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Fluxes, Hierarchies, and Metastable Vacua in Supersymmetric Field Theories

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Flüsse, Hierarchien und metastabile Vakua in supersymmetrischen Feldtheorien — Zusammenfassung: Diese Arbeit behandelt Themen sowohl im Bereich der effektiven Niederenergietheorien aus Typ IIB-Superstring-Flusskompaktifizierungen als auch der vierdimensionalen, global supersymmetrischen Eichtheorien. Wir diskutieren Flusskompaktifizierungen mit sogenannten "warped throat"-Regionen, die zu großen Skalenhierarchien in der vierdimensionalen effektiven Feldtheorie führen, und stellen den Zusammenhang zwischen einem speziellen solchen "warped throat" und einem fünfdimensionalen Randall-Sundrum-Modell vor. Wir zeigen, wie sich gewisse stringtheoretische Eigenschaften der Kompaktifizierung ins fünfdimensionale Bild übersetzen, etwa die Stabilisierung von Moduli durch Flüsse oder die Existenz eines unstabilisierten Kähler-Modulus. Wir erläutern die KKLT-Konstruktion für metastabile de Sitter-Vakua sowie einige mögliche Modifikationen durch spontane Supersymmetriebrechung mit F-Termen. In KKLT-artigen Modellen mit dem supersymmetriebrechenden Sektor innerhalb eines "warped throat" untersuchen wir die Vermittlung der Supersymmetriebrechung an den sichtbaren Sektor. Wir erklären den Mechanismus der kombinierten Vermittlung durch Moduli und Weyl-Anomalie und zeigen, dass Beiträge von derselben Größenordnung durch höherdimensionale Operatoren entstehen können. Wir behandeln schließlich das ISS-Modell der metastabilen dynamischen Supersymmetriebrechung in vier Dimensionen und präsentieren eine renormierbare Erweiterung, die eine große Skalenhierarchie in natürlicher Weise erzeugt. Wir zeigen auch, wie das ISS-Modell aus einem Typ IIB-Superstringmodell gewonnen werden kann.

Fluxes, Hierarchies, and Metastable Vacua in Supersymmetric Field Theories — Abstract: This thesis concerns topics both in low-energy effective field theories from type IIB superstring flux compactifications and in four-dimensional, rigidly supersymmetric gauge theories. We introduce flux compactifications with so-called "warped throat" regions, which lead to large hierarchies of scales in the effective four-dimensional theory. The correspondence between a particular such throat and a five-dimensional Randall-Sundrum-like model is established. We shown how certain string-theoretic features of the compactification, such as moduli stabilization by fluxes or the presence of an unstabilized Kähler modulus, are incorporated in the five-dimensional picture. The KKLT construction for metastable de Sitter vacua is reviewed, as well as some possible modifications involving spontaneous F-term supersymmetry breaking. For KKLT-like models with their hidden sector localized inside a throat, the mediation of supersymmetry breaking to the visible sector is investigated. We review the mechanism of mixed modulus-anomaly mediation, and show that there can be additional equally important gravity-mediated contributions. We finally turn to the ISS model of metastable dynamical supersymmetry breaking in four dimensions, and present a renormalizable extension which generates a large hierarchy naturally. We also recapitulate how the ISS model may be obtained from a type IIB superstring model.

Contents

Chapter 1

Motivation and overview

Quantum field theory is a universal framework in which the fundamental constituents of Nature and their interactions can be described. It comprises beautiful concepts and elegant mechanisms; and there is a particular quantum field theoretic model, the Standard Model of Particle Physics, whose predictions agree with almost all observational data to greatest accuracy.

But despite its successes, the Standard Model still suffers from serious shortcomings. On the observational side, for instance, it fails to incorporate dark matter, neutrino masses (at least in its minimal, renormalizable version), and suitable mechanisms for cosmic inflation or baryogenesis. On the theoretical side, it is plagued by the abundance of parameters, and by the need to fine-tune some of them to unnaturally small values in order to fit the data.

In the light of these problems, the view has emerged that the Standard Model is an effective theory, approximately valid at relatively low energies but to be superseded by a currently unknown, more complete theory above a certain energy scale. Whatever comes beyond the Standard Model might well be an effective intermediate-scale description itself, in the form of a different weakly coupled quantum field theory. However, in order to describe processes at energies near the Planck scale, the problem of quantum gravity must be addressed. For this purpose not only the Standard Model, but in fact the entire concept of perturbative QFT is inadequate.

The Standard Model does not incorporate gravity. Quantum gravity effects are negligibly weak in particle physics processes at energies at which the Standard Model has been probed, and indeed most likely at all energies that will ever be within the reach of particle physics experiments. Yet they do become important for processes at energy scales comparable to the Planck scale. Any theory that aims to be a complete description of Nature, valid up to arbitrarily high energies, must of course include gravity; but gravity as a perturbatively non-renormalizable theory cannot be treated in the conventional framework of perturbative QFT. It is a notoriously difficult problem to construct a realistic quantum theory of gravity, and currently none of the candidate theories is universally accepted.

The probably most promising candidate for such a theory of quantum gravity is superstring theory. One of the main advantages of superstring theory is that a single superstring model can in principle comprise both gravity and the Standard Model fields, thereby providing a truly unified description of all interactions (although no fully realistic model has been constructed as of now). Many classes of superstring models also naturally contain other features that have been proposed independently, in a purely field-theoretical context, in order to cure the above-mentioned shortcomings of the Standard Model. For example, concepts such as supersymmetry, extra spacetime dimensions, and grand-unified gauge groups are commonly encountered.

This thesis is concerned with the low-energy effective field theories that arise from certain superstring models, as well as with certain QFT models that are at least motivated by, and to a certain extent even derivable from, superstring theory. More precisely, we will discuss recent results in warped type IIB superstring flux compactifications[1] and in four-dimensional supersymmetric gauge theory[2].

One of the most appealing insights gained from string theory is that there is a deep relation between these two: The dynamics of certain type IIB compactifications can be equivalently described by four-dimensional supersymmetric gauge theories [6–8]. This "AdS/CFT correspondence" has been rigidly established only in special limiting cases, but is widely expected to hold in far more general circumstances. While the AdS/CFT duality is not the main topic of this work, it has been crucial to derive many of the results we will be making use of. It is also often illuminating to re-examine results found on one side of the duality from the other perspective, as we will see.

A central issue in all the models we will encounter is the appearance of large hierarchies of scales. This fits well with one of the key motivations for extending the Standard Model, namely the electroweak hierarchy problem. The problem is very simple to state: Assume that the Standard Model indeed arises as the low-energy effective theory of a fundamental theory which takes effect around the Planck scale, such as superstring theory. The most natural value for the only dimensionful parameter of the Standard Model, which is the Higgs mass, would then be the of the order of the Planck mass. However, its Standard Model value is about 16 orders of magnitude lower than this naive estimate. Numerically even more severe is another problem of the same kind: If we combine the Standard Model with classical gravity as a low-energy description of our universe, there is a second dimensionful parameter, the cosmological constant. The analogous naive estimate then

[1] Recent reviews of this topic include [1, 2].
[2] For reviews see e.g. [3–5].

turns out to be wrong by as much as 120 orders of magnitude. Whatever the fundamental theory is, it should ideally be able to provide an explanation for the smallness of these parameters; or, being less ambitious, it should least allow for tuning them to small values.

Even this latter requirement turns out to be quite nontrivial in superstring theory. Superstring theory has no continuous parameters to tune at all, and therefore obtaining a hierarchy of many orders of magnitude from a string model seems hardly possible at first sight. But a closer look reveals that the situation is actually very different. It is well-known that superstrings can be consistently quantized in ten spacetime dimensions only. To obtain a four-dimensional low-energy effective field theory, six dimensions should be compactified. The properties of the compactification geometry, along with possible nonvanishing vacuum expectation values for the fields and non-perturbative objects placed in the classical superstring background, are characterized by a set of discrete numbers. These eventually determine the four-dimensional phenomenology. Recent estimates (see, for instance, [9–11]) have shown that the number of such discrete parameters and their range of values are so vast that the resulting low-energy parameters can probably be tuned to arbitrary precision, for all practical purposes.

In fact, in type IIB superstring models the appearance of large hierarchies is even very common and natural.[3] A common feature of many type IIB compactifications are "warped throat" regions, which are regions that are strongly warped along a particular direction in the internal manifold. This means that there is a scale factor or "warp factor" multiplying the four-dimensional non-compact metric, with the value of the warp factor strongly depending on the position in the six-dimensional compact space. Different sectors of the low-energy effective field theory in four dimensions will have their dimensionful parameters exponentially redshifted, according to where the objects they arise from are localized in the internal space. This naturally provides a large hierarchy of scales.

In type IIB superstring theory, warped solutions can be realized as follows [12]. The low-energy limit of type IIB superstring theory is type IIB supergravity. Superstring backgrounds are usually constructed as classical supergravity solutions, possibly containing additional nonperturbative localized objects. In type IIB supergravity there are a number of differential form gauge fields, whose field strengths may be non-vanishing in such a background. Their vacuum expectation values or "fluxes" can be regarded as originating from string theory, since string theory includes non-perturbative localized objects, such as D-branes, which couple to and source the supergravity gauge fields. Including such objects in the compactification back-

[3]It has been conjectured that all superstring theories should be related to each other by string dualities. It is therefore not surprising that large hierarchies can be generated in the other string theories too (e.g. from gaugino condensates in the heterotic theory).

ground thus leads to solutions with flux. On the other hand, from the point of view of pure supergravity, one can just as well construct solutions to the field equations including fluxes without relying on any source objects. In that picture, what prevents the fluxes from decaying are topological obstructions. This is analogous to the Dirac monopole solution in electrodynamics, whose field strength can either be regarded as being sourced by a solitonic object, or as a topologically stable feature of the solution if the monopole is replaced by a puncture in spacetime. In the more general case of string-theoretic fluxes, if the submanifold of spacetime that is threaded by the flux represents a topologically nontrivial cycle, then the flux solution is topologically stable. Adding flux to some given spacetime background will deform the geometry due to backreaction, which will in general result in a warped spacetime.

Purely field-theoretic models with warped extra dimensions have been investigated in much detail without reference to any specific string models. Many essential features are, in fact, already captured by the simplest possible setup, the Randall-Sundrum-I model [13]. This model comprises a single extra dimension compactified on an interval (more precisely an $S^1/\mathbb{Z}_2$ orbifold) such that the metric is that of five-dimensional Anti-de Sitter space AdS_5. The interval boundaries are four-dimensional hypersurfaces of spacetime on which additional fields can be localized. There exists by now a plethora of models, ranging from simple toy models to sophisticated and potentially realistic extensions of the Standard Model, which are based on this compactification geometry.

A large part of this thesis is devoted to the low-energy effective field theories that are obtained from warped throats in type IIB superstring theory. The connection with the purely field-theoretic approach of Randall-Sundrum and others is an especially interesting aspect (see e.g. [14–16]): Is it possible to embed five-dimensional field-theoretic warped models in type IIB compactifications? Or, asking from the string-theoretic perspective, are there full string constructions that in the low-energy limit reduce to effective five-dimensional models, such that the appearance of a large hierarchy can already be described on the five-dimensional level?

We will in fact find that this intermediate step in dimensional reduction, compactifying from ten to five dimensions before ultimately arriving at four, is sometimes not only possible but even extremely useful. This is because the five-dimensional model may already capture effects that arise from spatial separation of different field-theoretic sectors in the warped internal space. On the other hand, a five-dimensional model tends to be technically much easier to deal with than a full ten-dimensional compactification.

A possible application of warped throat superstring backgrounds is analogous to the idea of the original Randall-Sundrum proposal: If the standard model fields, or even only the Higgs field, were localized in a region of sizeable redshift, then the Higgs mass would be naturally small and the electroweak hierarchy would emerge quite naturally. A rather different application concerns the tuning of the four-dimensional cosmological constant, within a setting which has become known as the KKLT construction [17].

In this scenario, the goal is to construct a background with all moduli fixed and a realistic value of the cosmological constant. Moduli are massless scalars which typically abound in the effective four-dimensional field theory of any superstring compactification. Their expectation values dictate the low-energy coupling parameters. In a realistic model, they should acquire a mass and effectively be frozen as far as the low-energy dynamics is concerned. In the KKLT construction, this is achieved by a combination of fluxes and non-perturbative effects. The resulting four-dimensional vacuum is Anti-de Sitter and supersymmetric. It has been argued that the cosmological constant can be tuned to a very small (negative) value, given the large choice of discrete parameters as explained above. Such tuning is required to retain computational control. However, in a fully realistic model, supersymmetry should be broken, and the cosmological constant should be close to zero but positive. Adding certain non-perturbative objects can achieve both of these goals, but their contribution to the vacuum energy density is generically string-scale, such that it seems impossible to cancel the hierarchically small negative cosmological constant and end up with a Minkowski vacuum (or, even more realistically, a de Sitter vacuum with a tiny cosmological constant). If, however, these objects are localized in a warped throat, their contribution to the four-dimensional vacuum energy density will be redshifted, and a realistic vacuum can be constructed. A vacuum obained in this manner will be only metastable but can be parametrically long-lived, with a lifetime far exceeding the age of the universe.

Imagining a particle physics model based on the KKLT construction, with the supersymmetry breaking sector inside a warped throat, it is now interesting to enquire how supersymmetry breaking is communicated to the Standard Model fields [18–21]. The latter would in this scenario be localized outside of the throat. We will in particular be interested in mediation effects from the warped throat background, since it turns out that the dominant mechanism of supersymmetry breaking mediation generally depends on the properties of the underlying compactification manifold, contrary to what one might expect in the first place.

Warped extra dimensions are not the only way to naturally obtain a large hierarchy of scales. Another class of models in particle physics, fundamentally or effectively four-dimensional, relies on dimensional transmutation. These models contain asymptotically free gauge groups. An asymptotically free gauge theory with moderately small gauge coupling at some fundamental energy scale will become strongly coupled at an exponentially smaller scale and may undergo a phase transition. This phenomenon is well-known from QCD, which becomes nonperturbative at a scale $\Lambda_{\mathrm{QCD}} \approx 200$ MeV, leading to chiral symmetry breaking and confinement.

In the light of the AdS/CFT correspondence mentioned earlier, it has become clear that these two mechanisms of generating large hierarchies can in fact be dual to each other. A weak form of the Maldacena conjecture [6], which underlies the AdS/CFT correspondence, is that there is an exact match between the generating functionals of two very different-looking theories: Classical type IIB supergravity compactified on $\mathrm{AdS}_5 \times S^5$ on one side, and strongly coupled 4d $\mathcal{N} = 4$ superconformal Yang-Mills theory with gauge group $\mathrm{SU}(N)$ at large N on the other. There is ample evidence that this duality continues to hold if "classical type IIB supergravity" is replaced by "quantum type IIB superstring theory" on the AdS side, and the requirements of large N and strong coupling are dropped on the CFT side. A particular radial direction in AdS space is distinguished in the string constructions by which the correspondence is usually motivated. A change along this direction corresponds to a change of the renormalization scale of the CFT (which, of course, does not affect either side of the duality by homogeneity of AdS and by conformality of the gauge theory). But it is widely believed that, if the compactification background is not exactly $\mathrm{AdS}_5 \times X_5$ but a deformation thereof (with X_5 some compact internal manifold, not necessarily S^5), there should exist a dual gauge theory in four dimensions which is only approximately conformal. If, in particular, the almost-AdS space terminates at some value of the radial direction, the dual gauge theory is expected to confine. This view is supported by some explicit examples for which both the supergravity solution and its gauge theory dual are known, notably the Klebanov-Strassler solution [22] whose supergravity formulation we will review in detail.

It has even been suggested that all Randall-Sundrum-I based models of particle physics, with the electroweak hierarchy obtained from warping, could be dual to strongly coupled technicolor-like models with the electroweak hierarchy generated by dimensional transmutation. However, it seems fair to say that such statements are quantitatively poorly founded, a situation which is unlikely to improve because strong coupling usually renders any putative dual gauge theory uncalculable.

Supersymmetric gauge theories in four dimensions which generate large hierarchies are not only interesting because of their potential relation to warped type IIB compactifications. In fact, they can be very useful as part of purely field-theoretical extensions of the standard model: Low-energy supersymmetry which is broken at a naturally small scale provides a solution to the electroweak hierarchy problem without any reference to string theory. Supersymmetric gauge theories may provide just this small scale, if they undergo non-perturbative supersymmetry breaking at strong coupling [23].

The last part of this thesis is devoted to the study of a model of this kind, the so-called ISS model of metastable dynamical supersymmetry breaking [24], and an extension of it by which the hierarchy of scales in the ISS model becomes fully natural [25]. As it happens, the ISS model and related models are again elegantly realized as four-dimensional effective field theories obtained from D-brane models in type IIB superstring theory [26–37], and as such can be useful ingredients in type IIB model-building when large hierarchies are essential.

Let us now give an overview of the present work. Parts of this thesis are based on research papers by the present author, namely [16] (with Arthur Hebecker and Enrico Trincherini), [21] (with Arthur Hebecker and Michele Trapletti), and [25].

In Chapter 2 we review some general aspects of type IIB flux compactifications which are important for our purposes. We will then recapitulate the construction and geometric properties of several examples of warped throat backgrounds, and the application of flux compactifications to moduli stabilization [12]. We will also revisit the KKLT construction [17], showing how four-dimensional metastable de Sitter vacua can be obtained. This chapter is mainly intended as a review to make the present work more self-contained.

In Chapter 3, following our paper [16], we will study the analogy between ten-dimensional and five-dimensional warped models in detail, focussing on a particular warped throat background, the Klebanov-Strassler throat [22]. We will explain how certain characteristic features of the underlying string construction may be understood in terms of a five-dimensional model. These include in particular the stabilization of the complex structure moduli and the dynamics of a universal light Kähler modulus.

In Chapter 4 we will review possible mechanisms of F-term supersymmetry breaking which, when incorporated in flux-stabilized type IIB backgrounds, lead to four-dimensional de Sitter backgrounds, thus generalizing the KKLT construction (see e.g. [21, 38–44]). We will explain why it is natural in this context to localize the supersymmetry breaking sector of the theory in a warped throat. Subsequently we will investigate supersymmetry breaking mediation within the effective four-dimensional field theory of warped throat compactifications, a subject pioneered in [18]. Many of the results of this chapter have been published in our paper [21].

Chapter 5 concerns a model with a large hierarchy of scales which can be understood outside of the context of type IIB superstring theory — in fact, it does not rely on string theory at all, although it can be constructed in D-brane model building. In that chapter, we will show how large hierarchies can be generated naturally within supersymmetric gauge theory, using as a simple renormalizable example an extension of the ISS model [24] which was introduced by the present author in [25]. This model has interesting applications both in string model building and in possible modifications of the Standard Model to include rigid supersymmetry, to name but a few. To make contact with type IIB superstring theory, we finally review a way to obtain a version of the ISS model from a D-brane construction in type IIB theory, following [26, 27].

We conclude in Chapter 6 with a summary of our results.

This thesis also includes four appendices: Appendix A contains remarks on notation and conventions, Appendix B gives more details on the geometry of the conifold which we will make extensive use of throughout the main text, and Appendix C recapitulates some properties and some useful notation related to five-dimensional Anti-de Sitter space. In Appendix D, some important results concerning the phases of $\mathcal{N} = 1$ supersymmetric QCD are summarized.

Chapter 2

Type IIB flux compactifications

In this chapter we discuss some aspects of warped flux compactifications of the type IIB superstring which will be relevant for the later discussion. Most of the material presented has been well-known for several years. It will be reviewed here to establish our notation and terminology, and to remind the reader of some important facts and concepts. For more exhaustive reviews see e.g. [1, 2].

There are several motivations to include fluxes in a compactification background (apart from the fact that there is no reason not to include them when writing down a generic model). Fluxes can serve to stabilize the complex structure moduli and the dilaton (see e.g. [12,45–48]), which would otherwise appear as massless scalar fields in the effective 4d field theory. They can also generate large hierarchies of scales [12], which may be used to eventually solve the electroweak hierarchy problem (as e.g. in the string models of [49]), or to fine-tune the 4d cosmological constant to a small positive value [17]. In this chapter, we will briefly review how all of this can be achieved.

2.1 General properties

In this section we closely follow [12]. The low-energy limit of type IIB superstring theory is type IIB supergravity, whose action is

$$\begin{aligned} S =& \frac{M_{10}^8}{2}\int d^{10}x\,\sqrt{-g}\left(\mathcal{R} - \frac{|\partial\tau|^2}{2(\operatorname{Im}\tau)^2} - \frac{|G_3|^2}{12\operatorname{Im}\tau} - \frac{\tilde{F}_5^2}{4\cdot 5!}\right) \\ &+ \frac{M_{10}^8}{8i}\int \frac{C_4\wedge G_3\wedge \overline{G}_3}{\operatorname{Im}\tau} + \text{fermion terms.} \end{aligned} \tag{2.1}$$

Here M_{10} is the 10d reduced Planck mass, and g is the 10d Einstein frame metric with Ricci scalar $\mathcal{R}$. τ is the axio-dilaton, formed from the Ramond-Ramond (RR) axion C_0 and the dilaton ϕ as $\tau = C_0 + ie^{-\phi}$. C_4 is the RR 4-form potential, whose field strength we will denote by F_5. The fields G_3 and $\tilde{F}_5$ are further constructed from the RR and Neveu-Schwarz (NS)

2-form potentials C_2 and B_2 and their respective field strengths F_3 and H_3 as follows:

$$\begin{aligned} G_3 &= F_3 - \tau H_3, \\ \tilde{F}_5 &= F_5 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3. \end{aligned} \tag{2.2}$$

$\tilde{F}_5$ is required to be self-dual, a condition which cannot be incorporated in the action and so must be imposed on the equations of motion.

It is possible to include additional localized sources of flux and energy density, such as D-branes or orientifold planes, in the background. Then the action (2.1) will be supplemented by a piece S_{loc} from these sources, containing the appropriate tensions and couplings to the p-form fields.

For a general compactification background which preserves 4d Poincaré invariance, the metric can be parametrized as

$$ds^2 = e^{2A(y)} \eta_{\mu\nu}\, dx^\mu\, dx^\nu + e^{-2A(y)} \tilde{g}_{mn}\, dy^m\, dy^n. \tag{2.3}$$

Here $\eta_{\mu\nu}$ is the 4d Minkowski metric, and the y^m are coordinates on a compact 6d internal space $\mathcal{M}$. The function $A(y)$ is called the warp factor. 4d Poincaré invariance also places some constraints on the background values of the other fields: The axio-dilaton can only depend on the internal coordinates, $\tau = \tau(y)$. G_3 can only have legs in the compact directions, and the self-dual $\tilde{F}_5$ must take the form

$$\tilde{F}_5 = (1 + *)(d\alpha(y) \wedge dx^0 \wedge dx^2 \wedge dx^2 \wedge dx^3) \tag{2.4}$$

for scalar function $\alpha(y)$ of the internal coordinates.

In a pure supergravity compactification, taking the trace over the Einstein equations then gives that all the fluxes must vanish and the warp factor must be constant [50, 51]. However, for backgrounds that contain localized sources with negative energy density such as orientifold planes or antibranes, it is possible to have both fluxes and non-trivial warping [12]. In the following we will assume that there are indeed such ojects, chosen such that the stringy consistency conditions are satisfied (e.g. tadpoles are cancelled), but we will not investigate their effects in detail. Note however that antibranes break supersymmetry completely, and that orientifold planes break the 4d $\mathcal{N} = 2$ SUSY, which is preserved by a pure Calabi-Yau compactification, to $\mathcal{N} = 1$. Throughout this work we will therefore tacitly assume that we are working with a Calabi-Yau orientifold (or a more general F-theory background) which is $\mathcal{N} = 1$ supersymmetric in 4d language. In the presence of additional antibranes, or for particular flux choices, SUSY may even be completely broken.

Poincaré invariant backgrounds may also contain other localized objects such as D3- and D7-branes which fill the noncompact dimensions, or Euclidean D3-brane instantons wrapping 4-cycles in the internal manifold. If all the localized objects in the background satisfy a certain BPS-like condition on their tensions [12] (which is actually the case for all classes of localized objects that we have mentioned and will be considering), then G_3 is imaginary self-dual with respect to the 6d internal metric,

$$*_6 G_3 = iG_3, \tag{2.5}$$

and the $\tilde{F}_5$ flux is related to the warp factor as $\alpha(y) = e^{4A(y)}$. Furthermore, to preserve 4d $\mathcal{N} = 1$ supersymmetry, G_3 must be a $(2,1)$-form on the internal manifold [52, 53].

In the 4d effective theory, G_3 flux on a Calabi-Yau compactification manifold $\mathcal{M}$ gives rise to a superpotential of Gukov-Vafa-Witten type [47],

$$W = \int_{\mathcal{M}} G_3 \wedge \Omega, \tag{2.6}$$

where Ω is the holomorphic $(3,0)$-form on $\mathcal{M}$. Since Ω depends on the complex structure moduli z^α, and G_3 depends on the dilaton, these fields may be stabilized by the fluxes. Note that there is no Kähler moduli dependence in (2.6), and therefore, to stabilize also the Kähler moduli, other mechanisms are needed.

Note that, starting from a Calabi-Yau (orientifold) as the internal manifold and then placing fluxes on its cycles, the resulting internal geometry will be affected by the flux backreaction and the resulting space will be merely conformally Calabi-Yau (as is evident from the ansatz (2.3)). However, one still retains much computational control, as opposed to e.g. the type IIA case where fluxes backreact such that the internal space, in general, ends up being not even a Kähler manifold any more.

2.2 The $\mathbf{AdS}_5 \times S^5$ throat

In type IIB string compactifications to four dimensions, a 'warped throat' refers to a region of the internal space where the warp factor is varying strongly along a particular direction. The simplest example is the geometry near a stack of D3-branes. Placing N coincident D3-branes in 10d flat spacetime will deform the metric to give

$$ds^2 = h(r)^{-1/2} \eta_{\mu\nu}\, dx^\mu\, dx^\nu + h(r)^{1/2}(dr^2 + r^2\, ds^2_{S^5}) \tag{2.7}$$

where

$$h(r) = 1 + \frac{R^4}{r^4}, \qquad R^4 = 4\pi \mathrm{g_s} N \alpha'^2 \frac{\pi^3}{\mathrm{Vol}\,(S^5)} = 4\pi \mathrm{g_s} N \alpha'^2. \tag{2.8}$$

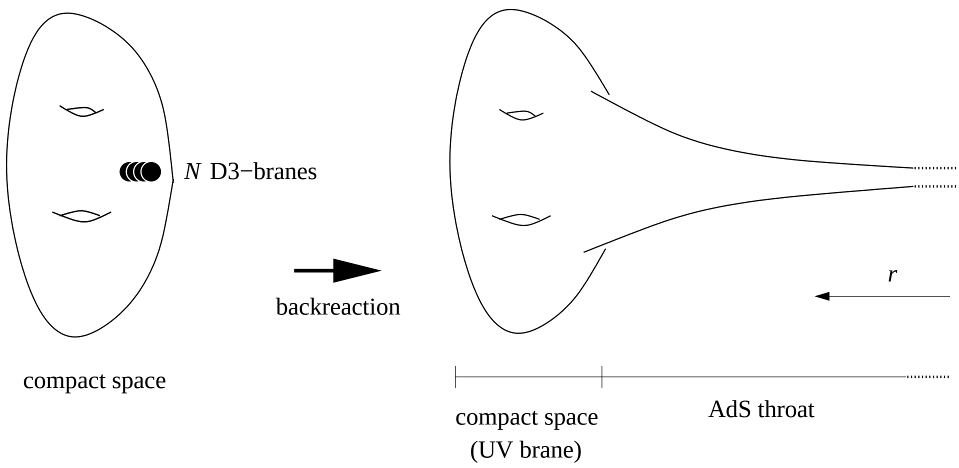

Figure 2.1: The $\mathrm{AdS}_5 \times S^5$ throat

This space is asymptotically flat as $r \to \infty$, since then $h(r) \to 1$. For small r, the second term in $h(r)$ dominates, and the metric becomes that of $\mathrm{AdS}_5 \times S^5$,

$$ds^2 = \frac{r^2}{R^2}\eta_{\mu\nu}\, dx^\mu\, dx^\nu + \frac{R^2}{r^2}dr^2 + R^2\, ds^2_{S^5}, \tag{2.9}$$

with the branes sourcing N units of $\tilde{F}_5$ flux through the internal S^5. The AdS/CFT correspondence states that, for large N, classical supergravity on this background is dual to strongly coupled 4d $\mathcal{N} = 4$ $SU(N)$ super-Yang-Mills theory [6–8]. The conformality of the 4d theory is reflected by translational invariance along the r direction of the 5d AdS space.

It has been pointed out [14] that this setup provides a stringy realization of the Randall-Sundrum-II model [54]: Placing N D3-branes at a generic point on a compact Calabi-Yau 3-fold will deform the geometry in its vicinity to an AdS_5 throat (at least if the internal manifold is stabilized at large volume, such that the curvature is small and the space is initially approximately flat in a suitable neighbourhood). The throat is terminated at large r by the remainder of the compact manifold, but extends all the way to infinity as $r \to 0$, as sketched in Figure 2.1. In terms of the AdS/CFT correspondence, the radial coordinate r of AdS_5 corresponds to a renormalization scale in 4d. The dual 4d field theory is exactly conformal in the infrared, for small r, but coupled to gravity in the UV where the throat ends in the compact manifold.

Disregarding the internal S^5, we can describe type IIB supergravity on this background as a 5d field theory on AdS_5 half-space, with the compact space serving as the Randall-Sundrum “UV brane”. There could be additional light fields localized on the compact space, coming from additional D-branes wrapping its cycles; in fact one could imagine an entire “Standard model" visible sector localized on the UV brane.

2.3 The Klebanov-Strassler throat

The $\mathrm{AdS}_5 \times S^5$ construction can be generalized as follows. Consider a IIB compactification on a Calabi-Yau 3-fold $\mathcal{M}$ which has a conical singularity, i.e. which near some point is given by a real cone $\mathcal{C}_{X_5}$ over a compact Einstein space X_5 (in the above example X^5 was S^5 and the 'cone' had deficit angle zero, so was just 6d flat space). Placing N D3-branes at the singular point will result in a similar deformation of the metric as above,

$$ds^2 = h(r)^{-1/2}\eta_{\mu\nu}\, dx^\mu\, dx^\nu + h(r)^{1/2}(dr^2 + r^2\, ds^2_{X_5}), \tag{2.10}$$

where

$$h(r) = 1 + \frac{R^4}{r^4}, \qquad R^4 = 4\pi \mathrm{g_s} N\alpha'^2 \frac{\pi^3}{\mathrm{Vol}\,(X_5)}. \tag{2.11}$$

Spacetime at small r becomes an $\mathrm{AdS}_5 \times X_5$ throat, whereas at large r it is given by $\mathbb{R}^{3,1} \times \mathcal{C}_{X_5}$ (eventually embedded in $\mathbb{R}^{3,1} \times \mathcal{M}$). In the throat, supergravity should be dual to some conformal field theory in 4d.

The warped throat we will mostly be concerned with is the Klebanov-Strassler (KS) solution or warped deformed conifold [22]. Its construction is motivated by considering the case of $X_5 = T^{1,1} = (SU(2) \times SU(2))\,/\,U(1)$. The cone $\mathcal{C}_{T^{1,1}}$ over $T^{1,1}$ is a non-compact singular Ricci-flat manifold which is called the conifold.[1] The metric near N D3-branes at a conifold singularity reads

$$ds^2 = h(r)^{-1/2}\eta_{\mu\nu}\, dx^\mu\, dx^\nu + h(r)^{1/2}(dr^2 + r^2\, ds^2_{T^{1,1}}), \tag{2.12}$$

with

$$h(r) = 1 + \frac{R^4}{r^4}, \qquad R^4 = \frac{27\pi}{4} \mathrm{g_s} N\alpha'^2, \tag{2.13}$$

and the solution has N units of $\tilde{F}_5$ flux through the internal $T^{1,1}$. It is sketched in Figure 2.2. The conformal field theory dual to this warped conifold background, or Klebanov-Witten solution, was found in [55].

$T^{1,1}$ is topologically $S^3 \times S^2$, and at the conifold singularity both the 3-cycle and the 2-cycle shrink to zero size. The singularity may be smoothed by either resolution or deformation, which leaves a finite-size S^2 or S^3 respectively.

We are interested in the deformed conifold, in which a 3-cycle at the tip is retained. The deformed conifold is a nonsingular, noncompact manifold

[1] We present some technical details concerning the conifold and its non-singular versions in Appendix B. In the main text we will merely state the relevant facts.

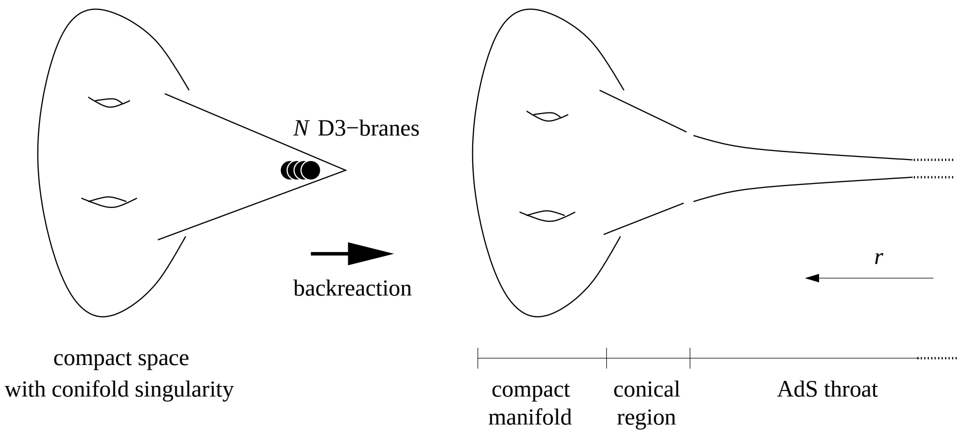

Figure 2.2: The warped conifold throat (note that the D3-branes are really placed at the singularity).

and also admits a Calabi-Yau metric. When placing M units of F_3 flux on the 3-cycle, the flux backreaction on the geometry gives the warped deformed conifold. It is possible to regard also this F_3 flux as sourced by D-branes, albeit in a more subtle way: If M D5-branes are wrapped around the collapsing 2-cycle of the singular conifold, they will be constrained to reside at the singularity and act effectively as "fractional" D3-branes. Their back-reaction on the geometry will deform the singularity and give rise to a warped metric, and they will source M units of F_3 flux threaded through the transversal S^3. This picture is especially useful to construct the AdS/CFT-dual gauge theory, which was achieved in [22, 56, 57].

For our purposes it is convenient to merely regard the warped deformed conifold with F_3 flux as a supergravity solution, dispensing momentarily with the D-brane picture. The singularity at the conifold tip is deformed, so that the throat now ends at finite r. The metric for the throat excluding the tip was found by Klebanov and Tseytlin (KT) [57]:

$$ds^2 = \tilde{h}(r)^{-1/2}\eta_{\mu\nu}\, dx^\mu\, dx^\nu + \tilde{h}(r)^{1/2}(dr^2 + r^2 ds^2_{T^{1,1}}), \tag{2.14}$$

where

$$\tilde{h}(r) = 1 + \frac{R^4_{\text{eff}}(r)}{r^4}, \quad R^4_{\text{eff}}(r) = \frac{27}{4}\pi \mathsf{g_s} N_{\text{eff}}(r)\alpha'^2, \quad N_{\text{eff}}(r) = \frac{3}{2\pi}\mathsf{g_s} M^2 \log\frac{r}{r_s}. \tag{2.15}$$

Here r_s is a parameter associated with the deformation size of the singularity.

The KT metric becomes singular for $r \to r_s$; in fact, it is no longer valid in the domain $r \lesssim r_s$, and the complete throat is perfectly smooth also at its tip. This can be inferred from studying its AdS/CFT dual [22]. However, the precise shape of the throat in the region near the tip, which we will call

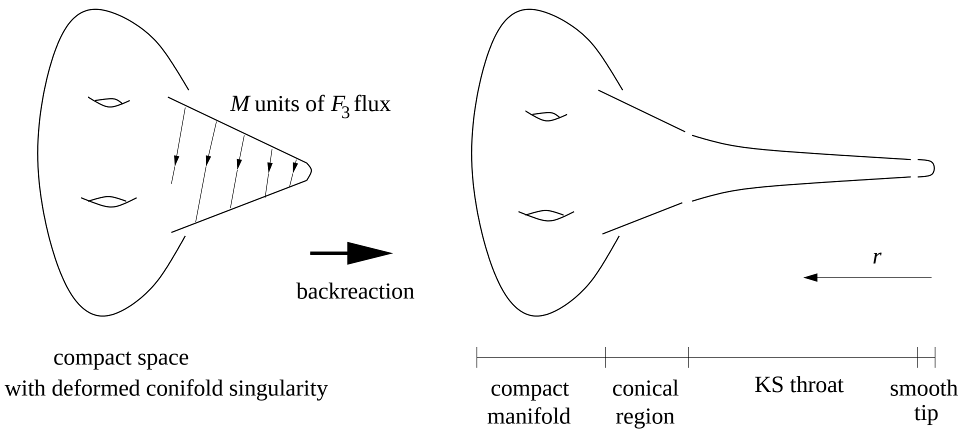

Figure 2.3: The Klebanov-Strassler throat or warped deformed conifold

the KS region, is not relevant for the following discussion. It will instead be sufficient to use the simpler KT metric.

As one goes along the throat, one finds $N_{\text{eff}}(r)$ units of $\tilde{F}_5$ flux through the internal $T^{1,1}$ at the radial coordinate r:

$$(4\pi^2\alpha')^2 N_{\text{eff}}(r) = \int_{T^{1,1}\,\text{at}\,r} \tilde{F}_5 = \left(\int_{S^3\,\text{at}\,r} F_3\right)\left(\int_{S^2\,\text{at}\,r} B_2\right). \tag{2.16}$$

The F_3 flux on the 3-cycle is quantized, but the B_2 potential integrated over the 2-cycle will vary continuously with r. The resulting space is only approximately $\text{AdS}_5 \times T^{1,1}$, since the radius of the internal space as well as the 'AdS curvature radius' depend weakly (logarithmically to be precise) on the radial coordinate.

With the general ansatz (2.14), this logarithmic dependence (2.15) of N_{eff} (and hence R_{eff}) on r can be derived as follows: For a finite segment of the throat, between r_1 and r_2 say, we have

$$\begin{aligned}(4\pi^2\alpha')^2 \left(N_{\text{eff}}(r_2) - N_{\text{eff}}(r_1)\right) &= \int_{T^{1,1}\,\text{at}\,r_2} \tilde{F}_5 - \int_{T^{1,1}\,\text{at}\,r_1} \tilde{F}_5 \\ &= \int_{T^{1,1}\times[r_1;r_2]} d\tilde{F}_5 = \int_{T^{1,1}\times[r_1;r_2]} H_3 \wedge F_3\end{aligned} \tag{2.17}$$

Since G_3 is imaginary self-dual, see (2.5), we have $H_3 = \text{g}_\text{s} *_6 F_3$. Here $*_6$ denotes the Hodge star with respect to the 6d metric

$$g_{6\,mn} dy^m dy^n = \tilde{h}(r)^{1/2}(dr^2 + r^2\, ds^2_{T^{1,1}}). \tag{2.18}$$

Note that $\tilde{h}(r)$ drops out of the expression for H_3. Since F_3 has no components in the r direction, we can thus write

$$H_3 \wedge F_3 = g_s \sqrt{g_6}\, F_{mnp} F^{mnp}\, d^6 y = \frac{g_s}{r} \sqrt{\bar{g}}\, F_{\bar{m}\bar{n}\bar{p}} F^{\bar{m}\bar{n}\bar{p}}\, dr\, d^5 \bar{y} \qquad (2.19)$$

where barred coordinates and indices refer to the $T^{1,1}$ metric

$$\bar{g}_{\bar{m}\bar{n}} d\bar{y}^{\bar{m}} d\bar{y}^{\bar{n}} = ds^2_{T^{1,1}}. \qquad (2.20)$$

By inserting into (2.17) and differentiating we obtain

$$\frac{dN_{\text{eff}}(r)}{dr} = \frac{g_s}{(4\pi^2\alpha')^2} \frac{1}{r} \int\limits_{T^{1,1}} d^5\bar{y}\, \sqrt{\bar{g}}\, F_{\bar{m}\bar{n}\bar{p}} F^{\bar{m}\bar{n}\bar{p}}. \qquad (2.21)$$

The quantization condition

$$\frac{1}{4\pi^2\alpha'} \int_{S^3} F_3 = M \qquad (2.22)$$

further implies the scaling $F_3 \sim M\alpha'$ for the non-vanishing components of F_3. We finally arrive at

$$N_{\text{eff}}(r) = a g_s M^2 \log(r/r_s) \qquad (2.23)$$

with an integration constant r_s and an $\mathcal{O}(1)$ numerical prefactor a. A detailed analysis, using the explicit conifold metric and flux forms, shows that in fact $a = 3/(2\pi)$.

The AdS/CFT dual of supergravity on the warped conifold background is a 4d $\mathcal{N} = 1$ superconformal gauge theory. The dual gauge theory of supergravity on the warped deformed conifold is no longer conformal, since the throat metric is not exactly AdS and in particular not homogenous with respect to the r direction (translations along which become scale transformations in the gauge theory). It is instead given by a cascading gauge theory [22, 58]: a gauge theory which repeatedly undergoes a series of Seiberg dualities [59] when changing the renormalization scale. Conformality is especially badly broken in the IR, where the throat terminates smoothly as described by the full KS solution, and where the dual field theory exhibits confinement and chiral symmetry breaking.

In the UV, the throat will end when $\tilde{h}(r)$ approaches unity, i.e. when $\alpha'^2 g_s N_{\text{eff}}(r) \simeq r^4$. Just from the knowledge of M and N_{eff} at a certain r, it is impossible to tell where the throat will end in the UV. From the dual 4d gauge theory perspective, this knowledge corresponds to information about higher-dimension operators, which is usually hard to access for the low-energy observer. In the conical region that follows at larger r, the integrated $\tilde{F}_5$ flux N_{eff} continues to grow with r as before, but the back-reaction is not strong enough to affect the geometry. At some still larger $r = R_c$, the approximate conifold geometry goes over smoothly to a compact Calabi-Yau orientifold geometry. The compactification radius can thus be approximately identified with R_c. Clearly, the total D3 charge of fluxes and localized sources in the bulk of the compact space has to compensate the $\tilde{F}_5$ flux present at the end of the conifold region at $r = R_c$.

The overall picture is sketched in Figure 2.3: The compact space has a conical region with non-vanishing $\tilde{F}_5$ flux. Going to smaller r, one reaches the throat region, where the back-reaction of the flux deforms the geometry significantly and which is finally smoothly terminated with a KS region.

It is possible to generalize this construction by allowing for both F_3 and F_5 flux, or equivalently, by considering the back-reaction of both fractional and integer D3-branes on the conifold background. If the throat contains, say, $N_{\rm D3}$ additional D3-branes, the $\tilde{F}_5$ flux changes from $N_{\rm eff}$ to $N_{\rm eff} + N_{\rm D3}$. In this case the throat ends in the infrared at some $r_{\rm IR} > r_s$ with a KS region containing the additional branes. In fact, they will generate a "throat within the throat", since of course the near-horizon geometry of $N_{\rm D3}$ explicit D3-branes will again be $\mathrm{AdS}_5 \times S^5$.

For the discussion of SUSY breaking mediation in the throat later on in Chapter 4, it is important to note that the KS solution has an SO(4) symmetry (see for instance [60]). The action of SO(4) is given in Appendix B.

Warped throats, and in particular the KS throat, have been argued to be a common phenomenon in the so-called type IIB "landscape" of possible compactification solutions [61–63]. In other words, if the compactification data such as the internal topology and flux quanta are randomly chosen, it appears that backgrounds containing warped throats are the generic outcome. This serves as another good reason to study warped throat solutions, besides the fact that, as discussed at length, they are useful ingredients in model-building.

2.4 Moduli stabilization in the Klebanov-Strassler throat

Following [12] we will now show how a complex structure modulus may be stabilized by fluxes and a large hierarchy of scales may be generated. We will consider the case of the Klebanov-Strassler background. Let us start with a Calabi-Yau orientifold whose complex structure at some point is nearly degenerate, in such a way that we are close to the conifold point in moduli space, so that locally the geometry is that of the deformed conifold. In the notation of Appendix B, the defining equation of the deformed conifold is

$$\sum_{i=1}^{4} w_i^2 = z. \tag{2.24}$$

z is the modulus which controls the size of the 3-cycle $\mathcal{A}$ at the tip of the throat, and thus eventually the throat length, or the hierarchy between the embedding manifold and the KS region. Taking z to be real and positive, $\mathcal{A}$ is given by the S^3 on which all w_i are real. In a compact space $\mathcal{A}$ has a dual cycle $\mathcal{B}$, here given e.g. by imaginary $w_{1,2,3}$ and real positive w_4 (this is a noncompact submanifold in the conifold case, but it will become part of a compact one once the conifold is embedded in a full compactification). The holomorphic 3-form is

$$\Omega = \frac{1}{2\pi^2}\frac{dw_2 \wedge dw_3 \wedge dw_4}{w_1}. \tag{2.25}$$

Placing M units of flux on $\mathcal{A}$ and K units of flux on $\mathcal{B}$, the superpotential (2.6) becomes

$$W = \int_{\mathcal{M}} G_3 \wedge \Omega = (2\pi)^2\alpha' \left(M \int_{\mathcal{B}} \Omega - K\tau \int_{\mathcal{A}} \Omega \right). \tag{2.26}$$

With the above parametrizations for $\mathcal{A}$ and $\mathcal{B}$, it is easily checked that

$$\int_{\mathcal{A}} \Omega = z, \tag{2.27}$$

and

$$\int_{\mathcal{B}} \Omega \equiv \mathcal{G}(z) = \frac{1}{2\pi i} z \log z + \text{holomorphic}, \tag{2.28}$$

where the holomorphic contributions are not calculable (because we have not specified the embedding geometry) but also not required for a leading-order analysis, as will become clear below. The expression for the $\mathcal{B}$ period integral also follows more generally from the Special Geometry of Calabi-Yau moduli spaces [64]. For the superpotential we obtain

$$W = (2\pi)^2\alpha' \left(M\mathcal{G}(z) - K\tau z \right). \tag{2.29}$$

Denote the Kähler potential for the moduli by $\mathcal{K}$. With the Kähler covariant derivative $D_z = \partial_z + (\partial_z \mathcal{K})$, the condition for a supersymmetric vacuum reads

$$0 = D_z W = (2\pi)^2\alpha' \left((M\partial_z \mathcal{G}(z) - K\tau + \partial_z\mathcal{K}(M\mathcal{G}(z) - K\tau z) \right). \tag{2.30}$$

The first two terms in this equation dominate for large $K/M\mathrm{g_s}$ and at small z. It is then solved approximately by

$$z \approx \exp(-2\pi K/M\mathrm{g_s}). \tag{2.31}$$

So z is indeed stabilized at an exponentially small value, for moderate values of the flux quanta K and M. To also stabilize the dilaton, additional fluxes on other cycles are needed, but this is straightforward to realize. In an analogous manner, the case of several complex structure moduli can be treated.

2.5 The KKLT construction

In [17] a scenario was proposed in which also the Kähler moduli of a IIB flux compactification are stabilized, using nonperturbative means. In the effective 4d theory, this yields an AdS supersymmetric vacuum. The negative cosmological constant is then cancelled by adding a stack of $\overline{\text{D3}}$-branes to the compactification background. The result is a metastable but parametrically long-lived SUSY-breaking vacuum in 4d, which could be either Minkowski or dS with a small positive cosmological constant. We will now review this model, the Kachru-Kallosh-Linde-Trivedi (KKLT) construction.

Let us start with a flux background as in the previous sections, with all complex structure moduli and the dilaton stabilized. Assuming that the compactification manifold has just one Kähler modulus T for simplicity (although this can be easily generalized). The low-energy effective 4d field theory is 4d supergravity, with all fields except T having acquired string-scale masses from fluxes and integrated out. They induce a constant superpotential W_0, which we take to be real without loss of generality. The modulus T has a no-scale Kähler potential:

$$\mathcal{K} = -3\log(T+\overline{T}). \tag{2.32}$$

Assume now that T controls the size of a 4-cycle in the compactification geometry. If this 4-cycle is wrapped by a stack of N_{D7} D7-branes (whose remaining 3 + 1 dimensions fill out the 4d non-compact spacetime), the effective theory on their world volume is known to be $SU(N_{\text{D7}})$ super-Yang-Mills theory, which at a scale Λ undergoes gaugino condensation. Λ is generally given in terms of the one-loop beta function coefficient b_0 as

$$\frac{\Lambda}{\mu} = e^{-\frac{2\pi}{b_0}\alpha(\mu)}, \tag{2.33}$$

where $\alpha(\mu) = g^2(\mu)/(4\pi)$, and $g(\mu)$ is the gauge coupling at scale μ. For the present case, $b_0 = 3N_{\text{D7}}$, and at the string scale the coupling is dictated by the brane dynamics. The gaugino condensate is then

$$\Lambda^3 = A\,e^{-\frac{2\pi}{N_{\text{D7}}}T}, \tag{2.34}$$

with some string-scale prefactor A. In the low-energy theory, there is thus a T-dependent piece Λ^3 in the superpotential, which serves to stabilize T: Define $a = \frac{2\pi}{N_{\rm D7}}$, then the superpotential reads

$$W = W_0 + A\, e^{-aT}. \tag{2.35}$$

With the Kähler potential (2.32), the condition for a supersymmetric vacuum $D_T W = 0$ gives

$$W_0 = -\left(\frac{a(T+\overline{T})}{3} + 1\right) A\, e^{-aT} \tag{2.36}$$

and accordingly for $\overline{T}$. It is expected that, given the large choice of topologies and fluxes available in the type-IIB landscape of vacua, W_0 can be tuned such that there is a solution at positive, moderately large T, which is required for the consistency of the ansatz. (T of order unity or smaller would correspond to string-scale compactification radii, where α'-corrections are uncontrolled and the supergravity approximation would break down.) Despite the fact that it is the sum of several string-scale contributions, W_0 is then negative and exponentially small.

The resulting 4d vacuum is an AdS minimum with unbroken supersymmetry. It is found from (2.36), with T minimizing the scalar potential

$$V_{\rm AdS} = \frac{a^2 A^2\, e^{-a(T+\overline{T})}}{3(T+\overline{T})} + \frac{2aA^2\, e^{-a(T+\overline{T})}}{(T+\overline{T})^2} + \frac{aAW_0\, e^{-a\overline{T}} + \text{h.c.}}{(T+\overline{T})^2}, \tag{2.37}$$

which results in a vacuum energy density

$$\langle V_{\rm AdS}\rangle = -3\frac{|W_0 + A\, e^{-aT}|^2}{(T+\overline{T})^3} = -\frac{a^2A^2\, e^{-a(T+\overline{T})}}{3(T+\overline{T})}. \tag{2.38}$$

In the final step of the KKLT construction, a stack of $\overline{\rm D3}$-branes is added, whose presence explicitly breaks supersymmetry, and whose positive energy density cancels the cosmological constant. The antibranes fill out the noncompact spacetime dimensions, whereas in the internal manifold they reside at a point. It turns out that, if the internal manifold contains a warped throat such as the KS solution, a stack of $\overline{\rm D3}$-branes at the tip of the throat represents also a metastable solution of the string theory [65]. The antibranes contribute a piece δV to the potential (2.37), which is given by [66]

$$\delta V = \frac{D}{(T+\overline{T})^2}. \tag{2.39}$$

Here D is a constant which depends on the number of antibranes and on the redshift factor at the tip of the throat. If the warping is sufficiently strong, it is possible to choose the number of antibranes such that their energy

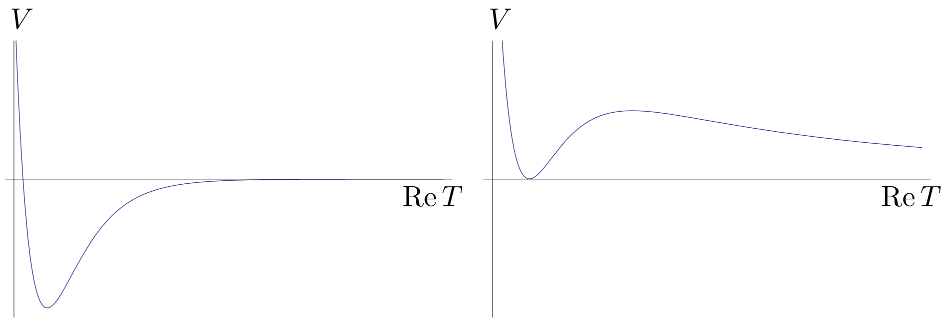

Figure 2.4: Sketch of the scalar potential with the AdS vacuum (left) and after the uplifting (right).

density cancels $\langle V_{\text{AdS}} \rangle$ in (2.38) with arbitrary precision. Without warping, it would have been impossible to achieve such a cancellation, because of the tinyness of $\langle V_{\text{AdS}} \rangle$ and the fact that a single brane would give a string-scale contribution to the potential.

The full scalar potential for T is now

$$V = V_{\text{AdS}} + \delta V. \tag{2.40}$$

We have sketched its shape in Figure 2.4, along with the pre-uplift potential (2.37). If D is fine-tuned as described above, the position of the uplifted minimum is nearly unchanged (hence a solution at moderately large volume will retain this property) , but it is no longer the global minimum. As is apparent from the potential, there is instead a runaway towards zero vacuum energy at $T \to \infty$, and the de Sitter minimum is metastable. Eventually the metastable state will decay by quantum tunneling into the true vacuum.

The lifetime of the metastable minimum may be estimated by computing the Coleman-de Luccia instanton action [67] for solutions interpolating between the false and the true vacuum. The decay width is then proportional to $e^{-S_{\text{bounce}}}$, where $S_{\text{bounce}} = S_{\text{inst}} - S_0$ is the difference between the instanton action and the action of the false vacuum solution. KKLT gave an estimate of the lifetime of their model based on the thin-wall approximation for the potential barrier, demonstrating that the false vacuum lifetime will exceed the lifetime of the universe by many orders of magnitude for a realistic choice of parameters.

We will revisit the KKLT construction and introduce alternative uplifting mechanisms in Chapter 4, when we will discuss how SUSY breaking might be communicated to a visible sector outside the throat.

Chapter 3

The Klebanov-Strassler throat as a Randall-Sundrum model

As discussed in Section 2.2, the infinite-length $\mathrm{AdS}_5 \times S^5$ throat produced by a stack of D3-branes, embedded in a full compactification, can be regarded as a realization of the Randall-Sundrum-II model in type IIB string theory. It is now an obvious question whether a compactification that contains a finite-length throat, such as the Klebanov-Strassler solution, in the same manner constitutes a stringy analogue of the Randall-Sundrum-I model [13]. In this chapter we will investigate in how far this is the case, recapitulating and slightly extending the analysis of [16]. We will identify the scales on which a warped throat compactification can be described by a 5d model. Furthermore, the 5d mechanism which is responsible for stabilizing the RS radius, corresponding to the length of the throat, will be discussed in detail. We will also explain how to incorporate the universal Kähler modulus, which is part of any string compactification of this type, in our 5d picture.

The picture of the KS throat as a RS-I-like model will be made use of in Chapter 4, where it will help us to find the correct ansatz for the 4d effective supergravity Lagrangian which is responsible for SUSY breaking mediation in the KKLT model. It has also recently been used in the literature [68] to demonstrate that in 5d models based on the KS throat, a successful thermal electroweak phase transition between the high-temperature (black hole) phase and the low-temperature (RS-I) phase can be achieved, as opposed to the case of a pure Randall-Sundrum background. Various other applications are conceivable: For instance, it would be interesting to find a string construction that comes as close as possible to the RS model with bulk-localized fermions [69–71] which has been proposed as a natural explanation of fermion mass hierarchies in the Standard Model. A common approach is to consider matter fields from D7-branes embedded in a warped throat (see e.g. [72, 73]). For the case of the KS throat, our considerations should be very useful to find the appropriate effective 5d description.

3.1 Preliminaries

In the warped conifold throat with geometry $\mathrm{AdS}_5 \times T^{1,1}$, the curvature radius R of AdS_5, which also measures the size of $T^{1,1}$, is constant along the radial direction. The geometry of the warped deformed conifold or KS throat is also approximately $\mathrm{AdS}_5 \times T^{1,1}$, but as we have shown, there is an effective curvature radius $R_{\mathrm{eff}}(r)$ which varies slowly with r.

In a neighbourhood of some radial position r, we can give an effective five-dimensional description on length scales $L \gg R_{\mathrm{eff}}(r)$ since at these scales excitations in the internal $T^{1,1}$ may be neglected. This implies that the curvature of our 5d background will always be relevant: There is no length scale at which flat 5d space would provide a good approximation. But as long as L is not too large, the *variation* of $R_{\mathrm{eff}}(r)$ will be insignificant on length scales L, so that the curvature is approximately constant and the 5d geometry is approximately AdS_5.

Just as in the $\mathrm{AdS}_5 \times S^5$ case, the compact manifold in which the throat is embedded serves as a RS ultraviolet brane.[1] For the effective 5d description to be valid, we also have to require that the size R_c of the compact space, the 'brane thickness', is smaller than L. The overall size of the internal manifold is governed by the universal Kähler modulus which we will turn to in Section 3.3; for now, we will assume that it has been fixed at an appropriate value by some dynamics that is not relevant in the throat region.

The analogue of the RS infrared brane is the Klebanov-Strassler region of small $r \lesssim r_s$, where the deviation from the Klebanov-Tseytlin metric (2.14) becomes significant and the throat is smoothly terminated. It is possible to place additional localized objects in this region, whose open-string fluctuations then correspond to fields localized on the IR brane in the 5d picture. This will become especially important in the next chapter, where we study supersymmetry breaking. In the KKLT construction, for instance, the $\overline{\mathrm{D3}}$-brane which breaks supersymmetry and uplifts the vacuum energy density to a positive value is localized in the infrared region of the throat. Alternatively, other, more complicated D-brane configurations in the KS region may give different uplifting mechanisms, a subject to which we will turn in Section 4.

The correspondence between the KS throat and the RS-I model is sketched in Figure 3.1.

[1] We adhere to the common terminology in RS model building, which is potentially confusing since we are also using similar terms from string theory: A "brane" in the context of the RS model is the four-dimensional boundary of the compactification interval. It is unrelated to string-theoretic D-branes (apart from the fact that these also constitute submanifolds of spacetime, of a completely different origin).

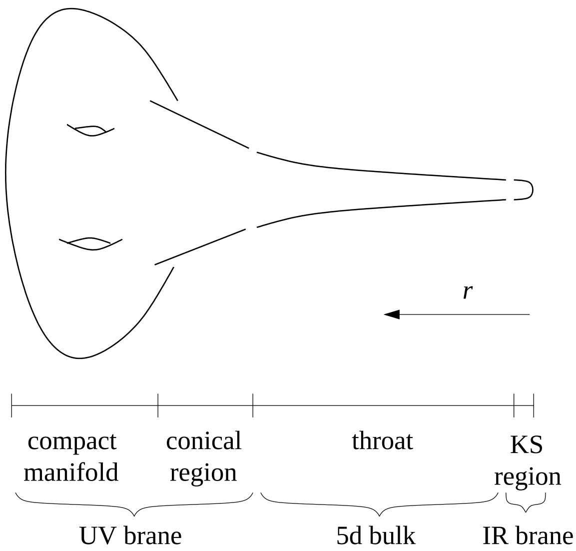

Figure 3.1: The KS throat as a RS-I model

3.2 Radius stabilization

We have seen in Section 2.4 how the complex structure modulus z of the KS throat can be stabilized by fluxes. The z modulus determines the hierarchy between the two ends of the throat, or, in the 5d picture, the distance between the two branes. This distance is commonly referred to as the "radius" of the RS geometry (since the original RS-I model was based on an $S^1/\mathbb{Z}_2$ orbifold with the S^1 radius becoming the interval length). The radius is a modulus itself in a pure RS-I model, but can be stabilized by additional dynamics. Let us investigate what these dynamics are if the 5d model is to be an effective description of the stabilized 10d throat.

While the 5d metric may locally be well approximated by AdS_5, the radial variation of R_{eff} has to be taken into account in order to characterize the throat as a whole. In other words, the negative 5d cosmological constant of AdS_5 has to be replaced by a vacuum energy density $V(H)$, which must be a function of at least one 5d scalar field H to allow for spatial variation. This field H must have a non-trivial profile $H(r)$ in the fifth dimension, which encodes the radial variation of the quantity N_{eff} (or equivalently R_{eff}) of the full 10d picture.

Working in a 5d Einstein frame with canonically normalized H,

$$\mathcal{L}_5 = \frac{1}{2}M_5^3\mathcal{R}_5 - \frac{1}{2}(\partial H)^2 - V(H) + \dots\,, \tag{3.1}$$

we can now enquire about the appropriate function $V(H)$. The profile $H(r)$ induced by this potential will give rise to a certain scalar-field energy density.

Its back-reaction has to modify the AdS_5 geometry in a way such as to reproduce the metric of (2.14).

To find the potential $V(H)$, it is convenient to first identify an alternative radial coordinate y which directly measures physical distances along the throat. An infinitesimal distance, measured in units of the 5d reduced Planck mass, should then be given by $M_5\, dy$. By contrast, a straightforward dimensional reduction of a model with the metric (2.14) to 5d would give rise to an r-dependent coefficient of the 5d Ricci scalar, which we call $M_{5,\,\text{eff}}^3(r)$. A model with the Lagrangian (3.1) could only result after a Weyl rescaling by an appropriate function of a radially varying scalar field. However, we can avoid this procedure by working with the r-dependent infinitesimal distance in units of $M_{5,\,\text{eff}}(r)$ and demanding

$$M_5\, dy = M_{5,\,\text{eff}}(r)\sqrt{g_{rr}}\, dr = [M_{10}^8 R_{\text{eff}}^5(r)\,\text{Vol}\,T^{1,1}\,]^{1/3}[R_{\text{eff}}(r)/r]\, dr\,. \quad (3.2)$$

Using $M_{10}^8 = 2/[(2\pi)^7\alpha'^4]$ and $\text{Vol}\,T^{1,1} = 16\pi^3/27$, this is further evaluated to give

$$M_5\, dy = \frac{1}{3}\left(3\text{g}_\text{s}^2 M^2/\pi^2\right)^{2/3}(\log(r/r_s))^{2/3}\, d(\log(r/r_s)), \quad (3.3)$$

which can be easily integrated. The constant of integration is conveniently fixed by choosing y as

$$y = \frac{\left(3\text{g}_\text{s}^2 M^2/\pi^2\right)^{2/3}}{5M_5}(\log(r/r_s))^{5/3} \equiv R_s\,(\log(r/r_s))^{5/3}, \quad (3.4)$$

or, in terms of the flux quanta,

$$y = R_s\left(\frac{2\pi N_{\text{eff}}(r)}{3\text{g}_\text{s} M^2}\right)^{5/3}. \quad (3.5)$$

R_s corresponds, up to $\mathcal{O}(1)$ factors, to the size of the $T^{1,1}$ in the infrared at $r = r_s$. In the following, we will treat y/R_s as parametrically large.

The 5d metric can now be written as

$$ds_5^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2\,, \quad (3.6)$$

where the warp factor, following from (2.14), (2.15) and (3.4) together with the Weyl rescaling used to go to the 5d Einstein frame, reads

$$A(y) = (y/R_s)^{3/5} + \mathcal{O}(\log(y/R_s)) + \text{const.} \quad (3.7)$$

Here the constant term is irrelevant since it can be absorbed in a rescaling of Minkowski space. We may also neglect the subleading logarithmic term, writing the warp factor as

$$A(y) = k(y)y, \qquad k(y) = R_s^{-1}(y/R_s)^{-2/5}\,. \quad (3.8)$$

Up to the slow variation of k, this choice of coordinates and parameters is as in (C.4) for AdS_5.

We are now looking for a potential $V(H)$ such that the back-reaction of the varying scalar H induces a varying curvature as in (3.8). In general, such an analysis requires the solution of the coupled equations of motion for the metric and H. However, in the present case, a simplified computation will be sufficient, assuming that both the warp factor and the profile of H will be slowly varying (the validity of this assumption will of course need to be checked on the solution afterwards). We use the equation of motion of a scalar field with potential $V(H)$ in a warped background (3.6),

$$\left(\partial_y^2 + 4A'(y)\partial_y\right) H - \frac{\partial V}{\partial H} = 0. \tag{3.9}$$

If the typical length scale for the variation of H is larger than the curvature radius $1/k$, we can neglect the second-derivative term. This gives

$$\frac{12}{5}\frac{1}{R_s}\left(\frac{y}{R_s}\right)^{-2/5} \partial_y H = \frac{\partial V}{\partial H}. \tag{3.10}$$

The profile of the warp factor is determined by an effective 5D cosmological constant coming mainly from the potential term with H set to its local VEV. From the trace of the Einstein equations for a slowly varying scalar field, we obtain a relation between the scalar curvature and the potential energy density similar to (C.3),

$$-\frac{3}{10}\mathcal{R} = \frac{V(H)}{M_5^3}, \tag{3.11}$$

which with the metric (3.6) and (3.7) becomes

$$V = \frac{54}{25}\left(\frac{y}{R_s}\right)^{-4/5} \frac{M_5^3}{R_s^2}. \tag{3.12}$$

Using the chain rule $\partial V/\partial y = (\partial V/\partial H)\, \partial_y H$, equations (3.10) and (3.12) give the profile of H as

$$H(y) = (2M_5)^{3/2} \left(\frac{y}{R_s}\right)^{3/10}. \tag{3.13}$$

It can now be easily verified that the conditions

$$|\partial_y^2 H| \ll |A'(y)\partial_y H| \qquad \text{and} \qquad (\partial_y H)^2 \ll |V|\,, \tag{3.14}$$

which justify our simplified treatment, are satisfied.

The desired functional dependence of V on H is finally obtained from (3.13) and (3.10):

$$V(H) = -\frac{864}{25}\frac{M_5^7}{R_s^2}\, H^{-8/3}. \tag{3.15}$$

Thus, we conclude that 5d gravity coupled to a scalar field H with the potential (3.15) reproduces the effective 5d geometry of the throat.

To describe the entire compactification, we need to add an IR and UV brane with specific tensions and boundary conditions for H to our 5d model. We assume the tensions to be positive and negative for the UV and IR brane respectively and the values to be such that both branes are static in an AdS space with curvature determined by the boundary values of H and $V(H)$. To discuss the boundary conditions on H explicitly, recall that H substitutes the parameter R_{eff}, or equivalently N_{eff}, of the 10d construction. The explicit relations are, cf. (3.5),

$$H = (2M_5)^{3/2}(R_{\text{eff}}/R_s)^2 = (2M_5)^{3/2}(N_{\text{eff}}/N_s)^{1/2} \qquad \text{with } N_s = \frac{3}{2\pi} \mathrm{g_s} M^2 \,. \tag{3.16}$$

Thus, the boundary condition $H(y_{\text{IR}}) = (2M_5)^{3/2}$ will reproduce the IR end corresponding to a KS region with M units of F_3 flux. Field-theoretically, such a boundary condition can be realized by an appropriate brane potential for H with an extremely steep minimum.

In the ultraviolet, we can define N as the number of $\tilde{F}_5$ flux units on the $T^{1,1}$ cycle at the UV end of the conical region. This number is determined by localized sources, e.g. O3-planes and D3-branes, and regions with 3-form flux within the remainder of the compact space.[2] In the conical region, this flux number changes according to (2.23). Assuming that the conical region is not too large, the change there is very small compared to the change that occurs within the throat, so that we can identify the $\tilde{F}_5$ flux N_{UV} at the UV end of the throat (the IR end of the conical region) with the flux number N defined above. Thus, the UV boundary condition of the 5d model reads $H(y_{\text{UV}}) = (2M_5)^{3/2}(N_{\text{UV}}/N_s)^{1/2} \simeq (2M_5)^{3/2}(N/N_s)^{1/2}$.

In summary, we have presented a 5d model, containing gravity plus a minimally coupled scalar field, which upon compactification on an interval with boundary conditions $H(y_{\text{IR/UV}}) = (2M_5)^{3/2}(N_{\text{IR/UV}}/N_s)^{1/2}$ provides the 5d description of the KS throat. The 5d bulk profile of H fixes, together with the boundary conditions, the throat length $y_{\text{UV}} - y_{\text{IR}}$. This is reminiscent of the Goldberger-Wise mechanism [74] in 5d Randall-Sundrum-I models: A scalar field with a nontrivial bulk profile and fixed boundary values, e.g. from steep brane-localized potentials, can stabilize the radius of the extra dimension. There is however an important difference: As opposed to the model of [74], in the conifold throat the back-reaction of the scalar field on the geometry is crucial. It describes the effect of the M units of F_3 flux – the duality cascade in the dual gauge theory.

[2] Note that in the literature $N = MK$ is frequently used to designate the effective D3 charge from M units F_3 flux on the S^3 cycle and K units of H_3 flux on its dual. This definition coincides with ours if the 3-form flux in question is mainly concentrated outside the 'compact manifold' of Figure 3.1.

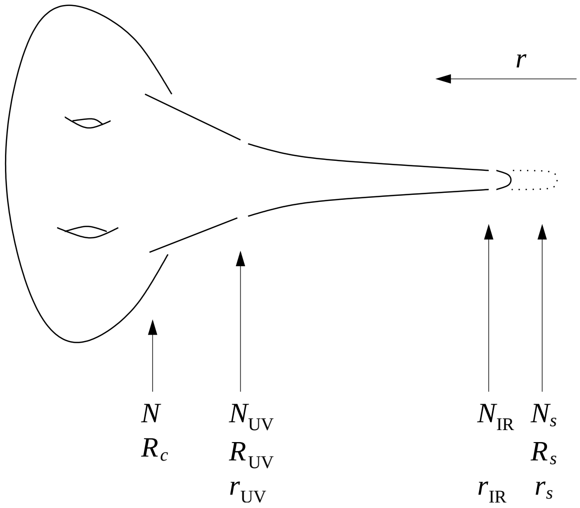

Figure 3.2: The throat with the values of $N_{\text{eff}}(r)$ and $R_{\text{eff}}(r)$ at several positions r. The dotted line indicates that, in the presence of D3-branes in the KS region, the throat may end at $r_{\text{IR}} > r_s$.

For reference, we have depicted the throat along with the values of $N_{\text{eff}}(r)$ and $R_{\text{eff}}(r)$ at several radial positions, as defined in the text, in Figure 3.2.

A related ansatz to characterize warped type IIB supergravity solutions with fluxes in terms of 5d scalars coupled to gravity was presented in [75], generalizing the methods employed by Klebanov-Tseytlin [57] (see also e.g. [76]). The fluctuations of the internal metric and of the gauge fields are parametrized by several scalar fields, such that part or all of the symmetries of the system are preserved. The bosonic action of type IIB then leads to a nonlinear sigma model for the scalar fields in five dimensions. For some systems such as the KS throat (or its simplified version, the KT solution), this model takes the form of a "fake supergravity", meaning that the potential can be derived from a simpler function which resembles a superpotential. This may considerably simplify the task of finding a solution in the first place.

The general nonlinear sigma model of [75] can be consistently truncated to a version which involves only four scalars and characterizes the KT solution. The fields are called q, f, Φ, T in [57]. q measures the $T^{1,1}$ volume and f the ratio of scales between the 2-cycle and the 3-cycle. Φ is the dilaton, and T measures the B_2 potential. With these fields, the 5d action is

$$S_5 = M_5^3 \int d^5x \left(\frac{1}{2}\mathcal{R}_5 - G_{ab}(\varphi)\partial\varphi^a\partial\varphi^b - V(\varphi) \right) . \tag{3.17}$$

with φ collectively denoting the dimensionless scalars (q, f, T, Φ), and

$$G_{ab}(\varphi)\partial\varphi^a\partial\varphi^b = 15(\partial q)^2 + 10(\partial f)^2 + \frac{1}{4}(\partial\Phi)^2 + \frac{1}{4}e^{-\Phi-4f-6q}(\partial T)^2,$$
$$V(\varphi) = e^{-8q}\left(e^{-12f} - 6e^{-2f}\right) + \frac{1}{8}P^2 e^{\Phi+4f-14q} + \frac{1}{8}(Q+PT)^2 e^{-20q}. \tag{3.18}$$

P and Q are constants, with P proportional to the number of 3-form flux quanta M. With a "warped" ansatz as in (3.6) for the 5d metric, a solution to the equations of motion is given by the KT background, with $f = \Phi = 0$ and the radial variation of the $T^{1,1}$ radius and B_2 field encoded in the nontrivial y-dependence of q and T. Explicitly, at large y,

$$e^{2q} \sim y^{1/5}, \qquad Q + PT \sim y^{3/5}. \tag{3.19}$$

In terms of the physical quantities we have been using, $PT + Q \sim N_{\text{eff}}$ and $e^{3q/2} \sim R_{\text{eff}}$. The leading contribution to the vacuum energy density, whose back-reaction determines the warp factor, is given by the first and last terms of the potential in (3.18), evaluated on the solution.

3.3 The universal Kähler modulus

In the last section we have presented a simple effective 5d model for the throat, consisting of a single scalar field coupled to gravity. The most important shortcoming of this model is the absence of the universal Kähler modulus common to such type IIB supergravity compactifications [12]. We have avoided this issue by simply assuming that the typical radius of the compact space at the UV end is somehow stabilized. In this section, we will relax this assumption and discuss the interplay of this degree of freedom with our 5d model of the throat.

There is always at least one Kähler modulus in a realistic compactification, which in the limit of zero warping is simply an overall scaling of the internal metric and hence corresponds to a change of the volume of the compact manifold. In the presence of warping the scaling behavior is more subtle [77]. In terms of the metric (2.14), the flat direction corresponds to a shift

$$\tilde{h}(r) \;\to\; \tilde{h}(r) + c - 1 \tag{3.20}$$

for an arbitrary value of the constant c. As in the unwarped case, this affects the volume of the manifold, but now obviously is no longer a simple rescaling.

This realization of the volume modulus can, in fact, be understood very easily: The metric (2.14) contains only two dimensionful parameters, α' and

r_s. A volume modulus, if present, can only change the ratio of these two scales. Indeed, a rescaling

$$r_s \to r_s c^{1/4} \tag{3.21}$$

corresponds, together with an appropriate rescaling of r and x^μ, to the shift (3.20) in $\tilde{h}$.

If c becomes extremely large, larger than $\tilde{h}(r_{\rm IR})$, the throat disappears and the variation of c corresponds to an overall scaling of the entire compact space. In this regime, the radius R_c is bigger than the length scale L at which our 5d effective description is defined. In other words, the "brane thickness" of the UV brane is so large that the 5d picture is lost.

Let us instead consider values of c such that $R_c < L$. In the compact space at the UV end of the throat, $\tilde{h}$ is approximately constant and the variation of c again corresponds to a simple scaling. In the throat, on the other hand, note that the $\tilde{F}_5$ flux N at the UV end of the conical region is fixed; it does not depend on the volume of the compact space. This is also approximately true for the $\tilde{F}_5$ flux $N_{\rm UV}$ at the UV end of the throat (i.e. at the IR end of the conical region). Furthermore, the F_3 flux is not affected by the volume scaling (and neither is the number of explicit D3-branes at the IR end of the throat, if we choose to include any). Thus, neither of the boundary conditions determined by $N_{\rm UV}$ and $N_{\rm IR}$ changes when c varies and therefore, as we discussed in Section 3.2, the length of the throat remains fixed. This means that, in the 5d description, the Kähler modulus plays the role of a massless UV-brane field while the 5d radion is already stabilized.

However, this picture is correct only at first approximation. The key to the c-independence of N was its definition as the flux at the transition point between the conical and the more general compact geometries. This definition does not depend on the overall scaling. By contrast, $N_{\rm UV}$ is defined at the transition point between conical and throat geometries. As we will now demonstrate, the location of this transition point has a non-trivial c-dependence, which is reflected in a weak c-dependence of $N_{\rm UV}$. The resulting effect on the length of the throat is small compared to the effect on the compact region, as we will show explicitly. Nevertheless, for extremely large c this effect will cut into the length of the throat such that, eventually, the throat disappears. This is consistent with the limit of weak warping discussed above.

From the 10d point of view, the RS UV brane (comprising the compact space and the conical region) is the area where the warp factor is, to a good approximation, constant. The universal Kähler modulus simply corresponds to an overall rescaling of this region. In particular, the flux number

$$N = N_{\rm eff}(R_c) = \frac{1}{(4\pi^2\alpha')^2} \int_{T^{1,1} \text{ at } r=R_c} \tilde{F}_5 \tag{3.22}$$

is invariant under this rescaling. From the point of view of the throat, it is determined completely by the localized sources and flux in the compact space.

We now focus on the conical region and the throat. If we choose our coordinates such that the warp factor in the conical region is unity, as in (2.14) and (2.15), then R_c can be identified with the universal Kähler modulus. Given the general r-dependence of $N_{\rm eff}$ in throat and conical region, (2.23), one finds the constraint

$$N = \frac{3}{2\pi} g_s M^2 \log(R_c/r_s)\,, \tag{3.23}$$

which fixes r_s in terms of R_c. This gives the warp factor (cf. (2.15)) to be

$$\tilde{h}(r) = 1 + \frac{27\pi}{4}\alpha'^2 g_s \frac{N - \frac{3}{2\pi} g_s M^2 \log(R_c/r)}{r^4}\,. \tag{3.24}$$

The boundary between the conical region and the throat, $r = r_{\rm UV}$, is then determined by the solution of the equation $\tilde{h}(r_{\rm UV}) = 1$. Assuming that, at this boundary, the logarithmic term in (3.24) is small relative to N and working to leading order in this small term, we find

$$r_{\rm UV}^4 = \frac{27\pi}{4}\alpha'^2 g_s \left[N - \frac{3}{8\pi} g_s M^2 \log\left(\frac{4\, R_c^4}{27\pi\, \alpha'^2 g_s N} \right) \right] \; + \; (\text{subleading terms}). \tag{3.25}$$

Thus, the conical region shrinks to zero size if R_c^4 takes the value

$$R_{c,\,\rm min}^4 = \frac{27\pi}{4}\,\alpha'^2 g_s N\,, \tag{3.26}$$

and our approximation remains valid as long as

$$(R_c/R_{c,\,\rm min}) \ll \exp(2\pi N/3 g_s M^2). \tag{3.27}$$

The RHS of this inequality is of the order of the inverse hierarchy, and is thus very large in the cases of interest to us. In other words, there is a large range in which the variation of the universal Kähler modulus R_c has very little effect on the throat length, as expressed by (3.25). In this domain, it is mainly just a scaling of the compact manifold at the UV end of the throat. Thus, we are led to the conclusion that, from the 5d point of view, the universal Kähler modulus is a field localized at the UV brane.

Let us now translate the above discussion to the 5d picture in a more quantitative way. From the 5d perspective, the fundamental scale is the reduced 5d Planck mass M_5. Near the UV brane, M_5 is related to M_{10} by $M_5^3 \simeq M_{10}^8 R_{\rm UV}^5$. For not too large values of R_c, we can identify $R_{\rm UV}$ with $R_{c,\,\rm min}$, with the result that

$$R_c M_5 \sim (g_s N)^{2/3} (R_c/R_{c,\,\rm min})\,. \tag{3.28}$$

We can think of this as of the UV brane thickness in units of M_5. In the same units, the physical length $L_{\rm th}$ of the throat is given by

$$L_{\rm th}M_5 \sim (y_{\rm UV} - y_{\rm IR})M_5 \sim (\mathsf{g_s}M)^{4/3}\left[\left(\frac{2\pi}{3}\frac{N_{\rm UV}}{\mathsf{g_s}M^2}\right)^{5/3} - \left(\frac{2\pi}{3}\frac{N_{\rm IR}}{\mathsf{g_s}M^2}\right)^{5/3}\right], \tag{3.29}$$

where $N_{\rm UV}$ is the flux at the IR end of the conical region or, equivalently, at the UV end of the throat. Our interest is in the dependence of $L_{\rm throat}$ on R_c. Hence we cannot simply identify $N_{\rm UV}$ with N, but rather we have to take care of this subtle distinction which is due to the running in the conical region:

$$N = N_{\rm UV} + \frac{3}{2\pi}\,\mathsf{g_s}M^2\log(R_c/R_{c,\,\rm min})\,. \tag{3.30}$$

We now assume that R_c grows by a factor $1+\epsilon$ (where $\epsilon \ll 1$). Then, on the one hand, the thickness of the UV brane in units of M_5 increases by $\sim \epsilon(\mathsf{g_s}N)^{2/3}R_c/R_{c,\rm min}$. On the other hand, the length of the throat, also measured in units of M_5, shrinks by $\sim \epsilon(\mathsf{g_s}M)^{4/3}(3N_{\rm UV}/2\pi\mathsf{g_s}M^2)^{2/3}$. The ratio of these two quantities is $\sim R_c/R_{c,\rm min} > 1$, i.e. the throat shrinks less than the brane thickness grows.

This can be turned into an even more explicit argument for the Kähler modulus being a brane field: From the 5d perspective, it is perfectly acceptable to define the throat length either by (3.29) or, including the UV brane thickness into the size of the 5d interval, by the sum of (3.28) and (3.29). When R_c grows, the throat length shrinks according to the first and grows according to the second definition. Thus R_c cannot be consistently identified with the length of the 5d interval. Instead, it has to be modelled by a field localized at the UV brane. Of course, because our 5d effective theory is valid only at length scales above L, we should be careful not to increase R_c above L. Otherwise, the 5d description of the UV end becomes meaningless.

3.4 The 5d effective action

We are now finally in a position to construct the 5d effective action including bulk and brane fields. This will be done mainly by consistency arguments, based on our results from the previous sections and on what is known about the effective action upon further compactification to four dimensions. An explicit dimensional reduction of the 10d action to 5d is not feasible because the internal space is too complicated and because we have not specified a UV embedding; see Section 3.5 for some more remarks on this.

For $R_c \gg R_{c,\rm min} \simeq R_{\rm UV}$, the integral over the compact space at the UV end of the throat contributes

$$\mathcal{L}_{\rm UV} = \frac{1}{2} M_{10}^8 R_c^6 \left(\mathcal{R}_4 + 30(\partial \log R_c)^2 + \ldots\right) \tag{3.31}$$

to the 4d effective action before Weyl rescaling to the 4d Einstein frame.[3] We can view this as a precise definition of R_c, which is chosen such that R_c^6 is the volume of the compact space.

The bulk part was already given in (3.1). Writing the 5d metric as

$$ds_5^2 = e^{2A(y)-2A(y_{\rm UV})} g_{\mu\nu} dx^\mu \, dx^\nu + dy^2 \,, \tag{3.32}$$

and integrating from $y_{\rm IR}$ to $y_{\rm UV}$, this contributes the following piece to the Einstein-Hilbert term of the 4d action:

$$\begin{aligned} \frac{1}{2}\left(M_5^3 \int_{y_{\rm IR}}^{y_{\rm UV}} dy \, \exp\left[2\left(\frac{y}{R_s}\right)^{3/5} - 2\left(\frac{y_{\rm UV}}{R_s}\right)^{3/5}\right]\right) \mathcal{R}_4 \\ \approx \frac{5}{12} M_5^3 R_s \left(\frac{y_{\rm UV}}{R_s}\right)^{2/5} \mathcal{R}_4 \,. \end{aligned} \tag{3.33}$$

Here $\mathcal{R}_4$ is to be evaluated with the 4d metric $g_{\mu\nu}$. The warp factor in (3.32) has been normalized to ensure consistency with the 4d metric in (3.31).

The relative normalization of the coefficients of the $\mathcal{R}_4$ and the $(\partial \log R_c)^2$ terms in (3.31) is due to the fact that R_c^4 is the real part of a superfield T [12], which is part of a no-scale supergravity model. It changes upon the addition of the 4d Einstein-Hilbert contribution of (3.33). However, this contribution is subdominant in the large-R_c limit in which (3.31) was derived. Corrections to (3.31) are indeed expected since, near the IR end of the conical region, R_c loses its interpretation as an overall scaling modulus of the compact space. To retain the 4d no-scale structure after including (3.33), the coefficient of the $\mathcal{R}_4$ term in (3.31) should to be modified according to

$$M_{10}^8 R_c^6 \to M_{10}^8 R_c^6 - \frac{5}{6} M_5^3 R_s \left(\frac{y_{\rm UV}}{R_s}\right)^{2/5} \,. \tag{3.34}$$

After these remarks we now give the full 5d action to the extent that it can be inferred from the present analysis. In doing so, it is convenient to absorb a factor $\mathsf{g}_{\rm s} M$ into the definition of the scalar field. Thus, we define

$$\tilde{H} \equiv \sqrt{\frac{3}{16\pi}} \mathsf{g}_{\rm s} M H = \sqrt{\mathsf{g}_{\rm s} N_s / 8} \; H, \tag{3.35}$$

[3] The prefactor 30 arises as $k(k-1)$, with $k = 6$ the number of compact dimensions.

where the prefactor has been chosen such that $\tilde{H}(y) = M_5^{3/2}\,(\mathrm{g_s}N_{\mathrm{eff}}(y))^{1/2}$. The action now reads

$$S_{5d} = \int d^5x\sqrt{-g_5}\left(\frac{1}{2}M_5^3\mathcal{R}_5 - \frac{8\pi}{3}\frac{1}{(\mathrm{g_s}M)^2}(\partial\tilde{H})^2 + cM_5^9\tilde{H}^{-8/3} + \dots\right)$$
$$+ \int\limits_{\text{UV brane}} d^4x\sqrt{-g_{4,\,\mathrm{UV}}}\,(K_{\mathrm{UV}} + \mathcal{L}_{\mathrm{UV}}) + \int\limits_{\text{IR brane}} d^4x\sqrt{-g_{4,\,\mathrm{IR}}}\,(K_{\mathrm{IR}} + \mathcal{L}_{\mathrm{IR}})\;,$$

where $K_{\mathrm{UV/IR}}$ is the trace of the extrinsic curvature (the Gibbons-Hawking surface term [78]) and $(g_{4,\,\mathrm{UV/IR}})_{\mu\nu}$ is the induced metric at each of the 4d boundaries. The constant c is given by $c = 27\cdot 2^{11/3}\pi^{4/3}$. The brane Lagrangians are

$$\begin{aligned}\mathcal{L}_{\mathrm{UV}} =& \frac{c_1}{2}M_5^2(\mathrm{g_s}N_{\mathrm{UV}})^{-10/3}\left[\left((R_cM_5)^6 - c_2(\mathrm{g_s}N_{\mathrm{UV}})^4\right)\mathcal{R}_4\right.\\ &\left.+30(R_cM_5)^6(\partial\log R_c)^2\right]\\ &- V_{\mathrm{UV}}(\tilde{H}) - \Lambda_{4,\,\mathrm{UV}} + \dots\end{aligned} \tag{3.36}$$

and

$$\mathcal{L}_{\mathrm{IR}} = -V_{\mathrm{IR}}(\tilde{H}) - \Lambda_{4,\,\mathrm{IR}} + \dots\;, \tag{3.37}$$

with numerical coefficients $c_1 = 32\pi^{1/3}/9$ and $c_2 = 3\cdot 2^{2/3}/(32\pi)$. Here V_{UV} and V_{IR} are steep potentials setting $\tilde{H}$ to its values at the UV and IR brane respectively, for example,

$$V_{\mathrm{UV/IR}} = \mu^2\left[\tilde{H} - M_5^{3/2}(\mathrm{g_s}N_{\mathrm{UV/IR}})^{1/2}\right]^2\;, \tag{3.38}$$

with a very large coefficient μ. The brane tensions or 4d brane cosmological constants Λ_{UV} and Λ_{IR} have values

$$\Lambda_{\mathrm{UV}} = +M_5^4\sqrt{6/c}(\mathrm{g_s}N_{\mathrm{UV}})^{-2/3} \qquad \text{and} \qquad \Lambda_{\mathrm{IR}} = -M_5^4\sqrt{6/c}(\mathrm{g_s}N_{\mathrm{IR}})^{-2/3}\,. \tag{3.39}$$

The fundamental dynamics of the throat can now be easily understood from the 5d action (3.36): The scalar field $\tilde{H}$ governs, via the potential term, the (approximately AdS) curvature and hence the warping. The rapidity with which the curvature changes as one moves along the 5th dimension is determined by the coefficient of the kinetic term for $\tilde{H}$. In the limit of vanishing M, no change is possible – this is the pure AdS_5 case. The boundary or brane values of $\tilde{H}$ are determined by steep brane potentials. The IR-brane potential models the way in which the Klebanov-Strassler region (or a more complicated corresponding geometry) determines the value of N_{eff} in the IR regime. The UV-brane potential models the way in which the various stringy and field-theoretic sources of D3-brane flux in the compact space determine N_{eff} in the conical region. The combined dynamics of UV/IR-brane and 5d bulk actions then stabilizes the length of the interval and fixes the hierarchy.

In the above 5d effective action, R_c appears as a brane field localized at the UV-boundary. However, it is a brane field of very peculiar type. In the 5d Einstein frame, R_c is part of the coefficient of the brane-localized Ricci-scalar and has a wrong-sign kinetic term. Of course, this can be remedied by performing an appropriate R_c-dependent Weyl rescaling of the 5d metric. However, in such a Weyl frame R_c would cease to be a UV-brane field. Note furthermore that R_c can easily be parametrically larger than its lower bound (in the present analysis) $R_{c,\,\mathrm{min}} \simeq R_{\mathrm{UV}}$. In this case, our 5d model develops a parametrically large gravitational brane-kinetic term, a scenario which can be very interesting for field-theoretic model building [79, 80].

Finally we would like to explicitly relate the most important parameters of our 5d description, the boundary scalar R_c and the 5d radion $\Delta y = y_{\mathrm{UV}} - y_{\mathrm{IR}}$, to the corresponding standard string moduli. Focussing on the universal Kähler modulus T (which is $T = -i\rho$ in the notation of [12]) and a single complex structure modulus z, and neglecting the warping for the moment, the 4d $\mathcal{N} = 1$ superfield action is determined by the Kähler potential

$$\mathcal{K}(T, z) = -3\log(T + \overline{T}) - \log\left(-i\int \Omega \wedge \bar{\Omega}\right), \tag{3.40}$$

and the superpotential

$$W(z) = \int G_3 \wedge \Omega\,. \tag{3.41}$$

The holomorphic (3,0) form Ω is normalized using some 3-cycle of the compact space at the UV end of the throat, and z is defined via the 3-cycle $\mathcal{A}$ in the throat as in Section 2.4,

$$z = \int_{\mathcal{A}} \Omega\,. \tag{3.42}$$

In the case of negligible warping, the universal Kähler modulus governs the compactification volume. More precisely, the 4d no-scale field T is related to R_c by

$$\mathrm{Re}\, T \sim R_c^4\,. \tag{3.43}$$

We can leave the constant of proportionality arbitrary since we do not intend to fix a possible additive constant in $\mathcal{K}$.

In [12] the relation of the complex structure modulus z to the relative warping between the UV and IR region is found to be

$$e^{A(r_{IR})-A(r_{\mathrm{UV}})} \simeq |z|^{1/3}\,. \tag{3.44}$$

Here $\exp[2A(r)] = \tilde{h}(r)^{-1/2}$ (cf. (2.14)) is the 10d warp factor, which differs from the 5d warp factor $\exp[2A(y)]$ of Eq. (3.6) by an insignificant (non-exponential) correction related to the 5d Weyl rescaling. The relative 5d warping is

$$e^{A(y_{IR})-A(y_{\rm UV})} \simeq \exp\left[-(\Delta y/R_s)^{3/5}\right] , \qquad (3.45)$$

which allows us to express z through the 5d radion:

$$|z|^{1/3} \simeq \exp\left[-\left(\frac{(5M_5\,\Delta y)^3}{(3{\rm g}_{\rm s}^2 M^2/\pi^2)^2}\right)^{1/5}\right] . \qquad (3.46)$$

This concludes our comparative discussion of R_c and Δy and the string moduli T and z. It would of course be most interesting to further identify the superfield description of the stabilized Randall-Sundrum model [81] with the moduli of the 10d flux compactification. In the next section we will offer some comments which may lead in this direction.

3.5 Towards a superfield action

Type IIB theory is maximally supersymmetric, i.e. there are 32 real supercharges. Compactification on $\mathrm{AdS}_5 \times S^5$ preserves all of the supersymmetry, so the theory of all the S^5-Kaluza-Klein fields on AdS_5 is 5d $\mathcal{N} = 4$ supersymmetric. This can also be understood from the dual gauge theory, which is $\mathcal{N} = 4$ supersymmetric in 4d and has four additional fermionic generators for the superconformal symmetry.[4]

Compactification on $\mathrm{AdS}_5 \times T^{1,1}$ breaks the supersymmetry to a quarter of the original SUSY. That is, the warped conifold throat has 5d $\mathcal{N} = 1$ SUSY, and the dual gauge theory is $\mathcal{N} = 1$ superconformal (this 4d $\mathcal{N} = 1$ superconformal symmetry is sometimes referred to as 4d $\mathcal{N} = 2$ SUSY in the literature concerned with the supersymmetric Randall-Sundrum model). More precisely, the corresponding 5d theory on AdS_5 should be an $\mathcal{N} = 1$ gauged supergravity, coupled to additional fields.

Adding 3-form flux gives the warped deformed conifold whose dual gauge theory has (non-conformal) 4d $\mathcal{N} = 1$ SUSY. We thus expect the 5d effective action for the KS throat not to be 5d supersymmetric. In fact, it was directly proven in [53] that the KS background admits only four supercharges. Still it should be possible to write the 5d action in a manifestly 4d $\mathcal{N} = 1$

[4] In the literature the supersymmetry of the $\mathrm{AdS}_5 \times S^5$ solution is sometimes called $\mathcal{N} = 8$ in reference to the eight 4d fermionic symmetry generators. We will always denote by $\mathcal{N}$ the number of supersymmetry generators proper in the respective dimensions, such that the number of supercharges is given by the number of real components of the minimal spinor times $\mathcal{N}$.

supersymmetric form, using superfields with an extra dependence on the fifth coordinate [82–86].

It would be desirable to derive the 5d effective action by explicit dimensional reduction of the 10d theory, rather than by consistency arguments as we have done. From dimensional reduction one could obtain the full set of light 5d fields and then properly identify the 4d multiplets. Unfortunately, dimensional reduction on a complicated space with varying warp factor does not seem to be technically feasible. The best one might hope for is to obtain the theory as some suitable flux-induced deformation of the effective 5d gauged supergravity of the $\mathrm{AdS}_5 \times T^{1,1}$ compactification. But in fact, not even this theory has been explicitly constructed. Let us nevertheless briefly summarize the state of the art there, in order to outline where the main difficulties are and where future investigations of this subject should be directed.

The Kaluza-Klein mode expansion of type IIB supergravity on $\mathrm{AdS}_5 \times T^{1,1}$ was performed in [87, 88]. Since $T^{1,1}$ is a homogenous space, it is possible to obtain the 5d spectrum using group-theoretic techniques (see e.g. [89]): The eigenfunctions of the Laplacian on a coset G/H, in this case $T^{1,1} = (\mathrm{SU}(2)\times \mathrm{SU}(2))/\mathrm{U}(1)$, are determined in terms of the matrix elements of unitary irreducible group representations. The fluctuations of the type IIB fields around the background, defined by the geometry and the $\tilde{F}_5$ flux, are expanded in these harmonics. Finally the resulting fields on AdS_5 are sorted into multiplets of the $\mathrm{SU}(2,2|1)$ superalgebra.

This procedure gives the full set of KK towers for all 5d fields, as well as their arrangement into AdS SUSY multiplets, but does not yet specify their interactions. However, these are of course important if one would like to write down an effective 5d action for the light modes. It should be pointed out here that we are not searching for a truncation of type IIB theory on $\mathrm{AdS}_5\times T^{1,1}$ to the 5d massless sector, which would amount to simply discarding the heavy states. It has in fact been shown that a consistent truncation does not exist [90]. What we would like to construct is a low-energy effective action in terms of the light fields, with the heavy ones not simply set to zero but integrated out. This will give rise to additional interactions between the light modes, suppressed by a mass scale of the order of the inverse $T^{1,1}$ radius.

In [91] the structure of the most general $\mathcal{N} = 1$ ($\mathcal{N} = 2$ in the language of that paper) gauged 5d supergravity coupled to vector and hypermultiplets was derived. The authors pointed out that, in particular, the scalar manifold must be the product of a "very special" [92] by a quaternionic Kähler manifold. They further identified the relevant degrees of freedom in the effective theory of type IIB on $\mathrm{AdS}_5 \times T^{1,1}$ as coming from a massless graviton multiplet, the seven massless vector multiplets corresponding to the SU(2)× SU(2)× U(1) isometry generators, and six hypermultiplets which were argued to appear by comparison with the operators of the dual CFT. Unfortunately this does not yet determine the scalar manifold, even when including all the additional information that can be extracted from the AdS/CFT correspondence. The interactions of the light fields thus remain unknown.

To summarize, even the effective theory on the warped singular conifold is not fully understood, not to mention its deformation upon the addition of 3-form flux which should give the effective warped deformed conifold theory.

One could try to pursue a different, more modest approach and start from the bottom up. To find a 5d superfield description which at least includes the degrees of freedom we have identified already would be analogous to what we did in the previous section without reference to supersymmetry. The simplest ansatz for this is based on the stabilized supersymmetric RS model. The essential quantity is the radion superfield t with $\mathrm{Re}\, t \sim \Delta y$. The Kähler potential in terms of t is expected to be [93] (see also [81, 83])

$$\mathcal{K}_{5d} \simeq -3 \log \left[\int_{y_{\mathrm{UV}} - \mathrm{Re}\, t}^{y_{\mathrm{UV}}} dy\, e^{2A(y) - 2A(y_{\mathrm{UV}})} \right] , \tag{3.47}$$

i.e. it is proportional to the logarithm of the coefficient of the Ricci scalar in the 4d effective action before Weyl rescaling. We now consider y_{UV} to be constant and focus exclusively on the t dependence entering through the lower integration limit $y_{\mathrm{IR}} = y_{\mathrm{UV}} - \mathrm{Re}\, t$. This t dependence corresponds to the z dependence in the language of 10d moduli (cf. (3.44) and (3.45)) so that we can write

$$\begin{aligned} \int_{y_{\mathrm{UV}} - \mathrm{Re}\, t}^{y_{\mathrm{UV}}} dy\, e^{2A(y) - 2A(y_{\mathrm{UV}})} &= \text{const.} - |z|^{2/3} \int_{-\infty}^{y_{\mathrm{IR}}} dy\, e^{2A(y) - 2A(y_{\mathrm{IR}})} \\ &\simeq \text{const.} - \frac{|z|^{2/3}}{2A'(y_{\mathrm{IR}})} . \end{aligned} \tag{3.48}$$

Since $A'(y_{\mathrm{IR}}) \sim (-\log |z|)^{-2/3}$, this implies for the z-dependent part of the Kähler potential

$$\mathcal{K}_{5\mathrm{d}} \simeq -3 \log \Big[\text{const.} - |z|^{2/3} (-\log |z|)^{2/3} \Big] \sim |z|^{2/3} (-\log |z|)^{2/3} , \tag{3.49}$$

where the prefactor and subdominant terms have been suppressed.

This is to be compared with the z-dependent part of the string moduli Kähler potential (3.40), which we already calculated in Section 2.4 for negligible warping. Following [94], we now account for the warping by replacing $\Omega \wedge \bar{\Omega}$ with $e^{-4A} \Omega \wedge \bar{\Omega}$ [95]. As in Section 2.4, the dominant z-dependent contribution comes from the tip of the throat and depends only on two period integrals. The relevant cycles of the compactification manifold are the conifold 3-cycle $\mathcal{A}$ with period z, cf. (3.42), and its dual $\mathcal{B}$ with period

$$\int_{\mathcal{B}} \Omega = \frac{z}{2\pi i} \log z + \text{holomorphic} \tag{3.50}$$

$\mathcal{B}$ will extend outside the throat into the compact manifold, whose precise form determines the holomorphic part. There will in general be other pairs of 3-cycles with period integrals that depend purely holomorphically on z. With the warp factor contribution at the tip given by $e^{-4A} \sim |z|^{-4/3}$, we obtain for the z-dependent part

$$\begin{aligned} -\log\left(-i\int e^{-4A}\,\Omega\wedge\bar{\Omega}\right) &= -\log\left[\text{const.} - |z|^{2/3}\log(z\bar{z}) + \ldots\right] \\ &\sim |z|^{2/3}(-\log|z|)\,. \end{aligned} \tag{3.51}$$

Here the ellipses stand for higher-order terms of the form $f(z)\bar{g}(\bar{z})$ with f, g holomorphic. As before, the prefactor and subdominant terms have been suppressed.

While the structure of (3.49) and (3.51) is very similar in the limit of small z, they do not agree completely. The failure to fully match the 10d string-theoretic with the 5d field-theoretic result is not unexpected in many ways. On the one hand, it may be necessary to account for subleading warping corrections on the 10d side. On the other hand, calculating the Kähler potential on the basis of (3.47) and using the naive identification of Δy in terms of $|z|$ may be too simplistic. Again, the supersymmetric RS model is a fairly crude approximation to the much more complicated situation at hand, where the varying warp factor plays a key role. It may thus be necessary to start with a 5d superfield Lagrangian which reproduces the correct 5d scalar potential governing the profile of the Goldberger-Wise scalar H and hence the warp factor. This could be an interesting direction for future work.

Chapter 4

Supersymmetry breaking and its mediation

There is a host of semi-realistic particle physics models based on the idea of the Randall-Sundrum-I model: a UV brane and an IR brane, connected by a slice of AdS_5, with several distinct sectors of the model localized on the branes or in the bulk. Having established the relationship between the KS throat and the RS-I model, it is now natural to investigate how the properties of such models are modified when the underlying theory is type IIB superstring theory. After all the RS-I model, with a bulk spacetime which is exactly AdS, terminated by two infinitely thin branes, and without an internal 5d compact manifold such as $T^{1,1}$, is a fairly crude approximation to a realistic compactification geometry as we have seen.

The main motivation to construct models with a warped extra dimension is of course the hierarchy of scales between the UV and the IR brane. As suggested in [13], this might be useful for solving the electroweak hierarchy problem: With the Standard Model (or even just the Higgs field) localized on the IR brane, it is easy to obtain an exponentially small electroweak symmetry breaking scale, since all dimensionful quantities in the IR are exponentially redshifted. In this chapter, however, we will consider models which use the warped hierarchy for a different purpose. As explained in Section 2.5, in the KKLT construction a large hierarchy of scales is required to be able to tune the vacuum energy density to a small positive value. It is thus the SUSY-breaking hidden sector, rather than the Standard Model fields, which must be located in a strongly warped region. The Standard Model fields should instead be localized on the UV brane, or equivalently, from the 10d point of view, should reside on D-branes somewhere in the compact space in which the throat is embedded.

The hidden and visible sectors are then "sequestered" in the sense of [96]: Direct cross-couplings in 4d are highly suppressed, as they can only be generated by the exchange of massive modes of the warped bulk. Separation in the internal space alone is in general not sufficient to guarantee sequestering in string models [97] (see also [98]). However, it has been shown that in minimal warped 5d field-theoretic models sequestering is achieved [81].

A more thorough analysis of string models has shown that, while in unwarped full string backgrounds sequestering is generally spoiled by the contributions from the compactification moduli, this is not the case for warped backgrounds such as the ones we are considering [99].

Sequestering is desirable from the phenomenological point of view, because it may allow for flavour-blind mediation mechanisms such as anomaly-mediated supersymmetry breaking [96,100] to become important. Anomaly-mediated SUSY breaking is an effect which is always present, but is subdominant in generic (non-sequestered) models with respect to gravity mediation. It gives rise to a calculable, flavour-independent, very distinctive pattern of soft terms. However, since minimal anomaly mediation leads to tachyonic sleptons, it is expected that some other mechanism must contribute to mediating SUSY breaking in any fully realistic model.

We will now proceed to investigate SUSY breaking and SUSY breaking mediation in the throat. We will revisit some proposals that have been made to model a SUSY breaking sector different from the original KKLT model; we will then show that in a minimal KKLT-like model SUSY breaking is communicated by both the light moduli and by anomaly mediation [18,101], and subsequently investigate in detail the case of the KS throat as a possibly realistic geometry [21]. It turns out that on a full string background the dominant mechanism may differ significantly from the minimal toy model.

4.1 Non-sequestered uplifts

Recall from Section 2.5 that after flux stabilization of the complex structure moduli and the dilaton, a possible way of stabilizing the Kähler moduli in a type IIB compactification is provided by gaugino condensation in strongly interacting gauge sectors. This mechanism, as we have reviewed, leads to a stable supersymmetric minimum in 4d with a large negative cosmological constant. To obtain a realistic phenomenology, SUSY must be broken and the vacuum energy density must be uplifted to give a metastable de Sitter minimum with a small positive cosmological constant.

The original KKLT proposal of uplifting by adding $\overline{\text{D3}}$-branes, which contribute a piece (2.39) to the scalar potential, explicitly breaks SUSY from the point of view of 4d supergravity. For a proper description of the mediation mechanism and an analysis of the soft terms it is then necessary to resort to nonlinearly realized SUSY [18].

It would be preferrable, however, to use an uplifting sector which breaks SUSY spontaneously, because this allows for greater computational control. The $\overline{\text{D3}}$-brane sector may also be modelled in this way, in the limit of a very steep breaking potential, analogous to the nonlinear sigma model limit for an ordinary spontaneously broken global symmetry. On the other hand, we can take an uplifting sector with spontaneous SUSY breaking seriously in its own right, since it could well appear within the low-energy effective field theory of a string compactification. For instance, in type IIB compactifications it could be realized by branes at singularities in the internal manifold.

We can break SUSY spontaneously by either F-terms or D-terms. An uplift by D-terms was first proposed in [102]: The superpotential piece responsible for stabilizing the Kähler moduli results from gaugino condensation on D7-branes. These branes wrap 4-cycles in the internal manifold, which themselves could contain nontrivial 2-cycles. If the volume of a wrapped 4-cycle is governed by Kähler modulus T, one may now gauge the symmetry $\mathrm{Im}\,T \to \mathrm{Im}\,T + \alpha$, which allows for 2-form gauge flux on the 2-cycle and eventually a nontrivial contribution to the D-term potential. The effect can be derived within the effective supergravity in a manifestly supersymmetric manner, and hence computational control is retained.

However, there are two main problems associated with this proposal, as has been pointed out e.g. in [18, 103, 104]. Firstly, the resulting D-terms cannot be used to uplift a supersymmetric AdS vacuum, because the D-terms generated this way will in the vacuum always be proportional to the F-terms. Secondly, the gauged symmetry which gives rise to the D-terms ceases to be a good symmetry once the T-stabilizing nonperturbative piece in the superpotential is included: Obviously Ae^{-aT} is not invariant under a shift in $\mathrm{Im}\,T$. One then must arrange for some fields that were originally integrated out and hidden in the coefficient A to remain light and to transform under the gauged symmetry, in order to cancel the transformation of T. Thus in the minimal scenario with just a single light field T, a D-term uplift is not possible.

Additional dynamics are therefore needed in any case, and in particular F-terms need to be present for a D-term uplift. This motivates looking for models where the uplift mainly or exclusively comes from the F-term contributions. In a type IIB model, the corresponding chiral superfields in 4d should be thought of as emerging from some D-brane configuration. Without specifying which particular string construction will give rise to such degrees of freedom, we can still write down some more or less generic models in effective field theory.

In [38] (see also [39–44]) the possibility of a non-seqestered hidden sector providing the F-terms was explored. Let us assume the simplest case of a single Kähler modulus T henceforth. The F-term uplift is realized by adding some SUSY breaking fields X_i, for which the effective supergravity is described by a Kähler potential and superpotential of the form

$$\mathcal{K} = -3\log(T+\overline{T})+\Delta\mathcal{K}(X_i,\overline{X}_i), \quad W = W_0+A(X_i)\,e^{-aT}+\Delta W(X_i). \tag{4.1}$$

The T-dependent part is as in the KKLT model, but there will now be couplings to the hidden sector fields X_i. Gravitational couplings will result from the exponentiated Kähler potential, even if A in the superpotential does not depend on the X_i — recall that the F-term potential is given by

$$V = e^{\mathcal{K}} \left(D_I W \, \overline{D_J W} \, \mathcal{K}^{I\bar{J}} - 3|W|^2 \right) . \tag{4.2}$$

The Kähler potential and superpotential for the X_i fields are then chosen such that they constitute one of the usual spontaneous F-term breaking models. A simple possibility with a single field X is the Polonyi model,

$$\Delta\mathcal{K}(X, \overline{X}) = |X|^2, \quad \Delta W(X) = -\mu^2 X, \quad A = \text{const.} \tag{4.3}$$

This model is however not stable at tree-level or in the global limit, and therefore the O'Raifeartaigh model is more appealing: With the SUSY-breaking fields $(X_i) = (X, X_1, X_2)$, take

$$\Delta\mathcal{K}(X_i, \overline{X_i}) = \sum_i |X_i|^2, \quad \Delta W(X_i) = M X_1 X_2 + (\lambda X_1^2 - \mu^2) X, \quad A = \text{const.} \tag{4.4}$$

If we assume $M \gg \mu$, then X_1 and X_2 will be stabilized at zero with a large mass. Upon integrating them out the model becomes similar to the Polonyi model above. However, loop corrections to the Kähler potential lead to an effective quartic stabilizing term, for suitably chosen parameters. Then, for small X, the effective Kähler potential and superpotential are

$$\Delta\mathcal{K}(X, \overline{X}) = |X|^2 - |X|^4/\Lambda^2, \quad \Delta W(X) = -\mu^2 X, \tag{4.5}$$

with $\Lambda^2 = 16\pi^2 M^2/\lambda^4$, up to factors of order one. The Polonyi-KKLT model and the O'KKLT model were analysed in detail in [43] and [44], respectively. It was found that the uplifting dynamics again does not significantly affect the position of the minimum, and that metastable dS vacua can be obtained just as in the case of a $\overline{\text{D3}}$ uplift. The gravitino masses can easily be tuned to be in the TeV range.

We should emphasize again at this point that one of the key requirements for the KKLT construction is a hierarchically small uplift energy density. This may be achieved by fine-tuning parameters in the models above, but then it is not clear why a generic string model should lead to such peculiar parameter values in the effective theory. A naturally small uplift may be achieved by having the SUSY breaking dynamics localized at the bottom of a warped throat as advertised earlier in this chapter, and as in the original KKLT scenario. In that case, however, the Kähler potential and superpotential will in general *not* take the form (4.1), because of the UV-localization of the Kähler modulus T in the underlying 5d or 10d picture. We will return to this issue in the next section, after a brief digression to a different interesting proposal that has been made to realize the hierarchy.

This proposal [42, 43] concerns models in which the uplift is achieved through dynamical supersymmetry breaking. In such models a small SUSY breaking scale is quite natural, so that the hidden sector need not be localized in an internal region with large redshift. Instead the hierarchy is generated dynamically by the renormalization group running of 4d gauge couplings. Traditional models of dynamical SUSY breaking tend to be hard to analyse, somewhat complicated and easily destabilized when coupled to other sectors. These drawbacks are overcome by the recently proposed model of Intriligator, Seiberg and Shih (ISS) [24], which can be neatly embedded in the KKLT model. It needs to be modified to naturally provide a small SUSY breaking scale, but that can be achieved without too much complication. This, however, is a subject which merits a separate discussion of its own. We will return to it, and to the technical details of the ISS model, in Chapter 5, since here we are only concerned with the KKLT embedding.

All that is needed for the subsequent discussion is the effective Kähler potential and superpotential of the ISS-KKLT model at low energies. They are given by

$$W = W_0 + Ae^{-aT} + W_{\rm ISS}, \quad \mathcal{K} = -3\log(T+\overline{T}) + \mathcal{K}_{\rm ISS}, \tag{4.6}$$

with

$$\begin{aligned} W_{\rm ISS} &= h \,\mathrm{tr}\, \tilde{\varphi}\Phi\varphi + \Lambda m \,\mathrm{tr}\,\Phi + W_{\rm np}(\Phi), \\ \mathcal{K}_{\rm ISS} &= |\varphi|^2 + |\tilde{\varphi}|^2 + |\Phi|^2. \end{aligned} \tag{4.7}$$

Here φ^i_c and $\tilde{\varphi}^c_i$ ($i = 1 \dots N_f$; $c = 1 \dots N$) are chiral superfields transforming in the $\mathbf{N}$ and $\overline{\mathbf{N}}$ of an SU(N) gauge symmetry ($N = N_f - N_c$ in the notation of Chapter 5), and in the $\overline{\mathbf{N_f}}$ and $\mathbf{N_f}$ of an SU(N_f) flavour symmetry, respectively. The model requires $N < N_f$. Φ^i_j is an uncharged chiral superfield transforming as $\overline{\mathbf{N_f}} \times \mathbf{N_f}$ under the flavour group. Λ is a dynamically generated scale which can be naturally small compared to the fundamental (e.g. string) scale. $m \ll \Lambda$ may be dynamically generated as well, see Chapter 5, and h is a dimensionless $\mathcal{O}(1)$ coupling. $W_{\rm np}(\Phi)$ is a non-perturbatively generated piece which is subdominant for small field values, i.e. for $\Phi \ll \Lambda$.

The model is such that, in the limit of global SUSY, $W_{\rm ISS}$ and $\mathcal{K}_{\rm ISS}$ alone would lead to a SUSY breaking, loop-stabilized minimum of the potential at $\Phi = 0$. More precisely, neglecting $W_{\rm np}$, it is easy to see that not all F-terms of Φ can vanish since the set of equations

$$0 = F_{\Phi^i_j} = h\,\tilde{\varphi}^c_i\varphi^j_c + \Lambda m\,\delta^j_i \tag{4.8}$$

is overconstrained if $N < N_f$. A more detailed analysis shows that there is a minimum of the potential at $\Phi = 0$, with the non-Goldstone flat directions lifted by the Coleman-Weinberg potential. The vacuum energy is given by

$$\langle V \rangle = \sum_{ij} \left| F_{\Phi^i_j} \right|^2 = (N_f - N) m^2 \Lambda^2, \tag{4.9}$$

This minimum would in fact be the global minimum if $W_{\rm np}(\Phi)$ were zero; taking into account $W_{\rm np}(\Phi)$ it turns out to be only metastable, and there are also supersymmetric minima at large values of Φ.

Embedding this model in supergravity and adding the T modulus, as in (4.6), the leading terms in an expansion of the F-term potential in $m\Lambda/M_4^2$ are

$$V = V_{\rm AdS}(T, \overline{T}) + \frac{1}{(T+\overline{T})^3} V_{\rm ISS}(\varphi, \tilde{\varphi}, \Phi) + \dots \tag{4.10}$$

Here $V_{\rm AdS}$ is the KKLT pre-uplift potential (2.37), and $V_{\rm ISS}$ is derived from $W_{\rm ISS}$ and $\mathcal{K}_{\rm ISS}$. To leading order, one may solve for the KKLT dynamics first, and then consider the added ISS sector as an uplifting sector. The uplifting energy density is approximately, from (4.9),

$$\langle \delta V \rangle = \frac{(N_f - N) m^2 \Lambda^2}{(T+\overline{T})^3}. \tag{4.11}$$

As advertised, it is hierarchically small if m and Λ are hierarchically small. That the latter is natural in certain versions of the ISS model will be shown in detail in Chapter 5. The detailed analysis of [42] shows that in this model neither the metastable vacuum of the uplifting sector nor the metastable vacuum that results from the KKLT embedding are destabilized.

Note that in principle we could have resorted to any other model of dynamical SUSY breaking for the uplifting sector, as elaborated on in [42]. Also, dynamical SUSY breaking models sometimes have an effective low-energy description in terms of simple O'Raifeartaigh-like models, which also justifies the discussion of e.g. the O'KKLT model in this context. Contrariwise, it may be possible to retrofit a given O'Raifeartaigh model by adding an additional sector which provides naturally small mass scales [105].[1]

4.2 Sequestered uplifts and modulus-anomaly mediation

We will now return to the idea of having the hidden sector localized in a warped throat in order to obtain the hierarchy. This is also the situation

[1]It has recently been proposed [106] that in the context of type IIB compactifications, another particularly natural way of obtaining a hierarchically small SUSY breaking scale is by means of D-brane instantons. We will not explain this mechanism in any detail, but it will be used in Section 5.3.

which is perhaps more attractive from the point of view of phenomenology for the reasons we have already mentioned: A sequestered hidden sector allows for a significant anomaly-mediated contribution to the visible sector soft parameters, which may solve the supersymmetric flavour problem in a natural way.

Let us start by rephrasing the KKLT model in the chiral compensator formalism. We will use string-scale units in this and the following sections and suppress factors of order one. It will be convenient to write the supergravity Lagrangian as

$$\mathcal{L} = \int d^4\theta\, \overline{\varphi}\varphi\, \Omega + \left(\int d^2\theta\, \varphi^3\, W + \text{h.c.} \right) , \tag{4.12}$$

with the kinetic function Ω (which is related to the Kähler potential as $\mathcal{K} = -3\log(-\Omega)$) and the chiral compensator $\varphi = 1 + \theta^2 F_\varphi$. For the KKLT model before uplifting, we have

$$\Omega = -(T + \overline{T}), \quad W = W_0 + e^{-T}, \tag{4.13}$$

and thus the scalar potential part of (4.12) is

$$\begin{aligned} \mathcal{L} \supset &- (T + \overline{T})\, |F_\varphi|^2 - (F_T F_{\overline{\varphi}} + \text{h.c.}) \\ &+ \Big[\big(3(W_0 + e^{-T})F_\varphi - e^{-T} F_T\big) + \text{h.c.} \Big]. \end{aligned} \tag{4.14}$$

The resulting equations of motion are (disregarding kinetic terms, since we are interested in constant solutions minimizing the potential from (4.14))

$$F_{\overline{\varphi}} : \quad - (T + \overline{T})F_\varphi - F_T + 3(W_0 + e^{-\overline{T}}) = 0, \tag{4.15}$$

$$F_{\overline{T}} : \quad - F_\varphi - e^{-\overline{T}} = 0, \tag{4.16}$$

$$\overline{T} : \quad - |F_\varphi|^2 - 3e^{-\overline{T}} F_{\overline{\varphi}} + e^{-\overline{T}} F_{\overline{T}} = 0. \tag{4.17}$$

Taking W_0 to be parametrically small (which may be justified by the exponentially large number of flux choices), it is easy to see that the above equations are solved for

$$F_T \sim F_\varphi \sim e^{-T} \sim W_0\,. \tag{4.18}$$

Note that here and below we focus on parametrically small factors $\sim e^{-T}$ but ignore factors $\sim 1/T$ (which are strictly speaking also parametrically small since T is moderately large, but to a much lesser degree). The vacuum energy density is negative and $\sim W_0^2$.

A solution of these equations of motion does not represent a true vacuum of the model unless the curvature scalar (which is multiplied by Ω) vanishes. This shortcoming will now be corrected.

Let us add a SUSY breaking hidden sector in the throat. The size of the hierarchy is characterized by an exponentially small redshift factor $\omega \ll 1$ for the hidden sector region. Generic hidden sector mass scales will be of the order ω in string units. In the KS background, for instance, we would have $\omega \approx z^{2/3}$ with z given by (2.31).

At this point the 5d picture developed in the previous chapter, which captures the essential properties of the throat, turns out to be very useful: We have seen in Section 3.3 that in this picture the universal Kähler modulus T is localized on the UV brane. The same will be true for the visible sector, which we take to reside at some other unspecified place in the UV manifold. On the other hand, the hidden sector at the bottom of the throat is an IR brane field. This means that the visible and hidden sector dynamics are effectively sequestered [96], unless other fields with unsuppressed couplings to both the UV and the IR brane are present. In the current minimal context this is not the case. Sequestering implies that the Kähler potential and superpotential (4.1) are inappropriate, because they include direct couplings between the UV brane field T and the IR brane fields X_i.

Instead, neglecting the visible sector for the moment, the requirement of sequestering leads to a kinetic function and superpotential of the form [81, 96, 107]

$$\begin{aligned} \Omega &= -(T+\overline{T}) + \omega^2 \Delta\Omega(X,\overline{X}), \\ W &= W_0 + e^{-T} + \omega^3 \Delta W(X). \end{aligned} \tag{4.19}$$

We have assumed for simplicity that there is just a single SUSY breaking field X. Furthermore, we have explicitly written the warp factor dependence, so that now the coefficients implicit in ΔW and $\Delta\Omega$ are of order unity. Sequestering forces the terms in X and T to be additive in the kinetic function rather than in the Kähler potential; this is one of the reasons why it is convenient to employ the chiral compensator formalism.

The T dependence is again such that without the non-perturbative exponential term in W, T would be a no-scale field. This is not sufficient to guarantee the form (4.19), since a Kähler-Weyl rescaling before adding $\Delta\Omega$ and ΔW could as well lead to

$$\begin{aligned} \Omega &= -(T+\overline{T})(T\overline{T})^{\alpha} + \omega^2 \Delta\Omega(X,\overline{X}), \\ W &= \left(W_0 + e^{-T}\right) T^{3\alpha} + \omega^3 \Delta W(X). \end{aligned} \tag{4.20}$$

However, α is fixed to be zero if we require the uplift energy density to scale as $(T+\overline{T})^{-2}$ as in (2.39), reproducing (4.19).

Neglecting for the moment the influence of F_φ on the X sector (this will be easy to justify a posteriori), the equation of motion for $F_{\overline{X}}$ reads

$$\omega^2 \Delta\Omega_{X\overline{X}} F_X + \omega^3 \Delta\overline{W}_{\overline{X}} = 0\,, \tag{4.21}$$

where the indices of ΔW and $\Delta\Omega$ denote partial derivatives.

If ΔW and $\Delta\Omega$ are such that, in the absence of warping, F_X would break SUSY at the string scale, then what we obtain upon taking warping into account is

$$F_X \sim \omega. \tag{4.22}$$

The vacuum energy density induced by the X sector is $\sim \omega^4$, with comparable contributions coming from ΔW and $\Delta\Omega$.

Therefore, to uplift the previously found negative vacuum energy density $\sim W_0^2$ to a realistic positive value (i.e. to zero, for all practical purposes), we need $W_0 \sim \omega^2$. Thus there is in fact only one small parameter in the model, which we can choose to be ω. It is also clear that, in this situation, the influence of the X sector on the previously found solution for F_φ (and hence on T and F_T) is of higher order in ω. Thus, (4.15) - (4.17) continue to be the right equations to solve. The X sector simply adds the necessary positive vacuum energy to promote the solutions of these equations to a physical vacuum with

$$F_T \sim F_\varphi \sim W_0 \sim \omega^2 \qquad \text{and} \qquad F_X \sim \omega\,. \tag{4.23}$$

We see that the vacuum F terms of the physical modulus T and the chiral compensator φ are of the same order of magnitude. Roughly speaking, the former will generically give the leading contribution to gravity-mediated SUSY breaking once a UV-localized visible sector is introduced, and the latter will be responsible for anomaly mediation [18]. Recall that, due to sequestering, F_X has no direct effect on soft terms in the visible sector.

A simple example for the X sector is realized by the model

$$\begin{aligned} \Omega &= -(T+\overline{T}) + \omega^2(|X|^2 - |X|^4), \\ W &= W_0 + e^{-T} + \omega^3 X. \end{aligned} \tag{4.24}$$

The part of the Lagrangian (4.12) relevant for the potential is now

$$\begin{aligned} \mathcal{L} \supset\ & -|F_\varphi|^2\,(T+\overline{T}) - (F_T F_{\overline{\varphi}} + \text{h.c.}) + \omega^2\,|F_\varphi|^2\,(|X|^2 - |X|^4) \\ & + \omega^2\,|F_X|^2 + \omega^2\left(F_{\overline{\varphi}} F_X \overline{X}\,(1-2|X|^2) + \text{h.c.}\right) - 4\omega^2\,|F_X|^2\,|X|^2 \\ & + \Big[\left(3F_\varphi(W_0 + e^{-T} + \omega^3 X) - F_T\,e^{-T} + \omega^3\,F_X\right) + \text{h.c.}\Big]. \end{aligned} \tag{4.25}$$

The equations of motion read (note that those for $F_{\overline{T}}$ and $\overline{T}$ are unchanged from (4.16) and (4.17))

$$\begin{aligned}
F_{\overline{\varphi}} :\quad & F_\varphi\,(T+\overline{T}) + F_T - \omega^2\,F_\varphi\left(|X|^2 - |X|^4\right) \\
& -\omega^2 F_X \overline{X}(1-2|X|^2) - 3W_0 - 3e^{-\overline{T}} - 3\omega^3\overline{X} = 0, && (4.26)\\
F_{\overline{T}} :\quad & F_\varphi + e^{-\overline{T}} = 0, && (4.27)\\
F_{\overline{X}} :\quad & \omega^2\,F_X + \omega^2\,F_\varphi X(1-2|X|^2) - 4\omega^2\,F_X\,|X|^2 + \omega^3 = 0, && (4.28)\\
T :\quad & |F_\varphi|^2 + 3F_\varphi\,e^{-T} - F_T\,e^{-T} = 0, && (4.29)\\
X :\quad & \omega^2\,|F_\varphi|^2\overline{X}(1-2|X|^2) + \omega^2\,F_\varphi F_{\overline{X}}(1-4|X|^2) \\
& -2\omega^2\,F_{\overline{\varphi}}F_X\overline{X}^2 - 4\omega^2\,|F_X|^2\overline{X} + 3\omega^3\,F_\varphi = 0. && (4.30)
\end{aligned}$$

As before, from (4.27) and the condition that the pre-uplift superpotential in the vacuum should be $\sim \omega^2$, one can immediately see that $F_\varphi \sim \omega^2$. From (4.29) it follows that $F_T \sim \omega^2$, and from (4.28) we can deduce that $F_X \sim \omega$. From (4.30) we obtain $X \sim \omega$.

4.3 Vector mediation

In the preceding section we have presented a minimal scenario for SUSY breaking mediation. In particular we have assumed that the only potentially relevant field for gravity mediation is the Kähler modulus T. However, this assumption is not correct in general. Despite the fact that all other fields will have acquired string-scale masses, there may be a sizeable effect from gravity mediation due to 4d vector multiplets. Its contribution to the visible sector soft paramters will be no more suppressed in powers of ω than the mixed modulus-anomaly mediated contributions. By contrast, contributions from string-scale massive chiral multiplets are truly suppressed to a higher degree.

Let us now study how sequestering is affected by a massive vector superfield in the throat. Whether or not there is such a field available in a given model depends on the compactification background. In the case that the throat admits a continuous isometry, this symmetry will become a gauge symmetry in the effective 4d field theory. If the isometry is not a symmetry of the entire internal manifold (in particular, of the UV end), this gauge symmetry is broken. Since we are imagining the UV end of the throat to be embedded in a Calabi-Yau manifold, which does not admit any isometries, this will generically be the case in realistic constructions. More precisely, we take the gauge symmetry to be nonlinearly realized at the UV end. The effect on the 4d vector multiplets is that they acquire a string-scale mass, which from the point of view of the 5d Randall-Sundrum-like model can be ascribed to a UV-brane mass operator.

The Klebanov-Strassler throat, for instance, has isometry group SO(4). Hence the 5d solution should contain six massless vector fields as discussed in Section 3.5. They will give rise to 4d string-scale massive vector multiplets due to the embedding of the throat in a full Calabi-Yau compactification.

Independently of the UV-scale breaking of the SO(4) gauge symmetry, we assume that the SUSY breaking sector at the bottom of the throat by itself also breaks this symmetry. In particular, a $\overline{\text{D3}}$ brane at the bottom of the throat already breaks part of the isometry. Clearly, as far as the mass of the 4d vector states is concerned, this IR-scale breaking cannot compete with the UV-scale breaking. We assume that the hidden sector breaks the isometry explicitly, since otherwise there will be no additional contribution to gravity mediation. (In the case of spontaneous breaking, the estimates of the current section will remain technically correct, but the soft parameters will be unaffected, as we will explain in Section 4.4.)

5d massive vector fields in the KKLT scenario have also been discussed in [20], where the effects of anomalous U(1) gauge groups on SUSY breaking mediation was investigated.

To estimate the dominant SUSY breaking effects, we introduce a single 4d vector superfield V (although the actual symmetry is non-abelian and several such fields are expected). Assume there is a term $\omega^2 V|X|^2$ in the kinetic function. This corresponds to the leading higher-dimensional operator that couples V to X as the dominant generic correction to a canonical kinetic function for X. Such a term is obviously not gauge invariant if X does not transform under the gauge symmetry, and hence leads to explicit gauge symmetry breaking.

V has the following component expansion:

$$V = C + \theta\sigma^\mu\overline{\theta}A_\mu + \frac{1}{2}\left(F_V\theta\theta + \text{ h.c.}\right) + \frac{1}{2}\theta\theta\overline{\theta}\overline{\theta}\left(D + \frac{1}{2}\partial^2 C\right) + \text{ fermions.} \tag{4.31}$$

Here F_V is complex, while A_μ, C, D are real. The UV-brane symmetry breaking (or non-linear realization of the gauge symmetry) is modelled by simply giving this vector superfield a string-scale mass term. A massive vector superfield can give rise to soft terms in two ways: it may develop F or D terms in the vacuum.[2]

The dominant effect on soft terms is easy to guess: Focus on the term CD (coming from the superfield mass term $\sim V^2$) and the term $\omega^2 C|F_X|^2$ (coming from the coupling $\omega^2 V|X|^2$). Varying these terms with respect to C one immediately finds

$$D \sim \omega^2|F_X|^2 \sim \omega^4\,, \tag{4.32}$$

[2]Note that for a massless vector superfield V, F terms are unphysical because the θ^2-components of V can be gauged away using Wess-Zumino gauge. This is no longer the case when V is massive.

which induces scalar masses $\sim \omega^4$ for standard model fields Q in the visible sector if there exists a coupling $V|Q|^2$ in the Kähler potential. Such a coupling is analogous as the one we have proposed for the hidden sector, representing the leading correction due to V to a canonical Kähler potential for Q. We note that the D term contribution to the vacuum energy density is negligible compared with $|F_X|^2$, which is responsible for the uplift.

To derive the above in more detail, we start with the Lagrangian

$$\begin{aligned}\mathcal{L} = &\int d^4\theta\, \overline{\varphi}\varphi \left[\Omega(T,\overline{T}) + \omega^2 \Delta\Omega(X,\overline{X},V) + V^2\right] \\ &+ \left(\int d^2\theta \left[\varphi^3 \left\{W(T) + \omega^3 \Delta W(X)\right\} + \frac{1}{4}\mathcal{W}^\alpha \mathcal{W}_\alpha\right] + \text{h.c.}\right),\end{aligned} \tag{4.33}$$

where $\mathcal{W}_\alpha$ is the field strength chiral superfield corresponding to V. The X dynamics will not be significantly disturbed since all expectation values of the components of V will be of higher order in ω, as is easily checked on the solution a posteriori. The most relevant term in the Lagrangian are now the mass term for V and the gauge-kinetic term, as well as the terms

$$\Omega = -(T+\overline{T}) + \omega^2\,(1+V)|X|^2 + \dots \qquad W = W_0 + e^{-T} + \dots \tag{4.34}$$

(as before we suppress any coefficients that are generically of order one). In components, the mass term contributes

$$\varphi\overline{\varphi}V^2|_{\theta^4} = CD + |F_V|^2 + A_\mu A^\mu + C\,(F_\varphi F_{\overline{V}} + \text{ h.c.}) + C^2|F_\varphi|^2, \tag{4.35}$$

and the gauge kinetic term gives

$$\mathcal{W}^\alpha \mathcal{W}_\alpha|_{\theta^2} = -\frac{1}{2}F_{\mu\nu}F^{\mu\nu} + D^2\,. \tag{4.36}$$

From the coupling of the gauge field to the SUSY breaking field X we get

$$\begin{aligned}\overline{\varphi}\varphi V\,|X|^2|_{\theta^4} =&|F_\varphi|^2 C\,|X|^2 + F_{\overline{\varphi}} C F_X \overline{X} + \text{ h.c.} + \frac{1}{2}F_{\overline{\varphi}} F_V\,|X|^2 + \text{ h.c.} \\ &+ C\,|F_X|^2 + \frac{1}{2}F_{\overline{V}} F_X \overline{X} + \text{ h.c.} + \frac{1}{2}D\,|X|^2.\end{aligned} \tag{4.37}$$

The equations of motion for the bosonic components of V are

$$\begin{aligned} C: \quad & D + (F_\varphi F_{\overline{V}} + \text{ h.c.}) + 2C\,|F_\varphi|^2 \\ & + \omega^2\left(|F_\varphi|^2|X|^2 + F_{\overline{\varphi}} F_X \overline{X} + \text{ h.c.} + |F_X|^2\right) = 0, \end{aligned} \tag{4.38}$$

$$F_{\overline{V}}: \quad F_V + CF_\varphi + \frac{1}{2}\omega^2\left(F_\varphi|X|^2 + F_X\overline{X}\right) = 0, \tag{4.39}$$

$$D: \quad C + D + \frac{1}{2}\omega^2|X|^2 = 0, \tag{4.40}$$

giving

$$D \sim \omega^4, \quad F_V \sim \omega^4, \quad C \lesssim \omega^4. \tag{4.41}$$

F_V is irrelevant for SUSY breaking mediation, because it is subdominant with respect to the other F-terms. D, however, will contribute significantly, because a possible coupling $\sim V|Q|^2$ to the visible sector will clearly induce soft scalar masses $m^2 \sim D \sim \omega^4$. This is just the same order of magnitude as we get from mixed modulus-anomaly mediation, so 'vector mediation' will compete with these effects. Of course, this can also be easily seen by focusing on the couplings $\sim \omega^2 V|X|^2$ and $\sim V|Q|^2$ and integrating out the heavy vector. The induced operator $\omega^2\,|X|^2\,|Q|^2$ provides soft masses $m^2 \sim \omega^2|F_X|^2 \sim \omega^4$.

4.4 Vector mediation and spontaneous gauge symmetry breaking

In this section we explain why vector mediation requires the hidden sector to break the gauge symmetry explicitly rather than spontaneously. We will demonstrate that, when a vector superfield acquires a mass (as e.g. the gauge field of a nonlinearly realized or spontaneously broken gauge symmetry), the D-term in the vacuum will inevitably vanish if all of its other couplings respect the linearly realized gauge symmetry. This can already be seen on the level of rigid SUSY.

The simplest way of coupling our bulk vector field to the SUSY breaking sector would of course be to have the SUSY-breaking fields charged under the gauge symmetry. This is obviously not possible with the hidden sector which we presented in Section 4.2, since we need several chiral superfields to write down a gauge-invariant superpotential. In the minimal case, there would be just two such fields X_1 and X_2 with equal and opposite charge.

As a side remark, it is not entirely straightforward to build a model in which a charged chiral superfield acquires an F-term in the vacuum. Let us assume that the superpotential is analytic around zero. By gauge invariance it cannot contain linear terms in the charged fields, and therefore there is always a supersymmetric vacuum at the origin. Thus, to have the supersymmetry-breaking fields charged under a gauge symmetry, one should look at models with local (metastable) minima. Since these may well describe a realistic physical system, this is however no serious obstruction to building a model.

Now consider a gauge superfield V (with gauge group U(1) for simplicity) coupled to charged chiral superfields. Turning on a mass term for V, the longditudinal polarization of the gauge field will become a dynamical degree of freedom. It can described by a chiral superfield U, the "eaten Goldstone superfield". It now turns out that the vacuum expectation value of U will eventually adjust such that $D = 0$ in the vacuum. This is most easily seen as follows: In our present rigid SUSY framework, the Lagrangian is

$$\begin{aligned}\mathcal{L} = &\int d^4\theta\, \mathcal{K}\left(X_i, e^{q_i V}\overline{X}_i\right) + \int d^4\theta\, V^2 \\ &+ \int d^2\theta\, W(X_i) + \text{h.c.} + \frac{1}{4}\int d^2\theta\, W_\alpha W^\alpha + \text{h.c.}\end{aligned} \tag{4.42}$$

We can now make a superfield redefinition, writing $V = \tilde{V} + U + \overline{U}$, where $\tilde{V}$ is in Wess-Zumino gauge and U is chiral. Further defining

$$Y_i = X_i e^{q_i U}, \tag{4.43}$$

the Lagrangian becomes (note that by gauge invariance neither the superpotential nor the gauge kinetic term depend on U)

$$\begin{aligned}\mathcal{L} = &\mathcal{L}_m(U, \tilde{V}) + \int d^4\theta\, \mathcal{K}\left(Y_i, e^{q_i \tilde{V}}\overline{Y}_i\right) \\ &+ \int d^2\theta\, W(Y_i) + \text{h.c.} + \frac{1}{4}\int d^2\theta\, W_\alpha W^\alpha + \text{h.c.}\end{aligned} \tag{4.44}$$

Here $\mathcal{L}_m$ denotes the gauge field mass term Lagrangian,

$$\mathcal{L}_m(U, \tilde{V}) = \int d^4\theta\, (\tilde{V} + U + \overline{U})^2, \tag{4.45}$$

containing the D-term of V (which is obviously also the D-term of $\tilde{V}$) as

$$\mathcal{L}_m(U, \tilde{V}) \supset 2\, D(U + \overline{U}). \tag{4.46}$$

The equation of motion for D is thus

$$D = -2\,(U + \overline{U}) + \frac{\partial}{\partial D}\int d^4\theta\, \mathcal{K}, \tag{4.47}$$

and since U does not appear anywhere except in the gauge field mass term $\mathcal{L}_m$ (and in particular does not have an F-term potential of its own), it can adjust its expectation value to guarantee $D = 0$. This is of course the vacuum configuration, by positivity of the D-term potential.

Let us now re-examine the situation we have envisaged for vector mediation: our throat vector fields are coupled, at the UV end of the throat, to fields which break the U(1) via string-scale dynamics, thus providing a mass term for V. Vector mediation now relies on the presence of a D-term for V induced by couplings to the hidden sector. We must conclude that these couplings cannot be gauge couplings. Thus there will be no vector mediation unless the hidden sector fields break the gauge symmetry *explicitly* rather than spontaneously (since there are no possible couplings except standard gauge couplings that are compatible with the symmetry).

Nevertheless we emphasize that, in the case of explicit breaking, vector mediation will in general take effect: It is easily checked that, for instance, with the toy model for the hidden sector which we gave in the previous section, the D-term in the vacuum is nonzero.

4.5 Other gravity-mediated contributions

For completeness, we will now derive that possible contributions to gravity mediation from string-scale massive chiral superfields are suppressed to a higher degree, and thus harmless to sequestering. We will work in supergravity in the chiral compensator formalism as in Section 4.3, and again use string-scale units.

Consider a chiral superfield Y with a string-scale mass term, such as might be produced by flux stabilization. We allow for direct couplings of Y to both the hidden and the visible sector. Let us estimate the F term of Y in the vacuum, since it may give SUSY breaking soft masses to visible sector fields via terms like $|Y|^2|Q|^2$.

Suppressing $\mathcal{O}(1)$ coefficients, the dominant terms in the kinetic function and superpotential are

$$\begin{aligned} \Omega &= -(T+\overline{T}) + |Y|^2 + \omega^2\,(X\overline{Y} + \text{h.c.}) + \dots \\ W &= W_0 + Y^2 + (1+Y)e^{-T} + \omega^3\,XY + \dots \end{aligned} \quad . \tag{4.48}$$

Since we imagine that Y contains fields propagating in the throat, we have allowed for the strongest possible couplings to the X sector. Furthermore, since Y does not represent a modulus of the fluxed Calabi-Yau, we have allowed for an unsuppressed mass term $\sim Y^2$ but excluded any leading-order linear term in Y or a mixing of Y and T. However, once non-perturbative effects (e.g. gaugino condensation) are incorporated, the clear separation between Y and T may be blurred, which motivates us to include the term $\sim Ye^{-T}$, as an example for such effects.[3] Note that we could have replaced $|Y|^2$ by $(T+\overline{T})|Y|^2$ without affecting the results of the following analysis.

Since F_φ would by itself not generate a non-zero F_Y, we will neglect its influence for the moment. Afterwards we will show that the backreaction of Y on F_φ is indeed negligible, hence this ansatz is fully self-consistent.

[3]This is a slight generalization of the otherwise similar analysis of [20]

We obtain the following Y- and F_Y-dependent terms in the bosonic Lagrangian:

$$\mathcal{L} \supset |F_Y|^2 + \omega^2 \left(F_X F_{\overline{Y}} + \text{h.c.}\right) + \left[2YF_Y + F_Y e^{-T} - Y e^{-T} F_T + \text{h.c.}\right] \\ + \omega^3 (F_X Y + X F_Y + \text{h.c.})\,. \tag{4.49}$$

Recall that $e^{-T} \sim \omega^2$ by assumption. This leads to the equation of motion for Y

$$2F_Y - \omega^2 F_T + \omega^3 F_X = 0\,, \tag{4.50}$$

hence

$$F_Y \sim \omega^2 F_T \sim \omega^3 F_X \sim \omega^4. \tag{4.51}$$

The equation of motion for $F_{\overline{Y}}$ reduces to

$$F_Y + \omega^2 F_X + 2\overline{Y} + \omega^2 + \omega^3 \overline{X} = 0\,, \tag{4.52}$$

thus

$$Y \sim \omega^2. \tag{4.53}$$

To ensure that this estimate is correct, we now need to prove that there are no contributions to F_φ and F_T of order ω^2. This is fairly obvious, however, since what we are adding to the Lagrangian by including Y is, in the vacuum, suppressed by sufficiently high powers of ω. For example, we can check that Eq. (4.16), the equation of motion for $F_{\overline{T}}$, now becomes

$$F_\varphi + (1+Y)e^{-T} = 0\,, \tag{4.54}$$

inducing a negligible correction to $F_\varphi \sim \omega^2$. (This remains correct if $|Y|^2$ is replaced by $(T+\overline{T})|Y|^2$. Similarly, it is easy to check that the vacuum values of T are not affected at leading order in ω.

In summary, we have seen that throat fields which are described by heavy chiral superfields in the 4d effective theory cannot contribute sizeably to SUSY breaking mediation because their F terms are always subdominant compared to F_φ and F_T.

Chapter 5

Hierarchies from metastable dynamical SUSY breaking

To naturally generate a large hierarchy of scales in a model of particle physics, there seem to be rather few conceptually distinct mechanisms available. One possibility is to use warped extra dimensions, which we have discussed at length in the context of type IIB superstring compactifications. A second, somewhat more long-standing, approach is based on dimensional transmutation: An asymptotically free gauge theory with a moderately small gauge coupling at some fundamental energy scale will become strongly coupled at an exponentially smaller scale and may undergo a phase transition.

By the AdS/CFT correspondence, these two mechanisms can in fact be dual to each other. This has been investigated in great detail for the KS solution: The gauge theory dual of the warped deformed conifold exhibits confinement and chiral symmetry breaking, at a scale which in the gravity picture corresponds to the tip of the throat. A large hierarchy of scales in the gravity theory, generated from warped extra dimensions, translates into a large hierarchy of scales in the gauge theory, dynamically generated by renormalization group running.

In this chapter we will investigate certain models implementing the second mechanism of generating large hierarchies, namely strong gauge dynamics. We will mainly focus on the field-theoretic aspects first, but in order to establish the connection to type IIB superstring theory, we will finally also review a related D-brane construction. The model we will mainly be concerned with is the Intriligator-Seiberg-Shih (ISS) model of metastable dynamical supersymmetry breaking [24]. We have already seen one of its applications in Chapter 4, as a possible uplifting sector for KKLT-type models. However, the ISS model per se is unrelated to string compactifications, and can be used for many other purposes just as well. An obvious application would be to explain the electroweak hierarchy of particle physics by taking the ISS model as a hidden supersymmetry breaking sector. It could then be coupled via messenger fields to a SUSY version of the Standard Model, or a subgroup of its global symmetry group could be gauged and identified with part or all of the Standard Model gauge group. Models along these lines have been constructed e.g. in [32, 108–112].

The condition that SUSY should be dynamically broken in the vacuum of a supersymmetric gauge theory is very restrictive, and thus traditional models of dynamical SUSY breaking tend to be quite contrived. Relaxing this condition by allowing for SUSY to be broken only in a metastable (sufficiently long-lived) state, there are many more and much simpler possibilities. The ISS model, in particular, is very simple, and easily coupled to messenger fields. It is also straightforward to deform it by operators which explicitly break the approximate R-symmetry in the metastable minimum, as required for phenomenology.

The ISS model in its simplest form does not generate all of its small mass scales dynamically. We will, however, show in detail how exponentially small scales can arise naturally when embedding the ISS model in a renormalizable field theory [25]. We will also point out their possible origin in superstring constructions.

We will make extensive use of the phase structure of $\mathcal{N} = 1$ supersymmetric QCD, and in particular of Seiberg duality. Some useful facts about $\mathcal{N} = 1$ SQCD are summarized in Appendix D.

5.1 The ISS model

Let us briefly review the analysis of ISS [24]. Consider $N = 1$ rigidly supersymmetric QCD with N_c colours and N_f flavours of massive quarks and antiquarks $q^i, \tilde{q}_i$ ($i = 1 \dots N_f$). Choose N_f and N_c such that $3N_c/2 > N_f > N_c$, which is the so-called "free magnetic range". Let us take the quark masses to be equal for simplicity and denote them by m (non-degenerate quark masses are possible in this model and will in fact be encountered in Section 5.3). The $SU(N_f)_L \times SU(N_f)_R$ flavour symmetry of the massless theory is then broken to a diagonal $SU(N_f)$, so that we have the following quantum numbers:

	SU(N_c)	SU(N_f) (global)
q	$\mathbf{N_c}$	$\mathbf{N_f}$
$\tilde{q}$	$\overline{\mathbf{N}}_\mathbf{c}$	$\overline{\mathbf{N}}_\mathbf{f}$

Assume also that $m \ll \Lambda$, where Λ is the strong-coupling scale of the gauge theory. The theory is asymptotically free. It has a dual description [59], not in the sense of gauge-gravity duality but rather in the sense of electric-magnetic duality, on scales much lower than Λ in terms of an IR free $SU(N_f - N_c)$ "magnetic" gauge theory.

The degrees of freedom of the dual theory are N_f dual quarks and anti-quarks $\varphi^i, \tilde{\varphi}_i$ and N_f^2 uncharged mesons Φ^i_j, transforming as

	$\mathrm{SU}(N_f - N_c)$	$\mathrm{SU}(N_f)$ (global)
φ	$\mathbf{N_f - N_c}$	$\mathbf{N_f}$
$\tilde{\varphi}$	$\overline{\mathbf{N_f - N_c}}$	$\overline{\mathbf{N_f}}$
Φ	$\mathbf{1}$	$\mathbf{N_f} \times \overline{\mathbf{N_f}}$

Near the origin of field space the dual Kähler potential is smooth and hence can be taken to be canonical to leading order (up to normalization factors of order one, which we drop). The infrared superpotential is, up to $\mathcal{O}(1)$ coefficients,

$$W = \tilde{\varphi}^c_i \Phi^i_j \varphi^j_c - m\Lambda \Phi^i_i + \left(\frac{\det \Phi}{\Lambda^{3N_c - 2N_f}} \right)^{\frac{1}{N_f - N_c}} \tag{5.1}$$

with $c = 1 \ldots N_f - N_c$, $i, j = 1 \ldots N_f$. At small field values, we can neglect the last term in W because of the Λ-suppression; then the F-terms of Φ are

$$F_{\Phi^i_j} = \tilde{\varphi}^c_i \varphi^j_c - m\Lambda \delta^j_i. \tag{5.2}$$

They cannot all vanish because $\tilde{\varphi}^c_i \varphi^j_c$ has rank $N_f - N_c$, whereas δ^j_i has rank N_f. It turns out that there is a SUSY breaking local minimum, the ISS vacuum, at

$$\Phi = 0, \qquad (\tilde{\varphi}^c_i) = (\varphi^j_c)^T = \begin{pmatrix} m \mathbb{1}_{N_f - N_c} \\ 0 \end{pmatrix}. \tag{5.3}$$

Here $\mathbb{1}_{N_f - N_c}$ denotes the $(N_f - N_c) \times (N_f - N_c)$ unit matrix. At tree-level, the potential still has several flat directions. Those that correspond to Goldstone directions from spontaneously broken global symmetries are unaffected by quantum corrections. The others are lifted by the one-loop Coleman-Weinberg potential, such that the ISS vacuum is indeed locally stable. In addition to the ISS vacuum there are supersymmetric vacua, which are found by taking into account also the determinant term in (5.1). However, they are well separated in field space from the ISS vacuum if m/Λ is sufficiently small, hence the ISS vacuum can be very long-lived. More precisely, in [24] the bounce action for overcoming the tunneling barrier and decaying into the proper vacuum was estimated to be

$$S_{\text{bounce}} \approx \left(\frac{\Lambda}{m} \right)^{\frac{6N_c - 4N_f}{N_c}}, \tag{5.4}$$

which shows that for $m \ll \Lambda$ the lifetime of the ISS vacuum is parametrically large.

5.2 Retrofitting the ISS model

As it stands, the ISS model does not yet provide a fully natural explanation for the small SUSY breaking scale. The reason is that, while the exponentially small scale Λ is indeed generated dynamically, the small dimensionful parameter m has been put in by hand. The SUSY breaking scale is then given by $\langle F \rangle \sim m\Lambda$. To obtain a fully natural model, we should also find a mechanism that explains the smallness of m. Extending SUSY breaking models by additional sectors in order to dynamically generate small dimensionful parameters has been dubbed "retrofitting" in [105].

A possible retrofitting mechanism for the modified ISS model of [109] has been put forward in [113]. The idea is to add an auxiliary sector which exhibits strong dynamics itself, and couple it to the ISS model via higher-dimensional operators. Specifically, if one takes as the auxiliary sector $SU(N_c')$ pure super-Yang-Mills theory, this theory will undergo gaugino condensation at a dynamically generated scale Λ'. The dimension 6 coupling between the auxiliary field strength W_α' and the ISS quarks $q, \tilde{q}$,

$$\mathcal{L} \supset \int d^2\theta \, \frac{\mathrm{tr}\, q\tilde{q}}{M^{*2}} \mathrm{tr}\, W_\alpha' W'^\alpha + \text{ h.c.} \tag{5.5}$$

leads to an effective ISS quark mass term $m \sim \Lambda'^3/M^{*2}$ at energies below Λ'. Here M^* is a fundamental scale at which the theory must be UV-completed, such as the GUT or Planck scale. In this model, $m \ll \Lambda$ can be easily accomplished, and thus the ISS analysis applies.

The drawback of this procedure is that one needs to rely on higher-dimensional operators, and hence indirectly on the physics of an unspecified UV completion. (Of course, when obtaining the model from string theory as we will in Section 5.3, the coefficients of these operators are calculable in principle, but one still does depend on the particular UV completion chosen.) When extending the model, it would certainly be desirable not to spoil one of its main advantages, the existence of a well-defined UV limit in terms of an asymptotically free electric gauge theory.

Let us therefore present another possibility to retrofit the ISS model in a fully renormalizable way. Again, we will introduce an auxiliary sector which generates a small mass scale dynamically, but now also an additional gauge singlet field which couples to both sectors and whose expectation value will ultimately become the ISS quark mass. This model was constructed in [25]; similar models were later considered in [114].

The auxiliary sector is now $\mathrm{SU}(N_c')$ SQCD with N_f' flavours of massless quarks and antiquarks $Q, \tilde{Q}$, where $N_c' > N_f'$. Couple this theory to an additional singlet S with tree-level superpotential

$$W_{\text{tree}} = \lambda' S \mathrm{tr}\, Q\tilde{Q} - \kappa S^3. \tag{5.6}$$

In the quantum theory, an additional contribution to the superpotential is generated nonperturbatively [115], which becomes relevant in the infrared:

$$W_{\rm np} = a\left(\frac{\Lambda'^{3N_c'-2N_f'}}{\det Q\tilde{Q}}\right)^{\frac{1}{N_c'-N_f'}} . \tag{5.7}$$

Here Λ' is the strong-coupling scale of the gauge theory, and a is a number of order one whose precise value is renormalization-scheme dependent. In [116] it was shown that by holomorphy and symmetry the exact low-energy effective superpotential is $W = W_{\rm tree} + W_{\rm np}$, in a range of parameters where S is the only light degree of freedom and the quarks are integrated out. It can be shown that $W = W_{\rm tree} + W_{\rm np}$ is indeed exact even in the general case (see [25]).

To analyse the IR behaviour of the theory, we introduce the meson fields

$$M^i_j = \frac{1}{\Lambda'}Q^i\tilde{Q}_j \tag{5.8}$$

(with a trace over colour indices implied), normalized by the $1/\Lambda'$ factor to have canonical dimension. In terms of the mesons and the singlet, the exact low-energy effective superpotential is then

$$W_{\rm eff} = \lambda'\Lambda' S {\rm tr}\, M - \kappa S^3 + a\left(\frac{\Lambda'^{3N_c'-2N_f'}}{\det M}\right)^{\frac{1}{N_c'-N_f'}} . \tag{5.9}$$

The equations for supersymmetric vacua,

$$\begin{aligned} 0 &= \lambda'\Lambda'{\rm tr}\, M - 3\kappa S^2, \\ 0 &= \lambda'\Lambda' S\,\delta^i_j - \frac{a}{N_c'-N_f'}\left(\frac{\Lambda'^{3N_c'-2N_f'}}{\det M}\right)^{\frac{1}{N_c'-N_f'}}\left(M^{-1}\right)^i_j , \end{aligned} \tag{5.10}$$

are solved by

$$\begin{aligned} S &= b\Lambda'\, e^{\frac{2\pi i n}{3N_c'-N_f'}}, \\ M &= c\Lambda'\, e^{\frac{4\pi i n}{3N_c'-N_f'}}\,\mathbb{1}_{N_f}, \end{aligned} \qquad (0 \le n < 3N_c'-N_f'), \tag{5.11}$$

where b and c are numerical constants given by

$$\begin{aligned} b &= \left[\left(\frac{N_f'}{3\kappa}\right)^{N_c'}(\lambda')^{N_f'}\left(\frac{a}{N_c'-N_f'}\right)^{N_c'-N_f'}\right]^{\frac{1}{3N_c'-N_f'}} , \\ c &= \left[\frac{3\kappa}{(\lambda')^3 N_f'}\left(\frac{a}{N_c'-N_f'}\right)^2\right]^{\frac{N_c'-N_f'}{3N_c'-N_f'}} . \end{aligned} \tag{5.12}$$

For simplicity, in the following we choose the couplings λ' and κ such that $b = c = 1$.

We now couple this model to an ISS sector, with the ISS quark mass coming from the expectation value of S. The fields and their quantum numbers are summarized in the following table.

	SU(N_c)	SU(N_c')	SU(N_f) (global)	SU(N_f') (global)
q	$\mathbf{N_c}$	$\mathbf{1}$	$\mathbf{N_f}$	$\mathbf{1}$
$\tilde{q}$	$\overline{\mathbf{N_c}}$	$\mathbf{1}$	$\overline{\mathbf{N_f}}$	$\mathbf{1}$
Q	$\mathbf{1}$	$\mathbf{N_c}'$	$\mathbf{1}$	$\mathbf{N_f'}$
$\tilde{Q}$	$\mathbf{1}$	$\overline{\mathbf{N}}_{\mathbf{c}}'$	$\mathbf{1}$	$\overline{\mathbf{N}}_{\mathbf{f}}'$
S	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$

The combined superpotential in the UV is

$$W = -\lambda\, S \mathrm{tr}\, q\tilde{q} + \lambda'\, S \mathrm{tr}\, Q\tilde{Q} - \kappa S^3. \tag{5.13}$$

We have deliberately omitted all possible operators with dimensionful couplings here: No scales are introduced by hand. The absence of linear and quadratic terms in W can be further justified by imposing an obvious discrete $\mathbb{Z}_3$ symmetry acting on the chiral superfields, which will be spontaneously broken by nonperturbative effects.

Assume now that $\lambda \ll 1$, such that also $\lambda\Lambda \ll \Lambda'$ and $\lambda\Lambda' \ll \Lambda$ (this can of course be achieved by, for instance, choosing the numbers of colours and flavours and the gauge couplings at the renormalization scale such that $\Lambda \approx \Lambda'$, and then setting $\lambda \ll 1$). Let us emphasize that this does not constitute an inacceptable fine-tuning: Firstly, it concerns a dimensionless parameter only, and secondly, we will see that λ of the order of a percent is sufficiently small for our purposes. The hierarchy between the fundamental and the SUSY breakings scale, comprising many orders of magnitude, is still generated dynamically.

The resulting model has various effective descriptions at different energy scales. In the far UV the appropriate superpotential is (5.13). The ISS and auxiliary sector then have effective descriptions at scales below their respective strong coupling scales Λ and Λ' (either of which can be the higher one): At scales around Λ we should pass to the Seiberg dual of the q sector, replacing

$$-\lambda S \mathrm{tr}\, q\tilde{q} \;\rightarrow\; -\lambda\Lambda S \mathrm{tr}\, \Phi + \mathrm{tr}\, \tilde{\varphi}\Phi\varphi + \left(\frac{\det \Phi}{\Lambda^{3N_c-2N_f}}\right)^{\frac{1}{N_f-N_c}}. \tag{5.14}$$

Here we anticipate that S, which is a dynamical field up to now, will eventually acquire an expectation value, such that the $\lambda S \mathrm{tr}\, q\tilde{q}$ term will become an ISS quark mass term. At scales below Λ' the Q sector together with S can be described by the exact superpotential (5.9), with the coupling to the q sector viewed as a small perturbation. We should therefore replace

$$\lambda' \, S \mathrm{tr}\, Q\tilde{Q} - \kappa S^3 \;\; \rightarrow \;\; \lambda' \Lambda' S \mathrm{tr}\, M - \kappa S^3 + a \left(\frac{\Lambda'^{3N_c'-2N_f'}}{\det M} \right)^{\frac{1}{N_c'-N_f'}} . \quad (5.15)$$

At scales much below Λ', M and S are massive and should be integrated out. Taking for definiteness the phases in (5.11) to vanish, we obtain

$$\langle S \rangle = \Lambda' \left[1 + \mathcal{O}\left(\frac{\lambda^2 \Lambda^2}{(\Lambda')^2} \right) \right] . \quad (5.16)$$

The correction terms of higher order in $\lambda\Lambda/\Lambda'$ are small by assumption.

In the IR, the only light degrees of freedom remaining are now the ISS mesons and dual quarks, whose interactions at low energies are governed by the superpotential (dropping again, as in Section 5.1, the irrelevant last term in (5.14))

$$W = -\lambda \langle S \rangle \Lambda \mathrm{tr}\, \Phi + \mathrm{tr}\, \tilde{\varphi} \Phi \varphi . \quad (5.17)$$

This is just the infrared superpotential of the ISS model from Section 5.1 with quark mass $m = \lambda\Lambda' + \mathcal{O}\left(\lambda^3\Lambda^2/\Lambda'\right)$, which is much smaller than Λ as required.

In summary, we have presented a simple renormalizable extension of the ISS model in which all small scales are dynamically generated. It does not rely on the presence of higher-dimensional operators and in this sense is independent of any specific UV-completion. In addition one of the main advantages of the ISS model, namely its simplicity, is more or less retained: all we have added is another SQCD sector and a singlet field.

It is easily checked that our constraints on the scales can be satisfied for reasonable values of the parameters: Take for instance $N_c = 5$, $N_f = 6$, $N_c' = 4$, $N_f' = 3$. Choose the gauge couplings at the Planck scale as $g^2/4\pi = 1/42$ and $g'^2/4\pi = 1/45$, to obtain $\Lambda \approx 1.8 \cdot 10^6$ GeV and $\Lambda' \approx 2.3 \cdot 10^5$ GeV. With $\lambda = 10^{-2}$, we then have $\lambda\Lambda/\Lambda' \approx 8 \cdot 10^{-2}$ and $\lambda\Lambda/\Lambda' \approx 10^{-3}$, hence both ratios are indeed small. A very crude estimate of the lifetime of the vacuum can be done with the bounce action

$$S_{\mathrm{bounce}} \approx \left(\frac{\Lambda}{m} \right)^{6/5} \approx 3 \cdot 10^3 . \quad (5.18)$$

With the decay width per unit voulume suppressed as

$$\frac{\Gamma}{V} \frac{1}{m^4} \sim e^{-S_{\mathrm{bounce}}} , \quad (5.19)$$

the minimal bounce action for our universe to survive for $\approx 10^{10}$ year in a metastable state is only $S_{\rm min} \approx 400$, so our vacuum is sufficiently long-lived. The SUSY-breaking scale is at about $6 \cdot 10^4$ GeV, of the right order of magnitude to be compatible with gauge mediation. Indeed it should be possible to couple our model to a messenger sector to obtain a simple gauge-mediated model similar to e.g. those of [109, 112].

5.3 The ISS model in type IIB

In this section we will review a way to obtain an ISS-like model from a D-brane construction in type IIB superstring theory, following Argurio, Bertolini, Franco and Kachru (ABFK) [26, 27].[1] The model is similar to the extension of the ISS model considered in [32], including a term which explicitly breaks the approximate R-symmetry of the ISS vacuum. (This is desirable if the model is to provide a hidden sector for the supersymmetric Standard Model, since without such a term it is difficult to obtain realistic gaugino masses.) It has $N_f = N_c + 1$, for which case the magnetic gauge group becomes a trivial $SU(1)$, and the dual quarks can be regarded as the baryons of the electric theory. Small quark masses are dynamically generated, partly from higher-dimensional operators and partly by a mechanism different from those we have discussed so far. The crucial ingredient here are D-brane instanton effects, which have recently been proposed as a particularly natural way of obtaining small scales in general type II string models with SUSY breaking [106].

The construction of [27] starts from a non-chiral $\mathbb{Z}_3$-orbifold of the conifold (see Appendix B for some details of the geometry). Orbifolding by a $\mathbb{Z}_N$ group leads to $2N-1$ types of independent fractional branes (i.e. D5-branes wrapped on the collapsing 2-cycle); in addition, there remains of course the possibility to place regular D3-branes at the conifold tip. A large number of these will place the model in a warped throat, thus providing an embedding in a weakly curved gravitational background. The gauge theory on the singularity should then emerge at the infrared end of a duality cascade, very similar to the KS case of the preceding chapters.

The gauge theory for fractional branes on the orbifolded conifold [117] is given by an appropriate orbifold of the Klebanov-Witten gauge theory [55]. For the $\mathbb{Z}_3$ orbifold it is conveniently described by a quiver diagram of the type depicted in Figure 5.1. This four-dimensional $\mathcal{N} = 1$ theory has six $SU(N_I)$ gauge groups ($I \in \mathbb{Z} \mod 6$), corresponding to the nodes, and six pairs of chiral superfields $X_{I,I+1}$ and $X_{I+1,I}$, corresponding to the arrows between them. The chiral superfields transform as bifundamentals

[1] For other work on string embeddings of the ISS model, see e.g. [28–37].

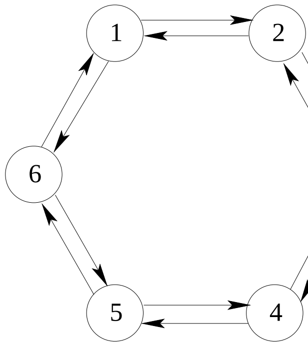

Figure 5.1: The quiver at a $\mathbb{Z}_3$ orbifold of the conifold

under the gauge groups of their respective nodes, so that $X_{I,I+1}$ transforms as $(\mathbf{N}_I, \overline{\mathbf{N}}_{I+1})$ and $X_{I+1,I}$ as $(\overline{\mathbf{N}}_I, \mathbf{N}_{I+1})$ under SU(N_I)× SU(N_{I+1}). The superpotential is

$$W = \frac{1}{M^*} \sum_{I=1}^{6} (-1)^I \text{tr} \left(X_{I,I+1} X_{I+1,I+2} X_{I+2,I+1} X_{I+1,I} \right). \tag{5.20}$$

Here M^* is the relevant UV-completion scale, which is the warped string scale if the model is realized in a warped throat. Since the combinations of fractional branes in this non-chiral quiver are not constrained by anomaly cancellation, the ranks of the gauge groups at the various nodes can be freely chosen. We choose the rank assignments $N_1 = N_2 = N_3 \equiv N_c$, $N_4 = 1$, and $N_5 = N_6 = 0$. The trivial SU(1) at node 4 means that there are fields X_{34} and X_{43} transforming in the fundamental and antifundamental of the SU(N_c) at node 3. The quiver is drawn in Figure 5.2.

The superpotential is

$$W = \frac{1}{M^*} \text{tr} \left(-X_{12} X_{23} X_{32} X_{21} + X_{23} X_{34} X_{43} X_{32} \right) - \mu X_{34} X_{43}. \tag{5.21}$$

The quadratic term is due to a D1-instanton wrapping node 5, which provides a naturally small μ. There can be a similar term $\sim X_{12} X_{21}$ from a D1-instanton at node 6, but this will not play a role for our choice of parameters.

Now assume that the strong-coupling scales on the various nodes satisfy

$$\Lambda_2 \ll \frac{\Lambda_1^2}{M^*} < \mu \ll \Lambda_3. \tag{5.22}$$

We will consider a very weakly gauged $SU(N_c)_2$, such that its gauge dynamics does not interfere with the other fields. We have effectively $N_f = N_c + 1$ flavours for $SU(N_c)_3$, which will become our ISS gauge group. With

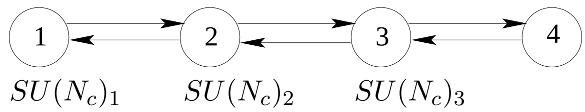

Figure 5.2: The ABFK quiver

$N_c < N_f < 3N_c/2$, it is in the free magnetic range (in fact, as mentioned above, the magnetic gauge group is trivial). From now on, we will explicitly retain only the node 2 "flavour" indices in our formulas, implying that the "colour" indices of nodes 1 and 3 are suitably contracted.

Let us examine the dynamics on node 1. $SU(N_c)_1$ has N_c flavours, and its low-energy dynamics is therefore described by the quantum-deformed moduli space constraint [118]

$$\det M - B\tilde{B} = \Lambda_1^{2N_c}. \tag{5.23}$$

Here M is the (unrescaled) meson field and $B, \tilde{B}$ are the baryons of $SU(N_c)_1$, formed from the elementary fields X_{12} and X_{21} as

$$\begin{aligned} M^i_j &= (X_{12})^i (X_{21})_j, \\ B^{i_1 \ldots i_{N_c}} &= (X_{12})^{i_1} \ldots (X_{12})^{i_{N_c}}, \\ \tilde{B}_{i_1 \ldots i_{N_c}} &= (X_{21})_{i_1} \ldots (X_{21})_{i_{N_c}}. \end{aligned} \tag{5.24}$$

The constraint (5.23) is solved by

$$M^i_j = \Lambda_1^2 \delta^i_j. \tag{5.25}$$

It can be checked on the solution afterwards that the metastable vacuum is not destabilized by turning on expectation values for the baryons.

Assuming that this is indeed the case, one can integrate out X_{12} and X_{21}, which gives the superpotential

$$W = -\frac{\Lambda_1^2}{M^*} \mathrm{tr}\, X_{23} X_{32} - \mu X_{34} X_{43} + \frac{1}{M^*} \mathrm{tr}\, X_{23} X_{34} X_{43} X_{32}. \tag{5.26}$$

Up to the quartic term, we thus obtain $SU(N_c)_3$ SQCD with $N_f = N_c + 1$ massive flavours and masses smaller than the strong-coupling scale Λ_3, by (5.22).

At energies below Λ_3, we use magnetic variables and the Seiberg dual theory. Denoting the dual quarks and antiquarks at node 3 by φ and $\tilde{\varphi}$, and defining the meson Φ by

$$\Phi = \frac{1}{\Lambda_3} \left(\begin{array}{c|c} \phi_{44} & \phi_{24} \\ \hline \phi_{42} & \phi_{22} \end{array} \right), \qquad \text{where } \phi_{IJ} = \langle X_{I3} X_{3J} \rangle, \tag{5.27}$$

the magnetic superpotential (disregarding irrelevant nonperturbative terms) is very similar to (5.1):

$$W = \mathrm{tr}\,\tilde{\varphi}\Phi\varphi - \Lambda_3 \mathrm{tr}\,\mathcal{M}\Phi + \frac{\Lambda_3^2}{M^*}\mathrm{tr}\,\phi_{24}\phi_{42}. \tag{5.28}$$

Here the effective quark mass matrix is given by

$$\mathcal{M} = \left(\begin{array}{c|c} \mu & 0 \\ \hline 0 & \frac{\Lambda_1^2}{M^*}\delta^i_j \end{array}\right). \tag{5.29}$$

The two main differences with the model of Section 5.1 are that that the eigenvalues of $\mathcal{M}$ are now non-degenerate and that there is an R-symmetry breaking quadratic coupling in the meson fields. Nevertheless the ensuing analysis is quite similar [32]. Since $\mu > \Lambda_1^2/M^*$, it will be the F-terms of the ϕ_{22}-mesons that are non-vanishing at the metastable minimum. The scalar components of these mesons are not fixed at tree-level, and while they are stabilized at zero by the Coleman-Weinberg potential in the ISS model, here they acquire a vacuum expectation value at one-loop. This is caused by the quadratic term in the mesons $\sim \phi_{24}\phi_{42}$, which otherwise does not disturb the metastable vacuum, provided that the scales satisty [32]

$$\Lambda_3^3 < \mu M^{*2}. \tag{5.30}$$

The non-vanishing vev for ϕ_{22} is crucial for ensuring the stability of the minimum against deformations towards the baryonic branch of node 1, which now can be checked a posteriori. It turns out that the tree-level coupling to the mesons of node 3 is the dominant contribution to the potential for the fields of node 1, and that (5.25) is a stable solution to the moduli space constraint (5.23) if the scales satisfy the additional condition

$$\Lambda_1 \ll \Lambda_3 < M^*, \qquad \mu < \Lambda_3. \tag{5.31}$$

This concludes our review of the ABFK model. To summarize, we have seen that the ISS model can be realized in type IIB superstring theory. It is thus justified to use it as an ingredient in building type IIB-based models on the field theory level, an example of which we have seen in Chapter 4.1.

Chapter 6

Summary

Let us recapitulate the main points of this thesis.

After summarizing some well-known facts about flux compactifications in type IIB superstring theory and supergravity, we have introduced a special class of solution, the warped throat solutions. We recapitulated the construction of the $AdS_5 \times S^5$ throat and the $AdS_5 \times T^{1,1}$ warped conifold throat, and discussed in detail the construction of the warped deformed conifold or Klebanov-Strassler solution. The KS solution is obtained by adding 3-form flux to the deformed conifold. This is the prime example of a warped throat of finite length, whose dual field theory is then non-conformal. We pointed out that on certain length scales, the KS solution is approximately $AdS_5 \times T^{1,1}$, but with radially varying curvature radius for both the AdS and the $T^{1,1}$ part, giving a simple derivation for its logarithmic variation.

We then demonstrated how the complex structure moduli of a type IIB superstring compactification can be stabilized by fluxes, using the example of the KS throat, where the modulus in question is the deformation size of the conifold singularity. Afterwards we reviewed the stabilization of Kähler moduli by non-perturbative means and the KKLT mechanism. This mechanism makes use of an additional uplifting sector in order to promote the fully stabilized vacuum, for which the four-dimensional background is AdS_4, to a metastable non-supersymmetric vacuum which is 4d Minkowski or dS_4. We pointed out that, in order to be in the domain of weak-curvature where one has computational control, the uplifting energy density should be hierarchically small. This is natural if the uplifting sector is localized in a strongly warped region such as the KS throat.

Following this review, we turned to the description of a IIB compactification containing a KS throat in terms of an effective 5d theory simliar to the Randall-Sundrum-I model. We presented the general idea of identifying the 10d bulk and an approximately conical region of the internal space with the RS ultraviolet brane, the actual throat with the RS bulk, and the tip of the throat where the deformation of the conifold singularity becomes relevant with the RS infrared brane. We further identified the scales on which a 5d description of the background by an almost AdS_5 geometry is viable, and showed that, for strongly warped throats, their range can be fairly wide.

Subsequently we turned to the description of radion stabilization in this 5d picture. We showed that the variation of the curvature scale in the KS throat can be modelled by a scalar field with a non-trivial profile in the background. The backreaction of the potential energy density on the geometry then deforms the internal space appropriately. Since the boundary conditions for this scalar degree of freedom are fixed at both ends of the throat, the length of the throat is also fixed and the 5d radion is stabilized. This can be viewed as a variation of the Goldberger-Wise mechanism for radion stabilization in the RS model, with back-reaction included. We calculated the potential for the Goldberger-Wise scalar to leading order from the known properties of the geometry. We also gave some remarks on the relation to the 5d model used by Klebanov and Tseytlin to construct the metric for the KS throat solution.

Additionally, we incorporated the unstabilized universal Kähler modulus, which is common to such type IIB compactifications, into our 5d picture. This modulus governs the size of the embedding manifold at the UV end of the throat, at least in the range of its values where the 5d description is applicable. We found the adequate description for this modulus in the effective RS model as UV brane field, whose contribution to the action we modelled such that upon further dimensional reduction to 4d, it becomes a no-scale field.

Including these degrees of freedom, we presented a 5d action of the resulting RS-I-like model. We concluded our discussion of the KS throat as a RS model by discussing the prospects for formulating this action in a manifestly supersymmetric way. Unfortunately the effective 5d supergravity even of type IIB supergravity on $\mathrm{AdS}_5 \times T^{1,1}$ is poorly understood, not to mention its deformation upon including 3-form flux. We pointed out, however, that it might be possible to construct a supersymmetric 5d action from the bottom up, by taking as the starting point the supersymmetrized RS model rather than attempting to dimensionally reduce from 10d.

We proceeded by turning to a different topic in type IIB compactifications, namely supersymmetry breaking mediation in the KKLT model. We focussed on F-term uplifting and summarized the main aspects of generic models, before turning to models which naturally realize the hierarchy required for the KKLT construction. One example of these is the ISS-KKLT model, in which the ISS model of dynamical metastable supersymmetry breaking is used to provide the uplifting piece of the scalar potential. However, it is by placing the uplifting sector in a warped throat that one obtains not only a natural hierarchy of scales, but also the desirable property of having the SUSY-breaking sector sequestered from the visible sector. Our focus was consequently mainly on sequestered models.

In a minimal sequestered setting, comprising only 4d supergravity, a nonperturbatively stabilized Kähler modulus, and a SUSY-breaking superfield, we showed that the leading contribution to communicating SUSY breaking are due to mixed modulus-anomaly mediation. We gave an explicit example of a sequestered hidden sector with F-term breaking. Finally, we investigated possible additional gravity-mediated contributions in a non-minimal setup, motivated by realistic throat backgrounds that might underly the KKLT model. It turned out that there can be a relevant contribution from string-scale massive vector multiplets which are present in the 4d effective theory if the throat admits an isometry, as is the case for the KS throat. Such an isometry is reflected in a gauge symmetry in 4d, which will generally be broken at the string scale because a Calabi-Yau manifold containing the throat will not admit any isometries. We showed that, if these vector fields have gauge-symmetry breaking couplings with both the SUSY-breaking and the visible sector, their contribution to the visible sector soft parameters can be equally important as that of mixed modulus-anomaly mediation. This is the case in spite of their string-scale masses. String-scale massive chiral multiplets, by contrast, contribute only subdominantly.

In the last part of the thesis, we reviewed the ISS model in more detail. We showed that it can be extended such that all its small parameters are generated dynamically. The retrofitted ISS model we presented is still very simple, consisting of two sectors of SQCD coupled by a singlet field. It is renormalizable and does not contain any dimensionful parameters: all scales are generated by dimensional transmutation. The ISS model and especially its retrofitted version has a wide range of applications in purely field-theoretical model building. However, it is also a useful ingredient for building models in string phenomenology, one of which we had already discussed in the context of the KKLT construction. We finally reviewed how the ISS model can be obtained as the low-energy field theory of a specific D-brane model in type IIB superstring theory, thus justifying its inclusion in models derived from type IIB compactifications.

Appendix A

Notation and conventions

Our conventions for type IIB superstring theory and supergravity in ten dimensions largely follow [119]. In particular, $\mathrm{g_s}$ denotes the string coupling and α' is the Regge slope or inverse string tension.

M_4, M_5, M_{10} are the reduced Planck masses in four, five and ten spacetime dimensions respectively. They are related to the corresponding physical Planck masses by factors of $\sqrt{8\pi}$, so that $M_4 = 1/\sqrt{8\pi}\, M_{4,\text{phys}}$ etc.

We use Greek letters for 4d spacetime indices, small Roman letters for extra dimensional indices, and capital Roman letters for both of them collectively, in either five or ten dimensions. Our metric has "mostly plus" signature.

We use script letters $\mathcal{R}, \mathcal{R}_{MN}$ to refer to the scalar curvature and the Ricci tensor. Roman R is reserved to generically denote a radius. Likewise, $\mathcal{L}$ is a Lagrangian and L a length scale.

In 4d $\mathcal{N} = 1$ supersymmetry, we stick to the common practice of denoting a superfield by the same symbol as its lowest component. Which of them is meant should always be clear from the context.

Appendix B

The conifold

Here we collect some useful material about the geometry of the conifold. Many of the derviations (which we have omitted) can be found in [120].

The conifold may be defined as the hypersurface in $\mathbb{C}^4$ given by

$$w_1^2 + w_2^2 + w_3^2 + w_4^2 = 0. \tag{B.1}$$

It is smooth except for the singularity at the coordinate origin. The conifold is a real cone over the Einstein manifold $T^{1,1}$. $T^{1,1}$ itself is $(SU(2) \times SU(2)')/U(1)$ with the $U(1)$ generated by $\sigma^3 + \sigma'^3$; it can also be thought of as a S^1 fibration over $S^2 \times S^2$ with topology $S^2 \times S^3$. At the apex of the conifold, both the S^2 and the S^3 shrink to zero size. The fibred structure can be seen from the Einstein metric of $T^{1,1}$, which is

$$ds^2 = \frac{1}{9}\left(d\psi + \sum_{i=1}^{2} \cos\theta_i d\phi_i\right)^2 + \frac{1}{6}\sum_{i=1}^{2}\left(d\theta_i^2 + \sin^2\theta_i \, d\phi_i^2\right). \tag{B.2}$$

Here $\psi \in [0; 4\pi)$ parametrizes the S^1 fibre, and (θ_i, ϕ_i) parametrize two S^2s in the standard way.

The singularity can be *deformed* by replacing the RHS of (B.1) by some nonzero z:

$$w_1^2 + w_2^2 + w_3^2 + w_4^2 = z. \tag{B.3}$$

Alternatively, one may introduce the matrix W,

$$W = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \tag{B.4}$$

where

$$a = w_3 + iw_4, \; b = -w_1 + iw_2, \; c = w_1 + iw_2, \; d = w_3 - iw_4. \tag{B.5}$$

In terms of these variables, the defining equation (B.1) becomes

$$\det W = 0, \tag{B.6}$$

and the singularity may be *resolved* by replacing (B.6) by the pair of equations

$$W\lambda = 0 \qquad\qquad (\lambda \in \mathbb{CP}^1). \tag{B.7}$$

In the resolved conifold, the singularity at the tip of the cone over $T^{1,1} \simeq S^2 \times S^3$ is replaced by a 2-cycle, with the S^3 shrunk to zero. In the deformed conifold, the roles of the two spheres are exchanged, such that the S^2 collapses and the S^3 is retained.

The defining equation (B.1) of the singular conifold is left invariant by an SU(2)× SU(2)× U(1) transformation, where the w_i transform as a vector under SU(2)× SU(2) ≃SO(4), and U(1) acts as $w_i \to e^{i\alpha} w_i$.[1] In the deformed conifold U(1) is broken and SO(4) remains as the unbroken isometry group of the deformed conifold, as is evident from (B.3). Note that this isometry acts on the transverse space only, as the radial coordinate

$$r = \sqrt{\frac{3}{2}} \left(\sum_{i=1}^{4} |w_i|^2 \right)^{1/3} \tag{B.8}$$

is manifestly invariant. It will therefore remain an isometry if we allow for a more general dependence of the transverse space on the radial position, in particular for warping.

The defining equation (B.6) may be used to construct the $\mathbb{Z}_N$ orbifold of the conifold considered in Section 5.3. The orbifold action acts on the variables of (B.5) as

$$a \to e^{2\pi i/N} a, \qquad d \to e^{-2\pi i/N} d. \tag{B.9}$$

In terms of the invariant variables $a' = a^N$ and $d' = d^N$, the equation describing the orbifolded conifold then becomes

$$a'd' - b^N c^N = 0. \tag{B.10}$$

Note that the orbifold group is a subgroup of the SU(2)× SU(2), and the U(1) factor of the isometry group is unaffected.

[1] Since the conifold is a real cone, it is also invariant under a real rescaling $w_i \to \lambda w_i$, but this symmetry is of course broken when the conifold is cut off and embedded into a compact space at some radial position. It is also broken by either deformation or resolution.

Appendix C

5d Anti-de Sitter space

Here give some coordinatizations and reference formulae for AdS_5 which are used on several occasions in the main text. For our purposes AdS_5 is the unique 5d spacetime with topology $\mathbb{R}^5$ and constant negative curvature $\mathcal{R}$ (this is actually the covering space $CAdS_5$ of AdS_5 in the strict sense, but we will not make this distinction since we will never deal with AdS_5 proper). It is an exact solution to the Einstein equations

$$\mathcal{R}_{MN} - \frac{1}{2}\mathcal{R}\, g_{MN} = -\frac{\Lambda}{M_5^3}\, g_{MN} \qquad (\Lambda < 0), \tag{C.1}$$

which arise from the variation of the Einstein-Hilbert action supplemented with a negative cosmological constant:

$$S = \int d^5x\, \sqrt{-g}\left(\frac{M_5^3}{2}\mathcal{R} - \Lambda\right). \tag{C.2}$$

The scalar curvature is related to Λ by

$$\mathcal{R} = 10\Lambda/3M_5^3 \tag{C.3}$$

AdS_5 is maximally symmetric, and in particular homogenous and isotropic, a fact which is not manifest in the common coordinatizations.

A convenient set of coordinates for Randall-Sundrum type models is (x^μ, y), with $\mu = 0\ldots 3$, in terms of which the metric is

$$ds^2 = e^{2ky}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2. \tag{C.4}$$

Here $\eta_{\mu\nu}$ is the 4d Minkowski metric, and $k = \sqrt{-\mathcal{R}/20} = \sqrt{-\Lambda/6}$. Beware that in these coordinates the sign of y is opposite to the widely used convention of [54]: the "warp factor" e^{2ky} *increases* with increasing y, so small y means large redshift. We adopt this somewhat nonstandard convention to avoid a large number of minus signs in the main text.

Another parametrization is frequently encountered when considering the near-horizon limit of a stack of D3 branes. Define the curvature radius R by $R = 1/k$, then the metric in terms of coordinates (x^μ, r) is

$$ds^2 = \frac{R^2}{r^2}dr^2 + \frac{r^2}{R^2}\eta_{\mu\nu}dx^\mu dx^\nu \qquad (r \in \mathbb{R}_+). \tag{C.5}$$

The transformation between the radial coordinates r and y is $y = R \log \frac{r}{R}$.

Finally, by setting $z = R^2/r$, one obtains the commonly used "Poincaré coordinates" (x^μ, z) with the metric

$$ds^2 = \frac{R^2}{z^2}(dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu) \qquad (z \in \mathbb{R}_+). \tag{C.6}$$

Appendix D

The phases of $\mathcal{N} = 1$ SQCD

In this appendix we summarize some well-known facts about 4d $\mathcal{N} = 1$ globally supersymmetric $SU(N_c)$ gauge theory with N_f massless flavours of quark and antiquark chiral superfields, q and $\tilde{q}$. We will omit $\mathcal{O}(1)$ prefactors throughout, and just state the results without proof (or without even giving supporting evidence — actually much of this material has not been rigorously proven so far). More comprehensive treatments can be found e.g. in [3, 4, 121].

The behaviour of SQCD under renormalization group evolution is dependent on the values of N_f and N_c. The one-loop beta function coefficient is

$$b_0 = 3N_c - N_f. \tag{D.1}$$

From the sign of b_0, we can read off the ultraviolet behaviour of the theory.

The theory is not asymptotically free if $N_f \geq 3N_c$. It has a trivial RG fixed point in the infrared and a Landau pole in the UV. In the marginal case $N_f = 3N_c$, the lack of asymptotic freedom is visible only at two-loop order.

For $N_f < 3N_c$, the theory is asymptotically free. Concerning the IR behaviour, five distinct cases have to be considered, all but the first of which are used at some point in the main text of this thesis:

1. $3N_c/2 < N_f < 3N_c$: The "electric" theory, whose degrees of freedom are the SQCD quarks and gauge fields, flows to a nontrivial (interacting) fixed point in the IR. It is dual [59] to an SU(N_c') "magnetic" gauge theory ($N_c' = N_f - N_c$) with N_f flavours of dual quarks and antiquarks, which also includes N_f^2 gauge singlets. Since $3N_c'/2 < N_f < 3N_c'$, the dual theory is in the same range of flavours and colours. Both the electric and the magnetic theory thus flow towards an interacting CFT at low energies.

2. $N_c + 1 \leq N_f \leq 3N_c/2$: This is the free magnetic range relevant for the ISS model. The gauge coupling for the electric theory diverges at low energies at a scale Λ. Its IR behaviour can nevertheless be described by a weakly coupled gauge theory: The dual magnetic theory [59] is given by an $\mathrm{SU}(N_c')$ gauge theory ($N_c' = N_f - N_c$) with N_f flavours φ and $\tilde{\varphi}$, including a colourless field Φ in the $\mathbf{N_f} + \overline{\mathbf{N}}_\mathbf{f}$ of the flavour group $\mathrm{SU}(N_f)_L \times \mathrm{SU}\,(N_f)_R$. The superpotential is

$$W = \tilde{\varphi}\Phi\varphi. \tag{D.2}$$

Since $N_f \geq 3N_c'$, this dual theory is infrared-free (hence the term "free magnetic range") and has a Landau pole at high energies, at a scale $\Lambda' \lesssim \Lambda$. Thus, in the IR, the magnetic theory is weakly coupled, and in the UV, the electric theory is weakly coupled. The dynamics can be described perturbatively in both ranges, in terms of the appropriate weakly coupled degrees of freedom. In the limiting case $N_f = N_c + 1$, the dual theory is not a gauge theory, but can still be constructed. The degrees of freedom are then similar to the meson, baryon and antibaryon fields defined below.

3. $N_f = N_c$: This theory has a moduli space of vacua parametrized by meson, baryon and antibaryon fields M, B and $\tilde{B}$, which classically are given by the gauge-invariant composites

$$\begin{aligned} M^i_j &= q^i_c \tilde{q}^c_j, \\ B^{i_1 \ldots i_{N_c}} &= \epsilon^{c_1 \ldots c_{N_c}} q^{i_1}_{c_1} \ldots q^{i_{N_c}}_{c_{N_c}}, \\ \tilde{B}_{i_1 \ldots i_{N_c}} &= \epsilon_{c_1 \ldots c_{N_c}} \tilde{q}^{c_1}_{i_1} \ldots \tilde{q}^{c_{N_c}}_{i_{N_c}}. \end{aligned} \tag{D.3}$$

The classical constraint

$$\det M - B\tilde{B} = 0 \tag{D.4}$$

is changed in the quantum theory [118] to

$$\det M - B\tilde{B} = \Lambda^{2N_c}, \tag{D.5}$$

where Λ is the strong-coupling scale. The vacuum structure is then dictated by this quantum-deformed moduli space constraint.

4. $0 < N_f < N_c$: The appropriate low-energy description for this theory is again in terms of the gauge-invariant meson fields M, constructed analogously to (D.3). (The baryons of (D.3) identically vanish for this range of $N - f$ and N_c.) The mesons are subject to the non-perturbative superpotential [115]

$$W = \left(\frac{\Lambda^{3N_c - N_f}}{\det M} \right)^{\frac{1}{N_c - N_f}}. \tag{D.6}$$

It follows that the theory does not have a stable vacuum but a runaway towards $M \to \infty$, unless it is coupled to additional degrees of freedom.

5. $N_f = 0$: Pure $\mathcal{N} = 1$ super-Yang-Mills theory exhibits gaugino condensation in the IR, below the strong-coupling scale Λ [122]. The gauge-kinetic term should be replaced by a constant below this scale when the theory is embedded in a larger model:

$$\int d^2\theta \, \mathrm{tr} \, W^\alpha W_\alpha \;\rightarrow\; \langle \mathrm{tr} \, \lambda_\alpha \lambda^\alpha \rangle = \Lambda^3. \tag{D.7}$$

A standard analysis shows that there are N_c supersymmetric vacua, related by discrete phase rotations in the condensate.

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Bibliography

[1] M. Graña, Phys. Rept. **423** (2006) 91 [arXiv:hep-th/0509003].

[2] M. R. Douglas and S. Kachru, Rev. Mod. Phys. **79** (2007) 733 [arXiv:hep-th/0610102].

[3] K. A. Intriligator and N. Seiberg, Nucl. Phys. Proc. Suppl. **45BC** (1996) 1 [arXiv:hep-th/9509066].

[4] M. E. Peskin, arXiv:hep-th/9702094.

[5] K. Intriligator and N. Seiberg, arXiv:hep-ph/0702069.

[6] J. M. Maldacena, Adv. Theor. Math. Phys. **2** (1998) 231 [Int. J. Theor. Phys. **38** (1999) 1113] [arXiv:hep-th/9711200].

[7] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, Phys. Lett. B **428** (1998) 105 [arXiv:hep-th/9802109].

[8] E. Witten, Adv. Theor. Math. Phys. **2** (1998) 253 [arXiv:hep-th/9802150].

[9] R. Bousso and J. Polchinski, JHEP **0006** (2000) 006 [arXiv:hep-th/0004134].

[10] M. R. Douglas, JHEP **0305** (2003) 046 [arXiv:hep-th/0303194].

[11] S. Ashok and M. R. Douglas, JHEP **0401** (2004) 060 [arXiv:hep-th/0307049].

[12] S. B. Giddings, S. Kachru and J. Polchinski, Phys. Rev. D **66** (2002) 106006 [arXiv:hep-th/0105097].

[13] L. Randall and R. Sundrum, Phys. Rev. Lett. **83** (1999) 3370 [arXiv:hep-ph/9905221].

[14] H. L. Verlinde, Nucl. Phys. B **580** (2000) 264 [arXiv:hep-th/9906182].

[15] C. S. Chan, P. L. Paul and H. L. Verlinde, Nucl. Phys. B **581** (2000) 156 [arXiv:hep-th/0003236].

[16] F. Brümmer, A. Hebecker and E. Trincherini, Nucl. Phys. B **738** (2006) 283 [arXiv:hep-th/0510113].

[17] S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, Phys. Rev. D **68** (2003) 046005 [arXiv:hep-th/0301240].

[18] K. Choi, A. Falkowski, H. P. Nilles and M. Olechowski, Nucl. Phys. B **718**, 113 (2005) [arXiv:hep-th/0503216].

[19] K. Choi, K. S. Jeong and K. i. Okumura, JHEP **0509** (2005) 039 [arXiv:hep-ph/0504037].

[20] K. Choi and K. S. Jeong, JHEP **0608** (2006) 007 [arXiv:hep-th/0605108].

[21] F. Brümmer, A. Hebecker and M. Trapletti, Nucl. Phys. B **755** (2006) 186 [arXiv:hep-th/0605232].

[22] I. R. Klebanov and M. J. Strassler, JHEP **0008**, 052 (2000) [arXiv:hep-th/0007191].

[23] E. Witten, Nucl. Phys. B **188**, 513 (1981).

[24] K. Intriligator, N. Seiberg and D. Shih, JHEP **0604** (2006) 021 [arXiv:hep-th/0602239].

[25] F. Brümmer, JHEP **0707** (2007) 043 [arXiv:0705.2153 [hep-ph]].

[26] R. Argurio, M. Bertolini, S. Franco and S. Kachru, JHEP **0701** (2007) 083 [arXiv:hep-th/0610212].

[27] R. Argurio, M. Bertolini, S. Franco and S. Kachru, JHEP **0706** (2007) 017 [arXiv:hep-th/0703236].

[28] S. Franco and A. M. Uranga, JHEP **0606** (2006) 031 [arXiv:hep-th/0604136].

[29] H. Ooguri and Y. Ookouchi, Phys. Lett. B **641** (2006) 323 [arXiv:hep-th/0607183].

[30] S. Franco, I. García-Etxebarria and A. M. Uranga, JHEP **0701** (2007) 085 [arXiv:hep-th/0607218].

[31] R. Tatar and B. Wetenhall, JHEP **0702** (2007) 020 [arXiv:hep-th/0611303].

[32] R. Kitano, H. Ooguri and Y. Ookouchi, Phys. Rev. D **75** (2007) 045022 [arXiv:hep-ph/0612139].

[33] I. García-Etxebarria, F. Saad and A. M. Uranga, JHEP **0705** (2007) 047 [arXiv:0704.0166 [hep-th]].

[34] T. Kawano, H. Ooguri and Y. Ookouchi, Phys. Lett. B **652** (2007) 40 [arXiv:0704.1085 [hep-th]].

[35] D. Malyshev, arXiv:0705.3281 [hep-th].

[36] R. Tatar and B. Wetenhall, arXiv:0707.2712 [hep-th].

[37] Y. Nakayama, M. Yamazaki and T. T. Yanagida, arXiv:0710.0001 [hep-th].

[38] O. Lebedev, H. P. Nilles and M. Ratz, Phys. Lett. B **636** (2006) 126 [arXiv:hep-th/0603047].

[39] A. Saltman and E. Silverstein, JHEP **0411**, 066 (2004) [arXiv:hep-th/0402135].

[40] M. Gómez-Reino and C. A. Scrucca, JHEP **0605**, 015 (2006) [arXiv:hep-th/0602246].

[41] M. Gómez-Reino and C. A. Scrucca, JHEP **0609**, 008 (2006) [arXiv:hep-th/0606273].

[42] E. Dudas, C. Papineau and S. Pokorski, JHEP **0702**, 028 (2007) [arXiv:hep-th/0610297].

[43] H. Abe, T. Higaki, T. Kobayashi and Y. Omura, Phys. Rev. D **75**, 025019 (2007) [arXiv:hep-th/0611024].

[44] R. Kallosh and A. Linde, JHEP **0702**, 002 (2007) [arXiv:hep-th/0611183].

[45] J. Polchinski and A. Strominger, Phys. Lett. B **388** (1996) 736 [arXiv:hep-th/9510227].

[46] J. Michelson, Nucl. Phys. B **495** (1997) 127 [arXiv:hep-th/9610151].

[47] S. Gukov, C. Vafa and E. Witten, Nucl. Phys. B **584** (2000) 69 [Erratum-ibid. B **608** (2001) 477] [arXiv:hep-th/9906070].

[48] K. Dasgupta, G. Rajesh and S. Sethi, JHEP **9908** (1999) 023 [arXiv:hep-th/9908088].

[49] J. F. G. Cascales, M. P. García del Moral, F. Quevedo and A. M. Uranga, JHEP **0402** (2004) 031 [arXiv:hep-th/0312051].

[50] B. de Wit, D. J. Smit and N. D. Hari Dass, Nucl. Phys. B **283** (1987) 165.

[51] J. M. Maldacena and C. Nuñez, Int. J. Mod. Phys. A **16** (2001) 822 [arXiv:hep-th/0007018].

[52] M. Graña and J. Polchinski, Phys. Rev. D **63** (2001) 026001 [arXiv:hep-th/0009211].

[53] S. S. Gubser, arXiv:hep-th/0010010.

[54] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999) [arXiv:hep-th/9906064].

[55] I. R. Klebanov and E. Witten, Nucl. Phys. B **536**, 199 (1998) [arXiv:hep-th/9807080].

[56] I. R. Klebanov and N. A. Nekrasov, Nucl. Phys. B **574**, 263 (2000) [arXiv:hep-th/9911096].

[57] I. R. Klebanov and A. A. Tseytlin, Nucl. Phys. B **578**, 123 (2000) [arXiv:hep-th/0002159].

[58] M. J. Strassler, arXiv:hep-th/0505153.

[59] N. Seiberg, Nucl. Phys. B **435**, 129 (1995) [arXiv:hep-th/9411149].

[60] C. P. Herzog, I. R. Klebanov and P. Ouyang, arXiv:hep-th/0108101.

[61] F. Denef and M. R. Douglas, JHEP **0405** (2004) 072 [arXiv:hep-th/0404116].

[62] A. Giryavets, S. Kachru and P. K. Tripathy, JHEP **0408** (2004) 002 [arXiv:hep-th/0404243].

[63] A. Hebecker and J. March-Russell, Nucl. Phys. B **781**, 99 (2007) [arXiv:hep-th/0607120].

[64] P. Candelas and X. de la Ossa, Nucl. Phys. B **355** (1991) 455.

[65] S. Kachru, J. Pearson and H. L. Verlinde, JHEP **0206**, 021 (2002) [arXiv:hep-th/0112197].

[66] S. Kachru, R. Kallosh, A. Linde, J. M. Maldacena, L. McAllister and S. P. Trivedi, JCAP **0310** (2003) 013 [arXiv:hep-th/0308055].

[67] S. R. Coleman and F. De Luccia, Phys. Rev. D **21** (1980) 3305.

[68] B. Hassanain, J. March-Russell and M. Schvellinger, arXiv:0708.2060 [hep-th].

[69] Y. Grossman and M. Neubert, Phys. Lett. B **474** (2000) 361 [arXiv:hep-ph/9912408].

[70] S. Chang, J. Hisano, H. Nakano, N. Okada and M. Yamaguchi, Phys. Rev. D **62** (2000) 084025 [arXiv:hep-ph/9912498].

[71] T. Gherghetta and A. Pomarol, Nucl. Phys. B **586** (2000) 141 [arXiv:hep-ph/0003129].

[72] T. Gherghetta and J. Giedt, Phys. Rev. D **74** (2006) 066007 [arXiv:hep-th/0605212].

[73] B. S. Acharya, F. Benini and R. Valandro, arXiv:hep-th/0612192.

[74] W. D. Goldberger and M. B. Wise, Phys. Rev. Lett. **83** (1999) 4922 [arXiv:hep-ph/9907447].

[75] G. Papadopoulos and A. A. Tseytlin, Class. Quant. Grav. **18** (2001) 1333 [arXiv:hep-th/0012034].

[76] M. Berg, M. Haack and W. Mück, Nucl. Phys. B **736** (2006) 82 [arXiv:hep-th/0507285].

[77] S. B. Giddings and A. Maharana, Phys. Rev. D **73** (2006) 126003 [arXiv:hep-th/0507158].

[78] G. W. Gibbons and S. W. Hawking, Phys. Rev. D **15** (1977) 2752.

[79] G. R. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B **485** (2000) 208 [arXiv:hep-th/0005016].

[80] H. Collins and B. Holdom, Phys. Rev. D **62** (2000) 105009 [arXiv:hep-ph/0003173].

[81] M. A. Luty and R. Sundrum, Phys. Rev. D **64** (2001) 065012 [arXiv:hep-th/0012158].

[82] N. Arkani-Hamed, T. Gregoire and J. G. Wacker, JHEP **0203** (2002) 055 [arXiv:hep-th/0101233].

[83] D. Martí and A. Pomarol, Phys. Rev. D **64** (2001) 105025 [arXiv:hep-th/0106256].

[84] A. Hebecker, Nucl. Phys. B **632** (2002) 101 [arXiv:hep-ph/0112230].

[85] F. Paccetti Correia, M. G. Schmidt and Z. Tavartkiladze, Nucl. Phys. B **709** (2005) 141 [arXiv:hep-th/0408138].

[86] F. Paccetti Correia, M. G. Schmidt and Z. Tavartkiladze, Nucl. Phys. B **751** (2006) 222 [arXiv:hep-th/0602173].

[87] A. Ceresole, G. Dall'Agata, R. D'Auria and S. Ferrara, Phys. Rev. D **61** (2000) 066001 [arXiv:hep-th/9905226].

[88] A. Ceresole, G. Dall'Agata and R. D'Auria, JHEP **9911** (1999) 009 [arXiv:hep-th/9907216].

[89] A. Salam and J. A. Strathdee, Annals Phys. **141** (1982) 316.

[90] P. Hoxha, R. R. Martinez-Acosta and C. N. Pope, Class. Quant. Grav. **17** (2000) 4207 [arXiv:hep-th/0005172].

[91] A. Ceresole and G. Dall'Agata, Nucl. Phys. B **585** (2000) 143 [arXiv:hep-th/0004111].

[92] B. de Wit and A. Van Proeyen, Commun. Math. Phys. **149** (1992) 307 [arXiv:hep-th/9112027].

[93] Z. Lalak, arXiv:hep-th/0109074.

[94] A. R. Frey, M. Lippert and B. Williams, Phys. Rev. D **68** (2003) 046008 [arXiv:hep-th/0305018].

[95] O. DeWolfe and S. B. Giddings, Phys. Rev. D **67** (2003) 066008 [arXiv:hep-th/0208123].

[96] L. Randall and R. Sundrum, Nucl. Phys. B **557** (1999) 79 [arXiv:hep-th/9810155].

[97] A. Anisimov, M. Dine, M. Graesser and S. D. Thomas, Phys. Rev. D **65** (2002) 105011 [arXiv:hep-th/0111235].

[98] A. Falkowski, H. M. Lee and C. Lüdeling, JHEP **0510** (2005) 090 [arXiv:hep-th/0504091].

[99] S. Kachru, L. McAllister and R. Sundrum, JHEP **0710** (2007) 013 [arXiv:hep-th/0703105].

[100] G. F. Giudice, M. A. Luty, H. Murayama and R. Rattazzi, JHEP **9812** (1998) 027 [arXiv:hep-ph/9810442].

[101] K. Choi, A. Falkowski, H. P. Nilles, M. Olechowski and S. Pokorski, JHEP **0411**, 076 (2004) [arXiv:hep-th/0411066].

[102] C. P. Burgess, R. Kallosh and F. Quevedo, JHEP **0310** (2003) 056 [arXiv:hep-th/0309187].

[103] E. Dudas and S. K. Vempati, Nucl. Phys. B **727** (2005) 139 [arXiv:hep-th/0506172].

[104] G. Villadoro and F. Zwirner, Phys. Rev. Lett. **95**, 231602 (2005) [arXiv:hep-th/0508167].

[105] M. Dine, J. L. Feng and E. Silverstein, Phys. Rev. D **74** (2006) 095012 [arXiv:hep-th/0608159].

[106] O. Aharony, S. Kachru and E. Silverstein, arXiv:0708.0493 [hep-th].

[107] T. Gregoire, R. Rattazzi, C. A. Scrucca, A. Strumia and E. Trincherini, Nucl. Phys. B **720**, 3 (2005) [arXiv:hep-th/0411216].

[108] T. Banks, arXiv:hep-ph/0606313.

[109] H. Murayama and Y. Nomura, Phys. Rev. Lett. **98** (2007) 151803 [arXiv:hep-ph/0612186].

[110] C. Csáki, Y. Shirman and J. Terning, JHEP **0705** (2007) 099 [arXiv:hep-ph/0612241].

[111] S. A. Abel and V. V. Khoze, arXiv:hep-ph/0701069.

[112] S. A. Abel, J. Jäckel and V. V. Khoze, arXiv:hep-ph/0703086.

[113] O. Aharony and N. Seiberg, JHEP **0702** (2007) 054 [arXiv:hep-ph/0612308].

[114] R. Essig, K. Sinha and G. Torroba, JHEP **0709** (2007) 032 [arXiv:0707.0007 [hep-th]].

[115] I. Affleck, M. Dine and N. Seiberg, Nucl. Phys. B **241** (1984) 493.

[116] N. Seiberg, Phys. Lett. B **318**, 469 (1993) [arXiv:hep-ph/9309335].

[117] A. M. Uranga, JHEP **9901** (1999) 022 [arXiv:hep-th/9811004].

[118] N. Seiberg, Phys. Rev. D **49** (1994) 6857 [arXiv:hep-th/9402044].

[119] J. Polchinski, "String theory. Vol. 2: Superstring theory and beyond," *Cambridge, UK: Univ. Pr. (1998) 531 p*

[120] P. Candelas and X. C. de la Ossa, Nucl. Phys. B **342**, 246 (1990).

[121] J. Terning, "Modern supersymmetry: Dynamics and duality," *Oxford, UK: Clarendon (2006) 324 p*

[122] G. Veneziano and S. Yankielowicz, Phys. Lett. B **113** (1982) 231.