dr. Arthur Hebecker
Prof. Dr. Luca Amendola

Axions, Wormholes and Inflation in the String Landscape

This thesis explores the string theory landscape in two distinct approaches. First, we study dark radiation predictions in Large Volume Scenarios (LVS). Specifically, we realise the visible sector by D7-branes wrapping a 4-cycle. In consequence, the tension between the predicted effective number of relativistic species and recent cosmological measurements is reduced, and further ameliorated by accepting fine-tuning. Next, we investigate whether large-field inflation is consistent with quantum gravity. We propose realisations of axion monodromy inflation and alignment inflation in string theory. Due to backreaction of moduli on the inflaton, axion monodromy inflation requires potentially severe fine-tuning, which we realise on Calabi-Yau 4-folds, whereas we find no-go results on Calabi-Yau 3-folds. The severity of the tunings is quantified by the diminution of the landscape vacua after imposing the fine-tuning conditions. Furthermore, alignment inflation is realised by a winding trajectory in the field space of two complex structure moduli. Severe fine-tuning seems avoidable and consistency with the mild Weak Gravity Conjecture possible, albeit not with its strong version. Finally, we investigate the role of gravitational instantons for axion inflation. Although Giddings-Strominger wormholes are shown to induce corrections to the axion potential, a derivation of relevant constraints on the axion field range remains challenging.

Axione, Wurmlöcher und Inflation in der String-Landschaft

Die vorliegende Arbeit untersucht die String-Landschaft durch zwei Methoden. Zuerst studieren wir Vorhersagen zu Dunkler Strahlung im Rahmen des "Large Volume Scenarios" (LVS). Insbesondere betrachten wir Modelle, in denen der Sektor des Standardmodells durch D7-Branes, die einen 4-Zykel umwickeln, realisiert wird. Daraus resultiert eine Verringerung der Abweichung theoretischer Vorhersagen bezüglich der effektiven Anzahl relativistischer Spezies von kosmologischen Messungen. Diese Diskrepanz kann durch Feinabstimmung weiter reduziert werden. Anschließend gehen wir der Frage nach, ob "Large-Field Inflation" mit Quantengravitation konsistent sein kann. Dazu schlagen wir Realisierungen von "Axion Monodromy Inflation" und "Alignment-Inflation" in Stringtheorie vor. Aufgrund von Rückkopplungseffekten von Modulifeldern auf das Inflaton erfordert "Axion Monodromy Inflation" potenziell starke Feinabstimmung, die wir auf Calabi-Yau 4-Falten realisieren. Auf Calabi-Yau 3-Falten ist die Möglichkeit dieser Feinabstimmungen weitgehend untersagt. Die Strenge dieser erforderlichen Bedingungen quantifizieren wir, indem die verbleibende Anzahl der String-Vakua nach Berücksichtigung der Feinabstimmungen abgeschätzt wird. Des Weiteren realisieren wir "Alignment-Inflation" auf Basis einer Trajektorie, die den Feldraum, aufgespannt durch zwei Parameter der komplexen Struktur der zugrunde liegenden Calabi-Yau 3-Falt, mehrfach umwindet. Dabei ist keine strenge Feinabstimmung erforderlich und Konsistenz mit der milden "Weak Gravity Conjecture", nicht aber mit der starken Version derselben, möglich. Schließlich untersuchen wir die Rolle gravitativer Instantone für Axion-Inflation. Wir stellen fest, dass Giddings-Strominger Wurmlöcher zwar Korrekturen zum Axion-Potential induzieren, sich jedoch nicht ohne Weiteres relevante Einschränkungen auf den Axion-Feldraum herleiten lassen.

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To my parents Christine & Paul

Contents

CHAPTER 1

Introduction and Basics of String Cosmology

The aim of this chapter is to make the reader familiar with the minimal basics of string cosmology and to summarise both the goals and results of the projects the thesis is based on. Therefore, we begin by a general introduction and motivation of the thesis. The subsequent sections summarise the basic concepts and notions of cosmological inflation, moduli stabilisation and the string landscape. Afterwards, a more technical and detailed summary of the contents of the thesis is provided.

1.1. Motivation of the Thesis

Theoretical and experimental high energy physicists arguably live in an exciting era of fundamental research. The progress in mathematical and theoretical physics has resulted in a rich toolbox for model building in particle physics and cosmology. At the same time, model builders and phenomenologists can expect even more guidance from collider physics experiments and astrophysical as well as cosmological observations in the near future.

Most prominently, the Large Hadron Collider (LHC) is able to probe energy scales of approximately 14 TeV and, therefore, recent data put already stringent constraints on various models of particle physics beyond the Standard Model (SM) or even exclude them. In particular, many physicists are/were hoping to find supersymmetry (SUSY) at the LHC, but at least up to now there is no experimental evidence for SUSY at the accessible energy scale. Not finding any superpartners at the TeV scale could force us to drop the hope to ameliorate the hierarchy problem via supersymmetry, and instead we might have to accept a fine-tuned Higgs mass. A possible conclusion could be that (bottom-up) naturalness is not always the guiding principle of nature. In this case, one would have to face the possibility of fine-tuned physical parameters.

Furthermore, recent measurements of the Cosmic Microwave Background (CMB) provide new data on cosmological parameters with unprecedented precision. For instance, the effective number of relativistic neutrino species is quoted to be $N_{\text{eff}} = 3.15 \pm 0.23$ (1σ; Planck TT+lowP+BAO) [1], which is to be compared with the value $N_{\text{eff;SM}} = 3.046$ expected from the Standard Model. Those measurements put tight constraints on models predicting an extra type of hidden relativistic species, which is (in analogy to "dark matter") called *dark* *radiation.* Moreover, the Planck collaboration can measure scalar perturbations predicted by inflation. They are quantified by the scalar spectral index n_s, which is measured to be $n_s = 0.968 \pm 0.006$ (Planck TT+lowP+lensing) [2]. Apart from scalar perturbations one also expects tensor perturbations from inflation, which are quantified by the tensor-to-scalar ratio r. So far, measurements could only provide an upper bound $r < 0.11$ (95% CL) (Planck TT+lowP+lensing) [2]. If the measurements by the Planck and BICEP2/Keck Array collaborations are combined [3], the upper bound is even reduced to $r < 0.07$ (95% CL). Due to the increasing sensitivity of planned experiments and measurements we can expect stricter bounds in the near future or, in the case of a detection of primordial gravitational waves, even a precise value of r. From a theoretical point of view it is important to know whether r is below or above the value $r \sim 0.01$, because $r \gtrsim 0.01$ requires super-Planckian field displacements of the inflaton field. Thus, fine-tuning of the Wilson coefficients of higher-dimensional operators to protect the inflaton-potential is no longer sufficient. Instead, a shift symmetry is needed to protect the flatness of the potential.

There are certainly more astrophysical measurements and missions to be mentioned here, for instance missions of detecting gravitational waves from black holes or from phase transitions in early epochs of the universe. Furthermore, missions probing the late-time evolution of the universe are expected to improve our understanding of dark energy and dark matter. In addition, such missions allow for further tests of general relativity at large scales. Discussing these topics in detail goes however beyond the scope of this thesis.

String theory is a prominent candidate for a unifying framework of all four fundamental forces (electromagnetism, strong force, weak force, and gravity) we observe in nature. It is thus exciting to understand the implications of string theory applied to particle physics and cosmology, e.g. in the context described above. This area of research is called *string phenomenology* and the (in itself very rich) subject of *string cosmology* comprises all questions related to applications of string theory to cosmology.

Crucially, (super-)string theory is a theory in *ten dimensions*.[1] In fact there are five consistent versions, namely type I, type IIA, type IIB, $SO(32)$ heterotic, and $E_8 \times E_8$ heterotic string theory, but they are related to each other via dualities (see e.g. [4] for a review).

Effective four-dimensional theories from string theory can be obtained as follows: Take a string theory, for example type IIB string theory, and consider its low-energy limit, which is (in our example) 10-dimensional type IIB supergravity. This theory needs to be compactified to four dimensions. The compactification geometries are typically *Calabi-Yau* (CY) manifolds (of complex dimension three).

However, given any CY 3-fold[2], one does not obtain a unique vacuum of the four-dimensional theory. Instead, one discovers a large but discrete set of vacua. This set of vacua is referred to as the *string theory landscape*. Qualitatively, one can understand its origin as follows: the size and shape of the CY geometry is accounted for by the appearance of scalar fields, so-called *moduli fields*, in the four-dimensional effective theory. Those moduli fields have to be fixed in order to obtain a (sufficiently) stable vacuum. There are well-known mechanisms of *moduli stabilisation* (see e.g. [5–7]). The stabilisation requires non-zero fluxes[3] and one finds that the vacuum is parameterised by a set of integers. For distinct choices of flux numbers one obtains

[1] There is also bosonic string theory, which requires 26 spacetime dimensions. Superstring theory, however, is consistent only for ten spacetime dimensions. In the following we mostly write "string theory", but actually mean "superstring theory".

[2] CY manifolds of complex dimension d are often referred to as CY d-fold.

[3] The fluxes occurring in type IIB string theory are analogous to the 2-form flux in electrodynamics; specifically in type IIB string theory, 3-form fluxes will be used for moduli stabilisation.

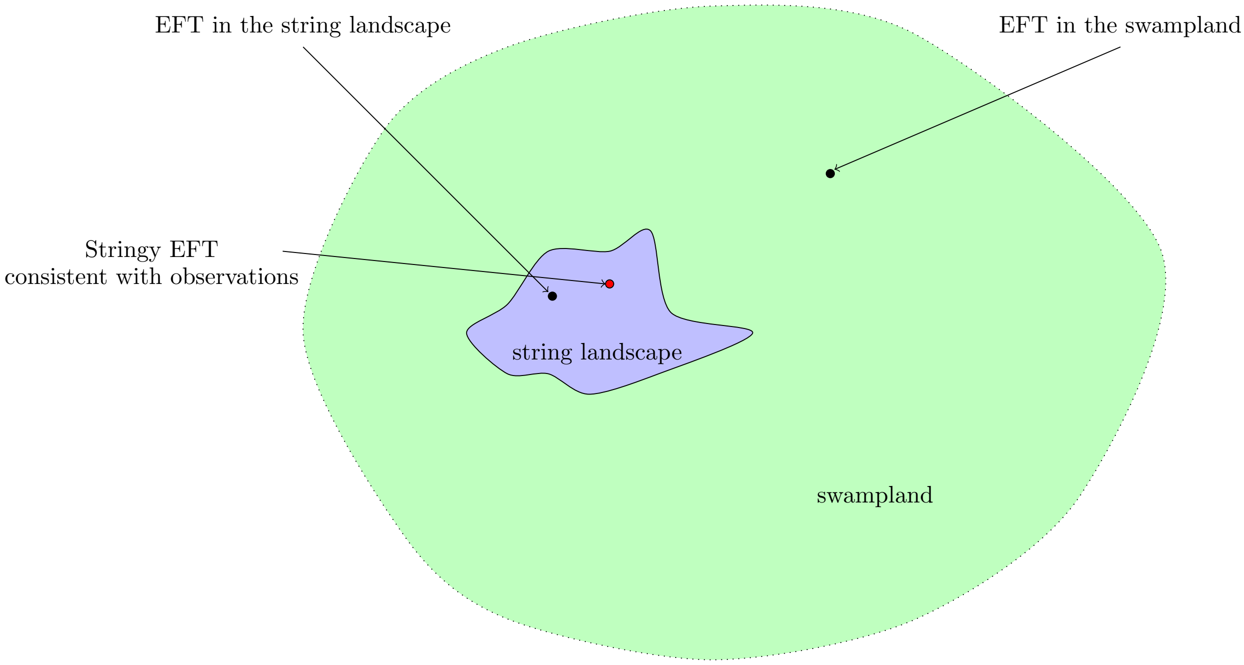


Figure 1.1.: Effective field theories (EFTs) which can be obtained from string compactifications are part of the string landscape (blue region). The rules of string theory (or quantum gravity) are expected to define the shape of the boundary of the string landscape within an even much larger set of EFTs, which cannot be obtained from string theory. This set is referred to as the swampland (green region). The red dot symbolises a stringy EFT, which is part of the string landscape and consistent with current data. This illustration is inspired from typical representations of the landscape within the swampland, such as in [20].

different vacua. Since the flux numbers are subject to a constraint (the tadpole-cancellation condition), the number of string vacua is believed to be finite, see [8–13] and e.g. [14–18] for reviews on the topic. In the past, the number of such vacua has been estimated and the results are often quoted as 10^{500} vacua [17].

This diversity of string vacua allows in principle to construct many models of particle physics and cosmology, each with distinctive features and parameters. In particular, the string landscape may support the concept of fine-tuned physical parameters as an alternative to the idea of naturalness. For instance, the small Higgs mass or the tiny cosmological constant [8] can be explained by the huge number of string vacua in combination with the anthropic principle.[4]

In this thesis, we want to understand how generic certain features of the string landscape are. We focus on applications to early-universe cosmology. In particular, the guiding questions of this thesis are:

- How generic is dark radiation in reheating models obtained from string compactifications?

[4] Reviews on the string landscape, anthropic arguments in physics and the multiverse can be found in [19].

- Are there no-go arguments for large-field inflation models in string theory?

The first question is addressed in Chapter 2 using the Large-Volume Scenario [7] as a moduli stabilisation scheme, in which the occurrence of dark radiation is generic due to the presence of an essentially massless axion. We demonstrate how and under which circumstances such models can still be consistent with the latest measurements $N_{\text{eff}} = 3.15 \pm 0.23$. While it is still possible to realise Large-Volume models consistent with current measurements of N_{eff} (accepting some fine-tuning in such models), these scenarios may easily be ruled out when error bars of the measurements continue to decrease and the central value converges towards the value predicted by the SM. In consequence, a large set of string vacua could easily be ruled out just because of the dark radiation excess.

We approach the second question in Chapters 3 to 5 by exploring the "shape" of the string landscape in the so-called *swampland* [21]. It is defined as the set of all effective field theories (EFTs), whose UV-completions are inconsistent with quantum gravity. Using string theory as a theory of quantum gravity, the string landscape is expected to be surrounded by a much larger swampland [21], see Figure 1.1, because string vacua are parametrised only by integral flux numbers. The boundaries of the string landscape within the swampland are investigated in three distinct approaches. We demonstrate how geometric constraints in type IIB string theory can complicate the explicit construction of a fine-tuned model of axion monodromy inflation in the string landscape. Under certain circumstances the required fine-tuning conditions cannot be met, see Chapter 3. Furthermore, various quantum gravity arguments suggest that string vacua must obey the so-called *Weak Gravity Conjecture* (WGC) which states the following: in any $U(1)$ gauge theory there must always be a charged particle whose charge-to-mass ratio is such that the gravitational attraction between two such particles is always subdominant over their repulsion due to the gauge theory [20]. Thus, there must be a particle of mass m and charge q such that $qM_p/m \gtrsim 1$, where M_p is the reduced Planck mass. Depending on details of its formulation, the WGC has the potential to rule out models of axion inflation. Hence, the WGC may provide explicit examples of EFTs of inflation residing in the swampland. In Chapter 4, we propose an example of axion inflation consistent with a "mild version" but in conflict with a "strong version" of the WGC (for details of these versions of the WGC see Section 1.4). Finally, in Chapter 5 we investigate whether gravitational instantons in theories of axions coupled to gravity are able to break their shift symmetry strongly enough, such that axion inflation can be constrained in a model-independent manner. We show, however, that effects of gravitational instantons are too weak to formulate general non-trivial constraints on axion inflation.

We conclude that certain corners of the string landscape can have common phenomenological features such as dark radiation, and thus are falsifiable already by present data. It is therefore crucial to work out and to improve our understanding of phenomenological features of the string landscape. Moreover, exploring the boundaries of the string landscape within the swampland is a promising path towards a tool to evaluate whether an EFT is part of the swampland or the landscape. Specifically, we hope to gain further constraints for inflationary model building.

1.2. Cosmological Inflation in String Theory – A Brief Overview

The framework of cosmological inflation [22–25] is widely accepted to complete the standard Big Bang cosmology by solving the horizon and flatness problems.[5] Additionally, quantum

[5]Furthermore, cosmic inflation automatically explains why no magnetic monopoles have been observed.

fluctuations during inflation [26–30] give rise to observable cosmological density perturbations. This section aims to briefly review the basics of slow-roll inflation and to argue why it is important to investigate inflation in string theory. Nice and detailed presentations of these topics can be found e.g. in [31; 32].

1.2.1. Slow-Roll Inflation

Most models of cosmological inflation are based on the idea of a scalar field ϕ with minimal coupling to Einstein gravity, which slowly rolls down its potential. This slow-roll phase ensures that the potential energy $V(\phi)$ of the scalar field dominates over its kinetic energy so that a controlled (approximate) exponential expansion of the universe can take place. This is the idea behind *slow-roll inflation*. The action for single-field slow-roll inflation is given by[6]

$$S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] , \tag{1.2.1}$$

where M_p is the (reduced) Planck mass and R the Ricci scalar for the metric $g_{\mu\nu}$. As a background we use the Friedman-Robertson-Walker (FRW) metric

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega_2^2\right) , \tag{1.2.2}$$

where $k = -1, 0, +1$ is the curvature parameter and $d\Omega_2^2$ the line element on S^2. The function $a(t)$ is the well-known scale factor. Variations of the action with respect to $g_{\mu\nu}$ and ϕ yield the Friedman equation (neglecting the spatial curvature term),

$$3M_p^2H^2 \simeq \frac{1}{2}\dot{\phi}^2 + V(\phi) , \tag{1.2.3}$$

and the Klein-Gordon equation,

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 , \tag{1.2.4}$$

where $H \equiv \dot{a}/a$ is the Hubble parameter and $V' \equiv \partial_\phi V$. Here, we assumed that ϕ is spatially homogeneous, i.e. $\phi = \phi(t)$, being consistent with the symmetries of the FRW metric.

Slow-roll inflation requires to find a solution to these equations of motion while satisfying the slow-roll conditions $V(\phi) \gg \dot{\phi}^2/2$ and $|\ddot{\phi}| \ll 3H\dot{\phi}$ (i.e. the dynamics is dominated by the friction term). One can show [31; 32] that these requirements are met as long as the two conditions

$$\boxed{\epsilon_V \equiv \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1 , \qquad |\eta_V| \equiv M_p^2\left|\frac{V''}{V}\right| \ll 1} \tag{1.2.5}$$

are satisfied. Inflation stops whenever at least one of those two conditions gets violated as ϕ, called the *inflaton*, moves.[7] Hence, eq. (1.2.5) requires one to have a sufficiently flat inflaton

[6] There are obvious extensions of single-field slow-roll inflation [31]: (i) One can consider multi-field inflation models. (ii) One can include higher-derivative corrections, so that $\mathscr{L} = F(\phi, X) - V(\phi)$ with $X \equiv \partial_\mu\phi\partial^\mu\phi$. (iii) We can also choose ϕ to couple non-minimally to gravity, e.g. by a term $f(\phi)R$. However, such theories can be rewritten as minimally-coupled theories upon field redefinition. (iv) One can also consider $f(R)$ gravity theories. But again, they can also be rewritten as theories with minimal coupling to Einstein gravity. For a review on inflation in modified gravity theories such as $f(R)$-gravity see e.g. [33] and references therein.

[7] Actually, the slow-roll conditions in (1.2.5) are approximate conditions. There are pathological cases, in which (1.2.5) can be violated, while the *exact* slow-roll conditions (see e.g. [31]) are still satisfied. Nevertheless, in practice, the approximate slow-roll conditions are good criteria for slow-roll inflation.

potential $V(\phi)$. After inflation stops, the inflaton oscillates around the minimum and decays into Standard Model particles. This process is called *(p)reheating* (see e.g [34] for a review on reheating).

While inflation takes place, the universe approximately expands exponentially, $a(t) \simeq e^{Ht}$. The horizon problem is solved if inflation drives the expansion by at least 40 to 60 e-folds (depending on the energy scale of inflation and the reheating temperature) [31; 32]. The rapid exponential expansion automatically solves the flatness problem, too.[8]

So far, this was the classical treatment of inflation. Taking into account quantum mechanics, spatially-dependent quantum fluctuations add to the classical dynamics and lead to the inflaton randomly jumping up and down the potential. These fluctuations in the field variable give rise to measurable density perturbations, which can be finally expressed in terms of the slow-roll parameters ϵ_V and η_V. Similarly, the metric is subject to quantum fluctuations. A rigorous computation of these density perturbations is rather involved and for details we refer to [31; 32]. However, there is a quick and intuitive approach to obtain the correct density perturbations up to numerical factors. The following is based on [29; 35] and on lectures by N. Arkani-Hamed [36].[9]

The size of the inflaton field fluctuations $\delta\phi(\mathbf{x})$ during slow-roll inflation depends on its wavelength k^{-1}. As the universe inflates, the wavelength gets stretched proportional to e^{Ht} until it is comparable to the Hubble scale H^{-1}. When the wavelength is larger than the Hubble horizon, the fluctuations get frozen (more technically, it can be shown that the comoving curvature perturbation is constant on super-horizon scales for adiabatic expansion). After the end of inflation the horizon gets larger again and the fluctuations eventually re-enter the horizon. At horizon exit the wavelength is $k^{-1} \sim H^{-1}$. Since H is the only dimensionful quantity the fluctuations could depend on, we expect by dimensional analysis

$$\delta\phi(\mathbf{x}) \sim H \ . \tag{1.2.6}$$

Note that due to the quantum fluctuations the time slice for the end of inflation fluctuates as well. Hence, in some regions reheating starts earlier and correspondingly in those regions the energy density is below average because of redshift. Analogously, regions in which reheating is delayed the energy density is above average. The fluctuations in time at which inflation ends scale as $\delta t(\mathbf{x}) \sim \delta\phi/\dot{\phi} \sim H/\dot{\phi}$. Since the density perturbations are then determined by the variations in the number of e-folds, defined by $dN \equiv Hdt$, we have

$$\Delta_s \equiv \left(\frac{\delta\rho}{\rho}\right)_\star \sim \delta N \sim H\delta t \sim \frac{H\delta\phi}{\dot{\phi}} \sim \frac{H^2}{\dot{\phi}} \ . \tag{1.2.7}$$

The index s reminds us that these are scalar perturbations at horizon crossing (denoted by $\star$). During slow-roll inflation $\dot{\phi} \sim -V'/H$ and $H^2 \sim V/M_p^2$ hold, and therefore it follows that

$$\Delta_s^2 \sim \frac{1}{M_p^6}\frac{V^3}{V'^2} \tag{1.2.8}$$

[8] Strictly speaking, a phase of exponential expansion is sufficient but not necessary to solve the horizon and the flatness problem. If the dynamics of the universe is governed by some matter content with equation of state $p = w\rho$, both problems are solved whenever $w < -1/3$, i.e. the strong-energy condition must be violated. Thus, the condition $w < -1/3$, or equivalently, a shrinking comoving horizon, $d((aH)^{-1})/dt < 0$, can be used as a general definition of inflation. Slow-roll inflation is then a special realisation of inflation with $w \simeq -1$ [31].

[9] A nice presentation of the estimation of density perturbations during slow-roll inflation can also be found in a blog post by J. Preskill [37].

at horizon crossing. The formula with numerical factors taken into account is

$$\Delta_s^2 = \frac{1}{12\pi^2 M_p^6} \frac{V^3}{V'^2} \ . \tag{1.2.9}$$

It is convenient to define the *spectral index* n_s:

$$n_s - 1 \equiv \frac{d \ln \Delta_s^2}{d \ln k} \ . \tag{1.2.10}$$

Hence, n_s is a measure of scale invariance of the scalar perturbations with perfect scale invariance for $n_s = 1$. This parameter can be directly related to the slow-roll parameters:

$$\boxed{n_s - 1 = (2\eta_V - 6\epsilon_V)_\star} \tag{1.2.11}$$

at horizon crossing. Therefore, given an inflaton potential $V(\phi)$, one can calculate explicitly the expected deviation from scale invariance. Since n_s is already known by measurements, one can immediately check whether the proposed model with potential $V(\phi)$ is consistent with present data.

Very similarly, we can estimate the quantum fluctuation δg of the gravitational field. Since $g_{\mu\nu}$ is dimensionless we have at horizon crossing $\delta g \sim H/M_p$. These *tensor perturbations* can observationally be distinguished from scalar perturbations. In analogy to Δ_s one can define Δ_t and obtain

$$\Delta_t^2 \sim \frac{H^2}{M_p^2} \sim \frac{V}{M_p^4} \ . \tag{1.2.12}$$

Accounting for all numerical factors one gets

$$\Delta_t^2 = \frac{2}{3\pi^2} \frac{V}{M_p^4} \tag{1.2.13}$$

at horizon crossing. It is convenient to introduce the *tensor-to-scalar ratio* r, which is directly given by ϵ_V at horizon exit:

$$\boxed{r \equiv \frac{\Delta_t^2}{\Delta_s^2} = 16\epsilon_{V\star}} \ . \tag{1.2.14}$$

Recent data collected by the Planck satellite [2] yields $\Delta_s \approx 4.7 \cdot 10^{-5}$ at a pivot scale of $k = 0.05$ Mpc^{-1}. Moreover, $n_s = 0.968 \pm 0.006$ (Planck TT+lowP+lensing), i.e. there is a small deviation from scale invariance, as predicted by slow-roll inflation. For r we currently only have upper bounds ranging from $r < 0.07$ (95% CL; Planck + BICEP 2) [3] to $r < 0.11$ (95% CL) (Planck TT+lowP+lensing) [2]. In the near future values below $r \sim 0.01$ are expected to be experimentally accessible.

For many reasons it is exciting to know the value of r as precisely as possible. For one, a measurement of inflationary tensor perturbations would give direct observational evidence that the gravitational field is also quantised. Additionally, the energy scale $V^{1/4}$ at which inflation took place, can be directly calculated [31; 32; 38]:

$$\boxed{V^{1/4} \simeq \left(\frac{r}{0.01}\right)^{1/4} \cdot 10^{16} \text{ GeV}} \ . \tag{1.2.15}$$

Note that for $r = \mathcal{O}(0.01)$ the energy scale is only two orders of magnitude below the Planck scale. Finally, the value of r tells by what distance in field space the inflaton ϕ had to move during inflation. The question of which lengths in field space an inflaton can traverse, is of theoretical interest, as we explain below. Equation (1.2.14) implies that

$$\frac{d\phi}{dN} = M_p \sqrt{\frac{r}{8}} \,. \tag{1.2.16}$$

The dependence of r on N is subleading. Knowing that at least 40 to 60 e-folds are required to solve the horizon problem, we obtain the Lyth bound [39]:

$$\boxed{\frac{\Delta\phi}{M_p} \gtrsim \mathcal{O}(1) \cdot \sqrt{\frac{r}{0.01}}} \,. \tag{1.2.17}$$

If $r > 0.01$ is measured in the near future, then the Lyth bound implies that $\Delta\phi > M_p$. Inflationary models with such trans-Planckian field ranges belong to the class of *large-field inflation* scenarios. So-called *small-field inflation* models require only $\Delta\phi < M_p$. The distinction between those two cases is crucial for inflation model building as we will discuss in the next subsection.

Besides measuring scalar and tensor fluctuations it is also important to search for primordial non-Gaussianities. Single-field inflation models based on (1.2.1) predict a three-point function for the fluctuations proportional to the deviation from scale invariance, $1 - n_s$. Hence, measuring a significant amount of primordial non-Gaussianities in the CMB can rule out such single-field models. In contrast, single-field models with higher-derivative interactions or multi-field inflation models can obtain support from detecting of non-Gaussianities. For more details on this topic including further possibilities to exclude certain classes of inflation models we refer to [32].

1.2.2. Why Inflation in String Theory?

The constraints on the potential $V(\phi)$ from the slow-roll conditions (1.2.5) raise the question of naturalness of slow-roll inflation. Indeed, we expect non-renormalisable contributions to the effective Lagrangian. Those arise from integrating out massive degrees of freedom (of mass Λ), to which the inflaton ϕ couples. Such massive degrees of freedom are expected to arise at or below the Planck scale in a UV-completion of gravity [32]. Naively, these corrections spoil slow-roll inflation and this failure is known as the *eta-problem* (the subsequent discussion follows [31; 32]). To see this, consider a classical Lagrangian

$$\mathscr{L} = -\frac{1}{2} g^{\mu\nu} \partial_\mu\phi \partial_\nu\phi - V_0(\phi) \tag{1.2.18}$$

with renormalisable potential V_0 suitable for slow-roll inflation. Integrating out heavy fields of masses $m \geq \Lambda$ is reflected by inclusion of non-renormalisable terms $c\,\mathcal{O}_\delta[\phi]/\Lambda^{\delta-4}$, where $\mathcal{O}_\delta$ is a higher-dimensional operator of dimension δ. Without further assumptions the *Wilson coefficient* c is expected to be $c \sim 1$. In principle, every allowed higher-dimensional operator term should be added to the Lagrangian. Let us specifically look at the correction with $\mathcal{O}_\delta = V_0 \phi^{\delta-4}$,

$$\Delta V \supset c V_0 \frac{\phi^{\delta-4}}{\Lambda^{\delta-4}} \,. \tag{1.2.19}$$

This term induces a significant change in the slow-roll parameter η:

$$\Delta\eta \simeq c(\delta-4)(\delta-5) \left(\frac{M_p}{\Lambda}\right)^2 \left(\frac{\phi}{\Lambda}\right)^{\delta-6} \,. \tag{1.2.20}$$

We see that the dimension-six operator yields $\Delta\eta = \mathcal{O}(1)$ even for $\Lambda = M_p$, because c is in general of order unity. Hence, this term prohibits successful slow-roll inflation. Terms with $\delta \gg 6$ are, in general, not dangerous for small-field inflation with $\Delta\phi < \Lambda \lesssim M_p$, because then $(\phi/\Lambda)^{\delta-6} \ll 1$. In contrast, for large-field inflation we have $\Delta\phi > M_p$ and, hence, arbitrarily many terms contribute to η during inflation.

The options to evade this severe problem are as follows: for small-field inflation there is in principle the option of making the Wilson coefficients sufficiently tiny by fine-tuning. However, this is rather unsatisfactory. More appealing is to assume a shift symmetry $\phi \to \phi + \text{const}$ in the UV-theory, which clearly implies an exactly flat potential. To obtain an appropriately small potential the shift symmetry must be broken by a small effect. Then the inflaton mass and the Wilson coefficients of dangerous terms are tiny due to the approximate shift symmetry. The inflation model is then said to be *technically natural*. In the case of large-field inflation such a symmetry is in fact mandatory since fine-tuning of infinitely many Wilson coefficients is not an option.

The origin of such a symmetry is expected to be understandable within a UV-complete theory of gravity, such as string theory. Indeed, string theory allows for a top-down approach towards inflationary models. If the idea of a string theory landscape within a much vaster swampland [21] is taken seriously, the construction of EFTs of inflation via bottom-up approaches can easily lead to swampland models. To give a concrete example for this statement, let us consider an axion θ, whose global shift symmetry $\theta \to \theta + c$ is non-perturbatively broken by a term $(\theta/f)F \wedge F$, where F is the 2-form field strength of some gauge theory and f is the axion decay constant. The shift symmetry is broken to a discrete shift symmetry $\theta \to \theta + 2\pi f$ and, thus, a periodic potential $V(\theta) \sim \cos(\theta/f)$ is generated. This model of *natural inflation* requires $f \gg M_p$ to be consistent with recent Planck data [2]. However, string theory provides many arguments [20; 40; 41] why super-Planckian axion decay constants are hard to realise. Hence, at this level natural inflation seems to be part of the swampland rather than the landscape. We explain this in more detail in Section 1.4.

However, this is not the end of inflationary model-building using axions. For instance, it is possible to construct inflation models (at least in EFT) with two or more axions, all having a sub-Planckian axion decay constant, but the "effective axion decay constant" is super-Planckian (see e.g. [42; 43]). Furthermore, the idea of axion monodromy inflation [44; 45] has been developed, where the axionic shift symmetry is weakly broken so that not even a discrete shift symmetry remains. In this way, a monomial potential term is generated. The size of f is then no longer important.

All the just mentioned models of axion inflation predict sizeable tensor-to-scalar ratios $r \gtrsim 0.01$ and can thus be tested in the near future. For this reason, and additionally with the hope to obtain a better understanding of the string theory landscape including all its constraints it may impose on EFTs, we find it interesting to study such inflationary scenarios in string theory.[10]

1.3. Basics of Moduli Stabilisation in Type IIB String Theory

We have argued that it is worthwhile studying inflation in string theory. In this thesis, we focus on the realisation of inflation in type IIB string theory, for which well-known methods for moduli stabilisation exist [5–7]. The origin of the string landscape, which is a core element

[10] A review of inflationary scenarios in string theory, both small- and large-field inflation models, can be found in [32].

of this thesis, is directly linked to moduli stabilisation: starting from the low-energy limit of type IIB string theory, we use bulk fluxes to freeze certain moduli, arising from a given Calabi-Yau manifold, at the SUSY minimum. Typically, a huge set of flux numbers is admissible to obtain a SUSY minimum. Every consistent choice of flux vectors then corresponds to a SUSY minimum. This is the origin of the string landscape (often also called flux landscape).

This section aims to briefly review the notation of type IIB string theory/supergravity and to outline the basics of moduli stabilisation. Everything what follows is textbook material and can be found e.g. in [14–18; 32; 46; 47], which are the main sources for this section.

The problem of moduli stabilisation arises in any theory with extra-dimensions. Given a Lagrangian of a d-dimensional manifold $\mathcal{M}_d$ with $d > 4$ and $\mathcal{M}_d = \mathcal{M}_4 \times \mathcal{M}_{d-4}$, our goal is to obtain an effective four-dimensional Lagrangian on $\mathcal{M}_4$. Upon dimensional reduction, metric moduli turn out to be typically massless, unless there is a mechanism (i.e. moduli stabilisation) yielding a potential for these moduli. Phenomenologically, massless scalar fields are, however, a problem. For instance, a modulus ϕ would couple gravitationally to ordinary matter, inducing a fifth force and thus modifying the $1/r^2$-behaviour of the gravitational force below length scales m_ϕ^{-1} (see also [48] for a review of the arguments). This contradicts experiments in the submillimetre regime (see e.g. [49]). Hence, moduli must not be massless (or they must be completely absent).

Independently of this issue, light moduli go along with further problems. If a modulus couples only gravitationally (this is the case for moduli in string compactifications), the decay rate is $\Gamma \sim m_\phi^3/M_p^2$. Hence, very light moduli have highly suppressed decay rates so that the moduli would have not decayed by today. In consequence these moduli would overclose the universe. Making the modulus slightly heavier will lead to a decay during or after Big Bang Nucleosythesis (BBN), resulting in deviations from the successful BBN predictions of light element abundances [50]. If, however, $m_\phi > \mathcal{O}(10)$ TeV, this so-called *cosmological moduli problem* [51–55] can be avoided. Further problems with massless moduli are described in [16].

Before heading towards moduli stabilisation in string theory, we want to demonstrate the origin of massless moduli in a simple toy example.

1.3.1. Toy Model: 6D Einstein-Maxwell Theory

Let us understand moduli stabilisation in six-dimensional Einstein-Maxwell theory (see also [56] and [14; 16] for reviews):

$$S_{6d} \sim \int d^6x \sqrt{-G} \left[R[G_{MN}] - F_{MN}F^{MN} \right] \tag{1.3.1}$$

on $\mathcal{M}_6 = \mathcal{M}_4 \times \mathcal{M}_2$ with metric G_{MN} and compact manifold $\mathcal{M}_2$. Capital indices run from $M, N = 0, ..., 5$, while Greek indices and lower-case indices run from $\mu, \nu = 0, ..., 3$ and $i, j = 4, 5$, respectively. $R[G_{MN}]$ is the Ricci scalar of the 6d metric and F_{MN} are the indices of the 2-form field strength F. Note that for the discussion of this toy-model we do not take into account numerical factors. We also set the fundamental scale $M_6 = 1$.

For the metric ansatz on $\mathcal{M}_6$ we choose

$$ds^2 = G_{MN}dx^M dx^N = g^{(4)}_{\mu\nu}dx^\mu dx^\nu + r^2(x)\tilde{g}^{(2)}_{ij}dy^i dy^j \ , \tag{1.3.2}$$

where $g^{(4)}_{\mu\nu}$ is the (Lorentzian) metric on $\mathcal{M}_4$ and $\tilde{g}^{(2)}_{ij}$ the metric on $\mathcal{M}_2$ normalised such that

$$\int_{\mathcal{M}_2} d^2y \sqrt{\tilde{g}^{(2)}} = 1 \ . \tag{1.3.3}$$

Our goal is to dimensionally reduce (1.3.1) to a 4d action S_{4d}. Let us first consider pure Einstein-Hilbert gravity in 6d, i.e. $F = 0$. We will show that $r = r(x)$ is to be interpreted as a scalar field in the 4d effective theory and that its potential $V(r)$ shows a runaway behaviour.

We can write

$$\begin{aligned} S_{6d;\mathrm{EH}} &\sim \int d^6x\sqrt{-G}R[G_{MN}] \sim \\ &\sim \int_{\mathcal{M}_4} d^4x\sqrt{-g^{(4)}}r^2(x)\left[R[g^{(4)}_{\mu\nu}]\int_{\mathcal{M}_2} d^2y\sqrt{\tilde{g}^{(2)}} + \frac{1}{r^2(x)}\int_{\mathcal{M}_2} d^2y\sqrt{\tilde{g}^{(2)}}R[\tilde{g}^{(2)}_{ij}]\right] + \ldots \\ &\sim \int_{\mathcal{M}_4} d^4x\sqrt{-g^{(4)}}r^2(x)\left[R[g^{(4)}_{\mu\nu}] + \frac{\chi(\mathcal{M}_2)}{r^2(x)}\right] + \ldots\ , \end{aligned} \tag{1.3.4}$$

where in the last line we used (1.3.3) and a standard formula for the Euler character $\chi(\mathcal{M}_2)$ of a 2d manifold $\mathcal{M}_2$. The ellipses stand for gradient terms of $r(x)$. This result is to be converted to Einstein frame by a rescaling $h_{\mu\nu} \equiv r^2(x)g^{(4)}_{\mu\nu}$, yielding

$$S_{\mathrm{EH},6d\to 4d} \sim \int_{\mathcal{M}_4} d^4x\sqrt{-h}\,[R[h_{\mu\nu}] - V(r)] + \ldots \tag{1.3.5}$$

with potential

$$V_{\mathrm{EH}}(r) \sim -\frac{\chi(\mathcal{M}_2)}{r^4(x)} = \frac{2g-2}{r^4(x)}\ , \tag{1.3.6}$$

with genus g of M_2. For $g = 0$ (sphere) we observe a runaway behaviour towards $r \to 0$, i.e. the two extra-dimensions collapse. For $g = 1$ (torus T^2) the potential vanishes identically and the modulus r is not fixed. For $g > 1$ we find that r is driven to $r \to \infty$ and thus the extra-dimensions decompactify. Thus, for $F = 0$ the scalar field r and hence the volume of the extra-dimensions is not stable. This result exemplifies the aforementioned problem of massless moduli arising in compactifications of higher-dimensional theories.

This is different for $F \neq 0$. Let us therefore turn on magnetic flux $F = (1/2)F_{ij}dy^i \wedge dy^j$. Assuming $dF = 0$ it holds

$$\int_{\mathcal{M}_2} F = n \in \mathbb{Z} \tag{1.3.7}$$

due to compactness of $\mathcal{M}_2$. (See also Appendix C.2 for a review of Dirac quantisation for a 3-form flux.) We then have

$$\begin{aligned} S_{6d;\mathrm{M}} &\sim -\int d^6x\sqrt{-G}G^{MP}G^{NQ}F_{MN}F_{PQ} \sim \\ &\sim -\int_{\mathcal{M}_4} d^4x\sqrt{-g^{(4)}}r^2(x)\int_{\mathcal{M}_2} d^2y\sqrt{\tilde{g}^{(2)}}\frac{1}{r^2(x)}\tilde{g}^{ik}\frac{1}{r^2(x)}\tilde{g}^{jl}F_{ij}F_{kl} \sim \\ &\sim -\int_{\mathcal{M}_4} d^4x\sqrt{-g^{(4)}}\frac{n^2}{r^2(x)} \sim -\int_{\mathcal{M}_4} d^4x\sqrt{-h}\frac{n^2}{r^6(x)}\ , \end{aligned} \tag{1.3.8}$$

where in the last step we used Weyl-rescaling $h_{\mu\nu} \equiv r^2(x)g^{(4)}_{\mu\nu}$ from above.

Consequently, the full 4d effective potential for r in our toy-model is

$$V_{\mathrm{EH+M}}(r) \sim \frac{2g-2}{r^4(x)} + \frac{n^2}{r^6} \tag{1.3.9}$$

up to normalisation factors. The second term, induced by non-vanishing fluxes, can now be used to stabilise r, although only for $g = 0$. In this case, i.e. $\mathcal{M}_2 = S^2$, the potential has a minimum and thus, runaway behaviour is avoided. For $g \geq 1$ we find, however, runaway behaviour towards $r \to \infty$. This simple toy-model demonstrates why in dimensionally reduced theories we often find unstable moduli, unless one can generate additional terms, e.g. by turning on fluxes, such that the potential can have a minimum. In string theory, there will be further positive contributions to $V_{\rm EH+M}(r)$ from D-branes and negative contributions from O-planes (orientifold planes).

1.3.2. Type IIB String Theory/Supergravity

In this subsection we summarise the basic ingredients of type IIB string theory/supergravity and introduce the common notation. The main reference for this subsection is [46].

Bosonic Type IIB Supergravity Action

For our purpose of deriving effective Lagrangians for cosmology from type IIB string theory, it is not necessary to take into account the whole string spectrum. Instead, it usually suffices to take the low-energy limit of type IIB string theory and only work with the massless modes. This limit is called type IIB supergravity.[11]

The massless bosonic field content of type IIB string theory is as follows [46]:

- In the NS-NS-sector ("NS" for "Neveu-Schwarz") we have the massless fields G_{MN}, B_{MN}, Φ (also in type IIA string theory). G_{MN} is the ten-dimensional metric, B_{MN} are the components of the NS-NS 2-form B_2, and Φ is the dilaton.

- In the R-R-sector ("R" for "Ramond") the massless states are C_p-forms with $p = 0, 2, 4$. (In type IIA we have $p = 1, 3, 5$.)[12]

The bosonic part of the type IIB supergravity action is given by [46]:

$$S_{\rm IIB} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left[e^{-2\Phi} \left(R[G_{MN}] + 4(\nabla\Phi)^2 - \frac{1}{2}|H_3|^2 \right) - \frac{1}{2}|F_1|^2 - \right. \\ \left. - \frac{1}{2}|\tilde{F}_3|^2 - \frac{1}{4}|\tilde{F}_5|^2 \right] - \frac{1}{4\kappa_{10}^2} \int C_4 \wedge H_3 \wedge \tilde{F}_3 \ . \tag{1.3.10}$$

In this equation, κ_{10} is the ten-dimensional gravitational coupling constant defined by $\kappa_{10}^2 = (4\pi^2\alpha')^4/(4\pi)$, where α' sets the string length $\ell_s \equiv 2\pi\sqrt{\alpha'}$ and the string mass $M_s \equiv 1/\ell_s$. Higher-order corrections in α' are neglected as we are interested in energy scales well below M_s. The differential forms $H_3, F_1, \tilde{F}_3$ and $\tilde{F}_5$ are defined as follows:

$$H_3 \equiv dB_2 \ , \tag{1.3.11}$$

$$F_1 \equiv dC_0 \ , \tag{1.3.12}$$

$$\tilde{F}_3 \equiv dC_2 - C_0 dB_2 \equiv F_3 - C_0 H_3 \ , \tag{1.3.13}$$

$$\tilde{F}_5 \equiv dC_4 - \frac{1}{2}C_2 \wedge dB_2 + \frac{1}{2}B_2 \wedge dC_2 \equiv F_5 - \frac{1}{2}C_2 \wedge H_3 + \frac{1}{2}B_2 \wedge F_3 \ , \tag{1.3.14}$$

[11] Note that in the literature "type IIB string theory" and "type IIB supergravity" are often used synonymously.

[12] Sometimes it is useful to switch to the "democratic formulation" of type IIB string theory. This requires additionally C_6 and C_8 as the duals to C_2 and C_0, respectively. The duality is constructed as follows: the kinetic term of a p-form gauge field C_p in d dimensions is given by $\int dC_p \wedge \star_d dC_p$. For the dual $\tilde{C}_q$ to C_p it holds $d\tilde{C}_q = \star_d dC_p$. Hence $q = d - p - 2$.

i.e. we also defined $F_3 \equiv dC_2$ and $F_5 \equiv dC_4$ (this is the usual definition of field strength tensors). We use the notation $|F_q|^2 = (1/q!)F_{M_1\ldots M_q}F^{M_1\ldots M_q}$. (For further details and conventions related to differential forms, see Appendix A.1.) Note that on top of (1.3.10) one has to impose the self-duality condition $\tilde{F}_5 = \star\tilde{F}_5$. Unfortunately, this cannot be done at the level of the action, because this self-duality condition would immediately imply $\int \tilde{F}_5 \wedge \star\tilde{F}_5 = 0$. Instead, the self-duality condition is to be imposed in addition to the equations of motion derived from (1.3.10).

It is convenient to express (1.3.10) in Einstein frame by a rescaling $G^{\mathrm{E}}_{MN} = e^{-\Phi/2}G_{MN}$. By defining

$$S \equiv C_0 + ie^{-\Phi} \ , \qquad G_3 \equiv \tilde{F}_3 - ie^{-\Phi}H_3 = F_3 - SH_3 \ , \tag{1.3.15}$$

we obtain (dropping the label E)

$$\begin{aligned} S_{\mathrm{IIB}} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G}\left[R - \frac{\partial_M S\partial^M\bar{S}}{2(\mathrm{Im}(S))^2} - \frac{1}{2}\frac{|G_3|^2}{\mathrm{Im}(S)} - \frac{1}{4}|\tilde{F}_5|^2\right] + \\ + \frac{1}{8i\kappa_{10}^2}\int \frac{1}{\mathrm{Im}(S)}C_4 \wedge G_3 \wedge \bar{G}_3 \ , \end{aligned} \tag{1.3.16}$$

where $\bar{G}_3$ is the complex-conjugate of G_3. We also define $|G_3|^2 = (1/3!)G_{MNP}\bar{G}^{MNP}$. The field S is called axio-dilaton.[13] The dilatonic VEV (vacuum expectation value) $\langle\Phi\rangle$ fixes the string coupling $g_s \equiv e^{\langle\Phi\rangle}$.

The type IIB supergravity action (1.3.16) is the starting point of type IIB flux compactification in analogy to the previously discussed 6D Einstein-Maxwell toy-model (1.3.1). However, there will be important contributions to (1.3.16) from localised objects such as D-branes and O-planes. Furthermore it is crucial to understand which compactification geometries with metric $\tilde{g}_{mn}$ are admissible. We go through these important points step by step.

D-Branes

Following [32], we only want to collect the most crucial facts about D-branes. Those are objects charged under the gauge symmetry of the R-R-fields C_{p+1}. More precisely, C_{p+1} is sourced by a p-dimensional object, called *Dp-brane*. The coupling term is given by the Chern-Simons term

$$S_{\mathrm{CS}} = \mu_p \int_{\Sigma_{p+1}} C_{p+1} \ , \tag{1.3.17}$$

where μ_p is the electric coupling and Σ_{p+1} the worldvolume of the Dp-brane. The idea is identical to classical electrodynamics, where a point-particle, a D0-brane, sources the gauge-field A_1. The coupling term is then given by $q\int_{\Sigma_1} A_1$, where q is the electric charge and Σ_1 the worldline described by the particle.

In string theory, open strings can end on a D-brane. One needs to impose Dirichlet boundary conditions on the transversal directions of the D-brane (this is where the letter "D" in "D-brane" comes from) together with Neumann boundary conditions along the spatial directions of the brane. Hence, endpoints of an open string can move along the D-brane, but they are fixed to this object.

One can quantise these open strings with endpoints on a D-brane. The massless spectrum consists of scalar fields parametrising the fluctuations of the D-brane position, a gauge field

[13] Be aware that in the literature such as [46] the axio-dilaton field S is often labeled by τ. We choose the symbol S, which we have also used in most of our papers this thesis is based on.

A_a living on the worldvolume of the D-brane, and their superpartners [32]. The effective action governing the dynamics of the massless field on a Dp-brane is given by

$$S_{\text{DBI}} = -g_s T_p \int d^{p+1}\sigma \sqrt{-\det\left(G_{ab} + \mathcal{F}_{ab}\right)} \ , \tag{1.3.18}$$

where G_{ab} is the pull-back of G_{MN} onto the Dp-brane worldvolume parametrised by $\sigma^0, ..., \sigma^p$:

$$G_{ab} = \frac{\partial X^M(\sigma)}{\partial \sigma^a} \frac{\partial X^N(\sigma)}{\partial \sigma^b} G_{MN} \ , \tag{1.3.19}$$

where X^M are the coordinates of the target space. Furthermore, $\mathcal{F}_{ab}$ is defined by

$$\mathcal{F}_{ab} \equiv B_{ab} + 2\pi\alpha' F_{ab} \ . \tag{1.3.20}$$

Therein, B_{ab} is again the pull-back of B_{MN} and $F_{ab} = 2\partial_{[a}A_{b]}$ the field strength tensor. The motivation for the definition of $\mathcal{F}_{ab}$ is to construct a gauge-invariant quantity. Finally, g_s is again the string coupling and T_p the brane-tension:

$$T_p = \frac{1}{(2\pi)^p g_s (\alpha')^{(p+1)/2}} \ . \tag{1.3.21}$$

We call (1.3.18) the Dirac-Born-Infeld (DBI)-action, because it combines the Born-Infeld action of non-linear electrodynamics together with the Dirac action for a Dp-brane (for more details, see [32]).

Finally, the CS-action (1.3.17) for a Dp-brane in the presence of background fields is modified to

$$S_{\text{CS}} = \mu_p \int_{\Sigma_{p+1}} \sum_n C_n \wedge e^{\mathcal{F}} \Big|_{(p+1)\text{-forms}} \ , \tag{1.3.22}$$

where the sum goes over all R-R-forms of the theory, and only the $(p+1)$-forms of this expression are to be integrated.

The action of a Dp-brane is then given by

$$S_{\text{D}p} = S_{\text{DBI}} + S_{\text{CS}} \ . \tag{1.3.23}$$

Finally, we remark that D-branes are also of interest in the context of higher-dimensional black hole physics. For instance, Dp-branes with $p > 0$, for which $\mu_p = g_s T_p$, are higher-dimensional generalisations of extremal black holes. Such Dp-branes are called BPS-branes, and are stable objects. For more details, see e.g. [57].

Before discussing O-planes, it is necessary to say some words about the geometry of the compact manifold X_6 when compactifying (1.3.16) to four dimensions.

Calabi-Yau Manifolds and Geometric Moduli

We will now introduce the basic language of geometry required to talk about compactifications from ten-dimensional type IIB supergravity (SUGRA) to four dimensions. The main reference is [32; 46].

First of all, the 10d type IIB SUGRA action (1.3.16) enjoys $\mathcal{N} = 2$ supersymmetry. However, solutions to the equations of motion do not have to preserve supersymmetry. For various reasons [32] mostly supersymmetric compactifications were studied. One motivation has been to address the hierarchy problem via $\mathcal{N} = 1$ supersymmetry broken near the electroweak scale rather than the compactification scale. Preserving supersymmetry in four dimensions puts constraints on the type of geometry for X_6 to be used. Choosing X_6 to be a so-called *Calabi-Yau 3-fold*, one still arrives at $\mathcal{N} = 2$ supersymmetry theory in 4d. By orientifold projections, which we discuss later, one can finally obtain $\mathcal{N} = 1$ SUSY. Hence, Calabi-Yau manifolds are frequently used as compactification geometries in type II string theories.

Calabi-Yau 3-folds are Ricci-flat *Kähler manifolds* of complex dimension three. Kähler manifolds themselves are defined as follows: A manifold, equipped with a Hermitian metric (i.e. vanishing components $g_{ab} = 0 = g_{\bar{a}\bar{b}}$ and the reality condition $g_{a\bar{b}} = \overline{g_{b\bar{a}}}$), is called Kähler manifold if $dJ = 0$, where

$$J \equiv i g_{a\bar{b}} dz^a \wedge d\bar{z}^b \tag{1.3.24}$$

is called the *Kähler form*. For more details on complex manifolds, Kähler manifolds and Calabi-Yau geometries, see Appendix A.3.

From the Ricci-flatness it follows that Calabi-Yau geometries are automatically solutions to the vacuum Einstein equations, i.e. to the equations of motion.

Moreover, using the Ricci-flatness, we can classify metric deformations into *Kähler moduli* and *complex structure moduli*. From the requirement that deformations of the metric should preserve Ricci-flatness,

$$R_{mn}(g + \delta g) = 0 \ , \tag{1.3.25}$$

one can show that metric fluctuations of the types $\delta g_{a\bar{b}}$ and $\delta g_{\bar{a}\bar{b}}$ decouple, i.e. they can be treated independently [15; 32; 46]. The former can be associated with $(1,1)$-forms

$$\delta J \equiv i \delta g_{a\bar{b}} dz^a \wedge d\bar{z}^{\bar{b}} \in H^{1,1}_{\bar{\partial}}(X_6, \mathbb{C}) \ , \tag{1.3.26}$$

and the latter with $(2,1)$-forms

$$\chi \equiv \Omega_{abc} g^{c\bar{d}} \delta g_{\bar{d}\bar{e}} dz^a \wedge dz^b \wedge d\bar{z}^{\bar{e}} \in H^{2,1}_{\bar{\partial}}(X_6, \mathbb{C}) \tag{1.3.27}$$

via the unique holomorphic $(3,0)$-form Ω on the Calabi-Yau 3-fold X_6 (see [15; 46]). The complex forms δJ and χ are harmonic and thus representatives of the Dolbeault cohomology classes $H^{1,1}_{\bar{\partial}}(X_6, \mathbb{C})$ and $H^{2,1}_{\bar{\partial}}(X_6, \mathbb{C})$, respectively. This leads directly to the definition of Kähler- and complex structure moduli.

We express the Kähler form J in a basis of harmonic $(1,1)$-forms $\omega^I \in H^{1,1}_{\bar{\partial}}(X_6, \mathbb{C})$, where $I = 1, ..., h^{1,1} \equiv \dim H^{1,1}_{\bar{\partial}}(X_6, \mathbb{C})$:

$$J = \sum_{I=1}^{h^{1,1}} t^I(x) \omega_I \ . \tag{1.3.28}$$

We call the $h^{1,1}$ real-valued moduli $t^I(x)$, which are functions of the 4d spacetime coordinates, *Kähler moduli*.

In analogy we can use a harmonic $(2,1)$-basis $\chi_A \in H^{2,1}_{\bar{\partial}}(X_6, \mathbb{C})$ with $A = 1, ..., h^{2,1} \equiv \dim H^{2,1}_{\bar{\partial}}(X_6, \mathbb{C})$ to express:

$$\Omega_{abc} g^{c\bar{d}} \delta g_{\bar{d}\bar{e}} = \sum_{A=1}^{h^{2,1}} u^A(x) (\chi_A)_{ab\bar{e}} \ , \tag{1.3.29}$$

where $u^A(x)$ are complex scalar fields in 4d spacetime. We call these $h^{2,1}$ fields the *complex structure moduli*.[14]

[14] In later parts of the thesis we will denote the complex structure moduli mostly by the variable z. Here, we refrain from doing so in order to avoid confusion with the labeling of our complex coordinates z.

Hence, given a Calabi-Yau 3-fold X_6 the dimensions of the Dolbeault cohomology classes $H^{1,1}_{\bar{\partial}}(X_6,\mathbb{C})$ and $H^{2,1}_{\bar{\partial}}(X_6,\mathbb{C})$ tell us that there are $h^{1,1}$ and $h^{2,1}$ Kähler and complex structure moduli arising in the 4d field theory after Calabi-Yau compactification.

On top of that there arise also scalar fields from the type IIB gauge potentials B_2, C_2 and C_4. Following [32] we expand these differential forms in a basis of corresponding harmonic forms:

$$B_2 = B_2(x) + \sum_{I=1}^{h^{1,1}} b^I(x)\omega_I \ , \tag{1.3.30}$$

$$C_2 = C_2(x) + \sum_{I=1}^{h^{1,1}} c^I(x)\omega_I \ , \tag{1.3.31}$$

$$C_4 = \sum_{I=1}^{h^{1,1}} \vartheta^I(x)\tilde{\omega}_I \ , \tag{1.3.32}$$

where $B_2(x) = (1/2)B_{\mu\nu}dx^\mu \wedge dx^\nu$ and $C_2(x) = (1/2)C_{\mu\nu}dx^\mu \wedge dx^\nu$ are the 4d space-time parts of the 10d 2-form fields B_2 and C_2. A similar, though much more complicated expression can also be included (see [58]) in the last equation, but we dropped these terms, because they are unimportant for our purposes. Note that ω_I denote again a harmonic basis of $H^{1,1}_{\bar{\partial}}(X_6,\mathbb{C})$. Since C_4 is a 4-form, we must introduce a harmonic basis $\tilde{\omega}_I$ of $H^{2,2}_{\bar{\partial}}(X_6,\mathbb{C})$. Its dimension is still $h^{1,1}$ because of the the relation $h^{2,2} = h^{1,1}$ on a CY 3-fold due to Poincaré duality (see Proposition A.24 of Appendix A.2).

Finally, the dilaton Φ and the zero-form C_0 (or in combination the previously defined axio-dilaton field S) also yield two scalars in the 4d theory.

Since the above field content obtained from CY-compactifications of type IIB supergravity is also part of the multiplets of $\mathcal{N} = 2$ supersymmetry in four dimensions, our 4d effective theory is indeed $\mathcal{N} = 2$ supersymmetric.

However, from a phenomenological point of view it is desirable to have $\mathcal{N} = 1$ supersymmetry in 4d rather than $\mathcal{N} = 2$, because the latter, for instance, forbids chiral fermions [32], which we need to describe the Standard Model of particle physics.

A good way to arrive at $\mathcal{N} = 1$ supersymmetry in 4d from type IIB string theory is by performing an *orientifold projection* of a type IIB Calabi-Yau compactification. The aforementioned *O-planes* as additional local sources contributing to (1.3.16) are an automatic consequence of this projection, which we want to describe now. We follow again [32; 46].

Orientifold Action and Orientifold-Planes

We restrict our description of orientifold actions $\mathcal{O}$ to the ones of the form

$$\mathcal{O} = (-1)^{F_L}\Omega_{\text{ws}}\sigma \ , \tag{1.3.33}$$

where Ω_{ws} is the (string-)worldsheet orientation reversal and F_L is the worldsheet fermion number in the left-moving sector (for details on the superstring see e.g. [15; 46]). The map $\sigma : \mathbb{R}^{1,3} \times X_6 \to \mathbb{R}^{1,3} \times X_6$ is an involution, i.e. $\sigma^2 = \text{id}$, and acts trivially on $\mathbb{R}^{1,3}$, but non-trivially on the Calabi-Yau manifold X_6 as follows: σ changes the sign of the holomorphic (3,0)-form Ω, i.e. $\sigma^*(\Omega) = -\Omega$ (σ^* denotes the pull-back of σ)[15], but it leaves the metric and complex structure invariant, $\sigma^*(J) = J$.

[15]It is also possible to have $\sigma^*(\Omega) = +\Omega$, but we do not consider this case any further.

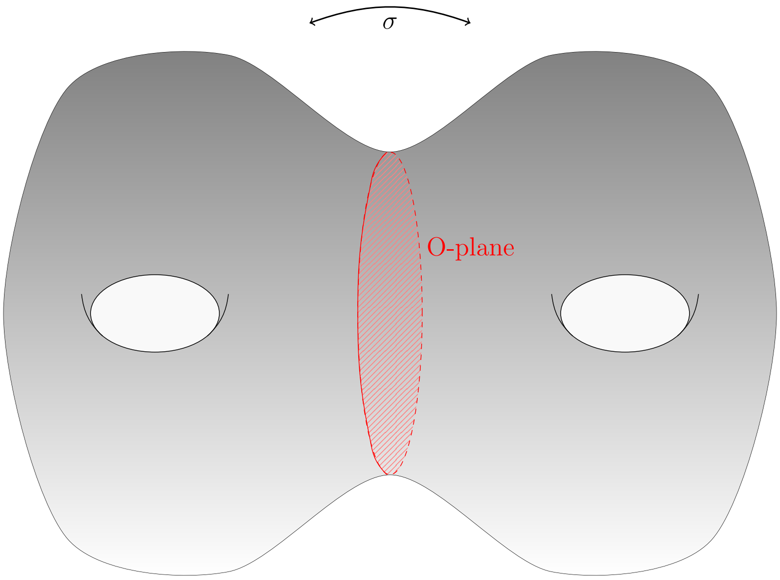


Figure 1.2.: This simplified picture shows a CY manifold. After orientifold projection there arises an O-plane depicted in red. It is defined by the set of all loci invariant under the involution σ. This picture is inspired from the lectures "Advanced String Theory" by Timo Weigand, summer term 2016, at Heidelberg University [59].

Orientfolds planes (O-planes) are precisely the fixed points of $\mathcal{O}$ (see Figure 1.2). Since σ acts trivially on $\mathbb{R}^{1,3}$, O-planes have at least three spatial dimensions. Due to $\sigma^*(\Omega) = -\Omega$, O-planes can fill either no or four further spatial dimensions in X_6. Hence, the O-planes we are considering have either three or seven spatial dimensional and we denote them as O3- and O7-planes, respectively.[16] As opposed to D-branes, these O-planes are non-dynamical objects, but they are also charged under R-R-gauge fields.

Upon the orientifold action $\mathcal{O}$ of (1.3.33) one finds that the Dolbeault cohomology class $H^{1,1}$ decomposes into two subspaces:

$$H^{1,1} = H^{1,1}_+ \oplus H^{1,1}_- \ . \tag{1.3.34}$$

All $(1,1)$-forms which are invariant under $\mathcal{O}$ belong to the subspace $H^{1,1}_+$ and correspondingly all $(1,1)$-forms, which obtain a minus sign are elements of $H^{1,1}_-$. In consequence, the basis ω^I of $H^{1,1}$ decomposes into $\omega^i \in H^{1,1}_+$ with $i = 1, ..., h^{1,1}_+$ and $\omega^\alpha \in H^{1,1}_+$ with $\alpha = 1, ..., h^{1,1}_-$. Likewise, all other cohomology classes are decomposed into a direct sum of even and odd subspaces: $H^{p,q} = H^{p,q}_+ \oplus H^{p,q}_-$.

From type IIB string theory and the definition of σ one obtains the behaviour under (1.3.33) of the 4d fields $t^I, u_A, b^I, c^I, \vartheta^I$, $B_2(x)$, $C_2(x)$, C_0, Φ, which we introduced previously. To do so, one has to analyse the behaviour under $(-1)^{F_L}\Omega_{\rm ws}$ separately, see [60]. One then finds that the even fields under $\mathcal{O}$ are $t^I, \vartheta^I, C_0, \Phi$. The remaining fields $u_A, b^I, c^I, B_2(x)$ and $C_2(x)$ are odd under the orientifold action [32]. Since $t^I(x)$ is even and $\sigma^*(J) = J$, the expansion of

[16] If we had chosen $\sigma^*(\Omega) = +\Omega$, one would obtain O5- and O9-planes.

J under orientifolding can only contain ω^i:

$$J = \sum_{i=1}^{h_+^{1,1}} t^i(x)\omega_i \ . \tag{1.3.35}$$

Likewise, B_2, C_2 and C_4 are expanded in terms of $b^\alpha(x)$, $c^\alpha(x)$, $\vartheta^i(x)$, respectively. Furthermore, the complex structure moduli $u^a(x)$ with $a = 1, ..., h_-^{2,1}$ remain in the orientifolded theory. The fields Φ and C_0 are already invariant under (1.3.33).

These invariant scalars now have to be combined to obtain the bosonic components of chiral multiplets of $\mathcal{N} = 1$ supersymmetry in 4d. One defines

$$S \equiv C_0 + ie^{-\Phi} \tag{1.3.36}$$

as the axio-dilaton and furthermore one combines b^α and c^α into

$$G_\alpha \equiv c_\alpha - Sb_\alpha \ . \tag{1.3.37}$$

Moreover, the volume $\mathcal{V}$ of X_6 is given by

$$\mathcal{V} = \frac{1}{6}\int_{X_6} J \wedge J \wedge J = \frac{1}{6}k_{ijk}t^i t^j t^k \ , \tag{1.3.38}$$

where (1.3.35) and the definition of the triple intersection numbers

$$k_{ijk} \equiv \int_{X_6} \omega_i \wedge \omega_j \wedge \omega_k \tag{1.3.39}$$

are used. The Kähler moduli t_i describe the 2-cycle volume. The 4-cycle volume τ_i (which we will be using mostly in the thesis) is obtained from

$$\tau_i = \frac{\partial \mathcal{V}}{\partial t_i} = \frac{1}{2}k_{ijk}t^j t^k \ . \tag{1.3.40}$$

The complexified 4-cycle volumes are given by [58; 60]:

$$T_i \equiv \frac{1}{2}k_{ijk}t^j t^k + i\vartheta_i + \frac{1}{4}e^\Phi k_{i\alpha\beta}G^\alpha(G - \bar{G})^\beta \ , \tag{1.3.41}$$

and very often the last term is omitted for practical purposes and we write:

$$T_i = \tau_i + i\vartheta_i \ . \tag{1.3.42}$$

The imaginary part ϑ_i is then given by the integral of C_4 over the corresponding 4-cycle Σ_4^i:

$$\vartheta_i = \int_{\Sigma_4^i} C_4 \ . \tag{1.3.43}$$

This field ϑ_i is an example of an *axion* arising in string compactifications from integrating gauge-potentials over corresponding cycles. Similarly, b_α and c_α can be identified as axions obtained in this way by integrating B_2 and C_2 over 2-cycles, respectively.

We conclude this digression by summarising the 4d field content after applying an orientifold projection on a type IIB CY-compactification. There are $h_+^{1,1}$ 4-cycle Kähler moduli T_i and $h_-^{2,1}$ complex structure moduli u^a. Furthermore, we have $h_-^{1,1}$ scalars G_α descending from 2-form fields. Last, we have the axio-dilaton S. In total we have $h_+^{1,1} + h_-^{2,1} + h_-^{1,1} + 1$ fields in the 4d theory obtained from a Calabi-Yau compactification after orientifold projection (1.3.33).

Knowing the field content of our $\mathcal{N} = 1$ supersymmetric 4d theory, the next logical step is to obtain an effective 4D Lagrangian obtained from orientifolded type IIB CY-compactifications.

1.3.3. Towards an $\mathcal{N}=1$ SUSY Lagrangian in 4d from Type IIB Compactifications

So far we saw that the choice of a Calabi-Yau geometry X_6 defines the 4d field content obtained from the corresponding type IIB CY-compactification. The effective Lagrangian in 4d is obtained from dimensional reduction of (1.3.16).

In the following we describe several important constraints that arise in the 10d type IIB theory (1.3.16) and state the result of the 4d effective Lagrangian. Much more details can be found in the original paper [5] and in many reviews such as [18; 35; 46], which are the main sources we follow closely.

10d Solutions and Tadpole Constraints

Starting point of this discussion is (1.3.16) and we now also include local sources, such as D3/D7-branes and O3/O7-planes:

$$S_{\text{IIB}} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G}\left[R - \frac{\partial_M S\partial^M \bar{S}}{2(\text{Im}(S))^2} - \frac{1}{2}\frac{|G_3|^2}{\text{Im}(S)} - \frac{1}{4}|\tilde{F}_5|^2\right] + \\ + \frac{1}{8i\kappa_{10}^2}\int \frac{1}{\text{Im}(S)} C_4\wedge G_3\wedge \bar{G}_3 + S_{\text{loc}}\ . \tag{1.3.44}$$

For instance, S_{loc} of a Dp-brane looks like (see (1.3.23) with no background fluxes)

$$S_{\text{loc}} = -T_p\int_{\mathbb{R}^{1,3}\times\Sigma_{p-3}} d^{p+1}\sigma\sqrt{-g} + \mu_p\int_{\mathbb{R}^{1,3}\times\Sigma_{p-3}} C_{p+1} \tag{1.3.45}$$

with Dp-brane tension T_p and charge μ_p. Moreover, Σ_{p-3} is a $(p-3)$-cycle in X_6. Henceforth we describe compactifications with no sources for the fluxes F_3 and H_3. Their Bianchi identities then read:

$$dF_3 = 0\ ,\qquad dH_3 = 0\ . \tag{1.3.46}$$

The fluxes F_3 and H_3 are then quantised (see e.g. Appendix C.2 for details):

$$\frac{1}{2\pi\alpha'}\int_{\Sigma_3^i} F_3 = 2\pi m_i \in 2\pi\mathbb{Z}\ ,\qquad \frac{1}{2\pi\alpha'}\int_{\Sigma_3^i} H_3 = 2\pi n_i \in 2\pi\mathbb{Z}\ , \tag{1.3.47}$$

for 3-cycles Σ_3^i. The factors $2\pi\alpha'$ arise from the string charges: e.g. $\mu_1\int_{\Sigma_3} F_3 \in 2\pi\mathbb{Z}$ with $\mu_1 = (2\pi\alpha')^{-1}$ [18; 46].

Moreover, $\tilde{F}_5$ satisfies the Bianchi identity

$$d\tilde{F}_5 = H_3\wedge F_3 + 2\kappa_{10}^2\mu_3\rho_{3,\text{loc}}\ . \tag{1.3.48}$$

The first term is straightforwardly obtained from (1.3.14) and the second term arises from the inclusion of D3-branes and O3-planes, which contribute to the localised source contribution. Since $\tilde{F}_5$ is self-dual, i.e. we have $\tilde{F}_5 = \star\tilde{F}_5$, the Bianchi identity can also be seen as an equation of motion.

Integrating the Bianchi identity over the compact manifold leads to a global constraint, which is called the *tadpole cancellation condition*. Since $\int_{X_6} d\tilde{F}_5 = 0$ it follows

$$\frac{1}{2\kappa_{10}^2\mu_3}\int_{X_6} H_3\wedge F_3 + Q_{3,\text{loc}} = 0\ , \tag{1.3.49}$$

where $Q_{3,\text{loc}}$ is the total charge of the local objects. In fact this logic is very familiar from electrodynamics on a compact space (see the below aside).

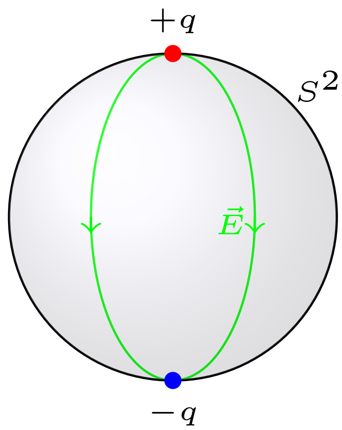
Aside: Electrodynamics on a compact space – Maxwell's equations for a gauge field A_1 in four dimensions are given by

$$d\star F_2 = \star j\ , \qquad dF_2 = 0\ ,$$

where j is the 1-form current of the sources. The second equation is simply the Bianchi identity. If we want to put charged particles on a compact 3-manifold, e.g. S^3, Gauss' law implies for the total charge Q_{tot}:

$$Q_{\text{tot}} = \int_{S^3} \star j = \int_{S^3} d\star F_2 = \oint_{\partial S^3} F_2 = 0\ .$$

Consequently, all charges must add up to zero. In particular, in the 2d case S^2 this is also pictorially clear. Putting a charge $+q$ on S^2 automatically implies to have a charge $(-q)$ at the antipodal point, as the green coloured field lines of the electric field $\vec{E}$ show. Clearly, $Q_{\text{tot}} = (+q) + (-q) = 0$.



For the very same reason the tadpole cancellation condition (1.3.49) arises: the charges of D-branes and O-planes have to cancel due to the compactness of X_6. In the case of D3-branes or O3-planes there is in addition the contributions from F_3 and H_3 to cancel.

Note that there are further contributions to (1.3.49) coming from curvature couplings on wrapped D7-branes, which contribute negative D3-charge $Q_{3,\text{D7}}$ [48]. This effect can be understood in the F-theory language, which uses elliptically fibred Calabi-Yau 4-folds, Y_8 (for a few more comments on F-theory, see Section 1.3.5). It can be shown that

$$Q_{3,\text{D7}} = -\frac{\chi(Y_8)}{24}\ , \tag{1.3.50}$$

where $\chi(Y_8)$ is the Euler character of the CY 4-fold Y_8. It follows

$$\frac{1}{2\kappa_{10}^2\mu_3}\int_{X_6} H_3\wedge F_3 + Q_{3,\text{loc}} = \frac{\chi(Y_8)}{24}\ . \tag{1.3.51}$$

This formula is of great importance. It limits the number of possibilities of choosing flux numbers m_i, n_i defined in (1.3.47). We return to this issue in Section 1.4.

For the 10d metric and $\tilde{F}_5$ the following ansatz is made [18]:

$$ds_{10}^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A(y)}\tilde{g}_{mn}(y)dy^m dy^n\ , \tag{1.3.52}$$

$$\tilde{F}_5 = (1+\star)\, d\alpha(y)\wedge dx^0\wedge dx^1\wedge dx^2\wedge dx^3\ , \tag{1.3.53}$$

where $e^{2A(y)}$ is called *warp factor* and $\alpha = \alpha(y)$ is a function of the extra-dimensional coordinates y^m. By plugging this ansatz into the Bianchi identity (1.3.48) and into Einstein's equations, one can obtain a relation between α and the warp factor. Subtracting the resulting equations from each other yields [5]:

$$\tilde{\nabla}^2(e^{4A}-\alpha) = \frac{e^{2A}}{6\mathrm{Im}(S)}\left|iG_3 - \star_6 G\right|^2 + e^{-6A}\left|\partial(e^{4A}-\alpha)\right|^2 + \\ + 2\kappa_{10}^2 e^{2A}\left[\frac{1}{4}\left(T^m_m - T^\mu_\mu\right)_{\mathrm{loc}} - \mu_3\rho_{3,\mathrm{loc}}\right] . \tag{1.3.54}$$

In this equation $\tilde{\nabla}^2$ is the Laplacian with respect to $\tilde{g}_{mn}$ and $\star_6$ is the Hodge-star operator on the CY 3-fold. Furthermore, T^μ_μ and T^m_m are the traces of the energy-momentum tensor derived from S_{loc} with respect to the 4d and the 6d components of the CY, respectively.

When integrating (1.3.54) over the compact CY-manifold X_6, the LHS vanishes due to Gauss' theorem. On the RHS the first two terms are non-negative. Hence, this equation imposes constraints on the choice of localised sources. We want to restrict to D3/D7-branes and O3-planes, for which the last term on the RHS of (1.3.54) vanishes identically. It then follows:

$$\star_6 G_3 = iG_3 \ , \qquad e^{4A(y)} = \alpha(y) \ . \tag{1.3.55}$$

The first equation is referred to as the ISD (imaginary self-dual)-condition. It fixes the moduli of X_6 which enter the ISD-condition through the Hodge-star operator $\star_6$ [18]. The issue of moduli stabilisation is however most easily studied in the 4d effective theory. Indeed, moduli stabilisation can be studied purely in 4d EFT for the following reason (see also [18]): As we just argued the moduli are stabilised by fluxes (this will be made more precise later), so we expect the moduli masses to be set by the scale of the local flux densities. Using (1.3.47) we expect

$$m_{\mathrm{mod}} \sim \frac{\alpha'}{R^3} \ , \tag{1.3.56}$$

where R is the typical radius of a 1-cycle in the compactification manifold. However, the compactification scale, i.e. the Kaluza-Klein scale, is $m_{\mathrm{KK}} \sim 1/R$. Hence, for rather large R we have $m_{\mathrm{mod}} \ll m_{\mathrm{KK}}$. Consequently, for the analysis of moduli stabilisation it suffices to restrict our attention to the 4d effective Lagrangian.

The Effective 4d Lagrangian

The 4d effective Lagrangian is then obtained by performing the dimensional reduction of the 10d type IIB supergravity action. The kinetic term of the 4d Lagrangian $\mathscr{L}$ is best obtained in the approximation of constant warp factor and vanishing $\tilde{F}_5$, which is justified in the limit of large radii [5]. The potential in the Lagrangian is induced by fluxes and requires the computation of

$$-\frac{1}{24\kappa_{10}^2}\int_{X_6} d^6y\sqrt{-\tilde{g}}\frac{G_{mnp}\bar{G}^{mnp}}{\mathrm{Im(S)}} \ , \tag{1.3.57}$$

where the indices of G_3 are to be raised an lowered with respect to $\tilde{g}_{mn}$. It is again convenient to work in the limit of large radii. We skip details of the rather tedious derivation of the 4d Lagrangian and only state the results. More explanations can be found in the appendix of [5].

The 4d $\mathcal{N}=1$ supergravity Lagrangian is given by

$$\mathscr{L} = -\mathcal{K}_{i\bar{j}}\partial_\mu\phi^i\partial^\mu\bar{\phi}^{\bar{j}} - V_F(\phi^i,\bar{\phi}^{\bar{i}}) \ , \tag{1.3.58}$$

where $\mathcal{K}_{i\bar{j}} = \partial_i\partial_{\bar{j}}\mathcal{K}$ (with $\partial_i \equiv \partial/\partial\phi^i$) is the *Kähler metric* corresponding to the *Kähler potential* $\mathcal{K}$. The potential V_F is the $\mathcal{N}=1$ F-term scalar potential. The field variable ϕ^i

summarises the moduli we introduced previously. In the following we set $M_p = 1$, except where otherwise stated.

At leading order in α' and string loop expansions, the Kähler potential $\mathcal{K}$ is given by [48]:

$$\mathcal{K} = -\ln\left[-i\left(S-\bar{S}\right)\right] - 2\ln\left(\mathcal{V}\right) - \ln\left(i\int_{X_6}\Omega_3\wedge\bar{\Omega}_3\right) \ . \tag{1.3.59}$$

At this level $\mathcal{K}$ consists of three separate sectors: the first is represented by a term containing the axio-dilaton S. The Kähler moduli enter the second term, see (1.3.38). The last term represents the complex structure sector. It can also be expressed in terms of the period vector defined by

$$\Pi^i = \int_{\Sigma_3^i}\Omega_3 \ , \tag{1.3.60}$$

where Σ_3^i is a 3-cycle. Choosing a symplectic basis of 3-cycles, it then holds [48]:

$$\mathcal{K}_{\text{cs}} = -\ln\left(-i\Pi^\dagger\cdot\Sigma\cdot\Pi\right) \ , \tag{1.3.61}$$

where Σ is a $2n\times 2n$-matrix, $n = h_-^{2,1}+1$:[17]

$$\Sigma = \begin{pmatrix} 0 & \mathbb{1}_n \\ -\mathbb{1}_n & 0 \end{pmatrix} \ . \tag{1.3.62}$$

Moreover, Gukov, Vafa and Witten have shown [61] that a potential

$$W = \int_{X_6} G_3\wedge\Omega_3 \tag{1.3.63}$$

is generated, where $G_3 = F_3 - SH_3$ as previously introduced. This result will obtain instantonic corrections, which we ignore for the moment, but become crucial later. Hence, we see that a non-vanishing flux G_3 induces a non-zero superpotential. Using the quantisation of F_3 and H_3 in (1.3.47), the Gukov-Vafa-Witten potential can be expressed as

$$W = (2\pi)^2\alpha'\left(m_i - Sn_i\right)\cdot\Pi^i(u) \ , \tag{1.3.64}$$

where we wrote $\Pi^i = \Pi^i(u)$ to emphasise the dependence on the complex structure moduli.

Knowing the Kähler- and the superpotential we can write down the $\mathcal{N}=1$ F-term scalar potential:

$$\boxed{V_F = e^{\mathcal{K}}\left[\mathcal{K}^{i\bar{j}}D_iW\overline{D_jW} - 3|W|^2\right]} \ , \tag{1.3.65}$$

where $D_i \equiv \partial_i + (\partial_i\mathcal{K})$ is the (Kähler-)covariant derivative. It is one of the most frequently used formulae in this thesis. For one, it is the basis for our construction of large-field inflation models. Furthermore, the best known methods of moduli-stabilisation are based on the F-term potential. The latter is discussed now.

[17] The dimension $2n = 2h_-^{2,1}+2$ is obtained from (1.3.60) using the fact that there are $h_-^{2,1}$ $(2,1)$-cycles and $h_-^{1,2} = h_-^{2,1}$ $(1,2)$-cycles. Moreover, there is one $(3,0)$-cycle and one $(0,3)$-cycle. Due to $h^{p,q} = h^{n-p,n-q}$ for CY n-folds, this gives in total $2h_-^{2,1}+2$ 3-cycles.

1.3.4. Moduli Stabilisation in Type IIB: An Overview

We are given many Kähler moduli T^i, complex structure moduli u^a, and the axio-dilaton S. Those fields need to be given a mass so that these moduli are stable. Mass terms can only arise from (1.3.65). We assume that the Kähler potential is given by (1.3.59) and the superpotential is generated only by fluxes via the Gukov-Vafa-Witten potential (1.3.63).

It follows that at tree-level the Kähler moduli are *not* stabilised. They enter the Kähler potential via (1.3.38). It holds [48]

$$\sum_{i=1}^{h_+^{1,1}} \mathcal{K}^{i\bar{\jmath}} \partial_i \mathcal{K} \partial_{\bar{\jmath}} \mathcal{K} = 3 \ , \tag{1.3.66}$$

where we sum over the Kähler moduli only. The Kähler moduli do also not occur in the superpotential W, at least not at tree-level. Thus, $D_i W = (\partial_i \mathcal{K}) W$ for all Kähler moduli. Then, using (1.3.66) we see that

$$\sum_{i,i=1}^{h_+^{1,1}} \mathcal{K}^{i\bar{\jmath}} D_i W \overline{D_j W} = 3|W|^2 \ , \tag{1.3.67}$$

called *no-scale structure*, where we sum again only over the Kähler moduli. Consequently, the Kähler moduli remain unfixed at tree-level. Quantum effects will eventually fix them.

In contrast, the stabilisation of the axio-dilaton and the complex structure moduli is straightforward. Since the Kähler moduli cancel the $-3|W|^2$-term, the remaining F-term potential reads

$$V_F = e^{\mathcal{K}} \mathcal{K}^{a\bar{b}} D_a W \overline{D_b W} \ , \tag{1.3.68}$$

where we implicitly sum only over the complex structure moduli and the axio-dilaton. The potential is positive definite and the minimum is obtained by solving the $h_-^{2,1} + 1$ complex equations

$$D_a W = 0 \ , \tag{1.3.69}$$

i.e. the F-terms have to vanish. Hence, the solution to (1.3.69) yields a 4d Minkowski vacuum – the cosmological constant is zero due to the no-scale structure. This 4d Minkowski vacuum does not have to be supersymmetric, because $D_{T^i} W \neq 0$ in general. But if we also require $D_{T^i} W = 0$, the vacuum would be supersymmetric. Note that (1.3.69) is directly related to the ISD-condition $\star_6 G_3 = i G_3$ in (1.3.55); for details see the below aside.

Aside: Minkowski vacua and the ISD-condition – For the stabilisation of moduli we demand $D_a W = 0$. This SUSY condition for the axio-dilaton S reads:

$$\begin{aligned} 0 = D_S W = D_S \int_{X_6} G_3 \wedge \Omega_3 = \partial_S \int_{X_6} G_3 \wedge \Omega_3 + \mathcal{K}_S \int_{X_6} G_3 \wedge \Omega_3 = \\ = - \int_{X_6} H_3 \wedge \Omega_3 - \frac{1}{S - \bar{S}} \int_{X_6} G_3 \wedge \Omega_3 = \frac{1}{\bar{S} - S} \int_{X_6} \bar{G}_3 \wedge \Omega_3 \ . \end{aligned}$$

Now, since Ω_3 is a $(3,0)$-form, it follows that the $(0,3)$-part of $\bar{G}_3$ must be zero; we write $\bar{G}_3|_{(0,3)} = 0$. Thus, we must have $G_3|_{(3,0)} = 0$.

Similarly, one obtains

$$0 = D_{u^a} W = \int_{X_6} G_3 \wedge \chi_a \ ,$$

where χ_a is some $(2,1)$-form. This follows from $\partial_{u^a} \Omega_3$ being $(3,0) + (2,1)$. Consequently, we then obtain the condition $G_3|_{(1,2)} = 0$.

Therefore, the requirement (1.3.69) implies that $G_3 = G_3|_{(2,1)} + G_3|_{(0,3)}$. But the ISD-condition $\star_6 G_3 = iG_3$ is equivalent to the statement that harmonic forms are of the type $(2,1) + (0,3)$, see e.g. [15]. Hence, the above construction of 4d Minkowski vacua indeed requires the ISD-condition.

For a SUSY vacuum one also needs $D_i W = 0$ with i labelling the Kähler moduli. Although this condition does not fix the Kähler moduli, it imposes the condition $W = 0$ at the SUSY minimum:

$$D_{T^i} W = 0 \qquad \Rightarrow \qquad \int_{X_6} G_3 \wedge \Omega_3 = 0 \ .$$

Thus, $G_3|_{(0,3)} = 0$. All three constraints then imply $G_3 = G_3|_{(2,1)}$.

For further details see e.g. [15; 18], which are the sources of this aside.

So far, this procedure stabilises the complex structure moduli and the axio-dilaton. The Kähler moduli are unfixed due to the no-scale structure. The procedure described in [5] and summarised above, is often referred to as GKP-stabilisation. However, quantum corrections lead to deviations from the no-scale structure and generate a potential for the Kähler moduli. Specifically, non-perturbative D-brane instanton effects give rise to a correction $\delta W = A(u)e^{-2\pi a T}$ of the superpotential. In this term, $A(u)$ is a holomorphic function of the complex structure moduli u and $a > 0$ is a model-dependent quantity [6; 18]. Such corrections were used in [6] to stabilise the Kähler moduli, and the setup proposed therein is now known as the KKLT-scenario. For the description of this model we follow [6; 18].

Let us consider a toy model with only one Kähler modulus T. At tree-level we have $\delta W = 0$, so T remains massless, while the complex structure moduli as well as the axio-dilaton receive masses of the order $m \sim \alpha'/R^3$, see (1.3.56). One can thus integrate out the complex structure moduli and the axio-dilaton. Effectively, we are left with a single-field model:

$$W = W_0 + Ae^{-2\pi a T} \ , \tag{1.3.70}$$

$$\mathcal{K} = -3 \ln \left(T + \bar{T} \right) \ . \tag{1.3.71}$$

Here, W_0 denotes the VEV of the Gukov-Vafa-Witten potential (1.3.63). If we had $W = W_0$, the potential would be exactly flat and thus the stabilisation of T would be impossible. Now, thanks to the instanton correction, a potential for T is generated. This field is stabilised by $D_T W = 0$. For simplicity, we take $T = \tau$ (and thus set the axionic part to zero) following [6]. One obtains [6; 18]

$$W_0 = -Ae^{-2\pi a \tau} \left(1 + \frac{4\pi a}{3} \tau \right) \qquad \text{and} \tag{1.3.72}$$

$$V_{\min} = -3e^{\mathcal{K}} |W|^2 = -\frac{2\pi^2 a^2 A^2 e^{-4\pi a \tau}}{3\tau} \ . \tag{1.3.73}$$

We observe that $V_{\min} < 0$, so we find an anti-de Sitter minimum in four dimensions, in short AdS_4. This vacuum is supersymmetric because we stabilised all fields by $D_i W = 0$, where i runs over all complex structure moduli, the axio-dilaton and the Kähler modulus. Furthermore, stabilising τ at moderately large values, which ensures that the supergravity approximation is valid and that α'-corrections are under control [6], implies that $|W_0| \ll 1$. To meet this requirement, a delicate fine-tuning is necessary. Due to the vast flux-landscape (see Section 1.4), at least some of the vacua are expected to allow for $|W_0| \ll 1$ [18; 62].

However, it would be desirable to obtain a de Sitter vacuum in four dimensions (dS_4) with small cosmological constant, instead of AdS_4. In [6] it was proposed to include anti-D3-branes

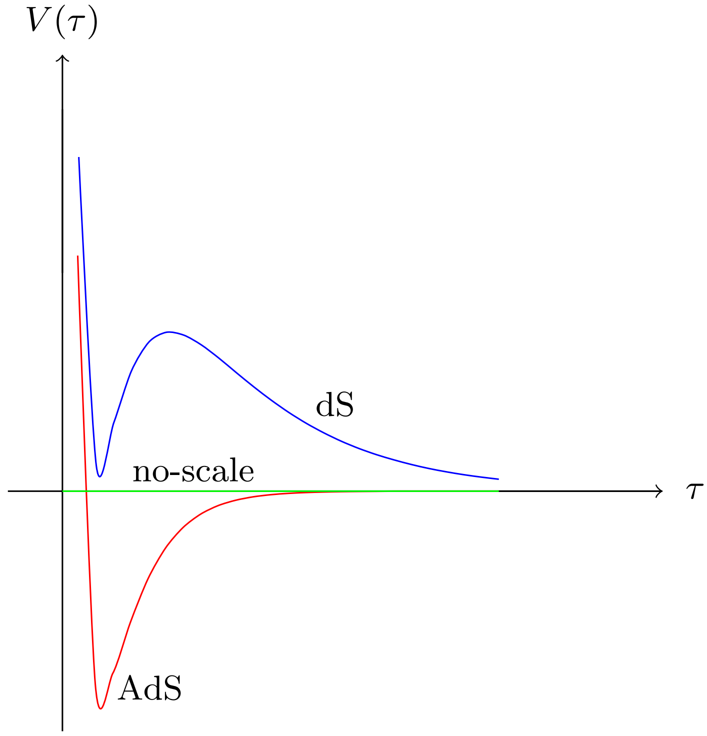


Figure 1.3.: The red curve presents the potential for τ giving rise to a SUSY AdS_4 minimum and the blue curve shows the potential after uplifting via $\overline{D3}$-branes, yielding a non-supersymmetric dS_4 minimum. Note that the curves are only schematic and do not present the precise potential. Without non-perturbative corrections the potential would vanish identically (green curve) due to the no-scale structure.

$\overline{D3}$ to arrive at de Sitter vacua. Note that they contribute negatively to $Q_{3,\text{loc}}$ in the tadpole cancellation constraint (1.3.49). Their extra contribution then needs to be compensated by "additional" fluxes. The F-term scalar potential obtains a correction [18]

$$\delta V = \frac{D}{(T+\bar{T})^3} , \tag{1.3.74}$$

where D is a model-dependent parameter. If the anti-D3-branes are put at (or dynamically driven to) the tip of a so-called KS-throat[18], D is a small number such that $V_{\text{tot}} \equiv V_{\text{min}} + \delta V$ has a minimum, at which V_{tot} is just above zero, i.e. we obtain dS_4 vacua with a tiny cosmological constant. This idea is often called *de Sitter uplift*.[19] These vacua are non-supersymmetric because SUSY is broken by the $\overline{D3}$-branes. For more on this, see e.g. [18]. The non-supersymmetric dS_4 vacuum we obtained are metastable (a sketch of the potentials

[18]These Klebanov-Strassler (KS) throats [63] typically occur in CY geometries with non-zero fluxes. See [64] for a statistical analysis.

[19]Note that this uplift to dS vacua via $\overline{D3}$-branes is controversially discussed in the literature (for a rather recent analysis and discussion see e.g. [65] and references therein).

giving rise to dS and AdS vacua is shown in Figure 1.3). In [6] it was demonstrated that they can have lifetimes much larger than 10^{10} years.

Consequently, even though Kähler moduli stabilisation is in general much more complicated than in the presented toy example, it demonstrates that the KKLT scenario can yield realistic 4d de Sitter vacua, at least in principle. For completeness, we want to remark that the so-called *uplift* to dS vacua via $\overline{\text{D3}}$-branes is controversially discussed in the literature (for a rather recent analysis and discussion see e.g. [65] and references therein).

Another scheme of Kähler moduli stabilisation is the *Large Volume Scenario* (LVS) [7; 66]. It avoids the tuning of $|W_0|$ to small values. The idea is to include leading α'-corrections to the Kähler potential. We briefly present the basics of the LVS following the toy example given in [7; 18]. The toy model uses a CY 3-fold $\mathbb{C}P^4_{(1,1,1,6,9)}[18]$ and has only $h^{1,1} = 2$ Kähler moduli and $h^{2,1} = 272$ complex structure moduli.[20] Again, we assume that all the complex structure moduli and the axio-dilaton are integrated out, so that a theory with only two Kähler moduli remains. We denote them by T_b and T_s. The real part of the former approximately measures the volume of X_6, i.e. $\text{Re}(T_b) \equiv \tau_b \simeq \mathcal{V}^{2/3}$, whereas the real part of the latter, τ_s, measures the size of a small 4-cycle in X_6 (see Figure 1.4). The Kähler potential with leading α'-corrections then reads [7; 18]:

$$\mathcal{K} = -2\ln\left[\frac{1}{9\sqrt{2}}\left(\tau_b^{3/2} - \tau_s^{3/2}\right) + \frac{\xi}{2g_s^{3/2}}\right] , \qquad \xi \equiv -\frac{\chi(X_6)\zeta(3)}{2(2\pi)^3} . \tag{1.3.75}$$

In our example we have $\chi = 2(h^{1,1} - h^{2,1}) = -540$ (see Appendix A.3, in particular (A.3.23)), so that $\xi > 0$, which is a necessary requirement for the stabilisation procedure. Moreover, we assume that the superpotential is given by

$$W = W_0 + A_s e^{-a_s\tau_s} . \tag{1.3.76}$$

Inserting this into the F-term scalar potential yields [18]

$$V \simeq \frac{\lambda a_s^2 |A_s|^2 \sqrt{\tau_s} e^{-2a_s\tau_s}}{\mathcal{V}} - \frac{\mu a_s W_0 A_s \tau_s e^{-a_s\tau_s}}{\mathcal{V}^2} + \frac{\nu\xi|W_0|^2}{g_s^{3/2}\mathcal{V}^3} , \tag{1.3.77}$$

where λ, μ and ν are $\mathcal{O}(1)$ factors. Let us assume that $\mathcal{V}$ is exponentially larger than τ_s: $\ln\mathcal{V} \sim a_s\tau_s$. For very large $\mathcal{V}$ the second term dominates so that the potential goes to zero from below. For smaller $\mathcal{V}$ the other two terms dominate and the potential is positive. Consequently, there exists a minimum at which $V < 0$. This minimum fixes $\mathcal{V}$ (and thus τ_b) and τ_s as follows [7; 18]:

$$\mathcal{V} \sim e^{a_s\tau_s} \qquad \text{and} \qquad \tau_s \sim \frac{\xi^{2/3}}{g_s} . \tag{1.3.78}$$

Note that the existence of this minimum does not require any tunings of W_0 (as is required in KKLT). It is again an AdS_4 vacuum but, as opposed to the KKLT scenario, it is not supersymmetric (on has in general $D_{T_b}W \neq 0$ and $D_{T_s}W \neq 0$). Hence, uplifts to dS_4 vacua are once again required, e.g. by $\overline{\text{D3}}$-branes.

[20] The CY manifold $\mathbb{C}P^4_{(1,1,1,6,9)}[18]$ is an example of a hypersurface in weighted complex projective space. Generally, $\mathbb{C}P^n_{(k_1,\dots,k_{n+1})}[p]$ is obtained as follows: First, consider $\mathbb{C}^{n+1} \setminus \{0\}$ with the identification $(z_1, \dots, z_{n+1}) \sim (\lambda^{k_1}z_1, \dots, \lambda^{k_{n+1}}z_{n+1})$, where $\lambda \in \mathbb{C} \setminus \{0\}$. Such spaces are compact and Kähler. Then, $\mathbb{C}P^n_{(k_1,\dots,k_{n+1})}[p]$ is a subspace defined by $f_p(z_1, \dots, z_{n+1}) = 0$, where f_p is a homogeneous polynomial of degree p. The subspace is CY if $p = k_1 + \dots + k_{n+1}$. These statements and more details can be found e.g. in [18].

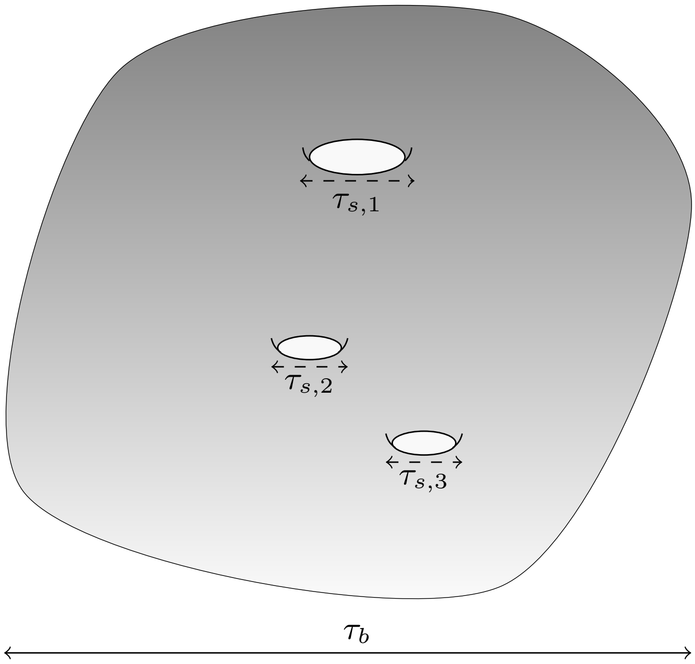


Figure 1.4.: Here we illustrate a Calabi-Yau 3-fold with several small 4-cycles, whose size is parametrised by τ_i, $i = 1, 2, 3$. The size of the whole CY 3-fold is measured by τ_b (corresponding to the volume of the bulk 4-cycle T_b). Due to the presence of the holes such CY 3-folds are often called Swiss-cheese CY manifolds. The toy model we are considering has just two Kähler moduli, so only one small 4-cycle would be present. This picture is inspired from the typical illustration of Swiss-cheese CY 3-folds, such as in [18].

Since the volume (and hence the bulk 4-cycle τ_b) is stabilised at large values, this mechanism is called Large Volume Scenario (LVS). It has the nice feature that the SUSY breaking effects due to $\overline{\text{D3}}$-branes are subdominant compared to the SUSY-breaking induced by the stabilisation scheme. The gravitino mass (which can be used as an order parameter of SUSY breaking) can be expressed in terms of $\mathcal{V}$ (measured in units of string-length) as follows [48]:

$$m_{3/2} \sim \frac{g_s^2 W_0}{\mathcal{V}} \ , \tag{1.3.79}$$

where we ignore numerical factors. For sufficiently large compactification volumes one can obtain gravitino masses at the TeV scale.[21] A large $\mathcal{V}$ also yields a relatively small string-scale

$$m_{\text{string}} \sim \frac{g_s}{\mathcal{V}^{1/2}} \ . \tag{1.3.80}$$

However, m_{string} at TeV scale is not possible to achieve, because this renders the Kähler moduli too light and one runs into the cosmological moduli problem. The masses of τ_b and

[21] While this possibility might not be too attractive anymore given the present LHC results, low-scale supersymmetry was certainly one of the interesting features of the LVS.

τ_s are given by [48]:

$$m_{\tau_b} \sim \frac{g_s^2 W_0}{\mathcal{V}^{3/2}} \ , \qquad m_{\tau_s} \sim \frac{a_s \tau_s g_s W_0}{\mathcal{V}} \ . \tag{1.3.81}$$

In the LVS, τ_b is the lightest modulus. Its SUSY partner, the axion a_b is even exponentially lighter:

$$m_{a_b} \sim e^{-\mathcal{V}^{2/3}} \ . \tag{1.3.82}$$

This is a generic feature of the LVS. Hence, a prediction of the LVS is a contribution to the effective number of relativistic species by the axion a_b acting as dark radiation (see Chapter 2).

It is fair to say that the KKLT scenario and the LVS are the most frequently used mechanisms to stabilise Kähler moduli, although there are further approaches such as Kähler uplifting [67].

1.3.5. A Few Comments on F-Theory

It is often useful and necessary to consider F-theory compactifications, which are more general than type IIB compactifications. In F-theory one considers elliptically fibred CY 4-folds X_8 and the axio-dilaton S varies over the six-dimensional base space. Instead of the fluxes F_3 and H_3 (which are combined to G_3) one has a 4-form flux G_4 in F-theory. The Gukov-Vafa-Witten superpotential is expressed as [17; 18; 61]

$$W = \int_{X_8} G_4 \wedge \Omega_4 \ , \tag{1.3.83}$$

where Ω_4 is the holomorphic $(4,0)$-form on X_8. Locally, one can describe an elliptically fibred CY 4-fold X_8 as a product of a CY 3-fold and a torus (except at singular fibres) [15; 18]. Then, G_4 and Ω_4 can be related to G_3 and Ω_3 as follows [17; 18]:

$$G_4 = \frac{G_3 \wedge d\bar{w}}{\bar{S} - S} + \text{h.c.} \ , \tag{1.3.84}$$

$$\Omega_4 = \Omega_3 \wedge dw \ , \tag{1.3.85}$$

where

$$dw \equiv dx + S dy \tag{1.3.86}$$

with x and y denoting the coordinates of the fibre. One can then show that (1.3.83) reduces to the type IIB expression (1.3.63). Similarly, the axio-dilaton part and the complex structure moduli part in (1.3.59) can be obtained from

$$\mathcal{K} = -\ln\left(\int_{X_8} \Omega_4 \wedge \bar{\Omega}_4\right) \ . \tag{1.3.87}$$

Of course, one can define a four-fold period vector Π^i similar to (1.3.60):

$$\Pi^i = \int_{\Sigma_4^i} \Omega_4 \ . \tag{1.3.88}$$

Here, Σ_4^i are the 4-cycles in X_8. It follows that W can be expressed in terms of the flux numbers N^i of G_4:

$$W = N_i \Pi^i(u) \ , \tag{1.3.89}$$

where u denotes all the complex structure moduli. Their number is given by $h^{3,1}$ (this can be seen by straightforwardly generalising the construction (1.3.29) using the holomorphic $(4,0)$-form Ω_4), whereas the number of Kähler moduli is still $h^{1,1}$. The potential for these fields is again given by the F-term scalar potential (1.3.65).

One advantage of using F-theory is that D7-brane position moduli in the type IIB language correspond to complex structure moduli of the CY 4-fold in F-theory [17; 18]. There are clearly many more interesting things to say about F-theory, but this goes beyond the scope of the thesis. For reviews on F-theory see for instance [17; 68].

1.4. An Introduction to the String-Landscape

The agenda of this section is as follows: First, following [17], we outline how to count the number of supersymmetric flux vacua given a Calabi-Yau geometry. Next, we summarise how the huge landscape can be applied to our observation of the smallness of the cosmological constant. Finally, we present the Weak Gravity Conjecture (WGC) as an argument that the string landscape must be landlocked by an even larger swampland.

1.4.1. Counting of Supersymmetric Flux Vacua in F-/Type IIB Theory

We already sketched the origin of the string landscape: Given a Calabi-Yau manifold/orientifold and a set of flux numbers (corresponding to F_3 and H_3 in type IIB string theory, or G_4 in F-theory), one can write down the effective F-term potential V_F. The supersymmetric minimum is defined by the vanishing of the F-terms, i.e. $D_iW = 0$ for all fields. Clearly, due to integrality of the flux numbers one obtains a discrete set of SUSY flux vacua. Crucially, the choice of flux numbers is subject to the tadpole cancellation constraint. For type IIB string theory the tadpole condition is (1.3.51), where we are working in the spirit of type IIB as the weak coupling limit of F-theory on an elliptically fibred Calabi-Yau 4-fold Y_8. One can write schematically

$$\eta_3(\vec{N}_F, \vec{N}_H) \leq L_\star \equiv \frac{\chi(Y_8)}{24} , \tag{1.4.1}$$

where η_3 is a quadratic form of the flux vectors $\vec{N}_F$ and $\vec{N}_H$ corresponding to F_3 and H_3. Note that $\eta_3(\vec{N}_F, \vec{N}_H)$ can be directly computed from $\int_{X_6} H_3 \wedge F_3$ by specifying the fluxes in terms of a basis of 3-cycles (η is then the intersection form of the 3-cycles). Compared with (1.3.51) the inequality is obtained by assuming $Q_{3,\mathrm{loc}} \geq 0$. In principle, one can also have $Q_{3,\mathrm{loc}} < 0$ by including anti-D3-branes, but then one would break supersymmetry.[22] We are interested in counting SUSY vacua and thus we have $Q_{3,\mathrm{loc}} \geq 0$ implying (1.4.1) (see also [10]). Note that the RHS of (1.4.1) contains topological data of Y_8, and hence implicitly also of the base space X_6.

For F-theory vacua we have an analogous tadpole constraint:

$$\eta_4(\vec{N}, \vec{N}) \leq L_\star \equiv \frac{\chi(Y_8)}{24} , \tag{1.4.2}$$

Here, η_4 is the intersection form of 4-cycles and $\eta_4(\vec{N}, \vec{N})$ is a quadratic expression in the components of the flux vector $\vec{N}$ corresponding to the G_4-flux. The expression $\eta_4(\vec{N}, \vec{N})$ is obtained from $\int_{Y_8} G_4 \wedge G_4$.

[22] Independently of this, note that the tadpole cannot be made arbitrarily large by including anti-D3-branes, because in a flux background a sufficiently large number of $\overline{\mathrm{D3}}$-branes decays into a state with flux and D3-branes only [12; 69; 70]

One can prove that for SUSY vacua one always has $\eta_3(\vec{N}_F, \vec{N}_H) > 0$ and $\eta_4(\vec{N}, \vec{N}) > 0$. We explain this in the aside below.

Aside: Positivity of the flux-contribution to the tadpole for SUSY vacua – We show that for SUSY vacua one has $\int_{X_6} H_3 \wedge F_3 \geq 0$. The idea is (see e.g. [18]) to use $G_3 = \star_6 i G_3$ (which necessarily holds for SUSY vacua) and $G_3 = F_3 - SH_3$ with $S = C_0 + i/g_s$ to obtain

$$F_3 = C_0 H_3 + \frac{1}{g_s} \star_6 H_3 \ .$$

It follows

$$\int_{X_6} H_3 \wedge F_3 = \frac{1}{g_s} \int_{X_6} H_3 \wedge \star_6 H_3 \geq 0$$

and equality holds if and only if fluxes are turned off.

In the case of F-theory one has $G_4 = \star_8 G_4$ for SUSY vacua (for a proof see [17]; this proof is based on deriving constraints on G_4 from the SUSY condition $DW = 0$ in the same spirit as we did for G_3 in the aside in Section 1.3.4). It follows immediately

$$\int_{Y_8} G_4 \wedge G_4 = \int_{Y_8} G_4 \wedge \star_8 G_4 \geq 0 \ . \tag{1.4.3}$$

Again, equality holds if and only if $G_4 \equiv 0$.

Hence, the task is to count how many choices for the integral flux numbers can be made such that (1.4.1) or (1.4.2) hold true together with $\eta_3(\vec{N}_F, \vec{N}_H) > 0$ or $\eta_4(\vec{N}, \vec{N}) > 0$ in the cases of type IIB or F-theory. Each viable choice yields a SUSY vacuum. The set of all vacua defines the string/flux landscape [9].[23]

A concrete systematics of counting SUSY flux vacua has been developed in [10; 12]. They are partially based on [8]. In the subsequent presentation we mostly follow [17], which presents the counting of SUSY flux vacua in F-theory. The approach can, however, also be applied to SUSY vacua in type IIB string theory.

Let us formulate the counting problem mathematically more precisely: Given an elliptically fibred CY 4-fold Y_8 and a G_4-flux with flux numbers $N^I \in \mathbb{Z}$, we want to solve

$$D_a W_N(z) = 0 \ , \qquad \text{where} \qquad W_N(z) = \int_{Y_8} G_4 \wedge \Omega_4 \ , \tag{1.4.4}$$

and

$$0 \leq \frac{1}{2} \eta_{IJ} N^I N^J \leq L_\star \equiv \frac{\chi(Y_8)}{24} \ , \qquad \text{where} \qquad \eta_{IJ} \equiv \int_{Y_8} \Sigma_4^I \wedge \Sigma_4^J \ , \tag{1.4.5}$$

where $a = 1, ..., h^{3,1}(Y_8) \equiv h$ and $I = 1, ..., b_4(Y_8)$. Recall that $b_4(Y_8)$ is the Betti-number of Y_8 and given by $b_4(Y_8) = 2h^{3,1}(Y_8) + h^{2,2}(Y_8)$, see also Appendix A.3. Moreover, z summarises all the complex structure moduli. Note that in the following we only count the number of SUSY vacua on the complex structure moduli space $\mathcal{M}$ and ignore the Kähler moduli sector. The Kähler moduli are unfixed at tree-level and should be stabilised by quantum effects. The

[23] The tadpole conditions together with $\eta_3(\vec{N}_F, \vec{N}_H) > 0$ or $\eta_4(\vec{N}, \vec{N}) > 0$ seem to suggest that there exists only a finite number of SUSY flux vacua. However, in [71] it was demonstrated that e.g. on $T^6/\mathbb{Z}_2$ one can find infinitely many inequivalent flux solutions to the D3-tadpole, but only finitely many lead to 4d vacua while the other vacua give rise to (partial) decompactification (see also [10]).

number N_{vac} of SUSY vacua in a subset $\mathcal{S} \subset \mathcal{M}$ can be formally written down as[24]

$$\mathcal{N}_{\text{vac}} = \sum_{\vec{N}} \int_{\mathcal{S}} d^{2h} z \delta^{2h}(DW_N) |\det(D^2 W_N)| \ , \tag{1.4.6}$$

where we only sum over flux vectors $\vec{N} \in \mathbb{Z}^{b_4}$ satisfying (1.4.5). One can show that the number $\mathcal{N}_{\text{vac}}$ of SUSY vacua respecting the tadpole condition can be approximated as [17]

$$\mathcal{N}_{\text{vac}} \simeq \frac{1}{\sqrt{\det \eta_{IJ}}} \frac{(2\pi L_\star)^{b_4/2}}{(b_4/2)!} \int_{\mathcal{S}} \frac{1}{\pi^h} \det\left(\mathcal{R} + J\mathbb{1}\right) \ , \tag{1.4.7}$$

where $\mathcal{R}$ is the curvature form on the holomorphic tangent bundle to $\mathcal{S}$, and J is the Kähler form on $\mathcal{S}$. To arrive at (1.4.7), one has to replace the sum over $\vec{N}$ in (1.4.6) by an integral $\int d^{b_4} N$, which is justified for a sufficiently dense distribution of the flux vacua. One then ends up with a Gaussian integral, which gives rise to the factor in front of the integral. Moreover, in the computation one drops the modulus in (1.4.6), assuming that there is no severe cancellation occurring. We omit the technical details, which can be found in [17] (see also Section 3.6, where we present technicalities of a similar computation taking into account fine-tuning conditions in the landscape). However, the prefactor in (1.4.7), which is most important for the estimation of the number of SUSY vacua, can be understood intuitively: The factor $(2\pi L_\star)^{b_4/2}/(b_4/2)!$ coincides with the volume of a b_4-dimensional ball of radius $\sqrt{2L_\star}$. (If η was the identity this is indeed what one would expect from (1.4.5).) For practical purposes it is usually enough consider the volume factor for a good estimation of $\mathcal{N}_{\text{vac}}$.

Let us look at the example of an elliptic fibration over $\mathbb{C}P^3$ analysed in [17]. The Hodge-numbers are $h^{1,1} = 2$, $h^{2,1} = 0$, $h^{3,1} = 3878$ and $h^{2,2} = 15564$ (the last Hodge-number can be computed from the previous ones by (A.3.24)). The Betti-number b_4 is then $b_4 = 23320$ and the Euler character $\chi = 23328$ (see (A.3.25)), thus $L_\star = 972$. Inserting these numbers into (1.4.7), one obtains from the volume factor:

$$\mathcal{N}_{\text{vac}} \sim 10^{1789} \ . \tag{1.4.8}$$

This is much more than the often quoted 10^{500} string vacua. This number rather arises in type IIB string compactifications. It can be obtained by considering the orientifold limit of the above example (see [17] for details on this). But now b_4 has to be replaced by $2b_3 = 600$ in (1.4.7). (Note that one has b_3 flux numbers for each F_3 and H_3.) Taking $L_\star = 972$ one finds

$$\mathcal{N}_{\text{vac, type IIB}} \sim 10^{522} \ . \tag{1.4.9}$$

Hence, there are much less type IIB than F-theory vacua. This is because in the counting of the latter the D7-brane degrees of freedom are taken into account [17].

Note that one has to carefully check whether (1.4.7) is really suitable to obtain a good estimate for $\mathcal{N}_{\text{vac}}$: If $b_4 > 4\pi e L_\star$, one has $\mathcal{N}_{\text{vac}} \to 0$ as $b_4 \to \infty$. This is clearly a wrong behaviour. In the 4-fold model above we are precisely in this critical regime, where $b_4 \gtrsim 4\pi e L_\star$. Following [72; 73] one can estimate $\mathcal{N}_{\text{vac}}$ by choosing n non-zero flux numbers out of b_4 flux numbers in total. Then, for each possibility we apply (1.4.7) (with b_4 being replaced by n, of course) and sum over all possibilities. We cut the sum at $n = L_\star$, because for $n > L_\star$ the

[24] To see this, recall that the zeros of a smooth function $f : \mathbb{R} \to \mathbb{R}$ can be counted by the expression $\int_{\mathbb{R}} dx \delta(f(x)) |f'(x)|$.

contributions to $\mathcal{N}_{\text{vac}}$ is negligible [72; 73]. One has (for $L_\star \ll b_4$):

$$\begin{aligned}\mathcal{N}_{\text{vac}} &\simeq \sum_{n=1}^{L_\star} \binom{b_4}{n} \frac{(2\pi L_\star)^{n/2}}{\left(\frac{n}{2}\right)!} \simeq \binom{b_4}{L_\star} \frac{(2\pi L_\star)^{L_\star/2}}{\left(\frac{L_\star}{2}\right)!} \simeq \\ &\simeq \exp\left[L_\star \ln\left(\sqrt{4\pi e}\frac{b_4}{L_\star}\right) + (b_4 - L_\star)\ln\left(1 + \frac{L_\star}{b_4}\right)\right] \ .\end{aligned} \tag{1.4.10}$$

For the above example the approximation yields $\mathcal{N}_{\text{vac}} \simeq 10^{2483}$ (computation of the whole sum gives $\mathcal{N}_{\text{vac}} \simeq 10^{2496}$) and hence somewhat more vacua than expected from (1.4.7).[25]

We would like to stress that $\mathcal{N}_{\text{vac}}$ depends strongly on the choice of CY geometry. In principle, $\mathcal{N}_{\text{vac}}$ can therefore be much smaller or much larger than in the aforementioned examples. But according to [74] there are many CY 3-folds with $b_3 = \mathcal{O}(100)$. Thus, obtaining $\mathcal{N}_{\text{vac}} \gg 10^{500}$ is not unusual. Recently, in [75] it was argued that virtually all F-theory compactifications arise from an elliptically fibred CY 4-fold, whose topological data give rise to $\mathcal{N}_{\text{vac}} \sim 10^{272000}$ vacua. Despite these huge numbers, the 4d vacua obtained from F-theory/type IIB compactifications are expected to be not at all arbitrary – we comment on this issue in Section 1.4.3.

1.4.2. Application to the Cosmological Constant

Now we briefly describe how the existence of a huge string landscape can be used to accommodate a small cosmological constant, about 120 orders of magnitudes below our expectations from quantum field theory. This section is not relevant for the thesis itself and can therefore be skipped. The following outline, which is based on [8; 17; 18], is included for completeness of the introduction to the string landscape.

Bousso and Polchinski famously proposed in [8] to consider a compactification, not necessarily in the context of string theory, and to take into account contributions to the vacuum energy density induced by non-zero fluxes:

$$V_N(z) = V_0(z) + \int_Z G \wedge \star G = V_0(z) + g_{IJ}(z) N^I N^J \ , \tag{1.4.11}$$

where Z is the compactification manifold and z the moduli of Z. The metric on the moduli space is denoted by $g_{IJ}(z)$. Moreover, G is a p-form flux with flux numbers $N^I \in \mathbb{Z}$, where $I = 1, ..., b$. To keep the discussion general, we neither specify the dimension of Z nor the rank of G. Furthermore, $V_0 < 0$ denotes the bare potential. One expects $|V_0| \sim M^4$, where M is the cutoff energy scale of the theory. Equation (1.4.11) clearly shows that one obtains a discrete set of vacuum energy densities.

It is therefore interesting to ask how many choices of flux numbers satisfy the condition

$$|V_0| \leq g_{IJ} N^I N^J \leq |V_0| + \Lambda_{\text{obs}} \ , \tag{1.4.12}$$

where Λ_{obs} denotes the value of the observed cosmological constant in our vacuum. This counting problem is similar to (1.4.5). The suitable sets of flux numbers are lying inside of a thin shell with thickness δR of a b-dimensional ball of radius $R = \sqrt{|V_0| + \Lambda}$. One therefore expects that the number $\delta\mathcal{N}_{\text{vac}}(\Lambda)$ of vacua with cosmological constant $\Lambda \pm \delta\Lambda$ is given by the volume of this thin shell:

$$\delta\mathcal{N}_{\text{vac}}(\Lambda) \simeq \frac{1}{\sqrt{\det g}} \frac{\pi^{b/2}(|V_0| + \Lambda)^{b/2-1}}{(b/2-1)!}\delta\Lambda \simeq \left[\frac{2\pi e(|V_0| + \Lambda)}{\mu^4}\right]^{b/2} \frac{\delta\Lambda}{|V_0| + \Lambda} \ , \tag{1.4.13}$$

[25] I would like to thank T. Watari for helpful email correspondence on this issue.

where $\mu^4 \simeq b(\det g)^{1/b}$ should be understood as the scale of the energy density of the flux part of the potential.

Thus, around $\Lambda \simeq 0$ one finds exponentially many flux choices consistent with (1.4.12). For instance, if $|V_0|/\mu^4 = \mathcal{O}(10)$ one gets

$$\delta\mathcal{N}_{\mathrm{vac}}(\Lambda \simeq 0) \simeq 10^b \frac{\delta\Lambda}{|V_0|} \tag{1.4.14}$$

Then, in examples where $|V_0| \simeq M_p^4$ and $\delta\Lambda = \Lambda_{\mathrm{obs}} = 10^{-120} M_p^4$, and $b = 300$ (like in the type IIB string theory example), one still has approximately 10^{180} vacua with cosmological constants $0 < \Lambda < \Lambda_{\mathrm{obs}}$.

In combination with the anthropic principle [76], the existence of a landscape of vacua, each having a different cosmological constant, can justify the unnatural smallness of Λ_{obs}. It is, however, necessary to have a mechanism that populates the whole landscape. An example for such a mechanism is eternal inflation [77; 78] (see also [19] for reviews). It leads to the concept of the *multiverse*. Of course, it is particularly tempting to aim for statistical predictions of typical parameters one would expect within any of the universes. Unfortunately, there are technical and conceptual issues, known as the *measure problem* (see for instance [19; 79]).

1.4.3. Landscape vs. Swampland: The Weak Gravity Conjecture

As we have seen, the number of supersymmetric flux vacua is typically very large, but probably finite. On the one hand, this result is appealing because the huge landscape could represent an alternative to naturalness [9]. On the other hand, one could be concerned about the predictivity of string theory. If there are so many 4d string vacua one could believe that the bottom-up approach by simply writing down any 4d effective field theory is the more efficient method for model building. However, there are several arguments that such bottom-up approaches have a good chance to fail. Indeed, according to [21] the string landscape is surrounded by an even larger swampland (see again Figure 1.1). The swampland contains (by definition) all effective field theories which cannot be UV-completed to a consistent quantum gravity theory. Assuming that string theory is a correct theory of quantum gravity, this means that EFTs in the swampland can never be obtained from string compactifications. Hence, it is crucial and exciting to understand how 4d EFTs from string compactifications differ from swampland EFTs.

An example of an EFT in the swampland could be as follows: EFTs with an exact continuous global symmetry are believed to be inconsistent with quantum gravity, and hence part of the swampland. One of the most solid arguments for this inconsistency of exact global continuous symmetries in quantum gravity comes from perturbative string theory: global symmetries on the string worldsheet always correspond to gauge symmetries in spacetime (see [80; 81]). However, it is desirable to also have general quantum gravity arguments against global symmetries, independently of string theory. Several rather general arguments against global symmetries in quantum gravity have been put forward in e.g. [20; 81–83]. One heuristic argument goes as follows [20; 83]: given an EFT with a continuous global symmetry, one can throw particles charged under this global symmetry into a black hole. An observer outside the black hole has no chance to measure how many of such particles have been falling into the black hole. Hence, it seems that conservation of this global charge is violated. (Note that the situation is different for local gauge symmetries. If particles charged under this gauge symmetry are thrown into the black hole, their total charge can still be determined from outside by Gauss' theorem.) There is further evidence supporting that there cannot be global continuous symmetries in quantum gravity. Take a $U(1)$ gauge symmetry with coupling constant g and consider the limit $g \to 0$, in which we recover a global continuous theory (gauge bosons decouple). It seems to be possible to construct black holes of arbitrary charge. However, this charge cannot be radiated away, although the black hole keeps losing mass due to Hawking evaporation. Hence, after the evaporation process one expects an arbitrarily large number of extremal black holes of a few Planck masses. Their number goes to infinity as $g \to 0$. This leads to problems [84] such as the violation of the covariant entropy bound [83; 85]. Therefore, quantum gravity should censor the limit $g \to 0$.

A possibility to avoid problems with those so-called black hole remnants is to *assume* that extremal black holes can decay. Thereby the limit $g \to 0$ is prohibited. We explain how this comes about. An extremal black hole of mass M and charge Q satisfies $M = \sqrt{2}gQ$ (extremality condition), where g is the electric coupling constant. (Note that by choosing appropriate normalisations, one can also formulate the extremality condition as $M = Q$, i.e. without coupling constant g and numerical factors. However, in this discussion it is convenient to keep g explicitly and follow the standard normalisation.) Since we want this extremal black hole to be able to decay, we need to impose charge and energy conservation:

$$Q = Nq \ , \qquad M > Nm \ , \tag{1.4.15}$$

where we assume that the black hole decays into N particles with charge q and mass m. Using the extremality condition $M = gQ$, these two equations yield

$$M = \sqrt{2}gQ = \sqrt{2}gNq > Nm \qquad \Rightarrow \qquad m < \sqrt{2}gq \ . \tag{1.4.16}$$

Note that a decay of an extremal black hole into subextremal particles ($m > \sqrt{2}gq$) and a superextremal ($M < \sqrt{2}gQ$) black hole is prohibited by the cosmic censorship hypothesis (see e.g. [86; 87] for reviews). We conclude that the assumption of unstable extremal black holes requires the existence of an elementary particle, whose charge-to-mass ratio is $\sqrt{2}gq/m > 1$. (Note that this result can also be obtained by allowing the black hole to decay into different particles.) In particular, we observe that the limit $g \to 0$ cannot be taken.

The result of (1.4.16) is the heart of the so-called *Weak Gravity Conjecture* (WGC), which represents one of the most prominent examples for a criterion to distinguish swampland theories from theories of the landscape.[26] It was motivated [20] precisely for the reasons outlined above. The name of this conjecture is easily understood: The inequality $m < \sqrt{2}gq$ simply means that the repulsion of the $U(1)$ force dominates over the gravitational force between two particles of mass m and charge q. Unfortunately, it is currently unclear, how the exact formulation of the WGC should look like. For instance, it might be necessary to demand that the *lightest* particle of mass m and charge q has to obey $m < \sqrt{2}gq$. This is often referred to as the *strong electric WGC*. In contrast, the *mild electric WGC* only requires that there is *some* particle in the spectrum satisfying $m < \sqrt{2}gq$. Further formulations are conceivable and supported by more or less solid evidence from Kaluza-Klein theories or string theory. A list of recent discussions can be found in [90–99]. Since in this thesis we will be applying only the mild and strong forms of the WGC, we want to draw our attention to those ones [20]:

[26] For other attempts to delineate the boundary between the landscape and the swampland, see also [88] and recently [89].

The electric Weak Gravity Conjecture – A $U(1)$ gauge theory with gauge coupling g, which is consistent with quantum gravity, contains a charged particle of electric charge q and mass m such that (in Planck units)

$$m < \sqrt{2} g q \ . \tag{1.4.17}$$

In the *mild form* it is sufficient if there is just one such state in the particle spectrum. The *strong form* requires that the state satisfying (1.4.17) must also be the *lightest* one.

Note that the mild WGC is sufficient to avoid the problems with black hole remnants. While the strong WGC seems unmotivated at this stage, it finds support by various concrete examples within string theory (see [20] for details). A Weak Gravity Conjecture can also be formulated for the dual objects to charged particles, namely magnetic monopoles. Assuming that also magnetically charged black holes should be able to decay, it follows for the mass of a magnetic monopole (ignoring $\mathcal{O}(1)$ factors)

$$m_{\text{mon}} < g_{\text{mag}} q_{\text{mag}} \sim \frac{1}{gq} \ . \tag{1.4.18}$$

This is not yet how the magnetic WGC is formulated traditionally. Rather, a further observation is used. The mass of the magnetic monopole is bounded from below by the energy density of its magnetic field, i.e.

$$m_{\text{mon}} \gtrsim \frac{(g_{\text{mag}} q_{\text{mag}})^2}{\Lambda^{-1}} \sim \frac{\Lambda}{g^2 q^2} \ , \tag{1.4.19}$$

where Λ is the cutoff of the $U(1)$ gauge theory. Together with (1.4.18) it follows $\Lambda < gq$, which is what the traditional formulation of the magnetic WGC states [20]:

The magnetic Weak Gravity Conjecture – A $U(1)$ gauge theory consistent with quantum gravity has a cutoff Λ with upper bound

$$\Lambda < gq \tag{1.4.20}$$

in Planck units.

Note that this inequality can also be obtained by demanding that the magnetic monopole should not be a black hole [20]. Once again, we observe that the limit $g \to 0$ yielding a continuous global symmetry leads to an immediate breakdown of the EFT.

The Weak Gravity Conjecture (both electric and magnetic) is expected to generalise to p-dimensional coupled to $(p+1)$-forms in $d > p$ dimensions. (The above WGC is stated for $p = 0$ (particle) and $d = 4$.) Heuristically, the generalisation follows from the requirement that extremal p-dimensional black branes should be able to decay. However, this heuristic argument fails for axions, i.e. 0-forms in d dimensions. These 0-form fields are sourced by instantons, which are localised in time, i.e. the black hole argument is not (directly) applicable. Nevertheless, there are various ways to motivate the WGC also for instantons coupled to axion fields, e.g. by string dualities (in combination with dimensional reduction) [100], Kaluza-Klein reduction [92], or by expressing the WGC in string compactifications in geometric terms [93].

Now, assuming that the WGC is applicable not only to particles/1-form gauge fields but also to instantons/axions, natural inflation can be shown to reside in the swampland: the instanton charge is given by the inverse of the axion decay constant f and its mass corresponds to the instanton action S. The (electric) WGC then tells us that $S < 1/f$. In the regime $S > 1$ (in string theory this is necessary to impose in order to have perturbative control) it then follows $f < 1$, which is the exact opposite of what is needed for successful natural inflation. Consequently, the WGC implies sub-Planckian axion decay constants. This finding is consistent with independent results from string theory [40; 41], see also the aside below.

Aside: Axion decay constants in string compactifications – We exemplify why axion decay constants of axions from string theory are expected to be sub-Planckian. As in [32] we consider the axion arising from the NS-NS-field B_2. In an O3/O7 orientifold compactification we can write $B_2 = b_\alpha(x)\omega^\alpha$, where $\alpha = 1, ..., h_-^{1,1}$. The type IIB SUGRA action (1.3.10) contains a term

$$S = \frac{1}{4\kappa_{10}^2 g_s^2}\int d^{10}x\sqrt{-G}|dB_2|^2 \supset \frac{1}{2}\int d^4x\sqrt{-g}\gamma^{\alpha\beta}\partial_\mu b_\alpha \partial^\mu b_\beta \ ,$$

where

$$\gamma^{\alpha\beta} = \frac{1}{2\cdot 3!\kappa_{10}^2 g_s^2}\int_{X_6}\omega^\alpha \wedge \star_6 \omega^\beta \ . \tag{$\star$}$$

We assume an isotropic compactification so that the volume of X_6 is given by $\mathcal{V} \simeq L^6$, where L is the characteristic length of the compactification manifold X_6. Since the basis ω^α is normalised such that $\int_{\Sigma_2^\beta}\omega^\alpha = \delta_\beta^\alpha$, where Σ_2^β is a 2-cycle (and thus scales like $\sim L^2$), one deduces a scaling $\omega^\alpha \sim 1/L^2$. Hence, in a diagonal basis the integral in ($\star$) scales as $\delta^{\alpha\beta}\mathcal{V}/L^4$. Taking into account that the Planck mass in 4d is given by $M_p^2 = \mathcal{V}/(\kappa_{10}^2 g_s^2)$, we get

$$\gamma^{\alpha\beta} = \frac{1}{2\cdot 3!}\frac{M_p^2}{L^4}\delta^{\alpha\beta} \ .$$

The axion decay constant f_α for axion $b_\alpha(x)$ is given by $f_\alpha^2 = \gamma^{\alpha\alpha}$. In order to have perturbative control, we need $L \gg M_p^{-1}$. Hence, f_α is sub-Planckian. More examples and details can be found in [40; 41].

Since the WGC censors trans-Planckian axion decay constants, more refined constructions of axion inflation models are needed, such as the KNP-mechanism (alignment inflation) [42] or models of N-flation [43]. However, there can be constraints of those models from the WGC extended to theories with multiple $U(1)$ gauge fields. In this case, the WGC requires that the convex hull of the set of charge-to-mass vectors $\vec{z}_i \equiv \pm\vec{q}_i/m_i$, $i = 1, ..., N$ (N number of gauge fields), has to contain the unit ball in $\mathbb{R}^N$ (the surface of this unit ball is defined by the charge-to-mass ratios of extremal black holes) [90]. This criterion is again motivated by avoiding black hole remnants. In [100; 101] this extended WGC was used to analyse constraints on axion inflation such as the KNP-mechanism. Indeed, attempts to realise the KNP-mechanism with two axions seems to runs afoul with the extended WGC. However, there is a loophole pointed out in [100; 101], which is based on introducing a "spectator instanton". In this way, consistency with the mild form of the WGC can be ensured. We do not go into further details in this introduction and refer to Chapter 4 for more remarks on the KNP-mechanism and the potential conflict with the WGC.

In summary, we learn that the WGC is an interesting tool, which can improve our understanding of the boundaries of the string landscape within the swampland (apart from applications of the WGC to inflation, the Weak Gravity Conjecture has recently brought into connection regarding the stability of non-supersymmetric vacua, see e.g. [102–105]).

1.5. Structure and Short Summary of the Thesis

Here we describe the structure of the next parts of the thesis. Thereby we provide a summary of the individual chapters and briefly present our conclusions and results. At the end of this section, we also explicitly list the publications, which constitute the heart of this thesis.

We begin our exploration of string landscape vacua by investigating whether the Large Volume Scenario (LVS) admits realistic reheating scenarios consistent with present cosmological measurements. The LVS possesses a light bulk volume modulus τ_b, whose mass is $m_{\tau_b} \sim 1/\mathcal{V}^{3/2}$, and an exponentially lighter axionic partner a_b of mass $m_{a_b} \sim \exp(-\mathcal{V}^{2/3})$. Since this axion is essentially massless, it gives rise to dark radiation and may therefore yield inconsistencies with recent measurements of the effective number of relativistic species. In Chapter 2 we review the dark radiation problem in reheating models based on the so-called sequestered LVS. Here, the Standard Model (SM) is realised by D3-branes at a singularity of the CY orientifold. Reheating occurs due to the decay of τ_b into SM particles, where the channel for decays into the Higgs sector is dominant. At the same time the coupling of τ_b to a_b leads to a decay into very light axions. Hence, dark radiation is produced and it contributes significantly to $\Delta N_{\text{eff}} = N_{\text{eff}} - 3.046$. Recent measurements allow only for small deviations from the SM value $N_{\text{eff}} = 3.046$. Therefore such LVS models can easily be in tension with present cosmological data. In Chapter 2 we go beyond the sequestered LVS. We propose to realise the visible sector by a stack of D7-branes wrapping a stabilised 4-cycle of the CY 3-fold. This 4-cycle can be stabilised in three different ways, either by D-terms, string-loop effects or non-perturbative effects. The first two possibilities enhance the decay into the SM sector by a new channel of decays of τ_b into gauge bosons of the Standard Model. Thereby, ΔN_{eff} can be reduced. The possibilities of lowering the prediction for ΔN_{eff} are, however, limited. The price to pay for an enhancement of the decay into the visible sector is a delicate fine-tuning of a parameter of the model. Nevertheless, in comparison with the sequestered LVS, our scenarios are more flexible in making ΔN_{eff} consistent with recent data. Only in the case of stabilising the visible sector by non-perturbative effects the decays into SM gauge bosons is not efficient enough to ameliorate the dark radiation excess. Our conclusion is as follows (up to some technicalities explained in Chapter 2): the two non-sequestered LVS models with the visible sector being stabilised by D-terms or string-loops are not yet ruled out. However, if the measurements of ΔN_{eff} keep decreasing toward zero, our models will be pushed to their limits and may finally be excluded. This would not imply that those models are in the swampland, but rather that we are most likely not living in an LVS vacuum. At the end of Chapter 2 we also comment on variants of the sequestered LVS equipped with flavour branes, as well as on further progress in this field beyond our work.

Afterwards, we continue the exploration of the string landscape based on the idea that 4d EFTs of the string landscape are expected to have particular features, which distinguish those from EFTs of the swampland [21]. In particular, we investigate whether models of large-field inflation can be realised in the string landscape. If this were not possible, large-field inflation models would reside in the swampland.

In Chapter 3 we propose a fine-tuned realisation of F-term axion monodromy inflation, where backreaction of the inflaton on the remaining moduli, which are stabilised at the SUSY locus, is under control. We take an axion associated with a D7-brane modulus or a complex structure modulus as the inflation and its potential is protected from quantum corrections by shift symmetry, which arises at the large complex structure (LCS) limit (we refer to the

LCS limit as the regime in which terms of the form $e^{-2\pi z}$ are highly suppressed due to the stabilisation of $\mathrm{Re}(z)$ at large values). An inflaton mass is generated by turning on fluxes appropriately so that the structure of the superpotential is given by $W = w_0(z) + a(z)u$, where u is the modulus corresponding to the inflaton and z labels the remaining moduli. In order to keep the inflaton mass light and to control backreaction of u on the other moduli z, it is necessary to make the coefficient $a(z)$ and its derivatives $\partial_z a(z)$ with respect to all moduli small by fine-tuning. The main emphasis of the presentation will be an analysis of the required geometry of the CY manifolds followed by a quantitative study regarding the severity of the required fine-tuning in the string landscape. We find that there is a no-go theorem for our proposed F-term axion monodromy inflation model on CY 3-folds, because it turns out that the necessary fine-tuning is prohibited in the perturbative regime. The basic idea of the no-go theorem is the observation that the coefficient $a(S)$ of the coupling term $a(S)u$, with axio-dilaton S, can never be tuned small. Indeed, one always has $a(S) \sim (n_1 + n_2 S)$, $n_1, n_2 \in \mathbb{Z}$ and $S = C_0 + i/g_s$. Hence, the tuning condition $|a(S)| \ll 1$ yields two requirements $|n_1 + n_2 C_0| \ll 1$ and $|n_2/g_s| \ll 1$, corresponding to the tuning of the real and imaginary part of $a(S)$, respectively. The latter condition can be achieved in the perturbative regime $g_s < 1$ only for $n_2 = 0$. In consequence, we then find $n_1 = 0$ and hence $a(S) \equiv 0$. By similar arguments one can show that all the coupling terms of the inflaton to the other moduli must vanish identically. Considering F-theory on Calabi-Yau 4-folds instead, this problem can be avoided. Their much richer geometrical structure allows to make the inflaton mass small by fine-tuning. Moreover, due to the fine-tuning the field space of the inflaton admits a long enough trajectory, along which quadratic inflation can take place. To complete the analysis, we study how many string vacua of the landscape remain after imposing the required tuning conditions. We show that a landscape of vacua remains as long as not too many complex structure moduli couple to the inflaton modulus u. If no such landscape were to remain, we would conclude that F-term axion monodromy inflation could not be realised in type IIB string theory. To summarise, we can put forward an explicit proposal for realising large-field inflation in string theory at the cost of accepting potentially severe fine-tuning. Currently, this scenario seems to be robust against strong quantum gravity constraints [93; 106; 107]. Hence, F-term axion monodromy inflation seems to be a viable candidate for the realisation of large-field inflation in the string landscape.

In Chapter 4 we take up the challenge of realising alignment inflation, for which quantum gravity constraints from the Weak Gravity Conjecture have been discussed in e.g. [100; 101; 108], in type IIB string theory. First, we briefly review the basic logic of alignment inflation and then explain the implementation in type IIB string theory. Here, we focus again on the geometric setup as well as on the consistency of our model with Kähler moduli stabilisation à la LVS. We consider two complex structure moduli u and v in the LCS limit (where u is assumed to be deeper in the LCS limit so that the non-perturbative corrections containing u are more suppressed than the ones for v). By flux choice we ensure that the leading order superpotential terms exhibit the structure $W = w_0(z) + f(z)(u - Nv)$, where N is a combination of certain flux numbers, which must be taken large, i.e. $N \gg 1$. Moreover, z again labels the remaining complex structure moduli. At the SUSY locus the variables $\mathrm{Im}(u)$, $\mathrm{Im}(v)$ and the combination $\mathrm{Re}(u - Nv)$ are stabilised. This leaves one unfixed and hence flat direction, which is a winding trajectory in the $\mathrm{Re}(u)$-$\mathrm{Re}(v)$-space. Therefore, the unfixed orthogonal combination to $\mathrm{Re}(u - Nv)$ is a good inflaton candidate (which can be identified with $\mathrm{Re}(u)$ up to $\mathcal{O}(1/N)$ corrections). A non-vanishing inflaton potential is then generated by including leading order instanton corrections for v. Taking into account backreaction of the inflaton modulus on other moduli, our setup yields a potential $V(\theta) \sim (1 - \cos(\theta/f_{\mathrm{ax}}))$ for the canonically normalised axion θ with axion decay constant $f_{\rm ax} \sim N/(4\pi)$, which becomes effectively trans-Planckian for sufficiently large N. Our scenario is therefore similar to the alignment mechanism in field theory proposed by Kim, Nilles and Peloso [42]. For a consistent realisation in type IIB string theory, Kähler moduli stabilisation is to be taken into account. We show that the magnitude of the energy scale of the inflaton potential can be well below the energy scale of Kähler moduli stabilisation. Thus, destabilisation of Kähler moduli during inflation is avoided. Furthermore, we address the question whether our inflation model is consistent with the Weak Gravity Conjecture (WGC) if it is applicable to our construction. We observe that our model precisely realises a loophole pointed out in [100; 101], which only exists if the mild WGC holds true. The existence of a heavier instanton with sufficiently large charge suffices to satisfy the mild WGC. The instanton we need is precisely the one that generates an instanton potential for u. Note that what we mean by "instantons" here are actually worldsheet instantons on the dual (type IIA) side, which give rise to "instanton" corrections on the type IIB side. (We therefore often refer to these "instanton" or "non-perturbative corrections" $e^{2\pi i u}$ or $e^{-2\pi u}$, depending on conventions, as instanton corrections.) Since u is, by construction, stabilised deeper in the LCS limit, the corresponding instanton is heavy enough to suppress any dangerous corrections to the inflaton potential. If, however, the strong WGC is valid, the instanton satisfying the WGC must also be the lightest one. Obviously, our model would then be censored by the strong WGC. Finally, we also comment on some further concerns with respect to the realisation of our model in string theory. For instance, we comment on possible challenges in obtaining $N \gg 1$ in the light of the D3-tadpole cancellation constraint. To conclude, our model, which we call "F-term winding inflation", realises alignment inflation in string theory. In order to answer the question whether this scenario is ruled out by the WGC, a deeper understanding of the Weak Gravity Conjecture is necessary.

It is moreover desirable to delineate the string landscape without relying on conjectures, ideally in a model-independent way. In [108] it was suggested that models of axion inflation, such as natural inflaton or alignment inflation, are constrained by effects induced by *gravitational instantons*. In our context, these are finite-action solutions of Euclidean Einstein-axion systems. The metric solution is a wormhole, which we henceforth refer to as the Giddings-Strominger wormhole [109]. It connects two distant regions of the asymptotically flat space by a throat. However, the Euclidean Einstein-axion system is just an effective theory, which reaches its limits of validity at very small distances or large curvatures. Consequently, the wormhole solutions can only be trusted up to a certain energy cutoff scale Λ. In particular, wormholes with throat radii $r_0 < \Lambda^{-1}$ cannot be trusted. In terms of the cutoff scale, the corrections of the axion potential are of order $\delta V(\theta) \sim \cos\theta e^{-S} \sim \cos\theta e^{-r_0^2} \sim \cos\theta e^{-1/\Lambda^2}$. Hence, such corrections are only relevant for inflation if Λ is just below the Planck scale, i.e. $\Lambda \lesssim 1$. However, in the context of string theory, there are many scales below the Planck scale at which the validity of our Einstein-axion system can be lost. A breakdown of the description is expected to occur at the Kaluza-Klein scale and possibly even much below at the stabilisation scale of the lightest moduli. In the latter case, the correction $\delta V(\theta) \sim \cos\theta e^{-1/\Lambda^2}$ would be totally irrelevant for axion inflation. Apart from this concern, doubts have been raised in [92], whether such wormhole contribute at all to the potential. Globally, a wormhole connecting two different regions of the same manifold does not lead to a net change in the instanton flux number, and hence one may wonder whether the presence of Giddings-Strominger wormholes can really break the axionic shift symmetry.

These two questions are treated in Chapter 5. Regarding the issue whether Giddings-Strominger wormholes can contribute at all to the axion potential, our approach is as follows:

we write down the partition function for any distribution of wormholes in Euclidean space and deduce the effective axion potential in some region of this Euclidean space. We find that there is indeed a contribution to the effective potential of the type $\cos\theta e^{-S}$, originating from a non-local effect. Locally, it looks like shift symmetry is broken, although it is preserved globally. This result is supported by a quantum mechanical toy-model, where we compute the energy eigenvalues of a Hamiltonian for a gas of paired instantons and anti-instantons. Thus, wormholes could spoil axion inflation if the generated potential is of the right magnitude. In the next part of the project, we therefore study the effective description of gravitational instanton solutions. To see whether this EFT breaks down due to destabilisation of light moduli, we consider Einstein-axion-dilaton systems, which occur frequently in string compactifications. As opposed to the Einstein-axion system, gravitational instanton solutions now come in three types [110]: there are again wormhole solutions, but also a flat space solution (which we call extremal gravitational instanton) and a solution with a singularity (cored gravitational instanton). The origin of these two solutions can be traced back to contributions from the kinetic energy of the dilaton to the energy momentum tensor. Note that wormhole solutions can only arise for a certain range of dilaton couplings, which are more difficult to obtain in string compactifications. All three solutions are expected to break down below some length scale Λ^{-1}. However, we show that the dilaton variations with respect to Euclidean time can be controlled and thus destabilisation of the dilaton is in general not an issue. Hence, the cutoff scale Λ can be taken up to the Kaluza-Klein (KK) scale. We push the KK-scale as close to the Planck scale as possible and find that there are no model-independent constraints on axion inflation. In particular, Giddings-Strominger wormholes with the smallest possible throat radius induce a potential, which is negligible in comparison with the potential during inflation. Therefore, we come to the conclusion that possible constraints on large-field inflation (if existent) must come from the “quantum part” of any quantum gravity theory rather than from semi-classical treatments. Furthermore, we make a couple of observations related to the Weak Gravity Conjecture. For instance, we note that the wormholes are precisely the instantons satisfying the WGC. Moreover, the WGC implies the instability of extremal and cored gravitational instantons, which can be obtained by dimensional reduction of extremal and sub-extremal five-dimensional Reissner-Nordström black holes, respectively. The WGC in 5d implies the instability of these black holes. Therefore, it is conceivable that the instability is inherited by the extremal and cored instantons.

Finally, Chapter 6 summarises the results of this thesis and discusses how they shape the direction of future work. In brief, we learn that despite the huge number of string vacua within the landscape, the properties of EFTs from string compactifications are by far not arbitrary. For instance, some corners of the string landscape predict dark radiation and thus, those parts of the landscape might face more stringent constraints from observations in the near future or even be ruled out. Furthermore, we find that large-field inflation has not yet been proven to reside in the swampland. Instead, we were able to put forward two different types of large-field inflation scenarios, which seem to work from the perspective of string theory modulo some technical details, which are still to be clarified in future work. We also conclude that gravitational instantons are not suitable for deriving generic and strong constraints on large-field inflation. Whether the WGC has to say more about large-field inflation can only be decided by making more progress in understanding the WGC itself.

Various appendices add technical details to the main chapters or contain some general background material.

List of Publications and Specification of Own Contributions

The contents of Chapter 2 to Chapter 5 are based on [111–114] (see also the list at the end of this thesis). All the papers are published in peer-reviewed journals. These publications were written in collaboration, in which I was involved in all aspects of each paper. In the following I describe, however, my main contributions to these publications.

Project 1: Dark Radiation predictions from general Large Volume Scenarios [111]

My main contributions to [111] are in section 3 of this work. Therein I was mostly responsible for the computation of decay rates in the presented models and the possibility of enhancing the decay rate into gauge bosons by fine-tuning. Hence, Chapter 2 is particularly based on those aspects and further parts of our joint work are summarised for completeness.

Project 2: Tuning and Backreaction in F-term Axion Monodromy Inflation [112]

My main contributions to [112] are the analysis of the geometric setup (sections 2.2 and 2.3) and the quantification of the tunings in the landscape (section 4). Furthermore, the observation that fine-tuning of the derivatives of $a(z)$ is required as well, is based on my computations at an early stage of this project. Hence, Chapter 3 is particularly based on those aspects and further parts of our joint work are summarised for completeness.

Project 3: Winding out of the Swamp: Evading the Weak Gravity Conjecture with F-term Winding Inflation? [113]

For this project I was in particular working on how the phenomenology of alignment inflation translates into a constraint on the flux number N. Furthermore, I analysed the geometry of the setup as well as the compatibility of our winding inflation model with Kähler moduli stabilisation. Concerning section D, I contributed in particular to the discussion on gravitational instantons, which we postpone to Chapter 5. Hence, Chapter 4 is particularly based on the described aspects and further parts of our joint work are summarised for completeness.

Project 4: Can Gravitational Instantons Really Constrain Axion Inflation? [114]

In this project I was the principal author of this paper. Hence, I include all of my results in this thesis so that Chapter 5 coincides with most of [114].

CHAPTER 2

Testing String-Vacua: Contraints from Observational Bounds on Dark Radiation

2.1. Motivation and Summary

The Large Volume Scenario (LVS) is arguably one of the most prominent proposals to stabilise the Kähler moduli of a Calabi-Yau 3-fold in the framework of type IIB string theory. One of its key features is that the mass of the axion a corresponding to the bulk-cycle is exponentially volume-suppressed, $m_a \sim \exp(-\mathcal{V}^{2/3})$, where $\mathcal{V}$ is the volume of the CY manifold. Hence, there will always be a light axion, which contributes to the effective number of relativistic species, N_{eff}. Such key features allow us to investigate the phenomenological compatibility of those corners of the string landscape with current observational data, such as from Planck 2015 [1], proposing a maximal $\Delta N_{\text{eff}} = 0.56$ (2σ; Planck TT+lowP+BAO), where ΔN_{eff} is defined by the deviation from the Standard Model value, i.e. $\Delta N_{\text{eff}} \equiv N_{\text{eff}} - 3.046$. This result does not leave much space for abundant dark radiation, and hence it is a challenge to realise a Large Volume model consistent with this finding. Note that, depending on the combination of available data, there is room for debate which value of ΔN_{eff} should be taken. For instance, when taking into account preliminary polarisation data, the current value of N_{eff} would reduce to $N_{\text{eff}} = 3.04 \pm 0.18$ (Planck TT, TE, EE+lowP+BAO) [1] consistent with the Standard Model prediction $N_{\text{eff}} = 3.046$. This measurement corresponds to $\Delta N_{\text{eff}} < 0.35$ at a 2σ level. However, there seems to be a mismatch between the preferred Hubble rate H_0 reported by Planck 2015, namely $H_0 = (67.8 \pm 0.9)$ km s^{-1}Mpc^{-1} (68% CL, Planck TT+lowP+lensing) [1], and the Hubble rate measured by local experiments. For instance, in [115] the "best estimate" is $H_0 = (73.24 \pm 1.74)$ km s^{-1}Mpc^{-1}. Those two results can be reconciled by allowing for a larger $\Delta N_{\text{eff}} \simeq 0.4 - 1$ [115]. In [116] the combination lowP+TT+CMB lensing allows for $\Delta N_{\text{eff}} < 0.77$ (95% CL) with $H_0 = 71.3^{+1.9}_{-2.2}$ km s^{-1}Mpc^{-1}. Alternatively, it is conceivable that tensions in the measurement of H_0 arise due to our location in an underdense region of the universe [117]. Clearly, there are some open questions left[1] and for the remainder of this chapter we will stick with the 'conservative' Planck 2015 result $\Delta N_{\text{eff}} = 0.56$ (2σ; Planck TT+lowP+BAO).

[1] I wish to thank Viraj Sanghai and Benjamin Wallisch for discussions on local vs. Planck measurements of H_0. In particular, I am grateful to Benjamin Wallisch for helpful email correspondence.

We will first review how dark radiation arises in the so-called sequestered Large Volume Scenario. In such models the visible sector is realised by a stack of D3-branes located at a singularity of the CY 3-fold and the most relevant decays during reheating are given by $\tau_b \to a_b a_b$ (decay of the bulk cycle modulus τ_b to its corresponding axion) and the decay into Higgs scalars $\tau_b \to H_u H_d$. The contribution to N_{eff} can be generally computed as follows[2]:

$$\Delta N_{\text{eff}} = \frac{43}{7}\left(\frac{10.75}{g_*(T_d)}\right)^{1/3} \frac{\Gamma(\tau_b \to \text{DR})}{\Gamma(\tau_b \to \text{SM})} \,, \tag{2.1.1}$$

where $g_*(T_d)$ is the effective number of particle species at the decay temperature (i.e. reheating temperature) T_d. The number 10.75 arises from $g_* = 10.75$ at neutrino decoupling. $\Gamma(\tau_b \to \text{DR})$ and $\Gamma(\tau_b \to \text{SM})$ are the decay rates of τ_b into dark radiation (DR) and Standard Model particles (SM), respectively. In the sequestered LVS the latter is dominated by the decay into the Higgs sector. We argue that the resulting branching ratio is typically too high to be consistent with present data.

Thus, we propose to reduce the branching ratio by considering more general Large Volume models. Specifically, we suggest to wrap a stack of D7-branes (representing the visible sector) on a stabilised 4-cycle of the CY 3-fold. In doing so, another decay channel of τ_b into Standard Model gauge bosons becomes relevant, lowering the branching ratio of the decay into dark radiation axions. The stabilisation of the 4-cycle can be achieved by D-terms, string-loop or non-perturbative corrections. All three cases are described in this chapter in the respective order. We point out explicitly that the decay into gauge-bosons can be enhanced by some fine-tuning of a flux-parameter. In this way consistency with data is improved. It remains an open question how much of this fine-tuning in the string landscape can be achieved.

Furthermore, we give a brief overview on other phenomenological issues and properties of our models. Our extensions of the sequestered LVS goes hand in hand with high-scale supersymmetry. Additionally, we briefly discuss whether the axion a_b can also be the QCD-axion at the same time.

To put our most important conclusions in a nutshell, we find that our rather generic Large Volume constructions described above are still compatible with recent measurements $N_{\text{eff}} = 3.15 \pm 0.46$ (2σ; Planck TT+lowP+BAO). However, if future measurements of N_{eff} continue to approach the Standard Model value $N_{\text{eff}} = 3.046$ (as suggested by including preliminary polarisation data), our proposed scenarios, and hence the corresponding parts of the string landscape, are on the verge of being ruled out.

This whole chapter is a summary of our published work [111] with the focus on non-sequestered LVS variants, whose presentation follows closely section 3 of this paper.

The paper [111] will also be discussed in [120] with particular attention to section 4 of our joint work.

2.2. Brief Review of Previous Work

We begin by reviewing dark radiation predictions within the sequestered LVS of the papers [119; 121; 122] that motivated our project. A similar summary can be found in our work [111].

The starting point of [121] is a so-called Swiss-cheese CY 3-fold, whose volume $\mathcal{V}$ can be expressed as

$$\mathcal{V} = \alpha\left(\tau_b^{3/2} - \sum_i \gamma_i \tau_{s,i}^{3/2}\right) \,, \tag{2.2.1}$$

[2]For a nice description of the origin of this formula we refer to [118] and [119].

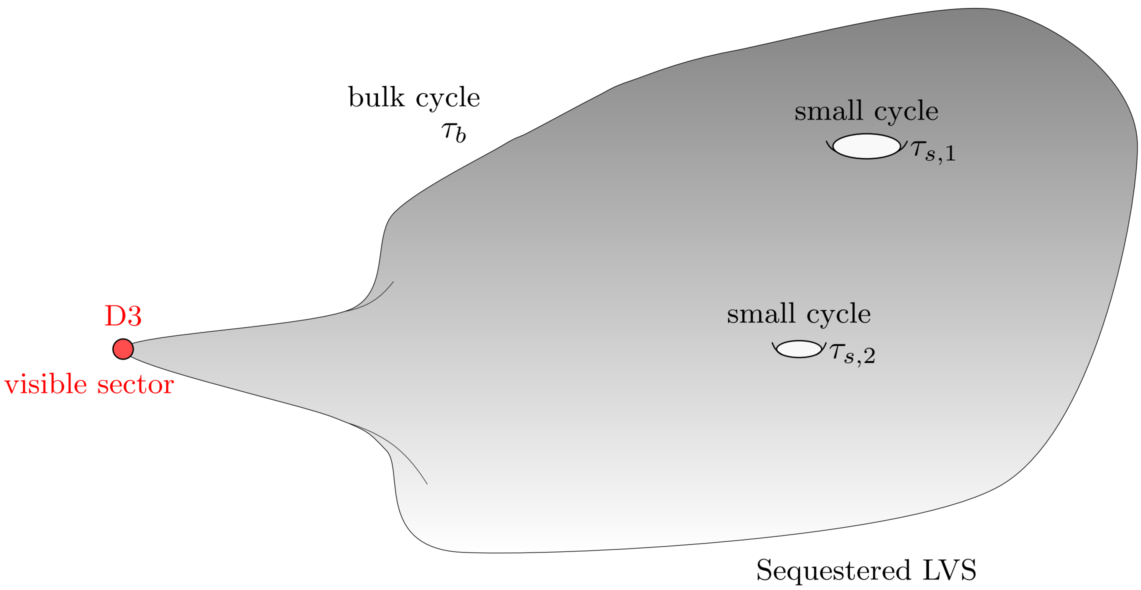


Figure 2.1.: This is an illustration of a Swiss-cheese Calabi-Yau 3-fold of the sequestered LVS. For simplicity, only two small cycles $\tau_{s,1}$ and $\tau_{s,2}$ are shown. In the sequestered LVS the visible sector (represented by a red dot) is realised by D3-branes at a singularity of the CY manifold.

with some parameters α and γ_i. $\mathcal{V}$ and at least one of the small cycles $\tau_{i,s}$ are fixed by the LVS scheme, such that $\mathcal{V} \simeq \alpha\tau_b^{3/2}$ is exponentially large (see also Section 1.3.4 for a brief review of the LVS). The LVS gives rise to a mass-hierarchy of the moduli. For our purpose, the masses of the lightest fields τ_b and its axion a_b are particularly interesting:

$$m_{\tau_b} \sim \frac{M_p}{\mathcal{V}^{3/2}} \,, \qquad m_{a_b} \sim M_p \; e^{-2\pi\mathcal{V}^{2/3}} \,. \tag{2.2.2}$$

In [121] the sequestered LVS was considered. As already mentioned, the idea is to realise the visible sector via D3-branes at a singularity of the CY manifold (see Figure 2.1). The sequestered LVS has the interesting feature to allow for TeV-scale supersymmetry. This can be seen as follows: the gaugino- and soft scalar masses scale as [121]

$$m_{1/2} \sim m_{\text{soft}} \sim \frac{M_p}{\mathcal{V}^2} \,, \tag{2.2.3}$$

whereas the gravitino mass goes as

$$m_{3/2} \sim \frac{M_p}{\mathcal{V}^{3/2}} \tag{2.2.4}$$

and therefore $m_{1/2} \sim m_{\text{soft}} \ll m_{3/2}$. Consequently, one can have soft masses at TeV-scale, while the gravitino and other moduli are sufficiently massive so that there is no conflict with the cosmological moduli problem (CMP) (see Section 1.3).

The decay rates for τ_b into dark radiation and Standard Model particles can be read off from the interacting terms of the Lagrangian $\mathscr{L}$. Everything we need are the relevant terms of the Kähler potential K and the tree-level gauge couplings f_a, which read

$$\begin{aligned} K =& -3\ln\left(T_b+\bar{T}_b-\frac{1}{3}\left[C^i\bar{C}^i+H_u\bar{H}_u+H_d\bar{H}_d+\{zH_uH_d+\text{h.c.}\}\right]\right)+\dots \quad (2.2.5)\\ =& -3\ln(T_b+\bar{T}_b)+\frac{C^i\bar{C}^i}{T_b+\bar{T}_b}+\frac{H_u\bar{H}_u+H_d\bar{H}_d}{T_b+\bar{T}_b}+\frac{zH_uH_d+\text{h.c.}}{T_b+\bar{T}_b}+\dots\ ,\\ f_a =& S+h_{a,k}T_{s_a,k}\ , \quad (2.2.6)\end{aligned}$$

for the sequestered LVS. We denote by $T_b=\tau_b+ia_b$ the bulk volume modulus superfield, by $T_{s_a,k}$ the blow-up modes and by S the axio-dilaton. C^i are the chiral matter superfields and H_u, H_d the Higgs bosons. z is an undetermined parameter. In the gauge-coupling f_a depends linearly on S and the blow-up modes $T_{s_a,k}$, multiplied by a parameter $h_{a,k}$.

From the part $K\supset -3\ln\left(T_b+\bar{T}_b\right)$ one obtains [121]

$$\mathscr{L}\supset\frac{3}{4\tau_b^2}\partial_\mu\tau_b\partial^\mu\tau_b+\frac{3}{4\tau_b^2}\partial_\mu a_b\partial^\mu a_b\ . \quad (2.2.7)$$

The kinetic terms of τ_b and a_b have to be canonically normalised by $\tau_b=\exp(\phi\sqrt{2/3})$ and $a_b=a\sqrt{2/3}$, where ϕ is the physical field corresponding to the bulk volume modulus. This yields the interacting term

$$\mathscr{L}\supset\sqrt{\frac{2}{3}}\phi\partial_\mu a\partial^\mu a\ , \quad (2.2.8)$$

and the decay rate for the process $\phi\to aa$ can be read off:

$$\Gamma(\phi\to aa)=\frac{1}{48\pi}\frac{m_\phi^3}{M_p^2}\ . \quad (2.2.9)$$

The relevant coupling-term of ϕ to the Higgs-bosons is given by [121]

$$\mathscr{L}\supset\frac{1}{\sqrt{6}}\left(zH_uH_d\Box\phi+\text{h.c.}\right)\ . \quad (2.2.10)$$

One can show that the decay rate for $\phi\to H_uH_d$ is

$$\Gamma(\phi\to H_uH_d)=\frac{2z^2}{48\pi}\frac{m_\phi^3}{M_p^2}\ . \quad (2.2.11)$$

As long as $z\gtrsim\mathcal{O}(1)$ the decay into the Standard Model sector is governed by $\phi\to H_uH_d$. For completeness we briefly argue why the remaining decay channels are subleading (relative to $\phi\to aa$ and $\phi\to H_uH_d$) [111]:

1. *Gauge bosons and gauginos* – In the sequestered LVS the decays into gauge bosons and gauginos are highly suppressed. This follows from the fact that in the sequestered LVS the gauge kinetic function is independent of the bulk Kähler modulus T_b at tree level. Therefore, decays of ϕ into gauge bosons can occur at loop level, i.e. $\Gamma\sim(\alpha_{\text{SM}}/4\pi)^2\times(m_\phi^3/M_p^2)$, where α_{SM} is the visible sector coupling. This can be computed from the loop-correction $\sim(\alpha_{\text{SM}}/4\pi)\phi F_{\mu\nu}F^{\mu\nu}$ [121].

2. *Matter scalars* – The decay rate for $\phi\to C^i\bar{C}^j$ into matter scalars C^i is given by $\Gamma\sim m_{\text{soft}}^2m_\phi/M_p^2$ and this channel is due to the term $\mathscr{L}\supset K_{i\bar{j}}C^i\bar{C}^{\bar{j}}$. In the sequestered LVS we have $m_{\text{soft}}\sim M_p/\mathcal{V}^2$ (see (2.2.3)). It follows $m_{\text{soft}}\ll m_\phi$ due to $m_\phi\sim M_p/\mathcal{V}^{3/2}$. Hence, this channel is negligible.

3. *Matter fermions and Higgsinos* – For the decay channel into fermions one obtains a chirality-suppressed decay rate $\Gamma \sim m_{\text{f}}^2 m_\phi / M_p^2$. Loop-induced contributions are at most of order $\Gamma \sim (\alpha_{\text{SM}}/4\pi)^2 \times (m_\phi^3/M_p^2)$ as in the case of gauge bosons.

Consequently, $\Gamma(\phi \to \text{SM}) \simeq \Gamma(\phi \to H_u H_d)$. We can then plug (2.2.9) and (2.2.11) into (2.1.1) to arrive at

$$\Delta N_{\text{eff}} = \frac{43}{7} \left(\frac{10.75}{g_*(T_d)}\right)^{1/3} \frac{1}{n_H z^2} \ , \tag{2.2.12}$$

where n_H counts the number of Higgs doublets in our model. Note the influence of the reheating temperature T_d on ΔN_{eff} through $g_*(T_d)$. One has

$$T_d \sim \sqrt{\Gamma_\phi M_p} \simeq \left(\mathcal{O}(10^{-2}) \frac{M_p^2}{\mathcal{V}^{9/2}}\right)^{1/2} = \mathcal{O}(10^{-1}) \frac{M_p}{\mathcal{V}^{9/4}} \ , \tag{2.2.13}$$

where (2.2.2) was used. Due to (2.2.3) the reheating temperature is essentially determined by the SUSY-breaking scale. In particular, the smaller the SUSY-breaking scale the smaller the reheating temperature and the more dark radiation.

For instance, supersymmetry at TeV-scale would imply $T_d \simeq 1$ GeV and thus $g_*(T_d) = 247/4$. According to Planck 2015 we have $\Delta N_{\text{eff}} < 0.56$ (2σ; Planck TT+lowP+BAO) [1], implying that $n_H z^2 > 6.1$. If our model contains only one pair of Higgs doublets, we require $z > 1.7$. It is however not yet clear which values of z are preferred in the string landscape. If, for example, $z \simeq 1$ is a typical value, it follows that more than six Higgs doublets are needed, i.e. further fields would have to be added to the Minimal Supersymmetric Standard Model (MSSM).

Since SUSY at TeV-scale has not yet been found, it is also an option to consider SUSY at a higher scale, e.g. with soft masses above 10 TeV, yielding $T_d > 100$ GeV so that $g_* = 106.75$ (this is the maximum value attained in the Standard Model). We then still find $n_H z^2 > 5.1$.

In summary, in the sequestered LVS one has only two options to limit the dark radiation contributions to N_{eff}: First, one can include sufficiently many pairs of Higgs doublets. Second, one can make the Giudice-Masiero coupling z large enough to avoid conflicts with current data. However, as already mentioned, it is an open question to determine preferred values of z from string theory. Let us be more precise about this point. One can see from (2.2.5) that there is a shift symmetry in the Higgs-sector for $z = 1$:

$$H_u \bar{H}_u + H_d \bar{H}_d + \{z H_u H_d + \text{h.c.}\} \xrightarrow{z=1} (H_u + \bar{H}_d)(\bar{H}_u + H_d) \ , \tag{2.2.14}$$

which is invariant under $H_u \to H_u + c$, $H_d \to H_d - \bar{c}$. Such a shift symmetry has been obtained within type IIB/F-theory (see [123–126]). Note, however, that in those cases, where the Higgs is contained in brane deformation moduli, the Kähler metric is independent of Kähler moduli. This would prohibit the decay of τ_b into the Higgs fields due to the absence of mixing of Kähler moduli with the Higgs fields, worsening the dark radiation problem.

2.3. Dark Radiation Predictions from Models Beyond the Sequestered Large Volume Scenario

In our project [111] we were seeking for alternatives to keep ΔN_{eff} sufficiently small without relying on too many Higgs doublets or large values of z. The easiest way to suppress the dark

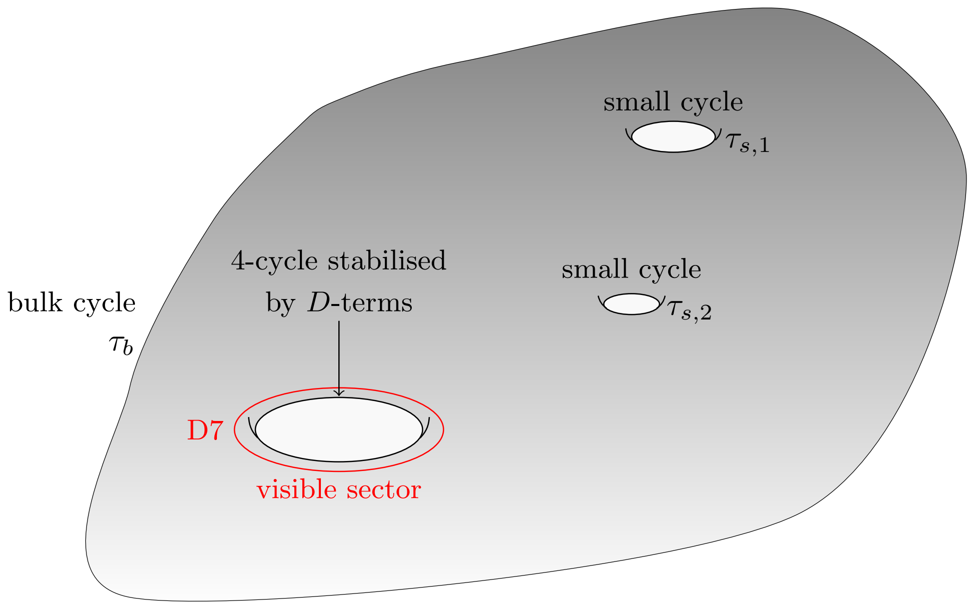


Figure 2.2.: This illustration shows a Swiss-cheese CY manifold where one 4-cycle is stabilised by D-terms. The visible sector (D7-brane) wraps this 4-cycle.

radiation abundance is to allow for further decay channels into SM particles. As we summarised in the previous section, decays into gauge bosons or matter particles are subleading (compared to the Higgs channel) in the sequestered LVS.

It is thus clearly interesting to study models beyond the sequestered LVS. Instead of realising the visible sector by a stack of D3-branes at a singularity we will rather wrap stacks of D7-branes on a 4-cycle of the Calabi-Yau manifold. This will the lead to a tree level coupling $\tau_b F_{\mu\nu} F^{\mu\nu}$ so that the decay channel into SM gauge bosons becomes relevant. In such scenarios decays into matter fermions are still chirality-suppressed [121]. Moreover, we do not know whether $m_{\text{soft}} \sim m_{\tau_b}$ can be obtained in LVS models in a natural way. In the non-sequestered LVS the soft masses turn out to be much larger than the mass of the lightest modulus (see e.g. [66; 127; 128]), i.e. decays into matter scalars are kinematically forbidden.

Hence, for the subsequent analysis we restrict ourselves to the additional decay channel to gauge bosons due to the realisation of the visible sector on a 4-cycle. It is, however, necessary to stabilise this 4-cycle. This can be done by gauge-flux induced D-terms, string-loop corrections or non-perturbative effects. We will discuss these three options in the respective order, following [111].

2.3.1. Stabilisation of the Visible Sector via D-Terms

The first scenario we propose is depicted in Figure 2.2. It shows the visible sector realised by a stack of D7-branes wrapped around a 4-cycle, which is stabilised by D-terms.

Before discussing the phenomenology of this scenario we first explain the idea behind the stabilisation of 4-cycles via D-terms. Most of the below description on D-term stabilisation follows our paper [111].

Cycle Stabilisation via D-Terms

In the following we consider a Calabi-Yau orientifold X with several 4-cycles D_i whose volumes are given by τ_i. For this discussion it will be convenient to also introduce the moduli t^i, which give the volumes of 2-cycles. In terms of these the overall volume can be written as $\mathcal{V} = k_{ijk}t^i t^j t^k/6$, where k_{ijk} are the triple intersection numbers of X (see also (1.3.38)). We also have the relation $\tau_i = \partial\mathcal{V}/\partial t^i = k_{ijk}t^j t^k/2$.

The goal is to stabilise the overall volume of X and one small 4-cycle using the LVS procedure, whereas all other 4-cycles will be stabilised by D-terms. One of those 4-cycles will be wrapped by D7-branes, realising the visible sector.

D-terms are induced due to fluxes on D7-branes wrapping these 4-cycles. The resulting D-term potential is given by [18]

$$V_D = \sum_i \frac{g_i^2}{2}\left(\sum_j c_{ij}|\phi_j|^2 - \xi_i\right)^2 , \tag{2.3.1}$$

where ξ_i are FI-terms and ϕ_j are open string states charged under the anomalous $U(1)$ giving rise to the D-term. The sum over i is over all 7-branes and the sum over j is over all charged open string states.

We assume that supersymmetric stabilisation can be achieved, without appealing to VEVs of charged fields, by the simultaneous vanishing of all FI-terms, i.e. $\xi_i = 0$ for all i.

The FI-terms can be expressed by an integral over X:

$$\xi_i = \frac{1}{4\pi\mathcal{V}}\int_X \hat{D}_i \wedge J \wedge \mathcal{F}_i = \frac{1}{4\pi\mathcal{V}} q_{ij}t^j , \tag{2.3.2}$$

where $\hat{D}_i$ are Poincaré dual 2-forms to the 4-cycles D_i, $J = t^i\hat{D}_i$ is the Kähler form and $\mathcal{F}_i = \tilde{f}_i^j\hat{D}_j$ is the gauge flux (for Poincaré duality see also proposition A.24). The $q_{ij} = \tilde{f}_i^k k_{ijk}$ are then the charges of the Kähler moduli T_i under the anomalous $U(1)$ [129; 130].

Clearly, every ξ_i can be expressed as a linear combination of the 2-cycle volumes t_i and imposing $\xi_i = 0\ \forall i$ in (2.3.2) yields a linear system of equations for the t_i:

$$\begin{aligned} \xi_1 = 0 \quad &\Leftrightarrow \quad 0 = q_{11}t^1 + q_{12}t^2 + \ldots + q_{1n}t^n , \\ \xi_2 = 0 \quad &\Leftrightarrow \quad 0 = q_{21}t^1 + \ldots . \\ &\vdots \end{aligned} \tag{2.3.3}$$

As a result D-terms fix volumes of some 2-cycles in terms of the volumes of other 2-cycles.

We apply this technique of D-term stabilisation to the LVS as follows. Let us consider a situation, where one of the 2-cycles, say t^1, does not appear in the expressions for the FI-terms, while all other 2-cycles t^j with $j \neq 1$ are fixed with respect to one another by the system of equations (2.3.3). We then arrive at the following situation: At this stage, t^1 remains unfixed, whereas all other 2-cycles can be expressed in terms of one other 2-cycle, say t^2. Further, we consider geometries where t^1 enters the volume in a diagonal way: $k_{1jk} = 0$ for $j, k \neq 1$ only, such that t^1 only contributes to the volume as $\mathcal{V} \supset \frac{1}{6} k_{111} (t^1)^3$. Consequently

$$\tau_1 = \frac{1}{2} k_{111} (t^1)^2 \ , \tag{2.3.4}$$

$$\tau_2 = \frac{1}{2} k_{2jk} (t^j t^k) \propto (t^2)^2 \ , \tag{2.3.5}$$

$$\text{for } i \geq 3: \quad \tau_i = \frac{1}{2} k_{ijk} (t^j t^k) \propto (t^2)^2 \quad \Rightarrow \quad \tau_i = c_i \tau_2 \ , \tag{2.3.6}$$

where c_i is a numerical factor. At the level of 4-cycles this leads to the desired result: D-terms stabilisation leaves two flat directions which we can parametrise by τ_1 and τ_2. All other 4-cycles are stabilised with respect to τ_2.

Thus, after D-term stabilisation the volume $\mathcal{V}$ of X depends on τ_1 and τ_2 only. Here, we consider geometries which lead to a volume of Swiss-Cheese form:

$$\mathcal{V} = \alpha (\tau_2^{3/2} - \gamma \tau_1^{3/2}) \ . \tag{2.3.7}$$

Without loss of generality we define $\tau_2 \equiv \tau_b$, $\tau_1 \equiv \tau_{np}$ and we fix them using the standard LVS procedure. Then τ_b remains the lightest modulus and its axionic partner a_b is a nearly massless dark radiation candidate, see (2.2.2). An explicit example for a construction of this type is given in [130].

How do we include the visible sector? The visible sector 4-cycle of volume τ_a is fixed by D-terms as $\tau_a = c_a \tau_b$. Ignoring flux contributions the gauge coupling of the visible sector branes is directly related to the VEV of τ_a, namely as $\alpha_{\text{SM}}^{-1} = \langle \tau_a \rangle \simeq 25$. Therefore, the parameter c_a has to be tuned appropriately in order to obtain the right value of $\langle \tau_a \rangle$. This potentially severe tuning of fluxes is unavoidable in our models we consider in the following.

Actually, the D-term stabilisation condition $\tau_a = c_a \tau_b$ should rather be formulated supersymmetrically, i.e. in terms of the superfields T: $T_a = c_a T_b$. This is because stabilisation via D-terms is achieved at the supersymmetric locus $V_D = 0$ so that the effective theory remains supersymmetric.

We are then in the position to understand why our model gives rise to a relevant interaction of τ_b with SM gauge bosons. The superfield Lagrangian for the visible sector gauge theory reads:

$$\mathscr{L} \supset \int \mathrm{d}^2\theta \ T_a W_\alpha W^\alpha = \int \mathrm{d}^2\theta \ c_a T_b W_\alpha W^\alpha = c_a \tau_b F_{\mu\nu} F^{\mu\nu} + c_a a_b F_{\mu\nu} \tilde{F}^{\mu\nu} \ . \tag{2.3.8}$$

As a result, the lightest modulus τ_b now couples to visible sector gauge bosons, which was the objective of the current construction. This result opens another channel for τ_b to decay into SM particles.

Phenomenology

Let us now discuss the implications of realising the visible sector on a 4-cycle stabilised by D-terms. We find that we can expect τ_b to reheat the Standard Model only by decays into the Higgs channel and into gauge bosons. Note that the decay of the volume modulus into matter scalars and the heavy Higgs is kinematically forbidden. This is because $m_{\text{soft}} \sim m_{3/2} \sim M_p/\mathcal{V}$ (mind the difference to (2.2.3) in the sequestered LVS), whereas $m_{\tau_b} \sim M_p/\mathcal{V}^{3/2}$. Hence, the bulk modulus can only decay into the light Higgs of the Higgs sector. Moreover, we necessarily have to consider high-scale supersymmetry in order to avoid the cosmological moduli problem. It imposes that all moduli masses satisfy $m_{\text{mod}} \gtrsim 10$ TeV and hence $m_{\text{soft}} \gtrsim$

10^6 TeV. Furthermore, the role of a_b is now slightly more complicated. From (2.3.8) we can see that a_b couples to the gauge sector via the topological term. Hence, QCD effects will generate a potential, making the axion massive. The decay of τ_b into a_b then gives rise to dark radiation and through the misalignment mechanism the axions become dark matter.

At first we have a look at the predictions for dark radiation. Again, we can compute the decay rates from the low energy effective Lagrangian. For our model the Kähler potential and the gauge kinetic function now read [111]

$$\begin{aligned} K = & -3\ln(T_b+\bar{T}_b)+ \\ & + \frac{(T_a+\bar{T}_a)^{1/2}}{T_b+\bar{T}_b}\left(H_u\bar{H}_u + H_d\bar{H}_d\right) + \frac{(T_a+\bar{T}_a)^{1/2}}{T_b+\bar{T}_b}\left(zH_uH_d + \text{h.c.}\right) + \dots \ , \end{aligned} \tag{2.3.9}$$

$$f_a = T_a + hS \ . \tag{2.3.10}$$

The visible cycle modulus T_a can be integrated out after D-term stabilisation by simply substituting $T_a = c_a T_b$. As in the sequestered LVS the decay rates can be directly computed from the relevant interaction terms of the Lagrangian. For the decays into dark radiation we still find

$$\Gamma(\phi \to aa) = \frac{1}{48\pi}\frac{m_\phi^3}{M_p^2} \ , \tag{2.3.11}$$

see (2.2.9). As opposed to the sequestered LVS the canonically normalised volume modulus ϕ can only decay into the light Higgs labelled by h and we obtain

$$\Gamma(\phi \to hh) = \frac{z^2}{96\pi}\frac{\sin^2(2\beta)}{2}\frac{m_\phi^3}{M_p^2} \ . \tag{2.3.12}$$

where β is defined such that $\tan\beta$ is the ratio of the two Higgs-VEVs. In addition, our non-sequestered LVS gives rise to a decay into gauge bosons with decay rate

$$\Gamma(\phi \to AA) = \frac{N_g}{96\pi}\gamma^2\frac{m_\phi^3}{M_p^2} \ , \tag{2.3.13}$$

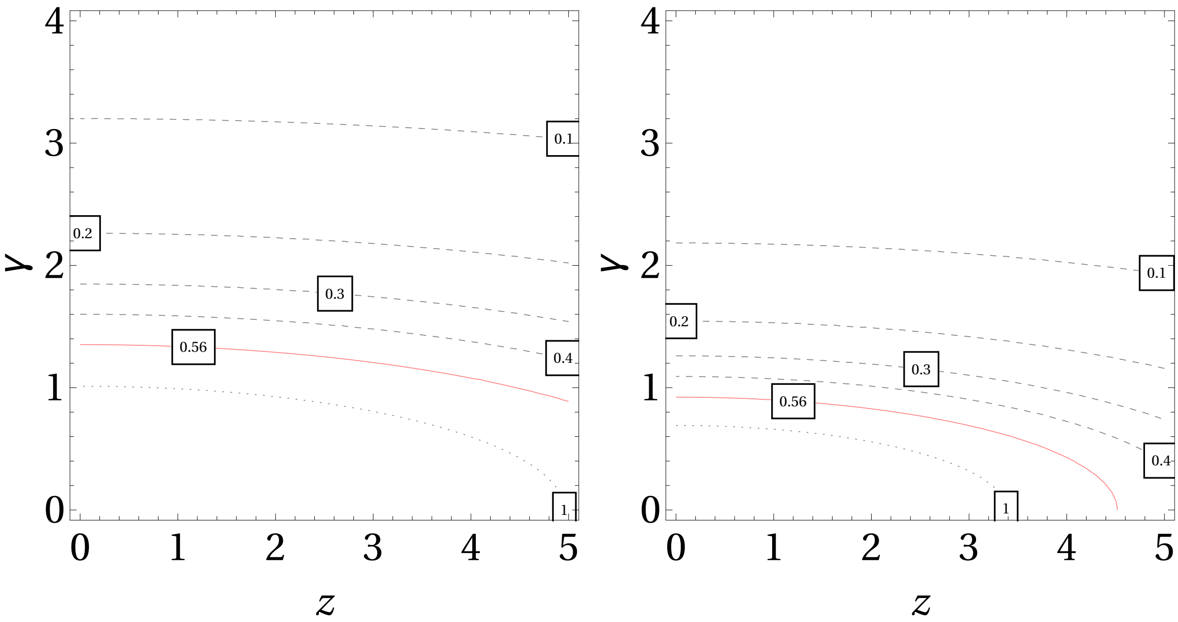
where N_g is the number of gauge bosons and the parameter γ is defined as

$$\gamma \equiv \frac{\tau_a}{\tau_a + h\text{Re}(S)} \ . \tag{2.3.14}$$

This factor γ^2 arises essentially from $\Gamma \sim K^{-1}_{T_b\bar{T}_b}|\partial_{T_b} f_{\text{vis}}|^2/(\text{Re}(f_{\text{vis}}))^2$, see Appendix B for a summary of formulae. Various values for γ correspond to the following regimes: For the case that gauge fluxes do not contribute to the gauge kinetic function, i.e. $h = 0$, one finds $\gamma = 1$. For $|\gamma| \ll 1$ the gauge kinetic function is dominated by the flux-dependent part $h\ \text{Re}(S)$. For $\gamma \gg 1$ we require a delicate cancellation between contributions from τ_a and $h\ \text{Re}(S)$. Since it is not known how much cancellation can be achieved in the string landscape, we cannot safely rely on arbitrarily large values of γ.

Now, using (2.1.1), we can write down ΔN_{eff} for our first scenario:

$$\boxed{\Delta N_{\text{eff}} = \frac{43}{7}\left(\frac{10.75}{g_*(T_d)}\right)^{1/3}\frac{1}{\frac{\sin^2(2\beta)}{4}z^2 + \frac{N_g}{2}\gamma^2}} \ . \tag{2.3.15}$$



(a) Predictions for $g_*(T_d) = 10.75$.

(b) Predictions for $g_*(T_d) = 106.75$.

Figure 2.3.: Dark radiation predictions for non-sequestered LVS with cycle-stabilisation via D-terms. Plots (a) and (b) of ΔN_{eff} vs. z and γ show the predictions for $g_*(T_d) = 10.75$ and $g_*(T_d) = 106.75$, respectively. For the plots we choose $\sin(2\beta) = 1$, but the plots are applicable to any value of β by simply redefining $\tilde{z}^2 = \sin^2(2\beta)z^2$ and relabieling $z \to \tilde{z}$ on the horizontal axis. If we take the Planck 2015 measurements $\Delta N_{\text{eff}} < 0.56$ (2σ) [1] at face value, the regions of the plots below the red curve are disfavoured at 2σ. In particular the dotted curve corresponding to $\Delta N_{\text{eff}} = 1$ is excluded. Those plots are based on fig. 1 in [111], but adapted and modified in the light of the new Planck data.

In the following we assume $N_g = 12$ of the SM gauge group $SU(3)\times SU(2)\times U(1)$, in order to keep the matter spectrum minimal. Furthermore, we choose $\sin(2\beta) = 1$ to minimise ΔN_{eff}. Since high-scale supersymmetry suggests $\sin(2\beta) = 1$ [124; 131–133], the choice is indeed well-motivated as our model requires high-scale SUSY.

Let us analyse the consequences of a 2σ bound $\Delta N_{\text{eff}} < 0.56$. For a rather low reheating temperature such that $g_*(T_d) = 10.75$ we obtain the condition

$$\frac{z^2}{4} + 6\gamma^2 > 10.9 \ . \tag{2.3.16}$$

Already a mild tuning $\gamma > 1.4$ is sufficient to fulfil this inequality, even if $z = 0$. For large enough temperatures such that $g_*(T_d) = 106.75$, we obtain the condition

$$\frac{z^2}{4} + 6\gamma^2 > 5.07 \ , \tag{2.3.17}$$

and $\gamma > 0.92$ already satisfies the bound even without Higgs-channel. Thus, we conclude that current bounds on dark radiation can be satisfied already if the gauge coupling is dominated by the contribution from τ_a. A delicate cancellation can evade even slightly stronger bounds. For instance, if future measurements suggest $\Delta N_{\text{eff}} < 0.1$ then we need $\gamma > 2.2$ for $g_*(T_d) = 106.75$ (or $\gamma > 3.2$ for $g_*(T_d) = 10.75$). This corresponds to a fine-tuning between τ_a and $h\text{Re}(S)$ of one part in two (or one part in three). Predictions for various choices of z and γ are shown in Figure 2.3.

As we pointed out already, the coupling of our axion a_b to QCD has to be taken into account. The coupling term $\sim a_b \text{tr}(F \wedge F)$ leads to a potential for the axion and via the misalignment mechanism the axion contributes to dark matter. The resulting amount of axion dark matter decisively depends on the coupling to QCD. In order to determine the axion decay constant we start from the relevant terms of the effective Lagrangian (see e.g. [134; 135]):

$$\mathscr{L} \supset K_{T_b \bar{T}_b} \partial_\mu a_b \partial^\mu a_b + \frac{c_a \tau_b}{4\pi} F_{\mu\nu} F^{\mu\nu} + \frac{c_a a_b}{4\pi} F_{\mu\nu} \tilde{F}^{\mu\nu} \ . \tag{2.3.18}$$

Canonical normalisation requires to replace $a_b \to a_b / \sqrt{2K_{T_b \bar{T}_b}}$ and $F_{\mu\nu} \to \sqrt{\pi/(c_a \tau_a)} F_{\mu\nu}$ in the above Lagrangian. We then arrive at

$$\mathscr{L} \supset \frac{g^2}{32\pi^2} \frac{c_a}{f_{a_b}} a_b F_{\mu\nu} \tilde{F}^{\mu\nu} \ , \tag{2.3.19}$$

where

$$g^{-2} = \frac{c_a \tau_b}{4\pi} \ , \qquad f_{a_b} = \frac{\sqrt{2K_{T_b \bar{T}_b}}}{2\pi} \ . \tag{2.3.20}$$

Note that there are observational bounds on the 'effective' axion decay constant f_{a_b}/c_a. For one, observation requires $f_{a_b}/c_a < 10^{12}$ GeV [136], which can however be relaxed if the initial misalignment angle is tuned. Moreover, to avert excessive cooling of stars an axion coupling to QCD also has to satisfy $f_{a_b}/c_a > 10^9$ GeV.

Let us now quantify the axion dark matter abundance (here we follow closely our work [111]). If the Pecci-Quinn (PQ) symmetry is broken before inflation, the initial misalignment angle $\theta_i = \frac{a_{b,\text{initial}}}{f_{a_b}/c_a} \in [-\pi, \pi)$ is homogeneous in our patch. The axion relic density is then (see e.g. [136]):

$$\Omega_a h^2 \sim 3 \times 10^3 \left(\frac{f_a/c_a}{10^{16} \text{GeV}} \right)^{7/6} \theta_i^2 \ . \tag{2.3.21}$$

The axions can contribute to the amount of cold dark matter, whose density parameter has been measured [1]: $\Omega_{\text{DM}} h^2 = 0.1197 \pm 0.0022$ (68% CL; Planck + lowP). Hence, for generic initial misalignment angles there is an overproduction of axion dark matter if $f_{a_b}/c_a \gtrsim 10^{12}$ GeV. Using (2.3.19), we obtain

$$\frac{f_{a_b}}{c_a} = \frac{\sqrt{2K_{T_b \bar{T}_b}} M_p}{2\pi c_a} = \frac{\sqrt{6} M_P}{4\pi c_a \tau_b} \sim 10^{16} \text{ GeV} \quad \text{for} \quad c_a \tau_b = \alpha_{\text{vis}}^{-1} \sim 25 \ , \tag{2.3.22}$$

where we used in the second step the leading order result $K_{T_b \bar{T}_b} = 3/(4\tau_b^2)$. Thus only a misalignment angle θ_i fine-tuned to sufficiently small values can evade an overabundance of axion dark matter. This tuning can be justified anthropically [137] and we find that $\theta_i \sim 10^{-2}$ is sufficient to evade dark matter bounds.

Bounds on isocurvature perturbations can lead to even stricter constraints for the QCD axion. If the tensor-to-scalar ratio r is of order $\mathcal{O}(0.1)$ the QCD axion candidate with $f_{a_b}/c_a \sim 10^{16}$ GeV will source excessive isocurvature perturbations [138]. In consequence, the scenario described in this section would be ruled out.

To avoid those problems it is conceivable to include another axion $\tilde{a}$ with a decay constant 10^9 GeV $< f_{\tilde{a}} < 10^{12}$ GeV which couples to QCD as follows:

$$\mathscr{L} \supset \frac{g^2}{32\pi^2}\left(\frac{\tilde{a}}{f_{\tilde{a}}} + \frac{a}{f_{a_b}/c_a}\right) F_{\mu\nu}\tilde{F}^{\mu\nu} \ . \tag{2.3.23}$$

The QCD axion is then mainly given by $\tilde{a}$, which can evade all bounds: its axion decay constant $f_{\tilde{a}}$ is small enough to evade the dark matter bounds without any fine-tuning of the misalignment angle. Further, $f_{\tilde{a}}$ is lower than the Hubble scale of inflation $H_I \sim 10^{14}$ GeV in case of large-field inflation, which implies that the PQ symmetry is intact during inflation. In this case the axion will not source excessive isocurvature perturbations. Beyond the QCD axion there will also be a combination of axions, dominantly given by a_b, which will remain light and contribute to dark radiation. This axion is then unaffected by isocurvature and dark matter bounds.

2.3.2. Stabilisation of the Visible Sector via String-Loop Corrections

In the following we make use of string-loop corrections to stabilise the 4-cycle wrapped by the visible sector. For this purpose we will consider fibred Calabi-Yau manifolds, for which examples of LVS models exist. The below presentation is a summary of our work [111].[3]

For our purpose we consider a CY 3-fold of volume

$$\mathcal{V} = \alpha\left(\sqrt{\tau_1}\tau_2 - \gamma_{\rm np}\tau_{\rm np}^{3/2}\right) \ . \tag{2.3.24}$$

The total volume $\mathcal{V}$ and $\tau_{\rm np}$ are stabilised via the LVS procedure, such that $\mathcal{V} \simeq \sqrt{\tau_1}\tau_2 \gg 1$.[4] Since $\mathcal{V} \simeq \sqrt{\tau_1}\tau_2$ is fix only as a combination, there is clearly one flat direction. This mode, which we denote as χ in the subsequent computations, has to be fixed by string-loop corrections as in [140].

Let us realise the visible sector by D7-branes on the fibre τ_1, see Figure 2.4. In order to obtain an appropriate physical gauge coupling, the fibre volume τ_1 has to be small compared to τ_2. This leads to the "anisotropic limit" $\tau_2 \gg \tau_1 \gg \tau_{\rm np}$, which is also discussed e.g. in [130; 141]. One could also realise the Standard Model by D7-branes on another cycle τ_a stabilised by D-terms so that τ_a is coupled to the size of the K3-fibre [141].[5] Then the constraint on the size of τ_1 can be relaxed, but one still has to ensure that τ_a is fixed with the correct size for the gauge coupling. Once again, a tuning is required, albeit potentially less severe than in our previous model. In any case, as the visible sector gauge coupling depends on τ_1 in both cases, our results in what follows will be the same for both situations.

As reheating proceeds via decays of the lightest modulus, we need to look carefully at the masses of the bulk volume $\mathcal{V}$ and the mode χ (orthogonal to $\mathcal{V}$) [143]:

$$m_{\mathcal{V}}^2 \sim \frac{M_p^2}{g_s^{3/2}\mathcal{V}^3 \ln \mathcal{V}} \quad \text{and} \quad m_\chi^2 \sim \frac{M_p^2}{\mathcal{V}^3\sqrt{\langle\tau_1\rangle}} \ . \tag{2.3.25}$$

[3] Our work [111] appeared simultaneously with [139], which also studies dark radiation in fibred LVS models. However, in that paper the visible sector was realised at a singularity, resulting in a severe dark radiation excess.

[4] It is possible to stabilise some of the cycles via D-terms as in the previous section. If this is the case, we interpret $\mathcal{V}$ as the volume after integrating out the moduli stabilised by D-terms.

[5] For a visible sector on two intersecting blow-up modes stabilised by both D-terms and string-loops see [142].

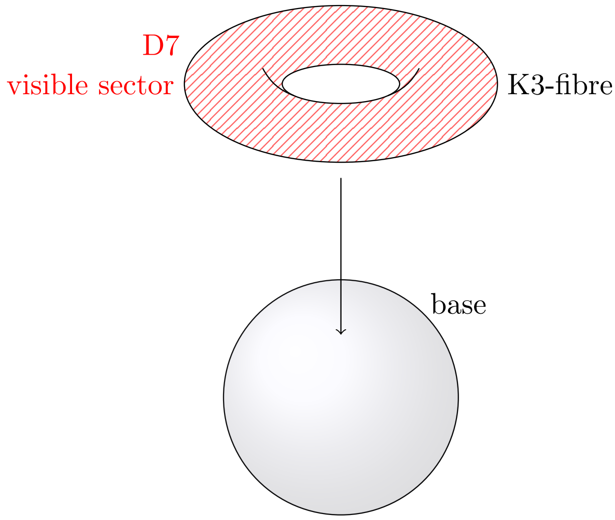


Figure 2.4.: This illustration shows a K3-fibration. The visible sector is realised by a D7-brane wrapping the K3-fibre, which is here naively represented as a complex torus. It is fibred over a base illustrated as S^2.

For simplicity we will assume that χ is lighter than the volume mode[6], such that the latter can be integrated out. Then, one can see immediately that τ_1 cannot be fixed too small.[7]

There is also another difference with respect to the previous model to be aware of. For every cycle τ_i stabilised by string-loop corrections the corresponding axion a_i will remain light. (Due to shift symmetry perturbative effects cannot generate an axion potential.) Hence, in our setup we will have to deal with two light axions $a_1 = \mathrm{Im}(T_1)$ and $a_2 = \mathrm{Im}(T_2)$. Both will contribute positively to ΔN_{eff}, but only a_1 will couple to QCD, because we realise the Standard Model on D7-branes wrapping the cycle τ_1. Thus, the axion a_1 will once again constrain our setup, while a_2 will only contribute to dark radiation.

Phenomenology

We begin by looking at the kinetic terms for τ_1, τ_2, a_1 and a_2, which can be obtained from the Kähler potential $K \supset -2\ln\mathcal{V}$. We find

$$\begin{aligned}\mathscr{L} \supset\ & \frac{1}{4\tau_1^2}\partial_\mu\tau_1\partial^\mu\tau_1 + \frac{1}{2\tau_2^2}\partial_\mu\tau_2\partial^\mu\tau_2 + \frac{\gamma_{\mathrm{np}}}{2}\frac{\tau_{\mathrm{np}}^{3/2}}{\tau_1^{3/2}\tau_2^2}\partial_\mu\tau_1\partial^\mu\tau_2 \\ & + \frac{1}{4\tau_1^2}\partial_\mu a_1\partial^\mu a_1 + \frac{1}{2\tau_2^2}\partial_\mu a_2\partial^\mu a_2 + \frac{\gamma_{\mathrm{np}}}{2}\frac{\tau_{\mathrm{np}}^{3/2}}{\tau_1^{3/2}\tau_2^2}\partial_\mu a_1\partial^\mu a_2\ .\end{aligned} \tag{2.3.26}$$

[6]If both masses were comparable, one would have to analyse the decay of both moduli simultaneously. In this case the analysis would be considerably more complicated.

[7]Another reason why τ_1 cannot be too small is the following: The presence of fluxes could generate a non-perturbative superpotential with the term e^{-aT_1}. Hence τ_1 should be large enough to suppress $e^{-a\tau_1}$ sufficiently.

To get the Lagrangian for χ we integrate out τ_2 by $\tau_2 = \alpha^{-1}\mathcal{V}\tau_1^{-1/2}$. Upon canonical normalisation (see also [144])

$$\tau_1 = e^{\frac{2}{\sqrt{3}}\chi} \,, \qquad a_1 = \sqrt{2}a_1' \,, \qquad a_2 = \frac{\mathcal{V}}{\alpha}a_2' \tag{2.3.27}$$

we obtain

$$\mathscr{L} \supset \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \frac{1}{2}e^{-\frac{4}{\sqrt{3}}\chi}\partial_\mu a_1'\partial^\mu a_1' + \frac{1}{2}e^{\frac{2}{\sqrt{3}}\chi}\partial_\mu a_2'\partial^\mu a_2' \,. \tag{2.3.28}$$

Henceforth we will drop the primes on the canonically normalised axions. From the Lagrangian we can now read off[8] the decay rates for $\chi \to a_1a_1$ and $\chi \to a_2a_2$:

$$\Gamma(\chi \to a_1a_1) = = \frac{1}{24\pi}\frac{m_\chi^3}{M_p^2} \tag{2.3.29}$$

$$\Gamma(\chi \to a_2a_2) = = \frac{1}{96\pi}\frac{m_\chi^3}{M_p^2} \,. \tag{2.3.30}$$

To calculate the decay rate into the Higgs fields we use

$$K \supset \frac{(T_1+\bar{T}_1)^{1/2}}{\mathcal{V}^{2/3}}\left(H_u\bar{H}_u + H_d\bar{H}_d\right) + \frac{(T_1+\bar{T}_1)^{1/2}}{\mathcal{V}^{2/3}}\left(zH_uH_d + \text{h.c.}\right) + \ldots \,. \tag{2.3.31}$$

The decay into the light Higgs h is then given by

$$\Gamma(\chi \to hh) = \frac{z^2\sin^2(2\beta)}{96\pi}\frac{m_\chi^3}{M_p^2} \,. \tag{2.3.32}$$

Finally, the decay into gauge bosons of the visible sector is due to the gauge kinetic function

$$f_{\text{vis}} = T_1 + hS \tag{2.3.33}$$

for the case of D7-branes wrapping the 4-cycle τ_1. It follows

$$\Gamma(\chi \to AA) = \frac{N_g}{48\pi}\gamma^2\frac{m_\chi^3}{M_p^2} \,, \tag{2.3.34}$$

where γ is defined in analogy to (2.3.14) by

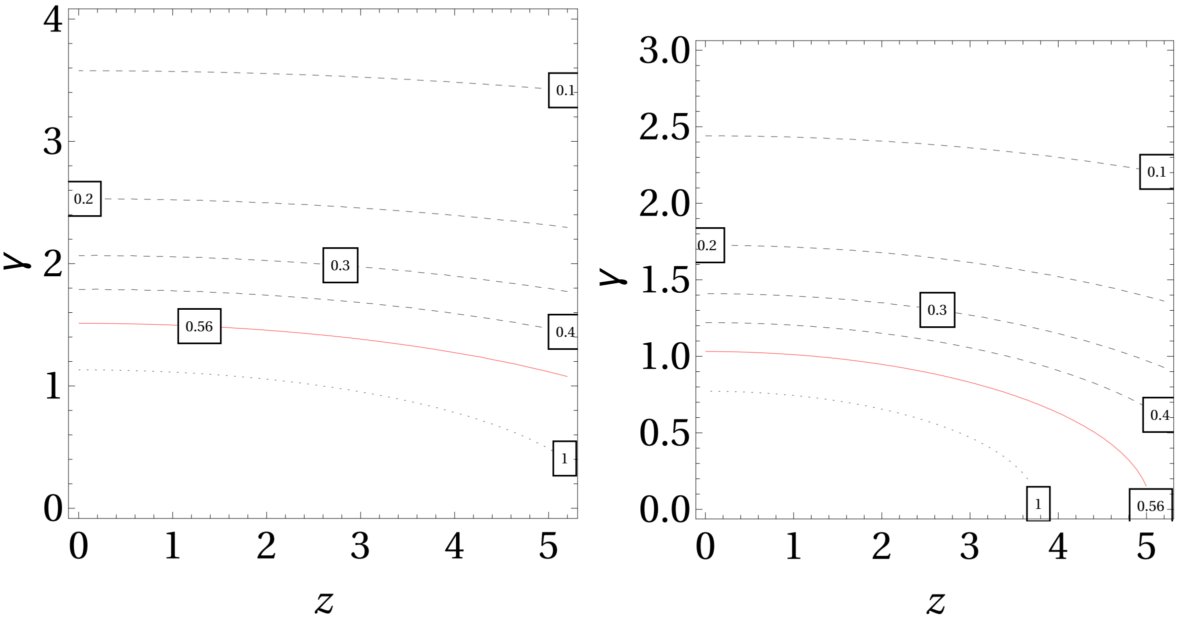
$$\gamma \equiv \frac{\tau_1}{\tau_1 + h\text{Re}(S)} \,. \tag{2.3.35}$$

Once again, $\gamma = 1$ means that the gauge coupling is primarily determined by τ_1 (geometric regime). For $|\gamma| < 1$ the gauge kinetic function is dominated by the flux-dependent part h Re(S). For $\gamma > 1$ we require a cancellation between contributions from τ_1 and h Re(S).

In this model ΔN_{eff} can be computed as follows:

$$\boxed{\Delta N_{\text{eff}} = \frac{43}{7}\left(\frac{10.75}{g_*(T_d)}\right)^{1/3}\frac{5}{z^2\sin^2(2\beta) + 2N_g\gamma^2} \,.} \tag{2.3.36}$$

[8] Indeed, the numerical factors can be obtained immediately by comparison with (2.2.9) obtained from a coupling term $\sqrt{2/3}\phi(\partial a)^2$. Since we have now $2/\sqrt{3}\chi(\partial a_1)^2$ and $1/\sqrt{3}\chi(\partial a)^2$, the corresponding decay rates are obtained by multiplying (2.2.9) by factors of $(\sqrt{2})^2 = 2$ and $(1/\sqrt{2})^2 = 1/2$, respectively.



(a) Predictions for $g_*(T_d) = 10.75$.

(b) Predictions for $g_*(T_d) = 106.75$.

Figure 2.5.: Dark radiation predictions for non-sequestered LVS with cycle-stabilisation via string-loop corrections. Again, plots (a) and (b) of ΔN_{eff} vs. z and γ show the predictions for $g_*(T_d) = 10.75$ and $g_*(T_d) = 106.75$, respectively. As in the previous plots we chose $\sin(2\beta) = 1$, but the plots are applicable to any value of β by simply redefining $\tilde{z}^2 = \sin^2(2\beta)z^2$ and relabieling $z \to \tilde{z}$ on the horizontal axis. If we take the Planck 2015 measurements $\Delta N_{\text{eff}} < 0.56$ (2σ) [1] at face value, the regions of the plots below the red curve are disfavoured at 2σ. In particular the dotted curve corresponding to $\Delta N_{\text{eff}} = 1$ is excluded. Those plots are based on fig. 2 in [111], but adapted and modified in the light of the new Planck data.

For a comparison with observational data we choose once again $N_g = 12$ and $\sin(2\beta) = 1$. Then, $\Delta N_{\text{eff}} < 0.56$ translates into

$$z^2 + 24\gamma^2 > 55 \tag{2.3.37}$$

for $g_*(T_d) = 10.75$ and

$$z^2 + 24\gamma^2 > 26 \tag{2.3.38}$$

for $g_*(T_d) = 106.75$. We can satisfy these inequalities without the Higgs sector if $\gamma > 1.5$ and $\gamma > 1.0$, respectively. Thus, a mild tuning allows to saturate observational bounds even without the Higgs-channel. However, much more fine-tuning is required if observational bounds become tighter in the future. For instance, if $\Delta N_{\text{eff}} < 0.1$ one needs $\gamma > 3.6$ (for $g_*(T_d) = 10.75$) and $\gamma > 2.4$ (for $g_*(T_d) = 106.75$). Our results for various choices of z and γ are presented in Figure 2.5.

Further constraints due to the coupling of a_1 to QCD will arise in analogy to the previous model. Once again, the misalignment angle has to be tuned to avoid overabundance of dark matter. This fine-tuning may be justified anthropically. Moreover, if primordial gravitational waves are detected with tensor-to-scalar ratio $r \sim 0.01 - 0.1$ the axion a_1 with $f_{a_1} \sim 10^{16}$ GeV will lead to excessive isocurvature perturbations and thus, our model would be ruled out. A possible loophole would then be again to include an additional axion that takes over the role of the QCD axion (see explanations in the previous section).

2.3.3. Stabilisation of the Visible Sector via Non-Perturbative Effects

We will now describe dark radiation predictions for the case where the visible sector cycle is stabilised by non-perturbative effects. Our conclusion will be that for those models dark radiation bounds are even more restrictive than in the sequestered scenarios. Correspondingly, our analysis here will be less detailed than our examinations in the previous sections. Much of the subsequent discussions follow our paper [111] with only few modifications.

Here, we will analyse a simple toy model which nevertheless exhibits all the necessary features. To be specific, we consider a compactification with a volume of Swiss-Cheese type:

$$\mathcal{V} = \eta_b \tau_b^{3/2} - \eta_s \tau_s^{3/2} \ . \tag{2.3.39}$$

Generalisations to setups with more than one cycle of type τ_s are straightforward. The visible sector will be realised by D7-branes wrapping τ_s. At the same time τ_s will be wrapped by an E3-instanton or D7-branes exhibiting gaugino condensation, thus giving a non-perturbative contribution $W_{\rm np}$ to the superpotential:

$$W = W_0 + W_{\rm np} = W_0 + A_s e^{-a_s T_s} \ . \tag{2.3.40}$$

The parameter a_s is model-dependent and depends on the non-perturbative effect wrapping the small cycle τ_s. For the case of an E3-instanton we have $a_s = 2\pi$ while for for gaugino condensation on a stack of N D7-branes we have $a_s = \frac{2\pi}{N}$. The prefactor A_s depends on the dilaton and the complex structure moduli and is a constant at this stage.

The moduli τ_b and τ_s will be stabilised by the standard LVS procedure. As before, the lightest modulus is τ_b and its axion partner is essentially massless (2.2.2).

Phenomenology

Here we will analyse the rates for decays of τ_b into axions a_b vs. SM fields.

First of all, realising the visible sector on τ_s leads to superpartners which are heavier than τ_b: $m_{1/2} \sim m_{soft} \sim M_P/\mathcal{V}$ (see e.g. [66]). Hence, decays of the canonically normalised bulk modulus field ϕ into matter scalars, the heavy Higgs and gauginos are kinematically forbidden.

As in our previous models, decays into the light Higgs will arise through the Giudice-Masiero term with a decay rate of the form

$$\Gamma(\phi \to hh) = \mathcal{O}(1) \frac{2z^2}{48\pi} \frac{\sin^2(2\beta)}{2} \frac{m_\phi^3}{M_p^2} \ . \tag{2.3.41}$$

Note that the exact expression for decay rate into the light Higgs will depend on the moduli dependence of the Kähler metric for Higgs fields. We do not give more detail as we do not expect any improvement compared to previous sections.

We will be most interested in the decay rate into gauge bosons, which can be calculated given the gauge kinetic function

$$f_{\rm vis} = T_s + hS \ . \tag{2.3.42}$$

To study decays of ϕ we need the effective theory for ϕ which is obtained by integrating out τ_s. By minimising the F-term potential w.r.t. τ_s one obtains (see e.g.[7; 145]):[9]

$$\tau_s = \frac{1}{a_s} \ln \left(\frac{4 a_s A_s}{3 \eta_s} \frac{\mathcal{V}}{W_0} \right) + \mathcal{O}(\ln \tau_s) \tag{2.3.44}$$

Most importantly, this introduces a dependence of the tree-level visible sector gauge coupling on $\ln \mathcal{V} \sim \ln \tau_b$. The rates for the decay channels of interest are then

$$\Gamma(\phi \to a_b a_b) = \frac{1}{48\pi} \frac{m_\phi^3}{M_p^2} \;, \tag{2.3.45}$$

$$\Gamma(\phi \to AA) = \frac{3 N_g}{128\pi} \; \frac{\gamma^2}{(a_s \tau_s)^2} \; \frac{m_\phi^3}{M_p^2} \;, \tag{2.3.46}$$

for decays of the canonically normalised field ϕ into axions a_b and gauge bosons, respectively. Once again, γ is defined by

$$\gamma = \frac{\tau_s}{\tau_s + h \ \mathrm{Re}(S)} \tag{2.3.47}$$

as in the previous sections, and N_g is the number of generators of the SM gauge group.

Let us analyse what we found: The decay rate into axions is unchanged compared to the sequestered scenario. Further, we find that the decay rate into Higgs fields is not parametrically different from the expressions found in previous sections. In contrast, the decay rate into SM gauge bosons is suppressed by $(a_s \tau_s)^{-2}$ compared to the cases where the visible sector cycle was stabilised by D-terms or string loops.

Therefore, it is easy to see that this construction is more constrained by dark radiation bounds than the setups with the visible sector being stabilised by D-terms or string loops. As the visible sector is realised on τ_s, we require $\tau_s \sim 25$ for an acceptable gauge coupling. In addition $a_s = 2\pi$ or $2\pi/N$ depending on the non-perturbative effect sourced by τ_s. Also, unless there is tuning between τ_s and h Re(S) we have $\gamma \sim 1$. Then it follows that the decay rate into gauge bosons is typically suppressed with respect to the decay rate into axions. This negative conclusion would only fail if we have $N \gtrsim 100$, corresponding to a gauge group $SU(N \gtrsim 100)$ on the stack of D7-branes exhibiting gaugino condensation. Alternatively, one could tune γ to adjust the amount of dark radiation. As the amount of dark radiation is not expected to have an effect on the development of life, such a tuning cannot be justified anthropically.

2.3.4. Comparison with Previous Work

In this section we briefly comment[10] on further extensions of LVS models which have been suggested in [146] and [147], where a non-sequestered LVS is combined with poly-instanton corrections to the superpotential. The authors consider a scenario with a Swiss-cheese CY with volume $\mathcal{V} = (\eta_1 \tau_1)^{3/2} - (\eta_2 \tau_2)^{3/2} - (\eta_3 \tau_3)^{3/2}$. Two separate stacks of D7-branes wrapping the 4-cycle τ_2 yield a superpotential with terms e^{-f_i} (race-track model), where the gauge-kinetic

[9]Here, the F-term potential is given by (see e.g. [145])

$$V = \frac{8 a_s^2 |A_s|^2 \sqrt{\tau_s} e^{-2 a_s \tau_s}}{3 \eta_s \mathcal{V}} - \frac{4 a_s |A_s| |W_0| \tau_s e^{-a_s \tau_s}}{\mathcal{V}^2} + \frac{3 \xi \gamma |W_0|^2}{4 g_s^{3/2} \mathcal{V}^3} \;, \tag{2.3.43}$$

where $\xi = -\frac{\zeta(3)\chi(X)}{2(2\pi)^3}$ parameterises the $(\alpha')^3$ correction to the Kähler potential: $K = -2\ln(\mathcal{V} + \frac{\xi}{2 g_s^{3/2}})$.

[10]This discussion follows sect. 3.4 of our paper [111].

function is of the form $f_i = T_2 + C_i e^{-2\pi T_3} + S$ due to gaugino condensations and Euclidean D3-instantons on the non-rigid cycle τ_3. Additionally, the VEV of the flux superpotential is assumed to be zero.

This race-track model allows to construct (large volume) minima by integrating out the heaviest modulus T_2 near the supersymmetric locus, $\partial_{T_2} W_{\rm np} = 0$. Hence, one is left with an effective superpotential $W_{\rm eff}$ which is small due to its exponential suppression by the VEV of τ_2. The stabilisation of T_1 and T_3 then proceeds as in the usual LVS.

In our work we do not consider this scenario any further since no substantial improvement of dark radiation bounds relative to more conventional LVS constructions is expected. The main hope for such an improvement is associated with modulus decays to gauginos which, according to [147], are very light. However, the corresponding rate $\Gamma_{1/2} \sim (M_{1/2}^2 m_\phi)/M_p^2$ is much smaller than the decay rate $\Gamma \sim m_\phi^3/M_p^2$ of the lightest modulus ϕ into axions due to the hierarchy $M_{1/2} \ll m_\phi$. This scaling of $\Gamma_{1/2}$ can be understood by expanding the generic gaugino Lagrangian

$$f(\mathcal{V})\lambda\not{\partial}\lambda + g(\mathcal{V})\lambda\lambda\,, \tag{2.3.48}$$

where $M_{1/2} = g/f$ is the physical gaugino mass, around the VEV of the volume: $\mathcal{V} = \mathcal{V}_0 + \delta\mathcal{V}$. We use the fact that both f and g do not depend on $\mathcal{V}$ more strongly than through some power, $f \sim \mathcal{V}^\alpha$ and $g \sim \mathcal{V}^\beta$. This implies that, at most $f'(\mathcal{V}) \sim f(\mathcal{V})/\mathcal{V}$, and similarly for g. Furthermore, one has to recall that the modulus $\mathcal{V}$ and the corresponding canonically normalised field ϕ are related by $\mathcal{V} \sim \exp(\phi/M_p)$. With the expansion $\phi = \phi_0 + \delta\phi$ one then finds the following parametric form of the Lagrangian relevant for the three-particle vertex and hence for the decay:

$$f(\mathcal{V})\frac{\delta\phi}{M_p}\lambda\not{\partial}\lambda + g(\mathcal{V})\frac{\delta\phi}{M_p}\lambda\lambda\,. \tag{2.3.49}$$

Now we canonically normalise, $\lambda \to \lambda/\sqrt{f}$, and use equations of motion in the first term, $i\not{\partial}\lambda = M_{1/2}\lambda$. This gives a contribution of the order of

$$\mathscr{L}_{\rm int} \supset \frac{M_{1/2}}{M_p}\delta\phi\lambda\lambda\,. \tag{2.3.50}$$

from both the first and second term above. From here, one can read off the decay rate (up to some numerical factors). Due to its suppression via the mass hierarchy, one can safely neglect this channel.

2.4. Further Comments and Conclusions

In the previous sections we discussed how to ameliorate the dark radiation excess in non-sequestered Large Volume Scenarios. In our work [111] we also proposed a sequestered LVS with flavour branes included. Here, we only briefly sketch the idea and results. More details can be found in [111] and in the thesis of F. Rompineve [120].

Flavour branes are 7-branes in the geometric regime passing through a singularity of a CY manifold with the Standard Model being realised at this singularity, see Figure 2.6.[11] Our motivation behind the inclusion of flavour branes is to obtain an indirect decay channel of the canonically normalised volume modulus ϕ into Standard Model gauge bosons in the

[11] Flavour branes have been known for a long time in the context of "model building at a singularity" [148]. For rather recent string model building using flavour branes see [149].

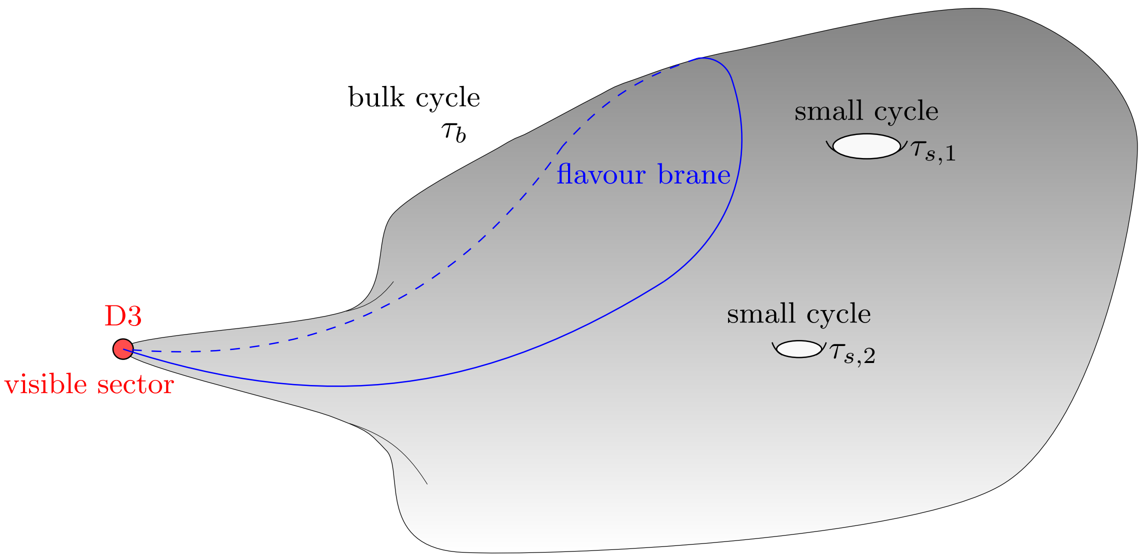


Figure 2.6.: This picture illustrates the realisation of the sequestered LVS with a flavour brane (shown in blue) included. It wraps the bulk cycle and intersects the singularity, in which a D3-brane for the realisation of the visible sector (in red) is located.

sequestered LVS. Due to the flavour branes, on which there live gauge bosons A_μ^{fl}, one has not only the decay channel $\phi \to a_b a_b$ into dark radiation, but also a new decay channel $\phi \to A_\mu^{\mathrm{fl}} A_\mu^{\mathrm{fl}}$. The Standard Model is then reheated by the subsequent decay $A_\mu^{\mathrm{fl}} \to \mathrm{SM}$. For this proposed scenario to work the mass m_A of the flavour gauge bosons must be below $m_{\tau_b}/2$ for the decay to be kinematically possible. One can show that there are no strong lower bounds on m_A (e.g. from imposing a lower bound on the reheating temperature of the Standard Model in order not to affect the standard Big Bang Nucleosynthesis (BBN)) [111]. Note that there are also consistency conditions from string theory on this model, which turn into a constraint on the number of flavour branes allowed [149]. An advantage of this model compared to our previously proposed non-sequestered Large Volume models is that the dark radiation axion does not couple to QCD and hence there are no constraints from dark matter overproduction or isocurvature bounds.

The decay rates into dark radiation and the Higgs sector are given by the formulae for the sequestered LVS, i.e. (2.2.9) and (2.2.11). The decay rate into the flavour gauge bosons is given by (2.3.13) with $N_g = N_f$ being the number of generators of the flavour gauge group and $\gamma = 1$ (geometric regime, i.e. $f_{\mathrm{fl}} = T_b$). It follows that for $\Delta N_{\mathrm{eff}} < 0.56$ one needs at least $\mathcal{O}(10)$ flavour gauge bosons. If the dark radiation bounds become tighter, this flavour brane scenario could turn out to be more constrained. It would be interesting to work out details of this model, especially how many flavour branes can be embedded at most. We leave this question for future work.

Further Comments on Recent Progress

In the recent work [150] the authors proposed to ameliorate the dark radiation excess within the sequestered LVS. The idea is to investigate decay channels of the bulk modulus into scalars in the context of split-SUSY. In such models the masses m_0 of squarks and sleptons are at the order of m_{τ_b}. For a decay of τ_b into such scalars to be possible one must have $m_{\tau_b} \geq 2m_0$. In [150] it was, however, shown that squarks and sleptons are typically, i.e. for most of the parameter space, heavier than τ_b. For those cases one would run into a severe dark radiation excess. The authors therefore proposed to include string-loop corrections to the Kähler potential, which then allow for a subset of parameter space in which decays into SUSY scalars are kinematically admissible. Furthermore, it was shown that this decay channel leads to a significant suppression of the branching ratio for the decay into dark radiation. Thereby it is possible to obtain $0.14 \lesssim \Delta N_{\text{eff}} \lesssim 1.6$ for their models, depending on the precise SUSY scalar masses [150].

Conclusions

In summary, we found that our proposed non-sequestered models as well as the combination of the sequestered LVS with flavour branes are more flexible in terms of evading observational bounds on dark radiation than the original sequestered LVS models of e.g. [121]. For instance, in the first two models we presented (stabilisation of the visible sector cycle by D-terms or string-loop corrections) we do not rely on a particularly huge number of Higgs doublets or sufficiently large Giudice-Masiero coupling z. Instead, we have a considerable suppression of ΔN_{eff} due to the decay channel to Standard Model gauge bosons. Depending on how much fine-tuning in the sense of a cancellation between the contributions from τ_{vis} and $h\text{Re}(S)$ is admissible, the dark radiation abundance can be below current bounds. To give an example for our model with D-term stabilisation Section 2.3.1, a tuning $\tau_{\text{vis}} : (\tau_{\text{vis}} + h\text{Re}(S)) = 2 : 1$ yields a prediction of $0.12 \lesssim \Delta N_{\text{eff}} \lesssim 0.26$, depending on the reheating temperature. This result is consistent with the results of Planck 2015 [1].

Another concrete prediction of those non-sequestered models is high-scale supersymmetry. The conditions for avoiding the cosmological moduli problem forces us to have soft masses far above 10 TeV.

Moreover, a feature of those models is the coupling of the dark radiation axion to QCD. This requires us to accept a fine-tuning of the initial misalignment-angle to $\theta_i \sim 10^{-2}$, otherwise one would end up with an overproduction of axionic dark matter. In case the tensor-to-scalar ratio r turns out to be $\mathcal{O}(0.1)$, isocurvature bounds would rule out our scenario. However, a possible loophole consists in including an additional axion with decay constant below the energy scale of inflation, which then takes over the role of the QCD-axion.

Finally, we also briefly mentioned sequestered scenarios with flavour branes. For $z \ll 1$ the amount of dark radiation ΔN_{eff} scales inversely with the number N_f of flavour gauge bosons. Details of the model will put an upper bound on N_f and therefore yield testable predictions for ΔN_{eff}.

In the context of the vast string landscape we learn the following: We investigated string compactifications where moduli are stabilised according to the Large Volume Scenario. Those corners of the type IIB string landscape exhibit a special feature. There is always a very light axion, the superpartner of the bulk volume modulus. It unavoidably contributes to ΔN_{eff}. Hence, due to cosmological measurements of increasing precision and sensitivity it is possible to rule out certain corners of the landscape, at least if the parameters of the concrete models in question are specified. Let us imagine that future measurements leave no room for dark radiation. Such a finding would rule out a whole popular framework of Kähler moduli stabilisation. One would then have to restrict to Kähler moduli stabilisation mechanisms such as the KKLT scenario, which – by construction – does not suffer from problems with dark radiation overproduction.

We conclude that the studies of dark radiation in string compactifications nicely underline that many string vacua can easily share several common and fairly generic features. This gives hope for projects aiming at a classification of string vacua according to phenomenological features. This is not only crucial for the understanding of the structure of the string landscape but also for bringing string theory in contact with experiments and observations.

CHAPTER 3

Tuning and Backreaction in F-term Axion Monodromy Inflation

3.1. Introduction

The aim of this chapter is to propose a model of F-term axion monodromy inflation as an example of large-field inflation in string theory and to analyse the price to pay for its realisation. The whole presentation is based on our work [112]. In this thesis we mainly follow sections 2.2, 2.3 and 4 of our paper, and summarise the remaining parts for completeness. A more detailed exposition of the backreaction analysis is given in our original paper and in the thesis of F. Rompineve [120].

Before going into details we briefly review the motivation of this project and summarise our findings.

3.1.1. Motivation and Summary

Several years ago a series of papers [44; 151; 152] have defined axion monodromy inflation in string and field theory. On the one hand, this is very exciting because models of axion monodromy inflation typically predict a tensor-to-scalar ratio of $r \simeq \mathcal{O}(0.1)$ and hence allow for testable predictions in this decade. On the other hand, the energy scale of inflation is thus very high (cf. (1.2.15)), putting lower bounds on the mass of the lightest modulus in string compactifications such as KKLT or LVS, in order to avoid destabilisation of the lightest moduli. As a result, a proper realisation of large-field inflation (and hence also axion monodromy inflation) in string theory has been considered to be at least difficult.

However, the excitement about the measurement of B-modes by the BICEP2 collaboration [153] triggered a large series of papers on large-field inflation in general and string theory in particular. Specifically, the idea of axion monodromy inflation was picked up. The paper [154] defined "F-term axion monodromy inflation" and the papers [155; 156], which appeared immediately after, suggested more concrete examples of F-term axion monodromy inflation with attention to the issue of moduli stabilisation.

Although the measurement $r = 0.20^{+0.07}_{-0.05}$ [153] by BICEP2 could finally not be confirmed, the interest in realising large-field inflation in string theory remained. The exciting question whether the existence of strong no-go theorems could prohibit large-field inflation in string theory became one of the main issues of recent research in string cosmology. We were therefore motivated to investigate further the possibility of realising or, by means of no-go theorems, ruling out F-term axion monodromy inflation within string theory, regardless of phenomenological aspects.

Being the starting point of our project [112], we briefly sketch the idea behind "D7-brane chaotic inflation" [155] as a model of F-term axion monodromy inflation. The crucial idea is to rely on the shift symmetry of a complex structure modulus or, equivalently a D7-brane modulus, of an F-theory CY 4-fold in the large complex structure (LCS) limit. The period vector Π, which determines the complex structure Kähler potential $\mathcal{K}$, and, given some flux, the superpotential W, contains non-perturbative corrections (corresponding to instantonic corrections in type IIA) of the form $\sim e^{-2\pi a z}$, where a is an $\mathcal{O}(1)$ number and z a complex structure modulus. Thus, instantonic corrections are suppressed as $\exp(-2\pi a \mathrm{Re}(z))$, i.e. they vanish as $\mathrm{Re}(z) \to \infty$, which defines the LCS limit. In the following we will refer to a complex structure modulus z to be stabilised in the LCS regime if $e^{-2\pi z}$ is sufficiently small for our purposes. This may already be the case for $\mathrm{Re}(z) = \mathcal{O}(1)$ due to the 2π-factor in the exponent. If the instanton corrections for z are negligible, the Kähler potential $\mathcal{K}$ depends on $z, \bar{z}$ only as $\mathcal{K}(z, \bar{z}) = \mathcal{K}(z - \bar{z})$, i.e. $\mathcal{K}$ is invariant under $z \to z + c$, $c \in \mathbb{R}$. We refer to this as *shift symmetry*. Moreover, if fluxes are turned off, z does not appear in the superpotential W. In consequence, there is no potential for $\mathrm{Re}(z)$. Such flat directions are of course appealing for inflationary model building. The shift symmetry can then be broken by turning on fluxes appropriately. This yields polynomial terms in the superpotential. In the following, u denotes a complex structure modulus in the LCS limit and z labels the remaining complex structure moduli, not necessarily stabilised in the large complex structure limit – we call this the "partial large complex structure" regime. It follows that

$$W(z, u) = w(z) + a(z)u + \frac{b(z)}{2!}u^2 + \dots \, , \tag{3.1.1}$$

and on CY 4-folds, W can be a polynomial up to power u^4. We do not focus on such higher powers as they can be removed by an appropriate flux-choice which, together with geometrical and topological properties of the underlying 4-fold, determines the structure of the coefficients $a(z)$ and $b(z)$. From the structure of the F-term scalar potential it can be seen that a necessary condition for a small inflaton mass is to make the coefficients $a(z)$ and $b(z)$ small by fine-tuning. This can in principle be achieved by playing with the VEVs of the complex structure moduli. As a result an embedding of chaotic inflation consistent with Kähler moduli stabilisation within the LVS was obtained [156]. Since u is 4-fold complex structure or D7-brane modulus, this model was dubbed "D7-brane chaotic inflation" [156]. Since fluxes are turned on to generate an F-term potential this model is an example of F-term axion monodromy inflation. This fine-tuning is to be understood in the context of the type IIB flux-landscape [8; 17]: Various terms, all depending on the moduli as well as the flux numbers, enter the coefficients $a(z)$ and $b(z)$. The VEVs of the complex structure moduli themselves also depend on the flux numbers. A sufficiently dense and vast flux-landscape is expected to contain sets of flux numbers, where the coefficients $a(z)$ and $b(z)$ are small enough to yield the correct inflaton mass for quadratic inflation.

Unfortunately, tuning the coefficients $a(z)$ and $b(z)$ small is not sufficient to get a flat potential. In addition to that, also every single derivative of a and b with respect to each modulus entering those coefficients have to be tuned small individually. This is necessary in order to control the field displacements of the complex structure moduli. Depending on the choice of Calabi-Yau geometries a severe amount of fine-tuning might have to be accepted.

Therefore, the main goal of this chapter is to understand if the realisation of F-term axion monodromy inflation in the flux-landscape is at all possible, and which price is to be paid due to the tuning.

The structure of this chapter is as follows: At first we explain in more detail how the fine-tuning requirements for our model of F-term axion monodromy inflation arise. While this fine-tuning controls the displacement of the complex structure moduli interacting with the inflaton, there is still some backreaction on the inflaton-potential to be expected. We comment on this issue in the section below.

Next, we show that Calabi-Yau 3-folds are generically not suited to support the fine-tuning of the coefficients to obtain F-term axion monodromy inflation for a model like (3.1.1) with u being a 3-fold complex structure modulus. The no-go-theorem we are able to formulate can, however, be circumvented by considering D7-brane moduli or complex structure moduli of a CY 4-fold. We find that such geometries allow for the required tuning, at least in principle. Afterwards, we provide a brief summary of the backreaction analysis of complex structure moduli on the inflation-potential. It turns out that backreaction affects the inflaton potential, but there are still suitable regions in field space of the inflaton in which the potential is quadratic, admitting chaotic inflation. Finally, we explain how to estimate the fraction of flux vacua with appropriate inflaton mass. Relying on toy-examples (see e.g. [12; 17]) for the estimation of the number of type IIB and F-theory flux vacua, our result is that after fine-tuning there can still remain a small landscape of suitable vacua, but only if the coefficients in the superpotential (3.1.1) do not depend on too many complex structure moduli. Finally, we summarise our findings and discuss them in the light of recent developments.

This chapter is based on [112] with the focus on the tuning issues and the geometrical realisation of this model. For details on the backreaction analysis see [112] or the thesis [120].

3.2. Problems of Tuning and Backreaction

In this section we describe the origin of the fine-tuning and backreaction problems in F-term axion monodromy inflation.[1] Thereby we define the setup and the notation for this chapter. The crucial ingredients are – as we already explained – a shift-symmetric Kähler potential and a superpotential, which then breaks this shift symmetry:

$$\mathcal{K}_{\rm cs} = \mathcal{K}_{\rm cs}(z, \bar{z}, u + \bar{u}) \ , \qquad W = w(z) + a(z)u + \ldots \ , \tag{3.2.1}$$

where z stands collectively for a set of moduli $\{z^i\}$. Since the imaginary part $y \equiv \mathrm{Im}(u)$ of the modulus u does not occur in $\mathcal{K}_{\rm cs}$, the Kähler potential is invariant under the shift $u \to u + ic$, $c \in \mathbb{R}$. This shift symmetry can be realised by stabilising u in the large complex structure (LCS) limit of the underlying type IIB orientifold or F-theory fourfold. Note that the shift symmetry was exact if $W = w(z)$, and in consequence the potential was flat. Since we want to create a potential for the inflaton, one needs to have a superpotential of the form (3.1.1). We will see that it is sufficient to restrict to a superpotential linear in u. This can be achieved by appropriate flux-choice. In case of type IIB string theory this is done by choosing the F_3- and H_3-flux vectors accordingly. In F-theory a suitable choice of the flux vector corresponding to the G_4-flux is required.

Further, slow-roll inflation requires a sufficiently flat inflaton potential. Additionally we need to avoid any interference with moduli stabilisation. Thus, shift symmetry should be

[1] In this section we stick closely to section 2.1 of [112].

broken only weakly. The breaking of shift symmetry is quantified by the parameter $a(z)$, which then needs to be made small at the SUSY locus $z = z_\star$ by fine-tuning.[2]

Unfortunately, this tuning of $a(z)$ is only necessary, but not sufficient. In the following we explain why further tunings are present in F-term axion monodromy inflation.

Starting from a type IIB orientifold or a F-theory setting, z and u can be identified as complex structure moduli (or D7-brane position moduli). Note that in the type IIB case the axio-dilaton is included in the labelling z as well.

The potential responsible for moduli stabilisation as well as inflation is the supergravity F-term scalar potential (see also (1.3.65))

$$V = e^{\mathcal{K}} \left(\mathcal{K}^{I\bar{J}} D_I W \overline{D_J W} + \mathcal{K}^{T_\gamma \bar{T}_\delta} D_{T_\gamma} W \overline{D_{T_\delta} W} - 3|W|^2 \right) \ , \tag{3.2.2}$$

where $\mathcal{K} = -2\ln\mathcal{V} + \mathcal{K}_{\mathrm{cs}}(z, \bar{z}, u+\bar{u})$, with $\mathcal{V}$ being the volume of the compact extra-dimensions. The index I runs over all moduli z as well as u. The Kähler moduli T_α are assumed to be stabilised via the Large Volume Scenario (LVS) [7]. Then, due to the no-scale structure in the Kähler moduli sector the last two terms cancel at leading order[3]. Thus,

$$V = e^{\mathcal{K}} (\mathcal{K}^{I\bar{J}} D_I W \overline{D_J W}) \ . \tag{3.2.3}$$

Since our aim is to study field displacements and backreaction of the moduli z during inflation, we do not integrate out all complex structure moduli at this stage. To get across the origin of the tuning-requirements, we restrict ourselves to a model with only two moduli fields z and u. The two F-terms entering (3.2.3) are then given by

$$D_u W = D_u w + a + \mathcal{K}_u a u \ , \tag{3.2.4}$$

$$D_z W = D_z w + (\partial_z a + \mathcal{K}_z a) u \ . \tag{3.2.5}$$

The SUSY-minimum is defined by the solution of the two equations

$$D_u W = 0, \quad D_z W = 0 \ . \tag{3.2.6}$$

Those two equations translate into conditions on the derivatives of the Kähler potential at the SUSY-minimum:

$$D_u W = 0 \Rightarrow \mathcal{K}_u|_{\mathrm{min}} = -\frac{a}{w + au}\Big|_{\mathrm{min}} \tag{3.2.7}$$

$$D_z W = 0 \Rightarrow \mathcal{K}_z|_{\mathrm{min}} = -\frac{\partial_z w + \partial_z a \cdot u}{w + au}\Big|_{\mathrm{min}} . \tag{3.2.8}$$

The idea is now to expand the potential around the SUSY-minimum $\{u = u_\star, z = z_\star\}$ in powers of $\Delta y \equiv y - y_\star$. One obtains:

$$V = e^{\mathcal{K}} \Big[\mathcal{K}^{u\bar{u}} |\mathcal{K}_u a|^2 + \mathcal{K}^{z\bar{z}} |\partial_z a + \mathcal{K}_z a|^2 + \mathcal{K}^{u\bar{z}} \mathcal{K}_u a (\overline{\partial_z a + \mathcal{K}_z a}) + \mathrm{h.c.} \Big]_{\mathrm{min}} \Delta y^2 + \dots \ , \tag{3.2.9}$$

where the ellipses summarise terms arising from backreaction of z. The details of backreaction are studied in our work [112] and we summarise the main results later. From (3.2.9) we can now read off that a small parameter $a(z)$ is not sufficient to ensure a small potential. Instead, we also require $|\partial_z a|$ to be sufficiently small. It is important to notice that small $|a|$ does not

[2] Alternatively, one could try to go to the regime $w(z) \gg 1$ [157]. A comparision of our approach with [157] is presented in appendix A of [112]. We also comment on this possibility later in this section.

[3] The LVS yields a contribution $V_{\mathrm{LVS}} \sim -|W|^2/\mathcal{V}^3$, which we ignore for the moment.

imply small $|\partial_z a|$. In the context of string theory compactifications with fluxes, parameters can be made small by tuning: various terms which are not small individually contribute to $a(z)$ and can be made to cancel up to a small remainder. However, this cancellation will generically not occur in $\partial_z a$. Requiring a small value for $|\partial_z a|$ hence introduces an additional tuning. The analysis can be easily generalised to the case of more than two moduli. For every additional modulus z^j entering a we also require $|\partial_{z^j} a|$ to be sufficiently small. Therefore, for n moduli z^i we have to tune $(n+1)$ quantities.

Indeed, one cannot get away with less tunings. To see this, we choose a basis in which the Kähler metric is diagonal. In such a basis the inflationary potential is a sum of positive terms (the mixed terms in (3.2.9) disappear). Therefore, in order to achieve a flat direction, each contribution has to be tuned small. One then has to tune $(n+1)$ different combinations of a and $\partial_{z^i} a$, $i = 1, \ldots, n$. Since these combinations involve elements of the original inverse Kähler metric as coefficients one could hope that these terms become small at special points of moduli space[4], where certain elements of the metric blow up. However, we do not know whether such situations can occur so that we do not consider this option in the following.[5] Thus, for the case of n moduli z^i entering the parameter $a(z)$, we require $|a|$, $|\partial_{z^1} a|$, $|\partial_{z^2} a|$, ..., $|\partial_{z^n} a|$ to be small.

If one admits higher powers of u in the superpotential, e.g. the term $b(z)u^2$ in (3.1.1), then one obtains the same tuning conditions as above, that is $|b(z)|$ as well as $|\partial_{z^i} b(z)|$ have to be small individually for all $i = 1, \ldots, n$. The same applies to all further coefficients in the superpotential.

Note that a loophole to circumvent the fine-tuning is to assume that a and b do not depend on any other moduli. This approach was chosen in [157]. Then, one cannot have parametrically small coefficients a and b as they are determined by integers (a combination of integral flux numbers and intersection numbers of the 4-fold). Instead, by flux choice one has to make the term $w(z)$ sufficiently large to control the backreaction of the complex structure moduli z with u.[6] While the remaining complex structure moduli can be made parametrically heavier than the inflaton, this is not the case for Kähler moduli. The mass of the lightest Kähler modulus scales as $m_{\mathcal{V}} \sim |W|/\mathcal{V}^{3/2}$, whereas the inflaton mass scales as $m_{\text{inf}} \sim 1/\mathcal{V}$ (this can be inferred from the factor $e^{\mathcal{K}}$ in the scalar potential). Since the inflaton mass cannot be tuned parametrically small the only option to obtain heavier Kähler moduli is to aim for large W. But there is a constraint $|W| \ll \mathcal{V}^{1/3}$ for LVS models due to $m_{3/2} \ll m_{\text{KK}}$. It then follows $m_{\mathcal{V}} \ll m_{\text{inf}}$. This observation makes a successful embedding of such an inflation model with non-tuned inflaton mass in the LVS difficult. Therefore, we find the option of moduli-dependent coefficients, allowing for a fine-tuning of the inflaton mass, more promising.

We conclude already at this stage that models of F-term monodromy inflation are more severely tuned than initially anticipated. One aim of this chapter is to estimate the number of string vacua consistent with the desirable properties for F-term axion monodromy inflation. While our estimate will be fairly rough, it will be sufficient to decide whether there is still a landscape of acceptable vacua. We will address this issue in Section 3.6.

F-term axion monodromy inflation faces yet another problem. The term $a(z)u \subset W$ generates the inflaton potential and the parameter a has to be small, which can only achieved

[4]See also [158] for attempts to realise inflation on special points of Calabi-Yau geometries other than the LCS regime.

[5]In fact, given that one complex structure modulus (the inflaton) must be stabilised in the large complex structure limit, we expect at least a few other complex structure moduli to be stabilised in the LCS regime as well. Then, $\mathcal{K}_{z\bar{z}} \sim (z+\bar{z})^{-2}$ is small in the LCS limit, which is the opposite behaviour.

[6]For details of this statement see e.g. appendix A of our work [112].

by allowing it to depend on other complex structure moduli. But this introduces a cross-coupling between z and u. The concern is then that for large field displacements of u, as required in models of large-field inflation, the moduli z could be significantly displaced from their values at the global minimum, possibly resulting in a destabilisation of the moduli. In the following we show that this backreaction is under control due to the fine-tuning of $a(z)$ and its derivatives (this part can be found in appendix A of our work [112]).

The superpotential we are interested in can be written more generally as

$$W = w_{\text{mass}}(z_i) + w_{\text{ax}}(z_i, u) \ , \tag{3.2.10}$$

i.e. we split W into one part which only depends on the complex structure moduli z_i and a part allowing for couplings to the modulus u containing the inflaton. $w_{\text{ax}}(z_i, u)$ may be written as in (3.1.1) or, by suitable flux-choice, simply as $a(z)u$. Then the F-term scalar potential splits into

$$V = V_{\text{mass}}(z_i) + V_{\text{mix}}(z_i, u) + V_{\text{ax}}(z_i, u) \ , \tag{3.2.11}$$

where

$$\begin{aligned} V_{\text{mass}} &= e^{\mathcal{K}}\mathcal{K}^{I\bar{J}} D_I w_{\text{mass}} \overline{D_J w_{\text{mass}}} \ , &(3.2.12)\\ V_{\text{mix}} &= e^{\mathcal{K}}\mathcal{K}^{I\bar{J}} (D_I w_{\text{mass}} \overline{D_J w_{\text{ax}}} + \overline{D_I w_{\text{mass}}} D_J w_{\text{ax}}) \ , &(3.2.13)\\ V_{\text{ax}} &= e^{\mathcal{K}}\mathcal{K}^{I\bar{J}} D_I w_{\text{ax}} \overline{D_J w_{\text{ax}}} \ , &(3.2.14) \end{aligned}$$

If $w_{\text{ax}} = 0$ the moduli z_i are stabilised at $D_I w_{\text{mass}} = 0$. In the following we will assess to what extent the moduli z_i will be destabilised if we turn on w_{ax} to generate an inflaton potential.

For simplicity we reduce the system to a setup with only two moduli u and z. Moreover, for this analysis we pretend that all quantities including those two fields are real variables. Although this is in general not realistic, the conclusions regarding the backreaction will be the same as in a more complete analysis.

The displacement δz of the complex structure modulus z is obtained from expanding the scalar potential up to second order in δz about the SUSY minimum. But first we need to expand the covariant derivatives $D_I w$:

$$\begin{aligned} D_I w_{\text{mass}}(z_\star + \delta z) &= \partial_z (D_I w_{\text{mass}})|_{z_\star} \delta z + \partial_z^2 (D_I w_{\text{mass}})|_{z_\star} (\delta z)^2 + \mathcal{O}((\delta z)^3) \ , &(3.2.15)\\ D_I w_{\text{ax}}(z_\star + \delta z, u) &= D_I w_{\text{ax}}(z_\star, u) + \partial_z (D_I w_{\text{ax}})|_{z_\star,u} \delta z + \mathcal{O}((\delta z)^2) \ . &(3.2.16) \end{aligned}$$

Note that $D_I w_{\text{mass}}(z_\star) = 0$ at the SUSY minimum, while for general $u \neq 0$ the term $D_I w_{\text{ax}}(z_\star, u)$ does not vanish. The F-term scalar potential reads

$$\begin{aligned} V = \ & V''_{\text{mass}}|_{z_\star} (\delta z)^2 + V'_{\text{mix}}|_{z_\star,u} \delta z + V''_{\text{mix}}|_{z_\star,u} (\delta z)^2 + \\ & + V_{\text{ax}}|_{z_\star,u} + V'_{\text{ax}}|_{z_\star,u} \delta z + V''_{\text{ax}}|_{z_\star,u} (\delta z)^2 \ . \end{aligned} \tag{3.2.17}$$

The displacement δz is obtained by minimising this potential with respect to δz. We get

$$\delta z \sim \frac{V'_{\text{mix}}|_{z_\star,u} + V'_{\text{ax}}|_{z_\star,u}}{2(V''_{\text{mass}}|_{z_\star} + V''_{\text{mix}}|_{z_\star,u} + V''_{\text{ax}}|_{z_\star,u})} \ . \tag{3.2.18}$$

As long as the numerator is small or the denominator large, the displacements δz are small and retrospectively justify our expansion of V. We show that we can keep δz sufficiently small by the above proposed tuning. We quantify the tuning by a parameter $\lambda \gg 1$ (used as a bookkeeping device for keeping track of terms which we tune small). We write

$$w_{\text{ax}}(z_\star, u) \to \lambda^{-1} w_{\text{ax}}(z_\star, u) \ , \quad D_I w_{\text{ax}}(z_\star, u) \to \lambda^{-1} D_I w_{\text{ax}}(z_\star, u) \ , \quad \lambda \gg 1 \ , \tag{3.2.19}$$

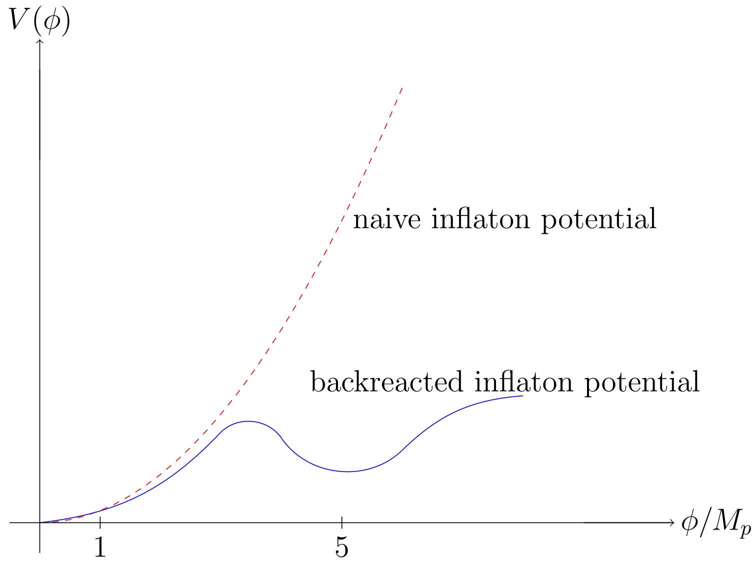


Figure 3.1.: The dashed (red) line shows the naively expected inflaton potential $V(\phi)$ for inflaton ϕ, and the solid (blue) line represents the actual inflaton potential after backreaction with other moduli is taken into account. This backreacted inflaton potential does not automatically have to come out sufficiently flat over super-Planckian field distances.

which represents our fine-tuning. Note that $\partial_z D_I w_{\text{ax}}|_{z_\star,u}$ is not necessarily small. It follows that $V_{\text{ax}}|_{z_\star,u} \to \lambda^{-2} V_{\text{ax}}|_{z_\star,u}$, while $V'_{\text{ax}}|_{z_\star,u} \to \lambda^{-1} V'_{\text{ax}}|_{z_\star,u}$ and $V'_{\text{mix}}|_{z_\star,u} \to \lambda^{-1} V'_{\text{mix}}|_{z_\star,u}$. The F-term scalar potential then reads

$$\begin{aligned} V = \; & V''_{\text{mass}}|_{z_\star} (\delta z)^2 + \lambda^{-1} V'_{\text{mix}}|_{z_\star,u} \delta z + V''_{\text{mix}}|_{z_\star,u} (\delta z)^2 + \\ & + \lambda^{-2} V_{\text{ax}}|_{z_\star,u} + \lambda^{-1} V'_{\text{ax}}|_{z_\star,u} \delta z + V''_{\text{ax}}|_{z_\star,u} (\delta z)^2 \; . \end{aligned} \tag{3.2.20}$$

By minimising this expression with respect to δz, we obtain

$$\delta z \sim \lambda^{-1} \frac{V'_{\text{mix}}|_{z_\star,u} + V'_{\text{ax}}|_{z_\star,u}}{2(V''_{\text{mass}}|_{z_\star} + V''_{\text{mix}}|_{z_\star,u} + V''_{\text{ax}}|_{z_\star,u})} + \mathcal{O}(\lambda^{-2}) \; . \tag{3.2.21}$$

As a result, we find that the size of displacements in δz can be controlled as long as $\lambda^{-1} \ll 1$ is ensured.[7] In other words, displacements are small if we tune $D_I w_{\text{ax}}|_{z_0,u}$ small.

However, note that δz depends non-trivially on u. This backreaction must be taken into account by inserting $\delta z = \delta z(u)$ into the scalar potential and the result is the effective potential for the inflaton. Due to the non-trivial u-dependence this effective potential might not be flat enough for a sufficiently long trajectory in field-space. Instead, after an initial rise one might encounter a series of local minima, see Figure 3.1. In [112] we computed the effective potential

[7]With the same methods we just presented, one can also show that displacements δz are small in the model [157], where w_{mass} is scaled up by flux choice: $w_{\text{mass}} \to \lambda w_{\text{mass}}$. Note that this is physically different from our proposed tuning and may be problematic in the context of Kähler moduli stabilisation – cf. previous discussion and appendix A of [112].

and figured out region in field-space where a quadratic potential for a sufficiently long field-distance is obtained thanks to the tuning we discussed. In this chapter, we will however only state the results of this analysis. A presentation of the details can found in our original paper or [120].

In the following we review how models of type (3.2.1) can arise in string compactifications. Additionally, we derive a no-go theorem for the realisation of the required tunings on type IIB orientifolds and present Calabi-Yau 4-fold geometries as a possible loophole. Finally, we will quantify how severe the tuning is in the context of the string landscape.

3.3. A No-Go-Theorem for Type IIB Orientifolds at Weak Coupling

This section follows sect. 2.2 of our work [112] with minor modifications.

As argued in Section 3.2, any successful large-field inflation model based on (3.2.1) with a complex structure modulus u in the large complex structure (LCS) regime requires a flux tuning of not only $|a|$ but also of all $|\partial_{z^i} a|$ and $|\partial_S a|$ at the minimum with $S = i/g_s + C_0$ being the axio-dilaton.[8] Let X be the orientifold on which we wish to realise large-field inflation. We denote by z^I, $I = 0, \dots, n$ the $h^{2,1}_-(X) = n+1$ complex structure moduli with the inflaton being $z^0 \equiv iu$. Throughout the whole chapter upper-case indices run from 0 to n, while lower-case indices run from 1 to n. In the orientifold case, the most general form of the superpotential W with u in the LCS limit is given by

$$W = w(S,z) + a(S,z)u + \frac{1}{2!}b(S,z)u^2 + \frac{1}{3!}c(S)u^3. \tag{3.3.1}$$

Here, z denotes all the z^i, $i = 1, \dots, n$. We now briefly show that a and b depend on S and the z^i, while only S enters c. One starts from the Gukov-Vafa-Witten potential [5]

$$W = \int_X (F_3 - SH_3) \wedge \Omega_3 \ , \tag{3.3.2}$$

where F_3 and H_3 are the type IIB three-form fluxes and Ω_3 is the holomorphic $(3,0)$-form on the threefold X. After flux quantisation one can write

$$W = (N_F - SN_H)^\alpha \Pi_\alpha \tag{3.3.3}$$

with the flux vectors N_F, N_H and the period vector Π, which is given by [159; 160]

$$\Pi_\alpha = \begin{pmatrix} 1 \\ z^I \\ \frac{1}{2}\kappa_{IJK} z^J z^K + f_{IJ} z^J + f_I + \sum_p A_{Ip} e^{-\sum_J b_{pJ} z^J} \\ -\frac{1}{3!}\kappa_{IJK} z^I z^J z^K + f_I z^I + g + \sum_p B_p e^{-\sum_J \tilde{b}_{pJ} z^J} \end{pmatrix} . \tag{3.3.4}$$

Here, κ_{IJK} $(I, J, K = 0, \dots, n)$ denote the triple intersection numbers of the 4-cycles of the mirror dual CY threefold $\tilde{X}$. Moreover, the flux index α runs from $\alpha = 1, \dots, 2h^{2,1}_-(X) + 2 = 2n + 4$ in our case. By stabilising u in the LCS limit, i.e. $\mathrm{Re}(u) \gtrsim \mathcal{O}(1)$, the (worldsheet) instanton terms $e^{-2\pi u}$ are suppressed. Instanton terms containing z^i but not u are not suppressed, but they only enter $w(S,z)$. Not much is known about the subleading terms f_{IJ}, f_I and g. In examples we are aware of, those terms turn out to be zero or half-integers (see

[8] In the orientifold case S enters the F-term scalar potential similarly to the complex structure moduli. Thus also $|\partial_S a|$ has to be tuned to a small value.

e.g. [160–163]) and hence, as we will explain, they will be irrelevant for the arguments below. Therefore we drop those terms in the following. Then, from (3.3.3) it follows that u enters W up to power three, as stated in (3.3.1). Clearly, S only appears linearly in W. In particular, c cannot depend on the z^i because only $\kappa_{000}u^3$ can contribute to c, and thus

$$c(S) \sim (m + nS) \tag{3.3.5}$$

with $m, n \in \mathbb{Z}$. Similarly, from (3.3.3) together with (3.3.4) one can easily see that a and b depend on the z^i and S as follows:

$$a(S, z) \sim (\alpha + \beta S + \gamma_i z^i + \lambda_i S z^i + \zeta_{ij} z^i z^j + \xi_{ij} S z^i z^j) \tag{3.3.6}$$

and

$$b(S, z) \sim (\tilde{\alpha} + \tilde{\beta} S + \tilde{\gamma}_i z^i + \tilde{\lambda}_i S z^i) \tag{3.3.7}$$

with integers $\alpha, \beta, \gamma_i, \lambda_i, \zeta_{ij}, \xi_{ij}, \tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}_i, \tilde{\lambda}_i$.

Note that for successful inflation we not only have to tune $|a|$ and its derivatives small (as explained in the previous section), but also $|b|, |\partial_{z^i} b|, |\partial_S b|, |c|, |\partial_S c|$ have to be small quantities at the minimum. This can be seen as follows. First of all, as we only want to break the shift symmetry in u weakly $|a|$, $|b|$ and $|c|$ need to be small. However, the scalar potential will receive further contributions which break the shift symmetry. In particular, we have $D_i W \supset ((\partial_{z^i} a)u + (\partial_{z^i} b)u^2/2)$ and $D_S W \supset ((\partial_S a)u + (\partial_S b)u^2/2 + (\partial_S c)u^3/3!)$, which do not obey the shift symmetry. Thus, the derivatives $|\partial_{z^i} a|, |\partial_S a|, |\partial_{z^i} b|, |\partial_S b|, |\partial_S c|$ indeed need to be tuned small as well. We will show in the following that these tunings either make it impossible to stabilise g_s in the perturbative regime or to stabilise $\mathrm{Re}(u)$ successfully.

For this, we first prove the following statement:

Statement 1: *In the perturbative regime one cannot tune $|c(S)|$ small, i.e. $|c(S)| < \epsilon$ with $\epsilon \ll 1$, as long as $c(S) \neq 0$.*

By (3.3.5), the tuning condition $|c(S)| < \epsilon$ translates into

$$|m + nS| < \epsilon, \tag{3.3.8}$$

where $m, n \in \mathbb{Z}$. Therefore, both the real and the imaginary part of $m + nS$ have to be as small as ϵ individually. Thus, $|n\ \mathrm{Im}(S)| < \epsilon$ for the imaginary part. However, because of $S = i/g_s + C_0$, it follows that

$$|n\ \mathrm{Im}(S)| = \frac{|n|}{g_s} < \epsilon. \tag{3.3.9}$$

If $n \neq 0$, then $g_s > |n|/\epsilon \gg 1$. Hence, in this case it is impossible to stabilise g_s in the perturbative regime. Since $n \in \mathbb{Z}$, one cannot simply tune n small. Therefore, one can only evade $g_s \gg 1$ if we choose $n = 0$. But then, $|c(S)| = |m| < \epsilon$, i.e. $m = 0$. This implies that $c(S)$ has to vanish identically.

This observation allows to go even one step further and to state and prove the following:

Statement 2: *On any CY threefold with $\kappa_{000} \neq 0$ or $\kappa_{i00} \neq 0$ for some z^i, the tuning requirements for large-field inflation imply that the string coupling is stabilised at $g_s \gg 1$.*

The proof is as follows. We have to tune all the parameters a, b, c and their derivatives as small as $\epsilon \ll 1$. Statement 1 shows that being in the perturbative regime requires $c \equiv 0$.

There are two possibilities to make c vanish identically. One could choose a CY threefold with $\kappa_{000} = 0$ or turn off the last entries of the flux vectors (i.e. choosing the flux numbers $(N_F)_{2n+4}$ and $(N_H)_{2n+4}$ to zero). Let us first consider the latter possibility. From (3.3.4) one can see that turning off these flux numbers indeed prevents $\kappa_{000}u^3$ from entering W. However, one would also simultaneously forbid the terms $\sim \kappa_{i00}z^i u^2$, i.e. $\tilde{\gamma}_i = \tilde{\lambda}_i = 0$ for all i. Thus, $b(S,z) = b(S)$ (see (3.3.7)), i.e. it then has the same moduli-dependence as c. In analogy, by using statement 1, we can then infer that $b \equiv 0$. Furthermore, $(N_F)_{2n+4} = (N_H)_{2n+4} = 0$ implies that $\zeta_{ij} = \xi_{ij} = 0$ for all i, j. We see that (3.3.6) becomes

$$a(S,z) \sim (\alpha + \beta S + \gamma_i z^i + \lambda_i S z^i), \tag{3.3.10}$$

and therefore $\partial_j a(S,z) \sim (\gamma_j + \lambda_j S)$, $\gamma_j, \lambda_j \in \mathbb{Z}$ for all j. Consequently, the tuning condition $|\partial_j a(S,z)| < \epsilon$ translates into $|\gamma_j + \lambda_j S| < \epsilon$ and again, by statement 1 we are forced to choose $\gamma_j = \lambda_j = 0$, and we are left with $a(S) \sim (\alpha + \beta S)$. Once more, $|a(S)| < \epsilon$ yields $a = 0$ by statement 1.[9]

However, even for $\kappa_{000} = 0$ we are forced to set $a = b \equiv 0$ in order to avoid $g_s \gg 1$. The requirement $|\partial_k b| < \epsilon$ yields $|\tilde{\gamma}_k + \tilde{\lambda}_k S| < \epsilon$ with $\tilde{\gamma}_k, \tilde{\lambda}_k \in \mathbb{Z}$. By statement 1 one must have $\tilde{\gamma}_j = \tilde{\lambda}_j = 0$ for all j. Then, again, the condition $|b| < \epsilon$ forces us to choose $\tilde{\alpha} = \tilde{\beta} = 0$ due to statement 1. Hence, b has to vanish identically, too. Since, by assumption, $\kappa_{i00} \neq 0$ for some z^i if $\kappa_{000} = 0$, one cannot avoid choosing $(N_F)_{2n+4}$ and $(N_H)_{2n+4}$ to be zero, otherwise $b \neq 0$. This then implies $\zeta_{ij} = \xi_{ij} = 0$. By repeating the above arguments, we find $a \equiv 0$ or $g_s > 1/\epsilon$. This proves statement 2.

Obviously, if we consider a CY threefold with $\kappa_{000} = 0 = \kappa_{i00}$ for all z^i (K3-fibrations admit such triple intersection numbers), then we have $b = c \equiv 0$, but generically $\zeta_{ij}, \xi_{ij} \neq 0$, because no fluxes have to vanish. In this case, it seems possible to stabilise g_s in the perturbative regime. However, it is then not clear how to stabilise $\mathrm{Re}(u)$ successfully. Note that a CY threefold with the above triple intersection numbers yields a Kähler potential of the form

$$\mathcal{K}_{cs} = -\ln\left(A(z) + B(z)(u + \bar{u})\right) \tag{3.3.11}$$

with A, B being functions of the remaining complex structure moduli. Then, the contribution

$$e^{\mathcal{K}_{cs}} V_{\mathrm{LVS}} \sim -e^{\mathcal{K}_{cs}} \frac{|W|^2}{\mathcal{V}^3} \tag{3.3.12}$$

from the LVS-potential, which dominates the F-term potential for $\mathrm{Re}(u)$, does not admit a minimum for $\mathrm{Re}(u)$ in the regime where $A + B(u + \bar{u}) > 0$, but rather shows a runaway behaviour. This issue is rooted in the simple structure of the Kähler potential. Note that an analogous problem occurs in inflation models with the universal axion, where the string coupling g_s needs to be stabilised. Consequently, large-field inflation with a complex structure modulus in the LCS limit cannot be realised on CY threefolds with $\kappa_{000} = 0 = \kappa_{i00}$ for all z^i. Together with statement 2, we summarise our findings as follows (and refer to it as the no-go theorem henceforth):

[9] If f_{IJ} is an integer or a half-integer it does not influence the argument. However, if there are cases in which f_{IJ} can be irrational or a sufficiently complicated fraction, there is a chance to evade the conclusion $a(S) = 0$. Instead one could use f_{IJ} to tune the whole expression $a(S)$ small. Since we are not aware of examples in which the terms f_{IJ} are irrational numbers or complicated fractions, we do not consider this possibility any further.

No-Go-Theorem – For any orientifold with at least one complex structure modulus u in the large complex structure limit, at least one of the following three conditions cannot be satisfied:

1. The coefficients in front of the inflaton field u in the superpotential W and their derivatives are tuned sufficiently small to allow for inflation.
2. The string coupling g_s is stabilised in the perturbative regime.
3. $\mathrm{Re}(u)$ can be stabilised using the classical supergravity F-term scalar potential.

Note that possible scenarios where only condition 3 is violated deserve more detailed investigation in future work. For instance, certain uplifting scenarios or a mild interference with Kähler moduli stabilisation could turn out to be a loophole concerning the problems in stabilising $\mathrm{Re}(u)$ that were outlined above.

This no-go theorem can be evaded by considering Calabi-Yau fourfolds as the starting point for the subsequent analysis.

3.4. Calabi-Yau Fourfolds in a Partial Large Complex Structure Regime

This section follows our work [112] (section 2.3) with minor modifications.

As explained, the no-go theorem forces us to work with Calabi-Yau *fourfolds* X, whose complex structure moduli are denoted by $u \equiv z^0$ and z^i, $i = 1, \ldots, n$, where $n = h^{3,1}(X) - 1$. Useful references for this section are [5; 17]. Again, u labels the complex structure modulus in the large complex structure regime which contains the inflaton field.

The superpotential W can be computed directly from the Gukov-Vafa-Witten potential [5]

$$W = \int_X G_4 \wedge \Omega_4 \ , \tag{3.4.1}$$

with G_4 and Ω_4 being the 4-form flux and the holomorphic 4-form on X, respectively. After flux quantisation this gives

$$W = N^\alpha \Pi_\alpha \ , \tag{3.4.2}$$

where N is flux vector and Π denotes the period vector with $\alpha = 1, \ldots, b_4(X)$. Schematically, Π has the following structure [159; 160; 164]:

$$\Pi_\alpha \sim \begin{pmatrix} 1 \\ z^I \\ \kappa_{IJKL} z^K z^L + \mathrm{Inst}(u,z) \\ \kappa_{IJKL} z^J z^K z^L + \mathrm{Inst}(u,z) \\ \kappa_{IJKL} z^I z^J z^K z^L + \mathrm{Inst}(u,z) \end{pmatrix} \ , \tag{3.4.3}$$

where κ_{IJKL} denote the intersection numbers of the 6-cycles of the mirror dual CY fourfold, and $\mathrm{Inst}(u, z)$ summarises various instanton terms, depending on u and all the z^i.

In general, W is a holomorphic function in u and the remaining complex structure moduli. In this work we wish to only consider superpotentials where u appears at most linearly:

$W = w + au$. The main motivation behind this restriction is to keep the analyses in the following chapters simple. In principle, the study of backreaction performed in this work should also be possible for models with a more complicated superpotential, but we leave this for future studies.

In the following we will argue how a superpotential linear in u can be obtained. One obstruction to this is the presence of non-perturbative terms of the form $\sim e^{-2\pi u}$ in Π. As before, by working in the LCS regime where u is large we can ensure that all non-perturbative terms containing u are exponentially suppressed. Note that we do not require that all moduli z^I need to be in the LCS regime: we only require a subset including u to be at LCS, which we refer to as 'partial large complex structure'. Then, at this stage, u can arise at most as u^4 in W.

In order to achieve a superpotential of the form $W = w(z) + a(z)u$, we assume X to have intersection numbers $\kappa_{0000} = 0 = \kappa_{i000}$ for all z^i. Hence, cubic or quartic terms in u are prohibited by the geometry of X. All terms which potentially give rise to quadratic terms in u need to be set to zero by a corresponding flux choice. For instance, the last component of Π contains $\kappa_{ij00}z^iz^ju^2$, which does not necessarily vanish, and thus the last component of N must be chosen to be zero. Since the Betti number $b_4(X)$ does not only receive contributions from $h^{3,1}(X)$ but also from $h^{2,2}(X)$, we expect that the available number of flux parameters exceeds the number of required tunings. For instance, if X is an elliptic fibration over $\mathbb{C}P^3$ one obtains $h^{3,1}(X) = 3878$, $h^{2,2}(X) = 15564$ and hence $b_4(X) = 23320$ [17]. Thus, in this example one has many more flux numbers than complex structure moduli.

We now want to write down the tuning conditions explicitly and argue that these requirements can be satisfied in principle. Using the notation $\vec{z} \equiv (z^1, \ldots, z^n)$, we can write $a(\vec{z})$ schematically as

$$a(\vec{z}) \sim (m + \vec{n}^t\vec{z} + \vec{z}^t\mathbf{N}\vec{z}) \tag{3.4.4}$$

with $m \in \mathbb{Z}$, $\vec{n} \in \mathbb{Z}^n$ and $\mathbf{N}$ being an integer valued matrix. The tuning condition on the derivatives of $a(\vec{z})$ is $|\nabla a| \lesssim \epsilon \simeq 0$. This gives two real equations

$$2\mathbf{N}\vec{v} \simeq -\vec{n}, \tag{3.4.5}$$

$$\mathbf{N}\vec{w} \simeq 0, \tag{3.4.6}$$

where $\vec{v} = \mathrm{Re}\vec{z}$ and $\vec{w} = \mathrm{Im}\vec{z}$. Inserting these results into $a(\vec{z})$, we find

$$a(\vec{z}) \sim (m + \frac{1}{2}\vec{n}^t\vec{v} + \frac{1}{2}i\vec{n}^t\vec{w}) \ . \tag{3.4.7}$$

As we need to tune $|a(\vec{z})| \lesssim \epsilon \simeq 0$ we also require

$$|\vec{n}^t\vec{w}| \simeq 0 \tag{3.4.8}$$

$$\left|m + \frac{1}{2}\vec{n}^t\vec{v}\right| \simeq 0. \tag{3.4.9}$$

A solution to these four conditions is as follows. Suppose $\det\mathbf{N} \neq 0$, then $\vec{w} \simeq 0$, i.e. the second and the third conditions are satisfied. The first condition (3.4.5) can be solved for $\vec{v}$ and plugged into (3.4.9) to get the requirement

$$m \simeq \frac{1}{4}\vec{n}^t(\mathbf{N}^t)^{-1}\vec{n}. \tag{3.4.10}$$

This can be satisfied easily if e.g. $\det\mathbf{N} = \pm 1$, since in this case $\mathbf{N}^{-1}$ is again integer valued. Of course, there can also be solutions to the tuning conditions for $\det\mathbf{N} = 0$, but we do not study them any further since our intention was to show that one can in principle satisfy the tuning requirements.

We now turn to the Kähler potential $\mathcal{K}_{\rm cs}$ for the complex structure moduli. It can be determined from the period vector Π as

$$\mathcal{K}_{cs} = -\ln\left(\Pi_\alpha(z,u)Q^{\alpha\bar\beta}\overline{\Pi_\beta}(\bar z,\bar u)\right) \tag{3.4.11}$$

with the intersection matrix $Q_{\alpha\bar\beta}$. Most importantly, since u is taken to be in the LCS regime, it appears only as $u+\bar u$ in the Kähler potential. Consequently, the Kähler potential for the complex structure moduli is indeed of the form $\mathcal{K}_{cs} = \mathcal{K}_{cs}(z,\bar z, u+\bar u)$, as stated in (3.2.1). From the structure of the period vector it is also evident that $\mathcal{K}_{cs}$ can in principle contain a polynomial in $(u+\bar u)$ of degree four (at most). Since, for simplicity, we consider $\kappa_{0000} = 0 = \kappa_{i000}$ for all z^i, we have in fact a quadratic polynomial in $(u+\bar u)$ in the logarithm of the Kähler potential. However, note that we do not rely on the specific structure of $\mathcal{K}_{cs}$ for the subsequent analysis. The crucial point is the existence of the shift symmetry of $\mathcal{K}_{cs}$ (under $u \to u + ic$, $c \in \mathbb{R}$), which is a necessary requirement to evade the η-problem.[10] Again, to arrive at a Kähler potential with one shift-symmetric direction, we do not require all complex structure moduli to be at LCS: only a subset of complex structure moduli containing u has to be large. As before, 'partial large complex structure' is sufficient. Overall, this leaves F-theory 4-folds as a promising starting point for models of F-term axion monodromy inflation.

For the sake of simplifying the notation, we henceforth abbreviate $f_I \equiv \partial_{z^I} f$, $I = 0,\dots,n$ and $f_i \equiv \partial_{z^i} f$, $i = 1,\dots,n$ for any function f.

3.5. Summary of the Backreaction Analysis

In Section 3.2 we explained that complex structure moduli get displaced due to their coupling to the modulus u as the inflaton field traverses large distances in field-space. We showed, however, that these displacements can be controlled by ensuring (via fine-tuning) that $|a(z)| < \epsilon$ and $|a_i(z)| < \epsilon$ with $\epsilon \ll 1$. Nevertheless, we still have to establish that backreaction of the complex structure moduli on the "naive" inflaton potential does not spoil inflation. A complete analysis can be found in [112] and in the thesis of F. Rompineve [120]. Here we only summarise and state the main results.

Our starting point is (3.2.1) and we choose the fluxes such that W is only linear in u. The first step is to calculate the F-term scalar potential

$$V = e^{\mathcal{K}}(\mathcal{K}^{I\bar J}D_I W\overline{D_J W}) \ , \tag{3.5.1}$$

where we can ignore the Kähler-moduli part (see discussion around (3.2.3)). Recall that our inflaton is the imaginary part of u and therefore we write out explicitly $u = x + iy$. Schematically, the F-term scalar potential then reads

$$V = A(x,z,\bar z) + B(x,z,\bar z)y + C(x,z,\bar z)y^2 \tag{3.5.2}$$

and the last two interaction terms reveal the origin of the backreaction problem. For large field displacements $\delta y \equiv y - y_\star$ of the inflaton from the minimum $y_\star$ the other moduli fields x,

[10] In addition, we require the existence of a point where $\partial_u\mathcal{K} = 0$. This will allow us to stabilise $\mathrm{Re}(u)$ through $\mathcal{K}$ and w only (see [156] for more detail). We can then ensure that $\mathrm{Re}(u)$ is parametrically heavier than the inflaton $\mathrm{Im}(u)$, which only acquires a mass through $au \subset W$. A Kähler potential with a quadratic polynomial in $(u+\bar u)$ inside the logarithm is sufficient to stabilise $\mathrm{Re}(u)$ through $\mathcal{K}$ and w only.

z^i are also displaced from their minimum. Even if these displacements are small, the dynamics backreacts and induces corrections to the "naive" inflaton potential

$$V_{\text{naive}} = e^{\mathcal{K}}\Big[\mathcal{K}^{u\bar{u}}|\mathcal{K}_u a|^2 + \mathcal{K}^{z\bar{z}}|\partial_z a + \mathcal{K}_z a|^2 + \mathcal{K}^{u\bar{z}}\mathcal{K}_u a(\overline{\partial_z a + \mathcal{K}_z a}) + \text{h.c.}\Big]_{\text{min}}\Delta y^2 \ , \tag{3.5.3}$$

which is obtained by expanding V about $y_\star$, while keeping the other fields at the SUSY minimum. For this computation it is important to recall that $\mathcal{K}$ does not depend on y due to the shift symmetry $u \to u + ic$. The result corresponds to the term $C(x, z, \bar{z})\Delta y^2$ in (3.5.2), with $A = 0 = B$ at the SUSY minimum (i.e. we neglect backreaction for the moment).

We see that $|a|$ and $|\partial_z a + \mathcal{K}_z a|$ must be small quantities at the SUSY minimum. For the tuning condition we propose two variants:

$$\text{Variant I:} \qquad \epsilon \equiv |a| \ll 1, \qquad |\partial_z a + \mathcal{K}_z a| \sim \epsilon^2 \ , \tag{3.5.4}$$

$$\text{Variant II:} \qquad \epsilon \equiv |a| \ll 1, \qquad |\partial_z a + \mathcal{K}_z a| \sim \epsilon \ . \tag{3.5.5}$$

Note that both variants imply in particular $|\partial_z a| \ll 1$. Clearly, the tuning in variant I is more severe than in II. Now, we analyse the effective potential for both variants individually.

Tuning Variant I

If we impose $\epsilon \equiv |a| \ll 1$ and $|\partial_z a + \mathcal{K}_z a| \sim \epsilon^2$, one can easily show that

$$V_{\text{naive}} \sim \epsilon^4 \Delta y^2 \ . \tag{3.5.6}$$

To see this, notice that the SUSY condition $D_u W = 0$ implies $\mathcal{K}_u \sim a$.

Let us now take into account backreaction. To do so, the potential V needs to be expanded up to quadratic order in the displacements δx and δz^i of the moduli. It is also convenient to write $z^i = v^i + iw^i$. Then, as a result one obtains a quadratic form in $\vec{\Delta} = \{\delta x, \delta v^i, \delta w^i\}$:

$$V = \frac{1}{2}\vec{\Delta}^t \mathcal{D}(\Delta y)\vec{\Delta} + (\vec{b}(\Delta y))^t \vec{\Delta} + \mu^2 \Delta y^2 \ , \tag{3.5.7}$$

where $\mathcal{D}$ and $\vec{b}$ are matrix- and vector-valued expressions. Although they are very complicated, they can in principle be computed explicitly, see [112] for details. Rather than calculating $\mathcal{D}$ and $\vec{b}$ explicitly, we only wish to outline the logic behind the computation to the corrections to the naive potential $V_{\text{naive}} \sim \mu^2 \Delta y^2 \sim \epsilon^4 \Delta y^2$.

Minimising (3.5.7) with respect to δx, δv^i and δw^i yields the effective potential $V_{\text{eff}}(\Delta y)$ for the inflaton:

$$V_{\text{eff}}(\Delta y) = -\frac{1}{2}(\vec{b}(\Delta y))^t \mathcal{D}^{-1}(\Delta y)\vec{b}(\Delta y) + \mu^2 \Delta y^2 \ . \tag{3.5.8}$$

Unfortunately, $V_{\text{eff}}(\Delta y)$ is a very complicated function. However, we can extract relevant information by looking at different values of Δy. If $\Delta y \ll 1$ then the displacements of the other moduli grow linearly in Δy, more precisely $\delta x \sim \delta z^i \sim \epsilon^2 \Delta y$. Hence, the displacements are tiny in this regime. More difficult to analyse is the regime $\Delta y = \mathcal{O}(1)$. While the displacements are still suppressed by ϵ^2, the functional dependence of $\delta x, \delta z^i$ on Δy is complicated. In particular we cannot expect inflation to work in this regime. But for $\Delta y \gg 1$ the displacements behave as $\delta z^i \sim \mathcal{O}(1)\epsilon^2$, whereas $\delta x \sim \mathcal{O}(1)\epsilon^2 \Delta y$. Requiring $\delta x \ll 1$ implies $\Delta y \ll 1/\epsilon^2$. Furthermore, in the regime $1 \ll \Delta y \ll 1/\epsilon$ (i.e. $\delta x \sim \epsilon$) the effective potential is quadratic in Δy:

$$V_{\text{eff}} \simeq \left(-\mathcal{O}(1)\epsilon^4 e^{\mathcal{K}} + \mu^2\right)\Delta y^2 \equiv \mu_{\text{eff}}^2 \Delta y^2 \ . \tag{3.5.9}$$

Hence, we find a flattening of the naive potential (see also [165; 166] for similar effects).

To summarise, the effective Lagrangian in the regime $1 \ll \Delta y \ll 1/\epsilon$, in which we want to study inflation, reads:

$$\mathscr{L}_{\text{eff}} = -\mathcal{K}_{u\bar{u}} \partial_\mu \Delta y \partial^\mu \Delta y - \mu_{\text{eff}}^2 \Delta y^2 \ , \tag{3.5.10}$$

where $\mu_{\text{eff}}^2 \sim \epsilon^4 e^{\mathcal{K}}$. The canonically normalised inflaton field φ is then given by $\varphi = \sqrt{2\mathcal{K}_{u\bar{u}}}\Delta y$ and the inflaton mass is $m^2 = \mu_{\text{eff}}^2 / \mathcal{K}_{u\bar{u}}$.

Tuning Variant II

We can also aim for less severe tuning: $\epsilon \equiv |a| \ll 1$ and $|\partial_z a + \mathcal{K}_z a| \sim \epsilon$. The analysis to be done is analogous to the previous case. One includes backreaction to the naive potential order by order in ϵ by calculating $\delta x, \delta v^i, \delta w^i$ as a function of Δy. As a result one can still obtain a quadratic effective potential in Δy, but only in the regime $\Delta y \gg 1/\epsilon$. Once again one obtains at leading order in ϵ that $\mu_{\text{eff}}^2 \sim e^{\mathcal{K}} \epsilon^4$ (but this time due to cancellations of terms quadratic in ϵ). However, because of the term $au \subset W$ the superpotential changes a lot during inflation (note that $\Delta W \sim a \Delta y \gg \epsilon \cdot 1/\epsilon = 1$). In consequence, the bulk Kähler modulus will also backreact on the inflaton potential (note that in the LVS the compactification volume $\mathcal{V}$ is stabilised at $\mathcal{V} \sim |W|$). An analysis of this inflation scenario therefore requires to add the LVS potential and an uplift-term to the potential $\mu_{\text{eff}}^2 \Delta y^2$. Nevertheless, one can obtain a monotonically rising potential in Δy, so that large-field inflation remains in principle possible, although details are left for future work.

In summary, we find that the two proposed tuning conditions can give rise to large-field inflation. Both the stronger tunings ($\epsilon \equiv |a| \ll 1$ and $|\partial_z a + \mathcal{K}_z a| \sim \epsilon^2$) and the less severe tuning conditions ($\epsilon \equiv |a| \ll 1$ and $|\partial_z a + \mathcal{K}_z a| \sim \epsilon$) lead to a quadratic potential in Δy for $1 \ll \Delta y \ll 1/\epsilon$ and $\Delta y \gg 1/\epsilon$, respectively. However, the latter possibility requires to take into account backreaction of the lightest Kähler modulus.

3.6. Fine-Tuning in the String Landscape

The goal of this section is to estimate the abundance of flux vacua remaining after imposing the tuning conditions for the realisation of our model of F-term axion monodromy inflation. We follow section 4 of our work [112].

The previous sections have shown the necessity of tuning of certain parameters, namely $a(z)$ and $a_i(z)$ with i running over all complex structure moduli entering a. In Section 3.5 we explained that backreaction of complex structure moduli can be controlled when we tune parameters as follows:

$$\begin{aligned} \text{Variant I:} \quad & |a| = \epsilon \ll 1 \qquad |a_i + \mathcal{K}_i a| \sim \epsilon^2 \ , \\ \text{Variant II:} \quad & |a| = \epsilon \ll 1 \qquad |a_i + \mathcal{K}_i a| \sim \epsilon \ . \end{aligned}$$

Let $J_t/2 - 1$ be the number of complex structure moduli which a depends on, i.e. $i = 1, \ldots, J_t/2 - 1$. Then J_t counts the required number of tunings in both cases (note that the tuning of one complex parameter results into two tuning conditions for real parameters). In this section we provide an estimate of the number of remaining supersymmetric F-theory flux vacua after imposing the tuning conditions. In particular, we wish to count the number of vacua where $|a|$ and $|a_i + \mathcal{K}_i a|$ are sufficiently small, i.e. $|a| < \epsilon$ and $|a_i + \mathcal{K}_i a| < \epsilon^2, \epsilon$ for tuning variants I and II, respectively.

We will closely follow [12] although the authors counted the number of SUSY flux vacua in the type IIB theory on a CY threefold Y, where $X = (T^2 \times Y)/\mathbb{Z}_2$.[11] However, due to our no-go theorem for complex structure inflation on CY threefolds, we actually do not want to consider threefolds. Nevertheless, we follow the computation in [12] and modify it appropriately in order to find the parametric dependence of the number of vacua on the tuning parameter ϵ, also in the fourfold case.[12]

Recall that in [12] the number of supersymmetric flux vacua satisfying the tadpole condition $L \leq L_\star \equiv \chi(X)/24$ on a CY fourfold X was estimated to be

$$\mathcal{N}(L \leq L_\star) = \frac{(2\pi L_\star)^{2m}}{(2m)!\sqrt{\det \eta}} \int_{\mathcal{M}} d^{2m}z \;\det(g)\;\rho(z), \tag{3.6.1}$$

where η is the intersection form on X and $m = h^{2,1}_{-}(Y) + 1$. $\mathcal{M}$ denotes the moduli space over which the density ρ of supersymmetric vacua (per unit volume of $\mathcal{M}$) is integrated. The authors arrived at this result by changing variables from the flux vector (of F-theory) to a set of variables $(X, Y, Z, \overline{X}, \overline{Y}, \overline{Z})$ defined by

$$X \equiv \int_X G_4 \wedge \Omega_4 = W, \qquad Y_A \equiv D_A W, \qquad Z_I \equiv D_S D_I W \tag{3.6.2}$$

in the orientifold limit. Using these variables, one can express ρ as follows:

$$\rho(z) = \pi^{-2m} \int d^2X d^{2m-2}Z e^{-|X|^2-|Z|^2}\,|X|^2 \left|\det\begin{pmatrix} \delta_{IJ}\overline{X} - \frac{Z_I\overline{Z}_J}{X} & \mathcal{F}_{IJK}\overline{Z}^K \\ \overline{\mathcal{F}}_{IJK}Z^K & \delta_{IJ}X - \frac{\overline{Z}_I Z_J}{\overline{X}}\end{pmatrix}\right| . \tag{3.6.3}$$

The tensor $\mathcal{F}_{IJK}$ has a purely geometric meaning and is defined by

$$\mathcal{F}_{IJK} = \int_Y D_I D_J D_K \Omega_4 \wedge \overline{\Omega}_4\,. \tag{3.6.4}$$

The prefactor in (3.6.1) will be modified if we impose the J_t tuning conditions. For tuning variant II discussed in Section 3.5 we require $|a_I| \lesssim \epsilon$ with $a_0 \equiv a$ and $a_i = \partial_i a$ for $i = 1, \ldots, J_t/2 - 1$ and $\epsilon \ll 1$. From the Gukov-Vafa-Witten potential it is clear that the a_I are linear functions of the F-theory flux vector components N_α with $\alpha = 1, \ldots, K = 4m - J_f$, where J_f counts the number of flux components chosen to be zero in order to construct a superpotential linear in u.

To see how $\mathcal{N} \equiv \mathcal{N}(L \leq L_\star, |a_I| \lesssim \epsilon)$ differs from $\mathcal{N}(L \leq L_\star)$ shown in (3.6.1), we redo the derivation in [12] and implement the tuning conditions by including factors $\Theta(\epsilon - |a_I|)$ for all I as follows:

$$\mathcal{N} = \frac{1}{2\pi i}\int_C \frac{d\alpha}{\alpha} e^{\alpha L_\star}\mathcal{N}(\alpha), \tag{3.6.5}$$

$$\mathcal{N}(\alpha) \simeq \int_{\mathcal{M}} d^{2m}z \int d^K N e^{-\frac{\alpha}{2}N\eta N}\delta^{2m}(DW)\left|\det D^2 W\right| \times \prod_{I=0}^{J_t/2-1}\Theta\left(\epsilon - |\tilde{a}_{I\alpha}N_\alpha|\right), \tag{3.6.6}$$

where the $\tilde{a}_{I\alpha}$ are the coefficients of the linear expansion of a_I in terms of the components of N, i.e. $a_I = \tilde{a}_{I\alpha}N_\alpha$. The curve C goes along the imaginary axis and passes the pole to the

[11] Notice however some minor differences in the notation. While derivatives with respect to the axio-dilaton are denoted by ∂_0 or D_0 in [12], we write ∂_S or D_S, respectively. The index 0 is reserved for the inflaton field.

[12] A more precise analysis would presumably be possible using the techniques of [72; 73], where the counting of vacua on fourfolds was discussed in the context of F-theory GUTs.

right. It is easy to show that this gives rise to a parametric behaviour $\mathcal{N}(\alpha) \sim \alpha^{-(K-J_t)/2}$. Indeed, one can write

$$\begin{aligned}\mathcal{N}(\alpha) &\simeq \int_{\mathcal{M}} d^{2m}z \int d^K N e^{-\frac{\alpha}{2}N\eta N} \delta^{2m}(DW) \left|\det D^2 W\right| \prod_{I=0}^{J_t/2-1} \Theta\left(\epsilon - |\tilde{a}_{I\alpha} N_\alpha|\right) \\ &= \int_{\mathcal{M}} d^{2m}z \int d^K \tilde{N} \alpha^{-K/2} e^{-\frac{1}{2}\tilde{N}\eta\tilde{N}} \delta^{2m}(DW) \left|\det D^2 W\right| \prod_{I=0}^{J_t/2-1} \Theta\left(\sqrt{\alpha}\epsilon - \left|\tilde{a}_{I\alpha}\tilde{N}_\alpha\right|\right),\end{aligned}$$

where we substituted $N = \tilde{N}/\sqrt{\alpha}$ and simultaneously rescaled the argument in the Θ-function by $\sqrt{\alpha}$. This rescaling clearly does not modify the result but it allows to read off the parametric dependence of $\mathcal{N}$ on α easily: we will justify in the steps from (3.6.10)-(3.6.12) that the $J_t/2$ Θ-factors give rise to an overall factor $\sim (\alpha\epsilon^2)^{J_t/2}$. Hence, the parametric dependence on α is indeed

$$\mathcal{N}(\alpha) \simeq \alpha^{-(K-J_t)/2}\mathcal{N}(\alpha = 1).$$

Note that without the rescaling of the argument of the Θ-functions the tuning conditions would have introduced factors of α into the terms $\delta^{2m}(DW)\left|\det D^2W\right|$, which makes it more difficult to find the overall parametric dependence on α. Now, the contour integral (3.6.5) can be readily evaluated:

$$\mathcal{N} = \frac{1}{2\pi i}\int_C \frac{d\alpha}{\alpha^{1+(K-J_t)/2}} e^{\alpha L_\star}\mathcal{N}(\alpha = 1) = \frac{L_\star^{2m-(J_f+J_t)/2}}{(2m-(J_f+J_t)/2)!}\mathcal{N}(\alpha = 1). \tag{3.6.7}$$

Consequently, the tuning conditions modify (3.6.1) as follows:

$$\begin{aligned}&\mathcal{N}(L \le L_\star, |a_I| \lesssim \epsilon) \\ \simeq\ & \frac{(2\pi)^{2m-J_f/2} L_\star^{2m-(J_f+J_t)/2}}{(2m-(J_f+J_t)/2)!\sqrt{\det\eta}} \int_{\mathcal{M}} d^{2m}z \det g\ \times \\ &\times \pi^{-2m-J_f/2}\int d^2X d^{2m-2-J_f/2}Z e^{-|X|^2-|Z|^2}|X|^2 \left|\det\begin{pmatrix}\delta_{IJ}\overline{X} - \frac{Z_I\overline{Z}_J}{X} & \mathcal{F}_{IJK}\overline{Z}^K \\ \overline{\mathcal{F}}_{IJK}Z^K & \delta_{IJ}X - \frac{\overline{Z}_I Z_J}{\overline{X}}\end{pmatrix}\right| \times \\ &\times \prod_{i=0}^{J_t/2-1}\Theta(\epsilon - |a_I(X,Z,z)|)\end{aligned} \tag{3.6.8}$$

Now one can make the following change of variables: we can express some X, Z, z by a_I, which introduces a factor $\left|\det\left(\frac{\partial(a_0,a_1,\ldots,a_{J_t/2-1})}{\partial(y_0,y_1,\ldots,y_{J_t/2-1})}\right)\right|$ with $\{y_0,\ldots,y_{J_t/2-1}\}$ being a subset of all the X, Z and z. We expect it to be neither particularly large nor small, since the components of the Jacobian are typically $\mathcal{O}(1)$. As a result, the number of remaining supersymmetric flux vacua is estimated to be

$$\mathcal{N}(L \le L_\star, |a_I| \lesssim \epsilon) \sim \frac{(2\pi)^{2m-(J_f+J_t)/2} L_\star^{2m-(J_f+J_t)/2}}{(2m-(J_f+J_t)/2)!} \cdot (\pi\epsilon^2)^{J_t/2}, \tag{3.6.9}$$

where we also neglected $\sqrt{\det\eta}$. The factor $(2\pi)^{-J_t/2}$ arises from integrating out the tuning conditions. The factor $(\pi\epsilon^2)^{J_t/2}$ can be understood by the following considerations:

First of all, we can rewrite the integral in eq. (3.6.8) symbolically as

$$\mathcal{N} \sim \int_{\mathcal{M}\times\mathbb{R}^{K/2}} d^d x f(\vec{x}) \prod_{I=0}^{J_t/2-1}\Theta(\epsilon - |a_I(\vec{x})|), \tag{3.6.10}$$

where the components of $\vec{x} \equiv (x^1, \ldots, x^d)$ with $d = 2m + K/2$ replace the variables z^I, Z_I and X. We assume that the combined zero locus of the a_I is a $(d - J_t)$-dimensional submanifold $\mathcal{R} \subset \mathcal{M} \times \mathbb{R}^{K/2}$. Without loss of generality we parametrize this submanifold by $x^1, \ldots, x^k$, $k = d - J_t$. The remaining variables $x^{k+1}, \ldots, x^d$ are traded for $J_t/2$ pairs of variables α_I, β_I, such that $a_I = \alpha_I + i\beta_I$. Thus, we have

$$\mathcal{N} \sim \int_{\mathcal{M}\times\mathbb{R}^{K/2}} d^k x d\alpha_0 d\beta_0 \ldots d\alpha_{J_t/2-1} d\beta_{J_t/2-1} \tilde{f}(x^1, \ldots, x^k, \vec{\alpha}, \vec{\beta}) \prod_{I=0}^{J_t/2-1} \Theta(\epsilon - \sqrt{\alpha_I^2 + \beta_I^2}), \tag{3.6.11}$$

where the determinant of the Jacobian for the transformation is absorbed into $\tilde{f}$. Next, it is convenient to introduce polar coordinates (r_I, ϕ_I) for every pair α_I, β_I. Hence, one has to evaluate

$$\begin{aligned} \mathcal{N} &\sim \int_{\mathcal{R}} d^k x \prod_{I=0}^{J_t/2-1} \int_0^\infty dr_I \int_0^{2\pi} d\phi_I r_I \Theta(\epsilon - r_I) \tilde{f}(x^1, \ldots, x^k, \vec{r}, \vec{\phi}) = \\ &= \int_{\mathcal{R}} d^k x \prod_{I=0}^{J_t/2-1} \int_0^\epsilon dr_I \int_0^{2\pi} d\phi_I r_I \tilde{f}(x^1, \ldots, x^k, \vec{r}, \vec{\phi}) \sim \left(\pi\epsilon^2\right)^{J_t/2}, \end{aligned} \tag{3.6.12}$$

where we assumed that $\tilde{f}$ is approximately constant inside the small region of size $\sim \epsilon$. Therefore, the number of remaining flux vacua is indeed suppressed by a factor of $\sim (\pi\epsilon^2)^{J_t/2}$.

We expect that (3.6.9) can be used to count the remaining F-theory flux vacua by simply replacing the dimension of the flux space in type IIB by the dimension of the F-theory flux space, which is given by the Betti number b_4 of X, from which we have to subtract the number J_f of flux components that had to be turned off in order to admit a linear superpotential in u and in order to allow for an F-theory limit. Thus, we use

$$\mathcal{N}(L \leq L_\star, |a_I| \lesssim \epsilon) \sim \frac{(2\pi L_\star)^{b_4/2-(J_f+J_t)/2}}{(b_4/2 - (J_f + J_t)/2)!} \cdot (\pi\epsilon^2)^{J_t/2}, \tag{3.6.13}$$

to estimate the number of flux vacua admitting large-field inflation with complex structure moduli, where tuning condition II, $|a_I| \lesssim \epsilon$, is applied.

In Section 3.5 we also considered the more severe tuning variant I. There we have $|a| \lesssim \epsilon$ and $|D_i a| = |a_i + \mathcal{K}_i a| \lesssim \epsilon^2$. Repeating the above analysis we find that the tuning of a introduces a factor of $(\pi\epsilon^2)$ into $\mathcal{N}$, while for every $|D_i a|$ which we tune small we get a contribution $(\pi\epsilon^4)$. In this case the counting formula is modified as

$$\mathcal{N}(L \leq L_\star, |a| \lesssim \epsilon, |D_i a| \lesssim \epsilon^2) \sim \frac{(2\pi L_\star)^{b_4/2-(J_f+J_t)/2}}{(b_4/2 - (J_f + J_t)/2)!} \cdot \pi^{J_t/2} \epsilon^{2J_t-2}. \tag{3.6.14}$$

Since the potential found in Section 3.5 is purely quadratic for sufficiently large Δy, we can now estimate the required size of the tuning parameter ϵ for successful chaotic inflation. The inflaton potential is given by $V_{\text{inf}} = \frac{1}{2} m^2 \varphi^2$, where $\varphi = \sqrt{2\mathcal{K}_{u\bar{u}}} \Delta y$ is the canonically normalised inflaton field. In order to have enough e-foldings (or equivalently in order to match the correct spectral index), chaotic inflation fixes the beginning of slow-roll inflation at $\varphi_{\max} \simeq 15$. Thus, the requirement $\Delta y < 1/\epsilon$ for variant I turns into an upper bound for ϵ:

$$\epsilon < \frac{1}{\varphi\sqrt{2}\text{Re}(u)} \simeq \frac{1}{15\sqrt{2}\text{Re}(u)}. \tag{3.6.15}$$

Together with (3.6.14) we find as an upper bound for the number of supersymmetric flux vacua with the required tuning:

$$\mathcal{N}(L \leq L_\star, |a| \lesssim \epsilon, |D_i a| \lesssim \epsilon^2) < \frac{(2\pi L_\star)^{b_4/2-(J_f+J_t)/2}}{(b_4/2-(J_f+J_t)/2)!} \cdot \pi^{J_t/2} \left(\frac{1}{15\sqrt{2}\mathrm{Re}(u)}\right)^{2J_t-2} . \quad (3.6.16)$$

Unfortunately we do not know J_f and J_t, i.e. the number of fluxes to be turned off and the number of tuning conditions. It is moreover not quite clear how large $\mathrm{Re}(u)$ can be. We therefore assume that $\mathrm{Re}(u) \sim \mathcal{O}(1)$.[13] This should be sufficient to suppress the instanton corrections which scale as $\sim e^{-2\pi u}$. Nevertheless, we try to give an estimate of how large J_t can be at most, assuming that $\mathrm{Re}(u) \simeq 1.2$ (equivalently $\epsilon < 0.04$). Then, for the study of one particular case with $L_\star = 972$, $b_4 = 23320$, $h^{3,1}(X) = 3878$ (see [17]), which gives rise to the famous number of 10^{1700} F-theory flux vacua, there will be a leftover of at most $\sim 10^{350}$ vacua if we require $J_t = 600$ tunings (i.e. the geometry of the CY fourfold is such that only ~ 300 out of the 3877 complex structure moduli (without u) enter a). In this estimate we ignored the variable J_f, but if it is small compared to b_4, this estimate should still be an appropriate approximation. Clearly, one cannot afford much more than 300 tuning conditions due to the severe suppression factor ϵ^{2J_t-2}. However, if it is possible to realise our inflation model on a fourfold along the lines of Section 3.5 via variant I, such that much less than 600 tunings are required, then there should still be a vast landscape of F-theory flux vacua left.[14] Note that in setups following Section 3.5 with variant II, where the tuning conditions are just $|a_I| \lesssim \epsilon$, the number of flux vacua is suppressed by ϵ^{J_t}, see (3.6.13), and hence the tuning is less severe (using the above numbers, i.e. $J_t = 600$ and $\epsilon = 0.04$, one has a leftover of 10^{1180} vacua).

It would be interesting to work out the required tuning conditions more specifically in the future by analysing specific CY fourfolds. This would allow us to determine J_f as well as J_t and hence to estimate the number of remaining flux vacua more explicitly.

Apart from the tuning conditions, the landscape will be further suppressed due to the stabilisation of $\mathrm{Re}(u)$ in the LCS limit. If, however, this requirement does not enforce too many other complex structure moduli to be stabilised in the LCS regime as well, then this constraint is not expected to be too severe. Scenarios in which all complex structure moduli are stabilised in the LCS limit are presumably difficult to realise in the string landscape.

3.7. Conclusion and Further Comments

In this chapter we presented crucial developments of the initial proposal of F-term axion monodromy inflation via four-fold complex structure moduli or D7-brane moduli [156]. The idea is to consider a complex structure modulus u in the partial large complex structure limit. Then, due to shift symmetry of the Kähler potential $\mathcal{K}$ the imaginary part $\mathrm{Im}(u)$ of this modulus (corresponding to the axionic part of the Kähler modulus of the mirror-dual type-IIA model) is absent in $\mathcal{K}$. Hence, it represents a periodic variable and it is a suitable inflaton candidate. Shift-symmetry and this periodicity are weakly broken by turning on fluxes. By appropriate flux-choice one can obtain a superpotential of the form $W = w + au$. The flux-induced term au gives rise to a monodromy.

[13] Interestingly, one can derive an upper bound on $\mathrm{Re}(u)$ from the energy scale of inflation. After canonical normalisation one obtains $V_{\mathrm{inf}} \simeq \epsilon^2\varphi^2/\mathcal{V}^2 \sim 0.5 \cdot 10^{-8}$. Using (3.6.15), one finds that $\mathrm{Re}(u) < 10^4/\mathcal{V}$.

[14] Furthermore, notice that for this chosen example, the integration over the flux space rather underestimates the correct value of the sum over the flux space due to the fact that the dimension of the flux space is very large. This indicates that there should be more vacua satisfying the tuning conditions left than estimated.

Since the inflaton mass has to be below the Kähler modulus stabilisation scale, this monodromy effect has to be sufficiently weak. Our aim was to make a, which generically depends on many other moduli, i.e. $a = a(z)$, small by flux-tuning. For this proposal the dependency of a on other moduli is crucial, because such a fine-tuning is impossible to achieve with integer flux-numbers only.

However, this dependency $a = a(z)$ comes with some further challenges to make this scenario work. As the inflaton field moves over large field distances, the other moduli are displaced due to the interaction term $a(z)u$. We argued that fine-tuning of the derivatives $\partial_z a(z)$ with respect to every complex structure modulus are required to control the displacements of the moduli z. However, there is significant backreaction on the inflaton potential, but there is a regime in which the potential remains sufficiently flat for large-field inflation.

Rather than discussing these backreaction issues in detail in this thesis, we focused on the realisation of such an F-term axion monodromy inflation model in the type IIB flux-landscape. We also paid particular attention to geometric constraints on the mandatory fine-tuning. First of all, we derived a no-go theorem for complex structure inflation on type IIB orientifolds. It turns out that the geometry of Calabi-Yau 3-folds are typically not rich enough to allow for the tuning of $a(z)$ and its derivatives in the perturbative regime. Next, we showed that this no-go theorem does not apply to Calabi-Yau 4-folds. Hence, F-term axion monodromy inflation via complex structure moduli should rather be addressed in F-theory (or by inflation with a D7-brane modulus in type IIB). Our analysis demonstrates that the tunings can be realised, at least in principle. It would nevertheless be interesting to realise such a tuning on concretely chosen 4-folds. This is most likely a computationally very challenging, but certainly interesting problem. Finally, we addressed the question whether the flux-landscape of F-theory is large enough to be consistent with our set of tuning conditions. Our philosophy of this issue is as follows: if, after imposing the tuning conditions, no or only few vacua of an initially large landscape remain, we conclude that the tuning is not realisable. However, taking a 4-fold with sufficiently large Euler character and many moduli, the initial landscape should typically be large enough to allow for the tuning, unless many moduli also enter the function $a = a(z)$.

Concerning the question of realising large-field inflation in string theory, our conclusion is therefore positive, even though the necessity of the potentially large amount fine-tuning might be disappointing at first sight. We demonstrated that our model of F-term axion monodromy inflation has a chance to work. To our present knowledge it resists stringent quantum gravity constraints, such as the Weak Gravity Conjecture (see e.g. [93; 100; 107], but also [106; 167] for constraints on relaxion models). Instead, we observe geometric constraints from string theory (such as moduli stabilisation and our no-go theorem) in combination with constraints due to the finiteness of the landscape. We therefore believe that the understanding of the fine-tuning on concrete Calabi-Yau geometries as well as possible developments of our presented landscape counting deserve further investigations in future work.

As to the realisation of our model in more concrete settings, it would be interesting to improve our understanding of the role of α'-corrections. The $\mathcal{N} = 2$ level α'-corrections on the type IIB side do not depend on complex structure moduli and therefore we expect these corrections to be irrelevant for our inflation scenario. On the type IIA side, such α'-corrections are reflected in the linear terms appearing in the period vector (3.3.4) in type IIB (see e.g. [168] and earlier works [169–171]). However, it would be important to account for α'-corrections arising from 7-branes.[15] In [172] it is demonstrated that on CY 4-folds

[15] See also our discussion in [112].

a certain class of F-theory α'-corrections does not modify the functional form of the Kähler potential (see also [173]). Additionally, in [154] it was argued that a more complete analysis of large-field inflation with D7-branes requires the incorporation of higher-derivative corrections to the 4d supergravity description coming from DBI-terms. This was also discussed in much more detail in [174], which appeared shortly after the first version of our work [112]. Further developments [175] on this indicate a flattening of the inflaton potential due to such DBI-induced α'-corrections, consistent with [176], where a flattening from Kähler moduli was observed as well. It would be interesting to analyse whether these flattening effects can relax the tuning conditions of our model.

Another hope in relaxing the tuning would consist in combining [157], where w was chosen to be large, with our fine-tuning of a and we leave such an analysis for future work.

Finally, the implications of the complicated behaviour of the potential close to the SUSY-minimum deserve further investigations. For instance, the non-monotonic behaviour of the potential could prevent the inflaton from rolling down to its minimum. Instead it could get stuck in a SUSY-breaking local minimum. This idea could provide a new F-term uplifting mechanism (similar to [177]).

CHAPTER 4

F-Term Winding Inflation: Realisation of Alignment Inflation in String Theory

4.1. Introduction and Summary

Using F-terms we were able to present in Chapter 3 a model of F-term axion monodromy inflation in supergravity language, taking into account backreaction. Although large-field inflation can in principle be achieved in this way, the required amount of fine-tuning may be disappointing. Furthermore, geometric constraints make a concrete realisation challenging, or – at least in the 3-fold case – even impossible.

In this chapter we therefore aim to establish a different, much less tuned, inflation model using F-terms. Specifically, we aim to realise alignment inflation, similar to the Kim-Nilles-Peloso (KNP)-mechanism [42], in string theory. This mechanism may, however, be in the swampland due to quantum gravity constraints such as the Weak Gravity Conjecture (WGC). We take this as an opportunity to construct an alignment model from string theory and to explore in which sense it is consistent or constrained by quantum gravity arguments. Thus, rather than trying to derive no-go theorems from some quantum gravity arguments, which finally may or may not hold true, we address the problem of establishing a viable model of alignment inflation by proof by existence.

The construction of natural inflation via complex structure moduli has been attempted in [178]. Our work [113] follows the spirit of [42; 179]: we try to achieve large field displacements by a winding trajectory in the compact field space of two complex structure "axions".[1] We therefore call our scenario "F-term winding inflation". More specifically, it is based on three observations (see also our introduction in [113]): First, as explained in Chapter 3, complex structure moduli exhibit an axionic shift symmetry in the large complex structure (LCS) limit. Second, bulk fluxes F_3 and H_3 lead to a non-vanishing F-term scalar potential for these axions. Appropriate flux choices allow to fix some of the axions such that only one flat axionic direction remains [157; 178]. In particular, one can achieve a flat direction by a long winding trajectory wrapping the compact field space multiple times (see e.g. Figure 4.2). Finally, corrections to the large complex structure geometry generate a potential along the previously

[1] Recently, in [180] it was proposed to realise alignment inflation in the spirit of our work [113] near a conifold locus of the complex structure moduli space.

flat direction. These corrections correspond to instantons in the mirror dual 3-fold. There are two types of relevant instantonic terms: those with a long period, corresponding to the full length of the trajectory, and those with a short period, potentially ruining inflation. We stabilise the saxions such that the long-period terms are reasonably small while the dangerous terms are completely negligible.

Our proposed model is subject to constraints from string theory and quantum gravity effects in general. First of all, it needs to be ensured that the Kähler moduli remain stabilised during inflation. Our analysis shows that this requirement is non-trivial but it can still be met in the Large Volume Scenario (LVS). Moreover, we learn that type IIB string theory puts an upper bound on the effective axion decay constant. This bound follows from the D3-tadpole cancellation condition. Nevertheless, at present this constraint is not strong enough to rule out our inflationary scenario. More stringent bounds on the axionic field displacement could be obtained from quantum gravity arguments such as the WGC. Indeed, if the particle satisfying the WGC must also be the lightest one (this is the "strong" form of the Weak Gravity Conjecture), our winding inflation model is ruled out. If, in contrast, the mild version holds true, there is a loophole [100; 101] in this argument and we can show that our model precisely fits this loophole. Moreover, [108] opened the question whether gravitational instantons, arising from a shift-symmetric axion coupled to Einstein gravity, can lead to dangerous corrections. We give a few remarks on this problem and postpone a detailed analysis to Chapter 5.

This chapter begins with a brief review of the realisation of alignment inflation via the KNP-mechanism. The remainder of this chapter is based on our paper [113]. Parts of this work, particularly the derivation of the inflaton potential including backreaction, will also be reviewed in the thesis [120] by coauthor F. Rompineve. In Section 4.3 we explain the technical details of our winding inflation proposal with focus on the geometric setup and the issue of Kähler moduli stabilisation. Most of Section 4.3.1 and Section 4.3.3 follows [113]. The issue with quantum gravity constraints is discussed in Section 4.4, where the explanations of the loophole [113] are extended by further details. Finally, we give a summary of our results and remark on recent progress in this field.

4.2. Review of Alignment Inflation in Effective Field Theory

We briefly sketch the idea of alignment inflation, known as the KNP-mechanism [42]. Our choice of notation anticipates what follows in the subsequent sections. The presentation is similar to some slides of a talk given by L.T. Witkowski [181]. For a more general description of the KNP-mechanism, we refer to the original work [42] and [32].

Consider a model in which a potential of the form

$$V = e^{-S_1}\left(1 - \cos\frac{v}{f_1}\right) + e^{-S_2}\left(1 - \cos\frac{u - Nv}{f_2}\right) \tag{4.2.1}$$

is generated for the axion fields u and v. We assume that $f_1, f_2 < 1$ so that there is no immediate conflict with the Weak Gravity Conjecture, or with negative results like [40] concerning large axion decay constants. Moreover we take $N \gg 1$, which will finally make the effective axion decay constant parametrically large. Furthermore, let us assume that the instanton generating the first term is heavier than the one yielding the second term, i.e. $S_2 \ll S_1$. It is convenient to express the potential in terms of a field basis defined by

$$\psi \equiv \frac{1}{N}(u - Nv)\ , \qquad \phi \equiv \frac{1}{N}(Nu + v)\ . \tag{4.2.2}$$

Hence, we have $u \simeq (\psi + N\phi)/N$ and $v \simeq (\phi - N\psi)/N$. It follows:

$$V = e^{-S_1}\left(1 - \cos\frac{\phi - N\psi}{Nf_1}\right) + e^{-S_2}\left(1 - \cos\frac{N\psi}{f_2}\right) . \tag{4.2.3}$$

By virtue of the hierarchy $S_2 \ll S_1$, the second term stabilises ψ at $\psi = (f_2/N)\pi \cdot \mathbb{Z}$. For simplicity we take $\psi = 0$. Then, integrating out ψ yields

$$V = e^{-S_1}\left(1 - \cos\frac{\phi}{f_{\text{eff}}}\right) \tag{4.2.4}$$

with effective axion decay constant $f_{\text{eff}} = Nf_1$. Since $N \gg 1$, the effective axion decay constant can be trans-Planckian, allowing to realise successful inflation.

Unfortunately, the WGC extended to multiple $U(1)$ gauge field rules out this model. This extended WGC demands that the convex hull spanned by the charge-to-mass vectors of each state has to span the unit ball, whose surface is given by the charge-to-mass vectors of an extremal black hole charged under these multiple $U(1)$'s. Applied to our case, we see that the convex hull spanned by the directions corresponding to v and $u - Nv$ can never contain the unit circle for $N \gg 1$. This problem is depicted in on the left of Figure 4.1. Hence, the alignment inflation scenario is in conflict with the extended WGC.

A way out is to add a term $e^{-S_3}(1 - \cos u/f_3)$ generated by a third "spectator instanton" (see again Figure 4.1) [100; 101] to (4.2.1). We assume that also $S_3 \gg S_2$. Again, ψ can be integrated out and hence

$$V = e^{-S_1}\left(1 - \cos\frac{\phi}{f_{\text{eff}}}\right) + e^{-S_3}\left(1 - \cos\frac{\phi}{f_3}\right) . \tag{4.2.5}$$

It still holds $f_{\text{eff}} = Nf_1$, but it must be guaranteed that the second term is highly suppressed, e.g. by $S_3 \gg S_1$. If the mild WGC holds true, this hierarchy turns out to be viable. We discuss this point in some more detail in Section 4.4.

4.3. Alignment via Winding Inflation in String Theory

Our inflation model is formulated in the effective supergravity description of type IIB string compactifications, where our inflaton arises from the complex structure moduli sector. The model is captured by the following Kähler potential and superpotential[2]

$$\mathcal{K} = -\log\left[\mathcal{A}(z, \bar{z}, u - \bar{u}, v - \bar{v}) + \left(\mathcal{B}(z, \bar{z}, v - \bar{v})e^{2\pi i v} + \text{c.c.}\right)\right] , \tag{4.3.1}$$

$$\mathcal{W} = w(z) + f(z)(u - Nv) + g(z)e^{2\pi i v} , \tag{4.3.2}$$

with $N \in \mathbb{Z}$ where we take $N \gg 1$. These are the necessary ingredients to compute the F-term scalar potential. The latter is minimised by imposing the conditions $D_I\mathcal{W} = 0$, where I runs over all moduli. Assuming that the exponential terms in (4.3.1) and (4.3.2) are suppressed, only the fields $\text{Im}(u)$ and $\text{Im}(v)$ as well as the combination $\text{Re}(u - Nv)$ are stabilised at the minimum. The fourth scalar component parametrises a flat direction which is closely aligned

[2]Mind the change of conventions with respect to Chapter 3, where a complex structure modulus u in the LCS limit entered the Kähler potential as $\mathcal{K} = \mathcal{K}(u + \bar{u})$. In this chapter we choose opposite conventions such that $\mathcal{K} = \mathcal{K}(u - \bar{u})$ in the LCS regime. This means that the inflaton should then be identified with $\text{Re}(u)$ instead of $\text{Im}(u)$ (as in the previous chapter).

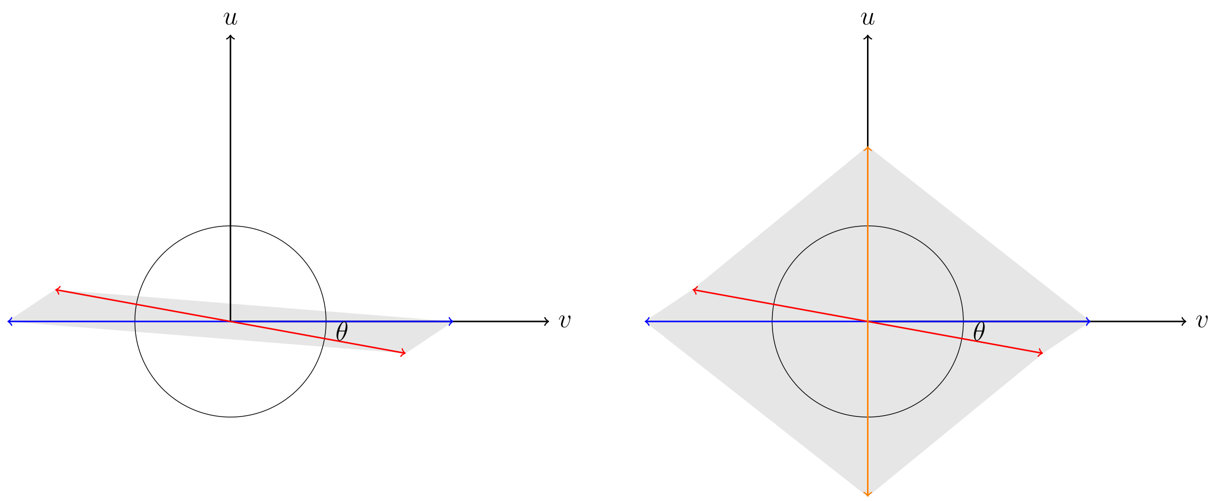


Figure 4.1.: The illustration on the left shows the charge vectors in alignment inflation. The red arrows correspond to the direction $\pm(-1, 1/N)$ of ψ and the blue arrows correspond to the direction $(\pm 1, 0)$ of v. Large N therefore corresponds to small angles θ, but also implies a large effective axion decay constant. The convex hull spanned by charge vectors along the described directions is depicted in gray. It does not contain the unit ball. On the right hand side, a "spectator instanton" is added. The corresponding directions (in orange) $(0, \pm 1)$ make sure that the unit ball is contained in the new convex hull.

with $\mathrm{Re}(u)$. As will be shown, the exponential terms lift this flat direction introducing a cosine-potential with a large period.

In the following we will examine in more detail how the structure of (4.3.1) and (4.3.2) arises from the geometry of Calabi-Yau 3-folds and argue that the resulting scalar potential is suitable for inflation.

4.3.1. Geometry, Fluxes and Instanton Corrections

We begin with a type IIB Calabi-Yau orientifold X with $h^{2,1}_{-}(X) = n$ complex structure moduli $\{z^i\}$. The quantum-corrected Kähler potential can be written as (see e.g. [160; 164])

$$\begin{aligned}\mathcal{K} = -\log\Bigg(&-\frac{i}{3!}\kappa_{ijk}(z^i - \bar{z}^i)(z^j - \bar{z}^j)(z^k - \bar{z}^k) + ic \\ &+ i \sum_{\substack{\beta \in H_2(\tilde{X},\mathbb{Z}) \\ \setminus\{0\}}} \sum_{m=1}^{\infty} \frac{2n_\beta[1 - \pi i m \beta_i(z^i - \bar{z}^i)]}{(2\pi i m)^3} \\ &\times \left[e^{2\pi i m \beta_i z^i} + e^{-2\pi i m \beta_i \bar{z}^i}\right]\Bigg) \,,\end{aligned} \tag{4.3.3}$$

where n_β are constants related to Gromov-Witten invariants, κ_{ijk} are intersection numbers of the dual 3-fold and $c = -\frac{i}{4\pi^3}\zeta(3)\chi(X)$. Summations over i, j, k run from 1 to $h^{2,1}_{-}(X)$.

The structure for $\mathcal{K}$ in (4.3.1) can be achieved when working in a regime of large complex structure (LCS) for some of the complex structure moduli. In particular, we consider two complex structure moduli which we label by u and v. We then assume that the F-term conditions stabilise u and v such that the following hierarchy is realised:

$$e^{-2\pi \mathrm{Im}(u)} \ll e^{-2\pi \mathrm{Im}(v)} \ll 1 \ . \tag{4.3.4}$$

As we will show later, inflation proceeds along a direction in which the condition (4.3.4) remains true. At LCS for u and v terms of the form $e^{2\pi i u}$ and $e^{2\pi i v}$ are suppressed and the leading term in the Kähler potential only depends on the shift-symmetric combinations $(u-\bar{u})$ and $(v-\bar{v})$. By enforcing (4.3.4) terms of the form $e^{2\pi i u}$ are subleading compared to $e^{2\pi i v}$ and are thus ignored in the following. We only retain the leading instantonic term in v. With the mild assumption that the dominant such term contributes as $e^{2\pi i v}$ (i.e. assuming $\beta_v = 1$) we obtain the term $\mathcal{B}e^{2\pi i v}$ in (4.3.1). The superpotential (4.3.2) is the Gukov-Vafa-Witten superpotential and takes the form

$$\mathcal{W} = (N_F - \tau N_H)^\alpha \Pi_\alpha \ , \tag{4.3.5}$$

where $N_F^\alpha, N_H^\alpha \in \mathbb{Z}$ are flux integers, τ is the axio-dilaton and Π is the period vector (see also (3.3.4)). It has entries (now writing down the structure of the instanton terms more explicitly)

$$\Pi = \begin{pmatrix} 1 \\ z^i \\ \frac{1}{2!}\kappa_{ijk}z^j z^k + \frac{1}{2!}a_{ij}z^j + b_i + \sum\limits_{\beta,m} \frac{n_\beta \ \beta_i}{(2\pi i m)^2} e^{2\pi i m \beta_i z^i} \\ -\frac{\kappa_{ijk}}{3!}z^i z^j z^k + b_i z^i + \frac{c}{2} + \sum\limits_{\beta,m} \frac{2n_\beta(1-\pi i m \beta_i z^i)}{(2\pi i m)^3} e^{2\pi i m \beta_i z^i} \end{pmatrix} . \tag{4.3.6}$$

The parameters a_{ij} and b_i are taken to be real (and, as we discussed in Section 3.3, they seem to be typically half-integers).

The structure observed in (4.3.2) can be recovered as follows. By choosing appropriate flux integers N_F^α, N_H^α, corresponding to the F_3- and H_3-fluxes, one can ensure that u and v only appear linearly in $\mathcal{W}$ at leading order. In particular, the last entry of the period vector will typically give rise to quadratic and cubic terms in u and v. We forbid these contributions by setting the last entry of the flux vector to zero. Choosing certain flux numbers large we can ensure that $N \gg 1$. As before, instantonic terms in u are ignored. We include the leading instantonic contribution in v which gives rise to $g(z)e^{2\pi i v}$ in (4.3.2).

In the following, we denote the remaining $(n-2)$ complex structure moduli (except u and v) as well as the axio-dilaton by z. All in all, we find that the structure of (4.3.1) and (4.3.2) can indeed be realised in the complex structure moduli sector of a Calabi-Yau 3-fold.

4.3.2. Moduli Stabilisation, Winding Trajectory and the Inflaton Potential

Moduli stabilisation and the generation of the inflationary potential then proceed as follows. Besides fixing complex structure moduli and the axio-dilaton, we also need to stabilise Kähler moduli, which we do according to the Large Volume Scenario (LVS) [7]. At leading order, the theory for Kähler moduli is of no-scale type giving rise to the no-scale cancellation in V (see also Section 3.2):

$$V = e^{\mathcal{K}} \left(\mathcal{K}^{I\bar{J}} D_I \mathcal{W} \overline{D_J \mathcal{W}} + \mathcal{K}^{T_\rho \bar{T}_\sigma} D_{T_\rho} \mathcal{W} \overline{D_{T_\sigma} \mathcal{W}} - 3|\mathcal{W}|^2 \right) \simeq e^{\mathcal{K}} \mathcal{K}^{I\bar{J}} D_I \mathcal{W} \overline{D_J \mathcal{W}} \ , \tag{4.3.7}$$

with I, J running over all complex structure moduli and the axio-dilaton. The potential is thus minimised for $D_I \mathcal{W} = 0$ for all I. Subleading terms due to α'- and non-perturbative corrections stabilise the Kähler moduli.

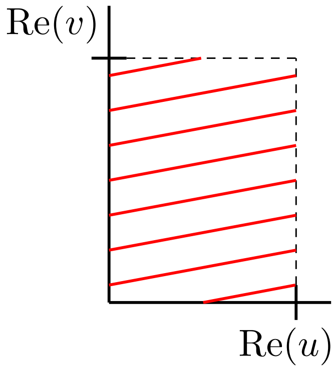


Figure 4.2.: Inflaton trajectory in Re(v)-Re(u)-plane. The winding trajectory is a result of stabilising one direction in Re(v)-Re(u)-space by an F-term potential due to bulk fluxes.

Ignoring instantonic corrections, our ansatz (4.3.1) and (4.3.2) is such that only $\mathrm{Re}(u) - N\mathrm{Re}(v)$ enters the F-term scalar potential, i.e. $V = f(\mathrm{Re}(u) - N\mathrm{Re}(v))$. Hence, after minimisation of V with respect to this combination, one obtains a linear equation that describes the flat direction in terms of the variables Re(u) and Re(v). This flat direction, depicted in Figure 4.2, is independent of the metric of the moduli space and it can be parametrised in any field basis. For convenience, we thus choose the set of complex variables ϕ and ψ defined by

$$\phi \equiv u \; , \qquad \psi \equiv u - Nv \; . \tag{4.3.8}$$

The flat direction is now parametrised by Re(ϕ) while Re(ψ) is fixed. (Note that the orthogonal direction to ψ is actually $\phi = u + v/N$, but for $N \gg 1$ this yields $\phi \simeq u$.)

The exponential term in these field variables reads:

$$e^{-2\pi \mathrm{Im}(v)} = e^{-2\pi \frac{\mathrm{Im}(\phi) - \mathrm{Im}(\psi)}{N}} \equiv \epsilon \ll 1, \tag{4.3.9}$$

where $\epsilon \equiv e^{-2\pi \mathrm{Im}(v_0)}$ is introduced to keep track of small corrections for the below computations. Since v is stabilised in the LCS regime, one has $\epsilon \ll 1$.

Furthermore, $\mathcal{K}$ and $\mathcal{W}$ can be expressed as:

$$\mathcal{K} = K(z, \bar{z}, \mathrm{Im}(\phi), \mathrm{Im}(\psi)) + \mathcal{O}(\epsilon) \; , \tag{4.3.10}$$
$$\mathcal{W} = w(z) + f(z)\psi + \mathcal{O}(\epsilon) \; . \tag{4.3.11}$$

The SUSY conditions $D_I \mathcal{W} = 0$ stabilise all the moduli z, as well as $\mathrm{Im}(\phi), \mathrm{Im}(\psi)$ and Re(ψ), while Re(ϕ) is unfixed.[3] Therefore, we see that the term $f(z)\psi$ in the superpotential results in a breakdown of the two shift-symmetries in the variables u, v such that only one shift symmetry remains. Hence, Re(ϕ), which does not appear in $\mathcal{K}$ and $\mathcal{W}$, is a suitable inflaton candidate.

Let us interpret Figure 4.2 in the ϕ-ψ-language. Since $\mathrm{Re}(\psi) = \mathrm{Re}(u) - N\mathrm{Re}(v)$ is fixed, we see that $\delta\mathrm{Re}(u) = N\delta\mathrm{Re}(v)$. In words, for $N \gg 1$ (achieved by flux-choice) the modulus u

[3]The existence of a flat direction is also clear from counting the SUSY-constraints. There are $2n + 2$ real moduli, but only $2n + 1$ real equations $D_I \mathcal{W} = 0$, because $D_\phi \mathcal{W} = K_\phi \mathcal{W} = 0$ yields only one real equation $K_\phi = 0$.

changes much more strongly than v. This reproduces Figure 4.2 with the depicted trajectory being parameterised by $\mathrm{Re}(\phi)$, which is closely aligned with $\mathrm{Re}(u)$.

The idea of using winding trajectories in the field space of two axions for inflation was originally developed in field theory in [42] (KNP-mechanism). Prior to the publication of our work [113] various different realisations of the KNP-mechanism in string theory have been proposed, see for instance [179; 182–187]. Our proposal is different: we establish a winding trajectory from an F-term potential, which is induced by non-vanishing fluxes.[4]

Note that at this stage, without taking into account instanton corrections, the potential of our inflaton candidate is exactly flat. To turn it around, the non-zero slope for inflation will be created by these instanton corrections. For the computation of the inflaton potential we therefore have to include the subleading term

$$e^{2\pi i v} = e^{2\pi i(\phi-\psi)/N} = \epsilon\ e^{2\pi i(\mathrm{Re}(\phi)-\mathrm{Re}(\psi))/N}\ , \tag{4.3.12}$$

where ϵ is defined in (4.3.9). Up to second order in ϵ the covariant derivative $D_I\mathcal{W}$ can be expressed as

$$\begin{aligned} D_I\mathcal{W} =& A_I(z,\bar{z},\psi,\bar{\psi},\phi-\bar{\phi})+ \\ &+\epsilon\left[B_I(z,\bar{z},\psi,\bar{\psi},\phi-\bar{\phi})e^{2\pi i\phi_1/N}+C_I(z,\bar{z},\psi,\bar{\psi},\phi-\bar{\phi})e^{-2\pi i\phi_1/N}\right]+\mathcal{O}(\epsilon^2)\ , \end{aligned} \tag{4.3.13}$$

where $\phi_1 \equiv \mathrm{Re}(\phi)$. The complex functions A_I, B_I and C_I depend on $z, \bar{z}, \psi, \bar{\psi}$ and $\mathrm{Im}(\phi)$; note that the phases $e^{-2\pi i\psi_1/N}$ and $e^{2\pi i\psi_1/N}$ have been absorbed into the complex functions B_I and C_I. Only this structure of $D_I\mathcal{W}$ is of relevance for the computation. Given a specific 3-fold one could calculate these functions explicitly.

One can show that the resulting inflaton potential V_{inf}, with backreaction of the moduli taken into account, is given by

$$V_{\mathrm{inf}} = e^K\epsilon^2\left(b'\cos\left(\frac{2\pi\phi_1}{N}\right)+c'\sin\left(\frac{2\pi\phi_1}{N}\right)\right)^2\ , \tag{4.3.14}$$

or

$$V_{\mathrm{inf}} = e^K\epsilon^2\lambda^2\left[\sin\left(\frac{2\pi\phi_1}{N}+\theta\right)\right]^2\ , \tag{4.3.15}$$

with some phase θ and constant λ, depending on b', c'. The coefficients b', c' themselves depend on the moduli $z, \bar{z}, \psi, \bar{\psi}$ and $\mathrm{Im}(\phi)$. To arrive at this result it is necessary to diagonalise the (leading order) Kähler metric by an orthogonal transformation. Furthermore, backreaction on the complex structure moduli is to be taken into account. It turns out that the displacements are of order $\mathcal{O}(\epsilon)$. Minimisation of the potential with respect to the displacements yields (4.3.14). Details of these computations are explained in our paper [113] and in the thesis [120].

4.3.3. Compatibility with Kähler Moduli Stabilisation

Finally, we analyse the compatibility of our inflation model with the LVS. Phenomenologically, it is identical to natural inflation. Using the definitions of u, v in terms of ϕ, ψ and $\partial\phi_1 = N\partial\psi_1$ we have

$$\mathscr{L} \supset \mathcal{K}_{u\bar{u}}(\partial\phi_1)^2+\mathcal{K}_{v\bar{v}}(\partial\phi_1)^2/N^2+\mathcal{K}_{u\bar{v}}(\partial\phi_1)^2/N+\mathrm{c.c.} \tag{4.3.16}$$

[4] For instance, in [183], the winding trajectory arises from non-perturbative effects.

Therefore, for large N, we have $\mathcal{K}_{\phi\bar{\phi}} = \mathcal{K}_{u\bar{u}} + \mathcal{O}(1/N)$ and the canonically normalised inflaton field is defined by: $\varphi \equiv \sqrt{2\mathcal{K}_{u\bar{u}}}\phi_1 + \mathcal{O}(1/N)$. In terms of φ, the potential (4.3.15) reads (after also using a trigonometric identity):

$$V_{\rm inf}(\varphi) \sim e^K \epsilon^2 \lambda^2 \left[1 - \cos\left(\frac{\varphi}{f} + 2\theta\right)\right] \ , \tag{4.3.17}$$

with the axion decay constant $f \sim N/(4\pi \mathrm{Im}(u))$ and the canonically normalised inflaton φ. Recent Planck results [2] impose a lower bound of $f > 6.8$ (at 95% CL), and thus our model has to satisfy the phenomenological constraint

$$\frac{N}{\mathrm{Im}(u)} > 85 \ . \tag{4.3.18}$$

Notice that the flux number N receives also an upper bound coming from the tadpole cancellation condition. If the tadpole $L_\star \equiv \chi(\mathrm{CY}_4)/24$ is not sufficiently large, it can turn out to be challenging to achieve $N = \mathcal{O}(100)$ to satisfy the above constraint. However, a detailed analysis could reveal that the above bound is too strict. So far we have not included backreaction of Kähler moduli which is expected to lead to a flattening of the inflaton potential [112; 165] (see in particular the recent analysis of [188]). In this case the bound on N would have to be modified which we leave for future work.

In the following, we demonstrate that the parameters $\lambda, \epsilon, \mathrm{Im}(u)$ and N can be arranged such that the Kähler moduli can in principle be stabilised successfully in the framework of the LVS.

In order to avoid destabilisation of the Kähler moduli during inflation, we require

$$V_{\rm inf} \sim 10^{-8} \ll V_{\rm LVS} \sim e^K \frac{|W|^2}{\mathcal{V}^3} \sim \frac{g_s |W|^2}{\mathcal{V}^3 \mathrm{Im}(u)^3} \ , \tag{4.3.19}$$

where we used that the energy density of the inflaton potential in natural inflation is $V_{\rm inf} \sim 10^{-8}$ in Planck units. Moreover, there is the well-known constraint on the ratio of the gravitino mass and the Kaluza-Klein scale [189]:

$$\frac{m_{3/2}}{m_{\rm KK}} \ll 1 \ . \tag{4.3.20}$$

This constraint can be translated into the condition[5]

$$\frac{m_{3/2}}{m_{\rm KK}} \sim \frac{\sqrt{g_s}|W|}{\mathrm{Im}(u)\mathcal{V}^{1/3}} \ll 1 \ . \tag{4.3.21}$$

Combining the constraints (4.3.19) and (4.3.21) yields

$$10^{-8} \ll \frac{1}{\mathrm{Im}(u)\mathcal{V}^{7/3}} \ . \tag{4.3.22}$$

In order to fulfil the bounds described above, we choose the following numerical example: $N = 150$, $\mathrm{Im}(u) = 1.5$, $\epsilon = 0.02$ (i.e. $\mathrm{Im}(v) \simeq 0.6$), $\mathcal{V} = 100$ and $\lambda \sim \mathcal{O}(10)$. The choices for

[5] One can derive this relation as follows: the gravitino mass is given by $m_{3/2} \sim e^{\mathcal{K}/2}|W| \sim \sqrt{g_s}|W|\mathcal{V}^{-1}\mathrm{Im}(u)^{-3/2}$. In anisotropic compactifications, e.g. in cases where one modulus is stabilised in the large complex structure limit, the Kaluza-Klein mass scales as $m_{\rm KK} \sim L^{-1}\mathcal{V}^{-1/2} \sim (\mathrm{Im}(u))^{-1/2}\mathcal{V}^{-2/3}$, where we parametrised the volume as $\mathcal{V} \sim L^3 R^3$ with $L \gg R$ and $\mathrm{Im}(u) \sim L/R$.

$\mathrm{Im}(u)$ and $\mathrm{Im}(v)$ ensure that terms of the form $e^{-2\pi \mathrm{Im}(v)}$ are small while the terms $e^{-2\pi \mathrm{Im}(u)}$ are negligible. At the same time it is ensured that inflation occurs at the correct energy scale $V_{\mathrm{inf}} \sim 10^{-8}$.

In this case, (4.3.18) and (4.3.22) are easily satisfied. If $g_s \sim 0.01$ and $W \lesssim \mathcal{O}(10)$, then (4.3.21) holds true as well. For this, we have to compensate the large contribution coming from $N = 150$ by a mild tuning of the coefficient $f(z) = \mathcal{O}(0.1)$. Since the LVS potential is $\mathcal{O}(10)$ times larger than the inflaton potential, there is no danger of destabilisation of the Kähler moduli.

4.4. Is F-Term Winding Inflation Part of the String-Landscape?

Finally, we discuss to what extent our inflation model might be censored by further constraints from string theory and by generic quantum gravity arguments. Each of these constraints shape the string landscape and hence winding inflation could still be an element of the swampland rather than the landscape. Possible concerns are the following:

Large N, the Tadpole and Landscape Vacua

As we already remarked, the choice of $N = \mathcal{O}(100)$ might clash with the D3-tadpole cancellation constraint (see (1.4.1))

$$\eta_3(\vec{m}, \vec{n}) \leq L_\star \ , \tag{4.4.1}$$

where η_3 denotes a quadratic form of the flux vectors $\vec{m}$ and $\vec{n}$ corresponding to F_3 and H_3, respectively.

Let us describe the potential problem in more detail. In type IIB string theory there are $2b$ flux numbers $m_1, ..., m_b$ and $n_1, ..., n_b$. The condition $N \gg 1$ will generally translate into an additional constraint $f(m_{i_1}, ...m_{i_p}, n_{j_1}, ..., n_{j_q}) \equiv N \gg 1$, where $1 \leq p, q \leq b$ and f is a model-dependent function of the flux numbers. The set of SUSY flux choices (yielding $\eta_3(\vec{m}, \vec{n}) \geq 0$, see Section 1.4.1) supporting our model is then given by solutions to

$$0 \leq \eta_3(\vec{m}, \vec{n}) \leq L_\star \qquad \text{and} \qquad f(m_{i_1}, ...m_{i_p}, n_{j_1}, ..., n_{j_q}) \equiv N \gg 1 \ . \tag{4.4.2}$$

Very naively, one could be concerned about terms like n_i^2 for some $1 \leq i \leq b$ occurring in $\eta_3(\vec{m}, \vec{n})$, specifically in cases where η_3 is positive definite (which is, however, not in general true). One could then conclude that there is an upper bound $|n_i| \lesssim \sqrt{2L_\star}$. In the example of $L_\star = 972$, we considered in Section 3.6, we obtain $|n_i| \lesssim 44$, which might hint at an inconsistency with (4.3.18) for our winding inflation model if our variable N is such a flux number. However, we should notice that in our model, N is a combination of flux numbers and the intersection numbers. Moreover, since η_3 is in general indefinite, it is conceivable to obtain a large N by choosing (almost) orthogonal vectors $\vec{m}$ and $\vec{n}$ with respect to η_3. This seems to imply that the length of the flux vectors can be arbitrarily large. But again, one should be cautious because this might lead to decompactification [10; 71]. Hence, to arrive at a final conclusion whether the tadpole constraint generically rules out our proposed scenario, further analysis using a concrete compactification geometry with sufficiently large $L_\star$ is needed.

Another concern is that $N \gg 1$ may reduce significantly the number of flux vacua of the string landscape consistent with our model. It is conceivable that the additional constraint $f(m_{i_1}, ...m_{i_p}, n_{j_1}, ..., n_{j_q}) \equiv N \gg 1$ leads to a drastic reduction of the number of flux vacua consistent with F-term winding inflation. For instance, in the worst (but also most naive) case, the effective tadpole number would be reduced to $L_\star^{\rm eff} \sim L_\star - N^2$, shrinking the landscape significantly.

Consequently, a final answer on how restrictive the tadpole condition finally is for our inflationary scenario, can only be obtained by a concrete case study. This problem certainly deserves attention in future work.

Stringy Instanton Corrections and String-Loop Effects

The superpotential written down in (4.3.2) is not quite complete. For a complete description we are missing the "small 4-cycle" instanton contributions of the LVS construction, $A\exp(-2\pi\tau_s)$. Note that it has a complex structure dependent prefactor $A(z,u,v)$. This term could in principle destroy the flatness of the inflaton potential.[6] We hope, based on the fact that this instanton is a highly local effect (associated with the small blow-up cycle), that there are models where the u-dependence of A is subdominant. However, we do not know whether this can be realised.

Similarly, there are loop corrections to the Kähler-moduli-part of the Kähler potential which, however, affect the scalar potential only at subleading order [140; 190; 191]. As the explicit string-loop results of [191] demonstrate, these effects depend in general on complex structure moduli and can ruin the inflationary potential in principle. One may however hope that (as shown in [192] in a special case) the dependence on certain complex structure moduli is exponentially suppressed in the LCS limit. Furthermore, the suppression by the $(\text{volume})^{1/3}$ factor relative to the leading-order LVS potential may be sufficient to control the effect.

We have to leave the detailed study of the above corrections to future work.

Gravitational Instantons and the Weak Gravity Conjecture (WGC)

We begin with brief comments on gravitational instantons. In [108] corrections of the inflaton potential due to gravitational instantons were considered. It was argued that they can constrain models of natural inflation (single axion and the KNP-mechanism), at least if the cutoff for the effective description of gravitational instantons can be pushed sufficiently close to the Planck-scale. Hence, F-term winding inflation could be affected as well. Taking string theory seriously, one might expect a breakdown of the effective description at a scale set by the mass of the lightest modulus, or at latest at the Kaluza-Klein (KK) scale. In [113] we assumed that due to displacements of the lightest modulus the effective description of gravitational instantons must break down. We then found that the generated instanton potential is many orders of magnitudes too week to constrain natural inflation. However, in [114] we showed that the description of gravitational instantons via axions coupled to Einstein gravity can be trusted even to the KK-scale. Nevertheless, even when pushing the KK-scale as close to the Planck-scale as possible, the effects turn out to be insufficient to generically constrain natural inflation. We postpone an extensive discussion on these topics to Chapter 5 of this thesis.

In contrast, the Weak Gravity Conjecture (WGC) can potentially rule out our proposed model. If the WGC is to be extended to instantons (for discussions see [100] and our paper [113]), we come to the following conclusions ([113]):

- If the strong version of the WGC is correct, all models with large axion decay constants in a calculable regime should be inconsistent.[7] In this case one should be able to rule

[6] Note that due to holomorphicity of the superpotential, the dependence of A on the complex structure moduli will definitely break shift symmetry.

[7] By calculable regime we mean situations where $S > 1$ and hence $\exp(-S)$ is small.

out our proposal: by studying our model in more detail we would then expect obstacles to large-field inflation to appear.

- On the other hand, if the WGC only holds in its mild form, there is a loophole [100; 101] which allows for simultaneously satisfying the mild WGC and realising trans-Planckian axion inflation. Very intriguingly, our model appears to fit precisely into this loophole as we now explain.

Let us describe this loophole pointed out in [100; 101][8]: To be consistent with the mild form of the WGC the relevant axion ϕ has to couple to two instantons giving rise to a potential of the structural form [100]

$$V = \Lambda_1^4 e^{-m} \left[1 - \cos\left(\frac{\phi}{f}\right)\right] + \Lambda_2^4 e^{-M} \left[1 - \cos\left(\frac{k\phi}{f}\right)\right] \ , \tag{4.4.3}$$

with $k \in \mathbb{Z}$. The first term is due to a light instanton of mass m and charge $q < m$ (note that $M_p = 1$), i.e. it does not satisfy the WGC. The axion decay constant is $f \sim 1/q$. The second term arises from a heavier instanton of mass $M > m$ and charge $Q = kq$ such that the WGC is satisfied: $M \lesssim Q$. Then, as long as f/k is sub-Planckian the mild form of the WGC is satisfied in virtue of the second term, even for a trans-Planckian f. The second term is not suitable for inflation while the first term can sustain inflation for trans-Planckian f. A successful model for inflation can then be achieved when $e^{-M} \ll e^{-m}$ such that contributions from the second term are suppressed.

We observe this structure in our setup. We begin with two instantons $e^{2\pi i u}$ and $e^{2\pi i v}$. We stabilise and integrate out the "saxions" $u_2 \equiv \text{Im}(u)$, $v_2 \equiv \text{Im}(v)$ such that $\exp(-2\pi u_2) \ll \exp(-2\pi v_2)$. In addition, we stabilise one axionic direction $\text{Re}(\psi) = \text{Re}(u) - N\text{Re}(v)$ such that we are left with one axion $\phi_1 = \text{Re}(u)$ coupling to two instantons. Canonically normalising $\varphi = \sqrt{2K_{\phi\bar{\phi}}}\ \phi_1$ and defining $f_{\text{eff}} = N\sqrt{2K_{\phi\bar{\phi}}}$, the axion φ couples to instantonic terms as

$$A_1 e^{-2\pi v_2} e^{2\pi i\varphi/f_{\text{eff}}} \quad \text{and} \quad A_2 e^{-2\pi u_2} e^{2\pi i N\varphi/f_{\text{eff}}} \ . \tag{4.4.4}$$

This is just as in the loophole presented above. Notice that we have here performed the change of basis only for the real parts of u and v , to make connection with (4.4.3). Choosing $N \gg 1$ one can achieve a trans-Planckian f_{eff}. The first term in (4.4.4) then gives rise to the inflaton potential, while the second instanton ensures that the mild form of the WGC is satisfied. By having stabilised $\exp(-2\pi u_2) \ll \exp(-2\pi v_2)$ we also prevent the second term from spoiling the inflaton potential.

We learn that it is fairly challenging to prove or disprove inconsistency of F-term winding inflation with string theory or quantum gravity. We conclude that the embedding of our scenario in a concrete string compactification could clarify whether the string landscape accommodates F-term winding inflation.

Unfortunately, it is still unclear which formulation, if any, of the WGC could be the correct one.[9] Therefore, it is too early to definitely decide whether F-term winding inflation is part of the swampland or the string landscape.

[8]Yet there is another loophole pointed out in [193], based on models with instanton action $S < 1$. In such cases $f > 1$ is consistent with the WGC. However, in general it is difficult to calculate the instanton potential in the regime $S < 1$ explicitly.

[9]Note that further investigation of our model in string theory can be rewarding: if F-term winding inflation can be shown to be consistent with string theory, then the strong version of the WGC cannot be true.

4.5. Conclusions

We introduced a new model of large-field inflation in string theory, which employs axionic fields arising from complex structure moduli of a Calabi-Yau 3-fold at large complex structure. The trans-Planckian field range is generated as a winding trajectory in the compact field space of two (or more) axions. One new aspect of this construction is that this winding trajectory can be generated by an F-term potential from bulk fluxes in type IIB string theory. Such a trajectory requires a hierarchy in the flux superpotential, and a certain combination of flux numbers must be chosen large. As the size of flux numbers is limited by the tadpole cancellation condition, there seems to be a natural cutoff for the axion field range, such that it cannot get arbitrarily large.

The inflaton potential is generated by instanton corrections, which sum up to an oscillatory potential. Phenomenologically, we end up with a realisation of natural inflation. We also demonstrated that destabilisation of Kähler moduli in the LVS can be avoided, without requiring excessive tuning. It would, however, be interesting to consider backreaction of the Kähler moduli as well, see e.g. [194]. This might result in a flattening of the potential and relax our requirement of having a large flux number N. Such effects could dispel any worry of inconsistency with tadpole cancellation or a decimation of the number of suitable flux vacua.

Further progress in the understanding of the Weak Gravity Conjecture is needed to finally reach a decisive conclusion whether our winding inflation model is censored by the WGC. We showed that, if only the “mild WGC” holds, then our model precisely realises the loophole [100; 101]. Given the current theoretical understanding, we thus believe that our construction is a valuable addition to the set of large-field models in string theory.

CHAPTER 5

Constraints on Axion Inflation from Gravitational Instantons?

This chapter follows our paper [114].

5.1. Introduction and Summary of Results

Slow-roll inflation relies on flat scalar potentials, making axion-like fields ideal inflaton candidates. This is especially true in the context of large-field inflation. The latter is of particular interest since, on the one hand, it is arguably the most natural form of inflation and, on the other hand, it will be discovered or experimentally ruled out in the foreseeable future.

The flatness of axion potentials (we denote the axion henceforth by θ) is protected by a shift symmetry which is only broken non-perturbatively, i.e. by instantons. However, possible problems with consistently embedding axionic models of inflation in quantum gravity are an issue of continuing concern [20; 40; 82; 90; 92–95; 100; 101; 106; 108; 113; 193; 195–207]. In particular, the focus has recently been on the Weak Gravity Conjecture [20]. In the context of axions, it states that with growing axion decay constant $f_{\rm ax}$ the action S of the 'lightest' instanton decreases, such that the flatness of the potential is spoiled by corrections $\sim \exp(-S)$.

However, the Weak Gravity Conjecture has not been firmly established. In particular, its validity remains unclear outside the domain of UV completions of quantum gravity provided by the presently understood string compactifications. This is even more true for the extension to axions. Moreover, the prefactors of the $\exp(-S)$ corrections mentioned above may be parametrically small, especially if SUSY or the opening up of extra dimensions come to rescue just above the inflationary Hubble scale.

Thus, it is useful to pursue the related but complementary approach of constraining axionic potentials on the basis of gravitational instantons. Indeed, the very fundamental statement that quantum gravity forbids global symmetries is, in the context of shift symmetries, explicitly realised by instantonic saddle points of the path integral of Euclidean quantum gravity. These are also known as Giddings-Strominger wormholes [109]. If, as proposed in [108], gravitational instantons yield significant contributions to the axion potential, some models of natural inflation would be under pressure (at least those with one or only few axions like alignment scenarios), while axion monodromy inflation models seem to be unaffected.[1] It is

[1] Natural inflation [208] with one axion requires a trans-Planckian axion field space. Ideas for realising natural

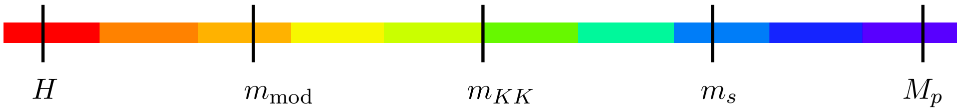


Figure 5.1.: Hierarchy of scales in a string model of inflation.

our goal to study the effect of Euclidean wormholes and that of related instantonic solutions in detail. In particular, in the spirit of what was said above, we want to be as model-independent and general as possible, ideally relying only on Einstein gravity and the additional axion. The goal is to constrain large classes of string models or even any model with a consistent UV completion. As we go along, we will however be forced to consider certain model-dependent features and take inspiration from the known part of the string theory landscape.

The aim of this chapter is thus to determine the strongest constraints on axion inflation due to gravitational instantons. One important aspect of our analysis is that – to be as model-independent as possible – calculations are performed in an effective 4-dimensional Einstein-axion(-dilaton) theory. However, this theory is only valid up to an energy-scale Λ and, for consistency, we have to make sure that our analysis only includes gravitational instanton solutions within the range of validity of our effective theory.

This leads to the following challenge pointed out in [113] (see also [82]) and which we will repeat here. Given an energy cutoff Λ, gravitational instantons within the range of theoretical control contribute at most as $\delta V \sim e^{-S} \sim e^{-M_p^2/\Lambda^2}$ to the axion potential. Then, gravitational instantons are dangerous for inflation if their contribution to the potential is comparable to the energy density in the inflationary sector, i.e. $\delta V \sim H^2$. If the cutoff Λ is not much above H gravitational instantons are clearly harmless. However, if Λ is close to M_p gravitational instantons can easily disrupt inflation. As a result, the importance of gravitational instantons for inflation hinges on a good understanding of the scale Λ where the 4-dimensional Einstein-axion(-dilaton) theory breaks down.

To arrive at a quantitative expression for Λ requires some knowledge about the UV completion of our theory. Here, we take string theory as our model of a theory of quantum gravity, i.e. we assume that the effective Einstein-axion(-dilaton) theory is derived from string theory upon compactification. String compactifications give rise to a hierarchy of scales as shown in Figure 5.1. Inflation is assumed to take place below the moduli scale m_{mod} where only gravity and one or more axions are dynamical. Above m_{mod} further scalars in the form of moduli become dynamical. As a result, if we want to work with a Einstein-axion theory the cutoff Λ is the moduli scale.

Here, we want to do better. An analysis using 4-dimensional gravitational instantons can in principle be valid up to the Kaluza-Klein (KK) scale m_{KK}, at which a description in terms of a 4-dimensional theory breaks down. However, to be able to go beyond m_{mod} we have to allow for dynamical moduli. Hence, for this purpose Einstein-axion theories are insufficient and we have to study gravitational instantons in Einstein-axion-moduli theories instead.

These considerations give rise to the following structure of this chapter. We start by re-

inflation in a subplanckian field space of multiple axions were proposed in [42; 43; 186; 209]. For models implementing these ideas see e.g. [113; 178; 180; 182–185; 187; 210–222]. Axion monodromy inflation was introduced in [44; 151] (for a field theory implementation see [45; 152]). A realisation of this idea with enhanced theoretical control is F-term axion monodromy [154–156]. For further work in this context see [112; 157; 166; 174; 179; 194; 223–230].

calling the Giddings-Strominger or Euclidean wormhole solution [109] in Section 5.2. This is a classical solution of the axion-gravity system which gives space-time a 'handle' with cross-section S^3. In fact, this solution can be interpreted as a real saddle point of the path integral only in the dual 2-form theory. We take some care to describe the relevant subtleties of the dualisation procedure in Section 5.2.1. Subsequently, we generalise to the case with an additional dilatonic scalar in Section 5.2.2. Now extremal as well cored instanton solutions [92; 110] also exist. The situation with a dilaton is important for us as a model of the realistic string-phenomenology case with light moduli. Section 5.2.3 focusses on the way in which cored and extremal gravitational instantons may arise from a Euclidean black 0-brane in an underlying 5d theory. In this way we obtain a UV-completion of cored and extremal gravitational instantons, which can then be understood by parameters of the 5d theory.

Section 5.3 is devoted to the crucial issue whether a scalar potential is induced by Euclidean wormholes. We will provide an explicit computation of the contributions to the axion potential from Euclidean wormholes. Thereby, we describe how to circumvent a recent counter-argument given in [92], suggesting that Euclidean wormholes could not break the axionic shift symmetry. Thus, we stress that Euclidean wormholes are by no means less important than cored or extremal gravitational instantons.

In Section 5.4 we calculate the instanton actions for Euclidean wormholes as well as for cored and extremal gravitational instantons. We also give a quantitative answer to the question which gravitational instantons can be trusted within our effective theory with cutoff Λ. The result is as follows. As in the case of gauge instantons one can associate gravitational instantons with an instanton number n. Given an energy cutoff Λ one can then only trust gravitational instantons with a sufficiently high instanton number $n \gg f_{\text{ax}} M_p/\Lambda^2$, where f_{ax} is the axion decay constant.[2]

In Section 5.5 we take first steps towards studying gravitational instantons in the presence of dynamical moduli. We argue that the case with one light modulus coupled to the Einstein-axion theory can be modelled by an Einstein-axion-dilaton theory with massless dilaton. For one, in Section 5.5.1 we show that for our purposes the modulus potential can be neglected if there is a sufficient hierarchy between the modulus mass and the cutoff Λ. The reason is that deep inside the 'throat' of a gravitational instanton the modulus mass only gives a subleading contribution to the stress-energy tensor, while curvature and gradient terms dominate. As this region is also the source of the dominant part of the instanton action, we conclude that the action obtained for a massless modulus will remain a good approximation even in the massive case. We then motivate our restriction to moduli with dilatonic couplings. This implies that the modulus φ is coupled to the axion θ through the kinetic term for the axion as $e^{\alpha\varphi}(\partial\theta)^2$. In Section 5.5.2 we review that dilatonic couplings arise frequently in string compactification.

In Section 5.6 we analyse possible constraints for inflation due to gravitational instantons. To this end we identify the instantons with the largest contributions to the axion potential in Section 5.6.1. We arrive at the strongest constraint if the cutoff Λ is as high as possible. In Section 5.6.2 we identify the highest possible cutoff Λ_{max} for an effective 4-dimensional theory arising from a string compactification. This is given by the KK scale of a compactification with smallest possible compactification volume, which we take as the self-dual volume under T-duality. Unfortunately, there is an ambiguity in this definition of Λ_{max} up to factors of π, which can be crucial. We then determine the maximal contribution δV to the axion potential due to gravitational instantons and compare this to the scale of inflation in models of large-field axion inflation. Our main result is as follows. We find that gravitational instantons do not

[2]Note that this implies that we neglect potentially more severe, but incalculable contributions due to instantons with low instanton numbers.

give rise to strong *model-independent* constraints on axion inflation. Extremal gravitational instantons may be important for inflation, but this is model-dependent, as the size of their contribution depends on the value of the dilaton coupling α.

Last, in Section 5.7 we record some observations regarding the Weak Gravity Conjecture (WGC) [20] in the context of gravitational instantons. We pick up the idea from [92] that extremal instantons play the role extremal charged black holes for the WGC. We then find that the WGC appears to be satisfied due to the existence of Euclidean wormholes. This either hints at a realisation of the WGC in the context of gravitational instantons, or implies a different definition of the WGC in the presence of wormholes.

We summarise our findings in Section 5.8 and point out directions for future work. Various appendices contain detailed computations on which some of our results are based, or clarify subtleties which are not absolutely essential for the understanding of the main body of this chapter.

Overall, our analysis leaves us with the following: a semi-classical approach to quantum gravity via gravitational instantons does not give rise to strong constraints for large-field inflation. Thus, if quantum gravity has anything to say about large-field inflation, the quantum part will have to speak.

5.2. Gravitational Instanton Solutions

A model of axion inflation will necessarily involve an axionic field coupled to gravity. One feature of such a system is that it may allow for *gravitational instantons*, i.e. finite-action solutions to the equations of motion of the Euclidean axion-gravity theory.

Our starting point is the Euclidean action for an axionic field θ coupled to gravity, which takes the form ($M_p = 1$)

$$S = \int d^4x\sqrt{g}\left[-\frac{1}{2}R + \frac{1}{2}K g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta\right] \ . \tag{5.2.1}$$

The prefactor K can in principle depend on further fields. In this section we ignore the Gibbons-Hawking-York boundary terms, because we will be focussing on the dynamics of the system. Instead of working with the axionic field θ, one can write the action in terms of the dual 2-form B and its field strength $H = dB$:

$$S = \int d^4x\sqrt{g}\left[-\frac{1}{2}R + \frac{1}{2}\mathcal{F} H_{\mu\nu\rho}H^{\mu\nu\rho}\right] \ , \tag{5.2.2}$$

where $\mathcal{F} = 1/(3!K)$. The field strength H is related to $d\theta$ via

$$H = K \star d\theta \ . \tag{5.2.3}$$

The dualisation from (5.2.1) to (5.2.2) must be done under the path integral using Lagrange multipliers. We will explain this in the following subsection.

In Euclidean space the theory of the 3-form H coupled to gravity (5.2.2) then has non-trivial solutions. In particular, *gravitational instantons* are rotationally symmetric solutions with metric

$$ds^2 = \left(1 + \frac{C}{r^4}\right)^{-1} dr^2 + r^2 d\Omega_3^2, \tag{5.2.4}$$

where the parameter C arises as a boundary condition or integration constant (see Appendix C.1). For $C < 0$ this is known as a *Giddings-Strominger or Euclidean wormhole*

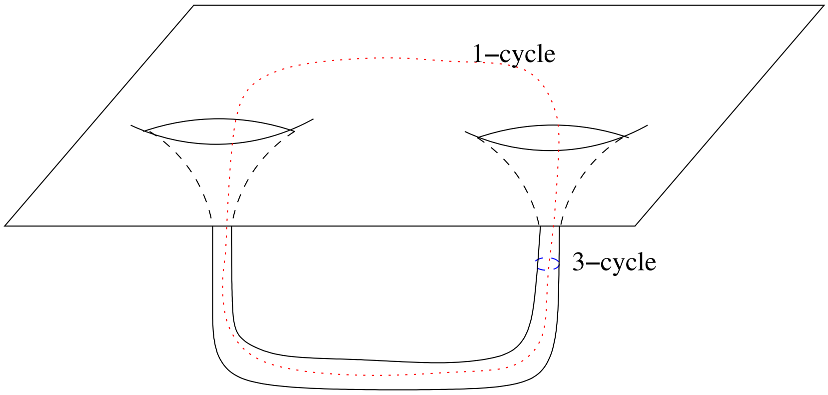


Figure 5.2.: This picture illustrates a Euclidean wormhole, whose two ends are connected to the *same* asymptotically flat space. Then there is a non-trivial 1-cycle (dotted line) passing through the wormhole. The cycle orthogonal to this 1-cycle is a S^3 (symbolised by the dashed line around the right-hand throat).

[109]: for large r it approaches flat space, while for decreasing r the geometry exhibits a throat with cross-section S^3. At $r = |C|^{1/4}$ one encounters a coordinate singularity, where another solution of this type can be attached (see e.g. Figure 5.2 and 5.3(a) for two possibilities). Gravitational instanton solutions for $C = 0$ and $C > 0$ can also be found if a dilaton-type field is included [92; 110].

Before we extend our system to dilaton-type couplings, we review and discuss several subtleties involved in the aforementioned dualisation between θ and B in Euclidean space.

5.2.1. Dualisation

For the sake of clarity, in this subsection we index the field variables by their rank, i.e. we write θ_0 and B_2. Those fields are sourced by an instanton and a microscopic string, respectively. We start from the two Euclidean actions in 4d:[3]

$$S[\theta_0] = \int_M \frac{1}{2g_\theta^2} F_1 \wedge \star F_1 + iQ_\theta \int_I \theta_0, \qquad F_1 = d\theta_0, \tag{5.2.5}$$

$$S[B_2] = \int_M \frac{1}{2g_B^2} H_3 \wedge \star H_3 + iQ_B \int_\sigma B_2, \qquad H_3 = dB_2, \tag{5.2.6}$$

where M denotes our 4-manifold, I the set of points where the instantons are located, and σ is the surface swept out by the string. One can identify the kinetic terms of (5.2.5) and (5.2.6) by imposing

$$H_3 = g_B^2 \star F_1 \tag{5.2.7}$$

and $g_B^2 = 1/g_\theta^2$. This now becomes a single theory with both strings and instantons allowed and either θ_0 or B_2 to be used locally as the appropriate field variable.

Note that the H_3-flux is quantised by

$$\int_{S^3} H = n \in \mathbb{Z}, \tag{5.2.8}$$

[3]The appearance of the i-factor in front of the coupling terms can be understood by writing these terms as $\int_M f_p \wedge j_{4-p}$ with p-form field f_p and source current j_{4-p}. One of the relevant tensor components of either f_p or j_{4-p} then always carries a zero-index and hence acquires an i-factor by Wick rotation.

as we review in Appendix C.2 in the context of the existence of fundamental strings and instantons.

We now couple the 1-form/3-form theory to gravity. It is well-known that choosing either θ_0 or B_2 as the fundamental field leads to Einstein equations differing by an overall sign [109]. Indeed, the action of (5.2.5) gives the energy-momentum tensor

$$T^{(\theta)}_{\mu\nu} = \frac{1}{g_\theta^2}\left(-\frac{1}{2}g_{\mu\nu}(\partial\theta_0)^2 + \partial_\mu\theta_0\partial_\nu\theta_0\right) , \tag{5.2.9}$$

while (5.2.6) leads to

$$\begin{aligned} T^{(B)}_{\mu\nu} &= \frac{1}{g_B^2}\left(-\frac{1}{2\cdot 3!}g_{\mu\nu}H_3^2 + \frac{1}{2}H_{\mu\rho\sigma}H_\nu{}^{\rho\sigma}\right) = \\ &= -\frac{1}{g_\theta^2}\left(-\frac{1}{2}g_{\mu\nu}(\partial\theta_0)^2 + \partial_\mu\theta_0\partial_\nu\theta_0\right) = -T^{(\theta)}_{\mu\nu} \, . \end{aligned} \tag{5.2.10}$$

In the second line we used (5.2.7) together with $g_B^2 = 1/g_\theta^2$.

The above sign difference implies that Euclidean wormholes exist in the B_2 but not in the θ_0 formulation. Technically, this is due to the Hodge star being introduced before or after the variation w.r.t. the metric. Also at the intuitive level the difference is clear: The H_3-flux on the transverse S^3, which is fixed due to the Bianchi identity, supports the finite-radius throat. By contrast, the dual quantity $\theta' \equiv \partial_r\theta$, i.e. the variation of θ along the throat, is *not* fixed by the dual Bianchi identity and the solution is lost.

We note that the *Minkowski-space* Einstein equations remain the same on both sides of the duality. However, we are interested in the path integral in the *Euclidean* theory with gravity, so this observation does not help.

Thus, one may wonder whether Giddings-Strominger wormholes do contribute to the action or whether the dual descriptions are really fully equivalent. This problem has been intensively investigated in the past, see e.g. [82; 109; 231–244] and our present understanding mainly derives from [241–243].

Indeed, it should be possible to resolve the problem by dualising under the Euclidean path integral and following the fate of the instanton solution. We review the dualisation following [235; 241; 242]. To be specific, let M be a cylinder, $M = S^3 \times I$, with an interval $I \subset \mathbb{R}$. This is the simplest relevant topology since the S^3 can carry H_3-flux, supporting a narrow throat somewhere within I.

Starting on the B_2-side, the partition function reads

$$Z \sim \int_{\text{b.c.}} d[B_2] \exp\left(-\int_M \frac{1}{2g_B^2} dB_2 \wedge \star dB_2\right) , \tag{5.2.11}$$

where "b.c." denotes the boundary conditions $B_2(S^3_I) \equiv B_2^{(I)}$ and $B_2(S^3_F) \equiv B_2^{(F)}$ at the initial and final boundaries S^3_I and S^3_F. The possibility of a non-trivial flux, $\int_{S^3} H_3 \neq 0$, can as usual be implemented by defining B_2 in patches over the transverse S^3 and choosing appropriate transition functions.

One can also express Z as a path integral over H_3, imposing $dH_3 = 0$ with the help of a Lagrange-multiplier θ_0:

$$Z \sim \int_{\text{b.c.}} d[H_3]d[\theta_0] \exp\left\{-\int_M \frac{1}{2g_B^2}\left(H_3 \wedge \star H_3 + 2ig_B^2\theta_0 dH_3\right)\right\} . \tag{5.2.12}$$

The previous B_2-boundary conditions now translate into boundary conditions on the pull-back[4] of H_3 to the initial and final boundary, i.e. $H_3(S^3_I) \equiv H_3^{(I)}$ and $H_3(S^3_F) \equiv H_3^{(F)}$. In this language, the information about a possible H_3-flux is simply part of the H_3 boundary conditions. The θ_0-integral is unconstrained. The i in front of the Lagrange-multiplier is needed to get a delta-functional $\delta(dH_3)$ in the path integral after integrating out θ_0. Hence, we have $dH_3 = 0$ and Stokes theorem yields $H_3^{(I)} = H_3^{(F)}$. In other words $Z \sim \delta(H_3^{(I)} - H_3^{(F)})$.

Equation (5.2.12) can be rewritten by integrating the second term by parts and completing the square:

$$Z \sim \int_{\text{b.c.}} d[H_3] d[\theta_0] \exp\left\{-i\int_{\partial M} \theta_0 H_3\right\} \tag{5.2.13}$$
$$\exp\left\{-\int_M \frac{1}{2g_B^2}\left[\left(H_3 - ig_B^2 \star d\theta_0\right)\wedge\star\left(H_3 - ig_B^2\star d\theta_0\right) + g_B^4 d\theta_0\wedge\star d\theta_0\right]\right\}.$$

According to [235; 241; 242] one can now shift the variable $H_3 \to \tilde{H}_3 \equiv H_3 - ig_B^2 \star d\theta_0$ and trivially perform the Gaussian integral. One may however also be concerned about this step since, for any fixed θ_0, the boundary conditions, e.g. $\tilde{H}_3(S^3_I) = H_3(S^3_I) - ig_B^2\star d\theta_0$, clash with the saddle point value $\tilde{H}_3 = 0$ of the Gaussian integral in the interior of M.

To make this issue more explicit, let us write $H_3 = \langle H_3\rangle + \delta H_3$, where $\langle H_3\rangle$ is constant along the S^3 but time dependent. Its boundary values are determined by the H_3-flux. Furthermore, decompose δH_3 into spherical harmonics on S^3. If the cylinder M were flat and gravity non-dynamical, we would now simply have a quantum mechanical system of infinitely many, independent oscillators. The dualisation process sketched above would correspond, as is well known from T-duality for a scalar field on the cylinder $S^1 \times \mathbb{R}$, to a canonical transformation $(p \leftrightarrow q)$ for each oscillator. In our case, the dual variables are coefficients of the spherical harmonic decomposition of θ_0.

Let us focus on the most interesting subsystem (see also the discussion in [243]) with the variable $\langle H_3\rangle \sim p$ and the dual variable $\langle\theta_0\rangle \sim q$. Thus, we first restrict our attention to the question whether it is correct to naively integrate out q in

$$Z \sim \int_{\text{b.c.}} d[p] \int d[q] \exp\left\{-\frac{1}{2}\int_{t_i}^{t_f} dt\, \left[(p - i\dot{q})^2 + \dot{q}^2\right]\right\}. \tag{5.2.14}$$

Based on an explicit, discretised calculation in Appendix C.3, we claim this is indeed the case. One can now argue that, also for the full system (5.2.13) including all oscillators and gravity, this formal manipulation with path integrals is correct. It will then also remain correct if, as argued in Appendix C.2, $\langle H_3\rangle$ is initially quantised, i.e. $\int_{S^3}\langle H_3\rangle = n \in \mathbb{Z}$. Indeed, this quantisation is 'neutralised' once the Lagrange multiplier is introduced and the now continuous variable $\langle H_3\rangle$ is integrated out as above.

As a result of all this the partition function can eventually be given as

$$Z \sim \int d[\theta_0] \exp\left(-\int_M \frac{1}{2g_\theta^2} d\theta_0 \wedge \star d\theta_0 - i\int_{\partial M}\theta_0 H_3\right), \tag{5.2.15}$$

where $g_\theta^2 = 1/g_B^2$. We emphasise that the sign of the kinetic term is the one required for a well-defined Euclidean path integral. This sign will become important below. We also note that this procedure can be straightforwardly generalised to any p-form in arbitrary dimensions

[4] This is *not* the same as H_3 at the position of the boundaries, which contains time-derivatives of B_2 and should not be constrained.

$d > p$. Moreover, we observe that despite the shift $H_3 \to H_3 - ig_B^2 \star d\theta_0$, the field θ_0 can be kept real (see also [242]).[5]

Varying the action in (5.2.15),

$$\delta S = \int_M \frac{1}{g_\theta^2} \delta\theta_0 d \star d\theta_0 - \int_{\partial M} \frac{1}{g_\theta^2} \delta\theta_0 \star d\theta_0 - i \int_{\partial M} \delta\theta_0 H_3 \stackrel{!}{=} 0\,, \tag{5.2.16}$$

we find the equation of motion $d \star d\theta_0 = 0$ in the bulk and

$$H_3(\partial M) = \frac{i}{g_\theta^2} \star d\theta_0(\partial M) \tag{5.2.17}$$

at the boundary. Thus, the θ_0 path integral has only complex saddle points [241; 242].[6] Indeed, the possibility of taking θ_0 imaginary at stationary points was discussed before, see e.g. [235; 236].

To summarise, dualisation leads to a Euclidean path integral in which θ_0 is a priori real and the kinetic term has the standard sign. However, a semi-classical evaluation is only possible on the basis of complex saddles. Crucially, the relevant field-theory solutions then also solve Einstein equations because imaginary θ_0 flips the sign of $T_{\mu\nu}^{(\theta)}$ (cf. (5.2.10)). Thus, one can expect gravitational instantons to contribute consistently both in the B_2 and the θ_0 formulation. Nevertheless, it is natural to use the B_2 path integral to keep the saddle points real [241; 242], and we will do so in what follows.

5.2.2. Gravitational Instantons in the Presence of a Massless Scalar Field

One goal of this work is to study the effect gravitational instantons can have on geometric moduli of string compactifications. In the 4-dimensional theory these moduli appear as scalar fields. Consequently, we will study systems of an axion θ and a scalar φ coupled to gravity.[7] The relevant Euclidean action then takes the form

$$S = \int d^4x \sqrt{g} \left[-\frac{1}{2} R + \frac{1}{2} K(\varphi) g^{\mu\nu} \partial_\mu \theta \partial_\nu \theta + \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi \right]. \tag{5.2.18}$$

Here we already canonically normalised the field φ. At 2-derivative level, the axion θ can only enter the action through a term $\partial_\mu \theta \partial^\mu \theta$ due to its shift symmetry. There is no such symmetry for φ and hence the kinetic term for θ can in general depend on φ. This situation is typically encountered in string compactifications, see Section 5.5.2 for examples. In this subsection we consider a massless scalar field φ and apply the subsequent results to the case of a massive scalar in Section 5.5.

As we are interested in gravitational instantons, we should consider the dual formulation of the above theory. The relevant Euclidean action is then

$$S = \int d^4x \sqrt{g} \left[-\frac{1}{2} R + \frac{1}{2} \mathcal{F}(\varphi) H^2 + \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi \right], \tag{5.2.19}$$

[5] In other references, e.g. [240; 244], the axion field was taken to be imaginary. Then, however, we do not see how to ensure $dH_3 = 0$ using (5.2.12).

[6] For a treatment of path integrals with complex phase space or complex saddles, see e.g. [245] and [246], respectively.

[7] A string compactification will typically give rise to many axionic fields and many geometric moduli. We focus here on one, potentially super-Planckian, light axion which may be identified with the inflaton. Similarly, the scalar can be identified with the lightest modulus. Note that the analysis in this subsection neglects any mass term for the modulus φ, which will be included only later in Section 5.5.

where $\mathcal{F} = 1/(3!K) = 1/(3!f_{\text{ax}}^2)$. Here f_{ax} is the φ-dependent analogue of the familiar axion decay constant.

In the following we will review explicit solutions of this system corresponding to gravitational instantons. Following [92] we will construct solutions to the equations of motion for the metric, the 3-form H and the scalar φ.

General solution

For completeness, let us recall the metric given in (5.2.4):

$$ds^2 = \left(1 + \frac{C}{r^4}\right)^{-1} dr^2 + r^2 d\Omega_3^2.$$

The derivation of the functional form of g_{rr} can be found in Appendix C.1. There we show that the equation of motion for g_{rr} decouples from the equations of motion of the massless fields φ and B. In particular, the form of the metric (5.2.4) is independent of the functional form of the kinetic terms of these fields. The constant C can a priori be negative, positive or zero. Depending on the sign of this parameter C, this solution has the following interpretations. Using the terminology of [92; 110] we can distinguish between three types of gravitational instantons (see Figure 5.3 for an illustration).

- *Euclidean wormholes* ($C < 0$):
  The case $C < 0$ leads to a geometry with a throat and we call this solution a Euclidean wormhole. The divergence of g_{rr} at $r = r_0 \equiv |C|^{1/4}$ is only a coordinate singularity. The Ricci scalar R is
  $$R = 6\frac{C}{r^6} \tag{5.2.20}$$
  and thus it is finite for all $r \geq r_0$. The locus $r = r_0$ can then be interpreted as the end of one wormhole throat. We can then attach another solution of this type at $r = r_0$ which can either be attached to our universe (see Figure 5.2) or a different universe (see Figure 5.3). In this work we will only consider wormholes which close again in our universe, i.e. we are dealing with pairs of holes each connected by a "handle".

- *Extremal instantons* ($C = 0$):
  The solution for $C = 0$ is called an extremal gravitational instanton [92; 110]. Even though space is flat in that case, the fields φ and θ still exhibit a nontrivial profile. This is possible due to a complete cancellation of terms in the energy-momentum tensor [247].

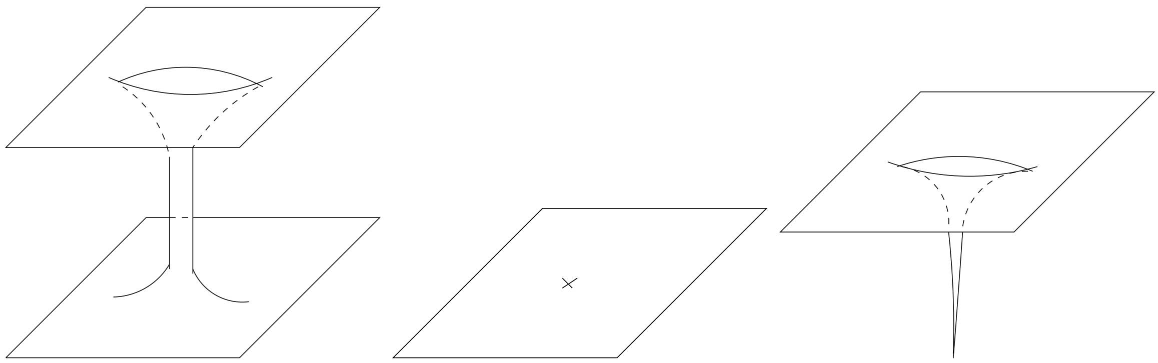
- *Cored gravitational instantons* ($C > 0$):
  The case $C > 0$ gives rise to a geometry with a curvature singularity at $r = 0$. Such solutions are called *cored gravitational instantons* [92].

Having reviewed the solution for the metric, we will now solve the equation of motion for H without specifying $\mathcal{F}(\varphi)$. From (5.2.19) we obtain the equation of motion:

$$d \star H = -\frac{\mathcal{F}'(\varphi)}{\mathcal{F}(\varphi)} d\varphi \wedge \star H. \tag{5.2.21}$$

We expect that solutions for φ and H should exist that respect the spherical symmetry of the background. We thus propose that $\varphi = \varphi(r)$. Similarly, following [109], we make the ansatz

$$H = h(r)\epsilon \tag{5.2.22}$$



(a) Wormhole connected to another universe (b) Extremal gravitational instanton (c) Cored gravitational instanton

Figure 5.3.: The three types of gravitational instantons are depicted. (a) Euclidean wormhole connecting two asymptotically flat spaces. It is also possible to connect both ends to the same space as shown in Figure 5.2. (b) Extremal gravitational instanton: in this case space is flat everywhere. The cross in the middle indicates the locus $r = 0$. (c) Cored gravitational instanton: there is a curvature singularity at $r = 0$.

with ϵ the volume form on S^3 such that

$$\int_{S^3} \epsilon = 2\pi^2 r^3 \ . \tag{5.2.23}$$

From (5.2.22) it follows that $\star H \sim h(r)dr$ and the LHS of (5.2.21) vanishes. As we have chosen $\varphi = \varphi(r)$ the RHS of (5.2.21) equally vanishes and the equation of motion for H is satisfied.

In addition, H also has to satisfy the Bianchi identity $dH = 0$. This enforces

$$h(r) = \frac{n}{Ar^3}, \tag{5.2.24}$$

with $A \equiv A(S^3) = 2\pi^2$ the area of the unit sphere. Charge quantisation (5.2.8) implies that $n \in \mathbb{Z}$.

In order to find the solution for φ it is sufficient to consider the rr-component of the Einstein equations, $G_{rr} = T_{rr}$, which can be shown to be equivalent to the Klein-Gordon equation for φ. It reads

$$\frac{1}{2}\left(1 + \frac{C}{r^4}\right)(\varphi')^2 - \frac{3\mathcal{F}(\varphi)n^2/A^2 + 3C}{r^6} = 0 \ , \tag{5.2.25}$$

where we already used the solution for H. We also defined $\varphi' \equiv \partial\varphi/\partial r$. The solution for φ can then be found by integrating this differential equation.

Model-dependent solutions

From (5.2.25) it is clear that explicit solutions for φ will depend on the functional form of the term $\mathcal{F}(\varphi)$. In this subsection we will restrict our attention to functions of the form $\mathcal{F}(\varphi) \sim \exp(-\alpha\varphi)$, where we choose without loss of generality $\alpha > 0$, as this functional form arises frequently in string compactifications. For example, this behaviour is observed when φ is identified with the dilaton. Similarly, the same functional form appears if φ corresponds to the volume modulus in setups with large compactification volume (e.g. [7]) or if φ is a complex structure modulus at large complex structure. We will study such examples in Section 5.5.2.

To be specific, we take

$$\mathcal{F}(\varphi) = \frac{1}{3! f_{\rm ax}^2} \exp(-\alpha\varphi) \ , \tag{5.2.26}$$

where $f_{\rm ax}$ is from now on a constant. The value of the parameter α will depend on the type of geometric modulus. We can assume $\lim_{r\to\infty}\varphi(r) = 0$ without loss of generality. Then $f_{\rm ax}$ will correspond to the asymptotic value of the axion decay constant.

In the following, we will summarise the explicit solutions for φ for the Euclidean wormhole, the extremal instanton and for the cored instanton. Further details can be found in Appendix C.4.

- *Euclidean Wormhole* ($C < 0$):
The analytical solution to (5.2.25) in this case is [109; 110]

$$e^{\alpha\varphi(r)} = \frac{1}{\cos^2(K_-)}\cos^2\left(K_- + \frac{\alpha}{2}\sqrt{\frac{3}{2}}\arcsin\left(\frac{\sqrt{|C|}}{r^2}\right)\right) \ . \tag{5.2.27}$$

Here, we already implemented the boundary condition $\lim_{r\to\infty}\varphi(r) = 0$, which also implies that

$$C = -\frac{n^2}{3! f_{\rm ax}^2 A^2}\cos^2(K_-). \tag{5.2.28}$$

The integration constant K_- is not a free parameter. This can be seen as follows. When the field reaches the wormhole throat at $r = r_0 \equiv |C|^{1/4}$, the factor $(1+C/r^4)$ in (5.2.25) vanishes, hence

$$3\mathcal{F}(\varphi(r_0))n^2/A^2 + 3C = 0 \ . \tag{5.2.29}$$

Using (5.2.28), this translates to

$$\cos^2\left(K_- + \frac{\alpha\pi}{4}\sqrt{\frac{3}{2}}\right) = 1, \tag{5.2.30}$$

and thus

$$K_- = -\frac{\alpha\pi}{4}\sqrt{\frac{3}{2}}. \tag{5.2.31}$$

Inserting this back into the solution yields

$$e^{\alpha\varphi(r)} = \frac{1}{\cos^2(\sqrt{3/2}\alpha\pi/4)}\cos^2\left(\frac{\alpha}{2}\sqrt{\frac{3}{2}}\arccos\left(\frac{\sqrt{|C|}}{r^2}\right)\right). \tag{5.2.32}$$

To see that one can take two wormhole solutions and glue them together, let us now change coordinates by writing $r = a(t)$ such that the metric becomes

$$ds^2 = dt^2 + a^2(t)d\Omega_3^2 \ . \tag{5.2.33}$$

One can show that $a(t)$ and $\varphi(t)$ are symmetric under $t \to -t$. This implies the existence of a "handle" as shown in Figure 5.2, assuming also that the two throats are very distant in $\mathbb{R}^4$.

Interestingly, not all values for α will lead to physically acceptable solutions. Note that $\varphi(r)$ is regular everywhere on $r \in [|C|^{1/4}, +\infty)$ only for dilaton couplings in the range $0 \le \alpha < 2\sqrt{2/3}$. For $\alpha > 2\sqrt{2/3}$ there is always a value of $r > |C|^{1/4}$, where $e^{\alpha\varphi(r)} = 0$, i.e. $\varphi(r) \to -\infty$. This is consistent with [82; 109; 110]. In our case the field φ

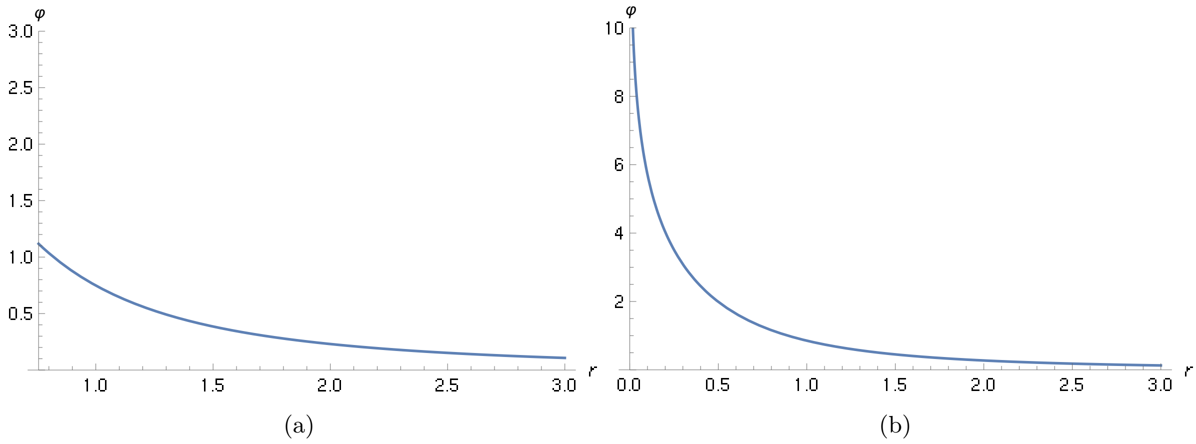


Figure 5.4.: Illustration of dilaton profiles. The values of r and φ are in Planck units.
(a) Euclidean wormhole ($C < 0$):
Here we choose n/f_{ax} such that $C = -\cos^2\left(\alpha\pi\sqrt{3/2}/4\right)$ and plot for $\alpha = 1$.
(b) Extremal instanton ($C = 0$) with $\alpha = 2\sqrt{2/3}$.

corresponds to the string coupling or a geometric modulus of the string compactification. A runaway behaviour $\varphi(r) \to -\infty$ is then pathological as it would correspond to a limit of decompactification or vanishing string coupling. In all these cases new light states will appear resulting in a loss of control over the effective theory. This pathology is avoided for $\alpha = 2\sqrt{2/3}$. However, in this case we obtain $C = 0$ which will be discussed next. Overall, we find that only the range $0 \leq \alpha < 2\sqrt{2/3}$ is physically allowed for Euclidean wormholes.

Last, note that the limit $\alpha \to 0$ can be identified with the Giddings-Strominger wormhole [109] which exhibits a constant dilaton profile.

- *Extremal Instanton* ($C = 0$):
For the case of an extremal instanton we find

$$e^{\alpha\varphi(r)} = \left(1 + \frac{\alpha n}{4Af_{\text{ax}}}\frac{1}{r^2}\right)^2, \tag{5.2.34}$$

which is valid for all $\alpha > 0$. (For $\alpha = 2\sqrt{2/3}$ this solution agrees with (5.2.32). A plot of the dilaton profile in this case can be found in Figure 5.4.). The result can be obtained most easily by solving (5.2.25) for $C = 0$. Notice that (5.2.34) with a minus sign in the bracket would in principle also be a solution (see Appendix C.4), but then there would again be a value of $r > 0$ so that $e^{\alpha\varphi} = 0$, leading to the same problems as described above. We hence exclude this possibility.

- *Cored gravitational instantons* ($C > 0$):
Finally, for the case of cored gravitational instantons $C > 0$ one finds [92; 110]

$$e^{\alpha\varphi(r)} = \frac{1}{\sinh^2(K_+)}\sinh^2\left(K_+ + \frac{\alpha}{2}\sqrt{\frac{3}{2}}\text{arcsinh}\left(\frac{\sqrt{C}}{r^2}\right)\right), \tag{5.2.35}$$

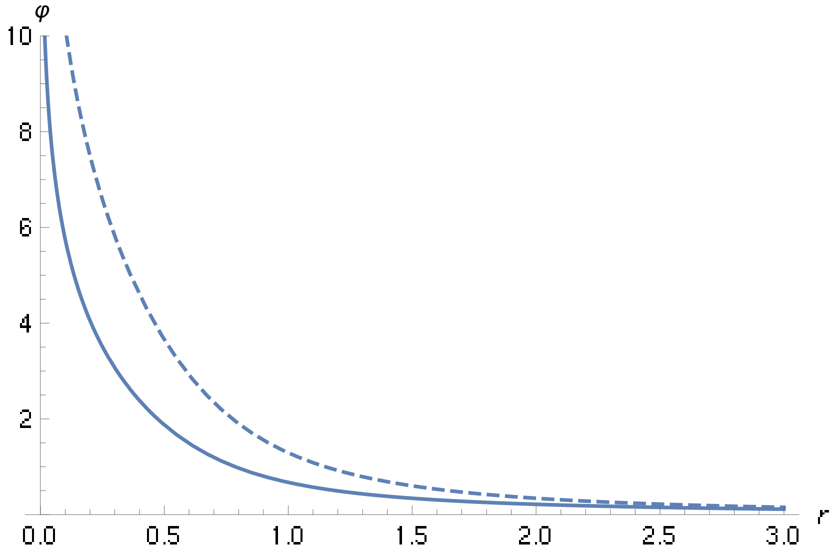


Figure 5.5.: This plot shows dilaton profiles for the cored gravitational instanton with $\alpha = 15$ (solid line) and $\alpha = 0.1$ (dashed line). Again, r and φ are given in Planck units. For the purpose of illustration we have chosen $K_+ = 0.5$ and n/f_{ax} such that $C/\sinh^2 K_+ = 1$.

where we again ensured $\lim_{r\to\infty} \varphi(r) = 0$ by demanding

$$C = \frac{n^2}{3! f_{\text{ax}}^2 A^2} \sinh^2(K_+) \ . \tag{5.2.36}$$

In Figure 5.5 two plots of the dilaton profile are presented. The integration constant K_+ should be positive in order to again avoid a divergence of φ for some $r > 0$, but is otherwise unconstrained. This is different compared to wormholes or extremal instantons, which do not exhibit a free parameter. From this 4d effective theory one is lead to believe that there exists a whole family of cored instanton solutions parametrised by K_+. However, by considering the microscopic origin of gravitational instanton solutions, one finds evidence that only certain values of K_+ are allowed, as we will now discuss.

5.2.3. Interpretation of the Integration Constant K_+

The integration constant K_+, or equivalently C, seems to be a free and continuous parameter giving rise to a family of solutions. We want to argue that this is not the case. Note that the cored gravitational instanton solutions are UV-sensitive and therefore a naive 4d field theory treatment is not sufficient. Instead, it is crucial to understand those solutions in a UV-complete theory, such as string theory. In this context the role of the integration constant K_+ becomes clear. Specifically, it was pointed out in [110] that the parameter C is determined by the mass M and charge Q of a dilatonic black brane wrapping internal cycles in a higher-dimensional theory, whose dimensionally reduced action coincides with (5.2.19). This holds true at least for some values of α. Consequently, we conjecture that C and K_+ generically take discrete and well-defined values determined by the underlying microscopic theory. Further, if the Weak Gravity Conjecture holds in 5d, cored gravitational instantons may not be stable.

We support this conjecture by providing a specific toy-example borrowed from [110]. Following their results, we can consider a five-dimensional model with Euclidean action in 5d Planck units

$$S = \int d^5x\sqrt{\hat{g}}\left[-\frac{1}{2}\hat{R} + \frac{1}{2}(\partial\hat{\phi})^2 + \frac{1}{4}e^{a\hat{\phi}}\hat{F}^2\right], \tag{5.2.37}$$

where $\hat{F} = d\hat{A}$ is a 2-form field strength tensor. For the dimensional reduction to a 4d theory we choose[8]

$$d\hat{s}^2_{(5)} = e^{2\beta_1\psi}d\tau^2 + e^{2\beta_2\psi}ds^2_{(4)} \tag{5.2.38}$$

together with $\hat{A} = \theta d\tau$ and $\hat{\phi} = \phi$, i.e. the fields θ and $\hat{\phi}$ do not depend on the extra-dimensional coordinate τ. In Einstein-frame with canonically normalised kinetic terms dimensional reduction fixes the constants β_1 and β_2 to

$$\beta_1 = -2\beta_2, \qquad \beta_2 = \frac{1}{\sqrt{6}}. \tag{5.2.39}$$

After redefining the fields ϕ and ψ via a rotation in the (ϕ, ψ)-plane we get

$$S = \int d^4x\sqrt{g}\left[-\frac{1}{2}R + \frac{1}{2}(\partial\tilde{\phi})^2 + \frac{1}{2}(\partial\tilde{\psi})^2 + \frac{1}{2}e^{\alpha\tilde{\phi}}(\partial\theta)^2\right], \tag{5.2.40}$$

where g denotes the metric corresponding to the 4d-line-element. Setting $\tilde{\psi} = 0$ in this action one obtains the model considered in (5.2.18) with dilatonic dependence in the kinetic term of θ. The 4d dilaton coupling α is related to the 5d dilaton coupling a via[9]

$$\alpha^2 = a^2 + \frac{8}{3}. \tag{5.2.41}$$

Therefore, the interpretation of the 4d theory in terms of a 5d theory is only possible if $\alpha \geq 2\sqrt{2/3}$.

Let us now explicitly relate the integration constant C to microscopic properties of a higher-dimensional theory. In [110] it was shown that for $\alpha = 2\sqrt{2/3}$ the solutions of the 4d model (5.2.40) can be uplifted to a five-dimensional Reissner-Nordström (RN) black hole solution

$$ds^2_{(5)} = g_+(\rho)g_-(\rho)d\tau^2 + \frac{d\rho^2}{g_+(\rho)g_-(\rho)} + \rho^2 d\Omega_3^2, \qquad \hat{F}_{\tau\rho} = \sqrt{6}\frac{Q}{\rho^3} \tag{5.2.42}$$

where

$$g_\pm(\rho) = 1 - \frac{\rho_\pm^2}{\rho^2} \tag{5.2.43}$$

with

$$\rho_\pm^2 = M \pm \sqrt{M^2 - Q^2}. \tag{5.2.44}$$

We take this as a simple toy-model to argue that C is generically fixed by properties of a black brane wrapping internal cycles. The RN black hole can be interpreted as N particles or 0-branes (or just one 0-brane wrapping the cycle N times) of total mass M and total

[8] For the purpose of compactification we switch to Euclidean time τ by Wick-rotation. For simplicity we choose the periodicity $\tau \sim \tau + 1$. Later in this subsection we allow the circumference of the S^1 to have length $\ell > 0$. This will then have to be taken into account in order to determine the axion decay constant.

[9] Notice that our normalisation of ϕ is such that the prefactors of the Ricci scalar R and the kinetic term $(\partial\phi)^2$ are equal, while in [110] the prefactor of the dilaton has a factor $1/2$ relative to R. This is why our dilaton-coupling α differs by a factor of $\sqrt{2}$.

charge Q. Note however that the ADM-mass M_{ADM} is related to the mass parameter M by $M_{\mathrm{ADM}} = 6\pi^2 M$. Nevertheless, we henceforth call M the mass of the RN-black hole. The charge Q is defined such that $M = Q$ sets the extremality bound. That is, $Q = N\hat{q}\sqrt{6}/(6\pi^2)$, where the charge $\hat{q}$ is defined by $N\hat{q} = 1/2 \int_{S^3} \star_5 \hat{F}$.[10]

Upon toroidal dimensional reduction along the coordinate τ with the identification $\tau \sim \tau + \ell$ and the circumference $\ell > 0$ of the compactified dimension, the 5d solution (5.2.42) turns into an instanton solution (5.2.4). Note that the coordinate singularity at $\rho = \rho_+$ of the 5d solution becomes a curvature singularity (at $r = 0$) in the 4d solution (5.2.4). In the subsequent computation we show that our integration constant C is simply given by $C = \ell^2(M^2 - Q^2)$ in 4d Planck units.

Denote by $g^{(5)}_{MN}$ the RN-metric (5.2.42), where M, N run over the coordinates of the 4d space and the extra-dimensional coordinate τ. Now, rescale the metric as follows: $\tilde{g}^{(5)}_{MN} = g^{(5)}_{MN}/(g_+ g_-)$. From the canonical Einstein-Hilbert term we then get:

$$\begin{aligned} \int d^5x \sqrt{g^{(5)}} R[g^{(5)}_{MN}] &= \int d^5x (g_+ g_-)^{3/2} \sqrt{\tilde{g}^{(5)}} R[\tilde{g}^{(5)}_{MN}] + \ldots = \\ &= \int d^4x \ell (g_+ g_-)^{3/2} \sqrt{\tilde{g}^{(4)}} R[\tilde{g}^{(4)}_{\mu\nu}] + \ldots \end{aligned} \tag{5.2.45}$$

The last term occurs in the compactified 4d theory using the identification $\tau \sim \tau + \ell$. We want to point out that for generic $\ell > 0$ there is a conical singularity at the outer horizon $\rho = \rho_+$. In principle, one could avoid such a conical singularity by choosing the periodicity of τ appropriately (it would be the inverse of the Hawking-temperature [249]), but this would mean to fix the compactification radius. Instead, we accept the conical singularity as a necessary feature of Euclidean branes wrapped on cycles of the compact space.[11] Since it is known that Euclidean branes wrapped on non-trivial cycles give rise to instantonic terms (see e.g. [250; 251]), we assume that the corresponding conical spacetimes are saddle-points of the Euclidean path integral.

We go to the Einstein frame (with 4d Planck mass $M_p = 1$) by rewriting the Einstein-Hilbert term using the rescaled metric $g^{(4)}_{\mu\nu} = \ell(g_+ g_-)^{3/2} \tilde{g}^{(4)}_{\mu\nu}$. The compactified 4d line-element then reads

$$ds^2_{(4)} = \ell \frac{d\rho^2}{\sqrt{g_+ g_-}} + \ell \sqrt{g_+ g_-} \rho^2 d\Omega_3^2. \tag{5.2.46}$$

For the comparison with the metric (5.2.4), the obvious coordinate transformation to be made is simply $r^2 = \rho^2 \ell \sqrt{g_+ g_-}$. Using the definitions of $g_\pm$ it follows

$$r dr = \ell^2 \frac{\rho(\rho^2 - M)}{r^2} d\rho. \tag{5.2.47}$$

Together with $(\rho^2 - M)^2 = r^4/\ell^2 + (M^2 - Q^2)$, this implies:

$$\ell \frac{d\rho^2}{\sqrt{g_+ g_-}} = \frac{dr^2}{1 + \ell^2(M^2 - Q^2)/r^4}. \tag{5.2.48}$$

[10] For the normalisation we found it useful to translate the conventions in [57; 248] to our situation.

[11] Note that in the so-called dual frame metric discussed in [110] non-extremal instantons with $\alpha = 2\sqrt{2/3}$ can be interpreted as sections of constant time of the RN black hole metric. In this frame one recovers a wormhole geometry connecting two asymptotically flat regions smoothly. One pays the price of rescaling by a divergent factor. The above is technically different from our approach of obtaining gravitational instantons by compactification of a 5d black hole solution on an S^1. In our case the RN black hole solution (5.2.42) in general yields conical singularities.

Hence, we find the simple relationship

$$C = \ell^2(M^2 - Q^2) \tag{5.2.49}$$

in 4d Planck units. Upon dimensional reduction of (5.2.37) and using the periodicity of the Wilson line $\hat{A}_\tau \cong \hat{A}_\tau + \pi/(\hat{q}\ell)$ one can easily check that the axion decay constant reads $f_{\text{ax}} = 1/(2\hat{q}\ell)$ for an axion θ with 2π-periodicity. It follows that

$$C = \frac{N^2}{24\pi^4 f_{\text{ax}}^2}\left[\left(\frac{M}{Q}\right)^2 - 1\right]. \tag{5.2.50}$$

We can compare this result to our previous expression (5.2.36). First, we can identify the wrapping number/number of 0-branes N with the flux number n. We then find that the integration constant K_+ in (5.2.36) is completely determined by the parameters M and Q describing black holes/branes in the 5d theory. An immediate result is that K_+ and hence C are not free parameters. The possible range of values is determined by the spectrum of black branes in the higher-dimensional theory. Furthermore, as M and Q are discrete quantities it follows that C can also only take discrete values (for a given value of f_{ax}). This property is only important as long as M and Q are small. In the macroscopic regime of large M and Q the value of C can be dialed to any positive value and it becomes effectively continuous. We come back to this in Section 5.4.

Notice that the case of $M = Q$, which gives $C = 0$, corresponds to an extremal Reissner-Nordström black hole. In this sense, the name *extremal instanton* for flat 4d solutions (5.2.4) is justified. In Section 5.4 we comment on how to express the extremal instanton action in terms of ℓ and M_{ADM}, consistent with, for instance, [92; 244; 252].

This example illustrates nicely how 4d cored or extremal instanton solutions can be obtained from black holes/branes with mass M and charge Q. Of course, one could also go beyond such simple toy-models we just discussed, allowing also for dilaton couplings $\alpha \neq 2\sqrt{2/3}$. We expect the relation $C = \ell^2(M^2 - Q^2)$ to be modified by the corresponding parameter $a \neq 0$ in this more general case. Furthermore, one would expect that after SUSY-breaking extremal objects in string theory would appear as non-extremal instantons in the 4d effective theory.

Last, let us remark on possible implications for cored gravitational instantons arising from the Weak Gravity Conjecture. In particular, if the Weak Gravity Conjecture holds in the 5d model we expect that objects with $M > Q$ can in principle decay. As cored gravitational instantons arise from such unstable objects upon dimensional reduction, one may wonder whether this instability is then inherited by the instantons. Here 'unstable instanton' means that two instantons exist which cause the same flux change but have smaller total action. In this sense, the contribution of cored instantons to the Euclidean path integral is subdominant if cored instantons are 'unstable'. This point will me made more precise in Section 5.7.

5.3. Instanton Potentials from Euclidean Wormholes

The goal of this section is to show that the one-instanton action, describing a Giddings-Strominger wormhole, gives rise to an instanton potential of the structure $\cos\theta e^{-S}$.

We begin with a brief review of Coleman's derivation [253; 254] of the energy eigenvalues for a simple one-dimensional quantum mechanical system with periodic potential V, e.g. $V(x) \sim \sin^2(2\pi x)$. These considerations can be applied to quantum field theory and in particular to our system as well.

The Hamiltonian is $H = p^2/2 + V(x)$. An instanton or an anti-instanton correspond to tunnelling events from x to $x+1$ or $x-1$, respectively. Using the dilute-gas approximation we can distribute instantons and anti-instantons freely in time. Let us introduce a basis of states $|j\rangle$ in which the particle is localised at $x \simeq j$. Then for some time interval $T > 0$, transition amplitudes are [253]

$$\langle j_+|e^{-HT}|j_-\rangle = \left(\frac{\omega}{\pi}\right)^{1/2} e^{-\omega T/2} \sum_{N=0}^{\infty} \sum_{\bar{N}=0}^{\infty} \frac{1}{N!\bar{N}!}(Ke^{-S_0}T)^{N+\bar{N}} \delta_{(N-\bar{N})-(j_+-j_-)}, \tag{5.3.1}$$

where j_- and j_+ are the positions of the initial and final state, respectively. N and $\bar{N}$ count the number of instantons and anti-instantons. Moreover, ω is defined by $\omega = V''(0)$. K is the familiar determinant factor, which depends on details of the potential V. S_0 denotes the instanton action. The Kronecker delta can be rewritten as

$$\delta_{ab} = \int_0^{2\pi} \frac{d\theta}{2\pi} e^{i(a-b)\theta}, \tag{5.3.2}$$

and thus, after performing the summation,

$$\langle j_+|e^{-HT}|j_-\rangle = \left(\frac{\omega}{\pi}\right)^{1/2} e^{-\omega T/2} \int_0^{2\pi} \frac{d\theta}{2\pi} e^{i(j_- - j_+)\theta} \exp\left(2KT\cos\theta e^{-S_0}\right). \tag{5.3.3}$$

From this we can read off that the system has an energy eigenbasis

$$|\theta\rangle = \sum_j e^{ij\theta}\,|j\rangle \tag{5.3.4}$$

with eigenvalues

$$E(\theta) = \frac{1}{2}\omega - 2K\cos\theta e^{-S_0}. \tag{5.3.5}$$

This derivation reveals the logic behind the famous contribution $\sim \cos\theta e^{-S}$ to the axion potential in quantum field theory, where the centres of the instantons are not distributed on a time interval but instead in a region of spacetime with volume $\mathcal{V}$. One then simply has to replace the variable T by the volume $\mathcal{V}$.

In the following we explain how this computation can be used to derive an instanton potential induced by Euclidean wormholes. In the previous Section 5.2 we reviewed that Euclidean wormholes exist in the presence of a non-vanishing 3-form flux H with quantised charge $n \in \mathbb{Z}$. An instanton would then correspond to a transition from n to $n+1$. By the logic of Coleman's computation above, this should induce a shift symmetry breaking potential.

In [92] this was questioned, because Euclidean wormholes appear as conduits and charges would not disappear. In other words, one always has an instanton and an anti-instanton, thus preserving n.

We argue that this issue is more subtle: the two ends of a Euclidean wormhole do not necessarily have to end at the same hypersurface of constant Euclidean time, but can also close on distant hypersurfaces. Similarly, the two ends can have very large spatial separation such that, from a local perspective, a potential à la Coleman should be induced. Then, a Minkowskian observer would only see either the instanton or anti-instanton part of the wormhole and thus find a change in the charge n, see Figure 5.6. This invalidates the reasoning

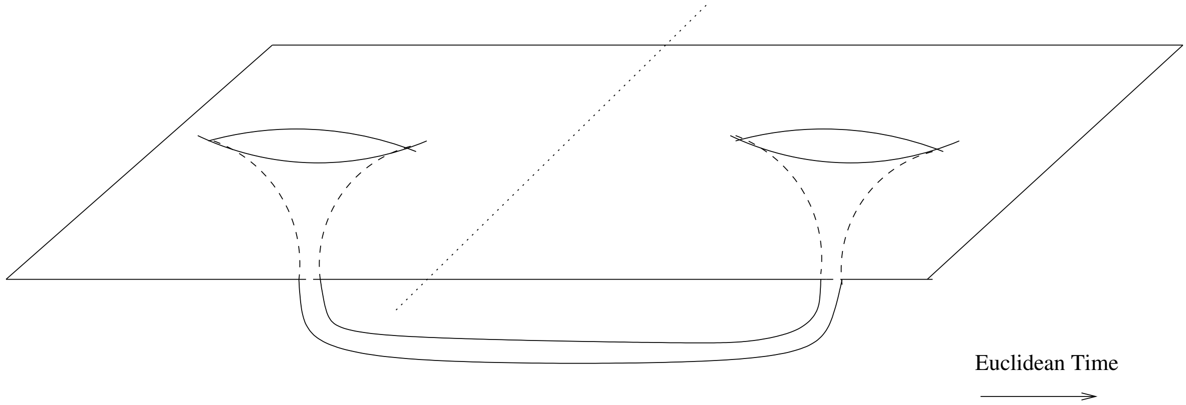


Figure 5.6.: This picture presents a wormhole which opens at some initial time t_i and closes at $t_f > t_i$. The dotted line indicates the separation of the two events.

in [92], and hence we do not see any argument against the breaking of the shift symmetry due to Euclidean wormholes.

We want to make this mathematically more precise. This requires to compute the path integral contribution of all possible wormhole configurations. This allows us to infer the effective potential $V_{\text{eff}}(\theta)$ for the axion field θ. The logic behind the computation of $V_{\text{eff}}(\theta)$ is the following. The expectation value of any observable $\mathcal{O}(\theta)$ is given by

$$\langle \mathcal{O}(\theta) \rangle \sim \int d[\theta] \mathcal{O}(\theta) \exp\left(-\frac{1}{2}\int d^4x f_{\text{ax}}^2 (\partial\theta)^2\right) Z_{\text{wh}}[\theta] \ , \tag{5.3.6}$$

where the path integral contribution of wormholes is schematically (i.e. no combinatioral factors included yet) given by

$$Z_{\text{wh}}[\theta] \sim \sum_w \prod_{n=1}^{w} \prod_{m=1}^{w} \int d^4x_n \int d^4x_m e^{-S} e^{i\theta(x_n)} e^{-S} e^{-i\theta(x_m)} \ . \tag{5.3.7}$$

Here w denotes the number of wormholes of a configuration. Note that the phase difference occurs because one factor is for the instantons, the other for anti-instantons. Those factors arise from the second term of (5.2.5), where $Q_\theta = \pm 1$ (+ for instantons, − for anti-instantons). We write out these factors explicitly, because they finally give rise to the cos-potential for θ. The contribution $Z_{\text{wh}}[\theta]$ induces a change $\delta S_{\text{ind}}(\theta)$ of the action for the axion and we expect

$$\langle \mathcal{O}(\theta) \rangle \sim \int d[\theta] \mathcal{O}(\theta) \exp\left(-\frac{1}{2}\int d^4x f_{\text{ax}}^2 (\partial\theta)^2 - \delta S_{\text{ind}}(\theta)\right) \ , \tag{5.3.8}$$

where $\delta S_{\text{ind}}(\theta)$ contains by definition the effective potential of θ plus higher derivative corrections:

$$\delta S_{\text{ind}}(\theta) = \int d^4x \left(V_{\text{eff}}(\theta) + \text{higher derivative terms}\right) \ . \tag{5.3.9}$$

Hence, the effective potential $V_{\text{eff}}(\theta)$ can be determined by computing Z_{wh} using any field configuration θ for which $V_{\text{eff}}(\theta)$ dominates all derivative terms. We choose a smooth version of the profile

$$\theta(x) = \begin{cases} \theta_0 & \text{for } x \in I \times \mathbb{R}^3 \\ 0 & \text{else} \ , \end{cases} \tag{5.3.10}$$

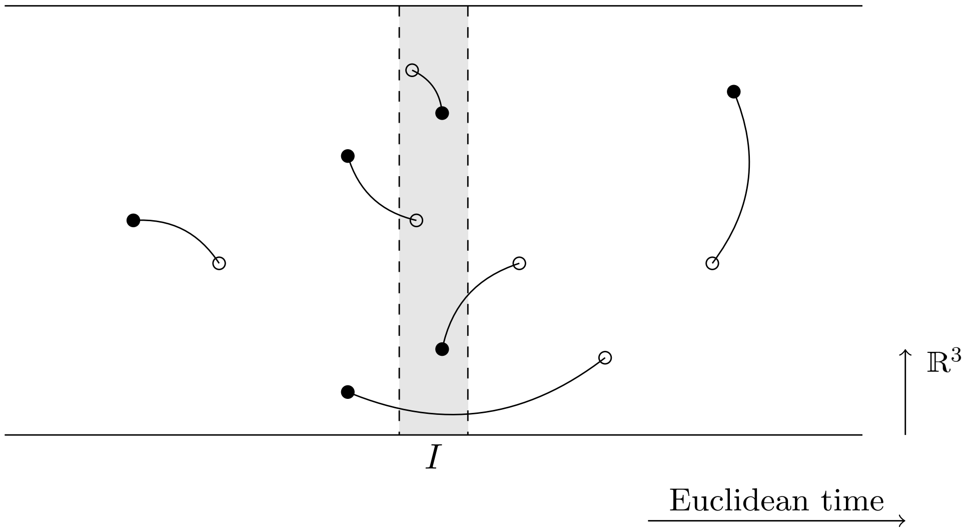


Figure 5.7.: This illustration shows pairs of connected black and white dots, each representing an end of a wormhole (black if the end corresponds to an instanton and white for an anti-instanton). Only few wormholes lie completely inside the shaded region $I \times \mathbb{R}^3$.

i.e. a profile which is only non-zero in a small Euclidean time interval I (see Figure 5.7) and goes to zero smoothly at the boundary of I. This is illustrated in Figure 5.8. The volumes of $I \times \mathbb{R}^3$ and of the remaining part of Euclidean space are denoted by $\mathcal{W}$ and $\mathcal{V}$, respectively. We assume that $\mathcal{W} \ll \mathcal{V}$ with $\mathcal{W}$ being large enough to typically contain many wormhole ends. We first check that the derivative terms can indeed be made subdominant with respect to the effective potential. For simplicity, we work near the minimum and use the approximation $V_{\text{eff}} \sim m^2\theta^2$. It is crucial that our axion profile at the boundary of I features a smooth transition of characteristic length ℓ from 0 to θ_0 with $\ell \ll L$, where L is the length of the Euclidean interval I. We then have $\partial\theta \sim \theta_0/\ell$ close to the boundary (and zero elsewhere) and the comparison of $\int d^4x f_{\text{ax}}^2(\partial\theta)^2$ with $\int d^4x V_{\text{eff}}$ should yield

$$\ell V_3 f_{\text{ax}}^2 \frac{\theta_0^2}{\ell^2} \ll L V_3 m^2 \theta_0^2 \ , \tag{5.3.11}$$

where V_3 denotes the corresponding 3-volume of the Euclidean spacetime regions we consider. It follows that

$$L \gg \frac{f_{\text{ax}}^2}{m^2 \ell} \tag{5.3.12}$$

has to be imposed. Note that ℓ cannot be arbitrarily small as we have to ensure that also higher-derivative terms must be subdominant. Comparing $f_{\text{ax}}^2(\partial\theta)^2$ with $(\partial\theta)^4$ yields

$$\ell \gg \frac{\theta_0}{f_{\text{ax}}} \sim \frac{1}{f_{\text{ax}}} \ . \tag{5.3.13}$$

It is not hard to see that these two conditions together with $L \gg \ell$ can be satisfied simultaneously. Thus, our field configuration (5.3.10) is suitable for the calculation of the effective

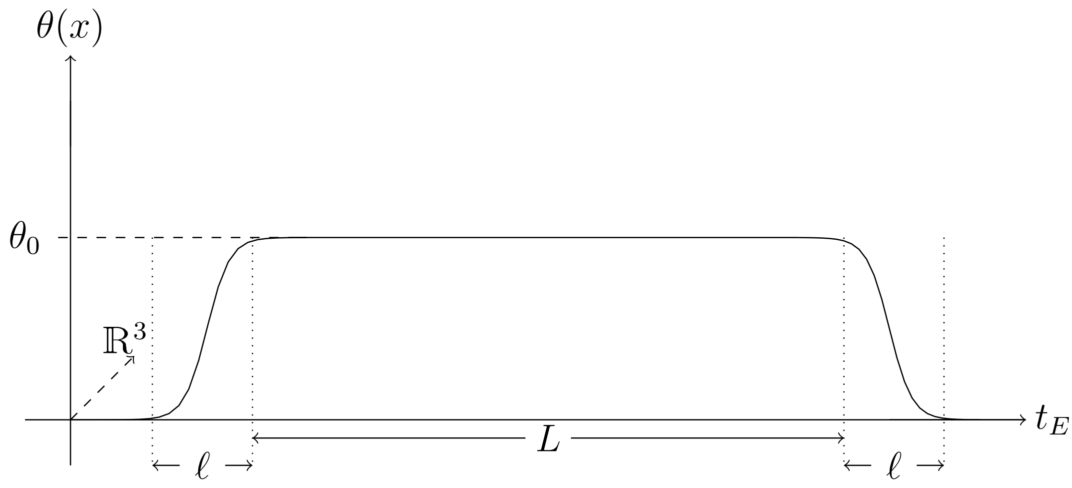


Figure 5.8.: We illustrate the smooth axion profile corresponding to the approximation in (5.3.10). In the Euclidean time interval of length L the axion field is constant (value θ_0) and outside this interval the field decays smoothly. The characteristic length of this transition from θ_0 to 0 is denoted by ℓ.

potential given below. For this computation we find it useful to group the sum in the above expression (5.3.7) according to the position of the wormhole ends, see again Figure 5.7. Let us assume that a wormhole corresponds, from the perspective of the θ-field theory, simply to an instanton-anti-instanton pair. Then, denoting by θ the field configuration of Figure 5.8, we can write:

$$Z_{\mathrm{wh}}[\theta] \sim \sum_n \frac{1}{n!n!} \prod_{j=1}^{n} e^{-2S} K^2 \left(\mathcal{V} + \mathcal{W} e^{i\theta_0}\right) \left(\mathcal{V} + \mathcal{W} e^{-i\theta_0}\right) . \tag{5.3.14}$$

Note that this differs from the toy model (5.3.1) in the sense that the sector with $N \neq \bar{N}$ is not contained in (5.3.14). But this is precisely our point: we want to find out whether a potential can still be generated if we impose $N = \bar{N}$, i.e. an equal number of instantons and anti-instantons. The combinatorial factor $1/(n!)^2$ is due to the indistinguishability of instantons and anti-instantons, respectively. The cross-terms $\mathcal{V}\mathcal{W}e^{\pm i\theta_0}$ correspond to wormholes where only one end is within $I \times \mathbb{R}^3$. The sum can be expressed as a Bessel function I_0

$$Z_{\mathrm{wh}} \sim I_0(x) \tag{5.3.15}$$

with

$$I_0(x) = \sum_{m=0}^{\infty} \frac{1}{m!m!} \left(\frac{x}{2}\right)^{2m} \tag{5.3.16}$$

$$x \simeq 2Ke^{-S}\mathcal{V}\left(1 + (\mathcal{W}/\mathcal{V})\cos\theta_0 + \mathcal{O}\left((\mathcal{W}/\mathcal{V})^2\right)\right) . \tag{5.3.17}$$

Furthermore, there is an integral expression for I_0:

$$I_0(x) = \frac{1}{2\pi} \int_0^{2\pi} d\phi e^{-x\cos\phi} . \tag{5.3.18}$$

Hence, we arrive at

$$Z_{\rm wh} \sim \frac{1}{2\pi} e^x \int_0^{2\pi} d\phi e^{-x(1+\cos\phi)} \simeq \frac{1}{2\pi} e^x \int_0^{2\pi} d\phi e^{-x(\phi-\pi)^2/2} \simeq \frac{1}{\sqrt{2\pi}} \frac{e^x}{\sqrt{x}} \,, \tag{5.3.19}$$

where we relied on the fact that in our case x is large (because $\mathcal{V}$ and $\mathcal{W}$ are large). Since we are interested in the effective potential $-\mathcal{W} V_{\rm eff}(\theta_0) \simeq \ln Z_{\rm wh}$, we can focus on the exponential factor:

$$Z_{\rm wh} \sim \exp\left[2K e^{-S} \left(\mathcal{V} + \mathcal{W}\cos\theta_0\right)\right] \,. \tag{5.3.20}$$

From here it is clear that, to explain the $\mathcal{W}\cos\theta_0$ term, the effective action for θ must contain a potential (which in our case contributes only in the region $I \times \mathbb{R}^3$):

$$V_{\rm eff}(\theta_0) \sim 2K e^{-S} \cos\theta_0 \,. \tag{5.3.21}$$

The θ-dependency is as in the case of an instanton-anti-instanton gas without any constraints imposed. Actually, the term generated by wormhole instantons, which looks like a potential term for the field configuration (5.3.10), is in fact a non-local interaction term. This can be seen by modifying (5.3.10) such that $\theta(x) = \theta_{\mathcal{V}} \neq 0$ for $x \in \mathcal{V}$. Then, $\cos(\theta_0)$ in (5.3.20) is replaced by $\cos(\theta_0 - \theta_{\mathcal{V}})$. Indeed the exponential now contains non-local terms. Globally, this non-local term preserves shift symmetry. Nevertheless, we observe a crucial effect induced by Giddings-Strominger wormholes: The change of the action due to a local fluctuation, $S[\theta + \delta\theta] - S[\theta]$, corresponds to that induced by a potential $V(\delta\theta) \sim 2K\exp(-S)\cos(\delta\theta)$.

This can also be seen by applying Coleman's computation [253] to our problem. We are then interested in computing the partition function $Z = \sum_n \langle n|e^{-HT}|n\rangle$. It is therefore sufficient to focus on transition functions $\langle n|e^{-HT}|n\rangle$, although transitions from $|m\rangle$ to $|n\rangle$ will occur as well (the Hamiltonian H is in general non-diagonal is this basis). Due to the trace we do not need to consider the latter in our calculation. The result of this computation has to be compared with the partition function $Z = \int_0^{2\pi} d\theta \, \langle\theta|e^{-HT}|\theta\rangle / (2\pi)$ in the θ-language.

In the free theory n is the dual variable to our axion field θ, which can be seen as follows: The free theory action for the axion is given by $S = \int d^4x f^2\dot{\theta}^2/2$ or, after integrating out the spatial directions, $S = \int dt A\dot{\theta}^2/2$ with $A \equiv f^2 V_3$, where V_3 is the 3-volume (θ is then the zero mode). The canonical momentum p is then given by $p = A\dot{\theta}$. As it is well known from quantum mechanics on S^1, p is quantised as $p = n \in \mathbb{Z}$. One can therefore relate $|\theta\rangle$-states to $|n\rangle$-states via

$$|\theta\rangle = \sum_n e^{in\theta} \, |n\rangle \tag{5.3.22}$$

in the free theory (see also [243]), where we chose the normalisation $\langle\theta|\theta'\rangle = 2\pi\delta(\theta - \theta')$. The Hamiltonian of the free theory is then given by $H = n^2/(2A)$ and in the free theory we have the transition amplitude

$$\langle n|e^{-HT}|n\rangle_{\rm free} = e^{-Tn^2/(2A)} \,. \tag{5.3.23}$$

Let us now return to interacting theory and take into account the effects of the wormhole gas induced by the coupling of θ to gravity. We assume that instantons and anti-instantons are randomly distributed and that wormhole ends can have arbitrarily long separation with no physical effect. By applying Coleman's formula (5.3.1) to our situation and taking into account (5.3.23), we find:

$$\langle n|e^{-HT}|n\rangle = e^{-Tn^2/(2A)} \sum_{N=0}^{\infty} \frac{1}{N!N!} (K e^{-S_0} T)^{2N}. \tag{5.3.24}$$

We emphasise once more that off-diagonal elements $\langle m|e^{-HT}|n\rangle$ are in general non-zero. For instance, if T corresponds to half of the time interval of Figure 5.6, the instanton number clearly changes by unity. However, such off-diagonal elements never appear explicitly in our calculation, which relies solely on the partition function.

We can once again express the sum in (5.3.24) by I_0 via (5.3.16) and then use the integral expression (5.3.18) with integration variable θ. We find:

$$\langle n|e^{-HT}|n\rangle = e^{-Tn^2/(2A)} \int_0^{2\pi} \frac{d\theta}{2\pi} \exp\left(-2KT\cos\theta e^{-S_0}\right) \ . \tag{5.3.25}$$

For the partition function $Z = \sum_n \langle n|e^{-HT}|n\rangle$ one then obtains

$$Z(T) \simeq \sqrt{\frac{2\pi A}{T}} \int_0^{2\pi} \frac{d\theta}{2\pi} \exp\left(-2KT\cos\theta e^{-S_0}\right) \ . \tag{5.3.26}$$

As we already mentioned, this should be compared with

$$Z = \int_0^{2\pi} \frac{d\theta}{2\pi} \langle\theta|e^{-HT}|\theta\rangle \ . \tag{5.3.27}$$

For a first naive comparison of (5.3.26) and (5.3.27) we ignore the non-exponential T-dependence in the prefactor of (5.3.26).[12] Then $|\theta\rangle$ is an eigenbasis of the Hamiltonian with eigenvalues $V(\theta)$. We see by comparison with (5.3.26) that $V(\theta) = 2\cos\theta K e^{-S_0}$.

We can, however, be more precise and understand also the prefactor. To do so we observe that (5.3.26) was derived on the basis of (5.3.24), and in this equation a non-trivial approximation was made: Indeed, we placed the factor $\exp(-Tn^2/(2A))$ outside the instanton sum. In general, that is not justified for the following reason. If we start at $t = 0$ with flux number n, and the first instanton occurs e.g. at $t = T_1$, we get a factor $\exp(-T_1 n^2/(2A))$ from the kinetic term. If then the next instanton occurs at T_2, we get a further factor $\exp(-T_2(n+1)^2/(2A))$ and so on. The times T_i have to be integrated over and these prefactors can not be extracted from the instanton sum. However, we can find conditions under which it is safe to approximate the $(n + N_i)^2$ in the exponents (where N_i is the number of instantons present at some given time) simply by n^2. To do so we note that, on the one hand, the instanton sum is dominated by instanton numbers of the order of

$$\langle N\rangle \sim K e^{-S_0} T \ . \tag{5.3.28}$$

On the other hand typical values of n dominating the sum over $\exp(-Tn^2/(2A))$ are of the order of $n \sim \sqrt{A/T}$. Thus, disregarding N_i relative to n in the $(n+N_i)^2$-terms will be justified if

$$\langle N\rangle \ll \sqrt{A/T} \qquad \text{or} \qquad A \gg \langle N\rangle^2 T \ . \tag{5.3.29}$$

Given that we anyway choose T large enough to ensure $\langle N\rangle \gg 1$, this implies in particular $A \gg T$.

With this in mind, we return to the corresponding quantum mechanical model. We conjecture that the instanton dynamics is captured by an effective potential $V(\theta) = 2\cos(\theta)Ke^{-S_0}$. To confirm this, we calculate the partition function

$$Z(T) \simeq \frac{1}{Z(0)} \int d\theta \int_{\tilde\theta(0)=\theta}^{\tilde\theta(T)=\theta} d[\tilde\theta] \exp\left[-\int_0^T dt \left(A\frac{\dot{\tilde\theta}^2}{2} + V(\tilde\theta)\right)\right] \ . \tag{5.3.30}$$

[12] Our parametrisation is then equivalent to $Z(T) = \int dE\rho(E)e^{-ET}$. Thus, we found the partition function. The latter characterises the system unambiguously.

It has to be compared to (5.3.26) to establish the correctness of the chosen effective description and, in particular, the potential. But working out (5.3.30) in the regime $A \gg T$ is easy. Indeed, if we first disregard the potential, we are simply dealing with a 1-dimensional system on the interval $(0, 2\pi)$ and a kinetic-term prefactor A. This prefactor sets the minimum time by which any wave packet unavoidably spreads to an $\mathcal{O}(1)$ width due to quantum dynamics. In addition, the potential has a maximal steepness $|V'| \sim Ke^{-S_0}$, leading to a displacement of $Ke^{-S_0}T^2/A \sim \langle N \rangle T/A$ during a time interval T. Our previously derived conditions on A, which underly our derivation of (5.3.26), are sufficient to ensure that the particle moves only by a distance $\Delta\theta \ll 1$ during the time T. Hence, in evaluating (5.3.30) we can approximate $V(\tilde{\theta}(t)) \simeq V(\theta)$. The path integral then becomes that of free particle, to be evaluated on times too short for the periodicity of the configuration space to be relevant. One obtains the well-known time-dependence $\sim \sqrt{A/T}$ of the amplitude, to be multiplied by the integral over $\exp(-TV(\theta))$. This is now in perfect agreement with (5.3.26).[13]

Thus, we find that Giddings-Strominger wormholes give rise to an effective potential $V(\theta) \sim 2Ke^{-S}\cos\theta$ in two independent approaches. We wish to remark that in both approaches we can be agnostic about details of the interpretation of wormholes connecting to baby-universes. Crucially, the axionic shift symmetry is broken locally even if the condition of having equally many instantons and anti-instantons is imposed on the global space-time.

Finally, we wish to remark that the correct choice of the combinatorial factors is a subtle issue. We interpreted a configuration of N wormholes as an instanton-anti-instanton-gas with (anti-)instantons randomly distributed. It is then plausible to include the combinatorial factor $1/(N!)^2$. However, one might argue that each instanton has a corresponding anti-instanton and therefore we should multiply by $N!$ to account for the number of possible pairings. If we assume that the right combinatorial factor is just $1/N!$, we can still do the computation starting with (5.3.7). We then still get $\cos\theta_0$, but this time the energy density in $I \times \mathbb{R}^3$ scales with

$$V(\theta_0) \sim K^2 \mathcal{V} e^{-2S} \cos\theta_0 \ , \tag{5.3.31}$$

which diverges as $\mathcal{V} \to \infty$. Possibilities to avoid this divergence were discussed in [255–258], mostly in the baby-universe interpretation of Giddings-Strominger wormholes. It is possible to express the partition function as an integral over a parameter α, which is an eigenvalue of a baby-universe operator [256].

We rather follow Preskill [257] to sketch the idea of how to evade the divergence: For a combinatorial factor $1/N!$ the partition function reads

$$Z \sim \sum_{N=0}^{\infty} \frac{C^N}{N!} = e^C \ , \tag{5.3.32}$$

where

$$C \sim \bar{z}z \ , \qquad z \equiv K\mathcal{V}e^{-S}e^{i\theta} \ . \tag{5.3.33}$$

Clearly, $C \sim \mathcal{V}^2$. But formally, we can write

$$Z \sim \int d\alpha d\bar{\alpha} e^{-\bar{\alpha}\alpha + \alpha\bar{z} + \bar{\alpha}z} \ . \tag{5.3.34}$$

[13] While our analysis establishes the quantum mechanical model with effective potential $V(\theta) = 2\cos(\theta)Ke^{-S_0}$ only for a certain range of T, we expect it to be valid also for $T \to \infty$.

If α is integrated out we obtain the divergent result (5.3.31). (To see $\cos\theta_0$ coming in one has to group terms carefully as in our first computation.) If, as suggested by Preskill [257], one has to fix α to a certain value, the energy density is simply given by[14]

$$\rho \sim \alpha e^{-S} \cos\theta \ . \tag{5.3.35}$$

In any case, no matter which combinatorial factor is correct and no matter how to interpret α, we always find that a term $\cos\theta$ arises in the effective action.

To summarise, we conclude that Euclidean wormholes are expected to induce an instanton potential $\sim \cos\theta e^{-S}$. Shift-symmetry appears to be broken locally. It would be interesting to study whether this can be seen more directly by building an analogy between gravitational and gauge instantons, where the role of the term $\theta\mathrm{Tr}(F\wedge F)$ is played by $\theta\mathrm{Tr}(R\wedge R)$.

In the following we apply the presented derivation of the instanton potential to cases of $S = nS_0$, giving rise to potentials of the form $\sum_n \cos(n\theta)e^{-nS_0}$.

5.4. The Limit of Validity of Gravitational Instanton Actions

In this section we summarise the instanton actions for all cases $C<0$, $C=0$ and $C>0$ and find limits for the validity of the computation. Qualitatively, we have

$$S \sim \frac{n}{f_{\mathrm{ax}}} \tag{5.4.1}$$

in all three cases. This is of course already known for Euclidean wormholes, see e.g. [108–110; 231–236] and also for $C \geq 0$, see e.g. [92; 110].

Furthermore, we address one concern raised in [110]: the cored gravitational instanton solutions have a singularity at $r=0$ and hence it is unclear whether these solutions can be trusted all the way to the limit $r\to 0$. In fact, we expect a breakdown of the solutions at some radius $r = r_c > 0$, which will be estimated in Section 5.6. We expect such a cutoff radius to be present in any extra-dimensional theory independently of whether a curvature singularity exists or not. Therefore, even the extremal instantons, which do not have singularities, should only be trusted down to $r = r_c$. The situation is different for the Euclidean wormhole solutions, where we can have full control over the solution as long as $r_0 \gtrsim r_c$, with $r_0 \equiv |C|^{1/4}$ being the radius of the wormhole throat at the centre.

The limit of validity affects the computation of the instanton action. In the case of $C\geq 0$ one would usually integrate from $r=0$ to infinity, but instead we can only rely on the contribution from the interval $(r_c, +\infty)$. Whenever a significant fraction of the action comes from $(0, r_c)$, we cannot trust the instanton actions computed in [92; 110] and we will discard these cases.

Thus, the initial task of this section is the evaluation of the on-shell contribution of the integral in (5.2.19). We proceed by using the equations of motion successively. Details of the computations are presented in Appendix C.5.

At first, by tracing Einstein's equations, we can express the Ricci scalar by the trace of the energy-momentum tensor:

$$R = -T. \tag{5.4.2}$$

[14] Nevertheless, the divergence remains disconcerting. For instance, the expectation value of the number of wormholes in a certain space-time region scales as $\langle N\rangle \sim \mathcal{V}^2$ and it is questionable whether the wormhole gas can be dilute in the limit $\mathcal{V}\to\infty$. See also discussions in e.g. [257; 259–261].

One can then rewrite (5.2.19) as

$$S = \int_M d^4x \sqrt{g} \mathcal{F}(\varphi) H^2. \tag{5.4.3}$$

However, this is not yet the full contribution to the instanton action, because the Gibbons-Hawking-York boundary term has to be taken into account. It is

$$S_{\text{GHY}} = -\oint_{\partial M} d^3x \sqrt{h}(K - K_0), \tag{5.4.4}$$

where h is the determinant of the induced metric on ∂M. K and K_0 are the traces of the extrinsic curvatures of ∂M in M and flat space, respectively.

Then, the instanton action is computed as

$$S_{\text{inst}} = S + S_{\text{GHY}} = \int_M d^4x \sqrt{g} \mathcal{F}(\varphi) H^2 - \oint_{\partial M} d^3x \sqrt{h}(K - K_0). \tag{5.4.5}$$

Henceforth, we restrict to the case $\mathcal{F}(\varphi) = \exp(-\alpha\varphi)/(3! f_{\text{ax}}^2)$. Using this together with the equation of motion (5.2.21) and (5.2.27), (5.2.34) or (5.2.35) depending on the choice of C, one can rewrite the first term, S, in the instanton action as an integral over φ. The contribution from S_{GHY} is computed by considering a surface of constant r, see [92] or Appendix C.5.

In the following we analyse the instanton action case by case:

Case $C = 0$:

Extremal instanton solutions go along with a flat metric ($C = 0$). Thus, we have

$$S_{\text{GHY}} = 0. \tag{5.4.6}$$

However, the fields φ and B have a non-trivial profile giving rise to non-vanishing contributions to the instanton action. The full contribution from $r = 0$ to $r = \infty$ is given by

$$S_{\text{inst}} = -\frac{n}{f_{\text{ax}}} \int_{\varphi(0)}^{\varphi(\infty)} d\varphi \exp(-\alpha\varphi/2) = \frac{2}{\alpha} \frac{n}{f_{\text{ax}}}. \tag{5.4.7}$$

As we explained previously, we cannot trust this computation for radii $r < r_c$. Nevertheless, as long as the main contributions to the action come from the regime $r > r_c$ the result (5.4.7) can still be used to estimate contributions to the instanton potential in Section 5.6. One should therefore compare the contribution ΔS from the regime $r < r_c$ with (5.4.7). Unfortunately, the contribution ΔS is UV-sensitive. We assume, however, that the actual UV-contribution ΔS can be parametrically estimated by the naive formula

$$\Delta S = -\frac{n}{f_{\text{ax}}} \int_{\varphi(0)}^{\varphi(r_c)} d\varphi \exp(-\alpha\varphi/2) = \frac{2}{\alpha} \frac{n}{f_{\text{ax}}} \left(1 + \frac{\alpha n}{4A f_{\text{ax}}} \frac{1}{r_c^2}\right)^{-1}, \tag{5.4.8}$$

where we used (5.2.34) in the second step. Demanding that $\Delta S \ll S_{\text{inst}}$ implies

$$\frac{\alpha n}{4A f_{\text{ax}}} \gg r_c^2, \tag{5.4.9}$$

which in turn can be rewritten as a lower bound on S_{inst}:

$$S_{\text{inst}} \gg \frac{8A}{\alpha^2} r_c^2. \tag{5.4.10}$$

This bound depends on the cutoff r_c and the dilaton coupling α. Interestingly, the bound gets weaker for larger α such that contributions from gravitational instantons become increasingly important with increasing α. However, as we will describe in Section 5.5.2, a regime of large dilaton coupling α may not be attainable in string theory. We find that only rather small values of $\alpha \sim \mathcal{O}(1)$ arise from the simplest string compactifications.

Before addressing the next case, we want to point out that (5.4.7) can be rewritten as

$$S_{\text{inst}} = \ell M_{\text{ADM}} \tag{5.4.11}$$

in the case of $\alpha = 2\sqrt{2/3}$, where M_{ADM} is the ADM-mass of our *extremal* 5d RN-black hole of Section 5.2.3, which is consistent with e.g. [92; 244; 252].[15]

Case $C > 0$:

The Gibbons-Hawking-York boundary term yields

$$S_{\text{GHY}} = -3Ar^2\left(\sqrt{1+\frac{C}{r^4}}-1\right)\Bigg|_{r_c}^{\infty} = 3Ar_c^2\left(\sqrt{1+\frac{C}{r_c^4}}-1\right), \tag{5.4.12}$$

where $A = 2\pi^2$.

The contribution from (5.2.19) is given by the integral

$$\begin{aligned} S &= -\frac{n^2}{Af_{\text{ax}}^2}\int_{\varphi(r_c)}^{\varphi(\infty)} \frac{\exp(-\alpha\varphi)}{\sqrt{n^2\exp(-\alpha\varphi)/(A^2f_{\text{ax}}^2)+6C}} = \\ &= \frac{2n}{\alpha f_{\text{ax}}}\sqrt{\exp(-\alpha\varphi)+\sinh^2 K_+}\Bigg|_{\varphi(r_c)}^{\varphi(\infty)}, \end{aligned} \tag{5.4.13}$$

where we used (5.2.36). Combining those two results and taking $r_c \to 0$, we obtain the instanton action

$$S_{\text{inst}} = \frac{2}{\alpha}\frac{n}{f_{\text{ax}}}\left(e^{-K_+} + \frac{\alpha}{2}\sqrt{\frac{3}{2}}\sinh K_+\right). \tag{5.4.14}$$

As before, we need to ensure that the integral from $r = 0$ to $r = r_c$ only gives a minor contribution to the full instanton action (5.4.14). This contribution is

$$\begin{aligned} \Delta S &\equiv (S + S_{\text{GHY}})|_{r=0}^{r=r_c} = \\ &= \frac{2}{\alpha}\frac{n}{f_{\text{ax}}}\left[\sqrt{\exp(-\alpha\varphi(r_c))+\sinh^2 K_+} - \left(1-\frac{\alpha}{2}\sqrt{\frac{3}{2}}\right)\sinh K_+\right] \\ &\quad - 3Ar_c^2\left(\sqrt{1+\frac{C}{r_c^4}}-1\right). \end{aligned} \tag{5.4.15}$$

In the limit $r_c^2/\sqrt{C} \ll 1$ this can be simplified to

$$\begin{aligned} \Delta S &= \frac{4n}{\alpha f_{\text{ax}}}\sinh K_+\left(\frac{r_c^2}{2\sqrt{C}}\right)^{\frac{\alpha}{\sqrt{2/3}}} + 3Ar_c^2 + \dots \\ &= \frac{2n}{\alpha f_{\text{ax}}}\sinh K_+\left[2\left(\frac{r_c^2}{\sqrt{C}}\right)^{\frac{\alpha}{2\sqrt{2/3}}} + \frac{\alpha}{2}\sqrt{\frac{3}{2}}\left(\frac{r_c^2}{\sqrt{C}}\right)\right] + \dots\,, \end{aligned} \tag{5.4.16}$$

[15] For the derivation of (5.4.11) we used that the black hole charge Q is related to n by $n = 2\pi^2\sqrt{6}Q$. This can be obtained by dimensional reduction of the term $1/(2\cdot 3!)\int(\star_5\hat{F})^2$ together with $\hat{F}_{\tau\rho} = \sqrt{6}Q/\rho^3$ and (5.2.8).

where omitted terms decrease as r_c^4/C. The condition $\Delta S \ll S_{\text{inst}}$ turns out to be self-consistent with the imposed regime $r_c^2/\sqrt{C} \ll 1$. More precisely, by choosing $\sqrt{C}$ sufficiently large one can always ensure that $\Delta S \ll S_{\text{inst}}$. According to (5.2.36) this is equivalent to choosing $(n \sinh K_+)/f_{\text{ax}}$ sufficiently large. This is very similar to the parametric situation encountered above for $C = 0$.

To determine the strongest constraints on inflation we are interested in identifying the instantons with the smallest action. For a given value of n/f_{ax} and at a fixed dilaton coupling α cored gravitational instantons correspond to a family of solutions parameterised by K_+ (see Section 5.2.2). We wish to identify the instanton with the smallest action in this family. As pointed out in Section 5.2.3, while K_+ is expected to take discrete values, it can be effectively treated as a continuous parameter in the limit of macroscopic objects. Hence we can determine the solutions with the smallest action by formally extremising (5.4.14) with respect to K_+ as it was done in [92]. For $\alpha \geq 2\sqrt{2/3}$ the action of cored instantons is always bigger than that of extremal instantons. If $0 < \alpha < 2\sqrt{2/3}$, the smallest cored instanton action is as big as the extremal instanton action for $\alpha = 2\sqrt{2/3}$. To summarise, we obtain

$$S_{\text{cored}}(\alpha) \geq \begin{cases} S_{\text{extremal}}(\alpha) & \text{for } \alpha \geq 2\sqrt{2/3} \\ S_{\text{extremal}}(\alpha = 2\sqrt{2/3}) & \text{for } \alpha < 2\sqrt{2/3} \end{cases}, \tag{5.4.17}$$

where the extremal instanton action was computed above in (5.4.7). The upshot is that the contributions to the axion potential due to cored gravitational instantons will always be sub-leading compared to the effects due to a suitable extremal instanton. As we are interested in determining the strongest constraints on axion inflation, we will hence neglect cored instantons in the following analyses and focus on extremal instantons and Euclidean wormholes instead.

Case $C < 0$:

For Euclidean wormholes the coordinate r is defined on $r \in [r_0, +\infty)$, where $r_0 \equiv |C|^{1/4}$ is the size of the wormhole at the centre. As long as $r_0 \gtrsim r_c$ one can safely integrate from $r = r_0$ to $r = \infty$. As $r_0 \equiv |C|^{1/4} \propto n/f_{ax}$ (see (5.2.28)) the condition $r_0 \gtrsim r_c$ can be fulfilled by choosing n/f_{ax} sufficiently large.

As pointed out in Section 5.2.2 we will only consider wormholes with dilaton couplings $\alpha < 2\sqrt{2/3}$ in order to have regular solutions for φ. We then proceed with calculating the action. The Giddings-Hawking-York boundary term vanishes [109],

$$S_{\text{GHY}} = 0, \tag{5.4.18}$$

since two asymptotically flat regions are connected by a handle and thus the integral gives zero. The on-shell contribution from (5.2.19) for only half of the wormhole[16] is given by

$$\begin{aligned} S_{\text{inst}} &= -\frac{n^2}{A f_{\text{ax}}^2} \int_{\varphi(r_0)}^{\varphi(\infty)} \frac{\exp(-\alpha\varphi)}{\sqrt{n^2 \exp(-\alpha\varphi)/(A^2 f_{\text{ax}}^2) - 6|C|}} = \\ &= \frac{2}{\alpha} \frac{n}{f_{\text{ax}}} \sin\left(\frac{\alpha\pi}{4}\sqrt{\frac{3}{2}}\right), \end{aligned} \tag{5.4.19}$$

[16] To get the instanton action, we have to divide the full wormhole action by two, as the wormhole represents a pair of instanton and anti-instanton. For more details, see Appendix C.5.

where we used the solutions for $C < 0$ from Section 5.2.2. Notice that the limit $\alpha \to 0$ corresponds to the Giddings-Strominger wormhole [109], and we have

$$S_{\text{inst}} = \frac{\pi\sqrt{6}}{4}\frac{n}{f_{\text{ax}}}. \tag{5.4.20}$$

Furthermore, in the limit $\alpha \to 2\sqrt{2/3}$ we find the instanton action of an extremal instanton with $\alpha = 2\sqrt{2/3}$.

Summary

We summarise our results for the instanton action. For one, the instanton action S_{inst} scales as $S_{\text{inst}} \sim n/f_{\text{ax}}$ for all three types of gravitational instanton. Results were obtained in an effective theory with a cutoff at a length scale r_c. The existence of this cutoff implies that not all gravitational instanton solutions can be trusted in the framework of the effective theory. One can derive a criterion for deciding which gravitational instantons to include. While numerical factors may vary, this condition exhibits the same parametric behaviour for all three types of gravitational instantons: given a cutoff at a length scale r_c one has to choose $n/f_{\text{ax}} \gg r_c^2$ for being able to trust the instanton action computed in the effective theory.

In order to determine the importance of such gravitational instantons it is crucial to estimate the size of the cutoff scale r_c. The first step is to see whether moduli stabilisation places a lower bound on r_c.

5.5. Gravitational Instantons and Moduli Stabilisation

We now want to make progress towards realistic string compactifications. The pure Einstein-axion system is relevant only below the moduli scale. Above that scale, moduli can play the role of an additional scalar φ with dilatonic coupling to the axion or 2-form kinetic term. We will make use of our detailed discussion of this extended system in Section 5.2 and Section 5.4.

5.5.1. Gravitational Instantons in the presence of a potential

We only consider the lightest modulus, which we will call φ. For instance, it could be the saxion associated with the axion θ. We will assume stabilisation at $\varphi = 0$. In the throat region of the instanton, the modulus will be typically driven away from this value. We will assume that this displacement is small enough so that the potential of the modulus can be approximated by a mass term, i.e. $V = m^2\varphi^2/2$.

The obvious extension of (5.2.2) is then

$$S = \int d^4x\sqrt{g}\left[-\frac{1}{2}R + \frac{1}{2}\mathcal{F}(\varphi)H^2 + \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + V(\varphi)\right]. \tag{5.5.1}$$

We take $\mathcal{F}$ to be exponential, which is the case discussed in detail earlier and which is typical for string-derived models (see Section 5.5.2). Nevertheless, due to the presence of the potential, solutions are more complicated than before. We make the most general ansatz respecting spherical symmetry

$$ds^2 = \lambda(r)dr^2 + r^2 d\Omega_3^2, \tag{5.5.2}$$

as in Appendix C.1. From the derivation therein it becomes clear that $\lambda(r)$ is no longer given by $(1 + C/r^4)^{-1}$. However, we will see that for $r \ll r_* \equiv 1/m$ the mass term is negligible and we can use the approximation $\lambda(r) \simeq (1+C/r^4)^{-1}$ (cf. the related discussion in [233]). Thus, the three types of gravitational instantons analysed above remain relevant.

The fact that the mass term is negligible close to the centre of the instanton is intuitively clear: The field strength contribution to the energy-momentum tensor increases as one approaches the centre and hence, for sufficiently small r, the contribution from the mass term becomes subdominant. This will become more explicit below.

Employing (5.5.2), the Einstein equation $G_{rr} = T_{rr}$ and the Klein-Gordon equation read

$$\frac{1}{2}(\varphi')^2 - \lambda(r)V(\varphi) + \frac{3}{r^2}(\lambda(r)-1) - 3\lambda(r)\mathcal{F}(\varphi)\frac{n^2}{A^2r^6} = 0 \tag{5.5.3}$$

$$\varphi'' + \left(\frac{3}{r} - \frac{\lambda'(r)}{2\lambda(r)}\right)\varphi' - \lambda(r)V'(\varphi) - 3\lambda(r)\mathcal{F}'(\varphi)\frac{n^2}{A^2r^6} = 0. \tag{5.5.4}$$

Here we also used (5.2.22) and (5.2.24), which specify the profile of H.

Approximation

As already sketched above, the strategy is as follows: Let $\varphi_0(r)$ and $\lambda_0(r) \equiv (1+C/r^4)^{-1}$ be the field and metric profiles for $V \equiv 0$. Then we work out the conditions under which

$$T_{rr}(\varphi_0, \lambda_0) \gg V(\varphi_0)\,. \tag{5.5.5}$$

This specifies the regime where we can expect the $\varphi_0(r)$ and $\lambda_0(r)$ to provide good approximations to the true solutions $\varphi(r)$ and $\lambda(r)$.

We now go into more detail: The full energy-momentum tensor of (5.5.1) reads

$$T_{\mu\nu} = -g_{\mu\nu}\left[\frac{1}{2}\mathcal{F}(\varphi)H^2 + \frac{1}{2}\partial_\rho\varphi\partial^\rho\varphi + V(\varphi)\right] + 3\mathcal{F}(\varphi)H_{\mu\rho\sigma}H_\nu{}^{\rho\sigma} + \partial_\mu\varphi\partial_\nu\varphi. \tag{5.5.6}$$

Taking $\varphi = \varphi_0$ and $\lambda = \lambda_0$ we find

$$T_{rr}(\varphi_0, \lambda_0) = \frac{3C}{r^6(1+C/r^4)} - \frac{V(\varphi_0)}{1+C/r^4}, \tag{5.5.7}$$

where (5.2.25) was used. We see that the potential is negligible compared to the curvature contributions if (for $C \neq 0$)

$$\left|\frac{3C}{r^6}\right| \gg \frac{1}{2}m^2\varphi_0^2\,. \tag{5.5.8}$$

Appealing again to (5.2.25), we first consider the regime $r \gg |C|^{1/4}$. Then $\varphi_0'(r) \sim 1/r^3$ and hence

$$\varphi_0(r) \sim \frac{1}{r^2}\,. \tag{5.5.9}$$

Here we treat n/A and C as '$\mathcal{O}(1)$ factors' and disregard them. We explain this below.

With this, (5.5.8) translates to

$$r \ll r_* \equiv \frac{1}{m}\,. \tag{5.5.10}$$

Now, our interest is in the case $m \ll 1$, i.e. in moduli much lighter than the Planck scale. This implies $r_* \gg 1$ so that $r_* \gg |C|^{1/4}$, giving us a large validity range for our approximation $\varphi_0 \sim 1/r^2$. Crucially, while $|C|$ also figured as a large parameter in other parts of this chapter, here the much stronger hierarchy $1/m \gg 1$ dominates and our crude approximation concerning '$\mathcal{O}(1)$ factors' is justified.

Next, we need to consider the region $r \lesssim |C|^{1/4}$. While here the profile $\varphi_0(r)$ is more complicated, we are now deeply inside the regime of large field strength. It is easy to convince oneself that the potential $\sim m^2\varphi^2$ remains subdominant. What is less obvious is whether the $m^2\varphi^2$ approximation remains justified, given that the field now moves significantly away from zero. This will be discussed later.

Finally, the extremal instanton with $C = 0$ requires an extra comment. In this case the energy-momentum tensor vanishes and the criterion (5.5.8) is no longer applicable. Instead, we require that the mass term in (5.5.3) should be subdominant compared to every other term in this equation, i.e.

$$m^2\varphi_0^2 \ll \frac{3\mathcal{F}(\varphi_0)n^2}{A^2r^6}, \tag{5.5.11}$$

which yields again the condition $r \ll 1/m$ (here we used that $\mathcal{F}$ is approximately constant for large r). Note that in this case the behaviour of φ_0 at large r is specified by (5.2.34) and the role of the 'largish' parameter $|C|^{1/4}$ is taken over by $n/f_{\rm ax}$.

To summarise, we have now argued rather generally that the gravitational solutions found in the absence of a potential are good approximations for $r \ll 1/m$. We will not need the behaviour of φ outside that region, at $r \to \infty$. Indeed, by redefining φ we can, as argued before, always ensure that the φ_0 asymptotically approaches the minimum of the potential at $\varphi = 0$. Thus, even while the actual profile of $\varphi(r)$ can significantly deviate from $\varphi_0(r)$ at $r \gg 1/m$, there is no doubt that the fundamental property of φ approaching zero at large r will be maintained. Crucially, since $1/m \gg |C|^{1/4}$ and the action integral is dominated by the region $r \lesssim |C|^{1/4}$, we can also trust the zero-potential solutions for evaluating the action, independently of the large-r region.

5.5.2. Dilaton Couplings from String Compactifications

The gravitational solutions in Section 5.2.2 were obtained for scalars with dilatonic couplings, i.e. where the prefactor of the kinetic term for the axion is given by $\mathcal{F}(\varphi) = e^{-\alpha\varphi}/(3!f_{\rm ax}^2)$. This form frequently occurs for effective theories obtained from string theory compactifications. The value of α will depend on the precise identification of the axion and scalar with the corresponding fields in the string compactifications. In the following, we will provide relevant examples.

The Axio-Dilaton

Let us first consider the case where both the axion and the scalar descend from the axio-dilaton field $S = C_0 + i/g_s$ with string coupling g_s and universal axion C_0. It appears in the Kähler potential as

$$\mathcal{K} = -\ln\left(-i(S - \bar{S})\right). \tag{5.5.12}$$

The kinetic term of the Lagrangian $\mathscr{L} \supset \mathcal{K}_{S\bar{S}}\partial_\mu S\partial^\mu \bar{S}$ then becomes

$$\mathscr{L} \supset \frac{g_s^2}{4}(\partial C_0)^2 + \frac{1}{4g_s^2}(\partial g_s)^2 \,. \tag{5.5.13}$$

Canonical normalisation of our saxion gives $g_s = g_s^0 \exp(\sqrt{2}\varphi)$. Thus, the field strength coupling reads

$$\mathcal{F}(\varphi) = \frac{1}{2\cdot 3!}\frac{1}{\mathcal{K}_{S\bar{S}}} = \frac{1}{3(g_s^0)^2}\exp(-2\sqrt{2}\varphi), \tag{5.5.14}$$

so in our notation the dilaton coupling α is $\alpha = 2\sqrt{2}$. Notice that $\varphi \to \infty$ corresponds to the strong coupling limit, while the weak coupling limit is given by $\varphi \to -\infty$.

Kähler Moduli at Large Volume

Let us now consider the Kähler moduli sector at large volume. In particular, consider the case where the volume is dominated by one Kähler modulus T. For example, this arises in the scheme of moduli stabilisation known as the Large Volume Scenario (LVS) [7]. The relevant part of the Kähler potential is

$$\mathcal{K} = -2\ln\mathcal{V} = -3(T+\bar{T}) + \dots \ . \tag{5.5.15}$$

Here we wish to identify the saxion with $\mathrm{Re}(T)$ and the axion with $\mathrm{Im}(T)$. The leading contribution to the kinetic term for the saxion and axion is then given by

$$\mathcal{K}_{T\bar{T}} = \frac{3}{(T+\bar{T})^2} \ . \tag{5.5.16}$$

Canonical normalisation gives

$$\mathrm{Re}(T) = \exp\left(-\sqrt{\frac{2}{3}}\varphi\right) , \tag{5.5.17}$$

and hence

$$\mathcal{F}(\varphi) \sim \exp\left(-2\sqrt{\frac{2}{3}}\varphi\right) \ . \tag{5.5.18}$$

The dilaton coupling is thus $\alpha = 2\sqrt{2/3}$.

Complex Structure Moduli in the Large Complex Structure Limit (LCS)

It is well-known that complex structure moduli in the LCS limit give rise to a shift-symmetric structure in the Kähler potential. Let u be a complex structure modulus in the LCS regime and z denote the remaining complex structure moduli. Then we have

$$\begin{aligned}\mathcal{K} = -\ln\Big(&\kappa_{uuu}(u+\bar{u})^3 + \kappa_{uui}(u+\bar{u})^2(z_i+\bar{z}_i) + \frac{\kappa_{uij}}{2!}(u+\bar{u})(z_i+\bar{z}_i)(z_j+\bar{z}_j) + \\ &+\frac{\kappa_{ijk}}{3!}(z_i+\bar{z}_i)(z_j+\bar{z}_j)(z_k+\bar{z}_k) + f(z_i)\Big) ,\end{aligned} \tag{5.5.19}$$

where the κ_{ijk} denote the intersection numbers of the mirror-dual Calabi-Yau three-fold and f is a function of the remaining complex structure moduli z_i and accounts for instantonic corrections to the Kähler potential. For the moment only u shall be stabilised in the LCS limit, i.e.

$$\mathrm{Re}(u) > 1 \ . \tag{5.5.20}$$

Thus, one obtains

$$\mathcal{K}_{u\bar{u}} = \frac{3}{(u+\bar{u})^2} \tag{5.5.21}$$

at leading order as long as $\kappa_{uuu} \neq 0$. Omitted terms scale as $(u+\bar{u})^{-3}$. Therefore, canonical normalisation yields

$$\mathrm{Re}(u) = \exp\left(-\sqrt{\frac{2}{3}}\varphi\right) , \tag{5.5.22}$$

and hence $\alpha = 2\sqrt{2/3}$.

In the situations studied so far the saxion and axion arose from the same complex scalar field. However, one may also consider the case where the saxion and axion originate from different moduli. To give just one example, let us again consider the complex structure sector of a CY threefold, but now we will assume that two complex structure moduli u, v are in the LCS regime. Further, we assume the following hierarchy

$$\mathrm{Re}(u) \gg \mathrm{Re}(v) \gg 1 \; . \tag{5.5.23}$$

We will now consider the axionic field $\mathrm{Im}(v)$ and study the coupling to the saxion $\mathrm{Re}(u)$. As before, the leading contribution to the kinetic term of the saxion is

$$\mathcal{K}_{u\bar{u}} = \frac{3}{(u+\bar{u})^2} \; , \tag{5.5.24}$$

and the canonically normalised saxion is given by (5.5.22). The leading contribution to the kinetic term for the axion is

$$\mathcal{K}_{v\bar{v}} \sim \frac{1}{(u+\bar{u})^2} \sim \exp\left(-2\sqrt{\frac{2}{3}}\varphi\right) \; , \tag{5.5.25}$$

where omitted terms decrease as $(u+\bar{u})^{-3}$. While both $\mathcal{K}_{u\bar{u}}$ and $\mathcal{K}_{v\bar{v}}$ scale as $(u+\bar{u})^{-2}$, this behaviour has different origins in the two cases. The leading contribution to $\mathcal{K}_{u\bar{u}}$ comes from $\kappa_{uuu}(u+\bar{u})^3$, whereas it is the terms $\kappa_{uvv}(u+\bar{u})(v+\bar{v})^2$ and $\kappa_{uuv}(u+\bar{u})^2(v+\bar{v})$ which contribute to $\mathcal{K}_{v\bar{v}}$ at leading order.

Despite these differences we again find

$$\mathcal{F}(\varphi) \sim \exp\left(-2\sqrt{\frac{2}{3}}\varphi\right) \; , \tag{5.5.26}$$

and $\alpha = 2\sqrt{2/3}$.[17]

Note that in all the cases examined above the dilaton coupling is just outside the range allowing for Euclidean wormhole solutions $0 \le \alpha < 2\sqrt{2/3}$. This observation and a possible way out have been pointed out in [243; 262]. The idea is as follows. Even if wormholes charged under individual axions do not exist, one can nevertheless find solutions which are charged under more than one axion (see also [263]).

We will conclude this section with an example that allows for the existence of Euclidean wormhole solutions and may be useful to illustrate and develop the above idea. Let us consider both the axio-dilaton sector and the complex structure moduli sector of a CY 3-fold at LCS:

$$\mathcal{K} = -\ln\left(-i(S-\bar{S})\right) - \ln\left(\kappa_{uuu}(u+\bar{u})^3\right) \; . \tag{5.5.27}$$

In the spirit of [243; 262; 263] we could now investigate Euclidean wormhole solutions charged under both the universal axion as well as the complex structure axion. Alternatively, we may assume that we can stabilise moduli such that $S = iu$. Then we effectively have the theory of one 4-fold complex structure modulus and we obtain (see Appendix C.6 for details)

$$\mathcal{K}_{u\bar{u}} = \frac{4}{(u+\bar{u})^2} \; . \tag{5.5.28}$$

[17] This would be different if κ_{uuu} and κ_{vvv} were the only non-vanishing intersection numbers. Then we would still have $\mathcal{K}_{u\bar{u}} \simeq 3/(u+\bar{u})^2$ but now $\mathcal{K}_{v\bar{v}} \sim (v+\bar{v})/(u+\bar{u})^3$ for $\mathrm{Re}(u) \gg \mathrm{Re}(v)$. Assuming that $\mathrm{Re}(v)$ is stabilised such that we can take it as constant, we would now find a dilaton coupling $\alpha = \sqrt{6}$.

Taking the saxion as $\text{Re}(u)$ and the axion as $\text{Im}(u)$ we now find $\alpha = \sqrt{2}$ which lies within the range allowing for wormholes. We leave it for future work to investigate whether this pattern of moduli stabilisation can be realised in a realistic compactification.

5.5.3. Maximal Field Displacements of Dilatonic Fields

In the previous sections we have made progress towards studying gravitational instantons in the presence of moduli. The results of Section 5.5.2 imply that a restriction to moduli with dilatonic couplings is well-motivated from string compactifications. We also made progress towards understanding the role of the potential stabilising the modulus in Section 5.5.1. In particular, in the regime $r \ll 1/m$ the potential can be ignored and gravitational instanton solutions for a massless dilaton will be good approximations.

There is another effect which we need to take into consideration. When approaching the core of a gravitational instanton, the value of the dilaton increases. In the case of a Euclidean wormhole it reaches a maximum at the wormhole throat, while for extremal and cored instantons the dilaton diverges for $r \to 0$. However, we cannot afford arbitrarily large field displacements, as this will take us outside the range of validity of our effective theory.

To be specific, consider the effective theory of the axio-dilaton at weak string coupling. When approaching the centre of a gravitational instanton solution the string coupling increases compared to its asymptotic value. If it becomes too large the supergravity regime breaks down and we cannot trust our solutions. A similar argument can be made for any effective dilaton-axion theory from string compactifications.

This gives us an additional criterion to decide which gravitational instantons to trust and which ones to disregard. We will analyse this condition focussing on Euclidean wormholes and extremal gravitational instantons. In the following, we will denote by φ_{max} the threshold value at which the effective theory breaks down.

Euclidean Wormholes

For Euclidean wormholes the displacement becomes maximal at the throat of the wormhole. Using our solution for $\varphi(r)$ (5.2.32) one finds:

$$\varphi(r_0) = -\frac{1}{\alpha} \ln \cos^2 \left(\frac{\alpha \pi}{4} \sqrt{\frac{3}{2}} \right) . \tag{5.5.29}$$

To trust the wormhole solution we require $\varphi(r_0) < \varphi_{\text{max}}$. The maximal displacement only depends on the dilaton coupling α, which is not a free parameter, but a property of the physical system studied. As a result the maximal displacement is model-dependent.

Recall that Euclidean wormhole solutions only exist for dilaton couplings in the range $0 \leq \alpha < 2\sqrt{\frac{2}{3}}$. The maximal displacement at the wormhole throat is smallest for $\alpha = 0$, grows when α is increased and eventually diverges for $\alpha \to 2\sqrt{\frac{2}{3}}$. To give just one example, the value $\alpha = \sqrt{2}$ yields a displacement $\varphi(r_0) - \varphi(\infty) = \varphi(r_0) \simeq 2.2$ in Planck units, which may already be critical.

Another important result from this section is that the maximal displacement $\varphi(r_0)$ is independent of the ratio n/f_{ax}, or, equivalently, the wormhole radius at the throat r_0. Hence we do not get any additional constraints on these quantities due to the displacement of the saxion.

Extremal Instantons

For extremal gravitational instantons the φ-profile exhibits a divergent behaviour for $r \to 0$. As laid out in Section 5.4, we will nevertheless trust such solutions as long as the dominant part of the instanton action arises from the region $r > r_c$, where r_c is the length cutoff of our effective theory. This cutoff will be discussed in more detail in Section 5.6.2. Here we will show that the displacement of the saxion gives an independent condition for the reliability of our action.

Let us be more precise. Given a threshold value φ_{max} beyond which our theory breaks down, we can identify a radius r_{min} at which the dilaton crosses this value: $\varphi(r_{\text{min}}) = \varphi_{\text{max}}$. This can be made explicit using (5.2.34):

$$e^{\alpha\varphi_{\text{max}}} = \left(1 + \frac{\alpha n}{4Af_{\text{ax}}}\frac{1}{r_{\text{min}}^2}\right)^2 . \tag{5.5.30}$$

To trust our solution we need to ensure that $\Delta S/S \ll 1$, where ΔS is the contribution to the instanton action from the region $r < r_{\text{min}}$. Using (5.4.7) and (5.4.8) we find

$$\frac{\Delta S}{S} = \left(1 + \frac{\alpha n}{4Af_{\text{ax}}}\frac{1}{r_{\text{min}}^2}\right)^{-1} = \exp\left(-\tfrac{\alpha}{2}\varphi_{\text{max}}\right) . \tag{5.5.31}$$

Hence the relevant condition is

$$\exp\left(-\tfrac{\alpha}{2}\varphi_{\text{max}}\right) \ll 1 , \tag{5.5.32}$$

which gives an additional (model-dependent) constraint.

Last, let us return to one aspect encountered for the case of Euclidean wormholes. There we observed that for $\alpha \to 2\sqrt{\frac{2}{3}}$ the saxion displacement at the wormhole throat grows without bound and would exceed any finite value φ_{max}. Note that this does not necessary constitute a pathology. Rather, the behaviour observed for a wormhole becomes similar to that of an extremal instanton. In fact, in the limit $\alpha \to 2\sqrt{\frac{2}{3}}$ the Euclidean wormhole becomes a pair of extremal instantons. We can then deal with the divergence of φ as in the case of extremal instantons and cut our solution off at some $r = r_{\text{min}}$.

5.6. Consequences for Large Field Inflation

In this section we will analyse to what extent gravitational instantons constrain axion inflation. The idea is as follows: we will check whether the contribution to the axion potential δV due to gravitational instantons can be large enough to disrupt inflation.

To be specific, gravitational instantons contribute as

$$\delta V = \mathcal{A}\cos(n\theta)e^{-S} , \tag{5.6.1}$$

where $S \sim n/f_{\text{ax}}$ (see Section 5.4). Whether such instanton corrections can have significant influence on the slow-roll dynamics clearly depends both on the size of the instanton action S and the prefactor $\mathcal{A}$. The latter is quoted to be of order M_p^4 [101; 108]. However, in Appendix C.7 we give arguments why the prefactor $\mathcal{A}$ can be significantly below the Planck scale in more realistic string compactification models. Specifically, we expect $\mathcal{A}$ to scale as $\mathcal{A} \sim \mathcal{V}^{-5/3}$ with compactification volume $\mathcal{V}$.

We then compare δV with the size of the axion potential during inflation. For large-field inflation the scale of inflation is of the order [2]

$$V_{\text{inflation}} \sim 10^{-8}. \tag{5.6.2}$$

Hence, whenever we find $\delta V \sim 10^{-8}$ we will conclude that the effects of gravitational instantons on the axion potential are in principle large enough to spoil inflation.

In what follows we compute δV only for the case of a single axion, but our results can be straightforwardly extended to models of N-flation, kinetic alignment and the Kim-Nilles-Peloso mechanism, see [108] for more details.

5.6.1. Action of the most 'dangerous' Gravitational Instantons

To check whether gravitational instantons are dangerous for inflation, we want to focus on the instantons with the smallest action. At the same time, we need to ensure that these are solutions which we can trust within the framework of our effective theory. In brief, we are interested in the most relevant instanton within the regime of validity of our theory.

The breakdown of our effective gravity theory is crucial in this context, because it will put a lower bound on the instanton action S. As explained in Section 5.4, in a theory with length cutoff r_c we can only trust gravitational instanton solutions with $n/f_{\rm ax} \gg r_c^2$. This translates into a lower bound on the instanton action as $S \sim n/f_{\rm ax}$.

To calculate the contributions of gravitational instantons to the axion potential we hence need to determine the cutoff r_c. In Section 5.6.2 we will estimate the smallest possible value of r_c at which the description in terms of a 4-dimensional theory may hold. Before doing this it will be instructive to check how large r_c can be so that gravitational instantons still induce a sizeable contribution to the inflaton potential.

Note that gravitational instanton solutions for the case of a massless dilaton will be sufficient for our analysis, despite the fact that we are interested in the case of massive dilaton fields. As described in Section 5.5.1 the non-zero potential does not affect the action significantly.

Euclidean Wormholes

For any n and $f_{\rm ax}$ the Euclidean wormhole action is computed in (5.4.19). At the same time the wormhole radius r_0 scales as $r_0 \sim (n/f_{\rm ax})^{1/2}$ according to (5.2.28). As we require $r_0 \gtrsim r_c$ we get

$$S_{\rm inst} \gtrsim (2\pi^2)\sqrt{6}\frac{2}{\alpha}\tan\left(\frac{\alpha\pi}{4}\sqrt{\frac{3}{2}}\right) r_c^2 \ . \tag{5.6.3}$$

On the allowed interval $0 \leq \alpha < 2\sqrt{2/3}$ the instanton action as a function of α increases monotonically. Therefore, the most dangerous wormhole corresponds to the Giddings-Strominger instanton with $\alpha = 0$. Hence

$$S_{\rm inst} \geq S_{\rm inst}(\alpha = 0) \gtrsim 3\pi^3 r_c^2. \tag{5.6.4}$$

Demanding that $e^{-S} \gtrsim 10^{-8}$ implies $r_c \lesssim 0.4$ (in Planck units).

In Section 5.5.2 we found that $\alpha = \sqrt{2}$ can be obtained from string compactifications and still lies in the allowed range of dilaton-couplings appropriate to allow for Euclidean wormholes. This example requires $r_c \lesssim 0.2$ in order to get a contribution of at least $\delta V \sim 10^{-8}$.

Note that the prefactor $\mathcal{A}$ (see Appendix C.7) may potentially lower the size of the contribution to the inflaton potential.

Extremal Gravitational Instantons

The action for extremal gravitational instantons is obtained from (5.4.7). However, we have to take into account the computability condition (5.4.9) for the action. It follows that

$$S_{\text{inst}} > \frac{8 \cdot (2\pi^2)}{\alpha^2} r_c^2 \,. \tag{5.6.5}$$

In string theory α cannot be chosen arbitrarily large. The largest α we could obtain from string compactifications was $\alpha = 2\sqrt{2}$. Extremal gravitational instantons then become relevant if $r_c \lesssim 1$. Hence, extremal gravitational instantons may turn out to be somewhat more dangerous for axion inflation than Euclidean wormholes.

We do not consider cored gravitational instantons, for which a similar analysis could be made. The reason is that their action is always larger than that of a suitable extremal instanton (see Section 5.4).

The question we want to address now is how small r_c can be in any controlled model of quantum gravity. Knowing that moduli displacements are not an issue, one would naively expect that $r_c \simeq 1$ can be problematic as we reach already Planck regime. Notice however that it is important to determine r_c as precisely as possible, because due to $\delta V \sim e^{-S} \sim e^{-r_c^2}$ the instanton contributions are very sensitive to every $\mathcal{O}(1)$-factorchange in the cutoff radius.

5.6.2. Estimating the Critical Radius r_c

Let us take string theory as our model of quantum gravity. String compactifications then yield a hierarchy of scales in the effective theory as depicted in Figure 5.1. We expect that going beyond the Kaluza-Klein scale will render our effective description insufficient. The reason is that the gravitational instanton solutions we consider are obtained in a 4-dimensional effective theory which arises from a more fundamental description upon compactification. For the 4-dimensional picture to remain valid, we require the length scale r_c associated with our 4-dimensional solution not to be smaller than the length scale associated to the compactified extra-dimensions.

But how small can this length scale be? In string theory it cannot be arbitrarily small. String compactifications exhibit a property termed T-duality which states that a compactification with a small volume describes the same physics as another compactification with large volume. This gives rise to the notion of a smallest length scale at the self-dual value of the compactification volume.

Putting everything together, we arrive at the smallest possible value r_c where we can trust our effective 4-dimensional analysis. We find that r_c should be related to the length scale of the compact dimensions at self-dual volume $\mathcal{V}_{\text{sd}}$ of the compactification space. In this way, we push the KK-scale as close to the Planck scale as possible, allowing us to consider the lightest gravitational instantons we can obtain within the regime of validity of our description.

What we mean by "related" is at this naive level ambiguous. There are at least two "canonical" possibilities to make the definition of r_c more precise. They differ by factors of π, which are unfortunately crucial when comparing e^{-S} with $V_{\text{inflation}}$. Given the volume $\mathcal{V}_{\text{sd}}$ of the six-dimensional compact space at the self-dual point we can define a length scale as $\mathcal{V}_{\text{sd}}^{1/6}$ and a 3-volume by $\mathcal{V}_{\text{sd}}^{1/2}$. Two possible definitions of r_c are then:

1. The volume of the S^3 of our wormhole solution should not be smaller than $\mathcal{V}_{\text{sd}}^{1/2}$, i.e.

$$2\pi^2 r_c^3 = \mathcal{V}_{\text{sd}}^{1/2}. \tag{5.6.6}$$

2. More generously, the great circle of S^3 should not be smaller than $\mathcal{V}_{\rm sd}^{1/6}$, i.e.

$$2\pi r_c = \mathcal{V}_{\rm sd}^{1/6} . \tag{5.6.7}$$

As a toy-model to compute $\mathcal{V}_{\rm sd}$ we take T^6 and apply T-duality six times for each S^1 to get $\mathcal{V}_{\rm sd}(T^6) = \ell_s^6 = (2\pi)^6(\alpha')^3$. To convert this into Planck units, recall that (see e.g. [48])

$$M_p^2 = \frac{4\pi\mathcal{V}}{g_s^2 \ell_s^8} . \tag{5.6.8}$$

In the following we also go to the S-self-dual point $g_s = 1$.

The first criterion (5.6.6) then gives $r_c M_p = \sqrt{4\pi} \cdot (2\pi^2)^{-1/3} \simeq 1.3$. Using (5.6.4) and (5.6.5), which are both in 4d Planck units, the contributions to the axion potential due to gravitational instantons are then:

$$\begin{aligned} \text{Giddings-Strominger wormhole:} \qquad & e^{-S} \simeq 10^{-68} \ , \\ \text{Extremal instantons:} \qquad & e^{-S} \lesssim 10^{-15} \quad \text{for } \alpha = 2\sqrt{2} \ . \end{aligned}$$

Hence, in both cases the gravitational instantons appear to be irrelevant for inflation.

If we apply the second criterion (5.6.7) we have $r_c M_p = 1/\sqrt{\pi} \simeq 0.56$. This yields

$$\begin{aligned} \text{Giddings-Strominger wormhole:} \qquad & e^{-S} \simeq 10^{-13} \ , \\ \text{Extremal instantons:} \qquad & e^{-S} \lesssim 10^{-3} \quad \text{for } \alpha = 2\sqrt{2} \ . \end{aligned}$$

Again, Euclidean wormholes contribute to the axion potential too weakly to interfere significantly with inflation. However, extremal instantons can in principle be important, but this will depend on the value of the dilaton coupling α. Note that for $\alpha = 2\sqrt{2/3}$ we still get $e^{-S} \lesssim 6 \cdot 10^{-9}$ for extremal instantons, which is marginal as far as the significance for inflation is concerned. However, we want to emphasise that our numerical results should be taken with a grain of salt. In particular, given a value of a length cutoff r_c we only have a lower bound (5.6.5) for the action of the most important trustworthy extremal instanton. However, δV is exponentially sensitive to the instanton action. Thus, unless the instanton action is close to saturating the inequality (5.6.5) the contributions from extremal instantons can quickly become irrelevant for inflation.

Of course, the instanton contribution $\delta V = \mathcal{A} e^{-S} \cos(n\theta)$ also involves the prefactor $\mathcal{A}$, which we estimate in Appendix C.7. We expect $\mathcal{A} \sim \mathcal{V}^{-5/3}$, which is $\mathcal{O}(1)$ at the self-dual point. Note that in more realistic scenarios away from the self-dual point (i.e. compactifications with a hierarchy of scales) it would suppress the gravitational instanton contributions even further.

Our results can be summarised as follows: overall, we find that gravitational instantons do not give rise to strong *model-independent* constraints on axion inflation, even if we push the KK-scale as close to the Planck-scale as possible. Extremal gravitational instantons may be important for inflation, but this is model-dependent, as the size of their contribution depends on the value of the dilaton coupling α.

5.7. Gravitational Instantons and the Weak Gravity Conjecture

Finally, we want to make further remarks on the relation between gravitational instantons and the Weak Gravity Conjecture (WGC) [20; 92; 108]. The original form of the WGC requires that the particle spectrum of a consistent, UV-complete gravitational theory with a $U(1)$ gauge field contains at least one particle whose charge-to-mass ratio is larger or equal to that of an extremal black hole [20]. There exists a straightforward generalisation to gravitational theories with an axion coupling to instantons. In the following we will argue in analogy with the WGC for particles with U(1) charges, i.e. we will treat instantons like particles with axion charge.

We start with the theory of an axion with an instanton-induced potential:

$$\mathscr{L} = \frac{1}{2}(\partial\theta)^2 - \sum_i \Lambda_i^4 e^{-S_i} \cos(\tfrac{n_i}{f_{\rm ax}}\theta) \ . \tag{5.7.1}$$

The WGC then requires the existence of an instanton with

$$z \equiv \frac{n_i/f_{\rm ax}}{S_i} > z_0 \ , \tag{5.7.2}$$

for some z_0 to be specified shortly.[18] The quantity z is the equivalent of the charge-to-mass ratio for the instanton, where the charge is given by $n/f_{\rm ax}$ and the mass corresponds to S. When working with black holes an object satisfying $z > z_0$ is referred to as superextremal, while a black hole with $z < z_0$ is termed subextremal. It will be useful to extend this nomenclature to the case of instantons. The WGC then requires the existence of superextremal instantons.

To define the WGC for instantons it is hence important to determine z_0. In the black hole case z_0 is the charge-to-mass ratio of an extremal RN black hole. By analogy, we will define z_0 as the charge-to-mass ratio of an extremal gravitational instanton as suggested in [92]. There is further support for this assertion. In Section 5.2.3 we saw that gravitational instantons in 4d are related to RN black holes in 5d. More specifically, the relation $C = \ell^2(M^2 - Q^2)$ implies that extremal black holes ($M^2 = Q^2$) are in one-to-one correspondence with extremal instantons ($C = 0$). It is thus plausible that extremal instantons play the role of extremal black holes in the WGC. Using our expression (5.4.7) for the action of an extremal gravitational instanton we find

$$z_0 = \frac{n/f_{\rm ax}}{S_{\rm extremal}} = \frac{\alpha}{2}. \tag{5.7.3}$$

Let us now compute the charge-to-mass ratio z for cored gravitational instantons and Euclidean wormholes to see how they fit into this picture. We begin with cored gravitational instantons. For fixed $n/f_{\rm ax}$ we have

$$S_{\rm cored}(\alpha) \geq \begin{cases} S_{\rm extremal}(\alpha) & \text{for } \alpha \geq 2\sqrt{2/3} \\ S_{\rm extremal}(2\sqrt{2/3}) & \text{for } \alpha < 2\sqrt{2/3}, \end{cases} \tag{5.7.4}$$

and thus

$$z_{\rm cored} \leq \begin{cases} z_0 & \text{for } \alpha \geq 2\sqrt{2/3} \\ \frac{2\sqrt{2/3}}{\alpha} z_0 & \text{for } \alpha < 2\sqrt{2/3} \end{cases} \tag{5.7.5}$$

We can make the following observation. For $\alpha \geq 2\sqrt{2/3}$ cored gravitational instantons are strictly subextremal and do not satisfy the WGC condition $z > z_0$. They hence play a role

[18] The WGC can be made more precise by adding further attributes to the condition $z > z_0$ [20]. A more careful definition becomes important when several U(1) group factors (or axion species) are present. See [90; 92; 94; 108; 197; 198] for more details.

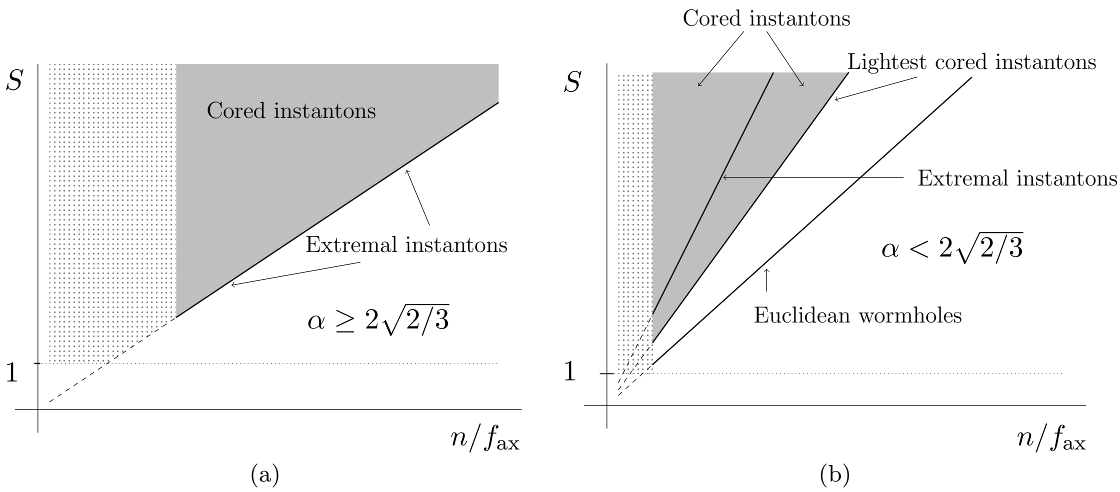


Figure 5.9.: Instanton action S vs. $n/f_{\rm ax}$ for **(a)** $\alpha \geq 2\sqrt{2/3}$ and **(b)** $\alpha < 2\sqrt{2/3}$.

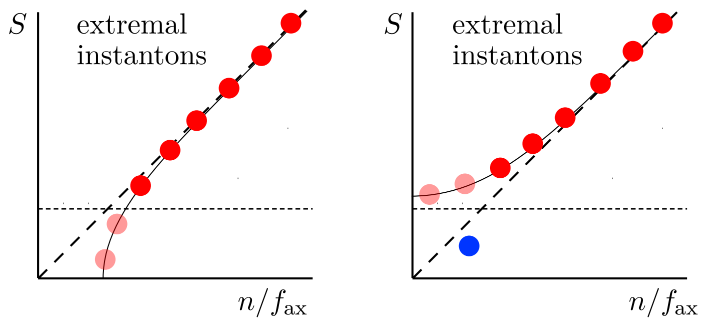


Figure 5.10.: Possible realisations of the WGC for gravitational instantons [264]. Red dots denote extremal gravitational instantons while blue dots correspond to additional superextremal instantons required by the WGC.

akin to subextremal black holes in the WGC for particles. This is consistent with the finding that for $\alpha \geq 2\sqrt{2/3}$ cored gravitational instantons are related to subextremal black holes in higher dimensions (see [110] and Section 5.2.3). The situation is different for $\alpha < 2\sqrt{2/3}$. The lightest cored instantons are now superextremal. We illustrate our findings in Figure 5.9.

Next, let us turn to Euclidean wormholes. From (5.4.19) we find

$$z_{\rm wh} = \frac{n/f_{\rm ax}}{S_{\rm wh}} = \frac{\alpha}{2\sin\left(\frac{\alpha\pi}{4}\sqrt{\frac{3}{2}}\right)} > \frac{\alpha}{2} = z_0 \tag{5.7.6}$$

for $0 \leq \alpha < 2\sqrt{2/3}$, which is the allowed range for wormhole solutions. We find that Euclidean wormholes are strictly superextremal. In addition, one can also show that $z_{\rm wh} > z_{\rm cored}$. This is displayed in Figure 5.9 (b).

What can one learn from these results about the WGC? We will discuss this question for the two cases $\alpha \geq 2\sqrt{2/3}$ and $\alpha < 2\sqrt{2/3}$ separately.

For $\alpha \geq 2\sqrt{2/3}$ the spectrum of gravitational instantons does not contain any superextremal objects that could satisfy the WGC. This is not surprising. Our analysis is restricted to macroscopic gravitational instantons, while it is expected that microscopic physics is responsible for satisfying the WGC. If the WGC is true, it could be realised in two different ways which are shown in Figure 5.10. For one, extremal gravitational instantons (red dots in Figure 5.10) could satisfy the WGC on their own. This occurs if quantum corrections decrease the instanton action for small n such that they naively become superextremal (LHS of Figure 5.10). If this is not the case (see RHS of Figure 5.10) the WGC requires the existence of additional superextremal instantons (blue dot). At the moment it is not clear which implementation of the WGC, if any, is realised. Unfortunately, our analysis is not suitable for resolving this issue.

Let us move on to $\alpha < 2\sqrt{2/3}$. Interestingly, the set of gravitational instanton solutions now contains superextremal objects in the form of wormholes and cored instantons. It thus seems that the WGC is satisfied in Einstein-axion-dilaton systems in virtue of cored instantons and Euclidean wormholes. Note that this is different to the situations shown in Figure 5.10. Here the WGC would be satisfied by an infinite tower of superextremal macroscopic objects.

Another interpretation of our findings is that the statement of the WGC has to be modified for $\alpha < 2\sqrt{2/3}$. In this regime the 'lightest' macroscopic object with given charge $n/f_{\rm ax}$ is not the extremal instanton but the wormhole. Also the correspondence between extremal instantons and extremal black holes in higher dimensions is lost for $\alpha < 2\sqrt{2/3}$. This may imply that the WGC condition is now set by the charge-to-mass ratio of the wormhole rather than that of the extremal instanton. To satisfy the WGC one would then require the existence of states with $z > z_{\rm wh}$. We leave further investigations on this topic for future work.

Last, there may be further implications for gravitational instantons if the WGC for axions is true: gravitational instantons may not be 'stable' in the following sense. To be specific, consider a cored instanton with action S and axion charge n in a theory with $\alpha > 2\sqrt{2/3}$. This corresponds to a tunnelling process between two configurations differing by n units of axion charge. Let us then assume that the WGC is true and implies the existence of instantons with charge-to-mass ratio $z > z_0$, where z_0 is the charge-to-mass of an extremal gravitational instanton as before. An immediate consequence is that a tunnelling process will then always be dominated by the instantons predicted by the WGC. For our example this works as follows. The instantons needed to satisfy the WGC have $z > z_0 \geq z_{\rm cored}$. Let two such instantons have n_1, S_1 and n_2, S_2, such that $n_1 + n_2 = n$. Since $z > z_{\rm cored}$ it follows that $S_1 + S_2 < S$ and tunnelling via two such instantons will dominate over tunnelling via the cored instanton.[19] Hence, we do not expect 'unstable' gravitational instantons to be relevant in the path integral computation of the instanton potential as the major contributions should arise from the instantons satisfying the WGC. We leave a more rigorous analysis of this issue for future work.

5.8. Conclusions and Outlook

It is of great interest to understand whether quantum gravity forbids periodic scalars with large field range and flat potential. The obvious way in which this can happen is via instanton-induced corrections. In detail, there are two specific options: On the one hand, quantum gravity may demand the presence of instantons with certain actions and charges, via a generalized weak gravity conjecture. This is rather indirect: One tries to show that certain things 'go wrong' unless the relevant particles (or instantons) exist.

[19] Note that this is equivalent to the statement that (sub-)extremal black holes can in principle decay if the WGC for particles holds.

There is, however, also a more direct approach: gravity itself supplies, in a rather direct or 'constructive' way the instantons which may lift the flat potential. In this chapter, we have tried to push this direct approach as far as possible, striving also for maximal model-independence.

Our results are as follows. We observe that in a pure axion-gravity system a potential for the axion is generated by Giddings-Strominger wormholes and that this potential is parametrically unsuppressed if the cutoff is at the Planck scale. Trying to be more precise about this, we encountered a surprise: If, as a model of high-cutoff quantum gravity, we take string theory at self-dual coupling and self-dual compactification radius, we are still left with a purely numerical suppression factor of $\exp(-3\pi^2) \simeq 10^{-13}$. Such a result makes it hard to hope for a strong constraint on inflation, even after further refining the analysis.

Furthermore, we continued to ask for generic 4d constraints, but assuming more concretely that the 4d theory arises from string theory with a potentially low moduli scale. First, we found that in this setting nothing too dramatic happens to gravitational instantons: One linear combination of the moduli acts as a 4d dilaton governing the axion coupling; the instantons become more diverse in that extremal and cored gravitational instantons exist in addition to wormholes; the calculation still breaks down only at the Kaluza-Klein scale, which can of course still be high.

Unfortunately, the predictions now become model dependent as the coupling strength of the 4d dilaton to the axion (an $\mathcal{O}(1)$ numerical factor) enters. Taking the highest value for this factor that we could obtain in the simplest models results in a less severe instanton suppression factor of $\exp(-2\pi) \simeq 10^{-3}$. This is of course highly relevant for inflation, but easily avoided by considering models with different dilaton coupling.

In both of the above approaches, the suppression factors start out small and further fall as $\exp(-r^2)$, with r an appropriately normalised compactification radius in 4d Planck units. As a result, while we do believe that gravitational instantons are the most fundamental and model-independent way to constrain field ranges, the numbers appear to allow for enough room for realistic large-field inflation.

Finally, we have attempted to connect our analysis of the various types of gravitational instantons, including their dependence on the axion-dilaton coupling, to the ongoing discussion of the weak gravity conjecture. In particular, we found a intriguing regime where wormholes are the objects with highest charge-to-mass ratio and may thus be sufficient to satisfy the instanton-axion weak gravity conjecture.

There are many directions for further investigations. By limiting our analysis to gravitational instantons in 4-dimensional Einstein-axion-dilaton theories we were unable to arrive at strong constraints on inflation. While this approach allows us to remain ignorant about the detailed UV completion, we are forced to neglect potentially more important contributions. These would arise from gravitational instantons with low instanton numbers, which are incalculable in the 4-dimensional Einstein-axion-dilaton theory. However, a quantitative analysis may be possible if one assumes that UV physics is described by string theory. It is expected that gravitational instantons will correspond to non-perturbative effects such as D-brane instantons in string theory. To arrive at stronger constraints a better understanding of non-perturbative effects in string theory is desirable. In particular, it is expected that poorly understood non-BPS instantons may become important during inflation [108].

There is a related question that is worthy of further examination. While more important instanton contributions to the axion potential may exist, it is possible that the overall effect on the axion potential vanishes once all such contributions are included. To calculate contributions from 'more important' instantons is equivalent to studying instantons in a theory at a higher energy scale. However, taking string theory as our UV completion, we would expect the theory to become supersymmetric and/or higher-dimensional at some scale. It is then possible that, once we work above the supersymmetry scale, there are cancellations between the various instanton contributions to the axion potential. This is somewhat analogous to the cancellation between boson and fermion loops in supersymmetric field theory. We regard it as important to determine whether such cancellations can occur.

While we were unable to arrive at strong model-independent constraints on inflation, gravitational instantons may be important for inflation in models where the dilaton coupling takes sufficiently large values. In the effective 4-dimensional theory the dilaton coupling is a free parameter. However, one would expect that its value is constrained by possible UV completions. Indeed, by considering simple axion-dilaton systems in string compactifications, we find that the dilaton coupling typically takes $\mathcal{O}(1)$ values, i.e. it can neither be very small nor very large. It would be interesting to examine to what extent these results are generic.

The upshot of these points is clear: It is imperative to understand the ultraviolet end of the instanton spectrum.

CHAPTER 6

Conclusions and Outlook

In this thesis we explored the string landscape by investigating models of reheating and large-field inflation. Even though predictions from string theory may seem to be arbitrary due to the huge number of type IIB string vacua, we demonstrated that string vacua can have common features, and that by far not any cosmological scenario can be embedded in string theory. In this context, we also investigated whether gravitational instanton effects are relevant with respect to the question whether axion inflation resides in the landscape or swampland. We go through the chapters of the thesis step by step and summarise our conclusions and glimpse at possible future work.

Our exploration of the string landscape begins in Chapter 2 by looking at reheating models in a specific corner of the type IIB landscape with Kähler moduli being stabilised within the Large Volume Scenario (LVS). It provides a first example for a generic prediction from a substantial subset of the landscape. A generic feature of the LVS is an essentially massless axion a_b, which is the supersymmetric partner to the bulk 4-cycle modulus τ_b. Hence, in simple "sequestered" models of reheating, where the Standard Model (SM) sector is realised by a D3-brane at a singularity of the Calabi-Yau manifold, τ_b does not only decay into the SM sector, dominantly through the decay into Higgs particles, but there is also the decay channel into axions a_b. Phenomenologically, these axions contribute as dark radiation to the effective number of relativistic species N_{eff}, thus implying $\Delta N_{\text{eff}} \equiv N_{\text{eff}} - 3.046 > 0$, where the value $N_{\text{eff}} = 3.046$ is the prediction by the SM. For instance, a sequestered model with high-scale supersymmetry and two Higgs doublets predicts $\Delta N_{\text{eff}} \simeq 1.4$, which is disfavoured by recent measurements $N_{\text{eff}} = 3.15 \pm 0.23$ (1σ; Planck+TT+lowP+BAO) [1] already at 5.6σ. One option is to include many more Higgs doublets or to dial a large Giudice-Masiero coupling z. However, it is not yet fully understood how large this coupling parameter z can be chosen in string theory. Hence, we aimed for more flexibility in ameliorating the dark radiation abundance without relying on this Higgs sector. We proposed to go beyond the sequestered LVS by realising the visible sector via D7-branes wrapping stabilised 4-cycles. This opens a decay channel of τ_b into SM gauge bosons. By some fine-tuning of a parameter related to the size of the 4-cycle, one can enhance this decay channel. Without any such tuning, our model with the 4-cycle stabilised by D-terms, would predict $\Delta N_{\text{eff}} = 0.48$ (ignoring the Higgs sector and assuming sufficiently high reheating temperatures), which deviates by 1.6σ from the central value measured by the Planck collaboration. Obtaining the central value itself requires a fine-tuning of about one part in two. Similarly, we considered also models in which the visible sector is stabilised by string loop corrections. This method yields slightly larger values of ΔN_{eff}. Furthermore, it is possible to stabilise the visible sector by non-perturbative corrections, but such constructions typically yield too large values of ΔN_{eff}. In addition, we also commented on a sequestered model including flavour branes, intersecting the singularity at which the visible sector is located. Thereby, the predictions for ΔN_{eff} can also be lowered to some extent. Hence, we find that the Large Volume Scenario can still be consistent with present data on N_{eff}. However, if ΔN_{eff} continues to decrease, LVS models will be more constrained or even be ruled out. Clearly, more progress can be made by investigating the following directions: it would be desirable to improve our understanding of the decay channel into the Higgs sector. Especially, it is important to understand which values of z are possible in string theory. In addition, it is unclear how much fine-tuning for the enhancement of the decay channel into SM gauge bosons can be realised in our non-sequestered LVS models. Further ways to reduce the dark radiation abundance, such as in [150] are conceivable and should be investigated in the future as well.

For the purpose of model building, it is not only of interest which vacua are consistent with present observations, but also which bottom-up effective field theories (EFTs) are consistent with quantum gravity, and, for the purpose of our thesis, with string theory in particular. Therefore, it is important to delineate the string landscape surrounded by a swampland of EFTs, admitting no UV-completion consistent with string theory. In particular, it is interesting to see whether large-field inflation resides in the swampland or the landscape.

We address this topic in three distinct chapters. In Chapter 3 we proposed a fine-tuned model of F-term axion monodromy inflation in string theory. The idea is to use D7-brane moduli or complex structure moduli of Calabi-Yau compactifications as an inflaton candidate. Crucially, such a complex structure modulus has to be stabilised in the large complex structure (LCS) limit so that in absence of fluxes there is no potential for the axionic part of the modulus due to shift symmetry. The inflaton potential is then generated by turning on fluxes appropriately so that the inflaton modulus occurs linearly in the superpotential. We showed that not only the coefficient of the shift symmetry breaking term has to be tuned small, but also its derivatives with respect to all moduli entering this coefficient. This ensures that the inflaton mass is small and that backreaction of other moduli is under control. We demonstrated that such a fine-tuning cannot be realised in orientifolded type IIB CY compactifications, at least if the inflaton descends from a complex structure modulus. Our no-go theorem has, however, loopholes and the most obvious one is to consider CY four-folds in the context of F-theory (alternatively, one can think of the inflaton as a D7-brane modulus in type IIB). In this case, the fine-tuning seems to be possible, although this needs to be checked in future work upon choosing concrete CY 4-folds. Accepting the fine-tuning as a necessary ingredient of F-term axion monodromy inflation, one can show that the backreaction of the other moduli reduces the naively expected inflaton mass by some amount. Thanks to the fine-tuning, a large enough field space for quadratic inflation can be achieved. Unfortunately, the required fine-tunings can in principle be very constraining. In the context of the string landscape, we quantified how severe the realisation of the tunings finally is. In a case study, we showed that the huge number of 10^{1700} F-theory vacua is decimated to a landscape of about 10^{300} vacua consistent with the tunings if only 10% of the 4-fold complex structure moduli couple to the inflaton modulus. This remaining landscape is expected to be large enough to accommodate further tunings for different contexts such as in the case of the cosmological constant or Standard Model parameters. If, however, many more complex structure moduli couple to the inflaton modulus such that no landscape remains after imposing the tunings, we conclude that our inflation model could not be realised in F-/type IIB theory. The number of required tuning conditions can only be determined in a very concrete setting. Nevertheless, we were able to put forward a viable model of F-term axion monodromy inflation in string theory, which seems to allow for large-field inflation, at least in principle. As we pointed out, a realisation of our proposed fine-tuned scenario in a concrete geometric setting would be desirable. Moreover, F-term axion monodromy inflation would clearly be more appealing without this potentially large amount of fine-tuning. Hence, combining our approach with [174–176], where corrections from the DBI-term are considered, could be fruitful. These could lead to further flattening of the potential and hence might relax the tuning conditions. Concerning potential conflicts with quantum gravity arguments, the current status is that axion monodromy inflation does not face problematic constraints [93; 106; 107], although obtaining parametrically large field ranges in string theory remains challenging (see e.g. [265] for a recent analysis of axion monodromy inflation; due to backreaction the physical field range is reduced, although appropriate tunings could delay this effect). Hence, F-term axion monodromy inflation demonstrates a realisation of large-field inflation in string theory, although at the expense of accepting large amount of fine-tuning. However, in case axion monodromy inflation turns out to be more constrained by quantum gravity or string theory than currently understood, it would be interesting to learn where exactly our construction could fail.

In Chapter 4 we proposed a realisation of alignment inflation in type IIB string theory. Our construction, called "F-term winding inflation", is based on a flux choice such that a combination $(u - Nv)$ occurs in the superpotential, where u and v are complex structure moduli in the LCS limit. Moreover, $N \gg 1$ is a combination of flux numbers, which needs to be chosen large. We showed that there is a flat direction, which is a winding trajectory in field space. The inflaton potential is generated by including non-perturbative corrections $e^{2\pi i v}$. As a result, one obtains the potential of natural inflation and the effective axion decay constant is enhanced proportional to N. In addition, we also showed consistency of our model with Kähler moduli stabilisation à la LVS. Furthermore, our proposal realises the loophole [100; 101] of certain no-go results for alignment inflation based on the mild version of the Weak Gravity Conjecture. More specifically, the instanton satisfying the WGC corresponds to $e^{2\pi i u}$. The mild WGC allows this instanton to be more massive than the one generating the inflaton potential. In this way, dangerous corrections to the potential are avoided. If, however, the strong WGC holds true, F-term winding inflation cannot be realised. Consequently, a better understanding of the WGC is crucial in order to answer the question whether F-term winding inflation resides in the swampland or not. If our model turns out to be censored by stringent quantum gravity constraints, it would be, once again, important to understand where our construction goes wrong. In fact, one can be concerned about a couple of technicalities our model is based on. For instance, it would be desirable to investigate whether the choice of large N might result in a conflict with the D3-tadpole cancellation condition. A constraint on N translates directly into an upper bound on the effective field range. It is also conceivable that in concrete settings the required hierarchy $\text{Im}(v) \ll \text{Im}(u)$ might finally not be feasible. Furthermore, taking into account non-perturbative corrections Ae^{-aT} for Kähler moduli, will modify the inflaton dynamics due to the dependency of A on the complex structure moduli. Hence, in future work it would be interesting to analyse our model in view of these corrections along the lines of [266].

Finally, we addressed the question of model-independent quantum gravity constraints on axion inflation due to gravitational instanton effects. If strong enough they would constrain e.g. "F-term winding inflation" [108]. In Chapter 5 we addressed several issues related to these gravitational instantons. First of all, we demonstrated that Giddings-Strominger wormholes induce corrections to the axion potential of the type $\cos\theta e^{-S}$, even though no charges disappear globally. However, locally shift symmetry appears to be broken. We support this result by deducing the effective potential from the partition function of a wormhole gas in a field theory model and a quantum mechanical toy-model. We addressed a subtlety in this computation [257; 261]: if the connections of instantons and anti-instantons by a wormhole tube are taken into account explicitly, the combinatorial factors in the partition function are modified such that there occurs a divergence of the energy density as the volume of the (sub-)manifold goes to infinity. In particular, the dilute gas approximation, which is used in such instanton computations, breaks down. We do not know whether the resolution of this problem may lie in the choice of different combinatorial factors (e.g. ignoring the pairing of instantons and anti-instantons) or in a more refined treatment of the path integral and the interpretation of these wormholes. However, independently of this problem, we always obtained corrections to the axion potential of the form $\cos\theta e^{-S}$. As to the question of constraints on axion inflation, we analysed Einstein-axion-dilaton systems, because they naturally arise in string compactifications. In such systems, three types of gravitational instantons occur: Euclidean wormholes, extremal and cored gravitational instantons. We found that the presence of a gravitational instanton does not threaten the stabilisation of the corresponding dilaton. Hence, the effective description of gravitational instantons can be trusted up to the Kaluza-Klein (KK) scale. To maximise the effects of gravitational instantons, we pushed the KK scale as close to the Planck scale as possible. Specifically, we evaluated the instanton corrections using a cutoff obtained from string theory at the self dual point (i.e. self-dual coupling and self-dual compactification radius). Model-independent constraints are derived from the presence of Giddings-Strominger (GS) wormholes, which can be obtained by integrating out the dilaton. We found that $e^{-S} < 10^{-13}$ for these GS wormholes. Hence, their effects are not dangerous for inflation. For the same reason, there is also no danger if Euclidean wormholes should be interpreted as bounce solutions rather than instantons [238; 239] (see also recent comments in [267]). For the two other types of gravitational instantons, it is unfortunately difficult to formulate model-independent constraints. The size of the corrections to the axion potential depends on the dilaton coupling, which is clearly model-dependent. Therefore, we find it overall hard to constrain axion inflation using gravitational instantons. However, more issues related to gravitational instantons need to be resolved. For instance, our approach is not suitable to calculate effects with low instanton flux numbers. Such a treatment needs to be done in a UV-complete theory such as string theory, where gravitational instantons should correspond to D-brane instantons. Clearly, these problems are interesting, and hence, should be addressed in future work. Finally, we investigated the relations between the Weak Gravity Conjecture and gravitational instantons. We observe that the wormhole instantons are the objects satisfying the WGC for instantons/axions. Moreover, since the extremal and cored gravitational instantons have an interpretation in terms of five-dimensional Reissner-Nordström black holes, the application of the WGC in the 5d theory might hint at an instability of extremal and cored instantons in 4d. We leave a detailed elaboration on these observations for future research projects.

In summary, we learned that some corners of the string landscape can lead to testable predictions, such as the prediction of dark radiation. By cosmological measurements, one can then constrain or rule out models in those parts of the landscape. We learned that the LVS construction is indeed constrained due to dark radiation production, albeit not ruled out. Concerning inflation, our conclusion is that large-field inflation, e.g. via F-term axion monodromy inflation or F-term winding inflation, can in principle be realised in string theory, at least to our current knowledge. However, we observed that the construction of large-field inflation in string theory often goes along with many challenges and obstructions, which need to be overcome. Apart from technical details of our models, which need to be clarified, there are also general quantum gravity arguments, which could censor our models. The direct approach by calculating gravitational instanton effects turned out to be not suitable for constraining large-field inflation. Hence, a better understanding of the Weak Gravity Conjecture, which might have more to say about inflation, remains an important goal for future research in string cosmology.

APPENDIX A

Mathematical Appendix

This appendix gives an overview of selected concepts, methods and formulae of Riemannian and complex geometry, which we frequently employ in this thesis, without the intention of being pedagogical or even complete. The only aim is to collect all important formulae and concepts at one place (without giving proofs), which hopefully improves the readability of this thesis. For a much more detailed or pedagogical introduction to mathematical aspects of topology, geometry and complex manifolds, see e.g. [15; 17; 18; 46; 268–271], which are the sources of this appendix.

A.1. Differential Forms on Riemannian/Lorentzian Manifolds: Notation and Useful Identities

Definition A.1 (p-form)**.** An antisymmetric $(0, p)$-tensor field A on a differentiable manifold is called p-form.

In what follows $\Omega^p(\mathbf{X})$ denotes the vector space of p-forms on a d-dimensional Riemannian or Lorentzian manifold $\mathbf{X}$ at a given point on this manifold.

Definition A.2 (wedge-product)**.** Let $A \in \Omega^p(\mathbf{X})$ and $B \in \Omega^q(\mathbf{X})$. The wedge-product $\wedge$ yields a $(p+q)$-form $A \wedge B$ as follows:

$$(A \wedge B)_{i_1 \dots i_p j_1 \dots j_q} \equiv \frac{(p+q)!}{p!q!} A_{[i_1 \dots i_p} B_{j_1 \dots j_q]} \ , \tag{A.1.1}$$

with anti-symmetrisation [ ].

Lemma A.3. The wedge-product obeys the following rules:

1. Let $A \in \Omega^p(\mathbf{X})$ and $B \in \Omega^q(\mathbf{X})$. Then

$$A \wedge B = (-1)^{pq} B \wedge A \ . \tag{A.1.2}$$

In particular it holds $A \wedge A = 0$ if p is odd.

2. Let C be another differential form, then

$$(A \wedge B) \wedge C = A \wedge (B \wedge C) \ . \tag{A.1.3}$$

Definition A.4 (Exterior derivative)**.** Let $A \in \Omega^p(\mathbf{X})$, then in a chosen coordinate basis the exterior derivative is defined as

$$(dA)_{i_1 \dots i_{p+1}} \equiv (p+1)\partial_{[i_1} A_{i_2 \dots i_{p+1}]} \ . \tag{A.1.4}$$

Lemma A.5. The exterior derivative obeys the following rules:

1. Let $A \in \Omega^p(\mathbf{X})$. Then:

$$d^2 A = d(dA) = 0 \ . \tag{A.1.5}$$

2. Let B be another differential form. Then:

$$d(A \wedge B) = dA \wedge B + (-1)^p A \wedge dB \ . \tag{A.1.6}$$

Definition A.6 (Hodge-star operator)**.** Let $A \in \Omega^p(\mathbf{X})$ and g_{ab} be a metric on $\mathbf{X}$. Then, the Hodge dual to this form is a $(d-p)$-form $\star A$:

$$(\star A)_{a_1 \dots a_{d-p}} \equiv \frac{1}{p!} \epsilon_{a_1 \dots a_{d-p} b_1 \dots b_p} A^{b_1 \dots b_p} \ , \tag{A.1.7}$$

where the volume form $\epsilon_{i_1 \dots i_n}$ is defined by

$$\epsilon_{i_1 \dots i_n} \equiv \sqrt{|g|} \cdot \mathrm{sign}(i_1 \dots i_n) \tag{A.1.8}$$

and g is the determinant of the metric.

Corollary A.7. From this definition it follows:

$$\star(\star A) = \pm(-1)^{p(d-p)} A \ , \tag{A.1.9}$$

$$(\star d \star A)_{a_1 \dots a_{p-1}} = \pm(-1)^{p(d-p)} \nabla^b A_{a_1 \dots a_{p-1} b} \ , \tag{A.1.10}$$

where ∇ denotes the covariant derivative on $\mathbf{X}$. For Riemannian (Lorentzian) manifolds '+' ('−') has to be chosen.

A useful formula to prove these identities is:

$$\epsilon^{a_1 \dots a_p c_{p+1} \dots c_d} \epsilon_{b_1 \dots b_p c_{p+1} \dots c_d} = \pm p!(d-p)! \delta^{a_1}_{[b_1} \dots \delta^{a_p}_{b_p]} \ . \tag{A.1.11}$$

Theorem A.8 (Stokes)**.** For a $(p-1)$-form A and a p-dimensional oriented manifold $\mathbf{X}$ it holds ("Stokes theorem"):

$$\int_{\mathbf{X}} dA = \int_{\partial \mathbf{X}} A \ , \tag{A.1.12}$$

where $\partial \mathbf{X}$ is the boundary of $\mathbf{X}$. Here, integration of a p-form B over $\mathbf{X}$ is defined by $\int_{\mathbf{X}} B \equiv \int dx^1 \dots dx^p B_{1 \dots p}$. For more details, see e.g. [268; 269].

Definition A.9 (Inner product)**.** Let $A, B \in \Omega^p(\mathbf{X})$. We define the inner product of A and B as:

$$(A, B) \equiv \int_{\mathbf{X}} A \wedge \star B = \frac{1}{p!} \int_{\mathbf{X}} \sqrt{|g|} A_{m_1 \dots m_p} B^{m_1 \dots m_p} dx^1 \dots dx^d \ . \tag{A.1.13}$$

The second equality follows from:

Lemma A.10. Let $A, B \in \Omega^p(\mathbf{X})$. Then,

$$\star A \wedge B = \frac{1}{p!} A_{m_1 \dots m_p} B^{m_1 \dots m_p} \star 1 \ , \tag{A.1.14}$$

where $\star 1 = d^d x \sqrt{|g|}$.

Definition A.11 (Closed and exact p-forms)**.** Let $A \in \Omega^p(\mathbf{X})$. We call A a *closed* p-form iff $dA = 0$. If there is a globally defined $(p-1)$-form B such that $A = dB$ then A is called *exact.* Any exact differential form is closed due to $d^2 = 0$. The converse holds only locally.

Definition A.12 (de Rahm Cohomology group)**.** Let $\mathcal{Z}^p$ be the space of closed p-forms and $\mathcal{B}^p$ the space of exact p-forms, where the coefficients of the forms are real. The p-th de Rahm Cohomology group of $\mathbf{X}$ is then the quotient space of the two sets,

$$H^p(\mathbf{X}, \mathbb{R}) \equiv \frac{\mathcal{Z}^p}{\mathcal{B}^p} \ , \tag{A.1.15}$$

The cohomology class of a closed p-form A is denoted by $[A]$.

Definition A.13 (Betti numbers and Euler characteristic)**.** We call

$$b^p(\mathbf{X}) \equiv \dim H^p(\mathbf{X}, \mathbb{R}) \tag{A.1.16}$$

the Betti number of $\mathbf{X}$. The Euler characteristic is defined by

$$\chi(\mathbf{X}) \equiv \sum_k (-1)^k b^k(\mathbf{X}) \ . \tag{A.1.17}$$

Both are topological invariants.

Definition A.14 (Harmonic forms)**.** We call $A \in \Omega^p(\mathbf{X})$ *harmonic* iff it satisfies

$$\Delta_p A = 0, \tag{A.1.18}$$

where $\Delta_p \equiv d^\dagger d + d d^\dagger$ with $d^\dagger = (-1)^{dp+d+1} \star d \star$ (and another factor of (-1) is to be included for Lorentzian manifolds). We call $d^\dagger : \Omega^p(\mathbf{X}) \to \Omega^{p-1}(\mathbf{X})$ the *adjoint exterior derivative.* It holds $d^{\dagger 2} = 0$. A is called *coclosed* iff $d^\dagger A = 0$ and *coexact* iff $\exists\ B \in \Omega^{p+1}(\mathbf{X})$ such that $A = d^\dagger B$ globally.

Theorem A.15. Let $\mathbf{X}$ be a compact, orientable manifold without boundaries, i.e. $\partial \mathbf{X} = 0$. For $A \in \Omega^p(\mathbf{X})$ and $B \in \Omega^{p-1}(\mathbf{X})$ it then holds

$$(dB, A) = (B, d^\dagger A) \ . \tag{A.1.19}$$

Corollary A.16. Δ is a positive operator on a compact Riemannian manifold $\mathbf{X}$, i.e.

$$(A, \Delta A) = (dA, dA) + (d^\dagger A, d^\dagger A) \geq 0 \ . \tag{A.1.20}$$

In particular, A is harmonic iff it is both closed and coclosed.

Theorem A.17 (Hodge decomposition theorem)**.** Let $\mathbf{X}$ be a compact, orientable Riemannian manifold without boundaries. Then, $\Omega^p(\mathbf{X})$ can be uniquely decomposed:

$$\Omega^p(\mathbf{X}) = d\Omega^{p-1}(\mathbf{X}) \oplus d^\dagger \Omega^{p+1}(\mathbf{X}) \oplus \mathrm{Harm}^p(\mathbf{X}) \ , \tag{A.1.21}$$

where $\mathrm{Harm}^p(\mathbf{X})$ denotes the space of harmonic p-forms on $\mathbf{X}$.

Hence, any p-form $\mathcal{D}_p \in \Omega^p(\mathbf{X})$ can always be rewritten as

$$\mathcal{D}_p = dA_{p-1} + d^\dagger B_{p+1} + C_p \tag{A.1.22}$$

with $A_{p-1} \in \Omega^{p-1}(\mathbf{X})$, $B_{p+1} \in \Omega^{p+1}(\mathbf{X})$ and $C_p \in \mathrm{Harm}^p(\mathbf{X})$.

A.2. Some Facts about Chains and Cycles

Introduction A.18. Chains are in some sense "dual" objects to p-forms. Conceptually, a p-chain a_p is a linear combination of p-dimensional submanifolds N_k^p of a manifold $\mathbf{X}$ with real coefficients: $a_p \equiv c_k N_k^p$, $c_k \in \mathbb{R}$ [18]. For an exact definition via simplices, see e.g. [268; 270]. We refrain from reproducing a rigorous introduction to the concept of p-chains, because those details are not important for the thesis itself.

In analogy to the exterior derivative operator d one can define a boundary operator ∂_p acting on a p-chain, such that the result is a $(p-1)$-chain. Moreover, in analogy to d^2 we have $\partial^2 \equiv 0$.

Using the boundary operator one can define p-cycles and p-boundaries by the same logic that leads to closed and exact p-forms.

Definition A.19 (p-cycles and p-boundaries)**.** A p-chain a_p is called p-cycle if and only if

$$\partial a_p = 0 \ . \tag{A.2.1}$$

If there exists a $(p+1)$-chain b_{p+1} such that

$$a_p = \partial b_{b+1} \tag{A.2.2}$$

then a_p is called p-boundary (or trivial).

Clearly, every p-boundary is a p-cycle, but the converse is in general false (e.g. for topologically non-trivial manifolds).

Definition A.20 (Homology groups)**.** Let $\mathcal{Z}_p$ be the space of p-cycles and $\mathcal{B}_p$ the space of trivial p-cycles. The pth homology group of $\mathbf{X}$ is then the quotient space of the two sets,

$$H_p\left(\mathbf{X}, \mathbb{R}\right) \equiv \frac{\mathcal{Z}_p}{\mathcal{B}_p} \ , \tag{A.2.3}$$

The homology class of a_p is denoted by $[a_p]$. The homology class defines a vector space. Its dimension is a Betti number,

$$b_p(\mathbf{X}) \equiv \dim H_p\left(\mathbf{X}, \mathbb{R}\right) \ . \tag{A.2.4}$$

Proposition A.21 (De Rahm duality)**.** We have $(H^p(\mathbf{X}, \mathbb{R}))^* = H_p(\mathbf{X}, \mathbb{R})$, i.e. $H_p(\mathbf{X}, \mathbb{R})$ is the dual space to $H^p(\mathbf{X}, \mathbb{R})$. This follows from the fact that the integration of a p-form over a p-cycle is a bilinear map $\mathcal{I} : H^p(\mathbf{X}, \mathbb{R}) \times H_p(\mathbf{X}, \mathbb{R}) \to \mathbb{R}$.

Corollary A.22. The de Rahm duality implies $b^p(\mathbf{X}) = b_p(\mathbf{X})$.

Remark A.23. Due to the de Rahm duality it is always possible to choose bases of cycles $\{(a_p)_i\}$ and forms $\{(A_p)_i\}$, $i = 1, ..., b_p$, such that

$$\int_{[(a_p)_i]} [(A_p)_j] = \delta_{ij} \ . \tag{A.2.5}$$

Proposition A.24 (Poincare duality)**.** We have the relation $H^p(\mathbf{X}, \mathbb{R}) \cong H_{d-p}(\mathbf{X}, \mathbb{R})$. This follows because $\int_{\mathbf{X}} A_p \wedge B_{d-p}$ defines a map $\mathcal{I} : H^p(\mathbf{X}, \mathbb{R}) \times H^{d-p}(\mathbf{X}, \mathbb{R}) \to \mathbb{R}$, thus $H^p(\mathbf{X}, \mathbb{R}) = \left(H^{d-p}(\mathbf{X}, \mathbb{R})\right)^*$. Using de Rahm duality the claim follows.

Corollary A.25. It holds $b_p(\mathbf{X}) = b_{d-p}(\mathbf{X})$.

Remark A.26 (Torsional homologies)**.** Note that one can also consider cycles of $H_p(\mathbf{X}, \mathbb{Z})$. This allows us to have so-called *torsional cycles*. They come about as follows: Let c_{p+1} be a $(p+1)$-chain with integer coefficients. Then, a p-cycle $z_p \in \mathcal{Z}_p(\mathbf{X}, \mathbb{Z})$ satisfying $z_p = \frac{1}{k}\partial c_{p+1}$ is non-trivial in $H_p(\mathbf{X}, \mathbb{Z})$, i.e. $z_p \notin \mathcal{B}_p(\mathbf{X}, \mathbb{Z})$, while kz_p is trivial in $H_p(\mathbf{X}, \mathbb{Z})$ (and of course also trivial in $H_p(\mathbf{X}, \mathbb{R})$) for $k \in \mathbb{Z}$. Such z_p is called a *k-torsion cycle*.

One can show that to the above introduced torsion cycle z_p one can associate a non-closed p-form $A_p \in H^p(\mathbf{X}, \mathbb{Z})$ such that $dA_p = kB_{p+1}$: Given a k-torsion cycle z_p such that $z_p = \frac{1}{k}\partial c_{p+1}$, we first normalise the differential forms A_p and B_{p+1} as follows:

$$\int_{z_p} A_p = 1 \ , \qquad \int_{c_{p+1}} B_{p+1} = 1 \ . \tag{A.2.6}$$

But then

$$\int_{c_{p+1}} B_{p+1} = 1 = \int_{z_p} A_p = \int_{\frac{1}{k}\partial c_{p+1}} A_p = \frac{1}{k}\int_{\partial c_{p+1}} A_p = \frac{1}{k}\int_{c_{p+1}} dA_p \ .$$

Hence, $dA_p = kB_{p+1}$.

Such torsion cycles were used in [154] to propose axion monodromy inflation with F-terms.

A.3. Facts about Complex Manifolds

Introduction A.27. Complex manifolds are defined in analogy to real manifolds. In brief, a complex manifold of dimension n is a $2n$-dimensional real manifold with coordinate charts $\phi_\alpha : \mathcal{O}_\alpha \to \mathcal{U}_\alpha \subset \mathbb{C}^n$ (open subset) and transition functions being holomorphic [32]. For more details see e.g. [268; 272], on which the following is based.

Remark A.28 (Relation between real and complex manifolds)**.** Every n-dimensional complex manifold can be identified with a $2n$-dimensional real manifold. The converse statement is not always true. However, if the real manifold admits a tensor $J^m_{\ n}$ such that

$$J_m^{\ n} J_n^{\ p} = -\delta_m^{\ p} \tag{A.3.1}$$

$$N^p_{\ mn} \equiv J_m^{\ q}\partial_{[q} J_{\ n]}^{p} - J_n^{\ q}\partial_{[q} J_{\ m]}^{p} = 0 \tag{A.3.2}$$

then a $2n$-dimensional real manifold can be identified with a n-dimensional complex manifold, too.

Notation A.29 (Notation of tangent spaces and differential forms)**.** Let $\mathbf{X}$ be a complex manifold of dimension $n = 2d$, then the tangent space $T_p\mathbf{X}^{\mathbb{C}}$ at $p \in \mathbf{X}$ is spanned by

$$T_p\mathbf{X}^{\mathbb{C}} = \left\{ \left.\frac{\partial}{\partial z^1}\right|_p, ..., \left.\frac{\partial}{\partial z^d}\right|_p, \left.\frac{\partial}{\partial \bar{z}^1}\right|_p, ..., \left.\frac{\partial}{\partial \bar{z}^d}\right|_p \right\} \ . \tag{A.3.3}$$

Analogously, the cotangent space of $\mathbf{X}$ at $p \in \mathbf{X}$ is spanned by

$$T^*_p\mathbf{X}^{\mathbb{C}} = \left\{ \left. dz^1\right|_p, ..., \left. dz^d\right|_p, \left. d\bar{z}^1\right|_p, ..., \left. d\bar{z}^d\right|_p \right\} \ . \tag{A.3.4}$$

In fact one can decompose these spaces into the direct sum of two real tangent spaces in the following sense:

$$T_p\mathbf{X}^{\mathbb{C}} = T_p\mathbf{X}^{(1,0)} \oplus T_p\mathbf{X}^{(0,1)} \ , \tag{A.3.5}$$

$$T^*_p\mathbf{X}^{\mathbb{C}} = T^*_p\mathbf{X}^{(1,0)} \oplus T^*_p\mathbf{X}^{(0,1)} \ . \tag{A.3.6}$$

Furthermore, the notion of differential forms on complex manifolds is a straightforward extension of differential forms on real manifolds. Due to the above structure of the tangent space it is, however, possible to separate a p-form into r-holomorphic and s-anti-holomorphic parts, with $p = r + s$, as follows. Let A be a p-form on $\mathbf{X}$, then we write $A \in \Omega^{(r,s)}(\mathbf{X}) \equiv \wedge^r T_p^* \mathbf{X}^{(1,0)} \otimes \wedge^s T_p^* \mathbf{X}^{(0,1)}$ and

$$A = A_{i_1 \dots i_r, \bar{\jmath}_1 \dots \bar{\jmath}_s} dz^{i_1} \wedge \dots \wedge dz^{i_r} \wedge d\bar{z}^{\bar{\jmath}_1} \wedge \dots \wedge d\bar{z}^{\bar{\jmath}_s} \ . \tag{A.3.7}$$

The exterior derivative acts on A as follows:

$$\begin{aligned} dA = & \frac{\partial A_{i_1 \dots i_r, \bar{\jmath}_1 \dots \bar{\jmath}_s}}{\partial z^{i_{r+1}}} dz^{i_{r+1}} \wedge dz^{i_1} \wedge \dots \wedge dz^{i_r} \wedge d\bar{z}^{\bar{\jmath}_1} \wedge \dots \wedge d\bar{z}^{\bar{\jmath}_s} \\ & + \frac{\partial A_{i_1 \dots i_r, \bar{\jmath}_1 \dots \bar{\jmath}_s}}{\partial \bar{z}^{\bar{\jmath}_{s+1}}} dz^{i_1} \wedge \dots \wedge dz^{i_r} \wedge d\bar{z}^{\bar{\jmath}_{s+1}} \wedge d\bar{z}^{\bar{\jmath}_1} \wedge \dots \wedge d\bar{z}^{\bar{\jmath}_s} \ . \end{aligned} \tag{A.3.8}$$

We often abbreviate this by $dA = \partial A + \bar{\partial} A$, where ∂ and $\bar{\partial}$ (Dolbeault operators) act as exterior derivatives on the holomorphic and anti-holomorphic part respectively, as in the case of differential geometry on real manifolds. It is easy to see that $d^2 = 0$ requires $\partial^2 = \bar{\partial}^2 = \left\{ \partial, \bar{\partial} \right\} = 0$.

In analogy of Def. A.12 we can define the Dolbeault cohomology:

Definition A.30 (Dolbeault cohomology)**.** Let $\mathbf{X}$ be a complex manifold. Then we define the (r,s)-th Dolbeault cohomology group as

$$H_{\bar{\partial}}^{(r,s)}(\mathbf{X}, \mathbb{C}) = \frac{\mathcal{Z}_{\bar{\partial}}^{r,s}}{\mathcal{B}_{\bar{\partial}}^{r,s}} \ , \tag{A.3.9}$$

where $\mathcal{Z}_{\bar{\partial}}^{r,s}$ and $\mathcal{B}_{\bar{\partial}}^{r,s}$ denote the spaces of $\bar{\partial}$-closed and $\bar{\partial}$-exact (r,s)-forms, respectively. This means that $A \in \mathcal{Z}_{\bar{\partial}}^{r,s}$ implies $\bar{\partial} A = 0$, and if $A \in \mathcal{B}_{\bar{\partial}}^{r,s}$ then there is a $(r, s-1)$-form B such that $A = \bar{\partial} B$.

In the following we drop the $\bar{\partial}$-symbol on $H_{\bar{\partial}}^{(r,s)}$ and only write $H^{(r,s)}$.

Definition A.31 (Metric)**.** Let $\mathbf{X}$ be a complex manifold. Call the map

$$g : T_p \mathbf{X}^{\mathbb{C}} \times T_p \mathbf{X}^{\mathbb{C}} \to \mathbb{C} \tag{A.3.10}$$

metric if the following properties are satisfied: Let $w_1, w_2 \in T_p \mathbf{X}^{\mathbb{C}}$, i.e. we can write $w_1 = r + is$, $w_2 = u + iv$ where $r, s, u, v \in T_p \mathbf{X}$, then we require linearity

$$g(w_1, w_2) = g(r, u) - g(s, v) + i \left[g(r, v) + g(s, u) \right] \ . \tag{A.3.11}$$

The components are defined by $g_{ij} = g\left(\frac{\partial}{\partial z^i}, \frac{\partial}{\partial z^j} \right)$ and $g_{i\bar{\jmath}} = g\left(\frac{\partial}{\partial z^i}, \frac{\partial}{\partial \bar{z}^j} \right)$. Clearly, it holds $g_{ij} = g_{ji}, g_{i\bar{\jmath}} = g_{\bar{\jmath}i}, \overline{g_{ij}} = g_{\bar{\imath}\bar{\jmath}}$ and $\overline{g_{i\bar{\jmath}}} = g_{\bar{\imath}j}$.

Definition A.32 (Hermitian metric)**.** Consider a metric g with the additional restrictions $g_{ij} = 0$ and (hence) $g_{\bar{\imath}\bar{\jmath}} = 0$. Then, g ist called a hermitian metric. One can then write

$$g = g_{i\bar{\jmath}} dz^i \otimes d\bar{z}^{\bar{\jmath}} + g_{\bar{\imath}j} d\bar{z}^{\bar{\imath}} \otimes dz^j \ . \tag{A.3.12}$$

Definition A.33 (Kähler form and Kähler manifolds)**.** Let g be a hermitian metric. Then define the $(1,1)$-form

$$J = i g_{i\bar{\jmath}} dz^i \wedge d\bar{z}^{\bar{\jmath}} \; . \tag{A.3.13}$$

Call J Kähler form iff $dJ = 0$. The corresponding complex manifold $\mathbf{X}$ is then a Kähler manifold.

Corollary A.34. Let g be a hermitian metric on a Kähler manifold. Then it holds

$$\frac{\partial g_{i\bar{\jmath}}}{\partial z^k} = \frac{\partial g_{k\bar{\jmath}}}{\partial z^i} \tag{A.3.14}$$

and there is an analogous relation with z and $\bar{z}$ interchanged.

Corollary A.35 (Kähler Potential)**.** Locally there is a scalar function $\mathcal{K}$ such that

$$g_{i\bar{\jmath}} = \frac{\partial^2 \mathcal{K}}{\partial z^i \partial \bar{z}^{\bar{\jmath}}} \; . \tag{A.3.15}$$

Thus, $J = i\partial\partial\mathcal{K}$.

Now we return to the Dolbeault cohomology group and define the Hodge numbers:

Definition A.36 (Hodge numbers)**.** Call

$$h^{r,s} \equiv \dim H^{r,s}(\mathbf{X}, \mathbb{C}) \tag{A.3.16}$$

the Hodge numbers of $\mathbf{X}$.

There is an analogy to the Hodge decomposition theorem:

Theorem A.37. The p-th complexified cohomology group $H^p(\mathbf{X})^{\mathbb{C}}$ of a Kähler manifold $\mathbf{X}$ can be rewritten as the direct sum of the Dolbeault cohomology groups as follows:

$$H^p(\mathbf{X})^{\mathbb{C}} = \bigoplus_{r+s=p} H^{r,s}(\mathbf{X}) \; . \tag{A.3.17}$$

For the proof it is useful to introduce the Laplacian operators $\Delta_\partial = \partial\partial^\dagger + \partial^\dagger\partial$, $\Delta_{\bar{\partial}} = \bar{\partial}\bar{\partial}^\dagger + \bar{\partial}^\dagger\bar{\partial}$, where $\partial^\dagger = - \star \bar{\partial} \star$ as well as $\bar{\partial}^\dagger = - \star \partial \star$. One can show that on Kähler manifolds $\Delta_d = 2\Delta_\partial = 2\Delta_{\bar{\partial}}$.

Corollary A.38. It follows that the Betti numbers are given by

$$b^p(\mathbf{X}) = \sum_{k=0}^{p} h^{k,p-k}(\mathbf{X}) \; . \tag{A.3.18}$$

It is useful to be aware that there is a similar result of Cor. A.25, which implies $h^{r,s}(\mathbf{X}) = h^{n-r,n-s}(\mathbf{X})$. For a Kähler manifold $\mathbf{X}$ we have, by taking the complex conjugate, $h^{r,s}(\mathbf{X}) = h^{s,r}(\mathbf{X})$.

Example A.39 (Kähler Manifolds)**.** Every Riemann surface is a Kähler manifold. This is because any 2-form on a manifold with complex dimension 1 is closed. Hence, $dJ = 0$ follows automatically.

Example A.40 (Kähler Manifolds)**.** The ordinary complex projective n-space $\mathbb{C}P^n$ is a Kähler manifold. The complex projective space is defined by an equivalence relation $(z_1, ..., z_{n+1}) \sim (\lambda z_1, ..., \lambda z_{n+1})$ where at least one $z_j \neq 0$ and $\lambda \in \mathbb{C}$. Let $z_j \neq 0$, then choose $\lambda = \frac{1}{z_j}$ and local coordinates are defined by

$$(\xi_j^1, ..., \xi_j^n) \equiv \left(\frac{z_1}{z_j}, ..., \frac{z_{j-1}}{z_j}, \frac{z_{j+1}}{z_j}, ..., \frac{z_{n+1}}{z_j} \right) . \tag{A.3.19}$$

Most importantly for the thesis, there are particular complex manifolds, which are called *Calabi-Yau manifolds.*

Definition A.41 (Calabi-Yau manifolds)**.** A Calabi-Yau (CY) manifold is a compact, complex Kähler manifold with $SU(d)$ holonomy. Equivalently, a CY manifold admits a Ricci flat metric. Another equivalent definition is that a CY manifold has a vanishing first Chern class, $C_1 = \frac{1}{2\pi} [\mathcal{R}] = 0$, where $\mathcal{R} = iR_{i\bar{j}} dz^i \wedge d\bar{z}^{\bar{j}}$ is the Ricci-form.

Theorem A.42 (Holomorphic d-form)**.** Let $\mathbf{X}$ be a d-dimensional compact Kähler manifold. It holds $C_1 = 0$ iff $\mathbf{X}$ admits a nowhere vanishing d-form Ω, the holomorphic d-form, i.e.

$$\Omega(z^1, ..., z^d) = f(z^1, ..., z^d) dz^1 \wedge ... \wedge dz^d . \tag{A.3.20}$$

Thus, CY d-folds always have a nowhere vanishing holomorphic $(d, 0)$-form.

Remark A.43 (Hodge-diamond of CY manifolds)**.** CY-manifolds exhibit a particular structure of the Hodge numbers. Generally those can be ordered in a diamond structure. Here we give some examples.

(i) The K3 surface, a CY 2-fold, has the structure:

$$\begin{array}{ccccc} & & h^{0,0} & & \\ & h^{1,0} & & h^{0,1} & \\ h^{2,0} & & h^{1,1} & & h^{0,2} \\ & h^{2,1} & & h^{1,2} & \\ & & h^{2,2} & & \end{array} \quad = \quad \begin{array}{ccccc} & & 1 & & \\ & 0 & & 0 & \\ 1 & & 20 & & 1 \\ & 0 & & 0 & \\ & & 1 & & \end{array} \tag{A.3.21}$$

(ii) Calabi-Yau 3-folds generally have the following structure:

$$\begin{array}{ccccccc} & & & h^{0,0} & & & \\ & & h^{1,0} & & h^{0,1} & & \\ & h^{2,0} & & h^{1,1} & & h^{0,2} & \\ h^{3,0} & & h^{2,1} & & h^{1,2} & & h^{0,3} \\ & h^{3,1} & & h^{2,2} & & h^{3,1} & \\ & & h^{3,2} & & h^{2,3} & & \\ & & & h^{3,3} & & & \end{array} \quad = \quad \begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & h^{1,1} & & 0 & \\ 1 & & h^{2,1} & & h^{2,1} & & 1 \\ & 0 & & h^{1,1} & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array} \tag{A.3.22}$$

The Euler character of such a CY 3-fold $\mathbf{X}_6$ is given by:

$$\chi(\mathbf{X}_6) = \sum_{p=0}^{6} (-1)^p b^p(\mathbf{X}_6) = 2 \left(h^{1,1}(\mathbf{X}_6) - h^{2,1}(\mathbf{X}_6) \right) , \tag{A.3.23}$$

where the Betti-numbers b^p can be computed by (A.3.18).

(iii) For CY 4-folds $\mathbf{X}_8$ a similar, but bigger Hodge diamond can be written down. There are only three independent Hodge numbers $h^{1,1}, h^{3,1}$ and $h^{2,1}$, characterising $\mathbf{X}_8$. They determine the Hodge number $h^{2,2}$ as follows [15; 17]:

$$h^{2,2}(\mathbf{X}_8) = 2\left(22 + 2h^{1,1}(\mathbf{X}_8) + 2h^{3,1}(\mathbf{X}_8) - h^{2,1}(\mathbf{X}_8)\right) \ . \tag{A.3.24}$$

As above, the Euler character is then given by:

$$\chi(\mathbf{X}_8) = \sum_{p=0}^{8}(-1)^p b^p(\mathbf{X}_8) = 6\left(8 + h^{1,1}(\mathbf{X}_8) + h^{3,1}(\mathbf{X}_8) - h^{2,1}(\mathbf{X}_8)\right) \ . \tag{A.3.25}$$

Finally, note that on CY 4-folds the number of complex structure moduli is given by $h^{3,1}(\mathbf{X}_8)$. The number of Kähler moduli remains $h^{1,1}(\mathbf{X}_8)$.

Given a CY-manifold, it is in general highly non-trivial to find the Hodge numbers. Examples and explanations can be found e.g. in [15; 17].

Remark A.44 (Mirror symmetry)**.** Another feature of complex geometry is closely related to T-duality in string theory. It is called *mirror symmetry* and the conjecture is as follows [15]: Let $\mathbf{X}_6$ be a CY 3-fold. Then, in almost every case there exists another CY 3-fold $\mathbf{Y}_6$ such that

$$H^{p,q}(\mathbf{X}_6) = H^{3-p,q}(\mathbf{Y}_6) \tag{A.3.26}$$

and thus $h^{1,1}(\mathbf{X}_6) = h^{2,1}(\mathbf{Y}_6)$ and vice versa.

In fact, this is related to T-duality in string theory. Let us look at a simple example, the 2-torus: $T^2 = S^1 \times S^1$, so the torus can be imagined as a S^1 fibration over S^1. Let

$$\tau = i\frac{R_2}{R_1} \ , \qquad \text{and} \qquad \rho = iR_1R_2 \tag{A.3.27}$$

be the complex structure and the Kähler modulus, respectively. Performing T-duality on the fibered circle, we exchange $R_1 \leftrightarrow \frac{1}{R_1}$ and hence $\tau \leftrightarrow \rho$. This corresponds precisely to the exchange of $h^{1,1}$ and $h^{2,1}$.

Moreover, since type IIA string theory becomes type IIB under T-duality, we can state that compactification of type IIA string theory on a CY 3-fold is equivalent to compactification of type IIB string theory on the mirror manifold (see also [18]).

APPENDIX B

Decay Rates in the Non-Sequestered LVS

B.1. An Overview of General Results

In this part of the appendix we state for completeness the relevant general formulae for the decay rates used in Chapter 2. The results can also be found in [122] and are thus not original.[1]

Decays into Axions

Let $T_b = \tau_b + ia_b$ be the bulk Kähler modulus. The decay of τ_b into the axions a_b arises due the kinetic term

$$\mathscr{L} = K_{T_b\bar{T}_b}|\partial T_b|^2 \supset f(\tau_b)(\partial a_b)^2 \ , \tag{B.1.1}$$

which yields, by expanding about the VEV $\tau_b = \langle\tau_b\rangle + \delta\tau_b$, the relevant interaction term:

$$\mathscr{L} \supset \partial_{\tau_b} f(\tau_b)|_{\tau_b=\langle\tau_b\rangle}\, \delta\tau_b(\partial a_b)^2 \ . \tag{B.1.2}$$

By canonical normalisation, $\phi = \sqrt{2K_{T_b\bar{T}_b}}\delta\tau_b$ and $a = \sqrt{2K_{T_b\bar{T}_b}}a_b$ we get

$$\mathscr{L} \supset \frac{1}{2\sqrt{2}}\frac{\partial_{\tau_b} f}{K^{3/2}_{T_b\bar{T}_b}}\phi(\partial a)^2 \ . \tag{B.1.3}$$

From here one can calculate the decay rate for the process $\phi \to aa$:

$$\Gamma(\phi \to aa) = \frac{1}{64\pi}\frac{(\partial_{\tau_b} f)^2}{K^3_{T_b\bar{T}_b}}\frac{m^3_\phi}{M^2_p} \ . \tag{B.1.4}$$

Plugging in $K = -3\ln(T_b + \bar{T}_b)$ the decay rate (2.2.9) follows.

[1] Mind the slightly different normalisation of the fields in the subsequent presentation.

Decays into the Higgs Sector

Starting from the Kähler potential

$$K \supset Z_u|H_u|^2 + Z_d|H_d|^2 + g(T_b + \bar{T}_b)(H_u H_d + \text{h.c.}) \;, \tag{B.1.5}$$

see [122], the leading interaction term in the Lagrangian is

$$\mathscr{L} \supset \frac{1}{2} \frac{\partial_{T_b} g}{\sqrt{2K_{T_b\bar{T}_b} Z_u Z_d}} m_\phi^2 \phi (H_u H_d + \text{h.c.}) \;, \tag{B.1.6}$$

where $H_{u,d}$ are normalised by $H_{u,d} \to \sqrt{2Z_{u,d}} H_{u,d}$. We dropped terms involving $\Box H$ because on-shell we have $\Box H = m_H^2 \ll m_\phi^2$ and hence the contributions to the decay rate are subleading (see also [118]). The decay rate is then

$$\Gamma(\phi \to H_u H_d) = \frac{1}{8\pi} \frac{(\partial_{T_b} g)^2}{K_{T_b\bar{T}_b} Z_u Z_d} \frac{m_\phi^3}{M_p^2} \;. \tag{B.1.7}$$

Again, the respective decay rates for the various scenarios can be computed. For the non-sequestered LVS we have to take into account that only decays into the light Higgs are possible and thus the above decay rate has to be multiplied by $\sin^2(2\beta)/2$. Furthermore, for the scenario where the visible cycle is stabilised by D-terms, one first has to integrate out T_a (Kähler modulus for the visible cycle) by $T_a = c_a T_b$.

Decays into Gauge Bosons

The interaction term is obtained from $\mathscr{L} \supset \int d^2\theta T_a W_\alpha W^\alpha$, see (2.3.8). By canonical normalisation, i.e. $F_{\mu\nu} \to \sqrt{f_{\text{vis}}} F_{\mu\nu}$, we get

$$\mathscr{L} \supset \frac{1}{8} \frac{1}{\sqrt{2K_{T_b\bar{T}_b}} \text{Re}(f_{\text{vis}})} \Big(\text{Re}(\partial_{T_b} f_{\text{vis}}) \phi \, \text{tr}(F_{\mu\nu} F^{\mu\nu}) - \text{Im}(\partial_{T_b} f_{\text{vis}}) \phi \, \text{tr}(F_{\mu\nu} \tilde{F}^{\mu\nu}) \Big) \;. \tag{B.1.8}$$

For N_g gauge bosons the decay rate for the process $\phi \to AA$ is given by

$$\Gamma(\phi \to AA) = \frac{N_g}{128\pi} \frac{|\partial_{T_b} f_{\text{vis}}|^2}{(\text{Re} f_{\text{vis}})^2} \frac{1}{K_{T_b\bar{T}_b}} \frac{m_\phi^3}{M_p^2} \;. \tag{B.1.9}$$

In the sequestered LVS $\partial_{T_b} f_{\text{vis}} = 0$ and the decay channel can only take place at one-loop, which is highly suppressed. For the non-sequestered LVS it is easy to see that the above result yields (2.3.13)

APPENDIX C

On Gravitational Instantons

C.1. Derivation of the Metric Structure of Gravitational Instantons

We present a derivation of the metric (5.2.4) following [243], which also shows that C arises as an integration constant. The most general 4d line element with rotational symmetry is

$$ds^2 = \lambda(r)dr^2 + r^2 d\Omega_3^2, \tag{C.1.1}$$

where $d\Omega_3^2$ represents the metric on S^3. Let us be more generic than in (5.2.1) and consider a set of moduli ϕ^I on moduli space with metric G_{IJ}:

$$S = \int d^4x\sqrt{g}\left[-\frac{1}{2}R + \frac{1}{2}G_{IJ}(\phi)g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J\right]. \tag{C.1.2}$$

Due to the rotational symmetry of our system we take $\phi^I = \phi^I(r)$. Variation of S with respect to ϕ^K yields the equation of motion

$$\left(\sqrt{g}g^{rr}G_{KJ}\phi'^J\right)' - \frac{1}{2}\sqrt{g}g^{rr}\partial_K G_{JL}\phi'^J\phi'^L = 0. \tag{C.1.3}$$

Here, the prime denotes the derivative with respect to the coordinate r and ∂_K the derivative with respect to ϕ^K. Let us introduce a new variable τ such that $dr/d\tau = \sqrt{g}g^{rr}$. The equation of motion above can then be rewritten as the geodesic equation on moduli space, i.e.

$$\partial_\tau^2\phi^I + \Gamma^I_{JL}\partial_\tau\phi^J\partial_\tau\phi^L = 0, \tag{C.1.4}$$

with Christoffel-symbols Γ^I_{JL} for the metric G_{IJ}. Along the geodesics we then have

$$\partial_\tau\left(G_{IJ}\partial_\tau\phi^I\partial_\tau\phi^J\right) = 0 \tag{C.1.5}$$

or, expressed in the coordinate r,

$$G_{IJ}\phi'^I\phi'^J = \frac{k}{(\sqrt{g}g^{rr})^2} = \frac{k\lambda(r)}{r^6}, \tag{C.1.6}$$

where we introduced a constant k and used (C.1.1).

Furthermore, the rr-component of the energy-momentum tensor is

$$T_{rr} = \frac{1}{2} G_{IJ} \phi'^I \phi'^J \overset{(\text{C.1.6})}{=} \frac{k\lambda(r)}{2r^6}. \tag{C.1.7}$$

The algebraic form of $\lambda(r)$ can now be read off from the rr-component of Einstein's equations. The rr-component of the Einstein tensor is

$$G_{rr} = \frac{3}{r^2}(1 - \lambda(r)) \tag{C.1.8}$$

and hence $G_{rr} = T_{rr}$ yields

$$\lambda(r) = \left(1 + \frac{C}{r^4}\right)^{-1}, \tag{C.1.9}$$

where $C = k/6$ is indeed an integration constant. It is interesting to note that the metric component g_{rr} is determined independently of the functional form of $G_{IJ}(\phi)$.

Also note that the metric is asymptotically flat, because $\lambda(r) \to 1$ as $r \to \infty$.

Finally, we want to remark that for the creation of Euclidean wormholes ($C < 0$) it is necessary to have $G_{IJ}\phi'^I\phi'^J < 0$ (see (C.1.6)). While one cannot simply put a wrong sign into the kinetic term of the scalar fields, one can instead consider a Lagrangian with a 2-form gauge field, whose dual field is an axion. According to our discussions of quantum mechanical dualisation in Section 5.2.1, this axion is imaginary at the saddle-point of the path integral and effectively obtains an opposite sign in the kinetic term. Moreover, for solutions with $C \geq 0$ one necessarily needs to include dynamical scalar fields so that $G_{IJ}\phi'^I\phi'^J > 0$.

C.2. Charge Quantisation

Let us first recall how flux and charge quantisation usually work in a B_2-/θ_0-theory with strings and fundamental instantons. For any 3-cycle S^3 we have

$$Q_B \int_{S^3} H_3 = 2\pi n \tag{C.2.1}$$

with integer n.[1] Analogously, for any 1-cycle S^1, we have

$$Q_\theta \int_{S^1} F_1 = 2\pi m, \tag{C.2.2}$$

$m \in \mathbb{Z}$. Obviously, n and m can only be non-zero if the relevant cycle is either non-trivial in M or if it encloses the appropriate charged object.

The above are just the familiar flux quantisation conditions. In order to derive charge quantisation, we temporarily go back to Minkowskian space and use the equations of motion of

$$S = -\int_M \frac{1}{2g_B^2} H_3 \wedge \star H_3 + Q_B \int_M B_2 \wedge j_2, \tag{C.2.3}$$

where j_2 is the current modelling the distribution of strings. It can be defined explicitly by $\int_\Sigma j_2 = N$, where N is the number of strings intersecting some surface Σ. Without loss of generality we choose $N = 1$. From the equation of motion for B_2,

$$d(1/g_B^2 \star H_3) = -Q_B j_2, \tag{C.2.4}$$

[1] This follows from assuming gauge invariance of the coupling term in (5.2.6), i.e. one can define B_2 with either the south- or north pole of S^3 removed, getting the same result in both cases. This is another argument to see the necessity of the i-factor in front of the coupling terms.

we find, using Stokes theorem:

$$Q_B = -\int_{\partial\Sigma} 1/g_B^2 \star H_3 = -\int_{\partial\Sigma} F_1 = \frac{2\pi m}{Q_\theta}. \tag{C.2.5}$$

In the last step we used F_1-flux quantisation. Thus, we see that

$$Q_B Q_\theta = 2\pi m, \tag{C.2.6}$$

which is the well-known Dirac quantisation condition. For the following, we take the freedom to choose $Q_\theta = 1$, i.e. the periodicity of the axion field is in this case $\theta_0 \to \theta_0 + 2\pi$. Then, combining (C.2.6) with $m = 1$ (here we assume that a string with smallest charge exists) and (C.2.1), we find that the quantisation condition on H_3 can simply be expressed as

$$\int_{S^3} H_3 = n. \tag{C.2.7}$$

This flux quantisation condition (C.2.7) is at the heart of gravitational instanton solutions.

Now, we are actually interested in potentials introduced by gravitational instantons, i.e., in shift symmetry breaking by quantum gravity. Hence, assuming the existence of fundamental instantons defeats the purpose. So let us see how far we get with the logic above if we abandon the source term in (5.2.5).

First, if we allow for geometries with non-trivial 3-cycles, the H_3 flux quantisation condition (C.2.1) can still be derived. All we need is the existence of strings coupled to B_2. This then also implies that Q_B is quantised. By contrast, (C.2.2) cannot be derived without assuming the existence of fundamental instantons. However, if we allow for geometries which also have non-trivial 1-cycles (see Figure 5.2), and if we postulate that the dual potential θ_0 is a globally defined function taking values on S^1 (i.e. $\theta_0 \equiv \theta_0 + 2\pi$)), then both (C.2.2) and charge quantisation, (C.2.6) and (C.2.7), follow.

C.3. Dualisation under the Path Integral

In Section 5.2.1 we are interested in computing

$$\langle H_3^{(F)}| \, e^{-HT} \, |H_3^{(I)}\rangle \sim \int_{\text{b.c.}} d[H_3] d[\theta_0] \exp\left\{-\int_M \frac{1}{2g_B^2}\left(H_3 \wedge \star H_3 + 2ig_B^2 \theta_0 dH_3\right)\right\}, \tag{C.3.1}$$

which is (5.2.12). Here, $T \equiv t_F - t_I$. At the end we want to obtain a path integral over the variable θ_0, i.e. (5.2.15). This is nothing but dualising from a set of canonical momentum variables to their generalised coordinates.

Thus, we illustrate the subtleties of the computation leading to (5.2.15) by considering the quantum-mechanical harmonic oscillator, i.e. $H = q^2/2 + p^2/2$. The momentum p then corresponds to the background flux $\langle H_3\rangle$ or, more precisely, to the quantised charge n, while the position variable q corresponds to θ_0. The transition amplitude from state $|q_I\rangle$ to $|q_F\rangle$ reads

$$\langle q_F| \, e^{-HT} \, |q_I\rangle = \int d[p] \int_{\text{b.c.}} d[q] \exp\left\{\int_{t_I}^{t_F} dt \, (ip\dot{q} - H(q,p))\right\}, \tag{C.3.2}$$

with boundary conditions $q(t_I) = q_I$ and $q(t_F) = q_F$ imposed.

In fact we rather want to compute $\langle p_F | e^{-HT} | p_I \rangle$, which is expressed similarly:

$$\begin{aligned} \langle p_F | e^{-HT} | p_I \rangle &= \int d[q] \int_{\text{b.c.}} d[p] \exp\left\{ \int_{t_I}^{t_F} dt \left(-iq\dot{p} - H(q,p) \right) \right\} \\ &= \int d[q] \int_{\text{b.c.}} d[p] \exp\left\{ \int_{t_I}^{t_F} dt \left(ip\dot{q} - H(q,p) \right) \right\} \exp\left\{ -i(q_F p_F - q_I p_I) \right\}, \end{aligned} \tag{C.3.3}$$

where we impose again $p(t_I) = p_I$ and $p(t_F) = p_F$. In the second step we integrated the first term of the exponential by parts. In our case we have $H = q^2/2 + p^2/2$ which allows us to complete the square. Integrating out p without worrying about the boundary conditions to be imposed yields the desired result

$$\langle p_F | e^{-HT} | p_I \rangle \sim \int d[q] \exp\left(-i(q_F p_F - q_I p_I) \right) e^{-S[q]}, \tag{C.3.4}$$

see also [273] for comments on the integration over the momentum. We wish to have a closer look at this decisive step.

To do so, we write the amplitude $\langle p_F | e^{-HT} | p_I \rangle$ as:

$$\langle p_F | e^{-HT} | p_I \rangle = \int dq_I dq_F \langle p_F | q_F \rangle \langle q_F | e^{-HT} | q_I \rangle \langle q_I | p_I \rangle . \tag{C.3.5}$$

Now let us assume that the two dual relations (C.3.2) and (C.3.3) hold. Then, in particular, (C.3.2) implies

$$\langle q_F | e^{-HT} | q_I \rangle = \int_{\text{b.c.}} d[q] e^{-S[q]}, \tag{C.3.6}$$

and the result (C.3.4) follows immediately (use $\langle p | q \rangle = e^{-ipq}$). The operation of integrating out p while disregarding its boundary conditions is thereby indirectly justified.

Finally, we can demonstrate this directly and explicitly by writing[2]

$$\begin{aligned} &\langle p_F | e^{-HT} | p_I \rangle \\ &= \int \prod_{m=0}^{N} dq_m \prod_{n=0}^{N-1} dp_n \langle p_F | q_N \rangle \langle q_N | e^{-H\epsilon} | p_{N-1} \rangle \langle p_{N-1} | q_{N-1} \rangle \langle q_{N-1} | e^{-H\epsilon} | p_{N-2} \rangle \dots \\ &\qquad \dots \langle p_1 | q_1 \rangle \langle q_1 | e^{-H\epsilon} | p_0 \rangle \langle p_0 | q_0 \rangle \langle q_0 | p_I \rangle , \end{aligned} \tag{C.3.7}$$

where $\epsilon \equiv T/(N+1)$ and $q_0 = q_I$, $q_N = q_F$. This becomes the discretised version of (C.3.3):

$$\begin{aligned} \langle p_F | e^{-HT} | p_I \rangle = \int \prod_{m=0}^{N} dq_m \prod_{n=0}^{N-1} dp_n \, e^{-iq_N (p_F - p_{N-1}) - H(q_N, p_{N-1})\epsilon} \dots \\ \dots e^{-iq_1 (p_1 - p_0) - H(q_1, p_0)\epsilon} e^{-iq_0 (p_0 - p_I)}. \end{aligned} \tag{C.3.8}$$

For the harmonic oscillator (and in fact for more general potentials $V(q)$) we can integrate out $p_0, ..., p_{N-1}$ (after completing the square for each p_m). As a result we find

$$\begin{aligned} \langle p_F | e^{-HT} | p_I \rangle \sim \int \prod_{m=0}^{N} dq_m \, \exp\left\{ -iq_N p_F \right\} \exp\left\{ -\frac{q_N^2}{2}\epsilon - \frac{(q_N - q_{N-1})^2}{2\epsilon} \right\} \dots \\ \dots \exp\left\{ -\frac{q_1^2}{2}\epsilon - \frac{(q_1 - q_0)^2}{2\epsilon} \right\} \exp\left\{ iq_0 p_I \right\} . \end{aligned} \tag{C.3.9}$$

This is precisely the discretised version of (C.3.4). Hence, integrating out the momenta from (C.3.3) to (C.3.4) without considering the boundaries is indeed justified.

[2] We are grateful to K.-M. Lee for pointing out this possibility and for further discussions on this issue. See also [236].

C.4. Analytical Solutions to Einstein's Equation

Einstein's equation (5.2.25) which follows from the action (5.2.19) can be solved analytically. We explain how to arrive at solutions (5.2.27), (5.2.34) and (5.2.35) for $C < 0$, $C = 0$ and $C > 0$, respectively.

First of all, (5.2.25) can be rewritten as:

$$\pm \int d\varphi \frac{1}{\sqrt{\mathcal{F}(\varphi)n^2/A^2 + C}} = \sqrt{6} \int dr \frac{1}{r^3\sqrt{1 + C/r^4}}. \tag{C.4.1}$$

Thus, integral representations can in principle be obtained for any $\mathcal{F}$. For $\mathcal{F}(\varphi) = 1/(3! f_{\mathrm{ax}}^2) \exp(-\alpha\varphi)$, explicit solutions exist.

For the RHS, one finds

$$\sqrt{6} \int dr \frac{1}{r^3\sqrt{1 + C/r^4}} = \begin{cases} -\sqrt{\frac{3}{2|C|}} \arcsin\left(\frac{\sqrt{|C|}}{r^2}\right) + \text{const} & \text{for } C < 0 \\ -\sqrt{\frac{3}{2C}} \operatorname{arcsinh}\left(\frac{\sqrt{C}}{r^2}\right) + \text{const} & \text{for } C > 0 \\ -\sqrt{\frac{3}{2}}\frac{1}{r^2} + \text{const} & \text{for } C = 0, \end{cases} \tag{C.4.2}$$

where we use the substitution $y = \sqrt{|C|}/r^2$ for $C \neq 0$. The integral on the LHS can be rewritten as

$$\pm \frac{1}{\sqrt{|C|}} \int d\varphi \frac{1}{\sqrt{k \exp(-\alpha\varphi) \pm 1}}, \tag{C.4.3}$$

with $k \equiv n^2/(3!|C|A^2 f_{\mathrm{ax}}^2)$. If $C > 0$ ($C < 0$), the positive (negative) sign under the square root applies. In the case of $C > 0$ we substitute

$$\sinh y = \frac{1}{\sqrt{k}} \exp(\alpha\varphi/2), \tag{C.4.4}$$

and for $C < 0$ we take

$$\sin y = \frac{1}{\sqrt{k}} \exp\left(\alpha\varphi/2\right). \tag{C.4.5}$$

Using appropriate identities for the hyperbolic or trigonometric functions, one arrives at

$$\begin{aligned} &\pm \int d\varphi \frac{1}{\sqrt{\exp(-\alpha\varphi)n^2/(3! f_{\mathrm{ax}}^2 A^2) + C}} = \\ &= \begin{cases} \pm\frac{2}{\sqrt{|C|}\alpha}\left[\arcsin\left(\frac{1}{\sqrt{k}}\exp\left(\alpha\varphi/2\right)\right) - \text{const}\right] & \text{for } C < 0 \\ \pm\frac{2}{\sqrt{C}\alpha}\left[\operatorname{arcsinh}\left(\frac{1}{\sqrt{k}}\exp(\alpha\varphi/2)\right) - \text{const}\right] & \text{for } C > 0 \\ \pm\frac{2\sqrt{6}A f_{\mathrm{ax}}}{n\alpha}\exp\left(\alpha\varphi/2\right) + \text{const} & \text{for } C = 0. \end{cases} \end{aligned} \tag{C.4.6}$$

From here one can read off the solutions, which can be rewritten as (5.2.27), (5.2.34) or (5.2.35).

C.5. Computation of the Instanton Action

We present further details of the computation of the instanton action in Section 5.4. The computation consists of determining the on-shell contribution from the action and the contribution coming from the Gibbons-Hawking-York boundary term. We begin by looking at the latter, where we follow [92].

Gibbons-Hawking-York boundary term:

The Gibbons-Hawking-York boundary term is

$$S_{\text{GHY}} = -\oint_{\partial M} d^3x\sqrt{h}(K - K_0), \tag{C.5.1}$$

as described around (5.4.4). Starting from our metric ansatz (5.2.4) we choose hypersurfaces of constant r. The normal unit vector n is then

$$n = \sqrt{1 + \frac{C}{r^4}}\frac{\partial}{\partial r}. \tag{C.5.2}$$

The trace of the extrinsic curvature is

$$K = \nabla_\mu n^\mu = \partial_\mu n^\mu + \Gamma^\mu_{\mu\nu} n^\nu \tag{C.5.3}$$

where ∇ is the Levi-Civita connection on M. One finds

$$\Gamma^\mu_{\mu r} = \frac{2C}{r^5}\left(1 + \frac{C}{r^4}\right)^{-1} + \frac{3}{r}, \tag{C.5.4}$$

and therefore

$$K = \nabla_\mu n^\mu = \frac{3}{r}\left(1 + \frac{C}{r^4}\right)^{1/2}. \tag{C.5.5}$$

By taking $C = 0$ we can also read off the trace of the extrinsic curvature of ∂M embedded in $\mathbb{R}^4$:

$$K_0 = \frac{3}{r}. \tag{C.5.6}$$

It then follows

$$S_{\text{GHY}} = -\oint_{\partial M} \epsilon_{S^3}(K - K_0) = -3Ar^2\left[\left(1 + \frac{C}{r^4}\right)^{1/2} - 1\right]\Bigg|_{\text{boundary}}, \tag{C.5.7}$$

with surface area $A = 2\pi^2$ of S^3. Recall that according to our conventions the volume form on S^3 contains a factor r^3. Clearly, for $C = 0$ we have $S_{\text{GHY}} = 0$. For $C > 0$ the boundary is at $r = 0$ and at $r = \infty$,

$$S_{\text{GHY}} = 3AC^{1/2}, \qquad C > 0. \tag{C.5.8}$$

In the case of $C < 0$ the integral vanishes, because we always consider instanton-anti-instanton pairs, so $S_{\text{GHY}} = 0$.

These are the results used in Section 5.4.

On-shell contribution:

We now evaluate the bulk action (5.2.19) on-shell, i.e. we plug in the equations of motion successively. As described in Section 5.4, the first step is to express the Ricci scalar R by the trace of the energy-momentum tensor using Einstein's equations:

$$R = -T. \tag{C.5.9}$$

The energy-momentum tensor $T_{\mu\nu}$ from the action (5.2.19) is

$$T_{\mu\nu} = -g_{\mu\nu}\left[\frac{1}{2}\mathcal{F}(\varphi)H^2 + \frac{1}{2}\partial_\rho\varphi\partial^\rho\varphi\right] + 3\mathcal{F}(\varphi)H_{\mu\rho\sigma}H_\nu{}^{\rho\sigma} + \partial_\mu\varphi\partial_\nu\varphi. \tag{C.5.10}$$

Consequently,

$$T = g^{\mu\nu} T_{\mu\nu} = \mathcal{F}(\varphi) H^2 - (\partial\varphi)^2, \tag{C.5.11}$$

and then (5.2.19) becomes simply

$$S = \int d^4x \sqrt{g} \mathcal{F}(\varphi) H^2 = A \int dr \frac{r^3}{\sqrt{1 + C/r^4}} \mathcal{F}(\varphi) H^2, \tag{C.5.12}$$

where we used the rotational symmetry of our system. Next, we plug in the solution to (5.2.21),

$$H = \frac{n}{Ar^3} \epsilon, \tag{C.5.13}$$

and restrict ourselves to $\mathcal{F}(\varphi) = \exp(-\alpha\varphi)/(3! f_{\text{ax}}^2)$, for which we know the analytical solutions:

$$S = \frac{n^2}{A f_{\text{ax}}^2} \int dr \frac{1}{r^3 \sqrt{1 + C/r^4}} \exp(-\alpha\varphi). \tag{C.5.14}$$

It is then convenient to rewrite the action as an integral over $d\varphi$ using Einstein's equation (5.2.25). We consider only regular solutions. They are monotonically decreasing and therefore we have $\varphi'(r) < 0$ everywhere. Hence,

$$S = -\frac{n^2}{A f_{\text{ax}}^2} \int d\varphi \frac{\exp(-\alpha\varphi)}{\sqrt{n^2 \exp(-\alpha\varphi)/(A^2 f_{\text{ax}}^2) + 6C}}. \tag{C.5.15}$$

The integral has to be evaluated case by case.

For extremal gravitational instantons with $C = 0$ we have

$$S = -\frac{n}{f_{\text{ax}}} \int_{\varphi(0)}^{\varphi(\infty)} d\varphi \exp(-\alpha\varphi/2) = \frac{2n}{\alpha f_{\text{ax}}}. \tag{C.5.16}$$

In the case of $C > 0$ we obtain

$$\begin{aligned} S &= -\frac{n^2}{A f_{\text{ax}}^2} \int_{\varphi(0)}^{\varphi(\infty)} d\varphi \frac{\exp(-\alpha\varphi)}{\sqrt{n^2 \exp(-\alpha\varphi)/(A^2 f_{\text{ax}}^2) + 6C}} \\ &= \frac{2n}{\alpha f_{\text{ax}}} \sqrt{\exp(-\alpha\varphi) + \sinh^2 K_+} \Bigg|_{\varphi(0)}^{\varphi(\infty)} = \frac{2n}{\alpha f_{\text{ax}}} e^{-K_+}, \end{aligned} \tag{C.5.17}$$

where we used (5.2.36) and took $K_+ > 0$. Combining this with the GHY boundary term yields the desired instanton action (5.4.14).

Finally, for Euclidean wormholes, i.e. for $C < 0$, we have

$$S = \int d^4x \sqrt{g} \mathcal{F}(\varphi) H^2 = 2 \times A \int_{r_0}^{\infty} dr \frac{r^3}{\sqrt{1 - |C|/r^4}} \mathcal{F}(\varphi) H^2, \tag{C.5.18}$$

where the factor of two occurs because the left integral is over the whole Euclidean space, and hence accounts for the whole wormhole and thus for the instanton and anti-instanton, while the integral on the RHS integrates from the centre of the wormhole to one end. The appearance of this factor may be seen more easily by evaluating the integral on the LHS using the t-coordinate (5.2.33) and then changing coordinates from t to r. As was noted in [197], this contribution has to be divided by two, because the instanton action S_{inst} should only take into account half of the full wormhole action. Consequently, using the equations of motion as in the previous cases,

$$\begin{aligned} S_{\text{inst}} &= -\frac{n^2}{Af_{\text{ax}}^2}\int_{\varphi(r_0)}^{\varphi(\infty)} d\varphi \frac{\exp(-\alpha\varphi)}{\sqrt{n^2\exp(-\alpha\varphi)/(A^2f_{\text{ax}}^2) - 6|C|}} \\ &= \frac{2n}{\alpha f_{\text{ax}}}\left|\sin\left(\frac{\alpha\pi}{4}\sqrt{\frac{3}{2}}\right)\right|. \end{aligned} \tag{C.5.19}$$

Hence, (5.4.19) follows, where we can drop the modulus due to the restriction to $0 \leq \alpha < 2\sqrt{2/3}$.

C.6. On the Kähler potential on a CY 3-fold with $S = iu$

We try to justify that

$$K = -\ln(S + \bar{S}) - 3\ln(U + \bar{U}) \tag{C.6.1}$$

effectively becomes

$$K = -4\ln(U + \bar{U}) \tag{C.6.2}$$

upon choosing $S = U$. Note that for simplicity we defined S such that the i-factor is removed.

Algebraically, this is clear. What is actually happening is the following. Let us define $\phi \equiv U$ and $\psi \equiv S - U$. The idea is to fix ψ at $\psi = 0$ and keep ϕ dynamical. In terms of ϕ, ψ the Kähler potential reads

$$K = -\ln(\psi + \bar{\psi} + \phi + \bar{\phi}) - 3\ln(\phi + \bar{\phi}). \tag{C.6.3}$$

Again, $K = -4\ln(\phi + \bar{\phi})$ follows for $\psi = 0$.

This level may still be too naive, hence let us choose another approach following [243][3]:

Once again, we start with (C.6.1). We write $S = s + i\tilde{\sigma}$ and $U = u + i\tilde{\nu}$. The line element on the Kähler manifold reads

$$d\ell^2 = \frac{1}{4s^2}ds^2 + \frac{1}{4s^2}d\tilde{\sigma}^2 + \frac{3}{4u^2}du^2 + \frac{3}{4u^2}d\tilde{\nu}^2. \tag{C.6.4}$$

Let us define φ, θ as $s = e^{-2\varphi}$ and $u = e^{-2\theta/\sqrt{3}}$. It follows:

$$d\ell^2 = d\varphi^2 + e^{4\varphi}d\sigma^2 + d\theta^2 + e^{4\theta/\sqrt{3}}d\nu^2, \tag{C.6.5}$$

where $\sigma = \tilde{\sigma}/2$ and $\nu = \tilde{\nu}/2$.

Later we wish to stabilise at $S = U$. This then implies $s = u$ and $\sigma = \nu$. The condition $s = u$ yields

$$\varphi = \frac{\theta}{\sqrt{3}}. \tag{C.6.6}$$

This motivates us to introduce ψ as follows:

$$\psi \equiv \frac{\sqrt{3}\varphi - \theta}{2}. \tag{C.6.7}$$

The orthogonal direction is parameterised by

$$\xi \equiv \frac{\varphi + \sqrt{3}\theta}{2}, \tag{C.6.8}$$

[3] I thank L.T. Witkowski for suggesting this computation.

as can be seen by the corresponding rewriting of the metric:

$$d\ell^2 = d\psi^2 + d\xi^2 + e^{2\sqrt{3}\psi+2\xi}d\sigma^2 + e^{-2\psi/\sqrt{3}+2\xi}d\nu^2. \tag{C.6.9}$$

Restriction to $\psi = 0$ gives

$$d\ell^2 = d\xi^2 + e^{2\xi}(d\sigma^2 + d\nu^2). \tag{C.6.10}$$

Furthermore, we have to account for $\sigma = \nu$. In the very same spirit as above we define

$$\rho \equiv \frac{\sigma - \nu}{\sqrt{2}} \tag{C.6.11}$$

and

$$\tau \equiv \frac{\sigma + \nu}{\sqrt{2}}. \tag{C.6.12}$$

Again, $S = U$ implies $\rho = 0$. Hence, at $S = U$:

$$d\ell^2 = d\xi^2 + e^{2\xi}d\tau^2. \tag{C.6.13}$$

But such a line element can also be obtained from $K = -4\ln(V + \bar{V})$ with $V = v + i\tau$:

$$d\ell^2 = \frac{1}{v^2}dv^2 + \frac{1}{v^2}d\tau^2. \tag{C.6.14}$$

For $v = e^{-\xi}$ this is consistent with (C.6.13).

C.7. Estimating the Size of the Prefactor $\mathcal{A}$ in the Instanton Potential

The contribution of gravitational instantons to the axion potential is given by $\delta V = \mathcal{A}e^{-S}\cos(n\theta)$. While it has been proposed e.g. in [101; 108] that $\mathcal{A} \sim 1$ (in Planck units), we attempt a somewhat more precise estimate. This is inspired by the analogies between gravitational instantons and instantons arising from Euclidean branes wrapping an internal cycle of the compactification manifold (see e.g. [92; 101; 108]). Let us start by recalling how the latter contributes to the supergravity F-term potential in a simple setup.

We consider a Euclidean brane instanton modifying the perturbative superpotential W_0 as

$$W = W_0 + A(z)e^{-aT}, \tag{C.7.1}$$

where z denotes the complex structure moduli and T is a Kähler modulus.

Then the supergravity F-term potential

$$V_F = e^K\left(K^{i\bar{j}}D_iWD_{\bar{j}}\overline{W} - 3|W|^2\right) \tag{C.7.2}$$

is corrected at leading order by

$$\delta V \sim e^K W_0 A(z)e^{-a\tau}\ , \tag{C.7.3}$$

where τ is the real part of T. Recall that $K = -2\ln\mathcal{V}+...$, which gives a suppression by $1/\mathcal{V}^2$. Furthermore, we rewrite the above expression in terms of the gravitino mass $m_{3/2} \sim W_0/\mathcal{V}$ and the KK-scale $m_{\text{KK}} \sim 1/\mathcal{V}^{2/3}$:

$$\delta V \sim \frac{1}{\mathcal{V}^{5/3}}\frac{m_{3/2}}{m_{\text{KK}}}A(z)e^{-a\tau}. \tag{C.7.4}$$

If we were allowed to compare this with (5.6.1) then, using $m_{3/2} \lesssim m_{\rm KK}$, we would conclude that

$$\mathcal{A} \lesssim \frac{A(z)}{\mathcal{V}^{5/3}} \tag{C.7.5}$$

in Planck units. Here we identified $\exp(-a\tau)$ with $\exp(-S)$ motivated by the obvious analogy: Indeed, the Euclidean brane action is proportional to the brane tension and the volume of the cycle. Similarly, the action of a cored gravitational instanton is proportional to the ADM tension of a black brane wrapping a cycle in a higher-dimensional version of the gravitational instanton system, see e.g. [92] for an example.

Nevertheless, our proposal to estimate $\mathcal{A}$ by (C.7.5) remains nontrivial. Indeed, we first need to consider a large wrapping number n to relate to the calculable regime on the gravitational side. This is unproblematic in the present case since these higher instantons will contribute to W analogously to (C.7.1). Next, we are *not* interested in Euclidean brane instantons (their effect is well-known) but in some possibly very different type of instanton arising in a string model and not having a simple microscopic description. The claim or proposal implicit in (C.7.5) is then that this instanton may, conservatively, also be suppressed by a factor $\mathcal{A}$ which becomes small as the KK-scale and SUSY breaking scales go down. This appears to be reasonable since, beyond the simple Euclidean brane case discussed here, higher-dimensional and SUSY-based cancellations are expected to occur above those scales.

Accepting the above proposal, compactification volumes in the range $\mathcal{V} \sim 10^2$ to 10^3 imply $\mathcal{A} \sim 10^{-4}$ and 10^{-5}, respectively, assuming that $A(z) = \mathcal{O}(1)$. Note that in order to avoid destabilisation of the Kähler moduli the compactification volume is at most of order $\mathcal{O}(10^3)$, see e.g. [112; 155; 156]. Nevertheless, the suppression by e^{-S} remains dominant in all regimes we considered.

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Own Publications

During my Ph.D. studies the following papers have been published:

- ◇ A. Hebecker, P. Mangat, F. Rompineve and L. T. Witkowski, “Dark Radiation predictions from general Large Volume Scenarios,” JHEP **1409** (2014) 140 arXiv:1403.6810 [hep-ph].
- ◇ A. Hebecker, P. Mangat, F. Rompineve and L. T. Witkowski, “Tuning and Backreaction in F-term Axion Monodromy Inflation,” Nucl. Phys. B **894** (2015) 456 arXiv:1411.2032 [hep-th].
- ◇ A. Hebecker, P. Mangat, F. Rompineve and L. T. Witkowski, “Winding out of the Swamp: Evading the Weak Gravity Conjecture with F-term Winding Inflation?,” Phys. Lett. B **748** (2015) 455 arXiv:1503.07912 [hep-th].
- ◇ A. Hebecker, P. Mangat, S. Theisen and L. T. Witkowski, “Can Gravitational Instantons Really Constrain Axion Inflation?,” JHEP **1702** (2017) 097 arXiv:1607.06814 [hep-th].

This thesis is based on these papers.