Department of Physics and Astronomy
University of Heidelberg

Master Thesis in Physics
submitted by

Torben Skrzypek

born March 1996 in Marburg (Germany)

October 2019

Obstacles to realizing Quintessence in String Theory

This Master Thesis has been carried out by Torben Skrzypek at the
Institute for Theoretical Physics
under the supervision of
Prof. Arthur Hebecker

Abstract

In this master thesis we analyze the construction of quintessence models by flux compactification of type IIB string theory. We focus on Kähler moduli as candidates for the quintessence field and briefly comment on other approaches. The large hierarchies required for simultaneously describing quintessence and the standard model pose a major challenge to model building and depend upon parametric control over the scalar potential, which is gained in the limit of large compactification volume. Further suppression of the quintessence scale can be achieved by an anisotropic compactification. However, by lowering the quintessence mass we also lower the masses of several other fields. As has been noticed before, the volume modulus becomes too light to avoid fifth-force constraints. We call this the "light volume problem". Furthermore, the masses of the SUSY partners of standard-model fields turn out too light as well, so we need a further source of SUSY breaking. Introducing an appropriate SUSY-breaking hidden sector on the standard-model brane then leads to a large positive F-term contribution to the scalar potential that cannot be canceled by the known negative terms and thus significantly raises the vacuum energy. To cancel the F-term, it would take an equally large negative contribution, which is currently unknown.

In the context of the de Sitter swampland conjecture, this "F-term problem" raises yet another question. If we manage to cancel the F-term with some large additional contribution, a tiny change of parameters in the SUSY-breaking sector could de-tune this cancellation and uplift the potential to de Sitter. Since the conjecture does not allow for such potentials, there has to be some mechanism preventing the uplift.

Zusammenfassung

In dieser Masterarbeit untersuchen wir die Konstruktion von Quintessenzmodellen durch Kompaktifizierung von Typ IIB Stringtheorie. Wir fokussieren uns auf Kählermoduli als Kandidaten für das Quintessenzfeld und kommentieren andere Ansätze kurz. Die großen Hierarchien, die die gleichzeitige Beschreibung von Quintessenz und Standardmodell benötigt, stellen eine besondere Herausforderung für die Modellbildung dar. Man benötigt für ihre Umsetzung parametrische Kontrolle über die Potenzialterme, die im Grenzwert großer Kompaktifizierungsvolumina gewährleistet ist. Eine weitere Unterdrückung der Quintessenzskala lässt sich durch anisotrope Kompaktifizierung erreichen. Allerdings verringern sich durch das Absenken der Quintessenzmasse auch die Massen anderer Felder. Bereits bekannt ist, dass der Volumenmodulus zu leicht wird, um den Beschränkungen an fünfte Kräfte zu entgehen. Wir nennen dies das “leichte Volumen Problem”. Außerdem werden die Massen der Superpartner des Standardmodells zu leicht, sodass eine weitere Quelle für Brechung der Supersymmetrie benötigt wird. Das Einführen eines angemessen SUSY-brechenden verborgenen Sektors auf der Standardmodellbrane führt zu einem großen positiven F-term-Beitrag zum Skalarpotential, der nicht durch die bekannten negativen Beiträge aufgehoben werden kann und so die Vakuumenergie beträchtlich hebt. Es bedarf eines ebenso großen negativen Beitrages, um diesen F-term aufzuheben. Ein solcher Beitrag ist bislang unbekannt.

Im Kontext der de Sitter-Sumpfland-Vermutung führt dieses “F-term Problem” zu einer weiteren Frage. Wenn eine Aufhebung des F-terms durch einen zusätzlichen großen Beitrag möglich ist, könnte eine kleine Veränderung der SUSY-brechenden Parameter zu einer Missabstimmung dieser Aufhebung führen und das Potential auf de Sitter-Niveau anheben. Da die Vermutung solche Potentiale nicht zulässt, muss es einen Mechanismus geben, der das Anheben des Potentials verhindert.

Contents

1 Introduction

Since the development of General Relativity (GR) and Quantum Field Theory (QFT) in the last century the most prominent question in theoretical physics has been whether and how a unification of both could be achieved. One promising contender for this unification is string theory, which consists of a quantum theory on a two-dimensional worldsheet embedded in an ambient spacetime. The coordinates in said spacetime are fields on the worldsheet. The theory has to fulfill certain self-consistency conditions, one of which is the vanishing of the beta-functions of the worldsheet theory. Computation of these beta-functions to first order yields the Einstein field equations in the ambient spacetime, bridging the gap between GR and QFT. This is promising from a conceptual point of view but from a "theory of everything" we expect more. We also want a successful description and prediction of experimental results, which is the task string phenomenology tries to solve.

The most surprising consistency condition of (super-)string theory is the need for ten spacetime dimensions.[1] Since we only experience four, the remaining six dimensions are assumed to be curled up into a small manifold, which has yet escaped our observations. We call this a compactification of six dimensions. The compactification manifold is not unique, in fact there is a plethora of choices, which opens up a whole landscape of string-theory solutions. Successful phenomenology now involves choosing a solution that fits best to the experiments.

Experiments in particle physics and cosmology test the highest and lowest energy scales that we are capable of measuring. The Standard Model (SM) of particle physics is believed to be realizable through specific combinations of geometrical objects called branes (see [4] for a review). On the other hand, the possibilities for implementing the standard model of cosmology (ΛCDM) into string theory are under lively discussion and will be the starting point of this thesis.

Since the discovery of the accelerated expansion of the universe [5, 6], cosmologists have been searching for an explanation [7–10]. The simplest model involves a cosmological constant Λ in the field equations, which corresponds to a non-vanishing vacuum energy density and drives the expansion. This constant can be calculated from cosmological data and takes the value 7.15×10^{-121} in reduced Planck units [11].[2] Although many other models were proposed in the last decades, leading to more or less agreement with data (see [9] for an extensive review), the cosmological constant still serves as the "null hypothesis" of cosmology, canonized in the ΛCDM model.

[1] Although there has also been research into non-critical string theory with other dimensionalities [2, 3], we will only work with the critical superstring.

[2] In this thesis, we will mainly work in (reduced) Planck units, where in addition to $\hbar = c = 1$ the reduced Planck mass $M_{\rm P} = 1/\sqrt{8\pi G}$ is set to unity as well. These are 4D units and we have to be careful when discussing 10D string theory. Any intrinsic geometry is measured in string units, which scale as $M_{\rm s}^2 = M_{\rm P}^2/\mathcal{V}$, where $\mathcal{V}$ is the compactification volume introduced later. Sometimes, we may reintroduce $M_{\rm P}$ for clarity and for conversion to experimentally more accessible energy units, such as electronvolts (eV). The Planck scale is approximately $M_{\rm P} \approx 2.4 \times 10^{27}$eV.

Because the cosmological constant is so small, we run into a problem when we try to compare it with the vacuum energy density of QFT. In a quantum field theory, the vacuum state receives closed-loop corrections such that the resulting vacuum energy is formally infinite. We therefore have to regularize the theory, introducing a cut-off at high energies. Since the modern point of view on any field theory is that of an effective field theory at a certain energy scale, the introduction of a cut-off scale is conceptually sound. When we approach a quantum theory of gravity, however, the natural cut-off scale is the Planck scale. We would thus expect a vacuum energy of order 1, which is $\mathcal{O}(10^{-121})$ far from the measured value. This is called the cosmological constant problem and requires an explanation or a fine-tuning mechanism. One approach is to introduce matter in pairs of bosons and fermions whose positive and negative loop contributions to the vacuum energy cancel exactly. However, collider experiments of the past decades have ruled out such supersymmetric (SUSY) models up to the TeV-scale [12, 13]. If SUSY is restored at higher energies, we still face a hierarchy problem between the SUSY scale and the cosmological constant of at least $\mathcal{O}(10^{-61})$. String theory can realize SUSY-breaking at appropriate scales by warped geometries, which redshift phenomena at one point in comparison to another point in the compactification [14]. The remaining hierarchy problem, however, has to be solved by some kind of cancellation, which is highly tuned and thus difficult to realize in explicit models.

Fortunately, string theory also allows for a huge number of solutions and if we assume their respective cosmological constants to be randomly distributed in the interval $[-1, 1]$, there might even exist a solution with cosmological constant sufficiently close to the observed one. More refined statistical arguments have been made in [15–17]. Furthermore, specific string-theory models of de Sitter spacetime (dS), i.e. spacetime with constant positive Λ, have been proposed [18,19]. Although these models were discussed a lot (see [20–41]), the decision on their validity is still pending. On the critical side, the de Sitter swampland conjecture has been put forth, which roughly states that no quantum gravity allows for a positive cosmological constant [42–44]. More precisely:

Conjecture. *A potential $V(\phi)$ for scalar fields in a low energy effective theory of any consistent quantum gravity must satisfy either,*

$$|\nabla V| \geq cV \tag{1}$$

or

$$\min\{\nabla_i \nabla_j V\} \leq -c'V \tag{2}$$

for some universal $\mathcal{O}(1)$ constants $c, c' > 0$ in Planck units, where $\min\{\nabla_i \nabla_j V\}$ is the minimal eigenvalue of the Hessian in an orthonormal frame [44].

If this conjecture proves to be true,[3] the cosmological constant has to be substituted by an-

[3] After the proposal of the conjecture, the discussion of dS models has been revitalized and criticism has been raised as to whether these constructions are consistent. See [45–49] for progress in refuting some of the criticism based on 10D considerations.

other cosmological model. In this thesis we will assume that the conjecture holds and take the simplest alternative to Λ, namely quintessence, as has been proposed already in [42, 50]. Quintessence models explain the accelerated expansion by a scalar field which slowly rolls down a potential and currently provides a vacuum energy comparable to the cosmological constant [51–53]. Realizing quintessence in string theory, however, is not easy to accomplish (see e.g. [31, 54–61] for discussions). The most promising candidates for stringy quintessence are moduli (see e.g. [62–64]) and axions (see e.g. [31,55,65–69]), which are both ubiquitous in string compactifications.

The question we ask ourselves now is whether building quintessence models in string theory is possible at all and if it is any easier than building dS spacetime. The quintessence scalar has an equation of state parameter

$$\omega = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}\,, \tag{3}$$

which has to be negative and sufficiently large to source the acceleration of the universe. This requires $\dot{\phi}^2$ to be small. We further need ω to remain stable for a long period of time to match the history of the universe. These slow-roll conditions lead to bounds on the potential $V(\phi)$ as well as on its first and second derivatives (as we will explain in section 3.1). From the measured expansion rate H of the universe, the mass of the quintessence field can be restricted to be lower than order $\mathcal{O}(10^{-60})$.

As for the cosmological constant it is not easy to achieve such low scales even in string theory, so we focus on a scenario where the scalar potential is naturally small and can be controlled parametrically. The scenario in question is the Large Volume Scenario (LVS) [70], which uses the framework of type-IIB flux compactification and the resulting no-scale supergravity (SUGRA) model. In this case the scalar potential is only generated by higher-order corrections, which are suppressed by powers of the compactification volume $\mathcal{V}$. Taking $\mathcal{V}$ to be large, we have control over the different contributions and may hope to achieve a sufficiently low scale. However, since a too large compactification volume would be observable by experiments, the possible suppression by large volume is limited, so we have to rely on further model-building ideas. A promising candidate is the suggestion of an anisotropic compactification presented in [62, 71].

Lowering the quintessence scale comes at the expense of simultaneously lowering various other masses in the theory. As has been observed already by the authors of [62, 71], the volume modulus becomes dangerously light and violates fifth-force constraints, which we shall call the "light volume problem". Another problem, which we pointed out in our paper [1], arises if we want SUSY to be broken at sufficiently high scale to match experimental constraints. The geometrical SUSY-breaking scale turns out much too low, which forces us to introduce another SUSY-breaking hidden sector by hand. The additional SUSY-breaking sector contributes a very large positive F-term to the potential, which is not canceled by no-scale or any known corrections and thus requires another very large but negative contribution to cancel. This new term δV_{new} may even stabilize the volume modulus at sufficiently high mass scale, solving the light volume problem. However, we would need some explanation for its appearance and why it cancels the F-term to great accuracy. If the cancellation were not precise, we would be able to uplift the quintessence potential by slightly changing the SUSY-breaking sector and thus to violate the dS swampland conjecture. We have suggested the name "F-term problem" for this issue. Coming back to the initial question, it seems that our current knowledge of stringy quintessence models lacks a major component. Whether finding this element or improving the dS constructions is more challenging remains to be shown.

We will begin our discussion in the second part by reviewing type IIB flux compactifications. Since it will be important for the following discussion, we will focus especially on the no-scale structure and the inclusion of higher-order corrections therein. In the third part, we will present phenomenological restrictions of string-theoretic quintessence models and rule out the volume modulus as a "natural" candidate for quintessence. Turning to the most promising models of [62, 71], we will present the "light volume problem" and the "F-term problem" in the fourth part, expanding on the discussion in our paper [1]. In the fifth part, we shall discuss apparent loopholes and review other proposed stringy quintessence or dark energy models. We will see that similar issues to our F-term problem arise there as well. Finally, we will conclude our discussion by looking back at the argument in summary as well as pointing out possible directions for future developments.

2 Preliminaries on String Theory and no-scale SUGRA

The theoretical background of superstring theory consists of a superconformal field theory on a two-dimensional worldsheet. The bosonic fields correspond to coordinates of an embedding of the worldsheet into the ambient spacetime. During the quantization procedure, the inner consistency of the theory has to be maintained, resulting in the critical dimension 10 as well as in a specific choice of spectra. It turns out that there are five consistent string theories, namely types I, IIA and IIB as well as two heterotic string theories. Since dualities relate the five string theories to each other and to a hypothetical M-theory, of which we only know the classical 11D SUGRA limit, it is assumed that these sectors are only limits of one universal theory, which in an abuse of naming is often called M-theory as well. For our goal of building phenomenologically successful models, we may thus choose the "corner" of this net of theories which is most suitable for constructing quintessence. Since type IIB string theory has the advantage of a particularly simple no-scale structure after flux compactification, most dS constructions and several approaches to quintessence model-building have used this framework. We will thus rely on type IIB for our discussion as well. Let us briefly review the main steps in the compactification procedure before we take a closer look at the resulting scalar potential. The following summary is based on [72, 73] and we refer to these works for further details.

2.1 Compactification of type IIB string theory

String theory only depends on one continuous free parameter: the length scale l_{s} of the strings, which corresponds to a mass scale M_{s}. Since we have not observed strings in our worlds yet, the string scale M_{s} is assumed to be larger than our currently available energies. M_{s} also sets the scale for excitations of the string, so we can restrict our attention to the low energy limit by only considering its first-order excitations, which are massless. Computing the massless spectrum of type IIB string theory, one ends up with type IIB SUGRA, which is the unique chiral $\mathcal{N} = 2$ SUGRA in 10 dimensions. Although higher-order corrections will become important later, the stringy nature of the theory will contribute only indirectly from this point on.

The field content of the 10D type IIB SUGRA is completely determined by supersymmetry. The bosonic sector comprises the graviton $g_{\mu\nu}$, an anti-symmetric Kalb-Ramond field $B_{\mu\nu}$, a dilaton φ and differential forms C_0, C_2 and C_4. The field-strength of C_4 has to fulfill a self-duality condition, which is a constraint not visible in the Lagrangian and enforced by hand. The fermionic sector comprises two left-handed Majorana-Weyl gravitinos and two right-handed Majorana-Weyl dilatinos. For completeness we shall give the low-energy 10D action, which consists of four parts

$$\mathcal{S} = \mathcal{S}_{\text{bosons}} + \mathcal{S}_{\text{Chern-Simons}} + \mathcal{S}_{\text{fermions}} + \mathcal{S}_{\text{local}} \tag{4}$$

with

$$\mathcal{S}_{\text{bosons}} = \frac{1}{2\kappa}\int \mathrm{d}^{10}x\sqrt{-g}\mathrm{e}^{-2\varphi}\left(\mathcal{R} + \partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}|H_3|^2 - \frac{1}{2}\big|\tilde{F}_1\big|^2 - \frac{1}{2}\big|\tilde{F}_3\big|^2 - \frac{1}{2}\big|\tilde{F}_5\big|^2\right) \tag{5}$$

$$\mathcal{S}_{\text{Chern-Simons}} = -\frac{1}{4\kappa}\int C_4 \wedge H_3 \wedge F_3\,, \tag{6}$$

where the field strength of the Kalb-Ramond field is denoted by H_3 and the field strength of C_i is denoted by F_{i+1}. In addition, the combinations

$$\tilde{F}_1 = \mathrm{e}^{\varphi}F_1\,, \qquad \tilde{F}_3 = \mathrm{e}^{\varphi}(F_3 - C_0 H_3)\,, \qquad \tilde{F}_5 = \mathrm{e}^{\varphi}\left(F_5 - \frac{1}{2}C_2 \wedge H_3 + \frac{1}{2}B_2 \wedge F_3\right) \tag{7}$$

are used. Since we search for classical solutions of the theory, the fermion fields will not be excited, so we do not need to explain their action $\mathcal{S}_{\text{fermions}}$ any further. Finally the term $\mathcal{S}_{\text{local}}$ introduces local objects like Dp-branes. Dp-branes are $p+1$-dimensional extended dynamical objects, to which the ends open strings are attached. They are similar to solitons in field theory. In type IIB string theory the only stable Dp-branes are those that have odd p, as they can be shown to be BPS states.[4]

As mentioned in the introduction, a description of real-world phenomena requires a compactification to our familiar four dimensions. This procedure is called Kaluza-Klein compactification and was originally an attempt to unify gravity and electrodynamics via compactification of a five-dimensional spacetime on a circle. In this scenario, the metric has to be split into a 4D metric, one scalar and one vector field. The scalar then parameterizes the radius R of the circle while the vector field was thought to be the electromagnetic four-potential. Additionally, the momentum in circle-direction is quantized due to single-valuedness of the wave function, similar to standing waves in a box. Thus, a tower of so-called Kaluza-Klein (KK) states can be excited, whose masses are given by n/R for any natural number n. In string theory another tower of states arises due to closed strings winding around the compact dimension. These towers play a crucial role in string-theory dualities, as they are exchanged during T-dualization but since we will remain in the IIB picture, we will not go into these details. Instead, let us further specify the compactification we want to perform.

To get to a homogeneous, isotropic 4D theory, the 10D spacetime should split like $\mathbb{R}^{1,3} \times \mathcal{X}$, where $\mathcal{X}$ is a 6D manifold, such that the metric decomposes into

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + g_{mn}\mathrm{d}y^m \mathrm{d}y^n\,. \tag{8}$$

In principle, we could choose any compact manifold $\mathcal{X}$, but the calculations get extremely difficult if we do not carry over some degree of symmetry from ten dimensions. The usual way

[4] Bogomol'nyi–Prasad–Sommerfield (BPS) states preserve some of the SUSY of the theory as they form a short representation of the SUSY algebra. One can connect this property to saturation of a "BPS bound" on their mass, yielding an alternative definition.

to achieve this is to look at the decomposition of the 10D Lorentz group

$$SO(1,9) \to SO(1,3) \times SO(6) \cong SO(1,3) \times SU(4)\,. \tag{9}$$

If we choose $\mathcal{X}$ with $SU(3)$ holonomy, there exists a Killing spinor on $\mathcal{X}$ and thus 8 of the initial 32 real supercharges are conserved. This will result in an $\mathcal{N}=2$ SUGRA in 4D. We will therefore restrict our attention to Calabi-Yau (CY) 3-folds, which are complex Kähler manifolds with SU(3) holonomy.[5] These are also characterized by Ricci-flatness, i.e. the vanishing of the Ricci form, which is the complex analogue of the Ricci tensor. Thus our CY 3-fold and therefore also our overall 10D space satisfies the vacuum Einstein field-equations. The Dolbeault cohomology $H^{i,j}$ of CY 3-folds is determined by two Hodge numbers $h^{1,1}=h^{2,2}$ and $h^{1,2}=h^{2,1}$, while $h^{0,0}=h^{3,0}=h^{0,3}=h^{3,3}=1$ and all other Hodge numbers vanish. Since these numbers determine all nontrivial form-field configurations on $\mathcal{X}$, we can decompose the form-fields B_2 and C_i accordingly. The dilaton remains unchanged, while the graviton has to be split up to match the split in $\mathbb{R}^{1,3}\times\mathcal{X}$. A graviton survives in the $\mathbb{R}^{1,3}$ part, while from the 4D point of view the degrees of freedom of the metric in $\mathcal{X}$ are scalar fields, called moduli. Since these will become our main playground, let us further analyze their field space.

The moduli are the dynamical degrees of freedom of the metric on $\mathcal{X}$. Since $\mathcal{X}$ is a CY 3-fold, the metric is fixed to Kähler form and only has entries $g_{m\bar{n}}$, where we use holomorphic and anti-holomorphic indices familiar from complex geometry. Looking at perturbations of the metric, we can either change the non-zero entries $g_{m\bar{n}} \to g_{m\bar{n}} + \delta g_{m\bar{n}}$ or add δg_{mn} to former zeros. In the first case this only changes the values of the metric itself and thus the Kähler form $J = -\mathrm{i} g_{m\bar{n}} \mathrm{d}y^m \wedge \mathrm{d}y^{\bar{n}}$ of the manifold. These perturbations are called Kähler moduli, accordingly. In the second case, the resulting metric is not Kähler anymore, so we need to deform the complex structure to get back to a Kähler metric. Therefore, these perturbations are called complex-structure moduli. If we also preserve the required Ricci-flatness, one can show that the metric perturbations are associated to closed forms and thus can be counted by the Hodge numbers. The Kähler moduli t^i are related to real (1,1)-forms decomposed in the $H^{1,1}$ cohomology, while the (2,0)-forms associated to the complex-structure moduli U^a can be related to $H^{1,2}$ by contraction with the unique (3,0)-form Ω in $H^{3,0}$.

Since $H^{1,1}$ also encloses wrapped form fields, we can complexify the Kähler moduli by matching them with form-field scalars. Finally, we end up with a fully characterized moduli space

$$\mathcal{M} = \mathcal{M}^{\mathrm{K}}_{h^{1,1}} \times \mathcal{M}^{\mathrm{cs}}_{h^{1,2}} \tag{10}$$

which is a subset of $\mathbb{C}^{h^{1,1}+h^{1,2}}$. One can show that both $\mathcal{M}^{\mathrm{K}}_{h^{1,1}}$ and $\mathcal{M}^{\mathrm{cs}}_{h^{1,2}}$ are special Kähler

[5] Although it may seem arbitrary to look at complex manifolds, Bergers classification of holonomies showed that all 2N-dimensional, simply-connected, Riemannian manifolds of holonomy SU(N) which are irreducible and nonsymmetric are Calabi-Yau N-folds. In the present context, all these requirements can either be motivated or dropped without further difficulties.

manifolds, endowed with a metric derived from the respective Kähler potential

$$K_{\mathrm{K}} = -\ln\Big(k_{ijk}t^i t^j t^k\Big) \qquad \text{or} \qquad K_{\mathrm{cs}} = -\ln\left(-\mathrm{i}\int_{\mathcal{X}} \Omega \wedge \bar{\Omega}\right) \tag{11}$$

with triple intersection-numbers k_{ijk} depending on the specific CY 3-fold. Since the real Kähler moduli t^i parameterize the size of different 2-cycles of the compactification, the combination $k_{ijk}t^i t^j t^k$ is proportional to the volume $\mathcal{V}$ of $\mathcal{X}$. One can also perform a coordinate change to 4-cycle volumes τ^i, which are dual to the t^i. Similar to the t^i they can be complexified by adding imaginary parts ρ^i which parameterize the wrapped form fields.

We have managed to compactify our 10D theory to 4D $\mathcal{N}=2$ SUGRA in a controlled way. However, for successful model building, we have actually preserved too much supersymmetry, which restricts our theory quite strongly. A well-studied alternative is the compactification on a Calabi-Yau orientifold, which follows the same line of development, but mods out a $\mathbb{Z}_2$-symmetry to break down the theory to 4D $\mathcal{N}=1$ SUGRA. This requires a careful analysis of all fields and their behavior under orientifolding, such that all anti-symmetric components are modded out. Since the procedure is rather technical and the specifics are irrelevant to our discussion, we will refer to [73,74] for a detailed description. Suffice it to say that in the end we can use the methods of 4D $\mathcal{N}=1$ SUGRA [75] and that there are new extended objects called O-planes. There is some arbitrariness in choosing the type of $\mathbb{Z}_2$-action, and we focus on CY orientifolds with O3/O7 planes, which are particularly well-studied (see e.g. [14,18,19,76]).[6]

In 4D $\mathcal{N}=1$ SUGRA, the potential for the scalars ϕ_i consists of an F-term potential and a D-term potential

$$V_F(\phi,\bar{\phi}) = \mathrm{e}^K\left(K^{i\bar{j}}D_iWD_{\bar{j}}\bar{W} - 3|W|^2\right) \qquad \text{and} \qquad V_D(\phi,\bar{\phi}) = \frac{1}{2}(\mathrm{Re}f)^{-1,ab}D_aD_b\,, \tag{12}$$

where all terms derive from the real-valued Kähler potential $K(\phi,\bar{\phi})$, the holomorphic superpotential $W(\phi)$ and the gauge-kinetic functions $f_a(\phi,\bar{\phi})$ via

$$K_i = \partial_i K\,,\quad K_{i\bar{j}} = \partial_i\partial_{\bar{j}}K\,,\quad K^{i\bar{j}} = (K_{i\bar{j}})^{-1}\,,\quad D_iW = K_iW + \partial_iW\,,\quad D_a = \frac{D_iW}{W}(T_a)^i_j\phi^j\,.$$

Here $(T_a)^i_j$ denotes the generators of the gauge group counted by the gauge index a. From our discussion of moduli space, we already know the form of the Kähler potential for the moduli. Fortunately, a suitable choice of orientifold mods out all other scalars except for the axio-dilaton $S = \mathrm{e}^{-\varphi} - \mathrm{i}C_0$, so we can write down the full Kähler potential as

$$K = -2\ln\Big(\mathcal{V}(T+\bar{T})\Big) - \ln\Big(S+\bar{S}\Big) - \ln\left(-\mathrm{i}\int_{\mathcal{X}}\Omega\wedge\bar{\Omega}\right)\,, \tag{13}$$

[6]The alternative orientifold action, which introduces O5 and O9 planes, is more involved. While for O3/O7 planes the influence of fluxes on the scalar potential can be written as a contribution to the superpotential, the O5/O9 case also introduces a D-term as well as an additional mass term for a linear multiplet [74]. We further expect difficulties in realizing chiral matter.

where T stands for the complexified 4-cycle Kähler moduli $T^i = \tau^i + \mathrm{i}\rho^i$ and where we have introduced the volume function $\mathcal{V}$ corresponding to the volume of the compactification manifold. This function is homogeneous of degree 3/2 in its arguments and encodes the cycle structure of the CY manifold.

In the simple case of CY compactification of type IIB string theory, the superpotential W vanishes and no D-term is induced, so the scalar potential vanishes as well. This has two direct implications. Firstly, the vacuum energy vanishes and thus we get a Minkowski vacuum. Secondly, the moduli are all massless scalars, which is the physical definition of "moduli" and thus a justification for calling them so in the first place.[7] Since such scalars are not found in nature, we have to stabilize them by turning on a potential that induces a mass and maybe a vacuum energy. We can do so by not only giving the graviton a non-trivial background, but also the other fields in the 10D action. This is referred to as "turning on flux" and the resulting compactification scheme is called flux compactification, which we will outline in the next section, following the influential analysis of [14].

2.2 Flux compactification and moduli stabilization

We hold on to our idea of compactifying to a flat 4D Minkowski space, however, we relax our condition of metric separation and allow for so-called "warped" compactification with

$$\mathrm{d}s^2 = \mathrm{e}^{2A(y)} g_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{e}^{-2A(y)} g_{mn} \mathrm{d}y^m \mathrm{d}y^n \,, \tag{14}$$

where the warp factor $A(y)$ only depends on the location in the inner manifold $\mathcal{X}$, which is now only conformally Calabi-Yau. In contrast to the Ricci-flat vacuum solution of the last subsection, we can now introduce energy densities which are due to non-trivial form-field configurations and localized objects.

Since the 4D sector should remain flat, we can only turn on field strengths on non-trivial cycles of $\mathcal{X}$ or on the entirety of the 4D space. For the first case, the available cycle dimensions 2, 3 and 4 can be read off the Hodge structure, while the only form fields in type IIB SUGRA whose field strengths match these dimensions are B_2 and C_2. The flux on any 3-cycle γ_3, which is Poincaré dual to an element of $H^{1,2}$ or $H^{2,1}$, is restricted by a Dirac quantization condition

$$\int_{\gamma_3} F_3 = m \in \mathbb{Z} \,, \qquad \int_{\gamma_3} H_3 = n \in \mathbb{Z} \tag{15}$$

and thus the number of possible flux configurations is countable. For field strengths on all of 4D space, the only viable candidate is $\tilde{F}_5 = (1 + \star)(\mathrm{d}\xi \wedge \mathrm{d}^4 x)$ with ξ being a function on $\mathcal{X}$.

[7] The term "modulus" derives from *modus*, the latin word for measure. It is used in mathematics in the sense of "parameter", especially for parameterizing different geometries. In QFT, different vacua are characterized by the vacuum expectation values of scalar fields. If a scalar has flat potential, its vacuum expectation value can be shifted continuously, much like a parameter of the theory, thus coining the expression "modulus" for massless scalar fields. Our moduli can actually be interpreted as mathematical and as physical moduli simultaneously.

Solving the equations of motion and the Bianchi identities for the fluxes (as is presented in [14]), two restrictions arise on the possible flux configurations.[8] The first one is imaginary self-duality of the specific combination $G_3 = F_3 - \mathrm{i}SH_3$ and the second one is the tadpole-cancellation condition

$$\frac{1}{(2\pi)^4\alpha'^2}\int_{\mathcal{X}} H_3 \wedge F_3 + Q_3^{\text{loc.}} = 0\,. \tag{16}$$

Here we have used the Regge-slope parameter $\alpha' = l_{\mathrm{s}}^2/2$ and $Q_3^{\text{loc.}}$, which is the C_4 charge from all localized sources. Upon solving these constraints, the warp factor is related to the 5-form field strength via $\mathrm{e}^{4A} = \xi$.

From (16), we can infer the possible number of D-branes for a given field configuration. Together with the quantization condition, the solution space appears to be countable. Furthermore, it has been shown [77] that in the absence of localized sources no non-trivial flux configuration can be achieved.

Now assuming a solution has been found, what are the implications for moduli space? After dimensional reduction to 4D, the fluxes contribute a superpotential of the Gukov-Vafa-Witten form [78]

$$W(S,U) = \int_{\mathcal{X}} G_3 \wedge \Omega\,. \tag{17}$$

Since this potential only depends on 3-cycles, it is independent of the Kähler moduli T^i. Returning to the F-term potential of (12) (while no D-term is induced), we can decompose it into

$$V = \mathrm{e}^K \left(K^{S\bar{S}} D_S W D_{\bar{S}}\bar{W} + K^{U\bar{U}} D_U W D_{\bar{U}}\bar{W} + (K^{T\bar{T}} K_T K_{\bar{T}} - 3)|W|^2\right)\,. \tag{18}$$

With our knowledge of $K = -2\ln\mathcal{V}(T+\bar{T})$ for the last term and the homogeneity of $\mathcal{V}$ we can apply Euler's homogeneous function theorem to derive

$$(T^i + \bar{T}^{\bar{\imath}})K_{T^i} = -3 \tag{19}$$

and its derivative w.r.t. $\bar{T}^{\bar{\jmath}}$

$$(T^i + \bar{T}^{\bar{\imath}})K_{T^i\bar{T}^{\bar{\jmath}}} + K_{\bar{T}^{\bar{\jmath}}} = 0\,. \tag{20}$$

Combining these equations leads to the no-scale property $K^{T^i\bar{T}^{\bar{\jmath}}}K_{T^i}K_{\bar{T}^{\bar{\jmath}}} = 3$, which exactly cancels the gravitational contribution $-3e^K|W|^2$ in (18).

The remaining potential only consists of the positive semi-definite F-terms of S and U and we can find a supersymmetric minimum by solving the equations

$$D_{U^a}W = 0 \qquad \text{and} \qquad D_S W = 0\,. \tag{21}$$

These equations stabilize the complex-structure moduli and the axio-dilaton. We have not introduced higher-order corrections to the potential yet but if we assume perturbative control

[8] To be precise, a BPS-like condition on the extended objects is assumed, as discussed in [14]. We only use O3/O7-planes as well as D3/D7-branes, so this condition is satisfied.

over these, the behavior of S and U is mainly determined by the classical potential with only minor changes from correction terms. However, due to the no-scale structure, the Kähler moduli are still unfixed and have to be stabilized by higher-order corrections alone. Thus for the discussion of Kähler moduli, we can integrate out S and U and set the superpotential to the constant

$$W_0 \sim \left\langle \int_{\mathcal{X}} G_3 \wedge \Omega \right\rangle , \tag{22}$$

where we absorb all arising contributions to the Kähler potential into the constant of proportionality. We have arrived at the no-scale model which was the motivation for our choice of type IIB. Let us collect our results.

The moduli space of the n complex valued Kähler moduli is itself a Kähler manifold. The potential is given by

$$V_F(T, \bar{T}) = \mathrm{e}^K \left(K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2 \right) \tag{23}$$

with $K = -2 \ln \mathcal{V}(T + \bar{T})$, where $\mathcal{V}$ is the real homogeneous volume-function of degree 3/2. At classical level, the superpotential $W = W_0$ is constant and the scalar potential vanishes. To stabilize the Kähler moduli, we have to include higher-order corrections.

The perturbation theory of string theory is an expansion in two parameters, the Regge slope α' and the string coupling constant $g_\mathrm{s} = \langle \mathrm{e}^\varphi \rangle$. Let us describe their origins and effects individually:

- **α' corrections**

  The α'-expansion is often described as an expansion in "stringiness". It parallels the expansion in M_P in quantum gravity and includes the effects of higher-derivative terms into the theory. The leading contribution arises from an $\alpha'^3 \mathcal{R}^4$ term in the 10D action. As shown in [79], the correction to the 4D theory can be captured by

  $$K \;\to\; K = -2 \ln(\mathcal{V} + \xi) \qquad \text{where} \qquad \xi = \frac{\chi(\mathcal{X})\zeta(3)}{2(2\pi)^3} , \tag{24}$$

  depending on the Euler characteristic $\chi(\mathcal{X})$ of the compactification manifold and the Riemann zeta function ζ.

- **String-loop corrections**

  The g_s expansion is an expansion in string loops and includes effects of higher-genus worldsheet topologies, which is parallel to a loop expansion in QFT, only for higher-dimensional objects. They are notoriously difficult to compute and have been explicitly derived only for simple toroidal compactifications [80]. For other compactifications, it has been estimated that loop corrections generally contribute a term δK_loop to the Kähler potential that looks like

  $$\delta K_\mathrm{loop} \sim \sum_{i=1}^{h^{1,1}} \left[\frac{\mathcal{C}_i^{KK}(U, \bar{U}) a_{ik} t^k}{\mathrm{Re}(S)\mathcal{V}} + \frac{\mathcal{C}_i^{W}(U, \bar{U})}{b_{ik} t^k (S) \mathcal{V}} \right] , \tag{25}$$

where the two contributions arise from KK and winding modes. $a_{ik}t^k$ and $b_{ik}t^k$ are linear combinations of 2-cycle Kähler moduli. Although these corrections would naively be the leading contribution to the scalar potential, it has been shown that, under reasonable assumptions, the first-order terms in the loop expansion cancel [80–83]. This extended no-scale structure renders them sub-leading.

- **Non-perturbative corrections**

  Next to these perturbative effects, there are also non-perturbative ones that arise due to instantons, which are Euclidean ED3-branes wrapped on 4-cycles, or due to condensation of gauginos on D7-branes. Usually, their effect is subleading to the perturbative corrections. However, SUSY protects the superpotential against perturbative corrections, which is a famous non-renormalization theorem [84, 85]. Thus, the non-perturbative effects are the only ones that can correct the superpotential:

$$W \;\to\; W = W_0 + A_i \mathrm{e}^{-a_i T^i} \,, \tag{26}$$

  where a_i is 2π for ED3-instantons and $2\pi/N$ for gaugino condensation of an SU(N) gauge group.

The two most prominent constructions of dS solutions of string theory use different combinations of these corrections to stabilize the Kähler moduli.
The KKLT scenario [18], named after the authors, only uses non-perturbative corrections. In the simplest model with one Kähler modulus T and volume function $\mathcal{V} = (T + \bar{T})^{3/2}$, T gets stabilized by gaugino condensation of a large gauge group at a supersymmetric AdS-minimum. This requires tuning of W_0 to small values.
The Large Volume scenario (LVS) [19] uses a combination of α' corrections and non-perturbative corrections on a small cycle. This involves two Kähler moduli, a big one, which is essentially the volume modulus, and a small one, which carries the non-perturbative corrections. The authors have shown that a non-supersymmetric AdS-minimum exists at very large volume. Since all correction terms are inversely proportional to some power of the volume (as we will see in the next section) the resulting potential is very shallow and flat. This makes the LVS scenario a promising starting point for quintessence.
Both scenarios stabilize the moduli at negative values of the potential. To get to a dS vacuum, they need a positive energy density which "uplifts" the potential to positive values. In [18], anti-D3-brane tension in a highly warped region was proposed as a possible uplift contribution. Since then, this aspect has been vividly discussed and is now attacked by the dS swampland conjecture (1).
We have not introduced D-branes yet. These are necessary to satisfy the tadpole-cancellation condition (16) as well as to introduce the SM, which can be realized on fractional D3-branes located at a singularity or on D7-branes wrapped on a blow-up cycle [70]. Since we chose the

orientifold action with O3/O7-planes, further D-branes of these dimensionalities D3/D7 can be introduced without further SUSY-breaking, while D5/D9-branes would break down SUSY entirely. Thus, we will only look at D3 and D7 branes. Both cases introduce new scalars X, which contribute to the superpotential as well as to the Kähler potential. The way they enter in the Kähler potential is fairly specific:

- **D3-branes**

  The D-brane moduli enter the Kähler sector of the Kähler potential via rewriting

$$T^i \ \rightarrow \ T^i + \frac{\mathrm{i}}{2\pi}(\omega^i)_{k\bar{l}}\mathrm{Tr}X^k \left(\bar{X}^{\bar{l}} - \frac{\mathrm{i}}{2}\bar{U}^{\bar{a}}(\bar{\chi}_{\bar{a}})^{\bar{l}}_{m}X^m\right) , \tag{27}$$

  where ω_i and χ_a are bases of $H^{1,1}$ and $H^{2,1}$, respectively. For our discussion, we simplify this expression by introducing appropriate real functions $k^i(X,\bar{X})$, such that

$$T^i + \bar{T}^{\bar{\imath}} \ \rightarrow \ T^i + \bar{T}^{\bar{\imath}} + k^i(X^a, \bar{X}^{\bar{a}}) \,. \tag{28}$$

- **D7-branes**

  The D-brane moduli enter the dilaton sector of the Kähler potential via rewriting

$$S + \bar{S} \ \rightarrow \ S + \bar{S} + 2\mathrm{i}L_{a\bar{b}}X^a\bar{X}^{\bar{b}} \,, \tag{29}$$

  where $L_{a\bar{b}}$ are certain geometrical quantities. Here again we abbreviate by introducing a function $k(X,\bar{X})$, such that

$$S + \bar{S} \ \rightarrow \ S + \bar{S} + k(X^a, \bar{X}^{\bar{a}}) \,. \tag{30}$$

In the next section we will take a closer look at the mathematical structure of the no-scale model and of the higher-order corrections. The goal is to review the general behavior and to introduce techniques and approximations that will be useful in the subsequent analysis of quintessence models.

2.3 No-scale model with higher-order corrections and branes

Although the complexification of the Kähler moduli is useful for a consistent treatment of complex manifolds and SUSY, we observe that the volume function $\mathcal{V}(T^i + \bar{T}^{\bar{\imath}})$ only depends on the real part τ^i of the T^i-moduli. Furthermore, we can decompose the kinetic term given by

$$\mathcal{L} \supset -K_{i\bar{\jmath}}\partial_\mu T^i \partial^\mu \bar{T}^{\bar{\jmath}} \tag{31}$$

into a kinetic term for the real moduli τ^i and a kinetic term for imaginary parts ρ^i, which are axions. In the following, we can thus focus on the real Kähler sector, postponing comments

on axions to part 5. As all derivatives of $\mathcal{V}$ and K act on the combination $2\tau^i = T^i + \bar{T}^{\bar{i}}$, the distinction between holomorphic and antiholomorphic indices can be dropped.
We are faced with a "real Kähler geometry" determined by the Kähler potential $K = -2\mathcal{V}(\tau^i)$, kinetic term $-K_{ij}\partial_\mu \tau^i \partial^\mu \tau^j$ and scalar potential

$$V = \mathrm{e}^K \left(K^{ij} D_i W \, \overline{D_j W} - 3|W|^2 \right) . \tag{32}$$

2.3.1 Volume-modulus separation and no-scale structure

We first note that due to the homogeneity of the volume function, there is one scaling degree of freedom that decouples from the others at the level of the Kähler structure and causes the no-scale structure. Let us perform a change of coordinates that isolates this modulus explicitly. The first step is to introduce a scale Ω and homogeneous coordinates $\chi^i = \tau^i/\Omega$, such that $\mathcal{V}(\tau^i) = \mathcal{V}(\chi^i)\Omega^{3/2}$. Now we can perform a coordinate transformation from $\{\tau^i\}$ to $\{\chi^k, \Omega\}$ by fixing χ^n through the constraint

$$\mathcal{V}(\chi^i) = \mathcal{V}(\chi^k, \chi^n(\chi^k)) = 1 \, , \tag{33}$$

where here and from now on we understand $i,j \in [1, ..., n]$ and $k,l \in [1, ..., n{-}1]$. This coordinate change is only possible if $\partial_n \mathcal{V} = \mathcal{V}_n \neq 0$, but we can always choose an appropriate χ^n locally and build an atlas from these local charts. This resembles the choice of spherical coordinates, where Ω is a global coordinate for $\mathbb{R}^n/\{0\}$, corresponding to the radius. Since Ω gives the scale of the overall volume of $\mathcal{X}$, we will call it the volume modulus.
Now the Kähler metric K_{ij} can be calculated explicitly:

$$K_{ij}(\tau) = \frac{2}{\mathcal{V}^2(\tau)} \left[\mathcal{V}_i(\tau)\mathcal{V}_j(\tau) - \mathcal{V}_{ij}(\tau)\mathcal{V}(\tau) \right] = \frac{2}{\Omega^2} \left[\mathcal{V}_i(\chi)\mathcal{V}_j(\chi) - \mathcal{V}_{ij}(\chi) \right] . \tag{34}$$

Here a subscript i denotes a partial derivative with respect to the i-th argument of the function, be it a τ or a χ. With this we can express the kinetic term solely in the new coordinates:

$$\mathcal{L} \supset -K_{ij}(\tau)\partial_\mu \tau^i \partial^\mu \tau^j = -\frac{2}{\Omega^2} \left[\mathcal{V}_i(\chi)\mathcal{V}_j(\chi) - \mathcal{V}_{ij}(\chi) \right] \partial_\mu(\Omega\chi^i)\partial^\mu(\Omega\chi^j) \, . \tag{35}$$

Now we need to apply the product rule to isolate the kinetic terms of the new coordinates and also resolve derivatives of χ^n via $\partial_\mu \chi^n = (\partial\chi^n/\partial\chi^k)\partial_\mu \chi^k$. For notational convenience we will denote $(\partial\chi^n/\partial\chi^k)$ by Γ_k. This results in the fully expanded expression

$$\mathcal{L} \supset -A_{kl}\partial_\mu \chi^k \partial^\mu \chi^l - \frac{2B_k}{\Omega}\partial_\mu \chi^k \partial^\mu \Omega - \frac{C}{\Omega^2}\partial_\mu \Omega \partial^\mu \Omega \, , \tag{36}$$

where A_{kl}, B_k and C are functions of the χ^k, independent of Ω and given by

$$
\begin{aligned}
A_{kl} &= 2\left[\mathcal{V}_k\mathcal{V}_l - \mathcal{V}_{kl} + \mathcal{V}_k\mathcal{V}_n\Gamma_l - \mathcal{V}_{kn}\Gamma_l + \mathcal{V}_n\mathcal{V}_l\Gamma_k - \mathcal{V}_{nl}\Gamma_k + \mathcal{V}_n\mathcal{V}_n\Gamma_k\Gamma_l - \mathcal{V}_{nn}\Gamma_k\Gamma_l\right] \\
B_k &= 2\left[\mathcal{V}_k\mathcal{V}_l\chi^l - \mathcal{V}_{kl}\chi^l + \mathcal{V}_k\mathcal{V}_n\chi^n - \mathcal{V}_{kn}\chi^n + \mathcal{V}_n\mathcal{V}_l\Gamma_k\chi^l - \mathcal{V}_{nl}\Gamma_k\chi^l + \mathcal{V}_n\mathcal{V}_n\Gamma_k\chi^n - \mathcal{V}_{nn}\Gamma_k\chi^n\right] \\
C &= 2\left[\mathcal{V}_i\mathcal{V}_j - \mathcal{V}_{ij}\right]\chi^i\chi^j .
\end{aligned}
$$

To simplify these expressions, we can use the fact that the constraint (33) has to hold for all χ^k. Thus we can take total differentials on both sides to get

$$
\frac{\mathrm{d}}{\mathrm{d}\chi^k}\mathcal{V}(\chi^l, \chi^n(\chi^l)) = 0 \quad \Rightarrow \quad \mathcal{V}_k + \Gamma_k\mathcal{V}_n = 0. \tag{37}
$$

Another simplification arises from the homogeneity of $\mathcal{V}(\chi)$ and Euler's homogeneous function theorem, which in this case states that

$$
\mathcal{V}_i\chi^i = \frac{3}{2}\mathcal{V} = \frac{3}{2}\,. \tag{38}
$$

This again has to hold for every choice of χ^k. Thus we can again take the total derivative, leading to

$$
\frac{\mathrm{d}}{\mathrm{d}\chi^k}(\mathcal{V}_i\chi^i) = 0 \quad \Rightarrow \quad \mathcal{V}_{kl}\chi^l + \mathcal{V}_{kn}\chi^n + \mathcal{V}_{nl}\Gamma_k\chi^l + \mathcal{V}_{nn}\Gamma_k\chi^n + \mathcal{V}_k + \mathcal{V}_n\Gamma_k = 0 \tag{39}
$$

or with (37) simply

$$
\mathcal{V}_{kl}\chi^l + \mathcal{V}_{kn}\chi^n + \mathcal{V}_{nl}\Gamma_k\chi^l + \mathcal{V}_{nn}\Gamma_k\chi^n = 0\,. \tag{40}
$$

Due to (37), (40), their second derivatives and homogeneity, the coefficients A_{kl}, B_k and C boil down to

$$
\begin{aligned}
A_{kl} &= -\frac{2}{\mathcal{V}_n^2}\left[\mathcal{V}_{kl}\mathcal{V}_n^2 - \mathcal{V}_{kn}\mathcal{V}_l\mathcal{V}_n - \mathcal{V}_{nl}\mathcal{V}_k\mathcal{V}_n + \mathcal{V}_{nn}\mathcal{V}_k\mathcal{V}_l\right] = 2\mathcal{V}_n\partial_k\partial_l\chi^n \\
B_k &= 0 \\
C &= 3\,.
\end{aligned} \tag{41}
$$

This shows that we can always isolate the volume modulus Ω from the residual moduli χ^k and get a diagonal metric and kinetic term, whose volume scaling can be read off the powers of Ω in

$$
\mathcal{L} \supset -A_{kl}\partial_\mu\chi^k\partial^\mu\chi^l - \frac{3}{\Omega^2}\partial_\mu\Omega\partial^\mu\Omega\,. \tag{42}
$$

As an aside, the no-scale property now follows directly. We can invert the matrix identity

$$
K_{ij} = \begin{pmatrix} A_{kl} & 0 \\ 0 & 3\Omega^{-2} \end{pmatrix} \frac{\partial(\chi_k, \Omega)}{\partial\tau^i}\frac{\partial(\chi_l, \Omega)}{\partial\tau^j} \tag{43}
$$

and split the expression

$$K^{ij}K_iK_j \to A^{kl}\tilde{K}_k\tilde{K}_l + \frac{\Omega^2}{3}\tilde{K}_\Omega\tilde{K}_\Omega \,. \tag{44}$$

Here we define

$$\tilde{K}_k = \frac{\partial\tau^i}{\partial\chi^k}K_i = \partial_k\tilde{K} \qquad \tilde{K}_\Omega = \frac{\partial\tau^i}{\partial\Omega}K_i = \partial_\Omega\tilde{K} \tag{45}$$

and re-express the Kähler potential in the new coordinates:

$$K(\tau) = \tilde{K}(\chi,\Omega) = -2\ln\Big(\mathcal{V}(\chi^i)\Omega^{3/2}\Big) \xrightarrow{\mathcal{V}(\chi^i)=1} -3\ln\Omega \,. \tag{46}$$

Thus $\tilde{K}_k = 0$ and $\tilde{K}_\Omega = -3/\Omega$ and

$$K^{ij}K_iK_j = A^{kl}\tilde{K}_k\tilde{K}_l + \frac{\Omega^2}{3}\tilde{K}_\Omega\tilde{K}_\Omega = 0 + \frac{\Omega^2}{3}\cdot\Big(-\frac{3}{\Omega}\Big)\cdot\Big(-\frac{3}{\Omega}\Big) = 3\,. \tag{47}$$

We see that in these coordinates, the scaling direction in moduli space decouples from the other degrees of freedom, which are restricted to a submanifold of constant volume-function. The only contribution to the scalar potential thus derives from the volume modulus Ω, making the no-scale property explicitly one-dimensional.

Next we will take a look at various deviations from this no-scale structure. First, we will analyze the effect of α' corrections and non-perturbative corrections on small cycles, since these are the leading contributions and are used for stabilizing the volume modulus in LVS. Second, because loop corrections are difficult to compute, we will provide an estimate of their order of magnitude. Finally, we will study the contributions of D-branes to the scalar potential.

2.3.2 Corrections to the volume function

As we have seen in the previous discussion, we can choose coordinates such that only one modulus Ω gives a non-zero contribution to the scalar potential, while the others are restricted to a submanifold of the moduli space where the volume $\mathcal{V}$ and by that also the Kähler potential $\tilde{K}$ is constant. Now we can analyze corrections that only influence the overall scaling of the moduli and do not disturb their relative proportions. Such corrections enter the Kähler potential via $\mathcal{V} \;\to\; \mathcal{V} + A$. Since the $n-1$ residual moduli χ^k enter in the same specific combination $\mathcal{V}(\chi^k, \chi^n(\chi^k))$ as before, the derivatives $\tilde{K}_l$ still vanish and the residual moduli decouple from the volume modulus. We can therefore generally treat such contributions in the same way as in the one-modulus case $K = -2\ln\Big(\Omega^{3/2} + A\Big)$.

The contribution we get from α' corrections is a constant $A = \xi$. Here we can directly compute the first-order no-scale-breaking contribution

$$K^{\Omega\Omega}K_\Omega K_\Omega = \frac{3}{1-\frac{\xi}{2\mathcal{V}}} = 3 + \frac{3}{2}\frac{\xi}{\mathcal{V}} + \mathcal{O}\left(\left(\frac{\xi}{\mathcal{V}}\right)^2\right) \tag{48}$$

and the resulting scalar potential

$$\delta V_{\alpha'} = \mathrm{e}^K \left[\frac{3}{2}\frac{\xi}{\mathcal{V}} + \mathcal{O}\left(\frac{\xi^2}{\mathcal{V}^2}\right)\right] |W|^2 = \frac{3}{2}\frac{\xi}{\mathcal{V}^3}|W|^2 + \mathcal{O}\left(\frac{\xi^2}{\mathcal{V}^4}|W|^2\right) . \tag{49}$$

On the contrary, non-perturbative corrections have been discussed to predominantly arise in the superpotential. How is that connected to A? Since the contribution $\delta W_{\mathrm{np}} = A_i \mathrm{e}^{-a_i T^i}$ is exponentially suppressed by the size of the 4-cycle involved, the largest effects come from small cycles. This has been used in LVS [19] to generate a potential from non-perturbative corrections on a small cycle τ_{s} that enters the volume function like $\mathcal{V} = {\tau_{\mathrm{b}}}^{3/2} - {\tau_{\mathrm{s}}}^{3/2}$. The big cycle τ_{b} is effectively the volume modulus. Therefore we will analyze the scalar potential contribution that arises from $A = -\tau_{\mathrm{s}}^{3/2}$ when τ_{s} has non-perturbative corrections to W depending on it. The discussion can easily be expanded to any homogeneous contribution $\mathcal{V} + f_{3/2}(\tau_m)$ of small moduli that preserve the no-scale structure of the Kähler potential but violate it in the superpotential. Since the combination $\mathcal{V} - \tau_{\mathrm{s}}^{3/2}$ is itself homogeneous, the only no-scale breaking arises from the terms

$$\delta V_{\mathrm{np}} = 2\mathrm{e}^K (K^{\mathrm{s}\Omega} K_\Omega + K^{\mathrm{ss}} K_{\mathrm{s}}) \mathrm{Re}(W \partial_{T_{\mathrm{s}}} \delta W) + \mathrm{e}^K K^{\mathrm{ss}} |\partial_{T_{\mathrm{s}}} \delta W|^2 . \tag{50}$$

If τ_{s} is assumed to be small, we can simplify the terms to yield

$$\delta V_{\mathrm{np}} \sim -6 |W A_{\mathrm{s}}| a_{\mathrm{s}} \frac{\tau_{\mathrm{s}}}{\mathcal{V}^2} \mathrm{e}^{-a_{\mathrm{s}} \tau_{\mathrm{s}}} + |A_{\mathrm{s}}|^2 a_{\mathrm{s}}^2 \frac{\sqrt{\tau_{\mathrm{s}}}}{\mathcal{V}} \mathrm{e}^{-2 a_{\mathrm{s}} \tau_{\mathrm{s}}} . \tag{51}$$

We see that the non-perturbative correction to τ_s stabilizes the small modulus and contributes to the volume modulus potential. As shown in [19], $e^{-a_{\mathrm{s}} \tau_{\mathrm{s}}}$ is stabilized at order $\mathcal{V}^{-1}$, such that the non-perturbative corrections scale as

$$\delta V_{\mathrm{np}} \sim \frac{\sqrt{\ln \mathcal{V}}}{\mathcal{V}^3} - \frac{\ln \mathcal{V}}{\mathcal{V}^3} . \tag{52}$$

This places α' and non-perturbative correction terms at equal scaling $\mathcal{V}^{-3}$ and their combined potential fixes the volume modulus Ω.
Obviously, the other moduli hidden in $\mathcal{V}$ are unaffected by α' and non-perturbative corrections. Generalizations to several small cycles with non-perturbative contributions are straight-forward. More intricate geometries will require more involved calculations, but the general picture is that every non-perturbative correction fixes a direction in moduli space and together with α' corrections also the volume modulus. The remaining moduli have to be stabilized by other effects but will generally receive a contribution from string-loop corrections, which we will discuss briefly in the next subsection.

2.3.3 Estimate on loop corrections

The string-loop corrections estimated in [80] to follow the structure of (25) consist of two contributions: δK^{KK} from KK modes, which is homogeneous of degree -1 in 4-cycle volumes, and δK^{W} from winding modes, which is homogeneous of degree -2. Naively, one would expect the leading contribution to the scalar potential (from terms like $\mathrm{e}^K K^{ij} K_i \delta K_j^{\mathrm{KK}}$) to be homogeneous of degree -4, which corresponds to a scaling with $\mathcal{V}^{-8/3}$. However, by treating the loop corrections as perturbations to the uncorrected Kähler potential K_0 and expanding the inverse Kähler metric in a Neumann series, the authors of [73,83] have shown that the first-order terms cancel, which is called an "extended no-scale structure". They further presented the first non-vanishing contribution to be

$$\delta V_{\mathrm{loop}} = \sum_{i=1}^{h^{1,1}} \left[\frac{(\mathcal{C}_i^{KK})^2}{\mathrm{Re}(S)^2} K_{ii}^0 - 2\delta K_i^{\mathrm{W}} \right] \frac{|W|^2}{\mathcal{V}^2} \,, \tag{53}$$

which now scales like $\mathcal{V}^{-10/3}$. This peculiar scaling renders the loop-corrections subdominant in the large volume limit. In [71,73] an interpretation in the language of QFT loop potentials has been proposed, which will become useful in our discussion of quintessence model-building. In the 4D picture, it seems natural to interpret the string loops as loops in the resulting field theory. These are described by the Coleman-Weinberg-potential [86], which has been generalized to SUSY theories [87]:

$$V = V_{\mathrm{tree}} + \frac{1}{64\pi^2} \mathrm{STr}\mathcal{M}^0 \cdot \Lambda^4 \log \frac{\Lambda^2}{\mu^2} + \frac{1}{32\pi^2} \mathrm{STr}\mathcal{M}^2 \cdot \Lambda^2 + \frac{1}{64\pi^2} \mathrm{STr}\mathcal{M}^4 \log \frac{\mathcal{M}^2}{\Lambda^2} + \dots \,. \tag{54}$$

where Λ is the cut-off of the theory, $\mathcal{M}$ is the mass matrix and STr denotes the supertrace. Due to the equal number of bosons and fermions in SUSY, the second term disappears. This has been interpreted as the field theoretic reason for the extended no-scale structure. The third term involves the supertrace $\mathrm{STr}\mathcal{M}^2$ of all fields running in the loops. In general 4D $\mathcal{N} = 1$ SUGRA, this supertrace is given by $\mathrm{STr}\mathcal{M}^2 = 2Qm_{3/2}^2$, where Q is a model dependent $\mathcal{O}(1)$ coefficient, while $m_{3/2}$ is the gravitino mass given by $|W|/\mathcal{V}$. The cut-off of the theory is assumed to be at the lowest KK scale, where the theory becomes effectively higher-dimensional.[9] This allows us to estimate the lowest-order loop corrections by

$$\delta V_{\mathrm{loop}} \sim A m_{\mathrm{KK}}^2 m_{3/2}^2 + B m_{3/2}^4 \sim A m_{\mathrm{KK}}^2 \frac{W_0^2}{\mathcal{V}^2} + B \frac{W_0^4}{\mathcal{V}^4} \tag{55}$$

with $\mathcal{O}(1)$ coefficients A and B. The mass of the lowest KK-mode in a compactification on S^1 is given by the inverse radius R^{-1}. This, however, is measured in string units and to go to 4D Planck units, we have to multiply by $M_{\mathrm{s}} = \mathcal{V}^{-1/2}$. In a compactification of six dimensions we

[9] This is a non-trivial assumption since loop corrections may, of course, also arise in higher-dimensional field theory or directly at the string level. In fact, one probably has to assume that the restoration of a sufficiently high level of SUSY above the KK scale cuts off the loop integrals. However, in the present case SUSY is broken by fluxes, and these penetrate not just the large-radius but *all* extra dimensions. So further scrutiny may in fact be required to justify the use of the *lowest* KK scale as a cutoff.

would expect to find six a priori different extended radii R_i and presumably a number of small radii from localized cycles inside the manifold. Due to their smallness, the associated KK-modes to these small cycles are heavy and since we are searching for the lightest KK-mode, we can drop them. The compactification volume is then simply $\mathcal{V} = R_1 R_2 ... R_6$. Naively, each radius scales as $\mathcal{V}^{1/6}$. However, depending on the geometry, the volume scaling might be anisotropic, generally allowing different scaling exponents $\mathcal{V}^{p_i}$ as long as the p_i add up to 1. In the most drastic case, all radii but one could remain at string scale, while one carries all volume scaling $\mathcal{V}^1$ alone. W.l.o.g. let R_1 be the largest radius (or one of several largest radii) such that its KK modes set the cut-off in (54). Its volume scaling shall be parameterized by $\mathcal{V}^{1/l}$ with $l \in [1, 6]$. This is motivated by the simple case of l equally large dimensions and $6-l$ dimensions at string scale. In case of intermediate scales $R_1 > R_i > R_j$, also non-integer values of l are possible. The KK-scale is then given by

$$m_{\mathrm{KK}} = \frac{M_{\mathrm{s}}}{\mathcal{V}^{1/l}} = \mathcal{V}^{-1/l-1/2} \tag{56}$$

and the loop corrections scale as

$$\delta V_{\mathrm{loop}} \sim A \frac{W_0^2}{\mathcal{V}^{3+2/l}} + B \frac{W_0^4}{\mathcal{V}^4} \,. \tag{57}$$

In the case of isotropic compactification $l = 6$ the first term gives precisely the familiar volume scaling $\mathcal{V}^{10/3}$ from (53). For anisotropic compactifications, however, the loop corrections might even be further volume suppressed. We see that for $l < 2$, the second term dominates and thus $l = 2$ is a "best-case" scenario. The only possibility to get even smaller loop corrections is by tuning W_0 hierarchically small.

This heuristic approximation has not taken into account the special structure of string theory compactifications yet. We note that the only available cycles in Calabi-Yau 3-folds are two- and four-cycles. It is thus questionable if we can manipulate the 6 dimensions individually or if we have to restrict our attention to models of the type $\mathcal{V} = R_1^2 R_2^2 R_3^2$. In this case, the best-case scenario $l = 2$ would also be the maximally anisotropic scenario.

2.3.4 Contributions from D-branes

Finally, we shall take a look at the F-term potential induced by D-branes. We will start with D3-branes.

As we mentioned in (28), the brane moduli X^a only appear in the Kählerpotential K through real valued functions $k^i(X^a, \bar{X^a})$ which are added to the original moduli like

$$\tau^i \;\to\; \tau'^i = \tau^i + k^i(X^a, \bar{X^a}) \,. \tag{58}$$

Also the superpotential W gets a generic X^a-dependent holomorphic contribution

$$W \;\to\; W' = W_0 + g(X^a) \,. \tag{59}$$

The derivatives of K with respect to the τ^i have precisely the same algebraic form as before, only substituting the new variables $\tau^i \to \tau'^i$, so in this sector, we still get $K^{ij}K_iK_j = 3$, where derivatives are now taken with respect to τ'^i. We have dropped holomorphic and anti-holomorphic indices for the τ^i moduli, but they could become important for the X^a, so we will keep them in this sector. The derivatives of K with respect to X^a are given by:

$$K_a = K_i \partial_a k^i \quad K_{\bar{a}} = K_i \partial_{\bar{a}} k^i \quad K_{aj} = K_{ij} \partial_a k^i \quad K_{i\bar{b}} = K_{ij} \partial_{\bar{b}} k^j \quad K_{a\bar{b}} = K_{ij} \partial_a k^i \partial_{\bar{b}} k^j + K_i \partial_a \partial_{\bar{b}} k^i \,. \tag{60}$$

In this section, indices i, j, k, l, m always denote the τ sector, while a, b, c, d are indices of the X sector. We will write I, J for the combination $I = (i, a)$, $J = (j, b)$ with bars acting only on a, b. Now the Kähler metric has the form

$$K_{I\bar{J}} = \begin{pmatrix} K_{ij} & K_{il}\partial_{\bar{b}} k^l \\ K_{kj}\partial_a k^k & K_{kl}\partial_a k^k \partial_{\bar{b}} k^l + K_m \partial_a \partial_{\bar{b}} k^m \end{pmatrix} . \tag{61}$$

This can be inverted by the block-matrix rule

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}^{-1} = \begin{pmatrix} A^{-1} + A^{-1}BS^{-1}CA^{-1} & -A^{-1}BS^{-1} \\ -S^{-1}CA^{-1} & S^{-1} \end{pmatrix} \tag{62}$$

with $S = D - CA^{-1}B$, which in our case is

$$S_{a\bar{b}} = K_{kl}\partial_a k^k \partial_{\bar{b}} k^l + K_m \partial_a \partial_{\bar{b}} k^m - K_{kj}\partial_a k^k K^{ij} K_{il} \partial_{\bar{b}} k^l = K_m \partial_a \partial_{\bar{b}} k^m \tag{63}$$

where we have used the fact that K^{ij} is the inverse of K_{ij}. The inverse Kähler metric is then given by

$$K^{\bar{I}J} = \begin{pmatrix} K^{ij} + S^{\bar{c}d}\partial_{\bar{c}} k^i \partial_d k^j & -S^{\bar{c}b}\partial_{\bar{c}} k^i \\ -S^{\bar{a}d}\partial_d k^j & S^{\bar{a}b} \end{pmatrix} . \tag{64}$$

Now we can determine the scalar potential from

$$\begin{aligned} V =& e^K (K^{\bar{I}J} D_{\bar{I}} \bar{W} D_J W - 3|W|^2) \\ =& e^K (K^{\bar{I}J} (K_{\bar{I}} \bar{W} + \partial_{\bar{I}} \bar{W})(K_J W + \partial_J W) - 3|W|^2) \\ =& e^K \Big[S^{\bar{c}d} \partial_{\bar{c}} k^i \partial_d k^j (K_i \bar{W})(K_j W) - S^{\bar{c}b} \partial_{\bar{c}} k^i (K_i \bar{W})(K_b W + \partial_b W) \\ & \quad - S^{\bar{a}d} \partial_d k^j (K_{\bar{a}} \bar{W} + \partial_{\bar{a}} \bar{W})(K_j W) + S^{\bar{a}b} (K_{\bar{a}} \bar{W} + \partial_{\bar{a}} \bar{W})(K_b W + \partial_b W) \Big] \\ =& e^K S^{\bar{a}b} \partial_{\bar{a}} \bar{W} \partial_b W \,, \end{aligned} \tag{65}$$

where the explicit forms of X-sector derivatives led to cancellations of the cross terms in the last line. This surprising simplicity arises because we only really perform a coordinate transformation in moduli space and couple this transformation to a nontrivial contribution to the superpotential.

The structure of this F-term is similar to the generic form $e^K K^{\bar{a}b} D_{\bar{a}} \bar{W} D_b W$. This similarity is an identity, when $\partial_a k^i = \partial_{\bar{a}} k^i = 0$, which is the case for k^i of $\mathcal{O}(X^2)$ and X^a stabilized at $X^a = 0$. Since linear terms in k^i can be absorbed into the definition of T^i the only relevant condition for this identity is the stabilization of X at 0.
We found that the insertion of D3-brane moduli does not disturb the no-scale cancellation of the Kähler-moduli sector. However, a non-trivial contribution to the superpotential will generate a positive F-term of the brane moduli in the scalar potential, which also depends on the Kähler moduli through K_m. Stabilization of the X moduli will generally arise already at this level. However, there is a possibility of flat directions in the X sector, which will be stabilized via interactions with other moduli and higher-order corrections.
In our discussion of quintessence models, we rely on D3-brane contributions to break SUSY at sufficiently high scale. A specific example will be computed in appendix B.
The D7-brane case is similar, although here we cannot invoke no-scale, but may use $D_S W = 0$. The computation of the Kähler metric is parallel to the D3-case, only the terms arising in the potential are slightly different. Still, if the X-sector contributes to W we again get an additional F-term potential, that takes the classic form $e^K K^{\bar{a}b} D_{\bar{a}} \bar{W} D_b W$ for k of $\mathcal{O}(X^2)$ and X stabilized at 0. It depends on the dilaton (which has been integrated out and can be treated as a constant) only through the Kähler metric, and again has no cross terms between $D_a W$ and $D_S W$.
This concludes our discussion of corrections and also our review of type IIB flux compactification. Having collected all necessary preliminaries and techniques, we can now turn our attention to quintessence model-building. What do we expect of a valid quintessence model? In the next section we shall present the phenomenological requirements that we want to fulfill.

3 Phenomenological Restrictions on Stringy Quintessence

The main challenge for quintessence models is the creation of sufficiently large hierarchies. If these can be achieved, further questions about decoupling from the SM and the correct evolution history arise. Although we will not go deep into these further issues, the coupling to the SM already eliminates one natural candidate for a quintessence field - the volume modulus. To understand this logic and to specify the challenge we face, we will start this section with a short summary of quintessence-model requirements and move on to eliminate the volume modulus. After that, we can break down the phenomenological restrictions to explicit bounds on four mass scales that have to be implemented in a viable model.

3.1 Quintessence and its requirements

The cosmological constant Λ as the driving force behind cosmic acceleration can describe cosmological observations quite well. However, its origin is unknown and the mere possibility of it arising has not been a convincing reason for its actual existence to everyone so the term "dark energy" was coined. Since the cosmological constant enters the Friedmann equations in linear combination with matter and radiation energy densities, the interpretation of Λ as an actual energy density ρ_Λ with negative pressure $P_\Lambda = -\rho_\Lambda$ and thus equation of state parameter $\omega = P_\Lambda/\rho_\Lambda = -1$ is only a matter of rewriting.

A different approach to dark energy has been promoted in the 80s [51–53] under the names of dynamical dark energy and later quintessence (see [88] for a review). The main idea is to introduce a scalar field with negative equation of state parameter, which is perhaps the next simplest explanation after introducing a constant by hand. The name either derives from the scalar field being a fifth fundamental force of nature or from it being the fifth component of energy beside baryonic matter, dark matter, neutrinos and radiation. A scalar field driving cosmic expansion has also been suggested to explain inflation at the early universe. The techniques are fairly similar, but the energy scales are very different. Indeed, the required low mass of the quintessence field will prove to be a major problem in model building. Let us explain this requirement by introducing a classical scalar field ϕ with Lagrangian

$$\mathcal{L}_\phi = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\,. \tag{66}$$

The stress-energy tensor is then given by

$$T_{\mu\nu} = \frac{-2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_\phi)}{\delta g^{\mu\nu}} = -2\frac{\delta(\mathcal{L}_\phi)}{\delta g^{\mu\nu}} + g_{\mu\nu}\mathcal{L}_\phi\,. \tag{67}$$

Since we still remain in a homogeneous and isotropic scenario, we can set the spatial derivatives of ϕ to zero and diagonalize the stress-energy tensor. The entries are

$$T_{00} = \rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \qquad \text{and} \qquad T_{ii} = P = \frac{1}{2}\dot{\phi}^2 - V(\phi) \tag{68}$$

resulting in the equation of state

$$\omega = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} . \tag{69}$$

If $\dot{\phi}^2$ is smaller than $V(\phi)$, Ω is smaller than $-1/3$, as is needed to realize the late-time cosmic acceleration. If we were to add matter and radiation, we would need an even more negative ω and if we want to match the observed acceleration over long enough timescales, we have to make certain that ω does not change to quickly. This can be rephrased as the conditions

$$\dot{\phi}^2 \ll V \qquad \text{and} \qquad \frac{\mathrm{d}}{\mathrm{d}t}(\dot{\phi}^2) \ll \frac{\mathrm{d}V}{\mathrm{d}t} \quad \Rightarrow \quad \ddot{\phi} \ll V' . \tag{70}$$

Using the Friedmann equations, one can translate these requirements to the scalar potential

$$\epsilon \equiv \frac{1}{3}\left(\frac{V'}{V}\right)^2 \ll 1 \qquad \text{and} \qquad |\eta| \equiv \left|\frac{V''}{V}\right| \ll 1 , \tag{71}$$

where we have introduced the “slow-roll parameters” ϵ and η. Furthermore, from the first Friedmann equation, we can infer that at dark energy domination $V \sim 3H^2$, where H is the Hubble constant. This leads to a bound on the scalar mass, which we shall define in the following as $m_\phi \equiv \sqrt{V''} < H$. With the measured value of today's Hubble constant of order $\mathcal{O}(10^{-60})$ the arising picture is one of very small energy scales: **A quintessence model needs an ultralight scalar field of mass $m_\phi < \mathcal{O}(10^{-60})$ with a potential of order $\mathcal{O}(10^{-121})$ and even smaller slope.**

Since the quintessence scalar is so light, we also have to keep in mind yet another difficulty. In string theory, the various degrees of freedom are generally intertwined with each other and in the end have to be separated into physical fields. This usually leads to coupling terms of all fields with each other. Even if we do not start with string theory, a field theory involving the SM and quintessence has a priori no reason to forbid vertices of quintessence and SM fields. However, if these operators appear, we would on the one hand expect SM-loop corrections to raise the quintessence mass to SM scales and on the other hand expect the quintessence scalar to convey a long-range interaction between matter fields. The former issue is a re-iteration of the cosmological constant problem and requires a fine-tuning mechanism to explain.[10] The latter issue is usually summarized as a collection of “fifth-force constraints”. Let us specify this last point.

[10] Probably an even higher amount of fine-tuning than for the cosmological constant is needed, since we have to make sure that the quintessence potential as well as its first and second derivative all remain small. A discussion of fine-tuning for some stringy quintessence models can be found in [59].

A scalar field of mass m coupled to other particles with coupling constant λ conveys a force leading to the classical Yukawa potential

$$V(r) = \lambda^2 \frac{e^{-mr}}{r} \, . \tag{72}$$

For sufficiently massive scalars, the exponential function suppresses the interaction such that no long-range forces are measurable. However, if the mass becomes too light, we expect the interaction to become accessible by experiments. Especially below the threshold of meV, corresponding to length scales of 100 μm, the scalar "fifth force" would become detectable by high-precision torsion-balance measurements of gravity [89]. The only way out would be to either stay above the meV threshold or suppress the interaction, making λ significantly smaller than the gravitational coupling. Since quintessence is much lighter then meV, only the second option is available (although we may fall back to the first option for other scalars appearing in our models). The required smallness of the coupling constants has been analyzed for several stringy quintessence models in [58] building on the partially related discussion of dilaton models in [90]. There are several ways in which a fifth force could become detectable and since experiments have not found signs of it yet, each one provides a bound on the coupling constants.

- **Post-Newtonian parameters**

  If the scalar couples universally to all fields, a small deviation from the gravitational potential may be observable. In standard GR, the perturbative expansion of the Einstein field equations leads to corrections to Newtons law of gravity. In this post-Newtonian expansion the coupling to rest mass can be parameterized (according to [91]) by the experimentally confirmed Eddington parameter $\gamma = 1$. If any other forces contribute, the resulting deviation $\gamma - 1$ is related to the coupling λ and the experimental evidence yields approx. $\lambda^2 < 10^{-5}$.

- **Violation of the equivalence principle**

  If the scalar couples differently (i.e. with different λ_i) depending on the matter field involved, its interaction violates the equivalence principle. This violation may even arise if the coupling λ' is universal on a UV scale but the operators involved run differently during renormalization to lower scales. Calculating the contributions of particle masses and binding energy to the overall energy of different atoms, violations of the equivalence principle can be estimated and compared with data. This restricts several ratios and differences of λ_i-s even stronger. In [58] a bound on the universal UV coupling is given by $\lambda'^2 < 10^{-13}$, where the UV scale is assumed to be the Planck scale. This has to be slightly weakened for lower UV-scales, so in the case of a TeV string scale for example, we would only get 10^{-11}. Still, violation of the equivalence principle turns out to be the most critical effect.

- **Varying fundamental constants**

  A time-dependent scalar field varies the fundamental constants, e.g. the strength of couplings in the SM part, which would be observable on cosmological time scales [92]. However, this effect turns out to be less constraining than the equivalence principle violations in [58].

For the specific stringy models given in [58], the authors have translated these restrictions to lower bounds on the volume of the compactification manifold. However, it will turn out in our discussion that in order to reliably achieve the required mass hierarchies in stringy models, we already need to go to very large compactification volume. Still, the fifth-force constraints will serve the purpose of eliminating the volume modulus as quintessence field in the next section.

3.2 Volume-modulus quintessence?

As we have seen in section 2.3.1, the volume modulus plays a special role in string compactifications. Even the minimal models have at least one volume modulus in the Kähler sector as for example in the simplest KKLT scenario. It is thus only natural to take it into consideration during our search for a viable quintessence scalar. However, the volume modulus acquires strong universal couplings to all fields when going to the Einstein frame. Let us illustrate this point with a simple toy model.
Suppose we have compactified a 10 dimensional gravity theory to four dimensions, where a scalar field Ω parameterizes the compactification volume $\mathcal{V} = \Omega^{3/2}$. Let us furthermore add a SM sector Lagrangian $\mathcal{L}_m$, which in string theory lives on localized branes and is thus independent from the bulk action. The only connecting component is the metric, which also measures the geometry on the branes. Thus, our 4D effective action takes the schematic form

$$S = \int \mathrm{d}^4x\sqrt{-g}\frac{\mathcal{V}}{2\kappa_0^2}\left[\mathcal{R} - k(\Omega)(\partial\Omega)^2 - V(\Omega)\right] + \sqrt{-g}\mathcal{L}_m(g^{\mu\nu})\,, \tag{73}$$

where $\mathcal{R}$ is the 4D Ricci scalar and the factor $\mathcal{V}$ arises by integrating out 6 of 10 dimensions. Now if we want to rewrite this action in a form with canonical Einstein-Hilbert term, we can expand Ω around its vacuum expectation value Ω_0, absorb $\mathcal{V}_0 = \Omega_0^{3/2}$ into $\kappa = \kappa_0/\sqrt{\mathcal{V}_0}$ and finally rescale the metric to get

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left[\mathcal{R} - k'(\Omega')(\partial\Omega')^2 - V'(\Omega')\right] + \frac{\mathcal{V}_0^2}{\mathcal{V}^2}\mathcal{L}_m\left(\frac{\mathcal{V}}{\mathcal{V}_0}g^{\mu\nu}\right)\right]\,. \tag{74}$$

The next step is the canonical normalization of Ω' such that we end up with the actual physical field with canonical kinetic term $-\partial_\mu\phi\partial^\mu\phi/2$. To this end we remember from our discussion of the volume modulus separation (42) that the volume modulus has kinetic term

$$\mathcal{L} \supset -\frac{3}{(\Omega_0 + \Omega')^2}\partial_\mu\Omega'\partial^\mu\Omega'\,. \tag{75}$$

The physical field ϕ is thus the logarithm of Ω and properly treating the expectation value Ω_0 we get the relation

$$\Omega' = \Omega_0 \left(\mathrm{e}^{\phi/\sqrt{6}} - 1\right) \tag{76}$$

which leads to

$$\frac{\mathcal{V}}{\mathcal{V}_0} = \mathrm{e}^{\sqrt{\frac{3}{8}}\phi}\,. \tag{77}$$

A Taylor expansion of the last term in (78) now yields the coupling of ϕ to the SM-fields:

$$\mathrm{e}^{-\sqrt{\frac{3}{2}}\phi}\mathcal{L}_m\left(\mathrm{e}^{\sqrt{\frac{3}{8}}\phi}g^{\mu\nu}\right) = \mathcal{L}_m - \frac{\sqrt{6}}{8}\phi\cdot T^\mu_\mu + \mathcal{O}\left(\phi^2\right), \tag{78}$$

so ϕ couples to the trace of the energy-momentum tensor with coupling constant $\lambda = \sqrt{6}/8 \approx 0.3$. Although ϕ couples universally to all matter fields and does not violate the equivalence principle, the coupling would be observable through the deviations from GR in the post-Newtonian expansion. We see that the volume modulus violates a fifth-force constraint and thus has to avoid them altogether. **Therefore the volume modulus has to be sufficiently heavy (above meV) and is ruled out as a candidate for quintessence.**

The process of rescaling the metric in this toy model is known as transition to the Einstein frame and is necessary in every string compactification. We thus expect this argument to rule out the volume modulus also in more elaborate models. Note that then also further couplings between volume modulus and SM fields might arise, for example through the gauge-kinetic function. In this case, the equivalence principle may be violated and even stronger constraints apply, as is discussed in [58].

3.3 Phenomenological restrictions enlisted

In the previous two subsections we have highlighted two phenomenological necessities: a light quintessence field and a heavy volume modulus. There are (at least) two further restrictions arising in string-theory model building.

Firstly, superstring theory is intrinsically supersymmetric and the string theory vacua that can be controlled reasonably well at least exhibit broken SUSY. Since collider experiments like the LHC have not found any SUSY partners in their energy range, the SUSY-breaking scale has to be sufficiently high so that the SUSY partners are heavier than the TeV scale currently achievable at the LHC. A particularly easy to compute SUSY-partner mass term is the gaugino mass which can be read of the 4D $\mathcal{N} = 1$ Lagrangian in [75] as

$$m_{1/2} = \frac{1}{2}\frac{\partial_i f}{\mathrm{Re} f}F^i\,, \tag{79}$$

where the gauge index of the gauge-kinetic function f has been dropped and F^i is given by

$$F^i = \mathrm{e}^{K/2} K^{i\bar{j}} D_{\bar{j}} \bar{W} \,, \tag{80}$$

so that the F-term potential can actually be written as an "F-term"

$$V_{F\text{-term}} = \mathrm{e}^K K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} = F^i K_{i\bar{j}} \bar{F}^{\bar{j}} \,. \tag{81}$$

Due to the simple form of its mass term, we will use the gaugino as representative of the SUSY partners but in the end they all have to be sufficiently heavy.
Secondly, as has been mentioned in the discussion of fifth-force constraints, 4D gravity has been tested with torsion balance experiments to hold below 0.2 meV $\sim$ 1 mm^{-1} [89]. This also implies that compactifications have to be of the order of 100 μm or smaller. Accordingly, the corresponding KK modes have to be heavier then meV. For isotropic compactifications, this also introduces a bound on the compactification volume but as we have discussed in section 2.3.3, anisotropic compactifications may become useful for suppressing loop corrections. In that case, the largest compact dimension has to be small and the associated KK-mode heavy enough. As this is the KK-mode setting the loop-correction cut-off in (54), the interplay of anisotropy, loop-correction suppression and KK-mass bound is highly restraining.
With this final requirements we can list all phenomenological restrictions we want to pose:

1. **Light quintessence modulus** ϕ with $m_\phi \lesssim 10^{-60} M_\mathrm{P}$.

2. **Heavy volume modulus** with $m_\mathcal{V} \gtrsim 10^{-30} M_\mathrm{P}$.

3. **Heavy superpartners** with $m_S \gtrsim 10^{-15} M_\mathrm{P}$.

4. **Heavy KK scale** with $m_\mathrm{KK} \gtrsim 10^{-30} M_\mathrm{P}$.

Of course, there are further conditions that have to be met for a truly realistic model, like the implementation of the SM, fifth-force considerations, a viable quintessence potential etc.. However, we will find that these four restrictions already lead to serious challenges for string-theoretic model building, so we will remain ignorant of these further complications.

4 Challenges of Stringy Quintessence

If we want to construct quintessence from string theory, we have to find a scalar degree of freedom with very low mass and flat potential. In the framework of type IIB flux compactification, we find axions and Kähler moduli as candidates for this role. Axions are naturally very light, but naive attempts of implementing slow-roll on their potential run into contradictions with another swampland conjecture, the (axionic) weak gravity conjecture (WGC). Therefore, we will turn our attention to Kähler moduli here, postponing comments on axion quintessence to part 5.

In section 2.3 we analyzed the scalar potential of the Kähler moduli, which has no-scale structure classically and only generates masses by higher-order corrections. We now have to find a sufficiently flat direction in this potential to serve as quintessence modulus. Since the volume modulus has been ruled out in the last section, we need at least a second modulus to play this role.

4.1 The light volume problem

For the huge hierarchies of quintessence, we need parametric control over the potential. As mentioned before, a large volume accomplishes just that, so we will rely on the LVS in this context. Let us recall the volume scaling of the various scalar-potential contributions. The no-scale cancellation occurs at the level of $V_{\text{no-scale}} \sim W_0^2/\mathcal{V}^2$. The higher-order corrections then scale as

$$\delta V_{\text{np}} \sim \frac{\sqrt{\tau_{\text{s}}}\mathrm{e}^{-2a_s\tau_{\text{s}}}}{\mathcal{V}} + \frac{W_0\tau_{\text{s}}\mathrm{e}^{-a_s\tau_{\text{s}}}}{\mathcal{V}^2} \;\to\; \frac{W_0^2}{\mathcal{V}^3}\log^{3/2}(W_0/\mathcal{V})\,, \qquad \delta V_{\alpha'} \sim \frac{W_0^2}{\mathcal{V}^3}\,, \qquad \delta V_{\text{loop}} \sim \frac{W_0^2}{\mathcal{V}^{10/3}}\,. \tag{82}$$

Here we have integrated out the small cycle τ_{s}, which is stabilized by the non-perturbative corrections at mass scale $m_{\text{s}} \sim W_0/\mathcal{V}$. As we will see, this is heavier then the volume modulus and thus τ_{s} is no candidate for quintessence. Although LVS only requires one small modulus, we allow for any number of small moduli that are all fixed by non-perturbative corrections at high mass. We integrate them out and retain a volume dependent correction. As shown in section 2.3.2, α' and non-perturbative corrections only fix these small moduli and the volume modulus at order

$$V_{\text{LVS}} \;\sim\; \delta V_{\text{np}} + \delta V_{\alpha'} \;\sim\; \frac{W_0^2}{\mathcal{V}^3}\,. \tag{83}$$

This potential can have an AdS minimum at exponentially large $\mathcal{V}_0$ with the exponent being $\sim \chi^{2/3}/g_{\text{s}}$ [19, 70, 73].

Due to the scaling of V_{LVS} the volume modulus is stabilized at mass $m_{\mathcal{V}} \sim W_0/\mathcal{V}^{3/2}$, which can be inferred from the known Kähler metric of section 2.3.1. In appendix A we will argue more generally that the moduli masses can be approximated by their contribution to the scalar potential δV via

$$m^2 = m^2 M_{\rm P}^2 \gtrsim \delta V\,. \tag{84}$$

Since the volume modulus mass is bounded by requirement 2, our quintessence modulus has to be stabilized by even further suppressed contributions. In the stabilization hierarchy given by (82), all remaining large moduli (large enough to make $e^{-a_i\tau^i}$ negligibly small) are fixed by loop corrections. By process of elimination, one of these moduli is the quintessence modulus τ_ϕ and thus receives a mass

$$m_\phi \sim \sqrt{\delta V_{\rm loop}} \sim \frac{W_0}{\mathcal{V}^{5/3}} \tag{85}$$

or heavier. Combining (85) with the required scales listed in the previous section, one finds

$$\mathcal{O}\Big(10^{30}\Big) \lesssim \frac{m_{\mathcal{V}}}{m_\phi} \sim \mathcal{V}^{1/6} \qquad \Rightarrow \qquad \mathcal{V} \gtrsim \mathcal{O}\Big(10^{180}\Big)\,. \tag{86}$$

This is a very large volume and will result in very small KK scales given by

$$m_{\rm KK} = \frac{M_{\rm s}}{R} \sim \frac{M_{\rm P}}{\mathcal{V}^{1/2+1/6}} \lesssim \mathcal{O}\Big(10^{-120}\Big) M_{\rm P}\,, \tag{87}$$

which is in conflict with requirement 4. Here we have used (56) in the least restricting isotropic $l = 6$ case.

The loop corrections involving the quintessence modulus thus have to be suppressed more strongly than by $\mathcal{V}^{-10/3}$. As suggested in [62, 71], anisotropic compactifications may provide the required suppression. The proposed model contains 3 Kähler moduli with a volume function of the form

$$\mathcal{V} = \sqrt{\tau^1}\tau^2 - (\tau^3)^{3/2}\,. \tag{88}$$

τ^3 corresponds to $\tau_{\rm s}$ in the standard LVS construction, but the bulk of the compactification is now a fibration over two large dimensions governed by the 2-cycle dual of τ^1 and a 4-cycle fiber, as for example a T^4 or $K3$ manifold. During canonical normalization, the volume modulus will decouple and a specific ratio of fiber and base modulus will be left unstabilized by $V_{\rm LVS}$. This ratio is then supposed to be the quintessence scalar. The authors move on to constructing a suitable quintessence potential from poly-instanton corrections, which are very small corrections that can generate a flat potential suitable for quintessence. However, these only dominate if the loop corrections are even smaller. We recall from the heuristic argument in section 2.3.3 that loop corrections may be described by 4D field theory loops in form of the Coleman-Weinberg potential (54) and effective scaling (57)

$$\delta V_{\rm loop} \sim A\frac{W_0^2}{\mathcal{V}^{3+2/l}} + B\frac{W_0^4}{\mathcal{V}^4}\,. \tag{89}$$

Thus, in this anisotropic scenario with $l = 2$, the quintessence scalar gets loop corrections only at order $\mathcal{V}^{-4}$ which in contrast to (85) induces a quintessence mass[11]

$$m_\phi \sim \sqrt{\delta V_{\text{loop}}} \sim \frac{W_0}{\mathcal{V}^2}\,. \tag{90}$$

Since requirement 4 bounds the volume to $\mathcal{V} \lesssim \mathcal{O}(10^{30})$ we can marginally source the right quintessence mass. However, using $m_\mathcal{V} \sim W_0/\mathcal{V}^{3/2}$ and m_ϕ from (90) together with our phenomenological requirements 1 and 2, we conclude

$$\mathcal{O}\Big(10^{-30}\Big) \gtrsim \frac{m_\phi}{m_\mathcal{V}} \sim \mathcal{V}^{-1/2} \sim m_{\text{KK}}^{1/2} \qquad \Rightarrow \qquad \mathcal{O}\Big(10^{-60}\Big) \gtrsim m_{\text{KK}}\,, \tag{91}$$

where in the last step, we see a contradiction with requirement 4 arising as the KK scale becomes too low. So even in the anisotropic case the required hierarchy cannot be achieved through the standard LVS approach.[12]

We will refer to this problem, which has already been noted in [62, 71], as the "light volume problem". To resolve it, one needs an extra contribution to the scalar potential, which gives the volume modulus a higher mass. This is already critical. However, as we will see momentarily, things get even more challenging if we take into account SUSY breaking according to requirement 3. This will provide an independent argument for a new scalar-potential term, fixing also its sign and prescribing a significant overall magnitude.

4.2 The F-term problem

It is necessary to ensure that the SM superpartners are sufficiently heavy (requirement 3). This will prove to be very challenging. We will turn to the gaugino mass $m_{1/2}$ given in (79) as a representative of this issue. If the SM gauge group is realized on D7-branes, $m_{1/2}$ scales as $|W|/\mathcal{V}$. For D3 realizations, the soft scale is suppressed more strongly [70] – so this does not help. Due to the aforementioned phenomenological requirements 1 and 3, the hierarchy between the quintessence field and the gaugino must fulfill

$$\frac{m_\phi}{m_{1/2}} \lesssim \mathcal{O}(10^{-45}). \tag{92}$$

We can furthermore use the first term in (89) to conclude that $m_\phi \gtrsim m_{\text{KK}} m_{3/2}$ and observe that $m_{3/2} \sim m_{1/2}$ in the present setting. This implies

$$m_{KK} \lesssim \frac{m_\phi}{m_{1/2}} \lesssim \mathcal{O}(10^{-45})\,, \tag{93}$$

[11] We again refer to appendix A for a justification of the formula $m_\phi \sim \sqrt{\delta V_{\text{loop}}}$.

[12] As mentioned in subsection 2.3.3, we can further suppress V_{loop} by choosing $l < 2$ and tuning W_0 small. The obvious possibility is $l = 1$ corresponding to one large and five small dimensions, but more complicated geometries can lead to values $1 \leq l \leq 2$ in the crucial formula (56) for m_{KK}. Either way, repeating the analysis which led to (91) one arrives at $m_{\text{KK}} \leq \mathcal{O}(10^{-30-15l})$ for general l. Thus, requirement 4 is always violated and the light volume problem cannot be resolved by going to $l \leq 2$.

in conflict with requirement 4. We conclude that the gaugino mass cannot be generated by the SUSY breaking of the Kähler moduli alone.

Instead, to obtain large enough gaugino masses, we need a further source of SUSY breaking. One can realize this on the SM brane through mediation from a hidden sector where SUSY is broken spontaneously by the non-vanishing F-term of a spurion field X. Without loss of generality, we will use the language of spontaneous SUSY breaking even in the case that this breaking is realized locally (at the same Calabi-Yau singularity) and directly at the string scale.[13]

According to our discussion of D-brane contributions in section 2.3.4, SM-brane SUSY breaking gives a positive contribution to the scalar potential, which is added on top of the zero potential resulting from the Kähler-moduli no-scale structure. Now consider a simple toy model with a single spurion field X and F-term F. Let SUSY breaking be mediated through higher-dimension operators suppressed by M, which we define to be the mediation scale of the flat SUSY limit. After canonical normalization of X and its F-term,[14] one has $m_{1/2} \sim F/M$ (and similarly for the other soft terms), which implies

$$\delta V_X \sim F^2 \sim M^2 m_{1/2}^2 \,. \tag{94}$$

Soft masses are phenomenologically constrained (requirement 3) to be at least $\sim \mathcal{O}(10^{-15})M_{\mathrm{P}}$. Moreover, M should be high enough to hide the SUSY-breaking sector. It is then natural to assume $M \gtrsim \mathcal{O}(10^{-15})M_{\mathrm{P}}$ and we will more carefully exclude lower values in the next subsection. This implies $\delta V_X \sim M^2 m_{1/2}^2 \gtrsim \mathcal{O}(10^{-60})M_{\mathrm{P}}^4$, which is of the same order of magnitude as the cancellation in the standard no-scale scenario, i.e. far larger than the first-order LVS corrections.[15] Thus δV_X raises the height of the scalar potential to very large positive values which cannot be canceled by the terms in V_{LVS} of (83).

4.2.1 Limits on δV_X

Since δV_X has emerged as a key issue for the most popular stringy quintessence models, we want to evaluate more carefully whether this hidden-sector contribution to the scalar potential can be consistently tuned to smaller values. Recall from (94) that it scales as $\delta V_X \sim m_{1/2}^2 M^2$. Since the gaugino mass should not be smaller than $\mathcal{O}(10^{-15})M_{\mathrm{P}}$, the only option is to reduce M and F at the same time, which implies a reduction of the gravitino mass. In the past, there have been many investigations that aimed at constraining the latter using data from electroweak colliders [94–101] like LEP or hadronic ones [13, 102–105] like the Tevatron. These bounds on

[13] In this case one may speak of non-linearly realized SUSY (see [93] for recent progress in this context). One may, however, also continue to use the language of e.g. F-term SUSY breaking in SUGRA, sending the masses of the fields in the SUSY-breaking sector to infinity.

[14] We carried out this procedure for an explicit example in appendix B.

[15] Indeed, as noted earlier $m_\phi \gtrsim m_{\mathrm{KK}} m_{3/2}$ so that the canceling terms in the no-scale potential are of order $V_{\mathrm{no-scale}} \sim m_{3/2}^2 \lesssim m_\phi^2/m_{\mathrm{KK}}^2 \lesssim 10^{-60} M_{\mathrm{P}}^4$, where we enforce requirements 1 and 4.

$m_{3/2}$ translate into lower limits of the SUSY-breaking scale, which typically constrain $\sqrt{F}$ to be larger than a few 100 GeV.

The most recent and stringent bounds result from missing-momentum signatures in pp collisions at the LHC. To understand the emergence of such bounds, let us consider an exemplary toy model where SUSY is spontaneously broken in a hidden sector through a non-vanishing F-term in the vacuum and mediated to the SM sector via the interaction terms

$$\mathcal{L}_{\text{int}} = \frac{a}{M^2}\int d^4\theta X^\dagger X \Phi^\dagger \Phi + \frac{b}{M}\int d^2\theta X W^\alpha W_\alpha + \text{h.c.}\,, \tag{95}$$

where Φ is a chiral superfield representing quarks q and squarks $\tilde{q}$ whereas W^α is the supersymmetric field-strength tensor of a vector superfield V representing gluons g and gluinos $\tilde{g}$. A non-zero F in the vacuum will generate soft masses for the squarks and gluinos, which are given by $m_{\tilde{q}}^2 = aF^2/M^2$ and $m_{\tilde{g}} \sim bF/M$, respectively. The hidden-sector field X contains the goldstino $\tilde{G}$, which gets eaten by the gravitino due to the super-Higgs mechanism. In the limit $\sqrt{s}/m_{3/2} \gg 1$, the helicity-1/2 modes dominate over the helicity-3/2 modes and, according to the gravitino-goldstino equivalence theorem [106,107], yield the same S-matrix elements as the goldstinos. Hence, in this simple discussion, we identify the gravitino with the goldstino. We are now interested in processes which turn two hadrons into a hadronic shower plus gravitinos, where the latter induce a missing-momentum signature. For instance, we can consider the process of two quarks in the initial state and two gravitinos in the final state with a gluon being eradiated from one of the initial quarks, resulting in a hadronic shower. The gluon radiation costs a factor $\sqrt{\alpha_S}$. Several beyond-SM processes contribute to the crucial qq-$\tilde{G}\tilde{G}$-amplitude. One of them is the direct 4-particle coupling from (95):

$$\sim \frac{a}{M^2}\bar{\tilde{G}}\tilde{G}\bar{q}q \subset \frac{a}{M^2}\int d^4\theta X^\dagger X \Phi^\dagger \Phi\,. \tag{96}$$

Due to the prefactor a/M^2, this vertex contributes a factor $1/F^2$ to the amplitude so that the cross section will be proportional to α_S/F^4. This F^{-4}-dependence of the cross section is typical for such processes and therefore the upper limits on them, provided by measurements at hadron colliders, translate into lower bounds on F.

In a recent experimental analysis of the ATLAS collaboration [13], the process $pp \to \tilde{G} + \tilde{q}/\tilde{g}$ is considered, whereupon the squark or gluino decays into a gravitino and a quark or gluon, respectively. Depending on the squark and gluino masses, as well as on their ratios, the authors derive lower bounds on the gravitino mass around $m_{3/2} \approx (1-5)\times 10^{-4}\,\text{eV}$ corresponding to SUSY-breaking scales $\sqrt{F} \approx (650-1460)\,\text{GeV}$.

In [108], not only the process $pp \to \tilde{G}+\tilde{q}/\tilde{g} \to 2\tilde{G}+q/g$ but also direct gravitino-pair production with a quark or gluon emitted from the initial proton as well as squark or gluino pair production with a following decay into gravitinos and quarks or gluons are considered. Taking into account all three processes, the authors of [108] use the model-independent 95% confidence-level upper limits by ATLAS [12] on the cross section for gravitino + squark/gluino production to constrain

$\sqrt{F} > 850\,\text{GeV}$. This is done for the case when the squark and gluino masses are much larger than those of the SM particles so that they can effectively be integrated out (in the paper, the value $m_{\tilde{q}/\tilde{g}} = 20\,\text{TeV}$ is used). In other scenarios, where one or both of these two types of superpartners have lower masses, the bound becomes even higher.

We conclude that, in accordance with the current experimental status, the mass scale of SUSY breaking $\sqrt{F}$ cannot be lowered significantly below $100\,\text{GeV} - 1\,\text{TeV}$ so that δV_X can be at most a few orders of magnitude below $\mathcal{O}(10^{-60})M_{\text{P}}^4$. Such a contribution cannot be canceled by any known term in our scenario as has been discussed already.

4.2.2 Need for a new contribution

We have seen that requirement 3 of heavy superpartners implies the presence of a large positive contribution δV_X to the scalar potential. This would raise the potential far above the observed energy density $\mathcal{O}(10^{-120})M_{\text{P}}^4$, rendering this whole scenario unviable. Since we do not know how to avoid this effect, it appears logical to assume the presence of a further negative contribution of equal magnitude, which fine-tunes V to a level consistent with observations. In the preferred case of $l = 2$ and for $W_0 \sim \mathcal{O}(1)$, the required magnitude is $\delta V_{\text{new}} \sim \mathcal{V}^{-2}$. Such a contribution may also solve the light volume problem (91). Indeed, if its volume dependence is generic, one expects an induced volume-modulus mass $m_{\mathcal{V}} \sim \mathcal{V}^{-1}$. This is just enough to build all required hierarchies.

We emphasize that this contribution is substantially hypothetical and that the nature of its generation and form is not understood. Possible effects suggested in [62,71] are loop corrections from open strings on the SM brane and the back-reaction of the bulk to the brane tension along the lines of the SLED models [109]. Open string loops may induce a Coleman-Weinberg potential with cutoff at the string scale $M_{\text{s}} \sim M_{\text{P}}/\sqrt{\mathcal{V}}$, such that the leading term scales as $M_{\text{s}}^4 \sim M_{\text{P}}^4/\mathcal{V}^2$. Although this is the correct order of magnitude for δV_{new}, the volume dependence appears to be too simple to allow for volume-modulus stabilization. Moreover, being a higher-order correction to the brane sector, we would assume it to already be part of the low-energy effective Kähler potential for X and the SM fields which we used to derive F-terms and induce superpartner masses. As such it could not contribute the required negative energy to cancel the critical F-term.

As mentioned above, a counteracting contribution could also be found in the bulk back-reaction. Since the SM-brane tension is the origin of the large F-term, a back-reaction to this tension from the bulk appears to be promising. Still, as our analysis shows, it remains a challenge to include this in the 4D effective theory, specifically in the 4D effective SUGRA, which we expect to arise at low energies in the string theoretic settings we consider (see also [62,71,110–112] for related discussions).

Finally, in the context of the de Sitter swampland conjecture (1), our F-term implies yet another difficulty. Even if the new term δV_{new} cancels the F-term to leave a sufficiently small potential, a small change in the SM or SUSY-breaking parameters can raise the F-term and with it the residual scalar potential to violate the conjecture. This is also problematic in other models and we will come back to this issue in the following sections.

5 Loopholes and alternative Approaches

There are several potential loopholes in our analysis. The first one is the possibility that the quintessence modulus is extremely light (i.e. the loop-induced potential is extremely flat) by fine-tuning.[16] However, this seems implausible for the following reason: The flatness must hold on a time scale of order H_0^{-1}. In quintessence models which respect the de Sitter conjecture (1), the scalar field has to run sufficiently far during such a period. Indeed, from the Klein-Gordon equation in Friedmann-Robertson-Walker background together with $|V'|/V \lesssim 1$ as required by (71) it follows that $\Delta\phi \sim \mathcal{O}(1)$ in one Hubble time. In a Taylor expansion of δV_{loop}, we therefore have to take into account all orders of $\Delta\phi$. It is thus not enough to fine-tune δV_{loop} at one point but we must tune an infinite number of derivatives to small values. This cannot be coincidental but has to be based on some mechanism or symmetry. Although in our specific model such a perfect decoupling of one Kähler modulus from the loop corrections seems implausible, there might of course be other constructions where the required sequestering can be achieved (see [58, 113] for discussions).
Another possibly critical point is the approximation of loop corrections through the Coleman-Weinberg potential (54) with m_{KK} as a cutoff. Here, one has to be concerned that no other, stronger corrections arise. This seems possible, for example, since the KK scale is far below the weak scale. Thus, when applying the formula, one has to do so in a setting where the SM brane (with SUSY broken at a higher scale) has already been integrated out. This needs further scrutiny. Another concern is that even in the bulk SUSY may not be fully restored above m_{KK} due to the effect of bulk fluxes. Still, we trust the formula to at least give a lower bound on loop corrections that cannot be neglected and thus makes our conclusions inevitable.

A number of alternative approaches to quintessence building from string theory have been proposed. Let us first comment on the possibility of axion quintessence. Based on the SUGRA scalar potential, one generically expects an axion potential

$$V = \Lambda^4 \cos\left(\frac{\phi}{f}\right) + a\,, \qquad \Lambda^4 \sim M_{\text{P}}^2 m_{3/2}^2 \mathrm{e}^{-\mathrm{S}_{\text{inst.}}}\,, \tag{97}$$

where $S_{\text{inst.}}$ is the instanton action inducing the potential. This could provide the required dark energy if ϕ is at the "hilltop" and, at the same time, satisfy the de Sitter conjecture (2) (assuming reasonably small c'). For simplicity, let us start the discussion taking $a = 0$. Then the slow-roll conditions (71), which we need phenomenologically, require a trans-Planckian axion decay constant f [67]. But this is in conflict with quantum-gravity expectations or, more

[16]For example, one could imagine a model where the two terms in (57) cancel to a very small residue.

concretely, the weak gravity conjecture for axions [114, 115]:

$$f \leq \mathcal{O}(1) M_{\mathrm{P}} \qquad \text{or} \qquad S_{\text{inst.}} \leq \alpha \frac{M_{\mathrm{P}}}{f} \,. \tag{98}$$

The conflict is strengthened if one recalls that the potential must be tiny, i.e. $M_{\mathrm{P}}^2 m_{3/2}^2 \mathrm{e}^{-\alpha M_{\mathrm{P}}/f} \lesssim 10^{-120} M_{\mathrm{P}}^4$. For $\alpha \sim \mathcal{O}(1)$, this implies $f \sim \mathcal{O}(10^{-2}) M_{\mathrm{P}}$, which is in conflict with slow-roll. As suggested in [31], one might hope to ease the tension by employing the constant contribution a to the potential (97).[17] If a is negative, the slow-roll condition is violated even more strongly. Positive a greater than Λ^4 leads to a violation of the de Sitter conjecture at the minimum. The best option is then $a = \Lambda^4$ which, however, does not help much: The slow-roll requirements on f change only by a factor $\sqrt{2}$, so f still needs to be at the Planck scale.

With this naive approach we would have to violate the weak gravity conjecture by assuming an unacceptably large $S_{\text{inst.}}$. However, the weak gravity conjecture is presumably on stronger footing than the de Sitter conjecture, so this is against the spirit of the swampland discussion. Instead, alternative elements of model building may be invoked to save axion quintessence. An option is the use of axion monodromy [67]. Another idea developed and discussed in [65, 69, 116–118] is a further suppression of the prefactor of the axion potential. A specific model with a highly suppressed axion potential for an electroweak axion has been developed in [65, 69]. We note that the most obvious suppression effects are related to high-quality global symmetries in the fermion sector, suggesting a relation between the weak gravity conjecture and global-symmetry censorship [118, 119].

If such models succeed in providing a sufficiently flat potential, we still have to account for large enough SUSY breaking in the full model to generate heavy SM superpartners. The large F-term required has to be canceled to allow for the flat axion potential to dominate. Assuming this cancellation to be implemented, we can again slightly change the SUSY-breaking contributions to shift the axion potential to positive values and violate the de Sitter conjecture at the minima. The full model would need to balance out these changes by some intricate mechanism.

An alternative approach to building a quintessence potential from KKLT-like ingredients has been taken in [64] where the quintessence field is given by the real part of a complexified Kähler modulus. This Kähler modulus runs down a valley of local axionic minima in the real direction. Since the universe is assumed to be in a non-supersymmetric non-equilibrium state today, it can evolve at positive potential energies. However, since the potential has to be sufficiently small to constitute a quintessence model, the superpotential has to be tuned to very small values, which results in a small gravitino mass. It appears that one needs further SUSY breaking and the F-term problem re-emerges.

An interesting alternative to quintessence has been introduced in [120]: The zero-temperature scalar potential is assumed to satisfy the de Sitter conjecture, but a thermally excited hidden

[17] Another idea to resolve the conflict would be to move away from the hilltop to a point in field space where both slow-roll conditions are as weak as possible. This turns out not to work.

sector stabilizes a scalar field at a positive-energy hilltop. The authors illustrate this idea using a simple Higgs-like potential $V = -m_\phi^2\phi^2/2 + \lambda\phi^4 + C$. Since the hidden sector must not introduce too much dark radiation, the temperature and hence also m_ϕ are bounded from above by today's CMB temperature, which is roughly 0.24 meV. This model does not need an approximate no-scale structure to ensure an extremely flat potential at large $\mathcal{V}$, so our F-term problem does not immediately arise.

However, it makes an indirect appearance as follows: Both the present toy model potential as well as more general models of this type are expected to have a minimum somewhere. In the present case, its depth is $m_\phi^4/16\lambda$, which is very small unless λ is truly tiny. Now, since some F-term effect δV_X must be present somewhere in the complete model, a small de-tuning of this δV_X will be sufficient to lift the model into the swampland. Thus, some form of conspiracy must again be at work for this model to describe our world and the de Sitter conjecture to hold simultaneously.

A way out is provided by assuming that $\lambda \sim \mathcal{O}(10^{-64})$ and available δV_X are bounded at $\sim$ TeV. Then the minimum is too deep to be lifted to de Sitter by de-tuning. Even then, one has to be careful to ensure that $|V''|/V$ does not become too small as one uplifts the model by de-tuning the SUSY-breaking effect. We approximate the possible de-tuning by the order of magnitude of the F-term itself: $\Delta(\delta V_X) \sim \delta V_X \sim F^2$. As a result $|V''|/\Delta(\delta V_X) \sim m_\phi^2/F^2 \sim \mathcal{O}((10^{-31})^2/10^{-60}) \sim \mathcal{O}(10^{-2})$, which is critical in view of the de Sitter conjecture. Thus, even in this rather extreme case, a version of the F-term problem can at best be avoided only marginally.

6 Conclusion

We reviewed the type IIB flux-compactification procedure with the goal of constructing a quintessence model from string theory. Following the need of huge hierarchies we identified the quintessence field as a Kähler modulus independent of the overall volume, which is unfixed by the leading-order corrections to the scalar potential. Further suppression of the quintessence scale then required the suppression of the next-to-leading order corrections, which involve string-loop corrections, and led to the proposal of anisotropic compactifications described in [62].
However, phenomenology restricts not only quintessence, but at least three further mass scales: volume modulus, SUSY breaking and KK scale. The hierarchies between those scales are restricted by experiments and we identified the difficulties they cause in model building. At first, we re-derived the "light volume problem" which manifests itself in a volume-modulus mass which is too light to avoid fifth-force constraints. This issue was already known to the authors of [62] and a possible resolution was hinted on in [71].
Moving on to SUSY breaking, we found that due to the low scales required by quintessence, the amount of SUSY breaking from the Kähler sector is much too small to allow for sufficiently high superpartner masses. This could be resolved by introducing an additional SUSY breaking sector on the SM brane. However, it turns out that this induces a positive F-term contribution to the scalar potential which is much bigger than the Kähler-sector contribution and lifts the potential far into the dS regime. Since neither phenomenology nor swampland conjectures allow for such potentials, we need a further contribution δV_{new} to the potential, which is negative and scales like $\mathcal{V}^{-2}$ to cancel the F-term. Although on one hand this term could also resolve the "light volume problem" by fixing the volume-modulus mass at order $\mathcal{V}^{-1}$, its inclusion could on the other hand upset the stabilization hierarchy of LVS, since it is of the same order as the no-scale canceling terms. We thus have to make sure that the new term only raises the mass of the volume modulus.
Although this is the essence of the "F-term problem" which we introduced in our paper, a further aspect of it arose in the context of the swampland conjecture. The required cancellation between F-term and δV_{new} has to be fine-tuned to yield a quintessence potential of the order of the cosmological constant, which is the old cosmological constant problem in disguise. If such a tuned situation could be found in the string landscape, a similar model with slightly altered SM or SUSY-breaking parameters should be possible as well. However, this can increase the F-term and lift the potential into dS, where it violates the dS swampland conjecture (1). The same argument is also applicable to the other models discussed in part 5. The canceling term δV_{new} thus has to be coupled to the F-term itself such that it counterbalances any change in parameters exactly. Some kind of back-reaction as proposed in [71] seems to be the most promising solution but has not been fully described in 4D yet.
Of course alternative approaches to quintessence could completely evade the effective 4D SUGRA logic that we used or study entirely different string-theoretic settings like type IIA

or heterotic string theory. Also other control parameters like large complex-structure or small string coupling could lead to very different scenarios. However, due to the various dualities linking the different corners of the string-theory landscape, these alternative approaches are limited and might turn out to be similar to our type IIB large-volume setup. Large complex-structure for example is the mirror-symmetry dual to large volume and thus not intrinsically different. Overall, we expect problems similar to the F-term problem to appear in any quintessence model from string theory. The hierarchy between SUSY breaking and quintessence is necessarily big and the required fine-tuning mechanism has to explain why a small de-tuning is impossible as suggested by the dS conjecture.

At this point, we have to raise the question whether quintessence is really a good way out of the dS conjecture. If it is always possible to turn a quintessence model into a dS model by tiny changes in the SM parameters, nothing would be gained by considering quintessence. Furthermore, the construction involves considerably more fine-tuning [59] and more involved constructions than dS models [31]. Only by resolving or bypassing the F-term problem we can make progress in understanding the advantage of quintessence over dS models and continue searching for a string vacuum in simultaneous agreement with experiments and the dS swampland conjecture. That is, if we are not taking these difficulties as a counterargument against the latter and return to dS constructions.

Appendix

A Estimating moduli masses from the potential

We will argue that under reasonable assumptions the mass scale of a physical modulus is usually set by the highest order term δV in the scalar potential that involves the respective modulus:

$$m^2 \gtrsim \delta V \,. \tag{99}$$

This is easy to see for the volume modulus but requires justification for the other moduli. Although heavier masses can easily arise for 'small-cycle' moduli which correspond to small terms in $\mathcal{V}$, much lighter masses require some kind of cancellation, which will generally involve tuning.

To illustrate the idea, consider the toy-model Lagrangian

$$\mathcal{L} = -\frac{\partial_\mu X \partial^\mu X}{2X^2} - V(X)\,, \qquad \text{where} \qquad V''(X) \sim \frac{V(X)}{X^2}\,. \tag{100}$$

The canonical field is introduced through $X = \exp(\phi)$. Then the physical mass squared is the second derivative of the potential w.r.t. ϕ. Given our assumption about $V''(X)$, this is of the same order of magnitude as the potential itself. Thus, suppressing $\mathcal{O}(1)$ coefficients, the approximation $m^2 \sim \delta V$ is justified.

For the volume modulus Ω the argument is basically as in the toy model above. In (42) we observed a prefactor $3/\Omega^2$, so the physical volume modulus is given by $\phi_{\mathcal{V}} = \sqrt{6}\ln\Omega$, as we have argued in our discussion of volume modulus quintessence in section 3.2.

For the other moduli, a closer examination of (41) reveals that in general A_{kl} scales as

$$A_{kl} \sim \mathcal{V}_{kl} \sim \frac{\mathcal{V}}{\chi_k \chi_l} \sim \mathcal{O}\Big(\chi_k^{-1}\chi_l^{-1}\Big). \tag{101}$$

Thus, it is natural to transform $\chi_k \sim \exp(\phi_k)$ as we did for the volume modulus. The result is a kinetic term of the form

$$-K_{ij}(\tau)\partial_\mu \tau_i \partial^\mu \tau_j = -A'_{kl}(\chi)\partial_\mu \phi_k \partial^\mu \phi_l - \partial_\mu \phi_{\mathcal{V}} \partial^\mu \phi_{\mathcal{V}} \tag{102}$$

with

$$A'_{kl} = 2\chi_k \chi_l \mathcal{V}_n \partial_k \partial_l \chi^n \,. \tag{103}$$

The entries of A'_{kl} now generally scale as $\mathcal{O}(1)$ coefficients, but hierarchies between the residual moduli and cancellations of different terms in $\mathcal{V}(\chi_i)$ might spoil this approximation. On the one hand, small moduli as for example $\tau_{\rm s}$ in the LVS scenario, will have small entries in A'_{kl}, so the canonical normalization will rescale these fields and increase their mass. On the other hand, we may end up with big entries in A'_{kl} whenever two terms in $\mathcal{V}(\chi_i)$ are of higher order then

$\mathcal{O}(1)$ but cancel against each other. Since this is nothing else than tuning, we will assume in the following that such cancellations do not occur, so that canonical normalization only mixes the residual moduli with $\mathcal{O}(1)$ coefficients and raises the masses of small moduli.
To extract mass scalings, we turn to the potential. The potential $V(\tau_i)$ expressed through $V(\phi_k, \phi_{\mathcal{V}})$ can be expanded in p powers of $\exp(\phi_{\mathcal{V}})$, which is parallel to an expansion in $\mathcal{V}$. The mass of each modulus ϕ_k is fixed by some contribution

$$\delta V = f(\mathrm{e}^{\phi_k})\mathrm{e}^{p\phi_{\mathcal{V}}}\,, \tag{104}$$

where f is usually some function homogeneous in its argument (at least at leading order and especially for string loop corrections according to (25)), which may also depend on other moduli except the volume modulus. Now due to f being homogeneous and the stabilizing contribution the mass term

$$m_k^2 = \mathrm{e}^{2\phi_k} f''(\mathrm{e}^{\phi_k})\mathrm{e}^{p\phi_{\mathcal{V}}} \sim f(\mathrm{e}^{\phi_k})\mathrm{e}^{p\phi_{\mathcal{V}}} \tag{105}$$

scales as $m_k^2 \sim \delta V$. Especially the volume scaling, which has been made explicit through $\exp\left(\frac{2}{3\sqrt{6}}\phi_{\mathcal{V}}\right) = \mathcal{V}$, can be read of directly. A similar procedure works out for the volume modulus itself, generally linking its mass to the highest appearing power of $\mathcal{V}$ in V. We find that assuming homogeneous potential contributions and no major cancellations, the estimation (99) holds at least with respect to volume scaling. We further note that the requirements are met in many simple cases, for example the models of [62, 71]. (99) is a lower bound due to the possible mass enhancement from A'_{kl}. Since the challenge in the main text was finding smallest possible quintessence modulus masses, this lower bound can be used as the best case scenario.

In our paper, we have not gone through the separation procedure that led to the decoupling of Ω, so we have argued this point more directly. Let us also give the direct argument for completeness:
We restrict our attention to the submanifold of constant $\mathcal{V}$ in the space of real moduli $\tau^1, ..., \tau^n$. We choose an arbitrary trajectory on this submanifold and parameterize it as

$$(\tau^1(\phi), ..., \tau^n(\phi)) = (\tau^1(0)\mathrm{e}^{\xi^1(\phi)\phi}, ..., \tau^n(0)\mathrm{e}^{\xi^n(\phi)\phi})\,. \tag{106}$$

We normalize our parameter ϕ so that it takes the value 0 at the point of interest $\tau^i \equiv \tau^i(0)$. The coefficient vector $\xi^i \equiv \xi^i(0)$ is chosen to be $\mathcal{O}(1)$ valued. Now the Lagrangian for motion along the trajectory contains the kinetic term

$$\mathcal{L} \supset \mathcal{L}_{\text{kin}} = \sum_{ij} K_{ij}\tau^i\tau^j\xi^i\xi^j\partial_\mu\phi\partial^\mu\phi\,. \tag{107}$$

We can compute the Kähler metric from the Kähler potential $K = -2\ln(\mathcal{V}(\tau^i))$ and since we are moving along the submanifold of constant volume we can use

$$\sum_i \mathcal{V}_i \tau^i \xi^i = 0 \qquad \text{such that} \qquad \mathcal{L}_{\text{kin}} = -2 \sum_{ij} \frac{\mathcal{V}_{ij}}{\mathcal{V}} \tau^i \tau^j \xi^i \xi^j \partial_\mu \phi \partial^\mu \phi \,. \tag{108}$$

Unless there is significant cancellation between terms in $\mathcal{V}$ we can assume

$$\mathcal{V}_{ij} \lesssim \frac{\mathcal{V}}{\tau^i \tau^j} \tag{109}$$

and since ξ^i was chosen $\mathcal{O}(1)$, the whole prefactor of $\partial_\mu \phi \partial^\mu \phi$ can be assumed to be $\mathcal{O}(1)$ or smaller. A small prefactor can arise from a small contribution in $\mathcal{V}(\tau^i)$ as for example in the standard LVS example of $\mathcal{V} = {\tau_\text{b}}^{3/2} - {\tau_\text{s}}^{3/2}$ where τ_s is a small modulus and gets a small prefactor in the kinetic term. The canonical normalization will thus either not change or even increase the order of magnitude of the modulus mass.

Turning to the potential, we see that, since we move along the submanifold, any contribution only involving the volume does not contribute to the mass, as for example V_{LVS} in (83). Turning to the leading-order contribution δV involving the other moduli (in our case string-loop corrections) we will rewrite the potential in the coordinates $(\mathcal{V}, \tau^1, ...\tau^{n-1})$ where we have solved the constraint of staying on the submanifold for a suitable τ^n. We introduce indices k and l which only run over $\{1, ..., n-1\}$ in contrast to i and j. The mass squared of our modulus is now determined by the Hessian of the potential contracted with the vector $\delta\tau^k$ corresponding to an infinitesimal shift in ϕ :

$$m^2 \sim \delta V_{kl} \frac{\delta \tau^k}{\delta \phi} \frac{\delta \tau^l}{\delta \phi} = \sum_{kl} \delta V_{kl} \tau^k \tau^l \xi^k \xi^l \sim \mathcal{O}(\delta V) \,. \tag{110}$$

Here we have to assume that after rewriting the potential in terms of $(\mathcal{V}, \tau^1, ...\tau^{n-1})$ it is still sufficiently well behaved to allow for an order of magnitude estimate $\delta V_{kl} \sim \delta V / \tau^k \tau^l$, resembling (109). Due to the homgeneous form of the loop corrections in (25) which is also required for the extended no-scale structure explained in [80–83], this condition is usually satisfied. Since the choice of trajectory was arbitrary, we assume a similar scaling for all moduli involved except for the volume modulus. Bearing in mind the possible mass enhancement from the canonical normalization, we estimate

$$m^2 \gtrsim \delta V \,. \tag{111}$$

B A simple F-term breaking model

In the main text, the necessity of a SUSY-breaking D-brane contribution arose. In section 2.3.4 we analyzed the F-term contribution of scalars from D3-branes to the scalar potential. Here we will give an explicit example with enough SUSY breaking to raise the gaugino mass (79) which is directly proportional to the F-term.

We motivate our model from flat 4D $\mathcal{N}=1$ SUSY, where a SUSY-breaking sector X is coupled to the field strength W of the vector multiplets via a mediation scale M. The Lagrangian is given by

$$\mathcal{L} \supset (Y_\theta \bar{Y}_\theta - c_1 (Y_\theta \bar{Y}_\theta)^2)|_{\theta^2\bar{\theta}^2} + c_2 Y_\theta|_{\theta^2} + \frac{c_{3(ab)}}{M} W^{(a)} W^{(b)} Y_\theta|_{\theta^2} + \text{h.c.} + \mathcal{L}_c \tag{112}$$

$$\supset -\partial_\mu \bar{Y} \partial^\mu Y - 2c_1 F_Y \bar{F}_{\bar{Y}} Y \bar{Y} - \frac{c_{3(ab)}}{2M} F^{(a)\mu\nu} F^{(b)}_{\mu\nu} Y \tag{113}$$

where $\mathcal{L}_c$ includes all further couplings to chiral fields and the superfield formalism has been used in the first equation with $Y_\theta = Y + \sqrt{2}\theta\psi_Y + \theta\theta F_Y$. We now need to find a SUGRA model, whose flat SUSY limit will lead back to this Lagrangian. To this end we introduce a D3-brane scalar X via the real functions

$$k_i(X,\bar{X}) = -a_i|X|^2 + b_i|X|^4 \tag{114}$$

in (28) and add a term linear in X to the superpotential and to the gauge-kinetic function:

$$W' = W + cX \tag{115}$$

$$f'_{(ab)} = f_{(ab)} + \frac{d_{(ab)}}{M'} X + \text{h.c.}\,, \tag{116}$$

which by application of (65) leads to the scalar potential

$$V = e^K \frac{|c|^2}{K_i(-a_i + 2b_i|X|^2)} \tag{117}$$

$$= -e^K \frac{|c|^2}{K_i a_i} + e^K \frac{|c|^2}{(K_i a_i)^2}\left[(K_i a_i)^2 - K_{ij} a_i a_j - 2K_i b_i\right]|X|^2 + \mathcal{O}\left(|X|^4\right). \tag{118}$$

If we now use $K = -2\ln\mathcal{V}$, we get

$$V = \frac{|c|^2}{2\mathcal{V}\mathcal{V}_i a_i} + \frac{|c|^2}{\mathcal{V}(\mathcal{V}_i a_i)^2}\left[\frac{1}{2\mathcal{V}}(\mathcal{V}_i a_i)^2 + \frac{1}{2}\mathcal{V}_{ij} a_i a_j + \mathcal{V}_i b_i\right]|X|^2 + \mathcal{O}\left(|X|^4\right). \tag{119}$$

With positive a_i, b_i we can stabilize X at value 0 with mass fully controllable by the b_i parameters. After this stabilization, we get a positive F-term potential from the first term alone. Identifying this term with the standard form $V = F^X K_{X\bar{X}} F^{\bar{X}}$ where by prescription of (63) the Kähler metric amounts to

$$K_{X\bar{X}} = \frac{2\mathcal{V}_i a_i}{\mathcal{V}}\,, \tag{120}$$

the resulting F-term is simply

$$F^X = \frac{\bar{c}}{2\mathcal{V}_i a_i}\,. \tag{121}$$

Now the gaugino receives the mass

$$m_{1/2,(a)} = \frac{1}{2\text{Re}f_{(a)}} F^X \frac{d_{(a)}}{M'} = \frac{1}{2\text{Re}f_{(a)}} \frac{\bar{c}d_{(a)}}{2M'\mathcal{V}_i a_i}\,, \tag{122}$$

where we have diagonalized the gauge-kinetic function and canonically normalized the gauge fields. The mediation scale M' does not correspond to M from (113), because the X field has yet to be canonically normalized. Fortunately, the Kählermetric is already block-diagonal, since the off-diagonal terms $K_{i\bar{X}}$ are linear in X, which has been stabilized at value 0. So we only need to make the transition

$$-\det(e) K_{X\bar{X}} \partial_m X \partial^m \bar{X} \to -\partial_\mu X \partial^\mu \bar{X}\,, \tag{123}$$

where we, by hand, constrain the metric to be Minkowski.
The relevant factor is $X_{\text{normalized}} = X \cdot \sqrt{K_{X\bar{X}}}$.
Matching (113) with the corresponding coupling in the SUGRA Lagrangian (taken from [75])

$$-\frac{c_{3(ab)}}{2M} F^{(a)\mu\nu} F^{(b)}_{\mu\nu} X_{\text{normalized}} \hat{=} -\frac{1}{4}\det(e)\frac{d_{(ab)}}{M'} F^{(a)mn} F^{(b)}_{mn} X\,, \tag{124}$$

we get the relation $M = M' \cdot \sqrt{K_{X\bar{X}}}$, which translates to a gaugino mass in SUSY terms of

$$m_{1/2,(a)} = \frac{1}{2\text{Re}f_{(a)}} \frac{\bar{c}d_{(a)}}{M\sqrt{2\mathcal{V}\mathcal{V}_i a_i}} \sim \mathcal{O}\left(\frac{c}{M\sqrt{\mathcal{V}\mathcal{V}_i a_i}}\right). \tag{125}$$

The F-term potential is therefore directly proportional to

$$V_{F\text{-term}} \sim (m_{1/2}M)^2\,. \tag{126}$$

Although this has been computed in a specific model, we want to emphasize that this model is a good representative of F-term breaking models in general. Even if other theories might involve a multitude of fields, couplings and an intricate brane-dynamical background, the SUSY-breaking effects have to be mediated to the SM-sector by some mechanism and raise the gaugino mass to sufficiently high scales. Thus, we believe that the scaling of the F-term potential (126) with mediation scale and gaugino mass is quite generic. Furthermore, since experimental bounds on both are known and discussed in section 4.2.1, the SUSY breaking F-term potential is bounded by phenomenology.

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Acknowledgments

I want to express my gratitude to Professor Arthur Hebecker under whose supervision I gained access to the field of string phenomenology and in whose collaboration the material of this thesis was developed. Always open to questions, Arthur would encourage me to broaden my horizon and build up a conception of modern physics.
I also want to thank Manuel Wittner, who was working with me on our paper and thus contributed greatly to this thesis, as well. It was in discussions with Arthur and Manuel that I got a clear picture of the topic and that progress was made. I am grateful for this pleasant and harmonious collaboration.
Receiving answers to my questions and enjoying frequent discussions, lunch break conversations and presentations in the group seminar, I benefited a lot from interaction with my research group. I want to thank Philipp Henkenjohann, Daniel Junghans, Sascha Leonhardt, Christian Reichelt, Andreas Schachner, Pablo Soler and Xu Fengjun for their help and their contributions to this atmosphere of shared curiosity.
During our analysis of his original works, I had the chance to speak to Michele Cicoli, whom I want to thank for providing input and expressing his opinion on the subject.
Finally, I want to thank my friend and fellow student Jannik Fehre for proofreading.

Declaration of Authorship

I hereby certify that this thesis has been composed by me, is based on my own work and that all used sources have been acknowledged as references.

Heidelberg, 28th of October 2019