

Handwritten text in Leonardo da Vinci's mirror-image script, likely a technical treatise on mechanics or geometry. The text is arranged in approximately 12 horizontal lines, with some words appearing to be underlined or grouped. The script is dense and characteristic of his late work.

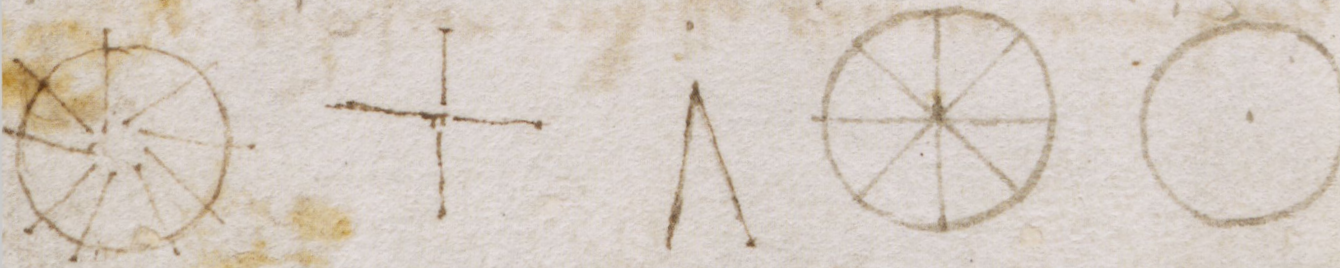


Fig. 1. Detail of Codex Arundel, f. 132r

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The Paradox of the Point

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Five diagrams in the Codex Arundel, f. 132r (cat. 37 and detail in fig. 1)¹ – at the bottom of the left part of this large bifolio – refer both to Leonardo's natural philosophy and his theory of the image. In their sequence from right to left, they show a circle with a centre point in the first diagram and a circle with eight radii in the second. The third drawing represents an acute vertical angle of *c.* 30 degrees. On the next diagram, two orthogonal lines appear to converge in a minuscule letter. The last sketch shows a circle, again intersected by radial lines from the circle's centre, like the spokes of a wheel. How can we interpret this sequence? The notes on this bifolio are concerned with definitions of point, line, nought (*nulla*) and border (*termine*). The point is of central interest here. Directly above the diagrams, Leonardo claims that the infinity of lines that converge in a circle meet only in a single point; that the infinity of the angles of these converging lines ends in an infinity of points that are equal to one single point; and that this identity of one and multitude defines nought (*il nulla*).²

Nulla ('Nought')

In the first paragraph of his unabridged, posthumously edited *Treatise on Painting*, written *c.* 1500–5,³ Leonardo claims that painting is a 'science' (*scientia*) because, like geometry, it is founded on an 'ultimate principle' (*ultimo principio*), the point.

Therefore, the point is the first principle of geometry, and no other thing can exist either in nature or in the human mind that would be more fundamental than the point. If you were to say that the creation of a point is the final contact made with the point of a stylus on a surface, this is not true; we would say such contact is a surface that surrounds a center, and in that centre is the location of the point.

All the points of a surface, even of the world, would not create 'more' than a single point, Leonardo continues. The point 'is not materially part of the picture surface' (*non è della materia di essa superfite*); nevertheless, it mysteriously creates the image. Leonardo seems to equate the point to 'nothingness', but the fact that the 'addition' of zero, as Leonardo puts it, changes the value of numbers – from 1 to 10 to 100 *ad infinitum* – provides an analogy to the dynamic qualities of the point, namely, its extensions into line and surface on a picture's plane.⁴ Lines and surfaces are created by the transit of the point (*la linia è il transito del punto*). Consequently, lines and surfaces also have no extension, they are 'something spiritual rather than substantial' (*cosa spirituale che sustantia*).⁵ The point is thus intimately connected to movement and therefore to Leonardo's categories of physics.

In light of the apparently abstract quality of the point, Leonardo's bold assertion that 'painting is a mental discourse' (*la pittura è mentale*)⁶ comes as little surprise, yet it is only one side of the coin. Leonardo also emphasises again and again the importance of visuality, leaning strongly towards an identification of sensory activity as a mental activity *per se*, thereby circumventing the traditional hierarchy between 'outer' and 'inner' senses.⁷ However, as is well known, Leonardo's remarks on the point as an invisible principle of painting contrast significantly with somewhat earlier statements by Leon Battista Alberti, Filarete, and Piero della Francesca. All three authors

describe the first element of painting as a *visual* point, as the smallest *perceptible* element on the picture surface.⁸ Leonardo, instead, seems to follow closely the authority of Euclid, who defines the point, at the very beginning of his *Elements*, as ‘that which has no part’.⁹ But the paradox in Leonardo’s argument is obvious. Identifying the first principle of painting as a mathematical point seems to be a negative answer to the question, which Leonardo posed in the aforementioned passage, as to whether or not painting is a science. He writes: ‘That mental discourse is termed science that originates in first principles beyond which nothing else can be found *in nature* as part of this science.’¹⁰ The question is: How can an ‘abstract’ quality like the point be part of nature and, therefore, belong to sensory experience?

Punto (‘Point’)

In a brilliant article, Fabio Frosini reconstructed the artist’s ideas about *nulla* (‘nought’), *punto* (‘point’) and *zero* (‘zero’), and located them in classical, medieval and contemporary discourses on mathematical entities, cosmology and metaphysics.¹¹ According to Frosini, Leonardo developed and accepted paradoxical formulas in his definitions of the point mainly for two reasons: firstly, to overcome the categorical difference between mathematics and nature; and secondly, to depart from this traditional juxtaposition in order to develop an ontology that interprets nature as a transition between being and non-being. In his meditations on the paradoxical nature of the point, Leonardo established an interpretive model that permits reflection upon the continuity and, at the same time, discontinuity of bodies. My short essay focuses on the *dynamic* qualities of the point and its connection with Leonardo’s theory of painting. My argument can be summarised in the following way: Leonardo conceives of the point not as a mathematical, non-dimensional entity, or the smallest visible sign, but rather as an infinitesimally small entity that motors the transition of non-being into being, and vice versa. This provides a striking parallel to Leonardo’s theory of painting as a surface oscillating between materiality and immateriality.¹²

In a series of fascinating meditations that can be dated – following Carlo Pedretti¹³ – around 1505–8, and which are found mostly in the Codex Arundel of the British Library (see cat. 37), Leonardo struggles with the paradoxical properties of point and ‘nothingness’. At the end of this breathtaking intellectual exploration, Leonardo defines the point as a third, liminal entity *between* nothing and something. ‘Nothing can be called smaller than the point, and it is the common border (*termine*) of nought and line, it is neither nought nor line, and it does not occupy any space between nought and line. Therefore, the end of nought [!] and the beginning of the line are in contact, but not connected. And in this contact the point is the divider between the continuity of nought and line.’¹⁴ There is not an identity, but an intimate relationship between point, the infinitely small, and nothingness. The point is the ‘brother of nought’ (*Del qual punto el nulla è fratello*).¹⁵

The point ensures, to put it somewhat differently, continuity and discontinuity *at the same time*. As the ‘extension’ of the point, the surface itself oscillates between nothing (pure absence) and something: ‘Air is conjoint with water, and the end of the one is shared with the other, in a way that it could be called continuous quantity because they are connected, and discontinuous because they have two different natures.’¹⁶ Leonardo conceived of the point as a liminal entity between ‘something’ and ‘nothing’, an entity that not only ensures continuity and discontinuity, but actively (*in atto*) oscillates between the two states. In other words, at the core of physical reality, an immanent principle constantly works against non-dimensional ‘unity’ in an effort to achieve ‘multitude’ and also, simultaneously, wears down the distinctions and plurality of the three-dimensional world. The point is, in this view, the chief agent of the continuous drama of a world that ‘contracts’ into the infinitely small, that collapses into the principle of unity, *and* that re-emerges at every infinitely small moment of time and at every infinitely small point of transparent

space. It is through his identification of the point with the principle of movement that Leonardo reintegrates 'nothingness' and nature in her capacity for transformation.

Implicitly critical of his own earlier attempt to ground the *scienza* of painting in the principle of the point as an invisible entity, Leonardo continues:

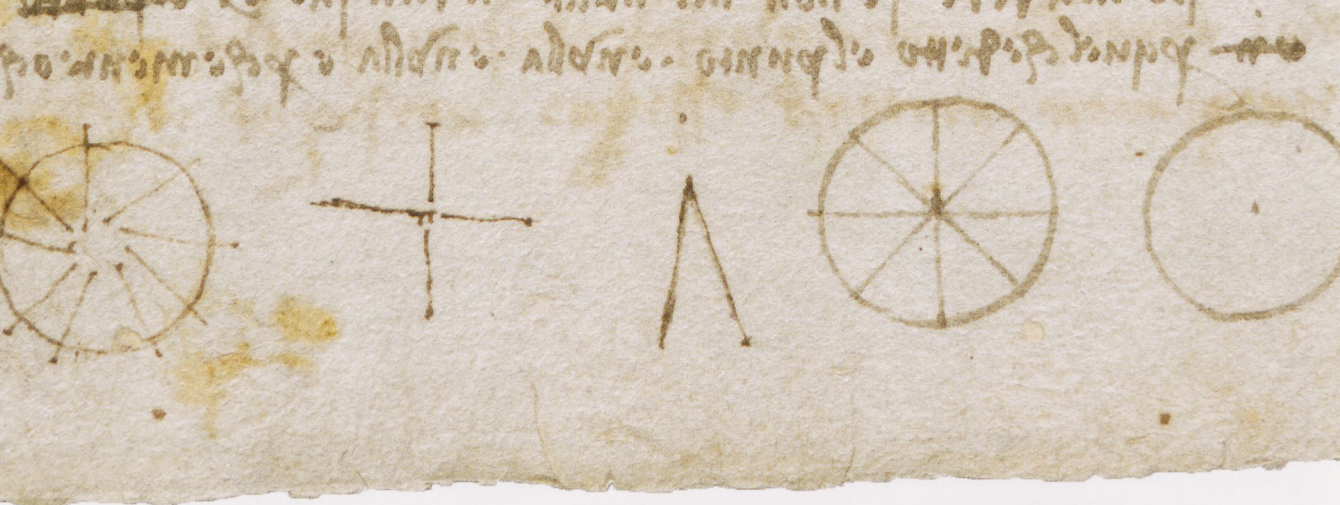
the point is nothing [*nulla*], but on the nothing one cannot build up any *scienza*. And to avoid this principle we will say: Nothing can be smaller than the point, and the line is created by the movement of the point, and its ends are two points, and the surface is generated by the transversal movement of the line, ... and the body is made of movement.¹⁷

But the converse of the argument also holds. In the latter case, privation (i.e. 'nought' in action) *reduces* the dimensionality of bodies, surfaces etc.¹⁸ Therefore, nothing and something are everywhere *gradually* connected by the infinitely small: 'Where nothing ends, the thing is born, and where the thing becomes less, nothing emerges.'¹⁹ Corporeal nature is entirely, permanently permeated by nothing, but *nulla* is – through the borderline activity of the point – continuously 'emanating' into space and time. It is the point that creates and negates the dimensions at the same time.

For Leonardo, the point is, in other words, the power of transition itself – a liminal entity connecting and dividing, a paradoxical being, the all-pervasive 'One', a motor that works against its own being, identical with itself only through permanent self-transcendence. One consequence of Leonardo's meditations on *punto* and *nulla* is quite obvious: without the infinitely small (the point, as a transitional element between being and non-being), there would be no distinction between objects in nature, but also no multitude in the visual field; objects would be indistinguishable, and therefore invisible. Only 'nought' allows for the possibility of vision.²⁰ At the same time, it is true that *no thing* is really visible because of the invisible nature and the lack of extension and quality in points, the final constituents of bodies and time: 'The body is covered by several surfaces, and the surfaces are encircled by lines, and the lines are limited by points... Something that has no borders, has no figure at all.'²¹

Pittura ('Painting')

It is important to realise that what appears to be a mere mind game on Leonardo's part establishes not only optics,²² but also painting itself. No stylistic trait is more characteristic of Leonardo than the blurred boundaries and transparent substances of his soft bodies and surfaces, his almost weightless draperies and vaporous atmospheres. *Sfumato* can indeed be related to Leonardo's observations regarding the complexity of the visual process, particularly in relation to his optical treatise (MS D, Institut de France, Paris) of about 1508. Leonardo adopted the standard conviction of traditional optics, namely that only the 'centric ray' emitting from or reaching to each eye establishes a 'sharp' perception of the object's form. However, following his ideas about the paradoxical status of the point and the non-existence of contour lines, Leonardo went on to challenge this formula. Around 1508, Leonardo came to understand the line of the centric ray as a liminal entity in a continuity of more or less sharp perceptions. More importantly, Leonardo observes that this liminal entity oscillates continuously in order to 'scan' the object's boundaries and internal differentiations of detail.²³ *Sfumato* exaggerates the fact that physical boundaries are 'nothing', paradoxical entities connecting and disconnecting bodies and their surroundings. In this revolutionary view, 'nothing' (the centric ray) meets 'nothing' (the contours of objects), creating a dynamic field of gradual differentiation and a negation of any positive location of forms.²⁴



Brian Rotman compellingly interpreted the organising principle of perspectival space, the vanishing point, as a non-sign and at the same time as the condition of every pictorial sign – like zero in post-medieval Western mathematics.²⁵ But Leonardo goes further. As we have seen, he defines the utmost principle of painting as the infinitely small point that creates the line. *Every* single area of the picture surface is marked by the point. Leonardo discovered one of the most fascinating principles of pictorial representation, namely, that every point of the picture plane is at the same time a positive, material element on a surface *and* transparent to the fictive appearance of represented objects, figures, spaces, etc. The material element of colour is thus added to a material surface in order to transgress material factualness, to become ‘nothing’ (*non è della materia di essa superfittie*) and vice versa.²⁶ This becomes even more evident if the depicted surfaces are themselves ‘nothing’, like Leonardo’s favourite objects of the *paragone* with sculpture – water, veils, dust, mist etc. – or the transitional states of movement.²⁷ In a more formal manner, one could call the point the active agent which distinguishes between *and* connects the two main constituents of any image, (1) its material basis and (2) the appearance of forms, objects, and space.²⁸ Of course, the transition from nought to being embraces, in this perspective, the paradigm of *creatio ex nihilo* and therefore complements Leonardo’s definition of the painter as *signore e dio* of his creations.²⁹

Representing the Point

How could the point be represented – the transition from the invisible to the visible, the place where the positive localisation of objects in space collapses in the regress of the infinitely small? Leonardo added the five diagrams of Codex Arundel f. 132r (cat. 37), mentioned at the beginning of this essay, to a discourse that struggles with the paradoxical identity of all the points of the world. This discourse ends with the assumption that the point as nought occupies no space.³⁰ Read in sequence from right to left, the diagrams reveal Leonardo’s breathtaking dynamics of thought; they are among the most fascinating documents of a historical topology of images.³¹ Leonardo accepts the challenge to represent a thing without extension, something ‘that has no figure at all’, being ‘there’ and ‘not there’ at the same time. The first diagram on the right marks the centre of the circle. As the point of intersection for virtually an infinite number of radii, this central point demonstrates the identity of infinity and unity in the point (second diagram). Each angle produced by any pair of radii converges at its apex in a point, as the third diagram demonstrates, whereby the invisible point of convergence seems to be represented (schematically and paradoxically) above the angle – a dissociation of place (the apex of the angle) and the infinitely small. The fourth diagram reveals that the intersection of lines – here, two orthogonals – marks an infinitely small element

that is at the same time 'part' and 'not ... part' of this picture surface. Therefore, Leonardo avoids the intersection of the lines and adds, instead, what appears to be a minuscule letter 'n', probably for *nulla* (or rather, *fratello del nulla!*). The fifth diagram is even more spectacular. It shows that the radii of the circle, after intersecting its circumference, converge towards an 'empty' centre from which they appear to be moved, or shifted like the spokes of a wheel. The infinite diminutiveness of the point resists any positive localisation and disturbingly annuls the characteristics of Euclidean space. The point is a motor of displacement, non-identity and movement – abyss, emergence and swerve.



1. See Pedretti 1960.
2. 'Se in uno circolo è solo uno punto al quale concorrano infinite linee, e infra ogni 2 linee s'include uno angolo, e ciascuno angolo separato termina 'n un punto, adunque molti angoli hanno molti punti i quali ritornati nel circolo sono equali a un sol punto, cietro d'esso circolo. Onde è manifesto che molti puntu sono equali a uno e uno a molti, la qual cosa non pò acadere se non nel nulla.'
3. See Leonardo da Vinci (Pedretti) 1964.
4. 'E questo si prova col zero over nulla, cioè la decima figura de la arismetrica, per la quale si figura un 0 per esso nullo, il quale, posto dopo la unità, il fa dire dieci [...] e così *infinitamente* crescerà.' Leonardo da Vinci (Pedretti) 1964, § 1 (my emphasis).
5. See Leonardo's drawing in the Royal Collection (RCIN 919151r, K/P 118rB, c. 1508); and Leonardo da Vinci, Codex Arundel, f. 131v (cat. 37). Marinoni 1974, p. 23 underlines the paradoxical properties that connect these geometrical elements, the optical pyramid, nature's imponderable forces, and the soul: "cose spirituali", *incorporee*'. His reading of Leonardo's oppositions from a Neoplatonic perspective has been convincingly challenged by Frosini (2003), esp. pp. 213 and 231.
6. Leonardo da Vinci (Pedretti) 1964, § 31c.
7. See Summers 1990.
8. On the history and philosophy of the point, see Federici-Vescovini 1965, pp. 213–37. See Alberti 2002, I, 2: 'Quae vero intuitum non recipiunt, ea nemo ad pictorem nihil pertinere negabit' (notice the hyperbolic triple negation!). See also Filarete 1890, ff. 173v, 175v; and Piero della Francesca 1984, preface.
9. Euclid (Heath) 1956, Def. 1. For an overview on the mathematics of Leonardo, see Bagni and D'Amore 2006.
10. Leonardo da Vinci (Pedretti) 1964, § 1 (my emphasis). See also Leonardo da Vinci, Codex Atlanticus, f. 784r, Milan, Biblioteca Ambrosiana: 'The point does not occupy any space, and it exists in nature (*si dà in natura*); it is mobile and generates the line'. See also Codex Madrid I, f. 60v, 109v, Madrid, Biblioteca Nacional
11. Frosini 2003.
12. For a significantly extended version of this essay, including the reverberations of Leonardo's speculations on the point in his science (optics, hydrogeology, impetus physics), see Fehrenbach 2015.
13. Pedretti 1960.
14. Leonardo da Vinci, Codex Arundel, f. 159v.
15. Leonardo da Vinci, Codex Arundel, f. 204r. On the genealogical relationship of the 'four powers', see for instance Leonardo da Vinci, MS A, f. 35v, Institut de France, Paris. On music as the sister of painting, see Leonardo da Vinci (Pedretti) 1964, § 29.
16. Leonardo da Vinci, Codex Arundel, f. 130r.
17. Leonardo da Vinci, Codex Arundel, f. 159r. The authoritative discussion of point and line, instant and time, with a focus on Zeno's paradoxes, is in the sixth book of Aristotle's *Physics*: 'impossibile est ut aliquod continuum sit compositum ex indivisibilis: et linea sit composita ex punctis' (*Physics*, Aristotle (Seidl) 1987–8, 231a; see 235b, 237b); for the analogy in time, see *Physics* 234a. On the continuum in Aristotle, see Wieland 1970, pp. 283–5, and Böhme 1974, pp. 99–158. Leonardo, however, seems to follow more closely Euclid, who states in Euclid (Heath) 1956, Def. 3: 'The extremities of a line are points'. On the line generated by the point in motion, see Aristotle (Seidl) 1995, I, 4, 409 a 4. A parallel is provided by Proclus who refers to line as 'flux of the point'; see Beierwaltes 1979, p. 173. Simplicius follows the same argument; see Heath's commentary in Euclid (Heath) 1956, Def. 1, p. 157.
18. 'La conversa. Il punto si genera dove manca la lunghezza della linea; la linea nasce dove finisce la larghezza o lunghezza della superficie, e la superficie è creata nel termine della larghezza, lunghezza e profondità del corpo', in Leonardo da Vinci, Codex Arundel, f. 159r.
19. Leonardo da Vinci, Codex Arundel, f. 159r.
20. See Frosini 2003, p. 222.
21. 'El corpo è vestito di più superficie, e le superficie son circondate di linee, e le linee son terminate da punti.... Ciò che non ha termine, non ha figura alcuna', in Leonardo da Vinci, Codex Arundel, f. 132r.
22. See Fehrenbach 2015, pp. 76–80. Leonardo's formula for the omnipresence of punctiform contractions of *spetie* ('images') in transparent media – *tutto per tutto e tutto in ogni parte* – has strong metaphysical connotations and refers to the indivisible presence of soul in every part of the body; see, with references to Plotinus, Augustine, Cusanus and Ficino, among others, Leinkauf 1993, pp. 58–60.
23. 'È la maestra dell'altre linee, dalle quali sempre essa è mossa d'eterminando quel che l'altre vedano e non cognoscano', in Leonardo da Vinci, MS D, f. 8v, Institut de France, Paris.
24. For a more extended discussion of this argument, see Fehrenbach 2002. For different views on *sfumato*, see esp. Nagel 1993; Prater 1999; Bell 2002.
25. Rotman 1993, pp. 14–22.
26. See the discussion of this aspect of the point in Boehm 2003.
27. On transitional objects in painting and their relationship to Leonardo's notion of the point, see Pedretti 1989.
28. On this distinction in current picture theory, see Pichler and Ubl 2018, pp. 20–42 ('Bildvehikel' versus 'Bildobjekt').
29. Leonardo da Vinci (Pedretti) 1964, § 13. The relationship between Leonardo's theory of the point and artistic creation is considered in Batkin 1988, pp. 167–82.
30. 'Adunque, per quel ch'è detto, el punto è nulla, e nulla è perché niente occupa.'
31. For topology in art, see Pichler and Ubl 2009.

- Werner Beierwaltes, *Proklos: Grundzüge seiner Metaphysik*, 2nd edn, Frankfurt am Main, 1979.
- Janis Bell, 'Sfumato, Linien und Natur', in Frank Fehrenbach (ed.), *Leonardo da Vinci: Natur im Übergang*, Munich: Fink, 2002, pp. 229–56.
- Gottfried Boehm, 'Der Topos des Anfangs: Geometrie und Rhetorik in der Malerei der Renaissance', in Ulrich Pfisterer and Max Seidel (eds), *Visuelle Topoi: Erfindung und tradiertes Wissen in den Künsten der italienischen Renaissance*, Munich: DKV, 2003, pp. 48–59.
- Gernot Böhme, *Zeit und Zahl: Studien zur Zeittheorie bei Platon, Aristoteles, Leibniz und Kant*, Frankfurt am Main: Klostermann, 1974.
- Euclid, *The Thirteen Books of Euclid's Elements*, Thomas L. Heath (ed.), 3 vols, 2nd edn, New York: Dover, 1956.
- Graziella Federici-Vescovini, *Studi sulla prospettiva medievale*, Turin, 1965.
- Frank Fehrenbach, 'Leonardo's Point', in Alina Payne (ed.), *Vision and its Instruments, c. 1350–1750*, University Park, PA: Penn State University Press, 2015, pp. 69–98.
- Frank Fehrenbach, 'Der oszillierende Blick: "Sfumato" und die Optik des späten Leonardo', in *Zeitschrift für Kunstgeschichte* 65(4), 2002, pp. 522–44.
- Filarete: *Antonio Averlino Filarete's Tractat über die Baukunst*, Wolfgang von Oettingen (ed.), Vienna, 1890.
- Fabio Frosini, 'Leonardo da Vinci e il "Nulla": Stratificazioni semantiche e complessità concettuale', in Arturo Calzona (ed.), *Il volgare come lingua di cultura dal Trecento al Cinquecento: Atti del convegno internazionale, Mantova, 18–20 ottobre 2001*, Florence: Olschki, 2003, pp. 209–32.
- Leonardo da Vinci, *Leonardo da Vinci on Painting: A Lost Book (Libro A)*, Carlo Pedretti (ed.), Berkeley: University of California Press, 1964.
- Thomas Leinkauf, *Mundus combinatus: Studien zur Struktur der barocken Universalwissenschaft am Beispiel Athanasius Kirchers SJ (1602–1680)*, Berlin: De Gruyter, 1993.
- Augusto Marinoni, 'L'essere del nulla', in Paolo Galluzzi (ed.), *Leonardo da Vinci, letto e commentato da Marinoni*, Lettura Vinciana I–XII (1960–72), Florence: Giunti, 1974, pp. 7–28.
- Alexander Nagel, 'Leonardo and "sfumato"', *Res* 24, 1993, pp. 7–20.
- Carlo Pedretti, 'Saggio di una Cronologia dei fogli del Codice Arundel di Leonardo da Vinci', in *Bibliothèque d'Humanisme et Renaissance*, 22, Geneva: Droz, 1960, pp. 172–7.
- Carlo Pedretti, 'A Poem to Sculpture', *Accademia Leonardi Vinci*, 2, 1989, pp. 11–39.
- Wolfram Pichler and Ralph Ubl (eds), *Topologie: Falten, Knoten, Netze, Stülpungen in Kunst und Theorie*, Vienna: Turia, 2009.
- Wolfram Pichler and Ralph Ubl, *Bildtheorie zur Einführung*, 3rd edn, Hamburg: Junius, 2018.
- Piero della Francesca, *De prospectiva pingendi*, G. Nicco-Fasola (ed.), Florence: Sansoni, 1984.
- Andreas Prater, 'Sehnsucht nach dem Chaos: Versuch über das Sfumato der Mona Lisa', in

Frank Fehrenbach

- Leon Battista Alberti, *Della pittura* [1435], ed. by Oskar Bätschmann as *Über die Malkunst*, Darmstadt: WBG, 2002.
- Aristotle, *Physics*, ed. by Hans Günter Sekl as *Physik: Vorlesung über die Natur*, 2 vols, Hamburg: Meiner, 1987–8.
- Aristotle, *De anima*, ed. by Horst Seidl as *Über die Seele*, Hamburg: Meiner, 1995.
- Giorgio Tomaso Bagni and Bruno D'Amore, *Leonardo e la matematica*, Florence: Giunti, 2006.
- Leonid Batkin, *Leonardo da Vinci*, Bari: Laterza, 1988.

Ellen Spickernagel et al. (eds), *Ikonomie und Didaktik: Begegnungen zwischen Kunstwissenschaft und Kunstpädagogik. Festschrift für Axel v. Criegern*, Weimar: VDG, 1999, pp. 89–105.

Brian Rotman, *Signifying Nothing: The Semiotics of Zero*, 2nd edn, Stanford, CA: Stanford University Press, 1993.

David Summers, *The Judgment of Sense: Renaissance Naturalism and the Rise of Aesthetics*, Cambridge: Cambridge University Press, 1990.

Wolfgang Wieland, *Die aristotelische Physik: Untersuchungen über die Grundlegung der Naturwissenschaft und die sprachlichen Bedingungen der Prinzipienforschung bei Aristoteles*, 2nd edn, Göttingen: Vandenhoeck und Ruprecht, 1970.