# Heidelberger Texte zur Mathematikgeschichte

# Gottfried Köthe, 1905–1989

### Joachim Weidmann

Dedicated to the memory of Professor Gottfried Köthe

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Gottfried Köthe Photo: Heidelberg University Archive

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Gottfried Maria Hugo Köthe, son of an engineer and salesman, was born on the 25th of December 1905 at Graz/Austria. After attending the «Volksschule» and the «Realgymnasium» he got his «Abitur» in 1923.

In the winter term 1923/24 he began to study Chemistry and Philosophy at the Philosophical Department of the Graz University. Just by accident during his vacations at the Wörthersee he met the philosopher Alfred Kastil from Innsbruck who noticed his interest for Philosophy. Köthe decided to change for his third term to Innsbruck in order to study Philosophy with Kastil. In 1960, in connection with his election as a member of the «Heidelberger Akademie» he reported on his studies of Franz Brentano's Philosophy: «Daß ich Mathematiker wurde, ist beinahe ein Zufall. Auf der Schule hatte ich zwei Interessen, denen ich ziemlich intensiv nachging. Das eine war die Chemie, das andere die Philosophie. Ich begann mit dem Studium der Chemie. Durch die Begegnung mit dem Innsbrucker Philosophen Kastil aus der Schule von Franz Brentano trat wieder die Philosophie in den Vordergrund. Da mich vor allem die Erkenntnistheorie und Logik, speziell die Paradoxien der Mengenlehre fesselten, schien es mir richtig, die Chemie aufzugeben und an ihrer Stelle Mathematik neben Philosophie zu studieren. Die Mathematik hat mich dann doch stärker angezogen als die Philosophie; ich fand in ihren Schlußweisen die Präzision und Sicherheit, die ich in der Philosophie gesucht, aber nicht gefunden hatte. Geblieben ist mir jedoch ein stetes Interesse an den Grenzfragen von Mathematik und Philosophie»<sup>1</sup>.

At the celebration colloquium in honour of his 80th birthday during the winter term 1985/86 he also remembered this period of his life and closed his short speech with the phrase «So ist also Franz Brentano, ein Mann aus der Frankfurter Umgebung, schuld daran, daß ich Mathematiker wurde»<sup>2</sup>.

After 4 years of studies, at an age of less than 22 years, Köthe received his doctor degree in October 1927. The title of this thesis was «Contributions to Finsler's foundation of set theory». This thesis was never published because of «gewisser Schwierigkeiten, die sich ergaben und die mit der Unsicherheit der Finslerschen Grundlagen zusammenhängen»<sup>3</sup>, as Emmy Noether wrote later in her letter of recommendation to Otto Toeplitz.

Köthe spent the winter term 1927/28 at Zürich, where he studied with Paul

<sup>&</sup>lt;sup>1</sup>«I became a mathematician almost by chance. At school I had two interests which I pursued rather intensively, one was chemistry, the other philosophy. At university I began with the study of chemistry. A meeting with Innsbruck philosopher Alfred Kastil, of the school of Franz Brentano, brought philosophy again into the foreground Since I was fascinated by epistemology and logic, in particular the paradoxes of set theory, it seemed best to give up chemistry and to study mathematics together with philosophy instead. It turned out then that mathematics attracted me more strongly than philosophy; in mathematical reasoning I found the precision and certainty which I had sought in philosophy, but in the end failed to find there. Nevertheless I have always retained an interest for the questions which lie at the borderline between mathematics and philosophy≫. (Cf. Jahresheft der Heidelberger Akademie der Wissenschaften 1960/61).

 $<sup>^2</sup>$ «So, in the end, Franz Brentano, a man from the Frankfurt area, is to blame that I became a mathematician».

 $<sup>^3 {&</sup>lt;\!\!\!<} {\rm certain}$  difficulties which arose, having to do with the uncertainties in Finsler's axiomatic System».

Finsler, Rudolf Fueter and Andreas Speiser, and among others attended a course on «Quantum Mechanics and Group Theory», given by Hermann Weyl. For the summer term 1928 he changed to Göttingen, supported by a fellowship from the «Notgemeinschaft Deutscher Wissenschaft» (later on called Deutsche Forschungsgemeinschaft, DFG). There he attended courses on Non-Commutative Algebra, given by Emmy Noether, and on Algebraic Numbers, given by B.L. van der Waerden. His field of interest changed to Algebra. But, as Emmy Noether writes in her above mentioned letter to Otto Toeplitz, «um eine pekuniäre Grundlage zu haben»<sup>4</sup>, he also registered for 7 courses on insurance affairs; one of these courses was Insurance Calculus, given by Paul Bernays.

Upon recommendation by Emmy Noether, in 1929 Köthe accepted the position of an assistant with Otto Toeplitz in Bonn for one year. Some years earlier, Toeplitz had been working on the theory of equations with infinitely many variables together with Ernst Hellinger. Now he had been searching for an algebraist in order to join him in continuing this work. This was the beginning of many years of intensive and extremely fruitful collaboration, which certainly had an important impact on Köthe's further work and carreer.

In 1930 Köthe became an assistant with Heinrich Behnke in Münster. There he got his habilitation in 1931 with a still purely algebraic thesis «Skew fields of infinite rank over the center». In connection with his habilitation Emmy Noether wrote to Toeplitz in 1930: «Ich würde mich sehr freuen, wenn Köthe sich bei Ihnen habilitieren könnte. Ich halte ihn für sehr begabt, und er arbeitet intensiv, trotz aller scheinbaren Faulheit. Mich hat das keinen Augenblick gestört oder getäuscht, aber Courant konnte über die (the quotation marks are also in the original) 'österreichische Schlampigkeit' nicht hinweg kommen. Insofern wird er auch nicht zu denjenigen gehören, die Courant - der ja eigentlich immer gefragt wird - in allererster Linie bei Berufungen empfiehlt»<sup>5</sup>.

Although Köthe was in Münster now, his collaboration with Toeplitz continued until 1939, when Toeplitz had to leave Germany (he went to Palestine where he died 1940). A large number of letters which have been exchanged between Toeplitz and Köthe witnesses for the intensity of this collaboration. (Actually Köthe lost all his letters during the war; but after the death of Toeplitz he got his collection; it seems the collection of Köthe's handwritten letters survived rather complete; a number of Toeplitz's letters exist as carbon copies.) These letters mainly present discussions of a variety of mathematical problems. But in the course of time they begin to contain more and more comments about actual political happenings, especially in connection with the fate of jewish mathematicians and the activities of sympathizers of the Nazi regime.

During some periods letters have been exchanged daily. For this reason, beside

<sup>&</sup>lt;sup>4</sup>«in order to have a pecuniarity basis»

<sup>&</sup>lt;sup>5</sup>«I would be very happy if Köthe could do his habilitation with you. I regard him as *very* gifted, and he works *intensively*, for all his apparent laziness. Personally, this hasn't bothered or disappointed me a bit, but Courant just couldn't get over his «Austrian slovenliness» (the quotation marks are also in the original). Thus he's probably not high on the list of those whom Courant, who is after all always asked, recommends for university positions».

the date there was also marked the day time when the letter was written. How fast letters have been delivered in those days, long before the age of electronic mail, can be imagined from a letter from Toeplitz in Bonn to Köthe in Münster dated April 8,1933: «Daher bitte ich Sie also, so schnell Sie mögen herzukommen. Falls Sie mir telephonieren wollen: ich bin vermutlich den ganzen Tag heute zu Hause, von 19 Uhr ab - eher können Sie diese Zeilen wohl nicht haben. Wenn Sie aber kommen, ist das gar nicht nötig, dann genügt eine einfache Postkarte, die ich morgen früh habe»  $^6$ .

About his first work together with Toeplitz, Köthe writes in his above mentioned presentation to the Heidelberger Akademie: «Wir haben dann gemeinsam die Theorie der vollkommenen Räume entwickelt, ein Gegenstück zur Theorie der Banachräume. Beide Theorien sind nach dem zweiten Weltkrieg in der allgemeinen Theorie der topologischen linearen Räume aufgegangen, die durch die französischen Mathematiker des Bourbakikreises ihre endgültige Gestalt erhielt, nachdem die allgemeine Topologie als Hilfsmittel genügend weit entwickelt war. Seit diesen ersten Arbeiten mit Toeplitz bin ich im wesentlichen diesem Problemkreis treu geblieben, der auch als das Gebiet der Funktionalanalysis bezeichnet wird und der methodisch als Durchdringung und Fortentwicklung der klassischen Analysis zu topologisch-algebraischen Begriffen gekennzeichnet werden kann»<sup>7</sup>.

In 1937 Köthe became an «außerordentlicher Professor» in Münster. In 1940 he received a call from the University of Gießen where he became again an «außerordentlicher Professor» in 1941 and an «ordentlicher Professor» (full professor) in 1943.

Like many other mathematicians in 1940 Köthe was drafted by the Foreign Office as a scientific assistant for the solution of decoding problems. I leamed from Köthe's widow that beside mathematicians also classical philologists, as she is one, had been considered to be intellectually qualified and suitably trained for this work; in connection with this function they met the first time.

In 1946, the year of reopening of this university, Köthe followed a call from the Johannes Gutenberg-Universität at Mainz. During many years as a Director of the Mathematical Institute he profoundly determined its development. From 1948 to 1950 he was Dean of the Science Department, from 1954 to 1956 he was Rector of the University. A memorial article in the Journal «JOGU» of the Johannes Gutenberg-University at Mainz shortly after Köthe's death in June 1989 says about this period: «In dieser Zeit führte er die Hochschule aus einer ihrer

<sup>&</sup>lt;sup>6</sup>«So please come as soon as you can. If you want to give me a phone call, I'll probably be home the whole of today, from 7 P.M. on; you can't very well get this before then. But if you do come, it's not necessary to call: just drop me a postcard and I'll get it by tomorrow morning».

<sup>&</sup>lt;sup>7</sup>«Together we developed the theory of perfect spaces, a counterpart to the theory of Banach spaces. After Second World War both theories were incorporated into the theory of linear topological spaces, which attained definitive form in the hand of French mathematicians of the Bourbaki school, after the apparatus of general topology had been sufficiently developed. Since these first papers with Toeplitz, I have remained more or less failhful to this area of mathematics, also known as functional analysis, which may be characterized as the penetration and further development of classical analysis with the help of topological-algebraical concepts».

kritischen Phasen heraus und es gelang ihm, ihre Entwicklung zu konsolidieren»<sup>8</sup>.

In fall 1957 Köthe followed a call from the Ruperto Carola University Heidelberg to take a chair for Applied Mathematics; at the same time he became a Director of the newly installed «Institut für Angewandte Mathematik». During the academic year 1960/61, just after finishing the first volume of the «Topologische lineare Räume», he was Rector of the Heidelberg University. It was about that time, during my fifth or sixth term, when I met Köthe first. In the following years I had the pleasure to attend his inspiring lectures on «Hilbert Space Theory», «Partial Differential Equations», «Game Theory» and especially about the field of his main interest «Topological Vector Spaces». The speech as a rector at the occasion of the immatriculation ceremony and the 574th anniversary of the Ruprecht-Karl-Universität in 1960 had the title «Game Theory, a New Branch of Applied Mathematics». He succeeded to produce a remarkably wide public resonance to a mathematical subject. Looking through his papers after his death I found out that actually the range of topics of his lectures during 40 years of teaching covered almost all fields of mathematics.

The Johann Wolfgang Goethe-Universität at Frankfurt offered Köthe a chair for Mathematics (Applied Mathematics). He accepted this call in the summer of 1965. After 6 years of teaching in Frankfurt he retired at 31st of March 1971 at the end of the term, when he reached the age of 65. The main reason for retiring at the earliest possible time was his wish to complete the second volume of the «Topologische Lineare Räume»; but it turned out that it was not before 1979 when this volume appeared in print. (The first volume was published in English translation «Topological Vector Spaces» in 1969; the second volume was published only in English).

The success of Köthe's teaching may be estimated by considering the large number of diplom and doctorial theses advised by him. As far as I could find out he was the Supervisor of one doctorial thesis in both, Münster and Mainz, five in Heidelberg and of fifteen during the short period of six years in Frankfurt. Looking through the list of doctorates, one notices that his students, originating from many different countries, now work in a large variety of very different positions all over the world. Many of them, mainly those who work at universities now, had a regular correspondence with him until his death. A large number of them attended the Memorial Colloquium, organized by the Fachbereich Mathematik of the Frankfurt University on October 28,1989.

Beside his engagement at different universities, Köthe also served the mathematical community in other important positions:

- 1957/58 he was chairman of the «Deutsche Mathematikervereinigung (DMV)»,
- 1959 he was one of the 15 leading mathematicians founding the «Gesell-schaft für Mathematische Forschung», which was created in order to have

 $<sup>^8</sup>$ «During this period he led the university out of a critical phase, and succeded in consolidating its development».

- a legal basement to run the «Mathematisches Forschungsinstitut Oberwolfach»,
- from 1959 to 1963 he was chairman of the «Fachausschuß» for mathematics in the German Science Foundation,
- in many places he acted as an editor or coeditor, e.g. for the *Mathematische Annalen* from 1959 to 1971, for the *Zentralblatt für Mathematik* from 1958 to 1988 and for *Mathematische Leitfäden* with the Teubner-Verlag.

For his scientific merits he was honoured by many institutions:

- As mentioned above, in 1960 he became a member of the Heidelberger Akademie der Wissenschaften. He expressed his thanks to the Akademie with the following words: «Meine Neugierde, zu erfahren, was in den anderen Disziplinen vorgeht, stand immer im Widerstreben mit dem Alleinherrschaftsanspruch der Mathematik. Ich bin sehr froh darüber, daß diese Neugierde damit auf eine legale Basis gestellt ist»<sup>9</sup>.
- 1961 he became a «Commandeur dans l'Ordre des Palmes Academiques»,
- 1968 he had been elected to be a member of the «Deutsche Akademie der Naturforscher Leopoldina» at Halle,
- the science departments of the universities of Montpellier (1965), Münster (1980), Mainz (1981) and Saarbrücken (1981) honoured him with «Doctor honoris causa».

Gottfried Köthe died, completely unexpected at that time, on April 30, 1989, scientifically active up to his last days: He was still acting as one of the editors of the *Mathematische Leitfäden*, he still had an extensive correspondence with a large number of mathematicians all over the world, and he was still publishing scientific papers. Let me only recall his thoroughly article about the Polish mathematician Stanislaw Mazur, which appeared in the *Mathematsche Annalen* in 1987; during the preparation of the manuscript he spent many hours in our library, and he wrote many letters in order to get additional Information. His last work «On complemented subspaces of convergence free spaces» will appear in this volume.

When I followed Köthe on his chair in 1971, I was hoping that he would continue his teaching to some extent. But to my regret he never again gave a regular course, and kept distance to the administrational buisiness of the Mathematical Institute; I think that this was, at least partially, his reaction to the turbulent years in the late sixties.

But anyhow, he watched the development of the Fachbereich Mathematik (which was founded in the summer of 1971 as one of the eight parts into which

 $<sup>^9 \</sup>sim$  My curiosity to find out what was going on in other areas of knowledge was always in conflict with the demands of mathematics for exclusive supremacy. I'm very glad to have my curiosity now put on a legal basis».

the Faculty of Natural Sciences was divided) with great interest. Quite regularly he came to the institute discussing with us all kinds of problems, scientific, administrative, political and private ones; we all enjoyed these discussions since he always mixed great earnestness with a good part of humour, and we gained a lot from his rich experience in science and administration. In the years 1976 through 1978, when I was Dean of the Fachbereich Mathematik, he often helped me to consider seemingly large problems much easier. I am quite sure that there are many colleagues who made similar experiences. We all will keep him an honourable memory.

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- [14] Hans-Jürgen Moschke, Elliptische Differentialoperatoren in unbeschränkten Gebieten, 26.06.1963<sup>10</sup>.
- [15] Dieter Keim, Über zwei Klassen topologischer Vektorverbände: Die Ordnungstopologie und ordnungstonnelierte Topologien, 19.02.1971.
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- [17] Peter Spuler, Lambda-Nukleare Räume, 16.12.1970.
- [18] Dieter Wich, Der Holmgrensche Eindeutigkeitssatz in der Theorie der Hyperfunktionen, 23.10.1970.
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- [21] Robert Wagner, Surjektivitätskriterien für lineare Abbildungen zwischen lokalkonvexen Räumen, 16.07 1971.
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- J. Weidmann FB Mathematik JWG - Universität Robert Mayer-Str. 6–10 D-6000 Frankfurt am Main 11 Germany

<sup>&</sup>lt;sup>10</sup>i.e. 1968 [G. Dörflinger]