Dissertation

submitted to the
Combined Faculties for the Natural Sciences and for Mathematics
of the Ruperto-Carola University of Heidelberg, Germany
for the degree of
Doctor of Natural Sciences

presented by

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Oral examination: February 10th 2005

5D Supersymmetric Orbifolds: Supergravity/Phenomenological Aspects

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Abstract

Five-dimensional braneworlds attracted much attention in recent years, be it for phenomenological, cosmological or theoretical reasons. In this work we study supersymmetric theories compactified on the orbifold $S^1/\mathbb{Z}_2$. We start with a short discussion of power-law unification, where 5D rigid super Yang-Mills theory is introduced in its superfield formulation. We develop a superfield description of 5D orbifold $\mathcal{N}=2$ supergravity coupled to vector and hyper multiplets. The basic building blocks are $\mathcal{N}=1$ supermultiplets obtained as reductions of the full multiplets of $\mathcal{N}=2$ conformal supersymmetry by Fujita, Kugo and Ohashi. After identifying the relevant superfields we build superspace actions for the vector, hyper and radion sectors. The couplings of these sectors to the 4D Weyl multiplet are obtained by the replacement of integrations over (flat) superspace by the F and D densities of 4D conformal supergravity. We then observe that a Weyl rescaling is enough to extend the formalism to warped geometries, and show how to consistently introduce brane-localized couplings. The superfield approach to 5D orbifold $\mathcal{N}=2$ supergravity is used to rederive the BPS conditions in the generalized Randall-Sundrum models without using the 4-form mechanism to introduce the odd couplings. It is noted that BPS conditions correspond to F and D flatness conditions, which are simple to obtain in this formalism. Next, a study of recent claims on supersymmetric radion stabilization leads us to speculate on a possible no-go theorem on this possibility. We then consider the supergravity embedding of tuned Fayet-Iliopoulos terms, show that they do not break supersymmetry even in warped geometries, and obtain new supersymmetric vacua with negative brane tensions and a bulk fat brane. We close with a study of supersymmetric models of gauge inflation.

Zusammenfassung

5D Braneworlds haben in den letzten Jahren große Aufmerksamkeit erzeugt, sei es aus phänomenologischen, kosmologischen oder theoretischen Gründen. Diese Arbeit befaßt sich mit supersymmetrischen Theorien kompaktifiziert auf dem Orbifold $S^1/\mathbb{Z}_2$. Wir beginnen mit eine kurze Diskussion der "Power-law" großen Vereinigung, wobei die 5D globale Super-Yang-Mills Theorie eingeführt wird. Wir entwickeln eine Superfeld-Beschreibung der 5D Orbifold $\mathcal{N}=2$ Supergravitation gekoppelt an Vektor- sowie Hypermultiplets. Die Bausteine sind $\mathcal{N}=1$ Supermultiplets, welche durch Reduzierung der vollen Multiplets der $\mathcal{N}=2$ konformen Supersymmetrie, von Fujita, Kugo, Ohashi erhalten wurden. Wir identifizieren die relevanten Superfelder und formulieren die Superraum-Wirkungen für die Vektor-, Hyper- sowie Radion-Sektoren. Die Kopplung dieser Sektoren an das 4D Weyl Multiplet erfolgt durch Ersetzen der Integrale über den Superraum durch F- und D-Dichten der 4D konforme Supergravitation. Wir erklären wie durch eine Weyl Reskalierung dieser Formalismus zu Geometrien mit "Warping" erweitert werden kann, und zeigen wie konsistente Brane-Kopplungen eingeführt werden können. Wir wenden dann diesen Formalismus an, um die BPS Bedingungen für verallgemeinerte Randall-Sundrum Modelle herzuleiten, wobei die ungeraden Kopplungen ohne 4-Form Mechanism eingeführt werden. Da die BPS Gleichungen aus den Bedingungen für F- und D-Flachheit folgen, sind sie einfach zu erhalten. Die Untersuchung aktueller Behauptungen zur SUSY-Radion-Stabilisierung führt zur Vermutung, daß dieses augeschlossen ist. Wir betrachten dann Fayet-Iliopoulos Terme in der 5D Supergravitation, zeigen, daß in Geometrien mit "Warping" Supersymmetrie nicht gebrochen ist, und finden neue BPS Lösungen. Wir schließen mit einer Untersuchung supersymmetrischer Modelle für Eich-Inflation.

Contents

Chapter 1

Introduction

In a certain analogy to the 3.000 years old desire to unify all the deities in the person of one sole god, modern physics has the goal of unifying all forces in one, following the creed that all interactions are just different manifestations of the same sole force. This search for simplicity has led in the past to the unification of the laws governing terrestrial gravity and the motion of the celestial bodies by Newton, to the theory of electromagnetism by Maxwell, but also to the unification of space and time, and matter and energy by Einstein. More recently, it was understood that the weak force responsible for the β-decay can only be described in the framework of a unified theory with spontaneously broken symmetry, and it turned out that electromagnetism is also a part of this unified theory. There is even strong evidence that unification of all known forces besides gravity takes place at energies of order 10^{16} GeV, even if the symmetry between these forces appears to be broken below this energy. Yet, despite all the progress that superstring theory underwent in the recent decades, a true understanding of the way gravity and the other fundamental forces unify is still lacking.

Early attempts to unify gravity with the other forces in nature can be traced back to the works of Nordström (1914), Kaluza (1921) and Klein (1926) [1–3], where in all these three cases the existence of *extra-dimensions* was to play a crucial rôle. Nordström considered a Maxwell theory in five dimensions, and observed that assuming the fields to be constant along the 5th dimension one obtains in addition to the 4D Maxwell theory a scalar theory that he identified with gravity. Eventually, with the advent of the theory of general relativity, his theory of gravity proved to be wrong and the attempt of using extra-dimensions for unification was forgotten. Later, Kaluza presented a model build upon Einstein theory in the same way as Nordström's model was built upon Maxwell theory. The result was a 4D unified theory of Einstein gravity and Maxwell electromagnetism. But it was not until the work of Klein that the necessity of compactifying the extra-dimensions was recognized, and that the constancy of the fields in the 5th direction was understood as a consequence of the small size of the extra-dimensions.

After these early attempts, and for many years, theories with extra-dimensions were everything but mainstream, until the emergence of string theory in the early 70's brought them back to the attention of at least part of the theoretical physics community. Indeed, it was then shown that the by now most promising quantum theory unifying gravity and the other fundamental forces, superstring theory, is only well-defined in 10 or 11 dimensions. This fact has motivated the interest in Kaluza-Klein theories until the mid-80's and in braneworlds starting in the mid-90's. The concept of braneworlds is a *variation* of the Kaluza-Klein idea, different in the fact that now different fields can be constrained to propagate in different subspaces of the higher-dimensional space-time. This picture emerged with the advent of D-branes [4] and the Hořava-Witten theory of heterotic M-theory [5]. It is the later that is mostly invoked as the inspiration for five-dimensional orbifold braneworlds, which are the object of this thesis.

What is heterotic M-theory? Hořava and Witten considered an eleven-dimensional set-up consisting of 11D supergravity compactified on an $S^1/\mathbb{Z}_2$ orbifold. 11D supergravity is a highly constrained theory, an eleven-dimensional generalisation of Einstein gravity endorsed with supersymmetry, the symmetry which unifies fermions and bosons. A compactification on the $S^1/\mathbb{Z}_2$ orbifold means that physical field quantities, but also the Lagrange density, must satisfy: (a) $\mathcal{L}(x^\mu, x^{11}) = \mathcal{L}(x^\mu, x^{11}+2\pi R)$, and (b) $\mathcal{L}(x^\mu, x^{11}) = \mathcal{L}(x^\mu, -x^{11})$, where x^{11} parametrizes the 11th direction of finite size $2\pi R$. The orbifold condition (b), identifies two special points $x^{11} = 0, \pi R$ which are the ten-dimensional boundaries of the physical 11D space-time. Hořava and Witten made the observation that to obtain a consistent (i.e. anomaly free) quantum theory, the 11D supergravity must be supplemented with two supersymmetric gauge theories with gauge group E_8, each localised at one of the boundary planes, and by suitable couplings between these and the 11D theory. The theory they obtained in this way was then related to the strong coupling limit of the $E_8 \times E_8$ heterotic string theory, whose weak coupling limit was already known.

To obtain a realistic phenomenology, the six additional extra-dimensions should be compactified. In particular, compactifications of the heterotic M-theory on 6D Calabi-Yau spaces [6] with size smaller than πR, result in effective five-dimensional orbifold ($\mathcal{N} = 2$) supergravities, coupled to ($\mathcal{N} = 1$) supersymmetric gauge theories localised at the 4D boundaries. It was this understanding that there could be an intermediate regime between the string and the weak scale where physics would be effectively five dimensional, that sparked the construction of innumerous *toy*-models, aiming to explain different aspects of low-energy physics with 5D mechanisms. From the many scenarios that appeared, the Randall-Sundrum I model [7] is perhaps the most eminent one. These authors observed that a shockingly simple set-up, consisting of a negative cosmological constant in the 5D bulk of the orbifold and tensions at the boundary branes could explain the exponential hierarchy between the 4D Planck scale and the masses of the known particles. For this to happen the *only* requirements are a tuning between the bulk cosmological constant and the brane tensions of opposite sign, and that the size of the extra-dimension be slightly larger than the 5D Planck size.

Even though it is not yet clear how much of the developed 5D scenarios could be embedded in a superstring framework, one cannot deny that 5D orbifold models are quite interesting on their own. The closer these models are to the *known* effective low-energy descriptions of string theories the higher is the probability that they provide some insight into the solutions which we expect will emerge from string theory. In this work we follow the view that 5D orbifold $\mathcal{N} = 2$ supergravities are close enough to the 5D heterotic M-theory to probe some of the problems and constrains of this theory, but retain enough freedom which allows one to search for phenomenologically realistic models. We think therefore that 5D orbifold supergravity is a very good terrain to check the pertinence of many of the proposals already made and to get insights for new proposals. Of course, in problems where gravity and the dynamics of light fields plays no rôle, one can consider the rigid limit of those theories to obtain 5D orbifold $\mathcal{N} = 2$ supersymmetric Yang-Mills theories.

The central piece of this thesis is the development - and also some applications - of a *superfield* description of five-dimensional $\mathcal{N} = 2$ supergravity on the $S^1/\mathbb{Z}_2$ orbifold. As pointed out by Mirabelli and Peskin [8] for the rigid case, the simplest way of introducing couplings between brane and bulk fields is to use an *off-shell* formulation of 5D supersymmetry. For supergravity this program was followed by Zucker [9] and independently by the group of Fujita, Kugo and Ohashi (FKO) [10–14]. In our view, it is the results of this latter group which are of greater interest as they reduce, on-shell, to the old 5D supergravity of Gunaydin, Sierra and Townsend (GST) [16], which can in some cases be obtained from Calabi-Yau compactifications of 11D supergravity. We should mention also the works of Falkowski et al. [17], and Altendorfer et al. [18], where supergravity couplings to the branes are introduced on-shell, making it difficult to include additional matter on the boundaries.

Both Zucker and FKO were motivated in their investigation by the idea of supersymmetrizing the Randall-Sundrum model. In fact, in a supersymmetric set-up the tuning of the brane and bulk cosmological constants is tantamount to the existence of supersymmetric vacua [20]. Therefore deformations of the Randall-Sundrum model should be found in the form of BPS vacua of orbifold 5D supergravities. The crucial difference between Zucker's and FKO's work lies in the use of a *superconformal* tensor calculus by the latter group, versus a *supergravity* tensor calculus by the former. It turns out that in the FKO approach the *old*-minimal 4D Poincaré supergravity [19] is induced on the boundary branes, while Zucker's 5D supergravity induces a non-minimal version. It is this feature and the nice fact that the old 5D supergravity of GST is recovered on-shell that brought us to search for a superfield description of the off-shell (conformal) supergravity of FKO.

The idea of using the $\mathcal{N} = 1$ superspace language to write down extra-dimensional actions can be traced back to the early 80's, to the work of Marcus et al. [21], where the ($\mathcal{N} = 4$) 10D super Yang-Mills theory was written in terms of 4D superfields. The advantage of such a formulation was also recognized recently for the 5D case by Arkani-Hamed et al. [22], Marti and Pomarol [23], and Hebecker [24], who presented superfield actions for rigid 5D SYM theories coupled to bulk hypermatter. A superfield description entails very compact expressions where only the $\mathcal{N} = 1$ supersymmetry is manifest, which survives the orbifold compactification. Very attractive is also the fact that gauge invariant brane localised couplings are rather simple to write down in terms of superfields. Upon integrating out the Kaluza-Klein towers one then obtains - in principle - 4D effective expressions written in a language already well-known to model-builders. Our goal in this work was to extend these successes to 5D supergravity coupled to super Yang-Mills theory. And indeed we found an almost complete superfield formulation of this theory, suitable to be applied to many different relevant problems, as will show in this thesis. It is specially well-suited to study supersymmetric vacuum configurations with warped geometry, in arbitrary brane-bulk setups.

This thesis is organized as follows:

Chapter 2 contains a short discussion of the issue of power-law unification in the orbifold case. As a starting point we present a loop calculation of the gauge coupling using the world-line method. We use the chance and introduce the superfield formulation of global 5D supersymmetric SYM theories and the concept of prepotential, needed for later chapters. We then review recent advances towards calculable power-law unification, and discuss the way this calculability emerges in the limit of large radius.

In chapter 3 we come to the central piece of this work, the superfield approach to gauge and hypermatter coupled to 5D orbifold supergravity [25]. We start with a brief overview of the superconformal approach to 5D SUGRA presenting the relevant multiplets of 5D local supersymmetry and their reduction to multiplets of $\mathcal{N} = 1$ supersymmetry. There we identify the supermultiplets which will be relevant to obtain a superspace action, and clarify the concept of radion superfield(s). We then write down the superspace actions for the vector, hyper and radion sectors which reproduce the supergravity expressions of FKO upon conformal gauge fixing. Performing the rigid limit, we are also able to reproduce known expressions for the global supersymmetric case. We come then to consider the way warped geometries should be taken into account, using a Weyl rescaling for this purpose. We close this chapter explaining how to introduce brane couplings, and the local supersymmetry meaning of the superspace integrals.

Equipped with the superfield formalism of chapter 3, we discuss in chapter 4 several set-ups with warped geometries. The starting point is the derivation of the action and BPS equations of the generalized supersymmetric Randall-Sundrum models, a demonstration of what we consider to be the straigthforwardness of our formalism. We then move to a discussion of the possibility of radion stabilisation without breaking supersymmetry, with a critical comment on some recent literature on the subject. The recent claim that tuned Fayet-Iliopoulos terms in warped geometries break supersymmetry is refuted, and the correct solutions found [26]. We close this chapter presenting a warped supersymmetic model with two negative brane tensions and a fat brane of positive energy localised in the bulk, induced by the localisation of hyperscalars due to Fayet-Iliopoulos terms.

In chapter 5 we come to a discussion of supersymmetric gauge inflation [27] on the orbifold. After reviewing the concept of gauge inflation, we present a supersymmetric realization of it on the orbifold. This is followed by a discussion of the interplay between inflation, SUSY breaking and radion stabilisation.

We conclude in chapter 6 with a brief outlook.

Chapter 2

Power-law Unification in the Orbifold Case

2.1 Introductory remarks

Extra-dimensional orbifold grand unified theories (GUTs) emerged not long ago as a possibility of solving some typical problems of their 4D counterparts by exploring the geometry of the extra-dimensions. In this context, the doublet-triplet problem - the Achilles heel of many 4D GUT's - can be solved by the same mechanism that breaks the gauge symmetry and provides proton stability, namely by assigning different orbifold parities to different submultiplets of the unified multiplets [28–32]. One can also use the localisation of the KK zero-modes in different places along the extra-dimensions to explain the mass hierarchies observed in the standard model fermion sector [33], the neutrinos included [34]. In addition, it is possible to use the topology of the extra-dimensions to stabilise the Higgs mass without supersymmetry, in case the Higgs boson is a Wilson-line descending from an extra-dimensional gauge field [35].

One may want, however, to go beyond using 5D mechanisms to explain the observed low-energy phenomenology, and investigate scenarios predicting new physics at near-to-low energy scales. One exciting possibility is that gauge unification is genuinely five-dimensional, in the sense that the compactification scale lies one or more orders of magnitude below the unification scale. As discussed in the literature, such a scenario could have interesting consequences, one of which is the possibility of having a low-scale unification [36], another an exponential hierarchy between the compactification and unification scales [37]. We can understand these scenarios using a naïve approach to obtain the running of the gauge coupling. The idea is rather simple, and consists of regarding a 5D theory with UV cut-off Λ and unification scale $M_G \ll \Lambda$ as a 4D theory with a finite number of Kaluza-Klein modes, this number given as $N_{KK} \approx M_G R$ in the case of a massless 5D field. Neglecting any thresholds other than the KK modes, one gets for the low energy (4D) gauge couplings

$$\alpha_i^{-1}(M_Z) = \alpha^{-1}(M_G) + \frac{b_i}{2\pi} \ln \frac{M_G}{M_Z} + \frac{\hat{b}_i}{2\pi} \sum_n^{N_{KK}} \ln \frac{M_G}{m_n}, \tag{2.1}$$

where the index i runs over the SM gauge groups $i = 1, 2, 3$, the b_i are the b-factors of the SM (MSSM), while the $\hat{b}_i$ are the b-factors of a *generation* of KK modes with masses $\{m_n\}$. In the following we will assume that $m_n^2 = n^2/R^2$, which means that we do not allow for any significant bulk or brane masses.

The idea behind the scenario of low-scale unification is to take $M_G R \gg 1$ which means that a large number N_{KK} of KK thresholds will contribute to the running of the gauge couplings between the compactification and the unification scale. A simple calculation shows that in this case the ratio between the weak scale and the unification scale is given as

$$\begin{aligned} \ln \frac{M_G}{M_Z} \simeq & \frac{2\pi}{b_1 - b_2}(\alpha_1^{-1} - \alpha_2^{-1}) - \frac{\hat{b}_1 - \hat{b}_2}{b_1 - b_2} R M_G \\ \simeq & \ln 10^{14} - \frac{\hat{b}_1 - \hat{b}_2}{b_1 - b_2} R M_G . \end{aligned} \tag{2.2}$$

Since in the SM $b_1 - b_2 > 0$ we see that to lower the unification scale below the usual 4D value of $M_G \sim 10^{16}$GeV we must have $\hat{b}_1 > \hat{b}_2$ and $RM_G > 0$. This shows in particular that any gauge groups containing the SU(5) cannot lead to a decrease of the scale of unification, as *full* SU(5) multiplets satisfy $\hat{b}_1 = \hat{b}_2 = \hat{b}_3$. In view of this fact, and pursuing the possibility of bringing down the GUT scale through the so-called particle desert, we are thus led to the conclusion that between the unification scale and the compactification scale the SU(5) multiplets must be incomplete, and these two scales must differ by at least one order of magnitude. The problem is now to understand which mechanism breaks these multiplets at that scale. This means that to make any predictions we need a detailed knowledge of what happens at the unification scale, unlike what happens in 4D GUTs. This is related to the well-known fact that in more than four dimensions gauge theory is in general not renormalizable.

One possibility that we had in mind in work published at the beginning of these PhD studies [38] was a step-by-step compactification. This would consist of starting with a semi-simple gauge group at 6D, for instance $\mathcal{G} = \mathrm{SU}(5)$, and compactify on a torus $T^2/\mathbb{Z}_2 \times \mathbb{Z}'_2$ with sides of unequal sizes $R' \ll R$. We would use the orbifold projections in such a way as to break the SU(5) already at the scale R'^{-1}, obtaining below this scale a 5D theory with the SM gauge symmetry. Taking the unification scale to be the 6D compactication scale, $M_G \approx R'^{-1}$, and suitably choosing the bulk field content, it would then be possible to get low-scale unification by tuning some brane masses (see [38]). This idea however still remains to be worked out, and it is unclear how it should work in detail.

Another, more attractive possibility is that in addition to the orbifold breaking of the semi-simple group, e.g. $\mathcal{G}$ =SU(5)→SM, a non-vanishing VEV of a bulk scalar field breaks $\mathcal{G}$ spontaneously also in the bulk. Since in this way the SU(5) multiplets are only complete above the bulk breaking scale, *calculable* power-law unification can take place below this scale [39, 40]. In conjugation with supersymmetry, this proposal turns out to be very robust, as the one-loop perturbative result is both UV insensitive and exact up to terms suppressed by powers of $M\,R$, where M is the symmetry breaking VEV. In the following sections we will present a brief study of this proposal, in particular displaying the way a number of contributions to the 1-loop gauge coupling disappear in the limit $M\,R \gg 1$, which corresponds to the regime of power-law unification. Since we will be dealing with supersymmetric Yang-Mills theories, we will take this chance to present the superfield formulation of 5D SYM theories, which will be usefull for later chapters.

2.2 The running gauge coupling in the 5D Orbifold

The starting point for our study of the proposal of [39, 40], will be a 1-loop calculation of the radiative corrections to the gauge coupling in a U(1) gauge theory due to a scalar field of charge e. For this purpose we will use the string inspired world-line formalism. There are several reasons for using this method [27, 42]:

(i) it allows one to appreciate the 5D structure [41] of the radiative corrections, i.e. the way these corrections locate in the orbifolded direction;

(ii) in contrast to other approaches, in the world-line formalism the radiative corrections are not organized as sums over Kaluza-Klein towers but rather as sums over winding modes. It turns out that UV divergences are due only to the lowest winding mode, and not to a sum over an infinite number of modes;

(iii) this method entails a natural gauge invariant UV regularisation, which corresponds to a minimal Schwinger proper-time $T = 1/\Lambda^2$ of the world-line (see below). This regularisation does not depend on the number of extra-dimensions or on their topology.

We start by recalling the scalar contribution to the one-loop gauge coupling in D non-compact flat dimensions, which for convenience is calculated in appendix A.1. We have:

$$\Delta S^{(1)} = \frac{e^2}{3(4\pi)^{D/2}} \int d^D x_0 \, \frac{1}{4} F_{NM} F^{NM} \cdot \int_0^\infty dT e^{-m^2 T} T^{1-\frac{D}{2}}. \tag{2.3}$$

Here, the need for regularization is evident, as one observes that for $D \geq 4$ the integral over the proper-time T diverges at $T \approx 0$. Clearly, as the proper time becomes smaller the world-line is probing smaller lengths and therefore senses possible UV divergences in this regime. It is sensible, and in fact gauge-invariant, to introduce a small proper time cut-off $T_\Lambda = \Lambda^{-2}$. Doing this we get for $D \geq 4$ the following divergent pieces

$$\int_{\Lambda^{-2}}^\infty dT e^{-m^2 T} T^{1-\frac{D}{2}} \approx \begin{cases} \ln(\Lambda^2/m^2) & , D = 4 \\ \frac{2}{D-4} \Lambda^{D-4} & , D \geq 5. \end{cases} \tag{2.4}$$

This result displays a well-known feature of extra-dimensional gauge theories, namely the fact that the one-loop gauge coupling is power-law divergent [43]. In other words, for $D > 4$ gauge theories are - apart from some exceptional cases - non-renormalizable and therefore, in principle, highly UV sensitive. This fact is both worrisome and interesting, as we will explain below.

Now that we demonstrated one of the basic characteristics of gauge coupling running in non-compact extra-dimensions, let us learn what is *new* in the *orbifold* case. For our purposes it is enough to consider the orbifold $\mathbb{R}^4 \times S^1/\mathbb{Z}_2$, obtained from $\mathbb{R}^4 \times S^1$ by modding out the Z_2 symmetry which acts on $y \in S^1$ as

$$\mathbb{Z}_2 : \, y \sim -y. \tag{2.5}$$

In the world-line formalism this means that the paths defined by $y(T) \sim y(0)$ do not need to start and end in the same point. To be concrete, we have now the following paths [27, 42],

$$y_k(\tau) = y_0 + k2\pi R \frac{\tau}{T} + \bar{y}(\tau), \tag{2.6}$$

and

$$y_k^{orb}(\tau) = y_0 \left(1 - 2\frac{\tau}{T}\right) + k2\pi R \frac{\tau}{T} + \bar{y}(\tau), \tag{2.7}$$

with $\bar{y}(T) = \bar{y}(0)$ and $y_0 \equiv y(0)$. While the first of these paths are closed ones which simply wind around the circle k times, the second ones wind around the circle only *almost* k times, ending at a point related to y_0 by the orbifold symmetry. Depending on the orbifold parity of the scalar field, the contribution of $y_k^{orb}(\tau)$ paths should be added or subtracted from the periodic paths. Let us now see how eq.(A-3) should be written in this case. We have

$$\begin{aligned} \int_0^T d\tau \, \frac{1}{4}(\dot{x}_M)^2 = & \int_0^T d\tau \, \frac{1}{4} \left((\dot{\bar{x}}_\mu)^2 + \dot{\bar{y}}^2\right) \\ & + \begin{cases} (k\pi R)^2/T & , \quad y(\tau) = y_k(\tau), \\ (k\pi R - y_0)^2/T & , \quad y(\tau) = y_k^{orb}(\tau), \end{cases} \end{aligned} \tag{2.8}$$

where x_μ is the 4D coordinate. We consider again a background with 4D constant field-strength, $A_\mu = \frac{1}{2} x^\nu F_{\nu\mu}$, $A_y = 0$ [1]. The terms in the path-integral involving the gauge field are purely 4D and therefore eq.(2.3) is only modified in its y_0-dependence,

$$\begin{aligned} \Delta S^{(1)} = & \frac{e^2}{6(4\pi)^{5/2}} \int d^4 x_0 \int_{-\pi R}^{\pi R} dy_0 \, \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \cdot \\ & \int_0^\infty \frac{dT}{T^{3/2}} \, e^{-m^2 T} \sum_{k \in \mathbb{Z}} \left[e^{-\frac{(k\pi R)^2}{T}} \pm e^{-\frac{(k\pi R - y_0)^2}{T}} \right], \end{aligned} \tag{2.9}$$

the $\pm$ sign staying for different orbifold parities.

We focus first on the *divergent* pieces. It is clear that these come from the contributions with $k = 0$ and $y_0 = k\pi R$. While the $k = 0$ contribution is (up to a factor of 2) exactly the same as we get in the uncompactified theory, as should be expected from taking the limit $R \to \infty$ in the

[1] To obtain a 4D constant field-strength one could also add constants to A_μ and A_y. While in the former case this would have *no* effect, because $\int_0^T d\tau \, \dot{x}_\mu = 0$, in the later case a non-vanishing Wilson-line A_y would have an observable effect due to the non-trivial topology of the orbifold. In fact we have $\int_0^T d\tau \, \dot{y}_k = k2\pi R$ and $\int_0^T d\tau \, \dot{y}_k^{orb} = k2\pi R - 2y_0$. This means that only Wilson-lines differing by $(eR)^{-1}$ are physically equivalent. We will leave a discussion of the 1-loop effective action of A_y to chapter 5.

above expression, the other divergences are localised at the fix-point branes. We can use now the fact that

$$\lim_{T\to 0} e^{-\frac{(k\pi R-y_0)^2}{T}} = \sqrt{\pi T}\delta(y_0-k\pi R)+\mathcal{O}(T^{\frac{3}{2}}), \tag{2.10}$$

to obtain the brane localised divergences. In this way, we get the following result

$$\begin{aligned}\Delta S^{(1)} =&\frac{e^2}{3(4\pi)^{5/2}}\int d^4x_0\int_{-\pi R}^{\pi R} dy_0\,\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\left(\Lambda\pm\sqrt{\pi}\ln\frac{\Lambda}{m}\left(\delta(y_0)+\delta(y_0-k\pi R)\right)\right)\\ &+\text{finite corrections,}\end{aligned} \tag{2.11}$$

which neatly displays the 5D divergence *structure* of the U(1) gauge coupling, with its bulk and brane contributions. It is interesting to compare this result with the one obtained in appendix A, using Pauli-Villars regularisation:

$$\Delta S^{(1)} = \frac{e^2}{3(4\pi)^{5/2}}\int d^4x_0\int_{-\pi R}^{\pi R} dy_0\,\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\left(\frac{\sqrt{\pi}c}{5}\Lambda\pm\sqrt{\pi}\ln\frac{\Lambda}{m}\left(\delta(y_0)+\delta(y_0-k\pi R)\right)\right), \tag{2.12}$$

with $c\approx 1$. It is notorious that the linear divergence appears with *different* coefficients in these two different calculations. This reflects the fact that gauge theories in 5D dimensions are UV sensitive and therefore different UV completions/regularisations can give completely different low-energy results. Of course, one could redefine Λ in such a way that the difference becomes only a logarithmic brane effect. However this does not change the fact that unless we know what the UV completion is, we cannot say what the relation is between the *cut-offs* of the different multiplets in the theory. This fact is even more evident if one recalls that in dimensional regularisation the linear divergence is absent (see e.g. [44]).

Apart from the divergent contributions that we displayed above, we are interested also on the m-dependent *finite* pieces that are going to play a crucial rôle in the subsequent discussion. We have to use

$$\int_0^\infty \frac{dT}{T^{3/2}}\,e^{-m^2T}\,e^{-\frac{(k\pi R-y_0)^2}{T}} = \frac{\sqrt{\pi}}{|k\pi R-y_0|}e^{-2|m|\,|k\pi R-y_0|}, \tag{2.13}$$

to show that the the finite contributions are:

$$\Delta g_{fin}^{-2} = \frac{e^2}{12(4\pi)^2}\left[-2|m|+\sum_{k\neq 0}\frac{1}{|k\pi R|}e^{-2|k\pi Rm|}\pm\sum_k\frac{1}{|k\pi R-y_0|}e^{-2|m|\,|k\pi R-y_0|}\right]. \tag{2.14}$$

The most interesting feature of this result is perhaps the fact that for a large (bulk) mass, $|m|R\gg 1$, or in the uncompactified limit, the dominant finite contribution is just

$$\lim_{|m|R\to\infty}\Delta g_{fin}^{-2} = -\frac{e^2}{6(4\pi)^2}|m|. \tag{2.15}$$

All the other terms are exponentially suppressed in this limit.

2.3 5D SUSY and the running of couplings

In the following we briefly describe 5D supersymmetric Yang-Mill theories coupled to matter. For more details please see [8, 22, 24]. The field content of such a theory consists of two types of multiplets of $\mathcal{N} = 2$ supersymmetry[2], namely vector multiplets, which we denote by $\mathbb{V}$, and hypermultiplets, which we denote by $\mathbb{H}$. An off-shell vector multiplet consists of a scalar M (also known as the vector scalar), an $\mathrm{SU}(2)_R$ doublet of fermions Ω^i, the 5D gauge boson A_M, and an $\mathrm{SU}(2)_R$ triplet of auxiliary scalars $\vec{Y}$:

$$\mathbb{V} = (M,\ \Omega^i,\ A_M,\ \vec{Y}). \tag{2.16}$$

All fields of the vector multiplet transform in the adjoint representation. In the same way, an off-shell hypermultiplet consists of an $\mathrm{SU}(2)_R$ doublet of scalars $\mathcal{A}_i$, a Dirac spinor ζ, and a doublet of auxiliary fields $\mathcal{F}_i$:

$$\mathbb{H} = (\mathcal{A}_i,\ \zeta,\ \mathcal{F}_i). \tag{2.17}$$

Since in orbifold compactifications at least one of the supersymmetries is broken, it is usefull to ensemble the components of the $\mathcal{N} = 2$ multiplets in supermultiplets of the unbroken $\mathcal{N} = 1$ supersymmetry. This is of great advantage if one has in view the introduction of couplings localised on the fix-point branes, where one has 4D $\mathcal{N} = 1$ supersymmetry (or less). Out of a 5D vector multiplet one obtains by reduction to the $\mathcal{N} = 1$ supersymmetry a vector superfield V plus a chiral superfield Σ [8, 22–24]:

$$V = -\theta\sigma^\mu\bar{\theta}A_\mu + \theta^2\bar{\theta}\,2i\bar{\omega}^2 - \bar{\theta}^2\theta\,2i\omega^2 + \frac{1}{2}\theta^2\bar{\theta}^2(2Y^3 - D_5 M), \tag{2.18}$$

$$\Sigma = \frac{1}{2}(M + iA_y) + \theta\,2i\omega^1 + \theta^2(Y^1 - iY^2), \tag{2.19}$$

where ω^1, ω^2 are two-component Weyl spinors obtained from the four component Majorana gauginos Ω^i (see appendix B.3), and $D_5 M$ is the gauge covariant derivative. Similarly, the 5D hypermultiplet reduces to two chiral superfields H and H^c, given by

$$H = \mathcal{A}_1 + \theta\,2i\eta^1 + \theta^2\,(D_5\mathcal{A}_2 + M\mathcal{A}_2 - i\mathcal{F}_2), \tag{2.20}$$

and

$$H^c = \mathcal{A}_2^+ - \theta\,2i\eta^2 + \theta^2\,(-D_5\mathcal{A}_1^+ + \mathcal{A}_1^+ M + i\mathcal{F}_1^+), \tag{2.21}$$

where η^1, η^2 are two-component Weyl spinors obtained from the four-component hyperino ζ. We will introduce here also the radion superfield

$$T = e_y^5 + i\kappa B_y + \theta\,\kappa\psi_y + \theta^2\kappa F_T, \tag{2.22}$$

[2]Since in flat uncompactified five dimensions $\mathcal{N} = 2$ corresponds to the minimal amount of supersymmetry, it is also common to speak of 5D $\mathcal{N} = 1$ supersymmetry.

even though it will play no rôle in the present analysis. Here $\kappa = M_5^{-\frac{3}{2}}$ and B_y, ψ_y are components of the 5D graviphoton and the 5D gravitino, and we take $e_y^5 = 1$.

As we said, the components of the 5D vector multiplet transform in the adjoint representation, therefore under a non-Abelian gauge transformation V and Σ transform as [22–24]

$$e^V \to e^{\Lambda} e^V e^{\Lambda^+}, \qquad \Sigma \to -e^{\Lambda}(\partial_y - \Sigma)e^{-\Lambda}. \tag{2.23}$$

Out of V and Σ one can build now the following superfield

$$\mathcal{V}_5 = 2\frac{\Sigma + e^V \Sigma^+ e^{-V} + e^V \partial_y e^{-V}}{T + T^+}, \tag{2.24}$$

which will transform as

$$\mathcal{V}_5 \to e^{\Lambda} \mathcal{V}_5 e^{-\Lambda}. \tag{2.25}$$

The point of introducing this new superfield is that even though it is not hermitian [24], $\mathcal{V}_5^+ = e^{-V}\mathcal{V}_5 e^V$, it can be used to write down the following hermitian gauge invariants,

$$\operatorname{tr} \mathcal{V}_5^n, \tag{2.26}$$

which will be usefull to obtain gauge invariant superspace actions. Note also that in the Abelian case $\mathcal{V}_5$ is itself hermitian and gauge invariant.

It is a well known fact that the interactions of the vector multiplet of $\mathcal{N} = 2$ SUSY are completely determined by a cubic function, the so-called prepotential [52]:

$$\mathcal{F}(M) = a + b_I\, M^I + \frac{1}{2} c_{IJ}\, M^I M^J + \frac{1}{6}\tilde{d}_{IJK}\, M^I M^J M^K, \tag{2.27}$$

where c_{IJ} and $\tilde{d}_{IJK}$ are totally symmetric coefficients. We will show later, in chapter 3, how prepotentials of rigid SYM theory is obtained in the $M_5 \to \infty$ limit of 5D supergravity (see section 3.2). In the case of a single non-Abelian gauge symmetry, the prepotential can also be written as

$$\mathcal{F}(M) = c \operatorname{tr} M^2 + \tilde{d} \operatorname{tr} M^3, \tag{2.28}$$

with $c_{IJ} = c\cdot\delta_{IJ}$ and $\tilde{d}_{IJK} = 6\tilde{d}\cdot d_{IJK}$, where $d_{IJK} = \frac{1}{2}\operatorname{tr}\{t_I, t_J\}t_K$. In terms of this prepotential, and in the Abelian case, the vector part of the 5D superspace Lagrangian reads

$$\begin{aligned}\mathcal{L}_V = &\frac{1}{4}\int d^2\theta\, \mathcal{F}_{IJ}(2\Sigma/T)\, T\, \mathcal{W}^{\alpha I}\mathcal{W}_\alpha^J + \text{h.c.} + \int d^4\theta\, (T + T^+)\, \mathcal{F}(\mathcal{V}_5)\\ &- \frac{1}{12}\int d^4\theta\, \mathcal{F}_{IJK}\, \partial_y V^I \Omega(V^J, V^K),\end{aligned} \tag{2.29}$$

where we introduced the Chern-Simons superfield (see [45] and references therein)

$$\Omega(V^J, V^K) = -(D^\alpha V^J\, \mathcal{W}_\alpha^K + \bar{D}_{\dot\alpha} V^J\, \bar{\mathcal{W}}^{\dot\alpha K} + V^J D^\alpha \mathcal{W}_\alpha^K). \tag{2.30}$$

The non-Abelian case differs solely in regard to the last term in the Lagrangian eq.(2.29), a term which unfortunately is still missing. (We suspect that it is of the form $\mathrm{tr}\{e^V \partial_y e^{-V} \cdot \Omega(V,V)\}$, with $\Omega(V,V)$ a superfield transforming as $\Omega \to e^{\Lambda}\,\Omega\, e^{-\Lambda}$.)

We can also write a superspace Lagrangian for a hypermultiplet charged under Abelian gauge symmetries, with charges $\{q_I\}$ (i.e. transforming as $H \to e^{q_I \Lambda^I} H$), and mass m:

$$\begin{aligned}\mathcal{L}_H = &\int d^4\theta\,(T+T^+)\,(H^+ e^{-q_I V^I} H + H^c e^{q_I V^I} H^{c+})\\ &+2\int d^2\theta\, H^c(\partial_y - q_I \Sigma^I - m\,T)H + \text{h.c.}\end{aligned} \tag{2.31}$$

The generalization to the non-Abelian case is obtained with the substitution $q_I V^I \to V = V^I T_I$, and similar for Σ.

It is clear from the vector Lagrangian that $\mathcal{F}_{IJ}$, i.e. the second derivative of the prepotential, determines the gauge coupling. We assume that it is possible to diagonalize c_{IJ}, so that we can write

$$\mathcal{F}_{IJ}(M) = \frac{1}{g_I^2}\delta_{IJ} + \tilde{d}_{IJK}\, M^K, \tag{2.32}$$

and the *holomorphic* gauge coupling is then

$$\mathcal{F}_{IJ}(2\Sigma)\big|_{\theta=0} = \frac{1}{g_I^2}\delta_{IJ} + \tilde{d}_{IJK}\,(M^K + iA_y^K). \tag{2.33}$$

It becomes now clear the similarity between this expression for the tree-level gauge coupling and the cut-off independent part of the 1-loop correction that we obtained in the previous section. In fact a non-vanishing VEV of a vector scalar M acts as a mass for the charged hypermultiplets, $|m| = |\frac{1}{2}qM|$, so that the finite correction due to a single scalar, given by eq.(2.15), reads now

$$\Delta\frac{1}{g^2_{scalar}} = -\frac{q^2}{48(4\pi)^2}|qM + 2m|. \tag{2.34}$$

Obviously, one has to take also the other hyperscalar and the hyperinos into account. If we do this we get[3]

$$\Delta\frac{1}{g^2_{hyper}} = -\frac{q^2}{8(4\pi)^2}|qM + 2m|, \tag{2.35}$$

which differs from the result of ref. [40] just by a factor of 2, due to the orbifolding. This corresponds to a 1-loop prepotential of the form

$$\mathcal{F}(M) = \mathcal{F}_{cl}(M) - \frac{1}{96\pi^2}|\tfrac{1}{2}q_I M^I + m|^3. \tag{2.36}$$

[3]This can be obtained by recalling that the contribution of a 5D hypermultiplet to the beta function is $2\cdot 3 = 6$ times the contribution of one of its two complex scalar components. This result can also be obtained by calculating the scalar 1-loop contributions to the renormalization of the term $D^2/4g^2$.

The above expression is only complete in the Abelian case. We are however interested in the 1-loop gauge coupling for a non-Abelian SYM theory with semi-simple gauge group $\mathcal{G}$ such as the SU(5), in the case it is broken spontaneously in the bulk by switching on the VEV of a vector scalar. This is the way we intend to get power-law corrections to the gauge couplings [39, 40]. Including the contributions of both the vector and hyper sectors, the full 1-loop prepotential in the *Coulomb branch* is [40, 46–49]

$$\mathcal{F}(M) = \mathcal{F}_{cl}(M) + \frac{1}{96\pi^2}\left[\sum_{\alpha} |\alpha_I M^I|^3 - \sum_f \sum_\lambda |\lambda_I M^I + m_f|^3\right], \qquad (2.37)$$

where by Coulomb branch one denotes the set of vacua where the adjoint scalars M^I in the 5D vector multiplet have non-zero VEVs in the Cartan sub-algebra, i.e. $M = M^I H_I$ with H_I the Cartan generators of $\mathcal{G}$. In this case the gauge symmetry group $\mathcal{G}$ is generally broken down to a $\mathrm{U}(1)^r$, $r = \mathrm{rank}(\mathcal{G})$. Some words on the notation: f runs over different *flavours*, α runs over different root vectors α_I of $\mathcal{G}$, and λ over the weight vectors. The classical piece, $\mathcal{F}_{cl}(M)$ is the one given in eq.(2.28):

$$\mathcal{F}_{cl}(M) = \frac{1}{g^2}\,\mathrm{tr}\,M^2 + \tilde{d}\,\mathrm{tr}\,M^3. \qquad (2.38)$$

Some remarks are now in order. The first concerns the observation that the classical and 1-loop pieces have the same form - both are cubic functions - but only locally. Due to the modulus-signs, the 1-loop prepotential is only locally holomorphic, the massive vector contribution being allways positive and the hyper contribution always negative. A second remark concerns the classical parameter $\tilde{d}$, which determines the strength of the Chern-Simons term. In the orbifold case, $\tilde{d}$ is going to be an odd parameter, $\tilde{d} = \gamma\epsilon(y)$, in case the orbifold transformations do not break other gauge symmetries than the VEV of the vector scalar does, in fact the case of interest. (γ can obviously also be zero for certain $\mathcal{G}$'s.) Since such a Chern-Simons term induces anomalies at the branes, the value of γ must be tuned to cancel the brane-localised anomalies. In other words γ is fixed by the anomalies constraints. Finally let us note that the above 1-loop prepotential is *quantum exact*, at least in the limit the radius is much larger than the symmetry breaking vector scalar VEV M_V, i.e. $RM_V \gg 1$. We will give a hand-waving argument for this in the following section.

We come now to the phenomenological interest of eq.(2.37), which is the reason for this analysis. As pointed out by Hebecker and Westphal [40], this expression gives (the leading effect of) the power-law running due to the non-vanishing VEV of a vector scalar. Let us be more precise and consider the SU(5) case. By orbifolding one can break the SU(5) down to the SM group $\mathrm{SU}(3)_c\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$. Under the $\mathbb{Z}_2$ orbifold symmetry the chiral superfields descending from the 5D vector multiplets will therefore have the following parities

$$\Sigma_c,\ \Sigma_L,\ \Sigma_{U(1)_Y} \sim -, \qquad \Sigma_X,\ \Sigma_Y \sim +. \qquad (2.39)$$

If we now turn on a VEV of $\Sigma_{U(1)_Y}$, the SU(5) will also be spontaneously broken down to the SM in the bulk, with the breaking scale given by that VEV. We can then use eq.(2.37) to get the exact power-law running, assuming that the symmetry breaking VEV is larger than the compactification scale. As it was shown in [39, 40], by a suitable choice of the bulk hypermultiplets and their masses it is not difficult to find scenarios with power-law *unification*.

The remaining question is to understand how the odd superfield $\Sigma_{U(1)_Y}$ gets its VEV. It is well known that brane-localised Fayet-Iliopoulos terms have the effect of inducing such odd vector scalar VEVs. The problem is that in the case of a bulk SU(5) it is not clear how to write such terms, even though the SU(5) is reduced to the SM at the branes, which includes an U(1) factor. The only possibilty would be that this $\mathrm{U}(1)_Y \subset \mathrm{SU}(5)$ would gauge an U(1) subgroup of the SU(2) R-symmetry which clearly is not possible. On the other hand, the above prepotential may already allow for a vacuum solution with non-vanishing $\Sigma_{U(1)_Y}$. In fact, the D-flatness condition implies

$$\partial_y \mathcal{F}_I(M) = 0, \tag{2.40}$$

and since for an odd M_I also $\mathcal{F}_I(M)$ is odd, we get

$$\mathcal{F}_{U(1)_Y}(M) = 0, \tag{2.41}$$

which has in general two solutions, a trivial one with $M_{U(1)_Y} = 0$, and a stepwise VEV $M_{U(1)_Y} = \epsilon(y)|M_{U(1)_Y}|$. It must be checked whether this solution is of phenomenological interest or not. If not one would have to rely on other mechanisms to give $M_{U(1)_Y}$ a VEV without breaking supersymmetry (see also the discussion in [40]).

2.4 Exact Results in 5D Supersymmetric Theories

One of the most striking aspects of supersymmetric theories is the existence of non-renormalization theorems and analytic results on their strong coupling behaviour. This is due to the holomorphicity of superpotentials and prepotentials, as functions of the chiral superfields and the (holomorphic) couplings, combined with the symmetries and dualities of the theories. The Seiberg-Witten solution of low-energy 4D $\mathcal{N} = 2$ super Yang-Mills (SYM) theory [50, 51] is a beautiful example of this. It partially relies upon the holomorphicity of the prepotential $\mathcal{F}(\Phi)$ which determines the couplings in the vector sector of the $\mathcal{N} = 2$ SYM theories. This very same fact lies on the base of the analysis by Intriligator, Morrison and Seiberg [47–49] of the exact effective action of 5D SYM theories.

As we already pointed out, in 5D *uncompactified* SYM theories, in the Coulomb branch, the exact quantum prepotential is determined at the one-loop level, recieving no non-perturbative corrections [46]. The fact that the 1-loop quantum prepotential is not corrected by non-perturbative effects, is the same as to say that there are no instanton contributions. Let us give a *hand waving* argument for this by uplifting the 4D result for *pure* $\mathcal{N} = 2$ SYM in the Coulomb branch to 5D dimensions compactified on a circle of size $2\pi R$. We consider the simplest gauge group, i.e. $\mathcal{G}$ =SU(2). The 4D result, with $M = \frac{1}{2} a \sigma_3$, is [52]

$$\tau^{1loop}(a) = \frac{1}{4\pi^2} \ln \frac{a^2}{\Lambda^2} + \sum_{k=1}^{\infty} \tau_k \left(\frac{\Lambda}{a}\right)^{4k}, \tag{2.42}$$

where the holomorphic coupling is

$$\tau(a) \equiv \frac{\partial^2 \mathcal{F}}{\partial a^2} = \frac{1}{g^2(a)} + i\frac{\theta(a)}{8\pi^2}. \tag{2.43}$$

As we pointed out above, $\tau^{1loop}(a)$ is the only quantum correction. It includes a perturbative piece and the instanton contributions, the k'th term in the sum being a contribution with k instantons. In a *tour de force* [50, 51], Seiberg and Witten have shown how to calculate the infinitely many coefficients τ_k that are different from zero. To uplift (2.42) to the 5D theory compactified in the circle, we must note that while doing this a new symmetry arises, namely

$$a \sim a + i\frac{n}{R}, \quad n \in \mathbb{Z}. \tag{2.44}$$

This symmetry corresponds to 5D non-periodic gauge transformations of the Wilson-line Im(a), which nevertheless respect the boundary conditions of the integrated out fields (see also sec.5.2). That means that the 5D $\tau(a)$ must be invariant under this shift symmetry:

$$\tau_{5D}^{1loop}(a + iR^{-1}) = \tau_{5D}^{1loop}(a). \tag{2.45}$$

On the other hand it must reduce to the 4D case in the $R \to 0$ limit. We achieve this by performing the following replacements:

$$\ln\frac{a^2}{\Lambda^2} \to \sum_{n=-\infty}^{+\infty} \ln\frac{\left(a + i\frac{n}{R}\right)^2}{\Lambda^2} - C = \ln\left[\frac{\sinh(\pi R a)}{\pi R \Lambda}\right]^2, \tag{2.46}$$

and

$$\sum_{k=1}^{\infty} \tau_k \left(\frac{\Lambda}{a}\right)^{4k} \to \sum_{n=-\infty}^{+\infty}\sum_{k=1}^{\infty} \tau_k \left(\frac{\Lambda}{a + i\frac{n}{R}}\right)^{4k} = \sum_{k=1}^{\infty} \tau_k \frac{\Lambda^{4k}}{(4k-1)!}\frac{d^{4k-2}}{da^{4k-2}}\left[\frac{\sinh(\pi R a)}{\pi R}\right]^{-2}, \tag{2.47}$$

where C is a suitably chosen constant. Note that by construction these functions are invariant under the shift symmetry. We have therefore

$$\tau^{1loop}(a) = \frac{1}{4\pi^2}\ln\left[\frac{\sinh(\pi R a)}{\pi R \Lambda}\right]^2 + \sum_{k=1}^{\infty} \tau_k \frac{\Lambda^{4k}}{(4k-1)!}\frac{d^{4k-2}}{da^{4k-2}}\left[\frac{\sinh(\pi R a)}{\pi R}\right]^{-2}. \tag{2.48}$$

That our approach is quite reasonable can be recognized from the fact that the perturbative part of $\tau^{1loop}(a)$ can be written as

$$\frac{1}{2\pi^2}\ln 2\sinh(\pi R a) = \frac{2\pi R}{2\pi^2}\left[2|a| - \sum_{k\neq 0}\frac{e^{-2\pi R|ka|}}{\pi R|k|}\right], \tag{2.49}$$

agreeing perfectly with the form of the perturbative result, eq.(2.14). There is naturally a difference in sign and prefactor, due to the fact that here we consider the vector sector of the SU(2), while there the charged scalar in the U(1) theory.

Since for $R \to 0$ we have

$$\lim_{R\to 0} \frac{\sinh(\pi R a)}{\pi R} = |a|, \tag{2.50}$$

one sees that in the zero radius limit we indeed recover the 4D result, as expected. On the other hand, in the decompactification limit, $R \to \infty$, we get

$$\tau_{5D}^{1loop} = \frac{\tau^{1loop}(a)}{2\pi R} \to \frac{|a|}{4\pi^2} + \sum_{k>1} \frac{\tau_k}{(4k-1)!} \frac{(2\pi R\Lambda)^{4k}}{2\pi R} e^{-2\pi R a} \to \frac{|a|}{4\pi^2}. \tag{2.51}$$

One sees that in this limit the instanton contributions decouple, and eq.(2.37) is recovered by integrating $\tau(a)$ twice. This result can be explained by the fact that there are no known instanton solutions in uncompactified 5D theories [53]: in the compactified theory, 4D instantons can be made to wrap the 5th dimension, but as R increases instantons smaller than R become unstable so that in the $R \to \infty$ limit the instantons eventually "evaporate".

Eq.(2.51) gives - in our view - a convincing argument for the fact that in 5D gauge SYM theories compactified on the circle perturbation theory is reliable for $a, \Lambda \gg R^{-1}$. In the orbifold case we expect the same result, i.e. that all non-perturbative and non-local effects disappear in this limit, as the orbifold differs from the circle only at low scales and brane effects should be suppressed by the large bulk. Yet, it would be interesting to study these issues for the orbifold case in more detail.

Chapter 3

Superfield Approach to 5D Conformal Supergravity

3.1 Off-Shell 5D Supergravity: an Overview

The construction of off-shell local supersymmetric 5D theories using the framework of conformal supersymmetry [10–14, 54, 55] proceeds in the following way: Instead of considering only local supersymmetry and local Poincaré transformations, one considers an enlarged set of local transformations, which is obtained by *grading* the algebra of conformal transformations. In this way, in addition to translations ($\mathbf{P}_a$) and Lorentz transformations ($\mathbf{M}_{ab}$), one has dilatations ($\mathbf{D}$) and special conformal transformations ($\mathbf{K}_a$), and besides supersymmetric transformations ($\mathbf{Q}^i$) ($i = 1, 2$) one has so-called *special* supersymmetric transformations ($\mathbf{S}^i$). There is also an $\mathrm{SU}(2)_R$ symmetry ($\mathbf{U}_{ij}$) under which the fermionic generators transform as doublets and the bosonic ones as singlets. The corresponding gauge fields are:

$$e_\mu^a,\ \omega_\mu^{ab},\ b_\mu,\ f_\mu^a,\ \psi_\mu^i,\ \phi_\mu^i,\ V_\mu^{ij}. \tag{3.1}$$

The number of bosonic degrees of freedom (d.o.f.) exceeds by far the number of fermionic ones. This, and the fact that the symmetries are internal symmetries which are in no connection with the reparametrizations of the manifold, makes it necessary to impose a set of constraints which make the fields ω_μ^{ab}, ϕ_μ^i, f_μ^a, dependent from the other gauge fields. These unconstrained fields, plus the auxiliary fields needed to close the algebra off-shell, build the so-called *Weyl*-multiplet:

$$(e_\mu^a,\ b_\mu,\ \psi_\mu^i,\ V_\mu^{ij},\ v_{ab},\ \chi^i,\ D), \tag{3.2}$$

where v_{ab} is a real anti-symmetric boson, χ^i is an $\mathrm{SU}(2)_R$ Majorana fermion, and D is a real scalar. Now, to build a physically consistent theory of 5D supergravity one needs $(48+48)$ off-shell degrees of freedom. To count the supergravity d.o.f. of the Weyl multiplet we must note that after breaking the dilatation invariance, $b_\mu = 0$. This multiplet has therefore only $(33+40)$ d.o.f. and compensator multiplets must be introduced, which account for the missing degrees of freedom. These are (in the minimal version) a $U(1)$ vector multiplet ($\mathbb{V}^0$) and a hypermultiplet ($\mathbb{H}^\alpha$). While the vector multiplet fixes the superconformal symmetries ($\mathbf{D}$, $\mathbf{K}_a$, $\mathbf{S}^i$) down to

Poincaré supersymmetry, the hypermultiplet fixes the $\mathrm{SU}(2)_R$ symmetry.

In addition to the compensator vector multiplet $\mathbb{V}^0$ one may couple n_V other vector multiplets $\mathbb{V}^I$ to SUGRA ($I = 1, \ldots, n_V$). An off-shell 5D vector multiplet consists of a scalar M, an $\mathrm{SU}(2)_R$ doublet of fermions Ω^i, a gauge field W_μ and an $\mathrm{SU}(2)_R$ triplet of auxiliary scalars Y^{ij}:

$$\mathbb{V}^I = (M,\ \Omega^i,\ W_\mu,\ Y^{ij})^I. \tag{3.3}$$

All fields transform in the adjoint representation of the gauge group G, so that for instance $M = M^I t_I$ where $\{t_I\}$ are the generators of the gauge group.[1] The fixing of the $\mathbf{D},\ \mathbf{S}^i,\ \mathbf{K}_a$ is achieved by imposing constraints on the scalars and on the gauginos (see also appendix B):

$$\mathcal{N}(M) = \kappa^{-2}, \quad \mathcal{N}_I(M)\Omega^I = 0, \quad \hat{\mathcal{D}}_a\mathcal{N}(M) = 0, \tag{3.4}$$

where $\kappa^{-2} \equiv M_5^3$ and the *norm function* $\mathcal{N}(M)$ is given by

$$\mathcal{N}(M) = \kappa c_{IJK} M^I M^J M^K. \tag{3.5}$$

Here $I, J, K = 0, \ldots, n_V$, and the coefficients c_{IJK} are real and totally symmetric.

The (off-shell) 5D hypermultiplet $\mathbb{H}^\alpha$ consists of two scalars $\mathcal{A}_i^\alpha$, a Dirac spinor ζ^α, and two auxiliary fields $\mathcal{F}_i^\alpha$:

$$\mathbb{H}^\alpha = (\mathcal{A}_i^\alpha,\ \zeta^\alpha, \mathcal{F}_i^\alpha)\ . \tag{3.6}$$

Here $i = 1, 2$ is the $SU(2)_R$ index. The superscript α has an even number of values, $\alpha = 1, 2, \cdots, 2r$, and describes the representation of a subgroup G' of the gauge group G to which $\mathbb{H}$ couples. G' includes the $U(1)$ gauge groups to which we will restrict for simplicity in our work. Among the considered $U(1)$ gauge groups there is the $U(1)_Z$ corresponding to the graviphoton gauge supermultiplet, $\mathbb{V}^0$, which gauges the central Z-charge. The hypermultiplet gives an infinite dimensional representation of the $U(1)_Z$. The index α is raised and lowered with a G' invariant tensor $\rho_{\alpha\beta}$ ($\rho^{\gamma\alpha}\rho_{\gamma\beta} = \delta^\alpha_\beta$). At least one hypermultiplet is unphysical - it is a compensator, needed for gravity to have canonical form and to fix the $SU(2)_R$ symmetry. In order to clarify the notation, let us consider the kinetic term for the lowest scalar components $D_\mu \mathcal{A}_i^{\bar\alpha} D^\mu \mathcal{A}_\alpha^i = D_\mu \mathcal{A}_i^\alpha d_\alpha^\beta D^\mu \mathcal{A}_\beta^i$, where d_α^β is a metric matrix. The scalar components satisfy the reality condition

$$(\mathcal{A}_{\alpha i})^* = \mathcal{A}^{\alpha i} = \rho^{\alpha\beta}\epsilon^{ij}\mathcal{A}_{\beta j}\ , \tag{3.7}$$

and similar for the $\mathcal{F}$ components. In the standard representation we have [56]

$$d = \mathrm{Diag}(\mathbf{1}_{2\mathrm{p}},\ -\mathbf{1}_{2\mathrm{q}})\ , \qquad \rho = \epsilon \otimes \mathbf{1}\ , \tag{3.8}$$

where $\mathbf{1}_{2p}$ corresponds to the compensators, while $\mathbf{1}_{2q}$ to the physical hypermultiplets. For the former (as FKO) we use the index $\underline{\alpha}$, for the latter $\tilde\alpha$. In this way the compensator hypermultiplet will be denoted by $\mathbb{H}^{\underline{\alpha}}$ and the physical one by $\mathbb{H}^{\tilde\alpha}$. With these conventions the kinetic term of the scalar components will have the form $D_\mu \mathcal{A}_i^{\bar\alpha} D^\mu \mathcal{A}_\alpha^i = -|D_\mu \mathcal{A}_{\underline{\alpha} i}|^2 + |D_\mu \mathcal{A}_{\tilde\alpha i}|^2$ and one sees that the compensators $\mathcal{A}_{\underline{\alpha} i}$ are unphysical because of their negative kinetic terms. As

[1]Here the t_I are hermitian. The results of FKO are obtained with $t^I = -it^I_{\mathrm{FKO}}$ and $[A, B]^I = -i[A, B]^I_{\mathrm{FKO}}$.

one can read from the Lagrangian, eq.(B-9) in appendix B, after integrating out the auxiliary field D', the coupling $D'(\mathcal{A}^2 + 2\mathcal{N})$ will impose the constraint $\mathcal{A}^2 = -2\mathcal{N} = -2\kappa^{-2}$ on the hypermultiplets. This VEV breaks the $SU(2)_R$ as advertised before.

The field content we just described can be found in appendix B, where the fields are classified according to their orbifold parities[2]. The off-shell component action for 5D SUGRA of FKO can be found in the same appendix.

3.2 Vector Multiplets: $\mathcal{N} = 1$ Supermultiplets and Superspace Action

It is a well known fact that from a 4D point of view 5D $\mathcal{N} = 1$ supersymmetry corresponds to $\mathcal{N} = 2$ supersymmetry and that multiplets of rigid 5D supersymmetry reduce to pairs of (rigid) 4D supermultiplets. This also means that in the rigid case the components of the 5D supermultiplets can be assembled in pairs of superfields and one can use all the power of superspace to write down 5D supersymmetric actions in a rather straightforward way, as was done in [21–24, 45]. On the other hand, a systematic study of the reduction of multiplets of (local) 5D conformal supersymmetry to 4D ones was given in [14], with the intention to formulate interaction terms on the branes. Here we recall the results for the vector multiplet and identify the *radion multiplet*. Using these $\mathcal{N} = 1$ multiplets we then write down an 5D action for the Abelian vector multiplets, including the radion multiplet. The hypermultiplet will be handled in section 3.3.

3.2.1 Reduction of the vector multiplet and radion multiplet

Before we proceed a word on the notation. In this paper we will use two different ways of representing the supermultiplets of $\mathcal{N} = 1$ supersymmetry. The one is a component notation where the fermions are four-component Majorana spinors. It has the advantage that it is the one used in refs. [14, 57], where rules are given for the multiplication of multiplets and the construction of actions invariant under 4D superconformal symmetries. The other is the superfield notation with two-component Weyl spinors, which is rather usefull in applications where one does not focus on 4D conformal gravity. These two notations are, of course, equivalent and the way one switches between them is explained in appendix B.

The 5D vector multiplet reduces to a 4D gauge multiplet $V^I \equiv (A^I_{\bar{\mu}}, \lambda^I, D^I)$ plus a chiral multiplet $\Sigma^I \equiv (\phi^I, \chi^I, F^I_\phi)$. The vector multiplet has Weyl and chiral weights $(w, n) = (0, 0)$ and is given by [14]

$$V^I = (W_{\bar{\mu}}, 2\Omega_+, 2Y^3 - \hat{\mathcal{D}}_5 M)^I, \tag{3.9}$$

[2]Besides the multiplets mentioned above, linear and tensor multiplets can be introduced in 5D SUGRA. These multiplets and their properties (allowing to embed vector and hypermultiplets into them) turn out to be very useful and powerful for building invariant actions [10–14, 54]. However, we do not need to consider these multiplets here.

where

$$\Omega_+ \equiv \Omega_R^1 + \Omega_L^2, \tag{3.10}$$

and the covariant derivative of M is

$$\hat{\mathcal{D}}_5 M = (\partial_5 - b_5)M - ig[W_5, M] - 2\kappa i\bar{\psi}_5\Omega. \tag{3.11}$$

(After fixing the dilatation symmetry one has $b_5 = 0$, see appendix B). Note that since we are going to consider Abelian vector multiplets only, the commutator in the last expression vanishes. In the rigid limit ($\kappa \to 0$) the gravitino term drops in (3.11) and V^I becomes the vector multiplet identified in [8].

The chiral multiplet $\Sigma^I = (\phi^I, \chi_R^I, F_\phi^I)$, with weights $(0,0)$, is given by [14]:

$$\begin{array}{rcl} \phi^I &=& \frac{1}{2}(e_y^5 M^I - iW_y^I), \\ \chi^I &=& 2e_y^5\gamma_5\Omega_-^I - 2i\kappa\psi_{y-}M^I, \\ F_\phi^I &=& -e_y^5(Y^1 + iY^2)^I - i\kappa M^I(V_y^1 + iV_y^2) + i\kappa\bar{\psi}_{y-}(1+\gamma_5)\Omega_-^I, \end{array} \tag{3.12}$$

where

$$\Omega_- \equiv i(\Omega_R^2 + \Omega_L^1). \tag{3.13}$$

A rather interesting object arises if one contracts Σ^I with $\mathcal{N}_I(M)$: $\Sigma_T \equiv (\mathcal{N}_I/3\kappa\mathcal{N})\Sigma^I$. Using the constraints $\mathcal{N} = \kappa^{-2}$, $\mathcal{N}_I\Omega^I = 0$ and the fact that $\mathcal{N}_I M^I = 3\mathcal{N}$, one gets for its components

$$\begin{array}{rcl} \phi_T &=& \frac{1}{2}(\kappa^{-1}e_y^5 - iB_y), \\ \chi_T &=& -i2\psi_{y-}, \\ F_{\Sigma_T} &=& e_y^5(t^1 + it^2) - i(V_y^1 + iV_y^2). \end{array} \tag{3.14}$$

Here $B_y \equiv (\mathcal{N}_I/3\kappa\mathcal{N})W_y^I$ and $\vec{t} \equiv -(\mathcal{N}_I/3\kappa\mathcal{N})\vec{Y}^I$. Following literature on the subject, this may be called the *radion supermultiplet* even though only in the $\kappa \to 0$ limit it becomes a supermultiplet (of weights $(-1,0)$). However, this does not need to bother us, as the couplings involving its components arise from the superspace action that we will present below without need to introduce a radion superfield separately. To see what happens in the $\kappa \to 0$ limit one must consider the norm function. After suitable redefinitions of the scalars M^I, the vacuum is given by $M^0 = (c_{000})^{-1/3}\kappa^{-1}$, $M^{I\neq 0} = 0$. One can therefore perturb around this vacuum, which corresponds to an expansion in powers of $\kappa M^{I\neq 0}$. It is then not difficult to find out that

$$\frac{\mathcal{N}_I}{3\kappa\mathcal{N}} = (c_{000})^{1/3}\delta_I^0 + \mathcal{O}(\kappa M^{I\neq 0}). \tag{3.15}$$

It is thus clear that in the $\kappa \to 0$ limit Σ^0 is the radion superfield. On the other hand, for $I \neq 0$, Σ^I reduces in the $\kappa \to 0$ limit to the chiral supermultiplet identified in [8] up to a pre factor e_y^5.

There is still another multiplet $\mathcal{V}_5^I$ that can be obtained out of the components of the 5D vector multiplet. This is a general type multiplet[3] with $(1,0)$ weights and is given by [14]

$$\mathcal{V}_5^I = (\ M, -2i\gamma_5\Omega_-,\ 2Y^1,\ 2Y^2,\ \hat{F}_{a5} + 2\kappa v_{a5}M,\ \lambda^{\mathcal{V}_5},\ D^{\mathcal{V}_5})^I, \tag{3.16}$$

[3] A general type superfield is of the form $\Phi = C + \theta\zeta + \bar{\theta}\zeta' + \theta^2 H + \bar{\theta}^2 K + \bar{\theta}\sigma^a\theta B_a + \bar{\theta}^2\theta\lambda + \theta^2\bar{\theta}\lambda' + \bar{\theta}^2\theta^2 D$.

where

$$\lambda^{\mathcal{V}_5} = -2\hat{\mathcal{D}}_5\Omega_+ + 2\kappa i\gamma^a v_{a5}\Omega_- - \frac{i}{4}\kappa\gamma_5\chi_+ M, \tag{3.17}$$

$$D^{\mathcal{V}_5} = \hat{\mathcal{D}}_5(\hat{\mathcal{D}}_5 M - 2Y^3) - \frac{1}{4}\kappa^2 DM + \kappa v_5^a(2\hat{F}_{a5} + \kappa v_{a5}M) + \frac{1}{2}\kappa\bar{\chi}_+\Omega_-. \tag{3.18}$$

Note the appearance of the auxiliary fields v_{ab}, χ, D of the Weyl multiplet. In the rigid limit all these fields drop out and $\mathcal{V}_5$ can be written as a simple combination of the vector and the chiral multiplets as

$$\mathcal{V}_5^I|_{\kappa=0} = (e_y^5)^{-1}(\Sigma + \Sigma^+)^I - \partial_5 V^I. \tag{3.19}$$

On the other hand, in the general (local) case to obtain $\mathcal{V}_5$ out of Σ and V one must lift up e_y^5 to a full multiplet $\mathbb{W}_y$ with $(-1, 0)$ weights. In ref. [14] such a multiplet was identified, consisting of fields from the 5D Weyl multiplet which do not participate in the 4D Weyl multiplet:

$$\mathbb{W}_y = (\ e_y^5, -2\kappa\psi_{y-}, -2\kappa V_y^2,\ 2\kappa V_y^1, -2\kappa v_{ay},\ \lambda^{\mathbb{W}_y},\ D^{\mathbb{W}_y}), \tag{3.20}$$

with

$$\lambda^{\mathbb{W}_y} = \tfrac{i}{4}\kappa e_y^5\gamma_5\chi_+ + 2\phi_{y+} + 2\kappa^2\gamma_5\gamma^b v_{b5}\psi_{y-}, \tag{3.21}$$

$$D^{\mathbb{W}_y} = \kappa^2 e_y^5\left[\tfrac{1}{4}D - (v_{a5})^2\right] - 2f_y^5 + \tfrac{i}{4}\kappa^2\bar{\chi}_+\gamma_5\psi_{y-}, \tag{3.22}$$

where ϕ_{y+} and f_y^5 are combinations of the gauge fields of the Weyl multiplet (as pointed out in section 2). We can use this now to write $\mathcal{V}_5$ in terms of the superfields V and Σ also in the local case

$$\mathcal{V}_5 = \frac{\Sigma + \Sigma^+ - \partial_y V}{\mathbb{W}_y} + \cdots \tag{3.23}$$

This expression misses contributions from some of those fields that belong to the 5D Weyl multiplet but have *negative* parity under orbifolding (see table 1). This means that they belong neither to the 4D Weyl multiplet nor to the radion multiplet. The ratio appearing in eq.(3.23) can be calculated with the usual superspace rules, or alternatively one may use the formulas for products of general multiplets of 4D conformal SUGRA as given in [14,57]. The later differ from the former in that all 4D derivatives become covariant in respect to 4D conformal supergravity. We should emphasize that $\mathcal{V}_5$ is invariant under the abelian gauge transformations and can be coupled directly to the orbifold fix-points since it also transforms trivially under the *odd* superconformal symmetries at the boundaries. This is in strong contrast to the behaviour of Σ and $\mathbb{W}_y$ which at the branes transform in a non-trivial way under the odd superconformal symmetries and, in the case of Σ, also under the gauge symmetries.

Finally let us mention the effects of orbifolding the 5th dimension. The requirement of invariance of the action under orbifold projections implies that $W_{\bar{\mu}}$ and W_y must have opposite orbifold parities, i.e.

$$\Pi(W_{\bar{\mu}}) = -\Pi(W_y). \tag{3.24}$$

This in turn means that

$$\Pi(V^I) = -\Pi(\Sigma^I) = -\Pi(\mathcal{V}_5^I). \tag{3.25}$$

The two possible choices, $\Pi(V^I) = +1$ and $\Pi(V^I) = -1$, give two essentially different physical pictures at low energies, in particular the second choice allows the breaking of an $U(1)$ gauge symmetry by orbifolding (see section 3.3). See table 1 for the detailed assignment of orbifold parities to the fields of the vector multiplet.

3.2.2 Superspace action

For the discussion of the effective 4D theory and model building a formulation in terms of 4D $\mathcal{N} = 1$ superfields is very usefull. In this section we give such a formulation for the *Abelian* vector part of the Lagrangian, including the radion multiplet.

As pointed out before, a 5D abelian vector multiplet reduces in 4D to a vector plus a chiral multiplet. The corresponding superfields, which we denote by V^I and Σ^I ($I = 0, \ldots, n$), transform under the (abelian) gauge transformation as

$$\delta V^I = \Lambda^I + \Lambda^{I+}, \qquad \delta\Sigma^I = \partial_y \Lambda^I. \tag{3.26}$$

In the rigid limit, out of these two superfields two independent super gauge invariant superfields can be constructed [22, 24, 45]

$$\mathcal{W}_\alpha^I = -\frac{1}{4}\bar{D}^2 D_\alpha V^I, \qquad \mathcal{V}_5^I = (\Sigma^I + \Sigma^{I+}) - \partial_5 V^I. \tag{3.27}$$

The local (superconformal) version of $\mathcal{V}_5$ has already been presented in the previous subsection (eq.(3.16)), we are thus left with $\mathcal{W}_\alpha$, which is a chiral multiplet of weights $(3/2, 3/2)$:

$$\mathcal{W}_\alpha = (-i2\Omega_{+\alpha}, -i(\hat{\not{F}})_\alpha^{\ \beta} + \delta_\alpha^\beta(2Y^3 - \hat{\mathcal{D}}_5 M),\ 2(\hat{\not{\mathcal{D}}}\Omega_+)_\alpha). \tag{3.28}$$

These invariant superfields can now be used to construct the 5D Lagrangian we are searching for. Let us for this purpose introduce the prepotential[4]

$$P(M) \equiv -\frac{1}{2}\mathcal{N}(M), \tag{3.29}$$

where $\mathcal{N}(M) = c_{IJK}M^I M^J M^K$ is the norm function introduced above, the real coefficients c_{IJK} being totally symmetric. Note that the prepotential is a cubic polynomial, in agreement with the requirement [47] that it be a gauge invariant *at most* cubic polynomial. We have now all the ingredients we need to write down the Lagrangian in terms of superfields, which turns out to be

$$\begin{aligned} e_{(4)}^{-1}\mathcal{L}_5 =& \frac{1}{4}\int d^2\theta \left(P_{IJ}(2\Sigma)\mathcal{W}^{I\alpha}\mathcal{W}_\alpha^J - \frac{1}{6}P_{IJK}\bar{D}^2(V^I D^\alpha \partial_y V^J - D^\alpha V^I \partial_y V^J)\mathcal{W}_\alpha^K \right) \\ &+ \text{h.c.} + \int d^4\theta\, \mathbb{W}_y\, 2P(\mathcal{V}_5). \end{aligned} \tag{3.30}$$

[4]We will keep using $\mathcal{N}$ just as a function of the scalars M^I, while P will be a function of the superfields Σ^I and $\mathcal{V}_5^I$.

This Lagrangian agrees with the 5D SUGRA Lagrangian of FKO [11,13] (eq.(B-1) and following) upon the use of the constraints $\mathcal{N}(M) = \kappa^{-2}$ and $\mathcal{N}_I(M)\Omega^I = 0$, and the replacement of the superspace integrations by F and D-densities of 4D conformal SUGRA (see section 3.6). Missing are only odd fields from the 5D Weyl multiplet. Note that even though we consider only Abelian vector multiplets, it should be straightforward to extend this to the non-Abelian case too.

A number of remarks is now in order. The first one concerns the Weyl weights: With $w(d^n\theta) = n/2$, one sees that the right hand side of the above expression has Weyl weight four, which indeed compensates for the transformation properties of $e_{(4)} \equiv \det e_{(4)}{}_\mu^a$ under dilatations since $w(e_{(4)}) = -4$. Another comment concerns the fact that $\mathbb{W}_y$ and Σ transform in a non-trivial way [14] under the *odd* 5D superconformal transformations, unlike, for instance, what happens with $\mathcal{V}_5$. To compensate for this non-trivial behaviour one may need to add terms to $\mathbb{W}_y$, which include derivatives of the corresponding gauge fields, build out of those components of the 5D Weyl multiplet which are odd under orbifold parity. In the same way, one expects the derivative ∂_y acting on V to be promoted to a superoperator including odd elements of the 5D Weyl multiplet, to ensure 5D superconformal invariance.

3.2.3 The rigid limit

Let us see now how the rigid supersymmetric Abelian gauge theory described in section 2.3 emerges in the $\kappa \to 0$ limit. For this purpose we consider the simple case that

$$\kappa^{-1}\mathcal{N} = M^{0^3} - \beta M^0 M^{1^2} - \gamma M^{1^3}. \tag{3.31}$$

A supersymmetric vacuum is given by the condition of D-flatness. From eq.(3.30) we get the following terms involving the D^I

$$e^{-1}\mathcal{L}_D = -\frac{1}{4}\mathcal{N}_{IJ}D^I D^J - \frac{1}{2}(\partial_5\mathcal{N}_I)D^I. \tag{3.32}$$

Additional couplings to the D^I may come from charged hypermultiplets, but we will postpone the discussion of such a case to chapter 4. The condition of D-flatness is therefore simply given by

$$\partial_5\mathcal{N}_I = 0. \tag{3.33}$$

Here, there is a major difference between the S^1 case and the $S^1/\mathbb{Z}_1$ orbifold case. In the circle, the solutions of (3.33) build a one-parameter family of solutions, and the corresponding parameter is an undetermined modulus. On the other hand, in the orbifold we must assign parities to Σ^0 and Σ^1. Since we are here interested in having a SYM theory surviving the orbifold compactification, we will take $\Pi(\Sigma^0) = 1$ and $\Pi(\Sigma^1) = -1$. Then, since $\mathcal{N}_1$ is odd, eq.(3.33) is solved by

$$\mathcal{N}_1 = 0. \tag{3.34}$$

This condition has only two solutions, namely

$$\text{(I)} \qquad M^1 = 0, \quad M^0 = \kappa^{-1} \tag{3.35}$$

and

$$\text{(II)} \qquad M^1 = -\frac{2\beta}{3\gamma} M^0, \quad M^0 = \frac{\kappa^{-1}}{[1 - 4\beta^3/27\gamma^2]^{\frac{1}{3}}} \tag{3.36}$$

Expanding around these configurations one gets

$$P(\varphi) \simeq -\frac{1}{2\kappa^2} + \frac{1}{2}\varphi^2 + \frac{\gamma}{2}\kappa\varphi^3 + \cdots , \tag{3.37}$$

where $\varphi = \mathcal{V}_5^1,\ 2\Sigma^1$, the dots indicate additional terms involving higher powers of κM^1 and couplings to fields of the gravitational sector, and we took[5]

$$\beta = \begin{cases} 1 & \text{(I)} \\ [1 + 4/(27\gamma^2)]^{-1/3} & \text{(II).} \end{cases} \tag{3.38}$$

One sees that the prepotential becomes the cubic function discussed by Seiberg in [47] and used in [45], which we presented in sec. 2.3 . In this way, in the $\kappa \to 0$ limit we obtain the results of [22, 45], the supersymmetric Chern-Simons term included. Note, however, that since we put the theory on the orbifold $S^1/\mathbb{Z}_2$, the coupling γ must be odd, i.e. $\gamma \sim \epsilon(y)$. This has the consequence that under a gauge transformation the above Lagrangian is not invariant, having a non-vanishing transformation on the branes:

$$\begin{aligned} \delta(e_{(4)}^{-1}\mathcal{L}_5) &\sim \kappa(\partial_y\gamma) \int d^2\theta \Lambda^1 \mathcal{W}^{1\alpha}\mathcal{W}_\alpha^1 + \text{h.c.} \\ &= 2\kappa|\gamma|\, [\delta(y) - \delta(y - \pi R)] \int d^2\theta \Lambda^1 \mathcal{W}^{1\alpha}\mathcal{W}_\alpha^1 + \text{h.c.} \end{aligned} \tag{3.39}$$

In fact, it is well known that this can be used to cancel anomalies arising at the fix-point branes [22].

To make a first analysis more simple, in eq.(3.37) we dropped the *radion sector*. Let us thus now see explicitely how - for $\kappa \to 0$ - the radion sector couples to the gauge sector. For this recall our remark in section 3.2 that in the $\kappa \to 0$ limit Σ^0 is (proportional to) the so-called radion chiral superfield, Σ_T. If we introduce $T \equiv \kappa 2\Sigma_T$ we have in this limit

$$\Sigma^0 = \frac{T}{2\kappa}. \tag{3.40}$$

On the other hand it is not difficult to recognize that there is some overlap between $\frac{1}{2}(T + T^+)$ and $\mathbb{W}_y$, the two superfields becoming identical if the auxiliary fields v_{ay}, $\lambda^{\mathbb{W}_y}$ and $D^{\mathbb{W}_y}$ as well as $\vec{Y}^0$ and $\hat{F}_{a5}^0$ (the graviphoton's field-strength) are set to zero[6]. We have thus

$$\mathbb{W}_y = \frac{T + T^+}{2} + \cdots \tag{3.41}$$

[5]One of the parameters β or γ can be fixed by the normalisation of M^1.

[6]The fields $\lambda^{\mathbb{W}_y}$ and $D^{\mathbb{W}_y}$ are Lagrange multipliers that can safely be put to zero if one imposes *by hand* the constraints that they imply.

In this case one also has $(\Sigma^0 + \Sigma^{0+} - \partial_y V^0) = (2\kappa)^{-1}(T + T^+)$ and therefore $\mathcal{V}_5^0 = \kappa^{-1}$. If we use all this we obtain (see eq.(3.37))

$$2\mathbb{W}_y P(\mathcal{V}_5) \simeq -\kappa^{-2}\frac{T+T^+}{2} + 2\frac{(\Sigma^1 + \Sigma^{1+} - \partial_y V^1)^2}{T+T^+} + 4\kappa\gamma\frac{(\Sigma^1 + \Sigma^{1+} - \partial_y V^1)^3}{(T+T^+)^2}. \qquad (3.42)$$

Note that the first term in the r.h.s. was given in [60], and the second term has the same form as the one first presented in ref. [23] to couple the vector multiplet with the radion multiplet. There is also a third term, part of the supersymmetric Chern-Simons term, whose coupling to the radion superfield was not considered before in the literature. Let us emphasize the difference between our results and the ones of Marti and Pomarol in [23]. The difference is that $(\Sigma^1 + \Sigma^{1+} - \partial_y V^1)$ also includes in its components elements of the radion multiplet, like the gravitino ψ_{y-} and the $V_y^{1,2}$, which give rise to additional couplings between the vector multiplet and the radion multiplet. These couplings (absent in [23]) can be *shifted away* by a redefinition of the 4D gauginos and other fields in Σ and V which makes $(\Sigma^1 + \Sigma^{1+} - \partial_y V^1)$ independent of any gravitational fields (apart from the *radion* e_y^5, which may be factored out). But, such redefinitions clearly break the $\mathrm{SU}(2)_R$ symmetry and should therefore be used only when we do not care about the 5D structure.

Finally, another coupling between the vector multiplet and the radion multiplet arises from the F-term coupling,

$$\frac{1}{4}\int d^2\theta P_{IJ}(2\Sigma)\mathcal{W}^{I\alpha}\mathcal{W}^J_\alpha = \frac{1}{4}\int d^2\theta\, T\,\mathcal{W}^{1\alpha}\mathcal{W}^1_\alpha + \cdots, \qquad (3.43)$$

which is a Chern-Simons like term and clearly has the same form as the one in [23].

3.3 Hypermultiplet Superspace Action

3.3.1 Reduction of the hypermultiplet

As is well known, it is convenient to split the 5D hypermultiplets $\mathbb{H}^\alpha = (\mathcal{A}_i^\alpha,\ \zeta^\alpha, \mathcal{F}_i^\alpha)$ into r pairs $(\mathbb{H}^{2\hat{\alpha}-1},\ \mathbb{H}^{2\hat{\alpha}})$, where $\hat{\alpha} = 1, 2, \cdots, r$ indicates the number of introduced 5D hypermultiplets. The reason for doing this is the reality condition, eq.(3.7), which now reads

$$(\mathcal{A}_2^{2\hat{\alpha}})^* = \mathcal{A}_1^{2\hat{\alpha}-1}\ , \qquad (\mathcal{A}_1^{2\hat{\alpha}})^* = -\mathcal{A}_2^{2\hat{\alpha}-1}\ , \qquad (3.44)$$

(and similarly for $\mathcal{F}$ components) and clearly relates $\mathbb{H}^{2\hat{\alpha}-1}$ with $\mathbb{H}^{2\hat{\alpha}}$. Therefore, for each $\hat{\alpha}$ only four real scalar components are independent[7]. For a given $\hat{\alpha}$, the 5D hypermultiplet decomposes into a pair of $\mathcal{N} = 1$ 4D chiral superfields with opposite orbifold parities [14]:

$$H = \left(\mathcal{A}_2^{2\hat{\alpha}},\ -2\mathrm{i}\zeta_R^{2\hat{\alpha}},\ (\mathrm{i}M_*\mathcal{A} + \hat{\mathcal{D}}_5\mathcal{A})_1^{2\hat{\alpha}}\right),$$

$$H^c = \left(\mathcal{A}_2^{2\hat{\alpha}-1},\ -2\mathrm{i}\zeta_R^{2\hat{\alpha}-1},\ (\mathrm{i}M_*\mathcal{A} + \hat{\mathcal{D}}_5\mathcal{A})_1^{2\hat{\alpha}-1}\right), \qquad (3.45)$$

[7] For a more detailed discussion about hypermultiplets see [11, 13].

where we introduced

$$M_* \mathcal{A}_i^\alpha = igM^I(t_I)^\alpha_\beta \mathcal{A}_i^\beta + \mathcal{F}_i^\alpha \ ,$$

$$\hat{\mathcal{D}}_5 \mathcal{A}_i^\alpha = \partial_5 \mathcal{A}_i^\alpha - igW^\alpha_{5\beta}\mathcal{A}_i^\beta - W_5^0 \frac{1}{\alpha}\mathcal{F}_i^\alpha - \kappa V_{5ij}\mathcal{A}^{\alpha j} - 2\kappa \mathrm{i}\bar{\psi}_{5i}\zeta^\alpha \ , \tag{3.46}$$

and $(t_I)^\alpha_\beta$ is the generator of gauge group G_I. Eq.(3.45) should be compared with the usual decomposition of the 5D hypermultiplet in global SUSY [8, 24], given in eqs.(2.20) and (2.21), when we take the limit $\kappa \to 0$. In the following we will use the notation

$$\mathbf{H} \equiv (\mathbf{H}_1,\ \mathbf{H}_2) = (H,\ H^c) \ , \tag{3.47}$$

keeping in mind that if we have more than one 5D hypermultiplet, i.e. for $r > 1$, the index $\hat{\alpha}$ should be present, $\mathbf{H}^{\hat{\alpha}} \equiv (\mathbf{H}_1,\ \mathbf{H}_2)^{\hat{\alpha}} = (H,\ H^c)^{\hat{\alpha}}$, but for legibility remains only implicit.

3.3.2 Superspace action

As we have seen in section 3.1, a 5D vector multiplet reduces to a 4D vector superfield V^I (eq.(3.9)) and a chiral superfield Σ^I (eq.(3.12)). We want to build a superspace action for the hypermultiplet $(\mathbf{H}_1,\ \mathbf{H}_2)$ interacting with a 5D gauge multiplet $(V,\ \Sigma)$. For this purpose we introduce

$$\mathbf{V}^{ab} = gV\vec{q}\cdot\vec{\sigma}^{ab} \ , \qquad \mathbf{\Sigma}^{ab} = g\Sigma\vec{q}\cdot\vec{\sigma}^{ab} \ , \quad \text{with} \ \ |\vec{q}| = 1 \ . \tag{3.48}$$

(Like in QED, the coupling constants only appear with matter and are written explicitely. In the gauge kinetic part, there are no gauge couplings.) This notation turns out to be convenient for constructing the action invariant under different orbifold parity prescriptions for the vector superfields. Here we use it for a general Abelian $U(1)$ gauge symmetry.

Now we are ready to write a superspace Lagrangian for hypermultiplets. In the case of one r-hypermultiplet and one gauge field it has the form

$$e_{(4)}^{-1}\mathcal{L}(\mathbf{H}) = \int d^4\theta\, \mathbb{W}_y 2\mathbf{H}_a^\dagger (e^{-\mathbf{V}})^{ab}\mathbf{H}_b - \int d^2\theta (\mathbf{H}\epsilon)_a \left(\hat{\partial}_y - \mathbf{\Sigma}\right)^{ab}\mathbf{H}_b + \text{h.c.} \tag{3.49}$$

where the superoperator $\hat{\partial}_y$ is obtained by promoting ∂_y to an operator containing odd (under orbifold parity) elements of the 5D Weyl multiplet, namely $\hat{\partial}_y = \partial_y + \Lambda^\alpha D_\alpha + \Lambda^{\overline{\mu}}\partial_{\overline{\mu}}$. The superfields Λ^α, $\Lambda^{\overline{\mu}}$ are such that $\hat{\partial}_y\mathbf{H}_a$ is a chiral superfield. For this Λ^α must be a chiral superfield, $\Lambda^\alpha = \overline{D}^2 L^\alpha$, with a spinor index ($L^\alpha$ is a general complex superfield). The superfield $\Lambda^{\overline{\mu}}$ is related to L^α: $\Lambda_{\alpha\dot{\alpha}} = 8i\overline{D}_{\dot{\alpha}}L_\alpha + \Omega_{\alpha\dot{\alpha}}$, where $\Omega_{\alpha\dot{\alpha}}$ is chiral superfield. With these conditions it is straightforward to check out that the $\hat{\partial}_y\mathbf{H}_a$ is chiral. This type of construction is important for obtaining the correct interactions of matter with the 5D SUGRA multiplet. The lowest component of Λ^α is $\Lambda^\alpha|_{\theta=0} = -\kappa(\psi_{y+})^\alpha_{\underline{L}} = -\kappa(\chi^2)^\alpha$ (see appendix B for conventions). This term is important for the cancellation of $\mathcal{F}\psi\zeta$ type terms, in order to recover the FKO Lagrangian. Higher components of Λ^α should be obtained by SUSY (and superconformal) transformations. One should note that also compensator hypermultiplets can couple to the gauge fields $(\mathbf{V},\ \mathbf{\Sigma})$, see [11]. Since they have negative kinetic terms, for compensators $(e^{-2\mathbf{V}})^{ab}$ should simply be replaced by $-(e^{-2\mathbf{V}})^{ab}$. As usual, the exponent of the first term in eq.(3.49) completes 4D derivatives promoting them to derivatives covariant under the gauge transformations. In the same way, an additional coupling to the 4D *Weyl* supermultiplet should be included, in order to covariantize the 4D derivatives of the hypermultiplets in respect to the superconformal symmetries. This can however be bypassed by using the (4D) superconformal-invariant D and F-term action formulas of [57, 58], see section 3.6. The superconformal covariant derivatives in the fifth direction, which appear in the kinetic terms, arise in part from the F-term coupling $[(\mathbf{H}\epsilon)_a(\hat{\partial}_y - \mathbf{\Sigma}^{ab})\mathbf{H}_b]_F$. While $\mathbf{\Sigma}^{ab}$ takes care of the gauge invariance, $\hat{\partial}_y$ induces some of the pieces of the superconformal covariant derivatives of $\mathcal{A}$ and ζ. The remaining terms are due to the coupling of $\mathbb{W}_y$ to the hypermultiplets in the D-term coupling $[\mathbb{W}_y\mathbf{H}^\dagger\mathbf{H}]_D$.

One can check that all the relevant (non gravitational) couplings of FKO [13] involving hypermultiplets are reproduced by expression (3.49). We consider now the effects of orbifolding the theory, where we can distinguish between the two following special cases:

1. Gauge field with positive orbifold parity: In this case the 4D gauge superfield $\mathbf{V}$ and its (5D) partner $\mathbf{\Sigma}$ transform under the Z_2 orbifold parity ($y \to -y$) as

$$Z_2: \quad \mathbf{V} \to \mathbf{V}\ , \quad \mathbf{\Sigma} \to -\mathbf{\Sigma}\ . \tag{3.50}$$

Therefore 4D $U(1)$ gauge invariance is unbroken at the orbifold fixed points. With $\mathbf{H}$ components' parities

$$Z_2: \quad H \to H\ , \quad H^c \to -H^c\ , \tag{3.51}$$

for an invariant action, we have to choose $q_1 = q_1 = 0$, $q_3 = 1$. In eq.(3.49) for this choice we have $(e^{-\mathbf{V}})^{ab} = \mathrm{Diag}(e^{-gV},\ e^{gV})^{ab}$ and (3.49) reduces to

$$\begin{aligned} e_{(4)}^{-1}\mathcal{L}_+(\mathbf{H}) = &\int d^4\theta\, \mathbb{W}_y 2\left(H^\dagger e^{-gV} H + H^{c\dagger} e^{gV} H^c\right) \\ &+ \int d^2\theta (H^c\hat{\partial}_y H - H\hat{\partial}_y H^c - 2gH^c\Sigma H) + \text{h.c.} \end{aligned} \tag{3.52}$$

In the rigid limit, this expression coincides with the superspace Lagrangian of [22, 24]. It is transparently invariant under the orbifold symmetry, with the parity prescriptions (3.50) and (3.51). The Lagrangian (3.49) however is more general and allows one to consider also the case of a gauge field with negative parity. Notice that eq.(3.52) is invariant also if we take for H negative parity and for H^c positive parity.

2. Gauge field with negative orbifold parity: In this case instead of (3.50) we have

$$Z_2: \quad \mathbf{V} \to -\mathbf{V}\ , \quad \mathbf{\Sigma} \to \mathbf{\Sigma}\ , \tag{3.53}$$

while for the hypermultiplet the orbifold parities are the same as before (eq.(3.51)). The requirement of invariance enforces now: $q_3 = 0, q_1 = \cos\hat{\theta}, q_2 = \sin\hat{\theta}$. With this and the superfield redefinition $H \to e^{-\mathrm{i}\hat{\theta}/2}H$, $H^c \to e^{\mathrm{i}\hat{\theta}/2}H^c$, eq.(3.49) takes the form

$$e_{(4)}^{-1}\mathcal{L}_-(\mathbf{H}) = \int d^4\theta \mathbb{W}_y 2\left((H^\dagger H + H^{c\dagger}H^c)\cosh(gV) - (H^\dagger H^c + H^{c\dagger}H)\sinh(gV)\right)$$

$$+\int d^2\theta \left(H^c \hat{\partial}_y H - H \hat{\partial}_y H^c + g\Sigma(H^2 - H^{c2}) \right) + \text{h.c.} \tag{3.54}$$

This expression can be also derived from eq.(3.52): With the parity prescription eq.(3.53) and with the modified boundary conditions for hypermultiplets $H(-y) = H^c(y)$, $H^c(-y) = H(y)$, eq.(3.52) is still invariant. Introducing the new combinations $H^c_+ = \frac{1}{\sqrt{2}}(H + H^c)$ and $H_- = \frac{1}{\sqrt{2}}(H - H^c)$ (with definite positive and negative parities respectively) and rewriting (3.52) in terms of H_-, H^c_+, we recover the Lagrangian eq.(3.54).

Note that the radion superfield and its corresponding 4D vector superfield also have the parity assignment (3.53). Therefore, eq.(3.54) can be used in order to derive couplings of the radion superfield components with 4D chiral matter. Alternatively we can introduce an odd gauge coupling $G(y) = \epsilon(y)g$ and use the Lagrangian of case **1**, eq.(3.52). This is the way FKO introduce the $U(1)_R$ gauging of SUGRA to obtain supersymmetric RS-like models [13, 20].

Let us now comment on the couplings of the hypermultiplets with the *radion* multiplet. As we pointed out in section 3.2, if one sets the auxilary fields v_{ay}, $\lambda^{\mathbb{W}_y}$ and $D^{\mathbb{W}_y}$ to zero, one has $2\mathbb{W}_y = T + T^+$. In this case the D-term coupling in eq.(3.49) can be rewritten as

$$\int d^4\theta \, \frac{T + T^+}{2} 2\mathbf{H}^\dagger_a \cdot (e^{-2\mathbf{V}})^{ab} \mathbf{H}_b. \tag{3.55}$$

Expression (3.55) has a form similar to the one which in ref. [23] describes the couplings between hypermatter and the *radion* multiplet. But, as in the case of the couplings of the vector with the radion multiplet, it differs in the fact that the components of **H** also include elements of the radion multiplet, see eq.(3.45). The terms not included in [23] ensure not only that the different auxiliary fields do not mix, but also that there is no gravitino/gaugino mixing.

3.3.3 SUSY breaking by the F-term of the radion superfield

To discuss the possible SUSY breaking by the F-term of the radion superfield, let us consider the case where the only hypermultiplet is a compensator which does not couple to any gauge multiplet. This is the same as to say that the R-symmetries are ungauged. We denote the compensator hypermultiplet by $(h,\ h^c)$ to distinguish from physical hypers denoted by $(H,\ H^c)$. One can show that from integrating out $\lambda^{\mathbb{W}_y}$ and $D^{\mathbb{W}_y}$ it follows that $\mathcal{A}^2 = -2\kappa^{-2}$ and $\zeta = 0$. This is in fact equivalent to integrating out χ' and D' in eq.(B-9). If we solve this with $\mathcal{A}^\alpha_i = \kappa^{-1}\delta^\alpha_i$ we obtain for the compensator chiral superfields (with the conventions of appendix B)

$$h = \kappa^{-1} + \theta^2 \left[\left(i + \frac{W^0_5}{\alpha} \right) \mathcal{F}^{2*}_1 + \kappa^{-1} F^*_T \right], \tag{3.56}$$

$$h^c = \theta^2 \left(i + \frac{W^0_5}{\alpha} \right) \mathcal{F}^{1*}_1, \tag{3.57}$$

where $e^5_y F_T$ is the F-component of T. One gets in this way the following Lagrangian

$$-\int d^4\theta \, \frac{T + T^+}{2} 2 \left(h^\dagger h + h^c h^{c\dagger} \right) = e^5_y \left[\left(1 + \frac{(W^0_5)^2}{\alpha^2} \right) \mathcal{F}^2 + 2M^3_5 |F_T|^2 \right], \tag{3.58}$$

where we used that for compensators $\mathcal{F}^2 = \mathcal{F}_i^\alpha d_\alpha^\beta \mathcal{F}_\beta^i = -\sum |\mathcal{F}_i^\alpha|^2$. Eq.(3.58) makes evident that due to the breaking of the $\mathrm{SU}(2)_R$ by the VEV of $\mathcal{A}_i^\alpha$, the F-term of the radion superfield, $F_T = -i\kappa(V_5^1 + iV_5^2)$, is not a flat direction. This means that it cannot be used to induce supersymmetry breaking in the way discussed in [61] (see also [23,62,63]), at least in the minimal (*ungauged*) version we just described. There is, however, the possibility of extending this by coupling the compensator hypermultiplet to a 5D U(1) vector multiplet [11] with orbifold parity $\Pi(V_R) = -\Pi(\Sigma_R) = -1$. It turns out that in this case the above potential for F_T becomes

$$-e_y^5 2M_5^3 \left|F_T - ge^{i\alpha}W_{R5}\right|^2, \tag{3.59}$$

where α is an arbitrary (constant) phase. One sees that if F_T is shifted as $F_T \to F_T + ge^{i\alpha}W_{R5}$, all fields that transform under the $\mathrm{SU}(2)_R$ will now couple to the Wilson-line W_{R5} [11]. Clearly, a non-vanishing VEV of W_{R5} will thus lead to supersymmetry breaking. There are now three possibilities: The first is that the vector multiplet is dynamical, which implies that the norm function depends on its scalar component. The second possibility is to couple $\mathbb{V}_R$ to a tensor multiplet to constrain W_{R5} to be a constant (up to a gauge transformation). This type of SUSY breaking is equivalent to the one discussed in [62, 63]. Finally, there is the possibility that $\mathbb{V}_R$ is neither constrained nor dynamical, i.e. that it does not couple to a tensor multiplet and the norm function is independent of M_R.

3.4 Warped backgrounds

In this section we show in detail how to deal with a warped background. To do this we will use the fact that the (component) Lagrangian of 5D SUGRA is invariant under *Weyl* rescalings of the fields, in case the conformal compensator also transforms. The Weyl transformations can then be used to reach a y-independent metric, for which the superspace actions presented in the previous sections can be used straightforwardly. In other words, we shift the y-dependence of the metric to the hyper, vector and radion sectors. To be precise let us consider the following metric:

$$ds^2 = e^{2\sigma(y)}dx_\mu dx^\mu - (e_y^5)^2 dy^2, \tag{3.60}$$

where e_y^5 is also a function of y. If we perform a Weyl transformation

$$e_\mu^a \to (e_\mu^a)' = e^{-\sigma(y)} e_\mu^a, \tag{3.61}$$

we obtain a y-independent $4D$ metric. Under this transformation, the chiral multiplets arising from the 5D hypermultiplets transform as

$$H \to H' = (\mathcal{A}',\ \lambda', F') = e^{3\sigma/2}\tilde{H}, \quad \text{with } \tilde{H} = (\mathcal{A},\ e^{\sigma/2}\lambda,\ e^\sigma F), \tag{3.62}$$

while for the chiral multiplet which comes from the gauge sector, Σ, we have

$$\Sigma \to \Sigma' = (\phi,\ e^{\sigma/2}\chi,\ e^\sigma F_\phi). \tag{3.63}$$

In a similar way we obtain:

$$V \to V' = e^{\sigma}\tilde{V}, \quad \text{with } \tilde{V} = (W_\mu,\ e^{\sigma/2}2\Omega_+,\ e^{\sigma}D), \tag{3.64}$$

$$\mathcal{W}_\alpha \to \mathcal{W}'_\alpha = e^{3\sigma/2}\tilde{\mathcal{W}}_\alpha, \quad \text{with } \tilde{\mathcal{W}}_\alpha = (-e^{\sigma/2}i2\Omega_{+\alpha}, e^{\sigma}\left\{-i(\hat{\not{F}})_\alpha^{\ \beta} + \delta_\alpha^\beta(2Y^3 - \hat{\mathcal{D}}_5 M)\right\}, \cdots), \tag{3.65}$$

and

$$\mathbb{W}_y \to \mathbb{W}'_y = e^{-\sigma}\tilde{\mathbb{W}}_y, \quad \text{with } \tilde{\mathbb{W}}_y = (\ e_y^5, -e^{\sigma/2}2\kappa\psi_{y-}, -e^{\sigma}2\kappa V_y^2,\ e^{\sigma}2\kappa V_y^1, \cdots). \tag{3.66}$$

We can use the Lagrangians presented in the previous sections replacing the *unprimed* fields by the *primed* ones. This procedure leads one to

$$\mathcal{L}_V = \frac{1}{4}\int d^2\theta \left(e^{3\sigma}P_{IJ}(\Sigma')\tilde{\mathcal{W}}^\alpha\tilde{\mathcal{W}}_\alpha + \cdots\right) + \text{h.c.} + \int d^4\theta\, e^{2\sigma}\tilde{\mathbb{W}}_y 2P(\tilde{\mathcal{V}}_5), \tag{3.67}$$

where

$$\tilde{\mathcal{V}}_5 = \frac{\Sigma' + \Sigma'^+ - \partial_y V'}{\tilde{\mathbb{W}}_y} + \cdots \tag{3.68}$$

Let us now consider the hypermultiplet Lagrangian. Without loss of generality, we will focus on case **1**, but case **2** is analog. We get

$$\begin{aligned}\mathcal{L}_H = \pm \int d^4\theta\, e^{2\sigma}\tilde{\mathbb{W}}_y\, 2\left(\tilde{H}^\dagger e^{-2gV'}\tilde{H} + \tilde{H}^{c\dagger}e^{2gV'}\tilde{H}^c\right) \\ - \int d^2\theta\, e^{3\sigma}\left(\tilde{H}^c\partial_y\tilde{H} - \tilde{H}\partial_y\tilde{H}^c - 2g\Sigma'\tilde{H}^c\tilde{H}\right) + \text{h.c.}\end{aligned} \tag{3.69}$$

Note that we introduced the pre-factor $\pm$, which depends on the hypermultiplet being physical or a compensator.

In the following we will show that the action of ∂_y on the several powers of $e^{\sigma(y)}$ present in the Lagrangians does not lead to any terms absent in [13]. To be more precise, the terms dependent on $\partial_y\sigma$ turn out to cancel out up to the terms arising from the 5D Ricci scalar. This will be shown for (part of) the bosonic terms in the action. Let us start with hypermultiplet Lagrangian, which in terms of components reads

$$\begin{aligned}\mathcal{L}_H = e^{4\sigma}e_y^5\Big[&\pm 2\left(|F|^2 + |F^c|^2 + 2gD(|\mathcal{A}^c|^2 - |\mathcal{A}|^2)\right) \\ &+ \Big(2F\left(\partial_y\mathcal{A}^c + \tfrac{3}{2}\mathcal{A}^c\partial_y\sigma + ge_y^5(M - iA_5)\mathcal{A}^c\right) \\ &+ (F \to F^c,\ \mathcal{A} \to -\mathcal{A}^c,\ g \to -g) + \text{h.c.}\Big)\Big].\end{aligned} \tag{3.70}$$

After integrating out the F's this can be rewritten as

$$\begin{aligned}\mathcal{L}_H = \pm\, e^{4\sigma}e_y^5\Big[&-2\Big(|\nabla_5\mathcal{A}|^2 + |\nabla_5\mathcal{A}^c|^2 + g^2M^2\left(|\mathcal{A}|^2 + |\mathcal{A}^c|^2\right) - g(D + \partial_5 M)\left(|\mathcal{A}^c|^2 - |\mathcal{A}|^2\right)\Big) \\ &- 3(\partial_5\sigma)\partial_5\left(|\mathcal{A}|^2 + |\mathcal{A}^c|^2\right) + 2gM(\partial_5\sigma)\left(|\mathcal{A}^c|^2 - |\mathcal{A}|^2\right) - \frac{9}{2}(\partial_5\sigma)^2\left(|\mathcal{A}|^2 + |\mathcal{A}^c|^2\right)\Big].\end{aligned} \tag{3.71}$$

Note that we assembled the $\partial_5\sigma$ terms in the second line of this equation. These terms are of two different types. There are two terms which are universal, i.e., they are equal for all hypermultiplets. After performing the sum over all hypermultiplets (including compensators) we get the following:

$$\mathcal{L}_H \supset -e^{4\sigma} e_y^5 \left(\frac{3}{2} (\partial_5 \sigma) \partial_5 + \frac{9}{4} (\partial_5 \sigma)^2 \right) \mathcal{A}^2, \tag{3.72}$$

where $\mathcal{A}^2 \equiv 2\sum(-1)^d(|\mathcal{A}|^2 + |\mathcal{A}^c|^2)$, with $d = 0, 1$, for physical and compensator hypermultiplets, respectively. The third term is non-universal, being present only for hypermultiplets charged under the symmetry gauged by W_μ. This term must be canceled by a contribution coming from the vector sector.

Let us consider now the vector sector. The Lagrangian contains

$$\mathcal{L}_V \supset -e^{4\sigma} e_y^5 \left[\frac{1}{4} \mathcal{N}_{IJ}(M) D^I D^J + (\partial_5 \sigma) \mathcal{N}_I(M) D^I + \frac{1}{2} \mathcal{N}_{IJ}(M) D^I \partial_5 M^J \right]. \tag{3.73}$$

Denote by $M^{I=i}$ the scalar of the vector multiplet $\mathbb{V}^{I=i}$ which couples to one of the hypermultiplets. The total Lagrangian involving the auxiliary fields D^I is obtained from eqs.(3.73) and (3.71)

$$\mathcal{L}_D = -e^{4\sigma} e_y^5 \left[\frac{1}{4} \mathcal{N}_{IJ}(M) D^I D^J + \frac{1}{2} D^I R_I \right], \tag{3.74}$$

with

$$R_I = \mathcal{N}_{IJ} \partial_5 M^J + 2(\partial_5 \sigma) \mathcal{N}_I - 4\delta_I^i g(|\mathcal{A}^c|^2 - |\mathcal{A}|^2). \tag{3.75}$$

After integrating out the D^I's one gets a term $\propto (\partial_5\sigma) M^i(|\mathcal{A}^c|^2 - |\mathcal{A}|^2)$ that exactly cancels the non-universal one in the second line of eq.(3.71). The total Lagrangian, which combines the vector and the hyper parts, is thus

$$\begin{aligned} \mathcal{L}_{H+V} \supset & e^{4\sigma} e_y^5 \Bigg[\frac{1}{4} \mathcal{N}_{IJ} \partial_5 M^I \partial_5 M^J - 2 \Big(|\nabla_5 \mathcal{A}|^2 + |\nabla_5 \mathcal{A}^c|^2 + g^2 (M^i)^2 \left(|\mathcal{A}|^2 + |\mathcal{A}^c|^2\right) \Big) \\ & + 4 g^2 \mathcal{N}^{ii} \left(|\mathcal{A}^c|^2 - |\mathcal{A}|^2\right)^2 - \left(\mathcal{N} - \frac{3}{2} \mathcal{A}^2 \right) \left(\frac{5}{2} (\partial_5 \sigma)^2 + \partial_5^2 \sigma \right) \Bigg]. \end{aligned} \tag{3.76}$$

We can use now the fact that on-shell $\mathcal{A}^2 = -2\mathcal{N}$. The last term in the equation becomes

$$e^{4\sigma} e_y^5 \left(-2\mathcal{N} \left(5(\partial_5 \sigma)^2 + 2\partial_5^2 \sigma \right) \right). \tag{3.77}$$

This term arises from the 5D Ricci scalar:

$$\mathcal{R} \supset 4 \left(5(\partial_5 \sigma)^2 + 2\partial_5^2 \sigma \right), \tag{3.78}$$

showing that in the superfield approach proposed in this chapter, the warp-factor is not just a background field but it is fully dynamical. This fact will become evident in the following chapter, where we will discuss the *generation* of supersymmetric generalizations of the Randall-Sundrum model and of other warped backgrounds.

3.5 Brane Couplings

One of the great advantages of having an off-shell description of 5D supergravity is the ease with which bulk-brane couplings can be introduced. This is specially true for the superfield formulation. We will here explain how brane-localised couplings involving brane and/or bulk fields must be written to be consistent with 5D local supersymmetry.

There are two rules which must be followed when writing down the superspace brane couplings, namely we require in addition to gauge invariance that F-densities have weights $w = (3, 3)$, while D-densities have $w = (2, 0)$. In other words, the brane couplings must have the form of the usual 4D old-minimal supergravity in its superconformal formulation [57–59]:

$$\mathcal{L}_{brane} = \sum_i \mathcal{L}^i \delta(y - y_i), \tag{3.79}$$

with

$$\mathcal{L}^i = \int d^4\theta \, (h^+ h)^{\frac{2}{3}} \, e^{K^i(S,S^+)} + \left(\int d^2\theta \, \left(f^i_{IJ}(S) \mathcal{W}^{I\alpha} \mathcal{W}^J_\alpha + h^2 \, W^i(S) \right) + \text{h.c.} \right), \tag{3.80}$$

where the chiral superfields have weights $(0, 0)$ and can be either genuine brane superfields, or be induced from bulk multiplets. In the last case we must ensure that S realy has $(0, 0)$ weights, if necessary by multiplying bulk chiral superfields with inverse powers of the compensator. (For example $S = h^{-1}H$ has correct weights.) In the same way, the field-strength superfields $\mathcal{W}^I_\alpha$ can correspond to gauge symmetries localised only in the brane or induced from the bulk.

We would like to emphasize that the brane Lagrangians must be invariant under any *local* symmetries induced at the branes by bulk local symmetries. Explicit brane breaking of such symmetries would lead to quantum inconsistencies [64]. This means, in particular, that when the *even* $U(1)_R$ subgroup of the $SU(2)_R$ symmetry is gauged, as is the case in the supersymmetric RS model and its generalisations (see sec.4.1), since the compensator h has charge $R = 1$, the superpotentials $W^i(S)$ must have $R = -2$. This fact will play a determinant rôle in sec.4.2, where we discuss some recent proposals for supersymmetric radion stabilization.

As an example of the effects of brane couplings let us consider the simplest brane couplings that one can imagine, namely constant brane superpotentials:

$$\delta(y - y_i) \int d^2\theta \, h^2 \, w_i + \text{h.c.}. \tag{3.81}$$

It is well known that such terms break supersymmetry [65,66]. To see this we note that eq.(3.81) can also be written as

$$\delta(y - y_i) \int d^4\theta \, h^+ h \, \tfrac{1}{2} \left(\bar{\theta}^2 w_i + \theta^2 w_i^* \right). \tag{3.82}$$

Here we used the fact that $\mathcal{A}_h$ is real. Now, if we compare this expression with the following term in the bulk Lagrangian,

$$- \int d^4\theta \, \mathbb{W}_y 2h^+ h, \tag{3.83}$$

we see that the presence of the constant superpotentials implies a shift of the θ^2 component of $\mathbb{W}_y$, in other words it induces a non-zero VEV of F_T with support in the two branes:

$$F_T = \frac{1}{4}\sum_i \delta(y-y_i) w_i^* . \tag{3.84}$$

This VEV in turn signals the breaking of supersymmetry for $w_i \neq 0$. Note that to obtain this result, apart from the compensator, we do not rely on any knowledge of the coupling of 4D gravity to the brane superpotentials[8]. The coupling to the 4D gravity (Weyl) multiplet will be discussed in the next section, but we anticipate the fact that eq.(3.81) must be replaced by (see eq. (3.87))

$$\delta(y-y_i)\left[\int d^2\theta\, h^2\, \mathcal{W}_i - 2\kappa^2 \mathcal{A}_h w_i\, \bar{\psi}^{(4)}\gamma\psi^{(4)} + \text{h.c.}\right] . \tag{3.85}$$

Clearly, this shows that non-vanishing constant brane superpotentials correspond to brane-localised gravitini masses. This should be expected since the gravitino is an $\mathrm{SU}(2)_R$ doublet and therefore should have a mass proportional to F_T, the field which gauges the $\mathrm{SU}(2)_R$.

3.6 4D Weyl Multiplet and its Couplings to Matter

In our formulations we have used superspace actions, which produce F and D terms of various operators upon integrating over $d^2\theta$ and $d^4\theta$ respectively. Obviously, this does not account for the couplings of matter (i.e. gauge multiplets and hypermultiplets) with 4D SUGRA. In order to achieve this, one should first obtain the 4D Weyl multiplet induced from the 5D Weyl super multiplet. The components of the former are [14]

$$e_\mu^{(4)a} = e_\mu^a \ , \quad \psi_\mu^{(4)} = \psi_{\mu+} \ , \quad b_\mu^{(4)} = b_\mu \ , \quad A_\mu^{(4)} = \frac{4}{3}(V_\mu^3 + v_{\mu 5}) \ . \tag{3.86}$$

Note that the composition of the 4D Weyl multiplet and of the radion supermultiplet are independent also with respect to the auxiliary fields entering in both multiplets. This is necessary since $(V,\Sigma)^{I=0}$ should transform independently of the 4D SUGRA [60]. Note also that the definition eq.(3.86) of the 4D Weyl multiplet (as well as that of compensators) is unique and does not allow any addition as emphasized by the authors of [14] (this is not the case for the approach of ref. [9]).

We could now assemble the components of the 4D Weyl multiplet into a superfield, which would than be coupled to the other superfields. This is well known [67] but its usefullness is not clear if one has in mind that one would have to calculate in some background (e.g. AdS). There is, however, an alternative formalism which gives the very same couplings we are searching for [57–59]. It consists of replacing the $d^2\theta$ and $d^4\theta$ integrations in the Lagrangian by F-terms and D-terms invariant under the 4D conformal SUGRA. Knowing this, the F-term of a chiral operator of weights $(3,3)$, $\mathcal{C} = [\mathcal{A},\ \chi_R,\ \mathcal{F}]^{\mathcal{C}}$ (which in general is a composite chiral superfield), should be understood as follows [57–59]

$$[\mathcal{C}]_{\mathbf{F}} = e^{(4)}\left(\int d^2\theta \mathcal{C} + \text{h.c.}\right) - e^{(4)}\left[i\kappa\bar{\psi}^{(4)}\cdot\gamma\chi_R^{\mathcal{C}} + 2\kappa^2\bar{\psi}_a^{(4)}\gamma^{ab}\psi_{Lb}^{(4)}\mathcal{A} + \text{h.c.}\right] \ . \tag{3.87}$$

[8] In fact, in going from eq.(3.81) to (3.82) we suppress couplings to gravity.

In the same way, the D-term of a real general operator $\mathcal{V} = [C, \zeta, H, K, B_a, \lambda, D]^{\mathcal{V}}$ (which in general is a composite real multiplet), with weights $(2, 0)$, must be understood as follows

$$\begin{aligned}[\mathcal{V}]_{\mathbf{D}} =& e^{(4)} \int d^4\theta\, 2\mathcal{V} + e^{(4)} \Big[-\kappa\bar{\psi}^{(4)} \cdot \gamma\gamma_5\lambda^{\mathcal{V}} + \kappa\frac{2}{3}i\bar{\zeta}^{\mathcal{V}}\gamma_5\gamma^{\mu\nu}\mathcal{D}_\mu\psi_\nu^{(4)} \\ &+ \frac{1}{3}C^{\mathcal{V}}(\mathcal{R}^{(4)} + 4\mathrm{i}\kappa^2\bar{\psi}_\mu^{(4)}\gamma^{\mu\nu\lambda}\mathcal{D}_\nu\psi_\lambda^{(4)}) + \mathrm{i}\kappa^2\epsilon^{abcd}\bar{\psi}_a^{(4)}\gamma_b\psi_c^{(4)}(B_d^{\mathcal{V}} - \kappa\bar{\psi}_d^{(4)}\zeta^{\mathcal{V}}) \Big] .\end{aligned} \tag{3.88}$$

(See [14] for the definitions of $\mathcal{R}^{(4)}$, $\mathcal{D}_\mu\psi_\lambda^{(4)}$ etc.) As we see, the first terms in (3.87), (3.88) are the contributions which we get upon integration over the θ superspace coordinates. The remaining terms give interactions with the components of the 4D Weyl multiplet. Therefore, for calculating all relevant couplings, in our superspace actions we should make the replacements

$$e^{(4)} \int d^2\theta(\cdots) + \text{h.c.} \to [\cdots]_{\mathbf{F}} , \qquad e^{(4)} \int d^4\theta(\cdots) \to \frac{1}{2}[\cdots]_{\mathbf{D}} , \tag{3.89}$$

and then use expressions (3.87) and (3.88) for their evaluation. Note also that when applying these expressions to composite multiplets, one should use the formulas for products of general multiplets of 4D conformal SUGRA given in [14, 57] to ensure that all 4D derivatives are covariant in respect to 4D conformal SUGRA.

As an illustration of the correctness of this procedure let us look at the D-terms present in our actions:

$$\frac{1}{2}\left[\mathbb{W}_y(2P(\mathcal{V}_5) + \mathbf{H}^+ \cdot \mathbf{H})\right]_{\mathbf{D}} \supset e^{(4)}e_y^5 \left(\frac{1}{8}D(2\mathcal{N} + \mathcal{A}^2) - \frac{1}{6}(\mathcal{N} - \mathcal{A}^2)\mathcal{R}^{(4)} \right) . \tag{3.90}$$

From a variation of the auxiliary field D it follows that $\mathcal{A}^2 = -2\mathcal{N}$ and therefore $\mathcal{N} - \mathcal{A}^2 = 3\mathcal{N}$. In this way one gets the correct coupling between the Ricci scalar and $\mathcal{N}$,

$$-\frac{1}{2}\mathcal{N}\mathcal{R}^{(4)}, \tag{3.91}$$

which becomes canonical after the conformal gauge fixing $\mathcal{N} = \kappa^{-2}$.

As we mentioned in the previous sections, what is missing in our superspace formulation is the odd (with respect to orbifold parity) part of the 5D Weyl multiplet. Additional effort is needed in order to embed these degrees of freedom into 4D superfields. Once this is done, we should be able to account for the full 5D covariance as well as for the couplings between the matter multiplets and that odd part induced from the 5D Weyl supermultiplet. However, this goes beyond the scope of this thesis.

Chapter 4

The Gauging of 5D Orbifold SUGRA and Fayet-Iliopoulos Terms

In this chapter we present a discussion of two technically related issues, namely the gauging of a subgroup of the $SU(2)_R$ symmetry which leads to supersymmetric versions of the Randall-Sundrum (I) model, and BPS Fayet-Iliopoulos terms in 5D orbifold SUGRA. What relates them is the fact that in both cases one has to introduce *stepwise* couplings. The presence of such couplings is only consistent with local supersymmetry if supplemented with suitable brane-localised couplings, needed to compensate the non-invariance of the bulk terms under (local) supersymmetry. A way of self-consistently introducing all these couplings was presented in ref. [20]. It relies on the addition of a auxiliary 4-form bulk field, coupled to a scalar $G(x, y)$ which is a singlet of supersymmetry. The integration of the 4-form implies that $G = g\epsilon(y)$. The off-shell version of this mechanism was presented in [11]. As we will see, in the superfield formalism the introduction of *stepwise* couplings can be consistently made without having to rely on the 4-form mechanism. In fact, the stepwise couplings introduced directly in the superspace action give rise to the correct brane-localized couplings upon suitable partial integrations. In addition, the BPS conditions correspond to the conditions of D-flatness and F-flatness, which as we will see are rather simple to write down within the superfield formalism.

4.1 The Gauging of 5D SUGRA and the RS Model

It is the purpose of this section to show how the gauging of 5D *orbifolded* SUGRA, necessary to obtain (local) supersymmetric generalizations of the RS model, can be performed in a very economic way, without having to resort to the 4-form mechanism of [20] and [13]. In particular, we consider the case of a single compensator and no physical bulk hypermultiplets. The gauging of 5D SUGRA proceeds - as it is usual in the superconformal formalism - by the coupling of the compensating hypermultiplet to a combination $\mathbb{V} \equiv V_I \mathbb{V}^I$ of $(n + 1)$ 5D vector multiplets $\mathbb{V}^I = (\Sigma^I,\ V^I)$, all with orbifold parities $\Pi(V^I) = -1$ [1]. As pointed out in the previous section,

[1] The reader less familiar with the superconformal formalism may wonder why we couple the vector multiplets to the compensator to gauge an R-symmetry and not directly to multiplets of the $SU(2)_R$. The reason is that if A_M is the gauge field in question, after imposing the conformal fixing $h = \kappa^{-1} + \cdots$, the covariant kinetic

if the gauge coupling is odd, $G(y) = g\epsilon(y)$, then we must use the Lagrangian of case **1**, eq.(3.52):

$$\begin{aligned} e_{(4)}^{-1}\mathcal{L} = &- \int d^4\theta \, \mathbb{W}_y 2 \left(h^+ e^{-G(y)V} h + h^{c+} e^{G(y)V} h^c \right) \\ &- \int d^2\theta (h^c \hat{\partial}_y h - h \hat{\partial}_y h^c - 2G(y) h^c \Sigma h) + \text{h.c.}, \end{aligned} \tag{4.1}$$

where we introduced $V \equiv V_I V^I$, $\Sigma \equiv V_I \Sigma^I$. Note that the $(n+1)$ *even* chiral superfields Σ^I will correspond to $(n+1)$ moduli, and in the minimal case of $n = 0$ the modulus is just the radion. As we will see below, the minimal case is the supersymmetric Randall-Sundrum model.

We assume in the following that the metric is of the warped type,

$$ds^2 = e^{2\sigma(y)} dx_\mu dx^\mu - (e_y^5)^2 dy^2, \tag{4.2}$$

where, depending on the gauge we use, the fünfbein's component e_y^5 will be y-dependent or not. As we explained in sec.3.4, the most straightforward way of taking into account the y-dependence of this metric is to perform a Weyl rescaling of all fields such that the 4D metric becomes flat, and then use eq.(4.1) for the rescaled fields. For the vielbein this means

$$e_\mu^a \to e^{-\sigma} e_\mu^a, \tag{4.3}$$

while for the compensating hypermultiplet we have (see eqs.(3.56),(3.57)),

$$h = e^{3\sigma/2}\kappa^{-1} + \theta^2 e^{5\sigma/2} F, \quad h^c = \theta^2 e^{5\sigma/2} F^c. \tag{4.4}$$

For the remaining superfields V, Σ, $\mathbb{W}_y$ we get

$$\Sigma = \frac{1}{2}(e_y^5 M^I + i A_y^I) + \theta^2 e^\sigma F_\Sigma + \cdots, \quad V = -\theta\sigma^\mu\bar{\theta}\, e^\sigma A_\mu + \frac{1}{2}\theta^2\bar{\theta}^2\, e^{2\sigma} D + \cdots, \tag{4.5}$$

where $M = V_I M^I$, $A_\mu = V_I W_\mu^I$, and

$$\mathbb{W}_y = e^{-\sigma} e_y^5 + \cdots. \tag{4.6}$$

(For a more detailed analysis see section 3.4.)

The bosonic part of the F-term Lagrangian becomes

$$\mathcal{L}_F \supset e^{4\sigma} F^c \kappa^{-1} \left[3\partial_y \sigma - G(y)(e_y^5 M + i A_y) \right] + h.c., \tag{4.7}$$

while the D-term Lagrangian is

$$\mathcal{L}_D \supset -e^{4\sigma} e_y^5\, 2 \left[|F|^2 + |F^c|^2 - \tfrac{1}{2}\kappa^{-2} G(y) D + \kappa^{-1}(F_T F^* + h.c.) \right], \tag{4.8}$$

term of the compensator becomes $\mathcal{L} = -M_5^3 |\kappa \vec{V}_M + g_R \vec{q} A_M|^2$. One sees that a shift of $\vec{V}_M$ will now induce the couplings of A_M with the $SU(2)_R$-multiplets.

where again $D = V_I D^I$. One can now integrate out the auxiliary fields F and F^c:

$$F = -\kappa^{-1} F_T^*, \quad F^c = -\frac{\kappa^{-1}}{2e_y^5}\left[G(y)(e_y^5 M - iA_y) - 3\partial_y\sigma\right], \tag{4.9}$$

to obtain

$$\mathcal{L}_H = e^{4\sigma} e_y^5 M_5^3 \left(2|F_T|^2 + G(y)D + \frac{1}{2}g^2M^2 - 3(\partial_5\sigma)G(y)M + \frac{9}{2}(\partial_5\sigma)^2\right) + \cdots \tag{4.10}$$

To obtain the additional contributions to the scalar potential of gauged SUGRA which appear from the vector sector, one has to consider the following terms:

$$\mathcal{L}_V \supset -e^{4\sigma} e_y^5 \left[\frac{1}{4}\mathcal{N}_{IJ}D^I D^J + \frac{1}{2}\mathcal{N}_{IJ}D^I\partial_5 M^J + (\partial_5\sigma)\mathcal{N}_I D^I\right]. \tag{4.11}$$

It is then straightforward to integrate out D^I:

$$D^I = \mathcal{N}^{IJ}\left(2M_5^3 G(y)V_J - e^{-2\sigma}\partial_5 e^{2\sigma}\mathcal{N}_J\right), \tag{4.12}$$

and finally we get

$$\begin{aligned} e^{-1}\mathcal{L}_{H+V} =& 6M_5^3(\partial_5\sigma)^2 - \frac{1}{4}\mathcal{N}_{IJ}\partial_a M^I \partial^a M^J \\ &+ M_5^3\left(\frac{1}{2}g^2M^2 + M_5^3 g^2 V_I V_J \mathcal{N}^{IJ} + e_5^y(\partial_y\epsilon(y))gM\right), \end{aligned} \tag{4.13}$$

where $\mathcal{N}^{IJ}$ is the inverse matrix of $\mathcal{N}_{IJ}$.

To put this in a better known form, let us introduce n even scalars, ϕ^i, to parameterise the *very special* manifold defined by $\mathcal{N}(M) = \kappa^{-2}$. The M^I $(I = 0, \ldots, n)$ are now functions of the ϕ^i $(i = 1, \ldots, n)$. We will use the following definitions:

$$g_{ij}(\phi) \equiv -\frac{1}{2}\mathcal{N}_{IJ}\frac{\partial M^I}{\partial\phi^i}\frac{\partial M^J}{\partial\phi^j}, \quad \mathcal{W}(\phi) \equiv M_5^3 g V_I M^I(\phi), \tag{4.14}$$

where $g_{ij}(\phi)$ is the sigma-model metric and $\mathcal{W}(\phi)$ the superpotential. We can rewrite eq.(4.13) as

$$e^{-1}\mathcal{L} = \frac{1}{2}g_{ij}(\phi)\partial_a\phi^i\partial^a\phi^j + 6M_5^3(\partial_5\sigma)^2 - V_B(\phi) + 2e_5^y[\delta(y) - \delta(y-\pi R)]\,\mathcal{W}(\phi), \tag{4.15}$$

where the bulk potential is now given as

$$V_B(\phi) = \frac{1}{2}g^{ij}\mathcal{W}_i\mathcal{W}_j - \frac{2}{3M_5^3}\mathcal{W}^2, \tag{4.16}$$

with $\mathcal{W}_i = \partial\mathcal{W}/\partial\phi^i$ and g^{ij} being the inverse of the metric g_{ij}.[2] These results agree precisely with the ones obtained with the 4-form mechanism [20], [13]. Note how supersymmetry relates the brane potentials to the bulk one. In particular, if we take no physical (bulk) vector

[2]To obtain this result we used the relation $g^{ij}\mathcal{N}_{I,i}\mathcal{N}_{J,j} = 4a_{IJ} - 2\mathcal{N}_I\mathcal{N}_J/3\mathcal{N}$.

multiplets, then we have $\mathcal{W}_i = 0$ and thus the brane tensions, $\tau = \pm 2\mathcal{W}$, are tuned with the bulk cosmological constant, with $\tau = \pm\sqrt{-6M_5^3 V_B}$, as in the Randall-Sundrum model [7]. The equation of motion for the warp-factor $\sigma(y)$ then follows from the Lagrangian, eq.(4.15), giving the solution $\sigma(y) = \pm\sqrt{-V_B/6M_5^3}|y|$.

In the general case, i.e. with an arbitrary number of *moduli* Σ^I, the BPS equations can be obtained from the F-flatness and D-flatness conditions, which read, for $A_y = 0$,

$$\partial_y \sigma = e_y^5 \frac{\epsilon(y)}{3M_5^3}\mathcal{W}, \tag{4.17}$$

$$\partial_y e^{2\sigma}\mathcal{N}_J = 2e^{2\sigma} e_y^5 \epsilon(y) M_5^3 g V_J. \tag{4.18}$$

Note that the second of these BPS equations can also be rewritten in terms of the ϕ^i as

$$\partial_y \phi^i = -g^{ij} e_y^5 \epsilon(y) \frac{\partial \mathcal{W}}{\partial \phi^j}, \tag{4.19}$$

but it is clear that eq.(4.18) is simpler to solve. Special solutions to these BPS equations can be found in ref. [20].

Closing this section, we want to emphasize how easy it is to write down the BPS equations using the superfield formalism. Usually, when working with a component Lagrangian, one has either to write down the Killing equations or put the Lagrangian in the form of a sum of perfect squares. This can be quite a bit of work. In the superfield approach, on the other hand, the BPS equations are just the requirements of D and F-flatness, which are very simple to obtain, even in the presence of arbitrary complicated brane terms.

4.2 Supersymmetric Radion Stabilization?

This short section is devoted to discuss some proposals in the literature for radion stabilization without SUSY breaking. In fact, the authors of [68] presented a mechanism of radion stabilization in the supersymmetric Randall-Sundrum model, which - they claimed - does not break supersymmetry. (Similar mechanisms were presented in [69] and very recently in [70].) Let us first recall their proposal and then explain why it does not work. The physical field content is the gravity sector plus a bulk hypermultiplet (H, H^c). In [68] the following brane-localised superpotentials were introduced:

$$\sum_i J^i \delta(y - y_i) \int d^2\theta \, hH + \text{h.c.}, \tag{4.20}$$

which clearly give new contributions to F_H. Now, to get a supersymmetric vacuum we require that $F_H = 0$. In the presence of the brane superpotentials, and assuming that $|\mathcal{A}_H|, |\mathcal{A}_H|^c \ll \kappa^{-1}$, this condition becomes:

$$\partial_y e^{3\sigma/2}\mathcal{A}_H^c + \frac{e^{3\sigma/2}}{2} M_5^{3/2} J^i \delta(y - y_i) \simeq 0. \tag{4.21}$$

As it was pointed out in [68], this equation has only a solution for tuned values of J^1, J^2. In fact, the odd quantity $e^{3\sigma/2}\mathcal{A}_H^c$ is *constant* in the bulk and therefore must have jumps in the branes of the same strength but opposite signs. This then implies a relation between the warp-factor and the constants J^i, which reads

$$J^1 = -J^2 \exp\left[\tfrac{3}{2}\left(\sigma(\pi R) - \sigma(0)\right)\right]. \tag{4.22}$$

Clearly, if R changes its value this condition is broken and we would expect that SUSY is also broken, generating a potential for the radion with a SUSY Minkowski minimum at the point defined by the ration of J_1 and J_2.

There are however several errors in this derivation. The crucial one is, in our view, that the brane superpotentials introduced by [68] are not allowed in gauged SUGRA, *unless* the physical hypermultiplet (H, H^c) is also charged under the $\mathrm{U}(1)_R$, as the compensator hypermultiplet is. In addition H (H^c) must have opposite charge to h (h^c), otherwise eq.(4.20) would not be invariant under the $\mathrm{U}(1)_R$ symmetry. Now, this implies that (H, H^c) now have masses proportional to the VEV of M^0, and therefore it is not $e^{3\sigma/2}\mathcal{A}_H^c$ that is constant in the bulk, but $\mathcal{A}_H^c$. Looking at eq.(4.21) it is then obvious that the true condition for unbroken SUSY is

$$J^1 = -J^2, \tag{4.23}$$

which does not depend at all on the size of the extra dimension.

Let us explain this in more detail. We will take the compensator to have $\mathrm{U}(1)_R$ charge $R = 1$, while the physical hyper has $R = -1$. The BPS equations are in this case

$$\left[\partial_y - \frac{e_y^5}{2}\epsilon(y) g_0 M^0\right] e^{3\sigma/2}\left\{1 + \kappa^2 |\mathcal{A}_H^c|^2\right\}^{\frac{1}{2}} = 0, \tag{4.24}$$

and

$$\left[\partial_y - \frac{e_y^5}{2}\epsilon(y) g_0 M^0\right] e^{3\sigma/2}\mathcal{A}_H^c + \frac{e^{3\sigma/2}}{2} M_5^{3/2} J^i \delta(y - y_i)\left\{1 + \kappa^2 |\mathcal{A}_H^c|^2\right\}^{\frac{1}{2}} = 0, \tag{4.25}$$

where for simplicity we assumed that $\mathcal{A}_H = 0$. Now, combining these two equations we get to the conclusion that $\mathcal{A}_H^c$ must be constant in the bulk, and

$$\left[\partial_y - \frac{e_y^5}{2}\epsilon(y) g_0 M^0\right] e^{3\sigma/2} = 0. \tag{4.26}$$

On the other hand, to satisfy eq.(4.25) also at the boundaries, we must have:

$$2e^{3\sigma(y_i)/2}\mathcal{A}_H^c + \frac{e^{3\sigma(y_i)/2}}{2} M_5^{3/2} J^i \left\{1 + \kappa^2 |\mathcal{A}_H^c|^2\right\}^{\frac{1}{2}} = 0, \tag{4.27}$$

which is fullfilled only with $J^1 = -J^2$, as already advertised.

We close this section wondering if it is possible to show that there is no such *supersymmetric* radion (or moduli) stabilisation mechanism in 5D orbifold SUGRA. Indeed, the fact that the known supersymmetric vacua are (4D) Minkowski would imply such a conclusion. The question should therefore be reformulated: Is it possible to obtain non-Minkowski (i.e. AdS) supersymmetric vacua in 5D orbifold SUGRA? As pointed out in ref. [20], in analogy with the vanishing of the Hamiltonian of a closed universe, the energy of any static y-dependent bosonic vacuum configuration vanishes locally. This seems to be a no-go theorem for finding supersymmetric moduli stabilisation.

4.3 BPS FI Terms

When FI terms where considered first, in the context of 4D supersymmetric theories [71], they were seen as a means of breaking supersymmetry and/or gauge symmetry. Later, their utmost relevance for cosmology was also recognized, as it became clear that they could be at the origin of de Sitter configurations, and more generally of inflationary scenarios [72]. While in global (4D) supersymmetric theories the introduction of FI terms is rather straightforward, it turns out that in supergravity this is not the case. In fact, the compatibility of local supersymmetry and FI terms requires the U(1) gauge symmetry in question to be an R-symmetry [73–75], and therefore the gravitino has to be charged. In addition, they only can be radiatively generated in the presence of a mixed U(1)-gravitational anomaly.

In five-dimensional orbifolds the situation gets another twist. In the rigid case, the FI terms can be consistently introduced at the 4D fix-point branes, but unlike in the 4D case they can be tuned in such a way that neither supersymmetry nor the U(1) gauge symmetry are broken [76–79]. As it was pointed out in [77], FI terms can be generated radiatively even in the case that the mixed anomaly is absent, but turn out to be of the *tuned* type that we just mentioned. The effect of such tuned FI terms is to induce a stepwise VEV of the (odd) scalar component of the U(1) vector multiplet, which leads to the localisation of zero-modes of charged hypermatter [78, 79]. On the other hand, if this U(1) symmetry is part of a larger bulk gauge symmetry $\mathcal{G}$, the VEV of the vector scalar will break $\mathcal{G}$ in the bulk while orbifolding breaks it at the boundary. The relevance of this for *calculable* power-law unification has been recently emphasized in [40].

A discussion of the embedding of (tuned) FI terms in 5D orbifold supergravity was first given in ref. [77], where it was pointed out that they are associated with a bulk Chern-Simons term with one $U(1)_{FI}$ gauge boson and two graviphotons. In particular, the strength of the FI terms is fixed by the strength of the stepwise coupling of the CS term. As in the rigid case, the tuned FI terms lead to a stepwise VEV of the vector scalar, and therefore to the localisation of charged hypermultiplets. This analysis was recently extended in [80] to orbifold SUGRA with warped geometry, i.e. these authors considered the possibility of gauging the $U(1)_R$ symmetry. They came to the, in our view, incorrect conclusion that in the presence of a warped geometry, unless hypermatter is introduced, SUSY is broken by non-vanishing (tuned) FI terms.

We will here show that tuned FI terms do not lead to the breaking of $\mathcal{N} = 1$ supersymmetry, even in a warped geometry. In other words, we will see that the BPS conditions have solutions in the presence of tuned FI terms, even if we gauge the $U(1)_R$ symmetry. For this reason we call this type of FI terms *BPS* FI terms. We consider here two different cases, namely with and without charged hypermultiplets, and in both cases we find SUSY vacua. While in absence of hypermultiplets we obtain a solution with a warp-factor of the Randall-Sundrum type and a stepwise VEV for the vector scalar, the inclusion of two bulk hypermultiplets with opposite $U(1)_{FI}$ charges allows for more general solutions. In particular, in the case the $U(1)_R$ is not gauged, we obtain warped solutions corresponding to the presence of negative brane tensions. These are induced by non-vanishing profiles of the two even hyperscalars, which are localised near opposite branes.

The rigid case. As we already pointed out, in 5D orbifolds brane-localised FI terms can be canceled by a stepwise VEV of the scalar component of the 4D chiral superfield $\Sigma = \frac{1}{2}(M+iA_y)+\cdots$ (we take $e_y^5 = 1$), which accompanies the 4D vector superfield V. Indeed, the derivative $\partial_y\Sigma$ can cancel the FI terms localized at the fixed point boundaries, in which case SUSY remains unbroken and M gets a stepwise VEV [76–79]. This cancelation takes only place in case the FI terms in the two boundaries are *tuned*, having opposite signs and equal absolute values at different branes. Using the superfield description of 5D rigid supersymmetry [22–24] presented in sec.2.3, these FI terms can be writen as

$$\mathcal{L}_{FI} = -4[\delta(y) - \delta(y-\pi R)] \int d^4\theta \xi V. \tag{4.28}$$

We now make the observation that in the rigid case the (tuned) FI term can be rewritten as follows

$$\mathcal{L}_{FI} = -2\int d^4\theta\xi(\partial_y\epsilon(y))V = 2\int d^4\theta\xi\epsilon(y)[\partial_y V - (\Sigma+\Sigma^+)] = -2\int d^4\theta\,\xi\epsilon(y)\mathcal{V}_y, \tag{4.29}$$

where we introduced the gauge invariant $\mathcal{V}_y \equiv \Sigma + \Sigma^+ - \partial_y V$. There is also a term in the Lagrangian, quadratic in $\mathcal{V}_y$, which is responsible for part of the kinetic terms [22]:

$$\mathcal{L} \supset \int d^4\theta\,(\mathcal{V}_y)^2. \tag{4.30}$$

This can be combined with eq.(4.29) to get

$$\mathcal{L} \supset \int d^4\theta\,(\mathcal{V}_y - \xi\epsilon(y))^2. \tag{4.31}$$

From this expression it becomes clear that the only effect of the FI terms is to shift the lowest component of Σ as $M \to M + \xi\epsilon(y)$, which does not break SUSY. The $U(1)_{FI}$ is also unbroken since Σ is neutral under this group.

BPS FI terms in 5D orbifold SUGRA. We assume in the following that the metric is of the warped type, i.e.

$$ds^2 = e^{2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu - (e_y^5)^2 dy^2, \tag{4.32}$$

where the fünfbein's component e_y^5 can also be y-dependent. Eventually, we will later on choose the gauges $e_y^5 = e^{-2\sigma}$ or $e_y^5 = \text{const.}$ for practical reasons. Note that the warp factor $\sigma(y)$ is not fixed *a priori* but will be determined from the equations of motion.

We now argue that in the case of SUGRA, a term similar to expression (4.29) is obtained with a *norm*-function of the following form (proposed in [77])

$$\kappa^{-1}\mathcal{N}(M) = (M^0)^3 - M^0(M^1)^2 + 2\kappa\xi\epsilon(y)(M^0)^2 M^1, \tag{4.33}$$

where, to ensure that $\mathcal{N}$ has even orbifold parity, M^0 and M^1 must have positive and negative parities, respectively. It is not hard to see that the last term in the norm-function contributes to the Lagrangian a term (see eq.(3.67))

$$-\int d^4\theta\, \mathbb{W}_y\, \mathcal{N}(\mathcal{V}_5) \supset -2(\kappa M^0)^2 \int d^4\theta\, e_y^5 e^\sigma\, \xi\epsilon(y)\mathcal{V}_5^1, \tag{4.34}$$

which indeed has the same form as the tuned FI term in rigid SUSY but also takes into account the warped geometry. One sees that here it is the vector multiplet $\mathbb{V}^{I=1}$ which gauges the $\mathrm{U}(1)_{FI}$ symmetry, for which there are FI terms. Due to its orbifold parity, brane localized FI terms involving V^0 are not possible.

In this section we will consider the case with no physical hypermultiplets, and only one compensator multiplet, leaving the more general case to the following section. We recall that the compensator hypermultiplet corresponds to a pair of chiral superfields $(h,\ h^c)$, where we take h to have positive orbifold parity, h^c to have negative. After the fixing of the conformal symmetries, we have

$$h = e^{3\sigma/2}\kappa^{-1} + \theta^2\, e^{5\sigma/2} F_h, \qquad h^c = \theta^2\, e^{5\sigma/2} F_h^c. \tag{4.35}$$

We will gauge an $\mathrm{U}(1)_R$ subgroup of the $\mathrm{SU}(2)_R$ by coupling the compensator hypermultiplet $(h,\ h^c)$ to the $\mathbb{V}^0$ vector multiplet with an *odd* gauge coupling, $g_0\epsilon(y)$, as we did in section 4.1. The D-term Lagrangian does not only arises from eq.(3.67), but also has a contribution from the compensator Lagrangian, which in this case reads

$$\begin{aligned}\mathcal{L}_{comp} = &-2\int d^4\theta\, \mathbb{W}_y\, (h^+ e^{-g_0\epsilon(y)V^0} h + h^{c+} e^{g_0\epsilon(y)V^0} h^c) \\ &\qquad - 2\left(\int d^2\theta\, h^c(\partial_y - g_0\epsilon(y)\Sigma^0)h + \text{h.c.}\right).\end{aligned} \tag{4.36}$$

The total D-term Lagrangian is thus

$$\mathcal{L}_D = e^{4\sigma} e_y^5 \left[-\frac{1}{4}\mathcal{N}_{IJ}(M) D^I D^J - \frac{e^{-2\sigma} e_5^y}{2}(\partial_y e^{2\sigma}\mathcal{N}_I(M))D^I + M_5^3 g_0\epsilon(y) D^0\right]. \tag{4.37}$$

As it was already pointed out, the BPS conditions correspond in the superfield approach to F-flatness and D-flatness. In particular we must have $D^I = 0$. Now, it follows from the Lagrangian above that

$$D^I = \mathcal{N}^{IJ}\left(2M_5^3 g_0 \epsilon(y)\delta_J^0 - e^{-2\sigma} e_5^y \partial_y e^{2\sigma}\mathcal{N}_J\right), \tag{4.38}$$

and so the BPS condition $D^I = 0$ becomes

$$\partial_y(e^{2\sigma}\mathcal{N}_J) = 2e^{2\sigma} e_y^5\, M_5^3 \epsilon(y) g_0 \delta_J^0. \tag{4.39}$$

Since $\mathcal{N}_1$ has negative parity, the BPS equation with $I = 1$ is solved by

$$\mathcal{N}_1 = 0 \Rightarrow -2M^0\, M^1 + 2\kappa\xi\epsilon(y)(M^0)^2 = 0, \tag{4.40}$$

that is

$$M^1 = \kappa\xi\epsilon(y)M^0. \tag{4.41}$$

The value of M^0 then follows readily from $\mathcal{N} = \kappa^{-2}$, being

$$M^0 = M_5^{3/2}(1 + (\kappa\xi)^2)^{-1/3}. \tag{4.42}$$

Finally, the metric is obtained by solving the BPS equation with $I = 0$. In the gauge $e_y^5 = e^{-2\sigma}$, we obtain

$$e^{2\sigma}\mathcal{N}_0 = t_0 + 2g_0 M_5^3 |y|, \tag{4.43}$$

where t_0 is an integration constant. We get

$$e^{2\sigma} = \frac{M^0}{3M_5^3}[t_0 + 2g_0 M_5^3|y|]. \tag{4.44}$$

Note that this result could also have been obtained from the F-flatness condition $F_h^c = 0$. If prefered, one can introduce a new coordinate z defined by $dz = e^{-2\sigma(y)}dy$. This is the Randall-Sundrum gauge. In terms of this variable the metric becomes

$$ds^2 = e^{2\sigma}dx^2 - dz^2, \quad \text{with} \quad e^{2\sigma(z)} = \exp\left(\tfrac{2}{3}g_0 M^0|z|\right). \tag{4.45}$$

It is clear from this discussion that the presence of the FI terms, even in a warped geometry, does not lead to supersymmetry breaking, due to the fact that the *odd* scalar M^1 *absorbs* the FI term, just as in the case of rigid SUSY. The authors of ref. [80] obtained the opposite result. The point is that these authors introduced an odd scalar field ϕ to parametrise the very special manifold defined by $\mathcal{N}(M) = \kappa^{-2}$. The M^J are then functions of ϕ, but the relation between M^1 and ϕ also involves $\epsilon(y)$. This means that

$$\partial_y M^1 = \frac{\partial M^1}{\partial\phi}\partial_y\phi + \frac{\partial M^1}{\partial\epsilon}\partial_y\epsilon(y), \tag{4.46}$$

but in [80] the second term on the r.h.s. was neglected, e.g. in going from the third equation in eqs.(22) to the third equation in eqs.(23) of [80]. This fact was recognised in a new version of [80] which appeared after our work was published. In this way they confirmed that the component approach, with the 4-form mechanism, is equivalent to the superfield approach used here. In addition their problems seem to show that the superfield formalism is more transparent and straightforward to use.

4.4 Charged Hypermultiplets and Localisation

In this section we discuss the consequences of introducing hypermultiplets charged under the $\mathrm{U}(1)_{FI}$. To be concrete let us consider in addition to the setup we had before a physical hypermultiplet $(H,\ H^c)$ with charge $q_1 = 1$ (we absorb the charge in the gauge coupling g_1). Here, the chiral superfield H will be taken to be even while H^c is odd. One consequence of this is that the scalar component of the even compensator chiral superfield is now a function of $\mathcal{A}_H$ and $\mathcal{A}_H^c$, the scalar components of H and H^c:

$$h = e^{3\sigma/2}\kappa^{-1}\left\{1+\kappa^2\left(|\mathcal{A}_H|^2+|\mathcal{A}_H^c|^2\right)\right\}^{\frac{1}{2}} + \theta^2 e^{5\sigma/2}F_h. \tag{4.47}$$

In addition there are new couplings involving H and H^c:

$$\begin{aligned}\mathcal{L}_H = 2\int d^4\theta\, \mathbb{W}_y\left(H^+e^{-g_1V^1}H + H^{c+}e^{g_1V^1}H^c\right)\\ -2\int d^2\theta H^c\left(\partial_y - g_1\Sigma^1\right)H + \text{h.c.}\end{aligned} \tag{4.48}$$

This leads to a new set of BPS conditions. From the conditions $F_h^c = 0 = F_H^c = F_H$ we get

$$\left[\partial_y - \frac{e_y^5}{2}\epsilon(y)g_0M^0\right]e^{3\sigma/2}\left\{1+\kappa^2\left(|\mathcal{A}_H|^2+|\mathcal{A}_H^c|^2\right)\right\}^{\frac{1}{2}} = 0, \tag{4.49}$$

$$\left[\partial_y - \frac{e_y^5}{2}g_1M^1\right]e^{3\sigma/2}\mathcal{A}_H = 0, \tag{4.50}$$

and

$$\left[\partial_y + \frac{e_y^5}{2}g_1M^1\right]e^{3\sigma/2}\mathcal{A}_H^c = 0, \tag{4.51}$$

while from $D^I = 0$ we obtain (instead of (4.39))

$$\partial_y e^{2\sigma}\mathcal{N}_J = 2M_5^3\, e^{2\sigma}e_y^5\,\epsilon(y)f_J(\mathcal{A}), \tag{4.52}$$

where

$$f_J(\mathcal{A}) \equiv g_J\cdot\begin{cases}\left(1+\kappa^2\left(|\mathcal{A}_H|^2+|\mathcal{A}_H^c|^2\right)\right) & ,J=0,\\ \epsilon(y)\kappa^2\left(|\mathcal{A}_H^c|^2-|\mathcal{A}_H|^2\right) & ,J=1.\end{cases} \tag{4.53}$$

Now, we can combine eqs.(4.49) to (4.51) to get an equation for the warp-factor $\sigma(y)$,

$$\partial_y \sigma = e_y^5 \frac{\epsilon(y)}{3M_5^3} \mathcal{W}, \tag{4.54}$$

where the *superpotential* $\mathcal{W}$ is defined as $\mathcal{W} \equiv M_5^3 f_I(\mathcal{A}) M^I$. Note that this very same equation follows by multiplication of eq.(4.52) with M^J upon the use of the constraint $\mathcal{N} = M_5^3$, showing that one just needs to solve four of the above five equations. This constraint defines a 1-dimensional scalar manifold which can be parametrized by a single scalar ϕ. In this way the scalars M^I become functions of ϕ. To obtain the equation of motion for ϕ we therefore have to contract eq.(4.52) with $(\partial M^J/\partial\phi)$. After some manipulations, we get (using $\partial_y \mathcal{N}_J = \partial_\phi \mathcal{N}_J \, \partial_y \phi + \partial_\epsilon \mathcal{N}_J \, \partial_y \epsilon$)

$$g_{\phi\phi} \partial_y \phi = -e_y^5 \epsilon(y) \frac{\partial \mathcal{W}}{\partial \phi} + \frac{1}{2} \frac{\partial M^J}{\partial \phi} \frac{\partial \mathcal{N}_J}{\partial \epsilon} \Big|_\phi \partial_y \epsilon, \tag{4.55}$$

where we introduced the sigma-model *metric*, $g_{\phi\phi}(\phi)$, defined by

$$g_{\phi\phi}(\phi) = -\frac{1}{2} \mathcal{N}_{IJ} \frac{\partial M^I}{\partial \phi} \frac{\partial M^J}{\partial \phi}. \tag{4.56}$$

Note that eq.(4.55) is independent of the way we choose to parametrize the very special manifold. In particular we can take ϕ to be an *even* scalar. This choice has the property that the second term at the r.h.s. of (4.55) vanishes, and we get

$$g_{\phi\phi} \partial_y \phi = -e_y^5 \epsilon(y) \frac{\partial \mathcal{W}}{\partial \phi}. \tag{4.57}$$

Solutions of the BPS equations. Let us now discuss the solutions of this new set of BPS equations. The first observation we make is that by integrating eq.(4.52) over the whole extra dimension we obtain the constraint

$$\oint dy \, e^{2\sigma} e_y^5 (|\mathcal{A}_H|^2 - |\mathcal{A}_H^c|^2) = 0. \tag{4.58}$$

On the other hand, from eq.(4.51) and the fact that $\mathcal{A}_H^c$ is odd, one gets that $\mathcal{A}_H^c = 0$. Otherwise eq.(4.51) would have singularities at the branes positions. It then readily follows that also $\mathcal{A}_H = 0$, and we are back to the case discussed in section 4.3 so that M^0 and M^1 are given by eqs.(4.41) and (4.42), and the warp-factor is the one given in that section.

Less trivial solutions, i.e. with non-vanishing hyperscalar VEVs, are possible if we add a second (bulk) hypermultiplet, $(\hat{H}, \hat{H}^c)$, with opposite charge, $q_1 = -1$. While the odd hyperscalars are still vanishing, $\mathcal{A}^c = \hat{\mathcal{A}}^c = 0$, the constraint (4.58) now gets replaced by

$$\oint dy \, e^{2\sigma} e_y^5 (|\mathcal{A}_H|^2 - |\hat{\mathcal{A}}_H|^2) = 0, \tag{4.59}$$

which allows for non-trivial profiles for $\mathcal{A}_H$ and $\hat{\mathcal{A}}_H$. In this case, even if $g_0 = 0$, the metric will be warped, as follows from eq.(4.49):

$$e^{3\sigma} = \frac{c_0}{1 + \kappa^2(|\mathcal{A}_H|^2 + |\hat{\mathcal{A}}_H|^2)}. \tag{4.60}$$

Note that we can obtain some additional knowledge about the solutions to the BPS equations by integrating eq.(4.52) for $J = 1$ over a small neighbourhood of the fix-point branes. In this way we learn that

$$M^1 = \frac{\xi\epsilon(y)}{[1 + (\kappa\xi)^2]^{1/3}} + \psi, \tag{4.61}$$

where ψ vanishes on the branes. This means that the value of M^1 near the branes is solely determined by the stength of the FI term.

Let us solve the BPS equations for the case with $g_0 = 0$ and non-trivial profiles of the even hyperscalars. To parametrize the 1-dimensional very special scalar manifold we introduce an even scalar ϕ in the following way:

$$M^1(\phi) = \kappa(\xi + \phi)\epsilon(y)M^0(\phi), \tag{4.62}$$

$$M^0(\phi) = \frac{\kappa^{-1}}{[1 + (\kappa\xi)^2 - (\kappa\phi)^2]^{1/3}}. \tag{4.63}$$

We will have to resort to some approximation. We thus assume that $\kappa|\phi| \ll 1$ and get:

$$\partial_y\phi \simeq e_y^5\epsilon(y)g_1(|\mathcal{A}_H|^2 - |\hat{\mathcal{A}}_H|^2)[1 + (\kappa\xi)^2]^{2/3}, \tag{4.64}$$

while from eq.(4.50) (and a similar equation for $\hat{\mathcal{A}}_H$) we obtain,

$$|\mathcal{A}_H|^2 \simeq |a|^2 \exp\left(e_y^5 g_1 r(y)\right), \qquad |\hat{\mathcal{A}}_H|^2 \simeq |\hat{a}|^2 \exp\left(-e_y^5 g_1 r(y)\right), \tag{4.65}$$

where we chose a gauge with constant e_y^5, and introduced $r(y) \equiv \int_0^y dy\, M^1$. In the bulk ($0 < y < y_\pi$), from eq.(4.64), we obtain the following equation for $r(y)$:

$$\partial_y^2 r = \frac{d}{dr}\left[|A|^2 \cosh(e_y^5 g_1(r - \bar{r}))\right], \tag{4.66}$$

where $|A|^2 = 2|a||\hat{a}|[1 + (\kappa\xi)^2]^{1/3}$, and $\bar{r} = (e_y^5 g_1)^{-1} \ln|\hat{a}/a|$.

Eq.(4.66) has a rather simple interpretation as being the equation of motion of a particle in a inverted cosh potential. The FI terms set boundary conditions at the two branes, $y = \{0, y_\pi\}$, which correspond, in the mechanical analogon, to fixing the start and end velocities: $\partial_y r(0) = \partial_y r(y_\pi) = \xi[1 + (\kappa\xi)^2]^{-1/3}$. In addition, the initial position is $r(0) = 0$, by definition. The fact that we have 3 boundary conditions implies that one of the parameters, $|A|$ or $\bar{r}$, is fixed by

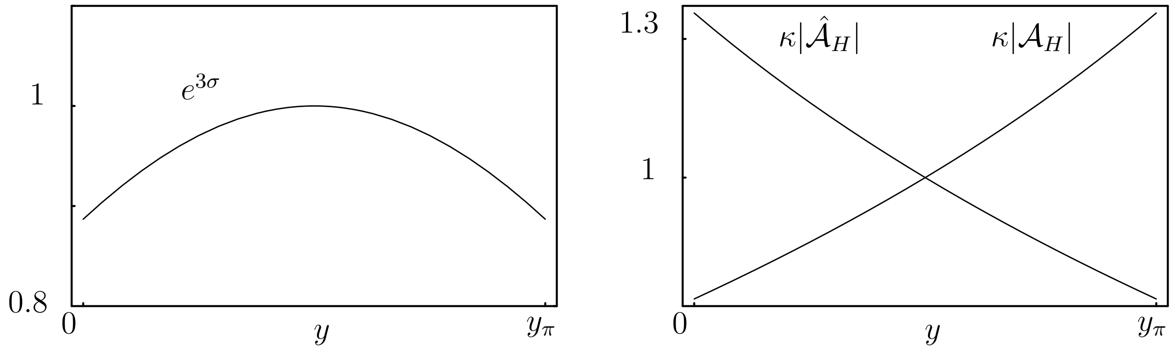

Figure 4.1: Profiles of $\exp(3\sigma)$ and $|\mathcal{A}_H|$, $|\hat{\mathcal{A}}_H|$, for $g_1\xi > 0$ and the parameters $|\hat{a}| = 1.36\kappa^{-1}$, $|a| = 0.74\kappa^{-1}$.

the other, the value of the FI terms and the size of the extra dimension . For special values of these parameters it is possible to solve eq.(4.66) analytically, and in this way to obtain the corresponding warp-factor. In particular, for $2|A|\cosh(g_1 e_y^5 \bar{r}/2) = |\xi|[1+(\kappa\xi)^2]^{-1/3}$ we get

$$\exp\left(-\tfrac{1}{2}e_y^5 g_1(r(y)-\bar{r})\right) = \frac{1+\tan\left(\frac{\xi}{4|\xi|}|A|g_1 e_y^5(y_\pi - 2y)\right)}{1-\tan\left(\frac{\xi}{4|\xi|}|A|g_1 e_y^5(y_\pi - 2y)\right)}, \tag{4.67}$$

where we used

$$\left|\frac{\hat{a}}{a}\right| = \tan^2\left(\frac{\pi}{4} + \frac{\xi}{4|\xi|}|A|g_1 e_y^5 y_\pi\right), \tag{4.68}$$

which follows from the boundary condition at $y = y_\pi$. To obtain the warp-factor and hyperscalars profiles, we can use eqs.(4.60) and (4.65). We ilustrate our findings in Fig.4.1, with plots for a specific choice of the parameters.

Perhaps the most salient feature of these solutions, and without the particular assumption we made above, is the fact that they correspond to vacua with the same *negative tension* in both branes. This can be recognized from eq.(4.54) by noting that $\partial_y\sigma(0^+) = -\partial_y\sigma(y_\pi^-) > 0$. To show this, we use again the mechanical analogon: since at the boundaries the *velocities* are equal, the potential must also be the same. This implies that $r(y_\pi) = 2\bar{r}$. From eq.(4.65) it follows then that $|\mathcal{A}_H(0^+)| = |\hat{\mathcal{A}}_H(y_\pi^-)|$ and $|\hat{\mathcal{A}}_H(0^+)| = |\mathcal{A}_H(y_\pi^-)|$, and therefore we obtain

$$\partial_y\sigma(0^+) = -\partial_y\sigma(y_\pi^-) = \xi\kappa^2(|\hat{a}|^2 - |a|^2)\,\frac{e_y^5 g_1}{3}[1+(\kappa\xi)^2]^{-1/3} > 0. \tag{4.69}$$

The origin of these negative brane tensions is simple to understand. In each brane, the FI terms induce localised mass terms for both hyperscalars, which have the same magnitude but opposite sign. The positive mass repulses the corresponding hyperscalar from the brane while the hyperscalar with negative mass is attracted. This clearly has the net effect of producing negative tensions at both branes. To compensate these negative tensions, there must positive curvature in the bulk. This is indeed the case, as can be recognized from the 5D Ricci scalar (see eq.(3.78)):

$$\mathcal{R} = 4\left(5(\partial_5\sigma)^2 + 2\partial_5^2\sigma\right) = \tfrac{16}{5} e^{-5\sigma/2}\,\partial_5^2\, e^{5\sigma/2}. \tag{4.70}$$

Looking at the plot of $e^{3\sigma}$ in Fig.4.1, it becomes clear that the Ricci scalar is negative all over the bulk, corresponding to a positive curvature (energy density) in the bulk. In addition, one can show that this has a maximum at the middle of the bulk, so that one may speak of a *fat brane* of positive *tension* sitting on bulk. All this reasoning leads us to expect the zero-mode of the graviton to be localised not on (one of) fix-point branes but in the bulk, on the positive tension fat brane. We should however emphasize that to trully obtain 4D gravity we must stabilize the moduli, i.e. the radion and the hyperscalars zero-modes.

Finally, we comment on the possibility that the FI term is renormalised by 1-loop corrections due to the charged hypermultiplets. This is not the case, as clearly the FI term involves both the $\mathbb{V}^0$ and the $\mathbb{V}^1$ vector multiplets, while the hypers only couple to $\mathbb{V}^1$. This is in agreement with the results in global orbifold SUSY, where it was shown that with a non-anomalous field content the 1-loop FI term is proportional to the hypers masses, which correspond in the local case to couplings to Σ^0. Note however that the calculation of 1-loop effects in 5D orbifold SUGRA might be tricky, in what concerns the divergence structure. The point is that to keep the conformal substructure of the theory one has to consider a regularization that respects both that structure [81] and the other symmetries of the theory. It is not clear if this is allways possible in the *orbifold* case.

(De)localisation of hypermatter. Let us now see the consequences of the odd VEV of M^1. In the case of rigid SUSY one knows that a hypermultiplet charged under the $\mathrm{U}(1)_{FI}$ can get localised [78, 79]. This is due to the fact that it gets an odd mass. Here the same happens. The hypermultiplet Lagrangian includes a term

$$-2\int d^2\theta H^c \left[\partial_y - \frac{1}{2}e_y^5 g_1 M^1(y)\right] H + \text{h.c.} \tag{4.71}$$

This shows that if there is a hyperscalar KK zero-mode $f_0(y)$, it must satisfy

$$\left[\partial_y - \frac{1}{2}e_y^5 g_1 M^1(y)\right] e^{3\sigma/2} f_0(y) = 0. \tag{4.72}$$

In the case that in the vacuum the physical hyperscalars vanish, $\mathcal{A}_H = 0 = \hat{\mathcal{A}}_H$, the solution is rather simple to obtain:

$$f_0 \propto \exp[\kappa\xi g_1/g_0 - 1]3\sigma/2. \tag{4.73}$$

For $\xi g_1 = 0$ the localisation is due only to the warped geometry, while for $\xi g_1 \neq 0$ the FI terms induce an additional amount of localisation in the same brane or localize the hyperscalar towards the other fix-point brane. Note that the best coordinate to evaluate this effect of (de)localisation is y, not z. In terms of y the kinetic term of the zero-mode is already canonically normalized.

In the case with hyperscalars developing non-zero VEVs, the solution to eq.(4.72) is just proportional to those VEVs. In the example we studied above, the scalar $\mathcal{A}_H$ is localised near one of the branes, $\hat{\mathcal{A}}_H$ near the other. The same happens with the zero modes.

Chapter 5

Gauge Inflation, SUSY & Orbifolds

5.1 Why inflation?

The paradigm of inflation was introduced [82] to solve the cosmological flatness, horizon, large scale structure and monopole problems. That means it should explain why in the early universe the energy density was so incredibly near the critical one; how could causally disconnected regions have such a highly correlated background temperature; how did the galaxies and galaxies' clusters form; and why there are no relic monopoles and other topological defects remaining from the early universe. inflation solves these puzzles with a period of exponential expansion of the early universe in which the size of the universe expanded $\geq e^{55}$ times.

This idea can be realized in models with scalar fields, during a period of potential energy domination. Since this potential energy domination must remain for a certain time, the scalar potential must satisfy flatness conditions $V' \ll V/M_{pl}$ and $|V''| \ll V/M_{pl}^2$. There are two known ways of protecting such flat potentials against radiative corrections, namely supersymmetry (SUSY) and non-linearly realized symmetries, in particular shift symmetries. But once supergravity (SUGRA) effects are taken into account, radiative corrections arise which typically become important for values of the fields of order the Planck scale and give a mass to the inflaton of order V/M_{pl}^2, spoiling slow-roll (see e.g. [84]). A way out of this problem is to consider shift symmetries, i.e. inflaton fields which are axion-like [83]: $\theta \sim \theta + 2\pi$. The canonically normalized field is $\phi_\theta = f\theta$, where f has mass dimension one, and the slow-roll conditions imply therefore $f \gg kM_{pl}$, where k is the number of the mode that dominates the Fourier decomposition of the periodic potential $V(\theta)$. Unfortunatly there is no way of explaining the presence of a mass scale which is larger than M_{pl} in the framework of 4D theories. However, if one considers extra-dimensional theories, then this picture may change. A realization of this idea, proposed by the authors of ref. [84, 85], and its SUSY orbifold version, is what we discuss in the following sections.

5.2 Compact extra-dimensions, gauge symmetry and non-locality

Consider a 5D abelian gauge theory, compactified in a circle of size $2\pi R$. While from a local point of view the gauge field A_M $(M = 1, \ldots, 5)$ does not know about the finiteness of the 5th dimension, globally the topology of the circle can be noticed in the form of boundary conditions. To be more precise, while the Lagrangian is invariant under any gauge transformation, the boundary conditions are not. Take for instance $A_5 = \Theta/2\pi R$ and use the gauge parameter $\Lambda(y) = -(\Theta/2\pi R)\, y$ to make it vanish. A charged scalar field in the fundamental representation transforms then as

$$\phi(y) \to e^{-iq(\Theta/2\pi R)\, y}\phi(y). \tag{5.1}$$

Clearly, the periodicity of $\phi(y)$ under $y \to y + 2\pi R$ is not respected, unless $q\Theta = \text{mod } 2\pi$. In general, gauge transformations with non-periodic (mod 2π) gauge parameter, change the boundary conditions on the charged matter fields. As a consequence, after integrating out these fields one obtains an effective action which is invariant only under *periodic* gauge transformations. This means that one can obtain a non-vanishing effective potential for the non-local quantity (Wilson-line) [86],

$$\Theta_5(x) \equiv \oint dy\, A_5(x, y), \tag{5.2}$$

which is invariant under periodic gauge transformations. In addition, it follows from the discussion following eq.(5.1) that $\Theta_5(x) \sim \Theta_5(x) + 2\pi/q$, i.e. the charged fields do not distinguish between $\Theta_5(x)$ and $\Theta_5(x) + 2\pi/q$. The effective potential $V_{eff}(\Theta)$ has thus exactly this very same symmetry, which is the shift symmetry mentioned in the previous section.

The precise form of the 1-loop effective potential, obtained by integrating out the scalar of charge $e = 1$ and mass$^2 = m^2$ in a background of constant $\Theta_5(x)$, is straightforward to obtain. We will use for this the world-line formalism described in appendix A and section 2.2. For our purposes what we need is eq.(A-3), with $A_M = \delta^5_M\, \Theta/2\pi R$:

$$V(\Theta) = -\int \frac{dT}{T}\, e^{-m^2 T} \int\limits_{x(T)\sim x(0)} \mathcal{D}^5 x \exp\left[-\int_0^T d\tau\, \left(\tfrac{1}{4}\dot{x}^2 + i\dot{y}\Theta/2\pi R\right)\right]. \tag{5.3}$$

Here $y(T) \sim y(0)$ defines closed paths that wind around the circle:

$$y_k(\tau) = y_0 + k 2\pi R \frac{\tau}{T} + \bar{y}(\tau), \tag{5.4}$$

with $\bar{y}(T) = \bar{y}(0)$. Using the results of appendix A we obtain

$$V(\Theta) = -\frac{1}{(4\pi)^{\frac{5}{2}}} \sum_{k\in\mathbb{Z}} \cos(k\Theta) \int_0^\infty \frac{dT}{T^{\frac{7}{2}}}\, e^{-m^2 T} e^{-\frac{(k\pi R)^2}{T}}, \tag{5.5}$$

which can be rewritten as

$$\begin{aligned} V(\Theta) = -\frac{3}{2^6 \pi^7 R^5} \sum_{k=1}^{+\infty} \frac{\cos(k\Theta)}{k^5} e^{-2\pi k R|m|}[1 + 2\pi k R|m| + \frac{1}{3}(2\pi k R m)^2] \\ + \mathcal{V}_{5D}(\Lambda, |m|), \end{aligned} \tag{5.6}$$

where the 5D cosmological constant $\mathcal{V}_{5D}(\Lambda, |m|)$ is only a function of the cut-off Λ and of the 5d mass m. One may wonder whether in a (broken) supersymmetric theory this divergent vacuum energy can be canceled out by the contribution of the fermionic superpartners. This is in general not always the case, but there are specific examples where such a cancelation happens, as we will discuss in the next section.

The virtues of the above potential for inflation become clear after one normalizes Θ_5 canonically [84],

$$\mathcal{L}_\Theta = \frac{1}{2}\frac{\partial_\mu \Theta \partial^\mu \Theta_5}{(2\pi R g_4)^2} - V_{eff}(\Theta_5) \to \frac{1}{2}\partial_\mu \phi_\Theta \partial^\mu \phi_\Theta - V_{eff}(\phi_\Theta/f). \tag{5.7}$$

($V_{eff}(\Theta) \equiv 2\pi R V(\Theta)$.) The canonically normalized inflaton is $\phi_\Theta = f\Theta_5$, where $f = (2\pi R g_4)^{-1}$. By suitably choosing the gauge coupling g_4 one can make the potential flat enough to get the needed amount of inflation (see section 5.5). In addition, due to the non-local nature of the Wilson-line, the potential is protected from gravitationaly induced non-renormalizable operators which could become relevant for $\phi_\Theta \sim M_{pl}$. Such operators can only arise from couplings of the Wilson-lines to extended gravitational objects of size comparable to R and are therefore suppressed by powers of $e^{-M_5 R}$ (M_5 is the 5D Planck mass) [84]. It is these properties that make gauge inflation so attractive. It is fair to mention, however, that there are some hints that such a scenario, with a very small g_4, is not obtainable from string theory. According to ref. [88], in this limit stringy non-perturbative effects will contribute to the periodic potential $V_{eff}(\Theta)$ in such a way that the convergence of the series will be slow, and higher harmonics with $k \sim f/M_{pl}$ will destroy the flatness of the potential.

Note also that if one has $|m|R \gg 1$ the model is suitable for quintessence [87], as in this case, with a small hierachy $|m|R \sim 30 - 50$, $V_{eff} \sim \frac{M^2}{R^2} e^{-2\pi|m|R}$ can easily have the correct magnitude $\sim (10^{-3}\text{eV})^4$.

5.3 SUSY gauge inflation in the orbifold case

Consider the 5D rigid supersymmetric U(1) gauge theory, consisting of a vector multiplet $\mathbb{V} = (V, \Sigma)$, and a charged hypermultiplet $\mathbb{H} = (\phi, \bar{\phi})$. In this section we use its formulation [22–24] in terms of (4D) $\mathcal{N} = 1$ superfields, which was described in section 2.3. The 5D Lagrangian, written in terms of these superfields turns out to be $\mathcal{L} = \mathcal{L}_V + \mathcal{L}_H$, where

$$\mathcal{L}_V = \frac{2}{g^2}\int d^4\theta \frac{1}{T+T^+}\left(\partial_y V - (\Sigma + \Sigma^+)\right)^2 + \left(\frac{1}{4g^2}\int d^2\theta\, T\, \mathcal{W}^2 + h.c.\right), \tag{5.8}$$

$$\mathcal{L}_H = \int d^4\theta \frac{T+T^+}{2}\left(\phi^+ e^V \phi + \bar{\phi} e^{-V}\bar{\phi}^+\right) + \left(\int d^2\theta\, \bar{\phi}(\partial_y + \Sigma)\phi + h.c.\right), \tag{5.9}$$

and is invariant under the following gauge transformations:

$$\begin{aligned} V \to V + \Lambda + \Lambda^+ \quad &; \quad \Sigma \to \Sigma + \partial_5 \Lambda \\ \phi \to e^{-\Lambda}\phi \quad &; \quad \bar{\phi} \to \bar{\phi} e^{\Lambda}. \end{aligned} \tag{5.10}$$

Now, upon compactification in the $S^1/\mathbb{Z}_2$, it is common to ascribe the following orbifold transformations:

$$\begin{aligned} V(y) = V(-y) \quad &; \quad \Sigma(y) = -\Sigma(-y) \\ \phi(y) = \phi(-y) \quad &; \quad \bar{\phi}(y) = -\bar{\phi}(-y). \end{aligned} \tag{5.11}$$

This means however that the ought-to-be Wilson-line $\oint A_y \, dy$ vanishes, as $\Sigma = \Phi + iA_y + \cdots$ is odd. We will take therefore $\bar{\phi}$ and ϕ to transform under orbifolding in a different way:

$$\bar{\phi}(y) = \phi(-y). \tag{5.12}$$

In this case the above Lagrangian is invariant if one has

$$V(y) = -V(-y), \quad \Sigma(y) = \Sigma(-y), \tag{5.13}$$

and the Wilson-line can be non-zero as now $A_5(y)$ is an even field. Let us note that one can define a new set of matter chiral fields $\phi' = \frac{1}{\sqrt{2}}(\bar{\phi} + \phi)$ and $\bar{\phi}' = \frac{1}{\sqrt{2}}(\bar{\phi} - \phi)$ which transform as $\phi'(y) = \phi'(-y)$, $\bar{\phi}'(y) = -\bar{\phi}'(-y)$ under orbifolding but mix under the U(1) gauge transformation:

$$\begin{pmatrix} \bar{\phi}' \\ \phi' \end{pmatrix} \rightarrow e^{\Lambda \sigma_1} \begin{pmatrix} \bar{\phi}' \\ \phi' \end{pmatrix}. \tag{5.14}$$

The reader may have noticed that the set-up described above corresponds to case **2** described in section 3.3.2.

To generate a non-vanishing effective potential for the Wilson-line, SUSY must be broken somehow. We assume that this is done by a non-vanishing VEV of the 5th component of the auxiliary field $\vec{V}_M$ which gauges the $SU(2)_R$ (see sec. 3.3.3). In superfield language this corresponds to a VEV for the F-component of the radion-superfield $T(x, y)$:

$$T = (e_y^5 + iB_y) + \theta^2 F_T, \tag{5.15}$$

where we take $e_y^5 = 1$, while $y \in [-\pi R, \pi R]$, and $T(y)$ is even, as usual. In this way, the terms quadratic in ϕ and $\bar{\phi}$ are (we assume that $\partial_5 \Phi = 0$)

$$\begin{aligned} \mathcal{L}_{quad} = & \bar{\phi}^*(\partial_5 - iA_5)^2 \bar{\phi} + \phi^*(\partial_5 + iA_5)^2 \phi \\ & + F_T \phi(\partial_5 - iA_5)\bar{\phi} - F_T^* \bar{\phi}^*(\partial_5 - iA_5)\phi^* \\ & - \left(\Phi^2 + \frac{|F_T|^2}{4} \right) (|\bar{\phi}|^2 + |\phi|^2), \end{aligned} \tag{5.16}$$

leading to the following KK spectrum

$$(m_n^\pm)^2 = \Phi^2 + \left(\frac{n}{R} - A_5 \pm \frac{1}{2}|F_T| \right)^2. \tag{5.17}$$

Note that there are two KK towers of real degrees of freedom, one with masses^2 $(m_n^+)^2$, the other with $(m_n^-)^2$. The hyperini KK spectrum is obtained from the hyperscalar spectrum by noting that the hyperini are singlets under the $SU(2)_R$. It must therefore coincide with eq.(5.17) for $F_T = 0$.

Let us now comment on the SUGRA embedding of this model. This not difficult to perform. Since we have one physical Abelian vector multiplet and one physical hypermultiplet, according to our discussion in chapter 3 we need to add one compensator vector multiplet $\mathbb{V}_0$ and one compensator hypermultiplet $\mathbb{h}$. The rigid Lagrangians that we presented above are correct in the limits $|\Phi| \ll M_5$, and $|\phi|, |\bar{\phi}| \ll M_5^{3/2}$, in case we take the following norm-function:

$$\mathcal{N} = (\Phi_0)^3 - \Phi_0 \Phi^2. \tag{5.18}$$

Note that the above spectrum is not deformed by the extension to SUGRA. It is only the sigma-model metric of the vector scalars and hyperscalars that is going to differ from 1 for scalar VEVs comparable with the 5D Planck scale. In the following we assume that this is not the case.

The second "complication" arising in the extension of this model to SUGRA is the fact that in its minimal version, due to the breaking of the $SU(2)_R$ down to nothing, the auxiliary field F_T is not a flat direction already at tree level. We discussed this in sec. 3.3.3. As we pointed out there, we must consider a non-minimal set-up which consists of adding an additional vector multiplet $\mathbb{V}_R$ and a linear multiplet $\mathbb{L}$. The compensator hypermultiplet must then be charged under a U(1) gauged by $\mathbb{V}_R$, with $q_3 = 0$. In this way there is a subgroup $U(1)_R \subset SU(2)_R \times U(1)$ which remains unbroken and F_T can be redefined in such a way as to become a flat direction, $F_T' = i g_R e^{i\alpha} W_{R5}$. In addition there is a coupling between $\mathbb{V}_R$ and $\mathbb{L}$, which acts as a Lagrange multiplier term imposing

$$Y_R^{ij} = 0, \ \bar{\Omega}_R^i = 0, \ M_R = 0, \ F_{\mu\nu} = 0. \tag{5.19}$$

The net result of this construction is thus that the *new* F_T, $F_T' = i g_R e^{i\alpha} W_{R5}$, is a flat direction and due to $F_{\mu\nu} = 0$ it must be a constant (up to a gauge transformation). This will be shown in detail in a publication to appear soon (second reference in [27]).

5.4 The Effective Potential and Radion Stabilization

Let us now calculate the 1-loop effective potential in four-dimensions, obtained after integrating out the hypermultiplet in the background of constant A_5 and Φ. Since we have half the number of d.o.f. that we have in the circle one gets half of the circle result [89], i.e.

$$\begin{aligned} V_{eff}(\omega,\theta,\varphi) = & \frac{3}{16\pi^6 R^4} \sum_{k=1}^{\infty} \frac{\sin^2(k\pi\omega)}{k^5} \cos(k\theta/f) e^{-k|\varphi|/f} \\ & \left[1 + k|\varphi|/f + \frac{1}{3}(k\varphi/f)^2\right], \end{aligned} \tag{5.20}$$

where $f \equiv (\sqrt{2\pi R} g_5)^{-1}$, $\omega \equiv R|F_T|/2$, $\theta \equiv 2\pi R f A_5$ is the canonically normalized Wilson-line and $\varphi \equiv \sqrt{2\pi R} g_5^{-1} \Phi$ is the canonically normalized Φ. This potential corresponds to a 1-loop correction to the Kähler potential of the form:

$$\propto \frac{1}{(T+T^{+})^2}\sum_{k=1}\frac{e^{-k2\pi R\Sigma}+e^{-k2\pi R\Sigma^{+}}}{k^3}+\cdots\,, \tag{5.21}$$

where this expression is only valid for $\Phi>0$ (i.e. inside a certain Weyl-chamber). For $\Phi<0$ we should replace $\Sigma\to-\Sigma$. Note that this Kähler potential can only give terms $\propto|F_T|^2$. The higher powers of $|F_T|$ which appear in (5.20) are non-local in superspace and should therefore be given by a superspace derivative expansion.

Several remarks are now in order. First remember that $\omega/R\propto|V_5^1+iV_5^2|$ and since V_M is an auxiliary, i.e. non-dynamical, field, ω must be set to the value which minimizes the above potential for given θ and φ. This means in particular that for $\cos(\theta/f)\approx-1$ one has $\omega=\frac{1}{2}$ and the potential is negative, while for $\cos(\theta/f)\approx1$ one has $\omega=0$ and the potential vanishes (SUSY is unbroken). For $-\epsilon(\varphi)/8<\cos(\theta/f)<\epsilon(\varphi)/8$, where $\epsilon(\varphi)\lesssim1$, ω interpolates between $\frac{1}{2}$ and 0. This behaviour of $\omega(\theta,\varphi)$ is however changed if one takes the contributions of the gauge and gravity sectors into account, where both the gauginos and the gravitino also get masses $\propto\omega/R$. The 1-loop effective potential is now [62]:

$$V_{eff}(\omega,\theta,\varphi)=-\frac{3}{16\pi^6R^4}\sum_{k=1}^{\infty}\frac{\sin^2(k\pi\omega)}{k^5}g_k(\theta,\varphi), \tag{5.22}$$

with

$$g_k(\theta,\varphi)=2+N_V-N_H-\tilde{N}_H\cos(k\theta/f)e^{-k|\varphi|/f}\left[1+\frac{k|\varphi|}{f}+\frac{1}{3}\left(\frac{k\varphi}{f}\right)^2\right]. \tag{5.23}$$

Here we have N_V 5D vector multiplets, $\tilde{N}_H$ 5D hypermultiplets which couple to the inflaton/Wilson-line θ, and N_H hypermultiplets which couple to no (dynamical) Wilson-line whatsoever.

Again, we must set ω to the value which minimizes $V_{eff}(\omega,\theta,\varphi)$. This value depends on N_V, $\tilde{N}_H$, N_H. In the case that $g_k(\theta,\varphi)\geq0$, $\omega=\frac{1}{2}$ and as the inflaton θ settles to the minimum one has $V_{eff}<0$, an undesirable property for an inflationary model. Note that one cannot solve this problem by simply adding a positive cosmological constant in the bulk, as such a counterterm is forbidden by SUSY. But one can introduce positive brane tensions T_0, T_π in addition to a AdS bulk cosmological constant g^2. To obtain a Minkowski vacuum at the end of inflation (at $\theta\approx f\pi$, $\varphi=0$), one can tune g^2, T_0+T_π and also the size of the 5th dimension R. However, as pointed out in [63, 90], in this *solution*, R is *not stabilized* being attracted towards either the origin or an AdS vacuum with finite R. To see this note that by including the bulk cosmological constant and the brane tensions the potential becomes [63, 90, 91]

$$V(R)=\frac{1}{R^2}\left(-a\,R+b-\frac{c}{R^4}\right), \tag{5.24}$$

where the extra prefactor R^{-2} is due to a rescaling of the 4D coordinates $x\sqrt{R}\to x$ to obtain a R-independent 4D Planck mass, $a=a(g^2)$, $b=b(T_0+T_\pi)$ and $c=c(\theta=f\pi,\varphi=0)$.

It is not difficult to show that there is only one (AdS) minimum at R_- with vacuum energy $V(R_-) \leq -a/(12R_-)$. In addition, this is a *false minimum* as for small R one has $V(R) \propto -cR^{-6}$.

If on the other hand one considers $g_k(\theta,\varphi) \leq 0$, one has $\omega = 0$ (SUSY is unbroken) and therefore there is no gauge inflation.

In [91] and [63] the authors proposed a solution to this problem, consisting of the introduction of hypermultiplets with supersymmetric bulk odd masses $M\epsilon(y)$. The effect of these fields is to modify the parameter c in eq.(5.24) in such a way that it gets a dependence on R as follows:

$$c(R < \alpha M^{-1}) = 0, \qquad c(R > \beta M^{-1}) = +const, \tag{5.25}$$

where α, β are $O(1)$. This behaviour is due to SUSY restauration ($\omega = 0$) for $R < \alpha M^{-1}$ and maximal SUSY breaking ($\omega = \frac{1}{2}$) for $R > \beta M^{-1}$. Now, in addition to the AdS vacuum there can be a second (false) vacuum tuned to have zero energy at $\theta = f\pi$ and for small R, $V(R) \sim bR^{-2}$ is repulsive. This hints at the possibility that during gauge inflation the radion is not stabilized and possibly plays an important rôle as proposed in [92]. It is however also possible that the radion overshoots the local maximum which separates the Minkowski local minimum from the AdS global minimum. Surely, this problem deserves a more detailed investigation.

5.5 Slow-roll and Spectral Properties of Density Perturbations

To check that the slow-roll conditions are satisfied we approximate the potential with the $k = 1$ contribution in (5.20) and add a constant to it so that at its minimum (in respect to ϕ_Θ) the vacuum energy vanishes. We also assume that the radion is stabilized. The slow-roll parameters for ϕ_Θ are

$$\varepsilon(\phi_\Theta) = \frac{q^2 M_{pl}^2}{2f^2}\frac{\sin^2(q\phi_\Theta/f)}{(1+\cos(q\phi_\Theta/f))^2}, \quad |\eta(\phi_\Theta)| = \frac{q^2 M_{pl}^2}{f^2}\left|\frac{\cos(q\phi_\Theta/f)}{1+\cos(q\phi_\Theta/f)}\right|, \tag{5.26}$$

while for φ_Φ one has,

$$\varepsilon(\varphi_\Phi) = \frac{q^2 M_{pl}^2}{18f^2}\left[\frac{|q\varphi|/f + (q\varphi/f)^2}{1+|q\varphi|/f + \frac{1}{3}(q\varphi/f)^2}\right]^2, \tag{5.27}$$

and

$$|\eta(\varphi_\Phi)| = \left|2\varepsilon(\varphi_\Phi) - \frac{q^2 M_{pl}^2}{3f^2}\frac{1+2|q\varphi|/f + \frac{2}{3}(q\varphi/f)^2}{(1+|q\varphi|/f + \frac{1}{3}(q\varphi/f)^2)^2}\right|. \tag{5.28}$$

Since $M_{pl}f^{-1} = 2\pi g_4 M_{pl} R$, for g_4 small enough and not too large hierarchy M_{pl},R^{-1} the slow roll conditions $\varepsilon, |\eta| \ll 1$ are easily satisfied if ϕ_Θ is sufficiently far from the minimum $\pi f/q$. Clearly it is ϕ_Θ that triggers the end of inflation, while φ_Φ always satisfies the slow-roll conditions. Finally, the number of e-folds is given by

$$N_e = \frac{\rho}{(\pi g_4 M_{pl} R)^2}, \quad \rho = O(1), \tag{5.29}$$

and is easily ≥ 55. It remains to be shown that gauge inflation is also successful after we treat the radion field properly. This issue will be the object of future work.

Chapter 6

Conclusions and Outlook

We close with a summary of the results contained in this thesis and discuss possible future research.

We started this thesis with a short discussion of the issue of power-law unification, recalling its problems and reviewing recent proposals for working scenarios. We took the chance and presented the superfield action for 5D global super Yang-Mills (SYM) theory coupled to charged hypermultiplets. The exactness of the 1-loop gauge coupling plays a crucial rôle in the new scenarios of power-law unification. For this reason, we inspected the way this exactness emerges in the limit of a large radius (in units of the unification scale), which is also the limit of power-law unification.

The main goal of this work was the development of a superfield description of orbifold 5D supergravity coupled to vector and hyper multiplets. For this purpose we had to identify the relevant $\mathcal{N} = 1$ supermultiplets, arising from the reduction of the multiplets of $\mathcal{N} = 2$ (conformal) supersymmetry. Here we made the interesting observation that there are not one but two objects that one could call *radion* superfield, namely T and $\mathbb{W}_y$. In fact one has $2\mathbb{W}_y = (T + T^+) + \cdots$, and the dots stay for several gravitational auxiliary fields, two of which are Lagrange multipliers. Using these building blocks we were able to write down gauge invariant superspace actions for both the vector and the hyper sectors, which are coupled to the 4D Weyl supermultiplet by the well-known extensions of the superspace integrals to F and D-densities. Missing in our formulation are the so-called odd Weyl multiplets, which contain 5D Weyl-multiplet fields odd under orbifolding. With the intention of recovering the global SYM theory reviewed in chapter 2, we performed the rigid limit, showing how the prepotential of the rigid theory is related to the norm function of the local theory, and how the interactions of the SYM theory to the radion superfield arise. We considered next the problem of taking warped geometries into account. For this purpose we used a Weyl rescaling, and observed that in addition to all the expected couplings of the warped factor to the different fields, one also obtains the kinetic term of the warp-factor. Last but not least, we explained how to introduce brane localized couplings in a way consistent with the bulk local symmetries (including supergravity).

We then applied the superfield formalism to discuss specific scenarios with warped geometry.

We started with a derivation of the generalized Randall-Sundrum models without having to rely on the 4-form mechanism to introduce the odd coupling. We showed how the well-known BPS conditions, governing the vacua of these models, are obtained in the form of F and D-flatness conditions, in the $\mathcal{N}=1$ language. This represents a nice advantage of the superfield approach over the component description. Since these models contain arbitrary many moduli, we investigated recently proposed mechanisms of moduli stabilization without SUSY breaking, and found them to be inconsistent. We pointed to the possible existence of a no-go theorem on this issue. As an additional application we considered tuned Fayet-Iliopoulos terms in warped geometries, taking into account their back-reaction on the geometry. We found that, contrary to earlier claims, the tuned FI terms do not force the breaking of supersymmetry. In the presence of charged hypermultiplets, such FI terms can induce *new* supersymmetric warped geometries, with negative tension branes and a positive-energy fat brane in the bulk, without being necessary to gauge an R-symmetry.

Finally, we presented a discussion of the gauge-inflation scenario in the orbifold case. We observed that - as it should be expected - the stabilization of the radion may be a problem, so that in the best case the radion will play a rôle during inflation, getting stabilized towards its end.

At the end of this work, there are several open issues and new questions which would be interesting to pursue. Let us name just a few:

- The first that we want to mention is the problem of the *odd* part of the *Weyl* multiplet. Even though often these fields are not relevant for the problems one has in mind (e.g. the scenarios we investigated in chapter 4), they can participate in brane interactions in the form of $(\text{odd})^2$ terms. This means that they might play a rôle in mediating SUSY breaking from one brane to the other, participating in bulk loops. It is therefore important to find also a way of organizing these fields in $\mathcal{N}=1$ supermultiplets.

- A second question, which surely is not specific for our scenario, regards the problem of *moduli stabilization* which we discussed in chapter 4, and also appeared in the gauge inflation scenario of chapter 5, which we presently further investigate. We think that the present formalism might be of help in searching for solutions to this problem, since it is simpler to use than the component formalism.

- Another technical issue is the calculation of *quantum loops* in this formalism, or to be more precise, the problem of finding regularization schemes that respect its conformal substructure, as we mentioned in section 4.4. This is the only way of ensuring that the form of our superspace Lagrangians is robust against quantum corrections. This is essential if one wants to explore the holomorphicity of the superpotential and prepotential to obtain exact results also in this formalism.

In fact, these three issues are interconnected. The two technical problems that we mentioned here - odd Weyl multiplets and loop calculations - must be solved if one wants to fully use the superfield formalism to explore the moduli stabilization problem or more generally to perform 1-loop calculations. As we explained in section 4.2, a supersymmetric moduli stabilization mechanism in the 5D orbifold does not seem to possible. Forgetting about possible non-perturbative effects, this means that we will have to rely on spontaneous supersymmetry breaking to find stabilizing potentials. In addition, the tree-level terms will hardly be enough for that purpose. The 1-loop corrections will play therefore an essential rôle in moduli stabilization. An example of this fact was provided in the discussion of the radion potential during gauge inflation, in section 5.4. As we mentioned there, gravity loops also contribute to the Kähler potential, and are relevant in determining the sign of the effective potential if there are not too many vector multiplets in the bulk. For this reason it is important to take the odd Weyl multiplets into account. Gravity 1-loop contributions to the low-energy effective Kähler potential have been discussed by a few groups for different brane terms, in flat and recently also in a warped RS background (see [93] and references therein). We think that our superfield formalism will be a good framework to reconsider and generalize these calculations, specially if one thinks that almost all these works rely on the non-minimal formalism of Zucker [9], instead of the more powerfull formalism of FKO which is the base of our work. In this respect we should emphasize the difference in the understanding of the radion superfield(s) between ours and the previous works. In particular, as we have seen in section 4.2, there can be arbitrary many moduli, in which case the low-energy concept of radion superfield T is vacuum-dependent, while the other radion superfield $\mathbb{W}_y$ is always a well-defined quantity. This understanding is missing in the previous works.

Also, to ensure that the analytic and conformal structure of the supergravity theory is retained at 1-loop level, we will have to envisage a regulator with the correct conformal weights. Even though we think this should be possible, it is clear that in the orbifold case we will find some problems in constructing one. For instance, one could think of a Pauli-Villars regularization, where the masses of the PV fields would be given by couplings to Σ^0 (or the radion superfield T in the rigid limit), so that this regulator would have the correct weights. The problem is that there is always a zero-mode PV field, regardless the magnitude of its bulk mass. This shows that to regularize the theory a more elaborated scheme is needed. At this point we should note however, that when calculating those finite effects which are due to the topology of the extra-dimension, regularization is not relevant. This means that the study of moduli stabilization is not affected by the above-mentioned problem.

All in all, we think this work paves the way for some interesting future research.

Appendix A

The Running Gauge Coupling

A.1 The running gauge coupling in the world-line formalism

We review here how the 1-loop correction $\Delta S^{(1)}$, due to a scalar of charge e, to the five-dimensional F_{MN}^2 term,

$$S[A_M] = \int d^5x \, \frac{1}{4g^2} F_{MN} F^{MN}, \tag{A-1}$$

is obtained in the world-line formalism. The 1-loop correction is given as:

$$\Delta S^{(1)} = -\frac{1}{2} \operatorname{tr} \ln \frac{-(\partial_M + ieA_M)^2 + m^2}{-\partial_M^2 + m^2} = \int_0^\infty \frac{dT}{T} \operatorname{tr} \exp\left[-T(-(\partial + ieA)^2 + m^2)\right], \tag{A-2}$$

where, at the r.h.s., we dropped an A_M independent piece. Note that we did not make yet any specific assumptions regarding the number of dimensions and the boundary conditions. The later are encoded on the way the *trace* is performed. The essential point in the world-line method is that the trace can be rewritten as a quantum mechanical path-integral, so that we get

$$\Delta S^{(1)} = \int \frac{dT}{T} \, e^{-m^2 T} \int\limits_{x(T)\sim x(0)} \mathcal{D}^D x \exp\left[-\int_0^T d\tau \left(\tfrac{1}{4}\dot{x}^2 + ie\dot{x}^M A_M\right)\right]. \tag{A-3}$$

Note that the path-integral is to be performed over all paths with $x(T) \sim x(0)$, i.e. with a final position *physically* equal to the initial position. In topologically non-trivial manifolds and orbifolds, this has interesting consequences [27, 42], as is explained in section 2.2.

We choose now a background configuration with constant field-strength, $A_M = \frac{1}{2} x^N F_{NM}$, which corresponds to the lowest order term in a derivative expansion. The one-loop contribution to the gauge coupling is obtained as the second order term in an expansion in powers of F_{NM}. Let us first consider the *non-compact* D-dimensional case:

$$\Delta S^{(1)} = \int d^D x_0 \frac{-e^2}{8} F_{NM} F_{KL} \int \frac{dT}{T} \, e^{-m^2 T} \int\limits_{\bar{x}(T)=\bar{x}(0)=0} \mathcal{D}^D \, \bar{x} e^{-\int_0^T d\tau \frac{1}{4}\dot{\bar{x}}^2} \int_0^T d\tau \dot{\bar{x}}^M \bar{x}^N \int_0^T d\tau' \dot{\bar{x}}^L \bar{x}^K . \tag{A-4}$$

The path-integral on the right-hand side can be obtained using the following well-known results:

$$\langle x_1^M x_2^N \rangle = \eta^{MN} \left(|\tau_1 - \tau_2| - \frac{(\tau_1 - \tau_2)^2}{T} \right) \langle 1 \rangle, \tag{A-5}$$

and

$$\int\limits_{\bar{x}(T)=\bar{x}(0)=0} \mathcal{D}^D \bar{x}\, e^{-\int_0^T d\tau \frac{1}{4}\dot{\bar{x}}^2} = (4\pi T)^{-D/2}. \tag{A-6}$$

In this way one finally gets the scalar contribution to the one-loop gauge coupling in D non-compact flat dimensions:

$$\Delta S^{(1)} = \frac{e^2}{3(4\pi)^{D/2}} \int d^D x_0 \, \frac{1}{4} F_{NM} F^{NM} \cdot \int_0^\infty dT e^{-m^2 T} T^{1-\frac{D}{2}}. \tag{A-7}$$

Clearly, for $D \geq 4$ the integral over the proper-time T diverges at $T \approx 0$. We introduce therefore a small proper time cut-off $T_\Lambda = \Lambda^{-2}$, and obtain for $D \geq 4$ the following divergent pieces

$$\int_{\Lambda^{-2}}^\infty dT e^{-m^2 T} T^{1-\frac{D}{2}} \approx \begin{cases} \ln(\Lambda^2/m^2) & , D = 4 \\ \frac{2}{D-4}\Lambda^{D-4} & , D \geq 5. \end{cases} \tag{A-8}$$

The same type of reasoning is used in sec.2.2 to obtain the 1-loop gauge coupling in the orbifold case.

A.2 Running of the gauge coupling with mixed propagators

We perform here a 1-loop calculation of the radiative corrections to the gauge coupling in a orbifold U(1) due to a scalar field of charge $Q = 1$, where instead of the world-line formalism we will use the so-called *mixed momentum-coordinate* formalism [94–96], a method inspired in finite temperature field theory calculations. It consists of the use of propagators which depend on both 4D momenta and the extra-dimensional coordinates. For example the 5-dimensional propagator for a scalar of mass M in the S^1/Z_1 orbifold turns out to be given by

$$G^{\pm}(p; y', y) = \frac{1}{4\chi} \sum_{n=-\infty}^{\infty} \{ e^{-\chi|y-y'+2n\pi R|} \pm e^{-\chi|y+y'+2n\pi R|} \}, \tag{A-9}$$

where $\chi^2 \equiv p^2 + M^2$, and $+(-)$ stays for positive (negative) orbifold parity. By summing over the *winding* modes one obtains, for $-\pi R \leq y, y' \leq \pi R$,

$$G^{\pm}(p; y', y) = \frac{\cosh \chi(\pi R - |y - y'|) \pm \cosh \chi(\pi R - |y + y'|)}{4\chi \sinh(\chi \pi R)}. \tag{A-10}$$

As expected, the orbifold symmetry is notorious, in particular $G^-(p; y', y)$ vanishes at the fix-points $y = 0, \pi R$. Note that in the UV limit $p \to \infty$, $y \to y'$, the $n = -1, 0$ winding modes dominate:

$$G^{\pm}(p; y, y) \to \frac{1}{4p}(1 \pm e^{-2p|y|} \pm e^{-2p|y-\pi R|}). \tag{A-11}$$

As we will see, these terms can lead to UV divergences localized in the Bulk and the two fix-points.

To obtain the 1-loop gauge coupling we turn to the one-particle irreducible (1PI) effective action, in particular to the terms quadratic in the gauge field. These are given by

$$S^{1-loop} = \int d^5 z \frac{1}{4g^2} F_{MN} F^{MN} + \Delta S[A_M], \tag{A-12}$$

where the 1-loop correction is

$$\begin{aligned} \Delta S = \int d^5 z G(z,z) A_M^2(z) - \frac{1}{2} \int d^5 z d^5 \bar{z} \{ (2iA \cdot \partial_z + i(\partial_z \cdot A)) G(z, \bar{z}) \} \\ \times \{ (2iA \cdot \partial_{\bar{z}} + i(\partial_{\bar{z}} \cdot A)) G(z, \bar{z}) \}. \end{aligned} \tag{A-13}$$

Let us consider now the piece proportional to $A_\mu A_\nu$ where μ, ν are 4D indices. It is

$$\frac{1}{2} \int dy\, dy' \int \frac{d^4 q}{(2\pi)^4} A_\mu(q,y) \Pi^{\mu\nu}(q^2; y, y') A_\nu(-q, y'), \tag{A-14}$$

where the vacuum polarization function is given by

$$\Pi^{\mu\nu}(q^2; y, y') \equiv q^\mu q^\nu \Pi^1(q^2; y, y') + q^2 \eta^{\mu\nu} \Pi^2(q^2; y, y'), \tag{A-15}$$

with

$$\Pi^1(q^2; y, y') = \frac{4}{3q^2} \int \frac{d^4 p}{(2\pi)^4} \left\{ p^2 + \frac{3}{4} q^2 - 4 \frac{(q \cdot p)^2}{q^2} \right\} G^\pm(p; y, y') G^\pm(|p+q|; y, y'), \tag{A-16}$$

and

$$\begin{aligned} \Pi^2(q^2; y, y') = & \frac{1}{q^2} \int \frac{d^4 p}{(2\pi)^4} \Bigg\{ 2G^\pm(p; y, y) \delta(y - y') \\ & - \frac{4}{3} \left(p^2 - \frac{(q \cdot p)^2}{q^2} \right) G^\pm(p; y, y') G^\pm(|p+q|; y, y') \Bigg\}. \end{aligned} \tag{A-17}$$

To obtain the 5-dimensional divergence structure one uses that in the UV limit, $p \gg \pi R$, one has $e^{-p|y|} \to 2\delta(y)/p + 2\delta''(y)/p^3 + \cdots$ (Here the dots stay for terms which do not contribute to the UV divergent part of $\Pi^{\mu\nu}$). Using this prescription one gets

$$\begin{aligned} \Pi^1(q^2; y, y') \to & \frac{\delta(y-y')}{48} \int \frac{d^4 p}{(2\pi)^4} \left\{ 6 \frac{1}{\chi^3} - 7 \frac{p^4}{\chi^7} \right\} \\ & \pm \delta(y - y')[\delta(y) + \delta(y - \pi R)] \int \frac{d^4 p}{(2\pi)^4} \left\{ \frac{1}{4\chi^4} - \frac{p^4}{3\chi^8} \right\}, \end{aligned} \tag{A-18}$$

and

$$\begin{aligned} \Pi^2(q^2; y, y') \to & \frac{\delta(y-y')}{2q^2} \int \frac{d^4 p}{(2\pi)^4} \left\{ \frac{1}{\chi} - \frac{p^2}{4\chi^3} - \frac{q^2}{48} \left(7 \frac{p^4}{\chi^7} - 9 \frac{p^2}{\chi^5} \right) \right\} \\ & \pm \frac{\delta(y-y')}{4q^2} [\delta(y) + \delta(y - \pi R)] \int \frac{d^4 p}{(2\pi)^4} \left\{ \frac{2}{\chi^2} - \frac{p^2}{\chi^4} + q^2 \left(\frac{p^2}{\chi^6} - \frac{2p^4}{3\chi^8} \right) \right\} \\ & + \text{higher order in the derivatives.} \end{aligned} \tag{A-19}$$

The terms of higher order in the derivatives which we dropped above contribute to the renormalization of the operators $(\partial_5 F_{\mu\nu})^2$, $\delta(y)(\partial_5 F_{\mu\nu})^2$ and also $\delta(y-\pi R)(\partial_5 F_{\mu\nu})^2$. Clearly, Π^1 is linearly divergent in the bulk and logarithmic divergent at the fix-points, while Π^2 is cubic divergent in the bulk and quadratic divergent in the fix-points. On the other hand gauge invariance implies that Π^1 and Π^1 must have the same divergent behaviour, so one expects that when using a *gauge invariant* regularization, both the cubic divergences in the bulk and the quadratic ones in the branes cancel. That is, we are left with a linear divergence in the bulk $\sim \Lambda F^2$ and logarithmic divergences in the branes $\sim (\delta(y)+\delta(y-\pi R))\ln\Lambda\ F^2$. We will use here Pauli-Villars (PV) regularization, which preserves gauge invariance [97,98]. It will turn out that a minimum of two PV fields with opposite statistics is needed to regularize the 1-loop terms.

We take thus a set of PV scalar fields with exactly the same orbifold parity as the physical scalar. These fields have masses Λ_i, U(1)-charges Q_i and are either bosonic scalars ($F_i = 0$) or fermionic scalars ($F_i = 1$). For $\Lambda_i^2 \gg M^2, q^2$ the regularized vacuum polarization function becomes:

$$\Pi^{\mu\nu} = \frac{\delta(y-y')}{16\pi^2}(q^\mu q^\nu - q^2\eta^{\mu\nu})\sum_i (-)^{F_i} Q_i^2 \left\{ \frac{\Lambda_i}{30} \pm \frac{\delta(y)+\delta(y-\pi R)}{6}\ln\Lambda_i \right\} + \text{finite one-loop corrections.} \tag{A-20}$$

The divergences which are absent in this expression are proportional to $1+\sum(-)^{F_i}Q_i^2$ and $\sum(-)^{F_i}Q_i^2\Lambda_i^2$, thus we are implicitly assuming that

$$1+\sum_i(-)^{F_i}Q_i^2 = 0, \text{ and } \sum_i(-)^{F_i}Q_i^2\Lambda_i^2 = 0. \tag{A-21}$$

Note that these conditions can be satisfied using one fermionic PV scalar of squared charge $Q_F^2 > 1$, mass Λ_F, plus one bosonic PV scalar of squared charge $Q_B^2 = Q_F^2 - 1$ and squared mass $\Lambda_B^2 = Q_F^2\Lambda_F^2/(Q_F^2-1)$. In this case the divergent part of eq.(A-20) can be rewritten as

$$\Pi^{\mu\nu} = -\frac{\delta(y-y')}{16\pi^2}(q^\mu q^\nu - q^2\eta^{\mu\nu})\left\{ \frac{c(Q_F^2)}{30}\Lambda \pm \frac{\delta(y)+\delta(y-\pi R)}{6}\ln\Lambda \right\}, \tag{A-22}$$

where $c(x) \equiv x[1-(1-x^{-1})^{\frac{1}{2}}]\cdot(1-x^{-1})^{\frac{1-x}{2}}$. One can check that for $\infty > x > 1$ one has $\frac{1}{2}e^{1/2} < c(x) < 1$. Note that one obviously can reabsorb c in the cut-off Λ, in which case it reappears as a finite contribution to the brane couplings.

To cancel the divergences we write the gauge coupling g as a sum of renormalized couplings and counterterms

$$\frac{1}{g^2} = \frac{1}{g_5^2(\mu)} + \frac{1}{24\pi^2}c\frac{\Lambda-\mu}{20} + \frac{\delta(y)}{g_0^2(\mu)} + \frac{\delta(y-\pi R)}{g_\pi^2(\mu)} \pm \frac{1}{24\pi^2}\frac{\delta(y)+\delta(y-\pi R)}{4}\ln\frac{\Lambda}{\mu}. \tag{A-23}$$

Here μ is the renormalization point.

Appendix B

The 5D Off-shell Supergravity of FKO

B.1 Field content and component supergravity action

The field content of the introduced supermultiplets and orbifold parity assignments are given in Table 1.

Z_2 parity	Field content
Weyl multiplet	
$+$	$e^a_\mu,\ e^5_y,\ \psi_{\mu+},\ \psi_{y-},\ b_\mu,\ V^3_\mu,\ V^{1,2}_y,\ v^{5a},\ \chi_+,\ D$
-	$e^5_\mu,\ e^a_y,\ \psi_{\mu-},\ \psi_{y+},\ b_y,\ V^3_y,\ V^{1,2}_\mu,\ v^{ab},\ \chi_-$
Vector multiplet	
Π_V	$M,\ W_y,\ \Omega_-,\ Y^{1,2}$
$-\Pi_V$	$W_\mu,\ \Omega_+,\ Y^3$
Hypermultiplet	
$\Pi_{\hat\alpha}$	$\mathcal{A}^{2\hat\alpha-1}_1,\ \mathcal{A}^{2\hat\alpha}_2,\ \zeta^{\hat\alpha}_-,\ \mathcal{F}^{2\hat\alpha-1}_1,\ \mathcal{F}^{2\hat\alpha}_2$
$-\Pi_{\hat\alpha}$	$\mathcal{A}^{2\hat\alpha-1}_2,\ \mathcal{A}^{2\hat\alpha}_1,\ \zeta^{\hat\alpha}_+,\ \mathcal{F}^{2\hat\alpha-1}_2,\ \mathcal{F}^{2\hat\alpha}_1$

Table B.1: Field Content and Z_2 Parities

We present now the Lagrangians obtained in [11,13] with explicit powers of the 5D Planck mass M_5. Note that here the conformal symmetries are already fixed. The gravity/vector part is, with $\kappa \equiv M_5^{-3/2}$,

$$\begin{aligned} e^{-1}\mathcal{L}_{GV} = & -\frac{1}{2\kappa^2}\mathcal{R}(\omega) - 2i\bar{\psi}_\mu\gamma^{\mu\nu\rho}\nabla_\nu\psi_\rho + \kappa^2\bar{\psi}_a\psi_b(\bar{\psi}_c\gamma^{abcd}\psi_d + \bar{\psi}^a\psi^b) \\ & + \mathcal{N}_I(g[\bar{\Omega},\Omega]^I + \kappa^2\frac{i}{4}F_{ab}(W)^I\bar{\psi}_c\gamma^{abcd}\psi_d) + a_{IJ}f_1^{IJ}(W,\Omega,M) \\ & - \mathcal{N}_{IJK}f_2^{IJK}(W,\Omega,M) + \frac{\kappa^2}{8}\left(2\bar{\psi}_a\psi_b + \bar{\zeta}^{\bar{\alpha}}\gamma_{ab}\zeta_\alpha + a_{IJ}\bar{\Omega}^I\gamma_{ab}\Omega^J\right)^2 \\ & + \kappa^2\frac{i}{4}\mathcal{N}_I F^{ab}(W)^I(2\bar{\psi}_a\psi_b + \bar{\zeta}^{\bar{\alpha}}\gamma_{ab}\zeta_\alpha + a_{IJ}\bar{\Omega}^I\gamma_{ab}\Omega^J) \\ & + \kappa^2(\mathcal{A}^{\bar{\alpha}i}\nabla_a\mathcal{A}^j_\alpha + ia_{IJ}\bar{\Omega}^{Ii}\gamma_a\Omega^{Jj})^2, \end{aligned} \tag{B-1}$$

where

$$\begin{aligned} f_1^{IJ} = & -\frac{1}{4}F(W)^I\cdot F(W)^J + \frac{1}{2}\nabla M^I\cdot\nabla M^J + 2i\bar{\Omega}^I\gamma\cdot\nabla\Omega^J \\ & + i\kappa\bar{\psi}_a(\gamma\cdot F(W) - 2\gamma\cdot\nabla M)^I\gamma^a\Omega^J \\ & - 2\kappa^2\{(\bar{\Omega}^I\gamma^a\gamma^{bc}\psi_a)(\bar{\psi}_b\gamma_c\Omega^J) - (\bar{\Omega}^I\gamma^a\gamma^b\psi_a)(\bar{\psi}_b\Omega^J)\}, \end{aligned} \tag{B-2}$$

$$\begin{aligned} f_2^{IJK} = & -\frac{i}{4}\bar{\Omega}^I\gamma\cdot F(W)^J\Omega^K + \frac{2\kappa}{3}(\bar{\Omega}^I\gamma^{ab}\Omega^J)(\bar{\psi}_a\gamma_b\Omega^K) \\ & + \frac{2\kappa}{3}(\bar{\psi}^i_a\gamma^a\Omega^{Ij})(\bar{\Omega}^J_{(i}\Omega^K_{j)}), \end{aligned} \tag{B-3}$$

and $\mathcal{N}_I$, $\mathcal{N}_{IJ}$, $\mathcal{N}_{IJK}$, a_{IJ} are functions of the scalar components of the vector multiplets, M_I. These functions are obtained through differentiation of the *norm function* $\mathcal{N}(M_I)$, a homogeneous cubic function of the M_I which characterizes the vector part of the system:

$$\mathcal{N}_I \equiv \frac{\partial\mathcal{N}}{\partial M^I}, \quad \mathcal{N}_{IJ} \equiv \frac{\partial^2\mathcal{N}}{\partial M^I\partial M^J}, \quad \mathcal{N}_{IJK} \equiv \frac{\partial^3\mathcal{N}}{\partial M^I\partial M^J\partial M^K}, \tag{B-4}$$

$$a_{IJ} = -\frac{1}{2\kappa^2}\frac{\partial^2}{\partial M^I\partial M^J}\ln(\kappa^2\mathcal{N}). \tag{B-5}$$

There is also a Chern-Simons Lagrangian,

$$\begin{aligned} e^{-1}\mathcal{L}_{C-S} = & \frac{\kappa}{8}c_{IJK}\epsilon^{\lambda\mu\nu\rho\sigma}W^I_\lambda\left(F^J_{\mu\nu}(W)F^K_{\rho\sigma}(W) + \frac{i}{2}g[W_\mu,W_\nu]^J F^K_{\rho\sigma}(W)\right. \\ & \left. -\frac{g^2}{10}[W_\mu,W_\nu]^J[W_\rho,W_\sigma]^K\right), \end{aligned} \tag{B-6}$$

where $c_{IJK} = \mathcal{N}_{IJK}/6$, and a hypermultiplet Lagrangian,

$$\begin{aligned} e^{-1}\mathcal{L}_{hyper} = & \nabla^a\mathcal{A}^{\bar{\alpha}}_i\nabla_a\mathcal{A}^i_\alpha - 2i\bar{\zeta}^{\bar{\alpha}}(\gamma\cdot\nabla + igM)\zeta_\alpha - \mathcal{A}^{\bar{\alpha}}_i(gM)^2{}_\alpha{}^\beta\mathcal{A}^i_\beta \\ & - 4i\kappa\bar{\psi}^i_a\gamma^b\gamma^a\zeta_\alpha\nabla_b\mathcal{A}^{\bar{\alpha}}_i - 2i\kappa^2\bar{\psi}^{(i}_a\gamma^{abc}\psi^{j)}_c\mathcal{A}^{\bar{\alpha}}_j\nabla_b\mathcal{A}_{\alpha i} \\ & + \mathcal{A}^{\bar{\alpha}}_i\left(-8g\bar{\Omega}^i_{\alpha\beta}\zeta^\beta + 4\kappa g\bar{\psi}^i_a\gamma^a M_{\alpha\beta}\zeta^\beta - 4\kappa g\bar{\psi}^{(i}_a\gamma^a\Omega^{j)}_{\alpha\beta}\mathcal{A}^\beta_j\right. \\ & \left. + 2\kappa^2 g\bar{\psi}^{(i}_a\gamma^{ab}\psi^{j)}_b M_{\alpha\beta}\mathcal{A}^\beta_j\right) + \kappa^2\bar{\psi}_a\gamma_b\psi_c\bar{\zeta}^{\bar{\alpha}}\gamma^{abc}\zeta_\alpha \\ & - \frac{\kappa^2}{2}\bar{\psi}^a\gamma^{bc}\psi_a\bar{\zeta}^{\bar{\alpha}}\gamma_{bc}\zeta_\alpha. \end{aligned} \tag{B-7}$$

Here we assumed that the hyperfields transform under the gauge group G $(I \neq 0)$ as

$$\delta\mathcal{A}_i^\alpha = i\omega^I (t_I)^\alpha_\beta \mathcal{A}_i^\beta. \tag{B-8}$$

Finally we present the so-called auxiliary Lagrangian, which appart from the Y-terms vanishes on-shell:

$$\begin{aligned} e^{-1}\mathcal{L}_{aux} &= D'(\kappa^2\mathcal{A}^2 + 2) - 8i\kappa\bar{\chi}'^i \mathcal{A}_i^{\bar{\alpha}} \zeta_\alpha + (Y\text{-terms}) + 2(v-\tilde{v})^{ab}(v-\tilde{v})_{ab} \\ &+ (V_\mu - \tilde{V}_\mu)^{ij}(V^\mu - \tilde{V}^\mu)_{ij} + \left(1 - \frac{(W_a^0)^2}{\alpha^2}\right)(\mathcal{F}_i^{\bar{\alpha}} - \tilde{\mathcal{F}}_i^{\bar{\alpha}})(\mathcal{F}_\alpha^i - \tilde{\mathcal{F}}_\alpha^i), \end{aligned} \tag{B-9}$$

where the Y-terms are given by

$$-\frac{1}{2}\mathcal{N}_{IJ} Y_{ij}^I Y^{Jij} + Y_{ij}^I \left[2i\mathcal{A}_\alpha^{(i}(gt_I)^{\bar{\alpha}\beta}\mathcal{A}_\beta^{j)} + i\mathcal{N}_{IJK}\bar{\Omega}^{Ji}\Omega^{Kj}\right], \tag{B-10}$$

and $\tilde{v}_{ab}$, $\tilde{\mathcal{F}}_\alpha^i$, $\tilde{V}_\mu^{ij}$, are combinations of non-auxiliary fields [11]:

$$\tilde{V}_a^{ij} = -\frac{\kappa}{2}(2\mathcal{A}^{\bar{\alpha}(i}\nabla_a \mathcal{A}_{\bar{\alpha}}^{j)} - i\mathcal{N}_{IJ}\bar{\Omega}^I\gamma_{ab}\Omega^J), \tag{B-11}$$

$$\tilde{v}_{ab} = -\frac{\kappa}{4}\left[\mathcal{N}_I F_{ab}^I - i(6\bar{\psi}_a\psi_b + \bar{\zeta}^\alpha\gamma_{ab}\zeta_\alpha - \frac{1}{2}\mathcal{N}_{IJ}\bar{\Omega}^I\gamma_{ab}\Omega^J)\right] \tag{B-12}$$

$$\tilde{\mathcal{F}}^\alpha = (gM^0 t_0)^\alpha{}_\beta \mathcal{A}_i^\beta \tag{B-13}$$

B.2 The constraints

The action, as it stands above, is the result of fixing the superconformal symmetries ($\mathbf{D}$, $\mathbf{S}^i$, $\mathbf{K}_a$) with the following constraints on the norm function:

$$\mathcal{N} = \kappa^{-2}, \quad \mathcal{N}_I\Omega^I = 0, \quad \hat{\mathcal{D}}_a\mathcal{N} = 0. \tag{B-14}$$

Consider first the last constraint $\hat{\mathcal{D}}_a\mathcal{N} = 0$. To do this, note that [14]

$$\hat{\mathcal{D}}_a M^I = (\partial_a - b_a)M^I - ig[W_a, M]^I - 2i\kappa\bar{\psi}_a\Omega^I, \tag{B-15}$$

where b_a is the gauge field of dilatation and ψ_a is the gravitino (the gauge field of susy). Now, $\mathcal{N}$ is a gauge invariant, has Weyl weight $w = 3$, therefore one has

$$\hat{\mathcal{D}}_a\mathcal{N} = (\partial_a - 3b_a)\mathcal{N} - 2i\kappa\bar{\psi}_a\frac{\partial\mathcal{N}}{\partial M^I}\Omega^I = (\partial_a - 3b_a)\mathcal{N} - 2i\kappa\bar{\psi}_a\mathcal{N}_I\Omega^I. \tag{B-16}$$

With the constraints $\mathcal{N} = \kappa^{-2}$ and $\mathcal{N}_I\Omega^I = 0$, it follows that

$$b_a = 0. \tag{B-17}$$

To solve the other two constraints one must precise the vector multiplets. Let us consider as an example in addition to the compensator vector multiplet V^0 just another $U(1)$ vector multiplet. In this case the most general norm function is

$$\kappa^{-1}\mathcal{N} = \alpha {M^0}^3 - \beta\alpha^{\frac{1}{3}} M^0 {M^1}^2 - \gamma {M^1}^3. \tag{B-18}$$

To obtain canonically normalized fields we set $\alpha = (2/3)^{\frac{3}{2}}$ and $\beta = 1$. Introducing $\phi_1 \equiv M^1/\alpha^{\frac{1}{3}}\kappa M^0$, one can rewrite the constraint $\mathcal{N} = \kappa^{-2}$ as

$$\kappa^{-1}\mathcal{N} = \alpha {M^0}^3(1 - \kappa^2\phi_1^2 - \gamma\kappa^3\phi_1^3) = \kappa^{-3}. \tag{B-19}$$

This is solved by

$$\kappa M^0(\phi_1) = (3/2)^{\frac{1}{2}} \left[1 - \kappa^2\phi_1^2 - \gamma\kappa^3\phi_1^3\right]^{-\frac{1}{3}} \simeq (3/2)^{\frac{1}{2}} \left[1 + \frac{1}{3}\kappa^2\phi_1^2 + \frac{1}{3}\gamma\kappa^3\phi_1^3 + \mathcal{O}\left((\kappa\phi_1)^4\right)\right] \tag{B-20}$$

Clearly we are assuming that in the vacuum $M^0 = (3/2)^{\frac{1}{2}}\kappa^{-1}$ and $M^1 = \phi_1 = 0$, and an expansion in powers of κM^1 is reasonable.

B.3 Conventions and superfield definitions

In this appendix we present some conventions and expressions for the superfields in four component Majorana spinors, as well as in two component Weyl spinors.

The 5D γ-matrices, satisfying the relations $\{\gamma^a, \gamma^b\} = 2\eta^{ab}$ with $\eta^{ab} = \mathrm{Diag}(1, -1, -1, -1, -1)^{ab}$, are

$$\gamma^0 = \begin{pmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{pmatrix}, \qquad \gamma^i = \mathrm{i}\begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad \gamma^4 = \mathrm{i}\begin{pmatrix} -\mathbf{1} & 0 \\ 0 & \mathbf{1} \end{pmatrix}, \qquad i = 1, 2, 3\ . \tag{B-21}$$

The charge conjugation matrix C_5, satisfying $C_5^T = -C_5$, $C_5^\dagger C_5 = \mathbf{1}$, $C_5\gamma_a C_5^{-1} = \gamma^T$, in (B-21) representation is $C_5 = \mathrm{Diag}(\epsilon, \epsilon)$. 5D spinors ψ^i, being doublets of $SU(2)_R$, can be represented through two component Weyl spinors

$$\psi^i = \left((\chi^i)_\alpha,\ \ (\bar{\xi^i})^{\dot\alpha}\right)^T\ , \tag{B-22}$$

where $\alpha, \dot\alpha = 1, 2$. The indices are lowered and raised by $SU(2)$ antisymmetric tensors: $\psi_i = \psi^j\epsilon_{ji}$, $\chi_\alpha = \epsilon_{\alpha\beta}\chi^\beta$, $\chi^\alpha = \chi_\beta\epsilon^{\beta\alpha}$ (similar for dotted indices), $\epsilon^{12} = \epsilon_{12} = 1$. Upon imposing the $SU(2)$ Majorana condition $\overline{\psi}^i \equiv (\psi_i)^\dagger\gamma^0 = (\psi^i)^T C_5$, it is easy to check out that

$$\psi^1 = \left((\chi^1)_\alpha,\ \ (\bar{\chi^2})^{\dot\alpha}\right)^T\ , \quad \psi^2 = \left((\chi^2)_\alpha,\ \ -(\bar{\chi^1})^{\dot\alpha}\right)^T\ ,$$

$$\overline{\psi}^1 = \left((\chi^1)^\alpha,\ \ -(\bar{\chi^2})_{\dot\alpha}\right)\ , \quad \overline{\psi}^2 = \left((\chi^2)^\alpha,\ \ (\bar{\chi^1})_{\dot\alpha}\right)\ . \tag{B-23}$$

We can use this notation for the parameterization of gravitinos. Similarly four component gauginos Ω^i can be written, with χ^i in (B-23) replaced by ω^i. Treating the hypermultiplet's fermionic components $(\zeta^{2\hat{\alpha}-1}, \zeta^{2\hat{\alpha}})$ - hyperinos - in analogy with (ψ^i, ψ^2), it is convenient to use the following parameterization:

$$\zeta^{2\hat{\alpha}-1} \equiv \zeta^1 = \left((\eta^1)_\alpha, \quad (\bar{\eta^2})^{\dot{\alpha}}\right)^T , \quad \zeta^{2\hat{\alpha}} \equiv \zeta^2 = \left((\eta^2)_\alpha, \quad -(\bar{\eta^1})^{\dot{\alpha}}\right)^T ,$$
$$\overline{\zeta}^{2\hat{\alpha}-1} = \overline{\zeta}^1 = \left((\eta^1)^\alpha, \quad -(\bar{\eta^2})_{\dot{\alpha}}\right) , \quad \overline{\zeta}^{2\hat{\alpha}} = \overline{\zeta}^2 = \left((\eta^2)^\alpha, \quad (\bar{\eta^1})_{\dot{\alpha}}\right) . \tag{B-24}$$

In 4D, the charge conjugation matrix C_4 satisfy $C_4^T = -C_4$, $C_4^\dagger C_4 = \mathbf{1}$, $C_4 \gamma_a C_4^{-1} = -\gamma_a^T$ and is given by $C_4 = \mathrm{Diag}(\epsilon, -\epsilon)$. The 4D Majorana condition $\overline{\psi} \equiv \psi^\dagger \gamma^0 = \psi^T C_4$ defines the 4D Majorana spinor $\psi = \left((\lambda)_\alpha, \quad (\bar{\lambda})^{\dot{\alpha}}\right)^T$. The 4D chirality matrix γ_5 is related with γ^4 by $\gamma_5 = -\mathrm{i}\gamma^4$ and defines left and right projection operators

$$\mathcal{P}_L = \frac{1}{2}(1 - \gamma_5) , \quad \mathcal{P}_R = \frac{1}{2}(1 + \gamma_5) , \quad \text{with} \quad \psi_L = \mathcal{P}_L \psi , \quad \psi_R = \mathcal{P}_R \psi . \tag{B-25}$$

From 5D spinors (B-23) one can build the combinations

$$\psi_+ = \psi_R^1 + \psi_L^2 = \left((\chi^2)_\alpha, \quad (\bar{\chi^2})^{\dot{\alpha}}\right)^T , \quad \psi_- = \mathrm{i}(\psi_L^1 + \psi_R^2) = \mathrm{i}\left((\chi^1)_\alpha, \quad -(\bar{\chi^1})^{\dot{\alpha}}\right)^T . \tag{B-26}$$

As we see, ψ_+ and $\mathrm{i}\gamma_5\psi_-$ are 4D Majorana spinors. This will be used for constructing superfield's fermionic components by the 5D spinors.

The superspace coordinates' fermionic component is the Majorana spinor $\Theta = \left(\theta_\alpha, \quad \bar{\theta}^{\dot{\alpha}}\right)^T$. The superfield H of eq.(3.45) reads

$$H = \phi_H + \overline{\Theta}_R(\psi_H)_R + \overline{\Theta}_R \Theta_R F_H , \tag{B-27}$$

where $\overline{\Theta}_R(\Psi_H)_R = \Theta_R^T C_4 (\Psi_H)_R = (\bar{\theta}\bar{\psi}_H)$ and therefore H is the chiral superfield with right chirality in two component notation. From it we can build the superfield with left chirality $H = \phi_H^* - \theta\psi_H - \theta^2 F_H^*$. With $(\phi_H, \psi_H) = (\mathcal{A}_2^{2\hat{\alpha}}, \ (\psi_H)_R = -2\mathrm{i}\zeta_R^{2\hat{\alpha}})$, we will have in two component notation

$$H = \mathcal{A}_2^{2\hat{\alpha}} + 2\mathrm{i}(\theta\eta^1) - \theta^2 F_H^* . \tag{B-28}$$

Similarly, from H^c we can build superfield with left chirality in two component notation:

$$H^c = \mathcal{A}_2^{2\hat{\alpha}-1} - 2\mathrm{i}(\theta\eta^2) - \theta^2 F_H^{c*} . \tag{B-29}$$

In the actions (3.30),(3.49) the superfield W_y is used which in 4-component notations is given by $(W_y)_{4-comp} = e_y^5 + 2\kappa\overline{\Theta}\gamma^4\psi_{y-} + \cdots$. In two component spinor notations, it is given by

$$W_y = e_y^5 \left(1 + 2\kappa(\theta\chi^1) + 2\kappa(\bar{\theta}\bar{\chi^1})\right) + \cdots \tag{B-30}$$

Here we also present the vector and (its partner) chiral superfields V, Σ in two component notations

$$V = -(\theta\sigma^{\bar{\mu}}\bar{\theta})W_{\bar{\mu}} + 2\mathrm{i}\theta^2(\bar{\theta}\bar{\omega^2}) - 2\mathrm{i}\bar{\theta}^2(\theta\omega^2) + \frac{1}{2}\theta^2\bar{\theta}^2\left(2Y^3 - \hat{D}_5 M - \mathrm{i}\partial_{\bar{\mu}} W^{\bar{\mu}}\right) ,$$

$$\Sigma = \frac{1}{2}(e_y^5 M + \mathrm{i}W_y) + 2e_y^5 \theta^\alpha \left(\mathrm{i}\omega_\alpha^1 + \kappa M \chi_\alpha^1\right) - \theta^2 F_\Phi^* \ . \tag{B-31}$$

From V the chiral superfield strength $W_\alpha = -\frac{1}{4}\overline{D}^2 D_\alpha V$ can be constructed. Also, the superfield $\mathcal{V}_5$ in two component notations should be constructed through the combination $\mathcal{V}_5 = (\Sigma + \Sigma^\dagger - \partial_y V)/W_y$ with superfields Σ, V, W_y taken in two component spinor notation.

In the superspace actions eqs.(3.30) and (3.49), all superfields are assumed to be in the two component notation given in this appendix.

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Acknowledgments

I would like to thank...

... Prof. Michael G. Schmidt for the possibility of working in this interessant field, the patient supervision, the many years of fruitful collaboration, and for being so tolerant with my peculiarities.

... Prof. Arthur Hebecker for being such an interested second referee.

... Zurab Tavartkiladze for the fruitful exchange of ideas which led to so many results, but specially for being such a good friend.

... Ralf Hofmann for a nice collaboration and his friendship.

... Thomas Konstandin for accepting the torture of reading the unfinished work.

... Thomas Konstandin, Bjorn Garbrecht and Timo Bergmann for the vivid atmosphere in these rooms, at least in the afternoon.

... All the other members of the institute, for the help in diverse matters, and a good atmosphere...

... My wife Joana specially for her solicitude, care and great patience, the good tips and everything else...

... My doughter Marta for being so eccentric...

... My parents Celso and Teresa, my brothers David, João, Francisco and sister Margarida, for their enormous patience and, of course, a noisy and happy youth.

... To my big family and all my friends, for all the great and happy moments in my life.