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Controlled derivation of semiclassical transport equations for electroweak baryogenesis

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Ableitung semiklassischer Transportgleichungen für elektroschwache Baryogenese

Zusammenfassung

In dieser Arbeit untersuchen wir einen wesentlichen Baustein für die Beschreibung der Baryogenese beim elektroschwachen Phasenübergang: wir führen eine strenge Herleitung semiklassischer Transportgleichungen für CP-verletzende Flüsse in einem System aus skalaren und fermionischen Teilchen in Anwesenheit eines langsam veränderlichen Hintergrundfeldes durch. Diese Situation liegt bei einem elektroschwachen Phasenübergang erster Ordnung vor, wo der Vakuumerwartungswert des Higgs-Feldes, der sich innerhalb der Phasengrenze ändert, die Rolle des Hintergrundfeldes übernimmt. Ausgehend von den exakten Bewegungsgleichungen für die Wigner Funktionen im Schwinger-Keldysh-Formalismus führen wir eine Entwicklung in Ableitungen des Hintergrundfeldes durch, wobei der Kollisionsterm mitbehandelt wird. Diese Entwicklung entspricht einer Entwicklung in Potenzen der Planck-Konstanten $\hbar$. Wir berücksichtigen alle Terme erster Ordnung in $\hbar$ und erhalten somit semiklassische Gleichungen, mit denen CP-Verletzung beschrieben werden kann. Sowohl im skalaren als auch im fermionischen Fall haben die Gleichungen eine spektrale Lösung, die es erlaubt die Plasma-Anregungen als Quasiteilchen zu behandeln. Während die Transportgleichung für die skalaren Teilchen eine gewöhnliche klassische Boltzmann-Gleichung ist, enthält die fermionische Gleichung Quantenkorrekturen, die als Quellen für Baryogenese dienen.

Controlled derivation of semiclassical transport equations for electroweak baryogenesis

Abstract

In this work we study a basic ingredient for the description of baryogenesis at the electroweak phase transition: we provide a controlled derivation of semiclassical transport equations for CP-violating flows for a system of scalar and fermionic fields in the presence of a slowly varying background field. This is the situation in a first order electroweak phase transition, where the background is given by the vacuum expectation value of the Higgs field that varies inside the phase transition front. Starting from the exact equations of motion for the Wigner functions in the Schwinger-Keldysh formalism we perform a systematic expansion in orders of gradients of the background field, including the collision term. This expansion is equivalent to an expansion in powers of the Planck constant $\hbar$, and by keeping all terms up to first order in $\hbar$ we obtain semiclassical transport equations that are adequate for the description of CP-violation. We find that for both scalar and fermionic fields the equations have a spectral solution which allow for an on-shell description of the plasma excitations. The transport equation for the scalar particles turns out to be a usual classical Boltzmann equation. In the fermionic equations we find quantum corrections that give rise to sources for baryogenesis.

Contents

Introduction

Physicists know that for any particle there exists a corresponding antiparticle that only differs in having opposite charges. If particles and antiparticles come into contact, they annihilate with emission of energetic γ-rays. In daily life, however, we fortunately don't experience the existence of these antiparticles at all, so that many, if not most, people believe antimatter to be an invention of science fiction authors. The only antiparticles observed on earth are either produced with great effort in accelerators like at CERN, or are antiprotons in the cosmic rays. These are mainly products of the reaction $pp \to 3p + \bar{p}$, taking place when cosmic protons hit the atmosphere with high energies. But not only the earth is dominated by matter. In fact we can exclude the existence of considerable amounts of antimatter within the entire visible universe: detailed studies of the γ-ray distribution haven't given any indication for the radiation that we would expect to be produced by the annihilation processes at the boundaries between large matter and anti-matter dominated areas [1]. Another signature that could in principle be used to detect the presence of large anti-matter domains would be a distortion of the spectrum of the cosmic microwave background. At present however, it is not yet possible to derive any strong bounds from this effect. We conclude that either matter and antimatter are separated on scales at least as big as the visible universe, or the whole universe is dominated by matter. Since it is very unlikely that the universe is inhomogeneous at such large scales and no explanation for this is in sight, it is commonly believed that the first possibility is not realized.

So, despite the symmetry between the properties of particles and their antiparticles, the universe displays an asymmetry in their numbers. This asymmetry is usually quantified by the difference of the densities of baryons and antibaryons divided by the entropy density of the universe,

$$\eta = \frac{n_B - \bar{n}_B}{s} \approx 2 - 7 \times 10^{-11} \,. \tag{1}$$

Since both baryon and entropy density scale with a^{-3}, where a is the cosmological scale factor, this ratio stays constant during the expansion of the universe. The quantity η is tightly linked to the production of light elements like deuterium and ^{3}He at the nucleosynthesis epoch, and its numerical value can be determined by the present abundances of these particles [2]. The first estimates of the baryon asymmetry were obtained by simply counting the number of visible objects in the universe, but as we now know these objects make up less than 10% of the total baryonic matter.

The basic question now is: what is the origin of the asymmetry between particles and antiparticles in the universe? To explain this as being an initial condition for the evolution of the universe is quite unnatural. Furthermore, in the accepted standard model of cosmology an inflationary era in the very early universe plays a central role [3]. This exponential expansion would dilute away any initial asymmetry between particles and antiparticles.

In 1967 A. Sakharov [4] discovered that in principle it is possible to dynamically produce a baryon asymmetry in particle reactions, provided three necessary conditions, called the Sakharov criteria, are satisfied:

- Presence of baryon number violating processes.
  This condition is obvious, because when starting from a baryon symmetric universe, $B = 0$, the only way to obtain $B \neq 0$ is to have baryon number violating processes.
- Presence of C (charge conjugation) and CP (charge conjugation and parity) violating processes.
  If C was conserved, the probability for a reaction $i \to f$ to take place would be the same as for the process with particles exchanged by antiparticles. Since the baryon number of $\bar{f}$ is just the negative baryon number of f, no net baryon number could be obtained. If CP was conserved, then T (time reversal) would have to be conserved, too, as stated by the CPT theorem. With T-invariance, however, the probability for the process $i \to f$ to take place is the same as the one for the reversed process $f \to i$, which again makes it impossible to create any net baryon number.
- Departure from thermal equilibrium.
  We know that in thermal equilibrium the number densities are given by the Fermi-Dirac or the Bose-Einstein distributions. They only depend on the mass, which is the same for particles and antiparticles.

It took some years until Sakharov's idea was implemented in concrete baryogenesis scenarios. Most of the models that have been suggested since then can very roughly be divided into two categories.

On the one hand there are models where the out-of-equilibrium criterion is satisfied by the expansion of the universe and the presence of heavy decaying particles. The expansion of the universe can drive these particles out of equilibrium if the characteristic times for their reactions are larger than the characteristic time of the expansion. If these particles then decay through a B-violating process, a net baryon number can be produced [5]. This type of baryogenesis scenario is typically worked out in the framework of Grand Unified Theories (GUT), where there are several ways of implementing B-, C- and CP-violating processes. Most of the publications concerned with baryogenesis work with this out-of-equilibrium decay scenario. The drawback of these models is that they involve physics at energy scales of about 10^{16} GeV, and therefore direct experimental tests are impossible.

On the other hand an out-of-equilibrium situation can be reached during a strong first order phase transition in the early universe, when a global or a gauge symmetry is broken. In this work we will be concerned with a situation of this type, the electroweak phase transition that might have taken place at a temperature of about 100 GeV. If this phase transition is of first order, then we have the necessary departure from thermal equilibrium. C- and CP-violation are present in the CKM-matrix for the quarks, and finally, even in the electroweak standard model there are baryon number violating processes due to the chiral anomaly, which was pointed out by 't Hooft in 1976 [6]. Today, when the universe has almost zero temperature, these processes are completely negligible. They proceed by tunneling between different vacua of the gauge part of the theory which are separated by large energy barriers and therefore are suppressed by a factor of $\mathrm{e}^{-4\pi/\alpha_w} \sim 10^{-160}$. At high temperatures, however, it is possible to have thermal transitions over the top of the barrier, so called sphaleron transitions, which become most effective at temperatures above 100 GeV, that is at the electroweak scale. On the other hand these sphaleron transitions must be ineffective after the completion of the phase transition, because otherwise any produced baryon asymmetry would be destroyed immediately. This is the case if the ratio of the Higgs vacuum expectation value in the broken phase to the temperature, which is a measure for the strength of the phase transition, is larger than one [7, 8]. For studies of the possibility of electroweak baryogenesis see for instance [9, 10].

The great benefit of electroweak baryogenesis is that it is based on physics at the electroweak scale, which is in reach of experiments, in contrast to GUT scenarios, which are almost completely speculative. Unfortunately, after all investigations that have been performed it is now clear that it doesn't work within the standard model of particle physics: there is no first order phase transition at all if the Higgs mass is greater than about 74 GeV [11–15], which is clearly below the current experimental lower bound for the Higgs mass. So in any case an extension of the standard model is required for the explanation of the baryon asymmetry. The solution which suggests itself is to consider supersymmetric extensions, like the Minimal Supersymmetric Standard Model (MSSM) or the MSSM with an additional singlet field, called Next-To-Minimal Supersymmetric Standard Model (NMSSM), which are quite popular also in other situations where the standard model is insufficient. Even with this modifications we stay with physics that is, at least partially, accessible to experiments in the near future. Indeed it has been found that electroweak baryogenesis is possible within the MSSM, although only within a rather restricted region of the parameter space [17–25]. In the NMSSM not only the possibility of electroweak baryogenesis is given in wide regions of the parameter space, but it can also have stronger CP violation, increasing the amount of the actually produced baryon asymmetry [26–28].

This work is not intended to be a further intense study of the produced baryon asymmetry in various regions of the parameter spaces of extensions of the standard model. It is rather intended to cure a basic shortcoming common to most of the electroweak baryogenesis calculations performed so far. A calculation of the baryon asymmetry produced in a first order phase transition requires an understanding of the behavior of the particles in the plasma at the passage of the phase transition front. In essence, one has to derive equations which describe the generation and the transport of CP-violating flows for particles with a varying mass. The works mentioned above use a heuristic way of finding the required transport equations: it is tried to somehow isolate the essential quantum features of the transport in the form of “sources”, and then these sources are inserted into classical Boltzmann equations [29–35]. With such methods electroweak baryogenesis has been studied already in the MSSM [33, 36–42] and in the NMSSM [43, 44]. However these works have found contradictory results.

The aim of this work is to provide a rigorous derivation of these transport equations relevant for baryogenesis, based on first principles. In order to achieve this aim we use an expansion in derivatives of the Higgs condensate, which is responsible for the mass generation, or equivalently, an expansion in powers of the Planck constant $\hbar$. We will truncate this expansion at the first nontrivial order of $\hbar$, which leads to semiclassical transport equations capable of describing CP-violating effects [45, 46].

The derivation of transport equations from the underlying theory is not only important for a consistent treatment of baryogenesis. Besides for applications in the field of heavy ion collisions, for instance, such a discussion is also of general theoretical interest. It is well known, for example, that the usual Boltzmann equation can be obtained as the classical limit of the Schwinger-Dyson equation. The kinetic equations for fermions in presence of a classical gauge field have been considered in gradient approximation in [47–49]. Kinetic equations with a pseudo-scalar mass term have been considered in [50, 51],but only in the classical limit. Up to now, however, no derivation of such equations for scalars or fermions has been performed in the presence of a varying background field, up to first nontrivial order in $\hbar$ and including a treatment of the collision term.

The outline of the work is as follows: in chapter one we first give a more detailed description of how baryogenesis in the electroweak phase transition works. It follows an introduction into the so called Schwinger-Keldysh formalism, which is suitable for the treatment of quantum field theoretical problems in out-of equilibrium situations. Here the basic quantities of our work, the non-equilibrium two-point functions, are defined. By making use of the 2PI formalism we derive the equations of motion for these functions in a model of fermionic and scalar fields, coupled to each other by Yukawa interactions. We write these equations in the Wigner representation, which is convenient for our situation with a slowly varying background field. Finally we use the gradient expansion to reduce the scalar equation of motion to a semiclassical transport equation. An important result here is that the equation admits a spectral solution to first order in $\hbar$.

This is also true for the fermionic equation of motion, which is studied in detail in the second chapter. The spinor structure of this equation makes the treatment much more involved than in the scalar case. A crucial point is the observation that the spin of the fermions in the direction perpendicular to the wall is conserved. The use of this symmetry leads to the essential simplifications that allow the derivation of semiclassical transport equations. We study the mixing of several fermionic particles via a mass matrix and give explicit examples for the MSSM and the NMSSM.

Chapter three contains an extensive treatment of the collision terms of both equations. Based on the results of the previous chapters we show how particle interactions can be consistently integrated into our work. The main focus here lies in the identification of collisional sources for baryogenesis. In the last part of the chapter we study the influence of mixing on the collision term.

In chapter four we use the fermionic transport equation to derive an equation for the CP-violating part of the distribution function. We discuss the sources that appear in the flow term of this equation in some detail and derive a set of fluid equations. Finally we summarize our results.

1. Equations of motion for the Green functions

1.1 Electroweak Baryogenesis

The weak and electromagnetic interactions are described by a gauge theory which today is spontaneously broken by the vacuum expectation value of the Higgs field. The symmetry is restored, however, at temperatures of above ~ 100 GeV in the early universe [52–54]. In the following we assume that the breaking of the symmetry happens in a first order phase transition. In a naive picture the vacuum expectation value of the Higgs field is governed by a temperature dependent effective potential which can be calculated in resummed perturbation theory. The potential contains a thermal mass and a negative cubic term that is essential for having a first order phase transition. In those cases where the perturbative calculation yields a strong transition, this picture seems to be supported by lattice calculations [13, 55]. At high temperatures the potential has a unique global minimum at vanishing Higgs expectation value $\langle H \rangle = 0$, which corresponds to the symmetric phase of the universe (see figure 1.1). With decreasing temperature a new local minimum at a non vanishing value of $\langle H \rangle$ develops. When the temperature reaches the critical temperature T_c, both minima are degenerate. Here both the symmetric ($\langle H \rangle = 0$) and the broken ($\langle H \rangle \neq 0$) phase are energetically equal but separated by a barrier. If the temperature drops further down to the nucleation temperature T_n, the volume energy of spontaneously nucleating bubbles of the broken phase, which is gained by the transition to the energetically favored new global minimum, is bigger than the surface tension. The bubbles will grow until the whole space is converted into the new phase.

When the wall of such a bubble passes a point in space, the Higgs vacuum expectation value at that point undergoes a rapid change. At that time the universe was not almost empty, as it is today, but filled with a hot plasma of particles. These particles are coupled to the Higgs vacuum expectation value - this is the way they obtain their masses - and so the changing $\langle H \rangle$ leads to a departure from equilibrium. Figuratively spoken, the particles in the plasma are thrown out of equilibrium by the passing wall. In the presence of CP-violating interactions this leads to a local separation of lefthanded particles from their antiparticles. Our task is the derivation of appropriate transport equations which are capable of describing these CP-violating flows. Since the mass of the particles is given by a coupling to the Higgs expectation value, $m = y\langle H \rangle$, the interaction with the wall is described by giving the particles a space-time-dependent mass. The asymmetry between the lefthanded particles and their antiparticles is subsequently translated into a net baryon asymmetry in front of the wall, where the sphaleron transition is fast. Finally this baryon asymmetry is transported into the bubble, where the sphaleron transition is frozen out and thus the asymmetry remains until the present day.

The motion of the bubble wall is influenced by two effects. On the one hand there is pressure inside the bubble which acts as an accelerating force on the wall. The origin of this pressure is just the volume energy gained by the transition from the symmetric to the broken phase. On the other hand there is a decelerating force, because the plasma exerts friction on the moving wall. After a short time there will be an equilibrium between the two forces and eventually the wall moves with constant velocity through the plasma. Numerical analyses have shown that in the MSSM the wall velocity v_w is of the order of 0.1 of the speed of light [56]. The wall velocity is an important parameter for baryogenesis. We expect that for extremely slow walls the produced baryon asymmetry is small, since we have only a very slight departure from equilibrium. On the other hand, for big wall velocities the baryon number will be small because the time available for the sphaleron process is too short. This behavior has indeed been observed in calculations of the baryon asymmetry (see for instance [40, 41]).

Since after a short period of expansion the size of a bubble is large compared to the thickness of its walls, we can neglect the curvature of the wall in the derivation of the transport equations. So it is effectively only dependent on one spatial coordinate,

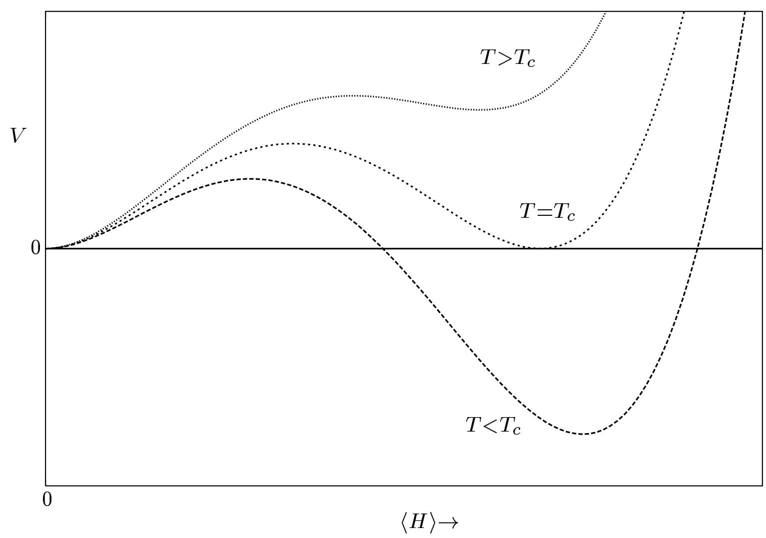

Figure 1.1: Effective potential for the Higgs vacuum expectation value during the phase transition.

which we choose to be the z-direction. The shape of the wall, which of course influences the way the plasma reacts on the passage of the wall and therefore influences the produced baryon asymmetry, can in the MSSM be very well described by a kink ansatz. If we denote both the absolute value of $\langle H \rangle$ and (the Higgs condensate may be complex) its phase generically by ϕ, we can write

$$\phi(z - v_w t) = \frac{\phi_{broken}}{2}\left[1 + \tanh\left(\frac{z - v_w t}{L_w}\right)\right] , \tag{1.1}$$

where ϕ_{broken} denotes the value inside the bubble and L_w is the width of the wall. The width lies in the range of about $6/T - 14/T$ in the MSSM [58, 59] and has about the same size in the NMSSM [60]. In the NMSSM the actual shape deviates slightly from (1.1), but for baryogenesis calculations this seems not to be important.

1.2 Schwinger-Keldysh Formalism

"Ordinary" vacuum quantum field theory is designed to describe particle scattering experiments, where the system is prepared to be in a definite in-state at $t \to -\infty$ and we ask for the probability of finding the system in a definite out-state at $t \to +\infty$ (in-out-formalism). In statistical physics, however, we are interested in the temporal evolution of a system. Starting with definite initial conditions, we ask for the expectation values of physical quantities at finite times. A theoretical framework for such problems was first suggested by Schwinger in 1961 [61] and then developed further by Keldysh [62] and others. An extension of field theory capable of dealing with non-equilibrium problems is obtained by defining the time arguments of all quantities on a path C that leads from $-\infty$ to $+\infty$ and then back to $-\infty$. All integrals and derivatives have then to be performed along that path, and the usual time ordering becomes time ordering T_C along C. This formalism is often also called Closed Time Path (CTP) formalism [63]. The definitions of the scalar and the fermionic Green functions are

$$\begin{aligned} \Delta(u, v) &= -i \left\langle T_C\, \phi(u)\phi^\dagger(v) \right\rangle , && (1.2)\\ G_{\alpha\beta}(u, v) &= -i \left\langle T_C\, \psi_\alpha(u)\bar{\psi}_\beta(v) \right\rangle . && (1.3) \end{aligned}$$

The contour C can now be split into a "+" branch from $-\infty$ to $+\infty$ and a "−" branch from $+\infty$ to $-\infty$. Denoting the branch on which a time argument lies by an index $a = \pm$, we can rewrite the formalism using ordinary time arguments. We then have

$$\begin{aligned} \int_C d^4u &\to \sum_a a \int_{-\infty}^{\infty} d^4u \\ \delta_C(u - v) &\to a\delta_{ab}\delta(u - v) \\ G(u, v) &\to G^{ab}(u, v) . \end{aligned} \tag{1.4}$$

The additional factors a in the integral and in the δ-function have their origin in the fact that the "$-$" branch runs backwards with respect to usual time. The Green functions have four different parts,

$$\begin{aligned}
G^{++}_{\alpha\beta}(u,v) &= G^{t}_{\alpha\beta}(u,v) = -i\left\langle T\psi_\alpha(u)\bar{\psi}_\beta(v)\right\rangle \\
G^{+-}_{\alpha\beta}(u,v) &= G^{<}_{\alpha\beta}(u,v) = i\left\langle \bar{\psi}_\beta(v)\psi_\alpha(u)\right\rangle \\
G^{-+}_{\alpha\beta}(u,v) &= G^{>}_{\alpha\beta}(u,v) = -i\left\langle \psi_\alpha(u)\bar{\psi}_\beta(v)\right\rangle \\
G^{--}_{\alpha\beta}(u,v) &= G^{\bar{t}}_{\alpha\beta}(u,v) = -i\left\langle \overline{T}\psi_\alpha(u)\bar{\psi}_\beta(v)\right\rangle ,
\end{aligned} \tag{1.5}$$

where $\overline{T}$ denotes anti-time ordering and the additional minus sign in the second line is due to the anticommutation property of fermionic fields. Right from these definitions one can see that $G^<$ and $G^>$ have the hermiticity property

$$\left(i\gamma^0 G(x,y)\right)^\dagger = i\gamma^0 G(y,x)\,. \tag{1.6}$$

For the scalar Green functions analogous expressions hold, where of course all four right hand sides start with a minus sign. The scalar hermiticity property reads

$$(i\Delta(x,y))^* = i\Delta(y,x)\,. \tag{1.7}$$

The four parts of (1.5) are not independent of each other, because with the definition of the time and anti-time ordering we can express G^t and $G^{\bar{t}}$ in terms of $G^<$ and $G^>$:

$$\begin{aligned}
G^t(u,v) &= \theta(u_0-v_0)G^>(u,v)+\theta(v_0-u_0)G^<(u,v)\,, \\
G^{\bar{t}}(u,v) &= \theta(u_0-v_0)G^<(u,v)+\theta(v_0-u_0)G^>(u,v)\,.
\end{aligned} \tag{1.8}$$

For a system in thermal equilibrium at a temperature T, explicit expressions for the Green functions can be obtained. In equilibrium they depend only on the relative coordinate $u-v$, so a Fourier transformation can be applied:

$$G_{eq}(k) = \int d^4(u-v)\mathrm{e}^{ik\cdot(u-v)}G_{eq}(u-v)\,. \tag{1.9}$$

The expectation values in the definition of the Green functions reduce to thermal expectation values, and they can be evaluated to [64]

$$G^t_{eq}(k) = \frac{\not{k}+m}{k^2-m^2+i\mathrm{sgn}(k_0)\epsilon} + 2\pi i(\not{k}+m)\delta(k^2-m^2)\mathrm{sgn}(k_0)n_{eq}(k_0) \tag{1.10}$$

$$G^{\bar{t}}_{eq}(k) = \frac{-\not{k}-m}{k^2-m^2+i\mathrm{sgn}(k_0)\epsilon} - 2\pi i(\not{k}+m)\delta(k^2-m^2)\mathrm{sgn}(k_0)(1-n_{eq}(k_0)) \tag{1.11}$$

$$G^<_{eq}(k) = 2\pi i(\not{k}+m)\delta(k^2-m^2)\mathrm{sgn}(k_0)n_{eq}(k_0) \tag{1.12}$$

$$G^>_{eq}(k) = -2\pi i(\not{k}+m)\delta(k^2-m^2)\mathrm{sgn}(k_0)(1-n_{eq}(k_0))\,, \tag{1.13}$$

where the equilibrium phase space distribution is the well-known Fermi-Dirac function

$$n_{eq}(k_0) = \frac{1}{\mathrm{e}^{\beta k_0}+1} \quad , \quad \beta = 1/T\,. \tag{1.14}$$

These expressions are given for the case of a real mass. They can be trivially extended to the case of a complex mass by replacing $\not k + m$ by $\not k + m_R + i\gamma^5 m_I$, where m_R and m_I are the real and imaginary part of the mass, respectively. The scalar equilibrium Green functions are

$$\begin{aligned}
\Delta^t_{eq}(k) &= \frac{1}{k^2-m^2+i\mathrm{sgn}(k_0)\epsilon} - 2\pi i\delta(k^2-m^2)\mathrm{sgn}(k_0)n^\phi_{eq}(k_0) && (1.15)\\
\Delta^{\bar t}_{eq}(k) &= -\frac{1}{k^2-m^2+i\mathrm{sgn}(k_0)\epsilon} - 2\pi i\delta(k^2-m^2)\mathrm{sgn}(k_0)(1+n^\phi_{eq}(k_0)) && (1.16)\\
\Delta^<_{eq}(k) &= -2\pi i\delta(k^2-m^2)\mathrm{sgn}(k_0)n^\phi_{eq}(k_0) && (1.17)\\
\Delta^>_{eq}(k) &= -2\pi i\delta(k^2-m^2)\mathrm{sgn}(k_0)(1+n^\phi_{eq}(k_0))\,. && (1.18)
\end{aligned}$$

Here the Bose-Einstein distribution function is used:

$$n^\phi_{eq}(k_0) = \frac{1}{\mathrm{e}^{\beta k_0}-1}\,. \tag{1.19}$$

The equilibrium Green functions satisfy the famous Kubo-Martin-Schwinger (KMS) relations:

$$G^>_{eq}(k) = -\mathrm{e}^{\beta k_0}G^<_{eq}(k) \quad , \quad \Delta^>_{eq}(k) = \mathrm{e}^{\beta k_0}\Delta^<_{eq}(k)\,. \tag{1.20}$$

1.3 Lagrange density and equation of motion

We study a system of a fermionic and a scalar particle, both having space-time dependent masses and being coupled to each other by a Yukawa interaction. The Lagrangian for the system is

$$\begin{aligned}
\mathcal{L} &= i\bar\psi \not\partial \psi - \bar\psi_L m\psi_R - \bar\psi_R m^*\psi_L \\
&\qquad + (\partial_\mu\phi)^\dagger(\partial^\mu\phi) - m^2_\phi\phi^\dagger\phi + \mathcal{L}_{\mathrm{int}}.
\end{aligned} \tag{1.21}$$

The masses of the particles arise from an interaction with a scalar field condensate. This is the case, for example, in a first order electroweak phase transition, where the particles are coupled to the Higgs field condensate, which varies at the boundary between the symmetric and the broken phase. In a way these varying masses can then be said to describe the interaction with the bubble wall. But we are not restricted to this specific scenario. Any system which has varying masses and which furthermore satisfies the restrictions we will make in the course of our treatment can be studied. We choose the fermion mass to be complex,

$$m(u) = m_R(u) + im_I(u) = |m(u)|\mathrm{e}^{i\theta(u)}, \tag{1.22}$$

which is a potential source of CP-violation. Here m_R and m_I denote the real and imaginary part of m, respectively. With the definition of the left- and right-handed fields

$$\psi_L = P_L \psi \quad , \quad \psi_R = P_R \psi, \tag{1.23}$$

where the chiral projection operators

$$P_L = \frac{1}{2}(1-\gamma^5) \quad , \quad P_R = \frac{1}{2}(1+\gamma^5) \tag{1.24}$$

with $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ are used, we can rewrite the fermionic mass term in a form where the pseudoscalar part is made explicit:

$$\begin{aligned} \bar{\psi}_L m \psi_R + \bar{\psi}_R m^* \psi_L &= \bar{\psi}\left(P_R\, m + P_L\, m^*\right)\psi \\ &= \bar{\psi}\left(m_R + i\gamma^5 m_I\right)\psi\,. \end{aligned} \tag{1.25}$$

The interaction part of the Lagrangian contains the Yukawa coupling term, which can also be rewritten with the help of the projection operators:

$$\begin{aligned} \mathcal{L}_{\text{int}} &= -y\phi\, \bar{\psi}_L \psi_R - y^*\phi^*\, \bar{\psi}_R \psi_L \\ &= -\bar{\psi}\left(P_R\, y\phi + P_L\, y^*\phi^*\right)\psi\,. \end{aligned} \tag{1.26}$$

In the next sections we will derive the equations of motion for the Green functions in the CTP-formalism, first for the scalar and then for the fermionic field.

1.3.1 Scalar field

We now derive the equation of motion for the scalar Green function in the CTP-formalism. If one neglects collisions, there is a very easy and straightforward way to obtain the desired equation. We just use the Lagrangian (1.21) to write down the field equation, which is of course the Klein-Gordon equation:

$$\left(\partial_u^2 + m_\phi^2(u)\right)\phi(u) = 0\,. \tag{1.27}$$

After multiplying this from the left with $-i\phi^\dagger(v)$ we take the expectation value and use the definition of the Green function $\Delta^<$:

$$\left(\partial_u^2 + m_\phi^2(u)\right)\Delta^<(u,v) = 0\,. \tag{1.28}$$

In a similar way one finds that the same equation holds for $\Delta^>$.

In order to obtain the equations for the Green functions including interactions we follow the approach of Calzetta and Hu [65], which uses the two-particle irreducible (2PI) representation of the CTP effective action. In this approach one works with an extended effective action Γ. This 2PI-effective action depends not only on the expectation value of the field $\langle\phi(u)\rangle$ but also on the two-point-function $\langle T_C\phi(u)\phi^\dagger(v)\rangle$, which is nothing else than the Green function $\Delta(u,v)$. The equation of motion for one of these quantities is then obtained by requiring the derivative of the effective action with respect to this quantity to vanish. Since we are not interested in an expectation value of the field itself we only deal with the Green functions and set the field expectation value to zero. Cornwall, Jackiw and Tomboulis [66] showed that the 2PI-effective action for a complex scalar field can be written as

$$\Gamma[\Delta] = i\mathrm{Tr}\Delta^{(0)^{-1}}\Delta + i\mathrm{Tr}\ln\Delta^{-1} + \Gamma_2[\Delta]\,. \tag{1.29}$$

The inverse free propagator $\Delta^{(0)^{-1}}$ for the scalar theory is defined by rewriting the classical effective action in the form

$$I[\phi] = \int_C d^4u\,\mathcal{L}(u) = \frac{1}{2}\int_C d^4u\,d^4v\,\phi(u)\Delta^{(0)^{-1}}(u,v)\phi(v) + I_{int}[\phi]\,, \tag{1.30}$$

where we only use the scalar part of the Lagrangian. It is easily seen to be

$$\Delta^{(0)^{-1}}(u,v) = -\left(\partial_u^2 + m_\phi^2(u)\right)\delta_C^4(u-v)\,. \tag{1.31}$$

The quantity Γ_2 is the sum of all two-particle irreducible vacuum graphs with vertices defined by I_{int} and propagators set equal to Δ. We take the functional derivative of the effective action with respect to Δ and obtain

$$\frac{\delta\Gamma[\Delta]}{\delta\Delta(v,u)} = +i\Delta^{(0)^{-1}}(u,v) - i\Delta^{-1}(u,v) + \frac{\delta\Gamma_2[\Delta]}{\delta\Delta(v,u)}\,. \tag{1.32}$$

The equation of motion is obtained by requiring this to be zero, and with the definition of the self energy

$$\Pi(u,v) = i\frac{\delta\Gamma_2[\Delta]}{\delta\Delta(v,u)} \tag{1.33}$$

we recognize (1.32) as the Schwinger-Dyson equation. Finally we multiply the equation from the right with the Green function Δ and find the equation of motion:

$$\left(\partial_u^2 + m_\phi^2(u)\right)\Delta(u,v) = -\delta_C^4(u-v) - \int_C d^4w\,\Pi(u,w)\Delta(w,v)\,. \tag{1.34}$$

Up to now we have written everything in contour notation. In index notation the self energy is

$$\Pi^{ab}(u,v) = i\frac{\delta\Gamma_2[\Delta]}{ab\,\delta\Delta^{ba}(v,u)}\,, \tag{1.35}$$

where the additional indices on the right hand side have their origin in the fact that on the "−" branch we have derivatives "in the negative direction". With this the equation of motion is

$$\left(\partial_u^2 + m_\phi^2(u)\right)\Delta^{ab}(u,v) = -a\delta_{ab}\delta^4(u-v) - \sum_c\int d^4w\,c\Pi^{ac}(u,w)\Delta^{cb}(w,v)\,. \tag{1.36}$$

The current density for the scalar field, which contains all information about the semiclassical particle densities, can be expressed in terms of the Wigner function $\Delta^<$:

$$\begin{aligned} \left\langle j^0(u)\right\rangle &= -(\partial_{u^0} - \partial_{v^0})\Delta^<(u,v)\Big|_{u=v}\,, \qquad (1.37)\\ \left\langle j^i(u)\right\rangle &= +(\partial_{u^i} - \partial_{v^i})\Delta^<(u,v)\Big|_{u=v}\,. \qquad (1.38)\end{aligned}$$

We therefore focus on the equation for $\Delta^<$, obtained by choosing $ab = +-$:

$$\left(\partial_u^2 + m_\phi^2(u)\right)\Delta^<(u,v) = -\int d^4w\,\left(\Pi^t(u,w)\Delta^<(w,v) - \Pi^<(u,w)\Delta^{\bar t}(w,v)\right)\,. \qquad (1.39)$$

We define the "hermitean" part of the Green function and of the self energy

$$\begin{aligned} \Delta_R &= \Delta^t - \frac{1}{2}\left(\Delta^> + \Delta^<\right) = -\Delta^{\bar t} + \frac{1}{2}\left(\Delta^> + \Delta^<\right) \qquad (1.40)\\ \Pi_R &= \Pi^t - \frac{1}{2}\left(\Pi^> + \Pi^<\right) = -\Pi^{\bar t} + \frac{1}{2}\left(\Pi^> + \Pi^<\right)\,, \qquad (1.41)\end{aligned}$$

and the collision term

$$\mathcal{C}_\phi(u,v) = \frac{1}{2}\int d^4w\left(\Pi^>(u,w)\Delta^<(w,v) - \Pi^<(u,w)\Delta^>(w,v)\right)\,. \qquad (1.42)$$

With these definitions we can rewrite the equation of motion:

$$\begin{aligned} &\left(\partial_u^2 + m_\phi^2(u)\right)\Delta^<(u,v) + \int d^4w\,\Pi_R(u,w)\Delta^<(w,v)\\ &\qquad + \int d^4w\,\Pi^<(u,w)\Delta_R(w,v) = -\mathcal{C}_\phi(u,v)\,. \qquad (1.43)\end{aligned}$$

The term $\Pi_R\Delta^<$ on the left hand side is the self energy contribution to the mass, and $\Pi^<\Delta_R$ essentially leads to a modification of the particle spectrum. In this work we neglect these two terms. We comment on this in the beginning of chapter 3. The collision term $\mathcal{C}_\phi(u,v)$ on the right hand side contains the gain and loss terms that usually lead to relaxation.

1.3.2 Fermionic fields

The derivation of the equation of motion for the fermionic Green function proceeds along the same lines as in the scalar case. Again there is a very easy way to obtain this equation when one neglects interactions, except of the interaction with the wall, starting from the Dirac equation for the spinor ψ. Here we directly go to the derivation in the 2PI approach. The fermionic 2PI generating functional is

$$\Gamma[G] = -i\mathrm{Tr}{G^{(0)}}^{-1}G - i\mathrm{Tr}\ln G^{-1} + \Gamma_2[G]\,, \qquad (1.44)$$

where Γ_2 is the same as in the scalar case, since the interaction part of the Lagrangian is common to both fields. The inverse free propagator is given by

$$G^{(0)^{-1}}(v,u) = -\left(i\not{\partial}_u + m_R + i\gamma^5 m_I\right)\delta^4_C(u-v)\,. \tag{1.45}$$

In order to obtain the equation of motion we this time have to take the functional derivative of (1.44) with respect to the fermionic Green function. The result reads

$$-iG^{(0)^{-1}ab}(u,v) + iG^{-1ab}(u,v) + ab\,\Sigma^{ab}(u,v) = 0\,, \tag{1.46}$$

where the definition of the fermionic self energy is

$$\Sigma^{ab}(u,v) = -i\frac{\delta\Gamma_2[G]}{ab\,\delta G^{ba}(v,u)}\,. \tag{1.47}$$

We multiply this equation with G, integrate over v, sum over a, multiply with b and use the explicit expression for the inverse free propagator to find

$$\left(i\not{\partial}_u - m_R - i\gamma^5 m_I\right)G^{ab}(u,v) = a\delta_{ab}\delta^4(u-v) + \sum_c \int d^4w\, c\,\Sigma^{ac}(u,w)G^{cb}(w,v)\,. \tag{1.48}$$

We are interested only in the equation for the component $G^< = G^{+-}$, which is

$$\left(i\not{\partial}_u - m_R - i\gamma^5 m_I\right)G^<(u,v) = \int d^4w\left(\Sigma^t(u,w)G^<(w,v) - \Sigma^<(u,w)G^{\bar{t}}(w,v)\right)\,. \tag{1.49}$$

The definitions of the hermitian part of the propagator and of the self energy as well as the definition of the collision term are analogous to the corresponding quantities in the scalar case. With these we then can write the fermionic equation in the form

$$\begin{aligned}\left(i\not{\partial}_u - m_R - i\gamma^5 m_I\right)G^<(u,v) &- \int d^4w\,\Sigma_R(u,w)G^<(w,v)\\ &- \int d^4w\,\Sigma^<(u,w)G_R(w,v) = \mathcal{C}_\psi(u,v)\,.\end{aligned} \tag{1.50}$$

Like in the scalar case we neglect the self energy correction to the mass as well as the part that influences the spectrum and only keep the collision term.

1.4 Wigner transformation and gradient expansion

In equilibrium the Green functions $\Delta(u,v)$ and $G(u,v)$, which contain complete information about the system, depend only on the relative coordinate $r = u - v$. This dependence corresponds to the internal quantum fluctuations that typically take place on microscopical scales. In a non-equilibrium situation, however, there is also a dependence on the average coordinate $X = (u+v)/2$. This dependence describes the systems behavior on large, macroscopical scales. For example, in a study of thermalization of a system one follows the evolution of the system with growing time X_0. Or, if the system experiences an external perturbation that has a macroscopical spatial variation, then this will show up in a corresponding dependence of the Green functions on the average coordinate. This is what we are doing, since the bubble wall is a large scale perturbation of the system. The idea is to separate the small scale fluctuations from the behavior on macroscopical scales by performing a Fourier transformation with respect to the relative coordinate r. This is called a Wigner transformation. The Green function in the Wigner representation, which is called Wigner function, is

$$\Delta(X,k) = \int d^4 r \, \mathrm{e}^{ik\cdot r} \, \Delta(X + r/2, X - r/2) \,, \tag{1.51}$$

and analogous for the fermionic G. Throughout this work we use the same symbol for a function and its Wigner transform. Which one is meant will be clear or indicated by the arguments. The hermiticity properties (1.6) and (1.7) in the Wigner representation become

$$\Big(i\gamma^0 G^{<,>}(X,k)\Big)^\dagger \;=\; i\gamma^0 G^{<,>}(X,k) \,, \tag{1.52}$$

$$\Big(i\Delta^{<,>}(X,k)\Big)^* \;=\; i\Delta^{<,>}(X,k) \,. \tag{1.53}$$

In order to transform the equation of motion, we write the first term of (1.43) as a convolution:

$$\Big(\partial_u^2 + m_\phi^2(u)\Big)\Delta^<(u,v) = \int d^4 w \, \Big[\Big(\partial_u^2 + m_\phi^2(u)\Big)\delta(u-w)\Big] \, \Delta^<(w,v) \,. \tag{1.54}$$

The Wigner transform of a general convolution is

$$\int d^4(u-v) \, \mathrm{e}^{-ik\cdot(u-v)} \int d^4 w A(u,w) B(w,v) = \mathrm{e}^{-i\diamond}\{A(X,k)\}\{B(X,k)\} \,, \tag{1.55}$$

where the average variable is $X = (u+v)/2$ and the diamond operator is defined by

$$\diamond\{.\}\{.\} = \frac{1}{2}\Big(\partial^{(1)} \cdot \partial_k^{(2)} - \partial_k^{(1)} \cdot \partial^{(2)}\Big)\{.\}\{.\} \,. \tag{1.56}$$

The superscripts (1) and (2) refer to the first and second argument, respectively. Here and in the following ∂ alway means the derivative with respect to X. The Fourier transform of the differential operator inside the square brackets in (1.54) is

$$\int d^4 r \, \mathrm{e}^{ik\cdot r} \Big(\partial_u^2 + m_\phi^2(u)\Big)\delta(u-v) = -k^2 + m_\phi^2(X) \,, \tag{1.57}$$

where again $r = u - v$ and $X = (u+v)/2$. This relation can be shown by replacing the δ-function by its Fourier representation. The equation of motion for the Wigner function then becomes

$$\mathrm{e}^{-i\diamond}\left\{-k^2 + m_\phi^2(X)\right\}\left\{\Delta^<(X,k)\right\} = -\mathcal{C}_\phi(X,k)\,. \tag{1.58}$$

The collision term in configuration space (1.42) has the form of a convolution, so in the Wigner representation it is simply given by

$$\mathcal{C}_\phi(X,k) = \frac{1}{2}\mathrm{e}^{-i\diamond}\Big(\left\{\Pi^>(X,k)\right\}\left\{\Delta^<(X,k)\right\} - \left\{\Pi^<(X,k)\right\}\left\{\Delta^>(X,k)\right\}\Big)\,. \tag{1.59}$$

This term will be studied in chapter 3 in detail. In an expansion of the exponentiated diamond operator on the left hand side of (1.58) there are terms that contain only $\partial^{(1)}\partial_k^{(2)}$ or $-\partial_k^{(1)}\partial^{(2)}$, which can be recombined into $\exp(-i\partial^{(1)}\partial_k^{(2)})$ and $\exp(i\partial_k^{(1)}\partial^{(2)})$, respectively. In addition there are mixed terms, but they always contain a part $\partial^\mu\partial_k^\nu(-k^2+m_\phi^2)$, which is obviously zero. So it is quite straightforward to see that (1.58) can be simplified to

$$\left(\frac{1}{4}\partial^2 - k^2 - ik\cdot\partial + m_\phi^2(X)\mathrm{e}^{-\frac{i}{2}\overleftarrow{\partial}\cdot\partial_k}\right)\Delta^<(X,k) = -\mathcal{C}_\phi(X,k)\,. \tag{1.60}$$

Here one can see explicitly how the space-time dependence of the mass affects the equation. Derivatives of the mass are combined with momentum derivatives of the Wigner function, which in a classical picture just means that a varying mass like a varying potential acts as a force on the particles and changes their momenta. The computation of the Wigner transform of the fermionic equation of motion (1.50) runs along the same lines. We first arrive at

$$\mathrm{e}^{-i\diamond}\big\{\not{k} - m_R(X) - i\gamma^5 m_I(X)\big\}\big\{G^<(X,k)\big\} = \mathcal{C}_\psi(X,k)\,, \tag{1.61}$$

and with the same arguments as above this can be simplified to give

$$\left(\not{k} + \frac{i}{2}\not{\partial} - \Big(m_R(X) + i\gamma^5 m_I(X)\Big)\mathrm{e}^{-\frac{i}{2}\overleftarrow{\partial}\cdot\partial_k}\right)G^<(X,k) = \mathcal{C}_\psi(X,k)\,. \tag{1.62}$$

The expression for the collision term is analogous to the scalar one, we only have to use the fermionic self energy and Wigner function instead of the scalar ones.

The basic assumption in this work is that the variation of the background field, which is responsible for the space-time-dependence of the mass, has a characteristic length scale that is big in comparison to the de Broglie wavelength of the particles in the plasma. Since the varying background field is responsible for the space-time dependence of all quantities in the Wigner representation, we can perform an expansion in derivatives with respect to the average coordinate X. The assumption of a slowly varying background field seems to be justified in our case: in the MSSM, for instance, the width of the bubble wall L_w, which is the scale of the variation of the background, is roughly $10/T$, where T is the temperature of the plasma. The typical momentum of a particle in the plasma is of the order T, so that the de Broglie wavelength $l_{dB} \sim 1/T$ is indeed small when compared to L_w.

Since the expansion in powers of derivatives is equivalent to an expansion in powers of the Planck constant $\hbar$, this procedure will lead to semiclassical equations. We expect that the leading order terms in the gradient expansion correspond to classical behavior, while taking into account higher order derivatives gives rise to quantum corrections.

1.5 Scalar fields

We first take a closer look on the scalar equation. The hermiticity property (1.53) of the Wigner function just states that $i\Delta^<(X,k)$ is a real quantity. So we can split the complex equation (1.60) into two real ones by taking the real part,

$$\left(-k \cdot \partial - m_\phi^2(X) \sin(\frac{1}{2} \overleftarrow{\partial} \cdot \partial_k)\right) i\Delta^<(X,k) = -\Re\, \mathcal{C}_\phi(X,k)\,, \tag{1.63}$$

and the imaginary part,

$$\left(-\frac{1}{4}\partial^2 + k^2 - m_\phi^2(X) \cos(\frac{1}{2} \overleftarrow{\partial} \cdot \partial_k)\right) i\Delta^<(X,k) = -\Im\, \mathcal{C}_\phi(X,k)\,. \tag{1.64}$$

Now we apply the gradient expansion. In equation (1.63) only odd powers of derivatives occur, so in order to get an equation that is correct up to second order in gradients it is sufficient to keep only the first order derivatives. Similarly, the imaginary part (1.64) contains only even powers of derivatives, and therefore throwing them all away still leaves us with an equation which is correct up to first order in gradients. This way we obtain

$$\left(-k \cdot \partial - \frac{1}{2}(\partial m_\phi^2(X)) \cdot \partial_k\right) i\Delta^<(X,k) = -\Re\, \mathcal{C}_\phi(X,k) \tag{1.65}$$

and

$$\left(k^2 - m_\phi^2(X)\right) i\Delta^<(X,k) = -\Im\, \mathcal{C}_\phi(X,k)\,. \tag{1.66}$$

In this form the meaning of the two equations becomes apparent. The first one is a kinetic equation, while the second one is an algebraic equation (for a moment we forget about the right hand side) which restricts the space of possible solutions of the kinetic equation to functions which are on-shell. This equation is called the constraint equation. When using the constraint equation this way, it becomes clear that it was consistent to keep only the first order derivatives there: any second order contribution to the constraint would lead, when used in the kinetic equation, to a term that is of third order in gradients, since all the terms in the kinetic equation already contain a derivative. But now we see immediately that neither the constraint equation nor the flow term of the kinetic equation for one scalar particle contain any CP-violating quantum corrections that could act as sources for the production of a baryon asymmetry. For a scalar particle we just find the usual Boltzmann equation with a classical force term.

If we include the term on the right hand side of the constraint equation, which is a functional of the scalar Wigner function, the equation is not algebraic any more. But we will see later that the collision term is at least of first order in gradients. So, when we use the constraint equation (1.66) to justify an on-shell ansatz for $i\Delta$ in (1.65), which contains only first order terms, then we can neglect the collision term, because it would lead to corrections that are second order in gradients and additionally suppressed by the coupling constant.

The interpretation of the second equation as a spectral constraint on the solutions of the kinetic equation works also beyond first order in gradients. But then things are much more complicated, because the constraint equation is not an algebraic equation that forces the particles on-shell any more, but a further differential equation that has to be solved simultaneously with the kinetic equation.

In models like the MSSM we are in a situation where there are several scalar fields coupled to each other by a mass matrix, which in our situation is space-time dependent. An example is the stop sector $\tilde{q} = (\tilde{t}_L, \tilde{t}_R)^T$ of the MSSM, where the mass matrix is

$$M_{\tilde{q}}^2 = \begin{pmatrix} m_Q^2 & y(A^* H_2 + \mu H_1) \\ y(A H_2 + \mu^* H_1) & m_U^2 \end{pmatrix} , \tag{1.67}$$

where m_Q^2 and m_U^2 denote the sum of the soft susy-breaking masses. The derivation of the equation of motion proceeds as shown in section 1.3, with the modification that the fields now have an index indicating the type of the particle. We find an equation which is obtained from (1.60) by replacing the real mass m_ϕ^2 by a hermitean mass matrix M_ϕ^2, and the Wigner function $i\Delta$ becomes a hermitean matrix, too.

The basic idea for the treatment of mixing is to diagonalize the mass. If the mass was constant, then after the diagonalization we would, on the left hand side, just obtain N separate equations, corresponding to the independent propagation of the N mass eigenstates. In the bubble wall, however, the mass matrix and therefore the definitions of the eigenstates vary, so we expect their propagation to be not any longer independent of each other, which is described by a non-diagonal Wigner function, and the propagation will also be directly influenced by the changing mass matrix. Since these effects are, due to their origin, at least of first order in gradients, we can treat them as small corrections.

The hermitean matrix M_ϕ^2 is diagonalized by a unitary, space-time dependent rotation matrix U:

$$M_\phi^2 = U^\dagger M_{\phi,d}^2 U \,. \tag{1.68}$$

We define the rotated Wigner function $i\Delta_d = Ui\Delta U^\dagger$, which in general is not diagonal, as explained above. By rotating the equation of motion we find

$$\left(\frac{1}{4}D^2 - k^2 - ik\cdot D + M_{\phi,d}^2(X)\mathrm{e}^{-\frac{i}{2}\overleftarrow{D}\cdot\partial_k}\right)\Delta_d^<(X,k) = -\mathcal{C}_{\phi,d}(X,k)\,, \tag{1.69}$$

where we used the "covariant" derivative $D_\mu = \partial_\mu - i[iU\partial_\mu U^\dagger, \cdot]$, arising from derivatives acting on the rotation matrix U, and defined $\mathcal{C}_{\phi,d} = U\mathcal{C}_\phi U^\dagger$. The constraint and the kinetic equation are this time obtained by taking the antihermitean and the hermitean part of (1.69), respectively. Like in the one field case we keep terms up to first order in gradients in order to get the constraint equation

$$k^2 i\Delta_d^< - \frac{1}{2}\{M_{\phi,d}^2, i\Delta_d^<\} + \frac{i}{2}[k\cdot D, i\Delta_d^<] = -\Im\,\mathcal{C}_{\phi,d}\,, \tag{1.70}$$

while in the kinetic equation we go up to second order:

$$-\frac{1}{2}\left\{k\cdot D + \frac{1}{2}\left(DM_{\phi,d}^2\right)\cdot\partial_k, i\Delta_d^<\right\} + \frac{i}{2}\left[M_{\phi,d}^2\left(1+\frac{1}{8}(\overleftarrow{D}\cdot\partial_k)^2\right), i\Delta_d^<\right] = -\Re\,\mathcal{C}_{\phi,d}\,, \tag{1.71}$$

where we can rewrite the anticommutators like $\{A,B\} = 2AB - [A,B]$. In the end we are only interested in the quasiclassical particle densities, which are described by the diagonal elements of $i\Delta_d^<$. We will now argue that in the equations for these diagonal elements all commutator terms can be neglected. This is obvious for the terms of the form $[M_{\phi,d}^2, i\Delta_d^<]$: they don't contribute to the equations for the diagonal elements, because the commutator of a diagonal matrix with any other matrix has no diagonal elements at all.

The discussion of the other terms is a bit more delicate. It is convenient to restore explicit powers of $\hbar$ here, which is done by the following recipe: assign a factor of $\hbar$ to each space-time-derivative, and then multiply the whole equation with that power of $\hbar$ that makes the classical terms order $\hbar^0$. According to this the kinetic equation, to which we will restrict our discussion here, has to be multiplied with $\hbar^{-1}$ in order to make the classical flow term explicitly of order $\hbar^0$. Since we are interested in the leading order quantum effects, we neglect all terms of order $\hbar^2$ in this equation.

The other commutators in the kinetic equation are of the form $\hbar^{-1}[i\hbar U\partial U^\dagger, i\Delta_d^<]$ or similar, so formally they are of the order $\hbar^0$. The diagonal part of this commutator contains only off-diagonal elements of $i\Delta_d^<$, however, and these are implicitly suppressed by $\hbar$ when compared to the diagonal elements. The reason is that the off-diagonal components of $i\Delta_d^<$ vanish in equilibrium and are only sourced by the diagonal elements via a term of order $\hbar$. To make things even more complicated, we are looking for CP-violating effects. In the end we will take the equation for particles and subtract the equation for antiparticles in order to obtain an equation for CP-violating densities. Therefore a term in our equation which contains no CP-violation is irrelevant even if it is of order $\hbar$. So what is the order of the CP-violating part of the off-diagonal elements of $i\Delta_d^<$? CP-violation is a quantum effect, and therefore the CP-violating contribution to the diagonal elements of $i\Delta_d^<$ has to be of order $\hbar$. The commutator term that mixes diagonal and off-diagonal elements itself is not CP-violating, so the CP-violating part of the diagonal elements of order $\hbar$ will source a CP-violating part of the off-diagonal elements of order $\hbar^2$. This means that the CP-violating part of the diagonal elements of the commutator $\hbar^{-1}[i\hbar U\partial U^\dagger, i\Delta_d^<]$ is implicitly of order $\hbar^2$ and therefore can be neglected. For the commutator terms in the constraint equation the same arguments hold.

We can conclude that in the kinetic and constraint equations for scalar particles, both in the non-mixing and in the mixing case, no CP violation is present at order $\hbar$, and therefore no source term for baryogenesis of order $\hbar$. We obtain purely classical dispersion relations and transport equations. In order to catch the lowest order source term we would in principle have to go one order higher. Fortunately we will see later that there are source terms of order $\hbar$ in the fermionic equations, so potential second order source terms in the scalar equations are subdominant and can be neglected.

The fact that we find classical equations which are correct to order $\hbar$ means that a WKB approach to the problem should lead to correct results. There it is simply assumed that the plasma can be treated as a collection of quasiparticles which can be described by wavefunctions. The Klein-Gordon equation for the wavefunctions leads to a dispersion relation which is inserted into a classical Boltzmann equation. In older WKB treatments a CP-violating force was found, however. The reason for this is that in those works the canonical momentum was used for the description of the particles. In [40] the kinetic momentum was used instead, leading to the correct results. For a brief comparison between the use of canonical and kinetic momentum in WKB see [46].

There are also other approaches to the problem that use field theoretical methods. There the coupling to the Higgs condensate is described as a part of the self energy and an expansion in Higgs mass insertions is performed in order to calculate the scalar current density or its divergence, respectively [34, 35, 37]. In these works CP-violating contributions to these quantities at order $\hbar$ are found, in contrast to our result.

2. Fermionic equation

We saw in the end of the last chapter that the equation of motion for the scalar Wigner function has no quantum corrections of order $\hbar$. In the non-mixing case the calculation was almost trivial. Due to the additional spin degree of freedom, formally displayed by the fact that we now have to deal with a matrix equation, the treatment of the fermionic equation of motion is considerably more complicated, so we devote an own chapter to this part.

The equation is covariant under Lorentz transformations, which enables us to write it right from the start in a system which is at rest with respect to the bubble wall. In this so called "wall frame" the mass doesn't depend on time, and making use of the symmetry of the wall discussed in section 1.1, we can adjust the coordinate system in such a way that the mass only depends on the z-component of the average coordinate X. The equation of motion in the wall frame is

$$\left(\not{k} + \frac{i}{2}\not{\partial} - \left(m_R(z) + i\gamma^5 m_I(z)\right)\mathrm{e}^{\frac{i}{2}\overleftarrow{\partial}_z\cdot\partial_{k_z}}\right) G^<(X,k) = \mathcal{C}_\psi(X,k)\,. \tag{2.1}$$

In the following all functions and variables are written in the wall frame, if not indicated otherwise.

A straightforward way to treat this problem is to make a decomposition of the matrix $G^<$ with real component functions and then to extract scalar equations for these components from the matrix equation (2.1). This is what we do in the first part of this chapter. Unfortunately the resulting equations are too complicated to be addressed directly. By an appropriate boost, however, we can transform into a system where a symmetry of the problem becomes obvious. We make an ansatz for the Wigner function which is adjusted to this symmetry, and this way we are able to derive semiclassical Boltzmann equations. In this part we omit the collision term on the right hand side for simplicity. In the second part we make use of the discovered symmetry right from the start and are lead to a quite elegant treatment of the equation. We relax some of the restrictions we placed in the first part and include the collisions.

2.1 Bad basis

First we have to choose a basis for the Clifford algebra. The matrices of the basis should be hermitean in order to obtain real component functions for the Wigner function. We write them as external products of Pauli matrices:

$$\begin{aligned} \mathbb{1} &= \mathbb{1}\otimes\mathbb{1} & \gamma^0\gamma^i\gamma^5 &= \sigma^i\otimes\mathbb{1} \\ \gamma^0 &= \mathbb{1}\otimes\rho^1 & \gamma^i\gamma^5 &= \sigma^i\otimes\rho^1 \\ -i\gamma^0\gamma^5 &= \mathbb{1}\otimes\rho^2 & -i\gamma^i &= \sigma^i\otimes\rho^2 \\ -\gamma^5 &= \mathbb{1}\otimes\rho^3 & -\gamma^0\gamma^i &= \sigma^i\otimes\rho^3\,. \end{aligned} \tag{2.2}$$

The σ describe the spin degree of freedom while the ρ are connected to the particle-antiparticle degree of freedom. Since we know that $i\gamma^0 G^<$ is hermitean, we decompose the Wigner function like

$$i\gamma^0 G^<(X,k) = -\frac{1}{4}\sigma^a\otimes\rho^b g_{ab}(X,k) \tag{2.3}$$

and obtain 16 real component functions g_{ab}. In the above expression a summation over a and b from 0 to 3 is understood and $\sigma^0 = \rho^0 = \mathbb{1}$. The basis (2.2) has already been used in former approaches to quantum transport theory [48, 49]. It has the advantage that the components g_{ab} can be directly related to the fermionic currents. These have the form

$$\left\langle \bar\psi(x)\Gamma\psi(x)\right\rangle\,, \tag{2.4}$$

$$\Gamma = \left(\mathbb{1}, \gamma^5, \gamma^\mu, \gamma^5\gamma^\mu, \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu,\gamma^\nu]\right) \tag{2.5}$$

and can be expressed in terms of the Wigner function $G^<$:

$$\begin{aligned} \left\langle \bar\psi(x)\Gamma\psi(x)\right\rangle &= -i\,\mathrm{Tr}\,\Gamma G^<(x,x) \\ &= -\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\Gamma\gamma^0\, i\gamma^0 G^<(x,k)\right]\,. \end{aligned} \tag{2.6}$$

When we insert the decomposition (2.3), we find for example for the vector current the expressions

$$\left\langle \bar\psi(x)\gamma^0\psi(x)\right\rangle = \int\frac{d^4k}{(2\pi)^4}g_{00}(x,k)\,, \tag{2.7}$$

$$\left\langle \bar\psi(x)\gamma^i\psi(x)\right\rangle = \int\frac{d^4k}{(2\pi)^4}g_{i3}(x,k)\,. \tag{2.8}$$

It is not hard to see that g_{01} is a scalar density, $-ig_{02}$ a pseudo-scalar density, and $(-g_{03}, -g_{i0})$ build a pseudo-vector density. The remaining functions g_{i1} and g_{i2} are the components of a tensor density (see also appendix A).

We insert the decomposition (2.3) into the equation of motion (2.1) and extract scalar equations for the g_{ab} by multiplying with the matrices of the basis and performing the trace. Taking real and imaginary parts finally leads to 2 sets of 16 real, coupled differential equations for the 16 real functions g_{ab}. They are shown in appendix B. There we already truncated the gradient expansion after the second order and omitted the collision term for simplicity. Since the mass in the wall frame doesn't depend on the time nor on x or y, it cannot cause any t-, x- or y-dependence of $G^<$. If we assume that the plasma was in thermal equilibrium before the wall passed by, then $G^<$ can only depend on z. In essence, by this assumption we forbid any non-stationary and non-symmetrical initial conditions for the Wigner function. In this case we can drop all space-time derivatives in the equations in appendix B, except of those with respect to z.

In the classical limit these equations lead to a usual Boltzmann equation for g_{00}, the zero-component of the vector-density. But we have to go beyond the classical limit, since only then the CP-violating effects essential for baryogenesis can be described. Though we put some effort to it, we didn't manage to reduce the system to only one kinetic equation in this case.

A way out can be found by going one step back to the matrix equation (2.1) for $G^<$ before inserting any decomposition:

$$\left(\gamma^0 k_0 - \gamma^1 k_x - \gamma^2 k_y - \gamma^3 (k_z - \frac{i}{2}\partial_z) - \left(m_R(z) + i\gamma^5 m_I(z)\right) e^{\frac{i}{2}\overleftarrow{\partial}_z \cdot \partial_{k_z}}\right) G^<(z,k) = 0\,. \tag{2.9}$$

Having a closer look at this equation and the explicit representation of the Clifford algebra (2.2), we observe that all matrices appearing in the differential operator, except of γ^1 and γ^2, are block-diagonal, that is diagonal in the spin part of the direct product. If the differential operator was completely block-diagonal, the 4×4 matrix equation would break into uncoupled 2×2 matrix equations, which would simplify the problem significantly. Since γ^1 and γ^2 in (2.9) are multiplied by k_x and k_y, respectively, we can get rid of them and achieve block-diagonality by performing a suitable boost in the x-y-plane.

2.1.1 The boost

We construct a k-dependent boost $\Lambda(k)$ in such a way, that when acting on the 4-vector k it sets the x- and y-components to zero without touching the z-component:

$$\begin{pmatrix} k_0 \\ k_x \\ k_y \\ k_z \end{pmatrix} \xrightarrow{\Lambda(k)} \begin{pmatrix} \tilde{k}_0 \\ 0 \\ 0 \\ k_z \end{pmatrix} \quad , \quad \tilde{k}_0^2 = k_0^2 - k_x^2 - k_y^2\,. \tag{2.10}$$

We use the notation $k^\mu = (k_0, \vec{k})$, $\vec{k} = (k_x, k_y, k_z)$. A general boost transformation with relative velocity $\vec{v}$ can be written as

$$\tilde{k}_0 = \gamma \left[k_0 - \vec{v} \cdot \vec{k} \right] \quad , \quad \tilde{\vec{k}} = \vec{k} + \vec{v} \left[(\gamma - 1) \frac{\vec{v} \cdot \vec{k}}{v^2} - \gamma k^0 \right] \tag{2.11}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2}} \, , \tag{2.12}$$

so that in our case we have to set

$$\vec{v} = \frac{1}{k_0} \vec{k}_{||} \quad \text{with} \quad \vec{k}_{||} = \begin{pmatrix} k_x \\ k_y \\ 0 \end{pmatrix} \, , \tag{2.13}$$

where $\vec{k}_{||}$ is the projection of $\vec{k}$ parallel to the bubble wall. Since we have to deal with fermions, we need the representation of this boost in spinor space. It is given by

$$L(k) = \frac{k_0 + \tilde{k}_0 + \gamma^0 \vec{\gamma} \cdot \vec{k}_{||}}{\sqrt{2 \tilde{k}_0 (k_0 + \tilde{k}_0)}} \, , \tag{2.14}$$

the inverse L^{-1} is obtained by reversing the sign of $\vec{k}_{||}$. Equation (2.10) doesn't specify the matrix L uniquely, since we could append an arbitrary rotation around the z-axis. But this would complicate things unnecessarily, and if we additionally demand to have a pure boost, then (2.14) is the only solution. Like every pure boost operator it is hermitean.

We already stated that we are dealing with a covariant equation, so we can immediately write it down in the boosted frame. Since there the x- and y-components of the momentum vanish by construction, the unpleasant matrices γ^1 and γ^2 are gone:

$$\left(\gamma^0 \tilde{k}_0 - \gamma^3 \big(k_z - \frac{i}{2} \partial_z \big) - \big(m_R(z) + i \gamma^5 m_I(z) \big) \mathrm{e}^{\frac{i}{2} \overleftarrow{\partial}_z \cdot \partial_{k_z}} \right) \tilde{G}^{<}(z, \tilde{k}) = 0 \, . \tag{2.15}$$

Boosting this equation is not completely trivial, because in general it is not possible to perform a k-dependent boost of a differential equation in the variable k. But in our case there is no problem, since the only momentum derivative is with respect to k_z, while the boost matrix depends only on k_0 and $\vec{k}_{||}$.

After the boost our equation has become effectively 1+1 dimensional, because only 0- and z-components have survived. We will therefore in the following refer to the boosted system as the "1+1 frame" and denote all quantities in this frame by a tilde. The original system is called the "3+1 frame". Looking at this equation it seems like allowing a time dependence would not destroy block-diagonality, since time derivatives multiply γ^0. Note however that a time dependence in the 3+1 frame is transformed into a dependence on $\tilde{t}$, $\tilde{x}$ and $\tilde{y}$ by the boost, leading again to terms with γ^1 and γ^2. Only very special dependences on t, x and y in the $3+1$ frame lead to a pure time dependence in the 1+1 frame, so we prefer to restrict ourselves here to having no such dependences at all. We will come back to this point in the second part of the chapter.

2.1.2 1+1 dimensional treatment

In the $1+1$ frame the differential operator in the equation of motion for the Wigner function is block-diagonal, or, to be more precise, diagonal in the spin-part of the basis. It commutes with

$$\tilde{S}_z = \gamma^0\gamma^3\gamma^5 = \sigma^3 \otimes \mathbb{1}\,, \tag{2.16}$$

which is the operator that measures spin in z-direction. Physically this means that the interaction with the wall conserves spin in z-direction.

Again we assume that the plasma was in thermal equilibrium before the phase transition took place, so the fermionic particles were described by the equilibrium Green functions (1.10)-(1.13). These are diagonal in spin in the 1+1 frame. Since the interaction with the bubble wall induces no spin-mixing, we make an ansatz for the Wigner function which is also diagonal in spin:

$$\tilde{G}^< = \sum_s \tilde{G}^{<s}\,, \tag{2.17}$$

$$i\gamma^0\tilde{G}^{<s} = -\frac{1}{4}(\mathbb{1} + s\sigma^3) \otimes \rho^a \tilde{g}_a^{<s}\,. \tag{2.18}$$

Because of the hermiticity property $(i\gamma^0 G^<)^\dagger = i\gamma^0 G^<$ of the Wigner function, the components $\tilde{g}_a^{<s}$ are real functions. The spin diagonal parts of $\tilde{G}^<$ correspond to the quasiclassical phase space densities of particles with spin up or down, respectively, while the off-diagonal parts describe quantum effects like mixing of or transitions between states with different spin. We labeled the functions with the spin quantum number s according to

$$\tilde{G}^{<s} = \tilde{P}_s \tilde{G}^< \tilde{P}_s\,, \tag{2.19}$$

where the spin projection operator is defined by

$$\tilde{P}_s = \frac{1}{2}(1 + s\tilde{S}_z)\,. \tag{2.20}$$

Of course interactions with other particles of the plasma, which are described by the collision term, will in general cause spin-mixing and so lead to a Wigner function which contains also off-diagonal parts. For the rest of this section we just assume that these effects are negligible and the spin diagonal ansatz for $G^<$ is sufficiently good. In the second part of the chapter we include off-diagonal parts in our treatment.

With this ansatz equation (2.15) has become completely block-diagonal, so that our original 4×4 matrix equation can trivially be separated into two uncoupled 2×2 matrix equations for the two different spin states. These equations are obtained by effecting the replacements

$$\gamma^0 \to \rho^1 \quad , \quad \gamma^3 \to is\rho^2 \quad , \quad \gamma^5 \to -\rho^3 \tag{2.21}$$

in (2.15) after inserting the ansatz, and we find

$$\left[\mathbb{1}\tilde{k}_0 - is\rho^2(k_z + \frac{i}{2}\partial_z) - (m_R - i\rho^3 m_I)\mathrm{e}^{\frac{i}{2}\overleftarrow{\partial}_z\cdot\partial_{k_z}}\right]\rho^1\rho^a\tilde{g}_a^{<s} = 0\,. \tag{2.22}$$

We extract scalar equations for the real functions $\tilde{g}_a^{<s}$ by multiplying with $\mathbb{1}$ and ρ^i, respectively, and taking the trace.The real and imaginary parts of these equations provide twice as many equations as independent functions are present. Hence one half of the equations must correspond to constraints on the solutions of the other half, which are kinetic equations. The constraint equations will be essential in order to derive a semiclassical transport equation from the kinetic equations as we will see in the following. The importance of the constraints was first pointed out in the context of kinetics of fermions in [51, 67].

Now recall that the mass is a slowly varying function of the average coordinate, and as a consequence the same is true for the components of the Wigner function. We truncate the gradient expansion at second order, which is the lowest order at which CP-violating effects can be discussed consistently. In our equation this leads to

$$m\,\mathrm{e}^{\frac{i}{2}\overleftarrow{\partial}_z\cdot\partial_{k_z}} = m + \frac{i}{2}m'\partial_{k_z} - \frac{1}{8}m''\partial_k^2\,. \tag{2.23}$$

We will from now on denote the derivative with respect to z with a prime. Even with spin-conservation and this truncation we have a problem consisting of eight coupled second order partial differential equations, which we want to reduce to a single equation governing the dynamics of the fermionic two-point function. In order to simplify the notation, we will from now on drop the arguments of the functions. Furthermore we will drop the index $<$, since we have to deal only with $G^<$ here.

Constraint equations

We first examine the constraint equations, which are the real parts of the traces of (2.22). It will become evident that it is sufficient to treat them only up to first order in derivatives, so we find:

$$-\tilde{k}_0\tilde{g}_0^s + sk^3\tilde{g}_3^s + m_R\tilde{g}_1^s + m_I\tilde{g}_2^s = 0 \tag{2.24}$$

$$-\tilde{k}_0\tilde{g}_1^s - \frac{1}{2}s\partial_z\tilde{g}_2^s + m_R\tilde{g}_0^s - \frac{1}{2}m_I'\partial_{k_z}\tilde{g}_3^s = 0 \tag{2.25}$$

$$-\tilde{k}_0\tilde{g}_2^s + \frac{1}{2}s\partial_z\tilde{g}_1^s + \frac{1}{2}m_R'\partial_{k_z}\tilde{g}_3^s + m_I\tilde{g}_0^s = 0 \tag{2.26}$$

$$-\tilde{k}_0\tilde{g}_3^s + sk^3\tilde{g}_0^s - \frac{1}{2}m_R'\partial_{k_z}\tilde{g}_2^s + \frac{1}{2}m_I'\partial_{k_z}\tilde{g}_1^s = 0\,. \tag{2.27}$$

We now use the constraint equations (2.25)-(2.27) iteratively in order to express the functions $\tilde{g}_i^s$ in terms of $\tilde{g}_0^s$:

$$\tilde{k}_0\tilde{g}_1^s = \left(m_R - \frac{s}{2\tilde{k}_0}(\partial_z m_I - m_I'\partial_{k_z}k^3)\right)\tilde{g}_0^s \tag{2.28}$$

$$\tilde{k}_0\tilde{g}_2^s = \left(m_I + \frac{s}{2\tilde{k}_0}(\partial_z m_R + m_R'\partial_{k_z}k^3)\right)\tilde{g}_0^s \tag{2.29}$$

$$\tilde{k}_0\tilde{g}_3^s = \left(sk^3 + \frac{1}{2\tilde{k}_0}|m|^2\theta'\partial_{k_z}\right)\tilde{g}_0^s\,. \tag{2.30}$$

Note that the derivatives inside the parentheses also act on $\tilde{g}_0^s$. Then we insert these expressions for $\tilde{g}_i^s$ into (2.24). It is remarkable that all terms containing derivatives acting on $\tilde{g}_0^s$ cancel and we are left with a purely algebraic equation:

$$\left(\tilde{k}_0^2 - k_z^2 - |m|^2 + \frac{s|m|^2\theta'}{\tilde{k}_0}\right)\tilde{g}_0^s = 0\,. \tag{2.31}$$

This is a very important result, since it shows that we can, to this order in gradients, work on-shell and have a picture of quasi-particles with an energy that is shifted by the interaction with the CP-violating bubble wall. Equation (2.31) gives the physical dispersion relation for particles and antiparticles of a given spin s. Due to the derivative corrections the spin degeneracy is lifted at first order in gradients, so that the varying background field leads to different accelerations for particles with different spin, as we will see below. Hoiwever note that this on-shell condition is only present up to first order in gradients. Although even at higher orders we still can describe the system by a single independent function (to be precise, two independent functions, because of the two possible spin directions), already at second order the constraint equation for this function will contain derivatives that don't allow an on-shell treatment.

Kinetic equations

The so far unused equations are kinetic equations. We will restrict our discussion here to the one obtained by taking the imaginary party of the trace of (2.22), the meaning of the other equations will be discussed in the second part of the chapter. This equation,

$$s\partial_z \tilde{g}_3^s - m'_R \partial_{k_z} \tilde{g}_1^s - m'_I \partial_{k_z} \tilde{g}_2^s = 0\,, \tag{2.32}$$

is correct up to second order in gradients. We again use the constraint equations to express $\tilde{g}_i^s$ in terms of $\tilde{g}_0^s$, where it is sufficient to have the relations correct up to first order in gradients since all the terms in (2.32) are already of first order. We obtain the following kinetic equation for $\tilde{g}_0^s$:

$$k_z \partial_z \tilde{g}_0^s - \left(\frac{1}{2}|m|^{2\prime} - \frac{s}{2\tilde{k}_0}(|m|^2 \theta')' \right) \partial_{k_z} \tilde{g}_0^s = 0\,. \tag{2.33}$$

In addition to this equation, the function $\tilde{g}_0^s$ has to satisfy the constraint equation (2.31) that forces it to be on-shell. Here we see explicitely that the force acting on a particle is spin-dependent and hence CP-violating. It is this very force that leads to the CP-violating density fluctuations which finally are transformed into a baryon asymmetry by the sphaleron transition.

Now we have managed, in the 1+1 frame, to simplify the problem significantly. Instead of dealing with a complex 4×4 matrix equation we were able to reduce the system to two real functions $\tilde{g}_0^s$, for which we have simple on-shell conditions and kinetic equations. Our next task is to make a connection with the initial, 3+1 dimensional problem.

2.1.3 Back to 3+1

The essential feature of the 1+1 treatment is the fact that the spin in z-direction is conserved, allowing us to decouple the equations. We can construct a conserved quantity in the 3+1 frame by just boosting the spin operator $\tilde{S}_z$ from the 1+1 frame back to the 3+1 frame:

$$\begin{aligned} S_z(k) &= L^{-1}(k)\tilde{S}_z L(k) \\ &= \frac{k_0}{\tilde{k}_0}\gamma^0\gamma^3\gamma^5 - i\frac{k_1}{\tilde{k}_0}\gamma^0\gamma^2 + i\frac{k_2}{\tilde{k}_0}\gamma^0\gamma^1\,. \end{aligned} \tag{2.34}$$

If $\tilde{S}_z$ commutes with the differential operator of the equation in the 1+1 frame, then the boosted version S_z obviously commutes with the differential operator in the 3+1 frame and therefore it is a conserved quantity. For simplicity we will continue to refer to this quantity, which now depends on the momentum, just as spin. The boosted Wigner function

$$G^s(k) = L^{-1}(k)\tilde{G}^s(k)L(k) \tag{2.35}$$

is of course not an explicitely block-diagonal matrix anymore, but is still diagonal in spin,

$$P_s(k)G^s(k)P_s(k) = G^s(k)\,, \tag{2.36}$$

where the projection operator is $P_s(k) = 1/2(\mathbb{1} + sS_z(k))$. We make a general decomposition of the spin diagonal parts G^s of the Wigner function in the basis (2.2):

$$i\gamma^0 G^s = -\frac{1}{4}\sigma^a \otimes \rho^b g^s_{ab}\,. \tag{2.37}$$

All we have to do now is to find the transformation relations between the component functions g^s_{ab} in the 3+1 frame and $\tilde{g}^s_a$ in the 1+1 frame. Then we can just boost the kinetic and constraint equations (2.33) and (2.31) back to the 3+1 system. Furthermore we will be able to express all the functions g^s_{ab} in terms of just two independent functions - one for each spin direction - by boosting the corresponding relations from the 1+1 frame.

The straightforward way to find the relations between the component functions in the two frames is just to use the transformation rule (2.35) and insert the decompositions of the Wigner functions in the 1+1 and 3+1 frame as well as the explicit form of he boost operator (2.14). A more elegant method is to consider the fermionic currents (2.6) expressed in terms of the component functions. Since the behavior of the currents under Lorentz transformations is known we can quite easily deduce the transformation properties of the component functions. To this end we make a general decomposition like (2.37) in the 1+1 frame,

$$i\gamma^0 \tilde{G}^s = -\frac{1}{4}\sigma^a \otimes \rho^b \tilde{g}^s_{ab}\,, \tag{2.38}$$

and by comparison with (2.18) we immediately see that

$$\tilde{g}^s_{0a} = s\tilde{g}^s_{3a} = \tilde{g}^s_a \quad , \quad \tilde{g}^s_{1,2a} = 0. \tag{2.39}$$

We already have identified the combination (g^s_{00}, g^s_{i3}) as the four-vector current density (2.7),(2.8), and since the integral measure d^4k is invariant we can simply read off the transformation relation

$$g^s_{00} = \gamma\left(\tilde{g}^s_{00} + v^i \tilde{g}^s_{i3}\right) = \gamma\tilde{g}^s_{00} = \gamma\tilde{g}^s_0\,, \tag{2.40}$$

where $\vec{v}$ is the boost velocity (2.13) and $\gamma = k_0/\tilde{k}_0$ is the boost factor. This way we proceed for all the functions and obtain

$$g^s_{ab} = \begin{pmatrix} \gamma\tilde{g}^s_0 & \tilde{g}^s_1 & \tilde{g}^s_2 & \gamma\tilde{g}^s_3 \\ \gamma v_x\tilde{g}^s_3 & \gamma v_y s\tilde{g}^s_2 & -\gamma v_y s\tilde{g}^s_1 & \gamma v_x\tilde{g}^s_0 \\ \gamma v_y\tilde{g}^s_3 & -\gamma v_x s\tilde{g}^s_2 & \gamma v_x s\tilde{g}^s_1 & \gamma v_y\tilde{g}^s_0 \\ s\tilde{g}^s_0 & \gamma s\tilde{g}^s_1 & \gamma s\tilde{g}^s_2 & s\tilde{g}^s_3 \end{pmatrix}. \tag{2.41}$$

Now it is a simple task to obtain the kinetic and constraint equation for g^s_{00} from those of $\tilde{g}^s_0$ (2.33, 2.31). We finally find

$$k_z\partial_z g^s_{00} - \left(\frac{1}{2}|m|^{2\prime} - \gamma\frac{s(|m|^2\theta')'}{2k^0}\right)\partial_{k_z} g^s_{00} = 0 \tag{2.42}$$

and

$$\left(k^2 - |m|^2 + \gamma \frac{s|m|^2\theta'}{2k^0}\right) g^s_{00} = 0 \,. \tag{2.43}$$

This is almost what one naively would have guessed if asked to extend the 1+1 results to the 3+1 frame. The one remarkable point is the boost enhancement of the CP-violating terms by the factor γ.

The algebraic constraint equation that we have found here again implies that the plasma can be treated as a collection of quasiparticles and therefore a WKB approach should be possible. Indeed, when the kinetic momentum is used to describe the particles, the correct results are found [40]. The WKB treatments, however, missed the boost factor γ multiplying the source term. There is a further, basic difference to our results: the authors of the works using the WKB method assumed that the helicity of the particles is a conserved quantity, and consequently the energy shift as well as the force term found by them are helicity dependent. We have shown that in fact the spin is conserved, leading to spin dependent energy and force. This has important consequences as we will see in chapter 4.

2.2 Good basis

In this section we give a more formal approach to the problem and now include collisions. The equation to examine is

$$D(X,k)G(X,k) = \mathcal{C}_\psi(X,k) \,, \tag{2.44}$$

where we introduced an abbreviation for the differential operator

$$\mathcal{D}(X,k) = \not{k} + \frac{i}{2}\not{\partial} - \big(m_R(z) + i\gamma^5 m_I(z)\big)\mathrm{e}^{\frac{i}{2}\overleftarrow{\partial}_z\cdot\partial_{k_z}} \,. \tag{2.45}$$

$\mathcal{C}_\psi$ denotes the collision term, which will be studied later in detail. In this section we still omit the index $<$ for the sake of notational simplicity.

The main point of the previous section is that we found a symmetry of the system under study: the differential operator commutes with the spin operator S_z, so the spin is conserved by the interaction with the bubble wall. In this section we exploit this symmetry right from the start in order to treat our problem in a more elegant way, including the collision term and avoiding the assumption that the Green function is diagonal in spin.

In the last section we assumed that the system is stationary and symmetric in the x-y-plane, so that the Green function only depends on the z-coordinate. This was suggested by the fact that the mass only had a z-dependence, too. Here we don't want to make any a priori assumptions on the dependences of G, but instead just check under which conditions the symmetry of the problem is maintained. So we take the full differential operator (2.45), including t-, x- and y-derivatives, and compute the commutator with the spin operator:

$$
\begin{aligned}
[\mathcal{D}, S_z(k)] &= \frac{i}{2}\left[\gamma^0\partial_0 + \gamma^1\partial_x + \gamma^2\partial_y\, ,\, S_z(k)\right] \qquad (2.46)\\
&= \frac{1}{\tilde{k}_0}\Big(\gamma^0\left(k_x\partial_y - k_y\partial_x\right) - \gamma^1\left(k_0\partial_y + k_y\partial_t\right) + \gamma^2\left(k_0\partial_x + k_x\partial_t\right)\Big)\,.
\end{aligned}
$$

Since we know from the previous section that the commutator vanishes in the stationary, x-y-symmetric case we only have to keep that part of $\mathcal{D}$ which contains the time and the x- and y-derivatives. This commutator will be zero if we impose the condition

$$
\vec{\partial}_{\|} = -\frac{\vec{k}_{\|}}{k^0}\partial_t\,, \tag{2.47}
$$

where this is understood as acting on G. It may be that there is a less restrictive form of the dependence of G on t, x and y if one takes into account the matrix structure of (2.46) and of the Wigner function, but here we will be content with this. It just means that G can have a dependence on time and the parallel coordinates of the form

$$
G(t, \vec{x}_{\|}, z; k) = G(t - \vec{v}\cdot\vec{x}_{\|}, z; k) \tag{2.48}
$$

So we can treat time dependent problems, which can be used to study how the system relaxes from some initial conditions to the stationary state, admittedly only for quite special forms of the initial condition. Another possibility would be not to demand stationarity and x-y-symmetry, but instead

$$
\left(\gamma^0\partial_0 + \gamma^1\partial_x + \gamma^2\partial_y\right)G = 0\,. \tag{2.49}
$$

In this case, however, we have two completely independent equations. The time-evolution doesn't affect the z-dependence of our problem and therefore is not interesting for us.

We make use of the symmetry of the problem by choosing a new basis for the Clifford algebra which has a simple behavior under the multiplication with the spin projection operator $P_s(k)$. The basis (2.2) we used in the last section certainly doesn't satisfy this requirement. It is not difficult to construct 8 matrices which commute with the projector, they are

$$
P_s(k)B \quad \text{where} \quad s = \pm 1\,,\; B \in \{\mathbb{1}, -i\gamma^3, -\gamma^5, \gamma^3\gamma^5\}\,. \tag{2.50}
$$

But which is the best choice for the rest of the basis? Let us go back to the 1+1 frame, where the spin operator is simply

$$
\tilde{S}_z = \gamma^0\gamma^3\gamma^5 = \sigma^3 \otimes \mathbb{1}\,. \tag{2.51}
$$

When one now remembers quantum mechanics, there are two operators which are "orthogonal" to the spin operator σ_z, namely the spin-flip operators $\sigma_\pm = \frac{1}{2}(\sigma_x \pm i\sigma_y)$. In our case they correspond to the matrices

$$\tilde{Q}_s = \frac{1}{2}(\gamma^0\gamma^1\gamma^5 + is\gamma^0\gamma^2\gamma^5)\,, \tag{2.52}$$

which in the 3+1 frame take the form

$$\begin{aligned} Q_s(k) &= S^{-1}(k)\tilde{Q}_s S(k) \\ &= \frac{1}{2\tilde{k}_0(k_0+\tilde{k}_0)}\Big[\gamma^0\gamma^1\gamma^5(k_0^2 - k_x^2 - isk_xk_y + k_0\tilde{k}_0) \\ &\qquad + is\gamma^0\gamma^2\gamma^5(k_0^2 - k_y^2 + isk_xk_y + k_0\tilde{k}_0) \\ &\qquad - i\gamma^0\gamma^3(k_0+\tilde{k}_0)(k_y - isk_x)\Big]\,. \end{aligned} \tag{2.53}$$

By showing the corresponding relations in the 1+1 frame and then boosting to the 3+1 frame it is not hard to find that $P_s(k)$ and $Q_s(k)$ satisfy the relations

$$\begin{aligned} P_s(k)P_{s'}(k) &= \delta_{s,s'}P_s(k)\,, \\ Q_s(k)P_{s'}(k) &= \delta_{s,-s'}Q_s(k)\,, \\ P_s(k)Q_{s'}(k) &= \delta_{s,s'}Q_s(k)\,, \\ Q_s(k)Q_{s'}(k) &= \delta_{s,-s'}P_s(k)\,, \end{aligned} \tag{2.54}$$

which is exactly the behavior we expect from a projector and an operator that flips spin from $-s$ to s. So Q_s has indeed a nice behavior under multiplication with P_s, and that's why we choose the rest of our basis to be

$$Q_s(k)B \quad \text{where} \quad s = \pm 1\,,\ B \in \{\mathbb{1}, -i\gamma^3, -\gamma^5, \gamma^3\gamma^5\}\,. \tag{2.55}$$

It remains to show that the 16 matrices P_sB and Q_sB really set up a basis by proving that they are not linearly dependent. Since B is invariant under the boost it suffices to show the linear independence of

$$\tilde{P}_sB\,,\ \tilde{Q}_sB\,. \tag{2.56}$$

When one writes down these matrices explicitly, the matrices of the old basis (2.2) appear, from which we know that they are independent.

Let us add a mathematical remark. The way we set up our basis makes the Clifford algebra a graded algebra. We have two disjunctive subsets of the Clifford algebra, the "commuting" subset spanned by P_sB and the "non-commuting" one spanned by Q_sB. Using the multiplication rules (2.54) and the fact that the matrices B either commute or anticommute with Q_s, it is easy to check that the product of two elements of the commuting subset as well as the product of two elements of the non-commuting subset are elements of the commuting subset, while the product of an element of the commuting with an element of the non-commuting subset falls into the non-commuting subset.

Now we use our new basis to construct a decomposition of the Green function that is custom-made to exploit the symmetry of the system:

$$\begin{aligned} G(k) &= \sum_s \left(G^{s,s} + G^{s,-s} \right) && (2.57) \\ G^{s,s} &= iP_s(k) \left[s\gamma^3\gamma^5 g_0^s - s\gamma^3 g_3^s + \mathbb{1} g_1^s - i\gamma^5 g_2^s \right] && (2.58) \\ G^{s,-s} &= iQ_s(k) \left[s\gamma^3\gamma^5 g_4^s - s\gamma^3 g_7^s + \mathbb{1} g_5^s - i\gamma^5 g_6^s \right] . && (2.59) \end{aligned}$$

The first, spin diagonal part describes particles propagating with a definite spin. Restricted to the 1+1 frame it just reduces to the one we used in the first part of this chapter (2.18), except of a different normalization. The second, non-diagonal part describes the transition from one spin state to the other. This is the part we neglected in the first section. The decomposition is constructed in order to satisfy

$$P_s(k) G(k) P_{s'}(k) = G^{s,s'}(k) . \qquad (2.60)$$

The hermiticity condition (1.52) for the Green function and the relations

$$\begin{aligned} P_s^\dagger(k)\gamma^0 &= \gamma^0 P_s(k) && (2.61) \\ Q_s^\dagger(k)\gamma^0 &= \gamma^0 Q_{-s}(k) && (2.62) \end{aligned}$$

allow us to deduce that the functions g_a^s are real for $a = 0, 1, 2, 3$, while the component functions of the non-diagonal part obey

$$g_a^{s*} = g_a^{-s} \quad , \quad a = 4, 5, 6, 7 . \qquad (2.63)$$

In order to obtain real, s-independent functions, we can define

$$g_a^s = g_a + ish_a \quad , \quad a = 4, 5, 6, 7 . \qquad (2.64)$$

Altogether we have 16 independent, real component functions, as it has to be.

The next task is to extract scalar equations for the component functions from the matrix equation of motion (2.44). This is done by multiplying this equation from the left with the matrices of the basis (2.50), (2.55) and then taking the trace. Here we get paid for the effort of constructing the new basis, because the equations decouple on the left hand side. When taking the traces with the matrices $P_s B$, only the spin-diagonal parts survive,

$$\mathrm{Tr} P_s(k) B \mathcal{D}(x,k) G(x,k) = \mathrm{Tr} B \mathcal{D}(x,k) G^{s,s}(x,k) , \qquad (2.65)$$

while the traces with $Q_s B$ lead to equations for the spin off-diagonal parts:

$$\begin{aligned} & \mathrm{Tr} Q_s(k) B\mathcal{D}(x,k) G(x,k) \\ = \; & \mathrm{Tr} P_s(k) Q_s(k) P_{-s}(k) B\mathcal{D}(x,k) G(x,k) \\ = \; & \mathrm{Tr} Q_s(k) B\mathcal{D}(x,k) G^{-s,s}(x,k)\,. \end{aligned} \tag{2.66}$$

This allows us to study the influence of the wall on the different parts of G separately, which is a significant simplification, as we have seen in the previous section. Instead of a set of 16 coupled equations (see appendix B) we get two sets of 4 equations for the spin diagonal parts and one set of 8 equations for the off-diagonal parts. The latter don't decouple further because in $G^{s,-s}$ both the g_a and the h_a occur. The impossibility to decouple the two off-diagonal parts also makes sense physically, since the process of flipping the spin from + to − should not be independent from the reversed process.

Of course the different sets of equations are coupled by the collision term, because particle reactions mix spin, in general. But the propagating states are defined by the quadratic part of the Lagrangian, and the basis we use reflects the symmetry of this term.

2.2.1 Spin-diagonal equations

The equations with the spin diagonal part on the left hand side are called spin diagonal equations. Taking the trace of the equation of motion (2.44) for the Wigner function after multiplying with $P_s B$ leads to

$$2i\hat{k}_0 g_0^s - 2i\hat{m}_R g_1^s - 2i\hat{m}_I g_2^s - 2is\hat{k}_3 g_3^s = \mathrm{Tr} P_s(k) \mathbb{1} \mathcal{C}_\psi \tag{2.67}$$

$$2i\hat{k}_0 g_1^s - 2i\hat{m}_R g_0^s + 2\hat{m}_I g_3^s - 2s\hat{k}_3 g_2^s = \mathrm{Tr} P_s(k) s\gamma^3\gamma^5 \mathcal{C}_\psi \tag{2.68}$$

$$2i\hat{k}_0 g_2^s - 2\hat{m}_R g_3^s - 2i\hat{m}_I g_0^s + 2s\hat{k}_3 g_1^s = \mathrm{Tr} P_s(k) \left(-is\gamma^3\right) \mathcal{C}_\psi \tag{2.69}$$

$$2i\hat{k}_0 g_3^s + 2\hat{m}_R g_2^s - 2\hat{m}_I g_1^s - 2is\hat{k}_3 g_0^s = \mathrm{Tr} P_s(k) \left(-\gamma^5\right) \mathcal{C}_\psi\,, \tag{2.70}$$

where we used the shorthand notations

$$\hat{k}_0 = \tilde{k}_0 + \frac{i}{2}\frac{k_0\partial_t + k_x\partial_x + k_y\partial_y}{\tilde{k}_0} \quad , \quad \hat{k}_z = k_z - \frac{i}{2}\partial_z\,, \tag{2.71}$$

and $\hat{m} = m\,\mathrm{e}^{\frac{i}{2}\overleftarrow{\partial}_z\cdot\partial_{k_z}}$. One always has to keep in mind that the dependence of the Wigner function on time and the parallel coordinates is restricted by (2.47). Since the functions g_a^s are real, we simply can take the real and imaginary part. The real parts are the kinetic equations

$$\begin{aligned}
\frac{1}{2\tilde{k}_0}(k_0\partial_t + \vec{k}_{\|}\cdot\vec{\partial})g_0^s - \frac{1}{2}m_R'\partial_{k_z}g_1^s - \frac{1}{2}m_I'\partial_{k_z}g_2^s + \frac{1}{2}s\partial_z g_3^s &= \mathcal{K}_0^s \quad (2.72)\\
\frac{1}{2\tilde{k}_0}(k_0\partial_t + \vec{k}_{\|}\cdot\vec{\partial})g_1^s - \frac{1}{2}m_R'\partial_{k_z}g_0^s + sk_z g_2^s - (m_I - \frac{1}{8}m_I''\partial_{k_z}^2)g_3^s &= \mathcal{K}_1^s \quad (2.73)\\
\frac{1}{2\tilde{k}_0}(k_0\partial_t + \vec{k}_{\|}\cdot\vec{\partial})g_2^s - \frac{1}{2}m_I'\partial_{k_z}g_0^s + sk_z g_1^s + (m_R - \frac{1}{8}m_R''\partial_{k_z}^2)g_3^s &= \mathcal{K}_2^s \quad (2.74)\\
\frac{1}{2\tilde{k}_0}(k_0\partial_t + \vec{k}_{\|}\cdot\vec{\partial})g_3^s + \frac{1}{2}s\partial_z g_0^s + (m_I - \frac{1}{8}m_I''\partial_{k_z}^2)g_1^s & \\
-(m_R - \frac{1}{8}m_R''\partial_{k_z}^2)g_2^s &= \mathcal{K}_3^s \quad (2.75)
\end{aligned}$$

and the imaginary parts are the constraint equations:

$$\begin{aligned}
-\tilde{k}_0 g_0^s + sk_z g_3^s + (m_R - \frac{1}{8}m_R''\partial_{k_z}^2)g_1^s + (m_I - \frac{1}{8}m_I''\partial_{k_z}^2)g_2^s &= \mathcal{C}_0^s \quad (2.76)\\
-\tilde{k}_0 g_1^s - \frac{1}{2}s\partial_z g_2^s + (m_R - \frac{1}{8}m_R''\partial_{k_z}^2)g_0^s - \frac{1}{2}m_I'\partial_{k_z}g_3^s &= \mathcal{C}_1^s \quad (2.77)\\
-\tilde{k}_0 g_2^s + \frac{1}{2}s\partial_z g_1^s + (m_I - \frac{1}{8}m_I''\partial_{k_z}^2)g_0^s + \frac{1}{2}m_R'\partial_{k_z}g_3^s &= \mathcal{C}_2^s \quad (2.78)\\
-\tilde{k}_0 g_3^s + sk_z g_0^s + \frac{1}{2}m_I'\partial_{k_z}g_1^s - \frac{1}{2}m_R'\partial_{k_z}g_2^s &= \mathcal{C}_3^s\,. \quad (2.79)
\end{aligned}$$

We introduced the following abbreviations for the traces of the collision term:

$$\begin{aligned}
\mathcal{K}_0^s &= -\frac{1}{2}\Re\,\mathrm{Tr}\,\mathbb{1}P_s\mathcal{C}_\psi \quad (2.80)\\
\mathcal{K}_1^s &= -\frac{1}{2}\Re\,\mathrm{Tr}\,\gamma^3\gamma^5 P_s\mathcal{C}_\psi \quad (2.81)\\
\mathcal{K}_2^s &= -\frac{1}{2}\Re\,\mathrm{Tr}\,(-i\gamma^3)P_s\mathcal{C}_\psi \quad (2.82)\\
\mathcal{K}_3^s &= -\frac{1}{2}\Re\,\mathrm{Tr}\,(-\gamma^5)P_s\mathcal{C}_\psi\,. \quad (2.83)
\end{aligned}$$

The $\mathcal{C}_a^s$ are defined analogously, just with the real part replaced by imaginary part. All these collisional contributions are at least of first order in gradients, since at zeroth order the collision term vanishes as a consequence of the KMS relation, as we will see later.

From here on we proceed like in the 1+1 case, just that the equations are a bit more complicated and we have a collision term. Iterative use of the constraint equations (2.77)-(2.79) allows us to express $g_{i=1,2,3}^s$ in terms of g_0^s. For the consistency of the system it is important to keep the second order terms and the collisional contributions here. We find

$$
\begin{aligned}
\tilde{k}_0 g_1^s &= \Bigg(m_R - \frac{s}{2\tilde{k}_0}\big(\partial_z m_I + m_I'\partial_{k_z} k_z\big) - \frac{1}{8} m_R'' \partial_{k_z}^2 \\
&\qquad - \frac{1}{4\tilde{k}_0^2}\big(m_I'|m|^2\theta'\partial_{k_z}^2 + \partial_z^2 m_R + \partial_z m_R' \partial_{k_z} k_z\big)\Bigg) g_0^s - \mathcal{C}_1^s
\end{aligned} \tag{2.84}
$$

$$
\begin{aligned}
\tilde{k}_0 g_2^s &= \Bigg(m_I + \frac{s}{2\tilde{k}_0}\big(\partial_z m_R + m_R'\partial_{k_z} k_z\big) - \frac{1}{8} m_I'' \partial_{k_z}^2 \\
&\qquad + \frac{1}{4\tilde{k}_0^2}\big(m_R'|m|^2\theta'\partial_{k_z}^2 - \partial_z^2 m_I + \partial_z m_I' \partial_{k_z} k_z\big)\Bigg) g_0^s - \mathcal{C}_2^s
\end{aligned} \tag{2.85}
$$

$$
\begin{aligned}
\tilde{k}_0 g_3^s &= \Bigg(s k_z + \frac{1}{2\tilde{k}_0}|m|^2\theta'\partial_{k_z} \\
&\qquad - \frac{s}{4\tilde{k}_0^2}\Big((m_R'^2 + m_I'^2)\partial_{k_z} k_z + (m_R'\partial_z m_R + m_I'\partial_z m_I)\partial_{k_z}\Big)\Bigg) g_0^s - \mathcal{C}_3^s\,.
\end{aligned} \tag{2.86}
$$

Note that all the derivatives inside the parentheses also act on g_0^s. The left hand side of the equation of motion for $G^>$ is the same as the one for $G^<$, so, except of different collisional parts, these equations hold for the component functions of $G^>$, too. We insert these expressions into the so far unused constraint equation (2.76) and obtain to first order in gradients

$$
\begin{aligned}
&\left(k^2 - |m|^2 + s\frac{1}{\tilde{k}_0}|m|^2\theta'\right) g_0^s \\
&\qquad = -\left(\tilde{k}_0\mathcal{C}_0^s + m_R\mathcal{C}_1^s + m_I\mathcal{C}_2^s + s k_z \mathcal{C}_3^s\right)\,.
\end{aligned} \tag{2.87}
$$

Except of the collisional contribution this is of course the same dispersion relation as found earlier (2.43). If we insert the relations (2.84)-(2.86) for g_i^s into equation (2.72) and consistently keep all terms up to second order in gradients, we obtain a kinetic equation for g_0^s alone:

$$
\frac{1}{2\tilde{k}_0}(k\cdot\partial)g_0^s - \frac{1}{2\tilde{k}_0}\left(\frac{1}{2}|m|^{2\prime}\partial_{k_z} g_0^s - \frac{1}{2\tilde{k}_0} s\left(|m|^2\theta'\right)'\partial_{k_z} g_0^s\right) = \mathcal{K}_0^s\,. \tag{2.88}
$$

In contrast to the constraint equation (2.87) we don't get collisional contributions from the g_i^s here, since the corresponding terms in (2.72) are already of first order in gradients.

Up to now we have used all constraint equations, but only one of the kinetic equations. If our treatment is consistent, then the remaining kinetic equations have to be equivalent to (2.88). This is indeed the case, even if at first sight after inserting (2.84)-(2.86) into (2.73)-(2.75) the equations look quite different. The explicit proof of the equivalence is given in appendix C.

The function g_0^s plays a special role. We cannot choose another function as basic function, because this would lead to singularities in the relations between the functions and in the kinetic equation.

2.2.2 Spin off-diagonal equations

In this subsection we examine the equations for those parts of the Wigner function which are off-diagonal in spin. We multiply the matrix equation (2.44) by the matrices $Q_s B$, perform the trace and take real and imaginary parts. Since the component functions g_a^s, $a = 4, 5, 6, 7$ of the off-diagonal part are not real, the resulting equations contain combinations of the form $g_a \pm ish_a$. We get rid of these combinations by finally summing or subtracting the equations for $s = \pm 1$, respectively. This yields 4 kinetic and 4 constraint equations for the functions g_a, and the same for h_a. As already mentioned above, it is not possible to decouple the equations for the g_a from those for the h_a. The full set of equations is listed in appendix D. When we talk about off-diagonal functions or equations in this section, then off-diagonal refers to spin and not to the mixing of different species of fermions as in the next section.

The strategy for the treatment of these equations is the same as for the diagonal ones: we try to reduce the number of independent functions by the use of the constraint equations and then derive kinetic and constraint equations for the remaining functions. The whole computation has to be performed consistently up to first order in $\hbar$. As we already have stated, the off-diagonal elements of the Wigner function describe the transition between different spin states and therfore are quantum effects. Consequently these off-diagonal elements are suppressed by one order of $\hbar$ when compared to the diagonal ones. Furthermore, by construction spin-mixing occurs only in the collision term, so the off-diagonal part is expected to be additionally suppressed by one order of the coupling constant with respect to the diagonal part. We can see this also explicitely in the equations: in thermal equilibrium the off-diagonal elements are zero, as can be seen for example by taking the off-diagonal projections of the equilibrium Wigner function (1.12): $P_s G^<_{eq} P_{-s} = 0$. The off-diagonal functions are sourced exclusively by the diagonal ones via the collision term, which is at least of first order in gradients, as we will see in the next chapter. In order to obtain a treatment which is consistent up to order $\hbar$ we can therefore work with equations that contain one order of gradients less than the corresponding ones for the diagonal functions.

The detailed computation is also given in the appendix D. Here we only want to state the results. It turns out that also in the off-diagonal part everything can be reduced to two basic functions. In contrast to the diagonal part, however, where g_0^s plays a special role, we have the choice to either use g_5, h_5 or g_6, h_6. We decided to choose g_5, h_5. The functions g_4, h_4 and g_6, h_6 are of the same order, while the functions g_7, h_7

are suppressed by one order of gradients. We find the following kinetic equation,

$$\begin{aligned} & \frac{1}{2\tilde{k}_0}(k\cdot\partial)g_5 - \frac{1}{4\tilde{k}_0}|m|^{2\prime}\partial_{k_z}g_5 \\ & \qquad + \frac{1}{2\tilde{k}_0}\frac{m_I'}{\tilde{k}_0^2 - m_I^2}\left(k_z m_I g_5 + \tilde{k}_0 m_R h_5\right) \\ = \;& \mathcal{K}_{g5} + \frac{m_I}{\tilde{k}_0}\mathcal{C}_{g7}\,, \end{aligned} \tag{2.89}$$

which is correct up to first order in gradients. The corresponding equation for h_5 is obtained by exchanging g and h and reversing the sign of m_R. In addition to the usual classical flow term we here have a term that couples the equations for g_5 and h_5 to each other. The constraint equation is, except of the collisional contribution, just the usual classical on-shell condition:

$$\begin{aligned} \left(k^2 - |m|^2\right) g_5 = & -(\tilde{k}_0 - \frac{m_I^2}{\tilde{k}_0})\mathcal{C}_{g5} \\ & - m_R(-\mathcal{C}_{g4} + \frac{m_I}{\tilde{k}_0}\mathcal{C}_{g6}) + k_z(-\mathcal{C}_{h6} + \frac{m_I}{\tilde{k}_0}\mathcal{C}_{h4})\,. \end{aligned} \tag{2.90}$$

The constraint equation for h_5 is obtained by exchanging g and h and replacing k_z by $-k_z$.

2.3 Mixing fermions

Before we turn our attention to the collision terms, we extend our analysis to the case of N fermionic species which are coupled by a mass matrix. This kind of system is present in the quark mixing in the standard model or in SUSY models, where the coupling to the Higgs field gives rise to such a mass matrix. We restrict our discussion to the spin diagonal part of the Wigner function.

The flavor degree of freedom is denoted by an additional index i to the spinor $\psi_{\alpha,i}(x)$, and the Green function becomes a matrix in the product space of spinor and flavor space:

$$G^<_{\alpha\beta,ij}(x,y) = i\left\langle\bar{\psi}_{\beta,j}(y)\psi_{\alpha,i}(x)\right\rangle\,, \tag{2.91}$$

for which the hermiticity property (1.52)

$$\left(i\gamma^0 G^<(x,y)\right)^\dagger = i\gamma^0 G^<(y,x) \tag{2.92}$$

still holds, where the hermitean conjugate now has to be taken in both spinor and flavor space. The Lagrangian for such a system is formally the same as in the non-mixing case

$$\mathcal{L} = i\bar{\psi}\,\partial\!\!\!/\,\psi - \bar{\psi}_L M\psi_R - \bar{\psi}_R M^\dagger\psi_L + \mathcal{L}_{\rm int}\,, \tag{2.93}$$

but here the mass M is a complex $N \times N$ matrix, whose nondiagonal elements couple fields of different flavor, and which in general is non hermitean. Its components are z-dependent, due to the varying Higgs vacuum expectation value. The interaction part of the Lagrangian is

$$\mathcal{L}_{\text{int}} = -\phi^l \bar{\psi}_L y^l \psi_R + \text{h.c.}\,, \tag{2.94}$$

where y^l is a matrix in flavor space and the index l denotes the different scalar particles. We can rewrite the mass term

$$\begin{aligned} \bar{\psi}_L M \psi_R + \bar{\psi}_R M^\dagger \psi_L &= \bar{\psi}\left(P_R \otimes M + P_L \otimes M^\dagger\right)\psi &(2.95)\\ &= \bar{\psi}\left(\mathbb{1} \otimes M_R + i\gamma^5 \otimes M_I\right)\psi\,, &(2.96) \end{aligned}$$

where we introduced the hermitean and anti-hermitean part of the mass matrix, respectively,

$$M_R = \frac{1}{2}(M + M^\dagger) \quad , \quad M_I = \frac{1}{2i}(M - M^\dagger)\,. \tag{2.97}$$

Here $\otimes$ denotes the external product of a matrix in spinor space with one in flavor space. Similarly we can rewrite the interaction part

$$\phi^l \bar{\psi}_L y^l \psi_R + \text{h.c.} = \bar{\psi}\left(P_R \otimes \phi^l y^l + P_L \otimes \phi^{l*} y^{l\dagger}\right)\psi\,. \tag{2.98}$$

The derivation of the equation of motion for the Green function works as in the one field case, except of additional indices denoting the flavor. We find

$$\left(\not{k} \otimes \mathbb{1} + \frac{i}{2}\not{\partial} \otimes \mathbb{1} - (P_R \otimes M + P_L \otimes M^\dagger)\mathrm{e}^{\frac{i}{2}\overleftarrow{\partial}_z \cdot \partial_{k_z}}\right) G^< = \mathcal{C}_\psi\,. \tag{2.99}$$

Like in the treatment of the scalar equation we diagonalize the mass. As we already have mentioned, M in general is not hermitean, so the diagonalization this time has to be done by a biunitary transformation:

$$M_d = U M V^\dagger\,. \tag{2.100}$$

The unitary matrices U and V diagonalize the hermitean matrices $MM^\dagger$ and $M^\dagger M$, respectively. The diagonalization of the flavor part of the complete mass matrix is done by

$$P_R \otimes M + P_L \otimes M^\dagger = \mathbf{M} = \mathbf{X}^\dagger \mathbf{M}_d \mathbf{Y}\,, \tag{2.101}$$

where the unitary rotation matrices in the full space are

$$\mathbf{X} = P_L \otimes V + P_R \otimes U \quad , \quad \mathbf{Y} = P_L \otimes U + P_R \otimes V\,. \tag{2.102}$$

The rotated Green function, which is not diagonal, is defined by

$$G_d = \mathbf{Y} G \mathbf{X}^\dagger\,. \tag{2.103}$$

Note that the component functions $g^s_{a,d}$ of the rotated Green function G^s_d, which are defined analogously to (2.58), are not just the flavor rotated components of G. This is a consequence of the fact that because of the axial contribution to the mass the complete mass term $M_R + i\gamma^5 M_I$ cannot be written as a direct product of spinor times flavor. The diagonalization of the mass then introduces a mixing of the two structures. We expect that the functions multiplying $\gamma^3\gamma^5$ and γ^3 mix with each other, as well as those multiplying $\mathbb{1}$ and γ^5. Indeed, inserting (2.58) into (2.103) leads to the following relations:

$$
\begin{aligned}
g^s_0 &= \frac{1}{2}\left(V^\dagger(g^s_{0,d} + g^s_{3,d})V + U^\dagger(g^s_{0,d} - g^s_{3,d})U\right) && (2.104)\\
g^s_3 &= \frac{1}{2}\left(-V^\dagger(g^s_{0,d} + g^s_{3,d})V + U^\dagger(g^s_{0,d} - g^s_{3,d})U\right) && (2.105)\\
g^s_1 &= \frac{1}{2}\left(U^\dagger(g^s_{1,d} - ig^s_{2,d})V + \text{h.c.}\right) && (2.106)\\
g^s_2 &= \frac{1}{2}\left(U^\dagger(g^s_{2,d} + ig^s_{1,d})V + \text{h.c.}\right) . && (2.107)
\end{aligned}
$$

Now we apply the diagonalization to the equation of motion (2.99). We replace G by $\mathbf{Y}^\dagger G_d \mathbf{X}$ and multiply the equation from the left with $\mathbf{X}$ and from the right with $\mathbf{X}^\dagger$:

$$
\begin{aligned}
&\left(\not{k} + \frac{i}{2}\not{\partial}\right) \otimes \mathbb{1}\; G_d - \mathbf{X}\mathbf{M}\mathrm{e}^{\frac{i}{2}\overleftarrow{\partial}_z\cdot\partial_{k_z}}\mathbf{Y}^\dagger G_d\\
&\qquad + \frac{i}{2}(\gamma^\mu \otimes \mathbb{1})\left((\mathbf{Y}\partial_\mu\mathbf{Y}^\dagger)G_d - G_d(\mathbf{X}\partial_\mu\mathbf{X}^\dagger)\right) = \mathbf{X}\mathcal{C}_\psi\mathbf{X}^\dagger . \qquad (2.108)
\end{aligned}
$$

We project this equation into flavor space by taking the spinorial traces as we did in section 2.2.1. The result is very similar to (2.67)-(2.70), but contains some extra terms, which are due to the fact that the derivatives of the differential operator also act on the rotation matrices:

$$
\begin{aligned}
&2i\hat{k}_0 g^s_{0,d} - 2i\hat{M}_R g^s_{1,d} - 2i\hat{M}_I g^s_{2,d} - 2is\hat{k}_3 g^s_{3,d} && (2.109)\\
&\qquad + is\left[\Delta_z, g^s_{0,d}\right] + is\left[\Sigma_z, g^s_{3,d}\right] = \mathrm{Tr}P_s(k)\mathbb{1}\mathbf{X}\mathcal{C}_\psi\mathbf{X}^\dagger\\
&2i\hat{k}_0 g^s_{1,d} - 2i\hat{M}_R g^s_{0,d} + 2\hat{M}_I g^s_{3,d} - 2s\hat{k}_3 g^s_{2,d} && (2.110)\\
&\qquad + is\left\{\Delta_z, g^s_{1,d}\right\} + s\left[\Sigma_z, g^s_{2,d}\right] = \mathrm{Tr}P_s(k)s\gamma^3\gamma^5\mathbf{X}\mathcal{C}_\psi\mathbf{X}^\dagger\\
&2i\hat{k}_0 g^s_{2,d} - 2\hat{M}_R g^s_{3,d} - 2i\hat{M}_I g^s_{0,d} + 2s\hat{k}_3 g^s_{1,d} && (2.111)\\
&\qquad + is\left\{\Delta_z, g^s_{2,d}\right\} - s\left[\Sigma_z, g^s_{1,d}\right] = \mathrm{Tr}P_s(k)\left(-is\gamma^3\right)\mathbf{X}\mathcal{C}_\psi\mathbf{X}^\dagger\\
&2i\hat{k}_0 g^s_{3,d} + 2\hat{M}_R g^s_{2,d} - 2\hat{M}_I g^s_{1,d} - 2is\hat{k}_3 g^s_{0,d} && (2.112)\\
&\qquad + is\left[\Delta_z, g^s_{3,d}\right] + is\left[\Sigma_z, g^s_{0,d}\right] = \mathrm{Tr}P_s(k)\left(-\gamma^5\right)\mathbf{X}\mathcal{C}_\psi\mathbf{X}^\dagger .
\end{aligned}
$$

Here we used again the shorthand notations (2.71) and defined

$$
\begin{aligned}
\Delta_z &= \frac{i}{2}\left(V\partial_z V^\dagger - U\partial_z U^\dagger\right) && (2.113)\\
\Sigma_z &= \frac{i}{2}\left(V\partial_z V^\dagger + U\partial_z U^\dagger\right) . && (2.114)
\end{aligned}
$$

The choice of the rotation matrices U and V is not unique. After a z-dependent phase redefinition $U \to wU$ and $V \to wV$, where w is a diagonal matrix with eigenvalues of absolute value 1, U and V still diagonalize M. This freedom to redefine the rotation matrices was the source of some problems in finding the correct physical source in the WKB approach. We will find, however, that in the end only the diagonal elements of Δ_z are of relevance, which are invariant under this reparametrization. The mass terms in the equations are:

$$\begin{aligned}
\hat{M}_{R,d} &= \frac{1}{2}\left(UM\mathrm{e}^{\frac{i}{2}\overleftarrow{\partial}_z\cdot\partial_{k_z}}V^\dagger + VM^\dagger\mathrm{e}^{\frac{i}{2}\overleftarrow{\partial}_z\cdot\partial_{k_z}}U^\dagger\right), &(2.115)\\
\hat{M}_{I,d} &= \frac{1}{2i}\left(UM\mathrm{e}^{\frac{i}{2}\overleftarrow{\partial}_z\cdot\partial_{k_z}}V^\dagger - VM^\dagger\mathrm{e}^{\frac{i}{2}\overleftarrow{\partial}_z\cdot\partial_{k_z}}U^\dagger\right). &(2.116)
\end{aligned}$$

Note that these masses are diagonal only in the leading order.

Like in the scalar case the constraint and kinetic equations are obtained by taking the hermitean and antihermitean parts of the equations (2.109)-(2.112), respectively. We are again only interested in the diagonal equations, describing the behavior of the quasiclassical particle densities. Therefore we can neglect all commutator terms appearing in these equations, the detailed argument for this is given in the treatment of the mixing scalar particles in section 1.5. The constraint equations correct up to first order in gradients are given by

$$\begin{aligned}
2\tilde{k}_0 g^s_{0,d} - 2sk_z g^s_{3,d} - 2M_R g^s_{1,d} - 2M_I g^s_{2,d} &= 0 &(2.117)\\
2\tilde{k}_0 g^s_{1,d} + 2s\Delta_z g^s_{1,d} - 2M_R g^s_{0,d} + (M_I' + 2M_R\Delta_z)\partial_{k_z} g^s_{3,d} + s\partial_z g^s_{2,d} &= 0 &(2.118)\\
2\tilde{k}_0 g^s_{2,d} + 2s\Delta_z g^s_{2,d} - 2M_I g^s_{0,d} - (M_R' - 2M_I\Delta_z)\partial_{k_z} g^s_{3,d} - s\partial_z g^s_{1,d} &= 0 &(2.119)\\
2\tilde{k}_0 g^s_{3,d} - 2sk_z g^s_{0,d} + (M_R' - 2M_I\Delta_z)\partial_{k_z} g^s_{2,d} - (M_I' + 2M_R\Delta_z)\partial_{k_z} g^s_{1,d} &= 0 &(2.120)
\end{aligned}$$

All the mass matrices here and in the following are the diagonalized ones, we omit the index d for simplicity. We know already from the non-mixing case that the collisional contributions to the constraint equations are irrelevant for the derivation of the dispersion relation and kinetic equation for $g^s_{0,d}$, so we just set them to zero here. We use equations (2.118)-(2.120) iteratively in order to express the functions $g^s_{i,d}$ in terms of $g^s_{0,d}$ consistently up to first order in gradients. The result is

$$\begin{aligned}
\tilde{k}_0 g^s_{1,d} &= \left(M_R - \frac{s}{2\tilde{k}_0}(\partial_z M_I + 2\Delta_z M_R) - \frac{s}{2\tilde{k}_0}(M_I' + 2\Delta_z M_R)\partial_{k_z} k_z\right) g^s_{0,d} &(2.121)\\
\tilde{k}_0 g^s_{2,d} &= \left(M_I + \frac{s}{2\tilde{k}_0}(\partial_z M_R - 2\Delta_z M_I) - \frac{s}{2\tilde{k}_0}(M_R' - 2\Delta_z M_I)\partial_{k_z} k_z\right) g^s_{0,d} &(2.122)\\
\tilde{k}_0 g^s_{3,d} &= \left(sk_z + \frac{1}{2\tilde{k}_0}\big(M_R M_I' - M_R' M_I + 2(M_R^2 + M_I^2)\Delta_z\big)\partial_{k_z}\right) g^s_{0,d}\,. &(2.123)
\end{aligned}$$

We insert these relations into the first constraint equation and use the definition

$(|M|^2\Theta')_d = M_R M_I' - M_R' M_I + 2(M_R^2 + M_I^2)\Delta_z$ to obtain

$$\Big(k^2 - |M|_d^2 + \frac{s}{\tilde{k}_0}(|M|^2\Theta')_d\Big) g_{0,d}^s = 0\,. \tag{2.124}$$

From the non-mixing case we furthermore know that it is sufficient to work only with the first kinetic equation, which is the hermitean part of (2.109). Inserting relations (2.121)-(2.123) leads finally to the kinetic equation

$$\begin{aligned}\frac{1}{2\tilde{k}_0} k\cdot\partial g_{0,d}^s - \frac{1}{2\tilde{k}_0}\left(|M^2|_d' - \frac{s}{\tilde{k}_0}\left(|M|^2\Theta'\right)_d'\right)\partial_{k_z} g_{0,d}^s \\ = \frac{1}{2}\left(\mathrm{Tr}P_s(k)\mathbf{X}\mathcal{C}_\psi\mathbf{X}^\dagger + \mathrm{h.c.}\right).\end{aligned} \tag{2.125}$$

The diagonal elements of $|M|_d^2$ are just the eigenvalues of $M^\dagger M$. Let us finally note that for the diagonal elements we can rewrite

$$(|M|^2\Theta')_d = -\frac{1}{2}\Im\left(U(MM'^\dagger - M'M^\dagger)U^\dagger\right)\,, \tag{2.126}$$

which will be convenient in the following.

2.3.1 MSSM

Now that we have a general expression for the semiclassical source in the case of mixing fermions, we want to study two explicit examples, which are of relevance for baryogenesis. First we compute the source in the transport equations for the chargino sector of the MSSM. The chargino mass term reads

$$\overline{\psi}_R\, M\, \psi_L + \mathrm{h.c.}\,, \tag{2.127}$$

where $\psi_R = (\tilde{W}_R^+, \tilde{h}_{1,R}^+)^T$ and $\psi_L = (\tilde{W}_L^+, \tilde{h}_{2,L}^+)^T$ are the chiral fields in the basis of winos. The mass matrix is

$$M = \begin{pmatrix} m_2 & gH_2^* \\ gH_1^* & \mu \end{pmatrix}\,, \tag{2.128}$$

where H_1 and H_2 are the Higgs field vacuum expectation values and μ and m_2 are the soft supersymmetry breaking parameters, which introduce CP-violation[1]. Since for a reasonable choice of parameters there is no transitional CP-violation in the MSSM, that is CP-violation that only occurs during the phase transition, we can take the Higgs expectation values H_1 and H_2 to be real [41, 42]. The matrix that diagonalizes $MM^\dagger$ can be parameterized as [40]

$$U = \frac{\sqrt{2}}{\sqrt{\Lambda(\Lambda+\Delta)}}\begin{pmatrix} \frac{1}{2}(\Lambda+\Delta) & a \\ -a^* & \frac{1}{2}(\Lambda+\Delta) \end{pmatrix}\,, \tag{2.129}$$

[1] We keep the possibility of complex Higgs vev's, because we will reuse the formulas in the NMSSM case.

where

$$\begin{aligned} a &= g(m_2 H_1 + \mu^* H_2^*) \\ \Delta &= |m_2|^2 - |\mu|^2 + g^2(h_2^2 - h_1^2) \\ \Lambda &= \sqrt{\Delta^2 + 4|a|^2}\,, \end{aligned} \tag{2.130}$$

and $h_i = |H_i|$. The mass eigenvalues of the charginos are given by

$$m_\pm^2 = \frac{1}{2}\left(|m_2|^2 + |\mu|^2 + g^2(h_1^2 + h_2^2)\right) \pm \frac{\Lambda}{2}\,. \tag{2.131}$$

Upon inserting (2.128) and (2.129),(2.130) into (2.126) it is straightforward to show that the source term for the charginos can be recast as

$$\left(|M|^2\Theta'\right)_{d,\pm} = \mp\frac{g^2}{\Lambda}\Im(\mu m_2)\,(h_1 h_2)'\ . \tag{2.132}$$

This result agrees with the one found in [40]. In [37], however, a different dependence on the Higgs fields was obtained.

2.3.2 NMSSM

In the NMSSM there is an additional singlet field S in the Higgs sector. One consequence of this extension is the possibility to have spontaneous transitional CP-violation, so we can no longer assume the Higgs vacuum expectation values to be real. The singlet couples to higgsinos, and therefore we obtain the mass matrix by generalizing the higgsino-higgsino component of the chargino mass matrix (2.128)

$$\mu \to \tilde{\mu} = \mu + \lambda S\,, \tag{2.133}$$

where λ is the coupling for the higgsino-higgsino-singlet interaction. The field content we will consider is the same as in the MSSM, so the mass matrix is

$$M = \begin{pmatrix} m_2 & gH_2^* \\ gH_1^* & \tilde{\mu} \end{pmatrix}\,. \tag{2.134}$$

This matrix is still diagonalized by U. We write the Higgs expectation values as

$$H_i = h_i \mathrm{e}^{i\theta_i} \quad , \quad i = 1, 2\,, \tag{2.135}$$

where only one phase is physical. With the gauge constraint [68]

$$h_1^2\theta_1' = h_2^2\theta_2' \tag{2.136}$$

we can write

$$\theta_1' = \frac{h_2^2}{h_1^2 + h_2^2}\theta' \quad , \quad \theta_2' = \frac{h_1^2}{h_1^2 + h_2^2}\theta'\,, \tag{2.137}$$

where $\theta = \theta_1 + \theta_2$ is the physical CP-violating phase. Now everything is prepared to write the NMSSM-source term. It can be divided into three contributions, which have to be added. The first one is a generalization of the chargino source (2.132)

$$\left(|M|^2\Theta'\right)_{d,h_1h_2,\pm} = \mp\frac{g^2}{\Lambda}\Im(\tilde{\mu}m_2 e^{i\theta})(h_1h_2)' \tag{2.138}$$

for the case involving a new scalar field S and complex higgs expectation values. In addition to this there are two new types of sources. One of them is proportional to a derivative of the CP-violating phase θ in the Higgs sector:

$$\left(|M|^2\Theta'\right)_{d,\theta,\pm} = -\frac{g^2\theta'}{\Lambda}\left(\left(\Lambda \pm (|m_2|^2 + |\tilde{\mu}|^2)\right)\frac{h_1^2h_2^2}{h_1^2+h_2^2} \mp \Re(\tilde{\mu}m_2 e^{i\theta})h_1h_2\right). \tag{2.139}$$

Finally there is a source that can be written as a derivative of the singlet condensate:

$$\begin{aligned}\left(|M|^2\Theta'\right)_{d,S,\pm} &= \pm\frac{\lambda g^2}{\Lambda}\Im(m_2H_1H_2S') \\ &\quad +\frac{\lambda g^2}{2\Lambda}\left(\Lambda \pm (|\tilde{\mu}|^2 + g^2(H_1^2+h_2^2) - |m_2|^2)\right)\Im(\tilde{\mu}^*S').\end{aligned} \tag{2.140}$$

The mass eigenvalues $m_\pm$, that is the diagonal elements of $|M|_d^2$, can be obtained from the corresponding expression (2.131) in the MSSM part with the replacement $\mu \to \tilde{\mu}$.

3. Collision term

In the previous part of the work we investigated the interaction of the particles in the plasma with the bubble wall, that is the direct influence of a spatially varying mass on the Wigner functions. This chapter is devoted to a detailed study of the interactions of the different particle species with each other, in our model this is the Yukawa coupling between the fermions and the scalars [69].

In chapter 1 we derived the equations of motion for bosonic and fermionic Green functions. In the Wigner representation the equation for bosons, for example, is given by

$$\begin{aligned} &\mathrm{e}^{-i\diamond}\left\{k^2-m^2\right\}\left\{\Delta^<\right\}-\mathrm{e}^{-i\diamond}\left\{\Pi_R\right\}\left\{\Delta^<\right\} \\ &\quad -\mathrm{e}^{-i\diamond}\left\{\Pi^<\right\}\left\{\Delta_R\right\}=\frac{1}{2}\mathrm{e}^{-i\diamond}\big(\{\Pi^>\}\{\Delta^<\}-\{\Pi^<\}\{\Delta^>\}\big)\,, \end{aligned} \tag{3.1}$$

where the last term is the collision term, which we denoted by $\mathcal{C}_\phi$ in the previous chapters. We already stated that we neglect the term $\mathrm{e}^{-i\diamond}\left\{\Pi_R\right\}\left\{\Delta^<\right\}$, the self energy correction to the mass. We expect that it can be treated in a similar manner as the collision term here. The other term, $\mathrm{e}^{-i\diamond}\left\{\Pi^<\right\}\left\{\Delta_R\right\}$, is more dangerous, because it essentially leads to a broadening of the spectrum and therefore to a breakdown of the quasiparticle picture. There are indications that in the equation for scalars this term has only contributions to the constraint equation and therefore can be neglected. We just assume that this is correct and also true for the fermionic equation and neglect these terms in our treatment. We furthermore assume that the Yukawa coupling is small, so that the dominant effect is the interaction with the wall. In this case it is justified to truncate the gradient expansion in the collision term already after the first order, since we have an additional suppression by the coupling constant.

When we expand the exponentiated diamond operator (1.56) in the collision term, we obtain two contributions, in the following referred to as zeroth and first order collision term, respectively (although the "zeroth order" contribution in fact is of first order in gradients, too, as we will see):

$$\begin{aligned} \mathcal{C}_\phi &= \mathcal{C}_\phi^{(0)}+\mathcal{C}_\phi^{(1)} \\ &= \frac{1}{2}\Big(\Pi^>\Delta^<-\Pi^<\Delta^>\Big) \\ &\quad -\frac{i}{2}\diamond\Big(\left\{\Pi^>\right\}\left\{\Delta^<\right\}-\left\{\Pi^<\right\}\left\{\Delta^>\right\}\Big)\,. \end{aligned} \tag{3.2}$$

First we point out that any contribution of the collision term is of first order in gradients, a fact that we already used several times. This is obvious for $\mathcal{C}_\phi^{(1)}$. In the case of $\mathcal{C}_\phi^{(0)}$ we note that this term vanishes if we insert the leading order Wigner functions of our problem. These leading order Wigner functions are simply the equilibrium expressions (1.15)-(1.18) with a varying mass. We will denote them in the following with Δ_{eq}, although they are not really Green functions in thermal equilibrium. It is not hard to check that they are the solution of our equations of motion to zeroth order in gradients. The fact that $\mathcal{C}_\phi^{(0)}$ vanishes is a well known consequence of the KMS relation (1.20), which also holds for the leading order Wigner functions, and can be seen as follows: the self energy Π consists of products of scalar and an even number of fermionic Wigner functions. At leading order we can use the KMS relation in order to reverse the directions of the greater and less symbols in each of the Wigner functions, but we get additional exponential factors. Since we have momentum conservation in the self energy, all these exponential factors can be combined to a single one containing the momentum of the self energy:

$$\Pi_{eq}^{>}(k) = \mathrm{e}^{\beta \bar{k}_0} \Pi_{eq}^{<}(k) \,. \tag{3.3}$$

That is, at leading order the self energy itself satisfies the KMS relation. We will see this in detail below. Note that we still work in the wall frame. Since the plasma is at rest in the plasma frame, the exponential factor in the KMS relation contains the momentum $\bar{k}_0$, see appendix E. But now we can use the KMS relation for $\Pi^>$ and $\Delta^<$ in the first term of $\mathcal{C}_\phi^{(0)}$:

$$\begin{aligned} \mathcal{C}_{\phi,eq}^{(0)} &= \frac{1}{2}\Big(\Pi_{eq}^{>}(k)\Delta_{eq}^{<}(k) - \Pi_{eq}^{<}(k)\Delta_{eq}^{>}(k)\Big) & (3.4)\\ &= \frac{1}{2}\Big(\mathrm{e}^{\beta\bar{k}_0}\Pi_{eq}^{<}(k)\mathrm{e}^{-\beta\bar{k}_0}\Delta_{eq}^{>}(k) - \Pi_{eq}^{<}(k)\Delta_{eq}^{>}(k)\Big) = 0\,. & (3.5)\end{aligned}$$

Indeed the zeroth order collision term vanishes at leading order in gradients. In thermal equilibrium the whole collision term has to vanish, this is part of the definition of thermal equilibrium. Because of the varying mass, however, the first order collision term is not zero when the leading order Wigner functions are inserted, which makes clear that they are not really equilibrium functions. Everything we said here holds as well in the fermionic case, of course.

The full scalar Green function deviates from the leading order expression,

$$\Delta^{<,>}(k) = \Delta_{eq}^{<,>}(k) + \delta\Delta^{<,>}(k) \,, \tag{3.6}$$

which is clearly a consequence of the interaction with the wall, and therefore the correction $\delta\Delta$ is at least of first order in gradients. The full fermionic Wigner function differs in two ways from the leading order expressions (1.12) and (1.13). First, there are derivative corrections in the equations (2.84)-(2.86) which relate the components g_i^s with g_0^s for $i = 1, 2, 3$. Second, the function g_0^s itself contains a correction to its leading order value

$$g_0^{<,>s}(X,k) = g_{eq}^{<,>}(X,k) + \delta g^{<,>s}(X,k) \tag{3.7}$$

like in the scalar case, and the components of the spin off-diagonal part are first order anyway, as we argued in 2.2.2. Only terms that contain one of these corrections will survive in the collision term, and therefore it is first order, as claimed above.

The computation of the zeroth order collision term will be done as follows: first we write down an "extended version" of the KMS relation for the full Green function, which will be the standard KMS relation plus corrections due to the above mentioned deviations from the leading order expressions:

$$\Delta^>(k) = \mathrm{e}^{\beta \bar{k}_0} \Delta^<(k) + \text{corrections} \tag{3.8}$$

and likewise for the fermionic G. Then we use these relations for the Wigner functions in the first term of $\mathcal{C}^{(0)}$. The uncorrected part will cancel against the second term, and we are left with the first order contributions. In the scalar case the "extended KMS" is quite simple:

$$\begin{aligned}
\Delta^>(k) &= \Delta_{eq}^>(k) + \delta\Delta^>(k) \\
&= \mathrm{e}^{\beta \bar{k}_0} \Delta_{eq}^<(k) + \mathrm{e}^{\beta \bar{k}_0} \delta\Delta^<(k) - \mathrm{e}^{\beta \bar{k}_0} \delta\Delta^<(k) + \delta\Delta^>(k) \\
&= \mathrm{e}^{\beta \bar{k}_0} \Delta^<(k) + \Big(\delta\Delta^>(k) - \mathrm{e}^{\beta \bar{k}_0} \delta\Delta^<(k) \Big) \,.
\end{aligned} \tag{3.9}$$

The fermionic case is slightly more complicated, since we have two sources for corrections. We will only look at the spin diagonal part, as stated above. The functions which appear in the decomposition

$$G^{<s,s} = iP_s(k) \left[s\gamma^3\gamma^5 g_0^{<s} - s\gamma^3 g_3^{<s} + \mathbb{1} g_1^{<s} - i\gamma^5 g_2^{<s} \right] \tag{3.10}$$

can all be related to $g_0^{<s}$ by

$$g_i^{<s} = (c_i^s + d_i^s) g_0^{<s} \quad , \quad i = 1, 2, 3 \,, \tag{3.11}$$

where c_i are numbers and d_i are first order derivatives, they can be read off from equations (2.84)-(2.86). For functions $g_i^{<s}$ that appear in the collision term we neglect the collisional contributions $\mathcal{C}_i$ to these relations, because they would lead to terms which are second order in the coupling constant and therefore can be neglected in our approximation.

For the basic function g_0^s we use the decomposition (3.7), where the leading order part satisfies the KMS relation, so when we relate $g_i^{>s}$ to $g_i^{<s}$ we have

$$\begin{aligned}
g_i^{<s} &= (c_i^s + d_i^s)(-\mathrm{e}^{-\beta\bar{k}_0} g^{>} + \delta g_0^{<s}) \\
&= -\mathrm{e}^{-\beta\bar{k}_0}(c_i^s + d_i^s) g^{>} - (d_i^s \mathrm{e}^{-\beta\bar{k}_0}) g^{>} + c_i^s \delta g_0^{<s} \\
&= -\mathrm{e}^{-\beta\bar{k}_0} g_i^{>s} + (d_i^s \beta\bar{k}_0)\mathrm{e}^{-\beta\bar{k}_0} g^{>} + c_i^s \left(\delta g_0^{<s} + \mathrm{e}^{-\beta\bar{k}_0}\delta g_0^{>s}\right) ,
\end{aligned} \tag{3.12}$$

where we neglected terms of the form $d_i \delta g$, because they are second order. The corrections in the second term arise when the derivatives in the relations between the g_i and the g_0 act on the exponential factor from the KMS relation. If we insert this into (3.10) we obtain

$$\begin{aligned}
G^{<s,s} = & -\mathrm{e}^{-\beta\bar{k}_0} G^{>s,s} \\
& + iP_s(k)\left(\left[-s\gamma^3 d_3^s + \mathbb{1} d_1^s - i\gamma^5 d_2^s\right]\beta\bar{k}_0\right)\mathrm{e}^{-\beta\bar{k}_0} g^{>} \\
& + iP_s(k)\left[s\gamma^3\gamma^5 - s\gamma^3 c_3^s + \mathbb{1} c_1^s - i\gamma^5 c_2^s\right]\left(\delta g_0^{s<} + \mathrm{e}^{-\beta\bar{k}_0}\delta g_0^{s>}\right) .
\end{aligned} \tag{3.13}$$

The first term here is the usual KMS-part. The second one describes the first order corrections because of the nontrivial relation between g_i^s and g_0^s, and the last one is due to first order corrections within g_0^s.

When we use the relations (3.9) and (3.13) in the calculation of the collision term we find two different effects. The terms that contain $\delta\Delta$ or δg are the usual gain and loss terms that lead to relaxation if the system is out of equilibrium. The other terms are present even if we set the deviations to zero, so they are source terms which may be CP-violating. They are called spontaneous baryogenesis sources [30].

One could raise the question if it is possible to choose the functions $\delta\Delta$ and δg in such a way that the usual KMS relation holds also for $\Delta^{<,>}$ in (3.6) and $G^{<,>}$ in (3.7). It is no problem to find a suitable $\delta\Delta$ that would make the correction in (3.9) vanish, for the function δg this is not possible, however. The reason is that the matrix structure of the fermionic Wigner function dictated by the constraint equations is not compatible with the KMS relation. This becomes immediately clear from equation (3.13): the matrices P_s, $P_s\gamma^3$, $P_s\gamma^5$ and $P_s\gamma^3\gamma^5$ are linearly independent, so the freedom to choose the scalar functions $\delta g^{<}$ and $\delta g^{>}$ is not enough to make the correction terms cancel each other. If this worked, then one could, just by redefinition of the leading order functions, move the sources from the collision term to the flow term. But with the above argument it is clear that this is not possible.

Now we want to calculate the collision terms. The self energies appearing in the scalar and fermionic collision terms are the functional derivatives of the two loop part Γ_2 of the 2PI effective action , so we first have to compute this quantity. Γ_2 is the sum of all two-particle irreducible vacuum graphs with vertices defined by the interaction

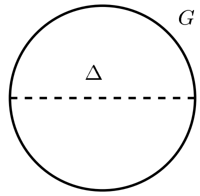

Figure 3.1: Two-loop contribution to the 2PI effective action. The solid line represents the fermionic propagator, the dashed line is the scalar Wigner function.

part of the classical effective action

$$\begin{aligned} I_{int} &= \int d^4z\mathcal{L}_{\rm int}(z)\,, \\ \mathcal{L}_{\rm int} &= -y\phi\,\bar{\psi}_L\psi_R - y^*\phi^*\,\bar{\psi}_R\psi_L \\ &= -\bar{\psi}\,(P_R\,y\phi + P_L\,y^*\phi^*)\,\psi \end{aligned} \tag{3.14}$$

and propagators set equal to Δ or G, respectively. We calculate Γ_2 in a loop expansion and truncate it after the first nonvanishing contribution, which is the two loop diagram shown in figure 3:

$$\begin{aligned} \Gamma_2(\Delta,G) &= -y^2\int_C d^4u\,d^4v\,\mathrm{Tr}\,[P_RG(u,v)P_LG(v,u)]\,\Delta(u,v) \\ &= -y^2\sum_{ab}\int d^4u\,d^4v\,ab\,\mathrm{Tr}\,\left[P_RG^{ab}(u,v)P_LG^{ba}(v,u)\right]\Delta^{ab}(u,v)\,. \end{aligned} \tag{3.15}$$

3.1 Scalar collision term

We begin our investigation with the collision term for the scalar equation of motion. To this end we need the scalar self energy Π, which is obtained by taking the functional derivative of Γ_2 with respect to the scalar Wigner function Δ. The result is

$$\begin{aligned} \Pi^{ab}(u,v) &= i\frac{\delta\Gamma_2[G,\Delta]}{ab\,\delta\Delta^{ba}(v,u)} \\ &= -iy^2\,\mathrm{Tr}\left(P_RG^{ba}(v,u)P_LG^{ab}(u,v)\right)\,. \end{aligned} \tag{3.16}$$

Since we are working in the Wigner representation, we have to transform this expression, and we furthermore choose $ab = +-/-+$:

$$\begin{aligned} \Pi^{<,>}(X,k) &= -iy^2\int\frac{d^4k'd^4k''}{(2\pi)^8}(2\pi)^4\delta^4(k+k'-k'') \\ &\qquad \mathrm{Tr}\,(P_RG^{>,<}(X,k')P_LG^{<,>}(X,k''))\,. \end{aligned} \tag{3.17}$$

We have a closer look at the trace in the end of this expression, where fermionic Wigner functions are sandwiched between chiral projectors. We only use the spin diagonal part of the Wigner function and insert the decomposition (3.10). Since the spin projectors in (3.10) commute with the chiral projectors $P_{L/R}$, we can move them to the front. Furthermore we note that the chiral projectors commute with γ^5, while moving P_L past γ^3 turns it into P_R and vice versa:

$$\begin{aligned} & P_R\left[s\gamma^3\gamma^5 g_0^{<s} - s\gamma^3 g_3^{<s} + g_1^{<s} - i\gamma^5 g_2^{<s}\right] P_L \\ = \; & P_R\left[s\gamma^3\gamma^5 g_0^{<s} - s\gamma^3 g_3^{<s}\right] \\ = \; & P_R s\gamma^3\left[-g_0^{<s} - g_3^{<s}\right] . \end{aligned} \tag{3.18}$$

So only the terms with γ^3 survive, and in the last line we used $P_R\gamma^5 = P_R$. The trace becomes

$$\begin{aligned} & \sum_{s',s''} \mathrm{Tr}\left(P_R G^{<s',s'}(X,k') P_L G^{>s'',s''}(X,k'')\right) \\ = & \sum_{s',s''} s's''\,\mathrm{Tr}\big(P_R P_{s'}(k') P_{s''}(k'')\big)\,[g_0^{<s'}(k') + g_3^{<s'}(k')][g_0^{>s''}(k'') - g_3^{>s''}(k'')] . \end{aligned} \tag{3.19}$$

Now we use equation (3.12) for the functions g_0 and g_3. We only keep terms up to first order in gradients and leave aside the terms containing δg. We will come back to them later.

$$\begin{aligned} & \sum_{s',s''} \mathrm{Tr}\left(P_R G^{<s',s'}(X,k') P_L G^{>s'',s''}(X,k'')\right) \\ = & \sum_{s',s''} s's''\,\mathrm{Tr}\big(P_R P_{s'}(k') P_{s''}(k'')\big) \mathrm{e}^{-\beta(\bar{k}_0' - \bar{k}_0'')} \\ & \Big([g_0^{>s'}(k') + g_3^{>s'}(k')][g_0^{<s''}(k'') - g_3^{<s''}(k'')] \\ & \quad - (1 + c_3^{s'}(k'))g_{eq}^{>}(k')(d_3^{s''}(k'')\beta\bar{k}_0'')g_{eq}^{<}(k'') \\ & \quad - (d_3^{s'}(k')\beta\bar{k}_0')g_{eq}^{>}(k')(1 - c_3^{s'}(k''))g_{eq}^{<}(k'')\Big) . \end{aligned} \tag{3.20}$$

Inserted back into (3.17) we can write this as a KMS relation for the self energy. The first term gives the usual KMS term, and the rest is the first order correction. In this correction term we perform the trace of the projection operators

$$\mathrm{Tr}\big(P_R P_{s'}(k') P_{s''}(k'')\big) = \left(1 + s's''\frac{k_0'k_0'' - \vec{k}_\parallel' \cdot \vec{k}_\parallel''}{\tilde{k}_0'\tilde{k}_0''}\right), \tag{3.21}$$

insert the explicit expressions for the c_3 (2.86), use

$$d_3^s(k)\beta\bar{k}_0 = \beta v_w \frac{|m|^2\theta'}{2\tilde{k}_0^2} \tag{3.22}$$

and finally perform the spin summations. The result is

$$\begin{aligned}\Pi^>(k) = \mathrm{e}^{\beta\tilde{k}_0}\Pi^<(k) \\ + iy^2\int\frac{d^4k'd^4k''}{(2\pi)^8}(2\pi)^4\delta^4(k+k'-k'')\mathrm{e}^{\beta\tilde{k}_0}g^>_{eq}(k')g^<_{eq}(k'') \\ \beta v_w|m|^2\theta'\frac{k'_0k''_0-\vec{k}'_\parallel\cdot\vec{k}''_\parallel}{\tilde{k}'_0\tilde{k}''_0}\left(\frac{1}{\tilde{k}'^2_0}+\frac{1}{\tilde{k}''^2_0}\right).\end{aligned} \tag{3.23}$$

Here we have an explicit example that the self energy itself satisfies the KMS relation in equilibrium, when the correction vanishes. Now we can insert this expression for the self energy in the first term of the scalar zeroth order collision term

$$\mathcal{C}^{(0)}_\phi(k) = \frac{1}{2}\left(\Pi^>(k)\Delta^<(k)-\Pi^<(k)\Delta^>(k)\right) \tag{3.24}$$

and use the (KMS like) relation (3.9) for the scalar Green function $\Delta^<$, leaving aside the $\delta\Delta$. As already explained above, the uncorrected part of the first term cancels against the second term and we finally find for

$$\begin{aligned}\mathcal{C}^{(0)}_\phi(k) &= \frac{i}{2}y^2\int\frac{d^4k'd^4k''}{(2\pi)^8}(2\pi)^4\delta^4(k+k'-k'')g^>_{eq}(k')g^<_{eq}(k'')\Delta^>_{eq}(k) \\ &\qquad \beta v_w|m|^2\theta'\frac{k'_0k''_0-\vec{k}'_\parallel\cdot\vec{k}''_\parallel}{\tilde{k}'_0\tilde{k}''_0}\left(\frac{1}{\tilde{k}'^2_0}+\frac{1}{\tilde{k}''^2_0}\right).\end{aligned} \tag{3.25}$$

Now we insert the explicit expressions for the functions $g^{<,>}_{eq}$. We extract them from the leading order Wigner functions (1.12) and (1.13) by taking the spin-projection and performing the trace with γ^0:

$$g^{<,>}_{eq}(k) = -\frac{1}{2}\frac{\tilde{k}_0}{k_0}\mathrm{Tr}\, i\gamma^0P_s(k)G^{<,>}_{eq}(k)P_s(k)\,. \tag{3.26}$$

Since (3.25) is already proportional to the small wall velocity v_w, we don't have to care here about the difference of the expressions for the leading order Wigner functions between the plasma frame and the wall frame. We find

$$\begin{aligned}g^<_{eq}(k) &= 2\pi\tilde{k}_0\delta(k^2-|m|^2)\mathrm{sgn}(k_0)n_0(k_0) \\ &= \pi\frac{\tilde{\omega}_0}{\omega_0}\left[\delta(k_0-\omega_0)f_0+\delta(k_0+\omega_0)(1-f_0)\right] \qquad (3.27) \\ g^>_{eq}(k) &= -2\pi\tilde{k}_0\delta(k^2-|m|^2)\mathrm{sgn}(k_0)(1-n_0(k_0)) \\ &= -\pi\frac{\tilde{\omega}_0}{\omega_0}\left[\delta(k_0-\omega_0)(1-f_0)+\delta(k_0+\omega_0)f_0\right]\,, \qquad (3.28)\end{aligned}$$

where $\omega_0 = \sqrt{\vec{k}^2 + |m|^2}$, $\tilde{\omega}_0 = \sqrt{k_z^2 + |m|^2}$ and $f_0 = n_{eq}(\omega_0)$. With the δ-functions we can perform the k_0' and k_0'' integrals:

$$
\begin{aligned}
\mathcal{C}_\phi^{(0)}(k) \;=\; & -\frac{i}{2} y^2 \beta v_w |m|^2 \theta' \int \frac{d^3k' d^3k''}{(2\pi)^6} (2\pi)^4 \delta^3(k + k' - k'') \Delta_{eq}^{>}(k) \\
& \frac{k_0' k_0'' - \vec{k}_{||}' \cdot \vec{k}_{||}''}{\tilde{k}_0' \tilde{k}_0''} \left(\frac{1}{\tilde{k}_0'^2} + \frac{1}{\tilde{k}_0''^2} \right) \frac{\tilde{\omega}_0' \tilde{\omega}_0''}{\omega_0' \omega_0''} \\
& \Big[\quad \delta(k_0 + \omega_0' - \omega_0'') f_0' (1 - f_0'') \\
& + \delta(k_0 - \omega_0' + \omega_0'')(1 - f_0') f_0'' \\
& + \delta(k_0 + \omega_0' + \omega_0'') f_0' f_0'' \\
& + \delta(k_0 - \omega_0' - \omega_0'')(1 - f_0')(1 - f_0'') \Big] .
\end{aligned} \tag{3.29}
$$

The first two terms in this expression can be interpreted as absorption and emission of a scalar particle, the latter two correspond to annihilation or creation of a fermion-antifermion pair, respectively. Note that this expression is invariant under $\vec{k} \to -\vec{k}$. When we now multiply with the scalar Wigner function

$$
i\Delta_{eq}^{>}(k) = \frac{\pi}{\omega_\phi} \left[\delta(k_0 - \omega_\phi)(1 + f_\phi) + \delta(k_0 + \omega_\phi) f_\phi \right] , \tag{3.30}
$$

where we have used the definitions $\omega_\phi = \sqrt{\vec{k}^2 + m_\phi^2}$ and $f_\phi = n_{eq}^\phi(\omega_\phi)$, we see that in the last two terms within the square brackets only the terms with $k_0 = -\omega_\phi$ or $k_0 = \omega_\phi$ can contribute, respectively. But it is simple to see that with $\omega_\phi = \omega_0' + \omega_0''$ we can write

$$
f_\phi (1 - f_0')(1 - f_0'') = (1 - f_\phi) f_0' f_0'' , \tag{3.31}
$$

and so these two terms can be combined into

$$
\delta(\omega_\phi - \omega_0' - \omega_0'') \, (1 - f_\phi) f_0' f_0'' \left(\delta(k_0 - \omega_\phi) + \delta(k_0 + \omega_\phi) \right) . \tag{3.32}
$$

For the treatment of the other two terms we first look at that part of the integral where $\omega_0' > \omega_0''$. Then in the first term only the contribution with $k_0 = -\omega_\phi$ survives, and in the second one we only have $k_0 = \omega_\phi$. In both cases there is $\omega_\phi'' = \omega_0' - \omega_0''$ and we can use the relation

$$
(1 + f_\phi) f_0' (1 - f_0'') = f_\phi (1 - f_0') f_0'' \tag{3.33}
$$

to combine the two terms and find again a factor $\delta(k_0 - \omega_\phi) + \delta(k_0 + \omega_\phi)$. In the case $\omega_0' < \omega_0''$ an analogous argument can be used. This means that the whole expression is proportional to $\delta(k_0 - \omega_\phi) + \delta(k_0 + \omega_\phi)$, and therefore it doesn't depend on the sign of k_0. We saw already at the beginning that it is invariant under $\vec{k} \to -\vec{k}$, so we can conclude that the zeroth order scalar collision term is the same for the scalar particle and its antiparticle. This simply means that this term cannot serve as a source for baryogenesis.

The calculation of the first order collision term

$$\mathcal{C}_\phi^{(1)}(k) = -\frac{i}{2} \diamond \Big(\{\Pi^>(k)\}\{\Delta^<(k)\} - \{\Pi^<(k)\}\{\Delta^>(k)\} \Big) \tag{3.34}$$

is comparatively simple. It is already of first order because of the derivative in the diamond operator, so we can just insert the leading order Green functions. We know that then the self energy Π_{eq} and the Green function Δ_{eq} satisfy the usual KMS relation, and use this for the first part of (3.34). Then only those terms survive, in which a derivative acts on the KMS-exponential factor:

$$\begin{aligned} \mathcal{C}_\phi^{(1)}(k) &= -\frac{i}{4}\Big(\mathrm{e}^{\beta\bar{k}_0}\big(\partial\Pi^<_{eq}(k)\big)\cdot\big(\partial_k \mathrm{e}^{-\beta\bar{k}_0}\big)\Delta^>_{eq}(k) \\ &\qquad - \big(\partial_k \mathrm{e}^{\beta\bar{k}_0}\big)\Pi^<_{eq}(k)\cdot \mathrm{e}^{-\beta\bar{k}_0}\big(\partial\Delta^>_{eq}(k)\big)\Big) \\ &= -\frac{i}{4}\big(\partial_k\beta\bar{k}_0\big)\cdot\partial\big(\Pi^<_{eq}(k)\Delta^>_{eq}(k)\big)\,. \end{aligned} \tag{3.35}$$

Using the fermionic hermiticity property (1.52) one can show that the scalar self energy (3.17) is imaginary. Furthermore we know that $i\Delta$ is real, and therefore the first order scalar collision term is imaginary. But this means that it contributes only to the constraint equation and hence it can be neglected, as we argued in the treatment of the scalar flow term.

We found already in section 1.5 that there is no CP-violating source term in the flow term of the scalar equation. Together with the results of this section this means that there is no CP-violating source term at all in the kinetic equation for scalar particles at order $\hbar$. In the course of the calculation we omitted the terms $\delta\Delta$ and δg. As we will see in the fermionic case, these terms lead to the usual relaxation terms. But without a source there is no point in writing the kinetic equations for scalars, so we don't need these terms.

3.2 Fermionic collision term

Now we turn our attention to the collision term for the fermionic equation. First we have to calculate the self energy by taking the functional derivative of Γ_2, this time with respect to G,

$$\begin{aligned} \Sigma^{ab}(u,v) &= -i\frac{\delta\Gamma_2[G]}{ab\,\delta G^{ba}(v,u)} \\ &= iy^2\left(P_L G^{ab}(u,v)P_R\Delta^{ba}(v,u) + P_R G^{ab}(u,v)P_L\Delta^{ab}(u,v)\right)\,, \end{aligned} \tag{3.36}$$

which in Wigner space and for the combinations $ab = +-/-+$ becomes

$$\begin{aligned}\Sigma^{<,>} &= iy^2 \int \frac{d^4k'\, d^4k''}{(2\pi)^8} (2\pi)^4 \delta^4(k-k'+k'') \\ &\quad [\Delta^{>,<}(k'') P_L G^{<,>}(k') P_R + \Delta^{<,>}(-k'') P_R G^{<,>}(k') P_L] \,. \end{aligned} \tag{3.37}$$

Again we distinguish between two parts of the collision term, the zeroth order and the first order collision term:

$$\begin{aligned}\mathcal{C}_\psi &= \mathcal{C}_\psi^{(0)} + \mathcal{C}_\psi^{(1)} \\ &= \frac{1}{2}\Big(\Sigma^> G^< - \Sigma^< G^>\Big) \\ &\quad -\frac{i}{2} \diamond \Big(\{\Sigma^>\}\{G^<\} - \{\Sigma^<\}\{G^>\} \Big) \,. \end{aligned} \tag{3.38}$$

Already in the beginning of this chapter we argued that all contributions for the collision term are at least of first order in gradients, where we used the scalar case as an example. Of course the same holds as well for the fermionic collision term. The procedure for the calculation is the same as in the scalar case: we use the extended KMS relations (3.9) and (3.13) for the Green functions in the first part of the collision term in order to reverse the direction of the $<$ and $>$. For the fermionic Wigner function we use only the spin diagonal part: we have seen in the last chapter that the spin off-diagonal functions are implicitly of first order in gradients and of first order in the coupling constant. They influence the diagonal equations only via the collision term, so this influence is suppressed by a further coupling constant and therefore beyond our approximation.

In the self energy there is a fermionic Wigner function between the two chiral projectors that can be simplified like in the scalar case:

$$\begin{aligned}& P_L \left[s\gamma^3\gamma^5 g_0^s - s\gamma^3 g_3^s + g_1^s - i\gamma^5 g_2^s \right] P_R \\ =\; & P_L \left[s\gamma^3\gamma^5 g_0^s - s\gamma^3 g_3^s \right] \,. \end{aligned} \tag{3.39}$$

The same holds for P_L exchanged with P_R. Since we know that in the scalar equation there is no source and therefore $\delta\Delta = 0$ we can replace the scalar propagator by its leading order expression. This one satisfies $\Delta_{eq}^>(k'') = \Delta_{eq}^<(-k'')$, which reflects the same property of the equilibrium distribution function as the KMS relation. Then we have

$$\Delta_{eq}^<(k'') P_L + \Delta_{eq}^>(-k'') P_R = \Delta_{eq}^<(k'') \tag{3.40}$$

and the self energy finally becomes

$$\begin{aligned}\Sigma^>(k) &= iy^2 \sum_{s'} \int \frac{d^4k'\, d^4k''}{(2\pi)^8} (2\pi)^4 \delta^4(k-k'+k'') P_{s'}(k') \Delta_{eq}^<(k'') \\ &\quad \Big(s'\gamma^3\gamma^5 g_0^{>s'}(k') - s'\gamma^3 g_3^{>s'}(k') \Big) \,. \end{aligned} \tag{3.41}$$

Now we insert this into the zeroth order collision term and use (3.12) and (3.13) in order to reverse the direction of the bigger and less symbols in the first term. Again the standard KMS part cancels against the second term and all that is left are the correction terms, where we consistently neglect derivatives higher than first order. The result can be divided into two pieces,

$$\mathcal{C}_{\psi}^{(0)} = \mathcal{C}_{\psi,src}^{(0)} + \mathcal{C}_{\psi,rate}^{(0)}\,, \tag{3.42}$$

according to the origin of the corrections. The term $\mathcal{C}_{\psi,src}^{(0)}$ contains the deviation from equilibrium arising from the nontrivial structure of the fermionic propagator. Since it is present even if we have vanishing δg and $\delta\Delta$ it is a source term. The other term leads to relaxation.

The calculation of the first order collision term is identical to the one in the scalar case. We can just take equation (3.35) and replace the scalar quantities by their fermionic counterparts:

$$\begin{aligned}\mathcal{C}_{\psi}^{(1)} &= -\frac{i}{2}\diamond\Big(\{\Sigma_{eq}^{>}\}\{G_{eq}^{<}\} - \{\Sigma_{eq}^{<}\}\{G_{eq}^{>}\}\Big)\\ &= -\frac{i}{4}\Big(\partial_{k_z}\,\beta\bar{k}_0\Big)\cdot\partial_z\Big(\Sigma_{eq}^{<}G_{eq}^{>}\Big)\,.\end{aligned} \tag{3.43}$$

Since only leading order Wigner functions appear, this term is a source term, too.

3.2.1 Collisional sources

We saw in the last chapter that for the spin diagonal part there were four different equations, obtained by taking the trace with P_sB, for the four component functions g_a^s. It turned out that it was possible to use the constraint equations to reduce the number of independent functions to a single one, so we had four kinetic equations for one function. We were able to show, however, that the left hand sides of these equations, that is the flow terms without collisions, are equivalent so that there indeed is only one kinetic equation. We complete this proof by showing that this equivalence is also given when the source part of the collision term is included in the second part of appendix C. After all, the equivalence of these equations including the collision term is a nontrivial consistency check of our approach and of our approximation scheme.

The source part of the collision term for the one kinetic equation left is given by

$$\mathcal{K}_{0,src}^{s} = -\frac{1}{2}\Re\,\mathrm{Tr}\,P_s\mathcal{C}_{\psi,src}\,. \tag{3.44}$$

The explicit expression for $\mathcal{C}_{\psi,src}$ can be found in appendix C. When taking the trace, the contribution from the first order collision term vanishes, and in the zeroth order expression most of the terms drop out:

$$\begin{aligned}
\mathcal{K}^s_{0,src} &= \frac{1}{4}y^2\int\frac{d^4k'\,d^4k''}{(2\pi)^8}(2\pi)^4\delta^4(k-k'+k'')i\Delta^>_{eq}(k'')g^<_{eq}(k')g^>_{eq}(k) \qquad (3.45)\\
&\sum_{s'}\Re\mathrm{Tr}P_{s'}(k')P_s(k)\Big[-s'c_3^{s'}(k')\Big(sd_3^s(k)\beta\bar{k}_0\Big)+\Big(s'd_3^{s'}(k')\beta\bar{k}'_0\Big)sc_3^s(k)\Big]\\
&= \frac{1}{4}y^2\beta v_w\int\frac{d^4k'\,d^4k''}{(2\pi)^8}(2\pi)^4\delta^4(k-k'+k'')i\Delta^>_{eq}(k'')g^<_{eq}(k')g^>_{eq}(k)\\
&\frac{1}{2}s|m|^2\theta'\sum_{s'}s'\left(1+ss'\frac{k_0k'_0-\vec{k}_\parallel\cdot\vec{k}'_\parallel}{\tilde{k}_0\tilde{k}_0}\right)\left(\frac{sk_z}{\tilde{k}_0\tilde{k}'^2_0}-\frac{s'k'_z}{\tilde{k}_0^2\tilde{k}'_0}\right). \qquad (3.46)
\end{aligned}$$

We use the expressions (3.30), (3.27) and (3.28) for the leading order functions $i\Delta^>_{eq}$, $g^<_{eq}$ and $g^>_{eq}$. We can neglect any v_w in these expressions since the source is already proportional to v_w. The definitions of the ω, $\tilde{\omega}$ and f are analogous to the scalar case. The δ-functions in the leading order expression lead to all possible combinations $k_0=\pm\omega_0$, $k'_0=\pm\omega'_0$ and $k''_0=\pm\omega''_\phi$ that have to be inserted into the energy part of the momentum conserving δ-function. But only one of these combinations leads to a non-vanishing contribution: it is not hard to convince oneself that

$$\omega_0\omega'_0-|m|^2\geq\vec{k}\cdot\vec{k}'\,, \qquad (3.47)$$

and after multiplying with -2 and adding $\vec{k}^2+\vec{k}'^2$ this can be rewritten as

$$(\omega_0-\omega'_0)^2\leq\left(\vec{k}-\vec{k}'\right)^2<\left(\vec{k}-\vec{k}'\right)^2+m_\phi^2\,, \qquad (3.48)$$

where we assumed $m_\phi>0$. Of course in the symmetric phase we have $m_\phi=0$, but there the whole collision term vanishes because of the prefactor $|m|^2\theta'$. With the spatial part of the momentum conserving δ-function we recognize the last term as the square of ω''_ϕ and finally conclude

$$\pm(\omega_0-\omega'_0)<\omega''_\phi\,. \qquad (3.49)$$

This means, however, that after performing the k'_0, k''_0 and $\vec{k}''$ integrals the contributions with $\delta(\omega_0-\omega'_0\pm\omega''_\phi)$ vanish as well as the ones which contain $\delta(\pm(\omega_0+\omega'_0+\omega''_\phi))$. After performing the spin summation we find

$$\begin{aligned}
\mathcal{K}^s_{0,src} &= \frac{1}{4}y^2s|m|^2\theta'\,\beta v_w\int\frac{d^3k'}{(2\pi)^3}\frac{\pi^2}{2}\frac{\tilde{\omega}_0\tilde{\omega}'_0}{\omega_0\omega'_0\omega''_\phi}\delta(\omega_0+\omega'_0-\omega''_\phi)\\
&\Big[\delta(k_0+\omega_0)f_0f'_0(1+f_\phi)-\delta(k_0-\omega_0)(1-f_0)(1-f'_0)f_\phi\Big]\\
&\left(\frac{k'_z}{\tilde{\omega}_0^2\tilde{\omega}'_0}+\frac{k_z}{\tilde{\omega}_0^2\tilde{\omega}'^3_0}(\omega_0\omega'_0+\vec{k}_\parallel\cdot\vec{k}'_\parallel)\right). \qquad (3.50)
\end{aligned}$$

With $\omega''_\phi = \omega_0 + \omega'_0$ we can write the relation

$$(1-f_0)(1-f'_0)f_\phi = f_0 f'_0(1+f_\phi)\,, \tag{3.51}$$

and after inserting this into (3.50) we immediately see that the collisional source in the fermionic case is CP-odd, in contrast to the scalar one:

$$\begin{aligned}\mathcal{K}^s_{0,src} \;=\;& \frac{1}{4}y^2 s|m|^2\theta'\,\beta v_w\,[\delta(k_0+\omega_0)-\delta(k_0-\omega_0)]\\ & \int\frac{d^3k'}{(2\pi)^3}\frac{\pi^2}{2}\frac{1}{\omega_0\omega'_0\omega''_\phi\tilde{\omega}_0}\delta(\omega_0+\omega'_0-\omega''_\phi)f_0f'_0(1+f_\phi)\\ & \left(k'_z + k_z\frac{\omega_0\omega'_0+\vec{k}_{||}\cdot\vec{k}'_{||}}{\tilde{\omega}'^2_0}\right)\,. \end{aligned} \tag{3.52}$$

Parts of the $\vec{k}'$-integration can be done analytically. The calculation is shown in appendix F. There we will find that this source contains a mass threshold: it is nonzero only if

$$m_\phi > 2|m|\,. \tag{3.53}$$

The reason for this threshold is that in the one loop self energy only absorption and emission processes are contained. Since the constraint equation forces us to put the participating particles on-shell, energy-momentum conservation simply leads to the above condition.

In the next chapter we derive fluid equations for the CP-violating part of the fermionic distribution function. For this we need the zeroth and first k_z-moment of the collisional source. In appendix F we show that the collisional source is odd under $k_z \leftrightarrow k'_z$, so the zeroth moment vanishes. The first moment

$$\mathcal{S}_\pm = \int_\pm \frac{d^4k}{(2\pi)^4}\frac{k_z}{\omega_0}\mathcal{K}^s_{0,src} = \pm v_w s|m|^2\theta' y^2\frac{\mathcal{I}}{64\pi^3 T|m|^2} \tag{3.54}$$

is nonzero, however. In figure 3.2.1 we plot $\mathcal{I}$ as a function of the fermion mass $|m|$, the scalar mass is chosen to be proportional to $|m|$.

3.2.2 Relaxation term

So far we have studied that part of the collision term that acts as a CP-violating source in the kinetic equation for g^s_0. This term is obtained by inserting the leading order expressions for g^s_0 and $i\Delta$ and is due to the nontrivial structure of the fermionic propagator in presence of the wall. Now we will have a brief look at the remaining terms, where the collision term is nonzero because of the deviation of g^s_0 from a thermal distribution. We will use a linear response approach: we only keep terms which are linear in the deviations $\delta\Delta$ and δg. This procedure is in the spirit of our gradient expansion, because the correction terms are implicitly of first order in gradients, and so we can neglect terms containing two or more of them in the collision term.

We have seen that there is no source term in the scalar equation itself and argued that the influence of $\delta\Delta$ on the fermionic equation is of of second order in the coupling constant, and therefore beyond our approximation. So we simply set $\delta\Delta = 0$ in the following. Then there are two terms left:

$$\begin{aligned}
\mathcal{K}^s_{0,rate} &= -\frac{1}{2}\Re\,\mathrm{Tr}\,P_s\mathcal{C}_{\psi,rate} \\
&= \frac{1}{4}y^2\,\mathrm{Tr}\sum_{s'}\int\frac{d^4k'\,d^4k''}{(2\pi)^8}(2\pi)^4\delta^4(k-k'+k'')i\Delta^>_{eq}(k'') \\
&\quad \left[g^<_{eq}(k')\left(\mathrm{e}^{\beta\bar{k}_0}\delta g_0^{<s}(k)+\delta g_0^{>s}(k)\right)+\left(\mathrm{e}^{-\beta\bar{k}'_0}\delta g_0^{>s'}(k')+\delta g_0^{<s'}(k')\right)g^>_{eq}(k)\right] \\
&\quad P_{s'}(k')P_s(k)\left(s'\gamma^3\gamma^5-s'\gamma^3c_3^{s'}(k')\right)\left(s\gamma^3\gamma^5-s\gamma^3c_3^s(k)+c_1^s(k)-i\gamma^5c_2^s(k)\right).
\end{aligned} \tag{3.55}$$

Upon taking the trace in the last line all terms except of those proportional to γ^3 drop out. We call the first part "local", because the deviation δg is outside of the

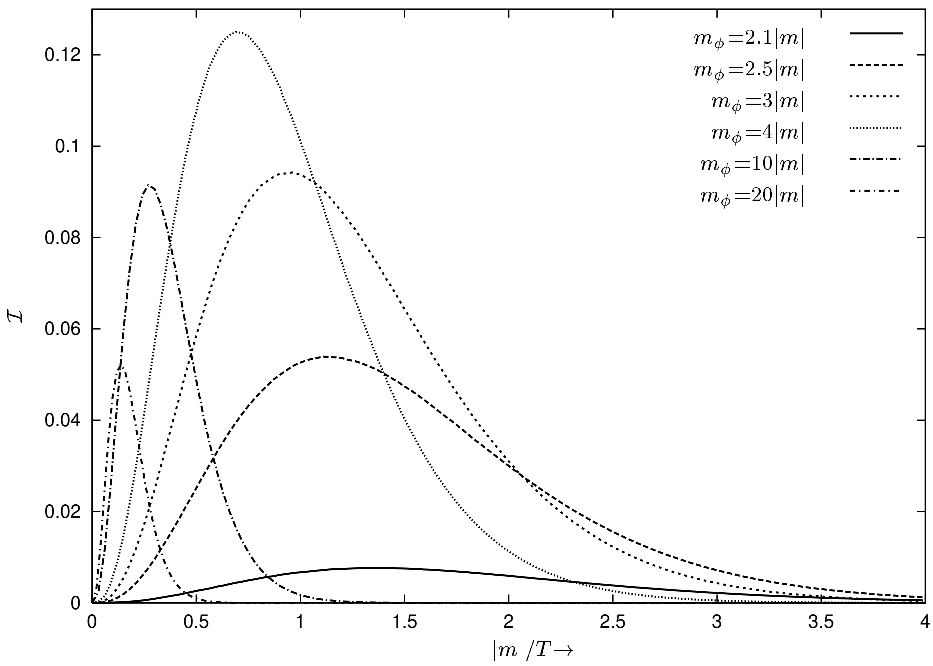

Figure 3.2: First moment of the collisional source (3.52) as a function of the rescaled mass $|m|/T$. The scalar mass is set to be a multiple of $|m|$.

integral. Here we can perform the spin summation:

$$\begin{aligned} \mathcal{K}^s_{0,loc} &= \frac{1}{4}y^2 \int \frac{d^4k'\, d^4k''}{(2\pi)^8}(2\pi)^4\delta^4(k-k'+k'')i\Delta^>_{eq}(k'') \\ &\qquad g^<_{eq}(k')\left(\mathrm{e}^{\beta\bar{k}_0}\delta g_0^{<s}(k) + \delta g_0^{>s}(k)\right)(-2)\frac{k\cdot k'}{\tilde{k}_0\tilde{k}'_0}\,. \end{aligned} \tag{3.56}$$

The second term, called "non-local" rate, is

$$\begin{aligned} \mathcal{K}^s_{0,nloc} &= \frac{1}{4}y^2 \int \frac{d^4k'\, d^4k''}{(2\pi)^8}(2\pi)^4\delta^4(k-k'+k'')i\Delta^>_{eq}(k'') \\ &\qquad \sum_{s'}\left(\mathrm{e}^{-\beta\bar{k}'_0}\delta g_0^{>s'}(k') + \delta g_0^{<s'}(k')\right)g^>_{eq}(k) \\ &\qquad\qquad \left(-ss' + ss'\frac{k_z k'_z}{\tilde{k}_0^2\tilde{k}_0'^2}(k_0k'_0 - \vec{k}_{||}\cdot\vec{k}'_{||}) + \frac{k\cdot k'}{\tilde{k}_0\tilde{k}'_0}\right)\,. \end{aligned} \tag{3.57}$$

In the local rate we could now perform the integrals, but in the nonlocal one this is not possible. In the next chapter we make a fluid ansatz for the fermionic distribution function. Then the momentum dependence of δg is explicitely given and the integrals can be performed. The results are shown in appendix F.2.

3.3 Mixing in the collision term

After having studied the collision term for the Yukawa coupling of a single scalar particle and a single fermion, we now have to include the possibility of several scalar and fermionic species. The interaction part of the Lagrangian in the mixing case was already given in section 2.3:

$$\mathcal{L}_{\rm int} = -\phi^l\bar{\psi}_L y^l\psi_R + \text{h.c.} = -\bar{\psi}\left(P_R\otimes\phi^l y^l + P_L\otimes\phi^{l*}y^{l\dagger}\right)\psi\,, \tag{3.58}$$

where y^l is a matrix in the fermionic flavor space and l denotes the different scalar particles. The calculation of the effective action Γ_2 doesn't change in principle, one just has to keep track of some more indices. We find

$$\Gamma_2 = -i\int_C d^4u d^4v\,\mathrm{Tr}\Big(P_R\otimes y^l G(u,v)P_L\otimes y^{l'\dagger}G(v,u)\Big)\Delta_{ll'}(u,v)\,, \tag{3.59}$$

where the trace now has to be performed in spinor and fermionic flavor space. In the Wigner representation the self energies calculated from this expression are given by

$$\begin{aligned}\Pi^{ab}_{ll'}(X,k) &= i\int \frac{d^4k'd^4k''}{(2\pi)^8}(2\pi)^4\delta^4(k+k'-k'') \\ &\quad \mathrm{Tr}\Big(P_R\otimes y^l G(X,k')P_L\otimes y^{l'\dagger}G(X,k'')\Big)\,, \qquad (3.60)\\ \Sigma^{ab}(X,k) &= i\int \frac{d^4k'\,d^4k''}{(2\pi)^8}(2\pi)^4\delta^4(k-k'+k'') \\ &\quad \Big(\Delta_{ll'}(X,k'')P_L\otimes y^{l'\dagger}G^{ba}(X,k')P_R\otimes y^l \\ &\quad + \Delta_{l'l}(X,k'')P_R\otimes y^{l'}G^{ba}(X,k')P_L\otimes y^{l\dagger}\Big)\,. \qquad (3.61)\end{aligned}$$

Mixing was handled on the left hand side of the equations of motion for the Wigner functions by performing a rotation into the basis where the mass is diagonal. Due to the varying mass this rotation was space-time dependent. It is clear that we have to perform the same rotation also in the collision term. We divide our investigation into two parts: first we examine the influence of the scalar mixing on both scalar and fermionic collision term, then we do the same for the fermionic mixing, which turns out to be more complicated.

3.3.1 Scalar Mixing

The scalar zeroth order collision term now has the form of a product of two matrices in the scalar flavor space:

$$\mathcal{C}^{(0)}_\phi(k) = \frac{1}{2}\left(\Pi^>(k)\Delta^<(k) - \Pi^<(k)\Delta^>(k)\right)\,. \qquad (3.62)$$

After the rotation in the scalar flavor space this form is maintained, but we have a rotated self energy, which corresponds to a redefinition of the couplings, and the Wigner function is now $i\Delta_d$, which is not diagonal in general. Since we want to have an equation for the semiclassical particle distribution function we are only interested in the diagonal components of the collision term, where we can neglect off-diagonal components of $i\Delta_d$ for the same reasons we gave in the treatment of the flow term. This means that we can simply treat the matrices appearing in the rotated collision terms as diagonal, which is the same as having a set of uncoupled equations which all have the same form as the scalar collision term in the non-mixing case. Then we can use the same arguments as above and finally state that also in the case of scalar mixing there is no source in the scalar collision term. This is true even if fermionic mixing is present, as we will see below.

In the first order collision term one has to be careful because of the derivatives acting on the rotation matrices. Nevertheless, after using the KMS relation for the first part of the collision term and neglecting all commutators, as we did in the flow term, one can see that the hermitean part of this expression vanishes. This means that there is only a contribution to the constraint equation, which can be neglected. In the calculation one has to make use of the antihermiticity of the scalar self energy , which is maintained even in the presence of fermionic and scalar mixing.

In the fermionic collision term scalar mixing only appears within the self energy. There we have an expression like

$$\Delta_{ll'}\, P_L \otimes y^{l'\dagger} G P_R \otimes y^l \,, \tag{3.63}$$

which has the form of a trace and therefore is invariant under unitary transformations. Since we can again neglect off-diagonal components of the scalar Wigner function, the effect of scalar mixing on the fermionic collision term is simple: it is a sum of usual fermionic collision terms, one for each scalar mass-eigenstate. Since the mixing only appears within the self energy, the derivatives in the first order collision term don't change this result.

3.3.2 Fermionic Mixing

Since we now know that scalar mixing has no effect on the form of the collision terms, we can work here with only one scalar particle. We again begin with the scalar collision term. Since the self energy (3.60) is a trace in the fermionic flavor space, its form stays invariant, we just have to replace the fermionic Wigner function and the coupling matrix by the rotated versions. For notational simplicity we omit the index d. Like in the treatment of the flow term we can neglect the non-diagonal elements of G, so if we use index notation in the flavor space we can write:

$$\Pi^{ab}(X,k) = i y_{ij} y^*_{ij} \int \frac{d^4k' d^4k''}{(2\pi)^8} (2\pi)^4 \delta^4(k+k'-k'') \mathrm{Tr}\Big(P_R G_j(X,k') P_L G_i(X,k'')\Big)\,.$$

The trace has to be taken in spinor space, and we used $G_i \equiv G_{ii}$. We can go through the same steps as in the non-mixing case, always keeping track of the indices attached to the fermionic functions. Remember that in the mixing case the relation (2.123) between g^s_3 and g^s_0 obtains an additional term from the rotation. The expression analogous to (3.25) is

$$\begin{aligned} \mathcal{C}^{(0)}_\phi(k) &= \frac{i}{2}\int \frac{d^4k' d^4k''}{(2\pi)^8}(2\pi)^4\delta^4(k+k'-k'') \\ &\quad \sum_{ij} |y_{ij}|^2 g^>_{eq,i}(k') g^<_{eq,j}(k'') \Delta^>_{eq}(k) \\ &\quad \beta v_w \frac{k'_0 k''_0 - \vec{k}'_{\|}\cdot \vec{k}''_{\|}}{\tilde{k}'_0 \tilde{k}''_0} \left(\frac{\big(|m|^2(\theta'+2\Delta_z)\big)_i}{\tilde{k}'^2_0} + \frac{\big(|m|^2(\theta'+2\Delta_z)\big)_j}{\tilde{k}''^2_0} \right)\,. \end{aligned} \tag{3.65}$$

The argument that finally lead us to the conclusion that the scalar zeroth order collision term is CP-even and therefore cannot serve as a source for baryogenesis holds also in the mixing case. In principle we just have to replace ω_0' by $\omega_{0,i}'$ and ω_0'' by $\omega_{0,j}''$, and the same for the f, but that doesn't change anything.

Like in the non-mixing case the first order collision term is imaginary and therefore contributes only to the constraint equation. The self energy stays imaginary, and the derivative from the diamond operator acts on the self energy as a whole and doesn't notice the rotation. So we can conclude that there is no source in the scalar self energy, neither in the non-mixing nor in the mixing case.

After the diagonalization, the fermionic zeroth order collision term reads

$$\begin{aligned}\left(\mathbf{X}\mathcal{C}_\psi^{(0)}\mathbf{X}^\dagger\right)_i &= iy^2\int\frac{d^4k'\,d^4k''}{(2\pi)^8}(2\pi)^4\delta^4(k-k'+k'') \\ &\quad \frac{1}{2}\sum_j\Big(\big[y_{ji}^*y_{ji}\Delta(X,k'')\,P_L\,G_j^{ba}(X,k')\,P_R \\ &\qquad + y_{ij}^*y_{ij}\Delta(X,k'')\,P_R\,G_j^{ba}(X,k')\,P_L\big]G_i^<(X,k) \\ &\qquad - (<\leftrightarrow>)\quad\Big),\end{aligned} \tag{3.66}$$

where y and G denote the rotated quantities. We already made use of the fact that all off-diagonal elements of G can be neglected and switched to the index notation. With the same steps as in the non-mixing case we arrive at the expression analogous to (3.45):

$$\begin{aligned}\mathcal{K}_{0,i}^s &= \frac{1}{4}\int\frac{d^4k'\,d^4k''}{(2\pi)^8}(2\pi)^4\delta^4(k-k'+k'')i\Delta_{eq}^>(k'') \\ &\quad \sum_j\frac{|y_{ij}|^2+|y_{ji}|^2}{2}g_{eq,j}^<(k')g_{eq,i}^>(k) \\ &\quad \sum_{s'}\Re\mathrm{Tr}P_{s'}(k')P_s(k)\Big[-s'c_{3,j}^{s'}(k')\Big(sd_{3,i}^s(k)\beta\bar{k}_0\Big)+\Big(s'd_{3,j}^{s'}(k')\beta\bar{k}_0'\Big)sc_{3,i}^s(k)\Big].\end{aligned} \tag{3.67}$$

Like in the non-mixing case the contributions from the leading order functions $g_{eq}^<$ and $i\Delta_{eq}^>$ vanish, if they lead to $\delta(\omega_{0,i}-\omega_{0,j}'\pm\omega_\phi'')$ or $\delta(\pm(\omega_{0,i}+\omega_{0,j}'+\omega_\phi''))$. We find

$$\begin{aligned}\mathcal{K}_{0,i}^s &= \frac{1}{4}\beta v_w\left[\delta(k_0+\omega_{0,i})-\delta(k_0-\omega_{0,i})\right] \\ &\quad \int\frac{d^3k'}{(2\pi)^3}\frac{\pi^2}{2}\sum_j\frac{|y_{ij}|^2+|y_{ji}|^2}{2}\frac{1}{\omega_\phi''}\delta(\omega_{0,i}+\omega_{0,j}'-\omega_\phi'')f_{0,i}f_{0,j}'(1+f_\phi) \\ &\quad \left(s\big(|m|^2(\theta'+\Delta_z)\big)_i\frac{k_z'}{\omega_{0,i}\tilde{\omega}_{0,i}\omega_{0,j}'}+s\big(|m|^2(\theta'+\Delta_z)\big)_j\frac{k_z}{\omega_{0,i}\tilde{\omega}_{0,i}\omega_{0,j}'}\frac{\omega_{0,i}\omega_{0,j}'+\vec{k}_\parallel\cdot\vec{k}_\parallel'}{\tilde{\omega}_{0,j}'^2}\right).\end{aligned} \tag{3.68}$$

From here on we can proceed as we did in the non-mixing case and perform some of the integrals. This is shown in appendix F.

In the first order collision term for the fermionic equation the diagonalization leads to extra terms with spatial derivatives acting on the rotation matrices. But when we take the appropriate trace in order to obtain the relevant kinetic equation for g_0^s, they all vanish.

4. Fluid equations

In the previous chapters we have shown that in the semiclassical limit the equations of motion for the scalar and fermionic Wigner functions can be reduced to on-shell conditions and single kinetic equations for the phase space densities of the corresponding particles. While in the scalar equation no correction of order $\hbar$ can be found, the fermionic equation contains two types of CP-violating sources. The first one, appearing in the flow term of the equation, is a force that directly results from the interaction with the background field. The second type of source was found in the collision term.

In the first part of this chapter we rewrite the flow term of the kinetic equation for fermions (2.88) as a Boltzmann equation for the on-shell distribution function. Then we derive an equation for the CP-violating part of the distribution function and study the sources that appear there in more detail. In the second part we make a fluid ansatz for this CP-violating part. With this ansatz the flow term of the equation obtains a quite simple form. Furthermore this ansatz allows us to write also the relaxation part of the right hand side of the kinetic equation (2.88) in the form of a collision term for a Boltzmann equation.

We already pointed out that the form of the kinetic equation found in chapter 2 is equal to the one obtained by the WKB approach in [40]. When we write the equation for the CP-violating quantity, however, we find a source term that was not taken into account in these works and that might become important. We differ from previous authors also in another important point: we identify spin states as quasiparticle states while they have used helicity states instead, which is not entirely correct since helicity is not a good quantum number for our problem. This leads to important consequences when writing down the fluid equations.

4.1 Boltzmann equation

In chapter 2 we found that the semiclassical limit of the equation of motion for the fermionic Wigner function is the following kinetic equation for g_0^s, which is correct up to order $\hbar$:

$$\frac{1}{2\tilde{k}_0}(k\cdot\partial)g_0^s - \frac{1}{2\tilde{k}_0}\left(\frac{|m|^{2\prime}}{2} - \frac{s\left(|m|^2\theta'\right)'}{2\tilde{k}_0}\right)\partial_{k_z} g_0^s = \mathcal{K}_0^s\,. \tag{4.1}$$

In addition to this kinetic equation, g_0^s has to satisfy the constraint equation (2.87). We take account of this constraint by making the ansatz

$$g_0^s(k) = 2\pi\tilde{k}_0\delta\left(k^2 - |m|^2 + \frac{s}{\tilde{k}_0}|m|^2\theta'\right)\mathrm{sgn}(k_0)n^s(k), \tag{4.2}$$

which is modeled according to the leading order solution (3.27). When we insert this ansatz into the kinetic equation, the terms with derivatives acting on the δ-function cancel, as it should be. The δ-function can be rewritten in the form

$$\delta\left(k^2 - |m|^2 + \frac{s}{\tilde{k}_0}|m|^2\theta'\right) = \sum_{\pm}\frac{\delta(k_0 \mp \omega_{s\pm})}{2\omega_{s\pm}Z_{s\pm}} \tag{4.3}$$

$$\omega_{s\pm} = \omega_0 \mp \frac{s|m|^2\theta'}{2\omega_0\tilde{\omega}_0} \quad , \quad Z_{s\pm} = 1 \mp \frac{s|m|^2\theta'}{2\tilde{\omega}_0^3}\,, \tag{4.4}$$

so the energies of particles and antiparticles are shifted by the same, spin-dependent amount, but in different directions. The kinetic equation for the distribution n^s becomes

$$\sum_{\pm}\frac{\pi}{2Z_{s\pm}}\delta(k_0 \mp \omega_{s\pm})\left[\frac{k\cdot\partial}{\omega_{s\pm}} + F_{\pm}^s\partial_{k_z}\right]n^s(k) = \mathcal{K}_0^s\,, \tag{4.5}$$

where we have defined the force term, consisting of the usual force in presence of a potential and the first order quantum correction, by

$$F_{\pm}^s = -\frac{|m|^{2\prime}}{2\omega_{s\pm}} \pm \frac{s\left(|m|^2\theta'\right)'}{2\omega_{s\pm}\tilde{\omega}_{s\pm}}\,. \tag{4.6}$$

We define the on-shell distribution functions for particles and antiparticles by

$$f_{s+} = n^s(\omega_{s+}, k_z) \tag{4.7}$$

$$f_{s-} = 1 - n^s(-\omega_{s+}, -k_z)\,, \tag{4.8}$$

respectively. In equilibrium they reduce to the usual thermal distributions. Now we integrate over positive and negative frequencies separately, and in the latter case additionally send $\vec{k}$ to $-\vec{k}$, to obtain the Boltzmann equation

$$\frac{1}{Z_{s\pm}}\left(\partial_t + \frac{\vec{k}}{\omega_{s\pm}}\cdot\vec{\partial} + F_{\pm}^s\partial_{k_z}\right)f_{s\pm} = \pm\frac{2}{\pi}\int_{\pm}dk_0\,\mathcal{K}_0^s\left(\vec{k}\to\pm\vec{k}\right)\,. \tag{4.9}$$

The on-shell distribution is then decomposed like $f_{s\pm} = f_{0s\pm} + \delta f_{s\pm}$. The first part is the usual thermal distribution function, but with the shifted energy, and taking into account the movement of the plasma in the wall frame:

$$f_{0s+} = n_{eq}(\omega_{s+} + v_w k_z) \tag{4.10}$$

$$f_{0s-} = 1 - n_{eq}(-\omega_{s-} - v_w k_z) = n_{eq}(\omega_{s-} + v_w k_z)\,. \tag{4.11}$$

Now we can insert this ansatz and subtract the equation for negative frequencies from that for positive ones in order to get an equation for the CP-violating quantity $\delta f_s = \delta f_{s+} - \delta f_{s-}$. Finally we insert the explicit expression for $\omega_{s\pm}$, neglect all terms higher than second order in gradients and make an expansion in the wall velocity v_w, keeping only the linear terms. We obtain

$$\begin{aligned}
&\left(\partial_t + \frac{1}{\omega_0}\vec{k}\cdot\partial - \frac{|m|^{2\prime}}{2\omega_0}\partial_{k_z}\right)\delta f_s \\
&\quad - \beta f_\omega(1-f_\omega)v_w\frac{s\left(|m|^2\theta'\right)'}{\omega_0\tilde{\omega}_0} \\
&\quad\quad + \beta f_\omega(1-f_\omega)v_w\frac{s|m|^{2\prime}|m|^2\theta'}{\omega_0\tilde{\omega}_0}\left(\frac{1}{\omega_0^2}+\frac{1}{\tilde{\omega}_0^2}+\frac{1}{\omega_0}\beta(1-2f_\omega)\right) \\
&\quad = \frac{2}{\pi}\left(\int_0^\infty dk_0\,\mathcal{K}_0^s + \int_{-\infty}^0 dk_0\,\mathcal{K}_0^s\left(\vec{k}\to-\vec{k}\right)\right),
\end{aligned} \tag{4.12}$$

where $f_\omega = n_{eq}(\omega_0)$. The terms in the second and third line of this equation are source terms: the first one reflects directly the quantum correction to the force, the second one is due to the energy shift appearing in the classical part of the force. This second contribution was not taken into account in the works that used the WKB method. For a static wall, described by a vanishing wall velocity v_w, these sources disappear, as well as the collisional source, and the equation is solved by $\delta f_{s\pm} = 0$. In this case the particles are described by the distribution function $f_{0s\pm}$: the interaction with the wall leads only to a shift of the particle energies, but doesn't change the form of the distribution, which is still thermal. Only a moving wall changes the form of the distribution.

When we integrate equation (4.12) over the spatial momenta, we find

$$\partial_t\rho_s + \vec{\partial}\cdot(\vec{v}\rho)_s + v_w(S_s^a + S_s^b) = 4\left(\int_+ \frac{d^4k}{(2\pi)^4}\mathcal{K}_0^s + \int_- \frac{d^4k}{(2\pi)^4}\mathcal{K}_0^s(\vec{k}\to-\vec{k})\right), \tag{4.13}$$

where we have defined

$$\begin{aligned}
\rho_s &= \int \frac{d^3k}{(2\pi)^3}\delta f_s\,, \\
(\vec{v}\rho)_s &= \int \frac{d^3k}{(2\pi)^3}\frac{\vec{k}}{\omega_0}\delta f_s\,.
\end{aligned} \tag{4.14}$$

This equation has the form of a continuity equation for the momentum-integrated CP-violating density ρ_s. The integrated sources are denoted by S_s^a and S_s^b, respectively. We can write them as

$$S_s^a = -s\left(|m|^2\theta'\right)'\frac{I_a}{(2\pi)^2} \tag{4.15}$$

$$S_s^b = s|m|^2\theta'|m|^{2\prime}\frac{I_b}{(2\pi)^2T^2}\,, \tag{4.16}$$

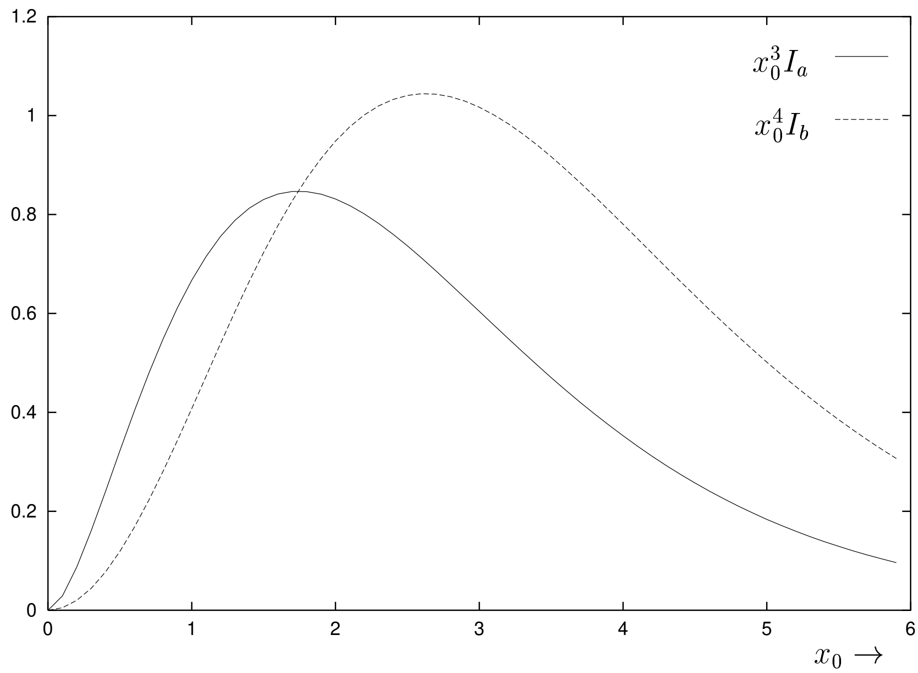

Figure 4.1: The integrals I_a and I_b as a function of the rescaled mass $x_0 = |m|/T$. We scaled I_a with x_0^3 because this is the way it appears in the source term, analogously I_b is scaled with x_0^4. Note that these contributions enter the total source with different signs.

where T is the temperature and $I_{a/b}$ are the dimensionless integrals

$$
\begin{aligned}
I_a &= 2 \int_{x_0}^{\infty} dx\, f_x(1-f_x) \ln \frac{\sqrt{x^2 - x_0^2} + x}{x_0} && (4.17) \\
I_b &= \int_{x_0}^{\infty} dx\, \frac{f_x(1-f_x)}{x} \\
&\qquad \left[\left(\frac{1}{x} + 1 - 2f_x \right) \ln \frac{\sqrt{x^2 - x_0^2} + x}{x_0} + \frac{\sqrt{x^2 - x_0^2}}{x_0^2} \right] && (4.18)
\end{aligned}
$$

with $f_x = 1/(\mathrm{e}^x + 1)$ and $x_0 = |m|/T$. In figure 4.1 we have plotted these integrals as a function of the mass. We see that S^b is bigger, but has its maximum at higher values of $|m|$ than S^a. So at small masses S^a is the dominant source, whereas at higher masses S^b becomes dominant. Note that these two sources have a different sign, so they cancel each other in part. Because of their different z-dependences it is difficult to see the result of this cancellation at this stage, however. To study the behavior of the sources in the bubble wall we simply model both the absolute value and the phase of the mass by

$$|m| = m_0 \frac{1}{2}\left(1 - \tanh\frac{z}{L_w}\right) \quad , \quad \theta = \theta_0 \frac{1}{2}\left(1 - \tanh\frac{z}{L_w}\right) . \tag{4.19}$$

For the width of the wall we choose $L_w = 10/T$, which is a typical value for the MSSM. In figure 4.1 we plotted the sources S^a and S^b as well as their sum as a function of the z-coordinate. For small values of the mass S^a dominates the source and it is justified to neglect S^b. For increasing masses S^b becomes more and more important. At $m_0 \approx 5T$ both contributions have about the same size, and because of their opposite signs they cancel each other partially. S^b can definitely not be neglected in this regime.

4.2 Fluid equations

The Boltzmann equation (4.9) is hard to solve, because it is an integro-differential equation in the momentum (see the relaxation part of the collision term in 3.2.2) and a differential equation in the spatial coordinates. In order to make progress, we can make a fluid ansatz for the distribution functions [32]:

$$f_{s\pm} = \frac{1}{\mathrm{e}^{\beta(\omega_{s\pm} + v_w k_z - \mu^s_\pm + u^s_\pm k_z)}} . \tag{4.20}$$

This form mimics the equilibrium distribution, but the chemical potential μ and the velocity perturbations u, which are only functions of the spatial coordinates, allow for local fluctuations in the density and velocity distribution. In the given form the velocity perturbation accounts only for particle movement in the z-direction, but because of the symmetry of the wall this is sufficient. With this ansatz all momentum dependences are explicit, so by integrating the Boltzmann equation over momentum only spatial degrees of freedom are left over. In particular we can evaluate the integrals in the expressions for the relaxation part of the collision term. This is done in appendix F.2.

The chemical potential and the velocity perturbation are caused by the interaction with the wall and hence are implicitly of first order in gradients. We expand the ansatz for δf_s and keep only terms linear in μ, u and v_w upon inserting into equation (4.12). Then we take the zeroth and first moment with respect to k_z, that is we multiply by 1 and k_z/ω_0, respectively, and then integrate over the spatial momentum. The zeroth

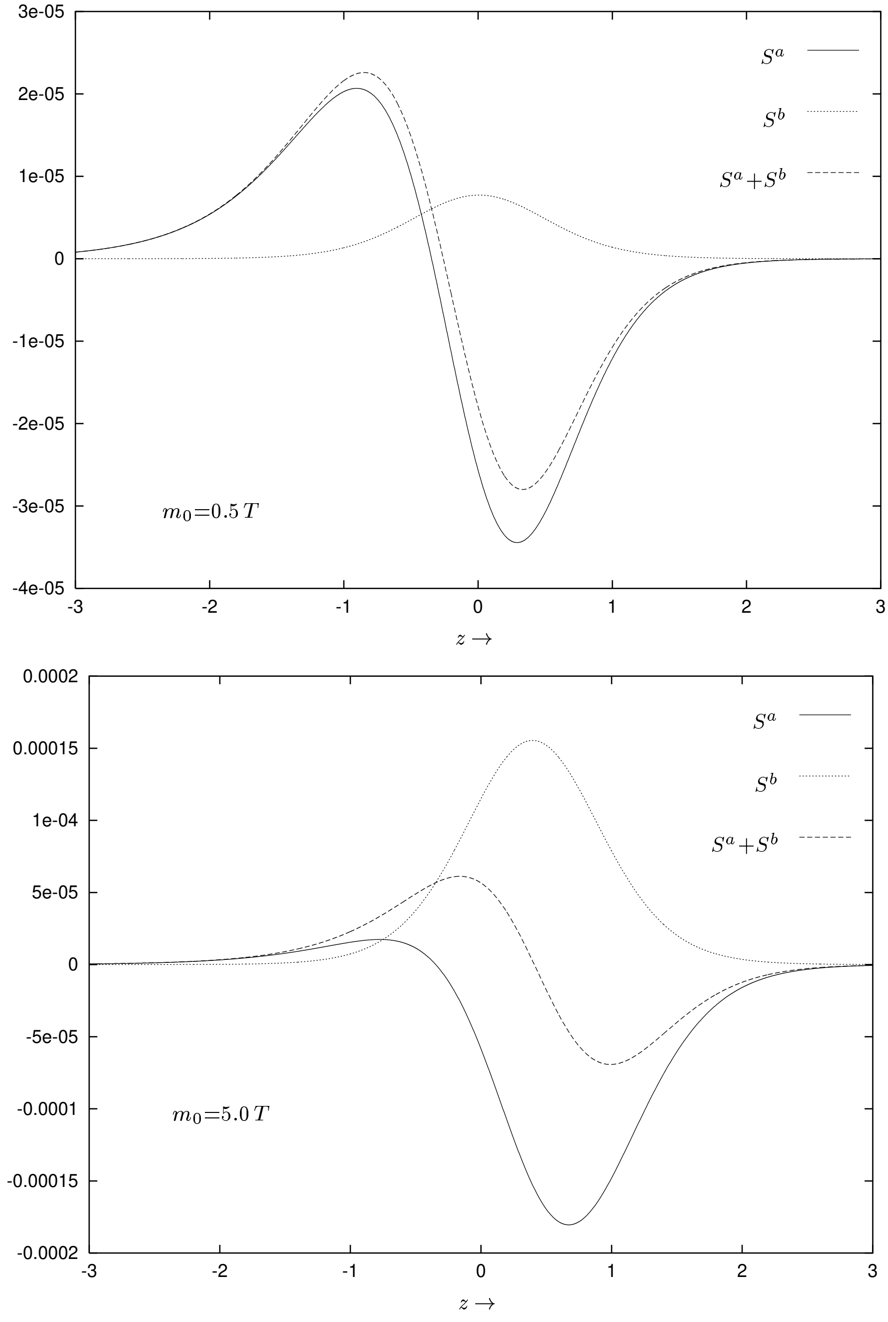

Figure 4.2: The behavior of the two contributions to the source and their sum as a function of z/L_w, we plotted the functions $S/(s\theta_0 T^4)$. For small masses S^a is clearly dominant, while with increasing mass the two contributions tend to cancel each other.

moment equation is:

$$
\begin{aligned}
& \dot{\mu}_s + v_w\beta\Big\langle(1-2f_\omega)k_z^2\Big\rangle\dot{u}_s - v_w\beta\Big\langle(1-2f_\omega)\frac{k_z^2}{\omega_0}\Big\rangle\mu_s' - \Big\langle\frac{k_z^2}{\omega_0}\Big\rangle u_s' \\
& +v_w\beta\Big\langle(1-2f_\omega)\frac{1}{\omega_0}\Big\rangle\frac{|m|^{2\prime}}{2}\mu_s + \frac{|m|^{2\prime}}{2}\Big\langle\frac{1}{\omega_0}\Big\rangle u_s + v_w(S_s^a + S_s^b) \\
= \quad & -\sum_{s'}\Gamma_{ss'}^{0\mu}\mu_{s'} - v_w\sum_{s'}\Gamma_{ss'}^{0u}u_{s'}\,.
\end{aligned}
\tag{4.21}
$$

Here dots denote the derivative with respect to time, primes are z-derivatives (except of s'), and we introduced the symbol

$$
\Big\langle\ldots\Big\rangle = \int\frac{d^3k}{(2\pi)^3}\beta f_\omega(1-f_\omega)\ldots\,. \tag{4.22}
$$

The source terms S_s^a and S_s^b have already been discussed in the previous section. The collisional source (3.54) is proportional to k_z and therefore drops out of this equation. $\Gamma_{ss'}^{0\mu}$ and $\Gamma_{ss'}^{0u}$ are the rates calculated from the relaxation part of the collision term, explicit expressions can be found in appendix F.2. Similarly one finds the first moment equation:

$$
\begin{aligned}
& \Big\langle\frac{k_z^2}{\omega_0}\Big\rangle\dot{u}_s + v_w\beta\Big\langle(1-2f_\omega)\frac{k_z^2}{\omega_0}\Big\rangle\dot{\mu}_s + \Big\langle\frac{k_z^2}{\omega_0^2}\Big\rangle\mu_s' + v_w\beta\Big\langle(1-2f_\omega)\frac{k_z^4}{\omega_0^2}\Big\rangle u_s' \\
& -v_w\beta\Big\langle(1-2f_\omega)\frac{k_z^2}{\omega_0^2}\Big\rangle|m|^{2\prime}u_s \\
= \quad & \mathcal{S}_s - v_w\sum_{s'}\Gamma_{ss'}^{1\mu}\mu_{s'} - \sum_{s'}\Gamma_{ss'}^{1u}u_{s'}\,.
\end{aligned}
\tag{4.23}
$$

This time the source from the flow term disappears and only the one from the collision term is present.

For several particles the form of these equations doesn't change, one simply has to provide an index denoting the type of the particle in addition to the spin index. The fluid equations above are well suited for a numerical treatment. They consist of of a system of first order differential equations, which is a comparable simple problem. Note that only the time and the z-derivative have survived. This is a consequence of the fluid ansatz, which in the above form effectively only allows spatial variations in the z-direction. Since in the wall frame the mass depends only on the z-coordinate, the problem is stationary in essence and we could drop the time derivative, too.

We argued in the beginning of chapter 2 that keeping the time derivative could allow a treatment of non-equilibrium initial conditions. But in order not to destroy the symmetry of the problem, these initial conditions have to have a quite special form, given by equation (2.48), so the physical relevance of such a possibility is questionable.

Keeping the time derivative could however be a convenient tool to solve the equations. At first sight it seems to be simpler to have only ordinary differential equations instead of partial ones. To solve these ordinary equations is tricky in fact, since boundary conditions at infinity have to be satisfied: at sufficiently large distances apart from the wall we expect the system to be in equilibrium. Starting with some initial values at one side of the wall usually leads not to the correct equilibrium at the other side, so the initial value has to be fine-tuned. A simpler procedure is to keep the time derivative and simulate the time-evolution of the system starting from some initial functions $\mu_0(z)$ and $u_0(z)$. Because of the interplay between the dissipative effects in the collision term and the sources this procedure leads to stable solutions quite fast.

The rates we obtain by the one-loop calculation of the self energy turn out not to be very physical. A numerical evaluation of the rates given in appendix F.2 has shown that they vanish if the mass goes to zero, which is not what collision rates in general are expected to do. In particular this would mean that all particle interactions are switched off in the symmetric phase. The reason for this behavior is simple: in the one loop self energy only absorption and emission processes are contained. Since the constraint equation forces us to put the participating particles on-shell, these processes cannot take place if the masses are zero. In the one-loop calculation we furthermore miss elastic scatterings which are essential for particle transport. So in order to obtain realistic rates in the collision term a two-loop calculation of the self energy is needed. Then elastic processes are taken into account, and there are diagrams with retarded or advanced Green functions on the internal lines. These have an off-shell contribution, so that they should show a weaker mass dependence.

We already stated that the source for the Boltzmann equation (4.9) that has been found in previous works by making use of the WKB method is formally the same as the one we have derived here. The authors of these works however assumed that the quasiparticle states of the system are helicity eigenstates. But when s denotes helicity, the sources in the flow term are odd in k_z and contribute to the first moment equation (4.21) rather than to (4.23). So in these works the correct form of the source has been obtained, except of the fact that S^b has been missed, but finally it appears in the wrong equation.

The fact that our source and the one calculated with helicity eigenstates appear in different equations makes a comparison rather complicated, in principle one has to solve the equations to see the differences. The same problem appears when we try to compare the collisional source, which appears in the first moment equation, with the one from the flow term contributing to the zeroth moment equation. But we can try to obtain at least a qualitative picture by converting the fluid equations for μ_s and u_s into a single second order differential equation for μ_s, the diffusion equation. This is a standard approximation for the solution of the fluid equations. In order to keep the notation simple we introduce the abbreviations

$$I_{ij} = \int \frac{d^3k}{(2\pi)^3} \beta f_\omega (1 - f_\omega) \frac{k_z^i}{\omega_0^j}, \tag{4.24}$$

$$J_{ij} = \int \frac{d^3k}{(2\pi)^3} \beta^2 f_\omega (1 - f_\omega)(1 - 2f_\omega) \frac{k_z^i}{\omega_0^j}. \tag{4.25}$$

When we drop the time derivative terms, the zeroth moment equation reads

$$\begin{aligned} & -v_w J21 \mu_s' - I_{21} u_s' + v_w J_{01} \frac{|m|^{2\prime}}{2} \mu_s + I_{01} \frac{|m|^{2\prime}}{2} u_s + v_w \langle F_s \rangle \\ = & -\sum_{s'} \Gamma^{0\mu}_{ss'} \mu_{s'} - v_w \sum_{s'} \Gamma^{0u}_{ss'} u_{s'} \end{aligned} \tag{4.26}$$

and the first moment equation is

$$\begin{aligned} & I_{22} \mu_s' + v_w J_{42} u_s' - v_w J_{22} |m| 2' u_s + v_w \langle F_s^{\text{hel}} \rangle \\ = & v_w \langle \mathcal{S} \rangle - v_w \sum_{s'} \Gamma^{1\mu}_{ss'} \mu_{s'} - \sum_{s'} \Gamma^{1u}_{ss'} u_{s'}. \end{aligned} \tag{4.27}$$

Here we introduced F_s^{hel}, which is the source we would have obtained by the use of helicity eigenstates, it has the same form as F_s. Then we take linear combinations of these equations so that we obtain an equation that only contains u_s' and one where only μ_s' occurs. The latter one is solved for u_s and then derived with respect to z, where we neglect all terms where the derivative acts on rates or masses. When we insert these expressions for u_s and u_s' into the other equation we can write it as

$$\begin{aligned} & \frac{D_s}{I_{22}} \mu_s'' + v_w \frac{D_s}{I_{22}} \frac{1}{I_{22}} \left[\langle F_s^{\text{hel}} - \mathcal{S} \rangle' - \frac{I_{01}}{I_{21}} \frac{|m|^{2\prime}}{2} \langle F_s^{\text{hel}} - \mathcal{S} \rangle \right] + v_w \langle F_s \rangle + \ldots \\ = & \ldots - \sum_{s'} \Gamma^{0\mu}_{ss'} \mu_{s'} + \ldots . \end{aligned} \tag{4.28}$$

We omitted most of the terms here and kept only those we need for the comparison of the different sources. D_s contains combinations of rates and the quantities I_{ij} and J_{ij}, but in the way the equation is written we can identify D_s/I_{22} as the diffusion constant for the particles under study, which can be obtained by other means. Now we can read off from this that a source from the first moment equation compared to one from the zeroth moment equation is multiplied with the diffusion constant and obtains an additional derivative. This means that in situations where transport is very efficient, the use of the helicity eigenstates leads to sources which are too big, while in the opposite case the true source is underestimated. In the same way the collisional source becomes important if transport is efficient.

In order to make a quantitative comparison one really has to solve the equations and compare the results. In any case we now know that in all previous works sources have been used that are more or less incorrect.

5. Conclusion

In this work we performed a controlled, first principle derivation of transport equations for CP-violating fluxes valid up to order $\hbar$ for a system of scalar and fermionic particles in presence of a slowly varying background field. We consistently included the collision term in our treatment and made a model calculation in leading order of the coupling constant. Such a derivation has been the main theoretical challenge of recent work on electroweak baryogenesis, since there is a controversial discussion in literature concerning the way to obtain the correct transport equations.

Starting point of our work have been the equations of motion for the Wigner functions in the Schwinger-Keldysh formalism, which is suited for the treatment of non-equilibrium situations. In order to reduce these exact equations to semiclassical transport equations we used an expansion in powers of gradients of the background field, which is a good approximation if the variation of the background field is slow, which is the case for the phase transition front in a first order electroweak phase transition. This expansion is equivalent to an expansion in powers of $\hbar$. The equations split into two types, constraint and kinetic equations. The constraint equations should provide spectral conditions which restrict the possible solutions of the kinetic equations.

A major result of our work is that indeed both for scalar and for fermionic particles the constraint equations are algebraic equations which allow a spectral solution for the Green functions. This essentially confirms the basic assumptions underlying WKB approaches to electroweak baryogenesis: the plasma can at this level indeed be described as a collection of quasi-particles with a classical phase space density. It has also become clear, however, that this spectral solution only holds up to first order in $\hbar$. At higher orders the constraint equation contains derivatives and no longer can be solved by on-shell Wigner functions.

In the scalar equations there are no quantum corrections at all at our level of approximation. They lead to a usual classical Boltzmann equation. This is in contrast to the results of earlier work using the WKB approach, where the particle momentum was identified with the canonical momentum. When using the kinetic momentum instead, in the WKB approach one finds no $\hbar$-deviations from classical behavior, either. With our work we therefore have shown that the kinetic momentum has to be used in a WKB approach. Our results differ from earlier works claiming to provide a controlled derivation of kinetic equations from the Schwinger-Dyson equation, which do find quantum corrections in the scalar equation.

In the fermionic case the Wigner function is a matrix in spinor space and consists of 16 independent functions which are governed by 32 coupled equations. By identifying a conserved quantity of the system, essentially the spin perpendicular to the wall, we were able to decouple these equations into three separate blocks. Two of them describe the propagation of particles with definite spin, while the third one describes spin-mixing. We saw that this spin mixing part has no effect at our level of approximation. Using the constraint equations we were able to show that both of the spin-diagonal parts are described by only one function, which again has to satisfy an on-shell condition. This function corresponds to the classical phase-space-density of the particles. Unlike in the scalar case, however, the energy of the fermionic particles is not given by the classical on-shell condition but has a correction due to the interaction with the wall. Correspondingly there is a semiclassical force term in the kinetic equation which is a CP-violating source. This source term in the kinetic equation has the same form as the one found in the WKB approach, provided the correct momentum is used there. Again our result is in contrast to other approaches to the problem.

We were also able to provide a consistent treatment of the collision term of the equations of motion for the Wigner functions within our formalism, now in contrast to the WKB based works. There the dispersion relation for the particles, which is modified by the interaction with the background field, is inserted into a classical Boltzmann equation and so leads to the semiclassical force. The rates appearing on the right hand side of the equation have to be calculated by other methods and then are inserted into the equation. For our model with a Yukawa interaction we calculated the collision term by an expansion in the coupling constant and only kept the leading order term, which corresponds to a calculation of the self energy to one loop. In addition to the usual gain and loss terms, which lead to relaxation, we found in the fermionic collision term a further source that is called spontaneous baryogenesis source. This source seems to be somewhat smaller than the one from the semiclassical force. By making use of a fluid ansatz we furthermore were able to compute the interaction rates in the relaxation part of the collision term within our formalism. Due to the fact that we restricted ourselves to a one-loop calculation of the self energy, only absorption and emission processes are taken into account, so we miss elastic scattering processes which are essential for transport. Furthermore the rates show a strong mass dependence, which is unphysical. A two loop calculation of the self energy is required in order to obtain realistic results for the rates. In this work we provide everything that is needed for such a calculation, where for example the treatment of the spin off-diagonal parts in the fermionic equation is necessary. Such a computation exceeds the one-loop calculation presented here by far, both in size and in technical complexity, but there are no basic problems.

Although the semiclassical force term we found in the kinetic equation for the fermions is formally the same as the one derived with the use of the WKB method, they differ in an important detail. The authors who used this approach assumed that the propagating states in the plasma are helicity eigenstates, while we have proved that the conserved quantity is spin rather than helicity. When deriving fluid equations, this has the consequence that the semiclassical source in these works appears in the first moment equation and not in the continuity equation, as it should be. Furthermore we have seen that in these former treatments a part of the semiclassical source has been missed that might be important. So although the WKB approach in principle is capable of providing the proper source, up to now no entirely correct calculation of electroweak baryogenesis is available. We do not expect that the result will change by orders of magnitude, but if one takes into consideration that electroweak baryogenesis in the MSSM only works within a rather restricted region of the parameter space anyway, a precise calculation is highly desirable.

With this work we have solved the basic problem of a rigorous derivation of the transport equations relevant for electroweak baryogenesis. Based on this, new calculations of the produced baryon asymmetry can be performed. Together with new experimental results that will be available in the near future this will help to decide whether electroweak baryogenesis is realized in nature or not.

Appendix A. Fermionic currents

In this appendix we list the expressions for the fermionic currents in terms of the component functions of $G^<$. The general expression is

$$\left\langle \bar{\psi}(x)\Gamma\psi(x)\right\rangle = -i\mathrm{Tr}\,\Gamma G^<(x,x) = -\int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[i\Gamma G^<(x,k)\right] , \tag{A.1}$$

where the matrix Γ is one of the following:

$$\Gamma = \left(\mathbb{1}, \gamma^5, \gamma^\mu, \gamma^5\gamma^\mu, \sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu,\gamma^\nu]\right) . \tag{A.2}$$

With the decompositions (2.3) and (2.58) we find:

- scalar

$$\left\langle \bar{\psi}(x)\psi(x)\right\rangle = \int \frac{d^4k}{(2\pi)^4} g_{10}(x,k) = 2\sum_s \int \frac{d^4k}{(2\pi)^4} g_1^s \tag{A.3}$$

- pseudoscalar

$$\left\langle \bar{\psi}(x)\psi(x)\right\rangle = \int \frac{d^4k}{(2\pi)^4} g_{20}(x,k) = -2i\sum_s \int \frac{d^4k}{(2\pi)^4} g_2^s \tag{A.4}$$

- vector

$$\left\langle \bar{\psi}(x)\gamma^0\psi(x)\right\rangle = \int \frac{d^4k}{(2\pi)^4} g_{00}(x,k) = 2\sum_s \int \frac{d^4k}{(2\pi)^4} \frac{k_0}{\tilde{k}_0} g_0^s \tag{A.5}$$

$$\left\langle \bar{\psi}(x)\gamma^j\psi(x)\right\rangle = \int \frac{d^4k}{(2\pi)^4} g_{3j}(x,k) = 2\sum_s \int \frac{d^4k}{(2\pi)^4} \frac{k^j}{\tilde{k}_0} g_0^s \tag{A.6}$$

$$\left\langle \bar{\psi}(x)\gamma^3\psi(x)\right\rangle = \int \frac{d^4k}{(2\pi)^4} g_{33}(x,k) = 2\sum_s \int \frac{d^4k}{(2\pi)^4} s g_3^s \tag{A.7}$$

- pseudo-vector

$$\left\langle \bar{\psi}(x)\gamma^5\gamma^0\psi(x)\right\rangle = \int \frac{d^4k}{(2\pi)^4} g_{30}(x,k) = -2\sum_s \int \frac{d^4k}{(2\pi)^4} \frac{k_0}{\tilde{k}_0} g_0^s \tag{A.8}$$

$$\left\langle \bar{\psi}(x)\gamma^5\gamma^j\psi(x)\right\rangle = \int \frac{d^4k}{(2\pi)^4} g_{0j}(x,k) = -2\sum_s \int \frac{d^4k}{(2\pi)^4} \frac{k^j}{\tilde{k}_0} g_3^s \tag{A.9}$$

$$\left\langle \bar{\psi}(x)\gamma^5\gamma^3\psi(x)\right\rangle = \int \frac{d^4k}{(2\pi)^4} g_{03}(x,k) = -2\sum_s \int \frac{d^4k}{(2\pi)^4} s g_0^s \tag{A.10}$$

- tensor, "electric", $i\gamma^0\gamma^i = \sigma^{0i}$

$$
\begin{aligned}
\langle\bar{\psi}(x)i\gamma^0\gamma^1\psi(x)\rangle &= \int\frac{d^4k}{(2\pi)^4}g_{21}(x,k) = -2\sum_s\int\frac{d^4k}{(2\pi)^4}\frac{k_y}{\tilde{k}_0}sg_1^s && \text{(A.11)}\\
\langle\bar{\psi}(x)i\gamma^0\gamma^2\psi(x)\rangle &= \int\frac{d^4k}{(2\pi)^4}g_{22}(x,k) = 2\sum_s\int\frac{d^4k}{(2\pi)^4}\frac{k_x}{\tilde{k}_0}sg_1^s && \text{(A.12)}\\
\langle\bar{\psi}(x)i\gamma^0\gamma^3\psi(x)\rangle &= \int\frac{d^4k}{(2\pi)^4}g_{23}(x,k) = 2\sum_s\int\frac{d^4k}{(2\pi)^4}\frac{k_0}{\tilde{k}_0}sg_2^s && \text{(A.13)}
\end{aligned}
$$

- tensor, "magnetic", $\gamma^0\gamma^i\gamma^5 = \frac{1}{2}\epsilon^{ijk}\sigma^{jk}$

$$
\begin{aligned}
\langle\bar{\psi}(x)\gamma^0\gamma^1\gamma^5\psi(x)\rangle &= \int\frac{d^4k}{(2\pi)^4}g_{11}(x,k) = 2\sum_s\int\frac{d^4k}{(2\pi)^4}\frac{k_y}{\tilde{k}_0}sg_2^s && \text{(A.14)}\\
\langle\bar{\psi}(x)\gamma^0\gamma^2\gamma^5\psi(x)\rangle &= \int\frac{d^4k}{(2\pi)^4}g_{12}(x,k) = -2\sum_s\int\frac{d^4k}{(2\pi)^4}\frac{k_x}{\tilde{k}_0}sg_2^s && \text{(A.15)}\\
\langle\bar{\psi}(x)\gamma^0\gamma^3\gamma^5\psi(x)\rangle &= \int\frac{d^4k}{(2\pi)^4}g_{13}(x,k) = 2\sum_s\int\frac{d^4k}{(2\pi)^4}\frac{k_0}{\tilde{k}_0}sg_1^s && \text{(A.16)}
\end{aligned}
$$

The spin off-diagonal parts don't contribute to the scalar, pseudo-scalar and the z-components of vector and pseudovector currents. The remaining components of the vector current obtain a contribution from g_7^s, the ones of the pseudo-vector from g_4^s. In the z-component of the "electric" part of the tensor as well as in the x- and y-components of the "magnetic" part of the tensor g_5^s contributes, in the other tensor components we find g_6^s.

Appendix B. Full equations without spin-projection

Insertion of the decomposition (2.3) into the equation of motion leads to the following 32 real equations. They can be grouped nicely if we use the notation $f_a = g_{0a}$ and $\vec{g}_a = (g_{1a}, g_{2a}, g_{3a})$. Furthermore we omit the right hand sides. The kinetic equations are

$$\begin{aligned}
&\frac{1}{2}\partial_t f_0 && +\frac{1}{2}\vec{\nabla}\times\vec{g}_3 + \frac{1}{2}\left(\partial_x m_R\right)\partial_k f_1 && +\frac{1}{2}\left(\partial_x m_I\right)\partial_k f_2 = \ldots\\
&\frac{1}{2}\partial_t f_1 + \vec{k}\times\vec{g}_2 && +\frac{1}{2}\left(\partial_x m_R\right)\partial_k f_0 && -(m_I - \frac{1}{8}\ldots)f_3 = \ldots\\
&\frac{1}{2}\partial_t f_2 - \vec{k}\times\vec{g}_1 && +\frac{1}{2}\left(\partial_x m_I\right)\partial_k f_0 && +(m_R - \frac{1}{8}\ldots)f_3 = \ldots\\
&\frac{1}{2}\partial_t f_3 && +\frac{1}{2}\vec{\nabla}\times\vec{g}_0 - (m_R - \frac{1}{8}\ldots)f_2 && +(m_I - \frac{1}{8}\ldots)f_1 = \ldots\\
&\frac{1}{2}\partial_t \vec{g}_0 - \vec{k}\times\vec{g}_3 && +\frac{1}{2}\vec{\nabla}f_3 + \frac{1}{2}\left(\partial_x m_R\right)\partial_k \vec{g}_1 && +\frac{1}{2}\left(\partial_x m_I\right)\partial_k \vec{g}_2 = \ldots\\
&\frac{1}{2}\partial_t \vec{g}_1 + \vec{k}f_2 && +\frac{1}{2}\vec{\nabla}\times\vec{g}_2 + \frac{1}{2}\left(\partial_x m_R\right)\partial_k \vec{g}_0 && -(m_I - \frac{1}{8}\ldots)\vec{g}_3 = \ldots\\
&\frac{1}{2}\partial_t \vec{g}_2 - \vec{k}f_1 && -\frac{1}{2}\vec{\nabla}\times\vec{g}_1 + \frac{1}{2}\left(\partial_x m_I\right)\partial_k \vec{g}_0 && +(m_R - \frac{1}{8}\ldots)\vec{g}_3 = \ldots\\
&\frac{1}{2}\partial_t \vec{g}_3 - \vec{k}\times\vec{g}_0 && +\frac{1}{2}\vec{\nabla}f_0 - (m_R - \frac{1}{8}\ldots)\vec{g}_2 && +(m_I - \frac{1}{8}\ldots)\vec{g}_1 = \ldots
\end{aligned}$$

The constraint equations read

$$\begin{aligned}
&-k^0 f_0 && +\vec{k}\cdot\vec{g}_3 && +(m_R - \frac{1}{8}\ldots)f_1 + (m_I - \frac{1}{8}\ldots)f_2 && = \ldots\\
&-k^0 f_1 && -\frac{1}{2}\vec{\nabla}\cdot\vec{g}_2 && +(m_R - \frac{1}{8}\ldots)f_0 + \frac{1}{2}\left(\partial_x m_I\right)\partial_k f_3 && = \ldots\\
&-k^0 f_2 && +\frac{1}{2}\vec{\nabla}\cdot\vec{g}_1 && -\frac{1}{2}\left(\partial_x m_R\right)\partial_k f_3 + (m_I - \frac{1}{8}\ldots)f_0 && = \ldots\\
&-k^0 f_3 && +\vec{k}\times\vec{g}_0 && +\frac{1}{2}\left(\partial_x m_R\right)\partial_k f_2 - \frac{1}{2}\left(\partial_x m_I\right)\partial_k f_1 && = \ldots\\
&-k^0 \vec{g}_0 && +\vec{k}f_3 + \frac{1}{2}\vec{\nabla}\times\vec{g}_3 && +(m_R - \frac{1}{8}\ldots)\vec{g}_1 + (m_I - \frac{1}{8}\ldots)\vec{g}_2 && = \ldots\\
&-k^0 \vec{g}_1 && +\vec{k}\times\vec{g}_2 - \frac{1}{2}\vec{\nabla}\cdot f_2 && +(m_R - \frac{1}{8}\ldots)\vec{g}_0 + \frac{1}{2}\left(\partial_x m_I\right)\partial_k \vec{g}_3 && = \ldots\\
&-k^0 \vec{g}_2 && -\vec{k}\times\vec{g}_1 + \frac{1}{2}\vec{\nabla}\cdot f_1 && +(m_I - \frac{1}{8}\ldots)\vec{g}_0 - \frac{1}{2}\left(\partial_x m_R\right)\partial_k \vec{g}_3 && = \ldots\\
&-k^0 \vec{g}_3 && +\vec{k}f_0 + \frac{1}{2}\vec{\nabla}\times\vec{g}_0 && +\frac{1}{2}\left(\partial_x m_R\right)\partial_k \vec{g}_2 - \frac{1}{2}\left(\partial_x m_I\right)\partial_k \vec{g}_1 && = \ldots
\end{aligned}$$

Appendix C. Consistency of the fermionic kinetic equations

In section 2.2.1 we obtained four kinetic equations, but because of the constraint equations we only have one independent function, namely g_0^s. Here we prove that these equations are equivalent, which means that our approach is consistent. We break the problem into two parts: first we show the equivalence of the flow terms and then we consider the collision term. We don't give the proof for all three equations, but instead restrict ourselves to (2.75), which is the kinetic equation for g_3^s:

$$\frac{1}{2\tilde{k}_0}(k_0\partial_t + \vec{k}_{\|}\cdot\vec{\partial})g_3^s + \frac{1}{2}s\partial_z g_0^s + (m_I - \frac{1}{8}m_I''\partial_{k_z}^2)g_1^s - (m_R - \frac{1}{8}m_R''\partial_{k_z}^2)g_2^s = \mathcal{K}_3^s\,. \quad \text{(C.1)}$$

The other two equations can be treated similarly.

C.1 Flow term

We insert the expressions (2.84)-(2.86) for g_i^s, where this time it is not sufficient to use these equations only to first order in gradients, because the kinetic equation for g_3^s contains zeroth order terms:

$$\begin{aligned}
&\frac{1}{\tilde{k}_0}\left(sk_z + \frac{1}{2\tilde{k}_0}|m|^2\theta'\partial_{k_z}\right)\frac{1}{2\tilde{k}_0}(k_0\partial_t + \vec{k}_{\|}\cdot\vec{\partial})g_0^s \qquad \text{(C.2)}\\
&\quad + \frac{1}{2}s\partial_z g_0^s + \frac{s}{2\tilde{k}_0^2}\Big(-|m|^{2\prime} - \frac{1}{2}|m|^{2\prime}k_z\partial_{k_z} - |m|^2\partial_z\Big)g_0^s\\
&\quad + \frac{1}{4\tilde{k}_0^3}\Big(-\frac{1}{2}|m|^{2\prime}|m|^2\theta'\partial_{k_z}^2 + (|m|^2\theta')' + 2|m|^2\theta'\partial_z\\
&\qquad + \big((|m|^2\theta')' + |m|^2\theta'\partial_z\big)(1 + k_z\partial_{k_z})\Big)g_0^s\\
=\;& \mathcal{K}_3^s + \frac{m_I}{\tilde{k}_0}\mathcal{C}_1^s - \frac{m_R}{\tilde{k}_0}\mathcal{C}_2^s\,.
\end{aligned}$$

Now we take the derivative of the constraint equation (2.87) with respect to z. The collisional contribution to this term can then be neglected. We find

$$-|m|^{2\prime}g_0^s + \frac{s}{\tilde{k}_0}(|m|^2\theta')'g_0^s + \left(k^2 - |m|^2 + \frac{s}{\tilde{k}_0}|m|^2\theta'\right)g_0^{s\prime} = 0 \qquad \text{(C.3)}$$

and use this relation to replace the term $-|m|^{2\prime} g_0^s$ in the second line of (C.2). After some cancellations this leads to

$$\begin{aligned}
&\frac{1}{\tilde{k}_0}\left(sk_z + \frac{1}{2\tilde{k}_0}|m|^2\theta'\partial_{k_z}\right)\frac{1}{2\tilde{k}_0}(k_0\partial_t + \vec{k}_{\|}\cdot\vec{\partial})g_0^s \\
&\qquad + \frac{s}{2\tilde{k}_0^2}\left(k_z^2\partial_z - \frac{1}{2}|m|^{2\prime}k_z\partial_{k_z}\right)g_0^s \\
&\qquad\qquad + \frac{1}{4\tilde{k}_0^3}\left(-\frac{1}{2}|m|^{2\prime}|m|^2\theta'\partial_{k_z}^2 + (|m|^2\theta')'k_z\partial_{k_z} + |m|^2\theta'\partial_z(1+k_z\partial_{k_z})\right)g_0^s \\
&= \mathcal{K}_3^s + \frac{m_I}{\tilde{k}_0}\mathcal{C}_1^s - \frac{m_R}{\tilde{k}_0}\mathcal{C}_2^s\,.
\end{aligned} \tag{C.4}$$

This equation can finally be reexpressed in the form

$$\begin{aligned}
&\frac{1}{\tilde{k}_0}\left(sk_z + \frac{1}{2\tilde{k}_0}|m|^2\theta'\partial_{k_z}\right) \\
&\qquad \times \frac{1}{2}\left[\frac{1}{\tilde{k}_0}k\cdot\partial - \frac{1}{2\tilde{k}_0}|m|^{2\prime}\partial_{k_z} + \frac{s}{2\tilde{k}_0^2}(|m|^2\theta')'\partial_{k_z}\right]g_0^s \\
&= \mathcal{K}_3^s + \frac{m_I}{\tilde{k}_0}\mathcal{C}_1^s - \frac{m_R}{\tilde{k}_0}\mathcal{C}_2^s\,,
\end{aligned} \tag{C.5}$$

which we recognize as something times the left hand side of the familiar kinetic equation for g_0^s. The treatment of the equations (2.73) and (2.74) is somewhat more complicated, but runs along the same lines. There we have to use the derivative of the constraint equation with respect to k_z and obtain

$$\frac{1}{k_0}\left(m_R - \frac{s}{2k_0}(\partial_z m_I + m_I'\partial_{k_z}k_z\right) \times \frac{1}{2}\Big[\ldots\Big]g_0^s = \mathcal{K}_1^s + \frac{m_I}{\tilde{k}_0}\mathcal{C}_1^s \tag{C.6}$$

and

$$\frac{1}{k_0}\left(m_I + \frac{s}{2k_0}(\partial_z m_R + m_R'\partial_{k_z}k_z\right) \times \frac{1}{2}\Big[\ldots\Big]g_0^s = \mathcal{K}_2^s - \frac{m_R}{\tilde{k}_0}\mathcal{C}_1^s\,. \tag{C.7}$$

Note that the terms that multiply the g_0^s-equation are exactly the ones that relate the functions g_i^s with g_0^s.

C.2 Collision term

We still have to prove the equivalence for the right hand side of (C.5). Here only the calculation for the source part of the collision term is shown, the other terms can be treated in the same way. The source part of the zeroth order collision term before taking the trace is

$$
\begin{aligned}
\mathcal{C}^{(0)}_{src} = -\frac{1}{2}y^2 \sum_{ss'} \int & \frac{d^4k'\,d^4k''}{(2\pi)^8}(2\pi)^4\delta^4(k-k'+k'') \\
& P_{s'}(k')P_s(k)\, i\Delta^{>}_{eq}(k'')g^{<}_{eq}(k')g^{>}_{eq}(k) \\
& \Big[\Big(s'\gamma^3\gamma^5 - s'\gamma^3 c_3^{s'}(k')\Big)\Big(-s\gamma^3 d_3^s(k) + d_1^s(k) - i\gamma^5 d_2^s(k)\Big)\beta\bar{k}_0 \\
& + \Big(s'\gamma^3 d_3^{s'}(k')\beta\bar{k}'_0\Big)\Big(s\gamma^3\gamma^5 - s\gamma^3 c_3^s(k) + c_1^s(k) - i\gamma^5 c_2^s(k)\Big)\Big],
\end{aligned}
\tag{C.8}
$$

and the corresponding contribution from the first order collision term is

$$
\begin{aligned}
\mathcal{C}^{(1)}_{\psi}(k) &= -\frac{1}{4}y^2\left(\partial_{k_z}\beta\bar{k}_0\right)\sum_{ss'}\int\frac{d^4k'\,d^4k''}{(2\pi)^8}(2\pi)^4\delta^4(k-k'+k'')\,P_{s'}(k')P_s(k) \\
& \quad \partial_z\, i\Delta^{>}_{eq}(k'')g^{<}_{eq}(k')g^{>}_{eq}(k)\Big(s'\gamma^3\gamma^5 - s'\gamma^3 c_3^{s'}(k')\Big) \\
& \quad\quad \Big(s\gamma^3\gamma^5 - s\gamma^3 c_3^s(k) + c_1^s(k) - i\gamma^5 c_2^s(k)\Big).
\end{aligned}
\tag{C.9}
$$

The traces one has to take in order to obtain the relevant terms are given in (2.80)-(2.83). If we put everything together we find

$$
\begin{aligned}
& \mathcal{K}_3^s + \frac{m_I}{\tilde{k}_0}\mathcal{C}_1^s - \frac{m_R}{\tilde{k}_0}\mathcal{C}_2^s \\
= \; & \frac{1}{4}y^2\beta v_w \int \frac{d^4k'\,d^4k''}{(2\pi)^8}(2\pi)^4\delta(k-k'+k'')i\Delta^{>}_{eq}(k'')g^{<}_{eq}(k')g^{>}_{eq}(k) \\
& \Bigg[-|m|^2\theta'\overline{kk'}\left(\frac{1}{\tilde{k}_0'^3\tilde{k}_0} - \frac{1}{\tilde{k}'_0\tilde{k}_0^3}\right) \\
& + \frac{m_I}{\tilde{k}_0}\left(-\frac{1}{\tilde{k}_0^2\tilde{k}_0'^3}\left(\tilde{k}_0'^2 k_z k'_z m'_R - \overline{kk'}|m|^2\theta' m_I\right) - \frac{1}{\tilde{k}_0^2\tilde{k}'_0}m'_R\overline{kk'}\right) \\
& - \frac{m_R}{\tilde{k}_0}\left(-\frac{1}{\tilde{k}_0^2\tilde{k}_0'^3}\left(\tilde{k}_0'^2 k_z k'_z m'_I + \overline{kk'}|m|^2\theta' m_R\right) - \frac{1}{\tilde{k}_0^2\tilde{k}'_0}m'_I\overline{kk'}\right)\Bigg].
\end{aligned}
\tag{C.10}
$$

The last terms in the last two lines are the contributions from the first order collision term. There are also contributions from this term with the derivative acting on the Wigner functions, but they cancel each other. In writing this equation we used the abbreviation $\overline{kk'} = k_0k'_0 - k_xk'_x - k_yk'_y$. The terms inside of the square brackets can be combined to

$$
|m|^2\theta'\left[-\frac{\overline{kk'}}{\tilde{k}_0'^3\tilde{k}_0} + \frac{k_zk'_z}{\tilde{k}_0^3\tilde{k}'_0} + |m|^2\frac{\overline{kk'}}{\tilde{k}_0^3\tilde{k}_0'^3}\right],
\tag{C.11}
$$

and by using the constraint equation to leading order, $|m|^2 = \tilde{k}_0^2 - k_z^2$, we find

$$
\ldots = \frac{sk_z}{\tilde{k}_0}\times s|m|^2\theta'\left(\frac{k'_z}{\tilde{k}_0^2\tilde{k}'_0} - k_z\frac{\overline{kk'}}{\tilde{k}_0^2\tilde{k}_0'^3}\right).
\tag{C.12}
$$

A comparison with (3.46) finally leads to

$$\mathcal{K}_3^s + \frac{m_I}{\tilde{k}_0}\mathcal{C}_1^s - \frac{m_R}{\tilde{k}_0}\mathcal{C}_2^s = \frac{sk_z}{\tilde{k}_0} \times \mathcal{K}_0^s \,. \tag{C.13}$$

This is the same factor as the one that appeared in the flow term, which means that the complete kinetic equation for g_3^s is some term times the equation for g_0^s. The same can be shown for the other two equations. With this we have now proven the consistency of the equations: there is indeed only one kinetic equation for g_0^s.

Appendix D. Spin off-diagonal equations

Taking the traces and real and imaginary parts of the off-diagonal equation of motion for the Green function leads to the following set of equations. The kinetic equations for the g-functions are

$$\frac{\tilde{k}_0}{2k_0}\partial_t g_4 + \frac{1}{2}m'_R\partial_{k_z} g_5 + \frac{1}{2}m'_I\partial_{k_z} g_6 - k_z h_7 = \mathcal{K}_{g4} \tag{D.1}$$

$$\frac{\tilde{k}_0}{2k_0}\partial_t g_5 + \frac{1}{2}m'_R\partial_{k_z} g_4 + m_I g_7 + \frac{1}{2}\partial_z h_6 = \mathcal{K}_{g5} \tag{D.2}$$

$$\frac{\tilde{k}_0}{2k_0}\partial_t g_6 + \frac{1}{2}m'_I\partial_{k_z} g_4 - m_R\partial_{k_z} g_7 - \frac{1}{2}\partial_3 h_5 = \mathcal{K}_{g6} \tag{D.3}$$

$$\frac{\tilde{k}_0}{2k_0}\partial_t g_7 - m_I g_5 + m_R g_6 - k_z h_4 = \mathcal{K}_{g7} \tag{D.4}$$

the kinetic equations for the h-functions are

$$\frac{\tilde{k}_0}{2k_0}\partial_t h_4 + \frac{1}{2}m'_R\partial_{k_z} h_5 + \frac{1}{2}m'_I\partial_{k_z} h_6 + k_z g_7 = \mathcal{K}_{h4} \tag{D.5}$$

$$\frac{\tilde{k}_0}{2k_0}\partial_t h_5 + \frac{1}{2}m'_R\partial_{k_z} h_4 + m_I h_7 - \frac{1}{2}\partial_z g_6 = \mathcal{K}_{h5} \tag{D.6}$$

$$\frac{\tilde{k}_0}{2k_0}\partial_t h_6 + \frac{1}{2}m'_I\partial_{k_z} h_4 - m_R\partial_{k_z} h_7 + \frac{1}{2}\partial_3 g_5 = \mathcal{K}_{h6} \tag{D.7}$$

$$\frac{\tilde{k}_0}{2k_0}\partial_t h_7 - m_I h_5 + m_R h_6 + k_z g_4 = \mathcal{K}_{h7}\,, \tag{D.8}$$

the constraint equations for the g

$$-\tilde{k}_0 g_4 - m_R g_5 - m_I g_6 + \frac{1}{2}\partial_z h_7 = \mathcal{C}_{g4} \tag{D.9}$$

$$-\tilde{k}_0 g_5 - m_R g_4 + \frac{1}{2}m'_I\partial_{k_z} g_7 + k_z h_6 = \mathcal{C}_{g5} \tag{D.10}$$

$$-\tilde{k}_0 g_6 - m_I g_4 - \frac{1}{2}m'_R\partial_{k_z} g_7 - k_z h_5 = \mathcal{C}_{g6} \tag{D.11}$$

$$-\tilde{k}_0 g_7 - \frac{1}{2}m'_I\partial_{k_z} g_5 + \frac{1}{2}m'_R\partial_{k_z} g_6 + \frac{1}{2}\partial_z h_4 = \mathcal{C}_{g7}\,, \tag{D.12}$$

and the constraint equations for the h

$$-\tilde{k}_0 h_4 - m_R h_5 - m_I h_6 - \frac{1}{2}\partial_z g_7 \;=\; \mathcal{C}_{h4} \tag{D.13}$$

$$-\tilde{k}_0 h_5 - m_R h_4 + \frac{1}{2}m_I'\partial_{k_z} h_7 - k_z g_6 \;=\; \mathcal{C}_{h5} \tag{D.14}$$

$$-\tilde{k}_0 h_6 - m_I h_4 - \frac{1}{2}m_R'\partial_{k_z} h_7 + k_z g_5 \;=\; \mathcal{C}_{h6} \tag{D.15}$$

$$-\tilde{k}_0 h_7 - \frac{1}{2}m_I'\partial_{k_z} h_5 + \frac{1}{2}m_R'\partial_{k_z} h_6 - \frac{1}{2}\partial_z g_4 \;=\; \mathcal{C}_{h7}\,. \tag{D.16}$$

We introduced abbreviations for the collisional traces:

$$\mathcal{K}_{g4} \;=\; \frac{1}{4}\sum_s \Re\,\mathrm{Tr}\,Q_{-s}\mathcal{C} \tag{D.17}$$

$$\mathcal{K}_{g5} \;=\; -\frac{1}{4}\sum_s s\,\Re\,\mathrm{Tr}\,Q_{-s}\gamma^3\gamma^5\mathcal{C} \tag{D.18}$$

$$\mathcal{K}_{g6} \;=\; -\frac{1}{4}\sum_s s\,\Re\,\mathrm{Tr}\,Q_{-s}(-i\gamma^3)\mathcal{C} \tag{D.19}$$

$$\mathcal{K}_{g7} \;=\; \frac{1}{4}\sum_s \Re\,\mathrm{Tr}\,Q_{-s}(-\gamma^5)\mathcal{C}\,. \tag{D.20}$$

The expressions for the $\mathcal{K}_{ha}$ are obtained from these by replacing $\Re$ by $s\Im$. By replacing $\Re$ by $\Im$ one finds the $\mathcal{C}_{ga}$ and finally the $\mathcal{C}_{ha}$ are obtained by replacing $\Re$ by $-s\Re$.

The constraint equation allow us again to reduce the set of independent functions. We can choose g_5, h_5 or g_6, h_6 as basic functions, respectively. From (D.12) and (D.16) it is obvious, that g_7 and h_7 are suppressed by one order of gradients in comparison to the other functions, and therefore cannot serve as basic functions. Since we expect all these off-diagonal functions to be themselves suppressed by one order compared to the diagonal ones, we will neglect g_7, h_7 from now on. We cannot choose g_4, h_4 as basic functions, either, because this would lead to $1/|m|^2$ terms in the relations between the functions, which is possibly singular. The reason for this is that for mass equal to zero the functions g_4, h_4 have to vanish, as can be seen from the constraint equations (D.9) and (D.13) (note that g_7, h_7 is already of first order in gradients).

We choose g_5, h_5 as basic quantities. Then we have

$$g_4 \;=\; \frac{1}{\tilde{k}_0^2 - m_I^2}\Big(-\tilde{k}_0 m_R g_5 + k_z m_I h_5 - \tilde{k}_0\mathcal{C}_{g4} + m_I\mathcal{C}_{g6}\Big) \tag{D.21}$$

$$h_4 \;=\; \frac{1}{\tilde{k}_0^2 - m_I^2}\Big(-\tilde{k}_0 m_R h_5 - k_z m_I g_5 - \tilde{k}_0\mathcal{C}_{h4} + m_I\mathcal{C}_{h6}\Big) \tag{D.22}$$

and

$$g_6 = \frac{1}{\tilde{k}_0^2 - m_I^2}\Big(m_R m_I g_5 - \tilde{k}_0 k_z h_5 - \tilde{k}_0 \mathcal{C}_{g6} + m_I \mathcal{C}_{g4}\Big) \tag{D.23}$$

$$h_6 = \frac{1}{\tilde{k}_0^2 - m_I^2}\Big(m_R m_I h_5 + \tilde{k}_0 k_z g_5 - \tilde{k}_0 \mathcal{C}_{h6} + m_I \mathcal{C}_{h4}\Big)\,. \tag{D.24}$$

We find a constraint equation for g_5, h_5 which is

$$-\tilde{k}_0\left(k^2 - |m|^2\right) g_5 = (\tilde{k}_0^2 - m_I^2)\mathcal{C}_{g5} + m_R(-\tilde{k}_0 \mathcal{C}_{g4} + m_I \mathcal{C}_{g6}) - k_z(-\tilde{k}_0 \mathcal{C}_{h6} + m_I \mathcal{C}_{h4})\,. \tag{D.25}$$

The constraint equation for h_5 is obtained by exchanging g and h and replacing k_z by $-k_z$.

The kinetic equation for g_5 is

$$\frac{\tilde{k}_0}{2k_0}\partial_t g_5 - \frac{1}{4\tilde{k}_0}|m|^{2\prime}\partial_{k_z} g_5 + \frac{1}{2\tilde{k}_0} k_z \partial_z g_5 + \frac{1}{2\tilde{k}_0}\frac{m_I'}{\tilde{k}_0^2 - m_I^2}\Big(k_z m_I g_5 + \tilde{k}_0 m_R h_5\Big) = \mathcal{K}_{g5} + \frac{m_I}{\tilde{k}_0}\mathcal{C}_{g7}\,. \tag{D.26}$$

The corresponding equation for h_5 is obtained by exchanging g and h and replacing m_R by $-m_R$.

It is important to observe that these equations have no source which is not collisional, in the sense that when one starts with thermal equilibrium, where the off-diagonal functions are zero, they will stay zero forever. They are only sourced by the diagonal functions via the collision term.

Appendix E. Fermionic Green function in the wall frame

Most of the calculations in this work are done in the wall frame. In order to obtain the leading order Wigner functions in that frame, we just have to perform a boost. The wall frame and the plasma frame are related by

$$\begin{aligned} t &= \gamma_w(\bar{t} - v_w \bar{z}) \\ z &= \gamma_w(\bar{z} - v_w \bar{t}) \\ x &= \bar{x} \\ y &= \bar{y}\,, \end{aligned} \tag{E.1}$$

where we denoted the coordinates in the plasma system with a bar and the ones in the wall frame without (so if v_w is positive, the wall moves in positive $\bar{z}$-direction). The boost of the fermionic Wigner function $G^<$ is

$$\begin{aligned} G^<_{eq}(k) &= L\bar{G}^<_{eq}(\bar{k})L^{-1} \qquad \text{(E.2)} \\ &= 2\pi i L(\bar{\not{k}} + m)L^{-1}\delta(\bar{k}^2 - |m|^2)\mathrm{sgn}(\bar{k}_0)n_{eq}(\bar{k}_0) \\ &= 2\pi i(\not{k} + m)\delta(k^2 - |m|^2)\mathrm{sgn}(\gamma_w(k_0 + v_w k_z))n_{eq}(\gamma_w(k_0 + v_w k_z))\,. \end{aligned}$$

Because k is on the mass-shell ($|k_z| < |k_0|$) and $v_w < 1$, we have

$$\mathrm{sgn}(\gamma_w(k_0 + v_w k_z)) = \mathrm{sgn}(k_0)\,. \tag{E.3}$$

We know that the wall is comparatively slow, so we can neglect the boost factor γ_w and finally obtain

$$G^<_{eq}(k) = 2\pi i(\not{k} + m)\delta(k^2 - |m|^2)\mathrm{sgn}(k_0)n_{eq}(k_0 + v_w k_z) \tag{E.4}$$

The same changes have to be made in $G^>_{eq}$ and in the scalar Wigner functions.

Appendix F. Fermionic Collision Integrals

F.1 Collisional Source

In this appendix we show the details of the calculation of the fermionic collisional source term, which we omitted in the main text. We show here the calculation in the mixing case, the nonmixing case is simply obtained by setting $|m|_i^2 = |m|_j^2 = |m|^2$. We begin with equation (3.68):

$$
\begin{aligned}
\mathcal{K}_{0,i}^s \; = \; & \frac{1}{4}\beta v_w \left[\delta(k_0+\omega_{0,i}) - \delta(k_0-\omega_{0,i})\right] \sum_j \frac{|y_{ij}|^2+|y_{ji}|^2}{2} \\
& \int \frac{d^3k'}{(2\pi)^3}\frac{\pi^2}{2}\frac{1}{\omega''_\phi}\delta(\omega_{0,i}+\omega'_{0,j}-\omega''_\phi) f_{0,i} f'_{0,j}(1+f_\phi) \\
& \left(s\big(|m|^2(\theta'+\Delta_z)\big)_i \frac{k'_z}{\omega_{0,i}\tilde{\omega}_{0,i}\omega'_{0,j}} + s\big(|m|^2(\theta'+\Delta_z)\big)_j \frac{k_z}{\omega_{0,i}\tilde{\omega}_{0,i}\omega'_{0,j}} \frac{\omega_{0,i}\omega'_{0,j} + \vec{k}_\| \cdot \vec{k}'_\|}{\tilde{\omega}'^2_{0,j}} \right) .
\end{aligned}
\tag{F.1}
$$

The $\vec{k}'$ integration is performed in cylindrical coordinates, $d^3k' = dk'_z k'_\| dk'_\| d\phi$, where ϕ is the angle between $\vec{k}_\|$ and $\vec{k}'_\|$, and $k_\|$ and $k'_\|$ are the absolute values of the parallel momenta, respectively. We substitute

$$
x = \vec{k}\cdot\vec{k}' = k_\| k'_\| \cos\phi \tag{F.2}
$$

and instead of integrating over the absolute value of the parallel momentum we integrate over $k' = |\vec{k}'|$:

$$
\int d^3k' = \int_{-\infty}^{\infty} dk'_z \frac{1}{2}\int_0^\infty dk'^2 \theta(k'^2 - k_z^2) \int_{-\infty}^{\infty} \frac{2\,dx}{\sqrt{(k_\| k'_\|)^2 - x^2}} \theta(k_\| k'_\| - |x|) . \tag{F.3}
$$

Since $\omega''_\phi = \left((\vec{k}-\vec{k}')^2 + m_\phi^2\right)^{-\frac{1}{2}} = \left(k^2 + k'^2 - 2x - 2k_z k'_z + m_\phi^2\right)^{-\frac{1}{2}}$ is a function of x we can rewrite the remaining δ-function

$$
\delta(\omega_0 + \omega'_0 - \omega''_\phi) = \omega''_\phi\, \delta\left(x - (\xi - k_z k'_z)\right) , \tag{F.4}
$$

where we used the abbreviation

$$
\xi = \frac{m_\phi^2 - |m_i|^2 - |m_j|^2}{2} - \omega_{0,i}\omega'_{0,j} . \tag{F.5}
$$

With the δ-function the x-integration can now be performed trivially. With a not so trivial calculation one can show that for arbitrary ξ the relation

$$\theta\big(k_{\|}k'_{\|} - (\xi - k_z k'_z)\big)\theta\big(k'^2 - k'^2_z\big) = \theta\big(k^2 k'^2 - \xi^2\big)\theta\big(k'_{z_2} - k'_z\big)\theta\big(k'_z - k'_{z_1}\big) \tag{F.6}$$

holds, where $k'_{z_{1,2}}$ are the roots of $(k_{\|}k'_{\|})^2 - (\xi - k_z k'_z)^2 = 0$:

$$k'_{z_{1,2}} = \frac{\xi k_z}{k^2} \mp \frac{k_{\|}}{k^2}\sqrt{k^2 k'^2 - \xi^2}\,. \tag{F.7}$$

The function $\theta\big(k^2k'^2 - \xi^2\big)$ contains a mass-threshold. It can be found by trying to determine those values of k' for which the θ-function is non-vanishing:

$$\xi^2 < k^2 k'^2 \Leftrightarrow -kk' < \xi < kk'\,. \tag{F.8}$$

The latter two conditions can only be satisfied if the quadratic equations

$$\begin{aligned} k'^2|m_i|^2 \mp k'k(m_\phi^2 - |m_i|^2 - |m_j|^2)^2 & \\ + k^2|m_j|^2 + |m_i|^2|m_j|^2 - \frac{1}{4}(m_\phi^2 - |m_i|^2 - |m_j|^2)^2 = 0\,, & \end{aligned} \tag{F.9}$$

have real solutions, which is only the case if

$$m_\phi > |m_i|^2 + |m_j|^2\,. \tag{F.10}$$

Now we can write the source as

$$\begin{aligned} \mathcal{K}^s_{0,i} \;=\; & \frac{1}{32\pi}\beta v_w \sum_j \frac{|y_{ij}|^2 + |y_{ji}|^2}{2}\left[\delta(k_0 + \omega_{0,i}) - \delta(k_0 - \omega_{0,i})\right]\frac{1}{\omega_{0,i}\tilde{\omega}_{0,i}} \\ & \int k'dk'\theta(k^2k'^2 - \xi^2)\frac{1}{\omega'_{0,j}} f_{0,i} f'_{0,j}(1 + f_\phi)\int_{k'_{z_1}}^{k'_{z_2}} \frac{1}{k\sqrt{(k'_{z_2} - k'_z)(k'_z - k'_{z_1})}} \\ & \Bigg[s\big(|m|^2(\theta' + \Delta_z)\big)_j k_z \frac{m_\phi^2 - |m_i|^2 - |m_j|^2}{2(k'^2_z + |m_j|^2)} \\ & \qquad + k'_z\Big(s\big(|m|^2(\theta' + \Delta_z)\big)_i - s\big(|m|^2(\theta' + \Delta_z)\big)_j \frac{k_z^2}{k'^2_z + |m_j|^2}\Big)\Bigg]\,. \end{aligned} \tag{F.11}$$

For the fluid equations the integral over all momenta is needed. With the observation that the integrand is odd under $k_z \to -k_z$ and $k'_z \to -k'_z$, we can immediately write

$$\int_\pm \frac{d^4k}{(2\pi)^4}\mathcal{K}^s_{0,i} = 0\,, \tag{F.12}$$

where the index $\pm$ denotes the integral over positive and negative frequencies, respectively. The integral with an additional factor $k_z/\omega_{0,i}$ can be evaluated further by making use of the integrals

$$
\begin{aligned}
\int_a^b \frac{dk'_z}{k'^2_z+|m_j|^2}\frac{1}{\sqrt{(b-k'_z)(k'_z-a)}} &= \frac{\pi}{|m_j|^2}\Re\left((|m_j|^2-ab)+i|m_j|(a+b)\right)^{-\frac{1}{2}}\\
\int_a^b \frac{k'_z dk'_z}{k'^2_z+|m_j|^2}\frac{1}{\sqrt{(b-k'_z)(k'_z-a)}} &= -\pi\Im\left((|m_j|^2-ab)+i|m_j|(a+b)\right)^{-\frac{1}{2}}\\
\int_a^b \frac{k'_z dk'_z}{\sqrt{(b-k'_z)(k'_z-a)}} &= \frac{\pi k_z\xi}{k^2}\\
ab = \frac{k'^2k_z^2+(\xi^2-k^2k'^2)}{k^2} &\quad , \quad a+b=\frac{\pi}{2}(a+b)\,.
\end{aligned}
\tag{F.13}
$$

We finally obtain

$$
\begin{aligned}
&\int_\pm \frac{d^4k}{(2\pi)^4}\frac{k_z}{\omega_{0,i}}\mathcal{K}^s_{0,i} \qquad\qquad (F.14)\\
=\;& \pm\frac{1}{64\pi^3}\beta v_w\sum_j\frac{|y_{ij}|^2+|y_{ji}|^2}{2}\int kdk\int k'dk'\theta(k^2k'^2-\xi^2)f_{0,i}f'_{0,j}(1+f_\phi)\\
&\int_{-k}^{k}dk_z\frac{k_z^2}{\omega_{0,i}^2\omega'_{0,j}\tilde{\omega}_{0,i}}\Big[s\big(|m|^2(\theta'+\Delta_z)\big)_i\frac{\xi}{k^3}\\
&+s\big(|m|^2(\theta'+\Delta_z)\big)_j\Big(k_z\Im\left(k^2(|m_j|^2+k'^2)-k_z^2k'^2-\xi^2+2i|m_j|\xi k_z\right)^{-\frac{1}{2}}\\
&\quad+\frac{m_\phi^2-|m_i|^2-|m_j|^2}{2|m_j|}\Re\left(k^2(|m_j|^2+k'^2)-k_z^2k'^2-\xi^2+2i|m_j|\xi k_z\right)^{-\frac{1}{2}}\Big)\Big]\,.
\end{aligned}
$$

F.2 Collisional Rates

In section 3.2.2 we found the following expressions for the relaxation part of the collision term:

$$
\begin{aligned}
\mathcal{K}^s_{0,loc} &= \frac{1}{4}y^2\int\frac{d^4k'\,d^4k''}{(2\pi)^8}(2\pi)^4\delta^4(k-k'+k'')i\Delta^>_{eq}(k'')\\
&\qquad g^<_{eq}(k')\left(\mathrm{e}^{\beta\bar{k}_0}\delta g_0^{<s}(k)+\delta g_0^{>s}(k)\right)(-2)\frac{k\cdot k'}{\tilde{k}_0\tilde{k}'_0}
\end{aligned}
\tag{F.15}
$$

and

$$
\begin{aligned}
\mathcal{K}^s_{0,nloc} &= \frac{1}{4}y^2\int\frac{d^4k'\,d^4k''}{(2\pi)^8}(2\pi)^4\delta^4(k-k'+k'')i\Delta^>_{eq}(k'')\\
&\qquad\sum_{s'}\left(\mathrm{e}^{-\beta\bar{k}'_0}\delta g_0^{>s'}(k')+\delta g_0^{<s'}(k')\right)g^>_{eq}(k)\\
&\qquad\left(-ss'+ss'\frac{k_zk'_z}{\tilde{k}_0^2\tilde{k}'^2_0}(k_0k'_0-\vec{k}_\parallel\cdot\vec{k}'_\parallel)+\frac{k\cdot k'}{\tilde{k}_0\tilde{k}'_0}\right)\,.
\end{aligned}
\tag{F.16}
$$

With the fluid ansatz (4.20) the momentum dependence of δg is completely specified.

The evaluation of the integrals runs along the same lines as for the collisional source in the first part of this appendix. We just give the results. The contribution to the zeroth moment equation is

$$\begin{aligned}4\left(\int_{k_0=0}^{\infty}\frac{d^4k}{(2\pi)^4}\mathcal{K}_0^s+\int_{k_0=-\infty}^{0}\frac{d^4k}{(2\pi)^4}\mathcal{K}_0^s\right)&\\ =-\sum_{s'}\Gamma^{0\mu}_{ss'}\mu_{s'}-v_w\sum_{s'}\Gamma^{0u}_{ss'}u_{s'}\,,&\end{aligned}\tag{F.17}$$

where the rates are given by

$$\begin{aligned}\Gamma^{0\mu}_{ss'} &= \frac{1}{4\pi^3}\beta\frac{y^2}{4}\int_0^\infty k\,dk\,k'\,dk'\,\theta(k^2k'^2-\xi^2)\frac{\omega_0\omega_0'+\xi}{\omega_0\omega_0'}f_0f_0'(1+f_\phi'')\delta_{ss'}\\
&-\beta\frac{1}{16\pi^4}\frac{y^2}{4}\int_0^\infty dk\,dk'\theta(k^2k'^2-\xi^2)k'f_0f_0'(1+f_\phi'')\\
&\quad\int_{-\infty}^{\infty}dk_z\,dk_z'\frac{\theta(k^2-k_z^2)\theta(k_{z_2}'-k_z')\theta(k_z'-k_{z_1}')}{\sqrt{(k_{z_2}'-k_z')(k_z'-k_{z_1}')}}\\
&\quad\times\left(\frac{\omega_0\omega_0'+\xi}{\omega_0\omega_0'}+\left(\frac{\tilde\omega_0\tilde\omega_0'}{\omega_0\omega_0'}+\frac{\omega_0\omega_0'+\xi-k_zk_z'}{\omega_0\omega_0'\tilde\omega_0\tilde\omega_0'}k_zk_z'\right)ss'\right)\end{aligned}\tag{F.18}$$

and

$$\begin{aligned}\Gamma^{0u}_{ss'} &= \frac{1}{4\pi^3}\beta^2\frac{y^2}{4}\int_0^\infty k\,dk\,k'\,dk'\,\theta(k^2k'^2-\xi^2)\frac{\omega_0\omega_0'+\xi}{\omega_0\omega_0'}f_0f_0'(1+f_\phi'')\\
&\quad\left[\left(1-f_0+f_\phi''\right)-\left(1-f_0'+f_\phi''\right)\frac{\xi}{k^2}\right]\frac{k^2}{3}\delta_{ss'}\\
&+\frac{1}{16\pi^4}\beta^2\frac{y^2}{4}\int_0^\infty dk\,dk'\theta(k^2k'^2-\xi^2)k'f_0f_0'(1+f_\phi'')\\
&\quad\int_{-\infty}^{\infty}dk_z\,dk_z'\frac{\theta(k^2-k_z^2)\theta(k_{z_2}'-k_z')\theta(k_z'-k_{z_1}')}{\sqrt{(k_{z_2}'-k_z')(k_z'-k_{z_1}')}}\\
&\quad\times\Bigg(\frac{\omega_0\omega_0'+\xi}{\omega_0\omega_0'}\left[(1-f_0+f_\phi'')k_zk_z'-(1-f_0'+f_\phi'')k_z^2\right]\\
&\qquad+\left(\frac{\tilde\omega_0\tilde\omega_0'}{\omega_0\omega_0'}+\frac{\omega_0\omega_0'+\xi-k_zk_z'}{\omega_0\omega_0'\tilde\omega_0\tilde\omega_0'}k_zk_z'\right)\\
&\qquad\quad\left[(1-f_0+f_\phi'')k_zk_z'-(1-f_0'+f_\phi'')k_z^2\right]ss'\Bigg)\,.\end{aligned}\tag{F.19}$$

In the first moment equation the collision term is

$$4\left(\int_{k_0=0}^{\infty}\frac{d^4k}{(2\pi)^4}\frac{k_z}{\omega_0}\mathcal{K}_0^s-\int_{k_0=-\infty}^{0}\frac{d^4k}{(2\pi)^4}\frac{k_z}{\omega_0}\mathcal{K}_0^s\right)$$
$$=\mathcal{S}_s-\sum_{s'}\Gamma^{1u}_{ss'}u_{s'}-v_w\sum_{s'}\Gamma^{1\mu}_{ss'}\mu_{s'}\,, \tag{F.20}$$

where besides the source term there are the rates

$$\begin{aligned}\Gamma^{1u}_{ss'} \;=\;& -\frac{1}{4\pi^3}\beta\frac{y^2}{4}\int_0^\infty k\,dk\,k'\,dk'\,\theta(k^2k'^2-\xi^2)\frac{\omega_0\omega_0'+\xi}{\omega_0\omega_0'}f_0f_0'(1+f_\phi'')\frac{k^3}{3\omega_0}\delta_{ss'}\\
&-\frac{1}{16\pi^4}\beta\frac{y^2}{4}\int_0^\infty dk\,dk'\theta(k^2k'^2-\xi^2)k'f_0f_0'(1+f_\phi'')\\
&\int_{-\infty}^{\infty}dk_z\,dk_z'\frac{\theta(k^2-k_z^2)\theta(k'_{z_2}-k'_z)\theta(k'_z-k'_{z_1})}{\sqrt{(k'_{z_2}-k'_z)(k'_z-k'_{z_1})}}\\
&\times\left(\frac{\tilde\omega_0\tilde\omega_0'}{\omega_0\omega_0'}\frac{k_zk_z'}{\omega_0}+\left(\frac{\tilde\omega_0\tilde\omega_0'}{\omega_0\omega_0'}+\frac{\omega_0\omega_0'+\xi-k_zk_z'}{\omega_0\omega_0'\tilde\omega_0\tilde\omega_0'}k_zk_z'\right)\frac{k_zk_z'}{\omega_0}ss'\right)\end{aligned}$$

and

$$\begin{aligned}\Gamma^{1\mu}_{ss'} \;=\;& -\frac{1}{4\pi^3}\beta^2\frac{y^2}{4}\int_0^\infty k\,dk\,k'\,dk'\,\theta(k^2k'^2-\xi^2)\frac{\omega_0\omega_0'+\xi}{\omega_0\omega_0'}f_0f_0'(1+f_\phi'')\\
&\left[\left(1-f_0+f_\phi''\right)-\left(1-f_0'+f_\phi''\right)\frac{\xi}{k^2}\right]\frac{k^3}{3\omega_0}\delta_{ss'}\\
&+\frac{1}{16\pi^4}\beta^2\frac{y^2}{4}\int_0^\infty dk\,dk'\theta(k^2k'^2-\xi^2)k'f_0f_0'(1+f_\phi'')\\
&\int_{-\infty}^{\infty}dk_z\,dk_z'\frac{\theta(k^2-k_z^2)\theta(k'_{z_2}-k'_z)\theta(k'_z-k'_{z_1})}{\sqrt{(k'_{z_2}-k'_z)(k'_z-k'_{z_1})}}\\
&\times\left(\frac{\omega_0\omega_0'+\xi}{\omega_0\omega_0'}\frac{1}{\omega_0}\left[(1-f_0+f_\phi'')k_z^2-(1-f_0'+f_\phi'')k_zk_z'\right]\right.\\
&+\left(\frac{\tilde\omega_0\tilde\omega_0'}{\omega_0\omega_0'}+\frac{\omega_0\omega_0'+\xi-k_zk_z'}{\omega_0\omega_0'\tilde\omega_0\tilde\omega_0'}k_zk_z'\right)\\
&\left.\frac{1}{\omega_0}\left[(1-f_0+f_\phi'')k_z^2-(1-f_0'+f_\phi'')k_zk_z'\right]ss'\right)\,.\end{aligned} \tag{F.21}$$

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