HOW TO OVERCOME POVERTY TRAPS BY EDUCATION

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Preface

The present dissertation thesis originated during my time at the Chair for Economic Policy I of Prof. Dr. Hans Gersbach at the Faculty of Economics and Social Studies of the Ruprecht-Karls-University Heidelberg. At this time I was member of the Graduate Program for “Environmental and Resource Economics” of the universities of Heidelberg and Mannheim. The thesis was submitted in December 2004.

I am very grateful to my supervisor Prof. Dr. Hans Gersbach. After my economics diploma he offered me this dissertation project and furthered my admission as scholar in the graduate program. I have learned very much from many helpful discussions with him. I am also grateful to Prof. Clive Bell, Ph.D., who was willing to act as second supervisor. He also gave me important hints and helpful comments.

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Ultimately, I want to thank my parents above all, to whom this thesis is dedicated. Without the long-term education that they provided me, and their financial support, my studies and this thesis would not have been possible. Thank you for all you have done for me!

Heidelberg, February 2005

Lars-H.R. Siemers


2Here I especially appreciated the seminar at the Centre for Economic and Business Research (CEBR) in Copenhagen in 2003.
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Chapter 1

Introduction

„The welfare of the people is the ultimate law.“

– Cicero (106 B.C. - 43 B.C.)

1.1 Objective of the Thesis

Poverty is one of the fundamental problems in the world. Though well-being depends on more than purely material aspects, increasing individual incomes is a necessary condition to improve the well-being of poor people. This requires economic growth. However, there is evidence that poverty has the tendency to persist. This suggests that economically backward countries (or regions or groups within a country) are caught in a locally stable, “adverse” equilibrium, a so-called poverty trap. The big-push theory states that a strong enough impulse can burst the local stability and hence economic development is realizable. Consequently, the task at hand is how to create a sufficiently strong “push”, that is, how the required economic growth, starting in a locally stable equilibrium, could be attained.

In the theory of economic growth, human capital is a major determinant. In a world in which the success of economies becomes more and more dependent on skills and markets increasingly globalized, human capital is considered to be of great importance for economic development. If backward countries cannot achieve the education of their societies, then the gap between developed and underdeveloped economies will increase rather than decrease. Therefore, the objective of this thesis is to analyze ways to achieve human capital accumulation in a society caught in a poverty trap, so that

1In Latin: „Salus Populi Suprema Est Lex.“
poverty will be overcome and economic development will begin. That is, the big push should be created by the education of the society. In doing so, the thesis tries to give answers to the following questions:

- Which instruments are suited for educating societies and overcoming poverty?
- Which particular problems and risks may prevent the escape from poverty traps and what can be done about it?
- Which instrument amongst all those which are identified as being suitable is the most effective?
- Which constitutional rules are necessary for particular policies to be realizable in democracies?
- Which transition process may economies implementing such policies pass through?

We provide answers to these questions within specific models. The thesis comprehensively discusses two different policy instruments to overcome poverty: subsidies (financed by foreign aid or taxes) and land reforms. In the next section, we give background information about evidence and theory on poverty, underdevelopment and growth.

1.2 Background and Literature

It is fairly surprising how serious the problem of poverty and malnutrition still is after decades of intensified efforts to fight them. Hunger, poverty and backwardness are widespread. Like Basu (2003), p. 3, emphasizes, “...privation is the norm rather the exception.”

1.2.1 Poverty: A Definition

Individual poverty has many faces: hunger, malnutrition and lack of access to a non-contaminated water supply; lack of education, lack of political representation, which exposes the poor to exploitation; landlessness; child labor; environmental degradation.\(^2\) Eventually, poverty is connected with broad fields of human dignity and welfare. We have to deal with intertwining multi-dimensional aspects. However, there is evidence

that growing incomes improve all facets of poverty. RAY (1998) demonstrates that there is good empirical evidence that an improvement of per-capita income is positively correlated with all the aspects usually used to measure human welfare, for instance by the human development index. He states that “... per capita GDP ... acts as a fairly good proxy for most aspects of development.” Thus, increasing per-capita incomes is a promising goal to overcome all problems connected with poverty. Moreover, this suggests that focusing on income is not a too narrow reduction of the complexity of poverty and human welfare. Consequently, we identify poverty with low income. In doing so, we will be able to explain lack of education, child labor, and landlessness, for example.

The World Bank identifies absolute poverty by poverty lines, i.e. by per-capita income-thresholds: an individual is poor if it lives on less than US-$2 a day. Extreme poverty is defined by living on less than US-$1 per day. Individual poverty of a country’s citizens directly translates into a low gross domestic product (GDP) and gross national product (GNP). We will call such a state of an economy underdevelopment or backwardness.

1.2.2 Poverty and Economic Growth: Some Evidence

Data from the World Bank indicates the enormous extent of poverty. In 1998, the number of people living on less than US-$1 a day (extreme poverty) was 1.175 billion, which represented 23.4% of the world’s population. Furthermore, 2.812 billion human beings lived on less than US-$2 a day. This means that more than half of the world’s population (56.1 %) lived in poverty. Additionally, there is an immense discrepancy in material well-being across the world: while the global average of per-capita income is US-$109.59 per day, half of the world’s population lives on less than US-$2 per day. Consequently, the second half of the world’s population, on average, lives on roughly

---

4 Nonetheless, one should not make the mistake to draw the conclusion that only economic growth allows for the existence of good social conditions. There are, of course, also poor countries that have done very well on social indicators, despite limited resources.
5 The thresholds have to be interpreted as US-dollars in 1985-prices, that is, the current per-day dollar-incomes have to be adjusted correspondingly. The following GDP-numbers are also given in purchasing power parity incomes on this basis. See the introduction of Ray (1998) for an excellent survey on the methods of exchange-rate basis versus purchasing power parity income.
US-$220 per day.

Individual poverty goes hand in hand with underdevelopment of single countries. The lowest per-capita GDPs can be found in Africa, which is the only continent where per-capita GDPs below US-$1000 (per year) are widespread; on average, then, a single human being has to live on less than US-$82 per month or US-$2.74 per day. The data, presented by Barro and Sala-i-Martin (1995), demonstrates that many poor countries, with low per-capita GDPs, did not make much progress in the 25 years from 1960 to 1985; the average growth rates of the poor countries are low or even negative. The data sets of the World Bank for 1997 and the survey in Ray (1998) show that this tendency still holds today. However, the World Development Indicators 2004 stress that the proportion of people living on less than US-$1 dropped by almost half between 1981 and 2001 due to dramatic progress in East Asia. At the same time, other countries experienced throwbacks: the number of poor increased and the GDPs in the Sub-Saharan region shrank by around 14%.

Ray (1998), p. 47, states that “A percentage point added to the growth rate can make the difference between stagnation and prosperity over the period of a generation”, and Ravallion and Chen (1997) found that countries in the process of economic growth experience a decline in the proportion of people below the line of extreme poverty. Consider Table 1.1. It contains information about historic data on GDP per capita and day in 1985-US-$ for selected developed and developing countries, where the year in which the first data is available (the number in parenthesis) differs from country to country. We then calculated the average growth rates in the period from the year of this first data and the year 1990.

In 1870, the United Kingdom (U.K.) was the wealthiest power. It grew on average with only 0.4043 percentage points less than the United States of America (U.S.A.), or 0.712 less than Germany. As a consequence of this lower growth rate of the U.K., in 1990, the U.S.A. was by far the wealthiest economy and countries like France, Germany, Japan and Switzerland had also overtaken the U.K. In 1890, Japan, with only US-$2.31 per capita per day, was by far the poorest of today’s economic powers listed in the table. According to this data, Japan was at a similar stage of development as many of the underdeveloped countries today. However, with an average growth rate of nearly 3% per year Japan became one of the wealthiest countries in the world. Chile, for instance, started in 1900 with a higher value (US-$2.62 per capita per day) than Japan did in 1890, but with only half of the average growth rate of Japan, Chile remained a developing country. Furthermore, we see that India, and even more so


Table 1.1: Historic per-day incomes and average growth rates of selected economic powers and four developing countries from South-America and Asia.
(Source: Own calculations on basis of Table 10.2 of Barro and Sala-i-Martin (1995, 1998))

Bangladesh, have on average experienced only negligible economic growth for 90 years. This demonstrates that poverty can be very persistent, which contradicts the former conventional wisdom that countries should grow faster, the poorer they are\textsuperscript{10} – actually, within the time period 1960-1999, most poor countries were falling behind instead of catching up to the rich countries\textsuperscript{11}. Hence, economic success is possible, but by no means guaranteed. It follows that elaborating on strategies to attain economic growth is essential.

The growth rates which are necessary to overcome poverty can be deduced from Table 1.2. It tells us the average per-day income a single individual of selected developing economies will have in 2015, if from 1990 to 2015 the average real growth rate amounts to 2% and alternatively, if it amounts to 5%. We see that an average growth rate of 2% would not suffice for the per-capita income in Bangladesh and Chad to cross the poverty line of US-$2 per-day in 2015. That is, a substantial fraction of the population will remain below this threshold. In the 5% scenario, the average citizen of the listed countries will earn more than US-$2. However, all but very few countries stay developing economies with less than US-$10 per day per capita. As poor households earn far less than the per-capita income and underdeveloped countries have experienced nega-

\textsuperscript{10} Cf., for instance, Easterly (2002), p. 59, on this traditional convergence argument of the neoclassic growth model.

Table 1.2: Potential per-capita incomes per day of selected developing countries of Africa, Asia and South-America for the year 2015, in the case of an average growth rate of 2% and 5%, respectively.
(Source: Own calculations on basis of Table 10.1 of Barro and Sala-i-Martin (1995,1998))

<table>
<thead>
<tr>
<th>Country</th>
<th>per-capita income (1985-US-$) per day in 1990</th>
<th>in 2015 at average growth rate of 2%</th>
<th>in 2015 at average growth rate of 5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bangladesh</td>
<td>1.027</td>
<td>1.686</td>
<td>3.479</td>
</tr>
<tr>
<td>Burundi</td>
<td>1.430</td>
<td>2.346</td>
<td>4.843</td>
</tr>
<tr>
<td>Cameroon</td>
<td>3.389</td>
<td>5.560</td>
<td>11.476</td>
</tr>
<tr>
<td>Chad</td>
<td>1.049</td>
<td>1.641</td>
<td>3.386</td>
</tr>
<tr>
<td>Chile</td>
<td>9.296</td>
<td>15.251</td>
<td>31.479</td>
</tr>
<tr>
<td>The Gambia</td>
<td>1.767</td>
<td>2.899</td>
<td>5.984</td>
</tr>
<tr>
<td>India</td>
<td>1.814</td>
<td>2.976</td>
<td>6.142</td>
</tr>
<tr>
<td>Kenya</td>
<td>2.496</td>
<td>4.095</td>
<td>8.452</td>
</tr>
<tr>
<td>Mali</td>
<td>1.427</td>
<td>2.342</td>
<td>4.834</td>
</tr>
<tr>
<td>Peru</td>
<td>6.521</td>
<td>10.698</td>
<td>22.081</td>
</tr>
<tr>
<td>Rwanda</td>
<td>1.814</td>
<td>2.976</td>
<td>6.142</td>
</tr>
<tr>
<td>Sierra Leone</td>
<td>2.282</td>
<td>3.744</td>
<td>7.728</td>
</tr>
<tr>
<td>Togo</td>
<td>1.668</td>
<td>2.800</td>
<td>5.780</td>
</tr>
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</table>

The fact that positive macro-shocks initiated by development policy often had only short-term effects and some developing economies in the long-term often tend to return to the initial position, suggests that the state of underdevelopment might be a locally stable equilibrium (we use the term stable in the sense that per-capita income

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12For example, Burundi (-2.2%), Congo (-4.8%), Sierra Leone (-4.4%). See <http://www.worldbank.org/data/wdi2003/tables/table4-1.pdf>.
remains constant). Accordingly, persistence of poverty can be explained as a locally stable equilibrium that can arise in models of economic growth where particular frictions in markets or institutions are present. Due to the local stability, model dynamics is such that, despite temporary success of a reform, underdeveloped countries can fall back to so-called low-level equilibria or poverty traps. The corresponding theories were developed by Young (1928), Rosenstein-Rodan (1943), Singer (1949), Nurkse (1953), Scitovsky (1954), Flemming (1955), or Nelson (1956). The characteristic of a poverty trap is that the rate of intensive growth stabilizes at zero, while the absolute level of GDP remains low. That is, the convergence argument of the neoclassical growth theory does not hold. The economic performance of Bangladesh and India in the period 1900-1990, given in Table 1.2, demonstrates this. However, as theory assumes that poverty traps are only locally stable equilibria, a strong enough positive shock can break the local stability of poverty trap equilibria and economic development sets in. This is the basic idea of a big push [see, for instance, Rosenstein-Rodan (1943) or Lewis (1954)].

Theory has provided many helpful insights as to why poverty and underdevelopment may persist. An excellent survey can be found in Basu (2003). Beginning with the classic Malthusian Population Theory, population growth can (over-)compensate for the growth of GDP, so that intensive growth does not occur. Based on the Harrod-Domar model and the neoclassical growth theory, a lack of physical capital can produce a supply-side vicious circle: capital scarcity leads to low output and income, which, in turn, causes low savings. Consequently, investments are low and capital remains scarce. Similarly, imperfect capital markets prevent poor households from undertaking highly remunerate investments: since they are poor, they have to raise credit to invest, but because of their poverty they do not have any securities to offer to the banks, and the households remain poor. Baland and Robinson (2000), Barham, Boadway, Marchand, and Pestieau (1995), Bell and Gersbach (2001), Galor and Zeira (1993), Ranjan (1999, 2001), for example, demonstrate that imperfect capital markets also produce poverty traps caused by human capital scarcity. Easterly

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15See Barro and Sala-i-Martin (1995), Section 1.3.5, on poverty traps. Galor and Ryder (1989) show that changing savings rates can also cause poverty traps.
17The strand of literature on micro-finance elaborates one way to overcome this trap scenario. See, for instance, Bell (2003), chapter 15; Braverman and Guasch (1985); Hoff, Braverman, and Stiglitz (1993); Hulme and Mosley (1996); Morduch (1999, 2000); Vogelgesang (2003).
Chapter 1. Introduction

(2002), Chapter 8, describes poverty traps caused by increasing returns. For instance, the rate of return to investment in new knowledge or education may depend on how much knowledge and education already exists. If in a backward economy the stock of knowledge and the average education level is initially low, then the rate of return of such investments is also low. Consequently, given this low rate of return is below the minimum required rate of return given by the discount rate, there is no incentive to invest in knowledge or education, and the economy may remain backward and unskilled.

The *balanced-growth* literature established the hypothesis that poverty and underdevelopment persist in backward countries as long as single sectoral expansions are not “coordinated” with each other. If a single sector performs economic expansion, the resulting income increase will only partly produce higher demand for the sector’s good, so that this *demand lack* will cause the sector to shrink again. Therefore, economic development requires a synchronized (or balanced) expansion in many sectors to produce a sufficient impulse for growth.\(^\text{18}\) Contrary to this, the *unbalanced growth* theory assumes that a large investment in one or only a few sectors can generate the scope for expansion in other sectors. The idea is that investments in one sector can serve as an initial ignition for economic growth via linkages between the sectors of an economy.\(^\text{19}\) That is, expansion in one sector that causes a strong enough impulse will generate the expansion of further sectors so that economic development begins.\(^\text{20}\) The *O-Ring Theory*\(^\text{21}\) of low productivity is related to the theories of balanced and unbalanced growth and was invented by Kremer (1993). This theory also attributes the persistence of poverty to complementarities between sectors and emphasizes the importance of the development of single sectors. The underdevelopment in one sector harms all the complementary sectors. A persistent backwardness of one sector thus causes all other complementary sectors to downgrade their productivities (*skill-clustering theorem*). Consequently, there exist multiple equilibria and a single backward industry can be responsible for the economy being caught in a low-income trap – like the malfunction of a small O-ring can cause the malfunction of a whole engine. Recent literature on industrialization and multiple equilibria is Eswaran and Kotwal (1996), Matsuyama (1992), and Murphy, Shleifer, and Vishny (1989).

In addition to these theories analyzing the direct economic environment of backward

\(^\text{18}\) Cf., for instance, Dagnino-Pastore (1963), Findlay (1959), Nurkse (1953), Rosenstein-Rodan (1943), Sheahan (1958).

\(^\text{19}\) Cf. Hirschman (1958).

\(^\text{20}\) Mathur (1966) demonstrates that the theories of balanced and unbalanced growth can be reconciled.

\(^\text{21}\) The name stems from the fall of the space shuttle “Challenger”. It turned out that the malfunction of small O-rings caused the explosion of the shuttle.
Chapter 1. Introduction

In economies, there is also a strand of literature that identifies political failure as a source of the persistence of poverty. In this literature, it is emphasized that the choice of the correct development objectives is not sufficient for success, but that development policies also have to be implemented efficiently. Easterly (2002), Hillman (2004b) and Svensson (2000, 2003) provide theories of political failure. In all of these theories, the authors state that development policy always involves typical principal-agent problems: donors pay money to local governments to implement a certain policy; governments, in turn, employ bureaucrats to support the poor. At all of these stages, one has to ensure that the individuals carrying out the developing policy have an incentive to work socially efficiently. Easterly (2002) argues that development policy often failed due to neglecting exactly these incentive effects. The governments of developing countries that received money from international donors had no incentive to apply growth promoting policies – and instead enriched themselves. Though financial aid was paid conditionally on policy adjustments towards growth promoting policies (growth-oriented adjustment programs), violations were not punished. Consequently, rent-seeking, corruption and fraud wasted massive parts of the foreign aid, and only few positive impulses for growth arose. Hillman (2004b) explains the failure of international efforts by Nietzschean behavior of the individuals who are in charge of policy-making in developing economies. He states that these people often do not have any incentive to adopt efficiency-enhancing policies because they benefit from the failure of development policies.

Overall, a per se correct development policy will fail if it cannot initiate a sufficiently strong impulse. An adverse incentive system that bears rent-seeking, corruption and fraud weakens the impulse and thus increases the likelihood of failure.

1.3 The Thesis and the Focus on Human Capital

We have emphasized that economic growth is essential for overcoming poverty and that growth does not occur when economies are imprisoned in poverty traps. In this thesis, we emphasize the role of human capital in the process of economic growth.


23At the 2004 annual meeting of the Public Choice Society in Berlin Arye Hillman also argued that in a Nietzschean world the weak are considered “good” and that the rich in the Western World do not want, or do not dare, to criticize the governments of these poor countries. Therefore, governments receive payments despite violating imposed warranties.
1.3.1 Human Capital and the Theory of Economic Growth

The theory of economic growth states three major sources of intensive economic growth at the production level:\textsuperscript{24}

i) accumulation of physical capital

ii) accumulation of human capital

iii) technological progress

The work of UZAWA (1965) and, most of all, LUCAS (1988) demonstrates the important role of human capital \textit{accumulation} (or skill acquisition) for growth.\textsuperscript{25} GYLFASON AND ZOEGER (2003) present recent evidence that education increases growth, and SYLWESER (2000) that human capital formation has a positive effect on economic growth.\textsuperscript{26} Furthermore, NELSON AND PHELPS (1966) and WELCH (1970) show how the \textit{stock} of human capital (skills and knowledge) positively affects the ability to generate technological progress (like process, product, technological, or organizational innovations, basic and applied research) and to adapt to new technologies, which accelerates the diffusion of technological progress. WELCH (1970) (p. 55) found that this “leverage” associated with schooling exists only for higher skills (college education).\textsuperscript{27} Hence we do not consider this aspect in the thesis. Nevertheless, one can state that human capital positively affects other major sources of growth: the full power of the accumulation of physical capital and technological progress will only unfold in economies that are endowed with sufficient human capital. Additionally, GALOR (2004) (p. 1) emphasizes the “increasing role of human capital in the production process.” CIGNO (2003) finds that international integration and pulling down trade barriers may harm countries with a largely uneducated workforce, whereas countries with educated workers may gain from globalization. This underlines the outstanding importance of human capital for growth, and that the significance of education might rise in an increasingly globalized world. Finally EASTERLY (2002) emphasizes that only those countries escaped poverty that were educated. It follows that the education of poor societies has to be one of the major objectives in fighting poverty in the future.

\textsuperscript{25} For a very good review see AGHION AND HOWITT (1998), Section 10.
\textsuperscript{26} Other studies, for instance PRITCHETT (2001), could not find any positive association between growth in education and growth of per-capita income. Of course, it is also possible that economic growth boosts education.
\textsuperscript{27} See also ROMER (2001), p. 149.
1.3.2 The Poverty Trap in the Thesis

In this thesis, we investigate how policy intervention could produce a sufficiently strong impulse to burst the local stability of low-development equilibria. Recent literature identifying low-development traps in growth models due to a lack of education (or human capital) comprises articles by Azariadis and Drazen (1990), Bell and Gersbach (2001), Galor and Zeira (1993) and Redding (1996). We follow this strand of literature.

We analyze a low-income trap that is driven by the failure to form human capital. The basic vicious circle runs as follows: poverty forces parents to send their children to work (inclusive housework) in order to ensure the survival of the family. As children have to work, they cannot attend school and stay illiterate, unskilled and miss any formal basic school education. Capital markets are imperfect because parents cannot solve this education problem by borrowing against the future returns of education. Therefore, when these children have grown up, they are only in a position to earn money as unskilled workers and will be poor just as their parents were due to the lack of education. It follows that the grown-up children face the same constraints as their parents did. Thus, the vicious circle is closed.

It becomes clear that our type of poverty trap is directly connected with the issue of child labor: poverty causes child labor, and child labor in turn, prevents the formation of human capital, which again causes poverty.

1.3.3 Human Capital in the Thesis: A Definition

So far, we have used the term human capital only abstractly. Hence, we will now define more precisely what is meant by this term. Human capital is defined as all knowledge, skills and education that can be utilized in the production process to obtain individual income or GDP (micro- and macroeconomic perspective). Romer (2001), p. 133, defines the stock of human capital as “the total amount of productive services supplied by workers” and entrepreneurs, which also includes raw labor.

In our context of developing countries caught in poverty traps, we focus first of all on the human capital that is meant by basic education: reading, writing, arithmetic.

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28 Estimates by the International Labour Office (ILO) say that 246 million children are child laborers. 73 million are less than ten years old and 22,000 die every year in work-related accidents. See Facts on Child Labour on <http://www.ilo.org/>.

29 This link between child labor, human capital and poverty was first explored by Bell and Gersbach (2001).

These fundamental skills create an essential foundation of knowledge and productivity for the individual. If workers cannot count, then the employer can cheat on them by asserting that, for instance, less units of goods have been produced than were actually made by the worker.\footnote{Cf. the example of a child worker given in Imhasly (2004), p. 20.} If individuals cannot read, then they are unable to read what their rights are, and therefore can neither refer to the decisive laws and rules, nor read contracts. If they cannot write, then they are not able to offer written contracts, send letters to complain, write down notes, or perform simple book-keeping. Furthermore, being illiterate, one cannot read books, for example about techniques of production to increase knowledge and productivity, and one cannot communicate with customers, component suppliers, trade creditors, or with governmental authorities, to build up business relations outside the narrow local region or to receive governmental support.\footnote{Notice that in poor regions there are no well functioning telecommunications networks, so that one has to rely on postal communication.} All these personal deficiencies lower individual incomes and enable other persons to exploit the poor. Summing up, the outlined consequences of a lack of basic skills demonstrate the outstanding importance of a basic education for economic and social development. In today’s world there are about 1 billion adults that are illiterate, which is approximately 25 percent of the world population. Moreover, over one hundred million children in the world have no access to school.\footnote{Cf. Ho (2003), Background Information.}

### 1.4 Political Embedding of the Thesis

In this section, we briefly deal with the political background of our approach. According to Article 26 of the Universal Declaration of Human Rights, literacy is one of the basic human rights.\footnote{Cf. Ho (2003), Background Information.} Furthermore, our focus on the link between poverty and education corresponds with the two most important of the eight \textit{Millennium Development Goals}\footnote{Cf. MDG (2000) or the United Nations Millennium Declaration (55/2, Sept. 18, 2000) [United Nations (2000)]. See also Collier and Dollar (2001).} stated at the Millennium Summit of the United Nations in September 2000. First, regarding poverty and nutrition, the proportion of people who live on less than US-$1$ a day and the proportion who suffer from hunger should be halved between 1990 and 2015 (goal 1). Second, concerning education, children everywhere, boys and girls alike, should be able to complete a full course of primary schooling by 2015 (goal 2). The World Development Indicators 2004 of the World Bank also support our approach of focusing on the two goals \textit{poverty} and \textit{education}, i.e. human capital: it is emphasized that only those countries have been successful in reducing poverty that
combined growth incentives with investments in education.

World Bank Group President James D. Wolfensohn states that “Since the time of Bretton Woods Conference, ... all confirm that the eradication of poverty is central to stability and peace.”\textsuperscript{36} Since the terror attacks on September 11, 2001, the West faces the threat of an (international) pseudo-Islamic terrorism. One theory claims that poverty, backwardness, and a lack of education within the underdeveloped world are sources of this terror. Therefore, fighting poverty by educating societies might also be a tool to attain stability and to fight international terrorism.\textsuperscript{37, 38}

In the next chapter, we explain the basic model of the thesis. It follows a brief welfare analysis where we identify an inter-generational externality of schooling. Then, in Part I, we comprehensively analyze how to overcome poverty by schooling via subsidy schemes. In Part II, we precisely demonstrate that land reforms are also an adequate instrument to overcome poverty by education. Additionally, we identify a potential transition process that economies will pass through when implementing successful land reforms. In doing so, we respectively address difficulties and potential risks that may arise in practice.

\textsuperscript{36}Cf. Wolfensohn (2004).

\textsuperscript{37}Cf., for instance, Wolfensohn (2004): “... together, we fight terror. We must. The danger, however, is that ... we lose sight of the long-term and equally urgent causes of our insecure world: poverty, frustration, and lack of hope.”

\textsuperscript{38}One constrictively has to say that, so far, the current threat of international terrorism solely stems from (pseudo-)Islamic individuals, and there is no empirical evidence for a significant correlation between terrorism and education or poverty [cf. Berrebi (2003), Krueger and Malečková (2003), Piazza (2003), Sandler and Enders (2001), and Sandler, Tsichrhart, and Cauley (1983)]. Hence, there is also ongoing discussion about Samuel Huntington’s hypothesis of a clash of civilizations. Cf. Huntington (1996).
Chapter 2

The Basic Model

The basic model of this thesis rests on Bell and Gersbach (2001). We consider an OLG model in which individuals live for two periods; these periods are labeled childhood and adulthood, respectively. Each generation consists of a continuum of households represented by the interval $[0, 1]$. There is no population growth. Each household, denoted by $i \in [0, 1]$, comprises one adult and one child, so that each adult gives birth to one child. All households are alike and we use time index $t$ to denote a period. The driving power in our growth model is human capital. Human capital has to be formed in childhood. Each child is endowed with one unit of time that can be used either for schooling or for child labor. Since the adult has the right to make decisions for the child, we use the so-called unitary model [see Becker (1964) or Basu and Van (1998)]. The proportion of child $i$’s time devoted to school education in period $t$ is denoted by $e_i^t \in [0, 1]$. Neglecting leisure time, the residual time, $1 - e_i^t$, is used for child labor. For simplicity, adults spend all their time working.

2.1 The Human Capital Technology

Human capital is assumed to be formed in childhood in a process that combines child-rearing with formal education in the following way: Let an adult $i$ in period $t$ possess $\lambda_i^t$ efficiency units of labor, where $\lambda_i^t \geq 1$ is a natural measure of her human capital. The condition $\lambda_i^t = 1$ represents the level of pure labor, that is the case when the

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1 Cf. also Uzawa (1965), Lucas (1988), Basu (1999), or the “Uzawa-Lucas-model” in Barro and Sala-i-Martin (1995), Section 5.2.2.
2 However, the adult nonetheless rears the child. This activity is assumed to claim a fixed amount of time of the adult’s and the child’s time. Using an endogenous labor-leisure choice of the adults can lead to multiple equilibria in the labor market, see Basu and Van (1998).
3 For simplicity, we assume that the adult is the mother.
adult $i$ is fully uneducated in period $t$. For the society as a whole, i.e. $\lambda_i^t = 1$ for all $i \in [0, 1]$, this can be thought of as a state of backwardness. In the course of rearing a child, the adult gives the child a certain capacity to build human capital for adulthood.\footnote{The empirical significance of parental human capital was, for instance, documented by \textsc{Becker and Tomes} (1986) or \textsc{Coleman, Campbell, Hobson, McPartland, Mood, Weinfeld, and York} (1966).} For all households $i \in [0, 1]$ alike, the amount of this contributing factor is assumed to be a fixed fraction, $z \in (0, 1]$, of the adult’s own endowment of efficiency units of labor. School education also increases human capital. This is expressed by the schooling function $h(e^t_i)$, with\footnote{Following the common notation of Lagrange, a prime symbol “$'$” stands for the first derivative, a second for the second derivative, and so on.} $h'(e^t_i) > 0$ and $h(0) = 0$. We assume, $h(\cdot)$ is a continuous, increasing and differentiable function.\footnote{\textsc{Colclough and Al-Samarrai} (2000) stress that a crucial point of accumulating human capital is the quality of schooling, involving the quality of learning utilities, the motivation and qualification of teachers etc. So the functional form of $h(e^t_i)$ also represents the schooling system’s quality. This issue of school quality is highlighted in Chapter 6.} Furthermore, we make the assumption that the adult’s gift by rearing, $z\lambda_i^t$, will not preserve the child from the state of $\lambda_{i+1}^t = 1$ as an adult, unless it is complemented by some formal education in school, which includes, first of all, the basic cultural skills of reading, writing and calculating.

Summarizing, the adult’s human capital of household $i$ in period $t+1$, denoted by $\lambda_{i+1}^t$, is a function of the human capital of the adult in period $t$, $\lambda_i^t$, and the time the adult spent in school during childhood, $e^t_i$. Hence, $\lambda_{i+1}^t$ is the human capital of the child in period $t$ of household $i$ when reaching adulthood in $t+1$:\footnote{\textsc{Plug and Vijverberg} (2001) estimate that a fraction of 0.65 of the ability effects relevant for school achievements can be attributed to genetic effects like IQ. Hence, the IQ is also a contributing factor and can be incorporated into $h(e^t_i)$ by adding some child-specific parameter.} \footnote{We use $h(e^t_i) \cdot (z\lambda_i^t)$ instead of $h(e^t_i) + (z\lambda_i^t)$, because of our assumption that a formal school education is essential, i.e., is required if $\lambda_{i+1}^t > 1$ should be feasible.}

$$\lambda_{i+1}^t = (z\lambda_i^t) \cdot h(e^t_i) + 1$$

(2.1)

Note that $e_i^t = 0$ leads to $\lambda_{i+1}^t = 1$.

\section*{2.2 The Output Technology}

We consider one aggregated consumption good that is produced by the input of labor in terms of human capital. We assume that there is a proportional relationship between output and the input of human capital. That is, the productivity of an efficiency
unit of labor is constant and we denote this productivity by \( \alpha > 0 \). Labor is paid by (marginal) productivity. The income of a household is thus proportional to the amount of efficiency units of labor that this household provides for production. In the case of child labor, the supply of efficiency units is at most one, since children are not educated at all. Therefore, the human capital of a child is some fraction of one, denoted by \( \gamma \in (0, 1] \). If the child attends school, this education time will diminish the labor supply of the child. The household’s total supply of efficiency units of labor is, therefore, \( \lambda_i^t + (1 - e_i^t) \gamma \). Applying the average productivity of an efficiency unit of labor yields

\[
y_i^t = \alpha \left[ \lambda_i^t + (1 - e_i^t) \gamma \right]
\]

as the income of household \( i \) in period \( t \), labeled \( y_i^t \).

### 2.3 The Household’s Behavior

Since we assume that all households are alike, we henceforth drop household index \( i \). As mentioned above, we assume that all household decisions lie in the hand of the adult. Capital markets in developing countries are imperfect. Especially in the context of education debts for children, there exists a lack of knowledge on the side of banks about the ability of a particular child and there arise associated enforcement problems when parents borrow while the children would have to pay back the credit (adverse selection, moral hazard). Especially in rural areas, there are often no opportunities to borrow sufficiently, particularly not for educational purposes (credit rationing). Informal credit markets with traditional money lenders dominate, which have an “... envious eye for any ... forms of wealth that might serve as collateral...” (Bell (2003), p. 415). Obviously the poorest households do not have any collateral. Formal credit institutions, on the other hand, are inefficient, “... plagued by high rates of default, and forced to resort to rationing in the face of heavy excess demand for loans...”, where “... traditional lenders ... enjoy implicit debt seniority.” (Bell (2003), p. 415, 416). Consequently, the foregone earnings of education \( \alpha \gamma e_i \) (or any other educational costs)

---

9 A standard method to establish constant productivity of inputs in models with multiple production factors can be found in Barro and Sala-i-Martin (1998), Chapter 5, pp. 202-203: Consider production function \( Y = AK^{a}H^{1-a} \), where \( K \) denotes physical and \( H \) human capital. Profit maximization demands \( K/H = a/(1-a) \). Hence, \( Y = A(a/(1-a))H \equiv \alpha H \) and the constant (marginal) productivity of an efficiency unit of labor is given by \( \alpha \). See also Aghion and Howitt (1998), Chapter 10, or Maussner and Klump (1996), pp. 243-56.

10 It is also conceivable that in special circumstances, e.g. when small hands are advantageous, children may be more productive than adults. However, on average, the assumed will be the case.

11 Though this form reminds us to a typical AK model, we show in Section 4.2.2 that long-term growth is just as possible as a neoclassic-like high-income steady state.
cannot be compensated for by debts [see_balirdand robinson_(2000); basu (2003), chapter 13; bell (2003), chapter 15; galor and zeira (1993); ranjan (2001); ray (1998), chapter 14]. On page 82, priya ranjan therefore emphasizes that “... borrowing against the future earnings of children is not possible.” accordingly, we assume:

**Assumption 2.1**

*poor households do not receive credit for the education of their children.*

Similarly, we assume that the adult cannot leave negative bequests to shift the costs of education to the child at death [cf. baland and robinson (2000)]. On page 665, baland and robinson remark that “... the importance of the nonnegativity constraints on both bequests and savings arises from capital market imperfection.”12 As the considered households are poor, we also assume that there are no positive bequests. Thus, equation (2.2) represents current real income, which is consumed completely. For the sake of simplicity, let the child’s consumption be a fixed fraction \( \beta \in (0, 1) \) of the adult’s, the latter denoted by \( c_t \). From equation (2.2) we then obtain the family’s budget constraint:

\[
(1 + \beta)c_t + \alpha \gamma e_t = \alpha (\lambda_t + \gamma)
\]

(2.3)

The budget set is illustrated by figure 2.1.13,14 The price for education is the foregone earnings per unit of time spent in school, \( \alpha \gamma \).

The household’s full income is given by \( \alpha (\lambda_t + \gamma) \). This maximum income is driven by the adult’s level of human capital \( \lambda_t \). For the case the adult \( i \) chooses \( e^i_t = 0 \), we define

\[
\bar{c}_t(\lambda_t) \equiv \frac{\alpha (\lambda_t + \gamma)}{1 + \beta}
\]

(2.4)

and for the case \( e^i_t = 1 \)

\[
\underline{c}_t(\lambda_t) \equiv \frac{\alpha \lambda_t}{1 + \beta}
\]

(2.5)

The effect of an increase of the adult’s human capital is illustrated in figure 2.1. An increase in \( \lambda_t \) shifts the budget line to the north-east, but leaves the relative price between \( e_t \) and \( c_t \) unchanged.15

12For an analysis with perfect capital markets see, for instance, galor and tsiddon (1997).
13Note that \( e_t \in [0, 1] \). Therefore, for \( \frac{\lambda_t + \gamma}{\lambda_t + \gamma} > 1 \), the budget “line” always has a kink.
14At this point, the author confesses that in all figures within this thesis the relation between adult’s full income \( \alpha \lambda_t \) and the child’s full income \( \alpha \gamma \) is unrealistic low. However, this allows to illustrate the effects much better.
15If the child’s productivity as a child-worker increases with her mother’s productivity, then the opportunity costs of \( e \) will increase with \( \lambda \) and there will be a substitution effect that works against sending the child to school. This possibility is not pursued here.
Chapter 2. The Basic Model

Figure 2.1: The budget line and the effect of an increase of adult’s human capital, where $\mu_t = 1 + \lambda_t / \gamma$ and the budget line’s slope is $-\frac{1+\beta}{\alpha \gamma}$.

We now turn to the preferences of the adults. We assume that preferences are determined by the size of consumption $c_t$ and of schooling-time $e_t$. Hence, an adult is altruistic towards her own child. Preferences are convex. The adult’s demand for consumption we denote by $c^*_t$ and the demand for the child’s education time by $e^*_t$. The gift of factor $z \lambda_t$ through rearing is one form of transfer *inter vivos*. A second form is sending the child to school at least part of the time ($e_t > 0$), which is necessary if the child is to enjoy $\lambda_{t+1} > 1$ as an adult. However, since current consumption is maximized by choosing $e_t = 0$, the adult’s sense of altruism towards the child must be sufficiently strong for her to choose $e_t > 0$. The central assumption of the model is the following:

**Assumption 2.2**

(a) An adult does not send the child to school as long as $(1 + \beta) \overline{c}(\lambda_t)$ does not cross a minimum level $c^S$:

$$(1 + \beta) \overline{c}(\lambda_t) \leq c^S \iff e_t = 0$$

19
(b) As soon as \((1 + \beta)\mathbb{g}(\lambda)\) reaches a level of \(c^a\), the adult is willing to give the child a full basic education:

\[
(1 + \beta)\mathbb{g}(\lambda) \geq c^a \iff e_t = 1
\]

That is, as long as the family as a whole is not able to consume more than a minimum consumption level \(c^S\), though the child contributes to the family’s income by full child labor, the adult is not willing to dispense with full child labor in favor of education;\(^{16}\) level \(c^S\) at least ensures that the family has enough to eat. However, once the adult’s income alone suffices to finance a family consumption level of \(c^a\), the adult dispenses with child labor fully, so that the child attends school full time.

The thresholds \(c^S\) and \(c^a\) are exogenously given by the adult’s preferences, and thus measures of the degree of altruism of the adult towards the child. We assume that full-time child labor is solely caused by poverty and hence that:

**Assumption 2.3**

\(c_t > c^S\) is equivalent to \(e_t > 0\), irrespective of the size of the slope of the budget line, given by \(-\frac{1+\beta}{\alpha\gamma}\).

Therefore, for a consumption level \(c_t > c^S\), education time \(e_t\) is essential for perceiving a higher utility than at locus \((0,7\lambda^S)\). That is, as long as \(c_t > c^S\), there are strictly convex indifference curves that never intersect the horizontal axis in Figure 2.1. Then, in this region there exists a substitutability between consumption \(c_t\) and education time \(e_t\). However, as long as \(c_t \leq c^S\) only consumption \(c_t\) determines the level of utility and there exists no substitutability between consumption \(c_t\) and \(e_t\).

Consider budget constraint (2.3). The household’s demands for consumption and education, \(c_t^o\) and \(e_t^o\), are determined by the household’s preferences, the relative price of education \(\alpha\gamma\), the size of \(\beta\) and by full income \(\alpha(\lambda_t + \gamma)\). Since \(\alpha, \gamma, \beta\) are constants, the demands are determined by the preferences and the adult’s level of human capital \(\lambda_t\): \(c^o(\lambda_t)\) and \(e^o(\lambda_t)\).\(^{17}\) Accordingly, there are two threshold values, \(\lambda^S\) and \(\lambda^a\) that correspond with the consumption thresholds \(c^S\) and \(c^a\). As long as \(\lambda_t \leq \lambda^S\), the adult chooses \(e_t^o(\lambda_t) = 0\), but as soon as \(\lambda_t \geq \lambda^a\), she chooses \(e_t^o(\lambda_t) = 1\).\(^{18}\) We

\(^{16}\)Empirical studies show that the incidence of child labor in rural areas is much higher than in urban ones. See, for instance, PALLAGE AND ZIMMERMANN (2001), p. 6.

\(^{17}\)In our model, the education decision is determined by full income, by the opportunity costs and the degree of altruism (preferences). In practice, the parents’ decision regarding school attendance may also be determined by the direct cost of education, the expected return of education, the access to and the regional facilities of education [cf. CIGNO, ROSATI, AND GUARCELLO (2002)]. In Chapter 6, we will extend our model to a part of these further determinants.

\(^{18}\)Besides BELL AND GERSBACH (2001), for instance, HAZAN AND BERDUGO (2002) also use a model with two such thresholds.
suppose that both goods are non-inferior, that is, the demand for consumption $c_t$ increases monotonically with full income and the demand for education $e_t$ monotonically in the interval $[c^S, c^a)$. Therefore, non-inferiority demands $\frac{\partial c_t}{\partial \lambda_t} > 0$ and $\frac{\partial e_t}{\partial \lambda_t} > 0$ for $\lambda_t \in [\lambda^S, \lambda^a)$. Hence, our central assumptions imply that the “income expansion path” takes the following form:

\[
(e_t, c_t) = \begin{cases} 
(0, \overline{c}(\lambda_t)) & \forall \lambda_t \leq \lambda^S; \\
(e^o(\lambda_t), c^o(\lambda_t)) & \forall \lambda_t \in (\lambda^S, \lambda^a); \\
(1, \underline{c}(\lambda_t)) & \forall \lambda_t \geq \lambda^a.
\end{cases}
\] (2.6)

where the locus $(e^o, c^o)$ is monotonically increasing in $\lambda_t$ for all $\lambda_t \in (\lambda^S, \lambda^a)$. Three possible cases are illustrated in Figure 2.2. Case a) implies that the household does

![Figure 2.2: The income-expansion path for a) $e''(\lambda) < 0$, b) $e''(\lambda) = 0$, and c) $e''(\lambda) > 0$ in the interval $\lambda_t \in [\lambda^S, \lambda^a)$.](image)

increase the child’s education strongly once the critical level $c^S$ is crossed, but that more and more consumption is then required to increase the child’s education ($e'' < 0$). Alternative b) covers the case of homothetic preferences where additional income does not change the “exchange rate” between child’s education and consumption ($e'' = 0$). Finally, c) is the case where once $c^S$ is crossed, the adult chooses more and more education per additional unit income ($e'' > 0$).
2.4 Dynamics

Inserting the optimal choice for education, given by (2.6), in (2.1) we obtain:

\[
\lambda_{t+1} = \begin{cases} 
1 & \forall \lambda_t \leq \lambda^S; \\
zh(e^*_{t}(\lambda_t))\lambda_t + 1 & \forall \lambda_t \in (\lambda^S, \lambda^a); \\
zh(1)\lambda_t + 1 & \forall \lambda_t \geq \lambda^a.
\end{cases} \tag{2.7}
\]

If \(\lambda^S < 1\), then this threshold is irrelevant (because \(\lambda_t \geq 1\)) and all children would attend school partly in any case. This is not what we observe and thus not an adequate description of reality. Accordingly, we state:

**Assumption 2.4**

\(\lambda^S > 1\)

Then, it follows from the first line of (2.7) that the state of backwardness (\(\lambda = 1\)) is a locally stable low-income steady state. This establishes the locally stable poverty trap that we have to escape from. It is clear that \(\lambda_{t+1} = 1\) for all \(\lambda_t \in [1, \lambda^S]\), and that \(\lambda_{t+1} = zh(1)\lambda_t + 1\) for all \(\lambda_t \geq \lambda^a\). In the interval \(\lambda_t \in (\lambda^S, \lambda^a)\), the model allows for different patterns of curvature of the trajectory \(\lambda_{t+1}(\lambda_t)\), which can cause the existence of multiple equilibria. In this area, we have

\[
\lambda_{t+1}(\lambda_t) = zh(e^*_{t}(\lambda_t))\lambda_t + 1. \tag{2.8}
\]

We obtain\(^{19}\)

\[
\lambda''_{t+1}(\lambda_t) = zh(e') \left[ 2 + \left( \frac{h''(e')}{h'} e'' \right) \right] \tag{2.9}
\]

where we use the following abbreviations: \(h' \equiv h'(e_t), h'' \equiv h''(e_t), e' \equiv e'(\lambda_t), e'' \equiv e''(\lambda_t)\). Assuming homothetic preferences (in the interval \(\lambda_t \in (\lambda^S, \lambda^a)\)) the level of income does not influence the first derivatives of the Marshallians: \(e'' = c'' \equiv 0\).

In this case, when additionally \(h(e)\) is concave in \(e\) (i.e. \(h'' < 0\)), term \(A\) is strictly negative. As long as \(A\) is bigger than \(-2\) (\(0 > A > -2\)), \(\lambda_{t+1}(\lambda_t)\) is strictly convex. If \(A < -2\), \(\lambda_{t+1}(\lambda_t)\) is strictly concave for all \(\lambda \in [\lambda^S, \lambda^a]\). Rearranging (2.9) and considering \(e''(\lambda_t) = 0\), one receives:

\[
\lambda''_{t+1}(\lambda_t) = zh(e') \left( 2 + \varepsilon_{h',e}(\lambda) \cdot \eta_{e,\lambda}(\lambda) \right) \tag{2.10}
\]

with the elasticities \(\varepsilon_{h',e}(\lambda) \equiv \frac{\partial h'(e(\lambda_t))}{\partial e_t} \frac{e(\lambda_t)}{h'(e(\lambda_t))}\) and \(\eta_{e,\lambda}(\lambda) \equiv \frac{\partial e(\lambda_t)}{\partial \lambda_t} \cdot \frac{\lambda_t}{e(\lambda_t)}\). Since the child’s education time is a non-inferior good, the income elasticity has to be positive;

\(^{19}\)See also Bell and Gersbach (2001), p. 11.
hence it is clear that also \( \eta_{e,\lambda}(\lambda) > 0 \). The interesting case is \( \varepsilon_{h',e}(\lambda) < 0 \), which requires that \( h(e) \) is concave.

**Proposition 2.1**
Suppose preferences are homothetic and \( \eta_{e,\lambda}(\lambda) > 0 \).

(a) If \( h(e) \) is convex \( (\varepsilon_{h',e}(\lambda) > 0) \), \( \lambda_{t+1}(\lambda_t) \) is strictly convex in the interval \([\lambda^S, \lambda^a]\).

(b) If \( h(e) \) is concave \( (\varepsilon_{h',e}(\lambda) < 0) \), there are four patterns to consider:

1. If \( 2 > - \left[ \varepsilon_{h',e}(\lambda) \cdot \eta_{e,\lambda}(\lambda) \right] \) for all \( \lambda_t \in [\lambda^S, \lambda^a] \) then \( \lambda_{t+1}(\lambda_t) \) is strictly convex in the interval \([\lambda^S, \lambda^a]\).
2. If \( 2 < - \left[ \varepsilon_{h',e}(\lambda) \cdot \eta_{e,\lambda}(\lambda) \right] \) for all \( \lambda_t \in [\lambda^S, \lambda^a] \) then \( \lambda_{t+1}(\lambda_t) \) is strictly concave in the interval \([\lambda^S, \lambda^a]\).
3. If \( 2 > - \left[ \varepsilon_{h',e}(\lambda) \cdot \eta_{e,\lambda}(\lambda) \right] \) for some interval \([\lambda^S, \lambda^S + \epsilon]\), \( \epsilon > 0 \), and \( 2 \leq - \left[ \varepsilon_{h',e}(\lambda) \cdot \eta_{e,\lambda}(\lambda) \right] \) for \( \lambda_t \in (\lambda^S + \epsilon, \lambda^a) \), then \( \lambda_{t+1}(\lambda_t) \) is, first, in the interval \([\lambda^S, \lambda^S + \epsilon]\), convex, but then concave in the interval \((\lambda^S + \epsilon, \lambda^a)\).

4. If \( 2 < - \left[ \varepsilon_{h',e}(\lambda) \cdot \eta_{e,\lambda}(\lambda) \right] \) for some interval \([\lambda^S, \lambda^S + \epsilon]\), \( \epsilon > 0 \), and \( 2 \geq - \left[ \varepsilon_{h',e}(\lambda) \cdot \eta_{e,\lambda}(\lambda) \right] \) for \( \lambda_t \in (\lambda^S + \epsilon, \lambda^a) \), then \( \lambda_{t+1}(\lambda_t) \) is, first, in the interval \([\lambda^S, \lambda^S + \epsilon]\), concave, but then convex in the interval \((\lambda^S + \epsilon, \lambda^a)\).

The patterns are depicted in the figures 2.3 to 2.5. In general, one cannot exclude any case. However, a plausible assumption is diminishing marginal returns of schooling, i.e., that \( h(e) \) is strictly concave. In this case, there would be an initial concave part of the trajectory for the following reason. Suppose \( \lambda_t \) is small and close to \( \lambda^S > 1 \). Then the optimal education time \( e^*_t \) is close to zero. Additionally, homothetic preferences imply that \( e'(\lambda_t) \) is constant. Hence, for small \( \lambda_t \), \( \eta_{e,\lambda}(\lambda_t) \) tends towards infinity. Due to diminishing marginal returns of schooling, we have alternating signs of the derivatives of \( h(e) \): \( h' > 0 \), \( h'' < 0 \), \( h''' > 0 \), and so on. For \( e_t \rightarrow 0 \), \( h'(e) \) will be very high and \( h''(e) \) will be strongly negative. Hence, it should hold that for small \( \lambda_t \) we initially have a concave shape of the trajectory, because \( \eta_{e,\lambda} \rightarrow \infty \) and \( \varepsilon_{h',e}(\lambda) < 0 \). For the same reason, case Proposition 2.1 (b) 3. is unlikely and we do not consider it hereafter.\(^{20,21}\)

In the area \( \lambda_t \geq \lambda^a \), this trajectory has a kink and becomes a line, that has its axis intercept at \( \lambda_{t+1} = 1 \) and a slope of \( zh(1) \). The size of this slope decides upon whether our growth model provides long-term growth. In case of \( z h(1) \geq 1 \), the line never

\(^{20}\)Nonetheless, Emerson and Souza (2000) and Dessy (2000) suppose that \( \lambda_{t+1} \) is initially convex and then concave.

\(^{21}\)We neglect the analysis of the more complex case where \( e'' \neq 0 \), since this does not bear more insights.
intersects the 45°-line for any \( \lambda_t \geq 1 \), and the level of human capital grows for all time once \( \lambda_t \geq \lambda^a \) (growth case). Then, human capital and household income grow (asymptotically) at rate \( zh(1) - 1 \), when \( \lambda_t \) tends to infinity.\(^{22}\) In the case of \( zh(1) < 1 \), the line will intersect the 45°-line at level \( \lambda = \frac{1}{1-zh(1)} \) in the interval \( (\lambda^a, \infty) \), and hence there is a locally stable high-income steady state in this interval, where the levels of human capital and income do not grow (no-growth case). Note that the first case corresponds with AK models, while the second is related to neoclassical\(^{23}\) growth models.

To deduce which steady states\(^{24}\) the elaborated patterns of curvature cause, we follow a graphical analysis.

**Remark 2.1**

\( zh(1) \geq 1 \) forces \( \lambda^a \) to be strictly higher than any possible stationary state \( (\lambda_{t+1} = \lambda_t) \), since, then, \( zh(1)\lambda^a + 1 > \lambda^a \) is always true.

**Remark 2.2**

The line \( zh(1)\lambda_t + 1 \) establishes an upper bound for all potential trajectories, i.e., no admissible trajectory crosses this line.

In our analyses in this thesis, we will mostly refer to Figure 2.3: in the interval \([\lambda^S, \lambda^a]\), the trajectory is convex and there is only the poverty trap at \( \lambda = 1 \) and an intermediate, unstable steady state at some \( \lambda^* \). The horizontal line in the interval \([1, \lambda^S]\) represents the area where the human capital is too low to achieve any education \( (e^o = 0): \lambda_{t+1} = zh(0) + 1 = 1 \). Then, for \( \lambda_t > \lambda^S \), education is achieved and increases in \( \lambda_t \), whereby \( \lambda_{t+1} \) increases. For all \( \lambda_t \geq \lambda^a \) the trajectory becomes a linear function with slope \( zh(1) \). Figure 2.3, case (a), for instance, depicts the situation where there is long-term growth, Figure 2.3, case (b) a situation where there is a high-income steady state at \( \lambda^{**} \).

As long as the trajectory runs under the 45°-line we have negative growth \( (\lambda_{t+1} < \lambda_t) \), and above the 45°-line we have positive growth. Hence, the poverty trap is locally stable in the interval \([1, \lambda^*]\). To escape the suction of the poverty trap, a household needs human capital strictly higher than \( \lambda^* \). It is clear that if \( zh(1) < 1 \) and \( \lambda^a \) displays the characteristic \( zh(1)\lambda^a + 1 < \lambda^a \), there is only one steady state, namely the poverty trap, since we have negative human capital growth for all levels of \( \lambda_t > 1 \). Then, the

\(^{22}\)Note that \( zh(1) = 1 \) can also establish long-term growth if \( zh(1)\lambda^a + 1 > \lambda^a \), but in that case the long-term growth rate will converge to zero as \( \lambda_t \) tends to infinity (because the constant term \( \lambda_{t+1} - \lambda_t \equiv 1 \) will be divided by a term that tends to infinity).

\(^{23}\)Cf. Solow (1956) and Swan (1956).

\(^{24}\)Note that a stationary state of human capital implies a corresponding stationary state of household income and therefore a steady state.
only way to escape poverty is to improve the technology of human capital.\textsuperscript{25} As the school quality initially is given exogenously, we assume:

Assumption 2.5
\[ zh(1)\lambda^a + 1 \geq \lambda^a \]

The case where the trajectory is strictly concave in the interval \([\lambda^S, \lambda^a]\) is depicted in Figure 2.4. Other possible scenarios are demonstrated by Figure 2.5. If one counts over the steady states, beginning with the poverty trap, each steady state with an even number is unstable and the odd numbers are (locally) stable. \(\lambda^*\) in Figure 2.5 (f) is a special case as this steady state occurs at a tangent point of the trajectory with the 45\(^\circ\)-line, wherefore this \(\lambda^*\) is locally unstable on the left-hand side, but locally stable on the right-hand side.\textsuperscript{26} How many steady states are in the interval \((\lambda^S, \lambda^a)\) and what the curvature of the trajectory looks like in this interval is not important for our policy analysis. What is important is that the following facts hold in any case:

1. At human capital level \(\lambda = 1\) there is a poverty trap that is locally stable in the interval \([1, \lambda^*]\), where \(\lambda^*\) represents the next steady state on the right of the poverty trap.

2. A household chooses full-time schooling as soon as \(\lambda_t \geq \lambda^a\), which, given Assumption 2.5, definitively leads to the escape from the poverty trap.

3. Condition \(zh(1) \geq 1\) in combination with Assumption 2.5, ensures long-term growth of human capital and household’s income if \(\lambda_t \geq \lambda^a\).

4. Condition \(zh(1) < 1\) in combination with Assumption 2.5, in contrast, means that, as soon as \(\lambda_t \geq \lambda^a\), the household will end up in the high-skill steady state at \(\lambda = \frac{1}{1-zh(1)}\), which is locally stable.

Accordingly, only a big shock leads to the transition from the poverty trap to long-term growth or the high-skill steady state, and once this transition has occurred, only big shocks can throw the household back to the poverty trap.

Finally, we establish some convention of termini used in this thesis. When the escape from the poverty trap is henceforth discussed, we mean that long-term growth (case \(zh(1) \geq 1\)) or the highest possible steady state level of human capital (case \(zh(1) < 1\)) is reached for all households of the society. A weaker alternative form is that we only

\textsuperscript{25}In Chapter 6, we will relinquish Assumption 2.5 and emphasize the importance of the school quality.

\textsuperscript{26}In this case, our rule referring to “odd” and “even” does not hold.
demand that at least some education occurs in each household in a non-fully backward steady state. In particular, referring to Figure 2.3, the escape from the poverty trap will only occur if all households are endowed with human capital strictly higher than $\lambda^*$. This is exactly the policy problem to solve. Notice that the assumptions imply $\lambda^a > \lambda^*$.  

$\lambda^a$ implies $e_t = 1$, and $h(1) \geq 1$ implies $h(1)\lambda^a + 1 > \lambda^a$.

$^27$This is because $\lambda^a$ implies $e_t = 1$, and $h(1) \geq 1$ implies $h(1)\lambda^a + 1 > \lambda^a$. 
Figure 2.3: Human capital technology in the case of (a) $zh(1) \geq 1$ and (b) $zh(1) < 1$ when $h''(e) > 0$. 
Figure 2.4: Human capital technology in the case of (c) $zh(1) \geq 1$ and (d) $zh(1) < 1$ when trajectory is concave for all $\lambda_t \in [\lambda^S, \lambda^a)$. 
Figure 2.5: Human capital technology in the case of (e) $zh(1) \geq 1$ and (f) $zh(1) < 1$ when the trajectory displays a turning point from concave to convex in the interval $[\lambda^S, \lambda^a]$. 
Chapter 3

Social Optimum and an Inter-Generational Externality

In this chapter we elaborate on the socially optimal allocation. We demonstrate that there exists an inter-generational externality of schooling that is not internalized by the decentralized solution. Accordingly, the decentralized solution is not efficient. We conclude that state intervention can not only be justified by aiming at overcoming the poverty trap but also by internalizing this externality. Furthermore, we show that the education of a society mitigates the negative effects of imperfect capital markets in underdeveloped countries.

3.1 The Socially Optimal Choice of Education

In the social optimum, social welfare is maximized. Determining social welfare finally remains subjective. Accordingly, what follows is a normative approach to measure social welfare. In our analysis, welfare is based on individual preferences. Therefore, we specify adults’ preferences more closely. For simplicity, suppose (within this chapter) that preferences can be described by a utility function. Following our basic model we assume that an adult’s preferences are determined both by the size of her consumption and the child’s schooling attainment. We assume that these preferences can be described by a concave function \( u(c_t, e_t) \) in \( c_t \) and \( e_t \). (The choice of this functional form is motivated and discussed in Appendix A.)

Let us assume society in period \( t = 0 \) is imprisoned in the poverty trap: \( \lambda_{i0} = 1 \) for all \( i \in [0, 1] \). A transition of all households from the poverty trap at \( \lambda = 1 \) to a level of human capital beyond \( \lambda^* \) will increase the level of consumption of all generations in the aftermath of this transition (due to rising incomes, consider Figure 2.3). Therefore,
overcoming the poverty trap is definitely part of the optimal path. Such a transition requires educating the society, i.e., the children have to attend school \((e_t > 0, t \geq 0)\). The question is, which specific allocation scheme \((e_t, c_t)\) is socially optimal in each period \(t\)? As we want to analyze policies for overcoming poverty traps by education, we are especially interested in the optimal level of schooling in the early periods, in which the social planner faces the problem of backwardness.

To find the socially optimal path of consumption \(c_t\) and education-time \(e_t\), one could search for the optimum that prevails in a perfect Arrow-Debreu world, which is characterized by spot and future markets for each single good and period. That is, there are perfect capital and insurance markets, for instance. This approach does not provide a realistic reference world for developing economies. Hence, we apply an approach reflecting the constraint that adults in developing countries cannot consume more than their incomes. For simplicity, we neglect the utility of children and assume \(\beta = 0\). According to the pattern of our basic model, we therefore directly use \(c_t = \alpha [\lambda_t + (1 - e_t) \gamma]\) in \(u(e_t, c_t)\).

The welfare optimum is characterized by the maximization of the society’s intertemporal utility. Applying inter-generational analyses, one has to decide whether or not to discount the utility of future generations. Since the social planner has to decide today, he knows that at some point in time man will be vanished from earth. Hence, the probability that mankind exists on earth decreases from generation to generation. Accordingly, we apply the discount factor \(\rho \in [0, 1)\) to take account of this fact. Let the welfare function be denoted by \(W\) and additively separable, so that welfare is measured by the sum of the discounted instantaneous utility functions \(u(c_t, e_t)\) (“felicity”), starting from period \(t = 0\) to \(t = \infty\):

\[
W(\{e_t\}_{t=0}^{\infty}) = \sum_{t=0}^{\infty} \rho^t u(\alpha [\lambda_t (e_{t-1}) + (1 - e_t) \gamma], e_t)
\] (3.1)

As all households are alike, we assume that all households of a single generation are treated equally by the social planner. In maximizing social welfare, the social planner chooses the path \(\{e_t\}_{t=0}^{\infty}\) subject to \(e_t \in [0, 1]\) for all \(t \in [0, \infty)\). With these preliminaries

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1Notice that \(\alpha \lambda_t\) represents the income throughout adulthood in our model. If all credit must be paid back until death, capital market access does not change the current value of this life-time income, and thus demand \((e_t, c_t)\) is not touched.

2Ramsey (1928), for instance, did not discount the utilities of future generations.

3Furthermore, discounting simplifies the analysis, because without it the welfare function could not converge.
settled, the Lagrangean to maximize by the social planner is given by:

\[
L \left( \{e_t, \kappa_t, \nu_t\}_{t=0}^\infty \right) = \sum_{t=0}^\infty \rho^t u \left( \alpha [\lambda_t(e_{t-1}) + (1 - e_t)\gamma], e_t \right) + \sum_{t=0}^\infty [(1 - e_t)\kappa_t] + \sum_{t=0}^\infty e_t\nu_t
\]

with the Lagrangean multipliers (or shadow prices) \(\kappa_t\) and \(\nu_t\), the technology of human capital \(\lambda_t(e_{t-1}) = h(e_{t-1})z\lambda_{t-1}(e_{t-2}) + 1\), \(\lambda_0 = 1\) and the “transversality condition” \(\lim_{t \to \infty} \lambda_t = 1\). The transversality condition demands that in the limit, when the very last period is reached, the value of the final stock of human capital is minimal, so that no consumption possibility foregoes. For \(\alpha\) being constant, this requires that human capital diminishes to its natural minimum, which is at \(\lambda = 1\), the inborn level of human capital of pure labor. We label the resulting optimal path of \(\{e_t\}_{t=0}^\infty\) socially optimal or \textit{first-best}.\(^4\) It is obvious that maximizing this Lagrangean only produces meaningful results if the Lagrangean converges. Therefore, we assume that the instantaneous utility function is bounded from above.\(^5\) Then, discounting guarantees that the sum of instantaneous utilities converges. To derive the socially optimal path of schooling, we deduce the corresponding Euler equation. Let \(V_t(\lambda_t)\) denote the value function of the Lagrangean (3.2):

\[
V_t(\lambda_t) = \max_{\{e_t\}} \left\{ \sum_{t'=t}^\infty \rho^{t'-t} u \left( \alpha [\lambda_{t'}(e_{t'-1}) + (1 - e_{t'})\gamma], e_{t'} \right) + \sum_{t'=t}^\infty [(1 - e_{t'})\kappa_{t'}] + \sum_{t'=t}^\infty e_{t'}\nu_{t'} \right\}
\]

s.t. \(\lambda_{t+1} = z\lambda_th(e_t) + 1\)

Deriving the corresponding Bellman equation, the dynamic problem reduces to:

\[
V_t(\lambda_t) = \max_{\{e_t\}} \left\{ u(\alpha[\lambda_t + (1 - e_t)\gamma], e_t) + [(1 - e_t)\kappa_t] + e_t\nu_t + \rho V_{t+1}(z\lambda_th(e_t) + 1) \right\}
\]

Differentiating with respect to \(e_t\), we arrive at the following Kuhn-Tucker first-order-

\(^4\)Of course, in a narrow understanding, we deduce a second-best social optimum. To distinguish the optimum of the Arrow-Debreu world from the social optimum derived here, we could label the latter \textit{super first-best}. Note that this approach is very common in economics, for instance, when “first-best” Pigou taxes on emissions are derived without endogenizing the number of firms.

\(^5\)For instance, the following instantaneous utility function is bounded from above at 2:

\[
u(c_t, e_t) = \left(1 - \frac{1}{c_t}\right)(1 + e_t)
\]
conditions for a global maximum:

\[ \frac{\partial V_t}{\partial e_t} = \frac{\partial u_t}{\partial e_t} - \frac{\partial u_t}{\partial c_t} \alpha \gamma - \kappa_t + \nu_t + \rho \frac{\partial V_{t+1}}{\partial \lambda_{t+1}} \lambda_t h'(e_t) \leq 0 \quad \text{with} \quad \frac{\partial V_t}{\partial e_t} \cdot e_t = 0 \quad (3.3) \]

\[ \frac{\partial V_t}{\partial \kappa_t} = 1 - e_t \geq 0 \quad \text{with} \quad \frac{\partial V_t}{\partial \kappa_t} \cdot \kappa_t = 0 \quad (3.4) \]

\[ \frac{\partial V_t}{\partial \nu_t} = e_t \geq 0 \quad \text{with} \quad \frac{\partial V_t}{\partial \nu_t} \cdot \nu_t = 0 \quad (3.5) \]

The marginal social benefit of education in period \( t \) for future generations is given by

\[ \rho \frac{\partial V_{t+1}}{\partial \lambda_{t+1}} z \lambda_t h'(e_t). \]

(One might get a better intuition of the functioning of the externality by looking at the “basic approach” elaborated on in Appendix A.) Thus there exists a positive externality of today’s education on the welfare of future generations. Moreover, if

\[ -\frac{\partial u_t}{\partial e_t} = \alpha \gamma \frac{\partial u_t}{\partial c_t} - \frac{\partial u_t}{\partial c_t} > 0 \]

represents today’s net investment cost of schooling. Let us now assume we knew solution \( e_t^{\ast} (\lambda_t) \), and hence \( e_t^{\ast} = \alpha [\lambda_t + (1 - e_t^{\ast} (\lambda_t)) \gamma] \). Then the Bellman equation changes to:

\[ V_t (\lambda_t) = u (\alpha [\lambda_t + (1 - e_t^{\ast} (\lambda_t)) \gamma], e_t^{\ast} (\lambda_t)) + \rho V_{t+1} (\lambda_{t+1}, e_t^{\ast} (\lambda_t)) \]

\[ + \rho V_{t+1} (\lambda_{t+1}, e_t^{\ast} (\lambda_t)) = 0 \quad \text{in optimum via (3.4) and (3.5)} \]

(3.6)

Let us, for a moment, assume \( e_t^{\ast} \in (0, 1) \). That is, in Condition (3.3) it must hold that \( \frac{\partial V_t}{\partial e_t} = 0 \) and \( \kappa_t = \nu_t = 0 \). Applying the envelope theorem we find:

\[ \frac{\partial V_t}{\partial \lambda_t} = \frac{\partial e_t^{\ast}}{\partial \lambda_t} \left( \frac{\partial u_t}{\partial e_t} - \alpha \gamma \frac{\partial u_t}{\partial c_t} + \rho \frac{\partial V_{t+1}}{\partial \lambda_{t+1}} \lambda_t h'(e_t) \right) + \alpha \frac{\partial u_t}{\partial c_t} + \rho \frac{\partial V_{t+1}}{\partial \lambda_{t+1}} z h(e_t) \]

\[ = 0 \quad \text{in optimum via (3.3)} \]

(3.7)

That is, in optimum, the marginal value of the stock of human capital in period \( t \) equals its marginal social welfare: one additional unit \( \lambda_t \) allows additional consumption of \( \alpha \), which produces additional welfare of \( \alpha \frac{\partial u_t}{\partial c_t} \); moreover, another unit of \( \lambda_t \) generates additional tomorrow’s human capital \( \lambda_{t+1} \) amounting to \( z h(e_t) \), which increases welfare by \( \rho \frac{\partial V_{t+1}}{\partial \lambda_{t+1}} z h(e_t) \). Via (3.3) we know that \( \rho z \frac{\partial V_{t+1}}{\partial \lambda_{t+1}} = \frac{1}{\lambda_t h'(e_t)} \left( \alpha \gamma \frac{\partial u_t}{\partial c_t} - \frac{\partial u_t}{\partial e_t} \right) \). Substituting this into (3.7), we obtain:

\[ \frac{\partial V_t}{\partial \lambda_t} = \alpha \frac{\partial u_t}{\partial e_t} + h(e_t) \left( \frac{\partial u_t}{\partial c_t} - \frac{\partial u_t}{\partial e_t} \right) \]

\[ = \frac{\partial V_t}{\partial \lambda_t} \left( \alpha \gamma \frac{\partial u_t}{\partial c_t} - \frac{\partial u_t}{\partial e_t} \right) \]

(3.8)

The first term on the right-hand-side is standard and describes the positive effect of schooling on the budget constraint \( c_t = \alpha (\lambda_t + \gamma) - \alpha \gamma e_t \). Additionally, we receive a term that expresses the utility change of increasing the marginal productivity of schooling via \( \lambda_{t+1} = 1 + z h(e_t) \lambda_t \), caused by schooling. Hence, we are able to identify

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\(^6\text{Cf. Chiang (1984), pp. 724-29.\)}}
two channels that produce marginal social benefits. Finally, using (3.8) in (3.3) one arrives at the Euler equation:

\[
\alpha \gamma \frac{\partial u_t}{\partial c_t} - \frac{\partial u_t}{\partial e_t} = \rho z \lambda_t h'(e_t) \left[ \alpha \frac{\partial u_{t+1}}{\partial c_{t+1}} + \frac{h(e_{t+1})}{\lambda_{t+1}h'(e_{t+1})} \left( \alpha \gamma \frac{\partial u_{t+1}}{\partial c_{t+1}} - \frac{\partial u_{t+1}}{\partial e_{t+1}} \right) \right]
\]

By rearrangements one finds the following condition:

\[
\frac{\alpha \gamma \frac{\partial u_t}{\partial c_t} - \frac{\partial u_t}{\partial e_t}}{\rho \left( \alpha \gamma \frac{\partial u_{t+1}}{\partial c_{t+1}} - \frac{\partial u_{t+1}}{\partial e_{t+1}} \right)} = z \lambda_t h'(e_t) \left( \frac{\alpha \gamma \frac{\partial u_{t+1}}{\partial c_{t+1}} - \frac{\partial u_{t+1}}{\partial e_{t+1}}}{\alpha \gamma \frac{\partial u_{t+1}}{\partial c_{t+1}} - \frac{\partial u_{t+1}}{\partial e_{t+1}}} \right) + zh(e_{t+1}) \frac{\lambda_t h'(e_t)}{\lambda_{t+1}h'(e_{t+1})}
\]

The term on the left-hand-side is the marginal rate of substitution (MRS) between today’s and tomorrow’s schooling (see Appendix A): \( \frac{de_{t+1}}{dc_t} \mid_{dW=0} = \frac{du_{t+1}}{du_t} \mid_{dW=0} \). The first term on the right-hand-side expresses the marginal rate of transformation (MRT) \( \frac{de_{t+1}}{dc_t} \mid_{dW=0} \) concerning the improvement of the marginal productivity of school investment (see Appendix A):

\[
-de_{t+1} = \left( \frac{\partial u_{t+1}}{\partial c_{t+1}} \right) \frac{\partial c_{t+1}}{\partial \lambda_{t+1}} \frac{\partial \lambda_{t+1}}{\partial c_t} \frac{\partial c_t}{\partial e_t}
\]

The second term is the marginal rate of transformation \( -\frac{de_{t+1}}{dc_t} \mid_{d(\lambda_{t+2})=0} \) of the technology of human capital (see Appendix A):

\[
\frac{\partial \lambda_{t+2}}{\partial \lambda_{t+1}} \left( \frac{\partial \lambda_{t+1}}{\partial e_t} \right) \left( \frac{\partial e_t}{\partial c_t} \right)
\]

Therefore, we obtain the typical condition of social optimum that the marginal rate of substitution has to equal the marginal rate of transformation. Applying \( -de_{t+1} = \frac{de_{t+1}}{\alpha \gamma} \), it follows that multiplying the marginal rates in (3.10) by \( \alpha \gamma \) generates derivative \( \frac{de_{t+1}}{dc_t} \), which tells us how much additional consumption of the society the social planner demands for being indifferent (MRS) and how much additional consumption society actually enjoys tomorrow (MRT), if the social planner invests another unit of the child’s time in schooling (given the technology of human capital, the budget constraint, the starting level of human capital, and the transversality condition). If for all \( e_t \in [0, 1] \) the marginal rate of substitution is at least as high as the marginal rate of transformation, then it is socially efficient to choose fulltime child labor: \( e_t^{\alpha} = 0 \) and \( e_t^{\gamma} = \alpha(\lambda_t + \gamma) \). In this case we obtain \( \nu_t \in [0, \alpha \gamma \frac{\partial u_t}{\partial e_t} - \frac{\partial u_t}{\partial e_t} - \rho z \lambda_t h'(e_t) \frac{\partial \lambda_{t+1}}{\partial \lambda_{t+1}}] \). That is, the utility loss of any level of education today outweighs all welfare gains. If, in contrast, the

---

7 The marginal rate of substitution between \( e_{t+1} \) and \( e_t \) transforms the additional consumption \( c_{t+1} \) into units of \( e_{t+1} \).
investment in human capital is remunerative for all levels of \( e_t \in [0, 1] \), i.e. the marginal rate of substitution demands always less than the marginal rate of transformation expresses, then it is socially optimal to choose fulltime schooling: \( e_t = 1 \) and \( c_t^* = \alpha \lambda_t \).

Furthermore, we obtain \( \kappa_t = \frac{\partial u_t}{\partial e_t} - \alpha \gamma \frac{\partial u_t}{\partial c_t} + \rho z \lambda_t h'(e_t) \frac{\partial V_{t+1}}{\partial \lambda_{t+1}} \), that is, the shadow price of an additional unit of time for the child equals in optimum the difference between the future revenue of this unit invested in the child’s education and the utility loss due to this investment. As time \( t \) tends to infinity, investing in education becomes inefficient, since in the very last period the efficient level of human capital is its natural minimum: \( \lim_{t \to \infty} \lambda_t = 1 \). If the social planner chooses \( e_t = 0 \) in \( t = \infty \), then \( \lim_{t \to \infty} \lambda_{t+1} = 1 \) is fulfilled for all levels of \( \lambda_t \) and thus socially efficient. Choosing \( e_t = 0 \) earlier in time would cause an unnecessary and hence inefficient reduction of consumption in all future periods.

Finally, the first-order conditions are sufficient for a maximum, if the functions \( u(c_t, e_t) \) and \( h(e_t) \) are concave with regard to \( e_t \) and \( \lambda_t \). Therefore, we assume that \( \frac{\partial^2 h(e_t)}{\partial e_t^2} \leq 0 \). Given \( \lambda_0 = 1 \) and \( \lim_{t \to \infty} \lambda_t = 1 \), one can, at least theoretically, find the socially optimal path of \( e_t \) recursively by applying specific functions for \( u(c_t, e_t) \) and \( h(e_t) \) (that fulfill the stated assumptions) in condition (3.10). However, it is uncertain whether solving this exercise is possible.

### 3.2 Laissez-faire

We now demonstrate how the laissez-faire solution, that is the allocation that prevails when the representative adult freely decides about education \( e_t \) (decentralized solution), differs from welfare optimum. In each period \( t \), the representative adult solves the following Lagrangean:

\[
L(e_t) = u(\alpha [\lambda_t + (1 - e_t) \gamma], e_t) + (1 - e_t) \tilde{\kappa}_t + e_t \tilde{\nu}_t \quad (3.11)
\]

The Kuhn-Tucker conditions are given by:

\[
\frac{\partial L_t}{\partial e_t} = \frac{\partial u_t}{\partial e_t} - \alpha \gamma \frac{\partial u_t}{\partial c_t} - \tilde{\kappa}_t + \tilde{\nu}_t \leq 0 \quad \text{with} \quad \frac{\partial L_t}{\partial e_t} \cdot e_t = 0 
\]

\[
\frac{\partial L_t}{\partial \tilde{\kappa}_t} = 1 - e_t \geq 0 \quad \text{with} \quad \frac{\partial L_t}{\partial \tilde{\kappa}_t} \cdot \tilde{\kappa}_t = 0 
\]

\[
\frac{\partial L_t}{\partial \tilde{\nu}_t} = e_t \geq 0 \quad \text{with} \quad \frac{\partial L_t}{\partial \tilde{\nu}_t} \cdot \tilde{\nu}_t = 0 
\]

\[\text{8} \text{ARROW AND KURZ (1970) state a theorem that shows that concavity of the objective function and the functions of the constraints is sufficient but not necessary. Cf. BARRO AND SALA-I-MARTIN (1998), appendix, section 3.6, p. 586. Further literature on dynamic optimization is, e.g., BLANCHARD AND FISCHER (1993), SARGENT (1987), and STOKEY, LUCAS, AND PRESCOTT (1989).}\]
Applying Assumption 2.2 of Chapter 2, it is clear that \((e_t, c_t) = (0, \tau(\lambda_t))\) as long as \(\tau_t \in [0, c^S]\). I.e., in this interval, the loss of marginal utility due to education (loss of consumption) is higher than marginal utility of education: \(\frac{\partial u_t}{\partial e_t} < \alpha \gamma \frac{\partial u_t}{\partial c_t}\). Consequently, we have \(e_t = 0\), \(\tilde{\kappa}_t = 0\), and \(\tilde{\nu}_t > 0\), as long as the adults are endowed with \(\lambda_t \leq \lambda^S\).

If at locus \((0, \tau(\lambda_t))\) we have \(\frac{\partial u_t}{\partial e_t} \geq \alpha \gamma \frac{\partial u_t}{\partial c_t}\), then the optimum involves \(e^o_t > 0\), \(\tilde{\nu}_t = 0\) and the following condition holds:

\[
\frac{\partial u_t}{\partial e_t} = \alpha \gamma \frac{\partial u_t}{\partial c_t} + \tilde{\kappa}_t \tag{3.15}
\]

As long as \(e^o_t \in (0, 1)\), \(\tilde{\kappa}_t = 0\) and the marginal utility loss due to education is equal to its marginal utility gain: \(\frac{\partial u_t}{\partial e_t} = \alpha \gamma \frac{\partial u_t}{\partial c_t}\). If \(\frac{\partial u_t}{\partial e_t} \geq \alpha \gamma \frac{\partial u_t}{\partial c_t}\) at \(e_t = 1\), we arrive at \(\tilde{\kappa}_t = \frac{\partial u_t}{\partial e_t} - \alpha \gamma \frac{\partial u_t}{\partial c_t} \geq 0\) and \(e^o_t = 1\).

Comparing the conditions of the social optimum (Equation (3.3)-(3.5)) to the laissez-faire conditions (Equation (3.12)-(3.14)), we see that the two scenarios will almost surely differ. This suggests that the decentralized solution is not socially efficient. The reason for the socially inefficient decentralized solution is that the positive externality \(\rho\frac{\partial V_{t+1}}{\partial \lambda_{t+1}}z\lambda_t h'(e_t)\) is not internalized. Hence, parents do not choose enough education for their children. However, one exception has to be mentioned: if the individually optimal demand \(e^o_t\) equals one, then, due to Assumption 2.5, there is \(e^o_t = 1\) for all \(t \geq 0\).

If this choice is individually optimal, then this also has to be socially optimal, because \(\rho \frac{\partial V_{t+1}}{\partial \lambda_{t+1}}z\lambda_t h'(e_t) \geq 0\) and \(e_t \leq 1\). It follows that in this special case the decentralized allocation is equal to the socially optimal one – nonetheless \(\kappa_t \geq \tilde{\kappa}_t\) holds for \(t\), because of \(\rho \frac{\partial V_{t+1}}{\partial \lambda_{t+1}}z\lambda_t h'(e_t) \geq 0\). But this will only be the case if \(\lambda_0 \geq \lambda^a\). However, the poor, uneducated households imprisoned in the poverty trap that we address, are endowed with \(\lambda_0 = 1\) and choose \(e_t = 0\), which is socially inefficient, since, given \(t < \infty\), \(\rho \frac{\partial V_{t+1}}{\partial \lambda_{t+1}}z\lambda_t h'(e_t) > 0\) should hold. Consequently, there is a legitimate reason for intervention.

### 3.3 Market Failure and State Intervention

Adults in underdeveloped countries are uneducated and poor, and hence choose no education for their children. However, education provides a positive externality to future

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9. Another, meaningless exception is the case when \(dV_{t+1}/de_t = 0\) for all \(t\).

10. Prolonging the time horizon of the adult will lead to more education, since the positive effects on the descendants that are covered by the adult’s time horizon increase the incentive to provide more education. In the dynastic model, the externality is internalized, but the utility of future generations is most likely discounted at another rate than the social planner does. Hence, the resulting allocation is still not socially efficient.
generations that ought\(^{11}\) to be internalized, it mitigates the negative effects of imperfect capital markets by increasing the household’s income, and is required in order to escape the poverty trap. A natural idea is to compensate today’s generations for financing human capital formation such that they provide the socially efficient level of schooling and to shift the burden of financing these compensations to future generations. I.e., the children and future generations that benefit from the externality have to pay back loans that were necessary to finance this compensation today. However, underdeveloped countries are often highly indebted, so that these economies have to rely on foreign aid. Perfect capital markets would even allow children to borrow against their future earnings, i.e. they even would not require their parents to finance education. However, as we will explain in the next chapter in more detail, capital markets (in developing countries) are imperfect. Moreover, it is an illusion to believe that state intervention could reform capital markets in developing countries so that parents or even children can borrow against future earnings, because even in developed countries this is not possible. Hence, the governments of underdeveloped countries have to intervene and to finance the compensation by taxes or by foreign aid from abroad.

In this thesis, we will elaborate on which means governments can use to produce the socially optimal level of education. The basic idea is that the marginal cost of education, \(\alpha \gamma \frac{\partial u}{\partial c} \), has to be lowered. This happens, for instance, by decreasing the marginal utility of consumption via increasing the consumption level by transfers (income effect). Alternatively, the net costs per unit of education can be lowered by paying subsidies per unit of time the child spends in school (substitution effect). It is clear that if governments can command households to establish the socially optimal level of education, for instance by a compulsory schooling law, then households will choose the socially efficient allocation. However, as we will see, compulsory schooling does in many cases not work in practice. Therefore, if compulsory schooling cannot be enforced, we have to apply other policy tools. In Part I of the thesis, we analyze how particular subsidy and tax-and-subsidy schemes must be applied to achieve the socially optimal level of education. Part II then demonstrates how land transfers within the framework of a land reform can also realize the objective of socially optimal education.

\(^{11}\)As mentioned above, welfare analyses are normative. Our approach bases on some kind of intergenerational public spirit, but one might ask why current generations should care so strongly for future generations. Finally that is a question of conviction and remains subjective.
Part I

Educational Subsidies to Overcome Poverty
Chapter 4

Human Capital Subsidies

4.1 Introduction

There is evidence for the result of Chapter 3 that households under-invest in education [see, for instance, Psacharopoulos (1994) or Tilak (1989)]. Psacharopoulos found that the potential marginal return of education exceeds the marginal cost in real world, wherefore the low stocks of human capital in developing economies represent inefficient under-investment in education, i.e. a waste of valuable human resources; the marginal return on education is much higher than it is on physical capital. Young people, especially children, or altruistic parents who are poor, cannot borrow money for educational purposes on the capital market because potential creditors face the problem of asymmetric information about the ability of the children. This is in particular true for developing economies, which are in the focus of this work. Additionally, parents face the moral hazard problem that they do not know whether their children will pay back informal credit for their old age. Potential solutions by contracts cannot be enforced in real world.\(^1\) Hence, there is not just socially inefficient under-investment in human capital due to an inter-generational externality, but inefficient under-investment arises also as a result of the combination of imperfect capital markets and poverty. Subsidization of poor families can be helpful in educating a society and promoting long-run growth [cf. Bell and Gersbach (2001); Jafarey and Lahiri (2000)].\(^2\) In this chapter, we examine the optimal design of such subsidization, under the premise that this policy is realizable in practice.

\(^1\)That is the reason why Baland and Robinson (2000) found that child labor is inefficiently high when bequests are zero. Parents would educate their child efficiently if bequests could be negative. Hazan and Berdugo (2002), Ranjan (2001) and Baland and Robinson (2000) discuss intergenerational contract enforcement problems in the context of credit constraints.

\(^2\)Acemoglu and Pischke (2001) recently found via a novel identification strategy large, robust income elasticities of the college enrollment decision in the USA.
Chapter 4. Human Capital Subsidies

Obviously the optimal design of subsidization in the combat against poverty involves a bundle of problems in real world situations. Besides the issue of how to finance subsidies, there exist three main areas of subsidization problems. First, the *targeting issue* must be solved. One has to identify the socio-economic indicators of the poor, their typical behavior and the like, to decide who qualifies for transfers and which regions should be supported. Second, after solving the targeting problem, the question of how subsidies should be designed need to be addressed, i.e. which *kind of subsidization* should be used to reach the aim efficiently. Third, once that decided who and how should be subsidized, institutional problems must be faced. One major problem is to ensure that especially the poorest, the main target, are actually reached by subsidies [see, e.g., Schubert and Balzer (1990)]. For instance, *inefficient institutional handling* and *corruption* diverts a large part of resources to others than targeted.

In this chapter, we abstract from targeting or institutional problems and concentrate upon the best subsidy policy after targeting has taken place. In Chapter 6, we will, beside other aspects, incorporate the issue of corruption. The empirical work of Evangelista de Carvalho Filho (2001) demonstrate that “cash redistribution policies may indeed generate desirable consequences beyond simply increasing consumption of the poor”, i.e., they could curb the incidence of child labor and increase the level of education. It is, however, stressed that significant improvements will be very costly (at least for the analyzed country Brazil). That is, the cost-effectiveness of the used instruments is very crucial. In real world, a bundle of different subsidy methods are practiced, but there is no theoretical work highlighting the issue of the design of such education subsidization yet. Furthermore, beyond a theoretical point of view, it is important to address the question whether the theoretically best instrument is realizable in practice.

Hence we begin by considering three subsidy types that we expect to be the candidates of a government with the highest probability to be chosen in practice: the simple lump-sum subsidy and two forms of *conditional* subsidization types. Conditional subsidies are only paid, if a particular school attendance requirement is fulfilled. We believe that in practice two types of conditional subsidies are of interest, namely *binary* and *continuous* conditional subsidies of the following design. Binary subsidization is characterized by

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3 See, for instance, Skoufias, Davis, and de la Vega (2001), Young (1995a) or Schubert and Balzer (1990) for descriptions of this kind of targeting issues.


5 The UN project “Food for Education” in Bangladesh is an example for such a conditional subsidy. Such a policy is also practiced in the Mexican programas de Educacion, Salud y Alimentacion (PROGRESA program), see Skoufias, Davis, and de la Vega (2001) – also for a valuation of PROGRESA’s targeting methods – or Gomez de León, Parker, and Hernandez (2000).
the condition that a family will only receive a fixed transfer if a certain prescribed level of school attendance is established by the child; otherwise the family does not receive a transfer. In the continuous subsidization regime, a family receives a subsidy payment proportional to the child’s established school attendance, i.e., the higher the school attendance, the higher the subsidy will be. Having done this comparison we generalize our analysis and allow for other possibilities to investigate whether there are even better candidates.

The remainder of the chapter is organized as follows: In Section 4.2, the model is specified. In Section 4.3, the instruments to analyze – unconditional, binary and continuous conditional subsidies – are presented for the situations of stark and of non-stark poverty, respectively. Section 4.4 compares the considered instruments concerning cost-effectiveness (static analysis) and Section 4.5 regarding to the speed of the education process (dynamic analysis). In Section 4.6, a specific example is given and the comparative statics is elaborated. Section 4.7 then generalizes and deepens the basic theoretical idea and discusses alternative methods to achieve a targeted level of education (non-linear subsidy tariffs). The robustness of the results is checked concerning different kinds of preferences, respectively, altruism, excluded before (Section 4.8). In Section 4.9, we work out under which circumstances a society can be educated within only one single generation, and when this education is sufficient to escape poverty traps. Section 4.10 concludes with a summary of the results and their implications. Finally, we discuss subsidization in the context of development policy in general.6

4.2 The Model

Consider the basic model of Section 2. We extend the basic model by specifying the underlain parental preferences in more detail. We also simplify the complex dynamics. In doing so, we set $\beta = 0$ without loss of generality. Accordingly, we arrive at $\tau(\lambda_t) = \alpha(\lambda_t + \gamma)$, $g(\lambda_t) = \alpha \lambda_t$, $c^S = \tau(\lambda^S)$, and $c^a = g(\lambda^a)$.

4.2.1 The Household’s Behavior

As all households are alike, we described the behavior of a single adult representative for all. The household’s utility in period $t$, labeled $U_t$, is determined by consumption $c_t$ and by the time the child spends in school, $e_t$. Remind that poor have no access to capital markets and leave no bequests at death. Household’s income is given by

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6Siemers (2003) represents a brief paper version of this chapter.
\[ y_t = \alpha[\lambda_t + (1 - e_t)\gamma] \] and thus at least as high as \( \alpha \). We neglect any cost of education except the opportunity cost \( \alpha \gamma \), indicating that schooling is free. Thus, as already mentioned in Chapter 3, the household’s utility maximization problem to solve is:

\[
\max_{\{c_t, e_t\}} U_t \quad \text{s.t.} \quad c_t + \alpha \gamma e_t \leq \alpha(\lambda_t + \gamma), \tag{4.1}
\]

using \( c_t \) as the numéraire good. The solution of problem (4.1) is given by the household’s optimal demands, labeled \( e^o(\lambda_t) \) and \( c^o(\lambda_t) \). We assumed standard behavior:

\[
\frac{\Delta e^o(\lambda_t)}{\Delta \lambda_t} \geq 0, \quad \frac{\Delta c^o(\lambda_t)}{\Delta \lambda_t} \geq 0
\]

We stated that the household’s demand follows the following pattern:

\[
(e_t, c_t) = \begin{cases} 
(0, \alpha(\lambda_t + \gamma)) & \forall \lambda_t \leq \lambda^S; \\
(e^o(\alpha \lambda_t), c^o(\alpha \lambda_t)) & \forall \lambda_t \in (\lambda^S, \lambda^a); \\
(1, \alpha \lambda_t) & \forall \lambda_t \geq \lambda^a.
\end{cases} \tag{4.2}
\]

Therefore, it is important to realize that as long as \( \lambda_t \leq \lambda^S \), the household maximizes utility by maximizing consumption, i.e. by choosing \( e_t = 0 \). That is, as long as a household suffers stark poverty, consumption is preferred and education is not chosen at all,\(^7\) so that all additional income that fills the gap to the threshold income

\[ c^S \equiv \alpha(\lambda^S + \gamma) \tag{4.3} \]

will exclusively be used for further consumption \( c_t \). Hence, we assume that as long as \( \lambda_t \leq \lambda^S \), the adult’s preferences are lexicographic. However, this also means that as long as the consumption level is not lowered, increases in child’s education \( e_t \) will lead to utility gains, although \( c_t \) is always preferred in this low income area. Thus, the altruism of the adult is active but only secondary below the critical level of consumption given by \( c^S \), as poverty forces full-time child labor \( (e_t = 0) \). For, we can refer to \( c^S \) as a measure of the degree of altruism: the lower is \( c^S \), the higher is the degree of altruism. Accordingly, the preferences in the area \( c_t \leq c^S \) are described by:

\[
(e_t, c_t) \succeq (e'_t, c'_t) \quad \text{if} \quad c_t \geq c'_t \quad \text{or} \quad (c_t = c'_t \land e_t \geq e'_t)
\]

As such preferences are not continuous and its indifference sets are singletons, lexicographic preferences cannot be described by a utility function. Nonetheless, for \( c_t \leq c^S \) it is clear that the adult demands \( c_t = \overline{\gamma}(\lambda_t) \), which establishes iso-consumption lines

\(^7\)This corresponds with Basu and Van’s (1998) description of children’s leisure as being a luxury good. Stark poverty makes the marginal utility of consumption of \( c_t \) very high so that non-labor activities, including education, does not occur: the child income is essential for the survival of the entire family. See also Jafarey and Lahiri (2000).
in the \(c_t-e_t\)-space. Alternatively, one could assume Stone-Geary preferences. This has the advantage that it allows to express preferences by a preference function. However, Stone-Geary preferences state that if \(c_t \leq c^S\), then solely consumption \(c_t\) spends utility and increases of the child’s education time \(e_t\) does not spend utility even if the level of consumption is not changed. This is unrealistic. Additionally, it will turn out that the fact that lexicographic preferences cannot be expressed by a preference function does not matter for our analysis. Therefore, we decide to suppose lexicographic preferences.\(^8\)

For levels of consumption of \(c_t > c^S\), we can state:

\[
U_t = \begin{cases} 
    u(e_t^t, c_t^t) & \text{if } c_t \in (c^S, c^a); \\
    u(1, \alpha \lambda_t) & \text{if } c_t \geq c^a,
\end{cases}
\]

with \(c^a \equiv \alpha \lambda^a\). Regarding the utility function \(u(e_t, c_t)\), remind our assumption about positive but decreasing marginal utility, non-satiation, quasi-concavity and that indifference curves never intersect the horizontal line \(e_t = 0\) in the area \(c_t > c^S\).

In the following, we will see that it is important to distinguish the case \(c_t \leq c^S\) from the case \(c_t > c^S\). Hence, we call the case with \(\lambda_t \leq \lambda^S\), respectively \(c_t \leq c^S\), the case of stark poverty. Note that for \(e_t = 1\) the level of consumption is \(\alpha \lambda_t \geq \alpha\), so that the budget “line” has a kink at \(\alpha \lambda_t\): it starts at locus \((e = 1, c = 0)\) as a horizontal line until \((e_t = 1, c_t = \alpha \lambda_t)\) is reached, then there occurs a downward kink as \(e_t\) decreases (see Figure 4.1, for example). At the intersection point with the horizontal line at \(e_t = 0\), we obtain \(c_t = \alpha(\lambda_t + \gamma)\).

### 4.2.2 Dynamics

For simplicity, we set \(z = 1\) and use:

\[
\lambda_{t+1} = h(e_t) \lambda_t + 1
\]

Applying (4.2) to (4.5), we obtain:

\[
\lambda_{t+1} = \begin{cases} 
    1 & \forall \lambda_t \leq \lambda^S; \\
    h(c^o(\alpha \lambda_t)) \lambda_t + 1 & \forall \lambda_t \in (\lambda^S, \lambda^a); \\
    h(1) \lambda_t + 1 & \forall \lambda_t \geq \lambda^a.
\end{cases}
\]

We described the dynamics of the model, given the optimal household’s behavior \(e^o_t(\lambda_t)\), in Chapter 2. Let us in this chapter assume that \(h(\cdot)\lambda_t\) is strictly convex in \(\lambda_t\) for all

\(^8\)In the welfare analysis in the previous chapter we implicitly excluded lexicographic preferences by using a utility function. One can easily use Stone-Geary functions, for instance.
\( \lambda_t \in [\lambda^S, \lambda^a] \);\(^9\)\(^10\) note that this implies not necessarily \( h''(e_t) > 0 \). By construction, the shape of \( \lambda_{t+1} \) is linear for all \( \lambda_t > \lambda^a \). We assume the growth case where \( h(1) \geq 1 \).

Consider henceforth Figure 2.3: due to \( \lambda^S > 1 \), the state of backwardness, \( \lambda = 1 \) and \( e^o(\alpha) = 0 \), is a locally stable low-income steady state (poverty trap). Furthermore, the considered dynamic system has an unstable, second steady state at \( \lambda^* \) and \( e^o(\alpha \lambda^*) \).

For all \( \lambda_t > \lambda^* \) human capital and income grow for all time as \( h(1) \geq 1 \). As will turn out, the pattern of the dynamics is not decisive for our results.\(^11\)

### 4.3 Conditional and Unconditional Subsidies

In this and the following sections, we examine how an education policy via subsidization should be practiced to overcome poverty traps and attain growth. The idea of subsidizing households for reasons of education is simple: the payment of transfers increases a household’s budget and thereby the household’s consumption and – given some degree of altruism – education. But different kinds of subsidies induce this positive effect with a different cost-effectiveness. We denote the level of education the policy maker wants to achieve by \( k \). However, in general, subsidization can involve also negative transfers, that is, taxes. The general idea is hence to manipulate the budget set of a household such that the adult chooses the desired level of schooling.

We start our analysis, as already outlined in the introduction, by analyzing the three subsidy types that, as we believe, one would expect to be chosen most likely in practice. Within this context, one has to distinguish two cases: the first case is given when the household’s income is sufficient for a consumption higher than \( c^S \): \( c_t > c^S \). Then the household chooses \( e_t > 0 \), but the child does not learn all the basic skills including reading, writing and arithmetic, and the family might nevertheless be in the transition towards the poverty trap (case of non-stark poverty). In this scenario, there exists a substitutability between consumption \( c_t \) and education time \( e_t \) (movements along the indifference curve). The second scenario is the case of stark poverty where the income of the household is so low so that the child must work full-time (\( e_t = 0 \), \( c_t \leq c^S \)) and there is no substitutability between the two goods \( c_t \) and \( e_t \) (a decrease

---

\(^9\)de Janvry and Sadoulet (1996) find increasing returns to education up to twelve years of schooling, and thus that economies of scale in human capital exist in the area of basic education.

\(^10\)It is plausible to assume that the lower the level of child labor, \( 1 - e_t \), the higher the ability to learn is, since the child is not as exhausted. As \( 1 - e_t \) falls in \( \lambda_t \) this is a justification for the convex form for low levels of human capital [see also Hazan and Berdugo (2002), footnote 19]. Another reason could be that with decreasing work the health of children may improve.

\(^11\)Considering the neoclassical frame instead would produce identical results concerning the valuation of the analyzed subsidy instruments.
Chapter 4. Human Capital Subsidies

of $c_t$ leads to utility losses, no matter how much additional education occurs). As will become clear, our three instruments allow only for non-negative subsidies and are offered to the poor, so that they can decide whether to accept or to reject the subsidy offer. Consequently, beneficiaries do not suffer utility losses. This restriction can be justified by our conjecture that the lack of education is caused by poverty and that the parents are providing positive external effects on future generations for which they ought to be compensated for rather than “punished”. At the end of the chapter we drop this restriction. Our basic analysis will be undertaken mainly graphically. However, afterwards we document our results by a formal analysis.

Finally, we denote the level of maximized household’s utility $U_t$ in the laissez-faire reference world by $U_t^{lf}$. In case of $\lambda_t \in (\lambda^S, \lambda^a)$, the bundle $(e^o(\alpha\lambda_t), c^o(\alpha\lambda_t))$ is chosen, and $(0, \alpha(\lambda_t + \gamma))$ in the case of stark poverty. We define:

$$\left(e_t^{lf}, c_t^{lf}\right) = \begin{cases} (e^o(\alpha\lambda_t), c^o(\alpha\lambda_t)) & \text{if } \lambda_t \in (\lambda^S, \lambda^a); \\ (0, \alpha(\lambda_t + \gamma)) & \text{if } \lambda_t \leq \lambda^S. \end{cases}$$

### 4.3.1 Unconditional Lump-Sum Subsidies

First, we want to examine the effect of lump-sum subsidies for education that are paid without any requirement for a change in the household’s behavior [as considered in Bell and Gersbach (2001)]. Graphically speaking, the increase in the household’s income causes an upwards parallel movement of the budget-line. The required level of subsidies to reach a level of education $k$ is denoted by $s^{uc}(k)$. As long as income is lower than, or equal to, $\alpha\lambda^S$ the household’s consumption $c_t$ does not cross the poverty threshold level $c^S$. Because of the locally lexicographic preferences, only the demand for $c_t$ is increased by the subsidy, and the household’s choice for $e_t$ stays at zero (case of stark poverty). Hence, to reach the schooling level $k$, we first have to increase household’s income until household’s consumption exceeds $c^S$, so that we reach the area where $U_t = u(e^o_t, c^o_t)$. In a second step, we have to increase the household’s income further until we obtain that:

$$c^o(\alpha\lambda_t + s^{uc}(k)) = k$$

The resulting level of utility, $u(k, c^o(\alpha\lambda_t + s^{uc}(k)))$, is labeled $U_t^{uc}$. The case for $c_t > c^S$ is shown in Figure 4.3 and the case of stark poverty in Figure 4.4. We now turn to the less investigated cases of binary and continuous conditional subsidies. In Section 4.7, we discuss conditional tariffs more generally.

\[^{12}\text{Nevertheless education spends utility if } c_t \text{ was not lowered by increased education.}\]
4.3.2 Binary Conditional Subsidies

We first analyze the case where a fixed subsidy, denoted by $s^{bc}$, is paid if, and only if, the considered parents send the child to school for at least a given amount of time, denoted by $k$. Otherwise the household has no right to obtain subsidies.\textsuperscript{13} We define

- \textit{Binary conditional subsidization (BCS)}: An entitled household is offered a subsidy according to the following rule:

\[ s(e_t) = \begin{cases} s^{bc} & \text{if } e_t \geq k \\ 0 & \text{otherwise.} \end{cases} \quad (4.7) \]

In the “Food for Education” program in Bangladesh, for instance, a child receives the in-kind transfer only if it attends school for at least 85 percent of all classes each month. The headmaster monitors this and the food distribution takes place each month.\textsuperscript{14} In the education, health and nutrition program PROGRESA in Mexico, cash benefits are only paid when children are sent to school and visit health centers on a regular basis.\textsuperscript{15}

We denote the resulting level of consumption in the case the household chooses $e_t = k$ by $c^k_t$. Obviously the parent chooses the bundle $(k, c^k_t)$ if, and only if,

\[ u(k, c^k_t(\alpha \lambda_t + s^{bc})) \geq u(e_t, c_t) \equiv U^{lf}_t. \quad (4.8) \]

where the use of $u(e_t, c_t)$ is solely a simplification of notation in the case of stark poverty; since preferences cannot be described by a function, the notation solely should express the utility given allocation $(e_t, c_t)$. The level of subsidies that induces the household to choose $k$ is labeled $s^{bc}_t(k)$. For simplicity, we assume that indifference suffices to move the adult to choose $e_t = k$; otherwise the required subsidy would have to be increased inframarginally.

4.3.2.1 The Case of Non-Stark Poverty

In case of $c_t > c^{S}$, the utility is given by $u(c_t, c_t)$. The required subsidy in this case can be computed by equating the indifference curve function at utility level $U^{lf}_t$, labeled $e(c_t, U^{lf}_t)$, and the budget line at the location $(k, c^k_t)$ given $U_t = U^{lf}_t = U^{bc}_t$ (see Figure

\textsuperscript{13}\textsuperscript{13}The school attendance can easily be monitored by teachers.

\textsuperscript{14}\textsuperscript{14}See Ravallion and Wodon (2000) for a detailed description of the program. Jafarey and Lahiri (2000) show that a food for education subsidy unambiguously lowers child labor and increases schooling.

\textsuperscript{15}\textsuperscript{15}Cf. Skoufias, Davis, and de la Vega (2001).


Figure 4.1: Household’s decision under BCS in case of $\lambda_t \in (\lambda^S, \lambda^f)$.

4.1):

$$
e(c^k_t, U^f_t) = \frac{\lambda_t}{\gamma} + 1 + \frac{s_{bc}^k(k) - c^k_t}{\alpha\gamma} = k \quad (4.9)$$

$$
s_{bc}^k(k) = \alpha\gamma k + c^k_t - \alpha(\lambda_t + \gamma) < \alpha\gamma k \quad (4.10)
$$

Notice that $k - a_t < k - e^f_t$, i.e. the required subsidy does not have to cover the full extra cost $\alpha\gamma(k - e^f_t)$. The necessary payment is lower than the opportunity cost $\alpha\gamma k$, since altruism establishes a substitutional relationship between $e_t$ and $c_t$. Consider Figure 4.1. The necessary subsidy for $e^t(\cdot) = k$ in terms of units of $e_t$ is equal to $k - a_t$.

In terms of consumption good $c_t$, we obtain:

$$
s_{bc}^k(k) = c(k, \alpha\lambda_t + s_{bc}^k) - c(k, \alpha\lambda_t)
$$

where $c(k, \cdot)$ labels the level of consumption given $e_t = k$ and the level of adult’s income with and without subsidy. The resulting utility $u(e^t(\alpha\lambda_t + s_{bc}^k(k)), c^t(\alpha\lambda_t + s_{bc}^k(k)))$ is denoted by $U^bc_t$ and is equal to the level of utility in the laissez-faire case, denoted by $U^f_t$. Note that without any educational requirement for the subsidization of the household, it would reach the fictional level of $U^f_t$ associated with a lower level of $e_t$ and a higher level of $c_t$.

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4.3.2.2 The Case of Stark Poverty

The case of stark poverty is illustrated in Figure 4.4. In the *laissez-faire* state, utility is *de facto* solely determined by $c_t$, as education is never preferred to consumption (as long as $c_t \leq c^S$). Therefore, the adult is only willing to send the child to school instead to work if the household is compensated such that it still enjoys the original level of consumption: $c_t = c^f_t = \alpha(\lambda_t + \gamma)$. I.e., the government has to pay the total amount of opportunity cost of education $\alpha \gamma k$ to the household in terms of the consumption good $c_t$:

$$s^{bc}(k) = \alpha \gamma k \quad \text{if} \quad c_t \leq c^S$$

(4.11)

The indifference curves are reduced to single points. It follows that there is no substitutional relationship between $c_t$ and $e_t$. As $c_t = \alpha(\lambda_t + \gamma)$ and $e_t > 0$, the household’s level of utility increases. Here, it is important to emphasize the effect of supposing lexicographic preferences. As long as consumption is not affected, an increase in education increases utility. That is, if we offer subsidy payments for education the adult will definitely accept this offer. If, contrary, preferences were of the Stone-Geary type, then the household was indifferent between education and no education, so that we do not know whether the adult will accept the offer; we would have to pay an inframarginal additional amount of subsidy or to assume that indifference suffices to choose $e_t = k$.

4.3.3 Continuous Conditional Subsidies

Now consider an alternative formulation of conditional subsidies:

- **Continuous conditional subsidization (CCS):** An entitled household is offered a transfer according to the following rule:

$$s_t = s^{cc}(e_t)$$

(4.12)

with

$$\frac{ds^{cc}(e_t)}{de_t} \geq 0, \forall e_t \in [0, 1); \quad s^{cc}(0) = 0; \quad s^{cc}(e_t) = S, \forall e_t \geq k.$$ 

Hence, the paid subsidy, $s^{cc}(e_t)$, is a continuously increasing function of the education-time $e_t$.\footnote{If schooling was not free, the schooling cost would to be paid as well.} Such subsidization does not go along with a shift, but with a rotation of

\footnote{One might say that it makes sense to state $s^{cc}(k - e^o(\alpha \lambda_t))$. However, in the case of non-stark poverty this would be adverse, because as long as a household is not offered a transfer, an incentive is created to keep the children out of school to maximize possible future transfers.}
the budget line, since CCS simply equals a “commodity subsidy” which changes the relative price ratio in favor of $e_t$. The implicit budget line is given by:

$$e_t = \frac{\lambda_t}{\gamma} + 1 + \frac{s^{cc}(e_t) - c_t}{\alpha \gamma}$$

Suppose the simple specification

$$s^{cc}(e_t) = \begin{cases} \sigma e_t & \text{for } e_t \leq k \\ \sigma k = S & \text{for } e_t \geq k \end{cases}$$

with $\sigma > 0$. For $e_t \leq k \in [0, 1]$, we obtain

$$e_t = \frac{\alpha (\lambda_t + \gamma)}{\alpha \gamma - \sigma} - \frac{1}{\alpha \gamma - \sigma} c_t$$

(4.13)

as the budget line, with $\sigma \neq \alpha \gamma$. At $\sigma = \alpha \gamma$ the budget line has a slope of $-\infty$, i.e. education is costless. Because of $s^{cc}(e_t) = S$ for all $e_t \geq k$, the budget contour is a doubled\textsuperscript{19} kinked function with a second sharp turn at $e_t = k$. The household’s maximization problem is given by:

$$\max \{e_t, c_t\} U_t \text{ s.t. } \alpha \gamma e_t + c_t \leq \alpha (\lambda_t + \gamma) + s^{cc}(e_t)$$

(4.14)

We denote the solution of problem (4.14) by $(e_{cc}^t, c_{cc}^t)$. An example for a policy close to the idea of continuous conditional subsidies could, for instance, be found in India where “on-site feeding” programs at school distributed food to the pupils [see Schubert and Balzer (1990), p. 28, or Subbaro (1989), p. 32, for more details].

### 4.3.3.1 The Case of Non-Stark Poverty

In case of $c_t > c^S$, utility is given by $u(e_t^o, c_t^o)$. To achieve schooling of level $k$, the functional form of $s^{cc}(e_t)$ has to be chosen such that the implicit function of the demand for education fulfills:\textsuperscript{21}

$$e_t^c = e^o (\alpha \lambda_t + s^{cc}(e_t)) = k$$

The case is shown in Figure 4.2. $U_t^{cc}$ denotes the utility at $e_t = k$ in case of CCS and non-stark poverty. The left, lower “budget line” limits the budget in the laissez-faire case with $U_t = U_t^f$. The necessary subsidy payment, measured in units of $e_t$,

\textsuperscript{18}Note that, as $e_t \in [0, 1]$, the vertical intercept, $\frac{\lambda_t}{\gamma} + 1$, is only the mathematical one of the line determining the budget.

\textsuperscript{19}Except for $k = 1$.

\textsuperscript{20}Kayiranga (2004) reports that pupils often face the problem that they do not get food when coming home from school at the evening. As food is important for physical development, health and learning aptitude, on-site feeding may be an important building block of educational subsidies.

\textsuperscript{21}Note that the relative price of consumption has also increased by the subsidization, wherefore $s^{cc}(e_t) \neq s^{bc}(e_t)$. 

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is given by \( k - a_t \) and equals \( p(c_t^{cc}(k) - c_t(k)) \equiv ps_t^{cc}(k) \), where \( c_t^{cc}(k) \) stands for \( c^0(\alpha\lambda_t + s_t^{cc}(k)) \) and \( c_t(k) \) for \( \alpha(\lambda_t + (1 - k)\gamma) \). So \( s_t^{cc}(k) \) equals the horizontal gap between the budget line without \( s_t^{cc} \) and the one with, at \( e_t = k \). Because of \( a_t > 0 \), we definitely have \( s_t^{cc}(k) = \alpha\gamma(k - a_t) < \alpha\gamma k \), i.e., via \( s_t^{cc}(k) = \sigma k \), we obtain \( \sigma < \alpha\gamma \). Since \( c_t^{cc} > c_t^{lf} \), the adult has to be overcompensated for the extra education, that is, \( s_t^{cc} = c_t^{cc} - \alpha(\lambda_t + (1 - k)\gamma) > \alpha\gamma(k - e_t^{lf}) \), respectively \( k - a_t > k - e_t^{lf} \). We directly conclude that BCS is more cost-effective than CCS.

\[
\begin{align*}
\text{Figure 4.2: Household's problem with CCS in case of } & \lambda_t \in (\lambda^S, \lambda^a) \text{ and } s^{cc}(e_t) = \sigma e_t. \\
& \text{With } a_t > 0, \text{ the adult has to be overcompensated for the extra education.}
\end{align*}
\]

### 4.3.3.2 The Case of Stark Poverty

In case of stark poverty, preferences are lexicographic and at the first level only consumption \( c_t \) matters. The *laissez-faire* solution lies at a point on the horizontal axis, where \( e_t = 0 \) and \( c_t = \tau(1) \). CCS will not overcome this boundary solution unless the costs of education are fully compensated by the transfer. Graphically this boundary solution prevails as long as the budget line is not rotated such that it is equal to a vertical line with a slope of minus infinity, that is, the price of education is zero. Given lexicographic preferences, the parent tries to maximize the level of education

\[\text{This is the required payment because given the resulting level of } c_t^{cc}(k), \text{ the difference between the level of } e_t \text{ given the old budget and the one given the new budget must be the amount of subsidy.}\]
as long as \( c_t = \alpha(\lambda_t + \gamma) \) is not prejudiced. It follows that \( s^{cc}(k) = \sigma k \) determines the subsidy ceiling: \( S = \sigma k \). Hence, the required marginal subsidy \( \frac{ds^{cc}(e_t)}{de_t} = \sigma \) has, in terms of \( c_t \), to be \( \alpha \gamma \) [see also Equation (4.13)]. The household remains on his laissez-faire-iso-consumption-line, but utility rises. We obtain:

\[
s^{cc}(k) = \alpha \gamma k \quad \text{if} \quad c_t \leq c^S \tag{4.15}
\]

As in the BCS scenario, one would need to pay a slightly higher subsidy, given Stone-Geary preferences. It directly follows that in case of stark poverty BCS and CCS are equally cost-effective.

In the following, we provide a comparison of the three subsidy types with respect to two aspects: cost-effectiveness and speed of the process of educating the society.

### 4.4 Cost-effectiveness: a Comparison

#### 4.4.1 The Case of Non-Stark Poverty

Figure 4.3 provides a simultaneous view on all three instruments, which allows us a comparison.\(^{23}\) It shows that to achieve a policy objective \( k \) by conditional subsidies is indeed more cost-effective than using unconditional lump-sum subsidies. At \( e_t = k \), we have \( e^{uc} > e^{cc} > e^{bc} \) and thus \( U^{uc} > U^{cc} > U^{bc} \). This directly implies that the income in the case of UCS is the highest and in case of BCS the lowest. It follows that conditional subsidies to attain an education level of \( k \) are lower, in particular is the binary subsidy the lowest. Accordingly, we state:

**Proposition 4.1**

In case of \( \lambda_t \in (\lambda^S, \lambda^a) \), the instrument of binary conditional subsidization (BCS) is more cost-effective in reaching a particular level of education \( k \) than the instrument of continuous conditional subsidization (CCS). The latter, CCS, in turn, is more cost-effective than the simple unconditional lump-sum subsidization:

\[
 s^{bc}_t(k) < s^{cc}(e_t) < s^{uc}_t(k)
\]

Therefore, the BCS method is the most cost-effective among the three types investigated.

---

\(^{23}\)See Appendix B.1 for details to Figure 4.3 regarding to budget functions and marginal rates of substitution.
4.4.2 The Case of Stark Poverty

We have found that the required transfer to achieve an educational objective $k$ is identical under BCS and CCS (Equation (4.11) and (4.15)):

$$s^b_t(k) = s^c_t(k)$$

Hence, in case of stark poverty, both instruments are equally cost-effective. However, the required unconditional subsidy payment is much higher, as illustrated in Figure 4.4. As long as $c_t \leq c^S$, all transfer payments are exclusively used for consumption $c_t$. Conditional payments prevent the use of transfers for consumption, so that the target $e_t = k$ can be reached without any transfers being used for consumption.

**Proposition 4.2**

*In case of stark poverty, both conditional subsidy instruments are equally cost-effective, whereas the unconditional subsidization is clearly least cost-effective:*

$$s^b_t(k) = s^c_t(k) < s^u_t(k)$$
4.5 The Speed of Educating a Society

To study the speed of the education process, we turn to the dynamic analysis of our model. We demonstrate that the cost-effectiveness is a crucial determinant of the number of periods the education process requires to educate a society. The dynamic analysis will provide further insights that the static analysis did not.

4.5.1 A Model Extension: Foreign Aid Financed Subsidies

Let us, for simplicity, assume that the policy maker disposes in each period $t$ of a particular amount of foreign aid\(^{24}\) that we denote by $F$. We neglect the possibility that individuals could be taxed.\(^{25}\) We again analyze both scenarios of poverty. That is, in the first period of subsidization, period $t = 0$, all households live either in a state of stark poverty and backwardness ($c_t \leq c^S$) or of non-stark poverty ($c_t > c^S$). A fraction $\delta_t$ of the parents $i \in [0, 1]$ will be offered a transfer in period $t$ for giving full-time schooling to their children. That is, we assume $k = 1$ is the socially optimal

\[^{24}\]Pallage and Zimmermann (2001) try to determine a donor’s optimal level of foreign aid.

\[^{25}\]Note that very poor households are hardly to tax. In rural areas, this is difficult alone on grounds of a lack of infrastructure and administration.
choice. The size of fraction $\delta_t$ is limited by the government’s resources of foreign aid $F$. It is plausible to assume, independent of the subsidy type, $\delta_t < 1$. Paying the necessary help to a household $i$ yields that $e^i_t = (e^i_{t0})^x = 1$ for all $i \in [0, \delta_t]$ and $e^i_t = e^i_{t0}(\alpha \lambda^i_t)$ for all $i \in (\delta_t^x, 1]$, with $x = \{uc, bc, cc\}$ representing the different types of instrument in question. Full-time schooling of supported households leads to human capital formation of the children and we obtain:

$$\lambda_{t+1}^i = h(1)\lambda_0^i + 1$$

If $\lambda_{t+1}^i < \lambda^*$, subsidizing one generation of household $i$ does not suffice to rid household $i$ from the poverty trap, because the household remains in the area $\lambda \in [1, \lambda^*)$. Therefore, the next generations of the household also have to be subsidized until the adult’s level of human capital is higher than $\lambda^*$. Once this happens, the household reaches the area of human capital growth and escapes the poverty trap. Therefore, if $\lambda_{t+1}^i \leq \lambda^*$, then repeated subsidization is necessary. The critical threshold of $\lambda_0$, labeled $\lambda^{\text{crit}}$, is determined by $h(1)\lambda^{\text{crit}} + 1 = \lambda^*$ and thus given by:

$$\lambda^{\text{crit}} = \frac{\lambda^* - 1}{h(1)}$$

**Proposition 4.3**

If $\lambda_0 \leq \lambda^{\text{crit}}$, repeated subsidization is necessary if a supported household should escape the poverty trap and display long-term growth of income. Otherwise, $\lambda_0 > \lambda^{\text{crit}}$, one-time subsidization suffices.

### 4.5.2 When One-Time Subsidization Suffices

If $h(1) + 1 > \lambda^*$, then $\lambda^{\text{crit}} < 1$ and any household, once subsidized, will escape the poverty trap. The children of such families will enjoy full-time schooling as soon as the endogenous human capital growth ($\lambda_{t+1}^i > \lambda_t, \forall \lambda_t > \lambda^*$) leads to an adult’s income of $\alpha \lambda^a$. We denote the time in which subsidization is necessary to educate the society such that it escapes the poverty trap as a whole by $T^x$, with $x = \{uc, bc, cc\}$; that is, in period $T$ the last subsidization takes place and in period $T + 1$ we have $\lambda_{T+1}^i > \lambda^*$ for all households $i \in [0, 1]$. 
Corollary 4.1
Consider \( h(1) + 1 > \lambda^* \).

(i) In the state of \( \lambda_0 > \lambda^S \), i.e. \( c_0 > c^S \), a society can be educated faster by a binary conditional subsidization (BCS) than by a continuous conditional subsidization (CCS). The slowest instrument in question is the unconditional lump-sum transfer.\(^{26}\)

\[
T^{uc} > T^{cc} > T^{bc}
\]

(ii) In the case of stark poverty, \( \lambda_0 \leq \lambda^S \), a society can be educated equally fast by BCS and CCS. The slowest instrument in question is the unconditional subsidy:

\[
T^{uc} > T^{cc} = T^{bc}
\]

Proof:

(i) We have identical households so that \( \lambda_0^i \) is the same at all households \( i \in [0, 1] \). All households require the same size of subsidy, given by \( s^x(1), x = \{uc, bc, cc\} \). Due to the fixed amount of foreign aid we have \( \delta^x_t = \delta^x = \frac{F}{s^x(1)} \), for all \( t \). It is possible to subsidize a particular fraction \( \delta^{uc} \) in each single period with the lump-sum subsidy necessary for full-time schooling, \( s^{uc}(1) \): \( e^{\alpha(\alpha \lambda_0 + s^{uc}(1))} = 1 \).

Without capital market, the government’s budget has to be balanced and we obtain: \( \delta^{uc} = \frac{F}{s^{uc}(1)} \). It follows that the time needed to educate the society is: \( T^{uc} = \frac{1}{\delta^{uc}} = \frac{s^{uc}(1)}{F} \). Similarly, the time needed under CCS, \( T^{cc} \), and under BCS, \( T^{bc} \), is: \( T^{cc} = \frac{1}{\delta^{cc}} = \frac{s^{cc}(1)}{F} \) and \( T^{bc} = \frac{1}{\delta^{bc}} = \frac{s^{bc}(1)}{F} \). From Proposition 4.1 we have \( s^{uc}(1) > s^{cc}(1) > s^{bc}(1) \) for \( \lambda_0 > \lambda^S \). It follows: \( T^{uc} > T^{cc} > T^{bc} \).

(ii) In case of stark poverty, we found that BCS and CCS are identically cost-effective (Proposition 4.2), so that \( \delta^{bc} = \delta^{cc} \). Furthermore, we found that the necessary payment at unconditional subsidization is higher. Thus, \( T^{uc} > T^{cc} = T^{bc} \).

\[\square\]
4.5.3 When Repeated Subsidization is Required

If $\lambda_0 \leq \lambda^{crit}$, repeated subsidization is necessary. Consequently, the cost-effectiveness in each single period of subsidization matters. Let us assume that a household must be subsidized $r$ times in a row to escape the poverty trap, i.e. to cross $\lambda^*$. The period in which subsidization starts is labeled by $t_s$.

**Corollary 4.2**
Suppose $\lambda_0 \leq \lambda^{crit}$.

(i) If $\lambda_{t_s+r-1} > \lambda^S$, then a society can be educated faster by BCS than by CCS. The slowest instrument is the unconditional subsidy:

$$T^{uc} > T^{cc} > T^{bc}$$

(ii) If $\lambda_{t_s+r-1} \leq \lambda^S$, Corollary 4.1 holds.

**Proof:**
If the level of human capital in the last period of required subsidization, $t = t_s + r - 1$, is higher than $\lambda^S$, then Proposition 4.1 tells us that at least the last necessary transfer is located in the area where BCS is more cost-effective than CCS.\(^{27}\)

There is an additional, second dynamic accelerating effect in favor of conditional subsidization we are able to identify. In Corollary 4.1, we assumed that there exists a fixed amount of foreign aid $F$, without itemizing the realistic possibility of simultaneously levying taxes upon citizens. Government resources were identical to $F$ under all regimes for all periods.

Consider the case $h(1) + 1 > \lambda^*$ where previously supported families will accumulate human capital rapidly and their incomes increase from period to period. Obviously, wanting all children to receive full-time education, yet supported lineages have to be left with an income of $\alpha \lambda^a$. Thus, all so far subsidized households could be taxed by $\alpha(\lambda_t - \lambda^a)$ as soon as $\lambda_t > \lambda^a$. Since $\delta^{uc}$ is smaller than those of BCS and CCS in each period, the conditional subsidization methods produce a bigger portion of the society with system inherent income growth in each single period, implying that the government, on average, can have more tax revenues in each period under BCS and CCS than under the unconditional alternative. The average additional tax revenue be labeled $R^x$, $x \in \{uc, cc, bc\}$. We obtain: $R^{uc} < R^{cc} \leq R^{bc}$. Using $T^x = \frac{\alpha^x(1)}{\delta + \alpha^x}$ in the

\(^{27}\)Note that $\lambda_{t_s} = \lambda_0$ and the level of human capital will increase once subsidization has started.
proofs of corollaries 4.1 and 4.2, demonstrates that including a domestic income tax improves the advantage of conditional subsidization. Let us call the first basic effect of accelerating the education process, identified in the propositions 4.1 and 4.2, the static expenditure effect, and the just found the dynamic revenue effect. We can conclude that using conditional subsidization allows foreign aid payments to be ended earlier than in the unconditional case. Thereafter, the developing country can rely on it’s own tax revenue. Hence, a self-financed policy is established earlier.\(^{28}\)

### 4.6 An Example

To illustrate, extend and to prove analytically our results derived from a graphical analysis, we provide a specific example in this section. To handle the case of stark poverty, we ignore the second stage of the lexicographic preferences and exclude education \(e_t\) from the determination of utility,\(^{29}\) so that we can use a utility function for the case of stark poverty as well. In doing so, we use a Stone-Geary-type of utility function and drop time index \(t\) for the static analysis. We introduce the level of subsistence consumption, labeled \(c_{sub}\). For all \(c < c_{sub}\) the household dies of hunger. Suppose the household’s utility is given by:\(^{30}\)

\[
U = \begin{cases} 
-\infty & \text{if } c < c_{sub}, \\
 c - c_{sub} & \text{if } c_{sub} \leq c \leq c^S, \\
 (c - c^S)e + c^S - c_{sub} & \text{if } c > c^S.
\end{cases} \quad (4.16)
\]

#### 4.6.1 Household’s Behavior and the Required Subsidies

The Marshallian demands are:

\[
e_o(\lambda) = \max \left\{ 0, \min \left\{ 1, \frac{\bar{v}(\lambda) - c^S}{2\alpha\gamma} \right\} \right\} \quad (4.17)
\]

\[
e_a(\lambda) = \max \left\{ 0, \min \left\{ 1, \frac{\bar{v}(\lambda) + c^S}{2} \right\} \right\} \quad (4.18)
\]

with \(\lambda^S = \frac{\bar{V}-\alpha\gamma}{\alpha}\) and \(\lambda^a = \frac{\bar{V}+\alpha\gamma}{\alpha}\). Calculating the elasticity of the Marshallian \(e(\lambda)\) referring to the adults human capital, \(\eta_{e^{a\lambda}} = \frac{\partial e^{a\lambda}}{\partial \lambda} \frac{\lambda}{e}\), we find

\(^{28}\)If we include the schooling costs of the state, labeled \(C\), but schooling is free, we obtain \(\delta^x = \frac{F - C}{s^{x\lambda}} > 0\), for all \(F > C\). Our results remain, but \(T\) will increase under each method.

\(^{29}\)For our analysis, this affects only the level of utility but not the optimal allocation to choose.

\(^{30}\)Note, \(c_{sub} \leq c^S\).
**Proposition 4.4**

Consider the interval \( \lambda \in [\lambda^S, \lambda^a] \).

(a) Elasticity \( \eta_{e,\lambda} \) is strictly bigger than 1:

\[
\eta_{e,\lambda} > 1
\]

(b) It holds \( \frac{\partial \eta_{e,\lambda}}{\partial \lambda} < 0 \), so that the highest value of the elasticity is displayed for the lowest possible relevant value of adult’s human capital: \( \lambda = \lambda^S \).

The proof can be found in the appendix. We consider the case of non-stark poverty: \( c > c^S \). Depending on the method of subsidization, utility maximization yields the following results:

\[
\begin{align*}
\{e^{uc}, c^{uc}\} &= \left\{ \frac{1}{2\alpha\gamma} \left( \bar{\tau}(\lambda) + s^{uc} - c^S \right) , \; \frac{1}{2} \left( \bar{\tau}(\lambda) + s^{uc} + c^S \right) \right\} \\
\{e^{bc}, c^{bc}\} &= \left\{ \begin{array}{ll}
{k} & \text{if } e = k, \\
\left( \frac{1}{2\alpha\gamma} [\bar{\tau}(\lambda) - c^S] , \; \frac{1}{2} [\bar{\tau}(\lambda) + c^S] \right) & \text{if } e < k;
\end{array} \right.
\end{align*}
\]

\[
\{e^{cc}, c^{cc}\} = \left\{ \frac{\bar{\tau}(\lambda) - c^S}{2(\alpha\gamma - \sigma)} , \; \frac{\bar{\tau}(\lambda) + c^S}{2} \right\}.
\]

Hence, the subsidies necessary for \( e = k \), in the case of non-stark poverty, amount to:

\[
\begin{align*}
s^{uc}(k) &= 2\alpha\gamma k + c^S - \bar{\tau}(\lambda) \quad \text{(4.19)} \\
s^{cc}(k) &= \sigma k = \frac{1}{2} \left( 2\alpha\gamma k + c^S - \bar{\tau}(\lambda) \right) = \frac{1}{2} s^{uc}(k) \quad \text{(4.20)} \\
s^{bc}(k) &= \frac{1}{4\alpha\gamma k} \left[ 2\alpha\gamma k + c^S - \bar{\tau}(\lambda) \right]^2 = \frac{1}{4\alpha\gamma k} [s^{uc}(k)]^2 = \frac{1}{\alpha\gamma k} [s^{cc}(k)]^2 \quad \text{(4.21)}
\end{align*}
\]

Obviously the CCS is twice as cost-effective as the unconditional subsidy. In the appendix, we prove that for all values of \( k \) the BCS is more cost-effective than CCS (in the case of non-stark poverty). Furthermore, the *comparative statics* of the relation between BCS and CCS concerning the cost-effectiveness is given there: the better cost-effectiveness of BCS is independent of the productivity of labor, \( \alpha \), and increases in the level of human capital, \( \lambda \) (respectively with the wealth of the households\(^{31}\)) and with the degree of altruism \( (c^S \text{ falls}) \). The advantage decreases with the level of human capital of the children, \( \gamma \), and with the height of the desired level of education, \( k \). The economic intuitions are given in the appendix.

The effect of an increase of \( k \) is easily explained by the assumption of quasi-concave preference functions, i.e., that an individual prefers mixtures of consumption bundles.

\(^{31}\)This result is obvious recalling that in the case of stark poverty both are identical.
Suppose case BCS. At subsidizing, we move along one and the same indifference curve. As we assume strong convexity of the upper contour set, the slope of the indifference curve becomes more and more negative (i.e., steeper) by moving to the left along the function. That is, each marginal increase of $k$ becomes increasingly expensive. At the extreme end, when the slope is $-\infty$, both instruments BCS and CCS are, marginally viewed, identically cost-effective.\footnote{Note the relationship of this extreme with the poverty case (despite the neglected lexicographic character of the preferences).} The advantage that under BCS the utility does not need to be increased to educate the society is then, in this area, fully compensated by the fact that the lower the level of $c$ becomes by increasing $e$, respectively $k$, the more $c$ becomes a relative scarce good for the household. The compensation payment thus increases in $k$. Hence, if at $e = 1$ the marginal rate of substitution is near $-\infty$, the difference between BCS and CCS is quite small. This scenario is especially in poor economies realistic, since the aimed level of education is relatively high compared to the level of consumption there.

A Numerical Example:
Consider the following constellation of parameter values:

$$\alpha = 0.1, \quad \gamma = 1, \quad c^S = 0.25, \quad k = 1, \quad c^{\text{sub}} = 0.1$$

In case of $c > c^S$, we assume $\lambda = 1.8$ and obtain $\lambda^S = 1.5$, $\lambda^c = 3.5$, $c^d = 0.265$, $U^d = 0.15225$ and:

<table>
<thead>
<tr>
<th></th>
<th>UCS</th>
<th>CCS</th>
<th>BCS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s(1)$</td>
<td>0.17</td>
<td>0.085</td>
<td>0.07225</td>
</tr>
<tr>
<td>$c$</td>
<td>0.35</td>
<td>0.265</td>
<td>0.25225</td>
</tr>
<tr>
<td>$U$</td>
<td>0.25</td>
<td>0.165</td>
<td>0.15225</td>
</tr>
</tbody>
</table>

In contrast, for the case of stark poverty with $\lambda = 1$ we obtain:\footnote{Remember that in the case of stark poverty, the conditional subsidies are equal to the term $\alpha \gamma k$.}

$$s^{uc}(1) = 0.25, \quad s^{uc}(1) = 0.1, \quad s^{bc}(1) = 0.1$$

We see that all three instruments are able to implement the socially optimal level of education, but that the unconditional subsidy comes along with more consumption per supported household, so that the utility of supported households and the required transfer are higher. Given scarce resources $F$, this directly means that less households enjoy utility increases and overcome the poverty trap. Notice that in our example $s^{uc}(1) = \sigma = 0.085 < 0.1 = \alpha \gamma$, i.e. CCS does not require a full compensation of the opportunity cost of education. The additional consumption of the unconditional.
subsidy per supported household (compared to laissez-faire) represents a waste of resources from a welfare point of view. This leads us to the dynamic task of proving our result of the graphically analysis that the education process takes a longer span of time when unconditional subsidies are used.

4.6.2 Dynamics

For the schooling function we underlie \( h(e_t) = (e_t)^\Theta, \Theta > 0. \)

Lemma 4.1

Underlying the specific difference equation \( \lambda_{t+1} = [e^\Theta(\lambda_t)]^\Theta \lambda_t + 1, \) the dynamics follow the following patterns:

(a) The schooling function \( h(e) \) displays the following curvature pattern:

\[
\Theta > 1 \iff h''(e) > 0 \quad (h(e) \text{ strictly convex in } e)
\]

\[
0 < \Theta < 1 \iff h''(e) < 0 \quad (h(e) \text{ strictly concave in } e)
\]

(b) The second derivative of the difference equation \( \lambda_{t+1}(\lambda_t) \) is strictly positive if \( \Theta > 1 - \frac{1}{\alpha_\lambda} > 0. \) Otherwise, the second derivative is negative.

(c) For a given scalar \( \Theta \in (0, 1) \) the trajectory displays a turning point in the interval \((\lambda^S, \lambda^a)\) at \( \tilde{\lambda} = \frac{\tilde{\lambda} - \lambda^a}{\alpha_\Theta} \) if \( \tilde{\lambda} \in (\lambda^S, \lambda^a) \). For all \( \Theta \geq 1 \) there exists no turning point in this region.

Proposition 4.5

1. Combining Proposition 2.1 (a) with Lemma 4.1 (a) and (c), the trajectory is strictly convex in the interval \([\lambda^S, \lambda^a]\) for all \( \Theta > 1 \).

2. The trajectory is definitely initially concave in the interval \([\lambda^S, \lambda^a]\) if \( \Theta \in (0, 1) \).

3. Combining Proposition 4.4 (b) with Lemma 4.1 (b) and (c), the trajectory is initially strictly concave in the interval \((\tilde{\lambda}, \lambda^a)\) and strictly convex in the interval \((\lambda^S, \tilde{\lambda})\) if \( \Theta \in (0, 1) \) and \( \tilde{\lambda} \in (\lambda^S, \lambda^a) \).

The proofs are given in the appendix. Figures 4.5 and 4.6 illustrate the trajectory for different \( \Theta \).
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1.5
2
2.5
3
3.5
4
4.5

Figure 4.5: The 45°-line, the $1 + h(1)\lambda_t$-line and the trajectory for our parameter value constellation and $\Theta = \frac{1}{3} < \frac{1}{2}$. $\lambda^* \approx 1.6$.

4.6.3 The Dynamic Analysis

We denote the period in which a household $i$, $i = [0, 1]$, is offered a subsidy by $t_s$. We obtain

$$\lambda_{t_s+1} = h(1) + 1 = 2.$$  

Note that $\lambda_{t_s+1} = 2 < 3.5 = \lambda^*$. The adverse threshold is given by $\lambda^* = \frac{1}{1-(\epsilon_t)^\Theta}$. Using $\epsilon_t^\alpha = \frac{\alpha(\lambda_t+\gamma)e^\beta}{2\alpha\gamma}$ and $\Theta = 1$ we find $\lambda^* = 2.7808 > 2 = \lambda_{t_s+1}$. For $\Theta \neq 1$, however, the general implicit solution is $\lambda^* = \frac{1}{1 - 5^\Theta(0.1\lambda^* - 0.15)^\Theta}$ and we cannot solve for $\lambda^*$. But using $\lambda_{t_s+2} = 2[\epsilon^\alpha(2\alpha)]^\Theta + 1$ we obtain that $\lambda_{t_s+2} > \lambda_{t_s+1}$ is equivalent to

$$[\epsilon^\alpha(2\alpha)]^\Theta > \frac{1}{2}. \quad (4.22)$$

Applying our parameter values stated above, Condition (4.22) holds for $\Theta < \log_4 2 = \frac{1}{2}$.

Thus, for $\Theta \in (0, \frac{1}{2})$ the education of the society succeeds: the schooling function is concave enough in $\epsilon_t$, that is, the schooling “technology” is (initially) very productive. If we assume that the society in question disposes of foreign aid\textsuperscript{34} $F = 0.05$, we can

\textsuperscript{34}Note that $i \in [0, 1]$ causes that all numbers are \textit{de facto} per capita.
Figure 4.6: The $45^\circ$-line, the $1 + h(1)\lambda_t$-line and the trajectory for our parameter value constellation and $\Theta = \frac{7}{10} > \frac{1}{2}$. $\lambda^* \approx 2.425$.

calculate the time a particular subsidization method needs to educate the society:

$$T^{uc} = 3.4, \quad T^{cc} = 1.7, \quad T^{bc} = 1.445$$

in the case of non-stark poverty, and in the case of stark poverty:

$$T^{uc} = 5, \quad T^{cc} = 2, \quad T^{bc} = 2$$

Hence, in our example, conditional subsidization allows the education of the society in both the non-stark poverty as well as in the stark poverty case, in at least half of the time that unconditional subsidization would require. As expected, in non-stark poverty, the binary conditional subsidy can be used to educate the society substantially faster than the continuous. Thus, our graphically derived results are confirmed by our analytical investigation.

### 4.7 Other Subsidy Schedules

So far we only have investigated the three types of subsidization that are most likely practiced. In this section, we discuss other subsidy forms that are also utilizable to
implement the socially optimal level of education. The allocation with the socially optimal level of education is dominated by the individually optimal \textit{laissez-faire} optimum. However, subsidy schedules can be implemented so that we change the budget set of the household such that the \textit{laissez-faire} optimum is, after intervention, not feasible anymore. We maintain the premise that the households ought not suffer utility losses. We focus on the case of non-stark poverty, because in stark poverty it is clear that we must pay the full opportunity cost \( \alpha \gamma k \). In the case of non-stark poverty, all methods that manipulate the budget set such that

1. the budget set touches the indifference curve of utility level \( U^{lf} \) at \( e_t = k \) and
2. the budget set is at all other loci \((e_t, c_t)\) located in the strictly lower contour set \( \{(e_t, c_t) : u(e_t, c_t) < U^{lf}_t\} \).

are optimal. BCS, for instance, is optimal because \( U^{bc} = U^{lf} \). Having identified optimal subsidy methods, we will evaluate these opportunities concerning administrative efficiency and realizability at the end of this section, because another important premise is that the proposed method is practicable in underdeveloped countries.

\subsection*{4.7.1 A Generalized Approach}

Hitherto we assumed a linear function for \( e \leq k \) in the CCS case. Let us briefly check whether concave or convex courses might work better than the linear tariff. Our CCS case can be generalized to:

\begin{equation}
\begin{aligned}
s(e) &= \begin{cases} 
\sigma e^\varsigma & \text{for } e \leq k \\
\sigma k^\varsigma &= S & \text{for } e > k
\end{cases} 
\end{aligned}
\end{equation}

with \( \varsigma \geq 0 \). The case \( \varsigma = 0 \) represents the case of unconditional subsidization, whereas \( \varsigma = 1 \) equals the afore studied linear CCS case. \( \varsigma \in (0, 1) \) establishes a regressive subsidization and \( \varsigma > 1 \) a progressive one. The idea for the non-stark poverty case is made vivid in Figure 4.7.

Using a regressive subsidy ends up in a convex budget set, wherefore this choice is, compared to the linear case, inefficient as it involves higher payments to achieve a level \( k < 1 \). However, using a progressive tariff leads to a strictly non-convex budget set that is a true subset of the budget set in the linear case. Hence, the progressive function might be an efficient choice. Since for all \( k \in (0, 1) \) the upper contour curve extends strictly below the budget line in the linear case, such an education target can
be achieved by a lower payment than in the case $s^c(e_t) = \sigma e_t$.\textsuperscript{35} However, the problem of the tariffs described by Equation (4.23) is that the tariffs are restricted such that $\frac{\partial s(e_t)}{\partial e_t} = \varsigma \sigma(e_t)^{\varsigma - 1} > 0$ for all $e_t > 0$, that is, as long as the child attends school, the household receives a positive payment and thus enjoys higher utility. This violates our condition that the budget set is at all loci $(e_t, c_t)$ located in set $\{(e_t, c_t) \mid u(e_t, c_t) < U_{lf}\}$, except in case of $e_t = k$. Over all, a simple progressive tariff might be more cost-effective than a linear tariff for targeting a $k < 1$, but it is never optimal.

However, when we drop the restriction $\frac{\partial s(e_t)}{\partial e_t} > 0$, then our BCS is one instance that produces the optimal outcome: as long as $e_t < k$, we have $\frac{\partial s(e_t)}{\partial e_t} = 0$.\textsuperscript{36} But the tariff does not have to be so restrictive. In the area $c_t > c_{lf}$, subsidy payments do not prevent the optimal outcome, as long as for $e_t < k$ the budget set remains in set $\{(e_t, c_t) \mid u(e_t, c_t) < U_{lf}\}$. Generally, the subsidy schedule can also involve $\frac{\partial s(e_t)}{\partial e_t} < 0$, which implies tax burdens. Nonetheless, the tariff is progressive in the area $e_t \leq k$: the subsidy increases in the level of $e_t$, but $\frac{\partial s(e_t)}{\partial e_t}$ has to be weakly negative in the area $e_t \in [0, e_{lf}]$. Furthermore, finally all these tariffs are equally cost-effective, since the actually paid transfer is always equal. Let the inverse indifference curve of utility level

\textsuperscript{35}In the non-stark poverty case, at $e_t = 1$, all three upper contour curves intersect, so that for $k = 1$ the choice does not matter either – we obtain identical subsidy payments for all $\varsigma \neq 0$.

\textsuperscript{36}In tax theory, BCS is comparable with the case of tax exemptions that establish an indirect tax progression effect.
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$U^H_t$ be given by $c(e_t, U^H_t)$. Then, any subsidy schedule with property

$$c_t = \begin{cases} 
\alpha(\lambda_t + (1 - e_t)\gamma) + s(e_t) < c(e_t, U^H_t) & \text{for all } e_t < k \\
\alpha(\lambda_t + (1 - e_t)\gamma) + s(e_t) = c(e_t, U^H_t) & \text{for } e_t = k 
\end{cases} \quad (4.24)$$

will be optimal. The size of the subsidy $s(e_t)$ that is actually paid is exactly equal to the horizontal gap between the laissez-faire budget set and the indifference curve of $U^H$ given $e_t = k$, that is, the optimally paid transfer is given by:

$$s^{opt}(k) = c(k, U^H_t) - \left( \alpha(\lambda_t + (1 - k)\gamma) \right)$$

Notice that, for instance, $s^bc(k) = s^{opt}(k)$. In the following, we show that the tariff choice involves particular building blocks. The linear continuous subsidy, e.g., represents a linear two-tariff choice: a linear tariff for $e_t < k$ and a tariff for $e_t = k$. The analyzed pro- and regressive tariffs are examples for non-linear tariffs. Compared to these multiple tariffs, the BCS represents a typical simple binary two-tariff choice: we have a lump-sum subsidy combined with a threshold.

Optimal non-linear tariffs display two building blocks. One building block consists of the marginal tax/subsidy rate, the other of a kind of threshold. Consider, for instance, Figures 4.8, 4.9 and 4.10. All depicted examples produce the optimal outcome. The simple idea is that the tariff includes some threshold $\tilde{e}$ or $\tilde{c}$ in the following way. For all levels of education lower than $\tilde{e}$, respectively levels of consumption higher $\tilde{c}$, the tariff demands a tax payment so that the budget is decreased in this area. However, for levels higher than this threshold $\tilde{e}$ one obtains subsidies and the budget is increased such that, given the preferences (the indifference curve at $U^H$), the household’s optimal choice is socially optimal: $e^*_t = k$.

The tariff for example (b) in Figure 4.9 with $k = 1$ is given by:

$$s(e_t) = c(k, U^H_t) - \alpha(\lambda_t + (1 - e_t)\gamma) \quad (4.25)$$

where $\tilde{c} = c(k, U^H_t)$ and $c_t = \alpha(\lambda_t + (1 - e_t)\gamma) + s(e_t)$. It follows that the budget set becomes a rectangle with the right upper corner located at the socially optimal allocation involving $e_t = k$.\footnote{Alternatively, one could simply tax away all child labor income above $\alpha\gamma(1 - k)$. Then, we arrive at the socially optimal allocation $(e_t, c_t) = (k, \alpha(\lambda + (1 - k)\gamma))$, but utility decreases.}

An interesting case is example (c), Figure 4.10, which is an extension of the CCS. The idea is again to distort the price ratio. But this time we tax the household per unit of $e_t$ lower than a threshold $\tilde{e}$ at a constant rate and subsidize at the same rate for levels higher than $\tilde{e}$. The linear tariff looks like:

$$s(e_t) = MRS(k, U^H_t) \alpha \gamma (e_t - \tilde{e})$$

67
where \( MRS(k, U_{lf}^f) \equiv \left( \frac{\partial u(k,c_t)}{\partial c_t} \right) \left( \frac{\partial u(k,c_t)}{\partial e_t} \right) \) stands for the marginal rate of substitution at \( e_t = k \) on the indifference curve for utility level \( U_{lf}^f \).  \( \bar{e} \) is the intersection point of the post-intervention budget line with the laissez-faire one. All presented tariffs in common is the fact that, even in contrast to BCS, the laissez-faire allocation is not feasible anymore so that the household is forced to comply. I.e., we do not need to assume that, given indifference, the household chooses to accept the offer, or to increase the transfer inframarginally.

Let us finally evaluate the new options. First of all, the alternative tariffs are equally cost-effective as BCS is. However, in contrast to BCS, they involve also subsidies or taxes as long as \( e_t \neq k \). Hence, they might require higher administrative expense than BCS. Of course, the tariffs can theoretically be levied such that the optimal level of education is chosen without taxes actually being paid, but in practice not all households would directly choose \( e_t = k \). Then, the government actually has to collect taxes and to enforce the tariffs one needs sophisticated public (civil) servants combined with a developed administrative infrastructure. Both is in developing countries not at hand. However, tariff (4.25) requires the same amount of information as BCS and is equally easy to implement, but has the advantage that the laissez-faire allocation is not feasible.
in the post-intervention scenario. Nonetheless, BCS is the most direct implementation of education target $k$; the headmaster or teachers are able to directly monitor the school attendance. While in case of (4.25) the size of subsidy or tax has to be determined by a comparison of $e_t$ and $\tilde{e}$, the size of $s^{bc}$ is clear and the household receives this transfer or not, which is easier. We therefore propose to use BCS for reasons of administrative efficiency and realizability. Nonetheless, more restrictive tariffs as given by (4.25) might have advantages compared to BCS – but also disadvantages.

### 4.8 Absolute and Relative Altruism

To check the robustness of our results, we consider other types of preferences that typically establish boundary solutions involving $e_t = 0$. So far, we assumed that children do not “enjoy” any school education because their parents do not earn enough or, more generally, are not wealthy enough. Hence we said that $e_t^o = 0$, if parents income is not higher than $\alpha \lambda^S$. However, once $\lambda_t > \lambda^S$, we assumed $e_t^o > 0$. Notice that this only holds in any case if $\alpha \gamma$ is constant. Therefore we assumed that the indifference curves do never intersect the horizontal for $\lambda_t > \lambda^S$. This type of altruism should be labeled *absolute altruism*, since absolute income and wealth is decisive. In
the following, we show that one has to distinguish between this absolute and a relative form of altruism.

Consider the type of preferences illustrated by Figure 4.11. The slope of the indifference curves at \( e_t = 0 \) is equal for all levels of consumption \( c_t \). Such preferences are quasi-linear with respect to education time \( e_t \) and could, for instance, be represented by preference function \( u(c_t, e_t) = v(e_t) + c_t = (a + e_t)^b + c_t, \) with \( a > 0 \) and \( b \in (0, 1) \). The indifference curves of this preference function are given by \( e_t = (u_t - c_t)^{1/b} - a \). The indifference curves’ slope is given by \( \frac{de_t}{dc_t}_{du_t=0} = -1/(b(a + e_t)^{b-1}) \), that is, at \( e_t = 0 \) the slope is \(-1/(ba^{b-1})\) for all levels of consumption \( c_t \). Underlying such preferences would imply that the only reason for a household’s choice of \( e_t = 0 \) is the fact that the price of education relative to consumption is too high. The marginal cost of education \( \alpha \gamma \) are higher than the maximum willingness to pay for it. If \( e_t = 0 \) is optimal for the household, then at locus \( (e_t, c_t) = (0, \bar{c}(\lambda_t)) \), the relative price level between consumption \( c_t \) and education \( e_t \) is always strictly lower (or at most equal to) the marginal rate of substitution (MRS) between the two goods:

\[
\frac{1}{\alpha \gamma} \leq \frac{\partial u(0, \bar{c}(\lambda_t))}{\partial c_t} \left/ \frac{\partial u(0, \bar{c}(\lambda_t))}{\partial e_t} \right. \equiv MRS
\]

Note that this holds for all levels of income. I.e., even the richest parents choose \( e_t = 0 \)
for their children. Hence, such kind of altruism we call relative altruism, since the relative price of the good (that concerns altruism, in our case education $e_t$) determines the behavior – and not (absolute) wealth. The higher the degree of this relative altruism is, the lower is the curvature of the indifference curve in the intersection points with the horizontal. That is, the MRS decreases. Therefore, the probability of an interior allocation with $e_t > 0$ increased. One can easily check that in this case the analysis for BCS and CCS in non-stark poverty holds, but that unconditional subsidies would be fully ineffective, since there are no income effects on $e_t$. Therefore, preferences that are quasi-linear with respect to education $e_t$ are not reasonable to explain the lack of education in poverty traps and are not supported by empirical facts. Consequently, we exclude this possibility.

A second type of preferences, involving indifference curves of the form $e_t = \frac{u_t}{c_t} - a$, root in utility functions like

$$u(e_t, c_t) = c_t(e_t + a), a > 0$$

and is illustrated by Figure 4.12. In contrast to the preferences that were quasi-linear with respect to $e_t$, the curvature at the point of intersection with the horizontal is not constant. At $e_t = 0$, the slope of the indifference curves is $-\frac{a^2}{u_t} < 0$ and thus increases in the level of utility $u_t$. The higher absolute income, respectively the level of
utility, the lower the curvature at locus \((0,\pi(\lambda_t))\) is. Increases in income are effective to increase the household’s level of education. Thus, such preferences involve absolute and relative altruism. Consequently, for such preferences our analysis of the case of non-stark poverty holds, with the following reservation. Underlying such preferences, we get:

\[
\lim_{u_t \to 0} MRS(e_t = 0, u_t) \to -\infty \\
\lim_{u_t \to \infty} MRS(e_t = 0, u_t) \to 0
\]

So for very low levels of utility, respectively wealth, the indifference curves are vertical lines, similar to our stark-poverty case with lexicographic preferences, where we had to deal with vertical isoconsumption lines. With increasing utility the slope of the indifference curves at \(e_t = 0\) becomes flatter. That is, the maximum willingness to pay for education increases in wealth. Hence, there does not exist this point of discontinuity \(c^S\) like in our analysis, so that we cannot separate between cases of stark-poverty and non-stark poverty – it’s a smoothing transition. Nevertheless, the core of our results continues to hold. We can conclude that the success of education policies using conditional subsidies are much less contingent than the unconditional subsidy policy on the underlain assumption for preferences. Consequently, this is a material advantage of conditional subsidization looking on the uncertainty about human beings’ preferences.
Finally, notice that preferences that are quasi-linear with respect to consumption \( c_t \), instead of \( e_t \), are instances for an altruism involving the relative and absolute type.\(^{38}\)

In general, a boundary solution \( e_t = 0 \) obeys the following economic intuition. First, if the marginal cost of education – the foregone earnings plus potential school cost shares – are everywhere strictly higher than the maximum marginal willingness to pay, we obtain a boundary solution with full-time child labor: \( e_t^o = 0 \); education is seen to be too expensive. Second, if the marginal cost of education for full-time schooling is always lower than the corresponding compensation requirements, determined by household’s preferences, the child will enjoy full-time education: \( e_t^o = 1 \).\(^{39}\)

### 4.9 Educating a Society within one Generation

In Chapter 3, we derived the socially efficient allocation in absence of foreign aid. Let us assume that in period \( t \) full-time schooling of the children is socially optimal, that is, the allocation \( e_t^{so} = 1 \) and \( c_t^{so} = \alpha \lambda_t \) is first-best. However, in general, scarce resources make it impossible to pay the required subsidies to all households. Then, the next-best policy maximizes the fraction of the society that displays the socially efficient education level. Thus, policy has to minimize the time required for the education of the society. The ideal solution is educating the society in one period. If a policy achieves this (without foreign aid) and establishes the socially efficient allocation, then this policy is first-best.

On first sight, there are some simple ways to achieve this. The most famous is the instrument of compulsory schooling. However, we know that compulsory schooling can be quite harmful to poor families as it might cut family income down below subsistence income, that is, \( c_t = \alpha \lambda_t < c_{sub} \). Consequently, in developing economies, compulsory schooling is often not realizable; due to poverty the society is often unable to adjust properly and hence does not comply. Therefore, as long as poverty and not parents’ ignorance causes child labor this instrument should not be taken into consideration, since at least in the short-term, it is not a promising tool of policy.\(^{40}\)

In this context, another discussed option is a ban on child labor: when child labor prevents schooling, then fighting child labor could be a tool to educate a society. However, experience shows that this can also be harmful to the poor, because children are forced to work illegally in much more dangerous occupations and prostitution to ensure

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\(^{38}\)An example is the preference function \( \sqrt{1 + e_t + c_t} + 1 \).

\(^{39}\)See also Cigno, Rosati, and Guarcello (2002).

\(^{40}\)See, for instance, López-Calva and Rivas (2000) or Dessy (2000).
household’s survival.\textsuperscript{41, 42} Schooling will therefore not arise and child labor continues even worse. A further unrealistic instrument could be the taxation of child incomes such that all child labor income is taxed away to force parents to send the children to school. All these policy proposals neglect the core of the problem: poverty. When the parents alone are not in a position to earn enough money so that their family survives, then children have to work if no member of the family should die hunger. We will see that this is the major problem of all tools that may allow for the education of a society in one period. In Appendix B.3, we discuss the option of a Pigouvian tax with and without refunding scheme.

4.9.1 Static Analysis of Tax-Financed Subsidies: Necessary Conditions

So far we always dealt with foreign aid of size $F$. Reflecting Section 4.5, it is easy to see that with foreign aid of size $F = s^x(k)$ we yield $T = 1$ (for achieving any education level $k$) for each single regime $x = \{bc, cc, uc\}$, respectively. However, the interesting case is the one with $F = 0$ where the society has to educate itself relying fully on its own resources. Then, the required transfers have to be financed by taxes levied on households.

The tax burden per household $i$ in period $t$ we label $\tau^i_t$. Realistically, we assume that the post-tax, respectively the net or disposal income has to be at least as high as the value of the subsistence consumption, labeled $c^\text{sub}$. A direct consequence of this constraint is that in a situation where households only dispose of an income that is at most as high to finance $c^\text{sub}$, these parents cannot be taxed at all, and the self-financed education is impossible.

Note that for the objective of educating the society in one generation we definitely have to force a balanced budget, since credit financed policies involve repayment and interest burdens for later generations.\textsuperscript{43} Hence, the per household tax burden $\tau^i_t$ has to be at least as high as per household subsidies $s^i_t$. A direct consequence is that the targeted allocation $(c^k_t, k)$, $c^k_t = c^0(\alpha \lambda_t + s^x_t - \tau_t)$, must be located in the laissez-faire budget set of the households. If all households in the starting period, $t = 0$, are alike, the net tax burden of each household has to be non-negative. So the policy has to achieve, by redistribution schemes, that the targeted allocation $(c^k_t, k)$, which is

\textsuperscript{41}The child unemployment bears the risk of malnutrition for the household as a whole.

\textsuperscript{42}The described also holds for the prohibition of imports of goods made partially by child labor, because this is a form of an indirect ban on child labor.

\textsuperscript{43}Given a credit financed program, one simply has to raise a high enough loan to educate the society in one generation.
dominated by \((c_l^f, e_l^f)\) without intervention, will dominate all other feasible allocations after intervention has taken place. The basic idea is that at first the household has to pay taxes. Then, the government pays (part of) these taxes back via subsidies that have to achieve \(c_t^e = k\), while ensuring consumption level \(c^{sub}\).

Note that the resulting allocation under unconditional subsidization is never feasible for the household, given the pre-intervention budget.\(^{44}\) Paying back taxes unconditionally, will definitely end up in the pre-intervention allocation, without (sufficient) education. Hence, like Bell and Gersbach (2001), page 25, examined with a different argumentation, using unconditional subsidization does not allow to educate a society within one generation, given \(F = 0\). For our conditional instruments this is, however, not necessarily the case. If \((c_t^k, k)\) is not element of the pre-intervention budget set, the end of reaching education level \(k\) for all households in one period will not be achievable, because the government is not able to transfer the required subsidies.

Let us first fix the following remarks:

**Remark 4.1**

(i) A society as a whole can attain a general education level \(k\) in one single period \(t\), if the following holds:

(a) \(\int_{i=0}^{1} \tau_t^i(i) \, di \geq \int_{i=0}^{1} s_t^i(k, i) \, di\)

(b) \((c_t^i)^k \geq c^{sub} \quad \forall i \in [0, 1]\)

(c) \(e_t^i \geq k \quad \forall i \in [0, 1]\)

where \(x = \{bc, cc, uc\}\).

(ii) Education level \(k\) (for the society as a whole) in period \(t\) will only enable the society to overcome poverty sustainable, if:

\[h(k)\lambda_t^i + 1 > \lambda^* \quad \forall i \in [0, 1]\]

In the case of stark poverty, we found \(s^{bc}(k) = s^{cc}(k) = \alpha \gamma k\). The households should in the end be in a position to consume \(c^{sub}\), despite paying tax \(\tau_t\) and losing child labor income \(\alpha \gamma k\). Therefore, we receive \(\tau_t - s_t = \tau_t - \alpha \gamma k \leq \alpha (\lambda_t + \gamma (1 - k)) - c^{sub}\). The net taxes have to be strictly non-negative, so that the education of the society within one generation is feasible if:

\[\alpha (\lambda_t + (1 - k)\gamma) \geq c^{sub}\]

\(^{44}(e^o(\alpha \lambda_t + s^{uc}), k)\) lies strictly in the north-east of \((e^o(\alpha \lambda_t), e^o(\alpha \lambda_t))\), which lies directly on the budget line.
with $\lambda_t \in [1, \lambda^S]$. Thus, the households’ pre-intervention income must be sufficiently high to cover the cost for subsistence consumption and foregone earnings of the child. Note that the maximum tax burden (corresponding with $c_t = c^{sub}$) is:

$$\tau^{max} = \alpha(\lambda_t + (1 - k)\gamma) - c^{sub} + s_t$$

Given $s_t = \alpha\gamma k$, we obtain:

$$\tau^{max} = \alpha(\lambda_t + \gamma) - c^{sub}$$

Obviously, the maximum possible education level that can be established is reached where all revenue is exactly utilized. That is, $k$ is so high that $\alpha(\lambda_t + (1 - k)\gamma) = c^{sub}$. This level we denote by $\overline{k}_t$:

$$\overline{k}_t = \frac{\overline{c}(\lambda_t) - c^{sub}}{\alpha\gamma}$$

So for all $k$ bigger than $\overline{k}_t(\lambda_t)$ the project is not feasible. If $\overline{c}(\lambda_t) \leq c^{sub}$ the household’s income is not sufficient to ensure the subsistence level of consumption even with full child labor. We obtain $\overline{k}_t \leq 0$ and the project of educating the society within one period is not feasible for any (positive) level of $k$. But if $\alpha\lambda_t \geq c^{sub} + \alpha\gamma$ the adult can afford to send the child full-time to school without suffering hunger. It follows that we obtain $\overline{k}_t(\lambda_t) \geq 1$, and each target can be implemented successfully within one generation. So the critical elements determining success or failure are the children’s level of human capital $\gamma$, the productivity of efficiency units of labor $\alpha$, and the subsistence level of consumption $c^{sub}$.

In a graphical analysis, one can see that the probability that the project is successful, increases with the size of the negative slope of the budget line (see Figure 4.13). The higher $\gamma$ the bigger is the pre-intervention budget set wherefore the project is more easily accomplished successfully due to an increasing tax margin.

In the case of non-stark poverty, we get the same result. The only difference is that the maximal tax revenue rises, since the level of human capital is higher: $\lambda_t > \lambda^S$. Consequently, higher education targets $\overline{k}_t$ are feasible: $\frac{\partial \overline{k}(\lambda_t)}{\partial \lambda_t} > 0$. 45

So overall, one has to utilize conditional subsidies, but it does not matter whether we use binary or continuous subsidies. In both cases, whether in stark poverty or not, both instruments require the same transfers, for $c^{sub} \leq c^S$. However, the higher the initial level of human capital, the higher the level of education one can reach within one generation. The general idea is that taxation reduces income to $c^{sub}$ and one afterwards offers to pay back (part of) the money, if education $k$ is established.

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45 Note that in case of non-stark poverty we only have the constraint of $c_t \geq c^{sub}$. 

76
Figure 4.13: The project “education for all in one generation” for different levels of human capital of the child, where \( p_1 = \frac{1}{\alpha_1} \gamma_1 > \frac{1}{\alpha_2} \gamma_2 = p_2 \), that is, \( \gamma_1 < \gamma_2 \).

### 4.9.2 Dynamic Analysis of Tax-Financed Subsidies: A Further Necessary Condition

What is left to be checked is whether this possible level of education, \( k(\lambda_t) \), is sufficient to escape the poverty trap, because only then we have educated the society as a whole sustainable. Given the quality of the schooling system, the success is sustainable if Remark 4.1 (ii) holds:

\[
h(k(\lambda_t)) > \frac{\lambda^* - 1}{\lambda_t}
\]

We denote the minimum education level of \( e_t \) that is required for escaping the poverty trap sustainable in period \( t \) by \( \overline{k} \). Note that if \( \overline{k} > 1 \), then the project is not feasible, because of a too unproductive technology of human capital. If we consider \( h(e_t) = (e_t)^{\Theta} \) the necessary condition becomes:

\[
\overline{k}(\lambda_t) > \left( \frac{\lambda^* - 1}{\lambda_t} \right)^{\frac{1}{\Theta}} \equiv \overline{k}(\lambda_t)
\]

**Proposition 4.6**

A sustainable education of a society within one single generation is possible if, but only if, the following holds:

\[
k(\lambda_t) < k \leq \overline{k}(\lambda_t)
\]
Chapter 4. Human Capital Subsidies

The preceding analysis highlighted the point that three considerations enter the determination of the question for the possibility of the project. First, the taxable capacity, which depends on $\tau(\lambda_t)$, i.e., on full income. Second, the burden of subsidies to be paid, which are given by the opportunity costs of education $\alpha \gamma k$ (as long as the supported adult’s resulting consumption level does not cross $c^S$). And third the productivity of schooling, given by $h(e_t)$.

If the budget is balanced all households display the socially efficient education and consumption level: $e_t = k_t = e_t^{so}$ and $c_t = c_t^{so} = \alpha (\lambda_t + (1 - e_t^{so}) \gamma)$. Thus, the conditional subsidies can attain first-best, regarding the case without foreign aid. Notice that due to $F = 0$ we were forced to drop the premise that supported households ought not suffer utility losses. Attaining “education for all” without foreign aid requires forcing households to choose another allocation than $(e_t^{lf}, c_t^{lf})$ without increasing their budget. Thus this objective has to cause a drop in utility. However, notice also that first-best policies indeed require that all children are educated fully in a single period, but not that the policy has to take place only in one period. If intervention in one period is not sufficient, then the policy has to continue to be first-best.

What conclusion are we able to draw from our analysis? First of all, we saw that the plan of educating a society within one period leads to utility losses for (most of) involved households, i.e., there exists the risk of unrest and non-compliance. The taxation of households in rural areas is in practice often a difficult task, so that these required tax revenues must be obtained in town areas where the necessary infrastructure is available.\footnote{On the difficulty of taxation and its effects on development in poor economies see, for instance, \textit{Burgess and Stern} (1993).} Furthermore, in democracies each policy proposal has to find support in order to form majorities. Thus, in democracies, the feasibility of the project is doubtful. But development policies face these problems generally. Hence, similar to Bell and Gersbach (2001), we must add further restrictions. In Appendix B.4 we therefore discuss extensions of our model covering differences in administrative cost of single subsidy methods, a constitutional or implicit political ceiling on the tax burden, and reelection constraints. A political economy analysis we provide in Chapter 5.
4.10 Conclusions, Evidence, and Future Research

4.10.1 Conclusions

We analyzed the simple, unconditional lump-sum subsidy and two types of conditional subsidization: binary (BCS) and continuous conditional subsidies (CCS). BCSs are paid lump-sum, but only if a prescribed education level is established. Contrary, CCSs are paid proportional to the established education level, so that the subsidy increases in the time a child spends in school.

Both analyzed types of conditional subsidy perform more cost-effectively than simple unconditional transfers. They therefore enable societies to be educated swifter. We have seen that the optimal design depends on the status quo of the society, that is, on whether the poor live in stark or less stark poverty. Overall, the binary conditional subsidy is a better policy choice than the continuous conditional, since it is equally cost-effective in case of stark poverty and more cost-effective in case of non-stark poverty. It follows that even in a society suffering from extreme poverty, it can allow to educate a society in a shorter span of time than the continuous one does. In an example we have shown that conditional subsidies allow for the education of a society in at least half of the time needed by the unconditional.

In the case of stark poverty, the necessary transfer must cover all opportunity costs of education, since the households cannot afford to renounce one single unit of income. However, in less poor economies the necessary subsidy is, in general, lower, because of the fact that the parents can afford to practice a certain degree of substitutability between consumption and education, resting on intrinsic altruism.

The reason for the supremacy of conditional subsidies is that paying subsidies only conditional on enrollment requirements endogenizes the policy objective directly to the household’s optimization problem, whereas simple lump-sum transfers do not. Binary conditional subsidization channels the subsidy only to the parents if the child attends school to a degree at least as high as government’s target. The continuous only creates the incentive of sending the child to school, but small attendance suffices to obtain entitlements. Furthermore, the continuous instrument works due to a price distortion so that the value of real income in terms of education increases. Hence, in contrast to BCS, the target is only reached if the involved households enjoy utility gains. It follows that this type of transfer has to be more expensive. Therefore, BCS is the optimal instrument, given our premise that households ought not suffer utility losses.

We have seen that there are multiple other forms of non-linear tariff choices that are equally cost-effective as BCS. The tariffs differ with respect to administrative efficiency.
and realizability. The simple form used in BCS appears to the author as being the best tool.

We have also shown that the success of conditional measures, in contrast to unconditional, seems to be robust concerning the form of preferences. Without altruism, there is no reason to believe that education can be achieved by just paying transfers to households. Under conditional subsidization, in contrast, the most that has to be paid to create the incentive to provide the socially optimal level of education are the opportunity costs. That is, the critical assumption that parents are altruistic is not required under conditional subsidies, whereas it is strictly essential under unconditional regimes. Being conscious of the uncertainty about the degree of altruism this is a material advantage. In this context, we distinguished between relative and absolute altruism. In case of relative altruism, the income of a household is not decisive for the household’s education decision, but the relative price of education to consumption goods. Then, only relative poverty is the source of the observed lack of education. In case of absolute altruism, the absolute wealth of a household determines the education decision, and only absolute poverty causes the lack of education. Thus, absolute altruism seem to explain the existence of poverty traps much better.

In a dynamic frame, the revenue effect was found as an additional advantage of conditional subsidization: a faster education process increases the tax potential of a government, and thus allows for a faster education process that, firstly, is accomplished earlier and, secondly, can earlier be managed independently of foreign aid.

We eventually elaborated that only conditional subsidies allow to educate a society within one single generation, given there is not a sufficient size of foreign aid. However, we demonstrated that there arise many constraints that have to be fulfilled. Hence, trying to attain the escape from poverty traps via education in one generation appears to be hardly feasible. Such an ambitious policy bears the risk that the transfers per household are too low and thus poverty could not be overcome sustainable. Moreover, without foreign aid, educating a society within one generation demands reducing consumption. If the society is very poor and there is not the possibility to tax an elite sufficiently, the situation of the present adult generation is strongly worsen. Therefore, overall, all instruments that cause massive income losses today are to evaluate as crucial, because they worsen the consumption situation of the poor that already suffer privation. Due to these potential counter-productiveness and/or missing realizability those proposals should be eliminated from the set of options. Therefore, subsidization policies should be implemented without massive burdens of the current poor, especially not in the light of our results in Chapter 3, where we identified that the current parents
generations’ education provides positive externalities.

4.10.2 Evidence

Ravallion and Wodon (2000) found a strong positive effect of the conditional subsidy established by the World Bank program “Food for Education” in Bangladesh on school attendance. Among participants, i.e. the fraction who were offered a subsidy, nearly full school attendance was achieved by support with a value considerably less than the mean child wage. Thus, although Bangladesh is a country in stark poverty, it was not necessary to pay the total opportunity cost as in our case of stark poverty. Consequently, parents’ altruism seems to be very strong. However, this enrollment increase was followed by an under-proportional curb of child labor. The authors therefore stress that a “subsidy increases schooling, but its effect on child labor is ambiguous.”

Therefore, much of the reached schooling developed at the expense of the children’s leisure. It follows that child labor disappeared not as strong as our model would predict (since we neglected the leisure time of children).

Anker and Melkas (1996) provide insights on different types of income replacement via in-kind payments like school lunches, providing books, write utilities, housing and the like in real world. Providing meals for the pupils does not necessarily achieve growing education. It ensures the nutrition of the children that will visit school; in many cases this is already a progress. But the households’ critical threshold, which we labeled $c^S$, however, may demand more than simply the nutrition cost for the children, so that the children won’t be sent to school. Thus, ensuring the nutrition of the children is the minimum a subsidy must achieve and can be seen as a minimum benchmark. In stark poverty, a child will only be sent to school if potential foregone earnings of the child will be substituted, just as our model predicts. This could be interpreted such that there is evidence for our assumption that the minimum level of consumption for sending a child to school, $c^S$, is higher than the subsistence level $c^{sub}$.

Even if subsidies can increase schooling, one could ask whether schooling actually spurs growth. This is discussed by Temple (2001). He analyses what he calls the “Pritchett hypothesis”, which says that educational attainment has done little to raise growth in less developed countries. He emphasizes that this was the case because the

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48 Pallagge and Zimmermann (2001), p. 5, cite empirical work of Canagarajah and Coulombe (1997) who found that, in Ghana, “poverty does not seem to be a good determinant of child labor, whereas the education of parents tends to reduce the incidence of child labor.” This implies that conditional subsidies are more effective than the unconditional in curbing the incidence of child labor and that it needs time until success is taking place.
success of schooling and subsidization is contingent to the necessary environment. For instance, schooling does not increase growth if there exists a lack of demand for skilled labor. Nevertheless, schooling often failed to spur growth because the school attendance has not produced sufficient human capital. He concludes that the occasionally found assertion that schooling is irrelevant for growth is wrong.\textsuperscript{49} What is essential, is the fact that a subsidy paid to a parent, or to any individual, has to involve (if possible) a level of human capital in the following period that is at least as high as the escape from poverty requires; in the notation of our model this is \( \lambda > \lambda^* \). Concentrating on simple enrollment ratios bears the danger of forgetting the fact that it is elementary to shift each single supported household out of the poverty trap.\textsuperscript{50} So besides the quantity of pupils enrolled, it is important not to neglect the quality aspect. The children must learn sufficient skills to be able to leave behind the poverty trap.\textsuperscript{51}

4.10.3 Future Research Issues

While we have concentrated on the design of optimal subsidy programs, a variety of extensions appear to be fruitful avenues for future research.

\textsc{Angrist, Bettinger, Bloom, King, and Kremer} (2002) report evidence from the Colombian PACES program.\textsuperscript{52} Here vouchers were only renewable if the pupils performed satisfactory, i.e. there is an additional requirement established, which not just involves an incentive to attend school but also to devote more effort to school. Although this largest school voucher program to date targets the (private) secondary school education,\textsuperscript{53} this can also be taken into consideration in the context of basic education. Hence, future research could extend our model towards the issue of the effort of the children. The introduction of such a second requirement for subsidy entitlement could spur the success of education programs additionally.

\textsc{Swaminathan} (1998) and \textsc{Ravallion and Wodon} (2000) stress that often growth in aggregated output goes hand in hand with an expansion of child labor as trade liberalization and government policies particularly push labor-intensive goods where

\textsuperscript{49}Bils and Klenow (2000) also state that the positive correlation between initial schooling and the per capita growth rate found by Barro (1991), Barro and Sala-i-Martin (1995), and others, should not be interpreted as the impact of schooling on growth.

\textsuperscript{50}See also Bell and Gersbach (2001).

\textsuperscript{51}There is another myopic policy that leads to solely short-term effects without any long-term success: if a politician officially wants to fight poverty, but in fact wants to ensure reelection by transfers to voters, a broad campaign using unconditional subsidies might be preferred, because it allows higher consumption. Cf. Bigsten and Levin (2000).

\textsuperscript{52}PACES stands for Programa de Ampliaci´on de Cobertura de la Educaci´on Secundaria.

\textsuperscript{53}For a recent controversy about vouchers in the context of higher education in the U.S.A. see Ladd (2002) and Neal (2002).
child labor is most rife. Cigno, Rosati, and Guarcello (2002) find no empirical
evidence that exposure to international trade and integration across national borders
raise the incidence of child labor, rather the opposite. But they argue that following
new trade theories, countries with a largely uneducated work force could be left out of
the globalization process as they miss, due to the lack of human capital, to take part
in. The induced negative income effect on education demand could rise the amount of
child labor. This controversy highlights a further open issue in the frame of child labor
and human capital accumulation investigation.

In Chapter 5, we analyze the political economy of educational redistribution via tax-
and-subsidy schemes, invented by Bell and Gersbach (2001). That is, we inves-
tigate which additional constraints arise in a democracy when the education of the
society should be reached by educational subsidies that have to be financed by taxes.

In Chapter 6, finally, we extend our model by implementing transaction costs of edu-
cation like bribes and transportation. Moreover, we extend the government’s portfolio
of educational expenditures by allowing for investments to increase the school density,
the schooling quality, the infrastructure, and to fight corruption.
Chapter 5

The Political Implementation

"Alle, die sich mit Politik befaßt haben, stimmen darin überein – und die Geschichte belegt es durch viele Beispiele –, daß wer einer Republik Verfassung und Gesetze gibt, davon ausgehen muß, daß alle Menschen schlecht sind und daß sie stets ihren bösen Neigungen folgen werden, sobald ihnen Gelegenheit dazu geboten wird."

– Niccolò Machiavelli (1469-1527)

5.1 Introduction

Since the fall of the Berlin Wall and the end of the Cold War, there has been ongoing discussion on whether to make democratization and democratic reforms a precondition for foreign aid. In this context, we ask whether democracy makes it possible to improve long-term welfare. As an example we use our AK growth model in which human capital accumulation is the source of growth. Can democracy educate a society that is caught in such a poverty trap? Or is a certain degree of dictatorship necessary to alleviate poverty?

Bell and Gersbach (2001) demonstrate how an adequate, dynamic scheme of taxes and subsidies can lift a society out of such a trap. At the beginning of Chapter 4, public revenues were simply given by foreign aid, but then we implemented the possibility of taxation as analyzed by Bell and Gersbach (2001). A crucial yet unanswered question is whether a policy scheme of taxes and subsidies can in fact be implemented in a democracy. We address this question and examine a political economy of the
education of a society. In particular, we ask which constitutional rules are required to induce the education of a society in a democracy. For this purpose, we embed our model in a political economy framework. Our main findings are as follows:

(i) A democracy with a benevolent but dictatorial agenda setter facing a simple majority rule can educate the society.

(ii) A democracy with equal, unrestricted agenda rights for all citizens and simple majority rules fails to educate the society and will remain in the poverty trap indefinitely.

(iii) The combination of flexible majority rules, where the size of the required majority depends on the tax differences of redistribution proposals, with a rotating agenda setting and agenda repetition can educate a society. The same effect can be obtained by a combination of simple majority rules, rotating agenda setting, agenda repetition and individual protection from excessive taxation via tax deductions.

(iv) Education of a society via a process of democracy will also be possible with simple majority voting and equal agenda setting rights, provided there is a subsidy ceiling and individual preferences are such that social concerns (with respect to child labor and poverty) do exist, but are lexicographically dominated by pure self-interest.

The overall conclusion of our normative analysis is that there are democratic constitutions that induce literacy and economic welfare. However, there is a variety of political failures that constitutions have to deal with.

The chapter is organized as follows. In the next section we survey the related literature. In Section 5.3, we briefly repeat the main features of our basic model. In Section 5.4 we explain the tax-and-subsidy scheme for educating a society and develop the political framework. Section 5.5 demonstrates that a democracy with a benevolent, dictatorial agenda setting can escape poverty via education. In Section 5.6, we first show that a democracy without constitutional constraints on the agenda setter cannot overcome child labor to escape poverty traps. Subsequently, we offer a variety of constitutional rules that can eliminate political failures, so that societies can, in principle, escape the poverty trap. Finally, we discuss potential political failures in Section 5.8. Section 5.9 concludes. In Appendix C.1, we explain how a successful tax-and-subsidy scheme must be designed so that a society can be educated within three periods, when the simple majority rule is employed.
5.2 Relation to the Literature

This chapter draws on different strands of the literature. We analyze the dynamic tax-and-subsidy scheme proposed by Bell and Gersbach (2001). However, in contrast to them, we do not assume that the schooling technology is so productive that a child of fully backward parents ($\lambda = 1$) that enjoys a full basic education ($e = 1$) will definitely choose full-time schooling for her/his child. That is, we do not assume that $zh(1) + 1 \geq \lambda^a$ essentially holds. Furthermore we add a political framework. Most of the relations to the literature we already discussed in Chapter 1. At this point, we hence concentrate upon the particular literature on political economy and public choice aspects.

In general, the issue of this chapter is linked with the literature dealing with the question whether democracy impedes economic growth. Theoretical and empirical investigations have come to contradictory results on the issue of whether democracy pushes growth or not. However, the comprehensive study of growth by Barro (1996) suggests a beneficial effect of democracy that may work through its positive impact on schooling. This is exactly the link we analyze.

The chapter is also broadly related to the political economy literature focusing upon redistribution policies. For an overview, see Hochman and Peterson (1974), Drazen (2000) or Persson and Tabellini (2000, 1997). However, most of these investigations deal with transfers from young to old in the social-security context, and not with transfers from adults to children to overcome poverty and backwardness.

Gradstein and Justman (1997) offer a political economy for the choice concerning the education system. The agents can choose between subsidies for privately purchased education and free uniform public provision. In contrast to our work, they do not offer a normative proposal focusing on developing economies. Moreover, in contrast to our model, the individuals do not propose how to educate the society, they simply have to choose between two exogenously given alternatives.

Acemoglu and Robinson (2000) argue that individuals who have political influence, and fear losing it, have an incentive to block changes (political loser hypothesis). They

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2See also Bénabou (1996) and Glomm and Ravikumar (1992).

3Cf. the economic losers hypothesis in Kuznets (1968).
conclude that thus the nature of political institutions is a crucial element.\footnote{Acemoglu and Robinson (1999) demonstrate that the initially disenfranchised poor in non-democratic societies controlled by a rich elite may force the elite to democratize by threatening social unrest or revolution. They show that asset redistribution such as educational reforms may be used as a strategic decision to consolidate both non-democratic and democratic regimes.} We elaborate constitutional rules that ensure the success of democratic reforms. While some rules prevent that wealthier households block necessary redistribution, other rules ensure that harmful policies can be blocked.

\textbf{Dessy} (1998) argues that in a democracy, when a government redistributes via expenditures on education, a negative relation between inequality and education driven growth emerges if the education expenditures of the government do not crowd out private ones. \textbf{Soares} (1998) develops a political economy of public funding of education. He emphasizes the importance of the effects of an education policy on the factor prices in determining the equilibrium level of policy. We examine whether democratic constitutions can induce a society to set up dynamic redistribution schemes in such a way that all individuals are provided at least with basic education and skills and which inequality implications such policy will have.

\textbf{Grossman and Helpman} (1998) argue that, when governments are unable to commit to a course of future redistributive policies, they cannot guarantee to keep promises to the young. If the current agenda setter suspects that transfers to the young will be reversed by future politicians, they will be tempted to cater to the old instead, which can be harmful to growth. They stress that constitutional constraints on the extent of politically motivated redistribution might help, but that it may be difficult to write a constitution that would distinguish political redistribution from well-intended redistribution. We highlight the fact that appropriate constitutional rules can lead to welfare-enhancing redistribution from the parent generation to the children, and hence to long-term welfare via the accumulation of human capital. But even small deviations from such rules can bring about inefficient redistribution.

Our constructive constitutional economics approach goes back to \textbf{Buchanan and Tullock} (1962). An excellent survey is provided by \textbf{van den Hauwe} (1999). Recent papers on constitutional design tradition are described in the following. \textbf{Aghion, Alesina, and Trebbi} (2002) endogenize the choice of political institution by analyzing in a five stage game, and given a veil of ignorance, how large the majority to pass legislation should be, when the voters do not know whether the leader that will be elected will promote a reform or expropriate. \textbf{Gersbach} (1999) gives a set of constitutional principles given the constraint of democracy. He elaborates on the social efficient constitution depending on the project being socially efficient or not, and
on the relative size of the project winner group. Such constitutions may include the
simple majority rule as well as super-majority rules, taxation constrained to majority
winners and half of the voting population, a ban on subsidies, and equal treatment
rules with respect to taxes and subsidies. ERLENMAIER and GERSBACH (1999) pro-
pose a so-called flexible majority rule for public good provision where the required
majority depends upon the proposal itself.\(^5\) In WICKSTRÖM (1984) the constitution
determines a set of possible income redistributions from which the agents choose one
by majority vote. In this chapter, we elaborate which constitutional rules are required
for applying our education policy to overcome poverty traps successfully. In doing so,
flexible majority rules turn out to be very helpful tools in deciding on redistribution
proposals.

5.3 The Model

We extend our basic model and embed it in a political-economic setting. Basically, we
again build on our basic model introduced in Chapter 2: Each generation consists of a
continuum of households represented by the interval \([0, 1]\). A household is indexed by
\(i\) or \(k\), where \(i, k \in [0, 1]\). The portion of childhood devoted to education in period \(t\)
remains \(e^i_t \in [0, 1]\), the residual portion being allocated to work. Adults spend all their
time working. We again consider the human capital technology

\[
\lambda^i_{t+1} = h(e^i_t)(z\lambda^i_t) + 1
\]

and household \(i\)’s income in period \(t\) is given by

\[
y^i_t = \alpha[\lambda^i_t + (1 - e^i_t)\gamma].
\]

The household’s behavior is summarized by:

\[
(c^i_t, e^i_t) =
\begin{cases}
(\pi(\lambda^i_t), 0) & \forall \lambda^i_t \leq \lambda^S; \\
(e^{i_o}_t, e^{i_o}_t) & \forall \lambda^i_t \in (\lambda^S, \lambda^a); \\
(\varnothing(\lambda^i_t), 1) & \forall \lambda^i_t \geq \lambda^a.
\end{cases}
\]

\(^5\)See also AGHION and BOLTON (1997) for a normative analysis of optimal majority rules. In
GERSBACH (2002) the legislative stage cannot observe individual utilities. Hence, flexible and ad-
ditional double majority rules concerning tax burden are as well needed as flexible agenda cost in
combination with a ban on subsidies to prevent vote buying. Following ERLENMAIER and GERSBACH
(2000) efficient public project provision may also ask for the agenda setter paying the highest tax.
YOUNG (1995b) searches for the optimal voting rules and propose the maximum likelihood method for
ranking alternatives in voting. POLBORN and MESSNER (2004) deals also with the selection of voting
rules over reforms when only the old incur the cost of the reform. WICKSTRÖM (1986b) reformulates
the theory of optimal majority for public decision concerning risk aversion.

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and the dynamics are described by:

\[
\lambda_{i+1}^i = \begin{cases} 
1 & \forall \lambda_i^i \leq \lambda^S; \\
z h (e^0 (\lambda_i^i)) \lambda_i^i + 1 & \forall \lambda_i^i \in (\lambda^S, \lambda^a); \\
z h (1) \lambda_i^i + 1 & \forall \lambda_i^i \geq \lambda^a. 
\end{cases}
\] (5.4)

For the sake of simplicity, we again concentrate on the growth case, where \(zh(1) > 1\) and \(h(e_i^i)\) is strictly convex in \(e_i^i\). Consider Figure 2.3 (a).

### 5.4 Education Policy and Democracy

#### 5.4.1 Redistribution via Taxation and Subsidization

The redistribution via taxation and subsidization is similar to our analysis in Section 4.9. We assume that the whole society is initially \((t = 0)\) in a state of poverty, i.e. all households \(i \in [0, 1]\) display \(\lambda_0^i = 1\), \(e_0^i = 0\) and \(c_0^i = \pi(1) = \alpha \frac{1 + \gamma}{1 + \beta}\). The broad objective of policy is to educate the whole society to enable all its members to escape from this backwardness. The instruments for this purpose are taxation and subsidization. We assume that only the income of adults is subject to taxation. This can be justified by the ease of tax evasion in connection with child labor income. Restricting taxation on adult’s income makes child labor more attractive, but as children work fulltime anyway, this does not make any difference. Therefore, it is unlikely that allowing for taxation of household income would change the main results of the chapter.\(^7\) Let \(\tau_i^t(\alpha \lambda_i^i)\) denote the tax levied in period \(t\) on the income \(\alpha \lambda_i^i\) of an adult in household \(i\). At the beginning of each period \(t\), some fraction \(\delta_t\) of the population, \(\delta_t \in [0, 1]\), will be subsidized from the ensuing tax revenue. We use \(s_i^t(\alpha \lambda_i^i)\) to denote the subsidy a household \(i\) will receive in period \(t\) if the adult has income \(\alpha \lambda_i^i\). That is, the education policy redistributes income via a tax-and-subsidy scheme as described in Bell and Gersbach (2001). Since the net income determines the demand \(c_i^o\) and \(e_i^o\), we henceforth change notation from \(e^0(\lambda_i^i)\) to \(e^o(\alpha \lambda_i^i + s_i^t - \tau_i^t)\). For reasons of efficiency, we assume that a household is either taxed or subsidized.

We label the subsidy that has to be paid to a household in a state of backwardness, \(\lambda_t = 1\), in order to achieve a human capital level of \(\lambda^a\) in the following period by \(s^a\),

\(^6\)Our analysis is the same if \(h(e_i^i) \lambda_i^i\) is concave in \([\lambda^S, \lambda^a]\) and the function for \(\lambda_{i+1}^i\), Equation (5.1), intersects just once with the 45° line. In all other cases, the results can easily be transferred from our analysis with small supplements, so that our analysis is robust.

\(^7\)Child labor is largely unofficial and informal, so that taxation of child labor incomes is de facto impossible.
which is determined by the subsequent implicit equation:

\[ zh(e^\alpha (\alpha + s^a)) + 1 = \lambda^a. \]  

(5.5)

Note that \( s^a \) only exists if \( zh(1) + 1 \geq \lambda^a \). The net income of household \( i \) in period \( t \), measured in units of output, is

\[ \bar{\tau}(\lambda^a_t) - \alpha \gamma e^i_t + s^i_t(\alpha \lambda^a_t) - \tau^i_t(\alpha \lambda^a_t) \equiv w^a_t + \alpha (1 - e^i_t) \gamma; \]  

(5.6)

where \( w^a_t \) denotes the net disposable income generated by the adult of household \( i \) in period \( t \). To simplify notation, we introduce \( v^i_t(\alpha \lambda^a_t) \) to identify the net tax burden (or negative subsidy transfer):

\[ v^i_t(\alpha \lambda^a_t) \equiv \tau^i_t(\alpha \lambda^a_t) - s^i_t(\alpha \lambda^a_t) \]  

(5.7)

I.e., the disposable income depends on whether the household is taxed \( (v^i_t(\alpha \lambda^a_t) = \tau^i_t(\alpha \lambda^a_t)) \), subsidized \( (v^i_t(\alpha \lambda^a_t) = -s^i_t(\alpha \lambda^a_t)) \) or none of both \( (v^i_t(\alpha \lambda^a_t) = 0) \). The adult chooses \( e^i_t \) based on the household’s net full income \( \alpha \lambda^a_t - v^i_t(\alpha \lambda^a_t) + \alpha \gamma \), or since \( \alpha \gamma \) is constant, based on \( w^a_t \). Therefore, the evolution of human capital accumulation and educational choice follows the same logic as in Equation (5.4) and is given by\(^8\)

\[ \lambda_t^{ia} = \begin{cases} 
1 & \forall w^a_t \leq \alpha \lambda^S; \\
zh(e^\alpha(w^a_t)) \lambda_t^i + 1 & \forall w^a_t \in (\alpha \lambda^S, \alpha \lambda^a); \\
zh(1) \lambda_t^i + 1 & \forall w^a_t \geq \alpha \lambda^a.
\end{cases} \]  

(5.8)

Moreover, the optimal educational choice \( e^\alpha(w^a_t) \) is monotonically increasing in adult income \( w^a_t \), with \( e^\alpha(\alpha \lambda^S) = 0 \) and \( e^\alpha(\alpha \lambda^a) = 1 \).

There is a subsistence level \( (1 + \beta) e^{sub} \) (for a household comprising one adult and one child) which must be ensured under all circumstances. Otherwise there is the risk that severe problems of morbidity and mortality will result from taxation. The taxation of a household \( i \) living in a state of backwardness is therefore assumed to be constrained by:

\[ \alpha (\lambda^i_t + \gamma) - \tau^i_t(\alpha \lambda^a_t) \geq (1 + \beta) e^{sub}. \]  

In particular, the tax must fulfill the following condition:

\[ \tau^i_t(\alpha) \leq \alpha (1 + \gamma) - (1 + \beta) e^{sub} \equiv \tau^{sub}, \]  

(5.9)

\(^8\)Using the definition of \( v^i_t \), human capital accumulation in (5.8) can be rewritten as

\[ \lambda_t^{ia} = \begin{cases} 
1 & \forall \lambda_t^i \leq \lambda^S + \frac{e^i_t}{\alpha}; \\
zh(e^\alpha) \lambda_t^i + 1 & \forall \lambda_t^i \in (\lambda^S + \frac{e^i_t}{\alpha}, \lambda^a + \frac{e^i_t}{\alpha}); \\
zh(1) \lambda_t^i + 1 & \forall \lambda_t^i \geq \lambda^a + \frac{e^i_t}{\alpha}.
\]
where it is plausible that $\tau^{\text{sub}}$ is small, since households with $\lambda_t = 1$ may already be close to the subsistence level $c^{\text{sub}}$. Let us denote the total government’s revenue in period $t$ by $R_t$. To formulate the government’s budget constraint, we reinterpret the indexation of households as a real valued function on $[0, 1]$, assigning every household its human capital in a particular period. Then, the budget constraint in a period $t$ is given by:

$$R_t = \int_0^1 \tau_t(\alpha \lambda_t(i), i) \, di \geq \int_0^1 s_t(\alpha \lambda_t(i), i) \, di$$

(5.10)

The tax or subsidy not only depends on household $i$’s income, but also on whether household $i$ has to pay taxes or receives a subsidy, so that $\tau$ and $s$ are also functions of $i$ itself. Requiring a balanced budget in each period, we exclude capital market-financed subsidies for education. It is obvious that a society that can be educated without access to capital markets can also be educated with access to them. In this sense, we analyze a worst-case scenario.

Similar to our analysis in Chapter 4, we might have to subsidize repeatedly to enable a household to escape the poverty trap. The education level in period $t + 1$, $e_{t+1}$, of a household that was supported with a subsidy in period $t$, can be lower as it was in period $t$, $e_t$. Whether this will be the case or not depends on the productivity of the technology of human capital. Let us denote the minimum size of subsidy $s_t$ that causes a fully uneducated adult ($\lambda = 1$) to choose fulltime schooling for the child by $\bar{s}$: $e^{\alpha}(\alpha + \bar{s}) = 1$. If such a household was supported by a subsidy in period $t$, then $e_{t+1} < e_t$ is equivalent to $\alpha z h(1) \leq \bar{s}$, because then the household’s income in period $t$ was higher than it is in period $t + 1$. A drop in education does not need to be crucial. It causes $h(e_t) > h(e_{t+1})$, but at the same time we have $\lambda_t < \lambda_{t+1}$. The crucial point is whether $\lambda_{t+1} = z \lambda_t h(e_t) + 1$ is higher than the poverty trap threshold at steady state $\lambda^*$, i.e. whether $\lambda_{t+2} > \lambda_{t+1}$. If this is the case, then the level of human capital will grow for all time (due to $zh(1) \geq 1$). Notice that this scenario is compatible with a temporary drop of education. However, if this drop is too strong, then the household cannot escape the poverty trap, despite initial full-time schooling, because $zh(1) + 1 \leq \lambda^*$. Thus, households that were subsidized in one period will only escape from the poverty trap if the productivity of the technology of human capital is sufficiently high. As we assume that $e = 1$ is socially efficient and targeted by policy, we additionally have to check whether the technology is productive enough to generate $zh(1) + 1 \geq \lambda^a$. In the following, we hence have to distinguish the insufficient-productivity case ($zh(1) + 1 < \lambda^a$) from the sufficient-productivity case ($zh(1) + 1 \geq \lambda^a$).
5.4.2 The Political Economy Framework

Buchanan and Tullock (1962) view political activity as a two-stage process. At the first, or constitutional stage, constitutions have to face the Wicksellian unanimity or consensus test [see Buchanan and Tullock (1962) and Wicksell (1896)]; this unanimity requirement serves as the basis of justification and is the ultimate criterion of efficiency [cf. van den Hauwe (1999), p. 612]. At the second stage, the individuals decide on politics, given the “rules of the game” stated by the constitution agreed upon in the first stage. In this chapter, we focus on this second stage where, in our context, the adults decide on education policy. Nonetheless, we briefly address the constitutional stage in the following.

The considered society initially is completely alike. As it turns out, all constitutional rules which we propose warrant that no particular subset of households is systematically favored. Moreover, as showed in the introduction and in Chapter 3, investing in basic education is socially profitable and leads to long-term growth. Consequently, every lineage can be generally better off over time. A priori, all alike adults are fully uncertain about their status in the future and therefore fulfill the characteristic of a veil of uncertainty, invented by Buchanan and Tullock (1962). Buchanan and Tullock’s work then suggests that the individuals unanimously agree on constitutional principles that ensure the education for the society as a whole. Hence, we assume that the constitutions that we will propose below will be accepted unanimously at the constitutional stage.

Constitutions usually restrict the political process, for instance, by agenda, agenda setting, agenda setter, campaign, decision and voting rules. The totality of all these rules, which represents the constitution, we denote by $C$. A proposal of subsidies and taxes for all households represents an agenda. Agenda rules may restrict the set of admissible agendas. Agenda setting rules determine how the agenda setter is to be found, and possible agenda setter rules constitute constraints on the agenda setter. Decision and voting rules describe how the society decides upon a proposal and when it is adopted. A proposal is constitutional if none of the rules stated are violated.

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9This unanimity requirement is closely related to the contractarian tradition in political philosophy, see van den Hauwe (1999). Buchanan (1987), p. 133, points out that if one remains within the presuppositions of methodological individualism, a polity and its “rules of the game” (the constitution) must ultimately be justified in terms of their potential for satisfying the desires of the individuals.

10The work of Buchanan and Tullock (1962) follows a long tradition started by Rousseau (1762). See also Harsanyi (1955), Mirrlees (1971), Rae (1969), Wicksell (1896, 1964), Rawls (1971). In reality, the idea of a veil of uncertainty must be modified as different wealthy individuals may favor different levels of majority to pass legislation. The availability of exit options in constitutional deliberation can substitute for a veil of uncertainty. Cf. Lowenberg and Yu (1992).
Let us consider the case of secret ballots in a *direct democracy* with a voting population consisting of the parent-generation. The modeled democratic election can be understood as a referendum. We assume that each voter has the same voting and agenda rights, i.e., in principle, every individual has the same chance of determining the agenda for a given period and the decision depends solely on the number of votes.

For the moment, we consider the simplest democratic process and leave the agenda setting stage unspecified. We assume that setting an agenda does not involve any costs, and that a tax-and-subsidy proposal denoted by $P_t = \{\tau(\alpha \lambda^t_i), s(\alpha \lambda^t_i)\}_{i=0}^1$ will be approved if at least half of the population support it, i.e., the political process is governed by the majority voting rule (MV). In doing so, we apply a closed rule, i.e. amendments are not possible.\footnote{Of course, the optimal size of required agreement can be deduced by applying the concept of *interdependence costs*, in Chapter 6 of *Buchanan and Tullock* (1962) (see also *Klick and Parisi* (2003)). However, in practice, democracies typically apply the simple majority rule.}

- **Majority voting rule (MV):** If a proposal receives a majority of $m = \frac{1}{2}$ of the citizens, it passes legislation.\footnote{*Aghion, Alesina, and Trebbi* (2002), p. 7, emphasize that closed rules “are associated with faster and more efficient fiscal reforms ...”. On p. 20 they conclude that the fact that it is harder to collect taxes in developing countries and to target truly deserving human beings for subsidization suggests that there shall be adopted systems with rare veto or amendment opportunities. On the other hand, this makes it easier to expropriate when being in office. We will forestall expropriation so that closed rules are to prefer.} Otherwise the status quo prevails.

Notice that *Helpman* (1995) stresses that although direct democracy is rarely applied, majority voting via direct democracy is a good approximation for outcomes in representative democracy as the results are reasonably close. We restrict the set of allowed proposals to one that satisfies the governmental budget constraint with an agenda rule:

- **Balanced budget (BB):** A constitutional proposal has to satisfy a balanced budget, i.e.

$$\int_{i=0}^1 \nu_i(i)di = 0, \quad \forall \ t.$$  

A weaker condition would be the requirement that aggregate subsidies must not exceed aggregate tax revenues. Referring to the voting behavior, voter $i$ supports proposal $P_t$, if $s(\alpha \lambda^t_i) > 0$ and rejects it, if $\tau(\alpha \lambda^t_i) > 0$. However, if $s(\alpha \lambda^t_i) = \tau(\alpha \lambda^t_i) = 0$, then the household will be indifferent between supporting and rejecting the proposal. For simplicity, we assume the following tie-breaking rule to cope with this indifference:

\begin{align*}
11 & \text{Of course, the optimal size of required agreement can be deduced by applying the concept of } \textit{interdependence costs}, \text{ in Chapter 6 of } \textit{Buchanan and Tullock} (1962) \text{ (see also } \textit{Klick and Parisi} (2003)). \text{ However, in practice, democracies typically apply the simple majority rule.} \\
12 & \text{*Aghion, Alesina, and Trebbi* (2002), p. 7, emphasize that closed rules “are associated with faster and more efficient fiscal reforms ...”. On p. 20 they conclude that the fact that it is harder to collect taxes in developing countries and to target truly deserving human beings for subsidization suggests that there shall be adopted systems with rare veto or amendment opportunities. On the other hand, this makes it easier to expropriate when being in office. We will forestall expropriation so that closed rules are to prefer.} \\
13 & \text{It is generally assumed that a proposal will be adopted if more than half of the citizens support it [see, e.g., *Mueller* (1979) or *Bernholz and Breyer* (1994)]. We could replace } m = \frac{1}{2} \text{ by } m = \frac{1}{2} + \epsilon. \text{ For sufficiently small } \epsilon, \text{ we obtain the same results as with } m = \frac{1}{2}. \\
\end{align*}
• **Tie-breaking rule (TR):** Voter $i$ supports the proposal $P_t$ if

$$s_i^t(\alpha \lambda^i_t) = \tau_i^t(\alpha \lambda^i_t) = 0.$$ 

The tie-breaking rule represents a standard assumption about voting behavior to break indifferences, which is not decisive for our results. Given tie-breaking rule TR and assuming that a proposal either levies taxes on individuals (including a tax rate of zero) or provides subsidies, a proposal is accepted if and only if the share of individuals not being taxed, denoted by $\phi_i$, is at least $\frac{1}{2}$. In reality, this decision is also determined by other aspects. For instance, the alleviation of child labor will reduce the labor supply which, in turn, might increase the wages of the adults, i.e., even if $\tau_i^t > 0$ it can be rational for adult $i$ to vote in favor of an education program.\footnote{However, we do not allow for labor mobility in the sense that taxed households may leave the country (cf. Tiebout (1956)). On the economy-wide level, this assumption is plausible for poor countries.} Furthermore, the decision may be influenced by some kind of inequality aversion and by envy etc. However, at the moment we neglect these aspects, but we will come back to this at the end of the chapter. We again use $T$ to denote the number of periods a democratic society needs to educate itself.

## 5.5 Democracy with a Benevolent Agenda Setter

We now investigate whether such a simple democratic process will enable the education of a society if the sequence of proposals or agendas is determined by a benevolent institution with the sole objective of educating the society. The institution is completely informed about technologies and the preferences of households. Furthermore, we do not highlight the legitimation of the institution. Nevertheless, the institution has to face elections whenever policy actions should be undertaken. Within this election a policy proposal needs to be legitimated by the required majority of votes stated in the constitution. Such a democracy is called a **democracy with dictatorial agenda setting (DA)**. Suppose that the government wants to educate the society in $T$ periods. On average, in each period there is a fraction $\frac{1}{T}$ of the society that must be subsidized in such a way that those households will choose full education for their child, i.e. $e^o(\alpha + s_t)$ equals unity. Accordingly, each supported household is paid subsidy $\overline{s} = \alpha(\lambda^a - 1)$.

We must distinguish three possible cases. The level of human capital an individual possesses in the period immediately after receiving $\overline{s}$, $z_h(1) + 1$, may be below, above, or equal to $\lambda^*$. If it is above $\lambda^*$ it may be below or above $\lambda^a$. We restrict our attention to proposals that either tax or subsidize a single adult. For a proposal to be accepted
in period $t$, the maximum fraction of the society to be taxed is $\frac{1}{2}$ because otherwise a majority would vote against the tax-and-subsidy policy. Thus, the fraction of taxed households, $1 - \phi_t$, is at most $\frac{1}{2}$. We construct a sequence of proposals $\{P_t\}_{t=0}^{T-1}$ such that the whole society can be educated. In the following, $\delta_t$ denotes the share of subsidized individuals in a period $t$. We turn first to the case $zh(1) + 1 > \lambda^a$, and obtain:

**Lemma 5.1**

A democracy with a constitution $C\{BB, DA, MV\}$ can educate a society in finite time, i.e. $T < \infty$, if $zh(1) + 1 > \lambda^a$.

**Proof:**

In period $t = 0$, all households display $\lambda = 1$. Consider the following agenda in $t = 0$:

$$P_0 = \begin{cases} 
\delta^i_0 = \bar{s} & \forall \ i \in [0, \delta_0]; \\
\upsilon^i_0(\alpha \lambda^i) = 0 & \forall \ i \in (\delta_0, \frac{1}{2}]; \\
\tau^i_0(\alpha \lambda^i) = \tau^{sub} & \forall \ i \in (\frac{1}{2}, 1].
\end{cases} \tag{5.11}$$

The tax revenue of the first period, $R_0$, then amounts to:

$$R_0 = \frac{1}{2} \tau^{sub} = \frac{1}{2} \left[ \alpha (1 + \gamma) - (1 + \beta) c^{sub} \right]$$

Due to the rule BB we obtain $\delta_0 = \frac{\tau^{sub}}{2\bar{s}}$ (balanced budget). In all following periods, already subsidized households can be taxed in period $t$ by $\tau_t = \alpha \lambda_t - \alpha \lambda^a$, so that they still choose full education for their children. The fraction of households that still live in a state of backwardness can be taxed by $\tau^{sub}$. Proposals will only be accepted if at least half of the households are not taxed. Accordingly, a benevolent agenda setter is always able to collect a strictly positive tax revenue by setting proposals that are accepted by the majority. This tax revenue amounts at least to:

$$R_t \geq \frac{1}{2} \min \left\{ \alpha (zh(1) + 1) - \alpha \lambda^a, \tau^{sub} \right\} \tag{5.12}$$

Therefore, in every period $t$, the share of subsidized individuals is bounded from below by:

$$\delta_t \geq \frac{\min \left\{ \alpha (zh(1) + 1) - \alpha \lambda^a, \tau^{sub} \right\}}{2\bar{s}} \tag{5.13}$$

Since the expression on the right hand side is greater than 0, the time required to educate the society is at the most

$$\frac{2\bar{s}}{\min \left\{ \alpha (zh(1) + 1) - \alpha \lambda^a, \tau^{sub} \right\}},$$

and thus finite, if $zh(1) + 1 > \lambda^a$. 

\[\square\]
The essential point of Lemma 5.1 is that a benevolent agenda setter can shift taxation and subsidization of households over time such that poor subsidized parents send their children to school fulltime, while wealthier taxed parents are not taxed excessively so that they send their children to school fulltime despite the tax burden. A concrete example of the case $T = 3$ is given in the appendix. It is easy to extend our analysis to the case $\lambda^* < zh(1) + 1 \leq \lambda^a$. Families, once subsidized, pass $\lambda^a$ in finite time if they are not taxed. We now denote the minimal number of periods by $r$, so that $\lambda_{t+r} > \lambda^a$ when $\lambda_t = zh(1) + 1$, and households are not taxed in the meantime. Then our argument applies for all periods $0, r, 2r, \ldots, (N - 1)r,^{15}$ and hence the time needed to educate the society is again finite. We summarize our observation in the following lemma.

**Lemma 5.2**

A democracy with a constitution $C\{BB, DA, MV\}$ can educate a society in finite time, i.e. $T < \infty$, if $zh(1) + 1 > \lambda^*$.

If we only consider one-time subsidization of a single household, and $\lambda^* \geq zh(1) + 1$, then the society is caught in the poverty trap or in the medium steady state at $\lambda^*$. Since the growth-rate of human capital is non-positive after one-off subsidization, the human capital of a lineage will decline toward backwardness over time, or –without the possibility of taxing these households– remain at $\lambda^*$. Multiple subsidizing of a single lineage, however, will accumulate the household’s human capital to a level higher than $\lambda^*$ in, say, $l$ periods. After $l$ periods, a single household crosses the threshold value $\lambda^*$, and Lemma 5.2 applies for all periods $0, l, 2l, \ldots, (N - 1)l$. We thus obtain the general result that the education of the society is possible in finite time, irrespective of $zh(1) + 1 \geq \lambda^*$:

**Proposition 5.1**

A democracy with a constitution $C\{BB, DA, MV\}$ can educate a society in finite time, i.e. $T < \infty$.

### 5.6 Democratic Agenda Setting

We now turn to democratic agenda setting. The first step to undertake is to determine the rule by which the agenda setter is chosen. We do not consider electoral competition in the Downsian sense of probabilistic voting. We are interested in the situation where

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15Here, $N$ means the number of periods needed to educate a society in the case where $zh(1) + 1 > \lambda^a$. 

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there is an agenda setter who makes a proposal that requires the constitutionally stated majority to pass legislation. Hence we specify a simple agenda setter selection:

- **Random Agenda Setter (RA):** In each period, every single adult has the same chance to make a proposal. Hence, the agenda setter of a period \( t \) is selected randomly from the population of adults.

Though random selection might appear unusual today, this kind of democracy goes back to the historical roots of Athenian democracy. Actually, Aristotle emphasized that it lies in the nature of democracy that decision makers are chosen by lot. Random selection is commonly seen as a decision rule that is generally accepted by individuals. In recent literature in political science and political economy selecting an individual at random to make a proposal is also common, see, for instance, BARON AND FEREJOHN (1989) and HARRINGTON (1986). Selecting the agenda setter by a lot represents a neutral recognition rule, that is, a rule that does not bias the result in favor of any member of society.

### 5.6.1 The Impossibility Result

The only agenda setting restriction we impose is that the agenda setter has to respect the subsistence level, the balanced budget rule, and the simple majority rule.

**Proposition 5.2**

A democracy with \( C \{BB,RA,MV\} \) can not educate a society in finite time, i.e. \( T = \infty \).

**Proof:**

If individual \( i \) is recognized as agenda setter in a particular period \( t \), he will tax half of the population as highly as possible in order to create the highest possible subsidies for himself. Then a winning majority is still ensured. Since there are no restrictions other than retaining a consumption level \((1 + \beta)c^{sub}\), half of the population entitled to vote is taxed: \( \tau_t(\alpha \lambda_t) = \alpha \lambda_t - (1 + \beta)c^{sub} \). It is rational to tax former subsidized households most heavily because they can pay the highest taxes. Therefore, children of...

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17See Parkinson (1958) and Rousseau (1762), Book IV, Chapter III (cf. Mueller, Tollison, and Willett (1972), p. 60)


19For a detailed analysis of selecting legislation at random from a voting population, see Mueller, Tollison, and Willett (1972) or Dahl (1970) (see also Bohm (1971), Mueller, Tollison, and Willett (1973), or Ward (1969)).

taxed households will not be educated at all, no matter how well-educated the parents are. Thus, in each single period $t$, half of the children do not attend school, i.e. $T = \infty$.

Proposition 5.2 is a dynamic variant of the general characteristic of majority voting rules to the effect that majorities can expropriate minorities ("tyranny of the majority").\textsuperscript{21} Bernholz and Breyer (1994), for example, show that the majority voting rule fails to produce just income distribution since the resulting majority (winning coalition) exploits the rest of the society.\textsuperscript{22} Mueller (1979) deals with Riker’s (1962) hypothesis that, in a zero-sum redistribution game, the majority voting rule implies one minimum-winning coalition, and another, one vote smaller, that is used as a losing coalition that pays.\textsuperscript{23}

In our context, this generates a large degree of dynamic inefficiency, since in the future every educated household will belong to a minority and therefore the society cannot be educated. Accordingly, Hayek (1960) and Buchanan and Tullock (1962) discussed the necessity of super-majority rules to protect minorities and to prevent excessive social costs.

### 5.6.2 Democratic Constitutions

In the last subsection, we saw that constraints on redistribution proposals are necessary to fully educate a society under random agenda setting. In the following subsections, we show how these problems can be solved in democracies.

Seizing the idea of super-majority rules, Erlenmaier and Gersbach (1999) and Gersbach (2004) introduced flexible majority rules for the provision of public goods. Under flexible majority rules the required majority depends on the proposal itself. Flexible majority rules can be used for achieving at least two targets. First, it can be utilized for applying the result of Wickström (1986b), which is that decisions of varying importance establish varying optimal majorities.\textsuperscript{24} Second, they can be utilized for protecting certain groups like minorities. In our context, we will use flexible majority rules to limit the taxation of educated households so that they do not relapse

\textsuperscript{21}E.g., this is the typical result in the Downs model, see Downs (1957) or Rodgers (1974).
\textsuperscript{22}See Bernholz and Breyer (1994), Subsection 11.3.
\textsuperscript{23}See Mueller (1979), Chapter 6, Section E, pp. 116-117.
\textsuperscript{24}See also Wickström (1986a) and Tullock (1986).
into poverty. We define

\[ \tau^\text{max}_t = \max_{i \in [0,1]} \tau^i_t. \]

- **Threshold flexible majority rule** (TFM(\(\tau^\text{max}_t, \tau\))): The required majority \(m(\tau^\text{max}_t, \tau)\) jumps from \(\frac{1}{2}\) to 1 if any household \(i\) is taxed higher than the threshold tax \(\tau\) stated in the constitution:

\[
m_t(\tau^\text{max}_t, \tau) = \begin{cases} 
\frac{1}{2} & \text{if } \tau^\text{max}_t \leq \tau; \\
1 & \text{if } \tau^\text{max}_t > \tau.
\end{cases}
\]

I.e., as soon as a citizen is adversely taxed (i.e., taxation prevents full-time basic schooling) the constitution demands a super-majority. To ensure that not a single household is taxed adversely and falls back into poverty, we must demand unanimity. Thus, our TFM rule combines the advantages of the majority rule and the unanimity rule and, at the same time, alleviates their difficulties in finding collective decisions.\textsuperscript{25}

An alternative constitutional principle suitable for overcoming the problem of excessive taxation could be to establish a taxpayer protection rule. Such protection has been broadly discussed in constitutional law in the context of the protection of property rights. Moreover, such taxpayer protection is ubiquitously provided by the existence of exemption levels and upper limits on marginal tax rates.\textsuperscript{26} In our context, the educated citizens must be protected to ensure that an income of \(\alpha \lambda\) is guaranteed. That is, we have to add a second exemption to ensure that full-time schooling is provided. Therefore, we introduce an education allowance of size \(\alpha \lambda\).\textsuperscript{27,28} In our model, this education allowance must be contingent on the education level of the household, as otherwise, initially, there would be no possibility of taxation. We define


\textsuperscript{26}In Germany, for instance, the “Halbteilungssgrundsatz” proposed by former constitutional judge Paul Kirchhof states that at most half of the income can be taken away by governmental policy as a whole. In March 1983, the Second Senate of the German Constitutional Court (Bundesverfassungsgericht) declared tax burdens that are excessive and basically impair wealth to be unconstitutional because of Article 14 of the German Basic Law (Grundgesetz) [see GG (1949) or Basic Law (1949)]. Already in 1891, Pope Leo XIII declared excessive taxation to be illegal in his encyclica “Rerum Novarum”. Cf. Reding and Müller (1999), Chapter 14.

\textsuperscript{27}Note that we so far implicitly used an exemption level of \(c\text{sub}\) as basic tax-free amount. This exemption can be increased by the education allowance.

\textsuperscript{28}In the German income tax, for instance, parents are guaranteed an education allowance for their children’s education by § 33a, clause 2. Cf. EStG (2004).
• **Claim on Education Allowance (CEA[Ε])**: Each household $i$ that can prove that it has completed basic education, that is $\lambda_i \geq \lambda^a$, has a claim on an education allowance amounting to $E > c_{sub}$.

Notice that tax allowances are working at two levels, the constitutional and legislative level. At the constitutional stage, the taxpayer protection may remain abstract in practice, while the detailed size of allowance is only written in specific laws. For laws are much easier to change, the protection from excessive taxation is weaker. Therefore, the constitutional rule ought to be stated more precisely in constitutions than it is the case today.

We moreover introduce agenda setting by coalitions representing interest groups, parties, or a single region of the country. We suppose that the fraction of households setting agenda, labeled $\Delta$, is constant in the course of time. We define

- **Rotating agenda setting (RoA)**: In each period $t$ a fraction $\Delta > 0$ of the adult society has the power to set the agenda. Once a household has joined a coalition it is excluded from the agenda setting process for all time.

I.e., lineages that have set the agenda in a particular period in time-interval $[0, t]$, are excluded from the agenda setting process in all future periods. In practice, this means that the number of allowed reelections is restricted, possibly to zero. In ancient Athens or the ancient Roman Republic, for instance, the constitutive principle of democracy was giving over power from citizen to citizen. BLEICKEN (1991), p. 192, finds that, due to this rotation rule, more or less all Athenians participated in the town’s sense-making process in the course of time. This is exactly the idea we follow. Given the RA rule, the fixed fraction $\Delta$ is selected randomly from the set of lineages which still have the right to set the agenda. It follows that the only period in which a household can expect to enjoy a subsidy is the period in which it has been selected to determine the agenda. Basically, it is plausible for a coalition of agenda setters to distribute tax revenues equally among themselves, and we will assume this in the...
following.\footnote{We do not explicitly analyze how the group decides upon an agenda; it suffices to know that they will maximize tax revenues for the group and divide them equally. For instance, one might think of the group as representing a party, an interest group, or simply one person.} We additionally assume that \( s_t = \frac{R_t}{\Delta} \) is at least as high as \( \overline{s} \). Otherwise, the size of \( \Delta \) must be reduced.

### 5.6.2.1 When the Technology of Human Capital is Sufficiently Productive

**Lemma 5.1**

A democracy with \( C\{BB, RA, TFM(\tau_t^{\max}, \overline{\tau}), RoA, MV\} \), where \( \Delta < \frac{R_t}{\overline{\tau}} \) and \( \overline{\tau} = \min\{\tau_{\text{sub}}, \alpha(zh(1) + 1 - \lambda^a)\} \), can educate a society in finite time, i.e. \( T < \infty \), if \( zh(1) + 1 > \lambda^a \).

**Proof:**

We construct the flexible majority rule as follows:

\[
m_t(\tau_t^{\max}, \overline{\tau}) = \begin{cases} \frac{1}{2} & \text{if } \tau_t^{\max} \leq \overline{\tau}; \\ 1 & \text{if } \tau_t^{\max} > \overline{\tau}; \end{cases}
\]

(5.14)

where \( m_t(\tau_t^{\max}, \overline{\tau}) \) denotes the required majority depending on the maximum tax rate levied on the households. With tax threshold \( \overline{\tau} \) at

\[
\overline{\tau} = \min\{\tau_{\text{sub}}, \alpha(zh(1) + 1 - \lambda^a)\}.
\]

(5.15)

the flexible majority rule guarantees that taxation that would prevent full-time schooling in any yet subsidized household is impossible,\footnote{Recall that \( \tau_{\text{sub}} \) is the highest taxation allowed for households in a state of backwardness, and that \( \alpha(zh(1) + 1 - \lambda^a) \) is the highest tax burden for an already subsidized household that does not endanger full-time schooling.} since adverse taxation of any yet subsidized household requires unanimity. Knowing tie-breaking rule TR, the coalition of agenda setters leave half of the society untaxed in order to form a winning coalition. They use all tax revenue for themselves and due to \( \Delta < \frac{R_t}{\overline{\tau}} \), the households of the agenda setting coalition receive a subsidy that cause fulltime schooling of the children of the coalition households. For \( zh(1) + 1 > \lambda^a \), rule TFM(\( \tau_t^{\max}, \overline{\tau} \)) guarantees sustainable, full-time education for the offspring of households that have set an agenda.

We know \( \Delta > 0 \). As re-nominations are not allowed (rule RoA), all households will have set agenda in finite time. It is easy to find the corresponding \( N < \infty \) that fulfills \( \Delta = \frac{1}{N} \). Consequently, after \( T = N \) periods the education of the society is attained in a finite span of time.

\[\square\]

Alternatively, we can utilize a tax allowance instead of the TFM rule:

\[\]
Lemma 5.2
A democracy with $C\{BB,RA,CEA[E],RoA,MV\}$, where $E = \alpha \lambda^a$ and $\Delta < \frac{R}{\tau}$, can educate a society in finite time, i.e. $T < \infty$, if $zh(1) + 1 \geq \lambda^a$.

Proof:
The CEA[E] rule establishes that an income of $\alpha \lambda^a$ is guaranteed as soon as a household has received subsidy $\tau$ in a former period. Therefore adverse taxation is not constitutional. The rest of the proof follows from the observations described in the proof of Lemma 5.1.

The upshot of Lemmata 5.2 and 5.1 is that exemptions or flexible majority rules both prevent adverse taxation, so that educated lineages cannot fall back into illiteracy. Note that $E$ and $\tau$ are determined by exogenous parameters. Hence, in practice, it should be possible to fix them by a precise constitutional rule.36

Flexible majority rules can also be more sophisticated. Suppose we define different thresholds for the subsidized households and for those in a state of backwardness. Given $zh(1) + 1 > \lambda^a$, there are only households displaying $\lambda_i = 1$ or $\lambda_i > \lambda^a$. Accordingly one can state:

$$\tau^i_t = \begin{cases} \tau^1_i = \alpha (\lambda^i_t - \lambda^a) & \forall i \text{ with } \lambda^i_t > \lambda^a; \\ \tau^2_i = \tau^{sub} & \forall i \text{ with } \lambda^i_t = 1; \end{cases} \quad (5.16)$$

Then the flexible majority rule is given by:

- **Flexible majority rule with multiple thresholds** (mTFM($\tau_t$, $\tau_t$))37:

$$m_t(\tau^i_t, \tau^j_t) = \begin{cases} \frac{1}{2} & \text{if } \tau^i_t \leq \tau^j_t \text{ for all } i \in [0,1]; \\ 1 & \text{if } \tau^k_t > \tau^l_t \text{ for any } k \in [0,1]; \end{cases} \quad (5.17)$$

for all $t$.

Under such flexible majority rules, the necessary majority is $\frac{1}{2}$ if the agenda setter does not tax any former subsidized household higher than $\alpha (\lambda^i_t - \lambda^a)$ and any not-yet-subsidized household higher than $\tau^{sub}$. Otherwise the constitution levies the unanimity requirement upon the agenda setter. In period 0, the society is poor and there is no major tax potential. But the tax potential is increasing due to education. Obviously, the tax revenue increases over time compared to the case with the TFM rule. In our setting, this would only increase the transfer per coalition member. However, if the

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36 An issue not pursued here is how to determine $\tau$ so as to minimize the time $T$ a society needs to educate itself.

37 $\tau_t = (\tau^i_t)_{i=0}^1$ and $\tau_t = (\tau^i_t)_{i=0}^1$. 

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size of coalition $\Delta$ could be augmented correspondingly, the multiple thresholds flexible majority rule allows for a quicker accomplishment of the education of the society.\footnote{Note that the inequality within a generation would also be lower, which decreases the probability of social unrest in the real world.} Suppose we extend the agenda setting $m$TFM by the following rule:

$$\Delta_t = \frac{R^{\text{max}}_t}{s},$$

where $R^{\text{max}}_t$ is the maximum tax revenue for the case with no unanimity requirement, i.e., the thresholds $\tau^i$ are not crossed for any $i$. Consequently, the corresponding $T$ will be lower than using only the TFM rule, as $R^{\text{max}}_t$ should be increasing in $t$: the society is educated earlier. This means that one must start with small “islands” (for instance a certain region or particular social group) that are given support. Then, over time, one can increase the islands in size when the tax revenue increases.

### 5.6.2.2 When the Technology of Human Capital is Not Sufficiently Productive

In the case where $zh(1) + 1 < \lambda^*$, the schooling technology is not productive enough for full-time schooling in one period to bear the required income necessary for full-time schooling in the next period following subsidization.

As long as $zh(1) + 1 > \lambda^*$, the once subsidized households do not fall back into the poverty trap, if they are not taxed so strongly that the adult income would fulfill $\alpha \lambda^*_t - \tau^i_t \leq \alpha \lambda^*$. The education allowance would save such households from taxation. In case of TFM, we have $\tau < 0$; this could be understood as a claim on subsidies. The modification that negative $\tau$ means that taxation is prohibited would also save the household from taxation, without any claim on subsidies via $\tau < 0$. In both cases the households would accumulate human capital and cross threshold $\lambda^a$ over time. Consequently, in principle, our results of the previous section hold. However, as we assume that $e^i_t = 1$ for all households is socially optimal, the policy maker should go on with subsidizing these households.

If $zh(1) + 1 \leq \lambda^*$, full-time schooling in one period does not allow escape from the poverty trap area $[1, \lambda^*]$. Consequently repeated subsidization is definitely required. Therefore the previously derived constitutions do not enable to escape the poverty trap, that is, Lemmata 5.2 and 5.1 do not hold. Hence, we need further constitutional principles. We use $r$ to denote the minimum number of periods a continuously subsidized household needs to accumulate human capital higher than $\lambda^a$, when the household receives subsidy $s_t$ in each period, beginning in period $t$. Given $s_t = \tau$ is the subsidy of
the agenda set accepted in period $t$, scalar $r$ is given implicitly by: \(^{39}\)

$$
\min_{r>0} \left\{ \lambda_{t+r} = \sum_{k=0}^{r} \left\{ zh[e(\alpha + s_t)] \right\}^k > \lambda^a \right\}
$$

Accordingly, we introduce:

- **Agenda repetition** ($AR(r)$): An agenda set in a period $t$ has to be repeated $r$ times in the subsequent periods. Therefore, an agenda setting coalition is only selected every $r$ periods.

It follows:

**Proposition 5.3**

A democracy with $C\{BB,RA,CEA[\mathcal{E}],RoA,MV,AR(r)\}$, in which $\mathcal{E} = \alpha\lambda^a$, can educate a society in finite time, i.e. $T < \infty$, irrespective of $zh(1) + 1 \gtrless \lambda^a$.

**Proposition 5.4**

A democracy with $C\{BB,RA,TFM(\tau_{max}^{t}, \tau),RoA,MV,AR(r)\}$, in which \(^{30}\)

$$
\tau = \min \{ \tau^{sub}, \alpha(\lambda_{t+r}^a - \lambda^a) \},
$$

can educate a society in finite time, i.e. $T < \infty$, irrespective of $zh(1) + 1 \gtrless \lambda^a$.

**Proof of Propositions 5.3 and 5.4:**

The constitutional rule $AR(r)$ transplants the idea of multiple subsidization, explained for Proposition 5.1, into a constitution: the subsidized households receive transfers as long as they do not have an income higher than $\alpha\lambda^a$. Lemmata 5.2 and 5.1, therefore, apply for all periods $0, r, 2r, \ldots, (T-1)r$. \(^{41}\) Hence, the society will overcome child labor and poverty through education in finite time. \(^{42}\)

\[^{39}\] Of course, the size of subsidy $s_t$ could be lowered from period to period, because the level of human capital of subsidized households increases. However, for a constitutional rule this might be too specific.

\[^{30}\] Time index $t$ represents the period in which an agenda set has been accepted.

\[^{41}\] Here $T$ means the number of periods needed to educate a society in the case of $zh(1) + 1 \geq \lambda^a$.

\[^{42}\] In the case of $\lambda^* < zh(1) + 1 < \lambda^a$, it would be sufficient to introduce a stop-over condition ensuring that the dynamic agenda setting process is interrupted as long as $\lambda_{t+r}^i < \lambda^a$ for any any-time subsidized household. However, then there are children that are not enjoying full-time schooling, which is, by assumption, not socially efficient.
Having established these results we can state which constitution allows for educating the society in a shorter span of time. In doing so, we assume that there is the constitutional rule that the size of the agenda setting coalition depends on the maximum tax revenue that is achievable, which we labeled $R_t^{\max}$.

**Proposition 5.5**

Suppose $\Delta_t = \frac{R_t^{\max}}{s}$. A democracy with $C\{BB, RA, RoA, MV, AR(r)\}$ and the additional rule $CEA[\alpha \lambda^a]$ can educate a society in a shorter span of time than with the additional rule $TFM(\tau_t^{\max}, \tau)$, in which $\tau = \min\{\tau^{\text{sub}}, \alpha(\lambda^a_{t+r} - \lambda^a)\}$.

**Proof:**

With rule $TFM(\tau_t^{\max}, \tau)$ and $\tau = \min\{\tau^{\text{sub}}, \alpha(\lambda^a_{t+r} - \lambda^a)\}$ the maximum tax revenue per adult is limited by $\tau$. In the case of the education allowance, this maximum revenue is limited by $\tau^{\text{sub}}$ for all not-yet subsidized households and by $\alpha(\lambda^a_{t+r} - \lambda^a)$ for all yet subsidized households. When revenue per capita rises, then $R_t^{\max}$ increases, so that more poor households can be subsidized per period, that is, coalition $\Delta_t$ rises. If $\tau^{\text{sub}} = \min\{\tau^{\text{sub}}, \alpha(zh(1) + 1 - \lambda^a)\}$, then the education allowance rule allows to tax yet subsidized households more strongly, while not-yet subsidized households are taxed equally. If contrary $\alpha(zh(1) + 1 - \lambda^a) = \min\{\tau^{\text{sub}}, \alpha(zh(1) + 1 - \lambda^a)\}$, the education allowance rule allows for heavier taxation of not-yet subsidized households, while yet subsidized households are at least taxed equally as much: if $\lambda^a_{t+r} = zh(1) + 1$ the corresponding households are taxed equally, and if $\lambda^a_{t+r} > zh(1) + 1$ the education allowance rule allows stricter taxation of the corresponding households. Ergo, the CEA($\alpha \lambda^a$) rule allows for the education of a society in a shorter span of time than $TFM(\tau_t^{\max}, \tau)$ does, because $\Delta_t = \frac{R_t^{\max}}{s}$ is, on average, bigger.

It is clear that agenda setters may propose unconstitutional policies and might even find a simple majority for them. Nevertheless, the application of such a policy is inadmissible. Hence, it is essential that an efficient constitutional court enforces the constitutional rules. The idea behind this is that citizens that are excessively taxed can sue for due consideration of their claim on educational allowance or, if rule $TFM$ is violated, for a ruling that the policy has been unconstitutionally established. This will (in most cases) force the agenda setter to accept the democratic principles and to re-establish the constitutional frame.
5.7 Other Concepts of Preferences

In this section, we briefly discuss empirical evidence for human preferences, which are not determined by pure self-interest, in material sense. We will then deduce the constitutional design required to educate a society in democracy, given that preferences weakly deviate from standard theory.

5.7.1 Evidence

Economists in general undertake investigations under the premise that individual behavior is solely motivated by self-interest. In most cases this premise is adequate and a very helpful simplification, as it allows us to concentrate on the relevant motivation of behavior in economic situations.

However, already the inventor of the assumption of narrow self-interest, John Stuart Mill, said that this description of men’s behavior “...does not treat of the whole of man’s nature by social state, nor of the whole conduct of man in society.”\(^{43}\) Accordingly, in sharp conflict with the narrow self-interest assumption, experiments like the ultimatum and dictator game provide a considerable amount of evidence that humans are willing to voluntarily share wealth with strange people who have no power to influence the outcome, though this means that their consumption possibilities diminish. Overall, it was found that *fairness, intention of actions, manners, altruism, social concerns, the desire to avoid social disapproval, reciprocity, and/or inequality aversion* determined the outcome additionally to self-interest [see Camerer and Thaler (1995), Charness and Rabin (2002), Charness (1998), Falk (2003), Falk and Fischbacher (2001), Fehr and Falk (2002), Fehr and Gächter (2000), Fehr and Henrich (2003), Fon and Parisi (2003, 2002), Kreps (1997), Persky (1995), Rabin (2002), Raut and Tran (2001), Segal and Sobel (1999), Sobel (2001), Tyran and Sausgruber (2002)]. As said in Fehr and Falk (2002), p. 688, taking these results “...into account one acknowledges human beings as social beings.” Nonetheless, the self-interested *homo oeconomicus* that is solely interested in increasing its payoff in terms of wealth is also found to be widespread. Hence, one developed models of *social preferences*, where people are self-interested, but are also concerned about others: *difference-aversion models, social-welfare models, reciprocity models.*\(^{44}\)

Concerning political actions, Bartels and Brady (2003) stress that narrow self-

interest is too weak to account for a great deal of political behavior. Experiments presented in Rabin (2002) demonstrate that individuals who have to decide upon allocating wealth amongst two other, unknown persons want to help these parties and care about both social efficiency and equality. Therefore, we will focus on the possibility of caring about others. In economics, it is natural to care about others’ well-being by assuming some degree of altruism. Following Rabin’s (2002) notation, altruism can be general, that is, one cares about all others, or specific, where one only cares about certain other groups or individuals. So far, we considered the specific altruism that the adult of a household cares about the child. We now additionally will assume that there is also a weak type of general altruism.

5.7.2 Lexicographically Dominated Social Concerns

We again assume that individuals are primarily self-interested and display a certain degree of altruism towards their children. However, adapted from the above mentioned results of the experiments in Rabin (2002), we assume that individuals have lexicographic preferences in the sense that while they are primarily interested in their family’s and own advantage, they also, secondarily, are interested in the education of the society as a whole (because this is socially efficient). We call this characteristic social concerns. For our purpose, we define lexicographically dominated social concerns as follows:

**Definition 5.1**

*Lexicographically dominated social concerns* prevail if an individual as agenda setter is interested in the education of the society as a whole, but this social concern is dominated by pure self-interest.

That is, we assume that in each individual there is the *good dictator* described in Plato’s Politeia, but that this benevolent agenda setter is suppressed by pure self-interest. Applied to our redistribution task, these lexicographically dominated social concerns mean that as soon as an adult who is an agenda setter cannot increase individual wealth, she neither endangers the success of the reform by taxing already subsidized

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45With reference to Adam Smith, one might also think of using the term *public spirit*, see Smith (1976, 1994b), but our assumption is weaker, since individuals only have social concerns when they cannot increase their own well-being any more.

46The Greek philosophers distinguished three types of constitutions: (i) autarchy (ii) dominion of the elite, and (iii) dominion of the people, which is respectively called (i) monarchy, (ii) aristocracy or (iii) democracy when it is good for the state and (i) tyrannis, (ii) oligarchy or (iii) ochlocracy when it is bad for the state. Of course, Plato’s first-best form of government was aristocracy and the good dictator ought to be a wise philosopher.
households to an extent where they would fall back into the poverty trap nor does she forego tax revenues just because she cannot channel them to herself. Moreover, all tax revenue that cannot be used for herself is then distributed to poor households so that these establish socially efficient full-time schooling and escape the poverty trap.

In our political economy analysis, we consider the case of random agenda setting. Hence, it is possible that some households will never set an agenda. Without subsidy ceiling, the agenda setter will use all tax revenues for herself, and the society might not be educated in finite time, that is, our impossibility result also holds when our type of social concerns prevails. Suppose we therefore constrain the subsidization possibilities by setting a subsidy ceiling as an agenda setter rule SC. We denote the agenda setter’s level of human capital in period $t$ by $\lambda_{ag}^t$.

- **Subsidy Ceiling (SC):** The agenda setter is not allowed to pay subsidies to herself that are higher than $s_{t}^{\text{max}}$, with

\[
s_{t}^{\text{max}} = \begin{cases} 
\alpha(\lambda^a - \lambda_{ag}^t) & \text{if } \lambda_{ag}^t < \lambda^a; \\
0 & \text{if } \lambda_{ag}^t \geq \lambda^a.
\end{cases}
\]

Therefore, the constitution allows the agenda setter to subsidize herself such that full-time schooling for the agenda setter’s child is ensured, but not more. It is clear that the tax revenue suffices for the agenda setter to receive subsidy $s_{t}^{\text{max}} \leq \bar{s}$, that is, $\Delta_t \bar{s} < R_t$ because $\Delta_t \to 0$. We thus obtain

**Proposition 5.6**

A democracy with constitution $C\{\text{BB,RA,MV,SC}\}$ and preferences according to Definition 5.1 can educate a society in finite time, i.e. $T < \infty$.

**Proof:**

Due to primary selfish preferences the agenda setter collects taxes and uses $s_{t}^{\text{max}}$ for herself. Having done that, the agenda setter cannot use subsidies for herself anymore. The level of selfish preferences is turned off and the secondary level of social concerns is activated. Therefore, she will collect taxes beyond ceiling $s_{t}^{\text{max}}$ as much as possible. In doing so, the agenda setter will, by Definition 5.1, not tax excessively and pays subsidy $\bar{s}$ to backward households. Thus, the fraction of educated households increases monotonously in the course of time. Therefore, after $T < \infty$ periods, the society is educated. If repeated subsidization is necessary, then this will be taken into account.

\[\square\]

\[47\]The case $\lambda_{ag} \geq \lambda^a$ resembles the “benevolent dictator” agenda setting discussed in Section 5.5, since social concerns would dominate the agenda setting.
Proposition 5.6 indicates that a weak deviation from the former type of preferences towards social concerns speeds up the education process and demands for a less restrictive constitutional design: a constraint on subsidies is sufficient to educate a society. That is, other characteristics of preferences might ask for another constitutional design and thus the required constitutional design is sensitive to deviations from the assumed preferences. If individuals are envious of the higher wealth of other people, for instance, a poor agenda setter might excessively tax somewhere along the way subsidized households. Then, again, constitutional rules like the threshold flexible majority rule or education allowances would solve this problem. When people are behaving reciprocally, for example, adults consider how particular households have dealt with them in the past. If households are “nice” to them, this will be rewarded and if households are “mean” to them, this will be punished, even if this involves a loss of wealth. In this context, reciprocal individuals take into account whether the household in question really had in mind to behave well or badly, or whether there was simply no choice to behave differently. Moreover, concerning people so far unknown, one has to consider expectations about how these people will behave in the future. Accordingly, analyzing the political economy of redistribution in a society of reciprocal adults is a very interesting but complex future research task.

The second broad field, difference-aversion and fairness, we address in the next session.

After all, this section was only a first small step in the research of the implications of preferences that deviate from the premise of, in a material sense, solely selfish households (that display altruism towards children). Our simple analysis has shown that such deviations might have strong implications, and that it is worthwhile to investigate them. Coming back to the introductory quote of Machiavelli, however, we should keep in mind that constitutions should cover the worst possible case. Many humans actually have social concerns and a public spirit, but it is not clear whether, at the end of the day, the public spirit is sacrificed in favor of self-interest or not, when, in real world situations, policies levy burdens on individuals. Furthermore, it is clear that there are also other humans that behave solely according to pure self-interest or, even worse, by intentions of negative reciprocity, envy, racism and the like. Consequently, all these patterns of behavior are able to cause the failure of the project and may therefore ask for additional constitutional rules.

48 Note that, without social concerns, the SC rule cannot be used to speed up the process of education, because within such an environment the agenda setter has no incentive to collect more taxes than required to receive the subsidy ceiling.

49 Recent research in the field of reciprocity and voting is Hahn (2004) and Hahn and Mühe (2004).
5.8 Sources of Political Failure

Our model identifies a variety of causes why the education of a society may fail. In this section, we discuss these and additional sources of political failures, and how constitutions might prevent the corresponding failure.

Expropriation of educated people

Adverse taxation of educated adults may take place, inducing those households to cut back on education; then human capital reverts to a state of backwardness. We have shown that allowances and threshold flexible majority rules solve this problem. As soon as an agenda setter suggests an adverse tax scheme, the constitution requires unanimous agreement which, de facto, makes such taxation impossible to implement. In the case of an educational allowance, adverse taxation is unconstitutional because the amount of income that is necessary for full-time basic schooling is free of taxation.

Ineffective subsidization

The subsidies poor adults receive might be too low to escape the poverty trap when the technology of human capital is too unproductive, so that even full-time schooling is not sufficient. We have shown that repeated subsidization of single households can solve this problem, so that we have to explicitly add the agenda repetition rule to the constitution. This ensures that a new agenda must wait until all supported households enjoy full-time schooling for their children. In the meantime, the old agenda is repeated to ensure that the supported households cross the adverse threshold $\lambda^*$. This demonstrates that the time-horizon of educational reforms might comprise generations.

In this context, it is also conceivable that the agenda setter tries to buy votes by paying small subsidies\textsuperscript{50} that do not suffice to leave behind the poverty trap.\textsuperscript{51} If the agenda is set by a coalition of individuals or groups, ineffective subsidization within the coalition is also possible, but will only occur if these individuals do not have the power to participate symmetrically.

Finally, ineffective subsidization may occur because the government wants to maximize school attendance rates and neglects the fact that sustainable success will only be achieved if the quality of schooling and the time individually spent in it is sufficient.

\textsuperscript{50}Facing democratic elections, government parties often use money for social or labor market programs, or the like, to influence voting behavior without any long-term effect.

\textsuperscript{51}In our framework, adults accept an agenda as long as they are not taxed, so that vote-buying is not necessary.
i.e. households accumulate human capital of size $\lambda^* + \epsilon$. Although such a myopic strategy is inefficient in the long run, it can be quite advantageous for a politician in the short term, since he satisfies more voters. This problem is widespread and occurs in all reform projects that involve costs at first but revenues mainly in the future.

*Incomplete subsidization*

There may be households that never enjoy any subsidization. One constitutional rule preventing incomplete subsidization is the rotating agenda setting, which means that each household that has set an agenda is excluded from the agenda setting process. In practice, this would mean limiting the number of allowed re-elections (not just of persons but also of interest groups). Hence, every household will be part of an agenda setting coalition at some point in time and, therefore, enjoy subsidies. It is also conceivable that one chooses by lot a region that is supported. Once the region has escaped poverty, there is no longer a reason to support this region and it is excluded from subsidization for all future periods. It follows that all indigent regions will be supported after a finite number of periods.

*Taxation is impossible*

All citizens may already live at or below the subsistence level, so that there is no taxable capacity to finance subsidies. In this case, the society is dependent upon foreign aid. Otherwise there is no escape from poverty. This foreign aid requirement, however, is only needed for an initial impetus to launch the tax-and-subsidy process.

*Quasi-monopoly agenda setting*

There may exist fixed costs for setting an agenda, representing an unsurmountable hurdle for some or even most of the citizens, if they are poor. This means that, although all people have the constitutional right to set an agenda, only a few rich people are actually in a position to do so. As a result, the same people always get subsidized. This problem can be explicitly dealt with by rotating agenda setting, i.e., by limiting the number of re-elections allowed. Additionally, the agenda setting costs must be covered by state intervention.

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52 This might also result from other asymmetric power relations within society, like unequal skills, influence etc.

53 In the constitution of the German state Hesse, for instance, it is stated in Artikel 76 (1) that everyone must be secured the opportunity of being elected for the Landtag (Hesse’s parliament), that everybody can follow her/his mandate unhinderedly and without disadvantage, cf. Hessische Verfassung (1946). In the German Constitution, the Grundgesetz, it is stated in Artikel 48 Entitlements
Chapter 5. The Political Implementation

Inequality Aversion, Fairness, Envy, and Negative Reciprocity

Unequal treatment of equal households can cause conflicts and thereby the failure of the policy. If certain policies are considered as “unfair”, then individuals might vote against them. Consequently, required redistributions to escape poverty traps might not be feasible in democracies, since they are rejected in elections. **Bell and Gersbach (2001)** addressed the issue of inequality aversion. They analyzed policy programs where the social planner has the constraint to educate the society subject to an upper bound on the degree of inequality the society is prepared to tolerate. They show that this constraint restricts the redistribution possibilities and thus increases the time needed to educate the society (**inequality-speed dilemma**). If the maximum tolerated inequality of incomes is too small, then the education of the society is impossible, because paying the minimal required subsidy and levying taxes to finance these transfer demands a minimum of inequality. In our political framework, the idea of thresholds of tolerance concerning inequality translates into the pattern of behavior that a voter $i$ will reject a proposal $P_t$ if it involves a degree of inequality above individual $i$’s threshold of tolerance. Just as in the work of **Bell and Gersbach (2001)**, the social planner or agenda setter has to respect this inequality aversion, for otherwise her agenda will not pass legislation. However, for democratic constitutions cannot dictate on citizens how to vote, democratic constitutional designs that could prevent political failure of this sort are difficult to construct. Hence, it might be necessary to implement a certain degree of dictatorship, especially if the individuals’ voting behavior prevents the feasibility of the education of a society. To cope with the issue of inequality and fairness, single households within one area shall not be treated too differently, but within one region all households shall be supported equally, whereas another is taxed.\(^{54}\) However, if the voters consider the inequality among the whole society, then this policy will most likely not be able to prevent political failure. But there is one tool that might be able to solve the problem in real world. One could offer a lottery in the following way: the agenda proposal states only the size of the subsidy and tax, contingent to the particular type of household (tariff). Who is taxed or subsidized is determined by a lottery. I.e., one states that the next drawn household have to pay the type-depended tax stated by the proposal or that it receives the type-depended subsidy; for instance, drawing households

\(^{54}\) Unequal treatment can be justified in practice by using ability tests: the uneducated with the highest potentials obtain subsidies. This would increase the efficiency of the program.

of Members]: (1) Every candidate for election to the Bundestag [the German Parliament] shall be entitled to the leave necessary for his election campaign. (2) No one may be prevented from accepting or exercising the office of Member of the Bundestag. No one may be given notice of dismissal or discharged from employment on this ground. (3) Members shall be entitled to remuneration adequate to ensure their independence. The latter point emphasizes an additional requirement in practice: the agenda setter’s independence of rich lobbies must be ensured. Cf. GG (1949) or Basic Law (1949).
that are taxed and those that are subsidized could alternate. This procedure repeats until all households are drawn. As long as the lottery is fair, most people would accept such a procedure. Accordingly, arising inequality is likely not to be considered as being unfair. It is clear that within this scenario the agenda setter has to propose agendas such that at least half of the society has an expected payoff that is non-negative. Which consequences this restriction involves has to be investigated in future research. However, there is no doubt that we again need the agenda repetition rule to ensure that supported households escape the poverty trap. To prevent adverse taxation, we also have to add our flexible majority rule or a tax allowance. Finally, without capital market we have the BB rule. In the previous section, we also emphasized that envy or negative reciprocity could cause political failures, because they might generate adverse taxation of educated households. However, it is clear that it does not matter how adverse taxation is motivated, flexible majority rules or tax allowances can prevent it.

There are a variety of other conceivable sources for political failure that do not directly stem from our model. At the most extreme level, corruption and rent-seeking by powerful clans or other interest groups may make it impossible to subsidize poor people sufficiently. At the other end, overcoming the incidence of child labor and achieving education might be in short-term conflict with other policy objectives. Furthermore, the supply side of schooling services has to be developed before any education can take place. Moreover, we have neglected, by construction, the demand for human capital in our model. Thus, we have implicitly assumed that those educated individuals are all able to transform their skills into higher income. On the labor market, this requires that the firms actually demand these higher skills, which is per se not ensured. Accordingly, it is also possible that the agenda setter pays educational subsidies to the firms to induce human capital accumulation on the firms’ side. Within such a framework, taxation of firms were also be conceivable. This would be an instance for solving the coordination problem described by Dessy and Pallage (2001).

Eventually, ineffective enforcement of constitutional rules can be a source of failure, because then there is no incentive to behave constitutionally. Hence, it is essential that conformity to each single constitutional rule is monitored by an institution and enforced by courts that function effectively. That is, an efficient working judiciary is imperative. Similarly, it is important to stress that, besides the enforcement of the

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55 As long as the lottery’s probability of drawing a particular name is equal for all names, this lottery will be considered as being fair.

56 Grossman and Helpman (1998) stress that, even if a constitution is well written, they fear that politicians will soon become adept at circumventing its constraints in order to foster their political ends.
constitution, the protection of the essential rules is a crucial point. If a majority wants to exploit a minority, this majority might want to change the constitution. Hence, it is important to ensure that constitutional changes require super-majorities.\textsuperscript{57}

\section{5.9 Conclusions}

We have shown that even when democracy works well, i.e. without corruption or organized rent-seeking, etc., the design of the constitution is crucial in deciding whether a society can escape poverty traps or not. Unconstrained agenda setting and simple majority rules will leave the society in poverty. However, appropriate democratic rules can enable a society to change things for the better. We hence propose that donor institutions levy pre-conditions on developing countries for aid payments or credits (as is done at growth-oriented adjustment programs of the World Bank in other contexts) that cover the identified necessary constitutional rules. However, we have demonstrated that the required constitutional design may not be robust to extents of the human preferences towards social preferences.

Our model could and should be extended in various directions. For instance, allowing for parties and interest groups more explicitly would bring the model closer to real-world situations. Moreover, we have neglected the fact that a deficient supply of schooling services or a conflict of policy aims may be a major barrier to education. These and other extensions, as set out in Section 5.8, could be useful for a better understanding of the way in which democratic institutions need to be constructed to help a society to extricate itself from a state of backwardness. Another interesting point for future research is to investigate the interdependent dual process of economic and political transition and transformation that many developing countries pass through: economic success leads to political stability and \textit{vice versa}. However, there also might be certain trade-offs. Eventually, extending human preferences to the possibility of inequality-aversion, reciprocity, social concerns \textit{etc.}, as the results of experimental economics suggest, may also highlight new, interesting sources of the failure of redistribution policies that the constitutional design has to cope with.\textsuperscript{58}

\textsuperscript{57}In the German constitution [see GG (1949) and Basic Law (1949)], for instance, Article 79 (2) establishes a protection mechanism. Article 79 (3) even prohibits changes of certain rules.

\textsuperscript{58}Cf. our discussion in Section 5.7.
Chapter 6

Multidimensional Education Policy

6.1 Introduction

In this chapter, we more closely look at the problems that parents face in taking the educational decision for their children. For instance, often children have long distances to travel to get to school. Teachers are often not showing up at schools (for whole weeks),\textsuperscript{1} are not motivated due to very low salaries, and hence there is large-scale cheating at examination, enabled by corruption.\textsuperscript{2} Moreover, the schools are in a very bad condition. Additionally, when poor have a claim on subsidies, or when pupils want to attend school, parents have to pay bribes for their official entitlement to bureaucrats or headmasters.\textsuperscript{3} Finally, we have also seen that the technology of human capital formation is a crucial determinant of the required policy. Therefore, we extend our basic model by the following aspects: quality level of the educational system, regional school density, traffic infrastructure, and corruption in the area of education. We then ask how to allocate state resources to improve this broader environment of education. In doing so, we will restrict ourselves to public schools.

This allows us to derive an optimal investment allocation in the sense of minimizing the time required to educate a society as a whole via educational subsidies. It turns out that a pre-subsidization phase may be required before paying subsidies makes sense.

\textsuperscript{1} Cf. Stern (2003), p. 17. This problem could be solved by involving parents in the governance of schools (like demonstrated by the District Primary Programme in India or by the EDUCO Program in El Salvador).


\textsuperscript{3} Saha (2001), for instance, deals with red tape and incentive bribes in providing subsidies in a principal agent model. Mauro (1998) finds that in the context of non-education expenditures the chance to collect bribes is higher, whereby corruption may lower educational expenditures in favor to others. Bell (1990), like others, discuss necessary side payments to get access to subsidized credits from government run banks.
Hence, we derive a "roadmap" for educating a society. This roadmap consists of a variety of expenditures to mitigate the problems connected with education: subsidies to poor households, investments in the infrastructure of schools that, firstly, improve the quality of schools and secondly, the regional school density, or efforts to fight corruption that levies extra costs for school attendance via side payments or decreases the fraction of educational subsidies that actually are received by the beneficiary.\textsuperscript{4,5} Infrastructure investments that target the improvement of transport systems are important to enable children actually to attend schools. Especially actions targeting at school quality are an alternative option to deal with the problem that the technology of human capital can be too unproductive: investments in the schooling system might be a better policy than to subsidize households repeatedly.

Furthermore, we will extend the parents' preferences such that we incorporate the fact that the parents' educational decision is affected by the result of school attendance, that is, by the school quality. Given this extension, investments that improve the broad environment of education send a signal that schooling pays. There arises also an obvious trade-off: when the school quality improves, then school attendance might be reduced, since one can attain a particular level of human capital with less education time. Colclough and Al-Samarrai (2000) stresses that an increased quality of schooling would diminish the number of repeated school years, which would in turn both reduce the cost of educating the society in future years and result in a higher final stock of human capital.

The remainder of the chapter is organized as follows. In Section 6.2, we explain the model extensions. In Section 6.3, we analyze an educational policy that combines subsidies with other educational investment forms. At first, we explain which investments the government can undertake to improve the educational level of the society. We respectively focus on the implications these investments have on the households' budgets, schooling productivity and the critical thresholds $\lambda^S$ and $\lambda^a$. We then deduce the optimal portfolio of expenditures and show that a subsidization policy may run through two phases: a pre-subsidization phase and a subsidization phase. In Section 6.4, we specify the preferences of the adults to generate deeper insights. In Section 6.5, we change the adults' preferences. We assume that the schooling quality also determines the optimal demand for education and elaborate on which effects this change has. In Section 6.6 we finally draw conclusions.

\begin{itemize}
\item \textsuperscript{4}Although officially there is free primary schooling, i.e. teaching material (notebooks etc.) is free of charge, teachers sell education resources. Cf. Easterly (2002), p. 83, who cites Narayan (2000).
\item \textsuperscript{5}Cf. Stern (2003), p. 18. Much may be achieved by increasing the transparency of transfers of public funds [see Reinikka (2001) for the success of this strategy in the Uganda expenditure tracking project].
\end{itemize}
6.2 The Model

Consider our basic model of Chapter 2.

6.2.1 The Technologies

6.2.1.1 The Human Capital Technology

Additional to the basic model we now extend the model by considering the quality state of the schools. The quality of the educational system in period $t$ is denoted by $Q_t$. The effect of schooling is represented by a continuously increasing and differentiable function $h(e_i^t, Q_t)$ on $e_i^t \in [0, 1]$ and $Q_t \geq 0$, with $h(0, Q_t) = 0$ for all $Q_t$ and $h(e_i^t, 0) = 0$ for all $e_i^t$. Hereafter we again drop index $i$. The case $Q_t = 0$ represents a state in which schools or teachers do not exist. Using these assumptions, the child’s endowment of efficiency units of labor on reaching adulthood at time $t + 1$ is given by the following technology:

$$\lambda_{t+1} = h(e_t^t, Q_t)(z\lambda_t) + 1 \quad (6.1)$$

Again, Equation (6.1) implies that rearing and formal education are both necessary if human capital is to be formed at all in the next generation. Additionally, as long as $Q_t = 0$, a formal school education is not feasible and the children will live in a state of backwardness.

6.2.1.2 The Output Technology and Household’s Income

We now extend our preceding model by including remoteness of schools. If, especially in rural regions, schools are remote, then the child spends time as well going to and from school, that we cannot neglect. The whole distance to and from school for the children in period $t$ will be represented by $d_t$. Let the average speed of a child on the way to or from school be $v_t$. For simplicity, we assume that if the child attends school, then it will do so every day, i.e. each day the child’s time is used for school, determined by $e_t$, and for child labor, determined by $1 - e_t$. It follows that a household supplies a total of $[\lambda_t + (1 - e_t - d_t/v_t)\gamma]$ efficiency units of labor to the production of the aggregate good. A direct consequence is: $e_t \in [0, 1 - d_t/v_t]$. Therefore, Equation (2.2) changes

\[\]
to:

\[ y_t = \alpha[\lambda_t + (1 - e_t - d_t/v_t)\gamma] \]

(6.2)

It follows that the opportunity cost of education increase by \( \alpha\gamma(d_t/v_t) \).

### 6.2.2 The Household’s Behavior

In this section, we will analyze the parents’ education decisions in a broader environment than hitherto. For the sake of simplicity, let again the child’s consumption be a fixed fraction of the adult’s, which can be neglected without loss of generality. Consider that due to corruption, each adult is expecting to be forced to pay bribes of the size \( \rho_t \) in period \( t \) for school enrollment and to obtain subsidies.\(^7\) Then the family’s budget constraint concerning \((c_t, e_t)\) changes to:

\[
\alpha(\lambda_t + \gamma) = \begin{cases} 
  c_t + \alpha\gamma(e_t + d_t/v_t) + \rho_t & \text{if } e_t > 0 \\
  c_t & \text{if } e_t = 0 
\end{cases} \tag{6.3}
\]

Reports often reveal that also school fees and costs for compulsory books and school uniforms, and the like, prevent schooling.\(^8\) Those costs also could be covered by \( \rho_t \).

For simplicity, we suppose those costs to be zero. We still define

\[ \tau_t(\lambda_t) \equiv \alpha(\lambda_t + \gamma) \]

(6.4)

but now

\[ \varphi_t(\lambda_t) \equiv \alpha\lambda_t - \rho_t, \]

(6.5)

where \( \varphi_t(\lambda_t) \) now corresponds to the adult choosing \( e_t = 1 - d_t/v_t \) and to a complete renunciation of child labor.

The adult decides on \( e_t \) and \( c_t \) on the basis of the maximal possible consumption level \( \tau_t(\lambda_t) \) and the fixed costs of education. The adult will choose \( e_t^a > 0 \) only if more than the consumption level \( c^S \) plus the fixed costs \( \rho_t + \alpha\gamma d_t/v_t \) are covered by \( \tau_t(\lambda_t) \). The higher \( \tau_t(\lambda_t) \) is, the higher the demands will be. The optimal choices for \( e_t \) and \( c_t \), are thus \( e_t^a = e^o(\lambda_t, d_t, v_t, \rho_t) \) and \( c_t^a = c^o(\lambda_t, d_t, v_t, \rho_t) \).

The two threshold values \( \lambda^S \) and \( \lambda^a \) follow from marginal utility comparisons. In our context, \( \lambda^S \) requires that at the locus \((e_t, c_t) = (0, \tau(\lambda^S) - \rho_t - \alpha\gamma(d_t/v_t))\), the marginal utility of consumption equals exactly the marginal utility of school attendance. Similarly, the upper threshold of human capital causes the equality of the marginal utilities

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\(^7\)As we analyze societies caught in a poverty trap, we assume that, ceteris paribus, education will only occur if the considered household is subsidized. Therefore, when we later on analyze subsidization, one can interpret \( \rho_t \) as the sum of grease payments involved with schooling and receiving the subsidy.

at the locus $(1 - (d_t/v_t), \varphi(\lambda^S))$. As the threshold of interest in this chapter corresponds with $e_t^c = 1 - \frac{d_t}{v_t}$, we introduce threshold $\lambda^{ac}$ to emphasize that this threshold only corresponds with the time-constraint level of education, and not with the higher fulltime schooling level like $\lambda^e$.\textsuperscript{9} It becomes directly clear that $\lambda^S = \lambda^S(\rho_t, (d_t/v_t))$ and $\lambda^{ac} = \lambda^{ac}(\rho_t, (d_t/v_t))$.

The lower the extras are, the lower the household’s income can be to afford school education. The consumption increases, whereby the marginal utility of consumption decreases and the marginal utility of education increases. Consequently the equality of marginal utilities forces $\lambda^S$ to decrease:

$$\frac{\partial \lambda^S(\cdot)}{\partial \rho_t}, \frac{\partial \lambda^S(\cdot)}{\partial d_t} > 0, \quad \text{and} \quad \frac{\partial \lambda^S(\cdot)}{\partial v_t} < 0 \quad (6.6)$$

Referring to $\lambda^{ac}$, an increase of $\rho_t$ lowers $\varphi(\lambda_t)$. The marginal utility of consumption increases and that of education decreases. Consequently $e_t^c = 1 - (d_t/v_t)$ requires $\lambda^{ac}$ to increase:

$$\frac{\partial \lambda^{ac}(\cdot)}{\partial \rho_t} > 0 \quad (6.7)$$

As long as $\lambda_t \geq \lambda^{ac}(\cdot)$, the highest possible level of schooling is chosen: $e_t^c(\lambda_t, d_t, v_t, \rho_t) = 1 - \frac{d_t}{v_t}$. If $(d_t/v_t)$ increases this directly corresponds with an increase of the requirements connected with $\lambda^{ac}$. The involved level of required education $c_t$ causes a reduction of the marginal utility of education. Therefore the marginal utility of consumption has to be risen by an increase of $\lambda^{ac}$:

$$\frac{\partial \lambda^{ac}(\cdot)}{\partial d_t} < 0 \quad \text{and} \quad \frac{\partial \lambda^{ac}(\cdot)}{\partial v_t} > 0 \quad (6.8)$$

Note that according to our assumed preferences, $\lambda^S$ and $\lambda^{ac}$ do not depend on the school quality $Q_t$.\textsuperscript{10} These thresholds can be directly translated into consumption thresholds. If the adult is endowed with income $\alpha \lambda^S$ the household’s level of consumption is $c_t = \tau(\lambda^S)$. Basically the adult is willing to send the child to school part-time as soon as $c_t = \tau(\lambda^S) - \rho_t - \alpha \gamma(d_t/v_t) \equiv \varphi$, but the extras prevent schooling. The income situation is still so precarious that $e_t^c = 0$ is chosen, but any increase in income bears education. Therefore, the extra cost $\rho_t + \alpha \gamma \frac{d_t}{v_t}$ augments the requirement for $e_t^c > 0$ to be chosen; the critical income threshold is $e^c + \rho_t + \alpha \gamma \frac{d_t}{v_t}$, and $\lambda^S = \frac{1}{\alpha} (c^S + \rho_t - \alpha \gamma (1 - d_t/v_t))$.

If $e_t^c = 1 - d_t/v_t$, then the household looses all income of the child. Thus the adult alone has to earn at least $\alpha \lambda^{ac} = \varphi(\lambda^{ac}) + \rho_t$. Hence $\lambda^{ac} \equiv \varphi(\lambda^{ac})$, and $\rho_t$ increases the

\textsuperscript{9}Nonetheless, when we address $e_t^c = 1 - (d_t/v_t)$ we will talk of fulltime schooling throughout the chapter.

\textsuperscript{10}Q_t does not determine $\lambda^S$ and $\lambda^{ac}$ because the households’ adults are assumed not to care about the quality of schools. In Section 6.2.2 we briefly extend our model by quality aspects.
requirements for \( e^o_t = 1 - (d_t/v_t) \), because to finance \( e^{ac} \) the adult needs an income of \( c(\lambda^{ac}) + \rho_t \).

We again come to:

\[
(e_t, e^o_t) = \begin{cases} 
(\lambda_t, 0) & \forall \lambda_t \leq \lambda^S(\cdot); \\
(e^o_t, e^o_t) & \forall \lambda_t \in (\lambda^S(\cdot) \lambda^{ac}(\cdot)); \\
(g(\lambda_t), 1) & \forall \lambda_t \geq \lambda^{ac}(\cdot).
\end{cases}
\]  

(6.9)

where the locus \((e^o_t, e^o_t)\) is monotonously increasing in \( \lambda_t \) for all \( \lambda_t \in (\lambda^S(\cdot), \lambda^{ac}(\cdot)) \).

6.2.3 Dynamics

Consider Equations (6.1), (6.9) and \( Q_t > 0 \). We deduce

\[
\lambda_{t+1} = \begin{cases} 
1 & \forall \lambda_t \leq \lambda^S(d_t, v_t, \rho_t); \\
zh(e^o(\lambda_t, d_t, v_t, \rho_t))\lambda_t + 1 & \forall \lambda_t \in (\lambda^S(d_t, v_t, \rho_t), \lambda^{ac}(d_t, v_t, \rho_t)); \\
zh(1 - d_t/v_t)\lambda_t + 1 & \forall \lambda_t \geq \lambda^{ac}(d_t, v_t, \rho_t).
\end{cases}
\]  

(6.10)

It is still plausible to assume that \( \lambda^S(d_t, v_t, \rho_t) > 1 \), for all the permutations of positive values of \((d_t, v_t, \rho_t)\).\(^{11}\) Hence the state of backwardness \((\lambda = 1)\) is once again a locally stable low-income equilibrium, where the society suffers stark poverty, illiteracy and child labor (poverty trap). The growth case is now characterized by \( zh(1 - d_t/v_t, Q_t) \geq 1 \). \( h(e^o(\lambda_t, d_t, v_t, \rho_t), Q_t)\lambda_t \) is assumed to be convex in \( \lambda_t \) within \([\lambda^S(d_t, v_t, \rho_t), \lambda^{ac}(d_t, v_t, \rho_t)]\). All other things are similar to the dynamics of our basic model. Therefore the reader can consider a dynamic system that has the two steady states \((\lambda^*(Q_t, d_t, v_t, \rho_t), e^o(\lambda^*_t, d_t, v_t, \rho_t))\) and \((1, 0)\). The growth-case would imply that \( \lambda^{ac}(d_t, v_t, \rho_t) > \lambda^*(d_t, v_t, \rho_t) \). In the no-long-term-growth-case, the highest-income steady state level of \( \lambda \) is implicitly given by

\[
\lambda^*(Q_t, d_t, v_t, \rho_t) = 1/(1 - zh(e^o(\lambda^*_t, d_t, v_t, \rho_t), Q_t)).
\]

An instance of a dynamic pattern is illustrated by Figure 6.1.

Finally, the state \( Q_t = 0 \) establishes a continuum of steady states, since \( h(e_t, 0) = 0 \) leads us directly to \( \lambda = 1 \) and \( e^o = 0 \). Hence the condition \( Q_t = 0 \) is not necessary but sufficient for the society to be caught in a poverty trap.

6.3 The Education of a Society

Bell and Gersbach (2001) demonstrate that paying lump-sum subsidies allow to educate a society. However, subsidies will not work if there are not enough schools,
teachers, books, and if extra payments are not taken into account.

The government can improve the educational system by investments denoted by $q^1_t$, and fight corruption by expenditures $q^2_t$ to lower side payments. Additionally, new schools can be built by investments denoted by $q^3_t$ to decrease $d_t$. Furthermore, the government can invest in traffic infrastructure, like supplying more bus lines for pupils to increase their average velocity $v_t$.\(^{12}\) These investments are denoted by $q^4_t$. Hence, there exists a bundle of channels through which the policy can influence the education level, so that the question arises how the optimal portfolio of these policy instruments should be designed.

The major problem is to give incentives to send the child to school. In poor societies this is mainly a question of income. But even if $e_t^0 = 1 - d_t/v_t$ the resulting human capital may not be sufficient to escape the poverty trap, and a major goal of the government must be an improvement of the quality of education (besides questions concerning the demand side of human capital\(^{13}\)). Otherwise repeated subsidization would be necessary.

\(^{12}\)Or improve the health system which might increase $v_t$ and the positive effect of schooling, but as well a child’s productivity $\gamma$. Health aspects are analyzed in Bell, Devarajan, and Gersbach (2003).

\(^{13}\)For instance the question of whether there is enough employment for higher educated individuals. If this is not the case, the assumed positive correlation between human capital and income has to be
To study the particular effects of the single instruments we will apply our results (6.6), (6.7) and (6.8).

As in the following one can easily lose overview of the investment types, we summarize them here, so that one can quickly consult the following list:

- $q_t^1$ Investments to improve the **quality of schools**
- $q_t^2$ Investments to fight **corruption**
- $q_t^3$ Investments to increase the density of **schools per region**
- $q_t^4$ Investments into the traffic infrastructure (**velocity**)

### 6.3.1 School Quality

The government is assumed to be able to improve education by making investments into the educational system. Such investments can be defined broadly, improve the educational facilities and increase the number of teachers, their skills and attendance.

Such investments in period $t$ will be labeled $q_t^1$.

$$Q_t = Q(q_t^1)$$  \hspace{1cm} (6.11)

with $\frac{\partial Q(q_t^1)}{\partial q_t^1} > 0$ and $\frac{\partial^2 Q(q_t^1)}{\partial q_t^1} \leq 0$. As one period comprises a generation – and thus many years – investments in period $t$ bears fruit already in the same period. We neglect depreciation so that it is plausible to assume $Q(q_t^1) = Q_{t-1}$ for $q_t^1 = 0$.

**Ceteris paribus**, an increase of $Q_t$ improves the effect of each single level of education, $e_t > 0$, on human capital formation, but has obviously neither an effect on $\lambda^S$ nor on $\lambda^{ac}$, since both depend solely on the preferences and on the extra cost of $\rho_t + \frac{d_t}{v_t}$.

As a quality improvement has no effect in the area $\lambda \in [1, \lambda^S]$ the coordinate $(\lambda_{t+1}, \lambda_t) = (1, \lambda^S(\rho_t, d_t, v_t))$ is fixed. This consequently corresponds with an upward turn of the trajectory in the turning axes at this point (see Figure 6.2). Note that

$$\lambda_{t+1} = h(e(\lambda_t, \cdot), Q(q_t^1)) (z\lambda_t) + 1,$$

and thus

$$\frac{\partial \lambda_{t+1}}{\partial q_t^1} = \frac{\partial h(\cdot)}{\partial Q_t} Q'(q_t^1) z \lambda_t > 0 \quad \text{for all} \quad \lambda_t > \lambda^S(\rho_t, d_t, v_t); \hspace{1cm} (6.12)$$

$$\frac{\partial^2 \lambda_{t+1}}{\partial q_t^1 \partial \lambda_t} = \frac{\partial h(\cdot)}{\partial Q_t} Q'(q_t^1) z > 0.$$
The power of the investment-effect increases in $\lambda_t$, $\frac{\partial^2 \lambda_{t+1}}{\partial q \partial \lambda_t} > 0$, since due to $\frac{\partial e^o}{\partial \lambda_t} > 0$ the children enjoy the improved school conditions for a longer span of time (similar to economies of scale). Additionally, the effect of child rearing increases due to the better school education (spillovers): $\frac{\partial \lambda_{t+1}}{\partial z} = h(e^o(\lambda_t, \cdot), Q(q^1_t))\lambda_t$. This effect also increases in $\lambda_t$. Therefore, the trajectory turns upwards.

The adverse threshold $\lambda_t^*$ shrinks, and it becomes easier to escape the poverty trap. The level of $\lambda^{ac}$ remains unchanged, since the consumption required for full-time schooling, $c^{ac}$, plus the required bribe $\rho_t$ remain unchanged. However, the starting point of the linear part is moved upwards, and its slope, $zh(1 - \frac{d_t}{\nu_t}, Q_t(q^1_t))$, increases too. The effects are illustrated in Figure 6.2. We thus infer that if $zh(1 - \frac{d_t}{\nu_t}, Q_t) < 1$ (no long-term growth), investments $q^1_t$ may produce long-term growth.

Figure 6.2: The effect of investments in the quality of schools.
Corruption increases the cost of education. This can occur twofold. First, parents often must pay side payments to school officials to enroll children in a school.\footnote{Cf. Friedman (2000), p. 216, referring to India.} Secondly, albeit having a claim to subsidies, the people who should enjoy the subsidies only receive the transfers if they pay bribes to the official who administers the subsidy payment. If the beneficiaries do not collude, they are menaced by “red tape”.\footnote{Cf. Saha (2001). Something near this occurs in the context of subsidized credits, see Bell (1990).}

Both forms of corruption channel parts of resources meant for the poor uneducated to others. Thus corruption reduces the effectiveness of subsidy policies, and is harmful for a society in numerous other ways.\footnote{Cf. Blackburn, Bose, and Haque (2002), Dreher and Siemers (2004), Lambsdorff (1999), Shleifer and Vishny (1993). One major drawback of corruption is that investments are shifted away from growth-enhancing projects like education because other expenditures offer better opportunities to collect bribes and better satisfy the demand for secrecy involved with corruption [see Ehrlich and Lui (1999), Mauro (1998) and Mauro (1997)].}

The level of bribes in a period $t$, $\rho_t$, is contingent on the effort in combating corruption:\footnote{$\rho$ could also be re-interpreted in the sense that it represents further extra costs like, e.g., compulsory school-uniforms. Then, paying parts of these costs were also investments $q_t^2$.}

$$\rho_t = \rho_t(q_t^2) \quad \text{with} \quad \frac{\partial \rho_t(q_t^2)}{\partial q_t^2} < 0, \quad \frac{\partial^2 \rho_t(q_t^2)}{\partial (q_t^2)^2} \geq 0$$

(6.13)

and $\rho(q_t^2) = \rho_{t-1}$ when $q_t^2 = 0$. For $e_t$ depends positively on $\alpha(\lambda_t + \gamma) - \rho_t(q_t^2) - \alpha \gamma \frac{d_t}{v_t}$, investments to extirpate corruption may increase the education level. As we have already seen in Subsection 6.2.2, $c^S + \rho_t + \alpha \gamma \frac{d_t}{v_t}$ is, referred to $\alpha(\lambda_t + \gamma)$, the critical income threshold to cross for $e_t^0 > 0$. For $e_t^0 = 1 - d_t/v_t$, $\alpha \lambda_t$ must be least as high as $c^{ac} + \rho_t$. Compared to our preceding analysis, the in this chapter considered extras hit a wedge of size $\rho_t + \alpha \gamma \frac{d_t}{v_t}$ (referring to $\lambda^S$) and $\rho_t$ (referring to $\lambda^{ac}$) between $c^S$, respectively $c^{ac}$, and the actual required income level of $c^S + \rho_t + \alpha \gamma \frac{d_t}{v_t}$, respectively $c^{ac} + \rho_t$. Hence, effort in fighting corruption lowers this wedge. Thus can parents afford a certain school attendance with lower human capital as they do not need to pay as high bribes as before, i.e. $\lambda^S$ and $\lambda^{ac}$ decrease. We find:

$$\frac{\partial \lambda^S(q_t^2)}{\partial q_t^2} < 0, \quad \frac{\partial \lambda^{ac}(q_t^2)}{\partial q_t^2} < 0$$

(6.14)

where we, from now on, abbreviate notation for more complex functions: for instance, $\lambda^S(q_t^2) \equiv \lambda^S(\rho(q_t^2))$, or $s(q_t) \equiv s(k(q_t^1, q_t^2, q_t^3, q_t^4), q_t^2, q_t^3, q_t^4)$; nonetheless we also will use the complex notation, when this is helpful.
The effect of investment $q_t^2$ is that the phase diagram is shifted to the left. The slope of the trajectory remains unaltered.\textsuperscript{18} Therefore it becomes easier to escape the poverty trap, since $\lambda^*_t$ decreases, and easier to reach full-time schooling as $\lambda^{ac}$ also decreases. The effects are made vivid by Figure 6.3.\textsuperscript{19}

![Figure 6.3: The effects of investments to fight corruption.](image)

### 6.3.3 Reachability of Schools

Education will only be feasible if children are actually in a position to attend school. Often means of transportation are missing, and when they exist they are too expensive.\textsuperscript{20} In rural areas, the remoteness of the next school thus can be a prohibitive hurdle for the education of the society, and the relative distance that children must travel to

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\textsuperscript{18}Note that we assume $\rho_t$ is lump-sum in fashion. If $\rho_t$ was increasing in $e_t$ and the marginal side payment decreases, the slope of the trajectory would change.

\textsuperscript{19}Coming back to the possibility that $\rho_t$ also could represent school fees, costs for school uniforms etc., investment $q_t^2$ would be lowering fees or making uniforms available costless.

get to school becomes an overall economic detriment to the society. In the extreme, the maximum achievable level of education $1 - \frac{d_t}{v_t}$ can be zero or even negative.\footnote{This means that the time needed to get to and back from school requires more than the time-endowment per day. Of course, education is nonetheless feasible if the children leave home and join a boarding-school. However, as this is quite expensive this is only possible if the state would pay for all costs. Furthermore, it remains open whether there is acceptance on the side of the parents for this alternative. Be this as it may, paying boarding-schools is another investment type in the context of time needed for education, and does not influence our results: children who join boarding schools enjoy $d_t/v_t \approx 0$ and the rest still face high $d_t/v_t$.}

Building new schools increases the density of schools per region, and reduces the distance to the next reachable school, so that education becomes more attractive. Investments of this kind are labeled as $q^3_t$ so that we state:

\begin{equation}
    d_t = d(q^3_t) \quad \text{with} \quad \frac{\partial d(q^3_t)}{\partial q^3_t} < 0, \quad \frac{\partial^2 d(q^3_t)}{\partial (q^3_t)^2} \geq 0
\end{equation}

and $d(q^3_t) = d_{t-1}$ when $q^3_t = 0$. A similar alternative to lower this restriction of remoteness is to improve the traffic infrastructure. For instance, a school bus can be organized so that the pupils do not have to walk long distances to get to school.\footnote{This may also lower fears of parents about the danger the child is exposed to on the way to and from school. Of course, countries like Kenya would lower their comparative advantage in the long-distance disciplines in athletics.} Such investments of type $q^4_t$ can greatly increase the velocity with which the school can be reached.

\begin{equation}
    v_t = v(q^4_t) \quad \text{with} \quad \frac{\partial v(q^4_t)}{\partial q^4_t} > 0, \quad \frac{\partial^2 v(q^4_t)}{\partial (q^4_t)^2} \leq 0
\end{equation}

and $v(q^4_t) = v_{t-1}$ when $q^4_t = 0$. Both investments reduce the time additionally needed for school, and thus the opportunity cost of schooling $\alpha \gamma d_t/v_t$: the education level $e_t$ increases for any given level of $\lambda \in (\lambda^S(\cdot), \lambda^{ac}(\cdot))$.

As in our explanation for the combat against corruption, $\lambda^S$ falls, as the opportunity cost $\alpha \gamma d_t/v_t$ is reduced. The consumption increases, whereby the marginal utility of $c_t$ decreases, and the marginal utility of $e_t$ increases. Thus has $\lambda^{ac}$ to rise. Additionally the slope of the linear part, $h(1 - \frac{d_t}{v_t}, Q_t)z$, increases, because full-time schooling now allows more schooling. The effect is illustrated by Figure 6.4. We thus have found:

\begin{equation}
    \lambda^S = \lambda^S(\rho(q^2_t), d(q^3_t), v(q^4_t)) \equiv \lambda^S(q^2_t, q^3_t, q^4_t)
\end{equation}

\begin{equation}
    \text{with} \quad \frac{\partial \lambda^S(q^2_t, q^3_t, q^4_t)}{\partial q^x_t} < 0 \quad \text{where} \quad x = \{2, 3, 4\},
\end{equation}

and

\begin{equation}
    \lambda^{ac} = \lambda^{ac}(\rho(q^2_t), d(q^3_t), v(q^4_t)) \equiv \lambda^{ac}(q^2_t, q^3_t, q^4_t)
\end{equation}
Figure 6.4 demonstrates that investments $q_t^3$ and $q_t^4$ might be able to change a no-long-term growth-case in the growth-case $zh(1 - (d_t/v_t), Q_t) \geq 1$.\(^{23}\)

### 6.3.4 Educational Subsidies

We now elaborate how the education of a society can be achieved when the set of instruments become extended by these investments in the educational infrastructure.

The government pays a lump-sum subsidy, so that $e_t^s > 0$: $\alpha(\lambda_t + \gamma) + s_t > c^S + \rho_t + \alpha \gamma \frac{d_t}{v_t}$.

The paid transfer $s_t$ causes $e^o > 0$ by increasing the household’s budget to cross the adverse threshold consumption of $c^S$ plus the extras $\rho_t + \alpha \gamma \frac{d_t}{v_t}$.

Let all households of the society initially be caught in the poverty trap at $\lambda = 1$, and

\(^{23}\)This will only be possible if $zh(1, Q_t) \geq 1$. Nonetheless long-term growth is also in this case reachable by improving schooling quality $Q_t$. 

the quality of the educational system strictly positive: \( Q > 0 \). Consider the status quo of the educational environment is given by tuple \((d/v, \rho, Q)\). The government wants to escape the poverty trap by achieving human capital growth via subsidization. A sight on Figure 6.1 shows that this goal requires a human capital level of a size bigger than \( \lambda^* (\cdot) \). Therefore the paid subsidy \( s_t \) has to cause the required human capital formation. Let \( s_t^* \) be determined by:

\[
zh[e^{c'}(s_t^*, d_t/v_t, \rho_t), Q_t] + 1 = \lambda^*(d_t/v_t, \rho_t, Q_t)
\]

(6.19)

with \( \lambda^*(d_t/v_t, \rho_t, Q_t) \) being implicitly defined by

\[
\lambda^*(d_t/v_t, \rho_t, Q_t) \cdot [1 - zh(e^{c'}(\lambda^*(d_t/v_t, \rho_t, Q_t), d_t/v_t, \rho_t, Q_t))] = 1.
\]

I.e., there is a particular level of education time \( e_t \) required to form human capital of a size of \( \lambda_t^* \), which we denote by \( e_t^* = e^{c'}(s_t^*, d_t/v_t, \rho_t). \)

The required subsidy thus is \( \tilde{s}_t^* = s_t^* + \varepsilon \).

Subsidy \( \tilde{s}_t^* \) increases household’s adult income so that, given the altruism towards the child, the education level \( e^{c'}(\tilde{s}_t^*) \) suffices to reach the area of human capital growth beyond \( \lambda^*(d_t/v_t, \rho_t, Q_t) \). Obviously \( s_t^* \) covers all side payments \( \rho_t \), variable opportunity cost \( \alpha \gamma e_t^* \), and the fixed opportunity cost \( \alpha \gamma \frac{d}{v_t} \).

Summarizing, we have found:

\[
e_t^* = e^{c'}(s_t, q_t^2, q_t^3, q_t^4)
\]

(6.20)

with \( \frac{\partial e^{c'}(s_t, q_t^2, q_t^3, q_t^4)}{\partial q_t^x} > 0 \), \( x = \{2, 3, 4\} \),

\[
\lambda_{t+1} = \lambda_{t+1}(q_t^1, e^{c'}(s_t, q_t^2, q_t^3, q_t^4))
\]

(6.21)

with \( \frac{\partial \lambda^{c'}(\cdot)}{\partial q_t^x} > 0 \), \( x = \{1, 2, 3, 4\} \),

\[
\lambda_t^{ac} = \lambda^{ac}(q_t^2, q_t^3, q_t^4)
\]

(6.22)

with \( \frac{\partial \lambda^{ac}(\cdot)}{\partial q_t^x} < 0 \), \( x = \{2, 3, 4\} \), and

\[
\lambda_t^{ac} = \lambda^{ac}(q_t^2, q_t^3, q_t^4)
\]

(6.23)

It is clear that state expenditures \( q_t \) decrease this necessary subsidy, since they lower \( \lambda^*(d_t/v_t, \rho_t, Q_t) \) by lowering \( d_t/v_t \) and \( \rho_t \), and improving \( Q_t \).

\( ^{24} \)Note that \( e_t^* \) is strictly higher than \( e^{c'}(\lambda_t^*, \cdot) \). In the first case, the households displays \( \lambda_t = 1 \), in the latter it displays \( \lambda_t = \lambda_t^* \). A comparison of \( zh \left( e^{c'}(s_t^*, \lambda_t = 1, \cdot)\right)1+1 = \lambda_t^* \) and \( zh \left( e^{c'}(\lambda_t^*, \cdot)\right)\lambda_t^* + 1 = \lambda_t^* \) demonstrates that \( e^{c'}(s_t^*, \cdot) > e^{c'}(\lambda_t^*, \cdot) \): the lower productivity of child rearing has to be compensated by \( e_t^* \) in comparison to \( e^{c'}(\lambda_t^*) \).
Proposition 6.1
The necessary subsidy to escape the poverty trap is a function \( s^*_t = s^*(\tilde{q}_t) \). Investments \( \tilde{q}_t \) lower this required subsidy payment:
\[
\frac{\partial s^*(\tilde{q}_t)}{\partial q^+_t} < 0, \quad \text{for all} \quad x = 1, 2, 3, 4
\]
The proof is given in the appendix, and uses Equation (6.19). Applying this equation, we receive
\[
s^*_t = s^* \left( Q(q^1_t), e^o[\rho(q^2_t), d(q^3_t), v(q^4_t)], \lambda^* \left[ Q(q^1_t), \rho(q^2_t), d(q^3_t), v(q^4_t) \right] \right) = s^* (\tilde{q}_t, \lambda^*(\tilde{q}_t))
\]
with
\[
\frac{ds^*(q^1_t, e^o(q^2_t, q^3_t, q^4_t), \lambda^*(\tilde{q}_t))}{dq^+_t} = \frac{\partial s^*_t(\cdot)}{\partial \lambda^*(\cdot)} \cdot \frac{\partial \lambda^*(\cdot)}{\partial q^+_t} + \frac{\partial s^*_t(\cdot)}{\partial q^+_t} < 0
\]
and
\[
\frac{ds^*_t(\cdot)}{dq^+_t} = \frac{\partial s^*_t(\cdot)}{\partial \lambda^*(\cdot)} \cdot \frac{\partial \lambda^*_t(\cdot)}{\partial q^+_t} + \frac{\partial s^*_t(\cdot)}{\partial e^o(\cdot)} \cdot \frac{\partial e^o(\cdot)}{\partial q^+_t} < 0, \quad x = \{2, 3, 4\}.
\]
The effect of \( q^1_t \) is a higher productivity of schooling for a given level of education \( e_t \), and the reduction of \( \lambda^*_t \). The required subsidy is hence lowered, since it suffices to produce lower levels of schooling \( e_t \); \( \frac{\partial s^*_t}{\partial q^+_t} < 0 \). The other investment types also lower \( \lambda^*_t \).

Additionally they increase the demand \( e^*_t \) and the resulting human capital formation increases. Consequently the required subsidy decreases.

The government has resources of size \( R_t \). To educate the society, as many households as possible should receive subsidy \( s^*_t = s^*(\tilde{q}_t) + \varepsilon \). Minimizing \( s^*_t \) therefore allows to support a maximal number of households, i.e. to educate the society as quick as possible. Optimal levels of variables are labeled by a small circle as superscript. For instance is \( (q^1_t)^o \) the optimal level of investment \( q^1_t \).

6.3.4.1 The Pre-Subsidization Phases

If no school is reachable given the time endowment, i.e. \( d_t/v_t - 1 \geq 0 \), and the households’ income is not sufficient to cover the fixed cost of education, i.e. \( \rho_t + \alpha\gamma(d_t/v_t) + c^S - \alpha(1 + \gamma) \geq 0 \), the households choose \( e^*_t = 0 \). The latter problem of a too low budget can be solved by subsidization, but the time constraint problem not. As long as the time constraint is hurt, subsidization is fully ineffective: \( s^*_t \rightarrow \infty \).

The subsidy has to cover \( \rho(q^2_t) + \alpha\gamma(d(q^3_t)/v(q^4_t)) + e^*_t + c^S - \alpha(1 + \gamma) \), and investments \( q^2_t \) and \( q^4_t \) have to reach \( d(q^3_t)/v(q^4_t) - 1 < 0 \). In doing so, two problems may force two pre-subsidization phases.
Phase 1 If resources $R_t$ do not suffice to establish $\frac{d}{v} - 1 < 0$, then investments $q_t^3$ and $q_t^4$ mitigate the time constraint, and at least help to move closer to the goal to establish $d(q_t^3)/v(q_t^4) - 1 < 0$. On this pre-stage of subsidization, the policy problem is therefore:  

$$
\min_{(q_t^3,q_t^4)} \frac{d(q_t^3)}{v(q_t^4)} - 1 \quad \text{s.t.} \quad q_t^3 + q_t^4 - R_t \leq 0, \quad q_t^x \geq 0, \forall x = 3, 4
$$

Be $\bar{q}_t = (q_t^3, q_t^4)$. The Lagrangian to minimize is

$$
L(\bar{q}_t) = \frac{d(q_t^3)}{v(q_t^4)} - 1 - \kappa_t^3 q_t^3 - \kappa_t^4 q_t^4 + \kappa_t^5 (q_t^3 + q_t^4 - R_t).
$$

**Proposition 6.2**

Suppose $1 - d(q_t^3)/v(q_t^4) < 0$ for all feasible investment plans $\bar{q}_t$, i.e. $\sum_{x=3}^4 q_t^x \leq R_t$ and $q_t^x \geq 0$ for all $x = 3, 4$. To establish the pre-conditions of a successful subsidy policy, it is optimal to use up all resources $R_t$. Contingent on the parameter value constellation, there are $2^2 - 1 = 3$ possible scenarios:

1. $(q_t^3)^o > 0$ and $(q_t^4)^o = 0$: Then $(q_t^3)^o = R_t$, $(\kappa_t^3)^o = 0$, $(\kappa_t^4)^o \geq 0$, and

$$
- \frac{d'(q_t^3)^o}{v((q_t^3)^o)} = (\kappa_t^5)^o > 0, \quad \frac{d[(q_t^3)^o]}{v((q_t^3)^o)^2} v'[(q_t^4)^o] + (\kappa_t^4)^o \leq - \frac{d'[q_t^3]}{v(q_t^4)^2} v'(q_t^4);
$$

2. $(q_t^3)^o = 0$ and $(q_t^4)^o > 0$: Then $(q_t^4)^o = R_t$, $(\kappa_t^3)^o \geq 0$, $(\kappa_t^4)^o = 0$, and

$$
- \frac{d'(q_t^3)^o}{v((q_t^3)^o)} + (\kappa_t^3)^o \leq \frac{d[(q_t^3)^o]}{v((q_t^4)^o)^2} v'[(q_t^4)^o], \quad \frac{d[(q_t^3)^o]}{v((q_t^4)^o)^2} v'[(q_t^4)^o] = (\kappa_t^5)^o > 0
$$

$$
0 \leq (\kappa_t^3)^o \leq \frac{d(q_t^3)}{v(q_t^4)^2} v'(q_t^4) + \frac{d'(q_t^3)}{v(q_t^4)} v'(q_t^4);
$$

3. $(q_t^3)^o > 0$ and $(q_t^4)^o > 0$: Then $(q_t^3)^o + (q_t^4)^o = R_t$, $(\kappa_t^3)^o = 0$, $(\kappa_t^4)^o = 0$, and

$$
- \frac{d'[q_t^3]^o}{v(q_t^4)} = \frac{d[(q_t^3)^o]}{v((q_t^4)^o)^2} v'[q_t^4]^o = (\kappa_t^5)^o > 0
$$

\footnote{The constraint follows from the fact that we neglect capital markets. The benefits of escaping the poverty trap are so high that the costs of investments $\bar{q}_t$ and of subsidization are negligible. However, the government cannot borrow against these huge benefits, and can only fall back on the period’s resources $R_t$.}
See for the proof in the appendix. If all resources are invested only in one investment type, say \( q_3 \), and the marginal contribution to lower the time constraint of this investment is still at least as high as that of the other, \( q_4 \), then it is optimal to invest all resources in \( q_3 \), and nothing in \( q_4 \), and vice versa. However, if this is neither the case for \( q_3 \) nor for \( q_4 \), then it is optimal to invest in both forms, where it is optimal to invest such that the marginal contribution to lower \( d_t/v_t \) are equal, as otherwise redistributing resources could lower \( d_t/v_t \) further. All these marginal contributions determine the shadow price of an additional unit of resource \( R_t \), labeled \( \kappa_5 \).

Finally the Lagrangian-multiplier of an investment form that is not undertaken, is positive but lower than the shadow price of the resources \( R_t \), even lower than the comparative advantage of the paying investment (see \( \kappa_3 \) and \( \kappa_4 \) in item 1. and 2., respectively). However, loosely speaking, the multiplier \( \kappa_x \) can be interpreted as measure of the comparative disadvantage of the investment form \( x \), because if an investment is actually remunerate, then its multiplier \( \kappa_x \) is zero.

In the cases where the optimum displays a boundary solution, it is possible that the marginal lowering of the positive investment, for instance \((q_3^o)^o > 0\), is exactly equal to the marginal lowering of investing a first unit of the zero-investment, \((q_4^o)^o = 0\). Then the boundary solution is a tangency solution like interior solutions are, and the shadow price of zero-investments becomes zero; in our instance, we had \((\kappa_4^o)^o = 0\) despite \((q_4^o)^o = 0\). The weak inequalities hence can turn into equalities.

**Phase 2** Once \( 1 - d(q_3^o)/v(q_4^o) > 0 \) is reached, subsidization is only successful if the time gap \( 1 - d_t/v_t \) is big enough to allow for the necessary education-time \( e^*(q_t) + \varepsilon \), which be abbreviated to \( e^*(q_t) \). Hence subsidization is only effective if constraint \( 1 - e^*(q_t) - d_t/v_t \geq 0 \) is fulfilled.

If investing all resources does not suffice to fulfill this constraint, subsidization is again ineffective, and the objective is:

\[
\max_{(q_t)} O(q_t) \equiv 1 - e^*(q_t) - \frac{d(q_3^o)}{v(q_4^o)} \quad \text{s.t.} \quad \sum_{x=1}^{4} q_t^x - R_t \leq 0, \quad q_t^x \geq 0, \forall x = 1, 2, 3, 4
\]

To describe the optimal strategy to educate a society, we define \( \mathcal{L} \) as the set of strict positive investments, \( \mathcal{L} = \{l \in \{1, 2, 3, 4\} \mid (q_l^o)^o > 0\} \), and \( \mathcal{K} \) as the set of zero-investments, \( \mathcal{K} = \{1, 2, 3, 4\} \setminus \mathcal{L} \), respectively \( \mathcal{K} = \{k \in \{1, 2, 3, 4\} \mid (q_k^o)^o = 0\} \), so that \( \mathcal{L} \cup \mathcal{K} = \{1, 2, 3, 4\} \).
Proposition 6.3
Suppose $1 - d(q_l^3)/v(q_l^4) \geq 0$, but $1 - \tilde{e}^*(\tilde{q}_l) - \frac{d(q_l^3)}{v(q_l^4)} \leq 0$ for all feasible investment plans $\tilde{q}_l$, i.e. $\sum_{x=1}^{4} q_l^x \leq R_t$ and $q_l^x \geq 0$ for all $x = 1, 2, 3, 4$. It is optimal to use up all resources $R_t$, and contingent on the parameter value constellation, there are $2^4 - 1 = 15$ possible scenarios, which follow the following general pattern:

$$\frac{\partial O(\tilde{q}_l)}{\partial q_l^l} = \kappa^5_l \text{ for all } l \in \mathcal{L}$$

$$\frac{\partial O(\tilde{q}_l)}{\partial q_l^k} + \kappa^k_l \leq \kappa^5_l \text{ for all } k \in \mathcal{K}$$

$$\frac{\partial O(\tilde{q}_l)}{\partial q_l^k} + \kappa^k_l \leq \frac{\partial O(\tilde{q}_l)}{\partial q_l^l} \text{ for all } k \in \mathcal{K}, \ l \in \mathcal{L}$$

Investments $\tilde{q}_l$ lower the required human capital for escaping the poverty trap, $\lambda^*(\tilde{q}_l)$. Investments $q_l^3$ and $q_l^4$ additionally diminish the time requirement $d_l/v_l$:

$$\frac{\partial O(\tilde{q}_l)}{\partial q_l^x} = -\frac{\partial \tilde{e}^*(\tilde{q}_l)}{\partial q_l^x} \text{ for } x = 1, 2$$

$$\frac{\partial O(\tilde{q}_l)}{\partial q_l^x} = -\frac{\partial \tilde{e}^*(\tilde{q}_l)}{\partial q_l^x} - \frac{\partial (d(q_l^x)/v(q_l^4))}{\partial q_l^x} \text{ for } x = 3, 4$$

The economic intuition is the same as in phase 1.

6.3.4.2 The Subsidization Phase

Once $d(q_l^3)/v(q_l^4) + \tilde{e}^*(\tilde{q}_l) - 1 < 0$ is achieved, the government is able to start successful subsidization. The goal is to maximize the fraction of households that can be supported by subsidy $\tilde{s}_l^*$. This fraction is labeled $\delta_l$. As we neglect the access to capital markets, the government’s budget will be balanced in the optimum: $\delta_l \tilde{s}^*(\tilde{q}_l) + \sum_{x=1}^{4} q_l^x = R_t$. Therefore is the government’s budget constraint fulfilled by construction. Subsidy $\tilde{s}^*(\tilde{q}_l)$ causes $e^*(\tilde{s}^*(\tilde{q}_l), q_l^2, q_l^3, q_l^4) \equiv e^*(\tilde{q}_l) \equiv \tilde{e}_l^*$, and hence the formation of human capital slightly higher than $\lambda_l^*$. The exercise is thus

$$\max_{\tilde{q}_l} \delta(\tilde{q}_l) = \frac{R_t - \sum_{x=1}^{4} q_l^x}{\tilde{s}^*(\tilde{q}_l)} \text{ s.t. } q_l^x \geq 0, \text{ for all } x = 1, 2, 3, 4. \quad (6.27)$$

Assumption 6.1

Maximization problem (6.27) fulfills the requirements for the utilization of the Kuhn-Tucker maximum conditions, i.e. $\delta(\tilde{q}_l)$ is concave.

A discussion of the assumption can be found in the appendix.
Proposition 6.4
Suppose the status quo is such that $1 - \tilde{c}^*_t - \frac{d}{dt} \geq 0$, and Assumption 6.1 holds. Then, it is optimal to use up all resources $R_t$. Contingent on the parameter value constellation, $2^4 - 1 = 15$ possible scenarios are possible in the optimum, all of which obey the following pattern:

$$\frac{R_t - \sum_{j=1}^{4} (\tilde{q}^*_t)^{\circ}}{(s^* (\tilde{q}^*_t))^2} \left( - \frac{\partial s^*(\tilde{q}^*_t)}{\partial q^*_t} \right) = \frac{1}{s^*(\tilde{q}^*_t)} \quad \text{and} \quad (\kappa^l_t)^{\circ} = 0 \quad \text{for all} \quad l \in \mathcal{L}$$

$$0 \leq (\kappa^k_t)^{\circ} \leq \frac{1}{s^* (\tilde{q}^*_t)} + \frac{R_t - \sum_{j=1}^{4} (\tilde{q}^*_t)^{\circ}}{(s^* (\tilde{q}^*_t))^2} \left( - \frac{\partial s^*(\tilde{q}^*_t)}{\partial q^*_t} \right) \quad \text{for all} \quad k \in \mathcal{K}$$

$$\frac{- \partial s^*(\tilde{q}^*_t)}{\partial q^*_t} > \frac{- \partial s^*(\tilde{q}^*_t)}{\partial q^*_t} \quad \text{for all} \quad k \in \mathcal{K}, \quad l \in \mathcal{L}$$

For the proof of Proposition 6.4 look in the appendix. The intuition is simple: a unit of any investment $q^x_t$ costs one unit of the scarce resources $R_t$, and lowers $\delta_t$ by factor $\frac{1}{s^* (\tilde{q}^*_t)}$. This marginal cost has to be compared with the marginal revenue in lowering $s^*_t$, which is $\frac{R_t - \sum_{j=1}^{4} q^j_t}{(s^*(\tilde{q}^*_t))^2} \left( - \frac{\partial s^*(\tilde{q}^*_t)}{\partial q^*_t} \right)$. As long as the net effect $\frac{R_t - \sum_{j=1}^{4} q^j_t}{(s^*(\tilde{q}^*_t))^2} \left( - \frac{\partial s^*(\tilde{q}^*_t)}{\partial q^*_t} \right) - \frac{1}{s^* (\tilde{q}^*_t)}$ is positive, the investment $q^x_t$ should be intensified. Thus, in the optimum we have the fundamental economic law ‘marginal revenue equals marginal cost’ for all remunerating investments. However, if right from the beginning the net effect of an investment form is negative, this instrument is inefficient, and hence not used in the optimum. Therefore, $- \frac{\partial s^*(\tilde{q}^*_t)}{\partial q^*_t} > - \frac{\partial s^*(\tilde{q}^*_t)}{\partial q^*_t}$. The shadow prices of the investment types measure the size of marginal lowering of $\delta_t$: $\frac{1}{s^* (\tilde{q}^*_t)} + \frac{R_t - \sum_{j=1}^{4} q^j_t}{(s^*(\tilde{q}^*_t))^2} \left( - \frac{\partial s^*(\tilde{q}^*_t)}{\partial q^*_t} \right) \geq 0$, for all $x = 1, 2, 3, 4$.\textsuperscript{26}

If we rearrange the optimum condition for the elements of $\mathcal{L}$, we find that the optimal fraction of the society that is lifted out of the poverty trap in a period $t$, $\delta(\tilde{q}^*_t)$, is determined by the marginal reduction of the required subsidy $\tilde{s}^*_t$ in the optimum:

$$\delta(\tilde{q}^*_t) = \left( - \frac{\partial \tilde{s}^*(\tilde{q}^*_t)}{\partial q^*_t} \right)^{-1}$$

Summarizing, we have found that even if the state’s resources (foreign aid) do not suffice to fulfill the pre-conditions for effective subsidization, the investment schemes described for phases 1 and 2 make sense, as they shift the society closer to the required starting condition of the subsidization process. If the subsidization program cannot start today, then those investments will enable the government to start effective subsidization some period later. So the education subsidy project might require an initial investment phase which prepares the pre-conditions for school subsidies.

\textsuperscript{26}At the edge, it is again possible that $- \frac{\partial s^*(\tilde{q}^*_t)}{\partial q^*_t} = - \frac{\partial s^*(\tilde{q}^*_t)}{\partial q^*_t}$, and the shadow price of zero-investments is also nil.
6.3.5 Discussion

In order to discuss the meaning of our results, we now give the model a little bit more structure. Consider

\[ Q_t = Q(q^1_t, Q_{t-1}) \quad \text{with} \quad \frac{\partial^2 Q_t}{\partial q^1_t \partial Q_{t-1}} < 0, \]

\[ \rho_t = \rho(q^2_t, \rho_{t-1}) \quad \text{with} \quad \frac{\partial^2 \rho_t}{\partial q^2_t \partial \rho_{t-1}} < 0, \]

\[ d_t = d(q^3_t, d_{t-1}) \quad \text{with} \quad \frac{\partial^2 d_t}{\partial q^3_t \partial d_{t-1}} \leq 0, \quad \text{and} \]

\[ v_t = v(q^4_t, v_{t-1}) \quad \text{with} \quad \frac{\partial^2 v_t}{\partial q^4_t \partial v_{t-1}} < 0. \]

The worse the situation is in one of the fields, the more effective are investments to improve the environment of education. Combining this structure with our previous results, we arrive at plausible conclusions. If the school quality is already very good, investments \( q^1_t \) are not very helpful in improving the situation, since then the school-quality is not the central detriment of the educational environment. Little improvements require the use of much of the scarce resources. Other investments in fields that are more crucial are thus more effective. In the described scenario, \( q^1_t \) is therefore a candidate for a zero-investment. Similarly, in countries where corruption is not widely spread, and the school density is low, efforts to defend corruption appear to be less helpful than investments \( q^3_t \) and \( q^4_t \).

Consequently we infer that the optimal strategy requires a tough analysis in front of any investment that endows the government with the necessary information about the status quo and the central drawbacks of the current situation (weak point analysis). Obviously each country has different weak points, and therefore requires other investment plans than other countries. Thus there is no general optimal strategy. Each country may have very particular characteristics that must be taken into account. It lies in the nature of the problem that our model does not cover all potential detriments for a successful education policy; for instance, cultural and religious peculiarities could prevent the success of policies that target the education of a society.\(^{27}\) However, the author hopes that the major general aspects were discussed.

Having deduced the general strategy for the education of a society, we now analyze a specific example in order to deepen the insights.

\(^{27}\)E.g., we neglected the special situation of female children.
6.4 A Preference Specification

Consider that as long as the household does not consume a particular level \( c^S \), the preferences are lexicographic, since consumption is so low that the household’s decisions are solely determined by current events, neglecting future aspects like education for the child. \( c^S \) is at least as high as the subsistence level of consumption \( c^{sub} \). Nevertheless, given a fixed level of consumption, more education for the child is preferred. So far this is equivalent to Chapter 4. As education now involves further burdens, however, we arrive at:

\[
\left\{ (e_t, c_t) \succeq (e'_t, c'_t) \mid c_t > c'_t \vee (c_t = c'_t \land e_t \geq e'_t) \right\} \quad \text{if} \quad c_t \leq c^S + \rho_t + \alpha\gamma\frac{d_t}{v_t}
\]

I.e. parents are nonetheless also altruistic for very low levels of consumption, but this altruism does virtually not operate, since the additional utility of the first unit of \( e_t \) is always lower than the additional utility of another unit of consumption. If \( e_t > 0 \) is chosen, the extra costs \( \rho_t + \alpha\gamma\frac{d_t}{v_t} \) diminish consumption below threshold \( c^S \).

As soon as the household’s income allows a consumption higher than \( c^S + \rho_t + \alpha\gamma\frac{d_t}{v_t} \), preferences are represented by a continuous, strictly quasi-concave Stone-Geary type utility function that is also twice differentiable and increasing:

\[
U(c_t, e_t) = (c_t - c^S) \cdot e_t + \overline{U} \quad \text{if} \quad c_t > c^S + \rho_t + \alpha\gamma\frac{d_t}{v_t}
\]

where \( \overline{U} \) is the level of utility of \( c_t = c^S + \rho_t + \alpha\gamma\frac{d_t}{v_t} \). \( c^S \) represents the degree of the household’s basically sense of altruism, namely the minimum level of consumption that is basically required before the adult is willing to send the child to school. This requirement increases by the extras \( \rho_t + \alpha\gamma\frac{d_t}{v_t} \).

6.4.1 The Household’s Demands

Note that if the household chooses no education no side payments \( \rho_t \) will be paid, and the household does not forgo the part \( \frac{d_t}{v_t} \) of the income of the child. We concentrate on the case \( e_t > 0 \), and assume non-satiation so that the household’s optimization problem is:

\[
\max_{\{e_t, c_t\}} \quad U(c_t, e_t) = (c_t - c^S) \cdot e_t + \overline{U} \\
\text{s.t.} \quad c_t + \alpha\gamma(e_t + d_t/v_t) + \rho_t = \alpha(\lambda_t + \gamma) + s_t \\
0 < e_t \leq 1 - d_t/v_t \\
c_t \geq 0
\]
The optimal demands are:

\[ e^o_t(s_t, \rho_t, \frac{d_t}{v_t}) = \min \left\{ 1 - \frac{d_t}{v_t}, \frac{1}{2\alpha \gamma} \left[ \alpha \left( \lambda_t + \gamma \left( 1 - \frac{d_t}{v_t} \right) \right) + s_t - c^S - \rho_t \right] \right\} \]  

(6.28)

\[ c^o_t(s_t, \rho_t, \frac{d_t}{v_t}) = \max \left\{ \alpha \lambda_t + s_t - \rho_t, \frac{1}{2} \left[ \alpha \left( \lambda_t + \gamma \left( 1 - \frac{d_t}{v_t} \right) \right) + s_t + c^S - \rho_t \right] \right\} \]

In case of \( \lambda_t = \lambda^S \), only the adult’s income \( \alpha \lambda_t \) plus full-time child labor income, \( \alpha \gamma \), could finance the household’s "altruism consumption" \( c^S \), plus the fixed cost of schooling, \( \rho_t + \alpha \gamma \frac{d_t}{v_t} \), whereby \( e^o = 0 \):

\[ \lambda^S(\rho_t, d_t, v_t) = \frac{1}{\alpha} (c^S + \rho_t) - \gamma(1 - d_t/v_t), \]  

(6.29)

To obtain the threshold \( \lambda^{ac} \) we must set \( \frac{1}{2\alpha \gamma} \left[ \alpha \left( \lambda_t + \gamma \left( 1 - \frac{d_t}{v_t} \right) \right) - c^S - \rho_t \right] \) equal to \( 1 - d_t/v_t \geq 0 \), whereat \( s_t = 0 \). This yields:

\[ \lambda^{ac}(\rho_t, d_t, v_t) = \frac{c^{ac} + \rho_t}{\alpha} = \frac{1}{\alpha} (c^S + \rho_t) + \gamma(1 - d_t/v_t) \]  

(6.30)

i.e. \( c^{ac} \) is a function \( c^{ac}(d_t, v_t) = c^S + \alpha \gamma (1 - d_t/v_t) \). In case of \( \lambda_t = \lambda^{ac}_t \), the child will not work at all. The adult’s income alone is sufficient to finance \( c^{ac}_t \) plus \( \rho_t \), where \( c^{ac}_t \) is \( c^S \) plus the direct forgone earnings of education without the forgone earnings of travelling from and to school. Hence, in contrast to \( c^S \), consumption level \( c^{ac}_t \) is endogenous, in the sense that the government is able to change it by investments \( \vec{q}_t \).

### 6.4.2 The Educational Subsidy

The adverse human capital threshold \( \lambda^*_t \) is implicitly given by

\[ \lambda^*(q^1_t, e^o(q^2_t, q^3_t, q^4_t)) = \frac{1}{1 - zh[e^o(\lambda^*_t, q^2_t, q^3_t, q^4_t), Q(q^1_t)]}. \]

(6.31)

We consider \( h(e_t, Q_t) = e_t \cdot Q_t \), so that

\[ \lambda_{t+1} = z \cdot e_t \cdot Q_t \cdot \lambda_t + 1. \]

If \( e_t \) is the interior solution of (6.28), then \( \lambda^* \) is implicitly determined by

\[ \lambda^*_t \cdot \left( 1 - zQ_t \left( \frac{\alpha(\lambda^*_t + \gamma) - c^S - \rho_t - \alpha \gamma \frac{d_t}{v_t}}{2\alpha \gamma} \right) \right) = 1, \]  

(6.32)

and we receive:

\[ \lambda^*(Q_t, \rho_t, d_t, v_t) = \frac{1}{2} \left( -A + \sqrt{A^2 - \frac{8\gamma}{zQ_t}} \right) \]  

(6.33)
with \( A = \gamma \left( 1 - \frac{d_t}{v_t} - \frac{2}{zQ_t} \right) - \frac{(e^S + \rho_t)}{\alpha} \) (the detailed calculation is given in the appendix).

It follows that the subsidy \( s_t^* \) has to cause human capital formation in the following way: \( z e^o(s_t^*, \lambda_t = 1, \cdot)Q_t + 1 = \lambda_t^* \). We conclude that \( e_t^o = \frac{\lambda_t^* - 1}{zQ_t} \leq 1 - \frac{d_t}{v_t} \). Setting this expression equal to the interior solution of \( e_t^o \), in which we have to set \( \lambda_t = 1 \), we find:

\[
\frac{d_t}{v_t} + \frac{2(\lambda_t^*(Q_t, \rho_t, d_t, v_t) - 1)}{zQ_t} - 1 \right) - 1 \right) (6.34)
\]

The subsidy \( s_t^* \) thus has to fill the gap between the income required for \( e_t^o = e_t^* \) and the laissez-faire income \( \alpha(1 + \gamma) \).

### 6.4.3 Comparative Statics

**Lemma 6.1**

Improvements of \( Q_t, \rho_t, d_t \) and \( v_t \) all decrease the threshold \( \lambda^* \):

\[
\frac{\partial \lambda^*(\cdot)}{\partial Q_t}, \quad \frac{\partial \lambda^*(\cdot)}{\partial v_t} < 0; \quad \frac{\partial \lambda^*(\cdot)}{\partial \rho_t}, \quad \frac{\partial \lambda^*(\cdot)}{\partial d_t} > 0
\]

A proof is in the appendix. With Lemma 6.1, we can state

**Proposition 6.5**

All investment types reduce the for the education of a household required subsidy \( s_t^* \), and thus allow the education of a society in a shorter span of time:

\[
\frac{\partial s_t^*(\cdot)}{\partial q_t^x} < 0, \quad x = 1, 2, 3, 4
\]

The proof is in the appendix. Overall, we state that all investment types are, without any doubt, helpful to expedite the proceedings of the education of a society.

### 6.5 A Preference Modification

So far we assumed that the time the child spends in school determines the adult’s utility. However, in reality parents are also interested in the consumption possibilities that this schooling time generates, i.e. on the resulting level of human capital of the child in adulthood, \( \lambda_{t+1} \). It is clear that \( \lambda_{t+1} \) depends, among other things, on the quality of schooling (cf. our discussion in Appendix 3). Let therefore utility \( u_t \) still be determined by consumption \( c_t \). But in contrast to our basic model, let us consider that

\[28\text{Note that the steady state level of } e \text{ equals } \frac{\lambda_t^*}{zQ_t}, \text{ which is lower due to } \lambda^* > 1 \text{ in the relevant cases. Remind also footnote 24.}\]
utility is not just determined by the time-fraction \( e_t \) but by the from the schooling time \( e_t \) resulting level of human capital \( \lambda_{t+1} \): \( u_t = u(c_t, \lambda_{t+1}(e_t)) \). Let all other things be exactly as in our former analysis in this chapter. Family’s budget constraint concerning \((c_t, e_t)\) is:

\[
\alpha(\lambda_t + \gamma) = \begin{cases} 
  c_t + \alpha \gamma(e_t + d_t/v_t) + \rho_t & \text{if } e_t > 0 \\
  c_t & \text{if } e_t = 0
\end{cases} \quad (6.35)
\]

The adult still chooses \( c_t^* \) and \( e_t^* \). This decision results from comparing marginal utility of consumption \( c_t \) and of education \( \lambda_{t+1} \). Assuming that preferences can be described by a differentiable utility function \( u(c_t, \lambda_{t+1}) \), standard theory teaches us that an interior extremum forces:

\[
\frac{\partial u(c_t, \lambda_{t+1}(e_t, Q_t))}{\partial c_t} = \frac{1}{\alpha \gamma} \frac{\partial u(c_t, \lambda_{t+1}(e_t, Q_t))}{\partial \lambda_{t+1}} \cdot \frac{\partial \lambda_{t+1}(e_t, Q_t)}{\partial e_t} \quad (6.36)
\]

where \( \lambda_{t+1}(e_t, Q_t) = h(e_t, Q_t)z\lambda_t + 1 \). Contrary to our previous analysis, we carefully have to check the second-order-conditions for a maximum, since due to \( \lambda_{t+1}(e_t) \), the utility function is not necessarily well-behaved. The interior extremum is a maximum, if the Hesse matrix of our utility function \( u(c_t, \lambda_{t+1}(e_t)) \) is negatively semi-definite, i.e.:

\[
\frac{\partial^2 u(c_t, \lambda_{t+1}(e_t))}{\partial c_t^2} < 0, \quad \text{and}
\]

\[
\frac{\partial^2 u(c_t, \lambda_{t+1}(e_t))}{\partial c_t^2} \cdot \frac{\partial^2 u(c_t, \lambda_{t+1}(e_t))}{\partial c_t^2} - \left( \frac{\partial^2 u(c_t, \lambda_{t+1}(e_t))}{\partial c_t \partial c_t} \right)^2 > 0
\]

The first condition holds due to our previous assumptions, but to fulfill the second, we suppose:

**Assumption 6.2**

We assume that the Hessian of \( u(c_t, \lambda_{t+1}(e_t)) \) is negatively semi-definite, i.e.:

\[
\frac{\partial^2 u_t}{\partial c_t^2} \left[ \frac{\partial^2 u_t}{\partial \lambda_{t+1}^2} + \frac{h''(e_t)}{z\lambda_{t+1}^2} \frac{\partial u_t}{\partial \lambda_{t+1}} \right] > \left( \frac{\partial^2 u_t}{\partial \lambda_{t+1} \partial c_t} \right)^2
\]

As long as the left hand side (l.h.s.) of Equation (6.36) is higher than the right hand side (r.h.s.), the adult increases \( c_t \) and decreases \( e_t \) to maximize utility. We conclude that the adult chooses boundary solution \((e_t^* = 0, c_t^* = \tau(\lambda_t))\) as long as:

\[
\frac{\partial u(\tau(\lambda_t) - \rho_t - \alpha \gamma(d_t/v_t), \lambda_{t+1}(0, Q_t))}{\partial c_t} \geq \frac{1}{\alpha \gamma} \left[ \frac{\partial u(\tau(\lambda_t) - \rho_t - \alpha \gamma(d_t/v_t), \lambda_{t+1}(0, Q_t))}{\partial \lambda_{t+1}} \cdot \frac{\partial \lambda_{t+1}(0, Q_t)}{\partial c_t} \right] \quad (6.37)
\]
For $\lambda_t = \lambda^S$ inequality (6.37) holds with equality, so that $c^S = \alpha(\lambda^S + \gamma) - \rho_t - \alpha\gamma(d_t/v_t)$. Similar we receive via $c^{ac} = \alpha \lambda^{ac} - \rho_t$:

$$
\partial u(c^{ac}, \lambda_{t+1}(1 - \frac{d_t}{v_t}, Q_t)) \frac{1}{\alpha \gamma} \left[ \frac{\partial u(c^{ac}, \lambda_{t+1}(1 - \frac{d_t}{v_t}, Q_t))}{\partial \lambda_{t+1}} \cdot \frac{\partial \lambda_{t+1}(1 - \frac{d_t}{v_t}, Q_t)}{\partial c_t} \right]
$$

(6.38)

i.e., that choosing the maximal possible level of education $e_t = 1 - (d_t/v_t)$ is optimal when the adult’s income allows consumption $c^{ac}$. If $c^{ac}$ is feasible the adult is willing to fully renounce child labor. Note that contrary to our previous chapters, it is important to emphasize that demand $e_t^o$ and $c_t^o$ depend on the quality of schools $Q_t$ (Equation (6.36)).

### 6.5.1 New Aspects of Investments in the Quality of Schools

We now study the effects of changes of $Q_t$, $\rho_t$, and $(d_t/v_t)$. Hitherto changes of the schooling quality $Q_t$ had no effects on the demands of the households, because independent from the quality, the child’s time spent in school determined utility.

To analyze the effect within our changed setup, we abbreviate the notation: derivatives are now abbreviated by a subscript at the function that should be differentiated with respect to this subscript. For instance is $U_{e_t}$ the marginal utility of consumption, or $h_{e_tQ_t}$ the second derivative of function $h(e_t, Q_t)$, where firstly $h(e_t, Q_t)$ is differentiated with respect to $e_t$, and secondly this derivative with respect to $Q_t$.

Using these abbreviations, the marginal rate of substitution is $MRS \equiv \frac{U_{c_t}}{U_{\lambda_t}} = \frac{U_{c_t}}{U_{\lambda_{t+1}} \lambda h_{e_t}}$.

A change of $Q_t$ does leave the budget unchanged. The MRS, i.e. the slope of the indifference curves, however, changes:

$$
\frac{\partial MRS}{\partial Q_t} = \frac{h_{Q_t}}{h_{e_t}} \left( \frac{U_{e_t \lambda_t}}{U_{\lambda_t}} - \frac{U_{e_t} U_{\lambda_t \lambda_t}}{(U_{\lambda_t})^2} \right) - \frac{h_{e_tQ_t} U_{c_t}}{z\lambda_t h_{e_t}} \left( \frac{U_{e_t}}{(U_{\lambda_t})^2} \right)
$$

(6.39)

Because of $\frac{h_{Q_t}}{h_{e_t}} \left( \frac{U_{e_t \lambda_t}}{U_{\lambda_t}} - \frac{U_{e_t} U_{\lambda_t \lambda_t}}{(U_{\lambda_t})^2} \right) > 0$ but $- \frac{h_{e_tQ_t} U_{c_t}}{z\lambda_t h_{e_t} (U_{\lambda_t})^2} < 0$ the sign of $\frac{\partial MRS}{\partial Q_t}$ remains open. It depends on the specific parameter value constellation whether the indifference curves become steeper or flatter when $Q_t$ rises. An increase of $Q_t$ increases $U_{c_t}$ and decreases $U_{\lambda_{t+1}}$, which tends to increase the MRS. Simultaneously the marginal productivity of schooling $z\lambda_t h_{e_t}$ increases, which tends to lower the MRS. There is a trade-off. On the one hand, human capital $\lambda_{t+1}$ becomes relatively less scarce, so that investment $e_t$ is less attractive. On the other hand, investment $e_t$ becomes more effective due to a higher productivity of $h(e_t, Q_t)$.

One would expect that an improvement of $Q_t$ will lower the MRS, because a reduction
of consumption \( c_t \) is then easier to compensate: \( \frac{\partial MRS}{\partial Q_t} < 0 \). This happens if

\[
\frac{U_{c_t}U_{\lambda_t}}{U_{c_t}U_{\lambda_t} - U_{c_t}U_{\lambda_t, \lambda_t}} > \frac{h_{Q_t}h_{e_t}z\lambda_t Q_t}{\varepsilon_{h_{Q_t}}h_{e_t}Q_t},
\]

i.e. if the elasticity of the productivity of the schooling function \( \varepsilon_{h_{Q_t}}h_{e_t}Q_t \equiv \frac{\partial h_{e_t}}{\partial Q_t}Q_t h_{e_t} > 0 \) is big enough.

We now have to apply this result in investigating the effect of changes of \( Q_t, \rho_t, \) and \( (d_t/v_t) \) on \( \lambda^S \) and \( \lambda^{ac} \). Note that movements of \( \lambda_t \) are analogous to those derived for \( Q_t \): \( \lambda_{t+1} \) becomes less scarce and the productivity of investment \( e_t \) rises. Additionally changes of \( \lambda_t \) involve movements of the budget. Consequently changes of \( \lambda_t \) directly involve simultaneous changes of the households’ budgets and of the indifference curves. It follows that we are not able to conclude general results. Therefore we restrict ourselves to a specific example.

### 6.5.2 A Specific Example

In order to study a specific example, we underlie:

\[
\lambda_{t+1} = e_t Q_t z\lambda_t + 1 \quad (6.40)
\]

Consider preference function:

\[
U = \begin{cases} 
-\infty & \text{if } \tau(\lambda_t) < c^{sub}; \\
\alpha(\lambda_t + \gamma) & \text{if } c^{sub} \leq \tau(\lambda_t) \leq c^{sub} + \rho_t + \alpha\gamma(d_t/v_t); \\
(c_t - c^{sub})^\alpha[e_t Q_t z\lambda_t + 1]^\beta + c^{sub} + \rho_t + \alpha\gamma(d_t/v_t) & \text{if } \tau(\lambda_t) > c^{sub} + \rho_t + \alpha\gamma(d_t/v_t). 
\end{cases} \quad (6.41)
\]

On principle, the adult is willing to send the child to school as soon as \( \alpha(\lambda_t + \gamma) > c^S = c^{sub} \). However, school attendance costs \( \rho_t \) and further opportunity cost \( \alpha\gamma(d_t/v_t) \). It becomes directly clear that \( e^o_t = \alpha(\lambda_t + \gamma) \) as long as \( \alpha(\lambda_t + \gamma) \leq c^{sub} + \rho_t + \alpha\gamma(d_t/v_t) \), that \( e^o_t > 0 \) as soon as \( \alpha(\lambda_t + \gamma) > c^{sub} + \rho_t + \alpha\gamma(d_t/v_t) \), and that therefore \( \lambda^S = \frac{1}{\alpha}[c^{sub} + \rho_t + \alpha\gamma(d_t/v_t)] - \gamma \). The quality of schools has no effect on \( c^S = c^{sub} \), because \( c^S \) is the maximum level of consumption the adult asks for before willing to renounce partly child labor. As soon as \( \alpha(\lambda_t + \gamma) > c^{sub} + \rho_t + \alpha\gamma(d_t/v_t) \), the adult can afford the extra costs for education and chooses \( e^o_t > 0 \). This is the interesting case for the policy maker. The budget constraint is \( \alpha(\lambda_t + \gamma) \geq c_t + \alpha\gamma(e_t + (d_t/v_t)) + \rho_t \).
Utility maximization yields:

\[
\frac{\partial U(c_t, \lambda_{t+1}(e_t, Q_t))}{\partial c_t} = \frac{\alpha[e_t Q_t z \lambda_t + 1]^\beta}{(c - c_{sub})^{1-\alpha}} \tag{6.42}
\]

\[
\frac{\partial U(c_t, \lambda_{t+1}(e_t, Q_t))}{\partial e_t} = \frac{\alpha}{(c - c_{sub})^{1-\beta}} \tag{6.43}
\]

and forces that the marginal utility of consumption and school attendance of the child must be equal if the optimum is an interior. We find that:

\[
c^o_t = \frac{\alpha}{\alpha + \beta} \left( \alpha(\lambda_t + \gamma) - \rho_t - \alpha \gamma \frac{d_t}{v_t} \right) + \frac{\beta}{\alpha + \beta} c_{sub} \tag{6.44}
\]

\[
e^o_t = \frac{\beta}{\alpha \gamma (\alpha + \beta)} \left( \alpha(\lambda_t + \gamma) - \rho_t - \alpha \gamma \frac{d_t}{v_t} - \left( \frac{\alpha}{\beta} \right) \frac{\alpha \gamma}{Q_t z \lambda_t} - c_{sub} \right) \tag{6.45}
\]

We can state: \( c^o_t = c^o_t(\lambda_t, Q_t, \rho_t, (d_t/v_t)) \) and \( e^o_t = e^o_t(\lambda_t, Q_t, \rho_t, (d_t/v_t)) \).\(^{29}\) Thus nothing changed except that the optimal demands are also a function of the school quality.

### 6.5.3 Comparative Statics

It can easily be verified that:

\[
\frac{\partial e^o_t}{\partial Q_t} = \frac{\alpha}{(\alpha + \beta) z \lambda_t (Q_t)^2} > 0
\]

\[
\frac{\partial e^o_t}{\partial \rho_t} = -\frac{\beta}{(\alpha + \beta) \alpha \gamma} < 0
\]

\[
\frac{\partial e^o_t}{\partial \left( \frac{d_t}{v_t} \right)} = -\frac{\beta}{\alpha + \beta} < 0
\]

From \( Q'(q^1_t) > 0, \rho'(q^2_t) < 0, \) and \( \frac{\partial MRS}{\partial Q_t} < 0, x = \{3, 4\}, \) we can conclude that the government is able to increase schooling by all investments \( q^1_t \) to \( q^4_t \). I.e., when the resulting adult’s human capital of the child spends utility, the government is, in contrast to the former preference specification, able to increase schooling by all four described investments, thus also by \( q^1_t \).

Since in our example \( \frac{\partial e^o_t}{\partial Q_t} > 0 \) and \( \frac{\partial e^o_t}{\partial \lambda_t} < 0, \) the indifference curves become flatter when the schooling quality improves; the same happens when \( \lambda_t \) increases:

\[
MRS = \frac{\alpha}{\beta} \left( \frac{1}{c - c_{sub}} \right) \left( e_t + \frac{1}{Q_t z \lambda_t} \right) \tag{6.46}
\]

\[
\frac{\partial MRS}{\partial Q_t} = -\frac{\alpha}{\beta} \left( \frac{z \lambda_t}{(c - c_{sub})(Q_t z \lambda_t)^2} \right) < 0 \tag{6.47}
\]

\[
\frac{\partial MRS}{\partial \lambda_t} = -\frac{\alpha}{\beta} \left( \frac{z Q_t}{(c - c_{sub})(Q_t z \lambda_t)^2} \right) < 0 \tag{6.48}
\]

\(^{29}\)There occur two contrary effects of an increase of the adult’s human capital \( \lambda_t \) on \( c^o_t \): for particular parameter constellations the consumption demand is inferior.
Chapter 6. Multidimensional Education Policy

In our example $\lambda^S$ and $\lambda^{ac}$ are determined by the optimum condition:

\[
\frac{\alpha}{\beta} \left( \frac{1}{e(\lambda^S)} - \rho_t - \alpha\gamma(d_t/v_t) - c^{sub} \right) \left( \frac{1}{Q_tz\lambda^S} \right) = \frac{1}{\alpha\gamma} \quad (6.49)
\]

\[
\frac{\alpha}{\beta} \left( \alpha\lambda^{ac} - \rho_t - c^{sub} \right) \left( 1 - \frac{d_t}{v_t} + \frac{1}{Q_tz\lambda^{ac}} \right) = \frac{1}{\alpha\gamma} \quad (6.50)
\]

One can easily verify that:

\[
\frac{\partial \lambda^S}{\partial \rho_t} > 0, \quad \frac{\partial \lambda^S}{\partial \left( \frac{d_t}{v_t} \right)} > 0, \quad \frac{\partial \lambda^S}{\partial Q_t} < 0 \quad (6.51)
\]

\[
\frac{\partial \lambda^{ac}}{\partial \rho_t} > 0, \quad \frac{\partial \lambda^{ac}}{\partial \left( \frac{d_t}{v_t} \right)} < 0, \quad \frac{\partial \lambda^{ac}}{\partial Q_t} < 0 \quad (6.52)
\]

Hence our qualitative results in (6.6), (6.7) and (6.8) remain valid. But as the quality aspect is considered by the parents, the role of $Q_t$ becomes more important, and thus investment $q^1_t$ is more effective.

6.6 Conclusions

The chapter performs two contributions. First, it deepens our understanding of educational decisions in underdeveloped countries. Second, resting on these new results, we have identified a trade-off. In an environment in which a policy maker is able to invest not only by subsidies but also by other investment types, it is decisive which investment bears the highest marginal return, given a particular objective function. Paying education subsidies is not efficient as long as the necessary pre-conditions for a successful subsidization are not fulfilled, yet. Therefore, we identified that, in the worst case, a subsidy policy that aims at the education of a society has to run through three phases. In the first phase, the goal is to ensure that parents are actually in a position to send children to school. Hence, the policy maker must increase the school density of the country to ensure a sufficient school supply. At the same time, the traffic infrastructure has to be improved to speed up the time for travelling to and from a school. As soon as all poor children actually are able to attend schools, phase two starts. In this phase, additionally, the quality of the schools (in a broad sense) has to be improved, and corruption has to be fought. This comprises, referring the school quality, school facilities, exercise books, chalk, and blackboards, but also well educated and motivated teachers. Fighting corruption comprises eliminating employees in the Civil Service who demand bribes for paying out subsidies to the beneficiaries and those at schools who demand bribes for accepting pupils at the school of their responsibility.
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The target of this second phase is to augment the demand for education (of the children) on the parents’ side, and to reach a higher effectiveness of schooling. Over all, the resulting time window for education per day should be widen and the for a successful subsidization required school-attendance time be lowered, so that a subsidy policy that enables the society to escape poverty becomes possible. Once the pre-conditions for a successful subsidization program are established, the final phase is, in principle, similar to the subsidy policy described in Bell and Gersbach (2001), but an additional, optimal investment plan accelerates the education of the society.

We demonstrated that the improvement of the school quality, of the school transportation system, and in defending corruption in the education sector, all lower the future necessary (conditional or unconditional) education subsidy payments that we discussed in previous chapters. Thus, these investments bear future returns in a twofold way. They directly increase the demand for education and they lower the future subsidy burden of the government. If investments are undertaken, one must weigh out which type of education policy (new schools, better quality of schools, subsidies, and fighting corruption) is, marginally viewed, most effective when compared to the costs (loss of one unit of resource). Therefore, it can be efficient to use some state resources for other educational investments than subsidies. This establishes an optimal investment portfolio of subsidies and investment plan $\vec{q}$. Finally, we demonstrated that our results are robust to the modification of the preferences that adults involve the quality of schooling in determining the education choice for their children, but their arise interesting additional effects.

In all three phases, those investments with the highest marginal (net) improvement of the respective objective function are used. In the optimum, all the marginal improvements of undertaken investments are equal to each other. If investments are too unproductive, they are not undertaken. Whether this is the case strongly depends on the particular environment of a single country viewed. We can conclude that there is no single common optimal policy for all underdeveloped countries. One very carefully has to distinguish the particular different circumstances of single underdeveloped countries. Hence, overall, we deduced a comprehensive strategy to educate a society.

Eventually, it is important to notice that there remains one drawback in our analysis, that we have not solved: we used “semi-static” objection functions that only covered one period. What is meant by this is, first, that we indeed covered the dynamic effects of the investment plan by reasonably assuming that these positive effects occur within the same period, since one period spans some 15-20 years. However, we were not allowing for the positive dynamic effects of subsidizing households when one-time
subsidization is not sufficient to enable beneficiaries to escape the poverty trap. It very well might be possible that repeated subsidization of single households is efficient when accounting for this missing effect. Consequently, subsidizing already in phase two, where the children are already in a position to attend school, might be part of the efficient strategy. For if some households receive subsidies in phase 2, these households form human capital, and consequently the future subsidy that is necessary to attain fulltime schooling diminishes. Hence, subsidies in phase two represent investment just as the other investments in phase two. Future research should definitely highlight this issue.

Moreover, in practice, educational investments are not undertaken efficiently: Prichett and Filmer (1999), for instance, found that spending on school materials has a rate of return that is much larger than additional spending on teachers. Future research should analyze why governments do not invest in the pattern that we have derived. Additionally, a careful analysis should elaborate on techniques to estimate the actual rates of return; this would be helpful to determine the optimal investments.

30 Cf. also Easterly (2002), p. 83. One reason for this inefficient spending is that politicians dispense teaching positions as patronage.
Part II

Land Reforms to Overcome Poverty
Chapter 7

Land Reforms and Economic Development

7.1 Introduction

We have seen that insufficient income and assets lead to the failure of human capital formation that perpetuates itself (poverty trap): poverty causes child labor, child labor missing education, and missing education again poverty. Perfect capital markets would enable parents to borrow against expected future earnings achieved by education and thus to invest in the human capital of their children. Poor parents in developing countries do not have access to capital markets, however, and children’s education must be financed by the household’s current earnings and assets. The most important asset and source of income in developing countries is land, because these are mostly agrarian economies. Kevin Cleaver, Director of the World Bank’s Rural Development Department, says “Since 75 percent of world’s poor live in rural areas, the battle against poverty will in large measure be fought and won there.”\(^1\) Rural poverty and lack of land ownership go hand in hand and the World Bank states that a widespread lack of land ownership is a major source of poverty.\(^2\) Therefore, land reforms are likely a fruitful path to fight poverty and the associated problems of child labor and education.

In many developing countries land is used inefficiently and distributed highly inequitable. For many poor have no (or not sufficient) access to land due to imperfect credit and land markets, land lies idle though it would be highly remunerate if those poor cultivated it. These conditions often cause violent conflicts, and considering population growth, these conflicts will become more acute rather than the reverse.\(^3\)

\(^1\)Cf. [http://www.weltbank.org](http://www.weltbank.org), feature stories, *Reaching the Rural Poor*.

\(^2\)Cf. for instance *Ravallion and Sen* (1994).

Chapter 7. Land Reforms and Economic Development

Hence, the objective of land reforms have to be enhancing equity. But, at the same time, it is important that land reforms improve efficiency and growth to overcome poverty. The inequality-growth literature suggests that improving equity might cause higher growth.\(^4\) Although land reforms were attempted in many places, most were not successful. Nevertheless, there is a political debate about land reforms – especially in African countries like Namibia and Zimbabwe –, so that land redistribution remains a top priority in the policy agenda of many countries.\(^5\) The major aim of these policy proposals is improving equity. We believe that the main goal of land reforms should be fighting poverty. Therefore, this chapter addresses how to design a land reform that allows a society to overcome poverty traps.

Moreover, we have learned that in developing countries individuals under-invest in human capital, that human capital becomes increasingly important in a future, increasingly globalized world, and that, for this reason, the World Bank stated the millennium goal that by 2015 all children should be able to complete a full course of primary schooling. Unfortunately, human capital formation and schooling is fully neglected in the discussion on land reforms. However, LUNDBERG AND SQUIRE (2003), p. 341, for instance, state “... expanded education and more equitable land redistribution will at least improve income distribution, and may also enhance growth.” Though for them the goals “education” and “land reform” are separated from each other, we will show that there might be an important nexus between the millennium goal regarding education and land reform policies, that was ignored so far.

We consider a two-sector economy with overlapping generations where each generation consists of a continuum of individuals. In the first sector, a consumption good is produced with land, labor (including child labor) and human capital. The second sector is similar to the technology used in Part I: output is produced with labor and human capital alone. Parents again have altruistic preferences regarding their children. They invest into the education of their children as soon as their income reaches a critical level. Land enables households to enter a higher income bracket which may ensure the education of children and relieve poverty. The experiences with the reforms in the Philippines, for instance, tend to support our model. The land reform there had a strong impact on investment in human and physical capital and on long-term growth of income, productivity, and investment [cf. DEININGER, OINTO, AND MAERTENS (2000), p. 12].


\(^5\) A dramatic example is the land dispute in Zimbabwe following a new Land Act Reform [see for instance Godwin (2003) and Waeterloos and Rutherford (2004)].

150
Our main results are as follows: first, the optimal land reform consists of a sequence of land transfers. In order to accumulate human capital, only a (small) part of the society should receive land transfers at a particular point in time; this enables beneficiaries to receive a sufficient size of land. Part of the land gift the households receive at one point in time can be kept as long as they use it for agriculture production. With the other part of the land gift they must, in the course of time, support the other poor households through future transfers. Second, allowing for open land market access, increases efficiency in (agriculture) production. However, we demonstrate that open land market access may induce land sales of beneficiaries too early, which causes a decline of human capital formation over time and thus can cause the reform to fail. Therefore, for reasonable parameter values, open access to land markets should be prohibited for beneficiaries of land reforms for some time.

The remainder of the chapter is organized as follows. In the next section, we discuss our findings in the context of related literature. In Section 7.3, we introduce the model and the corresponding dynamics. Section 7.4 gives a comprehensive analysis of how a successful land reform must be designed, when beneficiaries do not have access to the land market. The resulting distribution of land, human capital, and income is discussed. In Section 7.5, the implications of the access to land markets are identified. We then elaborate on the transition patterns that land reforms may induce. Section 7.6 concludes.

### 7.2 Relation to the Literature

The chapter is related to several strands of literature. We will not repeat the related literature already cited in Part I of this thesis, but concentrate on the chapter-specific related literature.

Related with Galor and Zeira (1993), Deininger and Olinto (2000) and Bigsten and Levin (2000) conclude for developing countries that there exists evidence for a negative impact of asset inequality on subsequent growth. A large inequality in asset distribution, for instance of land distribution, seems harmful for growth due to credit rationing. Our results suggest that temporary inequality of land holdings and income is necessary for inducing growth.\(^6\)

There exist only a few recent models on land reforms. Bell (2003) and Gersovitz (1976) provide models in which they focus on the effect of land redistribution upon aggregate output and factor prices. They demonstrate that different outcomes are possible. Contrary to our work, these analyses are static and do not incorporate long-term effects of a land reform. Within a cooperative game theory approach, Horowitz (1993) considers a model where the agents can decide to accept a reform proposal or enter a conflict. The optimal reform consists of a sequence of redistributions. Our model provides a dynamic perspective on an optimal sequence of land transfers and highlights the role of land markets.

Discussions of the main issues in the context of land reforms have been dealt with in excellent survey articles by Banerjee (1999) and Deininger (1999) [see also De Janvry and Sadoulet (1996), Lundberg and Squire (1999), Conning and Robinson (2001), Deininger and May (2000)]. This literature suggests that access to assets like land improves the access to credit markets, because land can be used as collateral. Moreover, it can provide benefits as an insurance to consumption fluctuations and enables the poor to undertake indivisible productive investments. Overall, land reforms should improve equity, efficiency and hence aggregate growth. Hence, in comparison to subsidy policies, land reforms might produce improvements that subsidies cannot attain. Our analysis suggests that only a sequence of partial land transfers with a restricted possibility of selling the land can deliver the gains associated with such a reform.

yet different types of land reforms are also discussed by Besley and Burgess (2000), Banerjee (1999), and Deininger (1999). The main causes of land reform failure have been imperfect capital, insurance and land markets which lead to insufficient investments, makes macroeconomic shocks very dangerous for land-based production and forces corresponding distress sales. Finally, a lack of knowledge of beneficiaries about agriculture reinforces the danger of failure. We show that even in a world without uncertainty adverse land sales can arise.

Finally, our work is also broadly related to Poutvaara (2003), who demonstrates that working adults may be voluntarily willing to finance public education if they hold land for old-age providence. As the value of land increases in the stock of human capital, education costs for the youth represent a paying investment for their retirement. Hence, land owners have an incentive to support land redistribution targeting on education.

### 7.3 The Model

The model of this chapter is a dual economy version of our basic model; it is related to the dual economy developed by Drazen and Eckstein (1988). In our investigation of land reforms in developing countries we continue to neglect capital markets in modelling the credit constraint faced by the poor (imperfect capital market). We also keep on considering an OLG structure in which individuals live for the two periods “childhood”, and “adulthood”. Each generation consists of a continuum of households represented by interval [0, 1]. There is no other form of bequest than land. Upon the decease of the adult, the household’s land is left to the child.\(^{10}\) The human capital technology remains

\[
\lambda_{it+1} = h(e_{it})(z\lambda_{it}) + 1. \tag{7.1}
\]

#### 7.3.1 The Consumption Good Technologies

Let there be one consumption good that is produced in two sectors, which are labeled by \(j = (1, 2)\).\(^{11}\) Sector 1 is a land-based sector, such as agriculture, producing the aggregated output good solely using land and effective labor (human capital). We

\(^{10}\)The land bequest is not endogenously motivated. We assume that the farms of the poor are family-based, and that it is out of question that the farm is left to the heirs. In a three period OLG model with a final retirement period this issue would seem more crucial. We expect our results to be robust with respect to an endogenous bequest motive in a model where each generation lives three episodes since bequest motives would increase the need to sequentially redistribute land in the society.

\(^{11}\)Similarly, one could argue that both sectors produce goods which are perfect substitutes for each other.
assume that all farms are family-based. Household \( i \)'s, \( i \in [0,1] \), possession in land in period \( t \) is denoted by \( n_{it} \) and its adult's level of human capital \( \lambda_{it} \). Each single child have human capital of \( \gamma \in (0,1) \). The output in period \( t \) per household \( i \), labeled by \( y_{it} \), is described by the following production function with constant returns to scale:\(^{12}\)

\[
y_{it} = A_1 [\lambda_{it}(1-e_{it})\gamma]^\alpha \cdot (n_{it})^{1-\alpha}
\]

(7.2)

with \( A_1 \) representing the technical status quo of the sector and \( \alpha \in (0,1) \) being the production elasticity of human capital.

The second sector is solely human capital-based and represents the technology in towns (industry sector). Let there, similar to the preceding chapters, be a proportional relationship between output and input of effective labor (human capital). \( A_2 \) represents the fixed productivity of a unit of effective labor (technical status quo). Thus, the output per household \( i \) in period \( t \), labeled \( y_{it}^2 \), is given by:

\[
y_{it}^2 = A_2 [\lambda_{it}(1-e_{it})\gamma]
\]

(7.3)

The entire value of output per household accrues to the household as income. The output of both sectors is homogeneous and is supplied in one and the same market, wherefore the output of both sectors costs the same price per unit; we normalize this price to one. We assume that a household works only in one of the sectors. Thus, neglecting any production costs, the income of a household \( i \) working at time \( t \) in sector \( j \) is \( y_{it}^j \).

### 7.3.2 The Household’s Behavior

#### 7.3.2.1 Consumption and Education

In principle, the household’s behavior remains the same as in the preceding chapters. But we have to extend our analysis to the household’s behavior in sector 1 and, as there are now two sectors, to the migration decision. To avoid confusion, we briefly repeat our preceding descriptions and embed the new aspects:

The households cannot borrow and there are no other bequests than land to children. However, the inter-generational transfer via child rearing, \( z\lambda_{it} \), and education \( e_{it} \) are other forms of gifts. All adults have identical convex preferences that satisfy the usual assumptions of positive but decreasing marginal utility and non-satiation referring to goods. The level of utility of adult \( i \) in period \( t \) is labeled by \( u_{it} \).

\(^{12}\)Deininger and Feder (1998), p. 16, report that a large number of empirical studies were unable to reject the hypothesis of constant returns to scale in agricultural production.
We assume that the adult \( i \)'s utility is determined by the period’s consumption of the aggregated good \( c_{it} \) – where it does not matter whether the good unit stems from sector 1 or 2 – and by the level of education of the child, i.e. \( u_{it} = u(c_{it}, e_{it}) \). The child’s consumption is again a fixed fraction of the adult’s and is without loss of generality neglected in our analysis. Furthermore, we assume that the land owned at death is left to the child.\(^{13}\) In order to opt for \( e_{it} > 0 \), the altruistic tie between child and parent (with regard to education) must be sufficiently strong. The household’s budget constraint in sector \( j \) (under consideration of non-satiation) is given by:

\[
c_{it} = y_{it}^j
\]

In sector 1, the household’s income is given by \( y_{it}^1 = y^1(n_{it}, \lambda_{it}, e_{it}) \) and in sector 2, we have \( y_{it}^2 = y^2(\lambda_{it}, e_{it}) \). Therefore, the resulting household’s demand, denoted by \( (e^o_{it}, c^o_{it}) \), is in sector 2 solely determined by the level of the adult’s human capital \( \lambda_{it} \), and in sector 1 additionally by the level of land ownership \( n_{it} \). Equations (7.2) and (7.3) manifest that schooling lowers household income. The marginal opportunity costs, i.e. the foregone earnings, of a single time unit of education are, in sector 1, equal to \( \alpha \gamma A_1 (\frac{n_{it}}{\lambda_{it} + (1 - e_{it})})^{1-\alpha} \), and in sector 2 equal to \( \gamma A_2 \). We can now state that the highest possible consumption level, \( \overline{c} \), (i.e. when \( e^o = 0 \)) and the lowest possible consumption, \( \underline{c} \), (i.e. when \( e^o = 1 \)) are given by:

\[
\overline{c}_{it} = \begin{cases} 
\tau^1(n_{it}, \lambda_{it}) = A_1[\lambda_{it} + \gamma]^{1-\alpha} & \text{if } j = 1 \\
\tau^2(\lambda_{it}) = A_2(\lambda_{it} + \gamma) & \text{if } j = 2 
\end{cases} 
\]

\[
\underline{c}_{it} = \begin{cases} 
\tau^1(n_{it}, \lambda_{it}) = A_1\lambda_{it}^{1-\alpha} n_{it}^{-\alpha} & \text{if } j = 1 \\
\tau^2(\lambda_{it}) = A_2\lambda_{it} & \text{if } j = 2 
\end{cases} 
\]

We assume that both goods are non-inferior. Hence, an increase in land property or in human capital, \textit{ceteris paribus}, increases a household’s income. As before, there are two consumption thresholds, denoted \( c^\alpha \) and \( c^S \), in the following way (see Assumption 2.2):

\[
(c_{it}, e_{it}) = \begin{cases} 
(\overline{c}_{it}, 0) & \text{if } \tau^a_{it} \leq c^S \\
(e^o_{it}, e_{it}) & \text{if } \tau^a_{it} > c^S \text{ but } \underline{c}_{it} < c^a \\
(\overline{c}_{it}, 1) & \text{if } \underline{c}_{it} \geq c^a 
\end{cases} 
\]

where \( j \) is equal to 1 or 2, depending on household \( i \)'s location at time \( t \), and \( e^o_{it} \in (0, 1) \).

\(^{13}\)As we analyze poorer families the land owned is seen as subsistence basis for the children and hence not subject to sales as long as land-based production is followed.
In sector 2 income is solely determined by the level of human capital. Therefore, just as in Part I of the thesis, we obtain:

\[
(c_{it}, e_{it}) = \begin{cases} 
(c^2(\lambda_{it}), 0) & \forall \lambda_{it} \leq \lambda^S \\
(c^0_{it}, e^0_{it}) & \forall \lambda^S < \lambda_{it} < \lambda^a \\
(c^2(\lambda_{it}), 1) & \forall \lambda_{it} \geq \lambda^a
\end{cases}
\]  

(7.7)

where the locus \((c^0_{it}, e^0_{it})\) is increasing in \(\lambda_{it}\) for all \(\lambda \in (\lambda^S, \lambda^a)\), and the thresholds are given by \(\lambda^S = \frac{c^S}{A^2} - \gamma\) and \(\lambda^a = \frac{c^a}{A^2}\).

In the first sector these two thresholds are simultaneously determined by the household’s level of \(n_{it}\) and \(\lambda_{it}\). Hence there exist certain levels of human capital with which the household must be endowed in order to choose \(e^0_{it} > 0\) or \(e^0_{it} = 1\), given a particular amount of land, \(n_{it}\). This is made vivid in the upper part of Figure 7.2. We state:

\[
(c_{it}, e_{it}) = \begin{cases} 
(c^1(n_{it}, \lambda_{it}), 0) & \forall \lambda_{it} \leq \lambda^S(n_{it}) \\
(c^0_{it}, e^0_{it}) & \forall \lambda^S(n_{it}) < \lambda_{it} < \lambda^a(n_{it}) \\
(c^1(n_{it}, \lambda_{it}), 1) & \forall \lambda_{it} \geq \lambda^a(n_{it})
\end{cases}
\]  

(7.8)

where \(\lambda^S(n_{it}) = \left(\frac{c^S}{A^1(n_{it})^{1-a}}\right)^{1/a} - \gamma\) and \(\lambda^a(n_{it}) = \left(\frac{c^a}{A^1(n_{it})^{1-a}}\right)^{1/a}\). The locus \((c^0_{it}, e^0_{it})\) increases in \(\lambda_{it}\) for all \(\lambda_{it} \in (\lambda^S(n_{it}), \lambda^a(n_{it}))\) and in \(n_{it}\), respectively ceteris paribus. Note that for sufficiently high \(n_{it}\), also for \(\lambda_{it} = 1\), the household’s consumption crosses \(c^S\) so that \(e^0_{it} > 0\) occurs. Hence, for \(\lambda^S(n_{it}) < 1\), no lower threshold exists. With high enough \(n_{it}\), \(e^0_{it} = 1\) is chosen for all levels of \(\lambda_{it} \geq 1\) (i.e. \(\lambda^a(n_{it}) \leq 1\)). We define the corresponding amounts of land, given a certain level of human capital, by \(n^S(\lambda_{it})\) respectively by \(n^a(\lambda_{it})\):\(^{14}\)

\[
n^S(\lambda_{it}) = \left[\frac{c^S}{A^1(\lambda_{it} + \gamma)^a}\right]^{\frac{1}{1-a}}
\]

(7.9)

\[
n^a(\lambda_{it}) = \left[\frac{c^a}{A^1(\lambda_{it})^a}\right]^{\frac{1}{1-a}}
\]

(7.10)

We obtain:

\[
\frac{dn^a(\lambda_{it})}{d\lambda_{it}} < 0, \quad \frac{dn^S(\lambda_{it})}{d\lambda_{it}} < 0
\]

\[
\frac{d\lambda^S(n_{it})}{dn_{it}} < 0, \quad \frac{d\lambda^S(n_{it})}{dn_{it}} < 0
\]

(7.11)

7.3.2.2 Location and Migration

Finally, we must analyze the household’s sector choice. We assume that this decision depends solely on the question of how much income is earned in each sector, given the

\(^{14}\)Note that agriculture might be more labor intensive and children are more likely to work on a received plot. However, this does not influence the levels \(c^S\) and \(c^a\).
household’s endowment of land, \( n_{it} \), and human capital, \( \lambda_{it} \). As a further simplification, we assume that households can move without cost between the sectors and that a household can only work in one sector within a single period. If a household does not possess any land, it must work in sector 2.\(^{15}\) For particularly small plots of land, agriculture output is very low. The fully uneducated only opt for agriculture if the following holds:\(^{16}\)

\[
n_{it} > \left( \frac{A_2}{A_1} \right)^{\frac{1}{1-\alpha}} (1 + \gamma) \equiv \bar{n} \tag{7.12}
\]

If a household possesses sufficient land, the child will enjoy a basic education (\( e_{it} = 1 \)) and the level of human capital will increase over time. Since income in sector 2 increases with the level of human capital, sector 2 may turn out to be an attractive alternative for educated households. Without land markets (or lease of land), and given \( e_{it} = 1 \), a household will opt for sector 2 as soon as the following condition is fulfilled:\(^{17},^{18}\)

\[
\lambda_{it} > \left( \frac{A_1}{A_2} \right)^{\frac{1}{1-\alpha}} \cdot n_{it} \equiv \tilde{\lambda}(n_{it}) \tag{7.13}
\]

That is to say, once a household has accumulated more than \( \tilde{\lambda}_{it} \), the human capital intensity per unit of land, \( \frac{\lambda_{it}}{n_{it}} \), is so high that a sector change becomes profitable because of diminishing marginal return of human capital in sector 1. The location decision is depicted in Figure 7.1. Finally, we introduce variable \( a_{it} \) to identify the sector location of household \( i \) in period \( t \), where \( a_{it} = 1 \) means that the household works in sector 1 and \( a_{it} = 0 \) that it works in sector 2.

### 7.3.3 Dynamics

We again exclude oscillating trajectories, for the sake of simplicity. The dynamics described here is equal for all households and we drop index \( i \).

#### 7.3.3.1 Sector 1

To establish the dynamics of human capital in sector 1, we have to analyze Equation (7.1) in the light of Equation (7.8):

\[
\lambda_{t+1} = \begin{cases} 
1 & \forall \lambda_t \leq \lambda^S(n_t); \\
zh(e^a(n_t, \lambda_t)) \lambda_t + 1 & \forall \lambda_t \in (\lambda^S(n_t), \lambda^a(n_t)); \\
zh(1) \lambda_t + 1 & \forall \lambda_t \geq \lambda^a(n_t).
\end{cases} \tag{7.14}
\]

\(^{15}\)In Chapter 8, we will extend our model and assume that all landless poor work for a landowner in agriculture.

\(^{16}\)Note that the poorest parents display \( \lambda = 1 \) and choose \( e = 0 \).

\(^{17}\)Later on it will become clear that we can restrict ourselves to the case where \( e = 1 \).

\(^{18}\)The impact of land markets is developed separately in Section 7.5.
Figure 7.1: The sector choice in dependence to the human capital-land ratio, where the threshold-line is given by \( \tilde{\lambda}(n_t) = \left( \frac{A_1}{A_2} \right)^{\frac{1}{1-\alpha}} n_t \).

Remark 2.2, and that:

**Remark 7.1**

\( zh(1) \geq 1 \) forces \( \lambda^o(n_t) \) to be strictly higher than any possible stationary state, where \( \lambda_{t+1} = \lambda_t \), since then \( zh(1)\lambda^o(n_t) + 1 > \lambda^o(n_t) \) is always true.

The size of estate \( n_t \) is determined exogenously by the policy maker and therefore assumed to be constant in the course of time. The complete description of the dynamic patterns associated with Equation (7.14) is drawn up in Propositions E.1 and E.2; since the propositions cover several pages, they are given in the appendix. The main paths of the difference equation are illustrated in Figures 7.2 to 7.6.

As our analysis over the last chapters has shown, the specific pattern of the trajectory is not relevant for our results. To understand the functioning of the dynamics consider, for instance, a household that possesses land of size \( n_t \) and displays \( e_t \) < \( c^S \). For, it chooses \( e_t = 0 \) and we arrive at \( \lambda_{t+1} = 1 \). Therefore, there is a poverty trap at \( \lambda = 1 \). Let there also be a medium steady state at some level \( \lambda^* \), so that the poverty trap area, from which we are trying to escape from, is the interval \( \lambda_t \in [1, \lambda^*(n_t)] \) (see, for instance, Figure 7.4). For our analysis, we assume:

\[ zh(1)\lambda^o(n_t) + 1 > \lambda^o(n_t) \]
Again, long-term growth is achievable if \( zh(1) \geq 1 \) (\textit{growth-case}). Otherwise, in case of \( zh(1) < 1 \), the economy converges, independent of its starting point, to a long-term steady state with a per-capita growth rate of zero. Consequently, all households participating in the growth promoting land reform eventually would end up at the high-income steady state at \( \lambda = \frac{1}{1 - zh(1)} \). The idea of the land reform will be to attain the socially efficient level of schooling. We assume fulltime schooling \((e = 1)\) is optimal, so that supported households, once they have received a plot of land, are in the dynamic pattern illustrated by 7.6.

### 7.3.3.2 Sector 2

The dynamics of sector 2 are the same as in our basic model (Chapter 2):

\[
\lambda_{t+1} = \begin{cases} 
1 & \forall \lambda_t \leq \lambda^S; \\
zh(e^o(\lambda_t))\lambda_t + 1 & \forall \lambda_t \in (\lambda^S, \lambda^a); \\
zh(1)\lambda_t + 1 & \forall \lambda_t \geq \lambda^a.
\end{cases}
\]

(7.15)

We concentrate on cases where there exists at least one medium steady state. Thus, the dynamic system has at least two steady states, namely \((\lambda^*, e^o(\lambda^*))\) and \((1, 0)\), where the former is unstable. The reader can, e.g., consider the convex trajectory illustrated by Figure 2.3. As in sector 1, \( zh(1) \) determines whether we can obtain long-term growth. In the case of \( zh(1) < 1 \), the highest stationary state is characterized by \( \lambda = \frac{1}{1 - zh(1)} \).

We denoted growth rates by \( g_k \) with \( k \) representing the variable considered. Suppose \( e_{it} = 1 \) for all \( t \). Then, it is clear that \( g_{\lambda_{it}} = zh(1) + \frac{1}{\lambda_{it}} - 1 \). In sector 2, we still have \( g_{\lambda_{it}} = g_{\lambda_{it}} \), because \( g_{\lambda_{it}} = \alpha \lambda_{it} \). In sector 1, we receive

\[
g_{\lambda_{it}} = \left( zh(1) + \frac{1}{\lambda_{it}} \right)^{\alpha} \left( \frac{n_{i(t+1)}}{n_{it}} \right)^{1-\alpha} - 1
\]

and therefore

\[
1 + g_{\lambda_{it}} = (1 + g_{\lambda_{it}})^{\alpha}(1 + g_{n_{it}})^{1-\alpha}.
\]

Overall, the growth patterns can be summarized as follows:

\footnote{In the intensive form with \( e = 1 \), we obtain in sector 1 \( \frac{n_{it}}{\lambda_{it}} = A_1(\frac{\lambda_{it}}{n_{it}})^{\alpha} \). If the individual land property \( n \) is fixed, we obtain a neoclassical growth model: \( \frac{n_{it}}{\lambda_{it}} = \tilde{A}_1 \cdot \lambda^\alpha \) with \( \tilde{A}_1 = \frac{A_1}{n_{it}} \). From a macroeconomic perspective, assume that the size of land is fixed at level \( N \). Then, decreasing marginal productivity of \( \lambda \) has the well-known consequence of a steady state at some level \( \lambda > 1 \). Due to human capital accumulation, the output per household and area soil can, however, increase indefinitely with growth rate \( g_{\lambda_{it}}(\lambda) = g_y - g_n = [zh(1)]^\alpha - 1 > 0 \) for \( [zh(1)]^\alpha > 1 \). This occurs since \( e \) is bounded from above at unity whereby \( [zh(e)]^\alpha \) becomes a constant. Thus the size of the term \( [zh(1)]^\alpha \) determines long-term growth (AK model) or a long-term steady state (Solow-Swan). If \( n \) is individually variable (via a land market) we obtain a two factor model with an optimal relation between \( \lambda \) and \( n \), similar to the broadly defined capital concept, cf. Barro and Sala-i-Martin (1995), Section 5.1.1. This case is dealt with in Section 7.5.}
Figure 7.2: Convex human capital technology in sector 1 for different levels of land that establish 1) $\lambda^S(n) < 1$, 2) $\lambda^S(n) = 1$, and 3) $\lambda^S(n) > 1$. 

$n_1 > n_2 > n_3$
Growth Patterns

(i) Consider a household $i$ with $e_{it} = 1$ in sector 1. If $zh(1) \geq 1$ the level of human capital per capita will grow asymptotically with $zh(1) - 1 > 0$ indefinitely. Agricultural output per capita will grow asymptotically with $[zh(1)]^a - 1 > 0$. Otherwise, $zh(1) < 1$, the household will end up in the steady state at $\lambda = \frac{1}{1-zh(1)}$ where both growth rates are equal to zero.

(ii) Consider a household $i$ with $e_{it} = 1$ in sector 2. If $zh(1) \geq 1$ the human capital per capita and the output per capita will grow asymptotically with $zh(1) - 1 > 0$. Otherwise, $zh(1) < 1$, the household will end up in the steady state at $\lambda = \frac{1}{1-zh(1)}$ where both growth rates are equal to zero.

7.4 Land Reforms without Land Markets

As emphasized in the introduction, land reforms may represent an effective tool to overcome poverty, especially in rural and agricultural areas. In this section, we analyze how land reforms can be designed in order to overcome under-development and to achieve growth due to human capital accumulation, when land markets do not exist.
We will demonstrate that land reforms allow for the amount of poverty, illiteracy and child labor to diminish.

Let the whole country’s endowment of suitable land be denoted by $N$. Initially all this land is owned by a social planner (representing the state) who is free to distribute land within the society. The aim is to educate the society in order to escape from the poverty trap; for this purpose the state targets to generate the socially optimal level of education in as many households as possible per period. The sequence of events is as follows: At the beginning of a period $t$, an adult $i$ is endowed with human capital $\lambda_{it}$ and land $n_{it}$. A household $i$ may or may not be selected as a beneficiary of the land reform. As a beneficiary the household receives a plot of land of size $n_{it} > 0$. All yet supported may be forced to donate land of size $n_{it}^{\tau}$ to the state for land distribution, i.e. $\pi_{it} < 0$. That is, at the beginning of each period the social planner determines the distribution of land by redistribution. After the redistribution of land all adults $i \in [0,1]$ decide in which sector they will work, on consumption $c_{it}$ and on the child’s education $e_{it}$. This cycle is repeated until the land reform is accomplished, i.e. until the society is educated.

Notice that all $i \in [0,1]$ initially own no land, that is, a household either owns no land
at all \((n_{it} = 0)\) because it has not yet received any land from the social planner, or it possesses some land, in which case the household was allocated with land by the state \((n_{it} > 0)\).

7.4.1 The Basic Idea and First Results

We again assume that \(e_{it} = 1\) is socially optimal for all \(i\) (as long as we have not reached the end of time). Thus, in order to attain the socially optimal level of education, the child of a supported household must attend school full-time: \(e^o(\lambda_{it}, \pi_{it}) = 1\). The necessary size of land, labeled \(n^a(\lambda_{it})\), is given by Equation (7.10) above. Hence, once household \(i\) has received land of size \(n^a(\lambda_{it})\) in a period labeled \(t\), this household decides to educate the child full-time, and child labor ceases. As the child obtains a full basic education, the household acquires skills. The next period’s level of human capital is given by:

\[
\lambda_{i(t+1)} = h(1)z\lambda_{It} + 1
\]  

(7.16)

Note that, in contrast to subsidization, we do not have to consider whether \(h(1)z\lambda_{It} + 1\) is higher than any other lower steady state like \(\lambda^*(\pi_{it})\) in Figure 7.4. The correct choice of \(\pi_{it} = n^a(\lambda_{it})\) means nothing else than turning \(\lambda_{it}\) into \(\lambda^a(\pi_{it})\), so that we end up in
a situation given in Figure 7.6. Since $zh(1)\lambda^a(n) + 1 > \lambda^a(n)$, we know that human capital accumulation is in all cases ensured.\footnote{In contrast to the instrument of subsidization, land transfers are not one-shot income streams, rather an increase in the stock of wealth that directly affects the income in the long-term.}

As the supported households’ level of efficiency units of labor grow over time, so does their income. Hence, for all households that already received plots, we obtain $y_{i'(t+1)} = c_{i'(t+1)} > c^a$, which allows for a “taxation” of size $c_{i'(t+1)} - c^a > 0$, or in general, of size $c_{it} - c^a$, for all households $i$ that have received plots of land. In each period, we can check how much land the already supported households still require for sustaining full-time schooling. The excessive land we can dispossess. This tax in the form of land is labelled $n^r_i$.\footnote{Note that $n_{it}$ is the size of land owned in period $t$ before expropriation takes place.}

$$n^r_i(\lambda_{it}) = \max \{0, n_{it} - n^a(\lambda_{it})\} = \max \left\{0, n_{it} - \left(\frac{c^a}{A_1\lambda_0^a}\right)^{\frac{1}{\alpha}}\right\} \quad (7.17)$$

where the case $n^r_i(\lambda_{it}) = 0$ holds for all the households not yet supported. For all other households, the remaining plot of land of size $n_{it} - n^r_i(\lambda_{it})$ is exactly equal to $n^a(\lambda_{it})$. The seized land is free to be redistributed to the poor anew. Note that $n^r_i$ is strictly positive because the human capital of beneficiaries continuously grows. The general
process of human capital formation through land transfers in a period $\bar{t}$ are described by:

$$\lambda_{it} = \left\{ \begin{array}{ll}
\sum_{k=0}^{t-\bar{t}} [zh(1)]^k & \text{for } t \geq \bar{t} \\
1 & \text{for } t < \bar{t}
\end{array} \right. \quad (7.18)$$

As the government adjusts the land ownership of beneficiaries so that each household $i$ always possesses $n^a(\lambda_{it})$, each beneficiary is endowed with an income of $c^a$. Consequently beneficiaries are indifferent between the two sectors when $A_2\lambda_{it} = c^a$. We obtain:

$$\tilde{\lambda} = \frac{c^a}{A_2} = \lambda^a \quad (7.19)$$

That is, at the migration threshold $\tilde{\lambda}$ households earn the same income in both sectors, namely $c^a$, so that $\tilde{\lambda}$ is equal to sector 2’s $\lambda^a$. Therefore, household $i$ will migrate to sector 2 in period $t$ if $\lambda_{it} > \lambda^a$.

**Proposition 7.1**

Each beneficiary of the land reform stays for $l$ periods in sector 1 before switching sectors, where $l$ is determined by:

$$\min_{l>0} \sum_{k=0}^{l} [zh(1)]^k > \frac{c^a}{A_2} = \lambda^a$$

Therefore, a group of land reform participants that received land in any period $\bar{t}$, changes to sector 2 in period $\bar{t} + l$.

### 7.4.2 The Exact Functioning of the Land Reform

Consider the worst case scenario where initially all households live in a state of backwardness, i.e., $\lambda_{i0} = 1$, and $n_{i0} = 0$, for all $i \in [0, 1]$. Each supported households has to be allocated with land of size $n^a(\lambda_{i0})$. In the first period, $t = 0$, all land $N$ can be distributed among the society, represented by households $i \in [0, 1]$. Given $N < n^a(1)$, only a fraction of the society can be allocated with $n^a(\lambda_{i0})$. $\delta_t$ denotes the fraction of the society entitled to land of size $n^a(\lambda_{i0})$ in a period $t$. To allocate as many households as possible, all land $N$ is distributed to the society in period $t = 0$. Accordingly, we obtain:

$$\delta_0 = \frac{N}{n^a(1)} \quad (7.20)$$

The land transfers can be summarized by:

$$\pi_{i0} = \left\{ \begin{array}{ll}
n^a(1) & \text{if } i \in [0, \delta_0] \\
0 & \text{else}
\end{array} \right. \quad (7.21)$$
The land transfers induce human capital formation that is described by:

$$\lambda_{i1} = \begin{cases} zh(1) + 1 & \forall \ i \in [0, \delta_0] \\ 1 & \text{else} \end{cases} \quad (7.22)$$

with $zh(1) + 1 > 1$. In the following period, the share $\delta_0$ can be expropriated according to $n_{i1}^\tau(\lambda_{i1}) = n^a(1) - n^a(zh(1) + 1)$. Human capital formation and expropriations will increase the human capital intensity. Therefore, according to Proposition 7.1, the group $\delta_0$ will switch sectors in period $l \geq 1$. We introduce the sector identification variable $a_1^{\delta_0}$ in the following way. In general, $a_{it}$ identifies the sector location of a household $i$ in period $t$. As all households are initially identical, households can be grouped in terms of the period in which they were entitled to land, labeled $\delta_t$. So for all $i \in (\delta_{t-1}, \delta_t]$ we have $a_{it} = a_t^{\delta_0}$. Now consider the group of households that were receiving a plot of land in period 0. If these households are in period 1 farmers in sector 1, they are displaying $a_1^{\delta_0} = 1$. If these families are located in sector 2, on the contrary, they are labeled with $a_1^{\delta_0} = 0$. Accordingly, we obtain that the group $\delta_0$ switches sectors if the households’ level of human capital in period 1 of the group, labeled $\lambda_1^{\delta_0}$, crosses $\tilde{\lambda}$:

$$a_1^{\delta_0} = \begin{cases} 1 & \text{if} \ \lambda_1^{\delta_0} \leq \tilde{\lambda} \\ 0 & \text{else} \end{cases} \quad (7.23)$$

Applying Equation (7.10), we obtain for group $i \in [0, \delta_0]$:

$$n_1^\tau(\lambda_{i1}) = \begin{cases} \left( \frac{c^a}{A_1} \right)^{1-\alpha} \cdot \left( 1 - \left( \frac{1}{zh(1)+1} \right)^\frac{1}{\alpha} \right) & \text{if} \ a_1^{\delta_0} = 1 \\ \left( \frac{c^a}{A_1} \right)^{1-\alpha} & \text{else} \end{cases} \quad (7.24)$$

so that beneficiaries who leave the land-based sector lose the claim to the received plot of land. For all $i \notin [0, \delta_0]$, of course, $n_1^\tau(\lambda_{i1}) = 0$. Thus, the government will have the following amount of land at its disposal in period 1:

$$\int_{i=0}^{1} n_1^\tau(\lambda_{i1}) \, di = \delta_0 \left[ a_1^{\delta_0} \left[ \left( 1 - \left( \frac{1}{zh(1)+1} \right)^\frac{1}{\alpha} \right) \left( \frac{c^a}{A_1} \right)^{1-\alpha} \right] + (1 - a_1^{\delta_0}) \left( \frac{c^a}{A_1} \right)^{1-\alpha} \right]$$

The resulting land redistribution scheme is:

$$\overline{n}_{i1} = \begin{cases} -n_{i1}^\tau(zh(1)+1) & \text{for} \ i \in [0, \delta_0] \\ n^a(1) & \text{for} \ i \in (\delta_0, \delta_0 + \delta_1] \\ 0 & \text{else} \end{cases} \quad (7.25)$$
where \( \delta_1 = \int_0^1 n^\tau_1(\lambda_1(i)) \, di \). We denote the measure of households in a society already entitled to land by \( \mu \): \( \mu_t = \sum_{k=0}^t \delta_k \). So \( \mu_0 = \delta_0 \) and \( \mu_1 = \delta_0 + \delta_1 \). Thus, within fraction \( \mu_1 \), all households display \( e^a = 1 \) and income \( c^a \) (unless the households of group \( \delta_0 \) display \( a_{i1} = 0 \)). Period 1’s land transfers have to fulfill the land constraint:

\[
\delta_1 n^a(1) = \int_0^1 n^\tau_1(\lambda_1(i)) \, di = \mu_0 n^\tau_1(zh(1) + 1)
\]

Therefore, \( \delta_1 = \frac{\mu_0 n^\tau_1(zh(1) + 1)}{n^a(1)} \). For the human capital levels in \( t = 2 \) we obtain:

\[
\lambda_{i2} = \begin{cases} 
  zh(1)(zh(1) + 1) + 1 & \text{for } i \in [0, \delta_0] \\
  zh(1) + 1 & \text{for } i \in (\delta_0, \mu_1] \\
  1 & \text{else}
\end{cases}
\] (7.26)

In general, in any period \( t \), land redistribution must take the following form:

\[
\pi_{it} = \begin{cases} 
  -n^a(\lambda_{it}) & \text{for } i \in [0, \mu_{t-1}] \\
  n^a(1) & \text{for } i \in (\mu_{t-1}, \mu_t] \\
  0 & \text{else}
\end{cases}
\] (7.27)

where \( n^a(\lambda_{it}) \) can be divided into the particular groups that were entitled in the same period, labeled \( \tau \), that is, in the groups \( \delta_\tau = \{ i \in [0, 1] \mid i \in (\mu_{\tau-1}, \mu_\tau) \} \). A group’s choice of location \( \delta_\tau \) can be described by:

\[
\delta^\tau_t = \begin{cases} 
  1 & \text{if } \lambda^\tau_t \leq \bar{\lambda} \\
  0 & \text{else}
\end{cases}
\] (7.28)

Neglecting migration, \( n^a(\lambda_{it}) \) is given by the term:

\[
\max \{ 0, n^a(\lambda_{it}(t-1)) - n^a(\lambda_{it}) \} = \max \left\{ 0, \left( \frac{c^a}{A_1 \lambda^a_{i(t-1)}} \right)^{1-\alpha} - \left( \frac{c^a}{A_1 \lambda^a_{it}} \right)^{1-\alpha} \right\}
\] (7.29)

However, we have to keep in mind the household’s choice of location. As soon as some supported groups choose to work in sector 2, the government obtains all the remaining land of these groups.

\[
n^\tau_{it} = \begin{cases} 
  \left( \frac{c^a}{A_1} \right)^{\frac{1}{1-\alpha}} \left[ \frac{1}{\sum_{k=0}^{\tau-1} (zh(1))^k} \right]^{\frac{\alpha}{1-\alpha}} - \left( \frac{c^a}{A_1 \lambda^a_{i(t-1)}} \right)^{\frac{\alpha}{1-\alpha}} & \text{if } a_{it} = 1 \\
  \left( \frac{c^a}{A_1 \lambda^a_{it}} \right)^{\frac{1}{1-\alpha}} \frac{1}{\sum_{k=0}^{\tau-1} (zh(1))^k} & \text{if } (a_{it} = 0 \text{ and } a_{i(t-1)} = 1)
\end{cases}
\] (7.30)
Hence, the proportion of the society that can be entitled to obtain \( n^a(1) \) in a period \( t \) is given by:

\[
\delta_t = \int_0^1 n^a_T(i) \, di
\]

The overall objective of educating the society as a whole is reached, obtaining long-term welfare without child labor, when all adults have acquired a full-time basic schooling during their childhood and are willing to send their children to school full-time, without any state intervention. The number of periods in which this target is reached is labelled by \( T \). Thus, \( \mu_t \) appears to be unity in period \( T - 1 \). Summarizing, to establish the socially optimal level of education \( (e_{it} = 1) \) in as many households as possible, the land reform redistribution sequence must not waste land, that is:

- \( \int_0^1 \bar{\pi}_t(i) \, di = 0 \quad \forall \, t = 1, \ldots, T - 1 \) and \( \int_0^1 \bar{\pi}_0(i) \, di = N \)
- \( \{ \bar{\pi}_{it} = n^a(\lambda_{it}) \quad \forall \, i \in (\mu_{t-1}, \mu_t) \} \quad \forall \, t \) with \( \mu_{t-1} \equiv 0 \)

The land reform is accomplished if \( e^a_{i(T-1)} = 1 \) holds forall \( i \in [0,1] \).

### 7.4.3 Migration Equilibrium

A migration equilibrium is established when no household wishes to migrate from one sector to another.\(^{23}\) The migration decision is determined by income comparison. In general, a migration equilibrium therefore requires the equality of all (expected) sectoral incomes. However, without capital and land markets, households without demesne cannot migrate to sector 1. Thus, in our setting, the migration equilibrium not necessarily forces \( y^1_{it} = y^2_{it} \). We find:

**Proposition 7.2**

Suppose the described land reform is applied. Then there exists a migration equilibrium in period \( t \) with:

\[
a^*_it = \begin{cases} 
1 & \text{if } \{ \lambda_{it} < \lambda^a \text{ and } n_{it} > 0 \} \\
0 \text{ or } 1 & \text{if } \{ \lambda_{it} = \lambda^a \text{ and } n_{it} > 0 \} \\
0 & \text{if } \lambda_{it} > \lambda^a \text{ or if } n_{it} = 0
\end{cases}
\]

for all \( i \in [0,1] \)

The migration equilibrium is unique, unless \( \lambda_{it} = \lambda^a \) for some \( i \in [0,1] \).

\(^{22}\)In the appendix we offer a general solution \( \delta_t(\delta_0) \) for the case where migration is neglected, i.e. when sector 2 does not exist.

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Proof:
Households with \( n_{it} = 0 \) are imprisoned in the poverty trap and display \( \lambda_{it} = 1 \). Hence, all non-beneficiaries display \( a_{it} = 0 \) and, due to \( y^1 = 0 \) for \( n = 0 \), have no incentive to switch sectors. Land reform beneficiaries own a plot of land of size \( n^a(\lambda_{it}) \), i.e., they earn an income of \( c^a \). Applying \( \tilde{\lambda}_{it} = \lambda^a \), it is clear that \( \lambda_{it} > \lambda^a \) leads to \( a^*_{it} = 0 \) in equilibrium and that \( \lambda_{it} < \lambda^a \) causes \( a^*_{it} = 1 \) in equilibrium. If \( \lambda_{it} = \lambda^a \), household \( i \) earns the same income in both sectors, and is therefore indifferent between sector 2 and sector 1, that is, it has no incentive to switch sectors. As in this case of indifference \( a_{it} = 0 \) as well as \( a_{it} = 1 \) is consistent with a migration equilibrium, the migration equilibrium is indeterminate. But equilibrium will only be indeterminate if a household \( i \) displays \( \lambda_{it} = \lambda^a \).

\[\Box\]

7.4.4 Land Reforms, Equality, and Transition

Land reforms are commonly seen as means of producing equality. In our model, at the beginning of the land reform all households are fully alike: all households \( i \) are landless and uneducated, i.e. \( n_i = 0 \) and \( \lambda_i = 1 \) for all \( i \in [0,1] \). As long as \( n^a(1) < N \) only a fraction \( \delta_0 < 1 \) can be allocated with land of size \( n^a(1) \) in the first period. Thus, the land reform generates inequality. One can lower inequality by lowering the land transfer. However, first, this lowers the targeted effect on education. Second, to escape the poverty trap in a sustainable way it is absolutely necessary that the land transfer generates an income \( y^1_{it}(\lambda_{it}, \pi_{it}) \) which guarantees that:

\[\lambda_{t+1} = 1 + zh\left(e(y^1_{it}(\lambda_{it}, \pi_{it}))\right) > \lambda^*(\pi_{it})\]

Hence, the creation of (temporary) inequality is a necessary condition to escape the poverty trap sustainable. However, in each succeeding period the land reform redistributes land to the poorest segments, which lowers inequality. Moreover, the land reform guarantees income equality among the beneficiaries at income level \( c^a \). Nonetheless, as we will see below, in our dual economy inequality is likely persistent. The long run distribution of income and human capital crucially depends on the transition process the society experiences. We demonstrate that the land reform induces the transition from a backward, poor economy towards a developed, human capital-based economy.


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7.4.4.1 Short- and Middle-Term Inequality

There is income equality within the group of the beneficiaries at income level \( c^a \) and within the not-yet supported poor at level \( \alpha \). Concerning the distribution of incomes, \( c^a > \alpha \) produces, at least temporary, inequality. In the last period in which land redistributions take place, \( t = T - 1 \), all households \( i \in [0, 1] \), that are still located in sector 1, have an equal income of \( c^a \), and the distribution of income within this segment of the society is equal. In the next period, period \( T \), all adults have enjoyed a full basic education, but it is clear that the distribution of human capital is not equal:

\[
\lambda_{iT} = \begin{cases} 
\sum_{k=0}^{T} (zh(1))^k & \text{for } i \in [0, \mu_0] \\
\sum_{k=0}^{T-1} (zh(1))^k & \text{for } i \in (\mu_0, \mu_1] \\
\vdots \\
\sum_{k=0}^{T-2} (zh(1))^k & \text{for } i \in (\mu_0, \mu_1] \\
zh(1) + 1 & \text{for } i \in (\mu_0, \mu_1] \\
\end{cases}
\]

(7.32)

with \( \mu_{T-1} = 1 \). As the resulting distribution of human capital is unequal, the income distribution after the completion of the land reform will become – already in period \( T \) – unequal:

\[
y_{iT} = \begin{cases} 
\delta_T^0 y^1 \left( \sum_{k=0}^{T} (zh(1))^k, n^a(1) - \sum_{k=0}^{T-1} n^a_{ik} \right) \\
+ (1 - \delta_T^0) y^2 \left( \sum_{k=0}^{T} (zh(1))^k \right) & \text{for } i \in [0, \mu_0] \\
\vdots \\
\delta_{T-1}^0 y^1 \left( zh(1) + 1, n^a(1) \right) \\
+ (1 - \delta_{T-1}^0) y^2 \left( zh(1) + 1 \right) & \text{for } i \in (\mu_0, \mu_1] \\
\end{cases}
\]

(7.33)

with \( \mu_{T-1} = 1 \). We assume that after the completion of the reform no land redistribution occurs anymore. In sector 2, where all beneficiaries with \( \lambda_{it} > \lambda^a \) work, income is solely determined by the level of human capital and the distribution of income is unequal. In sector 1, land redistribution stops and an identical income increase by human capital formation, given different levels of human capital and land, would be purely by accident. In the end, after the successful termination of the land reform, the resulting distribution of land, income, and human capital within society will – at least in the short- and the middle-term – display inequality, but economic welfare has improved, because society has escaped from poverty.

7.4.4.2 Transition

Whether this produced inequality persists over time depends on the transition process that is initiated by the land reform. We will demonstrate that the growth pattern is
important for the outcome. For the case $zh(1) < 1$, we denote the period in which the very last cohort of land receivers displays the steady state level of human capital $\lambda = \frac{1}{1-zh(1)}$ by $T^{ss}$; equally a superscript “ss” indicates variables corresponding to the steady state at $\lambda = 1/(1-zh(1))$. Assuming that the land reform is successful, we can conclude the following for the transition of the society:

**Proposition 7.3**

(i) Suppose $zh(1) < 1$, and that the land reform is successful, that is, each household ends up in the stationary state at $\lambda = \frac{1}{1-zh(1)}$. Then, we obtain in the steady state:

$$a^{ss}_i = \begin{cases} 1 & \text{if } \lambda_i(T-1) < \left[\lambda^a (1-zh(1))^{1-a}\right]^\frac{1}{a} \\ 0 & \text{if } \lambda_i(T-1) > \left[\lambda^a (1-zh(1))^{1-a}\right]^\frac{1}{a} \\ 0 \text{ or } 1 & \text{if } \lambda_i(T-1) = \left[\lambda^a (1-zh(1))^{1-a}\right]^\frac{1}{a} \end{cases}$$

(ii) Suppose $zh(1) \geq 1$, and that the land reform is successful, that is, the human capital of each household grows for all time. Then, all households will, asymptotically, leave sector 1 and end up in sector 2, that is, $a_{it} = 0$ for all $i \in [0,1]$, when $t \to \infty$.

**Proof :**

Independent of the size of $zh(1)$, a household $i$ will switch sectors towards sector 2 as soon as $\lambda_i/n_{it} > \left(\frac{A_1}{A_2}\right)^{1/(1-a)}$. If $zh(1) < 1$, each educated household will reach the stationary state at $\lambda = \frac{1}{1-zh(1)}$, where the household’s human capital no longer grows, in period $T^{ss}$. The land property of a household $i$ in period $T^{ss}$ is determined by $n_i(T-1)$, that is, by the property in the last period in which land redistribution had taken place: $n_{iT^{ss}} = n_{i(T-1)} = n^a(\lambda_i(T-1)) = \left(\frac{c^a}{A_1(\lambda_i(T-1))^a}\right)^\frac{1}{1-a}$

Moreover, we know that $\lambda_{iT^{ss}} = \frac{1}{1-zh(1)}$. Consequently, we have $a_{it} = 0$ in period $t = T^{ss}$ (and in all the following periods), if:

$$\left(\frac{1}{1-zh(1)}\right) \left(\frac{A_1(\lambda_i(T-1))^a}{c^a}\right)^\frac{1}{1-a} > \left(\frac{A_1}{A_2}\right)^{1/(1-a)}$$

Rearranging let us arrive at part (i) of the proposition. If $zh(1) \geq 1$, human capital grows indefinitely and the human capital land ratio will definitely cross the migration threshold $\left(\frac{A_1}{A_2}\right)^{1/(1-a)}$, so that all households will end up in sector 2. This proves part (ii) of the proposition.
The general calculus of Proposition 7.3 concerning migration is illustrated by Figure 7.7. We see that if the long-term level of human capital of a household is higher than \( \tilde{\lambda} \), then this household’s potential income in sector 2 is higher than its counterpart in sector 1, and the household will migrate to sector 2. Otherwise, the household earns a higher income in sector 1, and stays there. Furthermore, the threshold is given by \( \tilde{\lambda} = n_{i(T-1)} \left( \frac{A_1}{A_2} \right)^{1/(1-\alpha)} \), and thus dependent on estate \( n_{it} \). The higher is \( n_{i(T-1)} \), that is, the lower is \( \lambda_i(T-1) \), the more the intersection point is located to the right. It follows that the likelihood that the household moves to sector 2 decreases. It is clear that if we have the growth-case, i.e. \( zh(1) \geq 1 \), then \( \lambda_{it} \) will grow beyond any threshold \( \tilde{\lambda} \), and sector 1 will disappear. In case of \( zh(1) < 1 \), human capital formation stops at \( \lambda_{it} = 1/(1 - zh(1)) \), for all \( i \in [0, 1] \), and the size of land property \( n_{it} \) determines whether the members of a household \( i \) eventually enter sector 2 or remain farmers. Decisive is the question as to whether the use of the steady state level of human capital, \( 1/(1 - zh(1)) \), is more productively used in agriculture than in sector 2, given \( n_{i(T-1)} = n_{i(T-1)} \). During the land reform, beneficiaries were expropriated so much that, given their level of human capital, they were still sending the child to school full-time. It follows that, if households switch sectors towards sector 2, then the first cohorts of land reform beneficiaries start with migration, and the last cohorts least likely will emigrate.
Overall, our proposed land reform thus may induce the transition from an agriculture economy towards a human capital-based economy. If the growth pattern is like in an AK model ($zh(1) \geq 1$), then this transition will definitely occur and sector 1 disappears over time. In case of $zh(1) < 1$, agriculture (sector 1) may still exist in the long-term, because it is likely that the levels of human capital in period $t = T - 1$ of the very last cohorts is low compared to the steady state level $1/(1 - zh(1))$, that is: $\lambda_{i(T-1)} = \lambda^a(n_{i(T-1)}(T) < \left[\lambda^a(1 - zh(1))^{1-\alpha}\right]^{1/\alpha} < \lambda^a$. Hence these households still own a big fraction of the initial land transfer but are endowed with a comparable small level of human capital, wherefore they earn high incomes in agriculture relatively to their sector 2-incomes in the steady state.

### 7.4.4.3 Long-Term Equality

If $zh(1) < 1$ each single educated household will end up at the high-level stationary state at $\lambda = 1/(1 - zh(1))$, and the per household’s human capital growth is zero. Following the typical convergence argument, all households will be equal concerning human capital once they have reached the stationary state. Some households are still in sector 1, while others are in sector 2. In sector 2, it becomes directly clear that with even levels of human capital there is income equity among the households in sector 2. Moreover, in sector 1, the same level of human capital is combined with different sizes of land, unless there is only one cohort of beneficiaries in the sector. Thus, there is income inequality amongst the households in sector 1. Finally, one can calculate that the households in sector 1 only earn the same income as the households in sector 2, if they possess exactly land of size $n_{i(Tss)} = 1/(1 - zh(1)) \cdot (A_2/A_1)^{1/(1-\alpha)}$. Since the distribution of land is not equitable, this can only be the case for at most one cohort of the land reform. Thus, the inequality prevails also in the long run, unless sector 1 disappears in the long run.

If $zh(1) \geq 1$, then the growth rate, given by $zh(1) - 1 + \frac{1}{\lambda^t}$, will diminish when $\lambda^t$ rises (convergence). For $\lambda^t = \infty$ the growth rate is positive and equal for all households. It follows that the convergence process disappears not until $t \to \infty$, that is, the initial inequality diminishes from period to period. Eventually, income equity prevails because the growth rate of income of households with less human capital is strictly higher than the income growth rate of households with more human capital. Accordingly, we state:
**Proposition 7.4**

Suppose $T > 1$.

(a) Suppose $zh(1) < 1$. Successful land reforms produce a temporarily unequal distribution of human capital within the group of the poor, but the distribution of human capital is definitely equal in the long-term. The distribution of land and income is, respectively, unequal in both the short- and the long-term, unless $a_{iT} = 0$ for all households $i \in [0, 1]$.

(b) Suppose $zh(1) \geq 1$. Successful land reforms produce an unequal distribution of land within the group of the poor in the short- as well as in the long-term. Land reforms also cause a temporarily inequality of human capital and income amongst the poor, but in the long run both inequalities diminish and, finally, disappear.

### 7.5 Land Reforms with Access to Land Markets

We now elaborate on the effects that the access of beneficiaries to the land market may have. The purpose is to answer the question of whether or not to allow beneficiaries of land reforms access to the land market.

#### 7.5.1 The Demand for Land and Land Market Equilibrium

In households of land reform beneficiaries, land transfers induce full-time schooling. Consequently, child labor is extirpated in households that received land: $e_{it} = 1$. When beneficiaries have land market access, then they can sell or buy land at the given land market price, labeled $q_t$. The household optimization now involves the gross demand for land in sector 1, which we denote by $n_{it}^d$. Households’ demand for land is determined by utility maximization. Since land *per se* does not affect utility, it is clear that the utility maximizing level of land input $n_{it}^d$ is equivalent to the income maximizing level of $n_{it}^d$. For additional income is now possible via land market transactions, household’s income can differ from $y_{it}^j$. Hence, we denote household $i$’s income in sector $j$ and period $t$ by $w_{it}^j$. Note that $w_{it}^1$ is the lifetime-income of the adult $i$ in sector 1 in period $t$. As at the beginning of each period the social planner determines the distribution of land, there is no incentive for any land market transaction at the end of a period $t$. It follows that the value of land in a period $t$, $q_t$, is solely determined by its marginal productivity in that period. The optimal demand for land in sector 1 is determined by:

$$\max_{\{w_{it}^1\}} w_{it}^1 = A_1(\lambda_{it})^\alpha(n_{it}^d)^{1-\alpha} - q_t(n_{it}^d - n_{it})$$
We obtain:

\[ n^d(q_t, \lambda_{it}) = \left( \frac{(1 - \alpha)A_1}{q_t} \right)^{\frac{1}{\alpha}} \lambda_{it} \]  \hspace{1cm} (7.34)

\[ \frac{\partial n^d(q_t, \lambda_{it})}{\partial \lambda_{it}} > 0 \]  \hspace{1cm} (7.35)

If human capital is accumulated this lowers the marginal productivity of an efficiency unit of labor, and increases the productivity of land. In order to remain efficient, the farmer must adjust his input factor relation, \( \frac{n_{it}}{\lambda_{it}} \). There exists an optimal land human capital ratio that each single household will use (see Equation (7.34)): \( \frac{n_{it}}{\lambda_{it}} = \left( \frac{(1 - \alpha)A_1}{q_t} \right)^{1/\alpha} \). Notice that the optimal ratio for all farmers is identical, since they all use the same technology and face the same land market price. If a household’s level of human capital increases, it is, \textit{ceteris paribus}, optimal to buy additional land on the land market, and \textit{vice versa}.

Assumption 7.1

\textit{Poor households do not receive credit to purchase a plot of land.}

Assumption 7.1 is similar to Assumption 2.1 and follows from the same argumentation. Consequently, households that have not received land transfers are excluded from agriculture, and thus cannot migrate to sector 1. Then, the land market equilibrium price, denoted by \( q_t^* \), is found by the following approach:

\[ \int_{i=0}^{1} n^d(q_t^*, \lambda_t(i)) \, di = N \]  \hspace{1cm} (7.36)

which can be simplified to

\[ \int_{0}^{\mu_i} n^d(i) \, di = N, \]  \hspace{1cm} (7.37)

since only land reform beneficiaries are able to demand or offer land. Substituting (7.34) into the equilibrium condition and stating that the demand for land of families who are not involved in agriculture is zero, we obtain:

\[ q_t^*(A_1^1) = A_1(1 - \alpha) \left( \frac{A_1^1}{N} \right)^{\alpha} \]  \hspace{1cm} (7.38)

\[ ^{24}\text{It is clear that if a household wants to purchase additional land, then, in practice, it needs a loan, since production takes time. However, once the household owns a plot of land, it can be utilized as security. We neglect this complication and implicitly assume that the real interest rate equals zero.} \]

\[ ^{25}\text{The value of land as an asset is determined by the present value of all future returns one receives from land. In our case, the square meter price is thus exactly determined by the marginal productivity of land, referred to the lifetime income, which is maximized by } n^d_{it}. \text{ At the end of a period, the land redistribution at the beginning of the next period makes land worthless.} \]
where the stock of human capital supplied in sector 1, which is equal to the stock of human capital of adults in sector 1, labeled $\Lambda_1^t = \int_0^t a_t(i) \lambda_t(i) di$, is the explaining variable. Obviously $\Lambda_1^t$ depends on migration, and we therefore analyze this mutual relationship in the next section.

There is a direct positive correlation between the land price and human capital, given by $\frac{\partial q^*_t(\Lambda_1^t)}{\partial \Lambda_1^t} > 0$. Thus, all other things being equal, the education of the society via a land reform continuously raises the price of land. Substituting this equilibrium price in the demand for land, we find:

$$n^d_{it} = \frac{N}{\Lambda_1^t}$$  \hspace{1cm} (7.39)

The higher the individual $i$’s share of human capital stock in the land market, the higher the demand for land, since the productivity of land increases with the level of human capital: $\frac{\partial^2 y_{1it}}{\partial n_{it} \partial \lambda_{it}} > 0$. The education of the society increases the degree of relative land scarcity. Rearranging (7.39), we find that in equilibrium the fraction of the economy’s (productive) stock of land in the hand of a household $i$ must be equal to the fraction of sector 1’s stock of human capital in the hand of household $i$: $\frac{n^d_{it}}{N} = \frac{\lambda_{it}}{\Lambda_1^t}$.

### 7.5.2 Land-Market-Cum-Migration-Equilibrium

To identify how $\Lambda_1^t$ is determined, we must further elaborate on when a household will opt for a particular sector. Since only land reform beneficiaries are located in sector 1, we reconsider the case where $e_{it} = 1$ for all households in sector 1. In order for a household $i$ to leave the agriculture sector in favor of the human capital-based industry sector, the following condition must hold:

$$q_t > (1 - \alpha) \left( \frac{\alpha (A_1)^{1/\alpha}}{A_2} \right)^{(1-\alpha)/(1-\alpha)} = \tilde{q}$$  \hspace{1cm} (7.40)

At the switch-threshold, a particular household earns an identical income in both sectors,

$$w^1_{it} = A_1(\lambda_{it})^\alpha (n^d_{it})^{1-\alpha} - q_t(n^d_{it} - n_{it}) = A_2 \lambda_{it} + q_t n_{it} = w^2_{it},$$

so that the household is indifferent to stay in sector 1 or to migrate. The land reform guarantees $w^1_{it} \geq c^\alpha$ for all beneficiaries. Consequently, in the case without land market access, no beneficiary moves to town sector 2, as long as $\lambda_{it} < \lambda^a$. However, with land market access, additionally to income $A_2 \lambda_{it}$, the household receives income from the

\[26\] We implicitly assume that the market for land is well developed. In practice, there are departures from this case. The market for land as an asset is often thin, and a perfect land market may require a well functioning capital market (cf. Bell (2003), p. 399). Since our results will root in land sales this does not cast our results into doubt. If there are credit constraints, land sales are even more likely.
land sale, in case of emigration to sector 2. A migration equilibrium is established when no household wishes to migrate from one sector to another (see Section 7.4.3). The migration equilibrium thus requires \( q_t^* \leq \tilde{q} \). Substituting land market equilibrium price (7.38) in (7.40), we arrive at:

\[
\Lambda_t^1 > \left( \frac{A_1}{A_2} \right)^{\frac{1}{1-\alpha}} N \equiv \tilde{\Lambda}^1 \tag{7.41}
\]

That is, given the land market equilibrium, each single household opts to change to sector 2 as soon as the stock of human capital in sector 1 crosses a level \( \tilde{\Lambda}^1 \). A land-market-cum-migration equilibrium therefore demands:

\[
\frac{\Lambda_t^1}{N} \leq \left( \frac{A_1}{A_2} \right)^{\frac{1}{1-\alpha}}
\]

The land market equilibrium derived in Subsection 7.5.1 is dependent on the human capital stock in agriculture, which is, in turn, dependent on migration. If the stock of skills crosses \( \tilde{\Lambda}^1 \), households will move to the industry-sector. The process of migration continues until there is no longer an incentive to move: migration lowers the net demand for land, and the land price diminishes to establish a land market equilibrium.

A low enough land price, in turn, ends migration. This mutual adjustments of land market and migration equilibrium stops when both equilibria are established simultaneously. Accordingly, we define

**Definition 7.1**

A simultaneous land market and migration equilibrium is a tuple \( \{q_t^*, \{a_{it}^*\}_{i=1}^N\} \) such that

(i) \( \int_0^1 n^d(q_t^*, \lambda_t(i)) \, di = N; \)

(ii) for \( a_{it} = 1 \), \( n_{it}^* = n^d(q_t^*) \) and

for \( a_{it} = 0 \), \( n_{it}^* = 0; \)

(iii) \( a_{it}^* = 1 \) if \( w_{it}^{1}(\lambda_{it}, q_{it}^*, n_{it}^*) > w_{it}^{2}(\lambda_{it}, q_{it}^*, n_{it}^*) \);

\( a_{it}^* = 0 \) if \( w_{it}^{1}(\lambda_{it}, q_{it}^*, n_{it}^*) < w_{it}^{2}(\lambda_{it}, q_{it}^*, n_{it}^*) \); and

\( a_{it}^* = 0 \) or \( a_{it}^* = 1 \) if \( w_{it}^{1}(\lambda_{it}, q_{it}^*, n_{it}^*) = w_{it}^{2}(\lambda_{it}, q_{it}^*, n_{it}^*) \).

where \( w_{it}^{1}(\lambda_{it}, q_{it}^*, n_{it}^*) = A_1 \lambda_{it}^\alpha(n_{it}^*)^{1-\alpha} - q_{it}^*(n_{it}^* - n_{it}) \) and \( w_{it}^{2}(\lambda_{it}, q_{it}^*, n_{it}^*) = A_2 \lambda_{it} + q_{it}^* n_{it} \).

Part (i) demands that land demand equals land supply (land market equilibrium). Part (ii) simply states that in equilibrium, the optimal land ownership of households in sector 1 equals the optimal land input, \( n^d \), and in sector 2 equals zero, since land is useless in sector 2 and the level of consumption would be lowered if owned land is not
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sold. Finally part (iii) describes the necessary conditions for the migration equilibrium. If in equilibrium a household earns a higher income in sector 1 than in sector 2, then this household will work in sector 1, and *vice versa*. If it earns an identical income in both sectors, then the household is indifferent between working in sector 1 or 2.

We now introduce variable $\hat{\delta}_t$ as the fraction of households that can be endowed with land plots of size $n^a(1)$ in a period $t$, given the “normal” land dispossessions when no household migrates. That is,

$$\hat{\delta}_t = \int_0^1 a_{t-1}(i) n^r(\lambda_t(i)) di$$

when $n^r(\lambda_t(i)) = n^a(\lambda_{t(i-1)}) - n^a(\lambda_{a})$. Moreover, we define $\hat{\mu}_t = \mu_{t-1} + \hat{\delta}_t$. $\hat{\delta}_t$ describes a hypothetical scenario, and is not necessarily the actual $\delta_t$, since the government receives further land plots if some households decide to abandon agriculture.

**Proposition 7.5**

a) If $\int_0^{\hat{\mu}} \lambda_t(i) di \leq \tilde{\Lambda}^1$, there exists a land-market-cum-migration equilibrium in period $t$ with

$$q^*_t = (1 - \alpha)A_1 \left( \frac{\int_0^{\hat{\mu}} \lambda_t(i) di}{N} \right)^{\alpha} \leq \tilde{q} \quad \text{and} \quad \Lambda^1_t \leq \tilde{\Lambda}^1,$$

and

$$a^*_t = \begin{cases} 1 & \text{for all } i \in [0, \hat{\mu}_t] \\ 0 & \text{for all } i \in (\hat{\mu}_t, 1] \end{cases}$$

b) If $\int_0^{\hat{\mu}} \lambda_t(i) di > \tilde{\Lambda}^1$, there exists a land-migration equilibrium characterized by:

$$q^*_t = \tilde{q} = (1 - \alpha) \left( \frac{\alpha(A_1)^{1/\alpha}}{A_2} \right)^{1/\alpha} \quad \text{and} \quad \Lambda^1_t = \tilde{\Lambda}^1,$$

and a set of migration decisions $\{a^*_t\}_{i=0}^1$ such that

$$\int_0^1 a^*_t(i) \lambda_t(i) di = \tilde{\Lambda}^1.$$

**Proof:**

$\tilde{\Lambda}^1$ is the migration threshold given (partial) equilibrium in the land market [Condition (7.41)]. If $\int_0^{\hat{\mu}} \lambda_t(i) di \leq \tilde{\Lambda}^1$, there is no incentive to migrate in period $t$, and a land-market-cum-migration equilibrium is established at $q^*_t = A_1(1 - \alpha) \left( \frac{\int_0^{\hat{\mu}} \lambda_t(i) di}{N} \right)^{\alpha} < \tilde{q}$. Since, for all $k = \{1, 2, ..., t\}$, $\lambda_{ik} \geq \lambda_{i(k-1)}$ for all $i \in [0, \hat{\mu}_t]$, and $\lambda_{ik} = \lambda_{i(k-1)} = 1$.
otherwise, we conclude that \( a^*_{it} = 1 \) for all \( i \in [0, \mu_t] \), and \( a^*_{it} = 0 \) for all other poor households.

If \( \int_0^{\mu_t} \lambda_t(i) \, di > \tilde{\Lambda}^1 \), the partial land market equilibrium, given by (7.38), would lead to migration. If and only if \( \Lambda^1_t = \tilde{\Lambda}^1 \), a migration equilibrium results, and the land-market-cum-migration equilibrium is established at \( q^*_t = \tilde{q} \) and \( \Lambda^1_t = \tilde{\Lambda}^1 \), where no strict migration incentive prevails. Consequently the set \( \{a^*_{it}\}_{i=0}^1 \) has to fulfill\(^{27}\)

\[
\int_0^1 a^*_t(i) \lambda_t(i) \, di = \tilde{\Lambda}^1.
\]

\[\square\]

An immediate consequence is

**Corollary 7.1**

a) If \( \int_0^{\mu_t} \lambda_t(i) \, di \leq \tilde{\Lambda}^1 \), we uniquely find \( \{a^*_{it}\}_{i=0}^1 \), \( \delta_t = \tilde{\delta}_t \), and \( \mu_t = \tilde{\mu}_t \).

b) If \( \int_0^{\mu_t} \lambda_t(i) \, di > \tilde{\Lambda}^1 \), \( \{a^*_{it}\}_{i=0}^1 \), \( \delta_t \) and \( \mu_t \) are indeterminate.

**Proof :**

Part a) is obvious. Due to equilibrium condition \( \Lambda^1_t = \tilde{\Lambda}^1 \) in case b), set \( \{a^*_{it}\}_{i=0}^1 \) has to fulfill \( \int_0^1 a^*_t(i) \lambda_t(i) \, di = \tilde{\Lambda}^1 \). Therefore, there exist arbitrarily many measurable sets \( \{a^*_{it}\}_{i=0}^1 \) that fulfill the equilibrium condition, unless (i) \( \tilde{\Lambda}^1 = 0 \) (\( a_{it} = 0 \) for all \( i \in [0, 1] \)) or (ii) \( \int_0^1 \lambda_t(i) \, di = \tilde{\Lambda}^1 \) (\( a_{it} = 1 \) for all \( i \in [0, 1] \)). Since \( \tilde{\Lambda}^1 = \left( \frac{\alpha_1}{\alpha_2} \right)^{\frac{1}{1-\alpha}} N > 0 \), case (i) cannot occur, and case (ii) belongs to item a) of the corollary. Hence, \( \{a^*_{it}\}_{i=0}^1 \), and thus the size of land that is additionally available for redistribution due to migration, is indeterminate. Therefore, \( \delta_t \) and \( \mu_t \) are indeterminate.

\[\square\]

The non-migrating part accumulates a mass of human capital of \( \tilde{\Lambda}^1 \) and the migrating part represents the “excess mass” of human capital above \( \tilde{\Lambda}^1 \). Corollary 7.1 is rooted in the fact that the distribution of households between the two parts is not decisive. This result has a very crucial consequence. Migration occurs independently of individual-specific human capital: \(^{28}\) it is completely open as to who those migrating households are. Hence, there is a real threat to human capital accumulation if low-skilled persons

\(^{27}\) We assume that the set of non-migrating households \( \{i \in [0, 1] \mid a^*_{it} = 1 \} \) is measurable in the sense of Lebesgue.

\(^{28}\) Remind Condition (7.40).
migrate, since those households may be below the steady state human capital level $\lambda^*$ in sector 2.

The economic intuition of Proposition 7.5 is the following. If the stock of human capital in sector 1 (country side) is not too large there is no incentive to migrate, and we directly arrive at Proposition 7.5 a). However, as soon as the global sector productivity of human capital has shrunk too far due to accumulation, an incentive to change sector location arises. The land reform causes the land price to rise, since the relative scarceness of land increases. The incentive to sell land, respectively the opportunity cost of using land in production instead of selling it, becomes too high. All households want to change sectors, and migration occurs, which involves a further supply of land. The land price and the level of human capital in sector 1 fall until a simultaneous equilibrium of migration and land market is reached at $\tilde{\Lambda}_1$ and $\tilde{q}$.

One might wonder why the incentive to switch sectors arises irrespective of individual parameters (Conditions (7.40) and (7.41)). The reason for this is the constant return to scale technology for family farm production. Deininger and Feder (1998), p. 16, report that the hypothesis of constant returns to scale cannot be rejected for most agriculture production in developing countries. As a consequence, the scale of inputs, like the amount of human capital, does not change the relative productivity of an input.

In the appendix, we demonstrate that in the case of decreasing returns to scale the higher-skilled households leave the agriculture sector for the industry sector, whereas the opposite occurs for increasing returns to scale. However, as there is evidence for constant returns to scale in the agriculture of developing countries, Proposition 7.5 states what we should expect in reality. (In the Appendix to this chapter, we show that the identified danger of open access can be even higher when there does not prevail tâtonnement system stability.)

### 7.5.3 Access to Land Market: Pros and Cons

In this section, we will elaborate on the consequences of allowing beneficiaries of our land reform access to land markets. We begin by briefly discussing related statements found, for instance, in Platteau (1992) and Deininger and Feder (1998).\(^{29}\)

On the one hand, unrestricted access to land markets creates the risk that a short-term shock, for instance a bad crop will lead to distress sales, for instance of land, with the consequence of a loss of productive assets.\(^{30}\) Furthermore, investments required for the

\(^{29}\) Galal and Razzaz (2001) discuss the specific characterization of land markets, and which implications these have for reforming land markets.

\(^{30}\) Additionally, if it is a non-diversifiable macro-shock, all farmers will face the same situation, and
sustainable viability of the farm or expensive social events force temporary drops in current consumption. However, farmers often sell a part of their land, instead. In all these cases, the farmers can fall back into (or remain in) the poverty trap.

On the other hand, it is mentioned that if differences in skills and endowment of production factors exist, land markets allow the re-allocation of land in the direction of the overall highest productivity, and thus efficiency gains. However, land markets may decrease efficiency if the advantage to the larger farmers in accessing credit offsets this effect (credit market distortions). In this context, the additional efficiency gains due to an improved access to credit markets for land reform beneficiaries (bulk investments are possible since land can be used as collateral) is questionable for smaller farmers. Even with land as collateral, the high transaction costs connected with small credits may leave small farmers rationed in the credit, and hence in the land market. It follows that land market disadvantages of the poor remain. Therefore, the argument that these credit market distortions would be overcome, because beneficiaries then possess land as potential collateral, and that land market access thus causes efficiency gains, is not necessarily convincing.

We show that, even in a world without uncertainty, where distress sales cannot happen, unrestricted access to land markets may have yet another adverse effect. Nonetheless, we also identify reasons in favor of allowing beneficiaries access to land market.

7.5.3.1 Pros

Suppose again that, for simplicity, $\xi = 1$. Initially all members of the poor class are identical concerning human capital. In the first period of the reform, therefore, all beneficiaries obtain a plot of land of size $n^a(1)$. It follows that the land market equilibrium forces $n^d_{i0} = n^a(1)$ via a land market price adjustment.

At the beginning of the next period, however, the poor become heterogenous. The second cohort of beneficiaries is endowed with more land and less human capital than the first, thus the first cohort will buy land from the second at the equilibrium price. In any period, cohorts with a higher human capital than average will in general buy land from the cohorts with lower human capital (as long as they are located in the land sector), in order to establish the optimal factor relation of $N/\Lambda^1_t$. Note that in spite of the dispossessions, the households will use the land market for optimizing the factor allocation: each single household establishes the optimal factor intensity. This is clearly an advantage for the access to land markets.\(^{31}\)

\(^{31}\)We assume that land purchases are possible. But, as outlined in the introduction of this subsection, land must be sold at a low price because of a massive increase of land supply. Deininger and Feder (1998) report that 60% of land sales in Bangladesh were undertaken for food and medicine.
Given land market access, one can show that the required size of land for \( e = 1 \), \( \n_a(\cdot) \), becomes a function of the land price:

\[
\n_a(q_t, \lambda_{it}) = \frac{1}{q_t} \left( c^a - \alpha \lambda_{it} \left( A_1 \left( \frac{1 - \alpha}{q_t} \right)^{1-\alpha} \right)^{\frac{1}{\alpha}} \right) \tag{7.43}
\]

Substituting (7.38), we find:

\[
\n_a(A_1^t, \lambda_{it}) = \left( \frac{N}{(1 - \alpha)A_1^t} \right) \left[ \frac{c^a}{A_1} \left( \frac{A_1^t}{N} \right)^{1-\alpha} - \alpha \lambda_{it} \right] \tag{7.44}
\]

Thus, we find:

**Proposition 7.6**

With land market access of land reform beneficiaries, the required land transfer to a household \( i \) in period \( t \), \( \n_a^0 \), is lower than without land market access, if

\[
\lambda_{it} \neq c^a A_1^{\frac{1-2\alpha}{\alpha}} \left( \frac{A_1^t}{N} \right)^{1-\alpha} \equiv \tilde{\lambda}_t
\]

If \( \lambda_{it} = \tilde{\lambda}_t \), the required land transfer is equally large in size. That is,

\[
\n_a(A_1^t, \lambda_{it}) \begin{cases} < n_a(\lambda_{it}) & \text{if } \lambda_{it} \neq \tilde{\lambda}_t \\ = n_a(\lambda_{it}) & \text{if } \lambda_{it} = \tilde{\lambda}_t \end{cases}
\]

The proof is given in the appendix. We conclude that with the land market open to beneficiaries of the land reform, (static) efficiency increases, and the society might be educated in a shorter span of time. If land reform beneficiaries have access to the land market, they will maximize income by selling or buying land. If a household possesses no land and receives an amount \( \n_a(\lambda_{it}) = 1 \), this household is free to stay at \( \n_d = \n_a(1) \), so that the household’s consumption will at least be as high as \( c^a \), and full-time schooling is ensured. Consequently, for all \( \n_d^t \neq \n_a(\lambda_{it}) \) a household’s consumption will be strictly higher than \( c^a \), and the land transfer can be reduced. Therefore, with access to land markets, we may allocate each single beneficiary less land than without access to land markets, and the education of the society may be accomplished quicker.

Referring to static efficiency, “harmful” expropriations of higher-skilled households are “healed” by the land market, because the optimal factor relation can be established this crucially depends upon the premise that beneficiaries receive loans for land purchases, once they have been allocated with land as a potential security. If there remain market distortions, then this advantage will be less effective, or it will even be reversed, that is, land market access of beneficiaries would create disadvantages.
despite the redistribution of land — without reversing the targeted effect. Hence, the land market ensures the efficient production factor allocation. However, as we will demonstrate now, this increased static efficiency might be bought at the expense of dynamic efficiency.

7.5.3.2 Cons

Having identified the typical advantage of markets, we now show a potential risk of allowing access to the land markets. For this purpose, we label the period in which a household changes location by $\tilde{t}$.

Proposition 7.7

In the case of open access of beneficiaries, migration of a beneficiary household $i$ to sector 2 is adverse to the household’s level of education, i.e. $e_{i(\tilde{t}+1)}^o < e_{i\tilde{t}}^o$, if

$$\lambda_{i(\tilde{t}+1)} < \lambda^a.$$ 

Proof:

The choice $e_{i\tilde{t}}^o$ is determined by $w_{i\tilde{t}}^2 = A_2 \lambda_{i\tilde{t}} + q_{i\tilde{t}}(n_{i\tilde{t}} - n_{i\tilde{t}}^r) > w_{i\tilde{t}}^1 = c^a$, i.e. $e_{i\tilde{t}}^o = 1$. $e_{i(\tilde{t}+1)}^o$ is determined by $w_{i(\tilde{t}+1)}^2 = A_2(zh(1)\lambda_{i\tilde{t}} + 1) = A_2 \lambda_{i(\tilde{t}+1)}$. If $\lambda_{i(\tilde{t}+1)} < \lambda^a$, we obtain $w_{i(\tilde{t}+1)}^2 < c^a$, and hence $e_{i(\tilde{t}+1)}^o < e_{i\tilde{t}}^o$.

Note that it is not sufficient that $w_{i\tilde{t}}^2 > w_{i(\tilde{t}+1)}^2$. If this is the case, it is fully possible that $w_{i\tilde{t}}^2 > w_{i(\tilde{t}+1)}^2 \geq c^a$, and that therefore $e_{i(\tilde{t}+1)}^o = 1 = e_{i\tilde{t}}^o$. As long as $\lambda_{i(\tilde{t}+1)} > \lambda^*$, the household’s potential drop in education (described by Proposition 7.7) does not thrust the household back into the poverty trap. However, if $\lambda_{i(\tilde{t}+1)} < \lambda^*$, then the migrated household $i$ will end up in the poverty trap of sector 2 (for instance in urban slums), and adverse land sales wreck land reforms. Therefore,

Corollary 7.2

Beneficiaries of the land reform should be prohibited from selling land if, and only if, $zh(1) + 1 \leq \lambda^*$ in sector 2.

Due to Condition (7.41) the incentive to switch is present in all households, regardless of their levels of education. It is clear that in period $\tilde{t}$, beneficiaries display $e_{i\tilde{t}}^o = 1$. Hence, if $zh(1) + 1 < \lambda^*$, members of the latest group of land receivers that directly change sectors will stay in the poverty trap. If on the contrary $zh(1) + 1 \geq \lambda^*$, even sector switches by members of the latest group do not cause failure of the land reform.
However, if \( zh(1) + 1 = \lambda^* \), members of the last group will not slip back into poverty, but will remain at the instable equilibrium at \( \lambda^* \). This result is not satisfactory, especially because negative shocks would cause those households endowed with \( \lambda^* \) to slip back into the poverty trap. Additionally, households at the instable equilibrium will never enjoy a full basic education.

Of course, in practice, the land sale might bear such high revenue that bequests to the child may mitigate this effect. However, it is by no means ensured that the land sale revenues are so high that the loss of land is sufficiently compensated for.\(^{32}\) Even the group that has only been allocated land in the current period may change sectors despite their low skills. These backward households will (nearly) definitely stay in the poverty trap if \( zh(1) + 1 < \lambda^* \).\(^{33}\) Additionally, we have seen that under particular conditions the land-market-cum-migration equilibria do not display (tâtonnement) system stability. We infer that if there is no (tâtonnement) system stability, there will occur a permanent rural-urban migration movement due to ongoing land redistribution of the landholding of migrated households. Consequently, the risk of adverse sector changes of backward households increases: in case of \( zh(1) + 1 \leq \lambda^* \), the probability of adverse migration continuously rises. That is, a failure of the land reform becomes more likely.

One must carefully weigh the pros and cons of allowing land reform beneficiaries access to the land sale market. Even if the household does not fall back into poverty trap after a location switch, \( \lambda_i(t+1) > \lambda^* \), the potential drop in education, \( e_{i+1} < e_i \), might slow down the education of the society. To ensure the success of the reform, we may have to sacrifice the potential advantage of efficient land allocation through the land market. However, there is no reason to forbid land purchases, since these do not risk the success of the reform, but they do promote efficiency.

In practice, squires often tried to buy the land of land reform beneficiaries, be it due motives of speculation, own agricultural production or simply to buy back formerly owned land. This increases the demand for land and thus the equilibrium price. Thus, the supported households want to switch sectors even earlier, and the thus far neglected potential actions of squires might increase the demonstrated danger of land market access. Additionally, re-considering (7.38), the land price also increases if \( Hicks neutral \) technical progress, i.e. a rise of \( A_1 \), occurs in sector 1, or if human capital becomes more productive in sector 1 (a rise of \( \alpha \)), because then land will (indirectly) become more

---

\(^{32}\)In our experience one-time revenues, like land sale revenues, are most likely used for expensive consumption goods. Related, David (1995) and Islam (1991), for instance, find (for the Sahel respectively for Bangladesh) that major parts of remittances of migrated members of a family are used for “luxury goods” or status symbols.

\(^{33}\)This danger increases if there are increasing economies to scale in agriculture and decreases if the economies to scale are decreasing (cf. appendix, Proposition E.5).

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productive.\footnote{Via $\frac{\partial^2 U_1}{\partial n_{it} \partial \lambda_{it}}$.} If this is the case, technical progress in sector 1 would, \textit{ceteris paribus}, reinforce the threat to human capital accumulation and growth. If, on the other hand, technical progress also occurs in sector 2 the income that an adult, endowed with human capital of size $zh(1) + 1$, earns after migration towards sector 2 will also increase, so that the earlier migration to sector 2 not necessarily increases the danger of falling back into poverty.

### 7.5.4 The Dynamics of the Distribution of Land

The distribution of land during the land reform is continuously distorted by the land redistributions. The higher educated are expropriated so that they are left with an income of $c^a$, that is, land is redistributed towards the poor. The more interesting question concerns the dynamics of the distribution of land after the termination of the land reform. Hence, in this section we demonstrate the consequences of the access to land market of land reform beneficiaries on the distribution of land after the termination of the land reform, that is, when the redistribution of land has stopped. We demonstrate that the resulting dynamic pattern of land distribution crucially depends on the technology of human capital.

**Proposition 7.8**

Suppose that the land reform has been applied, and that all households in sector 1 choose $e = 1$. The re-allocation of land via the land market is then determined by the following pattern:

1. If $zh(1) > 1$, the higher skilled households will purchase land from the less skilled.
2. If $zh(1) < 1$, the less skilled households will purchase land from the higher skilled.
3. If $zh(1) = 1$, no land market interactions will occur unless sector switches take place.

**Proof:**

The optimal factor intensity is given by the average intensity $\frac{N}{A_i}$. Due to $e^o_i = 1$, we can write:

$$\lambda_{i+1} - \lambda_i = \lambda_i (zh(1) - 1) + 1 \quad (7.45)$$

If $zh(1) > 1$, we obtain $\frac{d (\lambda_{i+1} - \lambda_i)}{d \lambda_i} > 0$, that is, the increase of skills rises in $\lambda_i$. Hence, since the higher skilled will establish the highest increases in the level of human capital.
capital, their factor intensity falls short to the optimal, average intensity, given by $N/\Lambda_1^t$. Likewise, the factor intensity of the lower skilled is above average. It follows that the higher skilled buy land from the less skilled to establish the optimal intensity. If $zh(1) < 1$, the opposite is true, and the less skilled will purchase land from the higher skilled. If $zh(1) = 1$, then the level of $\lambda$ is irrelevant. As all adults establish identical changes in skills, they all directly establish the optimal level $N/\Lambda_1^t$.

Suppose, for example, a period’s optimal intensity $N/\Lambda_1^t$ runs up to 0.5, and is chosen by all households in sector 1. Then, let there be human capital accumulation due to schooling of the children. As the adults are heterogeneously endowed with human capital once the land reform has started, the change in the individual levels of human capital will differ (as long as $zh(1) \neq 1$). Suppose the optimal factor intensity $N/\Lambda_1^t$ decreases from 0.5 to 0.45 due to the growth of the stock of human capital. Human capital accumulation decreases the individual factor intensity. Some household’s factor intensity falls short of 0.45 and some display higher factor intensity. Hence, those households with a factor intensity higher than 0.45 establish a below-average increase of $\lambda$, and will therefore sell land to those who establish an above-average increase in skills. Whether a household displays below- or above-average increases of the level of human capital depends on the technology of human capital, that is, on the size of $zh(1)$. The land market redistributes land to the higher-educated households in case of $zh(1) > 1$ and to the less-educated in case of $zh(1) < 1$. In case of $zh(1) = 1$, the land market has no effect on the distribution of land.

Notice that in case of $zh(1) < 1$, all land market transactions stop as soon as the economy has reached the steady state at $\lambda = 1/(1 - zh(1))$, because then no more human capital accumulation occurs. As all households display the same level of human capital, they will all possess the same size of land, that is, in the steady state, equity in the distribution of land among the farmers in sector 1 prevails. Thus, the necessary temporary inequality eventually disappears. However, over the economy as a whole, the distribution remains unequal, because all the households that decide to live in town sector 2 possess no land. In case of $zh(1) \geq 1$, human capital formation continuous for all times. The households with higher levels of human capital will purchase more and more land from the lower skilled households, and the inequality rises. However, eventually, as we will demonstrate in the next section, land will become unimportant in production, and all households leave sector 1.
7.5.5 Transition

In this section we will highlight the structural change our land reform may induce in the case of open access to the land market. In the case of \( zh(1) < 1 \), we again denote a variable that is established in the steady state by the superscript \( ss \). For example, \( a^s_i \) identifies the sector in which household \( i \) works in the steady state.

**Proposition 7.9**

We assume that a land reform is implemented successfully.

(a) Suppose \( zh(1) < 1 \). A strictly positive fraction \( \int_0^1 a^{ss}(i)di \in [0, 1] \) remains in sector 1 indefinitely, while all other households are located in sector 2, where

\[
\int_0^1 a^{ss}(i)di = \min \left\{ 1, (1 - zh(1))\tilde{\Lambda}^1 \right\}
\]

(b) Suppose \( zh(1) \geq 1 \). The share of households ending up in sector 2 asymptotically approaches the whole society, that is, sector 1 disappears.

**Proof:**

As long as \( \Lambda^1_t < \tilde{\Lambda}^1 \), beneficiaries stay in sector 1 (see Condition (7.41)), and they are indifferent to switching sector if:

\[
\Lambda^1_t = \tilde{\Lambda}^1 = N (\alpha(A_1/A_2))^{1/(1-\alpha)} \tag{7.46}
\]

Initially, the society is backward and \( \Lambda^1_t \) is smaller than \( \tilde{\Lambda}^1 \) (right-hand-side (r.h.s.) of Equation (7.46)). The land transfers cause human capital accumulation and \( \Lambda^1_t \) moves towards the constant term \( N (\alpha(A_1/A_2))^{1/(1-\alpha)} \). Once \( \Lambda^1_t \) crosses this migration threshold, households move to sector 2 until the migration-cum-land-market equilibrium is established anew. Eventually, all households \( i \in [0, 1] \) receive land. If \( zh(1) \geq 1 \), human capital increases incessantly so that asymptotically the mass of households will leave sector 1: sector 1 disappears.

If \( zh(1) < 1 \), at skill level \( \lambda = \frac{1}{1 - zh(1)} \) the steady state is reached. For the migration-cum-land-market equilibrium demands \( \Lambda^1_t \leq \tilde{\Lambda}^1 \), the distribution of households between sector 1 and sector 2 in the steady state is determined by

\[
\tilde{\Lambda}^1 = \frac{1}{1 - zh(1)} \int_0^1 a^{s*}(i)di,
\]

if \( \int_0^1 \lambda^{ss}(i)di = \frac{1}{1 - zh(1)} \tilde{\Lambda}^1 \), while if \( \frac{1}{1 - zh(1)} \leq \tilde{\Lambda}^1 \), all households \( i \in [0, 1] \) stay in sector 1 in steady state.

\[\square\]
Both propositions concerning transition, Proposition 7.3 for the case without land market access and Proposition 7.9 for the case with, provide very similar results: if $zh(1) \geq 1$, there will be a transition of the society from a poverty trap to a high(er)-skilled economy. During transition the agriculture sector shrinks, because in the end (asymptotically) all households will have switched to the industry sector 2. In the case $zh(1) < 1$, agriculture will also exist in the long run, but most likely as a small, minor sector. In Appendix E.5, we extend our analysis to a non-constant returns to scale production function in sector 1, which gives further interesting insights.

Our results are in accord with Engel’s Law. We predict that when the income per capita rises over time, the modern industry sector 2 will grow relative to agriculture, and eventually there will occur a diminution of the relative importance of agriculture.\textsuperscript{35} While most papers assume exogenous technological progress as the driving power [for instance, Laitner (2000)], in our model human capital formation pushes the structural change.\textsuperscript{36}

7.6 Discussion and Conclusions

This chapter addressed a lot of the important issues concerning economic development in the context of land reforms: the required design of land reforms, the migration equilibrium, the transition process that the land reform induces, the dynamics of the distribution of human capital and income. After addressing these issues for the scenario without land market, we extended our model to the existence of a land market and examined which effects the access of land reform beneficiaries might have. We deduced the migration-cum-land-market equilibrium and analyzed which effects open access to the land market has on equilibrium, distribution, transition, and on the success of the reform.

Our major results are the following. There might be an important link between the objective of educating the society to overcome poverty and land reform policies that has so far been neglected in both theory and practice. Lack of land ownership and lack of human capital are two sides of the same medal: they are caused by poverty in combination with imperfect markets. Land reforms can be used to enhance both equity in land ownership and in the ownership of human capital, to attain economic


\textsuperscript{36}Standard literature in this field is, e.g., Lewis (1954) and Rostow (1962). See also Barro and Sala-i-Martin (1995), chapter 12.
growth. Land transfers should not just ensure a viable farm size, but also sufficient
education for the children of the beneficiaries in order to establish a sustainable human
capital accumulation. Then, it is possible to use land reforms as a means of inducing the
transition of a society caught in a poverty trap to a (more highly) developed, skill-based
economy where agriculture plays a minor role. The required land reform consists of a
sequence of land transfer episodes rather than simply only a one-off event. Therefore,
creating (temporary) inequality among the poor is unavoidable in the course of land
reforms. Whether equity arises in the long run depends on human capital technology.
Equity arises if the economy ends up in a steady state, which only happens if $zh(1) < 1$.

An important finding is that the land market access of beneficiaries must be restricted
for some time. With access to land markets, the incentive to migrate occurs irrespective
of the individual skill level. Hence, parents may prefer to sell the household’s land and
switch sectors too early, i.e. when they have not yet accumulated enough human
capital. This will result in the failure of the land reform as their descendants stay in
(or fall back into) the poverty trap. To prevent these, from a long-term perspective,
inefficient land sales, a prohibition of land sales for beneficiaries of the reform, for
instance for a time comprising two generations, seems necessary. Notice that this
result was derived in a world without uncertainty; that is, a world where distress sales
do not occur. However, land purchases should be allowed, since these can promote the
efficiency of countryside production and equality.\textsuperscript{37,38}

The experiences with cases like the applied land reform in the Philippines show that
simultaneous public investments (for instance in irrigation systems) increases the prob-
ability of success (see, for instance, \textsc{Bell} (2003), p. 406). Additionally, it is important
that beneficiaries are endowed with the specific husbandry skills and business knowl-
dge required to run a family-farm, because otherwise the initial harvests will be low.
It is clear that unexperienced beneficiaries will run through a learning phase that will
last some years and that new infrastructure has to be built up, so that an initial drop
in output is possible. Nonetheless, due to the education of the society, output will
definitely increase in the long run.

\textsuperscript{37}\textsc{Deaton and Laroque} (2001) and \textsc{Drazen and Eckstein} (1988) argue for different reasons
that land markets are inimical to growth: saving in the form of land crowds out growth-enhancing
capital formation [see also \textsc{Allais} (1947) and \textsc{Feldstein} (1977)]. \textsc{Deaton and Laroque} (2001)
demonstrate that the \textit{Golden Rule allocation} can be established by nationalizing land and “renting”
it out at no charge. This is related with our approach. However, the beneficiaries could not use the
land as collateral, and incentive problems arise.

\textsuperscript{38}\textsc{Díaz} (2000) examines a political economy for Latin America in which she concludes that the
landed elite, facing land expropriations, used their power to establish land reforms where the peasants
received land without full rights. They were especially not allowed to sell the land with the effect that
the abundant land became more scarce and hence more valuable when sold by the elite. In the light
of our analysis, there might have been good reason for prohibiting land sales.
A question open to future research is to what extent property rights should go to the beneficiaries of land reform. The advantage of doing so is that this increases the incentive of participants to develop and to make land more productive (effort and investment) and that land can be used as collateral. On the other hand, strict property right undermines the possibilities of further land redistribution. In our model, beneficiaries of the land reform can be given property rights as part of the transferred land while the remaining part is only given on a temporary base. Hence, the incentive effect of property rights can be functioning, but not to the full extent, and a collateral is at hand if a government follows a land redistribution scheme suggested in this chapter.

There are a variety of further productive extensions to our model that promise to yield further insights for the optimal design of land reforms. The most important extension is analyzing our model in a political economy framework, so that expropriations are endogenous, because in history most successful, large-scale land redistributions appeared after regime changes, but not in “normal” times. Therefore, a promising approach is a combination of our model with the model in Horowitz (1993), which highlights which scope for land redistribution exists, when social conflict should be prevented. Additionally, a deeper understanding of the effects of land reforms on the credit market access would be helpful. This could provide answers to the question for under which conditions beneficiaries actually are in a better position to raise a loan for investments. In this context, extending our model to physical capital as a production input in combination with an analysis of the credit supply of money lenders or banks in developing countries would be a promising path to follow to improve our understanding of both the realistic situation of beneficiaries and the interaction between human and physical capital. Moreover, the role of wage laborers employed by landlords and international trade in agricultural goods are important aspects that might further necessitate or warn against the large scale redistribution of land in poor societies. Moreover, our analysis neglects uncertainty while agriculture typically involves risk or uncertainty so that these aspects, and the role of imperfect insurance markets, might warrant further examination. An additional question is the effect of population growth on our results.

In the remaining two chapters that follow, we will, in the next chapter, highlight the effects that land reforms have on the social group of squires, on the rural labor...

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39 Cf. Bell (2003), Chap. 14.6. A survey on the nexus between land reforms and political systems arrives at the conclusion that a big-sized successful land reform might only be possible after a change of the political system [see Schrader (2004)].

40 We suggest that the likelihood for a successful land reform depends positively on the promotion of free trade when free trade generates higher farmer incomes in developing countries.

41 We conjecture that population growth may increase the land prices and thus the demonstrated danger of “hasty” land sales. Technical progress in agriculture might even enforce this effect, because it also rises the land price.
market, and which policy implications these effects have. In the final chapter, we extend the model to aspects as transportation cost, gathering costs, and highlight the nexus of environmental and economic sustainability, to draw conclusions concerning the necessary design of land reforms.
Chapter 8

Land Reforms and the Rural Labor Market

8.1 Introduction

In the previous chapter, we considered two sectors, the family farming sector 1 and the human-capital-based sector 2. The land reform allocated poor, illiterate households from sector 2 with a plot of land, the descendants visited school and received a full basic education. Human capital accumulation started and the households escaped poverty. Once the level of skills sufficed, these households moved back to sector 2. Interpreting sector 2 as a town sector, this could mean that the poor humans living in the slums of the cities were supported by the land reform.

In real world, poor households often also work on farms of squires. Then, the land reform allocate rural illiterate households with a plot of land, which then earn their income on their own family-farm. In cities, the wages are often regulated, and the supply of labor is big. However, the labor market in rural areas is much less regulated. Therefore, when a land reform opens a new family-farming sector, these might have effects on the rural labor market.

To analyze the nexus between rural labor market and land reforms, we modify our model from Chapter 7. We drop the town sector 2 and introduce a second rural sector representing the squires’ production to consider the rural labor market. This allows us to identify an interesting labor market equilibrium effect of land reforms. Additionally, we consider land as one form of asset in the squires’ portfolio of wealth. This allows us to offer an alternative land price determination. We then are able to address the possibility that squires buy back former owned land from land reform beneficiaries.

We will demonstrate that ongoing land redistribution leads to a situation in which more
and more poor day-laborers become independent from squires. Due to an increasing family-farm sector (caused by the land reform), the day-laborer reservoir of the squires melts. That is, in the squire sector labor becomes more scarce, and therefore the wage rate paid by squires may rise. Accordingly, even the non-beneficiaries’ welfare will rise in the course of a dynamic land reform. We will see that thus not all members of the group of the poor does have to be endowed with land gift \( n^a(1) \) to educate the society. However, as we will see, under certain conditions land reforms can produce exactly the opposite: the rural labor wage diminishes, and thus also the non-beneficiaries’ welfare diminishes.

### 8.2 The Model

Consider a small two-sector economy of a developing country with total land endowment of \( N \), as in the previous chapter. Additional to our family farming sector 1 there is a second rural sector, representing the production of the squires. We label this sector “sector 3” to avoid confusion in notation. For simplicity, we cancel sector 2.

There is a continuum of (initially) poor, unskilled and landless households, labeled by \( i \in [0, 1] \), as in the previous chapter. Consider the same OLG pattern and preferences as assumed in Chapter 7, and that these households are comprised of one adult and child. The adults decide upon the time fraction that their children spend in school, \( e_{it} \in [0, 1] \). Initially, all poor households display \( \lambda = 1 \) and \( n = 0 \). Additionally, to ensure perfect competition amongst squires, there is a non-small number of households of squires. Squires also live for the two periods “childhood” and “adulthood”. Children of squire-households enjoy, in any case, a very good education, so that all squires display skills by far beyond \( \lambda^a \).

### 8.2.1 The Technologies

The human capital technology is again given by Equation (2.1):

\[
\lambda_{it+1} = h(e_{it}) \cdot (z\lambda_{it}) + 1
\]

---

1. In the model of Basu and Van (1998), fighting child labor also rises the adults’ wages, since lower child labor corresponds with a labor supply shortage.
2. We neglect the possibility that a squire might be a monopsonist at his region’s labor market. This does not change our results, but the level of wages would be lower.
For simplicity, we assume the growth-case \((zh(1) \geq 1)\). Sector 1 is the family farming sector, already familiar from the previous chapter:

\[
y_{1it} = A_1[\lambda_{it} + (1 - e_{it})\gamma]^{\alpha}(n_{it})^{1-\alpha}
\]  

(8.1)

Moreover, there exists a second rural sector that represents the output production of the squires. The inputs of production in sector 3 are “pure labor”, labeled \(L_t\), and “land”. In sector 3, we denote the fixed amount of land by \(H_t\). As only squires have access to the land market, all land \(H_t\) has to be used in the sector of the squires, since otherwise land would lie idle, which cannot happen in equilibrium. All squires use the same technology. We assume that this technology displays constant returns to scale, so that the single squires choose the same labor-land ratio in cultivation. It follows that the production of the squires in sector 3 can be treated as one big farm.\(^3\) Therefore, we use a sectoral production function and the output of the squires in sector 3 in period \(t\), labeled \(Y^3_t\), is determined by the following technology:

\[
Y^3_t = A_3(L_t)^{\alpha_L}(H_t)^{1-\alpha_L}
\]

(8.2)

where \(0 < \alpha_L < 1\). Since \(H_t\) is fixed, only labor \(L_t\) is variable. Pure labor \(L_t\) may include child labor. The squires compete for day-laborers at the rural labor market. These day-laborers stem from the continuum of households distributed on \([0, 1]\). Again adults \(i \in [0, 1]\) spend all their time working. The pure labor of adults corresponds with the minimum level of human capital of adults, i.e. with \(\lambda = 1\). If adults are endowed with human capital beyond \(\lambda = 1\) this is irrelevant for their productivity in sector 3. Nonetheless, education of the child does spend utility, so that children might spend some time in school. Accordingly, the pure labor of children is given by \((1 - e_{it})\gamma\). Due to perfect competition on the labor market both laborers and squires consider the wage rate as given. The wage rate in sector 3 and period \(t\) we denote by \(\omega^3_t\). It is fully flexible so that there is full employment in equilibrium. Both sectors produce the same agricultural output. The output price is again normalized to one.

### 8.2.2 The Behavior of the Households

#### 8.2.2.1 Poor Households

Concerning the behavior of the poor households \(i \in [0, 1]\), everything deduced in Chapter 7 for sector 1 holds. We again state Assumptions 2.1, 2.2 and 7.1. Moreover, we denote the household’s wage income in sector 3 by \(y^3_{it}\):

\[
y^3_{it} = \omega^3_t(1 + (1 - e^\alpha(\omega^3_t))\gamma)
\]

(8.3)

\(^3\)Cf. also Bell (2003), p. 381.
Finally, we re-define the location identification variable $a_{it}$ from the previous chapter: $a_{it} = 0$ now means that household $i$ works in sector 3.

8.2.2.2 Squires

Squires are wealthy. They can invest their wealth in the international capital market or in the national land market.\(^4\) The international interest rate is denoted by $r$. We consider a small economy so that $r$ is exogenously given. Furthermore, they have to decide on how many day-laborers to employ.

We assume that the international capital market and the domestic land market is, from the perspective of the squires, perfect. *Fisher’s Separation Theorem* tells us that we can isolate the decisions on the production side (i.e. the decisions on the labor market and concerning investment) from the consumption side.\(^5\) Therefore, we are in a position to neglect the utility analysis for the squires. The sector output after deduction of wages accrues to the squires. This *residual income* we denote by $\Upsilon_t$. Profit maximization yields:

\[
\omega^3_t = \alpha_L A_3 \left( \frac{H_t}{L_t} \right)^{1-\alpha_L} \quad \text{(8.4)}
\]

\[
\Upsilon_t = (1 - \alpha_L) A_3 (L_t)^{\alpha_L} (H_t)^{1-\alpha_L} \quad \text{(8.5)}
\]

Therefore, the labor demand of the squires, labeled $L^d_t$, is equal to:

\[
L^d_t = H_t \left( \frac{\alpha_L A_3}{\omega^3_t} \right)^{1/(1-\alpha_L)} \quad \text{(8.6)}
\]

That is, in each period there is an optimal labor-land ratio, that is determined by the wage rate:

\[
\frac{L^d_t}{H_t} = \left( \frac{\alpha_L A_3}{\omega^3_t} \right)^{\frac{1}{1-\alpha_L}} \quad \text{(8.7)}
\]

The arbitrage equilibrium on the investment side demands that the agricultural revenue of land $H_t$, that is residual income $\Upsilon_t$, is equal to the revenue in case the land is sold and invested in the international capital market at interest rate $r$. That is,

\[
r \cdot q_t H_t \overset{!}{=} (1 - \alpha_L) A_3 (L_t)^{\alpha_L} (H_t)^{1-\alpha_L}.
\]

Therefore, by arbitrage, the land price is determined by:

\[
q_t = A_3 \left( \frac{1 - \alpha_L}{r} \right) \left( \frac{L_t}{H_t} \right)^{\alpha_L} = \frac{\Upsilon_t}{r H_t} \quad \text{(8.8)}
\]


Applying Equation (8.7), we obtain:

\[ q_t = A_3 \frac{(1 - \alpha_L)}{r} \left( \frac{\alpha_L A_3}{\omega_t} \right)^{\frac{\alpha_L}{1 - \alpha_L}} \]  

(8.9)

It follows that the land price falls when the wage rate rises, because when labor becomes more expensive, then the land rent falls, and thus it is less worthwhile to invest in land.

### 8.3 The Laissez-Faire Equilibrium

In the *laissez-faire* situation, there is no state intervention. This laissez-faire scenario is identified by the subscript \( L_f \), that is, for instance, \( \omega^3_{L_f} \) denotes the wage rate in sector 3 in the laissez-faire scenario. In this scenario, all land \( N \) is owned by the squires. Let us for simplicity normalize \( N \) to one, so that, in the laissez faire case, \( H_t = N = 1 \) for all \( t \). All poor households \( i \in [0, 1] \) are initially unskilled and landless. Sector 1 does not exist. Therefore, all these poor, unskilled households \( i \in [0, 1] \) supply their complete labor force of \( 1 + \gamma \) to squires in sector 3. Market clearing on the labor market requires that the complete continuum of poor households, including child labor, is employed by the squires. The labor supply runs up to \( 1 + \gamma \). Accordingly we find:

\[
\omega^3_{L_f} = \frac{\alpha_L A_3}{(1 + \gamma)^{1 - \alpha_L}} = \frac{\alpha_L Y^3_{L_f}}{1 + \gamma} 
\]

(8.10)

\[
L^3_{L_f} = 1 + \gamma 
\]

(8.11)

\[
Y^3_{L_f} = A_3 (1 + \gamma)^{\alpha_L} 
\]

(8.12)

\[
q_{L_f} = A_3 (1 + \gamma)^{\alpha_L} \frac{(1 - \alpha_L)}{r} 
\]

(8.13)

\[
\Upsilon_{L_f} = (1 - \alpha_L) A_3 (1 + \gamma)^{\alpha_L} 
\]

(8.14)

It is plausible to assume:

**Assumption 8.1**

The equilibrium wage rate in the laissez-faire scenario in sector 3, \( \omega^3_{L_f} \), does not allow consumption level \( c^S \). That is,

\[
(1 + \gamma) \omega^3_{L_f} < c^S. 
\]

It follows that all poor, landless households choose \( e_t = 0 \): there is full-time child labor in sector 3. Thus, all poor households are imprisoned in the poverty trap at \( \lambda = 1 \).
8.4 Labor Market Effects of the Land Reform

Suppose that the land reform proposal of Chapter 7 is implemented. Initially, the squires owned all land, that is, \( N = H_t = 1 \). However, in the first step of the land reform, the squires lose land of size \( \xi N = \xi \) to the state.\(^6\) Thus, we have \( H_t = (1 - \xi) \) for all \( t \).\(^7\)

In each period \( t \), a fraction \( \delta_t \) of the continuum of poor households is given a land gift of size \( n^a(1) \). We consider the case where beneficiaries do not have land market access. If a land reform offers a plot of land to poor beneficiaries, they compare their potential farming income in sector 1 with the household’s wage income in sector 3. In sector 3, an adult with \( \lambda = 1 \) chooses \( e_t = 0 \), but the land transfer guarantees consumption level \( c^a \) and \( e_t = 1 \), if sector 1 is chosen. Therefore, it is clear that land transfer \( n^a(1) \) guarantees that beneficiaries start family-businesses in sector 1.

8.4.1 The Labor Market Equilibrium

Each of the by the land reform supported households will start family-farming in sector 1 and display \( e = 1 \), as long as \( (1 + (1 - e^o(\omega^3_t))) \gamma ) \omega^3_t < c^a \). The fraction of already supported households, labeled \( \mu_t \), increases from period to period. The labor supply in sector 3 in each period \( t \), labeled \( L^*_t \), is equal to:

\[
L^*_t = (1 - \mu_t) \left[ 1 + (1 - e^o(\omega^3_t)) \gamma \right]
\]

(8.15)

Consequently, this labor supply decreases in the course of time due to the increase of \( \mu_t \). The diminishing labor supply will force the equilibrium day-laborer wage rate to rise. The equilibrium wage rate in sector 3 is implicitly given by:

\[
\omega^3_t = \alpha_L A_3 \left( \frac{1 - \xi}{(1 - \mu_t) \left[ 1 + (1 - e^o(\omega^3_t)) \gamma \right]} \right)^{1 - \alpha_L}
\]

(8.16)

Furthermore, equilibrium is described by:

\[
\Upsilon^*_t = (1 - \alpha_L) A_3 \left\{ (1 - \mu_t) \left[ 1 + (1 - e^o(\omega^3_t)) \gamma \right] \right\}^{\alpha_L} (1 - \xi)^{1 - \alpha_L}
\]

(8.17)

\[
g^*_t = A_3 \frac{(1 - \alpha_L)}{r} \left( \frac{(1 - \mu_t) \left[ 1 + (1 - e^o(\omega^3_t)) \gamma \right]}{1 - \xi} \right)^{\alpha_L}
\]

(8.18)

We define \( \eta^{e^o(\omega^3_t), \omega^3_t} \equiv \frac{\partial e^o(\omega^3_t)}{\partial \omega^3_t} : \frac{\omega^3_t}{e^o(\omega^3_t)} \), i.e., \( \eta^{e^o(\omega^3_t), \omega^3_t} \) is the income-elasticity of the Marshallian \( e^o_t \) of day-laborers.

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\(^6\) It is also possible that all plots used for the land reform are state-owned, and the squires do not lose any land. This is not crucial for the results in this chapter.

\(^7\) We assume that, despite dispossessions, \( e^o_t = 1 \) for the children of the squires persists.
Proposition 8.1
In course of the land reform, the wage rate $\omega_3^*$ increases from period to period, that is,
\[ \frac{d\omega_3^*}{d\mu_t} > 0, \]
as long as
\[ \eta e^o(\omega_3^*) \omega_3^* < \frac{1}{\gamma e^o(\omega_3^*) (1 - \alpha_L) (1 - \mu_t)^2 [1 + (1 - e^o(\omega_3^*)) \gamma]} \equiv \bar{\eta} \]
The proof is given in the appendix. The economic intuition is the following. The land reform takes day-laborers from sector 3 and locates them in sector 1. Hence, the labor supply in sector 3 decreases due to the land reform. Consequently the wage rate has to rise in equilibrium. However, if the income elasticity of $e^o_t$ is so high that an increase of the wage rate would decrease child labor more strongly than the wage rise decreases the squires’ labor demand, then labor market equilibrium requires the wage rate to decrease.

Figure 8.1 depicts the “normal” case. In period 1 labor market equilibrium is at $E_1$. Then, in period 2, the labor supply curve $L^s$ is shifted to the left, due to the land reform. The new equilibrium is located at $E_2$, where the wage rate has risen. Now take a look at Figure 8.2. In contrast to Figure 8.1, there exist multiple equilibria in period 1 and 2, only equilibrium $E_3$ in period 3 is unique. As we analyze developing countries in a poverty trap connected with full child labor, let $E_1$ represent the labor market equilibrium in period 1. Again the labor supply curve $L^s$ moves to the left in period 2, due to the land reform. $L^s_2$ represents this new labor supply curve. It does not matter in which of the three equilibria in period 2 we end up, the wage rate definitely increases by the land reform. However, imagine period 1 is a later period. Then the equilibrium in period 1 might be located at point $A$. In this case, it is possible that the labor market equilibrium in period 2 is at locus $B$ or $E_2$. Accordingly, the wage rate decreases from period 1 to period 2 due to the land reform. This is possible because, due to $\eta e^o(\omega_3^*) \omega_3^* > \bar{\eta}$, the slope of the middle part of the labor supply curve $L^s$ is flatter than the slope of the labor demand $L^d$. This produces multiple equilibria. If we assume that $\eta e^o(\omega_3^*) \omega_3^* < \bar{\eta}$ is always fulfilled, it is easy to obtain:

Corollary 8.1
Let $\eta e^o(\omega_3^*) \omega_3^* < \bar{\eta}$. The land reform causes the following additional equilibrium effects:
\[ \frac{d\Upsilon_t^*}{d\mu_t} < 0 \]
\[ \frac{dq_t^*}{d\mu_t} < 0, \quad \frac{dq_t^*}{d\xi} > 0 \]

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Figure 8.1: The “normal effect” of a land reform at the rural labor market, where \( A = (1 - \mu_1)(1 + \gamma), B = 1 - \mu_1, \) and \( C = 1 - \mu_2. \)

That is, the ongoing loss of day-laborers due to the land reform lowers the squires’ income from generation to generation. Consequently, the willingness of squires to pay for land diminishes and the land price falls over the course of the land reform. Furthermore, the initial dispossession of squires’ land increases the scarcity of land in sector 3, which causes an initial augment of the land price.

**8.4.2 Policy Implications for Land Reforms**

Let us assume that \( \eta_{t}(\omega_{t}) \omega_{t}^{3} < \overline{\eta}. \) As the wage rate \( \omega_{t}^{3*} \) grows from period to period, in some period labeled \( t^S, \omega_{t}^{3*} \) may cross level \( c^S, \) and in a later period, labeled \( t^a > t^S, \) even \( c^a. \) For simplicity, let us assume that on average a fraction \( \delta \) of the poor is given land gift \( n^a(1) \) in each period of the land reform.
Corollary 8.2
Suppose the land reform starts in period $t = 0$. Let $\eta(\omega^3), \omega^3 < \frac{\pi}{4}$.

(i) Day-laborers in sector 3 will educate their children partly, that is $e_{it} > 0$ for all $i \in (\mu, 1]$, once period $t = t^S$ is reached, where:

$$t^S = \frac{1}{\delta} \left[ 1 - \left( \frac{\alpha_L A_3}{c^S} \right)^{1/(1-\alpha_L)} \frac{1 - \xi}{1 + \gamma} \right] - 1 < \infty$$

and $\mu_t^S < 1$

(ii) Day-laborers in sector 3 will educate their children fully, that is $e_{it} = 1$ for all $i \in (\mu, 1]$, once period $t = t^a$ is reached, where:

$$t^S < t^a = \frac{1}{\delta} \left[ 1 - \left( \frac{\alpha_L A_3}{c^a} \right)^{1/(1-\alpha_L)} (1 - \xi) \right] - 1 < \infty$$

and $\mu_{t^a} < 1$
The development can be described as follows. Once we have passed period \( t = t^S \), day-laborers will start to send their children to school, and day-laborer households accumulate human capital.\(^8\) As soon as \( t = t^a \), beneficiaries of the land reform and day-laborers in sector 3 earn the same income, namely \( c^a \). If the government continues redistributing land in period \( t = t^a + 1 \), day-laborers will earn a higher income than the beneficiaries in sector 1 (Proposition 8.1). That is, \( \omega_3 > c^a \). No day-laborer would accept to start a family business with land gift \( n^a(1) \). However, once period \( t^a \) is reached, there is \( \omega_3 = c^a \). We draw the following conclusion.

**Corollary 8.3**

Let \( \eta_{t^a}(\omega_3^a) \omega_3^a < \eta_7 \). A land reform with the mission to educate the society and to overcome poverty is accomplished successfully in period \( t^a \), though \( \mu_{t^a} < 1 \).

Due to the dynamic labor market equilibrium effect of the land reform, land redistribution stops in period \( t^a \), although not all poor households have received a plot of land. Nonetheless, the success of the land reform is guaranteed: the wage rate of day-laborers has risen so strongly that also day-laborers are in a position to send their children to school full-time. As land redistribution stops, the wage rate stays at this level. It follows that the identified land reform’s equilibrium effect on the rural labor market accelerates the education process. Additionally, it demonstrates that land reforms not only improve the outcome of the direct beneficiaries, but also the welfare of not-supported poor households.

**8.5 When Beneficiaries have Land Market Access**

In this section, we will briefly come back to the issue of whether or not to allow beneficiaries land sell market access. In Chapter 7, we argued that land market access of beneficiaries bears the risk that these move to town sector 2 too early, i.e., when their level of human capital is not yet high enough to escape the poverty trap in town. Hence, we extend our model to the possibility that beneficiaries are in position to move to town sector 2.

For simplicity, suppose that there is no migration between sector 3 and 2. We derived that beneficiaries will switch sectors towards sector 2 as soon as \( q_t > \tilde{q} \). We argued that due to human capital accumulation in sector 1 the equilibrium price of land continuously will rise and therefore, at some point in time, cross threshold \( \tilde{q} \), so that

\(^8\)It is interesting to notice that day-laborers form human capital, although they cannot use it in the production process of sector 3. This is the case since schooling bears utility (altruism).
adverse migrations of uneducated families might occur that would cause the failure of the land reform.

In the light of our new results, it is, *a priori*, open whether the land market price will continuously rise in the course of time, and thus it is not clear whether the land price ever will cross threshold $\tilde{q}$. On the other hand, the land market price is, due to the demand for land of squires, high right from the beginning of the land reform. Especially in the first periods of the land reform the land rent $\Upsilon_t$ is still high. Therefore, even if the land price will not rise from period to period, it is very well possible that the land price is at levels above $\tilde{q}$ – the adverse migrations, described in Chapter 7, would be the consequence. We infer that our proposal to prohibit land sales temporarily continues to be reasonable. We additionally learned that this prohibition is especially important in the first periods of the land reform, when the squires still have a high willingness to pay for land.

### 8.6 Conclusions

In this chapter, we took a closer look at the interdependence between the rural actions of squires and the participants of land reforms. For this purpose, we added a squire sector to our basic land reform model of Chapter 7. We identified an *equilibrium effect* of land reforms. The ongoing land redistribution towards day-laborers of the squire sector is likely to increase the scarcity of labor for squires, and therefore the day-laborers wage rate rises in the course of a land reform. Consequently, land reforms improve not only the well-being of participants, but also the of the other poor groups in society, through wage increases. At some point in time, day-laborers income situation is improved so strongly that they, even without a land gift, are able to educate their children. Therefore, the identified land reform’s equilibrium effect allows the education of the society in an even shorter span of time than in Chapter 7. The empirical findings of Besley and Burgess (2000) support our results: they emphasize that land reforms also benefit the landless by raising agricultural wages. However, our model demonstrates that if the parents’ education decisions are highly sensitive with reference to the income level, i.e. the income elasticity of the demand for education is very high, then it is possible that the wages of the day-laborers actually *decrease* in equilibrium. That is, land reforms may cause the welfare of non-beneficiaries to diminish temporarily.

Squires hold their wealth in form of land and in assets, supplied at the international capital market. Their demand for land depends on the size of the land rent that they
earn in agriculture. When wages rise in the course of a land reform, caused by the land reform, this decreases the land rent. As well the squires demand for land as the price of land tend to fall. However, if the beneficiaries have land market access, the land price tends to rise because of the accumulation of human capital in the family-farming sector of the beneficiaries. Therefore, the development of the land price is contingent on the question which of the effects is stronger. With regard to the issue whether beneficiaries should be allowed land market access, this, on the one hand, means that the land price not necessarily will grow so strongly that adverse migration of uneducated households will occur. On the other hand, since the land rent of squires is, especially in the early periods of a land reform, high, so is their demand for land. It follows that already in these periods, before big-sized human capital formation of beneficiaries has started, adverse emigrations from beneficiaries to cities is possible, so that these household will drop back into the poverty trap. Consequently, our proposal to prohibit beneficiaries of land reforms to sell their received land in the following generations continues to be reasonable. Moreover, while our previous analysis only saw this danger for later periods of land reforms, our new results suggest that this danger already can occur in the very first periods of land reforms, that is, in any period. If the income elasticity of the demand for education is very high ("perverse" case), the land rent rises, and hence also the squires’ demand for land rises. Consequently the land price will increase and the danger of early migration of weakly educated beneficiaries would even increase, compared to Chapter 7.

Despite empirical studies that support our theoretical result, there are two building blocks, that we neglected, that should be discussed. First, population growth might mitigate our equilibrium effect on the wage rate of the day-laborers, because the labor supply increases and counteracts the loss of labor force caused by the land transfers. Strong population growth might even cause wage rate drops, in spite of the land reform effect. Second, we neglected the labor-leisure decision of the adults. The normal reaction is that wage rate increases cause a rise of the labor supply, which already is at the maximal level. Therefore, neglecting the labor-leisure decision seems fully reasonable. However, it is also possible that the adults decrease their labor supply, whereby the wage rate of the day-laborers would not rise as much in equilibrium.

Finally, our results concerning the value of land in the hand of the squires sheds a new light on the result of POUTVAARA (2003) and produces first, preliminary results for a political economy of land reforms. Poutvaara argues that land-possessing middle-

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9It also might be possible that day-laborers increase leisure and decrease their labor supply, because the adults are not anymore forced to work full-time, when the wage rate has risen sufficiently. Consequently, the day-laborers’ wages would reach the level of $c^a$ earlier, and the education of the society is accomplished quicker.
aged did have an incentive to support the education of the society, because it be
advantageous for them to give part of their land for this project: in his model, human
capital formation increases the value of their land over the course of time. We have
seen that the land rent of squires diminishes due to the land reform over the course of
time, as the labor supply of their sector decreases and thus labor cost increase. This
tends to lower the value of a plot of land. Applied to land reforms, Poutvaara implicitly
assumes that the productivity gains of human capital formation are for the benefit of
the squires. As our model demonstrates, this is by no means ensured. If the squires are
producing with technologies resting on the input of raw labor instead of skilled labor,
they do not benefit from the education of the society, but they loose. Consequently,
they will not support but fight land reform proposals in the political process. This is
what we actually often observe. However, Poutvaara’s argument completely applies to
the owners of skill-based firms (as in our sector 2), so that this social group has an
incentive to support a corresponding land reform with parts of their assets. This is an
important illumination in reference to lobbyism and the political process in the context
of land reforms. Their are, as always, project winners and losers.
Chapter 9

Land Reforms and Geography

9.1 Introduction

In agriculture, land is an important production factor. Land has certain particularities. In contrast to other factors, land is immobile and (nearly) not producible. Land is also often characterized by being “indestructible”. However, its economic value, for instance measured by the fertility of the soil, depends on certain circumstances as soil quality and can be destroyed, amongst other reasons, by adverse land use or by environmental damages and catastrophes. The soil of different plots of land is heterogenous in quality and the market access is different. In this chapter, we will highlight these particularities of agriculture and land, and extend our analysis of land reforms correspondingly.

9.1.1 The Authors of the Classic

The special character of land in agriculture was already studied by classical political economists (for instance, Mill, Ricardo, Smith, or von Thünen).\footnote{For a general overview on the Classic see, for instance, Blaug (1997), Johnson (1973), Hollander (1979), or Schneider (1970).} In the 18th and 19th century, the time of the Classic, European economies were still agrarian. The famous \textit{Ricardian Rent} demonstrates the peculiar value of land in agriculture resulting from heterogenous quality of soil when the \textit{Malthusian Population Growth Theory} is applied and fertile soil is scarce (differential rent). Those who claim property right to fertile plots of land accrue an increasing rent when the population is growing, because the quality advantage of fertile soil, compared to \textit{marginal soil} (also called \textit{marginal land}), augments.\footnote{See Hicks (1965) for a review of the agrarian growth models of Adam Smith and David Ricardo, or Smith (1994a) and Ricardo (1973).}
Even earlier, von Thünen (1826), in his famous *The Isolated State*, found that a rent appears due to geographic location advantages when transportation costs are taken into account (*von Thünenian Rent*). Plots closer to the market place (e.g. a city) display the advantage of lower transportation costs. These landholdings close to a town specialize on goods that are more expensive to transport. More remote plots specialize on goods which are cheaper to transport. He demonstrates that production becomes more extensive when the remoteness to the town increases. That is, the optimal labor-land relation decreases with increasing distance to the sales market. For our analysis this means that the optimal input of human capital per square meter of soil becomes a function of the geographic location of the considered farm and is thus no longer equal for all farmers. We will demonstrate that this also has an effect on migration and thus on our discussion of land market access.

While the *Ricardian theory* starts from the premise of heterogenous quality of soil of different plots, but neglects geographic issues, the *von Thünenian theory* emphasizes the meaning of geographic aspects and considers homogeneous soils. Consequently, the Ricardian rent is rooted in quality advantages of landholdings and the von Thünenian in advantages in market access. In both models, the value of a plot of land is determined exactly by the size of this rent. In a competitive land market equilibrium, the potential land purchasers offer a price that exactly equals the current value of the future expected rents. The price of plots which do not bear any expected future rents is zero (*marginal soil*).

### 9.1.2 Quintessence of the Classic

Overall, the classic authors agreed on the point that different plots of land are generally heterogenous (be it due geographic location or soil quality). Consequently, there cannot exist one common square meter price for acres. The approaches in Chapter 7 and 8 are based (implicitly) on the assumption of homogeneous soil quality and equal total transportation costs, so that the deduced equilibrium price, as the unit price per land, might be misleading.

Land cannot be (re)produced and must – similarly to the factor “man” – be seen as a special case (since they are natural resources). A differential rent can be caused by a variety of differences between parcels of land (and thinkable combinations of

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4Lösch (1940) demonstrated the spatial dimensions more generally. See Lösch (1954) for an English translation.
these): different distances to markets, different quality of soil, different supply of water, different endowment with infrastructure in general, differences in the political or social stability etc. All these differences in the economic, political and natural environment cause differences in the income that a plot of land of a certain size bears, and thus in land market prices and in the required land transfers of a land reform. Thus far, we have neglected this particularity of the factor “land” and of the land market in our analysis. In the following, we demonstrate some of the implications arising from these peculiarities of land in a von-Thünen-model.

The developing economies of today are, similar to the European economies studied by the Classic, strongly agricultural economies (although there remain striking differences to the European economies 200 years ago). Furthermore, agriculture and land reforms take place at the countryside, where the farm holdings are scattered. Hence, the von Thünen model, emphasizing geographic aspects and the peculiarity of land, appears to be a fruitful tool in analyzing development policy, especially in the context of land reforms.

9.2 The Model

Consider the basic model of Chapter 7. However, let us modify the production technology in sector 1 towards a model that is basically rooted in The Isolated State by Johann H. von Thünen.

Consider, for simplicity, that each land property can be described approximately by a

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5The International Water Management Institute has projected that by 2025 large regions of the earth will experience severe water scarcity [RUTTAN (2002), p. 171].

6One must also take into consideration the value of the land for the ecosystem, which is only partly covered by economic land prices. Hence, if a land reform should establish a sustainable development this definitely includes not just an isolated economic, but also an environmentally sustainable development. However, so far there exists no applicable method to find the correct eco-price of land, alone on grounds of the high complexity of the ecosystem and the therefore still insufficient knowledge of its functioning.

7Paul Samuelson emphasizes in his tribute-to-von-Thünen article (cf. SAMUELSON (1983)) that Johann Heinrich von Thünen “... not only created marginalism ... but also elaborated one of the first models of general equilibrium ...” (p. 1468) in his The Isolated State, and that he determined wages and rents before David Ricardo, Edward West or Robert Malthus. On page 1469, Paul Samuelson perceives that in von Thünen’s work the “... primitive implicit marginalism involved in classical Ricardian rent theory graduates into neoclassical marginal productivity.”

8Besides the references already cited above, see HARTWICK AND OLEWILER (1997), chapter 2. The SAMUELSON (1983) article unfortunately displays, from my point of view, mistakes at least in Equations (16), (17), (27) and (28) [Section Mathematical Derivation]. The correct final term for (16) is

\[
\frac{1}{f_0} u^*[1, p_1^0] = \frac{1}{w_1(r)} u^* \left[ \frac{\exp((a_0 + a_1)r)}{p_1^0}, 1 \right]
\]

---
concentric circle. The farm is located exactly in the middle of this circle. The quality of a single land property is homogeneous, while different parcels may be of heterogeneous soil quality. Different sizes of landholdings are represented by a different radius of the circle, which we denote by \( r \).

As each farm is endowed with homogeneous soil quality, the harvest is equally distributed over the landholding. To sell the output it has to be transported to the local sales market. We denote the distance between the farm of a household \( i \) and this sale market by \( d_i \). The transportation costs per unit of distance and unit of output (in terms of the output good) are \( c \), which are equal for all households \( i \in [0, 1] \).\(^9\) Hence, the transportation costs for bringing the harvest to the output market run up to \( c d_i y_{it}^1 \).\(^10\)

The structure of the model is illustrated in Figure 9.1.

As the soil of a single farm is of homogenous quality, the output per land unit is the same everywhere. Let the output per square meter of soil at the farm of household \( i \) in and consequently (17) is

\[
w_1(r) = f_0^* \left[ \frac{\exp((a_0 + a_1 r))}{p_i^*}, 1 \right].
\]

Therefore, Eq. (27) and (28) are also different.

\(^9\)Obviously this is a simplification, since transportation may require differing levels of effort. One instance is when one farmer has to transport goods up steep slopes while another has not to.

\(^10\)An alternative way to model the transportation costs is the iceberg model introduced by Samuelson (1954). This transportation pattern is also used in Samuelson’s JEL article from the year 1983, cited above. The idea was already noted by von Thünen himself: when, for instance, grain is moved by oxen, these oxen will eat part of the grain on the transport. Like an iceberg melts away, the gross output is lowered by this particular type of transportation costs, which can be described by an exponential function. In our model, the transportation costs would be given by \( \exp(c \cdot d_i) \cdot y_{it}^1 \).
period $t$ depend on the input of human capital per square meter soil and be given by:

$$A_{1t}^1 \left( \frac{\lambda_{it}}{n_{it}} \right)$$

(9.1)

where $A_{1t}^1$ is a technology parameter that represents the particular soil quality of the farm, and that may be contingent on period $t$.\textsuperscript{11,12} Though child labor plays a major role in this thesis, variable $e_{it}$ always disappeared in our formal analysis in Chapter 7, for the very reason that the land reform works toward eliminating child labor. Therefore, we already neglected child labor in the term for the output per square meter (and thenceforward), but keep in mind that the general form is $A_{1t}^1 [(\lambda_{it} + (1 - e_{it}) \gamma) / n_{it}]^\alpha$. The gross production function is given by

$$A_{1t}^1 \lambda_{it}^\alpha n_{it}^{1-\alpha},$$

(9.2)

which is derived in the appendix. Therefore, the overall production function has constant returns to scale. Equation (9.2) has been derived from plausible premises for agriculture and appears identical to (7.2). However, technology parameter $A_{1t}^1$ now additionally incorporates the particular soil quality of household $i$ and is thus household-specific and time-dependent.

We assume that gathering requires that the output has to be brought to a stable at the farm. Therefore, to arrive at the net production function we have to subtract the transportation costs. The intra-farm transportation costs are given by $cy_{it}^1$ (see appendix). That is, the intra-farm transportation costs increase in the size of landholding, because the output, and thus effort, increases. Using the output good as numéraire, the income of household $i$ in sector 1 can be expressed by:

$$w_{1it}^1 = A_{1t}^1 \lambda_{it}^\alpha n_{it}^{1-\alpha} \left[ 1 - c(1 + d_i) \right],$$

(9.3)

which is also derived in the appendix. It becomes evident that the household’s income decreases with the marginal transportation cost $c$ and also with the remoteness of the parcel of land to the output market $d_i$, and that, in addition to Chapter 7, the size of land has a second effect on the income which is negative, as production now involves longer distances for the peasants.

\textsuperscript{11}Compared to Chapter 7, we modify notation slightly by changing the sector index “1” from subscript to superscript.

\textsuperscript{12}In Friedman (2000), Chapter 5, example of a farmer, it is described how this information can be exactly determined by a combination of data of a sensor on a harvest machine, that is collecting the harvest quantity per square meter, with the position data of a GPS system. It follows that the farmer can obtain information about which square meter of soil needs how much water and which needs how much dung, respectively nitrates, \textit{etc. pp.} to optimize production.
9.3 Land Reforms without Land Market Access

The direct consequence for the land reform described in Chapter 7 is obvious. Referring to land transfers, Equation (9.3) tells us that the income augmenting effect of additional land, \( ceteris paribus \), is not just reduced by decreasing marginal returns, but also by increasing required efforts in gathering. Furthermore, the individual quality of the land, represented by \( A_1^{it} \), determines the household’s income, which also has to be taken into account when the size of a particular land transfer is decided upon.

The thresholds \( c^S \) and \( c^a \) stay at the same level for all households, but it turns out that the households need to take more effort to earn these threshold consumption levels. This additionally required effort varies from household to household depending on geographic location and soil quality. The critical thresholds \( n^a(\lambda_{ut}) \) and \( n^S(\lambda_{ut}) \) become individual-specific even for an identical level of human capital. The decisive function for land transfers \( n^a(\lambda_{ut}) \) changes to:

\[
n^a(\lambda_{ut}, d_i, A_1^{it}, c) = \left( \frac{c^a}{A_1^{it} \lambda_{it}^\alpha [1 - c(1 + d_i)]} \right)^{1/\alpha} \tag{9.4}
\]

The required land transfer \( n^a(1) \) changes in the transportation cost variables \( c \) and \( d_i \), and in the quality of soil (level of \( A_1^{it} \)). Compared to Chapter 7, \( n^S(\cdot) \) and \( n^a(\cdot) \) increase and therefore the education of a society requires more time.

It becomes clear that for parcels of very low quality (low \( A_1^{it} \) and/or high levels of \( d_i \)), the necessary land transfers tend to become quite large or, at the extreme end, infinity. This demonstrates that parcels that are far away from market places and that are not endowed with a sufficient infrastructure (and thus are simply not usable for competitive realization of income) shall be excluded from land reform transfers. All plots of land with \( d_i \geq \frac{1-c}{c} \) lead to income \( w_{1it} \leq 0 \). Hence, soil in distance \( \frac{1-c}{c} \) represents the marginal soil and is not usable for land reform transfers, because \( w_{1it} < c^a \) for all sizes of land \( n_{it} \).

Referring to the periodical expropriations, we obtain:

\[
n^a_{it} = \begin{cases} 
\left( \frac{c^a}{1-c(1+d_i)} \right)^{1/\alpha} \left[ A_1^{it} \left( \sum_{k=0}^{t} (zh(1))^k \right) - A_1^{i(t-1)} \left( \sum_{k=0}^{t-1} (zh(1))^k \right) \right]^{\alpha-1} & \text{if } a_t(i) = 1 \\
\left( c^a / \left( [1-c(1+d_i)]A_1^{it} \left( \sum_{k=0}^{t-1} (zh(1))^k \right) \right) \right)^{\frac{1}{\alpha}} & \text{if } (a_t(i) = 0 \text{ and } a_{t-1}(i) = 1) \\
0 & \text{else}
\end{cases} \tag{9.5}
\]
That is, disadvantages in location $d_i$, the soil quality $A_{it}^1$, and the transportation cost factor $c$ are relevant for the first land gift $n^a(1)$ as well as for the periodic expropriations. In Section 9.5, we will demonstrate that also changes in soil quality have to be taken into account and not just changes in the level of human capital.

Finally, our geographic extension to our former model emphasizes that it is likely that when $n^a(1)$ is very big, a family alone will, in practice, not be able to run the farm, but would require additional labor from the labor market. This is especially the case as the policy wants to ensure full-time schooling for the children, so that only the parents will work on the farm (beside the work that the children might do after school and homework). This would involve further production costs that have to be covered by the family-farming earnings.

### 9.4 The Case with Land Market Access Revisited

Let us now extend our model to open land market access of the beneficiaries. Household $i$ faces a land market price of $q_{it}$. Maximizing farmer $i$’s income given by Equation (9.3), we obtain:

$$n^d(q_{it}, c, d_i, A_{it}^1, \lambda_{it}) = \left(\frac{1 - \alpha}{q_{it}} A_{it}^1 [1 - c(1 + d_i)]\right)^{\frac{1}{\alpha}} \cdot \lambda_{it}$$

(9.6)

with

$$\frac{\partial n^d_{it}}{\partial \lambda_{it}} > 0, \quad \frac{\partial n^d_{it}}{\partial q_{it}} < 0, \quad \frac{\partial n^d_{it}}{\partial c} < 0, \quad \frac{\partial n^d_{it}}{\partial d_i} < 0, \quad \frac{\partial n^d_{it}}{\partial A_{it}^1} > 0.$$

That is, transportation cost, distance to markets, and soil quality influence also the land demand: a farmer extends land demand, if the individual skills increase, if the land price decreases, but also if the transportation costs diminish, and if the distance to the market diminishes, or when the soil quality improves.

Coming to the migration decision, the migration threshold $\tilde{q}$ changes to:

$$\tilde{q}_{it} = (1 - \alpha) \left[1 - c(1 + d_i)\right] \left\{\frac{\alpha + c(1 + d_i)(1 - \alpha)}{A_{it}^2} \left(\frac{A_{it}^1}{A_{it}^2}\right)^{\frac{1}{\alpha}}\right\}$$

(9.7)

Though the incentive to change sectors remains independent of the individual level of human capital, the particular height of the land price that produces a migration incentive is individual-specific, because it is dependent on $d_i$ and $A_{it}^1$. Therefore, in contrast to Chapter 7, in our new setting, a situation in which all households alike have an incentive to migrate does not occur. Referring to comparative statics, it is obvious that $\tilde{q}_{it}$ rises when the soil quality improves. We also find:

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Lemma 9.1
An increase of the costs \( c(1 + d_i) \) lowers the migration threshold, that is:

\[
\frac{\partial \tilde{q}_{it}}{\partial c(1 + d_i)} < 0
\]

Consequently, if we define Chapter 7’s variable \( A_1 \) as the average soil quality parameter, one can prove:

**Proposition 9.1**
Suppose \( A_{1t} = A_1 \). Then,\(^{13}\)

\[
\tilde{q}_{it} < \tilde{q}
\]

as long as \( c(1 + d_i) > 0 \).

The proofs are presented in the appendix. The result is obvious: increasing costs lower, *ceteris paribus*, farmers’ income relative to the potential sector 2 income. If we now allow \( A_{1t} \) to deviate from \( A_1 \), we have to distinguish two cases. If \( A_{1t} \) deviates downward, Proposition 9.1 is reinforced. However, if on the contrary it deviates upwardly, an effect occurs in the opposite direction. The better soil quality compensates, at least partly, for the costs, so that we cannot exclude \( \tilde{q}_{it} > \tilde{q} \). We infer, nevertheless, that the general problem revealed by Proposition 7.7 and Corollary 7.2 continues to exist, because the level of human capital is not decisive for the migration decision.

We already mentioned that including bequests might reduce the danger of access to the land markets, for the revenues of land sale might partly be transferred to the child; the descendant’s income would not be decreased as strongly. Paying respect to geographic aspects, we find another restriction: it is plausible to assume that a farmer searching for additional land will only buy land near his farm, because broadly dispersed land property involves higher costs via \( d_i \) and \( c \).\(^{14}\) Hence, a land market transaction might only occur if there are two farmers who come to terms within one and the same neighborhood.

Since the soils and the geographic location of different parcels can differ greatly in quality, the particular land prices per square meter will vary greatly from parcel to parcel and from region to region. There is no homogeneous equilibrium price for land, but rather a variety of square meter prices. As a consequence, predicting land prices becomes an involving, complex issue. Being aware of this, it follows that excluding beneficiaries from access to land *sell* markets is a policy option that definitely prevents

\(^{13}\)\( A_1 \) and \( \tilde{q} \) refer to Chapter 7.

\(^{14}\)Referring to the land market, a distance threshold should exist at which the potential land demander is indifferent between buying or leaving it.
adverse sales, whereas other options involve this danger. Nevertheless, there still seems to be no reason to prevent the access to land markets for purchasing parcels, since it improves efficiency. Moreover, when only squires are allowed to sell land, land purchases only could happen between beneficiaries and squires. It follows that permitting land purchases of beneficiaries further lowers the inequality in land ownership between the poor and the squires.

### 9.5 Overall Sustainable Land Reforms

In practice, the sustainability of the success of land reforms depends crucially on the sustainability of agriculture production (or, in general, of land-based businesses). This sustainability, in turn, is endangered by the loss of soil, degradation, water scarcity, salinity, pests, pathogens, hosts, and climate change. The land reform beneficiaries that must face these challenges need the knowledge that they can use fertilizers to compensate for the loss of nitrogen, that water logging and salinity result from excessive water use and poorly designed drainage systems, and so on.\(^\text{15}\) Since a large number of beneficiaries are not well-informed about agriculture in general, and about these geographic-ecological aspects, land reforms might even enforce these problems. By assuming that the productivity of land stays constant over time we neglected these important issues. In reality, this implicitly requires a sustainable form of agriculture. If the soil of the land given to beneficiaries is not run carefully, the content of nutrients of the soil will diminish over time and the soil may even become useless for agriculture.

Some forms of land require a very sensible form of land use. One example is the rain forest. Rain forest clearing for agriculture production involves the problem that in practice one can observe a quick decline of revenue; rain forest soil (often) displays only a very thin fertile stratum, which erodes, amongst other reasons, due to rain just after a few years of cultivation, once the trees have been removed.\(^\text{16}\) In our model, this corresponds with an \(A_{it}^1\) that diminishes from period to period. Consider, for instance, that the current way of cultivation is such that the soil quality looses fraction \(a\) of its current quality in each single period, once clearing is implemented. That is:

\[
A_{it(t+1)}^1 = (1 - a)A_{it}^1
\]

for all \(t \geq \bar{t}\), where \(\bar{t}\) is the period in which beneficiaries receive the plot of land. This corresponds with a quality loss at constant rate \(a\). Solving this difference equation we

\(^{15}\text{Cf. Ruttan (2002), p. 170, 171; Murgai, Mubarik, and Byerlee (2001).}\)

\(^{16}\text{Cf. Bremer (1999), Chapter 11.}\)
arrive at:

\[ A^1_{it} = (1 - a)^{t - \tau} A^1_{i\tau} \]  \hspace{1cm} (9.8)

for all \( t \geq \tau \), where we assume that the period of clearing is equivalent to the period in which cultivation starts at the corresponding parcel; in this period soil quality is \( A^1_{i\tau} \). Consequently, Equation (9.4) can be rearranged to:

\[
n^a(A_{it}, d_i, c, a, t) = \left( \frac{c^a}{(1 - a)^{t - \tau} A^1_{i\tau} \left( \sum_{k=0}^{t-\tau} z h(1)^k \right)^a \left[ 1 - c(1 + d_i) \right]} \right)^{1-\alpha} \]  \hspace{1cm} (9.9)

Obviously it is possible that the degradation of the soil cannot be compensated for by human capital accumulation. Thus, soil degradation might not only slow down the land redistribution possibilities over the course of the land reform, it might even force the government to transfer additional land gifts to beneficiaries to educate the society. Therefore, the former beneficiaries cannot support other poor with parts of their original land gifts: the land reform project would collapse. Therefore, our model is also able to address environmental and geographic issues.

So if countries with (huge) rain forest areas – for instance in South America – decide to use rain forest for land distribution this can have massive negative effects. If the government deforessts large areas of the rain forest to achieve free land for a land distribution, or the beneficiaries decide to do so, such a land reform will likely fail, since it might neither establish an economically nor ecologically sustainable development. Therefore, one cannot separate the economic development from the ecological. Adverse geographic and ecological effects will, in the end, be also harmful for the long-term economic performance.

Furthermore, the distributed land given to beneficiaries might become useless for future generations, for some time, because of the soil’s depletion. Deforestation of the rain forest may also destroy a whole ecosystem. The bio-diversity decreases as the forest has been the habitat for a multitude of species.\(^{17}\) This also represents a loss of natural resources.\(^{18}\) The whole water circulation system changes and may exhibit a sustained disturbance, so that even if the soil is dealt with carefully, the conservation can fail alone on the grounds of missing water. In turn, these negative effects can negatively influence other industries and people that originally were not involved in the land reform (negative externality). Eventually, the land reform may fail and the net effect of the reform might even be negative.

\(^{17}\)Especially in the rain forest there are species that only exist in certain areas of the forests.

\(^{18}\)For instance, a lot of species are (and might be even more in the future) important for discovering and developing new medicines.
Finally, rain forests are globally essential in binding \( CO_2 \) and producing \( O_2 \), and hence un-renounceable for the wellbeing of humankind. So one should be quite careful in changing existing landscapes for reasons of land reform. Experiences with such projects teach us that we (still) do not have enough knowledge about nature to understand precisely what is going on.\(^{19}\) This is in line with the result of geographers that emphasize that revenue and the realizability of sustainable development strongly depends on the way of cultivation.\(^{20}\) A sustainable land use requires a rotation of plots of land, where certain plots are cultivated, while others are not used in order that the not used plots recover (see Bremer (1999), p. 197, 205), the water resources must not be used too strong and erosion of soil has to be prevented by applying certain techniques (p. 210). After all, Bremer (1999) stresses that final conclusions about the development of particular types of soil are very difficult due to insufficient knowledge.

### 9.6 Conclusions

Building upon the work of the Classic, we elaborated on a von Thünen land reform model, taking account of transportation costs, gathering efforts, and heterogenous soil quality. Moreover, we allowed for the soil quality to deplete over the course of time.

We have seen that the land transfer that is required for the success of the land reform depends on a variety of additional, so far neglected, determinants. The land gift has to take into consideration heterogenous soil quality, sales market access, transportation and harvest cost, climate conditions, water supply, and the like. These particularities have also to be paid attention to in periodic land redistribution. Paying attention to these aspects we have seen that, contrary to Chapter 7, a single common market price for land does not exist, but rather a variety of prices. Prohibiting land sales to land reform participants remains an essential building block of a successful land reform.

Our spatial model also identifies another reason why households may be caught in poverty traps. In our model, we have identified a marginal soil. This remote soil bears no income. Therefore, owners of remote farms are caught in the poverty trap, since the transportation cost of bringing goods to the remote markets is so expensive that these farmers cannot earn enough income to send the children to school. We conclude, similar to our analysis in Chapter 6, that infrastructure investments which lower \( c \) and an economic policy that leads to the arising of new markets in remote areas, so that \( d_i \) diminishes, are promising tools to fight poverty.

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\(^{19}\)For environmental aspects of poverty and development see, e.g., Barbier (2002).  
In this context, we demonstrated how important a sustainable production form is and that adverse behavior of beneficiaries or the government can cause the failure of the land reform. Using rain forest land for a land reform bears the risk of failure, since the fertile stratum of soil is very thin and uneducated land reform beneficiaries possibly will not apply sustainable production techniques. We have discussed further risks that demonstrate that a land reform policy also has to focus on an ecological-sustainable development to reach an economic-sustainable overcoming of poverty, as both forms of sustainability intertwine. Overall, we emphasized how important it is to transfer knowledge about agriculture and ecology to land reform beneficiaries. Moreover, a lot of interdisciplinary research is necessary to understand the interaction of the economic and the ecological sphere of land reforms. What dynamic development of the productivity of the soil should we expect, given a certain geographic class of soil? What has to be done to achieve sustainability? What are the economic incentives of land reform beneficiaries that should apply techniques that allow for a sustainable production? How can we change the incentive scheme of beneficiaries so that they actually escape poverty in an overall sustainable way?
Chapter 10

Conclusions

"So eine Arbeit wird eigentlich nie fertig, man muß sie für fertig erklären, wenn man nach Zeit und Umständen das mögliche getan hat."

–JOHANN WOLFGANG VON GOETHE (1749-1832)

10.1 Contribution of the Thesis

The thesis makes a contribution to the question “How to overcome poverty traps by education”. We identified a positive inter-generational externality of education. By increasing individual incomes, education can mitigate the burden of imperfect capital markets in the course of time. Part I of the thesis adds a comprehensive dynamic analysis of subsidy policies to educate a society. This investigation covers aspects ranging from corruption, geography and school quality to political economy. In particular, conditional subsidies are often discussed and practiced, but, so far, they have not been analyzed theoretically (at least in the context of under-developed economies). We distinguish and compare different types of conditional subsidies. Moreover, we have demonstrated how important political economy issues are: the best design of a development policy will only bear fruits if this policy can be implemented within the current political system. This essential aspect is often neglected in development economics. We have derived constitutional rules that allow to attain human capital accumulation and growth. Finally, we have shown that a society might have to run through a pre-subsidization phase, since subsidizing poor households cannot be successful under certain circumstances.

Then, besides HOROWITZ (1993), Part II represents the only dynamic analysis of land
reforms. While Andrew Horowitz solely elaborates on the maximum amount of land that can be redistributed without social conflict, we, in contrast, offer a detailed investigation of land reforms aiming at overcoming poverty. We deduced the characteristic of a successful land reform, the resulting economic development (comprising issues like induced transition processes, equity, labor market effects etc.), and the consequences of beneficiaries’ open access to land market. In political debates, land reforms for the most part are seen as a means of lowering political pressure and to improve equity in an environment where the distribution of land ownership is considered as “unjust”. Our land reform approach, in contrast, suggests that land reforms could also be used as an effective tool to fight poverty, to educate a society and to foster economic growth. We highlight that there might exist an important nexus between human capital accumulation and land reforms that was disregarded within political and academic discussion on land reforms, so far.

Summarizing, the thesis makes the case that human capital formation within the group of the poor is an essential building block of a strategy that aims at abolishing poverty, underdevelopment, and child labor. In history, no country has become rich without being educated and skilled,\(^1\) and Gyfason and Zoega (2003) found evidence that education pushes growth.\(^2\) Hence, if underdeveloped countries are not able to educate their societies, they may, ceteris paribus, remain in poverty traps and the gap between them and the rich economies will rise in the future. However, it is clear that human capital accumulation is only a necessary pre-condition for overcoming poverty, but not a sufficient one. In this context Easterly (2002), p. 73, states that “If the incentives to invest in the future are not there, expanding education is worth little.” If e.g. no technology is in use that requires skilled, educated workers, then education cannot foster growth. Therefore, as we have argued in the thesis, the governments have to make further efforts to win the fight against poverty and backwardness.

Whatever policy is chosen, merely maximizing the enrollment rates is not advisable, but the policy maker has to maximize the enrollment rate subject to the constraint that the single transfers – be it money or land – must be sufficiently big in size to be able to snatch away the beneficiaries from the suction of the locally stable poverty trap. Otherwise, the supported households will enjoy only a temporary improvement before they sink into poverty again. This explains the tendency of poverty to persist and why, in the past, so many efforts in fighting poverty failed after short-term successes. The “big push” of a policy intervention has to produce a self-supporting education

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\(^1\) Cf. Easterly (2002), p. 84.
\(^2\) Other studies, as Pritchett (2001), could not find any positive association between growth in education and growth of per-capita income. Of course, it is also possible that economic growth pushes education.
and growth process. In the context of the Millennium Goal concerning education, the success of educational efforts is monitored by the net primary enrollment ratio.\textsuperscript{3} This indicator does not incorporate any aspect that would control for the quality of schooling. Hence, we have to keep in mind that this measurement method can be highly misleading. Moreover, our analysis demonstrated that, given scarce resources, temporary inequality among the poor is therefore unavoidable, irrespective of whether subsidies or land transfers are used.

\section*{10.2 Final Remarks}

The results of the single chapters were comprehensively explained in the final sections of the respective chapters. There, we also identified and discussed open issues and potential future research tasks. What we will do now is taking a step backward to adopt a broader perspective on all issues. This will allow us to arrive at conclusions on how single results of the chapters intertwine. In doing so, we will undertake a comparison of subsidization and land reforms, based on our results of the thesis. Having done this, we will outline interesting ideas for future research in a wider context.

\subsection*{10.2.1 Subsidies and Land Reforms}

In our models, both subsidies and land gifts induce economic growth via human capital accumulation due to an income increase. Therefore, at first sight, one might ask why one should choose the laborious path of land reforms, when subsidies directly raise income. Moreover, land transfers seem to be similar to unconditional lump-sum subsidies, which compared to conditional ones, have been proved to be inefficient. However, we have seen that land reforms have additional effects that subsidies cannot produce. Land property can be used as collateral and thus might enable the beneficiaries to raise loans. As a consequence, highly efficient investment opportunities, that were, due to a lack of access to credit, not realized in the past, can be undertaken. This may improve the agricultural and forestry productivity additionally\textsuperscript{4} and generates (accelerated) economic growth. Furthermore, if the hypothesis that large-scale production of squires is less productive than producing with smaller scales actually holds for most agriculture and forestry goods, then land redistribution might further increase efficiency and output by dividing up large farms into smaller units.

\textsuperscript{3}The net primary enrollment ratio is the ratio of the number of children of official school age (as defined by the national education system) who are enrolled in school to the population of the corresponding official school age. Cf. <http://www.developmentgoals.org/Definitions_Sources.htm>.

\textsuperscript{4}Additional to the positive effect on human capital formation that does improve productivity.
Although conditional subsidies produce the income effect required for human capital formation more easily and are more cost-effective, all these additional positive effects of land transfers cannot be attained by subsidies. Hence, there exists a trade-off, wherefore an evaluation of land reforms in comparison to subsidies is not trivial. In the remainder of the section, we combine our results of Part I and II in order to draw conclusions from an overall view. This will enable us to gain new insights and to find first, preliminary results with regard to the comparison of the two policy options.

First of all, applying our results of Chapter 4, conditional land transfers are equally possible as conditional subsidies. Therefore, to educate a society, it is best to use binary conditional land gifts, that is, the government offers a plot of land to a household, but the household only receives the plot, if it agrees on the targeted level of schooling for its child. If the household does not abide by the agreement, the plot of land can be dispossessed again.\(^5\) It follows that the disadvantage of land reforms in producing education, outlined above, is weaker.

Our political economy chapter of Part I on redistribution via tax-and-subsidy schemes (Chapter 5), in principle, also holds for our land redistribution scheme proposed in Part II. In the context of land reform, the agenda setter proposes land redistribution schemes and the constitution defines the majority required for adopting a proposal. Then, analogously to the subsidization case, self-interested agenda setters may have an incentive to expropriate former beneficiaries and squires excessively, namely, such that these fall back into the poverty trap. Consequently, to guarantee the success of the land reform, certain constitutional rules are required. For instance, a certain size of land ownership has to be protected from dispossession. This can be done directly by an “allowance” in terms of land ownership or by an accordingly modified flexible majority rule that demands unanimity if adverse expropriations should be carried out. As the income stream, generated by land gifts, flows not only for one period, as subsidies do, we do not need the repeated agenda setting rule in the framework of a land reform. This might be an advantage of land reforms compared to subsidies.

However, there is an interesting similarity between land reforms with land market access and subsidization policies, so that the latter only holds for land reforms without open access to land market. We demonstrated that land market access may lead to individually optimal migration decisions that are socially detrimental. If uneducated beneficiaries of a land reform migrate from their farm (in rural areas) to town, they may lose the income stream generated by the plot of land. Then, their future income is solely determined by their level of human capital. That is, they are in the same situation

\(^5\)Given that plots of land can be dispossessed if the agreement is not met, the advantage that land gifts can serve as collateral in the credit market is weaken.
as subsidy beneficiaries are. It turned out that, after one-time subsidization or after such a migration of land reform beneficiaries, supported households will fall back into the poverty trap if they are not endowed with a high enough level of human capital, that is, if the human capital technology is not sufficiently productive \( (zh(1) + 1 < 1) \). While this problem can easily be solved in case of subsidization by repeating aid payments, a corresponding method in the case of land reforms, that would work in practice, is more difficult to find.\(^6\) Hence, we concluded that a prohibition of land sales of beneficiaries is necessary if the society as a whole should escape poverty. This ban substitutes the repeated agenda setting rule of Part I in the context of a land reform with land market access.

Applying our results of Chapter 6 to the land reform analysis, we can conclude that additional investments (as education in husbandry and business management skills, providing credit facilities for investments, investments in the infrastructure etc.) can also improve the effectiveness of land reforms. This conclusion is supported by the literature on land reforms, as mentioned in our land reform chapters. In a case study presented at the 2004 World Bank conference on poverty reduction in Shanghai, an Indonesian rice farmer, for instance, claims that the building of a road reduced his cost of transporting rice to the market by some 50%.\(^7\) This example underlines the relevance of our spatial model that incorporates transportation costs (Chapter 9). Moreover, it is clear that investments in schooling quality and infrastructure as well as mitigating the extent of corruption etc. are also important in the setting of a land reform, since the household’s decision problem is the same as in the context of subsidization. If the pre-conditions for human capital formation are not yet fulfilled, the government ought to run through a pre-land-reform phase. That is, if most of the children of beneficiaries would not be able to send the child to school because there are no schools in the region, then the government, in a first step, should provide schools in sufficient number. It follows that, basically, our analysis in Chapter 6 also holds in the land reform context.

Comparing subsidies and land reforms, it is interesting to recognize that if a subsidy can be used for land purchases, then this covers exactly the idea of land-market assisted land reforms. Similarly, with a plot of land as collateral, land reform beneficiaries are able to raise loans, that is, they receive, like subsidy beneficiaries, money. Hence, there is a certain duality between subsidies and land reforms.

Going beyond the scope of our models, it is conjecturable that beneficiaries of land reforms face more problems than beneficiaries of subsidy policies do, because family-

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\(^6\)Of course, subsidies would work, which would be a mixture of land reform and subsidization.
farmers are independent contractors, while subsidy receivers often are employees, for instance, day-laborers. Farmers have to cope with all kinds of entrepreneurial problems and risks (for instance, the issues identified in Chapter 9, the risk of bad crops, the necessity to build up a constituency, to find component suppliers, to learn about farming and running a business etc.), while day-laborers “only” face the risk of becoming unemployed. This makes the success of land reforms more fragile than the success of subsidization policies, which might be an advantage of subsidization.

Moreover, land reforms should not be chosen by countries, in which agriculture and/or forestry are not major sectors of the economy or if the economy is specialized in agricultural and/or forestry goods with an increasing-returns-to-scale characteristic. Under these circumstances, subsidizing the poor is the better choice. However, in times of great or even hyper-inflation, the value of subsidies decreases dramatically from one day to another. Thus, subsidies cannot provide the required income increase and the subsidy would have to be adjusted every day. In contrast, the inflation-invariance of the real value of land prevents this adverse effect in the framework of a land reform.

Finally, to choose the suitable instrument, it is also decisive whether a land reform or a subsidization scheme is feasible, that is, whether a country actually disposes of sufficient (utilizable) land or financial resources. In respect thereof, a crucial potential drawback of land reforms is that, following for instance Bell (2003), Chap. 14.6, and Schrader (2004), land reforms might only be feasible after a change of the political system. In “normal times”, a big-sized redistribution of land via a land reform appears (very) difficult. Therefore, as long as there are no times of change and revolution, subsidization policies might be to handle more easily or even the only realistic remaining option (at least among the two options investigated in the thesis). However, a big-sized redistribution via a tax-and-subsidy scheme does not have to be less difficult, because, in the end, it does not matter whether an individual loses wealth in terms of land or in terms of income. In both cases the losers will fight a big-sized redistribution. Additionally, in the case of subsidization, losers react by capital flight and tax fraud. In the framework of a land reform, landlords, e.g., cloud their real estate by pooling it with family members and other men of straw. Thus, realizing big-sized redistributions in a political process and applying it in practice is difficult, irrespective of the instrument chosen.

The latter case is trivial: parceling out big farms inevitably causes decreasing productivity and output. In the first case, the agricultural and forestry sectors cannot absorb many of the poor. Large-scale land allocations will cause agricultural and forestry goods prices, due to excess supply, to fall dramatically. Thus, the sectors would not provide the necessary income increases. If, on the other hand, there is only small-sized redistribution of land, in order to prevent the decay of prices, educating society will take a long time.
10.2.2 Future Research Needs

The thesis offers multiple avenues for future research. A lot of the yet open issues, that are closely related to the interest of the thesis, were thoroughly discussed in the concluding sections of the single chapters. Therefore, here we only pose questions of research that are related to the thesis in a broader sense.

First of all, our comparison of subsidies and land reforms revealed that the analysis of land reforms should be extended to include the credit market. Then the described additional positive effects of land reforms, that we have not focused on in this thesis, could be covered. A respective extension of our welfare analysis in Chapter 3 then should allow for an accurate, final comparison of subsidization and land reform.

Moreover, we have demonstrated that there are interactions between rural and urban areas that are important for economic development, namely that adverse rural-to-urban migration can foil the success of land reform. However, there are also mutually beneficial linkages. Increasing urban markets, for instance, provide incentives to rural entrepreneurs (as farmers) to produce more and to achieve higher income. If rural investment increases, in turn, the demand for physical capital rises. Since this capital is mainly produced in urban areas, this has positive feedbacks on towns. Therefore, economic development in rural areas might reinforce the development in urban areas, and vice versa. Analogously, stagnation in one area can hamper economic development in the other. It is important that future research further contributes to our understanding of this economic rural-urban interaction, to develop an integrated model and to deduce a balanced and mutually supportive policy.\(^9\)

Another interesting extension would be to elaborate on the decision problem of the government facing a conflict of interests. For instance, leaders of developing countries might benefit from the population’s poverty. One example is described in William Easterly’s monograph *The Elusive Quest for Growth*:\(^10\) as long as the population is poor, the political leaders receive foreign aid payments, which they can utilize to enrich themselves. Another example could be the situation of the leaders in the Middle East. If they educated their societies and attained economic growth for big parts of the population, then people would probably demand more political participation. Hence, governments might prefer not to fight poverty and ignorance. One fruitful path to follow, therefore, is to deepen our understanding of the nexus between development efforts, corruption, fraud, and non-cooperation of political leaders and bureaucrats.

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\(^9\)The topic of the United Nations’ World Habitat Day 2004 also pointed to this direction: *Cities – engines of rural development.*

\(^10\)Cf. EASTERLY (2002).
There exists yet a further important conflict of interests. As the political leaders of backward countries might have no incentive to fight poverty, also the Western World has to weigh trade-offs. The protectionism of Northern America and the European Union in international trade hinders the economic development of developing countries. The reason for protectionism is to prevent a loss of jobs and influence that would occur in the Western World in case of free trade in particular sectors. The problem is well-known but the situation has not changed significantly for years. This might suggest that the current status quo is a kind of international political steady state, which strengthens the local stability of poverty traps. Developing a theoretical model that explains the persistence of protectionism more precisely and links trade policy with development policy might enable us to find policies that allow for the transition from this “protectionist steady state” to a steady state without protectionism that both sides can accept.

* * *

We hope to have extended the understanding of poverty traps and to have proposed new, promising ways out of backwardness. Of course, our broad discussions revealed a lot of yet open issues and, in practice, a lot of difficulties arise. Consequently, decades of efforts to overcome poverty have not achieved the anticipated breakthrough. However,

„The probability that we may fail in the struggle ought not to deter us from the support of a cause we believe to be just. “

–ABRAHAM LINCOLN (1809-1865)
Appendix A

Appendix to Chapter 3

A.1 The Choice of the Adult’s Instantaneous Utility Function

Referring to altruism, the dynasty-approach in Barro (1974), where all generations are effectively connected, is very common. In the dynasty model, utility is described by the functional form $u_t = u(c_t, u_{t+1})$. That is, the utility function of the child is substituted into the adult’s utility function and the utility function of the child, in turn, incorporates the utility function of the grand-child, and so on. Thus, a single adult while directly caring for her own consumption and the utility of her child, indirectly also takes into account the well-being of all of her descendants. We do not think that the dynasty model is appropriate for our task. In practice, parents do not know the utility perception of their descendants. They are only able to care about the consumption possibilities of their children, so that we, for instance, arrive at $u_t = u(c_t, c_{t+1})$. But if this is the case, the connection of all generations disappears, and will only prevail if we directly assume that an adult cares not only for the consumption level of the child, but also for that of all descendants: $u_t = u(c_t, c_{t+1}, c_{t+2}, \ldots)$. It is unlikely that decisions of poor parents are determined by considerations concerning future generations beyond their children and grand-children. If generations beyond affect decisions, we believe that these additional considerations are negligible. The poor in developing countries, which we address, live from hand to mouth, so that we assume a time horizon that comprises only the own child. As consumption $c_{t+1}$ is driven by period’s $t + 1$ full income $\alpha(\lambda_{t+1} + \gamma)$, we believe that the appropriate sort of altruism is represented by utility function $u(c_t, \lambda_{t+1})$. That is, a parent values the size of the child’s budget set as an adult.

However, $\lambda_{t+1}$ is determined by $e_t$ via the technology of human capital. Hence, we
Appendix A. Appendix to Chapter 3

simplify our analysis by modifying the utility function to \( u(c_t, e_t) \). Of course, we are aware that using \( u(c_t, e_t) \) means that the adult’s education decision for the child is independent of the resulting level of human capital, which is driven by the school quality, i.e. by \( h'(e_t) \). Consequently, education time \( e_t \) is per se utility augmenting, no matter how much human capital it produces. It is clear that the willingness to send a child to school may depend on the (subjective) expectation of how much education school attendance brings. Thus, the quality of schools may be an important signal for the adult’s decision. This aspect is neglected by \( u(c_t, e_t) \). Moreover, our approach simplifies the analysis because in case of \( u(c_t, \lambda_{t+1}(e_t)) \), depending on the curvature of the function \( \lambda_{t+1}(e_t) \), the second-order-conditions of the household’s maximization problem are not necessarily fulfilled. Nonetheless, as long as the technology of human capital remains unchanged, the qualitative statement of both functional forms is the same. Furthermore, these aspects do not have any effect on the qualitative results of the thesis, except in the analysis in Chapter 6. Therefore, we extend our approach there to cover all important effects.¹ In all other chapters, we will use the reduced form for reasons of simplicity. Finally, it is important to emphasize that our approach assumes that parents do not directly care for all future generations, but only for their children. Nevertheless, as we will see in the next section, today’s parents affect all future generations by their educational decisions.

A.2 Basic Approach

In this appendix we provide the basic solution to Lagrangean (3.2). We obtain the following first-order conditions:

\[
\frac{\partial L}{\partial e_t} = \rho \left[ (-\alpha \gamma) \frac{\partial u_t}{\partial c_t} + \frac{\partial u_t}{\partial e_t} \right] + \alpha \sum_{k=1}^{\infty} \rho^{t+k} \frac{\partial u_{t+k}}{\partial c_{t+k}} \frac{\partial \lambda_{t+k}}{\partial e_t} - \kappa_t + \nu_t \leq 0 \quad (A.1)
\]

with \( \frac{\partial L}{\partial e_t} \cdot e_t = 0 \quad \forall \ t \in [0, \infty] \)

\[
\frac{\partial L}{\partial \kappa_t} = 1 - e_t \geq 0 \quad \text{with} \quad \frac{\partial L}{\partial \kappa_t} \cdot \kappa_t = 0 \quad \forall \ t \in [0, \infty] \quad (A.2)
\]

\[
\frac{\partial L}{\partial \nu_t} = e_t \geq 0 \quad \text{with} \quad \frac{\partial L}{\partial \nu_t} \cdot \nu_t = 0 \quad \forall \ t \in [0, \infty] \quad (A.3)
\]

¹One could also incorporate the school quality effect in \( \frac{\partial u(c_t, e_t)}{\partial e_t} \), but this approach is “sloppy”.

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Appendix A. Appendix to Chapter 3

The marginal social benefit of education in period $t$ for future generations is

$$\alpha \sum_{k=1}^{\infty} \rho^{t+k} \frac{\partial u_{t+k}}{\partial c_{t+k}} \frac{\partial \lambda_{t+k}}{\partial e_t} \cdot$$

Thus there exists a positive externality of today’s education on the welfare of future generations. If the optimum involves $e_t \in (0, 1)$, i.e. $\kappa_t$ and $\nu_t$ are equal to zero, we find that social marginal revenue has to be equal to social marginal cost:

$$\frac{\partial u_t}{\partial e_t} + \alpha \sum_{k=1}^{\infty} \rho^k \frac{\partial u_{t+k}}{\partial c_{t+k}} \frac{\partial \lambda_{t+k}}{\partial e_t} = \alpha \gamma \frac{\partial u_t}{\partial c_t} \quad (A.4)$$

In the case that at $e_t = 1$ the social marginal revenue is still higher than marginal cost, we find $\kappa_t > 0$, and therefore $e_t = 1$ is welfare maximizing. Then the optimum condition is:

$$\frac{\partial u_t}{\partial e_t} + \alpha \sum_{k=1}^{\infty} \rho^k \frac{\partial u_{t+k}}{\partial c_{t+k}} \frac{\partial \lambda_{t+k}}{\partial e_t} = \alpha \gamma \frac{\partial u_t}{\partial c_t} + \kappa_t \rho^t \quad (A.5)$$

That is, the shadow price in period $t$ of an additional unit of time for schooling in period $t$, $\frac{\partial e_t}{\partial \rho}$, is the sum of the discounted value of the positive externality and marginal utility $\frac{\partial u_t}{\partial e_t}$, where $-\frac{\partial u_t}{\partial e_t} > 0$ represents the investment cost of schooling. In the case where $e_t = 0$ is socially efficient, we find $c_t = \alpha(\lambda_t + \gamma)$, $\nu_t > 0$ and $\kappa_t = 0$ in the optimum. Combining $d e_t = -\frac{1}{\alpha \gamma} d c_t$ and first-order-condition (A.1), we find that $\frac{d e_t}{\rho_t} \leq \alpha \left( \gamma \frac{\partial u_t}{\partial c_t} - \sum_{k=1}^{\infty} \rho^k \frac{\partial u_{t+k}}{\partial c_{t+k}} \frac{\partial \lambda_{t+k}}{\partial e_t} \right)$ holds in the social optimum. I.e., consuming full income improves welfare at least as much as when the child would attend school for the first unit of time.

A.3 Derivation of the Marginal Rate of Substitution

Given (3.1) and $d e_k = 0$ for all $k \neq t$ and $k \neq t + 1$, we obtain:

$$d W = \rho^t \left( -\alpha \gamma \frac{\partial u_t}{\partial c_t} + \frac{\partial u_t}{\partial e_t} \right) d e_t + \rho^{t+1} \left( -\alpha \gamma \frac{\partial u_{t+1}}{\partial c_{t+1}} + \frac{\partial u_{t+1}}{\partial e_{t+1}} \right) d e_{t+1}$$

Applying $d W = 0$ we arrive at the term given in Equation (3.10).

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2One can prove $\frac{\partial \lambda_{t+k}}{\partial e_{t+k}} = (\Pi_{k=1}^{n-1} h(e_{t+k})) z^n h'(e_t) \lambda_t$. Due to $z \in (0, 1)$ we obtain $\lim_{n \to \infty} z^n = 0$. Since $u_t$ is bounded from above, $\alpha \sum_{k=1}^{\infty} \frac{\partial u_t}{\partial c_{t+k}} \frac{\partial \lambda_{t+k}}{\partial e_t}$ is finite.
A.4 Derivation of the Marginal Rate of Transformation I

First of all, notice that:

\[ c_{t+1}(\lambda_{t+1}, e_{t+1}) = \alpha(\lambda_{t+1} + \gamma) - \alpha \gamma e_{t+1} \]  
\[ u_{t+1} = u(c_{t+1}(\lambda_{t+1}, e_{t+1}), e_{t+1}) \]  
\[ du_{t+1} = \frac{\partial u_{t+1}}{\partial e_{t+1}} \left( \frac{\partial c_{t+1}}{\partial \lambda_{t+1}} d\lambda_{t+1} + \frac{\partial c_{t+1}}{\partial e_{t+1}} de_{t+1} \right) + \frac{\partial u_{t+1}}{\partial e_{t+1}} de_{t+1} \]  
\[ \lambda_{t+1} = z\lambda_{t} h(e_{t}) + 1 \]  
\[ d\lambda_{t+1} = z\lambda_{t} h'(e_{t}) de_{t} \]  

The marginal rate of transformation concerning the untightening of the budget constraint (via mitigating the effect of an imperfect capital market) tells us how much investment \( e_{t+1} \) one saves if we invest one additional unit of \( e_{t} \) today, given the level of utility in period \( t+2 \) should be held constant. Using \( du_{t+1} = 0 \), \( \frac{\partial c_{t+1}}{\partial \lambda_{t+1}} = \alpha \), \( \frac{\partial c_{t+1}}{\partial e_{t+1}} = -\alpha \gamma \), and (A.10) in (A.8), we find:

\[ 0 = \frac{\partial u_{t+1}}{\partial c_{t+1}} \alpha z\lambda_{t} h'(e_{t}) de_{t} + \left( \frac{\partial u_{t+1}}{\partial e_{t+1}} - \alpha \gamma \frac{\partial u_{t+1}}{\partial c_{t+1}} \right) de_{t+1} \]

Hence we arrive at the first term on the r.h.s. of Equation (3.10): \( -\frac{de_{t+1}}{de_{t}} \bigg|_{dt+1=0} \).

A.5 Derivation of the Marginal Rate of Transformation II

Notice additionally that:

\[ \lambda_{t+2} = z\lambda_{t+1} h(e_{t+1}) + 1 \]  
\[ d\lambda_{t+2} = z\lambda_{t+1} h'(e_{t+1}) de_{t+1} + zh(e_{t+1}) d\lambda_{t+1} \]

The marginal rate of transformation concerning the technology of human capital tells us how much investment \( e_{t+1} \) one saves if we invest one additional unit of \( e_{t} \) today, given the level of human capital in period \( t+2 \) should be held constant. Therefore, we set \( d\lambda_{t+2} = 0 \) and arrive via (A.10) and (A.12) at:

\[ de_{t} = \frac{1}{z\lambda_{t} h'(e_{t})} d\lambda_{t+1} \]  
\[ -de_{t+1} = \frac{h(e_{t+1})}{h'(e_{t+1})\lambda_{t+1}} d\lambda_{t+1} \]

Accordingly we find the second term on the r.h.s. of Equation (3.10): \( -\frac{de_{t+1}}{de_{t}} \bigg|_{d\lambda_{t+2}=0} \).
Appendix B

Appendix to Chapter 4

In the following proof, we will fall back on the following trivial fact:

**Fact B.1**
\[ c^S = \alpha(\lambda^S + \gamma) > \alpha\gamma \]

**Proof of Proposition 4.4:**
In the interval \( \lambda_t \in [\lambda^S, \lambda^a] \), following Equation (4.17), the Marshallian is
\[ e^o(\lambda_t) = \frac{\alpha(\lambda_t + \gamma) - c^S}{2\alpha\gamma}. \tag{B.1} \]

(a) Via Equation (B.1) we receive \( e'(\lambda_t) = \frac{1}{2\gamma} \), and hence
\[ e'(\lambda_t) : \frac{\lambda_t}{e_t} \equiv \eta_{t,\lambda_t} = \frac{\alpha\lambda_t}{\alpha(\lambda_t + \gamma) - c^S}. \tag{B.2} \]

It is easy to prove that this term is strictly bigger than one, as long as \( c^S > \alpha\gamma \).
Due to Fact B.1, this is the case and we obtain \( \eta_{t,\lambda_t} > 1 \).

(b) Differentiating elasticity (B.2) with respect to \( \lambda_t \), we obtain:
\[ \frac{\alpha(\alpha\gamma - c^S)}{[\alpha(\lambda_t + \gamma) - c^S]^2} \]

Because of Fact B.1 this derivative is strictly negative. Thus, for the lowest value of \( \lambda_t \) – in the considered case of \( \lambda_t \geq \lambda^S \) this is \( \lambda^S \) – the elasticity \( \eta_t \) takes the highest value and declines for increasing \( \lambda_t \). 

\[ \square \]
Proof of the Better Cost-Effectiveness of BCS in Non-Stark Poverty:

We define

\[ r_t := \frac{s_t^{cc}(k)}{s_t^{bc}(k)} \]

as the relation between the necessary payment to achieve education level \( k \) under CCS and its equivalent under BCS. Applying (4.20) and (4.21), we come to:

\[ r_t = \frac{2 \alpha \gamma k}{2 \alpha \gamma k - [\alpha (\lambda_t + \gamma) - c^S]} \]

Thus, for all \( \alpha (\lambda_t + \gamma) > c^S \), we obtain \( r_t > 1 \) and hence, BCS is more cost-effective than CCS. We view the case \( c_t > c^S \). Therefore, because of \( c_t' > 0 \), it is true that \( \alpha (\lambda_t + \gamma) > c_t \). Using \( c^S = \alpha (\lambda^S + \gamma) \), yields

\[ r_t = \frac{2 \gamma k}{2 \gamma k - (\lambda_t - \lambda^S)} \]

and BCS is superior for all \( \lambda_t > \lambda^S \), ergo, all over the area of non-stark poverty.

Proof of Lemma 4.1:

(a) We underlie \( h(e_t) = (e_t)^\Theta \). Hence,

\[ h''(e_t) = \Theta (\Theta - 1)(e_t)^{\Theta - 2} \geq 0 \iff \Theta \geq 1 \]

with \( \Theta > 0 \).

(b) The difference equation for the human capital technology is

\[ \lambda_{t+1}(\lambda_t) = [e(\lambda_t)]^\Theta \lambda_t + 1. \]  

(B.3)

Twice differentiating with respect to \( \lambda_t \) yields

\[ \Theta e'[e(\lambda_t)]^{\Theta - 1} [1 + (\Theta - 1) \eta_{e_t, \lambda_t}] \]

The sign of this term solely depends on the sign of the term \( [1 + (\Theta - 1) \eta_{e_t, \lambda_t}] \). Thus,

\[ \lambda''_{t+1}(\lambda_t) \geq 0 \iff \Theta \geq \left(1 - \frac{1}{\eta_{e_t, \lambda_t}}\right). \]
(c) The existence of a turning point requires \( \lambda_{t+1}''(\lambda_t) = 0 \) for some \( \lambda_t \), which we label \( \tilde{\lambda} \). This asks for 
\[1 + (\Theta - 1)\eta_{t, \lambda_t} = 0.\] Plugging in \( e^\circ(\lambda_t) \) this becomes
\[1 + \frac{(\Theta-1)\alpha \lambda_t}{\alpha(\lambda_t+\gamma) - c_s} = 0.\] As we consider \( \lambda_t \in [\lambda^S, \lambda^a] \), the denominator is positive. It directly follows that a turning point does only exist for \( \Theta < 1 \). Solving for \( \lambda_t \) yields \( \tilde{\lambda} = \frac{c_s}{\alpha \Theta} \). Plugging in \( c_s = \alpha(\lambda^S + \gamma) \) we obtain:
\[\tilde{\lambda} = \frac{\lambda^S}{\Theta} \quad \text{(B.4)}\]
Again, for \( \Theta \geq 1 \) we end up with \( \tilde{\lambda} \leq \lambda^S \), and therefore \( \tilde{\lambda} \notin (\lambda^S, \lambda^a] \).

Proof of Proposition 4.5:

1. In Proposition 2.1 (a), we found that \( \lambda_{t+1}(\lambda_t) \) is strictly convex in the interval \( [\lambda^S, \lambda^a] \) if \( h(e_t) \) is convex. In Lemma 4.1 (a), in turn, we found that \( h(e_t) = [e(\lambda_t)]^\Theta \) is convex for all \( \Theta > 1 \). Thus, for \( \Theta > 1 \), \( \lambda_{t+1}(\lambda_t) \) is strictly convex in the interval \( [\lambda^S, \lambda^a] \). Lemma 4.1 (c) additionally proves this.

2. \( \eta_{t, \lambda_t} = \frac{\alpha \lambda_t}{\alpha(\lambda_t+\gamma) - c_s} \) and \( c_s = \alpha(\lambda^S + \gamma) \). Therefore, \( \lim_{\lambda_t \to \lambda^S} \eta_{t, \lambda_t} \) is infinity. We showed that \( \lambda_{t+1}(\lambda_t) \) is concave if \( \Theta < \left(1 - \frac{1}{\eta_{t, \lambda_t}}\right) \). Since \( \lim_{\lambda_t \to \infty} \left(1 - \frac{1}{\eta_{t, \lambda_t}}\right) = 1 \), the trajectory is definitely initially concave if \( \Theta < 1 \).

3. In Proposition 4.4, we proved \( \frac{\partial \eta_{t, \lambda_t}}{\partial \lambda_t} < 0 \), and that the highest value of \( \eta_t \), therefore, is reached at \( \lambda_t = \lambda^S \). Just proved, the trajectory is initially concave if \( 1 > \Theta > 0 \). If \( \tilde{\lambda} < \lambda^a \), thus, the trajectory’s curvature turns to convex within the interval \( (\lambda^S, \lambda^a) \).

\[\square\]

B.1 Details to Figure 4.3

In case of unconditional lump-sum subsidies, marked by \( uc \), and in the laissez-faire reference case, marked by \( lf \), utility maximization yields
\[MRS^{uc} = MRS^{lf} = \frac{1}{\alpha \gamma} = p\]

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as optimum condition, where the marginal rate of substitution, abbreviated by MRS, is defined as \(-\frac{\partial u_t}{\partial c_t} \big|_{U_t=0} = \frac{\partial u(c_t)}{\partial c_t} / \frac{\partial u(c_t)}{\partial e_t}\). In case of the binary conditional lump-sum subsidies, we obtain a corner solution, wherefore the MRS is not equal to the consumption price \(p\). Nonetheless, in all three cases the relative price of consumption, respectively the budget line’s slope, equals \(p\). In the unconditional subsidy case, the “budget line” is given by:

\[
e_t = \frac{\lambda_t + \gamma}{\gamma} + \frac{s^u e_t}{\alpha \gamma} - \frac{1}{\alpha \gamma} c_t
\]

In case of BCS, the budget line is given by:

\[
e_t = \begin{cases} \frac{\lambda_t + \gamma}{\gamma} + \frac{s^u e_t}{\alpha \gamma} - \frac{1}{\alpha \gamma} c_t & \text{if } e_t \geq k; \\ \frac{\lambda_t + \gamma}{\gamma} - \frac{1}{\alpha \gamma} c_t & \text{else}. \end{cases}
\]

In contrast, using CCS, we obtain:

\[
MRS^{cc} = \frac{1}{\alpha \gamma (1 - \frac{\sigma}{\alpha \gamma})} 
\]

and

\[
e_t = \frac{(\lambda_t + \gamma)}{\gamma (1 - \frac{\sigma}{\alpha \gamma})} - \frac{1}{\alpha \gamma (1 - \frac{\sigma}{\alpha \gamma})} c_t
\]

for the budget line in the special case of \(s^{cc}(e_t) = \sigma e_t\).

### B.2 The Comparison of BCS and CCS in Non-Stark Poverty: Comparative Statics

The derivations of \(r_t\) concerning \(\alpha, c^S, \lambda_t\) and \(\gamma\) are evaluated straightforward:

\[
\frac{\partial r_t}{\partial \alpha} = 0
\]

\[
\frac{\partial r_t}{\partial c^S} = \frac{-2 \alpha \gamma k}{[2 \alpha \gamma k - (\alpha (\lambda_t + \gamma) - c^S)]^2} < 0
\]

An increase of \(c^S\) represents a shrink in the degree of altruism. Hence, each unit of consumption must be compensated at a higher extent than before. It follows that BCS, using exactly this compensation channel, is directly affected. CCS is working via distorting the relative price of education and is therefore not directly affected by this. Our Stone-Geary preferences are homothetic in the sense that the slope of all indifference curves along any ray with origin at \((e_t = 0, c_t = c^S)\) are identical, i.e., income changes do not change the MRS. Hence, along a horizontal line, like the line \(e_t = k\), the slope of the indifference curves increase (become less negative) moving to
the right. Consequently, moving the origin of the rays to the right, while leaving the location of the budget frontier unchanged, means that the indifference curve of the resulting $U_t^0$ is now steeper at $e_t = k$. Hence, the indifference curve is tangent at a lower level of $e_t$ as before. Using both facts makes clear that $s_t^{bc}$ must increase strongly, because of the increased MRS at $e_t = k$. In contrast, for CCS the indifference curve’s slope at $e_t = k$ does matter as well but the increased distance between $e_t^o$ and $k$ has no influence.

$$\frac{\partial r_t}{\partial \lambda_t} = \frac{2\alpha^2 \gamma k}{[2\alpha \gamma k - (\alpha (\lambda_t + \gamma) - c^S)]^2} > 0$$

As low levels of income causes low levels of education, an increase of $\lambda_t$ works like an increase in the degree of altruism. As well in $s_t^{bc}$ as in $s_t^{cc}$ one finds $\lambda_t$ in the same term of the nominators, but in $s_t^{bc}$ the term is squared, so that $\lambda$’s decreasing effect lowers $s_t^{bc}$ stronger than $s_t^{cc}$. An increase of $\lambda_t$ does not affect the slope of the “budget line”. It shifts the horizontal part of the budget set to the right. The decreasing MRS along a horizontal line lowers the required CCS transfer. However, as $\lambda_t$ increases the adults income, the laissez-faire’s level of education increases as well. Hence, the gap between indifference curve and budget on the left of the laissez-faire allocation is becoming smaller, since the indifference curve does not run away too much from the budget set. So here we have the opposite case of the effect of changes in $e^S$. Here the budget frontier is moved while the bundle of indifference curves is unchanged.

$$\frac{\partial r_t}{\partial \gamma} = \frac{-2k(\lambda_t - \lambda^S)}{[2\gamma k - (\lambda_t + \lambda^S)]^2} < 0$$

The overall sign of $\frac{\partial r_t}{\partial \gamma}$ is clearly negative as $\lambda_t > \lambda^S$ in non-stark poverty. An increase of $\gamma$ increases the required compensation and lowers the laissez-faire level of $e_t$. Therefore, the distance in which the budget frontier and the indifference curve between $e_t^o$ and $k$ run away from each other increases. What is left to be done is to examine the effect of $k$ on $r_t$. We obtain:

$$\frac{\partial r_t}{\partial k} = \frac{2\alpha \gamma [c^S - \alpha (\lambda_t + \gamma)]}{[2\alpha \gamma k - (\alpha (\lambda_t + \gamma) - c^S)]^2} < 0$$

As we observe the case $c_t > c^S$, we have $e_t > 0$, and thus $\alpha (\lambda_t + \gamma) > c^S$. Therefore, $\frac{\partial r_t}{\partial k} < 0$. For an intuition see Section 4.6.

**B.3 The Pigouvian Tax**

In Chapter 3, we demonstrated that we have to revise an externality. The classic instrument to do so is to levy a Pigouvian tax.\(^1\) As all households alike would have to

pay the Pigouvian tax, it is clear that it could implement the education of the society within one generation.

We have seen in Chapter 3 that households do consume too much and give too less education to the child. Therefore, we levy a Pigouvian tax $\tau$ on consumption $c$. The budget line is therefore:

$$e = \frac{\bar{c}(\lambda)}{\alpha \gamma} - \frac{1 + \tau}{\alpha \gamma} c$$

Thus, the relative price of education, $\alpha \gamma / (1 + \tau)$, decreases in $\tau$, so that the demand for education increases: the budget line becomes steeper. If $k$ represents the socially optimal level of education, then it is implemented by the Pigouvian tax that fulfills equation (see Figure B.1 for the non-stark poverty case).\(^2\)

$$\frac{1 + \tau}{\alpha \gamma} = \frac{\partial u(k, c)}{\partial c} / \frac{\partial u(k, c)}{\partial e} \left( = MRS(k, c) \right)$$

Nonetheless, the resulting allocation is not first-best because the corresponding level of consumption is lower than the socially efficient level: $\frac{\alpha \lambda}{1 + \tau} < \alpha \lambda$. Neglecting administrative costs, the Pigouvian tax is first-best if the tax revenue is refunded lump-sum, so that $c = \alpha \lambda$.

\(^2\)The to $e = k$ corresponding consumption level is $c^k = \frac{\alpha \lambda (1 - k) \gamma}{1 + \tau}$. 

Figure B.1: The Pigouvian tax scenario in the case of non-stark poverty, where $A = \alpha \lambda$, $B = \frac{\alpha \lambda}{1 + \tau}$, $p^1 = \frac{1}{\alpha \gamma}$, and $p^2 = \frac{1 + \tau}{\alpha \gamma}$.
However, the Pigouvian tax fails to implement the socially optimal education level in case of stark poverty. As preferences are lexicographically, they prefer to consume all income in any case. Then, there is no trade-off between education and consumption, so that the relative price of education is not a determinant of the household’s decision. The tax would simply deteriorate the situation of the poor, and is insofar counter-productive. Moreover, it is not plausible anyway that an increase in consumer prices will stimulate education. We believe that the lack of education roots in poverty. Therefore, policies that increase the price of consumption goods to increase the education of children is not an adequate policy. Hence, the Pigouvian tax is not an option.

B.4 Extensions

In the preceding section, we assumed that all additional income above the level necessary for survival can, in the sense of net taxes, totally be taxed away from the households, implying marginal and average tax rates of 100 percent (for this income range). Like already mentioned in Bell and Gersbach (2001), this is politically not realizable in real world. Hence, tax revenues in real world situations can be distinctly smaller, lowering the feasibility of educating a society within one generation, but also quite generally.

There is also a political restriction. Politicians want to be reelected. The extreme redistribution by massive taxation lead, in spite of the subsidies, to utility losses of many voters. Hence, it is rather likely that the government will not be reelected. Consequently the government won’t take such a policy into consideration. Then, de facto, it is not feasible. The same accounts for the matter of fact that part of resources must be used for administrative cost like wages and the like. This item can also influence the comparison of our subsidy instruments. Suppose different methods involve different levels of cost besides the transfer itself. Then, the better cost-effectiveness of BCS might be over-compensated by those additional costs, and CCS is the overall better instrument.

B.4.1 Tax Burden Ceilings

Suppose the constitution of the society under consideration states that no citizen must, on average, not be taxed higher than by tax rate $\phi^{max}$. It follows that the maximum tax revenue becomes:

$$\tau^{max}(\lambda_t, k) = \min\{\phi^{max} \alpha(\lambda_t + \gamma) , \alpha[\lambda_t + \gamma] - c^{sub}\}$$
So if the ceiling is binding the necessary condition for the feasibility of education within one generation changes. Let $\bar{\phi}_i^t$ be the average tax rate of household $i$ in $t$.

**Corollary B.1**

A sustainable education of a society within one generation with consideration of the constitutional restriction $\bar{\phi}_i^t \leq \phi^{\text{max}}$, $\forall i = [0, 1]$ and $\forall t$, is possible if the following holds:

$$k(\lambda_t) < k \leq \frac{\lambda_t + \gamma}{\gamma} \cdot \phi^{\text{max}}$$

A direct consequence is:

**Corollary B.2**

The education of the society is impossible if the ceiling for the average tax burden, $\phi^{\text{max}}$, is equal to or smaller than:

$$\frac{\gamma}{\lambda_t + \gamma} \cdot k(\lambda_t) = \frac{\gamma}{\lambda_t + \gamma} \cdot \left(\frac{\lambda^* - 1}{\lambda_t}\right)^{\frac{1}{\alpha}}$$

### B.4.2 Reelection Constraints

Suppose the considered country is a democracy. To become elected a politician requires a majority of votes (majority rule), that is, one vote above half of all votes. As we view a continuum of households, let us assume half of all votes suffices to be elected. Let us further assume that there exists already an elected government which wants to be reelected in the next election.

If the reelection aspect is taken into account, the government will leave half of the households without any (net) tax burden. Note that there is no opportunity of compensating the loss of utility by the tax-and-subsidy scheme to educate the society via subsidies unless $F > 0$. This is the case because the government’s budget cannot display a deficit. The taxed adults will not vote pro the government unless the net tax burden is non-negative. Then, a compensation is not achievable, since half of society is not taxed but subsidized.

**Corollary B.3**

Suppose reelection requires half of votes (simple majority rule) and the government wants to be reelected. Moreover, there are no other projects that could be used to compensate for utility losses of voters. Then, a sustainable education of a society within one generation is possible if the following holds:

$$k(\lambda_t) < k \leq \frac{\overline{c}(\lambda_t) - e^{\text{sub}}}{2\alpha\gamma}$$
If we combine the reelection aspect with tax ceilings, we even obtain:

**Corollary B.4**

Suppose the assumptions of Corollary B.3 and that there is a constitutional tax ceiling of \( \phi_i^t \leq \phi_i^{\text{max}}, \forall i = [0, 1], \forall t \). Then, a sustainable education of a society within one generation is possible if the following holds:

\[
\frac{k(\lambda_i)}{\lambda_i + \gamma} > \frac{k(\lambda_i)}{\lambda_i + \gamma} \cdot \phi_i^{\text{max}}
\]

**B.4.3 Differences in Administrative Cost**

Administrative costs and the vulnerability to corruption are far too often neglected. In real world, these costs are sometimes the most crucial item. It just does not make sense to use a seemingly most efficient instrument when this instrument is connected with immense administrative cost and huge extent of corruption that eat big parts of the scarce resources. For instance can it happen that applying a certain instrument forces some fix cost block for building the necessary infrastructure and to employ specific skilled employees. Simultaneously, this instrument may be involved with a crucially more adverse incentive scheme concerning corruption; to prevent major corruption losses, further administrative costs would be necessary. If there exists a less efficient alternative that involves distinctly less cost it might over all be more efficient to use this alternative.

In our case, the unconditional subsidy has the clear advantage that the involved administrative cost is very low. The only thing that has to be done is paying the subsidies to the households. Although even this can be a problem this has to be solved in all the other regimes as well. But the conditional subsidies involve additional efforts, namely, the supervision of the educational performance of the households. This increases the cost and corruption probability substantially.

We denote the overall administrative costs per unit of paid subsidy by \( \psi^x, x = \{uc, cc, bc\} \). Under CCS the teachers and the headmaster of the school must simply check whether the child is present or not and pay, or occasion to pay, the subsidy. In practice, this could simply involve distributing food to pupils that are attending school. Under BCS the headmaster must in addition control for the total attendance of the children and only if the requirement is full-filled he is allowed to occasion the transfer. Hence, we can expect: \( \psi^{bc} > \psi^{cc} > \psi^{uc} \).

So concerning administrative costs, we obtain a diametral different result. The best instrument concerning the subsidy payment is the worst concerning administrative
cost, and vice versa; so there may be a trade-off. Therefore, the results so far derived stay valid only if the cost advantage in the transfers is not over-compensated by the administrative costs disadvantage:

**Proposition B.1**

*Considering administrative costs per unit paid subsidy running up to* $\psi^x$, $x = \{uc, cc, bc\}$, *the so far derived hierarchy of instruments remains only valid if:*

\[
\begin{align*}
\psi^{bc}s^{bc} - \psi^{cc}s^{cc} &< s^{cc} - s^{bc} \\
\psi^{cc}s^{cc} - \psi^{uc}s^{uc} &< s^{uc} - s^{cc}
\end{align*}
\]

It is clear that further costs diminishes the speed of the education process:

\[
T^x = \frac{(1 + \psi^x)s^x(k)}{F} > \frac{s^x(k)}{F}
\]
Appendix C

Appendix to Chapter 5

C.1 The Education of a Society in Three Periods

In this section, we turn to a concrete example and discuss the agenda setting designed to educate the society within three periods. To speed things up under a democratic regime, it may not be necessary to subsidize households so that they choose full-time schooling immediately. Therefore, the government may pay lower subsidies: $0 < e_i^o(\alpha + s_i^t) < 1$. In the example, we consider the growth case where $z_h(e_i^o(\alpha + s_i^t)) + 1 > \lambda^*$. Since the minimum coalition forming a majority is $\frac{1}{2}$, $\phi_t \geq \frac{1}{2}$ for all $t \in [0, T - 1]$.

We make two simplifications. First, we restrict ourselves to proposals $P$ providing identical subsidizing of households in period 0.\(^1\) A second simplification is the constraint that better-educated individuals will be taxed before taxes are levied on households that are either less well-educated or in a state of backwardness; for example, because they earn higher incomes and thus can be taxed higher. Then, first-period taxation is given as:

$$\tau^i_0 = \begin{cases} 0 & \forall \ i \in [0, \frac{1}{2}]; \\ \tau_{\text{sub}} & \forall \ i \in (\frac{1}{2}, 1]. \end{cases} \quad (C.1)$$

The tax revenue in period $t = 0$ amounts to $R_0 = \int_0^1 \tau_0(i) \ di = \frac{1}{2} \tau_{\text{sub}}$. The winning coalition allows subsidization for a fraction $\delta_0$ of the population. The budget has to be balanced and the size of subsidy is equal for all households that receive it. The subsidy per household in $t = 0$ is thus given by:

$$s^i_0 = \begin{cases} \frac{\tau_{\text{sub}}}{2\delta_0} & \forall \ i \in [0, \delta_0]; \\ 0 & \forall \ i \in (\delta_0, 1]. \end{cases} \quad (C.2)$$

\(^1\)We thus exclude the possibility of paying higher subsidies to some households in period 0 in order to create a potentially higher tax base in the future.
The program in \( t = 0 \) causes human capital accumulation:

\[
\lambda^i_1 = \begin{cases} 
zh(e^{\alpha}(\alpha + s^i_0)) + 1 & \forall \ i \in [0, \delta_0]; \\
1 & \forall \ i \in (\delta_0, 1].
\end{cases} \tag{C.3}
\]

We assume that better-educated individuals are taxed before less educated or uneducated individuals. Moreover, when half of the society was taxed in period 0, then it is obvious that \( \delta_0 \leq 1/2 \). It follows that all households subsidized in \( t = 0 \) are taxed in every period (except period 0). Note that these households have to be taxed in such a way that, in spite of the continuous taxation, they will reach full education in \( T = 3 \) periods, that is, in period \( t = 2 \).

![Figure C.1: The subsidized fractions of the society](image)

The fraction \((1 - \delta_0) \geq 1/2\) still remains in a state of backwardness at the end of period \( t = 0 \). In period \( t = 1 \), a further portion of the society, \( \delta_1 \), is subsidized. The situation is illustrated in Figure C.1. Again, only half of the households is taxed in order to create a winning majority coalition. Since \( \delta_0 \leq 1/2 \), a fraction \( \frac{1}{2} - \delta_0 \) of the \( 1 - \delta_0 \) still backward households are additionally taxed. Therefore, the distribution of the tax burden is:

\[
\tau^i_1 = \begin{cases} 
\tau^\delta_0_1 & \forall \ i \in [0, \delta_0]; \\
\tau^{\text{sub}}_1 & \forall \ i \in (\delta_0, \frac{1}{2}]; \\
0 & \forall \ i \in (\frac{1}{2}, 1].
\end{cases} \tag{C.4}
\]

The resulting total tax revenue then amounts to:

\[
R_1(\delta_0) = \int_0^1 \tau_1(i) \, di = \left( \frac{1}{2} - \delta_0 \right) \tau^{\text{sub}} + \delta_0 \tau^\delta_0_1
\]

Referring to subsidization in period 1, we divide the fraction \( \delta_1 \) of the society into two groups. The parents of both groups are subsidized so that in period \( t = 2 \) their offspring will enjoy full education. Let us assume that taxation of half of the society is necessary to finance the required subsidies in the last, third period. Then, if fraction \( \left( \frac{1}{2} - \delta_0 \right) > 0 \), households beyond fraction \( \delta_0 \) again have to be taxed in period 2; these households will stem from fraction \( \delta_1 \). Consequently, it is necessary to pay higher subsidies to these households than to the in period 2 untaxed part of fraction \( \delta_1 \), because only then these

\[^2\text{When initially the society is poor, then the tax revenue in period 0 is very small and } \delta_0 < 1/2 \text{ is even more obvious.}\]
At the beginning of $t$ households will earn (via a higher level of human capital) the additional income that will be taxed away in period 2. The restriction \( (1 - \delta_t - \delta_{t+1}) = 0 \) must fulfill:

\[
T = 3 \text{ is a solution of the considered policy problem, the human capital accumulation in the very next period, i.e. } s_2 = s^a.
\]

Therefore, the budget is balanced when

\[
R_1(\delta_0, \delta_1, s_1, s_2^1) = \left( \frac{1}{2} - \delta_0 \right) s_1^1 + \left( (\delta_0 + \delta_1) - \frac{1}{2} \right) s_2^1.
\]

At the beginning of $t = 2$, fraction \( (1 - \delta_2 - \delta_3) \) is still in a state of backwardness. If $T = 3$, the sum $\delta_1 + \delta_2 + \delta_3$ must be equal to one: $\delta_2 = 1 - \delta_1 - \delta_0$. Therefore, the government has to subsidize all the rest up to the income level necessary to bear $\lambda^a$ in the very next period, i.e. $s_2 = s^a$.

To finance these subsidies, fractions $\delta_0$ and $(\frac{1}{2} - \delta_0)$ are taxed adequately in $t = 2$:

\[
R_2(\delta_0, s_2^1) = \delta_0 \tau_2^0 + (\frac{1}{2} - \delta_0) \tau_2^1.
\]

Again, the restriction $s^a = \delta_0 \tau_2^0 + (\frac{1}{2} - \delta_0) \tau_2^1$ is taken into account. If $T = 3$ is a solution of the considered policy problem, the human capital accumulation must fulfill:

\[
\lambda_3^1 = \left\{ \begin{array}{ll}
zh(e^{(a l_2^1 - \tau_2^0)}) l_2^1 + 1 & \forall \ i \in [0, \delta_0]; \\
zh(e^{(a l_2^1 - \tau_2^1)}) l_2^1 + 1 & \forall \ i \in (\delta_0, \frac{1}{2}); \\
zh(e^{(a l_2^1)}) l_2^1 + 1 & \forall \ i \in (\delta_0 + \delta_1, 1].
\end{array} \right.
\]

\(3\)Of course, we implicitly assume that $s_1^1$ bears less than full-time schooling, because otherwise a higher subsidy would not cause a higher income in period 2.

\(4\)Recall that the corresponding subsidy, $s^a$, is given implicitly by $\lambda^a = zh(e^{(a + s^a)}) + 1$. Note that as long as $zh(1) + 1 < \lambda^a$, there is no subsidy $s^a$ and the society cannot be fully educated within 3 periods.
Appendix C. Appendix to Chapter 5

Over all, we hence can summarize the task as follows. The exogenous benevolent agenda setter must set the agenda with respect to balanced budgets. Moreover, the tax-and-subsidy scheme has to ensure that each single household will reach the level of human capital of $\lambda^a$ in period $T - 1$ (taken into account its taxation and subsidization over all periods). Applying variable $v$ in Equation (5.7), the general form of Condition (C.10) is:

$$
\lambda_i^3(v_i^0, v_i^1, v_i^2) = zh\left[e^\alpha (\alpha \lambda_i^2(v_i^0, v_i^1) - v_i^2)\right] \lambda_i^2(v_i^0, v_i^1) + 1 \geq \lambda^a \quad \forall \ i \in [0..1],
$$

with

$$
\begin{align*}
\lambda_i^2(v_i^0, v_i^1) &= zh \left[e^\alpha (\alpha \lambda_i^1(v_i^0) - v_i^1)\right] \lambda_i^1(v_i^0) + 1; \\
\lambda_i^1(v_i^0) &= zh \left[e^\alpha (\alpha - v_i^0)\right] + 1.
\end{align*}
$$

Whether or not there exists a solution $T = 3$ depends upon the tax potential the agenda setter is facing and on the productivity of the schooling system. If it is too low, the policy’s time horizon must be prolonged, but there will be a solution $T < \infty$, as we have shown. A simple example for a solution $T = 3$ is $\delta_t = \frac{1}{3}$ and $s_t = \frac{3\tau_{sub}}{2}$ for all $t = \{0, 1, 2\}$ if $zh(e^\alpha(\alpha + \frac{3\tau_{sub}}{2})) + 1 \geq \lambda^a + \tau_{sub}$.

A noteworthy result of this example is that as soon as $\lambda^* < zh(1) + 1 < \lambda^a$, the tax-and-subsidy policy cannot educate the society within three periods; however, the system’s inherent growth ensures the success of the education program within finite time.
Appendix D

Appendix to Chapter 6

D.1 Proofs

Proof of Proposition 6.1:
We have shown that $\frac{\partial \lambda}{\partial q} < 0$ for all $x = 1, 2, 3, 4$. It is obvious that $\frac{\partial \lambda}{\partial q} = z \frac{\partial h(Q_t)}{\partial q_t} \lambda_t > 0$ for all $x = 1, 2, 3, 4$. It follows that each point of the pre-investment trajectory lies strictly below the post-investment trajectory. The 45°-line is fixed. Therefore we find $\frac{\partial \lambda}{\partial q} = 0$ for all $x = 1, 2, 3, 4$. Applying Equation (6.19), we receive

$$s^*_t = s^*(Q(q^1_t), \rho(q^2_t), v(q^3_t)) = s^*(\tilde{q}_t, \lambda^*(\tilde{q}_t))$$

with $\frac{ds^*(\tilde{q}_t, \lambda^*(\tilde{q}_t))}{\partial q_t} = \frac{\partial s^*(\tilde{q}_t, \lambda^*(\tilde{q}_t))}{\partial q_t} + \frac{\partial s^*(\tilde{q}_t, \lambda^*(\tilde{q}_t))}{\partial \lambda_t} \frac{\partial \lambda_t}{\partial q_t} < 0$ for all $x = 1, 2, 3, 4$, because we additionally have $\frac{\partial e^o}{\partial q_t} > 0$. $\tilde{s}_t = s^*(\tilde{q}_t, \lambda^*(\tilde{q}_t)) + \epsilon$ completes the proof.

Proof of Proposition 6.2:
First note that the objective function is strictly convex, and the constraints are linear. Therefore, the Kuhn-Tucker minimum conditions will find the solution. We denote Lagrangean multipliers by $\kappa$. 

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The Kuhn-Tucker minimum conditions are:

\[
\begin{align*}
\frac{\partial L(\bar{q}_t)}{\partial q_t^3} &= \frac{d'(q_t^3)}{\nu(q_t^3)} - \kappa_t^3 + \kappa_t^5 \geq 0, \quad \frac{\partial L(\bar{q}_t)}{\partial q_t^3} \cdot q_t^3 = 0 \quad (D.1) \\
\frac{\partial L(\bar{q}_t)}{\partial q_t^4} &= -\frac{d(q_t^4)}{[v(q_t^4)]^2}v'(q_t^4) - \kappa_t^4 + \kappa_t^5 \geq 0, \quad \frac{\partial L(\bar{q}_t)}{\partial q_t^4} \cdot q_t^4 = 0 \quad (D.2) \\
\frac{\partial L(\bar{q}_t)}{\partial \kappa_t^3} &= -q_t^3 \leq 0, \quad \frac{\partial L(\bar{q}_t)}{\partial \kappa_t^3} \cdot \kappa_t^3 = 0 \quad (D.3) \\
\frac{\partial L(\bar{q}_t)}{\partial \kappa_t^4} &= -q_t^4 \leq 0, \quad \frac{\partial L(\bar{q}_t)}{\partial \kappa_t^4} \cdot \kappa_t^4 = 0 \quad (D.4) \\
\frac{\partial L(\bar{q}_t)}{\partial \kappa_t^5} &= q_t^3 + q_t^4 - R_t \leq 0, \quad \frac{\partial L(\bar{q}_t)}{\partial \kappa_t^5} \cdot \kappa_t^5 = 0 \quad (D.5)
\end{align*}
\]

If \((\kappa_t^5)\) = 0, then, for instance (D.1), would require \(-\frac{d'(q_t^3)}{\nu(q_t^3)} + \kappa_t^3 < 0\). As \(\kappa_t^3\) is a non-negative shadow price and \(-\frac{d'(q_t^3)}{\nu(q_t^3)} > 0\) by construction, this inequality cannot hold. The same logic is valid for (D.2). Thus \((\kappa_t^5)\) is always strictly positive, and all resources \(R_t\) are used up.

1. If \((\kappa_t^3) = 0\) and \((\kappa_t^4) = 0\), (D.3) tells us \((\kappa_t^3) = 0\), and thus, via (D.5), \((\kappa_t^3) = R_t\). Applying (D.1), we know \(\frac{\partial L(\bar{q}_t)}{\partial q_t^3} = 0\) and hence

\[
0 < -\frac{d'(q_t^3)}{\nu(q_t^3)} = \kappa_t^5. \quad \text{Finally using (D.2), we obtain} \quad -\frac{d'(q_t^3)}{\nu(q_t^3)} = \kappa_t^5 \geq \frac{d(q_t^3)}{[v(q_t^3)]^2}v'(q_t^4) + \kappa_t^4.
\]

We can conclude that \(0 \leq \kappa_t^4 \leq -\frac{d'(q_t^3)}{\nu(q_t^3)} - \frac{d(q_t^3)}{[v(q_t^3)]^2}v'(q_t^4)\).

2. If \((\kappa_t^3) = 0\) and \((\kappa_t^4) > 0\), we analogous receive \((\kappa_t^3) = 0\), \((\kappa_t^4) = 0\), and \((\kappa_t^4) = R_t\) due to (D.3)-(D.5), and via (D.1) and (D.2) \(0 < \kappa_t^5 \geq \frac{d(q_t^3)}{[v(q_t^3)]^2}v'(q_t^4) \geq \kappa_t^3 - \frac{d'(q_t^3)}{\nu(q_t^3)}\), so that \(0 \leq \kappa_t^3 \leq \frac{d(q_t^3)}{[v(q_t^3)]^2}v'(q_t^4) + \frac{d'(q_t^3)}{\nu(q_t^3)}\).

3. If the solution is interior, (D.3) to (D.5) express that \(\kappa_t^5 = \kappa_t^4 = 0\) and \(q_t^3 + q_t^4 = R_t\).

Consequently (D.1) and (D.2) force \(\kappa_t^5 = -\frac{d'(q_t^3)}{\nu(q_t^3)} = \frac{d(q_t^3)}{[v(q_t^3)]^2}v'(q_t^4) > 0\).

If an investment \(q_t^r\) is actually undertaken, i.e. \((q_t^r) = 0\), then \(\kappa_t^5 = 0, \frac{\partial L}{\partial q_t^r} = 0\), and thus \(\kappa_t^5 = -\frac{\partial (d/\nu)}{\partial q_t^r}\). Contrary an investment is not undertaken, i.e. \((q_t^r) = 0\), then \(\kappa_t^5 \geq 0, \frac{\partial L}{\partial q_t^r} \geq 0\), and therefore \(\kappa_t^5 \geq -\frac{\partial (d/\nu)}{\partial q_t^r} + \kappa_t^r\). Consequently \(\kappa_t^5 = -\frac{\partial (d/\nu)}{\partial q_t^r} \geq -\frac{\partial (d/\nu)}{\partial q_t^r} + \kappa_t^r\), and \(\kappa_t^k \leq -\frac{\partial (d/\nu)}{\partial q_t^k} + \frac{\partial (d/\nu)}{\partial q_t^k}\), where \(l\) represents a paying investment and \(k\) an investment that is not undertaken in the optimum.
APPENDIX D. APPENDIX TO CHAPTER 6

**Proof of Proposition 6.4:**

The Lagrangean to maximize is:

\[ L(q^*) = \delta(q^*) + \sum_{x=1}^{4} \kappa^*_x q^*_x \quad (D.6) \]

When Assumption 6.1 holds, the constrained maximum will be found by the Kuhn-Tucker maximum conditions, which are summarized by:

\[
\left( \frac{\partial L}{\partial q^*_l} \right) = \left( \frac{\partial \delta}{\partial q^*_l} \right) + \left( \frac{\partial}{\partial q^*_l} \right) \kappa^*_l \leq 0, \quad \left( \frac{\partial L}{\partial q^*_l} \right) \left( q^*_l \right) = \bar{0} \quad (D.7)
\]

\[
\left( \frac{\partial L}{\partial \kappa^*_l} \right) \geq \bar{0}, \quad \left( \frac{\partial L}{\partial \kappa^*_l} \right) \left( \kappa^*_l \right) = \bar{0} \quad (D.8)
\]

Condition (D.8) reveals that for \( l \in \mathcal{L} \), \( q^*_l > 0 \), \( \kappa^*_l = 0 \), and hence, due to (D.7), \( \frac{\partial s}{\partial q^*_l} = 0 \). Similarly we receive \( q^*_k = 0 \), \( \kappa^*_k \geq 0 \), and \( \frac{\partial s}{\partial q^*_l} + \kappa^*_k \leq 0 \) for all \( k \in \mathcal{K} \).

We conclude that \( \frac{\partial s^*(q^*_l)}{\partial q^*_l} \geq 0 \). Applying \( \frac{\partial s}{\partial q^*_l} = -\frac{1}{s_l} - \frac{R_i - \sum_{x=1}^{4} q^*_l \frac{\partial s}{\partial q^*_l}}{s^* - \sum_{x=1}^{4} q^*_l} \) and \( \frac{\partial s}{\partial q^*_l} = \frac{\partial s}{\partial q^*_l} \), we find:

\[
R_i - \frac{\sum_{x=1}^{4} (q^*_l)^o}{(s^* - \sum_{x=1}^{4} q^*_l)^2} - \frac{\partial s^*(q^*_l)}{\partial q^*_l}\frac{\partial s}{\partial q^*_l} = \frac{1}{s^* - \sum_{x=1}^{4} q^*_l} \quad \text{for all} \quad l \in \mathcal{L}
\]

\[
R_i - \frac{\sum_{x=1}^{4} (q^*_l)^o}{(s^* - \sum_{x=1}^{4} q^*_l)^2} - \frac{\partial s^*(q^*_l)}{\partial q^*_l} \geq -\frac{\partial s^*(q^*_l)}{\partial q^*_l} \quad \text{for all} \quad k \in \mathcal{K}
\]

\[
\frac{\partial s^*(q^*_l)}{\partial q^*_l} = 0
\]

\[
\square
\]

**Proof of Lemma 6.1:**

Rearranging \( A = \gamma \left( 1 - \frac{d}{v_t} - \frac{z(Q_t)}{z^2} \right) - \frac{(c^S + \rho_t)}{\alpha} \) - \( c^S - \rho_t \).

Because of \( \alpha < c^S + \rho_t \) and \( \gamma \leq 1 \) we definitely have \( A < 0 \). Additionally note that

\[
\frac{\partial A}{\partial Q_t} = \frac{2\gamma}{z(Q_t)^2} > 0 \quad (D.9)
\]

\[
\frac{\partial A}{\partial \rho_t} = -\frac{1}{\alpha} < 0 \quad (D.10)
\]

\[
\frac{\partial A}{\partial d_t} = -\frac{\gamma}{v_t} < 0 \quad (D.11)
\]

\[
\frac{\partial A}{\partial v_t} = \frac{dc^S}{(v_t)^2} > 0 \quad (D.12)
\]

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Therefore, due to $A < 0$ and (D.9) to (D.12), we receive via Equation (6.33):

$$
\frac{\partial \lambda^*(\cdot)}{\partial \rho_t} = -\frac{1}{2} \left( \frac{\partial A}{\partial \rho_t} \right) \left( 1 - \frac{A}{\sqrt{A^2 - \frac{8\gamma}{Q_t} \rho_t}} \right) > 0
$$

$$
\frac{\partial \lambda^*(\cdot)}{\partial \nu_t} = -\frac{1}{2} \left( \frac{\partial A}{\partial \nu_t} \right) \left( 1 - \frac{A}{\sqrt{A^2 - \frac{8\gamma}{Q_t} \nu_t}} \right) < 0
$$

$$
\frac{\partial \lambda^*(\cdot)}{\partial Q_t} = -\frac{1}{2} \left( \frac{\partial A}{\partial Q_t} \right) \left( 1 - \frac{A}{\sqrt{A^2 - \frac{8\gamma}{Q_t} \nu_t}} \right) + \frac{2\gamma}{z(Q_i)^2 \sqrt{A^2 - \frac{8\gamma}{Q_t}}} > 0
$$

Applying the implicit-function theorem to $\lambda_t^* = z e^\alpha(\lambda^*, \cdot) Q_t \lambda_t^* + 1$, we arrive at

$$
d \lambda_t^* = z e^\alpha(\lambda_t^*) \lambda_t^* 
d Q_t = 1 - z e^\alpha(\lambda_t^*) Q_t \left( 1 + \eta_{e^\alpha, \lambda_t^*} \right)
$$

where $\eta_{e^\alpha, \lambda} \equiv \frac{\partial e^\alpha(\lambda)}{\partial \lambda} \cdot \frac{\lambda^*}{e^\alpha(\lambda_t^*)}$, $\alpha$, $\alpha^*$. The derivative is (weakly) negative, if $1 \leq z e^\alpha(\lambda_t^*) Q_t \left( 1 + \eta_{e^\alpha, \lambda_t^*} \right)$, i.e. if $\eta_{e^\alpha, \lambda_t^*} \geq \frac{1}{z Q_i e^\alpha(\lambda_t^*)} - 1$. Due to $\frac{\partial \lambda_t^*}{\partial \lambda_t} = z e^\alpha(\lambda_t^*) Q_t \left( 1 + \eta_{e^\alpha, \lambda_t^*} \right)$ at the locus $\left( \lambda_t^*, \lambda_t^* \right)$, and the fact that, at this locus, this slope is always bigger than unity (as otherwise $\lambda_t^*$ does not exist), we arrive at $\frac{\partial \lambda_t^*}{\partial \lambda_t} < 0$; $h(e^\alpha(\lambda_t), Q_t) = e^\alpha(\lambda_t) \cdot Q_t$ completes the proof.

\[\square\]

**Proof of Proposition 6.5:**

Applying Lemma 6.1, and as only case $\lambda_t^* > 1$ is of interest, we find (via (6.34)):

$$
\frac{\partial s_t^*}{\partial Q_t} = -\left( \frac{2\alpha \gamma}{Q_t} \right) \left( \frac{\lambda_t^* - 1}{Q_t} - \frac{\partial \lambda_t^*}{\partial Q_t} \right) < 0 \quad (D.13)
$$

$$
\frac{\partial s_t^*}{\partial \rho_t} = 1 + \frac{2\alpha \gamma}{Q_t} \cdot \frac{\partial \lambda_t^*}{\partial \rho_t} > 0 \quad (D.14)
$$

$$
\frac{\partial s_t^*}{\partial \nu_t} = \frac{\alpha \gamma}{\nu_t} + \frac{2\alpha \gamma}{Q_t} \cdot \frac{\partial \lambda_t^*}{\partial \nu_t} > 0 \quad (D.15)
$$

$$
\frac{\partial s_t^*}{\partial \nu_t} = -\frac{\alpha \gamma}{\nu_t} + \frac{2\alpha \gamma}{Q_t} \cdot \frac{\partial \lambda_t^*}{\partial \nu_t} < 0 \quad (D.16)
$$

Finally, $s_t^* = s_t^* + \varepsilon$, and $Q'(q_t^1) > 0$, $\rho'(q_t^2) < 0$, $d'(q_t^3) < 0$ and $v'(q_t^4) > 0$.

\[\square\]
D.2 Discussion of Assumption 6.1

We have a classical maximization problem with four choice variables. Therefore Assumption 6.1 demands that the objective function \( \delta(q_t) = \frac{R_t - \sum_{i=1}^{s} q_t^i}{s(q_t)} \) is concave (second-order condition). The numerator of \( \delta(q_t) \) is linear. If \( 1/s(q_t) \) is (strictly) concave our objection function is also. This requires that \( s^*(q_t) \) is (strictly) convex, i.e. that

\[
\frac{\partial^2 s^*(q_t)(\cdot)}{\partial (q_t)^2} > 0,
\begin{vmatrix}
\frac{\partial^2 s^*(\cdot)}{\partial (q_t)^2} & \frac{\partial s^*(\cdot)}{\partial q_t} & \frac{\partial s^*(\cdot)}{\partial q_t} & \frac{\partial s^*(\cdot)}{\partial q_t} \\
\frac{\partial s^*(\cdot)}{\partial q_t} & \frac{\partial^2 s^*(\cdot)}{\partial (q_t)^2} & \frac{\partial s^*(\cdot)}{\partial q_t} & \frac{\partial s^*(\cdot)}{\partial q_t} \\
\frac{\partial s^*(\cdot)}{\partial q_t} & \frac{\partial s^*(\cdot)}{\partial q_t} & \frac{\partial^2 s^*(\cdot)}{\partial (q_t)^2} & \frac{\partial s^*(\cdot)}{\partial q_t} \\
\frac{\partial s^*(\cdot)}{\partial q_t} & \frac{\partial s^*(\cdot)}{\partial q_t} & \frac{\partial s^*(\cdot)}{\partial q_t} & \frac{\partial^2 s^*(\cdot)}{\partial (q_t)^2}
\end{vmatrix} > 0,
\]

and

\[
\frac{\partial^2 Q_t(q_t)}{\partial (q_t)^2} < 0 \text{ for } x = 1, 2, 3, 4, \text{ and it is plausible additionally to assume that initially school-quality improvements strongly can lower } s^*_t, \text{ but that this effect diminishes: } \frac{\partial^2 s^*(\cdot)}{\partial (q_t)^2} > 0. \text{ It follows that } s^*(\cdot) \text{ is not concave.} \]

Thus can \( s^*(\cdot) \) only be convex, or the Hessian is indefinite (which may involve saddle points at the point where the first order conditions hold).

The determinants of the (2x2)- and (4x4)-minors are always positive when the Hessian is definite, i.e. when \( s^*(q_t) \) is concave or convex. If these determinants are negative, then \( s^*(q_t) \) may at least have one saddle point. Therefore it is plausible to assume that the Kuhn-Tucker conditions do not identify a local minimum of \( \delta_t \), but they might identify a saddle point.

If the policy would have to fulfill an additional constraint, then there might occur a non-linear maximization problem, that would have to fulfill the requirements of the Arrow-Enthoven Sufficiency Theorem [cf. ARROW AND ENTHOVEN (1961), KUHN AND TUCKER (1951), TAKAYAMA (1974), and CHIANG (1984)].

---

1 That the Hessian is positive definite is not necessary but sufficient for a maximum. TAKAYAMA (1974), Chapter 1, endnote 12, p. 128, calls the used condition therefore second order sufficient condition.

2 Because \( s^*(q_t) \) is concave if the Hessian is negative definite. This requires \( \frac{\partial^2 s^*(\cdot)}{\partial (q_t)^2} < 0 \).

3 For the mathematics see CHIANG (1984), Chapter 11; SCHWARZE (1992), Chapter 13; or TAKAYAMA (1974), Chapter 1.
D.3 Calculation of $\lambda^*$ (Equation (6.33))

$\lambda^*$ is a steady state, so that $\lambda^* = zQ_t e_t^i(\cdot) \lambda^* + 1$. Plugging in the interior solution of $e^o(\cdot)$, Equation (6.28), we obtain:

$$
\lambda^* = \frac{zQ_t}{2\alpha \gamma} \left[ \alpha \left( \lambda^* + \gamma \left( 1 - \frac{d_t}{v_t} \right) \right) - c^S - \rho_t \right] \lambda^* + 1
$$

Rearranging yields

$$(\lambda^*)^2 + \left( \gamma \left( 1 - \frac{d_t}{v_t} \right) - \frac{(c^S + \rho_t)}{\alpha} - \frac{2\gamma}{zQ_t} \right) \lambda^* + \frac{2\gamma}{zQ_t} = 0$$

so that

$$
\lambda^*_{1,2} = -\frac{1}{2} \left( \gamma \left( 1 - \frac{d_t}{v_t} \right) - \frac{(c^S + \rho_t)}{\alpha} - \frac{2\gamma}{zQ_t} \right) \pm \sqrt{\frac{1}{4} \left( \gamma \left( 1 - \frac{d_t}{v_t} \right) - \frac{(c^S + \rho_t)}{\alpha} - \frac{2\gamma}{zQ_t} \right)^2 - \frac{2\gamma}{zQ_t}}
$$

Calculating $\frac{\partial^2 \lambda^*}{\partial \lambda^*}$ we receive $\frac{zQ_t}{\gamma} > 0$, so that the trajectory is strictly convex. Hence is the lower value of $\lambda^*$ not of interest. It follows:

$$
\lambda^*(Q_t, \rho_t, d_t, v_t) = -\frac{1}{2} \left( \gamma \left( 1 - \frac{d_t}{v_t} \right) - \frac{(c^S + \rho_t)}{\alpha} - \frac{2\gamma}{zQ_t} \right) \pm \sqrt{\frac{1}{4} \left( \gamma \left( 1 - \frac{d_t}{v_t} \right) - \frac{(c^S + \rho_t)}{\alpha} - \frac{2\gamma}{zQ_t} \right)^2 - \frac{2\gamma}{zQ_t}}
$$
Appendix E

Appendix to Chapter 7

E.1 Dynamics in Sector 1

Proposition E.1
Let the trajectory be strictly convex in the area \([\lambda^S(n_t), \lambda^a(n_t)]\).

(a) Let \(\lambda^S(n_t) > 1\) and \(zh(1) \geq 1\): There exists one unstable stationary state at a level \(\lambda^*(n_t)\) and the locally stable poverty trap stationary state at \(\lambda = 1\) with \(1 < \lambda^S(n_t) < \lambda^*(n_t) < \lambda^a(n_t)\).

(b) Let \(\lambda^S(n_t) > 1\) and \(zh(1) < 1\): There are three possible scenarios:

1. Let \(zh(1)\lambda^a(n_t) + 1 > \lambda^a(n_t)\): There exists an unstable, middle stationary state at a level \(\lambda^*(n_t)\), a second, locally stable, upper stationary state at a level \(\lambda^{**}(n_t)\), and the locally stable poverty trap at \(\lambda = 1\) with \(1 < \lambda^S(n_t) < \lambda^*(n_t) < \lambda^{**}(n_t)\).

2. Let \(zh(1)\lambda^a(n_t) + 1 = \lambda^a(n_t)\): There exists a stationary state at \(\lambda^a(n_t)\) of which stability depends upon the starting point. Only if \(\lambda_0 > \lambda^a(n_t)\) will \(\lambda\) converge to \(\lambda^a(n_t)\). Furthermore there exists the locally stable poverty trap at \(\lambda = 1\) with \(1 < \lambda^S(n_t) < \lambda^a(n_t)\).

3. Let \(zh(1)\lambda^a(n_t) + 1 < \lambda^a(n_t)\): There exists only the poverty trap as a stable stationary state.

(c) Let \(\lambda^S(n_t) = 1\) and \(zh(1) \geq 1\): There are two possible patterns:

1. Let \(\lim_{\lambda \to 1} \frac{d\lambda^{a+1}}{d\lambda} < 1\): There exists an unstable stationary state at a level \(\lambda^*(n_t)\) and a locally stable poverty trap at \(\lambda = 1\) with \(\lambda^S(n_t) < \lambda^*(n_t) < \lambda^a(n_t)\).
(2) Let \( \lim_{\lambda \to 1} \frac{d \lambda_{i+1}}{d \lambda_i} \geq 1 \): There exists only an unstable stationary state at \( \lambda = 1 \).

(d) Let \( \lambda^S(n_t) = 1 \) and \( zh(1) < 1 \): There are four possible patterns:

1. Let \( \lim_{\lambda \to 1} \frac{d \lambda_{i+1}}{d \lambda_i} \geq 1 \) and \( zh(1) \lambda^o(n_t) + 1 > \lambda^a(n_t) \): There exists a stable stationary state at a level \( \lambda^*(n_t) \) and an unstable stationary state at \( \lambda = 1 \) with \( \lambda^S(n_t) < \lambda^a(n_t) < \lambda^*(n_t) \).

2. Let \( \lim_{\lambda \to 1} \frac{d \lambda_{i+1}}{d \lambda_i} < 1 \) and \( zh(1) \lambda^o(n_t) + 1 > \lambda^a(n_t) \): There exist an unstable middle stationary state at a level \( \lambda^*(n_t) \), a second, locally stable, upper stationary state at a level \( \lambda^{**}(n_t) \), and a locally stable stationary state at \( \lambda = 1 \) establishing a poverty trap, with \( \lambda^S(n_t) < \lambda^*(n_t) < \lambda^a(n_t) < \lambda^{**}(n_t) \).

3. Let \( \lim_{\lambda \to 1} \frac{d \lambda_{i+1}}{d \lambda_i} < 1 \) and \( zh(1) \lambda^o(n_t) + 1 < \lambda^a(n_t) \): There exists only a globally stable poverty trap stationary state at \( \lambda = 1 \).

4. Let \( \lim_{\lambda \to 1} \frac{d \lambda_{i+1}}{d \lambda_i} < 1 \) and \( zh(1) \lambda^o(n_t) + 1 = \lambda^a(n_t) \): There exists one stationary state at \( \lambda^*(n_t) \) of which the stability again depends on the starting point, and a locally stable poverty trap state at \( \lambda = 1 \).

(e) \( \lambda^S(n_t) \) does not exist (respectively, formally, \( \lambda^S(n_t) < 1 \), \( \lambda^a(n_t) > 1 \), and \( zh(1) \geq 1 \). We have no lower threshold so that even at \( \lambda_t = 1 \) the resulting level of \( \lambda_{t+1} \) will be higher than unity but lower than \( zh(1) + 1 \). There are three possible cases:

1. There exists no stationary state and even for \( \lambda_0 = 1 \) continuous, sustainable human capital growth occurs. If \( \lim_{\lambda \to 1} \frac{d \lambda_{i+1}}{d \lambda_i} \geq 1 \) this is always the case.

2. Consider \( \lim_{\lambda \to 1} \frac{d \lambda_{i+1}}{d \lambda_i} < 1 \). There exists one point of tangency establishing a stationary state at some level \( \lambda^*(n_t) \) where stability depends upon the starting point with \( \lambda^*(n_t) < \lambda^a(n_t) \).

3. Consider \( \lim_{\lambda \to 1} \frac{d \lambda_{i+1}}{d \lambda_i} < 1 \). There exists a lower, locally stable stationary state at a level \( \lambda^*(n_t) \) and a second, unstable one at a level \( \lambda^{**}(n_t) \) with \( \lambda^*(n_t) < \lambda^{**}(n_t) < \lambda^a(n_t) \).

(f) \( \lambda^S(n_t) \) does not exist (respectively, formally, \( \lambda^S(n_t) < 1 \), \( \lambda^a(n_t) > 1 \), and \( zh(1) < 1 \). There are five potential patterns:

1. \( zh(1) \lambda^o(n_t) + 1 > \lambda^a(n_t) \) and there exists only one stable stationary state at a level \( \lambda^*(n_t) > \lambda^a(n_t) \). This case definitely occurs if \( \lim_{\lambda \to 1} \frac{d \lambda_{i+1}}{d \lambda_i} \geq 1 \).
Appendix E. Appendix to Chapter 7

Let the trajectory be strictly concave in the area $[\lambda^S(n_t), \lambda^a(n_t)]$.

(a) Let $\lambda^S(n_t) > 1$ and $zh(1) \geq 1$: There exists an unstable stationary state at $\lambda^*(n_t)$ and another locally stable one at $\lambda = 1$ establishing a poverty trap with $\lambda^S(n_t) < \lambda^*(n_t) < \lambda^a(n_t)$.

(b) Let $\lambda^S(n_t) > 1$ and $zh(1) < 1$: There are three patterns to distinguish:

1. Let $zh(1)\lambda^a(n_t) + 1 > \lambda^a(n_t)$: There exists a lower, unstable stationary state at $\lambda^*(n_t)$, another, locally stable at $\lambda^{**}(n_t)$, and a poverty trap state at $\lambda = 1$ with $\lambda^S(n_t) < \lambda^*(n_t) < \lambda^a(n_t) < \lambda^{**}(n_t)$.

2. Let $zh(1)\lambda^a(n_t) + 1 = \lambda^a(n_t)$:

   1. There is a stationary state at $\lambda^a(n_t)$ whose stability depends on the starting point and the poverty trap at $\lambda = 1$.
   2. There is a lower, unstable steady state at $\lambda^*(n_t)$ and locally stable steady states at $\lambda^a(n_t)$ and $\lambda = 1$ with $1 < \lambda^*(n_t) < \lambda^a(n_t)$.

3. Let $zh(1)\lambda^a(n_t) + 1 < \lambda^a(n_t)$:

   1. There is only the poverty trap at $\lambda = 1$.
   2. There exists a lower, unstable stationary state at $\lambda^*(n_t)$, another, locally stable at $\lambda^{**}(n_t)$, and a poverty trap state at $\lambda = 1$ with $\lambda^S(n_t) < \lambda^*(n_t) < \lambda^{**}(n_t) < \lambda^a(n_t)$.
(c) Let \( \lambda^S(n_t) = 1 \), and \( z_h(1) \geq 1 \): There exists only an instable stationary state at \( \lambda = 1 \), since \( \min \frac{\partial \lambda_{t+1}}{\partial \lambda_t} = z_h(1) \geq 1 \).

(d) Let \( \lambda^S(n_t) = 1 \), and \( z_h(1) < 1 \). There are three possibilities:

1. Let \( z_h(1) \lambda^a(n_t) + 1 \geq \lambda^a(n_t) \): There is a locally stable stationary state at \( \lambda^s(n_t) \leq \lambda^a(n_t) \), and another unstable one at \( \lambda = 1 \) with \( 1 < \lambda^a(n_t) < \lambda^s(n_t) \).

2. Let \( \lim_{\lambda \to 1} \frac{d \lambda_{t+1}}{d \lambda_t} > 1 \), and \( z_h(1) \lambda^a(n_t) + 1 \leq \lambda^a(n_t) \): There is a locally stable stationary state at \( \lambda^s(n_t) \leq \lambda^a(n_t) \), and an instable one at \( \lambda = 1 \).

3. Let \( \lim_{\lambda \to 1} \frac{d \lambda_{t+1}}{d \lambda_t} \leq 1 \): There is only the poverty trap at \( \lambda = 1 \), which is stable.

(e) \( \lambda^S(n_t) \) does not exist (respectively \( \lambda^S(n_t) < 1 \)), and \( z_h(1) \geq 1 \). No matter whether \( \lambda^a(n_t) \leq 1 \) or not, there exists no stationary state; sustainable growth of the household’s human capital stock occurs.

(f) \( \lambda^S(n_t) \) does not exist (respectively \( \lambda^S(n_t) < 1 \)), and \( z_h(1) < 1 \). No matter whether \( \lambda^a(n_t) \leq 1 \) or \( z_h(1) \lambda^a(n_t) + 1 \geq \lambda^a(n_t) \) or not, there is a stable stationary state at \( \lambda^s(n_t) > 1 \).

Note that the cases Proposition E.1 (d)(1), (f)(1), and Proposition E.2 (d)(1), (d)(2), and (f) are similar in structure to the neoclassical growth model.\(^1\) If \( \lambda^a(n_t) < 1 \), in other words does not exist, the trajectory is linear and there is no possibility of a poverty trap; if \( z_h(1) \geq 1 \) there is no steady state at all and if \( z_h(1) < 1 \), there exists a stable high level steady state.

Figure 7.2 illustrates Proposition E.1 (a) (left curve), (c)(1) (middle curve), and (e)(1) (right curve); Figure 7.3 illustrates Proposition E.1 (e)(2) and (e)(3), Figure 7.4 Proposition E.2 (a), Figure 7.5 Proposition E.2 (b)(3) 2., and Figure 7.6 Proposition E.1 (e)(1) and Proposition E.2 (e) for the special case where \( \lambda^a \leq 1 \), respectively.

### E.2 The Dynamics of the Land Constraint in a One-Sector Model

Consider that we neglect sector 2, and thus migration. In such a one-sector setting where there is only sector 1, we can state the following result:

\(^1\)Not all cases are covered by Propositions E.1 and E.2, since we have excluded oscillating trajectories.
Appendix E. Appendix to Chapter 7

Lemma E.1
The general land constraint in any period $t$ is given by:

$$\delta_t = \frac{1}{n^a(1)} \left\{ \left[ \sum_{j=1}^{t} \left( n^a \left( \sum_{k=0}^{t-j} (zh(1))^k \right) \right) - n^a \left( \sum_{k=0}^{t} (zh(1))^k \right) \right] \delta_{t-1} \right\}$$

Solving this complex difference equation iteratively we eventually arrive at:

Proposition E.3
The general solution of $\delta_t(\delta_0)$ is given by:

$$\delta_t(\delta_0) = \frac{1}{n^a(1)} \left\{ \left[ n^a \left( \sum_{k=0}^{t-1} [zh(1)]^k \right) - n^a \left( \sum_{k=0}^{t} [zh(1)]^k \right) \right] \delta_0 
+ \left[ n^a \left( \sum_{k=0}^{t-2} [zh(1)]^k \right) - n^a \left( \sum_{k=0}^{t-1} [zh(1)]^k \right) \right] \delta_1(\delta_0) 
+ \ldots + [n^a(1) - n^a(1 + zh(1))] \delta_{t-1}(\delta_0) \right\}$$

with $\delta_0 = \frac{\xi N}{n^a(1)}$, and

$$\delta_t(\delta_0) = \left( \frac{n^a(1) - n^a(1 + zh(1))}{n^a(1)} \right)^t \delta_0$$

for $t = \{0, 1\}$;

$$\delta_t(\delta_0) = \left\{ \sum_{j=0}^{t-2} \left[ n^a \left( \sum_{k=0}^{t-j-1} [zh(1)]^k \right) - n^a \left( \sum_{k=0}^{t-j} [zh(1)]^k \right) \right] \left[ \frac{n^a(1) - n^a(1 + zh(1))}{n^a(1)} \right]^{j+1} \right\} \delta_0$$

for $t = \{2, 3, 4\}$; and

$$\delta_t(\delta_0) = \left\{ \frac{n^a \left( \sum_{k=0}^{t-1} [zh(1)]^k \right) - n^a \left( \sum_{k=0}^{t} [zh(1)]^k \right)}{n^a(1)} \right\} \delta_0 
+ \sum_{j=2}^{3} j \left( \frac{1}{n^a(1)} \right)^j \sum_{l=0}^{t-1} \left[ \left( n^a \left( \sum_{k=0}^{l} [zh(1)]^k \right) - n^a \left( \sum_{k=0}^{l+1} [zh(1)]^k \right) \right) \right] \left( n^a(1) - n^a(1 + zh(1)) \right)^{j-2} 
\cdot \left( n^a \left( \sum_{k=0}^{t-j-l} [zh(1)]^k \right) - n^a \left( \sum_{k=0}^{t-j-l} [zh(1)]^k \right) \right) \left( n^a(1) - n^a(1 + zh(1)) \right)^{j-2} 
\cdot \left( n^a \left( 1 + zh(1) \right) - n^a \left( \sum_{k=0}^{2} [zh(1)]^k \right) \right) \left( n^a(1) - n^a(1 + zh(1)) \right)^t \delta_0$$

for $t = 5$. 

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E.3 Tâtonnement System Stability

In this section, we will analyze a potential adjustment process towards equilibrium (if at all the adjustment process works towards equilibrium). In practice, it is rather unrealistic to assume that all households alike will immediately try to sell their landholdings to migrate to sector 2, when $\Lambda^1_t > \tilde{\Lambda}^1$. It is much more plausible to assume that the households are diverse referring to their level of information and to their speed of reacting. Therefore, we assume that some households are earlier informed about migration incentives and/or react earlier than others following the information that moving to sector 2 is remunerative. This allows us to analyze the potential tâtonnement path more realistically.\(^2\)

In Corollary 7.1 b), we stated that the simultaneous equilibrium is not determinate. Mas-Colell, Whinston, and Green (1995), p. 779, find that in OLG models “a steady-state equilibrium is ... tâtonnement stable at any $t$ if and only if it is determinate.” Hence, we will analyze whether our equilibrium correspondence is stable. That is, we investigate whether $\Lambda^1_t$ converges towards the equilibrium level $\tilde{\Lambda}^1$ if $\int_0^t \mu(i) di > \tilde{\Lambda}^1$.

Definition E.1

Suppose $\int_0^t \mu(i) di > \tilde{\Lambda}^1$. There is (tâtonnement) system stability if, for any starting position $(\Lambda^1_t, \{a^*_{it}\}_{i=0}^1)$, the tuple $(\Lambda^1_t, \{a^*_{it}\}_{i=0}^1)$ converges to some equilibrium $(\tilde{\Lambda}^1, \{a^*_{it}\}_{i=0}^1)$.$^3$

The definition manifests that tâtonnement analyses are “fraught with difficulties.”$^4$ In particular, there appear two different “sorts of time”: we distinguish periods by time index $t$; these represent the length of childhood and adulthood. Nonetheless, Definition E.1 demands that within one period $t$, the tuple $(\Lambda^1_t, \{a^*_{it}\}_{i=0}^1)$ converges towards an equilibrium. Mas-Colell, Whinston, and Green emphasize that this adjustment time “cannot possibly be real time”, p. 621. Tâtonnement processes, in the sense of dynamic adjustment processes in disequilibrium, are not the actual evolution, “but rather ... a tentative trial-and-error process taking place in fictional time ...” (p. 621).

---

$^2$The investigated adjustment process is not a process typically considered in the literature. Hence we do not want to classify our analysis in terms of tâtonnement and non-tâtonnement models [cf. Varian (1992, 1994)]. The use of the term “tâtonnement” should simply emphasize that we analyze adjustment processes in disequilibrium. Refer to Arrow and Hahn (1971), and Hahn (1982) for a general review. Literature in the field of system stability is also Debreu (1974), Debreu and Scarf (1963), and Dierker (1972). The most famous contribution is Walras (1874).

$^3$Note that we restrict ourselves to the case $\Lambda^1_t > \tilde{\Lambda}^1$, since the problem of stability reduces to the issue of the stability of the land market equilibrium, when $\Lambda^1_t \leq \tilde{\Lambda}^1$.

It is clear that once $\Lambda_1^t > \hat{\Lambda}_1$, migration occurs. The remaining plot of land of migrating beneficiaries reverts to the state, and can additionally be distributed to further poor households. New households enter sector 1 and increase anew $\Lambda_1^t$, which may cause further migration of households, and so on. Therefore, there appears a row of adjustment rounds. The question is whether this row converges towards an equilibrium with $\Lambda_1^t = \hat{\Lambda}_1$, or rather explodes.

To distinguish the actual and fictional process, we label the fictional variables by a “ˆ”, as already has been done with $\hat{\mu}_t$ above. To denote the different rounds of adjustment, we add at each variable a subscript on the left side. For example, $\hat{\Lambda}_1$ is the starting fictional fraction of already supported poor households in period $t$ (first round), and $8\hat{\Lambda}_1^8$ is the total level of human capital in sector 1 in adjustment round 8. Furthermore, $\hat{\mathcal{M}}_t$ is the set of households migrating in period $t$ in round $r$, and $\hat{\mathcal{M}}^u_t$ is the union set of migrated households, inclusive round $r$, that is, $\hat{\mathcal{M}}^u_t = (\hat{\mathcal{M}}_t) \cup (\hat{\mathcal{M}}_{t-1}) \cup \ldots \cup (\hat{\mathcal{M}}_1)$.

Let us consider the situation $\int_0^{\hat{\mu}_t} \lambda_t(i) \, di > \hat{\Lambda}_1$, where $\hat{\mu}_t = \int_0^{\hat{\lambda}_t} \frac{a_{t-1}(i)n^a(i)di}{n^a(1)}$ with $a_{t-1}(i) = 1$ for all $i \in [0, \mu_{t-1}]$. Households will migrate until $\Lambda_1^t = \hat{\Lambda}_1$. That is, the in round 1 migrating households together display a mass of human capital of $\int_0^{\hat{\mu}_t} \lambda_t(i) \, di - \hat{\Lambda}_1$. We know that in period $t - 1$ each household displayed $n_{i(t-1)} = \frac{\Lambda_{i(t-1)}}{\Lambda_{t-1}^N} \lambda_{i(t-1)}$ (Condition (7.39)). Therefore, at the beginning of period $t$ there is $n_t = \frac{\Lambda_{i(t-1)}}{\Lambda_{t-1}^N} \lambda_{i(t-1)}$. Furthermore, we assume that the migration decision depends on the post-"normal"-expropriation situation. That is, in every round $r$ the landed property per unit of human capital of each single household equals $\frac{N}{\Lambda_{t-1}^N}$ minus the individual normal expropriation $n^a_t(\lambda_u) = n^a(\lambda_{i(t-1)}) - n^a(\lambda_u)$. Hence, in round 1, all migrating households display $1\hat{a}_u = 0$, and all those which stay $1\hat{a}_u = 1$, so that the government receives (due to migration) additional land of size

$$\left(1\hat{\Lambda}_1 - \hat{\Lambda}_1\right) \cdot \left(\frac{N}{\Lambda_{t-1}^N} - \frac{\int_0^{\hat{\mu}_t} [1 - (1\hat{\lambda}_t(i))] n^a_t(\lambda_t(i)) \, di}{1\hat{\Lambda}_1 - \hat{\Lambda}_1}\right)$$

with $1\hat{\Lambda}_1 = \int_0^{\hat{\mu}_t} \lambda_t(i) \, di$. As a single poor household must be allocated with land of size $n^a(\lambda_u)$ and is endowed with $\lambda_u = 1$, initial total human capital of sector 1 in round 2 is given by:

$$2\hat{\Lambda}_1 = \hat{\Lambda}_1 + \frac{1\hat{\Lambda}_1 - \hat{\Lambda}_1}{n^a(1)} \left[\frac{N}{\Lambda_{t-1}^N} - \frac{\int_0^{\hat{\mu}_t} [1 - (1\hat{\lambda}_u)] n^a_t(\lambda_t(i)) \, di}{1\hat{\Lambda}_1 - \hat{\Lambda}_1}\right].$$

Obviously, $\hat{\Lambda}_1$ moves towards its equilibrium level $\hat{\Lambda}_1$ if $(1\hat{\Lambda}_1) > (2\hat{\Lambda}_1)$. That is, if

$$\left[\frac{N}{\Lambda_{t-1}^N} - \frac{\int_0^{\hat{\mu}_t} [1 - (1\hat{\lambda}_u)] n^a_t(\lambda_t(i)) \, di}{1\hat{\Lambda}_1 - \hat{\Lambda}_1}\right]/n^a(1) < 1.$$
Appendix E. Appendix to Chapter 7

The average normal expropriation per unit of human capital we denote by \( \hat{n}_\tau \):

\[
\hat{n}_\tau = \sum_{j=1}^{\infty} \left[ \frac{\int_{\Lambda_l^1} n^\tau(\lambda_i(i))d\lambda_i}{j\Lambda_l^1 - \Lambda^1} \right]
\] (E.1)

Proposition E.4

If and only if

\[
\frac{N}{\Lambda_{l-1}^1} - \hat{n}_\tau < 1
\]

there is (tâtonnement) system stability.

Proof:

Applying \( \hat{n}_\tau \), we arrive at

\[
2\hat{\Lambda}_t^1 = \bar{\Lambda}^1 + \left( 1\hat{\Lambda}_t^1 - \bar{\Lambda}^1 \right) \frac{N}{\Lambda_{l-1}^1} - \hat{n}_\tau
\]

and thus for any round \( r \)

\[
r\hat{\Lambda}_t^1 = \bar{\Lambda}^1 + \left( 1\hat{\Lambda}_t^1 - \bar{\Lambda}^1 \right) \left( \frac{N}{\Lambda_{l-1}^1} - \hat{n}_\tau \right)^{r-1}
\] (E.2)

Obviously, in case of \( 1\hat{\Lambda}_t^1 > \bar{\Lambda}^1 \), \( r\hat{\Lambda}_t^1 \) converges towards \( \bar{\Lambda}^1 \) if and only if

\[
\lim_{r \to \infty} \left( \frac{N}{\Lambda_{l-1}^1} - \hat{n}_\tau \right)^{r-1} = 0.
\]

\( \frac{N}{\Lambda_{l-1}^1} - \hat{n}_\tau \) is the average size of additional land which the government receives per unit of “migrated human capital”. If \( \left( \frac{N}{\Lambda_{l-1}^1} - \hat{n}_\tau \right) / n^a(1) < 1 \), then, on average, per “migrated unit of human capital” less than one new unit of human capital enters sector 1 due to additional land redistribution. Consequently, the total level of human capital in sector 1 converges to its equilibrium level \( \bar{\Lambda}^1 \). However, if \( \left( \frac{N}{\Lambda_{l-1}^1} - \hat{n}_\tau \right) / n^a(1) > 1 \), then more than one new unit of human capital enters sector 1, and \( \hat{\Lambda}_t^1 \) increases more and more. That is, its remoteness to equilibrium increases: the row of adjustments explodes. A special case is \( \left( \frac{N}{\Lambda_{l-1}^1} - \hat{n}_\tau \right) / n^a(1) = 1 \). This case can be compared with
a satellite revolving the earth on an orbit: the level of $\hat{\Lambda}_1^t$ remains unchanged on its initial disequilibrium level, or is, per accident, initially exactly on its equilibrium level $\tilde{\Lambda}_1^t$.

The tâtonnement process can also be made vivid graphically. Applying (E.2), we obtain:

$$r_{t+1} \hat{\Lambda}_1^t = \left(1 - \frac{N}{n^a(1)} \hat{\Lambda}_1^t - \hat{n}^T\right) \tilde{\Lambda}_1^t + \left(\frac{N}{n^a(1)} \hat{\Lambda}_1^t - \hat{n}^T\right) (r_{t} \hat{\Lambda}_1^t)$$  \hspace{1cm} (E.3)

The stationary state of this difference equation is $\tilde{\Lambda}_1^t$. If $\left(\frac{N}{n^a(1)} - \hat{n}^T\right) / n^a(1) < 1$ the slope of the trajectory is smaller than unity and there is a negative intersection point with the vertical axis. Consequently, the dynamics are such that $\hat{\Lambda}_1^t$ will converge towards its equilibrium level if $r_{t} \hat{\Lambda}_1^t > \tilde{\Lambda}_1^t$ (see Figure E.1). If the slope is steeper than unity, the intersection point is positive (Figure E.2), and if the slope is exactly unity the trajectory is the 45°-line. In both cases, we do not arrive at $\tilde{\Lambda}_1^t$ when the starting point $r_{t} \hat{\Lambda}_1^t > \tilde{\Lambda}_1^t$. Therefore, the indeterminate equilibrium in Proposition 7.5 can but need not to be instable.
E.4 Proofs

Proof of Proposition 7.6:
Firstly, both \(n^a(\lambda_{it})\) and \(n^a(q_t, \lambda_{it})\) guarantee household \(i\) an income of \(c^a\):

\[
A_1(\lambda_{it})^\alpha (n^a(\lambda_{it}))^{1-\alpha} = c^a = A_1(\lambda_{it})^\alpha (n^d_{it})^{1-\alpha} + q_t(n^a(q_t, \lambda_{it}) - n^d_{it})
\]

with \(n^d_{it} = \lambda_{it} ((1 - \alpha) A_1 / q_t)^{1/\alpha}\). If \(n^d_{it} = n^a(q_t, \lambda_{it})\), then it is clear that \(n^a(q_t, \lambda_{it}) = n^a(\lambda_{it})\). This will be the case if

\[
n^d_{it} = \lambda_{it} \left( \frac{(1 - \alpha) A_1}{q_t} \right)^{\frac{1}{\alpha}} = \left( \frac{c^a}{A_1(\lambda_{it})^\alpha} \right)^{\frac{1}{1-\alpha}} = n^a(\lambda_{it}),
\]

that is, if \(\lambda_{it} = c^a \left( \frac{\left( \frac{q_t}{A_1} \right)^{1-\alpha}}{\lambda_{it}} \right)^{1/\alpha} \equiv \tilde{\lambda}_{it}\). Substituting the equilibrium level of land price, \(q_t = A_1(1 - \alpha) (\Lambda^1_r / N)^\alpha\), we arrive at \(\tilde{\lambda}_{it} = c^a A_1^{\frac{1}{1-\alpha}} (\Lambda^1_r / N)^{1-\alpha}\). If \(\lambda_{it} > \tilde{\lambda}_{it}\), then household \(i\) displays \(n^d_{it} > n^a(\lambda_{it})\). The household hence purchases additional land. Since \(n^d_{it}\) maximizes household \(i\)'s income, the household is endowed with an income higher than \(c^a\). It follows that there is no need to transfer as much land as \(n^a(\lambda_{it})\), and \(n^a(\Lambda^1_r; \lambda_{it}) < n^a(\lambda_{it})\). Similarly, if \(\lambda_{it} < \tilde{\lambda}_{it}\), then household \(i\) will sell part of the
transferred land. Since \( n_{it}^d \) maximizes income, household \( i \)'s income again will be higher than \( c^a \), and we obtain \( n^a(\Lambda_1^i, \lambda_{it}) < n^a(\lambda_{it}) \). \( \square \)

E.5 Non-Constant Returns to Scale

Proposition E.5
Consider the production function in sector 1 is \( y^1(\lambda_i, n_i) = A_1(\lambda_i)^\alpha (n_i)^\beta \) with \( \alpha, \beta > 0 \).\(^5\) For all \( \alpha + \beta \neq 1 \), the sector location decision depends upon the individual level of human capital \( \lambda_{it} \): \( \tilde{q}_i = \tilde{q}_i(\lambda_{it}) \).

1. If \( \alpha + \beta < 1 \) (decreasing returns to scale), we have \( \frac{\partial \tilde{q}(\lambda_i)}{\partial \lambda_i} < 0 \), so that, given a certain land price, the higher-skilled households leave sector 1 for sector 2 and the less-skilled stay.

2. If \( \alpha + \beta > 1 \) (increasing returns to scale), we have \( \frac{\partial \tilde{q}(\lambda_i)}{\partial \lambda_i} > 0 \), so that, given a certain land price, the less-skilled households leave sector 1 for sector 2 and the higher-skilled stay.

3. If \( \alpha + \beta = 1 \) (constant returns to scale), we have the case of Proposition 7.5.

Proof of Proposition E.5:
We have \( w_i(\lambda_i, n_i) = A_1 \lambda_i^\alpha (n_i^d)^\beta + q(n_i - n_i^d) \) with \( e_i = 1 \) and \( \beta > 0 \). Optimizing the income via the land demand \( n_i^d \), we obtain:

\[
n_i^d(q) = \left( \frac{\beta A_1 \lambda_i^\alpha}{q} \right)^{\frac{1 - \beta}{\beta}}
\]

A household \( i \) opts to switch location if the following holds:

\[
A_2 \lambda_i + qn_i > A_1 \lambda_i^\alpha (n_i^d(q))^{\beta} - qn_i^d(q) + qn_i
\]

Plugging in \( n^d(q) \) and rearranging yields:

\[
q > \left[ \frac{(A_1)^{\frac{1 - \beta}{\beta}} (\frac{1}{\lambda_i})^{\frac{\beta}{1 - \beta}}}{A_2} \right]^{\frac{\beta}{1 - \beta}} \left( 1 - \frac{1}{\lambda_i} \right)^{\frac{1 - \beta}{\beta}} =: \tilde{q}(\lambda_{it})
\]

Hence, we obtain individual \( \tilde{q}_i = \tilde{q}(\lambda_i) \) for each household. Depending upon the sign of \( 1 - \alpha - \beta \), \( \tilde{q}(\lambda_i) \) increases or decreases in \( \lambda_i \). For decreasing returns to scale\(^6\) the sector-switch land price level \( \tilde{q}(\lambda_i) \) decreases when a household accumulates human capital.

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\(^5\)Do not get confused with the \( \beta \) representing the child’s fraction of consumption in our basic model.

\(^6\)This is often assumed in empirical works.
wherefore the higher-skilled leave agriculture for the human capital sector over the course of time. If we, in contrast, have increasing returns to scale, then the opposite is true. For constant returns to scale, however, the last term with \( \lambda_i \) disappears.
Appendix F

Appendix to Chapter 8

F.1 Proofs

Proof of Proposition 8.1:

We rearrange Equation (8.16):

\[ f(\omega_t^{3*}, \mu_t) \equiv \omega_t^{3*} - \alpha L A_3 \left( \frac{1 - \xi}{(1 - \mu_t)[1 + (1 - e^o(\omega_t^{3*}))\gamma]} \right)^{1 - \alpha L} = 0 \]

Due to the implicit-function theorem, we know that:

\[ \frac{d \omega_t^{3*}}{d \mu_t} \left( \frac{\partial f(\omega_t^{3*}, \mu_t)}{\partial \mu_t} \right) \]

One can calculate:

\[ \frac{\partial f(\omega_t^{3*}, \mu_t)}{\partial \mu_t} = -\alpha L A_3 (1 - \alpha L) [1 + (1 - e^o(\omega_t^{3*}))\gamma]^{1 + \alpha L} \left( \frac{1 - \mu_t}{1 - \xi} \right)^{\alpha L} < 0 \]

\[ \frac{\partial f(\omega_t^{3*}, \mu_t)}{\partial \omega_t^{3*}} = 1 - \alpha L A_3 \gamma (1 - \alpha L) (1 - \mu_t)^{1 + \alpha L} (1 - \xi)^{1 - \alpha L} [1 + (1 - e^o(\omega_t^{3*}))\gamma]^{\alpha L} \frac{\partial e^o(\omega_t^{3*})}{\partial \omega_t^{3*}} \]

Via (8.16) we can rearrange this term to:

\[ \frac{\partial f(\omega_t^{3*}, \mu_t)}{\partial \omega_t^{3*}} = 1 - \eta e^o(\omega_t^{3*}) \omega_t^{3*} (1 - \alpha L) \gamma e^o(\omega_t^{3*}) (1 - \mu_t)^2 [1 + (1 - e^o(\omega_t^{3*}))\gamma] \]

It follows that:

\[ \frac{d \omega_t^{3*}}{d \mu_t} > 0 \quad \Leftrightarrow \quad \eta e^o(\omega_t^{3*}) < \frac{1}{\gamma e^o(\omega_t^{3*}) (1 - \alpha L) (1 - \mu_t)^2 [1 + (1 - e^o(\omega_t^{3*}))\gamma]} \]

\[ \square \]
Appendix F. Appendix to Chapter 8

Proof of Corollary 8.2:

(i) If $\omega^3_t \leq c^S$, we have $e^o_t = 0$, $\lambda_t = 1$, and the labor market clearing wage rate is given by:

$$\omega^3_t = \alpha L A_3 \left( \frac{1 - \xi}{(1 - \mu_t)(1 + \gamma)} \right)^{1-\alpha L}$$

Substituting $\mu_t = (1 + t)\delta$ and $\omega^3_t = c^S$, we obtain the term for $t^S$ stated in the corollary.

(ii) If $w^3_t = c^a$, the equilibrium wage rate is given by:

$$\omega^3_t = \alpha L A_3 \left( \frac{1 - \xi}{1 - \mu_t} \right)^{1-\alpha L}$$

Substituting $\mu_t = (1 + t)\delta$ and $\omega^3_t = c^a$, we obtain the term for $t^a$ stated in the corollary.

Finally, we have $\lim_{\mu_t \to 1} w^3_t = \infty$. Due to $c^S < c^a < \infty$, we infer $\mu_x < 1$ for $x = \{t^S, t^a\}$.

Proof of Corollary 8.3:

Once period $t^a$ is reached, all day-laborers $i \in (\mu_{t^a}, 1]$ send their children to school full-time, because of $\omega^3_{t^a} = c^a$. Moreover, all land reform beneficiaries $i \in [0, \mu_{t^a}]$ send their children to school full-time. Therefore, although $\mu_{t^a} < 1$, in period $t = t^a + 1$ the education of the society is attained, that is, the land reform is accomplished successfully in period $t^a$. 

\[\square\]
Appendix G

Appendix to Chapter 9

G.1 Derivation of the Global Production Function

Typically the size of a plot of land, as \( n_{it} \), is expressed by its area. The area of a circle, in turn, is given by:

\[
\pi r^2 = 2\pi \int_{x=0}^{r} x \, dx
\]

Each point within this circle produces output of

\[
A_{i1} \left( \frac{\lambda_{it}}{n_{it}} \right)^{\alpha}
\]

where \( n_{it} = \pi (r_{it})^2 \). Thus we obtain:

\[
y^{1}(\lambda_{it}, r_{it}, A_{i1}) = 2\pi A_{i1} \left( \frac{\lambda_{it}}{\pi (r_{it})^2} \right)^{\alpha} \int_{0}^{r_{it}} x \, dx
\]

Calculating that integral yields:

\[
y^{1}(\lambda_{it}, r_{it}, A_{i1}) = A_{i1} \left( \frac{\lambda_{it}}{\pi (r_{it})^2} \right)^{\alpha} \pi (r_{it})^2
\]

Applying \( n_{it} = \pi (r_{it})^2 \), one obtains Equation (9.2).

G.2 Derivation of the Global Income

The revenue is given by the total sell revenue of output: \( y^{1}(\lambda_{it}, n_{it}, A_{i1}) \). At each point of the land area output \( A_{i1} \left( \frac{\lambda_{it}}{\pi (r_{it})^2} \right)^{\alpha} \) is produced. Per unit output and distance costs of \( c \) arise. Thus, neglecting other production costs, we have production costs of:

\[
A_{i1} \left( \frac{\lambda_{it}}{\pi (r_{it})^2} \right)^{\alpha} c \int_{0}^{r_{it}} x \, dx = c \cdot A_{i1} \left( \frac{\lambda_{it}}{\pi (r_{it})^2} \right)^{\alpha} \pi (r_{it})^2
\]
Finally, the transportation costs for reaching the sale market are

\[ cd_i y_{it}^1 = cd_i A_{it} \lambda_{it}^\alpha \left( \pi(r_{it})^2 \right)^{1-\alpha}. \]

Hence, due to \( n_{it} = \pi(r_{it})^2 \), we arrive at Equation (9.3).

### G.3 Proofs

**Proof of Lemma 9.1:**
Following Equation (9.7), we find

\[
\frac{\partial \tilde{q}_{it}}{\partial c(1 + d_i)} = (1 - \alpha) \left\{ \left[ \alpha + c(1 + d_i)(1 - \alpha) \right] \frac{(A_{it}^1)^{1/\alpha}}{A_2} \right\}^{\frac{1}{1-\alpha}} \frac{\alpha[1 - c(1 + d_i)]}{\alpha + c(1 + d_i)(1 - \alpha) - 1}.
\]

\[
\frac{\alpha - \alpha c(1 + d_i)}{\alpha + (1 - \alpha)c(1 + d_i)} < 1 \text{ completes the proof.}
\]

\[ \square \]

**Proof of Proposition 9.1:**
Compare the term in Equation (9.7) with Equation (7.40) and set \( A_{it}^1 = A_1 \). Obviously, \( c(1 + d_i) = 0 \) causes \( \tilde{q}_{it} = \tilde{q} \). Thus, Lemma 9.1 completes the proof.

\[ \square \]
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