Contents

Chapter 1

Introduction and Overview

Investigating the issue of substitution has a very long tradition in production as well as in consumption theory. For the uncontroversial case of only two inputs, namely *capital* K and *labor* L, HICKS (1932) defined a unique substitution measure σ, called *the* "elasticity of substitution". Until today many possible generalizations have been suggested, e. g. in ALLEN and HICKS (1934), ALLEN (1938), UZAWA (1962), MCFADDEN (1968), BLACKORBY and RUSSELL (1975), and recently in DAVIS and SHUMWAY (1996), with HICKS' (HES), ALLEN's or ALLEN-UZAWA's (AES), HICKS-ALLEN's (HAES), MCFADDEN's (SES) and MORISHIMA's (MES) elasticities of substitution being the most prominent examples. However, even today there seems to be little agreement on how to define a concept measuring the "ease of substitution" of one production factor for another in a multifactor setting.

ALLEN partial elasticities of substitution (AES, ALLEN 1938) have been the most frequently used measures in empirical research on multifactor substitution (HAMERMESH 1993:35). Cross-price elasticities are well-known substitution measures as well. However, they have merely played a minor role in empirical studies, if at all. Both, AES and cross-price elasticities $\eta_{x_i p_j}$ are related by (see e. g. BERNDT and WOOD (1975:261))

$$\mathrm{AES}_{x_i p_j} = \frac{\eta_{x_i p_j}}{s_j}, \qquad \text{where } s_j = \frac{x_j p_j}{C} \tag{1.1}$$

is the cost share s_j of factor j. Yet, expression (1.1) is the "most compelling argument for ignoring the Allen measure in applied analysis ... The interesting measure is [$\eta_{x_i p_j}$] – why disguise it by dividing by a cost share? This question becomes all the more pointed when the best reason for doing so is that it yields a measure that can only be interpreted intuitively in terms of [$\eta_{x_i p_j}$]" (CHAMBERS 1988:95). Similarly, BLACKORBY and RUSSELL (1989:883) criticize that "as a quantitative measure, [AES] has no meaning; as a qualitative measure, it adds no more information to that contained in the (constant-output) cross-price elasticity". As a superior concept, BLACKORBY and RUSSELL (1989) suggest the MORISHIMA elasticity of substitution (MES), developed independently by MORISHIMA (1967) and BLACKORBY and RUSSELL (1975). Ever since BLACKORBY and RUSSELL's (1989) seminal article, the MES has been more and more employed by economists (DAVIS and SHUMWAY 1996:173).

Facing the variety of substitution measures – AES, MES, HES, HAES, SES, and, last but not least, cross-price elasticities – the central question arises which one of these measures should be employed in an empirical study. With respect to the particular issue of the substitutability of *capital* K and *energy* E, for instance, BERNDT and WOOD (1975, henceforth BW75) find *negative* estimates of the cross-price elasticities η_{EK} and η_{KE} as well as of AES$_{EK}$ for U.S. manufacturing (1965). Using BW75's data and estimates, THOMPSON and TAYLOR (1995) calculate *positive* estimates of MES$_{EK}$ and MES$_{KE}$. In this case, while being AES-complements, capital and energy have to be classified as MES-substitutes.

From this example it appears to be indispensable to state the concrete substitution measure employed when one argues that there is a substitution relationship between two production factors. From the perspective of the recipient of such information, the differences in the interpretation of substitution elasticities like MES, AES and cross-price elasticities are to be taken into account when drawing conclusions on the policy implications of such empirical estimates: Recent price shifts in energy by OPEC-cartel arrangements and/or by energy taxes, for instance, might give rise to different conclusions regarding the use of capital in U. S. manufacturing, depending upon whether these conclusions are based on estimates of AES$_{KE}$ or MES$_{KE}$.

By a collection of four papers, this thesis addresses the question of whether the real elasticity of substitution does exist at all and which one of the classical generalizations of HICKS' σ would be the candidate concept. Particularly, the focus is on AES, MES and cross-price elasticities, the trinity of classical substitution measures. It is argued, specifically, that not only AES, but also MES adds no more information to that already contained in cross- and own-price elasticities. A summary of classical substitution elasticities reveals that, first, all classical measures, i. e. , AES, MES, HES, HAES, and SES, build on constant-output cross-price elasticities. In fact, all these measures are mixtures of cross-price elasticities. Second, because constant-output cross-price elasticities *neglect output effects*, this is a common feature of AES, MES, HES, HAES, and SES as well. Thus, this thesis develops the concept of the *generalized cross-price elasticity* (GES), a measure of substitution which explicitly takes ouput effects into account. This concept is deliberately based on cross-price elasticities, since they are the principal ingredients of all classical substitution measures. All papers presented are both theoretically and empirically oriented. The papers are tied together by the common argument that the results of empirical studies are substantially determined by both the choice of the substitution measure employed and the estimation approach pursed. At least, this is true for translog approaches, which are used here exclusively.

The first paper, presented in Chapter 2, analyzes AES and MES in detail. This chapter explores a more general definition of MES than the one conceived by BLACKORBY and RUSSELL (1989), making the interpretation of MES more transparent. This definition provides also insight into the question why two arbitrary inputs are more frequently MES-substitutes than AES-substitutes, a fact already noticed by THOMPSON and TAYLOR (1995:566). Furthermore, the *technical elasticity of substitution* (TES) is suggested as an alternative short-run measure.

The purpose of TES is to appraise changes in the use of one production factor in response to an exogenous shock in the supply of another input, for instance, in the supply of labor due to migration, while all other inputs are fixed in the short term. Thus, estimates of TES should reflect short-run responses which can be gained by estimating production functions – the primal approach –, whereas for MES, for example, the dual approach of estimating dual cost functions is mandatory. The primal approach has the advantage that assumptions like cost minimization, indispensable in duality theory, do not have to be imposed *a priori*.

By reusing the U. S. manufacturing data of the classical study by BW75, estimates of AES, MES, and cross-price elasticities are compared to those of the TES. The results demonstrate that whenever one draws conclusions from empirical studies, for instance, on the particular question of capital-energy substitutability, it is absolutely necessary to clarify with regard to which measure capital and energy are being denoted as substitutes or complements.

The chapter concludes that the information given by cross-price elasticities, the common basis of AES and MES, suffices and that the cross-price elasticity is generally the best substitution measure empirical researchers have in hand so far. It is emphasized, however, that cross-price elasticities as well as AES and MES ignore output or scale effects. As long as a substitution measure, fulfilling such empirically relevant requirements, is not available, the question of BLACKORBY and RUSSELL (1989) has still to be posed: "Will the real elasticity of substitution please stand up?"

Building on Chapter 2's conclusion that, preferably, cross-price elasticities, rather than AES or MES should be the focus of empirical substitution studies, Chapter 3 deals with the well-known, still unresolved capital-energy controversy, in which BERNDT and WOOD (1975, 1979), GRIFFIN and GREGORY (1976, henceforth GG76), and PINDYCK (1979) are seminal studies, while YUHN (1991) and THOMPSON and TAYLOR (1995) are more recent examples. Concentrating on cross-price elasticities turns out to be the key to a reconciliation of the capital-energy debate.

After the oil crises economists have been increasingly interested in the question of capital-enregy substitutability. A substantial number of at least fifty empirical studies of capital-energy substitutability have appeared in the literature (see THOMPSON and TAYLOR 1995:565; for surveys, see KINTIS and PANAS 1989, and APOSTOLAKIS 1990). The overwhelming majority of empirical studies about capital-energy substitution involve the estimation of a translog cost function (SOLOW 1987:605). The results are notably contradictory: While time-series studies like BW75 and ANDERSON (1981) typically find that capital and energy are complements, panel studies like GG76 and PINDYCK (1979) typically classify both as substitutes.

GG76 argue that the major reason for this discrepancy is due to their use of the presumably superior panel data, whereas BW75's finding of complementarity is based on time-series data. That is, GG76 blame the nature of the data to be reason for the discrepancies observed: According to GG76, analyses using panel data should reflect long-run adjustments, while time-series investigations should tend to document short-run reactions. Specifically, short-run elasticity estimates concerning capital and energy are likely to show them as complements, since in the short-run it is not possible to design new equipment to achieve higher energy efficiency. By contrast, in the long-run energy and capital should be expected to be substitutes, leading to positive elasticities when estimated from panel data. Yet, despite many attempts in the literature at resolving this issue, the substitutability between capital and energy and the source of the discrepancies in the results still remain controversial after 25 years.

Chapter 3 offers a straightforward explanation for the capital-energy controversy: Using a static translog approach tends to reduce the issue of factor substitutability to a question of cost shares. Specifically, the magnitudes of energy and capital cost shares are of paramount importance for the sign of the energy-price elasticity η_{Kp_E} of capital. A review of a large number of static translog studies demonstrates that the cost-share argument is empirically far more relevant than the distinction between time-series and panel studies. The ample empirical evidence provided by this review reveals that estimates of η_{Kp_E} are generally located around (positive) cost shares s_E of energy and, typically, are the closer to s_E the higher is the cost share s_K of capital. Thus, a translog study will only under very particular circumstances be able to classify energy and capital as complements: Necessarily, cost shares of both factors have to be small.

In sum, under the cost-share perspective, there is in fact hardly any controversy between time-series studies on the one hand and cross-section and panel studies on the other hand. A somewhat pessimistic message, however, accompanies the cost-share argument: Static translog approaches are limited in their ability to detect a wide range of phenomena. Specifically, the data simply have no chance of displaying complementarity for energy and capital if the cost shares of these factors are sufficiently high. More generally, in any translog study, estimated cross-price elasticities $\eta_{x_ip_j}$ of any factor i with respect to the price p_j of another factor j are predominantly determined by the cost share of that factor j whose price is changing. In consequence, pursuing a translog approach will not be as flexible as one might hope. Rather, estimation results are predetermined more or less by given cost shares. Finally, in the light of this argument, differences in estimation techniques, in translog specifications or in data aggregation methods, typically blamed to cause the discrepancies across the opposing studies of the controversy, turn out to be of minor importance. Besides cost-share data, it is the translog approach which in fact determines the estimation results.

The question remains why this simple explanation has not been found earlier. The answer is that almost all studies involved in the controversy employ AES, while, in line with Chapter 2, Chapter 3 deliberately focuses on cross-price elasticities, specifically on η_{Kp_E}. From a closer inspection of the expression for the cross-price elasticity η_{Kp_E} for translog cost functions (see BW75),

$$\eta_{Kp_E} = \frac{\beta_{EK}}{s_K} + s_E, \tag{1.2}$$

one has to presume that η_{Kp_E} is close to the cost share of energy if the cost share of capital K is large relative to the second-order coefficient β_{EK}. If the translog cost function specializes to the COBB-DOUGLAS function, in particular implying $\beta_{EK} = 0$, $\eta_{K p_E}$ is even equal to the cost share of energy.

Additionally, the cost-share argument is supported by dropping data on materials use M and comparing elasticity estimates from a KLE-data base with those originating from KLEM-data. Since the cost shares of the other factors will change considerably if factor M is dropped from the analysis, the estimates of cross-price elasticities should be very sensitive towards inclusion or exclusion of data on materials use. In fact, all elasticity estimates presented in Chapter 3 unequivocally tend to increase upon the exclusion of M, specifically making a positive estimate of $\eta_{K p_E}$ and, hence, the finding of substitutability more likely.

The issue of in- or excluding a non-negligible factor – like M in Chapter 3's substitution study – is intimately related to the pivotal notion of *separability*. In empirical work, the principal purpose of an appropriate concept of separability is to justify the omission of variables for which data are of poor quality or even unavailable. Chapter 4 addresses the empirically relevant issue of whether the omission of a non-negligible factor such as energy after the oil crises affects the conclusions about the ease of substitution among remaining non-energy factors. Throughout, the intuition about separability pursued is that the ease of substitution between two factors should be unaffected by a third factor, from which those factors are assumed to be separable (see e. g. HAMERMESH 1993:34). This chapter develops a novel concept of separability which is – in line with Chapter 2 – based on the idea that the ease of substitution is preferably to be measured in terms of cross-price elasticities.

Due to the lack of (high-quality) data, empirical studies investigating the issue of factor substitution between K, L and M for German manufacturing, for example, typically do not incorporate the factor energy. RUTNER (1984), STARK (1988), KUGLER *et al.* (1989), and FLAIG and ROTTMANN (1998) are but a few examples. In order to justify the omission of energy, these authors typically invoke a standard notion of separability that has been researched thoroughly in economic production *theory*. There, the principal purpose of the notion of separability is to form a conceptual basis for the idea of sequential decision making. Inadvertently, though, those German studies implicitly build on an assumption of separability of energy from non-energy inputs which focuses on the conservation of the ease of substitution among non-energy inputs, rather than on sequential decision processes. When measuring the ease of substitution among non-energy inputs by estimating their cross-price elasticities, for example, these estimates should still remain correct in spite of omitting the factor energy. It transpires from this discussion that if we want to understand under what conditions energy, specifically, can be omitted safely from an empirical analysis, we need a clear notion of the empirical consequences involved in assuming separability.

The concept of separability is investigated in this chapter with respect to both theoretical and empirical aspects. A theoretical analysis provides clarification of the rigid nature of the classical separability definition formulated by BERNDT and CHRISTENSEN (1973, henceforth BC73). It is demonstrated that BC73's conditions lead to quite different implications regarding substitution issues in primal and dual contexts. In contrast to the previous literature, the chapter thus distinguishes *primal* from *dual* separability: Two factors i and j are *primally* (*dually*) BC73-separable from factor k if and only if their marginal rate of substitution (their input proportion x_i/x_j) is unaffected by the input level of k (the price of factor k).

However, rather than by marginal rates of substitution or input proportions, the overwhelming majority of empirical substitution studies analyzes the ease of substitution between two factors on the basis of AES or MES. In consequence, when empirical analysts – as in numerous studies – invoke the assumption of BC73-separability in order to justify the omission of a non-negligible input factor from their analysis, but then proceed to express their results in terms of, say AES, they base their empirical work inadvertently on an insufficient assumption. Therefore, this chapter criticizes BC73's separability definition to be of limited relevance for empirical studies – notwithstanding its important role in the conceptual justification of stepwise optimizing decisions in production theory – and suggests a practically more important definition of separability based on *cross-price elasticities*, which is called *empirical dual separability*.

Two factors i and j are defined to be empirically dual separable from factor k if and only if both cross-price elacticities, $\eta_{x_i p_j}$ and $\eta_{x_j p_i}$, are unaffected by the price of factor k. This definition incorporates the definition of dual BC73-separability, but is more restrictive. That means that even if K and L, for example, were BC73-separable from the factor energy, this would nevertheless not imply that the ease of substitution between K and L in terms of cross-price elasticities remains unaffected by E. Therefore, even if K and L were BC73-separable from E, omitting energy from the data base might be unjustified under empirical aspects. When omitting economically relevant, but not empirically separable factors like energy from the analysis, researchers generally risk to find incorrect cross-price elasticities $\eta_{K p_L}$ and $\eta_{L p_K}$.

By applying the definition of empirical dual separability to a translog cost function, it turns out that empirical dual separability of factors i and j from factor k holds globally if and only if

$$\beta_{ik} = \beta_{jk} = 0$$

for the second-order coefficients of the translog cost function. These conditions are the exact linear separability conditions which are sufficient, but not necessary for dual BC73-separability. Thus, DENNY and FUSS (1977) are perfectly right in claiming that exact linear separability conditions are more restrictive than necessary for dual BC73-separability. However, this chapter argues that only these restrictive conditions capture a notion of separability of factors i and j from factor k which has clear empirical content. Hence, by coining the notion of empirical dual separability, the exact linear separability conditions are rehabilitated.

In a concrete application of these concepts to German manufacturing data

(1978-1990), it is found that classical $[(K, L, M), E]$- as well as $[(K, L), (M, E)]$-separability according to BC73, and, hence, separability according to the definition of empirical dual separability has to be rejected across all models, approaches and scenarios employed. These results cast doubt on prior empirical KLM-studies for German manufacturing.

Chapter 5, finally, provides a summary of classical substitution elasticities and demonstrates that all these measures are, first, mixtures of constant-output cross-price elasticities and, second, ignore output effects. Therefore, in order to take ouput effects into account, Chapter 5 develops the concept of the *generalized cross-price elasticity* (GES), a measure of substitution which deliberately builds on constant-output cross-price elasticities, the principal ingredients of all classical substitution elasticities.

Because HICKS' σ served as a conceptual orientation, all of its classical generalizations retain the maintained hypothesis that output is constant. However, it is frequently problematic to ignore output effects. Oil price shifts, for instance, tend to have a severe impact on the level of economic activity. Thus, in general, any substitution measure with clear empirical content has to incorporate both pure (net) substitution *and* output effects. Correspondingly, any empirical study of factor substitutability which intends to predict the consequence of exogenous price shifts of one factor on the demand for another has to measure gross, rather than net substitution. Yet, emphasis in virtually all applied research has been on the conceptual characterization and estimation of net substitution, excluding output effects from the analysis despite their paramount importance.

Apparently, the factor ratio elasticity of substitution (FRES) derived by DAVIS and SHUMWAY (1996) has been the only empirical substitution measure so far developed which takes account of output effects. Unfortunately, the estimation of FRES requires industry data including profits, which are not easily available. This might be the major reason that FRES has been ignored in applied analysis. Yet, even FRES was only conceived as a generalization of MES, measuring the relative change of proportions of two factors due to a relative change in the price of one of these factors. An empirical assessment of output effects on factor demand was not intended. Because cross-price elasticities are often more relevant in terms of economic content than MES, the basis of FRES (see Chapter 2), the novel concept of the *generalized cross-price elasticity* (GES) is based on constant-output cross-price elasticities, which measure the relative change of one factor due to price changes of another one.

In an application to translog approaches, the empirical relevance of distinguishing between classical cross-price elasticities and their respective generalizations is checked on the basis of U.S. manufacturing data from the classical study by BW75. The concrete way for generalizing classical cross-price elasticities depends on the economic experiment to be described: Whether factor substitutability is to be estimated for profit-maximizing firms under perfect competition, for example, or for an industry which maximizes output subject to a constant-cost constraint requires different generalizations. Similar considerations (see MUNDLAK 1968:234) pertain to the question of which underlying demand function – the HICKSian or the MARSHALLian demand function – might be the appropriate basis for the GES.

MUNDLAK's point is exemplified by developing conrete analytical expressions of the GES for exactly those two artificial experiments. This supports FUSS, MCFADDEN and MUNDLAK (1978:241), who already formulated the obituary of an omnipotent substitution elasticity: "There is no unique natural generalization of the two factor definiton ... We conclude that the selection of a particular definition should depend on the question asked".

Chapter 2

Interpreting Allen, Morishima and Technical Elasticities of Substitution.

A Theoretical and Empirical Comparison

Abstract. Whereas the estimation of ALLEN elasticities of substitution (AES) has dominated the analysis of substitution possibilities between production factors such as capital and energy for a long time, MORISHIMA elasticities of substitution (MES) are the focus of more recent studies. This paper provides a theoretical summary of both measures and presents a more general definition of MES that makes its interpretation transparent and, therefore, allows a comparison with AES: Due to the very definitions of AES and MES two arbitrary inputs are more frequently MES-substitutes than AES-substitutes. The classical study of US manufacturing by BERNDT and WOOD (1975) classifies capital and energy as (AES-)complements. Their data are used here to illustrate the differences between AES and MES. Capital and energy, in particular, turn out to be (MES-)substitutes. Furthermore, to provide an alternative for the analysis of short-run effects technical elasticities of substitution (TES) are introduced as a two-dimensional quantity-oriented concept.

2.1 Introduction

Although substitution is a central issue in both consumer and production theory, "even today there appears to be little agreement about the way [this] concept [is] to be defined (FRENGER 1994:1). For the uncontroversial case of only two inputs, namely labor and capital, HICKS (1932) originally defined the unique substitution measure called "*the* elasticity of substitution". Since then many different generalizations of this fundamental concept up to an arbitrary number of inputs have been provided by e. g. ALLEN and HICKS (1934), ALLEN (1938), UZAWA (1962), MCFADDEN (1963), MORISHIMA (1967), BLACKORBY and RUSSELL (1975), and recently DAVIS and SHUMWAY (1996). Which one of these measures is employed in an empirical production study determines the type of substitution that will be captured.

This paper analyzes in detail the ALLEN (1938) partial elasticities of substitution (AES), the most used measures of substitutability in the production literature (FRENGER 1994:4, HAMERMESH 1993:35), and the MORISHIMA elasticities of substitution (MES), developed independently by MORISHIMA (1967) and BLACKORBY and RUSSELL (1975). This concept has been more and more employed by economists (DAVIS and SHUMWAY 1996:173). A more general definition of MES explored in this paper makes the interpretation of MES transparent. It specializes to the original definitions when the dual cost function approach is applied. This definition provides insight into the question why two arbitrary inputs are more frequently MES-substitutes than AES-substitutes, a fact already noticed by THOMPSON and TAYLOR (1995:566).

By reusing the U. S. manufacturing data of the classical study by BERNDT and WOOD (1975, henceforth BW75), for which energy and capital turn out to be (AES-)complements with statistical significance for the whole sample period (1947-1971), we find that energy and capital are indeed to be denoted as (MES-)substitutes over the entire period. Our study extends the note of THOMP-

SON and TAYLOR (1995:566), which derives this result merely for the single year 1965 and, moreover, only provides a point estimate of MES, but no standard error. Our results demonstrate that whenever one draws conclusions from empirical studies, it is indispensable to clarify with regard to which measure two inputs are being denoted as substitutes.

In addition, we compare estimates of AES, MES and cross-price elasticities to the *technical elasticity of substitution* (TES), suggested in this paper as a measure of technical substitution derived from the marginal rate of technical substitution. The purpose of TES is to appraise changes in the use of one production factor in response to an exogenous shock in the supply of another input, for instance in the supply of labor due to migration, while all other inputs are fixed in the short term. Thus, TES should reflect short-run responses. dictated solely by production technology, implying that estimates of TES can be gained by estimating production functions, whereas for MES and other measures the dual approach of estimating dual cost functions is mandatory. The primal approach has the advantage that assumptions like production cost minimization, indispensable in duality theory, do not have to be imposed *a priori*. A special appeal of the TES is its easy applicability in empirical studies in connection with translog production functions.

Section 2 proposes the new measure TES and compares it with the classical elasticity of substitution of HICKS, representing the basis of AES and MES. Section 3 comprises a survey of AES and MES and emphasizes their advantages and disadvantages. In Section 4, we compare the estimates of AES, MES, the TES and cross-price elasticities, calculated from the manufacturing data of BW75. Section 5 concludes.

2.2 Measures of Technical Substitution

Throughout Section 2.2 we assume that apart from two inputs all other production factors of a given technology, represented by a smooth production function $f(x_1, x_2, \ldots, x_n)$, are fixed and only the quantities of two inputs, say x_i and x_j, can be changed while holding output constant. This special "two-dimensional" case could be considered as a *short-run* response to varied production conditions. For example, for a capital-intensive branch of industry whose essential production factors are capital, labor and energy, it might be impossible to alter its capital endowment in a short period of time, but it is possible to vary its labor and energy use when it suddenly faces higher energy prices or energy scarcities.

Technical substitution measures, conceiving the problem of factor substitution rather as a technical issue, were formulated already long time ago: "Apparently the first published empirical paper attempting to measure substitution elasticities among inputs ... was an article by the Nobel Laureate RAGNAR FRISCH (1935), who sought to measure input substitution possibilities in the chocolate-manufacturing industry by estimating a substitution coefficient" (BERNDT 1991:452), known as the marginal rate of technical substitution r. This most simple measure of technical substitution is presented in the subsequent Section 2.2.1, the TES in Section 2.2.2, and σ, the classical elasticity of substitution of HICKS, the most common measure of technical substitution, is exhibited in Section 2.2.3.

2.2.1 The Marginal Rate of Technical Substitution

Ignoring any scale effects, pure effects of (technical) substitution are clearly determined by the shape of isoquants. Thus, one conceivable measure of *technical* substitution is the *slope* of an isoquant. Its negative value is well-known in the economic literature and denoted as *marginal rate of technical substitution* r (see

e. g. CHIANG 1984:419):

$$r := -\frac{\partial x_j}{\partial x_i}. \tag{2.1}$$

In general, the sign of the isoquant slope could be either positive or negative. In the two-dimensional case the sign of r is positive (see e. g. VARIAN 1992:12) and technical substitution is depicted as in Figure 2.1: A reduction in the quantity x_j of input j forces a rise in factor quantity x_i in order to hold output constant. This case, in which inputs i and j are technical substitutes, is frequently referred to as the "normal case" (see e. g. VARIAN 1992:12). Since (with positive prices) technical complementarity, i. e. $r < 0$, is outside the realm of possibilities, the only open question is that about its magnitude.

By differentiating the isoquant condition $f(x_1, \ldots, x_i(x_j), \ldots, x_n) = constant$ for x_j, an equivalent, and also well-known expression for r is obtained for the case that only factors i and j are variable:

$$f_{x_i}\frac{\partial x_i}{\partial x_j} + f_{x_j} = 0 \qquad \Longleftrightarrow \qquad r = -\frac{\partial x_i}{\partial x_j} = \frac{f_{x_j}}{f_{x_i}}. \tag{2.2}$$

In the next section, the TES is defined on the basis of the right expression of (2.2) .

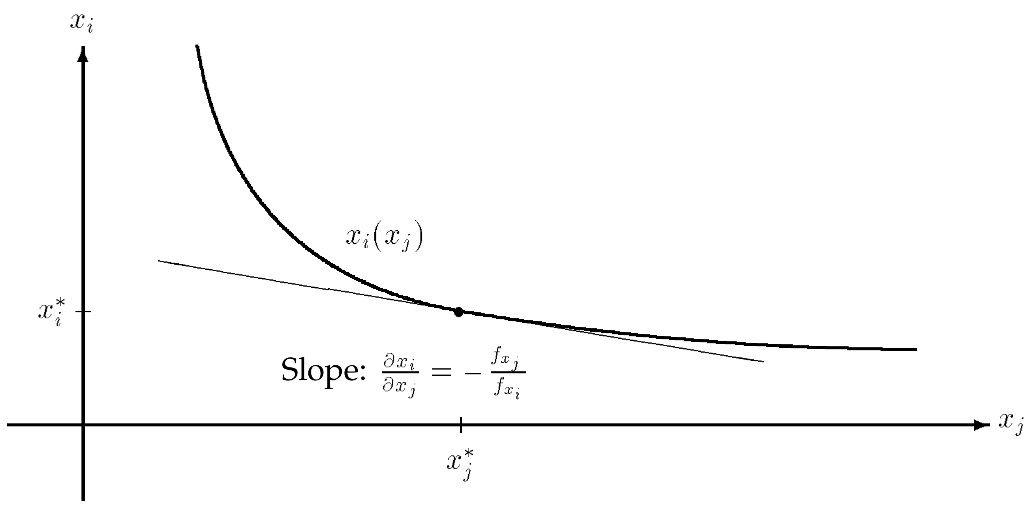


Figure 2.1: The marginal rate of technical substitution.

If, instead, three inputs i, j and k are variable, a slight generalization of $-\frac{\partial x_i}{\partial x_j}$ could be derived from $f(x_1, \ldots, x_i(x_j), x_j, x_k(x_j), \ldots, x_n) = constant$ by differentiating for x_j:

$$f_{x_i}\frac{\partial x_i}{\partial x_j} + f_{x_j} + f_{x_k}\frac{\partial x_k}{\partial x_j} = 0 \qquad \Longleftrightarrow \qquad -\frac{\partial x_i}{\partial x_j} = \frac{f_{x_j}}{f_{x_i}} + \frac{f_{x_k}}{f_{x_i}}\frac{\partial x_k}{\partial x_j}. \tag{2.3}$$

In contrast to the two-dimensional case, depending upon the magnitudes of the partial derivatives $f_{x_i}, f_{x_j}, f_{x_k}$, and upon whether $\frac{\partial x_k}{\partial x_j}$ is negative, a positive slope of the (projection) curve $x_i(x_j)$ can not be excluded, that is, $-\frac{\partial x_i}{\partial x_j}$ might also be negative. For example, an isoquant sphere is curved in Figure 2.2 such that the projection into the two-dimensional quantity plane of input i and j of a path from point x$^* = (x_i^*, x_j^*, x_k^*)$ to x̄ $= (\bar{x}_i, \bar{x}_j, \bar{x}_k)$ has a positive slope. The positive slope

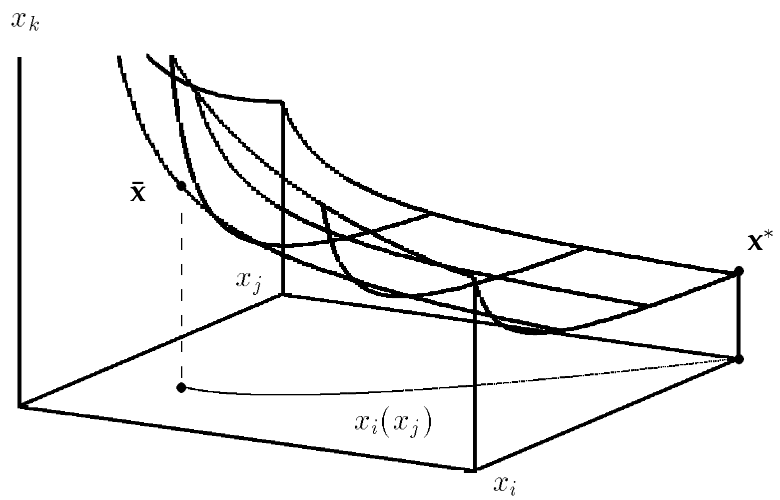


Figure 2.2: A case, where two inputs i and j behave as technical complements.

indicates that the quantity changes of input i and j are complementary (they both decline) when moving from x* to x̄. This is only possible because a compensating increase in input k holds output constant. But, instead of ending in point x̄ when production conditions are changing in x*, production might also take place in any other point of the isoquant sphere in Figure 2.2, perhaps in one such that the corresponding projection curve $x_i(x_j)$ has a negative slope and inputs i and j behave as substitutes.

For three or more variable inputs production technology alone does not determine uniquely where the new optimal production point is located under varied production conditions, but fixed output. Besides the technological framework, a further criterion like profit maximization or cost minimization is required, which eliminates ambiguity and provides information about the new optimal production point. For this reason, technical substitution measures solely based on the production technology are only appropriate in a two-dimensional short-run case.

2.2.2 The Technical Elasticity of Substitution

Multiplying the marginal rate of technical substitution in (2.2) by $\frac{x_j}{x_i}$ leads to the definition of the TES,

$$\text{TES}_{ij} := -\frac{x_j}{x_i} \cdot \frac{\partial x_i}{\partial x_j} = \frac{x_j}{x_i} \cdot \frac{f_{x_j}}{f_{x_i}}. \tag{2.4}$$

Rather than measuring *absolute* changes in two factor quantities like r, the TES quantifies *relative* changes, which is more desirable in most cases. Note that there is no question about the sign of TES in the two-dimensional "normal case" of Figure 1: TES_{ij} is positive, documenting the technical substitution relationship between two solely flexible inputs i and j.

The purpose of TES is to infer the short-run relative input change of a perfectly elastic production factor in response to an exogenous one percent shock in the supply of another input while all other inputs are fixed. For example, if labor supply to the market is perfectly elastic, then it is reasonable to ask what will happen to employment when the completely inelastic supply of a second factor, say energy, is changed exogenously while a third factor, say capital, is fixed. This would hardly be an unlikely scenario in the short term. Then, quantity-quantity effects for the two factors labor and energy can be appraised by TES solely from production technology without imposing additional assumptions like cost minimization. TES estimates therefore reflect the polar case in which substitution possibilities are dictated completely by technology, whereas estimates of cross-price elasticities from factor demand relations, implying optimality assumptions, reflect a case in which substitution happens under more flexible conditions, where more than two inputs may change and production takes place in accordance with both production technology and optimality goals.

To facilitate estimation, we transform definition (2.4) of TES into

$$\mathrm{TES}_{ij} = \frac{x_j}{x_i} \cdot \frac{f_{x_j}}{f_{x_i}} = \frac{x_j/f}{x_i/f} \cdot \frac{f_{x_j}}{f_{x_i}} = \frac{\dfrac{\partial \ln f}{\partial \ln x_j}}{\dfrac{\partial \ln f}{\partial \ln x_i}}. \tag{2.5}$$

This form will be particular suitable if the production technology is described by a translog (short for transcendental logarithmic) production function, originated in CHRISTENSEN *et. al.* (1971). It is commonly written in the shape (see e. g. GREENE 1993:209)

$$\ln f = \alpha_0 + \sum_{i=1}^{n} \alpha_i \cdot \ln x_i + \frac{1}{2} \cdot \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_{ij} \ln x_i \ln x_j, \tag{2.6}$$

where the following symmetry is imposed: $\alpha_{ij} = \alpha_{ji}$ for all i, j. Given these symmetry constraints for α_{ij}, the translog production function can readily be recognized as a second-order approximation of an arbitrary production function around the unit vector.[1] The translog model is a generalization of the COBB-DOUGLAS model, the most fundamental production model, which can be obtained from (2.6) as a special case by setting $\alpha_{ij} = 0$ for all i and j. The popularity of the translog concept builds on the advantage that it relaxes the COBB-DOUGLAS implications of an unitary elasticity of substitution σ (see the next section), and, furthermore, the CES-implication that all production factors have to be substitutes.

Using the last term of (2.5), the translog function (2.6) allows a comfortable calculation of TES_{ij}:

$$\mathrm{TES}_{ij} = \frac{\alpha_j + \alpha_{jj} \ln x_j + \sum_{k \neq j} \alpha_{kj} \ln x_k}{\alpha_i + \alpha_{ii} \ln x_i + \sum_{k \neq i} \alpha_{ki} \ln x_k}. \tag{2.7}$$

Note that TES is asymmetric not only in the special case (2.7), but in general: TES_{ij} will not be equal to TES_{ji}. As an example, the TES applied to the COBB-DOUGLAS

[1] Appendix A proves that whether the TAYLOR-series expansion is carried out around any point $(\bar{x}_1, \bar{x}_2, ..., \bar{x}_n)$ or around unity leaves the TES unchanged.

production function

$$f(x_1, x_2, ..., x_n) = A \cdot x_1^{\alpha_1} \cdot x_2^{\alpha_2} ... \cdot x_n^{\alpha_n} \qquad (A, x_1, x_2, ..., x_n > 0) \tag{2.8}$$

is constant, but generally not equal to unity:

$$\text{TES}_{ij} = \frac{\alpha_j}{\alpha_i}. \tag{2.9}$$

Intuitively, technical substitution possibilities between inputs i and j should be different from those between j and i and inversely dependent upon their output elasticities α_i, α_j. This intuition is confirmed by (2.9): If input j has an output elasticity α_j which is much larger than the output elasticity α_i of input i, in order to hold output constant, an exogenous 1 % reduction in the quantity of input j has to be compensated by an α_j/α_i percent increase of the quantity of input i, being much larger than 1 %.

When employing this translog approach in order to estimate the TES from empirical data in Section 2.4, the conditions of the following definition have to be tested: The (twice differentiable) translog production function (2.6) can be denoted as *well-behaved* if

a) $$\frac{\partial f}{\partial x_i} = \frac{f}{x_i}\frac{\partial \ln f}{\partial \ln x_i} = \alpha_i + \alpha_{ii} \ln x_i + \sum_{k \neq i} \alpha_{ki} \ln x_k > 0, \text{ (pos. monotonicity)} \tag{2.10}$$

b) $$A = (\frac{\partial^2 f}{\partial x_i \partial x_j}) \text{ is negative semidefinite}, \qquad \text{(concavity)} \tag{2.11}$$

where[2]

$$\begin{aligned}
\frac{\partial^2 f}{\partial x_i \partial x_j} &= \frac{f}{x_i x_j}\{(\alpha_i + \alpha_{ii} \ln x_i + \sum_{k \neq i} \alpha_{ik} \ln x_k)(\alpha_j + \alpha_{jj} \ln x_j + \sum_{k \neq j} \alpha_{jk} \ln x_k) + \alpha_{ij}\}, \\
\frac{\partial^2 f}{\partial x_i^2} &= \frac{f}{x_i^2}\{(\alpha_i + \alpha_{ii} \ln x_i + \sum_{k \neq i} \alpha_k \ln x_k)^2 - (\alpha_i + \alpha_{ii} \ln x_i + \sum_{k \neq i} \alpha_{ik} \ln x_k) + \alpha_{ii}\}.
\end{aligned}$$

For COBB-DOUGLAS functions the convexity of isoquants is an intrinsic quality due to their strict concavity. As well, positive monotonicity is globally given,

[2]Rather than well-behaved, BLACKORBY, PRIMONT and RUSSELL (1978:15,294) denote a production function f as regular if f is continuous, and positive monotonicity and quasi-concavity of f, as well ensuring convexity of isoquants, are fulfilled.

that is, for each production point. For functional forms such as the translog form (2.6), however, these properties can neither supposed to be valid *a priori* from the analytical form nor are they given globally, that is, they have to be verified for each observation vector.

By contrast to the TES, HICKS' substitution elasticity σ unambiguously implies a symmetric characterization of substitution possibilities. For instance, in the COBB-DOUGLAS case σ equals unity, irrespective of the concrete values α_i, α_j. HICKS' elasticity σ was the only one considered for a long time – and has therefore been called *the* elasticity of substitution. Since it is the fundamental basis of ALLEN's and MORISHIMA's partial elasticities of substitution, HICKS' substitution elasticity is now studied in detail.

2.2.3 The Elasticity of Substitution

The elasticity of substitution σ, originally introduced by HICKS (1932) for the analysis of factor shares of labor and capital, is defined as the ratio of the relative change in factor proportions to the relative change in the marginal rate of technical substitution r, that is, to the relative change in the slope of the isoquant:

$$\sigma := \frac{\frac{x_j}{x_i}\partial(\frac{x_i}{x_j})}{\partial r/r} = \frac{\partial \ln(\frac{x_i}{x_j})}{\partial \ln r}. \tag{2.12}$$

The change in slope is associated in turn with the isoquant's curvature κ, since κ is a multiple of the second derivative of the isoquant (the multiplier is $(1+r^2)^{-3/2}$). So, definition (2.12) implicitly contains an inversely proportional relationship between the substitution elasticity σ and the curvature κ, which is expressed explicitly in the equivalent formula (ALLEN 1934:342)

$$\sigma = r \cdot (1+r^2)^{-3/2} \cdot \frac{r \cdot x_j + x_i}{x_i \cdot x_j} \cdot \frac{1}{\kappa}. \tag{2.13}$$

According to (2.13), very 'shallow' isoquants will ceteris paribus exhibit large substitution effects, whereas very sharply curved isoquants will display relatively small substitution effects. On the basis of (2.13), the sign of σ is positive when an isoquant is convex to the origin, as it is the case for well-behaved production functions, and negative when the isoquant is concave, provided that the normal case $r > 0$ is given. For the sake of analyzing short-run (two-dimensional) substitution aspects, HEATHFIELD and WIBE (1987:112) suggest calculating σ with the help of translog production function (2.6).[3]

With $r = f_{x_j}/f_{x_i}$, definition (2.12) could be formulated alternatively as

$$\sigma = \frac{\partial \ln(\frac{x_i}{x_j})}{\partial \ln(\frac{f_{x_j}}{f_{x_i}})}. \tag{2.14}$$

For an arbitrary COBB-DOUGLAS function (2.8), σ is always equal to unity, since

$$\ln \frac{f_{x_j}}{f_{x_i}} = \ln \frac{\alpha_j}{\alpha_i} + 1 \cdot \ln \frac{x_i}{x_j}. \tag{2.15}$$

In order to construct possible generalizations of σ, it shall be redefined in a third way as the ratio of the relative change in factor proportions to the relative change in factor prices:

$$\sigma = \frac{\partial \ln(\frac{x_i}{x_j})}{\partial \ln(\frac{p_j}{p_i})} = \frac{\frac{x_j}{x_i}\partial(\frac{x_i}{x_j})}{\frac{p_i}{p_j} \cdot \partial(\frac{p_j}{p_i})}. \tag{2.16}$$

Under the assumptions of perfect competition and profit maximizing firms, $\frac{f_{x_j}}{f_{x_i}}$ equals relative factor prices p_j/p_i, and (2.14) and (2.16) are identical. In this two-dimensional case, the change in price proportions can be normalized: Without any loss of generality, (2.16) can be calculated as if only the price p_j changes and p_i is constant. A similar normalization will be applied for the derivation of MES in the n-dimensional case (see Section 2.3.2).

Definition (2.16) serves as a basis for AES and MES, the two most popular generalizations of σ, critically summarized in the next section. In particular, it will be specified clearly which variables are allowed to vary and which are held

[3] This approach would offer a comparison of the estimation results of the TES and σ, but it is not further pursued in this note.

constant, because "a great deal of confusion in the discussion about the elasticity of substitution has been arisen from [this] failure" (FRENGER 1994:5).

2.3 A Theoretical Comparison of AES and MES

In a multifactor setting, one needs to generalize the characterization of substitution possibilities to the case where more than two factors are adjusted at a time. Two prominent concepts proposed in the literature are AES and MES. Their development was inspired by HICKS' argument that a concept of substitution should reflect the "ease of substitution" between factors by measuring the curvature of a level surface such as the isoquant or the factor-price frontier (FRENGER 1994:1). However, following BLACKORBY and RUSSELL (1989) it is explained in Section 2.3.1 why AES fails this litmus test, because it is neither a measure of the curvature of isoquants nor of the factor-price frontier. Furthermore, it is demonstrated that while MES satisfies the first criterion, it fails to be a measure of the curvature of a factor-price frontier (Section 2.3.2). In order to make more transparent what is measured by MES and what aspect distinguishes it most from the AES, the MES will be embedded in a definition inspired by DAVIS and SHUMWAY (1996). It is more general compared to those of Morishima (1967) and BLACKORBY and RUSSELL (1975).

2.3.1 AES

HAMERMESH (1993:35), who provides an overview about essays in major economics journals from 1965 to 1990 addressing substitution issues, finds that "[t]he measure [AES_{ij}] has been used extensively in empirical research on multifactor substitution". With particular respect to capital-energy substitution, THOMPSON and TAYLOR (1995:565) claim that in virtually all studies about this issue over the last twenty years conclusions were based on estimates of AES, giving rise to controversy: While cross-sectional and panel studies suggest that capital and energy are substitutes, time-series studies suggest the converse.

For a profit-maximizing firm acting under perfect competition and for a two-factor production function ALLEN (1934:372-373) related HICKS' substitution elasticity σ to the cross-price elasticity $\eta_{ij} := \frac{\partial \ln x_i}{\partial \ln p_j}$,

$$\eta_{ij} = \frac{x_j \cdot p_j}{Y \cdot p} \cdot \sigma = s_j \cdot \sigma \ , \tag{2.17}$$

where Y is the output produced, p denotes the output price and $s_j = \frac{x_j \cdot p_j}{Y \cdot p}$ denotes factor j's share to total revenue. Specifically, for linear-homogeneous production functions s_j is equal to $\frac{x_j p_j}{C}$, the total cost share of factor j.

In a multifactor setting, the AES can then be developed as a potential generalization of σ (see e. g. SATO and KOIZUMI 1973:49): Using the dual approach via a cost function $C = C(Y, p_1, ..., p_n)$ by SHEPHARD's lemma, $C_i := \frac{\partial C}{\partial p_i} = x_i$, the factor demand elasticity η_{ij} can be transformed into

$$\eta_{ij} = \frac{\partial \ln x_i}{\partial \ln p_j} = \frac{p_j}{x_i} \cdot \frac{\partial x_i}{\partial p_j} = \frac{p_j}{x_i} \cdot \frac{\partial C_i}{\partial p_j} = \frac{p_j}{x_i} \cdot C_{ij}, \tag{2.18}$$

where C_{ij} is an abbreviation of the second partial derivative $\frac{\partial^2 C}{\partial p_i \partial p_j}$. Completing (2.18) by x_j and C and utilizing SHEPHARD's lemma again leads to

$$\eta_{ij} = \frac{p_j \cdot x_j}{C} \cdot \frac{C \cdot C_{ij}}{x_i \cdot x_j} = \frac{p_j \cdot x_j}{C} \cdot \frac{C \cdot C_{ij}}{C_i \cdot C_j} = s_j \cdot \frac{C \cdot C_{ij}}{C_i \cdot C_j}. \tag{2.19}$$

Comparing (2.19) and (2.17) yields the definition of AES in its dual form, introduced first by UZAWA (1962) and sometimes also called the ALLEN-UZAWA elasticity of substitution:

$$\text{AES}_{ij} := \frac{C \cdot C_{ij}}{C_i \cdot C_j}. \tag{2.20}$$

From definition (2.20), a simple interpretation can certainly not be perceived. Rather, the equation

$$\text{AES}_{ij} = \frac{\eta_{ij}}{s_j}, \tag{2.21}$$

obtained by combining (2.19) and (2.20), allows an interpretation. However, expression (2.21) is the "most compelling argument for ignoring the Allen measure in applied analysis ... The interesting measure is $[\eta_{ij}]$ – why disguise it by dividing by a cost share? This question becomes all the more pointed when the best reason for doing so is that it yields a measure that can only be interpreted intuitively in terms of $[\eta_{ij}]$" (CHAMBERS 1988:95).

As suggested by definition (2.20), AES$_{ij}$ can be estimated using the dual cost function approach, the popularity of which might be the reason why AES have been the standard statistics reported in empirical studies. However, only in the case of two inputs or of a CES production structure AES does serve as an appropriate measure of the curvature of isoquants, or the ease of substitution, it was defined for. In fact, in general it is not a measure of the curvature of any surface, be it a factor-price frontier, an isoquant or an indifference curve. A compelling three-dimensional example of BLACKORBY and RUSSEL (1989:883) demonstrates this fact for isoquants: For the two-stage LEONTIEF production function,

$$f(x_1, x_2, x_3) = \min\{x_1, \sqrt{x_2 \cdot x_3}\}, \tag{2.22}$$

input 1 is separable from the inputs 2 and 3. By the very construction of production function (2.22), any changes in x_2 and x_3, holding the 2-3 aggregate output $\sqrt{x_2 \cdot x_3}$ constant, should have no influence on x_1. Vice versa, the proportion x_2/x_3 must be insensitive to changes in x_1. Thus, any substitution elasticity measuring substitution effects between input 2 and 3 should be invariant with respect to altering prices of input 1 or changes in x_1.

This intuition, however, would not be confirmed by the computation of AES$_{23}$: The cost function dual to (2.22) consists of two parts[4],

$$C(Y, p_1, p_2, p_3) = Y \cdot p_1 + Y \cdot 2 \cdot \sqrt{p_2 \cdot p_3}, \tag{2.23}$$

[4] Given technology (2.22), production of an output Y necessitates an input $x_1 = Y$ of factor 1 costing the amount of $p_1 \cdot x_1$. Also, the output $\sqrt{x_2 \cdot x_3}$ of the COBB-DOUGLAS subaggregate has to be equal to Y costing at least $Y \cdot 2 \cdot \sqrt{p_2 \cdot p_3} = \min_{x_2, x_3}\{p_2 \cdot x_2 + p_3 \cdot x_3, Y = \sqrt{x_2 \cdot x_3}\}$.

and application of definition (2.20) provides

$$\text{AES}_{23} = \frac{1}{2} \cdot \frac{p_1}{\sqrt{p_2 \cdot p_3}} + 1. \tag{2.24}$$

AES_{23} is sensitive to the price of input 1 even though optimal quantities of all inputs should be completely insensitive to changes in p_1 for a given output.

Because inputs 2 and 3 are separable from input 1, the 2 - 3 COBB-DOUGLAS aggregator function $\sqrt{x_2 \cdot x_3}$ in (2.22) may be seen as an isolated two-dimensional production function, for which the elasticity of substitution σ, measuring the curvature of the COBB-DOUGLAS isoquants, equals unity. Any generalization of σ should therefore redisplay this value. Yet, AES_{23} is neither equal to unity nor even constant: "Hence, the AES cannot possibly be a measure of curvature, or the ease of substitution" BLACKORBY and RUSSELL (1989:884) conclude.

Consequently, "as a quantitative measure, it has no meaning (BLACKORBY and RUSSELL 1989:883). Qualitatively, as to be seen from construction (2.19), AES classifies pairs of inputs as complements or substitutes on the basis of its sign. Therefore, "it adds no more information to that contained in the cross-price elasticity" (BLACKORBY and RUSSELL 1989:883, see also HAMERMESH 1993:35). Moreover, in formula (2.20), output Y is implicitly held constant, since C minimizes costs for a given output. Therefore, altering the jth price, AES_{ij} generally does not hold cost constant and, hence, cannot measure the curvature of the factor-price frontier.

Assuming that the cost function C is twice differentiable, definition (2.20) is symmetric, an undesirable confinement of an n-dimensional substitution elasticity illustrated by the following example: Reducing energy use due to energy-price shocks might be compensated optimally by an additional use of a third factor, say labor, while capital remains constant. Yet, conversely, a further expansion in capital use due to lower capital prices may necessitate more energy for an economically optimal way of production. Thus, defining a symmetric elasticity of substitution means imposing *a-priori* constraints.

2.3.2 MES

As a superior alternative to AES, BLACKORBY and RUSSELL (1975) suggested the MORISHIMA elasticity of substitution[5]. For the derivation of this paper's definition of MES, which is intended to be another possible generalization of the HICKSian two-variable elasticity σ, we will start from the expression (2.16) for σ. Moving from the two-factor to a multifactor setting, but making the standard assumption that a change in p_j/p_i is solely due to a change in p_j, as BLACKORBY and RUSSELL (1989:883) implicitly did, (2.16) simplifies to the following definition of MES:

$$\frac{\partial \ln(\dfrac{x_i}{x_j})}{\partial \ln(\dfrac{p_j}{p_i})} = \frac{\partial \ln(\dfrac{x_i}{x_j})}{\dfrac{p_i}{p_j} \cdot \partial(\dfrac{p_j}{p_i})} = \frac{\partial \ln(\dfrac{x_i}{x_j})}{\dfrac{p_i}{p_j}\dfrac{1}{p_i} \cdot \partial p_j} = \frac{\partial \ln(\dfrac{x_i}{x_j})}{\partial \ln p_j} =: \mathrm{MES}_{ij}. \tag{2.25}$$

Thus, when the price of input j *alone* varies proportionately and all other prices are constant, MES$_{ij}$ measures the percentage change in the ratio of input i to input j, whereas AES$_{ij}$ measures under the same conditions merely changes in input i. Two factors are termed MES-substitutes (with respect to changes of the price p_j) if MES$_{ij} > 0$ and MES-complements if MES$_{ij} < 0$.

Instead of (2.25), BLACKORBY and RUSSELL (1981:147) define MES by the expression[6]

$$\frac{p_j C_{ij}}{C_i} - \frac{p_j C_{jj}}{C_j}\ , \tag{2.26}$$

where C is a cost function, meeting the so-called regularity conditions, that is, C has to be continous, nondecreasing, and linearly homogeneous and concave in

[5] BLACKORBY and RUSSELL named it in honor of M. MORISHIMA, who formulated it independently from them in 1967.

[6] MORISHIMA originally defines MES as (see BLACKORBY and RUSSELL 1981:147)

$$-\frac{\partial \log(\dfrac{C_i}{C_j})}{\partial \log(\dfrac{p_i}{p_j})}.$$

Taking into account "that meaningful variation in p_i/p_j is entirely attributable to variations in p_i" (BLACKORBY and RUSSELL 1981:148), they transform MORISHIMA's definition into (2.26).

prices. Taking SHEPHARD's Lemma into account, expression (2.26) equals

$$\frac{p_j C_{ij}}{C_i} - \frac{p_j C_{jj}}{C_j} = \frac{p_j}{x_i}\frac{\partial x_i}{\partial p_j} - \frac{p_j}{x_j}\frac{\partial x_j}{\partial p_j} = \frac{\partial \ln x_i}{\partial \ln p_j} - \frac{\partial \ln x_j}{\partial \ln p_j} = \frac{\partial \ln(\frac{x_i}{x_j})}{\partial \ln p_j} = \text{MES}_{ij}\ , \quad (2.27)$$

indicating that definition (2.25) of MES specializes to the formulation (2.26) if duality theory is applied. But this implies optimality assumptions (cost minimization) and requires the validity of regularity conditions, which *a priori* are not necessary for the more general definition (2.25).

For the example (2.22), definition (2.26) yields $\text{MES}_{23} = 1$, which is – contrary to AES – the same result that would be provided by the two-factor measure σ.[7] Hence, MES is rather a possible generalization for σ than AES and a measure of the curvature of isoquants (BLACKORBY and RUSSEL 1989:883). However, as well as AES, due to the very construction of (2.26) from a cost function C, MES can not measure the curvature of a factor-price frontier. In addition, "[t]he MES only captures the substitution (net) effects while ignoring the output [(scale)] effects", DAVIS and SHUMWAY (1996:181) conclude. Only if the technology is homothetic MES should be adequate, because then, factor ratios are independent of the scale of production, that is, output and substitution effects are separable from each other.

From (2.25) a relationship between cross-price and own-price elasticities on the one hand and MES on the other hand is straightforward:

$$\text{MES}_{ij} = \frac{\partial \ln(\frac{x_i}{x_j})}{\partial \ln p_j} = \frac{\partial(\ln x_i - \ln x_j)}{\partial \ln p_j} = \frac{\partial \ln x_i}{\partial \ln p_j} - \frac{\partial \ln x_j}{\partial \ln p_j} = \eta_{ij} - \eta_{jj}\ . \quad (2.28)$$

According to (2.28), the effect of variation in p_j on the quantity ratio x_i/x_j – holding output constant – divides into two parts, the proportional effect of altering p_j on x_i given by the cross-price elasticity η_{ij} and the proportional effect on x_j itself.

[7] BLACKORBY and RUSSEL (1981:149) prove that AES and MES are identical if and only if the production technology has a CES structure or an (explicit) COBB-DOUGLAS structure or if there are only two inputs. Hence, the two-stage LEONTIEF production function (2.22) is certainly not the only example where MES and AES differ from each other.

Inversely, the effect of a single price change of factor i on the ratio x_j/x_i is generally different from (2.28):

$$\text{MES}_{ji} = \frac{\partial \ln(\frac{x_j}{x_i})}{\partial \ln p_i} = \frac{\partial \ln x_j}{\partial \ln p_i} - \frac{\partial \ln x_i}{\partial \ln p_i} = \eta_{ji} - \eta_{ii} \; . \tag{2.29}$$

Thus, in contrast to AES, MES is asymmetric in general. Notice in particular that two factors i and j, being MES-complements with respect to changes of the price p_j ($\text{MES}_{ij} < 0$), nevertheless might be MES-substitutes with respect to changes of the price p_i ($\text{MES}_{ji} > 0$).

If interest is on a comparative static analysis about relative factor shares, then MES provides complete information:

$$\frac{\partial \ln(\frac{p_j x_j}{p_i x_i})}{\partial \ln p_j} = \frac{\partial \ln(p_j x_j)}{\partial \ln p_j} - \frac{\partial \ln(p_i x_i)}{\partial \ln p_j} = 1 + \frac{\partial \ln x_j}{\partial \ln p_j} - \frac{\partial \ln x_i}{\partial \ln p_j} = 1 - \text{MES}_{ij}. \tag{2.30}$$

However, the fact that $\text{MES}_{ij} > 0$, that is, the fact of a (MES-)substitutability of the inputs i and j with respect to changes of the price p_j does not necessarily imply that the income share of j decreases relatively to that of i when its own price increases: According to (2.30), the income share of j decreases relatively to that of i if and only if MES_{ij} is greater than one. Hence, the characterization (2.30) of the comparative statics of relative income shares mirrors the HICKSian idea that the effect of changes in capital or labor prices on the distribution of income (for a given output) is completely determined by a scalar measure of curvature of isoquant (BLACKORBY and RUSSEL 1989:882), but this relationship is not perfectly in line with the notion of substitutability ($\text{MES}_{ij} > 0$).

Applying $\eta_{ij} = s_j \cdot \text{AES}_{ij}$ and $\eta_{jj} = s_j \cdot \text{AES}_{jj}$ yields

$$\text{MES}_{ij} = \eta_{ij} - \eta_{jj} = s_j \cdot (\text{AES}_{ij} - \text{AES}_{jj}). \tag{2.31}$$

Equation (2.31) allows further insight into the relationship between AES and MES: Because $\text{AES}_{jj} < 0$ is always valid, two inputs being AES-substitutes ($\text{AES}_{ij} = \eta_{ij}/s_j > 0 \Leftrightarrow \eta_{ij} > 0$, since $s_j > 0$) are also inevitably MES-substitutes ($\text{MES}_{ij} > 0$), whereas AES-complements ($\text{AES}_{ij} < 0$) might be MES-substitutes as well.

Therefore, if one were to classify factors using MES, one would more frequently conclude that they are substitutes than if one were using AES. Consider again definition (2.25) of MES: When price p_j rises exogenously and for this reason quantity x_i increases to compensate the decrease of x_j, these inputs are both AES-substitutes and MES-substitutes. Yet, when quantity x_i decreases rather than it increases as a reaction to the rise in the price of factor j, but by less than the corresponding decrease in x_j, MES$_{ij}$ is positive. Thus, despite their classification as AES-complements, both factors are MES-substitutes.

In consequence, whether two factors are regarded as substitutes or as complements naturally depends upon the substitution measure employed. The following empirical example about the characterization of substitution possibilities between energy and nonenergy inputs will illustrate that the clarification of the concept employed is not only a theoretical issue, but that it is of considerable practical relevance.

2.4 An Empirical Comparison of AES and MES

For U. S. manufacturing data for 1947 - 1971, displayed in Tables 2.2 and 2.5 of Appendix B, BW75 report, among others, an ALLEN partial elasticity of substitution between energy and capital, AES$_{EK}$, of about - 3.2 and conclude that "energy and capital are complementary" (BW75:260). By reestimating their four-input translog cost function model and applying MES rather than AES and cross-price elasticities, we will find with statistical significance for the whole sample period that energy and capital may also be considered as being substitutes, namely MES-substitutes, a result derived already in the note of THOMPSON and TAYLOR (1995:566). Their note provides a point estimate for the single year 1965, but no standard error. Here, we will extend their note by considering the complete set of substitution possibilities in this four factor setting, and by deriving standard errors, both permitting assessment of the empirical relevance of this issue. In sum, the results demonstrate that whenever one draws conclusions from empirical studies, it is indispensable to clarify with regard to which measure two inputs are being denoted as substitutes.

For their four-input KLEM-model, including capital (K), labor (L), energy (E), and all other intermediate materials (M), BW75 employ a translog cost function,

$$\ln C = \ln \beta_0 + \ln Y + \sum_i \beta_i \ln p_i + \frac{1}{2} \sum_i \sum_j \beta_{ij} \ln p_i \ln p_j, \tag{2.32}$$

where symmetry of β_{ij} for $i, j = K, L, E, M$ and constant returns to scale are imposed, and Y is a given level of output. The cost function (2.32) is the dual counterpart to the translog production function (2.6).

Linear homogeneity in prices of a cost function C additionally requires the following restrictions for the coefficients of (2.32):

$$\beta_K + \beta_L + \beta_E + \beta_M = 1, \tag{2.33}$$

$$\beta_{Kj} + \beta_{Lj} + \beta_{Ej} + \beta_{Mj} = 0 \qquad \text{for } j = K, L, E, M. \tag{2.34}$$

Using SHEPHARD's Lemma, $\partial C / \partial p_i = x_i$, four cost-share equations can be obtained by differentiating the translog cost function (2.32) logarithmically:

$$\frac{\partial \ln C}{\partial \ln p_i} = \frac{p_i}{C} \frac{\partial C}{\partial p_i} = \frac{p_i \cdot x_i}{C} = s_i = \beta_i + \sum_j \beta_{ij} \ln p_j \qquad i, j = K, L, E, M. \tag{2.35}$$

In principle, the unknown parameters β_i and β_{ij} may be estimated from a stochastic version of the cost share equation system (2.35), each equation additionally containing a disturbance ε_i. However, since these four cost shares always sum to unity, and because of restrictions (2.33) and (2.34), the sum of the disturbances across the four equations is zero at each observation, implying the singularity of the disturbance covariance matrix. This problem is solved by dropping arbitrarily one of the four equations in (2.35). Like BW75, we estimate the equation system

$$\begin{aligned}
s_K &= \beta_K + \beta_{KK} \ln(\frac{p_K}{p_M}) + \beta_{KL} \ln(\frac{p_L}{p_M}) + \beta_{KE} \ln(\frac{p_E}{p_M}) + \varepsilon_K \\
s_L &= \beta_L + \beta_{KL} \ln(\frac{p_K}{p_M}) + \beta_{LL} \ln(\frac{p_L}{p_M}) + \beta_{LE} \ln(\frac{p_E}{p_M}) + \varepsilon_L \\
s_E &= \beta_E + \beta_{KE} \ln(\frac{p_K}{p_M}) + \beta_{LE} \ln(\frac{p_L}{p_M}) + \beta_{EE} \ln(\frac{p_E}{p_M}) + \varepsilon_E \ ,
\end{aligned} \tag{2.36}$$

where restrictions (2.34) are already imposed and the cost-share equation of M is left out.

This seemingly unrelated regressions (SUR) model, where equations are linked merely by disturbances, can be estimated efficiently and consistently by Generalized Least Squares (GLS). Unfortunately, the GLS parameter estimates will depend upon which equation is dropped in order to achieve a nonsingular equation system. Computing maximum-likelihood (ML-) estimates, however, ensures invariance with respect to the choice of the share equation dropped (BERNDT 1991:473). ML-parameter estimates of (2.36) are reported in Table 2.3 of Appendix B and compared to those obtained by BW75, who employ an iterative three-stage least-square (I3SLS) method, using 10 instruments, for example U.S. population. Table 2.3 shows that ML and I3SLS estimations are quite close. The check whether or not the translog cost function (2.32) is well-behaved, being analogous to that for translog production functions (see (2.10) and (2.11)), yields that positive monotonicity and concavity in prices are satiesfied at each annual observation: Fitted cost shares are always positive, providing global monotonicity according to (2.35), and the Hessian matrix, based on ML estimates, is always negative semidefinite, yielding global concavity.

Once the parameters β_{ij} have been estimated, AES can be computed as (see e. g. HAMERMESH 1993:41)

$$\text{AES}_{ij} = \frac{\beta_{ij}}{s_i s_j} + 1 \qquad \text{for} \qquad i \neq j, \tag{2.37}$$

$$\text{AES}_{ii} = \frac{\beta_{ii}}{s_i^2} - \frac{1}{s_i} + 1, \tag{2.38}$$

while the MES for the translog cost function approach (2.32) follows from (2.31) to be

$$\text{MES}_{ij} = \frac{\beta_{ij}}{s_i} - \frac{\beta_{jj}}{s_j} + 1 \qquad \text{for} \qquad i \neq j, \tag{2.39}$$

$$\text{MES}_{ji} = \frac{\beta_{ij}}{s_j} - \frac{\beta_{ii}}{s_i} + 1 \qquad \text{for} \qquad i \neq j. \tag{2.40}$$

From definitions (2.25) and (2.26), respectively, it follows that $\text{MES}_{ii} = 0$, whereas according to (2.38) $\text{AES}_{ii} \neq 0$ in general.

Both categories of elasticities, MES and AES, are nonlinear functions of the estimated parameters and, therefore, standard errors for their estimates cannot be calculated exactly. According to PINDYCK (1979:171), approximate estimates of the standard errors can be obtained under the assumption that the cost shares s_i are constant and equal to the means of their estimated values. Under this assumption, it is, asymptotically,

$$var(\widehat{\text{AES}}_{ij}) = var(\hat{\beta}_{ij})/(\hat{s}_i^2 \hat{s}_j^2), \tag{2.41}$$

$$var(\widehat{\text{MES}}_{ij}) = var(\hat{\beta}_{ij})/\hat{s}_i^2 + var(\hat{\beta}_{jj})/\hat{s}_j^2 - 2cov(\hat{\beta}_{ij}, \hat{\beta}_{jj})/(\hat{s}_i \hat{s}_j). \tag{2.42}$$

Table 1 reports estimates of AES, MES and cross-price elasticities for all conceivable combinations of the four inputs K, L, E, and M and, because estimates are rather stable, for five equidistant years, chosen for the sake of comparability to the BW75 study. Not surprisingly, the AES and cross-price elasticities displayed and calculated by applying our ML-parameter estimates on formulae (2.37) and (2.38) are quite similar to those BW75 report for the same data. According to Table 1, capital and energy, in particular, should be considered as (AES-)complements, since estimates of both AES_{EK} and cross-price elasticities η_{EK} and η_{KE} are significantly negative for the whole period of time. However, apart from the fact that the magnitude of AES carries no information, as proposed in Section 2.3.1, and, hence, listing estimates of AES_{EK} is redundant if estimates of η_{KE} or η_{EK} are indicated as well, on the basis of the MES estimates, capital and energy have to be denoted as substitutes: Both $\widehat{\text{MES}}_{EK}$ and $\widehat{\text{MES}}_{KE}$ are significantly positive for the whole period, with $\widehat{\text{MES}}_{KE}$ averaging around 0.37 and $\widehat{\text{MES}}_{EK}$ being roughly around 0.2.

Because $\widehat{\eta}_{EK}$ is about -0.2 and $\widehat{\eta}_{KK}$, not reported in Table 1, is roughly -0.4, and hence $\widehat{\text{MES}}_{EK} \approx -0.2 - (-0.4) = 0.2 > 0$, a 1 % increase in the price of capital leads to a 0.2 % reduction in the use of energy and a 0.4 % reduction for capital holding output constant: In comparison to capital more energy is used when capital gets more expensive. Thus, capital and energy are MES-substitutes, though the input of energy in fact shrinks, as indicated by $\widehat{\eta}_{EK} = -0.2$, that is, capital and energy are AES-complements. From this example it appears to be of minor importance which estimation method is applied, for instance, whether instruments are used or not. In a typical application, the variability of parameter estimates with respect to alterations in the specification and the estimation method might be considerable.

Yet, the differences in the qualitative conclusions regarding substitutability might rest to an even larger degree on the choice of the substitution concept: For example, shifting the price ceiling of energy, say by energy taxes, would tend to reduce energy and capital intensiveness and increase labor intensiveness, since $\widehat{\eta}_{KE}$ is significantly negative ($\widehat{\eta}_{KE} \approx -0.15$) and $\widehat{\eta}_{LE}$ is slightly positive ($\widehat{\eta}_{LE} \approx 0.03$), but capital intensiveness reduces not as much as energy intensiveness, as indicated by $\widehat{\text{MES}}_{KE} \approx 0.35$. Furthermore, comparing the reductions of energy and capital intensiveness due to higher energy taxes by MES_{KE} might be less interesting in this example than considering the seperate impacts of higher energy taxes on the input of capital, labor or energy, which are given by the cross-price elasticities η_{KE}, η_{LE} and η_{EE}, respectively.

As we would expect from last section's theoretical considerations – see equation (2.31) –, Table 1 displays that the magnitudes of MES are generally higher than those of the corresponding cross-price elasticities. Moreover, Table 1 contains estimates for TES. Analytically, the TES can be obtained from the translog production function (2.6), but, unfortunately, unlike for instance for a Cobb-Douglas production function, a translog production function can not be calculated via duality theory from its dual counterpart, as the translog cost function (2.32) has no self-dual (HEATHFIELD and WIBE 1987:110). Thus, a further estimation, now of a translog function of the quantities of *K, L, E* and *M* is necessary (Quantity indices for *K, L, E* and *M* are presented in Table 2.5 of Appendix B).

Table 1: Comparison of the Estimates of AES, Cross-Price Elasticities, MES and TES.

Year	AES_{EK}	η_{KE}	η_{EK}	MES_{EK}	MES_{KE}	TES_{KE}
1947	-3.71 (1.56)	-0.16 (0.07)	-0.19 (0.08)	0.18 (0.07)	0.36 (0.11)	0.79 (0.017)
1953	-3.95 (1.64)	-0.17 (0.07)	-0.18 (0.08)	0.14 (0.08)	0.35 (0.11)	0.84 (0.009)
1959	-2.62 (1.20)	-0.12 (0.05)	-0.16 (0.07)	0.29 (0.06)	0.42 (0.10)	0.86 (0.006)
1965	-3.55 (1.51)	-0.15 (0.06)	-0.19 (0.08)	0.21 (0.07)	0.36 (0.12)	0.78 (0.009)
1971	-3.88 (1.62)	-0.17 (0.07)	-0.18 (0.07)	0.14 (0.08)	0.36 (0.11)	0.79 (0.015)
Year	AES_{EL}	η_{LE}	η_{EL}	MES_{EL}	MES_{LE}	TES_{LE}
1947	0.58 (0.23)	0.02 (0.01)	0.14 (0.06)	0.59 (0.05)	0.54 (0.12)	0.16 (0.004)
1953	0.62 (0.21)	0.03 (0.01)	0.17 (0.06)	0.62 (0.05)	0.55 (0.12)	0.16 (0.002)
1959	0.65 (0.20)	0.03 (0.01)	0.18 (0.05)	0.63 (0.05)	0.57 (0.11)	0.17 (0.001)
1965	0.62 (0.21)	0.03 (0.01)	0.17 (0.06)	0.62 (0.05)	0.53 (0.13)	0.14 (0.002)
1971	0.66 (0.19)	0.03 (0.01)	0.19 (0.05)	0.64 (0.05)	0.57 (0.12)	0.14 (0.003)
Year	AES_{KL}	η_{LK}	η_{KL}	MES_{KL}	MES_{LK}	TES_{LK}
1947	0.97 (0.31)	0.05 (0.02)	0.24 (0.08)	0.69 (0.07)	0.42 (0.12)	0.22 (0.010)
1953	0.97 (0.31)	0.05 (0.01)	0.26 (0.08)	0.71 (0.08)	0.37 (0.13)	0.19 (0.004)
1959	0.98 (0.23)	0.06 (0.01)	0.27 (0.06)	0.72 (0.06)	0.52 (0.10)	0.20 (0.004)
1965	0.98 (0.25)	0.05 (0.01)	0.27 (0.07)	0.72 (0.07)	0.45 (0.11)	0.19 (0.004)
1971	0.97 (0.29)	0.05 (0.01)	0.28 (0.08)	0.73 (0.08)	0.36 (0.13)	0.18 (0.003)
Year	AES_{LM}	η_{ML}	η_{LM}	MES_{LM}	MES_{ML}	TES_{ML}
1947	0.57 (0.07)	0.14 (0.02)	0.37 (0.04)	0.57 (0.07)	0.59 (0.04)	0.41 (0.004)
1953	0.59 (0.06)	0.16 (0.02)	0.38 (0.04)	0.59 (0.07)	0.61 (0.04)	0.42 (0.002)
1959	0.58 (0.06)	0.16 (0.02)	0.36 (0.04)	0.59 (0.07)	0.61 (0.04)	0.44 (0.003)
1965	0.60 (0.06)	0.17 (0.02)	0.37 (0.04)	0.60 (0.07)	0.62 (0.04)	0.45 (0.004)
1971	0.61 (0.06)	0.16 (0.02)	0.38 (0.04)	0.60 (0.07)	0.63 (0.04)	0.48 (0.006)
Year	AES_{EM}	η_{ME}	η_{EM}	MES_{EM}	MES_{ME}	TES_{ME}
1947	0.85 (0.31)	0.04 (0.01)	0.56 (0.20)	0.76 (0.24)	0.55 (0.14)	0.07 (0.001)
1953	0.85 (0.31)	0.04 (0.01)	0.55 (0.20)	0.76 (0.23)	0.56 (0.13)	0.07 (0.001)
1959	0.85 (0.30)	0.04 (0.01)	0.53 (0.19)	0.76 (0.22)	0.58 (0.13)	0.07 (0.001)
1965	0.84 (0.33)	0.03 (0.01)	0.52 (0.21)	0.75 (0.24)	0.54 (0.14)	0.06 (0.001)
1971	0.85 (0.31)	0.04 (0.01)	0.53 (0.19)	0.76 (0.23)	0.57 (0.13)	0.07 (0.001)
Year	AES_{KM}	η_{MK}	η_{KM}	MES_{KM}	MES_{MK}	TES_{MK}
1947	0.43 (0.29)	0.02 (0.01)	0.28 (0.19)	0.48 (0.20)	0.39 (0.13)	0.08 (0.004)
1953	0.37 (0.33)	0.02 (0.02)	0.24 (0.21)	0.45 (0.21)	0.34 (0.14)	0.08 (0.002)
1959	0.50 (0.26)	0.03 (0.02)	0.31 (0.16)	0.54 (0.17)	0.49 (0.11)	0.09 (0.002)
1965	0.44 (0.29)	0.02 (0.02)	0.27 (0.18)	0.50 (0.19)	0.43 (0.12)	0.08 (0.002)
1971	0.34 (0.34)	0.02 (0.01)	0.21 (0.21)	0.44 (0.22)	0.33 (0.14)	0.08 (0.003)

Standard errors are given in parentheses.[8]

[8] Under the same assumptions as made before in order to approximate estimates of the stan-

To make things as comparable as possible, constant returns to scale and optimality behaviour (cost minimization) under perfect competition are assumed as well as in the translog cost approach. Under these assumptions, the output elasticity of any factor i,

$$\frac{x_i}{f}\frac{\partial f}{\partial x_i} = \frac{\partial \ln f}{\partial \ln x_i} = \alpha_i + \sum_j \alpha_{ij} \ln x_j \qquad \text{for } i, j \in \{K, L, E, M\}\ , \tag{2.43}$$

necessary to compute the TES according to (2.7) and obtained by differentiating the translog production function (2.6) logarithmically, equals factor i's total cost share $s_i = \frac{x_i \cdot p_i}{f \cdot p} = \frac{x_i \cdot p_i}{C}$. Then, translog production function parameters may be estimated from a three equation system similar to (2.36), with input prices being replaced by input quantities:

$$\begin{aligned} s_K &= \alpha_K + \alpha_{KK}\ln(\frac{x_K}{x_M}) + \alpha_{KL}\ln(\frac{x_L}{x_M}) + \alpha_{KE}\ln(\frac{x_E}{x_M}) + \nu_K \\ s_L &= \alpha_L + \alpha_{KL}\ln(\frac{x_K}{x_M}) + \alpha_{LL}\ln(\frac{x_L}{x_M}) + \alpha_{LE}\ln(\frac{x_E}{x_M}) + \nu_L \\ s_E &= \alpha_E + \alpha_{KE}\ln(\frac{x_K}{x_M}) + \alpha_{LE}\ln(\frac{x_L}{x_M}) + \alpha_{EE}\ln(\frac{x_E}{x_M}) + \nu_E\ . \end{aligned} \tag{2.44}$$

Linear homogeneity restrictions for $\alpha_{ij}(i, j = K, L, E, M)$ analogous to (2.34) are implicitly imposed in (2.44). Because of the singularity problem known from above, the share equation for M is dropped without any consequences: ML-estimation results, presented in Table 2.4 of Appendix B, are the same, no matter which equation is left out. Well-behavedness of the translog production function (see (2.10 and (2.11)) is checked on the basis of these estimates and is given globally.

Although the TES are not symmetric, for the sake of brevity and as both are related inversely, Table 1 reports merely one estimate of either TES$_{ij}$ or TES$_{ji}$. Without exception, the estimated TES, computed as the ratio of the corresponding output elasticities – see (2.4) –, are positive – and estimated output elasticities are positive, as required for a well-behaved production function. The largest

dard errors of AES and MES, we have, asymptotically, $var(\hat{\eta}_{ij}) = \hat{\beta}_{ij}/\hat{s}_i^2$ and $var(\hat{\eta}_{ii}) = \hat{\beta}_{ii}/\hat{s}_i^2$ (see PINDYCK 1979:171). Similarly, for the standard errors of the TES we obtain, asymptotically, $var(\widehat{\text{TES}}_{ij}) = var(\hat{\alpha}_j + \sum_k \hat{\alpha}_{jk} \ln x_k)/\hat{s}_i^2$, where the variance of the linear term $\hat{\alpha}_j + \sum_k \hat{\alpha}_{jk} \ln x_k$ can be calculated exactly.

difference between estimates of TES and cross-price elasticities display $\widehat{\text{TES}}_{EK}$ and $\widehat{\eta}_{EK}$: Because of $\widehat{\text{TES}}_{EK} \approx 1.2$, a 1 % decrease in the quantity of capital, for instance, due to a price shift for capital of 2.5 % (recall $\widehat{\eta}_{KK} \approx -0.4$) has to be compensated by a 1.2 % increase in the quantity of energy in order to ensure a given level of output, provided that the quantity of no other input is variable. This increase of 1.2 % in energy input, dictated by production technology constraints to hold output fixed, represents the inflexible short-run reaction. By contrast, $\widehat{\eta}_{EK} \approx -0.2$ tells us that, under more flexible circumstances, where the quantities of all other factors may change as well, the necessary quantity of energy may even decrease by 0.5 % ($= 0.2 \cdot 2.5\%$).

2.5 Conclusion

The summary of the basis and differences of AES and MES provided in this paper stresses the following facts: First, though being the most applied substitution measure, AES is not a measure of substitution, because it does not measure the curvature of any level surface. Qualitatively, AES yields the same results as cross-price elasticities. Second, the more general definition (2.25) of MES given here provides a deeper insight into the differences between MES on the one side and AES or cross-price elasticities on the other side, in particular, with respect to the classification of pairs of inputs as substitutes or complements. Generally, MES more frequently leads to a classification of two factors as substitutes than AES or cross-price elasticities. For instance, in Section 2.4's empirical example of U. S. manufacturing two factors being MES-substitutes are AES-complements. From this example it appears to be of minor importance, which estimation method is applied, but indispensable to state the concrete substitution measure when one argues that there is a substitution relationship between two production factors. From the persepective of the recipient of such information, the differences in the interpretation of substitution elasticities like MES, AES and cross price elasticities are to be taken into account when drawing conclusions on the policy implications of such empirical estimates: For example, price shifts in energy by energy taxes would tend to reduce capital intensiveness if $\widehat{\eta}_{KE} \approx -0.15$, as in our example, but would tend to increase it, if, say, $\widehat{\eta}_{KE} \approx 0.15$. In both cases, MES$_{KE}$ is positive if $\widehat{\eta}_{EE} \approx -0.5$, as in our example, implying that capital intensiveness would tend to rise relatively to that of energy. However, is this aspect that relevant? In other words, wouldn't the information given by cross-price elasticities, the common basis of both AES and MES, be sufficient enough?

Third, with definition (2.25) of MES in hand a common structure of the substitution definitions corresponding to the substitution measures discussed in this paper can be given:
Two factors i and j are termed

$$\text{AES-}\textit{substitutes} \text{ if } \frac{\partial \ln x_i}{\partial \ln p_j} > 0, \text{ AES-}\textit{complements} \text{ if } \frac{\partial \ln x_i}{\partial \ln p_j} < 0, \tag{2.45}$$

$$\text{MES-}\textit{substitutes} \text{ if } \frac{\partial \ln(\frac{x_i}{x_j})}{\partial \ln p_j} > 0, \text{ MES-}\textit{complements} \text{ if } \frac{\partial \ln(\frac{x_i}{x_j})}{\partial \ln p_j} < 0\ , \tag{2.46}$$

$$\text{TES-}\textit{substitutes} \text{ if } \frac{\partial \ln x_i}{\partial \ln x_j} < 0, \text{ and TES-}\textit{complements} \text{ if } \frac{\partial \ln x_i}{\partial \ln x_j} > 0\ . \tag{2.47}$$

The two former substitution definitions (2.45) and (2.46) are measuring the reaction in factor quantities and their ratio, respectively, due to changes of only one input price at a constant output level. They might therefore be categorized as *quantity-price* definitions. In definition (2.47) of the TES, the quantity x_j replaces the price p_j in (2.45), and, hence, (2.47) might be called a *quantity-quantity* definition. The TES, a substitution measure suggested in this paper to provide an alternative for the analysis of short-run effects, may be useful for infering short-run responses in the use of an input to exogenous shocks in the supply of another production factor.

Finally, "while the MES is a powerful tool for economists" (THOMPSON and TAYLOR 1996:173), and remains for many purposes a convenient substitution elasticity under the price-normalization condition that merely one input price changes, MES as well as cross-price elastities and AES ignore any output or scale effects. However, shrinking gross domestic products due to oil price shifts during oil crises exemplify that output effects often can not be ignored. Moreover, "prices in the real world rarely move in such a manner that only one of them change while the others remain constant or change proportionately" (FRENGER 1992:1). FRENGER (1994:1) criticizes also that MES is not a measure of the curvature of a factor-price frontier and by that criteria not a proper elasticity of substitution in his eyes. As long as a substitution measure, fulfilling such requirements, is not available, the question of BLACKORBY and RUSSELL (1989), "Will the real elasticity of substitution please stand up?", has still to be posed, and it is reasonable to employ several measures in an empirical study and to offer a comparison, as it is done here.

Appendix A: Proof.

Proposition: Output elasticities $\eta_{yj} = \frac{\partial \ln f}{\partial \ln x_k}$ do not depend upon the special translog-representation, i. e. whether the translog function is a TAYLOR-series approximation around the unit vector or around any other vector $(\bar{x}_1, \bar{x}_2, ..., \bar{x}_n)^T$.

Proof: Instead of the commonly applied second-order TAYLOR-expansion around the unit vector,

$$\ln f = \alpha_0 + \sum_{i=1}^{n} \alpha_i \cdot \ln x_i + \frac{1}{2} \cdot \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_{ij} \ln x_i \ln x_j, \tag{2.48}$$

alternatively, the following second-order TAYLOR-expansion around an arbitrary point $(\overline{x}_1, \overline{x}_2, ..., \overline{x}_n)^T$ is considered:

$$\ln f = \overline{\alpha}_0 + \sum_{i=1}^{n} \overline{\alpha}_i \cdot (\ln x_i - \ln \overline{x}_i) + \frac{1}{2} \cdot \sum_{i=1}^{n} \sum_{j=1}^{n} \overline{\alpha}_{ij} \cdot (\ln x_i - \ln \overline{x}_i) \cdot (\ln x_j - \ln \overline{x}_j). \tag{2.49}$$

The comparison of both expansions yields

$$\begin{aligned} \alpha_0 &= \overline{\alpha}_0 - \sum_i \overline{\alpha}_i \ln \overline{x}_i - \frac{1}{2} \cdot \sum_{i=1}^{n} \sum_{j=1}^{n} \overline{\alpha}_{ij} \ln \overline{x}_i \cdot \ln \overline{x}_j, \\ \alpha_i &= \overline{\alpha}_i - \sum_j \overline{\alpha}_{ij} \ln \overline{x}_j, \\ \alpha_{ij} &= \overline{\alpha}_{ij}. \end{aligned} \tag{2.50}$$

On the one hand, the output elasticity η_{yk} at $(\overline{x}_1, \overline{x}_2, ..., \overline{x}_n)^T$, gained from (2.48) by differentiation with respect to $\ln x_k$, is:

$$\eta_{yx_k}(\overline{x}_1, \overline{x}_2, ..., \overline{x}_n)^T = \frac{\partial \ln f}{\partial \ln x_k}(\overline{x}_1, \overline{x}_2, ..., \overline{x}_n)^T = \alpha_k + \alpha_{kk} \ln \overline{x}_k + \sum_{j \neq k} \alpha_{kj} \ln \overline{x}_j. \tag{2.51}$$

On the other hand, the output elasticity η_{yk} at any point $(x_1, x_2, ..., x_n)^T$ gained from (2.49) is:

$$\eta_{yx_k} = \frac{\partial \ln f}{\partial \ln x_k} = \overline{\alpha}_k + \overline{\alpha}_{kk} \ln \overline{x}_k + \sum_{j \neq k} \overline{\alpha}_{kj} (\ln x_j - \ln \overline{x}_j). \tag{2.52}$$

In particular, for $(x_1, x_2, ..., x_n)^T = (\overline{x}_1, \overline{x}_2, ..., \overline{x}_n)^T$ this is

$$\eta_{yx_k} = \frac{\partial \ln f}{\partial \ln x_k} = \overline{\alpha}_k + \overline{\alpha}_{kk} \ln \overline{x}_k + 0. \tag{2.53}$$

From (2.50), especially from $\overline{\alpha}_k = \alpha_k + \sum_{j \neq k} \alpha_{kj} \ln \overline{x}_j$ and $\overline{\alpha}_{ij} = \alpha_{ij}$, the same output elasticity (2.51) follows again:

$$\eta_{yx_k}(\overline{x}_1, \overline{x}_2, ..., \overline{x}_n)^T = \frac{\partial \ln f}{\partial \ln x_k}(\overline{x}_1, \overline{x}_2, ..., \overline{x}_n)^T = \alpha_k + \alpha_{kk} \ln \overline{x}_k + \sum_{j \neq k} \alpha_{kj} \ln \overline{x}_j. \quad (2.54)$$

Appendix B

Table 2.2: Input Prices and Cost Shares of Capital, Labor, Energy and other Intermediate Inputs – U.S. Manufacturing 1947 - 1971

	Cost Shares				Input Prices			
Year	K	L	E	M	p_K	p_L	p_E	p_M
1947	0.05107	0.24727	0.04253	0.65913	1.00000	1.00000	1.00000	1.00000
1948	0.05817	0.27716	0.05127	0.61340	1.00270	1.15457	1.30258	1.05525
1949	0.04602	0.25911	0.05075	0.64411	0.74371	1.15584	1.19663	1.06225
1950	0.04991	0.24794	0.04606	0.65609	0.92497	1.23535	1.21442	1.12430
1951	0.05039	0.25487	0.04482	0.64992	1.04877	1.33784	1.25179	1.21694
1952	0.04916	0.26655	0.04460	0.63969	0.99744	1.37949	1.27919	1.19961
1953	0.04728	0.26832	0.04369	0.64071	1.00653	1.43458	1.27505	1.19044
1954	0.05635	0.27167	0.04787	0.62411	1.08757	1.45362	1.30356	1.20612
1955	0.05258	0.26465	0.04517	0.63760	1.10315	1.51120	1.34277	1.23835
1956	0.04604	0.26880	0.04576	0.63940	0.99606	1.58186	1.37154	1.29336
1957	0.05033	0.27184	0.04820	0.62962	1.06321	1.64641	1.38010	1.30703
1958	0.06015	0.27283	0.04836	0.61866	1.15619	1.67389	1.39338	1.32699
1959	0.06185	0.27303	0.04563	0.61948	1.30758	1.73430	1.36756	1.30774
1960	0.05788	0.27738	0.04585	0.61889	1.25413	1.78280	1.38025	1.33946
1961	0.05903	0.27839	0.04640	0.61617	1.26328	1.81977	1.37630	1.34319
1962	0.05578	0.28280	0.04530	0.61613	1.26525	1.88531	1.37689	1.34745
1963	0.05601	0.27968	0.04470	0.61962	1.32294	1.93379	1.34737	1.33143
1964	0.05452	0.28343	0.04392	0.61814	1.32798	2.00998	1.38969	1.35197
1965	0.05467	0.27996	0.04114	0.62423	1.40659	2.05539	1.38635	1.37542
1966	0.05460	0.28363	0.04014	0.62163	1.45100	2.13441	1.40102	1.41878
1967	0.05443	0.28646	0.04074	0.61837	1.38617	2.20616	1.39197	1.42428
1968	0.05758	0.28883	0.03971	0.61388	1.49901	2.33869	1.43388	1.43481
1969	0.05410	0.29031	0.03963	0.61597	1.44957	2.46412	1.46481	1.53356
1970	0.05255	0.29755	0.04348	0.60642	1.32464	2.60532	1.45907	1.54758
1971	0.04675	0.28905	0.04479	0.61940	1.20177	2.76025	1.64689	1.54978

Source: BERNDT and WOOD (1975:263).

Table 2.3: Comparison of our ML-Parameter Estimates with BW75's I3SLS Parameter Estimates of their KLEM-Translog Cost Function – U.S. Manufacturing 1947 - 1971.

	ML			I3SLS	
	Estimates	Std. Error	t-Ratios	Estimates	t-Ratios
β_K	0.0570**	0.00136	41.976	0.0564**	36.571
β_L	0.2534**	0.00212	119.552	0.2539**	112.359
β_E	0.0443**	0.00088	50.119	0.0442**	38.078
β_M	0.6453**	0.00336	192.054	0.6455**	173.348
β_{KK}	0.0297**	0.00594	5.007	0.0254**	3.455
β_{KL}	-0.0003	0.00387	-0.095	0.0001	0.022
β_{KE}	-0.0102**	0.00339	-3.013	-0.0102**	-2.444
β_{KM}	-0.0191*	0.00989	-1.931	-0.0153*	-1.301
β_{LL}	0.0754**	0.00681	11.076	0.0739**	10.159
β_{LE}	-0.0044*	0.00244	-1.808	-0.0043*	-1.438
β_{LM}	-0.0706**	0.01077	-6.555	-0.0697**	-5.942
β_{EE}	0.0188**	0.00535	3.511	0.0214**	2.393
β_{EM}	-0.0041	0.00858	-0.478	-0.0068	-0.527
β_{MM}	0.0939**	0.02327	4.035	0.0918**	3.243

Source: I3SLS estimates: BERNDT and WOOD (1975:264). ML estimates of cost share system (2.35).

* Significant at the 5 %-level. ** Significant at the 1 %-level.

Table 2.4: ML-Parameter Estimates of our KLEM-Translog Production Function – U.S. Manufacturing 1947 - 1971.

	Estimates	Std. Error	t-Ratios
α_K	0.0532^{**}	0.00240	22.18096
α_L	0.2614^{**}	0.00265	98.69578
α_E	0.0422^{**}	0.00092	45.61503
α_M	0.6433^{**}	0.00329	195.37548
α_{KK}	-0.0032	0.01126	-0.28133
α_{KL}	0.0044	0.00776	0.56231
α_{KE}	0.0241^{**}	0.00556	4.33323
α_{KM}	-0.0252^{*}	0.01266	-1.99588
α_{LL}	-0.0871^{**}	0.01615	-5.39373
α_{LE}	0.0182^{**}	0.00273	6.64256
α_{LM}	0.0646^{**}	0.01935	3.33860
α_{EE}	0.0049	0.00811	0.60020
α_{EM}	-0.0471^{**}	0.00599	-7.86908
α_{MM}	0.0078	0.02620	0.29849

Source: ML estimates of equation system (2.44). * Significant at the 5 %-level. ** Significant at the 1 %-level.

Table 2.5: Total Cost and Quantity Indexes of Capital, Labor, Energy and other Intermediate Inputs – U.S. Manufacturing 1947 - 1971.

Year	Total Input Cost[a]	Input Quantities			
		K	L	E	M
1947	182.373	1.00000	1.00000	1.00000	1.00000
1948	193.161	1.14103	0.97501	0.92932	0.88570
1949	186.533	1.23938	0.92728	1.01990	0.94093
1950	221.710	1.28449	0.98675	1.08416	1.07629
1951	255.945	1.32043	1.08125	1.18144	1.13711
1952	264.699	1.40073	1.13403	1.18960	1.17410
1953	291.160	1.46867	1.20759	1.28618	1.30363
1954	274.457	1.52688	1.13745	1.29928	1.18144
1955	308.908	1.58086	1.19963	1.33969	1.32313
1956	328.286	1.62929	1.23703	1.41187	1.35013
1957	338.633	1.72137	1.23985	1.52474	1.35705
1958	323.318	1.80623	1.16856	1.44656	1.25396
1959	358.435	1.82065	1.25130	1.54174	1.41250
1960	366.251	1.81512	1.26358	1.56828	1.40778
1961	366.162	1.83730	1.24215	1.59152	1.39735
1962	390.668	1.84933	1.29944	1.65694	1.48606
1963	412.188	1.87378	1.32191	1.76280	1.59577
1964	433.768	1.91216	1.35634	1.76720	1.64985
1965	474.969	1.98212	1.43460	1.81702	1.79327
1966	521.291	2.10637	1.53611	1.92525	1.90004
1967	540.941	2.27814	1.55581	2.03881	1.95160
1968	585.447	2.41485	1.60330	2.08997	2.08377
1969	630.450	2.52637	1.64705	2.19889	2.10658
1970	623.466	2.65571	1.57894	2.39503	2.03230
1971	658.235	2.74952	1.52852	2.30803	2.18852

Source: BERNDT and WOOD (1975:263). [a] Billions of current dollar.

Chapter 3

The Capital-Energy Controversy: A Reconciliation.

Together with CHRISTOPH M. SCHMIDT

Abstract. Any serious empirical study of factor substitutability has to allow the data to display complementarity as well as substitutability. The standard approach reflecting this idea is a translog specification – this is also the approach used by the overwhelming majority of studies analyzing the substitutability of energy and capital. Yet, the substitutability between capital and energy and the source of discrepancies in the results still remain controversial. This paper offers a straightforward, but somewhat pessimistic explanation: Using a translog approach reduces the issue of factor substitutability to a question of cost shares. Our review of translog studies demonstrates that this argument is empirically far more relevant than the distinction between time-series and panel studies, being favored in the literature – all these studies can be reconciled with each other on the basis of the cost share argument alone.

3.1 Introduction

Since the first energy crisis of 1973 energy has been recognized as an important factor in the production of aggregate output. Specifically, economists have been interested in the possibility to substitute energy for other factors such as labor and, most importantly, capital. Most analysts would agree that any serious empirical study of factor substitutability has to allow the data to display complementarity as well as substitutability. The typical approach reflecting this idea is a static translog cost function specification (SOLOW 1987:605). It is used by the overwhelming majority of the large number of studies which have analyzed the question of capital-energy substitutability (for surveys, see THOMPSON and TAYLOR 1995, APOSTOLAKIS 1990, and KINTIS and PANAS 1989). In the more recent literature, dynamic, rather than static cost function approaches are pursued, though (e. g. MORRISON-PAUL and SIEGEL 1999).

The nature of data has apparently determined the results to a large extent: While time-series studies typically conclude that *capital* K and *energy* E are complements, analyses of panel data usually find the opposite. In particular, in the seminal time-series study by BERNDT and WOOD (1975, henceforth BW75) the energy-price elasticity of capital demand η_{Kp_E} is negative, indicating that capital and energy are complements, rather than substitutes. Their results, however, contrast with those of the will-cited cross-country panel studies by GRIFFIN and GREGORY (1976) and PINDYCK (1979), henceforth GG76 and P79, respectively. These authors find η_{Kp_E} to be significantly positive.

In their original contribution, GG76 argue that the major reason for this discrepancy would be the distinction between short-run and long-run adjustments. While analyses using cross-section or panel data should reflect long-run adjustments, time-series investigations should tend to document short-run reactions. Specifically, short-run (= time series) elasticity estimates concerning capital and energy are likely to show both as complements, since in the short-run it is not possible to design new equipment to achieve higher energy efficiency. In contrast, in the long-run energy and capital should be expected to be substitutes, leading to positive elasticities when estimated from panel data.

While this explanation seems quite convincing, a number of studies, for example TURNOVSKY *et al.* (1992), have diverged from this simple time-series versus cross-section/panel data dichotomy (see APOSTOLAKIS 1990:52-53). TURNOVSKY *et al.* (1992:62) conclude "that the different estimates of the elasticity of substitution between energy and capital cannot be reconciled simply on the basis of a long-run/short-run distinction based on the use of pooled or time series data". This issue has remained controversial ever since, despite considerable further effort being expended upon attempting to resolve the question of capital-energy substitutability and the sources of the discrepancies across the opposing studies. In the end, GRIFFIN's (1981:80) concerns seem to come true that "academics seem to prefer a varied diet opting to move on to new questions even if the old ones are not resolved. Will this be the fate of the energy-capital complementarity issue?"

This paper offers a straightforward, albeit somewhat pessimistic explanation for the observed discrepancies: Using a static translog approach tends to reduce the issue of factor substitutability to a question of cost shares. Specifically, the magnitudes of energy and capital cost shares are of paramount importance for the sign of the energy-price elasticity η_{Kp_E} of capital, regardless of whether a study is a time-series, cross-section or panel study. In any translog study, estimated cross-price elasticities $\eta_{x_ip_j}$ of any factor i with respect to the price p_j of another factor j are predominantly determined by the cost share of that factor j whose price is changing. Moreover, the estimate of the cross-price elasticity $\eta_{x_ip_j}$ tends to be the closer to the cost share s_j of factor j, the higher the cost share of factor i. Specifically, if cost shares of both capital and energy are relatively large, estimates of η_{Kp_E} will tend to be equal to the cost share s_E of energy and, hence, are substantially positive. On the other hand, if cost shares of capital and energy are small, estimates of η_{Kp_E} may happen to be positive as well as negative.

Our review of the static translog studies collected in the selective review by APOSTOLAKIS (1990) demonstrates that this cost-share argument is empirically far more relevant than the distinction between time-series and cross-section/panel studies. Static translog studies are the majority of studies listed in APOSTOLAKIS (1990). Irrespective of all the variation in estimated coefficients, all these studies can be reconciled with each other on the basis of our cost-share argument alone. For example, in all studies the cost shares s_M of the factor materials (M) are typically high, those of energy low. Correspondingly, estimates of elasticities $\eta_{x_i p_M}$ concerning price changes of materials have generally the largest values, whereas those of $\eta_{x_i p_E}$ are typically small in absolute values. Hence, input effects for capital (K), labor (L) and materials (M) due to changes of energy prices necessarily turn out to be small when they are estimated from static translog approaches.

Moreover, the estimates of cross-price elasticities should be very sensitive towards inclusion or exclusion of data on materials use into the analysis, as the cost shares of the other factors will change considerably if the factor M is dropped from the analysis. Therefore, on the basis of KLEM-panel data for German manufacturing, we finally support our cost-share argument by dropping data on materials use and comparing elasticity estimates from a KLE-data base with those originating from KLEM-data. In fact, all elasticity estimates unequivocally tend to increase upon the exclusion of M, making in particular a positive estimate of $\eta_{K p_E}$, that is, the finding of substitutability more likely.

Section 2 deals with the relationship of cost shares and cross-price elasticities within dual translog approaches. Section 3 offers a review of the translog studies cited by APOSTOLAKIS (1990). In Section 4, cross-price elasticity estimates from KLE-panel data for German manufacturing are compared to those obtained from KLEM-panel data. Section 5 concludes.

3.2 Cross-Price Elasticities Within Translog Studies

The overwhelming majority of studies analyzing the substitutability of capital and energy employs the classical dual translog approach (see e. g. APOSTOLAKIS 1990). This holds, in particular, for the seminal studies BW75 and GG76. In these studies, it is typically assumed that in manufacturing there exists a homothetic, twice differentiable aggregate translog cost function of the form (see TAKAYAMA 1985:148)

$$\ln C(p_1, ..., p_I, Y) = \beta_0 + \beta_Y \ln Y + \sum_{i=1}^{I} \beta_i \ln p_i + \frac{1}{2} \sum_{i,j=1}^{I,I} \beta_{ij} \ln p_i \ln p_j, \tag{3.1}$$

where p_i denotes the price of input i and Y aggregate output. Symmetry of β_{ij} is imposed *a priori*. If all second-order translog parameters β_{ij} are equal to zero, expression (3.1) specializes to the well-known COBB-DOUGLAS cost function. Linear homogeneity in prices, an inherent feature of any cost function, requires

$$\sum_{i=1}^{I} \beta_i = 1 \qquad \text{and} \qquad \sum_{i=1}^{I} \beta_{ij} = 0 \qquad \text{for } j = 1, ..., I. \tag{3.2}$$

Applying SHEPHARD's Lemma, $x_i = \frac{\partial C}{\partial p_i}$, and differentiating (3.1) logarithmically, one can derive a linear expression of the share of overall cost attributable to each factor i:

$$s_i = \frac{x_i p_i}{C} = \frac{\partial \ln C}{\partial \ln p_i} = \beta_i + \sum_{j=1}^{I} \beta_{ij} \ln p_j. \tag{3.3}$$

In the further analysis, this paper focuses on cross-price elasticities, specifically on η_{Kp_E}, the energy-price elasticity of capital. In the literature on the substitutability of energy and capital, however, empirical studies typically report ALLEN elasticities of substitution (AES), the most prominent measures of substitution (THOMPSON and TAYLOR 1995:565). Nevertheless, our decision does not at all limit our capacity to discuss the issue of energy and capital, since only the signs of either η_{Kp_E} or AES$_{KE}$ are of interest and, generally, both AES$_{ij}$ and $\eta_{x_ip_j}$ have always the same sign due to the fact that any cost share s_j is always positive and

$$\text{AES}_{ij} = \frac{1}{s_j} \cdot \eta_{x_ip_j} \qquad \text{for} \qquad i \neq j. \tag{3.4}$$

Moreover, with

$$\text{AES}_{ij} = \frac{\beta_{ij}}{s_i s_j} + 1 \qquad \text{for} \qquad i \neq j \tag{3.5}$$

for translog cost functions, the cross-price elasticity $\eta_{x_i p_j}$ of each factor i with respect to the change in the price of factor j reads

$$\eta_{x_i p_j} = \frac{\beta_{ij}}{s_i} + s_j \qquad \text{for} \qquad i \neq j. \tag{3.6}$$

Obviously, the cost shares s_i and s_j of both factors i and j affect the cross-price elasticity $\eta_{x_i p_j}$. From a closer inspection of expression (3.6), it is to be expected that, in general, the cross-price elasticity $\eta_{x_i p_j}$ will be close to the cost share of factor j if factor i's cost share is large relative to the second-order coefficient β_{ij}. If the translog cost function (3.1) specializes to the COBB-DOUGLAS function ($\beta_{ij} = 0$ for all i, j), $\eta_{x_i p_j}$ is even equal to the cost share of factor j. Moreover, estimates of $\eta_{x_i p_j}$ generally should tend to be the closer to the cost share s_j, the larger is the cost share s_i. With commonly small magnitudes of estimates of second-order coefficients β_{ij}, expression (3.6) is then clearly dominated by the cost share s_j of factor j.[1] The economic intuition behind this reasoning is: The larger the cost share s_i of factor i already is, the harder it is to substitute i for a factor j whose price is increasing, and input reactions of i depend upon the "importance" of factor j, measured in terms of its cost share s_j.

In our specific application to the capital-energy debate, we expect the cost share of energy to play a major role in the determination of $\eta_{K p_E}$.[2] Because the cost share attributable to energy is typically low, estimates of any elasticity $\eta_{x_i p_E}$, specifically those of $\eta_{K p_E}$, may be expected to be small in absolute value. As

[1] Table A1 of Appendix A, reporting parameter estimates for various translog models on the basis of German manufacturing data, reveals that such cases are rather the rule than the exception: Apart from $\widehat{\beta}_{EM} = 0.0638$, all estimates for parameters β_{ij} with $i \neq j$ are in the range of -0.033 to 0.033.

[2] Rather than considering both, the cross-price elasticities $\eta_{E p_K}$ and $\eta_{K p_E}$, it suffices to focus on $\eta_{E p_K}$ alone, since $\eta_{E p_K}$ and $\eta_{K p_E}$ have the same sign. This follows from (3.4) and the symmetry of AES. Finally, note that BLACKORBY and RUSSELL (1989:883) criticize AES to have no meaning as a quantitative measure and, qualitatively, to add no more information to that contained in the cross-price elasticity. The focus on $\eta_{K p_E}$, rather than on AES_{KE} has the additional advantage that it is relatively transparent under which conditions cost shares of the factor energy are a close approximation to the elasticity estimate. This will be difficult with a focus on AES, since in (3.5) products of cost shares are involved in the denominator.

an illustration, we contrast here the seminal studies by BW75 and GG76. BW75 included information on the use of materials, a factor with large cost shares in any of the years during the observation period (see Table 1). In accordance with our reasoning, these large cost shares correspond to large and positive estimates of elasticities $\eta_{x_i p_M}$ for any factor i, while the parameters associated with labor, s_L and $\eta_{x_i p_L}$, are in the second place of cost-share and elasticity rankings. On the other hand, small capital and energy shares are in agreement with low estimates of capital and energy price elasticities $\eta_{x_i p_K}$ and $\eta_{x_i p_E}$, respectively. In fact, estimates of $\eta_{K p_E}$ and $\eta_{E p_K}$ are even negative, implying BW75's well-known conclusion of capital-energy complementarity.

Table 1: Comparison of the Studies by BW75 and GG76.

	BERNDT & WOOD (1975) Time Series Data for the USA					GRIFFIN & GREGORY (1976) Panel Data for 1955, 1960, 1965 and 1969								
	1947	1953	1959	1965	1971	B	D	F	W-G	I	NL	NOR	UK	USA
Cost Shares						Cost Shares for 1965								
s_E	0.04	0.04	0.05	0.04	0.05	0.17	0.08	0.11	0.10	0.15	0.16	0.17	0.12	0.13
s_K	0.05	0.05	0.06	0.06	0.05	0.32	0.37	0.27	0.39	0.33	0.32	0.31	0.26	0.14
s_L	0.25	0.27	0.27	0.28	0.29	0.51	0.55	0.62	0.51	0.52	0.52	0.52	0.62	0.73
s_M	0.66	0.64	0.62	0.62	0.61	–	–	–	–	–	–	–	–	–
Cross-Price Elasticities						Cross-Price Elasticities for 1965								
$\eta_{K p_E}$	-0.14	-0.15	-0.14	-0.14	-0.16	0.17	0.08	0.11	0.10	0.15	0.16	0.17	0.12	0.13 (0.11)
$\eta_{L p_E}$	0.03	0.03	0.03	0.03	0.03	0.15	0.05	0.08	0.08	0.12	0.13	0.14	0.09	0.11 (0.02)
$\eta_{M p_E}$	0.03	0.04	0.03	0.03	0.03	–	–	–	–	–	–	–	–	–
$\eta_{E p_K}$	-0.17	-0.17	-0.18	-0.18	-0.17	0.32	0.39	0.27	0.40	0.33	0.32	0.33	0.27	0.15 (0.14)
$\eta_{L p_K}$	0.06	0.05	0.06	0.06	0.05	0.12	0.19	0.11	0.19	0.14	0.13	0.13	0.10	0.01 (0.05)
$\eta_{M p_K}$	0.03	0.03	0.03	0.03	0.02	–	–	–	–	–	–	–	–	–
$\eta_{E p_L}$	0.16	0.17	0.18	0.18	0.20	0.45	0.40	0.52	0.40	0.45	0.45	0.45	0.53	0.64 (0.10)
$\eta_{K p_L}$	0.26	0.27	0.28	0.29	0.30	0.20	0.29	0.26	0.26	0.23	0.22	0.21	0.25	0.05 (0.08)
$\eta_{M p_L}$	0.15	0.16	0.16	0.17	0.18	–	–	–	–	–	–	–	–	–
$\eta_{E p_M}$	0.49	0.49	0.47	0.46	0.46	–	–	–	–	–	–	–	–	–
$\eta_{K p_M}$	0.37	0.34	0.35	0.35	0.30	–	–	–	–	–	–	–	–	–
$\eta_{L p_M}$	0.37	0.37	0.37	0.37	0.37	–	–	–	–	–	–	–	–	–

Note: B: Belgium, D: Denmark, F: France, W-G: West Germany, I: Italy, NL: Netherlands, NOR: Norway. While BW75 do not provide any standard errors, GG76 report standard errors solely for the USA. Cost shares, not reported by GG76, are calculated from reported AES_{ij} and $\eta_{x_i p_j}$ by the authors on the basis of (3.4).

The elasticity estimates of the panel study by GG76 are based on 4 observations for 9 countries. Cost shares are higher as a consequence of the omission of materials, and elasticity estimates therefore resemble closely the pattern of cost shares: Table entries in the rows for s_E and for estimates of $\eta_{L p_E}$ are very close to each other, and those in the table rows for s_E and the estimates of η_{Kp_E} are even identical. This implies, in particular, that capital and energy are estimated as substitutes. Moreover, in the counterfactual situation in which GG76 had included materials with a cost share of about 2/3, as in BW75, and if they had estimated still the same second-order coefficient β_{KE} upon inclusion of M, estimated η_{Kp_E}'s would have been considerably smaller – much closer to the BW75 results than has been recognized.

3.3 The Capital-Energy Controversy Reviewed

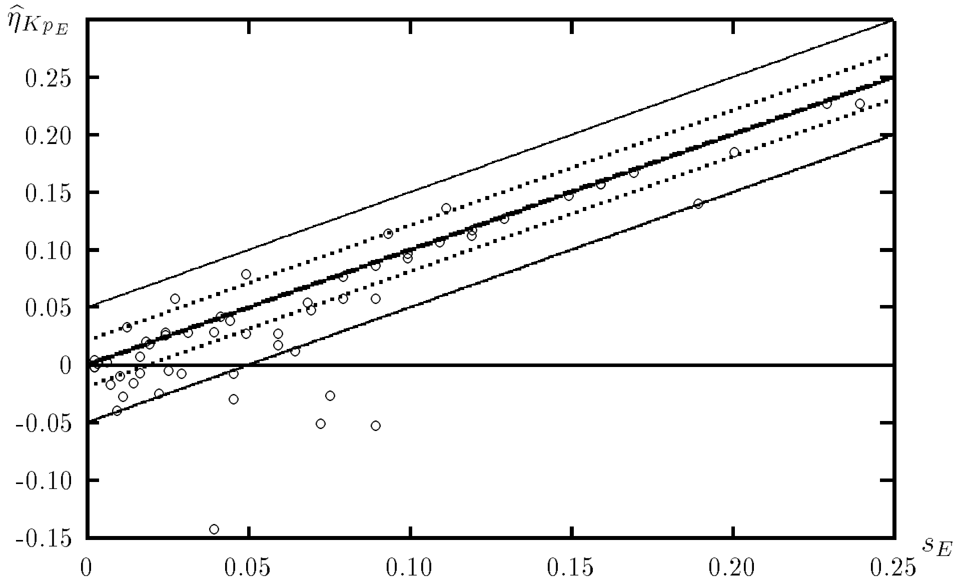
This section, analyzing all accessible static translog studies of APOSTOLAKIS' (1990) review of the capital-energy controversy presents further empirical evidence for our argument that factor cost shares s_j typically represent a good approximation to the related cross-price elasticities $\eta_{x_i p_j}$ and, moreover, that the approximation is the better, the larger is the cost share s_i. Hence, specifically, if cost shares of capital and energy are relatively large, estimates of η_{Kp_E} from translog studies will tend to be equal to the cost share s_E of energy and will therefore turn out to be substantially positive, indicating that capital and energy are substitutes. On the other hand, if cost shares of capital and energy are small, estimates of η_{Kp_E} may happen to be positive as well as negative, regardless of whether a study is a time-series, cross-section or a panel study.

Our cost-share argument is confirmed by Figure 1. It summarizes our detailed review by plotting 69 estimates of energy-price elasticities of capital against the corresponding cost shares s_E of energy.[3] Figure 1 reveals that the overwhelming number of η_{Kp_E}-estimates are located in a ± 0.05-corridor around the benchmark given by the cost share s_E of energy, regardless of the magnitude of the capital cost shares s_K. Only 5 out of 69 estimates display greater deviations from s_E in

[3] Because of limited variation in cost shares during the sample periods, only one pair of observations is taken from each times-series study. For example, from BW75 the entry ($s_E = 0.04, \widehat{\eta}_{Kp_E} = -0.14$) is chosen as being representative for the whole period.

absolute terms. These outliers all stem from studies in which capital cost shares s_K are relatively low. In BW75, $\widehat{\eta}_{Kp_E} = -0.14$ is far from $s_E = 0.04$ due to very low capital shares $s_K = 0.05$. Similarly, in ANDERSON (1980), $\widehat{\eta}_{Kp_E} = -0.05$ deviates strongly from $s_E = 0.09$ in absolute terms, as capital cost are as low as 9 % of overall cost. Capital cost shares of DARGAY's (1983) study, providing the remaining 3 out of the 5 outliers, are relatively low as well: s_K is in the range of 4.3% to 17.3 %.

Figure 1: The Relationship between η_{Kp_E} and Cost Shares of Energy – Empirical Evidence from the Capital-Energy Debate.



Given the numbers in Figure 1, for any empirical study, no matter whether a time-series, cross-section or panel study, one would expect a substantially positive estimate of η_{Kp_E} for both relatively large energy cost shares greater than 10 % and relatively large cost shares of capital greater than 30 %. Then, it is most unlikely that capital and energy turn out to be complements: In fact, on the basis of all estimates of β_{KE} from the studies involved in Figure 1 the estimates of η_{Kp_E} for $s_E = 0.1$ and $s_K = 0.3$ would turn out to be positive, except for the five southern European countries in APOSTOLAKIS (1987). Thus, simply the single knowledge of the magnitudes of capital and energy shares already allows a good prediction of whether any translog study will lead to the conclusion that these factors are substitutes or complements.

We now review in detail those of APOSTOLAKIS' (1990) studies which employ static translog approaches. This is not only the majority of the studies listed in APOSTOLAKIS (1990), but also a collection of all the prominent studies in this area. Our cost-share argument holds for all studies, in particular for those being exceptions from the dichotomy between times-series and (pooled) cross-section studies suggested by GG76, namely, the time-series studies by WALTON (1981), TURNOVSKY *et al.* (1982), DARGAY (1983) and APOSTOLAKIS (1987). These studies find positive, not negative estimates of the energy-price elasticity of capital (*substitutability*), while the regional panel study of FUSS (1977) is exceptional by concluding that capital and energy are *complements* for at least one region. In our review, we pay close attention to the question of whether a study analyzes a model for capital, labor and energy (KLE) or also incorporates materials (KLEM), a distinction which turns out to be quite important. The decisive factor, though, is the magnitude of the cost shares involved, not the number of factors employed in the estimation.

Cross-section and Panel Studies

The seminal panel study P79 analyzes cross-country manufacturing data over the period of 1963-1973. Table 2 demonstrates that the proportion of labor cost in this data is much larger than that of capital in all countries except for Japan and West-Germany, while the shares of energy are uniformly the lowest. This pattern is perfectly in line with these countries' hierachy of cross-price elasticities: Except for Japan and West-Germany, $\widehat{\eta}_{x_i p_L}$ are larger than $\widehat{\eta}_{x_i p_K}$, while estimates of $\eta_{x_i p_E}$ display the smallest magnitudes in all countries. Values of $\widehat{\eta}_{L p_E}$ and s_E are almost equal, those of $\widehat{\eta}_{K p_E}$ and s_E are very close, confirming our expectation that both coincide for sufficiently high s_K. Except for the UK, capital cost shares are in the range of 36 % up to 59 %, compared to about 5 % in BW75. These large shares might be partly due to missing M, since P79 estimates a KLE-model like GG76.

However, the cost-share structure of P79 differs substantially from that of GG76 for the same countries (compare Table 2 to Table 1). In particular, GG76's energy shares are in the range of 8 % to 17 %, whereas those in P79 are around 6 and 7 %. Accordingly, cross-price elasticities concerning energy prices estimated by P79 are much lower than those estimated by GG76.

Table 2: PINDYCK's (1979) Panel Study.

	Japan	France	West-Germany	Italy	Nether-lands	Norway	United Kingdom	Sweden
Cost Shares (own calculations)								
s_K	0.54	0.47	0.59	0.42	0.37	0.42	0.26	0.36
s_L	0.38	0.48	0.36	0.51	0.57	0.53	0.68	0.55
s_E	0.08	0.05	0.05	0.07	0.06	0.05	0.06	0.09
Cross-Price Elasticities								
η_{Kp_E}	0.06	0.03	0.03	0.05	0.03	0.03	0.02	0.06
η_{Lp_E}	0.09	0.06	0.06	0.08	0.07	0.06	0.07	0.10
η_{Ep_K}	0.41	0.26	0.38	0.27	0.20	0.21	0.09	0.24
η_{Lp_K}	0.38	0.34	0.42	0.30	0.26	0.30	0.17	0.25
η_{Ep_L}	0.43	0.57	0.45	0.57	0.64	0.62	0.75	0.60
η_{Kp_L}	0.26	0.35	0.25	0.36	0.40	0.38	0.44	0.38

Note: It remains unclear whether PINDYCK reports representative estimates or estimates for one of the 11 years. Cross-price elasticities are calculated from PINDYCK's AES_{ij}. Data for Canada and USA are pooled separately from those of the other countries. Thus, estimates for Canada and USA are not reported here.

In the KLEM-cross-section study by WILLIAMS and LAUMAS (1981) for Indian manufacturing in 1968 capital and energy are classified as substitutes in all industries. Although substitutability is found in the presence of materials, these results can be explained by the cost-share structure as well (see Table 3). Symptomatic for less developed countries like India, materials and especially labor cost are quite low in comparison to that in industrialized countries, whereas the cost shares of capital are as high as those to be found in the KLE-studies by P79 and GG76, except for machinery. In accordance with our arguments the corresponding estimates of η_{Kp_E} are very close to the energy cost shares s_E and thus significantly positive, even for industries where energy shares are small.

A study displaying a similar cost-share structure is the KLE-panel study of IQBAL (1986) for Pakistan's manufacturing (see the first panel in Table 4). While the cost share of capital is very high, the share of energy only amounts to 5

%. Consequently, the estimate of η_{KE} is positive, although $\widehat{\eta}_{Kp_E} = 0.082$ does not seem to be very close to $s_E = 0.05$. However, standard errors are quite high, qualifying a deviation of 0.032 as being clearly in the realm of sampling variability.

Table 3: WILLIAMS and LAUMAS' Cross-Section Study (1981).

	Manufacturing Industries of India in 1968.							
	Food	Textile	Chemical Products	Mineral Products	Machinery	Metal Products	Electrical Machinery	Transport
Cost Shares (own calculations)								
s_K	0.370	0.390	0.498	0.466	0.097	0.348	0.415	0.476
s_L	0.124	0.254	0.136	0.206	0.429	0.211	0.293	0.315
s_M	0.466	0.311	0.324	0.234	0.409	0.418	0.275	0.189
s_E	0.040	0.045	0.042	0.094	0.065	0.025	0.017	0.020
Selected Cross-Price Elasticities								
η_{Kp_E}	0.0314	0.0413	0.0448	0.1169	0.0149	0.0286	0.0104	0.0208
	(0.0070)	(0.0064)	(0.0060)	(0.0308)	(0.0242)	(0.0031)	(0.0044)	(0.0017)
η_{Lp_E}	-0.0509	0.0377	0.0336	0.1608	-0.0019	-0.0002	-0.0348	0.0007
	(0.041)	(0.0550)	(0.0328)	(0.0972)	(0.0428)	(0.0149)	(0.0450)	(0.0060)
η_{Mp_E}	0.0529	0.0590	0.0427	0.0206	0.0315	0.0299	0.0147	0.0229
	(0.0073)	(0.014)	(0.0071)	(0.0578)	(0.0185)	(0.0059)	(0.0034)	(0.0090)
η_{Ep_M}	0.6164	0.4069	0.3322	0.0507	0.1990	0.5013	0.2081	0.2576
	(0.0850)	(0.0951)	(0.0553)	(0.1421)	(0.2767)	(0.9992)	(0.0515)	(0.1018)
η_{Lp_M}	0.5651	0.2698	0.3630	0.2974	0.2119	0.5699	0.2686	0.0245
	(0.0817)	(0.00495)	(0.0569)	(0.0773)	(0.0926)	(0.0538)	(0.0314)	(0.0624)
η_{Kp_M}	0.5440	0.2301	0.2731	0.1548	0.3863	0.3611	0.2470	0.2077
	(0.0499)	(0.0427)	(0.0291)	(0.0436)	(0.0484)	(0.0392)	(0.0199)	(0.0172)

Note: Standard errors are reported in parentheses.

The KLEM-Panel Study by ÖZATALAY *et al.* (1979) for 7 nations (1963-1974) reports an η_{Kp_E}-elasticity estimate of 0.031 for the USA, which is very close to the cost share of energy ($s_E = 0.025$). This also holds for the elasticity estimates $\widehat{\eta}_{Lp_E} = 0.026$ and $\widehat{\eta}_{Mp_E} = 0.024$ (see the second panel of Table 4). FUSS' (1977) KLEM-study for Canadian manufacturing is often cited as a panel study providing evidence for capital and energy being complements, albeit not statistically significant (see the third panel of Table 4). The explanation for this outcome is simple, though: For such low cost shares of energy like $s_E = 1.7\%$, negative estimates of η_{KE} are in the realm of sampling variability, particularly with a cost share of capital which is not very large, either.

In HALVORSON and FORD's (1979) cross-section study for US manufacturing (last panel of Table 4), for 3 out of 4 sectors capital is found to substitute for energy, while both have to be classified as complements in the "Fabricated Metals"-sector. High standard errors do not allow outcomes to be statistically significant, though. Again, estimates of η_{KE} are very close to energy cost shares, except for "Fabricated Metals". There, s_K displays the smallest value among the 4 sectors. With both a low energy cost share ($s_E = 2.3\%$) and a relatively small share of capital, it is not unlikely that negative estimates of η_{KE} occur in this sector. In contrast, negative values seem to be rather unlikely in the chemicals producing sector, where both s_E and s_K are relatively high.

Table 4: The Panel Studies by IQBAL (1986), ÖZATALAY *et al.* (1979) and FUSS (1977), and the Cross-Section Study by HALVORSON and FORD (1979).

Panel-Study by IQBAL (1986) for Pakistanian Manufacturing (1960-1970).							
	Energy Price Elasticties			Cost Shares			
	η_{Kp_E}	η_{Lp_E}		s_E	s_K	s_L	
	0.082	-0.024		0.050	0.787	0.163	
	(0.014)	(0.127)					
Panel Study by ÖZATALAY *et al.* (1979) : USA reported only.							
Cross-Price Elasticties for 1965(own calculation)				Cost Shares			
	η_{Kp_E}	η_{Lp_E}	η_{Mp_E}	s_E	s_K	s_L	s_M
	0.031	0.026	0.024	0.025	0.210	0.240	0.525
FUSS' (1977) Panel Study for Canada (1961-1971): Ontario reported only							
Cross-Price Elasticties calculated at the mean values				Cost Shares			
	η_{Kp_E}	η_{Lp_E}	η_{Mp_E}	s_E	s_K	s_L	s_M
	-0.004	0.043	-0.0006	0.017	0.225	0.228	0.531
	(0.018)	(0.022)	(0.0106)				
US-Cross-Section Study by HALVORSON and FORD (1979) for 1974 *							
	Energy Price Elasticties			Cost Shares			
Sector	η_{Kp_E}	η_{Bp_E}	η_{Wp_E}	s_E	s_K	s_B	s_W
Food	0.031	-0.016	0.091	0.032	0.258	0.415	0.295
	(0.027)	(0.073)	(0.057)				
Chemicals	0.139	0.199	0.257	0.112	0.364	0.431	0.145
	(0.073)	(0.055)	(0.118)				
Primary	0.057	0.155	0.331	0.069	0.371	0.315	0.243
Metals	(0.078)	(0.044)	(0.093)				
Fabricated	-0.022	0.054	-0.042	0.023	0.207	0.530	0.240
Metals	(0.034)	(0.010)	(0.031)				

Note: All cost shares are calculated here on the basis of the published information.

Standard errors are given in parentheses. * B denotes blue-collar and W white-collar workers.

Finally, the cross-section study by FIELD and GREBENSTEIN (1990) is a KLE-study with two types of capital, physical and working capital. The authors argue that the divergent results of the studies by BW75 and GG76 are due to the way they handle capital input. While BW75 use a service price approach, in which the cost of reproducible capital is expressed as the product of the service price and the quantity of *physical* capital alone, GG76 pursue a value-added approach, where the total cost of capital, i. e., the sum of what they termed *physical* and *working* capital with cost shares s_P and s_W, respectively, is in the center of attention. This cost is constructed as the difference of value added and the payments of wages and salaries. Although standard errors are extremely high (see Table 5) due to very few degrees of freedom (in 3 industries merely one degree of freedom), they conclude that physical capital is complementary to energy, whereas working capital substitutes for energy. Applying our explanation pattern, different signs for η_{Pp_E} and η_{Wp_E} might also be due to low cost shares s_P and high shares s_W. Then, rather than η_{Pp_E}, η_{Wp_E} is much more likely to be close to s_E and, hence, positive. However, outcomes have to be interpreted with caution, since they are hardly significant.

Table 5: FIELD and GREBENSTEIN's Cross-Section Study (1980).

	Selected Sectors of US Manufacturing in 1971.					
	Energy Price Elasticities of Capital Inputs		Cost Shares (own calculations)			
Sector	η_{Pp_E}	η_{Wp_E}	s_E	s_P	s_W	s_L
Food	-1.410 (0.072)	0.075 (0.019)	0.028	0.088	0.517	0.367
Lumber	-0.648 (0.160)	0.373 (0.502)	0.045	0.123	0.354	0.478
Chemicals	-0.505 (0.173)	0.220 (0.062)	0.054	0.137	0.528	0.282
Petroleum	-0.005 (0.173)	0.355 (0.234)	0.069	0.165	0.594	0.172
Rubber	-0.201 (0.150)	0.063 (0.037)	0.028	0.099	0.458	0.415
Stone, Clay Glass	-0.236 (0.178)	0.090 (0.090)	0.072	0.130	0.392	0.405
Primary Metals	-0.536 (0.259)	0.410 (0.197)	0.076	0.167	0.281	0.476
Fabricated Metals	0.023 (0.156)	0.020 (0.023)	0.019	0.067	0.429	0.485
Transport	-0.342 (0.242)	0.170 (0.333)	0.014	0.059	0.467	0.460
Instruments	0.165 (0.169)	0.453 (0.377)	0.011	0.056	0.509	0.424

Note: P denotes physical and W working capital. Standard errors are given in parentheses.

The Time-Series Studies

The KLEM-study for Australian manufacturing (1946-1975) by TURNOVSKY *et* *al.* (1982:62) is apparantly "the first time-series study to show energy-capital as substitutes": The estimated energy-price elasticity of capital is $\widehat{\eta}_{Kp_E} = 0.06$, standard errors are not provided, though. This study fits very well into our cost-share argument. Cross-price elasticities concerning materials prices ($\widehat{\eta}_{Kp_M} = 0.40, \widehat{\eta}_{Lp_M} = 0.33$ and $\widehat{\eta}_{Ep_M} = 0.42$) are located in the vicinity of $s_M = 0.535$, whereas those concerning energy prices are around the cost share of energy: With $s_K = 0.196$ and a relatively small cost share of energy, $s_E = 0.028$, $\widehat{\eta}_{Kp_E}$ happens to be positive while, on the other hand, with $s_L = 0.241$ the energy-price elasticity of labor happens to be negative: $\widehat{\eta}_{Lp_E} = -0.07$. Finally, $\widehat{\eta}_{Mp_E} = 0.02$ reflects the high share of materials in the denominator of the corresponding elasticity.

APOSTOLAKIS' (1987) KLE-study for 5 southern european economies is also one of the few time-series analyses concluding that capital and energy are substitutes (see Table 6).

Table 6: APOSTOLAKIS' KLE-Time-Series Study (1987) for 1953-1984.

Year	η_{KE}	η_{LE}	η_{EK}	η_{LK}	η_{EL}	η_{KL}	s_E	s_K	s_L
				France					
1953	0.23	0.24	0.28	0.28	0.44	0.47	0.24	0.30	0.46
1965	0.15	0.17	0.33	0.34	0.44	0.46	0.17	0.37	0.46
1979	0.14	0.16	0.29	0.30	0.50	0.52	0.16	0.32	0.52
				Greece					
1953	0.12	0.19	0.21	0.30	0.48	0.51	0.19	0.31	0.50
1965	0.05	0.13	0.15	0.36	0.47	0.50	0.13	0.38	0.49
1979	0.17	0.23	0.19	0.25	0.51	0.54	0.22	0.25	0.53
				Italy					
1953	0.29	0.36	0.16	0.16	0.51	0.62	0.31	0.17	0.53
1965	0.14	0.19	0.30	0.35	0.43	0.51	0.17	0.37	0.46
1979	0.20	0.24	0.21	0.23	0.53	0.62	0.22	0.23	0.55
				Portugal					
1953	0.16	0.10	0.32	0.44	0.28	0.33	0.16	0.51	0.33
1965	0.14	0.09	0.28	0.41	0.36	0.41	0.14	0.45	0.40
1979	0.24	0.21	0.19	0.21	0.52	0.55	0.24	0.22	0.54
				Spain					
1953	0.23	0.19	0.33	0.37	0.33	0.36	0.23	0.41	0.36
1965	0.17	0.13	0.30	0.37	0.40	0.43	0.17	0.40	0.43
1979	0.21	0.18	0.21	0.23	0.53	0.55	0.21	0.24	0.55

Note: Standard errors are not provided.

This is hardly surprising: Without taking account of materials use, cost shares of capital and energy are relatively high, even though those of energy are decreasing over time. Therefore, the estimates of elasticities η_{Kp_E} and η_{Lp_E} should be expected to be in the vicinity of energy cost shares s_E, an expectation confirmed by the results of Table 6. This necessarily implies positive elasticities η_{Kp_E}, that is, capital and energy are to be classified as substitutes for all countries.

ANDERSON's (1981) time-series study uses exactly the same KLEM-price data for almost the same sample period as BW75, only data for 1947 are dropped (see Table 7).

Table 7: BW75's and ANDERSON's (1981) Times-Series Studies for US Manufacturing.

	BERNDT & WOOD			ANDERSON		
	1947	1959	1971	1948	1960	1971
Cost Shares (own calculations)						
s_K	0.05	0.06	0.05	0.09	0.09	0.07
s_L	0.25	0.27	0.29	0.41	0.44	0.45
s_E	0.04	0.05	0.05	0.09	0.08	0.08
s_M	0.66	0.62	0.62	0.41	0.39	0.40
Cross-Price Elasticities						
η_{Kp_E}	-0.14	-0.14	-0.16	-0.05	-0.05	-0.10
η_{Lp_E}	0.03	0.03	0.03	0.06	0.05	0.05
η_{Mp_E}	0.03	0.03	0.03	0.01	0.00	0.00
η_{Ep_K}	-0.17	-0.18	-0.17	-0.06	-0.06	-0.09
η_{Lp_K}	0.06	0.06	0.05	0.05	0.06	0.04
η_{Mp_K}	0.03	0.03	0.02	0.00	0.00	-0.02
η_{Ep_L}	0.16	0.18	0.20	0.31	0.33	0.34
η_{Kp_L}	0.26	0.28	0.30	0.24	0.28	0.28
η_{Mp_L}	0.15	0.16	0.18	0.34	0.37	0.38
η_{Kp_M}	0.37	0.35	0.30	-0.01	-0.02	-0.14
η_{Lp_M}	0.37	0.37	0.37	0.34	0.34	0.34
η_{Ep_M}	0.49	0.47	0.46	0.08	0.02	0.01

Note: Both studies do not report any standard errors.

However, it employs a net output, rather than a gross output concept. Therefore, the implied cost shares differ substantially from those of BW75: Cost shares of materials use are substantially lower, while those of all other factors are higher. Accordingly, all estimated elasticities with respect to changing materials prices are lower than those of BW75, while almost all other elasticities increase. Consistent with a higher cost share of energy, the finding of BW75 of a substantial capital-energy complementarity is weakened.

A particular characteristic of DARGAY's (1983) study for Sweden is that both energy and capital cost shares are relatively small in the majority of the 12 sectors investigated. Expression (3.6) indicates that in such a situation it is difficult to predict the sign of the estimated elasticity. In this study, apart from the estimate of η_{Kp_E} for "Sheltered Food", all other estimates of η_{Kp_E} are negative. Several of these estimates are not statistically significant, though.

Table 8: DARGAY's Time-Series Study (1983).

Swedish Manufacturing, 12 Sectors (1952-1976)								
	Energy Price Elasticities (own calculations)			Cost Shares (own calculations)				
Sector	η_{Kp_E}	η_{Lp_E}	η_{Mp_E}	s_E	s_K	s_L	s_M
4. Sheltered Food	0.036	0.004	0.000	0.013	0.043	0.115	0.829
5. Import-Competing Food	-0.025	0.013	0.007	0.012	0.072	0.137	0.779
6. Beverages & Tobacco	-0.005	-0.037	0.024	0.030	0.129	0.250	0.591
7. Textiles & Clothing	-0.037	0.003	0.021	0.010	0.084	0.340	0.566
8. Wood, Pulp & Paper	-0.027	0.001	0.004	0.046	0.122	0.224	0.608
9. Printing	-0.014	0.008	0.006	0.008	0.135	0.459	0.399
10. Rubber Products	-0.002	0.012	0.019	0.026	0.141	0.342	0.491
11. Chemicals	-0.005	-0.010	0.025	0.046	0.117	0.251	0.586
13. Mineral Products	-0.024	-0.018	0.102	0.076	0.173	0.336	0.415
14. Primary Metals	-0.048	-0.045	-0.012	0.073	0.148	0.231	0.548
15. Engineering	-0.013	0.000	0.019	0.015	0.084	0.357	0.544
16. Shipbuilding	-0.007	0.004	0.009	0.011	0.085	0.316	0.588

Note: It remains unclear for which of the 15 years elasticity estimates are reported or whether DARGAY reports representative estimates.

WALTON's (1981) time-series study for U. S. Middle Atlantic manufacturing reveals the same characteristics with its very small cost shares of energy inputs as DARGAY's study for Sweden (see Table 9). In addition, WALTON divides energy inputs into electric energy E and fossile fuels F. As a result, specifically, cost shares s_E of electric energy are extremly small and even lower than DARGAY's energy cost shares. Hence, one would expect capital and energy to be complements in some sectors. Contrary to DARGAY's study, though, cost shares of capital are relatively high across all five sectors. Thus, it is not surprising to find estimates for η_{Kp_E}

which are closer to s_E than in DARGAY's study and to find solely positive-signed elasticity estimates of η_{Kp_E} and η_{Kp_F}.

Table 9: WALTON's Time-Series Study (1981).

U. S. Middle Atlantic Manufacturing, 5 Sectors (1950-1973).									
	Energy Price Elasticities				Cost Shares (own calculations)				
Years	η_{Kp_E}	η_{Kp_F}	η_{Lp_E}	η_{Lp_F}	s_E	s_F	s_K	s_L	s_M
All other Manufacturing									
1953	0.004	0.006	0.01	0.001	0.004	0.005	0.308	0.230	0.453
1971	0.005	0.004	0.01	0.001	0.005	0.004	0.331	0.278	0.382
SIC 28: Chemicals and Allied Products.									
1953	0.001	0.006	0.05	-0.01	0.003	0.008	0.488	0.116	0.383
1971	0.004	0.005	0.04	-0.01	0.006	0.007	0.556	0.130	0.299
SIC 29: Petroleum and Coal Products.									
1953	0.007	0.012	0.03	-0.005	0.003	0.005	0.458	0.052	0.482
1971	0.010	0.017	0.04	-0.004	0.005	0.008	0.372	0.040	0.574
SIC 32: Stone, Clay and Glass Products.									
1953	0.005	0.025	0.05	-0.04	0.007	0.023	0.530	0.164	0.276
1971	0.008	0.020	0.05	-0.04	0.010	0.018	0.543	0.171	0.258
SIC 33: Primary Metals Industry.									
1953	0.001	0.003	0.07	-0.08	0.003	0.016	0.500	0.107	0.374
1971	0.003	0.002	0.07	-0.08	0.007	0.015	0.516	0.113	0.349

Note: E denotes electric energy and F fossile fuels. No standard errors are provided.

Finally, on the basis of 13 separate translog time-series estimations KIM and LABYS (1988:317) summarize in their study for the Korean industrial sector (1960-1980) that "there is not substantial scope for substitutability of ... capital ... for energy in the less energy-intensive industries, but that this is possible in the energy-intensive industries" such as non-metallic and basic metal industries (see Table 10). Using our cost-share argument, their results are not surprising at all. Since the cost shares of energy are particularly high in the energy-intensive industries, these industries display the highest estimates of η_{Kp_E}, whereas the η_{Kp_E}-estimates in the food- and paper-industries and machinery, the less energy-intensive industries, are the lowest among all η_{Kp_E}-estimates. Under this perspective, doubt is cast on KIM and LABYS' (1988:319) conclusion "that capital and energy are weak substitutes and ... policies that foster accelerated replacement investment will not significantly serve to reduce gross energy consumption".

Table 10: The Time-Series Study by KIM and LABYS (1988).

		Korean Industrial Sectors (1960-1980).								
		Cross-Price Elasticities						Cost Shares		
Sector	Years	η_{Kp_E}	η_{Lp_E}	η_{Ep_K}	η_{Lp_K}	η_{Ep_L}	η_{Kp_L}	s_E	s_K	s_L
Food	1970	0.06	-0.17	0.86	0.81	-0.38	0.12	0.06	0.82	0.12
	1980	0.09	-0.09	0.78	0.75	-0.15	0.15	0.09	0.76	0.15
Textile	1970	0.10	-0.12	0.60	0.55	-0.41	0.31	0.10	0.58	0.32
	1980	0.10	-0.12	0.60	0.54	-0.39	0.32	0.10	0.58	0.32
Wood	1970	0.12	-0.11	0.55	0.39	-0.28	0.21	0.12	0.57	0.31
	1980	0.14	-0.02	0.40	0.29	-0.07	0.30	0.14	0.39	0.47
Paper	1970	-0.02	-0.01	-0.37	0.62	-0.83	0.36	0.01	0.74	0.35
	1980	0.03	0.04	0.34	0.59	-0.28	0.34	0.05	0.59	0.36
Chemical	1970	0.11	-0.19	0.49	0.75	-0.17	0.16	0.15	0.70	0.15
	1980	0.15	-0.11	0.47	0.68	-0.09	0.18	0.20	0.63	0.17
Non-metallic	1970	0.27	0.08	0.49	0.75	0.04	0.20	0.33	0.50	0.17
	1980	0.37	0.13	0.47	0.68	0.04	0.17	0.43	0.42	0.15
Basic	1970	0.23	0.09	0.45	0.63	0.07	0.27	0.26	0.52	0.22
Metal	1980	0.28	0.06	0.47	0.69	0.03	0.20	0.31	0.53	0.16
Machinery	1970	0.04	-0.02	0.64	0.64	-0.13	0.34	0.04	0.68	0.28
	1980	0.05	-0.00	0.60	0.60	-0.04	0.38	0.04	0.60	0.36
Other	1970	0.03	-0.11	0.42	0.56	-0.89	0.33	0.04	0.61	0.35
Manufacturing	1980	0.06	-0.06	0.43	0.50	-0.33	0.36	0.07	0.53	0.40
Total	1970	0.09	-0.12	0.52	0.67	-0.22	0.22	0.11	0.67	0.22
Manufacturing	1980	0.19	-0.01	0.48	0.56	-0.01	0.23	0.22	0.56	0.22
Agriculture	1970	0.01	-0.08	0.73	0.94	-0.60	0.17	0.02	0.81	0.17
	1980	0.05	-0.04	0.71	0.89	-0.07	0.11	0.06	0.84	0.10
Construction	1970	0.06	0.03	0.33	0.42	0.23	0.59	0.07	0.39	0.54
	1980	0.04	0.02	0.21	0.32	0.22	0.72	0.06	0.30	0.64
Total	1970	0.03	-0.13	0.38	0.70	-0.42	0.19	0.05	0.76	0.19
Industry	1980	0.12	-0.33	0.44	0.55	0.57	0.25	0.15	0.58	0.27

Note: No standard errors are provided. Cost shares are calculated on the basis of published information.

This review of the capital-energy debate provides ample empirical evidence for our argument that estimated cross-price elasticities $\eta_{x_i p_j}$ are mainly the result of corresponding cost shares s_j. With particular respect to the capital-energy debate, this review reveals also that only under very particular circumstances will a translog study be able to classify capital and energy as complements – when cost shares of both factors are small, negative estimates of the second-order coefficients β_{EK} of the associated translog cost function might lead to a negative estimate of the energy-price elasticity η_{Kp_E} of capital. In all other cases, capital and energy will typically be classified as substitutes.

The omission of materials from the calculation of total variable cost often implies that the calculated cost shares of both capital and energy are high, leading to the conclusion of $K-E$ substitutability. For panel studies, specifically, consistent information on materials is frequently unavailable, with the effect that the majority of panel studies find $K-E$ substitutability. Ultimately, though, it remains quite irrelevant whether a translog study is based on panel data or incorporates materials – it is the cost shares which tie together the results of the literature. Other reasons for differences in elasticity estimates between empirical studies turn out to be of minor importance, differences in estimation techniques, for example.[4] Similarly, distinct aggregation methods of capital inputs, suggested by FIELD and GREBENSTEIN (1980) as a major reason for the discrepancies giving rise to the capital-energy debate, are only relevant in so far as the magnitudes of corresponding cost shares are concerned.

3.4 A KLEM- versus a KLE-study for Germany

Using panel data for German manufacturing, we now move the issue of sensitivity of elasticity estimates into the center of attention. First, by specifying various models within the classical dual translog approach, the effect of model variation on elasticity estimates is investigated. In line with our central argument, these variations do not matter for our qualitative conclusions. Second, rather than varying specifications, cost shares are varied artificially by omitting M. This alteration turns out to be decisive.

The Effect of Varying the Translog Specification

Because of data limitations for energy data, our data base relates to the short range of 1978-1990. Overall, we have $377 = 29 \times 13$ observations from 29 sectors

[4] On the basis of the data used by BW75, this view is supported by comparing translog parameter estimates from different estimation methods, namely Maximum-Likelihood (ML) and Iterated Three-Stage Least Squares (I3SLS). I3SLS is employed by BW75 in order to tackle endogeneity problems of input prices with the help of instruments. Without access to these instruments, FRONDEL (1999) employs ML to estimate the same parameters. Differences in both parameter and cross-price elasticity estimates are negligible (see FRONDEL 1999:22,31).

of German manufacturing. Data necessary for estimation include cost shares and price indices for K, L, E and M.[5] With the dummy variable D_s refering to the sector s and assuming that in German manufacturing there exists a twice differentiable, nonhomothetic aggregate translog cost function of the form

$$\ln C = \sum_{s=1}^{29} \sum_{i=K,L}^{M,E} \beta_{is} D_s \ln p_i + \sum_{i,j=K,L}^{M,E} \left(\frac{\beta_{ij}}{2} \ln p_i \ln p_j + \beta_{iT} \ln p_i T + \beta_{iY} \ln Y \ln p_i\right), \quad (3.7)$$

we follow P79, except that technical progress is additionally taken into account by including a linear time trend. As in the panel study by P79, first-order parameters β_{is} are allowed to be different across all industries of German manufacturing, while second-order parameters β_{ij} are assumed to be constant, as are the parameters β_{iT} and β_{iY}.

The resulting cost-share equation system for $i \in \{K, L, E, M\}$ are (*Model* I):

$$s_i = \sum_{s=1}^{29} \beta_{is} D_s + \sum_{j=K,L}^{M,E} (\beta_{ij} \ln p_j + \beta_{iT} T + \beta_{iY} \ln Y). \quad (3.8)$$

To investigate the robustness of estimation results for these parameters, we alternatively estimate a *homothetic* version of system (3.8) by setting $\beta_{iY} = 0$ for all i (*Model* II). Finally, we estimate a quite restrictive homothetic model with a common intercept for all industries (*Model* III), which is based on the assumption of a single aggregate cost function for the 29 industries of German manufacturing. Apart from the linear time trend, Model III forms the estimation basis for the time-series study by BW75 as well as for the panel study by GG76.

The four cost shares in (3.8) always sum to unity at each observation. Therefore and because of the linear homogeneity in prices, the sum of addititionally added disturbances ε_i across the four equations is zero at each observation, implying the singularity of the disturbance covariance matrix. This problem is solved by dropping arbitrarily one of the four equations. By dropping for example the cost-share equation for M, the equation system for Model III, our most restrictive

[5] The sources of data and methods for constructing series for prices are described in Chapter 4. Cost shares for selected industries are reported in Table A3 of the Appendix.

model, notes:

$$
\begin{aligned}
s_K &= \beta_K + \beta_{KK} \ln(\frac{p_K}{p_M}) + \beta_{KL} \ln(\frac{p_L}{p_M}) + \beta_{KE} \ln(\frac{p_E}{p_M}) + \beta_{KT} T + \varepsilon_K, \\
s_L &= \beta_L \; + \beta_{KL} \ln(\frac{p_K}{p_M}) \; + \beta_{LL} \ln(\frac{p_L}{p_M}) + \beta_{LE} \ln(\frac{p_E}{p_M}) + \beta_{LT} T + \varepsilon_L, \\
s_E &= \beta_E + \beta_{KE} \ln(\frac{p_K}{p_M}) + \beta_{LE} \ln(\frac{p_L}{p_M}) + \beta_{EE} \ln(\frac{p_E}{p_M}) + \beta_{ET} T + \varepsilon_E \; .
\end{aligned}
\tag{3.9}
$$

In the seemingly unrelated regressions system (3.9), where equations are linked merely by disturbances, restrictions by homogeneity in prices are already imposed. System (3.9) is estimated by maximum likelihood (ML) in order to ensure that results do not depend upon the choice of which share equation is dropped (BERNDT 1991:473).

Translog-parameter estimates for all models are given in Table A1 of the Appendix and are quite close across all models, as are the corresponding estimates of cross-price elasticities. This illustrates that, generally, elasticity estimates do not seem to depend heavily on the choice of which model is employed. Moreover, on the basis of Lagrange-Multiplier tests, our homothetic model with individual intercepts, Model II, turns out to be the most appropriate one among our nested models (see the results of the LM-tests in Table A1). In Table A2, we therefore report only estimates of cross-price elasticities originating from Model II.[6] Estimates of cross-price elasticities for German manufacturing suggest once again that the pattern among cross-price elasticities more or less reflects that among the input cost shares, which are displayed in Table A3.

The Effect of Omitting M

Without taking account of materials with commonly high cost shares, the weights

[6]Because estimation results are rather stable, estimates for 3 equidistant years are displayed only and, moreover, merely for those industries, which display negative own-price elasticities for energy (only 7 out of 29 industries). Since energy cost shares s_E for all other industries are smaller than 6 % and the parameter estimate common to all industries is approximately 0.06 (see Table A1 of Appendix A), the fraction $\frac{\beta_{EE}}{s_E}$ is necessarily higher than one. This implies that the estimate of $\eta_{Ep_E} = \frac{\beta_{EE}}{s_E} - 1 + s_E$ is positive. That is, the empirical fact that energy plays a minor role with respect to overall cost in most industries of German manufacturing, together with the cross-industry restrictions on estimated parameters endangers the theoretical requirement of negative own-price elasticities.

of the remaining production factors are undoubtly much larger in the structure of cost shares than before. Taking for example BW75's data for 1965, where data on materials are included, the share of labor merely amounts to 29 % and those of capital and energy are about 5 %. Omitting M would artificially increase cost shares of labor, capital and energy up to 76%, 13 % and 13 %, respectively, which coincide almost exactly with those used by GG76 for U. S. manufacturing (see Table 1).

Our argument that elasticities $\eta_{x_i p_j}$ scatter around related cost shares s_j leads to the conclusion that enlarging cost shares by excluding M will trigger a raise in cross-price elasticities. However, dropping M from the data base and, hence, enlarging cost shares implies a radical change of the data base, which of course will not conserve estimates for β_{ij} in general. Changes of estimates for β_{ij} may outweigh increases in cost shares such that in fact there will be no raise in cross-price elasticities. Moreover, omitting materials may cause misspecification and estimation results may be biased in either direction (*omitted-variable* bias) if the assumption of weak separability of materials from capital, labor and energy does not hold (GG76:852). Therefore, the question about the changes of elasticity estimates upon the exclusion of M can only be answered empirically.

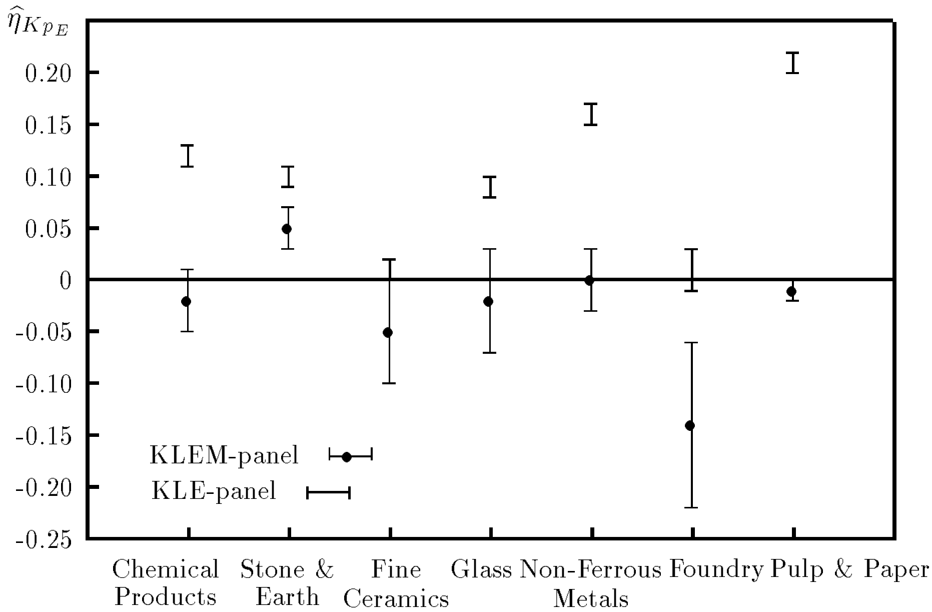
In order to find empirical evidence for an upward shift in cross-price elasticties, we now drop M from the KLEM-panel data of German manufacturing and estimate a translog KLE-model with different intercepts analogous to Model II. That is, we deliberately cause potential misspecification and estimate the followi-ng cost-share system, where the equation for E may be dropped arbitrarily (recall any equation may be dropped arbitrarily when estimating by ML):

$$
\begin{aligned}
s_K &= \sum_{s=1}^{29} \beta_{Ks} + \beta_{KK} \ln(\frac{p_K}{p_E}) + \beta_{KL} \ln(\frac{p_L}{p_E}) + \beta_{KT} \cdot T + \varepsilon_K, \\
s_L &= \sum_{s=1}^{29} \beta_{Ls} + \beta_{KL} \ln(\frac{p_K}{p_E}) + \beta_{LL} \ln(\frac{p_L}{p_E}) + \beta_{LT} \cdot T + \varepsilon_L.
\end{aligned}
\qquad (3.10)
$$

The result of a Lagrange-Multiplier-Test displayed in Table A1 reveals that Model II using four inputs is more appropriate than the restricted KLE-model (3.10). This answers the question of whether a KLE-model is misspecified. Nevertheless, estimates of remaining cross-price elasticities obtained from KLE-model (3.10) are reported in Table A4. Comparing these estimates with those in the upper half of Table A2, where M is taken into account, reveals an upward shift in all remaining elasticities. The upward shift is pronounced for the non-ferrous metals producing industry, where the contribution of materials cost to overall cost is the highest one among all industries of Table 3. By contrast, the shift is moderate for "fine ceramics", where the share of materials to overall cost is the lowest one. With particular regard to η_{Kp_E}, we find a significant sign reversal for the non-ferrous metals producing industry and for "foundry", indicating that capital and energy have to be classified as complements from the KLEM-model and as substitutes from the KLE-model.

Specifically, estimates of energy-price elasticities of capital, originating from a KLE-panel model and shown in Figure 2, are substantially higher than those resulting from the corresponding KLEM-panel model.

Figure 2: The Effect of Switching from a KLEM- to a KLE-Panel Model II.



Overall, for all industries selected Figure 2 validates our argument that elasticity estimates $\widehat{\eta}_{Kp_E}$ will be shifted upwards if cost shares s_E are increased artificially. This pattern is to be observed for all other industries not reported, too, less pronouced, though, as energy shares are lower than for those industries displayed in Figure 2. Obviously, manipulating cost shares changes elasticity estimates decisively, whereas varying the translog model or the estimation technique produces rather negligible changes.

3.5 Summary and Conclusion

Our review of the capital-energy debate provides ample empirical evidence for our argument that estimated cross-price elasticities $\eta_{x_i p_j}$ are mainly the result of corresponding cost shares s_j. As a first implication, typically, the estimates of cross-price elasticities concerning energy prices, $\eta_{x_i p_E}$, scatter around low energy shares and, hence, are small in absolute values. Input effects for K, L and M due to changes of energy prices will thus turn out to be small when estimated by a static dual translog approach.

Second, with respect to the capital-energy debate, a translog study will be able to classify capital and energy as complements only under very special circumstances: When cost shares of both factors are small and negative estimates of the second-order coefficients β_{EK} of the associated translog cost function might lead to a negative estimate of the energy-price elasticity η_{Kp_E} of capital. In all other cases, in particular, when cost shares of both capital and energy are relatively high, capital and energy will typically be classified as substitutes.

Third, the omission of materials from the calculation of total variable cost often implies that the calculated cost shares of both capital and energy are high, leading to the conclusion of $K - E$ substitutability. In panel studies, in particular, consistent information on materials is frequently unavailable, with the effect that the majority of panel studies find $K - E$ substitutability.

Ultimately, though, it remains quite irrelevant whether or not a translog study incorporates materials or whether it is a time-series, cross-section, or a panel study – it is the cost shares which tie together the results of the literature. BERNDT and WOOD (1979:352) are right that "the short-run, long-run $E - K$ substitutability-complementarity issue does not seem to be simply one of pooled time-series data". Yet, this issue is simply one of the cost shares of capital and energy. Other reasons for differences in elasticity estimates between empirical studies, for example, differences in estimation techniques or in translog model specifications are of minor importance, as our empirical KLEM-example for German manufacturing reveals.

In sum, this paper offers a straightforward explanation for the capital-energy debate: All studies reviewed can be reconciled with each other on the basis of our cost-share argument alone. Under this perspective, there is in fact hardly any controversy between time-series studies on the one hand and cross-section/panel studies on the other hand. However, a somewhat pessimistic message accompanies this explanation: Static translog approaches are limited in their ability to detect a wide range of phenomena. The data simply have no chance of displaying complementarity for two factors if the cost shares of these factors are sufficiently high. In consequence, pursuing a translog approach will not be as flexible as one might hope: In a translog-world – the maintained hypothesis for extracting the structural parameters from the data –, the answer to the question of whether two factors are complements or substitutes would be dominated by the cost shares. The most credible way out of this dilemma might be to use micro data at the firm level, enabling the analyst to model the relation between factor use and price variation without resorting to a parametric functional form.

Appendix Estimation Results

Table A1: ML-Parameter Estimates for Various Models of our Translog Cost Function Approach – German Manufacturing 1978 - 1990.

	homothetic KLEM-models				nonhomothetic		homothetic	
	Model III		Model II		KLEM-Model I		KLE-model (3.10)	
	common intercept		different intercepts		different intercepts		different intercepts	
β_K	0.0961**	(0.0118)	29DV	—	29DV	—	29DV	—
β_L	0.3004**	(0.0101)	29DV	—	29DV	—	29DV	—
β_E	0.1125**	(0.0074)	29DV	—	29DV	—	29DV	—
β_M	0.4910**	(0.0113)	29DV	—	29DV	—	29DV	—
β_{KK}	0.0395**	(0.0149)	0.0386**	(0.0023)	0.0396**	(0.0023)	0.0757**	(0.0041)
β_{KL}	-0.0161	(0.0123)	-0.0152**	(0.0017)	-0.0145**	(0.0017)	-0.0587**	(0.0036)
β_{KE}	-0.0030	(0.0087)	-0.0101**	(0.0022)	-0.0109**	(0.0023)	-0.0170**	(0.0024)
β_{KM}	-0.0204	(0.0138)	-0.0133**	(0.0032)	-0.0141**	(0.0033)	—	—
β_{LL}	0.0945**	(0.0336)	0.0445**	(0.0061)	0.0488**	(0.0060)	0.1382**	(0.0067)
β_{LE}	-0.1107**	(0.0218)	-0.0293**	(0.0039)	-0.0282**	(0.0039)	-0.0800	(0.0052)
β_{LM}	0.03240	(0.0250)	9.6e^{-6}	(0.0067)	-0.0060	(0.0069)	—	—
β_{EE}	0.0410*	(0.0225)	0.0595**	(0.0061)	0.0569**	(0.0062)	0.09647**	(0.0046)
β_{EM}	0.0638**	(0.0199)	-0.0201**	(0.0075)	-0.0177**	(0.0077)	—	—
β_{MM}	-0.0758**	(0.0306)	0.0334**	(0.0120)	0.0379**	(0.0126)	—	—
β_{KT}	0.0003**	(5.4e^{-5})	0.0005**	(0.0002)	0.0006**	(0.0002)	0.0036**	(0.0004)
β_{LT}	-0.0002**	(4.6e^{-5})	-0.0022**	(0.0002)	-0.0022**	(0.0002)	-0.0027**	(0.0003)
β_{ET}	-0.0003**	(3.2e^{-5})	-0.0012**	(0.0002)	-0.0013**	(0.0002)	-0.0009**	(0.0002)
β_{MT}	0.0002**	(5.2e^{-5})	0.0029**	(0.0003)	0.0030**	(0.0003)	—	—
β_{KY}	—	—	—	—	-0.0211**	(0.0070)	—	—
β_{LY}	—	—	—	—	-0.0120**	(0.0049)	—	—
β_{EY}	—	—	—	—	0.0189**	(0.0072)	—	—
β_{MY}	—	—	—	—	0.0142**	(0.0053)	—	—
Lagrange-Multiplier Tests								
Test-Values	1081		6.53		—	—	713	
Reference	KLEM-Model II		KLEM-Model I		—	—	KLEM-Model II	
Critical χ^2	$\chi^2_{0.01}(84) \approx 113$		$\chi^2_{0.10}(4) = 7.78$		—	—	$\chi^2_{0.01}(33) = 53.96$	

Note: * Significant at the 5 %-level. ** Significant at the 1 %-level. Standard errors are given in parentheses.

29DV indicates the use of separate dummy intercept variables for each industry.

Table A2: Estimates of Cross-Price Elasticities for Selected Industries of German Manufacturing (1978 - 1990) on the basis of translog model (3.8).

	Year	Chemical Products	Stone & Earth	Fine Ceramics	Glass	Non-Ferrous Metals	Foundry	Pulp & Paper
	1978	0.00 (0.02)	0.03 (0.01)	0.01 (0.02)	0.02 (0.02)	-0.03 (0.03)	-0.05 (0.03)	0.04 (0.02)
η_{Kp_E}	1984	0.06 (0.01)	0.03 (0.02)	0.01 (0.02)	0.07 (0.02)	-0.01 (0.02)	-0.08 (0.04)	0.04 (0.02)
	1990	0.01 (0.01)	0.01 (0.01)	-0.03 (0.02)	0.01 (0.01)	-0.06 (0.03)	-0.06 (0.03)	0.04 (0.02)
	1978	-0.05 (0.02)	-0.02 (0.02)	0.02 (0.01)	0.00 (0.01)	-0.10 (0.02)	0.00 (0.01)	0.00 (0.01)
η_{Lp_E}	1984	-0.05 (0.02)	-0.02 (0.02)	0.04 (0.01)	0.05 (0.01)	-0.13 (0.03)	0.00 (0.01)	-0.01 (0.02)
	1990	-0.08 (0.02)	-0.05 (0.02)	0.00 (0.01)	-0.04 (0.01)	-0.12 (0.03)	-0.01 (0.01)	-0.03 (0.02)
	1978	0.05 (0.01)	0.05 (0.01)	0.02 (0.03)	0.05 (0.02)	0.05 (0.01)	0.02 (0.02)	0.08 (0.01)
η_{Mp_E}	1984	0.09 (0.01)	0.07 (0.02)	0.04 (0.02)	0.10 (0.02)	0.05 (0.01)	0.05 (0.02)	0.11 (0.01)
	1990	0.04 (0.01)	0.05 (0.01)	0.01 (0.02)	0.04 (0.02)	0.04 (0.01)	0.02 (0.02)	0.09 (0.01)
	1978	0.01 (0.03)	0.05 (0.03)	0.02 (0.03)	0.03 (0.02)	-0.04 (0.03)	-0.06 (0.03)	0.04 (0.02)
η_{Ep_K}	1984	0.07 (0.02)	0.05 (0.02)	0.02 (0.02)	0.06 (0.02)	-0.06 (0.03)	-0.05 (0.02)	0.03 (0.02)
	1990	0.02 (0.03)	0.03 (0.03)	-0.05 (0.04)	0.02 (0.03)	-0.07 (0.03)	-0.07 (0.04)	0.05 (0.02)
	1978	0.06 (0.01)	0.10 (0.01)	0.11 (0.01)	0.09 (0.01)	-0.01 (0.01)	0.05 (0.01)	0.06 (0.01)
η_{Lp_K}	1984	0.06 (0.01)	0.08 (0.01)	0.08 (0.01)	0.07 (0.01)	-0.04 (0.01)	0.02 (0.01)	0.02 (0.01)
	1990	0.08 (0.01)	0.11 (0.01)	0.07 (0.01)	0.10 (0.01)	-0.02 (0.01)	0.05 (0.01)	0.05 (0.01)
	1978	0.11 (0.01)	0.13 (0.01)	0.10 (0.01)	0.11 (0.01)	0.07 (0.01)	0.06 (0.01)	0.10 (0.01)
η_{Mp_K}	1984	0.13 (0.01)	0.12 (0.01)	0.08 (0.01)	0.10 (0.01)	0.05 (0.01)	0.03 (0.01)	0.07 (0.01)
	1990	0.14 (0.01)	0.15 (0.01)	0.08 (0.01)	0.13 (0.01)	0.06 (0.01)	0.06 (0.01)	0.10 (0.01)
	1978	-0.13 (0.05)	-0.06 (0.04)	0.11 (0.05)	0.01 (0.04)	-0.20 (0.05)	-0.02 (0.06)	0.00 (0.03)
η_{Ep_L}	1984	-0.07 (0.03)	-0.03 (0.04)	0.16 (0.04)	0.09 (0.03)	-0.22 (0.05)	0.08 (0.04)	-0.01 (0.03)
	1990	-0.21 (0.05)	-0.17 (0.05)	-0.03 (0.06)	-0.11 (0.05)	-0.28 (0.06)	-0.08 (0.06)	-0.05 (0.03)
	1978	0.11 (0.01)	0.16 (0.01)	0.36 (0.01)	0.21 (0.01)	-0.01 (0.02)	0.24 (0.02)	0.12 (0.01)
η_{Kp_L}	1984	0.07 (0.01)	0.14 (0.01)	0.32 (0.01)	0.16 (0.01)	-0.09 (0.03)	0.13 (0.03)	0.03 (0.02)
	1990	0.10 (0.01)	0.15 (0.01)	0.30 (0.01)	0.18 (0.01)	-0.03 (0.02)	0.21 (0.02)	0.07 (0.01)
	1978	0.23 (0.01)	0.26 (0.01)	0.46 (0.02)	0.31 (0.02)	0.17 (0.01)	0.41 (0.02)	0.23 (0.01)
η_{Mp_L}	1984	0.16 (0.01)	0.24 (0.01)	0.45 (0.02)	0.28 (0.02)	0.14 (0.01)	0.39 (0.01)	0.19 (0.01)
	1990	0.20 (0.01)	0.24 (0.01)	0.44 (0.02)	0.28 (0.01)	0.16 (0.01)	0.38 (0.01)	0.19 (0.01)
	1978	0.32 (0.09)	0.27 (0.08)	0.07 (0.09)	0.23 (0.08)	0.41 (0.09)	0.13 (0.11)	0.35 (0.06)
η_{Ep_M}	1984	0.39 (0.06)	0.32 (0.07)	0.13 (0.07)	0.31 (0.05)	0.47 (0.09)	0.25 (0.08)	0.43 (0.05)
	1990	0.29 (0.10)	0.64 (0.10)	0.07 (0.12)	0.23 (0.10)	0.40 (0.11)	0.14 (0.12)	0.39 (0.06)
	1978	0.46 (0.02)	0.40 (0.02)	0.22 (0.02)	0.35 (0.02)	0.51 (0.04)	0.28 (0.04)	0.41 (0.03)
η_{Kp_M}	1984	0.47 (0.02)	0.42 (0.02)	0.22 (0.03)	0.33 (0.02)	0.51 (0.05)	0.23 (0.05)	0.43 (0.03)
	1990	0.49 (0.02)	0.45 (0.02)	0.27 (0.03)	0.41 (0.02)	0.52 (0.04)	0.31 (0.04)	0.46 (0.02)
	1978	0.56 (0.03)	0.49 (0.03)	0.31 (0.02)	0.45 (0.02)	0.66 (0.04)	0.43 (0.02)	0.52 (0.03)
η_{Lp_M}	1984	0.56 (0.04)	0.51 (0.03)	0.33 (0.02)	0.44 (0.02)	0.71 (0.05)	0.46 (0.02)	0.57 (0.04)
	1990	0.57 (0.03)	0.53 (0.03)	0.39 (0.02)	0.49 (0.02)	0.70 (0.04)	0.47 (0.02)	0.56 (0.04)

Note: Standard errors are given in parentheses. Under the assumption that cost shares are constant and equal to the means of their estimated values, approximate estimates of the standard errors of $\eta_{x_i p_j}$ are, asymptotically, $var(\hat{\eta}_{x_i p_j}) = \hat{\beta}_{ij}/\hat{s}_i^2$ and $var(\hat{\eta}_{x_i p_i}) = \hat{\beta}_{ii}/\hat{s}_i^2$ (PINDYCK 1979:171).

Table A3: Cost Shares for Selected Industries of German Manufacturing (1978 - 1990).

Cost Shares	Year	Chemical Products	Stone & Earth	Fine Ceramics	Glass	Non-Ferrous Metals	Foundry	Pulp & Paper
	1978	0.13	0.16	0.15	0.14	0.09	0.09	0.13
s_K	1984	0.15	0.14	0.12	0.13	0.07	0.06	0.10
	1990	0.16	0.17	0.11	0.15	0.08	0.09	0.13
	1978	0.23	0.26	0.46	0.32	0.17	0.42	0.24
s_L	1984	0.17	0.24	0.45	0.29	0.14	0.39	0.19
	1990	0.20	0.24	0.44	0.28	0.16	0.38	0.19
	1978	0.08	0.09	0.08	0.10	0.08	0.07	0.12
s_E	1984	0.12	0.11	0.10	0.15	0.08	0.10	0.15
	1990	0.07	0.07	0.06	0.08	0.07	0.06	0.12
	1978	0.56	0.49	0.31	0.45	0.66	0.43	0.52
s_M	1984	0.56	0.51	0.33	0.44	0.71	0.46	0.57
	1990	0.57	0.52	0.39	0.49	0.70	0.47	0.56

Table A4: Estimates of Cross-Price Elasticities for the dual KLE-translog Model (3.10).

Cost Shares	Year	Chemical Products	Stone & Earth	Fine Ceramics	Glass	Non-Ferrous Metals	Foundry	Pulp & Paper
	1978	0.13 (0.01)	0.12 (0.01)	0.04 (0.01)	0.10 (0.01)	0.17 (0.01)	0.01 (0.01)	0.18 (0.01)
η_{Kp_E}	1984	0.23 (0.01)	0.15 (0.01)	0.04 (0.01)	0.19 (0.01)	0.22 (0.01)	0.02 (0.02)	0.27 (0.01)
	1990	0.12 (0.01)	0.10 (0.01)	0.01 (0.01)	0.09 (0.01)	0.16 (0.01)	0.01 (0.02)	0.21 (0.01)
	1978	0.03 (0.01)	0.02 (0.01)	0.03 (0.01)	0.03 (0.01)	0.08 (0.01)	0.01 (0.01)	0.09 (0.01)
η_{Lp_E}	1984	0.07 (0.01)	0.05 (0.01)	0.04 (0.01)	0.11 (0.01)	0.12 (0.01)	0.07 (0.01)	0.16 (0.01)
	1990	-0.01 (0.01)	-0.01 (0.01)	0.01 (0.01)	0.05 (0.01)	0.07 (0.01)	0.02 (0.01)	0.09 (0.01)
	1978	0.20 (0.01)	0.22 (0.01)	0.07 (0.02)	0.15 (0.01)	0.19 (0.01)	0.01 (0.02)	0.19 (0.01)
η_{Ep_K}	1984	0.28 (0.01)	0.21 (0.01)	0.06 (0.02)	0.16 (0.01)	0.17 (0.01)	0.01 (0.01)	0.18 (0.01)
	1990	0.27 (0.01)	0.24 (0.01)	0.02 (0.02)	0.19 (0.01)	0.18 (0.01)	0.02 (0.02)	0.23 (0.01)
	1978	0.18 (0.01)	0.20 (0.01)	0.12 (0.01)	0.15 (0.01)	0.15 (0.01)	0.07 (0.01)	0.14 (0.01)
η_{Lp_K}	1984	0.19 (0.01)	0.17 (0.01)	0.09 (0.01)	0.11 (0.01)	0.11 (0.01)	0.03 (0.01)	0.09 (0.01)
	1990	0.24 (0.01)	0.23 (0.01)	0.10 (0.01)	0.20 (0.01)	0.15 (0.01)	0.08 (0.01)	0.16 (0.01)
	1978	0.09 (0.03)	0.07 (0.03)	0.11 (0.04)	0.11 (0.03)	0.16 (0.02)	0.06 (0.04)	0.17 (0.02)
η_{Ep_L}	1984	0.09 (0.02)	0.13 (0.02)	0.16 (0.04)	0.21 (0.02)	0.21 (0.02)	0.27 (0.03)	0.20 (0.02)
	1990	-0.01 (0.03)	-0.03 (0.03)	-0.03 (0.05)	0.02 (0.03)	0.16 (0.02)	0.08 (0.04)	0.14 (0.02)
	1978	0.32 (0.01)	0.32 (0.01)	0.39 (0.02)	0.34 (0.01)	0.28 (0.01)	0.35 (0.02)	0.26 (0.01)
η_{Kp_L}	1984	0.21 (0.01)	0.29 (0.01)	0.34 (0.02)	0.25 (0.02)	0.23 (0.01)	0.17 (0.02)	0.17 (0.01)
	1990	0.30 (0.01)	0.33 (0.01)	0.39 (0.02)	0.35 (0.01)	0.29 (0.01)	0.35 (0.02)	0.23 (0.01)

Note: Standard errors are given in parentheses.

Chapter 4

Facing the Truth about Separability: Nothing Works Without Energy.

Together with CHRISTOPH M. SCHMIDT

Abstract. Separability is a pivotal theoretical *and* empirical concept in production theory. BERNDT and CHRISTENSEN's (1973) classical definition of separability is primarily motivated by the desire to conceptualize production decisions as a sequential process. By contrast, the principal purpose of an appropriate concept of separability in empirical work is to justify the omission of variables for which data are of poor quality or even unavailable. This paper demonstrates that this empirical concept is more restrictive than the classical separability definition. Therefore, we suggest a novel definition of separability based on cross-price elasticities which has clear empirical content. As an application, we focus on the empirical question of whether the omission of energy affects the conclusions about the ease of substitution among non-energy factors. The classical and our separability concept are contrasted in a translog approach to German manufacturing data (1978-1990).

4.1 Introduction

When modeling factor substitution, e. g. the substitutability of capital and labor, it is generally impossible to focus on the bi- or multivariate relationship between the variables of interest. Two situations may arise, however, which could justify the isolated analysis of these factors. First, the omitted variable might be of limited quantitative relevance to the production process. Energy, for instance, accounted for a negligible share of production cost during the "the golden years" of economic growth in the early post-WW II era (CRAFTS and TONIOLO 1996). The analysis of the possibilities to substitute *capital* K, *labor* L, and *material inputs* M would not have been altered by the inclusion or exclusion of energy and its cost. Yet, in the aftermath of the energy crises of the seventies the production factor *energy* E became non-negligible and, consequently, gained prominence in empirical studies. Since then a large number of studies have appeared analyzing the substitution of energy and non-energy inputs. BERNDT and WOOD (1975, 1979), GRIFFIN and GREGORY (1976), PINDYCK (1979), and MAGNUS (1979) are seminal studies, more recent examples are YUHN (1991), THOMPSON and TAYLOR (1995), and RAMAIAH and DALAL (1996), with the cost shares of energy varying between 1 and 10 %.

Second, whether or not a non-negligible variable, such as energy after the oil crises, is included, might be irrelevant for inferences about the ease of substitution between non-energy inputs. For example, in spite of omitting the factor energy, estimates of cross-price elasticities, measuring the ease of substitution between non-energy inputs, may still remain correct. Such a notion of *separability* is particularly important when the data do not provide information on a non-negligible input factor, but interest is on the substitutability relations of observable factors. For German manufacturing, for instance, empirical studies investigating the issue of factor substitution between K, L and M do typically not incorporate the factor energy, RUTNER (1984), STARK (1988), KUGLER *et al.* (1989), and FLAIG and ROTTMANN (1998), for example.

As their justification for this omission, these authors typically invoke a standard notion of separability that has been researched thoroughly in economic production *theory*, there serving the principal purpose to form a conceptual basis for the idea of sequential decision making. Inadvertently, though, these studies implicitly build on an assumption of $[(K, L, M), E]$-separability, a property focusing on the ease of substitution among non-energy inputs, rather than on sequential decision processes. Such a separability assumption incorporates stronger requirements than those implied by the standard notion of separability, making it questionable whether this severe restriction on the process is palatable.

Moreover, there is ample reason to doubt the applicability of even the relatively mild standard form of separability. In their classic study, BERNDT and WOOD (1975), for example, provide empirical evidence that the similar assumption of $[(K, L), (M, E)]$-separability, that is, the assumption of separability of K and L on the one hand, and M and E on the other, is violated for U. S. manufacturing (1947-1971). Because $[(K, L), (M, E)]$-separability is an assumption necessarily required for value-added studies, which exclusively employ the inputs K and L, BERNDT and WOOD (1975:266) "call into question the reliability of [...] factor demand studies for U. S. manufacturing based on [...] value-added specification[s]". It clearly transpires from the strata of the discussion that if we want to understand the role of energy in production, specifically, under what conditions it can be omitted safely from the empirical analysis, we need a clear notion of the precise restrictions involved in assuming separability, and empirical tests of their validity.

This paper investigates both theoretical and empirical aspects of the concept of separability. Throughout, the intuition about separability pursued is that the ease of substitution between two factors should be *unaffected* by a third factor, from which those factors are assumed to be separable (see e. g. HAMERMESH 1993:34). In our theoretical analysis, we provide clarification of the rigid nature of the assumption of separability in empirical applications: First, we discuss theoretically in which sense the ease of substitution between two factors i and

j is unaffected by the factor k if both are separable from k according to the classical definition of separability formulated by BERNDT and CHRISTENSEN 1973, henceforth BC73). The structure of BC73's conditions is identical in both primal and dual contexts. However, it is demonstrated here that they lead to quite different implications regarding substitution issues. In contrast to the previous literature, we thus distinguish *primal* from *dual* separability: Two factors i and j are *primally* (*dually*) BC73-separable from factor k if and only if their marginal rate of substitution (their input proportion x_i/x_j) is unaffected by the input level of k (the price of factor k). Yet, characterizing substitution relationships between two factors in such ways is rather unusual in empirical studies. Therefore, we suggest a novel and more restrictive definition of (dual) separability which has clear empirical content.

Using KLEM-panel data for German manufacturing (1978-1990) and the prominent translog approach, it is then examined empirically whether $[(K, L, M), E]$- and $[(K, L), (M, E)]$-separability assumptions hold such that energy or even energy and materials may be omitted from the data base without affecting the estimated substitution relationships among K, L and M or K and L, respectively. Separability conditions according to both our and the milder BC73's classical definition are tested. The difference between primal and dual separability necessitates that a primal as well as a dual approach is pursued, whereas merely one of either approaches is usually employed in empirical studies. Moreover, by testing these separability assumptions for two different models and two alternative scenarios within each translog approach it is checked whether they are robust.

Section 2 investigates theoretically the notion of separability, in Section 3, both our and BC73's classical separability definition are applied to translog approaches. Separability test results for German manufacturing are presented in Section 4, indicating that energy should not be omitted. Section 5 concludes.

4.2 Separability and Substitution

In the received literature, considerations regarding to separability of production factors have their principal motivation in a theoretical issue, the possibility to conceptualize the optimization of production decisions by stages: If separability holds according to BC73's classical definition of separability (given in detail below), factor intensities can first be optimized within each separable subset. Then, optimal intensities can be attained by holding fixed the within-subset intensities and optimizing the between-subset intensities. For empirical work, though, one would like to determine whether the omission of non-negligible variables for which data are unavailable is justified.

As substitution is the center piece of any empirical study, the natural intuition of separability for empirical work is that the ease of substitution among observable factors should be unaffected by the variable omitted. It is demonstrated in this section that BC73-separability of two factors i and j from a third factor k does not suffice to justify the restrictive specifications pursued in the literature. Due to different implications in primal and dual contexts, in contrast to the previous literature, we distinguish primal from dual BC73-separability[1]. Neither of them lends itself to empirical application, though. Therefore, we proceed to develop a more restrictive concept of separability with clear empirical content.

Primal BC**73-Separability**

Quite naturally, empirical studies investigated substitution issues by estimating production functions. Consequently, the notion of primal separability was defined first. Along the lines of GOLDMAN and UZAWA (1964) and BC(73:404), two factors i and j of a twice differentiable production function $Y = F(x_1, x_2, ..., x_n)$ with nonvanishing first and second partial derivatives are generally defined to be

[1] Contrary to the literature we do not distinguish strong from weak BC73-separability, since the intuition regarding substitution issues is perfectly the same behind both concepts. Moreover, with respect to $[(K, L, M), E]$- and $[(K, L), (M, E)]$-separability for German manufacturing, this distinction is irrelevant: If merely two subsets of input factors are of interest, weak implies strong BC73-separability (BC73:404).

primally BC73-*separable* from factor k if and only if

$$\frac{\partial}{\partial x_k}\left(\frac{\frac{\partial F(x_1,\ldots,x_n)}{\partial x_i}}{\frac{\partial F(x_1,\ldots,x_n)}{\partial x_j}}\right) = 0 \qquad \Longleftrightarrow \qquad \frac{\partial^2 F}{\partial x_i \partial x_k}\frac{\partial F}{\partial x_j} - \frac{\partial^2 F}{\partial x_j \partial x_k}\frac{\partial F}{\partial x_i} = 0. \tag{4.1}$$

This property, which may locally hold at a point or globally, reads alternatively

$$\frac{\partial}{\partial x_k}\left(\frac{\partial x_j}{\partial x_i}\right) = 0, \tag{4.2}$$

with $-\frac{\partial x_j}{\partial x_i}$ denoting the marginal rate of substitution between i and j. Thus, primal BC73-separability of factor k from i and j implies that factor k's intensity does not affect the ease of substitution between i and j. However, this ease of substitution is exclusively measured in terms of the marginal rate of substitution, not by another, for empirical work more relevant, candidate concept such as cross-price elasticities or ALLEN's elasticities of substitution (AES), the most prominent measures of substitution.

Moreover, for a linear-homogeneous production function F definition (4.1) is equivalent to

$$\mathrm{AES}_{ik} := \frac{\frac{\partial F}{\partial x_i}\frac{\partial F}{\partial x_k}}{F\frac{\partial^2 F}{\partial x_i \partial x_k}} = \frac{\frac{\partial F}{\partial x_j}\frac{\partial F}{\partial x_k}}{F\frac{\partial^2 F}{\partial x_j \partial x_k}} =: \mathrm{AES}_{jk}. \tag{4.3}$$

That is, primal BC73-separability of factor k from i and j means that the ease of substitution between k and i – in terms of AES – equals that between k and j. However, it does not imply – in terms of this concept – that k does not affect the substitution of i for j or vice versa.

In practice, due to the fact that inputs of production functions may be endogenous and therefore estimators may be inconsistent, the classical way to overcome these endogeneity problems has been to apply dual cost function approaches (MUNDLAK 1996:431). Hence, besides definition (4.2) of primal BC73-separability a dual definition is indispensable.

Dual BC73-Separability

On the basis of a twice differentiable cost function $C(Y, p_1, p_2, ..., p_n)$ with nonvanishing first and second partial derivatives, two factors i and j are defined to be *dually* BC73-*separable* from factor k along the lines of BC(73:405) if and only if

$$\frac{\partial}{\partial p_k}\left(\frac{\frac{\partial C(Y,p_1,...,p_n)}{\partial p_i}}{\frac{\partial C(Y,p_1,...,p_n)}{\partial p_j}}\right)=0 \qquad \Longleftrightarrow \qquad \frac{\partial^2 C}{\partial p_i \partial p_k}\frac{\partial C}{\partial p_j}-\frac{\partial^2 C}{\partial p_j \partial p_k}\frac{\partial C}{\partial p_i}=0. \tag{4.4}$$

Since the structure of dual condition (4.4) and primal condition (4.1) is identical, at first glance, the notion of separability seems to be equal in primal and dual contexts. Yet, with particular respect to the interpretation of the ease of substitution between i and j both definitions differ: Using SHEPHARD's Lemma, $\frac{\partial C}{\partial p_i}=x_i$, definition (4.4) equals

$$\frac{\partial}{\partial p_k}\left(\frac{x_i(p_1,p_2,...,p_n)}{x_j(p_1,p_2,...,p_n)}\right)=0. \tag{4.5}$$

That is, two inputs i and j are *dually* BC73-*separable* from factor k if and only if the their *input proportion* x_i/x_j is independent of changes of factor k's price. The ease of substitution between i and j, in virtually every empirical study measured in terms of their cross-price elasticities, or by AES or the MORISHIMA elasticities of substitution (MES) is not at issue, though.

Moreover, the implications of dual separability so defined for cross-price elasticities, and for AES and MES are similiar to those of primal separability for AES given by equation (4.3): First, differentiating and multiplying (4.5) by p_k yields

$$\eta_{x_ip_k}:=\frac{p_k}{x_i}\frac{\partial x_i}{\partial p_k}=\frac{p_k}{x_j}\frac{\partial x_j}{\partial p_k}=:\eta_{x_jp_k}. \tag{4.6}$$

That is, under dual BC73-separability assumption (4.5) substitution reactions between i and k on the one hand and j and k on the other hand are restricted to be equal, when captured by cross-price elasticities $\eta_{x_ip_k}$. Second, AES and MES are related to cross-price elasticities by

$$\text{AES}_{x_ip_k}:=\eta_{x_ip_k}/s_k \qquad \text{and} \qquad \text{MES}_{x_ip_k}:=\eta_{x_ip_k}-\eta_{x_kp_k}, \tag{4.7}$$

where s_k denotes the cost share of factor k. By equation (4.6), AES and MES thus obey similar restrictions under dual separability of i and j from k:

$$\text{AES}_{x_ip_k}=\text{AES}_{x_jp_k} \qquad \text{and} \qquad \text{MES}_{x_ip_k}=\text{MES}_{x_jp_k}. \tag{4.8}$$

In sum, dual BC73-separability definition (4.5) only implies that the ease of substitution between factors i and j is unaffected by factor k when this ease is measured on the basis of the input proportion x_i/x_j. However, definiton (4.5) generally does not imply that this ease is unaffected by factor k when the ease is measured by cross-price elasticities, AES or MES, that is, those measures which are employed in empirical substitution studies almost without exception. In consequence, when empirical analysts – as in numerous studies – invoke the assumption of BC73-separability in order to justify the omission of a non-negligible input factor from their empirical analysis, but then proceed to express their results in terms of, say AES, they base their empirical work inadvertently on an insufficient assumption. Therefore, this paper suggests a new definition of dual separability with clear empirical content.

An Empirically Oriented Approach

Specifically, we define two factors i and j to be *empirically dually separable* from factor k if and only if

$$\frac{\partial}{\partial p_k}\eta_{x_i p_j} = 0 \qquad \text{and} \qquad \frac{\partial}{\partial p_k}\eta_{x_j p_i} = 0, \tag{4.9}$$

that is, if and only if the ease of substitution between i and j, measured by the cross-price elasticities involving both factors, is not affected by the price of factor k. While there was some choice of specific approach, we decided to build our separability definition (4.9) on the basis of cross-price elasticities, because alternative definitions based on AES or MES are more restrictive than our definition[2]: When substitution relationships are intended to be measured by AES, the requirements given by (4.9) do not assure that the ease of substitution concerning i and j is independent of the price of factor k, since

$$\frac{\partial}{\partial p_k}\text{AES}_{x_i p_j} = \frac{\partial}{\partial p_k}\left(\eta_{x_i p_j}/s_j\right) = \frac{1}{s_j}\frac{\partial}{\partial p_k}\eta_{x_i p_j} - \eta_{x_i p_j}\frac{1}{s_j^2}\frac{\partial s_j}{\partial p_k} = -\eta_{x_i p_j}\frac{1}{s_j^2}\frac{\partial s_j}{\partial p_k} \neq 0, \tag{4.10}$$

[2]Furthermore, FRONDEL (1999:24) concludes that AES and MES do not provide any economically meaningful information beyond that given by cross-price elasticities, which form the common basis of both AES and MES (see (4.7)).

in general. Rather, beyond both conditions of definiton (4.9) changes in the price of factor k must also not affect both cost shares s_i and s_j in order to guarantee

$$\frac{\partial}{\partial p_k}\text{AES}_{x_i p_j} = 0 \qquad \text{and} \qquad \frac{\partial}{\partial p_k}\text{AES}_{x_j p_i} = 0. \tag{4.11}$$

It is difficult to imagine that this is possible in actual applications. Correspondingly, even if the requirements given by our definition (4.9) do hold, it is

$$\frac{\partial}{\partial p_k}\text{MES}_{x_i p_j} = \frac{\partial}{\partial p_k}\eta_{x_i p_j} - \frac{\partial}{\partial p_k}\eta_{x_j p_j} = -\frac{\partial}{\partial p_k}\eta_{x_j p_j} \neq 0, \tag{4.12}$$

in general. Similar conditions to those of (4.9) and (4.11) will hold for MES only if (4.9) is valid and if, additionally, own-price elasticities of both factors i and j are unaffected by the price of factor k.

Using SHEPHARD's Lemma, $\frac{\partial C}{\partial p_i} = x_i$, and the definitions of $\eta_{x_i p_j}$ and $\eta_{x_j p_i}$ given in (4.6), our definition (4.9) may be written alternatively as

$$\frac{\partial^2 C}{\partial p_i \partial p_k}\frac{\partial^2 C}{\partial p_i \partial p_j}/\frac{\partial C}{\partial p_i} = \frac{\partial^3 C}{\partial p_i \partial p_j \partial p_k} \quad \text{and} \quad \frac{\partial^2 C}{\partial p_j \partial p_k}\frac{\partial^2 C}{\partial p_i \partial p_j}/\frac{\partial C}{\partial p_j} = \frac{\partial^3 C}{\partial p_i \partial p_j \partial p_k}. \tag{4.13}$$

When both these conditions hold, definition (4.4) of dual BC73-separability is fulfilled, but not vice versa. Hence, BC73's dual separability condition (4.4) is necessary, but not sufficient for separability definition (4.9) to hold, and hence represents a weaker requirement.[3]

The relevance of the conceptual arguments raised in this section is exemplified in Sections 3 and 4 in a concrete application to German manufacturing data, where the factors whose substitution relationships are of interest are capital, labor, materials and the nuisance factor energy, which is possibly unavailable in the data. The central question is whether energy can safely be omitted when analyzing substitution relations between the other three production factors. To this end, we employ both our own, more restrictive dual separability definition (4.9) and the classical concepts of primal and dual BC73-separability, whose violation would

[3] BLACKORBY, DAVIDSON, and SCHWORM (1991) coin the notion of implicit separability. Implicit separability contains BC73-separability as a special case and hence is a more general form of separability as well. In contrast to our defintion, however, the focus of implicit separability is on theoretical, rather than on empirical issues (see also BLACKORBY and RUSSELL 1994).

already put the issue at rest. On the other hand, omitting a factor might be unjustified even when BC73-separability is satisfied.

4.3 Separability Conditions for Translog Approaches

Presenting our arguments with a focus on a concrete econometric specification, this section constitutes the formal part of our investigation on the separability of energy from non-energy inputs for German manufacturing. Throughout, we retain the assumption that technology can be modeled sucessfully by a four-input translog production function *and* its dual translog cost function, because it is not simply a matter of indifference whether to choose either of them: Separability test results based on the translog cost function may not be used as conclusive evidence on functional separability of the translog production function, since both functions are – contrary to COBB-DOUGLAS and CES functions - not necessarily self-dual (YUHN 1991:232). Apart from the different implications of primal and dual BC73-separability, this is the reason why one has to pursue both a primal and a dual translog approach if the issue of separability is intended to be investigated seriously.

By testing separability assumptions for two different models and two alternative scenarios within each translog approach, we check the sensitivity of separability test results: Model I assumes a single aggregate production/cost function for all German industries, while Model II relaxes this assumption. Moreover, DENNY and FUSS (1977, henceforth DF77), and BLACKORBY *et al.* (1977) point out that *exact separability tests*, based on the assumption that the translog form is an exact representation of the true underlying production structure, are more restrictive than necessary and place severe restrictions on the functional form of the translog functions. Hence, it cannot be excluded that the rejection of either of these restrictive functional forms may lead to the rejection of the separability hypotheses (YUHN 1991:242).

DF77 and NORSWORTHY and MALMQUIST (1983) overcome these problems by designing *approximate separability tests*, which are based on the assumption that the translog production (cost) function is a second-order approximation to an arbitrary production (cost) function in the neighborhood of any given expansion point. Therefore, we establish $[(K,L,M),E]$- as well as $[(K,L),(M,E)]$- separability conditions in detail for two alternative scenarios, where it is assumed that translog functions are either exact images of the production technology in German manufacturing or merely approximations.

Primal and Dual Translog Models

Following the seminal contribution of CHRISTENSEN *et al.* (1971), we assume that there exists a twice-differentiable *aggregate* translog production function for all industries of German manufacturing, relating gross output (Y) to the services of capital (x_K), labor (x_L), all other intermediate materials (x_M) and energy (x_E) (Model I):

$$\ln Y = \ln \alpha_0 + \sum_{i=K,L}^{M,E} \alpha_i \cdot \ln x_i + \frac{1}{2} \sum_{i,j=K,L}^{M,E} \alpha_{ij} \cdot \ln x_i \cdot \ln x_j + \sum_{i=K,L}^{M,E} \alpha_{iT} \ln x_i \cdot T. \quad (4.14)$$

Technological progress is taken into account by the last part in (4.14), where α_{iT} determines the bias of productivity growth for factor i, defined as the change of this factor's cost share with respect to time (JORGENSON *et al.* 1987).

Unknown parameters might be estimated directly from a stochastic version of (4.14), but resulting estimates are well-known to show large standard errors. Yet, efficiency gains can be realized by estimating a system of cost-share equations (YUHN 1991:238). To this end, though, optimality behavior under perfect competition has to be assumed and constant returns to scale has to be imposed, requiring a set of adding-up conditions:

$$\alpha_K + \alpha_L + \alpha_E + \alpha_M = 1, \quad (4.15)$$

$$\alpha_{Kj} + \alpha_{Lj} + \alpha_{Ej} + \alpha_{Mj} = 0 \quad \text{for } j = K, L, E, M, \quad (4.16)$$

$$\alpha_{KT} + \alpha_{LT} + \alpha_{ET} + \alpha_{MT} = 0. \quad (4.17)$$

Under these assumptions, the following factor-share equation system can be obtained from (4.14),

$$s_i = \frac{\partial \ln Y}{\partial \ln x_i} = \alpha_i + \sum_{j=K,L}^{M,E} \alpha_{ij} \ln x_j + \alpha_{iT} \cdot T \qquad \text{for } i = K, L, E, M, \tag{4.18}$$

and the unknown parameters α_i, α_{ij} may be estimated from a stochastic version of it. However, since cost shares always sum to unity, and because of restrictions (4.15), (4.16) and (4.17), the sum of the disturbances ε_i across the four equations is zero at each observation, implying singularity of the disturbance covariance matrix. This problem is solved by dropping arbitrarily one of the four equations in (4.18), the cost-share equation for materials (M), for example:

$$\begin{aligned}
s_K &= \alpha_K + \alpha_{KK} \ln(\frac{x_K}{x_M}) + \alpha_{KL} \ln(\frac{x_L}{x_M}) + \alpha_{KE} \ln(\frac{x_E}{x_M}) + \alpha_{KT} \cdot T + \varepsilon_K \\
s_L &= \alpha_L + \alpha_{KL} \ln(\frac{x_K}{x_M}) + \alpha_{LL} \ln(\frac{x_L}{x_M}) + \alpha_{LE} \ln(\frac{x_E}{x_M}) + \alpha_{LT} \cdot T + \varepsilon_L \\
s_E &= \alpha_E + \alpha_{KE} \ln(\frac{x_K}{x_M}) + \alpha_{LE} \ln(\frac{x_L}{x_M}) + \alpha_{EE} \ln(\frac{x_E}{x_M}) + \alpha_{ET} \cdot T + \varepsilon_E \ ,
\end{aligned} \tag{4.19}$$

Herein, restrictions (4.16) and (4.17) are already imposed. The seemingly unrelated regressions (SUR) model (4.19), where equations are linked merely by disturbances, is preferably estimated by maximum likelihood (ML) in order to ensure that results do not depend upon the choice of which share equation is dropped (BERNDT 1991:473).

In a second primal specification (Model II), we relax Model I's assumption of a single aggregate production function for all S industries and allow for (first-order) heterogeneity between industries through the use of industry intercept dummy variables D_s in all share equations:

$$s_i = \frac{\partial \ln Y}{\partial \ln x_i} = \alpha_i + \sum_{s=2}^{S} \alpha_{is} D_s + \sum_{j=K,L}^{M,E} \alpha_{ij} \ln x_j + \alpha_{iT} \cdot T \qquad \text{for } i = K, L, E, M. \tag{4.20}$$

Allowing both the set of first- and second-order coefficients to vary across industries is equivalent to estimating a separate model for each industry, but this would require more data than are available.

The dual counterpart to translog production function (4.14) is (Model I)

$$\ln C = \ln \beta_0 + \ln Y + \sum_{i=K,L}^{M,E} \beta_i \cdot \ln p_i + \frac{1}{2} \sum_{i,j=K,L}^{M,E} \beta_{ij} \ln p_i \ln p_j + \sum_{i=K,L}^{M,E} \beta_{iT} \ln p_i \cdot T, \quad (4.21)$$

where Y is a given level of output, and symmetry of β_{ij} and constant returns to scale are imposed *a priori*. Linear homogeneity in prices, an inherent feature of any cost function, requires the same adding-up conditions for the β_i, β_{ij} and β_{iT} as for the coefficients of translog production function (4.14), when additionally constant returns to scale are assumed by (4.15), (4.16) and (4.17). As well as in the primal approach, a second specification relaxes the assumption of a single aggregate cost function for all industries and allows for first-order heterogeneity among industries (dual Model II).

BC73-**Separability and Translog Approaches**

Here, we present separability conditions merely for translog production function (4.14), because dual separability conditions read identically due to the same structure of primal and dual BC73-separability. Because of $\frac{\partial \ln F}{\partial x_i} = \frac{1}{x_i} \frac{\partial \ln F}{\partial \ln x_i}$ and $\frac{\partial^2 \ln F}{\partial x_i \partial x_k} = \frac{1}{x_i x_k} \frac{\partial^2 \ln F}{\partial \ln x_i \partial \ln x_k}$, primal BC73-separability condition (4.1) can alternatively be written as

$$\frac{\partial}{\partial x_k} \left(\frac{\frac{\partial \ln F}{\partial x_i}}{\frac{\partial \ln F}{\partial x_j}} \right) = 0 \iff \frac{\partial^2 \ln F}{\partial \ln x_i \partial \ln x_k} \frac{\partial \ln F}{\partial \ln x_j} - \frac{\partial^2 \ln F}{\partial \ln x_j \partial \ln x_k} \frac{\partial \ln F}{\partial \ln x_i} = 0. \quad (4.22)$$

Applied on translog production function (4.14), condition (4.22) implies that two factors i and j are BC73-separable from factor k if and only if

$$\alpha_j \alpha_{ik} - \alpha_i \alpha_{jk} + \sum_{l=K,L}^{M,E} (\alpha_{jl} \alpha_{ik} - \alpha_{il} \alpha_{jk}) \cdot \ln x_k + (\alpha_{jT} \alpha_{ik} - \alpha_{iT} \alpha_{jk}) \cdot T = 0, \quad (4.23)$$

that is, if and only if

$$s_j \alpha_{ik} - s_i \alpha_{jk} = 0, \quad (4.24)$$

where s_i and s_j denote the shares of factor i and j, respectively (see (4.18)).

Equation (4.23) holds for all values x_k and for any point T of time if and only if the following set of *exact nonlinear separability conditions* is satisfied:

$$\alpha_j \alpha_{ik} - \alpha_i \alpha_{jk} = 0,$$

$$\begin{aligned} \alpha_{jl}\alpha_{ik} - \alpha_{il}\alpha_{jk} &= 0 \qquad \text{for } l = K, L, M, E, \text{ and} \\ \alpha_{jT}\alpha_{ik} - \alpha_{iT}\alpha_{jk} &= 0. \end{aligned} \tag{4.25}$$

For $l = k$, of course, $\alpha_{jl}\alpha_{ik} - \alpha_{il}\alpha_{jk} = 0$ is redundant. Obviously, system (4.25) is always satisfied if the following *exact linear separability conditions* (see BERNDT and WOOD 1975:266) do hold:

$$\alpha_{ik} = \alpha_{jk} = 0. \tag{4.26}$$

DF77 demonstrate that tests based on either the exact nonlinear conditions (4.25) or the exact linear separability conditions (4.26) are not just tests of the hypothesis of separability. Rather, these tests examine the *joint* hypothesis of separability *and* a particular functional form: Imposing nonlinear separability conditions on (4.14) yields a translog function of COBB-DOUGLAS subaggregates (DF77:406). When imposing e. g. linear $[(K, L, M), E]$-constraints on translog production function (4.14), that is $\alpha_{KE} = \alpha_{LE} = \alpha_{ME} = 0$, it degenerates to a COBB-DOUGLAS function of translog subaggregates, where $\ln G$ and $\ln H$ are translog subaggregates:

$$\begin{aligned} \ln Y = & \underbrace{\ln \alpha_0 + \sum_{i \neq E} \alpha_i \cdot \ln x_i + \frac{1}{2} \cdot \sum_{i \neq E} \alpha_{ij} \ln x_i \ln x_j + \sum_{i \neq E} \alpha_{iT} \ln x_i \cdot T +}_{\ln G} \\ & \underbrace{\alpha_E \cdot \ln x_E + \frac{1}{2}\alpha_{EE}(\ln x_E)^2 + \alpha_{ET} \ln x_E \cdot T}_{\ln H}. \end{aligned} \tag{4.27}$$

Thus, it cannot be excluded that the rejection of either of these restrictive functional forms, the COBB-DOUGLAS function of translog aggregates or the translog function of COBB-DOUGLAS aggregates, may lead to the rejection of the corresponding separability hypotheses (YUHN 1991:242). In fact, the flexible translog form as an exact representation is "separability-inflexible" (BLACKORBY *et al.* 1977:195). DIEWERT and WALES (1995) therefore propose functional forms based on the normalized quadratic functional form, which are flexible indeed.

Alternatively, along the lines of DF77, we overcome this problem by interpreting translog production function (4.14) now as an approximation of any production function around $(x_K^*, x_L^*, x_M^*, x_E^*, T^*) = (1,1,1,1,0)$. Under this assumption, rather than the whole set of nonlinear BC73-separability conditions (4.25), the first line of (4.25),

$$\alpha_j \alpha_{ik} - \alpha_i \alpha_{jk} = 0, \tag{4.28}$$

suffices as an *approximate separability condition* for factors i and j to be BC73-separable from factor k: By setting $x_k^* = 1$ in (4.23), condition (4.28) assures local separability of i and j from k at the single point $(1,1,1,1,0)$. In addition, BC73-separability of i and j from k is even given globally according to DF77 (1977:408) if condition (4.28) holds and translog function (4.14) is considered as approximation.

Empirical Dual Separability and Translog Cost Functions

Our definition (4.9) of empirical dual separability of i and j from k necessitates that the cross-price elasticities $\eta_{x_i p_j}$ and $\eta_{x_j p_i}$ do not depend upon changes of the price of k. For translog cost functions such as (4.21) the cross-price elasticity $\eta_{x_i p_j}$ is (see e. g. PINDYCK 1979:171)

$$\eta_{x_i p_j} = \frac{\beta_{ij}}{s_i} + s_j, \tag{4.29}$$

where e. g. the cost share s_i is given by

$$s_i = \beta_i + \sum_l \beta_{il} \ln p_l + \beta_{iT} T. \tag{4.30}$$

The first condition of definition (4.9) implies

$$0 = \frac{\partial}{\partial \ln p_k}\left(\eta_{x_i p_j}\right) = -\frac{\beta_{ij}}{s_i^2}\frac{\partial s_i}{\partial \ln p_k} + \frac{\partial s_j}{\partial \ln p_k} = -\frac{\beta_{ij}}{s_i^2}\beta_{ik} + \beta_{jk}, \tag{4.31}$$

which is equivalent for $\beta_{jk} \neq 0$ to

$$s_i = \sqrt{\frac{\beta_{ij}\beta_{ik}}{\beta_{jk}}}. \tag{4.32}$$

For symmetry reasons, $0 = \frac{\partial}{\partial \ln p_k}\eta_{x_j p_i}$, our definition's second condition requires

$$0 = -\frac{\beta_{ij}}{s_j^2}\beta_{jk} + \beta_{ik} \tag{4.33}$$

and

$$s_j = \sqrt{\frac{\beta_{ij}\beta_{jk}}{\beta_{ik}}} \qquad \text{for } \beta_{jk} \neq 0. \tag{4.34}$$

The factors i and j are *locally* separable from factor k merely at those points where both conditions (4.32) and (4.34) are fulfilled[4]. Yet, this is neither a rather likely situation to arise in practice nor relevant for the question of whether a non-negligible factor k can safely be omitted when substitution relations in terms of cross-price elasticities between i and j are intended to be estimated reasonably.

For all practical purposes, the property of interest is *global* separability of i and j from k in the sense of our definition (4.9), holding for any price vector and for any point of time. Only global separability ascertains that factor k can safely be omitted. Because s_i and s_j vary across observations, on the basis of conditions (4.31) and (4.33), global separability holds if only if

$$\beta_{ik} = \beta_{jk} = 0. \tag{4.35}$$

These conditions are perfectly equal to the exact linear separability conditions which are sufficient, but not necessary for BC73-separability (see (4.26)). In consequence, in the special case of translog approaches it is immediatedly obvious that empirical dual separability of two factors i and j implies their dual BC73-separability from factor k, a fact already generally noted for arbitrary cost functions. That is, if two factors i and j are globally empirically dual separable from factor k, which means that the ease of substitution between i and j, in terms of cross-price elasticities, is unaffected by the price of k, their input proportion x_i/x_j is independent of the price of p_k as well.

By contrast, though, dual BC73-separability of two factors i and j from factor k does not imply their empirical dual separability: BC73-separability is a property only necessitating the validity of the exact nonlinear separability conditions (4.25), rather than the exact linear separability conditions. Thus, DF77 are perfectly

[4]As we have already noted for arbitrary cost functions, combining condition (4.32) and (4.34) of our defintion yields $s_i\beta_{jk} = s_j\beta_{ik}$, i. e. , BC73's dual condition (4.24) of (local) separability for translog cost functions.

right in claiming that the linear separability conditions (4.35) are more restrictive than necessary for dual BC73-separability. However, we argue that only these restrictive conditions capture a form of separability of factors i and j from factor k which has clear empirical content. Hence, by coining the notion of empirical dual separability, the linear separability conditions are rehabilitated. Finally, the necessary and sufficient conditions (4.35) for empirical dual separability are sufficient for *approximate* dual BC73-separability. Now, in Section 4, it is examined whether energy is empirically or at least BC73-separable from non-energy inputs in German manufacturing.

4.4 Empirical Test Results

Because of data limitations for energy data, our data base relates to the short range of 1978-1990. Overall, we have $377 = 29 \times 13$ observations from $S = 29$ sectors of German manufacturing. Data necessary for estimation include cost shares, price and quantity indices for K, L, E and M. The sources of data and methods for constructing series for prices and quantities are described in Appendix A. Cost shares for selected industries are reported in Table A1 of Appendix A.

Empirical Comparison of Approaches and Models

Parameter estimates for both the primal and the dual approach are reported in Table B1 and B2 of Appendix B, respectively. Among all models of both approaches dual Model II generally displays the lowest standard errors for second-order coefficients, with only one parameter estimate being insignificant. Moreover, by performing LAGRANGE-multiplier tests, it is examined whether dual Model I and II are statistically different, that is, whether or not there exists an aggregate cross-industry cost function. On the basis of the LM-test values for the primal and the dual approach reported in Table B1 and B2, respectively, the hypothesis of common intercepts across all industries is rejected for both approaches at all significance levels. Consequently, estimates of cross-price elasticities are computed on the basis of parameter estimates of dual KLEM-Modell II.

Before performing separability tests, it is checked whether the translog cost function of KLEM-Modell II is well-behaved, that is, if

$$\text{a)} \quad (\text{positivity}) \quad \frac{\partial C}{\partial p_i} = \frac{C}{p_i} s_i > 0, \tag{4.36}$$

$$\text{b)} \quad (\text{concavity}) \quad (\frac{\partial^2 C}{\partial p_i \partial p_j}) \text{ is negative semidefinite}, \tag{4.37}$$

where

$$\frac{\partial^2 C}{\partial p_i \partial p_j} = \frac{C}{p_i p_j}\{s_i s_j + \beta_{ij}\}, \frac{\partial^2 C}{\partial p_i^2} = \frac{C}{p_i^2}\{s_i^2 - s_i + \beta_{ii}\}.$$

Specifically, $\frac{\partial^2 C}{\partial p_i^2} \leq 0$ is a necessary condition for concavity. Cost share estimates $\hat{s}_i$ are significantly positive without exception. Hence, positivity is globally given for all industries, whereas concavity can not be rejected for merely 7 out of the 29 industries: All other industries display positive estimates for both $\frac{\partial^2 C}{\partial p_E^2}$ and own-price elasticities for energy due to small energy cost shares.[5]

$[(K, L, M), E]$)- and $[(K, L), (M, E)]$-**Separability Conditions**

Exact tests of BC73-separability consist of two parts, linear and nonlinear exact separability tests. Upon rejection of the sufficient linear $[(K, L, M), E]$)- and $[(K, L), (M, E)]$-separability hypotheses,

$$\alpha_{KE} = \alpha_{LE} = \alpha_{ME} = 0 \tag{4.38}$$

and

$$\alpha_{KE} = \alpha_{LE} = \alpha_{KM} = \alpha_{LM} = 0, \tag{4.39}$$

respectively, nonlinear $[(K, L, M), E]$- and $[(K, L), (M, E)]$-separability hypotheses may still hold and have to be examined. For $[(K, L, M), E]$-separability, 7

[5] Since energy cost shares s_E for these 22 industries are smaller than 6 % and the estimate of parameter β_{EE} common to all industries is approximately 0.06 (see Table B2 of Appendix B), the fraction $\frac{\beta_{EE}}{s_E}$ is necessarily higher than one. This implies positive estimates for both $\eta_{Ep_E} = \frac{\beta_{EE}}{s_E} - 1 + s_E$ as well as $\frac{\partial^2 C}{\partial p_E^2} = \frac{C}{p_E^2} \cdot s_E \cdot \eta_{Ep_E}$. That is, the empirical fact that energy plays a minor role with respect to overall cost in most industries of German manufacturing, together with the cross-industry restrictions on estimated parameters endangers the theoretical requirements of negative own-price elasticities and concavity.

nonlinear restrictions, derived in Appendix C, are necessary and sufficient:

$$\alpha_L = (\alpha_E - 1)\frac{\alpha_{KL}}{\alpha_{KE}},\ \alpha_{LL} = \frac{\alpha_{KL}^2}{\alpha_{KE}^2}\cdot\alpha_{EE},\ \alpha_{LE} = \frac{\alpha_{KL}}{\alpha_{KE}}\cdot\alpha_{EE},\ \alpha_{LT} = \frac{\alpha_{KL}}{\alpha_{KE}}\cdot\alpha_{ET}, \quad (4.40)$$

$$\alpha_K = (\alpha_E - 1)\frac{\alpha_{KE}}{\alpha_{EE}},\ \alpha_{KK} = \frac{\alpha_{KE}^2}{\alpha_{EE}},\ \alpha_{KT} = \frac{\alpha_{KE}}{\alpha_{EE}}\cdot\alpha_{ET}\ . \quad (4.41)$$

For $[(K, L), (M, E)]$-separability merely 4 nonlinear restrictions are required:

$$\alpha_L = \alpha_K\frac{\alpha_{KL}}{\alpha_{KK}},\ \alpha_{LL} = \frac{\alpha_{KL}^2}{\alpha_{KK}},\ \alpha_{LE} = \frac{\alpha_{KL}}{\alpha_{KK}}\cdot\alpha_{KE},\ \alpha_{LT} = \frac{\alpha_{KL}}{\alpha_{KK}}\cdot\alpha_{KT}\ . \quad (4.42)$$

Note that replacing α_K, α_{KK} and α_{KT} in (4.42) by (4.41) reproduces (4.40).

Less restrictive than exact tests are approximate separability tests, which specifically require the following constraints for $[(K, L), (M, E)]$- and $[(K, L, M), E]$)-separability, respectively,

$$\alpha_K\alpha_{LE} = \alpha_L\alpha_{KE} \quad \text{and} \quad \alpha_K\alpha_{LM} = \alpha_L\alpha_{KM}, \quad (4.43)$$

and

$$\alpha_K\alpha_{LE} = \alpha_L\alpha_{KE},\ \alpha_K\alpha_{ME} = \alpha_M\alpha_{KE}, \quad \text{and} \quad \alpha_L\alpha_{ME} = \alpha_M\alpha_{LE}. \quad (4.44)$$

As a combination of the first and the second equation, the last equation in (4.44) is superflous.

Separability Test Results

The classical $[(K, L), (M, E)]$-separability results reported in Table 1 cast doubt on all prior value-added studies. These separability conditions, necessary for value-added approaches, are most likely to be violated. With particular respect to the separability of energy from non-energy inputs, test results for classical and, consequently, $[(K, L, M), E]$-separability in the sense of this paper's definition has to be rejected at all significance levels, irrespective of the model estimated. Still imposing e. g. linear $[(K, L, M), E]$-separability conditions (4.38) on dual KLEM-model II causes misspecification. This is confirmed by the result of the LAGRANGE-multiplier test reported in Table B2.

Table 1: Separability Tests – German Manufacturing 1978 - 1990.

Class of Separability Tests	degrees of freedom	Dual Approach Model I	Dual Approach Model II	Primal Approach Model I	Primal Approach Model II
Exact Tests		linear separability conditions			
$[(K, L, M), E]$	3	32.5**	190.7**	487.5**	99.7**
$[(K, L), (M, E)]$	4	40.0**	362.1**	578.0**	125.4**
		nonlinear separability conditions			
$[(K, L, M), E]$	7	60.9**	303.0**	152.8**	215.4**
$[(K, L), (M, E)]$	4	30.1**	196.5**	81.6**	72.6**
Approximate Tests		approximate separability conditions			
$[(K, L, M), E]$	2	302.5**	71.9**	333.4**	303.9**
$[(K, L), (M, E)]$	2	301.6**	449.5**	5.3*	6.0*

Note: * (**) denotes significance at the 5 %- (1 %)-level. Results for nonlinear and approximate tests refer to the Chemical industry, but are very similar for the other industries.

Such results call into question those prior empirical studies for German manufacturing, which – in contrast to e. g. UNGER (1986) and FALK and KOEBEL (1999) – have abstained from the factor energy. Estimates of cross-price elasticities for K, L and M provided by those studies may hardly be expected to be reliable. Of course, this also holds for AES and MES, which build on cross-price elasticities.

The Effects of Omitting E and of Imposing Linear $[(K, L, M), E]$-separability

In their review of the capital-energy debate, FRONDEL and SCHMIDT (2000) find ample empirical evidence for their argument that in static translog approaches estimated cross-price elasticities $\eta_{x_i p_j}$ are mainly the result of corresponding cost shares s_j. Estimates of cross-price elasticities for the most energy-intensive German manufacturing industries, the sole industries displaying negative own-price elasticities for energy, are reported in Table B3 of Appendix B. These estimates suggest again that the pattern among cross-price elasticities more or less reflects that among the input cost shares (compare Tables B3 and C1).

Without taking account of energy with its commonly very low cost shares, cost shares of K, L and M remain almost unchanged. Hence, along the lines of FRONDEL and SCHMIDT's (2000) cost-share argument, remaining cross-price elasticities are expected to change moderately after omitting energy from the data base. In order to find empirical evidence for this expectation, energy is now dropped from the KLEM-panel data of German manufacturing and a translog KLM-model with different intercepts analogous to dual Model II is estimated. That is, we deliberately cause potential misspecification and estimate the following cost-share system, where the equation for M is dropped arbitrarily (recall that any equation may be dropped arbitrarily when estimating by ML):

$$\begin{aligned} s_K &= \sum_{s=1}^{29} \beta_{Ks} + \beta_{KK} \ln(\frac{p_K}{p_M}) + \beta_{KL} \ln(\frac{p_L}{p_M}) + \beta_{KT} \cdot T + \varepsilon_K, \\ s_L &= \sum_{s=1}^{29} \beta_{Ls} + \beta_{KL} \ln(\frac{p_K}{p_M}) + \beta_{LL} \ln(\frac{p_L}{p_M}) + \beta_{LT} \cdot T + \varepsilon_L. \end{aligned} \tag{4.45}$$

The result of the LAGRANGE-multiplier test displayed in Table B2 reveals that dual KLME-Model II, using four inputs, is more appropriate than KLM-model (4.45), where energy E is dropped. That is, KLM-model (4.45) is misspecified, which is perfectly in line with the rejection of $[(K, L, M), E]$-separability.

Nevertheless, estimates of remaining cross-price elasticities obtained from KLM-model (4.45) are quite close to those of the more general KLME-model (compare Tables B3 and B4). This comparison as well as the comparison of Tables B3 and B5 confirms FRONDEL and SCHMIDT's (2000) cost-share argument. In Table B5 elasticity estimates on the basis of dual KLME-Model II are reported, but with linear $[(K, L), (M, E)]$-separability restrictions (4.38) imposed. Again, changes of remaining cross-price elasticities are rather moderate, although $[(K, L), (M, E)]$-separability does not hold indeed. But, rather than rehabilitating prior German KLM-studies, these results cast doubt on static translog approaches and support FRONDEL and SCHMIDT's (2000:24) somewhat pessimistic message that "[s]tatic translog approaches are limited in their ability to detect a wide range of phenomena".

4.5 Summary and Conclusion

With particular respect to substitution issues, the natural intuition of two factors i and j being separable from a factor k is that this factor k should not affect the ease of substitution among the former (see e. g. HAMERMESH 1993:34). According to BC73's classical definition, primal (dual) separability of factor i and j from factor k implies that in primal (dual) approaches their marginal rate of substitution (their input proportion x_i/x_j) is unaffected by the input level of k (the price of factor k). However, rather than by marginal rates of substitution or input proportions, the overwhelming majority of empirical substitution studies analyzes the ease of substitution between two factors on the basis of cross-price elasticities, AES or MES.

In consequence, when empirical analysts – as in numerous studies – invoke the assumption of BC73-separability in order to justify the omission of a non-negligible input factor from their empirical analysis, but then proceed to express their results in terms of, say AES, they base their empirical work inadvertently on an insufficient assumption. This paper therefore criticizes BC73's separability definition to be of limited relevance for empirical studies – notwithstanding its important role in the conceptual justification of stepwise optimizing decisions in production theory – and suggests a practically more important definition of separability based on cross-price elasticities, which we call *empirical dual separability*.

We define two factors i and j to be empirically dual separable from factor k, if and only if both the cross-price elacticities $\eta_{x_i p_j}$ and $\eta_{x_j p_i}$ are unaffected by the price of factor k. This definition incorporates the definition of dual BC73-separability, but is more restrictive. That means, specifically, that even if K and L were BC73-separable from the factor energy, this would nevertheless not imply that the ease of substitution between K and L in terms of cross-price elasticities remains unaffected by E. Therefore, even if K and L were BC73-separable from E, omitting energy from the data base might be unjustified under empirical aspects.

When omitting economically relevant, but not empirically separable factors like energy from the analysis, researchers generally risk to find incorrect cross-price elasticities η_{Kp_L} and η_{Lp_K}.

By applying our definition of empirical dual separability to a translog cost function, it turns out that empirical dual separability of factors i and j from factor k holds globally if and only if

$$\beta_{ik} = \beta_{jk} = 0$$

for the second-order coefficients of the translog cost function. These conditions are the exact linear separability conditions which are sufficient, but not necessary for dual BC73-separability. Thus, DF77 are perfectly right in claiming that exact linear separability conditions are more restrictive than necessary for dual BC73-separability. However, we argue that only these restrictive conditions capture a notion of separability of factors i and j from factor k which has clear empirical content. Hence, by coining the notion of empirical dual separability, the exact linear separability conditions are rehabilitated.

In a concrete application of these concepts to German manufacturing data (1978-1990), it is found that classical $[(K, L, M), E]$- as well as $[(K, L), (M, E)]$-separability according to BC73, and hence according to our definition has to be rejected across all models, approaches and scenarios employed. These results cast doubt on prior empirical KLM-studies for German manufacturing. However, on the basis of our theoretical considerations that empirical dual separability is a more restrictive definition than the classical one of BC73, and, hence, empirical dual separability may be seldomly given, our conclusion is much more general: Rather than being ignored or omitted, energy should be taken into account in all studies aiming at the estimation of substitution possibilities between non-energy inputs.

Appendix A Data

Data necessary for estimation include cost shares, price and quantity indices for K, L, E and M. These data originate from two sources, Input-Output Tables and National Accounts, because data on energy are not available in the National Accounts. Energy expenditures and quantities based on the Input-Output classification (1978-90, unpublished data) have been provided by the Federal Statistical Office. We use this information for splitting up gross materials into energy and (non-energy) materials. Data from both sources are not directly comparable, though. For this reason, we are forced to the same adjustments described by FALK and KOEBEL (1999) to make energy data based on Input-Output Tables consistent with data stemming from National Accounts. Because of data limitations for energy data, our data base relates to the short range of 1978-1990. Overall, we have 377 $=$ 29 $\times$ 13 observations from $S =$ 29 sectors of German manufacturing. Unfortunately, for 3 of a total of 32 sectors of German manufacturing not all data necessary have been available.

Cost shares

Labor cost shares s_L are the sum of wages and salaries yearly paid in each industry in relation to gross production values generated in the corresponding industries. Capital cost shares s_K are the differences between gross value added and labor cost shares of each industry. Energy cost shares s_E are each industry's energy expenditures related to its gross production value. Cost shares for M result from the differences between gross production values and energy cost shares. For the energy-intensive industries of German Manufacturing, cost shares are displayed in Table A1.

Quantity and Price Indices

Dividing labor cost by the average number of employees which are occupied in each industry yields the average price of labor for each year and, by normalizing to one in 1978, the corresponding price indices p_L for labor. Capital K is measured by gross fixed capital formation at prices of 1985. Then, capital price indices are obtained by dividing capital cost, the residual of gross value added and labor cost, by K and normalization to one in 1978. Energy price indices are constructed similarly on the basis of energy cost and energy quantities E (in terajoule), both given by the Input-Output Tables. Finally, real gross production values, that is gross production values at constant prices, are calculated with the help of producer price indices. Then, quantities for M are constructed by subtracting real gross value added from real gross production values. The deflator for (non-energy) materials p_M is calculated by dividing materials expenditures by their respective quantities.

Table A1: Cost Shares for Energy-Intensive Industries of German Manufacturing (1978 - 1990).

Cost Shares	Year	Chemical Products	Stone & Earth	Fine Ceramics	Glass	Non-Ferrous Metals	Foundry	Pulp & Paper
	1978	0.13	0.16	0.15	0.14	0.09	0.09	0.13
s_K	1984	0.15	0.14	0.12	0.13	0.07	0.06	0.10
	1990	0.16	0.17	0.11	0.15	0.08	0.09	0.13
	1978	0.23	0.26	0.46	0.32	0.17	0.42	0.24
s_L	1984	0.17	0.24	0.45	0.29	0.14	0.39	0.19
	1990	0.20	0.24	0.44	0.28	0.16	0.38	0.19
	1978	0.08	0.09	0.08	0.10	0.08	0.07	0.12
s_E	1984	0.12	0.11	0.10	0.15	0.08	0.10	0.15
	1990	0.07	0.07	0.06	0.08	0.07	0.06	0.12
	1978	0.56	0.49	0.31	0.45	0.66	0.43	0.52
s_M	1984	0.56	0.51	0.33	0.44	0.71	0.46	0.57
	1990	0.57	0.52	0.39	0.49	0.70	0.47	0.56

Appendix B Estimation Results

Table B1: ML-Parameter Estimates of the KLEM Translog Production Function Approach – German Manufacturing 1978 - 1990.

	Model I common intercept		Model II different intercepts	
α_K	0.4467**	(0.0443)	29DV	—
α_L	0.1409**	(0.0271)	29DV	—
α_E	-0.3143**	(0.0234)	29DV	—
α_M	0.7267**	(0.0442)	29DV	—
α_{KK}	0.0174	(0.0138)	-0.0068	(0.0055)
α_{KL}	-0.0147**	(0.0057)	0.0044	(0.0027)
α_{KE}	-0.0364**	(0.0041)	0.0171**	(0.0051)
α_{KM}	0.0337**	(0.0124)	-0.0147**	(0.0050)
α_{LL}	0.1098**	(0.0042)	0.0694**	(0.0075)
α_{LE}	-0.0064**	(0.0024)	-0.0082	(0.0075)
α_{LM}	-0.0886**	(0.0048)	-0.0656**	(0.0065)
α_{EE}	0.0501**	(0.0023)	0.0385**	(0.0142)
α_{EM}	-0.0073*	(0.0041)	-0.0473**	(0.0115)
α_{MM}	0.0622**	(0.0144)	0.1276**	(0.0126)
α_{KT}	1.1e^{-5}	(5.9e^{-5})	0.0011**	(0.0003)
α_{LT}	-0.0003**	(2.4e^{-5})	0.0002	(0.0002)
α_{ET}	-3.8e^{-6}	(2.2e^{-5})	-0.0004	(0.0005)
α_{MT}	0.0003**	(6.5e^{-5})	0.0001	(0.0002)
Lagrange-Multiplier Test				
Test-Value	934			
Critical χ^2	$\chi^2_{0.01}(84) \approx 113$			

Note: ** (*) denotes significant at the 1 %- (5 %)-level. 29DV indicates the use of separate dummy intercept variables for each industry. Standard errors are given in parentheses. The LAGRANGE-multiplier (LM) test builds on the following test statistic, $N \cdot tr[\hat{M}_r^{-1} \cdot (\hat{M}_r - \hat{M}_u)]$, where N is the number of observations, and $\hat{M}_r$ and $\hat{M}_u$ denote the estimated residual cross-product matrices of the constrained (Model I) and the unconstrained model (Modell II), respectively. The LM-test statistic is distributed asymptotically as a chi-square random variable, with degrees of freedom equal to the number of restrictions implemented in the constrained model. In comparison to its reference, Model II, Model I incorporates $3 \times 28 = 84$ restrictions, whence the LM-test statistic is asymptotically chi-square distributed with 84 degrees of freedom. One might also use the

Likelihood-ratio (LR) test statistic, $-N \cdot (\ln|\hat{M}_u| - |\hat{M}_r|)$, or the WALD test statistic $N \cdot tr[\hat{M}_u^{-1} \cdot (\hat{M}_r - \hat{M}_u)]$. But, whenever the null hypothesis does not hold exactly in the sample, the WALD, LR and LM test statistics are subject to the following inequality: WALD > LR > LM (BERNDT 1991:467). This implies that in practice there will always be a level of significance for which these three test procedures will yield conflicting statistical significance. Yet, in choosing the LM test statistic, we know that a rejection of the null hypothesis on this basis will be supported by the WALD and the LR test statistics as well.

Table B2: ML Parameter Estimates for Various Models of the KLEM Translog Cost Function Approach – German Manufacturing 1978 - 1990.

	Model I common intercept KLME-model		Model II different intercepts KLME-model		(KLM)E-separability		KLM-model	
β_K	0.0961**	(0.0118)	29DV	—	29DV	—	29DV	—
β_L	0.3004**	(0.0101)	29DV	—	29DV	—	29DV	—
β_E	0.1125**	(0.0074)	29DV	—	29DV	—	29DV	—
β_M	0.4910**	(0.0113)	29DV	—	29DV	—	29DV	—
β_{KK}	0.0395**	(0.0149)	0.0386**	(0.0023)	0.0345**	(0.0022)	0.0386**	(0.0024)
β_{KL}	-0.0161	(0.0123)	-0.0152**	(0.0017)	-0.0126**	(0.0016)	-0.0114**	(0.0016)
β_{KE}	-0.0030	(0.0087)	-0.0101**	(0.0022)	0	—	—	—
β_{KM}	-0.0204	(0.0138)	-0.0133**	(0.0032)	-0.0219**	(0.0026)	-0.0272**	(0.0025)
β_{LL}	0.0945**	(0.0336)	0.0445**	(0.0061)	0.0309**	(0.0059)	0.0348**	(0.0059)
β_{LE}	-0.1107**	(0.0218)	-0.0293**	(0.0039)	0	—	—	—
β_{LM}	0.03240	(0.0250)	$9.6e^{-6}$	(0.0067)	-0.0184**	(0.0056)	-0.0234**	(0.0073)
β_{EE}	0.0410*	(0.0225)	0.0595**	(0.0061)	0	—	—	—
β_{EM}	0.0638**	(0.0199)	-0.0201**	(0.0075)	0	—	—	—
β_{MM}	-0.0758**	(0.0306)	0.0334**	(0.0120)	0.0402**	(0.0062)	0.0506**	(0.0120)
β_{KT}	0.0003**	($5.4e^{-5}$)	0.0005**	(0.0002)	0.0004*	(0.0002)	0.0003	(0.0002)
β_{LT}	-0.0002**	($4.6e^{-5}$)	-0.0022**	(0.0002)	-0.0022**	(0.0002)	-0.0023**	(0.0002)
β_{ET}	-0.0003**	($3.2e^{-5}$)	-0.0012**	(0.0002)	-0.0014**	(0.0003)	—	—
β_{MT}	0.0002**	($5.2e^{-5}$)	0.0029**	(0.0003)	0.0033**	(0.0003)	0.0020**	(0.0003)
Lagrange-Multiplier Tests								
Test-Values	1081		—	—	81		112	
Critical χ^2	$\chi^2_{0.01}(84) \approx 113$		—	—	$\chi^2_{0.01}(3) = 11.34$		$\chi^2_{0.01}(33) = 53.96$	

** (*) denotes significant at the 1 %- (5 %)-level. 29DV indicates the use of separate dummy intercept variables for each industry. Standard errors are given in parentheses. The LM-Tests use KLEM-Model II as the unrestricted model.

Table B3: Estimates of Cross-Price Elasticities for Selected Industries of German Manufacturing (1978 - 1990) on the basis of the dual KLEM-Model II.

	Year	Chemical Products	Stone & Earth	Fine Ceramics	Glass	Non-Ferrous Metals	Foundry	Pulp & Paper
	1978	0.00 (0.02)	0.03 (0.01)	0.01 (0.02)	0.02 (0.02)	-0.03 (0.03)	-0.05 (0.03)	0.04 (0.02)
η_{Kp_E}	1984	0.06 (0.01)	0.03 (0.02)	0.01 (0.02)	0.07 (0.02)	-0.01 (0.02)	-0.08 (0.04)	0.04 (0.02)
	1990	0.01 (0.01)	0.01 (0.01)	-0.03 (0.02)	0.01 (0.01)	-0.06 (0.03)	-0.06 (0.03)	0.04 (0.02)
	1978	-0.05 (0.02)	-0.02 (0.02)	0.02 (0.01)	0.00 (0.01)	-0.10 (0.02)	0.00 (0.01)	0.00 (0.01)
η_{Lp_E}	1984	-0.05 (0.02)	-0.02 (0.02)	0.04 (0.01)	0.05 (0.01)	-0.13 (0.03)	0.00 (0.01)	-0.01 (0.02)
	1990	-0.08 (0.02)	-0.05 (0.02)	0.00 (0.01)	-0.04 (0.01)	-0.12 (0.03)	-0.01 (0.01)	-0.03 (0.02)
	1978	0.05 (0.01)	0.05 (0.01)	0.02 (0.03)	0.05 (0.02)	0.05 (0.01)	0.02 (0.02)	0.08 (0.01)
η_{Mp_E}	1984	0.09 (0.01)	0.07 (0.02)	0.04 (0.02)	0.10 (0.02)	0.05 (0.01)	0.05 (0.02)	0.11 (0.01)
	1990	0.04 (0.01)	0.05 (0.01)	0.01 (0.02)	0.04 (0.02)	0.04 (0.01)	0.02 (0.02)	0.09 (0.01)
	1978	0.01 (0.03)	0.05 (0.03)	0.02 (0.03)	0.03 (0.02)	-0.04 (0.03)	-0.06 (0.03)	0.04 (0.02)
η_{Ep_K}	1984	0.07 (0.02)	0.05 (0.02)	0.02 (0.02)	0.06 (0.02)	-0.06 (0.03)	-0.05 (0.02)	0.03 (0.02)
	1990	0.02 (0.03)	0.03 (0.03)	-0.05 (0.04)	0.02 (0.03)	-0.07 (0.03)	-0.07 (0.04)	0.05 (0.02)
	1978	0.06 (0.01)	0.10 (0.01)	0.11 (0.01)	0.09 (0.01)	-0.01 (0.01)	0.05 (0.01)	0.06 (0.01)
η_{Lp_K}	1984	0.06 (0.01)	0.08 (0.01)	0.08 (0.01)	0.07 (0.01)	-0.04 (0.01)	0.02 (0.01)	0.02 (0.01)
	1990	0.08 (0.01)	0.11 (0.01)	0.07 (0.01)	0.10 (0.01)	-0.02 (0.01)	0.05 (0.01)	0.05 (0.01)
	1978	0.11 (0.01)	0.13 (0.01)	0.10 (0.01)	0.11 (0.01)	0.07 (0.01)	0.06 (0.01)	0.10 (0.01)
η_{Mp_K}	1984	0.13 (0.01)	0.12 (0.01)	0.08 (0.01)	0.10 (0.01)	0.05 (0.01)	0.03 (0.01)	0.07 (0.01)
	1990	0.14 (0.01)	0.15 (0.01)	0.08 (0.01)	0.13 (0.01)	0.06 (0.01)	0.06 (0.01)	0.10 (0.01)
	1978	-0.13 (0.05)	-0.06 (0.04)	0.11 (0.05)	0.01 (0.04)	-0.20 (0.05)	-0.02 (0.06)	0.00 (0.03)
η_{Ep_L}	1984	-0.07 (0.03)	-0.03 (0.04)	0.16 (0.04)	0.09 (0.03)	-0.22 (0.05)	0.08 (0.04)	-0.01 (0.03)
	1990	-0.21 (0.05)	-0.17 (0.05)	-0.03 (0.06)	-0.11 (0.05)	-0.28 (0.06)	-0.08 (0.06)	-0.05 (0.03)
	1978	0.11 (0.01)	0.16 (0.01)	0.36 (0.01)	0.21 (0.01)	-0.01 (0.02)	0.24 (0.02)	0.12 (0.01)
η_{Kp_L}	1984	0.07 (0.01)	0.14 (0.01)	0.32 (0.01)	0.16 (0.01)	-0.09 (0.03)	0.13 (0.03)	0.03 (0.02)
	1990	0.10 (0.01)	0.15 (0.01)	0.30 (0.01)	0.18 (0.01)	-0.03 (0.02)	0.21 (0.02)	0.07 (0.01)
	1978	0.23 (0.01)	0.26 (0.01)	0.46 (0.02)	0.31 (0.02)	0.17 (0.01)	0.41 (0.02)	0.23 (0.01)
η_{Mp_L}	1984	0.16 (0.01)	0.24 (0.01)	0.45 (0.02)	0.28 (0.02)	0.14 (0.01)	0.39 (0.01)	0.19 (0.01)
	1990	0.20 (0.01)	0.24 (0.01)	0.44 (0.02)	0.28 (0.01)	0.16 (0.01)	0.38 (0.01)	0.19 (0.01)
	1978	0.32 (0.09)	0.27 (0.08)	0.07 (0.09)	0.23 (0.08)	0.41 (0.09)	0.13 (0.11)	0.35 (0.06)
η_{Ep_M}	1984	0.39 (0.06)	0.32 (0.07)	0.13 (0.07)	0.31 (0.05)	0.47 (0.09)	0.25 (0.08)	0.43 (0.05)
	1990	0.29 (0.10)	0.64 (0.10)	0.07 (0.12)	0.23 (0.10)	0.40 (0.11)	0.14 (0.12)	0.39 (0.06)
	1978	0.46 (0.02)	0.40 (0.02)	0.22 (0.02)	0.35 (0.02)	0.51 (0.04)	0.28 (0.04)	0.41 (0.03)
η_{Kp_M}	1984	0.47 (0.02)	0.42 (0.02)	0.22 (0.03)	0.33 (0.02)	0.51 (0.05)	0.23 (0.05)	0.43 (0.03)
	1990	0.49 (0.02)	0.45 (0.02)	0.27 (0.03)	0.41 (0.02)	0.52 (0.04)	0.31 (0.04)	0.46 (0.02)
	1978	0.56 (0.03)	0.49 (0.03)	0.31 (0.02)	0.45 (0.02)	0.66 (0.04)	0.43 (0.02)	0.52 (0.03)
η_{Lp_M}	1984	0.56 (0.04)	0.51 (0.03)	0.33 (0.02)	0.44 (0.02)	0.71 (0.05)	0.46 (0.02)	0.57 (0.04)
	1990	0.57 (0.03)	0.53 (0.03)	0.39 (0.02)	0.49 (0.02)	0.70 (0.04)	0.47 (0.02)	0.56 (0.04)

Note: Standard errors are given in parentheses. Under the assumption that cost shares are constant and equal to the means of their estimated values, approximate estimates of the standard errors of $\eta_{x_i p_j}$ are, asymptotically, $var(\hat{\eta}_{x_i p_j}) = \hat{\beta}_{ij}/\hat{s}_i^2$ and $var(\hat{\eta}_{x_i p_i}) = \hat{\beta}_{ii}/\hat{s}_i^2$ (PINDYCK 1979:171).

Table B4: Estimates of Cross-Price Elasticities for the KLM-translog model II.

	Year	Chemical Products	Stone & Earth	Fine Ceramics	Glass	Non-Ferrous Metals	Foundry	Pulp & Paper
	1978	0.08 (0.01)	0.11 (0.01)	0.13 (0.01)	0.11 (0.01)	0.01 (0.01)	0.08 (0.01)	0.08 (0.01)
η_{Lp_K}	1984	0.08 (0.01)	0.10 (0.01)	0.10 (0.01)	0.09 (0.01)	-0.02 (0.01)	0.05 (0.01)	0.04 (0.01)
	1990	0.10 (0.01)	0.13 (0.01)	0.09 (0.01)	0.12 (0.01)	0.00 (0.01)	0.08 (0.01)	0.07 (0.01)
	1978	0.08 (0.01)	0.10 (0.01)	0.06 (0.01)	0.08 (0.01)	0.05 (0.01)	0.03 (0.01)	0.07 (0.01)
η_{Mp_K}	1984	0.10 (0.01)	0.09 (0.01)	0.03 (0.01)	0.06 (0.01)	0.03 (0.01)	0.00 (0.01)	0.05 (0.01)
	1990	0.11 (0.01)	0.14 (0.01)	0.04 (0.01)	0.10 (0.01)	0.04 (0.01)	0.03 (0.01)	0.08 (0.01)
	1978	0.14 (0.01)	0.19 (0.01)	0.38 (0.01)	0.24 (0.01)	0.04 (0.02)	0.28 (0.02)	0.15 (0.02)
η_{Kp_L}	1984	0.10 (0.01)	0.16 (0.01)	0.35 (0.01)	0.20 (0.01)	-0.04 (0.02)	0.19 (0.03)	0.07 (0.03)
	1990	0.13 (0.01)	0.17 (0.01)	0.33 (0.01)	0.20 (0.01)	0.01 (0.02)	0.25 (0.02)	0.10 (0.02)
	1978	0.19 (0.01)	0.21 (0.01)	0.38 (0.02)	0.27 (0.01)	0.13 (0.01)	0.36 (0.01)	0.19 (0.01)
η_{Mp_L}	1984	0.13 (0.01)	0.20 (0.01)	0.38 (0.02)	0.23 (0.01)	0.11 (0.01)	0.34 (0.01)	0.14 (0.01)
	1990	0.16 (0.01)	0.21 (0.01)	0.38 (0.01)	0.24 (0.01)	0.13 (0.01)	0.33 (0.01)	0.15 (0.01)
	1978	0.46 (0.03)	0.40 (0.02)	0.26 (0.01)	0.37 (0.02)	0.52 (0.03)	0.37 (0.02)	0.42 (0.03)
η_{Lp_M}	1984	0.42 (0.03)	0.41 (0.02)	0.28 (0.01)	0.36 (0.02)	0.54 (0.04)	0.40 (0.02)	0.44 (0.03)
	1990	0.45 (0.02)	0.43 (0.02)	0.34 (0.01)	0.41 (0.02)	0.55 (0.04)	0.41 (0.02)	0.44 (0.03)
	1978	0.36 (0.02)	0.31 (0.02)	0.11 (0.02)	0.25 (0.02)	0.36 (0.03)	0.12 (0.03)	0.30 (0.02)
η_{Kp_M}	1984	0.38 (0.02)	0.32 (0.02)	0.10 (0.02)	0.22 (0.02)	0.30 (0.04)	-0.01 (0.04)	0.29 (0.03)
	1990	0.40 (0.02)	0.36 (0.02)	0.14 (0.03)	0.31 (0.02)	0.35 (0.04)	0.15 (0.03)	0.35 (0.02)

Table B5: Estimates of Cross-Price Elasticities, when linear $[(K,L,M),E]$-separability restrictions (4.38) are imposed on dual KLEM-model II.

	Year	Chemical Products	Stone & Earth	Fine Ceramics	Glass	Non-Ferrous Metals	Foundry	Pulp & Paper
	1978	0.07 (0.01)	0.11 (0.01)	0.12 (0.01)	0.10 (0.01)	0.01 (0.01)	0.06 (0.01)	0.07 (0.01)
η_{Lp_K}	1984	0.08 (0.01)	0.09 (0.01)	0.09 (0.01)	0.08 (0.01)	-0.03 (0.01)	0.03 (0.01)	0.03 (0.01)
	1990	0.10 (0.01)	0.12 (0.01)	0.08 (0.01)	0.11 (0.01)	-0.00 (0.01)	0.05 (0.01)	0.04 (0.01)
	1978	0.09 (0.01)	0.11 (0.01)	0.08 (0.01)	0.10 (0.01)	0.06 (0.01)	0.04 (0.01)	0.08 (0.01)
η_{Mp_K}	1984	0.11 (0.01)	0.10 (0.01)	0.05 (0.01)	0.08 (0.01)	0.04 (0.01)	0.01 (0.01)	0.06 (0.01)
	1990	0.12 (0.01)	0.15 (0.01)	0.05 (0.01)	0.11 (0.01)	0.05 (0.01)	0.04 (0.01)	0.07 (0.01)
	1978	0.13 (0.01)	0.18 (0.01)	0.37 (0.01)	0.23 (0.01)	0.03 (0.02)	0.27 (0.02)	0.14 (0.01)
η_{Kp_L}	1984	0.08 (0.01)	0.15 (0.01)	0.34 (0.01)	0.18 (0.01)	-0.05 (0.02)	0.17 (0.03)	0.06 (0.02)
	1990	0.12 (0.01)	0.16 (0.01)	0.32 (0.01)	0.20 (0.01)	0.00 (0.02)	0.24 (0.02)	0.09 (0.01)
	1978	0.19 (0.01)	0.22 (0.01)	0.40 (0.02)	0.28 (0.01)	0.14 (0.01)	0.37 (0.01)	0.20 (0.01)
η_{Mp_L}	1984	0.14 (0.01)	0.21 (0.01)	0.40 (0.02)	0.24 (0.01)	0.11 (0.01)	0.35 (0.01)	0.15 (0.01)
	1990	0.17 (0.01)	0.22 (0.01)	0.39 (0.01)	0.24 (0.01)	0.13 (0.01)	0.34 (0.01)	0.15 (0.01)
	1978	0.50 (0.02)	0.42 (0.02)	0.27 (0.01)	0.39 (0.02)	0.55 (0.03)	0.39 (0.01)	0.44 (0.02)
η_{Lp_M}	1984	0.45 (0.03)	0.43 (0.02)	0.29 (0.01)	0.38 (0.02)	0.58 (0.04)	0.41 (0.01)	0.47 (0.03)
	1990	0.48 (0.03)	0.45 (0.02)	0.35 (0.01)	0.43 (0.02)	0.60 (0.04)	0.41 (0.01)	0.49 (0.03)
	1978	0.39 (0.02)	0.35 (0.02)	0.16 (0.02)	0.29 (0.02)	0.42 (0.03)	0.18 (0.03)	0.34 (0.02)
η_{Kp_M}	1984	0.41 (0.02)	0.36 (0.02)	0.14 (0.02)	0.27 (0.02)	0.38 (0.04)	0.08 (0.05)	0.34 (0.03)
	1990	0.43 (0.02)	0.39 (0.02)	0.19 (0.02)	0.35 (0.02)	0.42 (0.03)	0.21 (0.03)	0.39 (0.02)

Note: Standard errors are given in parentheses.

Appendix C Nonlinear Separability Conditions

Nonlinear $[(K,L),(M,E)]$-separability constraints

Necessary and sufficient conditions for separability of two factors i and j from k, derived in Section 4.4, yield exactly the same set of conditions for both $k = M$ and $k = E$ while $i = K, j = L$:

$$\frac{\alpha_L}{\alpha_K} = \frac{\alpha_{KL}}{\alpha_{KK}} = \frac{\alpha_{LL}}{\alpha_{KL}} = \frac{\alpha_{LE}}{\alpha_{KE}} = \frac{\alpha_{LM}}{\alpha_{KM}} = \frac{\alpha_{LT}}{\alpha_{KT}} \tag{4.46}$$

These 5 restrictions are equivalent to the set of four equations (4.42) of Section 4.4, as one condition in (4.46) is superfluos when the constant resturns to scale restrictions (4.16) and the first three equations of (4.46) are applied:

$$\frac{\alpha_{LM}}{\alpha_{KM}} = \frac{\alpha_{LL}+\alpha_{LE}+\alpha_{LK}}{\alpha_{KK}+\alpha_{KL}+\alpha_{KE}} = \frac{\frac{\alpha_L}{\alpha_K}\alpha_{KL}+\frac{\alpha_L}{\alpha_K}\alpha_{KE}+\frac{\alpha_L}{\alpha_K}\alpha_{KK}}{\alpha_{KK}+\alpha_{KL}+\alpha_{KE}} = \frac{\alpha_L}{\alpha_K} \quad . \tag{4.47}$$

Nonlinear $[(K,L,M),E]$-separability constraints

In addition to (4.46), obtained for $i = K, j = L$ and $k = E$, $[(K,L,M),E]$-separability requires that

$$\frac{\alpha_M}{\alpha_K} = \frac{\alpha_{KM}}{\alpha_{KK}} = \frac{\alpha_{LM}}{\alpha_{KL}} = \frac{\alpha_{ME}}{\alpha_{KE}} = \frac{\alpha_{MM}}{\alpha_{KM}} = \frac{\alpha_{MT}}{\alpha_{KT}} \quad . \tag{4.48}$$

This results for $i = K, j = M$ and $k = E$ from the set of nonlinear conditions (4.25).

For $i = L, j = M$ and $k = E$, one can not gain further information from (4.25) than already given by (4.46) and (4.48). As above, one of the five conditions in (4.48) is superfluos due to constant returns of scale requirements. Moreover, equation

$$\frac{\alpha_{KM}}{\alpha_{KK}} = \frac{\alpha_{LM}}{\alpha_{KL}} \quad \Longleftrightarrow \quad \frac{\alpha_{LM}}{\alpha_{KM}} = \frac{\alpha_{KL}}{\alpha_{KK}} \tag{4.49}$$

is already contained in (4.46). By these $[(K,L,M),E]$-separability constraints, the number of 12 independent parameters, which are common to Model I and II, is reduced to 5: $\alpha_E, \alpha_{KE}, \alpha_{EE}, \alpha_{ET}$ and α_{KL}.

In order to derive the set of seven equations (4.40) - (4.41), displayed in Section 4.4, we depart from (4.46), using three conditions in the middle of (4.46) and the constant resturns to scale restrictions (4.16):

$$\alpha_{LE}\cdot\alpha_{MK} = \alpha_{KE}\cdot\alpha_{LM} = -\alpha_{KE}(\alpha_{LL}+\alpha_{LE}+\alpha_{LK}) = -\alpha_{KE}(\frac{\alpha_{KL}^2}{\alpha_{KK}}+\frac{\alpha_{KL}}{\alpha_{KK}}\alpha_{KE}+\alpha_{LK}). \tag{4.50}$$

Combining $\frac{\alpha_{LE}}{\alpha_{KL}} = \frac{\alpha_{KE}}{\alpha_{KK}}$ of (4.46) and $\frac{\alpha_{KE}}{\alpha_{KK}} = \frac{\alpha_{ME}}{\alpha_{MK}}$ of (4.48) and using (4.16) again, yields

$$\alpha_{LE}\cdot\alpha_{MK} = \alpha_{ME}\cdot\alpha_{KL} = -\alpha_{KL}(\alpha_{EE}+\alpha_{KE}+\alpha_{LE}) = -\alpha_{KL}(\alpha_{EE}+\alpha_{KE}+\frac{\alpha_{KL}}{\alpha_{KK}}\alpha_{KE}). \tag{4.51}$$

By equating (4.50) and (4.51), we have the second constraint of (4.41),

$$\alpha_{KK} = \frac{\alpha_{KE}^2}{\alpha_{EE}} \quad . \tag{4.52}$$

Next, when using (4.15) and (4.16), the first condition in (4.48),

$$\frac{\alpha_M}{\alpha_K} = \frac{1-\alpha_K-\alpha_L-\alpha_E}{\alpha_k} = -\frac{\alpha_{KK}+\alpha_{KL}+\alpha_{KE}}{\alpha_{KK}} = \frac{\alpha_{KM}}{\alpha_{KK}} \quad , \tag{4.53}$$

is equivalent to

$$\alpha_K = (\alpha_E-1)\frac{\alpha_{KK}}{\alpha_{KE}} \quad , \tag{4.54}$$

when $\frac{\alpha_L}{\alpha_K} = \frac{\alpha_{KL}}{\alpha_{KK}}$ is applied. By expresssion (4.52), this is the same as the first condition in (4.41),

$$\alpha_K = (\alpha_E-1)\frac{\alpha_{KE}}{\alpha_{EE}} \quad . \tag{4.55}$$

By considering $\frac{\alpha_L}{\alpha_K} = \frac{\alpha_{KL}}{\alpha_{KK}}, \frac{\alpha_{LT}}{\alpha_{KT}} = \frac{\alpha_{KL}}{\alpha_{KK}}$ and (4.55), the following equation of (4.48),

$$\frac{\alpha_M}{\alpha_K} = \frac{1-\alpha_E-\alpha_K-\alpha_L}{\alpha_K} = -\frac{\alpha_{KT}+\alpha_{LT}+\alpha_{ET}}{\alpha_{KT}} = \frac{\alpha_{MT}}{\alpha_{KT}} \quad , \tag{4.56}$$

leads to the third condition of (4.41),

$$\alpha_{KT} = \frac{\alpha_{KE}}{\alpha_{EE}}\cdot\alpha_{ET} \quad . \tag{4.57}$$

Finally, the expressions (4.55), (4.52) and (4.57) for α_K, α_{KK} and α_{KT}, respectively, substituted in the set of four nonlinear conditions (4.42) for $[(K,L),(M,E)]$-separability yield the remaining four nonlinear constraints (4.40) for $[(K,L,M),E]$-separability.

Chapter 5

The Real Elasticity of Substitution – An Obituary.

Together with CHRISTOPH M. SCHMIDT

Abstract. Apart from the factor ratio elasticity of substitution (FRES) derived by DAVIS and SHUMWAY (1996), all generalizations of HICKS' elasticity of substitution for a multifactor setting retain the hypothesis that output is constant. Hence, they measure *pure* substitution effects. Oil-price shocks however indicate that it is frequently problematic to ignore output effects in empirical studies of factor substitution. Therefore, in general, any substitution measure with clear empirical content has to incorporate both pure (net) substitution *and* output effects. As a preferable alternative to FRES, this paper suggests the concept of *generalized cross-price elasticities* (GES) in order to measure gross, rather than net substitution. GES is based on constant-output cross-price elasticities, the principal ingredients of all classical substitution elasticities.

5.1 Introduction

For the uncontroversial case of only two production factors, HICKS (1936) defined *the* elasticity of substitution σ in a seminal paper. This measure unambiguously reflects substitution relationships between both factors under the assumption that *output* is *constant*. HICKS was interested in the development of factor shares in a growing economy, so holding the output constant was primarily a device for isolating the phenomenon of interest. By measuring substitution possibilities along a fixed isoquant, σ thus exclusively quantifies *pure* substitution effects: "[T]he constant output ES [= elasticity of substitution] is in spirit the purest measure of substitution" (MUNDLAK 1968:234).

It is well-known that there is no unambiguous way for generalizing this concept to a multifactor setting. Until today many possible generalizations have been suggested e. g. in ALLEN and HICKS (1934), UZAWA (1962), MCFADDEN (1968), BLACKORBY and RUSSELL (1975), and recently DAVIS and SHUMWAY (1996), with HICKS' (HES), ALLEN's or ALLEN-UZAWA's (AES), MCFADDEN's (SES), MORISHIMA's (MES) and cross-price elasticities of substitution being the most prominent examples. Because σ served as a conceptual orientation, all these elasticities retain the maintained hypothesis that output is constant. Thus, they all measure pure substitution effects when the prices of one or, at most, two inputs change.

However, it is frequently problematic to ignore output effects. Oil-price shocks, for instance, tend to have a severe impact on the level of economic activity. Therefore, in general, any substitution measure with clear empirical content has to incorporate both pure (net) substitution *and* output effects. Correspondingly, any empirical study of factor substitutability which intends to predict the consequence of exogenous price shifts of one factor on the demand for another has to measure gross, rather net substitution. Yet, emphasis in virtually all applied research has been on the conceptual characterization and estimation of net substitution, excluding output effects from the analysis despite their paramount importance.

Apparently, the factor ratio elasticity of substitution (FRES) derived by DAVIS and SHUMWAY (1996) has been the only empirical substitution measure so far developed which takes output effects into account. Unfortunately, the estimation of FRES requires industry data including profits, which are not easily available. This might be the major reason that FRES has been ignored in applied analysis. Yet, even FRES was only conceived as a generalization of MES measuring the relative change of proportions of two factors due to a relative change in the price of one of these factors. An empirical assessment of output effects on demand was not intended. Moreover, constant-cost elasticities of substitution are typically not considered, although they are a viable alternative in the implicit construction of a counterfactual situation, at equal footing with constant-output elasticities.

This paper develops the concept of the *generalized cross-price elasticity* (GES) which explicitly takes account of ouput effects. This straightforward generalization deliberately builds on constant-output cross-price elasticities, since they are the principal ingredients in all classical substitution elasticities. Moreover, since cross-price elasticities measure the relative change of one factor due to price changes of another one, they are often more relevant in terms of economic content than MES (see FRONDEL 1999:24). That this choice is preferable in terms of economic interpretation is emphasized by our summary of classical substitution elasticities in the following section of the paper.

The concrete way for generalizing classical cross-price elasticities depends on the economic experiment to be described: Whether factor substitutability is to be estimated for profit-maximizing firms under perfect competition, for example, or for an industry which maximizes output subject to a constant-cost constraint requires different generalizations. Similar considerations pertain to the question of which underlying demand function, the HICKSian or the MARSHALLian demand function (see MUNDLAK 1968:234), might be the appropriate basis for the GES. This point is exemplified here by developing conrete analytical expressions of GES for exactly those two artificial experiments. Finally, in a concrete application to translog approaches, the empirical relevance of distinguishing between classical cross-price elasticities and their respective generalizations is checked on the basis of U.S. manufacturing data from the classical study by BERNDT and WOOD (1975, henceforth BW75).

Section 2 provides a summary of classical substitution elasticities. In Section 3, we develop the concept of GES. It is empirically applied to translog approaches in Section 4. Section 5 concludes.

5.2 A Summary of Classical Substitution Elasticities

The elasticity of substitution σ, originally introduced by HICKS (1932) for the analysis of only two factors, say x_1 and x_2, measures the relative change in factor proportions due to the relative change in the marginal rate of technical substitution f_{x_2}/f_{x_1} while output Y is held constant:

$$\sigma = \frac{d\ln\left(\frac{x_1}{x_2}\right)}{d\ln\left(\frac{f_{x_2}}{f_{x_1}}\right)}. \tag{5.1}$$

Because relative changes of factor proportions are measured for a fixed isoquant, σ measures pure (net) substitution. By contrast, gross substitution would by definition take output effects into account. Furthermore, σ is an inverse measure of the curvature of the isoquant (see e. g. CHAMBERS 1988:30): Very 'shallow' isoquants will exhibit large substitution effects, whereas very sharply curved isoquants will display relatively small effects. Finally, with more than two factors being flexible, the marginal rate of technical substitution f_{x_2}/f_{x_1} would not be determined uniquely (SILBERBERG 1990:322).

To avoid ambiguities in a multifactor setting further assumptions are necessary, leading to an alternative definition of σ in the two-dimensional case, which BLACKORBY and RUSSELL (1989) call HICKS' elasticity of substitution (HES): Under the assumptions of perfect competition and profit maximization, f_{x_2}/f_{x_1} equals relative factor prices p_2/p_1, whence

$$\text{HES} = \frac{d\ln\left(\frac{x_1}{x_2}\right)}{d\ln\left(\frac{p_2}{p_1}\right)}. \tag{5.2}$$

Definition (5.2) serves as a basis for all generalizations of σ for a multifactor setting. The literature's consensus of an ideal concept of multifactor substitution is to report optimal adjustment in relative inputs x_i/x_j when the relative input price of two arbitrary factors i and j changes with all inputs being flexible and cost minimization for fixed output (see e. g. THOMPSON 1997:125). This measure is often called HICKS-ALLEN elasticity of substitution (HAES), where

$$\text{HAES}_{ij} = \frac{\partial\ln\left(\frac{x_i}{x_j}\right)}{\partial\ln\left(\frac{p_j}{p_i}\right)} = \frac{\partial\ln x_i}{\partial\ln\left(\frac{p_j}{p_i}\right)} - \frac{\partial\ln x_j}{\partial\ln\left(\frac{p_j}{p_i}\right)} \tag{5.3}$$

and only the relative price of two factors i and j changes. If apart from i and j all other factors are assumed to be constant, HAES_{ij} is in fact HICKS' elasticity of substitution, which can be interpreted as a short-run elasticity measuring the degree of substitution between i and j, the sole flexible factors in the short-run. The most general measure of substitution on the basis of (5.2) would be a concept of *total substitution*, where besides p_i and p_j all other prices are flexible as well. According to MUNDLAK (1996:232), however, "[a]s a concept it may have little to contribute".

Cross-price elasticities, AES and MES, the trinity of classical substitution measures, are just special cases of basis expression (5.3): First, the cross-price elasticity

$$\eta_{x_i p_j} := \frac{\partial\ln x_i}{\partial\ln p_j} \tag{5.4}$$

focuses merely on the relative change of factor i – that is, the second term on the right-hand side of (5.3) has to be ignored – due to a sole change of the price of factor j, while output and all other prices are fixed, that is, $\partial\ln p_i$ in (5.3) has to be set to zero. Thus, according to MUNDLAK's (1968) classification, $\eta_{x_i p_j}$ is a *one-price-one-factor* elasticity of substitution.

Second, AES is related to $\eta_{x_ip_j}$ by (see e. g. BERNDT and WOOD (1975:261))

$$\text{AES}_{x_ip_j} = \frac{\eta_{x_ip_j}}{s_j}, \qquad \text{where } s_j = \frac{x_j p_j}{C}. \tag{5.5}$$

$\text{AES}_{x_ip_j}$ is therefore a one-price-one-factor elasticity of substitution as well, since it is the cross-price elasticity $\eta_{x_ip_j}$ divided by the cost share s_j of factor j. However, expression (5.5) is the "most compelling argument for ignoring the Allen measure in applied analysis ... The interesting measure is [$\eta_{x_ip_j}$] – why disguise it by dividing by a cost share? This question becomes all the more pointed when the best reason for doing so is that it yields a measure that can only be interpreted intuitively in terms of [$\eta_{x_ip_j}$]" (CHAMBERS 1988:95). Nevertheless, AES has been the elasticity of substitution extensively used in empirical studies (see e. g. HAMERMESH (1993:35) or THOMPSON and TAYLOR (1995:565)).

Third, MES, most generally defined by

$$\text{MES}_{x_ip_j} := \frac{\partial \ln(x_i/x_j)}{\partial \ln p_j} = \frac{\partial \ln x_i}{\partial \ln p_j} - \frac{\partial \ln x_j}{\partial \ln p_j} \tag{5.6}$$

(see BLACKORBY and RUSSELL (1989), FRONDEL (1999:14)), is a *two-factor-one-price* elasticity, where solely the price of factor j is flexible, again with all other prices being fixed.

The *two-factor-two-price* elasticity HAES_{ij} is in turn a weighted average of $\text{MES}_{x_ip_j}$ and $\text{MES}_{x_jp_i}$ (see CHAMBERS 1988:97): Using the chain rule, we have

$$\frac{\partial \ln x_i}{\partial \ln(\frac{p_j}{p_i})} = \eta_{x_ip_i} \frac{\partial \ln p_i}{\partial \ln(\frac{p_j}{p_i})} + \eta_{x_ip_j} \frac{\partial \ln p_j}{\partial \ln(\frac{p_j}{p_i})} \tag{5.7}$$

and

$$\frac{\partial \ln x_j}{\partial \ln(\frac{p_j}{p_i})} = \eta_{x_jp_i} \frac{\partial \ln p_i}{\partial \ln(\frac{p_j}{p_i})} + \eta_{x_jp_j} \frac{\partial \ln p_j}{\partial \ln(\frac{p_j}{p_i})}, \tag{5.8}$$

because merely the prices p_i and p_j are flexible, and, hence,

$$\begin{aligned} \text{HAES}_{ij} &= \frac{\partial \ln(\frac{x_i}{x_j})}{\partial \ln(\frac{p_j}{p_i})} = \frac{\partial \ln x_i}{\partial \ln(\frac{p_j}{p_i})} - \frac{\partial \ln x_j}{\partial \ln(\frac{p_j}{p_i})} \\ &= \underbrace{(\eta_{x_ip_j} - \eta_{x_jp_j})}_{\text{MES}_{x_ip_j}} \frac{\partial \ln p_j}{\partial \ln(\frac{p_j}{p_i})} - \underbrace{(\eta_{x_jp_i} - \eta_{x_ip_i})}_{\text{MES}_{x_jp_i}} \frac{\partial \ln p_i}{\partial \ln(\frac{p_j}{p_i})}, \end{aligned} \tag{5.9}$$

where the weights add to unity:

$$\frac{\partial \ln p_j}{\partial \ln(p_j/p_i)} + (-\frac{\partial \ln p_i}{\partial \ln(p_j/p_i)}) = \frac{\partial \ln(p_j/p_i)}{\partial \ln(p_j/p_i)} = 1. \tag{5.10}$$

The weighted sum in (5.9) reflects the fact that there is an infinite number of changes of prices p_i and p_j which lead to the same change of price ratio p_j/p_i. There are two polar cases: If only p_j changes and p_i is fixed, σ equals MES$_{x_i p_j}$, while, vice versa, σ specializes to MES$_{x_j p_i}$.

To complete our summary, we prove that MCFADDEN's *shadow elasticity of substitution* SES, which *additionally* holds cost constant, is both a weighted average of MES$_{x_i p_j}$ and MES$_{x_j p_i}$ and a special case of basic expression (5.3) as well. Since SES fixes cost C and only two prices p_i and p_j are supposed to change, it follows by using SHEPHARD's Lemma, $\frac{\partial C}{\partial p_i} = x_i$:

$$0 = \frac{\partial C}{\partial(\frac{p_j}{p_i})} = \frac{\partial C}{\partial p_i}\frac{\partial p_i}{\partial(\frac{p_j}{p_i})} + \frac{\partial C}{\partial p_j}\frac{\partial p_j}{\partial(\frac{p_j}{p_i})} = x_i\frac{\partial p_i}{\partial(\frac{p_j}{p_i})} + x_j\frac{\partial p_j}{\partial(\frac{p_j}{p_i})}. \tag{5.11}$$

By multiplying (5.11) by p_i and p_j and dividing it by C, it follows that

$$0 = p_j\underbrace{\frac{p_i x_i}{C}}_{s_i}\frac{\partial p_i}{\partial(\frac{p_j}{p_i})} + p_i\underbrace{\frac{p_j x_j}{C}}_{s_j}\frac{\partial p_j}{\partial(\frac{p_j}{p_i})} \text{ and thus } 0 = s_i\frac{\partial \ln p_i}{\partial \ln(\frac{p_j}{p_i})} + s_j\frac{\partial \ln p_j}{\partial \ln(\frac{p_j}{p_i})}. \tag{5.12}$$

Combining equations (5.10) and the right equation of (5.12) yields

$$\frac{\partial \ln p_i}{\partial \ln(\frac{p_j}{p_i})} = -\frac{s_j}{s_i + s_j} \quad \text{and} \quad \frac{\partial \ln p_j}{\partial \ln(\frac{p_j}{p_i})} = \frac{s_i}{s_i + s_j}. \tag{5.13}$$

By substituting both derivatives of (5.13) into the right-hand side of (5.9), we finally have (see CHAMBERS 1988:97)

$$\text{SES}_{ij} = (\frac{s_i}{s_i + s_j})\text{MES}_{x_i p_j} + (\frac{s_j}{s_i + s_j})\text{MES}_{x_j p_i}. \tag{5.14}$$

In sum, the common feature of AES, MES, HAES and SES is that all these elasticities are, first, mixtures of cross-price elasticities and, second, ignore output effects. SES, specifically, measures substitution relationships under the assumption that, in addition to fixed output, cost are constant. We now generalize classical cross-price elasticities, the common basis for all other concepts of this summary, by explicitly allowing for changes of output. For this reason, along the lines of DAVIS and SHUMWAY (1996), the subsequent section develops relations which are well-established – in consumption, rather than in production theory, though.

5.3 Generalized Cross-Price Elasticities (GES)

With $p_1, ..., p_n$ being exogenous factor prices, conditional general factor demand functions $f_j(p_1, ..., p_n, \phi)$ for any factor j are obtained by postulating that a certain objective function is maximized or minimized subject to a conditioning variable ϕ. Utility U, output Y or cost C are prominent examples for the conditioning variable ϕ, which may either be exogenous as well or a choice variable.

For exogenous $\phi = Y$, for instance, the classical constant-output cross-price elasticities $\eta_{x_j p_i}$ can be derived from the HICKSian factor demand function $x_j(p_1, ..., p_n, Y)$. If ϕ is a choice variable, the output $Y(p_1, ..., p_n, p)$ to be produced facing given factor prices and an output price p, for example, ϕ represents then an indirect function $\phi = \phi(p_1, ..., p_n, z)$, where $z = p$ is the additional exogenous variable besides factor prices. At the optimal point, the *unconditional* factor demand function $g_j(p_1, ..., p_n, z)$ equals the *conditional* factor demand function $f_j(p_1, ..., p_n, \phi)$:

$$g_j(p_1, ..., p_n, z) = f_j(p_1, ..., p_n, \phi(p_1, ..., p_n, z)). \tag{5.15}$$

Taking the logarithm of fundamental identity (5.15), differentiating the result logarithmically with respect to factor price p_i and using the chain rule yields the decomposition of $\theta_{g_j p_i} := \frac{\partial \ln g_j}{\partial \ln p_i}$, the cross-price elasticity of the *unconditional* factor demand g_j:

$$\theta_{g_j p_i} = \varepsilon_{f_j p_i} + \varepsilon_{f_j \phi} \cdot \theta_{\phi p_i}. \tag{5.16}$$

In decomposition (5.16), $\varepsilon_{f_j p_i} := \frac{\partial \ln f_j}{\partial \ln p_i}$ denotes the conditional demand elasticity of factor j with respect to the price p_i holding ϕ constant, $\varepsilon_{f_j \phi} := \frac{\partial \ln f_j}{\partial \ln \phi}$ represents the demand elasticity of factor j with respect to the conditioning variable ϕ, and

$\theta_{\phi p_i} := \frac{\partial \ln \phi}{\partial \ln p_i}$ is the elasticity of the conditioning variable ϕ with respect to price p_i. Decomposition (5.16) is a generalization of the popular SLUTZKY equation, where utility represents the conditioning variable ϕ (see e. g. TAKAYAMA 1985:247).

This general approach will now be applied to two artificial economic experiments, differing in the variables which are held constant. The first experiment describes profit-maximizing firms in a competitive environment, where the output price is given exogenously, whereas the second aims at an output-maximizing industry with a constant-cost constraint.

GES Under Profit Maximization and Perfect Competition

When the objective function is profit maximization under perfect competition, and, hence, input and output prices are given exogenously, the output price p represents the exogenous variable z and the supply function $Y(p_1, ..., p_n, p)$ plays the role of $\phi(p_1, ..., p_n, z)$. At the optimum, the demand x_j for factor j conditional on a desired output Y is identical to the unconditional factor demand u_j:

$$u_j(p_1, ..., p_n, p) = x_j(p_1, ..., p_n, Y(p_1, ..., p_n, p)). \tag{5.17}$$

Equation (5.17) is a special case of fundamental identity (5.15). In analogy to general decomposition (5.16), the *unconditional* price elasticity of the demand for factor j for given prices, $\theta_{u_j p_i} := \frac{\partial \ln u_j}{\partial \ln p_i}$, comprises two parts,

$$\theta_{u_j p_i} = \eta_{x_j p_i} + \eta_{x_j Y} \cdot \theta_{Y p_i}, \tag{5.18}$$

where

$$\begin{aligned}
\eta_{x_j p_i} &:= \frac{\partial \ln x_j}{\partial \ln p_i} = \text{ the classical \textit{conditional} constant-output cross-price elasticity},\\
\eta_{x_j Y} &:= \frac{\partial \ln x_j}{\partial \ln Y} = \text{ the output-elasticity of the demand for factor } j,\\
\theta_{Y p_i} &:= \frac{\partial \ln Y}{\partial \ln p_i} = \text{ the price-elasticity of output with respect to price } p_i.
\end{aligned}$$

While the first part of (5.18) measures the pure substitution effect for constant output Y, output effects are taken into account by the second term. Consequently,

$\theta_{u_j p_i}$ is a specific example of GES, the substitution concept suggested in this paper in order to take account of output effects, whereas DAVIS and SHUMWAY (1996) suggest FRES, a measure which has largely been ignored in the applied literature so far. We argue that GES should be prefered to FRES in applied analyses: As a generalization of MES, FRES measures the relative change of proportions of two factors, due to a relative change in the price of one of these factors, rather than measuring the relative change of only one factor due to price changes of the other factor, which is often more relevant in terms of economic content (FRONDEL 1999:24). Moreover, for the special case of a homothetic technology, relative output effects are the same for each factor, whence factor ratios do not change, and, hence, FRES and the constant-output measure MES are identical. Even for homothetic technologies, however, GES and classical constant-output cross-price elasticities are different, in general.

The GES $\theta_{u_j p_i}$ equals the classical cross-price elasticity $\eta_{x_j p_i}$ if and only if the last term of (5.18) is zero. This holds if either a) the output Y does not change when the price p_i changes, that is, there is *no output effect* or b) $\eta_{x_j Y}$ is zero. Condition a) reflects exactly the definition of the classical elasticity $\eta_{x_j p_i}$, where the output is required to be constant. Condition b) means that the demand x_j for factor j is in fact independent of the output, that is, j is a fixed factor. Then, of course, $\theta_{u_j p_i}$ and $\eta_{x_j p_i}$ are equal, namely zero.

$\theta_{u_j p_i}$ is lower than $\eta_{x_j p_i}$ according to (5.18) if and only if $\eta_{x_j Y} \cdot \theta_{Y p_i} < 0$. This holds if either factor i and j are both normal or both are inferior: Whenever a factor i is normal, i. e. $\eta_{x_i Y} > 0$, an increase in p_i will cause profit-maximizing output to decline, that is $\theta_{Y p_i} < 0$ (see e. g. CHAMBERS 1988:133). Correspondingly, $\theta_{Y p_i} > 0$ if factor i is inferior. Thus, a necessary condition for $\theta_{u_j p_i}$ being lower than $\eta_{x_j p_i}$, or, in other words, a necessary condition for two factors being net substitutes[1], but gross complements is that either both are normal or both are inferior.

[1] Inputs i and j are *gross* substitutes (complements) if $\theta_{u_j p_i} > 0$ ($\theta_{u_j p_i} < 0$), whereas they are *net* substitutes (complements) if $\eta_{x_j p_i} > 0$ ($\eta_{x_j p_i} < 0$).

Finally, in contrast to $\theta_{u_j p_i}$ and $\theta_{Y p_i}$, neither $\eta_{x_j p_i}$ nor $\eta_{x_j Y}$ can be derived directly from the underlying profit-maximization function. Yet, taking the logarithm of identity (5.17), differentiating the result logarithmically with respect to the output price p and using the chain rule again provides, first, an expression for $\eta_{x_j Y}$,

$$\theta_{u_j p} = \eta_{x_j Y} \cdot \theta_{Y p}, \tag{5.19}$$

where $\theta_{u_j p} := \frac{\partial \ln u_j}{\partial \ln p}$ denotes the output-price elasticity of the *unconditional* factor demand u_j and $\theta_{Y p} := \frac{\partial \ln Y}{\partial \ln p}$ represents the own-price elasticity of the output, which is nonnegative, since supply is nondecreasing in p. Second, substituting $\theta_{u_j p}/\theta_{Y p}$ from equation (5.19) for $\eta_{x_j Y}$ of equation (5.18) yields a formula for the calculation of the classical cross-price elasticity $\eta_{x_j p_i}$ (see e. g. CHAMBERS 1988:135):

$$\eta_{x_j p_i} = \theta_{u_j p_i} - \frac{\theta_{u_j p}}{\theta_{Y p}} \cdot \theta_{Y p_i}. \tag{5.20}$$

As an application, in the subsequent section the constant-output elasticity $\eta_{x_j p_i}$ is calculated via (5.20) for the specific flexible form of a translog profit function and is compared to the GES $\theta_{u_j p_i}$. In empirical applications, where factors are typically normal, $\eta_{x_j p_i}$ is an upper bound for $\theta_{u_j p_i}$.

GES Under Output Maximization for Constant Cost

When the objective function is output maximization under the assumption of constant cost, the MARSHALLian demand function $a_j(p_1, ..., p_n, C)$ provides the optimal demand for factor j, with $C(p_1, ..., p_n, Y)$ being the cost function. At the optimum, the HICKSian demand $x_j(p_1, ..., p_n, Y)$ for factor j is identical to the factor demand a_j conditioned on the minimal cost for producing the output Y:

$$x_j(p_1, ..., p_n, Y) = a_j(p_1, ..., p_n, C(p_1, ..., p_n, Y)). \tag{5.21}$$

This is another a special case of fundamental identity (5.15). By differentiating identity (5.21) with respect to output Y and price p_i, it is, respectively,

$$\frac{\partial \ln x_j}{\partial \ln Y} = \frac{\partial \ln a_j}{\partial \ln C} \cdot \frac{\partial \ln C}{\partial \ln Y}, \tag{5.22}$$

$$\frac{\partial \ln x_j}{\partial \ln p_i} = \frac{\partial \ln a_j}{\partial \ln p_i} + \frac{\partial \ln a_j}{\partial \ln C} \cdot \frac{\partial \ln C}{\partial \ln p_i}, \tag{5.23}$$

where by SHEPHARD's Lemma,

$$\frac{\partial \ln C}{\partial \ln p_i} = \frac{p_i \frac{\partial C}{\partial p_i}}{C} = \frac{p_i x_i}{C} = s_i, \tag{5.24}$$

which is the cost share of factor i. Combining (5.22) and (5.23) by eliminating $\frac{\partial \ln a_j}{\partial \ln C}$ and using (5.24) yields

$$\theta_{a_j p_i} := \frac{\partial \ln a_j}{\partial \ln p_i} = \frac{\partial \ln x_j}{\partial \ln p_i} - \frac{\frac{\partial \ln x_j}{\partial \ln Y}}{\frac{\partial \ln C}{\partial \ln Y}} \cdot s_i = \eta_{x_j p_i} - \frac{\frac{\partial \ln x_j}{\partial \ln Y}}{\frac{\partial \ln C}{\partial \ln Y}} \cdot s_i. \tag{5.25}$$

$\theta_{a_j p_i}$ is a second example of GES. On the basis of a specific cost function $C(p_1, ..., p_n, Y)$, all ingredients for the calculation of $\theta_{a_j p_i}$ via (5.25) are available. The classical cross-price elasticity $\eta_{x_j p_i}$ is different from $\theta_{a_j p_i}$ unless the cost share s_i of factor i is zero, that is, factor i is economically not relevant, or $\frac{\partial \ln x_j}{\partial \ln Y}$ vanishes, that is, j is a fixed factor which does not vary with output Y.

Since cost shares are always positive and cost functions are nondecreasing in output, that is $\frac{\partial \ln C}{\partial \ln Y} \geq 0$, $\theta_{a_j p_i}$ is lower than $\eta_{x_j p_i}$ if and only if factor j is normal, that is $\frac{\partial \ln x_j}{\partial \ln Y} > 0$. Similarly, $\theta_{a_i p_j}$ is lower than $\eta_{x_i p_j}$ if and only if factor i is normal. Hence, overall, in the scenario of output maximization for fixed cost, a necessary condition for two factors being net substitutes, but gross complements is that both are normal.

5.4 GES Within Translog Approaches

In this section, in a concrete empirical application to translog approaches, both concepts of GES described in the previous section are compared to constant-output cross-price elasticities.

GES Under Profit Maximization and Perfect Competition

For the translog profit function

$$\begin{aligned} \ln \pi(p_1, ..., p_n, p) \quad &= \quad \beta + \beta_Y \ln p + \frac{1}{2}\beta_{Yp}(\ln p)^2 + \sum_{i=1}^{n} \beta_i \ln p_i \\ &\quad + \frac{1}{2} \sum_{i,j=1}^{n} \beta_{ij} \ln p_j \ln p_i + \sum_{i=1}^{n} \beta_{Yi} \ln p_i \ln p, \end{aligned} \tag{5.26}$$

by HOTELLING's Lemma ($\frac{\partial\pi}{\partial p_i} = -u_i$, $\frac{\partial\pi}{\partial p} = Y$), the following equation system for $j = 1, ..., n$ can be obtained,

$$\xi_j \quad := \quad -\frac{u_j p_j}{\pi} = \frac{\frac{\partial\pi}{\partial p_j}p_j}{\pi} = \frac{\partial\ln\pi}{\partial\ln p_j} = \beta_j + \beta_{Yj}\ln p + \sum_{i=1}^{n}\beta_{ij}\ln p_i \tag{5.27}$$

$$\xi_Y \quad := \quad \frac{Yp}{\pi} = \frac{\frac{\partial\pi}{\partial p}p}{\pi} = \frac{\partial\ln\pi}{\partial\ln p} = \beta_Y + \beta_{Yp}\ln p + \sum_{j=1}^{n}\beta_{Yj}\ln p_j, \tag{5.28}$$

where ξ_Y denotes the inverse profit-revenue ratio. Due to $Y = \pi\xi_Y/p$ and $u_j = -\pi\xi_j/p_j$, it follows from (5.26) to (5.28) and the exogeneity of prices that

$$\theta_{Yp_i} \quad = \quad \frac{\partial\ln Y}{\partial\ln p_i} = \frac{\partial\ln\pi}{\partial\ln p_i} + \frac{\partial\ln\xi_Y}{\partial\ln p_i} - \frac{\partial\ln p}{\partial\ln p_i} = \xi_i + \frac{\beta_{Yi}}{\xi_Y}, \tag{5.29}$$

$$\theta_{Yp} \quad = \quad \frac{\partial\ln Y}{\partial\ln p} = \frac{\partial\ln\pi}{\partial\ln p} + \frac{\partial\ln\xi_Y}{\partial\ln p} - \frac{\partial\ln p}{\partial\ln p} = \xi_Y + \frac{\beta_{Yp}}{\xi_Y} - 1, \tag{5.30}$$

$$\theta_{u_jp_i} \quad = \quad \frac{\partial\ln u_j}{\partial\ln p_i} = \frac{\partial\ln\pi}{\partial\ln p_i} + \frac{\partial\ln(-\xi_j)}{\partial\ln p_i} - \frac{\partial\ln p_j}{\partial\ln p_i} = \xi_i + \frac{\beta_{ij}}{\xi_j}, \tag{5.31}$$

$$\theta_{u_jp} \quad = \quad \frac{\partial\ln u_j}{\partial\ln p} = \frac{\partial\ln\pi}{\partial\ln p} + \frac{\partial\ln(-\xi_j)}{\partial\ln p} - \frac{\partial\ln p_j}{\partial\ln p} = \xi_Y + \frac{\beta_{Yj}}{\xi_j}. \tag{5.32}$$

Substituting (5.29) to (5.32) into expression (5.20) yields the classical constant-output cross-price elasticity $\eta_{x_jp_i}$ for this translog profit approach (5.26):

$$\eta_{x_jp_i} = \theta_{u_jp_i} - \frac{\theta_{u_jp}}{\theta_{Yp}}\cdot\theta_{Yp_i} = \xi_i + \frac{\beta_{ij}}{\xi_j} - \frac{\xi_Y + \frac{\beta_{Yj}}{\xi_j}}{\xi_Y + \frac{\beta_{Yp}}{\xi_Y} - 1}(\xi_i + \frac{\beta_{Yi}}{\xi_Y}). \tag{5.33}$$

Obviously, the constant-output elasticity $\eta_{x_jp_i}$ and the GES $\theta_{u_jp_i}$ generally differ from each other. For a concrete application of the concept of $\theta_{u_jp_i}$, the GES under profit maximization, empirical firm or industry data including profits are necessary in order to estimate a profit-share system like (5.27) and (5.28). But, unfortunately, such data are not easily available.

The Classic Study of BERNDT and WOOD (1975)

As an alternative, we now use data for U. S. Manufacturing (1947-1971) provided by the seminal study by BW75 and estimate $\theta_{a_jp_i}$ from their translog cost function in order to appraise the corresponding values of $\theta_{u_jp_i}$: Because production factors are typically normal in empirical studies, according to the previous section, the GES $\theta_{u_jp_i}$ are always lower than classical cross-price elasticities $\eta_{x_jp_i}$, that is, $\eta_{x_jp_i}$ is an upper bound for $\theta_{u_j p_i}$, whereas, intuitively, $\theta_{a_j p_i}$ should be a lower bound, since the adjustment to an increase in price p_i given by $\theta_{a_j p_i}$ overstates the adjustment which would occur, if cost were free to adjust, that is, the adjustment given by $\theta_{u_j p_i}$.

On the basis of a homothetic translog cost function with constant returns to scale and symmetry imposed,

$$\ln C(p_K, p_L, p_M, p_E, Y) = \beta + \ln Y + \sum_{i=K,L}^{M,E} \beta_i \ln p_i + \frac{1}{2} \sum_{i,j=K,L}^{M,E} \beta_{ij} \ln p_j \ln p_i, \quad (5.34)$$

and the corresponding cost-share system

$$s_j = \frac{\partial \ln C}{\partial \ln p_j} = \beta_j + \sum_{i=K,L}^{M,E} \beta_{ij} \ln p_i \quad \text{for } j = K, L, M, E, \quad (5.35)$$

BW75 estimate classical constant-output cross-price elasticities. Because of $x_j = s_j C/p_j$, classical cross-price elasticities $\eta_{x_j p_i}$ may be derived from translog function (5.34) as follows:

$$\eta_{x_j p_i} = \frac{\partial \ln x_j}{\partial \ln p_i} = \frac{\partial \ln s_j}{\partial \ln p_i} + \frac{\partial \ln C}{\partial \ln p_i} - \frac{\partial \ln p_j}{\partial \ln p_i} = \frac{1}{s_j} \frac{\partial s_j}{\partial \ln p_i} + s_i = \frac{\beta_{ij}}{s_j} + s_i. \quad (5.36)$$

By reusing BW75's data, we estimate $\theta_{a_j p_i}$, the GES for output maximization under cost constraints, and compare our results to BW75's estimates of the classical cross-price elasticities. For BW75's translog function (5.34), for which $\frac{\partial \ln C}{\partial \ln Y}$ equals 1, it follows again from $x_j = s_j C/p_j$ that

$$\frac{\partial \ln x_j}{\partial \ln Y} = \frac{\partial \ln s_j}{\partial \ln Y} + \frac{\partial \ln C}{\partial \ln Y} - \frac{\partial \ln p_j}{\partial \ln Y} = 1, \quad (5.37)$$

because price p_j is exogenous and $\frac{\partial \ln s_j}{\partial \ln Y} = 0$ (see (5.35)). Hence, by (5.36), expression (5.25) of $\theta_{a_j p_i}$ simplifies for translog function (5.34) to

$$\theta_{a_j p_i} = \eta_{x_j p_i} - s_i = \frac{\beta_{ij}}{s_j}. \quad (5.38)$$

Comparing (5.36) and (5.38), the formulae for constant-ouput cross-price elasticity $\eta_{x_j p_i}$ and for the GES $\theta_{a_j p_i}$, reveals that net (pure) substitution effects due to increases in prices p_i on the input level of factor j are more positive than gross substitution effects, because cost shares s_i are always positive. Table 1 documents this theoretical fact empirically: BW75's estimates of classical constant-output elasticities $\eta_{x_j p_i}$ are considerably higher as our estimates of $\theta_{a_j p_i}$. Without exception our estimates of $\theta_{a_j p_i}$ are negative, not always significantly, though. According to our estimation results, nearly all factors are gross complements. That is, the impacts of factor price increases are almost uniformly negative with respect to the input levels of other factors.

Table 1: Comparison of BW75's estimates of $\eta_{x_i p_j}$ and our estimates of $\theta_{a_j p_i}$.

	BERNDT & WOOD (1975)					Our Estimates of GES				
	1947	1953	1959	1965	1971	1947	1953	1959	1965	1971
Cross-Price Elasticities $\eta_{x_i p_j}$						Generalized Cross-Price Elasticities $\theta_{a_j p_j}$				
Kp_E	-0.14	-0.15	-0.14	-0.14	-0.16	-0.18 (0.06)	-0.20 (0.06)	-0.18 (0.06)	-0.18 (0.06)	-0.21 (0.07)
Lp_E	0.03	0.03	0.03	0.03	0.03	-0.02 (0.01)	-0.02 (0.01)	-0.02 (0.01)	-0.02 (0.01)	-0.02 (0.01)
Mp_E	0.03	0.04	0.03	0.03	0.03	-0.02 (0.01)	-0.02 (0.01)	-0.02 (0.01)	-0.02 (0.01)	-0.02 (0.01)
Ep_K	-0.17	-0.17	-0.18	-0.18	-0.17	-0.23 (0.08)	-0.22 (0.07)	-0.23 (0.08)	-0.24 (0.08)	-0.22 (0.07)
Lp_K	0.06	0.05	0.06	0.06	0.05	-0.01 (0.06)	-0.01 (0.06)	-0.01 (0.06)	-0.01 (0.06)	-0.01 (0.06)
Mp_K	0.03	0.03	0.03	0.03	0.02	-0.03 (0.02)	-0.03 (0.02)	-0.03 (0.02)	-0.03 (0.02)	-0.03 (0.02)
Ep_L	0.16	0.17	0.18	0.18	0.20	-0.10 (0.07)	-0.10 (0.06)	-0.10 (0.06)	-0.10 (0.07)	-0.10 (0.06)
Kp_L	0.26	0.27	0.28	0.29	0.30	-0.01 (0.06)	-0.01 (0.06)	-0.01 (0.06)	-0.01 (0.06)	-0.01 (0.06)
Mp_L	0.15	0.16	0.16	0.17	0.18	-0.11 (0.02)	-0.11 (0.02)	-0.11 (0.02)	-0.11 (0.02)	-0.11 (0.02)
Ep_M	0.49	0.49	0.47	0.46	0.46	-0.09 (0.19)	-0.09 (0.18)	-0.09 (0.20)	-0.10 (0.20)	-0.09 (0.19)
Kp_M	0.37	0.34	0.35	0.35	0.30	-0.34 (0.17)	-0.37 (0.19)	-0.34 (0.18)	-0.33 (0.17)	-0.39 (0.20)
Lp_M	0.37	0.37	0.37	0.37	0.37	-0.28 (0.04)	-0.26 (0.04)	-0.26 (0.04)	-0.25 (0.04)	-0.24 (0.04)

Note: Standard errors are given in parentheses. Under the assumption that cost shares are constant and equal to the means of their estimated values, approximate estimates of the standard errors of $\eta_{x_i p_j}$ are, asymptotically, $var(\hat{\eta}_{x_i p_j}) = \hat{\beta}_{ij}/\hat{s}_i^2$ and $var(\hat{\eta}_{x_i p_i}) = \hat{\beta}_{ii}/\hat{s}_i^2$ (PINDYCK 1979:171).

This evidence suggests that the detrimental effect of factor-price increases on output leads in turn to generally lower input requirements for all factors. So, taking output effects into account by using the GES $\theta_{a_j p_i}$ results in drastically different estimates for the effects of price changes of factor i on the use of factor j than in the case where output effects are ignored, that is, for classical cross-price elasticities $\eta_{x_i p_j}$.

5.5 Conclusion

HICKS' old venerable σ is the dominating elasticity of substitution for the two-factor case. Since σ serves as a conceptual orientation for its prominent generalizations AES, HAES, SES, MES, and classical cross-price elasticities, the search for a superior concept measuring the elasticity of substitution is the prevailing idea in the literature of multifactor substitution. This paper however reiterates FUSS, MCFADDEN and MUNDLAK's (1978:241) conclusion and argues that an omnipotent measure can not exist at all: "There is no unique natural generalization of the two factor definiton ... We conclude that the selection of a particular definition should depend on the question asked".

Without any doubt, in theoretical contexts, MES, the concept suggested by BLACKORBY and RUSSEL (1989) in their seminal article is superior to all other substitution elasticities conceived to measure the curvature of isoquants, that is, pure substitution effects. In particular, MES is superior to AES, the most criticized, but still most employed measure of substitution. Rather than AES, MES "does preserve the salient characteristics of the original *Hicks*ian concept ... [and] is a measure of curvature, or ease of substitution" (BLACKORBY and RUSSEL 1989:883). Yet, while solely measuring pure substitution effects, which is in line with HICKS' σ, AES, HAES, SES, MES and classical cross-price elasticities ignore output effects.

In empirical contexts, the idea of measuring pure substitution effects – notwithstanding its fundamental role in production theory – has to be buried: Because it is most likely that output shrinks when the price of a factor such as energy rises, elasticities capturing gross substitution effects, that is, pure substitution and output effects, are to be prefered in any empirical study. Building on classical constant-output cross-price elasticities, the common basis for AES, HAES, SES and MES, this paper thus suggests an *empirical* measure of substitution, termed generalized cross-price elasticity, which explicitly takes output effects into account.

The argument that an omnipotent elasticity of substitution cannot exist is further supported by the fact that the concrete way in which we have to generalize classical cross-price elasticities depends on the economic experiment to be described: Estimating factor substitutability for example for profit-maximizing firms under perfect competition or for an output-maximizing industry under constant-cost constraints requires different generalizations. Already (MUNDLAK 1968:234) was it who formulated the obituary of an omnipotent substitution elasticity, though: "For different problems, different demand functions may be pertinent and consequently different measures of substitution."

References

ALLEN RGD, HICKS JR (1934): A Reconsideration of The Theory of Value, II. *Economica*, Vol. 1:196-219.

ALLEN RGD (1938): *Mathematical Analysis for Economists*. MacMillan & Co., Ltd, London.

ANDERSON RG (1981): Conditional Factor Demand Functions. In: BERNDT ER and FIELD BC: *Modeling and Measuring Natural Resource Substitution*. MIT Press, Cambridge, MA, pp 119-144.

APOSTOLAKIS BE (1987): The Role of Energy in Production Functions for Southern European Economies. *Energy*, Vol. 12, No. 7:531-541.

APOSTOLAKIS BE (1990): Energy-Capital-Substitutability/Complementarity. *Energy Economics*, Vol. 12, No. 1:48-58.

ARROW KJ, CHENERY HB, MINHAS B, SOLOW RM (1961): Capital-Labor Substitution and Economic Efficiency. *Review of Economics and Statistics*, Vol. 43:225-50.

BERNDT ER, CHRISTENSEN LR (1973): The Internal Structure of Functional Relationships: Separability, Substitution and Aggregation. *Review of Economic Studies*, Vol. 40:403-10.

BERNDT ER, WOOD DO (1975): Technology, Prices, and the Derived Demand for Energy. *Review of Economics and Statistics*, Vol. 57:259-68.

BERNDT ER, WOOD DO (1979): Engineering and Econometric Interpretations of

Energy-Capital Complementarity. *American Economic Review*, Vol. 69, No. 3:342-54.

BERNDT ER (1991): *The Practice of Econometrics: Classic and Contemporary*. Addison-Wesley, New York.

BLACKORBY C, RUSSELL R (1975): *The Partial Elasticity of Substitution*. Discussion Paper No. 75-1, Economics Department, University of California, San Diego.

BLACKORBY C, PRIMONT D, RUSSELL R (1977): On Testing Separability Restrictions with Flexible Functional Forms. *Journal of Econometrics*, Vol. 5:195-209.

BLACKORBY C, PRIMONT D, RUSSELL R (1978): *Duality, Separability, and Functional Structure: Theory and Economic Applications.* In: Dynamic Economics: Theory and Applicants. North-Holland, New York.

BLACKORBY C, RUSSELL R (1981): The Morishima Elasticity of Substitution; Symmetry, Constancy, Separability, and its Relationship to the Hicks and Allen Elasticities. *Review of Economic Studies*, Vol. 158: 147-158.

BLACKORBY C, RUSSELL R (1989): Will the Real Elasticity of Substitution Please Stand Up? A Comparison of the Allen/Uzawa and Morishima Elasticities. *American Economic Review*, Vol. 79: 882-888.

BLACKORBY C, DAVIDSON R, SCHWORM W (1991): Implicit Separability: Characterisation and Implications for Consumer Demands. *Journal of Economic Theory*, Vol. 55:364-399.

BLACKORBY C, RUSSELL R (1994): The Conjunction of Direct and Indirect Separability. *Journal of Economic Theory*, Vol. 62:480-498.

CHAMBERS RG (1988): *Applied Production Analysis: A Dual Approach.* Cambridge University Press, Cambridge.

CHRISTENSEN LR, JORGENSEN DW, LAU LJ (1971): Conjugate Duality and the Transcendental Production Function. *Econometrica*, Vol. 39:255-256.

CHIANG AC (1984): *Fundamental Methods of Mathematical Economics*. McGraw-Hill, New York.

CRAFTS N, TONIOLO G (1996): *Economic Growth in Europe since 1945*. Cambridge University Press, New York.

DARGAY JM (1983): The Demand for Energy in Swedish Manufacturing Industries. *Scandinavian Journal of Economics*, Vol. 85, No. 1:37-51.

DAVIS GC, SHUMWAY CR (1996): To Tell the Truth about Interpreting the Morishima Elasticity of Substitution. *Canadian Journal of Agricultural Economics*, Vol. 44:173-182.

DENNY M, FUSS M (1977): The Use of Approximation Analysis to Test for Separability and the Existence of Consistent Aggregates. *American Economic Review*, Vol. 67: 404-18.

DIEWERT WE, WALES TJ (1995): Flexible Functional Forms and Tests of Homogeneous Separability. *Journal of Econometrics*, Vol. 67:259-302.

FALK M, KOEBEL B (1999): *Curvature Conditions and Substitution Patterns among Capital, Energy, Materials and Heterogeneous Labour*. Discussion paper No. 99-06, Centre for European Economic Research.

FIELD BC, GREBENSTEIN C (1980): Capital-Energy Substitution in U. S. Manufacturing. *Review of Economics and Statistics*, Vol. 62, No. 2:207-212.

FLAIG G, ROTTMANN H (1998): *Faktorpreise, technischer Fortschritt und Beschäftigung. Eine empirische Analyse für das westdeutsche Verarbeitende Gewerbe*. Ifo discussion papers No. 59, ifo-Institut für Wirtschaftsforschung, München.

FRENGER P (1992): *Properties of the Directional Shadow Elasticity of Substitution*. Discussion paper No. 14-147, Department of Economics. University of Oslo.

FRENGER P (1994): *The Directional Direct Elasticity of Substitution and Duality*. Discussion paper No. 14-186, Department of Economics. University of Oslo.

FRISCH R (1935): The Principle of Substitution: An Example of Its Application in the Chocolate Industry. *Nordisk Tidsskrift for Teknisk Okonomi*, 1:1, 12 - 27.

FRONDEL M (1999): *Interpreting Allen, Morishima and Technical Elasticities of Substitution: A Theoretical and Empirical Comparison*. Discussion paper No. 292, Department of Economics. University of Heidelberg.

FRONDEL M, SCHMIDT CM (2000): *The Capital-Energy Controversy: A Reconciliation*. Discussion paper No. 318, Department of Economics. University of Heidelberg.

FRONDEL M, SCHMIDT CM (2000b): *Facing the Truth about Separability: Nothing Works Without Energy*. Discussion paper No. 331, Department of Economics. University of Heidelberg.

FRONDEL M, SCHMIDT CM (2000c): *The Real Elasticity of Substitution – An Obituary*. Discussion paper No. 341, Department of Economics. University of Heidelberg.

FUSS MA (1977): The Demand for Energy in Canadian Manufacturing. *Journal of Econometrics*, Vol. 5, No. 1:89-116.

FUSS M, MCFADDEN D, MUNDLAK Y (1978): A Survey of Functional Forms in the Economic Analyses of Production. In: *Production Economics: A Dual Approach to Theory and Applications.* FUSS M, MCFADDEN D. (editors). North-Holland, Vol. 1:219-68, Amsterdam.

FUSS M, MCFADDEN D (1978): *Production Economics: A Dual Approach to Theory and Applications, Vol. 1: The Theory of Production, Vol. 2: Applications of the Theory of Production*. North-Holland, New York.

GOLDMANN SM, UZAWA H (1964): A Note on Separability in Demand Analysis. *Econometrica*, Vol. 32:387-98.

GREENE WH (1993): *Econometric Analysis*. Second edition. MacMillan, New York.

GRIFFIN JM, GREGORY PR (1976): An Intercountry Translog Model of Energy Substitution Responses. *The American Economic Review*, Vol. 66:845-858.

GRIFFIN JM (1981): The Energy-Capital Complementarity Controversy: A Progress Report on Reconcilitation Attempts. In BERNDT ER, FIELD BC (editors): *Modeling and Measuring Natural Resource Substitution*, pp 70-80. MIT Press, Cambridge, MA.

HALVORSON R, FORD J (1979): Substitution among Energy, Capital and Labor Inputs in US Manufacturing. In: PINDYCK R: *Advances in the Economics of Energy and Resources*, Vol. 1:51-75. JAI Press, Greenwich, CN.

HAMERMESH DS (1993): *Labor Demand*. Princeton University Press, New Jersey.

HEATHFIELD DF, WIBE S (1987): *An Introduction to Cost and Production Functions*. MacMillan & Co., Ltd, London.

HICKS JR (1932): *Theory of Wages*. MacMillan, London.

HUDSON EA, JORGENSON DW (1974): U. S. Energy Policy and Economic Growth, 1975-2000. *The Bell Journal of Economics and Management Science*, Vol. 5:461-514.

IQBAL M (1986): Substitution of Labour, Capital and Energy in the Manufacturing Sector of Paktistan. *Empirical Economics*, Vol. 11:81-95.

JORGENSON DW, GOLLOP FM, FRAUMENI B (1987): *Productivity and U. S. Economic Growth.* In: Contributions to Economic Analysis. North-Holland, New York.

KIM BC, LABYS WC (1988): *Application of the Translog Model of Energy Substitution to Developing Countries. Energy Economics*, Vol. 10:313-323.

KINTIS A, PANAS EE (1989): The Capital-Energy Controversy: Further Results. *Energy Economics*, Vol. 11:201-212.

KUGLER P, MÜLLER U, SHELDON G (1989): Non-Neutral Technical Change, Capital, White-Collar and Blue-Collar Labor. *Economics Letters*, Vol. 31:91-94.

MAGNUS JR (1979): Substitution Between Energy and Non-Energy Inputs in the Netherlands. *International Economic Review*, Vol. 20, No. 2:465-483.

McFadden D (1963): Constant Elasticity of Substitution Production Functions. *Review of Economic Studies*, Vol. 30:73-83.

Morishima M (1967): A Few Suggestions on the Theory of Elasticity. *Keizai Hyoron (Economic Review)*, Vol. 16:145-150.

Morrison Paul CJ, Siegel DS (1999): Scale Economies and Industry Agglomeration Externalities: A Dynamic Cost Function Approach. *American Economic Review*, Vol. 89:272-290.

Mundlak Y (1968): Elasticities of Substitution and the Theory of Derived Demand. *Review of Economic Studies*, Vol. 35:225-236.

Mundlak Y (1996): Production Function Estimation: Reviving the Primal. *Econometrica*, Vol. 64, No. 2:431-438.

Norsworthy JR, Malmquist DH (1983): Input Measurement and Productivity Growth in Japanese and U.S. Manufacturing *American Economic Review*, Vol. 73:947-967.

Özataly S, Grubaugh S, Long TV (1979): Energy Substitution and National Energy Policy. *American Economic Review*, Vol. 69, No. 2:369-371.

Pindyck RS (1979): Interfuel Substitution and the Industrial Demand for Energy: An Intertemporal Comparison. *Review of Economics and Statistics*, Vol. 61:169-179.

Ramaiah KC, Dalal AJ (1996): Functional Separability and the Existence of Consistent Aggregates in U. S. Manufacturing: Comment. *International Economic Review*, Vol. 37:253-258.

Rutner D (1984): *Faktorsubstitution in den Produktionssektoren der Bundesrepublik Deutschland: Eine ökonometrische Analyse des Translog-Modells.* Schriften zur angewandten Ökonometrie, Band 12.

Sato R, Koizumi T (1973:49): On the Elasticities of Substitution and Complementarity. *Oxford Economic Papers*, Vol. 25:44-56.

SILBERBERG E (1990): *The Structure of Economics: A Mathematical Analysis*. Second edition. McGraw-Hill, New York.

STARK E (1988): *Faktoreinsatzverhalten im Verarbeitenden Gewerbe: Eine sektorale Analyse unter Verwendung eines Translog-Modells.* DIW-Vierteljahresheft 1/2-88.

SOLOW JL (1987): The Capital-Energy Complementarity Debate Revisited. *American Economic Review*, Vol. 77:605-614.

TAKAYAMA A (1985): *Mathematical Economics*. Second edition. Cambridge University Press, Cambridge, MA.

TURNOVSKY M, FOLIE M, ULPH A (1982): Factor Substitutability in Australian Manufacturing with Emphasis on Energy Inputs. *The Economic Record*, Vol. 58:61-72.

THOMPSON H (1997): Substitution Elasticities with Many Inputs. *Applied Mathematical Letters*, Vol. 10, No. 3:123-127.

THOMPSON P, TAYLOR TG (1995): The Capital-Energy Substitutability Debate: A New Look. *Review of Economics and Statistics*, Vol. 77:565-569.

UNGER R (1986): *Messung der totalen Faktorproduktivität für 28 Sektoren der Bundesrepublik Deutschland, 1960 - 1981.* Verlag Peter Lang, Frankfurt am Main.

UZAWA H (1962): Production Functions with Constant Elasticities of Substitution. *Review of Economic Studies*, Vol. 29:291-9.

VARIAN HR (1992): *Microeconomic Analysis*. Third edition. Norton, New York.

WILLIAMS M, LAUMAS P (1981): The Relation between Energy and Non-Energy Inputs in India's Manufacturing Industries. *The Journal of Industrial Economics*, Vol. 30, No. 2:113-122.

YUHN K (1991): Functional Separability and the Existence of Consistent Aggregates in U. S. Manufacturing. *International Economic Review*, Vol. 32:229-250.

Acknowledgements

It is a pleasure to express my gratitude to my supervisor, Professor CHRISTOPH M. SCHMIDT, for his careful reading of all drafts of these chapters. I am highly indebted to him for his detailed and invaluable comments. Furthermore, I am grateful to my colleagues BORIS AUGURZKY, MARTIN BIEWEN, MICHAEL FERTIG, JOCHEN KLUVE, and RALPH WUERTHWEIN for their helpful feed-back to single chapters. I have benefitted very much from the productive, co-operative and cheerful atmosphere and the intellectual capacity of Professor SCHMIDT's team.