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CP-Violation and Baryogenesis on Electroweak Scales:

Cosmological Predictions for the Standard Model and Beyond

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CP-Verletzung und Baryogenese bei der Elektroschwachen Skala:

Zusammenfassung

Wir untersuchen verschiedene Aspekte von CP-Verletzung auf der elektroschwachen Skala und damit zusammenhängend Quantentransport bei der elektroschwachen Baryogenese. Zuerst konzentrieren wir uns auf CP-Verletzung vom CKM Typ wie sie im Standard Modell vorliegt. Wir erklären zugrundeliegende Konzepte und Ursachen für die Kleinheit von CP-Verletzung in direkten Beobachtungen und führen theoretische Schranken für CP-Verletzung im Standard Modell ein. Wir diskutieren die Voraussetzungen der Jarlskog Determinante und geben Beispiele in denen diese Voraussetzungen in einem kosmologischen Kontext nicht erfüllt sind. Als Ursache hierfür finden wir raum-zeitabhängige Massen oder nicht-perturbative Effekte. Eines der Beispiele bezieht sich auf effektive Wirkungen von chiralen Eichtheorien und wir präsentieren einen Formalismus zum Ermitteln von effektiven Wirkungen basierend auf der Weltlinien Repräsentation von Pfadintegralen.

Der Hauptteil der Arbeit beschäftigt sich mit der Transporttheorie von Quantensystemen. Beruhend auf der Herleitung von Transportgleichungen für Systeme mit einem *flavour* verallgemeinern wir auf Systeme mit *mixing* und mehreren *flavours*. Eine herausragende und neue Eigenschaft unseres Formalismus ist das Auftreten von Oszillationen der Nebendiagonalelemente der Dichtematrix analog zu Neutrinooszillationen. Im konkreten Fall des MSSM finden wir, dass mit generischen Parametern elektroschwache Baryogenese nicht ausreicht, um die beobachtete Baryonasymmetrie zu erklären.

CP-Violation and Baryogenesis on Electroweak Scales:

Abstract

In this work we study different aspects of CP-violation on electroweak scales and in this context quantum transport in electroweak baryogenesis. First we focus on CP-violation of CKM type as it is present in the Standard Model. We explain basic concepts and the reason for the smallness of CP-violation in direct observations and introduce theoretical bounds on CP-violation in the Standard Model. We discuss the prerequisites of the Jarlskog determinant and give some examples where these assumptions are not satisfied in a cosmological setting. We find that these bounds can be dissatisfied due to space-time dependent masses or non-perturbative effects. One of the examples is linked to effective actions of chiral gauge theories and we present a formalism to determine effective actions based on the worldline representation of path integrals.

The main part of the present work deals with transport theory of quantum systems. Based on the derivation of quantum transport equations for one flavour we general-

ize to mixing systems with several flavours. One prominent and novel feature of our formalism is the appearance of oscillations of the off-diagonal densities analogous to neutrino oscillations. Using our approach in the concrete case of MSSM we find that electroweak baryogenesis is insufficient to explain the observed value of baryon asymmetry generically.

Contents

Chapter 1

Introduction

One of the masterly achievements of quantum field theory (QFT) was the prediction of the positron made by Dirac in 1928 [1]. Since then it is well understood that the discovery of every particle assures the existence of a particle with opposite quantum numbers, its antiparticle. That this insight required the invention of QFT is due to the fact that our environment, as we experience it in everyday observations, only consists of matter and not antimatter. Reason for this discrepancy is the fact that if a particle and its antiparticle come in contact they annihilate and emit their energy as γ-radiation. Not only our earth or solar system consist solely of matter but in fact the whole visible universe does not show a trace of a considerable amount of anti-matter. Detailed studies of the γ-ray distribution have not indicated the existence of any anti-matter dominated areas because that would lead to annihilation effects on the boundary to a matter dominated region [2].

How can it happen that on one hand the theory attests a very high degree of symmetry between particles and anti-particles and on the other hand this is not reflected by our observations? Part of the answer is that the observed matter is only the remnant of a tiny mismatch between matter and anti-matter in the hot plasma of the early universe. This asymmetry is normally quantified by the expression

$$\eta = \frac{n_B - \bar{n}_B}{s} = 0.87 \pm 0.04 \times 10^{-10}, \tag{1.1}$$

where n_B and $\bar{n}_B$ denote the density of baryons and anti-baryons and s the entropy density. All three quantities scale with a^{-3} in the expanding universe, where a is the cosmological scale factor, such that η will be constant during the evolution of the universe. The numerical value of η can be determined based on an analysis of the production of light elements at the epoch of nucleosynthesis [3] or examinations of the cosmic microwave background [4]. In this light, the asymmetry between matter and anti-matter is not as large as it seemed on a first glance.

To explain the deviation of η from zero one could procrastinate the problem and lay

the blame on the initial values generated by the Big Bang. However this is not very satisfactory and in addition it is not in accordance with the present picture of Standard Cosmology. An appealing cosmological model has to explain naturally the flatness and homogeneity of our universe and the absence of super heavy relic particles as for example magnetic monopoles. This is normally attained by prepending an epoch of exponential expansion, called inflation, to the history of our universe. After inflation all matter-antimatter asymmetries are diluted such that the baryon asymmetry has to be created after inflation. This process of baryon generation is usually called baryogenesis.

Already in 1967, Sakharov pointed out the criteria for a viable baryogenesis mechanism [5]: a) Violation of parity (P) and charge parity (CP) symmetry. b) Violation of baryon (B) number conservation. c) Departure from equilibrium. The first two criteria are obvious since all these transformations relate baryons with anti-baryons while departure from equilibrium is only necessary if the CPT symmetry is intact (T denotes time inversion). This is required, since thanks to CPT conservation the mass of particles and antiparticles are equal and hence they have the same distribution function in equilibrium.

In the present thesis we deal with baryogenesis and CP-violation on electroweak scales, a topic that started with the seminal work of Kuzmin, Rubakov and Shaposhnikov [6, 7, 8]. An appealing property of electroweak baryogenesis is that the relevant physics is in principle testable by the next generation of experiments. This does not only render this scenario falsifiable but also makes it very predictive.

At electroweak scales the only source of B-violation is the weak anomaly as discovered by 't Hooft [9]. Nowadays processes that contribute to this anomaly are highly suppressed by a factor $10^{-4\pi/\alpha_w} = 10^{-120}$, while in the early universe thermally induced anomalous processes contributed, the so called sphaleron transitions. They are especially efficient before the electroweak phase transition when the W-bosons are still massless. Since the anomaly only couples to the left-handed particles, the sphaleron breaks in addition to the baryon number the charge conjugation C maximally.

The next ingredient is the departure from equilibrium. In most of the baryogenesis scenarios that are operative on energy scales far beyond the electroweak scale, the hot plasma is driven out of equilibrium by the expansion of the universe. At the time when the plasma of the universe has a temperature of several 100 GeV, the expansion is too slow to compete with the relevant interaction time scales, and thus one needs another origin of the departure from equilibrium. This is provided by an electroweak phase transition when the Higgs field acquires its vacuum expectation value (*vev*). There are several possibilities how the phase transition can proceed. One case is the cross-over, where the Higgs *vev* smoothly and homogeneously changes from the symmetric phase to the broken phase. A cross-over is not able to drive the plasma out of equilibrium.

Another possibility is that the Higgs potential contains a barrier between the sym-

metric and the broken potential minimum (realized by a cubic term in the case of a strong phase transition). In this case the Higgs *vev* will change during the phase transition only if this potential barrier is surmounted by thermal or quantum fluctuations. This will happen first for small regions by bubble nucleation and as soon as these bubbles reach a critical size they will start to expand instead of collapsing. This way the bubble wall profile will deliver a space and time dependent Higgs *vev* that is able to drive the plasma out of equilibrium. This kind of phase transition is denoted as of *first order*. The phase transition is one of the reasons why electroweak baryogenesis is ruled out in the Standard Model. It turns out that the Standard Model does not contain enough bosonic matter coupled to the Higgs-field to generate a strong cubic term, and the phase transition appears as cross-over or second order phase transition [10, 11].

In addition, to make electroweak baryogenesis viable, the phase transition has to fulfill the Shaposhnikov bound on the Higgs *vev* [8, 12]. This criterion states that the Higgs *vev* has to be large enough after the phase transition to suppress the sphaleron effects sufficiently. Otherwise the net baryon number produced during the phase transition would be washed out again instantly. A phase transition that fulfills this requirement is usually called a *strong* first order phase transition. Whether these special conditions are met depends on the model under consideration. We comment on this in chapter 4.

The last prerequisite for baryogenesis is CP-violation, and this is one central object of investigation in the present text. Depending on the model a large variety of CP-violating sources exists. In the Standard Model the only source is the CKM matrix, and it is common lore that this source is to weak to account for the observed value of baryon asymmetry. This prejudice is usually based on an argument by Jarlskog [13, 14]. We elaborate on this point and scrutinize under which conditions in a cosmological setting the assumptions of Jarlskog are not satisfied and Jarlskog's bound on CP-violation can be circumvented. One of the key observations is that Jarlskog's arguments fail if non-perturbative effects from the quark masses are relevant or if the masses are space-time dependent. Motivated by this insight we discuss examples in a cosmological context where these prerequisites are met. Furthermore, we present a formalism that makes it possible to determine the effective action of chiral theories by 'integrating out' the quark degrees of freedom in a non-perturbative and very efficient manner. In addition, this effective action takes into account the space-time dependence of the background fields.

In the main part of this work, we address the problem of quantum transport. One feature of electroweak baryogenesis is that if the quark density is driven out of equilibrium only locally, the sphalerons cannot effectively generate a net baryon number [15, 16]. Hence the CP-violating particle density has to be transported into the symmetric phase by diffusion. Based on the former work of Prokopec, Schmidt and Weinstock [17, 18, 19, 20] we derive new basis independent transport equations for

mixing fermions from first principles. The basis independence of our transport equations resolves some problems and controversies in the literature concerning transport in electroweak baryogenesis calculations and identification of CP-violation. Former approaches had to specify the flavour basis and to project on the diagonal entries, since transport was taken into account by *classical* Boltzmann or diffusion equations. These classical equations describe only the dynamics of decoupled quasi-particle densities, thus neglecting mixing between different particle species. A consequence of mixing effects are oscillations as observed for neutrinos; we expect the appearance of similar effects in our transport equations, since in electroweak baryogenesis scenarios the mass eigenbasis and the interaction eigenbasis do not in general coincide.

Finally we apply our transport equations to the concrete case of MSSM electroweak baryogenesis. We derive some analytical results in limiting cases, present a numerical analysis of electroweak baryogenesis in the MSSM and discuss our results and its phenomenological implications.

Chapter 2

CP-Violation in the Standard Model

Among the Sakharov conditions, CP-violation has a prominent position. In the Standard Model the only source of CP-violation is the CP-phase in the Cabibbo-Kobayashi-Maskawa matrix. Even though this phase is of order one, the resulting CP-violating effects in observables are only tiny. In addition, new sources of CP-violation are restricted by measurements of electric dipole moments, such that there are strict constraints on CP-violation in extensions of the SM.

In this chapter we will explain why in the laboratory the CP-violation of the Standard Model is so small and question if this is necessarily valid in the early universe. The basis of our argument is the Jarlskog determinant that produces an upper bound of CP-violation in the first case. It will be introduced in the first section. The second section will present two ideas how Jarlskog's argument can be circumvented in a cosmological setting.

2.1 The Jarlskog determinant

In the Standard Model the Jarlskog determinant [13, 14] is believed to produce an upper bound on CP-violating effects. The basis for this bound is the following reasoning: Suppose the SM Lagrangian contains two non-hermitian mass matrices for the up and down quarks (they are denoted by $\tilde{m}_u$ and $\tilde{m}_d$ and are due to the Yukawa coupling to the Higgs field) while the coupling of the left-handed quarks to the W bosons is still proportional to unity in flavour space.

Using four unitary flavour matrices for the left/right-handed up/down quarks (U_u^L, U_u^R, U_d^L, U_d^R) the Lagrangian can be rewritten in terms of the mass eigenstates by diagonalizing the mass matrices $m_u = U_u^{L\dagger}\tilde{m}_u U_u^R$, $m_d = U_d^{L\dagger}\tilde{m}_d U_d^R$. The unitary

matrices for the right handed quarks have no physical significance, while the product of the left-handed up/down matrices lead to the CKM matrix C in the coupling term between left-handed quarks and W bosons ($C = U_d^{L\dagger}\, U_u^L$).

The Lagrangians in the mass and in interaction basis are not in one-to-one correspondence: If we would start with mass matrices that need the same left-handed but different right-handed transformation matrices to be diagonalized, we would end up with the same CKM matrix, and the same diagonal mass matrices. If we express now our measurable quantities by the primary non-diagonal mass matrices of the interaction basis, only combinations are allowed that do not include the right-handed transformation matrices after diagonalization.

This observation can be recast by counting the number of parameters. The Lagrangian in the interaction basis has 36 parameters in the two complex quark mass matrices. In the mass eigenbasis this reduces to six real mass parameters and four parameters in the CKM matrix. Hence in observables only special combinations of the initial 36 parameters can appear.

In the SM the combinations of lowest dimension, that do not depend on the right-handed transformation matrices are $\tilde{m}_u\, \tilde{m}_u^\dagger$ and $\tilde{m}_d\, \tilde{m}_d^\dagger$, and it turns out that the first CP-sensitive contribution is the Jarlskog determinant

$$\begin{aligned} &\mathrm{Im}\left(\det[\tilde{m}_u\, \tilde{m}_u^\dagger, \tilde{m}_d\, \tilde{m}_d^\dagger]\right) \\ &= \mathrm{Im}\,\mathrm{Tr}(C m_u^4 C^\dagger m_d^4 C m_u^2 C^\dagger m_d^2) \\ &\approx -2J\; m_t^4 m_b^4 m_c^2 m_s^2, \end{aligned} \tag{2.1}$$

and has dimension 12.

Here J denotes a specific combination of the angles of the CKM matrix [14, 21]. For example, in the Kobayashi-Maskawa parametrization [22]

$$C = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 \mathrm{e}^{i\delta} & c_1 c_2 s_3 + s_2 c_3 \mathrm{e}^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 \mathrm{e}^{i\delta} & c_1 s_2 s_3 - c_2 c_3 \mathrm{e}^{i\delta} \end{pmatrix}, \tag{2.2}$$

it is given by [21]

$$J \;=\; s_1^2 s_2 s_3 c_1 c_2 c_3 \sin(\delta) = (3.0 \pm 0.3) \times 10^{-5}\,, \tag{2.3}$$

where $s_i \equiv \sin(\vartheta_i)$ and $c_i \equiv \cos(\vartheta_i)$ $(i = 1, 2, 3)$.

To derive this special form of the CKM matrix the unobservable phases of the quarks have been adjusted, namely for the up and down quark fields the following transformations have been used

$$u_{L,i} \to e^{i\theta_i^u} u_{L,i}, \quad d_{L,i} \to e^{i\theta_i^d} d_{L,i}, \tag{2.4}$$

resulting into

$$C^{ij} \to e^{i(\theta_j^d - \theta_i^u)} C^{ij} \tag{2.5}$$

for the CKM matrix.

The parameterization (2.2) is not unique and other choices of the phases θ_i^u, θ_i^d lead to different parametrizations with three angles and one complex phase. From this fact one can again infer that only special combinations of the entries in the CKM matrix can appear in observables. These combinations are called *rephasing invariants*. The largest CP-odd *rephasing invariant* is J as given in (2.3).

We already noticed that the quantity (2.1) has mass dimension 12. To make it dimensionless, one can reexpress it in quark Yukawa couplings, what results in a suppression by the 12th power of the W boson mass that is proportional to the Higgs *vev*. In a thermal system this suppression can be weaker, but has to be due to dimensional reasons at least the 12th power of the temperature. On these grounds the first physical CP-violating effect would be of order [23, 24]

$$\left(\tfrac{g_W^2}{2M_W^2}\right)^7 J\, m_t^6 m_b^4 m_c^2 m_s^2 \sim 10^{-22}, \tag{2.6}$$

what is too small for a viable baryogenesis mechanism, as can be seen from Eq. (1.1).

2.2 Applicability of Jarlskog's Determinant

Jarlskog's argument is based on two assumptions: a) That physics does not depend on the chosen basis in flavour space. b) The observable under consideration is perturbative in the quark Yukawa couplings and weak interactions.

However these assumptions are not always satisfied, as can be seen by considering the only experiment with directly measurable CP-violating effects, namely the decay of the neutral Kaon system. The observed CP-violation in the branching ratios is of order 10^{-3} (a detailed discussion can be found in [25]) and clearly not in accordance with Jarlskog's determinant. The reason is, that the initial and final states, the Kaons and Pions, are not invariant under flavour basis changes. Using a basis transformation one could *e.g.* rotate a K-meson into a B-meson, which are of course distinguishable particles in the laboratory. In this light the possibility to single out states with a special flavour content affords the opportunity to circumvent Jarlskog's argument. In last instance, this can be traced back to a violation of the assumption b) since the different masses of the quarks lead to the various particles in the non-perturbative low energy spectrum of hadronic matter.

On the other hand the argument of rephasing invariance (2.5) is still valid, since the quark phases are of course unobservable. This leads to the fact, that CP-violation in the Kaon-system is suppressed by the factor $J = (3.0 \pm 0.3) \times 10^{-5}$.

In the following we will depict two ideas how Jarlskog's argument can be circumvented in a cosmological setting.

2.2.1 Standard Model CP-violation in the Early Universe

In this section we will analyze a CP-violating quantity that appears in a cosmological setting and is relevant for baryogenesis calculations. The discussion will follow the analysis given in [26].

In the early universe the presence of the hot plasma makes is much more intricate to single out some special flavour structure in an observable. In the plasma the particles of all flavours are more or less equally present, and the complete dependence on flavour seems to be perturbative in the quark Yukawa couplings especially for the light quarks.

One way to question the validity of the Jarlskog bound is to look for new quantities that are invariant under flavour basis changes. Electroweak baryogenesis is based on the electroweak phase transition by which the Standard Model particles acquire their masses. Since the masses are proportional to the Higgs *vev*, the masses obtain during a first order phase transition a dependence on space and time. This dependence can be used to construct new rephasing invariant quantities as *e.g.*

$$\mathrm{Tr}\,\left(m\partial_\mu m^\dagger - \partial_\mu m m^\dagger\right) \tag{2.7}$$

that is odd under CP. In [17] it is was proven that in the one flavour fermionic case a space-time dependent complex phase in the mass will lead to a phase shift in the dispersion relation of the form (s denotes the spin of the particle as will be explained in chapter 4)

$$\frac{\delta\omega}{\omega} = s\frac{(m\partial_z m^* - \partial_z m m^*)}{2k_0^3}, \tag{2.8}$$

and will result in CP-violating deviations from thermal equilibrium that are needed for baryogenesis. This pole shift is proportional to $(m\partial_\mu m^* - \partial_\mu m m^*)$ and the generalization of this expression to several flavours (2.7) will be used in the following as an indicator for the magnitude of CP-violation in the hot plasma.

In the Standard Model this quantity vanishes on the tree level since the mass matrices are proportional to their derivatives,

$$m \sim \partial_\mu m, \tag{2.9}$$

such that one has to include self-energy loop corrections to the mass, what will be discussed in the following. The hermitian part of the thermal self-energy of the quarks in the Standard Model reads [27]

$$\begin{aligned}\Sigma_R &= \not{k}\,(K_L\,P_L + K_R\,P_R)\\ &\quad +\not{u}\,(U_L\,P_L + U_R\,P_R)\\ &\quad +M\,P_L + M^\dagger\,P_R,\end{aligned} \tag{2.10}$$

with K_L, K_R, U_L, U_R hermitian 3×3 matrices, M an arbitrary 3×3 matrix, all depending on X_μ, the external energy $\omega = u\cdot k$, and the external momentum $\kappa = \sqrt{\omega^2 - k^2}$ in the rest-frame of the plasma. P_L, P_R denote the left/right-handedness projection operators and u^μ the plasma four vector that in the plasma frame reads $u^\mu = (1,0,0,0)$.

In general all these coefficients can contain CP-violating contributions, but we will focus on the mass term, since it leads to a direct CP-violating shift in the dispersion relation [19, 20]. The mass part of the thermal self-energy of the down quarks in the mass eigenbasis has the form

$$\begin{aligned} M_d &= h_1 m_d + \alpha_w C \frac{m_u^2}{m_W^2} h_2\, C^\dagger m_d \\ &+ \alpha_w^2 \int C \frac{m_u^2}{m_W^2} h_3\, C^\dagger \frac{m_d^2}{m_W^2} h_4\, C \frac{m_u^2}{m_W^2} h_3\, C^\dagger m_d \\ &+ O(\alpha_w^3), \end{aligned}$$

where h_1 and h_4 depend only on m_d^2, while h_2 and h_3 depend on m_u^2. The integral is performed over the energies and momenta of the particles in the loop. The terms including the CKM matrices result only from the loops of the charged Higgs bosons and are displayed in fig. (2.1). Since the derivatives of the mass matrices are proportional to

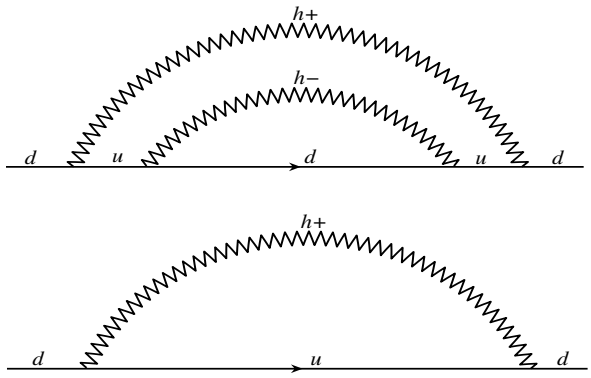

Figure 2.1: Leading contributions to the non-diagonal term in the self-energy at one and at two loop level.

the mass matrices themselves, in the combination Im $(\mathrm{Tr}[M_d \partial M_d^\dagger])$ only the derivatives of the h functions will contribute. Furthermore the first CP-sensitive term has to include at least four CKM matrices due to the relations [14]

$$\begin{aligned} \mathrm{Im}\,\mathrm{Tr}(C X_1 C^\dagger X_2) &= 0, \\ \mathrm{Im}\,\mathrm{Tr}(C X_1 C^\dagger X_2 C X_3 C^\dagger X_4) &= -2J \sum_{ij} \epsilon_{ikl} X_1^k X_3^l \epsilon_{jmn} X_2^m X_4^n, \end{aligned} \qquad (2.11)$$

for diagonal matrices X with the entries X^i and J as in (2.6). Hence we find the following CP-violating contributions (in the following prime denotes differentiation with

respect to the Higgs *vev*)

$$\begin{aligned}
&\frac{1}{2}\,\mathrm{Tr}\,(M_d M_d^{\dagger\prime} - M_d' M_d^\dagger) \\
= \;& \frac{\alpha_w^3}{m_W^8}\int \mathrm{Tr}(C^\dagger m_d^2\, h_4\, C\, m_u^2\, h_3\, C^\dagger m_d^4\, C\, m_u^4\, h_3\, h_2') \\
&+\frac{\alpha_w^3}{m_W^8}\int \mathrm{Tr}(C^\dagger m_d^2\, C\, m_u^2\, h_3'\, C^\dagger m_d^2\, h_4\, C\, m_u^4\, h_3\, h_2) \\
&+\frac{\alpha_w^3}{m_W^8}\int \mathrm{Tr}(C^\dagger m_d^2\, C\, m_u^2\, h_3\, C^\dagger m_d^2\, h_4'\, C\, m_u^4\, h_3\, h_2) \\
&+\frac{\alpha_w^3}{m_W^8}\int \mathrm{Tr}(C^\dagger m_d^2\, C\, m_u^2\, h_3\, C^\dagger m_d^2\, h_4\, C\, m_u^4\, h_3'\, h_2) - \mathrm{h.c} \\
+ \;& O(\alpha_w^4).
\end{aligned}$$

We do not attempt to calculate the two loop contribution, but give qualitative arguments how the enhancement of CP-violating terms appearing in h_2 result from the one loop calculation.

The thermal propagators for the up quarks $S(p)$ and the Higgs bosons $D(p)$ in the Feynman gauge are given by (see [27] for details of the calculation)

$$\begin{aligned}
S(p) &= (\not{p} + m_u)\left(\frac{1}{p^2 - m_u^2 + i\epsilon} + i\Gamma_F(p)\right), \\
D(p) &= \frac{1}{p^2 - m_h^2 + i\epsilon} - i\Gamma_B(p),
\end{aligned}$$

with the thermal parts

$$\begin{aligned}
\Gamma_F &= 2\pi\delta(p^2 - m_u^2) f_F(p\cdot u), \\
\Gamma_B &= 2\pi\delta(p^2 - m_h^2) f_B(p\cdot u),
\end{aligned}$$

and the fermionic and bosonic distribution functions

$$\begin{aligned}
f_F(p\cdot u) &= \frac{1}{\exp\,(p\cdot u/T) + 1}, \\
f_B(p\cdot u) &= \frac{1}{\exp\,(p\cdot u/T) - 1}.
\end{aligned}$$

The $T = 0$ contributions undergo renormalization and are absorbed into the bare parameters of the Lagrangian. The remaining hermitian terms lead to the following form of h_2

$$h_2(\omega,\kappa) = \int \frac{d^4p}{(2\pi)^3}\left(\frac{\Gamma_B(p)}{(p+k)^2 - m_u^2} - \frac{\Gamma_F(p+k)}{p^2 - m_h^2}\right),$$

and after three elementary integrations to

$$\begin{aligned}
h_2(\omega,\kappa) = \frac{1}{\kappa}\int_0^\infty \frac{d|\mathbf{p}|}{2\pi}\Big(&\frac{|\mathbf{p}|}{\epsilon_h} L_2(\epsilon_h, |\mathbf{p}|) f_B(\epsilon_h) \\
&- \frac{|\mathbf{p}|}{\epsilon_u} L_1(\epsilon_u, |\mathbf{p}|) f_F(\epsilon_u)\Big).
\end{aligned}$$

The functions L_1 and L_2 are defined by

$$\begin{aligned} L_{1/2}(\epsilon, |\mathbf{p}|) &= \log\left(\frac{\omega^2 - \kappa^2 \pm \Delta + 2\epsilon\omega + 2\kappa|\mathbf{p}|}{\omega^2 - \kappa^2 \pm \Delta + 2\epsilon\omega - 2\kappa|\mathbf{p}|}\right) \\ &\quad + \log\left(\frac{\omega^2 - \kappa^2 \pm \Delta - 2\epsilon\omega + 2\kappa|\mathbf{p}|}{\omega^2 - \kappa^2 \pm \Delta - 2\epsilon\omega - 2\kappa|\mathbf{p}|}\right), \end{aligned}$$

where ω and κ are the energy and the momentum of the external particle in the rest-frame of the plasma, $\epsilon_h = \sqrt{\mathbf{p}^2 + m_h^2}$, $\epsilon_u = \sqrt{\mathbf{p}^2 + m_u^2}$ and $\Delta = m_u^2 - m_h^2$.

For $|\omega^2 - k^2 - \Delta| > 2|m_u|$ or $|\omega^2 - k^2 + \Delta| > 2|m_h|$ both particles in the loop can be on-shell, whilst otherwise not. This makes the function h_2 strongly dependent on the two loop masses. In the fig. (2.2)-(2.4) h_2 is plotted as a function of the Higgs *vev* for a set of fixed external energies ω and momenta k and three different masses m_u of the quark in the loop. This strong dependence on the Higgs *vev* $\langle\Phi\rangle$ results in large

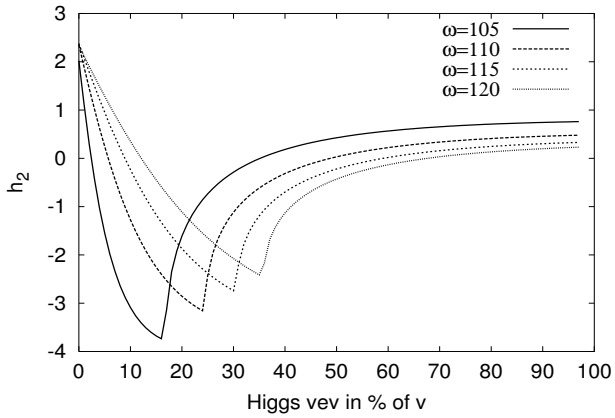

Figure 2.2: Dependence of h_2 on the Higgs *vev* $\langle\Phi\rangle$ in % of its value $v = 246$ GeV at $T = 0$. The external energies and momenta are fixed at $\omega = 105$ GeV to $\omega = 120$ GeV, $k = 100$ GeV, the mass of the quark in the loop is $m_u = 100$ GeV.

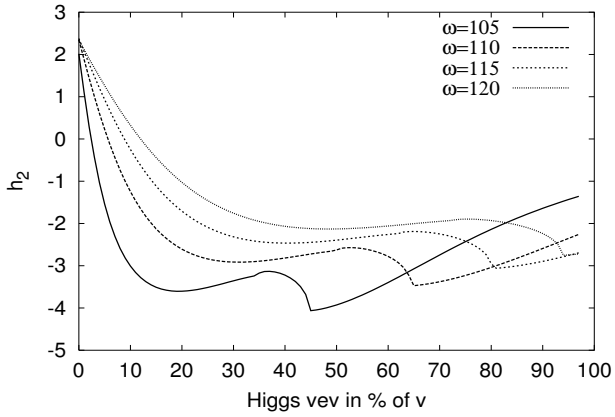

Figure 2.3: Same as fig. (2.2); mass of the quark in the loop is $m_u = 10$ GeV.

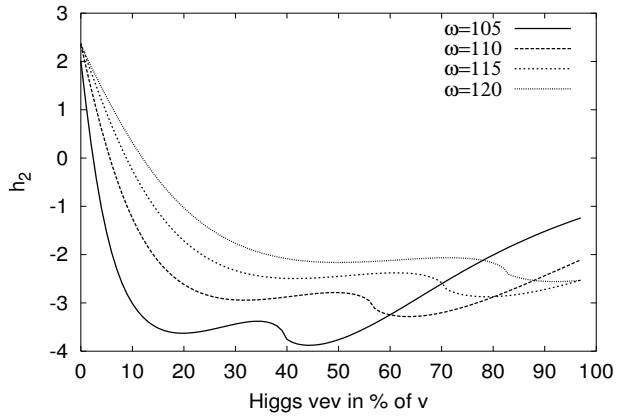

Figure 2.4: Same as fig. (2.2); mass of the quark in the loop is $m_u = 1$ GeV.

derivatives of h_2 due to the wall profile. In fig. (2.5) h_2' is plotted versus the Higgs *vev* for different internal quark masses. Note that the first derivative of h_2 is in a broad

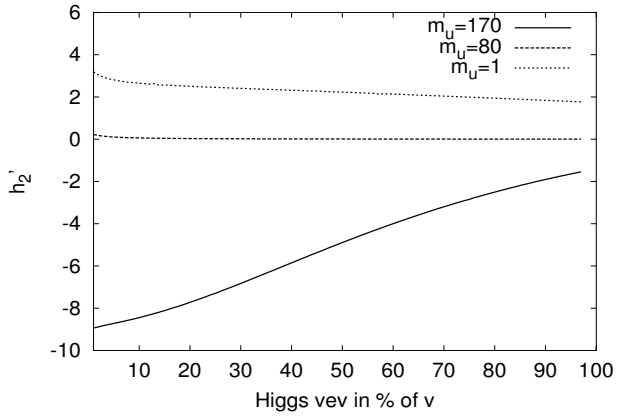

Figure 2.5: Dependence of h_2' on the Higgs *vev* with an on-shell external quark of mass $m_e = 4$ GeV and an internal quark mass in the range 1 GeV to 170 GeV.

range of parameter space $\{\langle\Phi\rangle, m_u\}$ of order *unity* or even larger. Furthermore in the limit of vanishing external mass the sign of h_2' changes in the range, where the internal quark mass agrees with the mass of the charged Higgs boson $m_h = m_W = 80$ GeV. We would like to emphasize that we have chosen the external momentum in all plots to be of the same order of magnitude as the temperature, such that the observed effect is not related to convergence problems in the IR of the thermal QFT.

The self-energy behaves non-perturbatively in the sense that, when expanded in the mass of the internal quark, the main contributions come from higher powers of m_u/m_W. In fig. (2.6) the derivative h_2' is plotted versus the mass of the up quark in the loop. Here it is obvious that the effect is based on a resonance in the loop and cannot be increased arbitrarily by increasing the mass of the quark in the loop.

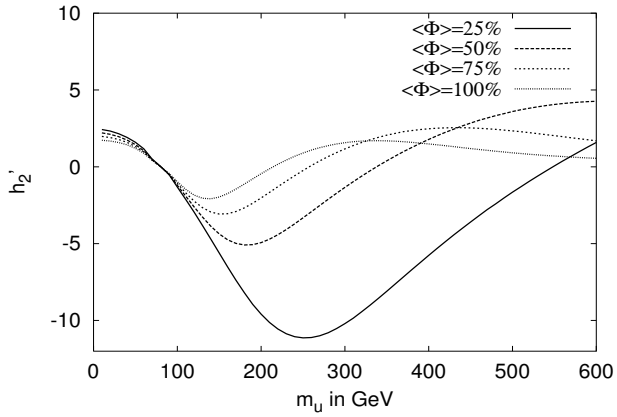

Figure 2.6: Dependence of h_2' on the mass of the quark in the loop with an on-shell external quark of mass $m_e = 4$ GeV. The Higgs *vev* is chosen in a range of 25% to 100% of its value in the broken phase at $T = 0$.

It is reasonable to expect the functions h_3 and h_4 to be after integration effectively of order one as well and not proportional to unity in flavour space. This allows an estimate of the CP-violating pole dependence of the down quarks [26]

$$\frac{\delta\omega}{\omega} \sim J\, m_t^4 m_s^2 m_b^2 m_c^2 \frac{\alpha_w^3 h_2'}{m_W^8 l_w T^3} \sim 10^{-15},$$

(we have used the value $l_w T \approx 20$ and that most of the particles carry a momentum of the order of the temperature) which is seven orders of magnitude larger than the constraint (2.6), but still much too small to account for the BAU.

In summary, we have seen that in a cosmological setting the Jarlskog bound can be circumvented due to a) new rephasing invariants that contain derivatives acting on a classical background and b) non-perturbative effects in the Yukawa coupling as *e.g.* in resonant loop contributions. However, the CKM matrix CP-violation presumably is still by many orders too small to be sufficient for baryogenesis, at least in the concrete example worked out here.

2.2.2 Mild Extensions of the Standard Model

We have seen, that in spite of the enhancement of the axial current, the CP-violating source due to the CKM matrix is too weak to account for the BAU. Thus we will discuss in this subsection further possibilities to generate terms of the form (2.8) in extensions of the SM.

One attractive alternative is the extension to supersymmetric models. In this case one has a new form of CP-violation in the so called chargino sector that makes the operator in Eq. (2.7) already effective on the tree level. We dedicate a chapter to this possibility and will discuss the MSSM at length in chapter 4.

Besides supersymmetric models, more general two Higgs doublet models are most appealing in extending the SM to explain the BAU *via* electroweak baryogenesis. According to Ref. [28], in a certain region of parameter space, the phase transition is of first order and the sphaleron bound is fulfilled (the sphaleron bound [8, 12] has to be fulfilled to avoid a wash-out of the baryon asymmetry after the phase transition). This derivation assumed the two Higgs *vev*s to be proportional to each other, thus simplifying the calculation, while the general case has not been completely studied so far. Baryogenesis is not compatible with this assumption, since if the quotient of the two Higgs *vev*s is constant, it is not possible to generate an axial fermion current *via* Eq. (2.8) on the tree level, and we would rely again on loop contributions as in the preceding subsection. Therefore the character of the phase transition has to be examined in every specific model separately.

Since these models are not subject to the stringent restrictions of supersymmetry, there are several possibilities to introduce new sources of CP-violation. One feasible approach is to avoid flavour changing neutral currents (FCNC) by construction, leading to the so called type I and type II models (for a comprehensive discussion see Ref. [29]). An additional source of CP-violation in this context is the complex phase between the two Higgs fields [30, 31, 32, 33, 34, 35, 36]. These models can, with a reasonable choice of parameters, just marginally explain the generated BAU to be consistent with primordial nucleosynthesis [37] and fail to be in accordance with more recent insights from the WMAP experiment [4] that predict a larger baryon to entropy ratio.

Another possibility, even if less attractive because of minor predictivity, is to admit FCNCs at the tree level, called two Higgs doublet models type III. Due to the large parameter space these models still resist to be ruled out by experiments (for some implications on experimental bounds see [38]) even with quite natural choices for the new parameters and impressive experimental lower bounds on FCNC processes. The rich phenomenology can even account for deviations from the SM as for example the difference between the measurement of the $g-2$ muon factor and its SM prediction [39].

The main difference to models without FCNC and particularly the SM is that during the electroweak phase transition the derivatives of the mass matrices are not necessarily proportional to the mass matrices themselves. This gives the possibility to construct CP-odd rephasing invariants of the form (2.7) on the tree level and even with just two flavours.

The Lagrangian under consideration for the Yukawa couplings of the Higgs fields to the quarks is of the form

$$\begin{aligned} \mathcal{L}_Y^{(III)} &= \eta_{ij}^U \bar{Q}_{i,L} \tilde{\phi}_1 U_{j,R} + \eta_{ij}^D \bar{Q}_{i,L} \phi_1 D_{j,R} \\ &\quad + \xi_{ij}^U \bar{Q}_{i,L} \tilde{\phi}_2 U_{j,R} + \xi_{ij}^D \bar{Q}_{i,L} \phi_2 D_{j,R} + h.c.\,, \end{aligned} \tag{2.12}$$

where we used the standard notation: $Q_{i,L}$ denote the left-handed quark doublets,

$U_{j,R}$ and $D_{j,R}$ $(i,j = 1,2,3)$ the up and down quark singlets, and ϕ_1, ϕ_2 are the two Higgs doublets. To fulfill the experimental bounds, it is sufficient to assume a hierarchy between the couplings $\eta^{U,D}$ and $\xi^{U,D}$. In the basis where only the Higgs field ϕ_1 acquires a *vev* and after diagonalization of the fermion masses the Yukawa couplings are parametrized [38] as

$$\begin{aligned} \hat{\eta}^{U,D}_{ij} &= \frac{m_i \delta_{ij}}{v}, \\ \hat{\xi}^{U,D}_{ij} &= \lambda_{ij} \frac{\sqrt{m_i m_j}}{v}, \end{aligned} \tag{2.13}$$

and $|\lambda_{ij}| \lesssim 10^{-1}$ is needed to suppress $D^0 - \bar{D}^0$ and $B^0 - \bar{B}^0$ mixing sufficiently. Note that a change in the quotient of the two Higgs *vev*s in the mass eigenbasis of the Higgs fields leads to a change in the Yukawa couplings η and ξ in the above used basis with only one Higgs *vev* and at the same time to terms of the form (2.7). The effect can be for example quite large in a two-stage phase transition, as it was seen in Ref.[40, 41].

The resulting pole shift will be of order

$$\frac{\delta\omega}{\omega} \approx |\lambda| \frac{m^2}{4k_0^3 l_w}, \tag{2.14}$$

and for the top quark this is approximately

$$\frac{\delta\omega}{\omega} \sim \frac{|\lambda|}{T l_w}, \tag{2.15}$$

with T the temperature and l_w the wall thickness. The high degree of arbitrariness in these models opens this way a large window for electroweak baryogenesis.

Another idea related to a more complex Higgs sector is to allow for dimension six operators that operate on the TeV scale. By a suitable choice of these dimension six operators, it is possible to make the phase transition strong of first order even in the case of just one Higgs doublet [42, 43]. This can also lead to new sources of CP-violation, viable for baryogenesis [42]. Similar to the two Higgs doublet models, these models lead to FCNCs and new contributions to the electric dipole moments of the SM particles. We will not go into the details of these models and switch to another topic.

2.2.3 Cold Electroweak Baryogenesis

Besides extending the Standard Model, recently there appeared another potential possibility to make baryogenesis viable in the SM. One of the requirements for Jarlskog's argument is that the relevant physics is perturbative in the Yukawa couplings of the quarks. One way to approach the problem of dealing with the complicated non-perturbative physics is to reduce the Standard Model to its bosonic degrees of freedom

by 'integrating out the fermions' in a non-perturbative manner. The CP-violation in the bosonic theory will be represented by higher dimensional operators. In cosmological scenarios the simplest of these CP-violating operators under consideration is of the form [44, 7]

$$\frac{3\,\delta_{CP}}{16\pi^2 M^2}\,\phi^\dagger\phi\ \mathrm{Tr}\ (F^{\mu\nu}\tilde{F}_{\mu\nu}), \tag{2.16}$$

where $F_{\mu\nu}$ denotes the field strength, $\tilde{F}_{\mu\nu}$ its dual, M some mass scale depending on the context, ϕ the Higgs field and δ_{CP} a dimensionless constant characterizing the strength of the CP-violation. In the case of the electroweak phase transition, a natural choice for M would be the temperature of the plasma and for δ_{CP} the Jarlskog determinant

$$\big(\frac{g_W^2}{2M_W^2}\big)^7 J\ m_t^6 m_b^4 m_c^2 m_s^2 \sim 10^{-22}. \tag{2.17}$$

This would render the operator (2.16) useless, since it will not be able to explain a baryon asymmetry of order $\eta_B/s \approx 10^{-10}$. The conclusion would be that CKM matrix CP-violation cannot be the only source of CP-violation in electroweak baryogenesis.

If the relevant physical processes are non-perturbative in the Yukawa couplings Jarlskog's argument will be reduced to its weaker version of rephasing invariance where the relevant quantity is

$$J = s_1^2 s_2 s_3 c_1 c_2 c_3 \sin(\delta) = (3.0 \pm 0.3) \times 10^{-5}. \tag{2.18}$$

As shown in numerical simulations [45] a CP-violating phase δ_{CP} of order J would be enough to explain the observed value of baryon asymmetry in the framework of cold electroweak baryogenesis. In this scenario the electroweak phase transition is not induced by the decreasing temperature in the expanding universe but by an electroweak scale inflation. The Higgs field is coupled to the inflaton, such that at the end of inflation a tachyonic transition of the symmetric Higgs phase to the broken Higgs phase takes place and can lead to a viable mechanism for baryogenesis [46, 47, 48, 49].

These ideas are supported by calculations of the effective action in the covariant derivative expansion. In [50, 51] the effective action was derived for operators up to dimension four in four euclidean dimensions. The surprising result is that in this approximation scheme, the transformation behaviour under flavour basis transformations is explicit and the appearing operators are homogeneous functions in the quark masses and have finite limits in the case of degenerate masses. This suggests that as long as the covariant derivative expansion is valid, the expression (2.18) could be the correct estimate of CP-violation. On the other hand, in [51] only the operators up to dimension four have been determined and it can be shown [52] that up to this order, the appearing CP-violating operators vanish in the Standard Model. The calculation of the effective action in the covariant derivative expansion as presented in [51] is a rather cumbersome task. In chapter 3 we will present a formalism based on the worldline representation

of the path integral that aims to determine the strength of the CP-violation dimension six operators (2.16) or similar operators in chiral gauge theories.

Chapter 3

CP-Violation and Chiral Effective Actions

In this chapter, we present a formalism that allows to calculate the one-loop effective action of chiral gauge theories in a very elegant way. The fermion in the loop is coupled to arbitrary space-time dependent external fields. These background fields include the Higgs field, that is space-time dependent during the phase transition, and the chirally coupled gauge fields. These background fields and their derivatives lead to *rephasing invariant* contributions in the effective action that are in general not restricted to be small by Jarlskog's argument as discussed in the last chapter.

Foundation of our method is the worldline representation of the path integral. The worldline representation of S-matrix elements has first been derived from the Polyakov path integral of string theory in the limit of infinite string tension by Bern and Kosower [53, 54]. Shortly after, Strassler proved that the worldline representation can be obtained from ordinary QFT in the one-loop approximation [55] and without reference to string theory. This formalism then was generalized and used to calculate effective actions and form factors [56, 57, 58, 59] in abelian and non-abelian gauge theories.

The main advantage of the worldline formalism is that it automatically combines different contributions of Feynman diagrams to gauge covariant structures. In addition, it avoids the momentum integration and Dirac algebra, as we will see in one of the following sections.

The chapter is organized as follows. In the first section we review the results of Salcedo [51], who derived the effective action of general chiral gauge theories up to fourth order in covariant derivative expansion. In the subsequent section we analyze the implications of this effective action in the light of CP-violation in the Standard Model. In the last section we rederive the result of Salcedo using the worldline approach and comment on the contributions of sixth order of the effective action that could be

relevant for CP-violation.

3.1 Effective Actions in the Covariant Derivative Approach

In [51] Salcedo calculated the fermion contribution to the euclidean effective action in a derivative expansion up to fourth order in gauge covariant derivatives. The euclidean effective action of n Dirac fields is in the one loop approximation formally given by

$$W = -\mathrm{Tr}(\ln D), \tag{3.1}$$

where D is the Dirac operator of the form

$$\begin{aligned} D &= D_\mu^R \gamma_\mu P_R + D_\mu^L \gamma_\mu P_L + m_{LR} P_R + m_{RL} P_L, \\ P_R &= \tfrac{1}{2}(1+\gamma_5), \quad P_L = \tfrac{1}{2}(1-\gamma_5), \end{aligned} \tag{3.2}$$

with $\gamma_\mu = \gamma_\mu^\dagger \quad (\mu = 1 \ldots 4)$, and $\gamma_5 = -\gamma_1\gamma_2\gamma_3\gamma_4$. The covariant derivatives $D_\mu^{L,R} = \partial_\mu + v_\mu^{L,R}$ depend on chiral $U(n) \times U(n)$ gauge fields $v_\mu^{L,R} = -(v_\mu^{L,R})^\dagger$ and m_{LR} and $m_{RL} = m_{LR}^\dagger$ are $n \times n$ matrix scalar fields.

The field strength and the covariant derivative are defined in the usual way

$$\begin{aligned} F_{\mu\nu}^{L,R} &= \left[D_\mu^{L,R}, D_\nu^{L,R}\right], \\ \hat{D}_\mu m_{LR} &= \partial_\mu m_{LR} + v_\mu^L m_{LR} - m_{LR} v_\mu^R, \end{aligned} \tag{3.3}$$

and analogously for $\hat{D}_\mu m_{RL} = (\hat{D}_\mu m_{LR})^\dagger$.

We are interested in the imaginary part of the effective action W^- that transforms odd under the pseudo-parity transformation $v^L \leftrightarrow v^R$, $m_{LR} \leftrightarrow m_{RL}$, since these terms contain the CP-odd contributions to W.

The imaginary part of the effective action W^- incorporates a chiral anomaly, because the fermion fields form an anomalous representation of the $U(n) \times U(n)$ gauge group. It can be written in the form

$$W^- = \Gamma_{gWZW} + W_c^-, \tag{3.4}$$

where Γ_{gWZW} denotes the so called extended gauged Wess-Zumino-Witten action and W_c^- the gauge invariant remainder. This remainder W_c^- is invariant under the $U(n) \times U(n)$ gauge group. The WZW action Γ_{gWZW} will be gauge invariant if the gauge group is restricted to the anomaly free representation of the Standard Model $U(1) \times SU(2) \times SU(3)$.

Before we shortly review the method used by Salcedo to determine the effective action, we will quote the final results. Using the notation introduced by Salcedo [51],

the chiral invariant remainder can be written in the from

$$W_c^-[v,m] = \frac{1}{48\pi^2}\int d^4x\,\epsilon_{\kappa\lambda\mu\nu}\mathrm{Tr}\,\Big[N_{123}\hat{D}_\kappa\mathbf{m}\hat{D}_\lambda\mathbf{m}\,\mathbf{F}_{\mu\nu} + N_{1234}\hat{D}_\kappa\mathbf{m}\hat{D}_\lambda\mathbf{m}\hat{D}_\mu\mathbf{m}\hat{D}_\nu\mathbf{m}\Big]. \quad (3.5)$$

Here N_{123} is a function of $\mathbf{m}_1$, $\mathbf{m}_2$ and $\mathbf{m}_3$, while N_{1234} is a function of $\mathbf{m}_1$, $\mathbf{m}_2$, $\mathbf{m}_3$ and $\mathbf{m}_4$. The notation of *labeled operators* is used in the sense that the subscript at the masses denotes the position where the masses are inserted in the trace. Furthermore, the boldface letters depict chiral quantities, whose subscript denoting the chirality has been dropped but can be recovered by the context. The constituents $\mathbf{m}$, $\hat{D}$ and $\mathbf{F}$ have to be replaced such that a pseudo parity odd and gauge invariant expression results. As an example consider the first term with $N_{123} = \mathbf{m}_1\,\mathbf{m}_2^3\,\mathbf{m}_3^2$:

$$\begin{aligned}\mathrm{Tr}\,&\Big[N_{123}\,\hat{D}_\kappa\mathbf{m}\hat{D}_\lambda\mathbf{m}\,\mathbf{F}_{\mu\nu}\Big]\\ &= \mathrm{Tr}\,\Big[\mathbf{m}_1\,\mathbf{m}_2^3\,\mathbf{m}_3^2\,\hat{D}_\kappa\mathbf{m}\hat{D}_\lambda\mathbf{m}\,\mathbf{F}_{\mu\nu}\Big]\\ &= \mathrm{Tr}\,\Big[\mathbf{m}\,\hat{D}_\kappa\mathbf{m}\,\mathbf{m}^3\,\hat{D}_\lambda\mathbf{m}\,\mathbf{m}^2\,\mathbf{F}_{\mu\nu}\Big]\\ &= \mathrm{Tr}\,\Big[m_{RL}\,(\hat{D}_\kappa m_{LR})\,m_{RL}m_{LR}m_{RL}\,(\hat{D}_\lambda m_{LR})\,m_{RL}m_{LR}\,F^R_{\mu\nu}\\ &\qquad - \quad (L\leftrightarrow R)\quad\Big].\end{aligned} \quad (3.6)$$

By convention, the positive term starts with a right-handed subscript. This convention leads to the uncommonly appearing relation

$$\mathrm{Tr}\,(m^q\,X) = (-)^q\,\mathrm{Tr}\,(X\,m^q). \quad (3.7)$$

More complex expressions, for example with masses in the denominator, are dealt with by performing a gauge rotation to the mass eigenbasis where both mass matrices are diagonal

$$d := U_L m_{LR} U_R^\dagger = U_R m_{RL} U_L^\dagger. \quad (3.8)$$

In this basis the following expression results

$$\begin{aligned}\mathrm{tr}\,&\Big[N_{123}\,\hat{D}_\kappa m\hat{D}_\lambda m F_{\mu\nu}\Big]\\ &= \sum_{ijk} d_i\,d_j^3\,d_k^2 (U_L\,\hat{D}_\kappa m_{LR}U_R^\dagger)_{ij}(U_L\,\hat{D}_\lambda m_{LR}\,U_R^\dagger)_{jk}(U_R\,F^R_{\mu\nu}U_R^\dagger)_{ki}\\ &\qquad - \quad (L\leftrightarrow R).\end{aligned} \quad (3.9)$$

The functions N_{123} and N_{1234} have been derived in [51] and are quoted in the appendix A.

In the following we will shortly illustrate how Salcedo calculated the functions N_{123} and N_{1234}. The expression we are interested in is the fermionic contribution to the effective action, where the bosonic gauge and Higgs fields are treated as a classical

background. In the picture of Feynman diagrams this corresponds to the one loop contribution with an arbitrary number of external field insertions.

$$\begin{aligned} W &= -\log \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \, \exp(-\int d^d x \, \bar{\psi} D \psi) \\ &= -\mathrm{Tr} \log D, \end{aligned} \tag{3.10}$$

where D again denotes the Dirac operator from Eq. (3.2).

$$\begin{aligned} D = D^R_\mu \gamma_\mu P_R + D^L_\mu \gamma_\mu P_L + m_{LR} P_R + m_{RL} P_L, \\ P_R = \tfrac{1}{2}(1+\gamma_5), \quad P_L = \tfrac{1}{2}(1-\gamma_5). \end{aligned} \tag{3.11}$$

However, to compute this quantity is a cumbersome task, especially for the the imaginary part. Reason for that is that a consistent calculation has to reproduce the anomaly and therefore the calculation cannot be manifestly gauge invariant in each step. One possibility is to break the gauge invariance explicitly to derive a worldline representation of the effective action. This was done in [60, 61] by introducing an additional integration variable α. However the resulting worldline representation is rather complex and to finally arrive at gauge invariant expressions requires subtle cancellations between different contributions.

Therefore it is advisable to choose an indirect approach to determine the effective action as it was done in [62]. The main idea is that instead of the imaginary part of the effective action, we will determine its Schwinger functional derivative

$$\delta W^- = \delta v \, J^-_v + \delta m \, J^-_m. \tag{3.12}$$

The important observation is now that the expression for the Schwinger functional derivative of (3.10) has a covariant representation

$$-\mathrm{Tr} \frac{\delta D}{D} = \delta v \, J^-_{v,cov} + \delta m \, J^-_m. \tag{3.13}$$

This seems to be in contradiction with the observation that W^- is not gauge invariant. The reason is that Eq. (3.13) is not really the derivative of Eq. (3.10) and in fact it is not the derivative of anything. This can be seen if the regularization scheme is made explicit. In the approach by Fujikawa [63] the expression (3.10) is regularized by an additional factor $\exp(-\epsilon \not{D}^2)$. If this factor is introduced *after* the functional derivative is performed, terms are missing which would have resulted from the derivative acting on $\exp(-\epsilon \not{D}^2)$. But these terms are exactly the terms that will lead to the anomaly and break gauge covariance. Therefore one has the possibility to follow an indirect approach. In a first step one determines the covariant currents in Eq. (3.13). Next the current resulting from the Wess-Zumino-Witten action is calculated. This current can be uniquely split into a gauge covariant part and a part that represents the anomaly and does not depend on the masses but only on the gauge fields and the field strengths. The

covariant part that is already taken into account in the Wess-Zumino-Witten action is subtracted from the covariant current (3.13) and one finally obtains a consistent current that is the derivative of the chiral invariant remainder as it appears in Eq. (3.4).

In order to compute the covariant current (3.13) Salcedo used a method introduced by Pletnev and Banin[64]. The method can be summarized as follows: Let $f(m, D)$ be an operator constructed out of the mass m and the covariant derivative D, then the trace over $f(m, D)$ can be reexpressed as

$$< x|f(m,D)|x >= \int \frac{d^d p}{(2\pi)^d}\, f(\bar{m}, \bar{D}) \tag{3.14}$$

with the expansions

$$\begin{aligned} \bar{m} &= m - \hat{D}_\mu m\, \partial^p_\mu + \frac{1}{2!}\hat{D}_\nu \hat{D}_\mu\, m\, \partial^p_\nu \partial^p_\mu + \frac{1}{3!}\hat{D}_\nu \hat{D}_\mu \hat{D}_\lambda\, m\, \partial^p_\nu \partial^p_\mu \partial^p_\lambda + \ldots, \\ \bar{D}_\mu &= p_\mu - \frac{1}{2!}F_{\nu\mu}\partial^p_\nu + \frac{2}{3!}\hat{D}_\lambda F_{\nu\mu}\partial^p_\nu \partial^p_\lambda + \frac{3}{4!}\hat{D}_\kappa \hat{D}_\lambda F_{\nu\mu}\partial^p_\nu \partial^p_\lambda \partial^p_\kappa + \ldots \quad . \end{aligned} \tag{3.15}$$

These expressions are evaluated for $f(m, D) = \frac{\delta D}{D}$. In this expansion the covariant derivatives are treated to be small in comparison to the scale of the mass, what was the motivation to coin the term covariant derivative expansion. Finally one arrives at an expression that appears as a momentum integral that could have been generated by Feynman diagram methods. In two dimensions *e.g.*

$$J^{d=2}_{v,cov}\,\delta v = -2\,\eta_2 \int \frac{d^2 p}{4\pi^2}\,\mathrm{Tr}\,\left[\left(\frac{\mathbf{m}_2}{\Delta_1\Delta_2} - p^2\frac{\mathbf{m}_1+\mathbf{m}_2}{\Delta_1^2\Delta_2}\right)\hat{D}\mathbf{m}\,\delta v\right], \tag{3.16}$$

with $\Delta_i = p^2 + m_i^2$, η_2 a dimension dependent constant, and the conventions of Salcedo. To arrive at this expression is a rather cumbersome task since one has to perform calculations involving the Dirac algebra of the γ matrices. Finally one has to carry out the momentum integration introducing Feynman parameters.

Before we derive how to determine the covariant current by worldline methods, thus avoiding the Dirac algebra and the momentum integrals, we will first show in the next section that up to fourth order in covariant derivative expansion no CP-violating operators appear in the effective action of the Standard Model.

3.2 The Standard Model up to Fourth Order

The discussion in this section basically rederives the results of [52] but in a somewhat different disguise. In [52] it is shown that the operators up to fourth order in the derivative expansion of the effective action of the Standard Model do not contain CP-violation. While the author of [52] excludes CP-violating operators rather by inspection, we will use more general arguments based on rephasing invariants as discussed in chapter 2.

As we have seen in the last section, the covariant remainder of the effective action has up to fourth order in derivative expansion the form

$$W_c^-[v,m] = \frac{1}{48\pi^2}\int d^4x\epsilon_{\kappa\lambda\mu\nu}\mathrm{Tr}\left[N_{123}\hat{D}_\kappa\mathbf{m}\hat{D}_\lambda\mathbf{m}\mathbf{F}_{\mu\nu} + N_{1234}\hat{D}_\kappa\mathbf{m}\hat{D}_\lambda\mathbf{m}\hat{D}_\mu\mathbf{m}\hat{D}_\nu\mathbf{m}\right]. \quad (3.17)$$

To scrutinize this expression in the light of CP-violation, we transform to the mass eigenbasis by a convenient gauge transformation. The electroweak gauge field and the corresponding field strength can be expanded in the usual degrees of freedom $W_\mu^{+,-,3}$ and $W_{\mu\nu}^{+,-,3}$. Due to this transformation, the flavour changing quantities $W_\mu^+, W_\mu^-, W_{\mu\nu}^+, W_{\mu\nu}^-$ will not enter anymore proportional to unity in flavour space but the CKM matrix and its hermitian conjugate will appear.

Using the relation Eq. (2.11) of chapter 2

$$\begin{aligned}\mathrm{Tr}(CX_1C^\dagger X_2) &= 0,\\ \mathrm{Tr}(CX_1C^\dagger X_2CX_3C^\dagger X_4) &= -2J\sum_{ij}\epsilon_{ikl}X_1^kX_3^l\epsilon_{jmn}X_2^mX_4^n,\end{aligned} \quad (3.18)$$

we see that we need at least four factors of the CKM matrix and its hermitian conjugate to generate a rephasing invariant. Hence we see that the first term in Eq. (3.17) cannot contribute since it contains at most three factors C and $C^\dagger$. The second term has the potential to contain C and $C^\dagger$ twice each. However in this case the expression would be proportional to

$$\epsilon_{\kappa\lambda\mu\nu}\,W_\kappa^+\,W_\lambda^+\,W_\mu^-\,W_\nu^- = 0 \quad (3.19)$$

and hence would vanish. As pointed out in [52], the first operator in the covariant derivative expansion that potentially violates CP is the term of sixth order

$$\epsilon_{\kappa\lambda\mu\nu}\mathrm{Tr}\left[N'_{1234}\mathbf{F}_{\kappa\lambda}\mathbf{F}_{\mu\nu}\,\hat{D}_\rho\mathbf{m}\,\hat{D}_\rho\mathbf{m}\right]. \quad (3.20)$$

This expression could contain a term proportional to

$$\epsilon_{\kappa\lambda\mu\nu}\,W_{\kappa\lambda}^+\,W_\rho^+\,W_{\mu\nu}^-\,W_\rho^-, \quad (3.21)$$

but this requires a special structure of N'_{1234} since the W-fields only appear coupled to the left-handed quarks. Hence the determination of N'_{1234} will show whether the corresponding CP-violating operator will support the scenario of cold electroweak baryogenesis to be a viable candidate to explain the baryon asymmetry of the universe. Notice that this operator is not exactly the operator used in cold electroweak baryogenesis (2.16).

In the next section we present a worldline approach to calculate the covariant current. This simplifies the calculation done by Salcedo in the covariant derivative expansion enormously.

3.3 Worldline Approach to the Effective Action

In this section we derive the worldline representation of the real part of the effective action and the covariant current of the imaginary part of the effective action. A worldline representation of the effective action with general external fields for one flavour was first derived in the work [62]. In [60, 61] the worldline representation for several flavours was presented using a more concise notation. The method presented there yields a manifestly gauge covariant expression for the real part. On the other hand, in the representation of the imaginary part, gauge covariance is broken by introducing an additional parameter α that is integrated over. This makes calculations of the imaginary part extremely unhandy. We will follow the approach of Salcedo [51] and calculate not directly the imaginary part of the effective action but the Schwinger functional derivative and will reconstruct the effective action by matching. Since our formalism is based on the worldline method we will use the notation of [60]. We will first rederive the expression of the real part of the effective action as found in [60] before we proceed to the covariant current of the imaginary part.

3.3.1 Real Part of the Effective Action

In the following we will shortly reproduce the worldline representation of the real part of the effective action as given in [60]. We start with the expression

$$W^+ = \frac{1}{2}\log(\mathrm{Det}\mathcal{O} + \mathrm{Det}\mathcal{O}^\dagger) \tag{3.22}$$

where $\mathcal{O} = \not{p} - i\Phi - \gamma_5\Pi - \not{A} - \gamma_5\not{B} + \gamma_\mu\gamma_\nu K_{\mu\nu}$ is the Dirac operator in euclidean space. In addition to the chiral gauge fields and the mass terms we included a spin coupling $K_{\mu\nu}$ and thus are considering the general case. Using the definition of the operator Σ as

$$\Sigma = \begin{pmatrix} 0 & \mathcal{O} \\ \mathcal{O}^\dagger & 0 \end{pmatrix} \tag{3.23}$$

this can be transformed into

$$W^+ = \frac{1}{2}\log(\mathrm{Det}\Sigma) = \frac{1}{4}\log(\mathrm{Det}\Sigma^2) = \frac{1}{4}\mathrm{Tr}\log\Sigma^2. \tag{3.24}$$

The next step is to use the Schwinger integral trick to transform the logarithm into an integral over an exponential expression. This identity is only valid in a strict sense for positive operators and thus we squared the Σ operator in the expression of the effective action. Using the Schwinger integral representation of the logarithm we obtain

$$W^+ = \int_0^\infty \frac{dT}{T}\,\mathrm{Tr}\,\exp(-T\Sigma^2). \tag{3.25}$$

Even though the quantity W^+ is gauge invariant this characteristic is not explicit in the expression Eq. (3.25). To illuminate the gauge invariance one has to perform some γ matrix gymnastics. Introducing six-dimensional hermitian euclidean Γ matrices, satisfying $\{\Gamma_A, \Gamma_B\} = 2\delta_{AB}, (A, B = 1..6)$

$$\Gamma_\mu = \begin{pmatrix} 0 & \gamma_\mu \\ \gamma_\mu & 0 \end{pmatrix}, \quad \Gamma_5 = \begin{pmatrix} 0 & \gamma_5 \\ \gamma_5 & 0 \end{pmatrix}, \quad \Gamma_6 = \begin{pmatrix} 0 & i\mathbb{1}_4 \\ -i\mathbb{1}_4 & 0 \end{pmatrix},$$
$$\Gamma_7 = \begin{pmatrix} \mathbb{1}_4 & 0 \\ 0 & -\mathbb{1}_4 \end{pmatrix} = -i\Gamma_1\Gamma_2\Gamma_3\Gamma_4\Gamma_5\Gamma_6 \quad , \tag{3.26}$$

the Σ matrix reads

$$\Sigma = \Gamma_\mu(p_\mu - A_\mu) + \Gamma_6\Phi - \Gamma_5\Pi - i\Gamma_\mu\Gamma_5\Gamma_6 B_\mu - i\Gamma_\mu\Gamma_\nu\Gamma_6 K_{\mu\nu}. \tag{3.27}$$

To obtain a manifestly chiral covariant expression for Σ one has to perform a basis change as described in [60]. The choice of basis is motivated by the fact that the operator $i\Gamma_5\Gamma_6$ can be relabeled as an internal quantum number

$$M^{-1} i\Gamma_5\Gamma_6 M = \begin{pmatrix} \mathbb{1}_4 & 0 \\ 0 & -\mathbb{1}_4 \end{pmatrix}, \quad M = \begin{pmatrix} \mathbb{1}_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbb{1}_2 \\ 0 & 0 & \mathbb{1}_2 & 0 \\ 0 & \mathbb{1}_2 & 0 & 0 \end{pmatrix}. \tag{3.28}$$

In this basis, Σ takes the form

$$\tilde{\Sigma} = M^{-1}\Sigma M = \begin{pmatrix} \gamma_\mu(p_\mu - A^L_\mu) & \gamma_5(-iH + \frac{1}{2}\gamma_\mu\gamma_\nu K^s_{\mu\nu}) \\ -\gamma_5(-iH^\dagger + \frac{1}{2}\gamma_\mu\gamma_\nu K^{s\dagger}_{\mu\nu}) & \gamma_\mu(p_\mu - A^R_\mu) \end{pmatrix}$$

and is manifestly chiral covariant and the transformation behaviour of the different entries is explicit.

We use the notation $A^L = A + B$, $A^R = A - B$, $H = \Phi + i\Pi$ and $K^s = K + i\tilde{K}$ (self-dual). With the definition of the enlarged background fields

$$\mathcal{A}_\mu = \begin{pmatrix} A^L_\mu & 0 \\ 0 & A^R_\mu \end{pmatrix}, \quad \mathcal{H} = \begin{pmatrix} 0 & iH \\ -iH^\dagger & 0 \end{pmatrix}, \quad \mathcal{K}_{\mu\nu} = \begin{pmatrix} 0 & iK^s_{\mu\nu} \\ -iK^{s\dagger}_{\mu\nu} & 0 \end{pmatrix}, \tag{3.29}$$

one arrives at the final expression

$$\tilde{\Sigma} = \gamma_\mu(p_\mu - \mathcal{A}_\mu) - \gamma_5\mathcal{H} - \frac{1}{2}\gamma_\mu\gamma_\nu\gamma_5\mathcal{K}_{\mu\nu} \tag{3.30}$$

and

$$W^+ \;=\; \int_0^\infty \frac{dT}{T} \,\mathrm{Tr}\, \exp(-T\tilde{\Sigma}^2). \tag{3.31}$$

The concept of the worldline approach is now to derive from this expression a one-dimensional path integral representation. In the usual approach this is done by inserting

complete sets of the Hilbert space leading effectively to the Legendre-transformation of the operator in question. In our case Σ contains Γ matrices and thus one has to construct a Hilbert-space for the Γ matrices too. This is done using the coherent state formalism as proposed in [65].

In appendix B we shortly review the fundamentals of the coherent state formalism and its application in the derivation of fermionic path integrals. The final result is that, as expected, in the construction of the path integral the Legendre transformation of the operator $\tilde{\Sigma}$ appears

$$i\dot{x}\cdot p+\frac{1}{2}\psi_\mu\dot{\psi}_\mu+\frac{1}{2}\psi_5\dot{\psi}_5+\frac{1}{2}\psi_6\dot{\psi}_6-\tilde{\Sigma}^2(\Gamma_A\Gamma_B\to\psi_A\psi_B). \tag{3.32}$$

The substitution of the Γ_A matrices by the fields ψ_A has only to be done in the quadratic terms and for $A\neq B$.

The path integral of the real part of the effective actions reads

$$W^+ \;=\; \frac{1}{8}\,\mathrm{Tr}\int_0^\infty\frac{dT}{T}\mathcal{N}\int\mathcal{D}x\int_{AP}\mathcal{D}\psi\,\mathcal{P}\;e^{-\int_0^T d\tau\mathcal{L}(\tau)}, \tag{3.33}$$

where $\mathcal{L}$ denotes the Lagrange density given by

$$\begin{aligned}\mathcal{L}(\tau) \;=\;& \frac{\dot{x}^2}{4}+\frac{1}{2}\psi_\mu\dot{\psi}_\mu+\frac{1}{2}\psi_5\dot{\psi}_5-i\dot{x}_\mu\mathcal{A}_\mu+\mathcal{H}^2-\frac{1}{2}\mathcal{K}_{\mu\nu}\mathcal{K}_{\mu\nu}+2i\psi_\mu\psi_5(\mathcal{D}_\mu\mathcal{H}+i\dot{x}_\nu\mathcal{K}_{\mu\nu})\\ &+i\psi_\mu\psi_\nu(\mathcal{F}_{\mu\nu}+\{\mathcal{H},\mathcal{K}_{\mu\nu}\})-2\psi_\mu\psi_\nu\psi_\rho(\psi_5\mathcal{D}_\mu\mathcal{K}_{\nu\rho}+\frac{1}{2}\psi_\sigma\mathcal{K}_{\mu\nu}\mathcal{K}_{\rho\sigma}).\end{aligned} \tag{3.34}$$

We used the normalization

$$\mathcal{N} \;=\; \int\mathcal{D}p\;e^{-\int_0^T d\tau p^2(\tau)} \tag{3.35}$$

satisfying

$$\mathcal{N}\int\mathcal{D}x\;e^{-\int_0^T d\tau\frac{\dot{x}^2}{4}}=(4\pi T)^{-D/2}\int d^Dx. \tag{3.36}$$

The path integral (3.33) corresponds to a one dimensional 'quantum field theory' (corresponding to relativistic quantum mechanics). The operator $\mathcal{P}$ denotes path ordering and the subscript AP reminds that the boundary conditions of the fermionic fields are anti-periodic as explained in appendix B. The remaining trace is over the internal degrees of freedom, namely the $U(n)\times U(n)$ gauge structure.

The fields $\psi(\tau)$ and $x(\tau)$ depend on the Schwinger proper time $\tau\in[0,T]$. In particular, the field $x(\tau)$ fulfills periodic boundary conditions and thus contains a zero mode of the free field operator $\frac{d^2}{d\tau^2}$. Hence one has to split the field $x(\tau)$ into a constant part and a deviation parametrized as $x(\tau)=y(\tau)+x_0$, $\partial_\tau x_0=0$ and $\int_0^T d\tau\,y(\tau)=0$. At the same time the measure has to be split into $\mathcal{D}x=\mathcal{D}y\,d^Dx_0$, already used in the normalization (3.36). The Green function for the deviation must be defined on the subspace orthogonal to the zero modes

$$\frac{1}{2}\frac{d^2}{d\tau^2}g_b(\tau-\tau')=\delta(\tau-\tau')-\frac{1}{T}\;, \tag{3.37}$$

such that $\frac{1}{2}\frac{d^2}{d\tau^2}\int d\tau' \, g_b(\tau-\tau')\, x(\tau') = y(\tau)$ due to the property $\int_0^T d\tau \, y(\tau) = 0$ and $\partial_\tau x_0 = 0$. Taking into account the condition that $g_b(\tau-\tau')$ has to fulfill periodic boundary conditions,

$$g_b(0,\tau') \;\; = \;\; g_b(T,\tau'), \tag{3.38}$$

and this leads to the Green function (neglecting an arbitrary constant)

$$g_b(\tau-\tau') = -\frac{(\tau-\tau')^2}{T} + |\tau-\tau'|. \tag{3.39}$$

In the following we will comment on the chiral gauge invariance of this expression. While the gauge invariance in Eq. (3.31) was manifest, this feature is not present in the Lagrangian (3.34). There are different possibilities to acquire gauge covariant expressions for the final effective action. One way is to use the Fock-Schwinger gauge [66] as used in [57]. This gauge is defined by

$$y^\mu \mathcal{A}^a_\mu(x_0 + y(\tau)) = 0 \tag{3.40}$$

and can be used to express the gauge field $\mathcal{A}_\mu$ through its field strength $\mathcal{F}_{\mu\nu}$

$$A_\mu(x_0 + y(\tau)) = \int d\eta \, \eta \, \mathcal{F}_{\mu\nu}(x_0 + \eta y(\tau)) \, y^\nu. \tag{3.41}$$

Using this relation one will end up again with gauge covariant expressions that are valid in each gauge and not just in the Fock-Schwinger gauge chosen. Another way to argue that the final expression is gauge invariant is to identify the term $\dot{x}\cdot A$ in the Lagrangian as a Wilson line. The important point is now that this Wilson line will restore the gauge covariance in the derivative expansion. Using the splitting of the space-time coordinate $x(\tau) = x_0 + y(\tau)$, we can expand all background fields as

$$X(x_0 + y(\tau)) = \exp(y(\tau)\cdot\partial_{x_0}) X(x_0). \tag{3.42}$$

This expression again breaks gauge covariance explicitly since the exponential contains an ordinary derivative instead of a covariant derivative. In a next step we will expand the exponential of the Lagrangian

$$e^{-\int_0^T d\tau \mathcal{L}(\tau)} \approx e^{-\int_0^T d\tau \mathcal{L}_0(\tau)} \, (1 + \text{higher orders in the derivative expansion}) \tag{3.43}$$

and just treat the masses non-perturbatively

$$\mathcal{L}_0(\tau) = \frac{\dot{x}^2}{4} + \frac{1}{2}\psi_\mu\dot{\psi}_\mu + \frac{1}{2}\psi_5\dot{\psi}_5 + \mathcal{H}^2(x_0). \tag{3.44}$$

This expansion corresponds to the covariant derivative expansion in the work of Salcedo as described earlier by using the one dimensional rules to generate Feynman graphs. *E.g.* if an operator gives some contribution of higher order

$$\mathcal{P}\int d\tau_1 \ldots y(\tau_1)\cdot\partial_{x_0} X(x_0,\tau_1)\ldots, \tag{3.45}$$

then at the same order of the derivative expansion the following term appears

$$\mathcal{P}\int d\tau_2 d\tau_1 \ldots \dot{y}(\tau_2)\cdot\mathcal{A}(x_0)X(x_0,\tau_1)\ldots. \tag{3.46}$$

Taking account of the periodic boundary conditions in $y(\tau)$ and the path ordering operator $\mathcal{P}$ this will after partial integration lead to

$$\mathcal{P}\int d\tau_1 \ldots [y(\tau_1)\cdot\mathcal{A}, X(x_0,\tau_1)]\ldots \tag{3.47}$$

and thus complete the ordinary derivative in (3.45) to a covariant derivative. To check this picture in higher order becomes tedious but it should be clear from the Fock-Schwinger gauge argument given above that the resulting effective action will be gauge covariant at all orders. There we see, that the first contribution from the Wilson loop term that is not 'eaten up' by completing ordinary derivatives to covariant derivatives is of order $\dot{y}_\mu y_\nu \mathcal{F}_{\mu\nu}$ and will not enter in the calculation that will be presented in section 3.4. Thus we can neglect the Wilson loop term and substitute the ordinary derivatives in the Taylor expansion by covariant derivatives.

As already mentioned, the fermionic functions fulfill antiperiodic boundary conditions and hence the fermionic propagator is given by

$$g_f(\tau-\tau') = \frac{1}{2}\mathrm{sign}(\tau-\tau'). \tag{3.48}$$

This propagator is specified by the conditions

$$\begin{aligned} \partial_\tau g_f(\tau,\tau') &= \delta(\tau-\tau'), \\ g_f(0,\tau') &= -g_f(T,\tau'). \end{aligned} \tag{3.49}$$

Since the boundary conditions of the fermionic fields are antiperiodic, the measure does not include an integration over zero modes. This will be different in the imaginary part of the effective action, where an additional factor of Γ_7 will change the boundary conditions into periodic boundary conditions. In this case the fermionic integration has to be split into an ordinary integration and the integration over the zero-modes as we present in the next section.

3.3.2 Worldline Path Integral for the Imaginary Part of the Effective Action

As explained earlier, for the imaginary part of the action it is not possible to find a chiral covariant expression due to the anomaly. Therefore, we will determine the Schwinger functional derivative of the imaginary part, which in contrast has a chiral invariant description. Thus our starting point is the expression

$$\delta W^- = \frac{1}{2}\delta\log(\mathrm{Det}\mathcal{O}-\mathrm{Det}\mathcal{O}^\dagger) = \frac{1}{2}Tr(\delta\mathcal{O}\frac{1}{\mathcal{O}}-\delta\mathcal{O}^\dagger\frac{1}{\mathcal{O}^\dagger}) \tag{3.50}$$

where $\mathcal{O} = \not{p} - i\Phi - \gamma_5\Pi - \not{A} - \gamma_5\not{B} + \gamma_\mu\gamma_\nu K_{\mu\nu}$ is the Dirac operator in euclidean space.

The functional derivative can be written in chiral invariant form since we regularize after performing the functional derivative, such that (3.50) is strictly speaking not the derivative of the regularized effective action. Indeed it will not be a derivative of anything. However, to recover the effective action from this expression can be accomplished by adding a (unique) local polynomial as described in [67].

In the following we will recast (3.50) into a worldline path integral. A convenient procedure is to square first the expression, such that it contains only terms with an even number of γ matrices. This is necessary in order to use the coherent state formalism, and is achieved by

$$\delta W^- = \frac{1}{2} Tr \begin{pmatrix} 0 & \delta\mathcal{O} \\ -\delta\mathcal{O}^\dagger & 0 \end{pmatrix} \begin{pmatrix} 0 & 1/\mathcal{O}^\dagger \\ 1/\mathcal{O} & 0 \end{pmatrix}, \tag{3.51}$$

what can be rewritten as

$$\delta W^- = \frac{1}{2} Tr\, \chi\, \delta\Sigma\, \Sigma^{-1}, \tag{3.52}$$

with the definitions

$$\Sigma = \begin{pmatrix} 0 & \mathcal{O} \\ \mathcal{O}^\dagger & 0 \end{pmatrix}, \quad \chi = \begin{pmatrix} \mathbb{1}_4 & 0 \\ 0 & -\mathbb{1}_4 \end{pmatrix}. \tag{3.53}$$

Introducing again six-dimensional hermitian euclidean Γ matrices, satisfying

$$\{\Gamma_A, \Gamma_B\} = 2\delta_{AB}, \quad (A, B = 1..6) \tag{3.54}$$

as in Eq. (3.26), the Σ matrix is again of the form

$$\Sigma = \Gamma_\mu(p_\mu - A_\mu) + \Gamma_6\Phi - \Gamma_5\Pi - i\Gamma_\mu\Gamma_5\Gamma_6 B_\mu - i\Gamma_\mu\Gamma_\nu\Gamma_6 K_{\mu\nu}. \tag{3.55}$$

As in [60] and the last subsection, we perform a basis change, such that the expression can again be written in a manifestly chiral covariant way. A basis is chosen in which $i\Gamma_5\Gamma_6$ can be relabeled as an internal quantum number and in this basis, Σ takes again the form (3.29) and is manifestly chiral invariant. We use again the notation $A^L = A + B$, $A^R = A - B$, $H = \Phi + i\Pi$ and $K^s = K + i\tilde{K}$ (self-dual) and the definition of the enlarged background fields (3.29).

Finally one arrives at the final expression

$$\tilde{\Sigma} = \gamma_\mu(p_\mu - \mathcal{A}_\mu) - \gamma_5\mathcal{H} - \frac{1}{2}\gamma_\mu\gamma_\nu\gamma_5\mathcal{K}_{\mu\nu}, \tag{3.56}$$

and χ is transformed into

$$\tilde{\chi} = M^{-1}\chi M = \begin{pmatrix} -\gamma_5 & 0 \\ 0 & \gamma_5 \end{pmatrix}, \tag{3.57}$$

and we note $\{\Sigma, \chi\} = \{\delta\Sigma, \chi\} = 0$. To get a worldline action that is even in γ matrices, we write the effective action in the following symmetric form

$$\begin{aligned}
\delta W^- &= \frac{1}{4}\mathrm{Tr}\,(\tilde{\chi}\,\delta\tilde{\Sigma}\tilde{\Sigma} + \tilde{\Sigma}\chi\,\delta\tilde{\Sigma})\tilde{\Sigma}^{-2} && (3.58)\\
&= \frac{1}{4}\mathrm{Tr}\,\tilde{\chi}\,\left[\delta\tilde{\Sigma}, \tilde{\Sigma}\right]\tilde{\Sigma}^{-2} && (3.59)\\
&= \frac{1}{4}\mathrm{Tr}\int_0^\infty dT\,\tilde{\chi}\,\left[\delta\tilde{\Sigma}, \tilde{\Sigma}\right]\exp(-T\tilde{\Sigma}^2) && (3.60)
\end{aligned}$$

where $\tilde{\Sigma}^2$ results as

$$\begin{aligned}
\tilde{\Sigma}^2 &= (p-\mathcal{A})^2 + \mathcal{H}^2 + \frac{1}{2}\mathcal{K}_{\mu\nu}\mathcal{K}_{\mu\nu} + \frac{i}{2}\gamma_\mu\gamma_\nu(\mathcal{F}_{\mu\nu} + \{\mathcal{H}, \mathcal{K}_{\mu\nu}\} + i\,[\mathcal{K}_{\mu\rho}, \mathcal{K}_{\rho\nu}])\\
&\quad + i\gamma_\mu\gamma_5(\mathcal{D}_\mu\mathcal{H} + \{p_\nu - \mathcal{A}_\nu, \mathcal{K}_{\mu\nu}\}) - \frac{1}{2}\gamma_{\mu\rho\sigma}\gamma_5\mathcal{D}_\mu\mathcal{K}_{\rho\sigma}\\
&\quad - \frac{1}{4}\gamma_{\mu\nu\rho\sigma}\mathcal{K}_{\mu\nu}\mathcal{K}_{\rho\sigma}, && (3.61)
\end{aligned}$$

and $\gamma_{\mu_1..\mu_k}$ denotes the antisymmetric product of $k\,\gamma$-matrices and

$$\mathcal{F}_{\mu\nu} = i\,[\mathcal{D}_\mu, \mathcal{D}_\nu]\,, \quad \mathcal{D}_\mu = \partial_\mu - i\,[\mathcal{A}_\mu, .]\,. \tag{3.62}$$

New in comparison to the real part of the effective action is only the insertion $\left[\delta\tilde{\Sigma}, \tilde{\Sigma}\right]$. To calculate the covariant current, we specify the functional derivative as $\delta\tilde{\Sigma} = \gamma_\mu\delta\mathcal{A}_\mu$ and get

$$\left[\delta\tilde{\Sigma}, \tilde{\Sigma}\right] = -\frac{1}{2}\gamma_{\mu\nu}\,\{\delta\mathcal{A}_\mu, p_\nu - \mathcal{A}_\nu\} - \mathcal{D}_\mu\delta\mathcal{A}_\mu + \gamma_\mu\gamma_5\,\{\delta\mathcal{A}_\mu, \mathcal{H}\}\,. \tag{3.63}$$

The next step is to transform this into a path integral, using the coherent state formalism as introduced in appendix B (see also [65]).

Key observation is that using the relation $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$, one would lose the property that the exponential in the worldline action is even in γ matrices. Therefore, it is better to enlarge the matrices using $\Gamma = \gamma \otimes \left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right)$. Notice that the complete expression including the insertion has an odd number of γ matrices (remember the γ_5 in the basis transformed $\tilde{\chi}$ in (3.57)), such that one can compensate by an additional factor of $\frac{i}{2}\Gamma_7\Gamma_6 = \mathbb{1}_4 \otimes \frac{1}{2}\left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right)$. Now we can use the standard method to transform this into a worldline path integral. The result reads

$$J^\mu_{v,cov}\delta\mathcal{A}_\mu = \frac{1}{8}\int_0^\infty dT\,\mathcal{N}\int\mathcal{D}x\int_P\mathcal{D}\psi\,\mathrm{Tr}\,\mathcal{P}\,\chi\,\omega(T)\,e^{-\int_0^T d\tau\mathcal{L}(\tau)}, \tag{3.64}$$

with the Lagrangian as in 3.34

$$\begin{aligned}
\mathcal{L}(\tau) &= \frac{\dot{x}^2}{4} + \frac{1}{2}\psi_\mu\dot{\psi}_\mu + \frac{1}{2}\psi_5\dot{\psi}_5 - i\dot{x}_\mu\mathcal{A}_\mu + \mathcal{H}^2 - \frac{1}{2}\mathcal{K}_{\mu\nu}\mathcal{K}_{\mu\nu} + 2i\psi_\mu\psi_5(\mathcal{D}_\mu\mathcal{H} + i\dot{x}_\nu\mathcal{K}_{\mu\nu})\\
&\quad + i\psi_\mu\psi_\nu(\mathcal{F}_{\mu\nu} + \{\mathcal{H}, \mathcal{K}_{\mu\nu}\}) - 2\psi_\mu\psi_\nu\psi_\rho(\psi_5\mathcal{D}_\mu\mathcal{K}_{\nu\rho} + \frac{1}{2}\psi_\sigma\mathcal{K}_{\mu\nu}\mathcal{K}_{\rho\sigma}), && (3.65)
\end{aligned}$$

the insertion

$$\omega(\tau) \;=\; -4i\psi_5\psi_\mu\psi_\nu\delta\mathcal{A}_\mu\dot{x}_\nu - 4\psi_5\mathcal{D}_\mu\delta\mathcal{A}_\mu + 2\psi_\mu\left\{\delta\mathcal{A}_\mu,\mathcal{H}\right\}, \tag{3.66}$$

and the normalization

$$\mathcal{N} \;=\; \int \mathcal{D}p\; e^{-\int_0^T d\tau p^2(\tau)} \tag{3.67}$$

satisfying

$$\mathcal{N}\int \mathcal{D}x\; e^{-\int_0^T d\tau \frac{\dot{x}^2}{4}} = (4\pi T)^{-D/2}\int d^D x. \tag{3.68}$$

Here some comments are in order. The matrix Γ_7 has been absorbed into the boundary conditions, such that the fermions have periodic boundary conditions as indicated by the letter P under the integral. A short derivation why the Γ_7 changes the boundary conditions of the path integral can be found in appendix B.

The measure $\mathcal{D}\psi = \mathcal{D}\psi_\mu\mathcal{D}\psi_5$ includes the integration over zero modes, that appear due to the periodic boundary conditions of the fermionic fields. The integration over ψ_6 has been performed since it reduces to a single zero-mode integration over the ψ_6 factor in the insertion. The χ appearing in front of the insertion is not the $\tilde{\chi}$ including γ_5 but as in the definition (3.53). Notice that this factor χ leads to the fact that the imaginary part of the effective action is odd under the pseudo-parity transformation $v^L \leftrightarrow v^R$, $m_{LR} \leftrightarrow m_{RL}$.

The resulting bosonic and fermionic Green functions are

$$\frac{1}{2}g_b(\tau-\tau') = \frac{1}{2}|\tau-\tau'| - \frac{(\tau-\tau')^2}{2T}, \tag{3.69}$$

$$g_f(\tau-\tau') = \frac{1}{2}\dot{g}_b(\tau-\tau') = \frac{1}{2}\mathrm{sign}(\tau-\tau') - \frac{(\tau-\tau')}{T}, \tag{3.70}$$

where a dot indicates a derivative with respect to the first variable. Note that due to the boundary conditions, the fermionic Green function differs from the Green function used in the real part.

We will see, that this description is better suited to perform higher order calculations than the approach of [51], since the integrals reduce to simple τ integrations and do not lead to complicated momentum integrations and tedious Dirac algebra calculations. An inspection of the covariant derivative expansion shows that in addition, the worldline formalism automatically resums certain contributions appearing there. In comparison to the worldline expression in [60] we got rid of an additional integration over the parameter α that had to be introduced to break the chiral invariance and a lengthy insertion in exchange for the term matching, that has to be done after the calculation of the covariant current.

3.4 The Lowest Order Results

In this section we will reproduce some results accomplished in [51] using the derivative expansion of the heat kernel. To make the calculation comparable to this work, we will use the notation of labeled operators as explained in the first section of this chapter.

However, we will not use the same normalization. In [51] the used normalization is

$$\langle X\rangle_D = \frac{\eta_D (D/2)!}{(2\pi)^{D/2} D!}\int d^D x = \begin{cases} \frac{i}{4\pi}\int D^2 x, & D=2 \\ \frac{1}{48\pi^2}\int D^4 x, & D=4 \end{cases} \tag{3.71}$$

where $\eta_D = \pm i^{D/2}$ is deduced from $\gamma_5 = \eta_D \gamma_0 ... \gamma_{D-1}$, while we use the normalization (3.68)

$$\langle X\rangle_D = \frac{1}{(4\pi)^{D/2}}\int d^D x = \begin{cases} \frac{1}{4\pi}\int D^2 x, & D=2 \\ \frac{1}{16\pi^2}\int D^4 x, & D=4 \end{cases} \tag{3.72}$$

which is more convenient for our calculation. The integrals used are given in appendix C.

3.4.1 Lowest Order Result in Two Dimensions

First, we drop the dependence on the spin field $\mathcal{K}^{\mu\nu}$ and just reproduce the results of Salcedo [51]. Furthermore we note, that the path integral includes in two dimensions the integral over three zero modes. Therefore the expansion in background fields has to produce at least the three factors $\psi_1\psi_2\psi_5$. Additional factors of the fields ψ and x have to come in pairs, such that they can be contracted using the Green functions. Let us start with the first term in the insertion (3.66) $\psi_5\psi_\mu\psi_\nu\delta\mathcal{A}_\mu\dot{x}_\nu$. It contains already the necessary ψ factors to saturate the integration over the zero modes and it is of order one in the derivative expansion (because of $\delta\mathcal{A}_\mu$). The field x has to be contracted with another field coming from the Taylor expansion of the background fields. Hence the only combination that leads to a contribution to second order in derivative expansion is a term whose integrand is

$$-4i\chi\psi_5\psi_\mu\psi_\nu\dot{x}_\nu\delta\mathcal{A}_\mu(T)x_\rho\mathcal{D}_\rho\mathcal{H}^2(\tau)\, e^{-\int_0^T d\tau\mathcal{L}_0(\tau)}. \tag{3.73}$$

The integration over the zero modes will lead to

$$\int \mathcal{D}\psi_\mu\mathcal{D}\psi_\nu\mathcal{D}\psi_5\, \psi_\mu\psi_\nu\psi_5 = \epsilon_{\mu\nu}, \tag{3.74}$$

thus yielding the expression

$$-\frac{i}{2}\mathrm{Tr}\int_0^\infty \frac{dT}{T}\,\mathcal{P}\int d\tau_1\,\epsilon_{\mu\nu}\dot{g}_b(T,\tau_1)\chi\delta\mathcal{A}_\mu(T)\mathcal{D}_\nu\mathcal{H}^2(\tau_1)\, e^{-\int_0^T d\tau\frac{\dot{x}^2}{4}+\mathcal{H}^2(x_0)}. \tag{3.75}$$

The calculation of this path integral involves the integral $\mathcal{I}_1^b$ that is given in appendix C and finally we arrive at the contribution

$$-\frac{i}{2}\left\langle \mathcal{I}_1^b(\mathcal{H}_1^2,\mathcal{H}_2^2)(\mathcal{H}_1+\mathcal{H}_2)\epsilon^{\mu\nu}\chi\delta\mathcal{A}_\mu\mathcal{D}_\nu\mathcal{H}\right\rangle. \tag{3.76}$$

Here we used the *labeled operator* conventions of Salcedo to make the non-perturbative dependence on the masses explicit that are given by the background field $\mathcal{H}$.

A second contribution comes from the last term in the insertion (3.66) while the second term does not contribute in the second order of the gradient expansion. The integrand reads

$$4i\chi\psi_\mu\left\{\delta\mathcal{A}_\mu,\mathcal{H}\right\}(T)\psi_\nu\psi_5\mathcal{D}_\nu\mathcal{H}(\tau)\,e^{-\int_0^T d\tau\mathcal{L}_0(\tau)}. \tag{3.77}$$

So in total we reproduce the result of [51]

$$\begin{aligned}\delta W_{d=2}^- &= \frac{i}{2}\left\langle\left(\mathcal{I}_1^a(\mathcal{H}_1^2,\mathcal{H}_2^2)(\mathcal{H}_1-\mathcal{H}_2)-\mathcal{I}_1^b(\mathcal{H}_1^2,\mathcal{H}_2^2)(\mathcal{H}_1+\mathcal{H}_2)\right)\epsilon^{\mu\nu}\chi\delta\mathcal{A}_\mu\mathcal{D}_\nu\mathcal{H}\right\rangle\\ &= \frac{i}{2}\left\langle\frac{2(\mathcal{H}_1^2-\mathcal{H}_2^2)-2\mathcal{H}_1\mathcal{H}_2\log\left(\mathcal{H}_1^2/\mathcal{H}_2^2\right)}{(\mathcal{H}_1-\mathcal{H}_2)(\mathcal{H}_1^2-\mathcal{H}_2^2)}\epsilon^{\mu\nu}\chi\delta\mathcal{A}_\mu\mathcal{D}_\nu\mathcal{H}\right\rangle.\end{aligned} \tag{3.78}$$

Notice that we did not need to perform any Dirac algebra to perform this calculation since it is hidden in the contraction of the fermionic fields. Up to this order this contraction was trivial due to the zero modes. For higher orders this feature will simplify the calculation tremendously. In addition the necessary integrations are elementary and do not require the introduction of Feynman parameters as promised earlier.

3.4.2 Lowest Order Result in Four Dimensions

The calculation in four dimensions will go along the same lines as the calculation in two, but this time the lowest order contribution will be in fourth order in gradient expansion and we will just quote some intermediate steps. For the necessary integrals we again refer to appendix C. First we calculate the contributions containing $\mathcal{F}_{\mu\nu}$. They are

$$\begin{aligned}&4\chi\psi_\mu\left\{\delta\mathcal{A}_\mu,\mathcal{H}\right\}(T)\psi_\lambda\psi_\kappa\mathcal{F}_{\lambda\kappa}(\tau_2)\psi_\nu\psi_5\mathcal{D}_\nu\mathcal{H}(\tau_1)\,e^{-\int_0^T d\tau\mathcal{L}_0(\tau)},\\ &-4\chi\psi_5\psi_\mu\psi_\nu\dot{x}_\nu\delta\mathcal{A}_\mu(T)\psi_\lambda\psi_\kappa\mathcal{F}_{\lambda\kappa}(\tau_2)x_\rho\mathcal{D}_\rho\mathcal{H}^2(\tau_1)\,e^{-\int_0^T d\tau\mathcal{L}_0(\tau)},\\ &-4\chi\psi_5\psi_\mu\psi_\nu\dot{x}_\nu\delta\mathcal{A}_\mu(T)\psi_\lambda\psi_5x_\kappa\mathcal{D}_\lambda\mathcal{D}_\kappa\mathcal{H}(\tau_1)\psi_\rho\psi_5\mathcal{D}_\rho\mathcal{H}(\tau_1)\,e^{-\int_0^T d\tau\mathcal{L}_0(\tau)},\end{aligned}$$

and similar expressions with $\mathcal{F}_{\mu\nu}$ on the last position that can be generated by hermitian conjugation.

Using $g_f(t_1,t_2)+g_f(t_2,t_3)+g_f(t_3,t_1)=\pm\frac{1}{2}$ (the sign depends on the time-ordering of the times t_1, t_2, t_3) and $(\mathcal{D}_\lambda\mathcal{D}_\kappa-\mathcal{D}_\kappa\mathcal{D}_\lambda)\mathcal{H}=[\mathcal{F}_{\kappa\lambda},\mathcal{H}]$, we arrive at the expression

$$\begin{aligned}&\frac{1}{2}\left\langle\left(\mathcal{I}_2^a(\mathcal{H}_1^2,\mathcal{H}_2^2,\mathcal{H}_3^2)(\mathcal{H}_1-\mathcal{H}_3)+\mathcal{I}_2^b(\mathcal{H}_1^2,\mathcal{H}_2^2,\mathcal{H}_3^2)(\mathcal{H}_2+\mathcal{H}_3)\right.\right.\\ &\left.\left.+\mathcal{I}_2^c(\mathcal{H}_1^2,\mathcal{H}_2^2,\mathcal{H}_3^2)(\mathcal{H}_1-\mathcal{H}_2)\right)\epsilon^{\mu\nu\lambda\kappa}\chi\delta\mathcal{A}_\mu\mathcal{F}_{\lambda\kappa}\mathcal{D}_\nu\mathcal{H}\right\rangle.\end{aligned}$$

The terms producing $(\mathcal{D}\mathcal{H})^3$ are

$$\begin{aligned}
&8i\chi\psi_\mu \left\{\delta\mathcal{A}_\mu, \mathcal{H}\right\}(T)\psi_\nu\psi_5\mathcal{D}_\nu\mathcal{H}(\tau_3)\psi_\kappa\psi_5\mathcal{D}_\kappa\mathcal{H}(\tau_2)\psi_\lambda\psi_5\mathcal{D}_\lambda\mathcal{H}(\tau_1)\\
&-8i\chi\psi_5\psi_\mu\psi_\nu\dot{x}_\nu\delta\mathcal{A}_\mu(T)\Big(\psi_\kappa\psi_5\mathcal{D}_\kappa\mathcal{H}(\tau_3)\psi_\lambda\psi_5\mathcal{D}_\lambda\mathcal{H}(\tau_2)x_\rho\mathcal{D}_\rho\mathcal{H}^2(\tau_1)\\
&\quad + \text{ 2 permutations }\Big)\, e^{-\int_0^T d\tau\mathcal{L}_0(\tau)},
\end{aligned} \tag{3.79}$$

yielding

$$\begin{aligned}
\Big\langle\Big(&\mathcal{I}_3^a(\mathcal{H}_1^2,\mathcal{H}_2^2,\mathcal{H}_3^2,\mathcal{H}_4^2)(\mathcal{H}_1-\mathcal{H}_4)+\mathcal{I}_3^b(\mathcal{H}_1^2,\mathcal{H}_2^2,\mathcal{H}_3^2,\mathcal{H}_4^2)(\mathcal{H}_3+\mathcal{H}_4)\\
&-\mathcal{I}_3^c(\mathcal{H}_1^2,\mathcal{H}_2^2,\mathcal{H}_2^2,\mathcal{H}_3^2)(\mathcal{H}_2+\mathcal{H}_3)+\mathcal{I}_3^d(\mathcal{H}_1^2,\mathcal{H}_2^2,\mathcal{H}_3^2,\mathcal{H}_4^2)(\mathcal{H}_1+\mathcal{H}_2)\Big)\\
&\epsilon^{\mu\nu\lambda\kappa}\chi\delta\mathcal{A}_\mu\mathcal{D}_\nu\mathcal{H}\mathcal{D}_\kappa\mathcal{H}\mathcal{D}_\lambda\mathcal{H}\Big\rangle\,.
\end{aligned}$$

These expressions agree with the results stated in [51] up to our differently chosen normalization.

We will in the following not comment on the matching procedure that makes it possible to calculate the gauge covariant remainder of the imaginary part of the effective action out of these currents. An exhaustive treatment of this topic can be found in the work of Salcedo [51], and we hope that the reader appreciates the efficiency of the worldline formalism to reproduce the result of the covariant current elaborated in the covariant derivative expansion there.

3.5 Comments on the Effective Action to Sixth Order

In the following we will comment on the procedure to calculate and match the contributions of order six. There are, due to the different possibilities to contract the Lorentz indices, around twenty contributions to the effective action and the covariant current. Unfortunately the different contributions mix such that we can determine the operator (3.20) not by calculating only one term in the covariant current. The most efficient way to do the matching procedure is by starting with the term

$$\epsilon^{\alpha\beta\gamma\delta}\delta\mathcal{A}_\mu\mathcal{D}_\mu\mathcal{F}_{\alpha\beta}\mathcal{F}_{\gamma\delta} \tag{3.80}$$

since it is only generated by the two expressions

$$\begin{aligned}
&\epsilon^{\alpha\beta\gamma\delta}\mathcal{D}_\mu\mathcal{F}_{\alpha\beta}\mathcal{D}_\mu\mathcal{F}_{\gamma\delta},\\
&\epsilon^{\alpha\beta\gamma\delta}\mathcal{F}_{\mu\alpha}\mathcal{F}_{\mu\beta}\mathcal{F}_{\gamma\delta}.
\end{aligned} \tag{3.81}$$

The first contribution to the current of this kind is given by the integrand

$$\begin{aligned}
&4\chi\psi_5\mathcal{D}_\mu\delta\mathcal{A}_\mu(T)\psi_\alpha\psi_\beta\mathcal{F}_{\alpha\beta}(\tau_1)\psi_\gamma\psi_\delta\mathcal{F}_{\gamma\delta}(\tau_2)\, e^{-\int_0^T d\tau\mathcal{L}_0(\tau)}\\
&\quad\sim \epsilon_{\alpha\beta\gamma\delta}\delta\mathcal{A}_\mu\left(\mathcal{D}_\mu\mathcal{F}_{\alpha\beta}\,\mathcal{F}_{\gamma_\delta}+\mathcal{F}_{\alpha\beta}\,\mathcal{D}_\mu\mathcal{F}_{\gamma_\delta}\right).
\end{aligned} \tag{3.82}$$

The problem with this term is that it is not the derivative of anything. Comparing it with the derivatives of (3.81) we see that the two terms have the wrong relative sign to be matchable with the effective action. One possibility is that this contribution to the current is removed by another contribution. However it turns out that there is only one additional contribution of this form and the sum does not vanish identically, even in the case of only one flavour.

For the terms of fourth order, a similar problem appears. There the contributions from the Wess-Zumino-Witten term, that is needed to take account of the anomaly, canceled the dangerous contributions in the currents. At sixth order we do not expect any anomalous terms and hence this problem seems to remain. The integrability of the covariant current should be more transparent if the regularization is made explicit. However this problem is beyond the scope of the present text and will be addressed elsewhere.

3.6 Summary

We reviewed the derivation of the effective action of chiral gauge theories in the covariant derivative expansion as presented by Salcedo in [51]. In addition the statement of [52] that up to fourth order CP is conserved in the effective action of the SM has been reproduced using general arguments instead of direct inspection.

Subsequently, we derived a worldline representation of the Schwinger functional of the effective action with general couplings and several flavours and used it to reproduce Salcedo's results. The formalism presented here can easily be generalized to include spin-couplings or to derive the covariant density that is the functional derivative with respect to the background mass field.

The detailed calculation of the terms of sixth order including the cosmological relevant operator (3.20) is left for future work.

Chapter 4

Electroweak Baryogenesis in the MSSM

In this chapter a first principle approach to determine the BAU in an electroweak baryogenesis scenario as described in the introduction is discussed. It contains a formalism to derive the baryon asymmetry in the MSSM and a numerical analysis connecting to MSSM phenomenology. This chapter constitutes the main part of the present text and we would like to emphasize that even though we apply in this chapter the derived transport equations to the concrete example of MSSM, the developed techniques are rather general and can be used not only to treat electroweak baryogenesis in other specific models but also in almost every semi-classical situation where transport plays a role.

As we have noticed in the introduction, transport is necessary to carry the CP-violating particle densities from the wall into the symmetric phase, such that the sphaleron can efficiently transform the CP-violation into a baryon asymmetry. Usually transport is described by a set of classical equations as diffusion or Boltzmann equations that are derived for a macroscopic, classical and statistical system. CP-violation on the other hand is a microscopic quantum effect in particle physics based on flavour mixing. The main problem in determining the BAU in an electroweak setting consists in the quest to fill this gap between macroscopic and microscopic physics and to incorporate both aspects into one formalism.

We have seen in the first chapter that in the Standard Model, CP-violation is rather small due to rephasing invariance and that the electroweak phase transition is only a cross-over. In the first section of this chapter an explanation is given why contrary to the Standard Model, electroweak baryogenesis could be feasible in the Minimal Supersymmetric Standard Model. Especially the new source of CP-violation in the chargino sector is discussed.

In the second section a description of the Kadanoff-Baym equations is given that

are the analogon to the Schwinger-Dyson equations in a statistical framework. They present the starting point of the derivation of quantum transport equations.

The third section discusses two former approaches that filled the gap between the quantum source of CP-violation and the classical transport equations by using hand-waving arguments. Some inconsistencies arise related to the question of the correct flavour basis to be chosen. This gives the main motivation for deriving basis independent quantum transport equations as will be done in the subsequent section.

The last section consists of the numerical analysis of the resulting baryon asymmetry from the quantum transport equations of the CP-violating chargino densities in the MSSM.

4.1 Necessary Ingredients for MSSM-EWB

In this section we will describe in which sense the two reasons for the failure of electroweak baryogenesis in the minimal Standard Model are disposed in the MSSM. The first problem is the lack of CP-violation which is solved by an additional source in the chargino sector as will be explained in the next subsection. The second problem is that instead of a strongly first order phase transition the Standard Model provides only a cross-over. In a second subsection a discussion of the phase transition in the MSSM will be given along the lines of the publication [68].

4.1.1 CP-Violation in the Chargino sector

In the MSSM electroweak baryogenesis scenario, there are in principle two new sources of CP-violation during the electroweak phase transition. Reason for this is that the winos and Higgsinos, that are the fermionic superpartners of the W- and Higgs-bosons, carry the same quantum numbers and mix due to Higgs-interactions. The mixed mass eigenstates are called either neutralinos or charginos depending on their electric charge.

Part of the mass matrix is generated by the Yukawa coupling to the Higgs fields that acquire a *vev* during the phase transition. In addition there need to be supersymmetry breaking mass terms to make the masses of the supersymmetric particles consistent with experimental bounds from collider physics. *E.g.* in the chargino sector these terms result in the following mass matrix for the two chargino Dirac fermions

$$m(x_\mu) = \begin{pmatrix} M_2 & gH_2^*(x_\mu) \\ gH_1^*(x_\mu) & \mu_c \end{pmatrix}. \tag{4.1}$$

The Higgs fields H_1 and H_2 will acquire a *vev* during the phase transition and will give rise to a space-time dependent contribution to the mass matrix. By a suitable

choice of gauge, these Higgs *vevs* can be made real, such that only the SUSY breaking parameters M_2 and μ_c carry a CP-violating complex phase.

The key observation is that this mass matrix can give rise to CP-violation already at the tree level. The reason for this is that the Higgsinos couple differently to the quarks and squarks than the winos such that not only rephasing invariant quantities of their mass matrix are relevant but also for example the quantity

$$\mathrm{Tr}\left(\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\left[mm^\dagger, (\partial_\mu mm^\dagger)\right]\right) = 4g^2\,\mathrm{Im}(M_2\mu_c^*)(\partial_\mu H_1 H_2). \tag{4.2}$$

The neutralino sector that consists of four Dirac fermions could give rise to a similar source of CP-violation. However this source does not generate the same amount of baryon asymmetry, since the CP-violation in the neutralino sector is hardly communicated to the quark sector [69]. Usually electroweak baryogenesis calculations are restricted to the source in the chargino sector for simplicity.

4.1.2 The Phase Transition in the MSSM

The second reason that makes electroweak baryogenesis in the Standard Model futile is the electroweak phase transition, since in the Standard Model the phase transition is not strong of first order [11, 70]. This is required since if the phase transition is of second order or just a cross-over no bubble nucleation takes place and the hot plasma is not driven sufficiently out of equilibrium which is one of the Sakharov conditions to generate a baryon asymmetry. It turns out that for a first order phase transition, light bosonic matter coupled to the Higgs fields is important. In the Standard Model the relevant particles are the gauge bosons while in the MSSM there is a whole plethora of additional bosonic particles, namely the superpartners of the fermionic SM matter content. It turns out that in the MSSM the phase transition is indeed of first order if the stop is relatively light [71, 68, 72] or if some additional bosonic gauge singlet is added [73] (this case is the so called NMSSM = Next to Minimal Supersymmetric Standard Model). Furthermore the phase transition has to be *strongly* of first order. This means that shortly after the phase transition the Higgs *vev* needs to be already large enough to make the W-bosons sufficiently heavy to avoid a wash-out of the baryon asymmetry by the weak sphaleron process. This sphaleron suppression is a rather generic feature of the MSSM and NMSSM not depending on the fine tuning of the involved parameters. As was shown in [68] the two Higgs *vev*s in the MSSM can be parametrized during the phase transition by

$$H_1(z) = H(z)\sin(\beta(z)), \quad H_2(z) = H(z)\cos(\beta(z)) \tag{4.3}$$

and

$$H(z) = \frac{1}{2}v(T)\left(1-\tanh\left(\alpha\left(1-\frac{2z}{L_w}\right)\right)\right), \tag{4.4}$$

$$\beta(z) = \beta_\infty - \frac{1}{2}\Delta\beta\left(1+\tanh\left(\alpha\left(1-\frac{2z}{L_w}\right)\right)\right). \tag{4.5}$$

The parameters used turn out to be for a generic light stop scenario

$$T_c = 110 \text{ GeV}, \quad v(T) = 175 \text{ GeV}, \quad \alpha = \frac{3}{2}, \quad L_w = 20/T_c. \tag{4.6}$$

These parameters and the bubble wall profile will be the main input to our baryogenesis analysis. The parameter $\Delta\beta$ can be determined as well in the phase transition analysis and depends strongly on the SUSY breaking parameter m_A. We will see that $\Delta\beta$ (or equivalently m_A) will only marginally affect our results. Three representative values are

$$\Delta\beta = 0.02200, \quad m_A = 100\text{GeV},$$
$$\Delta\beta = 0.01077, \quad m_A = 200\text{GeV},$$
$$\Delta\beta = 0.00258, \quad m_A = 400\text{GeV},$$

and those will be used in the numerical analysis.

4.2 The Kadanoff-Baym Equations

The Kadanoff-Baym equations are the statistical analogon to the Schwinger-Dyson equation. Usually in quantum field theory, one is interested in S-matrix elements. These are determined using the in-out formalism in which states are prepared at the times $-\infty$ and $+\infty$ and the interactions take place in between and are switched on and off adiabatically. On the other hand for a statistical system one is not interested in S-matrix elements but in the time evolution of the density matrix. Since the density matrix can be expanded in a basis of ket-bra states $|i><j|$ it is more appropriate to use the in-in formalism. In this scheme both states $|i>$ and $<j|$ are evolved into the far future where they are projected on a complete set of states. This procedure is the so called Closed-Time-Path (CTP) formalism or Schwinger-Keldysh formalism. Thus effectively the CTP formalism leads to doubling of the degrees of freedom compared to the Schwinger-Dyson equation and an additional 2×2 matrix structure. However, of these four entries only two entries in the Green function are independent.

The second main difference to the Schwinger-Dyson equations is that the Green functions and self-energies not only depend on the relative-coordinates, that is defined as $r_\mu = x_\mu - y_\mu$, but as well on the average coordinate, $X_\mu = (x_\mu + y_\mu)/2$. Therefore by

Fourier transformation from coordinate space to momentum space, the convolution will not result in a normal product but in a Moyal star product. This space in which the two-point functions depend on momentum and average coordinates is the Wigner-space.

In the bosonic case the Kadanoff-Baym equations read

$$\begin{aligned} e^{-i\Diamond}\{k^2-\mathcal{M}^2\}\{\Delta^<\} - e^{-i\Diamond}\{\Pi_h\}\{\Delta^<\} - e^{-i\Diamond}\{\Pi^<\}\{\Delta_h\} &= \mathcal{C}_\phi, \\ e^{-i\Diamond}\{k^2-\mathcal{M}^2\}\{\Delta_\mathcal{A}\} - e^{-i\Diamond}\{\Pi_h\}\{\Delta_\mathcal{A}\} - e^{-i\Diamond}\{\Pi_\mathcal{A}\}\{\Delta_h\} &= 0, \\ \mathcal{C}_\phi = \tfrac{1}{2}\, e^{-i\Diamond}\left(\{\Pi^>\}\{\Delta^<\} - \{\Pi^<\}\{\Delta^>\}\right), & \end{aligned} \tag{4.7}$$

where the diamond operator in the Moyal star product coming from the transformation into Wigner space is defined by the Poisson bracket

$$\Diamond\{a\}\{b\} \equiv \frac{1}{2}\Big((\partial_{X^\mu} a)\partial_{k_\mu} b - (\partial_{k_\mu} a)\partial_{X^\mu} b\Big). \tag{4.8}$$

The Δ in (4.7) denotes the Green function, Π the self-energy of the bosons and $\mathcal{C}_\phi$ the collision term that drives the system usually to equilibrium. All quantities are $N \times N$ matrices in flavour space and depend on the average coordinate X_μ and the momentum variable k_μ. The superscripts $<, >$ and the subscripts h and $\mathcal{A}$ denote the additional 2×2 matrix structure as usual in the Kadanoff-Baym formalism

$$\Delta = \begin{pmatrix} \Delta^{++} & \Delta^{+-} \\ \Delta^{-+} & \Delta^{--} \end{pmatrix},$$

$$\begin{aligned} \Delta^< = \Delta^{+-}, \quad \Delta^> = \Delta^{-+}, \quad \Delta^t = \Delta^{++}, \quad \Delta^{\bar{t}} = \Delta^{--}, \\ \Delta_h = \Delta^t - \tfrac{1}{2}(\Delta^< + \Delta^>), \quad \Delta_\mathcal{A} = \tfrac{1}{2}(\Delta^> - \Delta^<). \end{aligned} \tag{4.9}$$

As we have seen in the last section, the dominating CP-violating source is expected to come from the space-time dependent mass, what in the chargino case is already feasible without interactions. Hence we will assume that CP-violating effects based on interactions are subdominant due to weak couplings and neglect all self-energy contributions besides the collision term. To keep the collision term is important since it will act as a damping force and ensure that our solution will approach equilibrium far away from the wall. However we will not determine it using loop calculations but parametrize it finally by physical reasoning. In principle the collision terms could lead to additional CP-violation, but a one-loop calculation [74] shows that they are phase space suppressed with respect to our tree level sources.

Since we neglect self-energies, the equations for $\Delta^<$ and $\Delta_\mathcal{A}$ decouple and we obtain

$$e^{-i\Diamond}\{k^2-\mathcal{M}^2\}\{\Delta^<\} = \mathcal{C}_\phi, \tag{4.10}$$

$$e^{-i\Diamond}\{k^2-\mathcal{M}^2\}\{\Delta_\mathcal{A}\} = 0. \tag{4.11}$$

These two equations will determine the dynamics of the Green function during the phase transition. If these equations are solved, the CP-violating particle densities can

be determined. *E.g.* in thermal equilibrium the Green function for a quasiparticle with mass m is

$$\begin{aligned} i\Delta^{<}_{\rm eq}(k_\mu) &= 2\pi\,\delta(k^2-m^2)\ \mathrm{sign}(k_0) f_{BE}(k_0), \\ i\Delta^{\mathcal{A}}_{\rm eq}(k_\mu) &= 2\pi\,\delta(k^2-m^2)\ \mathrm{sign}(k_0), \end{aligned}$$

with the Bose-Einstein distribution function

$$f_{BE}(k_0) = \frac{1}{e^{\beta k_0}-1}. \tag{4.12}$$

In general the particle density can be deduced from the Green function using

$$j_\nu(X_\mu) = 2i\int_{k_0>0}\frac{d^4k}{(2\pi)^4}\,k_\nu\,\Delta^{<}(X_\mu,k_\mu). \tag{4.13}$$

In the fermionic case analogous reasonings hold and the final equations are

$$e^{-i\diamondsuit}\{\not{k}-\mathcal{M}\}\{S^{<}\} = \mathcal{C}_\psi, \tag{4.14}$$

$$e^{-i\diamondsuit}\{\not{k}-\mathcal{M}\}\{S_{\mathcal{A}}\} = 0,$$

$$\mathcal{C}_\phi = \tfrac{1}{2}\,e^{-i\diamondsuit}\left(\{\Sigma^{>}\}\{S^{<}\}-\{\Sigma^{<}\}\{\S^{>}\}\right), \tag{4.15}$$

and the Wightman function S contains an additional spinor structure. In thermal equilibrium the fermionic Green functions for a quasiparticle with mass m are

$$\begin{aligned} iS^{<}_{\rm eq}(k_\mu) &= 2\pi(\not{k}-\mathcal{M})\,\delta(k^2-m^2)\ \mathrm{sign}(k_0) f_{FD}(k_0), \\ iS^{\mathcal{A}}_{\rm eq}(k_\mu) &= 2\pi(\not{k}-\mathcal{M})\,\delta(k^2-m^2)\ \mathrm{sign}(k_0), \end{aligned} \tag{4.16}$$

with the Fermi-Dirac distribution function

$$f_{FD}(k_0) = \frac{1}{e^{\beta k_0}+1}. \tag{4.17}$$

The fermionic vector and axial particle density is determined using

$$j_\nu(X_\mu) = 2i\int_{k_0>0}\frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}\left(\gamma_\nu\,S^{<}(X_\mu,k_\mu)\right), \tag{4.18}$$

$$j^5_\nu(X_\mu) = 2i\int_{k_0>0}\frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}\left(\gamma_\nu\gamma_5\,S^{<}(X_\mu,k_\mu)\right). \tag{4.19}$$

Even though the neglection of the self-energies simplified the Kadanoff-Baym equations considerably the resulting equations are still a coupled system of partial differential equations and to find a solution is far from trivial. Hence as a further approximation we will use the gradient expansion which is based on the diamond operator as expansion parameter. Important for the fast convergence of this expansion is the fact that the diamond operator is a product of momentum and coordinate derivatives. The corresponding relevant physical scales during the phase transition are the temperature T_c

and the width of the bubble wall l_w. Using the result from phase transition calculations that in the MSSM the wall width is of order $l_w \approx 20/T_c$, we conclude that it is sufficient to expand the Moyal star product

$$e^{i\diamondsuit} \approx 1 + i\diamondsuit - \frac{1}{2}\diamondsuit^2 \dots . \tag{4.20}$$

We will see later, that in the fermionic one flavour case the leading CP-violating contribution in the transport equation will be of second order, while in the multi-flavour mixing case already the first order in gradient expansion will contribute.

4.3 Former Approaches

As explained in the introduction, the challenge in electroweak baryogenesis calculations is to incorporate two fundamentally different features into the analysis. The first is CP-violation what makes it unavoidable to leave classical physics and to take account of quantum effects. In quantum mechanical systems mixing is important and can be decisive for the relevant physical phenomenon. The second is transport what makes it important to include statistical aspects. The first who realized the importance of transport in electroweak baryogenesis have been Cohen, Kaplan and Nelson [15, 16]. However their formalism is only applicable to the thin wall regime, while the phase transition in the MSSM generates thick bubble walls that allow for the gradient expansion.

Usually transport is described by Boltzmann or diffusion equations that are classical by nature. In particular classical physics neglects mixing and the degrees of freedom are equal to the number of flavours. In the following we discuss two formalisms that aim to fill the gap between CP-violation and transport theory by extracting some CP-sensitive information in a quantum treatment and to include this information into a classical transport equation.

4.3.1 The Work of Cline, Joyce and Kainulainen

The work of Cline, Joyce and Kainulainen[75, 76, 69] uses the WKB method and is a further development of the techniques derived in [77, 78, 35, 79] and their application to the MSSM. The WKB method is used to determine the dispersion relations $E(\vec{p}, z)$ of fermions in a space-time dependent background that is given by the Higgs *vev* generated mass term during the electroweak phase transition.

Remarkably, the dispersion relation depends on a spin quantum number and is sensitive to the change of the CP-violating complex phases in the mass background.

This CP-violating dispersion relation is then used in the Boltzmann equation of the form

$$\frac{dE}{dz}\partial_{p_z} f + \frac{dE}{dp_z}\partial_z f = \text{Coll.}, \tag{4.21}$$

where Coll. denotes some collision terms that drive the system back to equilibrium and f is the particle distribution function. Due to the CP-violating traces in the dispersion relation, the particle distribution functions develop some CP-violating deviations from equilibrium. These deviations are then communicated by a set of diffusion equations from the charginos to the quarks and finally bias the weak sphaleron process producing thus a net baryon number.

Clearly, in this context the only source of CP-violation is the dispersion relation that is determined by diagonalizing the mass of the chargino sector. Thus mixing is neglected [1] and the system is projected on the two diagonal flavour degrees of freedom in the mass eigenbasis before the classical Boltzmann equations are solved. We will see in a later section in which we reproduce the results of Cline *et al.* that in the Boltzmann equations the resulting CP-violating forces are of second order in the gradient expansion.

4.3.2 The Work of Carena, Moreno, Quiros, Seco and Wagner

Carena, Moreno, Quiros, Seco and Wagner [80, 81] aimed to include mixing effects into the electroweak baryogenesis analysis. Their formalism is based on the calculation of local sources as it was performed in [82]. The main idea is to expand the mass locally

$$m(x) = m(z) + (x - z)\, m'(z) + \cdots \tag{4.22}$$

on the level of Lagrange densities and to treat the first derivative of the mass as an interaction. Then the deviation of the Green function is calculated perturbatively and interpreted as a source for a diffusion equation of the form

$$v_w\, n' - D\, n'' = \text{Coll.} + \frac{1}{\tau}\int d^4p\, p_z \delta S, \tag{4.23}$$

where $n(z)$ denotes the particle number densities, v_w the velocity of the wall, D a diffusion constant, Coll. again some collision term, δS the perturbatively calculated deviation of the Green function and τ a thermalization time.

The previous approach of Riotto [82] treated in addition the Higgs *vev* only as a perturbation and had problems related to the renormalization of the new effective operator proportional to the derivative of the mass. In the work of Carena *et al.* [80, 81] this problem was resolved by resumming the effects of the Higgs fields to all orders and

[1] In the strict sense the effect in the dispersion relation comes from mixing between left-handed and right-handed parts of the Dirac fermion and mixing is neglected between different flavours.

to restrict to the Green function $S^<$ in the Kadanoff-Baym equations that is not plagued by infinities due to renormalization.

The perturbative calculation of the source was done using the Kadanoff-Baym equations and by taking account of the full 2×2 structure of the chargino sector and thus including mixing at least in the determination of the source. This source was then projected on the two diagonal entries in the interaction basis of the flavour space arguing that these degrees of freedom will later communicate with the weak sphalerons via the quarks. These projected sources delivered CP-violation for the classical diffusion equations at first order in gradients.

The most important result of the work of Carena *et al.* [80, 81] is that in the region of mass degeneracy of the charginos the BAU is enhanced.

4.3.3 A Qualitative Comparison

We will shortly compare the two approaches presented in the last two subsections. First difference is that the formalism of Carena *et al.* included mixing effects in the sources (but not in the transport) in contrast to Cline *et al.* where mixing was completely neglected. A quantitative analysis shows that these mixing effects are larger than the effects coming from the dispersion relation and this is plausible since they are only of first order in the gradient expansion and thus of lower order. In the treatment of Carena *et al.* especially the enhancement in the case of mass degeneracy has to be emphasized and that the approach of Cline *et al.* is not valid in this case since it is expected that the neglected mixing effects can be substantial there. However this first order effect would disappear if Carena *et al.* would have evaluated their source in the mass eigenbasis and one could argue that the flavour eigenstates are not the right degrees of freedom to be treated in a classical transport equation. In addition the approach of Carena *et al.* is much more phenomenological due to the diffusion constant D and the thermalization term τ that are hard to determine from the parameters of the underlying quantum field theory. To resolve these problems, in particular the question of the right choice of flavour basis, one needs to derive quantum transport equations that describe mixing not only in the sources but also in the transport equations. The resulting formalism should be basis independent and contain sources and transport genuinely without the ambiguity coming from phenomenological parameters. Target of this chapter is a derivation of quantum transport equations from a first principle approach starting with the Kadanoff-Baym equations.

4.4 Transport Equations for Fermionic Systems with One Flavour

The analysis in this section follows the work [17] in which quantum transport equations have been derived for the first time from first principles in the fermionic one flavour case. The bosonic one flavour case is trivial in the sense that it does not include CP-violation. For fermions the main issue is how to decouple the spinor structure what will be done by a consistent iterative approach.

In the derivation of transport equations in this and the following section the same approximations are used. We assume planar symmetry and boost into a frame with stationary wall, such that in this wall frame the mass background depends only on one space coordinate z that denotes the direction in that the wall propagates. This assumption is justified when the bubbles become sufficiently large [83]. In addition we will use the gradient expansion as explained above.

Our starting point for the fermionic one-flavour case are the equations (4.15) generalized to complex masses

$$e^{-i\Diamond}\{\not{k} - m_R - i\gamma_5 m_I\}\{S^<\} = \mathcal{C}_\psi. \tag{4.24}$$

Working in the wall frame, it is not hard to show that the tree level Dirac kinetic operator $\mathcal{D}$ defined by this equation

$$\mathcal{D}iS^< \equiv \Big(\not{k} + \frac{i}{2}\not{\partial} - m_R \mathrm{e}^{\frac{i}{2}\overleftarrow{\partial_z}\partial_{k_z}} - i\gamma^5 m_I \mathrm{e}^{\frac{i}{2}\overleftarrow{\partial_z}\partial_{k_z}}\Big) iS^< = \mathcal{C}_\psi\,, \tag{4.25}$$

commutes with the spin operator,

$$\begin{aligned} S_z &= \frac{1}{\tilde{k}_0}\big(\gamma^0 k_0 - \gamma^1 k_x - \gamma^2 k_y\big)\gamma^3\gamma^5, \\ \tilde{k}_0 &= \mathrm{sign}(k_0)(k_0^2 - k_x^2 - k_y^2)^{1/2}, \end{aligned} \tag{4.26}$$

provided the coordinate dependences of the Wightman functions are of the form

$$iS^{<,>} = iS^{<,>}\big(k, t - (k_x x + k_y y)/k_0, z\big). \tag{4.27}$$

Equivalently to this complicated spin-structure, one can boost to a frame in which the momentum of the particle is perpendicular to the wall (only $k_z \neq 0$) such that the problem reduces to a $1+1$ dimensional one. In this frame the spin operator is given by $S_z = \gamma^0\gamma^3\gamma^5$.

Having found a conserved quantity, we can write the solution of (4.25) in a block-diagonal form in spinor space (diagonal in spin) [18],

$$\begin{aligned} iS^< &= \sum_{s=\pm 1} iS_s^<, \\ iS_s^< &= -P_s\big[s\gamma^3\gamma^5 g_0^{s<,>} - s\gamma^3 g_3^{s<,>} + \mathbb{1} g_1^{s<,>} - i\gamma^5 g_2^{s<,>}\big]\,, \end{aligned} \tag{4.28}$$

where $P_s = \frac{1}{2}(\mathbb{1} + sS_z\ )$ is the spin projector, $P_sP_{s'} = \delta_{ss'}P_s$ $(s, s' = -1, 1)$, and g_0^s, g_1^s, g_2^s and g_3^s denote vector, scalar, pseudo-scalar, and pseudo-vector densities of spin s on eight-dimensional phase space $\{k, x\}$, respectively.

Upon multiplying Eq. (4.25) by $\{P_s\mathbb{1}, P_s\gamma_0, -P_sis\gamma_3, -P_s\gamma_5\}$ and tracing over the spinor structure, one finds

$$
\begin{aligned}
2i\tilde{k}_0 g_0^s - (2isk_z + s\partial_z)\, g_3^s - 2im_R \mathrm{e}^{\frac{i}{2}\overleftarrow{\partial_z}\overrightarrow{\partial_{k_z}}} g_1^s - 2im_I \mathrm{e}^{\frac{i}{2}\overleftarrow{\partial_z}\overrightarrow{\partial_{k_z}}} g_2^s &= 0, && (4.29)\\
2i\tilde{k}_0 g_1^s - (2sk_z - is\partial_z)\, g_2^s - 2im_R \mathrm{e}^{\frac{i}{2}\overleftarrow{\partial_z}\overrightarrow{\partial_{k_z}}} g_0^s + 2m_I \mathrm{e}^{\frac{i}{2}\overleftarrow{\partial_z}\overrightarrow{\partial_{k_z}}} g_3^s &= 0, && (4.30)\\
2i\tilde{k}_0 g_2^s + (2sk_z - is\partial_z)\, g_1^s - 2m_R \mathrm{e}^{\frac{i}{2}\overleftarrow{\partial_z}\overrightarrow{\partial_{k_z}}} g_3^s - 2im_I \mathrm{e}^{\frac{i}{2}\overleftarrow{\partial_z}\overrightarrow{\partial_{k_z}}} g_0^s &= 0, && (4.31)\\
2i\tilde{k}_0 g_3^s - (2isk_z + s\partial_z)\, g_0^s + 2m_R \mathrm{e}^{\frac{i}{2}\overleftarrow{\partial_z}\overrightarrow{\partial_{k_z}}} g_2^s - 2m_I \mathrm{e}^{\frac{i}{2}\overleftarrow{\partial_z}\overrightarrow{\partial_{k_z}}} g_1^s &= 0\,, && (4.32)
\end{aligned}
$$

where we have dropped the superscript $<$ of g_a^s $(a = 0, 1, 2, 3)$.

Finally we are interested in the two quantities g_0^s and g_3^s since these densities will enter in the fermionic vector and axial currents (4.19). To decouple these equations, the exponentials of the differential operators are expanded up to second order and then the equations are split into hermitian and anti-hermitian parts. The resulting eight real equations can then be used to finally arrive at the following three equations that are sufficient to discuss the behaviour of the vector and axial currents.

$$
\begin{aligned}
\left(k_z\partial_z - (m^2)'\partial_{k_z}\right) g_3^s &= \mathcal{C}_\psi, && (4.33)\\
\left(k_z\partial_z - (m^2)'\partial_{k_z} - s\frac{(m^2\theta')'}{k_0}\partial_{k_z}\right) g_0^s &= \mathcal{C}_\psi, && (4.34)\\
\left(k_0^2 - k_z^2 - m^2 - s\frac{m^2\theta'}{k_0}\right) g_0^s &= 0, && (4.35)
\end{aligned}
$$

where the mass is rewritten in the form $m_R + i\, m_I = m\, e^{i\theta}$. Equation (4.35) is the so called constraint equation. It determines the dispersion relation and leads to a δ function in the spectrum. The effect in the dispersion relation is analogous to the effect in the WKB approach as derived by Cline *et al.* [78]. Since θ changes sign under CP-conjugation, we see, that g_3^s acquires no CP-violating contribution, while g_0^s has a CP-odd source that is of second order in gradients. In addition we see, that CP-odd terms always come with a spin factor s, such that we can infer that the CP-violation in g_0^s will be odd in the spin s. Hence only the three component of the axial vector current $j_3^5 = g_0^+ - g_0^-$ obtains a contribution and not the zero component of the vector current $j_0 = g_0^+ + g_0^-$. Our calculation has been performed in the wall frame, such that in the plasma frame the CP-violating axial particle density will be proportional to

$$
j_0^{5,plasma-frame} = \gamma_{v_w}\, v_w\, j_3^{5,wall-frame}, \qquad (4.36)
$$

where v_w denotes the wall velocity. Notice that the Eq. (4.35) is necessary since the Eq. (4.34) depends explicitely on k_0; in addition the resulting CP-violating particle

density vanishes if the wall is not moving ($v_w = 0$) as expected. This is required since for a stationary wall time reflection and hence CP should not be violated.

To determine the whole Green function we need in addition two more equations out of the set of eight equations. The remaining three equations are then fulfilled automatically [17] what shows that our procedure to decouple the equations was consistent.

The kinetic equation (4.34) can now be solved numerically for arbitrary wall profiles $m(z), \theta(z)$. Under the assumption that mixing is suppressed, these results can be directly generalized to the case of several flavours as done in [19] what will lead to results close to the analysis of Cline *et al.*

4.5 Transport Equations and Flavour Mixing

In this section we will derive the explicit transport equations for systems with several flavours. In the bosonic multi-flavour case the first time oscillations of the off-diagonal densities will appear that will turn out to be crucial for the discussion of CP-violation of the first order sources. Subsequently we will address the relevant case for EWB in the MSSM: multi-flavour transport equations for a fermionic system, *e.g.* the chargino sector. More technical details of the derivations in the fermionic case can be found in [84, 85].

4.5.1 Bosonic Multi-Flavour systems

In this section we will derive transport equations for mixing bosons from the Kadanoff-Baym equations and the resulting CP-violating particle densities as presented in [85]. In the fermionic case, the spinor structure complicates the decoupling of the system of equations, but the bosonic case given here will already support the main conclusions of [84] concerning oscillations without the technical issues coming from the spinor structure.

Kadanoff-Baym Equations and the Approximation Scheme

Starting point are the bosonic Kadanoff Baym equations (4.11)

$$e^{-i\Diamond}\{k^2 - \mathcal{M}^2\}\{\Delta^<\} = \mathcal{C}_\phi. \tag{4.37}$$

The mass squared matrix $\mathcal{M}^2$ is space-time dependent and hermitian. During the electroweak phase transition, the bosonic particles relevant for baryogenesis are the squarks whose mass matrix is given by

$$\mathcal{M}^2 = \begin{pmatrix} m_Q^2 + h_t^2 H_2^2(X_\mu) & h_t(A_t H_2(X_\mu) - \mu_c^* H_2(X_\mu)) \\ h_t(A_t^* H_2(X_\mu) - \mu_c H_2(X_\mu)) & m_U^2 + h_t^2 H_2^2(X_\mu) \end{pmatrix}. \tag{4.38}$$

As in the fermionic one-flavour case we will perform the calculation in the limit of planar bubble walls and in the bubble wall frame such that $\mathcal{M}^2$ depends only on z. In addition we work again in the thick wall regime, what makes a gradient expansion reasonable. The system expanded up to first order in gradients reads (prime denotes derivatives with respect to z):

$$\left(k^2 + ik_z\partial_z + \frac{1}{4}\partial_z^2 - \mathcal{M}^2 - \frac{i}{2}\mathcal{M}^{2\prime}\partial_{k_z}\right)\Delta^< = \mathcal{C}_\phi. \tag{4.39}$$

Using the hermiticity condition $\Delta^{<\dagger} = -\Delta^<$, this equation can be split into its hermitian and anti-hermitian parts

$$\left(k^2 + \frac{1}{4}\partial_z^2\right)\Delta^< - \frac{1}{2}\left\{\mathcal{M}^2, \Delta^<\right\} - \frac{i}{4}\left[\mathcal{M}^{2\prime}, \partial_{k_z}\Delta^<\right] = 0, \tag{4.40}$$

$$k_z\partial_z\Delta^< + \frac{i}{2}\left[\mathcal{M}^2, \Delta^<\right] - \frac{1}{4}\left\{\mathcal{M}^{2\prime}, \partial_{k_z}\Delta^<\right\} = \mathcal{C}_\phi, \tag{4.41}$$

where $[\cdot,\cdot]$ and $\{\cdot,\cdot\}$ denote commutators and anticommutators. In the following we refer to these two equations as the constraint and kinetic equation.

Lowest Order Solution

Let us first discuss Eqs. (4.40–4.41) for a two-dimensional mass matrix that is constant in space and time. The mass matrix can be diagonalized by a unitary transformation and the equation in this basis reads ($\mathcal{M}_d^2$ denotes the diagonalized mass matrix and Δ_d the corresponding Green function that is non-diagonal in general)

$$\left(k^2 + \frac{1}{4}\partial_z^2\right)\Delta_d^< - \frac{1}{2}\left\{\mathcal{M}_d^2, \Delta_d^<\right\} = 0, \tag{4.42}$$

$$k_z\partial_z\Delta_d^< + \frac{i}{2}\left[\mathcal{M}_d^2, \Delta_d^<\right] = \mathcal{C}_{\phi,d}\,. \tag{4.43}$$

The question is, in which sense these equations can recover the solution in thermal equilibrium (4.12). We expect that the Kubo-Martin-Schwinger (KMS) equilibrium condition is then satisfied, such that $\mathcal{C}_{\phi,d} = 0$. We can use the derivative of the second equation to obtain

$$k^2\Delta_d^< - \frac{1}{16k_z^2}\left[\mathcal{M}_d^2, \left[\mathcal{M}_d^2, \Delta_d^<\right]\right] - \frac{1}{2}\left\{\mathcal{M}_d^2, \Delta_d^<\right\} = 0, \tag{4.44}$$

$$k_z\partial_z\Delta_d^< + \frac{i}{2}\left[\mathcal{M}_d^2, \Delta_d^<\right] = 0\,. \tag{4.45}$$

The constraint equation (4.44) is algebraic, and it determines the spectrum of the quasi-particles in the plasma. At this point it is helpful to introduce two projection operators

$$P^T X = \frac{1}{\Lambda^2}\left[\mathcal{M}_d^2, \left[\mathcal{M}_d^2, X\right]\right], \quad P^D = 1 - P^T, \tag{4.46}$$

where

$$\Lambda := \sqrt{\mathrm{Tr}\mathcal{M}^2 - 4\ \mathrm{Det}\mathcal{M}^2} = \mathrm{Tr}\,(\sigma_3 \mathcal{M}_d^2) \tag{4.47}$$

denotes the difference of the eigenvalues of $\mathcal{M}_d^2$, and σ_i $(i = 1, 2, 3)$ are the Pauli matrices. The properties of the projection operators

$$(P^T)^2 = P^T, \quad (P^D)^2 = P^D, \quad P^T + P^D = 1 \tag{4.48}$$

can be easily checked.

In the mass eigenbasis $P^T \Delta_d^<$ corresponds to the complex off-diagonal entries, while $P^D \Delta_d^<$ corresponds to the two real diagonal entries. If we split $\Delta_d^<$ in its transverse and diagonal parts $\Delta_d^T := P^T \Delta_d^<$, $\Delta_d^D := P^D \Delta_d^<$ and using the relations

$$\left\{Y^D, X^D\right\} = 2 Y^D\, X^D, \qquad \left\{Y^D, X^T\right\} = 2(\mathrm{Tr}\ Y) X^T, \tag{4.49}$$

$$P^D \mathcal{M}_d^2 = \mathcal{M}_d^2, \qquad P^T \mathcal{M}_d^2 = 0\,, \tag{4.50}$$

the constraint equations (4.44) for the diagonal and transverse parts of $\Delta_d^<$ decouple

$$\left(k^2 - \mathcal{M}_d^2\right) \Delta_d^D = 0, \tag{4.51}$$

$$\left(k^2 - \frac{\Lambda^2}{16 k_z^2} - \frac{1}{2}\mathrm{Tr}\ \mathcal{M}_d^2\right) \Delta_d^T = 0\,. \tag{4.52}$$

Both diagonal and transverse constraint equations are algebraic, and thus the solutions are given by the appropriate δ-functions, which represent sharp on-shell projections. The diagonal shell is given by the standard dispersion relation, whose frequencies are, $k_0^2 \equiv \omega_i^2 = \vec{k}^2 + m_i^2$, where m_i^2 are the eigenvalues of $\mathcal{M}^2$. The transverse parts fulfill a different on-shell condition, which can be easily obtained from (4.52).

The kinetic equation (4.45) reveals another difference between diagonal and transverse parts. The projected kinetic equations read

$$k_z \partial_z \Delta_d^D = 0, \tag{4.53}$$

$$k_z \partial_z \Delta_d^T + \frac{i}{2}\left[\mathcal{M}_d^2, \Delta_d^T\right] = 0\,. \tag{4.54}$$

The diagonal parts are constant in space and time, while the transverse parts rotate in flavour space with the frequency $\sim \Lambda / k_z = (m_1^2 - m_2^2)/k_z$.

In the equilibrium solution (4.12) the transverse entries vanish everywhere, but it is clear that this oscillation dominates the dynamics of the transverse parts as soon as they are sourced by higher order contributions in the gradient expansion.

Alternatively, oscillations can be induced by the initial conditions. This is, for example, the case for neutrino oscillations. Neutrinos are created as flavour eigenstates, and hence, from the point of view of the mass eigenbasis, a mixture of diagonal and transverse states. Since in most environments the damping of neutrinos is very small, neutrino oscillations persist for a long time. In our case, particle densities are subject to damping by interaction with the heat bath, such that the oscillations will not appear as a long range effect.

First order solution and CP-violation

Let us consider again the Kadanoff-Baym equations (4.40–4.41) but this time to first order in gradients.

In the last section we saw that in lowest order the spectrum can be separated into the diagonal and transverse contributions. One can show that in the first order system (4.40–4.41) however, the different quasi-particles start to mix and the spectral functions acquire a finite width. This is reflected in the fact that, at first order in gradients, the constraint equation is not any more algebraic.

Fortunately we do not need any information about the spectrum to solve the kinetic equation (4.41), since it does not explicitely contain any k_0 dependence. When transformed into the mass eigenbasis, the kinetic equation reads

$$\begin{aligned} k_z\partial_z\Delta_d^< + \frac{i}{2}\left[\mathcal{M}_d^2,\Delta_d^<\right] + k_z\left[\Sigma,\Delta_d^<\right] \\ -\frac{1}{4}\left\{\mathcal{M}_d^{2\prime} + \frac{1}{2}\left[\Sigma,\mathcal{M}_d^2\right],\partial_{k_z}\Delta_d^<\right\} = \mathcal{C}_{\phi,d}, \end{aligned} \tag{4.55}$$

with

$$\Sigma = U^\dagger U' \tag{4.56}$$

and the matrix $U(z)$ diagonalizes $\mathcal{M}^2$, $\mathcal{M}_d^2 = U^\dagger\mathcal{M}^2U$.

The next step is to determine the CP-violating contributions to the particle densities. By definition the CP-conjugation acts in the mass eigenbasis as

$$\begin{aligned} \Delta_d^{\mathcal{CP}}(X,k) \equiv \mathcal{CP}\ \Delta_d(X,k)\ \mathcal{CP} = \Delta_d^*(\bar{X},-\bar{k}), \\ \bar{X}^\mu = (X_0,-X_i), \quad \bar{k}^\mu = (k_0,-k_i). \end{aligned} \tag{4.57}$$

Note that by CP-conjugation of bosonic degrees of freedom we mean the CP-conjugation inherited from the fermionic sector due to supersymmetry. Namely this CP-conjugation relates right-handed and left-handed squarks that are *a priory* not related by normal parity conjugation.

The transformation (4.57) is in our equation (4.55) equivalent to

$$U \to U^*, \quad \Sigma \to \Sigma^*\,. \tag{4.58}$$

Now suppose that as in the chargino case our particles do not directly couple to the sphaleron process. Then the CP-violating particle density has to be communicated to another species via interactions. In this case we are rather interested in the CP-violating densities in the diagonal matrix elements of the Green function in the interaction eigenbasis. These are given by

$$\begin{aligned} \mathrm{Tr}\,[\Delta^< - \mathcal{CP}\,\Delta^<\mathcal{CP}] &= \mathrm{Tr}\,\left[U\Delta_d^<U^\dagger - U^*\,\Delta_d^{<\mathcal{CP}}U^{\dagger *}\right] \\ &= \mathrm{Tr}\,\left[U(\Delta_d^< - \Delta_d^{<\mathcal{CP}*})U^\dagger\right], \end{aligned} \tag{4.59}$$

and

$$\begin{aligned}\mathrm{Tr}\,\left[\sigma_3\Delta^< - \sigma_3\mathcal{CP}\,\Delta^<\mathcal{CP}\right] &= \mathrm{Tr}\,\left[\sigma_3 U\Delta_d^< U^\dagger - \sigma_3 U^*\,\Delta_d^{<\mathcal{CP}}U^{\dagger *}\right]\\ &= \mathrm{Tr}\,\left[\sigma_3 U(\Delta_d^< - \Delta_d^{<\mathcal{CP}*})U^\dagger\right], \end{aligned} \tag{4.60}$$

where the latter equality in both cases follows from the fact that $\Delta_d^{<\mathcal{CP}}$ is hermitian. Henceforth we consider in the mass eigenbasis the equation for $\Delta^{<\mathcal{Q}} := \Delta^{<\mathcal{CP}*}$. This Q-conjugation coincides with CP-conjugation on the diagonal, but it is in addition basis independent for the diagonal entries, since it commutes with the diagonalization matrix.

The equation for $\Delta^{<\mathcal{Q}}$ is given by (notice that Σ is anti-hermitian)

$$\begin{aligned}k_z\partial_z\Delta_d^{<\mathcal{Q}} + k_z\left[\Sigma,\Delta_d^{<\mathcal{Q}}\right] - \frac{i}{2}\left[\mathcal{M}_d^2,\Delta_d^{<\mathcal{Q}}\right]\\ -\frac{1}{4}\Big\{\mathcal{M}_d^{2\prime} + \frac{1}{2}\left[\Sigma,\mathcal{M}_d^2\right],\partial_{k_z}\Delta_d^{<\mathcal{Q}}\Big\} = \mathcal{C}_{\phi,d}\,.\end{aligned} \tag{4.61}$$

The only change with respect to the original equation of $\Delta_d^<$ is a sign-change in the oscillation term $\left[\mathcal{M}_d^2,\Delta_d^{<Q}\right]$. If we include higher order terms in the gradient expansion additional Q-breaking terms will appear. Since in leading order CP violation is based on the oscillation effect, one has to solve only the equation for the transverse parts and its Q-conjugate. Collecting terms that are at most first order in gradients (deviations from equilibrium $\delta\Delta_d = \Delta_d - \Delta_{\mathrm{eq}}$ are counted as of order one in the gradient expansion, $\mathcal{M}_d^{2\prime}$ and Σ are explicitely of first order in gradients) we obtain for the transverse deviations

$$\begin{aligned}k_z\partial_z\delta\Delta_d^T + \frac{i}{2}\left[\mathcal{M}_d^2,\delta\Delta_d^T\right] - \mathcal{C}_{\phi,d} &= \mathcal{S}_d,\\ k_z\partial_z\delta\Delta_d^{T\mathcal{Q}} - \frac{i}{2}\left[\mathcal{M}_d^2,\delta\Delta_d^{T\mathcal{Q}}\right] - \mathcal{C}_{\phi,d} &= \mathcal{S}_d\,,\end{aligned} \tag{4.62}$$

with the source term

$$\mathcal{S}_d = -k_z\left[\Sigma,\Delta_{\mathrm{eq}}^<\right]^T + \frac{1}{4}\left\{\mathcal{M}_d^{2\prime} + \left[\Sigma,\mathcal{M}_d^2\right],\partial_{k_z}\Delta_{\mathrm{eq}}^<\right\}^T. \tag{4.63}$$

These equations can be solved numerically using an *ansatz* for a flow solution as will be described in a later section.

Since $\Delta_d^{<\mathcal{CP}}$ and $\Delta_d^{<\mathcal{Q}}$ differ only by transposition, this calculation in addition shows that the diagonal entries in the mass eigenbasis will be CP-even up to first order in gradients and that the CP-violation in the squark sector will not give rise to a baryon asymmetry up to this order.

4.5.2 Fermionic Multi-Flavour systems

This section will give the generalization of the fermionic one-flavour case to several flavours (details can be found in [84]). As in the one flavour case we start with

$$e^{-i\Diamond}\{\not{k} - m_h - i\gamma_5 m_a\}\{S^<\} = \mathcal{C}_\psi, \tag{4.64}$$

where m_h and m_a denote the hermitian and anti-hermitian parts of a general $N \times N$ mass matrix. The Green function $S^<$ carries of course the same flavour structure.

Since the spinor structure of the Dirac operator did not change in comparison to the one flavour case, the considerations about spin projection are still valid and performing the same steps we arrive at (neglecting the collision term)

$$\begin{aligned}
2i\tilde{k}_0 g_0^s - (2isk_z + s\partial_z)\, g_3^s - 2im_h \mathrm{e}^{\frac{i}{2}\overleftarrow{\partial_z}\overrightarrow{\partial_{k_z}}} g_1^s - 2im_a \mathrm{e}^{\frac{i}{2}\overleftarrow{\partial_z}\overrightarrow{\partial_{k_z}}} g_2^s &= 0, && (4.65)\\
2i\tilde{k}_0 g_1^s - (2sk_z - is\partial_z)\, g_2^s - 2im_h \mathrm{e}^{\frac{i}{2}\overleftarrow{\partial_z}\overrightarrow{\partial_{k_z}}} g_0^s + 2m_a \mathrm{e}^{\frac{i}{2}\overleftarrow{\partial_z}\overrightarrow{\partial_{k_z}}} g_3^s &= 0, && (4.66)\\
2i\tilde{k}_0 g_2^s + (2sk_z - is\partial_z)\, g_1^s - 2m_h \mathrm{e}^{\frac{i}{2}\overleftarrow{\partial_z}\overrightarrow{\partial_{k_z}}} g_3^s - 2im_a \mathrm{e}^{\frac{i}{2}\overleftarrow{\partial_z}\overrightarrow{\partial_{k_z}}} g_0^s &= 0, && (4.67)\\
2i\tilde{k}_0 g_3^s - (2isk_z + s\partial_z)\, g_0^s + 2m_h \mathrm{e}^{\frac{i}{2}\overleftarrow{\partial_z}\overrightarrow{\partial_{k_z}}} g_2^s - 2m_a \mathrm{e}^{\frac{i}{2}\overleftarrow{\partial_z}\overrightarrow{\partial_{k_z}}} g_1^s &= 0\,, && (4.68)
\end{aligned}$$

However, this time we cannot easily decouple this system of equations since extracting the hermitian and anti-hermitian parts will lead to additional commutator terms in flavour space that inhibit the decoupling.

A cause for the complexity of these equations stem from a bad choice of spinor representation. The present basis is useful in the one flavour case (as well as in the mixing case with well separated mass eigenvalues), since at order $\hbar$ in gradient expansion the vector density g_0 obeys an algebraic constraint equation, from which one obtains the dispersion relation with a spin dependent CP-violating shift appearing at order $\hbar$. An important implication of this result is that, at order $\hbar$, the quasiparticle picture of the plasma is preserved [17, 18]. When inserted into the kinetic equation for g_0^s, and upon integration over the positive and negative frequencies, one arrives at the Boltzmann-like kinetic equation for the distribution function for particles and antiparticles, respectively, which at second order in gradients (first order in $\hbar$) exhibits a spin-dependent CP-violating force.

In the case of several flavours however, the basis g_a^s leads to mixing between different g_a^s's already at the classical (leading order) level. Moreover, g_a^s's do not transform in a definite manner under flavour rotations [19]. A more appropriate basis to describe fermion mixing is the chiral basis

$$g_R^s = g_0^s + g_3^s\,, \qquad g_L^s = g_0^s - g_3^s\,, \tag{4.69}$$

and the following densities,

$$g_N^s = g_1^s + ig_2^s \qquad {g_N^s}^\dagger = g_1^s - ig_2^s\,. \tag{4.70}$$

These densities do transform in a definite way under mass diagonalization (flavour rotation). Motivated by the diagonalization of the complex masses

$$\begin{aligned}
m = m_h + im_a\,, &\qquad m^\dagger = m_h - im_a,\\
m \to m_d = UmV^\dagger\,, &\qquad m^\dagger \to m_d^\dagger = Vm^\dagger U^\dagger = m_d,\\
mm^\dagger \to m_d^2 = Umm^\dagger U^\dagger\,, &\qquad m^\dagger m \to m_d^2 = Vm^\dagger mV^\dagger,
\end{aligned} \tag{4.71}$$

where the unitary transformation matrices U and V are chosen such that $m_d = m_d^\dagger$ are diagonal mass matrices with real eigenvalues, the chiral densities transform as

$$\begin{aligned} g_L^s \to g_{Ld}^s &= U g_L^s U^\dagger, \qquad g_R^s \to g_{Rd}^s = V g_L^s V^\dagger, \\ g_N^s \to g_{Nd}^s &= U g_N^s V^\dagger, \quad {g_N^s}^\dagger \to {g_{Nd}^s}^\dagger = V {g_N^s}^\dagger U^\dagger. \end{aligned} \tag{4.72}$$

From Eqs. (4.65–4.68) we easily find

$$2i\tilde{k}_0 g_R^s - s\,(2ik_z + \partial_z)\, g_R^s - 2im^\dagger \hat{E} g_N^s = 0, \tag{4.73}$$

$$2i\tilde{k}_0 g_L^s + s\,(2ik_z + \partial_z)\, g_L^s - 2im\hat{E} {g_N^s}^\dagger = 0, \tag{4.74}$$

$$2i\tilde{k}_0 g_N^s + s\,(2ik_z + \partial_z)\, g_N^s - 2im\hat{E} g_R^s = 0, \tag{4.75}$$

$$2i\tilde{k}_0 {g_N^s}^\dagger - s\,(2ik_z + \partial_z)\, {g_N^s}^\dagger - 2im^\dagger \hat{E} g_L^s = 0\,, \tag{4.76}$$

where we introduced the following notation

$$\hat{E} \equiv \exp\Big(\frac{i}{2}\overleftarrow{\partial}_z \overrightarrow{\partial}_{k_z}\Big), \qquad \hat{E}^\dagger \equiv \exp\Big(-\frac{i}{2}\overleftarrow{\partial}_{k_z} \overrightarrow{\partial}_z\Big)\,. \tag{4.77}$$

Definite transformation properties of these equations are apparent. Indeed, Eqs. (4.73–4.76) transform just as the densities g_L^s, g_R^s, g_N^s and ${g_N^s}^\dagger$ in the Eq. (4.72).

From the anti-hermitian parts of Eqs. (4.73–4.74) we get the corresponding constraint equations for g_R^s and g_L^s, while the constraint equation for g_N^s is obtained by taking a hermitian conjugate of (4.76) and subtracting the result from (4.75),

$$(2\tilde{k}_0 - 2sk_z) g_R^s - m^\dagger \hat{E} g_N^s - {g_N^s}^\dagger \hat{E}^\dagger m = 0, \tag{4.78}$$

$$(2\tilde{k}_0 + 2sk_z) g_L^s - m\hat{E} {g_N^s}^\dagger - g_N^s \hat{E}^\dagger m^\dagger = 0, \tag{4.79}$$

$$(2\tilde{k}_0 - is\partial_z) g_N^s - m\hat{E} g_R^s - g_L^s \hat{E}^\dagger m = 0, \tag{4.80}$$

$$(2\tilde{k}_0 + is\partial_z) {g_N^s}^\dagger - m^\dagger \hat{E} g_L^s - g_R^s \hat{E}^\dagger m^\dagger = 0\,. \tag{4.81}$$

Note that the constraint equation for ${g_N^s}^\dagger$ is simply a hermitian conjugate of (4.80).

Analogously, the kinetic equations for g_R^s and g_L^s are obtained from the hermitian parts of Eqs. (4.73–4.74), while the kinetic equation for g_N^s is obtained by adding a hermitian conjugate of (4.76) to (4.75),

$$-s\partial_z g_R^s - im^\dagger \hat{E} g_N^s + i{g_N^s}^\dagger \hat{E}^\dagger m = 0, \tag{4.82}$$

$$s\partial_z g_L^s - im\hat{E} {g_N^s}^\dagger + ig_N^s \hat{E}^\dagger m^\dagger = 0, \tag{4.83}$$

$$2isk_z g_N^s - im\hat{E} g_R^s + ig_L^s \hat{E}^\dagger m = 0, \tag{4.84}$$

$$-2isk_z {g_N^s}^\dagger - im^\dagger \hat{E} g_L^s + ig_R^s \hat{E}^\dagger m^\dagger = 0. \tag{4.85}$$

As above, the equation for ${g_N^s}^\dagger$ is a hermitian conjugate of (4.84). The collision terms in the kinetic equations that have been neglected will be finally parametrized by phenomenological damping terms.

The kinetic and constraint equations represent an exact tree-level description of fermionic dynamics in the presence of bubble walls with planar symmetry. We will now show how to solve equations (4.78–4.81) and (4.82–4.85) in a gradient expansion, but without performing flavour rotations before we decouple partially the equations, in contrast to what was done in Refs. [19].

Constraint equations

To get an idea about the classical quasiparticle limit of our solutions, we first consider the constraint equations to lowest (classical) order, which are obtained from (4.78–4.81) by taking the limit $\hat{E} \to 1$ and $\hat{E}^\dagger \to 1$,

$$\begin{aligned} 2(k_0 - sk_z)g_R - m^\dagger g_N - g_N^\dagger m &= 0, && (4.86)\\ 2(k_0 + sk_z)g_L - m g_N^\dagger - g_N m^\dagger &= 0, && (4.87)\\ 2k_0 g_N - is\partial_z g_N - m g_R - g_L m &= 0\,, && (4.88) \end{aligned}$$

where for notational simplicity here and in the subsequent text we drop the superscript spin index s. In order to solve these equations to lowest order, it is convenient to make use of the self-consistency of the system of equations (4.78–4.81) and (4.82–4.85) and use the solution of the kinetic equations (4.84–4.85) to lowest order,

$$\begin{aligned} g_N &= \frac{1}{2sk_z}\Big(m g_R - g_L m\Big), && (4.89)\\ {g_N}^\dagger &= \frac{1}{2sk_z}\Big(g_R m^\dagger - m^\dagger g_L\Big)\,. && (4.90) \end{aligned}$$

When these equations are inserted in (4.86–4.87), one gets

$$\begin{aligned} \Big(\tilde{k}_0 - sk_z - \frac{1}{4sk_z}\{m^\dagger m, \cdot\}\Big)g_R + \frac{1}{2sk_z} m^\dagger g_L m &= 0, && (4.91)\\ \Big(\tilde{k}_0 + sk_z + \frac{1}{4sk_z}\{m m^\dagger, \cdot\}\Big)g_L - \frac{1}{2sk_z} m g_R m^\dagger &= 0\,, && (4.92) \end{aligned}$$

where $\{a, b\}$ denotes the anticommutator. These equations can be decoupled by multiplying (4.92) by m from the right and by $m^\dagger$ from the left, and inserting the solution into (4.91) (and performing an analogous procedure for the other equation). The result is

$$\begin{aligned} \Big(k^2 - \frac{1}{2}\{m^\dagger m, \cdot\} - \frac{1}{16k_z^2}\Big[m^\dagger m, [m^\dagger m, \cdot]\Big]\Big)g_R &= 0, && (4.93)\\ \Big(k^2 - \frac{1}{2}\{m m^\dagger, \cdot\} - \frac{1}{16k_z^2}\Big[m m^\dagger, [m m^\dagger, \cdot]\Big]\Big)g_L &= 0\,, && (4.94) \end{aligned}$$

where $k^2 = \tilde{k}_0^2 - k_z^2 = k_0^2 - \vec{k}^2$, and we made use of $\big\{a, \{a, f\}\big\} - 4afa = \big[a, [a, f]\big]$.

These constraint equations are easily solved by transforming to the diagonal basis, that is by applying on the first (second) equation V (U) from the left and $V^\dagger$ ($U^\dagger$) from

the right, we find that the mass shells of g_R and g_L are identical,

$$\Big(k^2 - \frac{1}{2}(m_i^2 + m_j^2) - \frac{1}{16k_z^2}(m_i^2 - m_j^2)^2\Big)(g_{R/L_d})_{ij} = 0, \tag{4.95}$$

where m_i^2 $(i = 1, .., N)$ are the diagonal entries of the matrix

$$m_d^2 \equiv \mathrm{diag}(m_1^2, m_2^2, .., m_N^2) = V m^\dagger m V^\dagger = U m m^\dagger U^\dagger. \tag{4.96}$$

These constraints have exactly the same form as the corresponding equations in the bosonic case (4.44). Again one infers that, while the diagonal densities are projected on the standard classical shells, the shells of the off-diagonal densities are in principle different for each element of the Green function in mass eigenbasis.

A particularly simple case and at the same time the relevant case for MSSM electroweak baryogenesis is when there are only two fermionic flavours and only one off-diagonal shell. In this case we would like to follow the same strategy as in the bosonic case to project out diagonal and transverse parts. For fermions there are however in the flavour basis two projectors needed to decompose g_L and g_R into their diagonal and transverse densities as follows,

$$g_R = g_R^D + g_R^T\,, \qquad g_L = g_L^D + g_L^T\,, \tag{4.97}$$

where

$$g_R^D = P_R^D g_R\,, \qquad g_L^D = P_L^D g_L\,, \qquad g_R^T = P_R^T g_R\,, \qquad g_L^T = P_L^T g_L\,. \tag{4.98}$$

The projection operators are defined as

$$P_R^T = \frac{1}{4\Lambda^2}\big[m^\dagger m, [m^\dagger m, \cdot]\big]\,, \qquad P_R^D = 1 - P_R^T, \tag{4.99}$$

$$P_L^T = \frac{1}{4\Lambda^2}\big[m m^\dagger, [m m^\dagger, \cdot]\big]\,, \qquad P_L^D = 1 - P_L^T, \tag{4.100}$$

where

$$\begin{aligned} \Lambda^2 &:= [\mathrm{tr}(m^\dagger m)/2]^2 - \det(m^\dagger m) \\ &= [\mathrm{tr}(m m^\dagger)/2]^2 - \det(m m^\dagger). \end{aligned} \tag{4.101}$$

This can be checked expanding the functions in terms of the Pauli matrices, $\sigma^a = (\mathbb{1}, \sigma^i)$, $[\sigma^i, \sigma^j] = 2i\epsilon^{ijl}\sigma^l$, and using the fact that Λ is an invariant. In the frame in which $m^\dagger m$ ($m m^\dagger$) is purely diagonal, g_R^D (g_L^D) are diagonal, and g_R^T (g_L^T) are off-diagonal and thus transverse, which explains the notation in (4.97–4.98). These projectors are not the same as defined in the bosonic case, since the projector in (4.46) was defined with the diagonalized mass matrices, while here we perform the projection in interaction basis. Let us emphasize that in both cases the operators project out the degrees of freedom that correspond to the diagonals and off-diagonals in mass eigenbasis and

that the projection operators in mass and interaction eigenbasis are related by basis transformation. We will not introduce new notation to distinguish between these two cases, since it should be clear which projectors are to be used in which cases.

Using the projectors $P^D_{R/L}$ and $P^T_{R/L}$, the constraints for the diagonal and transversal parts decouple and read

$$(k^2 - mm^\dagger) g^D_L = 0, \tag{4.102}$$

$$\Big(k^2 - \mathrm{Tr(mm^\dagger)} - \frac{\Lambda^2}{4\mathrm{k_z^2}}\Big)\mathrm{g^T_L} = 0. \tag{4.103}$$

In deriving these equations we used

$$[mm^\dagger, g^{sD}_L] = 0, \tag{4.104}$$

$$\{mm^\dagger, g^T_L\} = \mathrm{tr}(mm^\dagger) g^T_L = 2\mathrm{Tr(mm^\dagger)g^T_L}, \tag{4.105}$$

$$\big[mm^\dagger, [mm^\dagger, g^T_L]\big] = 4\Lambda^2 g^T_L. \tag{4.106}$$

Similarly, we have

$$(k^2 - m^\dagger m) g^D_R = 0, \tag{4.107}$$

$$\Big(k^2 - \mathrm{Tr(m^\dagger m)} - \frac{\Lambda^2}{4\mathrm{k_z^2}}\Big)\mathrm{g^T_R} = 0. \tag{4.108}$$

Since both $(m^\dagger m)$ and $mm^\dagger$ are hermitian, $(m^\dagger m)^0 = (mm^\dagger)^0$, and the dispersion relations for the L- and R- chiralities are identical at the leading order in gradients.

As in the bosonic case, an analysis of the constraint equations (4.78–4.81) shows that at higher order in gradients the diagonal and transverse shells mix in a manner which includes the derivative ∂_{k_z}, leading to non-algebraic constraints for the Wightman functions, and thus seemingly breaking the quasiparticle picture of the plasma, which questions the validity of any on-shell description of the dynamics of CP-violating densities, which necessarily involve higher order gradients. The situation is more complex however than this simple argument seems to indicate. As we show in the next section, in spite of this problem with the constraint equations, one can solve the tree-level kinetic equations to an arbitrary high order in gradients, thanks to the fact that, in stationary situations, the kinetic equations (4.82–4.85) do not involve k_0, and thus the tree-level dynamics of the Wightman functions g_R, g_L and g_N and the corresponding on-shell densities (obtained by k_0-integration) are identical.

Additional importance of the leading order analysis of the constraint equations presented here stems from the fact that it allows for the on-shell projection of the collision term and self-energies at leading order in gradients, such that it is essential for a self-consistent derivation of the kinetic equations for mixing fermions, provided one aims to determine the collision term at leading order in gradients what we will not do in the present text.

Kinetic Equations to Lowest Order

Using (4.89–4.90) in (4.82) and (4.83) to lowest order, and working in the stationary limit, we get

$$\partial_z g_R + \frac{i}{2k_z}\Big[m^\dagger m, g_R\Big] = 0, \tag{4.109}$$

$$\partial_z g_L + \frac{i}{2k_z}\Big[mm^\dagger, g_L\Big] = 0\,. \tag{4.110}$$

From the solutions of these equations (the commutator in the exponential has to be understood in our notation as a series in nested commutators)

$$g_R(k_z,z) \simeq \exp\Big(-\frac{i}{2k_z}\int_0^z dz'[m^\dagger m(z'),\cdot]\Big)g_R(k_z,0)\,, \tag{4.111}$$

$$g_L(k_z,z) \simeq \exp\Big(-\frac{i}{2k_z}\int_0^z dz'[mm^\dagger(z'),\cdot]\Big)g_L(k_z,0)\,. \tag{4.112}$$

We see that the diagonal and off-diagonal densities, when viewed in the diagonal basis where $g_{Rd} = V g_R V^\dagger$, $g_{Ld} = U g_L U^\dagger$, exhibit a qualitatively different behavior. The diagonal densities do not evolve, while the off-diagonals exhibit the oscillations, well known from the neutrino studies and already observed in the bosonic multi-flavour case. Note the identical evolution of the L and R chiralities, when viewed in the diagonal basis and that the transverse densities rotate in flavour space with the frequency $|\vec{\omega}| = |\Lambda/k_z|$ in the z-direction.

Kinetic Equations to Second Order

In the wall frame one can rewrite the system of kinetic Eqs. (4.82–4.85) in terms of the chiral densities g_R and and g_L only, valid to all orders in gradient expansion,

$$\begin{aligned}\partial_z g_R &+ \frac{i}{2}\Big(m^\dagger\hat{E}\frac{1}{k_z}(m\hat{E}g_R)\Big) - \frac{i}{2}\Big(\frac{1}{k_z}(g_R\hat{E}^\dagger m^\dagger)\hat{E}^\dagger m\Big)\\ &-\frac{i}{2}\Big(m^\dagger\hat{E}\frac{1}{k_z}(g_L\hat{E}^\dagger m)\Big) + \frac{i}{2}\Big(\frac{1}{k_z}(m^\dagger\hat{E}g_L)\hat{E}^\dagger m\Big) = 0,\end{aligned} \tag{4.113}$$

$$\begin{aligned}\partial_z g_L &+ \frac{i}{2}\Big(m\hat{E}\frac{1}{k_z}(m^\dagger\hat{E}g_L)\Big) - \frac{i}{2}\Big(\frac{1}{k_z}(g_L\hat{E}^\dagger m)\hat{E}^\dagger m^\dagger\Big)\\ &-\frac{i}{2}\Big(m\hat{E}\frac{1}{k_z}(g_R\hat{E}^\dagger m^\dagger)\Big) + \frac{i}{2}\Big(\frac{1}{k_z}(m\hat{E}g_R)\hat{E}^\dagger m^\dagger\Big) = 0\,.\end{aligned} \tag{4.114}$$

Note first that the chiral densities g_R and g_L couple through derivative terms only, which justifies the use of the chiral densities in writing the kinetic equations for mixing fermions. Next, equations (4.113) and (4.114) are transformed into each other by the following replacements, $R \leftrightarrow L$, $m \leftrightarrow m^\dagger$ and $s \leftrightarrow -s$ (see *e.g.* Eqs. (4.91–4.92)), defining thus the symmetry, which relates the dynamics of the chiral densities g_R^s to g_L^s. Furthermore, we have arrived at Eqs. (4.113–4.114) without using the constraint

equations (4.78–4.81). This procedure has the advantage that k_0 appears nowhere in Eqs. (4.113–4.114), implying that the kinetic equations for the distribution functions f_{Rs} and f_{Ls}, defined as the positive frequency integrals of g^s_R and g^s_L, have exactly the same form as (4.113–4.114), resolving thus the problem of closure of the on-shell kinetic equations. We emphasize that the (tree-level) closure is thus achieved, even though the constraint equations are non-algebraic. One consequence of the non-algebraic nature of the constraint equations is a coupling between the off-diagonal and diagonal densities, which is nevertheless implemented in a self-consistent manner into the kinetic equations (4.113–4.114) (through the higher derivative terms), without ever referring to the on-shell structure of the system. If one had attempted to further decouple the equations for g_R and g_L, one would have found out that this could be achieved by making use of the constraint equations (4.78–4.81), which would reintroduce the dependencies on k_0 and s, which is not explicit in equations (4.113–4.114).

Upon expanding $\hat{E}$ and $\hat{E}^\dagger$ in (4.77) to second order in gradients,

$$\begin{aligned}\hat{E} &= 1+\frac{i}{2}\overleftarrow{\partial}_z\overrightarrow{\partial}_{k_z}-\frac{1}{8}(\overleftarrow{\partial}_z\overrightarrow{\partial}_{k_z})^2+..\\ \hat{E}^\dagger &= 1-\frac{i}{2}\overleftarrow{\partial}_{k_z}\overrightarrow{\partial}_z-\frac{1}{8}(\overleftarrow{\partial}_{k_z}\overrightarrow{\partial}_z)^2+..\end{aligned} \tag{4.115}$$

we can write the chiral kinetic equations (4.113–4.114), truncated at second order in gradients as follows

$$\begin{aligned}&k_z\partial_z g_R+\frac{i}{2}\big[m^\dagger m,g_R\big]-\frac{1}{4}\big\{(m^\dagger m)',\partial_{k_z}g_R\big\}\\ &\quad+\frac{1}{4k_z}\big(m^{\dagger\prime}mg_R+g_Rm^\dagger m'\big)-\frac{1}{4k_z}\big(m^{\dagger\prime}g_Lm+m^\dagger g_Lm'\big)\\ &\quad-\frac{i}{16}\big[(m^\dagger m)'',\partial^2_{k_z}g_R\big]+\frac{i}{8k_z}\big[m^{\dagger\prime}m',\partial_{k_z}g_R\big]\\ &\quad+\frac{i}{8}\Big(m^{\dagger\prime\prime}m\partial_{k_z}(\frac{g_R}{k_z})-\partial_{k_z}(\frac{g_R}{k_z})m^\dagger m''\Big)\\ &\quad-\frac{i}{8}\Big(m^{\dagger\prime\prime}\partial_{k_z}(\frac{g_L}{k_z})m-m^\dagger\partial_{k_z}(\frac{g_L}{k_z})m''\Big)\simeq 0\,.\end{aligned} \tag{4.116}$$

The kinetic equation for g_L is obtained from (4.116) simply by exacting the replacements, $g_R\leftrightarrow g_L$ and $m\leftrightarrow m^\dagger$.

CP-violating sources

Let us define CP symmetry as the following transformations of the Dirac spinors (up to an irrelevant phase) in the mass eigenbasis,

$$\begin{aligned}\psi^{cp}(u) &\equiv \mathcal{CP}\psi(u)(\mathcal{CP})^\dagger=i\gamma^2\bar{\psi}^T(\bar{u}),\\ \bar{\psi}^{cp}(u) &\equiv \mathcal{CP}\bar{\psi}(u)(\mathcal{CP})^\dagger=\psi^T(\bar{u})i\gamma^2.\end{aligned} \tag{4.117}$$

Then the Wightman functions transform as (the subscript d denotes the mass eigenbasis)

$$S_d^{<,>}(k,x) \xrightarrow{\mathcal{CP}} -\gamma^2 S_d^{>,<T}(-\bar{k},\bar{x})\gamma^2 \equiv S_d^{cp<,>}(k,x). \tag{4.118}$$

Using this transformation property in our kinetic equation in the wall frame, in which $m_{h,a} = m_{h,a}(z)$, the CP transformation of the Green function is in our context equivalent to the transformations

$$m_h \to m_h^*\,, \qquad m_a \to -m_a^*\,, \qquad (m \to m^*,\ \ m^\dagger \to m^T)\,, \tag{4.119}$$

leaving *e.g.* $k_z^{\pm 1}\partial_z$ and $\partial_z\partial_{k_z}$ invariant. From these rules the CP transformed equation (4.116) can be written as follows,

$$\begin{aligned}
&k_z\partial_z g_R^{cp} + \frac{i}{2}\big[m^T m^*, g_R^{cp}\big] - \frac{1}{4}\big\{(m^T m^*)', \partial_{k_z} g_R^{cp}\big\} \\
&+\frac{1}{4k_z}\big(m^{T\prime} m^* g_R^{cp} + g_R^{cp} m^T m^{*\prime}\big) - \frac{1}{4k_z}\big(m^{T\prime} g_L^{cp} m^* + m^T g_L^{cp} m^{*\prime}\big) \\
&-\frac{i}{16}\big[(m^T m^*)'', \partial_{k_z}^2 g_R\big] + \frac{i}{8k_z}\big[m^{T\prime} m^{*\prime}, \partial_{k_z} g_R\big] \\
&+\frac{i}{8}\Big(m^{T\prime\prime} m^* \partial_{k_z}\big(\frac{g_R}{k_z}\big) - \partial_{k_z}\big(\frac{g_R}{k_z}\big) m^T m^{*\prime\prime}\Big) \\
&-\frac{i}{8}\Big(m^{T\prime\prime}\partial_{k_z}\big(\frac{g_L}{k_z}\big) m^* - m^T \partial_{k_z}\big(\frac{g_L}{k_z}\big) m^{*\prime\prime}\Big) \simeq 0\,.
\end{aligned} \tag{4.120}$$

Our primary goal is to identify the CP-violating sources in the system. A naïve way of doing that would be to subtract Eq. (4.121) from Eq. (4.116), and identify the terms in the equation for $\delta g_R^{cp} = g_R - g_R^{cp}$ which involve a CP-violating operator acting on $\bar{g}_R = (g_R + g_R^{cp})/2$, thus representing mixing of CP-odd and CP-even densities. This procedure leads to equations with indefinite transformation properties under flavour rotations, which is a consequence of the indefinite transformation properties of the newly defined densities δg_R^{cp} and $\bar{g}_R$, making it difficult to disentangle the genuine CP-violating densities from the apparent, but possibly spurious, CP-violating densities.

This problem is analogous to the same situation in the bosonic case. CP-conjugation does normally depend on the basis, since the change from the mass eigenbasis to the flavour eigenbasis does not commute with CP-conjugation. Hence CP-conjugation is defined in the mass eigenbasis, where it has the physical interpretation of changing particles into antiparticles. In the flavour eigenbasis, the CP-conjugation inherited from the mass eigenbasis has to be used taking into account the nontrivial behaviour of the transformation matrices U and V under CP. The problem is that we are interested in CP-violating densities in the flavour eigenbasis, since the CP-violation has to be communicated to the SM quarks. On the other hand CP-violation is defined in the mass eigenbasis and this basis is in addition much more convenient to perform a numerical analysis of our differential equations. As in the bosonic case a simple solution to this problem is to solve for Q-violating densities in the mass eigenbasis that will lead to

CP-violation in the diagonal entries of the flavour eigenbasis. In summary the Q-conjugation is just a convenient tool that takes into account the behaviour of the transformation matrices U and V under CP-conjugation that are used to transform to the flavour basis after the numerical calculation in the mass eigenbasis has been worked out.

We define

$$\mathcal{Q}\, g_L(k,x)\,(\mathcal{Q})^\dagger = \mathcal{CP}\, g_L^T(k,x)\,(\mathcal{CP})^\dagger = g_R(-\bar{k},\bar{x}), \tag{4.121}$$

$$\mathcal{Q}\, g_R(k,x)\,(\mathcal{Q})^\dagger = \mathcal{CP}\, g_R^T(k,x)\,(\mathcal{CP})^\dagger = g_L(-\bar{k},\bar{x}). \tag{4.122}$$

Using this definition, we look for the transformation properties of the kinetic equation under Q-conjugation and observe that all terms that come from an even order of the gradient expansion acquire an additional minus sign relative to the odd contributions from the gradient expansion. As a consequence, in the one flavour case one has to take second order terms into account in order to break Q (in this case Q is of course equivalent to CP), since there are no zeroth order terms. This leads to semiclassical force induced baryogenesis as in the case of Cline *et al.* as we have shown already in the one flavour case.

In the multi flavour case, an expansion up to first order is sufficient as long as the zeroth order contributes (otherwise the Green function is everywhere diagonal in the mass eigenbasis and the problem reduces to the one flavour case). The second order terms will finally be treated analogously to the one flavour case to reproduce the correct results for regions in parameter space where mixing is suppressed. As we will see later this is, as expected, the region where the eigenvalues of the mass matrix are far from degeneracy.

Let us recall the kinetic equation of the right-handed density to first order

$$\begin{aligned} k_z\partial_z g_R + \frac{i}{2}\big[m^\dagger m, g_R\big] - \frac{1}{4}\big\{(m^\dagger m)', \partial_{k_z} g_R\big\} \\ + \frac{1}{4k_z}\big(m^{\dagger\prime} m g_R + g_R m^\dagger m'\big) \\ - \frac{1}{4k_z}\big(m^{\dagger\prime} g_L m + m^\dagger g_L m'\big) \simeq 0. \end{aligned} \tag{4.123}$$

Then the Q-conjugate equation is

$$\begin{aligned} k_z\partial_z g_R^Q - \frac{i}{2}\big[m^\dagger m, g_R^Q\big] - \frac{1}{4}\big\{(m^\dagger m)', \partial_{k_z} g_R^Q\big\} \\ + \frac{1}{4k_z}\big(m^{\dagger\prime} m g_R^Q + g_R^Q m^\dagger m'\big) \\ - \frac{1}{4k_z}\big(m^{\dagger\prime} g_L^Q m + m^\dagger g_L^Q m'\big) \simeq 0, \end{aligned} \tag{4.124}$$

such that as mentioned earlier only the sign of the second (zeroth order commutator) term is affected.

To solve this equation, we will first determine the lowest order solution and then expand around it. The best way is to determine it in the mass eigenbasis, since in this basis the 'direction' of the mass in flavour is fixed. The choice of the lowest order is given by the requirement, that for infinite damping, the solution should be in local equilibrium, which means that the solution is given by the usual equilibrium expressions but is space-time dependent due to the space-time dependence in the mass.

This is simply given by

$$g_R^{(0)} = \Big(1 + \frac{sk_z}{\tilde{k}_0}\Big) V^\dagger g_{0d}^{(0)} V \,, \tag{4.125}$$

where $g_{0d}^{(0)}$ is the diagonal vector density in the mass-eigenbasis of the spectral form,

$$g_{0d}^{(0)} = 2\pi |\tilde{k}_0| \delta(k^2 - |m_d|^2) n_{FD} \,, \tag{4.126}$$

and $n_{FD}(k_0) = 1/[\exp(k_0/T) + 1]$ is the Fermi-Dirac distribution function. Using $V\big\{(m^\dagger m)', \partial_{k_z} g_R^{(0)}\big\}^D V^\dagger = 2(m_d^2)' \partial_{k_z} g_{Rd}^{(0)}$, it can be shown that this lowest order solution fulfills the lowest order kinetic equation in interaction basis

$$\begin{aligned} k_z \partial_z g_R^{(0)} + \frac{i}{2}\big[m^\dagger m, g_R^{(0)}\big] - \frac{1}{4}\big\{(m^\dagger m)', \partial_{k_z} g_R^{(0)}\big\}^D \\ -k_z \Big[V^{\dagger\prime} V, g_R^{(0)}\Big] = 0 \,, \end{aligned} \tag{4.127}$$

where the first commutator term vanishes, since there is no source for the transversal parts. The last term in the equation compensates for a flavour basis change from the mass eigenbasis to the interaction basis. We emphasize that in our expansion all influences of the changing background are then negligible and the Green function depends only locally on the mass what is a reasonable requirement in the case of infinite damping.

The transverse part of the deviation from $g_R^{(0)}$ is in the next order given by

$$k_z \partial_z g_R^{T(1)} + \frac{i}{2}\big[m^\dagger m, g_R^{T(1)}\big] + k_0 \Gamma\, g_R^{T(1)} - k_z \Big[V^{\dagger\prime} V, g_R^{T(1)}\Big] = S_R^{(1)}, \tag{4.128}$$

where we defined the source

$$\begin{aligned} S_R^{(1)} \;\equiv\; & -k_z \Big[V^{\dagger\prime} V, g_R^{(0)}\Big]^T + \frac{1}{4}\Big\{(m^\dagger m)', \partial_{k_z} g_R^{(0)}\Big\}^T \\ & - \frac{1}{8k_z}\Big[m^{\dagger\prime} m - m^\dagger m', g_R^{(0)} - \hat{g}_L^{(0)}\Big]^T \\ & - \frac{1}{8k_z}\Big\{(m^\dagger m)', g_R^{(0)} - \hat{g}_L^{(0)}\Big\}^T , \end{aligned} \tag{4.129}$$

and $\hat{g}_L^{(0)} \equiv V^\dagger U g_L^{(0)} U^\dagger V$. The definition of $\hat{g}_L^{(0)}$ is motivated by the relation

$$2\, m\, \hat{g}_L^{(0)} m^\dagger = \Big\{m m^\dagger, g_L^{(0)}\Big\} , \tag{4.130}$$

such that $\hat{g}_L^{(0)}$ transforms as a right-handed quantity, what simplifies the expression in the sources (4.129).

Here we have introduced the damping rate Γ that helps fulfill boundary conditions at infinity ($g_R^{(1)} \to 0$ for $z \to \pm\infty$) and substitute phenomenologically the collision terms that have been neglected. The lowest order solution is recovered in the case of infinite damping. At the same time this damping helps to cure the infrared divergencies in the sources. For simplicity, in this work we assume that the damping Γ is flavour blind, *i.e.* we take it to be proportional to the unity matrix in flavour space.

The solution of equation (4.129) in the mass eigenbasis is formally given by

$$g_{Rd}^{T(1)}(z) = \int_{-\infty}^{+\infty} W(z,z') S_{Rd}^{(1)}(z') dz', \tag{4.131}$$

with the kernel

$$\begin{aligned} W(z,z') &= \frac{1}{k_z}\big[\theta(k_z k_0)\theta(z-z') - \theta(-k_z k_0)\theta(z'-z)\big] \\ &\quad \times \exp\Big(-\frac{k_0}{k_z}\Gamma(z-z')\Big)\, \exp\left(-\frac{i}{2k_z}\int_{z'}^{z}\big[m_d^2(y),\,\cdot\,\big]\,dy\right). \end{aligned} \tag{4.132}$$

The exponential function is understood as the power series in nested commutators, and the source is rotated into the mass eigenbasis $S_{Rd}^{(1)} = V\, S_R^{(1)} V^\dagger$.

From this we can deduce the part of g_R that breaks the Q-symmetry ($g_R^{\not Q} = g_R - g_R^{Q}$)

$$g_{Rd}^{\not Q}(z) = \int_{-\infty}^{+\infty} W_{\not Q}(z,z') S_{Rd}^{(1)}(z') dz', \tag{4.133}$$

with

$$\begin{aligned} W_{\not Q}(z,z') &= \frac{1}{k_z}\big[\theta(k_z k_0)\theta(z-z') - \theta(-k_z k_0)\theta(z'-z)\big] \\ &\quad \times \exp\Big(-\frac{k_0}{k_z}\Gamma(z-z')\Big)\, \sin\left(\frac{-i}{2k_z}\int_{z'}^{z}\big[m_d^2(y),\,\cdot\,\big]\,dy\right). \end{aligned} \tag{4.134}$$

The CP-violating diagonal part of g_R in the flavour basis is given by Tr $\left(\sigma_3\, V g_{Rd}^{\not Q} V^\dagger\right)$.

First Order Contributions in the Local Approximation

In the following paragraphs we will discuss the case of very large damping. This is one of the limiting cases in which one can obtain analytical results. In addition this situation is close to the approach of Carena *et al.* who calculate in a first step sources in the local approximation. Since the large damping Γ will suppress long range effects, we approximate the integral (4.133) by expanding all functions around the position z and keep only the first Taylor coefficients. This procedure is justified provided $\Gamma\, L_w \gg 1$. For the MSSM this leads to $\Gamma \gg L_w^{-1} \simeq T_c/20$ [68], with the expansion parameter of the gradient expansion $T_c\, L_w \simeq 20$. Since the off diagonal entries are coherent

superpositions of particle states, Γ characterizes the inverse decoherence length. On physical grounds we expect Γ to be at least as large as the thermalization rate, which we take to be of the order the thermalization rate for the W bosons, $\Gamma \sim \Gamma_W \simeq \alpha_w T_c$. To get its detailed form and magnitude would require a quantitative analysis of the collision term for mixing fermions however, which is beyond the scope of this work. Here we take Γ to be of the order of the thermalization rate and proportional to unity in flavour space.

Making a leading order approximation of the sine function in Eq. (4.134), the expression contributing at leading (first) order in gradients acquires the following simple local form,

$$g_R^{T(1)} = \left(k_0\Gamma - \frac{i}{2}\left[m^\dagger m, \cdot\right]\right)^{-1} S_R^{(1)}\,, \tag{4.135}$$

such that the Q-breaking part reads

$$g_R^{T\not{Q}} = \frac{\frac{i}{2}\left[m^\dagger m, \cdot\right]}{k_0^2\Gamma^2 + \frac{1}{4}\left[m^\dagger m, \left[m^\dagger m, \cdot\right]\right]} S_R^{(1)}\,. \tag{4.136}$$

Since only transverse sources lead to Q-breaking terms we rewrite the expression (4.129) in the more compact form

$$\begin{aligned} S_R^{(1)} &= -k_z\left[V^{\dagger\prime}V, g_R^{(0)}\right] + \frac{1}{4}\left\{(m^\dagger m)', \partial_{k_z} g_R^{(0)}\right\}^T \\ &\quad -\frac{s}{4\tilde{k}_0}\left[m^{\dagger\prime}m - m^\dagger m', \hat{g}_0^{(0)}\right]^T - \frac{s}{4\tilde{k}_0}\left\{(m^\dagger m)', \hat{g}_0^{(0)}\right\}^T\,, \end{aligned} \tag{4.137}$$

where $\hat{g}_0^{(0)} \equiv V^\dagger g_{0d}^{(0)} V = (g_R^{(0)} + V^\dagger U g_L^{(0)} U^\dagger V)/2$ transforms right-handedly, and we made use of the leading order constraint equation, $\hat{g}_3^{(0)} = s(k_z/\tilde{k}_0)\hat{g}_0^{(0)}$, with $\hat{g}_0^{(0)} \equiv V^\dagger g_{0d}^{(0)} V$ and

$$g_{0d}^{(0)} = 2\pi|\tilde{k}_0|\delta(k^2 - |m_d|^2)n\,, \tag{4.138}$$

where in equilibrium and for a static wall the occupation number reduces to the Fermi-Dirac distribution, $n \to n_0 = (\exp(k_0/T_c) + 1)^{-1}$.

Here we can establish for the first time the important fact that in our treatment a static wall does not induce any CP-violating charge densities. In the local approximation (4.136) the following integrals are relevant for the calculation of the CP-violating source current

$$\begin{aligned} j_R^{T\not{Q}} &= -\frac{s}{4}\int\frac{d^4k}{(2\pi)^4}\frac{\frac{i}{2}\left[m^\dagger m, \cdot\right]}{k_0^2\Gamma^2 + \frac{1}{4}\left[m^\dagger m, \left[m^\dagger m, \cdot\right]\right]}\frac{k_0}{\tilde{k}_0^2} \\ &\times \left(4k_z^2\left[V^{\dagger\prime}V, \hat{g}_0^{(0)}\right] + \left[m^{\dagger\prime}m - m^\dagger m', \hat{g}_0^{(0)}\right] + \left\{(m^\dagger m)', \hat{g}_0^{(0)}\right\}^T\right). \end{aligned} \tag{4.139}$$

The left-handed source current $j_L^{T\not{Q}}$ is obtained simply by the substitutions $s \to -s$, $m \leftrightarrow m^\dagger$ and $\hat{g}_0^{(0)} = V^\dagger g_{0d}^{(0)} V \to U^\dagger g_{0d}^{(0)} U$. The second term in Eq. (4.137) does not contribute to the current (4.139), since the integral over the momenta vanishes for this term.

Second Order Contribution in the Local Approximation and Domination by Diagonal Parts

For special choices of the mass matrix, or in the limit where the oscillations suppress the off-diagonal contributions, the first order contributions to the CP-violation, produced by the oscillations of the off-diagonal terms, are negligibly small. When viewed in the mass eigenbasis, the problem then reduces to the diagonal case, such that the first CP-violating contributions come from the second order semiclassical force in the kinetic equation. This approach was originally pursued in [19], and we summarized its main results in the section on the one flavour case section 4.4.

We have seen that the first order terms are for large damping Γ or fast oscillation Λ suppressed as $(k_0^2\Gamma^2 + \Lambda^2)^{-1}$. In this section we pose the question: How are the second order terms suppressed in these limits?

Since Γ is large we integrate the Taylor expansion of the source using the Green function method and notice that the first coefficient gives no contribution (since it is odd in k_z) and the second term gives

$$g_0^{(2)D} = \frac{isk_z}{8k_0^2\Gamma^2\tilde{k}_0}\left\{m^{\dagger\prime\prime}m - m^\dagger m'', \partial_{k_z}\hat{g}_0^{(0)}\right\}^{\prime D}. \tag{4.140}$$

The term (4.140) is suppressed by Γ^2 as the contributions in the first order terms, but in addition by two more orders in the gradient expansion. Therefore we cannot infer that the second order terms dominate for large damping. Rather the region in parameter space where Λ is large leads to dominance of the diagonal terms since the second order terms are not afflicted by the oscillation and hence not suppressed by Λ. However, the second order terms can yield CP-violation in the trace of the Green function, while the first order terms are always traceless. Therefore the second order terms could be more important for generating a baryon asymmetry, depending on which contribution is more efficiently transformed into the BAU in the model under consideration.

Local Contributions to the Currents in the MSSM

In this section we give the explicit expressions for the CP-violating currents in the MSSM in the local approximation. Since in the MSSM we expect the damping Γ to be less than $T_c/20$, we are in the regime where transport is important, such that the main intention is to make our approach comparable with Carena *et al.*, in which local sources for diffusion equations have been derived [80, 81] in the MSSM.

The most important contribution to the BAU in the MSSM is determined by the mass matrix of the chargino-Higgsino sector with complex M_2, μ_c and real H_1, H_2,

$$m = \begin{pmatrix} M_2 & gH_2^* \\ gH_1^* & \mu_c \end{pmatrix}. \tag{4.141}$$

The procedure how to diagonalize m is outlined in appendix D.

Using this parametrization we can evaluate the CP-violating chiral source currents $j_R^{T\mathcal{Q}}$ and $j_L^{T\mathcal{Q}}$ (4.139) in the interaction basis. Since these sources are traceless, the relevant quantities are $\mathrm{Tr}\big(\sigma^3 j_R^{T\mathcal{Q}}\big)$ and $\mathrm{Tr}\big(\sigma^3 j_R^{T\mathcal{Q}}\big)$, where $\sigma^3 = \mathrm{diag}(1,-1)$ in flavour space.

The traces can be easily evaluated by making use of appendix D, such that we find for the three different source terms in (4.139)

$$\begin{aligned}
\mathrm{Tr}\big[(V\sigma^3V^\dagger)^T(VV^{\dagger\prime})^T\big] &= 4i\frac{1}{\Lambda^2}\Im(M_2\mu_c)\big(u_1'u_2-u_1u_2'\big),\\
\mathrm{Tr}\big[(V\sigma^3V^\dagger)^T\big(V(m^{\dagger\prime}m-m^\dagger m')V^\dagger\big)^T\big] &= -4i\frac{\bar\Delta}{\Lambda^2}\Im(M_2\mu_c)\big(u_1u_2\big)',\\
\mathrm{Tr}\big[(V\sigma^3V^\dagger)^T\big(\sigma^3V(m^\dagger m)'V^\dagger\big)^T\big] &= -4i\frac{1}{\Lambda}\Im(M_2\mu_c)\big(u_1'u_2-u_1u_2'\big),
\end{aligned} \tag{4.142}$$

where we used $\bar\Lambda=\Lambda$. The form of the chiral source can then be written as (we reinsert the spin superscript)

$$\begin{aligned}
\mathrm{Tr}\big(\sigma^3 j_R^{s\mathcal{Q}}\big) &= s\frac{\Im(M_2\mu_c)}{T_c^2}\frac{\bar\Delta}{T_c^2}\big(u_1u_2\big)'\eta^3_{(0)}\\
&\quad -s\frac{\Im(M_2\mu_c)}{T_c^2}\big(u_1'u_2-u_1u_2'\big)\big(\eta^0_{(0)}+4\eta^3_{(2)}\big),
\end{aligned} \tag{4.143}$$

$$\begin{aligned}
\mathrm{Tr}\big(\sigma^3 j_L^{s\mathcal{Q}}\big) &= s\frac{\Im(M_2\mu_c)}{T_c^2}\frac{\Delta}{T_c^2}\big(u_1u_2\big)'\eta^3_{(0)}\\
&\quad +s\frac{\Im(M_2\mu_c)}{T_c^2}\big(u_1'u_2-u_1u_2'\big)\big(\eta^0_{(0)}+4\eta^3_{(2)}\big)\,,
\end{aligned} \tag{4.144}$$

where $\bar\Delta = |M_2|^2-|\mu_c|^2+(u_1^2-u_2^2)$, $\Delta=|M_2|^2-|\mu_c|^2-(u_1^2-u_2^2)$, and we defined the integrals

$$\begin{aligned}
\eta_{(n)1/2} &\equiv T_c^{2-n}\int_{k_0>0}\frac{d^4k}{(2\pi)^3}\frac{k_0}{\tilde k_0}k_z^n\frac{n(k^\mu,m^2_{1/2})}{k_0^2\Gamma^2+(\Lambda/2)^2}\delta(k^2-m^2_{1/2}),\\
\eta^0_{(n)} &\equiv \frac{1}{2}\big(\eta_{(n)1}+\eta_{(n)2}\big)\,,\qquad \eta^3_{(n)}\equiv\frac{T_c^2}{2\Lambda}\big(\eta_{(n)1}-\eta_{(n)2}\big).
\end{aligned} \tag{4.145}$$

The functions $\eta^0_{(n)}$ and $\eta^3_{(n)}$ are dimensionless and depend only weakly on Λ in the region where $|k_0|\Gamma\geq\Lambda$, but generate a behaviour $\propto T_c^4/\Lambda^2$ in the limit $\Gamma\to 0$.

Several comments are now in order. Eqs. (4.143–4.144) represent the CP-violating sources, which were calculated by solving iteratively the quantum kinetic equations for two mixing charginos of the MSSM. These sources are absent in the single fermion case, in which case the diagonal semiclassical force source dominates. Both of the chiral source currents are proportional to spin, and hence when summed over spin the sources vanish,

$$\sum_s j_R^{s\mathcal{Q}}=0\,,\qquad \sum_s j_L^{s\mathcal{Q}}=0\,. \tag{4.146}$$

Non-vanishing sources are obtained only when a weighted sum over spin is performed, $\sum_s s j_R^{s\mathcal{Q}}$ and $\sum_s s j_L^{s\mathcal{Q}}$. Using the spin projectors as described in the section about the one flavour case, these sums will lead to z-components of the vector- and axial-currents. This is in contrast to the one flavour case, where just the axial current was sourced. The currents in the plasma frame can be obtained by boosting the present solutions from the wall frame into the plasma frame. In the case of the first order contributions there are covariant expressions for the resulting four-currents.

From Eqs. (4.143–4.144) we then easily get for the four-currents

$$\begin{aligned}
\mathrm{Tr}\big(\sigma^3 j_{5\,\mu}^{\mathcal{Q}}\big) &= \mathrm{Tr}\sigma^3 \sum_s \frac{s}{2}\big(j_{R\mu}^{s\mathcal{Q}} + j_{L\mu}^{s\mathcal{Q}}\big) = \mathcal{S}_\mu^a, \\
\mathcal{S}_\mu^a &:= 2\frac{\Im(M_2\mu_c)}{T_c^2}\frac{|M_2|^2 - |\mu_c|^2}{T_c^2}\big[\partial_\mu\big(u_1 u_2\big)\big]\eta_{(0)}^3,
\end{aligned} \tag{4.147}$$

and

$$\begin{aligned}
\mathrm{Tr}\big(\sigma^3 j_\mu^{\mathcal{Q}}\big) &= \mathrm{Tr}\sigma^3 \sum_s \frac{s}{2}\big(j_{R\mu}^{s\mathcal{Q}} - j_{L\mu}^{s\mathcal{Q}}\big) = \mathcal{S}_\mu^b + \mathcal{S}_\mu^c, \\
\mathcal{S}_\mu^b &:= 2\frac{\Im(M_2\mu_c)}{T_c^2}\frac{u_1^2 - u_2^2}{T_c^2}\big[\partial_\mu(u_1 u_2)\big]\eta_{(0)}^3, \\
\mathcal{S}_\mu^c &:= \frac{\Im(M_2\mu_c)}{T_c^2}\big(u_2\partial_\mu u_1 - u_1\partial_\mu u_2\big)\big(\eta_{(0)}^0 + 4\eta_{(2)}^3\big)\,,
\end{aligned} \tag{4.148}$$

such that the sources in the local approximation neatly split into the plus and minus contributions, $\propto \partial_\mu(u_1 u_2)$ and $\propto u_2\partial_\mu u_1 - u_1\partial_\mu u_2$, respectively. The axial current is sourced by the plus contribution only (just like in the case of the second order semiclassical force), while the vector current is sourced by both plus and minus contributions. In the non-local case, both plus and minus terms contribute to $j_\mu^{\mathcal{Q}}$ and $j_{5\mu}^{\mathcal{Q}}$. These results have a similar structure to the sources found in Refs. [80] and [81] by Carena *et al.* They differ however, when a detailed quantitative comparison is made.

Finally, we quote the second order diagonal source calculated in the local approximation (4.140) and the wall frame:

$$\mathrm{Tr}\big(\mathbb{1}\, j_{5\,\mu}^{(2)}\big) = \quad \mathcal{S}_\mu^d \quad := 2\frac{\Im(M_2\mu_c)}{T_c^4}\partial_\mu\Big(u_1'' u_2 + u_1 u_2''\Big)\zeta_{(0)}^3 \tag{4.149}$$

with the definitions

$$\begin{aligned}
\zeta_{(n)1/2} &\equiv T_c^{2-n}\int_{k_0>0}\frac{d^4k}{(2\pi)^3}\frac{k_0}{\tilde{k}_0}k_z^n\frac{n(k^\mu, m_{1/2}^2)}{k_0^2\Gamma^2}\delta(k^2 - m_{1/2}^2), \\
\zeta_{(n)}^3 &\equiv \frac{T_c^2}{2\Lambda}\big(\zeta_{(n)1} - \zeta_{(n)2}\big).
\end{aligned} \tag{4.150}$$

Note again that, in contrast to η, ζ is not suppressed for large Λ, such that the second order terms dominate in the local regime for large values of Λ.

Nonlocal Transport

To evaluate (4.133) for small Γ we could just solve the integral numerically. However this would involve some technical and physical shortcomings. First, the integrand is oscillating with a frequency $\omega \sim \Lambda/k_z$, which makes numerical evaluation hard. Second, since we have parametrized the collision terms in the kinetic equation by just one parameter, our solution does not show the expected behaviour in certain regions of parameter space. *E.g.* we expect that collisions help to isotropize the deviation from equilibrium, while the solution to equation (4.133) has a strong k_z dependence, but almost no $k_{||}$ dependence ($k_{||}$ denotes the momentum parallel to the wall and will only be introduced when the boost is performed that made our problem effectively $1+1$ dimensional, see (4.26)).

Another feature which may play an important role is diffusion, by which particles get transported typically to distances

$$\ell_{\rm diff} \simeq \frac{2D}{v_w + (v_w^2 + 4\Gamma D)^{1/2}} \tag{4.151}$$

in front of the wall, where D denotes the diffusion constant, v_w the wall velocity, and Γ the damping. The effect is basically, that the damping in front and behind the wall is not symmetric. Our transport equations depend not explicitly on the wall velocity, such that this effect cannot be generated by our equations. To mimic such an effect that normally would be introduced by an explicit calculation of damping induced by the collision terms, we will choose an *ansatz* that displays an explicit v_w-dependence.

Only in systems with small Γ and/or large D, such that $v_w \gg 4\Gamma D$ is satisfied, the diffusion tail may be large, $\ell_{\rm diff} \simeq D/v_w$. Since for charginos of the MSSM, the diffusion constant and the wall velocity are rather small, $D \sim 10/T$, $v_w \leq 0.1$ and the damping quite large, $\Gamma \sim \alpha_w T$, the condition $v_w^2 \ll 4\Gamma D$ is amply fulfilled, and we can estimate the diffusion 'tail' to be $\ell_{\rm diff} \simeq (D/\Gamma)^{1/2} \sim 15/T$. Since diffusion in our case extends only to distances of the order the wall thickness, we expect it to be captured reasonably well by our simple model of damping. The sole implication of this artificial introduction of diffusion is an additional suppression in the case of large wall velocities and effective diffusion and will affect our numerical results only marginally.

To cure these shortcomings we shall solve the kinetic equation (4.129) by a fluid *ansatz* in the mass eigenbasis and the wall frame

$$\begin{aligned} g^{(1)}_{Rdij} &= 2\pi \sum_{a=0}^{N} T_c^{-a} \mu_{aij}(z)\,(k_z - v_w k_0)^a\,|k_0| \\ &\qquad \times \big[\partial_{k_0} n_0\big(\gamma(k_0 - v_w k_z)\big)\big]\,\delta(k_0^2 - \omega_{ij}^2)\,, \end{aligned} \tag{4.152}$$

where ω_{ij} are given by the lowest order on-shell conditions (4.95), and n_0 is the Fermi-Dirac distribution function, $n_0(x) = 1/[\exp(x/T_c) + 1]$.

If one now takes the first N momenta of the kinetic equation (4.129), defined as

$$\int_{k_0>0} [d^4k/(2\pi)^4](k_0/\tilde{k}_0)(k_z/T_c)^l\, X, \quad (l=0,..,N), \tag{4.153}$$

one gets a matrix equation of the form (here and below we suppress the i,j indices):

$$A\partial_z\mu + \frac{i}{2}B\big[m_d^2,\mu\big] + \Gamma C\mu = D\,. \tag{4.154}$$

A, B, C are matrices and μ and D vectors in the a,b space ($a,b\in\{0..N-1\}$), with

$$\begin{aligned}
A_{ab} &= T_c^{-a-b}\int_{k_0>0}\frac{d^4k}{(2\pi)^3}|k_0|(k_z-v_wk_0)^a k_z^{b+1} \\
&\qquad \big[\partial_{k_0}n_0(\gamma(k_0-v_wk_z))\big]\delta(k_0^2-\omega^2), \qquad (4.155)\\
B_{ab} &= T_c^{-a-b}\int_{k_0>0}\frac{d^4k}{(2\pi)^3}|k_0|(k_z-v_wk_0)^a k_z^{b} \\
&\qquad \big[\partial_{k_0}n_0(\gamma(k_0-v_wk_z))\big]\delta(k_0^2-\omega^2), \\
C_{ab} &= T_c^{-a-b}\int_{k_0>0}\frac{d^4k}{(2\pi)^3}|k_0|(k_z-v_wk_0)^a k_0k_z^{b} \\
&\qquad \big[\partial_{k_0}n_0(\gamma(k_0-v_wk_z))\big]\delta(k_0^2-\omega^2), \\
D_a &= -\frac{s}{4T_c^a}\int_{k_0>0}\frac{d^4k}{(2\pi)^3}\frac{k_0}{|\tilde{k}_0|}k_z^a\delta(k_0^2-\omega^2) \\
&\quad\times\left(4k_z^2\left[VV^{\dagger\prime},n_0\right]+\left[V(m^{\dagger\prime}m-m^\dagger m')V^\dagger,n_0\right]^T+\left\{V(m^\dagger m)'V^\dagger,n_0\right\}^T\right).
\end{aligned}$$

The summation in Eq. (4.154) runs over b, while a, i and j are held fixed.

The eigenvalues γ_i of the matrix

$$\gamma = A^{-1}\left(\frac{i}{2}\,B[m_d^2,\cdot]+\Gamma\,C\right) \tag{4.156}$$

determine the damping and oscillatory behaviour of the solution. Due to the form of the source, the first few momenta dominate the solution, what has been checked numerically. In case of only two momenta, the system can be converted in one differential equation of second order and a mathematical system of diffusion equation type results. If the source has a compact support, we can deduce that outside this compact region, μ is a superposition of damped harmonic oscillations with the frequencies $\Im(\gamma_i)$ and damping rates $\Re(\gamma_i)$. The amplitude of these oscillations is then suppressed by $|\gamma_i|^{-1}$, such that fast oscillating modes give smaller contributions to the current.

In the following we will present numerical results of the fluid *ansatz*. The Higgs *vevs* and the β angle are parametrized by $H_1(z)=H(z)\sin(\beta(z))$, $H_2(z)=H(z)\cos(\beta(z))$ and

$$H(z)=\frac{1}{2}v(T)\left(1-\tanh\left(\alpha\left(1-\frac{2z}{L_w}\right)\right)\right), \tag{4.157}$$

$$\beta(z)=\beta_\infty-\frac{1}{2}\Delta\beta\left(1+\tanh\left(\alpha\left(1-\frac{2z}{L_w}\right)\right)\right), \tag{4.158}$$

as motivated in section 4.1.

The parameters used in the plots are

$$T_c = 110 \text{ GeV}, \ v(T_c) = 175 \text{ GeV}, \ \alpha = \tfrac{3}{2}, \tag{4.159}$$

$$\tan(\beta_\infty) = 10, \ T_c\, l_w = 20, \ \Gamma = \alpha_W T_c \tag{4.160}$$

and the complex phase is chosen maximally $\Im(M_2\mu_c) = |M_2\mu_c|$. Furthermore, the value of $\Delta\beta = 0.0108$ is deduced from [68] by using the value $m_A = 200$ GeV.

The wall velocity is taken to be $v_w = 0.05$ and the plots are evaluated for the first six momenta. The currents j^0, j_5^0 and the second order term ${j_5}^{(2)}$ are evaluated in the plasma frame, thus the expressions are linear in v_w.

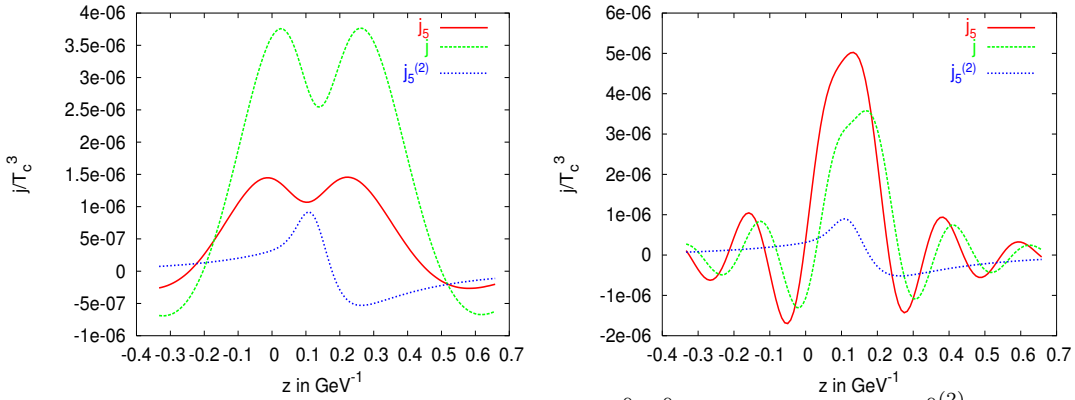

Figure 4.1: The CP-violating currents of first order j^0, j_5^0 and of second order $j_5^{0(2)}$. $M_2 = 200$ GeV and $m_A = 200$ GeV.
left plot: $\mu_c = 205$ GeV, $\Lambda/T_c^2 \in [0.22, 4.0]$, right plot: $\mu_c = 220$ GeV, $\Lambda/T_c^2 \in [0.93, 4.2]$.

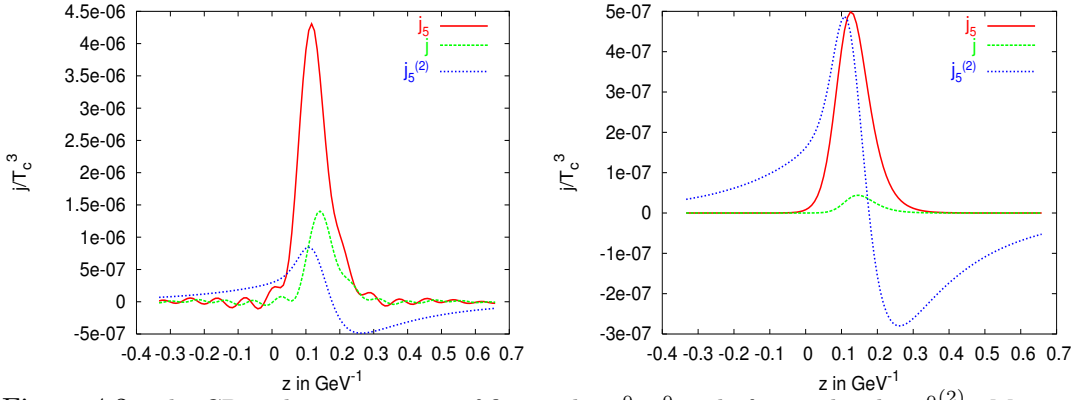

Figure 4.2: The CP-violating currents of first order j^0, j_5^0 and of second order $j_5^{0(2)}$. $M_2 = 200$ GeV and $m_A = 200$ GeV.
left plot: $\mu_c = 250$ GeV, $\Lambda/T_c^2 \in [2.4, 5.1]$, right plot: $\mu_c = 450$ GeV, $\Lambda/T_c^2 \in [18.0, 19.2]$.

The plots Fig. 4.1 and Fig. 4.2 show all three currents for the four chosen values $\mu_c = \{205, 220, 250, 450\}$ GeV and $M_2 = 200$ GeV. For small Λ, respectively $M_2 \simeq \mu_c$, the solution is oscillating and has rather large amplitudes. These oscillations are, as

expected, suppressed for larger values of Λ and a local contribution remains. For $\Lambda \sim 20\,T_c^2 \sim T_c^3 L_w$ the second order contribution, which shows a weaker dependence on Λ, begins to dominate. When Λ is large, the first order currents are suppressed due to efficient decoherence. In the BAU, the second order terms start to dominate earlier since, for the first order terms, the oscillations and inefficient transport prevent in part an efficient source conversion to baryon asymmetry, while the second order terms are transported more efficiently and without oscillations, such that they give a truly non-local contribution.

The term $\mathcal{S}^c$, resulting from the combination $u_1\partial_\mu u_2 - u_2\partial_\mu u_1 \sim \Delta\beta$, is suppressed due to the fact that $\Delta\beta$ is small. In addition, the terms that include η^3 are smaller than the terms including η^0.

The axial vector current j_5^0, that is normally the largest contribution to the source, is suppressed for small Λ due to the factor $|M_2|^2 - |\mu_c|^2$ in Eq. (4.147) and in this region the vector current j^0 becomes the most important one.

In the next section we will analyze how this CP-violation in the chargino sector is communicated to the Standard Model fermions such that the weak sphaleron process is biased and a net baryon number generated.

4.6 Diffusion to the Standard Model Particles

Using our formalism, we can deduce the CP-violating particle densities in the chargino sector. To evaluate the baryon asymmetry in the broken phase, we need to compute the density of left-handed quarks and leptons n_L in front of the wall. These densities couple to the weak sphaleron and produce a net baryon number.

To determine how the CP-violating currents are transported from the charginos to the left-handed quarks and leptons we use a system of coupled diffusion equations as derived in [86, 87], and later adapted in [80] and [76]. The diffusion equations are

$$\begin{aligned}
v_w\, n_Q' &= D_q\, n_Q'' - \Gamma_Y \left[\frac{n_Q}{k_Q} - \frac{n_T}{k_T} - \frac{n_H + n_h}{k_H}\right] - \Gamma_m \left[\frac{n_Q}{k_Q} - \frac{n_T}{k_T}\right] \\
&\quad -6\,\Gamma_{ss}\left[2\,\frac{n_Q}{k_Q} - \frac{n_T}{k_T} + 9\,\frac{n_Q + n_T}{k_B}\right], && (4.161)\\
v_w\, n_T' &= D_q\, n_T'' + \Gamma_Y \left[\frac{n_Q}{k_Q} - \frac{n_T}{k_T} - \frac{n_H + n_h}{k_H}\right] + \Gamma_m \left[\frac{n_Q}{k_Q} - \frac{n_T}{k_T}\right] \\
&\quad +3\,\Gamma_{ss}\left[2\,\frac{n_Q}{k_Q} - \frac{n_T}{k_T} + 9\,\frac{n_Q + n_T}{k_B}\right], && (4.162)\\
v_w\, n_H' &= D_h\, n_H'' + \Gamma_Y \left[\frac{n_Q}{k_Q} - \frac{n_T}{k_T} - \frac{n_H + n_h}{k_H}\right] - \Gamma_h \frac{n_H}{k_H}, && (4.163)\\
v_w\, n_h' &= D_h\, n_h'' + \Gamma_Y \left[\frac{n_Q}{k_Q} - \frac{n_T}{k_T} - \frac{n_H + n_h}{k_H}\right] - (\Gamma_h + 4\Gamma_\mu)\frac{n_h}{k_H}, && (4.164)
\end{aligned}$$

where n_T denotes the density of the left-handed top and stop particles, n_Q the remaining left-handed quarks and squarks and n_H and n_h the sum and difference of the two Higgsino densities n_{H_1} and n_{H_2}. The quantities k_i are statistical factors defined by $n_i = k_i\,\mu_i \frac{T_c^2}{6}$ (μ_i denotes the chemical potential of species i). For light, weakly interacting particles $k_i \approx 2$ (bosons) or $k_i \approx 1$ (fermions), while for particles much heavier than T_c, k_i is exponentially small. We use the values

$$k_Q \approx 6, \quad k_T \approx 3, \quad k_B \approx 3, \quad k_H \approx 12, \tag{4.165}$$

taken from [86, 87], and the diffusion constants are [35]

$$D_q \sim 6/T_c, \quad D_h \sim 110/T_c. \tag{4.166}$$

For the particle number changing rates we take [35, 78, 86, 87],

$$\Gamma_y \approx \frac{1}{10}T_c, \quad \Gamma_m \approx \frac{1}{10}T_c, \quad \Gamma_h \approx \frac{1}{20}T_c, \quad \Gamma_\mu \approx \frac{1}{10}T_c \tag{4.167}$$

and for the sphaleron rates [88]

$$\Gamma_{ss} \approx 1.5 \times 10^{-2}\,T_c, \quad \Gamma_{ws} \approx 6.0 \times 10^{-6}\,T_c. \tag{4.168}$$

The diffusion equations (4.161–4.164) are derived under the assumptions [86, 87, 76] that (a) the supergauge interactions, which are of the weak strength, are in equilibrium; (b) the chargino asymmetry gets transported to the quark sector via the strong top Yukawa interactions, while the wino asymmetry does not contribute; (c) the gaugino helicity-flip interactions are in equilibrium, implying that the chemical potentials for particles and their supersymmetric partners are equal. These approximations imply that the main channel for baryon production is the conversion of the chargino asymmetry into the top sector, which then bias electroweak sphalerons.

The solution of Eqs. (4.161–4.164) is performed in several steps. First we use the transport equations in the chargino sector as described in the last section to determine n_H and n_h. The result is used as an input in the equations (4.161) and (4.162). From these equations the left-handed particle density $n_L = 5\,n_Q + 4\,n_T$ can be determined and used as a source for the weak sphaleron process as described in [80]. The net baryon density is given by

$$n_B = -\frac{3\,\Gamma_{ws}}{v_w}\int_{-\infty}^{0} dz\; n_L(z) \exp\left(z\frac{15\Gamma_{ws}}{4\,v_w}\right), \tag{4.169}$$

and finally the baryon-to-entropy ratio is determined via

$$\eta = \frac{n_B}{s}, \quad s = \frac{2\pi^2}{45} g_{\rm eff} T_c^3 \approx 51.1\; T_c^3. \tag{4.170}$$

To check whether our solution of the diffusion equation is consistent, we used the densities n_Q and n_T as input for the equations (4.163) and (4.164) and determined the

back-reactions on the Higgsino densities. The resulting deviations in the Higgsino densities never exceed 5% of the original densities. This is due to the fact that the Higgsino diffusion constant D_h is rather large and that the oscillation partially suppresses an efficient transport to the quarks and squarks. In this light the equations of the Higgsinos decouple, since the oscillation provides the shortest time-scale and the helicity flip rate Γ_h suppresses the Higgsino densities.

4.7 Numerical Results

In this section we will present numerical results of the transport and diffusion equations as published in [85]. The Higgs *vevs* and the β angle are parametrized as in (4.157) and (4.158).

The parameters used are again

$$T_c = 110 \text{ GeV}, \quad v(T) = 175 \text{ GeV}, \quad \alpha = \frac{3}{2}, \ \tan(\beta_\infty) = 10, \quad L_w = 20/T_c, \quad (4.171)$$

and the complex phase is chosen maximally

$$\Im(M_2\mu_c) = |M_2\mu_c| \,. \quad (4.172)$$

The values of $\Delta\beta$ are deduced from [68] for the different values of m_A. The wall velocity is taken to be $v_w = 0.05$ and the transport equations are evaluated using the fluid *ansatz* for the first six momenta. The parameters of the diffusion equations are given in the last section.

The plot Fig. 4.3 supports the claim that within our approximations and for our choice of parameters, the back-reaction of left-handed quarks and squarks, n_Q, n_T, on the charginos can be neglected. The amplitude of the Higgsino densities coming from the back-reaction is always smaller than 3% and leads never to corrections of the baryon-to-entropy ratio larger than 5%.

In Fig. 4.4 we plot the first order sources $\mathcal{S}^a_\mu$, $\mathcal{S}^b_\mu$, $\mathcal{S}^c_\mu$ and the second order source (semiclassical force) $\mathcal{S}^d$, as defined in Eqs. (4.147),(4.148) and (4.149). The first order sources are roughly of the same magnitude, and they peak when $|\mu_c| \simeq |M_2|$, where they also switch the sign. The second order source varies slowly with $|\mu_c|$ and tends to dominate when the difference $|\mu_c| - |M_2|$ becomes large. Note that when the damping is small, the first order sources become more peaked around $|\mu_c| = |M_2|$, but the amplitude of the baryon asymmetry does change by less than a factor 2. On the other hand, the second order source is about an order of magnitude larger in the lower plot, implying that in the limit of a small damping the second order source (semiclassical force) may result in a viable baryogenesis. Since our damping term is phenomenological and flavour blind, it would be premature to conclude that the second order source

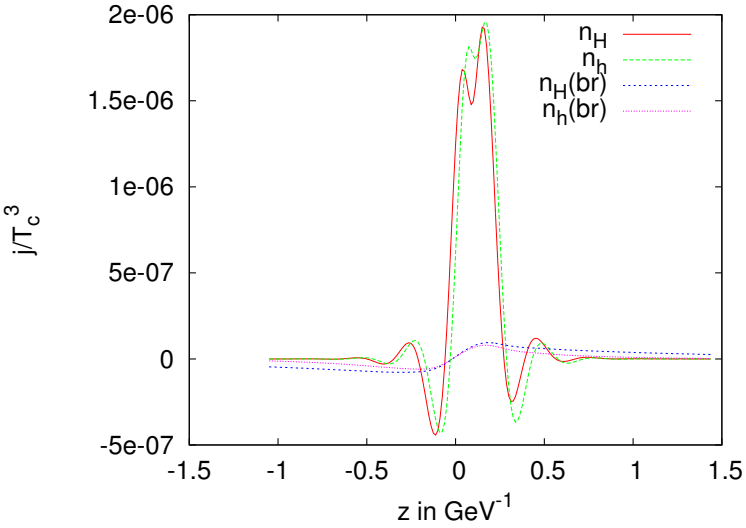

Figure 4.3: The original Higgsino densities and the corresponding back-reactions depending on the coordinate z. The parameters of the plot are $\mu_c = 200$ GeV, $M_2 = 180$ GeV, $m_A = 200$ GeV

cannot lead to a viable baryogenesis until a more quantitative analysis of the damping term is performed. However in the MSSM, a damping significantly smaller than $\alpha_w T_c$ is not expected. The parameters chosen in figures 4.5 and 4.6 are similar to the ones chosen in the work of Carena *et al.* [81], in order to facilitate comparison. In plot Fig. 4.5 the parameters m_A and M_2 are fixed while μ_c is varied. The maximum is not exactly at $\mu_c = M_2$ as in [81], but rather close to $\mu_c \approx M_2 + 20$ GeV. The reason for this difference is that in our case all sources in (4.147) and (4.148) are of similar order, while in [81], the baryon asymmetry is completely dominated by a source term of form $\mathcal{S}_\mu^c$ in (4.148) that is proportional to $\Delta\beta$ in the parametrization (4.158) and hence suppressed for large values of m_A as shown in [68]. Another difference is that our plot shows the suppression for $\mu_c \gg M_2$ what is expected since in this case the quasi-particles have highly separated on-shell conditions and mixing should be suppressed. In Fig. 4.6 the baryon asymmetry is plotted near the maximal value $\mu_c \approx M_2 + 20$ GeV. The maximum is reached near $\mu_c \approx 80$ GeV in contrast to [81] where the maximum was $\mu_c \approx 250$ GeV.

Finally in Fig. 4.7 and 4.8, two contour plots are shown with regions in the (M_2, μ_c) parameter space for the baryon asymmetry expressed in terms of $\eta_{10} \equiv 10^{10} \times \eta$. In these units the observed value due to WMAP [4] is close to unity, $\eta_{10,\mathrm{obs}} = 0.8 - 0.9$. If the absolute value of the determined value is larger than the observed baryon asymmetry, $|\eta_{10}| > \eta_{10,\mathrm{obs}}$, the observed value can be attained simply by adjusting the complex phase, which is in our calculation chosen to be maximal. The two plots correspond to the choices $m_A = 200$ GeV and $m_A = 400$ GeV.

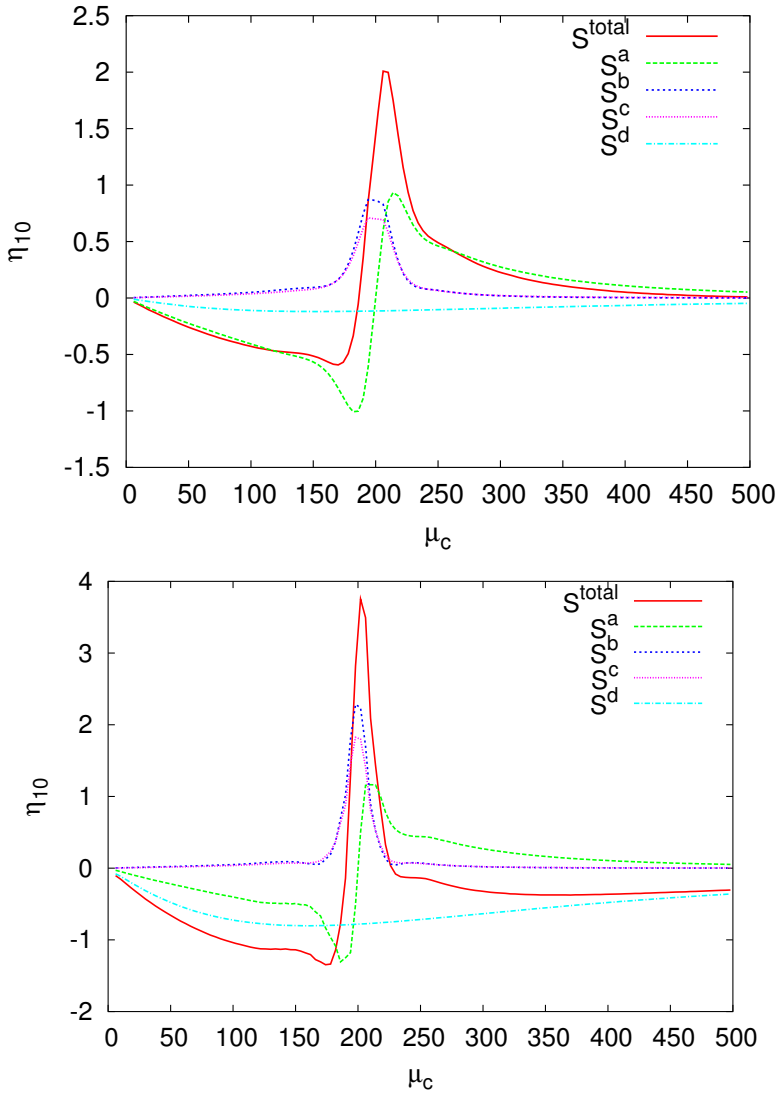

Figure 4.4: This plot shows the first and second order sources as a function of μ_c with $M_2 = 200$ GeV. The upper plot displays the sources with the damping $\Gamma = \alpha_w T_c$, while the lower plot $\Gamma = 0.25\,\alpha_w T_c$.

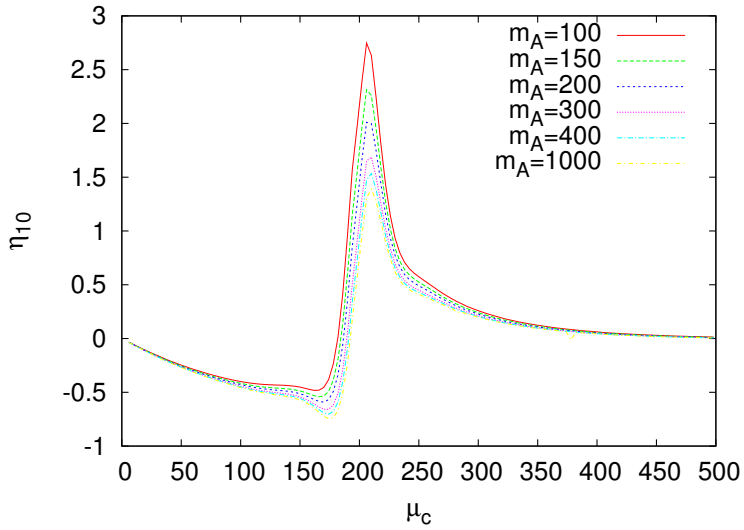

Figure 4.5: This plot shows $\eta_{10} = 10^{10}\eta$ as a function of μ_c with $M_2 = 200$ GeV and for several values of m_A in GeV.

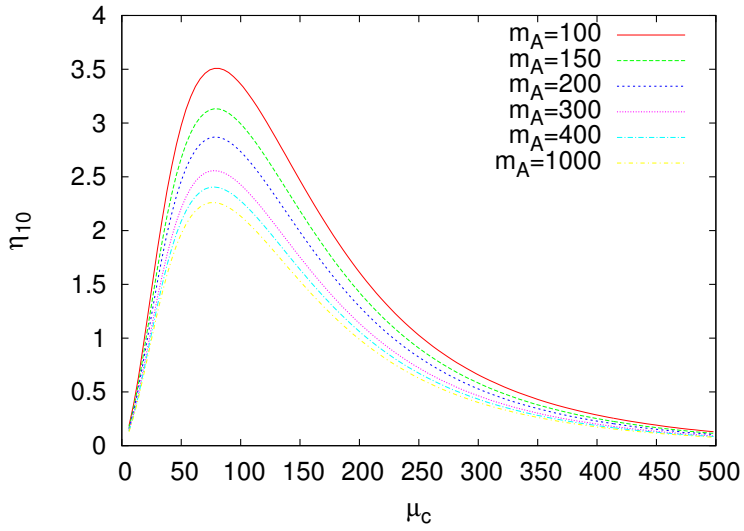

Figure 4.6: This plot shows $\eta_{10} = 10^{10}\eta$ as a function of μ_c, $M_2 = \mu_c - 20$ GeV and for several values of m_A (in GeV) .

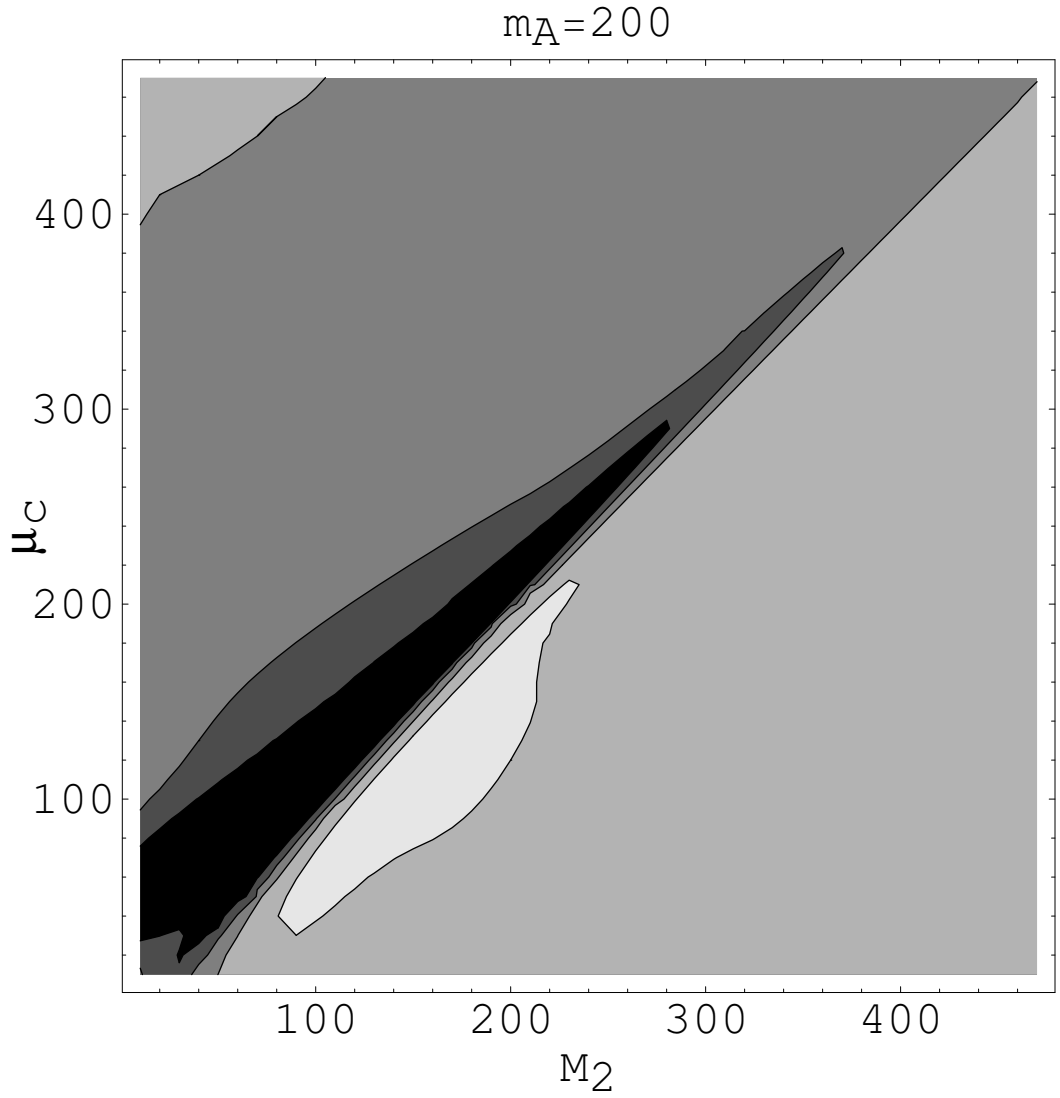

Figure 4.7: The baryon-to-entropy ratio $\eta_{10} = 10^{10} \times \eta$ in the (M_2, μ_c) parameter space from (0 GeV,0 GeV) to (400 GeV,400 GeV). The value $m_A = 200$ GeV is used. The black region denotes $\eta_{10} > 1$, where baryogenesis is viable. The other four regions are bordered by the values of η_{10}, $\{-0.5, 0, 0.5, 1\}$, beginning with the lightest color. A region with $\eta_{10} < -1$ is not present.

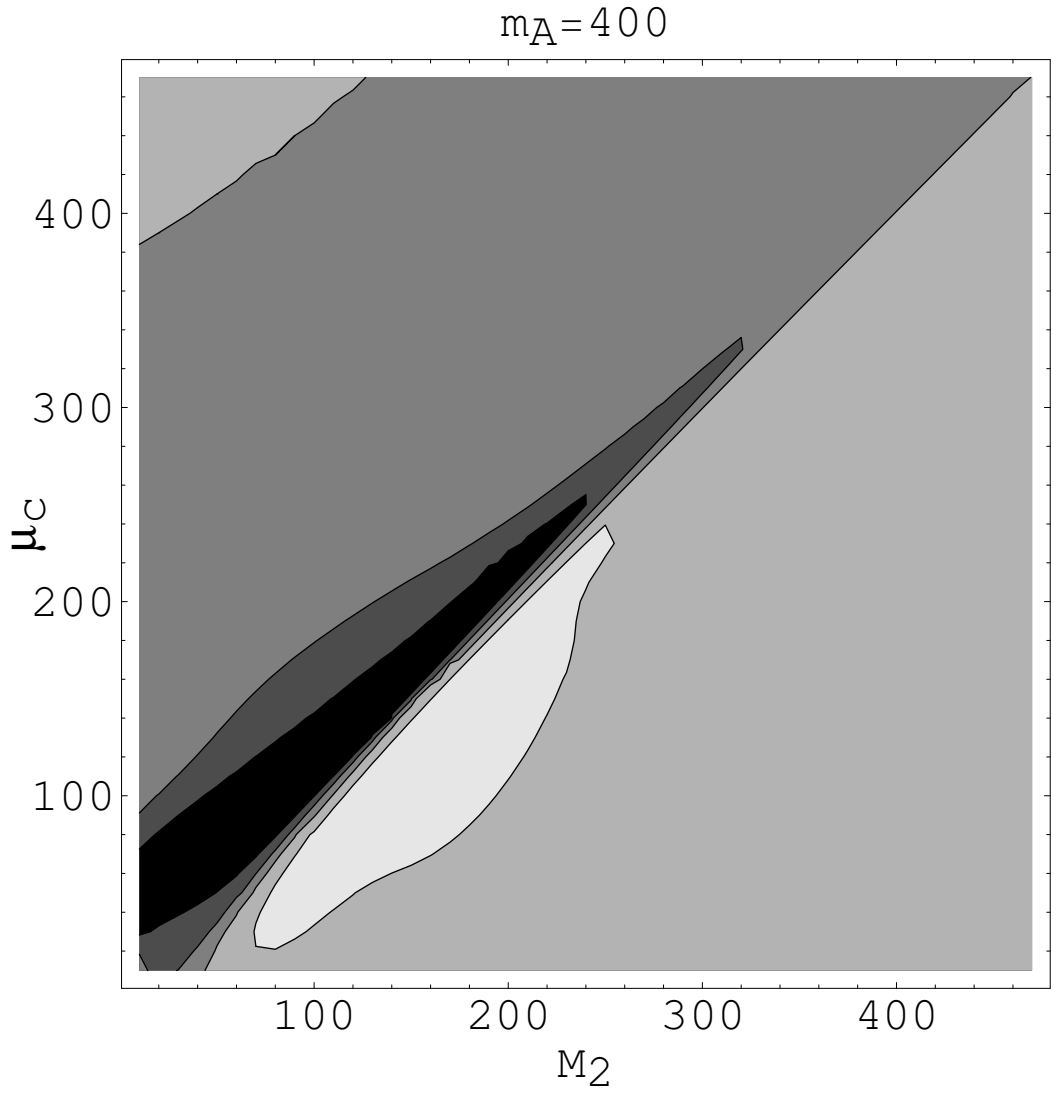

Figure 4.8: Same as Fig 4.7 but with $m_A = 400$ GeV.

4.8 Discussion

We attained our goal to derive basis independent transport equations from first principles. Essential in our approach is the use of the Wigner transform, that leads to a phase space construction with well defined semi-classical approximation given by the gradient expansion. The most important discovery is the occurrence of oscillations of the transverse densities that are decisive for the dynamics of CP-violating densities of first order in gradient expansion.

In the following we will comment on differences between the formalism used in this thesis and the work of Carena *et al.* and Cline *et al.* The first observation is that our results agree with the method of Cline *et al.* in the case of suppressed mixing (fast oscillations and large $\Lambda/T_c^2 \gg 20$). In this case the second order source dominates that corresponds to a shift in the dispersion relation. However in our model this region is not interesting from a phenomenological point of view since the generated baryon asymmetry is insufficient to explain the observed value. In other models this effect could be viable.

In comparison to the work of Carena *et al.* our basis independent formalism shows that using flavour eigenstates in classical transport equations is misleading. We have seen that the dynamics of the transverse densities is fundamentally altered by the occurrence of oscillations. This is important since these transverse parts are the only densities that lead to CP-violation up to first order in gradients in our analysis and likewise in the work of Carena *et al.* First new implication of these oscillations is that the baryon asymmetry is suppressed far from the chargino mass degeneracy (large Λ). This is physically expected since in this case mixing effects should be negligible, and that makes it possible to recover in this limit the results of Cline *et al.* Second implication is that one has additional suppression in the case of small oscillations since only the oscillating densities contribute in a CP-violating way. This can be seen most easily in the Green function solution (4.134) where the additional suppression in the sine function is explicit. This effect antagonizes the resonant enhancement found in Carena *et al.* in the chargino mass degenerate region of parameter space and makes in comparison our numerical result a factor 5-10 in magnitude smaller depending on the parameters. This enhancement was also claimed earlier in [82] and recently in [89].

Finally we will comment on the phenomenological implications of our numerical analysis similar to the discussion in [91]. Besides the requirement of sufficient BAU generation, viable baryogenesis models are subject to constraints on CP-violation coming from electromagnetic dipole moments (EDM) of the Standard Model particles. In particular the EDM of the electron fulfills the experimental constraint [92]

$$|d_e| \lesssim 1.6 \times 10^{-27} \mathrm{e\,cm}. \tag{4.173}$$

These dipole moments are enhanced by interactions with the charginos as seen in Fig.

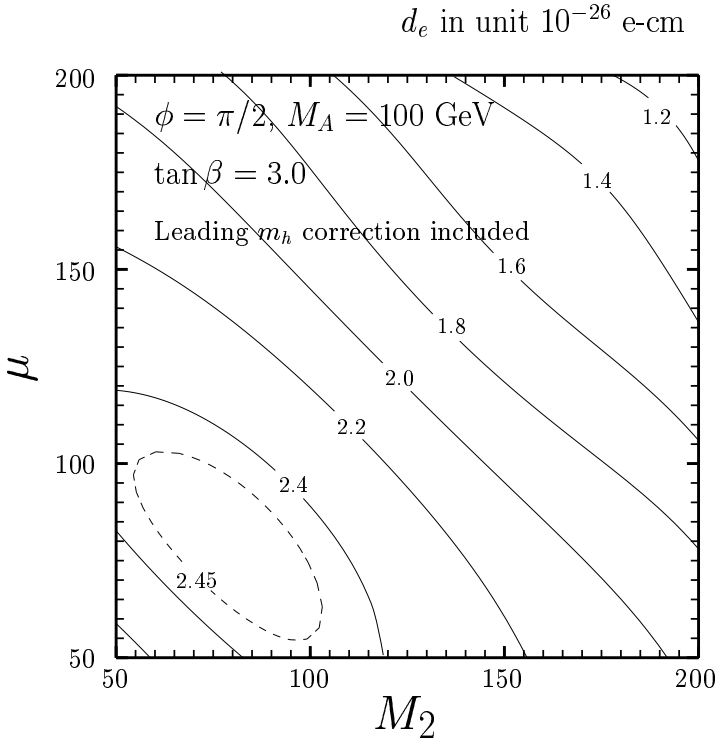

Figure 4.9: The electron EDM contour plot versus M_2 and μ_c for the case $\tan\beta = 3$, $m_A = 100$ GeV and a maximal complex phase taken from [90].

4.9 taken from [90]. Another analysis [93] comes to slightly different results but that will not affect our main conclusion.

Besides the dependence on the chargino masses the electron EDM increases with increasing $\tan\beta$ while the baryon asymmetry will in this limit decrease. However $\tan\beta$ is due to the phase transition analysis and collider experiments restricted to the region [94]

$$5 \lesssim \tan\beta \lesssim 10. \tag{4.174}$$

For the limiting case $\tan\beta = 5$ and parameters that maximize the baryon asymmetry ($\mu_c \sim 120$ GeV, $M_2 \sim 100$ GeV, $m_A \sim 100$ GeV) we find that due to the electron EDM the CP-violating phase in the chargino mass matrix has to be smaller than 1/25 and in this case the chargino mediated baryon asymmetry will be to small by a *factor 5*. Notice that we have maximized the baryon asymmetry by the choice of our parameters. All supersymmetric masses have been chosen on the edge to experimental discovery and the used bubble wall parameters tend to maximize the BAU. Even if the used system of diffusion equations gives rise to uncertainties of a *factor 4*, we do not find a region in parameter space that is in accordance with observation. This is in strong contrast to the results of Carena *et al.* [81], who claim a large region in parameter space with viable electroweak baryogenesis in the MSSM. However notice that fortuitous cancellations

between different EDM contributions could relax the EDM constraints[2] and could lead to enough CP-violation.

Our conclusion is that the chargino mediated electroweak baryogenesis in the MSSM is insufficient to explain the observed value of baryon asymmetry in the light of the electron EDM constraints as long as the EDMs are not diminished by fortuitous cancellations.

[2]C.E.M. Wagner, private communication.

Chapter 5

Conclusions

In this thesis we illuminated different aspects of CP-violation on electroweak scales.

First we payed attention to CP-violation of the CKM type as it appears in the electroweak sector of the Standard Model. We discussed the Jarlskog determinant that produces an upper bound for CP-violation in the SM and is based on the chiral nature of the electroweak interactions. We scrutinized the prerequisites of Jarlskog's argument and discovered that:

- Due to a classical background during a phase transition new rephasing invariants can be constructed.
- The hot plasma in the early universe can lead to effects that are resonant and non-perturbative in the Yukawa couplings and are hence not restricted to be small because of the Jarlskog bound.

The CP-violation in our concrete example, the quark-selfenergies, is indeed several orders larger than Jarlskog's determinant but still to small to be sufficient for a viable baryogenesis mechanism [26].

The following chapter addressed the topic of effective actions of chiral gauge theories. We presented a formalism based on the worldline representation that is in principle efficient enough to determine the coefficients of CP-violating operators in the effective action that are of sixth order in the covariant derivative expansion or higher. However in the present text we only reproduced the already known result of fourth order and left the higher order calculation for future work.

The final chapter constitutes the main part of the present text. Based on former work, that dealt with transport equations for one fermionic flavour, we derived new transport equations for several flavours in the bosonic and fermionic case [84]. As semi-classical approximation, our approach is based on the gradient expansion, but we would like to remark that our intermediate result Eqs. (4.114) and (4.113) are valid to all orders in gradient expansion. In this light, our formalism is rather general and the

presented techniques can be applied to all systems where at the same time transport and quantum mechanical effects are of importance.

Our approach is by construction completely basis independent, what resolves some controversies existing in the literature concerning CP-violation and the classical limit. We would like to emphasize that the crucial device in our derivation is the Wigner transform since it makes a phase space interpretation possible and thus leads to a well defined semi-classical limit. Furthermore, the derived transport equations are completely unambiguous and no additional phenomenological input is needed (our only phenomenological parameter is the damping term Γ that could in principle be determined by calculating the collision term).

One of the novel key feature of our transport equations is the appearance of oscillations in the off-diagonal densities in the mass eigenbasis. Since CP-violation is at this order of the gradient expansion only present *via* the off-diagonal terms, these oscillations lead to several physical expected properties in our solutions that have been missing in former approaches:

- Mixing effects are only relevant close to mass degeneracy (see Fig. 4.5).
- All effects are exponentially suppressed if both chargino masses become large (see Fig. 4.6).

Hence the oscillations make it possible to connect with former results that are valid far from degeneracy. We found that in this limit our results agree qualitatively with the semi-classical force result of the WKB approach by Cline *et al.* [76].

Furthermore, we observe no resonant enhancement close to the mass degeneracy as claimed by Carena *et al.* [81] and the authors of [82, 89]. This finding is especially important since it leads to numerical results near the degeneracy that are by a factor 5-10 smaller compared to Carena *et al.*, depending on the parameters.

Finally, a quantitative analysis in the concrete case of the MSSM was given [85]. We concluded that in the light of EDM constraints the produced baryon asymmetry is insufficient to explain the observed BAU as long as the EDMs are not diminished by fortuitous cancellations. This statement is in clear opposition to the former approach by Carena *et al.* [81] that claimed viable electroweak baryogenesis in the MSSM for large regions in parameter space.

Some aspects of MSSM electroweak baryogenesis have been beyond the scope of the present thesis and will be addressed in the future.

One of our approximations is the phenomenological treatment of damping. This damping term can be in principle predicted by determination of the collision term in Eq. (4.64). Even though CP-violation in these collision terms is phase space suppressed compared to the tree level sources [19], the explicit calculation of the damping rate can be of importance, since contributions that are not flavour blind could alter the

dynamics. Besides, a damping much smaller than our assumed thermalization scale $\alpha_w T_c$ could change the proportions between second order sources and first order mixing sources as mentioned in section 4.7.

In addition, because CP-violation in the MSSM seems to be insufficient to be in accordance with the observed magnitude of BAU, the analysis of MSSM extensions could be the focus of further work. Interesting models in the context of electroweak baryogenesis are *e.g* the nMSSM [95] or the $U(1)'$ extension of the MSSM [96, 97]. These models afford the opportunity to relax constraints on the MSSM parameters and on cosmological requirements, as *e.g.* possible dark matter candidates and the electroweak phase transition, at the cost of introducing additional degrees of freedom.

Altogether electroweak baryogenesis has the strong advantage that it can be worked out in any detail and that it can be tested in the near future in high energy particle physics experiments.

Appendix A

The Effective Action

For completeness, we will quote in this appendix the imaginary part of the effective action up to second order in the two dimensional case and up to fourth order in the four dimensional case, as it was derived in [50]. The chiral invariant remainder of the imaginary part of the effective actions can be parametrized as

$$\begin{aligned} W^-_{c,d=2} &= \left\langle N_{12}\,\mathbf{m}'^2 \right\rangle, \\ W^-_{c,d=4} &= \left\langle N_{1234}\mathbf{m}'^4 + N_{123}\mathbf{m}'^2\,\mathbf{F} \right\rangle. \end{aligned} \tag{A.1}$$

In addition to these terms, there could be additional terms of the Form $\langle M_1\,\mathbf{F}\rangle$ or $\left\langle M_{12}\,\mathbf{F}^2\right\rangle$, but these are redundant since they can be reduced to terms of the form as given in (A.1).

In two dimensions the effective action is given by

$$N_{12} = -\frac{\mathbf{m}_1\mathbf{m}_2}{\mathbf{m}_1^2-\mathbf{m}_2^2}\left(\frac{\log(\mathbf{m}_1^2/\mathbf{m}_2^2)}{\mathbf{m}_1^2-\mathbf{m}_2^2} - \frac{1}{2}\left(\frac{1}{\mathbf{m}_1^2}+\frac{1}{\mathbf{m}_2^2}\right)\right). \tag{A.2}$$

In four dimensions, the function N_{123} can be written as

$$N_{123} = N^R_{123} + N^L_{123}\log(\mathbf{m}_1^2/\mathbf{m}_2^2) + N^L_{321}\log(\mathbf{m}_2^2/\mathbf{m}_3^2), \tag{A.3}$$

with

$$\begin{aligned} N^R_{123} &= \frac{1}{2\mathbf{m}_1\mathbf{m}_2\mathbf{m}_3(\mathbf{m}_1^2-\mathbf{m}_2^2)(\mathbf{m}_2^2-\mathbf{m}_3^2)(\mathbf{m}_3^2-\mathbf{m}_1^2)} \\ &\quad\times\big(4\mathbf{m}_1\mathbf{m}_2\mathbf{m}_3(\mathbf{m}_1+\mathbf{m}_3)(2\mathbf{m}_1^2-3\mathbf{m}_1\mathbf{m}_3+2\mathbf{m}_3^2-\mathbf{m}_2^2) \\ &\quad+\mathbf{m}_2^2(\mathbf{m}_1^4+10\mathbf{m}_1^3\mathbf{m}_3-18\mathbf{m}_1^2\mathbf{m}_3^2+10\mathbf{m}_1\mathbf{m}_3^3+\mathbf{m}_3^4) \\ &\quad 3\mathbf{m}_1^2\mathbf{m}_3^2(\mathbf{m}1-\mathbf{m}_3)^2-\mathbf{m}_2^4(\mathbf{m}_1+\mathbf{m}_3)^2\big) \end{aligned} \tag{A.4}$$

and

$$\begin{aligned} N^L_{123} &= \frac{2}{(\mathbf{m}_1^2-\mathbf{m}_2^2)^2(\mathbf{m}_1^2-\mathbf{m}_3^2)(\mathbf{m}_1-\mathbf{m}_3)} \\ &\quad\times\big(\mathbf{m}_1^4(\mathbf{m}_2-2\mathbf{m}_3)+\mathbf{m}_1^2(\mathbf{m}_2^3+\mathbf{m}_3^3)+\mathbf{m}_2^2\mathbf{m}_3^2(\mathbf{m}_2+\mathbf{m}_3) \\ &\quad\mathbf{m}_1^3(\mathbf{m}_2^2-3\mathbf{m}_2\mathbf{m}_3-\mathbf{m}_3^2)\mathbf{m}_1\mathbf{m}_2\mathbf{m}_3(\mathbf{m}_2^2-\mathbf{m}_3^2)\big). \end{aligned} \tag{A.5}$$

Likewise the coefficient N_{1234} can be written

$$N_{1234} = N^R_{1234} + N^L_{1234}\log(\mathbf{m}_1^2) + N^L_{234\underline{1}}\log(\mathbf{m}_2^2) + N^L_{34\underline{12}}\log(\mathbf{m}_3^2) + N^L_{4\underline{123}}\log(\mathbf{m}_4^2). \quad \text{(A.6)}$$

Here we used the notation of [50] that underlined parameters acquire a minus sign, such that for example

$$N_{34\underline{12}} = N(\mathbf{m}_3, \mathbf{m}_4, -\mathbf{m}_1, -\mathbf{m}_2). \quad \text{(A.7)}$$

The coefficients N^R_{1234} and N^L_{1234} are

$$\begin{aligned} N_{1234} \;=\; & \frac{1}{4}\Bigg(\frac{2(2\mathbf{m}_2+\mathbf{m}_3)}{(\mathbf{m}_1^2-\mathbf{m}_2^2)(\mathbf{m}_2^2-\mathbf{m}_3^2)(\mathbf{m}_2-\mathbf{m}_4)} \\ & - \frac{2(2\mathbf{m}_2+\mathbf{m}_1)}{(\mathbf{m}_1^2-\mathbf{m}_2^2)(\mathbf{m}_2^2-\mathbf{m}_3^2)(\mathbf{m}_2+\mathbf{m}_4)} \\ & - \frac{3(\mathbf{m}_2\mathbf{m}_3-\mathbf{m}_1(\mathbf{m}_2+\mathbf{m}_3))}{\mathbf{m}_3(\mathbf{m}_1^2-\mathbf{m}_3^2)(\mathbf{m}_2^2-\mathbf{m}_3^2)(\mathbf{m}_3-\mathbf{m}_4)} \\ & + \frac{3(\mathbf{m}_1\mathbf{m}_2-\mathbf{m}_3(\mathbf{m}_1+\mathbf{m}_2))}{\mathbf{m}_1(\mathbf{m}_1^2-\mathbf{m}_2^2)(\mathbf{m}_1^2-\mathbf{m}_3^2)(\mathbf{m}_1+\mathbf{m}_4)} \\ & - \frac{\mathbf{m}_2\mathbf{m}_3+\mathbf{m}_1(\mathbf{m}_2+\mathbf{m}_3)}{\mathbf{m}_1(\mathbf{m}_1^2-\mathbf{m}_2^2)(\mathbf{m}_1^2-\mathbf{m}_3^2)(\mathbf{m}_1-\mathbf{m}_4)} \\ & + \frac{\mathbf{m}_2\mathbf{m}_3+\mathbf{m}_1(\mathbf{m}_2+\mathbf{m}_3)}{\mathbf{m}_3(\mathbf{m}_1^2-\mathbf{m}_3^2)(\mathbf{m}_2^2-\mathbf{m}_3^2)(\mathbf{m}_3+\mathbf{m}_4)} \\ & + \frac{1}{\mathbf{m}_1\mathbf{m}_2\mathbf{m}_3\mathbf{m}_4}\Bigg) \end{aligned} \quad \text{(A.8)}$$

and

$$\begin{aligned} N^L_{134} \;=\; & \frac{1}{2(\mathbf{m}_1^2-\mathbf{m}_2^2)^2(\mathbf{m}_1^2-\mathbf{m}_3^2)^2(\mathbf{m}_1^2-\mathbf{m}_4^2)^2} \\ & \times\Bigg(6\mathbf{m}_1^7\mathbf{m}_3 + (\mathbf{m}_2-\mathbf{m}_4)(\mathbf{m}_2^2\mathbf{m}_3^3\mathbf{m}_4^2 + 3\mathbf{m}_1^6\mathbf{m}_3) \\ & -\mathbf{m}_1\mathbf{m}_2\mathbf{m}_3^3\mathbf{m}_4(\mathbf{m}_2-\mathbf{m}_4)^2 \\ & +\mathbf{m}_1^2\mathbf{m}_3^2(\mathbf{m}_2^3(2\mathbf{m}_4+\mathbf{m}_3) - \mathbf{m}_4^3(2\mathbf{m}_2+\mathbf{m}_3)) \\ & -\mathbf{m}_1^4\Big[\mathbf{m}_2-\mathbf{m}_4)(2\mathbf{m}_2^2(\mathbf{m}_3+\mathbf{m}_4) + \mathbf{m}_2\mathbf{m}_4(\mathbf{m}_3+2\mathbf{m}_4) \\ & \quad + 2\mathbf{m}_3(\mathbf{m}_3^2+\mathbf{m}_4^2)\Big] \\ & +\mathbf{m}_1^3\Big[-\mathbf{m}_3^2\mathbf{m}_4^3 + \mathbf{m}_2^2\mathbf{m}_4^2(2\mathbf{m}_3+\mathbf{m}_4) + \mathbf{m}_2\mathbf{m}_3\mathbf{m}_4(2\mathbf{m}_3^2+\mathbf{m}_4^2) \\ & \quad + \mathbf{m}_2^3(\mathbf{m}_3^2+\mathbf{m}_3\mathbf{m}_4+\mathbf{m}_4^2)\Big] \\ & -\mathbf{m}_1^5\Big[\mathbf{m}_2^2(4\mathbf{m}_3+\mathbf{m}_4) + \mathbf{m}_2(-\mathbf{m}_3+2\mathbf{m}_3\mathbf{m}_4+\mathbf{m}_4^2) \\ & \quad + \mathbf{m}_3(2\mathbf{m}_3^2-\mathbf{m}_3\mathbf{m}_4+4\mathbf{m}_4^2)\Big]\Bigg). \end{aligned} \quad \text{(A.9)}$$

We apologize that we could not simplify the result to a concise expression.

Appendix B

Coherent States and Fermionic Path Integrals

In the following we will review fundamentals and properties of the coherent state formalism and show how it can be made operative in fermionic path integrals. The discussion will be close to the concrete case of the worldline path integral of the real part of the effective action as defined in chapter 3.

B.1 Coherent State Formalism

To handle the Γ matrices in the operator $\tilde{\Sigma}$, the coherent state formalism shall be employed. The key observation is, that the matrices a_r^+ and a_r^- defined by

$$a_r^\pm = \frac{1}{2}(\Gamma_r \pm i\,\Gamma_{r+3}), r = 1\ldots 3 \tag{B.1}$$

satisfy the Fermi-Dirac anticommutator relations

$$\left\{a_r^+, a_r^-\right\} = \delta_{rs}, \quad \left\{a_r^+, a_r^+\right\} = \left\{a_r^-, a_r^-\right\} = 0 \tag{B.2}$$

and thus a_r^+ and a_r^- are creation and annihilation operators for a Hilbert space with a vacuum defined by $a_r^-|0>=0$ and $<0|\,a_r^+ = 0$. To construct coherent states, one introduces six Grassmann variables θ_r and $\bar{\theta}_r$ which anticommute with each other and with the Fermi operators a_r^+ and a_r^-. The coherent states are then defined by

$$\begin{aligned}
<\theta| &= i<0|(\theta_1 - a_1^-)(\theta_2 - a_2^-)(\theta_3 - a_3^-),\\
|\theta> &= \exp\left(-\sum_{r=1}^{3}\theta_r a_r^+\right),\\
|\bar{\theta}> &= i(\bar{\theta}_1 - a_1^+)(\bar{\theta}_2 - a_2^+)(\bar{\theta}_3 - a_3^+)|0>,\\
<\bar{\theta}| &= <0|\exp\left(-\sum_{r=1}^{3}\bar{\theta}_r a_r^-\right).
\end{aligned} \tag{B.3}$$

This definition leads to the useful property that these vectors are eigenvectors of the creation and annihilation operators a_r^- and a_r^+

$$\begin{aligned} &< \theta|a_r^- =< \theta|\theta_r, \quad < \bar{\theta}|a_r^+ =< \bar{\theta}|\bar{\theta}_r, \\ &a_r^-|\theta >= \theta_r|\theta >, \quad a_r^+|\bar{\theta} >= \bar{\theta}_r|\bar{\theta} > . \end{aligned} \tag{B.4}$$

In addition, the coherent states fulfill orthogonality conditions and a relation that displays something like a 'canonical conjugate' behaviour between $|\theta >$ and $|\bar{\theta} >$

$$< \theta|\theta >= 1, \qquad < \bar{\theta}|\bar{\theta} >= 1, \tag{B.5}$$

$$< \theta|\bar{\theta} >= \exp\left(\sum_{r=1}^{3} \theta_r \bar{\theta}_r\right), \qquad < \bar{\theta}|\theta >= \exp\left(\sum_{r=1}^{3} \bar{\theta}_r \theta_r\right). \tag{B.6}$$

An additional ingredient in deriving fermionic path integrals is the Grassmann integration, defined by

$$\int \theta \, d\theta = \int \bar{\theta} \, d\bar{\theta} = i. \tag{B.7}$$

Using this definition the following *completeness relation* can be defined

$$1 = \int |\theta >< \theta| \, d^3\theta = \int d^3\bar{\theta} \, |\bar{\theta} >< \bar{\theta}|, \tag{B.8}$$

with the integration measures $d^3\theta = d\theta_3 d\theta_2 d\theta_1$ and $d^3\bar{\theta} = d\bar{\theta}_1 d\bar{\theta}_2 d\bar{\theta}_3$ and the differentials are also Grassmann valued anticommuting quantities. All these relations are proven in [65]. In the same work, the following expression for the trace over the fermionic sector is derived

$$\mathrm{Tr}\,(U) = \int d^3\theta < -\theta|U|\theta > . \tag{B.9}$$

To derive this relation one has to realize that a complete basis of the Hilbert space is already given by the 2^3 states of the form

$$a_{j_1}^\dagger a_{j_2}^\dagger \ldots a_{j_N}^\dagger |0 >, \quad \text{with}\, 1 \leq j_1 < j_2 < \cdots < j_N \leq 3 \tag{B.10}$$

and by use of the *completeness relations*.

The whole formalism of coherent states is based on the fact that one starts with an even number of Γ matrices. In our context we have a Dirac algebra of six elements but a derivation using just four elements could be carried out analogously. A special role plays the operator Γ_7. It can be expressed as the product of the six Γ matrices and this will lead to the relation

$$\Gamma_7 = \prod_{r=1}^{3}(1 - 2a_r^+ a_r^-) =: \prod_{r=1}^{3}(1 - 2\mathcal{F}_r) = (-1)^{\sum_{r=1}^{3}\mathcal{F}_r} =: (-1)^{\mathcal{F}}, \tag{B.11}$$

where we defined the *fermion number counter* $\mathcal{F}_r = a_r^+ a_r^-$ and $\mathcal{F} = \sum_{r=1}^{3}\mathcal{F}_r$ (the unfortunate name *fermion number counter* is a little misleading since $\mathcal{F}_r$ counts spin

and helicity of the single loop fermion rather than fermions). The explicit expression of the *fermion number counter* can now be used to derive the relation

$$< -\theta|\Gamma_7 =< -\theta|(-1)^{\mathcal{F}} = - < \theta|, \tag{B.12}$$

such that a Γ_7 in the trace will change the antiperiodic boundary conditions in (B.9) into periodic boundary conditions. We will make use of this relation in the derivation of the imaginary part of the effective action.

B.2 The Worldline Path Integral

This knowledge can now be used to deduce a path integral expression for the trace in Eq. (3.31). Starting point is

$$\mathrm{Tr}\exp(-T\tilde{\Sigma}^2) \;=\; \mathrm{Tr}\int d^4x\, d^3\theta \; < x, -\theta|\exp(-T\tilde{\Sigma}^2)|x, \theta > . \tag{B.13}$$

The next step is to insert an infinite number of complete sets of states into this trace to turn it into a path integral. The elementary relation that is needed to handle now the Γ matrices in Σ is

$$< \theta^i|\Gamma_A\Gamma_B|\theta^j >= -\int d^3\bar{\theta}^l < \theta^i|\bar{\theta}^l >< \bar{\theta}^l|\theta^j > \; 2\,\psi_A^{i,l}\psi_B^{j,l}, \quad A \neq B \tag{B.14}$$

with

$$\psi_r^{a,b} = \frac{1}{\sqrt{2}}(\theta_r^a + \bar{\theta}_r^b), \quad \psi_{r+3}^{a,b} = \frac{i}{\sqrt{2}}(\theta_r^a - \bar{\theta}_r^b). \tag{B.15}$$

Thus in the path integral, the Γ matrices will be replaced by the field $\psi(\tau)$ where τ denotes the Schwinger time $\tau \in [0, T]$. Using the 'canonical conjugate' relation Eq. (B.6), we see that the factor $< \theta^i|\bar{\theta}^k >< \bar{\theta}^l|\theta^j >$ will lead to the well known Legendre transformation term $\exp\int_0^T d\tau \sum_r (\frac{1}{2}\dot{\theta}_r\bar{\theta}_r - \frac{1}{2}\theta_r\dot{\bar{\theta}}_r)$ of the fermionic degrees of freedom and we obtain

$$\begin{aligned}\mathrm{Tr}\exp(-T\tilde{\Sigma}^2) \;=\; & \mathrm{Tr}\int \mathcal{D}p \int \mathcal{D}x \int_{AP} \mathcal{D}\theta\mathcal{D}\bar{\theta}\,\mathcal{P}\, \exp\left\{\int_0^T d\tau \right. \\ & \left. \left[i\dot{x}\cdot p + \sum_r \left(\frac{1}{2}\dot{\theta}_r\bar{\theta}_r - \frac{1}{2}\theta_r\dot{\bar{\theta}}_r\right) - \Sigma^2(\Gamma_A\Gamma_B \to \psi_A\psi_B)\right]\right\},\end{aligned} \tag{B.16}$$

where $\mathcal{P}$ denotes path ordering and AP reminds of the anti-periodic boundary conditions of the fermionic fields. The remaining trace is over the internal degrees of freedom, namely the $U(n) \times U(n)$ gauge structure. Using again the relations (B.15) one can transform the path integral representation into an integral over the six Grassmann fields ψ_A

$$W^+ \;=\; \frac{1}{8}\,\mathrm{Tr}\int_0^\infty \frac{dT}{T}\mathcal{N}\int \mathcal{D}x \int_{AP} \mathcal{D}\psi\,\mathcal{P}\; e^{-\int_0^T d\tau\mathcal{L}(\tau)} \tag{B.17}$$

where $\mathcal{L}$ denotes the Lagrange density given by

$$\begin{aligned}\mathcal{L}(\tau) &= \frac{\dot{x}^2}{4} + \frac{1}{2}\psi_\mu\dot{\psi}_\mu + \frac{1}{2}\psi_5\dot{\psi}_5 - i\dot{x}_\mu\mathcal{A}_\mu + \mathcal{H}^2 - \frac{1}{2}\mathcal{K}_{\mu\nu}\mathcal{K}_{\mu\nu} + 2i\psi_\mu\psi_5(\mathcal{D}_\mu\mathcal{H} + i\dot{x}_\nu\mathcal{K}_{\mu\nu}) \\ &\quad + i\psi_\mu\psi_\nu(\mathcal{F}_{\mu\nu} + \{\mathcal{H}, \mathcal{K}_{\mu\nu}\}) - 2\psi_\mu\psi_\nu\psi_\rho(\psi_5\mathcal{D}_\mu\mathcal{K}_{\nu\rho} + \frac{1}{2}\psi_\sigma\mathcal{K}_{\mu\nu}\mathcal{K}_{\rho\sigma}). \qquad \text{(B.18)}\end{aligned}$$

The quadratic integration over the momentum p has been carried out using

$$i\dot{x}\cdot p - (p-\mathcal{A})^2 = -(p - \mathcal{A} - \frac{i\dot{x}}{2})^2 - \frac{\dot{x}^2}{4} + i\dot{x}\cdot\mathcal{A} \qquad \text{(B.19)}$$

and the normalization

$$\mathcal{N} = \int \mathcal{D}p\; e^{-\int_0^T d\tau p^2(\tau)} \qquad \text{(B.20)}$$

satisfying

$$\mathcal{N}\int \mathcal{D}x\; e^{-\int_0^T d\tau \frac{\dot{x}^2}{4}} = (4\pi T)^{-D/2}\int d^D x_0. \qquad \text{(B.21)}$$

The proof that the non-trivial shift (B.19) does not change the path integral can be found in [60].

Appendix C

Integrals in the Worldline Formalism

The following integrals are used in the calculation. The subscript indicates the number of τ-integrations involved.

C.1 Integrals in Two Dimensions

In two dimensions the following two integrals are relevant

$$\begin{aligned} \mathcal{I}_1^a(x,y) &= \int_0^\infty \frac{dT}{T} \int_0^T d\tau\, e^{-xT-(y-x)\tau} \\ &= \frac{\log(x/y)}{(x-y)} \end{aligned} \tag{C.1}$$

$$\begin{aligned} \mathcal{I}_1^b(x,y) &= \int_0^\infty \frac{dT}{T} \int_0^T d\tau\, \dot{g}_b(T,\tau)\, e^{-xT-(y-x)\tau} \\ &= \frac{(x+y)\log(x/y) - 2(x-y)}{(x-y)^2} \end{aligned} \tag{C.2}$$

C.2 Integrals in Four Dimensions

The subsequent integrals are needed in four dimensions

$$\begin{aligned} \mathcal{I}_2^a(x,y,z) &= \int_0^\infty \frac{dT}{T^2} \int_0^T d\tau_2 \int_0^{\tau_2} d\tau_1\; e^{-xT-(y-x)\tau_2-(z-y)\tau_1} \\ &= \frac{x(z-y)\log x}{(x-y)(y-z)(z-x)} \quad + \quad 2 \text{ permutations} \end{aligned} \tag{C.3}$$

$$\begin{aligned}
\mathcal{I}_2^b(x,y,z) &= \int_0^\infty \frac{dT}{T^2}\int_0^T d\tau_2 \int_0^{\tau_2} d\tau_1\, \dot g_b(T,\tau_1)\, e^{-xT-(y-x)\tau_2-(z-y)\tau_1} \\
&= \frac{zx(y-z)^2\log x - zy(z-x)^2\log y - z(x-y)(xy-z^2)\log z}{(x-y)(y-z)^2(z-x)^2} \\
&+ \frac{z}{(y-z)(z-x)} \qquad\qquad (C.4)
\end{aligned}$$

$$\begin{aligned}
\mathcal{I}_2^c(x,y,z) &= \int_0^\infty \frac{dT}{T^2}\int_0^T d\tau_2 \int_0^{\tau_2} d\tau_1\, \dot g_b(T,\tau_2)\, e^{-xT-(y-x)\tau_2-(z-y)\tau_1} \\
&= -\mathcal{I}_2^b(y,z,x) \qquad\qquad (C.5)
\end{aligned}$$

$$\begin{aligned}
\mathcal{I}_3^a(x,y,z,w) &= \int_0^\infty \frac{dT}{T^2}\int_0^T d\tau_3 \int_0^{\tau_3} d\tau_2 \int_0^{\tau_2} d\tau_1\; e^{-xT-(y-x)\tau_3-(z-y)\tau_2-(w-z)\tau_1} \\
&= \frac{x\log x}{(x-w)(x-y)(x-z)} \quad + \quad 3 \text{ permutations} \qquad\qquad (C.6)
\end{aligned}$$

$$\begin{aligned}
\mathcal{I}_3^b(x,y,z,w) &= \int_0^\infty \frac{dT}{T^2}\int_0^T d\tau_3 \int_0^{\tau_3} d\tau_2 \int_0^{\tau_2} d\tau_1\, \dot g_b(T,\tau_1) \\
&\qquad \times \quad e^{-xT-(y-x)\tau_3-(z-y)\tau_2-(w-z)\tau_1} \\
&= \frac{xw\log x}{(x-w)^2(x-y)(x-z)} + \frac{wy\log y}{(y-w)^2(y-x)(y-z)} \\
&+ \frac{wz\log z}{(z-x)(z-y)(z-w)^2} + \frac{w^3(x+y+z) - 2w^4 - xyzw\log w}{(w-x)^2(w-y)^2(w-z)^2} \\
&+ \frac{w}{(w-x)(w-y)(w-z)} \qquad\qquad (C.7)
\end{aligned}$$

$$\begin{aligned}
\mathcal{I}_3^c(x,y,z,w) &= \int_0^\infty \frac{dT}{T^2}\int_0^T d\tau_3 \int_0^{\tau_3} d\tau_2 \int_0^{\tau_2} d\tau_1\, \dot g_b(T,\tau_2) \\
&\qquad \times \quad e^{-xT-(y-x)\tau_3-(z-y)\tau_2-(w-z)\tau_1} \\
&= \frac{x(x^2-wz)\log x}{(x-w)^2(x-y)(x-z)^2} + \frac{y(y^2-wz)\log y}{(y-w)^2(y-x)(y-z)^2} \\
&+ \frac{z(xy-z^2)\log z}{(z-x)^2(z-y)^2(z-w)} + \frac{w(xy-w^2)\log w}{(w-x)^2(w-y)^2(w-z)} \\
&+ \frac{xy-wz}{(w-x)(w-y)(z-x)(z-y)} \qquad\qquad (C.8)
\end{aligned}$$

$$\begin{aligned}
\mathcal{I}_3^d(x,y,z,w) &= \int_0^\infty \frac{dT}{T^2}\int_0^T d\tau_3 \int_0^{\tau_3} d\tau_2 \int_0^{\tau_2} d\tau_1\, \dot g_b(T,\tau_3) \\
&\qquad \times \quad e^{-xT-(y-x)\tau_3-(z-y)\tau_2-(w-z)\tau_1} \\
&= \mathcal{I}_3^b(w,x,y,z) \qquad\qquad (C.9)
\end{aligned}$$

Appendix D

The Chargino-Higgsino mass matrix

The chargino-higgsino mass matrix is given by

$$m = \begin{pmatrix} M_2 & gH_2^* \\ gH_1^* & \mu_c \end{pmatrix} \tag{D.1}$$

The mass matrix m is diagonalized by the biunitary transformation

$$m_d = UmV^\dagger\,, \tag{D.2}$$

with

$$\begin{aligned} U &= \left(\frac{2}{\bar{\Lambda}(\bar{\Lambda}+\bar{\Delta})}\right)^{\frac{1}{2}} \begin{pmatrix} \frac{1}{2}(\Lambda+\Delta) & a \\ -a^* & \frac{1}{2}(\Lambda+\Delta) \end{pmatrix}, & \text{(D.3)} \\ a &= g(M_2H_1 + \mu_c^* H_2^*)\,, \qquad \Delta = |M_2|^2 - |\mu_c|^2 - (u_1^2 - u_2^2)\,, & \text{(D.4)} \\ \Lambda &= (\Delta^2 + 4|a|^2)^{\frac{1}{2}} \end{aligned}$$

and

$$\begin{aligned} V &= \left(\frac{2}{\bar{\Lambda}(\bar{\Lambda}+\bar{\Delta})}\right)^{\frac{1}{2}} \begin{pmatrix} \frac{1}{2}(\bar{\Lambda}+\bar{\Delta}) & \bar{a} \\ -\bar{a}^* & \frac{1}{2}(\bar{\Lambda}+\bar{\Delta}) \end{pmatrix}, & \text{(D.5)} \\ \bar{a} &= g(M_2^* H_2^* + \mu_c H_1)\,, \qquad \bar{\Delta} = |M_2|^2 - |\mu_c|^2 + (u_1^2 - u_2^2)\,, & \text{(D.6)} \\ \bar{\Lambda} &= (\bar{\Delta}^2 + 4|\bar{a}|^2)^{\frac{1}{2}} = \Lambda, \end{aligned}$$

where we defined $u_{1,2} = |gH_{1,2}|$. Note that $\bar{a}$ and $\bar{\Delta}$ can be obtained from a and Δ by the replacements, $M_2 \leftrightarrow M_2^*$, $\mu \leftrightarrow \mu^*$ and $H_1 \leftrightarrow H_2^\dagger$, such that $\bar{\Lambda} = \Lambda$, as indicated in (D.5). The mass eigenvalues-squared are given by

$$m_{d_{1/2}}^2 = \frac{1}{2}\big(|M_2|^2 + |\mu_c|^2 + (u_1^2 + u_2^2)\big) \pm \frac{\Lambda}{2} \tag{D.7}$$

and can be calculated quite simply by noting that

$$Umm^\dagger U^\dagger = m_d^2 = Vm^\dagger m V^\dagger. \tag{D.8}$$

Bibliography

[1] P. A. M. Dirac, "The quantum theory of electron," *Proc. Roy. Soc. Lond.* **A117** (1928) 610–624.

[2] A. G. Cohen, A. De Rujula, and S. L. Glashow, "A matter-antimatter universe?," *Astrophys. J.* **495** (1998) 539–549, `astro-ph/9707087`.

[3] K. A. Olive, G. Steigman, and T. P. Walker, "Primordial nucleosynthesis: Theory and observations," *Phys. Rept.* **333** (2000) 389–407, `astro-ph/9905320`.

[4] **WMAP** Collaboration, D. N. Spergel *et al.*, "First year Wilkinson microwave anisotropy probe (WMAP) observations: Determination of cosmological parameters," *Astrophys. J. Suppl.* **148** (2003) 175, `astro-ph/0302209`.

[5] A. D. Sakharov, "Violation of CP invariance, C asymmetry, and baryon asymmetry of the universe," *Pisma Zh. Eksp. Teor. Fiz.* **5** (1967) 32–35.

[6] V. A. Kuzmin, V. A. Rubakov, and M. E. Shaposhnikov, "On the anomalous electroweak baryon number nonconservation in the early universe," *Phys. Lett.* **B155** (1985) 36.

[7] M. E. Shaposhnikov, "Baryon asymmetry of the universe in standard electroweak theory," *Nucl. Phys.* **B287** (1987) 757–775.

[8] M. E. Shaposhnikov, "Possible appearance of the baryon asymmetry of the universe in an electroweak theory," *JETP Lett.* **44** (1986) 465–468.

[9] G. 't Hooft, "Symmetry breaking through Bell-Jackiw anomalies," *Phys. Rev. Lett.* **37** (1976) 8–11.

[10] B. Bergerhoff and C. Wetterich, "The strongly interacting electroweak phase transition," *Nucl. Phys.* **B440** (1995) 171–188, `hep-ph/9409295`.

[11] K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, "A non-perturbative analysis of the finite T phase transition in SU(2) x U(1) electroweak theory," *Nucl. Phys.* **B493** (1997) 413–438, `hep-lat/9612006`.

[12] G. R. Farrar and M. E. Shaposhnikov, "Baryon asymmetry of the universe in the standard electroweak theory," *Phys. Rev.* **D50** (1994) 774, hep-ph/9305275.

[13] C. Jarlskog, "A basis independent formulation of the connection between quark mass matrices, CP violation and experiment," *Z. Phys.* **C29** (1985) 491–497.

[14] C. Jarlskog and R. Stora, "Unitarity polygons and CP violation areas and phases in the standard electroweak model," *Phys. Lett.* **B208** (1988) 268.

[15] A. G. Cohen, D. B. Kaplan, and A. E. Nelson, "Progress in electroweak baryogenesis," *Ann. Rev. Nucl. Part. Sci.* **43** (1993) 27–70, hep-ph/9302210.

[16] A. G. Cohen, D. B. Kaplan, and A. E. Nelson, "Diffusion enhances spontaneous electroweak baryogenesis," *Phys. Lett.* **B336** (1994) 41–47, hep-ph/9406345.

[17] K. Kainulainen, T. Prokopec, M. G. Schmidt, and S. Weinstock, "First principle derivation of semiclassical force for electroweak baryogenesis," *JHEP* **06** (2001) 031, hep-ph/0105295.

[18] K. Kainulainen, T. Prokopec, M. G. Schmidt, and S. Weinstock, "Semiclassical force for electroweak baryogenesis: Three- dimensional derivation," *Phys. Rev.* **D66** (2002) 043502, hep-ph/0202177.

[19] T. Prokopec, M. G. Schmidt, and S. Weinstock, "Transport equations for chiral fermions to order h-bar and electroweak baryogenesis," *Ann. Phys.* **314** (2004) 208–265, hep-ph/0312110.

[20] T. Prokopec, M. G. Schmidt, and S. Weinstock, "Transport equations for chiral fermions to order h-bar and electroweak baryogenesis. II," *Ann. Phys.* **314** (2004) 267–320, hep-ph/0406140.

[21] **Particle Data Group** Collaboration, K. Hagiwara *et al.*, "Review of particle physics," *Phys. Rev.* **D66** (2002) 010001.

[22] M. Kobayashi and T. Maskawa, "CP violation in the renormalizable theory of weak interaction," *Prog. Theor. Phys.* **49** (1973) 652–657.

[23] G. R. Farrar and M. E. Shaposhnikov, "Baryon asymmetry of the universe in the minimal standard model," *Phys. Rev. Lett.* **70** (1993) 2833–2836, hep-ph/9305274.

[24] G. R. Farrar and M. E. Shaposhnikov, "Note added to 'Baryon asymmetry of the universe in the standard model'," hep-ph/9406387.

[25] J. F. Donoghue, E. Golowich, and B. R. Holstein, "Dynamics of the standard model," *Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol.* **2** (1992) 1–540.

[26] T. Konstandin, T. Prokopec, and M. G. Schmidt, "Axial currents from CKM matrix CP violation and electroweak baryogenesis," *Nucl. Phys.* **B679** (2004) 246–260, hep-ph/0309291.

[27] C. Quimbay and S. Vargas-Castrillon, "Fermionic dispersion relations in the standard model at finite temperature," *Nucl. Phys.* **B451** (1995) 265–304, hep-ph/9504410.

[28] J. M. Cline and P.-A. Lemieux, "Electroweak phase transition in two Higgs doublet models," *Phys. Rev.* **D55** (1997) 3873–3881, hep-ph/9609240.

[29] J. F. Gunion, H. E. Haber, G. Kane, and S. Dawson, "The Higgs hunter's guide," *Frontiers in Physics Lecture Note Series* **No. 80** (2000).

[30] L. D. McLerran, M. E. Shaposhnikov, N. Turok, and M. B. Voloshin, "Why the baryon asymmetry of the universe is approximately 10**-10," *Phys. Lett.* **B256** (1991) 451–456.

[31] N. Turok and J. Zadrozny, "Electroweak baryogenesis in the two doublet model," *Nucl. Phys.* **B358** (1991) 471–493.

[32] N. Turok and J. Zadrozny, "Phase transitions in the two doublet model," *Nucl. Phys.* **B369** (1992) 729–742.

[33] M. Dine, P. Huet, J. Singleton, Robert, and L. Susskind, "Creating the baryon asymmetry at the electroweak phase transition," *Phys. Lett.* **B257** (1991) 351–356.

[34] M. Dine, P. Huet, and J. Singleton, Robert, "Baryogenesis at the electroweak scale," *Nucl. Phys.* **B375** (1992) 625–648.

[35] M. Joyce, T. Prokopec, and N. Turok, "Nonlocal electroweak baryogenesis. Part 1: Thin wall regime," *Phys. Rev.* **D53** (1996) 2930–2957, hep-ph/9410281.

[36] W. Bernreuther, "CP violation and baryogenesis," *Lect. Notes Phys.* **591** (2002) 237–293, hep-ph/0205279.

[37] J. M. Cline, K. Kainulainen, and A. P. Vischer, "Dynamics of two Higgs doublet CP violation and baryogenesis at the electroweak phase transition," *Phys. Rev.* **D54** (1996) 2451–2472, hep-ph/9506284.

[38] D. Atwood, L. Reina, and A. Soni, "Phenomenology of two Higgs doublet models with flavor changing neutral currents," *Phys. Rev.* **D55** (1997) 3156–3176, hep-ph/9609279.

[39] A. Diaz Rodolfo, R. Martinez, and J. A. Rodriguez, "Bounds for lepton flavor violation using g-2 muon factor in the two Higgs doublet model type III," hep-ph/0010339.

[40] D. Land and E. D. Carlson, "Two stage phase transition in two Higgs models," *Phys. Lett.* **B292** (1992) 107–112, hep-ph/9208227.

[41] A. Hammerschmitt, J. Kripfganz, and M. G. Schmidt, "Baryon asymmetry from a two stage electroweak phase transition?," *Z. Phys.* **C64** (1994) 105–110, hep-ph/9404272.

[42] D. Bodeker, L. Fromme, S. J. Huber, and M. Seniuch, "The baryon asymmetry in the standard model with a low cut- off," *JHEP* **02** (2005) 026, hep-ph/0412366.

[43] C. Grojean, G. Servant, and J. D. Wells, "First-order electroweak phase transition in the standard model with a low cutoff," *Phys. Rev.* **D71** (2005) 036001, hep-ph/0407019.

[44] M. E. Shaposhnikov, "Structure of the high temperature gauge ground state and electroweak production of the baryon asymmetry," *Nucl. Phys.* **B299** (1988) 797.

[45] A. Tranberg and J. Smit, "Baryon asymmetry from electroweak tachyonic preheating," *JHEP* **11** (2003) 016, hep-ph/0310342.

[46] L. M. Krauss and M. Trodden, "Baryogenesis below the electroweak scale," *Phys. Rev. Lett.* **83** (1999) 1502–1505, hep-ph/9902420.

[47] J. Garcia-Bellido, D. Y. Grigoriev, A. Kusenko, and M. E. Shaposhnikov, "Non-equilibrium electroweak baryogenesis from preheating after inflation," *Phys. Rev.* **D60** (1999) 123504, hep-ph/9902449.

[48] J. Garcia-Bellido, M. Garcia-Perez, and A. Gonzalez-Arroyo, "Chern-Simons production during preheating in hybrid inflation models," *Phys. Rev.* **D69** (2004) 023504, hep-ph/0304285.

[49] E. J. Copeland, D. Lyth, A. Rajantie, and M. Trodden, "Hybrid inflation and baryogenesis at the TeV scale," *Phys. Rev.* **D64** (2001) 043506, hep-ph/0103231.

[50] L. L. Salcedo, "Derivative expansion for the effective action of chiral gauge fermions: The normal parity component," *Eur. Phys. J.* **C20** (2001) 147–159, hep-th/0012166.

[51] L. L. Salcedo, "Derivative expansion for the effective action of chiral gauge fermions: The abnormal parity component," *Eur. Phys. J.* **C20** (2001) 161–184, hep-th/0012174.

[52] J. Smit, "Effective CP violation in the standard model," *JHEP* **09** (2004) 067, hep-ph/0407161.

[53] Z. Bern and D. A. Kosower, "Efficient calculation of one loop QCD amplitudes," *Phys. Rev. Lett.* **66** (1991) 1669–1672.

[54] Z. Bern and D. C. Dunbar, "A mapping between Feynman and string motivated one loop rules in gauge theories," *Nucl. Phys.* **B379** (1992) 562–601.

[55] M. J. Strassler, "Field theory without Feynman diagrams: One loop effective actions," *Nucl. Phys.* **B385** (1992) 145–184, hep-ph/9205205.

[56] M. J. Strassler, "Field theory without Feynman diagrams: A demonstration using actions induced by heavy particles,". SLAC-PUB-5978.

[57] M. G. Schmidt and C. Schubert, "On the calculation of effective actions by string methods," *Phys. Lett.* **B318** (1993) 438–446, hep-th/9309055.

[58] D. Fliegner, M. G. Schmidt, and C. Schubert, "The higher derivative expansion of the effective action by the string inspired method. Part 1," *Z. Phys.* **C64** (1994) 111–116, hep-ph/9401221.

[59] D. Fliegner, P. Haberl, M. G. Schmidt, and C. Schubert, "Inverse mass expansions from worldline path integrals: Higher order coefficients and ordering problems," hep-th/9505077.

[60] E. D'Hoker and D. G. Gagne, "Worldline path integrals for fermions with scalar, pseudoscalar and vector couplings," *Nucl. Phys.* **B467** (1996) 272–296, hep-th/9508131.

[61] E. D'Hoker and D. G. Gagne, "Worldline path integrals for fermions with general couplings," *Nucl. Phys.* **B467** (1996) 297–312, hep-th/9512080.

[62] M. Mondragon, L. Nellen, M. G. Schmidt, and C. Schubert, "Axial couplings on the world-line," *Phys. Lett.* **B366** (1996) 212–219, hep-th/9510036.

[63] K. Fujikawa, "Path integral measure for gauge invariant fermion theories," *Phys. Rev. Lett.* **42** (1979) 1195.

[64] N. G. Pletnev and A. T. Banin, "Covariant technique of derivative expansion of one-loop effective action," *Phys. Rev.* **D60** (1999) 105017, hep-th/9811031.

[65] Y. Ohnuki and T. Kashiwa, "Coherent states of fermi operators and the path integral," *Prog. Theor. Phys.* **60** (1978) 548.

[66] M. A. Shifman, "Wilson loop in vacuum fields," *Nucl. Phys.* **B173** (1980) 13.

[67] H. Leutwyler, "Chiral fermion determinants and their anomalies," *Phys. Lett.* **B152** (1985) 78–82.

[68] J. M. Moreno, M. Quiros, and M. Seco, "Bubbles in the supersymmetric standard model," *Nucl. Phys.* **B526** (1998) 489–500, hep-ph/9801272.

[69] J. M. Cline, M. Joyce, and K. Kainulainen, "Supersymmetric electroweak baryogenesis. (Erratum)," hep-ph/0110031.

[70] K. Kajantie, M. Laine, K. Rummukainen, and M. E. Shaposhnikov, "Is there a hot electroweak phase transition at m(H) $>$ approx. m(W)?," *Phys. Rev. Lett.* **77** (1996) 2887–2890, hep-ph/9605288.

[71] D. Bodeker, P. John, M. Laine, and M. G. Schmidt, "The 2-loop MSSM finite temperature effective potential with stop condensation," *Nucl. Phys.* **B497** (1997) 387–414, hep-ph/9612364.

[72] M. Laine and K. Rummukainen, "The MSSM electroweak phase transition on the lattice," *Nucl. Phys.* **B535** (1998) 423–457, hep-lat/9804019.

[73] S. J. Huber, P. John, M. Laine, and M. G. Schmidt, "CP violating bubble wall profiles," *Phys. Lett.* **B475** (2000) 104–110, hep-ph/9912278.

[74] K. Kainulainen, T. Prokopec, M. G. Schmidt, and S. Weinstock, "Some aspects of collisional sources for electroweak baryogenesis," hep-ph/0201245.

[75] J. M. Cline, M. Joyce, and K. Kainulainen, "Supersymmetric electroweak baryogenesis in the WKB approximation," *Phys. Lett.* **B417** (1998) 79–86, hep-ph/9708393.

[76] J. M. Cline, M. Joyce, and K. Kainulainen, "Supersymmetric electroweak baryogenesis," *JHEP* **07** (2000) 018, hep-ph/0006119.

[77] M. Joyce, T. Prokopec, and N. Turok, "Efficient electroweak baryogenesis from lepton transport," *Phys. Lett.* **B338** (1994) 269–275, hep-ph/9401352.

[78] M. Joyce, T. Prokopec, and N. Turok, "Electroweak baryogenesis from a classical force," *Phys. Rev. Lett.* **75** (1995) 1695–1698, hep-ph/9408339.

[79] M. Joyce, T. Prokopec, and N. Turok, "Nonlocal electroweak baryogenesis. Part 2: The classical regime," *Phys. Rev.* **D53** (1996) 2958–2980, hep-ph/9410282.

[80] M. Carena, J. M. Moreno, M. Quiros, M. Seco, and C. E. M. Wagner, "Supersymmetric CP-violating currents and electroweak baryogenesis," *Nucl. Phys.* **B599** (2001) 158–184, hep-ph/0011055.

[81] M. Carena, M. Quiros, M. Seco, and C. E. M. Wagner, "Improved results in supersymmetric electroweak baryogenesis," *Nucl. Phys.* **B650** (2003) 24–42, hep-ph/0208043.

[82] A. Riotto, "Towards a nonequilibrium quantum field theory approach to electroweak baryogenesis," *Phys. Rev.* **D53** (1996) 5834–5841, hep-ph/9510271.

[83] P. Y. Huet, K. Kajantie, R. G. Leigh, B.-H. Liu, and L. D. McLerran, "Hydrodynamic stability analysis of burning bubbles in electroweak theory and in QCD," *Phys. Rev.* **D48** (1993) 2477–2492, hep-ph/9212224.

[84] T. Konstandin, T. Prokopec, and M. G. Schmidt, "Kinetic description of fermion flavor mixing and CP- violating sources for baryogenesis," hep-ph/0410135.

[85] T. Konstandin, T. Prokopec, M. G. Schmidt, and M. Seco, "MSSM electroweak baryogenesis and flavour mixing transport equations,". in preparation.

[86] P. Huet and A. E. Nelson, "CP violation and electroweak baryogenesis in extensions of the standard model," *Phys. Lett.* **B355** (1995) 229–235, hep-ph/9504427.

[87] P. Huet and A. E. Nelson, "Electroweak baryogenesis in supersymmetric models," *Phys. Rev.* **D53** (1996) 4578–4597, hep-ph/9506477.

[88] D. Bodeker, G. D. Moore, and K. Rummukainen, "Hard thermal loops and the sphaleron rate on the lattice," *Nucl. Phys. Proc. Suppl.* **83** (2000) 583–585, hep-lat/9909054.

[89] C. Lee, V. Cirigliano, and M. J. Ramsey-Musolf, "Resonant relaxation in electroweak baryogenesis," hep-ph/0412354.

[90] D. Chang, W.-F. Chang, and W.-Y. Keung, "New constraint from electric dipole moments on chargino baryogenesis in MSSM," *Phys. Rev.* **D66** (2002) 116008, hep-ph/0205084.

[91] C. Balazs, M. Carena, A. Menon, D. E. Morrissey, and C. E. M. Wagner, "The supersymmetric origin of matter," *Phys. Rev.* **D71** (2005) 075002, hep-ph/0412264.

[92] B. C. Regan, E. D. Commins, C. J. Schmidt, and D. DeMille, "New limit on the electron electric dipole moment," *Phys. Rev. Lett.* **88** (2002) 071805.

[93] A. Pilaftsis, "Higgs-mediated electric dipole moments in the MSSM: An application to baryogenesis and Higgs searches," *Nucl. Phys.* **B644** (2002) 263–289, hep-ph/0207277.

[94] M. Carena, M. Quiros, and C. E. M. Wagner, "Electroweak baryogenesis and Higgs and stop searches at LEP and the Tevatron," *Nucl. Phys.* **B524** (1998) 3–22, hep-ph/9710401.

[95] A. Menon, D. E. Morrissey, and C. E. M. Wagner, "Electroweak baryogenesis and dark matter in the nMSSM," *Phys. Rev.* **D70** (2004) 035005, hep-ph/0404184.

[96] J. Kang, P. Langacker, T.-j. Li, and T. Liu, "Electroweak baryogenesis in a supersymmetric U(1)' model," *Phys. Rev. Lett.* **94** (2005) 061801, hep-ph/0402086.

[97] T. Han, P. Langacker, and B. McElrath, "The Higgs sector in a U(1)' extension of the MSSM," *Phys. Rev.* **D70** (2004) 115006, hep-ph/0405244.

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