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Testing and Extending Swampland Conjectures

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Testing and Extending Swampland Conjectures

In this thesis we study landscape constructions to challenge existing swampland conjectures and extend established ones in order to expand our understanding of quantum gravity and its possible consequences for phenomenology.

We construct a new class of axions in Klebanov-Strassler throats including a four-dimensional supergravity formulation. We show that the axion and associated instantons violate the axionic weak gravity conjecture in its simplest form and that the instantons consistent with it populate very coarse charge sub-lattices.

Based on the winding scenario, we show that light axionic directions of field space can posses tunable, positive minima in their scalar potential. This gives a controlled supersymmetry-breaking and uplifting mechanism which may be applied in known AdS vacua. We discuss stability of these solutions.

We propose a bound on the quality of global symmetries that are derived from gauge symmetries. This involves a Stückelberg axion as well as instantons coupling to it. Integrating over instanton insertions leads to global-symmetry-violating operators. Using the axionic weak gravity conjecture the coefficient of the operator may be bounded from below.

In the spirit of the weak gravity conjecture, we constrain attractive forces in the absence of gauge charges. We claim that the minimal size of a bound state is governed by gravitational forces alone and arrive at a bound state conjecture: In an EFT, there is a universal lower bound on the typical radius of any bound state.

Tests und Weiterentwicklungen von Swampland Vermutungen

In dieser Arbeit überprüfen wir mittels *Landscape* Konstruktionen *Swampland* Vermutungen und erweitern bestehende Vermutungen um unser Verständnis von Quantengravitation und deren phänomenologischen Auswirkungen zu verbessern.

Wir konstruieren eine neue Klasse von Axionen in *Klebanov-Strassler Throats* zusammen mit ihrer Beschreibung in Supergravitation. In seiner einfachsten Form wird das *Axionic Weak Gravity Conjecture* verletzt. Die Instantone, die es erfüllen, leben auf einem sehr groben Ladungsuntergitter.

Basierend auf dem *Winding* Szenario zeigen wir, dass für leichte axionische Richtungen des Feldraums abstimmbare, positive Potentialminima existieren. Dies liefert einen kontrollierten Mechanismus von Supersymmetrie-Brechung und *Uplifting* für bekannte AdS Vakua. Wir diskutieren die Stabilität solcher Lösungen.

Wir schlagen eine Schranke für die Qualität der von Eichsymmetrien abstammenden globalen Symmetrien vor. Dies erfordert ein Stückelberg Axion sowie daran koppelnde Instantone. Durch Ausintegrieren dieser Instantone erhält man diese Symmetrie verletzende Operatoren. Durch die *Axionic Weak Gravity Conjecture* erhält man eine untere Schranke für die Koeffizienten der Operatoren.

Im Sinne des *Weak Gravity Conjectures* schränken wir attraktive Kräfte in Abwesenheit von Eichladungen ein. Wir fordern, dass die minimale Größe eines gebundenen Zustands durch Gravitation bestimmt ist und mutmaßen: In EFTs besitzt jeder gebundene Zustand eine allgemeine untere Schranke an seinen Radius.

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Contents

Chapter 1

Introduction

1.1 String Phenomenology

1.1.1 Beyond the Standard Model of Particle Physics and Cosmology

With the standard model of particle physics (SM) and general relativity (GR) we have two theories that explain – to unprecedented precision – most of the phenomena associated to particle interactions via the strong and electroweak interaction and gravity. In recent years, these theories were confirmed once again by the measurement of the Higgs boson [1], completing the standard model, and the measurement of gravitational waves produced by the collapse of binary systems [2] as predicted by GR. The latter gives justified hope that we will soon be able to complete the cosmological standard model ΛCDM (cosmological constant Λ + Cold Dark Matter) with a more concrete scenario of inflation [3]. Depending on the concrete realization, this already touches the one big outstanding problem in modern theoretical physics: At some energy scale, both these theories become relevant for the description of nature and therefore the two theories have to be combined into a single theory of quantum gravity (QG), which becomes, for example, relevant in the description of black holes. Since GR however is perturbatively non-renormalizable as a quantum field theory (QFT), the framework for QG may have to be of a different nature.[1] To this day, the best understood candidate for a theory of QG is (super)string theory, in which one goes away from the idea of fundamental objects being point-like particles and rather considers strings, extended one-dimensional objects, as the building blocks of nature.

[1] Whether GR is non-perturbatively renormalizable is an open problem. There are attempts to consistently combine SM and GR within the framework of QFT.

There have been many successes in the study of string theory. One of the most outstanding outcomes is that GR, or rather its supersymmetric (SUSY) completion supergravity (SUGRA), is part of the low-energy effective field theory (EFT). In this framework, including some purely stringy effects, a *Landscape of String Vacua* has been found. We give a short introduction to string vacua in Sect. 1.1.2 while postponing details on the terminology and technology required to Sect. 2.1. The sheer amount of vacua seems to suggest that anything goes. That is, within string theory any desired universe can be constructed. In Sect. 1.2 and in more detail in Sect. 2.2 we will see that this is not the case. There are certain criteria that a low-energy theory has to fulfill in order to fit into the framework of string theory. These so-called *Swampland Conjectures* will be the main focus of this thesis. We will highlight in particular the *Weak Gravity Conjecture* (WGC) in a number of formulations, as it is on the one hand one of the most established criteria and on the other hand one of the most useful ones when talking about hands-on phenomenology. As briefly explained in Sect. 1.1.3, (natural) inflation may be such a phenomenological example that is constrained by the WGC. In Sect. 1.3, we will give an overview over the research topics in this thesis and how they relate to the general ideas just discussed.

1.1.2 String Vacua

While the advantages of string theory include the prediction of the number of spacetime dimensions, one immediately also comes to find that this prediction is ten dimensions. To make contact with the four-dimensional world we see around us, one has to compactify six of these extra-dimensions, the idea being to make them so small that we cannot see them in any experiment carried out so far.

Upon compactifying one is still faced with the problem of choosing by hand the compact manifold that reduces the number of large dimensions from ten to four. A particularly popular class of manifolds are Calabi-Yau manifolds (CY). Compactifying on CYs leads to some desirable properties, such as preserving supersymmetry in lower dimensions and solving the vacuum Einstein equations. Because they possess a lot of structure, they are in some ways also the most easy to deal with. They are described by a handful of integer numbers that are relevant for the low-energy theory. Two of these numbers count the possible deformations of size and shape of a given CY that do not spoil any of its properties. As any dimensionless parameter in string theory, these parameters are dynamical fields. In this specific case they are so-called moduli, fields that do not have any potential: Changing the value of these fields doesn't spoil a vacuum solution. Together with other moduli coming from the string spectrum in ten dimensions, Ramond and Neveu-Schwarz fields, there is an enormous number of fields in the low-energy spectrum. Many of these fields are actually stabilized by a potential due to internal fluxes of the higher-dimensional fields. While it is expected that most, if not all, moduli are actually stabilized by a potential once all stringy effects are taken into account, there is still a very large number of possible vacua and EFTs [4]. This set of vacua is known as the (string) landscape.

Interestingly, these EFTs generically contain axion-like particles [2], or axions for short, which have a non-perturbatively generated, small potential [5]. It remains a challenge to distinguish all vacua including light axions and see how we might test whether one of them describes our universe with the axions possibly being relevant as dark matter, the QCD axion or the inflaton.

1.1.3 An Example: Cosmic Inflation

One test of string phenomenological models often considered because of its simplicity as an EFT is cosmic inflation. Inflation is the most popular explanation for the open problems of the ΛCDM model. It is a phase of almost exponential expansion of the early universe. In this phase quantum fluctuations such as gravitational waves grow in wave-length. We should in principle still see the imprint of such primordial gravitational waves from quantum fluctuations in the polarization of the cosmic microwave background, see Fig. 1.1. The strength of gravitational waves from inflation is usually given as the tensor-to-scalar ratio r, which is so far only bounded from above to $r < 0.10$ at 95% CL by the most recent results of the Planck Collaboration [6].

Simply considering the so-called Lyth bound [8]

$$\Delta\phi \lesssim \sqrt{r/0.01}M_{\mathrm{P}}\,, \tag{1.1}$$

where $\Delta\phi$ is the distance the inflaton field traverses in field space, we see that future experiments will be able to distinguish whether so-called large- or small-field models, in which the field traverses super- or sub-Planckian distances in field space respectively, are favored. Therefore, when constructing inflationary models in string theory, models in which the inflaton field range is larger than M_{P} are particularly interesting.

To make this more concrete, we take a closer look at natural inflation [9]. When trying to find candidate fields for inflation one is led to consider axions,

[2] We always consider axions to have discrete shift symmetries since non-perturbative effects typically break any continuous shift symmetry to a discrete subgroup.

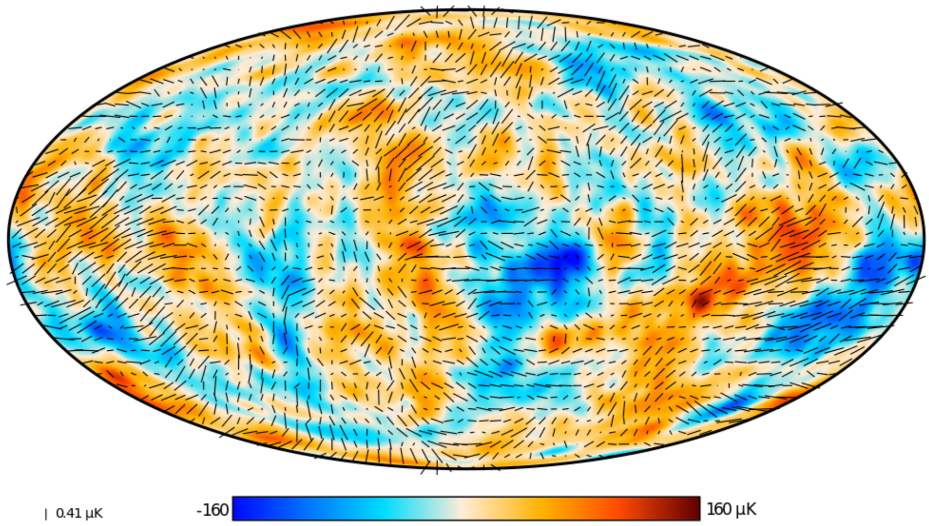

Figure 1.1: The cosmic microwave background as measured by the Planck Collaboration. Shown are scalar temperature deviations as well as polarization of the spectrum. Certain polarization modes are an indicator for primordial gravitational waves. Taken from [7].

since their shift symmetry allows for small masses that are not subject to large radiative corrections. The scalar potential is usually parameterized as

$$V(\phi) = V_0 \left[1 - \cos\left(\frac{\phi}{f}\right)\right] , \tag{1.2}$$

where f is the axion decay constant. In order to find a consistent inflationary phase, one requires $f > M_{\mathrm{P}}$ [9], allowing for large r at large field excursions $\Delta\phi \sim M_{\mathrm{P}}$. For more details and a plethora of different string motivated models see [10].

While string theory gives rise to many axions upon compactifying, one finds that these have periodicities that are generically sub-Planckian when staying in the regime of perturbative control. Such a statement can also be derived from a version of the WGC as discussed in Sects. 1.2.2 and 2.2.3. So one of the questions that motivate us to further test these kinds of swampland conjectures is: Could we even *in principle* construct a model of natural inflation in string theory or is there an intrinsic property of QG that forbids such a construction?

1.2 The Swampland Paradigm

1.2.1 Landscape vs. Swampland

While constructions in the string landscape give rise to many vacua among which we may find our universe, the whole procedure is very complicated. Even though for any fixed CY, there is a finite number of flux vacua, there is an unknown number of CYs to choose from. It seems an impossible task to go through all models even in this possibly small corner of quantum gravity. For this reason the swampland program was started [11, 12]. The idea is, rather than considering top-down constructions of models one looks for criteria that all of these models fulfill: From either general quantum gravity arguments or broad experience of known constructions one phrases criteria that any low-energy theory that is derivable from a theory of quantum gravity or string theory is expected to fulfill. Then one may simply write down a desired model and check whether it is consistent with the catalog of properties. If it is not, one says that it belongs to the swampland: Consistent looking EFTs, that cannot be completed in the ultraviolet (UV) to a theory of quantum gravity, see Fig. 1.2.

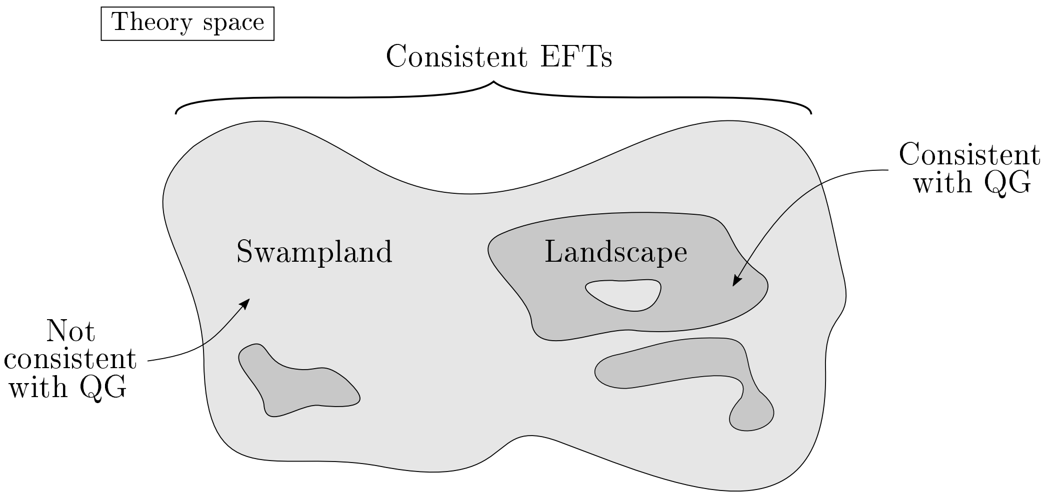

Figure 1.2: Consistent-looking EFTs in theory space. Within this set, there are islands of theories that are UV-completable to theories of QG, the landscape. The complement is known as the swampland.

Since we do not know all general properties of QG, the criterea we are able to formulate are conjectures. They are to be tested by either looking for constructions in string theory that go against them, or by experiments in the real world: We know that the GR+SM EFT is necessarily consistent with any QG describing our universe. While the first approach seems straight forward, it is hard to argue for the absolute validity of all assumptions used in a given landscape model. The latter approach may be used in, e.g., models of inflation: If future measurements of the tensor-to-scalar ratio seem to favor the standard scenario of large-field inflation, we may question swampland conjectures forbidding this or look for ways of avoiding the specific prerequisites that led to the formulation of a given conjecture and by this broaden our understanding of QG.

1.2.2 Gravity as the Weakest Force

In order to understand how to work with and expand our knowledge of the swampland and therefore the landscape, we introduce some conjectures that will reappear throughout this thesis. A quantitative motivation for these conjectures will be given in Sect. 2.2. Reviews can be found in [13, 14].

We start by stating the *No Global Symmetries Conjecture*, which has been introduced long before the idea of a swampland [15, 16]: An EFT that can be consistently embedded in a theory of QG cannot possess exact global symmetries. The standard argument for general theories of quantum gravity independent of any specific setup uses black hole evaporation, in which a global charge hidden behind an event horizon simply disappears or leads to an infinite number of remnant states.

A possible way to quantify this is the *Weak Gravity Conjecture* [17]. In its simplest form it states that, given a gauge theory with coupling g, there is always a particle of mass m and charge q subject to

$$m/M_{\mathrm{P}} \leq qg\,. \tag{1.3}$$

We see that there is an obstruction to taking the limit $g \to 0$ which would give rise to a global symmetry. The name of the conjecture derives from the fact that the l.h. side gives (the square-root of) the strength of the long-range gravitational force between two particles of the same species, while the r.h. side measures (the square-root of) the long-range Coulomb force between them. The inequality reads 'gravity is weaker than gauge interaction'. Originally, also this bound was motivated by black hole physics by proposing that no non-supersymmetric black hole should be stable. Applying the conjecture to magnetic monopoles one finds an inequality constraining the cut-off Λ of the EFT (which is introduced to regularize the energy stored in the field of the monopole),

$$\Lambda \lesssim gM_{\mathrm{P}}\,. \tag{1.4}$$

This is known as the *Magnetic Weak Gravity Conjecture*.

These ideas have been generalized in a number of ways. The *Weak Gravity Conjecture with Scalar Fields* [18] adds an additional attractive force mediated by massless scalars with dimensionful coupling parametrized as $\mu\, m$ to the particles. One may check that the corresponding long distance force has dimensionless coupling μ . The claim is then that the sum of attractive forces is smaller than the repulsive force,

$$(m/M_{\rm P})^2 + \mu^2 \leq q^2 g^2 \,. \tag{1.5}$$

The *Axionic Weak Gravity Conjecture* [17] is, as the name suggests, an inequality constraining axionic interaction. The gauge coupling is given by (the inverse of) the axion decay constant $1/f$, while the mass of a particle is replaced by the action S_I of the objects charged under axions – instantons. Rearranging terms one arrives at

$$S_I \leq q M_{\rm P}/f \,. \tag{1.6}$$

With this, we now see the use of such conjectures going back to our motivation of models of inflation in Sect. 1.1.3: In controlled stringy scenarios, that is $S_I \gg 1$, the axionic WGC demands $f \ll M_{\rm P}$. So, if this conjecture turns out to be true, it seems like natural inflation would be ruled out. We may again constrain the magnetically charged objects, strings, to find the *Magnetic Version of the Axionic Weak Gravity Conjecture* [19]

$$\Lambda \lesssim \sqrt{f M_{\rm P}} \,. \tag{1.7}$$

1.3 Contribution of this Thesis

In this thesis, we explore both routes – the landscape and the swampland perspective – to the physics of string vacua. In this we are motivated by improving our understanding of the imprint of quantum gravity or string theory on low-energy theories and their phenomenology. While axion-like particles, which we call axions for short, and supersymmetry (breaking) play a significant role in of the following, we are not interested in building complete phenomenological models. Rather, we attempt to draw and sharpen the boundary between the landscape and the swampland.

Ultralight Axions in Klebanov-Strassler Throats

We start by considering landscape constructions of ultralight axions in Ch. 3. The chapter is based on [A]. We are motivated on the one hand by the relevance of axions for beyond-standard-model phenomenology [20,21] and string phenomenology in particular [22–24, 5, 25]. On the other hand axions play an important role in the swampland debate [26–39]. The model is inspired by [40], where it was proposed that one may arrive at a model of axion monodromy [41, 42]. As such models are also interesting to study for both reasons just discussed [43–48], a further motivation is to clarify whether a monodromy actually arises.

We consider so-called Klebanov-Strassler throats [49], warped regions of CYs which arise due to back-reaction of large flux numbers stabilizing the geometry. Such regions are thought to be generic features of string orientifold compactifications [50, 4, 51]. More specifically, we require the setup of a double (multi) throat where two (multiple) throats are topologically connected by sharing a common three-cycle, see Fig. 1.3. The genericity of this more specific setup has recently been analyzed in [52].

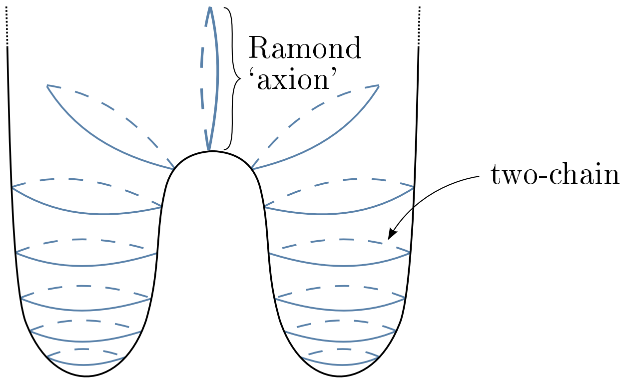

Figure 1.3: A lower-dimensional sketch of the double throat: Two throats share a three-cycle. When cutting off the tips, a two-cycle emerges. On this, we define the Ramond axion.

Cutting off the tips of both throat, the topology possesses a two-cycle which can support the Ramond two-form of supergravity. This gives rise to an axion in four dimensions. However, considering the full double throat, the strongly warped infrared (IR) regions, the two-cycle disappears as it becomes a boundary of some three-chain describing the tips of the throat. This necessarily gives an obstruction to the axion's shift symmetry as an excitation of the axion induces energy densities of the corresponding Ramond-field strength at the tips of the throats. While this was argued to lead to a model of axion monodromy in [40], it turns out that by back-reaction on the geometry in the explicit setting, a larger periodicity is restored: The deformation of the throat due to the would-be axion happens to coincide with a discrete symmetry of the geometry – leading also to a restored discrete shift symmetry of the axion.

We work out the EFT describing this mode and find that its mass is unexpectedly light compared to its decay constant. The low-energy description is extended by a proposal for the super- and Kähler potential of the mode including its saxion partner. We check that our findings are consistent with the conjectured existence of the dual Klebanov-Strassler gauge theory [49].

Finally, we check consistency with the axionic WGC [17]. We draw two conclusions: First, we find that multi throat systems in CYs are surprisingly weakly stabilized as the saxion mode has a low mass and can therefore transfer energy density from one throat to another via only a tiny energy barrier. Secondly, we find that the simplest versions of the axionic WGC may not hold in general. When taking the (super)potential proposed seriously and attributing the terms to instanton processes [3], the axion decay constant can parametrically violate (1.6). One may instead consider a lattice version of the inequality [53,54], that is one requires that only a sub-lattice of the (fully filled) charge lattice fulfills the inequality. Then, we find that the sub-lattice subject to (1.6) is unusually coarse compared to other known examples.

Winding Uplifts – Parametrically Small SUSY Breaking

Ch. 4 is based on [D]. We use an axion potential consisting of multiple competing terms of different periodicities to uplift both to de Sitter (dS) vacua from known anti-de Sitter (AdS) vacua as well as to non-supersymmetric AdS vacua starting from known supersymmetric lower-lying vacua. Such a model may be used as a counterexample to the dS conjecture [55] and SUSY AdS Conjecture [56, 57]. The former has sparked a recent debate on the stability of existing solutions [58–70].

Concretely, we consider the real parts of complex structure moduli in the large field limit. These fields are axions since a discrete shift symmetry in the large-complex-structure point of the moduli space is manifest [71]. By choice of fluxes some direction in the axion field space remains unstabilized by the leading flux super- and Kähler potential. Once sub-leading corrections are taken into account, also this direction is stabilized. By the shift symmetry these corrections are periodic. This is known as the winding scenario [32].

By tuning the imaginary parts of the corresponding complex structure moduli appropriately, multiple sub-leading terms of different periodicities in the axion of interest can be relevant at the same time. Choice of flux ratios results in a potential given by a large-periodicity, dominant oscillation modulated by smaller-periodicity, subdominant terms. Such a potential can possess positive minima which are tunable independently of the height of the

[3] This is motivated by the fact that the superpotential of the dual gauge theory is generated by genuine instantons.

potential barrier protecting a vacuum solution for the axion from decaying into a lower-lying minimum. With this, we have found a mechanism of controlled SUSY breaking in string compactifications. We apply the mechanism in uplifting scenarios based on the non-supersymmetric large volume scenario (LVS) [72,73] as well as supersymmetric Kachru-Kallosh-Linde-Trivedi (KKLT) [58] and DeWolfe-Giryavets-Kachru-Taylor (DGKT) [74] vacua. We may uplift LVS vacua to de Sitter and break supersymmetry in the DGKT vacuum. For KKLT vacua we point out the problems that interfere with a straightforward application of the mechanism.

A Swampland Global Symmetry Conjecture

We change our perspective in Chs. 5 and 6. There, we explore possible extensions of commonly accepted swampland conjectures to study their implications on low-energy theories.

In Ch. 5 we turn to global symmetries. The chapter is based on [C]. While it seems generally accepted that exact global symmetries cannot arise in theories of QG [15, 75–81, 16, 82], it is an outstanding problem to quantify how strong of a violation of would-be global symmetries is expected at any given energy scale. In the context of symmetry-breaking by wormholes this has been discussed for a long time [76–78], there is however still a lack of more general arguments. Recently, [83] has given a proposal for a bound on the violation based on black hole physics in a thermal bath.

We attempt to quantify the strength of global symmetry violation for $U(1)$ symmetries by employing the WGC for axions in the following way. A global symmetry can arise as a broken gauge symmetry by a Stückelberg mechanism which gives a mass to the gauge boson. Such a mass term is introduced in a gauge-invariant way by coupling the theory to an axion which is charged under the gauge symmetry. Now however, the global symmetry is intimately related to the axion theory which comes with charged instantons. This manifests itself in the fact that worldlines of charged particles have to end on instantons by gauge-invariance. This very intuitively describes how global charges necessarily seize to exist without contradicting the underlying gauge symmetry.

As usual, instantons as intrinsically quantum objects are to be integrated out in the path integral (by summing over all possible instanton insertions) when computing the effective action of a low-energy theory valid below some cut-off Λ. By this, effective operators for the fields coupling to instantons are generated which, once the axion is gauge-fixed, violate the would-be global symmetry. These operators are suppressed by the instanton action S_I, which is subject to the WGC for axions. To be precise, the coefficients of the operators may be estimated as $\exp(-S_I)$. Applying the axionic WGCs discussed above, both the electric and magnetic [17, 19] version, one may estimate the coefficient to be exponentially suppressed as $\exp(-M_\mathrm{P}^2/\Lambda^2)$.

While such a bound has been discussed before [83], we give a derivation for a large class of global symmetries given the WGC as a basis. We give the prime example of a fermion theory with a global $U(1)$ symmetry. By coupling to the axion, gauge instantons induce a global-symmetry-violating 't Hooft vertex. Taking a QG perspective, we discuss how the bound is satisfied in the standard string theory example of brane instantons coupling to global symmetries coming from a brane gauge theory. Finally, we show how wormholes lead to a violation of global symmetries in the same way by constructing an appropriate wormhole solution with charged particles passing the wormhole throat. We discuss possible extensions of our derivation to a conjecture covering all global symmetries including global symmetries from tuning operators and possibly accidental ones.

A Conjecture on the Minimal Size of Bound States

Finally, Ch. 6 is based on [B]. We shift our focus away from axions. Instead, we are motivated by the question how the idea of the WGC may be extended to uncharged particles. We explain in which ways previous attempts to answer this, see in particular [18, 84], are unsatisfactory. The original WGC may be understood as forbidding an infinite set of bound states [17]. It does so by stating that the long-range repulsive force of gauge interaction is always stronger than the long-range attractive force of gravitational interaction. Therefore, two particles of mass m and charge q will have no net attraction and will not form a bound state. This idea has been extended to include scalar interaction in the WGC with scalar fields. Just like gravitational interaction, the interaction mediated by a scalar field is attractive. Therefore the sum of attractive forces, gravitational and scalar, is conjectured to be subdominant to the repulsive gauge interaction [18] (see also [85–88, 84, 89–92]).

If the WGC is supposed to be understood as the statement of attractive forces being constrained to not allow for bound states, the question that now arises is: How can it be phrased for particles that are not charged at all under a gauge boson? We claim that rather than forbidding bound states altogether the proper way forward is to constrain them. Specifically, we propose that the minimal typical radius of any stable bound state is bounded from below by the corresponding purely gravitationally bound state. That is, independent of the kind and strength of attractive interaction, the constituents of a bound state cannot be bound arbitrarily tightly. Interestingly, taking bosonic particles of mass m and considering gravity as the only interaction, the minimal radius a bound state will have is, parametrically, $R \sim 1/m$. This happens at some critical mass or particle number of the bound state where it collapses to a black hole. We see that this is actually independent of the Planck scale and we are able to decouple gravity by taking $M_{\rm P} \to \infty$ to end up with a interesting non-gravitational conjecture: Any stable bound state in an EFT with heaviest stable particle m has a typical radius of $R \gtrsim 1/m$. We test this conjecture in a number of non-trivial examples and see that whenever this threshold would be crossed an instability arises.

We conclude in Ch. 7 with a summary of our findings and an outlook.

This thesis is based on the publications [A-D]. I wrote most of these papers with the notable exceptions of Sects. 3 and 4 of [A] written by Jakob Moritz and Sect. 3 of [C] written by Tristan Daus. The ideas and conceptual discussions that can be found in these papers were a joint effort of all co-authors including myself.

Chapter 2

The Landscape and the Swampland

This chapter gives a summary of topics and quantities that will reappear throughout this thesis. It is not intended as a pedagogical introduction but rather as a reminder for the reader who has come across these topics before. We start with top-down constructions of low-energy effective theories from string theory in Sect. 2.1. For a general introduction see, e.g., [93, 94]. In Sect. 2.2, we give a short introduction to the swampland paradigm with focus on the weak gravity conjecture. A detailed review can be found in [14].

2.1 The Landscape

2.1.1 Compactification

We will start with the basic terminology of compactifications. A short review with a string theory motivation can be found in [93]. We will be able to see all basic features in a 5d to 4d compactification, where we assume the spacetime to be flat four-dimensional Minkowski space times a compact circle of radius R, $\mathbb{R}^{1,3} \times S^1_R$. We will split the coordinates x^M, $M = 0, \ldots, 4$, into *external* coordinates x^μ, $\mu = 0, \ldots, 3$, and *internal* coordinate $y = x^4$.

Now consider a complex scalar field $\phi(x^\mu, y)$. For consistency with the spacetime symmetry $y \simeq y + 2\pi R$ the scalar field has to have the same periodicity in its y-dependence. We can decompose the field into harmonics

$$\phi(x^\mu, y) = \sum_{n\in\mathbb{Z}} \phi_n(x^\mu) e^{i\frac{ny}{R}} . \tag{2.1}$$

Inserting this ansatz into the action of a free scalar field gives

$$\begin{aligned} S &= \int_{\mathbb{R}^{1,3}\times S^1_R} \mathrm{d}^4x\mathrm{d}y \left[-\frac{1}{2}\partial_M\phi(x,y)\partial^M\phi^*(x,y)\right] \\ &= (2\pi R)\int_{\mathbb{R}^{1,3}} \mathrm{d}^4x \sum_{n=-\infty}^{\infty}\left[-\frac{1}{2}\partial_\mu\phi_n(x)\partial^\mu\phi_n^*(x) - \frac{1}{2}\frac{n^2}{R^2}\phi_n(x)\phi_n^*(x)\right] . \end{aligned} \tag{2.2}$$

The four-dimensional theory contains a so-called Kaluza-Klein (KK) tower of states of masses $m_n = n/R$. The low-energy theory below the *KK scale* $m_{\mathrm{KK}} \equiv m_1 = 1/R$ contains only the massless field ϕ_0.

We can repeat this exercise for a vector field $A_M(x,y)$. An analysis of the kinetic term tells us that there is a tower of states with one massless field $A_{0,M}$ in four dimensions. We may split the vector into

$$A_{0,M}(x) = \begin{pmatrix} A_\mu(x) \\ a(x) \end{pmatrix} . \tag{2.3}$$

In the language of forms, $A = A_M\mathrm{d}x^M$, we may equally define $a(x) = \frac{1}{2\pi R}\int_{S^1} A$. The action is

$$S = \int \mathrm{d}^4x\mathrm{d}y\left[-\frac{1}{4}F_{MN}F^{MN}\right] \supset \int \mathrm{d}^4x\left[-\frac{1}{2}f_a^2\partial_\mu a\partial^\mu a\right] , \tag{2.4}$$

where f_a is the *axion decay constant*, $f_a^2 = 2\pi R$. From the gauge field's gauge symmetry, the pseudo-scalar field $a(x)$ inherits a shift symmetry, $a \to a + \mathrm{const}$. Fields with such a symmetry are called *axions*.

Finally, we want to consider the graviton $G_{MN}(x,y)$ and the 5d Einstein-Hilbert action. We will find a KK tower of symmetric two-tensors. The massless mode $G_{0,MN}(x)$ splits into

$$G_{0,MN}(x) = e^{\sigma(x)/3}\begin{pmatrix} g_{\mu\nu}(x) + e^{-\sigma(x)}A_\mu(x)A_\nu(x) & e^{-\sigma(x)}A_v(x) \\ e^{-\sigma(x)}A_\mu(x) & e^{-\sigma(x)} \end{pmatrix} . \tag{2.5}$$

Under this decomposition the action reads

$$\begin{aligned} S &= \frac{M_5^3}{2}\int \mathrm{d}^4x\mathrm{d}y\sqrt{-G}\,R[G] \\ &\supset (2\pi R)\frac{M_5^3}{2}\int \mathrm{d}^4x\sqrt{-g}\,\left(R[g] - \frac{1}{4e^{\sigma(x)}}F_{\mu\nu}F^{\mu\nu} - \frac{1}{6}\partial_\mu\sigma\partial^\mu\sigma\right) . \end{aligned} \tag{2.6}$$

The field stength $F_{\mu\nu} = \partial_{[\mu}A_{\nu]}$ defines a $U(1)$ gauge theory, which is inherited from the circle's local reparametrization invariance $y \to y + \lambda(x)$. The scalar field σ describes deformations of the internal space

$$\mathrm{Vol}(S^1) = \int_0^{2\pi R}\mathrm{d}y\sqrt{G_{44}} = 2\pi Re^{-\sigma(x)/3} . \tag{2.7}$$

Such deformation parameters that do not change the topology of the internal manifold are called *geometric moduli*. They are dynamical fields in the presence of gravity. Note, that one more generally uses the term *modulus* for scalar fields without a potential.

2.1.2 Type IIB Orientifold Compactifications

Type IIB Supergravity

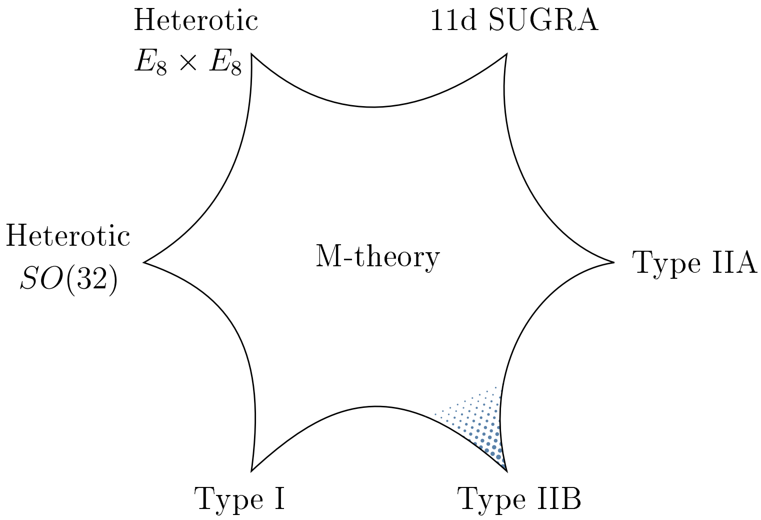

Figure 2.1: The M-theory star with its perturbative corners. There are known dualities between neighboring corners, such as T-duality between Type IIB and Type IIA string theory. We consider the shaded type IIB SUGRA corner.

In this section, we will take a closer look at the construction of low-energy effective theories. We do not possess the tools to analyze string theory, or rather its completion into M-theory, in full detail. Rather, we can only describe perturbatively different limits of string theory, which are illustrated in Fig. 2.1, and add some non-perturbative effects. We will deal with one of the most studied corners of string theory: Type IIB SUGRA on CY manifolds. In what follows one should keep in mind that we only cover a small part of the possible landscape of low-energy theories. We focus on this corner as it is well-explored and thought to be fruitful for finding realistic models of nature. A full account of string theory including what follows can be found in the literature, see, e.g., [94].

We will consider only the bosonic field content of the closed string. It is made up by the Neveu-Schwarz-Neveu-Schwarz (NSNS) sector consisting of a symmetric 2-tensor, the Kalb-Ramond 2-form and the dilaton

$$G_{MN}\,, \quad B_2\,, \quad \Phi\,, \tag{2.8}$$

as well as the Ramond-Ramond (RR) sector consisting of the p-forms

$$C_0\,, \quad C_2\,, \quad C_4\,. \tag{2.9}$$

The p-forms are gauge potentials with gauge invariant field strengths

$$\begin{aligned}
H_3 &= \mathrm{d}B_2\,,\\
F_1 &= \mathrm{d}C_0\,,\\
\widetilde{F}_3 &= \mathrm{d}C_2 - C_0\mathrm{d}B_2 = F_3 - C_0 H_3\,,\\
\widetilde{F}_5 &= \mathrm{d}C_4 - \frac{1}{2}C_2\wedge\mathrm{d}B_2 + \frac{1}{2}B_2\wedge\mathrm{d}C_2 = F_5 - \frac{1}{2}C_2\wedge H_3 + \frac{1}{2}B_2\wedge F_3\,.
\end{aligned} \tag{2.10}$$

The full effective field theory describing the interactions of these fields has infinitely many terms. These, however, can be organized in inverse powers of the string scale $M_s = 1/\sqrt{\alpha'}$. We will, for now, only consider what is known as Type IIB SUGRA, the leading terms of Type IIB string theory in the α'-expansion:

$$\begin{aligned}
S_{\mathrm{IIB}} \;&= \frac{1}{2\kappa_{10}^2}\int \mathrm{d}^{10}x\sqrt{-G}\left[R - \frac{|\partial\tau|^2}{2(\mathrm{Im}\,\tau)^2} - \frac{|G_3|^2}{2\mathrm{Im}\,\tau} - \frac{\left|\widetilde{F}_5\right|^2}{4}\right]\\
&+ \frac{1}{8i\kappa_{10}^2}\int \frac{C_4\wedge G_3\wedge \widetilde{G}_3}{\mathrm{Im}\,\tau} \quad + \quad \sum S_{\mathrm{loc}}\,,
\end{aligned} \tag{2.11}$$

where $\kappa_{10}^2 = \frac{(4\pi^2\alpha')^4}{4\pi}$ and the squares of p-forms are $|F_p|^2 = \frac{1}{p!}F_{M_1\ldots M_P}F^{M_1\ldots M_p}$. In this, only the combinations

$$\tau = C_0 + ie^{-\Phi}\,, \quad G_3 = \widetilde{F}_3 - ie^{-\Phi}H_3 = F_3 - \tau H_3\,, \tag{2.12}$$

appear. The complex scalar τ is the *axio-dilaton*. The vacuum expectation value (VEV) of the dilaton $\langle\Phi\rangle$ defines the *string coupling* $g_s = e^{-\langle\Phi\rangle}$. It appears as the expansion parameter in the loop-expansion of string diagrams. When talking about vacua, it is common to use the parameter g_s straight away, thinking about the dilaton already being stabilized to some value.

The localized contribution describes sources for p-form fields: *D-branes* and *O-planes*. A Dp-brane has an action

$$S_{\mathrm{loc}}^{(p)} = -T_p\int_{\mathcal{W}}\mathrm{d}^{p+1}\xi\sqrt{-g}\,e^{-\Phi} + \mu_p\int_{\mathcal{W}} C_{p+1}\,. \tag{2.13}$$

Here, $T_p = 2\pi(4\pi^2\alpha')^{-\frac{p+1}{2}}$ is the brane tension and g and ξ are induced metric and coordinates on the worldvolume $\mathcal{W}$. The coupling μ_p to the gauge potential C_{p+1} differs in sign for brane, $\mu_p = T_p$, and anti-brane, $\mu_p = -T_p$. Non-dynamical O-planes have a similar action, but, importantly, their tension is negative, $T_{\mathrm{O}p} = -\frac{1}{2}T_p$. From their action we can see that Dp-branes are a generalization of charged point-particles (0-branes). We will come back to the origin of O-planes below.

Calabi-Yau Orientifold Compactifications

From here on, one may explicitely compactify on a six-dimensional manifold to make contact with the four-dimensional world. We choose to do this on a compact, complex Kähler manifold of $SU(3)$-holonomy - a *Calabi-Yau 3-fold*. These are Ricci-flat manifolds, so a compactification to Minkowski space will automatically fulfill the vacuum Einstein equations. A first step in explicitely compactifying is choosing a basis of p-forms in which we decompose our fields. Here, a CY X comes in handy: They are characterized by the non-zero *Hodge numbers*

$$\begin{gathered} h^{i,j} = \dim_{\mathbb{R}} H^{i,j}(X)\,, \\ h^{0,0} = h^{3,3} = 1\,, \quad h^{3,0} = h^{0,3} = 1\,, \\ h^{1,1} = h^{2,2}\,, \quad h^{1,2} = h^{2,1}\,. \end{gathered} \tag{2.14}$$

We see, there is a unique volume form and its Hodge dual as well as a unique *holomorphic* 3*-form* Ω and its complex conjugate. Hodge duality and complex conjugation also relate the other Hodge numbers, leaving only two independent Hodge numbers usually chosen to be $h^{1,1}$ and $h^{1,2}$.

If one were to compactify on a CY to four dimensions, one would find, that of the 32 supercharges, corresponding to $\mathcal{N}=2$ SUSY in ten dimensions, 8 supercharges would remain unbroken. That is, we find a $\mathcal{N}=2$ SUGRA theory in four dimensions. To get closer to realistic models, with, e.g., chiral fermions, and to allow for fluxes, see below, we will reduce this to only 4 remaining supercharges and $\mathcal{N}=1$ SUGRA. For this we mod out with the orientifolding action $\Omega_{\mathrm{ws}}\sigma$, where Ω_{ws} is a parity operator on the string worldsheet and σ a target space involution that acts non-trivially on the compact part. This makes sense if the action of $\Omega_{\mathrm{ws}}\sigma$ is a symmetry of string theory on a given target space such as CY (orientifolds). The fixed point locus of the involution is called an orientifold plane, or O-plane for short. Note that, unlike D-branes, these O-planes are not dynamical, but rather they a property of spacetime. To end up with four-dimensional Minkowski space, we choose the orientifold action to act trivially on the external space.

Under this involution σ, p-cycles and p-forms can be even or odd. This allows us to split the Hodge numbers into

$$h^{1,1} = h^{1,1}_{+} + h^{1,1}_{-}\,, \quad h^{1,2} = h^{1,2}_{+} + h^{1,2}_{-}\,. \tag{2.15}$$

Counting Moduli

We highlight a further property of CY manifolds: Yau's theorem tells us that there is a unique Ricci-flat metric $g_{i\bar{\jmath}}$ for a CY with holomorphic 3-form Ω and Kähler 2-form J. Here, we split the indices using the complex structure into holomorphic i and anti-holomorphic $\bar{\jmath}$. However: There are deformations of this metric that can be shown to maintain all the properties of a CY including Ricci-flatness! The deformation parameters are moduli, just like the volume modulus of the circle in Sect. 2.1.1. Since there is no energy density coming from additional curvature when deforming it is clear that they do not have any potential. We can parameterize deformations as

$$g_{i\bar{\jmath}}\mathrm{d}z^i\mathrm{d}z^{\bar{\jmath}} \to g_{i\bar{\jmath}}\mathrm{d}z^i\mathrm{d}z^{\bar{\jmath}} + \delta g_{i\bar{\jmath}}\mathrm{d}z^i\mathrm{d}z^{\bar{\jmath}} + \delta g_{ij}\mathrm{d}z^i\mathrm{d}z^j + \text{h.c.}\,. \tag{2.16}$$

Due to Yau's theorem we know: These deformations must be accompanied by a change of either the Kähler 2-form J or the holomorphic 3-form Ω to maintain uniqueness. To see this, we introduce the harmonic $(1,1)$-forms ω_α, $\alpha = 1,\ldots,h^{1,1}$, that form a basis of $H^{1,1}$ and the harmonic $(1,2)$-forms χ_a, $a = 1,\ldots,h^{1,2}$, that form a basis of $H^{1,2}$. Then the Kähler form can be expanded as

$$J \equiv i g_{i\bar{\jmath}}\mathrm{d}z^i\mathrm{d}z^{\bar{\jmath}} = t^\alpha(x)\omega_\alpha\,. \tag{2.17}$$

A deformation $\delta g_{i\bar{\jmath}}$ is induced by a change of *Kähler moduli* $t^\alpha(x)$. A complex structure deformation δg_{ij} can be expanded using the holomorphic 3-form

$$\delta g_{ij} = \frac{i}{||\Omega||^2} z^a(x)(\chi_a)_{i\bar{k}\bar{l}}\Omega_{jkl}g^{k\bar{k}}g^{l\bar{l}}\,, \tag{2.18}$$

so such a derformation is induced by a change of *complex structure moduli* $z^a(x)$.

There are non-geometric moduli as well: We can have non-trivial configurations of the p-forms in the internal space. We may parameterize the internal components of the p-forms, focussing only on scalar fields, as

$$\begin{gathered} C_2 = c^\alpha(x)\omega_\alpha\,, \quad B_2 = b^\alpha(x)\omega_\alpha \\ C_4 = \tilde{c}^\alpha(x)\widetilde{\omega}_\alpha\,. \end{gathered} \tag{2.19}$$

Here, we also introduced a basis of harmonic 4-forms $\widetilde{\omega}_\alpha$, $\alpha = 1,\ldots,h^{1,1}$. The fields $c^\alpha(x)$, $b^\alpha(x)$ and $\tilde{c}^\alpha(x)$ are real and, just like the axion in Sect. 2.1.1, they inherit a shift symmetry - they are axions. We may combine them with the geometric moduli into

$$\begin{aligned} &h^{1,1} \text{ complexified Kähler moduli} \quad T^\alpha = t^\alpha + i\tilde{c}^\alpha\,, \\ &h^{1,1} \text{ two-form moduli} \quad G^\alpha = c^\alpha - \tau b^\alpha\,, \\ &h^{1,2} \text{ complex structure moduli} \quad z^a\,, \\ &1 \text{ axio-dilaton} \quad \tau\,. \end{aligned} \tag{2.20}$$

Finally, we consider CY orientifolds and keep only the part of the supergravity spectrum that is invariant under the orientifold action. We also have to take into account the action of the involution on the geometry: $\sigma(J) = J$ and $\sigma(\Omega) = -\Omega$ [1]. After carefully studying this action, one finds the only remaining moduli are

$$\begin{aligned} &h^{1,1}_+ \text{ complexified Kähler moduli} \quad T = t + i\tilde{c}\,, \\ &h^{1,1}_- \text{ two-form moduli} \quad G = c - \tau b\,, \\ &h^{1,2}_- \text{ complex structure moduli} \quad z\,, \\ &1 \text{ axio-dilaton} \quad \tau\,. \end{aligned} \tag{2.21}$$

While they will not be of relevance for this thesis, note that there are non-scalar fields as well. These come for example from expanding C_4 in a basis of 3-forms, giving rise to an external vector, or from purely external 2-form components B_2. All fields will be arranged in SUSY multiplets.

2.1.3 The Low-Energy Theory

$\mathcal{N} = 1$ 4d SUGRA

One can show by studying the orientifold action that the low-energy four-dimensional theory possesses $\mathcal{N} = 1$ supersymmetry. This tells us that the complex, scalar moduli (2.21) are arranged in chiral multiplets Φ^i. The form of the $\mathcal{N} = 1$ SUGRA Lagrangian is well known. The scalar components ϕ^i of the chiral multiplets Φ^i are subject to the Lagrangian of the general form

$$\mathcal{L} = -K_{i\bar{\jmath}}\,\partial\phi^i \cdot \partial\bar{\phi}^{\bar{\jmath}} - V_F(\phi^i, \bar{\phi}^{\bar{\imath}})\,, \tag{2.22}$$

where the *supergravity scalar potential* is given by

$$V_F = e^K \left[K^{i\bar{\jmath}} D_i W \overline{D_j W} - 3\,|W|^2\right]\,. \tag{2.23}$$

[1] One may also choose $\sigma(\Omega) = +\Omega$ leading to a different spectrum, e.g., $h^{1,2}_+$ instead of $h^{1,2}_-$ complex structure moduli. Instead of O3- and O7-planes, one will have O5-planes in the CY orientifold.

Lower indices K_i and $K_{\bar{\jmath}}$ indicate derivatives $\partial K/\partial\phi^i$ and $\partial K/\partial\bar{\phi}^{\bar{\jmath}}$ and $K^{i\bar{\jmath}}$ is the inverse of the matrix $K_{i\bar{\jmath}}$. The covariant derivative is defined as $D_i = \partial_i + K_i$.

Given the action (2.11), one finds the Kähler potential

$$\begin{aligned} K &= K_{\mathcal{V}} + K_\tau + K_{\text{cs}} \\ &= -2\ln(\mathcal{V}) - \ln(-i(\tau-\bar{\tau})) - \ln\left(-i\Pi^\dagger\Sigma\Pi\right) . \end{aligned} \tag{2.24}$$

Here, the *volume* $\mathcal{V}$ is a function of the Kähler moduli, $\mathcal{V} = \frac{1}{6}\int J\wedge J\wedge J$. To define the quantities in the last term, we have to introduce a basis of odd 3-cycles $\{A^i, B_j\}$, $i,j = 0,\ldots,h^{1,2}_-$ [2]. We choose them in such a way that the symplectic intersection matrix Σ of the cycles A^i and B_j has the normalized form

$$\Sigma = \begin{pmatrix} A^i\cap A^j & A^i\cap B_j \\ B_i\cap A^j & B_i\cap B_j \end{pmatrix} = \begin{pmatrix} 0 & \mathbb{1}^i_j \\ -\mathbb{1}^j_i & 0 \end{pmatrix} . \tag{2.25}$$

Then, the $(2h^{1,2}_- + 2)$-dimensional *period vector* Π takes the form

$$\Pi = \begin{pmatrix} z^i = \int_{A^i}\Omega \\ G_j(z) = \int_{B_j}\Omega \end{pmatrix} , \quad i,j = 0,\ldots,h^{1,2}_- , \tag{2.26}$$

where $z^0 = 1$ defines the normalization of Ω. The remaining $h^{1,2}_-$ z^a's are the complex structure moduli and $G_j(z^a)$ are functions of the complex structure moduli, whose form depends on the CY under consideration.

The Kähler potential above fulfills an important property:

$$\sum_{\alpha,\bar{\beta}\in h^{1,1}_+} K^{\alpha\bar{\beta}}K_\alpha K_{\bar{\beta}} = 3 . \tag{2.27}$$

We see that $D_aW = 0$ for $a = 1,\ldots,h^{1,2}_-$ already implies that the scalar potential vanishes identically in the absence of non-vanishing derivatives of the superpotential! This property of the Kähler potential is called *no-scale*.

We did not include the 2-form moduli yet. On a perturbative level, they appear only in the Kähler potential [95], but the exact form will not be relevant here.

Flux Stabilization

The (perturbative) superpotential is only non-vanishing in the presence of fluxes, non-trivial quantized field configurations of the 3-form field strengths

[2] Note that the 3-cycles dual to the odd holomorphic and anti-holomorphic three-forms are also odd, thereby explaining why there are $2(h^{1,2}_- + 1)$ odd 3-cycles.

F_3 and H_3. A major step in establishing a general mechanism for finding vacua has been done in [96]. There it was shown that general solutions to the equations of motion derived from the action (2.11) exist for certain fluxes and localized objects using a *warped* metric ansatz

$$\mathrm{d}s^2 = e^{2A(y)}\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + e^{-2A(y)}g_{mn}\mathrm{d}y^m\mathrm{d}y^n\,, \tag{2.28}$$

where g_{mn} is a CY metric. These solutions include *imaginary self-dual fluxes*, $F_3 = -\tau\star_6 H_3$, and their back-reaction. The superpotential, derived from the 10d flux potential $\int \mathrm{d}^{10}x\,|G_3|^2$, in the effective four-dimensional description is the Gukov-Vafa-Witten (GVW) superpotential [97]

$$W = \int_X \Omega \wedge G_3\,, \tag{2.29}$$

where $G_3 = F_3 - \tau H_3$. This now generates a positive semi-definite potential $V(z^a) \sim |D_aW|^2 + |D_\tau W|^2$ for the complex structure moduli z^a and the axio-dilaton τ. The stabilized values are in the supersymmetric vacuum if and only if they are F-term solutions

$$D_aW = 0\,, \quad D_\tau W = 0\,. \tag{2.30}$$

By choice of fluxes, their value can be tuned. The Kähler moduli remain unstabilized at this stage.

Note that the fluxes on 3-cycles,

$$\int_{A^i} F_3 = (2\pi)^2\alpha' M^i\,, \quad \int_{B_j} H_3 = -(2\pi)^2\alpha' K_j\,, \tag{2.31}$$

are subject to a *tadpole cancellation condition.* Integrating the Bianchi identity for $\widetilde{F}_5$ over the compact manifold yields

$$\begin{aligned} 0 &= N_{\mathrm{D3}} - \frac{1}{2}N_{\mathrm{O3}} + \frac{1}{2\kappa_{10}^2 T_3}\int_X H_3 \wedge F_3 \\ &= N_{\mathrm{D3}} - \frac{1}{2}N_{\mathrm{O3}} + \sum_i M^i K_i\,, \end{aligned} \tag{2.32}$$

where the first two contributions come from localized sources modifying the Bianchi identity, $\mathrm{d}\widetilde{F}_5 \supset \sum_{\mathrm{D3/O3}}\delta^{(6)}(y_{\mathrm{D3/O3}})$, and contain both positive (D3) and negative (O3) contributions. The last term is the flux contribution. In the sum on the right-hand side we only include pairs of fluxes (M^i, K_j) on symplectic dual cycles, see (2.25). We see that O-planes and therefore orientifolding is a necessary step in turning on fluxes stabilizing moduli. In F-theory, D7-branes (and O7-planes) give an additional contribution in the form of the $\chi(Y)/24$ on the left-hand side, where $\chi(Y)$ is the Euler number of a CY 4-fold $Y \to X$ which is an elliptic fibration over a 3-fold X. One may then cancel tadpoles without considering orientifolds.

2.1.4 Stabilizing All Moduli

There are some drawbacks to the setup of vacua so far: We end up with a supersymmetric Minkowski solution that has only taken into account the leading order perturbative theory and has unstabilized moduli. We will now see, that including pertubative and non-perturbative corrections will actually generically take care of these problems. Whether the final solution can actually be non-supersymmetric de Sitter, as we find in nature, is currently under debate [98]. We will content ourselves with presenting the basic ideas behind the scenarios without much scrutiny.

We are going to add perturbative and non-perturbative corrections to the Kähler and superpotential

$$\begin{aligned} K &= K_0 + K_{\mathrm{p}} + K_{\mathrm{np}} \approx K_0 + K_{\mathrm{p}} \, , \\ W &= W_0 + W_{\mathrm{np}} \, . \end{aligned} \tag{2.33}$$

In this we assume that the first perturbative correction dominates over non-perturbative corrections. The superpotential can be shown to be free of perturbative corrections, so we only have to deal with non-perturbative terms.

More specifically, the non-perturbative corrections come from for example Euclidean branes wrapping internal cycles. Since we know that the superpotential is holomorphic and the imaginary parts of Kähler moduli T_i have an exact shift symmetry in string theory (they are after all axions $\sim \int C_4$), the only way these corrections can appear are in exponentials of the form.

$$W_{\mathrm{np}} = \sum_{\alpha=1,\ldots,h_+^{1,1}} \sum_{n\in\mathbb{N}} A_{\alpha,n} e^{-a_\alpha n T_\alpha} \, , \tag{2.34}$$

where A_α are prefactors depending on other moduli in general and a_α are numerical coefficients depending on the origin of the correction. In scenarios with controlled corrections, $\left|e^{-a_\alpha T_\alpha}\right| \ll 1$, it suffices to consider the leading terms, $n = 1$.

The leading order correction in the α'-expansion is [99]

$$K_{\mathrm{p}} = -\frac{\xi}{g_s^{3/2}\mathcal{V}} \, , \tag{2.35}$$

where $\xi = -\frac{\chi(X)\zeta(3)}{2(2\pi)^3}$ depends on the Euler characteristic $\chi(X) = 2(h^{1,1} - h^{2,1})$ of the CY.

From the general structure, we immediately find the Dine-Seiberg problem [100] that we will explain by only considering a single Kähler modulus, $\mathcal{V} = (T - \bar{T})^{3/2}$: We see that at infinite volume the potential becomes 0 as all the corrections as well as the uncorrected potential go with inverse powers or negative exponentials of this modulus. This means, there are two possibilities, illustrated in Fig. 2.2: If the leading correction has a positive sign we find a runaway behavior with the modulus $\mathcal{V}$ going to ∞. If the sign is negative, the volume will go to small values and there is no reason to only consider the leading order perturbative or non-pertubative corrections; all corrections become relevant. To avoid this, one needs to achieve that at least two corrections become relevant at the same time. Only then can one find non-trivial minima at finite values of the modulus.

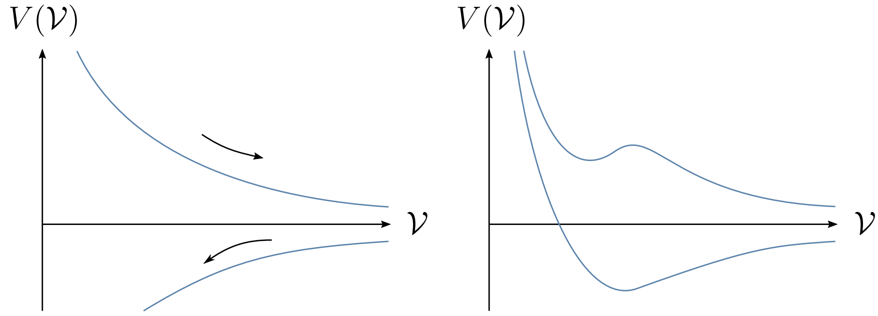

Figure 2.2: The Dine-Seiberg problem and its resolution.

Using (2.23), one finds a structure [101]

$$V = V_0 + V_{\mathrm{p}} + V_{\mathrm{np}} + \dots\,, \tag{2.36}$$

where

$$V_0 \sim W_0^2\,, \quad V_{\mathrm{p}} \sim K_{\mathrm{p}} W_0^2\,, \quad V_{\mathrm{np}} \sim W_{\mathrm{np}}^2 + W_0 W_{\mathrm{np}}\,, \tag{2.37}$$

There are now two famous mechanisms of stabilizing the Kähler moduli and thereby contructing an AdS solution, which can then be uplifted to a de Sitter space, breaking all supersymmetry in the process.

The first one, the KKLT scenario [58], uses only the two terms in the potential V_{np} by assuming that W_0 is tuned to a small number comparable to $W_{\mathrm{np}} \sim e^{-T}$ by an appropriate choice of fluxes. Then V_p is negligible and one can find finite solutions for $W_{\mathrm{np}}^2 \sim W_0 W_{\mathrm{np}}$. Since the scalar potential is of the form $|D_\alpha W|^2$, one finds a minimum in the F-term solution $D_\alpha W = 0$. Hence, the vacuum is given by a supersymmetric AdS space with $V_{\mathrm{AdS}} = -|W_0|^2$.

The second one, the large volume scenario [73], balances perturbative and non-perturbative terms at generic values of W_0. This requires at least two Kähler moduli. The two terms being balanced are $K_{\mathrm{p}} W_0^2 \sim W_0 W_{\mathrm{np}}$. The minimum of the scalar potential will not be a F-term solution and therefore supersymmetry is broken in the AdS minimum with $V_{\mathrm{AdS}} = -\ln^{1/2}(\mathcal{V})/\mathcal{V}^3$.

Finally, there may be additional positive potential terms that will uplift the minima described to values $V_{\mathrm{dS}} \gtrsim 0$. Whether this is actually possible without generating instabilities is still under debate [98, 55].

2.2 The Swampland

2.2.1 No Global Symmetries and Gravity as the Weakest Force

After having introduced the swampland idea already in Sect. 1.2, we want to give more quantitative details on how some of the conjectures are formulated.

As a first introduction to swampland conjectures, we may go back to the example of higher-dimensional gravity in Sect. 2.1.1. We use the parametrization (2.5) while following [14]. We rescale the fields $\sigma \to \widetilde{\sigma} = \sigma M_{\mathrm{P}}/\sqrt{3}$ and $A_\mu \to \widetilde{A}_\mu = A_\mu M_{\mathrm{P}}/\sqrt{2}$ to bring the 4d action to the standard form with dimensionful fields. We drop the $\widetilde{\cdot}$ to write

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{M_{\mathrm{P}}^2}{2}R[g] - \frac{1}{4g^2}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma\right) , \tag{2.38}$$

where now

$$M_{\mathrm{P}}^2 = 2\pi R\, M_5^3 \,, \quad g^2 = e^{\sqrt{3}\sigma/M_{\mathrm{P}}} \,, \quad \mathrm{Vol}(S^1) = 2\pi R\, e^{-\sigma/(\sqrt{3}M_{\mathrm{P}})} \,. \tag{2.39}$$

Let us set the dimensionful radial parameter to $R = 1/M_{\mathrm{P}}$ and at the same time set $M_{\mathrm{P}} = 1$. For any given VEV of σ we then have the internal volume in Planck units

$$\mathrm{Vol}(S^1) = 2\pi e^{-\sigma/\sqrt{3}} \,. \tag{2.40}$$

Any KK tower of a 5d field will now be charged and the KK scale depends on the VEV of σ

$$m_n = n\, m_{\mathrm{KK}} = n\, e^{\sqrt{3}\sigma/2} \,, \quad q_n = n \,. \tag{2.41}$$

We find the useful relation

$$m_n = g q_n \,. \tag{2.42}$$

Interestingly, we find that we cannot make the gauge symmetry global by simply taking the limit $g \to 0$, as this would require $\sigma \to -\infty$, $\mathrm{Vol}(S^1) \to \infty$ and $m_n \to 0 \,, \forall n$. The 4d EFT description breaks down, as an infinite tower of states becomes relevant and the actual description is revealed to be a five-dimensional theory as the circle is decompactified.

This is a simple example of how gravity forbids global symmetries, which is the oldest of the swampland conjectures: An EFT that can be consistently embedded in a theory of QG cannot possess exact global symmetries [15, 79–81, 16, 82]. To understand this independent of any specific model, consider a theory with a global continuous symmetry and imagine a globally charged black hole. By semi-classical calculations one comes to find that any such black hole will evaporate by emission of particles. These particles are created via Schwinger pair production close to the horizon. If the black hole possesses a gauge charge, the electric field outside the horizon will affect the production in such a way that the black hole is slowly discharged. If there is no field, however, as is the case for a global charge, both particles of charge and anti-charge are emitted with equal probability. Therefore the black hole will not lose its charge - albeit losing mass. One then has trouble making sense of how global symmetries and charges can be consistently implemented in a quantum theory of gravity even on a semi-classical level. Once the black hole reaches the quantum gravity regime, we do not know what happens. A possibility is the existence of remnants, black holes which are stable due to there left-over global charge. This idea leads to the possibly problematic idea of having an infinite number of absolutely stable black hole states of (the same) finite mass [102, 16]. Consequently, one the oldest proposed swampland conjecture is that global symmetries simply do not exist in EFTs derived from a theory of QG.

In fact, the limitation on the smallness of gauge couplings g is subject of the weak gravity conjecture [17]: In any EFT compatible with QG, there exists a state of mass m and charge q that fulfills

$$m \leq gqM_{\mathrm{P}}\,. \tag{2.43}$$

In above example any of the KK states in (2.42) saturates this bound (up to $\mathcal{O}(1)$-factors that can be made more precise). In its original proposal this bound was motivated also by black hole physics: There should always be particles in the spectrum that allow for black holes to evaporate such that no non-supersymmetric black holes are stable.

Since both the gravitational and gauge force are of Coulomb type, one may derive the WGC by claiming that for any Abelian gauge force equal-charge particles are more strongly repelled by the gauge force than they are attracted by gravity, see Fig. 2.3. By definition, this excludes a gravitational bound state of two or more such particles. Interestingly, it also implies that charged black holes, even extremal ones, can kinematically always decay, which was one of the original motivations of the WGC [17].

From (2.42), we can read off another feature that generically reappears in string compactifications: Taking the limit $g \to 0$ leads to the breakdown

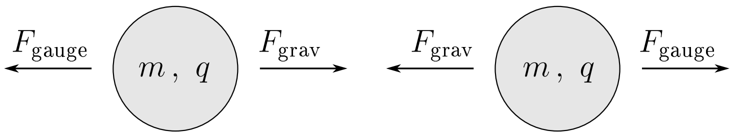

Figure 2.3: The WGC (2.43) doesn't allow for bound states as the repulsive Coulomb force F_{gauge} is at least as big in magnitude as the attractive Coulomb force F_{grav} .

of the EFT. In above example the infinite tower of KK states becomes light and the theory unveils its five-dimensional origin. The appearance of such towers leads to the formulation of the magnetic weak gravity conjecture: A $U(1)$ gauge theory with coupling g coupled to gravity has a cut-off which is bounded from above

$$\Lambda \lesssim g M_{\mathrm{P}} \,. \tag{2.44}$$

The name of this conjecture originates from a different derivation: Applying the WGC to the magnetically dual theory, one finds that the mass of a magnetic monopole is bounded by $m_{\text{magn}} \leq g_{\text{magn}} M_{\mathrm{P}} = M_{\mathrm{P}}/g_{\text{el}}$. At the same time we may calculate the mass of the monopole by the energy stored in its magnetic field. Regularizing this field energy by introducing a cut-off Λ gives $m_{\text{magn}} \sim \Lambda/g_{\text{el}}^2$. Hence, we arrive at $\Lambda \lesssim g_{\text{el}} M_{\mathrm{P}}$.

2.2.2 The Weak Gravity Conjecture and Scalars

It has long been believed that there might be a problem in theories of QG if there is an infinite tower of bound states arising [103–106]. It has therefore been claimed that the above logic of forbidding bound states should also hold for additional attractive interactions mediated by a scalar [18]. For an interaction of Coulomb type between charged scalars ϕ and a massless force carrier χ ,

$$\mu m \left|\phi\right|^2 \chi \,, \tag{2.45}$$

the inequality forbidding bound states should read

$$\left(\frac{m}{M_{\mathrm{P}}}\right)^2 + \mu^2 \leq g^2 q^2 \,. \tag{2.46}$$

An interaction of the above form appears whenever there is a modulus χ determining the mass $m(\chi)$ of ϕ , i.e., we may interpret $\mu = \partial_\chi m$ such that $\mu m = \frac{1}{2}\partial_\chi m^2$.

Another route to explore is to interpret the inequality (2.43) as the statement 'gravity is always the weakest force in a theory of QG', which of course gives rise to the name of the conjecture. Taking this to be the fundamental principle behind the conjecture, one is led to the scalar weak gravity conjecture [18], which restricts scalar interaction of the above form between uncharged scalars

$$\left(\frac{m}{M_{\mathrm{P}}}\right)^2 \leq \mu^2 \,. \tag{2.47}$$

2.2.3 The Axionic Weak Gravity Conjecture

A final set of weak gravity conjectures we want to explore to get back to the example of inflation as an EFT is the generalized version of (2.43) for p-form gauge theories. It deals, instead of charged point-particles of mass m, with p-branes of tension T_p coupling to gauge potentials C_{p+1}. The conjecture reads (in four dimensions) [17, 53]

$$\frac{p(2-p)}{2} T_p^2 \leq q_p^2 g_p^2 (M_{\mathrm{P}})^2 \,. \tag{2.48}$$

Since axions take a prominent place in the string landscape, the limiting case of $p=-1$ (the axionic version of the WGC [17]) is of particular interest

$$S_I \leq \frac{M_{\mathrm{P}}}{f} \,, \tag{2.49}$$

where S_I is the instanton action of the 0-dimensional object coupling to the axion of decay constant f. To motivate this, let us consider the action of a p-brane (2.13) now including the kinetic term for the field strength $F_{p+2} = \mathrm{d}C_{p+1}$

$$S_p = -\frac{1}{2g_p^2} \int F_{p+2} \wedge \star F_{p+2} - T_p \int_{\mathcal{W}} \star 1 + \int_{\mathcal{W}} C_{p+1} \,, \tag{2.50}$$

where we have absorbed the coupling constant μ_p in the field C_{p+1}. We compare this to the action of an axion coupling to gauge instantons

$$S_{-1} = -\frac{f^2}{2} \int \mathrm{d}\phi \wedge \star \mathrm{d}\phi - \frac{1}{2g_{\mathrm{YM}}} \int \mathrm{tr}\, F_2 \wedge F_2 + \frac{1}{8\pi^2} \int \phi \mathrm{tr}\, F_2 \wedge F_2 \,. \tag{2.51}$$

An instanton solution F_2^I will give an instanton action S_I which is the analogue of the worldvolume action of a Euclidean p-brane and a localized, 0-dimensional coupling between axion ϕ and gauge instanton via the last term.

The axionic WGC also has a magnetic formulation [19]. The magnetically charged objects of interest are light strings. The string tension gives a cut-off

$$\Lambda \lesssim \sqrt{f M_{\mathrm{P}}} \,. \tag{2.52}$$

2.2.4 The dS and SUSY AdS Conjectures

As already alluded to in Sect. 2.1.4, it is still under debate whether proposed realizations of de Sitter vacua in string theory are actually fully controlled. This discussion on the technical difficulties in de Sitter constructions ranges from no-go theorems for specific setups (see, e.g., [107] for an early no-go theorem) to possible loss of parametric control in others (see [98] and [108] for opposing viewpoints on some constructions). It gave rise to the *(Refined) de Sitter Conjecture* [55] which quantifies how de Sitter solutions have to be unstable: The scalar potential of an EFT coupled to gravity must satisfy either of the following bounds

$$|\nabla V| \geq \frac{c}{M_{\mathrm{P}}} V \,, \quad \min(\nabla_i \nabla_j V) \leq -\frac{c'}{M_{\mathrm{P}}^2} V \,. \tag{2.53}$$

Here, c and c' are constants of order one and where the minimum eigenvalue of the Hessian appears in the second inequality. Note that this is not in direct contradiction to the observed universe, as quintessence models [109] are a feasible explanation of the current epoch of accelerated expansion [110].

It has also been proposed that non-supersymmetric anti-de Sitter spaces are unstable [56, 57]. In flux-supported AdS vacua (compactifications to p-dimensional AdS space with non-vanishing field strengths F_p along the external dimensions) branes coupling to the gauge potential C_{p-1} can nucleate [111, 112]. Such a nucleated brane then serves as a domain wall between two AdS solutions which differ in flux number by the charge of the brane. In general, the brane may expand until it reaches the boundary of AdS within a finite time. Thereby, it reduces the flux of the original solution. A possible new solution of reduced flux suffers from the same instability. The solution can only be stable if the tension of the brane can compete with the Coulomb repulsion due to the flux. The argument of [56] is now based on a sharpened version of the WGC: The only objects saturating the WGC inequality (2.43) or its generalization to p-branes (2.48) are BPS objects in supersymmetric theories. According to this sharpened version, the stability of AdS therefore requires supersymmetry. This idea has been promoted to a conjecture about all non-supersymmetric AdS spaces being unstable in quantum gravity.

Chapter 3

Thraxions: Ultralight Throat Axions

3.1 Introduction

In this chapter, we consider axion-like particles, axions for short, from a landscape perspective. That is, we construct an EFT of axions in so-called Klebanov-Strassler throats [49], which may be derived from ten-dimensional type IIB supergravity by employing the techniques discussed in Sect. 2.1. The aim is twofold: On the one hand, as briefly discussed for the example of inflation in Sect. 1.1.3, these particles allow for interesting phenomenology in beyond-the-standard-model physics in general [20, 21] and in string phenomenology in particular [22–24, 5, 25]. On the other hand, we want to explore the boundaries of the landscape as defined by swampland conjectures. In this case we want to scrutinize the axionic weak gravity conjecture, see Sect. 2.2.3.

We present a novel type of ultralight axion which, as we argue, is generically present in the type-IIB part of the landscape, building on a proposal made in [40]. Its extreme lightness, both in absolute terms and in relation to its decay constant (i.e., compared to the scale $M_\mathrm{P}^4 \exp(-M_\mathrm{P}/f)$ of generic non-perturbative potentials) lets it stand out among the many other stringy axions.

Before turning to the details, we want to explain our central and, in our opinion, rather surprising, parametric results: Consider type-IIB Calabi-Yau orientifold or F-theory models stabilized by fluxes and non-perturbative effects [96, 58, 73]. It is generally accepted that Klebanov-Strassler (KS) throats [49] with warp factor $w_\mathrm{IR} \ll 1$ will be present in an order-one fraction of such models [50, 4, 51]. This warp factor can easily be *exponentially* small, such that it is justified to focus for the moment only on the dependence on $w_{\rm IR}$. In other words, let us for now set $R_{\rm CY} \sim M_{\rm string}^{-1}$ and $N_{\rm flux} \sim \mathcal{O}(1)$, thus ignoring all parameters except for the warp factor. Naively, the lightest states are then the glueballs (or warped-throat KK modes) with mass $\sim w_{\rm IR}$ (in Planck units). By contrast, we claim that an ultralight axion with mass $\sim w_{\rm IR}^3$ is frequently present in such settings. To be more precise, this happens at least in all cases where the fluxes stabilize the complex structure moduli near a conifold transition point in moduli space.

Moreover, our axion has a decay constant $f \sim \mathcal{O}(1)$ in the simplest models [1], which can be enhanced by products of flux numbers to super-Planckian values in more general settings, and an effective potential which can be much smaller than the naive expectation $V \sim \exp(-1/f)\,\cos(\phi/f)$ (again in Planck units). Clearly, this has potentially many interesting applications, from the WGC for axions to inflation and uplifting.

The chapter is organized as follows. We start with the background solution in Sect. 3.2.1. We consider a Calabi-Yau with a conifold point in complex structure moduli space at which multiple three-cycles degenerate simultaneously. We explain why this is a generic feature of Calabi-Yaus. Concentrating on the case of two degenerate three-cycles, we introduce separate deformation parameters z_i with phases $\varphi_i = \arg z_i$, $i = 1, 2$, for the two deformed conifold regions. Crucially, the two conifolds, specifically the S^3-cycles describing the apices, are related in homology. As a result, the Calabi-Yau condition ensures that only one complex structure modulus, $z = z_1 = z_2$, is present. Deformations with $z_1 \neq z_2$ are massive. We then introduce fluxes stabilizing the complex structure modulus z near the conifold point $|z| \ll 1$. The resulting geometry is illustrated in Fig. 3.1. One can see that the so-called $\mathcal{B}$-cycle is an S^3 which can be thought of as a family of S^2's. This S^2 family reaches into both throats such that the S^2's collapse at the apices. The corresponding dual $\mathcal{A}$-cycle is an S^3 over every point of the double throat in Fig. 3.1.

In Sect. 3.2.2, we introduce the axion [2] $c \sim \int_{S^2} C_2$, called *thraxion* from now on. An excursion of the thraxion generates non-zero opposite values of

[1] Note that despite the fact that strongly warped throats are needed to generate a small scalar potential for the axion, the decay constant is not suppressed by warping effects. This is because its internal field-profile is *not* localized at the bottom of the throats, in contrast to some examples that have appeared in the literature [113, 114].

[2] This may seem like a misnomer since the shift symmetry is completely broken: The 10d gauge invariance $C_2 \sim C_2 + {\rm d}\Lambda_1$ does not imply a 4d shift symmetry because the two-sphere is trivial in homology $S^2 = \partial\Sigma_3$, $\int_{S^2} {\rm d}\Lambda_1 = \int_{\Sigma_3} {\rm d}^2\Lambda_1 = 0$. However, from the unwarped UV-perspective the two-sphere is non-trivial and the field is a proper axion, with a monodromy created only by the warped-down IR region.

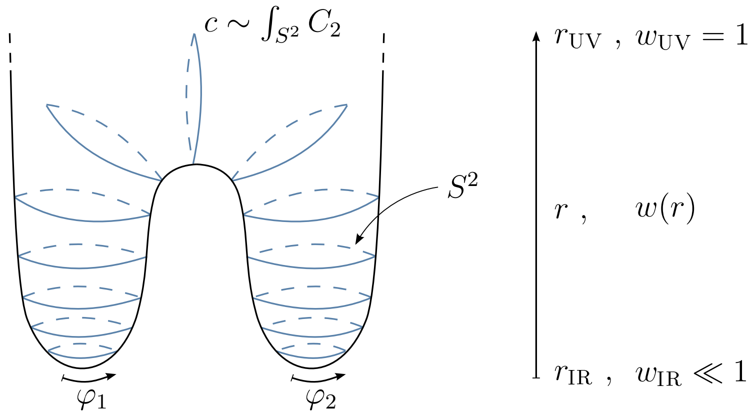

Figure 3.1: An illustration of the setup of the double throat including the phases φ_i and the axion c. The phases φ_i describe physical rotations of each throat. We have not drawn the S^3 over every point of the double throat.

the RR-field strength F_3 at the ends of the two throats. Local back-reaction of the resulting energy density then deforms the two throats independently: While the phase φ_1 of the local deformation parameter of one throat is displaced by fluxes, the phase φ_2 of the other throat is displaced in the opposite direction by anti-fluxes. This breaks the constraint $\varphi_1 = \varphi_2$ coming from the CY condition and the homology relation between the two throats. In Sect. 3.2.3, we calculate the potential induced by non-vanishing 10d Ricci curvature that stabilizes the two deformation parameters against each other. After integrating out heavy degrees of freedom, the result is an effective potential for the thraxion with the properties described above and discussed in Sect. 3.2.4. We discuss how the analogue B_2-field back-reacts on the double throat in Sect. 3.2.5.

The exact equations of motions, their results and corresponding calculations for all parameters appearing are presented in Sect. 3.3.

Sect. 3.4 rederives the effective axion potential from a proposed generalization of the Gukov-Vafa-Witten superpotential [97] that includes the axion. We thereby reproduce the results in 4d supergravity language, and identify the *saxion* partner of the axion. In Sect. 3.4.2 we generalize our results to general multi throat systems where one or more ultralight thraxions appear.

We discuss the consistency of our results with the holographic dictionary in the KS context [115, 49] in Sect. 3.5 by matching the enhancement of the decay constant of our axion $\int C_2$ with gaugino condensation on the gauge theory side. Applications and implications of these results are the content of Sect. 3.6. We consider as an explicit example the quintic threefold stabilized near a conifold transition point. We study the scalar potential for judicious choices of flux quanta. Interestingly, the overall monodromy enhancement is given by the least common multiple of all the flux quanta which can easily become parametrically super-Planckian. However, the presence of sub-Planckian modulations generically prevents successful slow-roll inflation. The underlying idea is that a large monodromy is generated by unsynchronized phases (of monodromies of individual throats) drifting away from one another. We call this mechanism *drifting monodromies.* This mechanism can also be thought of as the well-known beat phenomenon in accoustics, in which the interference of harmonics with slightly different small wavelengths leads to large wavelength oscillations. For related alternative possibilities of generating large decay constants see [32,116–121] [3]. We also describe a clash with the WGC: The effective Euclidean instanton action determined from the scale of the effective potential violates the axionic WGC ($S \lesssim qM_{\mathrm{P}}/f_{\mathrm{eff}}$) parametrically. After commenting on the relevance of our findings to light axion phenomenology, we finally consider interesting possibilities for uplifting to de Sitter vacua. We draw our conclusions in Sect. 3.7.

3.2 Back-Reacted Potential of the Thraxion from 10d

3.2.1 Geometric and Flux-Background

Geometric Features of Generic CYs

First we will explain the basic geometric requirements for our discussion to apply. We will explain why we expect them to be *generically* met.

Let us consider compactifications of type IIB string theory on a CY threefold, which leads to an effective $\mathcal{N} = 2$ supergravity theory in four dimensions. There is a moduli space of vacua parameterized by the $h^{2,1}$ complex structure moduli and $h^{1,1}$ Kähler moduli, cp. Sect. 2.1.2. There are special points in complex structure moduli space called conifold points where the CY develops conical singularities, and one or more three-cycles degenerate to zero volume [122,123]. We will consider a CY near such a conifold point, where *multiple* three-cycles degenerate.

[3] Note in particular the following two references: The work of [119] is closely related to ours in making use of the conifold complex structure modulus z to create super-Planckian decay constants, while on the technical level the approach is very different. The authors of [120] define the 5d axion $\int B_2$ on the Klebanov-Tseytlin background, analogously to our thraxion. There, the geometric back-reaction via the 5d breathing mode allows for monodromy-induced super-Planckian field ranges to be explored in an anisotropic and inhomogeneous 5d spacetime.

To understand in what sense this is generic, we consider cases in which it is also possible to *resolve* conifold singularities while preserving the CY condition. This does not restore the degenerate three-cycles to finite size, but rather produces non-trivial two-cycles. In going through this so-called *conifold transition*, a new CY threefold with Hodge numbers $\tilde{h}^{1,1} > h^{1,1}$ and $\tilde{h}^{2,1} < h^{2,1}$ is produced [124]. Whenever two CY threefolds are connected via a conifold transition, at the conifold transition point two or more three-cycles $\mathcal{A}_i$ that are related in homology shrink to zero size. This is a consequence of demanding Kählerity on the resolved side of the transition [125]. It is widely believed that a *generic* CY threefold is related to other CY threefolds via conifold transitions [126, 124]. While research on this is still on-going, there are large classes of CY's for which this has been shown [127–132] [4]. Therefore, a generic CY has loci in complex structure moduli space where multiple three-cycles $\mathcal{A}_i$ degenerate together. We expand on this in Sect. 3.4. Being related in homology, the number of homology classes is smaller than the number of collapsing three-cycles. For now we focus on the case of precisely two cycles $\mathcal{A}_{1,2}$ that degenerate. From the above it immediately follows that they are related in homology $[\mathcal{A}] \equiv [\mathcal{A}_1] = [\mathcal{A}_2]$. There is a symplectic dual three-cycle $\mathcal{B}$ connecting the two singular points. We will call this system a *double conifold*. Its complex structure will be denoted by z and the double conifold singularity develops in the limit $|z| \to 0$.

We introduce the fields z_1 and z_2 as illustrated in Fig. 3.2. These fields may be thought of as 'local complex structure deformations' $z_i = \int_{\mathcal{A}_i} \Omega$, with the holomorphic three-form Ω of the CY, and describe independent *local* deformations of the manifold near one of the two apices. Thus, in the vicinity of either apex of the double conifold we want to describe the manifold by embedding it into $\mathbb{C}^4$ via

$$w_1^2 + w_2^2 + w_3^2 + w_4^2 = z_i\,, \quad w \in \mathbb{C}^4\,. \tag{3.1}$$

While the homology relation $[\mathcal{A}_1] = [\mathcal{A}_2]$ enforces $z = z_1 = z_2$ on complex structure moduli space [5], we will also consider deformations of the manifold such that $z_1 \neq z_2$, i.e., deformations away from complex structure moduli space [6] (i.e., $\mathrm{d}\Omega \neq 0$). It is important to note that a deformation of the

[4] Note that this does not mean that every conifold singularity (or even a generic one) is also such a transition point. For example, the mirror quintic threefold at vanishing complex structure has a single shrunken three-cycle. Hence there is no resolved CY geometry [125].

[5] This is because the difference $\mathcal{A}_1 - \mathcal{A}_2$ is the boundary of a 4-chain $\mathcal{C}$. Therefore, one has $z_1 - z_2 = \int_{\mathcal{A}_1} \Omega - \int_{\mathcal{A}_2} \Omega = \int_{\partial\mathcal{C}} \Omega = \int_{\mathcal{C}} \mathrm{d}\Omega$. On complex structure moduli space one has $\mathrm{d}\Omega = 0$, and hence $z_1 = z_2$.

[6] For similar considerations with deformations away from Kähler moduli space, i.e., $\mathrm{d}J \neq 0$, see [133].

local phases φ_i of z_i becomes an isometry far away from the tip of either conifold: The cross-section of either deformed conifold at a given radial coordinate r possesses a spontaneously broken $U(1)_R$ isometry [7] realized as a rotation of the local deformation parameter z_i [134], see Fig. 3.2. In the limit of large radial coordinates, $r^3/|z_i| \to \infty$, the deformed conifold becomes indistinguishable from the singular conifold. Therefore, the symmetry remains unbroken in this limit. We make this explicit in App. A.1.

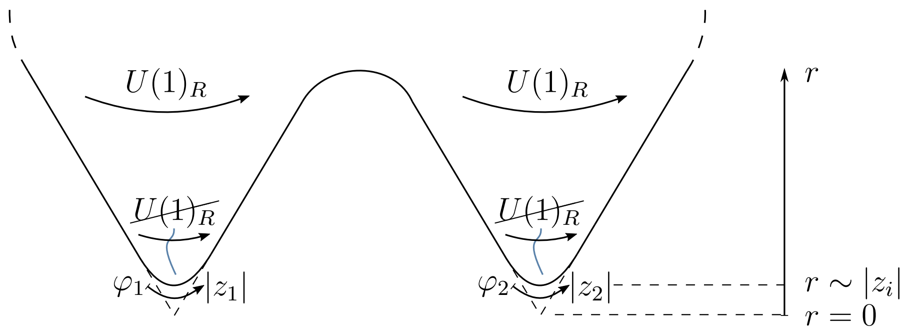

Figure 3.2: The double conifold with asymptotic $U(1)_R$-symmetric regions. The tips of the throats break this symmetry completely.

To complete the discussion of the geometric setting, we note that one may go beyond the simplest case with exactly two collapsing three-cycles. Such multi conifold situations are analyzed in Sect. 3.4. Furthermore, for reasons of tadpole cancellation, see Sect. 2.1.3, we are interested in CY threefolds which are orientifolded such that O3/O7-planes arise. This projection should leave the conifold transition intact and preserve the key ingredient of a $\mathcal{B}$-cycle reaching down into several conifold regions. In the double throat case, this is realized if two originally present pairs of throats are mapped to each other by the orientifold projection, see Fig. 3.3. This is completely analogous to the widely-discussed double throat system of the oldest axion-monodromy models, see, e.g., [42,135] (just simpler, since we need no 2-cycle for the NS5 brane and can hence use standard KS throats). More generally, F-theory solutions with the analogous geometric properties can be considered. Here the tadpole cancellation relies on the fourfold Euler number and no orientifolding is required. Either way, we do not expect that the orientifolding condition or the fourfold embedding endangers the generality of our setting. This has recently been confirmed by a systematic search for CICY orientifolds that fulfill the necessary properties on the resolved side of the conifold transition,

[7] The index R is due to this symmetry group being the R-symmetry of the dual supersymmetric gauge theory. While this is of no importance to us, we keep it for notational consistency.

cp. Sect. 3.4.1: In [52] some 300.000 CICY orientifold candidates have been found.

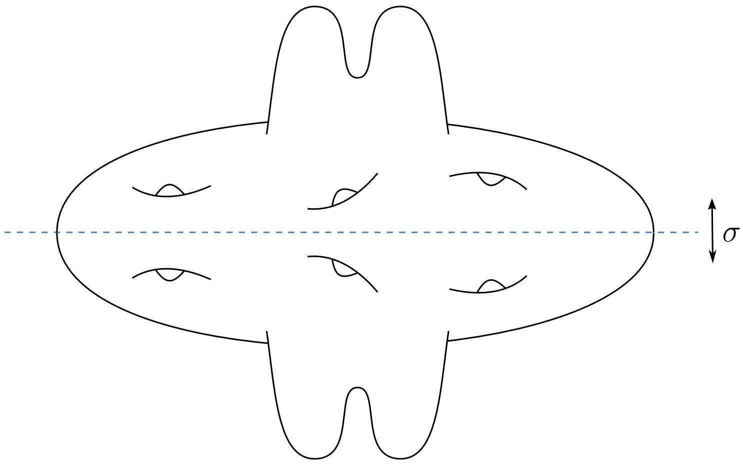

Figure 3.3: A sketch of the the orientifold projection σ. It maps the two originally independent double throat cycles $\mathcal{B}$ and $\mathcal{B}'$ onto one another.

Moduli Stabilization by Fluxes

As was shown in [96], generic choices of three-form flux quanta stabilize the axio-dilaton as well as all the complex structure moduli. Whenever the flux quanta $K \equiv -\frac{1}{(2\pi)^2\alpha'}\int_{\mathcal{B}} H_3$ and $M \equiv \frac{1}{(2\pi)^2\alpha'}\int_{\mathcal{A}_1} F_3 = \frac{1}{(2\pi)^2\alpha'}\int_{\mathcal{A}_2} F_3$ satisfy $K \gg g_s M$ (where g_s is the string coupling), our complex structure modulus $z = |z|\, e^{i\varphi}$ is stabilized near the conifold point

$$|z| \propto \exp\left(-2\pi \frac{K}{g_s M}\right) \ll 1\,. \tag{3.2}$$

Separating the equation $D_z W = 0$ into real and imaginary parts (see Sect. 3 of [96]), we also find that the phase is stabilized. Its value is set by the RR-3-form flux $Q \equiv \frac{1}{(2\pi)^2\alpha'}\int_{\mathcal{B}} F_3$

$$\varphi = 2\pi \frac{Q}{M}\,. \tag{3.3}$$

Locally, we can always set φ to 0 by an appropriate redefinition of the angle. Conversely, without loss of generality, we will choose $Q = 0$.

Moreover, back-reaction of fluxes leads to the formation of warped throats (or Klebanov-Strassler throats). Within these, the metric is well approximated by the Klebanov-Tseytlin (KT) solution [8] [115]

$$\begin{aligned} \mathrm{d}s^2 &= w(r)^2 \eta_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu + w(r)^{-2} (\mathrm{d}r^2 + r^2 \mathrm{d}s^2_{T^{1,1}}) \,, \\ w(r)^2 &\sim \frac{r^2}{g_s M \alpha'} \ln(r/r_{\mathrm{IR}})^{-\frac{1}{2}} \,, \end{aligned} \tag{3.4}$$

with radial coordinate r and warp factor $w(r)$. The radial coordinate is cut off in the IR by the Klebanov-Strassler region and in the UV by the gluing into the bulk CY. One has $w_{\mathrm{IR}} \equiv w(r_{\mathrm{IR}}) \sim r_{\mathrm{IR}}/r_{\mathrm{UV}} \sim |z|^{1/3}$ giving rise to an exponential hierarchy à la Randall-Sundrum [136, 49, 96]. As we have explained, in the vicinity of a conifold transition point a *double throat* (or even *multi throat*) forms [9], see Fig. 3.1.

The three-cycle $\mathcal{B}$ can be thought of as an S^2 fibered over the radial direction of the conifold [123]. The S^2 collapses at the two *tips* of the deformed conifolds. As discussed in [40], there exists a 4d mode $c(x)$ on the double throat background that can be thought of as the integral of the RR 2-form C_2 over the S^2 as measured far away from the tips of the double throat. A non-trivial field excursion leads to the creation of regions with flux on the two respective ends of the cycle $\mathcal{B}$, and hence a red-shifted potential $V(c) = \frac{1}{2} m^2 c^2 + \ldots$, with $m^2 \sim w_{\mathrm{IR}}^4$. Back-reaction of the geometry was neglected in [40].

In the rest of this section, we will establish the following points:

- The fields z_1 and z_2 of the two respective throats adjust to the flux/anti-flux pair in such a way that within the two throats supersymmetry is restored locally.

- This adjustment of z_1 and z_2 takes us away from the complex structure moduli space, which is characterized by $z_1 = z_2$ (cf. Fig. 3.4). For $z_1 \neq z_2$, the CY condition is broken and a scalar potential is generated. This potential is of the order $|z|^2 \sim w_{\mathrm{IR}}^6$ and receives its dominant contributions from the bulk CY.

- The back-reacted scalar potential is periodic in c with periodicity $2\pi M$. Hence, the naive 2π periodicity of the c-axion is enhanced by a *finite* factor M. While this does *not* allow for a super-Planckian effective

[8] Near the bottom of the throat, it has to be replaced by the full Klebanov-Strassler solution [49].

[9] It may not seem obvious that the units of NS-flux on the $\mathcal{B}$-cycle are split democratically so that *each* conifold region is replaced by a warped throat. In fact we will see that there is a light dynamical field that controls this relative distribution (see Sect. 3.4). In the vacuum however this field is stabilized such that fluxes are indeed distributed democratically.

axion decay constant $f \gg M_{\mathrm{P}}$, approximately Planckian values are possible (see however Sect. 3.6 for a way to generate large axion periodicities).

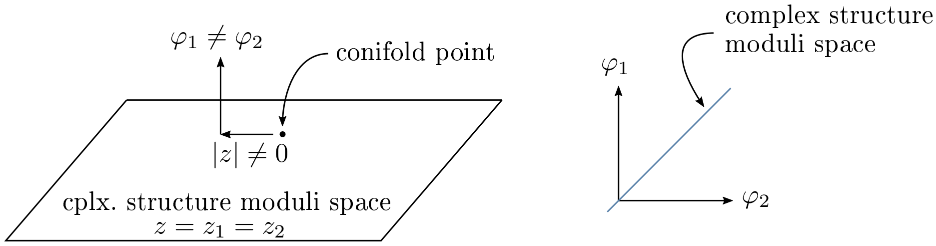

Figure 3.4: Illustration of the z_1-z_2 deformation space. The complex structure moduli space is the subspace $z_1 = z_2$. We only consider deformations away from $z_1 = z_2$ outside the conifold point.

3.2.2 Local Back-Reaction in the Throat

We start by discussing how a single throat reacts locally to a finite field excursion c. Since the outcome will be that the throat almost perfectly adjusts to produce a locally supersymmetric configuration, we are entitled to use the 4d description in terms of the GVW superpotential [97,96] for two KS throats. As far as (say) the first local throat is concerned, a non-vanishing field excursion c cannot be distinguished from additional flux $P \equiv c/2\pi$ on the local portion of the $\mathcal{B}$-cycle [10], see Fig. 3.5,

$$c = \frac{1}{2\pi\alpha'} \int_{S^2} C_2 = \frac{1}{2\pi\alpha'} \int_{\frac{1}{2}\mathcal{B}} \mathrm{d}C_2 = 2\pi\, P\,. \tag{3.5}$$

Considering a single throat with complex structure modulus $z_1 \equiv |z_1|\, e^{i\varphi_1}$, the arguments of Giddings, Kachru and Polchinski (GKP) [96] show that there are SUSY configurations for

$$\varphi_1 = 2\pi \frac{P}{M} = c/M\,, \tag{3.6}$$

compare (3.3). Hence, the throat can locally relax the SUSY breaking induced by the extra RR-flux by adjusting the phase of the deformation parameter z_1. However, the second throat sees the field excursion c as the flux

[10] We will use the term flux also for the non-quantized integral $\int F_3$ over some region.

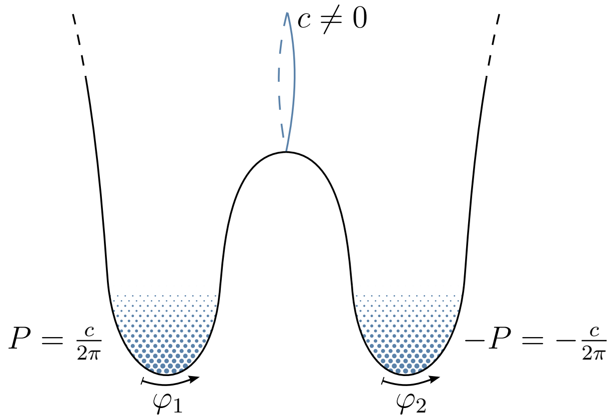

Figure 3.5: Fluxes induced by non-vanishing c are localized at the tips of the throats.

$-P = -c/2\pi$ on the $\mathcal{B}$-cycle for which there exists a locally supersymmetric configuration with $\varphi_2 = -c/M$.

Since there is no additional flux that would lead to $|z_1| \neq |z_2|$, we keep $|z| = |z_1| = |z_2|$ fixed at the stabilized value (3.3) for what follows. These two modes decouple from the discussion at hand.

We can encode the discussion above in a 4d EFT potential. To quadratic order, the discrepancy between the local fluxes and local deformations induces a potential

$$V_{\text{flux}}(c, \varphi_1, \varphi_2) = \frac{1}{2}\mu^4(M\varphi_1 - c)^2 + \frac{1}{2}\mu^4(M\varphi_2 + c)^2\,, \tag{3.7}$$

We have $\mu \sim w_{\text{IR}}$ since the potential is generated locally near the tip of the throats.

The fact that only the combinations $M\varphi_i \pm c$ appear in the scalar potential can be derived also via ten-dimensional considerations. As shown in App. A.2, in the local throats, the combined transformation $\varphi_{1,2} \longrightarrow \varphi_{1,2} \pm \delta$, $c \longrightarrow c + M\delta$ is a diffeomorphism acting on the KS solution. Hence, only the invariant combinations $M\varphi_{1,2} \mp c$ can appear in the scalar potentials that are generated locally at the bottom of the throats.

The potential derived so far possesses a flat direction which we parameterize by c. This flat direction is given by

$$\varphi_1 = -\varphi_2 = c/M\,. \tag{3.8}$$

3.2.3 The CY Breaking Potential

In the preceding section we have argued that the individual throats react to the field excursion c by adjusting their local deformation parameters z_1 and z_2, more specifically their phases φ_1 and φ_2 respectively. Since the corresponding CY has only one complex structure modulus $z \equiv z_1 \equiv z_2$, the mode z_1/z_2 or rather $\varphi_1 - \varphi_2$ must be massive already before fluxes are turned on. This eliminates the remaining flat direction in the potential.

We parameterize the part of the scalar potential that is due to the breaking of the CY condition as

$$V_{\text{CY-breaking}} = \Lambda^4(1 - \cos(\varphi_1 - \varphi_2))\,, \tag{3.9}$$

with a yet undetermined scale Λ. In writing this we have assumed the following:

1. The potential is a function of the difference $\varphi_1 - \varphi_2$ only.
2. It satisfies $V_{\text{CY-breaking}}(\varphi_1 - \varphi_2) = V_{\text{CY-breaking}}(\varphi_1 - \varphi_2 + 2\pi)$.
3. The lowest harmonic dominates.

Condition a) must hold because only the local fluxes of the throats stabilize $\varphi_{1,2}$ individually and, without the flux potential, the complex structure modulus $\varphi = (\varphi_1 + \varphi_2)/2$ should be a flat direction. We expect condition b) to hold because we see no reason for a monodromy. Condition c) is a rather unimportant assumption that we make for ease of exposition.

We combine (3.7) and (3.9) and integrate out $\varphi_{1,2}$ under the assumption $\Lambda^4 \ll \mu^4$ (to be justified below). This corresponds to imposing (3.8). The effective potential takes the form

$$V_{\text{eff}}(c) = \Lambda^4\left(1 - \cos(2c/M) + \mathcal{O}(\Lambda^4/\mu^4)\right)\,. \tag{3.10}$$

The height of this potential can be estimated using the 10d solution. To do so, we need to develop a clear picture of how field profiles and 10d geometry change if we excite c. Recall that c is originally defined by a particular 'Wilson line' VEV of C_2 in the UV of the two throats (as well as in the piece of the CY connecting them). Turning on this VEV and focusing on one throat only, we observe a back-reaction of the throat geometry which maintains SUSY and corresponds to the motion along a flat direction in 4d field space. This is independently true for the second throat, which back-reacts in the opposite way: $\varphi_1 = -\varphi_2 = c/M$.

Now, the crucial point is that these rotations, defined by the IR parameters $\varphi_{1,2}$, must by continuity be accompanied by a corresponding r-dependent rotational profile along the whole throat. We encode this in a five-dimensional field $\phi(x^\mu, r)$ that interpolates between $\varphi_1 = c/M$ and $\varphi_2 = -c/M$ at the respective ends of the throats. This is illustrated in Fig. 3.6, which also displays the expected symmetry: The solution should be antisymmetric under the exchange of the two throats.[11]

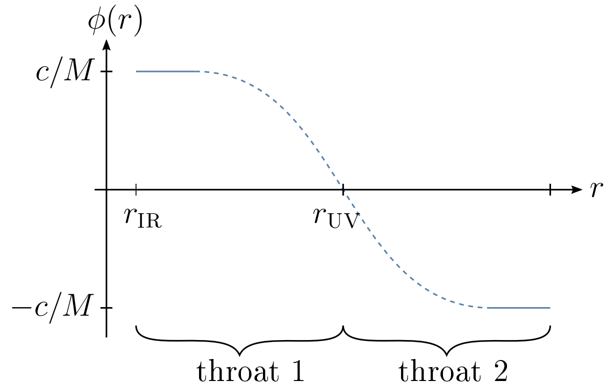

Figure 3.6: The expected profile of the 10d/5d mode along the radial direction.

For computational simplicity, we model the transition region between the throats by a single point, $r = r_{\mathrm{UV}}$[12]. In doing so, we ignore effects of the unwarped CY region (accepting an $\mathcal{O}(1)$ error). The field ϕ must be zero at this point for symmetry reasons. This symmetry also ensures that we may limit our attention to one of the two throats when computing the energy density associated with an excursion of c.

The key point is that, after these preliminaries, we are actually able to estimate this energy. It is given by the gradient energy of ϕ, which accounts precisely for the clash between the opposite rotations of φ_1 and φ_2. The relevant action for $\phi = \phi(x^\mu, r)$ is obtained by dimensionally reducing the 10d Ricci scalar to quadratic order on the warped conifold background (see Sect. 3.3):

$$\begin{aligned} S[\phi] &= \frac{M_{10\mathrm{d}}^8}{2} \int \mathrm{d}^4x\, \mathrm{d}r\, \sqrt{-g_{5\mathrm{d}}} r^5\, w(r)^{-5}\, \epsilon(r)^2 \, \left(-\frac{1}{2} g_{5\mathrm{d}}^{MN} \partial_M \phi \partial_N \phi \right) \\ &= \frac{M_{10\mathrm{d}}^8}{2} |z|^2 \int \mathrm{d}^4x \int_{r_{\mathrm{IR}}}^{r_{\mathrm{UV}}} \frac{\mathrm{d}r}{r} \left(-\frac{1}{2} (\partial_r \phi)^2 - \frac{1}{2} w(r)^{-4} (\partial_\mu \phi)^2 \right) . \end{aligned} \tag{3.11}$$

[11] By a slight abuse of notation we stick with the familiar variable r, although according to our figure this variable must now be growing as one goes down the second throat.

[12] In fact, the exact UV geometry and UV fluxes are irrelevant as long as we do not consider perturbative and non-perturbative corrections [137].

Considering the metric (3.4), this form of the 5d action is easily understood. The metric naturally splits into a 5d part $g_{5\mathrm{d}}$ in the external and radial direction and an angular part $\propto g_{T^{1,1}}$. The latter contributes the r-dependent terms $\sqrt{g_{T^{1,1}}} \propto r^5 w(r)^{-5}$ to the metric determinant. The function $\epsilon(r)$ encodes the degree of $U(1)_R$ symmetry breaking, compare Fig. 3.2. Since a field excursion ϕ is obtained by acting with a $U(1)_R$ transformation, any terms in the action that contain the field must be multiplied by the factor $\epsilon(r)^2$. This symmetry breaking is due to the deformation near the tip and takes the form $\epsilon(r) \sim |z|/r^3$ far from the tip of the throat, see App. A.1.

We now apply the static approximation (i.e., disregard the $(\partial_\mu\phi)^2$ in (3.11)), derive the equation of motion and solve it for the boundary conditions $\phi(x^\mu, r_{\mathrm{UV}}) = 0$ and $\phi(x^\mu, r_{\mathrm{IR}}) = \varphi_1 = c/M$. This gives

$$\phi(x^\mu, r) = c/M \frac{r_{\mathrm{UV}}^2 - r^2}{r_{\mathrm{UV}}^2 - r_{\mathrm{IR}}^2} \tag{3.12}$$

which, inserting back in (3.11), leads to a 4d potential $V \sim |z|^2 c^2$. This is a result at quadratic order in c but, comparing to (3.10), this is sufficient to infer that $\Lambda^4 \sim |z|^2$. Finally inserting the stabilized value $|z| \propto w_{\mathrm{IR}}^3$ [96], we arrive at $\Lambda^4 \sim w_{\mathrm{IR}}^6$. Our assumption $\mu^4 \gg \Lambda^4$ is now *a posteriori* justified. It is also apparent that the effective mass of our ultralight field is $m_c \sim w_{\mathrm{IR}}^3$.

Let us justify the use of the static approximation above. Really, we should have supplemented (3.11) by the kinetic term $S_{\mathrm{kin}}[c] \sim \int \mathrm{d}^4x\, (\partial_\mu c)^2$, imposed the constraint $\phi(x^\mu, r_{\mathrm{UV}}) = c(x^\mu)/M$, and determined the mass of the lowest-lying KK mode of the resulting 5d action. However, it is intuitively clear that the UV-dominated kinetic term of c is much more important than the warped-down 4d-gradient term $(\partial_\mu\phi)^2$ in (3.11). Thus, c is the most inert part of the system and it is an excellent approximation to assume that the ϕ-profile extremizes just the 5d-gradient-part of the action. To make this quantitative, one may substitute c on the r.h. side of (3.12) with the plane wave $c = \exp(ikx)$ (with $k^2 = -m_c^2$) and check that the resulting $(\partial_\mu\phi)^2$ contribution from (3.11) is negligible compared to $S_{\mathrm{kin}}[c]$.[13]

More details are given in Sect. 3.3.

[13] Note that there is no contribution from $S_{\mathrm{pot}}[c] \sim \int \mathrm{d}^4x \mathrm{d}r\, (\partial_r c)^2$. When exciting c in the UV, the local deformation parameter adjusts as explained above. The profile $c(r)$ is now stabilized in turn, for energies $\Lambda < E < \mu$, to the radially constant value of the phase of the deformation parameter $\pm M\varphi_i$. We expand on this in App. A.2.

3.2.4 Discussion of Results

The information we have gathered can be summarized in an effective Lagrangian [14],

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}f_\varphi^2(\partial\varphi_1)^2 - \frac{1}{2}f_\varphi^2(\partial\varphi_2)^2 - \frac{1}{2}f_c^2(\partial c)^2 \\ &- \frac{1}{2}\mu^4(M\varphi_1 - c)^2 - \frac{1}{2}\mu^4(M\varphi_2 + c)^2 - \Lambda^4(1-\cos(\varphi_1-\varphi_2))\,,\end{aligned} \tag{3.13}$$

with coefficients

$$\begin{aligned} f_\varphi^2 &\sim \ln\left(w_{\rm IR}^{-1}\right)^{-3/2} w_{\rm IR}^2\; M_{\rm P}^2\,, \\ f_c^2 &\approx \frac{2}{9M^2}\ln\left(w_{\rm IR}^{-1}\right)^{-1}\; M_{\rm P}^2\,, \\ \Lambda^4 &\sim \frac{g_s^2}{(g_sM)^4}\ln\left(w_{\rm IR}^{-1}\right)^{-7/2} w_{\rm IR}^6\; M_{\rm P}^4\,, \\ \mu^4 &\sim \frac{g_s^4}{(g_sM)^6}\ln\left(w_{\rm IR}^{-1}\right)^{-3} w_{\rm IR}^4\; M_{\rm P}^4\,. \end{aligned} \tag{3.14}$$

The above expressions are valid for the special case of the bulk CY having a single characteristic length-scale, $R_{\rm CY}^6 \sim {\rm Vol(CY)}$, and where the throats marginally fit into the bulk, i.e., $R_{\rm CY}^4 \sim R_{\rm throat}^4 \sim g_sMK\alpha'^2$. For the general case see Sect. 3.3.3. We briefly pause to explain what is meant by the requirement that the throats fit into the bulk marginally: On the one hand the bulk CY has an overall size $R_{\rm CY}$ that is set by a combination of the Kähler moduli. On the other hand the throats can really be thought of as objects of a characteristic physical size $R_{\rm throat}$ embedded into the bulk CY. This size $R_{\rm throat}^4$ is well known to be set by the local D3 brane charge stored in the fluxes of the throat [49, 96], independently of the size of the bulk CY. For this to be a geometrically consistent configuration, we should require $R_{\rm CY} > R_{\rm throat}$. Taking $R_{\rm CY} \sim R_{\rm throat}$ is the case where the throats fit into the bulk CY only *marginally.*[15]

Far below the scale $w_{\rm IR}M_{\rm P}$ we may integrate out $\varphi_{1,2}$, to obtain the effective Lagrangian

$$\mathcal{L}' = -\frac{1}{2}f_c^2(\partial c)^2 - \Lambda^4(1-\cos(2c/M))\,. \tag{3.15}$$

We would like to highlight the following points,

[14] For ease of exposition we have written down a diagonal kinetic matrix. This is not quite the case but is not relevant for our discussion. See App. A.2 for details.

[15] For a recent discussion on this see [69].

- Our simplification $R_{\text{throat}} = R_{S^2} = R_{\text{CY}}$ gives the largest possible value for the decay constant $f_{\text{eff}} = Mf_c$; any hierarchy $R_{\text{throat}} < R_{S^2} < R_{\text{CY}}$ suppresses its value. Taking into account logarithmic corrections, the maximal periodicity one can achieve is $\mathcal{O}(M_{\text{P}}/\sqrt{\ln w_{\text{IR}}^{-1}})$ (see Sect. 3.3.3). A large hierarchy $w_{\text{IR}} \ll 1$ suppresses the periodicity only very mildly. By taking $g_s M$ and g_s^{-1} to be large, the 10d perturbative expansion becomes better controlled without affecting the axion periodicity. In this sense, our axion can be made approximately Planckian.

- The mass of the axion is $\mathcal{O}(w_{\text{IR}}^3)$ which is parametrically smaller than both the warped Kaluza-Klein scale ($\mathcal{O}(w_{\text{IR}})$), and the estimate of [40], where back-reaction of the local geometry was not taken into account ($\mathcal{O}(w_{\text{IR}}^2)$). The mass-spectrum is essentially *gapped*.

- As pointed out before, the scale of the effective potential is set by the $U(1)_R$ breaking induced by the deformation of the conifold as measured in the UV, $\epsilon^2(r_{\text{UV}}) \propto |z|^2$. Strictly speaking this is *not* a warp factor suppression, although for moderate CY volumes $|z|$ and w_{IR}^3 are of the same order [16].

The following caveats should be noted: The effective Lagrangians (3.13) and (3.15) are incomplete: We have worked in the regime of classical type IIB solutions so at least the universal Kähler modulus T is not yet stabilized. Moreover, we have not included the b-axion that complexifies c. Finally, there is no parametric separation between the mass scale of the complex structures and the warped Kaluza-Klein scale. Hence, the Lagrangian (3.13) does not define a useful effective field theory in the Wilsonian sense. Equation (3.15) however *does* give rise to a Wilsonian effective Lagrangian once it is completed by the b-axion and the Kähler modulus T.

3.2.5 The B_2-axion

In the preceding sections we have focused on the ultralight c-axion that can be thought of as the integral of the RR two-form C_2 over a sphere between the two throats. Similarly, we can define a b-axion by integrating the NS two-form B_2 instead,

$$b \equiv \frac{1}{2\pi\alpha'} \int_{S^2} B_2 \,. \tag{3.16}$$

[16] One might for instance be tempted to consider the large volume limit where warping becomes negligible. In this case the scale of the potential would still be given by $|z|^2 \ll 1$.

By the same arguments as before (see (3.5)) a non-vanishing field excursion induces a pair of H_3 flux/anti-flux on the portions of the $\mathcal{B}$-cycle that reach down into the two throats. Now, in the vacuum the $\mathcal{B}$-cycle is already filled with quantized H_3-flux,

$$K \equiv K_1 + K_2 \equiv \frac{1}{(2\pi)^2\alpha'} \left(\int_{\mathcal{B}_1} H_3 + \int_{\mathcal{B}_2} H_3 \right) . \tag{3.17}$$

Here $\mathcal{B}_1$ and $\mathcal{B}_2$ are the three-chains that reach into the respective throats and are bounded by the sphere between the throats, so that $\mathcal{B} = \mathcal{B}_1 + \mathcal{B}_2$. Clearly the continuous field excursion of the b-axion does not change the quantized flux integer K. However, by Stokes' theorem it *does* change the relative flux distribution,

$$K_1 \longrightarrow K_1 + \frac{b}{2\pi} , \quad K_2 \longrightarrow K_2 - \frac{b}{2\pi} . \tag{3.18}$$

By definition, K_1 and K_2 are the (non-quantized) H_3 fluxes that reside in the respective throats. Again, treating the local throat deformation parameters $z_{1,2}$ as independent it is clear that the throats can restore supersymmetry by an appropriate adjustment [96]:

$$|z_{1,2}| \sim \exp\left(-2\pi \frac{K_{1,2}}{g_s M}\right) \quad \longrightarrow \quad \exp\left(-2\pi \frac{K_{1,2} \pm b/2\pi}{g_s M}\right) . \tag{3.19}$$

Thus, the discussion of the previous section applies also to the b-axion if one replaces the phases of the local deformation parameters by $\ln|z_i|$. In other words, while the c-axion rotates the throats against each other, the b-axion makes one throat longer and the other shorter (see Fig. 3.7).

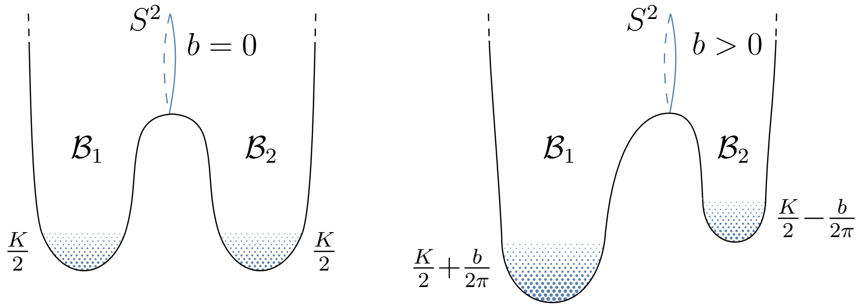

Figure 3.7: The physical effect of a field excursion of the b-axion in the double throat system. One throat becomes shorter, whereas the other becomes longer.

Expanding on the above, we can now comment on the interesting difference between the 5d and 4d perspective on excitations of $\int_{S^2} B_2$ in the local throat. This will in particular facilitate comparison with the related discussion in [120].

We start by noting that, instead of the 4d field b defined by an integral inbetween the two throats, one may also consider the 5d field $b(r) \sim \int_{S^2} B_2$ within either of the throats. Here r is the radial location of the relevant S^2.[17] Away from the tip of the throat and from the bulk CY, where the 5d language can be used, one has a continuum of solutions to the 10d supergravity equations [115]. In particular, there is a continuum of solutions for $b(r)$, parameterized by z via the boundary condition $b\left(r = |z|^{1/3}\right) = 0$. Two such solutions are plotted in Fig. 3.8. The relevant 5d equation for a static solution is, symbolically,

$$\left[\partial_r^2 - f(r)\partial_r - m^2(r)\right] b(r) = 0\,, \tag{3.20}$$

where $m(r)$ is the 5d mass. One immediately sees that the non-trivial (non-constant) profile of $b(r)$ is enforced by the non-zero $m^2(r)$. This non-zero potential comes from the Chern-Simons term $\int F_5 \wedge B_2 \wedge F_3$. One might be concerned that such a potential clashes with the presence of an ultralight (in the present approximation massless) 4d mode of b. But such concerns are unfounded: Indeed, the 4d flat direction parameterized by b is also present in the 5d description, in spite of the non-zero 5d potential. It corresponds to a change of the whole $b(r)$-profile within the available continuum of solutions accompanied by a change of boundary condition, e.g., $z \to z'$, cf. Fig. 3.8. The variable z is, of course, not accessible to a local observer in 5d at some fixed position r_*.

Finally, when gluing together throats to create a double throat one expects a small potential for the b-axion due to the misalignment of the *magnitudes* of the two deformation parameters, compare the discussion of Sect. 3.2.3 in the C_2-case. We will make a quantitative statement in Sect. 3.4.

3.3 Derivation and Solution of the 5d Equations of Motion

This section contains more detailed calculations to supplement the qualitative arguments of the previous section. The main result is the action (3.11) and the solutions of the corresponding equations of motion.

[17] In order to avoid introducing additional symbols, by a slight abuse of notation, we will denote by $b(r)$ the 5d mode while b without radial argument denotes the 4d mode.

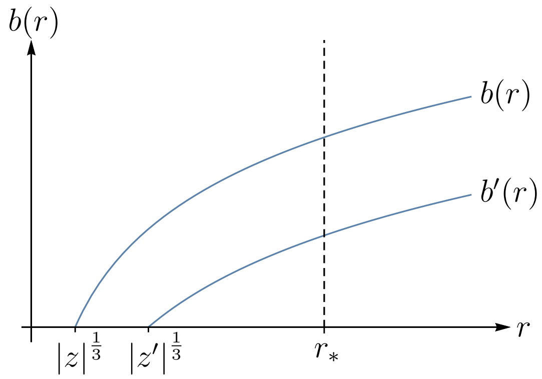

Figure 3.8: Two solutions $b(r) \sim \int B_2$ to the supergravity equations of motion in the Klebanov-Tseytlin throat. The 4d flat direction of the potential corresponds to a change of solution.

3.3.1 The 5d Action of the Interpolating Mode ϕ

In App. A.1 we explain how the phase φ of the complex structure modulus z enters the metric to leading order in $|z|$. In this subsection we now want to explicitly derive how we arrive at the action (3.11) from this. The 10d metric (A.15) can be decomposed into the external and radial five-dimensional metric and the internal angular metric

$$\begin{aligned} G_{MN} &= \left(w^2\eta_{\mu\nu} \oplus w^{-2}g_{rr}\right) \oplus \left(w^{-2}r^2 g_{\Omega,mn}\right)\,, \quad m,n = \phi_{1.2},\theta_{1,2},\psi\,, \\ g_{\Omega,mn} &= g_{T^{1,1},mn} + \epsilon(r)h_{mn}\,, \\ g_{T^{1,1}}{}_{mn}dy^m dy^n &\equiv d\Omega^2_{T^{1,1}}\,, \quad h_{mn}\mathrm{d}y^m\mathrm{d}y^n = d\Omega^2_5(\varphi)\,, \end{aligned} \tag{3.21}$$

where $\epsilon(r) = |z|/r^3$ is the $U(1)_R$-breaking parameter. As explained in Sect. 3.2.3, we promote φ to a dynamical field $\varphi \to \phi(x^\mu, r)$. While this field extends into multiple throats, we make use of its antisymmetry in order to reduce the problem to a single throat. The corresponding boundary conditions are given below.

We expand the 5d action for ϕ in ϵ via the expansion of the 10d Ricci scalar about the $U(1)_R$-preserving metric

$$\begin{aligned} R(G) &= R\left(G_{U(1)_R}\right) + \epsilon(r)\nabla_m\nabla_n h^{mn} - \Box\left(\epsilon(r)\,g^{mn}_{T^{1,1}}h_{mn}\right) + \mathcal{O}(\epsilon^2)\,, \\ G_{U(1)_R} &= w^2\eta \oplus w^{-2}g_{rr} \oplus w^{-2}r^2 g_{T^{1,1}}\,, \\ G_{5\mathrm{d}} &= w^2\eta \oplus w^{-2}g_{rr}\,. \end{aligned} \tag{3.22}$$

We know that for $\phi = \varphi =$ const. we simply describe a deformed conifold with general complex deformation parameter $z = |z|\,e^{i\varphi}$, which is by construction Ricci-flat. Therefore, we have $R(G) = 0$ in every order in ϵ, such that the only non-vanishing contribution can come from derivative terms of ϕ

$$\begin{aligned}\Box(\epsilon(r)\, g_{T^{1,1}}^{mn} h_{mn}) &= \epsilon(r)\, g_{T^{1,1}}^{mn}\, G_{5\mathrm{d}}^{ij}\, \partial_i\partial_j h_{mn}(\phi(x^\mu, r)) \\ &= \epsilon(r)\, g_{T^{1,1}}^{mn} \left[\partial_\phi h_{mn}\, G_{5\mathrm{d}}^{ij}\, \partial_i\partial_j\phi + \partial_\phi^2 h_{mn}\, G_{5\mathrm{d}}^{ij}\, \partial_i\phi\, \partial_j\phi\right] .\end{aligned} \tag{3.23}$$

We can write

$$g_{T^{1,1},mn}\mathrm{d}x^m\mathrm{d}x^n = \frac{1}{6}\left(\left(g^1\right)^2 + \left(g^2\right)^2 + \left(\left(g^3\right)^2 + \left(g^4\right)^2\right)\right) + \frac{1}{9}\left(g_5\right)^2 . \tag{3.24}$$

With this and looking at the perturbation h_{mn} above, one finds that to order ϵ, there is no contribution to the 5d action. This is due to the fact that there is no non-zero contraction $g_{T^{1,1}}^{mn}\partial_\phi h_{mn}$ or $g_{T^{1,1}}^{mn}\partial_\phi^2 h_{mn}$. There is no term $g^1g^4 - g^2g^3$ in $g_{T^{1,1}}$ and the terms coming from contractions of $\left(g^1\right)^2 + \left(g^2\right)^2$ and $\left(g^3\right)^2 + \left(g^4\right)^2$ cancel due to the different signs between these terms in $g_{T^{1,1}}$ and h [18] The first non-vanishing order is ϵ^2.

Using $\sqrt{G} \approx \sqrt{G_{U(1)_R}} = w^{-2}r^5$, the following terms may now arise up to order ϕ^2 after integrating out the angular directions in the 10d action

$$S_{5\mathrm{d}} = \frac{1}{g_s^2\kappa_{10}^2}\int \mathrm{d}^4x\, \mathrm{d}r\; r^5 w(r)^{-2}\left(R_{5\mathrm{d}} + \Lambda(r) - \frac{1}{2}\epsilon(r)^2\, G_{5\mathrm{d}}^{ij}\partial_i\phi\partial_j\phi\right) . \tag{3.25}$$

Here, we have implemented 5d Lorentz symmetry and used the fact, that there are no linear terms in ϕ once we go to order ϵ^2. Importantly there a no non-derivative terms for ϕ, because $\phi = \text{const.}$ is a point in complex moduli space. Dropping 5d gravity, we arrive at

$$S_{5\mathrm{d}} = \frac{1}{g_s^2\kappa_{10}^2}\int \mathrm{d}^4x \int_{r_{\mathrm{IR}}}^{r_{\mathrm{UV}}} \mathrm{d}r\, \epsilon(r)^2 r^5\left(-\frac{1}{2}w(r)^{-4}\left(\partial_\mu\phi\right)^2 - \frac{1}{2}\left(\partial_r\phi\right)^2\right) . \tag{3.26}$$

Explicity calculating $R(G)$ with $\varphi \to \phi(x^\mu, r)$ and expanding to first order in $|z|$ and second order in ϵ confirms this explicitly.

3.3.2 Schrödinger Equations and Exact Solutions for Free Fields

Using the previous subsection and the results of [40], we collect the two actions [19] that will give us all relevant parameters. Note that we derived the

[18] In other words: The term in $g_{T^{1,1}}$ is symmetric under the exchange of the $SU(2)$'s defining the cone, $(\phi_1, \theta_1) \leftrightarrow (\phi_2, \theta_2)$ in coordinates, while the term in h is antisymmetric.

[19] For ease of exposition we work with only two real fields although strictly speaking they are both real components of complex fields (see Sect. 3.4).

action for ϕ for $c = 0$ and that the one for c assumes $\phi = 0$. We deal with the form of interaction terms in App. A.2.

$$\begin{aligned} S_\varphi &= \frac{1}{g_s^2 \alpha'^4} \int \mathrm{d}^4 x \int \frac{\mathrm{d}r}{r}\, \epsilon(r)^2\, r^6 \left(-\frac{1}{2} w(r)^{-4} \left(\partial_\mu \phi_\varphi\right)^2 - \frac{1}{2} \left(\partial_r \phi_\varphi\right)^2 \right) , \\ S_c &= \frac{1}{\alpha'^2} \int \mathrm{d}^4 x \int \frac{\mathrm{d}r}{r}\, r^2\, w(r)^4 \left(-\frac{1}{2} w(r)^{-4} \left(\partial_\mu \phi_c\right)^2 - \frac{1}{2} \left(\partial_r \phi_c\right)^2 \right) , \end{aligned} \tag{3.27}$$

where we use a general notation with fields $\phi_i(x^\mu, r)$, $i = \varphi, c$. The equations of motion using the plane wave ansatz $\phi_i(x^\mu, r) = e^{i p_i x} \chi_i(r)$, with $p_i^2 = -m_i^2$, read

$$\begin{aligned} w(r)^4 \epsilon(r)^{-2} r^{-5} \partial_r \left(\epsilon(r)^2 r^5 \partial_r \chi_\varphi(r) \right) &= -\, m_\varphi^2 \chi_\varphi(r) , \\ r^{-1} \partial_r \left(r w^4 \partial_r \chi_c(r) \right) &= -\, m_c^2 \chi_c(r) . \end{aligned} \tag{3.28}$$

We insert the warp-factor (A.14), dropping logarithmic dependencies, and the $U(1)_R$-breaking factor to arrive at

$$\begin{aligned} r^5 \partial_r \left(\frac{1}{r} \partial_r \chi_\varphi(r) \right) &= -\, R^4\, m_\varphi^2 \chi_\varphi(r) , \\ \frac{1}{r} \partial_r \left(r^5 \partial_r \chi_c(r) \right) &= -\, R^4\, m_c^2 \chi_c(r) , \end{aligned} \tag{3.29}$$

where we introduced the abbreviation $R = \sqrt{g_s M \alpha'}$ for the normalization of the warp factor of KT and KS. Using the reparameterization

$$\begin{aligned} x_\varphi &= \frac{R^2\, m_\varphi}{r} , \qquad g_\varphi(x_\varphi) = \frac{1}{r} \chi_\varphi(r(x_\varphi)) , \\ x_c &= \frac{R^2\, m_c}{r} , \qquad g_c(x_c) = r^2 \chi_c(r(x_c)) , \end{aligned} \tag{3.30}$$

we arrive at the following form of the equations of motion

$$x_i^2 g_i''(x_i) + x_i g_i'(x_i) + \left(x_i^2 - \alpha_i^2 \right) g_i(x_i) = 0 , \tag{3.31}$$

where $i = \varphi, c$ and where $\alpha_\varphi = 1$ and $\alpha_c = 2$. This is the standard form of the Bessel equation. Plugging back in our reparameterization, we write down the general solutions to the equations of motion

$$\begin{aligned} \chi_\varphi(r) &= r \left(A\, J_1 \left(\frac{R^2\, m_\varphi}{r} \right) + B\, Y_1 \left(\frac{R^2\, m_\varphi}{r} \right) \right) , \\ \chi_c(r) &= \frac{1}{r^2} \left(A\, J_2 \left(\frac{R^2\, m_c}{r} \right) + B\, Y_2 \left(\frac{R^2\, m_c}{r} \right) \right) , \end{aligned} \tag{3.32}$$

with Bessel functions J_{α_i} and Y_{α_i} of first and second kind respectively.

We now have to apply the boundary conditions

$$\begin{aligned} \chi_\varphi(r_{\mathrm{UV}}) &= 0\,, & \partial_r\chi_\varphi(r_{\mathrm{IR}}) &= 0\,, \\ \partial_r\chi_c(r_{\mathrm{UV}}) &= 0\,, & w_{\mathrm{IR}}\partial_r\chi_c(r_{\mathrm{IR}}) &= \frac{\delta}{R}\chi_c(r_{\mathrm{IR}})\,, \end{aligned} \tag{3.33}$$

for some $\mathcal{O}(1)$-number δ depending on how we model the flux distribution in the IR. While the UV boundary conditions are easily implemented to fix the constants A and B up to an overall normalization, the ansatz $m_i \propto \frac{w_{\mathrm{IR}}^2}{R}$ gives consistent solutions to the IR boundary conditions when using the small argument expansions of the Bessel functions

$$\begin{aligned} m_\varphi^2 &= 8\frac{w_{\mathrm{IR}}^4}{R^2}\,, \\ m_c^2 &= \frac{8\delta}{4+\delta}\frac{w_{\mathrm{IR}}^4}{R^2}\,, \end{aligned} \tag{3.34}$$

where we now normalized the warp factor $w_{\mathrm{UV}} = 1$. We plot the solutions with $\delta = 1$, $w_{\mathrm{IR}} = e^{-10/3} \sim 10^{-2}$ and m_i as above in Fig. 3.9.

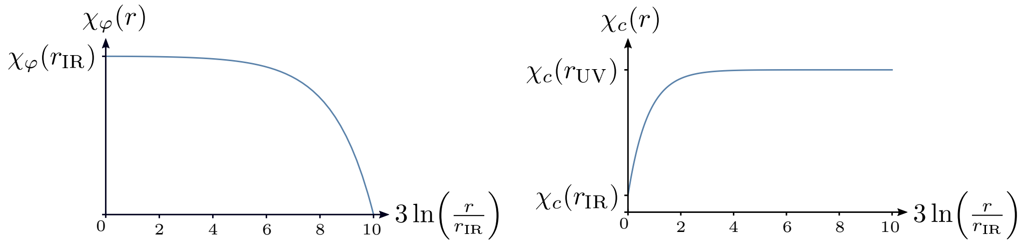

Figure 3.9: The lowest lying radial eigenmodes for φ and c for $w_{\mathrm{IR}} \sim 10^{-2}$ plotted in the physical radial coordinate $t = 3R\ln(r/r_{\mathrm{IR}})$.

Referring back to the discussion in Sect. 3.2.3, we plot the expected solutions over the entirety of the double throat in Fig. 3.10, where we simply mirrored one throat at the UV end with appropriate symmetry, so both $r = r_{\mathrm{IR}}$ and $r = 2r_{\mathrm{UV}} - r_{\mathrm{IR}}$ correspond to IR regions.

We are left with calculating the 4d kinetic terms. From Fig. 3.9, we would guess (correctly) that φ is an IR mode, while c is a UV mode. We derive this by considering the lowest KK mode $\phi_0(x^\mu, r) = f_0(x^\mu)\chi_0(r)$, where f_0 is now an arbitrary superposition of plane waves, to find the kinetic terms induced by the actions (3.27)

$$\begin{aligned} f_\varphi^2 &= \frac{1}{g_s^2\alpha'^4}\int_{r_{\mathrm{IR}}}^{r_{\mathrm{UV}}}\frac{\mathrm{d}r}{r}\,\epsilon(r)^2\,r^6\,w(r)^{-4}\chi_\varphi(r)^2 \approx \frac{R^6}{g_s^2\alpha'^4}w_{\mathrm{IR}}^2\,, \\ f_c^2 &= \frac{1}{\alpha'^2}\int_{r_{\mathrm{IR}}}^{r_{\mathrm{UV}}}\frac{\mathrm{d}r}{r}\,r^2\,\chi_c(r)^2 \approx \frac{R^2}{\alpha'^2}\,. \end{aligned} \tag{3.35}$$

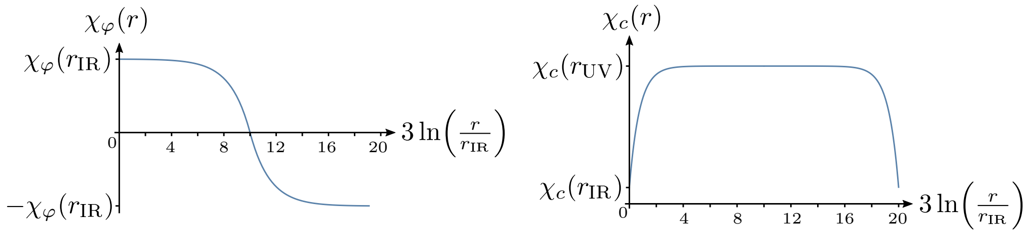

Figure 3.10: The eigenmodes for φ and c on the double throat. Here, $t < 10$ corresponds to the first throat, $t > 10$ corresponds to the second, with $t = 0$ and $t = 10$ corresponding to the respective ends of the throats.

Here, we have again expanded the solution in $w_{\mathrm{IR}} \ll 1$, and neglected logarithmic corrections. Furthermore we have normalized the internal field profiles such that $\chi_c(r_{\mathrm{UV}}) = \chi_\varphi(r_{\mathrm{IR}}) = 1$.

Comparing with the quadratic terms in c in the potential (3.7) and the quadratic terms in φ_i in the potential (3.9), we find the parameters μ and Λ

$$\mu^4 = m_c^2\, f_c^2 \propto w_{\mathrm{IR}}^4\,, \qquad \Lambda^4 = m_\varphi^2\, f_\varphi^2 \propto w_{\mathrm{IR}}^6\,. \tag{3.36}$$

3.3.3 Axion Decay Constant and Potential Parameters

In this subsection we extend the results by including logarithmic corrections to the first-order approximations and by taking care of the different length scales that may appear in the parameters (3.14) of the effective potential (3.10).

We compute the axion decay constant (without accounting for monodromy factors) for the c-axion from the 10d action, paying attention to numerical factors of $\mathcal{O}(1)$. We may dimensionally reduce the $|F_3|^2$ term of the 10d type IIB SUGRA and plug in $C_2 = 2\pi\alpha' c(x)\omega_\Sigma$, where ω_Σ is the associated (quasi-)harmonic form. This leads to

$$\frac{1}{(2\pi)^7\alpha'^4}\left(-\frac{1}{2}\int *\,|F_3|^2\right) = \int \mathrm{d}^4x\left(-\frac{1}{2}f^2(\partial c)^2\right), \tag{3.37}$$

with

$$\begin{aligned} f^2 &= (2\pi)^2 g_s^2\,\frac{\int_{\mathrm{CY}}\mathrm{d}^6y\,\sqrt{g_6}\,w^2\,\alpha'^2\,|\omega_\Sigma|^2}{\int_{\mathrm{CY}}\mathrm{d}^6y\,\sqrt{g_6}\,w^2}\,M_{\mathrm{P}}^2 \\ &\equiv (2\pi)^2 g_s^2\,\frac{\alpha'^2}{(\mathrm{Vol}(S^2)|_{\mathrm{UV}})^2}\,M_{\mathrm{P}}^2\,, \end{aligned} \tag{3.38}$$

where g_s is the string coupling, g_6 is the internal 6d metric, w^2 is the warp factor and M_{P} is the four-dimensional Planck mass. Due to the appearance of the warp factor, the integrals are dominated by UV contributions. Thus, for large values of the overall volume $\mathcal{V}$ we have $f^2 \sim g_s \mathcal{V}^{-2/3}$ in Planck units, where N_{D3} is the total D3 brane charge stored in fluxes (and mobile D3 branes). We expect to be allowed to make the string frame volume as small as $\mathcal{O}((g_s N_{\text{D3}})^{3/2})$. At even smaller volume back-reaction of fluxes becomes significant throughout the CY and we lose the notion of an unwarped bulk CY [96]. For simplicity we now take into consideration only the D3 brane charge stored in a single throat, i.e., $N_{\text{D3}} = MK$. In this regime the top of the throat marginally fits into the bulk CY and we expect the ratio of integrals in (3.38) to be well approximated by the value of $\alpha'^2 \, |\omega_\Sigma|^2$ at the top of the KS throat. Since the hierarchy is set by $\ln w_{\text{IR}}^{-1} \sim \frac{K}{g_s M}$ it follows immediately that f scales as $f \sim \sqrt{\frac{g_s}{MK}} M_{\text{P}} \sim \frac{1}{M} \ln(w_{\text{IR}}^{-1})^{-1/2} M_{\text{P}}$.

We can compute this in more detail by using the asymptotic form of the throat metric far away from the deformation, i.e., the KT metric (3.4)

$$
\begin{aligned}
\mathrm{d}s_6^2 &= w^{-2}(\mathrm{d}r^2 + r^2 \mathrm{d}s^2_{T^{1,1}}) \,, \quad \mathrm{d}s^2_{T^{1,1}} = \frac{1}{9}(g^5)^2 + \frac{1}{6}\sum_{i=1}^{4}(g^i)^2 \,, \\
w^2 &= \frac{2\sqrt{2}}{9}\frac{r^2}{g_s M \alpha'} \ln(r/r_{\text{IR}})^{-\frac{1}{2}} \,, \quad \omega_\Sigma = \frac{1}{8\pi}(g^1 \wedge g^2 + g^3 \wedge g^4) \,.
\end{aligned}
\tag{3.39}
$$

We see, that the physical radius at the top of the throat is given by

$$
R^2_{\text{throat}} = \frac{9}{2\sqrt{2}} g_s M \alpha' \sqrt{\ln r_{\text{UV}}/r_{\text{IR}}} \approx \frac{9}{2\sqrt{2}} g_s M \alpha' \sqrt{\ln w_{\text{IR}}^{-1}} \,, \tag{3.40}
$$

where in the last step we use the normalized warp factor with $w_{\text{UV}} = 1$. Taking into account angular dependencies of $T^{1,1}$, we arrive at $\alpha' \, |\omega_\Sigma| = (3\pi g_s M)^{-1} \ln(w_{\text{IR}}^{-1})^{-\frac{1}{2}}$, and hence

$$
f \approx \frac{2}{3M} \ln(w_{\text{IR}}^{-1})^{-1/2} M_{\text{P}} \,. \tag{3.41}
$$

Actually, we are interested in the case where two or more throats each store only part of the total D3 brane charge N_{D3}. We will assume that the this total charge is not parametrically larger than the fluxes (M_i, K^i) stored in a given throat i to write

$$
f \lesssim \frac{2}{3M_i} \ln((w^i_{\text{IR}})^{-1})^{-1/2} M_{\text{P}} \,, \tag{3.42}
$$

and this expression is correct when evaluated for any $i = 1, ..., n$. The upper bound is replaced by an (approximate) equality when introducing the ratio of fluxes in the throat to the overall charge.

In the double throat case with evenly distributed fluxes, $(M_1, K^1) = (M_2, K^2)$, and assuming that these fluxes dominate the total charge, $N_{\text{D3}} \approx M_1 K^1 + M_2 K^2$, the effective (monodromy-)potential (3.10) reads

$$V(c)/M_{\text{P}}^4 \sim w_{\text{IR}}^6 \left(1 - \cos\left(3\sqrt{\ln(1/w_{\text{IR}})}\frac{c}{M_{\text{P}}}\right)\right) . \tag{3.43}$$

Even in this marginal case, the decay constant of c is slightly sub-Planckian in regimes of parametric control since then $w_{\text{IR}} \ll 1$.

We may also derive the quantities given in (3.14): Using (3.27) the kinetic term of φ is given by

$$f_\varphi^2 \sim \frac{(g_s M\alpha')^6}{g_s^2 \alpha'^4} w_{\text{IR}}^2 \sim \frac{(g_s M)^3}{\mathcal{V}} w_{\text{IR}}^2 M_{\text{P}}^2 , \tag{3.44}$$

where we use that the integral for the kinetic term (3.35) is dominated by the IR, such that we may insert the IR throat radius, $R^2 \sim g_s M\alpha'$, and where we have made use of the fact that $M_{\text{P}}^2 \sim g_s^{-2}\alpha'^{-1}\mathcal{V}$, with the (bulk) CY volume $\mathcal{V}$ as measured in string units. Neglecting geometric back-reaction of the double throat system in the UV, the flux energy density $M^2\mu^4$ can be understood as a pure IR effect: It is induced by an excursion of $\varphi_{1,2}$ in the IR whilst keeping c fixed (for the factor of M^2 compare (3.7)). Therefore the resulting mass must be of the order of the warped KK-scale $m_{\text{wKK}}^2 \sim R^{-2} w_{\text{IR}}^2 \sim (g_s M\alpha')^{-1} w_{\text{IR}}^2$

$$\mu^4 \sim \frac{1}{M^2} f_\varphi^2 m_{\text{wKK}}^2 \sim \frac{g_s^4}{\mathcal{V}^2} w_{\text{IR}}^4 M_{\text{P}}^4 . \tag{3.45}$$

Finally, the scale Λ^4 is dominated by contributions from the bulk CY (which we again assume to have only one length-scale, $R_{\text{CY}}^2 \sim \mathcal{V}^{1/3}\alpha'$), so

$$\begin{aligned}\Lambda^4 &\sim \underbrace{\frac{1}{g_s^2}\frac{1}{\alpha'^4}}_{\sim M_{\text{P},10\text{d}}^8} \quad \underbrace{\mathcal{V}\alpha'^3}_{\sim \int \mathrm{d}^6 y \sqrt{g_{\text{CY}}}} \quad \underbrace{\epsilon^2(r_{\text{UV}})}_{\sim \text{UV tail of deformation}} \quad \underbrace{\frac{1}{\mathcal{V}^{1/3}\alpha'}}_{\sim \mathcal{L}_{10\text{d}} \supset (\nabla\phi)^2 \propto 1/R_{\text{CY}}^2} \\ &\sim \frac{g_s^2}{\mathcal{V}^{4/3}} \epsilon^2(r_{\text{UV}}) M_{\text{P}}^4 \sim \frac{g_s^2}{\mathcal{V}^{4/3}} \ln\left(w_{\text{IR}}^{-1}\right)^{-3/2} w_{\text{IR}}^6 M_{\text{P}}^4 .\end{aligned} \tag{3.46}$$

Here, we have made use of the fact that the symmetry breaking coefficient in the UV is $\epsilon^2(r_{\text{UV}}) \sim (r_{\text{IR}}/r_{\text{UV}})^6 \sim w_{\text{IR}}^6 \ln\left(w_{\text{IR}}^{-1}\right)^{-3/2}$. The scale Λ^4 determines the axion potential.

Going to the limit where the throats marginally fit into the bulk CY means taking $\mathcal{V} \sim (g_s N_{\mathrm{D3}})^{3/2} \sim (g_s M)^3 \ln\big(w_{\mathrm{IR}}^{-1}\big)^{3/2}$. In this limit,

$$\begin{aligned}
\Lambda^4/M_{\mathrm{P}}^4 &\sim \frac{g_s^2}{(g_s M)^4} \ln\big(w_{\mathrm{IR}}^{-1}\big)^{-7/2} w_{\mathrm{IR}}^6 \,, \\
\mu^4/M_{\mathrm{P}}^4 &\sim \frac{g_s^4}{(g_s M)^6} \ln\big(w_{\mathrm{IR}}^{-1}\big)^{-3} w_{\mathrm{IR}}^4 \,, \\
f_\varphi^2/M_{\mathrm{P}}^2 &\sim \ln\big(w_{\mathrm{IR}}^{-1}\big)^{-3/2} w_{\mathrm{IR}}^2 \,.
\end{aligned} \tag{3.47}$$

3.4 Four-Dimensional SUGRA Completion

So far we have discussed how the c-axion back-reacts on the phases of the local deformation parameters of the throats. In this section we propose a completion of the model in the language of 4d supergravity. The C_2-axion pairs with the analogous B_2-axion into a complex field $\mathcal{G} = c - \tau b$. The b-axion back-reacts on the *magnitudes* of the deformation parameters in a way that is analogous to the back-reaction of the c-axion on their phases.

3.4.1 Counting Moduli Through the Conifold Transition

Throughout Sect. 3.2 we have focused on the case of two S^3-cycles related in homology, i.e., $[\mathcal{A}_1] = [\mathcal{A}_2]$. In general we denote by n the number of collapsing three-spheres $\mathcal{A}^i$, $i = 1, ..., n$ and by m the number of homology relations between them $\sum_{i=1}^n p_i^I [\mathcal{A}^i] = 0$, $I = 1, ..., m$.

Before fluxes are turned on and orientifold projections are imposed the physical degrees of freedom assemble into $\mathcal{N} = 2$ multiplets. The $n-m$ complex structure moduli z^i are the scalar components of $n-m$ vector multiplets. The z^i parameterize the Coulomb branch of the gauge theory. Whenever some of the three-cycles shrink to zero size, charged hypermultiplets (*Strominger black holes*) become massless and have to be 'integrated in' [138]. These can be thought of as D3-branes that wrap the shrunken cycles. At the origin of the Coulomb branch there are hence n massless charged hypermultiplets and n singular nodes have developed in the CY threefold. There exists an m-dimensional Higgs-branch where the singular nodes are resolved into m (homologically independent) $\mathbb{P}^1$'s [139]. On this branch, the $n-m$ vector multiplets eat $n-m$ black brane hypermultiplets and become massive. Geometrically speaking this is the resolution of the conifold [123, 124].

In the $\mathcal{N} = 1$ flux compactification that we are considering the tips of the conifolds become strongly red-shifted. Moreover back-reaction of fluxes ensures that even at tiny complex structure the S^3's stay at finite size so that the Strominger black holes play no role. However, since the deformed and the resolved conifold differ only by their strongly red-shifted tip geometries we expect to recover some remnant of the resolved phase of the conifold theory in the light spectrum. As outlined in Sect. 3.2 we expect the 'local complex structures' to decouple from one another so that all the n local deformation parameters $z_1, \ldots, z_n$ become equally light. In other words there are m additional light geometric modes. Moreover, on the resolved side of the transition there would be m massless axion modes. Since the obstruction for them to be massless is also localized at the tips of the conifolds where the would-be two-cycles collapse we also expect m complex light axionic modes $\mathcal{G}^I$. As we will argue in the next section, these modes indeed appear quite naturally in the discussion of the flux superpotential.

3.4.2 The Thraxion Superpotential

In this section we make a proposal for the 4d supergravity completion of the Lagrangians (3.13) and (3.15) for a general number of throats n with m homology relations among the shrinking cycles. Throughout this section we work in units $M_{\mathrm{P}} = 1$.

The GVW Superpotential of a Multi Conifold System

As a starting point we consider the GVW flux superpotential for a multi conifold system. All the necessary ingredients are derived in [140] and summarized in App. A.3. We choose to treat the redundant set of the n complex structure parameters z_i associated with the n vanishing cycles $\mathcal{A}^i$ democratically, and impose the m *CY conditions* via a set of Lagrange multipliers λ_I, $I = 1, .., m$. The superpotential reads

$$W(z) = \sum_{i=1}^{n} \Big(M_i \frac{z_i}{2\pi i} \ln(z_i) + M_i g^i(z) - \tau K^i z_i \Big) + \sum_{I=1}^{m} \lambda_I P^I + \hat{\hat{W}}_0(z) \,. \quad (3.48)$$

The m homology relations among the vanishing cycles $\sum_{i=1}^{n} p_i^I \mathcal{A}^i = \partial \mathcal{C}^I$, $I = 1, ..., m$ lead to the following m CY conditions for the $z_i \equiv \int_{\mathcal{A}^i} \Omega$,

$$\begin{aligned} 0 \overset{\mathrm{d}\Omega=0}{=} \int_{\mathcal{C}^I} \mathrm{d}\Omega = \int_{\partial \mathcal{C}^I} \Omega = \sum_{i=1}^{n} p_i^I \int_{\mathcal{A}^i} \Omega \\ = \sum_{i=1}^{n} p_i^I z_i \equiv P^I \,, \quad I = 1, ..., m \,. \end{aligned} \quad (3.49)$$

In this language, the m CY conditions $P^I = 0$ are equivalent to the F-term equations of the Lagrange multipliers λ_I, $\partial_{\lambda_I} W \overset{!}{=} 0$. For details we refer the reader to App. A.3.[20]

Here, the M_i and K^i are the flux numbers associated to the $\mathcal{A}$- and $\mathcal{B}$-cycle of the i-th throat, and the holomorphic function $\hat{\hat{W}}_0(z)$ denotes contributions to the flux superpotential from other cycles. The $M_i \in \mathbb{Z}$ cannot all be chosen independently but must comply with the m homology conditions

$$\sum_{i=1}^{n} p_i^I M_i = 0\,, \quad I = 1, ..., m\,. \tag{3.50}$$

The K^i can be chosen independently but there is an m-fold redundancy in their definition because we may transform $K^i \longrightarrow K^i + \sum_I \alpha_I p_i^I$ for any $\alpha \in \mathbb{C}^m$ leaving the superpotential invariant upon imposing the constraint equations[21]. Furthermore, there are n unknown functions $g^i(z)$ defined on complex structure moduli space that are holomorphic near the origin.

[20] Note that we restrict ourselves to regions in complex structure moduli space close to the conifold transition point, where all throats degenerate simultaneously, compare the discussion in Sect. 3.2.1. This might be more restrictive than is needed for our analysis: If the matrix p_i^I is block-diagonal, we can separate the multi throat system into smaller multi throats whose deformations are independent of one another. In this case we can go through a conifold transition by local degeneration of the throats of a smaller system. Even away from the trivial case of multi throats factorizing, one might be able to achieve small thraxion masses by 'freezing' individual throats with larger deformation z. Given a multi throat with some large z's one has to check the thraxion potential as proposed in this section for flat directions. We leave a more thorough analysis of this possibility for future work.

[21] The $n-m$ physical H_3 flux quantization conditions can be stated as $K^a - \sum_{I=1}^m p_a^I K^{n-m+I} \in \mathbb{Z}$, $a = 1, \ldots, n-m$. This is because we can always choose the first $n-m$ of the shrinking cycles to correspond to integral basis elements $[\mathcal{A}^1], ..., [\mathcal{A}^{n-m}]$ in homology. The Lagrange constraints can be stated as $0 = P^I = \sum_{a=1}^{n-m} p_a^I z_a + z_{n-m+I}$, i.e., $z_{n-m+I} = -\sum_{a=1}^{n-m} p_a^I z_a$. In the superpotential the terms that multiply $z_1, ..., z_{n-m}$ are given by the above combination of K^i and correspond to the integer flux numbers on the cycles $\mathcal{B}_1, ..., \mathcal{B}_{n-m}$. Alternatively, one may demand the sufficient but not necessary conditions that $K^i \in \mathbb{Z}$ for $i = 1, \ldots, n$. In this more restrictive but democratic formulation the i-th throat carries K^i units of flux. We can still reach all possible integer values for flux numbers on the cycles $\mathcal{B}_a$.

The Kähler potential is given by

$$
\begin{aligned}
K_{\mathrm{cs}}(z_i, \bar{z}_i) &= -\ln\Big(-i\int \Omega \wedge \bar{\Omega}\Big) \\
&= -\ln\left(ig_K(z) - i\overline{g_K(z)} + \sum_{a=1}^{n-m} i\bar{z}_a G^a + c.c.\right) \\
&= -\ln\Big(ig_K(z) - i\overline{g_K(z)} \\
&\qquad + \sum_{i=1}^{n}\left[\frac{|z_i|^2}{2\pi}\ln(|z_i|^2) + i\bar{z}_i g^i(z) - iz_i\overline{g^i(z)}\right]\Big) ,
\end{aligned}
\tag{3.51}
$$

where the holomorphic function $g_K(z)$ encodes contributions from other cycles. We would like to stress that despite the fact that we have written the unknown functions g^i, g_K and $\hat{\hat{W}}_0$ as functions of all the z_i, $i = 1, ..., n$, knowledge of the periods of the various cycles (and the flux quanta) only determines their behavior *along* complex structure moduli space and not beyond.

The Thraxion as a Stabilizer Field

We are now ready to formulate a proposal for the thraxion superpotential. First, we note the following. By expanding the Lagrange multiplier terms, one may rewrite the superpotential (3.48) as

$$
\begin{aligned}
W(z) = &\sum_{i=1}^{n}\left(M_i \frac{z_i}{2\pi i}\ln(z_i) + M_i g^i(z) + \left[-\tau K^i + \sum_{I=1}^{m}\lambda_I p_i^I\right] z_i\right) \\
&+ \hat{\hat{W}}_0(z) .
\end{aligned}
\tag{3.52}
$$

One observes immediately that the combinations $\sum_{I=1}^{m}\lambda_I p_i^I$ can be interpreted as an *additional*, unquantized contribution to the complex three-form flux $G_3 = F_3 - \tau H_3$ on the (local portion $\tilde{\mathcal{B}}^i$ of the) $\mathcal{B}$-cycle of the i-th conifold. But we know that such a flux is detected by a boundary integral

$$
\hat{\mathcal{G}}_i \equiv c_i - \tau b_i \equiv \frac{1}{2\pi\alpha'}\int_{S^2|_{i\text{-th throat}}}(C_2 - \tau B_2) = \frac{1}{2\pi\alpha'}\int_{\tilde{\mathcal{B}}^i}(F_3 - \tau H_3) \tag{3.53}
$$

over the S^2 at the top of the i-th throat. Crucially, the variables $\hat{\mathcal{G}}_i$ define axionic field excursions as measured near the entrance of the i-th throat.

We would like to interpret (a subset of) these as light physical degrees of freedom. This is motivated by the fact that there are m light axions on the other side of the conifold transition that correspond to the integrals of $C_2 - \tau B_2$ over the independent resolution 2-cycles. Indeed, the counting is correct. A consistent axionic field excursion must not induce any overall flux on *any* of the global $\mathcal{B}$-cycles (see Fig. 3.11). There are hence $n-m$ *no-flux* conditions, one for each linearly independent $\mathcal{B}$-cycle, leaving only m physical axions. These can be parameterized as $\hat{\mathcal{G}}^i = \sum_{I=1}^m p_i^I \mathcal{G}_I$ and we are led to the following conjecture:

> *The Lagrange multipliers λ_I must be promoted to m light axionic degrees of freedom, $\lambda_I \to \frac{\mathcal{G}_I}{2\pi}$. Moreover, the z^i are promoted to n physically independent degrees of freedom.*

The normalization factor 2π is chosen such that locally in the i-th throat a shift of axionic field excursion $\mathcal{G}_I$ by 2π (or $2\pi\tau$) for some I is indistinguishable from an increase of the F_3-flux (respectively H_3-flux) on the $\mathcal{B}$-cycle of the throat by an integer amount p_I^i.

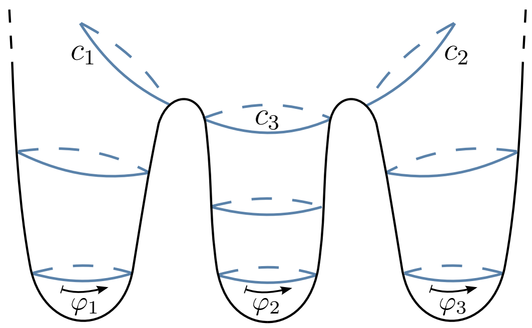

Figure 3.11: A cartoon of the $\mathcal{B}$-cycle of the triple throat, $n = 3$ and $m = 2$. The local c-axion excursions c_1, c_2 and c_3, $c_i = \mathrm{Re}\,\hat{\mathcal{G}}_i$, must be chosen so that no overall flux is generated on $\mathcal{B}$, i.e., $0 \overset{!}{=} c_1 + c_2 + c_3$ [22] or rather $\sum_i \hat{\mathcal{G}}^i = 0$.

Thus, our proposal for the superpotential is

$$W = \sum_{i=1}^{n} \left(M_i \frac{z_i}{2\pi i} \ln(z_i) + M_i g^i(z) - \tau K^i z_i \right) - \sum_{I=1}^{m} \frac{\mathcal{G}_I}{2\pi} P^I + \hat{\hat{W}}_0(z)\,. \quad (3.54)$$

We find it interesting to note that the axions $\mathcal{G}^I$ now serve as the *stabilizer fields* for the combinations of the local deformation parameters that break

[22] Compare this to Fig. 3.5: The 'no-flux' condition in the double throat setup amounts to $c_1 = -c_2$. The two axions c_1 and c_2 are actually identified, up to a sign due to different orientation of the two-sphere in the definition. This is why we only had one axion c to begin with in the 10d analysis of Sect. 3.2.

the m CY conditions $P^I = 0\,,\ I = 1, ..., m\,.$ This form of the dynamical thraxion superpotential is fairly unique in that it preserves the set of *discrete* shift-symmetries

$$\begin{gathered} z_i \longrightarrow z_i e^{\frac{2\pi i}{M_i}\sum_I \eta_I p_i^I}\,, \quad \mathcal{G}_I \longrightarrow \mathcal{G}_I + 2\pi\eta_I \\ \forall \eta \in \mathbb{C}^m\ : \sum_I \eta_I p_i^I \in M_i\mathbb{Z} \quad \forall i\,. \end{gathered} \tag{3.55}$$

Our proposal for the Kähler potential is

$$K(\mathcal{G}_I, \bar{\mathcal{G}}_{\bar{I}}, T, \bar{T}, z, \bar{z}) = K_1(\mathcal{G}_I - \bar{\mathcal{G}}_{\bar{I}}, T + \bar{T}) + K_{\rm cs}(z, \bar{z})\,, \tag{3.56}$$

where $K_{\rm cs}$ is the Kähler potential (3.51) and K_1 is the Kähler potential of the m axions (and Kähler moduli T) on the other side of the conifold transition as derived in [141]

$$K_1 = K_0 - 3\ln\!\left(T + \bar{T} - \frac{3i}{4(\tau - \bar{\tau})}\kappa_{1IJ}(\mathcal{G} - \bar{\mathcal{G}})^I(\mathcal{G} - \bar{\mathcal{G}})^J\right)\,, \tag{3.57}$$

where K_0 contains a constant part and the Kähler potential of the axio-dilaton and κ_{1IJ} are triple intersection numbers.[23]

We expect (3.51) and (3.54) to hold even when we break the CY condition $P^I \neq 0$ with the important subtlety that the domain of the holomorphic functions g^i, g_K and $\hat{\hat{W}}_0$ must be extended *beyond* complex structure moduli space. We find it reasonable to expect that such an extension *exists* although even full knowledge of the CY periods would not determine their behavior away from the moduli space. The detailed form of these functions will be of no importance in what follows. Moreover, we expect that using the potential K_1 gives an excellent approximation because the kinetic term of the axions is dominated by contributions from the UV where the deformation or resolution of the conifold plays but a tiny role. Note, that the behavior of the kinetic terms of the complex structure moduli is dominated by the logarithmic terms in (3.51). In particular, the functions $g^i(z)$ and $g_K(z)$ contribute to kinetic terms only at sub-leading order.

[23] For ease of exposition we have stated the Kähler potential for the case $h^+_{1,1} = 1\,.$ For $\tau =$ const. the Kähler potential fulfills the no-scale relation $(\partial_T K_1)(K_1^{-1})^{T\bar{T}}(\partial_{\bar{T}} K_1) = 3\,,$ and $(K_1^{-1})^{\mathcal{G}^I \bar{X}^{\bar{j}}}\partial_{\bar{X}^{\bar{j}}} K_1 = 0$ where $X^i = (T, \mathcal{G}^I)\,.$

Since we are interested in small z_i we Taylor-expand

$$\begin{aligned} g^i(z) =& g_0^i + \sum_{j=1}^{n} g_1^{ij} z_j + \mathcal{O}(z^2)\,, \\ \hat{\hat{W}}(z) =& g_{W,0} + \sum_{i=1}^{n} g_{W,1}^i z_i + \mathcal{O}(z^2)\,, \\ g_K(z) =& g_{K,0} + \sum_{i=1}^{n} g_{K,1}^i z_i + \mathcal{O}(z^2)\,. \end{aligned} \tag{3.58}$$

This should really be understood as a Taylor expansion in n *independent* variables z_i and makes our conjectured extension of the domain of these functions beyond the complex structure moduli space manifest.

We absorb all $\mathcal{O}(z^0)$ terms in the superpotential in the definition $\hat{W}_0 \equiv g_{W,0} + \sum_{i=1}^{n} M_i g_0^i$. The coefficients in (3.58) should all be viewed as independent of the flux quanta that thread the cycles of the multi throat system, and only $(g_{W,0}, g_{W,1}^i)$ depend on fluxes on other cycles.

The Double Throat: $n = 2$, $m = 1$

It is clear that to obtain the effective superpotential for the $\mathcal{G}$-fields we should integrate out the local deformation parameters. Before we discuss this in full generality it is instructive to first consider the simplest case of the double throat, i.e., $n = 2$ and $m = 1$. There are two $\mathcal{A}$-cycles $\mathcal{A}^1$ and $\mathcal{A}^2$ and we choose the homology relation to be $\mathcal{A}^1 \sim \mathcal{A}^2$. Hence, there are two deformation parameters z_1 and z_2 and one axion $\mathcal{G}$. For ease of exposition we assume that of all the coefficients defined in (3.58), only $g_{W,0}$ and $g_{K,0}$ are non-vanishing, in other words, we choose $\hat{W}_0$ as well as all non-logarithmic terms in the Kähler potential to be constant. In doing so we accept an $\mathcal{O}(1)$ error in all expressions, in particular in the resulting superpotential W_{eff} for $\mathcal{G}$. This simplifying assumption will be dropped when we generalize the discussion to the multi throat case in Sect. 3.4.2.

We must set $M_1 = M_2 \equiv M$ due to the homology relation between the shrinking cycles, and we choose $K^1 \equiv K/2 \equiv K^2$ which results in flux $K^1 + K^2 = K$ on the $\mathcal{B}$-cycle. All choices of the pair (K^1, K^2) that satisfy $K^1 + K^2 = K$ are physically equivalent to this choice and can be brought back to the symmetric choice via a linear redefinition of $\mathcal{G}$ (compare the discussion below (3.50)). The superpotential takes the form

$$\begin{aligned} W(z_1, z_2, \mathcal{G}) = &\sum_{i=1}^{2} \left(\frac{z_i}{2\pi i} \ln(z_i) M - \frac{1}{2} K \tau z_i \right) - \frac{\mathcal{G}}{2\pi}(z_1 - z_2) \\ &+ \hat{W}_0 + \mathcal{O}(z_i^2)\,. \end{aligned} \tag{3.59}$$

First, the F-terms F_{z_i} are given by

$$\begin{aligned} D_{z_i} W = \partial_{z_i} W + (\partial_{z_i} K) W &= \frac{\ln(z_i) + 1}{2\pi i} M - \frac{K}{2}\tau \mp \frac{\mathcal{G}}{2\pi} + \mathcal{O}(z_i) \\ &= \frac{M}{2\pi i}\left(\ln(z_i/z_0) \mp i\mathcal{G}/M\right) + \mathcal{O}(z_0)\,, \end{aligned} \tag{3.60}$$

where

$$z_0 = e^{-1} \exp\left(2\pi i \tau \frac{K/2}{M}\right) + \mathcal{O}\left(e^{-4\pi \frac{K/2}{g_s M}}\right) = \mathcal{O}\left(e^{-2\pi \frac{K/2}{g_s M}}\right)\,. \tag{3.61}$$

As usual, with $K/2 > g_s M$ one obtains $|z_0| \ll 1$ with universal dependence on the flux numbers. Following Sect. 3.2 we may integrate out the local deformation parameters, which yields

$$z_1 = z_0 e^{i\mathcal{G}/M}\,, \quad z_2 = z_0 e^{-i\mathcal{G}/M}\,. \tag{3.62}$$

The effective superpotential for the axion $\mathcal{G}$ reads

$$\begin{aligned} &W_{\text{eff}}(\mathcal{G}) = 2\epsilon\left(1 - \cos(\mathcal{G}/M)\right) + W_0 + \mathcal{O}(z_0^2)\,, \\ \text{with} \quad &\epsilon \equiv M\frac{z_0}{2\pi i}\,, \quad \text{and} \quad W_0 \equiv \hat{W}_0 - \frac{z_0}{\pi i} M\,. \end{aligned} \tag{3.63}$$

This is the expression we were after. Crucially, it is consistent with the results of Sect. 3.2: Using the Kähler potential for the Kähler moduli and $\mathcal{G}$-axions (3.57), one may show that $V(\mathcal{G}, \bar{\mathcal{G}}) \propto |\partial_{\mathcal{G}} W(\mathcal{G})|^2$ [24]. If we restrict to $\mathcal{G} = c \in \mathbb{R}$, we reproduce the periodic potential (3.10) with the correct scaling $|\epsilon|^2 \sim |z_0|^2 \sim w_{\text{IR}}^6$.

Note that because we have made use of the *unwarped* Kähler potential we do not reproduce the correct mass-scale of the local deformation parameters z_i. Here, this is of no importance because all degrees of freedom that are related to strongly warped regions are integrated out supersymmetrically. In particular, the potential energy induced by a non-vanishing field excursion of the field $\mathcal{G}$ receives its dominant contributions from the bulk CY where warping plays no role. Because in going from weak to strong warping, the

[24] Where we use that K_1 is of no-scale type.

solutions of the complex structure F-terms are left invariant [97], and because the z_i are parametrically heavier than $\mathcal{G}$ even when the appropriate red-shift factors are introduced in the scalar potential, this procedure is justified.

We are now ready to expand on the conclusions we have drawn in Sect. 3.2. First of all, the kinetic term of the full complex field $\mathcal{G}$ lives in the bulk. This implies that the squared mass of $\mathcal{G}$ is of order $|z_0|^2 \ll 1$. Since the Kähler potential is independent of $\mathrm{Re}(\mathcal{G})$ a discrete shift-symmetry $\mathcal{G} \longrightarrow \mathcal{G}+2\pi M$ is manifest [25], while the IR superpotential breaks the shift-symmetry corresponding to $\mathrm{Im}(\mathcal{G})$ completely.

While in principle the target space distance traversed by $\mathrm{Im}(\mathcal{G})$ can be made large, the scalar potential grows exponentially as a function thereof as is common for saxionic directions in field space. In particular, this direction in field space is of little use for (slow-roll) inflation. There is a *critical* field excursion $|\mathrm{Im}(\mathcal{G})_{\mathrm{crit}}| \lesssim 3M\ln(w_{\mathrm{IR}}^{-1})$ beyond which one side of the double throat is entirely pulled up into the bulk CY, $z_1 \sim 1$ or $z_2 \sim 1$. Near this field excursion we no longer know the form of the potential because we work to lowest order in $|z_1|, |z_2|$. Moreover, there is a tower of warped KK-modes with masses that scale as

$$m_n^2 \sim n^2 w_{\mathrm{IR}}^2 \exp\left(-2\left|\mathrm{Im}(\mathcal{G})\right|/3M\right) , \tag{3.64}$$

where the warp factor $w^2 \sim |z_i|^{2/3}$ now depends on $\mathrm{Im}(\mathcal{G})$. Since these modes have been integrated out, the ratio $\Lambda/m_{\mathcal{G}}$ of the cut-off of the $\mathcal{G}$-EFT (i.e., the smallest KK-mass) over the mass-scale of $\mathcal{G}$, comes down as

$$\Lambda/m_{\mathcal{G}} \sim w_{\mathrm{IR}}^{-2} e^{-\frac{4}{3M}|\mathrm{Im}(\mathcal{G})|} \leq w_{\mathrm{IR}}^{-2} \exp\left(-\frac{\phi_b}{M_{\mathrm{P}}}\right) , \tag{3.65}$$

at large field excursion, consistent with a *distance conjecture* [12, 48, 142, 86, 143, 87, 88, 144]. Here ϕ_b measures the canonical field distance from the origin along the imaginary $\mathcal{G}$-axis [26].

At strong warping the maximal allowed field excursion is $|\mathrm{Im}(\mathcal{G})_{\mathrm{max}}| \sim \frac{3}{2}M\ln(w_{\mathrm{IR}}^{-1})$ before the 4d EFT description breaks down. Near this field excursion, a large fraction of the reservoir of fluxes of one of the throats has been transferred to the other one and the mass scale of the $\mathcal{G}$-field is of the same order as the warped KK-scale of the longer throat. At this point, contributions to the scalar potential from non-vanishing F-terms $D_{z_i}W$ start

[25] In an exact no-scale background the scalar potential even has periodicity πM. However, any no-scale breaking effects will break it to the periodicity of the superpotential.

[26] See Sect. 3.3.3 for the conversion rule between $\mathcal{G}$ and the canonical distance in field space ϕ_b. At any point in field space, $g_{\mathcal{G}\overline{\mathcal{G}}} < M_{\mathrm{P}}^2/M^2$. Hence, $\phi_b = \int_0^{\mathrm{Im}(\mathcal{G})} \mathrm{d}\,\mathrm{Im}(\mathcal{G})' \sqrt{g_{\mathcal{G}\overline{\mathcal{G}}}} < \mathrm{Im}(\mathcal{G})\, M_{\mathrm{P}}/M$.

to play a significant role, or from a 10d point of view, the potential energy sinks down into the longer throat.

The General Multi Throat

For general m and n that satisfy $n - m > 0$ homology relations, there are n local deformation parameters $z_1, ..., z_n$ that have to be integrated out. We are left with an effective supergravity theory of m axions $\mathcal{G}^I$. The computational steps are analogous to the double throat case that was laid out in detail. Hence, we only state the effective axion superpotential

$$W_{\text{eff}}(\mathcal{G}^I) = -\sum_{i=1}^{n} \epsilon_i \, \exp\left(i \sum_{I=1}^{m} \frac{p_I^i \mathcal{G}^I}{M_i} \right) + \hat{W}_0 \,, \tag{3.66}$$

and we have defined

$$\begin{aligned} \epsilon_i &\equiv \frac{M_i}{2\pi i} z_{0,i}(1 - 2\pi \bar{\tilde{g}}_0^i \hat{W}_0/(aM_i)) \,, \\ \tilde{g}_0^i &\equiv g_0^i - \overline{g_{K,1}^i} \,, \quad a \equiv -2\text{Im}(g_{K,0}) \ , \end{aligned} \tag{3.67}$$

and

$$\begin{aligned} z_{0,i} = e^{-1} \exp\left(-\frac{2\pi i}{M_i} \left(\sum_j M_j g_1^{ji} + g_{W,1}^i + i\frac{\bar{\tilde{g}}_0^i \hat{W}_0}{a} \right) \right) \exp\left(2\pi i \frac{K^i \tau}{M_i} \right) \\ + \mathcal{O}\left(e^{-4\pi \frac{K^i}{g_s M_i}} \right) \,. \end{aligned} \tag{3.68}$$

It is important to note that the $z_{0,i}$ as defined above can in general *not* be interpreted as the values of the local deformation in the vacuum. The physical local deformation parameters are given by

$$z_{\text{ph.},i} \equiv z_{0,i} \exp\left(i \sum_I p_I^i \mathcal{G}^I / M_i \right) \ , \tag{3.69}$$

where in the vacuum the $\mathcal{G}^I$ need not vanish in general.

3.4.3 Comments on the b-Axion

In the above supergravity completion we have 'complexified' the c-axion by pairing it with the analogous b-axion. We have outlined the 10d back-reaction of the b-axion already in Sect. 3.2.5. Now that we have addressed the scalar potential of the b-axion quantitatively, in this section we would like to comment on a potential worry and how to resolve it: We recall that the effect of a non-vanishing field excursion of the b-axion is the creation of a pair of fluxes of the NS field strength H_3. Since both throats are filled up with H_3-fluxes already in the vacuum one should think about this process more properly as a transfer of H_3-flux from one throat to the other. Since the magnitudes of the local deformation parameters (and the associated hierarchy) are set by the ratio of local H_3-flux (on the $\mathcal{B}$-cycle) and F_3-flux (on the $\mathcal{A}$-cycle) it is clear that these will back-react when the H_3-fluxes are redistributed, see (3.19).

However, it is also clear that when the local H_3-fluxes are changed, the circumference of the throat at the UV end is affected strongly. This is because it is set by the total D3-charge that is stored in the throat which is itself proportional to the amount of H_3-flux [49], compare Fig. 3.7. Hence, naively one might worry that such considerable change at the UV ends of the throats could lead to a large potential energy. One may convince one-self that this is not the case as follows. Starting from the supersymmetric situation we can redistribute a small amount of fluxes from one throat to the other, so that throat A has δ units of H-flux more than throat B. We can now proceed to convert the extra fluxes into a number of D3-branes by going through the Kachru-Pearson-Verlinde transition [145]. From the UV perspective this process is only detected by a change in the throat complex structure which is a tiny perturbation far from the tip of the throat. Now we are back to an even flux distribution with a number of mobile D3-branes. These can be moved out of the throat at no cost in energy so the situation with the mobile branes should be a vacuum again. In other words, the redistribution of fluxes creates an energy density that is *only* due to the misalignment of local deformation parameters and the change of size of the throats at their UV ends does not generate an extra contribution to the potential. We reiterate that the situation is analogous to the back-reaction of the c-axion with the phases of the local deformation parameters replaced by the logarithms of their magnitude.

Finally, note that in the Kähler potential (3.57) the b-axion appears explicitly, while the approximate c-axion shift symmetry is manifest. One might suspect that the small scale of the b-axion is therefore accidental due to our use of tree level supergravity. This conclusion would be incorrect: The target space manifold with Kähler metric derived from (3.57) is shift symmetric also in the b-direction [146,147]. In general we expect both shift symmetries to be preserved to all orders in the perturbative expansion with explicit breaking only due to the superpotential (3.66). When moreover *non-perturbative* Kähler moduli dependent terms are generated in the superpotential such as the ones considered in [58], we expect the b-axion mass to be lifted to the scale of Kähler moduli stabilization while the c-axion can remain parametrically lighter.

3.4.4 A Possible $\mathcal{N} = 2$ Extension

As already mentioned in Sect. 3.4.1, we expect that before turning on fluxes the setup should be consistent without orientifolding and should therefore obey $\mathcal{N} = 2$ supersymmetry. That is, when setting the fluxes M_i and K^i as well as fluxes away from the throat to zero in (3.54),

$$W_{\text{no flux}}(\mathcal{G}_I) = -\sum_{I=1}^{m} \frac{\mathcal{G}_I}{2\pi} P^I \,, \tag{3.70}$$

we should find a superpotential consistent with the higher amount of supersymmetry and with the complex scalars $\mathcal{G}_I$ now part of a larger multiplet.

A complex scalar in $\mathcal{N} = 2$ SUSY may be part of hyper- or vectormultiplets. As the m light fields $\mathcal{G}_I$ considered become genuine axions on the resolved side of the conifold transition, we expect them to be part of the $h^{1,1} + m$ hypermultiplets. In this, they should be paired with the complexified Kähler moduli T^I of the m 4-cycles emerging under the transition. The superpotential of a hypermultiplet with complex scalars H_1 and H_2 does allow for a mass term of the form [27]

$$W_{\text{hyper}} = M\, H_1 H_2 \,. \tag{3.71}$$

The interpretation that arises is: Under the conifold transition, the m originally massless hypermultiplets with scalars $(\mathcal{G}_I, T^I)$ become massive, with the functions $P^I(z)$ of all local deformation parameters z to be interpreted as T^I. Indeed, enforcing $P^I = 0$ should result in well-known throats with (complexified) two- and four-cycle volumes forced to 0 (they are however not actual parameters in this description).

As discussed around (3.52), going to the fluxed case requires $M = 1/(2\pi)$ to allow for the consistent interpretation of the fields $\mathcal{G}_I$ as measures of local fluxes. This fact remains mysterious to us. Fully expanding the setup consistently to $\mathcal{N} = 2$ supersymmetry is beyond the scope of this work.

[27] This mass M is given by a VEV $g\langle v\rangle$, with g the coupling and v the scalar of a vector multiplet under which the hypermultiplet is charged [148].

3.5 The Axion Potential and Gauge/Gravity Correspondence

We have derived the axion potential via a classical computation within 10d SUGRA, and proposed a 4d SUGRA description that matches it. Since the local throats are believed to have a dual description in terms of KS gauge theories [49], it is useful to give an alternative derivation of our results on the gauge theory side of the correspondence. The KS gauge theory is a $SU(N+M) \times SU(N)$ gauge theory with (classical) global symmetry group $SU(2) \times SU(2) \times U(1)_R$. It contains matter in bi-fundamental representations $(\square, \overline{\square})$ and $(\overline{\square}, \square)$ of the gauge group that transform as doublets under the first respectively second global $SU(2)$ factor. The holomorphic gauge couplings of the two gauge theory factors τ_{YM} and $\tilde{\tau}_{YM}$ have been argued to be set by [134, 149]

$$\tau_{\mathrm{YM}} + \tilde{\tau}_{\mathrm{YM}} = \tau \,, \quad \tau_{\mathrm{YM}} - \tilde{\tau}_{\mathrm{YM}} = -\tau + \mathcal{G}/\pi \mod 2(m - n\tau)\,, \tag{3.72}$$

with $(m, n) \in \mathbb{Z}^2$. The radial running of the $\mathcal{G}$-field together with $\tau = \text{const.}$ matches the RG-running of the gauge theory coupling constants. Throughout this section $\mathcal{G}$ takes values in its suitable fundamental domain.

As the KS gauge theory flows to the infrared, it undergoes repeated steps of Seiberg dualities that reduce the ranks of the gauge groups according to

$$\begin{aligned} SU(N_0 + M) \times SU(N_0) &\longrightarrow SU(N_1 + M) \times SU(N_1) \\ &\longrightarrow \cdots \longrightarrow SU(N_k + M) \times SU(N_k)\,, \end{aligned}$$

with $N_k \equiv N - kM$, $k \in \mathbb{N}$. If we start with $N = KM$, after K steps in the duality cascade the gauge group is $SU(M)$. Since, roughly speaking, it corresponds to the first gauge group factor in $SU(M) \equiv SU(N_K + M) \times SU(N_K)$, its holomorphic scale is given by $\Lambda^{3M} = \mu_{\mathrm{IR}}^{3M} \exp\left(2\pi i \tau_{\mathrm{YM}}(\mu_{\mathrm{IR}})\right) = \mu_{\mathrm{IR}}^{3M} \exp(i\mathcal{G})$, where μ_{IR} is the infrared scale of the throat and where we make use of the KS dictionary $\tau_{\mathrm{YM}} \simeq \mathcal{G}/2\pi$. Gaugino condensation leads to an effective Affleck-Dine-Seiberg (ADS) superpotential [150, 151, 49]

$$\begin{aligned} W_{\mathrm{ADS}}(\mathcal{G}) = M\Lambda^3 &\sim M\mu_{\mathrm{IR}}^3 \exp(2\pi i \tau_{\mathrm{YM}}/M) \\ &\sim M\mu_{\mathrm{UV}}^3 \exp\left(\frac{2\pi i}{M}\left(\tau K + \frac{\mathcal{G}}{2\pi}\right)\right)\,, \end{aligned} \tag{3.73}$$

where we have used that the IR-scale is related to the UV-scale by $\mu_{\mathrm{IR}}^3 = \mu_{\mathrm{UV}}^3 \exp(-2\pi \frac{K}{g_s M})$.

The superpotential that we have proposed on the gravity side of the correspondence (3.66) indeed takes this form,

$$W \propto \sum_{i=1}^{n} M_i A_i \exp\left(\frac{2\pi i}{M_i}\left(\tau K^i + \frac{\hat{\mathcal{G}}^i}{2\pi}\right)\right) + \hat{W}_0\,, \tag{3.74}$$

with $\hat{\mathcal{G}}^i = \sum_{I=1}^{m} p_I^i \mathcal{G}^I$, and [28]

$$\begin{aligned} A_i \equiv \exp\Bigg(-\frac{2\pi i}{M_i}\Bigg(&\sum_{j=1}^{n} M_j g_1^{ji} + g_{W,1}^i + i\bar{\tilde{g}}_0^i \hat{W}_0/a\Bigg) \\ &+\ln\left(1-\frac{2\pi\bar{\tilde{g}}_0^i}{aM_i}\hat{W}_0\right)\Bigg)\,. \end{aligned} \tag{3.75}$$

From the gauge theory perspective we should interpret the appearance of the constants $g_1^{ji}, g_{W,1}^i$, $\tilde{g}_0^i$ and $\hat{W}_0$ as a parameterization of threshold corrections near the UV cut-off [29]. Indeed, as they are taken to zero the A_i become unity.

It is now obvious that the M-fold extension of the periodicity of the c-axion is related to gaugino condensation in the KS gauge theory [30] [115, 49, 154,149]: As usual there is a $U(1)_R$ symmetry that is broken to $\mathbb{Z}_{2M}$ by gauge theory instantons. Gaugino condensation spontaneously breaks $\mathbb{Z}_{2M} \to \mathbb{Z}_2$, so there are M gauge theory vacua. As we transform $c \to c + 2\pi$, we move from one gauge theory vacuum to the next, and the gaugino condensate (which corresponds to the local deformation parameters on the gravity side) picks up a phase $\exp(2\pi i/M)$. This is as in Sect. 3.2.2 where we learned that the M different vacua are reached by dialing the RR flux quanta on the $\mathcal{B}$-cycle $Q = 0, \ldots, M-1$ (see (3.3)).

3.6 Applications

3.6.1 Thraxions on the Quintic: Drifting Monodromy

In this section we will give a concrete example of a string compactification where a light thraxion appears. Along the way we identify concrete setups in which parametrically super-Planckian racetrack-type axion periodicities are possible. We choose the CY to be the quintic threefold which is defined

[28] Note that in (3.66) we have set $M_{\rm P} = 1$. Therefore we identify $\mu_{\rm UV} \sim M_{\rm P}$.

[29] Of course these are in general functions of all other complex structure moduli that do not control the infrared regions of the throats and are frozen at a high scale.

[30] Related observations were made in the non-compact flux-less multi-node setting of [152, 153].

as the vanishing locus of a homogeneous polynomial P of rank five in $\mathbb{P}^4$. Following [140], it can be brought to a conifold transition point by choosing the complex structures such that

$$P(X^0, ..., X^4) = X^3 f_4(X^0, ..., X^4) + X^4 g_4(X^0, ..., X^4)\,, \tag{3.76}$$

where f_4 and g_4 are generic homogeneous rank four polynomials of the projective coordinates $\{X^0, ..., X^4\}$ of $\mathbb{P}^4$. The conditions $P = 0, \mathrm{d}P = 0$ are satisfied whenever

$$0 = X^3 = X^4 = f_4(X^0, X^1, X^2, 0, 0) = g_4(X^0, X^1, X^2, 0, 0)\,. \tag{3.77}$$

Since f_4 and g_4 are chosen to be generic polynomials of rank 4, there exist $4 \cdot 4 = 16$ distinct solutions. These are 16 conifold points. Hence, there are 16 vanishing three-cycles $\mathcal{A}^i$, $i = 1, ..., 16$. Because the solution set lies on a $\mathbb{P}^2$ submanifold of $\mathbb{P}^4$, there is precisely one homology relation among them,

$$\sum_{i=1}^{16} [\mathcal{A}^i] = 0\,. \tag{3.78}$$

Hence, we have a multi throat system with $n = 16$ and $m = 1$ so there is one light axion.

Let us give two examples that differ by choices of flux numbers. In both examples we set the coefficients A_i defined in (3.75) to unity. Generically we expect these to be of order one. Inserting $\mathcal{O}(1)$ factors below does not change the physical outcome.[31]

Example 1: A simple thraxion potential

$$M_i = (-1)^{i+1} M\,, \quad K^i = (-1)^{i+1} K\,, \tag{3.79}$$

with $K/g_s M \gg 1$. Then we have $\epsilon_i \equiv (-1)^{i+1}\epsilon$, and

$$W_{\text{eff}}(\mathcal{G}) = -16 i \epsilon \sin 2\mathcal{G}/M + \hat{W}_0 = 16 i \epsilon (1 - \cos 2\mathcal{G}'/M) + W_0\,, \tag{3.80}$$

with $W_0 = \hat{W}_0 - 16\epsilon$, $\mathcal{G}' = \mathcal{G} - \pi M/4$, and small $|\epsilon| \propto \exp(-2\pi K/g_s M)$. Up to the numerical prefactor this is exactly what we found for $n = 2$ and $m = 1$.

Example 2: Drifting Monodromy We now slightly detune the F_3 fluxes from one another:

$$M_1 = M\,, \quad M_2 = M + 1\,, \quad M_3 = -M\,, \quad M_4 = -(M + 1)\,, \tag{3.81}$$

[31] If some coefficients can be tuned parametrically smaller than others, new qualitative features might arise. We leave an investigation of this possibility to future research.

and $M_{i+4} = M_i$, with $K^i \equiv \mathrm{sign}(M_i)\,|K|$, and again $K/g_sM \gg 1$. In this case

$$W_{\text{eff}}(\mathcal{G}) \approx -8iz_0(M\sin(\mathcal{G}/M) + (M+1)\sin(\mathcal{G}/(M+1))) + \hat{W}_0\,, \tag{3.82}$$

with $z_0 \sim \exp(2\pi i K\tau/M)$. Additionaly to the previous simplification, we have also neglected order one prefactors that arise from the fact that the ratios K^i/M_i are not all exactly equal. Again, this is of no consequences for our purposes.

The superpotential (3.82) is a *racetrack*-type superpotential[32] for $\mathcal{G}$. The axion periodicity is now given by $2\pi M(M+1)$. Crucially, this implies another M-fold extension of the axion field range *on top* of the one already discussed in the simpler examples of the double throat and the first example of this section. Clearly, one may take this even further to periodicities such as $2\pi M\cdots(M+3)$.[33] Since we still only have to fulfill the requirement that the throats fit into the bulk CY, this implies the existence of a simple, concrete and explicit mechanism in string theory that can generate huge super-Planckian axion periodicities. In general the full periodicity of the superpotential is given by the least common multiple of the different RR flux numbers M_i. We dub this mechanism of generating a parametrically large axion monodromy *drifting monodromies* since it relies on a frequency drift within a set of several finite-order monodromy effects. This is related but different from the winding idea, where a constraint forces the effective axion on a long trajectory in a multi-axion moduli space [156, 157, 32, 117, 118]. Here, by contrast, one may think of a single fundamental axion extended by several small, finite-order monodromy effects. The result of this can still be large as explained above. The intended outcome, namely to realize an effective large-f axion accepting a short-wavelength oscillatory potential, is of course the same (see in particular the recent analysis of [121]).

The minima of the potential $V \propto |\partial_{\mathcal{G}} W|^2$ are located along the slice $\mathrm{Im}(\mathcal{G}) = 0$ where it takes the form

$$\begin{aligned} V(c) &\propto [\cos(c/M) + \cos(c/(M+1))]^2\,, \quad c \equiv \mathrm{Re}\,\mathcal{G}\,, \\ &\propto \cos^2\!\left(\frac{2M+1}{2M(M+1)}c\right)\cdot\cos^2\!\left(\frac{1}{2M(M+1)}c\right)\,, \end{aligned} \tag{3.83}$$

which has $2M+1$ distinct Minkowski vacua (see Fig. 3.12).

We note that despite the long $2\pi M(M+1)$ periodicity the scalar potential oscillates on shorter wavelengths of order $2\pi M$. This is essentially due

[32] For a discussion of racetrack superpotentials in connection with the WGC, see [155].

[33] But not further because we have to respect the orientifold action.

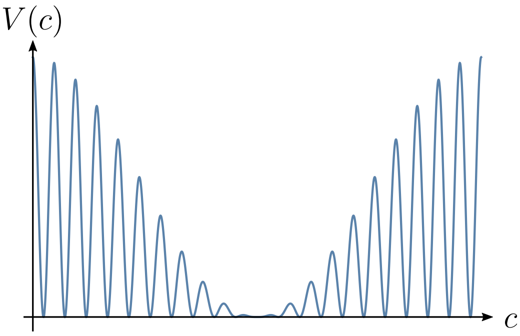

Figure 3.12: The axion potential of example 2 for the case $M = 10$. There are sub-Planckian oscillations within a long super-Planckian envelope.

to the rank condition (3.50) which forces us to introduce flux numbers of both signs.[34] We have not shown in general that suppressing such shorter wavelength oscillations in order to produce a smooth super-Planckian axion potential is impossible. At this point we only note that the condition (3.50) presents a severe obstacle towards this.

These examples also serve to illustrate that by scanning over flux numbers one may obtain a vast number of possible effective superpotentials and axionic potentials.

3.6.2 A Clash with the Weak Gravity Conjecture

In this section we would like to point out that the axion potential we have derived clashes with the weak gravity conjecture for axions [17, 30, 33], see Sect. 2.2.3. We have computed the axion potential via a classical supergravity calculation. However, one may equally well associate it to non-perturbative effects in the KS gauge theory (namely gaugino condensation), as argued in Sect. 3.5. As such, (if true) the weak gravity conjecture should apply to our construction.

In its form adapted to axions and instantons the conjecture states that there should exist an instanton with Euclidean action S and axionic charge q such that

$$S \leq \mathcal{O}(1)\, q\, M_{\mathrm{P}}/f_{\mathrm{eff}}\,. \tag{3.84}$$

Such instantons (if they contribute to the superpotential) generically induce terms in the scalar potential of the form

$$V(\phi) \supset e^{-S}(1 - \cos(q\,\phi/f_{\mathrm{eff}}))\,, \tag{3.85}$$

[34] This condition is *global* in the sense that it need not hold in a non-compact CY where gravity is decoupled.

where ϕ is the canonically normalized axion. Thus, $2\pi f_{\text{eff}}/q$ is the canonically normalized periodicity of the term in the (super)potential that is generated by a given instanton.

By comparison with the above we may associate an (effective) Euclidean instanton action to each of the leading exponentially suppressed terms in the axion (super)potential [35].

$$S^i_{\text{eff}} \approx 3\ln(1/w^i_{\text{IR}}) \approx 2\pi \frac{K^i}{g_s M_i}\,. \tag{3.86}$$

As computed in Sect. 3.3.3, in the regime where the throats marginally fit into the bulk CY, the periodicities f_{eff}/q^i of the dominant terms in the superpotential associated to each throat $i = 1, ..., n$ read

$$f_{\text{eff}}/q_i \approx \frac{2}{3}\ln(1/w^i_{\text{IR}})^{-1/2} M_{\text{P}} \approx \frac{2}{3}\left(\frac{2\pi}{3}\frac{K^i}{g_s M_i}\right)^{-1/2} M_{\text{P}}\,, \tag{3.87}$$

Hence,

$$S^i_{\text{eff}} \cdot f_{\text{eff}}/q^i \sim 2\sqrt{\ln(1/w^i_{\text{IR}})}\, M_{\text{P}} \approx 2\sqrt{\frac{2\pi}{3}\frac{K^i}{g_s M_i}} M_{\text{P}}\,. \tag{3.88}$$

In the regime $w_{\text{IR}} \ll 1$ (i.e., $K^i \gg g_s M_i$) the r.h. side is parametrically larger than $\mathcal{O}(1)$ so the objects that generate the relevant terms in the superpotential do not satisfy a weak gravity conjecture bound.

Of course as is always true in string theory compactifications [158] there *does* exist a tower of instantons that satisfies the weak gravity bound (3.84) but generates no monodromy.[36] It is also apparent that these instantons occupy a sub-lattice of the full charge lattice. This sub-lattice corresponds to all the possible wrapping numbers of a Euclidean D1-string. However, in our setup this sub-lattice can be made parametrically *coarse*.[37] Let us illustrate this with a concrete example: We consider a variant of the *drifting monodromies* example given in Sect. 3.6.1, with flux numbers $M_i \in \{5, 6, 7, 8, -5, -6, -7, -8\}$. The axion decay constant is enhanced by the least common multiple of $5, 6, 7, 8$ which is 840. The instantons that satisfy (3.84) respect the periodicity of the axion *before* monodromy. Thus the possible charges take values in $840\mathbb{Z} \subset \mathbb{Z}$. Clearly, a lattice WGC is parametrically violated, while a sub-lattice WGC [53, 54] (see also [159]) is always satisfied but with parametrically coarse sub-lattice. Note that generically these instantons only give rise to sub-leading corrections to the scalar potential (if they contribute at all), compare Sect. 3.6.4.

[35] Taking the correspondence with instantons seriously, these are ($\frac{1}{2}$BPS-)instantons.

[36] In our case, these are Euclidean D1-strings wrapping representative S^2's in the UV, compare Sect. 3.6.4.

[37] Hence we seem to realize explicitly the loophole mentioned in footnote 25 of [54].

3.6.3 Axion Phenomenology

We have identified a string theory axion with remarkable properties. It is parametrically lighter than the tower of states that is usually associated to strongly warped regions $m_{\text{tower}} \propto w_{\text{IR}} M_{\text{P}}$. The axion mass can be tuned almost independently of the periodicities of the dominant oscillations in the scalar potential, since we have $m \propto w_{\text{IR}}^3 M_{\text{P}}$, while the oscillation period f_{eff}/q of the scalar potential depends only weakly on the warp factor $f_{\text{eff}}/q \sim M_{\text{P}}/\sqrt{\ln\left(w_{\text{IR}}^{-1}\right)}$. Conversely, the mass scales unusually strongly with the oscillation wavelength,

$$\frac{m^2}{M_{\text{P}}^2} \propto w_{\text{IR}}^6 \approx e^{-2S_{\text{eff}}} \approx \exp\left(-\alpha\left(\frac{qM_{\text{P}}}{f_{\text{eff}}}\right)^2\right), \tag{3.89}$$

with $\alpha = \mathcal{O}(1)$.

In contrast most other stringy axions usually satisfy the relation [5]

$$\frac{m^2}{M_{\text{P}}^2} \sim \exp\left(-\alpha\frac{qM_{\text{P}}}{f_{\text{eff}}}\right), \tag{3.90}$$

As such the thraxion assumes a rather special place in the string theory landscape. This is potentially interesting for axion phenomenology. We refer the reader to [5] for a range of phenomenological applications for different axion mass scales.

We have to emphasize that at least in the simplest setups our axion is not a generic inflaton candidate as it was briefly discussed in Sect. 1.1.3 because of the generic presence of dominant sub-Planckian wavelength modes in the scalar potential, *despite* the large monodromy enhancement of the effective axion decay constant.

3.6.4 Uplifting

We would like to briefly comment on some possible scenarios of uplifting. In Ch. 4 we discuss how a superpotential of the form (3.66) can allow for SUSY-breaking vacua by generating a scalar potential in which multiple periodic terms in a single axion of comparable amplitude interact. Concretely, by tuning the amplitudes (and phases) of the individual periodic terms in a drifting-monodromies potential, see, e.g., (3.83), non-zero local minima of the potential may arise. In this section we will only discuss how non-zero minima can arise in non-tuned cases. Both ideas presented here are based on the idea of adding an oscillating potential of different wavelength to the known thraxion potentials of the form (3.10) or (3.83) which on their own only possess minima at 0.

Uplifting requires as a precondition, that a full mechanism of Kähler moduli stabilization is in place. Stabilizing the Kähler moduli by definition breaks the no-scale property of GKP-type flux compactifications. Hence, in a full setup of our multi throat system there are two sources of no-scale breaking – the Calabi-Yau breaking potential of our C_2-axion(s), and also the scalar potential that stabilizes Kähler moduli. In general we expect these two sources of no-scale breaking to mix non-trivially, and we leave a detailed analysis of this for future research. For the rest of this discussion we now assume that these subtleties get resolved for both KKLT- and LVS-type setups of Kähler moduli stabilization.

We now wish to look at situations where c-dependent corrections to the Kähler potential may become relevant. This is certainly the case in the regime $|W_0| \sim 1$, leading us to consider LVS-like moduli stabilization [73]. Potentially interesting corrections may arise from Euclidean D1-brane instantons that wrap members of the family of two-spheres that vanish at the tips of the conifold. Since the cycle is trivial in homology we expect no corrections to the superpotential but at most corrections to the Kähler potential of the form

$$\delta e^{-K_1} \sim \mathcal{C} e^{-S_{\text{DBI}} - iS_{\text{CS}}} + \text{c.c.}\,, \tag{3.91}$$

with $\mathcal{C} = \mathcal{O}(1)$ and Dirac-Born-Infeld (DBI) and Chern-Simons (CS) actions (cf. Sect. 2.1.2)

$$\begin{aligned} S_{\text{DBI}} &= \frac{1}{g_s} \frac{\text{Vol}(S^2)|_{\text{UV}}}{2\pi\alpha'}\,, \\ S_{\text{CS}} &= \frac{1}{2\pi\alpha'} \int_{S^2} \left(\sum_p C_p \right) \wedge e^B \big|_{2-\text{form}} = \text{Re}\,\mathcal{G}\,. \end{aligned} \tag{3.92}$$

Here, we have evaluated the DBI action on a representative sphere in the UV, i.e., in the bulk CY. This is because we expect such a representative to give the dominant contribution: As explained in App. A.2, there are different two-spheres at a given radial coordinate in the throat that are labeled by a $U(1)$ phase and that all share the same volume. As we scan over this phase, the corresponding integrals of C_2 at a given radial coordinate pass through their fundamental domain. Therefore, integrating over all Euclidean brane instantons on the two-spheres should cancel all contributions due to the oscillatory behavior of the correction (3.91). This is consistent with the fact that after accounting for back-reaction of the phases of the throat deformations the C_2 field excursion cannot be measured in the local throats.

In the analysis of Sect. 3.4, we extended this result to $\mathrm{Re}(\mathcal{G})$, i.e., to $C_2 - C_0\, B_2$. In passing towards the UV, our description of the throat breaks down. In particular, we do not expect the different sphere representatives to all share the same volume. Thus, we expect non-vanishing instanton corrections.

Using $\mathrm{Vol}(S^2)|_{\mathrm{UV}} \gtrsim R^2_{\mathrm{throat}} \propto (g_s MK)^{1/2}\alpha'$, this leads to corrections to the scalar potential of the form

$$\delta V \lesssim e^{-\alpha\sqrt{\frac{KM}{g_s}}}(1 - \cos(\mathrm{Re}\,\mathcal{G}))\,, \tag{3.93}$$

with $\alpha = \mathcal{O}(1)$. Assuming that the exponentially small prefactors of the classical warping suppressed potential (3.10) (or that of example 1 of Sect. 3.6.1) and the non-perturbative correction terms are of the same order, it is feasible that additional local minima appear in the scalar potential that could in principle lift to meta-stable non-supersymmetric minima, possibly even de Sitter vacua.

The exponential terms are of comparable magnitude when

$$\sqrt{\frac{K}{g_s M}} \gtrsim M\,. \tag{3.94}$$

In F-theory models with large Euler characteristic we do not see an immediate obstacle to realizing this.

We may turn this around and add large-wavelength corrections to shorter-wavelength oscillations such as those of the example of *drifting monodromies* given in Sect. 3.6.1. On the large scale of $f_{\mathrm{eff}} \sim MM_{\mathrm{P}}$ there are several Minkowski vacua of the potential (3.83), compare Fig. 3.12. It is conceivable that these are uplifted once further corrections to the potential are taken into account. This might happen automatically when the no-scale properties of the Kähler potential (3.57) get broken by perturbative or non-perturbative corrections, since we know that the existence of Minkowski vacua strongly depends on the cancellation of different terms in the scalar potential. We expect these scalar potential corrections to follow the periodicity of the superpotential, which is given by the super-Planckian decay constant f_{eff}. An optimistic sketch of this is illustrated in Fig. 3.13. A different approach might be to consider *drifting monodromies* in which we allow for hierarchies between fluxes M_i of individual throats.

We leave a more thorough investigation of these uplifting ideas for future research.

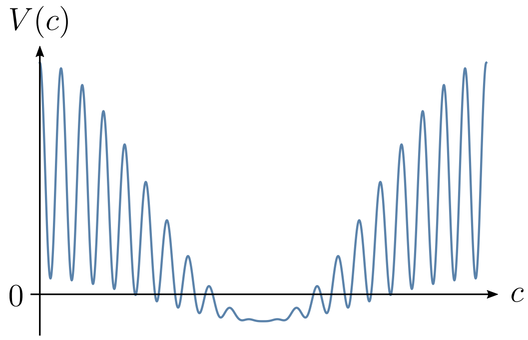

Figure 3.13: The axion potential of Fig. 3.12 with an additive correction $\delta V(c) \propto \text{const.} + \cos(c/f_{\text{eff}})$ that shares the periodicity of the superpotential.

3.7 Conclusion

We have shown that a novel type of axion-like particle is present in many flux compactifications of type IIB string theory. While its exponentially small mass is due to the existence of strongly red-shifted regions in the compactification manifold (warped throats), it is parametrically lighter than the red-shifted tower of KK-modes that is usually associated with such throats. We would like to emphasize that for fixed value of the axion decay constant the *thraxion* mass is far smaller than all other stringy axions that we are aware of. Moreover, we are able to find explicit models with even parametrically super-Planckian axion decay constants (but with generically dominant sub-Planckian oscillations in the scalar potential). As such the thraxion assumes a rather special place in the string theory landscape that is potentially interesting for axion phenomenology.

The existence of this type of ultralight axion is intimately linked to so-called conifold transitions between (topologically) distinct CY manifolds. Light thraxions arise whenever fluxes drive a CY close to the transition point. Moreover they come paired with light scalar degrees of freedom ('saxions') which are the relevant light degrees of freedom that control the global stabilization of the multi throat system that arises near such a point in moduli space. At the perturbative level the saxion is as light as the axion while non-perturbative Kähler moduli stabilization effects would generically lift this degeneracy. The extremely low (s)axion mass implies that multi throat systems are surprisingly weakly stabilized.

We now summarize the key steps in the derivation of the (s)axion potential. Throughout most of this chapter we have focused on the case of a double throat system. As shown in [40] in such a setting there exists a light axion mode that can be thought of as the integral of the RR two-form

C_2 over a family of spheres that degenerate at the infrared ends of the two throats. When holding the geometry fixed, a finite field excursion leads to the formation of a flux/anti-flux pair at the two ends, so an appropriately red-shifted potential energy $V \sim w_{\mathrm{IR}}^4$ is induced where w_{IR} is the infrared warp factor of the throats. One of the key observations made in this work is that when the throats are long there exist *two* light complex scalars z_1, z_2 that control the infrared geometry of the individual throats. This is despite the fact that only a diagonal combination of the two is an actual complex structure modulus of the CY. A finite field excursion of the axion mode drives z_1 and z_2 away from complex structure moduli space, i.e., to $z_1 \neq z_2$. After this geometrical back-reaction is accounted for, locally in each throat supersymmetry is almost perfectly restored. We have determined the scale of the remaining scalar potential by finding the higher dimensional field profile that interpolates the 4d modes z_1 and z_2 between the two throats. It turns out to be dominated by tiny contributions from the bulk CY where the mismatch between the two throats is detected. The final potential energy scales as w_{IR}^6 which is parametrically smaller than the estimate of [40]. Since the axion kinetic term is dominated by contributions from the bulk geometry, the axion mass is of order w_{IR}^3 which is parametrically smaller than the infrared mass-scale of the local throats. Furthermore, the final axion potential turns out to be periodic, and the periodicity is enhanced by a flux number M via monodromy. In other words, the available amount of axion monodromy after back-reaction is finite. The low scale of the axion potential implies that throughout the compact axion field space the local throats are essentially frozen supersymmetrically.

We have cross-checked and expanded on the 10d double throat calculation in several ways: First, we have shown that a natural proposal for the extension of the GVW flux superpotential leads to an axion potential that matches precisely our 10d conclusions. Moreover, in this framework it is immediate that the axion is paired with a saxion that can be thought of as the integral of the NS two-form B_2 over the same family of spheres. Together they form the complex scalar component of a chiral multiplet. Furthermore we have extended the discussion to the case of a general multi throat system where n three-cycles degenerate near a conifold transition point, with m homology relations among them. There are n 'local complex structure moduli' z_i that are relevant for the discussion of which only $n - m$ linear combinations correspond to complex structure moduli space. In this setup there are m families of two-spheres that can be used to define m (s)axions. Again, a finite field excursion of the (s)axions drives the local deformation parameters z_i away from complex structure moduli space and a small periodic potential remains which is of order w_{IR}^6 . Since the conjectured coupling between the (s)axions and the deformation parameters is independent of fluxes, we would expect it to be present in $\mathcal{N} = 2$ compactifications. We have briefly checked, albeit far from established, that such a term might exist. We have checked our conclusions also from the perspective of the Klebanov-Strassler gauge theory [115, 49] that is believed to be the gauge theory dual of the throat solution. The (s)axions set a combination of gauge couplings at the UV-ends of the throats and receive a non-perturbative superpotential from gaugino condensation in the infrared. This superpotential again matches precisely the one we have obtained from *classical* 10d/4d supergravity. As is common in the gauge/gravity duality, a classical effect on one side matches a non-perturbative quantum effect on the other.

Our construction can be used to investigate a vast number of axion-models, by scanning over different conifold transition points of CY threefolds and three-form flux quanta. We have illustrated this by giving two examples based on a well-studied conifold transition point of the quintic threefold. With one of these examples we exhibit a mechanism that may be able to generate parametrically super-Planckian axion periodicities. This happens when several throats carry different flux numbers M_i that each give rise to a finite monodromy enhancement. The overall enhancement is given by the least common multiple of all the flux numbers. In the regime considered, the validity of the effective field theory is *not* undermined by the appearance of a large number of light states below or near the axion mass-scale as is usually expected for large f [158, 53, 54] or as parametrically large geodesics in field space are traversed [12, 48, 142, 86, 143, 87, 88, 144]. We call this mechanism to generate large axion periodicities *drifting monodromies*. Nevertheless, we identify a global constraint among the flux numbers that presents a serious obstacle to actually realizing a scalar potential with parametrically sub-dominant sub-Planckian oscillations.

Clearly, the examples we have given form only a tiny subset of possible *thraxion* models. Moreover, these models are a promising playground for testing swampland conjectures such as the weak gravity conjecture for axions. The simplest models already suggest that the simplest form of the conjecture need not hold in general. We have briefly commented on thraxion-related ideas for uplifting. Whether the ideas presented in this chapter can be realized in a controlled way is left for future work. This would require a detailed understanding of the interplay between the thraxions and no-scale breaking effects such as gaugino condensation on seven-branes [58] or perturbative α' corrections [99], which are needed for full moduli stabilization [58, 73]. We expand on a related uplifting idea in the following Ch. 4.

Chapter 4

Winding Uplifts – Parametrically Small SUSY Breaking in String Compactifications

4.1 Introduction and Summary

It is well-known that long-lived de Sitter vacua are hard to realize in string theory. When constructing models of such vacua, cf. Sect. 2.1.4, the typical starting point are known and established AdS solutions such as the KKLT [58] or LVS [72] vacuum. Then, one may add positive contributions that come from either D-terms [160–170] or F-terms [171–186]. While there is a plethora of models, none of the proposed uplifting mechanism is fully satisfactory as it is usually hard to argue for the validity of all assumptions made (see [98] for many points of criticism) or because the ability to tune to achieve a small cosmological constant is limited. Because of its supersymmetry the KKLT AdS vacuum is arguably the most accepted starting point for models of uplifting in type-IIB models. Recently, there has been a lot of discussion about the simple uplifting mechanism via anti-D3-branes described in [58], see [59–70]. This seems to indicate that part of the problem might be the difficulty of implementing meta-stable SUSY breaking which is necessarily involved in any de Sitter construction. Thus, parametrically weak SUSY breaking in models with finite Planck mass might be a challenge in itself. This has been formalized in the SUSY anti-de Sitter conjecture [56] and the de Sitter conjecture [55], see Sect. 2.2.4. In this chapter we propose to use the tuning-power of the complex-structure-based flux landscape to face this challenge. Our method of choice are the multi-cosine-shaped axion potentials which arise if a long winding trajectory of a 'complex-structure axion' appears in the large-complex-structure limit of a Calabi-Yau orientifold. This has been studied in the inflationary context as 'Winding Inflation' [32] (see also [187,188,119,189]), but the potential of this method for realizing weak SUSY breaking with long lifetimes has not been analyzed in detail.

Let us summarize the main idea presented in this chapter: We consider type-IIB CY orientifold compactifications with the complex structure moduli u and v near the large-complex-structure point. There, the Kähler potential only depends on the imaginary parts $\mathrm{Im}\, u$ and $\mathrm{Im}\, v$ such that a shift symmetry in $\mathrm{Re}\, u$ and $\mathrm{Re}\, v$ is manifest. It is only broken by the flux superpotential. We may choose fluxes M and N in such a way that only the linear combination $Mu + Nv$ appears in the superpotential. Then, integrating out the complex structure moduli supersymmetricly leaves one axionic direction, parametrized by $\varphi \equiv N/M\, \mathrm{Re}\, v$, unstabilized.

Adding sub-leading corrections in the large-complex-structure moduli to the periods and therefore to the super- and Kähler potential induces a scalar potential for φ. These corrections are subject to the shift symmetry and therefore of the form $\exp(iu)$ and $\exp(iv)$. Both terms depend on the unstabilized axion φ and their magnitude is governed by the stabilized values of the saxions, $\exp(iu) \propto \exp(-\mathrm{Im}\, u_0 - i\varphi)$ and $\exp(iv) \propto \exp(-\mathrm{Im}\, v_0 + iM/N\, \varphi)$. We may tune the saxion values $\mathrm{Im}\, u_0$ and $\mathrm{Im}\, v_0$ in such a way that the two terms appear in the F-term with comparable amplitude, $\epsilon \equiv \exp(-\mathrm{Im}\, u_0) \sim \exp(-\mathrm{Im}\, v_0)$. Their relative magnitude is then measured by the parameter $\alpha \equiv \exp(\mathrm{Im}\, u_0 - \mathrm{Im}\, v_0) = \mathcal{O}(1)$.

The resulting F-term scalar potential of the axion is of the simple form

$$V(\varphi) = e^K \epsilon^2 \left(\sin(\varphi) + \alpha \sin(M/N\, \varphi)\right)^2 . \tag{4.1}$$

By tuning $1 \gtrsim \alpha \gtrsim N^2/M^2$, we find non-zero, local minima of the F-term potential. For these, we may parametrically separate the value of the potential minimum $\Delta V \sim \epsilon^2(1-\alpha)^2$ and the potential barrier $V_{\mathrm{wall}} \sim \epsilon^2$ to lower-lying vacua by choice of α, see Fig. 4.1.

Let us turn to the application of the mechanism just described in more concrete scenarios. It is straightforward to apply it in the large volume scenario. Tuning the value of $\epsilon(1-\alpha)$ against the value of the LVS AdS cosmological constant, one may consistently uplift the vacuum to de Sitter. For KKLT SUSY AdS vacua the minimal setup just discussed seems to fail. We encounter the problem of having to deal with small W_0 in order to find a supersymmetric solution in the Kähler moduli. This spoils the stabilization of the saxions as discussed above. Finally, we consider supersymmetric AdS vacua in type IIA as studied by DGKT [74]. This setting naturally gives rise to unstabilized axions in an otherwise fully supersymmetrically stabilized

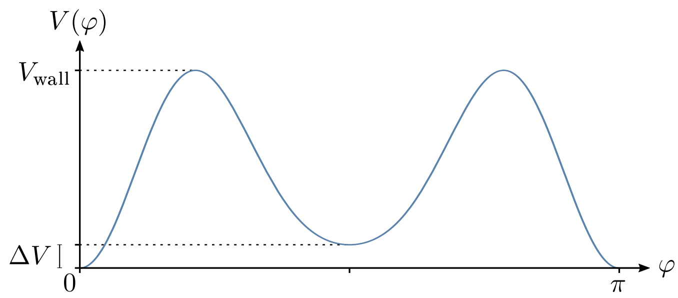

Figure 4.1: The axion potential (4.1) for $M/N = 3$. There is a minimum at $\varphi_* = \pi/2$ with $\Delta V \equiv V(\varphi_*) \propto \epsilon^2(1-\alpha)^2 > 0$, while the potential scales as ϵ^2 in general.

background. Only a single linear combination of RR axions is fixed. The superpotential resulting from non-perturbative corrections may be directly compared to the winding scenario with multiple axions. We expect that generally we have enough tuning power to find tunable non-zero local minima of the scalar potential with parametrically high barriers. While the uplifts are necessarily small, we may have found a way of consistently breaking supersymmetry in a stable AdS vacuum.

We want to highlight that the use of 'instantonic' terms is a common idea in uplifting scenarios. Such periodic terms have been used in, e.g., racetrack models or the STU model [190–198] (see however [155] for some possible issues of racetrack models). In these, the periodic terms are of non-perturbative origin and a positive contribution to the scalar potential is generated by the interplay of the exponential terms with polynomial terms from, e.g., perturbative corrections. In [199] a racetrack model with a vanishing perturbative superpotential has been used to find a small superpotential coming from the interplay of instantonic terms.

The interplay of different periodic terms in the potential has recently been discussed in other contexts: The 'drifting monodromies' scenario in compactifications involving multi throat systems as presented in Sect. 3.6.1 gives rise to a superpotential of the same form as we use in this chapter. There, we did not tune the amplitudes of terms of different periodicities but we expect to be able to reproduce scalar potentials of the form (4.1). The authors of [200] give a pure IR argument on how the QCD pion potential at general θ-angle generates a multi-cosine-shaped scalar potential which possesses non-zero minima. They discuss how this may naturally uplift the IR theory.

The chapter is organized as follows. Sect. 4.2 presents the SUSY-breaking mechanism just discussed in detail, including a short introduction to the winding idea, a discussion of all sub-leading corrections relevant and an analysis of the axion potential induced. We present how this may be used to uplift different AdS vacua of various origins in Sect. 4.3. Finally, we conclude in Sect. 4.4.

4.2 The Uplifting Potential

4.2.1 Winding Setup

We briefly introduce the winding scenario [32]. We consider complex structure moduli u and v at the large-complex-structure point. At this special point it has been noticed that a continuous shift symmetry for the real parts of the moduli arises in the Kähler potential [71]. This shift symmetry is only broken by the flux superpotential. By a certain choice of flux, we arrive at a super- and Kähler potential for the complex structure moduli of the form

$$\begin{aligned} W &= \widetilde{W}_0(z) + f(z)(Mu + Nv) + W_{\text{sub}}(z,u,v)\,, \\ K &= K_{\mathcal{V}} + K_\tau - \ln(k(z,\bar{z},\operatorname{Im} u, \operatorname{Im} v)) + K_{\text{sub}}(z,\bar{z},u,\bar{u},v,\bar{v})\,, \end{aligned} \tag{4.2}$$

where u and v are at large complex structure and z describes all other complex structure moduli [1] which appear generically. Here, $K_{\mathcal{V}}$ and K_τ describe the Kähler potential of the Kähler moduli and axio-dilaton respectively and k is some function. The contributions W_{sub} and K_{sub} denote terms that are sub-leading to

$$W_0(z,u,v) \equiv \widetilde{W}_0(z) + f(z)(Mu + Nv)\,, \tag{4.3}$$

as well as

$$K_0 \equiv K_{\mathcal{V}} + K_\tau - \ln(k)\,. \tag{4.4}$$

The sub-leading terms arise from corrections to the periods $\int \Omega$ of the large-complex-structure geometry. We will specify these sub-leading terms in Sect. 4.2.2. Importantly, only the linear combination $Mu + Nv$ appears in W_0, with M and N being flux numbers.

To analyze the F-terms in leading order, $F_{0,i} \equiv (\partial_i + K_{0,i})W_0$, it is convenient to introduce

$$\psi \equiv Mu + Nv\,, \quad \phi \equiv v\,. \tag{4.5}$$

[1] We will always include the axio-dilaton when simply using 'z'. Only in the Kähler potential we sometimes distinguish complex structure, $K_{\text{cs}} = -\ln(k(\operatorname{Im} z))$, and axio-dilaton, $K_\tau = -\ln(-2\operatorname{Im}\tau)$.

We then have

$$\begin{aligned} F_{0,z} &= (\partial_z K_0)W_0 + \widetilde{W}_{0,z} + (\partial_z f)\psi \overset{!}{=} 0\,, \\ F_{0,\psi} &= (\partial_\psi K_0)W_0 + f \overset{!}{=} 0\,, \\ F_{0,\phi} &= (\partial_\phi K_0)W_0 \overset{!}{=} 0\,. \end{aligned} \tag{4.6}$$

While the equations for z and ψ are (in general) complex, the equation for ϕ is real (up to an overall phase). The SUSY condition $F_{0,i} = 0$ therefore fixes

$$z = z_0\,, \quad \psi = \psi_0\,, \quad \mathrm{Im}\,\phi = \phi_0\,. \tag{4.7}$$

That is, importantly, $\mathrm{Re}\,\phi$ remains unstabilized. Expressed in the original fields we have stabilized

$$z = z_0\,, \quad \mathrm{Im}\,u = u_0\,, \quad \mathrm{Im}\,v = v_0\,, \quad M\mathrm{Re}\,u + N\mathrm{Re}\,v = \mathrm{Re}\,\psi_0\,. \tag{4.8}$$

For notational simplicity later on, we will shift the fields z, ψ and ϕ by the solution found above, such that the F-Terms (4.6) vanish in $z = 0$, $\psi = 0$ and $\mathrm{Im}\,\phi = 0$.

4.2.2 Adding Sub-Leading Corrections

Having analyzed the leading-order F-terms, we now add the first sub-leading corrections. This will stabilize $\mathrm{Re}\,\phi$ at a lower scale and will back-react on the leading-order solutions given in the previous section.

For the superpotential, we have [2]

$$\begin{aligned} W &= W_0 + W_{\mathrm{sub}} = W_0(z,u,v) + A(z)e^{iu} + B(z)e^{iv} + \dots \\ &= W_0(z,\psi) + A(z)e^{-u_0}e^{i\frac{\mathrm{Re}\,\psi_0}{M}}e^{i\frac{\psi-N\phi}{M}} + B(z)e^{-v_0}e^{i\phi} + \dots \\ &= W_0(z,\psi) + \frac{A(z)}{A(0)}\,\epsilon\,\left[e^{i\left(\frac{\psi-N\phi}{M}+\delta_1\right)} + \frac{B(z)}{B(0)}\,\alpha\,e^{i(\phi+\delta_2)}\right] + \mathcal{O}(\epsilon^2)\,, \end{aligned} \tag{4.9}$$

where

$$\begin{aligned} &\epsilon \equiv |A(0)|\,e^{-u_0}\,, \quad \alpha \equiv |B(0)/A(0)|\,e^{-(v_0-u_0)}\,, \\ &\delta_1 \equiv \arg(A(0)) + \frac{\mathrm{Re}\,\psi_0}{M}\,, \quad \delta_2 \equiv \arg(B(0))\,. \end{aligned} \tag{4.10}$$

[2] 'Instantonic' corrections in z may be included in $W_0(z)$ from now on. A possible dependence on u/v or ϕ/ψ will always be periodic in the respective axion, that is $C(u,v)e^{iz}$ can be written and expanded as $C(e^{iu},e^{iv})e^{iz} \sim (C(0,0) + \epsilon(e^{i(\psi-N\phi)/M}\partial_1 + \alpha e^{i\phi}\partial_2)C(0,0) + \dots)e^{iz}$. Any such term is therefore already accounted for in the sub-leading terms for u and v. The same reasoning holds for the Kähler potential.

The large complex structure regime implies $\epsilon \ll 1$. Furthermore we assume that both corrections in u and v are comparable, i.e., $u_0 \sim v_0$ or $\alpha = \mathcal{O}(1)$. We make the required hierarchy more precise below.

We also add the relevant corrections to the Kähler potential

$$\begin{aligned} K &= K_0 + K_{\text{sub}} = K_0(z, \bar{z}, \operatorname{Im} u, \operatorname{Im} v) + K_{\text{sub}}(z, \bar{z}, \phi, \overline{\phi}, \psi, \overline{\psi}) \,, \\ K_{\text{sub}} &= \Big(\widetilde{A}(z, \bar{z}, \operatorname{Im} u, \operatorname{Im} v) e^{iu} + \widetilde{B}(z, \bar{z}, \operatorname{Im} u, \operatorname{Im} v) e^{iv} + \text{c.c.} \Big) + \ldots \\ &= \frac{\widetilde{A}(\ldots)}{A(0)} \, \epsilon \left[e^{i\left(\frac{\psi - N\phi}{M} + \delta_1\right)} + \frac{\widetilde{B}(\ldots)}{B(0)} \, \alpha \, e^{i(\phi + \delta_2)} + \text{c.c.} \right] + \mathcal{O}(\epsilon^2) \,, \end{aligned} \tag{4.11}$$

4.2.3 The Axion Potential

We now turn to the scalar potential induced by the corrections. We will calculate the back-reaction on the leading-order solutions, similar to the analysis in [32]. We will include ϕ and ψ in the set of complex structure moduli denoted by z^i, $i = 1, \ldots, n$.

The scalar potential has the form

$$V = e^K K^{i\bar{\jmath}} F_i \overline{F}_{\bar{\jmath}} \,, \tag{4.12}$$

where at zeroth order in ϵ we have fixed $F_{0,i} = 0$ at $z = 0$ (4.6). At linear order in ϵ, F_i receives a correction δF_i, coming both from corrections to K and W. Due to this, z back-reacts, leading to a further correction which may be estimated by Taylor expanding $F_{0,i}$ in z. Hence

$$F_i \quad \longrightarrow \quad F_{ij} z^j + F_{i\bar{\jmath}} \bar{z}^{\bar{\jmath}} + \delta F_i \,, \tag{4.13}$$

where $F_{ij} = \partial F_{0,i} / \partial z^j$ and similarly for $F_{i\bar{\jmath}}$. The back-reaction is small, $z^i \sim \epsilon$, since $\delta F_i \sim \epsilon$. The only field excursion allowed to take $\mathcal{O}(1)$ values is that of $\operatorname{Re}\phi$, since this field is not stabilized at leading order. It appears in δF_i and only there. Moreover, the dependence of δF_i on the other z^i is irrelevant since this would be sub-leading in ϵ. Similarly, the z^i dependence of $\exp(K)$ and $K^{i\bar{\jmath}}$ in (4.12) may be disregarded.

To proceed, let us view (4.12) as the length squared of the complex vector F_i. At the expense of doubling the index range and appropriately redefining the metric, we may view this as the length squared of a real vector:

$$V = G^{ab} f_a f_b \qquad \text{with} \quad f_a = k_{ab} x^b + \delta f_a(x^1) \quad \text{and} \quad z^i = x^i + i x^{i+1} \,. \tag{4.14}$$

Here, we set $x^1 \equiv \operatorname{Re}\phi$ such that the vector k_{a1} vanishes by leading-order shift symmetry. The index range is $a, b = 1, \ldots, 2n$. The quantities G^{ab}, f_a,

δf_a and k_{ab} follow from (4.12) and (4.13) by a simple rewriting in real and imaginary components.

Our potential as a function of x^1 follows from (4.14) by integrating out $x^2, \ldots, x^{2n}$, which is straightforward: The first term in f_a generically takes values in a $(2n-1)$-dimensional subspace of the $\mathbb{R}^{2n}$ in which f_a and δf_a live. This is a result of the missing dependence on x^1. Let us call the unit vector orthogonal to that 'allowed' subspace $\hat{e}_a$. It is then clear that, when minimizing in $x^2, \ldots, x^{2n}$ at fixed x^1, the vector $k_{ab}x^b$ will take a value annihilating as much as possible of δf_a. Since the $\hat{e}_a$-subspace is not accessible to $k_{ab}x^b$, the projection of δf_a on that subspace cannot be compensated by the minimization. One finds

$$V = \left(\hat{e}^a\, \delta f_a(x^1)\right)^2 . \tag{4.15}$$

The elements of $\hat{e}^a = G^{ab}\hat{e}_b$ may be calculated in terms of the vacuum values of K_0 and W_0. Given what we know about the functional form of δF_i, the potential then takes the explicit form

$$V(\mathrm{Re}\,\phi) = e^{K_0}\tilde{\kappa}\tilde{\epsilon}^2 \left[\sin(N\mathrm{Re}\,\phi/M + \delta_1) - \tilde{\alpha}\sin(\mathrm{Re}\,\phi + \delta_2)\right]^2 , \tag{4.16}$$

where $\tilde{\kappa}$ captures derivatives of the Kähler potential evaluated in $z = 0$ [3]. The constants $\tilde{\epsilon}$ and $\tilde{\alpha}$ as well as phases δ_1 and δ_2 have been defined to absorb $\mathcal{O}(1)$ coefficients.[4]

Finally, we insert the Kähler potential K_0 and define $\kappa \equiv \tilde{\kappa}/(2\,k(0))$. For better illustration, we parameterize

$$\varphi \equiv \frac{N}{M}\mathrm{Re}\,\phi\,, \tag{4.17}$$

to arrive at

$$V(\varphi) = \frac{g_s}{\mathcal{V}^2}\kappa\tilde{\epsilon}^2 \left[\sin(\varphi + \delta_1) - \tilde{\alpha}\sin\!\left(\frac{M}{N}\varphi + \delta_2\right)\right]^2 . \tag{4.18}$$

For appropriate values of $\tilde{\alpha}$, $N^2/M^2 \lesssim \tilde{\alpha} \lesssim 1$, this potential has several local minima in φ as illustrated in Fig. 4.2. The value in a local, non-zero

[3] These derivatives come from the matrix $K^{i\bar{\jmath}}$ which, after splitting into real and imaginary parts, ultimately defines the components of the vector $\hat{e}$ in (4.15).

[4] To be precise, the perturbations $\delta F_i(\mathrm{Re}\,\phi)$ in complex notation contain periodic terms $\propto e^{-iN\mathrm{Re}\,\phi/M}$ with coefficients $\partial_{z^i}K_0A$, iA/N, $\partial_{z^i}A$, $W_0 i\widetilde{A}/N$ and $W_0\partial_{z^i}\widetilde{A}$ as well as periodic terms $\propto e^{i\mathrm{Re}\,\phi}$ with coefficients $\partial_{z^i}K_0B$, iB, $\partial_{z^i}B$, $W_0 i\widetilde{B}$ and $W_0\partial_{z^i}\widetilde{B}$. All coefficients are assumed to be $\mathcal{O}(1)$ when evaluated in the leading-order solution $z^i = 0$.

minimum may be written as

$$
\begin{aligned}
V(\varphi_*) &= \frac{g_s}{\mathcal{V}^2}\kappa\left(\tilde{\epsilon}\sin(\varphi_*+\delta_1)\right)^2\left(1-\frac{\sin(M/N\,\varphi_*+\delta_2)}{\sin(\varphi_*+\delta_1)}\tilde{\alpha}\right)^2 \\
&\equiv \frac{g_s}{\mathcal{V}^2}\kappa\,\epsilon^2\,(1-\alpha)^2\,.
\end{aligned}
\tag{4.19}
$$

In the last line, we introduced ϵ and α to absorb the factors $\sin(\varphi_*+\delta_1) = \mathcal{O}(1)$ and $\sin(M/N\,\varphi_*+\delta_2) = \mathcal{O}(1)$.

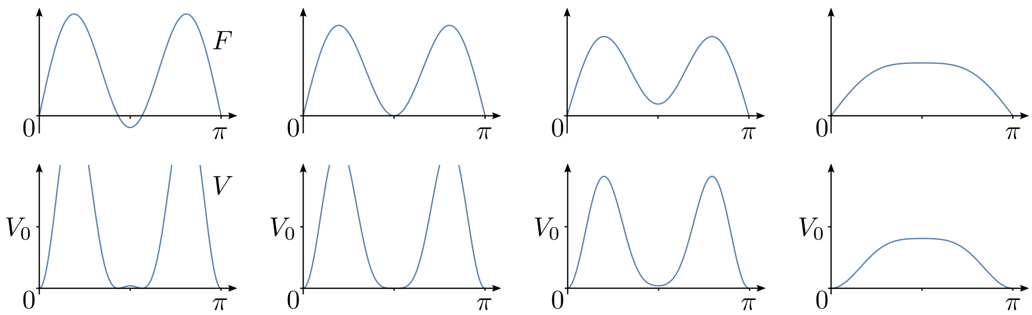

Figure 4.2: The F-term $\propto [\sin(\varphi) - \tilde{\alpha}\sin(3\varphi+\pi)]$ (upper panels) and corresponding scalar potential $V_0[\sin(\varphi) - \tilde{\alpha}\sin(3\varphi+\pi)]^2$ (lower panels) for $\tilde{\alpha} = 1.2\,,\ 1\,,\ 0.8\,,\ 0.1$ from left to right: By tuning $\tilde{\alpha}$ we find arbitrarily small non-zero, local minima (third column). If $\tilde{\alpha}$ becomes too small the minima go away (fourth column).

The decay constant and mass of φ are given by

$$
\begin{aligned}
f_\varphi^2 &= \frac{N^2}{M^2}K_{\phi\bar{\phi}} = \mathcal{O}(1)\,\frac{N^2}{M^2}\,, \\
m_\varphi^2 &= V''(\varphi_*)/f_\varphi^2 = \mathcal{O}(1)\,\frac{g_s}{\mathcal{V}^2}\frac{M^4}{N^4}\,\epsilon^2\alpha(1-\alpha)\,.
\end{aligned}
\tag{4.20}
$$

In the last equality we assumed $\alpha \gg N^2/M^2$, that is we are far from the destabilized situation of the fourth column in Fig. 4.2.

With this we have arrived at the main conclusion of this chapter: We have given a concrete tunable origin of an F-term that may uplift known vacua.

4.2.4 Winding in a Multi Axion Field Space

We may of course generalize to multiple complex structure moduli u^i, $i = 0,\ldots,m$, in the large-complex-structure limit, such that

$$
W_0(z,u^i) = \widetilde{W}_0(z) + f(z)\left(\sum_{i=0}^{m} N_i u^i\right)\,. \tag{4.21}
$$

Defining

$$\psi \equiv \sum_{i=0}^{m} N_i u^i \quad \text{and} \quad \phi^i \equiv u^i \quad \text{for} \quad i = 1, \ldots, m\,, \tag{4.22}$$

we find leading-order F-term equations

$$F_{0,\psi} = (\partial_\psi K_0) W_0 + f(z) \stackrel{!}{=} 0\,, \quad F_{0,\phi^i} = (\partial_{\phi^i} K_0) W_0 \stackrel{!}{=} 0 \tag{4.23}$$

The same reasoning as in Sect. 4.2.1 leads to all imaginary parts being stabilized, $\operatorname{Im} u^i = u_0^i$, while only one real direction is fixed, $\operatorname{Re}\psi = \operatorname{Re}\psi_0$. The remaining m axions are not stabilized in leading order. Shifting by leading-order solutions, we arrive at the corrected superpotential of the form

$$\begin{aligned} W &= W_0 + A_0\, e^{-u_0^0}\, e^{i\frac{\operatorname{Re}\psi_0}{N_0}}\, e^{-i\sum_{i=1}^m \left(\frac{N_i}{N_0}\phi^i\right)} + \sum_{i=1}^{m} A_i e^{-u_0^i}\, e^{i\phi^i} + \ldots \\ &\equiv W_0 + \epsilon \left[e^{-i\sum_{i=1}^m \left(\frac{N_i}{N_0}\phi^i\right) + i\delta_0} + \sum_{i=1}^{m} \alpha_i\, e^{i\phi^i + i\delta_i} \right] + \mathcal{O}(\epsilon^2)\,. \end{aligned} \tag{4.24}$$

The Kähler potential is of a similar form. The resulting F-terms have the same form as the ones in Fig. 4.2 in a higher-dimensional field space as long as all coefficients $A_i\, e^{-u_0^i}$, $i = 0, \ldots, m$, are of the same order. It is clear that, while complicating the actual calculation, we increase the amount of possible tuning this way.

4.3 Uplifting AdS Vacua

We now turn to uplifting known AdS vacua to higher-lying AdS and dS vacua. In this, we are interested in testing the conjectures and criticism against such constructions made in [56, 57, 55].

We do not possess knowledge of exact coefficients, but only about parametric scaling of both the potential (4.18), $V \sim \epsilon^2$, and its value in the minimum (4.19), $V_{\text{min}} \sim \epsilon^2(1-\alpha)^2$. Parametrically separating the potential difference between vacua, ΔV, from the height of the potential barrier, V_{wall}, suffices to stabilize a vacuum solution against Coleman-de Luccia decays [201]. In our setup we find $\Delta V / V_{\text{wall}} \sim (1-\alpha)^2$ between vacua. In scenarios that allow us to tune α such that $(1-\alpha)^2 \ll 1$, we therefore expect the uplifted vacuum to be (meta-)stable and long-lived.[5]

[5] Stability against Brown-Teitelboim decays [111] can be achieved by simply tuning ϵ (cp. simple estimates done in [202]). This brane-nucleation mechanism in non-supersymmetric flux vacua was suggested in [56] (based on [112]). Bubbles of nothing [203] are a possible non-perturbative instability which we, in general, may not be able to control by any tuning [204].

4.3.1 Large Volume Scenario

In Sect. 4.2, the potential (4.18) was derived in a setting where fluxes stabilize the complex structure moduli supersymmetrically such that $|W_0| = \mathcal{O}(1)$. This is the setting in which the large volume scenario [72] was studied. We are therefore naturally led to consider our uplifting mechanism with the addition of (at least) two Kähler moduli T_b and T_s. By assuming a hierarchy of the corresponding four-cycle volumes, $\tau_b \equiv \operatorname{Re} T_b \gg \operatorname{Re} T_s \equiv \tau_s$, we are justified in considering only the leading non-perturbative correction $W_{\mathrm{np}} \propto e^{-a_s T_s}$. We also add the leading perturbative α'-correction [99].

The axion does not mix (in leading order) with the fields T_s and T_b other than via the overall volume prefactor, $\mathcal{V} \approx \tau_b^{3/2}$, in (4.18). Therefore, we may stabilize the axion in the SUSY-breaking minimum discussed in Sect. 4.2.3 before stabilizing the Kähler moduli. We are then in the standard situation [73] where we add a term

$$V_{\mathrm{uplift}} \propto \frac{g_s \epsilon^2 (1-\alpha)^2}{\mathcal{V}^2}\,, \tag{4.25}$$

to the usual LVS potential. Without this uplifting term the latter possesses a minimum in [72]

$$\begin{aligned} V_{\mathrm{AdS}} \propto -g_s\,|W_0|^2\,\frac{\ln^{1/2}\mathcal{V}_{\mathrm{LVS}}}{\mathcal{V}^3_{\mathrm{LVS}}} \propto -g_s\,|W_0|^2\,\frac{\sqrt{\tau_{s,\mathrm{LVS}}}}{\mathcal{V}^3_{\mathrm{LVS}}}\,, \\ \tau_{s,\mathrm{LVS}} \sim \frac{1}{g_s}\,, \quad \mathcal{V}_{\mathrm{LVS}} \sim \sqrt{\tau_{s,\mathrm{LVS}}}\,e^{a_s \tau_{s,\mathrm{LVS}}}\,. \end{aligned} \tag{4.26}$$

A discussion of the viable parameter ranges of $\epsilon(1-\alpha)$ for which (4.26) can be consistently uplifted to de Sitter can be found in [205].[6] Part of the result is that a consistent uplift to $V \gtrsim 0$ is realized for values

$$\epsilon^2(1-\alpha)^2 \gtrsim |W_0|^2\,\frac{\sqrt{\tau_{s,\mathrm{LVS}}}}{\mathcal{V}_{\mathrm{LVS}}}\,. \tag{4.27}$$

Since uplifting only requires us to tune the combination $\epsilon(1-\alpha)$, we are free to tune the potential barrier $\propto \epsilon^2$ in the φ-direction independently[7] to make this vacuum long-lived. We here have to assume stability of the LVS vacuum.[8]

[6] In [205] (based on [206, 207]) a non-vanishing F-term $F = \epsilon\,|W_0|$ due to non-supersymmetric fluxes was discussed. Although the origin may be different, the analysis leads to a similar scalar potential, $V_F = g_s \epsilon^2/\mathcal{V}^2$, as we find in our model.

[7] The assumption of large complex structure of course still requires $\epsilon \ll 1$.

[8] That is, we have to assume validity of the EFT description. Within the EFT description the LVS vacuum is stable as it is a global minimum in the τ_s-τ_b plane [202].

4.3.2 The KKLT AdS Vacuum

So far we considered non-tuned $\mathcal{O}(1)$-valued $|W_0|$. We now consider the supersymmetric type-IIB KKLT vacuum [58] at small $|W_0|$ as a starting point for uplifting. Notice that compared to [58], in our notation the superpotential of the complex structure moduli reads $W_{\text{cs}} \equiv W_0 + W_{\text{sub}}$. That is, when trying to stabilize a Kähler modulus T by balancing non-perturbative terms $\propto e^{-aT}$ against a small perturbative superpotential, $e^{-a\text{Re}\,T} \sim |W|$, we have to consider W_{cs} rather than (what we called) W_0.

We notice that there is a problem in implementing the simple mechanism as described in Sect. 4.2.3: We consider the complex structure moduli u and v near a large-complex-structure point. In this limit, the CY can be thought of as a T^3-fibration over an S^3 with the fiber volume becoming singular [208–210]. As discussed in [211], if we assign a typical radius R to the torus-fiber and radius L to the base three-sphere, we may understand a complex structure modulus v as measuring $\text{Im}\,v = L/R$. At the same time, two-cycle volumes are measured by $R \cdot L$. Geometric control requires us to have $R > 1$ in string units. Then, the limit of $\text{Im}\,v \to \infty$ implies that the radius and volume of the base three-sphere goes to infinity along with the two-cycle volumes. Correspondingly, the four-cycle volumes have to go to infinity as

$$\text{Re}\,T \sim (\text{Im}\,v)^2 . \tag{4.28}$$

For the realization of the KKLT scenario in type IIB, this implies

$$|W_{\text{cs}}| = |W_0 + W_{\text{sub}}| \sim e^{-a\text{Re}\,T} \ll e^{-\text{Im}\,v} \sim \epsilon . \tag{4.29}$$

A similar inequality holds for $\text{Im}\,u$, that is $|W_{\text{cs}}| \ll e^{-\text{Im}\,u} \sim \epsilon\alpha$. Therefore, the perturbative calculation of Sect. 4.2 breaks down as it requires $\epsilon \lesssim |W_0|$.

Realizing the hierarchy (4.29) in a minimum necessarily requires tuning: We may either cancel W_{sub} ($= \mathcal{O}(\epsilon)$ for generic values of ϕ) against W_0 of the same order by tuning W_0 or tune W_0 and W_{sub} to small values, $\ll \epsilon$, indepependently. The latter requires tuning $|Ae^{-v_0}| \sim |Be^{-u_0}|$ such that the two terms cancel each other. In either case, the result is $|W_{\text{cs}}| \ll \epsilon$ in the vicinity of a desired stabilized value of ϕ.

Actually, (4.29) implies that *two* real directions are not stabilized in leading order, as can be seen by considering the F-term

$$D_\phi W = K_\phi W + \partial_\phi W_{\text{sub}} \lesssim \mathcal{O}(\epsilon) . \tag{4.30}$$

Here, we inserted the full superpotential $W = W_{\text{cs}}(\phi) + W_{\text{np}}(T)$. Since we stabilize T such that $\mathcal{O}(|W_{\text{np}}|) = \mathcal{O}(|W_{\text{cs}}|)$, we also have $W \ll \epsilon$.[9]

[9] A possibly non-vanishing term $\partial_\phi W_{\text{np}}$ (due to ϕ-dependent prefactors $A_T(\phi)e^{-aT}$) is also small, $\partial_\phi W_{\text{np}} \propto e^{-a\text{Re}\,T} \sim |W_{\text{cs}}|$.

Having discussed the failure of the perturbative approach, we now analyze the idea from scratch for a hierarchy $1 \gg \epsilon \gg W_{\rm cs}$. Compared to Sect. 4.2 the situation after integrating out the heavy moduli z is actually slightly simpler than before: The full complex modulus ϕ remains light. Therefore, we expect a consistent supergravity description to exist for only the light moduli ϕ and T. The superpotential reads

$$W = W_0 + W_{\rm sub}(\phi) + W_{\rm np}(T)\,, \quad W_{\rm np}(T) = A_T\, e^{-aT} + \dots \tag{4.31}$$

The Kähler potential is still given by (4.2) (with the heavy complex structure moduli z and $\psi = Mu + Nv$ fixed). Even though $\operatorname{Im}\phi$ is not stabilized in this scenario, we continue to use the parameters ϵ and α. We assume ϵ to take small values near a desired (large) minimum value of $\operatorname{Im}\phi$.

The scalar potential reads

$$V = e^K \left[K^{T\bar{T}}\, |D_T W|^2 + K^{\phi\bar{\phi}}\, |D_\phi W|^2 - 3\, |W|^2\right] \tag{4.32}$$

Stabilizing ϕ supersymmetrically, $D_\phi W = 0$, results in the KKLT minimum upon also fixing T supersymmetrically, $D_T W = 0$. The minimum is characterized by (assuming a to be real) [58]

$$|W_{\rm cs}| \sim |A_T|\; e^{-a\operatorname{Re}T}\,, \quad V_{\rm AdS} \sim -\, |W_{\rm cs}|^2\,. \tag{4.33}$$

We may now check the F-term potential of ϕ for non-zero minima to find higher-lying AdS vacua of (4.32). For this, we consider the F-term

$$F(\phi) = K_\phi(\operatorname{Im}\phi)\, W(\phi) + \partial_\phi W(\phi)\,, \tag{4.34}$$

where $\partial_\phi W(\phi)$ is a holomorphic function.

As discussed below (4.30), the first term in (4.34) is negligible as we assumed $|W| \ll |\partial_\phi W(\phi)| = \mathcal{O}(\epsilon)$. In the spirit of Fig. 4.2, an uplifted minimum of the scalar potential requires some minimal value of the modulus of the F-term in some neighborhood of ϕ_* in which $\partial_\phi W(\phi)$ is non-zero

$$0 < |\partial_\phi W(\phi_*)| \leq |\partial_\phi W(\phi)|\,. \tag{4.35}$$

By the minimum modulus principle such a value cannot exist for a holomorphic function if it is not constant. Therefore, the F-term (4.34) stabilizes $\partial_\phi W(\phi_*) = 0$ as long as $|W_{\rm cs}| \ll \epsilon$.

To overcome this, we may break holomorphicity by tuning $|\partial_\phi W(\phi)| \sim |K_\phi(\operatorname{Im}\phi) W(\phi)|$ near a desired minimum. This now allows in principle for non-zero F-term minima but a few problems arise.

While the F-term is of the shape as illustrated in Fig. 4.1 in the $\mathrm{Re}\,\phi$-direction, near the minimum it might differ. At $\mathcal{O}(|W|)$, the terms coming from $A_T(\phi)$ generally spoil the periodicity in $\mathrm{Re}\,\phi$. It is unclear to us whether these coefficients respect the shift symmetry of the geometry. Thus, these terms may spoil the simple potential structure of (4.18). Furthermore, the actual scalar potential is of the form $e^K \left(K^{\phi\bar{\phi}} |F|^2 - 3 |W|^2 \right)$. While the full potential still only contains terms of periodicity 2π and $2\pi N/M$ (if this is also true for A_T), considering only the F-term scalar potential as in Sect. 4.2.3 does not suffice. Finally, while the potential in the $\mathrm{Re}\,\phi$-direction allows for parametrically high potential barriers if local minima do exist, $V_{\mathrm{wall}}/\Delta V \sim \epsilon^2/|W|^2 \gg 1$, we do not know the general behavior and stability along the $\mathrm{Im}\,\phi$-direction. Analyzing these problems is beyond the scope of this work as they depart from the basic question of this chapter of deriving and analyzing the winding potential (4.18).

4.3.3 DGKT-type Vacua

While we did derive the winding potential with a type-IIB setting in mind, there is no reason not to consider it in a type-IIA scenario. Specifically, we want to consider DGKT vacua [74]. We follow the notation of [212,116]. Both Kähler moduli T^i and complex structure moduli $S = s + i\sigma$ and $U_\lambda = u_\lambda + i\nu_\lambda$ appear in the perturbative flux superpotential (in the large volume limit)

$$\begin{aligned} W_{\mathrm{flux}} &= W_{\mathrm{K}}(T^i) + W_{\mathrm{cs}}(S, U_\lambda)\,, \\ W_{\mathrm{cs}}(S, U_\lambda) &= -ih_0 S - iq^\lambda U_\lambda\,, \end{aligned} \tag{4.36}$$

where h_0 and q^λ are independent H_3-flux numbers. The Kähler potential is given by

$$K = -\ln(8\mathcal{V}) - \ln(S + \bar{S}) - 2\ln(\mathcal{V}')\,, \tag{4.37}$$

where $\mathcal{V}(\mathrm{Im}\,T^i)$ is the CY volume defined via the Kähler moduli and

$$\mathcal{V}' \equiv \frac{d_{\lambda\rho\sigma}}{6} v^\lambda v^\rho v^\sigma\,. \tag{4.38}$$

Here, the v^λ are related to two-cycle volumes in type IIB via mirror duality. As such, their relation to the mirror duals of four-cycle volumes u_λ is

$$u_\lambda = \partial_{v^\lambda} \mathcal{V}'\,. \tag{4.39}$$

The $d_{\lambda\rho\sigma}$ are rational coefficients. We see that $\mathcal{V}'$ is a homogeneous function of degree $\frac{3}{2}$ in u^λ.

The F-terms for S and U_λ stabilize

$$\begin{aligned} 2h_0 s &= -\mathrm{Im}\, W\,, \\ K_{u_\lambda} &= -\frac{q^\lambda}{h_0 s}\,, \\ h_0\sigma + q^\lambda \nu_\lambda &= -\mathrm{Re}\, W_{\mathrm{K}}\,. \end{aligned} \tag{4.40}$$

We may also fix the Kähler moduli supersymmetrically. The volume $\mathcal{V}$ may be considered a free parameter as it is set by unconstrained 4-form fluxes. For large volumes we then find the scaling behavior with the volume

$$\begin{aligned} &|W_0| \sim \mathrm{Im}\, W_{\mathrm{K}} \sim \mathcal{V}\,, \quad e^K \sim \mathcal{V}^{-5} \\ &\Rightarrow V_{\mathrm{AdS}} \sim -e^K\, |W_0|^2 \sim -\mathcal{V}^{-3}\,. \end{aligned} \tag{4.41}$$

We conclude two important facts about flux-stabilized type-IIA solutions and specifically (4.40): First, the real parts of complex structure moduli, s and u_λ, are stabilized entirely by fluxes. The ratios $\partial_{u_\lambda}\mathcal{V}'/\partial_{u_\rho}\mathcal{V}'$ are determined by H_3-flux ratios q^λ/q^ρ which in turn also determine the ratios u_λ/u_ρ to be some function of fluxes. Second, only a single linear combination of imaginary parts, axions σ and ν_λ, is fixed.

We now add non-perturbative corrections (from, e.g., E2-branes) to stabilize the remaining directions in the axion field space. We arrive at a potential of the form (4.24).[10] In this, we identify

$$\begin{array}{rcll} u_\lambda & \longleftrightarrow & u_0^i & \text{for} \quad i=\lambda=1,\dots,m \\ s & \longleftrightarrow & u_0^0 & \\ \nu_\lambda & \longleftrightarrow & \phi^i & \text{for} \quad i=\lambda=1,\dots,m \\ \sigma = -\mathrm{Re}\, W_{\mathrm{K}} - \displaystyle\sum_{\lambda=1}^m \frac{q^\lambda}{h_0}\nu_\lambda & \longleftrightarrow & \mathrm{Re}\, u^0 = \dfrac{\mathrm{Re}\,\psi_0}{N_0} - \displaystyle\sum_{i=1}^m \frac{N_i}{N_0}\phi^i & \end{array} \tag{4.42}$$

The stabilized values u_λ for a given flux choice depend on the (mirror) CY at hand via the function $\mathcal{V}'(u_\lambda)$. Note that the 3-form flux is free of tadpole-constraints. By fixing the u_λ, the quantities ϵ and α_i in (4.24) can be tuned.

As discussed in [116], we may limit the dimension of the axion field space in two ways: First, we may choose a CY with a small number of complex structure moduli. Second, setting by choice of fluxes some hierarchy in the

[10] Compare (2.22) of [116]: $W = W_0 + \sum_I A_I \exp\left(-a_0^I S - \sum_\lambda a_\lambda^I U_\lambda\right)$, where I runs over instanton insertions and the coefficients a_0^I/a_λ^I specify the cycles wrapped by the instanton I. We may expand this in large s and u_λ assuming that instantons wrapping individual cycles, e.g., $a_0^I = 1$ and $a_\lambda^I = 0$, exist. We arrive at $W = W_0 + A_0 \exp(-S) + \sum_\lambda A_\lambda \exp(-U_\lambda) + \dots$

values u_λ, say $s, u_1 \gg u_\lambda$ for $\lambda > 1$, we achieve $\alpha_i \gg 1$ for $i = 2, \ldots, m$. We may then (supersymmetrically) stabilize ν_λ for $\lambda > 1$. Only the lightest axion(s), ν_1 in our case, will remain in the effective winding potential (4.24). We then arrive at a potential of the simpler form (4.18).

With this, we expect that non-zero minima of the potential arise for appropriate choice of fluxes and CY. We should be able to parametrically separate $\Delta V \sim e^K \epsilon^2 (1 - \sum \alpha_i)^2$ and $V_{\text{wall}} \sim e^K \epsilon^2$ [11]. The uplift for controlled values $\epsilon \ll |W_0| \sim \mathcal{V}$ remains small. We may have found a mechanism of breaking supersymmetry in the AdS vacuum (4.41) in a controlled way.

4.4 Conclusion

In this chapter we have presented a mechanism of controlled SUSY breaking. Based on the winding scenario of [32] we presented how the interplay of multiple periodic terms in the super- and Kähler potential can be used to construct F-terms that allow for non-zero minima of the scalar potential. We argued that these minima can be parametrically small such that the resulting vacua are long-lived. We applied the mechanism in the LVS and DGKT vacuum and pointed out problems that arise in the KKLT AdS vacuum due to the requirement of having a small value for the perturbative superpotential.

The status of the KKLT uplift and SUSY-breaking mechanism remains unclear. We showed that the simple setup of Sect. 4.2 is not sufficient to consistently uplift the AdS vacuum. This is because the full complex field ϕ remains light in leading order. While we suggested how the mechanism may still be applied by a specific tuning of W_0 and α, we left the question of explicitly calculating the minimum value and stability in the $\text{Im}\,\phi$-direction for future work.

For supersymmetric DGKT AdS vacua the uplift seems very robust. Compared to the KKLT uplift the construction requires less tuning. Actually, the DGKT construction naturally gives rise to a multi axion potential which also comes with more tunable parameters. With this, we may have found stable non-supersymmetric AdS solutions which serve as counterexamples to the SUSY AdS conjecture [56].

Finally, we showed that the application in the LVS vacuum is straightforward. It does however rely on the existence of the non-supersymmetric AdS solution which is in itself already in conflict with the SUSY AdS conjecture.

[11] As in Sect. 4.2.3, we made some redefinitions of the quantities ϵ and α_i: From the superpotential (4.24) one arrives at values $\tilde{\epsilon}$ and $\tilde{\alpha}_i$ by diagonalizing, cf. (4.18). In the final expression for the uplifting term, cf. (4.19), we absorbed further prefactors in the quantities ϵ and α_i. These only differ from the original definitions by $\mathcal{O}(1)$ factors.

For a discussion on the stability of the LVS vacuum see [213]. Assuming its stability we are able to uplift to long-lived de Sitter vacua. Thus, we may have found a new class of counterexamples to the de Sitter conjecture [55] which rely on few ingredients for the uplifting mechanism. We may tune the value of the cosmological constant to high precision as this relies only on the tuning-power of the complex-structure-based flux landscape which is expected to be high.

In all scenarios discussed, the implementation of the SUSY-breaking mechanism relies on tuning the parameter $\alpha \sim e^{u_0 - v_0}$ or generalizations thereof in the multi axion case. We require specific values of $\mathcal{O}(1)$ for α [12] and therefore need to precisely tune u_0 against v_0. The next step in analyzing the suggested vacua is therefore a full landscape study of the amount and precision of tuning that is possible for the values of complex structure moduli in the large-complex-structure limit.

[12] The precise values required depend on the parameters which appear implicitly in the definition of $\tilde{\alpha}$ in (4.18).

Chapter 5

Towards a Swampland Global Symmetry Conjecture using Weak Gravity

5.1 Introduction

As discussed in Sects. 1.2.2 and 2.2.1, it is common lore that a quantum field theory, if consistently embedded in quantum gravity, will not possess exact global symmetries [15,75–81,16,82]. But it is not straightforward to translate this to a quantitative statement about symmetry-breaking operators in the low-energy effective theory. In this chapter, we attempt to address this question in an important class of models: Those possessing a linearly realized, approximate global symmetry which derives from a $U(1)$ gauge theory.

Of course, the size of coefficients of global-symmetry-violating operators has been discussed for a long time on the basis of wormholes or, more generally, gravitational instantons [76–78]. Moreover, in the case of a spontaneously broken global $U(1)$, an axion exists. Symmetry breaking is then encoded in the instanton-induced axion potential, which is constrained using the axionic version of the WGC [17]. By contrast, our focus here is on linearly realized global symmetries, which can, e.g., be used to protect some type of particle number in the low-energy effective field theory. In specific cases, relevant constraints deriving from the WGC have recently been given in [214, 83]. Additionally, a general bound, independent of the WGC but rather motivated by black hole effects in a thermal plasma, has been conjectured in [83]. Since it is likely that gauge symmetries are also constrained by swampland arguments, e.g., the total rank of the gauge group, and our interest is in global symmetries, we here adopt the terminology *Swampland*

Global Symmetry Conjecture for our statements and bounds, but we will argue that the precise formulation and underpinning of the conjecture is yet to be determined. We will make a corresponding suggestion.

Our main technical result goes beyond previous work as follows: First, we claim that given a slight, natural generalization of the WGC and the completeness hypothesis our constraint can actually be derived. Second, while not completely general, it addresses a very large class of constructions which play a central role in model building in general and in particular in string compactifications [215]. The models we want to consider have an underlying gauged $U(1)$ symmetry. If this $U(1)$ is non-linearly realized, the vector and the Nambu-Goldstone boson or axion are removed from the spectrum.[1] The axion may be a fundamental periodic scalar or the phase of a complex Higgs, though there may be some differences as we explain below. Importantly if some of the originally $U(1)$-charged particles survive in the low-energy effective theory, they will transform under a global $U(1)$. The latter is linearly realized, in spite of the fact that the high-scale gauge $U(1)$ is removed. The reason is simply that the axion is not part of the low-energy theory.

As we argue in the following, crucially, the axion should couple to some form of instanton – this is required by the completeness hypothesis [219, 16]. A mild generalization of the WGC to this case further constrains their action [17] and, in its magnetic form for axions [19],[2] provides a relation to the UV cut-off of the 4d effective theory. Moreover, as will be argued in full generality below, these instantons necessarily induce EFT operators which violate the global $U(1)$. This leads to the desired quantitative bound.

As an interesting fact we note that, while very different in their motivation and range of applicability, all of the above bounds on symmetry-violating operator coefficients have the parametric form $\exp(-M_{\rm P}^2/\Lambda^2)$. In all cases, from wormholes to instantons to black holes in a thermal plasma, one may argue that the parametrics are necessarily the same: The exponent is simply the Einstein-Hilbert action of some localized object, with $\int \mathrm{d}^4x\, R$ replaced by $1/\Lambda^2$, where Λ is the UV cut-off. Specifically in the wormhole context, we find a generalization of the well-known Giddings-Strominger solution to the case of a $U(1)$ gauge-derived global symmetry where a globally charged particle passes through the wormhole.

Finally, we note, and will discuss in more detail below, that the above

[1] Here we are interested in the case where the vector mass is not parametrically below the cut-off of the theory, so we are not considering the limit of small $U(1)$ gauge coupling, and the connection with the physics of light vector states with Stückelberg masses [216, 217], or anomalies [218].

[2] This bound has also been used to constrain a Stückelberg mass [216], which is however not our interest in the present work.

parametric similarity suggests a simplicity which might be misleading. First, it is essential whether just one or all symmetry-violating operators must respect the parametric bound above. Second, it may be that different types of approximate global symmetries (to be specified momentarily) call for different bounds.

The rest of the chapter is organized as follows. In Sect. 5.2, we review our main idea based on the WGC for axions gauged under a $U(1)$ symmetry and instanton-induced operators, which are symmetry-violating and suppressed by a factor of $\exp(-M_{\rm P}^2/\Lambda^2)$. Sect. 5.3 demonstrates this in simple 4d and 5d toy models involving, respectively, fermions and gauge instantons, and a purely bosonic 5d theory compactified to 4d. We discuss some explicit quantum gravity realizations of our bound in Sect. 5.4, including wormholes and the Euclidean brane instantons of string models. We also comment on recent arguments based on black hole effects in a thermal plasma. Limitations of our approach, in particular a possible loophole related to the numerical coefficient in the exponent, are discussed in Sect. 5.5. We briefly discuss a stronger bound that may be concluded from our arguments and how this bound is consistent with the examples of the previous section. Moreover, we discuss how our results may combine in a general swampland global symmetry conjecture. We conclude in Sect. 5.6.

5.2 Basic Argument

5.2.1 Definitions and Classification

Let us start by defining some basic terminology, without any claim to novelty or originality: We will say that an EFT possesses an *approximate global symmetry* if among all possible processes, $P(\{i\} \to \{j\})$, allowed by all space-time and gauge conservation laws there is a subset $P_{\rm gsv}$ with rates that are parametrically smaller, *and that this subset is distinguished by the violation of an otherwise conserved additive or multiplicative quantum number.*[3] Here

[3] We hope the reader finds the notion 'parametrically smaller rates' intuitively clear! To precisely define what one means by this is in general involved as can be illustrated by the following example: Consider a 4d $U(1)$ gauge theory with two types of bosons of charge, say, ± 1 and ± 11. Then the leading gauge-invariant operator that connects the two types of matter in a way that violates the individual (particle - antiparticle) numbers N_1 and N_{11} is $\phi_{11}(\phi_1^*)^{11}/\Lambda^8 + \text{h.c.}$ This leads to, e.g., a $\Delta N_1 = -11$, $\Delta N_{11} = 1$, $2 \to 10$ particle scattering process with cross section parametrically going as $\sigma_{\Delta N}(E) \sim E^{14}/\Lambda^{16}$, where here we are assuming the center-of-mass scattering energy $E \ll \Lambda$ is much greater than the masses of both ϕ_1 and ϕ_{11}. On the other hand there are $\Delta N_1 = \Delta N_{11} = 0$, $2 \to 10$, particle scattering processes starting with exactly the same initial states which

$\{i\}, \{j\}$ label the set of all possible multi-particle initial and final states of the EFT degrees of freedom. Note that it is important that *all* rates associated with the violation of the relevant additive or multiplicative quantum number are small, for there can be circumstances where individual processes in a theory can be small without there being a good notion of an approximate global symmetry. (Alternatively, for theories such as conformal field theories which do not have a well-defined notion of particle, we can consider all possible correlation functions of the theory and apply a similar definition.) In addition we emphasize that the gauge conservation laws may not be associated to long-range massless gauge bosons, as the theory could, and in general will, possess discrete gauge symmetries, either Abelian or non-Abelian which will restrict the allowed processes [220–224]. These discrete gauge symmetries can be distinguished from exact global symmetries by long-range Aharonov-Bohm-type scattering experiments.

Moreover, the global symmetries that are of interest to us in this work are associated with, in the continuous case, conventional Noether currents, and, more generally, group action operators faithfully realizing a continuous or discrete group that satisfy certain locality properties. Such global symmetries are 'splittable' in the terminology of the AdS/CFT proof of Harlow and Ooguri [82]. Of course, in the approximate global symmetry case the unitary operators enacting the would-be symmetry only approximately commute with the Hamiltonian of the theory and, if we are concerned with a continuous global symmetry, the Noether currents are only approximately conserved.

Corresponding to this definition the operators in an EFT action describing a theory with an approximate global symmetry may be divided into two disjoint classes: the singlets and the non-singlets with respect to the would-be global group action. Moreover, the non-singlets should either be irrelevant in the Wilsonian sense or have 'small' coefficients (we will later refine the meaning of 'small' and define a notion of a *high-quality* approximate global symmetry).

There are different reasons why an EFT might possess an approximate global symmetry. For example, approximate global symmetries may be

(1) **Gauge-derived.** With this term we would like to refer to global symmetries following from a non-linearly realized gauge symmetry. Specifically,

have rates not smaller than $\sigma_{2\to 10} \sim \alpha^{10}/E^2$, where α is the $U(1)$ fine structure constant. Thus for $E \ll \alpha^{5/8}\Lambda$ the rate of otherwise similar $\Delta N = 0$ and $\Delta N \neq 0$ processes is parametrically different. In addition there are many $\Delta N_1 = \Delta N_{11} = 0$, $2 \to k$ $(k < 10)$ processes with cross sections $\sigma_{2\to k}(E) \gg \sigma_{\Delta N}(E)$, so $\Delta N_{1,11}$ violation is slow. The issue of almost global symmetries appearing in the EFT due to large ratios of the gauge charges of the light states will be discussed further below.

in the case of a $U(1)$ gauge symmetry Higgsed by an axion both the vector and the pseudoscalar become heavy. Yet, any charged state which for whatever reason remains light will now be subject to an approximate global $U(1)$ where the coefficients of all symmetry-violating terms in the EFT are small. We will further explore the physics of such gauge-derived global symmetries as this case will be the main focus of our work.

(2) **Accidental.** Here, spacetime and gauge symmetries, continuous or discrete, forbid all relevant and marginal symmetry-violating operators constructed out of the light field content of the EFT.[4] An interesting WGC-derived bound on how high the mass-dimension of excluded operators can become in simple models has recently appeared in [83] (though it has also been noted that this can be avoided at the price of larger field content). Moreover, in, e.g., the gauged $\mathbb{Z}_N$ case the power of this idea clearly grows with N. This N, however, may be constrained using black-hole arguments [225] or, even more strongly, using also the WGC for 1- and 2-forms [226, 227, 83].

(3) **Fine-tuned.** By this we mean that the coefficients of all relevant and marginal operators that transform under the would-be global symmetry are 'small' by a landscape-type tuning. This option is limited in cases where, as expected in string theory, the landscape of EFTs with cut-off $\gtrsim \Lambda$ is finite [228, 229]. One may try to quantify this by arguing how the number of vacua grows with M_P/Λ. It is even conceivable that our bound, already advertised in the Abstract and Introduction, is valid for such type-(3) approximate global symmetries for the reason just explained. In this work, however, we will not be concerned with a quantitative analysis of this interesting possibility.

Given these definitions, we can usefully refine the notion of a 'small' violation: Suppose one has an EFT with cut-off Λ where all spacetime and gauge symmetries of the system have been identified. Then we define a *high-quality approximate global symmetry* to be one where the dimensionless coefficients (namely after appropriate powers of the cut-off have been extracted) of all

[4] Cf. B and L-symmetry in the Standard Model. At the level of the relevant and marginal operators there are no terms violating these global symmetries that can be written in the Lagrangian consistent with SM gauge symmetries and Lorentz invariance. There do exist potential irrelevant operators violating these symmetries. Moreover, given the SM field content there must be irrelevant operators present violating $(B+L)$, but not $(B-L)$, due the 't Hooft vertex interaction implied by the $U(1)_{B+L}$-$SU(2)_w^2$ anomaly. Note that potential Majorana mass terms for the neutrinos, which would violate L but not B, appear to be dimension-three operators in the far IR. However, above the weak scale they can be seen to arise from dimension five operators involving the SM Higgs doublet. This illustrates that the presence of accidental global symmetries depends on the energy scale at which one studies a given model.

symmetry-violating operators are *exponentially small.* The swampland global symmetry conjecture will bound just how high-quality any would-be global symmetry can possibly be, at least in the gauge-derived, type-(1) case.

An important comment concerning 'fine-tuned' or 'type-(3)' global symmetries has to be made: We do not use the word 'tuning' in 't Hooft's sense [230] since, of course, by its very definition the smallness of a coefficient is technically natural if its vanishing implies a global symmetry. Our main point is simply that an approximate symmetry might be present in the low-energy theory without a deep structural reason. Nevertheless, a relation to fine-tuning in the technical sense of 't Hooft exists. Indeed, in the absence of an underlying gauge symmetry, there may and in general will be irrelevant operators violating the desired approximate symmetry. Thus, while having finitely many operator coefficients small at the irrelevant and marginal level does not need tuning in the low-energy EFT, the full theory will tend to correct those coefficients on the basis of its symmetry-violating UV structure. In this sense, a fine tuning will indeed be needed and the name 'fine-tuned global symmetry' may be suitable in spite of the apparent clash with 't Hooft's nomenclature.

On a more general note, we should emphasize that the whole idea of conjecturing a quantum-gravity-derived minimal size of symmetry-violating effects goes against 't Hooft's technical naturalness. The latter is a concept of QFT in non-dynamical spacetime and assumes that it is possible to arrange things so that the couplings of global-symmetry violating operators renormalize only multiplicatively. We go beyond this by claiming that an unavoidable *additive* non-perturbative correction to the coefficients of such operators is always present.

5.2.2 Deriving the Bound

Our focus will be on type-(1) or gauge-derived global symmetries. To explain our general logic, recall first the gauging of a p-form gauge theory by a $(p+1)$-form gauge theory (or equivalently the 'Higgsing' of the latter by the former), see, e.g., [16]:

$$\frac{1}{g_p^2}|\mathrm{d}A_p|^2+\frac{1}{g_{p+1}^2}|\mathrm{d}A_{p+1}|^2 \qquad \to \qquad \frac{1}{g_p^2}|\mathrm{d}A_p+A_{p+1}|^2+\frac{1}{g_{p+1}^2}|\mathrm{d}A_{p+1}|^2\,. \tag{5.1}$$

In the Higgsed version on the RHS, the charged $(p-1)$-branes of the p-form theory cease to exist as independent objects for lack of gauge invariance. They can only appear as boundaries of the p-branes charged under A_{p+1}:

$$S \supset \int_{B_p} A_{p+1} + \int_{\partial B_p} A_p\,. \tag{5.2}$$

Only this combination is invariant under the gauge symmetry $\delta A_{p+1} = \mathrm{d}\chi_p$, $\delta A_p = -\chi_p$ of the Higgsed model. (For simplicity, we ignore for now the option of introducing a relative integer factor between $\mathrm{d}A_p$ and A_{p+1} on the RHS of (5.1). We comment on this below.)

Applied to our case of a 1-form Higgsed by a 0-form, this implies that instantons can only exist as the origin or endpoint of a worldline of a charged particle (see Fig. 5.1):

$$S \supset \int_{B_1(x_*)} A_1 \; + \; \phi(x_*) \,. \tag{5.3}$$

Here x_* is the location of an instanton and B_1 is the worldline of a light charged particle ending on it. It follows that the usual local (as far as the EFT is concerned) operator induced by the instanton sum also changes:

$$e^{-S_I + i\phi} \qquad \to \qquad \Phi\, e^{-S_I + i\phi} \,. \tag{5.4}$$

Here, we for simplicity assumed that our light charged particle is a complex scalar Φ, transforming as $\delta\Phi = \Phi e^{i\chi}$ for $\delta A = \mathrm{d}\chi$ and $\delta\phi = -\chi$.

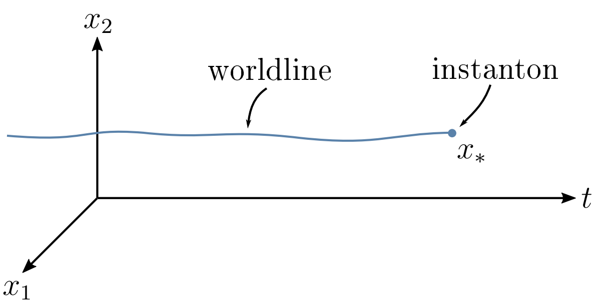

Figure 5.1: A worldline of a charged particle ending at an instanton at x_*. Integrating over instanton positions x_* induces a global-charge-violating operator.

Now the instanton action S_I is constrained by the WGC as $S_I \lesssim M_\mathrm{P}/f$ [17]. Furthermore, as also argued in [17], the WGC in general also has a magnetic version, constraining the cut-off. Specifically in the axionic case and for parametrically small $f \ll M_\mathrm{P}$, both this magnetic WGC as well as a black-hole evaporation argument suggest that $\Lambda \lesssim \sqrt{f M_\mathrm{P}}$ [19]. This gives $S_I \lesssim M_\mathrm{P}^2/\Lambda^2$ and hence the desired bound of the coefficient α of the global-symmetry-violating operator in (5.4):

$$\alpha \sim \exp(-S_I) \gtrsim \exp\left(-\frac{M_\mathrm{P}^2}{\Lambda^2}\right) \,. \tag{5.5}$$

To be more precise, the factor $\exp(i\phi)$ makes the operator on the RHS of (5.4) gauge-invariant. But after gauge fixing to $\phi = 0$, which is natural in the low-energy EFT, one is left with an instanton-suppressed global-$U(1)$-violating operator $\sim \alpha\,\Phi$, with the exponentially small coefficient α displayed above. Crucially, independently of any UV details, the WGC constrains the strength of the violation in terms of the cut-off of the 4d theory.

At this point, an important comment has to be made: When gauging the 0-form theory with the $U(1)$, the axionic degree of freedom merges with the 1-form to produce a massive vector in the familiar manner. More generally, according to (5.1) the p-form degrees of freedom merge with those of the $(p+1)$-form theory in the process of gauging. A priori it could be that the WGC does not apply to the two theories independently in their respective forms when they couple in this way. We here assume that it *does*, similar to the use of the WGC in, e.g., [216, 46]. Strictly speaking this represents an additional assumption mildly generalizing the minimal WGC.

We did not make a possible numerical coefficient in the exponent in (5.5) manifest since, as long as we do not make precise what we mean by the cut-off Λ, such a coefficient can always be absorbed in the latter. However, as discussed in more detail in [19], the present cut-off is associated with the tension of strings (coupling to the 2-form dual to the axion) going to zero. It hence in general represents a much more fundamental breakdown of the EFT than just a finite set of new particle states appearing at some scale. This situation also allows one in principle to make things more quantitative through replacing Λ^2 by the string tension T_1, such that $\exp(-M_{\rm P}^2/\Lambda^2) \;\to\; \exp(-c\,M_{\rm P}^2/T_1)$, where the $\mathcal{O}(1)$ coefficient c could now in principle be determined. This would require fixing the electric and magnetic versions of the WGC underlying our derivation exactly.[5] In the EFT, one might want to define the cut-off as $\Lambda = \min(m_A, \Lambda_A)$. Here, m_A is the photon mass after Higgsing, $m_A = g \cdot f$ (see Sect. 5.3), and Λ_A is the cut-off set by the magnetic weak gravity conjecture, $\Lambda_A \lesssim M_{\rm P}/g$ [17] for strong $U(1)$-coupling g and therefore weak magnetic coupling $\widetilde{g} = 1/g$. (At weak coupling g one finds $m_A \lesssim \Lambda_A = g M_{\rm P}$ by the WGC for axions.) One then finds $\Lambda \lesssim \sqrt{T_1}$ and therefore $\exp(-M_{\rm P}^2/\Lambda^2) \lesssim \exp(-M_{\rm P}^2/T_1)$. The strength of violation claimed in (5.5), using this more general cut-off Λ, is therefore even weaker than the one we explicitly derived.

[5] Precisely in the present case this is in fact non-trivial: On the one hand, it is not clear which object defines the WGC bound on the instanton side [53, 231, 232] (options include the Giddings-Strominger wormhole [75] or extremal instantons, which however involve a dilaton). On the other hand, the field-strength contribution to the tension of a charged string diverges in the IR, making also this side of the conjecture quantitatively more complicated [53].

Importantly, we expect that if an instanton-induced operator violates the global charge by one unit and is constrained as above, multi-instanton effects of instanton number k will induce operators violating the global symmetry by k units (here we are assuming for simplicity that the would-be conservation law is associated to a $U(1)$ and there is an additive quantum number) and be constrained to have coefficient above $\exp(-k\, M_{\mathrm{P}}^2/\Lambda^2)$. This structure of coefficients is consistent under renormalization group evolution of the EFT to lower scales. In addition if an operator with charge violation by k units is induced by an instanton, loops will induce all other operators with the same degree of charge violation unless there are secretly further symmetries.[6] If we now assume that the couplings of the theory not involved with the high-quality approximate global symmetry are not exponentially small, loop-suppression is non-exponential, and all operators violating the symmetry must appear with an overall structure of coefficients set by $\exp(-k\, M_{\mathrm{P}}^2/\Lambda^2)$ factors.

We have so far only focused on the exponential suppression of symmetry-violating operators. On top of it, there can be polynomial suppression by the cut-off. This is for example the case, when symmetry-violating operators of a field Φ , a $SU(N)$-singlet, are only loop-induced via coupling to a field ψ , a doublet under $SU(N)$ and which therefore couples to gauge instantons directly. We expect all operators that are not forbidden by an additional (hidden) global symmetry to be loop-induced. This could for example exclude fermion mass terms, as these are usually protected by an additional global flavor symmetry respected by the 't Hooft vertex[7]. We comment on this further in Sect. 5.6.

If we introduce an integer coefficient n in the coupling of the two gauge sectors in (5.1), $|\mathrm{d}\phi + nA_1|^2$, an unbroken $\mathbb{Z}_n \subset U(1)$ discrete gauge theory remains [220–222, 16]. This gauge symmetry strongly constrains the allowed operators. Given that the lowest $U(1)/\mathbb{Z}_n$-charge can be normalized to 1 , we can bound the dimension of the smallest operator that breaks the global $U(1)$-symmetry to $d \leq n$: $\Phi^n e^{-S_I}$. If there are multiple fields with higher charges, already operators of smaller dimension can respect the $\mathbb{Z}_n$ gauge symmetry.

Finally, we note that a $U(1)$ global symmetry may be broken, e.g., by a Higgs VEV, to a global discrete symmetry $\mathbb{Z}_n \subset U(1)$. In such cases, our bound (5.5) will of course apply to the latter.

[6] In the supersymmetric case this statement potentially requires some modification as there can be selection rules due to, e.g., holomorphy.

[7] See however [233, 234], where instanton-induced operators generate fermion mass terms via loops at large gauge couplings.

5.3 Simple Models

5.3.1 A Four-Dimensional Example

We illustrate the above argument with a simple, explicit example that is UV-complete in 4d. By this we mean that our instantons are conventional gauge instantons, such that no point-like 0-dimensional objects need to be introduced.

As above our two ingredients are a $U(1)$ gauge theory with charged fermions ψ on the one hand and an axion coupled to an $SU(N)$ gauge theory (and hence to instantons) on the other hand:

$$\begin{aligned} S_1 &= \int \mathrm{d}^4x \left(-\frac{1}{e^2}F^2 + \overline{\psi} i \not{D} \psi \right) , \\ S_2 &= \int \mathrm{d}^4x \left(-f^2(\partial\phi)^2 - \frac{1}{g^2}\mathrm{tr}\, G^2 + \frac{\phi\, \mathrm{tr}\, G\widetilde{G}}{8\pi^2} \right) . \end{aligned} \tag{5.6}$$

We now gauge the axion, which so far only possesses the discrete gauged shift symmetry $\phi \to \phi + 2\pi$, under the $U(1)$. Naively, one would simply replace $\partial_\mu \phi \to D_\mu \phi \equiv \partial_\mu \phi + A_\mu$. However, this is inconsistent due to the non-invariance of the last term in S_2 under gauge transformations $\delta\phi = \chi$. As explained in the general case, our gauging requires that charged worldlines end on instantons. In the case at hand, this can be realized by gauging the $U(1)$ charged fermions additionally under $SU(N)$.[8]

The theory is then defined by

$$\begin{aligned} S = \int \mathrm{d}^4x \Big(&-\frac{1}{e^2}F^2 - \frac{1}{g^2}\mathrm{tr}\, G^2 - f^2(D\phi)^2 \\ &+ \bar{\psi}_L i \not{D}_L \psi_L + \bar{\psi}_R i \not{D}_R \psi_R + \frac{\phi\, \mathrm{tr}\, G\widetilde{G}}{8\pi^2} \Big) . \end{aligned} \tag{5.7}$$

We have rewritten the Dirac fermion ψ in terms of a l.h. and a r.h. spinor. Moreover, each of these has been promoted to an $SU(N)$ fundamental multiplet. For the theory to be free of a mixed $U(1)SU(N)^2$-anomaly, we impose the condition $q_L - q_R = 1$ on the $U(1)$-charges q_L and q_R of the left- and right-handed fermion.[9]

[8] We anyway expect that, as in the case of the WGC for multiple $U(1)$'s [26], there should be light states charged under both gauge groups.

[9] Alternatively, we could have multiplied the $\phi G\widetilde{G}$ term by $(q_L - q_R)$.

We end up with a consistent theory [10] of fermions charged under $U(1) \times SU(N)$. Below the mass scale of the photon A_μ, whose mass is induced by $f^2(D\phi)^2 \supset f^2 A^2$, the $U(1)$ appears only as a global symmetry.

The $SU(N)$-instanton sum induces a 't Hooft operator [237], involving fermions and suppressed by $\exp(-S_I) = \exp(-8\pi^2/g^2)$, as part of the effective Lagrangian. In our case, it reads

$$\mathcal{O} = e^{-S_I}\, \bar{\psi}_L \psi_R \, e^{i\phi} + \text{h.c.} \tag{5.8}$$

This operator is of course invariant under the $U(1)$ gauge symmetry thanks to the shift in the axion. However, once we gauge-fix the axion to $\phi = 0$ and remove it from the effective theory, the operator explicitly violates the global $U(1)$ which would have otherwise survived.

5.3.2 Comments on a Possible Relation to an Effective Axion

Until now we focused on fundamental axions with a coupling $\phi G\widetilde{G}$ in the microscopic Lagrangian. It is clearly interesting to ask whether our arguments, leading to the bound of (5.5), can also be made in the case of an effective axion representing the phase of a complex scalar H. Indeed, let H have an Abelian-Higgs-model potential, enforcing a non-zero VEV: $H = ve^{i\phi}$. Then the low-energy EFT only contains the effective axion ϕ. The underlying global symmetry may be broken by operators $\alpha H + \bar{\alpha}\bar{H} \to \alpha v e^{i\phi} + \bar{\alpha} v e^{-i\phi}$, such that the full EFT partition function reads

$$\begin{aligned} \mathcal{Z} &= \int \mathcal{D}\phi \, \exp\left\{ -S_0[\phi] + \int \left(\alpha v e^{i\phi} + \bar{\alpha} v e^{-i\phi} \right) \right\} \\ &= \int \mathcal{D}\phi \; e^{-S_0[\phi]} \sum_{n,\bar{n}=0}^{\infty} \frac{1}{n!\bar{n}!} \left(\int \alpha v e^{i\phi} \right)^n \left(\int \bar{\alpha} v e^{-i\phi} \right)^{\bar{n}} . \end{aligned} \tag{5.9}$$

Here the second line can be interpreted as a sum over n-instanton/$\bar{n}$-anti-instanton sectors, which one would naturally expect to come with a fundamental axion.[11] One then identifies $v\alpha$ with e^{-S_I}, such that the WGC

[10] $U(1)$-gravity and $U(1)^3$ anomalies can be canceled by further Stückelberg terms (not involving the $SU(N)$) or by adding extra fermions, charged only under the $U(1)$ (for recent related work see [218]). In both cases our main points below are not affected. Note that gravitational instantons contributing to the effective action via the $\phi R\widetilde{R}$-coupling [235,236] will parametrically give the same result as gauge instantons, see Sect. 5.4.1.

[11] Note that these terms do not originate in a $\phi G\widetilde{G}$ coupling to gauge instantons. We simply rearranged the perturbative terms in the path integral in a suggestive, instanton-like manner.

for axions places a lower bound on the operator coefficient α. This logic extends to the gauged case as follows: Include a $U(1)$ gauge theory which, according to the WGC, comes with a charged particle Φ. To gauge H, we have to replace the 'instanton-type' operator αH by $\alpha H\Phi^\dagger$. After integrating out the massive vector and axion, the low-energy EFT now contains the operator $\alpha v\Phi^\dagger$, corresponding to the destruction or creation of Φ-particles (cf. Fig. 5.1), with a coefficient bounded from below by $e^{-M_\text{P}/f}$.

However, this Higgs-derived axion case may be different since our logic places only an exponentially small bound on operator coefficients which may be naturally $\mathcal{O}(1)$. In general, our Higgs field $H = v\,\exp(i\phi)$ is available for the construction of all kinds of operators in the high-scale theory. Thus, even in the fermionic case (where we previously had a symmetry reason for a light $U(1)$-charged particle) we must now allow for the operator $y\,H\bar{\psi}_L\psi_R$ to be present. Then no low-energy global $U(1)$ survives in the first place unless we can ensure that $y \ll 1$ [12]. In our present understanding, also more involved Higgs-based models of this type (with more fermions and other Higgs-charge) generically have the same feature: The survival of a global $U(1)$ before non-perturbative effects are included requires the choice of a small operator coefficient.[13]

Let us pause to spell out the difference between the fundamental and effective axion cases at energies below the physical Higgs scale and before the axion is gauged by the 1-form $U(1)$ symmetry: Both cases are by definition built on a scalar with gauged discrete shift symmetry $\mathbb{Z}$. In both cases, shifts $\phi \to \phi + 2\pi\epsilon$ with ϵ non-integer are *not* gauged and hence need *not* be respected by all terms in the Lagrangian. In the effective case, this simply means that one must suppress all higher-dimension operators violating such shifts by non-integer ϵ. This requires additional tools, e.g., tuning or extra symmetries. In the fundamental case, the standard form of the leading-order gauge theory action excludes non-derivative couplings of the axion. Indeed, our axion is viewed as a 0-form potential, the kinetic term is $|\mathrm{d}\phi|^2$, and any further appearance of ϕ arises only in combination with charged objects. These are the instantons, allowing contributions with ϕ evaluated at their location, but only at the price of a factor $\exp(-S_I)$. A more fundamental reason for why this basic gauge theory structure cannot be broken may be

[12] A small value of y may be technically natural in the 't Hooft sense since the coupling y can be forbidden by chiral symmetry. However, this further global chiral symmetry now comes in as an extra assumption. Our results limit how small y may become. Fortunately, the condition that the charged particle appears in the low-energy effective theory is simply that y is parametrically, not exponentially, small.

[13] Exceptions of the Frogatt-Nielsen-type are possible at the price of having only a highly-charged field in the low-energy EFT (as discussed in [83] in the present context).

given as follows: We declare that a proper gauge theory must allow for both an electric and a magnetic formulation. Hence, an axion is *weakly-coupled fundamental* only if a dual 2-form description exists in which the instantons (now viewed a 0-dimensional defects enclosed by a quantized 3-form-field-strength integral) have action $S_I \gg 1$. In this dual formulation, local operators providing non-derivative couplings of ϕ cannot be written down.

5.3.3 A Simple Five-Dimensional Example

We now sketch a simple 5d toy model which contains some features of the string models quoted in Sect. 5.4.2.

Consider a $U(1)$ gauge theory on $\mathbb{R}^4 \times S^1/\mathbb{Z}_2$. Let a charged scalar ρ be localized on boundary 1 and, similarly, a scalar σ on boundary 2. In addition, the WGC for the bulk $U(1)$ requires the existence of a charged bulk field Φ (see Fig. 5.2). Let Φ be a scalar for simplicity. We will return to the case where the WGC particle is a fermion at the end of this section. A VEV of σ, $\langle\sigma\rangle = ve^{i\theta} \neq 0$, gives a mass $m_A^2 = g_5^2 v^2/R$ to the photon, leaving a global symmetry under which ρ is charged at low energies.

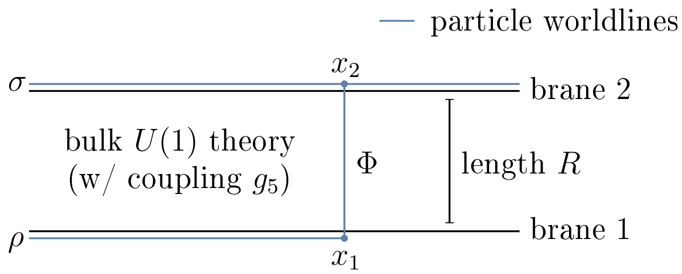

Figure 5.2: A 5d toy model with charged scalars confined to the two boundary-branes. In the presence of a σ-VEV, the bulk $U(1)$ gauge symmetry is broken but a global $U(1)$ survives.

We assume that the couplings of Φ to both σ and ρ allowed by locality and gauge invariance are present. As a result, there are instanton-like processes in which charged ρ-particles disappear from their brane, see Fig. 5.2. Summing over all such 'E0-brane instantons' induces a corresponding operator in the 4d effective action. It comes with a suppression factor $\exp(-S_\Phi)$, where $S_\Phi = m_\Phi \int_{\mathrm{E0}} \mathrm{d}y\sqrt{-g} \propto m_\Phi R$ is the action of a Euclidean 0-brane stretched over the interval. Moreover, the coupling of Φ to the gauge field, $i\int_{\mathrm{E0}} A \equiv i\phi$, gives a factor of $\exp(i\phi)$. All in all, this gives an effective gauge-invariant operator of the form

$$\rho\, e^{-S_\Phi} e^{i\phi}\, \sigma = \rho\, e^{-S_\Phi} e^{i(\phi+\theta)}\, v\,. \tag{5.10}$$

We may use the residual gauge symmetry to set $\phi + \theta = 0$, which leaves us with a global-symmetry-breaking tadpole operator for ρ:

$$\mathcal{O} \sim v e^{-S_\Phi} \rho\,. \tag{5.11}$$

From the 5d version of the WGC we have

$$S_\Phi \sim m_\Phi R \lesssim g_5 M_5^{3/2} R \lesssim M_5 R\,, \tag{5.12}$$

where the last estimate uses the perturbativity requirement $g_5 \ll 1/\sqrt{M_5}$. At first sight a reasonable cut-off of the 4d EFT is the compactification scale $\Lambda = m_{\rm KK} \sim 1/R \ll M_5$. Then, using $R \sim M_4^2/M_5^3$, we find

$$S_\Phi \lesssim M_5 R \sim \frac{M_4^2}{M_5^2} \ll \frac{M_4^2}{\Lambda^2}\,, \tag{5.13}$$

which agrees with our general bound. We may try to go beyond this by raising the cut-off above $m_{\rm KK}$ and incorporating the (weakly coupled) tower of KK modes in the effective 4d description. A 4d EFT perspective may be maintained until the growing number of KK modes overcomes their weak coupling and one loses perturbative control. Unsurprisingly, this happens at the quantum gravity cut-off scale $\Lambda \sim M_5$ (see, e.g., [238]). With this, the inequality on the r.h. side of (5.13) is saturated.

Finally, let us comment on the possible case that the WGC particle in the bulk is a fermion Ψ of unit $U(1)$ charge. Since we only have a scalar on the brane, it is not possible to introduce a coupling between these two fields which would allow one unit of charge to be carried away from brane 1. However, a coupling $\sim (\rho^*)^2 \overline{\Psi}\Psi^c$ permits the removal of two units of charge and, correspondingly, an instanton-like process suppressed by two rather than one massive brane-to-brane propagators. More coupling options arise if one invokes the sub-lattice WGC and hence the presence of fermions with other charges. One may also postulate and apply a generalized version of the completeness conjecture using the charge lattice of $Spin(4,1) \times U(1)$. Here $Spin(4,1)$ acts on the bulk tangent bundle as prescribed by 5d relativity. Based on this, one can argue that both a charged scalar *and* a charged fermion must always be present at the 5d Planck scale, which is sufficient to derive (5.12). The existence of charged bulk scalars and fermions also follows from the stronger conjecture of [239] that supersymmetry should always be present at the energy scale set by the WGC. Either way, this logic makes our derivation of global-symmetry violation more general.

5.4 Direct Quantum Gravity and Black Hole Arguments

5.4.1 Gravitational Instantons

General Gravitational Instantons

Our arguments so far were indirect in that we used quantum gravity to support the WGC and the latter to argue for global symmetry violation. A more direct approach is the inclusion of gravitational instantons in the path integral. Most generally, we here mean contributions (ideally Euclidean solutions) with non-trivial 4d topology, as pioneered in [240, 241]. Among the many possible topological fluctuations (see, e.g., [242, 243]) the gluing of a K3 surface into $\mathbb{R}^4$ might be particularly interesting since its effect on fermions is quite analogous to the 't Hooft vertex discussed earlier [214]. This induces global symmetry violation, with the relevant operator suppressed by $\exp(-S_I) \sim \exp(-M_\mathrm{P}^2/\Lambda^2)$. The last expression follows simply from the facts that M_P^2 multiplies the Einstein action and that the integral over instanton-sizes is dominated by the smallest objects allowed by the cut-off.

A more general and maybe more intuitive way to violate global symmetries through topology change are Euclidean wormholes [75–78, 81], which can be interpreted as a pair of gravitational instantons (each corresponding to the emission or absorption of a baby universe). This issue has in particular been recently revived in the context of the violation of global shift symmetries [29, 32, 53, 231, 244, 232].

A parametric analysis of the Einstein-Hilbert action gives a suppression factor

$$\exp(-S_I) \;\sim\; \exp(-M_\mathrm{P}^2 L^2) \tag{5.14}$$

for wormhole-induced global symmetry violation. Here L is the typical wormhole radius. The smaller the wormhole, the smaller is the action and the weaker the suppression in (5.14). Requiring the wormhole to be controlled in the EFT implies $L \gtrsim 1/\Lambda$, which saturates the bound (5.5).

A Wormhole Solution with Localized Charged Particles

Let us now look more specifically at a gauge-derived global symmetry and its possible violation by a wormhole with topology $S^3 \times I$, where $I \subset \mathbb{R}$ is an interval. The worldline of our unit $U(1)$-charged particle passing through the wormhole fills out I and is a point in S^3. Let the particle sit at the north pole of S^3. The wormhole dynamics is best understood using the magnetic dual description of the axion theory. The magnetic coupling is f and the (Euclidean) action in our gauged case is

$$S = S_{\rm EH} + \int \left(\frac{1}{f^2} H_3 \wedge \star H_3 + e^2 \mathcal{F}_2 \wedge \star \mathcal{F}_2 \right) . \tag{5.15}$$

Here $\mathcal{F}_2 = \widetilde{F}_2 + B_2$ is a gauge-invariant magnetically dual $U(1)$ field strength (with magnetic coupling e^{-1}), cf. (5.1) for $p = 1$. Away from electrically charged particles, we have $\mathrm{d}\mathcal{F}_2 = H_3$.

Consider, in complete generality, a smooth patch of $\mathbb{R}^4$ and a worldline of a unit-charge particle passing through it. This charge is measured by

$$\int_{S^2_\epsilon} \mathcal{F}_2 = \int_{S^2_\epsilon} \widetilde{F}_2 = 1 , \tag{5.16}$$

where S^2_ϵ is a sphere of infinitesimal radius ϵ threaded by the worldline. Here the first equality follows since by assumption B_2 is smooth in our patch.

Now, consider a 3-sphere of our wormhole with an infinitesimal ball B_ϵ, centered on the north pole, cut out: $S^3 \backslash B_\epsilon$. We find

$$\int_{S^3 \backslash B_\epsilon} H_3 = - \int_{S^2_\epsilon} \mathcal{F}_2 = -1 , \tag{5.17}$$

where the sign signals the different orientation of the boundary of $S^3 \backslash B_\epsilon$ relative to S^2_ϵ.

The interpretation of this is as follows: If a $U(1)$ gauge theory is Higgsed by an axion, then a $U(1)$-charged particle traveling through a wormhole must be accompanied by an appropriate H_3 flux. That is, the field $\mathcal{F}_2$ induced by the particle is compensated for by flux on the rest of the sphere. This analysis appears to support the persistence of the Giddings-Strominger solution [75] in the Higgsed case.

However, the above arguments were purely topological. To understand the dynamical solution, let us write the relevant part of the action on $S^3 \backslash B_\epsilon$ in the form

$$S \supset \int \frac{1}{f^2} \Big[|H_3|^2 + e^2 f^2 |\mathcal{F}_2|^2 \Big] = \int \frac{1}{f^2} \Big[|\mathrm{d}B_2|^2 + m_B^2 |B_2|^2 \Big] . \tag{5.18}$$

Here in the last expression we have chosen the gauge $\widetilde{F}_2 = 0$. This is always possible on $S^3 \backslash B_\epsilon$ since there is no charged particle. Let us view the geometry as fixed and of typical size L and try to understand the B_2 solution. Since we work classically, the overall prefactor $1/f$ of the action may be ignored. Then our problem has two terms whose ratio is governed by the mass parameter

$m_B = m_A = ef$. We expect the term suppressed by m_B to be irrelevant in a geometry of size L if $m_B \ll 1/L$. But this last relation holds in our regime of interest where the electric coupling is weak, $e \lesssim 1$, and $L \sim 1/\sqrt{fM_P}$. Thus, the H_3 flux spreads out approximately homogeneously on the sphere, as in the pure Giddings-Strominger case. The perturbation of the field profile by the charged particle is negligible.[14]

In the opposite regime, $m_B \gg 1/L$, the first term in (5.18) would dominate such that the field B_2 should strongly localize around the particle at the cost of large field-gradients. A quantitative discussion of this regime goes beyond the scope of this work.

Wormholes with Smeared Charged Particles

Finally, the arguments above rely on the presence of a localized charged-particle worldline in the wormhole. But in our regime of interest the particle's Compton wavelength is larger than the wormhole radius, $m \ll 1/L$. So a better model may be that of an electric charge distribution j_3 smeared homogeneously over the transverse S^3 . While an action principle for the magnetic fields $\widetilde{F}_2 = \mathrm{d}\tilde{A}_1$ in the presence of a smooth electric current is notoriously hard to formulate, the equations of motion are easy to write down:

$$\mathrm{d}\widetilde{F}_2 = j_3 \, , \qquad \mathrm{d} \star \widetilde{F}_2 = 0 \, . \tag{5.19}$$

Here we assume j_3 to be proportional to the volume form on S^3 . It is immediately clear that $B_2 = -\widetilde{F}_2$, $\mathcal{F}_2 = 0$, extremizes the action (5.15) together with a harmonic 3-form flux H_3 that is homogeneously distributed as in the solution by Giddings and Strominger:[15]

$$\int_{S^3} H_3 = \int_{S^3} \mathrm{d}B_2 = -\int_{S^3} \mathrm{d}\widetilde{F}_2 = -\int_{S^3} j_3 = -1 \, . \tag{5.20}$$

In summary, a Euclidean wormhole solution supported by 3-form flux persists even if B_2 is Higgsed and the charged particle passing the wormhole is light. We highlight that this is an extension of the Giddings-Strominger solution by a charged particle which couples to the Giddings-Strominger axion as specified implicitly in (5.15). This solution therefore differs from the extensions found in [77,78], where the axion was coupled to only an additional real field to form a complex scalar field.

[14] We recall, for the convenience of the reader, that the wormhole radius then follows by demanding that the flux energy density, $|H_3|^2/f^2 \sim 1/(f^2L^6)$, equals the gravitational energy density, $M_\mathrm{P}^2 R \sim M_\mathrm{P}^2/L^2$. Solving this for L gives $L \sim 1/\sqrt{fM_P}$, as quoted earlier.

[15] Note that both B_2 and $\widetilde{F}_2$ are gauge dependent and can only be defined locally with $B_2 = -\widetilde{F}_2$ on every patch. We refrain from introducing patches in (5.20).

Comments on Gravitational Instantons

Finally, we note that there are open fundamental questions related to Euclidean wormholes. In particular, there may be problems with the definition of the Euclidean path integral for gravity in general as well as deep conceptual issues with the summation over baby universe states in particular (see [232] for a review and [245–247] for recent developments). We also note, that unlike gauge theories where cluster decomposition can be used to argue that a sum over non-trivial gauge topologies must be included in the path integral, there is no analogous definitive argument that gravitational instantons, or more general configurations of non-trivial topology must be part of the gravitational path integral [248]. Nevertheless, we view the agreement between the old wormhole/gravitational-instanton logic and the WGC-based derivation noteworthy.

5.4.2 String Constructions and Euclidean Branes

If quantum gravity is defined by string theory, one may appeal to the precise (though not general) arguments against exact global symmetries of [15]. The situation is even better in AdS space: Inconsistency of global symmetries can be proven using properties of the dual CFT [82]. Clearly, it is non-trivial to map this to realistic string (or AdS/CFT derived) models with non-perturbative effects, broken supersymmetry and positive vacuum energy. Nevertheless, explicit constructions of global symmetries (e.g., [249–251]) support what was said in Sects. 5.2 and 5.3:

For global symmetries arising from gauge symmetries on branes in string compactifications [16], it has been established that Euclidean D-brane instantons induce symmetry-violating operators [252, 251, 253, 215] [17]. These operators are governed by a coefficient

$$\begin{aligned}\exp(-S_{\mathrm{E}p}) &\sim \exp(-T_p \mathrm{Vol}(\Sigma_{p+1})) \\ &\sim \exp\left(-\frac{1}{g_s}\left(\frac{R}{l_s}\right)^{p+1}\right) \sim \exp(-M_{\mathrm{P}}/f)\,.\end{aligned} \tag{5.21}$$

Here Σ_{p+1} is the cycle wrapped by the Ep-brane (with tension T_p), R is a typical compactification scale and we have suppressed all numerical coefficients.

[16] Concretely, $U(1)$-anomalies of D-branes gauge theories are canceled by a 4d version of the original Green-Schwarz mechanism. The gauge boson acquires a Stückelberg mass and the symmetry survives as a perturbatively exact global symmetry. It may then only be violated by non-perturbative effects.

[17] See [254, 255] for constructions in M- and F-theory.

The last relation involves estimating the decay constant f of a C_{p+1}-axion coupling to the Ep-brane.

Taking the cut-off Λ to be the KK-scale, $m_{\mathrm{KK}} \sim 1/R$, we arrive at

$$\exp(-S_{\mathrm{E}p}) \sim \exp\left(-\frac{M_{\mathrm{P}}^2}{\Lambda^2}\, g_s \left(\frac{l_s}{R}\right)^{7-p}\right) . \tag{5.22}$$

We see that in the perturbative regime, $g_s \ll 1$ and $l_s/R \ll 1$ (and $p \leq 5$ for purely internal Euclidean branes), the suppression factor in the exponent is generically much smaller and the violation therefore much stronger than (5.5). This is because we typically find $\Lambda \ll \sqrt{f M_{\mathrm{P}}}$, that is, the magnetic version of the axionic WGC is not saturated by m_{KK}. Of course, this is not surprising: While m_{KK} is the obvious and maybe most reasonable cut-off to apply to the 4d EFT the cut-off may also be raised by including KK modes in the 4d description (see the discussion around (5.13)). Therefore it is expected that m_{KK} as a cut-off does not saturate the fundamental WGC. We also recall that, in the stringy context, there is a well known smooth transition between brane instantons and the pure gauge-theory instantons we discussed in our earlier toy model.

We close by emphasizing that our generic bound does not become uninteresting just because stringy models have well-understood brane instantons. Indeed, one could try to perfection a global symmetry by considering very special geometries and brane arrangements, hoping to achieve an arbitrarily high quality of the symmetry from the 4d EFT perspective. If our 4d derivation can be established, such attempts would be known a priori to be futile.

5.4.3 Black Hole Effects in a Thermal Bath

While black holes are maybe the origin of our conviction that quantum gravity violates global symmetries, it is not obvious how to relate their effect to the desired operator coefficients. A recent suggestion made in [83] is based on a 'local rate bound'. The latter says that in a thermal bath with $T \lesssim \Lambda$ the violation rates of a global symmetry should obey

$$\Gamma_{\mathrm{BH}} \lesssim \Gamma_{\mathrm{EFT}} . \tag{5.23}$$

Here, Γ_{BH} and Γ_{EFT} are the charge violation rates induced by thermal black hole fluctuations (dominated by black holes with $R_{\mathrm{BH}} \sim \Lambda$) and by local operators explicitly included in the EFT, respectively.

A possible conjecture (different from [83] – see below) is then that *any EFT coupled to gravity with cut-off* Λ *and possessing an approximate global* *symmetry should satisfy* (5.23). While the motivation of (5.23) remains mysterious to us,[18] such a conjecture would be intriguing by its simplicity and attractive implications: One easily derives from it a bound of the type

$$\alpha \gtrsim \exp(-M_{\mathrm{P}}^2/\Lambda^2) \tag{5.24}$$

for the coefficient α of the operator which dominates (5.23) [83]. This is in fact immediately obvious if one considers the thermal black hole abundance $\sim \exp(-M_{\mathrm{BH}}/T) \sim \exp(-M_{\mathrm{BH}}/\Lambda)$ together with the smallest allowed black hole mass $M_{\mathrm{BH}} \sim M_{\mathrm{P}}^2 R \sim M_{\mathrm{P}}^2/\Lambda$. Moreover, one may write the action for a black hole propagating for a time τ as

$$M_{\mathrm{BH}}\,\tau \;\sim\; \int \mathrm{d}^3x \int_0^\tau \mathrm{d}t\, M_{\mathrm{P}}^2 \sqrt{-g}\mathcal{R} \;\sim\; R^3\,\tau\, M_{\mathrm{P}}^2\,\frac{1}{R^2}\,, \tag{5.25}$$

and use this as an estimate of the black hole mass. Then it becomes apparent that the above derivation of (5.24) fits perfectly in the scheme underlying all bounds discussed in this chapter:

In the end, in all cases the number in the exponent is just the factor M_{P}^2 of the Einstein-Hilbert action, with the $1/\Lambda^2$ supplied on dimensional grounds. One way or the other, one appears to rely on a topology fluctuation of size $1/\Lambda$. In the WGC-version, this is hidden in the WGC bound on instantons, but it is secretly still present in that wormholes saturate that bound.

Unfortunately, things are not that simple and the conjecture proposed in [83] is in fact much weaker. In our interpretation, it says that in any EFT with cut-off Λ_1 it should be possible to raise the cut-off to a scale $\Lambda_2 \geq \Lambda_1$ such that (5.23) holds. One reason for this is the existence of clockwork-style N-field gauge theories in which all operators up to mass dimension $\sim e^N$ respect a certain global $U(1)$. Given a species-bound-motivated cut-off $\Lambda \sim M_{\mathrm{P}}/\sqrt{N}$, it is clear that the symmetry-violation rates scale as $\exp[-\exp(M_{\mathrm{P}}^2/\Lambda^2)\ln(\Lambda)]$, such that the local rate bound can be violated. One way out is to accept that one may have to raise the cut-off, such that the full lattice [53] of charged states comes into play. Then symmetry-violating operators of lower mass-dimension become accessible and the rate bound is respected. Another option would be to conjecture that the required clockwork-style [117, 118] models will not be found in the landscape. Crucially, a conjecture for which one may need to raise the cut-off has limited use for the low-energy observer. It may also be possible to break it by adding a sector with light strings at some scale between Λ_1 and Λ_2, which formally stops one from raising the 4d cut-off.

[18] Arguments in favor of this bound which assume particle-like objects in the energy domain above the cut-off have recently been discussed in v2 of [83].

5.5 Synthesis of Results

5.5.1 Comments on Further Effects

Of the three types of approximate global symmetries we discussed (gauge-derived, accidental and fine-tuned), our focus was on the first case. Here, we provided a general argument bounding the exponential suppression of symmetry-violating operators (cf. (5.5) of Sect. 5.2.2). However, we did not address the important issue of non-exponential prefactors. Indeed, a (simplified) generic form of a symmetry-violating operator is

$$\mathcal{O} = C\, e^{-c\, M_{\mathrm{P}}^2/\Lambda^2}\, M_{\mathrm{P}}^4 \left(\frac{\Lambda}{M_{\mathrm{P}}}\right)^k \left(\frac{\Phi}{M_{\mathrm{P}}}\right)^d , \tag{5.26}$$

with real numbers C and c as well as integers k and d. While we exclude $C \ll 1$ by demanding that the operator is not fine-tuned, the suppression by hierarchies in scales can be strong. Our ignorance of non-exponential coefficients derives not only from instanton prefactors, but also from the loop effects to which we appeal when claiming that operators with different dimensions and field-content can be loop-generated on the basis of a single instanton-induced operator.

5.5.2 A Possible Loophole and Resolution

A Lattice of Charged Fields

Furthermore, there is a loophole (related to potentially large numerical coefficients in the exponent) whose resolution might come with interesting new insights into the nature of instantons or the weak gravity conjecture: Let us assume that in our underlying $U(1)$ gauge theory (which obeys the completeness hypothesis) all fields with charges $q = 1, \ldots, k-1$ are heavy, $m_i \sim \Lambda$, and only the field Φ_k with charge k is light. Then any EFT operator violating the global symmetry must do so by k units. It hence derives from a k-instanton effect and is correspondingly suppressed: $\Phi_k \exp(-k\, S_I)$. For the observer in the EFT, the global symmetry is much more precise than expected since he cannot know that Φ_k is highly charged in the underlying gauge theory. Possibly, this is resolved once one includes gravitational instantons: Since all fields couple to gravity, there will be operators suppressed by the gravitational instanton action $\exp(-S_I)$ without any further parameters. Alternatively, it may be impossible to make all the lower-charge fields parametrically heavier than Φ_k. This option is interesting since it also works towards inhibiting the method of breaking of the WGC in the low-energy EFT by Higgsing [32, 39].

We note that such a model is subject to strong consistency constraints if the spectrum is fermionic as in our prime example of Sect. 5.3: Hierarchies in fermion masses affect the available fermion spectrum that has to cancel $U(1)$-gravitational and $U(1)^3$ anomalies [218]. This might further constrain the cut-off of the anomaly-free theory, or, turning the argument around, the fermions that we are allowed to make heavy without the low-energy theory becoming inconsistent.

But maybe the most straightforward way of dealing with this loophole is by insisting that we are in the setting of non-tuned, 'type-(1)' global symmetries: In other words, we have to insist on a symmetry reason for the lightness of Φ_k. This requires that Φ_k transforms in a non-trivial representation of some group G. If G is identical to our gauged $U(1)$, underlying the global $U(1)$ we are discussing, then Φ_1 is made light by the same argument as Φ_k. This is what happens for the chiral fermions of Sect. 5.3. By contrast, if G is some further gauged or (gauge-derived) global symmetry, then we expect by completeness that a charged field Φ_1' exists. This field should have unit charge under our basic $U(1)$ and transform under G just like Φ_k. Then we expect that the symmetry argument keeping Φ_k light also applies to Φ_1'. As a result, the low-energy observer would see symmetry breaking effects associated with Φ_1' and suppressed only by $\exp(-S_I)$, closing the potential loophole.

Realization of the Loophole via Seiberg Duality

It has been found in [256] that there is a duality reminiscent of electromagnetic duality in the confined phase of $SU(N_c)$ SQCD. The theory comes with a flavor symmetry $SU(N_f)_L \times SU(N_f)_R$ with quarks and anti-quarks in the $(N_f, 1)$ and $(1, \overline{N}_f)$ respectively. The dual theory is described by the gauge group $SU(N_f - N_c)$ with N_f flavors. The fundamental (anti-)quarks of the dual theory are in the $(1, N_f)$ and $(\overline{N}_f, 1)$ under the global flavor symmetry. This duality, Seiberg duality, holds in the IR region of the theories as long as $N_f > N_c + 1$. Importantly, Seiberg duality has been realized in string theory [257, 258]. Apart from the global flavor symmetry there is also a global baryonic $U(1)_B$ symmetry. Normalizing the original quarks to have charge ± 1, the dual solitonic objects have charges $\mp N_c/(N_f - N_c)$.

We may consider the above $SU(N_c)$ SQCD as a realization of our loophole: In the low-energy theory, the unit-charged quarks confine and disappear from the spectrum while the solitonic objects, the fundamental fields of the dual theory, remain.[19] These may have baryonic charges as high as $N_c/2$ for

[19] This idea and the following possible general resolution are based on discussions with

$N_f = N_c + 2$. The low-energy observer may think of these fields as fundamental objects and would bound the violation of the global baryonic $U(1)_B$ by $\exp(-S_I)$, while considering the construction of the low-energy theory, it may be as small as $\exp(-N_c\, S_I/2)$, with S_I the action of instantons coupling to an originally gauged $U(1)_B$. There seems to be a priori no obstruction to taking large color indices to parametrically violate our conjecture (5.5).

There are however two arguments on how this violation is avoided: First, the low-energy observer is constrained by the confinement scale as his cut-off, $\Lambda = \mu e^{-1/\lambda}$, where $\lambda = N_c\, g^2$ is the 't Hooft coupling. So while the exponent of the coefficient of the symmetry-violating operator in our conjecture naively goes linearly with the color index, the cut-off corrects for this by exponentially decreasing. Second, even without knowing that the theory derives from a confined one, the low-energy observer may count the number of species in his theory. It is given by $N_s = N_f^2 > N_c^2$. Therefore, the assumed cut-off by swampland-type arguments may already be as low as the species scale $\Lambda_s < M_{\mathrm{P}}/\sqrt{N_s} < M_{\mathrm{P}}/N_c$ [106,259]. We see that the cut-off decreases in high charges N_c faster (with $1/N_c$) than required (with $1/\sqrt{N_c}$) to not violate our conjecture at low energies.

A Possible General Resolution

Given the above example of possibly realizing the loophole, an idea to be explored in the future is that high (global) charges always come at the cost of lowered cut-offs (if no additional symmetries are added). This is manifest in examples of simple towers of particles which are prevalent in landscape constructions. A charge $q \gg 1$ comes with a whole tower and the species scale serves as a necessarily lowered cut-off $\Lambda \sim M_{\mathrm{P}}/\sqrt{q}$. In QCD-like examples, the confinement scale usually comes down exponentially in the charge as we have seen in above example. One may also consider the simple example of (highly charged) baryons (consisting of N_c quarks of unit charge) in confined theories to arrive at the same conclusion. We leave an exploration of this idea to future work.

5.5.3 Comment on a Stronger Constraint

As already briefly discussed in Sect. 5.2.2, there are reasonable cut-offs to consider other than the string tension appearing in the magnetic version of the axionic WGC. Concretely, when staying in the regime of perturbative couplings, $e \lesssim 1$, the photon mass $m_A = ef$ is a general cut-off to a theory

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with a gauge-derived global symmetry that lies beneath the string tension $\sqrt{fM_{\rm P}}$.[20] Using this one arrives at the estimated strength of global symmetry violation

$$e^{-S_I} \gtrsim e^{-\frac{M_{\rm P}}{f}} \gtrsim e^{-\frac{M_{\rm P}}{m_A}} \gtrsim e^{-\frac{M_{\rm P}}{\Lambda}} \,. \tag{5.27}$$

This of course gives a much stronger bound than our original conjecture of Sect. 5.2 which agrees with bounds derived previously [83] and the simple parametric analysis of the Einstein action. A complete study of this bound is beyond the scope of this work. A derivation (still assuming the electric version of the WGC for axions) requires justifying the assumption of perturbativity of the coupling at the energy scale associated to the Stückelberg mechanism.

We reconsider the two explicit quantum gravity examples above: This stronger bound is of course consistent with wormholes for $e \lesssim 1$. We derived this, including the corresponding wormhole solution, in Sect. 5.4.1. Rather than imposing the magnetic version of the axionic WGC on the instanton action, we insert the photon mass as a cut-off to arrive at

$$\exp(-S_{\rm WH}) \sim \exp\big(-M_{\rm P}^2 L^2\big) \sim \exp\left(-\frac{M_{\rm P}}{f}\right) \gtrsim \exp\left(-\frac{M_{\rm P}}{\Lambda}\right) \,. \tag{5.28}$$

Turning to Euclidean branes as in Sect. 5.4.2, we expand the parametric estimates by assuming that the gauge theory is given by the worldvolume theory on a D(8$-p$)-brane othogonal to the Euclidean brane. Then, $1/e^2 = T_{8-p}\mathrm{Vol}(\Sigma_{5-p}) \sim (R/l_s)^{5-p}/g_s$ which is generally in the perturbative regime in a controlled supergravity calculation (and $p \leq 5$)[21]. We therefore find (cp. (5.21))

$$\exp(-S_{{\rm E}_p}) \sim \exp\left(-\frac{M_{\rm P}}{f}\right) > \exp\left(-\frac{M_{\rm P}}{\Lambda}\right) \,, \tag{5.29}$$

where $\Lambda \lesssim m_A$ as well as $g_s < 1$ and $l_s/R < 1$.

Interestingly, considering the loophole in Sect. 5.5.2 and its possible resolution as discussed, we would now have to claim that the cut-off comes down as $\Lambda \sim 1/q$. Interestingly, the Seiberg realization of the loophole does fulfill this precisely when only considering the species scale, as the number of species grows quadratically in the charge. Baryon-like examples with an exponentially small confinement scale are also save from the loophole. However, the tower of charges discussed is not, as the number of species grows only linearly in the charge. As discussed, such a model does potentially

[20] This idea is based on discussions with Arthur Hebecker.

[21] A controlled EFT also requires $m_A \lesssim m_{\rm KK}$ which is true for $p \geq 1$ in these simple parametric estimates.

come with additional symmetries only, leading to the closure of the loophole independently of a lowered cut-off.

5.5.4 Towards a General Swampland Global Symmetry Conjecture

Our argument could be the starting point for a derivation of a swampland global symmetry conjecture, but there are a number of caveats. To see this, we need to recall our classification of global symmetries in three categories: gauge-derived, accidental, and fine-tuned. There are now different possible conjectures to be made.

We could be satisfied with the fact that our constraint applies only to global symmetries of the gauge-derived type. Then, we could recall that our logic (allowing also for loop effects) in fact suggests that *all* operators are affected by our constraint: Any operator violating the gauge-derived $U(1)$ charge by n units comes with a prefactor generically not smaller than $\exp(-nS_I) \sim \exp(-nM_{\mathrm{P}}^2/\Lambda^2)$.

In a finite landscape, the derived bound cannot be beaten by landscape-type tuning for all operators at the same time. Therefore, we may more generally conjecture: An EFT where, for almost all n, at least one operator violating a global $U(1)$ by n units has a coefficient below $\exp(-nM_{\mathrm{P}}^2/\Lambda^2)$ is in the swampland. Conversely, an EFT in the landscape possesses only a finite amount of operators violating a gauge-derived or fine-tuned global symmetry with coefficients smaller than $\exp(-nM_{\mathrm{P}}^2/\Lambda^2)$, where n is the $U(1)$-charge of the operator. This is very general but also very weak since finitely many low-mass-dimension operators may not be restricted at all.

We could take an operator-focused approach: One may hope that a stronger conjecture holds for gauge-derived and fine-tuned symmetries: There exists some Λ_0 such that no EFT with $\Lambda < \Lambda_0$ has any operator violating a global symmetry by n units with coefficient below $\exp(-nM_{\mathrm{P}}^2/\Lambda^2)$. Clearly, establishing this requires knowledge about the tuning-power of the landscape and its growth with M_{P}/Λ.[22]

Finally, we could maintain the above form of the conjecture for gauge-derived and tuned symmetries without excluding accidental symmetries from consideration. In this case, we would have to postulate a separate bound for symmetry violation in accidental global symmetries, as suggested in [83] using

[22] To be more precise, one would have to allow for a prefactor $f_n(M_{\mathrm{P}}/\Lambda)$ and constrain its form. Moreover, it is clearly conceivable that the truth lies somewhere in between demanding that at least one symmetry-violating operator or all such operators satisfy our bound.

a simple model and the WGC. Unfortunately, the suggested bound on the maximal mass-dimension up to which all operators can be forbidden may be evaded by clockwork-type EFT constructions. One would then need to hope that the latter are in the swampland.

We also note that a universal statement about global symmetries of all types has been suggested [83] on the basis of a 'local rate bound' in a thermal plasma (see our Sect. 5.4.3). To avoid the above problem of 'clockworked accidental-symmetries', the authors formulate their conjecture in a fairly weak, UV-sensitive way (the cut-off may need to be raised to see that a certain EFT satisfies the bound). As we just explained, both in the gauge-derived case and possibly more generally, we would like to claim a stronger bound, purely in the low-energy EFT. Finally, we recall the possibly stronger bound that may be applied in all scenarios just discussed: When taking the photon mass instead of the string tension as the universal cut-off the general bounds on coefficients are replaced by the stronger lower bound $\exp(-nM_{\rm P}/\Lambda)$ for an operator violating the global symmetry by n units.

5.6 Conclusion

We summarize our fundamental point as presented in Sect. 5.2.2: A gauge-derived global $U(1)$ symmetry can arise if a gauged $U(1)$ is Higgsed by an axion. This only requires that some charged particles survive below the Stückelberg mass scale. Now, since the axion unavoidably couples to instantons and the latter, equally unavoidably, violate global $U(1)$ charge, we can quantify the global-symmetry violation in the low-energy EFT. More precisely, the electric and magnetic form of the WGC for axions constrain the instanton action in terms of the cut-off, leading to an upper bound $\exp(-S_I) \gtrsim \exp(-M_{\rm P}^2/\Lambda^2)$ for the relevant dimensionless operator coefficients. Moreover, the cut-off Λ is related to the tension of the string associated with our axion theory. We briefly commented in Sect. 5.5.3 that one may replace the string tension by the photon mass as the generic cut-off appearing. For weakly-coupled theories one then finds a bound $\exp(-S_I) \gtrsim \exp(-M_{\rm P}/\Lambda)$.

Our argument could be the nucleus for a derivation of a swampland global symmetry conjecture, and we demonstrated a number of supporting examples in Sects. 5.3 and 5.4, but there are a number of caveats and potential loopholes, as discussed in Sect. 5.5. We also presented in Sect. 5.2 a classification of global symmetries in three categories: gauge-derived, accidental, and fine-tuned. It could logically be the case that our bound applies only to gauge-derived global symmetries without the accidental or fine-tuned mechanisms also being operative. However our logic suggests a number of other possibilities for a general swampland global symmetry conjecture, as we briefly discussed in Sect. 5.5.4.

There are further interesting questions left open by our analysis. For example, in Sect. 5.4.1 we presented a generalization of the Giddings-Strominger wormhole solution to the case of a gauge-derived $U(1)$ global symmetry where a charged particle passes through the wormhole. This leads to a violation of the global symmetry of size in accord with our bound. However the (possible) role of wormholes and other topologically non-trivial configurations in a quantum theory of gravity is still far from settled and merits much more study.

In summary, while we believe to have made progress in developing a swampland global symmetry conjecture, there are clearly many interesting open issues that remain to be resolved.

Chapter 6

A Conjecture on the Minimal Size of Bound States

6.1 Introduction

In this chapter, we attempt to generalize the weak gravity conjecture to all forces. This will lead us in an unexpected direction, making claims about the non-existence of certain bound states in quantum field theory (with little or no relation to gravity). As discussed in Sect. 2.2.1, one may formulate the WGC by saying that, for any abelian gauge force, equal-charge particles are more strongly repelled by the gauge force than they are attracted by gravity.[1] By definition, this excludes a gravitational bound state of two or more such particles. Interestingly, it also implies that charged black holes, even extremal ones, can kinematically always decay. One may hence hope that the conjecture is actually more general, forbidding (under certain assumptions) the existence of stable bound states.

Indeed, it has been tried to extend the WGC along these lines to other forces, specifically to interactions mediated by a light scalar [18]. This is still fairly straightforward in the context of states charged under a gauge force, where the scalar force is merely an addition. One can then demand that two such particles are more strongly repelled by the gauge force than they are attracted by gravity together with the scalar force (which is always attractive).

If one starts talking about uncharged states, there is a problem: In this case, practically stable[2] bound states, such as planets, clearly exist. Hence,

[1] In fact, this is oversimplified – one can only try to claim that a particle species exists for which the above statement holds.

[2] Stability relies on the conserved baryon number which, as an accidental global sym-

we will try not to forbid bound states in general but to constrain them. A benchmark case of a bound state is a boson star [3]. Specifically, consider a collection of N free complex scalars, with the number N protected by a global $U(1)$. Such stars become smaller as N grows and collapse to black holes at a critical (minimal) radius $R \sim 1/m$, with m the mass of the elementary bosons.

Based on this, a possible conjecture about forces is the following: They should not allow stable bound states with a radius that is smaller than what gravity can achieve. In other words, attractive forces cannot lead to arbitrarily compact stars that can then collapse to arbitrarily small black holes. As a result, adiabatically produced black holes have a minimal size. An alternative formulation reads: In a quantum field theory where the heaviest elementary state has mass m, no bound state with radius below the scale $1/m$ set by the Compton wavelength exists. Thus, gravity may be dropped, the connection to the standard notion of the swampland and the scalar WGC becomes remote, but we may have an interesting statement about quantum field theories. It appears plausible given its similarity to the uncertainty relation. Yet, it is still non-trivial since, in principle, a strong external force can of course confine a state on an arbitrarily short length scale. The claim is that in a consistent, fully dynamical, power-counting renormalizable field theory this cannot happen.

If such a bound were proven to be true, a very interesting conclusion about the interplay of IR and UV physics in QFT may follow: There is no efficient, adiabatic, IR method of generating UV excitations. That is, we cannot adiabatically collect particles in the IR theory to gather enough energy to create particles of a UV completion of that theory. For this, one would have to collect the IR particles with mass m in a spatial region of size $\ll 1/m$. The smallness is necessary to ensure a non-negligible transition rate of our collection of many light particles to few heavy particles of mass $M \gg m$. Thus, it may be possible to read our conjecture as forbidding such an 'IR-to-UV transformer'.

We focus on the case of $3+1$ dimensions, commenting briefly on the extension to other dimensions in the discussion section.

The rest of the chapter is organized as follows. In Sect. 6.2, we briefly review ideas on how to constrain scalar interactions in the spirit of the swamp-

metry, should be weakly violated. By 'practically stable' we refer to the extremely large proton lifetime $\gtrsim 10^{33}$ y. We are after a practically useful generalization of the WGC and are therefore not satisfied with the possible and fairly obvious statement that all bound states not protected by gauge invariance must decay.

[3] For a review on boson stars (updated in 2017) see [260]. A selection of recent work can be found in [261–276].

land. We point out a possible counterexample to a recent conjecture and argue for a slightly different path towards promising constraints. The *Bound State Conjecture* is motivated and quantified in Sect. 6.3. As explained above, we base our discussion on the minimal size of a black hole built purely through gravitational attraction. We then formulate an inequality which bound states with general interaction should satisfy such that smaller black holes cannot be constructed. Finally, we decouple gravity and arrive at a formulation of the bound state conjecture for general QFTs. Evidence for the conjecture is collected in Sect. 6.4. For this we consider both non-gravitational theories (a model with scalar-modulus coupling and ϕ^4-theory) and gravitational theories (boson stars, where the scalar interaction adds to the gravitational attraction). We discuss our results and collect open questions in Sect. 6.5.

6.2 Constraining Scalar Interactions

6.2.1 From the WGC with Scalars to Scalar WGCs and Beyond

Consider BPS states in an $\mathcal{N}=2$ supergravity theory with vector multiplets (see, e.g., [277]). Such states are (in general electrically and magnetically) charged under the vectors. They are also coupled to the vector-multiplet scalars or moduli χ_i. This latter coupling can be understood on the basis of the moduli dependence of the mass squared, $m_\phi^2 = m_\phi^2(\chi_i)$, of the BPS state ϕ. The effective Lagrangian term $m_\phi^2\phi^2/2$ then induces the trilinear coupling $\mu_i m_\phi \chi_i \phi^2$ with $\mu_i \equiv \partial_i m_\phi$ [18]. In this language, the BPS relation of [277] takes the form

$$\text{mass}^2 + \sum \text{(scalar interaction couplings)}^2 = \sum \text{charges}^2\,. \tag{6.1}$$

Here the 'mass' is to be taken in Planck units and the 'scalar interaction couplings' are the dimensionless quantities μ_i. Given that the scalar mediator induces an attractive Coulomb force, this relation can be interpreted as force neutrality between (asymptotically separated) BPS particles. Repulsive gauge interaction and attractive gravitational plus scalar interaction cancel exactly.

A non-supersymmetric generalization of the above was proposed in [18]:

$$\text{mass}^2 + \sum \text{(scalar interaction couplings)}^2 \leq \sum \text{charges}^2\,. \tag{6.2}$$

This is the *Weak Gravity Conjecture with Scalar Fields*. By the force argument above, imposing such an inequality on charged particles forbids them to form stable bound states.[4] In [85] it was shown that, in a simple toy model, the WGC in 5d is equivalent to this inequality after compactification to 4d.

Next, one may wonder whether a bound of the form (6.2) exists for uncharged particles. The obvious candidate is

$$\text{mass}^2 + \sum (\text{scalar interaction couplings})^2 \leq 0\,. \tag{6.3}$$

This simply says that stable, uncharged, massive particles are forbidden.[5] Such a statement makes sense since long-lived particles are usually protected by global symmetries. But the latter are at best approximate, making the lifetime of the corresponding particles always finite. However, in practice global symmetries may be of excellent quality, as in the case of baryon number and the corresponding proton lifetime of about $10^{24}\times$ (Age of the Universe). Some uncharged particles may hence be 'practically stable' (we will refer to them as stable for simplicity), making a conjecture of the type of (6.3) somewhat boring.

In [18] it is speculated that the correct (and less obvious) ansatz for a bound on uncharged states is the *Scalar Weak Gravity Conjecture* (SWGC)

$$\text{mass}^2 \leq \sum (\text{scalar interaction couplings})^2\,. \tag{6.4}$$

This can again be motivated by $\mathcal{N} = 2$ models, where a specific subclass of BPS states satisfies (6.4) with an equal sign. The obvious interpretation of the inequality (6.4) relates nicely to other versions of the WGC: Gravity is always the weakest force and is thus, in particular, also weaker than scalar attraction. Since both the left- and right-hand side characterize attractive forces, there is no interpretation in terms of a net repulsive force or a bound state argument. The conjecture may also appear problematic for another reason: In the effective theory below the mass scale of the exchange scalar (which will be non-zero without $\mathcal{N} = 2$ SUSY), the conjecture cannot even be formulated. Thus, this conjecture is on weaker footing.

In [84], it is suggested to apply (6.4) to scalar particles with self-interactions and to interpret it as an inequality for derivatives of the potential. In

[4] To be precise, one only demands that *some* of the charged particles must satisfy the inequality. As a result, a finite number of bound states may still be allowed.

Furthermore, one needs the scalar fields to be exactly massless for this interpretation. However, without $\mathcal{N} = 2$ SUSY such scalars presumably do not exist. It is then not completely clear how to formulate a fundamental principle supporting a 'WGC with scalars' since the scalar force effect disappears at asymptotically large distances. We do not discuss this further since our suggested scalar conjecture will in any case be rather different.

[5] Particles charged under a *discrete* gauge symmetry can certainly be stable and should be allowed together with $U(1)$-charged particles.

this case, the mass squared corresponds to V'' and the trilinear coupling to $V'''/\sqrt{V''}$. Thus, an inequality of the type $(V'')^2/M_{\rm P}^2 \leq 2(V''')^2$ results. Since a quartic interaction with negative sign also represents an attractive force, the authors supplement the above by a corresponding 4th-order derivative term,

$$(V'')^2/M_{\rm P}^2 \;\leq\; 2(V''')^2 - V''''\,V''\,. \tag{6.5}$$

This is proposed as the *strong Scalar Weak Gravity Conjecture* (sSWGC). Recently, a *generalized Scalar Weak Gravity Conjecture* based on the sSWGC has been put forward and applied to early universe dynamics [92].

6.2.2 Possible Counterexamples

It is easily seen that the sSWGC leads to very strong constraints in some cases. For example, for the potential

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \lambda\phi^4\,, \tag{6.6}$$

with positive mass squared, it implies that

$$m^2/M_{\rm P}^2 \leq -\lambda\,, \tag{6.7}$$

if one analyzes the point $\phi = 0$ [6]. This would allow only for attractive interactions ($\lambda < 0$) between massive ($m^2 > 0$) scalars described by such a potential. However, we expect this potential to be in the landscape for any sign of the interaction, as it is the most general power-counting renormalizable potential for a scalar with a $\mathbb{Z}_2$ symmetry $\phi \to -\phi$.

Indeed, and this is our first new technical point, a dilute gas of scalar atoms (e.g., helium-4) provides such a counterexample: Such light atoms are characterized by mass and radius on the order of

$$m \approx 10^3\,\text{MeV}\,, \qquad R \approx 1\,\text{nm} \approx 10^3/\text{MeV}\,, \tag{6.8}$$

respectively. Below the energy scale $1/R$, they behave as point-like particles with a 2-to-2 repulsive δ-function interaction. Such a system can be described by an effective quantum field theory based on the Lagrangian $\mathcal{L}(\phi,\dot{\phi}) = (\partial\phi)^2/2 - V(\phi)$, where the potential is as in (6.6), with positive coupling $\lambda > 0$. Here a real field is sufficient if only particles (and no antiparticles) are present and the energy scale $1/R$ is too low for pair production.

[6] For non-vanishing but small field values $\phi^2 \ll m^2/|\lambda|$, we can write the bound as $\frac{m^2}{M_{\rm P}^2} \leq -\lambda\left(1 + \mathcal{O}\left(\frac{\phi^2}{m^2/|\lambda|}\right)\right)$.

We start by a first estimate of the violation. The scattering cross section of this QFT system is then $\sigma_{2\to 2} \sim \lambda^2/m^2$, to be compared with the hard-sphere value $\sigma_{\rm geom} \sim R^2$. Identifying the cross sections $\sigma_{2\to 2}$ and $\sigma_{\rm geom}$ gives $\lambda \sim Rm \sim 10^6$. In the regime of small ϕ the conjecture demands (6.7). The explicit violation then reads

$$10^{-36} \not\lesssim -10^6 \,. \tag{6.9}$$

It arises simply because at low density and energy, corresponding to $\phi \approx 0$, we have $\lambda > 0$.

In spite of the large values of the coupling constant λ , we argue that at low energies and density (6.6) is a perfectly fine quantum field theory which is realized in nature (e.g., by helium-4 atoms, or, as extensively demonstrated in experiments, by dilute alkali gases which have scattering lengths of similar size) and hence provides a counterexample to (6.5). To be more precise about the density requirement, we note that a gas with mean particle distance l has an energy density m/l^3 . If the state is sufficiently homogeneous and coherent to allow for a classical field description, this density can be identified with $m^2\phi^2$. Thus, the field value is set by $\phi^2 \sim 1/ml^3$. We now see that perturbativity (in the sense that $m^2\phi^2$ dominates over $\lambda\phi^4$) is guaranteed when the gas is dilute (in the sense that $l \gg R$) [7]:

$$\frac{\lambda\phi^4}{m^2\phi^2} \sim \frac{\lambda/m^2l^6}{1/ml^3} \sim \frac{\lambda}{(ml)^3} \sim \frac{(R/l)^3}{(mR)^2} \ll (R/l)^3 \,. \tag{6.10}$$

In other words, diluteness (and small energies) implies $\phi^2 \ll m^2/\lambda$, which gives rise to the approximation used in calculating (6.7).

Alternatively, and maybe more appropriately in this atomic-physics context, one can approximate the corresponding equation of motion of this $\lambda\phi^4$-theory by the non-relativistic Gross-Pitaevskii (GP) equation [278, 279]

$$i\partial_t \varphi = \left(-\frac{\nabla^2}{2m} + g\,|\varphi|^2 \right) \varphi \,. \tag{6.11}$$

Here $\varphi \simeq (m\phi + i\pi)/\sqrt{2m}$ (with $\pi \equiv \dot{\phi}$) is the non-relativistic, normalized complex scalar field and $g = \lambda/8m^2$ is the GP coupling. The 'radius' of the interacting atoms in a quantum mechanical treatment corresponds to the (s-wave) scattering length a entering the low-energy field theory through $a = mg/(4\pi) = \lambda/(32\pi m)$. a is defined via the phase shift between incoming

[7] This is true for either sign of λ . The equation can be read with λ replaced by $|\lambda|$. Instead of a hard-shell radius R one should insert the absolute value of the (possibly negative) s-wave scattering length a , see below.

and outgoing wave in the low-energy limit [280]. Applied to hard spheres of radius R, this definition indeed gives $a \equiv R$. With this more precise description, one finds, for helium-4 atoms in the ground state, that $a \approx 8\,\mathrm{nm}$ [281, 282]. The relativistic coupling is then positive and thus repulsive, with $\lambda \approx 2 \cdot 10^9$ (using $m_{(^4\mathrm{He})} \approx 4\mathrm{GeV}$).

Let us comment on a possible modification of the statement of the sSWGC (6.5). In the spirit of the original SWGC (6.4) one might expect the inequality to be valid only for massless exchange particles. For a self-interacting theory of massive particles, this would correspond to requiring that the inequality is satisfied only at energy scales far beyond the particle's mass. In this way, the non-relativistic atomic gas described above would not be subject to the modified conjecture.

There still seem to be theories that are expected to be in the landscape but are in tension with even this modified sSWGC. Take again the self-interacting theory (6.6). As already alluded to in [84], one can check that near the vacuum configuration, $\phi^2 \ll |m^2/\lambda|$, the sSWGC implies that the product λm^2 is negative. One expects though that a generic supersymmetric theory with soft mass terms and quartic interactions from D-terms can have both positive m^2 and λ since the parameters can arise from independent sectors.

6.3 Bound State Conjecture

6.3.1 An Alternative Approach to Constraining Scalar Interactions

As we have tried to explain, we perceive the idea of constraining attractive scalar forces as very promising but are not fully convinced by the corresponding conjectures proposed until now. More concretely, there exists a straightforward logic taking us from the WGC, via the BPS condition (6.1), to (6.2). However, the next steps are less clear: While (6.3) could be called trivial, (6.4) and (6.5) appear problematic. Especially the crucial sign-flip of the scalar force between (6.2) and (6.4) may lack motivation.

Thus, we return to the WGC with scalars, as quantified by (6.2), and generalize it in an entirely different way. We propose that, in the absence of charges, the conjecture still constrains the strength of attractive forces and hence bound states, but in a less trivial way than (6.3). Namely, we accept that uncharged states *can* be bound by gravity (as it happens with the practically stable neutral atoms of the real world). However, we conjecture that any additional attractive force *cannot* enhance this binding parametrically.

A way to quantify this is through the size of the smallest black hole which can be adiabatically built from a given particle species. Indeed, stronger gravity (smaller M_{P} or larger particle mass) allows for building smaller black holes. Attractive QFT forces can hence be constrained by imposing a lower bound on their radius. We phrase this as the *Bound State Conjecture:*

> *In a theory where the heaviest stable particle has mass m, it is impossible to construct adiabatically a black hole that is parametrically smaller than the black hole that can be built from free scalars of the same mass m.*

Let us argue more carefully why this constrains scalar forces: Consider a theory with just gravity and a free scalar field. Furthermore, consider a cloud of gravitationally bound scalar particles (a boson star) and a quasi-stationary process in which particles are added. In this process, the cloud becomes smaller and denser, eventually collapsing to a black hole. A very weak, additional attractive force does not change this picture qualitatively but makes the star at each stage even smaller, eventually leading to a smaller black hole. Our claim is that this road to smaller bound objects is severely limited: The purely gravitational case cannot be beaten parametrically. In fact, we would like to claim this in full generality, allowing for multiple particle species, gauge and scalar forces, quartic and any other interactions: In any given field theory coupled to gravity, no bound state parametrically smaller than the collapse size of a purely gravitational boson star made of the heaviest stable particle species[8] can be constructed. Due to gravity, there is, however, one exception: Following a black hole collapse Hawking evaporation sets in. So we can indeed create black holes smaller than $1/m$, but our conjecture is precisely about this *only* being possible due to gravity and Hawking evaporation (see also Sect. 6.3.2). Additional attractive forces within a QFT cannot achieve this.

The possible bearing of the swampland idea on boson stars has recently been discussed in [283, 284]. In particular, the swampland distance and de Sitter conjectures attempt to constrain scalar dynamics and are hence relevant to what type of boson stars may exist [284]. At the moment, we do not see a direct connection to our approach, but this may change with further research.

[8] Here, a particle is any state with mass m and radius $\lesssim 1/m$. By radius we mean the length scale above which no sub-structure can be resolved. Concretely, one may think of gravitons scattering off the state and determining above which energy scale the form factor becomes non-trivial.

6.3.2 Quantifying the Bound State Conjecture

To make our conjecture more explicit, we need to know the mass of the smallest black hole that can be adiabatically constructed in a theory of free (up to gravitational interactions) bosons. It is simplest to use complex free bosons, where a global $U(1)$ symmetry ensures approximate particle number conservation (up to very small non-perturbative gravitational effects). In this setting, we want to determine the size of a cloud of N gravitationally bound bosons of mass m. Assuming them to be confined in a sphere of radius R, we can estimate the energy (ignoring $\mathcal{O}(1)$-factors) by

$$E_{\text{tot}} = E_{\text{grav}} + E_{\text{loc}} \sim -\frac{M^2}{M_{\text{P}}^2 R} + N\,\frac{p^2}{m} \sim -\frac{M^2}{M_{\text{P}}^2 R} + \frac{N}{mR^2}\,. \tag{6.12}$$

Here we have added naive parametric estimates of gravitational binding and localization energy, the latter encoding what is also known as 'quantum pressure'. We use the uncertainty-principle value $p = p(R) \sim 1/R$.

Our expectation is that the equilibrium situation corresponds to roughly the minimum of (6.12) as a function of R. Using $N = M/m$, this 'free boson star radius' is found to be

$$R_{\text{FB}}(M) \sim \frac{1}{M}\left(\frac{M_{\text{P}}}{m}\right)^2, \tag{6.13}$$

in agreement with numerical results [285, 286]. Thus, as we keep adding particles and thereby increasing N and M, the boson star becomes denser (cf. Fig. 6.1). At some point, it reaches the critical radius[9] of black-hole collapse $R_{\text{FB}} \sim R_{\text{BH}} = M/M_{\text{P}}^2$. As a consequence, the density falls off again as even more mass is added, as is common for black holes. Of course, once we deal with a black hole, Hawking radiation also allows for moving downwards on the straight 'black hole' line in Fig. 6.1, as already stated in Sect. 6.3.1.

With this we can now quantify the bound state conjecture in the following way. A stable boson star built by a quasi-stationary process in any (in general interacting) theory gives rise to a curve $R(M)$. The intersection of $R_{\text{BH}}(M)$ with $R(M)$ in a graph analogous to Fig. 6.1 defines a minimum size R^{min}. At this radius black-hole collapse sets in. The conjecture demands that the resulting black hole is not parametrically smaller than the minimal size of a black hole built from free scalars of the same mass. This amounts to requiring

$$R^{\text{min}} \gtrsim R_{\text{FB}}^{\text{min}} \sim \frac{1}{m}\,. \tag{6.14}$$

[9] Inserting more correctly the Buchdahl limit or similar bounds results in further numerical $\mathcal{O}(1)$-factors only. We ignore this subtlety.

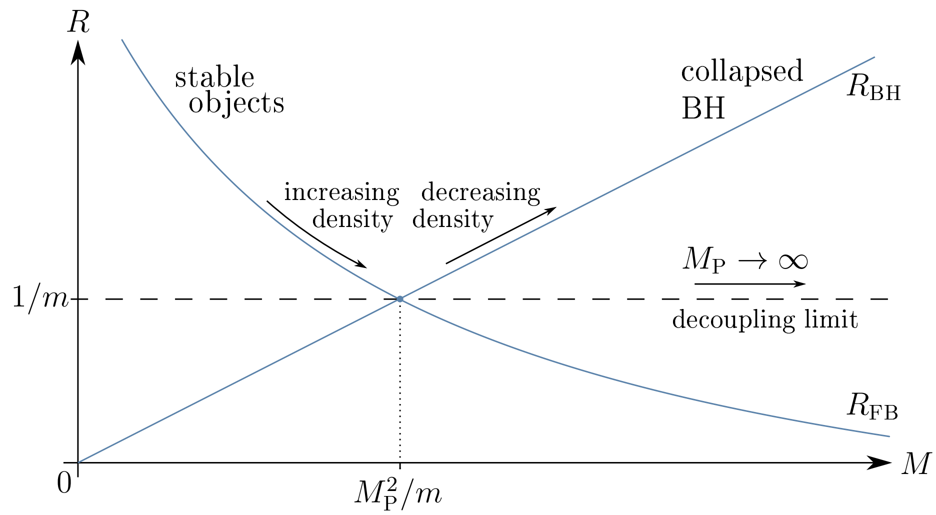

Figure 6.1: The mass-radius plot for the free boson star $R_{\text{FB}}(M)$ and a black hole $R_{\text{BH}}(M)$. As gravity is decoupled ($M_{\text{P}} \to \infty$) the intersection point of the two curves moves on the dashed line.

In other words, the intersection of $R(M)$ with $R_{\text{BH}}(M)$ cannot lie parametrically below the dashed line in Fig. 6.1.

Finally, we should comment on the definition of the radius $R(M)$ of a boson star. In a field theory, this radius can of course only be a typical length scale since field profiles are infinitely extended. In numerical calculations this scale is usually taken to be R_{99}, the radius which contains 99% of the mass. Since we are working only at the parametric or $\mathcal{O}(1)$-level, an analogously defined radius R_{50} is presumably more useful in our context.

6.3.3 Limit of Decoupled Gravity: General Bound State Conjecture

It is clear from the constraint on the size of bound states in (6.14) that our conjecture also makes sense in the limit of decoupled gravity, $M_{\text{P}} \to \infty$, since the dashed line in Fig. 6.1 is independent of M_{P}. It then says that any scalar configuration of arbitrary mass cannot be more localized than $R(M) \sim 1/m$ parametrically.

With this, and postponing a discussion of non-scalar bound states to Sect. 6.4.4, we are now in a position to quantify the (generalized) bound state conjecture:

Bound State Conjecture: *The typical radius R of a stable bound state in a power-counting renormalizable effective field theory, valid below some scale Λ, is bounded from below by*

$$R \gtrsim \frac{1}{m} . \tag{6.15}$$

Here, the scale m ($\ll \Lambda$) is the mass of the heaviest stable particle.

We consider only renormalizable theories, as we are looking to constrain infrared physics far below the cut-off scale Λ. Any higher-order term in an effective Lagrangian will be suppressed by this high scale. We will consider a non-renormalizable example in Sect. 6.4.2, which will provide an a posteriori justification for this restriction in our conjecture.

Let us note that our conjecture does not constrain cases where particles with mass m combine into a state of smaller mass, $M < m$. In this case, if $R(M) \lesssim 1/m$, then the newly formed object is, by our definition, a particle rather than a bound state. Furthermore, if $M > m$ one might worry that very small bound state radii, $R(M) \lesssim 1/M \lesssim 1/m$, would be allowed since, once again, the bound state must be viewed as a particle. However, this situation is excluded by the assumptions of our conjecture, which state that the particle of mass m is the heaviest stable particle of our theory.

We have to be careful with the notion of the 'heaviest stable particle' when going beyond perturbative and purely scalar theories. In a theory with only massless fundamental fields, like QCD (with massless quarks), states with finite radius and non-zero mass can certainly exist. A particularly simple example would be the lightest scalar glueball in pure Yang-Mills theory. Given some additional gauge or scalar force, such objects can form bound states of non-zero radius. So it would be wrong to demand that our conjecture holds with $m = 0$. Indeed, we were careful to define the term 'particle' as a state which appears point-like below its own mass scale. This is in contrast to bound states, which are extended objects. Then, e.g., a glueball or hadron can be viewed as a stable particle [10] whose mass sets the scale of the conjecture.

The bound states our conjecture applies to can, in general, carry higher spin. Recently, constraints have been put forward on the existence of higher-spin, composite states of mass M and size R which appear point-like or elementary in the sense that $R \lesssim 1/M$ [287, 288]. To make contact with our conjecture, which is specifically about non-elementary bound states, a

[10] Note again that our use of the word 'stable' includes objects protected by an approximate global symmetry.

generalization of the causality arguments of [287, 288] would be needed. Note that our constraint is stronger as it excludes bound states of size $R \lesssim 1/m$, with m the mass of the heaviest stable constituent particle.

It may be possible to formulate our conjecture as a field-theoretic realization of a resource theory [289, 290]. To explain this, let us for simplicity consider a model with light fundamental fields of mass $\leq m$ and one heavy field of mass M. We can now introduce an artificial cut-off Λ, with $m \ll \Lambda \ll M$, and focus on the effective theory below Λ. Our conjecture forbids the adiabatic construction of small bound states from particles with mass $\leq m$, which are the only ones available in the IR effective theory. Here 'small' means smaller than the inverse of the maximum mass, $1/m$. We expect that, in the full theory, this implies that we cannot efficiently create particles with mass M, point-like on length scales $\gtrsim 1/M$, from particles with mass $\leq m$ by an adiabatic process. The reason is that the transition rate from an extended object to a heavy fundamental particle is exponentially suppressed, even if quantum numbers allow the process in principle. For example, even if baryon number would be strongly violated, a small crystal of mass M_{P} would certainly have a negligible transition rate to a Planck-scale fundamental particle. That would change if one could confine enough light particles at the required small length scale $\ll 1/m$, but this is precisely what our conjecture forbids. In the language of resource theory, we may hence define the free states as those normally available to an IR experimenter, i.e., the particles with mass $\leq m$. The free operations would be adiabatic processes with those particles. Within this formulation and under these conditions, our conjecture is that a heavy particle with mass M is a resource which cannot be created by the IR experimenter.

Consider, e.g., a free stable boson star with radius (6.13) and mass $M \ll M_{\mathrm{P}}^2/m$ which, in mean-field approximation, is expected to be well described by a (semi-classical) coherent state of $N \sim M/m$ bosons in a single spatial field mode. Forcing the particles with mass m to localize at a smaller length scale R, with $R_{\mathrm{BH}}(M) \ll R \ll 1/m$ requires a deformation of the energy function which leads to a strong squeezing of the phase-space distribution of each particle and thus of the whole star. The external modification must in particular allow for anti-squeezing perpendicular to the squeezed spatial extent of the Wigner distribution. This is similar to what a larger value of m would achieve in the energetic balance (6.12), except that the squeezed particles assume relativistic momenta. Given that the modification is achieved by some in general non-linear interactions, a proof of our conjecture could then include an answer to the question why squeezing into the regime of relativistic momenta is inhibited, based on the practical impossibility to create the entanglement associated with the squeezed many-particle state.

One may summarize this in the *Bound State Conjecture (Resource-Theory Version)*: In a power-counting renormalizable effective theory where free states are those with particles of mass $\leq m$ and free operations are adiabatic processes involving these particles, heavy particles with mass $M \gg m$ represent a resource.

6.4 Evidence from Examples

We now discuss three different classes of examples all of which, as it turns out, appear to respect the conjecture: Attractive scalar-modulus couplings, quartic self-interactions of a complex scalar, and the attractive interaction implicit in axionic potentials.

6.4.1 Non-Gravitational Scalar-Modulus Coupling

Consider a theory with scalar interaction of the type discussed in [18, 85],

$$V(\phi) = \frac{m^2}{2}\,|\phi|^2 + \frac{\widetilde{m}^2}{2}\chi^2 + \mu m\,|\phi|^2\,\chi\,, \tag{6.16}$$

where ϕ is a $U(1)$-symmetric complex scalar field [11] and the real field χ is very light, $\widetilde{m} \ll m$. We will think of χ as a modulus, mediating a long-range, attractive force which is capable of binding ϕ particles. In this spirit, we will neglect $\widetilde{m}$ whenever it gives only sub-leading corrections.[12] Note that, instead of m and the dimensionless coupling μ, we may also use the two dimensionful parameters m and $\gamma \equiv \mu m$ to characterize the theory.

We introduced ϕ as a complex field so that we can make use of the conserved global $U(1)$ charge N (and accordingly particle number conservation). Using the standard stationary ansatz $\phi(t,\mathbf{x}) = \phi(\mathbf{x})e^{-i\omega t}$ with $\phi(\mathbf{x})$ real (cf. [291, 292]), the particle number is given by

$$N = i\int \mathrm{d}^3x\,(\phi^\dagger\dot{\phi} - \dot{\phi}^\dagger\phi) \sim \omega\int \mathrm{d}^3x\,\phi(\mathbf{x})^2\,. \tag{6.17}$$

[11] Note that we use the same symbol ϕ for both complex and real scalar fields throughout the chapter. We will clearly state in each section whether we deal with a real or complex theory.

[12] We did not set $\widetilde{m} = 0$ since, without $\mathcal{N} = 2$ SUSY, this is probably inconsistent. Moreover, it would be inconvenient to work in a model where only the boundary condition at spatial infinity sets the vacuum value of χ.

Non-Relativistic Estimate

As already discussed in the case of a gravitationally bound boson star, bound states with small N are large and become smaller as N grows. We start our analysis in the regime where $m \gg 1/R \gg \widetilde{m}$. This allows for a simple controlled calculation since we are on the one hand safely non-relativistic and can, on the other hand, neglect the finite range of the force inside the bound state. Indeed, the χ-exchange gives rise to an effective attractive potential of the Yukawa type, $V(r) \sim (\mu^2/r)\, e^{-\widetilde{m}r}$, for the ϕ particles. The effective binding energy inside a boson star of N particles is therefore estimated as [13]

$$E_{\text{scalar}} \sim -\frac{N^2\mu^2}{R} e^{-\widetilde{m}R} \approx -\frac{N^2\mu^2}{R}\,. \tag{6.18}$$

In addition, we have the localization energy (cf. (6.12))

$$E_{\text{loc}} \sim N \cdot \frac{p^2}{m} \sim \frac{N}{mR^2}\,. \tag{6.19}$$

Minimizing the total energy as we did in the free case (6.12), we find the radius of this bound state with scalar force to be

$$R_{\text{SF}}(N) \sim \frac{1}{N\mu^2 m}\,. \tag{6.20}$$

Given that the configuration is assumed to be spherically symmetric and localized inside a radius R as well as using the non-relativistic frequency $\omega \sim m$,[14] we have according to the general result (6.17)

$$N \sim m \int_0^R \mathrm{d}r\, r^2\, \phi(r)^2\,. \tag{6.21}$$

We expect that a $\phi(r)$-configuration sources a corresponding stationary profile $\chi(r)$. The latter is determined by the equation of motion

$$(\nabla^2 - \widetilde{m}^2)\chi(r) = \mu m\, \phi(r)^2\,. \tag{6.22}$$

[13] One can replace ϕ in $\mu m\,|\phi|^2\,\chi$ with the non-relativistic field $\varphi \simeq (m\phi + i\dot{\phi})/\sqrt{2m} \sim \sqrt{2m}\phi$. Here, in the last equality we used the exponential ansatz with $\omega \sim m$. Now the interaction term takes the form $\mu\,|\varphi|^2\,\chi$, explaining that only μ^2 and no m^2 appears in (6.18).

[14] Assuming the frequency $\omega \sim m$ one finds a solution to the equation of motion $(\nabla^2 - m^2 - \mu m\chi)\phi(r) = -\omega^2\phi(r)$ for large R (small gradients) and negligible interaction. The latter assumption can be checked a posteriori given the magnitude of back-reaction on the field $\chi(r)$.

The Green's function for the operator on the l.h. side is

$$G(r) = -\frac{e^{-\widetilde{m}r}}{4\pi r}\,, \tag{6.23}$$

leading to

$$\chi(r) \sim -\mu m \int_0^R \mathrm{d}r'\, r'^2\, \frac{e^{-\widetilde{m}(r-r')}}{r-r'} \phi(r')^2\,. \tag{6.24}$$

We assume that the source term is significant only inside a region of size R. Hence, focusing on the χ profile outside the star, we can use $r \gg R \geq r'$. This implies

$$\int_0^R \mathrm{d}r'\, r'^2 \frac{e^{-\widetilde{m}(r-r')}}{r-r'} \phi(r')^2 \approx \frac{e^{-\widetilde{m}r}}{r} \int_0^R \mathrm{d}r'\, r'^2 \phi(r')^2 \sim \frac{e^{-\widetilde{m}r}}{r} \frac{N}{m}\,, \tag{6.25}$$

where we used (6.21) in the last step. This gives

$$\chi(r) \sim -\frac{N\mu}{r}\, e^{-\widetilde{m}r}\,. \tag{6.26}$$

From this we cannot determine the solution inside the star as this needs precise knowledge of the $\phi(r)$-profile. However, it is reasonable to assume that it is not parametrically larger than (6.26) at the boundary $r = R$ (cf. Fig. 6.2). Thus,

$$\chi(r \leq R) \sim \chi(R) \sim -\frac{N\mu}{R}\, e^{-\widetilde{m}R} \approx -\frac{N\mu}{R}\,. \tag{6.27}$$

With this, we may reverse the logic and consider the effective potential for ϕ, induced in part by the χ profile which we just determined:

$$\begin{aligned} V(\phi) &\sim m^2\, |\phi(t,r)|^2 \left(1 + \frac{\mu}{m}\chi(r)\right) \sim m^2\, |\phi(t,r)|^2 \left(1 - \frac{N\mu^2}{mR}\right) \\ &\sim |\phi(t,r)|^2 \left(m^2 - \frac{1}{R^2}\right)\,. \end{aligned} \tag{6.28}$$

In the last two expressions, we assumed $r \lesssim R$ and made use of (6.20). This reveals a potential tachyonic instability. We see that the potential is only safely stable as long as

$$R_{\mathrm{SF}} \gtrsim \frac{1}{m}\,. \tag{6.29}$$

That is, for large R the calculation can be trusted and stable bound states exist. By contrast, for $R \sim 1/m$ or smaller, the effective potential for ϕ in (6.28) tends to become tachyonic. This appears to support our conjecture. However, at the same time our non-relativistic approach breaks down. In the next subsection, we address this issue.

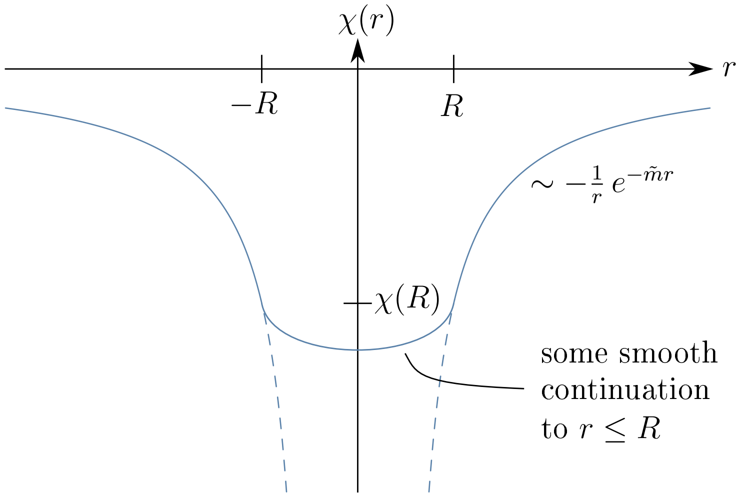

Figure 6.2: The $\chi(r)$-profile induced by the localized source $|\phi(r)|^2$. For better illustration, we extend the profile to $r < 0$ using spherical symmetry.

Analysis of the Instability

There are two caveats to the above conclusion. The first one is that our non-relativistic calculation breaks down close to the regime of interest $R_{\rm SF} \sim 1/m$. To remedy this non-relativistic assumption, we will repeat the calculation without specifying the oscillator frequency ω of the mode described by the mean field $\phi(t, r)$. The second is that, even if we continued to trust our calculation, the potential tachyonic instability found above does not necessarily lead to unstable bound states: While we found that *locally* the effective potential $m^2(r)\phi(r)^2 \sim m^2[1 + \mu\chi(r)/m]\phi(r)^2$ becomes tachyonic, this does not automatically lead to tachyonic modes. The oscillator frequency squared ω^2 receives also a *positive* contribution from the field gradient, which can potentially maintain stability, i.e., $\omega^2 > 0$. To find the conditions for instability, $\omega^2 < 0$, we analyze more precisely the ϕ-solutions in the χ-background created in the stable regime.

To start, we again assume some localized ϕ-configuration that sources a χ-profile. The calculation is the same as before, now however, we insert (6.21) for general frequency ω to find

$$\chi(r) \sim -\frac{N\mu m}{\omega r} e^{-\tilde{m}r} \,. \tag{6.30}$$

As before, we take the χ-profile for $r < R$ to be some smooth continuation of the calculable exponential profile at $r > R$. In particular, $\chi(r < R) \sim$

$-\frac{N\mu m}{\omega R}$, such that the effective ϕ-potential for $r \lesssim R$ is

$$V(\phi) = \frac{m^2}{2}|\phi|^2 + \mu m \chi(r)|\phi|^2 \sim m^2\left(1 - \frac{N\mu^2}{\omega R}\right)|\phi|^2 . \tag{6.31}$$

We will simplify this even further by using a step function approximation for the effective mass squared $m^2(r) = m^2[1 + \mu\chi(r)/m]$, see Fig. 6.3.

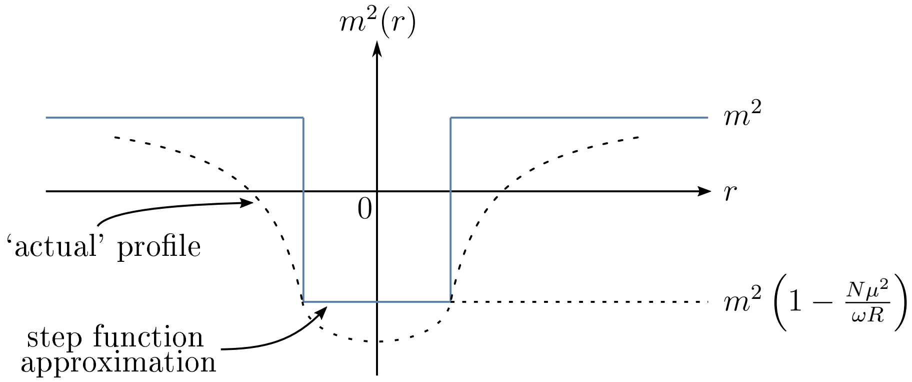

Figure 6.3: The effective mass squared for ϕ induced by the back-reaction on $\chi(r)$. We again extend the profile to $r < 0$ using spherical symmetry.

To find the radius that minimizes the energy, we equate the absolute values of the localization energy

$$E_{\text{loc}} \sim \int d^3x\, |\nabla\phi|^2 \sim \frac{1}{R^2}\int d^3x\, \phi^2 \sim \frac{N}{\omega R^2} \tag{6.32}$$

and the binding energy

$$E_{\text{scalar}} \sim \int d^3x\, \mu m \chi\, |\phi|^2 \sim \mu m\, \chi(r \lesssim R) \int d^3x\, \phi^2 \sim -\frac{N^2\mu^2 m^2}{\omega^2 R} . \tag{6.33}$$

The resulting radius of the scalar-force bound state scales as

$$R_{\text{SF}}(N) \sim \frac{\omega}{N\mu^2 m^2} . \tag{6.34}$$

As we have $\omega = \omega(R)$ and minimize the energy, we have to assume that the derivatives $\omega'(R)$ are sufficiently well-behaved to arrive at (6.34). In principle, we should also have taken into account

$$E_{\text{mass}} \sim \int d^3x\, m^2\, |\phi|^2 \sim \frac{m^2 N}{\omega} . \tag{6.35}$$

This term however, is just a constant offset in the non-relativistic regime, where $\omega \sim m$, and becomes negligible compared to E_{loc} in the relativistic regime, once $R \lesssim 1/m$.

Finally, we check the results obtained so far for consistency: We insert our calculated χ-profile, which depends on ω, into the equation of motion for ϕ and check whether this indeed produces a stable lowest-energy localized ϕ-mode with oscillator frequency ω. The stationary ansatz $\phi(t,r) = e^{-i\omega t}\phi(r)$ together with the back-reacted effective mass squared lead to the equation of motion

$$\left[\omega^2 + \nabla^2 - m^2(r)\right]\phi(r) = 0\,. \tag{6.36}$$

The instability expected in the previous subsection is realized if we find negative eigenvalue solutions, $\omega^2 < 0$. Such solutions would signal tachyonic modes which are not stationary and do not give rise to stable bound states.

At this point, we should highlight that we are actually doing a quantum rather than a purely classical calculation: Since we are dealing with bosons, we expect all the particles to populate the lowest (quantum) mode. The coherent state of bosons in the ground state has the interpretation of a classical field configuration described by the underlying wave function. This wave function, the lowest lying solution of the above classical equation of motion, must have positive 'energy' ω^2 for this standard interpretation to apply. But if ω^2 is negative, our bosons do not populate a positive-frequency-squared but rather a negative-frequency-squared oscillator. This is the quantum interpretation of the possible instability.

We notice that (6.36) represents a 3d Schrödinger-type equation when identifying

$$E \equiv \omega^2 - m^2 + \frac{\alpha}{R^2}\ , \qquad V(\mathbf{x}) = V(r) \equiv \frac{\alpha}{R^2}\theta(r - \beta R)\,. \tag{6.37}$$

Here, we explicitly reintroduced unknown $\mathcal{O}(1)$-coefficients α and β appearing in the χ-profile, $V_0 = \alpha/R^2$, and the radius, $R_{\text{actual}} = \beta R$. We can reduce this to a one-dimensional problem by introducing $u(r) = r\phi(r)$, which fulfills the Schrödinger equation

$$\left[E + \partial_r^2 - V(r)\right]u(r) = 0 \tag{6.38}$$

on $\mathbb{R}^+$ and is subject to the boundary condition $u(0) = 0$. The potential $V(r)$ vanishes for $r \leq \beta R$ and has a finite height $V_0 = \alpha/R^2$ otherwise. The energies corresponding to bound-state solutions obey $0 \leq E \leq V_0$ or equivalently $m^2 - \frac{\alpha}{R^2} \leq \omega^2 \leq m^2$.

An elementary textbook-style analysis of (6.38) shows that the energy eigenvalues fulfill

$$\begin{aligned}\sqrt{m^2-\omega^2} \;=\; -\, &\sqrt{\omega^2-m^2+\alpha/R^2} \\ &\times \cot\left((\beta R)\sqrt{2(\omega^2-m^2+\alpha/R^2)}\right)\,.\end{aligned} \tag{6.39}$$

A necessary condition for a solution is that the zero of the l.h. side of (6.39) is larger than the first zero of the r.h. expression. That is

$$m^2 \geq m^2 + \left(\frac{\pi^2}{8\beta^2}-\alpha\right)\frac{1}{R^2}\,. \tag{6.40}$$

Depending on the unknown coefficients this may or may not hold. Our discussion in Sect. 6.4.1 showed that solutions exist in the non-relativistic regime $R \gg 1/m$. When moving into the regime $R \sim 1/m$, the coefficients α and β stay roughly the same and we expect that not all solutions disappear. A physical reason that a stable localized solution should prevail even for $R \ll 1/m$ is as follows: Let us bring a particle (adiabatically, i.e., without kinetic energy) close to the localized configuration, or rather the potential well created by it. The latter corresponds to an attractive force. If, say, N particles have been bound in this way, there is no reason for the $(N+1)$st particle coming from infinity not to feel an attractive potential and be bound as well.

We parameterize the energy of the lowest mode by

$$E = \gamma V_0 = \gamma\frac{\alpha}{R^2}\,, \qquad \omega^2 = m^2 + (\gamma-1)\frac{\alpha}{R^2}\,, \tag{6.41}$$

with some factor $0 < \gamma \leq 1$ which depends on α, β and is fixed by (6.39). The value $\gamma = 1$ corresponds to marginal binding, when the equality applies in (6.40).

We see from the 2nd equation in (6.41) that, in the non-relativistic regime $R \gg 1/m$, any value of γ leads to a solution with $\omega^2 \sim m^2$. In the relativistic regime, $R \ll 1/m$, our analysis allows for two scenarios: On the one hand the solution can become tachyonic. This happens if γ takes generic values $0 < \gamma < 1$. On the other hand, the value of γ could always come out to be very close to unity, such that we continue to have a positive ω^2 solution and the bound state remains stable. In fact, the level of tuning required for this grows with decreasing R: To have a non-tachyonic solution, we need $0 \leq 1-\gamma < m^2R^2/\alpha$. The solutions in this case are known. Outside the finite potential well, the wave function $u(r)$ drops off exponentially, i.e., $\phi(r) = u(r)/r \propto \exp(-\kappa r)/r$, on the length scale

$$\frac{1}{\kappa} \sim \frac{1}{\sqrt{V_0-E}} = \frac{R}{\sqrt{\alpha(1-\gamma)}} > \frac{1}{m}\,. \tag{6.42}$$

We see that we cannot localize the ϕ-profile in a region smaller than $1/m$ without an instability.

Finally, we note that higher interaction terms might be introduced to cure the tachyonic potential problem above. An example would be a positive $\lambda\phi^4$ term. If this term is large enough to cure a possible instability, we would also expect its repulsive effect to make the bound state larger. Thus, we expect that our conjecture cannot be avoided by extending the model in this way. However, more complex models of this type clearly require a detailed analysis, which we have to leave to future research.

6.4.2 Quartic Interactions

We consider the $U(1)$-invariant theory of a complex scalar field with quartic potential,

$$V(\phi) = \frac{m^2}{2}\,|\phi|^2 + \lambda\,|\phi|^4\,, \tag{6.43}$$

taking the interaction to be attractive, $\lambda < 0$. We want to think of this as an effective theory below some cut-off Λ. This cut-off may be high enough to allow for $|\phi|$-values for which $V(\phi) < 0$. In this case the vacuum at $\phi = 0$ is only metastable.[15] In Sect. 6.4.2, we will introduce a $|\phi|^6$-term that bounds the potential from below and restricts the validity of (6.43) to the region below some maximal $|\phi|$-value. One may view this as an explicit implementation of a cut-off.

In the above setting, one could in principle imagine bound states to occur based on the attractive scalar interaction alone, on gravity, or on a combination of both. We will start our discussion with the gravitationally coupled case. It is clear that for sufficiently dilute systems gravity dominates and stable boson stars exist. Moving from there into a regime of high density and small radius, where the ϕ^4-attraction becomes important, one might naively expect that much denser objects can form. Instead, an instability develops. This suggests that small bound states, which could violate our conjecture, are excluded. In the case without gravity, we argue that bound states do not exist at all. Finally, we will allow for higher-order self-interactions, removing the instability at large $|\phi|$. Now bound states in the form of Q-balls are possible. However, they are always large enough to respect our conjecture, at least in the regime where they can be created adiabatically.

[15] The lifetime of the $|\phi| = 0$ vacuum has an exponential $1/\lambda$-dependence and a power-like dependence on the cut-off Λ. The latter comes from the running of λ and from the prefactor of the exponential tunneling suppression [293]. We accept that the lifetimes of the vacuum and of bound states may be only finite.

Gravitationally Bound States

At small particle numbers and correspondingly large bound-state radii $R \gg 1/m$, we expect the total attractive force to be dominated by gravity. To confirm this, we estimate the relative importance of the binding energy from self-interactions. We introduce the conserved particle number (6.17) and, using the mean field approach with $\phi(t,\mathbf{x}) = e^{-i\omega t}\phi(\mathbf{x})$, express it in terms of the average field excursion $\bar{\phi}^2 = \int \mathrm{d}^3x \, |\phi(\mathbf{x})|^2 / R^3$ of a localized solution:

$$N \sim m\bar{\phi}^2 R^3 . \tag{6.44}$$

Here we also used that we are in the non-relativistic regime, $\omega \sim m$. The binding energy from self-interactions may then be expressed in terms of the boson star mass $M \sim mN$:

$$E_{\text{self-int}} \sim -\int \mathrm{d}^3x \, |\lambda| \, |\phi|^4 \sim -|\lambda| \, R^3 \bar{\phi}^4 \sim -\frac{|\lambda| \, N^2}{m^2 R^3} \sim -\frac{|\lambda| \, M^2}{m^4 R^3} . \tag{6.45}$$

It is subdominant to both the gravitational binding energy $\propto 1/R$ and the quantum pressure $\propto 1/R^2$, cf. (6.12). Hence the radius depends on the mass just as in the familiar free-boson case, cf. $R(M)$ of (6.13).

As we increase the particle number and mass of the boson star, its size decreases and the attractive self-interaction becomes more relevant. Such a gravitationally bound system with additional self-interaction has been studied numerically in [261, 268] (along with some analytical estimates) using the Gross-Pitaevskii-Poisson equations. As expected, the stable radius of a bound state is, for small particle numbers, the same as in (6.13). The additional attractive force only affects the $\mathcal{O}(1)$-prefactor:

$$R_{\text{G+SI}}(M) \sim R_{\text{FB}}(M) \sim \frac{1}{M}\left(\frac{M_{\text{P}}}{m}\right)^2 . \tag{6.46}$$

We sketch the two curves $R_{\text{G+SI}}(M)$ and $R_{\text{FB}}(M)$ in Fig. 6.4.

As is also sketched in Fig. 6.4, there exists a maximum mass at which the curve $R_{\text{G+SI}}(M)$ ends [261, 268],

$$M^{\text{max}}_{\text{G+SI}} \sim \frac{M_{\text{P}}}{\sqrt{|\lambda|}} . \tag{6.47}$$

Above this mass there is no regular solution of the equations of motion that can be found using the numerical approach. Depending on the coupling strength $|\lambda|$, we distinguish two scenarios: For small couplings $|\lambda| \lesssim m^2/M_{\text{P}}^2$, self-interactions remain weak and the relation (6.46) is valid all the way to

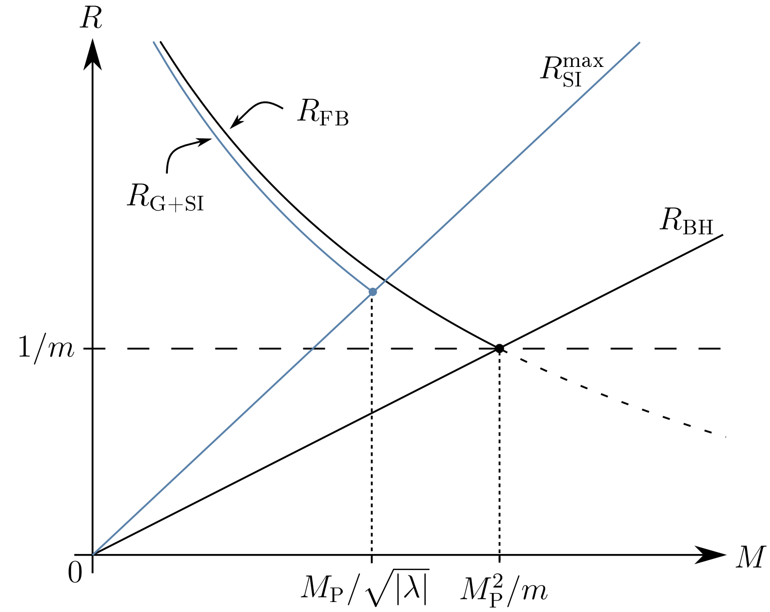

Figure 6.4: The M–R-plots of gravitationally bound boson stars. For the free-boson case, the radius is denoted as $R_{\mathrm{FB}}(M)$, for the case with additional self-interactions as $R_{\mathrm{G+SI}}(M)$. Also shown are the black-hole radius $R_{\mathrm{BH}}(M)$ as well as the radius $R_{\mathrm{SI}}^{\max}(M)$ at which self-interactions start to dominate the total energy, see subsection 6.4.2 .

the mass at which the star collapses to a black hole of size $R_{\mathrm{BH}} \sim 1/m$. For stronger coupling, $|\lambda| \gtrsim m^2/M_{\mathrm{P}}^2$, the solution breaks down before the critical radius of gravitational collapse is reached,

$$R_{\mathrm{G+SI}}(M_{\mathrm{G+SI}}^{\max}) \sim \sqrt{|\lambda|}\,\frac{M_{\mathrm{P}}}{m^2} \gtrsim \frac{1}{m}\,. \tag{6.48}$$

We will analyze in a moment what happens at this point. Throughout the rest of this section, our focus will be on the interesting case of large coupling (or weak gravity) $|\lambda| \gtrsim m^2/M_{\mathrm{P}}^2$.

Note that, according to the quoted numerical results, the curve $R_{\mathrm{G+SI}}(M)$ in the plot never falls *parametrically* below $R_{\mathrm{FB}}(M)$ in the stable bound-state regime. Hence, $R \gtrsim 1/m$ and we are justified in the use of $\omega \sim m$ for all bound states discussed so far.

Instability from Self-Interactions

The total energy of a bound state of non-relativistic constituents is the sum of the localization energy

$$E_{\mathrm{loc}} \sim \int \mathrm{d}^3x\ |\nabla\phi|^2 \sim R\bar{\phi}^2 \sim \frac{N}{mR^2}\,, \tag{6.49}$$

the binding energy from self-interactions, cf. (6.45),

$$E_{\text{self-int}} \sim -\int \mathrm{d}^3x\, |\lambda|\, |\phi|^4 \sim -|\lambda|\, R^3 \bar{\phi}^4 \sim -\frac{|\lambda|\, N^2}{m^2 R^3}\,, \tag{6.50}$$

and the gravitational energy

$$E_{\text{grav}} \sim -\frac{m^2 N^2}{M_{\text{P}}^2 R}\,. \tag{6.51}$$

Here, we used (6.44). By the same arguments as in Sect. 6.4.1 below (6.35), the energy associated with the mass term in the potential, $E_{\text{mass}} \sim Nm^2/\omega$, is not relevant. We sketch the total energy in Fig. 6.5.

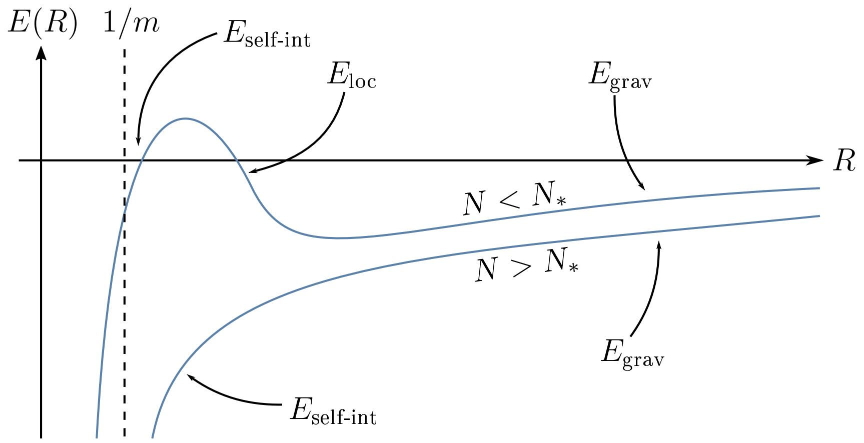

Figure 6.5: The total energy $E_{\text{tot}} = E_{\text{loc}} + E_{\text{grav}} + E_{\text{self-int}}$ as a function of R within the non-relativistic approximation. There is a local minimum for small N, which disappears at large N. While the minimum is always to the right of the vertical line $R \sim 1/m$, the maximum can be on either side. For sufficiently large particle number, $N \gtrsim 1/|\lambda|$, the latter is also to the right of $1/m$, as for the upper curve in the sketch.

For small N, we find a non-trivial minimum arising from the interplay of the gravitational energy and the localization energy. The resulting stable radius, as explained above, is (6.46). At smaller radii, there is a maximum of the total energy at

$$R_{\text{SI}}^{\text{max}}(N) \sim \frac{N\,|\lambda|}{m} \sim \frac{M\,|\lambda|}{m^2}\,. \tag{6.52}$$

We can see that there exists a particle number N_* at which the local minimum disappears. This happens at

$$R_{\text{SI}}^{\text{max}}(N_*) \sim R_{\text{G+SI}}(N_*) \qquad \Leftrightarrow \qquad N_* \sim \frac{M_{\text{P}}}{\sqrt{|\lambda|}\, m}\,, \tag{6.53}$$

in agreement with the maximum particle number or mass (6.47) found numerically, $N^{\max} \sim N_*$. Above this particle number, the stabilizing effect of quantum pressure is not strong enough to overcome the attractive self-interaction, cf. Fig. 6.5. The intersection of the curves $R_{\mathrm{SI}}^{\max}$ and $R_{\mathrm{G+SI}}$ can also be seen in Fig. 6.4.[16]

We should consider what happens to a bound state at the maximum particle number $N^{\max}$ where the energy minimum disappears. If we keep adding particles, the configuration might collapse to a small black hole of particle number $N \sim N^{\max}$ and radius $R_{\mathrm{BH}}(M \sim M^{\max}) \lesssim 1/m$. However, assuming that in the collapse process the mean-field approximation still holds, one can check that the average field value $\bar{\phi}$ inside the configuration at fixed mass $M \sim M^{\max}$ and particle number $N \sim N^{\max}$ necessarily exceeds the value $m^2/|\lambda|$.[17] That is, the true vacuum at $|\phi| \neq 0$ becomes classically accessible and, as long as there are no stabilizing higher-order terms in the potential, we expect vacuum decay to set in. In this way, our conjecture would not be violated. Clearly, we cannot be certain of the validity of this mean-field logic during the potentially violent collapse process. More scrutiny is needed to establish the result of the collapse.

Comments on the Non-Gravitational Theory

Finally, we want to consider a bound state from self-interaction alone, $M_{\mathrm{P}} \to \infty$. Instead of gravitationally building up a large bound state until self-interaction takes over, we want to consider the self-bound few-particle case.

The energy barrier that can be anticipated from the many-particle mean field calculation is sketched in Fig. 6.5 (without the gravitational tail at large R). It can also be derived in the few-particle scenario: Calculating the energy of a two-particle state with Hamiltonian H corresponding to (6.43), where the particles are Gaussian wave packets of width R, one finds[18]

$$E_{\mathrm{tot}}(R) = \langle H \rangle_{\text{2-particle}} \sim \begin{cases} m + \frac{1}{mR^2} - |\lambda| \frac{1}{m^2R^3} & R \gg \frac{1}{m}\,, \\ \frac{1}{R} - |\lambda| \frac{1}{R} & R \ll \frac{1}{m}\,. \end{cases} \tag{6.54}$$

[16] Note that the curve $R_{\mathrm{SI}}^{\max}(N)$ or $R_{\mathrm{SI}}^{\max}(M)$ characterizes the position of the maximum of $E_{\mathrm{loc}}(R) + E_{\text{self-int}}(R)$. As such, it does not describe a stable state and its only meaning is to specify the point where the curve $R_{\mathrm{G+SI}}(M)$ ends.

[17] The relevant equation is $M \sim \int \mathrm{d}^3x \left(\left|\dot{\phi}\right|^2 + |\nabla\phi|^2 + V(\phi) \right) \sim N^2/(\bar{\phi}^2 R^3) + R\bar{\phi}^2 + R^3 V(\bar{\phi})$. Since the collapse sets in within the non-relativistic regime, we can fix the mass at $M = mN$.

[18] We use the state $|2\rangle = \int \mathrm{d}^3p\, \mathrm{d}^3q\, f_R(\mathbf{p}) f_R(\mathbf{q})\, |\mathbf{p}, \mathbf{q}\rangle$ with $f_R(\mathbf{p}) = \mathcal{N} \exp\left(-\frac{1}{2}\mathbf{p}^2 R^2\right)$ with some normalization $\mathcal{N}$. The Fourier transform gives spatial Gaussian wave packets of width R.

The non-relativistic expression contains the mass contribution m, the kinetic term $p^2/m \sim 1/(mR^2)$, and a contribution $\propto 1/R^3$ associated with the scalar attraction.[19] The relativistic result can only depend on the scale $1/R$, as the mass m becomes irrelevant. This is sufficient to explain the second line of (6.54).

Let us start our analysis in the perturbative regime, $|\lambda| \lesssim 1$, and at large R. Here, the first line of (6.54) is applicable and the repulsive quantum pressure dominates. When moving to smaller R, this remains true until, at $R \sim 1/m$, we reach the applicability range of the second line of(6.54). But here, again, repulsion dominates. Thus, we have repulsion for all R and no binding is possible.[20]

For strong coupling, $|\lambda| \gtrsim 1$, tunneling to large ϕ-values becomes fast since the $\exp(-1/\lambda)$-suppression is ineffective. Hence we are forced to set a low cut-off $\Lambda \lesssim m/\sqrt{|\lambda|} \ll m$ to avoid this fast instability. Bound states exist, but they can as a matter of principle not be smaller than $1/\Lambda$, which is above the scale $1/m$.

Finally, we return to $|\lambda| \lesssim 1$ but allow for N particles. We expect the interaction energy to scale as N^2 (every particle interacts with every other particle) and the kinetic energy to scale as N. For sufficiently small N, the two-particle discussion above will suffice. However, for $N \gtrsim 1/|\lambda|$, the energy maximum of the non-relativistic regime comes to lie at $R > 1/m$ and hence becomes trustworthy. At smaller values of R, left of this maximum, the energy falls first as $1/mR^2$ and then as $1/R$, in the relativistic regime. So at best we can hope for a singular bound state, with the same problem of vacuum decay as above.

Higher-Order Self-Interactions and Q-Balls

We now want to explore the limitations of our conjecture: Does it still hold if we allow for higher-order, non-renormalizable terms in the effective Lagrangian? As we argued above, gravitational bound states may be driven to instability, where $|\phi|$ grows and explores the potential in the regime dominated by the negative $\lambda\,|\phi|^4$ term. This instability may be cured by higher-

[19] For a quantum mechanical derivation see [294]: A well-known result is that the 2-particle interaction induced by a ϕ^4-term is described in the Schrödinger equation by a potential $V(\mathbf{x}) = -|\lambda|/m^2\delta^{(3)}(\mathbf{x})$. Smearing out the delta-potential over a region R^3 results in an energy eigenvalue $E_\delta \sim -|\lambda|/(m^2R^3)$. This can be understood since $|\lambda|/m^2$ comes from $V(\mathbf{x})$ and, with it, $1/R^3$ on dimensional grounds.

[20] There have been some attempts at constructing two-particle bound states in ϕ^4-theory using different techniques, see [295–299] for an incomplete list. Conversely, it has also been claimed that such bound states do not exist [300]. Our simple scaling argument supports the latter option.

order terms, e.g., a repulsive $|\phi|^6$ interaction. In this case, a different, not yet discussed form of a stable or metastable bound state may arise. The key feature of the potential on which this relies is the second minimum at $|\phi| \neq 0$.

Let us start with the simple example

$$V(\phi) = \frac{1}{2} m^2 |\phi|^2 \left(1 - \frac{|\phi|^2}{{\phi_0}^2}\right)^2 , \tag{6.55}$$

which corresponds to the potential in (6.43), if $\lambda = -m^2/\phi_0^2$, with an additional $|\phi|^6$-term. The coefficient of the latter is adjusted to ensure that the second minimum at $|\phi| = \phi_0$ is degenerate with the minimum at $\phi = 0$.[21] Models of this type are well-studied and their bound states are known as Q-balls [301] or, more generally, non-topological solitons, see e.g., [302,303,292].

The existence of bound states is easily understood analytically within a thin-wall approximation: In the inner region of the Q-ball of radius R one has $\phi = \phi_0 \exp(-i\omega t)$. This region is surrounded by a wall of thickness $D \ll R$, in which the field transits from the $|\phi| = \phi_0$ to the $\phi = 0$ minimum. It is easy to show that $D \sim 1/m$ and that the wall tension is $T \sim D\, m^4/|\lambda|$. The expression $N \sim \omega \phi_0^2 R^3$ for the particle number, together with $\lambda = -m^2/\phi_0^2$, can be used to express ω in terms of N, λ, and R. With this, one can write the total energy, which comes from the inner region and the bubble wall, as a function of R:

$$E \;=\; E_{\text{inner}} + E_{\text{wall}} \sim \omega^2 \phi_0^2 R^3 + T R^2 \sim \frac{N^2 |\lambda|}{m^2 R^3} + \frac{m^3 R^2}{|\lambda|} . \tag{6.56}$$

The Q-ball radius follows by minimization,

$$R \sim \frac{(|\lambda|\, N)^{2/5}}{m} , \qquad \text{such that} \qquad E \sim \frac{mN}{(|\lambda|\, N)^{1/5}} . \tag{6.57}$$

Since the Q-ball can decay to N free particles if $E \geq mN$, we must have $N \gtrsim N_c \sim 1/|\lambda|$ as a stability requirement. But this implies that $R \gtrsim 1/m$, so our conjecture cannot be violated in the thin-wall approximation. If we modify the potential such that the ϕ_0-minimum has positive vacuum energy, the decay is facilitated and N_c increases. This leads to an increase of R and hence our conjecture is even more safe.

We also note that, according to (6.53), Q-balls of this type can be adiabatically produced by driving boson stars beyond the boundary of stability:

[21] Writing the $|\phi|^6$-term as $|\phi|^6/\Lambda^2$, we find that $\Lambda^2 = 2\phi_0^2/|\lambda|$, thus fixing the cut-off of the model.

Indeed, since $M_{\mathrm{P}}/m > 1/\sqrt{|\lambda|}$ in the region of interest, we always have $N > N_c$ and the resulting Q-balls are stable.

We now turn to the possibly more critical case in which the vacuum energy at the second minimum is negative:

$$V(\phi) = \frac{1}{2}m^2\,|\phi|^2 - |\lambda|\,|\phi|^4 + \frac{1}{\Lambda^2}\,|\phi|^6\,, \qquad \text{with} \qquad \Lambda^2 > \frac{2m^2}{\lambda^2}\,. \tag{6.58}$$

While the $\phi = 0$ vacuum and hence any possible Q-balls are now at best metastable, they can certainly be very long-lived. Q-balls relying on such a negative-energy second minimum are frequently called Q-bubbles. As long as the depth of the second minimum is parametrically small compared to the barrier height, the thin-wall approximation is useful and an analytical treatment is possible [304]. Unsurprisingly, our previous discussion of the case with degenerate minima still applies and the bound state conjecture cannot be violated. However, it also follows from the discussion above that bound states which are smaller by $\mathcal{O}(1)$ factors are now conceivable.

Thus, the critical question is whether by going to the regime of a *deep* second minimum one can find parametrically small Q-bubbles, violating the conjectured bound $1/m$. Unfortunately, we were not able to extract an unambiguous answer from the partial analytical results of [305]. Also from the numerical results of [306, 307] the answer is at least not obvious: The authors use the dimensionless quantities $\widetilde{\omega} = \omega/(|\lambda|\,\Lambda)$ and $\widetilde{m} = m/(|\lambda|\,\Lambda)$ as well as an analogously rescaled particle number or charge $\widetilde{N} = |\lambda|\,N$. Making use of the fact that a parametrically deep second minimum is located at $\phi_0 \sim \sqrt{|\lambda|}\,\Lambda$, one can estimate the radius in terms of these parameters as follows:

$$R \sim \left(\frac{N}{\phi_0^2\omega}\right)^{1/3} \sim \frac{1}{m}\left(\frac{\widetilde{m}^3\widetilde{N}}{\widetilde{\omega}}\right)^{1/3}\,. \tag{6.59}$$

Inserting concrete numerical values for $\widetilde{\omega}$, $\widetilde{m}$ and $\widetilde{N}$ from [306], we only arrive once again at $\mathcal{O}(1)$ coefficients multiplying $1/m$. Presumably, a dedicated numerical study would be needed to settle our question about small Q-bubbles.

We note once again that, even if small Q-bubbles exist, creating them adiabatically is problematic since boson stars collapse only at $N \gtrsim 1/|\lambda|$. Hence small Q-bubbles are presumably out of reach in the model discussed above. However, once we allow for a $|\phi|^6$-term, nothing stops us from also adding $|\phi|^8$- or $|\phi|^{10}$-terms etc. Then one may as well 'draw scalar potentials by hand'. In such a general setting, it is easy to imagine that Q-balls of size $\ll 1/m$ may, after all, both exist and be constructed adiabatically.

Thus, we prefer not to claim that parametrically small Q-balls do not exist or cannot be built. Instead we have, partially with a view on this subsection, formulated our conjecture as a statement about power-counting renormalizable effective field theories. A possible extension beyond this set of models is left to future research.[22]

Finally, another possible concern is the existence of Q-balls in renormalizable models [309]. This requires more fields and an analysis of our conjecture in this context goes beyond the scope of the present work.[23] It would however be important to understand whether such Q-balls can be small compared to all mass scales governing the theory in the original vacuum and whether they can be constructed adiabatically. A positive answer may force us to search for stronger constraints than renormalizability.

6.4.3 Axions

We now turn to the example of axion bound states. The relevant potential is

$$V(\phi) = m^2 f^2 \left[1 - \cos(\phi/f)\right] = \frac{m^2}{2}\phi^2 - \frac{m^2}{f^2}\phi^4 + \mathcal{O}\left((\phi/f)^6\right) . \qquad (6.60)$$

As ϕ is real, we are lacking the notion of the exactly conserved particle number used in Sects. 6.4.1 and 6.4.2. Nevertheless, at low energies approximate particle number conservation holds and long-lived axion stars (or, more generally 'oscillatons') exist. Moreover, detailed numerical simulations of the coupled Klein-Gordon and Einstein equations are available. In the following, we will consider these simulations and ask for small bound states, potentially violating our conjecture. For recent work on axion stars see, e.g., [311, 312] and refs. therein.

Specifically, we will rely on the 'phase diagram' obtained in [272, 275] and sketched in Fig. 6.6. It includes the curve $f_{\min}(M)$, representing the minimum axion decay constant for which an axion star with mass M is stable. Note that it is customary to use the dimensionless variable $Mm/M_{\rm P}^2$

[22] Let us note that a different idea for probing UV physics with Q-balls appeared in [308]. It does not aim at small bound-state radii but rather employs large VEVs to catalyze certain UV-scale-suppressed transitions between light particles.

[23] An analysis based on the toy model $V(\phi) = m^2 |\phi|^2 - A|\phi|^3 + \lambda|\phi|^4$ mimicking renormalizable couplings between multiple fields suggests the following: A thin-wall calculation for degenerate vacua gives a minimal charge N for stable Q-balls that leads to $R \gtrsim 1/m$, similar to the discussion following (6.57). The thick-wall limit at small charges and very non-degenerate vacua gives $R \sim 1/(\epsilon m)$ with $\epsilon \ll 1$ [310]. Of course, a model with multiple fields still requires a proper, independent analysis.

to characterize the mass of the star. In our context, it may be more intuitive to move through this plot horizontally, at fixed f. Then the curve $f_{\min}$ specifies the maximum mass up to which the star remains stable. We will discuss the different regions of the diagram in turn.

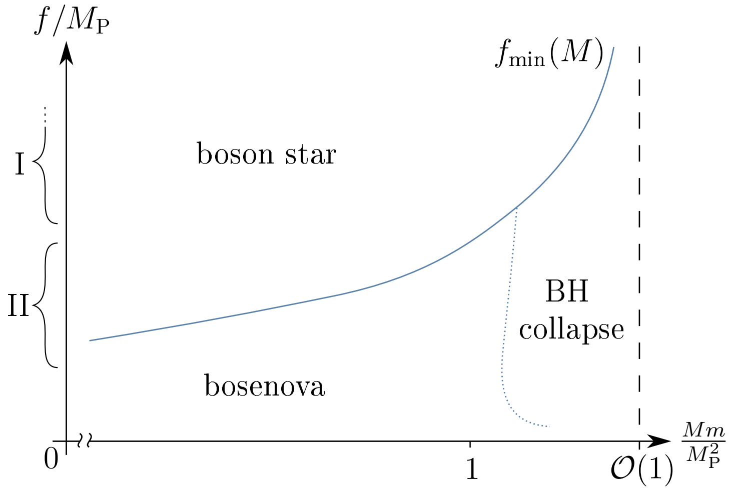

Figure 6.6: Sketch of the axion star phase diagram of [272, 275]. Region I contains the free limit, $f/M_{\mathrm{P}} \gg 1$. In the intermediate region II, the theory is approximated by an attractive ϕ^4-theory in the dilute regime. Below region II, corresponding to large ϕ/f, higher-order terms of the potential (6.60) are relevant. The dashed vertical line characterizes the mass at which a free boson star collapses to a black hole.

We start with region I, containing the free limit $f \to \infty$. Here, $f_{\min}(M)$ approaches the vertical line $Mm/M_{\mathrm{P}}^2 = \mathcal{O}(1)$, where a free boson star would collapse to a black hole. As long as $f/M_{\mathrm{P}} \gtrsim 1$, the curve $f_{\min}$ deviates from the free-boson vertical line only by an $\mathcal{O}(1)$ factor. Thus, no parametrically small black holes can form and our conjecture is safe.

In region II, the graph is approximately linear:

$$\frac{f_{\min}(M)}{M_{\mathrm{P}}} \sim \frac{mM}{M_{\mathrm{P}}^2}\,. \tag{6.61}$$

One may worry that a star which becomes unstable by crossing this line collapses into a small black hole. After all, we are now far below the critical value $M \sim M_{\mathrm{P}}^2/m$ distinguished by our conjecture. However, as already indicated in Fig. 6.6, this is not what happens [272,275]: The instability manifests itself through the emission of relativistic axions in form of a 'bosenova' [275, 313]. No small black hole is formed.

In more detail, the fate of axion stars becoming unstable in region II is as follows [269]: After shedding an outer shell in a bosenova, a dense axion star remnant forms. It is stabilized by higher-order (in the expansion of the cosine potential) repulsive interactions. This new dense object is then stable up to a total mass $\sim 10^5 M_{\mathrm{FB}}^{\mathrm{max}}$, for scalar masses $m \sim 10^{-4}\,\mathrm{eV}$ (typical of a QCD axion). So instead of creating a small black hole violating our bound, the object goes through an unstable transition to a different stable configuration, where it may reach larger masses before collapsing to a black hole, cf. Fig. 6.7. The dense branch satisfies (6.15) for all masses. Similar results were reported in [314] and very recently additional dense branches have been found in [315]. The radii on these branches and at small f can be even closer to the black hole radius. Analytical approximations were provided in [316]. See also [273] for a discussion on the stability and lifetime of the axion star in the dense regime.

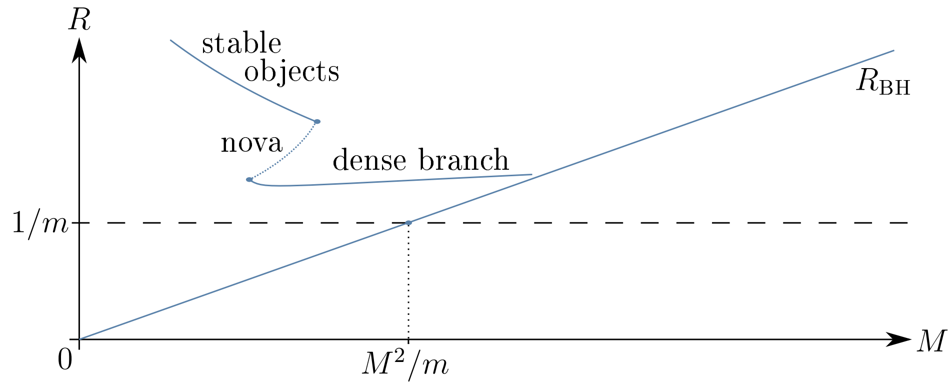

Figure 6.7: Sketch of an axion star's trajectory in the M–R-plane as discussed in [269]. In drawing this for general m, we extrapolate from the graph given for $m \sim 10^{-4}\,\mathrm{eV}$.

It would be interesting to include an analysis of the size of non-gravitational bound states of real fields, i.e., oscillons [317]. We were not able to extract enough information from the largely numerical studies to repeat our semi-quantitative axion-star discussion for the oscillon case. However, we expect the interesting regime of small oscillons to experience the same instability through violent particle production as seen in the gravitational case: According to [318], the metastability of purely field-theoretic bound states of real scalar fields is due to the approximate $U(1)$-symmetry of their effective low-energy description through a complex scalar. In other words, the finiteness of the lifetime comes from the explicit breaking of this symmetry at high energies. We expect the regime of $R \lesssim 1/m$, the relativistic regime, to break this $U(1)$-symmetry significantly. Particle production would then become efficient and prevent the existence of small, long-lived oscillons. Nevertheless, this is only an expectation and more work is needed to establish it.

6.4.4 Bound States Involving Non-Scalar Particles

We have so far only discussed scalar particles bound by scalar forces. The reason is that we view such bound states as most critical in terms of providing a counterexample to our conjecture. Here is a short list of other possibilities which we consider less dangerous:

First, free, massive vectors can form non-topological solitons [319] or gravitationally bound Proca stars [267]. The parametric behavior appears to be similar to the corresponding scalar objects.

Second, when binding fermions one faces additional repulsion due to Fermi pressure. This leads to the Chandrasekhar limit $\sim M_\mathrm{P}^3/m^2$ for the mass of a fermion star, exceeding the critical boson star mass by M_P/m. The radius exceeds $1/m$ by the same large factor.[24]

Finally, the binding may be due to a vectorial (i.e., gauge) force rather than to a scalar force. In the abelian case, one is limited to two constituents since the charges have to be opposite. The size is $1/(g^2 m)$, consistent with our conjecture at weak coupling.[25] At strong coupling, it is natural to use the dual, weakly coupled description. Thus, we have to discuss binding magnetic monopoles in a weakly coupled electric theory. Such monopoles are extended with a size comparable to their inverse mass, enhanced by the strong magnetic coupling. So our conjecture appears to be safe. In the non-abelian case, the confinement scale sets both the size and mass of particles and bound states. Again, we see no prospects for violating our conjecture parametrically, except maybe if one involves the rank of the gauge group as a large parameter. It could be worthwhile to further study this.

6.5 Conclusion

Our analysis of swampland ideas for constraining scalar interactions has led us to a novel proposal: the bound state conjecture (6.15). It differs from the conventional swampland framework in that it remains non-trivial when gravity is completely decoupled. Hence, it may turn out to be a provable feature of a class of non-gravitational QFTs.

[24] Mixed fermion-boson stars have been studied (in gravitationally bound systems) in [320] and more recently in, e.g., [321].

[25] One can compare this to the size of a bound state bound by a scalar mediator (6.20) extrapolated to small N.

We have described previous approaches in Sect. 6.2. Their main idea is to construct an inequality that quantifies the statement 'gravity is the weakest force' in the presence of scalar interactions. We have also presented problems, or possible counterexamples, we see with these proposals. Our approach, as discussed in Sect. 6.3, takes a different route: We do not attempt to forbid bound states by requiring that repulsive forces outweigh gravitational attraction. Neither do we try to claim that scalar attractive forces must act more strongly than gravity. Instead, our premise is that bound states should not be forbidden but constrained. Specifically, there should be a minimal size for bound states. We have quantified this by stating that the smallest black hole that can be built adiabatically from individual particles in an interacting theory is not parametrically smaller than the one built from free scalar particles. This does not give rise to a 'weak gravity' conjecture. Rather, it claims that 'attractive forces cannot be parametrically stronger than gravity alone'. By calculating the size of a black hole that can be adiabatically built from free scalars, one finds that this statement is actually (maybe surprisingly) independent of the strength of gravity: The resulting radius and hence the minimal size is $R \gtrsim 1/m$, with m the mass of the free scalar. We stress that this result is independent of M_{P}. We have put the resulting *Bound State Conjecture* to the test in Sect. 6.4. In all examples it has turned out that either the model becomes tachyonic or a bosenova-type instability of the bound state develops if one tries to beat the conjectured minimal radius $R \sim 1/m$.

We conclude with a short list of open problems and comments:

An obvious open problem is to determine a general, purely field-theoretic origin of the bound. The non-gravitational formulation should, if correct, allow for a proof using the well-understood framework of QFT. Employing the uncertainty relation $p \sim 1/R$, in essence, this would involve proving that relativistic particles cannot be bound. Another way forward might be to consider causality constraints in scattering processes involving the bound states, as it was done in [287, 288]. In this context we should also note that the decoupling of gravity remains peculiar. That is, we do not see any a priori reason why M_{P} should drop out of the bound. In other words, why do the two limitations for small bound states, one from field-theoretic instabilities and one from horizon formation, both coincide and give the value $R \sim 1/m$?

Next, we want to highlight that the critical radius $R \sim 1/m$ is independent of the spacetime dimension d: To see this, one repeats the original derivation, where we asked for the radius at which the localization energy $E_{\mathrm{loc}} \sim (M/m)/(mR^2)$ and the gravitational energy $E_{\mathrm{grav}} \sim M^2/(M_{\mathrm{P}}^{d-2}R^{d-3})$ coincide. Then, requiring that this radius is also the black-hole radius corresponding to M, one arrives at $R \sim 1/m$ for general d. However, this result is only formal since in $d \geq 6$ dimensions, the energy $E_{\text{loc}} + E_{\text{grav}}$ has a maximum rather than a minimum. Thus, particles cannot be bound by gravity at large distances at all. The case $d = 5$ is special since E_{loc} and E_{grav} scale identically with R. There is then a critical particle number $N_c \sim (M_{\text{P}}/m)^3$ for which the particles can be brought close together at no energy cost to form a black hole of size $1/m$.

Furthermore, we should warn the reader that, when moving outside the domain of power-counting renormalizable theories, small bound states appear conceivable. This is suggested by our discussion of small 'Q-bubbles' relying on a deep true vacuum in Sect. 6.4.2. Of course, the price to pay is that now our basic vacuum is a false one, being hence only long-lived rather than stable. We must also admit that we have neither established that such spiky Q-bubbles really exist nor do we see how to construct them adiabatically. The problem of small bound states in more involved multi-field models has also not yet been studied by us. These two open questions appear to be the most promising routes to either disprove the conjecture, to limit its generality, or, on the contrary, to collect highly non-trivial evidence in its favor.

Finally, let us point out a possible version of our conjecture related to resource theory. In (quantum) resource theory one defines so-called free states, which are readily available, and free operations, which the experimenter can perform. In our context, these would be light particles (of mass $\sim m$) and adiabatic processes involving them. The resources are then states that have special value since they cannot be produced from the above. These, in our case, would be small bound states or, possibly equivalently, fundamental heavy particles with mass $M \gg m$. The latter would become available through unsuppressed transition amplitudes from sufficiently heavy and localized bound states. The special feature setting these resource states apart may be some kind of strong entanglement involving constituent particles at relativistic momenta. It would be interesting to establish a formulation of our conjecture in these terms more carefully. The conjecture might then represent an obstacle to building an 'IR-to-UV transformer', i.e., a device that is fed light particles and which produces a heavy, UV-scale, fundamental particle after enough energy has been supplied in this IR channel.

Chapter 7

Summary and Outlook

In this thesis we approached the swampland program from two directions. To more sharply draw the boundary between landscape and swampland, we gave explicit landscape constructions of axions and considered implications of known swampland conjectures. The latter were on the one hand direct implications of the axionic WGC and on the other hand more speculative extensions of the idea motivating the WGC. In this we were mostly guided by two basic questions: What is the correct formulation and what are the consequences of the WGCs as introduced in Sect. 2.2? We now summarize our results.

In Ch. 3, we considered the complexified two-form $C_2 + \tau B_2$ on the two-sphere of the Klebenov-Strassler throat. We assumed that the throat is part of a multi throat system. While manifestly the usual shift-symmetry of a resulting axion is broken by the triviality of the cycle due to the IR geometry [40], we found that by geometric back-reaction a larger periodicity is restored. The periodic back-reaction is inherited from a weakly broken $U(1)_R$ isometry of the throat. We proposed a superpotential for this throat axion, 'thraxion', based on the interpretation of axion-excitations as unquantized flux of the corresponding G_3 field strength, which now includes the the saxion-partner of the axion. We were able to map this superpotential and enhancement of the periodicity to the KS gauge theory. Finally, we considered a simple explicit scenario on the quintic threefold. We found that given certain flux choices, the decay constant can easily be super-Planckian, with subdominant, sub-Planckian oscillations always being present. We dubbed the scenario, which generalizes to any multi throat setup, drifting monodromies.

We continued our landscape analysis of axions in Ch. 4. There, we considered axions in the winding scenario: By choice of fluxes, a certain direction of the multi axion field space remains lighter than all other complex structure moduli. Including sub-leading terms in the superpotential does stabilize this field as well. By appropriate tuning, we can achieve that the light direction in axion field space receives two potential contributions of different periodicities. Similar to the drifting monodromies scenario above, we found enhanced periodicities of the axion modulated by shorter oscillations. By tuning positive minima of the oscillatory potential arise. We found that this F-term potential may then be used to uplift vacua in the large volume scenario to de Sitter vacua. The potential allows for enough tuning to make this vacuum meta-stable. We found that in the KKLT scenario we cannot uplift consistently with the minimal setup described. We examined the problems that arise and proposed on how to further develop the winding uplifting idea in this background. We discussed how the DGKT solution in type IIA naturally gives rise to a similar scenario. We argued how the corresponding supersymmetric AdS vacuum can be uplifted to stable, non-supersymmetric ones.

A quantitative analysis of the no global symmetries conjecture was the goal of Ch. 5. We focused on global $U(1)$ symmetries that derive from gauged symmetries at a higher energy scale. A mass to the gauge boson is introduced by the Stückelberg mechanism which involves gauging an axion's shift symmetry under the $U(1)$ gauge symmetry. By this, the instantons originally charged in the axion theory can now serve as endpoints of worldlines of $U(1)$-charged particles. By integrating out instanton insertions coupled to charged fields, one arrives at the usual operators as in the 't Hooft vertex in $SU(N)$ gauge theories, suppressed by the instanton action S_I as $\exp(-S_I)$. These manifestly gauge-invariant operators break the global symmetry once the gauge redundancy is completely fixed by setting the axion to 0. Applying the WGC for axions in its electric and magnetic version, we bounded the coupling of the global-symmetry-violating operator from below by $\exp(-M_{\mathrm{P}}^2/\Lambda^2)$, where Λ is the cut-off of the EFT.

In Ch. 6, we turned to one of the possible motivations behind the WGC: Long-range repulsive forces should dominate over attractive ones to not allow for an infinite tower of bound states. Rather than forbidding bound states altogether, we proposed to constrain them in the absence of repulsive gauge forces: Attractive forces should not be stronger than gravity alone. In order to quantify this statement, we considered the minimal radius gravitationally bound states formed from particles of mass m can reach and claimed that this is the lower limit on the radius of any bound state formed from the same particle species. This minimal radius is achieved at the critical radius $R \sim 1/m$ of black hole collapse for scalar particles of mass m. Since this turns out to be independent of M_{P}, one may take the decoupling limit $M_{\mathrm{P}} \to \infty$. This led to the bound state conjecture: In any renormalizable effective field theory the typical radius R of a stable bound state is bounded from below by $R \gtrsim 1/m$, where the scale m is the mass of the heaviest stable particle. We gave a number of non-trivial examples to see how bound states become unstable once this threshold is seemingly crossed.

Finally, we conclude by embedding our results in the general context of the swampland program and by posing research questions that emerged from the work done in this thesis.

Concerning the WGC for axions, we found axions with large decay constants in Ch. 3 violating the simplest versions of the WGC, see Sect. 3.6.2. While it seems a generally accepted viewpoint that only a sub-lattice version is expected to hold [53, 54], we also found that only a very coarse sub-lattice is subject to the WGC, which goes against the general expectation. This is based, however, on two assumptions: First, the existence of Euclidean brane instantons on the same two-cycle as the thraxion in the UV. These instantons would be subject to the WGC but would have very large charges. Second, we assumed that the WGC should be applicable to the effective instantons corresponding to exponential terms in the superpotential. We expect this to be true, as the classical effect of geometric back-reaction on the supergravity side corresponds to actual instantonic effects on the dual gauge theory side. Whether these two assumptions are correct is to be scrutinized in future work.

There are two interesting open questions relating to the proposed thraxion super- and Kähler potential: We lack a proper motivation of a possible $\mathcal{N} = 2$ completion of the thraxion superpotential as well as the inclusion of non-perturbative effects used in a full moduli stabilization scheme. The latter usually lifts the mass degeneracy between axion and saxion partners. As KS throats are prevalent in string phenomenology and we expect a large portion to be part of multi throat systems [52], it is important to clarify to which energy scales multi throat systems can be assumed to be stable. This includes a necessary analysis of the possible interplay of the saxion with other modes of a given model.

Putting the focus on the landscape, the phenomenology of the thraxion is still to be explored in concrete setups. A possible application of the drifting monodromies scenario has been found in [322] in hybrid inflation.

In Ch. 4 we gave a concrete model of non-SUSY AdS and dS spaces. The conjectured instability of such solutions is to be tested for the concrete setups given. We expect the results to be very robust as they use few ingredients. However, whether the proposed models can be realized depends on the amount of tuning possible. We do expect to have a lot of tuning power by choice of CY and fluxes, allowing for almost arbitrarily small uplifting potentials that break supersymmetry. This is to be checked by a landscape study of the parameters involved in tuning the potential as desired.

As for the KKLT AdS vacuum, we were not successful in establishing that SUSY-breaking winding uplifts exist. While we proposed how this could be analyzed, we did not give a complete derivation of the existence of SUSY-breaking minima at small values of the superpotential. This is to be studied in future work.

Moving on to global symmetries in Ch. 5, we believe that we have made major progress towards establishing a quantitative statement about the non-existence of global symmetries which goes far beyond the statement of the WGC forbidding small or vanishing coupling constants. While there have been quantum gravity arguments to establish the same parametric estimate before, we gave a derivation of the strength of violation based on commonly accepted conjectures. As briefly discussed in Sect. 5.5.3, one might even consider a much stronger conjecture in the regime of perturbative EFTs by not relying on the magnetic version of the axionic WGC to constrain the cut-off but rather by imposing the energy scale at which the global symmetry reveals itself to be of gauged origin as the cut-off. We see that we have been conservative in our estimates and one might by able to conclude stronger constraints from our general arguments. The status of the loophole posed in Sect. 5.5.2 remains to be clarified.

Our argument could lead to a proper derivation of a swampland global symmetries conjecture. It is not clear yet which form it will take. Since we expect the landscape to be finite, there are far from enough tuning parameters to make all symmetry-violating operators of arbitrary mass dimension exponentially small. We therefore expect the conjecture to extend to fine-tuned global symmetries as well. As discussed in more detail in Sect. 5.5.4, whether all or only a subset of operators is to be restricted is as of yet unclear. If we include accidental symmetries in the discussion, we expect symmetry-violating operators that are forbidden by gauge symmetries in lower-dimensional operators to appear at some operator mass dimension. A possible extended conjecture could include a bound on the smallest mass dimension in which a symmetry-violating operator appears. An extension of our derivation to the case of non-abelian and discrete symmetries (not inherited from a broken $U(1)$) also remains to be discussed. There could be interesting phenomena arising from having multiple $U(1)$ symmetries with multiple axions coupling in more complicated ways (to multiple $U(1)$'s at the same time with different charges).

We highlight two current lines of research of relevance: Currently, the role of gravitational instantons is debated. There are claims that (some of) these are pure gauge redundancies [245–247]. Also, there is ongoing research on global symmetries in the context of generalized global symmetries [323] (see [324] for work in this direction), where a global symmetry is defined in purely topological terms. In this language, we have considered continuous 0-form symmetries.

It is interesting to further explore the purely quantum field theoretical origin of the bound state conjecture of Ch. 6. While it is motivated by gravitational arguments, it remains non-trivial in the gravitational decoupling limit. We saw that it does in fact stand up to simple estimates of non-gravitational systems. With this in mind, and its similarity to the uncertainty relation, a proper proof or construction of counterexamples seems achievable. Even without a fundamental derivation, further exploring examples away from renormalizable theories seems to be a fruitful way forward as we saw when considering Q-bubbles in Sect. 6.4.2.

If the conjecture turns out to be true, we may draw an interesting conclusion: There are no IR-to-UV transformers. That is, one may not be able to adiabatically collect energy density in the form of many low-mass particles to facilitate transitions to UV particles. The energy density may be too wide-spread and the transition rates may remain exponentially suppressed. If this turns out to be true, colliders seem to be the only way towards probing UV physics.

Appendix A

An Introduction to Conifold Geometry

A.1 Deformed Conifold for General Complex Structure Modulus

The unwarped internal geometry before back-reaction by fluxes is that of the deformed conifold [123]. It can be embedded in $\mathbb{C}^4$ via

$$\sum_i w_i^2 = z\,, \tag{A.1}$$

where w_i are complex coordinates and z is some complex parameter. For now, we set $z = |z| \in \mathbb{R}^+$. One can check, that the five-dimensional hypersurface at a given fixed radial coordinate $\rho^2 \equiv \sum |w_i|^2 > |z|$ allows for a transitive $SU(2) \times SU(2)$ action [1] with isotropy group $U(1) \subset SU(2) \times SU(2)$. The space can therefore be written as the homogeneous space $\frac{SU(2)\times SU(2)}{U(1)}$. At $\rho^2 = |z|$, the isotropy group is enhanced to a diagonal $SU(2) \subset SU(2) \times SU(2)$, therefore leading to the homogeneous space $\frac{SU(2)\times SU(2)}{SU(2)} \sim \frac{SO(4)}{SO(3)} = S^3$. Given that topologically $\frac{SU(2)\times SU(2)}{U(1)} = S^3 \times S^2$, we arrive at the picture of a family of two-spheres along some radial coordinate fibered over a three-sphere, where the two-sphere vanishes or collapses at the apex, while the three-sphere stays finite everywhere.

[1] Acting as $SO(4)$ on the vector $(w_1, w_2, w_3, w_4)^T$ in the obvious way [325]. It is more convenient to parametrize the embedding via the matrix $T = \frac{1}{\sqrt{2}} \sum w_i \sigma_i$, with Pauli matrices σ_i. General solutions to $\det T = -\frac{|z|}{2}$ and $\rho^2 = \operatorname{tr} T^\dagger T$ then take the form $T = L T_0 R$, with a specific solution $T_0(|z|, \rho)$ and $(L, R) \in SU(2) \times SU(2)$. The $SU(2) \times SU(2)$ action is now the obvious one $(L, R) \to (U_L L, U_R R)$ [326].

Following [326], we may derive the most general $SU(2) \times SU(2)$-invariant metric, that is also invariant under a $\mathbb{Z}_2$ exchange of the two $SU(2)$-factors. It takes the form

$$\begin{aligned} \mathrm{d}s^2 = &\left(B^2 + H^2\right) \left(\left(g^1\right)^2 + \left(g^2\right)^2\right) + C^2 \left(\left(g^3\right)^2 + \left(g^4\right)^2\right) \\ &+ D^2 \left(\mathrm{d}\rho^2 + \left(g^5\right)^2\right) + 2CH \left(g^1 g^4 - g^2 g^3\right) . \end{aligned} \tag{A.2}$$

Here, we used the vielbein

$$\begin{aligned} g^1 &= \frac{e^1 - e^3}{\sqrt{2}} , \quad g^2 = \frac{e^2 - e^4}{\sqrt{2}} , \\ g^3 &= \frac{e^1 + e^3}{\sqrt{2}} , \quad g^4 = \frac{e^2 + e^4}{\sqrt{2}} , \quad g^5 = e^5 , \end{aligned} \tag{A.3}$$

with

$$\begin{aligned} e^1 = -\sin\theta_1 \mathrm{d}\phi_1 , \quad e^2 = \mathrm{d}\theta_1 , \quad e^3 = \cos\psi \sin\theta_2 \mathrm{d}\phi_2 - \sin\psi \mathrm{d}\theta_2 , \\ e^4 = \sin\psi \sin\theta_2 \mathrm{d}\phi_2 + \cos\psi \mathrm{d}\theta_2 , \quad e^5 = \mathrm{d}\psi + \cos\theta_1 \mathrm{d}\phi_1 + \cos\theta_2 \mathrm{d}\phi_2 . \end{aligned} \tag{A.4}$$

The real coefficients B , C , D and H are functions of ρ and $|z|$. We impose the constraints on the coefficients from the embedding (A.1) and insert the definition of the radial coordinate, as well as demand Ricci-flatness of the metric (A.2). The resulting CY-metric with $SU(2) \times SU(2)$-isometry is now best expressed in the coordinate $\tau \geq 0$ defined via $\rho^2 = |z| \cosh\tau$ [326],

$$\begin{aligned} \mathrm{d}s^2_{\mathrm{dc}} = &B^2_{\mathrm{dc}}(\tau) \left(\left(g^1\right)^2 + \left(g^2\right)^2\right) + C^2_{\mathrm{dc}}(\tau) \left(\left(g^3\right)^2 + \left(g^4\right)^2\right) \\ &+ D^2_{\mathrm{dc}}(\tau) \left(\mathrm{d}\tau^2 + \left(g^5\right)^2\right) , \end{aligned} \tag{A.5}$$

with

$$\begin{aligned} B^2_{\mathrm{dc}}(\tau) &= |z|^{\frac{2}{3}} \frac{K(\tau)}{2} \sinh^2(\tau/2) , \quad C^2_{\mathrm{dc}}(\tau) = |z|^{\frac{2}{3}} \frac{K(\tau)}{2} \cosh^2(\tau/2) , \\ D^2_{\mathrm{dc}}(\tau) &= |z|^{\frac{2}{3}} \frac{1}{6K(\tau)^2} , \quad \text{where} \quad K(\tau) \equiv \frac{\left(\sinh(2\tau) - 2\tau\right)^{\frac{1}{3}}}{2^{\frac{1}{3}} \sinh(\tau)} , \end{aligned} \tag{A.6}$$

and $H_{\mathrm{dc}} \equiv 0$.

We now generalize this metric to arbitrary values of $z = |z|\, e^{i\varphi} \in \mathbb{C}$. This is most easily done by considering the action of $e^{i\varphi} \in U(1)_R$ on the coordinates

$$w_i \to w_i \, e^{i\varphi/2} . \tag{A.7}$$

Looking at the embedding (A.1), this gives the rotation $|z| \to |z|\, e^{i\varphi}$ we are after. After some calculation, one can reinterpret the rotation of coordinate w_i as an action on the coefficients B, C, D and H. The new coefficients after applying a $U(1)_R$ rotation to second order in φ read

$$\begin{aligned} B &= B_{\mathrm{dc}} + \varphi^2 \frac{B_{\mathrm{dc}}}{C_{\mathrm{dc}}} \frac{C_{\mathrm{dc}}^2 - B_{\mathrm{dc}}^2}{8C_{\mathrm{dc}}} \,, \\ C &= C_{\mathrm{dc}} + \varphi^2 \frac{B_{\mathrm{dc}}^2 - C_{\mathrm{dc}}^2}{8C_{\mathrm{dc}}} \,, \\ H &= \varphi \frac{C_{\mathrm{dc}}^2 - B_{\mathrm{dc}}^2}{2C_{\mathrm{dc}}} \,. \end{aligned} \tag{A.8}$$

In the full result (i.e., beyond quadratic order in φ) the periodicity $\varphi \longrightarrow \varphi + 2\pi$ is of course preserved.

We now want to determine the metric at large $\rho^2 \gg |z|$. For $\varphi = 0$ one may express it in the form

$$\mathrm{d}s^2 = f_0(r)^2 \mathrm{d}r^2 + r^2 \left(\frac{1}{9} f_5(r)^2 (g^5)^2 + \frac{1}{6} \sum_{i=1}^{4} f_i(r)^2 (g^i)^2 \right) , \tag{A.9}$$

where we have introduced the asymptotic conifold radial coordinate $r = \sqrt{\frac{3}{2}} \rho^{2/3}$. Defining $\epsilon(r) \equiv \frac{|z|}{r^3}$, at order ϵ^0 one has $f_i \equiv 1$ which corresponds to the metric of the singular conifold. However, also far away from the deformation $\epsilon \ll 1$, we see some remnant of the apex geometry

$$\begin{aligned} f_0^2(r) = f_5^2(r) &= \frac{9}{r^2} D^2\left(\tau(r)\right) = 1 + \mathcal{O}(\epsilon^2) \,, \\ f_1^2(r) = f_2^2(r) &= \frac{6}{r^2} B^2\left(\tau(r)\right) = 1 + \left(\frac{3}{2}\right)^{3/2} \epsilon + \mathcal{O}(\epsilon^2) \,, \\ f_3^2(r) = f_4^2(r) &= \frac{6}{r^2} C^2\left(\tau(r)\right) = 1 - \left(\frac{3}{2}\right)^{3/2} \epsilon + \mathcal{O}(\epsilon^2) \,. \end{aligned} \tag{A.10}$$

Neglecting numerical $\mathcal{O}(1)$-factors, we arrive at

$$\begin{aligned} \mathrm{d}s^2 =& \mathrm{d}r^2 + r^2 \left\{ \mathrm{d}\Omega_{T^{1,1}}^2 + \epsilon(r) \left[\left(g^1\right)^2 + \left(g^2\right)^2 - \left(g^3\right)^2 - \left(g^4\right)^2 \right] \right\} \\ &+ \mathcal{O}(\epsilon^2) \,. \end{aligned} \tag{A.11}$$

Using the transformation behaviour of the coefficients of the angular terms (A.8), it is now straightforward to write down the same asymptotic expansion for non-zero φ

$$\begin{aligned} \mathrm{d}s^2 &= \mathrm{d}r^2 + r^2 \left\{ \mathrm{d}\Omega^2_{T^{1,1}} + \epsilon(r) d\Omega^2_5(\varphi) \right\} . \\ d\Omega^2_5(\varphi) &\equiv (1+\varphi^2) \left(\left(g^1\right)^2 + \left(g^2\right)^2 - \left(g^3\right)^2 - \left(g^4\right)^2 \right) \\ &\quad + \varphi \left(g^1 g^4 - g^2 g^3 \right) \end{aligned} \tag{A.12}$$

Finally, we consider the full 10d metric found as a solution to 10d SUGRA with M units of F_3-flux on the three-sphere described above and some H_3-flux (unquantized in the non-compact setting) on the dual cycle [49]

$$\mathrm{d}s^2 = w^2(r)\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + w^{-2}(r)\mathrm{d}s^2_{\mathrm{dc}} \,, \tag{A.13}$$

where the warp factor goes like

$$w(r)^2 = \frac{r^2}{g_s M \alpha'} \frac{1}{\sqrt{\ln\left(\frac{r^3}{|z|}\right)}} \,, \tag{A.14}$$

for $\frac{r^3}{|z|} \gg 1$. For arbitrary φ, we will work with the asymptotic metric

$$\mathrm{d}s^2 = w^2(r)\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + w^{-2}(r) \left[\mathrm{d}r^2 + r^2 \left(\mathrm{d}\Omega^2_{T^{1,1}} + \epsilon(r)\mathrm{d}\Omega_5(\varphi)^2 \right) \right] . \tag{A.15}$$

A.2 The Axion Potential in the Local Throat

In the main text we have repeatedly made use of the fact that the C_2- and B_2-axions c and b can only enter the scalar potential that is generated in the local throat in certain combinations with the 'local complex structure' of the throat, namely the real and imaginary part of

$$M \ln(z) - i\mathcal{G} \,, \tag{A.16}$$

where $\mathcal{G} = c - \tau b$, and z is the 'local complex structure'. Here, we would like to derive this without using 'local flux stabilization' as in Sect. 3.2.3 but rather rely only on asymptotic properties of the KS/KT solution [115, 49]. For simplicity we will set the RR zero form to zero, i.e., $\tau = i g_s^{-1}$.

We cut off the throat at a radial coordinate r_{UV} and define the b-axion at that value of the radial coordinate. Since the B_2 profile runs along the radial direction [327], changing $b \longrightarrow b + \delta b$ can be realized by choosing a *different* UV-cut-off r'_{UV}. Since the absolute value of the complex structure is defined in units of the UV-cut-off it scales as

$$z \longrightarrow e^{-\frac{\delta b}{g_s M}} z \,. \tag{A.17}$$

Since this is just a coordinate transformation from the perspective of the local KS throat, the combination $g_s M \ln(|z|) + b$ cannot appear in the scalar potential that is generated within the throat. It acquires physical meaning only if the throat is cut off at fixed, finite r_{UV}.

Similarly, in the limit $|z|/r^3 \to 0$ the RR three form takes the form

$$F_3 = 2\pi\alpha' \, M \left(g^5 + \mathrm{d}c/M\right) \wedge \omega_\Sigma \,, \tag{A.18}$$

where $g^5 = \mathrm{d}\psi + ...$ is given by (A.3), and ω_Σ is the normalized harmonic 2-form of $T^{1,1}$. The field $c(x)$ transforms like a Goldstone boson under (local) coordinate transformations [149]

$$\psi \longrightarrow \psi + 2\omega(x)\,, \quad c(x) \longrightarrow c(x) - 2M\omega(x)\,. \tag{A.19}$$

Shifting along this angular direction is an isometry of the asymptotic KS solution (called $U(1)_R$). Near the IR this is not the case precisely because (by definition) the phase of the complex structure also transforms like a Goldstone boson, see App. A.1

$$\arg z \longrightarrow \arg z - 2\omega\,. \tag{A.20}$$

Again, in the local throat the combination $M \arg z + c$ has no physical meaning as it is eaten via the Higgs mechanism. Only when the throat is glued into the CY space at finite radial coordinate r_{UV} does the c-axion gain its independent physical meaning because the $U(1)_R$ symmetry is badly broken by the CY geometry. Putting together the real and imaginary part of $\mathcal{G} = c - \tau b$, we arrive at the conclusion that only the combination (A.16) is physical when considering a single throat. A second degree of freedom only becomes physical by finiteness of the throat, i.e., by breaking the asymptotic $U(1)_R$ symmetry. This also implies that the kinetic terms for the fields $\varphi_{1,2}$ stated in (3.13) actually take the form $[\partial(\varphi_{1,2} \pm c/M)]^2$ since they arise from local throat physics. We have disregarded some unimportant off-diagonal terms in the kinetic matrix.

The transformation behavior of $c(x)$ under a $U(1)_R$ can also be calculated directly [328]. For this, we choose some S^2-submanifold of the cross-section $T^{1,1}$ of the deformed conifold far in the UV and apply the $U(1)_R$ action on the coordinates as in App. A.1. One can show that the manifold X defined by the $U(1)_R$-orbit of S^2-submanifolds is an element of $H^3(T^{1,1})$, and therefore a multiple of the S^3-cycle of $T^{1,1}$. It turns out that the multiplicity is 2. Denoting a member of the family of two-spheres as $S^2(\omega)$, $\omega \in U(1)_R$, we therefore find by applying Stokes' theorem

$$\int_{S^2(0)} C_2 - \int_{S^2(2\pi)} C_2 = \int_X F_3 = 2\int_{S^3} F_3 = 2(2\pi)^2\alpha' M\,. \tag{A.21}$$

Using homogeneity of the flux distribution we extrapolate to general transformations $\int_{S^2(0)} C_2 - \int_{S^2(\omega)} C_2 = 4\pi\alpha' M\omega$. In terms of c defined on (say) $S^2(0)$, we arrive at the transformation law as stated above. Interpreting $\int_{S^2} C_2$ as a generalized Wilson line, we find the usual behavior under deforming the integration path over some region with non-vanishing associated field-strength F_3. Under $\pi \in U(1)_R$, the family $S^2(\omega)$ sweeps out S^3 completely; accordingly the Wilson line picks up the flux $\int_{S^3} F_3 = (2\pi)^2\alpha' M$.

A.3 Background on Multi Conifolds

In this appendix we discuss preliminaries that are important for Sect. 3.4. We follow mainly Chapter 8 of [140].

For a general CY threefold M we can choose $2h^{2,1}+2$ three-cycles $\mathcal{A}^a$, $\mathcal{B}_a$, $a = 1, ..., h^{2,1}+1$ as a symplectic basis of $H^3(M)$, i.e.

$$\int_{\mathcal{A}^b} \alpha^a = \int_M \alpha^a \wedge \beta_b = \delta^a_b\,, \quad \int_{\mathcal{B}_b} \beta_a = \int_M \beta_a \wedge \alpha^b = -\delta^b_a\,, \tag{A.22}$$

where α^a and β_a are the harmonic three-forms that are Poincaré dual to $\mathcal{B}_a$, respectively $\mathcal{A}^a$. One may define the periods

$$z_a = \int_{\mathcal{A}^a} \Omega\,, \quad G^a = \int_{\mathcal{B}_a} \Omega\,, \tag{A.23}$$

where Ω is the holomorphic three-form. The z_a form a set of projective coordinates on complex structure moduli space and the G^a are functions of them. We are interested in what happens when n cycles γ^i with m homology relations among them shrink at a conifold point in moduli space. The Picard-Lefschetz formula states that upon encircling a conifold point in moduli space, a three-cycle δ undergoes the monodromy [329, 330, 139]

$$\delta \longrightarrow \delta + \sum_{i=1}^{n} (\delta \cap \gamma^i)\gamma^i\,. \tag{A.24}$$

Knowing this monodromy transformation is enough to determine that

$$\int_\delta \Omega = \frac{1}{2\pi i} \sum_{i=1}^{n} (\delta \cap \gamma^i) \int_{\gamma^i} \Omega \,\ln(\int_{\gamma^i} \Omega) + \text{single-valued}\,. \tag{A.25}$$

We may choose $n-m$ of the degenerating cycles as part of the basis $\mathcal{A}^i = \gamma^i$ for $i = 1, \ldots, n-m$, while the remaining m are integer linear combinations

$\gamma^i = \sum_{a=1}^{n-m} c_a^i \mathcal{A}^a$ for $i = n-m+1,\ldots,n$. By applying (A.24) to the cycles $\mathcal{B}_a$ we arrive at

$$G^a = \int_{\mathcal{B}_a} \Omega = \frac{1}{2\pi i} z_a \ln(z_a) + \frac{1}{2\pi i} \sum_{i=n-m+1}^{n} c_a^i z_i \ln(z_i) + g^a(z)\,, \qquad a = 1,...,n-m\,, \tag{A.26}$$

where $g^a(z)$ are $n-m$ holomorphic functions. Here, we have defined $z_i \equiv \sum_{a=1}^{n-m} c_a^i z_a$ for $i = n-m+1,\ldots,n$, i.e., $z_i \equiv \int_{\gamma^i} \Omega$ when applying (A.24) [2]. At frozen values of z^a, $a = n-m+1,...,h^{2,1}+1$ the periods associated to other cycles, $G^a = \int_{\mathcal{B}_a} \Omega$, with $a = n-m+1,...,h^{2,1}+1$, are holomorphic in the complex structures that parametrize the multi conifold deformations, i.e., in the z^i, with $i = 1,...,n-m$. In what follows we denote by z^a only the multi conifold deformation parameters.

We may now evaluate the GVW superpotential $W = \int_M G_3 \wedge \Omega$ where we choose flux quanta M_a and K^a according to $G_3 = -\sum_{a=1}^{n-m}(M_a \alpha^a - \tau K^a \beta_a)$. Using that

$$\int_M \alpha^a \wedge \Omega = -\int_{\mathcal{B}_a} \Omega = -G^a\,, \qquad \int_M \beta_a \wedge \Omega = -\int_{\mathcal{A}^a} \Omega = -z_a\,, \tag{A.27}$$

one obtains

$$W(z_a) = \sum_{a=1}^{n-m} \frac{M_a}{2\pi i} z_a \ln(z_a) + \sum_{i=n-m+1}^{n} \frac{M_i}{2\pi i} z_i \ln(z_i) + \sum_{a=1}^{n-m} M_a g^a(z) - \tau \sum_{a=1}^{n-m} K^a z_a + \hat{\hat{W}}_0(z_a)\,, \tag{A.28}$$

where we have *defined* $M_i \equiv \sum_{a=1}^{n-m} c_a^i M_a$, and the holomorphic function $\hat{\hat{W}}_0(z_a)$ parametrizes the contributions from fluxes on other cycles. We may use the z_a and z_i with $i = n-m+1,...,n$ on the same footing by interpreting our definition of the z_i as m constraint equations

$$0 = P^I \equiv \sum_{i=1}^{n} p_i^I z_i \equiv z_{n-m+I} - \sum_{a=1}^{n-m} c_a^{n-m+I} z_a\,, \quad I = 1,...,m\,. \tag{A.29}$$

Here, the $m \times n$ matrix p_i^I is implicitely defined as

$$p_i^I = \begin{cases} -c_i^{n-m+I}\,, & i = 1,\ldots,n-m\,, \\ \delta_i^{n-m+I}\,, & i = n-m+1,\ldots,n\,. \end{cases} \tag{A.30}$$

[2] When using a local expression for the holomorphic three-form Ω in the vicinity of smoothed conical singularity described by (3.1) one can calculate $\int_{\gamma^i} \Omega = z_i$ [154]. This identifies the z_i defined here with the local deformation parameter of the i-th throat.

We may now write the superpotential as

$$\begin{aligned} W(z_i) = &\sum_{i=1}^{n} \Big(M_i \frac{z_i}{2\pi i} \ln(z_i) + M_i g^i(z) - \tau K^i z_i \Big) \\ &+ \sum_{I=1}^{m} \lambda_I P^I + \hat{\hat{W}}_0(z_i)\,, \end{aligned} \tag{A.31}$$

with m Lagrange multipliers λ_I. The homology relation is now enforced via the F-term of the fields λ_I, compare [154] where Lagrange multipliers in the superpotential are also used to impose additional constraints on chiral superfields. In doing so, we have defined g^i to be zero for $i > n-m$.

The F_3-flux on γ^i is given by M_i. By the definition of M_i for $i = n-m+1, \ldots, n$, the flux numbers automatically fulfill $\sum_{i=1}^{n} p_i^I M_i = 0$ for all I. In democratic terms, the n flux numbers M_i must be chosen in compliance with the m homology constraints $\sum_{i=1}^{n} p_i^I M_i = 0$. The H_3-flux on $\mathcal{B}_a$ is given by $K^a + \sum_{I=1}^{m} c_a^{n-m+I} K^{n-m+I}$, as this is the coefficient appearing in front of z_a. In other words the $n-m$ flux quantization conditions read $K^a + \sum_{I=1}^{m} c_a^{n-m+I} K^{n-m+I} \in \mathbb{Z}$. Note that we may transform $K^i \to K^i + \sum_I \alpha_I p_i^I$ for any $\alpha \in \mathbb{C}^m$ because the superpotential is left invariant upon imposing the constraint equations, that is to say, we can undo such a transformation by also shifting the Lagrange multipliers $\lambda^I \to \lambda^I + \tau\alpha^I$. Of course, the flux quantization conditions are invariant under these shifts. Finally, the Kähler potential is given by

$$\begin{aligned} K_{\text{cs}}(z_i, \bar{z}_i) &= -\ln\Big(-i \int \Omega \wedge \bar{\Omega} \Big) \\ &= -\ln\left(i g_K(z) - i\overline{g_K}(z) + \sum_{a=1}^{n-m} i \bar{z}_a G^a(z) + \text{c.c.} \right) \\ &= -\ln\Big(i g_K(z) - i\overline{g_K(z)} \\ &\qquad + \sum_{i=1}^{n} \left[\frac{|z_i|^2}{2\pi} \ln(|z_i|^2) + i\bar{z}_i g^i(z) - i z_i \overline{g^i(z)} \right] \Big)\,, \end{aligned} \tag{A.32}$$

where $g_K = \sum_{a=n-m+1}^{h^{2,1}+1} \overline{z_a} G^a(z)$ encodes contributions from other cycles. It is holomorphic in z_a, $a = 1, ..., n-m$. Note that despite the democratic formulation, the Kähler and superpotential are strictly defined only along complex structure moduli space, where $P^I = 0$. As explained in Sect. 3.4 we propose to extend the domain of these functions to the deformation space parametrized by *all* z_i by introducing general Taylor expansions of $g^i(z_i)$, $g_K(z_i)$ and $\hat{\hat{W}}_0(z_i)$ in (3.58).

Own Publications

This thesis is based on the following papers.

[A] A. Hebecker, S. Leonhardt, J. Moritz and A. Westphal, *Thraxions: Ultralight Throat Axions*,
JHEP **04** (2019) 158 [1812.03999].

[B] B. Freivogel, T. Gasenzer, A. Hebecker and S. Leonhardt, *A Conjecture on the Minimal Size of Bound States*,
SciPost Phys. **8** (2020) 058 [1912.09485].

[C] T. Daus, A. Hebecker, S. Leonhardt and J. March-Russell, *Towards a Swampland Global Symmetry Conjecture using Weak Gravity*,
Nucl. Phys. B **960** (2020) 115167 [2002.02456].

[D] A. Hebecker and S. Leonhardt, *Winding Uplifts: Small SUSY Breaking in the Flux Landscape*,
to appear.

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