MATHEMATICAL ANALYSIS OF THE TIME-DEPENDENT MOTION OF A FLUID THROUGH A TUBE WITH FLEXIBLE WALLS

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ABSTRACT. We study the motion of a Stokes fluid through an elastic cylinder. The fluid is driven by a small time-dependent pressure drop between the outflow and the inflow ends of the tube. We consider small displacements of the elastic structure, thus the domains involved are not moving in time. We prove existence and uniqueness of a weak solution for this three dimensional fluid-elastic structure interaction problem.

1. INTRODUCTION

We consider a time-dependent fluid-structure interaction problem in 3D: a viscous incompressible fluid flows through an elastic tube with thickness. The flow is driven by the difference of the pressures at the ends of the tube (as in Jäger & Mikelić [JäMi98], Conca, Murat & Pironneau [CMP94] or Čanić & Mikelić [ČaMi03]). We suppose that the pressure drop between the inflow and the outflow ends of the tube is small and that the viscous effects of the fluid are strongly predominant when compared to the inertial ones. The displacements of the elastic wall are assumed to be small, so that we can consider the fluid-structure interface (and thus the involved domains) as being fixed. We thus model the fluid by the Stokes equations, the behavior of the elastic structure is described by the Lamé equations for linearized elasticity and we deal with cylindrical domains. Concerning the boundary conditions, the coupling is expressed by the equilibrium of surface forces and by the continuity of velocities at the interface. The elastic wall is considered to be clamped on its entire boundary, excepting the interface between the two media and boundary conditions involving the

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pressure are taken for the fluid at the ends of the tube. These are non-standard boundary conditions for a fluid flow (for other references on this type of conditions, though in different contexts, see e.g., [ABC02], [Bern02] and [Bern04], [Luka97] and [Luka98]). We show the existence of a unique weak solution to the coupled problem described above.

Fluid-structure interaction problems arise in many practical applications and are often encountered in literature. One of the most popular topics is haemodynamics. The problem we study here seems to be a reasonable model for blood flow in smaller arteries (for the characteristics of blood flow in this type of vessels see e.g., [Fung96]).

In the following we give a short overview of some related works on mathematical analysis of fluid-structure interaction problems. Starting with *the stationary case*, we refer to [Gran98] and [BCCV04] for a 2D fluid interacting with a 1D elastic structure and to [Gran02], [Suru04a], [Suru04b] and [Suru04c] for 3D models. In *the time dependent case* we distinguish between models dealing with cylindrical domains and models where the domains are moving in time. For the former ones see for instance [ČaMi03] (handling a problem similar to the one considered here, however in a 2D/1D setting). Concerning the latter case, in [CDEG02] is studied the interaction between a Navier-Stokes fluid contained in a cavity with an elastic plate as cover and having the rest of the boundary fixed and rigid (3D/2D problem). In [Suru04c] similar problems are treated, namely a Navier-Stokes fluid flowing through a box with an elastic cover and having inflow and outflow sections (with boundary conditions involving the pressure), respectively a Navier-Stokes fluid moving in a cylinder bounded by a thin elastic shell and with prescribed velocities at the tube's ends (both in the 3D/2D setting). Other time-dependent fluid-structure interaction problems with time moving domains were considered for instance by Errate, Esteban and Maday [EEM94] (1D fluid, 1D structure), Litvinov [Litv96], Prouse [Prou71] and Beirao da Veiga [BdV04] (2D fluid, 1D structure) or by Desjardins, Esteban et al. [DEGL01] for the 3D case of a fluid interacting with an elastic structure having a finite number of elastic modes. Rigid bodies interacting with a fluid are studied for instance by Desjardins and Esteban [DeEs99] and [DeEs00] or Takahashi [Taka03].

2. Problem setting

Let $\Omega_f := D(0,r) \times (0,L) \subset \mathbb{R}^3$ the fluid domain, where $D(0,r)$ is the disk centered at 0 and having radius r and L denotes the length of the cylinder. The domain occupied by the elastic structure is $\Omega_s :=$

$(D(0,R)-D(0,r))\times(0,L)$, with $R>r$. Together, these subdomains form the domain $\Omega := \Omega_f \cup \Omega_s$. Let Γ_{fs} denote the fluid-structure interface, $\Gamma_{f,ends,k}$ $(k=1,2)$ be the fluid boundaries at the ends of the tube, Γ_{ext} be the exterior lateral boundary of the elastic cylinder and $\Gamma_{s,ends} = \Gamma_{s,ends,1} \cup \Gamma_{s,ends,2}$ be its boundaries at the tube's ends.

As we said in the previous section, we characterise the elastic structure with the aid of the Lamé system for linearized elasticity:

$$\partial_{tt}\mathbf{u} - \text{div }(\lambda \text{trace }\mathbf{e}(\mathbf{u})\mathbf{I} + 2\mu\mathbf{e}(\mathbf{u})) = \mathbf{g} \text{ in } (0,T)\times\Omega_s.$$

This can also be written in the equivalent form:

$$\partial_{tt}\mathbf{u} - (\lambda+\mu)\nabla(\text{div }\mathbf{u}) - 2\mu\nabla\cdot\mathbf{e}(\mathbf{u}) = \mathbf{g} \text{ in } (0,T)\times\Omega_s. \tag{1}$$

Here $\lambda,\ \mu>0$ are the Lamé constants for the St. Venant-Kirchhoff elastic material we consider, $\mathbf{e}(\mathbf{u}) = \frac{1}{2}(\nabla\mathbf{u} + \nabla\mathbf{u}^t)$ is Green's linear strain tensor and $\mathbf{g}$ is the given loading force.

The elastic structure is supposed to be clamped on its entire boundary, excepting the interface with the fluid, thus to (1) we add the boundary conditions:

$$\mathbf{u} = 0 \text{ on } (0,T)\times(\Gamma_{ext}\cup\Gamma_{s,ends}) \tag{2}$$

and we also have to take some initial conditions for the displacement $\mathbf{u}$ and its velocity:

$$\mathbf{u}(0) = 0, \quad \partial_t\mathbf{u}(0) = \mathbf{u}_{01} \text{ in } \Omega_s \tag{3}$$

(we assume here for simplicity of further writing that there is no initial displacement, however there is no problem with handling the case with $\mathbf{u}(0)\neq 0$).

For the Stokes flow we consider the system:

$$\begin{array}{rcl} \partial_t\mathbf{v} - \nu\Delta\mathbf{v} + \nabla p &=& \mathbf{f} \text{ in } (0,T)\times\Omega_f \\ \text{div }\mathbf{v} &=& 0 \text{ in } (0,T)\times\Omega_f \\ \mathbf{v}\times\mathbf{n} &=& 0 \text{ on } (0,T)\times\Gamma_{f,ends} \\ p &=& 0 \text{ on } (0,T)\times\Gamma_{f,ends,1} \\ p &=& P(t) \text{ on } (0,T)\times\Gamma_{f,ends,2} \\ \mathbf{v}(0) &=& \mathbf{v}_0 \text{ in } \Omega_f, \end{array} \tag{4}$$

where $\mathbf{v}$ stands for the velocity of the fluid, p for the pressure, $\mathbf{f}$ is a given body force and $P(t)$ is the time dependent pressure drop between the inflow and outflow sections.

We also have to add the coupling conditions, illustrating the equilibrium of surface forces and the continuity of velocities at the interface:

$$\begin{aligned}(\lambda \text{trace } \mathbf{e}(\mathbf{u})\mathbf{I} + 2\mu\mathbf{e}(\mathbf{u})) \cdot \mathbf{n}_s &= p \cdot \mathbf{n}_f - \nu(\nabla \times \mathbf{v}) \times \mathbf{n}_f \text{ on } (0,T) \times \Gamma_{fs} \\ \partial_t \mathbf{u} &= \mathbf{v} \text{ on } (0,T) \times \Gamma_{fs}.\end{aligned} \tag{5}$$

Remark 2.1.

- *We will take the pressure drop $P(t)$ in (4) as being as regular as we need in all our further considerations.*
- *Here we consider the case of a fixed fluid-structure interface. This can be done when assuming that the displacements of the structure (thus of the interface) are small enough; this is not the case for large displacements. However, for the viscous fluid sticking to the interface we could not consider a homogeneous Dirichlet condition , since even if the displacements are small, there is no guarantee that their velocity is small, too.*

Now, having set the equations, the problem is the following:

Problem 1. *Determine a solution $(\mathbf{u}, \mathbf{v})$ in $(0,T) \times \Omega$ of the system (1)-(3) and (4), together with the coupling conditions (5).*

3. Weak formulation and main result

In this section we give the weak formulation of the coupled problem and state the main result.

We consider the following function spaces:

$$\begin{aligned}\mathcal{V} := \{\boldsymbol{\varphi} \in \mathcal{D}(\bar{\Omega}) \ : \ \text{div } \boldsymbol{\varphi} = 0 \text{ in } \Omega_f, \\ \boldsymbol{\varphi} \times \mathbf{n} = 0 \text{ on } \Gamma_{f,ends}, \ \boldsymbol{\varphi} = 0 \text{ on } \Gamma_{ext} \cup \Gamma_{s,ends}\} \\ \mathbf{H}(\Omega) = \overline{\mathcal{V}}^{(\mathbf{L}^2(\Omega),(\cdot,\cdot)_{f,s})}, \ \mathbf{V}(\Omega) = \overline{\mathcal{V}}^{\mathbf{H}^1(\Omega)}, \\ \mathbf{V}_f := \{\mathbf{v} \in \mathbf{H}^1(\Omega_f) \ : \ \text{div } \mathbf{v} = 0 \text{ in } \Omega_f, \ \mathbf{v} \times \mathbf{n} = 0 \text{ on } \Gamma_{f,ends}\}.\end{aligned}$$

We denote by $(\boldsymbol{\xi}, \boldsymbol{\varphi})_{f,s}$ the $\mathbf{L}^2$-inner product

$$(\boldsymbol{\xi}, \boldsymbol{\varphi})_{f,s} := (\boldsymbol{\xi}, \boldsymbol{\varphi})_{\Omega_f} + (\boldsymbol{\xi}, \boldsymbol{\varphi})_{\Omega_s}, \ \forall \boldsymbol{\xi}, \boldsymbol{\varphi} \in \mathbf{L}^2(\Omega).$$

The norm in $\mathbf{L}^2(\Omega)$ is equivalent to the norm generated by this inner product.

Assume now that

$$\begin{aligned}\mathbf{g} \in \mathbf{L}^2(0,T;\mathbf{L}^2(\Omega_s)) \quad \text{and} \quad \mathbf{f} \in \mathbf{L}^2(0,T;\mathbf{L}^2(\Omega_f)), \\ \mathbf{v}_0 \in \mathbf{V}_f, \ \mathbf{u}_{01} \in \mathbf{H}^1_{0,\Gamma_{ext}\cup\Gamma_{s,ends}}(\Omega_s) \quad \text{with} \quad \mathbf{v}_0 = \mathbf{u}_{01} \text{ on } \Gamma_{fs}.\end{aligned} \tag{6}$$

One can prove (like in e.g., [CMP94], see also [GiRa86] ch.I, S.3) that the following coercivity condition involving the curl of the fluid's velocity is satisfied:

$$(7) \quad \exists C_{curl} > 0 \;\; : \;\; \forall \mathbf{v} \in \mathbf{L}^2(0,T;\mathbf{V}_f),\ |\nabla \times \mathbf{v}|^2_{\Omega_f} \geq C_{curl}||\mathbf{v}||^2_{\Omega_f}.$$

A weak formulation of problem (1)-(5) is obtained by testing the equations for the fluid and those for the structure by $\boldsymbol{\varphi} \in \mathbf{V}(\Omega)$. The weak problem obtained is the following:

Problem 2. *Find* $(\mathbf{u}, \mathbf{v}) \in \mathbf{L}^2(0,T;\mathbf{H}^1_{0,\Gamma_{ext}\cup\Gamma_{s,ends}}(\Omega_s)) \times \mathbf{L}^2(0,T;\mathbf{V}_f)$ *such that*

$$(8) \quad \frac{d}{dt}((\partial_t \mathbf{u}, \boldsymbol{\varphi})_{\Omega_s} + (\mathbf{v}, \boldsymbol{\varphi})_{\Omega_f}) + a(\mathbf{u}, \boldsymbol{\varphi}) + \nu(\nabla \times \mathbf{v}, \nabla \times \boldsymbol{\varphi})_{\Omega_f}$$

$$= (\mathbf{g}, \boldsymbol{\varphi})_{\Omega_s} + (\mathbf{f}, \boldsymbol{\varphi})_{\Omega_f} - \int_{\Gamma_{f,ends,2}} P(t)\varphi_3,\ \forall \boldsymbol{\varphi} \in \mathbf{V}(\Omega),$$

where $a(\mathbf{u}, \boldsymbol{\varphi})$ *is the continuous, bilinear form (see* [Ciar88]*):*

$$a(\mathbf{u}, \boldsymbol{\varphi}) := \lambda(div\ \mathbf{u}, div\ \boldsymbol{\varphi})_{\Omega_s} + 2\mu(\mathbf{e}(\mathbf{u}), \mathbf{e}(\boldsymbol{\varphi}))_{\Omega_s}$$

$$\partial_t \mathbf{u}(t=0) = \mathbf{u}_{01},\ \mathbf{v}(t=0) = \mathbf{v}_0 \text{ and } \int_0^t \mathbf{v}(s)ds = \mathbf{u}(t) \text{ a.e. } t \text{ on } \Gamma_{fs}.$$

Definition 3.1. $(\mathbf{u}, \mathbf{v}) \in \mathbf{L}^2(0,T;\mathbf{H}^1_{0,\Gamma_{ext}\cup\Gamma_{s,ends}}(\Omega_s)) \times \mathbf{L}^2(0,T;\mathbf{V}_f)$ *with* $\mathbf{u}' \in \mathbf{L}^2(0,T;\mathbf{L}^2(\Omega_s))$, $\mathbf{u}'' \in \mathbf{L}^2(0,T;\mathbf{H}^{-1}(\Omega_s))$ *and with* $\mathbf{v}' \in \mathbf{L}^2(0,T;\mathbf{L}^2(\Omega_f))$ *is called a weak solution of Problem 1 if for all* $\boldsymbol{\varphi} \in \mathbf{V}(\Omega)$ *the variational formulation in (8) is satisfied in the sense of distributions (in* $\mathcal{D}'((0,T))$*).*

Next, as in [DEGL01] we define a global velocity, together with its corresponding initial condition and a global exterior force. This will allow us to treat the problem as a whole, unlikely in e.g., [BdV04], [Gran02] or [Suru04a], [Suru04b], where it was splitted in the two subproblems (one for the fluid and one for the elastic structure), each of them being handled separately and eventually realising the coupling by a fixed-point procedure.

With the following notations (χ_{Ω_s}, respectively χ_{Ω_f} stand for the characteristic functions of Ω_s, respectively Ω_f):

$$\boldsymbol{\omega} := \partial_t \mathbf{u}\chi_{\Omega_s} + \mathbf{v}\chi_{\Omega_f}, \quad \boldsymbol{\omega}_0 := \mathbf{u}_{01}\chi_{\Omega_s} + \mathbf{v}_0\chi_{\Omega_f} \text{ and } \mathbf{G} := \mathbf{g}\chi_{\Omega_s} + \mathbf{f}\chi_{\Omega_f},$$

we obtain the problem (equivalent to Problem 2):

Problem 3. *Find* $\boldsymbol{\omega}$ *such that*

$$< \partial_t \boldsymbol{\omega}, \boldsymbol{\varphi} >_{f,s} + a(\int_0^t \boldsymbol{\omega}(s) ds, \boldsymbol{\varphi}) + \nu(\nabla \times \boldsymbol{\omega}, \nabla \times \boldsymbol{\varphi})_{\Omega_f} \tag{9}$$

$$= (\mathbf{G}(t), \boldsymbol{\varphi})_{f,s} - \int_{\Gamma_{f,ends,2}} P(t)\varphi_3, \ \forall \boldsymbol{\varphi} \in \mathbf{V}(\Omega) \ a.e. \ t \in [0, T],$$

$$\boldsymbol{\omega}(0) = \boldsymbol{\omega}_0 \ in \ \mathbf{V}'(\Omega) \tag{10}$$

and

$$\int_0^t \boldsymbol{\omega}(s) \chi_{\Omega_s} ds = \int_0^t \boldsymbol{\omega}(s) \chi_{\Omega_f} ds \ on \ \Gamma_{fs}, \ a.e. \ t, \tag{11}$$

where $\mathbf{V}'(\Omega)$ *is the dual space of* $\mathbf{V}(\Omega)$.

We have denoted by $< .,. >_{f,s}$ the duality pairing between $\mathbf{V}'(\Omega)$ and $\mathbf{V}(\Omega)$, that is generated from the inner product $(.,.)_{f,s}$. Having in mind the assumptions made on the data of the problem, $\boldsymbol{\omega}_0$ as defined above satisfies $\boldsymbol{\omega}_0 \in \mathbf{V}(\Omega)$ and the initial condition on $\boldsymbol{\omega}$ is equivalent to

$$< \boldsymbol{\omega}(0), \boldsymbol{\varphi} >_{f,s} = (\boldsymbol{\omega}_0, \boldsymbol{\varphi})_{f,s}, \ \forall \boldsymbol{\varphi} \in \mathbf{V}(\Omega).$$

Remark 3.1. *Choosing condition (11) instead of*

$$\boldsymbol{\omega}(t) \chi_{\Omega_s} = \boldsymbol{\omega}(t) \chi_{\Omega_f} \ on \ \Gamma_{fs}, \ a.e. \ t$$

was imposed by the regularity we get in virtue of the a priori estimates (see Sections 4.2 and 4.3 below).

We can now state the main result:

Theorem 3.2. *Under the assumptions in (6), there exists a unique weak solution of Problem 1.*

4. Proof of the existence

4.1. Galerkin approximations.

For the proof we use the method of Galerkin (like in Evans [Evan98], Section 7.2., where it is applied to general linear second order hyperbolic problems with Dirichlet homogeneous boundary conditions). This means that we build a weak solution of the problem by first constructing solutions of certain finite dimensional approximations and then passing to limits. We therefore take the functions $\mathbf{w}_k = \mathbf{w}_k(\mathbf{x})$ $(k = 1, 2, ...)$ such that

$$\{\mathbf{w}_k\}_k \text{ is a basis of } \mathbf{V}(\Omega). \tag{12}$$

We take $\{\mathbf{w}_k\}_k$ to be the complete set of eigenfunctions of the eigenvalue problem

$$\mathbf{w} \in \mathbf{V}(\Omega) \ : \ ((\mathbf{w}, \boldsymbol{\varphi}))_{f,s} = \alpha(\mathbf{w}, \boldsymbol{\varphi})_{f,s}, \ \forall \boldsymbol{\varphi} \in \mathbf{V}(\Omega),$$

where $((\mathbf{w}, \boldsymbol{\varphi}))_{f,s} := (\nabla \times \mathbf{w}, \nabla \times \boldsymbol{\varphi})_{\Omega_f} + (\nabla \mathbf{w}, \nabla \boldsymbol{\varphi})_{\Omega_s}$ and also assume that $\{\mathbf{w}_i\}_{i=1,2...}$ is orthonormalized with this $\mathbf{H}^1(\Omega)$-inner product $((.,.))_{f,s}$. Moreover, observe that $\{\mathbf{w}_k\}_k$ is orthogonal w.r.t. the L^2-inner product $(.,.)_{f,s}$.

Now fix a positive integer m and write

$$\boldsymbol{\omega}_m(t) := \sum_{k=1}^{m} c_{km}(t)\mathbf{w}_k, \tag{13}$$

where the coefficients $c_{km}(t)$ $(0 \leq t \leq T,\ k = 1, \ldots, m)$ are taken such that

$$(\boldsymbol{\omega}_m(0), \mathbf{w}_k)_{f,s} = (\boldsymbol{\omega}_0, \mathbf{w}_k)_{f,s} \tag{14}$$

be satisfied.

The Galerkin approximation corresponding to (9) writes $(0 \leq t \leq T$, $k = 1, ..., m)$:

$$\begin{aligned}(\partial_t \boldsymbol{\omega}_m(t), \mathbf{w}_k)_{f,s} \ &+ \ a(\int_0^t \boldsymbol{\omega}_m(s)ds, \mathbf{w}_k) + \nu(\nabla \times \boldsymbol{\omega}_m(t), \nabla \times \mathbf{w}_k)_{\Omega_f} \\ &= \ (\mathbf{G}(t), \mathbf{w}_k)_{f,s} - \int_{\Gamma_{f,ends,2}} P(t) w_{k,3}.\end{aligned} \tag{15}$$

The compatibility condition (11) is clearly satisfied for the Galerkin approximation defined in (13), i.e. we have

$$\int_0^t \boldsymbol{\omega}_m(s)\chi_{\Omega_s} ds = \int_0^t \boldsymbol{\omega}_m(s)\chi_{\Omega_f} ds \text{ on } \Gamma_{fs}, \text{ a.e. } t. \tag{16}$$

Now denoting the right hand side in (15) by $F_k(t)$ observe that the system (14), (15) can be written in the form of a linear ODE system of first order for the Galerkin coefficients $c_{km}(t)$ and for $d_{km}(t) :=$

$\int_0^t c_{km}(s)ds$:

$$\sum_{l=1}^{m}(\mathbf{w}_l,\mathbf{w}_k)_{f,s}c'_{lm}(t)+\nu\sum_{l=1}^{m}(\nabla\times\mathbf{w}_l,\nabla\times\mathbf{w}_k)_{\Omega_f}c_{lm}(t)$$
$$+\sum_{l=1}^{m}a(\mathbf{w}_l,\mathbf{w}_k)d_{lm}(t)=F_k(t), \tag{17}$$

with

$$d'_{km}(t)=c_{lm}(t),\ l=1,\dots,m$$

and with the initial conditions

$$\begin{aligned}\sum_{l=1}^{m}(\mathbf{w}_l,\mathbf{w}_k)_{f,s}c_{lm}(0) &= (\boldsymbol{\omega}_0,\mathbf{w}_k)_{f,s},\\ d_{lm}(0) &= 0,\ l=1,\dots,m.\end{aligned}$$

By the classical theory of this kind of systems and using the properties of $\{\mathbf{w}_k\}_k$ it follows that there exists a unique solution $(c_{1m},\dots,c_{mm}, d_{1m},\dots,d_{mm})\in C^1((0,T))$ of (17) with the conditions above. This leads to the existence of a unique solution $\boldsymbol{\omega}_m$ for the system (15) together with the compatibility condition (16).

4.2. **Energy estimates.**

We intend to pass to the limit with $m\to\infty$ in (15) and for this we need some estimates that should be uniform in m. These are given by the following

Proposition 4.1. *There exists a constant $C>0$ such that*

$$\sup_{0\le t\le T}\left(\|\boldsymbol{\omega}_m(t)\|^2_{f,s}+\|\int_0^t\boldsymbol{\omega}_m(s)ds\|^2_{\mathbf{H}^1(\Omega_s)}\right)$$
$$+\|\boldsymbol{\omega}_m\|^2_{\mathbf{L}^2(0,T;\mathbf{H}^1(\Omega_f))}+\|\boldsymbol{\omega}'_m\|^2_{\mathbf{L}^2(0,T;\mathbf{V}'(\Omega))} \tag{18}$$
$$\le C(\|G\|^2_{\mathbf{L}^2(0,T;\mathbf{L}^2(\Omega))}+\|P\|^2_{L^2(0,T;L^\infty(\Gamma_{f,ends,2}))}+\|\mathbf{u}_{01}\|^2_{\mathbf{H}^1(\Omega_s))}+\|\mathbf{v}_0\|^2_{\mathbf{H}^1(\Omega_f)}).$$

The constant C depends on the fixed $T>0$, on r, C_{curl}, ν and on the constants C_{trace} and C_{Korn} in the inequalities (with ζ in the corresponding spaces):

$$\|\zeta\|_{L^2(\Gamma_{f,ends,2})}\le C_{trace}\|\zeta\|_{H^1(\Omega_f)}\ \textit{(Sobolev embeddings)}$$

$$\|\zeta\|^2_{H^1(\Omega_s)}\le C_{Korn}a(\zeta,\zeta)\ \textit{(by Korn's inequality, see [Ciar88])}. \tag{19}$$

Proof. Multiply (15) by $c_{km}(t)$. Upon summing up after $k = 1, \dots, m$ and taking into account (13), we get:

$$(\partial_t \boldsymbol{\omega}_m(t), \boldsymbol{\omega}_m(t))_{f,s} + a(\int_0^t \boldsymbol{\omega}_m(s)ds, \boldsymbol{\omega}_m(t)) + \nu(\nabla\times\boldsymbol{\omega}_m(t), \nabla\times\boldsymbol{\omega}_m(t))_{\Omega_f}$$

$$= (\mathbf{G}(t), \boldsymbol{\omega}_m(t))_{f,s} - \int_{\Gamma_{f,ends,2}} P(t)\omega_{m,3}(t),$$

from which we deduce

$$\frac{1}{2}\frac{d}{dt}\Big[\|\boldsymbol{\omega}_m(t)\|^2_{f,s} + a(\int_0^t \boldsymbol{\omega}_m(s)ds, \int_0^t \boldsymbol{\omega}_m(s)ds)\Big] + \nu\|\nabla\times\boldsymbol{\omega}_m(t)\|^2_{\Omega_f} \tag{20}$$

$$\leq \frac{1}{2}\|\mathbf{G}(t)\|^2_{f,s} + \frac{1}{2}\|\boldsymbol{\omega}_m(t)\|^2_{f,s} + \frac{\delta}{2}\pi r^2\|P(t)\|^2_{L^\infty(\Gamma_{f,ends,2})} + \frac{C_{trace}}{2\delta}\|\boldsymbol{\omega}_m(t)\|^2_{\mathbf{H}^1(\Omega_f)},$$

with the constant δ chosen such that $\delta \geq \frac{C_{trace}}{2\nu C_{curl}}$.

Upon using (7) and applying the differential form of Gronwall's inequality it follows that:

$$\|\boldsymbol{\omega}_m(t)\|^2_{f,s} + a(\int_0^t \boldsymbol{\omega}_m(s)ds, \int_0^t \boldsymbol{\omega}_m(s)ds)$$

$$\leq e^T(\|\boldsymbol{\omega}_m(0)\|^2_{f,s} + \|\mathbf{G}\|^2_{\mathbf{L}^2(0,T;\mathbf{L}^2(\Omega))} + \delta\pi r^2\|P\|_{L^2(0,T;L^\infty(\Gamma_{f,ends,2}))}).$$

Now using (19) we obtain:

$$\|\boldsymbol{\omega}_m(t)\|^2_{f,s} + C_{Korn}\|\int_0^t \boldsymbol{\omega}_m(s)ds\|^2_{\mathbf{H}^1(\Omega_s)} \tag{21}$$

$$\leq e^T(\|\boldsymbol{\omega}_m(0)\|^2_{f,s} + \|\mathbf{G}\|^2_{\mathbf{L}^2(0,T;\mathbf{L}^2(\Omega))} + \delta\pi r^2\|P\|_{L^2(0,T;L^\infty(\Gamma_{f,ends,2}))}).$$

Integrate in time in (20) and use again (7) and (21) to deduce that

$$\|\boldsymbol{\omega}_m\|^2_{\mathbf{L}^2(0,T;\mathbf{H}^1(\Omega_f))} \leq const(T, r, C_{curl}, C_{Korn}, C_{trace}, \nu)\cdot \tag{22}$$

$$\cdot(\|\mathbf{v}_0\|^2_{\mathbf{L}^2(\Omega_f)} + \|\mathbf{u}_{01}\|^2_{\mathbf{L}^2(\Omega_s)} + \|\mathbf{G}\|^2_{\mathbf{L}^2(0,T;\mathbf{L}^2(\Omega))} + \|P\|_{L^2(0,T;L^\infty(\Gamma_{f,ends,2}))}).$$

In order to obtain (18), we still need some estimate for the time derivative of $\boldsymbol{\omega}_m$. In order to do that, let us fix any $\boldsymbol{\zeta} \in \mathbf{V}(\Omega)$ with $||\boldsymbol{\zeta}||_{\mathbf{H}^1} \leq 1$ and write $\boldsymbol{\zeta} = \mathbf{P}_m\boldsymbol{\zeta} + (\mathbf{I} - \mathbf{P}_m)\boldsymbol{\zeta}$, where $\mathbf{P}_m$ is the projection from $\mathbf{L}^2(\bar{\Omega})$ onto span $\{\mathbf{w}_k\}_{k=1,\dots,m}$, i.e. $\forall\, \boldsymbol{\zeta} \in \mathbf{L}^2(\bar{\Omega})$ it is $(\mathbf{P}_m\boldsymbol{\zeta}, \mathbf{w})_{f,s} = (\boldsymbol{\zeta}, \mathbf{w})_{f,s}$, $\forall \mathbf{w} \in$ span $\{\mathbf{w}_k\}_{k=1,\dots,m}$.

Since $\boldsymbol{\omega}'_m(t) \in \text{span } \{\mathbf{w}_k\}_{k=1,\dots,m}$, we have

$$(23)\quad <\boldsymbol{\omega}'_m(t), \boldsymbol{\zeta}>_{f,s} \;=\; (\boldsymbol{\omega}'_m(t), \mathbf{P}_m\boldsymbol{\zeta})_{f,s} + (\boldsymbol{\omega}'_m(t), (\mathbf{I} - \mathbf{P}_m)\boldsymbol{\zeta})_{f,s} \;=\; (\boldsymbol{\omega}'_m(t), \mathbf{P}_m\boldsymbol{\zeta})_{f,s}.$$

Then we can write:

$$<\boldsymbol{\omega}'_m(t), \boldsymbol{\zeta}>_{f,s} = (\mathbf{G}(t), \mathbf{P}_m\boldsymbol{\zeta})_{f,s} - \int_{\Gamma_{f,ends,2}} P(t)(\mathbf{P}_m\boldsymbol{\zeta})_3$$

$$-a(\int_0^t \boldsymbol{\omega}'_m(s)ds, \mathbf{P}_m\boldsymbol{\zeta}) - \nu(\nabla \times \boldsymbol{\omega}_m(t), \nabla \times \mathbf{P}_m\boldsymbol{\zeta})_{\Omega_f},$$

from which it follows that

$$(24)\qquad \|\boldsymbol{\omega}'_m\|^2_{\mathbf{L}^2(0,T;\mathbf{V}'(\Omega))} \le C(T, r, \nu, C_{curl}, C_{Korn}, C_{trace})\cdot$$

$$\cdot(\|\mathbf{v}_0\|^2_{\mathbf{H}^1(\Omega_f)} + \|\mathbf{u}_{01}\|^2_{\mathbf{H}^1(\Omega_s)} + \|\mathbf{G}\|^2_{\mathbf{L}^2(0,T;\mathbf{L}^2(\Omega))} + \|P\|_{L^2(0,T;L^\infty(\Gamma_{f,ends,2}))}),$$

upon using (7), (20), (22) and the fact that $\|\mathbf{P}_m\boldsymbol{\zeta}\|_{\mathbf{H}^1} \le \|\boldsymbol{\zeta}\|_{\mathbf{H}^1} \le 1$.

Now it is clear that we obtain (18) from (21), (22) and (24). □

4.3. **Existence of a weak solution.**

We now pass to limits (for $m \to \infty$) in our Galerkin approximations.

The estimate (18) implies that:

$$(25)\qquad (\boldsymbol{\omega}_m)_m \text{ is bounded in } \mathbf{L}^\infty(0,T;\mathbf{L}^2(\Omega))$$

$$(26)\qquad (\int_0^t \boldsymbol{\omega}_m(s)ds)_m \text{ is bounded in } \mathbf{L}^\infty(0,T;\mathbf{H}^1(\Omega_s))$$

$$(27)\qquad (\boldsymbol{\omega}_m)_m \text{ is bounded in } \mathbf{L}^2(0,T;\mathbf{H}^1(\Omega_f))$$

and

$$(28)\qquad (\boldsymbol{\omega}'_m)_m \text{ is bounded in } \mathbf{L}^2(0,T;\mathbf{V}'(\Omega)).$$

Consequently, there exists a subsequence $(\boldsymbol{\omega}_{m_k})_k \subset (\boldsymbol{\omega}_m)_m$ and a function $\boldsymbol{\omega} \in \mathbf{L}^2(0,T;\mathbf{L}^2(\Omega))$ with $\int_0^t \boldsymbol{\omega}(s)\chi_{\Omega_s}ds \in \mathbf{L}^2(0,T;\mathbf{H}^1(\Omega_s))$, $\boldsymbol{\omega}\chi_{\Omega_f} \in \mathbf{L}^2(0,T;\mathbf{H}^1(\Omega_f))$ and $\boldsymbol{\omega}' \in \mathbf{L}^2(0,T;\mathbf{V}'(\Omega))$ such that

$$(29)\qquad \boldsymbol{\omega}_{m_k} \stackrel{k\to\infty}{\rightharpoonup} \boldsymbol{\omega} \text{ in } \mathbf{L}^2(0,T;\mathbf{L}^2(\Omega))$$

$$\int_0^t \boldsymbol{\omega}_{m_k}(s)\chi_{\Omega_s} ds \overset{k\to\infty}{\rightharpoonup} \int_0^t \boldsymbol{\omega}(s)\chi_{\Omega_s} ds \text{ in } \mathbf{L}^2(0,T;\mathbf{H}^1(\Omega_s)) \tag{30}$$

$$\boldsymbol{\omega}_{m_k}\chi_{\Omega_f} \overset{k\to\infty}{\rightharpoonup} \boldsymbol{\omega}\chi_{\Omega_f} \text{ in } \mathbf{L}^2(0,T;\mathbf{H}^1(\Omega_f)) \tag{31}$$

and

$$\boldsymbol{\omega}'_{m_k} \overset{k\to\infty}{\rightharpoonup} \boldsymbol{\omega}' \text{ in } \mathbf{L}^2(0,T;\mathbf{V}'(\Omega)). \tag{32}$$

We now fix an integer N and choose a function $\boldsymbol{\varphi} \in \mathbf{C}^1(0,T;\mathbf{V}(\Omega))$ of the form

$$\boldsymbol{\varphi}(t) := \sum_{k=1}^{N} \alpha_k(t)\mathbf{w}_k, \tag{33}$$

where $\{\alpha_k\}_{k=\overline{1,N}}$ are smooth functions. We choose N such that $N \le m$, multiply (15) by $\alpha_k(t)$, sum after $k = 1, ..., N$ and integrate with respect to time to obtain:

$$\int_0^T \Big[< \boldsymbol{\omega}'_m(t), \boldsymbol{\varphi}(t) >_{f,s} + a(\int_0^t \boldsymbol{\omega}_m(s)ds, \boldsymbol{\varphi}(t)) + \nu(\nabla\times\boldsymbol{\omega}_m(t), \nabla\times\boldsymbol{\varphi}(t))_{\Omega_f}\Big]$$

$$= \int_0^T \Big[(\mathbf{G}(t), \boldsymbol{\varphi}(t))_{f,s} - \int_{\Gamma_{f,ends,2}} P(t)\varphi_3(t)\Big] dt. \tag{34}$$

Now we may pass to the limit in the above identity, in virtue of (29)-(32) (set $m = m_k$); we obtain:

$$\int_0^T \Big[< \boldsymbol{\omega}'(t), \boldsymbol{\varphi}(t) >_{f,s} + a(\int_0^t \boldsymbol{\omega}(s)ds, \boldsymbol{\varphi}(t)) + \nu(\nabla\times\boldsymbol{\omega}(t), \nabla\times\boldsymbol{\varphi}(t))_{\Omega_f}\Big] dt$$

$$= \int_0^T \Big[(\mathbf{G}(t), \boldsymbol{\varphi}(t))_{f,s} - \int_{\Gamma_{f,ends,2}} P(t)\varphi_3(t)\Big] dt. \tag{35}$$

Observe that (35) holds for all functions $\boldsymbol{\varphi} \in \mathbf{L}^2(0,T;\mathbf{V}(\Omega))$, since functions of the form (33) are dense in this space. It also follows from (35) that

$$< \boldsymbol{\omega}'(t), \boldsymbol{\varphi}(t) >_{f,s} + a(\int_0^t \boldsymbol{\omega}(s)ds, \boldsymbol{\varphi}(t)) + \nu(\nabla \times \boldsymbol{\omega}(t), \nabla \times \boldsymbol{\varphi}(t))_{\Omega_f}$$

$$= (\mathbf{G}(t), \boldsymbol{\varphi}(t))_{f,s} - \int_{\Gamma_{f,ends,2}} P(t)\varphi_3(t),$$

for all $\boldsymbol{\varphi} \in \mathbf{V}(\Omega)$ and a.e. $0 \leq t \leq T$. Also notice that $\boldsymbol{\omega} \in \mathbf{C}(0, T; \mathbf{H}^{-1}(\Omega))$.

The compatibility condition (11) follows by passing to the limit in (16) and using (26), (27), as well as the convergences (in $\mathbf{L}^2(0, T; \mathbf{H}^{1/2}(\Gamma_{fs}))$) for the respective traces on the interface Γ_{fs} .

The existence result is proved if we verify that

$$\boldsymbol{\omega}(0) = \boldsymbol{\omega}_0 \text{ in } \Omega. \tag{36}$$

We therefore choose any function $\boldsymbol{\varphi} \in \mathbf{C}^1(0, T; \mathbf{V}(\Omega))$ with $\boldsymbol{\varphi}(T) = 0$ and integrate by parts in time in (35) to obtain

$$\begin{aligned} &\int_0^T \Big[-(\boldsymbol{\omega}(t), \boldsymbol{\varphi}'(t))_{f,s} + a(\int_0^t \boldsymbol{\omega}(s)ds, \boldsymbol{\varphi}(t)) + \nu(\nabla \times \boldsymbol{\omega}(t), \nabla \times \boldsymbol{\varphi}(t))_{\Omega_f} \Big] dt \\ &= \int_0^T \Big[(\mathbf{G}(t), \boldsymbol{\varphi}(t))_{f,s} - \int_{\Gamma_{f,ends,2}} P(t)\varphi_3(t) \Big] dt - (\boldsymbol{\omega}(0), \boldsymbol{\varphi}(0))_{f,s}. \end{aligned} \tag{37}$$

From (34) we deduce in an analogous way that

$$\begin{aligned} &\int_0^T \Big[-(\boldsymbol{\omega}_m(t), \boldsymbol{\varphi}'(t))_{f,s} + a(\int_0^t \boldsymbol{\omega}_m(s)ds, \boldsymbol{\varphi}(t)) + \nu(\nabla \times \boldsymbol{\omega}_m(t), \nabla \times \boldsymbol{\varphi}(t))_{\Omega_f} \Big] \\ &= \int_0^T \Big[(\mathbf{G}(t), \boldsymbol{\varphi}(t))_{f,s} - \int_{\Gamma_{f,ends,2}} P(t)\varphi_3(t) \Big] dt - (\boldsymbol{\omega}_m(0), \boldsymbol{\varphi}(0))_{f,s}. \end{aligned} \tag{38}$$

We set again $m = m_k$ and deduce from (14) and (29)-(32) (after passing to the limit) that

$$\begin{aligned} &\int_0^T \Big[-(\boldsymbol{\omega}(t), \boldsymbol{\varphi}'(t))_{f,s} + a(\int_0^t \boldsymbol{\omega}(s)ds, \boldsymbol{\varphi}(t)) + \nu(\nabla \times \boldsymbol{\omega}(t), \nabla \times \boldsymbol{\varphi}(t))_{\Omega_f} \Big] \\ &= \int_0^T \Big[(\mathbf{G}(t), \boldsymbol{\varphi}(t))_{f,s} - \int_{\Gamma_{f,ends,2}} P(t)\varphi_3(t) \Big] dt - (\boldsymbol{\omega}_0, \boldsymbol{\varphi}(0))_{f,s}. \end{aligned} \tag{39}$$

Compare now the identities (37) and (39) to deduce (36), since $\boldsymbol{\varphi}(0)$ was arbitrary.

5. Proof of the uniqueness

In this section we prove the uniqueness of the weak solution found in Section 4. In order to do that, it suffices to show that the only weak solution of Problem 3 with $P(t) \equiv 0$ and $\mathbf{G}(t) \equiv \mathbf{0}$ for all $0 \leq t \leq T$ is

$$\boldsymbol{\omega} \equiv \mathbf{0}. \tag{40}$$

Thus, we know that $\boldsymbol{\omega} \in \mathbf{L}^\infty(0,T;\mathbf{L}^2(\Omega))$, $\boldsymbol{\omega}\chi_{\Omega_f} \in \mathbf{L}^2(0,T;\mathbf{H}^1(\Omega_f))$, $\boldsymbol{\omega}' \in \mathbf{L}^2(0,T;\mathbf{V}'(\Omega))$, $\int\limits_0^t \boldsymbol{\omega}(s)\chi_{\Omega_s} ds \in \mathbf{L}^2(0,T;\mathbf{H}^1(\Omega_s))$ and

$$< \boldsymbol{\omega}', \boldsymbol{\varphi} >_{f,s} + a(\int_0^t \boldsymbol{\omega}(s)ds, \boldsymbol{\varphi}) + \nu(\nabla \times \boldsymbol{\omega}, \nabla \times \boldsymbol{\varphi})_{\Omega_f} = 0, \tag{41}$$

for all $\boldsymbol{\varphi} \in \mathbf{V}(\Omega)$ and with the initial condition

$$\boldsymbol{\omega}(0) = \mathbf{0}. \tag{42}$$

Let us denote $\boldsymbol{\psi}(t) := \int\limits_0^t \boldsymbol{\omega}(s)ds$. Then notice that

$$< \boldsymbol{\psi}'', \boldsymbol{\varphi} >_{f,s} + a(\boldsymbol{\psi}(t), \boldsymbol{\varphi}) + \nu(\nabla \times \boldsymbol{\psi}'(t), \nabla \times \boldsymbol{\varphi})_{\Omega_f} = 0, \tag{43}$$

for all $\boldsymbol{\varphi} \in \mathbf{V}(\Omega)$,

$$\boldsymbol{\psi}(0) = \mathbf{0}, \quad \boldsymbol{\psi}'(0) = \mathbf{0} \tag{44}$$

and

$$\boldsymbol{\psi} \in \mathbf{L}^2(0,T;\mathbf{V}(\Omega)), \ \boldsymbol{\psi}' \in \mathbf{L}^2(0,T;\mathbf{H}(\Omega)), \ \boldsymbol{\psi}'' \in \mathbf{L}^2(0,T;\mathbf{V}'(\Omega)).$$

Fix $0 \leq s \leq T$ and take

$$\boldsymbol{\zeta}(t) := \begin{cases} \int\limits_t^s \boldsymbol{\psi}(\tau)d\tau & \text{if} \quad 0 \leq t \leq s \\ 0 & \text{if} \quad s \leq t \leq T. \end{cases} \tag{45}$$

Then (43) can be written:

$$\int_0^s [(\boldsymbol{\psi}''(t), \boldsymbol{\zeta}(t))_{f,s} + a(\boldsymbol{\psi}(t), \boldsymbol{\zeta}(t)) + \nu(\nabla \times \boldsymbol{\psi}'(t), \nabla \times \boldsymbol{\zeta}(t))_{\Omega_f}]dt = 0,$$

since $\boldsymbol{\zeta}(t) \in \mathbf{V}(\Omega)$, $\forall t \in (0,T)$, by the regularity of $\boldsymbol{\psi}$.

Upon integrating by parts with respect to time, it follows that (observe that $\boldsymbol{\zeta}'(t) = -\boldsymbol{\psi}(t)$ for $0 \leq t \leq s$):

$$-\int_0^s [(\boldsymbol{\psi}'(t), \boldsymbol{\zeta}'(t))_{f,s} + \int_0^s a(\boldsymbol{\zeta}'(t), \boldsymbol{\zeta}(t)) + \nu \int_0^s (\nabla \times \boldsymbol{\psi}(t), \nabla \times \boldsymbol{\zeta}'(t))_{\Omega_f}]dt = 0,$$

thus

$$\int_0^s (\boldsymbol{\psi}'(t), \boldsymbol{\psi}(t))_{f,s} dt - \int_0^s a(\boldsymbol{\zeta}'(t), \boldsymbol{\zeta}(t)) dt + \nu \int_0^s |\nabla \times \boldsymbol{\psi}(t)|^2_{\Omega_f} dt = 0.$$

It follows that

$$\frac{1}{2}\frac{d}{dt}\int_0^s [\|\boldsymbol{\psi}(t)\|^2_{f,s} - a(\boldsymbol{\zeta}(t), \boldsymbol{\zeta}(t))] dt = -\nu \int_0^s |\nabla \times \boldsymbol{\psi}(t)|^2_{\Omega_f} dt \leq 0,$$

thus

$$\frac{1}{2}[\|\boldsymbol{\psi}(t)\|^2_{f,s} + a(\boldsymbol{\zeta}(0), \boldsymbol{\zeta}(0))] \leq 0$$

and after applying (19) it follows that $\boldsymbol{\psi}(s) = \mathbf{0}$. Now, since s was arbitrary, this implies that $\boldsymbol{\psi} \equiv \mathbf{0}$ in $(0, T) \times \Omega$ and the conclusion follows. ∎

6. Conclusion

In this paper we have considered a 3D/3D fluid-elastic structure interaction problem. The viscous, incompressible fluid was moving through an elastic tube with flexible and thick walls. We considered very small displacements, in order to be able to assume that the domains involved were cylindrical. For the coupled problem with the fluid behavior described by the Stokes equations with boundary conditions involving the pressure (at the in- and outflow) and with the Lamé equations for linearized elasticity characterising the behavior of the deformable structure, we have shown the existence of a unique weak solution (velocity and displacement). The method seems not to be directly adaptable to the case of a Navier-Stokes fluid. However, we believe that the problem might be treated with the aid of other methods; this will make the object of a future work. Furthermore, allowing for larger displacements of the fluid-structure interafce would normally lead to dropping the assumption of cylindrical domains. By our knowledge, this kind of time-dependent problems has not been treated yet in the 3D/3D case, but only for 3D fluid/2D structure interactions ([CDEG02], [Suru04c]) or in lower dimensions.

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