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The universe from a string-theoretic and cosmological perspective

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[1] **Transient weak gravity in scalar-tensor theories.**
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[2] **The F-term Problem and other Challenges of Stringy Quintessence.**
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[3] **Axions in String Theory and the Hydra of Dark Radiation.**
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Abstract

One major challenge of modern-day theoretical physics concerns the extension of our standard models for particle physics and cosmology. A motivation therefor arises from several problems and tensions that these models face and which suggest their modification. In this thesis we utilise distinct approaches to tackle some of these issues from different perspectives. In a classical field-theoretic approach, we first consider an extension to the ΛCDM model called coupled dark energy. Characteristic for this scalar-tensor theory, which has been shown to possess the ability to alleviate the (in-)famous Hubble tension, is an intrinsic coupling within the dark sector of the universe. We will demonstrate that under certain conditions this model can give rise to a novel transient regime of weak gravity. This may help to solve or at least alleviate the σ_8 tension of the ΛCDM model. We then shift to the paradigm of string theory, which presumably provides the required ultraviolet completion of gravity including the other fundamental forces, and assess its consequences on two extensions of standard-model physics. The first is constituted by the postulation of a new particle, the QCD axion, which arguably represents the most prominent way to solve the strong CP problem. In string theory, there are many candidates for this new particle and we investigate phenomenological consequences of such a stringy realisation of the axion. In particular, we find a novel way to seemingly solve the notorious issue of too much dark radiation, which is a generic prediction of these constructions, via a fast decay channel of the internal volume into standard-model Higgses. Even though we ascertain that eventually the dark radiation problem presumably re-appears due to the altered cosmological setting, we are confident that our results will prove to be helpful for future constructions. The second extension to standard-model physics we consider is a cosmological one, namely quintessence. With regard to the recently postulated de Sitter-swampland conjecture, we analyse the realisability of such a dynamical form of dark energy in a stringy context. Taking into account several phenomenological requirements, we identify two major challenges that need to be overcome: a so-called light-volume problem implying a very light internal-volume modulus, that would give rise to inadmissible fifth forces, and a novel F-term problem, which emerges from the fact that the required supersymmetry breaking scale raises the resulting scalar potential and hence the effective vacuum energy to a value that is parametrically above the observed one.

Zusammenfassung

Eine große Herausforderung der modernen theoretischen Physik besteht aus der Erweiterung der Standardmodelle der Teilchenphysik und Kosmologie. Dies ist motiviert durch einige Probleme und Spannungen, welchen diese Modelle ausgesetzt sind und die eine Modifikation Letzterer erfordern. In dieser Dissertation betrachten wir unterschiedliche Ansätze, um einige der obigen Herausforderungen aus verschiedenen Blickpunkten anzugehen. In einem klassischen, feldtheoretischen Ansatz beschäftigen wir uns zunächst mit einer Erweiterung des ΛCDM-Modells namens gekoppelte, dunkle Energie. Bezeichnend für diese Skalar-Tensor-Theorie, für welche gezeigt wurde, dass sie die berüchtigte Hubble-Spannung abmildern kann, ist eine intrinsische Kopplung innerhalb des dunklen Sektors des Universums. Wir werden demonstrieren, dass dieses Modell unter gewissen Voraussetzungen zu einem neuartigen, vorübergehenenden Bereich schwacher Gravitation führen kann. Dies könnte dabei helfen, die σ_8-Spannung des ΛCDM-Modells zu lösen oder zumindest abzuschwächen. Anschließend wechseln wir unsere Anschauung hin zur Stringtheorie, von welcher vermutet wird, dass sie die benötigte UV-Vervollständigung der Gravitation inklusive der anderen Grundkräfte bereitstellt, und untersuchen ihre Folgen für zwei Erweiterungen der Standardmodellphysik. Erstere besteht aus der Vorhersage eines neuen Teilchens, des QCD-Axions, welches wohl die bekannteste Möglichkeit das starke CP-Problem zu lösen darstellt. In der Stringtheorie gibt es viele Kandidaten für dieses neue Teilchen und wir untersuchen die phänomenologischen Folgen einer solchen, stringtheoretischen Umsetzung des Axions. Insbesondere finden wir eine neuartige Möglichkeit das berüchtigte Problem von zu viel dunkler Strahlung, welches eine allgemeine Vorhersage solcher Konstruktionen ist, mithilfe eines schnellen Zerfalls des inneren Volumens in Standardmodell-Higgse scheinbar zu lösen. Obwohl wir herausfinden, dass das Dunkle-Strahlungsproblem aufgrund des veränderten kosmologischen Umfelds letztlich vermutlich wieder auftaucht, sind wir zuversichtlich, dass unsere Ergebnisse sich als hilfreich für zukünftige Konstruktionen herausstellen werden. Die zweite Erweiterung der Standardmodellphysik, die wir betrachten, ist eine kosmologische, nämlich Quintessenz. Im Hinblick auf die kürzlich aufgestellte de Sitter-Sumpfland-Vermutung, erforschen wir die Realisierbarkeit einer solchen dynamischen Form der dunklen Energie in einem stringtheoretischen Kontext. Unter Berücksichtigung einiger phänomenologischer Bedingungen identifizieren wir zwei große Herausforderungen, die überwunden werden müssen: Ein sogenanntes Leichtes-Volumen-Problem, welches einen leichten Volumenmodulus vorhersagt, der zu unerlaubten fünften Kräften führen könnte, und ein neuartiges F-Term-Problem, welches daher auftritt, dass die benötigte SUSY-Brechungsskala das resultierende Skalarpotenzial und damit die effektive Vakuumsenergie auf einen Wert erhöht, welcher parametrisch über dem beobachteten liegt.

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Conventions, abbreviations and acronyms

In this dissertation, we will abbreviate typical terms related to the document structure, namely ‘part’, ‘chapter’, ‘section’, ‘equation’ and ‘table’, by the short versions ‘Pt.’, ‘Chpt.’, ‘Sec.’, ‘Eq.’ and ‘Tab.’, respectively.

Most of the time, we will work in natural units where $c \equiv k_\mathrm{B} \equiv \hbar \equiv 1$ unless stated otherwise. We will mostly spell out explicitly the reduced Planck mass $M_\mathrm{P} \equiv \sqrt{\hbar c/8\pi G} \approx 2.4 \times 10^{18}$ GeV except in Chpt. 6 and App. A.4, where we set M_P to unity for brevity. For the metric tensor, will typically use the mostly-plus convention, with the Minkowski metric given by $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, \cdots, 1)$, although the terms concerning the large volume scenario in Sec. 4.2.4 and Chpts. 5 and 6 use the mostly-minus convention. We furthermore denote the imaginary unit by i.

Basic, string-theoretic quantities like the string length and mass are given by $l_\mathrm{s} = 2\pi\sqrt{\alpha'} = 2\pi/M_\mathrm{s}$, where α' is the Regge slope. In Pt. II, we typically denote the original moduli and axion fields as τ_i and θ_i and their canonically normalised versions as ϕ_i and a_i, respectively. On the other hand, in Pt. I ϕ will usually denote a general scalar field, which is not necessarily canonically normalised.

In Pt. I, we will often times abbreviate the partial derivative of a quantity y w.r.t. a variable x by $y_{,x}$ and the covariant derivative by $y_{;x}$. However, we will also employ the operators ∂_μ and ∇_μ for partial and covariant derivatives, respectively. Furthermore, a dotted quantity usually denotes a derivative w.r.t. cosmic time t and a prime the derivative w.r.t. the e-folds number $N = \ln a$.

We will also employ the following interpretations of relation operators:

$\equiv$	‘is defined by’
$\approx$	‘is (numerically) approximate to’
$\propto$	‘is proportional to’
$\sim$	‘is in principle similar to’
$\simeq$	‘is (functionally) approximate to’

Moreover, we will mostly interpret $\mathcal{O}(y)$ as the order of magnitude of a quantity y, where the implied range of values depends on the context. However, an exception to this interpretation is made in Chpt. 3, where $\mathcal{O}(y)$ will explicitly also include values that can be significantly lower than y.

Abbreviation	Meaning
BBN	Big Bang Nucleosynthesis
CC	Cosmological Constant
CS	Chern-Simons
CY	Calabi-Yau
CDE	Coupled Dark Energy
CDM	Cold Dark Matter
CMB	Cosmic Microwave Background
DBI	Dirac-Born-Infeld
DE	Dark Energy
DM	Dark Matter
DR	Dark Radiation
dS	de Sitter
EFT	Effective Field Theory
GR	General Relativity
GUT	Grand Unified Theory
KK	Kaluza-Klein
LVS	Large Volume Scenario
MG	Modified Gravity
NS	Neveu-Schwarz
PQ	Peccei-Quinn
QCD	Quantum Chromodynamics
RR	Ramond-Ramond
SM	Standard Model
SUGRA	Supergravity
SUSY	Supersymmetry
UV	Ultraviolet
VEV	Vacuum Expectation Value
w.l.o.g.	without loss of generality
w.r.t.	with respect to

Table 1.: List of abbreviations and acronyms.

1. Philosophical motivation

The nature of the universe and the fundamental laws governing it constitute one of the oldest scientific questions of humankind. In the modern era, this question can be concretised to a non-exhaustive list of unanswered problems, for instance: What is the field-theoretical nature of the Cosmological Constant (CC) or some dynamical form of Dark Energy (DE) and of Dark Matter (DM)? How do these components interact with each other? How can Einstein's theory of General Relativity (GR) be modified? What causes the Hubble tension and how can it be resolved? How can the Standard Model (SM) of particle physics be extended to solve the strong CP problem? Is there Supersymmetry (SUSY) and if so, what is its energy scale? Which consequences would string theory imply on concepts like DE or axions? In this thesis, we want to approach some of these questions from two different perspectives: a classical modified-gravity (MG) approach and a string-phenomenological one.

In the MG approach, we use classical field theory or geometry to extend the concordance model of cosmology, the ΛCDM model, hoping that the newly obtained model can describe observations better. Such an extension can include a modification of the underlying theory of gravity or of the matter content that is considered. Importantly, at this level of model building often times no underlying, fundamental theory is assumed to justify the given model. Instead, it is designed to provide an effective description of the universe on cosmological scales. Here the fields involved are usually classical and the model does not necessarily possess a UV completion. Guiding principles for this approach arise mostly from phenomenological consistency and from what are considered reasonable modifications of already established models. Typical criteria for a 'reasonable' cosmological setup include for example Lorentz invariance, the absence of ghosts and other instabilities, the resemblance to GR on solar-system scales and last but not least the correct prediction of observations. There are many facets to finding a good cosmological model and a lot of freedom in what one can do. This has led to a plethora of different MG theories, which include scalar-tensor theories, vector-tensor theories, massive and bimetric gravity, higher-order gravity such as $f(R)$ theories and many more. Here it remains a future task to further constrain their parameter spaces via observations so that hopefully some of them can be falsified, even though the vast amount of possibilities will always leave open a fair quantity of theories that will escape such bounds.

In the string-phenomenological approach, we assume an underlying UV completion of GR and the SM, namely string theory. This is motivated by the successes of this theory to presumably provide a way to unify all forces of nature while remaining UV finite in doing so. In this approach, phenomenological predictions are considered to result from the low-energy limit of a more fundamental, stringy theory. Within this paradigm, a vast amount of such low-energy effective descriptions arises from the many geometrical and topological possibilities in which string theory can be implemented in a ten-dimensional spacetime. This allows again for a lot of freedom for cosmological model building, which however is more restricted than by only phenomenological requirements since the effective model must stem from the underlying string theory. It is therefore a current and future task to find criteria to distinguish low-energy Effective Field Theories (EFT) which have a string-theoretical or Quantum Gravitational (QG) UV completion from those which have not. The former are said to be part of the 'landscape' whereas the latter are said to 'live in the swampland'. However, it should be mentioned that many results from this swampland programme possess the status of conjectures and are not vigorously proven. Nevertheless, there are promising setups of low-energy EFTs that can be derived from string theory and which might resemble our real world.

It is clear that both approaches have advantages and disadvantages and hence deserve attention. Obviously, the MG approach leaves open a much larger playground for model building and extensions to the concordance model. For instance, considerations like 'which are the consequences of adding higher-order terms to the GR action?' or 'what does a coupling between DM and DE imply?' cannot be ruled out per se and are hence interesting and justified on their own. Furthermore, in the MG approach one can easily think of scenarios which would stand in apparent contradiction to string theory but are nevertheless worthy of investigation and might represent a valid description of reality. One example is any theory which leads to a de Sitter (dS) solution and is forbidden by the so-called de Sitter-swampland conjecture stating that dS solutions belong to the swampland. On the other hand, it is widely accepted that the unification of forces into a theory of everything is highly desirable, both from an aesthetic point of view as well as from an experiential one. In the past, we have learned that electric and magnetic forces can be unified in Maxwell's theory of electrodynamics or that quantum electrodynamics and the weak interaction allow for a unified electroweak description at high enough energies. It seems reasonable that more forces will be united at higher energies. Therefore, it might be considered a disadvantage not to consider such unifications or at least take a possible UV completion as a criterion for a valid effective low-energy theory. Regarding this point, it is of course absolutely acceptable to argue that the unification of fundamental forces and the UV completion of gravity is not necessarily the main focus of cosmological reasearch one does. A cosmological model that can deal with current problems like the Hubble tension or the σ_8 tension is in any case highly valuable where the issue of a UV completion can be left to future research.

The advantage of the string-phenomenological approach lies definitely in the fact that the UV completion is already at hand. String theory provides a powerful framework to create interesting models that include gauge theories of higher rank, which can be useful for the construction of Grand Unified Theories (GUT), scalar-tensor theories, which can be used to establish theories for inflation or DE, and supersymmetry, which allows for physics beyond the SM or yields DM candidates. The biggest challenge for string theory remains to find a connection to reality. Calculational control in string theory is only ensured in very specific scenarios and limits so that a large subset of it remains inaccessible to us till this day. Hence it is is very difficult to create realistic settings. For example, often times a four-dimensional, low-energy EFT stemming from string theory is accompanied with a plethora of light scalar fields which contradict observations and need some handling. Finally, it is of course also possible that string theory, though being a very elegant UV completion, is not the true theory of everything but that nature has 'chosen' another way. Thus, no matter if we can find total consistency within this paradigm, the string-phenomenological approach remains speculative and might turn out wrong. In a sense, the MG approach can be considered to be the more agnostic one whereas the string-phenomenological one seems more idealistic albeit this statement should be taken with a grain of salt since it is a broad generalisation and many MG theories contain idealistic aspects as well.

In this thesis we do not dare to assess the two approaches against each other but will pursue them both. In part I, we will focus on the MG approach by considering a specific model called *Coupled Dark Energy* (CDE), which falls into the class of scalar-tensor theories. It is characterised by a non-minimal coupling between DE and DM, which can be useful in the search for a solution of the H_0 and the σ_8 tensions. Of particular interest will be the noteworthy behaviour of this theory for non-standard choices of the free functions in it. In part II, we will adopt the language of string theory and assume that this is the correct description of nature at high energies. We will first address the early universe and explore the consequences of the stringy paradigm on phenomenological properties of the QCD axion, inflation and the subsequent reheating of the SM. Finally, we will again consider the late-time universe and the nature of DE from a stringy perspective. More precisely, we will be working within the swampland programme of string theory and investigate the realisability of quintessence within it.

Part I.

The modified gravity approach

2. Introduction

2.1. General relativity and the ΛCDM model

The concordance model of cosmology is outstanding from every other known model which aims at a scientific description of the universe by one aspect: simplicity. It unites the well known components of the universe, baryonic matter, the lesser known yet proven to exist component, Cold Dark Matter (CDM), and the most successful theory for gravity, Einstein's general relativity, into one remarkably accurate model. Renownedly, these components contribute to today's energy density with approximately 4.9% baryonic matter, 26.6% CDM and 68.3% DE [4], where the latter is given by the famous cosmological constant Λ. The resulting, so-called ΛCDM model has shown great successes [5], which among others include an explanation for the accelerated late-time expansion of the universe, first measured via type Ia supernovae. Moreover, it offers a highly accurate description of the Cosmic Microwave Background (CMB) TT, TE and EE power spectra, measured for instance by WMAP and Planck, as well as the correct prediction of a peak in the correlation function of the galaxy distribution in the universe due to Baryonic Acoustic Oscillations (BAO), which has first been measured by SDSS. Due to its successes and simplicity – after all it uses only well known and established concepts plus an effective description of the dark sector – it is not suprising that the ΛCDM model has been the standard model of cosmology for decades.

Nevertheless, there are several aspects to it, which give reason to modify this model or to reconsider its validity: First, it goes without saying that the microscopic nature of DM and DE remain completely unspecified within the ΛCDM model. DM is typically described and defined as a non-relativistic, non-interacting fluid whereas the CC is merely given as a geometric object which is compatible with general covariance and thus remains as a free parameter of GR. Obviously, as an effective theory for the cosmological evolution of the universe, it is not the ambition of the ΛCDM model to provide such a particle-physics description. However, not knowing the microscopic nature of DM and DE is related to two famous, unsolved problems: the CC problem [6, 7] and the coincidence problem [8, 9]. The former is one of two fine-tuning problems that afflict our current understanding of the universe with the hierarchy problem of the Higgs sector being the other one. As is well known, the effective, measurable CC takes on a very small value in Planck units

$\Lambda_{\text{eff}} \sim 10^{-120}\, M_{\text{P}}^2$ where $M_{\text{P}} \equiv \sqrt{\hbar c / 8\pi G_{\text{N}}}$ is the reduced Planck mass. The effective CC is composed of a bare, geometrical contribution Λ_{bare}, which represents an additional term that is allowed by general covariance in GR, and the vacuum polarisation of all matter fields, which when evaluated at a cutoff of order Planck scale will contribute $\Lambda_{\text{UV}} \sim \mathcal{O}(1)\, M_{\text{P}}^2$. It is this discrepancy between the effective CC and the vacuum polarisation $\Lambda_{\text{eff}}/\Lambda_{\text{UV}} \sim 10^{-120}$, which requires a precise cancellation between Λ_{bare} and Λ_{UV} – two quantities that are a priori completely non-related to each other – up to the very small value of Λ_{eff}. This is a tremendous fine-tuning problem. The coincidence problem, on the other hand, describes the fact that the energy-densities of the two dark components, DM and DE, are both of the same order of magnitude. Since the energy densities of these two components evolve very differently, with CDM becoming diluted with time whereas Λ remains constant, it seems to be a remarkable coincidence that the two values resemble each other during the epoch of human life. Using the e-folds number $N = \ln a$ as a time scale, such a quantitative similarity of the DM and DE energy densities occurs only during a very narrow band, which raises the question why we exist inside this band and not, for instance, very far in the future. The ΛCDM model leaves this question unanswered. Anthropic reasons, i.e. that human life can only exist during this very special epoch of cosmic evolution, might be a possible explanation; however, it is also conceivable that DM and DE are coupled in a way that preserves some $\mathcal{O}(1)$ ratio of their energy densities [5].

Apart from these deeply theoretical issues, at which many models fail, there are also some very concrete problems with the ΛCDM model, which are related to increasing discrepancies between its theoretical predictions and the precision data provided by observations of modern experiments: the σ_8 tension [10, 11] and the famous Hubble tension [4, 12–14]. The latter is arguably one of the most discussed topics in current cosmology and concerns a discrepancy between model-independent, local measurements of the Hubble constant H_0 and measurements of CMB anisotropies, which assume the ΛCDM model to calculate H_0. This tension has now reached a value of about $5.0\,\sigma$ at $68\,\%$ CL [4, 13–16] and hints at new physics beyond the cosmological SM. Likewise, the σ_8 tension describes a deviation between Large Scale Structure (LSS) measurements, e.g. weak lensing or reshift surveys, and CMB measurements in the $\sigma_8 - \Omega_{\text{m}}$ parameter plane where σ_8 is the amplitude of the matter power spectrum and Ω_{m} is the amount of matter [17]. We will elaborate on these two tensions in the following sections after a very short introduction to GR as well as the background and linear perturbation dynamics of the ΛCDM model.

2.1.1. General relativity in a nutshell

Although older than a century, Einstein's theory of general relativity still remains our most elementary approach to understanding gravity. This is justified by its simplicity, its elegance and especially by its capability to describe gravitational phenomena at solar system scales with high accuracy. Since gravity is the only force relevant on cosmological scales and since GR is well tested and confirmed by experiments, it is not suprising that it represents the underlying theory of the ΛCDM model. Let us now briefly introduce the most basic concepts. GR is the unique, general-covariant theory of a massless spin-2 field. It is formulated on a four-dimensional, pseudo-Riemannian manifold on which the line element $\mathrm{d}s^2 \equiv g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu$ is preserved by general, differentiable coordinate transformations: $x^\mu \to x'^\mu(x^\nu)$. The so-called Einstein-Hilbert action of the theory reads [18, 19]

$$S_{\text{EH}} = \frac{M_{\text{P}}}{2} \int \mathrm{d}^4x\sqrt{-g}R, \tag{2.1}$$

where $g \equiv \det g_{\mu\nu}$ and R is the Ricci tensor. The action, and in particular the integral measure $\mathrm{d}^4x\sqrt{-g}$, is constructed in such a way that it is invariant under the same coordinate transformations as well. Furthermore, we can add a CC term and a matter sector without breaking this general covariance and thus arrive at the generic action of GR:

$$S_{\text{GR}} = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\text{P}}^2}{2}(R - 2\Lambda) + \mathcal{L}_{\text{m}}\right], \tag{2.2}$$

where $\mathcal{L}_{\text{m}}$ is the Lagrangian of matter minimally coupled to gravity. In principle, one could write down the whole Lagrangian of the SM of particle physics for $\mathcal{L}_{\text{m}}$; however, in the context of cosmology, we will almost always consider an effective description of matter as a perfect fluid. Varying the above action w.r.t. the metric, leads to the famous Einstein field equations:

$$\frac{\delta S_{\text{GR}}}{\delta g^{\mu\nu}} = 0, \tag{2.3}$$

$$\Leftrightarrow \quad R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = \frac{8\pi G_{\text{N}}}{c^4}T_{\mu\nu}, \tag{2.4}$$

where $R_{\mu\nu}$ is the Ricci tensor and

$$T_{\mu\nu} \equiv \frac{-2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L})}{\delta g^{\mu\nu}} \tag{2.5}$$

the energy-momentum tensor. Eq. (2.4) describes the interaction between matter and spacetime. Of further importance is the fact that the energy-momentum tensor is covariantly conserved:

$$\nabla_\mu T^{\mu\nu} = 0, \tag{2.6}$$

where ∇_μ is the covariant derivative. This is the GR analogue to energy and momentum conservation. Before we move to the introduction of the ΛCDM model, let us for the sake of completeness quote the geodesic equation, which governs the movement of a free particle inside a curved spacetime:

$$\frac{\mathrm{d}u^\mu}{\mathrm{d}\lambda} = -\Gamma^\mu_{\alpha\beta} u^\alpha u^\beta, \tag{2.7}$$

where λ is a quantity parametrising the trajectory of the particle, which in the case of a massive particle could be the proper time τ, $u^\mu \equiv \mathrm{d}x^\mu/\mathrm{d}\lambda$ is the derivative of the coordinate, i.e. the four-velocity, and $\Gamma^\mu_{\alpha\beta}$ is the Levi-Civita connection.

2.1.2. Background evolution in the ΛCDM model

One obvious application of GR – and the one we will be most interested in – is to describe the evolution of the universe (for an introduction to cosmology see for example [5, 20]). It is well established that the universe becomes continuously more isotropic and homogeneous when considering increasingly larger length scales. On scales as large as $l \gtrsim \mathcal{O}(100\,\mathrm{Mpc})$, this approximation becomes good enough to neglect any inhomogeneities and anisotropies, which is also known as the cosmological principle. This allows to consider a background evolution of the universe as a whole, whose validity is limited to these length scales. On shorter scales, the neglected inhomogeneities re-appear, which can be treated as linear perturbations as long as they are small. The requirement of homogeneity and isotropy leads to the so-called Friedmann-Lemaître-Robertson-Walker (FLRW) metric, whose line element can be written as

$$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + a^2(t)\left(\frac{\mathrm{d}r^2}{1-Kr^2} + r^2\mathrm{d}\Omega^2\right), \tag{2.8}$$

where $a(t)$ is the scale factor, K is the curvature parameter quantifying the spatial curvature of the universe and $\mathrm{d}\Omega^2 \equiv \mathrm{d}\theta^2 + \sin^2(\theta)\mathrm{d}\varphi^2$ is the infinitesimal solid-angle element. To satisfy the cosmological principle also in the matter sector while retaining simplicity, we treat the different matter species, labelled by i, each as a perfect fluid in the comoving frame, whose energy-momentum tensor reads

$$T^i_{\mu\nu} = \mathrm{diag}(\rho_i c^2, p_i, p_i, p_i), \tag{2.9}$$

where $\rho_i c^2$ is the respective fluid's energy density and p_i its relativistic pressure. Inserting Eqs. (2.8) and (2.9) into Eq. (2.4), one obtains the famous Friedmann equations:

$$H^2 = \frac{8\pi G_\mathrm{N}}{3}\sum_i \rho_i - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}, \tag{2.10}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_\mathrm{N}}{3}\left(\sum_i \left(\rho_i + \frac{3p_i}{c^2}\right) + \frac{\Lambda c^2}{3}\right), \tag{2.11}$$

where $H \equiv \dot{a}/a$ is the Hubble function. For i, we will typically use the labels r, m, b and c for radiation, matter, baryonic matter and cold dark matter, respectively. We can obtain another equation by plugging Eqs. (2.8) and (2.9) into Eq. (2.6):

$$\dot{\rho}_i + 3H\left(\rho_i + \frac{p_i}{c^2}\right) = 0, \tag{2.12}$$

which is called the continuity equation and describes the dilution of energy density due to the expansion of the universe. Note that only two of the three Eqs. (2.10), (2.11) and (2.12) are independent so that the acceleration equation (2.11) is typically omitted. Defining the equation-of-state parameter $w_i \equiv p_i/(\rho_i c^2)$, we can solve Eq. (2.12):

$$\rho_i = \rho_{i0} a^{-3(1+w)}, \tag{2.13}$$

where ρ_{i0} is an integration constant chosen to be the energy density today. Introducing the critical density $\rho_\mathrm{crit} \equiv 3H^2/(8\pi G_\mathrm{N})$ as well as the dimensionless density parameters $\Omega_i \equiv \rho_i/\rho_\mathrm{crit}$ and making the scaling of ρ_i explicit for matter and radiation via Eq. (2.13), $w_\mathrm{m} = w_\mathrm{b} = w_\mathrm{c} = 0$ and $w_\mathrm{r} = 1/3$, we can rewrite Eq. (2.10) into its most common form [5],

$$H^2 = H_0^2\left(\Omega_{\mathrm{m}0}a^{-3} + \Omega_{\mathrm{r}0}a^{-4} + \Omega_{K0}a^{-2} + \Omega_{\Lambda 0}\right), \tag{2.14}$$

where the subscript '0' denotes that a quantity is evaluated today, i.e. at $a = a_0 \equiv 1$, and we have defined $\Omega_{K0} \equiv -Kc^2/(H_0^2)$ and $\Omega_\Lambda \equiv \Lambda c^2/(3H^2)$. Knowing the free parameters H_0, $\Omega_{\mathrm{m}0} = \Omega_{\mathrm{b}0} + \Omega_{\mathrm{c}0}$, $\Omega_{\mathrm{r}0}$ and $\Omega_{\Lambda 0}$ then allows us to calculate the full background evolution of the universe. Notice that Ω_{K0} is redundant due to the relation $\Omega_{K0} = 1 - \Omega_{\mathrm{m}0} - \Omega_{\mathrm{r}0} - \Omega_{\Lambda 0}$. These parameters can all be measured by Planck and are given in Tab. 2.1.2.

The radiation density can be easily estimated using Planck's value for the redshift at radiation-matter equality $z_\mathrm{eq} = 3407 \pm 31$ [4], namely we have:

$$\Omega_{\mathrm{r}0} \approx \frac{\Omega_{\mathrm{m}0}}{1 + z_\mathrm{eq}} \approx 9.29 \times 10^{-5}. \tag{2.15}$$

The density parameter for spatial curvature turns out to be almost zero so that we

Parameter	Value at 68 % CL
$H_0\,[\mathrm{km\,s^{-1}\,Mpc^{-1}}]$	67.27 ± 0.60
Ω_{m0}	0.3166 ± 0.0084
$\Omega_{\mathrm{b0}} h^2$	0.02236 ± 0.00015
$\Omega_{\mathrm{c0}} h^2$	0.1202 ± 0.0014
$\Omega_{\Lambda 0}$	0.6834 ± 0.0084

Table 2.: Hubble constant and density parameters according to 2018 Planck data [4] using TT,TE,EE+lowE. Here, $h \equiv H_0/(100\,\mathrm{km\,s^{-1}\,Mpc^{-1}})$ is the dimensionless Hubble parameter.

can conclude to live in a nearly flat universe, even though there seems to be some recent indication that Planck data favour a K which is slighty negative thus leading to a positive curvature and a closed universe [21–23].

Quoting Planck results on the Hubble constant and density parameters, this is an appropriate moment to elaborate on the issue of the Hubble tension [5]. As previously mentioned, the Hubble tension describes the discrepancy between local measurements of H_0 and the value, which is inferred from CMB anisotropy measurements assuming the ΛCDM model. Let us briefly sketch how the latter works. The CMB, although being almost isotropic, possesses small temperature anisotropies of the order $\delta T/T \sim \mathcal{O}(10^{-5})$. Several effects come into play, e.g. the (non-integrated) Sachs-Wolfe effect, silk damping and acoustic oscillations. The former describes the gravitational redshift of photons due to potential wells at the surface of last scattering and is dominant at large angular scales. Silk damping occurs at small angular scales, in particular at length scales smaller than the mean free path of photons. Due to photon diffusion, any anisotropies are diluted leading to a suppression of the CMB power spectrum at small angular scales. Finally, acoustic oscillations are the consequence of two opposing forces: pressure and gravity. While overdense regions tend to attract more and more matter gravitationally, the increasing photon pressure therein due to the rising plasma temperature counteracts gravity and pushes the baryon-photon plasma outside. This results in acoustic waves traversing the primordial plasma with a sound-speed $c_{\mathrm{s}}^2 \equiv \delta p/\delta\rho$ that is of the same order of magnitude as the speed of light, $c_s \sim c$. The distance these waves have travelled until recombination defines the so-called *sound horizon*, which constitutes a resonant distance in the CMB. This is the case because said acoustic waves are generated in all overdense regions of the primordial plasma so that the overall CMB contains a statistical superposition of these waves, which have on average travelled a distance of the sound horizon. This allows us to extract the sound horizon from measurements of CMB anisotropies by using that acoustic waves enter the CMB power spectrum in the form of acoustic peaks. From the position of these peaks in the power spectrum, the observed angular scale of the sound horizon can be inferred at high precision, $\theta_* \approx 0.6^\circ \approx 10^{-2}$ rad [4].

From this, the Hubble parameter H_0 can be determined assuming that the underlying cosmological model is the ΛCDM model. Let us illustrate this by a (dramatically) simplified version of the analysis by Planck. To this end, we note that the observed angular scale is given by [4]

$$\theta_* = \frac{r_{*,\text{com}}}{d_{A,\text{com}}} \,, \tag{2.16}$$

where $r_{*,\text{com}}$ is the comoving sound horizon evaluated at recombination and $d_{A,\text{com}}$ the comoving angular diameter distance to the surface of last scattering. Assuming a specific model, which for this case is taken to be ΛCDM, both of the above quantities can be calculated,

$$r_{*,\text{com}} = \int_0^{t_\text{rec}} \frac{c_\text{s}\mathrm{d}t}{a} = \int_0^{a_\text{rec}} \frac{c_\text{s}\mathrm{d}a}{a^2 H} \,, \tag{2.17}$$

$$d_{A,\text{com}} = \int_{t_\text{rec}}^{t_0} \frac{c\mathrm{d}t}{a} = c\int_{a_\text{rec}}^{1} \frac{\mathrm{d}a}{a^2 H} \,, \tag{2.18}$$

where in the second line we assumed that the spatial curvature of the universe is zero, in which case the comoving angular diameter distance is simply given by the comoving distance.

To get an estimate for $r_{*,\text{com}}$, let us disregard the radiation-dominated period and assume that the universe is dominated by matter for the entire pre-recombination era. In this case, the Hubble function is given by

$$H^2 \approx \frac{\Omega_{\text{m}0}h^2}{a^3}\left(\frac{100\,\text{km}}{\text{s}\cdot\text{Mpc}}\right)^2 \,. \tag{2.19}$$

Since the quantity $\Omega_{\text{m}0}h^2$ is constrained by other features of the CMB spectrum, the integral in $r_{*,\text{com}}$ can be easily evaluated if we use that $a_\text{rec} \approx 1/1090$ and for the sake of simplicity assume a constant c_s. To evaluate the integral in the comoving angular diameter distance $d_{A,\text{com}}$, we assume a universe dominated by matter and a CC, so that

$$H^2 \approx H_0^2\left(\Omega_{\text{m}0}a^{-3} + (1-\Omega_{\text{m}0})\right) = \Omega_{\text{m}0}h^2\left(\frac{100\,\text{km}}{\text{s}\cdot\text{Mpc}}\right)^2 \times \left(a^{-3}-1\right) + H_0^2 \,. \tag{2.20}$$

Putting now everything together, the Hubble constant H_0 is the only unknown quantity in the Eq. (2.16) so that its value can be inferred. Within the much more involved analysis by Planck, this leads to the best-fit value cited in Tab. 2.1.2, $H_0 = (67.27 \pm 0.60)\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ [4]. This has to be compared to measurements from local observations using cosmic-distance-ladder methods, for instance by SH0ES [24] or H0LiCOW [25], which respectively find a best-fit value of $H_0 = (73.5 \pm 1.4)\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ and $H_0 = 73.3^{+1.7}_{-1.8}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. Note that the most recent measurement by SH0ES involves a best-fit value with even smaller confidence intervalls, $H_0 = (73.04 \pm 1.04)\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ [14], increasing the tension further. In general, while local, late-time measurements imply a larger value of the order $H_0 \approx (73-74)\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, early-time measurements which assume the ΛCDM model agree with a lower value of the order $H_0 \approx (67-68)\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. This discrepancy has by now reached a tension of $5.0\,\sigma$ at 68% CL [4, 13–16], which is the famous Hubble tension. A recent summary of H_0 measurements and possible solutions to the Hubble tension can be found in [15]. Since local measurements are independent of a cosmological model, whereas early-time measurements assume a model, an obvious solution to this tension, besides shedding possible systematic errors, is to modify the ΛCDM model. For instance, models which result in a smaller comoving sound horizon $r_{*,\mathrm{com}}$ at recombination imply a stronger expansion of the universe in order to explain the observed angular scale θ_s, which as a consequence leads to a larger inferred value for H_0. We will later consider a model called coupled dark energy [26], which can in principle lead to a decreased sound horizon [27], although our focus will not be on the Hubble tension but rather on structure formation.

2.1.3. Linear perturbations in the ΛCDM model

The assumption of a homogeneous universe pertains only when applied on large enough scales $l \gtrsim \mathcal{O}(100\,\mathrm{Mpc})$. On smaller scales, inhomogeneities arise, which constitute the basis of cosmological structure formation resulting in filaments and voids, galaxy clusters and of course galaxies themselves. The seeds of these inhomogeneities, which lead to the phenomenologically rich universe that we observe today, are assumed to originate ultimately from quantum fluctuations during the inflationary period, which will be explained in Sec. (4.3.1). At large enough scales, the violation of homogeneity is only mild; that is, the relative discrepancy of the density from mean density (i.e. the spatial average) will be small. This allows for an analysis in the linear regime, where higher orders of perturbations are neglected.

In this subsection, we will collect the main formulae of linear perturbation theory, referring to one of the many standard and introductory works in cosmology and perturbations [5, 20, 28, 29] for derivations and further reading.

The basic idea of perturbation theory is to split all dynamical variables into a homogeneous background contribution, which follows the background evolution equations as illustrated in the previous subsection, and a non-homogeneous perturbation, that generally depends on time and space and whose evolution equations will be described in this subsection. As mentioned above, on large enough scales the perturbated quantities will be small compared to their respective background values, which establishes the validity of linear perturbation theory.

In detail, we will always denote the background part of a quantity with a bar and its perturbation with a δ in front. Let us start with the metric tensor, which we write as

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu} \ . \tag{2.21}$$

The background metric $\bar{g}_{\mu\nu}$ may be any spatially homogeneous solution to the Einstein equations, as for example the flat Minkowski or de Sitter space; however, we will mostly be interested in an FLRW background, as given in Eq. (2.8), which we write as

$$\mathrm{d}\bar{s}^2 = \bar{g}_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = a^2(\tau)\left(-\mathrm{d}\tau^2 + \delta_{ij}\mathrm{d}x^i\mathrm{d}x^j\right) \ , \tag{2.22}$$

where we have assumed a spatially flat universe, $K = 0$, and introduced conformal time $\mathrm{d}\tau = \mathrm{d}t/a$. Using Helmholtz's theorem, the perturbed metric $\delta g_{\mu\nu}$ can be decomposed into four scalar, four vectorial and two tensorial degrees of freedom. Since the scalar, vector and tensor components decouple from each other at linear order and we are mainly interested in the former, we will disregard the latter two. Moreover, we will always work in the Newtonian gauge, where two of the four scalar degrees of freedom are eliminated. The resulting, perturbed metric reads [29]

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = a^2(\tau)\left[-(1+2\Psi)\mathrm{d}\tau^2 + (1-2\Phi)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j\right] \ , \tag{2.23}$$

where $\Psi, \Phi \ll 1$. Here Ψ corresponds to the Newtonian potential whereas Φ can be considered as a perturbation of spatial curvature [5]. In a similar manner, one decomposes the dynamical variables that describe the matter content of the universe into background and perturbed contributions,

$$\rho_i = \bar{\rho}_i(\tau) + \delta\rho_i(\tau, \vec{x}) \ , \tag{2.24}$$

$$p_i = \bar{p}_i(\tau) + \delta p_i(\tau, \vec{x}) \ , \tag{2.25}$$

$$u_i^\mu = \bar{u}_i^\mu(\tau) + \delta u_i^\mu(\tau, \vec{x}) \ , \tag{2.26}$$

where again the index i denotes the kind of species that is described by the respective fluid. Here u^μ is the fluid four-velocity, which could be transformed away in the background evolution described in the previous subsection but now needs to be included. The following steps include a calculation of the perturbed Einstein tensor and a perturbed energy-momentum tensor, which ultimately results in perturbed

Einstein equations [29]

$$\delta G^{\mu}_{\nu} = 8\pi G_{\mathrm{N}} \delta T^{\mu}_{\nu} \ . \tag{2.27}$$

We skip a detailed illustration of the above equations in real space and directly jump to the version in Fourier space, where all of the above perturbed quantities (or some alterations of these) have been expanded in Fourier modes,

$$X \sim \int \mathrm{d}^3 k \mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}} X_k \ . \tag{2.28}$$

Here X represents an arbitrary perturbed field, e.g. Ψ or Φ, while X_k is a corresponding Fourier mode. In what follows, we will drop the subscript 'k' and implicitly assume that all perturbed quantities represent a Fourier modes.

For a single fluid, the perturbed Einstein equations are then given by [29]

$$k^2\Phi + 3\mathcal{H}^2(\Phi' + \Psi) = 4\pi G_{\mathrm{N}} a^2 \rho\delta \ , \tag{2.29}$$

$$k^2\mathcal{H}(\Phi' + \Psi) = 4\pi G_{\mathrm{N}} a^2 (1+w)\rho\theta \ , \tag{2.30}$$

$$\Psi = \Phi \ , \tag{2.31}$$

$$\Phi'' + (2+\xi)\,\Phi' + \Psi' + (1+2\xi)\,\Psi = \frac{4\pi G_{\mathrm{N}} a^2 c_{\mathrm{s}}^2 \rho\delta}{\mathcal{H}^2} \ . \tag{2.32}$$

Here the following definitions have been made: $\mathcal{H} \equiv aH$ is the conformal Hubble function; the prime denotes the derivative w.r.t. the e-folds number $N \equiv \ln a$, which plays the role of a dimensionless time variable; $\xi \equiv \mathcal{H}'/\mathcal{H}$ describes the dimensionless change of the Hubble function; $\delta \equiv \delta\rho/\rho$ is the fluid's density contrast and $\theta \equiv \mathrm{i}\vec{k}\cdot\vec{v}$ its velocity divergence in Fourier space with $\vec{v}$ being the three-velocity of the fluid, i.e. the spatial component of u^μ. To arrive at the above equations, we have again assumed a perfect fluid, for which any heat fluxes, anisotropic stresses or the bulk viscosity vanish. Eq. (2.29) originates from the 00-component of the Einstein field equations and represents the Poisson equation in an expanding universe. Eq. (2.30) corresponds to the $0i$-components and is called the *velocity equation*. The so-called *anisotropic-stress equation* (2.31) stems from the off-diagonal, spatial ij-components with $i \neq j$ and implies that the two potentials Ψ and Φ are equal if and only if no anisotropic stresses are present. At last, the diagonal, spatial ii-components lead to the *pressure equation* given in Eq. (2.32), whose name arises because it describes the generation of gravitational effects due to a non-vanishing pressure perturbation $\delta p = c_{\mathrm{s}}^2 \rho\delta$.

The matter equations are obtained analogously to the background evolution from the conservation of the energy-momentum tensor, as given by Eq. (2.6), however, by using the perturbed version. The 0-component leads to the continuity equation [29],

$$\delta' + 3\left(c_{\mathrm{s}}^2 - w\right)\delta = -(1+w)\left(\frac{\theta}{\mathcal{H}} - 3\Phi'\right) \,, \tag{2.33}$$

and the spatial components to the Euler equation,

$$\theta' + \left[(1-3w) + \frac{w'}{1+w}\right]\theta = \frac{k^2}{\mathcal{H}}\left(\frac{c_{\mathrm{s}}^2}{1+w}\delta + \Psi\right) \,, \tag{2.34}$$

which describe the local conservation of energy and momentum, respectively.

Focusing now on perturbations of pressureless matter at sub-horizon scales, $k \gg \mathcal{H}$, one can combine the above equations to derive an evolution equation for the matter density contrast,

$$\delta_{\mathrm{m}}'' + (1+\xi)\delta_{\mathrm{m}}' - \frac{3}{2}\left(\Omega_{\mathrm{m}}\delta_{\mathrm{m}} + \sum_{i\neq \mathrm{m}}\Omega_i\delta_i\right) = 0 \,, \tag{2.35}$$

where the sum runs over all other species which are present, in particular radiation during the radiation-dominated era. Famously, the above equation gives rise to a solution of growing modes, $\delta_{\mathrm{m}} \propto a$, during matter domination ($\Omega_{\mathrm{m}} \approx 1$), whereas during radiation domination ($\Omega_{\mathrm{m}} \approx 0$, $\Omega_{\mathrm{r}} \approx 1$) the solution is logarithmic, implying that the growth of perturbations is almost frozen, $\delta_{\mathrm{m}}' \approx 0$. At late times, when radiation can be disregarded but the CC gives a non-negligible contribution to the overall energy density, Eq. (2.35) turns into [29]

$$\delta_{\mathrm{m}}'' + (1+\xi)\delta_{\mathrm{m}}' - \frac{3}{2}\Omega_{\mathrm{m}}\delta_{\mathrm{m}} = 0 \,, \tag{2.36}$$

where a Λ-dependent term is missing because the CC is not subject to perturbations. It is illuminating to re-write the matter-density parameter as

$$\Omega_{\mathrm{m}} = \frac{\rho_{\mathrm{m}}}{\rho_{\mathrm{crit}}} = \frac{8\pi G_{\mathrm{N}}\rho_m a^2}{3\mathcal{H}^2} \,. \tag{2.37}$$

We will later see that the inclusion of a scalar field can alter the factor in front of δ_{m} in Eq. (2.35). According to Eq. (2.37), this may be interpreted as an effective modification of the gravitational constant G_{N}.

Related to the evolution of matter perturbations is the aforementioned σ_8 tension, which we want to briefly illustrate in the following. Let us start by defining the parameter σ_8. We recall that the matter power spectrum is defined by the two-point correlation function of the density contrast in Fourier space,

$$\left\langle \delta(\vec{k})\delta(\vec{k}') \right\rangle \equiv (2\pi)^3 P(k)\delta_{\mathrm{D}}(\vec{k}-\vec{k}') \ , \tag{2.38}$$

and corresponds to the Fourier transform of the real-space autocorrelation function. σ_8 is then given by the variance of the density contrast up to scales of $8h^{-1}$ Mpc [29],

$$\sigma_8^2 = \frac{1}{2\pi^2}\int \mathrm{d}k k^2 P(k) W_8^2(k) \ , \tag{2.39}$$

where W_8 is a so-called *window function*, which mods out length scales larger than $8h^{-1}$ Mpc. The parameter σ_8 is an observable and also serves as a measure for the amplitude of the matter power spectrum $P(k)$. In general, a stronger clustering of matter implies an increased σ_8 parameter.

Analogously to the Hubble tension, the σ_8 tension describes a discrepancy between early-time measurements, for instance by Planck, and late-time observations like measurements of cosmic shear and galaxy clustering in the $\sigma_8 - \Omega_{\mathrm{m}0}$ parameter plane. A recent summary of the current experimental status and theoretical possibilities for a solution can be found in [17]. To quantify the tension within one parameter, one often defines $S_8 \equiv \sigma_8\sqrt{\Omega_{\mathrm{m}0}/0.3}$, which is especially useful for weak-lensing probes. While Planck finds a value $S_8 = 0.834 \pm 0.016$ assuming the ΛCDM model [4], large-scale structure probes imply a value of $S_8 = 0.766^{+0.020}_{-0.014}$ using a combination of KiDS-1000, BOSS, and 2dFLenS data [30], where both measurements use 68 % CLs. This corresponds to a tension of more than $3\,\sigma$, called the σ_8 or S_8 tension. As we can see, it is milder than the Hubble tension.

2.2. Scalar-tensor theories and coupled dark energy

In the previous section, we have explained the accordance model of cosmology, the ΛCDM model, despite being very successful suffers from several issues, most notably the H_0 and σ_8 tensions. In this section, we therefore describe a prominent way to modify the ΛCDM model by the inclusion of an additional, scalar degree of freedom. We will focus on some important aspects while referring to [29, 31] for further reading.

2.2.1. Overview of scalar-tensor theories

A modification of GR, which is the underlying gravitational theory of the ΛCDM model, is subject to the famous Lovelock's theorem [32, 33], which states that such a modification must involve either a change of the number of dimensions, the inclusion of higher-order derivatives, a breaking of the assumption of locality or the addition of further degrees of freedom. Scalar-tensor theories imply at least the latter criterion in the form of a new, dynamical scalar field. Arguably, the simplest and most prominent example of a scalar-tensor theory in the context of the late-time universe is quintessence [34, 35], which describes a canonical scalar field slowly rolling down a potential and thus resembling a CC.

While quintessence represents an interesting alternative to the ΛCDM model, it turns out that it is insufficient to solve the aforementioned tensions consistently without further ingredients (for a recent review on possible solutions of the Hubble tension see [36]). This motivates the interesting question for the nature of the most general, viable scalar-tensor theory. Here an important restriction arises from Ostrogradsky's theorem, according to which any theory with time derivatives of higher-than-second order in the Lagrangian implies a so-called *Ostrogradsky instability* due to a Hamiltonian that is unbounded from below, unless this Lagrangian is degenerate [37–40]. Here 'degeneracy of a Lagrangian' means that the Hessian matrix containing all second-order derivatives of this Lagrangian w.r.t. the highest-order time derivatives of all dynamical fields is not invertible. A prominent way to avoid Ostrogradsky instabilities is to consider scalar-tensor theories which belong to the class of Horndeski theories [41–44], constituting the most general scalar-tensor theory in four dimensions with second-order equations of motion. Even though there are more general ghostfree scalar-tensor theories, as for example beyond-Horndeski [45–47] or even more general DHOST theories [48, 49] (see also [50] for a review of DHOST theories), in this thesis we will stick to the framework of Horndeski theories and, in fact, only consider a specific example thereof.

The Lagrangian for a general Horndeski theory is given by [31]

$$\mathcal{L}_{\mathrm{H}} = \sum_{i=2}^{5} \mathcal{L}_i \;, \tag{2.40}$$

where the individual contributions read

$$\mathcal{L}_2 = G_2(\phi, X) \;, \tag{2.41}$$

$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi \;, \tag{2.42}$$

$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4,X}\left[(\Box\phi)^2 - \phi^{;\mu\nu}\phi_{;\mu\nu}\right] \;, \tag{2.43}$$

$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\phi^{\,\mu\nu} - \frac{G_{5,X}}{6}\left[(\Box\phi)^3 - 3\Box\phi\phi^{;\mu\nu}\phi_{;\mu\nu} + 2\phi^{;\mu}_{;\nu}\phi^{;\nu}_{;\lambda}\phi^{;\lambda}_{;\mu}\right] \;, \tag{2.44}$$

where $X \equiv -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi/2$ is kinetic term of scalar field ϕ. By choosing a specific form of the free functions $G_i(\phi, X)$, different theories can be constructed. For example, the ΛCDM model corresponds to the choice $G_2 = -\Lambda M_\mathrm{P}^2$, $G_4 = M_\mathrm{P}^2/2$ and $G_3 = G_5 = 0$, whereas quintessence is obtained by setting $G_2 = X - V(\phi)$, $G_4 = M_\mathrm{P}^2/2$ and $G_3 = G_5 = 0$, where V is the quintessence potential. To avoid instabilities, the functions G_i are subject to further restrictions, which in the case of quintessence for instance simply read

$$\frac{XG_{2,X}}{H^2} = \frac{X}{H^2} > 0 \ , \tag{2.45}$$

implying that the kinetic term $\propto \dot{\phi}^2$ enters with a positive sign. For a review of Horndeski theories, we refer to either [31] with a focus on theoretical aspects or to [51] with a focus on phenomenological ones, whereas we merely want to mention a few important aspects here.

First, in [52] it has been shown that the evolution of linear perturbations in a Horndeski theory is completely determined by four functions α_M, α_K, α_B and α_T of the free Horndeski functions G_i when accompanied with an arbitrary background evolution $H(t)$ as well as values for the constants Ω_{m0} and Ω_{K0}. The labels of the α-functions denote 'mass', 'kineticity', 'braiding' and 'tensor speed excess' as they can be interpreted to respectively quantify a time-dependent change of the effective Planck mass, direct kinetic energy of scalar perturbations, braiding or mixing between the scalar field and the metric, and a deviation of the speed of gravitational waves from the speed of light. In other words, two Horndeski theories with the same α_i, $H(t)$, Ω_{m0} and Ω_{K0} are degenerate in a sense and can never be distinguished by their background evolution and linear power spectra so that other probes are necessary for that purpose.

Second, modern-day gravitational-wave experiments like LIGO and Virgo have imposed tight constraints on a deviation of the gravitational-wave speed c_GW from the speed of light c [53],

$$-3 \times 10^{-15} \leq \frac{c_\mathrm{GW} - c}{c} \leq 7 \times 10^{-16} \ . \tag{2.46}$$

This forces $\alpha_T = c_\mathrm{GW}^2/c^2 - 1$ to be very small either, which on the other hand implies that a large subclass of Horndeski theories is excluded. In particular, the quartic and quintic terms are severely constrained by $G_{4,X} \approx 0$ and $G_5 \approx 0$ [54].

Third, one crucial property of Horndeski theories for our purposes is the fact that they are form-invariant under special disformal transformations [55]. That is, if one is given a Horndeski scalar-tensor theory A, described by a Lagrangian $\mathcal{L}_\mathrm{A}(\phi, g_{\mu\nu}) \supset \mathcal{L}_\mathrm{H}$, and applies a metric transformation

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = C(\phi) g_{\mu\nu} + D(\phi) \partial_\mu \phi \partial_\nu \phi \ , \tag{2.47}$$

where C and D are free functions, the resulting theory B will again be a Horndeski theory, $\mathcal{L}_{\mathrm{B}}(\phi, \tilde{g}_{\mu\nu}) \supset \mathcal{L}_{\mathrm{H}}$. In other words, the second-order nature of the equations of motions is preserved under a special disformal transformation (2.47).

We are now ready to discuss the scalar-tensor theory, which we will mostly be interested in in this thesis: coupled dark energy. Due to its specific way of construction, this theory will immediately turn out to be a Horndeski theory so that its stability is in principle guaranteed.

2.2.2. Coupled dark energy

An interesting approach to a modification of the cosmological concordance model, which also bears a rich accumulation of phenomenological features, makes use of a coupling within the dark sector of the universe [26, 56]. To be specific, we consider dark energy in the form of a scalar field, which possesses a non-vanishing coupling to the dark matter fluid. In principle, one could also couple the visible, baryonic sector to DE; however, this would likely result in fifth forces, which are ruled out by observations, unless some screening mechanism is at work. In this thesis, we will therefore only consider couplings to DM.

A major effect of such a coupling is a net flow of energy between the DM and DE sector. Hence, while the energy-momentum tensor for baryons follows the usual conservation equation,

$$\nabla^{\mu} T^{\mathrm{b}}_{\mu\nu} = 0 \ , \tag{2.48}$$

dark-sector is conserved only as a whole [57],

$$\nabla^{\mu} \left(T^{\phi}_{\mu\nu} + T^{\mathrm{c}}_{\mu\nu} \right) = 0 \ , \tag{2.49}$$

whereas the individual components obey the relation [29]

$$-\nabla^{\mu} T^{\phi}_{\mu\nu} = \nabla^{\mu} T^{\mathrm{c}}_{\mu\nu} = Q T^{\mathrm{c}} \partial_{\nu} \phi \ . \tag{2.50}$$

Here $T^{\phi}_{\mu\nu}$ is the DE energy-momentum tensor, Q is the so-called *coupling function* and $T^{\mathrm{c}} = g^{\mu\nu} T^{\mathrm{c}}_{\mu\nu}$ the trace of the DM energy-momentum tensor.

On the other hand, the Einstein equations are of the usual form [58]

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G_{\mathrm{N}}}{c^4} \left(T^{\mathrm{b}}_{\mu\nu} + T^{\mathrm{c}}_{\mu\nu} + T^{\phi}_{\mu\nu} \right) \ , \tag{2.51}$$

where the CC term from the ΛCDM model is missing but instead the DE energy-momentum tensor is present.

From the above (non-)conservation and Einstein field equations, one can now calculate the background evolution equations for our system in analogy to our discussion of the ΛCDM model in the previous section. While the Friedmann and baryon continuity equation are standard,

$$3M_{\rm P}^2\mathcal{H}^2 = a^2(\rho_{\rm b} + \rho_{\rm c} + \rho_\phi) \ , \tag{2.52}$$

$$\rho_{\rm b}' + 3\rho_{\rm b} = 0 \ , \tag{2.53}$$

the dark sector continuity equations exhibit the expected coupling behaviour

$$\rho_\phi' + 3(1 + w_\phi)\rho_\phi = Q\rho_{\rm c}\phi' \ , \tag{2.54}$$

$$\rho_{\rm c}' + 3\rho_{\rm c} = -Q\rho_{\rm c}\phi' \ . \tag{2.55}$$

Clearly, depending on the signs of Q and ϕ', energy will either flow from DE to DM or vice versa.

The latter behaviour can be used to alleviate the Hubble tension, which we briefly want to illustrate in the following. To this end, let us assume that Q is a positive constant and that the DE potential is chosen so that ϕ rolls from small to large field values, i.e. $\phi' > 0$. In this case, Eq. (2.55) can be easily solved by

$$\rho_{\rm c} = {\rm e}^{Q(\phi_0 - \phi)}\rho_{\rm c0}a^{-3} \ , \tag{2.56}$$

where ϕ_0 is the field value of ϕ today. This corresponds to the usual dilution of matter in an expanding universe but with an additional, exponential decay due to a positive net flow of energy from DM to DE. On the other hand, this implies that, in order to arrive at the measured value of today's DM density $\rho_{\rm c0}$, there must have been more DM at earlier times as compared to the ΛCDM model. Since the era of recombination and a great extent of the previous dynamics in the primordial plasma occurred during the period of matter domination, this results in an increased early-time Hubble value,

$$H^2 \approx \frac{8\pi G_{\rm N}}{3}\rho_{\rm c} \ . \tag{2.57}$$

According to Eq. (2.17), this will lead to a smaller comoving sound horizon $r_{*,\rm com}$ and as a consequence to a larger H_0 inferred from CMB measurements, as explained in Sec. 2.1.2. Thus the Hubble tension may be alleviated. In a recent quantitative analysis of CDE using a DE potential of Peebles-Ratra type, an increased best-fit value for the Hubble constant is found, which depending on the precise data set approximately reads $H_0 \approx 69\,\rm km\,s^{-1}\,Mpc^{-1}$ [27]. This mitigates the tension slightly, although it does not resolve it. Moreover, from an analysis of Bayesian evidence ratios, it is found that the ΛCDM model is preferred to this simple version of CDE. From this we can anticipate that CDE models possess in principle the capability to alleviate the Hubble tension but that further research and effort is required to do so.

Before we close this section, let us also have a look at an important feature of linear perturbation theory in CDE models. In particular, via a similar calculation as in the previous subsection and for typical CDE settings, in particular a constant coupling function Q, one arrives at the following evolution equation for the DM density contrast in the sub-horizon limit $k \gg \mathcal{H}$ [58],

$$\delta_{\mathrm{c}}'' + F\delta_{\mathrm{c}}' - \frac{3}{2}\Omega_{\mathrm{c}}\,(1+Y)\,\delta_{\mathrm{c}} = 0\;, \tag{2.58}$$

which should be compared to Eq. (2.36). Here $F = 1 + \xi - Q\phi'$ is the modified friction term, whereas Y is given by

$$Y = 2Q^2 M_{\mathrm{P}}^2 \frac{k^2}{k^2 + m_\phi^2}\;, \tag{2.59}$$

with m_ϕ being the mass of the DE field ϕ. Comparing this to Eq. (2.37), Y may be interpreted as a scale-dependent modification of the gravitational constant, which effectively becomes

$$G_{\text{eff}} = G_{\mathrm{N}}(1+Y)\;. \tag{2.60}$$

In real space, this leads to a so-called *Yukawa correction* of the gravitational potential, which becomes

$$V(r) = -\frac{G_{\mathrm{N}}m}{r}\left(1 + 2Q^2 M_{\mathrm{P}}^2 \mathrm{e}^{-m_\phi r}\right)\;, \tag{2.61}$$

where m is the mass of some gravitating object, for instance a galaxy cluster, and r the distance of some observer to it. Since Y as given in Eq. (2.59) is strictly positive, this will generally imply an increase of the gravitational constant on small length scales [59]. One may hence expect a stronger clustering of cosmological structures, thus worsening the σ_8 tension. This may be avoided in a regime of phantom dark energy with $w_\phi < -1$, which for instance can be achieved via non-standard kinetic terms; however, the latter results in instabilities in the model [60].

3. Transient weak gravity in coupled dark energy

This chapter represents one of the main works in this thesis. The entire content is based on [1] unless stated otherwise.

3.1. Motivation and outline

In the previous section, we have described the scalar-tensor theory coupled dark energy, whose characteristic property is a coupling of the DE field ϕ to DM. While this theory appears to be promising at alleviating the Hubble tension [27], it typically comes with an increased effective gravitational coupling constant as a generic feature due to the positive Yukawa correction Y, as given in Eqs. (2.58) and (2.59). This fifth-force enhanced, attractive gravity is expected to induce a stronger clustering of DM (which of course implies a stronger clustering of baryonic matter, which tends to follow the gravitational pull of DM), thus leading to an increased σ_8 and an exacerbated σ_8 tension.

In this work, we try to tackle this issue from a model-building perspective by investigating non-standard forms of one of the free functions in the model. In detail, we consider a standard kinetic and mass term for the DE scalar field ϕ and a DM sector that is conformally coupled to ϕ via a DM metric with conformal factor $C(\phi)$. In previous constructions, C has been chosen to be an exponential function of ϕ, in which case the coupling function Q becomes constant and we arrive at the scenario described in Sec. 2.2.2 leading to a strengthened gravitational coupling. In our approach, we consider more general functional forms for $C(\phi)$, which allow for a non-constant Q. We find that, in the regime where ϕ is close to a minimum of C, the derivative of the coupling function $Q_{,\phi}$ constitutes an additional mass scale besides the scalar field mass. If this new mass scale is non-negligble compared to k^2, this implies some interesting consequences. First, the friction term F, as indicated in Eq. (2.58), obtains a dependency on the scale k in the quasi-static approximation (QSA). The latter indicates the limit of considering length scales which are small compared to the Hubble horizon and the scalar-field sound horizon, i.e. $k \gg \mathcal{H}$ and $k \gg \mathcal{H}/c_s^{(\phi)}$ [51, 61]. In our scenario, the latter two conditions will be approximately equivalent since the scalar-field sound speed $c_s^{(\phi)}$ is approximately unity, so that the QSA simply corresponds to the sub-horizon limit. As an additional, arguably more interesting consequence of the new mass scale we find a Yukawa correction Y to the gravitational coupling which may become negative. Even though this negativity is only given for a transient period of time, this weakening of gravity may prove to be useful for alleviating the σ_8 tension [62–65], although we do not explore this possibility any further in this work. In general, the duration and strength of the weak gravity regime will depend on the specific model.

For the sake of simplicity and concreteness we will also illustrate our findings for a specific choice of the conformal function $C(\phi) = \exp(\phi^2/m_C^2)$, where m_C is a characteristic mass scale that sets the size of the derivatives of C.

3.2. Specification of the model

Let us start by defining the model explicitly, which will lead to the described coupling within the dark sector as described in Sec. 2.2.2. Following [66], we consider an action

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_\mathrm{P}^2}{2}R + \mathcal{L}_\phi(g_{\mu\nu},\phi) + \mathcal{L}_\mathrm{SM}(g_{\mu\nu},\psi_\mathrm{SM})\right] + \int \mathrm{d}^4x\sqrt{-\tilde{g}}\tilde{\mathcal{L}}_\mathrm{c}(\tilde{g}_{\mu\nu},\psi_\mathrm{c}) \; . \tag{3.1}$$

We have a gravitational sector that is given by the usual Einstein-Hilbert action and a DE sector represented by a scalar field ϕ whose dynamics are given by

$$\mathcal{L}_\phi = -\frac{1}{2}g_{\mu\nu}\partial^\mu\phi\partial^\nu\phi - V(\phi) \; , \tag{3.2}$$

where $V(\phi)$ is some potential. The SM sector is indicated by $\mathcal{L}_\mathrm{SM}$, where ψ_SM symbolically represents SM fields, in particular a baryonic-matter fluid or radiation. Both $\mathcal{L}_\phi$ and $\mathcal{L}_\mathrm{SM}$ are minimally coupled to the Einstein-frame metric $g_{\mu\nu}$. On the other hand, DM fields ψ_c are minimally coupled to an effective geometry described by the metric $\tilde{g}_{\mu\nu}$, which is given by a special disformal transformation of $g_{\mu\nu}$, as given in Eq. (2.47).[1] Since baryons are decoupled from the scalar field, they are not subject to fifth forces and experience a standard gravity so that no screening mechanism is needed in order to respect local constraints of gravity. Moreover, observational bounds on the speed of gravitational waves, which we discussed in Sec. 2.2.1, are not expected to be an issue [53].

[1] Note that we will usually drop the explicit dependence of C, D and V on ϕ.

The energy-momentum tensors of the individual sectors are defined by

$$T^{\phi}_{\mu\nu} \equiv \phi_{,\mu}\phi_{,\nu} - g_{\mu\nu}\left(\frac{1}{2}g^{\rho\sigma}\phi_{,\rho}\phi_{,\sigma} + V\right) \,, \tag{3.3}$$

$$T^{\mathrm{SM}}_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_{\mathrm{SM}})}{\delta g^{\mu\nu}} \,, \tag{3.4}$$

$$T^{\mathrm{c}}_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-\tilde{g}}\tilde{\mathcal{L}}_{\mathrm{c}})}{\delta g^{\mu\nu}} \,. \tag{3.5}$$

As usual, the equations of motion for this model are then constituted of the Einstein field equations, as given in Eq. (2.51), and the (non-)conservation equations of the energy-momentum tensors, in accordance to Eqs. (2.48) – (2.50). Here the coupling function is given by [57]

$$QT^{\mathrm{c}} = \frac{C_{,\phi}}{2C}T^{\mathrm{c}} + \frac{D_{,\phi}}{2C}T^{\mathrm{c}}_{\mu\nu}\nabla^{\mu}\phi\nabla^{\nu}\phi - \nabla^{\mu}\left(\frac{D}{C}T^{\mathrm{c}}_{\mu\nu}\nabla^{\nu}\phi\right) \,, \tag{3.6}$$

where the subscript $,\phi$ denotes the derivative w.r.t. ϕ. Additionally, the scalar field obeys the modified Klein-Gordon equation

$$\Box\phi = V_{,\phi} - QT^{\mathrm{c}} \,. \tag{3.7}$$

For the rest of this work, we will focus on the evolution of ϕ and matter. Here we will generally neglect baryons and radiation, assuming that they contribute only marginally to the overall energy density, and only consider DM.

3.2.1. Background evolution equations

To obtain the relevant background evolution equations, we will again assume a spatially flat FLRW metric as in Eq. (2.22). Then, without baryonic matter and radiation, the Friedmann equation is simply given by

$$3M_{\mathrm{P}}^2\mathcal{H}^2 = (\rho_{\mathrm{c}} + \rho_{\phi})a^2 \,, \quad \Omega_{\mathrm{c}} + \Omega_{\phi} = 1 \,. \tag{3.8}$$

Remember that the density parameters Ω_i, as defined in Sec. 2.1.2, are

$$\Omega_i = \frac{\rho_i}{\rho_{\mathrm{crit}}} = \frac{\rho_i a^2}{3\mathcal{H}^2 M_{\mathrm{P}}^2} \,. \tag{3.9}$$

For pressureless DM, we have $T^{\mathrm{c}} = \rho_{\mathrm{c}}$ and the coupling function becomes [66]

$$Q \equiv -\frac{a^2 C_{,\phi} - 2D(3\mathcal{H}^2\phi' + a^2 V_{,\phi} + C_{,\phi}\mathcal{H}^2\phi'^2/C) + D_{,\phi}\mathcal{H}^2\phi'^2}{2[a^2 C + D(a^2\rho_{\mathrm{c}} - \mathcal{H}^2\phi'^2)]} = -\frac{B}{2A} , \tag{3.10}$$

where A and B are defined as

$$A \equiv a^2 C + D(a^2\rho_{\mathrm{c}} - \mathcal{H}^2\phi'^2) , \tag{3.11}$$
$$B \equiv a^2 C_{,\phi} - 2D(3\mathcal{H}^2\phi' + a^2 V_{,\phi} + C_{,\phi}\mathcal{H}^2\phi'^2/C) + D_{,\phi}\mathcal{H}^2\phi'^2 . \tag{3.12}$$

While for DM the background evolution is governed by Eq. (2.55), the scalar field follows the dynamics described by the background Klein-Gordon equation

$$\phi'' + (2+\xi)\phi' + \frac{a^2 V_{,\phi}}{\mathcal{H}^2} = \frac{Q\rho_{\mathrm{c}}a^2}{\mathcal{H}^2} = 3QM_{\mathrm{P}}^2\Omega_{\mathrm{c}} . \tag{3.13}$$

At last, let us also note that the scalar-field energy density and pressure are given by the usual expressions

$$\rho_\phi = \frac{\phi'^2\mathcal{H}^2}{2a^2} + V(\phi) , \qquad p_\phi = \frac{\phi'^2\mathcal{H}^2}{2a^2} - V(\phi) , \tag{3.14}$$

which generally induces a time-dependent equation-of-state parameter $\omega_\phi = p_\phi/\rho_\phi$.

3.2.2. Evolution equations of linear perturbation

The perturbation equations are obtained in analogy to the ΛCDM case described in Sec. 2.1.2. Again we consider only scalar perturbations and work in the Newtonian gauge, where the metric is given by Eq. (2.23). While the anisotropic-stress equation stays the same as in Eq. (2.31), the relativistic Poisson and pressure equations are respectively given by

$$\frac{k^2}{\mathcal{H}^2}\Phi + 3(\Phi' + \Phi) = -\frac{3}{2}\Omega_{\mathrm{c}}\delta_{\mathrm{c}} - \frac{1}{2M_{\mathrm{P}}^2}\left(\phi'\delta\phi' - \Phi\phi'^2 + \frac{a^2 V_{,\phi}\delta\phi}{\mathcal{H}^2}\right) , \tag{3.15}$$
$$\Phi'' + (3+\xi)\,\Phi' + (1+2\xi)\,\Phi = \frac{1}{2M_{\mathrm{P}}^2}\left(\phi'\delta\phi' - \Phi\phi'^2 - \frac{a^2 V_{,\phi}\delta\phi}{\mathcal{H}^2}\right) . \tag{3.16}$$

Comparing the above two equations to Eqs. (2.29) and (2.32), we notice that they basically differ by a respective, additional contribution on the right-hand side due to the scalar field ϕ. Note that the pressure equation (3.16) does not contain a DM source term since the latter is pressureless. The perturbed continuity and Euler equation for DM are given by [66]

$$\delta_c' = -\frac{\theta_c}{\mathcal{H}} + 3\Phi' + Q\phi'\delta_c - \phi'\delta Q - Q\delta\phi' \;, \tag{3.17}$$

$$\theta_c' = -\theta_c + \frac{k^2\Phi}{\mathcal{H}} + Q\phi'\theta_c - \frac{Q}{\mathcal{H}}k^2\delta\phi \;. \tag{3.18}$$

Finally, perturbations of the scalar field are governed by the perturbed Klein-Gordon equation,

$$\delta\phi'' + (2+\xi)\,\delta\phi' + \frac{k^2 + a^2 V_{,\phi\phi}}{\mathcal{H}^2}\delta\phi = 4\phi'\Phi' - \frac{2a^2(V_{,\phi} - \rho_c Q)\Phi}{\mathcal{H}^2} + \frac{a^2\rho_c\delta Q}{\mathcal{H}^2} \;, \tag{3.19}$$

with the perturbed coupling function

$$\delta Q = -\frac{1}{A}\left(\mathcal{B}_1\delta_c + \mathcal{B}_2\Phi' + \mathcal{B}_3\Phi + \mathcal{B}_4\delta\phi' + \mathcal{B}_5\delta\phi\right) \;. \tag{3.20}$$

Here the coefficients $\mathcal{B}_i$ are given by [66]

$$\mathcal{B}_1 \equiv \frac{B}{2} + a^2 D Q\rho_c \;, \tag{3.21}$$

$$\mathcal{B}_2 \equiv 3\mathcal{H}^2 D\phi' \;, \tag{3.22}$$

$$\mathcal{B}_3 \equiv 6\mathcal{H}^2 D\phi' + 2\mathcal{H}^2 D\phi'^2\left(\frac{C_{,\phi}}{C} - \frac{D_{,\phi}}{2D} + Q\right) \;, \tag{3.23}$$

$$\mathcal{B}_4 \equiv -3\mathcal{H}^2 D - 2\mathcal{H}^2 D\phi'\left(\frac{C_{,\phi}}{C} - \frac{D_{,\phi}}{2D} + Q\right) \;, \tag{3.24}$$

$$\begin{aligned}\mathcal{B}_5 \equiv\; & \frac{a^2 C_{,\phi\phi}}{2} - D(k^2 + a^2 V_{,\phi\phi}) - a^2 D_{,\phi}V_{,\phi} - 3\mathcal{H}^2 D_{,\phi}\phi' \\ & - \mathcal{H}^2 D\phi'^2\left[\frac{C_{,\phi\phi}}{C} - \left(\frac{C_{,\phi}}{C}\right)^2 + \frac{C_{,\phi}D_{,\phi}}{CD} - \frac{D_{,\phi\phi}}{2D}\right] \\ & + (a^2 C_{,\phi} + a^2 D_{,\phi}\rho_c - \mathcal{H}^2 D_{,\phi}\phi'^2)Q \;. \end{aligned} \tag{3.25}$$

Before we derive an evolution equation for δ_c in analogy to Eq. (2.58), in the following section we will impose several approximations in order to simplify the system.

3.3. Approximations

It turns out that the following three assumptions suffice to derive a second-order differential equation in which δ_c enters as the only perturbed quantity besides other background quantities.

- **Approximation 1:** $k/\mathcal{H} \gg 1$. This accounts for the fact that we take into account only sub-horizon scales. As in our case the scalar-field sound speed is equal to unity, this also corresponds to the sub-sound-horizon limit and hence the quasi-static approximation.

- **Approximation 2:** We demand that $\Phi'' \sim \Phi' \sim \Phi$ and $\Psi \sim \Psi'$. Due to the anisotropic-stress equation, $\Psi = \Phi$, this implies that all these scales are of the same order. Moreover, we consider $\delta\phi'' \sim \delta\phi' \sim \delta\phi$. These requirements forbid a fast growth of the gravitational potentials or scalar-perturbations, thus avoiding instabilities.

- **Approximation 3:** For the background quantities, we require that $\phi'' \sim \phi'$ and $|Q|M_{\rm P} = \mathcal{O}(1)$. Here, in the latter relation it is understood that also small values of Q are included; in particular, it may also vanish.

Let us now use these approximations to simplify the dynamical system described by the equations of the previous section.

We start by noting that Eq. (3.8) restricts the DE density parameter (just like the DM density parameter) to be $\Omega_\phi \leq 1$. From Eqs. (3.9) and (3.14), we find the corresponding expression for Ω_ϕ and, as a result, the constraints

$$\frac{\phi'^2}{M_{\rm P}^2} \leq 1 \,, \quad \frac{a^2 V}{\mathcal{H}^2 M_{\rm P}^2} \leq 1 \,. \tag{3.26}$$

Due to Approximation 1, both of the above expressions are much smaller than $k/\mathcal{H}$. Moreover, using the relation

$$\xi = -\frac{1}{2} - \frac{3}{2} w_\phi \Omega_\phi \tag{3.27}$$

and that $|w_\phi| = \mathcal{O}(1)$, we infer $|\xi| = \mathcal{O}(1)$.

Applying now Approximation 2 and 3 on Eq. (3.13), one has

$$\frac{a^2 V_{,\phi}}{M_{\rm P} \mathcal{H}^2} = \mathcal{O}(1) \,, \tag{3.28}$$

and, using all three approximations on Eqs. (3.16) and (3.15), we find

$$\Phi \sim \delta\phi / M_{\rm P} \,, \qquad \frac{k^2}{\mathcal{H}^2} \Phi = -\frac{3}{2} \Omega_{\rm m} \delta_{\rm m} \,. \tag{3.29}$$

At last, we can use the above approximations and relations that we derived from them to simplify the perturbed Klein-Gordon equation (3.19) as

$$\left(k^2 + M^2\right)\delta\phi = a^2\rho_{\mathrm{c}} Q\delta_{\mathrm{c}} - \frac{a^2\rho_{\mathrm{c}}}{A}\left(a^2 DQ\rho_{\mathrm{c}}\delta_{\mathrm{c}} + \mathcal{B}_3\Phi + \mathcal{B}_4\delta\phi' + \mathcal{B}_5\delta\phi\right) \ , \tag{3.30}$$

where we have defined $M^2 \equiv a^2 V_{,\phi\phi}$.

In the following section, we will focus on a pure conformal coupling, which will simplify the dynamical system further.[2] As it turns out, together with the above approximations this still leads to the interesting behaviour of weak gravity. Note that our claim is only that the three approximations are sufficient to achieve a transient weak-gravity regime when paired with an appropriate conformal function C, whereas we do not assess the question whether they are all necessary. In particular, we do not investigate the question, whether these approximations may possess some dependencies but instead assume them to be independent.

3.4. The pure conformal coupling case and a new mass scale

Considering a pure conformal coupling amounts to setting $D = 0$ in Eq. (2.47). According to Eq. (3.10), this implies a rather simple expression for the coupling function

$$Q = -\frac{C_{,\phi}}{2C} \ . \tag{3.31}$$

Moreover, from Eqs. (3.21) – (3.25) we infer that the coefficients $\mathcal{B}_i$ simplify dramatically, especially that $\mathcal{B}_2 = \mathcal{B}_3 = \mathcal{B}_4 = 0$. With this, Eq. (3.30) can be written as

$$\left(k^2 + M^2 - a^2\rho_{\mathrm{c}} Q_{,\phi}\right)\delta\phi = a^2 Q\rho_{\mathrm{c}}\delta_{\mathrm{c}} \ . \tag{3.32}$$

Therefrom, we identify a new mass scale besides the scalar-potential mass M^2, which is given by

$$\mathcal{M}^2 \equiv -a^2\rho_{\mathrm{c}} Q_{,\phi} = -3\mathcal{H}^2 M_{\mathrm{P}}^2\Omega_{\mathrm{c}} Q_{,\phi} = 3\mathcal{H}^2 M_{\mathrm{P}}^2\Omega_{\mathrm{c}}\left(\frac{C_{,\phi\phi}}{2C} - \frac{C_{,\phi}^2}{2C^2}\right) \ . \tag{3.33}$$

Note that the size of this new mass-scale is essentially set by the derivatives of the conformal function C up to second order. Due to the factor $\mathcal{H}^2$ in Eq. (3.33), $\mathcal{M}^2$ will generally be negligible compared to k^2 unless $|Q_{,\phi} M_{\mathrm{P}}^2|$ is large enough to

[2] For the inclusion of a disformal coupling, we refer to the appendix of [1], where it was found that the emergence of a new mass scale $\mathcal{M}^2$ requires a subdominant disformal contribution compared to the conformal one. In this sense, we can therefore consider the conformal case as sufficiently general.

compensate for the suppression of order $\mathcal{H}^2/k^2$. Naively, such a large $Q_{,\phi}$ seems unnatural because Approximation 3 requires $QM_{\rm P} = \mathcal{O}(1)$, implying that $C_{,\phi}/C$ cannot be too large. Hence for a non-negligible $\mathcal{M}^2$, the first derivative of C needs to be small whereas the second derivative must be large. This condition can be satisfied if ϕ is at a locus in the vicinity of an extremum of C.

Before we consider a concrete example for C, let us comment on some aspects of the new scale $\mathcal{M}^2$. First, we note that, depending on the functional form of C and V, $Q_{,\phi}$ and hence $\mathcal{M}^2$ will be either positive or negative. The latter may even lead to a cancellation of M^2 and result in interesting phenomenological features; however, we will only regard the case when $\mathcal{M}^2$ is positive in this work. Second, we mention that the case $\mathcal{M}^2$ is equivalent to a constant coupling function Q, which in turn is equivalent to an exponential conformal factor $C(\phi) = \exp(\phi/m_C)$. That is, in this standard case, which as been extensively studied in the literature [26, 35, 56, 66, 67], no new mass scale arises. Third, as argued above, a proper condition for a non-negligible new mass scale $\mathcal{M}^2$ is a locus in field space close to an extremum of C. Hence, we should check under which circumstances this condition is fulfilled. An obvious possibility is a dynamical system, which naturally leads ϕ towards a minimum of C. In detail, we write the background Klein-Gordon equation (3.13) as

$$\phi'' + (2+\xi)\phi' + \frac{a^2}{\mathcal{H}^2} V_{{\rm eff},\phi} = 0 \, , \tag{3.34}$$

where we defined the effective potential as

$$V_{\rm eff} \equiv V - \rho_{\rm c} \int d\phi Q(\phi) = V + \rho_{\rm c} \log \sqrt{C} \, . \tag{3.35}$$

Clearly, the above equation drives ϕ to the minimum of $V_{\rm eff}$. If the latter coincides with a minimum of C, the condition for large $\mathcal{M}^2$ might be fulfilled. However, one must take into account that, depending on the specific form of C and V, the condition of being close to the minimum of C may only last for a very short time so that the effect of the new mass scale $\mathcal{M}^2$ on phenomenology is only marginal. One may also think about the possibility to achieve a non-negligible mass scale $\mathcal{M}^2$ by an extremely flat potential $V_{\rm eff}$, where ϕ' is so small that the resulting slow-roll effectively keeps ϕ close to the minimum of C for a significant period of time. However, as we will see later, in the limit of slow-roll we return to the standard scenario, described by Eq. (2.59). In general, in this work we will not bother with establishing a realistic cosmological background evolution but instead focus on the consequences of the new mass scale $\mathcal{M}^2$ on linear perturbations, while assuming that it is non-negligible.

To illustrate the above discussion, let us choose a concrete functional form for the conformal factor,

$$C(\phi) = \mathrm{e}^{\phi^2/m_C^2} \ , \tag{3.36}$$

whose minimum lies at $\phi = 0$ and where m_C provides a typical mass scale for the derivatives of C. According to the Eqs. (3.31) and (3.33), with the above C we obtain

$$Q = -\frac{\phi}{m_C^2} \ , \quad Q_{,\phi} = -\frac{1}{m_C^2} \ , \quad \mathcal{M}^2 = 3\Omega_\mathrm{c}\frac{\mathcal{H}^2 M_\mathrm{P}^2}{m_C^2} \ . \tag{3.37}$$

Note that in this scenario one has $Q_{,\phi} < 0$ and hence $\mathcal{M}^2 > 0$, as demanded above. Now, respecting the requirements $|Q|M_\mathrm{P} = \mathcal{O}(1)$ and $|Q_{,\phi}|M_\mathrm{P}^2 \gg 1$ implies that

$$\frac{|\phi|M_\mathrm{P}}{m_C^2} = \mathcal{O}(1) \ , \quad \frac{M_\mathrm{P}^2}{m_C^2} \gg \frac{1}{3\Omega_\mathrm{c}} \ . \tag{3.38}$$

The above two relations can easily be fulfilled close to the minimum of C, where $\phi \approx 0$. Since $\Omega \leq 1$, the second relation in Eq. (3.38) then implies

$$\mathcal{M}^2 \gg \mathcal{H}^2 \ , \tag{3.39}$$

so that $\mathcal{M}^2$ is not negligible in the quasi-static approximation.

Finally, let us impose an explicit scalar-field potential of simple, quadratic form $a^2V = M^2\phi^2/2$. The resulting effective potential, as given in Eq. (3.35), becomes $a^2V_\mathrm{eff} = \frac{1}{2}(M^2 + \mathcal{M}^2)\phi^2$, which indeed drives ϕ to its minimum at $\phi = 0$. We mention again that V_eff does not entail a realistic background evolution; in particular, it lacks a late-time accelerated expansion. We will nevertheless employ it to illustrate the mechanism leading to the regime of transient weak gravity.

3.4.1. The evolution equation for the DM density contrast

In this section, we derive the perturbation equations for the above scenario. Particularly, we are interested in the evolution equation for δ_c, in analogy to Eq. (2.58), which will show new, interesting behaviour of the friction term F and gravitational coupling Y. Of special interest will be their respective scaling with k, in particular a novel quartic k-dependence in Y, which ultimately can lead to a regime of transient weak gravity.

We start by casting the simplified perturbed Klein-Gordon equation (3.32) into the form

$$\delta\phi = \frac{a^2 Q\rho_\mathrm{c}}{k^2 + M^2 + \mathcal{M}^2}\delta_\mathrm{c} \ . \tag{3.40}$$

Moreover, due to the above approximations and after some simplifications, the perturbed continuity equation (3.17) becomes

$$\delta_{\mathrm{c}}' = -\frac{\theta_{\mathrm{c}}}{\mathcal{H}} - Q'\delta\phi \,. \tag{3.41}$$

Here the latter term is in general non-negligible because $Q_{,\phi}$ is assumed to be large. Hence, depending on the size of ϕ', Q' may be large as well. For instance, for the above, exemplary model given by Eq. (3.36), one has

$$|Q'|M_{\mathrm{P}} = \frac{|\phi'|M_{\mathrm{P}}}{m_C^2} \,. \tag{3.42}$$

Even though $|\phi|/M_{\mathrm{P}}$ must be small because we are close to the minimum at $\phi = 0$, its derivative may be large, $|\phi'|/M_{\mathrm{P}} = \mathcal{O}(1)$, so that $|Q'|$ can indeed be large as well.

Combining now Eq. (3.40) with Eq. (3.41) and the Euler equation (3.18), yields

$$\delta_{\mathrm{c}}' = -\frac{\theta_{\mathrm{c}}}{\mathcal{H}} + \frac{\mathcal{M}^2}{k^2 + M^2 + \mathcal{M}^2} Q\phi'\delta_{\mathrm{c}} \,, \tag{3.43}$$

$$\frac{\theta_{\mathrm{c}}'}{\mathcal{H}} = -\frac{\theta_{\mathrm{c}}}{\mathcal{H}} - \frac{3}{2}\Omega_{\mathrm{c}}\delta_{\mathrm{c}} + Q\phi'\frac{\theta_{\mathrm{c}}}{\mathcal{H}} - 3M_{\mathrm{P}}^2 Q^2 \Omega_{\mathrm{m}} \frac{k^2}{k^2 + M^2 + \mathcal{M}^2}\delta_{\mathrm{c}} \,. \tag{3.44}$$

We can than differentiate Eq. (3.43) w.r.t. our time-coordinate $N = \ln a$ and use Eq. (3.44) to eliminate θ_{c}', whereupon θ_{c} can be eliminated using Eq. (3.43) itself. This results in the evolution equation for the DM density contrast δ_{c},

$$\begin{aligned} 0 = \delta_{\mathrm{c}}'' &+ \left(1 + \xi - Q\phi' - Q\phi'\frac{\mathcal{M}^2}{k^2 + M^2 + \mathcal{M}^2}\right)\delta_{\mathrm{c}}' \\ &- \frac{3}{2}\Omega_{\mathrm{c}}\left[1 + \frac{2M_{\mathrm{P}}^2 Q^2 k^2 + \frac{(1+\xi - Q\phi')Q\phi' + (Q\phi')'}{3\Omega_{\mathrm{c}}/2}\mathcal{M}^2}{k^2 + M^2 + \mathcal{M}^2} + \left(\frac{\mathcal{M}^2}{k^2 + M^2 + \mathcal{M}^2}\right)' \frac{Q\phi'}{3\Omega_{\mathrm{c}}/2}\right]\delta_{\mathrm{c}} \,. \end{aligned} \tag{3.45}$$

Note that the above equation does not assume a specific functional form of C or V but results from the above, three approximations and $D = 0$. Comparing this to Eq. (2.58), we infer

$$F = 1 + \xi - Q\phi' - Q\phi'\frac{\mathcal{M}^2}{k^2 + M^2 + \mathcal{M}^2} \,, \tag{3.46}$$

$$Y = 2M_{\mathrm{P}}^2 Q^2 \frac{k^2 + g\mathcal{M}^2}{k^2 + M^2 + \mathcal{M}^2} + \left(\frac{\mathcal{M}^2}{k^2 + M^2 + \mathcal{M}^2}\right)' \frac{2Q\phi'}{3\Omega_{\mathrm{c}}} \,, \tag{3.47}$$

where

$$g \equiv \frac{(1+\xi - Q\phi')Q\phi' + (Q\phi')'}{3\Omega_{\rm m} M_{\rm P}^2 Q^2} \ . \tag{3.48}$$

We can immediately identify the novel scale dependence in the friction term F due to the new mass scale $\mathcal{M}^2$, as mentioned above. Furthermore, we can make the quartic k-dependence of Y explicit by re-writing it as

$$Y = 2M_{\rm P}^2 Q^2 \frac{k^4 + \alpha_2 k^2 + \alpha_0}{(k^2 + M^2 + \mathcal{M}^2)^2} = 2M_{\rm P}^2 Q^2 \frac{k^4 + \alpha_2 k^2 + \alpha_0}{k^4 + \beta_2 k^2 + \beta_0} \ , \tag{3.49}$$

where we have defined

$$\alpha_2 \equiv \frac{\phi'}{3\Omega_{\rm c} M_{\rm P}^2 Q} (\mathcal{M}^2)' + \bar{M}^2 + g\mathcal{M}^2 \ , \tag{3.50}$$

$$\alpha_0 \equiv \frac{\phi'}{3\Omega_{\rm c} M_{\rm P}^2 Q} \left[(\mathcal{M}^2)' M^2 - (M^2)' \mathcal{M}^2 \right] + \mathcal{M}^2 \bar{M}^2 g \ , \tag{3.51}$$

$$\beta_2 \equiv 2\bar{M}^2 \ , \qquad \beta_0 \equiv \bar{M}^4 \ , \tag{3.52}$$

and introduced the combined mass scale

$$\bar{M}^2 \equiv M^2 + \mathcal{M}^2 \ . \tag{3.53}$$

From Eqs. (3.49) – (3.52) and the expression for g, we deduce that both the slow-roll limit, in which ϕ' and ϕ'' are small, and the limit of vanishing new mass scale $\mathcal{M}^2 = 0$, which corresponds to a constant Q, take us back to the standard scenario, where Y is given by Eq. (2.59).

3.4.2. Weak gravity

At last, in this subsection we want to use the above new quartic k-dependence of Y to establish the anticipated transient-weak-gravity regime. Besides the fact that Y is now a ratio of fourth-order polynomials in k, we must further examine the function g, in particular, the term $\propto Q'$. As argued in the previous subsection, Q' can be large without contradicting any of the assumptions we made and we will use that to obtain the desired behaviour of Y. In detail, since all other quantities in the numerator of Eq. (3.48) are in general of order $\mathcal{O}(1)$, we make the assumption that g is indeed dominated by this very term proportional to Q so that

$$g \approx \frac{Q'\phi'}{3\Omega_{\rm m} M_{\rm P}^2 Q^2} \gg 1 \ . \tag{3.54}$$

Moreover, to simplify the functions α_2 and α_0, we note that the derivative of the new mass scale squared is readily obtained by differentiating Eq. (3.33),

$$(\mathcal{M}^2)' = \left(2\xi + \frac{\Omega_\mathrm{m}'}{\Omega_\mathrm{m}} + \frac{Q_{,\phi\phi}\phi'}{Q_{,\phi}}\right)\mathcal{M}^2 \, . \tag{3.55}$$

Since all terms in the brackets are generally of $\mathcal{O}(1)$, we conclude that the derivative of the new mass scale is of the same order of magnitude as that mass scale itself, $(\mathcal{M}^2)' \sim \mathcal{M}^2$. Assuming a similar behaviour for the scalar-field mass, i.e. $(M^2)' \sim M^2$, and using that $g \gg 1$, the above coefficients simplify to

$$\alpha_2 \simeq M^2 + g\mathcal{M}^2 \, , \tag{3.56}$$

$$\alpha_0 \simeq g\mathcal{M}^2\bar{M}^2 \, , \tag{3.57}$$

With that, the effective gravitational coupling becomes

$$Y \simeq 2M_\mathrm{P}^2 Q^2 \frac{k^4 + (M^2 + g\mathcal{M}^2)k^2 + g\mathcal{M}^2\bar{M}^2}{(k^2 + \bar{M}^2)^2} \, , \tag{3.58}$$

which may be further simplified by utilizing that for a large scalar-field derivative $\phi' \approx M_\mathrm{P}$ one has $g\mathcal{M}^2 k^2 \gg k^4 + M^2 k^2$. This results in

$$Y \simeq 2M_\mathrm{P}^2 Q^2 g\mathcal{M}^2 \frac{1}{k^2 + \bar{M}^2} = -2M_\mathrm{P}^2 (Q')^2 \frac{1}{k^2 + \bar{M}^2} \, . \tag{3.59}$$

Since we imposed a positive $\mathcal{M}^2$, the above Y is strictly negative, implying a weakening of the effective gravitational coupling. Finally, the corresponding expression in the specific case of a quadratic exponential conformal function, as given in Eq. (3.36), reads

$$Y \simeq -2M_\mathrm{P}^2 \frac{\mathcal{H}^2(\phi')^2}{m_C^2} \frac{1}{k^2 + \bar{M}^2} = -2M_\mathrm{P}^2 Q^2 \frac{\mathcal{H}^2(\phi')^2}{\phi^2} \frac{1}{k^2 + \bar{M}^2} \, . \tag{3.60}$$

This implies a real-space gravitational potential

$$V(r) = -\frac{G_\mathrm{N} m}{r}\left[1 - \frac{2M_\mathrm{P}^2 (Q')^2}{\bar{M}^2}(1 - e^{-\bar{M}r})\right] \, , \tag{3.61}$$

which should be compared to Eq. (2.61). While for small scales, the whole additional term cancels and one obtains the standard Newton potential, for large scales the exponential becomes small so that the negative term remains. This results in a weaker gravity on large scales, which may perhaps even become repulsive.

For a simple numerical treatment of the above scenario, we refer to the appendix of [1]. There we solve the background and perturbation equations, finding that the QSA is indeed valid and that the anticipated weakening of gravity appears when the appropriate, aforementioned conditions are met. As a reminder, the latter are given by $|Q_{,\phi}|M_{\rm P}^2 \gg 1$ and $|Q'|M_{\rm P} \gg 1$ as well as the approximations from Sec. 3.3.

3.5. Summary and discussion

Summarising this work, we have investigated a CDE model with a canonical gravity, scalar-field and baryonic sector but with a DM sector whose geometry is constituted by a disformally transformed metric tensor. This induces a coupling between the DE scalar ϕ and the DM fluid, which will generally have an effect on the background evolution and linear perturbations. Focusing on a pure conformal coupling, we find a novel behaviour in the case when the coupling function Q is non-constant. The origin thereof is the emergence of a new mass scale $\mathcal{M}^2$, which is associated to the derivative of the coupling constant in field space, $Q_{,\phi}$, which on the other hand is given by the derivatives of the conformal function C up to second order. We find that an $\mathcal{M}^2$ which is not negligible in the quasi-static approximation requires $Q_{,\phi}M_{\rm P}^2 \gg 1$, which can be achieved close to a minimum of $C(\phi)$. The new mass scale $\mathcal{M}^2$ enters the evolution equation for the DM density contrast in such a way that the friction term F receives a novel k-dependence. Moreover, the effective gravitational coupling Y becomes a ratio of fourth-order polynomials in k. We have shown that if $|Q'|M_{\rm P} \gg 1$, this implies a weakening of gravity on large scales, which may help to alleviate the σ_8 tension.

An interesting question that arises is how this model will look like in the (dark-matter) Jordan frame, which would allow to compare it to the class of Horndeski theories. Such a transformation has been indicated in the appendix of [1] and exhibits seemingly unnatural requirements; however, a detailed treatment of this model in the Jordan frame is left to future work. Let us nevertheless recall that, as explained in Sec. 2.2.1, Horndeski theories are form-invariant under special disformal transformations (2.47), which is why we can be confident that the above model is indeed a Horndeski theory so that no Ostrogradsky instabilities are expected.

Future research on the transient-weak-gravity regime should aim at connecting the above discussion to observations. Most importantly, this requires the inclusion of a DE sector which can drive the accelerated expansion of the universe. This may constitute a difficult challenge requiring significant model building, especially, since ϕ' and ϕ'' are required to be large during the transient-weak gravity regime. A next step would be a systematic analysis with observational data and an examination on how much the σ_8 and perhaps also the Hubble tension can be alleviated. Finally, obvious extensions of the above scenario may include a general disformal transformation, where C and D depend also on the kinetic term X of the scalar field

ϕ. This may result in a beyond-Horndeski or DHOST coupled-dark-energy theory, and include further phenomenological implications.

Part II.

The stringy universe

4. Introduction

4.1. String theory and supersymmetry in a nutshell

Summarising Chpt. 1, string theory comes with two major advantages: providing a presumably UV-finite candidate for quantum gravity or a theory of everything and doing so while retaining a certain mathematical beauty in the sense that many field-theoretical aspects obtain a geometrical meaning. A priori these advantages come at the only cost of accepting the idea that the fundamental objects in our world are not zero-dimensional particles but one-dimensional strings. Obviously, the dynamics of such strings, which possess the ability to vibrate, is much richer than that of particles. In this section, we want to summarise the basic concepts of bosonic string theory, supersymmetry and supergravity as well as superstring theory, which can be studied in more detail in several introductory standard textbooks, for instance [68–74] for bosonic- and superstring theory and [75–77] for SUSY and SUGRA (see also [78] for a very brief and focused introduction to the bosonic string as well as [79] for an introduction to string phenomenology including all of the aforementioned topics).

4.1.1. The bosonic string

In this section we give a brief summary on the bosonic string, which can be considered as the simplest string theory, that can nevertheless be utilised to introduce many important core concepts. Most content in this subsection is based on [70–73, 78–80].

Let us start by considering the action for a free point particle with mass m following a trajectory γ,

$$S = -mc \int_\gamma \mathrm{d}s \ , \tag{4.1}$$

where $\mathrm{d}s \equiv \sqrt{-\eta_{\mu\nu}\mathrm{d}X^\mu \mathrm{d}X^\nu}$ is the infinitesimal line-element of that particle in its so-called target space We assume that the latter has D dimensions and is parameterised by the coordinates X^μ, with $\mu \in [0, D-1]$. Just like this action measures the length of the one-dimensional worldline of a point particle, the analogous, so-called

Nambu-Goto action of a string propagating through the target space is obtained via measuring the surface area of that string's worldsheet:

$$S_{\text{NG}} = -T \int_{\Sigma} \mathrm{d}A \ , \tag{4.2}$$

where T is the string tension and $\mathrm{d}A$ is the infinitesimal surface element of the worldsheet Σ. The string tension T, with $[T] = [\text{mass}^2]$, is the only dimensionful parameter in string theory and is related to the so-called *Regge slope* α' as well as the theory's fundamental length and mass scale, i.e. the string length and string mass,

$$\alpha' = \frac{1}{2\pi T} \ , \quad l_{\text{s}} = 2\pi\sqrt{\alpha'} \ , \quad M_{\text{s}} = \frac{1}{\sqrt{\alpha'}} \ , \tag{4.3}$$

where the precise prefactors depend on conventions. As it is common, we adopted the latter from [68, 69].

Introducing worldsheet coordinates $\xi^a = (\tau, \sigma)$, where $\tau \in \mathbb{R}$ parameterises the time-like and $\sigma \in [0, l]$ the space-like direction of the worldsheet with l setting the length of the string[1] and the induced metric:

$$G_{ab} \equiv \eta_{\mu\nu} \frac{\partial X^\mu}{\partial \xi^a} \frac{\partial X^\nu}{\partial \xi^b} \ , \tag{4.4}$$

we can write the infinitesimal surface element as

$$\mathrm{d}A = \mathrm{d}^2\xi \sqrt{-G} \ , \tag{4.5}$$

with $G \equiv \det G_{ab}$ and thus obtain

$$S_{\text{NG}} = -T \int_{\Sigma} \mathrm{d}^2\xi \sqrt{-G} \ . \tag{4.6}$$

As is known, the Nambu-Goto action is classically equivalent to the *Polyakov action*:

$$S_{\text{P}} = -\frac{T}{2} \int_{\Sigma} \mathrm{d}^2\xi \sqrt{-h} h^{ab} G_{ab} \ , \tag{4.7}$$

where the worldsheet metric h_{ab} is a new degree of freedom and $h \equiv \det h_{ab}$. Upon using the equations of motion, h_{ab} and G_{ab} are proportional. Rendering the induced

[1] One should not confuse l_{s}, which represents the physical length of the string and sets its mass scale, with l, that merely parameterises the endpoint of the string in the σ coordinate and is often chosen as π or 2π for an open or closed string, respectively.

metric explicit, the Polyakov action becomes

$$S_{\mathrm{P}} = -\frac{T}{2}\int_{\Sigma} \mathrm{d}^2\xi \sqrt{-h}h^{ab}\eta_{\mu\nu}\frac{\partial X^{\mu}}{\partial \xi^{a}}\frac{\partial X^{\nu}}{\partial \xi^{b}} , \tag{4.8}$$

which is just the theory of D free scalar fields X^{μ} which live on a two-dimensional space Σ with a cylindrical topology.

The above action possesses three symmetries:

1) diffeomorphism invariance of the worldsheet coordinates $\xi^a \to \tilde{\xi}^a(\xi)$,

2) Poincare invariance of the target-space fields $X^{\mu} \to \Lambda^{\mu}_{\nu}X^{\nu} + A^{\mu}$ with $\Lambda \in SO(1, D-1)$,

3) Weyl symmetry of the worldsheet metric $h_{ab}(\xi) \to \mathrm{e}^{2\Omega(\xi)}h_{ab}(\xi)$.

By using the first and third symmetry, the worldsheet metric can be gauge fixed into the form

$$h_{ab} = \eta_{ab} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} , \tag{4.9}$$

which is called the *flat gauge*. Since this promotes the dynamical field h_{ab} to a static object η_{ab}, in order not to lose any information, we impose the equation of motion for h_{ab} manually,

$$T_{ab} \equiv \frac{4\pi}{\sqrt{-h}}\frac{\delta S_{\mathrm{P}}}{\delta h^{ab}} = 0 , \tag{4.10}$$

which is also called the *Virasoro constraint*. In flat gauge and using light-cone coordinates, defined by

$$\xi^{\pm} \equiv \tau \pm \sigma , \quad \partial_{\pm} \equiv \frac{1}{2}(\partial_{\tau} \pm \partial_{\sigma}) , \tag{4.11}$$

the Polyakov action then becomes

$$S_P = T\int_{\Sigma} \mathrm{d}\xi^{+}\mathrm{d}\xi^{-}\partial_{+}X^{\mu}\partial_{-}X_{\mu} . \tag{4.12}$$

The resulting equation of motion, which is given by

$$\partial_{+}\partial_{-}X^{\mu} = 0 , \tag{4.13}$$

implies that the fields, which represent the target-space coordinates, can be decomposed into a ξ^{+}-dependent and a ξ^{-}-dependent part, which correspond to left- and right-moving waves, respectively:

$$X^\mu = X^\mu_{\mathrm{L}}(\xi^+) + X^\mu_{\mathrm{R}}(\xi^-) \; . \tag{4.14}$$

For the sake of brevity and simplicity, let us only quote the solution for closed strings here, which will be our main focus, and refer the interested reader to the aforementioned textbooks for the treatment of open strings. Closed strings fulfill periodic boundary conditions:

$$X^\mu(\tau, \sigma) = X^\mu(\tau, \sigma + l) \; , \tag{4.15}$$

which, together with Eq. (4.13), implies that the general solution for X^μ is given by a Fourier series in ξ^+ and ξ^- plus integration constants. This so-called *mode decomposition* reads:

$$X^\mu_{\mathrm{L}}(\xi^+) = \frac{1}{2}x^\mu + \frac{1}{2}\frac{2\pi\alpha'}{l}p^\mu\xi^+ + \mathrm{i}\sqrt{\frac{\alpha'}{2}}\sum_{n\in\mathbb{Z}\setminus\{0\}}\frac{1}{n}\tilde{\alpha}^\mu_n \mathrm{e}^{-\frac{2\pi}{l}\mathrm{i}n\xi^+} \; , \tag{4.16}$$

$$X^\mu_{\mathrm{R}}(\xi^-) = \frac{1}{2}x^\mu + \frac{1}{2}\frac{2\pi\alpha'}{l}p^\mu\xi^- + \mathrm{i}\sqrt{\frac{\alpha'}{2}}\sum_{n\in\mathbb{Z}\setminus\{0\}}\frac{1}{n}\alpha^\mu_n \mathrm{e}^{-\frac{2\pi}{l}\mathrm{i}n\xi^-} \; . \tag{4.17}$$

Here, the first terms in both mode decompositions enter via integration constants and describe the center-of-mass position of the string x^μ at time $\tau = 0$ whereas the second terms correspond to the zero modes of the Fourier series and describe the string's motion with center-of-mass momentum $p^\mu \equiv \sqrt{2/\alpha'}\tilde{\alpha}^\mu_0 = \sqrt{2/\alpha'}\alpha^\mu_0$. Finally, the last terms describe left- and right-moving waves with mode number n, respectively. Note that reality of the target-space coordinates X^μ implies $(\tilde{\alpha}^\mu_n)^* = \tilde{\alpha}^\mu_{-n}$ and $(\alpha^\mu_n)^* = \alpha^\mu_{-n}$.

Quantisation

To quantise the theory, we follow [78] and perform a so-called *light-cone quantisation*, in which we implement the Virasoro constraint (4.10) at the classical level before quantisation. First, we transform two target-space coordinates, the time-direction and one arbitrary spatial direction, into light-cone coordinates:

$$X^\pm \equiv \frac{1}{\sqrt{2}}\left(X^0 \pm X^{D-1}\right) \; , \tag{4.18}$$

whereas the other $D-2$ spatial directions remain unchanged. We then use the crucial fact that the flat gauge (4.9) does not completely fix ξ but leaves a residual gauge freedom of the form

$$\xi^{\pm} \to \tilde{\xi}^{\pm}(\xi^{\pm}) \ . \tag{4.19}$$

With the freedom to transform ξ^+ and ξ^- into arbitrary functions that again depend only on ξ^+ or ξ^-, respectively, we can eliminate all oscillator modes of X^+, which then simplifies to

$$X^+ = x^+ + \alpha' p^+ \tau \ . \tag{4.20}$$

Moreover, the Virasoro constraints (4.10) imply a dependency between the oscillator modes of X^- and of the other X^i:

$$\tilde{\alpha}_n^- = \frac{1}{\sqrt{2\alpha'}p^+} \sum_{m=-\infty}^{\infty} \tilde{\alpha}_{n-m}^i \tilde{\alpha}_m^i \ , \tag{4.21}$$

$$\alpha_n^- = \frac{1}{\sqrt{2\alpha'}p^+} \sum_{m=-\infty}^{\infty} \alpha_{n-m}^i \alpha_m^i \ . \tag{4.22}$$

That is, the directions X^+ and X^- do not carry any physical, oscillatory degrees of freedom. Thus in flat gauge and light-cone coordinates (4.18), the action (4.8) becomes

$$S_{\mathrm{P}} = \frac{1}{4\pi\alpha'} \int_\Sigma \mathrm{d}\tau \mathrm{d}\sigma \left[(\partial_\tau X^i)^2 - (\partial_\sigma X^i)^2 - 2\alpha' p^+ \partial_\tau X^- \right] \equiv \int \mathrm{d}\tau L_{\mathrm{P}} \ . \tag{4.23}$$

Defining now the quantity

$$q^- \equiv \frac{1}{l} \int_0^l \mathrm{d}\sigma X^- \ , \tag{4.24}$$

the canonical variables and their conjugate momenta are given by

$$X^i \ , \quad \Pi_i \equiv \frac{\partial L_{\mathrm{P}}}{\partial(\partial_\tau X^i)} = \frac{\partial_\tau X_i}{2\pi\alpha'} \ , \tag{4.25}$$

$$q^- \ , \quad p_- \equiv \frac{\partial L_{\mathrm{P}}}{\partial(\partial_\tau q^-)} = -\frac{l}{2\pi} p^+ \ . \tag{4.26}$$

The quantisation of this theory can now be performed by promoting these fields and their corresponding conjugate momenta to operators which fulfill some commutation relations. Crucially, the oscillator modes $\tilde{\alpha}_n^i$ and α_n^i are also promoted to operators with the commutation relations

$$[\alpha_m^i, \alpha_n^j] = [\tilde{\alpha}_m^i, \tilde{\alpha}_n^j] = m\delta_{m+n,0}\delta_{ij} \ . \tag{4.27}$$

Thus, the oscillator modes become creation and annihilation operators for the wave modes of the string where α_n^i creates a wave mode for $n < 0$ and annihilates one for $n > 0$ and likewise for $\tilde{\alpha}^i_n$. As a Hilbert space, on which these operators can act, we consider only the subspace for a fixed momentum p^μ. The corresponding groundstate is then labelled and defined as

$$\alpha^\mu_n|0;p\rangle = \tilde{\alpha}^\mu_n|0;p\rangle = 0 \quad \forall n > 0 \; . \tag{4.28}$$

The overall Fock space consists of the span of an arbitrary number of creation operators $\alpha^\mu_{-|n|}$ or $\tilde{\alpha}^\mu_{-|n|}$ acting on this groundstate.

Criticality

Before we continue by constructing physical, closed-string states, we want to mention the topic of criticality. By the choice of light-cone coordinates, we have singled out a certain direction in the target space. While the choice of specific a coordinate system is unproblematic classically, we have to make sure that Lorentz invariance is also satisfied at the quantum level; that is, we have to avoid the appearance of an anomaly. A cumbersome calculation then implies that the absence of such an anomaly requires the target space dimension to be $D = 26$. Such theories, which fulfill this requirement, are called *critical* and will be the only interest in this part.

In order to analyse the physical states of closed strings, we notice that the mass of a state is given by [78]

$$M^2 = -p^2 = 2p^+p^- - p^ip^i = \frac{4}{\alpha'}(N_\perp - 1) \; . \tag{4.29}$$

To arrive at the last expression, we have used the Virasoro constraint (4.10), which also implies a level matching condition $N_\perp = \tilde{N}_\perp$, as well as the fact that a so-called *normal-ordering constant*, which is needed due to the ambiguity of normal ordering of the zero mode α^i_0, is forced to $a = 1$ by criticality. Here we have defined the number operator

$$N_\perp \equiv \sum_{n=1}^{\infty} : \alpha^i_{-n}\alpha^i_n : \; , \tag{4.30}$$

which measures the level of excitement of right-moving, transverse waves. Here, the normal-ordering symbol is defined by

$$: \alpha^i_m\alpha^i_n :\equiv \begin{cases} \alpha^i_m\alpha^i_n & \text{for } m \leq n \; , \\ \alpha^i_n\alpha^i_m & \text{for } m > n \; , \end{cases} \tag{4.31}$$

that is, large-mode creation operators are sorted to the most left and large-mode annihilation operators to the most right. Small-mode operators are sorted inbetween.

Physical States

We can now simply sort the physical states by their level of excitement [78]:

$N = 0$: The mass shell condition (4.29) yields $M^2 = -4/\alpha'$. This state is tachyonic and therefore very likely renders bosonic string theory unstable. This is one of the motivations to consider superstring theory where this state is projected out.

$N = 1$: The mass of these states is $M^2 = 0$. They are constructed via

$$\xi_{ij}\tilde{\alpha}^i_{-1}\alpha^j_{-1}|0;p\rangle \ , \tag{4.32}$$

where ξ_{ij} is the so-called *polarisation tensor*, which is an arbitrary $(D-2)\times(D-2)$ matrix. We can decompose it into a symmetric traceless, antisymmetric and trace part:

$$\xi_{ij} = \xi_{(ij)} + \xi_{[ij]} + \xi_0 \ . \tag{4.33}$$

The symmetric traceless part contains $(D-2)\cdot(D-1)/2-1$ degrees of freedom, which gives two degrees of freedom for $D = 4$. These states correspond to one-particle graviton states $g_{\mu\nu}$ in the D-dimensional target space. The states built from the antisymmetric part give rise to a new two-form field, called *Kalb-Ramond field* $B_{[\mu\nu]}$. It plays a crucial role in the fixing of some degrees of freedom of the extra-dimensional space. Finally, the trace part corresponds to a single scalar field Φ, which is called the *dilaton* and determines the strength of string coupling via the important relation $g_\mathrm{s} = exp(\langle\Phi\rangle)$.

$N > 1$: These states are very massive with $M^2 > 4/\alpha' \propto 4M_\mathrm{s}^2$ and are thus integrated out in the low-energy EFT.

Low Energy Action

For the sake of completeness, let us quote the low-energy action for the massless modes. This is achieved in the limit of small-curvature backgrounds, i.e. in spaces with a curvature radius much larger than the string length $l_\mathrm{s} \propto \sqrt{\alpha'}$, via an expansion in α'. The leading-order result reads [68]

$$S_\text{eff} = \frac{1}{2\kappa_0^2}\int \mathrm{d}^D x\sqrt{-g}\mathrm{e}^{-2\Phi}\left(R - \frac{1}{12}H_{\mu\nu\lambda}H^{\mu\nu\lambda} + 4\partial_\mu\Phi\partial^\mu\Phi + \mathcal{O}(\alpha')\right) \ , \tag{4.34}$$

where $H \equiv \mathrm{d}B$ is the exterior derivative of the Kalb-Ramond field, i.e. its field strength tensor, and the constant κ_0 is unspecified because it can be absorbed into a shift of Φ. Importantly, the same action arises later in the case of superstring theory in the closed-string bosonic sector with the only differences being another value than 26 for the number of dimensions D and further terms present. The above action is in the so-called *string frame* and can be brought into the Einstein frame via a Weyl transformation of the target space metric

$$g_{\mu\nu} = \mathrm{e}^{-\frac{4\phi}{D-2}} \tilde{g}_{\mu\nu} \,, \tag{4.35}$$

so that it becomes

$$S_{\text{eff}} = \frac{1}{2\kappa^2} \int \mathrm{d}^D x \sqrt{-\tilde{g}} \left(\tilde{R} - \frac{1}{12} \mathrm{e}^{-\frac{8\Phi}{D-2}} H_{\mu\nu\lambda} H^{\mu\nu\lambda} - \frac{4}{D-2} \partial_\mu \Phi \partial^\mu \Phi + \mathcal{O}(\alpha') \right) \,, \tag{4.36}$$

where indices are now raised and lowered with the Einstein-frame metric $\tilde{g}_{\mu\nu}$. This is completely analogous to a transformation between the Jordan and Einstein frame as it is common in MG theories. In the Einstein frame, the parameter κ is fixed by the choice of the dilaton background and represents the D-dimensional gravitational coupling constant.

In conclusion, bosonic string theory contains Einstein gravity in the closed-string sector as well as gauge theories, which arise from open string modes living on branes, which is well known even though we did not show it here explicitly. The obvious drawbacks are a tachyonic zero mode, which likely renders the theory unstable, and the absence of fermions. The inclusion of supersymmetry gives rise to superstring theory, which remedies both of these problems.

4.1.2. Supersymmetry

The main focus of this section is to introduce the most important concepts and the language of supersymmetry at a level which is needed for the following chapters. A customary standard reference for formulae in SUSY and SUGRA is Ref. [76] but we also want to refer to [75, 77] for further reading. Most parts in this subsection are based on [76, 79, 81]. Supersymmetry is a symmetry which assigns a bosonic partner to each fermion and vice versa. On the practical and phenomenological side, SUSY can be motivated as a tool that has the potential to solve or at least alleviate the hierarchy problem of the Higgs sector and the CC problem. This is achieved through the cancellation of many loop contributions as long as SUSY is present. On the theoretical and aesthetic side, SUSY represents a promising candidate towards unification and a theory of everything. This is especially the case since SUSY represents the unique way to generalise the spacetime symmetry of a relativistic QFT. To be more specific: According to the *Coleman-Mandula theorem* [82], the only way to combine the Poincaré symmetry with an internal symmetry of a field, is the trivial one, i.e. by a direct sum. This restriction can be softened by the inclusion of fermionic spacetime generators, which satisfy anticommutator relations. These form a so-called *supersymmetry algebra*, which is a non-trivial, and according to the *Haag-Lopuszański-Sohnius theorem* [83] unique, extension of the Poincaré algebra.

The Supersymmetry Algebra

The total superalgebra is given by three sets of generators P_μ, $M_{\mu\nu}$ and Q_α where μ and ν are spacetime indices whereas α is a spinorial index that runs from 1 to 2. P_μ and $M_{\mu\nu}$ correspond to the spacetime momentum and the Lorentz generators, which generate spacetime translations and rotations, respectively, and together build the Poincaré algebra [79],

$$[P_\mu, P_\nu] = 0 \ , \tag{4.37}$$

$$[M_{\mu\nu}, P_\lambda] = \mathfrak{i}\left(\eta_{\mu\lambda}P_\nu - \eta_{\nu\lambda}P_\mu\right) \ , \tag{4.38}$$

$$[M_{\mu\nu}, M_{\rho\sigma}] = \mathfrak{i}\left(\eta_{\mu\rho}M_{\nu\sigma} - \eta_{\nu\rho}M_{\mu\sigma} - \eta_{\mu\sigma}M_{\nu\rho} + \eta_{\nu\sigma}M_{\mu\rho}\right) \ . \tag{4.39}$$

This is now extended by the inclusion of the fermionic generators Q_α, which satisfy the following commutator and anticommutator relations:

$$[P_\mu, Q_\alpha] = 0 \ , \tag{4.40}$$

$$[M_{\mu\nu}, Q_\alpha] = \mathfrak{i}(\sigma_{\mu\nu})_\alpha{}^\beta Q_\beta \ , \tag{4.41}$$

$$\left\{Q_\alpha, \bar{Q}_{\dot\alpha}\right\} = 2(\sigma^\mu)_{\alpha\dot\alpha}P_\mu \ , \tag{4.42}$$

$$\{Q_\alpha, Q_\beta\} = 0 \ , \tag{4.43}$$

$$\left\{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\right\} = 0 \ , \tag{4.44}$$

with $\sigma^\mu \equiv (-\mathbb{1}_2, \vec\sigma)$ where $\mathbb{1}_2$ is the two-dimensional unity matrix and $\vec\sigma$ contains the three Pauli matrices as entries and $\sigma_{\mu\nu} \equiv -\left(\sigma_\mu\bar\sigma_\nu - \sigma_\nu\bar\sigma_\mu\right)/4$. We see that the superalgebra extends the Poincaré algebra in a non-trivial way.

Superspace

We can now introduce the notion of so-called *superspace*, which generalises the familiar Minkowski space by the inclusion of some extra fermionic dimensions. This superspace is parameterised by the coordinates $(x^\mu, \theta_\alpha, \bar\theta_{\dot\alpha})$ with $\alpha, \dot\alpha = 1, 2$. The additional coordinates are Grassmann valued, that is they satisfy the relations

$$\theta_1^2 = \theta_2^2 = 0 \ , \quad \theta_1\theta_2 = -\theta_2\theta_1 \ , \quad \int \mathrm{d}\theta_\alpha = 0 \ , \quad \int \mathrm{d}\theta_\alpha\theta_\alpha = \frac{\partial}{\partial\theta_\alpha}\theta_\alpha = 1 \ , \tag{4.45}$$

where in the last equation no sum is intended and analogous relations hold for the complex conjugate coordinates $\bar\theta_{\dot\alpha}$. Just like the P_μ generate translations along spacetime, the Q_α and $\bar{Q}_{\dot\alpha}$ generate translations along the superspace where the explicit forms

$$Q_\alpha = \partial_\alpha - \mathrm{i}\,(\sigma^\mu)_{\alpha\dot{\alpha}}\,\bar{\theta}^{\dot{\alpha}}\partial_\mu\;,\quad \bar{Q}_{\dot{\alpha}} = -\bar{\partial}_{\dot{\alpha}} + \mathrm{i}\theta^\alpha\,(\sigma^\mu)_{\alpha\dot{\alpha}}\,\partial_\mu\;, \tag{4.46}$$

contain terms $\propto \partial_\mu$ in order to satisfy Eq. (4.42). In order to construct SUSY invariant actions, we make use of so-called *superfields*, which are fields that depend on the position in superspace. They correspond to supermultiplets, i.e. a representation of the supersymmetry algebra whose components are familiar quantum fields. These components are merely the prefactors in an expansion of the superfield in the coordinates θ_α and $\bar{\theta}_{\dot{\alpha}}$. The number of components is finite due to Eqs. (4.45) and they correspond to fields that are superpartners w.r.t. each other or auxiliary fields. The application of a SUSY transformation on a superfield is therefore equivalent to a rotation of its individual components into each other, thus transforming bosons into their fermionic superpartners and vice versa.

Chiral Superfields

The simplest and most prominent example are chiral superfields, which are a subrepresentation of a general superfield. They are constructed by imposing the condition $\bar{D}_{\dot{\alpha}}\Phi = 0$ for chiral or $D_\alpha\bar{\Phi} = 0$ for antichiral superfields where [81]

$$D_\alpha \equiv \frac{\partial}{\partial\theta_\alpha} + \mathrm{i}\,(\sigma^\mu)_{\alpha\dot{\alpha}}\,\bar{\theta}^{\dot{\alpha}}\partial_\mu\;,\quad \bar{D}_{\dot{\alpha}} \equiv -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - \mathrm{i}\theta^\alpha\,(\sigma^\mu)_{\alpha\dot{\alpha}}\,\partial_\mu \tag{4.47}$$

are the SUSY covariant derivatives. Hence in terms of their components, chiral superfields are given by

$$\Phi = \phi(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y)\;, \tag{4.48}$$

where $y^\mu \equiv x^\mu + \mathrm{i}\theta\sigma^\mu\bar{\theta}$ and an analogous expression holds for $\bar{\Phi}$, which depends on $\bar{\theta}$ but not on θ. Here ϕ is the superfield's scalar component, ψ its fermionic superpartner and F a complex-valued scalar auxiliary field. Constructing SUSY invariant actions, it is crucial that the highest component of a chiral superfield, the so-called *F-Term*, only changes by a total derivative. This motivates the idea to construct such actions as the F-terms of chiral superfields. Generically, holomorphic functions of chiral superfields are again superfields. Furthermore, the highest components of general superfields, which are $\propto \theta^2\bar{\theta}^2$ and called *D-terms*, are SUSY invariant up to a total derivative as well. For that reason, the most general, second-derivative Lagrangian that depends on a collection of chiral superfields Φ_i is given by

$$\mathcal{L} = \int \mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\,\mathcal{K}\left(\Phi_i, \bar{\Phi}_{\bar{\imath}}\right) + \int \mathrm{d}^2\theta\, W\left(\Phi_i\right) + \int \mathrm{d}^2\bar{\theta}\,\overline{W}\left(\bar{\Phi}_{\bar{\imath}}\right)\;. \tag{4.49}$$

Here $\mathcal{K}$ is a general superfield called the *Kähler potential* and W a holomorphic function of the Φ_i, which is also a chiral superfield and is called the *superpotential*. The integrations over the fermionic coordinates θ and $\bar{\theta}$ project out the highest components of $\mathcal{K}$ and W, i.e. the D-term and F-term, respectively, thus rendering this Lagrangian SUSY invariant. Typically, $\mathcal{K}$ contains kinetic terms for the fermions and bosons whereas W contains fermionic mass terms as well as Yukawa-like interaction terms. Furthermore, as already mentioned, the F-terms F_i of the individual chiral fields Φ_i are auxiliary fields, that is they have no kinetic terms. Hence these fields can be integrated out and thus give rise to a potential for the scalar components ϕ_i, the so-called *F-term potential*

$$V_F(\phi_i) = \sum_i |F_i|^2 \ , \tag{4.50}$$

where the F_i are eliminated by inserting their respective equations of motion.

Vector Superfields

Another possibility to obtain an irreducible piece from a reducible, general superfield is via the reality constraint $V = \bar{V}$. This gives rise to real superfields or vector superfields. In the so-called *Wess-Zumino gauge*, they can be written in component form as [81]

$$V = -\theta\sigma^\mu\bar{\theta}A_\mu(x) + \mathrm{i}\theta^2\bar{\theta}\bar{\lambda}(x) - \mathrm{i}\bar{\theta}^2\theta\lambda(x) + \frac{1}{2}\theta^2\bar{\theta}^2 D(x) \ . \tag{4.51}$$

Here A_μ represents a familiar gauge field, λ_α is a spinorial field called gaugino and D is another auxiliary field, which is just the D-term. To obtain a Lagrangian that is SUSY and gauge invariant, one defines a field-strength superfield, which is given by

$$W_\alpha = -\frac{1}{4}\bar{D}^2 D_\alpha V \ , \quad \bar{W}_{\dot{\alpha}} = -\frac{1}{4}D^2\bar{D}_{\dot{\alpha}}V \tag{4.52}$$

for an abelian gauge symmetry and by

$$W_\alpha = -\frac{1}{4}\bar{D}^2 \mathrm{e}^{-V} D_\alpha \mathrm{e}^V \ , \quad \bar{W}_{\dot{\alpha}} = -\frac{1}{4}D^2\mathrm{e}^{-V}\bar{D}_{\dot{\alpha}}\mathrm{e}^V \tag{4.53}$$

for a non-abelian gauge symmetry. W_α is again a chiral superfield so that a SUSY invariant Lagrangian can be constructed as [81]

$$\mathcal{L}_{\text{gauge}} = \frac{1}{4}\mathrm{tr}\left(\int \mathrm{d}^2\theta W^\alpha W_\alpha + \int \mathrm{d}^2\bar{\theta}\bar{W}^{\dot{\alpha}}\bar{W}_{\dot{\alpha}}\right) = \mathrm{tr}\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \mathrm{i}\lambda\sigma^\mu D_\mu\bar{\lambda} + \frac{1}{2}D^2\right) \ , \tag{4.54}$$

which are just the canonical kinetic terms of the gauge field A_μ and the fermionic gaugino λ plus the auxiliary field D. Just like in the case of the F-term, integrating out the D field gives rise to a so-called *D-term potential* $V_D(\phi_i)$. In the case of a $U(1)$ gauge field, it is also possible to include the D-term of the vector superfield V itself to the SUSY invariant Lagrangian, which is the so-called *Fayet-Iliopoulos* (FI) term:

$$\mathcal{L}_{\text{FI}} = \xi \int \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta} V \ , \tag{4.55}$$

which also contributes to the D-term scalar potential. Interactions between vector and chiral superfields can be realised by modifying the Kähler potential, e.g. as

$$\mathcal{K} = \bar{\Phi} \mathrm{e}^V \Phi \ , \tag{4.56}$$

or by multiplying the gauge kinetic terms $\mathcal{L}_{\text{gauge}}$ with a holomorphic *gauge kinetic function* $f(\Phi_i)$:

$$\mathcal{L}_{\text{gauge}} = \frac{1}{4} \left[\int \mathrm{d}^2\theta f(\Phi_i) \mathrm{tr}\left(W^\alpha W_\alpha\right) + \int \mathrm{d}^2\bar{\theta} \bar{f}(\bar{\Phi}_{\bar{i}}) \mathrm{tr}\left(\bar{W}^{\dot{\alpha}} \bar{W}_{\dot{\alpha}}\right) \right] \ . \tag{4.57}$$

SUSY Breaking

Finally, it is important to discuss the issue of SUSY breaking. Since modern particle-accelerator experiments have not found any evidence for supersymmetry up to an energy scale of $\sim \mathcal{O}(\text{TeV})$, it is clear that a realistic scenario must have broken supersymmetry at the low-energy limit. Favourable are models which restore SUSY above the so-called *SUSY scale* $M_{\text{SUSY}} > \mathcal{O}(\text{TeV})$ and are subject to spontaneous symmetry breaking below that scale, very much like in the case of electroweak symmetry breaking.

On the technical level, this means that the action of a theory with spontaneous SUSY breaking should retain SUSY invariance but its ground state should break it:

$$Q_\alpha |0\rangle \neq 0 \ . \tag{4.58}$$

From Eq. (4.42), we can write down the Hamiltonian of the theory, which is just the zero-component of the four-momentum [81],

$$\mathcal{H} = P_0 = \frac{1}{4} \left(\bar{Q}_1 Q_1 + Q_1 \bar{Q}_1 + \bar{Q}_2 Q_2 + Q_2 \bar{Q}_2 \right) \ . \tag{4.59}$$

Crucially, SUSY invariance of the vacuum state $Q_\alpha |0\rangle = 0$ is equivalent to a vanishing vacuum energy $\langle 0|\mathcal{H}|0\rangle = 0$. The latter is simply given by the vacuum expectation value of the scalar potential generated by all F-terms and D-terms (and, if present, FI terms),

$$\langle 0|\mathcal{H}|0\rangle = \langle 0|V|0\rangle = \langle 0|V_{\mathrm{F}} + V_{\mathrm{D}}|0\rangle = \sum_i |F_i|^2 + \sum_a |D_a|^2 \,. \tag{4.60}$$

Depending on whether some of the F-terms or D-terms acquire a non-vanishing vacuum expectation value, one speaks of F-term or D-term breaking of the theory where it is obviously also possible that a combination of F- or D-terms leads to SUSY breaking. We conclude that for globally supersymmetric models, a vacuum state that retains SUSY corresponds to a vanishing cosmological constant, i.e. a Minkowski space.

Supergravity

Until now, supersymmetry has been treated as a global symmetry on a flat space. The generalisation to a local symmetry on a curved space is called supergravity. Here the spacetime graviton $g_{\mu\nu}$ is the component of another supermultiplet, which besides some auxiliary fields contains another physical field: the gravitino ψ^μ_α, which carries spin $3/2$ and represents the graviton's superpartner. The total, four-dimensional SUGRA action has been derived in [76], in the superspace approach as well as in component form, and is rather involved, which is why we only quote some important terms and features of it.

A supergravity theory is completely specified by the three functions $\mathcal{K}$, W and f_{ab}, which are the Kähler potential, the superpotential and the gauge kinetic function, respectively. It is invariant under so-called *Kähler transformations*

$$\mathcal{K}(\Phi_i, \bar{\Phi}_{\bar{\imath}}) \to \mathcal{K}(\Phi_i, \bar{\Phi}_{\bar{\imath}}) + F(\Phi_i) + \bar{F}(\bar{\Phi}_{\bar{\imath}}) \,, \tag{4.61}$$

where F is an arbitrary holomorphic function of the chiral superfields Φ_i. In component form, the action of a four-dimensional, $\mathcal{N} = 1$ SUGRA, where $\mathcal{N}$ denotes the number of SUSY generators, gives rise to the following terms [76, 79, 81]:

- An Einstein-Hilbert term for $g_{\mu\nu}$:

$$\mathcal{L}_{\mathrm{SUGRA}} \supset \frac{\sqrt{-g}}{2} R \,. \tag{4.62}$$

- A so-called *Rarita-Schwinger Lagrangian*; that is, a kinetic and mass term for the gravitino ψ^μ_α:

$$\mathcal{L}_{\mathrm{SUGRA}} \supset \sqrt{-g}\left[\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\bar{\sigma}_\nu\mathcal{D}_\rho\psi_\sigma - \mathrm{e}^{\mathcal{K}/2}\left(\bar{W}\psi_\mu\sigma^{\mu\nu}\psi_\nu + W\bar{\psi}_\mu\bar{\sigma}^{\mu\nu}\bar{\psi}_\nu\right)\right] \,, \tag{4.63}$$

where $\mathcal{D}_\rho$ is a covariant derivative w.r.t. the underlying gauge group, diffeomorphisms and Kähler transformations (4.61).

- Kinetic terms for the scalars ϕ_i and their fermionic superpartners χ_i:

$$\mathcal{L}_{\text{SUGRA}} \supset \sqrt{-g}\mathcal{K}_{i\bar{j}} \left[(\mathcal{D}_\mu\phi^i)(\mathcal{D}^\mu\bar{\phi}^{\bar{j}}) + \mathrm{i}\bar{\chi}^{\bar{j}}\bar{\sigma}^\mu\mathcal{D}_\mu\chi^i\right] \ , \tag{4.64}$$

  where the so-called *Kähler metric* $K_{i\bar{j}} \equiv \partial^2\mathcal{K}/(\partial\Phi_i\partial\bar{\Phi}_{\bar{j}})$ acts as a kinetic mixing matrix.

- Kinetic terms for the gauge bosons $A^{(a)}_\mu$ and their fermionic superpartners $\lambda^{(a)}$ called gauginos as well as axionic couplings of the gauge bosons

$$\frac{\mathcal{L}_{\text{SUGRA}}}{\sqrt{-g}} \supset \text{Re}(f_{ab})\left[-\frac{1}{4}F^{(a)}_{\mu\nu}F^{(b)\mu\nu} - \mathrm{i}\bar{\lambda}^{(a)}\bar{\sigma}^\mu\mathcal{D}_\mu\lambda^{(b)}\right] - \frac{1}{4}\text{Im}(f_{ab})F^{(a)}_{\mu\nu}\tilde{F}^{(b)}_{\mu\nu} \ . \tag{4.65}$$

- A scalar potential for the ϕ_i, consisting of an F-term and a D-term potential:

$$\frac{\mathcal{L}_{\text{SUGRA}}}{\sqrt{-g}} \supset V_F = \mathrm{e}^{\mathcal{K}}\left[\left(\mathcal{K}^{-1}\right)^{i\bar{j}}(D_iW)(\bar{D}_{\bar{j}}\bar{W}) - 3|W|^2\right] \ , \tag{4.66}$$

$$\frac{\mathcal{L}_{\text{SUGRA}}}{\sqrt{-g}} \supset V_D = \frac{1}{2}\left[\text{Re}(f^{-1})\right]^{ab} D_aD_b \ , \tag{4.67}$$

  where $(K^{-1})^{i\bar{j}}$ is the inverse of the Kähler metric $K_{i\bar{j}}$. Here one needs to take care not to confuse the D_i in V_F, which is a Kähler covariant derivative $D_iW = \partial_iW + W\partial_iK$, with the D_a in V_D, which are the D-terms.

Besides the above terms, there are many other ones in $\mathcal{L}_{\text{SUGRA}}$, which induce couplings between the scalars ϕ_i and fermions χ_i from the chiral multiplets Φ_i, the gauge bosons $A^{(a)}_\mu$ and gauginos $\lambda^{(a)}$ as well as the gravitino ψ^μ_α.

As in the case of global supersymmetry, spontaneous SUSY breaking in SUGRA occurs by F- or D-terms which acquire a non-vanishing vacuum expectation value. In SUGRA, the F-terms are given by [79]

$$F_i = \mathrm{e}^{\mathcal{K}/2}\left(\mathcal{K}^{-1}\right)^{i\bar{j}}\bar{D}_{\bar{j}}\bar{W} \ . \tag{4.68}$$

Comparing Eqs. (4.50) with (4.66) and (4.68), we notice that, analogously to global SUSY, the SUGRA F-term potential is again given by the F-terms but with an additional term $-3\exp(\mathcal{K})|W|^2$, which is inherent in SUGRA and has no counterpart in global SUSY. This term is responsible for the crucial fact that in SUGRA a supersymmetric vacuum state does not necessarily represent a Minkowski space but can also and often will be anti-de Sitter. Another interesting feature becomes apparent by inspecting Eq. (4.63), from which we can read off that the gravitino mass is given by

$$m_{3/2}^2 = \left\langle e^{\mathcal{K}} |W|^2 \right\rangle \ , \tag{4.69}$$

where the brackets $\langle \cdot \rangle$ denote the vacuum expectation value. With that, we can write the vacuum expectation value of the F-term potential as

$$\langle V_F \rangle = \sum_i | \langle F_i \rangle |^2 - 3m_{3/2}^2 \ . \tag{4.70}$$

Since the observed vacuum energy density in the late-time universe is very small compared to the natural values of F and $m_{3/2}$, this implies a precise cancellation between the two terms so that the SUSY breaking scale is $F \sim m_{3/2}$. As we will see later, this represents a huge challenge for string phenomenology and the implementation of a stringy dark energy, which is one of the main findings of this work.

4.1.3. Superstring theory

As mentioned before, bosonic string theory has two major drawbacks: the absence of fermions and a tachyonic zero mode. We take this as a motivation to quote the basic concepts of superstring theory, which solves both of these drawbacks by implementing SUGRA into string theory. Our main references for this subsection are [71, 72, 79]. There are two approaches to do so: In the so-called *Ramond-Neveu-Schwarz formalism*, SUSY is manifest on the two-dimensional worldsheet theory, whereas in the *Green-Schwarz formalism* it is so in the target space. It is a non-trivial but nevertheless true statement that both formalisms are equivalent and we will only quote the former in here.

We start by promoting the two-dimensional worldsheet to a superspace, which besides the worldsheet coordinates $\xi^a = (\tau, \sigma)$ is parameterised by two Grassman coordinates θ^α. Likewise, the target-space coordinates X^μ, which as we remember are merely a collection of D massless scalar fields living on the worldsheet, are promoted to the scalar components of superfields:

$$Y^\mu(\xi, \theta) = X^\mu(\xi) + \bar{\theta}\psi^\mu(\xi) + \frac{1}{2}\bar{\theta}\theta B^\mu(\xi) \ , \tag{4.71}$$

where the ψ^μ are the fermionic superpartners and B^μ is an auxiliary field. Note that Y^μ are not chiral superfields but depend on θ as well as $\bar{\theta}$ and that the ψ^μ fields also carry a spinorial index, which is suppressed and is contracted with $\bar{\theta}$. The generator of SUSY transformations is given by [72]

$$Q_\alpha = \frac{\partial}{\partial \bar{\theta}^\alpha} + \mathrm{i}(\rho^a \theta)_\alpha \frac{\partial}{\partial \xi^a} \ , \tag{4.72}$$

where ρ^a are the two dimensional dirac matrices defined by the anticommutator relations

$$\left\{\rho^a, \rho^b\right\} = -2\eta^{ab} \ , \tag{4.73}$$

with η^{ab} the two-dimensional Minkowski metric. A possible representation is

$$\rho^0 = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix} \ , \quad \rho^1 = \begin{pmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix} \ . \tag{4.74}$$

A SUSY transformation then corresponds to an infinitesimal translation in the superspace, which is generated by the Q_α:

$$\delta\xi^a = [\bar{\epsilon}Q, \xi^a] = \mathrm{i}\bar{\epsilon}\rho^a\theta \ , \tag{4.75}$$

$$\delta\theta^\alpha = [\bar{\epsilon}Q, \theta^\alpha] = \epsilon^\alpha \ , \tag{4.76}$$

where ϵ_α is a constant, anticommuting, spinorial infinitesimal parameter. Likewise, the superfields are subject to the transformation [72]

$$\delta Y^\mu = [\bar{\epsilon}Q, Y^\mu] = \bar{\epsilon}QY^\mu \ , \tag{4.77}$$

which implies that the components transform as

$$\delta X^\mu = \bar{\epsilon}\psi^\mu \ , \tag{4.78}$$

$$\delta\psi^\mu = -\mathrm{i}\rho^a\epsilon\partial_a X^\mu + B^\mu\epsilon \ , \tag{4.79}$$

$$\delta B^\mu = -\mathrm{i}\bar{\epsilon}\rho^a\partial_a\psi^\mu \ . \tag{4.80}$$

In order to write down the SUSY equivalent of the Polyakov action (4.8), we must be able to construct supersymmetric kinetic terms, which is why we need a SUSY-covariant derivative:

$$D_\alpha = \frac{\partial}{\partial \bar{\theta}^\alpha} - \mathrm{i}(\rho^a \theta)_\alpha \frac{\partial}{\partial \xi^a} \ . \tag{4.81}$$

Acting this derivative on a superfield will again yield a superfield. Thus we can give the action of a superstring in flat space [71]:

$$S = \frac{\mathrm{i}}{4\pi} \int \mathrm{d}^2\xi \mathrm{d}^2\theta \left(\bar{D}^\alpha Y^\mu\right) \left(D_\alpha Y_\mu\right) \ , \tag{4.82}$$

which expanded into component form reads

$$S = -\frac{1}{2\pi} \int \mathrm{d}^2\xi \left(\partial_a X^\mu \partial^a X_\mu - \mathrm{i}\bar{\psi}^\mu \rho^a \partial_a \psi_\mu - B^\mu B_\mu \right) \ . \tag{4.83}$$

As we can see, the auxiliary fields B^μ have no dynamics and can be integrated out as $B^\mu = 0$.

Until now we have treated SUSY as a global worldsheet symmetry on a flat space. In order to promote it to a local symmetry, ϵ is turned from a constant parameter to a general function on the worldsheet $\epsilon(\xi)$. As in the case of SUGRA, this requires the incorporation of the worldsheet metric h_{ab} and its SUSY partner χ^a_α, the gravitino, where a is a vectorial index parameterising the worldsheet and α is a spinorial one. Note that in contrast to Section 4.1.2, here we have adopted the notation of Ref. [72] where χ denotes the gravitino and ψ other fermionic felds. Furthermore, it is common not to work with the metric h_{ab} directly but with zweibein fields, defined by

$$h_{ab} = e^m{}_a e^n{}_a \eta_{mn} \ . \tag{4.84}$$

As a first ansatz, we can now generalise Eq. (4.83) to a curved space by replacing η_{ab} by the general worldsheet metric h_{ab} and the partial derivative ∂_a by a covariant one ∇_a. The latter is built from the usual Levi-Civita connection Γ and a spin connection ω and is defined to vanish when applied to the zweibein [79],

$$\nabla_a e^m{}_b = \partial_a e^m{}_b + (\omega_a)^m{}_n e^n{}_b - \Gamma^c_{ab} e^m{}_c \equiv 0 \ . \tag{4.85}$$

As it is mentioned in Ref. [72], in the case of two-dimensional Majorana spinors, the connection does not contribute to the action so that we can simply write

$$S_2 = -\frac{1}{2\pi} \int \mathrm{d}^2\xi e \left(h^{ab} \partial_a X^\mu \partial_b X_\mu - \mathrm{i}\bar{\psi}^\mu \rho^a \partial_a \psi_\mu \right) \ , \tag{4.86}$$

where $e \equiv \det(e^m{}_a)$ and we have integrated out B^μ. The subscript '2' denotes the second-order nature of this action in the fields. This specificity is appropriate due to the fact that Eq. (4.86) is not the only contribution to the total superstring action. Namely, under the SUSY transformations (4.78) – (4.80) but with ϵ a function of ξ, the action is not invariant but receives a term [72, 79]

$$\delta_\epsilon S_2 = \frac{2}{\pi} \int \mathrm{d}^2\xi \sqrt{-h} \left(\nabla_a \bar{\epsilon} J^a \right) \ , \tag{4.87}$$

due to the derivative of $\bar{\epsilon}$ where

$$J^a = \frac{1}{2} \rho^b \rho^a \psi^\mu \partial_b X_\mu \tag{4.88}$$

is the so-called *supercurrent*, which is conserved under global SUSY transformations. This variation of the action can be cancelled by the addition of a third-order counter term

$$S_3 = -\frac{1}{\pi} \int \mathrm{d}^2\xi e \bar{\chi}_a \rho^b \rho^a \psi^\mu \partial_b X_\mu \ , \tag{4.89}$$

where the gravitino transforms as

$$\delta \chi_a = \nabla_a \epsilon \ . \tag{4.90}$$

Although with the term S_3 the variation $\delta_\epsilon S_2$ can be compensated, another contribution $\propto \nabla\epsilon$ is induced by this due to the variation of X_μ in S_3. The latter is balanced by an additional, fourth-order counter term:

$$S_4 = -\frac{1}{4\pi} \int \mathrm{d}^2\xi e \bar{\psi}_\mu \psi^\mu \bar{\chi}_a \rho^a \rho^b \chi_b. \tag{4.91}$$

The total superstring action is then given by [72, 79]

$$S_{\text{string}} = S_2 + S_3 + S_4 \ , \tag{4.92}$$

and is invariant under local SUSY transformations [72]

$$\delta X^\mu = \bar{\epsilon}\psi^\mu \ , \tag{4.93}$$

$$\delta \psi^\mu = -\mathrm{i}\rho^a \epsilon \left(\partial_a X^\mu - \bar{\psi}^\mu \chi_a \right) \ , \tag{4.94}$$

$$\delta e^m{}_a = -2\mathrm{i}\bar{\epsilon}\rho^a \chi_a \ , \tag{4.95}$$

$$\delta \chi_a = \nabla_a \epsilon \ , \tag{4.96}$$

with $\epsilon = \epsilon(\xi)$. Here it is noteworthy that $\delta\psi$ has undergone a further modification as compared to Eq. (4.79) in order to ensure local SUSY invariance. Moreover, S_{string} possesses, analogously to the bosonic Polyakov action (4.8), a local Weyl symmetry

$$\delta X^\mu = 0 \ , \tag{4.97}$$

$$\delta \psi^\mu = -\frac{1}{2}\Omega(\xi)\psi^\mu \ , \tag{4.98}$$

$$\delta e^m{}_a = \Omega(\xi) e^m{}_a \ , \tag{4.99}$$

$$\delta \chi_a = \frac{1}{2}\Omega(\xi)\chi_a \ , \tag{4.100}$$

as well as a local fermionic symmetry

$$\delta\chi_a = \mathrm{i}\rho_a\eta\ , \tag{4.101}$$
$$\delta X^\mu = \delta\psi^\mu = \delta e^m{}_a = 0 \tag{4.102}$$

where η is an arbitrary Majorana spinor, i.e. it fulfills the reality condition $\eta = \eta^*$. With the three symmetries (4.93) – (4.102) together, this theory is called *superconformal*.

As in the case of bosonic string theory, we can now use the diffeomorphism invariance of the worldsheet together with the local Weyl symmetry (4.97) – (4.100) to go to the flat-gauge worldsheet metric $h_{ab} = \eta_{ab}$. This corresponds to the elimination of the zweibein field $e^m{}_a = 0$. Additionally, the local SUSY (4.93) – (4.96) together with the local fermionic symmetry (4.101) and (4.102) allow us to eliminate the gravitino χ_a. Thus, the superstring action simplifies again to [79]

$$S_{\text{string}} = -\frac{1}{2\pi}\int \mathrm{d}^2\xi\,\left(\partial_a X^\mu \partial^a X_\mu - \mathrm{i}\bar{\psi}^\mu \rho^a \partial_a \psi_\mu\right)\ , \tag{4.103}$$

which, as in the case of bosonic string theory, has to be accompanied with the equations of motion for $e^m{}_a$ and χ_a [72]

$$T_{ab} = \partial_a X^\mu \partial_b X_\mu + \frac{\mathrm{i}}{2}\bar{\psi}^\mu \rho_{(a}\partial_{b)}\psi_\mu - \frac{1}{2}\eta_{ab}\left(\partial_c X_\mu \partial^c X^\mu + \frac{\mathrm{i}}{2}\bar{\psi}^\mu \rho^c \partial_c \psi_\mu\right) = 0\ , \tag{4.104}$$

$$J_a \equiv -\frac{\pi}{2e}\frac{\delta S}{\delta\chi^a} = \frac{1}{2}\rho^b \rho_a \psi^\mu \partial_b X_\mu = 0\ , \tag{4.105}$$

which are the so-called *super-Virasoro constraints*. For this simplified action, we can then again perform a mode decomposition, however, this time not only for the bosonic coordinates X^μ but also for the fermionic ones ψ^μ. Since the bosonic sector behaves just as before, let us focus on the fermionic one. Writing the components of the spinor explicitly

$$\psi^\mu = \begin{pmatrix} \psi_-^\mu \\ \psi_+^\mu \end{pmatrix}\ , \tag{4.106}$$

and changing to worldsheet light-cone coordinates (4.11), the fermionic sector of Eq. (4.103) can be written as [79]

$$S_{\text{string}} \supset S_{\text{F}} = \frac{\mathrm{i}}{\pi}\int \mathrm{d}\xi^+ \mathrm{d}\xi^-\,\left(\psi_-^\mu \partial_+ \psi_-^\mu + \psi_+^\mu \partial_- \psi_+^\mu\right)\ . \tag{4.107}$$

The equations of motion

$$\partial_\mp \psi^\mu_\pm = 0 \; , \tag{4.108}$$

imply that $\psi^\mu_+(\xi^+)$ and $\psi^\mu_-(\xi^-)$ are left- and right-moving waves, respectively. Since Lorentz invariance requires that fermions appear quadratically in any observable, a change of sign in the ψ^μ is not detectable so that their boundary conditions allow for a for a factor $+1$ or -1 after one string length l:

$$\psi^\mu_-(\tau,\sigma) = \pm\psi^\mu_-(\tau,\sigma+l) \; , \quad \psi^\mu_+(\tau,\sigma) = \pm\psi^\mu_+(\tau,\sigma+l) \; , \tag{4.109}$$

which should be compared to Eq. (4.15). Note that the sign for the left- and right-moving waves can be chosen indepedently where boundary conditions with no sign change represent the so-called *Ramond (R) sector* and those with a sign change represent the *Neveu-Schwarz (NS) sector*. The corresponding mode expansions are given by [80]

$$\psi^\mu_-(\xi^-) = \sqrt{\frac{2\pi}{l}} \sum_{n\in\mathbb{Z}} \beta^\mu_n \mathrm{e}^{-\frac{2\pi}{l}\mathrm{i}n\xi^-} \; , \tag{4.110}$$

$$\psi^\mu_+(\xi^+) = \sqrt{\frac{2\pi}{l}} \sum_{n\in\mathbb{Z}} \tilde{\beta}^\mu_n \mathrm{e}^{-\frac{2\pi}{l}\mathrm{i}n\xi^+} \; , \tag{4.111}$$

for the R sector and by

$$\psi^\mu_-(\xi^-) = \sqrt{\frac{2\pi}{l}} \sum_{r\in\mathbb{Z}+1/2} \beta^\mu_r \mathrm{e}^{-\frac{2\pi}{l}\mathrm{i}r\xi^-} \; , \tag{4.112}$$

$$\psi^\mu_+(\xi^+) = \sqrt{\frac{2\pi}{l}} \sum_{r\in\mathbb{Z}+1/2} \tilde{\beta}^\mu_r \mathrm{e}^{-\frac{2\pi}{l}\mathrm{i}r\xi^+} \; , \tag{4.113}$$

for the NS sector. Due to the independent sign choice of left- and right-moving waves, there are four possible combinations for closed strings, representing different sectors: NS–NS, R–NS, NS–R and R–R. For reasons that we will explain further below, the NS–NS and R–R sectors contain bosonic physical states whereas the R–NS and NS–R sectors contain fermionic ones.

Let us now again perform a light-cone quantisation by rotating the temporal and one spatial direction of the fermionic fields

$$\psi^\pm \equiv \frac{1}{\sqrt{2}}(\psi^0 \pm \psi^{D-1}) \; . \tag{4.114}$$

It is important not to confuse ψ^+ and ψ^-, where the target-space directions μ have been rotated to $+$ and $-$, with ψ_- and ψ_+, which are the spinorial components of ψ^μ_α as given in Eq. (4.106).

In the bosonic case, a residual gauge freedom on the worldsheet coordinates ξ allowed us to eliminate the oscillatory modes of X^+ (cf. Eq. (4.20)). Furthermore, the Virasoro constraint $T_{ab} = 0$ has been used to relate the modes of X^- to the transverse ones (cf. Eqs. (4.21) and (4.22)). In a similar manner, there is a residual freedom of local SUSY transformations which do not spoil the chosen flat gauge. With this residual gauge freedom, we can completely eliminate the component

$$\psi^+ = 0 \ . \tag{4.115}$$

Moreover, with the super-Virasoro constraints (4.104) and (4.105), we can again express the oscillatory modes of X^- and ψ^- in terms of the transversal ones; however, in contrast to the bosonic case, there is now a mixing of target-space and fermionic modes [72]

$$\alpha^-_n = \frac{1}{\sqrt{2\alpha' }p^+}\left(\sum_{m=-\infty}^{\infty} \alpha^i_{n-m}\alpha^i_m + \sum_{r=-\infty}^{\infty}\left(r-\frac{n}{2}\right) b^i_{n-r}b^i_r\right) \ , \tag{4.116}$$

$$b^-_r = \sqrt{\frac{2}{\alpha'}}\frac{1}{p^+}\sum_{s=-\infty}^{\infty}\alpha^i_{r-s}b^i_s \ . \tag{4.117}$$

This should be compared to Eq. (4.22). Of course, there are analogous relations for the left-moving wave modes $\tilde{\alpha}^-_n$ and $\tilde{\beta}^-_r$. We see that, as in the bosonic case, only the transverse modes are physical degrees of freedom. In order to quantise the theory, we promote these modes to operators. Their corresponding anticommutator relations read

$$\{\beta^i_r, \beta^j_s\} = \{\tilde{\beta}^i_r, \tilde{\beta}^j_s\} = \delta_{r+s,0}\delta_{ij} \ , \tag{4.118}$$

which have to be complemented with the commutator relations (4.27) for the α^i_n and $\tilde{\alpha}^i_n$. Obviously, due to the (anti-)commutator relations, the order of the operator products in Eqs. (4.116) and (4.117) becomes relevant, which is taken care of via normal ordering and the introduction of a normal ordering constant.

Again, it is important to ensure the criticality of the theory, that is the absence of a quantum anomaly of the Lorentz algebra. This can be achieved by choosing the target-space dimensionality to be $D = 10$.

In order to construct physical states, we need to define a ground state and let the operators α^i_n and β^i_r with $|n| < 0$ and $|r| < 0$, representing creation operators, act on it. The left- and right moving waves of a closed string individually behave like those of an open string. Except for a level-matching condition that ensures the same mode number of left and right movers, they are independent from each other [71]. Hence, the states of a closed string correspond to a tensor multiplication of open-string states for which we will analyse the spectrum separately in the NS and R sector.

The mass of an NS state is given by [71]

$$M_{\text{NS}}^2 = \frac{1}{\alpha'}\left(N_\perp^{(X)} + N_\perp^{(\psi,\text{NS})} - \frac{1}{2}\right) , \tag{4.119}$$

where

$$N_\perp^{(X)} = \sum_{n=1}^{\infty} \alpha_{-n}^i \alpha_n^i , \tag{4.120}$$

$$N_\perp^{(\psi,\text{NS})} = \sum_{r=1/2}^{\infty} r\beta_{-r}^i \beta_r^i , \tag{4.121}$$

are the number operators that count the level of excitation of transverse X and ψ modes, respectively. The ground state, defined by

$$\alpha_n^i|0;p\rangle_{\text{NS}} = \beta_r^i|0;p\rangle_{\text{NS}} = 0 \quad \forall n,r>0 , \tag{4.122}$$

has excitation level zero and therefore a mass

$$M_{\text{NS}}^2 = -\frac{1}{\alpha' 2} . \tag{4.123}$$

It is therefore again tachyonic, which in contrast to bosonic string theory however, can be remedied as explained below. The first excited level, $N_\perp^{(X)} = 0$ and $N_\perp^{(\psi,\text{NS})} = 1/2$, is obtained by acting the operator $b_{-1/2}^i$ on the ground state:

$$\beta_{-1/2}^i|0;p\rangle_{\text{NS}} . \tag{4.124}$$

The ground state does not carry any target-space indices and is therefore a spacetime scalar whereas the operaor β_n^i carries one index i. Hence, the state (4.124) is a spacetime vector with mass $\alpha' M_{\text{NS}}^2 = 1/2 - 1/2 = 0$.

In the R sector, the mass of a state is given by [71]

$$M_{\text{R}}^2 = \frac{1}{\alpha'}\left(N_\perp^{(X)} + N_\perp^{(\psi,\text{R})}\right) , \tag{4.125}$$

with

$$N_\perp^{(\psi,\text{R})} = \sum_{n=1}^{\infty} n\beta_{-n}^i \beta_n^i . \tag{4.126}$$

The ground state is again defined by

$$\alpha_n^i|0;p\rangle_{\mathrm{R}} = \beta_n^i|0;p\rangle_{\mathrm{R}} = 0 \quad \forall n > 0 \ , \tag{4.127}$$

however, since the zero modes β_0^i do not appear in the number operator $N_\perp^{(\psi,\mathrm{R})}$, we can act them on the ground state

$$\beta_0^i|0;p\rangle_{\mathrm{R}} \tag{4.128}$$

without changing its mass. Therefore the Ramond-sector ground state is degenerate. According to Eq. (4.118), the zero modes fulfill the Clifford algebra

$$\{\beta_0^\mu, \beta_0^\nu\} = \eta^{\mu\nu} \ , \tag{4.129}$$

so that the ground state becomes a target-space spinor that carries a spinorial index

$$|0;p;\alpha\rangle_{\mathrm{R}} \ , \tag{4.130}$$

with α ranging from 1 to 32. Excited states can now be constructed by acting the bosonic or fermionic creation operators $\alpha_{-|n|}^i$ and $\beta_{-|n|}^i$ on this state. However, according to Eq. (4.125), already the first excited level, which is obtained by acting either α_{-1}^i or β_{-1}^i on $|0;p;\alpha\rangle_{\mathrm{R}}$, is heavy. It is worth mentioning that all states in the Ramond sector represent spacetime spinors. This is the case because the ground state is a spacetime spinor whereas the creation operators are spacetime vectors (β^i only carries a worldsheet-spinor index but not a target-space one) so that the spinorial structure remains unaffected.

The GSO Projection

The key ingredient to discard the tachyonic NS ground state is the so-called Gliozzi-Scherk-Olive (GSO) projection, which maps the set of allowed physical states to a subset, in which the tachyon is absent. Crucial is the observation that $|0;p\rangle_{\mathrm{NS}}$ has an even number of fermionic excitation modes, namely zero. The operator [71]

$$G_{\mathrm{NS}} \equiv (-1)^{F_{\mathrm{NS}}+1} \ , \tag{4.131}$$

where $F_{\mathrm{NS}} = \sum_{r=1/2}^{\infty} \beta_{-r}^i \beta_r^i$ is the fermion number, assigns a positive sign to NS states with an odd number of fermionic oscillatory modes and a negative sign to states with an even one. The respective operator for the R sector is defined as

$$G_{\mathrm{R}} \equiv \left(\prod_{i=0}^{9} \Gamma_i \right) (-1)^{F_{\mathrm{R}}} \ , \tag{4.132}$$

where $F_{\mathrm{R}} = \sum_{n=1}^{\infty} \beta^i_{-n}\beta^i_n$ and Γ_i are the ten-dimensional Dirac matrices. Keeping now only states with positive G_{NS}, i.e. odd fermion number in the NS sector, the tachyonic NS ground state is projected out whereas the first excited level $\beta^i_{-1/2}|0;p\rangle_{\mathrm{NS}}$ and the R ground state $|0;p;\alpha\rangle_{\mathrm{R}}$ are retained. In fact, to get rid of the tachyon, it is indeed only necessary to project to positive G states in the NS sector whereas the R sector can be projected to either positive or negative states. For closed strings, this projection can now be performed independently for left- and right-moving states. Together with some other consistency conditions, this gives rise to only two possible, tachyon-free theories: type IIA and type IIB string theory [71]. In the former, left- and right-moving R states are projected to different signs under the G_{R} operator whereas in the latter the signs are the same. That is, the four closed-string sectors of type IIA string theory are NS_+–NS_+, R_-–NS_+, NS_+–R_+ and R_-–R_+ and of type IIB string theory NS_+–NS_+, R_+–NS_+, NS_+–R_+ and R_+–R_+ where the subscript denotes the sign under action of the G operator. Both theories are subject to a so-called *extended supersymmetry* with $\mathcal{N} = 2$ SUSY generators, which explains the number ‘II’ in their name. However, since in the low-energy effective theories we will be studying this extended SUSY is broken down to the usual $\mathcal{N} = 1$, we will not bother to elaborate on this topic. The main difference between the two theories is that type IIB is a chiral theory while type IIA is a non-chiral one. The reason for this is that in type IIA the different signs for left and right movers in the R sectors imply an opposite chirality of these fermionic states whereas in type IIB the equal signs imply the same chirality.

We will from now on only be dealing with type IIB theory because it is arguably the theory that allows for the most calculational control for phenomenological studies. To close this section, let us list the massless states of the type IIB closed-string sectors:

- NS–NS: This sector contains the states that we already know from bosonic string theory: a target-space graviton $g_{\mu\nu}$, an antisymmetric Kalb-Ramond p-form field $B_{[\mu\nu]}$ and a scalar field Φ called dilaton.

- R–NS and NS–R: The fermionic states in this sector consist of one spin 3/2 gravitino and one spin 1/2 dilatino per sector. The existence of two gravitini in total corresponds to the extended $\mathcal{N} = 2$ SUSY.

- R–R: In this sector there are novel p-form fields where p is even valued: C_0, C_2 and C_4.

4.2. String phenomenology

In the last section, we have introduced the most important basics of superstring theory, which led us to the definition of type IIB string theory. The latter constitutes the most extensively studied sector of string theory, which is why we will only be dealing with this theory in what follows. The goal is now to make contact with our four-dimensional world. In this procedure, we will start by quoting the low-energy limit of type IIB string theory, which is a ten-dimensional $\mathcal{N} = 2$ SUGRA theory. Then a *Kaluza-Klein compactification* of the internal, six-dimensional space is applied, which results in an effective four-dimensional $\mathcal{N} = 2$ SUGRA, where many degrees of freedom from the internal space arise in the form of a plethora of massless scalar fields called *moduli*. Here in a so-called *orientifold projection*, the extended, four-dimensional $\mathcal{N} = 2$ SUSY is broken down to $\mathcal{N} = 1$. For reasons of consistency, that is in order to fulfill a so-called *tadpole cancellation*, this also requires the inclusion of D-branes as will be explained later. Since a large amount of massless scalar fields would imply a significant violation of observations, it will be of special interest to stabilise the moduli fields, i.e. to generate a scalar potential for them. This will ultimately lead us to the so-called *Large Volume Scenario* (LVS), which makes use of a large internal volume to achieve some level of calculational control by expanding in the inverse of the volume, that represents a small parameter. The LVS constitutes the overall setting of the work presented in this part.

4.2.1. Type IIB ten-dimensional supergravity

We begin the path to our four-dimensional world with a formulation of the low-energy limit of type IIB string theory in action form where we will focus on the bosonic sector. Our main references for this section are [68, 69, 71, 79]. As mentioned before, the massless spectrum consists of the ten-dimensional metric $g_{\mu\nu}$, the antisymmetric Kalb-Ramond field $B_{[\mu\nu]}$ and the dilaton Φ in the NS–NS sector, of the p-form fields C_0, C_2 and C_4 in the R–R sector, and of fermions in the mixed sectors R–NS and NS–R. These degrees of freedom also constitute the field content of the resulting ten-dimensional SUGRA that represents the low-energy limit. The bosonic sector of the action can be classified into three parts [68, 69, 71]

$$S_{\text{IIB}} = S_{\text{NS}} + S_{\text{R}} + S_{\text{CS}} \ . \tag{4.133}$$

The Neveu-Schwarz part is given by [69]

$$S_{\text{NS}} = \frac{1}{2\kappa_{10}^2} \int \mathrm{d}^{10}x\sqrt{-g}\mathrm{e}^{-2\Phi} \left(R + 4\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}|H_3|^2 \right) \ , \tag{4.134}$$

where κ_{10} is the ten-dimensional gravitational coupling, given by $2\kappa_{10}^2 = (2\pi)^7\alpha'^4 = l_\text{s}^8/(2\pi) = (2\pi)^7 M_\text{s}^{-8}$, and $H_3 \equiv \mathrm{d}B_2$ is the exterior derivative of the 2-form Kalb-Ramond field B_2, i.e. its field-strength tensor, and can be written as

$$|H_3|^2 = \frac{1}{3!} H_{\mu\nu\rho} H^{\mu\nu\rho}. \tag{4.135}$$

This part of the action corresponds to the massless states of the bosonic string and should be compared to Eq. (4.34). Additionally, there is the Ramond part, which has no counterpart in bosonic string theory and is given by [69]

$$S_\text{R} = -\frac{1}{4\kappa_{10}^2} \int \mathrm{d}^{10}x \sqrt{-g} \left(|F_1|^2 + |\tilde{F}_3|^2 + \frac{1}{2}|\tilde{F}_5|^2 \right) , \tag{4.136}$$

where

$$\tilde{F}_3 = F_3 - C_0 H_3 , \tag{4.137}$$

$$\tilde{F}_5 = F_5 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3 , \tag{4.138}$$

with $\wedge$ being the totally antisymmetric tensor product. Here the F_{p+1} are the exterior derivatives of the p-form fields C_p, respectively, i.e. their field-strength tensors, which are given by

$$|F_n|^2 = \frac{1}{n!} F_{\mu_1 \cdots \mu_n} F^{\mu_1 \cdots \mu_n} . \tag{4.139}$$

The composition via the tilde field strengths is required to retain gauge invariance. Furthermore, the 5-form field strength fulfills a self-duality condition that has to be imposed on the equations of motion by hand:

$$\tilde{F}_5 = \star \tilde{F}_5 , \tag{4.140}$$

where $\star$ is the Hodge star operator, which maps a p-form on a D-dimensional manifold to a $D-p$-form. Finally, the so-called *Chern-Simons (CS) term* corresponds to the integral of a p-form over a whole D-dimensional manifold with $p = D$, which does not depend on the metric and, in the case of type IIB theory, is given by [69]

$$S_\text{CS} = -\frac{1}{4\kappa_{10}^2} \int C_4 \wedge H_3 \wedge F_3 . \tag{4.141}$$

Note that due to the nilpotency of the exterior derivative, that is $\mathrm{d}(\mathrm{d}\omega_p) = 0$ for any p-form ω_p, and the antisymmetry of the wedge product $\wedge$, there is no other combination to construct a 10-form from B_2, the C_p and their field strengths H_3 and F_{p+1} other than the one in Eq. (4.141).

Additionally to the bosonic sector S_{IIB}, the theory can contain local objects, which are so-called Dp-branes.[2] Here the letter 'D' stands for 'Dirichlet' because these branes act as hypersurfaces to which open strings with Dirichlet boundary conditions are attached to whereas the p denotes the spatial dimensionality of the brane. Importantly, Dp-branes are not only abstract solutions for the trajectories of the endpoints of open strings but rather represent dynamical objects that traverse the ten-dimensional spacetime.

One crucial insight about brane phenomenology is the fact that open strings with both ends attached to a brane give rise to a $U(1)$ gauge theory living on this brane in the low-energy limit. Furthermore, one can consider a whole stack of N parallel Dp-branes of equal dimensionality. There will then be a degeneracy at the endpoints of open strings resulting from the circumstance that each endpoint can be attached to a separate brane of the stack, respectively. This degeneracy leads to an extension of the corresponding gauge group from $U(1)$ to $U(N)$. If one now adds a second stack of M branes which intersects the first one, open strings that start on one stack and end on the other are confined to the intersection space and behave as fermions that are charged under the combined gauge group $U(N) \times U(M)$ in the low-energy limit. This is a key ingredient in many attempts to construct GUT-like theories or the SM gauge group in string phenomenology. Generally, the dynamics of a stack of p-branes in an ambient space and the corresponding gauge theory living on it is described by the so-called *Dirac-Born-Infeld* (DBI) action [69]:

$$S_{\mathrm{DBI}} = -T_p \int_{\mathbb{R}^{1,p}} \mathrm{d}^{p+1}\xi \, \mathrm{Tr}\left[\mathrm{e}^{-\Phi}\sqrt{-\det\left(G_{ab} + B_{ab} + 2\pi\alpha' F_{ab}\right)}\right] , \tag{4.142}$$

where ξ are brane coordinates, the integral goes over the brane worldvolume $\mathbb{R}^{1,p}$, $T_p = 2\pi/l_{\mathrm{s}}^{p+1}$ is the brane tension, the trace runs over the generators of the gauge group, F_{ab} is the field strength of the gauge group, and G_{ab} and B_{ab} are the pullbacks of the spacetime metric and the Kalb-Ramond field to the brane worldvolume:

$$G_{ab} = \frac{\partial X^\mu}{\partial \xi^a}\frac{\partial X^\nu}{\partial \xi^b} g_{\mu\nu} , \tag{4.143}$$

$$B_{ab} = \frac{\partial X^\mu}{\partial \xi^a}\frac{\partial X^\nu}{\partial \xi^b} B_{\mu\nu} . \tag{4.144}$$

Expanding the square root in Eq. (4.142) gives rise to the effective action of the Yang-Mills theory living on the brane stack, which at quadratic order in α' reads [84]

$$S_{\mathrm{DBI}} \supset S_{\mathrm{YM}} = -\frac{1}{4(2\pi)g_{\mathrm{s}}l_{\mathrm{s}}^{5+p}} \int \mathrm{d}^{p+1}\xi \sqrt{-g}\, \mathrm{Tr}\mathcal{F}_{ab}\mathcal{F}^{ab} , \tag{4.145}$$

[2]Depending on the context, one often also speaks of 'D-branes', 'p-branes' or simply 'branes'.

where $\mathcal{F}_{ab} \equiv F_{ab} + B_{ab}/(2\pi\alpha')$ and we used that the string coupling is determined by the dilaton: $g_s = \exp(\langle\Phi\rangle)$.

Besides the DBI action, Dp-branes also allow for CS terms where $p+1$-form fields are integrated over the brane. The corresponding, general action reads [68, 69]

$$S_{\text{brane,CS}} = \mathrm{i}\mu_p \int_{\mathbb{R}^{1,p}} \mathrm{Tr}\left[\exp\left(2\pi\alpha' F_2 + B_2\right) \wedge \sum_q C_q\right] , \tag{4.146}$$

where $\mu_p = 2\pi/l_s^{p+1}$ describes the charge of the p-brane under C_{p+1}. Here the sum runs formally over all R–R p-form fields where through the integral the fitting terms are picked out. This action contains the simple integral of the $p+1$-form field C_{p+1} over the total worldvolume $\mathbb{R}^{1,p}$ of the p-brane:

$$S_{\text{brane,CS}} \supset \mu_p \int_{\mathbb{R}^{1,p}} C_{p+1} , \tag{4.147}$$

This constitutes a natural coupling between any p-brane and a $p+1$-form field C_{p+1}, which can be understood as a generalisation of an electron-photon coupling where a 0-brane (particle) is charged under a 1-form field. Moreover, $S_{\text{brane,CS}}$ induces the integral [84]

$$S_{\text{brane,CS}} \supset 2\pi \int_{\mathbb{R}^{1,q+3}} C_q \wedge \frac{1}{8\pi^2} \mathrm{Tr}\, F_2 \wedge F_2 . \tag{4.148}$$

over the $q+3$-brane, which represents the instanton action for axions as we will see later.

In summary, the total action of the type IIB theory is given by

$$S_{\text{tot,IIB}} = S_{\text{IIB}} + S_{\text{DBI}} + S_{\text{brane,CS}} \tag{4.149}$$

plus further fermionic terms stemming from the R–NS and NS–R sectors and with the self-duality condition (4.140) imposed by hand.

4.2.2. Kaluza-Klein compactification, Calabi-Yau manifolds and moduli spaces

The action (4.149) describes a ten-dimensional SUGRA with localised objects, namely Dp-branes. However, as a matter of fact, on macroscopic scales the universe is effectively four-dimensional. The most evident proof for this are the experiences in our everyday life but this has also been tested by measurements of Newton's inverse-square law for gravity [85]. The latter imply that our universe remains effectively four-dimensional down to $\mathcal{O}(\text{mm})$ length scales, which naively seems to be a surprisingly large bound. This can be understood by keeping in mind that gravity, being a very weak force, is rather difficult to measure at small length scales. In contrast, the other known forces due to SM gauge bosons are much stronger but are typically constituted by open strings, as explained above, which are attached to branes. and hence do not 'feel' the higher-dimensional bulk space [70].

As a consequence of our macroscopically four-dimensional universe, an implementation of a ten-dimensional SUGRA as a UV completion of the SM requires that six of nine spatial directions must be compact and small enough to evade the aforementioned bounds. In this subsection, we will mostly use the references [71, 79]. We also recommend [86] for a detailed, mathematical treatment of the following discussion about complex manifolds as well as homology and cohomology groups.

Kaluza-Klein theory

The original idea of a compactified internal space stems from the Kaluza-Klein (KK) theory [87–89], which describes general relativity on a $4+1$-dimensional spacetime with one of the spatial directions being compactified to a circle; that is the theory lives on the manifold $\mathcal{M} = \mathbb{R}^{1,3} \times S^1$. For illustrative reasons, let us add a scalar field to the five-dimensional Einstein-Hilbert action [79],

$$S_{\text{KK}} = \int \mathrm{d}^5 x \sqrt{-g_{(5)}} \left(\frac{M_{\text{P},(5)}^3}{2} R_{(5)} + \frac{1}{2} \partial_M \phi \partial^M \phi \right) , \tag{4.150}$$

where the subscript '(5)' indicates five-dimensional quantities and the capital-letter index takes on the values $M = 0 \cdots 4$. In order to find the effective, four-dimensional theory, we have to integrate out the internal dimension, i.e. we have to replace

$$\int_{\mathcal{M}} \mathrm{d}^5 x \to \int_{\mathbb{R}^{1,3}} \mathrm{d}^4 x \int_{S^1} \mathrm{d}y , \tag{4.151}$$

and perform the integral over $y \equiv x_4$. Parameterising the internal direction by $y \in [0, 2\pi R]$, where R is the typical length-scale of the internal dimension or the 'radius' of the circle S^1, it is clear that the scalar field – just like any other quantity – must be periodic in the internal coordinate:

$$\phi(x,y) = \phi(x, y + 2\pi R) . \tag{4.152}$$

As such, we can expand ϕ in sines and cosines of y [79]:

$$\phi(x,y) = \sum_{n=0}^{\infty} \phi_n^c(x) \cos(ny/R) + \sum_{n=1}^{\infty} \phi_n^s(x) \sin(ny/R) . \tag{4.153}$$

We see that the zero-mode ϕ_0^c does not come with a factor that depends on y whereas the other modes are multiplied by trigonometric functions. For this reason, when one inserts the above expansion into Eq. (4.150) and integrates out y, ϕ_0^c is just multiplied with $2\pi R$ and will act as a massless scalar field whereas the higher modes obtain mass terms with $m_n^2 = n^2/R^2$ and build a so-called *Kaluza-Klein tower*. That is, from a four-dimensional perspective below the KK scale $m_{\mathrm{KK}} = 1/R$, the five-dimensional theory for ϕ appears as one four-dimensional, massless scalar mode accompanied by an infinite tower of massive scalars. On the other hand, above the KK scale the theory becomes effectively five-dimensional.

Furthermore, the five-dimensional Einstein-Hilbert action can be brought to a low-energy 4D effective theory. Then the five-dimensional metric $g_{(5)\mu\nu}$ splits up into the usual 4D metric $g_{\mu\nu}$, a $U(1)$ gauge field A_μ, whose symmetry is a consequence of 5D diffeomorphism invariance and which is associated with the components $g_{(5)\mu 4}$, and a scalar field φ, which is associated with the component $g_{(5)44}$. The latter is also called a *radion* and parameterises the size of the internal dimension $\varphi \sim R$ with the exact prefactor depending on conventions.

Calabi-Yau manifolds

Now we want to generalise the above ideas in order to compactify six out of nine spatial dimensions in type IIB superstring theory. Obviously, moving from one compactified dimension to six complicates things tremendously. One important aspect to consider is that the 10D spacetime should allow for vacuum solutions of the Einstein equations. The latter implies 10D Ricci flatness, which on the other hand implies that the 6D internal space should be Ricci flat as well. Moreover, when compactifying to four dimensions, we ask for a certain amount of SUSY breaking. While retaining too many SUSY generators contradicts observations, a total breaking of all SUSY would imply a loss of calculational control over loop and α' corrections, which will be further explained later. These requirements justify the choice of *Calabi-Yau (CY) manifolds* as the internal space, which are by definition Ricci flat and break 3/4 of the ten-dimensional SUSY [79].

To define a CY manifold, let us start with defining a *complex manifold* of complex dimension n as a real manifold of real dimension $2n$, whose transition functions between charts as well as their inverses are holomorphic [71]. This motivates to use complex coordinates z^a and corresponding complex conjugates $\bar{z}^{\bar{a}}$, which do not mix with each other by a change of coordinates. Moreover, a complex manifold implies the existence of a globally defined, mixed tensor, the so-called *complex structure*, which fulfills $J^2 = -\mathbb{1}$ and whose components in the complex coordinates are given by[3]

$$J_a{}^b = \mathrm{i}\delta_a{}^b\ , \quad J_{\bar{a}}{}^{\bar{b}} = -\mathrm{i}\delta_{\bar{a}}{}^{\bar{b}}\ , \quad J_a{}^{\bar{b}} = J_{\bar{a}}{}^b = 0\ . \tag{4.154}$$

Equipping a complex manifold with a metric, yields a *complex Riemannian manifold* and if that metric can be locally written as the second derivative of a function

$$g_{a\bar{b}} = \frac{\partial}{\partial z^a}\frac{\partial}{\partial \bar{z}^{\bar{b}}}\mathcal{K}(z,\bar{z})\ , \tag{4.155}$$

one speaks of a *Kähler manifold* where the function $\mathcal{K}$ is called the Kähler potential.[4] The general line element of a complex manifold can be written as [71]

$$\mathrm{d}s^2 = g_{ab}\mathrm{d}z^a\mathrm{d}z^b + g_{a\bar{b}}\mathrm{d}z^a\mathrm{d}\bar{z}^{\bar{b}} + g_{\bar{a}b}\mathrm{d}\bar{z}^{\bar{a}}\mathrm{d}z^b + g_{\bar{a}\bar{b}}\mathrm{d}\bar{z}^{\bar{a}}\mathrm{d}\bar{z}^{\bar{b}}\ , \tag{4.156}$$

but for a Kähler manifold, due to hermiticity of the metric as is implied by Eq. (4.155), the elements $g_{ab} = g_{\bar{a}\bar{b}} = 0$ vanish. Moreover, in order for the metric to be real, $g_{a\bar{b}}$ must be the complex conjugate of $g_{\bar{a}b}$. The hermiticity condition also allows us to define the so-called *Kähler form*

$$J = \mathrm{i}g_{a\bar{b}}\mathrm{d}z^a \wedge \mathrm{d}\bar{z}^{\bar{b}}\ , \tag{4.157}$$

which is obtained by lowering one index of the complex structure with $g_{a\bar{b}}$ and will become important later. Crucially, we see that the Kähler metric $g_{a\bar{b}}$ and Kähler form J are closely related and in fact imply each other, once a specific complex structure is given. Finally, one way to define a Calabi-Yau manifold is to identify it as a compact Kähler manifold that is Ricci flat.

Since type IIB string theory and its corresponding low-energy $\mathcal{N} = 2$ SUGRA lives in ten dimensions, we need to compactify a total of six thereof. We are therefore especially interested in the properties of CY 3-folds. While there are only very few examples of compact CY manifolds in one or two complex dimensions, namely the two-torus T^2 for $n = 1$ and the four-torus T^4 and the K3 surface for $n = 2$, there is a plethora of CY 3-folds and it remains an unsolved problem whether their number is even finite [71]. We will not elaborate deeply on the construction of CY n-folds for $n > 2$ but merely mention that such a construction is possible as sub-

[3]Note that the existence on a complex structure on a real manifold is just a necessary but not a sufficient condition for it to be a complex manifold. Only if additionally the so-called *Nijenhuis tensor* vanishes, is it given that the transition functions are holomorphic and that the manifold is complex.

[4]Note that the equality of the name 'Kähler potential' compared to the context of supergravity is not accidental: The first term in Eq. (4.49) induces non-canonical kinetic terms for the chiral superfields Φ^i of the form $K_{i\bar{\jmath}}\partial_\mu\Phi^i\partial^\mu\bar{\Phi}^{\bar{\jmath}}$ where $K_{i\bar{\jmath}} \equiv \partial_{\Phi^i}\partial_{\bar{\Phi}^{\bar{\jmath}}}\mathcal{K}$. Thus the field space of the Φ^i can be interpreted as a Kähler manifold with Kähler metric $K_{i\bar{\jmath}}$ derived from the Kähler potential $\mathcal{K}$.

manifolds of so-called *complex projective spaces*. Basically, an $n+1$-dimensional complex projective space $\mathbb{C}P^{n+1}$ is a compact manifold defined as $\mathbb{C}^{n+1}/\{0\}$ with the identification

$$(z^1, \ldots, z^{n+1}) \sim (\lambda z^1, \ldots, \lambda z^{n+1}) \;, \quad \lambda \in \mathbb{C}/\{0\} \;. \tag{4.158}$$

If we now constrain $\mathbb{C}P^{n+1}$ via a polynomial equation

$$\text{Pol}(z^1, \ldots, z^{n+1}) = 0 \;, \tag{4.159}$$

where Pol is a homogeneous polynomial of degree $n+2$, i.e.

$$\text{Pol}(\lambda z^1, \ldots, \lambda z^{n+1}) = \lambda^{n+2}\text{Pol}(z^1, \ldots, z^{n+1}) \;, \tag{4.160}$$

then the resulting submanifold of $\mathbb{C}P^{n+1}$ is a CY n-fold. Several generalisations are possible, for instance by starting with a so-called *weighted complex projective space* instead, for which the identification in Eq. (4.158) is weighted with different powers of λ for the individual z^i [71, 79].

The topological structure of a CY n-fold can be characterised by its homology and cohomology groups, which we will quickly summarise here. On a real manifold $\mathcal{M}$ we can define p-forms A_p, which are antisymmetric rank-p tensors, and their corresponding exterior derivatives $\text{d}A_p$, which are $p+1$-forms. A p-form A_p is called *closed* if its exterior derivative vanishes $\text{d}A_p = 0$ and is called *exact* if it constitutes the exterior derivative of a $p-1$-form $A_p = \text{d}B_{p-1}$. Crucially, the exterior derivative is nilpotent, i.e. $\text{d}(\text{d}A_p) = 0$ for any p-form. We can then define the p-th *de Rham cohomology group* of $\mathcal{M}$ as the quotient space of all closed p-forms divided by all exact p-forms [79, 86],

$$H^p(\mathcal{M}) = \frac{\{A_p | \text{d}A_p = 0\}}{\{A_p | A_p = \text{d}B_{p-1} \text{ for some } B_{p-1}\}} \;. \tag{4.161}$$

In other words, $H^p(\mathcal{M})$ consists of equivalence classes of all closed p-forms on $\mathcal{M}$ with those p-forms which differ only by an exact p-form lying in the same class.

In a similar fashion, we can analyse submanifolds of $\mathcal{M}$. A linear superposition of p-dimensional submanifolds is called p-chain. The boundary of a p-chain is again a chain but of reduced dimensionality $p-1$. A p-chain which has no boundary is called a p-cycle. Noting that every boundary does not have a boundary itself, i.e. is a cycle, one can now in analogy to the above treatment of p-forms define the quotient space of p-cycles and p-chains which are a boundary. This is called the

p-th *simplicial homology group*:

$$H_p(\mathcal{M}) = \frac{\{p\text{-cycles}\}}{\{p\text{-dimensional boundaries}\}} \ . \tag{4.162}$$

Intuitively, $H_p(\mathcal{M})$ contains all p-cycles of $\mathcal{M}$ where those p-cycles which differ by a boundary are identified with each other. One can show that the cohomology and homology groups of a given manifold have the same dimension for each p. These dimensionalities are called the *Betti numbers*:

$$b_p \equiv \dim H^p(\mathcal{M}) = \dim H_p(\mathcal{M}) \ , \tag{4.163}$$

and they represent important topological data to characterise a manifold $\mathcal{M}$.

One important result, that we briefly want to mention here, is the fact that there exists an isomorphism $H^p(\mathcal{M}) \simeq H_{d-p}(\mathcal{M})$ with d the dimensionality of $\mathcal{M}$. This is known as *Poincaré duality* [79, 86].

In order to generalise the above discussion to the case of a complex manifold X, we first note that differential forms can now carry holomorphic and antiholomorphic indices so that we may speak of (p,q)-forms [90],

$$A_{p,q} = \frac{1}{p!q!} A_{i_1 \cdots i_p \bar{j}_1 \cdots \bar{j}_q} \mathrm{d}z^{i_1} \wedge \cdots \wedge \mathrm{d}z^{i_p} \wedge \mathrm{d}\bar{z}^{\bar{j}_1} \wedge \cdots \wedge \mathrm{d}\bar{z}^{\bar{j}_q} \ . \tag{4.164}$$

The complex analogue to the exterior derivative are the *Dolbeault operators*

$$\partial = \mathrm{d}z^i \frac{\partial}{\partial z^i} \ , \quad \bar{\partial} = \mathrm{d}\bar{z}^{\bar{j}} \frac{\partial}{\partial \bar{z}^{\bar{j}}} \ , \tag{4.165}$$

which add up to the exterior derivative, $d = \partial + \bar{\partial}$, but individually either act on the holomorphic indices of $A_{p,q}$ or on the antiholomorphic ones, respectively. Thus, ∂ maps $A_{p,q}$ to a $(p+1,q)$-form and $\bar{\partial}$ to a $(p,q+1)$-form. Both Dolbeault operators are nilpotent like the exterior derivative. Therefore, we can define a separate cohomology group for each Dolbeault operator in analogy to the de Rham cohomology group. For instance, for the antiholomorphic operator $\bar{\partial}$, we have [79]

$$H^{p,q}_{\bar{\partial}}(X) \frac{\{A_{p,q} | \bar{\partial} A_{p,q} = 0\}}{\{A_{p,q} | A_{p,q} = \bar{\partial} B_{p,q-1} \text{ for some } B_{p,q-1}\}} \ . \tag{4.166}$$

It turns out that in the case of Kähler manifolds and hence also CY n-folds, the Dolbeault cohomology groups are identical for the respective operators ∂ and $\bar{\partial}$ so that a distinction between the two is superfluous. The complex analogue to the Betti numbers are the so-called *Hodge numbers*, which count the dimensionality of the

Dolbeault cohomology group

$$h^{p,q} \equiv \dim H^{p,q}(X) \ , \tag{4.167}$$

and provide a finer distinction w.r.t. the complex structure than the Betti numbers. In fact, the two are related by

$$b_k = \sum_{p=0}^{k} h^{p,k-p} \ . \tag{4.168}$$

Just like the Betti numbers provide important topological data for real manifolds, so do the Hodge numbers in the case of complex ones. In order to provide a well-structured depiction, they are often arranged in a so-called *Hodge diamond*, which for even complex dimension n is given by

$$\begin{array}{ccccc} & & h^{n,n} & & \\ & \cdot^{\cdot^{\cdot}} & \vdots & \ddots & \\ h^{n,0} & \cdots & h^{n/2,n/2} & \cdots & h^{0,n} \\ & \ddots & \vdots & \cdot^{\cdot^{\cdot}} & \\ & & h^{0,0} & & \end{array} \tag{4.169}$$

and for odd n by

$$
\begin{array}{ccccccc}
 & & & h^{n,n} & & & \\
 & & & \vdots & & & \\
 & & & h^{(n+1)/2,(n+1)/2} & & & \\
h^{n,0} & \cdots & h^{(n+1)/2,(n-1)/2} & & h^{(n-1)/2,(n+1)/2} & \cdots & h^{0,n} \\
 & & & h^{(n-1)/2,(n-1)/2} & & & \\
 & & & \vdots & & & \\
 & & & h^{0,0} & & &
\end{array}
\tag{4.170}
$$

Here the dotted lines represent other Hodge numbers $h^{p,q}$ inbetween.

In the case of CY manifolds, the topological structure is highly constrained so that the Hodge diamond takes on a very specific form. For complex dimension $n = 3$, which we will be mostly interested in, it is given by [71, 79]

$$
\begin{array}{ccccccc}
 & & & h^{3,3} & & & \\
 & & h^{3,2} & & h^{2,3} & & \\
 & h^{3,1} & & h^{2,2} & & h^{1,3} & \\
h^{3,0} & & h^{2,1} & & h^{1,2} & & h^{0,3} \\
 & h^{2,0} & & h^{1,1} & & h^{0,2} & \\
 & & h^{1,0} & & h^{0,1} & & \\
 & & & h^{0,0} & & &
\end{array}
=
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & h^{1,1} & & 0 & \\
1 & & h^{2,1} & & h^{2,1} & & 1 \\
 & 0 & & h^{1,1} & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 1 & & &
\end{array}
$$

From this, we can see that there are only two independent Hodge numbers, $h^{1,1}$ and $h^{2,1}$, which are not fixed in a CY 3-fold. We furthermore note that $h^{0,0} = 1$, which describes closed zero-forms, i.e. constant functions. This a universal property of every compact connected Kähler manifold [90]. Finally, $h^{3,0}$ and $h^{0,3}$ corresponds to a unique, harmonic three-form Ω and its antiholomorphic counterpart $\bar{\Omega}$ with

$$
\Omega = \frac{1}{6}\Omega_{ijk}(z)\mathrm{d}z^i \wedge \mathrm{d}z^j \wedge \mathrm{d}z^k \ , \tag{4.171}
$$

which plays an important role for the stabilisation of moduli fields as will be explained below. By *harmonic* we mean that

$$\Delta\Omega = 0 \,, \tag{4.172}$$

where $\Delta \equiv \mathrm{d}\delta + \delta\mathrm{d}$ is the Laplace-de Rham operator with $\delta \equiv (-1)^p \star^{-1} \mathrm{d}\star$ being the co-differential. The latter maps a p-form to a $p-1$-form.

Metric moduli

We are now in the position to discuss deformations of the CY metric, which will lead us to the introduction of metric moduli and their associated moduli space. In an effective 4D description, these moduli will represent a priori massless scalar fields, which gain a potential by including fluxes or perturbative and non-perturbative effects and which will be the main players for the dynamical systems that we consider in the rest of this thesis.

To start from the beginning, we want to consider deformations of the metric $g_{mn} \to g_{mn} + \delta g_{mn}$ so that Ricci flatness is retained, $R_{mn}(g) = R_{mn}(g+\delta g) = 0$, where the indices m and n can take on either holomorphic or anti-holomorphic values. As an additional requirement to Ricci flatness of the perturbed metric, we want to make sure that these deformations are 'physical' and not merely a change of coordinates, so that we fix the gauge via [90]

$$\nabla^m \delta g_{mn} = \frac{1}{2} \nabla_n g^{mp} \delta g_{mp} \,. \tag{4.173}$$

We can then perturb the metric by either $\delta g_{a\bar{b}}$ or δg_{ab} and their respective hermitian conjugates. Solving the vacuum Einstein equations for the perturbed metric together with the gauge-fixing condition (4.173), it turns out that the above two sorts of deformations decouple.

The former ones, $\delta g_{a\bar{b}}$, are clearly related to a change of the Kähler form

$$\delta J_{a\bar{b}} = \mathrm{i} \delta g_{a\bar{b}} \,. \tag{4.174}$$

From Eq. (4.173) it also follows that the deformations and hence the Kähler form are harmonic, which motivates to expand them in a basis of harmonic $(1,1)$-forms [79],

$$J = t^i \omega_i \,, \quad i \in \{1, \cdots, h^{1,1}\} \,. \tag{4.175}$$

The prefactors t^i of the expansion are the volumes of 2-cycles Σ_2^i and are called *Kähler moduli*. They are measured by integrating the Kähler form over their corresponding 2-cycle,

$$t^i = \int_{\Sigma_2^i} J \ , \tag{4.176}$$

and their number corresponds to the number of the cohomologically distinct $(1,1)$-forms ω_i, which is given by $h^{1,1}$. We can also integrate the square of the Kähler form over the 4-cycle Σ_4^i that is Poncaré dual to ω_i to obtain its volume [79]

$$\tau_i = \frac{1}{2}\int_{\Sigma_4^i} J \wedge J = \frac{1}{2}\kappa_{ijk} t^j t^k \ , \tag{4.177}$$

where κ_{ijk} are the so-called *triple intersection numbers*

$$\kappa_{ijk} \equiv \int_X \omega_i \wedge \omega_j \wedge \omega_k \ . \tag{4.178}$$

Finally, by integrating the cube of the Kähler form over the whole manifold X, we obtain the overall internal volume in string units in the Einstein frame,

$$\mathcal{V} = \frac{1}{6}\int_X J \wedge J \wedge J = \frac{1}{6}\kappa_{ijk} t^i t^j t^k \ , \tag{4.179}$$

from which we also infer the useful relation $\tau_i = \partial\mathcal{V}/\partial t^i$. Since the metric is positive-definite, the quantities t^i, τ_i and $\mathcal{V}$ are positive as well [90]. Instead of the t^i, we will most of the time use the τ_i to analyse and describe the resulting dynamics, which contain the same information as the t^i and to which we will also refer as 'Kähler moduli'.

One important aspect is the fact that when we compactify ten-dimensional type IIB string theory on a CY 3-fold, the 4-cycle moduli τ_i combine with the integrals of the R–R 4-form C_4 over the respective 4-cycles into supermultiplets [79],

$$T_i = \tau_i + \mathrm{i}\theta_i \ , \quad \theta_i = \int_{\Sigma_4^i} C_4 \ , \tag{4.180}$$

which is called *complexification* of the moduli. The θ_i are intrinsically periodic and represent *axions* with the above defined Kähler moduli τ_i being their corresponding *saxions*. In a similar manner, the R-R 0-form field C_0 and the dilaton combine into the so-called *axio-dilaton*

$$S = C_0 + \mathrm{i}e^{-\Phi} \ , \tag{4.181}$$

which is generically present in type IIB string theory. To recapitulate, the T_i and S constitute superfields in the effective low-energy, four-dimensional SUGRA, which is why their dynamics are governed by the respective terms described in Sec. 4.1.2.

The other kind of metric deformation, δg_{ab} and its hermitian conjugate, violates the hermiticity of the metric (4.156), which implies that g_{ab} and $g_{\bar{a}\bar{b}}$ are zero. For this reason, these deformations involve a change of the complex structure. Again the gauge-fixing condition (4.173) implies harmonicity of these deformations; however, unlike the Kähler deformations $\delta g_{a\bar{b}}$, the δg_{ab} cannot be expanded in $(2,0)$-forms. The reason is simply that $h^{2,0} = h^{0,2} = 0$ for a CY 3-fold, which is why there are none present. Nevertheless, one can use the unique 3-form Ω in order to expand in $(1,2)$-forms $\bar{\chi}_\alpha$ [79, 90],

$$\delta g_{ab} = \frac{i}{||\Omega||^2} \bar{U}^\alpha (\bar{\chi}_\alpha)_{a\bar{c}\bar{d}} \Omega_{bmn} g^{m\bar{c}} g^{n\bar{d}} \,, \tag{4.182}$$

where the norm of Ω is given by

$$||\Omega||^2 = \frac{1}{3!} \Omega_{abc} \bar{\Omega}_{\bar{m}\bar{n}\bar{p}} g^{a\bar{m}} g^{b\bar{n}} g^{c\bar{p}} \,. \tag{4.183}$$

The parameters $\bar{U}^\alpha$ (or U^α for deformations $\delta g_{\bar{a}\bar{b}}$) are called *complex-structure moduli* and there are in total $h^{2,1}$ of them. Analogously to the t^i, they measure the relative sizes of 3-cycles, whose volumes are given by integrating Ω over the respective cycles.

Before we conclude this subsection, let us briefly mention another subtlety on the journey to a four-dimensional EFT. The natural brane content in type IIB string theory are odd-dimensional p-branes that couple to the even-rank R–R $p+1$-form fields according to Eq. (4.147). The latter coupling, however, implies that these branes are charged under C_{p+1}, which in a compact space, must be accompanied by another contribution so that the overall charge cancels to zero. This is also called *tadpole cancellation* [79]. Moreover, the compactification on a CY 3-fold still preserves too much four-dimensional SUSY, namely $\mathcal{N} = 2$ even though we would require $\mathcal{N} = 1$ in order to make contact with phenomenology (for instance, there would be no scalar potential in $\mathcal{N} = 2$ SUGRA). Both issues are remedied by the inclusion of so-called *orientifold planes* (O-planes). Those are higher dynamical objects; however, as opposed to branes they do not possess any dynamics. Instead they are the product of a so-called orientifold projection, which mods out a discrete group from the internal space as well as worldsheet parity, i.e. the orientability of the strings. For further details and an introduction on orientifold projections we refer to [91], whereas we only collect their most important implications here. As already mentioned, the inclusion of O-planes allows us to break SUSY further to a phenomenologically viable level. To be specific, in type IIB string theory compactified on a CY manifold, an $\mathcal{N} = 1$ SUSY is retained if either O3 and O7-planes or O5 and O9-planes are included, whereas other combinations will break SUSY completely [79]. Due to the resulting lack of calculational control, the latter are less favourable and we will disregard them. Moreover, we will also not consider the combination of O5 and O9-planes but merely the case with only O3 and O7-planes. The reason for that will become clear shortly. Importantly, let us note that an O7-plane carries an R–R charge of opposite sign than a D7-brane. Therefore, by placing stacks of O7-planes and D7-branes parallel to each other, we can always ensure that the corresponding R–R charges are locally cancelled. On the other hand, a cancellation of D3 R–R charges appears more difficult and is currently debated [92]. Generally, we infer that due to the tadpole cancellation Op-planes are typically accompanied by Dp-branes. Consequently, in the scenarios we consider there will only be D3 and D7-branes, which allow for a relatively simple embedding of the SM sector, which either lives on a non-compact spacetime filling D3-brane or a D7-brane, which wraps one or several internal 4-cycles while also filling the residual non-compact dimensions. This justifies our choice of O3 and O7-planes.

To conclude this subsection, we summarise its most important aspects and try to provide some further intuition. In order to make contact with our four-dimensional world, we must compactify six spatial dimensions, which turns the ten-dimensional low-energy type IIB SUGRA into a four-dimensional SUGRA. Reasonable criteria on the internal space, as for instance Ricci flatness, suggest that CY 3-folds are good candidates to this end. The latter also break the ten-dimensional $\mathcal{N} = 2$ SUSY to four-dimensional $\mathcal{N} = 2$. The topology of a CY 3-fold X can be characterised by the two Hodge numbers $h^{1,1}$ and $h^{2,1}$, which count the numbers of homologically distinct $(1, 1)$ and $(2, 1)$-cycles, respectively, i.e. lower-dimensional, boundary-less submanifolds of X. The sizes of these submanifolds are the so-called moduli, which span a *moduli space*. The latter encompasses a family of many geometrically distinct CY 3-folds, which are continuously parameterised by besaid moduli. They can be classified into Kähler moduli, t^i, which measure the sizes of $(1, 1)$-cycles, and complex-structure moduli, U^α, measuring the sizes of $(2, 1)$-cycles. Instead of the former, we will often use τ_i, which are the sizes of 4-cycles and which are complexified to include axionic superpartners θ_i. After compactification to four dimensions, the moduli correspond to massless scalar fields, which require stabilisation due to phenomenological reasons and whose dynamics we want to analyse in what follows.

4.2.3. Flux compactifications and moduli stabilisation

In the last subsection we have learned that compactification of the ten-dimensional type IIB low-energy $\mathcal{N} = 2$ SUGRA on a CY 3-fold X combined with an orientifold projection yields a four-dimensional $\mathcal{N} = 1$ SUGRA. Here the moduli fields, i.e. the sizes of boundary-less submanifolds of X become four-dimensional scalar fields and are complexified into supermultiplets with their axionic partners. Let us start by defining this resulting SUGRA, i.e. by giving expressions for the Kähler potential, superpotential and gauge-kinetic function. The formulae presented in this subsection are based on [79], although we also recommend [93] for a detailed introduction to flux compactifications.

The Kähler potential for the Kähler moduli simply reads

$$\mathcal{K}_{\text{K}} = -2 \ln \mathcal{V} \,, \tag{4.184}$$

with the volume $\mathcal{V}$ given by Eq. (4.179). Note that the t^i-moduli dependence of $\mathcal{V}$ can straightforwardly be translated into a dependence of the τ_i-moduli so that generically $\mathcal{V}$ is a function of the latter. Since $\tau_i = (T_i + \bar{T}_i)/2$, the volume can also be taken as a function of the complexified moduli, $\mathcal{V} = \mathcal{V}(T_i, \bar{T}_i)$.

For the complex-structure moduli, the Kähler potential is of the form

$$\mathcal{K}_{\text{cs}} = -\ln\left(\mathrm{i}\int_X \Omega \wedge \bar{\Omega}\right) \,. \tag{4.185}$$

An explicit expression in terms of the complex-structure moduli U^α can be obtained by expanding Ω in a symplectic basis of $H_3(X)$ where the U^α appear inside holomorphic functions, which appear as prefactors in this expansion. Since we will mainly be interested in the dynamics of the Kähler moduli, we refer the interested reader to [90, 93, 94] for further details on the complex-structure moduli.

At last, there is also a term for the axio-dilaton, which is given by [79]

$$\mathcal{K}_S = -\ln\left(-\mathrm{i}(S - \bar{S})\right) \tag{4.186}$$

and is often absorbed into the definition of $\mathcal{K}_{\text{cs}}$. The total Kähler potential is then given as the sum of the individual contributions

$$\mathcal{K}_{\text{tot}} = \mathcal{K}_{\text{K}} + \mathcal{K}_{\text{cs}} + \mathcal{K}_S \,. \tag{4.187}$$

Before we discuss the form of the superpotential, we need to address the effect that generates it, namely the presence of so-called *fluxes*. To be specific, we speak of *turning on the fluxes* if the integral of the 3-form field strengths, F_3 and H_3 (the latter should not be confused with the 3-form cohomology group $H_3(X)$), of the 2-form R–R and NS–NS fields C_2 and B_2 over a 3-cycle Σ_3^i do not vanish. That is, the expressions [79]

$$\mathcal{F}_{A,i} = \int_{\Sigma_3^i} F_3 \,, \quad \mathcal{F}_{B,i} = \int_{\Sigma_3^i} H_3 \,, \tag{4.188}$$

are non-zero. These fluxes are quantised and take on discrete values, which is why $\mathcal{F}_{A,i}$ and $\mathcal{F}_{B,i}$ are, up to their normalisation, given by an integer number. The amount of choices of these integer numbers for all the individual fluxes, the many possibilities of different CY manifolds, and the specific setting of D-branes and O-planes give rise to a vast number of theories, estimated to be larger than 10^{500}, so that one also speaks of the *flux landscape of string vacua* [93]. Clearly, the presence of non-vanishing $\mathcal{F}_{A,i}$ and $\mathcal{F}_{B,i}$ will induce an interaction of C_2 and B_2 with the complex-structure moduli U^α that measure the sizes of the 3-cycles Σ_3^i. This interaction leads to a potential for the U^α and S that fixes them. Specifically, these fluxes result in the generation of a so-called *Gukov-Vafa-Witten (GVW) superpotential*, which is given by [95]

$$W_{\text{GVW}} = \int_X G_3 \wedge \Omega \ , \tag{4.189}$$

where $G_3 \equiv F_3 - SH_3$. For the explicit expression, we again refer to the original works or [79]. This superpotential has been used in the renowned paper by *Giddings, Kachru and Polchinski (GKP)* [96] to show the possibility of the aforementioned stabilisation of complex-structure moduli via a proper choice of fluxes. With the above Kähler potential and W_{GVW} the relevant terms that determine the dynamics of the moduli and the axio-dilaton are given by the usual SUGRA Lagrangian. In particular, the kinetic terms and F-term scalar potential are the corresponding expressions in Eqs. (4.64) and (4.66).

Crucially, we note that the GVW superpotential does not depend on the Kähler moduli. Furthermore, the volume $\mathcal{V}$ is a homogeneous function of the T_i of degree $3/2$. One can show that, as a consequence of these two facts, in the F-term scalar potential

$$V_F = \mathrm{e}^{\mathcal{K}} \left[\sum_{S,U} (K^{-1})^{\alpha\bar{\beta}} (D_\alpha W)(\bar{D}_{\bar{\beta}} \bar{W}) + \sum_{T} (K^{-1})^{i\bar{j}} (D_i W)(\bar{D}_{\bar{j}} \bar{W}) - 3|W|^2 \right] \ , \tag{4.190}$$

the latter two expressions cancel exactly [97]. Here K^{-1} is the inverse of the Kähler metric K, which for the Kähler moduli reads $K_{i\bar{j}} \equiv \partial^2 \mathcal{K}_K / (\partial T_i \partial \bar{T}_{\bar{j}})$, and the first sum runs over S and all the complex-structure moduli, whereas the second sum includes all Kähler moduli. This cancellation is called the *no-scale* property of the F-term potential and implies that the Kähler moduli are massless without including additional effects. Obviously, this is inconsistent from a phenomenological point of view. First, in string theory the vacuum expectation values of these moduli fields determine the parameters of the resulting, low-energy EFT as for example its gauge-coupling constants. Unstabilised Kähler moduli would therefore imply that these parameters would rapidly vary in time, which contradicts observations. Second, the absence of measurable fifth forces on solar system scales forbids a plethora of massless scalar fields, as long as we assume that not all of them underly some screening mechanism. The latter assumption seems reasonable since the dynamics of the field space, defined by the Kähler metric $K_{i\bar{j}}$, will generically be very complicated so that such a screening would require a tremendous conspiracy. It is therefore clear that a mechanism is required which can provide a potential for the Kähler moduli and thus stabilise them.

Two prominent examples for Kähler moduli stabilisation are the so-called *KKLT scenario* [98] and the large volume scenario (LVS) [99, 100]. We will almost exclusively focus on the latter and therefore dedicate the next subsection to it; however, even though we will not pursue it further, we want to stress that the KKLT scenario represents important progress in string phenomenology.

Before we conclude this subsection, let us write down the gauge-kinetic function of the gauge theory that lives on a stack of D7-branes wrapping a 4-cycle τ_{gauge}. In a (semi-)realistic scenario such a configuration could be used to establish the SM or a GUT model on these D7-branes with the corresponding gauge group. The relevant Lagrangian, corresponding to Eq. (4.57) in the superfield formalism and to Eq. (4.65) in component form, is given by [101]

$$\mathcal{L}_{\text{gauge}} = \frac{1}{4}\int \mathrm{d}^2\theta T_{\text{gauge}} W^{\alpha} W_{\alpha} + \frac{1}{4}\int \mathrm{d}^2\bar{\theta}\bar{T}_{\text{gauge}} \bar{W}^{\dot{\alpha}} \bar{W}_{\dot{\alpha}} \,, \tag{4.191}$$

$$= -\frac{1}{4}\tau_{\text{gauge}} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4}\theta_{\text{gauge}} F_{\mu\nu} \tilde{F}^{\mu\nu} \,. \tag{4.192}$$

From this, we can read off that the gauge kinetic function is $f_{ab} = T_{\text{gauge}}$. In the presence of gauge fluxes, this can be shifted to $f_{ab} = T_{\text{gauge}} + hS$ with h a function of the fluxes; however, we will ignore such a shift most of the time when it is irrelevant for our analysis. Crucially, from the above expressions we see that the size of the 4-cycle at the minimum, i.e. its vacuum expectation value, determines the coupling constant of the respective gauge theory in the UV, $\langle\tau_{\text{gauge}}\rangle \propto \alpha_{\text{UV}}^{-1} \propto g_{\text{UV}}^{-2}$. Moreover, a direct coupling between τ_{gauge} and the gauge fields as well as an axionic coupling term are induced.

4.2.4. The large volume scenario

The core idea of the LVS [99, 100] is the inclusion of certain corrections to the scalar potential that will remove its flatness in the Kähler moduli directions. To this end, one assumes that the complex-structure moduli have already been fixed as explained in the previous subsection and integrated out, which is possible because they are typically heavier than the Kähler moduli. In particular, $\mathcal{K}_{\text{cs}}$ together with $\mathcal{K}_S$ as well as the GVW superpotential are now fixed at their respective vacuum expectation values und hence treated as constants. For convenience, we define $W_0 \equiv \langle W_{\text{GVW}}\rangle$. The corrections to V_F can be classified into perturbative and non-perturbative ones. The former include

- α'-corrections: They stem from the fact that in the derivation of the ten-dimensional low-energy SUGRA action (4.149) only the massless string modes were used. α'-corrections account for the respective higher-dimensional operators that have been neglected there and which are suppressed by powers of $M_\mathrm{s}^{-1} = \sqrt{\alpha'}$. In the effective, four-dimensional SUGRA they manifest as an alteration of the Kähler potential for the moduli fields (4.184), which at cubic order in α' becomes [90, 99]

$$\mathcal{K}_\mathrm{K} = -2\ln\left(\mathcal{V} + \frac{\xi}{2}\right) \ , \quad \text{with } \xi \equiv \frac{\chi(X)\zeta(3)}{2(2\pi)^3 g_\mathrm{s}^{3/2}} \ , \tag{4.193}$$

where $\chi(X)$ is the Euler characteristic of X, i.e. a topological invariant, and $\zeta(3) \approx 1.2$ is the Riemann zeta function.

- string loop corrections: They have been explicitly calculated on the torus $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ [102] and, based on this calculation, their expected effect has been investigated on more general CY spaces [92, 103, 104]. In summary, the Kähler potential is modified by two contributions

$$\delta\mathcal{K}_{(g_\mathrm{s})} = \delta\mathcal{K}_{(g_\mathrm{s})}^\mathrm{KK} + \delta\mathcal{K}_{(g_\mathrm{s})}^\mathrm{W} \ . \tag{4.194}$$

Here the former results from loops of closed strings with Kaluza-Klein momentum between different branes and the latter from winding-mode contributions at the intersection of branes wrapped around different cycles.

The non-perturbative corrections include instanton corrections and gaugino condensation. The former result from D3-branes which are wrapped around an internal 4-cycle in a closed loop. They can be considered as tunneling events at a fixed position x^μ in the non-compact spacetime. On the other hand, the corrections due to gaugino condensation are a non-perturbative confinement effect in the $\mathcal{N} = 1$ super-Yang Mills theory living on the stack of D7-branes that wraps a corresponding 4-cycle. On a technical level, they both modify the superpotential by

$$W = W_0 + \sum_i A_i \mathrm{e}^{-\mathfrak{a}_i T_i} \ . \tag{4.195}$$

Here A_i is a complex constant whose value depends on the fixed complex-structure moduli and $\mathfrak{a}_i = 2\pi/N$ where the integer N is unity for D3-brane instantons and $N > 1$ for gaugino condensation. In the latter case, N corresponds to the number of branes in the stack which wraps the 4-cycle τ_i and determines the gauge group that lives on these branes, which is $U(N)$ as elaborated in Sec. 4.2.1.

In order to realise the LVS, one makes use of non-perturbative corrections with the above superpotential and leading α'-correction where the total Kähler potential is given by[5] [79]

$$\mathcal{K} = -2\ln\left(\mathcal{V} + \frac{\xi}{2}\right) - \ln\left(-\mathrm{i}(S - \bar{S})\right) + \mathcal{K}_{\rm cs} \,. \tag{4.196}$$

At this point we have to explicitly state the form of the volume (4.179) in terms of the complexified Kähler moduli T_i. A typical scenario includes one so-called *big cycle* with volume $\tau_{\rm b}$ and one or several *small cycles* of size $\tau_{{\rm s},i}$, whose labelling is chosen because $\tau_{\rm b} \gg \tau_{{\rm s},i}$ as it will become clear later in this subsection. The volume is then given by

$$\mathcal{V} = \tau_{\rm b}^{3/2} - \sum_i \gamma_i \tau_{{\rm s},i}^{3/2} = \left[\frac{1}{2}\left(T_{\rm b} + \bar{T}_{\rm b}\right)\right]^{3/2} - \sum_i \gamma_i \left[\frac{1}{2}\left(T_{{\rm s},i} + \bar{T}_{{\rm s},i}\right)\right]^{3/2} \,, \tag{4.197}$$

where the γ_i are positive $\mathcal{O}(1)$ constants that parameterise the structure of the internal manifold and depend on the intersection numbers κ_{ijk}.[6] The above form for the volume is also called *swiss-cheese scenario* because in a sense it compares to a cheese of size $\tau_{\rm b}$ with holes of sizes $\tau_{{\rm s},i}$. For illustrative reasons, we will now consider a setting with only one small cycle $\tau_{\rm s}$ and a volume given by

$$\mathcal{V} = \tau_{\rm b}^{3/2} - \gamma_{\rm s}\tau_{\rm s}^{3/2} \,. \tag{4.198}$$

Using Eq. (4.66) together with the above expressions for W and $\mathcal{K}$ to calculate the Kähler moduli F-term potential, one obtains [99, 100]

$$V_{\rm F} = \frac{8\mathfrak{a}_s^2|A_s|^2\sqrt{\tau_{\rm s}}\mathrm{e}^{-2\mathfrak{a}_s\tau_{\rm s}}}{3\gamma_s\mathcal{V}} - \frac{4|A_{\rm s}W_0|\mathfrak{a}_{\rm s}\tau_{\rm s}\mathrm{e}^{-\mathfrak{a}_{\rm s}\tau_{\rm s}}}{\mathcal{V}^2} + \frac{3\xi|W_0|^2}{4\mathcal{V}^3} \,. \tag{4.199}$$

In order to arrive at this expression, we have absorbed the constant $\mathcal{K}_{\rm cs}$ and $\mathcal{K}_S$ into a re-definition of A_I and W_0, absorbed the complex phases of A_I and W_0 into a re-definition of the axion $\theta_{\rm s}$ and integrated out the $\theta_{\rm s}$. One can now calculate the minimum of this potential, which provides us the values at which the two moduli are fixed,

$$\langle\tau_{\rm s}\rangle = \left(\frac{\xi}{2\gamma_{\rm s}}\right)^{2/3} \,, \quad \langle\mathcal{V}\rangle = \frac{3\gamma_{\rm s}|W_0|\sqrt{\langle\tau_{\rm s}\rangle}\mathrm{e}^{\mathfrak{a}_{\rm s}\langle\tau_{\rm s}\rangle}}{4\mathfrak{a}_{\rm s}A_{\rm s}} \,, \tag{4.200}$$

[5] Note on convention: We will typically count the Kähler moduli with numerical indices $i \in \{1, \cdots, h^{1,1}\}$ and identify these numbers with labels. For instance, in a two-moduli case we would have $\tau_1 = \tau_{\rm b}$ and $\tau_2 = \tau_{\rm s}$. In that sense, the numerical indices are interchangable with the alphabetical labels and it will be clear from the context which cycle is meant.

[6] Note that $\tau_{\rm b}^{3/2}$ may also be multiplied with a prefactor, which after a redefinition of the γ_i corresponds to an additive constant coming with $\ln\mathcal{V}$. This constant can then be absorbed into $\mathcal{K}_{\rm cs}$.

where the limit $\mathfrak{a}_\mathrm{s}\tau_\mathrm{s} \gg 1$ has been taken, which is required for calculational control. From the latter of the above two equations, we see that at the minimum $\mathcal{V} \approx \exp(\mathfrak{a}_\mathrm{s}\tau_\mathrm{s}) \gg \tau_\mathrm{s}$. This also implies that the big cycle de facto determines the size of the volume and is indeed much bigger than the small one, $\tau_\mathfrak{b} \approx \mathcal{V}^{2/3} \gg \tau_\mathrm{s}$. For this reason, the big cycle is often times simply called the *volume cycle* and its size denoted by $\tau_\mathcal{V}$.

Let us stress one crucial point: Since the size of τ_s is at least unity in terms of string units – otherwise the description as an effective low-energy SUGRA would break down – we have found that the volume of the internal manifold measured in l_s^6 must be exponentially large. This constitutes the name of the LVS and provides a very small quantity, $\mathcal{V}^{-1}$, which is an excellent parameter to control further calculations.

To analyse the potential for the volume further, one can integrate out τ_s from Eq. (4.199), which leads to [79]

$$V_\mathrm{F} \simeq \frac{3\xi|W_0|^2}{4\mathcal{V}^3} - \frac{3\gamma_\mathrm{s}|W_0|^2}{2\mathcal{V}^3\mathfrak{a}_\mathrm{s}^{3/2}} \log^{3/2}\left(\frac{4\mathfrak{a}_\mathrm{s}|A_\mathrm{s}|\mathcal{V}}{3\gamma_s|W_0|}\right) . \tag{4.201}$$

Crucially, the above potential possesses a global minimum at a large value for $\mathcal{V}$ and a negative one for V_F. The resulting spacetime is therefore an anti-de Sitter one with a negative cosmological constant. In order to obtain a Minkowski or a de Sitter space with vanishing or positive CC, respectively, we therefore need to *uplift* the potential. In the literature, several mechanisms for such an uplift have been discussed. We merely name two prominent possibilities and describe their effect on V_F, referring to the original works for further details. One possibility is the inclusion of anti-D3-branes in a strongly warped region, i.e. a so-called *Klebanov-Strassler throat* [105]. Due to their negative tension, this leads to an uplift of the potential where the warping allows for downscaling the uplift, which would be to strong otherwise. In doing so, one also has to ensure that the total charge of D3- and anti-D3-branes, O3-planes and 3-form fluxes cancel, which constitutes the aforementioned tadpole cancellation. The other possibility is the generation of a D-term potential through fluxes of gauge fields on D7-branes [106, 107]. Technically, both proposals will add another contribution to the potential (4.201)

$$\delta V_\mathrm{uplift} = \frac{C}{\mathcal{V}^\alpha} , \tag{4.202}$$

where C is a constant that includes the warp factor, which is tuned so that the minimum corresponds to Minkowski (or de Sitter), and α is $4/3$ or 2 for an uplift with anti-D3-branes or D-terms, respectively. In other words, after the uplift there will still be a minimum at large $\mathcal{V}$ but it can be at a zero or positive value for $V_\mathrm{F} + \delta V_\mathrm{uplift}$. There is one major difference to the original AdS minimum, however:

the new minimum is not a global one. For very large values of $\mathcal{V}$, a potential barrier will be exceeded and the potential leads to a runaway of $\mathcal{V}$ to infinity resulting in a de-compactification of the internal space. In what follows, we will always implicitly assume that a proper uplift mechanism can and will be established even though we want to mention that this is a critical and controversial aspect [108, 109].

Before we conclude this section, let us provide the kinetic terms and the typical volume scaling of the masses for the Kähler moduli and their respective axions. The former, in correspondence to Eq. (4.64), are given by

$$\mathcal{L}_{\text{kin}} = \sum_{i,\bar{j}} K_{i\bar{j}} \partial_\mu T_i \partial^\mu \bar{T}_{\bar{j}} = \sum_{i,j} \frac{1}{4} \frac{\partial^2 \mathcal{K}_{\text{K}}}{\partial \tau_i \partial \tau_j} \left(\partial_\mu \tau_i \partial^\mu \tau_j + \partial_\mu \theta_i \partial^\mu \theta_j \right) \ . \tag{4.203}$$

The masses of the volume modulus and of small-cycle moduli that are stabilised non-perturbatively as well as their respective axions are qualitatively given by the simplistic formulae [100]

$$m_{\tau_{\text{b}}} \sim \frac{|W_0| M_{\text{P}}}{\mathcal{V}^{3/2}} \ , \quad m_{\theta_{\text{b}}} \sim \mathrm{e}^{-\mathfrak{a}_{\text{b}} \mathcal{V}^{2/3}} M_{\text{P}} \sim 0 \ , \quad m_{\tau_{\text{s},i}} \sim m_{\theta_{\text{s},i}} \sim \frac{|W_0| M_{\text{P}}}{\mathcal{V}} \ , \tag{4.204}$$

where we ignored prefactors that cannot be parametrically small or large compared to $\mathcal{V}$ and even possible logarithmic factors of $\mathcal{V}$. The exponential suppression of the volume-axion mass originates from the fact that its potential is created by non-perturbative effects on the big cycle, which scale like $\sim \exp(-\mathfrak{a}_{\text{b}} \tau_{\text{b}})$. For all relevant purposes, we can therefore consider θ_{b} to be exactly massless. We want to stress that the small-cycle-moduli masses are modified if they are not stabilised non-perturbatively but instead via, e.g., loop effects. With $\mathcal{L}_{\text{kin}}$ and the generalisation of V_F to more than one small cycle before integrating out the $\theta_{\text{s},i}$ given, the dynamics of the Kähler moduli fields and axions in this most simplistic scenario are determined, although other effects can have important influences beyond it.

The LVS will be the basic setting for what follows, which is why we want to summarise its most important aspects: An interplay of the α'-correction to the Kähler potential and non-perturbative corrections to the superpotential on a small cycle result in Kähler moduli stabilisation with an exponentially large volume. Its inverse is very small and can be used as an expansion parameter in order to obtain calculational control. The resulting minimum is an AdS vacuum but can be lifted to Minkowski or dS by an uplift mechanism.

4.3. Physics beyond the standard model in string theory

We want to dedicate this section to some selected, theoretical basics of two specific beyond the SM scenarios and their stringy realisations. One is inflation, which modifies the ΛCDM model and the other are axion-like particles, especially the QCD axion, which modify the SM of particle physics but also have cosmological implications. In the following, we merely want to provide a quick summary of the most important concepts and key formulae of these topics that are importent for this work. By no means do we aim at completeness or usability as a comprehensive introduction. For the latter purposes, we refer to the appropriate literature.

4.3.1. Inflation and reheating – overview

Despite a plethora of successes of the ΛCDM model, there are aspects where it struggles to explain the observed universe. Two of those, which among others represent standard examples in the literature, are related to the early times of the universe: the horizon problem and the flatness problem. A prominent way to solve them both is inflation, of which some important concepts will be introduced in this subsection. For further reading, we refer to [5, 110–114], which are also the main references for this subsection.

The horizon problem

This problem describes the issue that observations of the CMB imply correlations between patches in the universe that are too far away from each other to have ever been in causal contact. For an illustration, let us borrow an example from [5] and consider the proper particle horizon of a point in the universe during recombination, i.e. the proper distance that light could have travelled from this point between the big bang and the time of recombination t_{rec}:

$$d_{\mathrm{p}}(0, t_{\mathrm{rec}}) = \int_0^{t_{\mathrm{rec}}} c\,\mathrm{d}t = c\int_0^{a_{\mathrm{rec}}} \frac{\mathrm{d}a}{\dot{a}} = c\int_0^{a_{\mathrm{rec}}} \frac{\mathrm{d}a}{aH(a)}\ . \tag{4.205}$$

Inserting for $H(a)$ the Friedmann equation for a radiation- and matter-dominated universe, this becomes

$$\begin{aligned} d_{\mathrm{p}}(0, t_{\mathrm{rec}}) &= c\int_0^{a_{\mathrm{rec}}} \frac{\mathrm{d}a}{aH_0\sqrt{\Omega_{\mathrm{m}0}a^{-3} + \Omega_{\mathrm{r}0}a^{-4}}}\ , \\ &= \frac{2c}{3H_0\Omega_{\mathrm{m}0}^2}\left[(\Omega_{\mathrm{m}0}a_{\mathrm{rec}} - 2\Omega_{\mathrm{r}0})\sqrt{\Omega_{\mathrm{m}0}a_{\mathrm{rec}} + \Omega_{\mathrm{r}0}} + 2\Omega_{\mathrm{r}0}^{3/2}\right]\ . \end{aligned} \tag{4.206}$$

To translate this 'physical size' of patches, within which there has been causal contact, into the angle θ under which they appear on the sky, we should divide by the angular diameter distance between us and the decoupling of CMB photons. For a flat universe ($\Omega_K = 0$), this distance is given by

$$d_A(t_{\text{rec}}, t_0) = a_{\text{rec}} \int_{t_{\text{rec}}}^{t_0} \frac{c\,\mathrm{d}t}{a} = a_{\text{rec}} c \int_{a_{\text{rec}}}^{1} \frac{\mathrm{d}a}{a\dot{a}} = a_{\text{rec}} c \int_{a_{\text{rec}}}^{1} \frac{\mathrm{d}a}{a^2 H(a)} \ . \tag{4.207}$$

Since during recombination the universe was dominated by matter, inserting the respective Friedmann equation leads to

$$\begin{aligned} d_A(t_{\text{rec}}, t_0) &= a_{\text{rec}} c \int_{a_{\text{rec}}}^{1} \frac{\mathrm{d}a}{a^2 H_0 \sqrt{a^{-3}}} \ , \\ &= \frac{2c}{H_0} a_{\text{rec}} (1 - \sqrt{a_{\text{rec}}}) \ . \end{aligned} \tag{4.208}$$

In the latter calculation we made the simplificication of turning off the CC so that the late time universe is purely dominated by matter. This does not change the result significantly, however. The angle under which causally connected patches appear is now given by

$$\theta = \frac{d_{\text{p}}(0, t_{\text{rec}})}{d_A(t_{\text{rec}}, t_0)} \approx 0.017\,\text{rad} \approx 0.97^\circ \ . \tag{4.209}$$

Here we have used the values $\Omega_{\text{m}0} = 0.27$, $\Omega_{\text{r}0} = 4.7 \cdot 10^{-5}$ and $a_{\text{rec}} = 1/1100$. We conclude that according to the standard ΛCDM model, areas in the CMB farther apart than approximately 1° should never have been in causal contact. This contradicts with observations showing an almost isotropic temperature distribution with fluctuations being only of the order $\delta T/T \sim \mathcal{O}(10^{-5})$ and correlated structures spanning over much larger areas.

The flatness problem

Another issue of standard cosmology is the fact that the observed flatness of today's universe would require an immense fine-tuning of flatness at early times. To see this, let us look at the curvature parameter Ω_K, which measures the deviation of the content of the universe from the critical density [5]

$$\Omega_K \equiv 1 - \Omega_{\text{tot}} = 1 - \Omega_{\text{m}} - \Omega_r - \Omega_\Lambda = \frac{-Kc^2}{a^2 H^2} \ . \tag{4.210}$$

The curvature today is measured to be rather small $\Omega_{K0} = -Kc^2/H_0^2 \lesssim \mathcal{O}(10^{-3} - 10^{-2})$, although there is a trend to small-magnitude negative values [21], indicating a closed universe. Since during radiation- and matter-domination Ω_K scales like

$\sim a^2$ and $\sim a$, respectively, this implies that Ω_K had to be uncomfortably small at the beginning of the universe when $a \to 0$. If there had been a tiny deviation from the fine-tuned flatness in the early times, we would observe a drastically different, non-zero value for Ω_{K0} today. Since there is no a priori reason why the universe has to be flat, these circumstances ask for a deeper explanation.

Inflation as a solution

Both, the horizon and the flatness problem, can be elegantly explained through the postulation of a so-called inflationary period. During that, the universe underwent a de Sitter-like expansion with constant Hubble factor H_I whereby the scale factor increased exponentially

$$a \propto \mathrm{e}^{H_\mathrm{I} t} \,. \tag{4.211}$$

Such an expansion could solve the horizon problem because the constant Hubble horizon c/H_I during inflation implies a shrinking comoving Hubble horizon $c/(aH_\mathrm{I})$. Conversely, however, this means that the comoving Hubble horizon grows bigger if we go further into the past when $a \to 0$ so that at the beginning of inflation, structures could have had causal contact which now seem to be too far away from each other for that to be the case. In other words, at the beginning of inflation, structures were small enough to have causal contact and where then enlarged by the expansion so that they look too big now to have been in causal contact.

Furthermore, inflation provides us an explanation for the flatness problem since during the de Sitter expansion the curvature parameter scales like $\sim a^{-2}$. Thus any potential deviation of the curvature from zero will be strongly suppressed by the end of inflation.

Apart from the horizon and flatness problem, inflation can explain the origins of structure formation, which are given by early quantum fluctuations that grow to macroscopic scales during the inflationary period and represent the seeds of inhomogeneities in the Universe.

Inflationary models and reheating

In typical models, the exponential expansion is achieved by a scalar field which slowly rolls down a very flat potential, thus resembling a CC. One may distinguish *large-field models*, where the distance in field space that the inflaton traverses is larger than unity in Planck units, from *small-field models*, where the same distance is smaller than unity [113]. The inflationary period ends, when the inflaton field does not resemble a CC anymore but instead begins to decay into SM degrees of freedom – and possibly other exotic particles – in a process that is called *reheating*.[7] To quantify the end of inflation, we define the slow-roll parameters

$$\epsilon \equiv \frac{1}{2}\left(\frac{V_{,\phi}}{V}\right)^2 M_{\mathrm{P}}^2 \ , \quad \eta \equiv \frac{V_{,\phi\phi}}{V} M_{\mathrm{P}}^2 \ , \tag{4.212}$$

where $V(\phi)$ is the potential of the inflaton ϕ and $V_{,\phi}$ and $V_{,\phi\phi}$ are its first and second derivatives, respectively. Thus the inflationary period is characterised by both slow-roll parameters being $\epsilon, |\eta| \ll 1$ and ends when this condition does not hold anymore. During inflation, the scale factor increases according to Eq. (4.211) by several e-folds, whose number is given by [112]

$$N_{\mathrm{e}} \equiv \log\left(\frac{a(t_{\mathrm{end}})}{a(t_{\mathrm{init}})}\right) = -\frac{1}{M_{\mathrm{P}}^2}\int_{\phi(t_{\mathrm{init}})}^{\phi(t_{\mathrm{end}})} \frac{V}{V_{\phi}} \mathrm{d}\phi \ , \tag{4.213}$$

where t_{init} and t_{end} are the times of the onset and end of inflation, respectively.

The background evolution of the inflaton is typically described by the equation of motion

$$\ddot{\phi} + 3H\dot{\phi} + \Gamma_{\phi}\dot{\phi} + V_{,\phi} = 0 \ , \tag{4.214}$$

where Γ_{ϕ} describes a small coupling of the inflaton to another matter species resulting in the decay of the inflaton. During inflation, the friction term $3H\dot{\phi}$ dominates over $\ddot{\phi}$ and $H \gg \Gamma_{\phi}$, which leads to the above slow-roll conditions. At the end of inflation, the decay rate becomes comparable to the Hubble function, $\Gamma_{\phi} \sim H$, and the inflaton fulfills damped oscillations about the minimum of the potential. Assuming that the inflaton decays into relativistic degrees of freedom, the energy density after reheating is given by the typical formula for a radiation-dominated Universe

$$\rho_{\mathrm{r}} = \frac{\pi^2 g_*(T_{\mathrm{r}})}{30} T_{\mathrm{r}}^4 = 3H^2 M_{\mathrm{P}}^2 \ , \tag{4.215}$$

where $g_*(T)$ is the number of relativistic degrees of freedom at temperature T, which is typically of order $g_* \simeq \mathcal{O}(10-100)$. This can be solved for the T_{r}, which leads to the standard formula for the reheating temperature [114]

$$T_{\mathrm{r}} = \left(\frac{90}{\pi^2 g_*(T_{\mathrm{r}})}\right)^{1/4} \sqrt{\Gamma_{\phi} M_{\mathrm{P}}} \ . \tag{4.216}$$

The value of T_{r} is highly model dependent but is forced to fulfill the bound $T_{\mathrm{r}} \gtrsim$

[7] In some scenarios, reheating does not occur directly after inflation and by the inflaton itself but is preceded by the excitation of intermediate species of matter, which will then be responsible for the reheating of the SM. We will consider such a scenario in the work of this thesis.

$\mathcal{O}(\mathrm{MeV})$ in order not to spoil a successful Big Bang Nucleosynthesis (BBN).

The investigation of inhomogeneities during inflation can be done in an analogous way as for structure formation in the late-time universe, namely via linear perturbation theory. We take the perturbed metric in Newtonian gauge as given in Eq. (2.23) and also perturb the inflaton as

$$\phi = \phi_0(\tau) + \delta\phi(\tau, \vec{x}) \ . \tag{4.217}$$

One can then define the gauge-invariant comoving curvature perturbation [5],

$$\mathcal{R} = -\Phi - \frac{\mathcal{H}}{\phi_0'}\delta\phi \ , \tag{4.218}$$

where $'$ represents the derivative w.r.t. conformal time τ. The inflaton perturbations $\delta\phi$ are quantum fluctuations about the background value ϕ_0 and will be pushed outside the Hubble horizon due to the rapid expansion. Clearly, these quantum fluctuations will lead to fluctuations of the curvature $\mathcal{R}$, whose power spectrum is given by [5]

$$\Delta^2_{\mathcal{R}} = \left.\frac{H^2}{8\pi^2 M_{\mathrm{P}}^2 \epsilon}\right|_{k=aH} \ . \tag{4.219}$$

When these curvature fluctuations re-enter the horizon at a later time, that is after the inflationary period and reheating, they imprint their characteristic structure on the current matter distribution at that time thus representing the seeds for any inhomogeneities [114].

Likewise, the power spectrum of tensor modes reads

$$\Delta^2_{h} = \left.\frac{H^2}{\pi^2 M_{\mathrm{P}}^2}\right|_{k=aH} \ , \tag{4.220}$$

whose existence depends on the specific model, however. We can also extract the powerlike k-dependence and benchmark the power spectra on a selected pivot scale k_*, which is typically chosen as $0.002\,\mathrm{Mpc}^{-1}$ or $0.05\,\mathrm{Mpc}^{-1}$, so that we obtain [5]

$$\Delta^2_{\mathcal{R}} = A_{\mathrm{s}} \left(\frac{k}{k_*}\right)^{n_{\mathrm{s}}-1} \ , \quad \Delta^2_{h} = \frac{A_{\mathrm{t}}}{2} \left(\frac{k}{k_*}\right)^{n_{\mathrm{t}}} \ , \tag{4.221}$$

where A_{s} and A_{t} are the spectral amplitudes, whereas n_{s} and n_{t} are the scalar and tensor spectral indices, respectively. The latter characterise the scale dependence of the two power spectra with $n_{\mathrm{s}} = 1$ implying that the curvature power spectrum is scale-invariant. One can show that the spectral indices are related to the slow-roll parameters,

$$n_{\mathrm{s}} = 1 - 4\epsilon + 2\eta \ , \quad n_{\mathrm{t}} = -2\epsilon \ . \tag{4.222}$$

Since during inflation both ϵ and η are very small, one can infer from the above formulae that the scalar and tensor power spectrum are both indeed almost scale-invariant.

An important quantity is the tensor-to-scalar ratio, which relates the two power spectra at the chosen pivot scale [5],

$$r \equiv \left.\frac{2\Delta_h^2}{\Delta_{\mathcal{R}}^2}\right|_{k=k_*} = \frac{A_{\mathrm{t}}}{A_{\mathrm{s}}} = 16\epsilon \ , \tag{4.223}$$

where the factor 2 stems from the two possible polarisation states of the graviton.

The scalar power spectrum is observationally constrained, for instance by Planck [115],

$$n_{\mathrm{s}} = 0.9649 \pm 0.0042 \ , \tag{4.224}$$

$$\ln\left(10^{10} A_{\mathrm{s}}\right) = 3.044 \pm 0.014 \ , \tag{4.225}$$

at 68 % CL and pivot scale $k_* = 0.05\,\mathrm{Mpc}^{-1}$, respectively. In particular, note that a scalar-spectral index $n_{\mathrm{s}} = 1$ is practically ruled out. From Planck, we also obtain an upper bound on the tensor-to-scalar ratio,

$$r < 0.10 \ , \tag{4.226}$$

at 95 % CL and pivot scale $k_* = 0.002\,\mathrm{Mpc}^{-1}$.

Let us also mention that the curvature power spectrum is measured as well [4],

$$\Delta_{\mathcal{R}}^2 = (2.101^{+0.031}_{-0.034}) \times 10^{-9} \ , \tag{4.227}$$

at 68 % CL and pivot scale $k_* = 0.05\,\mathrm{Mpc}^{-1}$. Thus for any given inflation model, $\Delta_{\mathcal{R}}^2$ is basically fixed up to small corrections and needs to be consistently predicted by that model. If one then compares two such models that predict the correct value for $\Delta_{\mathcal{R}}^2$, the tensor-to-scalar ratio depends only on the inflation scale H_{I}. This can be easily seen from Eq. (4.223) after inserting the expression for Δ_h^2 und using that during inflation, the Hubble parameter takes on an almost constant value H_{I},

$$r = \frac{2H_{\mathrm{I}}^2}{\pi^2 M_{\mathrm{P}}^2 \Delta_{\mathcal{R}}^2} \ . \tag{4.228}$$

To provide a bit more intuition about the relation between $r \propto \epsilon \propto H_{\mathrm{I}}^2$ for a fixed scalar amplitude, we notice that ϵ by definition parameterises the relative steepness of the the inflaton potential normalised to the absolute value of the potential. Through the Friedmann equations, the latter is simply given by the inflation scale $V \propto H_I^2$. The size of a scalar mode after it has been enhanced by inflation will generally depend on the number of e-folds N_{e} before inflation ends. Writing Eq. (4.213) as

$$N_{\rm e} = \frac{1}{\sqrt{2}M_{\rm P}} \int_{\phi_{\rm init}}^{\phi_{\rm end}} \epsilon^{-1/2} {\rm d}\phi \ , \tag{4.229}$$

where we have cancelled the overall minus sign with the minus sign from $V_{,\phi}$, and using that ϵ is almost constant during inflation, the above equation can be cast into the form

$$\Delta N_{\rm e} = \frac{1}{\sqrt{2\epsilon}} \frac{\Delta\phi}{M_{\rm P}} \ , \tag{4.230}$$

where $\Delta\phi = \phi_{\rm end} - \phi_{\rm init}$, although this equation will also yield the number of e-folds between arbitrary field values. This tells us that if we want to retain the same amount of e-folds, a larger slow-roll parameter must be compensated by a larger distance in field space that the inflaton has to traverse. Naively speaking, this makes sense because a larger ϵ implies a steeper potential and thus a faster rolling field. In order to obtain the same amount of inflation, the distance in field space should hence be larger.

One other important aspect during reheating is the generation of so-called *Dark Radiation* (DR). This refers to the production of any relativistic degrees of freedom which are not photons. In the SM, DR consists solely of the three neutrino generations, which is why one uses the effective number of neutrino species $N_{\rm eff}$ to parameterise the amount of DR. It is defined by the contribution of DR to the current energy density of radiation [116]

$$\rho_{\rm rad} = \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\rm eff}\right] \rho_\gamma \ , \tag{4.231}$$

with ρ_γ being the density of CMB photons at temperature $T \approx 2.7\,{\rm K}$. The SM value is given by $N_{\rm eff,SM} = 3$ during BBN and is slightly higher during CMB due to neutrino production from electron-positron annihilation. Crucially, the parameter $N_{\rm eff}$ does also account for any other additional relativistic particles besides neutrinos. Of special interest is therefore the excess of DR, given by

$$\Delta N_{\rm eff} \equiv N_{\rm eff} - N_{\rm eff,SM} \ . \tag{4.232}$$

This parameter is constrained by several measurements, e.g. Planck [4], to be $\Delta N_{\rm eff} \lesssim 0.3$. To predict this value for a given model, we have to consider the decays during reheating. In particular, we have [116–120]

$$\Delta N_{\rm eff} = \frac{43}{7}\left(\frac{10.75}{g_*^4 g_{*,S}^{-3}}\right)^{1/3} \frac{\rho_{\rm DR}}{\rho_{\rm SM}}\bigg|_{T=T_{\rm r}} = \frac{43}{7}\left(\frac{10.75}{g_*^4 g_{*,S}^{-3}}\right)^{1/3} \frac{\Gamma_{\phi\to DR}}{\Gamma_{\phi\to SM}}\bigg|_{T=T_{\rm r}} \ , \tag{4.233}$$

where g_* and $g_{*,S}$ are the number of relativistic degrees of freedom defined via energy density and entropy density, respectively.

4.3.2. Stringy inflation models

In Sec. 4.2 we explained that the four-dimensional low-energy SUGRA derived from type IIB string theory involves a plethora of scalar fields in the form of metric moduli. One might ask the natural question whether one or several of these can play the role of a stringy inflaton. There are many candidates of models with exactly this intention; however, we want to have a closer look at only two of those, namely *blow-up inflation* [121] and *fibre inflation* [122], which will be our main concern in this work and both of which are established within the LVS. For other stringy constructions of inflation or a review, we refer the interested reader to [123–125].

Blow-up inflation

In this setup, first suggested in [121], the inflaton field corresponds to the size of one small blow-up cycle.[8] In detail, the volume is taken as

$$\mathcal{V} = \tau_{\mathrm{b}}^{3/2} - \sum_{i=1}^{N_{\mathrm{s}}} \gamma_i \tau_{\mathrm{s},i}^{3/2} - \gamma_{\mathrm{I}} \tau_{\mathrm{I}}^{3/2} \,, \tag{4.234}$$

where τ_{I} is the size of the inflaton cycle. The other N_{s} small cycles $\tau_{\mathrm{s},i}$ are needed in order to establish the LVS as described in Sec. 4.2.4 and to stabilise the dynamics during inflation as will be explained below. The inflaton cycle does not intersect with any other cycle, i.e. its only non-vanishing intersection number is κ_{III}. Moreover, τ_{I} can be considered as a usual small cycle that only differs from the $\tau_{\mathrm{s},i}$ in the fact that it is displaced from its vacuum expectation value to a small size, whereas the other cycles are stabilised at the LVS minimum. The generalisation of the F-term potential (4.199) to more than one small cycle reads

$$V_F = \sum_{i}^{N_{\mathrm{s}}+1} \left(\frac{8\mathfrak{a}_i^2 |A_i|^2 \sqrt{\tau_{\mathrm{s},i}} \mathrm{e}^{-2\mathfrak{a}_i \tau_{\mathrm{s},i}}}{3\gamma_i \mathcal{V}} - \frac{4|A_i W_0| \mathfrak{a}_i \tau_{\mathrm{s},i} \mathrm{e}^{-\mathfrak{a}_i \tau_{\mathrm{s},i}}}{\mathcal{V}^2} \right) + \frac{3\xi |W_0|^2}{4\mathcal{V}^3} \,, \tag{4.235}$$

where we included τ_{I} in the sum as the $N_{\mathrm{s}}+1$-th small cycle, i.e. $\tau_{\mathrm{s},N_{\mathrm{s}}+1} \equiv \tau_{\mathrm{I}}$. If all these small cycles, including τ_{I}, sat at their respective vacuum expectation values,

[8] The original name for this setting was ‘Kähler inflation’; however, it is customary to call it ‘blow-up inflation’ to distinguish it from other inflationary scenarios that make use of a Kähler modulus as well which is not a blow-up cycle.

the resulting generalisation for the volume potential (4.201) would be given by

$$V_F(\mathcal{V}) = \frac{3\xi W_0^2}{4\mathcal{V}^3} - \frac{3W_0^2 \log^{3/2}(\mathcal{V}/W_0)}{2\mathcal{V}^3} \sum_{i=1}^{N_s+1} \frac{\gamma_i}{\mathfrak{a}_i^{3/2}} , \tag{4.236}$$

where we neglected some $\mathcal{O}(1)$-factors inside the logarithm. If we now consider the case that $\tau_{\rm I}$ is displaced to a large value, its contribution to the overall potential (4.235) will be suppressed due to the exponential functions. Hence also in Eq. (4.236) its contribution $\gamma_{\rm I}/\mathfrak{a}_{\rm I}^{3/2}$ inside the sum will not be present. However, if n_s is large enough, this missing contribution will be negligible compared to the overall sum so that the potential for $\mathcal{V}$ is practically unaffected from this displacement. Thus the volume remains stable even during inflation, which is the reason why many small cycles have been included instead of only one.

The potential for the $\tau_{\rm I}$ during inflation is also given through the F-term potential (4.235) by [121]

$$V_{\rm I}(\tau_{\rm I}) = \frac{\beta W_0^2}{\langle\mathcal{V}\rangle^3} - \frac{4|A_{\rm I}W_0|\mathfrak{a}_{\rm I}\tau_{\rm I}\mathrm{e}^{-\mathfrak{a}_{\rm I}\tau_{\rm I}}}{\langle\mathcal{V}\rangle^2} . \tag{4.237}$$

To arrive at this expression, the term $\propto \exp(-2\mathfrak{a}_{\rm I}\tau_{\rm I})$ in Eq. (4.235) has been neglected because it is suppressed for large $\tau_{\rm I}$, whereas all the the other terms that do not depend on $\tau_{\rm I}$ have been summarised into the term $\propto \beta\langle\mathcal{V}\rangle^{-3}$ taking into account their correct volume scaling.

The canonically normalised inflaton field is given by

$$\phi_{\rm I} = \sqrt{\frac{4\gamma_{\rm I}}{3\langle\mathcal{V}\rangle}}\tau_{\rm I}^{3/4} . \tag{4.238}$$

Writing $V_{\rm I}$ in terms of the canonical field, this leads to the appearance of a volume factor inside the exponential function,

$$\exp(-\mathfrak{a}_{\rm I}\tau_{\rm I}) = \exp\left(-\mathfrak{a}_{\rm I}\left(\frac{3\langle\mathcal{V}\rangle}{4\gamma_{\rm I}}\right)^{2/3}\phi_{\rm I}^{4/3}\right) , \tag{4.239}$$

which results in an extremely flat potential. Indeed, the first slow-roll parameter during inflation is given by [121]

$$\epsilon = \frac{32\langle\mathcal{V}\rangle^3}{3\beta^2|W_0|^2}\mathfrak{a}_{\rm I}^2|A_{\rm I}|^2\sqrt{\tau_{\rm I}}(1-\mathfrak{a}_{\rm I}\tau_{\rm I})^2\mathrm{e}^{-2\mathfrak{a}_{\rm I}\tau_{\rm I}} , \tag{4.240}$$

and since $\tau_{\rm I}$ is displaced to a large value, one has that $\exp(-\mathfrak{a}_{\rm I}) \ll \langle\mathcal{V}\rangle^{-2}$ so that

$\epsilon \ll 1$. In the numerical analysis performed in [121] taking into account the measured value for the amplitude of the scalar power spectrum, it has been shown that $\epsilon < 10^{-12}$ in blow-up inflation, rendering it a typical small-field model with low inflation scale H_I according to the discussion above.

Before closing this subsection, we want to discuss the dynamics at the end of inflation. Even though the volume modulus is rather unaffected during inflation, effects from the rolling inflaton become important when it reaches a low field value that is close to its vacuum expectation value. In [126] it has been argued that an effect called *vacuum misalignment* will lead to a violent and non-perturbative production of volume-modulus particles. The reason is that the at the end of inflation, τ_I sits at the edge of a potential well. This induces a quasi-stable vacuum expectation value for the volume which is shifted from the true vacuum expectation value, that corresponds to the inflaton being stabilised at the bottom of the potential well. When the inflaton finally falls into the well and begins to oscillate in it, the volume modulus will still have a value corresponding to the shifted vacuum expectation value, whereas its potential is now equipped with the true vacuum expectation value [127]. Additionally, the oscillations of τ_I will imply that the mass of τ_b oscillates leading to parametric resonances. All in all, this results in an excitation of the volume modulus as well as other moduli and coherent oscillations thereof. These coherent oscillations of τ_I, τ_b et cetera quickly become decoherent after a few oscillations and represent quanta of the respective moduli, i.e. particles. Once all oscillations have ceased and the moduli have reached a plateau at their respective vacuum expectation values together with the decoherent quanta $\delta\tau_i$, the system can be considered perturbatively as it has been done in [128]. The volume modulus, which is the longest-lived modulus and behaves like matter, quickly comes to dominate the universe. Finally, after this early-matter-dominated period, reheating into the SM occurs when the τ_b decays.

Fibre inflation

Another approach to stringy inflation has been suggested in [122]. Since the major work of the relevant chapter in this thesis will be performed in the setting of blow-up inflation, we will only quickly outline the most important ideas and properties of fibre inflation referring to the original paper or [129–132] for further details.

The setting of fibre inflation is still the LVS but the volume is not given by a single volume modulus yet instead exhibits the structure of a fibre bundle. That is, the volume locally looks like the product space of two Kähler moduli and is given by

$$\mathcal{V} = t_1\tau_1 - \gamma_{\rm s}\tau_{\rm s}^{3/2} \,, \tag{4.241}$$
$$= \frac{1}{2}\sqrt{\tau_1}\tau_2 - \gamma_{\rm s}\tau_{\rm s}^{3/2} \,, \tag{4.242}$$

where t_1 is the two-dimensional base and τ_1 the four-dimensional fibre. In accord with the usual LVS procedure, a potential is generated for $\mathcal{V} \approx \sqrt{\tau_1}\tau_2/2$ and $\tau_{\rm s}$; however, one combination of τ_1 and τ_2 remains unfixed and completely massless. This massless direction in field space is taken to be the inflaton and its potential is generated by the inclusion of additional loop effects to the Kähler potential as outlined in Eq. (4.194). The resulting inflaton potential during the slow-roll phase reads [122]

$$V_{\rm I} = \frac{C}{\langle\mathcal{V}\rangle^{10/3}}\left(3 - 4{\rm e}^{-\phi/(\sqrt{3}M_{\rm P})}\right) \,, \tag{4.243}$$

where C is an $\mathcal{O}(1)$ constant and ϕ is the canonically normalised inflaton field. Crucially, we see that in contrast to blow-up inflation, no volume factor appears in the exponential function, implying that the potential is much steeper in fibre inflation. It therefore represents a large-field inflation model with a high inflation scale $H_{\rm I}$ and a large tensor-to-scalar ratio $r \sim \mathcal{O}(10^{-3}) - \mathcal{O}(10^{-2})$.

4.3.3. Axions

One famous issue of the SM of particle physics is the so-called *strong CP problem*, which describes the fact that Quantum Chromodynamics (QCD) for some a priori miraculous reason is invariant under a simultaneous charge conjugation and parity transformation, i.e. it is CP symmetric. Arguably the most prominent solution to this problem is the introduction of beyond-the-SM physics in the form of a new scalar field, the axion. Besides solving the strong CP problem, the axion turns out to represent a viable DM candidate in cosmology, thus promising to resolve two major problems of modern-day physics at the cost of only one additional degree of freedom. In this subsection we first elaborate on the QCD axion and the historic *Peccei-Quinn (PQ) mechanism* before we consider some important cosmological properties and at last recapitulate the origin of axion-like particles in type IIB string theory. This subsection is based on [133–135] for the discussions of the field-theoretic axion as well as axion cosmology and on [84, 136, 137]. We also recommend [138, 139] for further reading.

The strong CP problem and the QCD axion

A promising route to field-theoretical model building is to take a given gauge symmetry and consider all renormalisable operators that are allowed by this symmetry. Doing so in QCD, whose gauge symmetry is $SU(3)$, the gauge-kinetic sector consists of two terms, the standard kinetic term for gluons and the so-called *topological term* [135],

$$\mathcal{L}_{\text{QCD,gauge}} = -\frac{1}{4}\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right) + \frac{\alpha_{\mathrm{s}}}{8\pi}\theta_{\text{eff}}\,\mathrm{Tr}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right) \;, \tag{4.244}$$

where α_{s} is the strong coupling constant, the trace runs over all generators of the gauge symmetry and $\tilde{F}^{\mu\nu} \equiv \epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}/2$ is the dual field-strength tensor. As is well-known, the θ_{eff} term is a possible source of CP violation and generates an electric dipole moment for the neutron. Measurements of the latter have shown that it must be vanishingly small, $|d_n| < 1.8 \times 10^{-26}\,\mathrm{e\,cm}$ [140], indicating that the θ_{eff} parameter has to be very small as well,

$$|\theta_{\text{eff}}| \lesssim 10^{-10} \;. \tag{4.245}$$

So far, we have treated θ_{eff} as a free parameter, which has been measured to be zero, which is not necessarily an issue. However, in a similar manner as for the CC problem described in Sec. 2.1, things become problematic when we consider the fact that θ_{eff} is an effective, physical parameter which is composed of two non-related contributions. One is the bare value θ_{bare} for the topological term and the other is a shift of this bare value resulting from the complex phase of the quark mixing matrix M after rendering the quark masses real via a chiral transformation. That is, the physical θ_{eff} parameter is given by

$$\theta_{\text{eff}} = \theta_{\text{bare}} + \arg\det M \;, \tag{4.246}$$

where both of these terms have to cancel to zero, which reveals that the strong CP problem is actually a fine-tuning problem.

An elegant solution has been proposed in [141, 142]. The key ingredient is a new complex, scalar field that is charged under a global, anomalous $U(1)_{\text{PQ}}$ symmetry, the so-called *Peccei-Quinn symmetry*, and which possesses a potential very similar to the Higgs field with vacuum expectation value $f_a/\sqrt{2}$, where f_a is called the *axion decay constant*. Below an energy scale f_a, the $U(1)_{\text{PQ}}$ symmetry spontaneously breaks and the axion $\tilde{\theta}$ appears as the pseudo-Nambu-Goldstone boson associated with this symmetry. Its relevant terms read

$$\mathcal{L}_{\text{axion}} = -\frac{f_a^2}{2}\partial_\mu\tilde{\theta}\partial^\mu\tilde{\theta} + \frac{\alpha_{\mathrm{s}}}{8\pi}\tilde{\theta}\,\mathrm{Tr}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right) \;. \tag{4.247}$$

$\tilde{\theta}$ is periodic and is defined to take on values $\tilde{\theta} \in [0, 2\pi\}$. The canonically normalised axion is given by $\tilde{a} = f_a\tilde{\theta}$, which can take on values $\tilde{a} \in [0, 2\pi f_a\}$ and whose relevant terms are

$$\mathcal{L}_{\text{axion}} = -\frac{1}{2}\partial_\mu \tilde{a}\partial^\mu \tilde{a} + \frac{\alpha_{\text{s}}}{8\pi}\frac{\tilde{a}}{f_a}\,\text{Tr}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right) \ . \tag{4.248}$$

Combining the gauge-kinetic QCD with the axion Lagrangian, we can combine the two topological terms into one,

$$\mathcal{L}_{\text{QCD,gauge}} + \mathcal{L}_{\text{axion}} = -\frac{1}{4}\text{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right) - \frac{1}{2}\partial_\mu a\partial^\mu a + \frac{\alpha_{\text{s}}}{8\pi}\frac{a}{f_a}\,\text{Tr}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right) \ , \tag{4.249}$$

where we defined $a \equiv f_a\theta \equiv f_a\theta_{\text{eff}} + \tilde{a}$, which from now on will be the field we consider as 'the axion'. Stemming from a Goldstone boson together with a constant shift, one would expect this axion to be massless and possess a continuous shift symmetry; however, since the $U(1)_{\text{PQ}}$ symmetry is anomalous, a potential is created due to the coupling between a and QCD instantons, which is induced via the last term of Eq. (4.249). This potential can schematically be given by

$$V_{\text{axion}} \simeq m_a^2(T) f_a^2 \left[1 - \cos\left(\frac{a}{f_a}\right)\right] \ , \tag{4.250}$$

although the detailed expression is more complicated (see e.g. [143]). Here m_a is the axion mass generated by this potential, which generically depends on the temperature T,

$$m_a(T < T_{\text{QCD}}) \approx m_{a,0} \ , \quad m_a(T \gtrsim T_{\text{QCD}}) \propto \left(\frac{T_{\text{QCD}}}{T}\right)^4 m_{a,0} \ , \tag{4.251}$$

where $T_{\text{QCD}} \simeq 150\,\text{MeV}$ is the QCD temperature and [143]

$$m_{a,0} = 5.7\,\text{meV}\left(\frac{10^9\,\text{GeV}}{f_a}\right) \tag{4.252}$$

the zero-temperature mass of the axion.

Due to the potential V_{axion}, the continuous shift symmetry breaks to a discrete one and the field a, which now determines the physical θ parameter that can be measured by experiments, is dynamically driven to its zero vacuum expectation value, $\langle a \rangle = 0$. This constitutes a solution to the strong CP problem that does not involve any fine-tuning.

In the original Peccei-Quinn mechanism, f_a was chosen to be at the electroweak scale, which has been ruled out by particle accelerator experiments. Modern experiments constrain the axion decay constant to lie in the window $f_a \sim \mathcal{O}(10^9\,\text{GeV}) - \mathcal{O}(10^{12}\,\text{GeV})$, where the lower bound is an astrophysical one related for example to the cooling of supernovae [138, 144], whereas the upper one is of cosmological nature and accounts for abidance of bounds on the DM abundance [133, 145]. While the former one is rather hard, the latter can be somewhat relaxed if one allows for some tuning of the so-called *initial misalignment angle* θ_i, which will be introduced shortly. The lower bound on f_a also implies that the axion mass is very small, $m_{a,0} \lesssim \mathcal{O}(\text{meV})$.

Axions in cosmology

The QCD axion, or more generally speaking axion-like particles, have often times shown to constitute good DM candidates. By axion-like particles we mean other particles than the QCD axion which share typical properties like periodicity, i.e. a discrete shift symmetry, but for which the relation (4.252) stemming from QCD does generally not hold. We have already encountered axion-like particles in the low-energy limit of type IIB string theory, which were given in Eq. (4.180) as the integral of the R-R 4-form field C_4 over 4-cycles. This kind of axion-like particles will be a major focus of the work in this thesis – indeed we will also identify the QCD axion with such a stringy axion – so that we will simply call them 'axions' instead of 'axion-like particles'.

To understand how axions can represent DM, we will now outline the so-called *misalignment mechanism*. Immediately after the PQ symmetry has broken, the axion field θ takes on a random value in the interval $[0, 2\pi\}$ and remains massless at first.[9] However, when the temperature reaches a value close to T_QCD, the axion potential V_axion is generated, which implies that θ is initially displaced from the vanishing vacuum expectation value. This value of this displaced is called the *initial misalignment angle* θ_i has important phenomenological consequences.

It is important to differentiate between two scenarios: In the *post-inflationary scenario*, the PQ symmetry is never broken during inflation or is restored afterwards, whereas in the *pre-inflationary scenario* it is broken during inflation and not restored afterwards [133]. The condition that determines which of the two scenarios takes place is basically given by the relation between the PQ breaking scale f_a and the Gibbons-Hawking temperature associated to the inflation scale, $T_\text{GH} \propto H_\text{I}$, or the maximal temperature which the universe will reach after inflation. If f_a is

[9] This is intuitively clear in analogy to the Higgs field, which after electroweak symmetry breaking can fall down into any direction of the Mexican-hat potential and thus take on any value in the flat directions of the degenerate vacuum. Obviously, in the Higgs mechanism the three flat directions, which correspond to the Goldstone bosons of the spontaneous symmetry breaking $SU(2) \times U(1)_Y \to U(1)_\text{em}$, are eaten by the massive gauge bosons, which is where this analogy ceases to work.

higher than T_{GH} and this maximal temperature, we will be in the pre-inflationary scenario and otherwise in the post-inflationary one. In the latter scenario, θ_{i} will take on a different, random value in different small regions of the universe so that on large scales θ_{i} will be the statistical average over these regions. Hence, the overall initial misalignment angle cannot be tuned so that this scenario is less suitable to establish axionic DM. In this thesis we will exclusively work in the pre-inflationary scenario where f_a is large enough that the PQ symmetry is broken during and not restored after inflation. In this case, the region with a given initial value for θ_{i} will be stretched and smoothened out by inflation and thus be everywhere the same in our Hubble patch. We can hence treat it as a free parameter.

When the temperature of the universe reaches a value T_{osc} such that $H(T_{\mathrm{osc}}) \sim m_a(T_{\mathrm{osc}})$, the gradient of the potential overcomes the expansion and θ begins to roll down the potential starting from θ_{i} and fulfills damped oscillations. Since the potential is almost harmonic, the oscillating axion field has an equation of state like matter and represents a coherent state of axionic DM particles. The resulting axion relic density today depends on the axion decay constant f_a and the initial alignment angle θ_{i} and is given by [133, 145, 146]

$$\Omega_a h^2 \approx 0.2 \left(\frac{f_a}{10^{12}\,\mathrm{GeV}} \right)^{7/6} \theta_{\mathrm{i}}^2 \ . \tag{4.253}$$

To arrive at this equation, the relation (4.252) has been used as well as the two assumptions that the cosmological history is a standard one, where the onset of oscillations occurs during radiation domination, and that the comoving axion number density is conserved. Obviously, the axion relic density must not exceed observational constraints on the DM density [115],

$$\Omega_a h^2 \leq \Omega_{\mathrm{c}0} h^2 \approx 0.12 \ . \tag{4.254}$$

We will come back to the above formula later in order to constrain the stringy axion.

It will also become important to consider the case where the assumption of a standard cosmological history is abandoned. Indeed, as we have explained in Sec. (4.3.2), blow-up inflation as well as many other stringy realisations come with a period of early matter domination [137, 147]. If the onset of the axion oscillations occurs during this period, the scenario is significantly changed [145–148]. Let us assume that some modulus field ϕ is responsible for the early matter domination and that the same modulus is responsible for reheating the SM when it decays with decay rate Γ_ϕ. Then the modified axion relic density is approximately given by [145, 146][10]

$$\Omega_a h^2 = 6 \times 10^{-5} \left(\frac{f_a}{10^{12}\,\mathrm{GeV}} \right)^{3/2} \left(\frac{T_{\mathrm{end}}}{10\,\mathrm{MeV}} \right)^2 \theta_{\mathrm{i}}^2 \ , \tag{4.255}$$

where T_{end} is the temperature at which ϕ-domination breaks off, defined by $\Gamma_\phi = H(T_{\mathrm{end}})$. It is customary to refer to T_{end} as the reheating temperature, $T_{\mathrm{r}} \equiv T_{\mathrm{end}}$.

Another important constraint, that is exclusive to the pre-inflationary scenario, comes from the generation of isocurvature modes [133, 134, 149, 150]. These are small perturbations of the otherwise homogeneous axion field that are generated during inflation due to quantum fluctuations

$$\left\langle |\delta a(k)|^2 \right\rangle = \left(\frac{H_{\mathrm{I}}}{2\pi} \right)^2 \frac{2\pi^2}{k^3} \ , \tag{4.256}$$

with k the mode number. Since the energy contribution of the axion is negligible compared to the inflaton, these perturbations are uncorrelated with the curvature spectrum $\Delta^2_{\mathcal{R}}$. Their power spectrum is given by

$$\Delta_a^2 = \left\langle \left(\frac{\delta \rho_a}{\rho_a} \right) \right\rangle \Bigg|_{t_{\mathrm{CMB}}} \approx \left(\frac{\gamma H_{\mathrm{I}}}{\pi f_a \theta_{\mathrm{i}}} \right)^2 \ , \tag{4.257}$$

where γ is a factor that takes into account dispersive effects, which is typically taken as $\gamma = 2$ [149]. Crucially, isocurvature modes are constrained by Planck in terms of the so-called *isocurvature fraction* [115]

$$\beta_{\mathrm{iso}} = \frac{\Delta_a^2(k_*)}{\Delta_a^2(k_*) + \Delta^2_{\mathcal{R}}(k_*)} < 0.038 \ , \tag{4.258}$$

at 95 % CL and pivot scale $k_* = 0.050\,\mathrm{Mpc}^{-1}$.

Stringy axions

Let us briefly collect the relevant terms of type IIB string theory which represent the origin of stringy axions. We have argued that axions are the integrals of the R-R 4-form C_4 over 4-cycles, as given in Eq. (4.180). More generally, axions arise as the KK modes of an R-R p-form wrapping an internal p-cycle. Due to the compactness of these cycles, these closed-moduli axions are inherently shift symmetric. To arrive at the four-dimensional EFT, one starts with the ansatz [84]

$$C_p(x, y) = \theta(x) \omega_p(y) \ , \tag{4.259}$$

[10]Note that in order to arrive at this formula, the number of degrees of freedom at the oscillation temperature has been set to $g_*(T_{\mathrm{osc}}) = 70$.

where ω_p is a harmonic p-form and x and y represent the coordinates of the four-dimensional spacetime $\mathbb{R}^{1,3}$ and six-dimensional internal space X, respectively. The relevant four-dimensional terms in the Lagrangian can now easily be produced via by integrating the corresponding ten-dimensional terms given in Sec. 4.2.1 over the internal manifold. We give a quick outline for the most important terms. Kinetic terms for the axion are obtained from the dimensional reduction of the C_p kinetic terms [84],

$$S_R \supset -\frac{1}{4\kappa_{10}^2}\int \mathrm{d}^{10}x\sqrt{-g}|\mathrm{d}C_p|^2 \sim -\frac{1}{\kappa_{10}^2}\int_X \omega_p \wedge \star\omega_p \int \mathrm{d}^4x\sqrt{-g_4}\partial_\mu\theta\partial^\mu\theta \ . \tag{4.260}$$

Comparing the right-hand side to

$$\mathcal{L}_{\text{axion}} \supset -\frac{f_a^2}{2}\partial_\mu\theta\partial^\mu\theta \ , \tag{4.261}$$

we notice that

$$f_a^2 \propto \frac{1}{l_\mathrm{s}^8}\int_X \omega_p \wedge \star\omega_p \ , \tag{4.262}$$

which can be further simplified as shown in App. A.1.

Finally, axionic couplings to gauge instantons on a $p+3$-brane (or a stack of $p+3$-branes) wrapping a p-cycle Σ_p are obtained from the brane CS term (4.148) [84],

$$S_{\text{brane,CS}} \supset 2\pi\int_{\mathbb{R}^{1,p+3}} C_p \wedge \frac{1}{8\pi^2}\mathrm{Tr}\,F_2 \wedge F_2 \sim \int_{\Sigma_p}\omega_p\int_{\mathbb{R}^{1,3}}\mathrm{d}^4x\theta\,\mathrm{Tr}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right) \ . \tag{4.263}$$

Together with the gauge-kinetic terms (4.145) for the gauge theory living on the brane(s), Eqs. (4.260) and (4.263) represent the corresponding four-dimensional terms (4.249) of the field-theoretical axion.

5. Axions in string theory and the Hydra of dark radiation

This chapter is dedicated to one of the major works in this thesis. Every statement is based on [3] unless stated otherwise.

5.1. Motivation and outline of approach

In Sec. 4.3.3 we have explained how the strong CP problem is resolved by a field-theoretic axion [141, 142, 151, 152]. By 'field-theoretic' we mean that the axion represents the angular component of a complex scalar field with a spontaneously broken $U(1)$ symmetry of sufficiently high quality and featuring the right couplings to the SM. 'High quality' in essence refers to the property that the resulting axion potential should not be spoiled by other contributions than QCD instantons.

In usual field theory, axions come with the downside of adding complexity and one more degree of freedom to the SM. In string theory, on the other hand, gauge couplings are determined by the vacuum expectation values of moduli fields; furthermore, the presence of axions, which inherently possess many wanted properties like a shift symmetry or high quality, is a necessary fact. It used to be difficult to obtain a viable phenomenology [84]; however, the appearance of the type IIB landscape [96, 98] and the LVS [99, 100] has supported the realisability of a realistic QCD axion in a broad class of string compactifications [136, 137, 153–159].

One important aspect of many stringy realisations of cosmology is the generic presence of a significant amount of DR [101, 116–118, 130, 160–167]. This represents one of the major issues that we aim to tackle in this work. To be more specific, we try to establish a QCD axion together with a realistic cosmology, in particular an inflationary sector, in the setting of string theory. The goal to avoid too much DR in doing so will turn out to be a conandrum. That is, we have found a novel way to ameliorate the original DR problem that is caused by volume axions – this represents severring the Hydra's first head. However, as a consequence a new source of DR via the inflaton is created – the Hydra's second, regrown head. The key ingredient to solve the original DR problem are couplings of the volume axion to the SM Higgs. While the Higgs mass is fine-tuned in SUSY constructions, the relevant coupling terms turn out to be much larger than naively suspected. This allows us to boost the decay of the volume modulus into the SM, which avoids DR due to the volume axion. However, the original assumption of a long-lived volume modulus is broken, resulting in a new setting where the inflaton itself reheats the SM while re-introducing the DR problem.

5.2. General properties of stringy QCD axions

5.2.1. Achieving a small axion decay constant

A small axion decay constant, $f_a \ll M_{\mathrm{P}}$, is needed for several reasons. One is the aforementioned high quality of the $U(1)$ symmetry, which refers to the avoidance of non-QCD-related corrections to the axion potential, $\Delta V \sim \exp(-M_{\mathrm{P}}/f_a)$ [159, 168–174]. Another, more important reason is the adherence of observational bounds on the DM density. According to Eqs. (4.253) and (4.255), the axion relic density increases for larger f_a, which implies an upper bound $f_a \lesssim 10^{13}\,\mathrm{GeV}$. We will elaborate on these kinds of bounds in Sec. 5.3. The most straightforward way to obtain such a small axion decay constant is achieved via a large volume of the internal space [136] where the axion is given by the integral of a p-form over a small cycle that determines the SM gauge coupling.

Alternative approaches to realising small f_a make use of strongly warped regions [175–178], heterotic string theory [84], heterotic M-theory [178–180] or field-theoretic axions from open strings [181–183]. We will not pursue these approaches since they come with several, different caveats as explained in [3].

To understand how a large volume leads to, and in fact is needed for, a small f_a [136], we consider such a large internal CY manifold together with a small p-cycle. On this cycle, a stack of D$(p+3)$-branes is wrapped, on which the SM is located. In particular, we assume that QCD lives on this stack of branes and we identify the axion that arises from integrating C_p over this cycle with the QCD axion. Let us suppose that the string coupling is $g_{\mathrm{s}} \sim \mathcal{O}(1)$ and the small cycle has a typical size $\tau_{\mathrm{s}} \sim \mathcal{O}(10)$ in string units l_{s}^4 to produce the correct gauge coupling. Then the axion decay rate will be of the same order of magnitude as the only dimensionful quantity in this geometric region: the string scale $M_{\mathrm{s}} \propto 2\pi/l_{\mathrm{s}}$. Thus we have

$$f_a \sim M_{\mathrm{s}} \sim \frac{M_{\mathrm{P}}}{\sqrt{\mathcal{V}_{\mathrm{s}}}}\,, \tag{5.1}$$

where $\mathcal{V}_{\mathrm{s}}$ is the internal volume in the string frame measured in string units l_{s}^6.

A more careful analysis of this in the context of general type IIB string theory can be found in App. A.1 where we explicitly incorporate the parametric dependence on g_s and the size τ_s of the SM cycle. The result reads

$$\frac{f_{a,\min}^2}{M_P^2} \sim \frac{g_s \alpha_{s,\mathrm{UV}}}{\mathcal{V}_s} \sim \frac{\alpha_{s,\mathrm{UV}}}{\sqrt{g_s}} \frac{1}{\mathcal{V}} , \tag{5.2}$$

where $\mathcal{V} = \mathcal{V}_s / g_s^{3/2}$ is the CY volume in the ten-dimensional Einstein frame and $\alpha_{s,\mathrm{UV}} \propto \tau_s^{-1}$ is the high-scale value of the strong coupling parameter α_s. We ascertain that an optimal suppression of f_a is achieved for $g_s \sim 1$[1] and a small $\alpha_{s,\mathrm{UV}}$. The latter, however, cannot be used for an arbitrarily strong suppression since $\alpha_{s,\mathrm{UV}}$ needs to be consistent with the low-scale α_s. Since both of the above ways of suppression are limited, we reassert that a small axion decay constant is necessarily correlated to a large compactification volume.

This strongly points towards the LVS [99] as described in Sec. 4.2.4 as the optimal setting to study phenomenologically consistent stringy axions, which represents the only class of models with large internal volume that is understood well enough. Let us hence re-derive the result (5.2) in the LVS context. To this end, we consider a setting where one of the small blow-up cycles $\tau_{s,i}$ is wrapped by a stack of branes on which QCD lives. We will later see that we are driven to stabilise this SM cycle by loop effects, which is why we label it by τ_L for 'Loop'.

The size of this cycle will determine the $SU(3)$ gauge coupling, $\alpha_{s,\mathrm{UV}}^{-1} = 2\tau_L$.[2] We take a volume as given in Eq. (4.197) and a Kähler potential given by Eq. (4.193) excluding the parameter ξ,

$$\mathcal{K} = -2 \log \mathcal{V} , \quad \mathcal{V} = \tau_b^{3/2} - \gamma_s \tau_s^{3/2} - \gamma_L \tau_L^{3/2} , \tag{5.3}$$

where we included one more small cycle τ_s for illustrative reasons. The kinetic terms for the axions, according to Eq. (4.203), are given by

$$\mathcal{L}/M_P^2 \supset K_{i\bar{j}} \partial_\mu \theta_i \partial^\mu \theta_{\bar{j}} , \tag{5.4}$$

where a sum over $i, \bar{j} \in \{\mathrm{b}, \mathrm{s}, \mathrm{L}\}$ is implied and the axions' periodicity is set by $\theta_i = \theta_i + 1$. After a rotation to a diagonal basis θ'_i, we obtain for the QCD axion θ'_L

$$\mathcal{L}/M_P^2 \supset \lambda_L \partial_\mu \theta'_L \partial^\mu \theta'_L , \tag{5.5}$$

where λ_L is the adequate eigenvalue of $K_{i\bar{j}}$. The canonically normalised axion $a_L/M_P = \sqrt{2\lambda_L}\theta'_L$ then adheres to $a_L = a_L + \sqrt{2\lambda_L} M_P$.[3] Demanding that a_L has a

[1]Note that values of g_s larger than unity endanger the calculational control due to string-loop corrections or can be brought back to values smaller than unity by considering a dual theory.

[2]Due to gauge fluxes, this may be corrected by some $\mathcal{O}(1)$ factor depending on the details of model building

[3]Remember that we denominate the canonically normalised moduli and axion fields as ϕ_i and a_i,

periodicity of $2\pi f_{a_{\rm L}}$, we can read off the axion decay constant (see also [184] and references therein)

$$f_{a_{\rm L}} = \frac{\sqrt{2\lambda_{\rm L}} M_{\rm P}}{2\pi} \,. \tag{5.6}$$

To obtain the eigenvalue $\lambda_{\rm L}$, we make use of the fact that in the large-volume limit $\mathcal{V} \approx \tau_{\rm b}^{3/2} \gg \tau_{\rm s}^{3/2} \sim \tau_{\rm L}^{3/2}$, the Kähler metric is approximately diagonal. To see this, we note that

$$K_{i\bar{j}} = \frac{1}{4}\frac{\partial^2 \mathcal{K}}{\partial\tau_i \partial\tau_{\bar{j}}} = -\frac{1}{2}\frac{\partial^2 \log \mathcal{V}}{\partial\tau_i \partial\tau_{\bar{j}}} \sim \begin{cases} \tau_{\rm b}^{-2} \approx \mathcal{V}^{-4/3} & \text{for } i = \bar{j} = \text{b} \\ \mathcal{V}^{-1}\tau_{\rm b}^{-1} \approx \mathcal{V}^{-5/3} & \text{for } i = \text{b}, \bar{j} \neq \text{b or } i \neq \text{b}, \bar{j} = \text{b} \\ \mathcal{V}^{-2} & \text{for } i \neq \text{b}, \bar{j} \neq \text{b}, i \neq \bar{j} \\ \mathcal{V}^{-1} & \text{for } i = \bar{j} \neq \text{b} \end{cases} \,. \tag{5.7}$$

Clearly, the diagonal components, $i = \bar{j}$, have the least suppression by volume factors so that the eigenvalue can be approximated by (cf. Eq. (A.32))

$$\lambda_{\rm L} \approx K_{\rm LL} = \frac{3\gamma_{\rm L}}{8\sqrt{\tau_{\rm L}}\mathcal{V}} \,. \tag{5.8}$$

We then obtain for the QCD axion decay constant[4]

$$\frac{f_{a_{\rm L}}^2}{M_{\rm P}^2} \approx \frac{3\gamma_{\rm L}}{16\pi^2\sqrt{\tau_{\rm L}}\mathcal{V}} \simeq \frac{\mathcal{O}(1)}{2\pi^2\sqrt{\tau_{\rm L}}\mathcal{V}} \,, \tag{5.9}$$

where in the last step we have absorbed the numerical factors in $\lambda_{\rm L}$ into the $\mathcal{O}(1)$ prefactor. This is consistent with the estimated lower-bound (5.2) except for the factor $\tau_{\rm L}^{-1/2} \propto \sqrt{\alpha_{\rm s,UV}}$ in Eq. (5.9) compared to $\alpha_{\rm s,UV}$ in Eq. (5.2). This minor difference does not come as a surprise because the latter factor assumes a very optimistic structure of the harmonic form in App. A.1 to suppress f_a as much as possible.

5.2.2. Embedding of stringy axions into inflation

In Sec. (4.3.3) we have explained that there is a crucial difference between the post- and pre-inflationary setting relating to the question whether the PQ symmetry is unbroken at some point after inflation or not. The former case implies several important consequences like the absence of isocurvature fluctuations [133, 149, 150,

respectively.

[4]Note that models of fibre inflation, which have been explained in Sec. 4.3.2 and where $\tau_{\rm L}$ is a factor in the overall volume $\mathcal{V}$, will generally imply a decay constant of the same order of magnitude or larger.

185], the impossibility to tune the initial misalignment angle θ_i [149, 150, 186], axion miniclusters [187–190] or topological defects like axion strings and domain walls [191–195].

In the case of stringy axions that originate from the integral of p-form fields over internal cycles of the CY space, the situation is slightly different from a field-theoretic axion that results from the breaking of the PQ symmetry. As explained in Sec. (4.3.3), such stringy axions are inherently shift symmetric at the fundamental level so that there is no PQ symmetry that has to be broken first. Accordingly, this kind of axions will automatically be present during inflation and hence always establish a pre-inflationary scenario. Especially and most important for our purposes, stringy axions are bound to isocurvature constraints and allow for the tuning of θ_i.

One caveat to this is the fact that, for typical inflationary scenarios that make use of moduli fields, one or several of the latter are displaced from their respective vacuum expectation values. For instance, this is also exactly the case in blow-up inflation as explained in Sec. 4.3.2. Such a displacement dramatically changes the geometry of the internal space, which could, at least in principle, have effects on the properties of the axion, e.g. a decay constant that evolves during inflation. Obviously, this would imply important phenomenological consequences for the axion, e.g. regarding its initial conditions or isocurvature fluctuations. We believe that such modifications in a quantitative analysis cannot be ruled out per se and deserve further study; however, we do not expect them to be relevant for our analysis and conclusions. In particular, as argued in Sec. 4.3.2 in the setting of blow-up inflation, which we will mostly be dealing with, all cycles other than the inflaton cycle remain practically stable during inflation so that an evolution of f_a seems unlikely.

Other complications might arise from higher-temperature effects, on which we elaborate in [3] and which we expect to be irrelevant in this work as well.

5.3. Cosmological constraints

In Sec. 4.3.3 we have argued that axions have important cosmological consequences. Especially noteworthy are their contribution to the overall DM density and the production of primordial isocurvature fluctuations. In this section, we use corresponding cosmological constraints (for detailed studies cf. [133, 134, 145, 146, 148]) to restrict the parameter space of the axion and inflation, which will impose crucial implications on the required setting for a realistic scenario. Since there are some analytical approximations and dropped $\mathcal{O}(1)$ factors involved, the results of this section should be taken as order-of-magnitude estimates and an indication of the different qualitative regimes.

5.3.1. Assuming a standard cosmology for the expansion history

Dark matter abundance

In the previous section we have argued that stringy axions are established in a pre-inflationary scenario. This implies that the whole discussion in Sec. 4.3.3 applies, especially the facts that θ_i can be tuned and that isocurvature perturbations are generated. Moreover, axionic dark matter is produced via the usual misalignment mechanism [196–198], which under the assumption of a standard cosmological history, i.e. an onset of axion oscillations during radiation domination, results in an axion relic density as given in Eq. (4.253). The requirement that this does not lead to an overproduction of DM, i.e. that the inequality in Eq. (4.254) is respected, imposes a bound on the initial misalignment angle in terms of the axion decay constant,

$$\Omega_a \leq \Omega_{\mathrm{c}0} \quad \Rightarrow \quad \theta_\mathrm{i} \leq 0.8 \left(\frac{10^{12}\,\mathrm{GeV}}{f_a} \right)^{7/12} \,. \tag{5.10}$$

Saturating the observed dark matter abundance

Since axions represent a viable DM candidate, it is obviously tempting to have them not only contribute to but instead account for all of DM. This requires that the bound (5.10) is saturated. Since the initial misalignment angle cannot take on arbitrarily large values, $\theta_\mathrm{i} \leq \pi$ (this corresponds to the maximum of the axion potential), this suggests that f_a cannot be too small and has a lower limit that can yield the required DM abundance. For example, we notice that according to Eq. (5.10), an initial angle $\theta_\mathrm{i} \sim 1$ already requires $f_a \sim 10^{12}\,\mathrm{GeV}$. If we take $\theta_\mathrm{i} \leq 3$ as an upper limit for the initial angle that does not require significant tuning, one obtains

$$f_a \gtrsim 1 \times 10^{11}\,\mathrm{GeV} \tag{5.11}$$

to saturate the DM density. Allowing for tuning and including corrections due to the anharmonicity of the potential at large values of θ [134], the axion decay constant can become as low as

$$f_a \gtrsim 10^{10.3}\,\mathrm{GeV} \quad \text{for} \quad H_\mathrm{I} \gtrsim 10^4\,\mathrm{GeV}\,, \tag{5.12}$$

where the dependence on H_I arises due to fluctuations during inflation that might interfer with the tuning. In the following, we will mostly assume that the QCD axion constitutes all of dark matter, i.e. that the bound (5.10) is saturated; however, we will avoid tuning the misalignment angle but instead take the bound $\theta_\mathrm{i} \leq 3$ seriously.

Isocurvature constraints

Another observational bound relevant for axions in the pre-inflationary scenario is the one on isocurvature perturbations [133, 134, 149, 150], which have been introduced in Sec. (4.3.3). Combining Eq. (4.257) with Eq. (4.258) and inserting the measured value of the curvature power spectrum $\Delta^2_{\mathcal{R}}$ as given in Eq. (4.227), one finds an upper bound on the inflation scale in terms of the axion parameters,

$$H_{\rm I} \lesssim 1.4 \times 10^{-5} f_a \theta_{\rm i} \ . \tag{5.13}$$

Implications for the string scenario

We are now in the position that we have three parameters (f_a, $\theta_{\rm i}$ and $H_{\rm I}$) and two relations (Eqs. (5.10) and (5.13)), both of which also apply to a field-theoretic axion in the pre-inflationary setting. Let us now add the fact that we are considering stringy axions, i.e. that the axion decay constant scales with the volume $\mathcal{V}$ as given in Eq. (5.9). Combining these three relations and assuming that the bound from the DM abundance, i.e. Eq. (5.10) is saturated, we arrive at

$$H_{\rm I} < \frac{2 \times 10^9\,\text{GeV}}{\mathcal{V}^{5/25}} \ . \tag{5.14}$$

Considering that the volume is a very large parameter in the LVS, $\mathcal{V} \gg 1$, this implies a very low inflation scale and, according to Eq. (4.228), also a very low tensor-to-scalar ratio.[5]

While the above bound is independent of any assumption about the inflationary potential, we can also derive an explicit bound by using knowledge about the inflation scale in typical string-theoretic constructions. In general, $H_{\rm I}$ should be comparable to the natural magnitude of the scalar potential. For higher values, there is the danger of moduli de-stabilisation, whereas lower values generally necessitate more tuning. The natural scale of the LVS scalar potential, as given in Eq. (4.199), is given by $V_{\rm F} \sim M_{\rm P}^4/\mathcal{V}^3$ so that we expect the inflation scale in a typical LVS setting to be

$$H_{\rm I}^2 \simeq \beta \frac{W_0^2 M_{\rm P}^2}{\mathcal{V}^3} \ , \tag{5.15}$$

[5] One might argue, that the bound (5.13) from isocurvature constraints can be loosened or even evaded by considering a scenario where the QCD axion constitutes only a minor fraction of the total DM density. This would indeed allow for a relaxation of the isocurvature bounds; however, achieving such a small axion relic density requires that either θ_i or f_a are very small. The former cannot be tuned to arbitrarily low values because the emergence of quantum fluctuations would spoil such a tuning, whereas a smaller f_a in the LVS context is only achieved by an even larger volume, which would again imply a smaller inflation scale. We therefore believe that a small H_I is a general und hardly circumvented feature of a stringy axion.

where β is a model-dependent parameter of order $\mathcal{O}(1)$. Comparing this to Eq. (4.237), we see that for blow-up inflation this estimate indeed accurate.

We can now use Eq. (5.15) to solve Eq. (5.14) explicitly for $\mathcal{V}$, thus obtaining an estimate for a lower bound on $\mathcal{V}$, which translates into upper bounds on H_{I} and f_a. In order to also estimate opposite bounds, we demand that the axion relic density satiates the DM density without a fine-tuned $\theta_{\mathrm{i}} \approx \pi$. That is, we saturate the bound given by Eq. (5.10) for $\theta_{\mathrm{i}} \leq 3$, which leads to a lower bound on f_a that can again be translated into corresponding bounds on $\mathcal{V}$ and H_{I}. We obtain

$$
\begin{aligned}
(\kappa^{24/31})\, 1 \times 10^{7} &\lesssim \mathcal{V} \lesssim 9 \times 10^{12}\,, && (5.16)\\
(\kappa^{-5/31})\, 7 \times 10^{7}\,\mathrm{GeV} &\gtrsim H_I \gtrsim 0.1\,\mathrm{GeV}\,\kappa\,, && (5.17)\\
(\kappa^{-12/31})\, 9 \times 10^{13}\,\mathrm{GeV} &\gtrsim f_a \gtrsim 1 \times 10^{11}\,\mathrm{GeV}\,, && (5.18)\\
(\kappa^{7/31})\, 0.1 &\lesssim \theta_i \lesssim 3\,, && (5.19)
\end{aligned}
$$

where we defined $\kappa^2 \equiv \beta W_0^2$ and in the penultimate line used Eq. (5.9) with the $\mathcal{O}(1)$ factor taken to be equal to unity and $\tau_{\mathrm{L}} = 1/(2\alpha_{\mathrm{s,UV}}) = 25/2$. Here, the left-hand side represents the bounds from isocurvature constraints and the right-hand side those from a non-fine-tuned DM saturation.

5.3.2. Assuming an early matter domination for the expansion history

As we have argued in Sec. 4.3.3, stringy early-universe constructions often times entail a period of early matter domination due to a late-decaying modulus, which is typically but not necessarily the volume modulus. This modifies the axion relic density so that in an analogous analysis to above we have to replace Eq. (4.253) by Eq. (4.255).

Isocurvature constraints

On the other hand, we expect the bound (5.13) from isocurvature constraints to still be valid because the respective CMB modes that we observe had left the horizon already before the early-matter-domination phase and re-entered it only close to DM-radiation equality. They have thus never experienced the period of early matter domination and are hence unaffected by it. Therefore, we can again derive an upper bound for the inflation scale in terms of the volume by combining the formula for axionic DM abundance Eq. (4.255) with the isocurvature constraints Eq. (5.13) and the volume scaling of a stringy axion decay constant Eq. (5.9) while using the saturation condition, $\Omega_a h^2 = \Omega_{c0} h^2 = 0.12$. This yields

$$H_\mathrm{I} < \frac{1\times 10^{10}\,\mathrm{GeV}}{\mathcal{V}^{1/8}} \left(\frac{10\,\mathrm{MeV}}{T_\mathrm{end}}\right) , \tag{5.20}$$

which as before represents a comparatively low value for H_I and r. Note that T_end is bounded from below, $T_\mathrm{end} \gtrsim \mathcal{O}(1\,\mathrm{MeV})$, in order to guarantee a successful BBN [199–210].

Implications in the string scenario

To eliminate the volume from the above formula, we use again that the inflation scale in the LVS is typically given by Eq. (5.15). As before, this implies a lower bound on $\mathcal{V}$ and, equivalently, upper bounds on H_I and f_a. Moreover, as above we estimate opposing bounds by imposing a non-fine-tuned initial misalignment angle, $\theta_\mathrm{i} \leq 3$. We obtain

$$\left(\frac{\kappa T_\mathrm{end}}{10\,\mathrm{MeV}}\right)^{8/11} 9\times 10^5 \;\lesssim \mathcal{V} \;\lesssim\; 6\times 10^7 \left(\frac{T_\mathrm{end}}{10\,\mathrm{MeV}}\right)^{8/3} , \tag{5.21}$$

$$\kappa^{-1/11}\left(\frac{10\,\mathrm{MeV}}{T_\mathrm{end}}\right)^{12/11} 3\times 10^9\,\mathrm{GeV} \;\gtrsim H_I \gtrsim\; 5\times 10^6\,\mathrm{GeV} \left(\frac{10\,\mathrm{MeV}}{T_\mathrm{end}}\right)^{4} \kappa\, , \tag{5.22}$$

$$\left(\frac{10\,\mathrm{MeV}}{\kappa T_\mathrm{end}}\right)^{4/11} 3\times 10^{14}\,\mathrm{GeV} \;\gtrsim f_a \;\gtrsim\; 4\times 10^{13}\,\mathrm{GeV} \left(\frac{10\,\mathrm{MeV}}{T_\mathrm{end}}\right)^{4/3} , \tag{5.23}$$

where again the left-hand side represents bounds from isocurvature constraints and the right-hand side from DM saturation without fine-tuning. Note that the window which is spanned by the above bounds becomes narrower for smaller T_end and might even close if κ is large enough.

Let us eliminate $T_\mathrm{end} = T_\mathrm{r}$ and make the above bounds more explicit. For that purpose we use the fact that, according to Eq. (4.216), the decay of the longest-lived modulus determines the reheating temperature. In typical scenarios, this is the volume modulus, whose decay rate is qualitatively given by

$$\Gamma_{\tau_\mathrm{b}} \sim \frac{m_{\tau_\mathrm{b}}^3}{M_\mathrm{P}^2} , \tag{5.24}$$

where m_{τ_b} is given in Eq. (4.204) and we again ignored prefactors that are not relevant for the qualitative volume scaling. We can then use this to obtain an expression for the reheating temperature with an explicit volume dependence,

$$T_{\rm r} = \left(\frac{90}{g_* \pi^2}\right)^{1/4} \frac{|W_0|^{3/2} M_{\rm P}}{\mathcal{V}^{9/4}} \, . \tag{5.25}$$

With that we can eliminate $T_{\rm r}$ from the above bounds. As it turns out, the resulting bounds due to isocurvature describe a regime with very high reheating temperature, which violates the assumption that the axion oscillations begin during the early-matter-dominated period. Instead, this regime belongs to the standard, radiation-dominated case discussed above, which is why we only give the bounds due to DM saturation,

$$\mathcal{V} \lesssim 7 \times 10^8 \left(W_0^{4/7}\right) \, , \tag{5.26}$$

$$H_I \gtrsim 1 \times 10^5 \, \mathrm{GeV} \left(\beta^{1/2} W_0^{1/7}\right) \, , \tag{5.27}$$

$$f_a \gtrsim 1 \times 10^{13} \, \mathrm{GeV} \left(W_0^{-2/7}\right) \, , \tag{5.28}$$

$$T_r \gtrsim 30 \, \mathrm{MeV} \left(W_0^{3/14}\right) \, . \tag{5.29}$$

Bounds from BBN

As already mentioned, the reheating temperature is bounded from below, $T_{\rm r} \gtrsim \mathcal{O}(1\,\mathrm{MeV})$, in order to not spoil a succesful BBN [199–210]. Imposing this on Eq. (5.25) provides another upper bound on $\mathcal{V}$, which can be translated into bounds on $H_{\rm I}$ and f_a,

$$\mathcal{V} \lesssim 3 \times 10^9 \left(|W_0|^{2/3}\right) \, , \tag{5.30}$$

$$H_{\rm I} \gtrsim 1 \times 10^4 \, \mathrm{GeV} \left(\beta^{1/2}\right) , \tag{5.31}$$

$$f_a \gtrsim 5 \times 10^{12} \, \mathrm{GeV} \left(|W_0|^{-1/3}\right) . \tag{5.32}$$

Since the only assumption used to derive these bounds is that the volume modulus reheats the SM, they are independent of the exact time when the axion begins to oscillate. Hence these bounds are valid for both the standard scenario and the early-matter-domination scenario. While for the latter of the two scenarios the bounds from DM saturation are stronger, Eqs. (5.30) – (5.32) provide more restrictive bounds for the former scenario. Moreover, the BBN bounds are harder than those from DM saturation in the sense that their violation would immediately imply a phenomenologically unviable theory, although this should be taken with a grain of salt due to the ignored prefactors in their derivation.

We want to stress that the bounds in Eqs. (5.26) – (5.29) and (5.30) – (5.32) are based on the assumption that the volume modulus reheats the SM, which will be challenged in this work. In the next section, we will see that the volume modulus can possibly decay into SM degrees of freedom via another, very fast decay channel. This not only increases the reheating temperature and thus resembles more the standard scenario than the early-matter-domination one but also modifies the cosmological setting. That is, not the volume modulus but the inflaton cycle will be the longest-lived modulus and hence responsible for reheating. Obviously, this also dramatically changes the above bounds.

5.4. The old dark radiation problem and its new resolution by Higgs-mass-mediated decays

A ubiquitous prediction of many cosmological LVS constructions is the production of DR [101, 116–118, 130, 160–167]. As explained in Sec. 4.3.1, the amount of DR ΔN_{eff} is observationally constrained, which imposes a major challenge for the phenomenological viability of such models. The main reason for this so-called *dark radiation problem* is the volume modulus, which represents the longest-lived modulus and decays into its own axion with a branching fraction of $\mathcal{O}(1)$ [116, 117]. Since the volume axion is practically massless, as can be seen from Eq. (4.204), and its production occurs thermally, this constitutes a major contribution to DR. There are proposals to ameliorate the problem by either boosting the decays to light superpartners [161] or using a large flux on the cycle that carries the SM [130]; however, both of them are not compatible with the establishment of a stringy QCD axion. The former uses a so-called *sequestered setting*; that is, the SM does not live on a stack of 7-branes that wrap a 4-cycle but instead on a stack of 3-branes which represent a singular point in the internal space. In this case there is no candidate for a QCD axion present. The latter of the two proposals is based on fibre inflation, which as explained in Sec. 4.3.2 predicts a rather large tensor-to-scalar ratio and inflation scale and is therefore incompatible with our derived bounds from the previous section.

In the rest of this section, we will summarise the usual DR problem and explain a novel mechanism that seemingly solves it.

5.4.1. Decays of taub into its axion ab and the SM

Following [116, 117], in order to explain the DR problem, we first need to calculate the decay of τ_{b} into its own axion θ_{b}. The relevant operators originate from the kinetic term (4.203) for $T_{\text{b}} = \tau_{\text{b}} + \mathrm{i}\theta_{\text{b}}$. Starting from the usual, leading order Kähler potential (4.184), we can calculate the component K_{bb} of the Kähler metric and expand it up to leading order in the small parameter $\epsilon \equiv \tau_{\mathrm{b}}^{-1/2} \approx \mathcal{V}^{-1/3}$. This yields for the kinetic term

$$\mathcal{L}/M_{\mathrm{P}}^2 \supset K_{\mathrm{bb}} \partial_\mu T_{\mathrm{b}} \partial^\mu \bar{T}_{\mathrm{b}} = \frac{3}{4\tau_{\mathrm{b}}^2} \partial_\mu \tau_{\mathrm{b}} \partial^\mu \tau_{\mathrm{b}} + \frac{3}{4\tau_{\mathrm{b}}^2} \partial_\mu \theta_{\mathrm{b}} \partial^\mu \theta_{\mathrm{b}} \; . \tag{5.33}$$

The canonically normalised volume modulus field is given by $\phi_{\mathrm{b}}/M_{\mathrm{P}} \equiv \sqrt{3/2} \ln \tau_{\mathrm{b}}$, leading to

$$\mathcal{L} \supset \frac{1}{2} \partial_\mu \phi_{\mathrm{b}} \partial^\mu \phi_{\mathrm{b}} + \frac{3}{4} \exp\left(-2\sqrt{\frac{2}{3}} \frac{\phi_{\mathrm{b}}}{M_{\mathrm{P}}} \right) \partial_\mu \theta_{\mathrm{b}} \partial^\mu \theta_{\mathrm{b}} M_{\mathrm{P}}^2 \; . \tag{5.34}$$

Linearly expanding about the vacuum expectation value, $\phi_{\mathrm{b}} = \langle \phi_{\mathrm{b}} \rangle + \delta\phi_{\mathrm{b}}$, we obtain

$$\begin{aligned} \mathcal{L} &\supset \frac{1}{2} \partial_\mu \phi_{\mathrm{b}} \partial^\mu \phi_{\mathrm{b}} + \frac{1}{2} \exp\left(-2\sqrt{\frac{2}{3}} \frac{\langle \phi_{\mathrm{b}} \rangle}{M_{\mathrm{P}}} \right) \left(\frac{3}{2} - \sqrt{6} \frac{\delta\phi_{\mathrm{b}}}{M_{\mathrm{P}}} \right) \partial_\mu \theta_{\mathrm{b}} \partial^\mu \theta_{\mathrm{b}} M_{\mathrm{P}}^2 \; , &(5.35) \\ &= \frac{1}{2} \partial_\mu \phi_{\mathrm{b}} \partial^\mu \phi_{\mathrm{b}} + \frac{1}{2} \partial_\mu a_{\mathrm{b}} \partial^\mu a_{\mathrm{b}} - \sqrt{\frac{2}{3}} \frac{\delta\phi_{\mathrm{b}}}{M_{\mathrm{P}}} \partial_\mu a_{\mathrm{b}} \partial^\mu a_{\mathrm{b}} \; , &(5.36) \end{aligned}$$

where the canonical volume axion is defined as

$$a_{\mathrm{b}}/M_{\mathrm{P}} \equiv \sqrt{\frac{3}{2}} \exp\left(-\sqrt{\frac{2}{3}} \langle \phi_{\mathrm{b}} \rangle \right) \theta_{\mathrm{b}} = \sqrt{\frac{3}{2}} \frac{\theta_{\mathrm{b}}}{\langle \tau_{\mathrm{b}} \rangle} \; . \tag{5.37}$$

The last term in Eq. (5.36) induces a trilinear coupling between one volume modulus and two volume axion particles, leading to the decay rate [116, 117]

$$\Gamma_{\phi_{\mathrm{b}} \to a_{\mathrm{b}} a_{\mathrm{b}}} = \frac{1}{48\pi} \frac{m_{\tau_{\mathrm{b}}}^3}{M_{\mathrm{P}}^2} \; . \tag{5.38}$$

To obtain the amount of DR, ΔN_{eff}, we have to relate $\Gamma_{\phi_{\mathrm{b}} \to a_{\mathrm{b}} a_{\mathrm{b}}}$ to the decay rate of τ_{b} into the SM. Depending on the specific setting, different channels are possible; however, one major contribution is typically the decay into Higgs fields. In many supersymmetric extensions of the SM, including most prominently the so-called Minimal Supersymmetric Standard Model (MSSM), the Higgs sector is constituted by two chiral superfields, H_u and H_d. After SUSY breaking, one linear combination of these superfields and their hermitian conjugates represents the SM $SU(2)$ Higgs doublet. The coupling terms are obtained by extending the Kähler potential as [101, 211]

$$\mathcal{K} = -3\ln\left[T_\mathrm{b} + \overline{T}_\mathrm{b} - \frac{1}{3}\left(H_u\overline{H}_u + H_d\overline{H}_d + zH_uH_d + z\overline{H}_u\overline{H}_d\right)\right] , \tag{5.39}$$

where the small cycles have been omitted and z is an $\mathcal{O}(1)$ constant. Note that if z is exactly unity, the Kähler potential and thus the theory is shift-symmetric in the Higgs sector [212, 213]. Defining the small parameter

$$x \equiv -\frac{H_u\overline{H}_u + H_d\overline{H}_d + zH_uH_d + z\overline{H}_u\overline{H}_d}{3\left(T_\mathrm{b} + \overline{T}_\mathrm{b}\right)} \tag{5.40}$$

and expanding according to $\ln(1+x) \approx x$, we obtain

$$\mathcal{K} \approx -3\ln\left(T_\mathrm{b} + \overline{T}_\mathrm{b}\right) + \frac{H_u\overline{H}_u + H_d\overline{H}_d + zH_uH_d + z\overline{H}_u\overline{H}_d}{T_\mathrm{b} + \overline{T}_\mathrm{b}} \tag{5.41}$$

$$\approx -2\ln\mathcal{V} + \frac{H_u\overline{H}_u + H_d\overline{H}_d + zH_uH_d + z\overline{H}_u\overline{H}_d}{2\tau_\mathrm{b}} . \tag{5.42}$$

The resulting Lagrangian for the canonically normalised fields reads [116, 117]

$$\begin{aligned}\mathcal{L} \supset\ & \frac{1}{2}\partial_\mu\phi_\mathrm{b}\partial^\mu\phi_\mathrm{b} + \partial_\mu H_u\partial^\mu\overline{H}_u + \partial_\mu H_d\partial^\mu\overline{H}_d \\ & + \frac{1}{\sqrt{6}}\left[\phi_\mathrm{b}\left(\overline{H}_u\Box H_u + \overline{H}_d\Box H_d\right) + z\Box\phi_\mathrm{b}\left(H_uH_d + \overline{H}_u\overline{H}_d\right)\right] ,\end{aligned} \tag{5.43}$$

which induces a trilinear coupling leading to a decay rate [116, 117]

$$\Gamma_\mathrm{SM} \sim \Gamma_{\phi_\mathrm{b}\to H_uH_d} = \frac{z^2}{24\pi}\frac{m_{\tau_\mathrm{b}}^3}{M_\mathrm{P}^2} . \tag{5.44}$$

It is uncertain whether a $z \gg 1$ is possible so that the decay rate into the SM can be enhanced. Although the above decay rate applies most straightforwardly to the sequestered setting and low-scale SUSY, its qualitative implications generalise to other cases [101], e.g. to SUSY breaking at a higher scale or the non-sequestered case, which we are considering and which includes additional decays to SM gauge bosons.

Since $\Gamma_\mathrm{DR} \sim \Gamma_\mathrm{SM}$, there will be a significant amount of DR, as can be seen from Eq. (4.233). In conclusion, the DR problem is a generic issue of cosmological LVS constructions whose solution is widely accepted to be non-trivial.

In what follows, we will argue that there is an additional decay channel of the volume modulus into the SM Higgs in the regime of high-scale SUSY breaking, whose decay rate is parametrically larger than the one into volume axions, and thus the DR problem is seemingly solved.

5.4.2. Mass-term-induced, rapid decays of the volume modulus into Higgses

The underlying idea of this novel mechanism is based on the fact that the mass of the Higgs field depends on the volume modulus τ_{b}. Due to this term, small perturbations $\delta\tau_{\mathrm{b}}$ about the vacuum expectation value will induce a trilinear coupling that leads to the decay of a volume-modulus particle into two Higgs particles. Since the latter have a mass at the electroweak scale, which is usually much smaller than $m_{\tau_{\mathrm{b}}}$, one would naively expect that this coupling is suppressed by a factor $|m_H^2|/m_{\tau_{\mathrm{b}}}^2$ w.r.t. Γ_{DR}. Nevertheless, this expectation becomes disproven once we take into account that the small Higgs mass in high-scale SUSY models is not at its natural scale but instead the result of fine-tuning. This fine-tuning, which is adjusted such that the vacuum expectation value of the τ_{b}-dependent Higgs-mass parameter corresponds to the SM Higgs mass, is easily broken once we consider fluctuations $\delta\tau_{\mathrm{b}}$. The resulting trilinear coupling is therefore at the natural scale of the supersymmetric Higgs mass, which is essentially given by the SUSY breaking scale, resulting in a much larger decay rate.

In detail, we take the KK scale m_{KK} of the stack of SM branes as a UV cut-off. Above this scale, the four-dimensional supersymmetric EFT breaks down and becomes higher dimensional, whereas below it we can run down the Higgs mass matrix of H_u and H_d to the SUSY breaking scale, which is given by the gravitino mass $m_{3/2}$. The latter is determined by the F-terms of the Kähler moduli, $m_{3/2}/M_{\mathrm{P}} \sim F_T/T$. Since the natural value of the Higgs mass is set by the SUSY breaking scale, there will be entries in the Higgs mass matrix at the order of $m_{3/2}^2$. Moreover, further contributions to this mass are present, for instance due to loop corrections of virtual gauginos, which add a term $\sim c_{\mathrm{loop}} m_{1/2}^2 \ln(m_{\mathrm{KK}}/m_{3/2})$ [211–215]. Here the logarithm originates from running from the KK scale down to the SUSY breaking scale, while $c_{\mathrm{loop}} \sim 1/(16\pi^2) \ll 1$ is a loop factor and $m_{1/2} \sim m_{3/2}$ is the gaugino mass. The latter relation is a well known fact that we illustrate by a short calculation in App. A.2.[6]

Below the scale $m_{3/2}$, SUSY breaks and one linear combination of the scalar

[6] In fact, if a cycle that is wrapped by a stack of branes is stabilised non-perturbatively, the resulting gauge theory that lives on this stack is characterised by gauginos with a mass that is suppressed w.r.t. the gravitino mass by a factor $m_{3/2}/m_{1/2} \sim \ln(M_{\mathrm{P}}/m_{3/2}) \sim \ln \mathcal{V}$. This is due to a leading-order cancellation of the F-terms [216]. However, in the next section we will argue that the relevant cycle that carries the SM should not be stabilised non-perturbatively but rather by loop corrections. In this case, such a suppression of the gaugino mass does not occur so that the latter is indeed comparable to the gravitino mass [153, 217].

components in the chiral superfields H_u and $\overline{H}_d$ is removed. The remaining combination constitutes the SM Higgs doublet and its mass is fine-tuned between all contributions, which are naturally of the order $\mathcal{O}(m_{3/2})$ as we explained, down to the electroweak scale. Schematically, this can be written as

$$m_H^2 \sim m_{3/2}^2 \left[c_0 + c_{\text{loop}} \ln \left(\frac{m_{\text{KK}}}{m_{3/2}} \right) \right] , \tag{5.45}$$

where c_0 is an $\mathcal{O}(1)$ constant that entails the uncorrected, natural value of the Higgs. Since $|m_H^2| \ll m_{3/2}^2$, the two terms in brackets have to cancel with high precision. Let us now insert concrete expressions for the relevant mass scales. The SM lives on one (or several intersecting) stacks of D7 branes that wrap the small cycle τ_{L}. To produce a correct gauge coupling (cf. Eq. (4.192)), the size of this stack is rather constrained to a value of $\mathcal{O}(1-10)$ in string units. The resulting KK scale is therefore $m_{\text{KK}} \sim M_{\text{s}} \sim M_{\text{P}} \mathcal{V}^{-1/2}$. According to Eq. (4.69), together with Eq. (4.184), the gravitino mass is given by $m_{3/2}/M_{\text{P}} \sim W_0/\mathcal{V}$, so that we obtain

$$m_H^2 \sim \left(\frac{W_0}{\mathcal{V}} \right)^2 \left[c_0 + c_{\text{loop}} \ln \left(\frac{\mathcal{V}^{1/2}}{W_0} \right) \right] M_P^2 \, . \tag{5.46}$$

Now we want to perturb the volume by small values about its vacuum expectation value. More precisely, we insert $\mathcal{V} \approx \tau_{\text{b}}^{3/2}$ as well as the corresponding canonical field $\sqrt{3/2} \ln \tau_{\text{b}} = \phi_{\text{b}}/M_{\text{P}}$ and write the latter as $\phi_{\text{b}} = \langle \phi_{\text{b}} \rangle + \delta\phi_{\text{b}}$. Expanding then m_H^2 up to linear order in $\delta\phi_{\text{b}}$ yields

$$\begin{aligned}
\mathcal{L} \supset \sim\ & W_0^2 \exp\left(-\sqrt{6} \frac{\phi_{\text{b}}}{M_{\text{P}}} \right) \left[c_0 + \frac{c_{\text{loop}}}{2} \left(\sqrt{\frac{3}{2}} \frac{\phi_{\text{b}}}{M_{\text{P}}} - 2 \ln W_0 \right) \right] h^2 M_{\text{P}}^2 && (5.47) \\
\approx\ & W_0^2 \exp\left(-\sqrt{6} \frac{\langle \phi_{\text{b}} \rangle}{M_{\text{P}}} \right) \left[c_{\text{fine}} (1 - \sqrt{6} \frac{\delta\phi_{\text{b}}}{M_{\text{P}}}) + \frac{c_{\text{loop}}}{2} \sqrt{\frac{3}{2}} \frac{\delta\phi_{\text{b}}}{M_{\text{P}}} \right] h^2 M_{\text{P}}^2 && (5.48) \\
\approx\ & \frac{W_0^2}{\langle \mathcal{V} \rangle^2} \left[c_{\text{fine}} (1 - \sqrt{6} \frac{\delta\phi_{\text{b}}}{M_{\text{P}}}) + \frac{c_{\text{loop}}}{2} \sqrt{\frac{3}{2}} \frac{\delta\phi_{\text{b}}}{M_{\text{P}}} \right] h^2 M_{\text{P}}^2 \, , && (5.49)
\end{aligned}$$

where h is the Higgs scalar and we have defined

$$c_{\text{fine}} \equiv c_0 + \frac{c_{\text{loop}}}{2} \left(\sqrt{\frac{3}{2}} \frac{\langle \phi_{\text{b}} \rangle}{M_{\text{P}}} - 2 \ln W_0 \right) , \tag{5.50}$$

which is fine-tuned to a very small value. From Eq. (5.49) we see that the resulting Higgs mass as well as a contribution $\propto \sqrt{6} \delta\phi_{\text{b}}/M_{\text{P}}$ to the trilinear coupling is proportional to $c_{\rm fine}$ and hence very small. Crucially, there is another contribution to the trilinear coupling,

$$\mathcal{L} \supset \sim m_{3/2}^2 \frac{c_{\rm loop}}{2} \sqrt{\frac{3}{2}} h^2 \frac{\delta\phi_{\rm b}}{M_{\rm P}} , \tag{5.51}$$

which originates from the expansion of the logarithmic function and is therefore not subject to the fine-tuning of the mass parameter. That is, this term is not multiplied with $c_{\rm fine}$ and therefore remains at the natural scale of the Higgs mass matrix aside from the minor suppression by $c_{\rm loop}$. This induces a decay of one volume modulus into two SM Higgs particles, whose decay rate is parametrically given by

$$\Gamma_{\phi_{\rm b}\to hh} \sim \frac{m_{3/2}^4}{m_{\tau_{\rm b}}} \frac{c_{\rm loop}^2}{M_{\rm P}^2} \sim (c_{\rm loop}\mathcal{V})^2 \frac{m_{\tau_{\rm b}}^3}{M_{\rm P}} \gg \Gamma_{\phi_{\rm b}\to a_{\rm b}a_{\rm b}} . \tag{5.52}$$

Here we used Eqs. (4.204) and (5.38) and assumed $\mathcal{V} \gg 1/c_{\rm loop} \sim 16\pi^2$.

In summary, we have found a new decay channel of $\tau_{\rm b}$ into the SM, which is much stronger than into volume axions. This implies that the produced amount of DR, $\Delta N_{\rm eff} \sim \Gamma_{\tau_{\rm b}\to a_{\rm b}a_{\rm b}}/\Gamma_{\phi_{\rm b}\to hh}$ is neglible so that the standard DR problem seems to be solved. We have thus severred the Hydra's first head. There is one major issue, however, that we have not considered so far. The original discussion about the DR problem is based on the assumption that the volume modulus is the longest-lived modulus so that it is responsible for reheating the SM. This has been reasonable because the typical decay rate of a modulus scales as

$$\Gamma_{\tau_i} \sim \frac{m_{\tau_i}^3}{M_{\rm P}^2} , \tag{5.53}$$

which is lowest for the lightest modulus $\tau_{\rm b}$. However, in light of the above discussion about an increased decay into Higgses, this assumption needs re-evaluation. In particular, we need to investigate other moduli, which may have longer lifetimes, and their respective decay channels into DR, especially axions. Since such moduli could be related to the specific inflationary mechanism, we will dedicate the next section to the concrete establishment of the latter before resuming the analysis of DR in Sec. 5.6.

5.5. Combining the QCD axion with a suitable inflation model

So far, we have argued that the LVS constitutes a promising setting for the realisation of a stringy QCD axion and now we want to equip this with a proper and concrete mechanism for inflation and reheating. The two most prominent LVS based examples of inflationary models are blow-up [121] and fibre [122] inflation, which both have already been discussed in Sec. 4.3.2. As we have argued in Sec. 5.3, a stringy QCD axion requires a very small inflation scale H_I due to isocurvature constraints, which is why fibre inflation does not qualify for our needs. For the rest of this work, we will hence focus on blow-up inflation, whose volume is typically of the form given in Eq. (4.234).

As a quick reminder, in blow-up inflation the inflaton cycle τ_I corresponds to an ordinary blow-up cycle, just like the other small cycles $\tau_{\mathrm{s},i}$ needed for the LVS mechanism, which is only distinguished by its initial displacement to a large value. Both the inflaton and the small cycles are stabilised by non-perturbative effects, which modify the superpotential by a term $\propto \exp(-\mathfrak{a}_i\tau_i)$. During inflation, the corresponding correction for the inflaton is very small due to the displacement and the potential is very flat allowing for a slow-roll phase. After inflation, τ_I rolls to its minimum and is stabilised in the usual LVS manner.

Obviously, we also have to discuss which non-perturbative effects are used to stabilise the inflaton and small cycles. Possible candidates are D3-brane instantons that wrap the respective cycles or gaugino condensation on stacks of N D7-branes wrapping the cycles. As explained below Eq. (4.195), the former imply $\mathfrak{a}_\mathrm{I} = 2\pi$ and the latter $\mathfrak{a}_\mathrm{I} = 2\pi/N$. Gaugino condensation is disfavoured because of two reasons: First, additional branes that wrap a cycle other than the SM cycle would imply a dark sector due to the gauge theory on these branes, thus representing a potential danger of DR. Second, it has been argued in [121, 128] that loop effects, which have not yet been included into the analysis and which scale as

$$\delta V_{\text{inflaton,loop}} \sim \frac{M_\mathrm{P}^4}{\sqrt{\tau_\mathrm{I}}\mathcal{V}^3} \,, \tag{5.54}$$

have a tendency to spoil the flatness of the slow-roll potential. We will therefore focus on D3-brane instantons as the non-perturbative effect that stabilises τ_I and the other $\tau_{\mathrm{s},i}$. Unfortunately, this comes with a caveat as well because according to Eq. (4.200), the volume is stabilised at a value

$$\langle\mathcal{V}\rangle \sim \exp\left(\frac{\mathcal{O}(1)\mathfrak{a}_\mathrm{I}}{g_\mathrm{s}}\right) \,, \tag{5.55}$$

where the explicit dependence on g_s originates from the factor $g_\mathrm{s}^{-3/2}$ that has been absorbed into ξ in Eq. (4.193). For small values of g_s, the volume can become very large leading to a de-compactification of the internal space. While this is not necessarily harmless and deserves further investigation, we believe that the hypothetical danger of de-compactification per se is not reason enough to rule out a stabilisation via D3-brane instantons a priori and that large enough values for g_s in the range $\mathcal{O}(0.1-1)$ can be achieved, as is common in F-theory models.

After we have discussed the inflationary setting and the LVS stabilisation, we still need to implement the SM sector together with a QCD axion. The naive setting of wrapping a stack of D7-branes on a small cycle that is stabilised by non-perturbative effects is ruled out because the axion, which is the SUSY partner of the corresponding Kähler modulus, would obtain the same mass, $m_{\theta_\mathrm{s}} \sim m_{\tau_\mathrm{s}} \sim |W_0| M_\mathrm{P}/\mathcal{V}$, as is given in Eq. (4.204). This is parametrically of the same order as the SUSY breaking scale $m_{3/2}$ and hence incompatible with the QCD axion, which according to Eq. (4.252) is very light, $m_{a,0} \lesssim \mathcal{O}(\mathrm{meV})$. For that reason, we adapt an idea from Sec. 4.3 of [137] to our case. Here the SM sector is realised with the help of two intersecting small cycles. Via D-terms, one combination of the two corresponding Kähler moduli is stabilised, which can than be integrated out at a high mass. The other combination is stabilised by loop effects [102, 103, 218, 219] and acts as an effective, single modulus that carries the SM on intersecting stacks of branes that wrap the two moduli. Effectively, the volume can be written as

$$\mathcal{V} = \tau_\mathrm{b}^{3/2} - \gamma_\mathrm{s}\tau_\mathrm{s}^{3/2} - \gamma_\mathrm{I}\tau_\mathrm{I}^{3/2} - \gamma_\mathrm{L}\tau_\mathrm{L}^{3/2} \;, \tag{5.56}$$

where the index 'L' refers to 'Loop'. This constitutes the given form of the volume that we will consider for the rest of this work and from which we derive the relevant dynamics. Note that the term $\propto \tau_\mathrm{s}^{3/2}$ represents a sum over many small cycles that are needed for the stability of the volume during inflation, as explained in Sec. (4.3.2), although for most future purposes we can treat it as a single modulus or even ignore it. We also assign the numbers '1', '2', '3' to the labels 'b', 'I', 'L', respectively.

As argued in [137], the resulting loop potential for τ_L plausibly reads

$$V_\mathrm{loop} = \left(\frac{\mu_1}{\sqrt{\tau_\mathrm{L}}} - \frac{\mu_2}{\sqrt{\tau_\mathrm{L}} - \mu_3}\right)\frac{W_0^2 M_P^4}{\mathcal{V}^3} \;, \tag{5.57}$$

where the μ_i are positive constants. In detail, μ_1 and μ_2 are determined by the vacuum expectation values of the complex-structure moduli, whereas μ_3 is related to the size of a small cycle τ_s. That is, we suppose that the internal geometry will generate a loop correction as given above with $\mu_3 = c\sqrt{\langle\tau_\mathrm{s}\rangle}$ at a low enough energy at which τ_s has been integrated out. If μ_1 and μ_2 take on similar values and c is a constant in the range $\mathcal{O}(1-10)$, the resulting loop cycle will be fixed at a value that is compatible with the SM gauge coupling,

$$\langle \tau_{\rm L} \rangle = \frac{\mu_1 \mu_3^2}{(\sqrt{\mu_1} + \sqrt{\mu_2})^2} \sim \mathcal{O}(10) \; . \tag{5.58}$$

One important aspect that we want to stress is that the potential generated by the perturbative effects does not depend on the axionic partner $\theta_{\rm L}$. The latter remains therefore light until its potential is finally generated by QCD effects at a much lower energy. For that reason, $\theta_{\rm L}$ constitutes a suitable candidate for the QCD axion.

Having established a concrete inflationary scenario and identified our QCD axion, we want to return to the dynamics at the end of inflation, reheating and the issue of DR in the following section.

5.6. The new dark radiation problem due to problematic contributions from early decays

We have argued in Sec. 5.4 that the decay of the volume modulus $\phi_{\rm b}$ is greatly enhanced due to an additional channel into SM Higgses via the volume-dependent mass term. The resulting decay rate, as given by Eq. (5.52) scales like

$$\Gamma_{\phi_{\rm b}} \sim c_{\rm loop}^2 \frac{M_{\rm P}}{\mathcal{V}^{5/2}} \; . \tag{5.59}$$

Such a fast decay rate implies that the lifetime of the volume modulus is dramatically decreased, which challenges the standard assumption that this modulus comes to dominate the universe before it decays and reheats the SM. We therefore have to re-evaluate the dynamics at the end of inflation taking into account other moduli fields, in particular the inflaton modulus, and examine whether they decay more slowly than $\phi_{\rm b}$. In the latter case, one can consider any decay of such a longer-lived modulus into the volume modulus to be followed by a practically instantaneous decay of $\phi_{\rm b}$ into the SM; that is, decays into $\phi_{\rm b}$ may be treated as direct decays into the SM. The resulting reheating dynamics, especially w.r.t. the question about DR, are therefore determined by the branching fractions of the longest-lived modulus into $\phi_{\rm b}$, other moduli, SM particles and DR, specifically axions.

As we explained in the last section, we consider the scenario of blow-up inflation, with a volume given by Eq. (5.56), and want to focus on the dynamics after inflation. Therefore, the relevant fields that participate in the dynamics are the inflaton $\phi_{\rm I}$, the volume modulus $\phi_{\rm b}$, the loop-stabilised SM-cycle modulus $\phi_{\rm L}$ and the small-cycle moduli ϕ_s, which are needed for stability during inflation, as well as the axionic partners of all the respective moduli, a_I, a_b, a_L and a_s. In Sec. 4.3.2, it is described that after inflation, due to vacuum misalignment and parametric resonances, the volume modulus is excited in a violent process causing it to perform coherent oscillations [126]. We do not have anything new to add regarding this mechanism but instead consider the period afterwards, when the coherent oscillations have been damped after a few oscillations and can be considered as small, decoherent fluctuations about the minimum, i.e. particles, so that perturbation theory can be applied [128].

It is conceivable and perhaps even likely that the inflaton axion a_I is excited in this very process as well [126, 127, 220]; however, we will not take it into account in our dynamical analysis for two reasons: First, since its mass is the same as of the inflaton, as indicated in Eq. (4.204), a perturbative decay into it is kinematically forbidden. Second, if there is any energy stored in a_I due to the aforementioned coherent oscillations, a_I will tend to decay into lighter axions a_b and a_L at least as fast as ϕ_b because axionic decays always involve another axion as decay product, as we will argue below. Hence, an inclusion of a_I into the analysis would only strengthen our final results, which state that too much DR is produced.

We will also exclude the small-cycle moduli ϕ_s and their respective axions a_s from the dynamical analysis because in the perturbative regime their treatment and decay rates are identical to those of ϕ_I. Moreover, since their mass is comparable to the inflaton mass m_{τ_I}, it is questionable whether a decay $\phi_\mathrm{I} \to \phi_\mathrm{s}\phi_\mathrm{s}$ is kinematically allowed.

This leaves us with the fields ϕ_I, ϕ_b, ϕ_L, a_b and a_L. It will turn out that the inflaton is the longest-lived particle, which decays into the SM via ϕ_b and ϕ_L and in equal parts into DR in the form of a_b and a_L. In the following, we will illustrate the derivation of the relevant decay rates and present the most important findings, whereas we refer to Apps. A.3 and A.4 for more detailed calculations.

5.6.1. Decay rates

The underlying mass hierarchy

Let us first elaborate on the mass hierarchy of the relevant fields. The Kähler potential is given by Eq. (4.193) with $\mathcal{V}$ given in Eq. (5.56), where we ignore the small cycles τ_s as argued above. Without considering these small cycles, the superpotential is only corrected by non-perturbative corrections on the inflaton cycle, thus taking on the form

$$W = W_0 + A_\mathrm{I}\mathrm{e}^{-\mathfrak{a}_\mathrm{I} T_\mathrm{I}} \; . \tag{5.60}$$

The resulting scalar potential is the combination of the usual LVS F-term potential for the inflaton, $V_\mathrm{LVS}^{(\mathrm{I})}$, as given by Eq. (4.199), and the loop potential, V_loop, given in

Eq. (5.57), so that we have

$$V = V_{\mathrm{LVS}}^{(\mathrm{I})}(\tau_{\mathrm{I}}, \theta_{\mathrm{I}}, \mathcal{V}) + V_{\mathrm{loop}}(\tau_{\mathrm{L}}, \mathcal{V}) \ . \tag{5.61}$$

The former term explicitly reads

$$V_{\mathrm{LVS}}^{(\mathrm{I})}/M_P^4 = \mathcal{V}^{-2}\left[\frac{8\tau_{\mathrm{b}}^{3/2}\sqrt{\tau_{\mathrm{I}}}}{3\gamma_{\mathrm{I}}}\mathfrak{a}_{\mathrm{I}}^2|A_{\mathrm{I}}|^2\mathrm{e}^{-2\mathfrak{a}_{\mathrm{I}}\tau_{\mathrm{I}}} + 4\mathfrak{a}_{\mathrm{I}}\tau_{\mathrm{I}}\mathrm{e}^{-\mathfrak{a}_{\mathrm{I}}\tau_{\mathrm{I}}}|A_{\mathrm{I}}W_0|\cos\left(\mathfrak{a}_{\mathrm{I}}\theta_{\mathrm{I}}\right)\right] + \frac{3|W_0|^2\xi}{4\mathcal{V}^3} \ , \tag{5.62}$$

where at the minimum, the inflaton axion fulfills

$$\cos(\mathfrak{a}_{\mathrm{I}}\left\langle\theta_{\mathrm{I}}\right\rangle) = -1 \ . \tag{5.63}$$

With the Kähler potential and the scalar potential given, we can calculate the masses of all the relevant fields, which can be also found in the literature [99, 100]. In Tab. 3, all the explicit expressions as well as the volume scalings are summarised.

Field	m_i^2 scaling	m_i^2 explicit
ϕ_{b}	$\sim \mathcal{V}^{-3}M_P^2$	$\frac{-m_{13}m_{22}m_{31}-m_{12}m_{21}m_{33}+m_{11}m_{22}m_{33}}{m_{22}m_{33}}\mathcal{V}^{-3}M_P^2$
ϕ_{I}	$\sim \mathfrak{a}_{\mathrm{I}}^2\tau_{\mathrm{I}}^2\mathcal{V}^{-2}M_P^2$	$\frac{4\lvert W_0\rvert^2\mathfrak{a}_{\mathrm{I}}^2\tau_{\mathrm{I}}^2}{\mathcal{V}^2}M_P^2$
ϕ_{L}	$\sim \tau_{\mathrm{L}}^{-2}\mathcal{V}^{-2}M_P^2$	$\frac{W_0^2\left(3\tilde{\mu}^3\mu_1+\mu_2\tau_{\mathrm{L}}\left(-\mu_3+3\sqrt{\tau_{\mathrm{L}}}\right)\right)}{3\gamma_{\mathrm{L}}\tilde{\mu}^3\tau_{\mathrm{L}}^2\mathcal{V}^2}M_P^2$
a_{b}	0	0
a_{I}	$\sim \mathfrak{a}_{\mathrm{I}}^2\tau_{\mathrm{I}}^2\mathcal{V}^{-2}M_P^2$	$\frac{4\lvert W_0\rvert^2\mathfrak{a}_{\mathrm{I}}^2\tau_{\mathrm{I}}^2}{\mathcal{V}^2}M_P^2$
a_{L}	0	0

Table 3.: Masses of canonical moduli and axion fields. The parameters $\tilde{\mu}$ and m_{ij} are defined below Eq. (A.45) and in Eqs. (A.54) − (A.62).

For the resulting hierarchy, we have

$$m_{\tau_{\mathrm{I}}} \approx m_{a_{\mathrm{I}}} \gg m_{\tau_{\mathrm{L}}} \gg m_{\tau_{\mathrm{b}}} \gg m_{a_{\mathrm{L}}} \sim m_{a_{\mathrm{b}}} \sim 0 \ . \tag{5.64}$$

In particular, the volume and loop-cycle axions, a_{b} and a_{L} are essentially massless and hence constitute DR. Note that this is not in contradiction with the general discussion in Secs. 4.3.3 and 5.3, where we considered the QCD axion a_{L} to represent a DM candidate. Namely, when discussing reheating dynamics, a_{L}-particles are created thermally, mostly via decays of the inflaton, and at a very high temperature at which the axion mass is basically zero (cf. Eq. (4.251)). On the other hand, when discussing axionic DM, $a_{\rm L}$-particles are produced non-thermally due to the misalignment mechanism and originate from a coherent state that behaves like matter. Moreover, this happens at a much lower temperature when QCD effects create a potential for $a_{\rm L}$.

Decay rate of the loop-cycle modulus

Knowing the relevant mass hierarchy, we are ready to consider decays. Due to mixing effects, the corresponding calculations are rather involved and we will focus on qualitative derivations of the major results in the following subsections, referring to the Apps. A.3 and A.4 for details.

Since the inflaton is the heaviest particle and we assume that a large portion of the energy after inflation is stored in the form $\phi_{\rm I}$-particles, we are specifically interested in its decays. Its natural decay products are the respective lighter fields and axions. We have already argued that $\phi_{\rm b}$ dominantly decays into the SM, whereas the two axions $a_{\rm b}$ and $a_{\rm L}$, which are practically stable since they are massless, constitute DR. Let us therefore focus on the decays of $\phi_{\rm L}$ before we consider $\phi_{\rm I}$ in the next subsections.

The decay rates of $\tau_{\rm L}$, which determines the SM gauge couplings, are derived in a detailed calculation in App. A.3. One could also think about an even more exact calculation analogous to that presented in App. A.4 for the decays of $\tau_{\rm I}$. In this subsection, we give a very short, intuitive argument that the loop-cycle modulus decays predominantly into SM degrees of freedom. Since $\tau_{\rm L}$ is given by a 'largish' $\mathcal{O}(1)$ number in string units, we can make the assumption that the volume decouples, i.e. that $\mathcal{V}^{1/6}$ is larger than all local length scales. In that case, the relevant operator that is responsible for the decay into SM gauge bosons, $\phi_{\rm L}F_{\mu\nu}F^{\mu\nu}$, will be suppressed by a factor $M_{\rm s}^{-1}$ due to reasons of dimensionality, which characterises the local length scale. This implies that the decay rate into SM gauge bosons A_μ is given by

$$\Gamma_{\phi_{\rm L}\to AA} \sim \frac{m_{\tau_{\rm L}}^3}{M_{\rm s}^2} \sim \frac{M_{\rm P}}{\mathcal{V}^2}\,, \tag{5.65}$$

where the factor $m_{\tau_{\rm L}}^3$ arises so that the mass dimensions match and it has been used that $m_{\tau_{\rm L}} \sim M_{\rm P}/\mathcal{V}$, as given in Tab. 3, and $M_{\rm s} \sim M_{\rm P}/\mathcal{V}$. The above decay rate implies that the loop-cycle modulus decays much faster than the inflaton, which as we will see has a decay rate $\Gamma_{\phi_{\rm I}} \sim M_{\rm P}/\mathcal{V}^4$, so that we can already identify the inflaton as the longest-lived particle that will reheat the SM.

Now we must investigate the respective decay products of $\phi_{\rm L}$; in particular, we must check how much DR due to decays into its own axion $a_{\rm L}$ is created. Both the decay into SM gauge bosons and into loop-cycle axions originate from the respective kinetic terms and their dependence on $\tau_{\rm L}$ (cf. Eqs. (4.192) and (4.203)),

$$\mathcal{L} \supset\sim \tau_{\rm L}{\rm Tr}F_{\mu\nu}F^{\mu\nu} \ , \quad \mathcal{L} \supset\sim \frac{1}{\sqrt{\tau_{\rm L}}\mathcal{V}}\partial_\mu\theta_{\rm L}\partial^\mu\theta_{\rm L} \ . \tag{5.66}$$

The trilinear coupling terms are then obtained by perturbing the loop-cycle modulus about its vacuum expectation value, $\tau_{\rm L} = \langle\tau_{\rm L}\rangle + \delta\tau_{\rm L}$. The terms proportional to $\langle\tau_{\rm L}\rangle$ constitute the free kinetic terms and must be canonically normalised, whereas those proportional to $\delta\tau_{\rm L}$ lead to the couplings and resulting decays. Since the $\tau_{\rm L}$-dependence in the axionic kinetic term comes inside a square-root, the resulting amplitude for the decay rate is suppressed by a factor $1/2$ compared to the one into gauge-bosons,

$$\frac{|\mathcal{A}_{\phi_{\rm L}\to AA}|}{|\mathcal{A}_{\phi_{\rm L}\to a_{\rm L}a_{\rm L}}|} = 2 \ . \tag{5.67}$$

Moreover, since the resulting gauge bosons can have one of two possible polarisations and there are at least $N_g = 1+3+8 = 12$ gauge bosons, we obtain

$$\frac{\Gamma_{\phi_{\rm L}\to AA}}{\Gamma_{\phi_{\rm L}\to a_{\rm L}a_{\rm L}}} = 2\times 2^2 N_g = 8N_g \gg 1 \ . \tag{5.68}$$

In App. A.3 we perform the above calculation in more detail and also take into account a possible enhancement of decays to the SM sector due to the $\tau_{\rm L}$-dependence in the Higgs mass, in analogy to the discussion in Sec. 5.4.2. We conclude that $\phi_{\rm L}$ decays very fast, i.e. instantaneously when compared to the decay rate of the inflaton, into the SM without exacerbating the DR problem.

Decay rate of the inflaton to the volume modulus

In the previous subsections, we have illustrated that the inflaton modulus is the longest-lived particle after inflation and that any decay into $\phi_{\rm b}$ or $\phi_{\rm L}$ is followed by a quasi-immediate decay into SM degrees of freedom, whereas decays into the axions $a_{\rm b}$ and $a_{\rm L}$ manifest as DR. The next goal is therefore to analyse the branching ratios of $\phi_{\rm I}$ into the respective particles. It will turn out that decays due to kinetic terms will dominate over potential-induced decays and that the former lead to an equal decay rate into moduli fields and their respective axions. This will indicate the re-emergence of a DR problem due to inflaton decays. As before, we refer to App. A.4 for detailed calculations.

We benchmark all of the following relevant decay rates to the decay of one inflaton into two volume-modulus particles, which is arguably the simplest one to analyse. Let us start with a simplified form of the Kähler potential that ignores the loop-cycle modulus,

$$\mathcal{K} = -2\ln\left(\tau_{\mathrm{b}}^{3/2} - \gamma_{\mathrm{I}}\tau_{\mathrm{I}}^{3/2}\right) \,. \tag{5.69}$$

The resulting diagonal part of the kinetic terms, as given in Eq. (4.203), reads

$$\mathcal{L}/M_{\mathrm{P}}^2 \supset \sim \frac{\tau_{\mathrm{b}}}{\mathcal{V}^2}(\partial\tau_{\mathrm{b}})^2 + \frac{1}{\sqrt{\tau_{\mathrm{I}}}\mathcal{V}}(\partial\tau_{\mathrm{I}})^2 \,, \tag{5.70}$$

where we have ignored $\mathcal{O}(1)$ factors for brevity.

Perturbing the inflaton modulus about its vacuum expectation value, $\tau_{\mathrm{I}} = \langle\tau_{\mathrm{I}}\rangle + \delta\tau_{\mathrm{I}}$, and expanding the first of the above terms in leading order in $\delta\tau_{\mathrm{I}}$, one findes the trilinear coupling term

$$\mathcal{L}/M_{\mathrm{P}}^2 \supset \sim \frac{\tau_{\mathrm{b}}\sqrt{\tau_{\mathrm{I}}}}{\mathcal{V}^3}\delta\tau_{\mathrm{I}}(\partial\tau_{\mathrm{b}})^2 \,. \tag{5.71}$$

After inserting appropriate factors due to canonical normalisation and replacing the derivatives ∂^2 by $m_{\tau_{\mathrm{I}}}^2$ using the free Klein-Gordon equation (cf. Eq. (A.94)), the decay amplitude can approximately given by

$$|\mathcal{A}_{\phi_{\mathrm{I}}\to\phi_{\mathrm{b}}\phi_{\mathrm{b}}}|M_{\mathrm{P}} \sim \frac{\tau_{\mathrm{b}}\sqrt{\tau_{\mathrm{I}}}}{\mathcal{V}^3}\frac{\mathcal{V}^2}{\tau_{\mathrm{b}}}\sqrt{\mathcal{V}}\tau_{\mathrm{I}}^{1/4}m_{\tau_{\mathrm{I}}}^2 \sim \frac{\tau_{\mathrm{I}}^{3/4}}{\sqrt{\mathcal{V}}}m_{\tau_{\mathrm{I}}}^2 \,, \tag{5.72}$$

which results in the decay rate

$$\Gamma_{\phi_{\mathrm{I}}\to\phi_{\mathrm{b}}\phi_{\mathrm{b}}} \sim \frac{|\mathcal{A}_{\phi_{\mathrm{I}}\to\phi_{\mathrm{b}}\phi_{\mathrm{b}}}|^2}{m_{\tau_{\mathrm{I}}}} \sim \frac{\tau_{\mathrm{I}}^{3/2}}{\mathcal{V}}\frac{m_{\tau_{\mathrm{I}}}^3}{M_{\mathrm{P}}^2} \sim \frac{\tau_{\mathrm{I}}^{9/2}}{\mathcal{V}^4}M_{\mathrm{P}} \,, \tag{5.73}$$

where $m_{\tau_{\mathrm{I}}} \sim \tau_{\mathrm{I}}M_{\mathrm{P}}/\mathcal{V}$ has been used. A more precise derivation is given in App. A.4 where we not only include all the ignored $\mathcal{O}(1)$ factors but also diagonalise the system, thus taking care of mixing effects that lead to a modification of many results by $\mathcal{O}(1)$ factors. For that purpose, we follow the procedure explained in [128]; however, we extend the analysis by the inclusion of trilinear couplings that originate from the kinetic Lagrangian. Finally, let us mention that we always work at leading order in the large quantity $\tau_{\mathrm{I}} \sim \ln\mathcal{V}$. A more precise treatment might be appropriate for better quantitative statements; however, we do not expect any different outcome concerning our qualitative conclusions.

Dominance of kinetic over potential terms in the decay of the inflaton to the volume modulus

While the decay rate of the previous subsection originated from an expansion of the kinetic Lagrangian, in a full analysis we need to take into account trilinear terms stemming from an expansion of both the kinetic Lagrangian and the potential. A natural question to ask is which of the two results in a stronger contribution. It turns out that the answer is clearly the former, i.e., the kinetic terms provide the dominant contributions. To provide some intuition for this, let us compare the two kinds of terms disregarding mixing effects and suppressing $\mathcal{O}(1)$ factors.

The kinetic trilinear coupling terms are obtained by a linear expansion of the Kähler metric, i.e. a small shift of a modulus field $\delta\tau_i$ away from its vacuum expectation value, in the kinetic terms (4.203). This leads to

$$\mathcal{L}/M_{\mathrm{P}}^2 \supset \sim (\partial_{\tau_i} K_{jk})\delta\tau_i \partial_\mu \delta\tau_j \partial^\mu \delta\tau_k \, . \tag{5.74}$$

One of the above fields represents the inflaton modulus τ_{I} and the other two are the respective decay products. It turns out that the precise choice will change the result by only a possible $\mathcal{O}(1)$ factor and minus sign so that we w.l.o.g. assign the index $i = \mathrm{I}$ to the inflaton, which gives us

$$\mathcal{L}/M_{\mathrm{P}}^2 \supset \sim (\partial_{\tau_{\mathrm{I}}} K_{jk})\delta\tau_{\mathrm{I}} \partial_\mu \delta\tau_j \partial^\mu \delta\tau_k \sim m_{\tau_{\mathrm{I}}}^2 (\partial_{\tau_{\mathrm{I}}} K_{jk})\delta\tau_{\mathrm{I}} \delta\tau_j \delta\tau_k \, , \tag{5.75}$$

where in the last relation we again replaced the derivatives by the masses using the free Klein-Gordon equation. Ignoring mixing effects, the mass is given by the second derivative of the potential w.r.t. the canonical inflaton field evaluated at the minimum,

$$\begin{aligned} m_{\tau_{\mathrm{I}}}^2 \sim \frac{1}{M_{\mathrm{P}}^2} \frac{\partial^2 V}{(\partial\phi_{\mathrm{I}})^2}\bigg|_{\phi_{\mathrm{I}}=\langle\phi_{\mathrm{I}}\rangle} &= \frac{1}{M_{\mathrm{P}}^2}\left(\frac{\partial V}{\partial\tau_{\mathrm{I}}}\frac{\partial^2\tau_{\mathrm{I}}}{(\partial\phi_{\mathrm{I}})^2} + \frac{\partial^2 V}{\partial\tau_{\mathrm{I}}\partial\phi_{\mathrm{I}}}\frac{\partial\tau_{\mathrm{I}}}{\partial\phi_{\mathrm{I}}}\right)\bigg|_{\phi_{\mathrm{I}}=\langle\phi_{\mathrm{I}}\rangle} \\ &= \frac{1}{M_{\mathrm{P}}^2}\left(\frac{\partial V}{\partial\tau_{\mathrm{I}}}\frac{\partial^2\tau_{\mathrm{I}}}{(\partial\phi_{\mathrm{I}})^2} + \frac{\partial^2 V}{(\partial\tau_{\mathrm{I}})^2}\left(\frac{\partial\tau_{\mathrm{I}}}{\partial\phi_{\mathrm{I}}}\right)^2\right)\bigg|_{\phi_{\mathrm{I}}=\langle\phi_{\mathrm{I}}\rangle} \\ &\sim \frac{\partial_{\tau_{\mathrm{I}}}\partial_{\tau_{\mathrm{I}}} V}{K_{\mathrm{II}} M_{\mathrm{P}}^2} \, , \end{aligned} \tag{5.76}$$

where in the last line we used that $\partial V/\partial\tau_{\mathrm{I}} = 0$ at the minimum and that the canonically normalised inflaton field is approximately given by $\phi_{\mathrm{I}} \sim \sqrt{K_{\mathrm{II}}}\tau_{\mathrm{I}}$.

In analogy to Eq. (5.75), the potential trilinear coupling terms are obtained by expanding the potential to third order in the moduli fields,

$$\mathcal{L} \supset \sim (\partial_{\tau_{\mathrm{I}}}\partial_{\tau_j}\partial_{\tau_k} V)\delta\tau_{\mathrm{I}}\delta\tau_j\delta\tau_k \, . \tag{5.77}$$

To compare the decay rates of the inflaton into two volume-modulus particles resulting from Eqs. (5.75) and (5.77) to each other, we set $\tau_j, \tau_k \to \tau_{\mathrm{b}}$ and use the easily derived, approximate relations

$$\partial_{\tau_\mathrm{I}}\partial_{\tau_\mathrm{I}}V \sim \mathfrak{a}_\mathrm{I}^2 V \ , \quad \partial_{\tau_\mathrm{I}}\partial_{\tau_\mathrm{b}}\partial_{\tau_\mathrm{b}}V \sim \frac{\mathfrak{a}_\mathrm{I}}{\tau_\mathrm{b}^2}V \ , \quad \partial_{\tau_\mathrm{I}}K_\mathrm{bb} \sim \frac{\sqrt{\tau_\mathrm{I}}}{\tau_\mathrm{b}^{7/2}} \ , \quad K_\mathrm{II} \sim \frac{1}{\tau_\mathrm{b}^{3/2}\sqrt{\tau_\mathrm{I}}} \ . \tag{5.78}$$

The first two relations can be understood from the functional form of the potential, given in Eq. (5.62), which depends primarily exponentially on τ_I but power-like on τ_b. With the above relations and Eq. (5.76), we obtain for the estimated ratio of the decay amplitudes due to kinetic and potential coupling terms

$$\frac{|\mathcal{A}^\mathrm{kin}_{\tau_\mathrm{I}\to\tau_\mathrm{b}\tau_\mathrm{b}}|}{|\mathcal{A}^\mathrm{pot}_{\tau_\mathrm{I}\to\tau_\mathrm{b}\tau_\mathrm{b}}|} \sim \frac{m_{\tau_\mathrm{I}}^2 M_\mathrm{P}^2(\partial_{\tau_\mathrm{I}}K_\mathrm{bb})}{(\partial_{\tau_\mathrm{I}}\partial_{\tau_\mathrm{b}}\partial_{\tau_\mathrm{b}}V)} \sim \frac{(\partial_{\tau_\mathrm{I}}\partial_{\tau_\mathrm{I}}V)(\partial_{\tau_\mathrm{I}}K_\mathrm{bb})}{K_\mathrm{II}(\partial_{\tau_\mathrm{I}}\partial_{\tau_\mathrm{b}}\partial_{\tau_\mathrm{b}}V)} \sim \mathfrak{a}_\mathrm{I}\tau_\mathrm{I} \gg 1 \ . \tag{5.79}$$

Dominance of the kinetic over the potential term in the decay of the inflaton to the loop modulus

Having discussed the ratio of kinetic versus potential decays into the volume modulus, we want to perform an analogous analysis for the decay $\delta\tau_\mathrm{I} \to \delta\tau_\mathrm{L}\delta\tau_\mathrm{L}$ into the loop-cycle modulus. To this end, we will now include τ_L in the Kähler potential,

$$\mathcal{K} = -2\ln\left(\tau_\mathrm{b}^{3/2} - \gamma_\mathrm{I}\tau_\mathrm{I}^{3/2} - \gamma_\mathrm{L}\tau_\mathrm{L}^{3/2}\right) \ . \tag{5.80}$$

The corresponding kinetic trilinear coupling terms are simply given by Eq. (5.75) after assigning $\tau_j, \tau_k \to \tau_\mathrm{L}$. In doing so, we need to estimate the expression

$$\partial_{\tau_\mathrm{I}}K_\mathrm{LL} \sim \frac{\sqrt{\tau_\mathrm{I}}}{\tau_\mathrm{b}^3\sqrt{\tau_\mathrm{L}}} \ . \tag{5.81}$$

Likewise, the potential coupling terms are obtained from the corresponding expression in Eq. (5.77) after the same assignment of fields. Crucially, we now have to take into consideration both contributions, $V_\mathrm{LVS}^{(\mathrm{I})}$ and V_loop, to the scalar potential. In order to compare their respective effects, we will analyse them separately.

Regarding the contribution from $V_\mathrm{LVS}^{(\mathrm{I})}$, there is a minor issue that slightly complicates things: While $V_\mathrm{LVS}^{(\mathrm{I})}$ depends explicitly on τ_I, the moduli fields τ_b and τ_L enter the potential only inside the expression for the overall volume, $\mathcal{V}(\tau_\mathrm{b}, \tau_\mathrm{I}, \tau_\mathrm{L})$.[7] This implies that there is in fact a flat direction, which corresponds to fluctuations of the loop-cycle modulus τ_L after the system has been diagonalised. Excitations in this flat direction represent particles of the loop-stabilised modulus and naively one might expect that the potential would not yield any contribution due to this very

[7] Note that the seemingly explicit dependence on τ_b in Eq. (5.62) is merely the result of a leading order expansion and disappears for the full expression.

flatness. As a consequence, any potential coupling to loop-cycle particles would vanish. To scrutinise this further, note that an excitation in the exactly flat direction is specified by the two conditions $\delta\mathcal{V} = 0$ and $\delta\tau_{\rm I} = 0$, which simply imply that the excitation is not directed in the non-flat directions of $\mathcal{V}$ and $\tau_{\rm I}$. At linear level, the conditions are given by

$$\delta\mathcal{V} = (\partial_{\tau_{\rm b}}\mathcal{V})\delta\tau_{\rm b} + (\partial_{\tau_{\rm I}}\mathcal{V})\delta\tau_{\rm I} + (\partial_{\tau_{\rm L}}\mathcal{V})\delta\tau_{\rm L} = (\partial_{\tau_{\rm b}}\mathcal{V})\delta\tau_{\rm b} + (\partial_{\tau_{\rm L}}\mathcal{V})\delta\tau_{\rm L} = 0 \ , \quad (5.82)$$

where in the second step we used $\delta\tau_{\rm I} = 0$.

That is, in order to be aligned with the flat direction, an excitation of $\delta\tau_{\rm L}$ must always go together with a small excitation of $\delta\tau_{\rm b}$,

$$\delta\tau_{\rm b} = \delta\tau_{\rm b}(\delta\tau_{\rm L}) = \frac{\gamma_{\rm L}\sqrt{\tau_{\rm L}}}{\sqrt{\tau_{\rm b}}}\delta\tau_{\rm L} \ . \quad (5.83)$$

Even though this ensures that the direction $\mathcal{V}$ is not excited at linear order, a non-vanishing, second-order fluctuation will nevertheless emerge,

$$\delta\mathcal{V}(\delta\tau_{\rm L}) = \frac{1}{2}\frac{\partial^2\mathcal{V}}{\partial\tau_{\rm b}^2}\delta\tau_{\rm b}^2 + \frac{\partial^2\mathcal{V}}{\partial\tau_{\rm b}\partial\tau_{\rm L}}\delta\tau_{\rm b}\delta\tau_{\rm L} + \frac{1}{2}\frac{\partial^2\mathcal{V}}{\partial\tau_{\rm L}^2}\delta\tau_{\rm L}^2 \quad (5.84)$$

$$= \left(\frac{3\gamma_{\rm L}^2\tau_{\rm L}}{8\tau_{\rm b}^{3/2}} - \frac{3\gamma_{\rm L}}{8\tau_{\rm L}^{1/2}}\right)\delta\tau_{\rm L}^2 \ . \quad (5.85)$$

Since $\tau_{\rm b} \gg \tau_{\rm L}$, the former term is negligible and the second-order fluctuation in $\mathcal{V}$-direction takes on the same form that we would naively expect from an excitation in the non-diagonal $\tau_{\rm L}$-direction,

$$\delta\mathcal{V} = \frac{1}{2}\frac{\partial^2\mathcal{V}}{\partial\tau_{\rm L}^2}\delta\tau_{\rm L}^2 = -\frac{3\gamma_{\rm L}}{8\tau_{\rm L}^{1/2}}\delta\tau_{\rm L}^2 \ . \quad (5.86)$$

Hence, despite the presence of a flat direction at linear level, the potential yields a non-vanishing contribution at the quadratic level so that the resulting potential coupling to loop-cycle particles is non-zero.

We may therefore proceed analogously to the decay into the volume modulus and use Eq. (5.77) with $\tau_{\rm I} \to \tau_{\rm L}\tau_{\rm L}$. Naively, one would expect that

$$\partial_{\tau_{\rm I}}\partial_{\tau_{\rm L}}\partial_{\tau_{\rm L}}V_{\rm LVS}^{\rm (I)} \sim \frac{\mathfrak{a}_{\rm I}}{\tau_{\rm b}^{3/2}\sqrt{\tau_{\rm L}}}V_{\rm LVS}^{\rm (I)} \ ; \quad (5.87)$$

however, it turns out that those terms where the $\tau_{\rm I}$-derivative acts directly on the two exponential functions, $\sim \partial_{\tau_{\rm I}}\exp(-2\mathfrak{a}_{\rm I}\tau_{\rm I})$ and $\sim \partial_{\tau_{\rm I}}\exp(-\mathfrak{a}_{\rm I}\tau_{\rm I})$, cancel exactly. For that reason, the resulting leading-order contribution from the LVS potential is further suppressed and reads

$$\partial_{\tau_\mathrm{I}}\partial_{\tau_\mathrm{L}}\partial_{\tau_\mathrm{L}} V_{\mathrm{LVS}}^{(\mathrm{I})} \sim \frac{1}{\tau_\mathrm{b}^{3/2}\tau_\mathrm{I}\sqrt{\tau_\mathrm{L}}} V_{\mathrm{LVS}}^{(\mathrm{I})} \; . \tag{5.88}$$

Since the loop potential V_{loop} depends explicitly on τ_L, a resulting coupling term due to fluctuations of the loop-cycle modulus are apparent and the corresponding expression is readily obtained as

$$\partial_{\tau_\mathrm{I}}\partial_{\tau_\mathrm{L}}\partial_{\tau_\mathrm{L}} V_{\mathrm{loop}} \sim \frac{\sqrt{\tau_\mathrm{I}}}{\tau_\mathrm{b}^{3/2}\tau_\mathrm{L}^2} V_{\mathrm{loop}} \; . \tag{5.89}$$

We can now compare the two contributions. From Eqs. (5.62) and (5.57), we ascertain that $V_{\mathrm{LVS}}^{(\mathrm{I})} \sim \tau_\mathrm{I}^{3/2}\sqrt{\tau_\mathrm{L}} V_{\mathrm{loop}}$, which together with Eqs. (5.88) and (5.89) implies

$$\frac{\partial_{\tau_\mathrm{I}}\partial_{\tau_\mathrm{L}}\partial_{\tau_\mathrm{L}} V_{\mathrm{LVS}}^{(\mathrm{I})}}{\partial_{\tau_\mathrm{I}}\partial_{\tau_\mathrm{L}}\partial_{\tau_\mathrm{L}} V_{\mathrm{loop}}} \sim \tau_\mathrm{L}^2 \gg 1 \; . \tag{5.90}$$

We see that even though V_{loop} generates the mass for τ_L and hence stabilises it, the dominant contribution to potential-induced decays into the loop-cycle modulus originates from $V_{\mathrm{LVS}}^{(\mathrm{I})}$.

With this, let us now compare the decay amplitudes due to kinetic terms and potential terms. We find that the former are dominant,

$$\frac{|\mathcal{A}^{\mathrm{kin}}_{\tau_\mathrm{I}\to\tau_\mathrm{L}\tau_\mathrm{L}}|}{|\mathcal{A}^{\mathrm{pot}}_{\tau_\mathrm{I}\to\tau_\mathrm{L}\tau_\mathrm{L}}|} \sim \frac{(\partial_{\tau_\mathrm{I}}\partial_{\tau_\mathrm{I}} V)(\partial_{\tau_\mathrm{I}} K_{\mathrm{LL}})}{K_{\mathrm{II}}(\partial_{\tau_\mathrm{I}}\partial_{\tau_\mathrm{L}}\partial_{\tau_\mathrm{L}} V)} \sim \mathfrak{a}_\mathrm{I}^2\tau_\mathrm{I}^2 \gg 1 \; . \tag{5.91}$$

Before closing this subsection, we also compare the decay amplitudes into the loop-cycle modulus to those into the volume modulus. From Eqs. (5.75) and (5.77), one easily obtains

$$\frac{|\mathcal{A}^{\mathrm{kin}}_{\tau_\mathrm{I}\to\tau_\mathrm{b}\tau_\mathrm{b}}|}{|\mathcal{A}^{\mathrm{kin}}_{\tau_\mathrm{I}\to\tau_\mathrm{L}\tau_\mathrm{L}}|} \sim \frac{(\partial_{\tau_\mathrm{I}} K_{\mathrm{bb}})\delta\tau_\mathrm{I}\delta\tau_\mathrm{b}\delta\tau_\mathrm{b}}{(\partial_{\tau_\mathrm{I}} K_{\mathrm{LL}})\delta\tau_\mathrm{I}\delta\tau_\mathrm{L}\delta\tau_\mathrm{L}} \sim \frac{K_{\mathrm{LL}}(\partial_{\tau_\mathrm{I}} K_{\mathrm{bb}})}{K_{\mathrm{bb}}(\partial_{\tau_\mathrm{I}} K_{\mathrm{LL}})} \sim \mathcal{O}(1) \; , \tag{5.92}$$

$$\frac{|\mathcal{A}^{\mathrm{pot}}_{\tau_\mathrm{I}\to\tau_\mathrm{b}\tau_\mathrm{b}}|}{|\mathcal{A}^{\mathrm{pot}}_{\tau_\mathrm{I}\to\tau_\mathrm{L}\tau_\mathrm{L}}|} \sim \frac{(\partial_{\tau_\mathrm{I}}\partial_{\tau_\mathrm{b}}\partial_{\tau_\mathrm{b}} V)\delta\tau_\mathrm{I}\delta\tau_\mathrm{b}\delta\tau_\mathrm{b}}{(\partial_{\tau_\mathrm{I}}\partial_{\tau_\mathrm{L}}\partial_{\tau_\mathrm{L}} V)\delta\tau_\mathrm{I}\delta\tau_\mathrm{L}\delta\tau_\mathrm{L}} \sim \frac{K_{\mathrm{LL}}(\partial_{\tau_\mathrm{I}}\partial_{\tau_\mathrm{b}}\partial_{\tau_\mathrm{b}} V)}{K_{\mathrm{bb}}(\partial_{\tau_\mathrm{I}}\partial_{\tau_\mathrm{I}}\partial_{\tau_\mathrm{I}} V)} \sim \mathfrak{a}_\mathrm{I}\tau_\mathrm{I} \gg 1 \; , \tag{5.93}$$

where the factor $K_{\mathrm{LL}}/K_{\mathrm{bb}} \sim \sqrt{\tau_\mathrm{b}/\tau_\mathrm{L}}$ arises from the transformation of $\delta\tau_\mathrm{b}$ and $\delta\tau_\mathrm{L}$ into canonically normalised fields. In summary, we find that the decays of the inflaton-cycle modulus into lighter moduli fields are characterised by the following hierarchy of decay amplitudes:

$$|\mathcal{A}^{\mathrm{kin}}_{\tau_{\mathrm{I}}\to\tau_{\mathrm{b}}\tau_{\mathrm{b}}}| \sim |\mathcal{A}^{\mathrm{kin}}_{\tau_{\mathrm{I}}\to\tau_{\mathrm{L}}\tau_{\mathrm{L}}}| \gg |\mathcal{A}^{\mathrm{pot}}_{\tau_{\mathrm{I}}\to\tau_{\mathrm{b}}\tau_{\mathrm{b}}}| \sim \frac{|\mathcal{A}^{\mathrm{kin}}_{\tau_{\mathrm{I}}\to\tau_{\mathrm{b}}\tau_{\mathrm{b}}}|}{\mathfrak{a}_{\mathrm{I}}\tau_{\mathrm{I}}} \gg |\mathcal{A}^{\mathrm{pot}}_{\tau_{\mathrm{I}}\to\tau_{\mathrm{L}}\tau_{\mathrm{L}}}| \sim \frac{|\mathcal{A}^{\mathrm{kin}}_{\tau_{\mathrm{I}}\to\tau_{\mathrm{b}}\tau_{\mathrm{b}}}|}{\mathfrak{a}_{\mathrm{I}}^2\tau_{\mathrm{I}}^2}\ . \tag{5.94}$$

From this, we conclude that the corresponding decay rates due to kinetic and potential terms obey

$$\frac{\Gamma^{\mathrm{kin}}_{\tau_{\mathrm{I}}}}{\Gamma^{\mathrm{pot}}_{\tau_{\mathrm{I}}}} \sim \mathfrak{a}_{\mathrm{I}}^2\tau_{\mathrm{I}}^2 \sim (\ln\mathcal{V})^2 \gg 1\ . \tag{5.95}$$

A more careful and accurate analysis can be found in App. A.4 where we take into account mixing effects and show that decays of the inflaton into two different decay products are suppressed by powers of τ_{b}^{-1} and therefore negligible.

Equality of the decay rates into axions and saxions

After we have argued in the previous subsection that kinetic-term-induced decay rates are much stronger than the potential-term-induced ones, we now focus on the former. Another crucial observation is the fact that the decays of the inflaton via the kinetic terms have an equal branching ratio into moduli fields and their corresponding axions, respectively. This appears natural given the fact that moduli fields and their axions are superpartners; however, we want to dedicate this subsection to provide some intuition on the technical aspect of this result. We will again neglect mixing effects for the moment and consider the decays $\tau_{\mathrm{I}} \to \tau_i\tau_i$ and $\tau_{\mathrm{I}} \to \theta_i\theta_i$ where $i \in \{\mathrm{b},\mathrm{L}\}$. The relevant trilinear coupling terms are

$$\mathcal{L}^{\mathrm{kin}}_{\tau_{\mathrm{I}}\to\tau_i\tau_i}/M_P^2 = \partial_{\tau_{\mathrm{I}}}K_{ii}\delta\tau_{\mathrm{I}}\partial_\mu\tau_i\partial^\mu\tau_i + \partial_{\tau_i}K_{\mathrm{I}i}\delta\tau_i\partial_\mu\tau_{\mathrm{I}}\partial^\mu\tau_i + \partial_{\tau_i}K_{i\mathrm{I}}\delta\tau_i\partial_\mu\tau_i\partial^\mu\tau_{\mathrm{I}}\ , \tag{5.96}$$

$$\mathcal{L}^{\mathrm{kin}}_{\tau_{\mathrm{I}}\to\theta_i\theta_i}/M_P^2 = \partial_{\tau_{\mathrm{I}}}K_{ii}\delta\tau_{\mathrm{I}}\partial_\mu\theta_i\partial^\mu\theta_i\ , \tag{5.97}$$

where no sum over i is implied. As we can see, the coupling terms of the inflaton to the light moduli fields consist of three contributions because the Kähler metric depends on the τ_i. On the other hand, the coupling terms to light axion fields comprise only a single contribution because $K_{i\bar{j}}$ does not depend on the θ_i in order to respect their shift symmetry. A priori, it is therefore surprising that both decay amplitudes turn ot to be equal in magnitude.

To verify that this is indeed the case, we again use Eq. (A.94) to replace the derivatives ∂^2 by the inflaton mass squared $m^2_{\tau_{\mathrm{I}}}$ where we neglect the masses of the other moduli fields. Thus we obtain

$$\mathcal{L}^{\text{kin}}_{\tau_{\text{I}}\to\tau_i\tau_i}/M_P^2 = \frac{m^2_{\tau_{\text{I}}}}{2}\left(\partial_{\tau_{\text{I}}}K_{ii}\delta\tau_{\text{I}}\delta\tau_i\delta\tau_i - \partial_{\tau_i}K_{\text{I}i}\delta\tau_i\delta\tau_{\text{I}}\delta\tau_i - \partial_{\tau_i}K_{i\text{I}}\delta\tau_i\delta\tau_i\delta\tau_{\text{I}}\right)$$
$$= -\frac{m^2_{\tau_{\text{I}}}}{2}\partial_{\tau_{\text{I}}}K_{ii}\delta\tau_{\text{I}}\delta\tau_i\delta\tau_i \;, \tag{5.98}$$
$$\mathcal{L}^{\text{kin}}_{\tau_{\text{I}}\to\theta_i\theta_i}/M_P^2 = \frac{m^2_{\tau_{\text{I}}}}{2}\partial_{\tau_{\text{I}}}K_{ii}\delta\tau_{\text{I}}\delta\theta_i\delta\theta_i \;. \tag{5.99}$$

Here we also used that $\partial_{\tau_i}K_{jk} = \partial_{\tau_i}\partial_{\tau_j}\partial_{\tau_k}\mathcal{K}/4$ is invariant under permutations of i, j and k. We see that the trilinear coupling terms for both the moduli fields and axions have the same prefactors in front of the non-canonical fields, respectively, up to a minus sign. From Eq. (4.203) we observe that the standard kinetic terms for $\delta\tau_i$ and $\delta\theta_i$ are both multiplied by the same field metric so that the respective canonical fields follow fron an identical rescaling, $\delta\phi_i \sim \sqrt{K_{ii}}\delta\tau_i$ and $\delta a_i \sim \sqrt{K_{ii}}\delta\theta_i$. Hence, the overall decay amplitudes are the same in magnitude leading to identical decay rates for light moduli fields and axions.

The question whether this result is robust after we include mixing between the moduli and axion fields and their respective canonically normalised fields is of course a non-trivial one since naively one would at least expect $\mathcal{O}(1)$ corrections to the decay rates. Let us summarise some intuition gained from the detailed analysis of the mixed system in App. A.4. All moduli and axion fields will be a superposition of the respective canonical fields which diagonalise the system, i.e. $\delta\tau_i = \delta\tau_i(\delta\phi_j)$ and $\delta\theta_i = \delta\theta_i(\delta a_j)$. Since the system is approximately diagonal, i.e. $\delta\tau_i \approx \delta\phi_i/\sqrt{2K_{ii}}$ and $\delta\theta_i \approx \delta a_i/\sqrt{2K_{ii}}$ with contributions from other canonical fields suppressed, we can always identify a specific canonical field with a corresponding modulus or axion. Hence, when speaking of decays of the inflaton, we actually mean the decays of the canonical field $\delta\phi_{\text{I}}$ which gives the largest contribution to $\delta\tau_{\text{I}}$. Such a decay in the canonical frame, e.g. $\delta\phi_{\text{I}} \to \delta\phi_i\delta\phi_i$, will gain contributions from all possible combinations of trilinear $\sim \delta\tau_i\delta\tau_j\delta\tau_k$ vertices, which will be of different strength. A crucial insight is given by the fact that the dominant contributions to a decay of the inflaton are those terms for which the canonical inflaton $\delta\phi_{\text{I}}$ gets contributions from all moduli $\delta\tau_i$, whereas the canonical decay products $\delta\phi_i$ only get a contribution from the one respective modulus $\delta\tau_i$ that is associated to them. In other words, we can schematically write the dominant contributions to an inflaton decay as

$$\mathcal{L} \supset \sim \underbrace{c_1\delta\phi_{\text{I}}}_{\sim\sum_j\delta\tau_j}\underbrace{c_2\delta\phi_i}_{\sim\delta\tau_i}\underbrace{c_2\delta\phi_i}_{\sim\delta\tau_i} \;, \tag{5.100}$$

where c_1 and c_2 are constants which contain the information about which moduli contribute to the respective canonical field. An analogous conclusion can be drawn for the decay of the inflaton into canonical axion fields δa_i with dominant contributions

$$\mathcal{L} \supset \sim \underbrace{d_1 \delta\phi_{\mathrm{I}}}_{\sim\sum_j \delta\tau_j} \underbrace{d_2 \delta a_i}_{\sim\delta\theta_i} \underbrace{d_2 \delta a_i}_{\sim\delta\theta_i} . \tag{5.101}$$

With this, the precise equality of the decay rates into moduli fields and axions can easily be understood: We have $c_1 = d_1$ because the decaying field $\delta\phi_{\mathrm{I}}$ is the same and we have $c_2 = d_2$ because only the one associated field contributes and moduli and axion fields have the same normalisation constant $c_2\delta\phi_i \sim \delta\phi_i/\sqrt{K_{ii}}$ and $d_2\delta a_i \sim \delta a_i/\sqrt{K_{ii}}$.

Finally, we want to comment on the question whether potential terms can enhance the decays to axions. The answer is generally negative because for both light axions the natural scale of the potential is much lower. For the volume axion θ_{b} a potential is generated by non-perturbative effects on the volume cycle, which modify the superpotential by a term $\sim \exp(-\mathfrak{a}_{\mathrm{b}} T_{\mathrm{b}})$, leading to a negligibly small contribution. The potential of the QCD axion θ_{L} is by construction $\sim T^4_{\mathrm{QCD}}$, which is again very small in the present context. Furthermore, decays of the inflaton into its own axion are kinematically forbidden as argued before. At last, decays into axions θ_{s} of the other small-cycle moduli τ_{s} are technically possible, depending on the model-dependent parameters γ_{I}, A_{I},γ_{s} and A_{s}. Nevertheless, in our minimalist setting, w.l.o.g. we can assume that the inflaton is light enough so that such a decay is also kinematically forbidden.

Summary of decay rates

Let us recapitulate our main results of this section. We have ascertained that the decays of the inflaton are dominated by kinetic-term-induced couplings and that these very couplings result in equal branching ratios into light moduli fields and their respective axions. We summarise all relevant decay rates in Tab. 4 where we normalise them to two benchmark channels for the sake of clarity,

$$\Gamma_1 \equiv \Gamma^{\mathrm{kin}}_{\phi_{\mathrm{I}}\to\phi_{\mathrm{b}}\phi_{\mathrm{b}}} \approx \frac{3\gamma_{\mathrm{I}}|W_0|^3\mathfrak{a}_{\mathrm{I}}^3\tau_{\mathrm{I}}^{9/2}}{64\pi\mathcal{V}^4} M_{\mathrm{P}} , \tag{5.102}$$

$$\begin{aligned}\Gamma_2 &\equiv \Gamma^{\mathrm{pot}}_{\phi_{\mathrm{I}}\to\phi_{\mathrm{L}}\phi_{\mathrm{L}}} \\ &\approx \frac{3\gamma_{\mathrm{I}}\sqrt{\tau_{\mathrm{I}}}\left[-3|W_0|^2\gamma_{\mathrm{L}}\tilde{\mu}^4\tau_{\mathrm{L}}^2 + W_0^2\left(-4\mu_1\tilde{\mu}^4 + \mu_2(\mu_3^2 - 4\mu_3\sqrt{\tau_{\mathrm{L}}} + 4\tau_{\mathrm{L}})\tau_{\mathrm{L}}\right)\right]^2}{64\pi\gamma_{\mathrm{L}}^2|W_0|\tilde{\mu}^8\mathfrak{a}_{\mathrm{I}}\tau_{\mathrm{L}}^4\mathcal{V}^4} M_{\mathrm{P}} .\end{aligned} \tag{5.103}$$

Note that $\Gamma_1/\Gamma_2 \sim \mathfrak{a}_{\mathrm{I}}^4\tau_{\mathrm{I}}^4 \sim (\ln\mathcal{V})^4 \gg 1$.

Decay rate	scaling	explicit
$\Gamma^{\text{kin}}_{\phi_\text{I}\to\phi_\text{b}\phi_\text{b}}$	$\sim (\ln\mathcal{V})^{9/2}\mathcal{V}^{-4}M_\text{P}$	Γ_1
$\Gamma^{\text{pot}}_{\phi_\text{I}\to\phi_\text{b}\phi_\text{b}}$	$\sim (\ln\mathcal{V})^{5/2}\mathcal{V}^{-4}M_\text{P}$	$4\Gamma_1/(\mathfrak{a}_\text{I}\tau_\text{I})^2$
$\Gamma^{\text{kin}}_{\phi_\text{I}\to\phi_\text{L}\phi_\text{L}}$	$\sim (\ln\mathcal{V})^{9/2}\mathcal{V}^{-4}M_\text{P}$	$4\Gamma_1$
$\Gamma^{\text{pot}}_{\phi_\text{I}\to\phi_\text{L}\phi_\text{L}}$	$\sim (\ln\mathcal{V})^{1/2}\mathcal{V}^{-4}M_\text{P}$	Γ_2
$\Gamma^{\text{kin}}_{\phi_\text{I}\to a_\text{b}a_\text{b}}$	$\sim (\ln\mathcal{V})^{9/2}\mathcal{V}^{-4}M_\text{P}$	Γ_1
$\Gamma^{\text{kin}}_{\phi_\text{I}\to a_\text{L}a_\text{L}}$	$\sim (\ln\mathcal{V})^{9/2}\mathcal{V}^{-4}M_\text{P}$	$4\Gamma_1$
$\Gamma^{\text{kin}}_{a_\text{I}\to\phi_\text{b}a_\text{b}}$	$\sim (\ln\mathcal{V})^{9/2}\mathcal{V}^{-4}M_\text{P}$	$2\Gamma_1$
$\Gamma^{\text{kin}}_{a_\text{I}\to\phi_\text{L}a_\text{L}}$	$\sim (\ln\mathcal{V})^{9/2}\mathcal{V}^{-4}M_\text{P}$	$8\Gamma_1$

Table 4.: Decay rates of inflaton into moduli and axion fields. The explicit decay rates are defined as $\Gamma_1 \equiv \Gamma^{\text{kin}}_{\phi_\text{I}\to\phi_\text{b}\phi_\text{b}}$ and $\Gamma_2 \equiv \Gamma^{\text{pot}}_{\phi_\text{I}\to\phi_\text{L}\phi_\text{L}}$ where $\Gamma_1 \gg \Gamma_2$.

5.6.2. The dark radiation problem re-emerges

From Tab. 4 we see that the inflaton decays with a total decay rate $\Gamma^{\text{tot}}_{\phi_\text{I}} \approx 10\Gamma_1 \sim (\ln\mathcal{V})^{9/2}\mathcal{V}^{-4}M_\text{P}$, which is much slower than the decay rate of the volume modulus ϕ_b into Higgses, as given by Eq. (5.52), or of the loop-cycle modulus into SM gauge bosons, given in Eq. (5.65). This confirms our initial claim that the inflaton itself is the longest-lived modulus so that its decay channels dictate the amount of the energy density which after reheating is transferred into the SM sector and DR.

Based on Tab. 4 and considering the fact that decays into ϕ_b and ϕ_L result in an immediate, subsequent decay into SM degrees of freedom, we find

$$\frac{\Gamma_{\phi_\text{I}\to\text{DR}}}{\Gamma_{\phi_\text{I}\to\text{SM}}} \approx 1\,. \tag{5.104}$$

According to Eq. (4.233), this leads to an excessive effective number of neutrino species

$$\Delta N_\text{eff} \approx 1.5\left(\frac{100}{g_*^4 g_{*,S}^{-3}}\right)^{1/3}, \tag{5.105}$$

which is in tension with observational bounds, $\Delta N_\text{eff} \leq 0.3$ [4]. Thus the DR problem re-emerges.

A potential solution one may think of is a direct coupling of τ_I to the Higgs in order to boost the branching ratio into the SM sector, very much in analogy to our discussion in Sec. 5.4.2 about enhancing the decay rate of τ_b into Higgses. The latter was realised through a fluctuation of the canonical volume field ϕ_b, which again induced a fluctuation of the logarithmic term in Eq. (5.46),

$$\delta \ln \mathcal{V} \sim \delta\phi_\mathrm{b}/M_\mathrm{P} \,. \tag{5.106}$$

Similarly, since $\mathcal{V}$ also depends on the inflaton cycle, a fluctuation of ϕ_I will result in a fluctuation of the same logarithmic term, which is given by

$$\delta \ln \mathcal{V} \sim \delta \ln(\tau_\mathrm{b}^{3/2} - \gamma_\mathrm{I}\tau_\mathrm{I}^{3/2}) \sim (\sqrt{\tau_\mathrm{I}}/\mathcal{V})\, \delta\tau_\mathrm{I} \sim (\tau_\mathrm{I}^{3/4}/\sqrt{\mathcal{V}})\, \delta\phi_\mathrm{I}/M_\mathrm{P} \,. \tag{5.107}$$

Comparing Eqs. (5.106) and (5.107), we can infer that the decay amplitudes between the decay of the volume modulus to two Higgses and the inflaton into two Higgses differ by a factor

$$\frac{|\mathcal{A}_{\phi_\mathrm{b}\to hh}|}{|\mathcal{A}_{\phi_\mathrm{I}\to hh}|} \sim \frac{\sqrt{\mathcal{V}}}{\tau_\mathrm{I}^{3/4}} \,. \tag{5.108}$$

Moreover, the resulting decay rates differ by an additional factor $m_{\tau_\mathrm{I}}/m_{\tau_\mathrm{b}} \sim \tau_\mathrm{I}\sqrt{\mathcal{V}}$, leading to

$$\frac{\Gamma_{\phi_\mathrm{b}\to hh}}{\Gamma_{\phi_\mathrm{I}\to hh}} \sim \frac{|\mathcal{A}_{\phi_\mathrm{b}\to hh}|^2}{|\mathcal{A}_{\phi_\mathrm{I}\to hh}|^2}\frac{m_{\tau_\mathrm{I}}}{m_{\tau_\mathrm{b}}} \sim \frac{\mathcal{V}^{3/2}}{\sqrt{\tau_\mathrm{I}}} \,. \tag{5.109}$$

Combined with Eq. (5.59), we find that

$$\Gamma_{\phi_\mathrm{I}\to hh} \sim c_\mathrm{loop}^2 \frac{\sqrt{\tau_\mathrm{I}}}{\mathcal{V}^4} M_\mathrm{P} \,, \tag{5.110}$$

which is suppressed w.r.t. the decay rate of the inflaton into axions by a factor $\sim c_\mathrm{loop}^2/\tau_\mathrm{I}^4 \ll 1$ so that the DR problem is not avoided. We also want to mention that the idea of a drastically enhanced decay rate of ϕ_I to the SM is limited a priori because if the inflaton decays too fast, we expect the inflaton axion to become the longest-lived particle, which naturally tends to decay into DR. This is due to the fact that axionic coupling terms exclusively originate from kinetic terms, which necessarily involve exactly two axions so that decays of the inflaton axion always have one light axion as a decay product.

5.7. Resulting axion dark matter cosmology

After the discussion of the previous sections, it is clear that the cosmological scenario has been significantly altered – mostly due to the fast decays of the volume modulus, which render the inflaton ϕ_{I} the longest-lived particle. Therefore, we dedicate this section to a re-evaluation of our main findings from Sec. 5.3. regarding axion phenomenology and cosmology. In doing so, we will once more utilise the approximate analytical formulae from [145, 146] in order to derive qualitative order-of-magnitude estimates for most relevant quantities.

As explained in Sec. 5.3, the axion relic density is strongly dependent on the onset of the axion oscillations, in particular if this occurs before reheating in an early-matter-dominated phase or after. One of the key quantities of interest for that matter is the reheating temperature T_{r}, which is determined by the decay rate of the longest-lived modulus, which as we argued is the inflaton ϕ_{I}. From Tab. 4 we see that its total decay rate is

$$\Gamma^{\mathrm{tot}}_{\phi_{\mathrm{I}}} \approx 10\Gamma_1 = \frac{15\alpha}{64\sqrt{2}\pi^{5/2}}\mathcal{V}^{-4}\left(\log[\mathcal{V}/W_0]\right)^{9/2} M_{\mathrm{P}} \,, \tag{5.111}$$

where we defined

$$\alpha \equiv (2\pi)^{3/2}\frac{\gamma_I|W_0|^3\mathfrak{a}_I^3\tau_I^{9/2}}{(\log[\mathcal{V}/W_0])^{9/2}} \,. \tag{5.112}$$

The latter is chosen so that in the large-volume limit $\mathcal{V} \gg 1$, where according to Eq. (4.200) one has $\mathfrak{a}_{\mathrm{I}}\tau_{\mathrm{I}} \approx \ln(\mathcal{V}/W_0)$, and for $\mathfrak{a}_{\mathrm{I}} = 2\pi$ one simply obtains $\alpha \approx \gamma_{\mathrm{I}}|W_0|^3$; that is, α naturally takes on values of order unity.

For the axion decay constant and inflation scale, we use the expressions

$$f_a = \sigma\frac{M_{\mathrm{P}}}{\sqrt{2}\pi\tau_{\mathrm{L}}^{1/4}\sqrt{\mathcal{V}}} \,, \quad H_{\mathrm{I}} = \kappa\frac{M_{\mathrm{P}}}{\mathcal{V}^{3/2}} \,, \tag{5.113}$$

respectively, which are obtained from Eqs. (5.9) and (5.15) after defining $\sigma \equiv \sqrt{3\gamma_{\mathrm{L}}/8}$ and $\kappa^2 \equiv \beta W_0^2$. In the following, we will set $\tau_{\mathrm{L}} = 1/(2\alpha_{\mathrm{s,UV}}) = 25/2$. Moreover, in order to obtain explicit numerical estimates for the relevant cosmological quantities, we will at first set all model-dependent parameters to unity, $\alpha = \sigma = \kappa = W_0 = 1$. Afterwards, to illustrate the approximate influence of these parameters, we restore the dependence on them while neglecting all logarithmic effects.

In detail, after setting the model-dependent parameters to unity, the reheating temperature due to inflaton decays as given by Eq. (4.216) reads

$$T_{\mathrm{r}} \sim \left(\frac{90}{g_\star\pi^2}\right)^{1/4}\sqrt{\Gamma^{\mathrm{tot}}_{\phi_{\mathrm{I}}}M_{\mathrm{P}}} \sim 1\,\mathrm{GeV}\left(\frac{80}{g_\star}\right)^{1/4}\left(\frac{1.3\times10^{10}}{\mathcal{V}}\right)^2\left(\frac{\log(\mathcal{V})}{\log(1.3\times10^{10})}\right)^{9/4} \,, \tag{5.114}$$

which we have benchmarked to a value of the volume that corresponds to $T_{\mathrm{r}} = 1\,\mathrm{GeV}$. This is the formula that we use to derive the numerical values for our bounds. Now to obtain the power-like scalings of the model-dependent parameters, we disregard the logarithmic dependence in Eq. (5.114), thus finding that the reheating temperature scales as

$$T_{\mathrm{r}} \sim \sqrt{\alpha}/\mathcal{V}^2 \,. \tag{5.115}$$

Together with the relations $f_a \sim \sigma/\mathcal{V}^{1/2}$ and $H_{\mathrm{I}} \sim \kappa/\mathcal{V}^{3/2}$ from Eq. (5.113), we can use this to equip the resulting, numerical bounds in the following subsections with approximate scalings in the parameters α, σ and κ.

Depending on the reheating temperature, we want to distinguish now the two scenarios where the axion starts its oscillations either after reheating during a standard, radiation-dominated universe or before reheating during a period of early matter domination. Here the former scenario is characterised by a high and the latter by a low reheating temperature. The corresponding calculations in the next two subsections are essentially analogous to the respective subsections in Sec. 5.3.

5.7.1. High reheating temperatures and a standard radiation-dominated cosmology

For a reheating temperature $T_{\mathrm{r}} \gg 1\,\mathrm{GeV}$ the onset of axion oscillations occurs during a standard, radiation-dominated phase [145, 146]. From Eq. (5.114) we see that the above condition of high reheating temperature in order to be in the standard regime implies an upper boundary for $\mathcal{V}$. Moreover as argued in Sec. 5.3, if axions constitute all of DM, isocurvature constraints imply a small inflation scale, which again imposes a lower bound on $\mathcal{V}$. Using the appropriate equations of Sec. 5.3.1, we can translate the two bounds on the volume into corresponding bounds for the other relevant parameters, which yields

$$(\sigma^{-10/31}\kappa^{24/31})\,1\times 10^{7} \lesssim \mathcal{V} \lesssim 1\times 10^{10}\,(\alpha^{1/4}) \tag{5.116}$$
$$(\sigma^{36/31}\kappa^{-12/31})\,9\times 10^{13}\,\mathrm{GeV} \gtrsim f_a \gtrsim 3\times 10^{12}\,\mathrm{GeV}\,(\alpha^{-1/8}\sigma) \tag{5.117}$$
$$(\sigma^{-36/31}\kappa^{12/31})\,6\times 10^{-8}\,\mathrm{eV} \lesssim m_{a,0} \lesssim 2\times 10^{-6}\,\mathrm{eV}\,(\alpha^{1/8}\sigma^{-1}) \tag{5.118}$$
$$(\alpha^{1/2}\sigma^{20/31}\kappa^{-48/31})\,6\times 10^{5}\,\mathrm{GeV} \gtrsim T_{\mathrm{r}} \gtrsim 1\,\mathrm{GeV} \tag{5.119}$$
$$(\sigma^{15/31}\kappa^{-5/31})\,7\times 10^{7}\,\mathrm{GeV} \gtrsim H_{\mathrm{I}} \gtrsim 2\times 10^{3}\,\mathrm{GeV}\,(\alpha^{-3/8}\kappa), \tag{5.120}$$
$$(\sigma^{-21/31}\kappa^{7/31})\,0.1 \lesssim \theta_{\mathrm{i}} \lesssim 0.4\,(\alpha^{7/96}\sigma^{-7/12}). \tag{5.121}$$

Here the left-hand side represents constraints due to isocurvature fluctuations, whereas the right-hand side gives the requirement for a high reheating temperature and hence does not constitute an actual, observational limit. Remember that a natural choice is to set α, σ and κ to order unity, although other values may also be plausible.

5.7.2. Low reheating temperatures and axion oscillations during a matter-dominated phase

We proceed in analogy to the previous subsection. Since a small reheating temperature is required in order to be in a regime where the onset of axion oscillations occurs after reheating during an early-matter-dominated phase [145, 146], we assume $T_\mathrm{r} \ll 300\,\mathrm{MeV}$. This implies a lower boundary on the volume, whereas we obtain an upper bound by requiring that the axion relic density saturates the DM density for an initial misalignment angle that is not tuned large, $\theta_\mathrm{i} \leq 3$. Using the proper equations from Sec. 5.3.2, we obtain

$$\begin{aligned}
(\alpha^{1/4})\, 2\times 10^{10} &\lesssim \mathcal{V} &&\lesssim 5\times 10^{10}\,(\alpha^{4/19}\sigma^{6/19}) && (5.122)\\
(\alpha^{-1/8}\sigma)\, 2\times 10^{12}\,\mathrm{GeV} &\gtrsim f_a &&\gtrsim 1\times 10^{12}\,\mathrm{GeV}\,(\alpha^{-2/19}\sigma^{16/19}) && (5.123)\\
(\alpha^{1/8}\sigma^{-1})\, 3\times 10^{-6}\,\mathrm{eV} &\lesssim m_{a,0} &&\lesssim 4\times 10^{-6}\,\mathrm{eV}\,(\alpha^{2/19}\sigma^{-16/19}) && (5.124)\\
300\,\mathrm{MeV} &\gtrsim T_\mathrm{r} &&\gtrsim 100\,\mathrm{MeV}\,(\alpha^{3/38}\sigma^{-12/19}) && (5.125)\\
(\alpha^{-3/8}\kappa)\, 600\,\mathrm{GeV} &\gtrsim H_\mathrm{I} &&\gtrsim 200\,\mathrm{GeV}\,(\alpha^{-6/19}\sigma^{-9/19}\kappa), && (5.126)\\
(\alpha^{3/32}\sigma^{-3/4})\, 0.9 &\lesssim \theta_\mathrm{i} &&\lesssim 3\,, && (5.127)
\end{aligned}$$

where the left-hand side corresponds to the requirement of being in the regime of low T_r, whereas the right-hand side gives the limits for DM saturation without fine-tuning θ_i.

5.8. Discussion

In this work we tried to establish a stringy QCD axion together with a phenonemologically viable cosmology, in particular including a concrete construction of inflationary dynamics, and failed. Our logic has led us to consider LVS compactifications and to identify the QCD axion with the axionic superpartner of a small cycle which carries the SM sector on D7-branes wrapped around it. This allows for a small axion decay constant in the observationally favourable window. As in such a simple stringy scenario the axion is typically already present during inflation, the satisfaction of isocurvature constraints requires that the inflation scale is rather low, which directs us towards the setting of blow-up inflation. As many other LVS early-universe constructions, the latter is generally plagued by a DR problem due to decays of the volume modulus, being the longest-lived modulus, into its own axion. We have found a novel way to sever this head of the DR Hydra via an enhanced decay rate of the volume modulus into SM Higgses, which is induced by fluctuations of the respective, fine-tuned Higgs mass term. However, this led to an altered scenario where the inflaton itself represents the longest-lived modulus, which eventually comes to dominate the universe and reheats the SM. We have shown that this is accompanied with a significant branching ratio into light axions, which result in too much DR, thus re-growing the Hydra's second head.

At this point, two caveats are due concerning the validity of our conclusions. First, as discussed in more detail in [3], if the volume is stabilised at a very large value, especially at the upper bound of the low-temperature case described in Sec. 5.7.2, the volume-modulus mass might be so small that the decay into two Higgses is kinematically forbidden. Instead a mixing effect between the Higgs and volume modulus takes over (see also [221, 222]), which depending on the exact value of the volume-modulus mass may endanger a successful BBN. This deserves further investigation to determine the precise limits at which our conclusions of a fast decaying volume modulus fail and the scenario falls back to the standard case where the latter one is the longest-lived modulus.

Second, we want to mention that the phenomenologically viable range for the volume that we found is only marginally consistent with constraints due to the normalisation of CMB scalar perturbations in blow-up inflation [121], which we do not consider a major problem, however. On the one hand the lower volume regions, where the inflation scale is not as low and the potential not as flat as for higher volumes, still appear to be easily accessible by a mild adjustment of the model-dependent parameters. On the other hand, the higher volume regions may also be realised by invoking more severe fine-tuning, possibly by including more than one instanton or an interplay with loop effects. A detailed analysis including constraints from CMB scalar perturbations is appropriate for future work in order to understand this better. Nevertheless, we want to stress that our conclusions about the viability of a QCD axion in blow-up inflation without too much DR are negative in their nature anyway, which constitutes a more serious problem. All in all, our other implications should be taken with a grain of salt in the higher volume regime, which may suffer from other issues besides too much DR.

To put our findings into perspective, though being negative, they nevertheless represent progress towards a solution of the DR conundrum. While previously DR has been the result of the omnipresent volume modulus and its seemingly inevitable, large branching ratio into light axions, the new DR problem arises directly due to the inflationary sector. Therefore, it appears promising to pursue further inflationary model building in order to find a more suitable implementation of inflation. We believe that any scenario where the inflationary sector is sequestered from the SM will tend to suffer from too much DR. This is because any particle that is responsible for reheating and has only feeble interactions with the SM is likely to produce a significant amount of DR due to kinetic-term-induced, $\mathcal{O}(1)$ couplings to light axions, as we explicitly showed for the case of blow-up inflation.

Hence, it appears to be favourable to consider settings where the inflationary sector is directly coupled to the SM, for instance through some form of hybrid inflation [223–226] (for stringy attempts, see also [227–230]). Due to the inherently strong decays of the inflaton into the SM in such scenarios, branching ratios into DR are usually small. Obviously, this again involves the danger of a longest-lived volume modulus that leads to the usual DR problem; however, the channel of rapid decay into Higgses that we found in this work will greatly help to remedy this problem and to bring such a scenario to work. Here it is of course important to consider possible decays of other long-lived particles into DR, especially of the inflaton axion, as well. We leave the details of such an implementation to future work, which may finally lead to a permanent death of the DR Hydra. Moreover, having a concrete implementation of a stringy QCD axion together with a viable inflationary setting will likely provide us with rather predictive constraints on axionic and cosmological phenomenology, thus putting experimental searches into the focus of attention and allowing for some level of falsifiability in this specific area of string theory.

Finally, we want to mention that very recently some indications emerged[8] that there may probably be another decay channel of the inflaton into SM degrees of freedom. This might represent a solution to the DR problem even within the very model that we considered in this work. The idea is to utilise the coupling of the inflaton-cycle modulus into SM gauge bosons through the coupling term (A.19). Naively, only the loop-cycle modulus couples to the gauge bosons living on it but taking into account mixing effects between the inflaton and loop modulus may lead to a direct coupling of the inflaton to these gauge bosons. By a naive estimate, this results in a decay rate $\Gamma_{\phi_{\mathrm{I}}\to AA} = 8N_g\Gamma_1$, where N_g is the number of gauge bosons. This may be just enough to avoid the DR problem due to the decay $\Gamma_{\phi_{\mathrm{I}}\to\mathrm{DR}} = 5\Gamma_1$. We will leave this possibility for future investigations, which may result in the severing of another head of the DR hydra. Nevertheless, the next head is perhaps already waiting to regrow in the form of the inflaton axion, which may become the new longest-lived particle due to the faster decay rate of the inflaton and which could re-introduce the DR problem via its decays to light axions.

[8]These involve personal communication with Michele Cicoli.

6. Quintessence in string theory and the F-term problem

The entire content of this chapter represents one of the major works in this thesis and is based on [2] unless stated otherwise.

6.1. Motivation and outline

In the previous chapter, we considered early-universe models in the context of string theory and used one of the Kähler moduli fields, which are abundantly present in string compactifications, to represent a slow-rolling inflaton field. It appears to be obvious and natural to transfer the same logic to the late-time universe by identifying the corresponding late-time expansion as a result of underlying moduli dynamics. At the same time, we already elaborated in Chpt. 1 on the swampland programme of string theory, which tries to identify criteria for low-energy EFTs to separate those theories that possess a stringy UV completion from those that do not. Finally, in Sec. 4.2.3 we discussed the important topic of moduli stabilisation, which is an essential part of any string compactification. In particular, the stabilisation of Kähler moduli represents a crucial issue and the arguably most prominent constructions are the aforementioned KKLT scenario [98] and the LVS [99], which constitutes the main setting of the previous and the current chapter. In the following work, the above three aspects, namely stringy dark energy, moduli stabilisation as well as the swampland programme, and their interplay with each other are to be studied.

More precisely, we investigate the viability of a positive cosmological (quasi) constant in the context of moduli stabilisation and the swampland programme. One may distinguish two possible realisations of the late-time, de Sitter-like expansion of the universe: a non-dynamic, 'true' CC and a dynamic, 'quasi' CC, which actually results from a scalar field that slowly rolls down a flat potential resembling a CC. As a matter of fact, in string theory all free parameters except for the fundamental string scale l_s arise dynamically from the vacuum expectation values of effective fields. As such, also a 'true' CC can be considered to be merely the vacuum expectation value of the overall scalar potential. For instance, ignoring possible contributions from D-terms, Eq. (4.66) may imply a positive CC arising from the F-term potential if the corresponding terms cancel in the right way when evaluated at the minimum.

Despite several attempts, it has turned out to be notoriously difficult to find stringy constructions in which all moduli fields are stabilised and which allow for a 'true' CC, especially if SUSY is to be broken in an acceptable manner. Even in KKLT and the LVS, stable de Sitter vacua[1] can only be achieved with the help of fine-tuning and controversial mechanisms so that their realisability is highly debated in the literature [231–261]. In the context of the swampland programme, this has resulted in the so-called *de Sitter-swampland conjecture*, which suggests that de Sitter vacua are generically not achievable in string-derived scenarios. Technically, in the case of only one modulus, the conjecture claims that in order to belong to the landscape the scalar potential must obey one of the relations [262–264]

$$|V'| \geq c \cdot V \quad \text{or} \quad V'' \leq -c'V \ , \tag{6.1}$$

where c and c' are $\mathcal{O}(1)$ numbers.[2]

As pointed out in [262, 265], if we take the conjecture and the assumption of a string-theoretic UV completion seriously, this suggests that the late-time expansion of the universe originates from some stringy implementation of dynamic DE, e.g. quintessence [35, 269, 270]. Even though the latter has been proven to be difficult to establish as well [242, 271–278], in this work we want focus on this very idea of stringy quintessence. Natural candidates are the many moduli [279–281] and axion [242, 272, 282–286] fields that arise as scalar degrees of freedom in the effective four-dimensional theory after compactification, where we turn our attention especially to the former.

In detail, in this work we will argue that a stringy version of quintessence is not necessarily on a stronger footing than a de Sitter vacuum. Especially, two effects are worth mentioning, which turn out to be problematic. First, there is an enormous hierarchy between different mass scales due to observational constraints, in particular between the very light quintessence modulus and the comparatively heavy scales related to beyond-the-SM physics, that is the volume-modulus mass, the KK mass scale and the mass of superpartners. Since all these scales are controlled by the volume parameter $\mathcal{V}$, inconsistencies arise from the fact that a very large volume is needed in order to achieve such a light quintessence mass [279, 287], which renders the volume modulus inadmissibly light. We call this the *light-volume problem*. Second, in order to achieve SM-superpartner masses at an acceptably high scale, a

[1] When we speak of 'stable vacua', we also include quasi-stable ones, which are false vacua whose tunnelling probability into the true vacuum is so small that their lifetime is large compared to the age of the universe.

[2] Note that the $\mathcal{O}(1)$ magnitude of c and c' is not definitely specified and may also include 'smallish' $\mathcal{O}(1)$ numbers, in order to satisfy observational restrictions [265–268]. Moreover, we set $M_\mathrm{P} \equiv 1$ here and in most of the other equations in this chapter.

dedicated SUSY breaking sector needs to be introduced, whose F-term constitutes a very large contribution to the scalar potential, which is many orders of magnitude larger than the observed CC. We refer to this as the *F-term problem*, which represents our main finding in this work. Bringing the potential back down to a phenomenologically allowed value would require a new, negative contribution of unknown origin and involve severe fine-tuning.

6.2. Preliminaries and phenomenological requirements

In this section we set the scene and introduce requirements on the different, relevant mass scales, which will establish the aforementioned hierarchy.

We will again work in the setting of type IIB string-theoretic compactifications on CY orientifolds with O3- and O7-planes. This setting is advantageous because it allows for a considerable amount of calculational control [93, 96, 98, 99] and is subject to the leading-order no-scale cancellation of the scalar potential as discussed in Sec. 4.2.3. As a result, a potential for the Kähler moduli is only generated at a much lower scale through the same quantum corrections that have been discussed in Sec. 4.2.4. In particular, we have

$$V = \delta V_{\text{np}} + \delta V_{\alpha'} + \delta V_{\text{loop}} \neq 0 \,, \tag{6.2}$$

where δV_{np} are non-perturbative corrections due to D3-instantons or gaugino condensation on D7-branes, $\delta V_{\alpha'}$ results from α' corrections on the Kähler potential and δV_{loop} are loop corrections, which are subdominant to the other two due to an extended no-scale cancellation [102, 103, 218, 288]. The combination of the former two corrections leads us again to the LVS. With the help of a large volume, the resulting potential can be made parametrically small. Indeed, while the no-scale cancellation eliminates all terms that scale like $\sim \mathcal{V}^{-2}$, the remaining terms generated by the above quantum corrections are suppressed by a factor $\sim \mathcal{V}^{-3}$ or an even smaller one. In principle, if the volume is large enough, this can yield a small enough potential and, associated to that, moduli masses in order to obtain the desired properties of the quintessence field.

Let us now specify and justify the phenomenological requirements for a stringy quintessence scenario.

1. **Light quintessence modulus** ϕ with $m_\phi \lesssim 10^{-60} M_{\text{P}}$. This requirement originates from the cosmological premise that the quintessence scalar is subject to the conditions of slow-roll (cf. Eq. (4.212)). Defining the quintessence mass as the second derivative of the potential w.r.t. the canonical field,

$m_\phi = \sqrt{V''}$, cosmology imposes the constraint $|m_\phi| \lesssim H_0 \approx 10^{-33}\,\text{eV} \sim \mathcal{O}(10^{-60})M_\text{P}$ [289].

2. **Heavy superpartners** with $m_\text{S} \gtrsim 10^{-15}M_\text{P}$. As discussed in Chpt. 5, the SM can be included to the scenario either via D3-branes, which constitute a singular point in the internal space, or via D7-branes wrapping a 4-cycle [100]. Typically, this yields a supersymmetric extension of the SM where the SUSY breaking scale is constrained by collider experiments to be $\gtrsim \mathcal{O}(\text{TeV}) \sim 10^{-15}M_\text{P}$.

3. **Heavy KK scale** with $m_\text{KK} \gtrsim 10^{-30}M_\text{P}$. A heavy KK scale is needed so that our universe remains effectively four-dimensional at low energies. While a gauge theory resulting from open strings attached to D-branes is generally not sensitive to large extra dimensions, gravity, which effectively arises from closed strings that are not bound to branes, certainly is. Hence the above requirement on the KK scale results from tests of the standard, four-dimensional Newtonian gravity down to length scales of the order $\mathcal{O}(\text{mm})$, corresponding to an energy $\sim 0.2\,\text{meV} \sim \mathcal{O}(10^{-30})M_\text{P}$ [85].

4. **Heavy volume modulus** with $m_\mathcal{V} \gtrsim 10^{-30}M_\text{P}$. This requirement is obtained from the fact that the volume modulus couples to all matter fields with approximately gravitational strength. This coupling results from the factor $\mathcal{V}$ in the Einstein-Hilbert action after compactifying to four dimensions. After a conformal transformation into the Einstein frame, the corresponding coupling between $\mathcal{V}$, or the canonical field associated to it, and the matter fields arises. This would induce fifth-force effects, which are ruled out [85, 275, 290], thus requiring the volume modulus to be sufficiently heavy.

Combining requirement 1 and 4, it follows directly that the quintessence modulus ϕ cannot be the volume modulus. In the next section, we will therefore assume that ϕ is instead given by the relative size of different 4-cycles, which can be much lighter. While fifth-force constraints are still an issue for such Kähler moduli due to violations of the equivalence principle [275, 290], they are less severe as for the volume modulus and we will focus on other, more problematic aspects in the rest of this work.

6.3. Mass hierarchies and the light-volume problem

As explained before, the Kähler moduli are stabilised by the quantum corrections given in Eq. (6.2). Let us therefore sketch their overall volume scaling suppressing potential $\mathcal{O}(1)$ and $\ln \mathcal{V}$ factors (cf. Sec. 4.2.4 and [291] for example):

$$\delta V_{\text{np}} \sim \frac{\mathrm{e}^{-2\mathfrak{a}_s\tau_s}}{\mathcal{V}} + \frac{|W_0|\mathrm{e}^{-\mathfrak{a}_s\tau_s}}{\mathcal{V}^2} \sim \frac{|W_0|^2}{\mathcal{V}^3} , \quad \delta V_{\alpha'} \sim \frac{|W_0|^2}{\mathcal{V}^3} , \quad \delta V_{\text{loop}} \sim \frac{W_0^2}{\mathcal{V}^{10/3}} . \tag{6.3}$$

The former two are responsible for the stabilisation of small cycles that are subject to non-perturbative effects as well as the overall volume in the usual LVS manner. On the other hand, δV_{loop} may be used to stabilise the residual directions in moduli space that remain flat after the usual LVS stabilisation. The quintessence field is assumed to correspond to such a residual direction, which can be any combination of Kähler moduli which is not the overall volume and not so small that non-perturbative effects $\sim \exp(-\mathfrak{a}\tau)$ might give a non-negligible contribution. To provide some intuition about said flat directions, let us mention that we have already introduced a comparable scenario in our discussion about fibre inflation in Sec. 4.3.2. Here the overall volume modulus has been effectively a fibration of a four-dimensional fibre over a two-dimensional base. While the overall volume was fixed in the LVS manner, some combination of the base and fibre remained flat, which was only stabilised by loop effects and represented the inflaton.

We are especially interested in the respective masses that arise due to the above potential terms. From Eq. (4.204), we know that the small-cycle moduli are heavy, $m_{\tau_s} \sim |W_0|/\mathcal{V}$, and that one of the remaining 'large cycles', which corresponds to the overall-volume modulus, has a mass $m_{\mathcal{V}} \sim |W_0|/\mathcal{V}^{3/2}$. All the other cycles obtain their masses from δV_{loop}.[3] As shown in the appendix of [2], after integrating out the small cycles τ_s and the axions, the masses of the remaining moduli can be estimated to be equal to the square root of the respective potential term that generates it. This is a consequence of the specific structure of the Kähler metric K_{ij}. Hence, using Eq. (6.3), we arrive parametrically at (cf. [100])

$$m_{\mathcal{V}} \sim \sqrt{\delta V_{\alpha'}} \sim \frac{|W_0|}{\mathcal{V}^{3/2}} , \qquad m_{\tau_i} \sim m_\phi \sim \sqrt{\delta V_{\text{loop}}} \sim \frac{W_0}{\mathcal{V}^{5/3}} , \tag{6.4}$$

where we identified one of the remaining large-cycle volumes with the quintessence field ϕ.

We are now ready to use our phenomenological requirements of Sec. 6.2 in combination with the above volume-scalings of the masses to derive further bounds. Using requirements 1 and 4 together with Eq. (6.4), one finds

$$\mathcal{O}(10^{30}) \lesssim \frac{m_{\mathcal{V}}}{m_\phi} \sim \mathcal{V}^{1/6} \quad \Rightarrow \quad \mathcal{V} \gtrsim \mathcal{O}(10^{180}) . \tag{6.5}$$

[3]Actually, other contributions to the potential, for instance through poly-instanton corrections [287], are also possible; however, we will only focus on loop corrections because they are always present and hence can be considered to provide a lower limit on moduli masses.

Clearly, such a large internal volume will imply a decompactification and result in a very small KK scale. In detail, assuming that the internal space is isotropic with a typical radius R in string units l_s, the volume is given by $\mathcal{V} \sim R^6$. The KK scale is then the mass scale associated to the inverse radius R^{-1}, i.e.

$$m_{\mathrm{KK}} \sim \frac{M_s}{R} \sim \frac{M_{\mathrm{P}}}{\mathcal{V}^{1/2+1/6}} \lesssim \mathcal{O}(10^{-120}) M_{\mathrm{P}} \ , \tag{6.6}$$

where the string scale M_s arises because R is given in string units and in the last step we used the above bound (6.5) on the volume. Such a low KK scale is in severe conflict with requirement 3, which leads us to the conclusion that the above setting of an isotropic compactification and loop corrections that scale as $\delta V_{\mathrm{loop}} \sim \mathcal{V}^{-10/3}$ is not viable.

Let us therefore now turn to a suggestion from [279, 287], according to which the above loop corrections can be further suppressed by assuming an anisotropic compactification of the internal space. To this end, we first need to understand how δV_{loop} is generated [287, 291]: From the perspective of a four-dimensional EFT, one identifies a cutoff Λ at which the EFT breaks down, and which we assume to be equal to the lowest KK scale. The latter assumption is indeed a non-trivial one as we further explain in Sec. 6.5. The loop corrections then correspond to all loop contributions up to this cutoff. The resulting potential is given by the SUSY version of the Coleman-Weinberg potential [292, 293],

$$V = V_{\mathrm{tree}} + V_{\mathrm{loop}} \ , \tag{6.7}$$

with

$$V_{\mathrm{loop}} = \frac{1}{64\pi^2} \mathrm{STr}\mathcal{M}^0 \cdot \Lambda^4 \log \frac{\Lambda^2}{\mu^2} + \frac{1}{32\pi^2} \mathrm{STr}\mathcal{M}^2 \cdot \Lambda^2 + \frac{1}{64\pi^2} \mathrm{STr}\mathcal{M}^4 \log \frac{\mathcal{M}^2}{\Lambda^2} + \cdots \ . \tag{6.8}$$

Using that the first term in V_{loop} vanishes due to SUSY and ignoring numeric prefactors and logarithmic factors, the loop correction becomes

$$\delta V_{\mathrm{loop}} \sim A m_{\mathrm{KK}}^2 m_{3/2}^2 + B m_{3/2}^4 \ , \tag{6.9}$$

where A and B are $\mathcal{O}(1)$ constants. To arrive at the above expression, we utilised that in four-dimensional $\mathcal{N} = 1$ SUGRA the supertrace is given by $\mathrm{STr}\mathcal{M}^2 = 2Q m_{3/2}^2$ with Q being a model-dependent $\mathcal{O}(1)$ constant. Inserting the well-known expression for the gravitino mass, given in Eq. (4.69), we find

$$\delta V_{\mathrm{loop}} \sim A m_{\mathrm{KK}}^2 \frac{W_0^2}{\mathcal{V}^2} + B \frac{W_0^4}{\mathcal{V}^4} \ , \tag{6.10}$$

where we took the approximate expression for the Kähler potential given in Eq. (4.184).

One now easily ascertains that for the isotropic case, where the KK scale is given by Eq. (6.6), the loop contribution to the potential scales as $\delta V_{\text{loop}} \sim \mathcal{V}^{-10/3}$. Let us now assume an anisotropic compactification with l dimensions of size R_1 and $6-l$ dimensions of size R_2 in string units l_{s}, respectively. The overall volume is then given by $\mathcal{V} \sim R_1^l R_2^{6-l}$. This results in the fact that we have two different KK scales associated to the overall volume: one corresponds to the inverse of R_1 and the other to the inverse of R_2. The lower of the two KK scales represents the aforementioned cutoff Λ at which our four-dimensional theory becomes effectively higher dimensional. To reduce the cutoff and hence the KK scale in Eq. (6.10) as much as possible, one set of dimensions should be as large as possible, implying that the other set is very small for a given volume $\mathcal{V}$. Concretely and w.l.o.g., we set the $6-l$ dimensions to the string length, $R_2 \sim 1$, which implies that the volume is essentially given by the other l large dimensions of size $R \sim \mathcal{V}^{1/l}$. The two KK scales are thus hierarchically different, where the heavy KK modes correspond to the small dimensions of size R_2 and have a mass at the string scale M_{s}, whereas the lighter KK modes related to the large dimensions of size R_1 have masses of order

$$m_{\text{KK}} \sim \frac{M_{\text{s}}}{R_1} \sim \frac{M_{\text{P}}}{\mathcal{V}^{1/2+1/l}} \,. \tag{6.11}$$

We see that a lower l implies a stronger suppression of m_{KK} and hence of δV_{loop}; however, below a value $l = 2$ the second term in Eq. (6.10), which is independent of l, becomes parametrically larger and prohibits a further suppression by reducing l. We will therefore for the moment consider the case where $l = 2$ to yield the the smallest δV_{loop} and hence to be optimal for our purposes.

The corresponding loop corrections are then of the order $\delta V_{\text{loop}} \sim \mathcal{V}^{-4}$ so that the mass of the quintessence field can be estimated as

$$m_\phi \sim \sqrt{\delta V_{\text{loop}}} \sim \frac{W_0}{\mathcal{V}^2} \,, \tag{6.12}$$

where we again used the formula from [2] to approximate moduli masses by the square-root of the potential. Considering only requirement 3, which constrains the volume to $\mathcal{V} \lesssim \mathcal{O}(10^{30})$ to avoid decompactification, the quintessence mass given by the above formula can marginally fit lightness imposed by requirement 1. However, including requirement 4 of a light volume modulus, we obtain

$$\mathcal{O}(10^{-30}) \gtrsim \frac{m_\phi}{m_{\mathcal{V}}} \sim \mathcal{V}^{-1/2} \sim m_{\text{KK}}^{1/2} \quad \Rightarrow \quad \mathcal{O}(10^{-60}) \gtrsim m_{\text{KK}} \,, \tag{6.13}$$

where we used Eq. (6.4) for $m_{\mathcal{V}}$. The latter relation contradicts with requirement 3. We hence conclude, while the anisotropic case helps ameliorating the incompatibility of the requirements 1, 3 and 4, there is still a tremendous discrepancy between them all by many orders of magnitude. The required hierarchy between the relevant mass scales cannot be achieved via the standard LVS approach.

One may have the idea to suppress δV_{loop} further by fine-tuning W_0 to small values. Due to the quartic dependency on W_0 of the second term in Eq. (6.10), this very term may become neglibile compared to the first one for values of l even smaller than 2. The latter may be realised by choosing one large and five small dimensions, corresponding to $l = 1$, or through a more complex geometry with large, small and intermediate dimensions, which could be treated an effective non-integer scaling $1 < l < 2$. Nevertheless, this also does not help to resolve the contradictions. To illustrate this, let us simply ignore the second term and only use the first one for a general l. With Eq. (6.11), we then have

$$\delta V_{\text{loop}} \sim \frac{W_0^2}{\mathcal{V}^{3+2/l}} \quad \Rightarrow \quad m_\phi \sim \sqrt{\delta V_{\text{loop}}} \sim \frac{W_0}{\mathcal{V}^{3/2+1/l}} \, . \tag{6.14}$$

Combining requirements 1 and 4, this leads to

$$\mathcal{O}(10^{-30}) \gtrsim \frac{m_\phi}{m_\mathcal{V}} \sim \mathcal{V}^{-1/l} \quad \Rightarrow \quad \mathcal{O}(10^{-30-15l}) \gtrsim m_{\text{KK}} \, . \tag{6.15}$$

Therefore, requirement 3 will always be violated and our conclusions above appear to be inevitable.

The problem discussed above has already been noted in [279, 287] and we refer to it as the 'light-volume problem'. A possible way to resolve it is to increase the volume modulus mass $m_\mathcal{V}$ by another contribution to the scalar potential, which is however questionable.

6.4. The F-term problem

So far, we have not taken into account our second requirement of heavy supertpartner masses. In this section, which can be considered to be the major finding of this work, we will argue that providing the SM superpartners with a heavy mass will turn out to be difficult because the fine-tuned value of the CC tends to get spoilt. We will call this issue the 'F-term problem'. Let us start by considering gaugino masses, which are given by [294]

$$m_{1/2} = \frac{1}{2} \frac{F^m \partial_m f}{\text{Re} f} \, , \tag{6.16}$$

where f is the gauge-kinetic function and F^m the appropriate F-term of the SM modulus. If the SM lives on a stack of D7-branes, the gaugino mass scales parametrically like the gravitino mass (cf. Sec. A.2), $m_{1/2} \sim m_{3/2} \sim |W|/\mathcal{V}$, whereas if the SM is realised on D3-branes, the gaugino mass will be lighter [100]. We hence focus on the former case. Combining the requirements 1 and 2, one finds

$$\frac{m_\phi}{m_{1/2}} \lesssim \mathcal{O}(10^{-45}) \ . \tag{6.17}$$

Moreover, from Eq. (6.9) we deduce that $m_\phi \sim \sqrt{\delta V_{\text{loop}}} \gtrsim m_{\text{KK}} m_{3/2}$. Since in our context the gravitino mass scales like the gaugino mass, the above ratio must fulfill $m_\phi/m_{1/2} \gtrsim m_{\text{KK}}$, which is again in conflict with requirement 3. This implies that the standard procedure of spontaneous SUSY breaking via Kähler moduli F-terms is not enough to generate large enough gaugino masses.

In lieu thereof, one is advised to establish an additional source of SUSY breaking via a hidden sector where SUSY is spontaneously broken through the non-vanishing F-term of a spurion field X. This breaking is then mediated to the SM sector by one of the usual mechanisms (see [81] for an introduction to SUSY mediation).

If the hidden sector is realised in the form of D3 branes, the corresponding moduli X_α, which represent the position of the D3 branes in the internal CY manifold, modify the usual Kähler potential $\mathcal{K}(T+\overline{T})$ in the form of the replacement [100]

$$2\tau_i = T_i + \bar{T}_{\bar{i}} \quad \to \quad 2\tau_i' \equiv T_i + \bar{T}_{\bar{i}} + k_i(X_\alpha, \bar{X}_{\bar{\alpha}}) \ , \tag{6.18}$$

where the $k_i(X_\alpha, \bar{X}_{\bar{\alpha}})$ are real-valued functions, which depend quadratically or by a higher order on the X_α because any linear dependence may be either absorbed into a re-definition of the T_i or removed through a Kähler transformation (4.61). We will label the new Kähler potential, where the τ_i have been replaced by τ_i', by $\mathcal{K}'$. One can then calculate the F-term potential via the usual SUGRA formula (4.66) using this new Kähler potential and summing over the Kähler moduli T_i as well as the D3 moduli X_α. It turns out that, due to the no-scale property, the F-terms of the Kähler moduli T_i cancel again with the term $\propto |W|^2$ so that only a corresponding F-term contribution due to the X_α variables remains. In detail, assuming that $\langle X_\alpha \rangle = 0$, this remaining contribution reads

$$V \supset \delta V_X = K'_{\alpha\bar{\beta}} F_X^\alpha \bar{F}_X^{\bar{\beta}}; , \tag{6.19}$$

with

$$K'_{\alpha\bar{\beta}} = \frac{\partial^2 \mathcal{K}'}{\partial X_\alpha \partial \bar{X}_{\bar{\beta}}} = K_i \partial_\alpha \partial_{\bar{\beta}} k_i \quad \text{and} \quad F_X^\alpha = \mathrm{e}^{\mathcal{K}'/2} (K^{-1})'^{\alpha\bar{\beta}} \partial_{\bar{\beta}} \overline{W} \ , \tag{6.20}$$

where a summation over the index i is implied. One may easily verify these findings in the case where X_α is only given by a single field X and for the concrete choice

$k \sim X\bar{X} - a(X\bar{X})^2$ and $W = bX$ with a and b being constants. We conclude that the above term δV_X yields an additional, positive contribution to the scalar potential, which raises its zero-value due to the no-scale cancellation to an effective, positive CC. The crucial question is now what magnitude this CC takes on.[4]

To answer it, we consider a simple toy model with only one spurion field X, whose F-term is $F_X \equiv F$. SUSY breaking is mediated to the SM sector via higher-dimensional operators, which are suppressed by a mass scale M defined as the mediation scale in the limit of flat SUSY. As is usual for models with mediation of SUSY breaking [79], the masses of the superpartners are then given by the F-term and mediation scale, e.g. for the gaugino one obtains $m_{1/2} \sim F/M$ so that

$$\delta V_X \sim F^2 \sim M^2 m_{1/2}^2 \, . \tag{6.21}$$

Generally speaking, both $m_{1/2}$ and M should be of the order $\mathcal{O}(\text{TeV}) \sim \mathcal{O}(10^{-15}) M_\text{P}$ or higher. The former is constrained by collider experiments whereas the latter cannot be too small lest the SUSY breaking sector is not hidden anymore. We thus obtain a contribution $\delta V_X \sim \mathcal{O}(10^{-60}) M_\text{P}^4$. This is of the same order as the terms that cancel due to the no-scale property and hence much larger than the quantum corrections (6.2) which establish the LVS potential. This can be seen by using the aforementioned relation $m_\phi \gtrsim m_\text{KK} m_{3/2}$ together with requirements 1 and 3, so that we arrive at $V_\text{no-scale} \sim m_{3/2}^2 \lesssim m_\phi^2 / m_\text{KK}^2 \lesssim 10^{-60} M_\text{P}^4$. Therefore, the quantum corrections (6.2) cannot be used to cancel δV_X.

6.4.1. Limits on the F-term contribution

Now we want to answer the question whether δV_X, which we identified as a major issue in the previous subsection, can be made significantly smaller in more detail. From Eq. (6.21) and due to the fact that $m_{1/2}$ cannot be smaller than $\mathcal{O}(10^{-15}) M_\text{P}$, we infer that the only option for a smaller δV_X is a simultaneous reduction of F and M. This is not easy to achieve in general and even for the case of five-dimensional constructions involves problems [295–297].

As we explained in Sec. 4.1.2, the SUSY-breaking scale is strongly correlated to the gravitino mass so that a reduction of F implies a reduction on $m_{3/2}$. The latter has been constrained by many experiments, for instance by electroweak colliders [298–305] like LEP or hadronic ones like the Tevatron [306–310]. In summary,

[4] Note that in the case where the hidden SUSY breaking sector is realised in the form of D7-branes, the axio-dilaton experiences an analogous replacement in the Kähler potential as the Kähler moduli in the D3 case, namely $S + \bar{S} \to S + \bar{S} + k(X, \bar{X})$ [100]. However, since S is stabilised by fluxes and integrated out, one can consider it as a constant. The resulting contribution to the scalar potential therefore reads $V \supset |D_X W|^2$, which represents the analogous expression to Eq. (6.19).

these bounds on the gravitino mass imply lower limits on the SUSY-breaking scale of the order $\sqrt{F} \gtrsim \mathcal{O}(100-1000)$ GeV.

The most recent and stringent bounds arise from the LHC in the form of missing-momentum signatures in proton-proton collisions. In order to provide some insight into the emergence of such bounds, let us consider an exemplary toy model where SUSY is spontaneously broken in a hidden sector via a non-vanishing F-term in the vacuum. This SUSY breaking is mediated to the SM sector through the interaction terms (cf. Eqs. (4.49) and (4.57))

$$\mathcal{L} \supset= \frac{a}{M^2}\int d^4\theta X^\dagger X \Phi^\dagger \Phi + \frac{b}{M}\int d^2\theta X W^\alpha W_\alpha + \text{h.c.}\,, \tag{6.22}$$

where X is the chiral superfield responsible for SUSY breaking, Φ is another chiral superfield representing quarks q and squarks $\tilde{q}$ and W^α is the supersymmetric field-strength tensor of a vector superfield V representing gluons g and gluinos $\tilde{g}$. The SM superpartners, i.e. squarks and gluinos, have their soft masses generated by a non-vanishing vacuum F-term, resulting in $m_{\tilde{q}}^2 = aF^2/M^2$ and $m_{\tilde{g}} \sim bF/M$, respectively. Moreover, after SUSY breaking, the gravitino obtains its mass in the so-called *super-Higgs mechanism* [81]. Here the goldstino $\tilde{G}$, which is the fermionic component of the SUSY-breaking field X, gets eaten by the a priori massless gravitino and provides two additional degrees of freedom resulting in the massive, spin-3/2 gravitino. This is analogous to the standard Higgs mechanism, with SUSY breaking corresponding to EW symmetry breaking, the goldstino corresponding to the Goldstone bosons, the gravitino corresponding to the massive gauge bosons and the non-vanishing Higgs vacuum expectation value corresponding to the non-vanishing F-term [81]. One can show that in the limit $\sqrt{s}/m_{3/2} \gg 1$, the helicity-1/2 modes of the gravitino dominate over its helicity-3/2 modes and that, according to the gravitino-goldstino equivalence theorem [311, 312], the resulting S-matrix elements for them are equal to those for the goldstinos. In this simplified discussion, we can therefore identify the gravitino with the goldstino. The relevant processes in the LHC involve two hadrons, which turn into a hadronic shower plus gravitinos, where the latter are not detected and hence induce a missing-momentum signature. To be explicit, let us consider the process of two quarks in the initial state and two gravitinos in the final state with a gluon being eradiated from one of the initial quarks. The latter results in a hadronic shower. The vertex associated to the gluon radiation will give a factor $\sqrt{\alpha_\mathrm{s}}$. The important qq-$\tilde{G}\tilde{G}$-amplitude results from several contributions of beyond-SM processes, one of which is the direct 4-particle coupling due to Eq. (6.22),

$$\sim \frac{a}{M^2}\bar{\tilde{G}}\tilde{G}\bar{q}q \subset \frac{a}{M^2}\int d^4\theta X^\dagger X \Phi^\dagger \Phi\,. \tag{6.23}$$

Due to the above expression for the squark masses $m_{\tilde{q}}^2$, the prefactor a/M^2 in the 4-particle vertex contributes a factor $1/F^2$ to the amplitude, leading to a cross section $\propto \alpha_s/F^4$. The corresponding measurements at the LHC then provide upper limits for this cross section, which translate into lower bounds on F.

Several, different analyses with ATLAS data that make use of the above or similar processes [310, 313, 314] come to the result that the gravitino mass has a lower bound around $m_{3/2} \gtrsim \mathcal{O}(10^{-4})\,\mathrm{eV}$ and the SUSY-breaking scale at about $\sqrt{F} \gtrsim \mathcal{O}(100 - 1000)\,\mathrm{GeV}$ (see [2] for further details). We conclude that, according to the current experimental status, δV_X can at most be a few orders of magnitude below $\mathcal{O}(10^{-60})M_{\mathrm{P}}^4$. A contribution so high to the scalar potential cannot be cancelled by any known term in our scenario.

6.4.2. Need for a new contribution

The additional contribution δV_X to the scalar potential is many orders of magnitude higher than the observed energy density $\rho_\Lambda \sim \mathcal{O}(10^{-120})M_{\mathrm{P}}^4$ and hence indeed requires some mechanism of cancellation in order for this whole scenario to be viable. Since, as argued above, there is no known term in the usual LVS setting that can achieve such a cancellation, we now want to explore some more exotic possibilities. Essentially, we need a new, negative contribution to the scalar potential, which is of the order $\delta V_{\mathrm{new}} \sim \mathcal{V}^{-2}$ and which cancels with δV_X precisely to a very small, fine-tuned value. Such a contribution would not only be helpful to solve the F-term problem but may also provide a larger mass to the volume modulus. If our above formula for estimating moduli masses is applicable, one would obtain $m_{\mathcal{V}} \sim \sqrt{\delta V_{\mathrm{new}}} \sim \mathcal{V}^{-1}$, which could indeed ameliorate or even solve the light-volume problem. However, let us emphasize that such a negative contribution is of hypothetical nature and its generation not understood. As suggested in [279, 287], possible effects that might give rise to δV_{new} are loop corrections from open strings attached to the SM brane(s) as well as the back-reaction of the bulk to the brane tension along the lines of models with super-large extra dimensions [315]. However, as further elaborated in [2], both effects are problematic so that their applicability remains questionable (see also [279, 287, 316–318] for related discussions).

6.5. Summary and discussion

In this work, we have investigated the realisability of stringy quintessence motivated by the conjecture that true de Sitter-vacua belong to the swampland. To this end we have imposed several phenomenological requirements associated to the hierarchies between the relevant mass scales, i.e. the quintessence mass, SM-superpartner masses, the KK scale and the volume-modulus mass. Working in the setting of type IIB string theory, the scenario of [279] seems favoured, where the quintessence field corresponds to a combination of large Kähler moduli rolling down a potential at fixed volume. Two major problems regarding the aforementioned hierarchies arise: Combining the requirements of a light quintessence modulus and heavy KK scales implies first an unacceptably light volume modulus and second a weak SUSY-breaking by the F-terms of Kähler moduli, i.e. SM-superpartner masses which are too small. We called the former the light-volume problem and it requires some new ingredient to raise the volume-modulus mass, as suggested in [287]. The latter, which we refer to as the F-term problem, requires an additional SUSY-breaking sector, which induces a significant uplift contribution to the scalar potential. To be consistent with observations, this contribution must be cancelled by a further, negative contribution; however, since this cancellation must appear at the level of the LVS no-scale cancellation, there is no obvious effect that can achieve it. Hence, some less-known and speculative effect is required to produce the negative term of the order $\delta V_{\text{new}} \sim \mathcal{V}^{-2}$. Such a term might also provide a mass to the volume modulus, thus solving the light-volume problem.

However, such a term is nevertheless problematic because the fine-tuning is generally not robust against small changes in the SM or SUSY-breaking parameters. That is, according to the above discussion the observed vacuum energy density requires a precise cancellation of two terms with a very large magnitude, namely δV_X and δV_{new}. One can now imagine models where the aforementioned SM or SUSY-breaking parameters are slightly shifted, inducing a small change of δV_X. However, such a small change will appear gigantic compared to the tiny fine-tuned vacuum energy density, thus leading to its de-tuning. This may raise the residual scalar potential to a very large value while retaining a tiny slope and thereby violate the de Sitter-swampland conjecture (6.1). The fact that a miniscule change in the SM parameters can decide whether a model lies in the swampland or not seems unnatural and deserves further scrutiny.

Concerning the light-volume problem, two possible loopholes to evade it deserve further attention. First, one may think of a quintessence mass that is not light at its natural scale but due to fine-tuning, e.g. by cancelling the two terms in Eq. (6.9) against each other. Thus requirement 1 may be fulfilled at a much smaller volume $\mathcal{V}$ so that the KK scale and volume-modulus mass can remain large enough. However, this is problematic as well. In general, the flatness of the quintessence potential must be maintained for a time scale of the order of the age of the universe, i.e. $\mathcal{O}(H_0^{-1})$. If we respect the de Sitter conjecture (6.1), this flatness cannot be arbitrarily small so that the distance in field space which the quintessence field traverses needs to be sufficiently far. This can be easily seen from the Klein-Gordon equation in an FRW background together with the condition $|V'|/V \lesssim 1$, which implies that this very distance must be $\Delta\phi \sim \mathcal{O}(1)$ in one Hubble time. It is hence not enough to fine-tune the potential δV_{loop} at only a specific point in time and field space but instead one needs to consider the whole range which is covered during the slow-roll phase. Since this range is of order unity, one has to take into account all orders of $\Delta\phi$ in a Taylor expansion, which implies that an infinite number of derivatives must be tuned to small values. It seems implausible that such a decoupling of the quintessence modulus from the loop potential occurs in our scenario by coincidence; however, in other settings there may be a mechanism that can result in the required sequestering [275, 319].

Second, the choice of the lowest KK scale m_{KK} as cutoff in the Coleman-Weinberg potential (6.8) is a non-trivial one and needs to be scrutinised. According to requirement 3, the bulk KK scale is rather low, i.e. even much lower than the electroweak scale. Therefore, when applying the Coleman-Weinberg potential, we assume that the SM brane has already been integrated out, which involves the danger of stronger corrections at a higher scale. Moreover, due to bulk fluxes, which are fluxes on the large cycles that constitute the bulk of the internal space, SUSY may still be broken at a scale above m_{KK} even in the bulk itself. In this case, once more stronger contributions to the loop potential are expected. Nevertheless, the assumption of m_{KK} as cutoff can be considered to yield a lower bound on δV_{loop} whereby other contributions would just require an even larger $\mathcal{V}$ and hence amplify the severity of the light-volume problem. In this sense, our conclusions are inevitable.

Let us also mention some other approaches to stringy quintessence, where we refer to [2] for more detailed discussions.

One possibility is to identify the quintessence field with one of the many axions, that are abundantly present in type IIB compactifications (see e.g. [242] for a discussion of stringy axion quintessence). The slow-roll regime is then established close to the hilltop of the typical, cosine-like axion potential. However, it turns out that in the most naive constructions, the trans-Planckian axion decay constant [284], which is needed to fulfill the slow-roll conditions, is in conflict with another proposed requirement to avoid the swampland, the so-called *weak gravity conjecture* for axions [172, 320]. More sophisticated scenarios like axion monodromy [284] or models with a highly suppressed axion potential [282, 286, 321–324] may respect the weak gravity conjecture but achieving heavy SM-superpartners while retaining a flat and low enough quintessence potential remains a delicate issue due to the large F-terms that are needed for SUSY breaking. This requires further scrutiny and intricate model building.

Another approach is constructed along the lines of the KKLT scenario [281] and identifies the quintessence field with the real part of a complexified Kähler modulus. The latter is trapped inside a valley of local axionic minima and forced to roll down in the real direction. Problematically, in order to account for the smallness of the observed CC, the superpotential must be tuned to very small values, which implies a very light gravitino. Hence, we again face the problem of the requirement for an additional SUSY-breaking sector, which re-introduces the F-term problem.

At last, an interesting alternative proposal [325] establishes a scenario where the de Sitter conjecture is satisfied at zero temperature, whereas a thermally excited hidden sector generates a stable locus in the potential for a scalar field at positive energy. However, since the zero-temperature, negative-energy minimum is naturally very close to the line of Minkowski space, i.e. vanishing CC, and since SUSY breaking nevertheless requires a fine-tuned F-term contribution δV_X, there is again the danger of de-tuning, which can easily raise the minimum to positive energies. This would imply a de Sitter vacuum and thus a violation of the de Sitter conjecture.

To conclude, let us elaborate on the future perspective of realising the late-time expansion of the universe in string theory. All in all, there seem to be three possibilities: First, the de Sitter conjecture could turn out to be false and stringy de Sitter vacua may indeed be feasible. In this case, one could imagine that a CC is achieved at the positive-energy minimum of a KKLT-like construction or within the LVS including some uplift mechanism. Both cases generally require significant fine-tuning (see also [261] for a recent discussion about the calculational control of KKLT). Second, it is of course always a possibility that string theory is not the correct UV completion to describe our universe. While the capability to draw such a conclusion seems to be far in the future, it is generally thinkable that the swampland programme may eventually find proof that some crucial aspect of our universe lies in the swampland, thus falsifying string theory. Third, if we assume that both the de Sitter conjecture and the stringy nature of the universe are true, some sort of dynamic DE, as for example quintessence, appears to be the most promising candidate to describe the observed late-time expansion. In this case, the findings of this work take effect, namely that the light-volume and especially the F-term problem constitute major challenges which have to be overcome in order to establish a stringy quintessence scenario. We have argued that a new, negative contribution δV_{new} is needed to overcome both issues. The investigation of the detailed nature and effect of this term is left to future work. Alternatively, one may try to construct models, which completely evade the logic of an effective, four-dimensional SUGRA, or consider other string-theoretic settings than type IIB. Possible examples are type IIA and heterotic string theory or utilising the running of complex-structure moduli and the dilaton towards large and small values, respectively. Both of the latter two may nevertheless bear other issues, e.g. a large volume at large complex-structure moduli due to mirror symmetry or a string scale below the KK scale for small dilaton values. In summary, we believe that the F-term problem remains a generic issue in many scenarios, which re-appears in some form or another due to the necessity of heavy SM-superpartners.

Part III.

Conclusions

7. Summary

Understanding the underlying principles of our universe is an ever-evolving, continuous process, that may likely never stop. As described at the very beginning of this thesis, some of the big scientific questions of our time concern the nature of the dark sector and its implications on cosmology as well as the possibilities to extend the SM of particle physics in a meaningful way. In this thesis, we described different instances of progress towards the ultimate goal of answering these questions.

In Chpt. 3 we considered a model of coupled dark energy, which generates interactions within the dark sector via a conformally transformed DM metric. Our approach was to take a non-conventional choice of the conformal function $C(\phi)$, which leads to a non-constant coupling function Q. We were then able to show that under certain assumptions, of which the most important one is the quasi-static approximation, a regime of transient weak gravity on large scales is possible. This represents a novel result, which is opposed to the usual assumption that such a coupling can merely enhance gravity, and may imply a possible decrease of the clustering strength, thus alleviating the σ_8 tension. As discussed in Chpt. 3, arguably the biggest challenge will be to implement the above behaviour into a realistic cosmological model including the correct accelerated expansion behaviour of the universe. Afterwards, a next step will be to confront this model with real data and evaluate the resulting goodness of fit against the ΛCDM model as well as other extensions thereof – preferably by utilising the Bayesian evidence ratio.

We then adopted the language of string theory and proceeded with the assumption that the latter represents the correct description of the universe on a fundamental level. In Chpt. 5, we investigated the realisability and phenomenological consequences of a prominent extension to the SM of particle physics, namely the QCD axion, in a stringy context. We identified the branch of type IIB string theory and, in particular, the large volume scenario as a favourable setting to obtain the required small axion decay constant f_a. Due to further cosmological constraints related to the DM abundance and isocurvature bounds, we were led to consider models with low inflation scale H_{I}. A prominent representative of the latter in string theory is blow-up inflation, which like many other LVS models is plagued by a DR problem due to decays of the volume modulus into its own axion. With the help of a novel fast decay channel into SM Higgses, we were able to seemingly solve this DR problem; however, the altered cosmological setting, where the inflaton itself is the longest-lived particle before reheating, re-introduces it via decays into light axions with an $\mathcal{O}(1)$ branching ratio. While there may still be possibilities to fix this new DR problem within the considered model by utilising direct decays of the inflaton into SM gauge bosons, our findings could even turn out to be useful in other scenarios, too. This is because now the DR problem does not arise from the omnipresent volume modulus anymore but due to the inflationary sector. The latter allows for a lot more freedom in the form of inflationary model building so that this new DR problem may be solved by a direct coupling between the inflaton and the SM, as for example in hybrid inflation models.

Finally, in Chpt. 6 we again addressed the implementation of a dynamical DE, however, this time within the paradigm of string theory. Motivated by the de Sitter-swampland conjecture, according to which true dS vacua are disallowed in string-theoretic constructions, we collected phenomenological requirements and challenges concerning a stringy realisation of quintessence. We concluded that the most promising setting for the latter is the LVS where the quintessence field is given by a combination of large Kähler moduli, which roll down a small, flat and loop-generated potential while the overall volume $\mathcal{V}$ remains fixed. This implies two major challenges, which we called the 'light-volume problem' and the 'F-term problem'. The former describes the fact that the requirements for a light quintessence mass and a heavy KK scale imply a very light volume-modulus mass, which would result in inadmissible fifth forces. On the other hand, the novel F-term problem indicates that the same two requirements also imply a very small SUSY-breaking scale. This can be remedied through the addition of a dedicated SUSY-breaking sector, which is hidden and mediates the spontaneous symmetry breaking to the SM sector. However, as we have argued, this implies a very large contribution δV_X to the vacuum energy density, which again must be cancelled via a new contribution δV_{new} of unknown origin. While the latter may also solve the light-volume problem by increasing the volume-modulus mass, its nature remains highly speculative and there are several problems associated to the required fine-tuning between δV_X and δV_{new} down to the observed vacuum energy density.

8. Outlook

Future research is expected to shed more light on all of the above topics. With regard to contemporary cosmological tensions, CDE models are on a strong footing because they have revealed their ability to alleviate both the H_0 [27] and, as we have shown in our analysis, the σ_8 tension. However, it remains challenging to solve both tensions simultaneously and is questionable whether this is possible without further ingredients. At this point, we want to mention a class of CDE models similar to the ones considered in this thesis but with the additional feature of a velocity-dependent coupling within the dark sector [326–331]. The latter implies a momentum transfer between DE and the DM fluid, which can also result in a decreased gravitational coupling constant [332–334] so that these models may alleviate the σ_8 tension [335, 336] as well as the Hubble tension – possibly even at the same time [333, 337, 338]. In light of several future observational experiments, for instance the square kilometre array [339] or the Euclid satellite [340–342], we may expect further insight on the large scale structure of the universe, which could increase the σ_8 tension to a level as uncomfortable as the Hubble tension today. It is therefore all the more important to conduct theoretical model-building to tackle both of these tensions and we can conclude that CDE models have the potential to do so.

A consistent unification of an inflationary mechanism together with a QCD axion, which can also play the role of DM, within the stringy paradigm would represent a stupendous success of string-phenomenological model building. Not only would this provide a mechanism to answer a handful of important theoretical questions simultaneously but also imply some predictive power, that could be used to put string theory, or at least a some part of it, closer to the experimental side and test it. After our findings, this is obviously still a long road to go and requires a significant amount of model-building input; however, we have achieved some progress into the right direction. A crucial issue that requires further scrutiny is the DR problem, which we have extensively elaborated on in this thesis and whose solution appears to be closer after our findings than before in the form of inflationary model building. In particular, we want to recall that according to the newest indications a possible solution is at hand even within the model of blow-up inflation due to direct inflaton decays into SM gauge bosons. Nevertheless, there are other theoretical problems that need to be taken care of. First, as already mentioned in Sec. 5.5, there is still the danger of loop effects, which are not fully understood and may spoil the required flatness of the inflaton potential. This deserves further scrutiny. Second, it is still currently debated whether the LVS provides the needed calculational control [108, 109], in particular w.r.t. an uplift to a dS spacetime via anti-D3-branes as explained in Sec. 4.2.4. This may pose a danger for the entire setting and put the LVS into the swampland; however, as argued in [109], the uplift can be brought under control by the abidance of a constraint on the D3 tadpole. In a realistic scenario, this imposes another challenge to overcome. Assuming that such theoretical issues are redressed, observations may imply interesting consequences. First and foremost, as we have argued, stringy axions embedded into a realistic inflation model require a very low inflaton scale and hence tensor-to-scalar ratio r. Therefore, any future detection of primordial gravitational waves, for instance by the square kilometre array [339], would immediately rule out the considered scenario in Chpt. 5. Second, one may hope for a direct detection of axions or further constraints from current or future experiments (see also [343] or for a more recent review [135]). Worth mentioning are *light-shining-through-walls* experiments, e.g. ALPS II [344], *haloscopes* like ADMX [345–347] or *helioscopes* as for example IAXO [348, 349]. Moreover, a recent excess in the XENON1T detector [350] has attracted much attention; however, as argued in [351] this cannot be explained by solar axions. Nevertheless, depending on its precise properties, a detection of the QCD axion may be feasible within the next few years, and even for a negative result, we can expect a further constrained parameter space to test the above scenario against.

As we already elaborated in Sec. (6.5), under the assumption that both string theory and the dS-swampland conjecture are true some sort of dynamical DE is required to explain the late-time expansion of the universe. In some way or another, this seems to imply a huge challenge in the form of the light-volume and the F-term problem. Further issues of stringy quintessence have been examined in [352, 353] and one may conclude that dynamical DE models are not necessarily on a stronger footing than constructions with a ‘true’ CC. All in all this gives rise to a severe tension between string theory and observations, which casts some doubt on the correctness of the dS conjecture. Further research is required to either prove the latter (or at least provide solid indications for its validity) or to find counter examples, which violate the dS conjecture while retaining enough calculational control to be regarded as disproof. Here as mentioned above, a critical point in many LVS settings concerns the uplift to a dS space, which is as of yet controversial [108, 109].

To conclude, we want to emphasize that both the modified-gravity and the string-theoretic approach constitute important as well as necessary tools to garner insight on the nature of our universe and hence deserve attention. The former can be considered as the means of choice to address modern-day cosmological tensions due to high-precision observations, where string theory, which even struggles to reproduce a consistent and controllable background evolution of the universe, seems to be too restricted. On the other hand, string theory is well established and represents the best understood and most prominent UV completion of the SM including gravity that we have. As such, it naturally serves as a playground for both top-down and a bottom-up approaches to model building. To this end, the above mentioned restrictions can turn out to be advantageous and provide guidance. Eventually, whilst string theory certainly constitutes a very elegant candidate for a theory of everything, a classical, field-theoretical approach to cosmology remains without alternative.

A. Appendices

A.1. Realising small f_a in general type II string theory

In this section, we use the conventions of [96] and additionally set $l_\text{s} \equiv 1$. An axion θ derived from a R-R p-form C_p of type II string theory originates from the ansatz given by Eq. (4.259). Due to the CS term (4.147), the C_p field couples to D$(p-1)$-brane instantons wrapped around the p-cycle Σ_p, as described by the action

$$S \supset 2\pi \int_{\Sigma_p} C_p = 2\pi\theta \int_{\Sigma_p} \omega_p \ , \tag{A.1}$$

where $\theta \equiv \theta + 1$ if ω_p is chosen integral. We now want to estimate the size of the axion decay constant f_a on rather general grounds. The relevant terms of the ten-dimensional Lagrangian are the Einstein-Hilbert term (from Eq. (4.134)) and the kinetic term for C_p (from Eq. (4.136)), which in the string frame read

$$S \supset 2\pi \int \mathrm{d}^4x\mathrm{d}^6y\sqrt{-g}\left\{\frac{1}{g_\text{s}^2}R - \frac{1}{2}|\mathrm{d}C_p|^2\right\} \ , \tag{A.2}$$

where we used that the stabilised dilaton determines the string coupling,

$$g_\text{s} = \mathrm{e}^{\langle\Phi\rangle} \ . \tag{A.3}$$

Ignoring $\mathcal{O}(1)$ constants, we can read off (cf. Eq. (4.262))

$$\frac{f_a^2}{M_\text{P}^2} \sim \frac{g_\text{s}^2}{\mathcal{V}_\text{s}} \int_X \omega_p \wedge \star\omega_p \sim \frac{g_\text{s}^2}{\mathcal{V}_\text{s}} \int_X \mathrm{d}^6y\sqrt{-g}(\omega_p)_{m_1\cdots m_p}(\omega_p)_{n_1\cdots n_p} g^{m_1n_1}\cdots g^{m_pn_p} \ , \tag{A.4}$$

where we used $V_X g_\text{s}^{-2} l_\text{s}^{-8} \sim \mathcal{V}_\text{s} g_\text{s}^{-2} M_\text{s}^2 \sim M_\text{P}^2$ with V_X being the volume of the internal CY manifold X and $\mathcal{V}_\text{s}$ the very same volume in the string frame and in string units l_s^6.

We are interested in preferably small axion decay constants f_a, so let us construct a setting in favour of this. We assume that ω_p has support only in a tubular neighbourhood of Σ_p with diameter d and that Σ_p has a typical length scale L. Evaluating the above integral, one obtains

$$\frac{f_a^2}{M_{\mathrm{P}}^2} \sim \frac{g_{\mathrm{s}}^2}{\mathcal{V}_{\mathrm{s}}} \frac{d^{6-p}}{L^p} \ . \tag{A.5}$$

We now set the diameter of the tubular neighbourhood to the string scale, i.e. $d \sim 1$, for optimal suppression and use that, according to the DBI action, the UV gauge coupling is given by $\alpha_{\mathrm{s,UV}} \sim g_{\mathrm{s}}/L^p$ (cf. Eq. (4.145)). This gives us

$$\frac{f_{a,\mathrm{min}}^2}{M_{\mathrm{P}}^2} \sim \frac{g_{\mathrm{s}}\alpha_{\mathrm{s,UV}}}{\mathcal{V}_{\mathrm{s}}} \sim \frac{\alpha_{\mathrm{s,UV}}}{\sqrt{g_{\mathrm{s}}}} \frac{1}{\mathcal{V}} \ , \tag{A.6}$$

where $\mathcal{V} = \mathcal{V}_{\mathrm{s}}/g_{\mathrm{s}}^{3/2}$ is the volume of the internal CY manifold in the Einstein frame and in string units.

A.2. Gaugino mass for the loop-stabilised cycle

In this appendix, we want to illustrate by a short calculation that the mass of gauginos, associated to the super Yang-Mills theory on the branes that wrap τ_{L}, is of the same size as the gravitino mass, $m_{3/2} \sim |W_0| M_{\mathrm{P}}/\mathcal{V}$. The F-term for τ_{L} is given by

$$\begin{aligned}
F^{\mathrm{L}} &= \mathrm{e}^{\mathcal{K}/2} \sum_{\bar{i}} \left(K^{-1}\right)^{\mathrm{L}\bar{i}} \overline{D}_{\bar{i}} \overline{W} && \text{(A.7)}\\
&= \mathrm{e}^{\mathcal{K}/2} \sum_{\bar{i}} \left(K^{-1}\right)^{\mathrm{L}\bar{i}} \left(\overline{W} \partial_{\bar{i}} \mathcal{K} + \partial_{\bar{i}} \overline{W}\right) && \text{(A.8)}\\
&= \mathcal{V}^{-1} \left[-2\tau_{\mathrm{L}} \left(\overline{W}_0 + \sum_{\bar{j} \neq \mathrm{L}} \overline{A}_{\bar{j}} \mathrm{e}^{-\mathfrak{a}_{\bar{j}} \overline{T}_{\bar{j}}} \right) - 4 \sum_{\bar{j} \neq \mathrm{L}} \mathfrak{a}_{\bar{j}} \overline{A}_{\bar{j}} \tau_{\mathrm{L}} \tau_{\bar{j}} \mathrm{e}^{-\mathfrak{a}_{\bar{j}} \overline{T}_{\bar{j}}} \right] \ , && \text{(A.9)}
\end{aligned}$$

where in the last line we used $\mathrm{e}^{K/2} = \mathcal{V}^{-1}$ and the relation $\sum_i K^{i\bar{j}} \partial_{\bar{j}} \mathcal{K} = -2\tau_i$ as well as the fact that W does not depend on τ_{L} and the fact that at leading order $(K^{-1})^{i\mathrm{L}} = 4\tau_i \tau_{\mathrm{L}}$, as can be seen from Eq. (A.32). Clearly, the dominating contribution is the term proportional to $\overline{W}_0$ so that the F-term is given by

$$F^{\mathrm{L}} \approx -2 \frac{\tau_l \overline{W}_0}{\mathcal{V}} \ . \tag{A.10}$$

If no fluxes are present, the gauge-kinetic function is simply given by $f = T_{\rm L}$. The gaugino mass then becomes [294]

$$m_{1/2} = \frac{M_{\rm P}}{2}\frac{|F^{\rm L}\partial_{\rm L} f|}{{\rm Re}f} = \frac{|W_0|}{\mathcal{V}}M_{\rm P} \tag{A.11}$$

and is therefore of the same size as $m_{3/2}$.

A.3. Decays of $\tau_{\rm L}$

In this appendix we estimate the rates of the most relevant decay channels of $\tau_{\rm L}$ in order to check whether a significant amount of dark radiation is produced by this cycle. We only consider a simplified subsystem comprising $\tau_{\rm b}$ and $\tau_{\rm L}$ (and their corresponding axionic partners) assuming that all other cycles are close to their respective minima and ignoring their perturbations. The decays of $\tau_{\rm L}$ that we consider are into its own axion and into SM gauge fields, which live on stack(s) of branes wrapping $\tau_{\rm L}$, as well as into SM Higgses via the Higgs mass term. For the sake of simplicity, we also assume that, due to the almost diagonal structure of the Kähler metric, we obtain sufficiently meaningful results without diagonalising the system. The kinetic terms according to Eq. (4.203) at leading order in the small parameter $\epsilon \equiv \tau_{\rm b}^{-1/2}$ are

$$\mathcal{L} \supset \frac{3}{4\tau_{\rm b}^2}\partial_\mu\tau_{\rm b}\partial^\mu\tau_{\rm b} + \frac{3\gamma_{\rm L}}{8\tau_{\rm b}^{3/2}\sqrt{\tau_{\rm L}}}\partial_\mu\tau_{\rm L}\partial^\mu\tau_{\rm L} + \frac{3}{4\tau_{\rm b}^2}\partial_\mu\theta_{\rm b}\partial^\mu\theta_{\rm b} + \frac{3\gamma_{\rm L}}{8\tau_{\rm b}^{3/2}\sqrt{\tau_{\rm L}}}\partial_\mu\theta_{\rm L}\partial^\mu\theta_{\rm L}\ . \tag{A.12}$$

We canonically normalise the fields as

$$\tau_{\rm b} = \exp\left(\sqrt{\frac{2}{3}}\phi_{\rm b}\right)\ , \quad \tau_{\rm L} = \left(\frac{3\mathcal{V}}{4\gamma_{\rm L}}\right)^{2/3}\phi_{\rm L}^{4/3}\ , \tag{A.13}$$

$$\theta_{\rm b} = \sqrt{\frac{2}{3}}\mathcal{V}^{2/3}a_{\rm b}\ , \quad \theta_{\rm L} = \frac{2\sqrt{\mathcal{V}}\,\langle\tau_{\rm L}\rangle^{1/4}}{\sqrt{3\gamma_{\rm L}}}a_{\rm L}\ ,$$

where it is understood that $\mathcal{V}$ denotes the volume at the minimum. The relevant terms for the decay of $\tau_{\rm L}$ into axions read

$$\mathcal{L} \supset K_{\rm bb}\partial_\mu\theta_{\rm b}\partial^\mu\theta_{\rm b} + 2K_{\rm bL}\partial_\mu\theta_{\rm b}\partial^\mu\theta_{\rm L} + K_{\rm LL}\partial_\mu\theta_{\rm L}\partial^\mu\theta_{\rm L} \tag{A.14}$$

$$\supset \frac{15\gamma_{\rm L}\tau_{\rm L}^{3/2}}{8\tau_{\rm b}^{7/2}}\partial_\mu\theta_{\rm b}\partial^\mu\theta_{\rm b} - \frac{9\gamma_{\rm L}\sqrt{\tau_{\rm L}}}{4\tau_{\rm b}^{5/2}}\partial_\mu\theta_{\rm b}\partial^\mu\theta_{\rm L} + \frac{3\gamma_{\rm L}}{8\tau_{\rm b}^{3/2}\sqrt{\tau_{\rm L}}}\partial_\mu\theta_{\rm L}\partial^\mu\theta_{\rm L}\ , \tag{A.15}$$

where we used that $15\gamma_{\rm I}\tau_{\rm L}^{3/2}/(8\tau_{\rm b}^{7/2})$ is the leading-order $\tau_{\rm L}$-dependent term in $K_{\rm bb}$. Fixing $\tau_{\rm b}$ at its vacuum expectation value and inserting the canonical fields for $\tau_{\rm L}$, $\theta_{\rm b}$ and $\theta_{\rm L}$, we obtain

$$\mathcal{L} \supset \frac{15}{16}\phi_{\rm L}^2\partial_\mu a_{\rm b}\partial^\mu a_{\rm b} - \frac{3^{4/3}\gamma_{\rm L}^{1/6}\left\langle\tau_{\rm L}\right\rangle^{1/4}}{2^{7/6}\left\langle\tau_{\rm b}\right\rangle^{1/4}}\phi_{\rm L}^{2/3}\partial_\mu a_{\rm b}\partial^\mu a_{\rm L} + \left(\frac{\gamma_{\rm L}}{6}\right)^{1/3}\sqrt{\frac{\left\langle\tau_{\rm L}\right\rangle}{\left\langle\tau_{\rm b}\right\rangle}}\phi_{\rm L}^{-2/3}\partial_\mu a_{\rm L}\partial^\mu a_{\rm L} \ . \tag{A.16}$$

We can now perturb $\phi_{\rm L} \equiv \left\langle\phi_{\rm L}\right\rangle + \delta\phi_{\rm L}$ about its vacuum expectation value to obtain the trilinear couplings to axions:

$$\mathcal{L} \supset \frac{5\sqrt{3\gamma_{\rm L}}\left\langle\tau_{\rm L}\right\rangle^{3/4}}{4\left\langle\tau_{\rm b}\right\rangle^{3/4}}\delta\phi_{\rm L}\partial_\mu a_{\rm b}\partial^\mu a_{\rm b} - \sqrt{\frac{3}{2}}\delta\phi_{\rm L}\partial_\mu a_{\rm b}\partial^\mu a_{\rm L} - \frac{\left\langle\tau_{\rm b}\right\rangle^{3/4}}{2\sqrt{3\gamma_{\rm L}}\left\langle\tau_{\rm L}\right\rangle^{3/4}}\delta\phi_{\rm L}\partial_\mu a_{\rm L}\partial^\mu a_{\rm L} \ . \tag{A.17}$$

Clearly, since $\left\langle\tau_{\rm b}\right\rangle \gg \left\langle\tau_{\rm L}\right\rangle$, the last term dominates, which is unsurprisingly the coupling of $\tau_{\rm L}$ to its own axion. Since we assume that the potential for $a_{\rm L}$ is generated by QCD instantons, its mass is much smaller than that of $\phi_{\rm L}$. The resulting decay rate is then given by

$$\Gamma_{\phi_{\rm L}\to a_{\rm L}a_{\rm L}} = \frac{1}{384\pi\gamma_{\rm L}}\left(\frac{\left\langle\tau_{\rm b}\right\rangle}{\left\langle\tau_{\rm L}\right\rangle}\right)^{3/2}\frac{m_{\phi_{\rm L}}^3}{M_P^2} \ . \tag{A.18}$$

This has to be compared to the rate of decays into SM gauge bosons A_μ living on the branes that wrap $\tau_{\rm L}$. The induced coupling is given by

$$\mathcal{L} \supset \tau_{\rm L}{\rm Tr}F_{\mu\nu}F^{\mu\nu} \ . \tag{A.19}$$

The field strength can be canonically normalised by replacing $F_{\mu\nu} \to F_{\mu\nu}/(2\sqrt{\left\langle\tau_{\rm L}\right\rangle})$. With that and Eq. (A.13) we obtain

$$\mathcal{L} \supset \frac{1}{4\left\langle\tau_{\rm L}\right\rangle}\left(\frac{3\mathcal{V}}{4\gamma_{\rm L}}\right)^{2/3}\phi_{\rm L}^{4/3}F_{\mu\nu}F^{\mu\nu} \ . \tag{A.20}$$

Perturbing $\phi_{\rm L}$ about its vacuum expectation value, this becomes

$$\mathcal{L} \supset \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{3}\frac{\delta\phi_{\rm L}}{\left\langle\phi_{\rm L}\right\rangle}F_{\mu\nu}F^{\mu\nu} \tag{A.21}$$

$$= \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\left\langle\tau_{\rm b}\right\rangle^{3/4}}{2\sqrt{3\gamma_{\rm L}}\left\langle\tau_{\rm L}\right\rangle^{3/4}}\delta\phi_{\rm L}F_{\mu\nu}F^{\mu\nu} \ . \tag{A.22}$$

The resulting decay rate is

$$\Gamma_{\phi_{\rm L}\to AA} = \frac{N_g}{48\pi\gamma_{\rm L}}\left(\frac{\langle\tau_{\rm b}\rangle}{\langle\tau_{\rm L}\rangle}\right)^{3/2}\frac{m_{\phi_{\rm L}}^3}{M_{\rm P}^2}\ , \tag{A.23}$$

where N_g is the number of gauge bosons and we have neglected the contribution of brane-fluxes to the gauge-kinetic function. Realising that $\Gamma_{\phi_{\rm L}\to AA}/\Gamma_{\phi_{\rm L}\to a_{\rm L}a_{\rm L}} = 8N_g \gg 1$, one can infer that the decays of $\phi_{\rm L}$ have only a negligible branching ratio into DR.

Besides the decays into SM gauge fields, one may wonder whether $\tau_{\rm L}$ experiences an enhanced production of Higgses via a $\tau_{\rm L}$-dependent Higgs mass, in analogy to the discussion about the enhanced decay rate of the volume modulus in Sec 5.4.2. The relevant expression is given by Eq. (5.45),

$$m_H^2 \sim m_{3/2}^2\left[c_0 + c_{\rm loop}\ln\left(\frac{m_{\rm KK}}{m_{3/2}}\right)\right]\ . \tag{A.24}$$

Restoring the explicit dependence of the Kaluza-Klein scale on the SM cycle, $m_{\rm KK} \sim M_{\rm s}/\tau_{\rm L}^{1/4} \sim M_P\mathcal{V}^{-1/2}\tau_{\rm L}^{-1/4}$, this becomes

$$m_H^2 \sim \left(\frac{W_0}{\mathcal{V}}\right)^2\left[c_0 + c_{\rm loop}\ln\left(\frac{\mathcal{V}^{1/2}}{W_0\tau_{\rm L}^{1/4}}\right)\right]\ . \tag{A.25}$$

Inserting the canonical field for $\tau_{\rm L}$ as given by Eq. (A.13) and perturbing about the vacuum expectation value, we obtain the trilinear coupling

$$\mathcal{L}\ \supset \sim\ m_{3/2}^2 c_{\rm loop}\frac{\langle\tau_{\rm b}\rangle^{3/4}}{2\sqrt{3\gamma_{\rm L}}\,\langle\tau_{\rm L}\rangle^{3/4}}\delta\phi_{\rm L}h^2\ . \tag{A.26}$$

Parametrically, the resulting decay rate is given by

$$\Gamma_{\phi_{\rm L}\to hh} \sim \frac{c_{\rm loop}^2 m_{3/2}^4}{m_{\phi_{\rm L}}M_P^2}\frac{\langle\tau_{\rm b}\rangle^{3/2}}{\langle\tau_{\rm L}\rangle^{3/2}}\ . \tag{A.27}$$

Using typical values, $c_{\rm loop} \simeq (16\pi^2)^{-1}$, $\tau_{\rm L} \sim \mathcal{O}(10)$ and $N_g = 12$, this is smaller than the rate into SM gauge fields by a factor

$$\frac{\Gamma_{\phi_{\rm L}\to hh}}{\Gamma_{\phi_{\rm L}\to AA}} \sim \frac{c_{\rm loop}^2}{N_g}\frac{m_{3/2}^4}{m_{\phi_{\rm L}}^4} \sim \frac{c_{\rm loop}^2\tau_{\rm L}^4}{N_g} \sim \mathcal{O}(10^{-2})\ , \tag{A.28}$$

where we used that according to Eq. (A.65) the mass of the loop modulus scales like

$$m_{\phi_{\rm L}} \sim \frac{W_0}{\tau_{\rm L}\mathcal{V}} \sim \frac{m_{3/2}}{\tau_{\rm L}} \, . \tag{A.29}$$

We conclude by stating that the decay into SM gauge bosons is indeed the dominant channel of $\phi_{\rm L}$ and that only a negligible amount of DR is produced.

A.4. Dynamics of the three-moduli system $\tau_{\rm b}$, $\tau_{\rm I}$ and $\tau_{\rm L}$

In this appendix, we estimate the decay rates of the inflaton and its axion into the volume and the loop-cycle modulus as well as their respective axionic superpartners. Since this appendix involves many cumbersome technicalities, which do not provide a lot of physical insight, we adopt it in almost one-to-one correspondence from [3]. In the same reference, one can also find a simpler analysis involving only the 2-moduli system comprised of $\tau_{\rm b}$ and $\tau_{\rm I}$. Note that we only consider a simplified system here as well, which does not take into account the additional small cycles $\tau_{{\rm s},i}$, which must be present to ensure the stability of the volume during inflation. However, as we will argue below, we do not expect that the inclusion of said small cycles, which play a role very similar to the inflaton $\tau_{\rm I}$ except that they are not initially excited, would change our findings of this section significantly. This analysis follows the methodology of [128] adapted to our purposes: We expand the potential V and (going beyond [128]) the Kähler potential $\mathcal{K}$ up to third order in fluctuations of the $\delta\tau_i$ and $\delta\theta_i$ about their respective LVS vacuum expectation values. We then diagonalise and canonically normalise the fields so that we obtain trilinear coupling terms and can read off the respective decay rates. Throughout this appendix, we set $M_{\rm P} = 1$.

A.4.1. Basic definitions

The total Kähler potential is given by Eq. (4.187); however, we are only interested in the Kähler moduli Kähler potential modified by α' corrections, which is given in Eq. (4.193). Since we ignore the small cycles $\tau_{{\rm s},i}$, the volume and relevant Kähler potential are of the form

$$\mathcal{V} = \tau_{\rm b}^{3/2} - \gamma_{\rm I}\tau_{\rm I}^{3/2} - \gamma_{\rm L}\tau_{\rm L}^{3/2} \, , \quad \mathcal{K} = -2\ln\left(\mathcal{V} + \frac{\xi}{2}\right) + \mathcal{K}_S + \mathcal{K}_{\rm cs} \, . \tag{A.30}$$

The resulting Kähler metric and its inverse at leading order in the small parameter $\epsilon \equiv \tau_{\rm b}^{-1/2}$ are given by

$$K_{ij} = \frac{\partial^2 K}{\partial T_i \partial \bar{T}_j} \approx \begin{pmatrix} \frac{3}{4\tau_{\rm b}^2} & -\frac{9\gamma_{\rm I}\sqrt{\tau_{\rm I}}}{8\tau_{\rm b}^{5/2}} & -\frac{9\gamma_{\rm L}\sqrt{\tau_{\rm L}}}{8\tau_{\rm b}^{5/2}} \\ -\frac{9\gamma_{\rm I}\sqrt{\tau_{\rm I}}}{8\tau_{\rm b}^{5/2}} & \frac{3\gamma_{\rm I}}{8\sqrt{\tau_{\rm I}}\tau_{\rm b}^{3/2}} & \frac{9\gamma_{\rm I}\gamma_{\rm L}\sqrt{\tau_{\rm I}\tau_{\rm L}}}{8\tau_{\rm b}^3} \\ -\frac{9\gamma_{\rm L}\sqrt{\tau_{\rm L}}}{8\tau_{\rm b}^{5/2}} & \frac{9\gamma_{\rm I}\gamma_{\rm L}\sqrt{\tau_{\rm I}\tau_{\rm L}}}{8\tau_{\rm b}^3} & \frac{3\gamma_{\rm L}}{8\sqrt{\tau_{\rm L}}\tau_{\rm b}^{3/2}} \end{pmatrix} , \tag{A.31}$$

$$(K^{-1})^{ij} \approx \begin{pmatrix} \frac{4\tau_{\rm b}^2}{3} & 4\tau_{\rm b}\tau_{\rm I} & 4\tau_{\rm b}\tau_{\rm L} \\ 4\tau_{\rm b}\tau_{\rm I} & \frac{8\sqrt{\tau_{\rm I}}\tau_{\rm b}^{3/2}}{3\gamma_{\rm I}} & 4\tau_{\rm I}\tau_{\rm L} \\ 4\tau_{\rm b}\tau_{\rm L} & 4\tau_{\rm I}\tau_{\rm L} & \frac{8\sqrt{\tau_{\rm L}}\tau_{\rm b}^{3/2}}{3\gamma_{\rm L}} \end{pmatrix} , \tag{A.32}$$

where $T_i = \tau_i + \mathrm{i}\theta_i$ with $i \in \{\mathrm{b}, \mathrm{I}, \mathrm{L}\}$. Since the model is constructed so that $\tau_{\rm L}$ is not stabilised by non-perturbative effects, the superpotential is only corrected by D3-brane instantons on $\tau_{\rm I}$ and hence is given by

$$W = W_0 + A_{\rm I} \mathrm{e}^{-\mathfrak{a}_{\rm I} T_{\rm I}} , \tag{A.33}$$

with $\mathfrak{a}_{\rm I} = 2\pi$. The axio-dilaton S and the complex-structure moduli are fixed by fluxes so that $\mathcal{K}_S$ and $\mathcal{K}_{\rm cs}$ represent constants, which we absorb into a redefinition of $A_{\rm I}$ and W_0.

The total scalar potential consist of the typical LVS contribution as given in Eq. (5.62), which is generated through the usual interplay of non-perturbative and α' corrections as explained in Sec. (4.2.4), and a contribution induced by loop effects, as given in Eq. (5.57),

$$V = V_{\rm LVS}^{(\rm I)}(\mathcal{V}, \tau_{\rm I}, \theta_{\rm I}) + V_{\rm loop}(\mathcal{V}, \tau_{\rm L}) . \tag{A.34}$$

The individual contributions read [99, 100, 137]

$$V_{\rm LVS}^{(\rm I)} = \mathcal{V}^{-2} \left[\frac{8\tau_{\rm b}^{3/2}\sqrt{\tau_{\rm I}}}{3\gamma_{\rm I}} \mathfrak{a}_{\rm I}^2 |A_I|^2 \mathrm{e}^{-2\mathfrak{a}_{\rm I}\tau_{\rm I}} + 4\mathfrak{a}_{\rm I}\tau_{\rm I}\mathrm{e}^{-\mathfrak{a}_{\rm I}\tau_{\rm I}} |A_I W_0| \cos(\mathfrak{a}_{\rm I}\theta_{\rm I}) \right] + \frac{3|W_0|^2\xi}{4\mathcal{V}^3} , \tag{A.35}$$

$$V_{\rm loop} = \left(\frac{\mu_1}{\sqrt{\tau_{\rm L}}} - \frac{\mu_2}{\sqrt{\tau_{\rm L}} - \mu_3} \right) \frac{W_0^2}{\mathcal{V}^3} , \tag{A.36}$$

where the complex phases $\arg A_{\rm I}$ and $\arg W_0$ have been absorbed into a redefinition of $\theta_{\rm I}$. According to Eq. (4.200), the minimum of this potential is defined by the relations

$$\xi = 2\gamma_{\rm I}\langle\tau_{\rm I}\rangle^{3/2} \ , \quad \mathrm{e}^{\mathfrak{a}_{\rm I}\langle\tau_{\rm I}\rangle} = \frac{4\langle\mathcal{V}\rangle|A_I|\mathfrak{a}_{\rm I}}{3\gamma_{\rm I}|W_0|\sqrt{\langle\tau_{\rm I}\rangle}} \ , \quad \cos(\mathfrak{a}_{\rm I}\langle\theta_{\rm I}\rangle) = -1 \ . \tag{A.37}$$

Both the kinetic term as well as the scalar potential can now be expanded about this minimum. The relevant Lagrangian for us is the truncation of this expansion at cubic order,

$$\begin{aligned}\mathcal{L} = &\langle K_{ij}\rangle\partial_\mu\delta\tau_i\partial^\mu\delta\tau_j + \langle\partial_{\tau_i}K_{jk}\rangle\delta\tau_i\partial_\mu\delta\tau_j\partial^\mu\delta\tau_k + \langle K_{ij}\rangle\partial_\mu\delta\theta_i\partial^\mu\delta\theta_j \\ &+ \langle\partial_{\tau_i}K_{jk}\rangle\delta\tau_i\partial_\mu\delta\theta_j\partial^\mu\delta\theta_k - \langle V\rangle - \frac{1}{2}\left\langle\frac{\partial^2 V}{\partial\tau_i\partial\tau_j}\right\rangle\delta\tau_i\delta\tau_j \\ &- \frac{1}{6}\left\langle\frac{\partial^3 V}{\partial\tau_i\partial\tau_j\partial\tau_k}\right\rangle\delta\tau_i\delta\tau_j\delta\tau_k - \frac{1}{2}\left\langle\frac{\partial^2 V}{\partial\theta_i\partial\theta_j}\right\rangle\delta\theta_i\delta\theta_j \\ &- \frac{1}{2}\left\langle\frac{\partial^3 V}{\partial\tau_i\partial\theta_j\partial\theta_k}\right\rangle\delta\tau_i\delta\theta_j\delta\theta_k \ .\end{aligned} \tag{A.38}$$

Note that $\theta_{\rm I}$ enters the potential only inside a cosine $\propto \cos(\mathfrak{a}_{\rm I}\theta_{\rm I})$. The global minimum of the potential lies at a locus where the cosine is at an extremum. Therefore, any odd-order derivative of V w.r.t. $\theta_{\rm I}$ will be $\partial^{2n+1}_{\theta_{\rm I}}V \propto \sin(\mathfrak{a}_{\rm I}\theta_{\rm I}) = 0$ at the extremum. Thus, any mixing between moduli and axions appears only at cubic order in the perturbed fields.

A.4.2. Decay into moduli fields

Diagonalisation of fields

The next step is to diagonalise and canonically normalise the system. Usually, one would need to find a transformation, that simultaneously diagonalises both the kinetic and mass matrix followed by a field rescaling in order to canonically normalise the kinetic term. Practically, this can be done by first diagonalising only the kinetic matrix $\langle K\rangle$, then rescaling the fields to render the kinetic terms canonical and afterwards rotating the fields by an orthogonal transformation, which retains the canonical form of the kinetic terms, so that the mass matrix diagonalises as well.

In this analysis, we follow another approach presented in [128], which yields a diagonalised, canonical system as well. The basic idea is to define the matrix $(M^2)_{ij} \equiv \langle (K^{-1})_{ik}\, V_{kj}\rangle/2$. One can show that the eigenvectors $\vec{v}_i$ of this matrix constitute a field transformation,

$$\begin{pmatrix} \delta\tau_{\rm b} \\ \delta\tau_{\rm I} \\ \delta\tau_{\rm L} \end{pmatrix} = \begin{pmatrix} \\ \vec{v}_{\rm b} \\ \\ \end{pmatrix} \frac{\delta\phi_{\rm b}}{\sqrt{2}} + \begin{pmatrix} \\ \vec{v}_{\rm I} \\ \\ \end{pmatrix} \frac{\delta\phi_{\rm I}}{\sqrt{2}} + \begin{pmatrix} \\ \vec{v}_{\rm L} \\ \\ \end{pmatrix} \frac{\delta\phi_{\rm L}}{\sqrt{2}} \,, \tag{A.39}$$

which diagonalises and canonically normalises the system after imposing the normalisation condition

$$\vec{v}_i^{\sf T} \cdot \langle K \rangle \cdot \vec{v}_j = \delta_{ij} \,. \tag{A.40}$$

Moreover, the eigenvalues of M^2 turn out to be the squared masses of the canonical fields.

Let us illustrate how this works in detail. We define P_{ij} as the matrix that contains the eigenvectors $\vec{v}_j$ as columns. The field transformations then read $\delta\tau_i = P_{ij}\delta\phi_j/\sqrt{2}$ and the normalisation condition (A.40) becomes

$$\vec{v}_i^{\sf T} \cdot \langle K \rangle \cdot \vec{v}_j \equiv P_{ki}\langle K_{kl}\rangle P_{lj} = \delta_{ij} \,. \tag{A.41}$$

Applying this transformation on the kinetic and mass terms, one has

$$\begin{aligned}
\mathcal{L} \supset\ & \langle K_{ij}\rangle \partial_\mu \delta\tau_i \partial^\mu \delta\tau_j - \frac{1}{2}\langle V_{ij}\rangle \delta\tau_i\delta\tau_j \\
&= \langle K_{ij}\rangle \partial_\mu \left(\frac{P_{ik}\delta\phi_k}{\sqrt{2}}\right) \partial^\mu \left(\frac{P_{jl}\delta\phi_l}{\sqrt{2}}\right) - \frac{1}{2}\langle V_{ij}\rangle \left(\frac{P_{ik}\delta\phi_k}{\sqrt{2}}\right)\left(\frac{P_{jl}\delta\phi_l}{\sqrt{2}}\right) \\
&= \frac{1}{2} P_{ik}\langle K_{ij}\rangle P_{jl} \partial_\mu\delta\phi_k\partial^\mu\delta\phi_l - \frac{1}{4}\langle V_{ij}\rangle P_{ik}P_{jl}\delta\phi_k\delta\phi_l \\
&= \frac{1}{2}\delta_{kl}\partial_\mu\delta\phi_k\partial^\mu\delta\phi_l - \frac{1}{4}\langle V_{ij}\rangle \delta_{im}P_{mk}P_{jl}\delta\phi_k\delta\phi_l \\
&= \frac{1}{2}\partial_\mu\delta\phi_i\partial^\mu\delta\phi_i - \frac{1}{4}\langle V_{ij}\rangle \left(K^{-1}\right)_{ni} K_{mn}P_{mk}P_{jl}\delta\phi_k\delta\phi_l \,,
\end{aligned} \tag{A.42}$$

where $V_{ij} \equiv \partial_{\tau_i}\partial_{\tau_j} V$. From (A.41), we know that $P_{ki}K_{kj} = \left(P^{-1}\right)_{ij}$ and, using the definition of $\left(M^2\right)_{ij}$, we obtain

$$\mathcal{L} \supset \frac{1}{2}\partial_\mu\delta\phi_i\partial^\mu\delta\phi_i - \frac{1}{2}\left(M^2\right)_{nj}\left(P^{-1}\right)_{kn} P_{jl}\delta\phi_k\delta\phi_l \,. \tag{A.43}$$

Since P contains the eigenvectors $\vec{v}_i$ of M^2 as columns, the above expression diagonalises:

$$\begin{aligned}\mathcal{L} \supset \frac{1}{2}\partial_\mu \delta\phi_i \partial^\mu \delta\phi_i - \frac{1}{2} m_k^2 \delta_{kl} \delta\phi_k \delta\phi_l \\ = \frac{1}{2}\partial_\mu \delta\phi_i \partial^\mu \delta\phi_i - \frac{1}{2} m_i^2 \delta\phi_i^2 \; .\end{aligned} \tag{A.44}$$

The diagonalisation and canonical normalisation of the axion fields works in principle the same way; however, one has to take into account that the second-derivative matrix of the potential looks different.

Let us apply this method to the above 3-moduli system. We first need to calculate the second-derivative matrix V_{ij}, which at leading order in the small parameter $\epsilon \equiv 1/\sqrt{\tau_\mathrm{b}}$ is given by

$$\langle V_{ij} \rangle \approx \begin{pmatrix} v_{11}\epsilon^{13} & v_{12}\epsilon^{11} & v_{13}\epsilon^{11} \\ v_{12}\epsilon^{11} & v_{22}\epsilon^{9} & v_{23}\epsilon^{12} \\ v_{13}\epsilon^{11} & v_{23}\epsilon^{12} & v_{33}\epsilon^{9} \end{pmatrix} , \tag{A.45}$$

where the v_{ij} are expressions which do not depend on τ_b and which are given by

$$v_{11} = \frac{9(11 W_0^2(\mu_1\tilde{\mu} + \mu_2\sqrt{\tau_\mathrm{L}}) + 3|W_0|^2\gamma_\mathrm{I}\tau_\mathrm{I}^{3/2}\sqrt{\tau_\mathrm{L}}\tilde{\mu}}{4\sqrt{\tau_\mathrm{L}}\tilde{\mu}} , \tag{A.46}$$

$$v_{12} = -\frac{9|W_0|^2\gamma_\mathrm{I}\mathfrak{a}_\mathrm{I}\tau_\mathrm{I}^{3/2}}{2} , \tag{A.47}$$

$$v_{13} = \frac{9W_0^2(\mu_1\tilde{\mu}^2 - \mu_2\tau_\mathrm{L})}{4\tilde{\mu}^2\tau_\mathrm{L}^{3/2}} , \tag{A.48}$$

$$v_{22} = 3|W_0|^2\gamma_\mathrm{I}\mathfrak{a}_\mathrm{I}^2\tau_\mathrm{I}^{3/2} , \tag{A.49}$$

$$v_{23} = \frac{9\gamma_\mathrm{I}\sqrt{\tau_\mathrm{I}}(W_0^2(\mu_2\tau_\mathrm{L} - \mu_1\tilde{\mu}^2) - 3|W_0|^2\gamma_\mathrm{L}\tau_\mathrm{L}^2\tilde{\mu}^2)}{4\tilde{\mu}^2\tau_\mathrm{L}^{3/2}} , \tag{A.50}$$

$$v_{33} = \frac{W_0^2(3\mu_1\tilde{\mu}^3 - \mu_2(\mu_3 - 3\sqrt{\tau_\mathrm{L}})\tau_\mathrm{L})}{4\tilde{\mu}^3\tau_\mathrm{L}^{5/2}\tau_\mathrm{b}^{9/2}} , \tag{A.51}$$

$$\tag{A.52}$$

with $\tilde{\mu} \equiv \mu_3 - \sqrt{\tau_\mathrm{L}}$. Here we used the relations (A.37) after applying the second derivatives.

Next we have to calculate the eigenvectors $\vec{v}_j$. The M^2 matrix at leading order is given by,

$$\left(M^2\right)_{ij} \approx \begin{pmatrix} m_{11}\epsilon^9 & m_{12}\epsilon^7 & m_{13}\epsilon^7 \\ m_{21}\epsilon^8 & m_{22}\epsilon^6 & m_{23}\epsilon^9 \\ m_{31}\epsilon^8 & m_{32}\epsilon^9 & m_{33}\epsilon^6 \end{pmatrix} , \tag{A.53}$$

where the m_{ij} are given by

$$m_{11} = \frac{3\left[-6|W_0|^2\tilde{\mu}^2\sqrt{\tau_{\rm L}}\gamma_{\rm I}\mathfrak{a}_{\rm I}\tau_{\rm I}^{5/2} + W_0^2\left(14\tilde{\mu}^2\mu_1 + 11\tilde{\mu}\mu_2\sqrt{\tau_{\rm L}} - 3\mu_2\tau_{\rm L}\right)\right]}{2\tilde{\mu}^2\sqrt{\tau_{\rm L}}} , \tag{A.54}$$

$$m_{12} = 6|W_0|^2\gamma_{\rm I}\mathfrak{a}_{\rm I}^2\tau_{\rm I}^{5/2} , \tag{A.55}$$

$$m_{13} = \frac{W_0^2\left(6\tilde{\mu}^3\mu_1 - 3\tilde{\mu}\mu_2\tau_{\rm L} + \mu_2\tau_{\rm L}\left(-\mu_3 + 3\sqrt{\tau_{\rm L}}\right)\right)}{2\tilde{\mu}^3\tau_{\rm L}^{3/2}} , \tag{A.56}$$

$$m_{21} = -6|W_0|^2\mathfrak{a}_{\rm I}\tau_{\rm I}^2 , \tag{A.57}$$

$$m_{22} = 4|W_0|^2\mathfrak{a}_{\rm I}^2\tau_{\rm I}^2 , \tag{A.58}$$

$$\begin{aligned} m_{23} = &-\frac{W_0^2\tau_{\rm I}\left[-12\tilde{\mu}^3\mu_1 + \mu_2\tau_{\rm L}(\mu_3 - 3\sqrt{\tau_{\rm L}}) + 3\tilde{\mu}(2\tilde{\mu}^2\mu_1 + \mu_2\tau_{\rm L})\right]}{2\tilde{\mu}^3\tau_{\rm L}^{3/2}} \\ &- 9|W_0|^2\gamma_{\rm L}\sqrt{\tau_{\rm L}}\tau_{\rm I} , \end{aligned} \tag{A.59}$$

$$m_{31} = \frac{3W_0^2\left(\tilde{\mu}^2\mu_1 - \mu_2\tau_{\rm L}\right)}{\gamma_{\rm L}\tilde{\mu}^2\tau_{\rm L}} , \tag{A.60}$$

$$m_{32} = \frac{3\gamma_{\rm I}\left[2|W_0|^2\gamma_{\rm L}\tau_{\rm L}^2\tilde{\mu}^2\mathfrak{a}_{\rm I}^2\tau_{\rm I}^{5/2} + W_0^2\sqrt{\tau_{\rm I}}\left(\mu_2\tau_{\rm L} - \mu_1\tilde{\mu}^2\right)\right]}{\gamma_{\rm L}\tilde{\mu}^2\tau_{\rm L}} , \tag{A.61}$$

$$m_{33} = \frac{W_0^2\left(3\tilde{\mu}^3\mu_1 + \mu_2\tau_{\rm L}\left(-\mu_3 + 3\sqrt{\tau_{\rm L}}\right)\right)}{3\gamma_{\rm L}\tilde{\mu}^3\tau_{\rm L}^2} . \tag{A.62}$$

The eigenvalues and eigenvectors of M^2 at leading order in ϵ are given by

$$m_{\tau_b}^2 = \frac{-m_{13}m_{22}m_{31} - m_{12}m_{21}m_{33} + m_{11}m_{22}m_{33}}{m_{22}m_{33}}\epsilon^9, \quad \vec{v}_1 = \begin{pmatrix} -\frac{m_{33}}{m_{31}}\epsilon^{-2} \\ \frac{m_{21}m_{33}}{m_{22}m_{31}} \\ 1 \end{pmatrix}, \tag{A.63}$$

$$m_{\tau_I}^2 = m_{22}\epsilon^6, \qquad \vec{v}_2 = \begin{pmatrix} \frac{m_{12}(m_{22}-m_{33})}{m_{12}m_{31}+m_{22}m_{32}}\epsilon^{-2} \\ \frac{m_{22}(m_{22}-m_{33})}{m_{12}m_{31}+m_{22}m_{32}}\epsilon^{-3} \\ 1 \end{pmatrix}, \tag{A.64}$$

$$m_{\tau_L}^2 = m_{33}\epsilon^6, \qquad \vec{v}_3 = \begin{pmatrix} \frac{m_{13}}{m_{33}}\epsilon \\ \frac{m_{13}m_{21}+m_{23}m_{33}}{m_{33}(-m_{22}+m_{33})}\epsilon^3 \\ 1 \end{pmatrix}. \tag{A.65}$$

Note that $m_{\tau_I}^2/m_{\tau_L}^2 = m_{22}/m_{33} \sim \mathfrak{a}_I^2\tau_I^2\tau_L^2 \gg 1$. To fulfill the normalisation conditions (A.41), we rescale the above eigenvectors,

$$\vec{v}_b \equiv \frac{\vec{v}_1}{\sqrt{\vec{v}_1^{\mathsf{T}} \cdot \langle K\rangle \cdot \vec{v}_1}} \approx -\frac{2m_{31}}{\sqrt{3}m_{33}}\vec{v}_1 = \begin{pmatrix} \frac{2\tau_b}{\sqrt{3}} \\ -\frac{2m_{21}}{\sqrt{3}m_{22}} \\ -\frac{2m_{31}}{\sqrt{3}m_{33}} \end{pmatrix}, \tag{A.66}$$

$$\vec{v}_I \equiv \frac{\vec{v}_2}{\sqrt{\vec{v}_2^{\mathsf{T}} \cdot \langle K\rangle \cdot \vec{v}_2}} \approx \frac{4(m_{12}m_{31}+m_{22}m_{32})\tau_I^{1/4}}{\sqrt{6\gamma_I}m_{22}(m_{22}-m_{33})\tau_b^{3/4}}\vec{v}_2 = \begin{pmatrix} \frac{4m_{12}\tau_b^{1/4}\tau_I^{1/4}}{\sqrt{6\gamma_I}m_{22}} \\ \frac{4\tau_b^{3/4}\tau_I^{1/4}}{\sqrt{6\gamma_I}} \\ \frac{4(m_{12}m_{31}+m_{22}m_{32})\tau_I^{1/4}}{\sqrt{6\gamma_I}m_{22}(m_{22}-m_{33})\tau_b^{3/4}} \end{pmatrix}, \tag{A.67}$$

$$\vec{v}_L \equiv \frac{\vec{v}_3}{\sqrt{\vec{v}_3^{\mathsf{T}} \cdot \langle K\rangle \cdot \vec{v}_3}} \approx \frac{4\tau_b^{3/4}\tau_L^{1/4}}{\sqrt{6\gamma_L}}\vec{v}_2 = \begin{pmatrix} \frac{4m_{13}\tau_b^{1/4}\tau_I^{1/4}}{\sqrt{6\gamma_L}m_{33}} \\ \frac{4(m_{13}m_{21}+m_{23}m_{33})\tau_L^{1/4}}{\sqrt{6\gamma_L}m_{33}(m_{33}-m_{22})\tau_b^{3/4}} \\ \frac{4\tau_b^{3/4}\tau_L^{1/4}}{\sqrt{6\gamma_L}} \end{pmatrix}, \tag{A.68}$$

where we used $m_{22} > m_{33}$ to specify some signs.

Coupling terms

The kinetic and potential trilinear coupling terms are respectively given by

$$\mathcal{L}_{\text{int,kin}} = K_{mnp}\delta\tau_m(\partial_\mu\delta\tau_n)(\partial^\mu\delta\tau_p), \tag{A.69}$$

$$\mathcal{L}_{\text{int,pot}} = -\frac{1}{6}V_{mnp}\delta\tau_m\delta\tau_n\delta\tau_p\,. \tag{A.70}$$

The third-order derivatives $K_{ijk} \equiv \langle\partial_{\tau_i}K_{jk}\rangle$ and $V_{ijk} \equiv \langle\partial_{\tau_i}\partial_{\tau_j}\partial_{\tau_k}V\rangle$ at leading order read

$$K_{\rm bbb} = -\frac{3}{2\tau_{\rm b}^3}\,, \tag{A.71}$$

$$K_{\rm bbI} = \frac{45\gamma_{\rm I}\sqrt{\tau_{\rm I}}}{16\tau_{\rm b}^{7/2}}\,, \tag{A.72}$$

$$K_{\rm bbL} = \frac{45\gamma_{\rm L}\sqrt{\tau_{\rm L}}}{16\tau_{\rm b}^{7/2}}\,, \tag{A.73}$$

$$K_{\rm bII} = -\frac{9\gamma_{\rm I}}{16\sqrt{\tau_{\rm I}}\tau_{\rm b}^{5/2}}\,, \tag{A.74}$$

$$K_{\rm bIL} = -\frac{27\gamma_{\rm I}\gamma_{\rm L}\sqrt{\tau_{\rm I}\tau_{\rm L}}}{8\tau_{\rm b}^4}\,, \tag{A.75}$$

$$K_{\rm bLL} = -\frac{9\gamma_{\rm L}}{16\sqrt{\tau_{\rm L}}\tau_{\rm b}^{5/2}}\,, \tag{A.76}$$

$$K_{\rm III} = -\frac{3\gamma_{\rm I}}{16\tau_{\rm I}^{3/2}\tau_{\rm b}^{3/2}}\,, \tag{A.77}$$

$$K_{\rm IIL} = \frac{9\gamma_{\rm I}\gamma_{\rm L}\sqrt{\tau_{\rm L}}}{16\sqrt{\tau_{\rm I}}\tau_{\rm b}^3}\,, \tag{A.78}$$

$$K_{\rm ILL} = \frac{9\gamma_{\rm I}\gamma_{\rm L}\sqrt{\tau_{\rm I}}}{16\sqrt{\tau_{\rm L}}\tau_{\rm b}^3}\,, \tag{A.79}$$

$$K_{\rm LLL} = -\frac{3\gamma_{\rm L}}{16\tau_{\rm L}^{3/2}\tau_{\rm b}^{3/2}}\,, \tag{A.80}$$

$$V_{\rm bbb} = -\frac{9\left[143W_0^2\left(\mu_1\tilde{\mu}+\mu_2\sqrt{\tau_{\rm L}}\right)+72|W_0|^2\gamma_{\rm I}\tilde{\mu}\tau_{\rm I}^{3/2}\sqrt{\tau_{\rm L}}\right]}{8\tilde{\mu}\sqrt{\tau_{\rm L}}\tau_{\rm b}^{15/2}}\,, \tag{A.81}$$

$$V_{\rm bbI} = \frac{99\gamma_{\rm I}|W_0|^2\mathfrak{a}_{\rm I}\tau_{\rm I}^{3/2}}{4\tau_{\rm b}^{13/2}}\,, \tag{A.82}$$

$$V_{\rm bbL} = -\frac{99W_0^2\left(\mu_1\tilde{\mu}^2-\mu_2\tau_{\rm L}\right)}{8\tilde{\mu}^2\tau_{\rm L}^{3/2}\tau_{\rm b}^{13/2}}\,, \tag{A.83}$$

$$V_{\rm bII} = -\frac{27\gamma_{\rm I}|W_0|^2\mathfrak{a}_{\rm I}\sqrt{\tau_{\rm I}}}{2\tau_{\rm b}^{11/2}}\,, \tag{A.84}$$

$$V_{\rm bIL} = -\frac{27\gamma_{\rm I}\left[2|W_0|^2\gamma_{\rm L}\tilde{\mu}^2\mathfrak{a}_{\rm I}\tau_{\rm I}^{3/2}\tau_{\rm L}^2+2W_0^2\sqrt{\tau_{\rm I}}\left(\mu_2\tau_{\rm L}-\mu_1\tilde{\mu}^2\right)\right]}{4\tilde{\mu}^2\tau_{\rm L}^{3/2}\tau_{\rm b}^7}\,, \tag{A.85}$$

$$V_{\rm bLL} = -\frac{9W_0^2\left[3\mu_1\tilde{\mu}^3+\mu_2(-\mu_3+3\sqrt{\tau_{\rm L}})\tau_{\rm L}\right]}{8\tilde{\mu}^3\tau_{\rm L}^{5/2}\tau_{\rm b}^{11/2}}\,, \tag{A.86}$$

$$V_{\rm III} = -\frac{9\gamma_{\rm I}|W_0|^2\mathfrak{a}_{\rm I}^3\tau_{\rm I}^{3/2}}{\tau_{\rm b}^{9/2}}\,, \tag{A.87}$$

$$V_{\rm IIL} = \frac{9\gamma_{\rm I}\left[8|W_0|^2\gamma_{\rm L}\tilde{\mu}^2\mathfrak{a}_{\rm I}^2\tau_{\rm I}^2\tau_{\rm L}^2-W_0^2(\mu_1\tilde{\mu}^2-\mu_2\tau_{\rm L})\right]}{8\tilde{\mu}^2\sqrt{\tau_{\rm I}}\tau_{\rm L}^{3/2}\tau_{\rm b}^6}\,, \tag{A.88}$$

$$V_{\rm ILL} = \frac{9\gamma_{\rm I}\left[-3|W_0|^2\gamma_{\rm L}\tilde{\mu}^3\sqrt{\tau_{\rm I}}\tau_{\rm L}^2+W_0^2\left(3\mu_1\sqrt{\tau_{\rm I}}\tilde{\mu}^3+\mu_2\sqrt{\tau_{\rm I}}\tau_{\rm L}(-\mu_3+3\sqrt{\tau_{\rm L}})\right)\right]}{8\tilde{\mu}^3\tau_{\rm L}^{5/2}\tau_{\rm b}^6}\,, \tag{A.89}$$

$$V_{\rm LLL} = -\frac{3W_0^2\left[5\mu_1\tilde{\mu}^4-\mu_2\tau_{\rm L}\left(\mu_3^2-4\mu_3\sqrt{\tau_{\rm L}}+5\tau_{\rm L}\right)\right]}{8\tilde{\mu}^4\tau_{\rm L}^{7/2}\tau_{\rm b}^{9/2}}\,. \tag{A.90}$$

Here we have again used the relations (A.37), however, this time only after forming the third derivatives.

To obtain the kinetic couplings, we can insert the canonical fields $\delta\tau_i = P_{ij}\delta\phi_j/\sqrt{2}$ into (A.69),

$$\mathcal{L}_{\rm int,kin} = \frac{1}{2^{3/2}}K_{mnp}P_{mi}P_{nj}P_{pk}\delta\phi_i(\partial_\mu\delta\phi_j)(\partial^\mu\delta\phi_k). \tag{A.91}$$

To eliminate the derivatives, let us consider a trilinear kinetic coupling of three arbitrary fields φ_1, φ_2 and φ_3. Then by performing several integrations by parts, one obtains

$$\begin{aligned}\varphi_1\partial_\mu\varphi_2\partial^\mu\varphi_3 &= -\varphi_3(\partial^\mu\varphi_1\partial_\mu\varphi_2 + \varphi_1\Box\varphi_2) + \text{b.t.}\\ &= \varphi_2(\varphi_3\Box\varphi_1 + \partial_\mu\varphi_3\partial^\mu\varphi_1) - \varphi_1\varphi_3\Box\varphi_2 + \text{b.t.}\\ &= \varphi_2\varphi_3\Box\varphi_1 - \varphi_1(\varphi_2\Box\varphi_3 + \partial_\mu\varphi_3\partial^\mu\varphi_2) - \varphi_1\varphi_3\Box\varphi_2 + \text{b.t.}\ ,\end{aligned} \tag{A.92}$$

where $\Box \equiv \partial^\mu\partial_\mu$ and 'b.t.' stands for 'boundary terms'. From the last line, we than obtain

$$\varphi_1\partial_\mu\varphi_2\partial^\mu\varphi_3 = \frac{1}{2}(\varphi_2\varphi_3\Box\varphi_1 - \varphi_1\varphi_2\Box\varphi_3 - \varphi_1\varphi_3\Box\varphi_2) + \text{b.t.}\ . \tag{A.93}$$

As long as the φ_i are close to their respective VEVs, i.e. for small perturbations, we can then use the free Klein-Gordon equation to replace the box operators by the respective masses, thus arriving at

$$\varphi_1\partial_\mu\varphi_2\partial^\mu\varphi_3 = \frac{1}{2}\varphi_1\varphi_2\varphi_3\left(m^2_{\varphi_1} - m^2_{\varphi_2} - m^2_{\varphi_3}\right)\ . \tag{A.94}$$

Eliminating the derivatives in Eq. (A.91) via the relation (A.94), we obtain

$$\mathcal{L}_{\text{int,kin}} = \frac{1}{2^{5/2}}K_{mnp}P_{mi}P_{nj}P_{pk}\left(m_i^2 - m_j^2 - m_k^2\right)\delta\phi_i\delta\phi_j\delta\phi_k. \tag{A.95}$$

For the potential couplings, after inserting the canonical fields into (A.70), we have

$$\mathcal{L}_{\text{int,pot}} = -\frac{1}{12\sqrt{2}}V_{mnp}P_{mi}P_{nj}P_{pk}\delta\phi_i\delta\phi_j\delta\phi_k. \tag{A.96}$$

We can now calculate the individual coupling terms:

- Decay $\delta\phi_{\rm I} \to \delta\phi_{\rm b}\delta\phi_{\rm b}$:
  Relevant are those terms in $\mathcal{L}_{\text{int,kin}}$ for which one of the three indices i, j, k is an 'I' while the other two are a 'b'. As in the previous section, from (A.95), we see that all factors in $\mathcal{L}_{\text{int,kin}}$ are invariant under permutation of these indices except for the factor $(m_i^2 - m_j^2 - m_k^2)$. Again, $m^2_{\tau_{\rm I}} \gg m^2_{\tau_{\rm b}}$, and hence this factor is dominated by $m^2_{\tau_{\rm I}}$. Therefore, it only changes by a minus sign under permutation of i, j and k, depending on which of the three indices takes on the value 'I'. Summing up the three terms, two of which have a minus sign, we obtain,

$$\mathcal{L}^{(\phi_{\rm I}\to\phi_{\rm b}\phi_{\rm b})}_{\text{int,kin}} = -\frac{1}{2^{5/2}}K_{mnp}P_{m\rm I}P_{n\rm b}P_{p\rm b}m^2_{\tau_{\rm I}}\delta\phi_{\rm I}\delta\phi_{\rm b}\delta\phi_{\rm b}. \tag{A.97}$$

The contraction is given by

$$K_{mnp}P_{m\mathrm{I}}P_{n\mathrm{b}}P_{p\mathrm{b}} \approx K_{\mathrm{bbb}}P_{\mathrm{bI}}P_{\mathrm{bb}}P_{\mathrm{bb}} + P_{\mathrm{Ibb}}P_{\mathrm{II}}P_{\mathrm{bb}}P_{\mathrm{bb}} \tag{A.98}$$

$$\approx \frac{\sqrt{6\gamma_{\mathrm{I}}}\tau_{\mathrm{I}}^{3/4}}{2\tau_{\mathrm{b}}^{3/4}} \,. \tag{A.99}$$

Inserting this and (A.64) into (A.97), we obtain

$$\mathcal{L}_{\text{int,kin}}^{(\phi_{\mathrm{I}}\to\phi_{\mathrm{b}}\phi_{\mathrm{b}})} \approx -\frac{\sqrt{3\gamma_{\mathrm{I}}}|W_0|^2\mathfrak{a}_{\mathrm{I}}^2\tau_{\mathrm{I}}^{11/4}}{2\tau_{\mathrm{b}}^{15/4}}\delta\phi_{\mathrm{I}}\delta\phi_{\mathrm{b}}\delta\phi_{\mathrm{b}}. \tag{A.100}$$

For the potential coupling, only those terms from (A.96) contribute where one of the indices i, j, k takes on the value 'I' while the other two take on the value 'b'. Following the same argument as before there are in total three such terms that are all equal and can be accounted for by a factor of 3,

$$\mathcal{L}_{\text{int,pot}}^{(\phi_{\mathrm{I}}\to\phi_{\mathrm{b}}\phi_{\mathrm{b}})} = -\frac{1}{4\sqrt{2}}V_{mnp}P_{m\mathrm{I}}P_{n\mathrm{b}}P_{p\mathrm{b}}\delta\phi_{\mathrm{I}}\delta\phi_{\mathrm{b}}\delta\phi_{\mathrm{b}}. \tag{A.101}$$

The contraction reads

$$V_{mnp}P_{m\mathrm{I}}P_{n\mathrm{b}}P_{p\mathrm{b}} \approx V_{\mathrm{Ibb}}P_{\mathrm{II}}P_{\mathrm{bb}}P_{\mathrm{bb}} + V_{\mathrm{III}}P_{\mathrm{II}}P_{\mathrm{Ib}}P_{\mathrm{Ib}} \tag{A.102}$$

$$= \frac{4\sqrt{6\gamma_{\mathrm{I}}}|W_0|^2\mathfrak{a}_{\mathrm{I}}\tau_{\mathrm{I}}^{7/4}}{\tau_{\mathrm{b}}^{15/4}} \,. \tag{A.103}$$

Inserting this into (A.101), we arrive at

$$\mathcal{L}_{\text{int,pot}}^{(\phi_{\mathrm{I}}\to\phi_{\mathrm{b}}\phi_{\mathrm{b}})} = -\frac{\sqrt{3\gamma_{\mathrm{I}}}|W_0|^2\mathfrak{a}_{\mathrm{I}}\tau_{\mathrm{I}}^{7/4}}{\tau_{\mathrm{b}}^{15/4}}\delta\phi_{\mathrm{I}}\delta\phi_{\mathrm{b}}\delta\phi_{\mathrm{b}} \,. \tag{A.104}$$

From this we conclude

$$\frac{\mathcal{L}_{\text{int,kin}}^{(\phi_{\mathrm{I}}\to\phi_{\mathrm{b}}\phi_{\mathrm{b}})}}{\mathcal{L}_{\text{int,pot}}^{(\phi_{\mathrm{I}}\to\phi_{\mathrm{b}}\phi_{\mathrm{b}})}} \approx \frac{\mathfrak{a}_{\mathrm{I}}\tau_{\mathrm{I}}}{2} \gg 1. \tag{A.105}$$

- Decay $\delta\phi_{\mathrm{I}} \to \delta\phi_{\mathrm{L}}\delta\phi_{\mathrm{L}}$:
  Analogously to the decay $\delta\phi_{\mathrm{I}} \to \delta\phi_{\mathrm{b}}\delta\phi_{\mathrm{b}}$, now those terms from $\mathcal{L}_{\text{int,kin}}$ contribute for which one of the three indices i, j, k is an 'I' while the other two are an 'L'. With $m_{\tau_{\mathrm{I}}}^2 \gg m_{\tau_{\mathrm{L}}}^2$, the factor $(m_i^2 - m_j^2 - m_k^2)$ is again dominated

by $m_{\tau_{\mathrm{I}}}^2$. Thus we arrive at

$$\mathcal{L}_{\text{int,kin}}^{(\phi_{\mathrm{I}}\to\phi_{\mathrm{L}}\phi_{\mathrm{L}})} = -\frac{1}{2^{5/2}} K_{mnp} P_{m\mathrm{I}} P_{n\mathrm{L}} P_{p\mathrm{L}} m_{\tau_{\mathrm{I}}}^2 \delta\phi_{\mathrm{I}} \delta\phi_{\mathrm{L}} \delta\phi_{\mathrm{L}}. \tag{A.106}$$

The contraction is given by

$$K_{mnp} P_{m\mathrm{I}} P_{n\mathrm{L}} P_{p\mathrm{L}} \approx K_{\mathrm{bLL}} P_{\mathrm{bI}} P_{\mathrm{LL}} P_{\mathrm{LL}} + K_{\mathrm{ILL}} P_{\mathrm{II}} P_{\mathrm{LL}} P_{\mathrm{LL}} + K_{\mathrm{LLL}} P_{\mathrm{LI}} P_{\mathrm{LL}} P_{\mathrm{LL}} \tag{A.107}$$

$$\approx -\frac{\sqrt{6\gamma_{\mathrm{I}}}\tau_{\mathrm{I}}^{3/4}}{\tau_{\mathrm{b}}^{3/4}}, \tag{A.108}$$

where we used that $\mathfrak{a}_{\mathrm{I}}\tau_{\mathrm{I}}, \tau_{\mathrm{L}} \gg 1$ and assumed that there is no fine-tuning of the parameter $\tilde{\mu} = \mu_3 - \sqrt{\tau_{\mathrm{L}}}$. Inserting this and (A.64) into (A.106), we obtain

$$\mathcal{L}_{\text{int,kin}}^{(\phi_{\mathrm{I}}\to\phi_{\mathrm{L}}\phi_{\mathrm{L}})} \approx \frac{\sqrt{3\gamma_{\mathrm{I}}}|W_0|^2 \mathfrak{a}_{\mathrm{I}}^2 \tau_{\mathrm{I}}^{11/4}}{\tau_{\mathrm{b}}^{15/4}} \delta\phi_{\mathrm{I}} \delta\phi_{\mathrm{L}} \delta\phi_{\mathrm{L}}. \tag{A.109}$$

For the potential coupling, we obtain analogously to (A.101),

$$\mathcal{L}_{\text{int,pot}}^{(\phi_{\mathrm{I}}\to\phi_{\mathrm{L}}\phi_{\mathrm{L}})} = -\frac{1}{4\sqrt{2}} V_{mnp} P_{m\mathrm{I}} P_{n\mathrm{b}} P_{p\mathrm{b}} \delta\phi_{\mathrm{I}} \delta\phi_{\mathrm{L}} \delta\phi_{\mathrm{L}}. \tag{A.110}$$

The contraction reads

$$\begin{aligned} V_{mnp} P_{m\mathrm{I}} P_{n\mathrm{L}} P_{p\mathrm{L}} \approx & \frac{2\sqrt{6\gamma_{\mathrm{I}}}\tau_{\mathrm{I}}^{3/4} W_0^2 \left[-4\mu_1\tilde{\mu}^4 + \mu_2(\mu_3^2 - 4\mu_3\sqrt{\tau_{\mathrm{L}}} + 4\tau_{\mathrm{L}})\tau_{\mathrm{L}}\right]}{\gamma_{\mathrm{L}}\tilde{\mu}^4\tau_{\mathrm{L}}^2\tau_{\mathrm{b}}^{15/4}} \\ & - \frac{6\sqrt{6\gamma_{\mathrm{I}}}|W_0|^2\tau_{\mathrm{I}}^{3/4}}{\tau_{\mathrm{b}}^{15/4}} \end{aligned} \tag{A.111}$$

The potential coupling is then given by

$$\begin{aligned} &\mathcal{L}_{\text{int,pot}}^{(\phi_{\mathrm{I}}\to\phi_{\mathrm{L}}\phi_{\mathrm{L}})} = \\ &- \frac{\sqrt{3\gamma_{\mathrm{I}}}\tau_{\mathrm{I}}^{3/4} \left[-3|W_0|^2\gamma_{\mathrm{L}}\tilde{\mu}^4\tau_{\mathrm{L}}^2 + W_0^2\left(-4\mu_1\tilde{\mu}^4 + \mu_2(\mu_3^2 - 4\mu_3\sqrt{\tau_{\mathrm{L}}} + 4\tau_{\mathrm{L}})\tau_{\mathrm{L}}\right)\right]}{2\gamma_{\mathrm{L}}\tilde{\mu}^4\tau_{\mathrm{L}}^2\tau_{\mathrm{b}}^{15/4}} \\ &\times \delta\phi_{\mathrm{I}} \delta\phi_{\mathrm{L}} \delta\phi_{\mathrm{L}}. \end{aligned} \tag{A.112}$$

Note that the term $\sim |W_0|^2$, which stems from V_{LVS}, is larger than the term $\sim W_0^2$, which stems from V_{loop}, by a factor $\sim \tau_{\mathrm{L}}^2$. This confirms the correctness of our estimation (5.90). Again, the kinetic decay dominates the potential one,

$$\frac{\mathcal{L}_{\text{int,kin}}^{(\phi_\text{I}\to\phi_\text{L}\phi_\text{L})}}{\mathcal{L}_{\text{int,pot}}^{(\phi_\text{I}\to\phi_\text{L}\phi_\text{L})}} \sim \mathfrak{a}_\text{I}^2\tau_\text{I}^2 \gg 1. \tag{A.113}$$

Furthermore, we see that the potential couplings into the volume modulus and loop modulus differ by a factor

$$\frac{\mathcal{L}_{\text{int,pot}}^{(\phi_\text{I}\to\phi_\text{b}\phi_\text{b})}}{\mathcal{L}_{\text{int,pot}}^{(\phi_\text{I}\to\phi_\text{L}\phi_\text{L})}} \sim \mathfrak{a}_\text{I}\tau_\text{I} \gg 1. \tag{A.114}$$

- Decay $\delta\phi_\text{I} \to \delta\phi_\text{b}\delta\phi_\text{L}$:

  For this decay, the relevant terms are those with the indices $i = \text{I}$, $j = \text{b}$ and $k = \text{L}$ as well as all permutations thereof. In total, there are $3! = 6$ permutations, which have all the same absolute value but with four of them coming with a minus sign compared to the other two. Thus, w.l.o.g. we fix $i = \text{I}$, $j = \text{b}$ and $k = \text{L}$ and assign a factor $2 - 4 = -2$,

  $$\mathcal{L}_{\text{int,kin}}^{(\phi_\text{I}\to\phi_\text{b}\phi_\text{L})} = -\frac{1}{2^{3/2}} K_{mnp} P_{m\text{I}} P_{n\text{b}} P_{p\text{L}} m_{\tau_\text{I}}^2 \delta\phi_\text{I}\delta\phi_\text{b}\delta\phi_\text{L}. \tag{A.115}$$

  The contraction scales as

  $$K_{mnp} P_{m\text{I}} P_{n\text{b}} P_{p\text{L}} \sim \tau_\text{b}^{-3/2}, \tag{A.116}$$

  so that the total coupling term scales like

  $$\mathcal{L}_{\text{int,kin}}^{(\phi_\text{I}\to\phi_\text{b}\phi_\text{L})} \sim \tau_\text{b}^{-9/2} \delta\phi_\text{I}\delta\phi_\text{b}\delta\phi_\text{L}. \tag{A.117}$$

  This, the kinetic decay $\delta\phi_\text{I} \to \delta\phi_\text{b}\delta\phi_\text{L}$ is suppressed compared to the kinetic decays $\delta\phi_\text{I} \to \delta\phi_\text{b}\delta\phi_\text{b}$ and $\delta\phi_\text{I} \to \delta\phi_\text{L}\delta\phi_\text{L}$.

  For the potential coupling, all 6 permutations of $i = \text{I}$, $j = \text{b}$ and $k = \text{L}$ are the same so that we obtain:

  $$\mathcal{L}_{\text{int,pot}}^{(\phi_\text{I}\to\phi_\text{b}\phi_\text{L})} = -\frac{1}{2\sqrt{2}} V_{mnp} P_{m\text{I}} P_{n\text{b}} P_{p\text{L}} \delta\phi_\text{I}\delta\phi_\text{b}\delta\phi_\text{L}. \tag{A.118}$$

  Calculating the contractions, it turns out that we have

  $$\mathcal{L}_{\text{int,pot}}^{(\phi_\text{I}\to\phi_\text{b}\phi_\text{L})} \sim \tau_\text{b}^{-9/2} \delta\phi_\text{I}\delta\phi_\text{b}\delta\phi_\text{L}, \tag{A.119}$$

  which is also suppressed compared to the potential decays $\delta\phi_\text{I} \to \delta\phi_\text{b}\delta\phi_\text{b}$ and $\delta\phi_\text{I} \to \delta\phi_\text{L}\delta\phi_\text{L}$.

Note that the inclusion of the other small cycles $\tau_{s,i}$ does not alter the results for the couplings to moduli fields because it only changes the expression for ξ in (A.37), which then becomes a sum over all small cycles including $\tau_{\rm I}$. However, ξ appears only in the components $V_{\rm bb}$, $(M^2)_{11}$ and $V_{\rm bbb}$ at leading order. Even though this induces a slight shift of the volume modulus mass (A.63), none of these three components enters the trilinear coupling terms and hence they remain unaltered.

A.4.3. Decay into axion fields

Diagonalisation of fields

For the decay into the volume axion, we proceed analogously as for the decay into volume modulus. The second-derivative matrix w.r.t. the axions at leading order is given by

$$\langle V_{ij}^{(\theta)} \rangle \equiv \left\langle \frac{\partial^2 V}{\partial\theta_i \partial\theta_j} \right\rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{3\gamma_{\rm I}|W_0|^2 \mathfrak{a}_{\rm I}^2 \tau_{\rm I}^{3/2}}{\tau_{\rm b}^{9/2}} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{A.120}$$

where we have, again, used the relations (A.37) after applying the second derivatives. The transformation to canonical fields is given by

$$\begin{pmatrix} \delta\theta_{\rm b} \\ \delta\theta_{\rm I} \\ \delta\theta_{\rm L} \end{pmatrix} = \begin{pmatrix} \\ \vec{w}_{\rm b} \\ \\ \end{pmatrix} \frac{\delta a_{\rm b}}{\sqrt{2}} + \begin{pmatrix} \\ \vec{w}_{\rm I} \\ \\ \end{pmatrix} \frac{\delta a_{\rm I}}{\sqrt{2}} + \begin{pmatrix} \\ \vec{w}_{\rm L} \\ \\ \end{pmatrix} \frac{\delta a_{\rm L}}{\sqrt{2}} \tag{A.121}$$

or $\delta\theta_i = Q_{ij}\delta a_j/\sqrt{2}$ where Q is the matrix that contains the vectors $\vec{w}_j$ as columns. They are the eigenvectors of the matrix $(M^2_{(\theta)})_{ij} \equiv \langle (K^{-1})_{ik} V_{kj}^{(\theta)} \rangle /2$ whose eigenvalues are the axion masses. The eigenvectors fulfill the normalisation condition

$$\vec{w}_i^{\mathsf{T}} \cdot \langle K \rangle \cdot \vec{w}_j \equiv Q_{ki} \langle K_{kl} \rangle Q_{lj} = \delta_{ij}. \tag{A.122}$$

The $M^2_{(\theta)}$ matrix at leading order is given by

$$(M^2_{(\theta)})_{ij} \approx \begin{pmatrix} 0 & \frac{6\gamma_{\rm I}|W_0|^2 \mathfrak{a}_{\rm I}^2 \tau_{\rm I}^{5/2}}{\tau_{\rm b}^{7/2}} & 0 \\ 0 & \frac{4|W_0|^2 \mathfrak{a}_{\rm I}^2 \tau_{\rm I}^2}{\tau_{\rm b}^3} & 0 \\ 0 & \frac{6\gamma_{\rm I}|W_0|^2 \mathfrak{a}_{\rm I}^2 \tau_{\rm I}^{5/2}\tau_{\rm L}}{\tau_{\rm b}^{9/2}} & 0 \end{pmatrix}. \tag{A.123}$$

The corresponding eigenvalues and eigenvectors are

$$m^2_{\theta_\mathrm{b}} = 0, \qquad \vec{w}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \tag{A.124}$$

$$m^2_{\theta_\mathrm{I}} = \frac{4|W_0|^2 \mathfrak{a}_\mathrm{I}^2 \tau_\mathrm{I}^2}{\tau_\mathrm{b}^3}, \qquad \vec{w}_2 = \begin{pmatrix} \tau_\mathrm{b}/\tau_\mathrm{L} \\ \frac{2\tau_\mathrm{b}^{3/2}}{3\gamma_\mathrm{I}\sqrt{\tau_\mathrm{I}}\tau_\mathrm{L}} \\ 1 \end{pmatrix}, \tag{A.125}$$

$$m^2_{\theta_\mathrm{L}} = 0, \qquad \vec{w}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}. \tag{A.126}$$

After rescaling to fulfill the normalisation condition (A.122), the normalised eigenvectors read

$$\vec{w}_\mathrm{b} \equiv \frac{\vec{w}_1}{\sqrt{\vec{w}_1^\mathsf{T} \cdot \langle K \rangle \cdot \vec{w}_1}} \approx \frac{2\tau_\mathrm{b}}{\sqrt{3}} \vec{w}_1 = \begin{pmatrix} \frac{2\tau_\mathrm{b}}{\sqrt{3}} \\ 0 \\ 0 \end{pmatrix}, \tag{A.127}$$

$$\vec{w}_\mathrm{I} \equiv \frac{\vec{w}_2}{\sqrt{\vec{w}_2^\mathsf{T} \cdot \langle K \rangle \cdot \vec{w}_2}} \approx \frac{\sqrt{6\gamma_\mathrm{I}}\tau_\mathrm{I}^{3/4}\tau_\mathrm{L}}{\tau_\mathrm{b}^{3/4}} \vec{w}_2 = \begin{pmatrix} \sqrt{6\gamma_\mathrm{I}}\tau_\mathrm{I}^{3/4}\tau_\mathrm{b}^{1/4} \\ \frac{2\sqrt{2}\tau_\mathrm{I}^{1/4}\tau_\mathrm{b}^{3/4}}{\sqrt{3\gamma_\mathrm{I}}} \\ \frac{\sqrt{6\gamma_\mathrm{I}}\tau_\mathrm{I}^{3/4}\tau_\mathrm{L}}{\tau_\mathrm{b}^{3/4}} \end{pmatrix}, \tag{A.128}$$

$$\vec{w}_\mathrm{L} \equiv \frac{\vec{w}_3}{\sqrt{\vec{w}_3^\mathsf{T} \cdot \langle K \rangle \cdot \vec{w}_3}} \approx \frac{2\sqrt{2}\tau_\mathrm{L}^{1/4}\tau_\mathrm{b}^{3/4}}{\sqrt{3\gamma_\mathrm{L}}} \vec{w}_3 = \begin{pmatrix} 0 \\ 0 \\ \frac{2\sqrt{2}\tau_\mathrm{L}^{1/4}\tau_\mathrm{b}^{3/4}}{\sqrt{3\gamma_\mathrm{L}}} \end{pmatrix}. \tag{A.129}$$

Coupling terms

The kinetic and potential trilinear coupling terms are respectively given by

$$\begin{aligned}
\mathcal{L}_{\text{int,kin},(\theta)} &= \langle \partial_{\tau_m} K_{np} \rangle \delta\tau_m \partial_\mu \delta\theta_n \partial^\mu \delta\theta_p \\
&= \frac{1}{2^{3/2}} K_{mnp} P_{mi} Q_{nj} Q_{pk} \delta\phi_i \partial_\mu \delta a_j \partial^\mu \delta a_k, \qquad \text{(A.130)} \\
\mathcal{L}_{\text{int,pot},(\theta)} &= -\frac{1}{2} \left\langle \frac{\partial^3 V}{\partial\tau_m \partial\theta_n \partial\theta_p} \right\rangle \delta\tau_m \delta\theta_n \delta\theta_p \\
&= -\frac{1}{2^{5/2}} \left\langle \frac{\partial^3 V}{\partial\tau_m \partial\theta_n \partial\theta_p} \right\rangle P_{mi} Q_{nj} Q_{pk} \delta\phi_i \delta a_j \delta a_k \,. \qquad \text{(A.131)}
\end{aligned}$$

Let us first argue that the potential couplings to the volume and loop axions vanish: Since V does not depend on θ_{b} or θ_{L} but only on θ_{I}, the indices n and p in (A.131) must both take on the value 'I'. However, the components Q_{Ib} and Q_{IL} vanish, so that there are no potential couplings $\sim \delta\phi_{\mathrm{I}} \delta a_{\mathrm{b}} \delta a_{\mathrm{b}}$ or $\sim \delta\phi_{\mathrm{I}} \delta a_{\mathrm{L}} \delta a_{\mathrm{L}}$.

The individual coupling terms are then calculated as:

- Decay $\delta\phi_{\mathrm{I}} \to \delta a_{\mathrm{b}} \delta a_{\mathrm{b}}$:
Eliminating the derivatives by using (A.94), the kinetic coupling term becomes

$$\mathcal{L}_{\text{int,kin},(\theta)}^{(\phi_{\mathrm{I}} \to a_{\mathrm{b}} a_{\mathrm{b}})} = \frac{1}{2^{5/2}} K_{mnp} P_{m\mathrm{I}} Q_{n\mathrm{b}} Q_{p\mathrm{b}} m_{\tau_{\mathrm{I}}}^2 \delta\phi_{\mathrm{I}} \delta a_{\mathrm{b}} \delta a_{\mathrm{b}}. \qquad \text{(A.132)}$$

Since $Q_{\mathrm{Ib}} = Q_{\mathrm{Lb}} = 0$, the indices n and p must take on the value b, so that we have

$$\begin{aligned}
\mathcal{L}_{\text{int,kin},(\theta)}^{(\phi_{\mathrm{I}} \to a_{\mathrm{b}} a_{\mathrm{b}})} &= \frac{1}{2^{5/2}} K_{mbb} P_{m\mathrm{I}} Q_{\mathrm{bb}} Q_{\mathrm{bb}} m_{\tau_{\mathrm{I}}}^2 \delta\phi_{\mathrm{I}} \delta a_{\mathrm{b}} \delta a_{\mathrm{b}} \qquad \text{(A.133)} \\
&= \frac{1}{2^{5/2}} (K_{\mathrm{bbb}} P_{\mathrm{bI}} + K_{\mathrm{Ibb}} P_{\mathrm{II}} + K_{\mathrm{Lbb}} P_{\mathrm{LI}}) Q_{\mathrm{bb}} Q_{\mathrm{bb}} m_{\tau_{\mathrm{I}}}^2 \delta\phi_{\mathrm{I}} \delta a_{\mathrm{b}} \delta a_{\mathrm{b}} \\
&\qquad \text{(A.134)} \\
&\approx \frac{\sqrt{3\gamma_{\mathrm{I}}} |W_0|^2 \mathfrak{a}_{\mathrm{I}}^2 \tau_{\mathrm{I}}^{11/4}}{2\tau_{\mathrm{b}}^{15/4}} \delta\phi_{\mathrm{I}} \delta a_{\mathrm{b}} \delta a_{\mathrm{b}}. \qquad \text{(A.135)}
\end{aligned}$$

- Decay $\delta\phi_{\mathrm{I}} \to \delta a_{\mathrm{L}} \delta a_{\mathrm{L}}$:
Analogously to before we have

$$\mathcal{L}_{\text{int,kin},(\theta)}^{(\phi_{\mathrm{I}} \to a_{\mathrm{L}} a_{\mathrm{L}})} = \frac{1}{2^{5/2}} K_{mnp} P_{m\mathrm{I}} Q_{n\mathrm{L}} Q_{p\mathrm{L}} m_{\tau_{\mathrm{I}}}^2 \delta\phi_{\mathrm{I}} \delta a_{\mathrm{L}} \delta a_{\mathrm{L}}. \qquad \text{(A.136)}$$

Since $Q_{\mathrm{bL}} = Q_{\mathrm{IL}} = 0$, the indices n and p must take on the value 'L', so that we have

$$\mathcal{L}_{\text{int,kin},(\theta)}^{(\phi_{\mathrm{I}}\to a_{\mathrm{L}}a_{\mathrm{L}})} = \frac{1}{2^{5/2}} K_{m\mathrm{LL}} P_{m\mathrm{I}} Q_{\mathrm{LL}} Q_{\mathrm{LL}} m_{\tau_{\mathrm{I}}}^2 \delta\phi_{\mathrm{I}} \delta a_{\mathrm{L}} \delta a_{\mathrm{L}} \tag{A.137}$$

$$= \frac{1}{2^{5/2}} (K_{\mathrm{bLL}} P_{\mathrm{bI}} + K_{\mathrm{ILL}} P_{\mathrm{II}} + K_{\mathrm{LLL}} P_{\mathrm{LI}}) Q_{\mathrm{LL}} Q_{\mathrm{LL}} m_{\tau_{\mathrm{I}}}^2 \delta\phi_{\mathrm{I}} \delta a_{\mathrm{L}} \delta a_{\mathrm{L}} \tag{A.138}$$

$$= -\frac{\sqrt{3\gamma_{\mathrm{I}}} |W_0|^2 \mathfrak{a}_{\mathrm{I}}^2 \tau_{\mathrm{I}}^{11/4}}{\tau_{\mathrm{b}}^{15/4}} \delta\phi_{\mathrm{I}} \delta a_{\mathrm{L}} \delta a_{\mathrm{L}}. \tag{A.139}$$

- Decay $\delta\phi_{\mathrm{I}} \to \delta a_{\mathrm{b}} \delta a_{\mathrm{L}}$:
  For this decay, we have

$$\mathcal{L}_{\text{int,kin},(\theta)}^{(\phi_{\mathrm{I}}\to a_{\mathrm{b}}a_{\mathrm{L}})} = \frac{1}{2^{3/2}} K_{mnp} P_{m\mathrm{I}} Q_{n\mathrm{b}} Q_{p\mathrm{L}} m_{\tau_{\mathrm{I}}}^2 \delta\phi_{\mathrm{I}} \delta a_{\mathrm{b}} \delta a_{\mathrm{L}}, \tag{A.140}$$

where we have also assigned a factor 2 because their are two possibilities how $\delta\phi_i \delta a_j \delta a_k$ can contribute to this decay. Again, since $Q_{\mathrm{Ib}} = Q_{\mathrm{Lb}} = Q_{\mathrm{bL}} = Q_{\mathrm{IL}} = 0$, the indices are forced to take on the values $n = \mathrm{b}$ and $p = \mathrm{L}$ so that we have

$$\mathcal{L}_{\text{int,kin},(\theta)}^{(\phi_{\mathrm{I}}\to a_{\mathrm{b}}a_{\mathrm{L}})} = \frac{1}{2^{3/2}} K_{m\mathrm{bL}} P_{m\mathrm{I}} Q_{\mathrm{bb}} Q_{\mathrm{LL}} m_{\tau_{\mathrm{I}}}^2 \delta\phi_{\mathrm{I}} \delta a_{\mathrm{b}} \delta a_{\mathrm{L}} \tag{A.141}$$

$$= \frac{1}{2^{3/2}} (K_{\mathrm{bbL}} P_{\mathrm{bI}} + K_{\mathrm{IbL}} P_{\mathrm{II}} + K_{\mathrm{LbL}} P_{\mathrm{LI}}) Q_{\mathrm{bb}} Q_{\mathrm{LL}} m_{\tau_{\mathrm{I}}}^2 \delta\phi_{\mathrm{I}} \delta a_{\mathrm{b}} \delta a_{\mathrm{L}} \tag{A.142}$$

$$\approx 0. \tag{A.143}$$

Note that this zero only holds at leading order under the approximation that $\mathfrak{a}_{\mathrm{I}}\tau_{\mathrm{I}}, \tau_{\mathrm{L}} \gg 1$ and that there is no fine-tuning of the parameter $\tilde{\mu} = \mu_3 - \sqrt{\tau_{\mathrm{L}}}$. At the next-to-leading order, we would get a contribution that scales as,

$$\mathcal{L}_{\text{int,kin},(\theta)}^{(\phi_{\mathrm{I}}\to a_{\mathrm{b}}a_{\mathrm{L}})} \sim \tau_{\mathrm{b}}^{-9/2} \delta\phi_{\mathrm{I}} \delta a_{\mathrm{b}} \delta a_{\mathrm{L}}, \tag{A.144}$$

which is suppressed compared to $\delta\phi_{\mathrm{I}} \to \delta a_{\mathrm{b}} \delta a_{\mathrm{b}}$ and $\delta\phi_{\mathrm{I}} \to \delta a_{\mathrm{L}} \delta a_{\mathrm{L}}$.

A.4.4. Decays of the inflaton axion

The trilinear couplings of the inflaton axion always involve exactly one other axion and one modulus field. The relevant coupling terms are given in (A.130) and (A.131). In analogy to the argument above, the potential coupling terms (A.131) vanish because the indices n and p must both take on the value 'I' while on of the indices j

and k must either take on the value 'b' or 'L'. This gives rise to either a factor 'Q_{Ib}' or 'Q_{IL}', both of which are zero.

From the kinetic coupling terms of the inflaton axion are induced from (A.130). There are always two possibilities how $\delta\phi_i\partial_\mu\delta a_j\partial^\mu\delta a_k$ can contribute to a decay of a_{I} corresponding to $j = \mathrm{I}$ or $k = \mathrm{I}$. Eliminating the derivatives using (A.94), the individual coupling terms are given as follows:

- Decay $\delta a_{\mathrm{I}} \to \delta\phi_{\mathrm{b}}\delta a_{\mathrm{b}}$:
  Here we have

$$\mathcal{L}^{(a_{\mathrm{I}}\to\phi_{\mathrm{b}}a_{\mathrm{b}})}_{\text{int,kin},(\theta)} = -\frac{1}{2^{3/2}}K_{mnp}P_{m\mathrm{b}}Q_{n\mathrm{I}}Q_{p\mathrm{b}}m^2_{\theta_{\mathrm{I}}}\delta\phi_{\mathrm{b}}\delta a_{\mathrm{I}}\delta a_{\mathrm{b}}. \tag{A.145}$$

  Since $Q_{\mathrm{Lb}} = Q_{\mathrm{Ib}} = 0$, the index p is forced to take on the value 'b' so that we obtain

$$\begin{aligned}
\mathcal{L}^{(a_{\mathrm{I}}\to\phi_{\mathrm{b}}a_{\mathrm{b}})}_{\text{int,kin},(\theta)} &= -\frac{1}{2^{3/2}}K_{mn\mathrm{b}}P_{m\mathrm{b}}Q_{n\mathrm{I}}Q_{\mathrm{bb}}m^2_{\theta_{\mathrm{I}}}\delta\phi_{\mathrm{b}}\delta a_{\mathrm{I}}\delta a_{\mathrm{b}} && \text{(A.146)}\\
&\approx -\frac{1}{2^{3/2}}\left(K_{\mathrm{bbb}}P_{\mathrm{bb}}Q_{\mathrm{bI}} + K_{\mathrm{bIb}}P_{\mathrm{bb}}Q_{\mathrm{II}}\right)Q_{\mathrm{bb}}m^2_{\theta_{\mathrm{I}}}\delta\phi_{\mathrm{b}}\delta a_{\mathrm{I}}\delta a_{\mathrm{b}} && \text{(A.147)}\\
&\approx -\frac{\sqrt{3\gamma_{\mathrm{I}}}|W_0|^2\mathfrak{a}_{\mathrm{I}}^2\tau_{\mathrm{I}}^{11/4}}{\tau_{\mathrm{b}}^{15/4}}\delta\phi_{\mathrm{b}}\delta a_{\mathrm{I}}\delta a_{\mathrm{b}}\,. && \text{(A.148)}
\end{aligned}$$

- Decay $\delta a_{\mathrm{I}} \to \delta\phi_{\mathrm{b}}\delta a_{\mathrm{L}}$:
  This decay is given by

$$\mathcal{L}^{(a_{\mathrm{I}}\to\phi_{\mathrm{b}}a_{\mathrm{L}})}_{\text{int,kin},(\theta)} = -\frac{1}{2^{3/2}}K_{mnp}P_{m\mathrm{b}}Q_{n\mathrm{I}}Q_{p\mathrm{L}}m^2_{\theta_{\mathrm{I}}}\delta\phi_{\mathrm{b}}\delta a_{\mathrm{I}}\delta a_{\mathrm{L}}. \tag{A.149}$$

  Since $Q_{\mathrm{IL}} = Q_{\mathrm{bL}} = 0$, the index p is forced to take on the value 'L' so that we have

$$\begin{aligned}
\mathcal{L}^{(a_{\mathrm{I}}\to\phi_{\mathrm{b}}a_{\mathrm{L}})}_{\text{int,kin},(\theta)} &= -\frac{1}{2^{3/2}}K_{mnp}P_{m\mathrm{b}}Q_{n\mathrm{I}}Q_{\mathrm{LL}}m^2_{\theta_{\mathrm{I}}}\delta\phi_{\mathrm{b}}\delta a_{\mathrm{I}}\delta a_{\mathrm{L}} && \text{(A.150)}\\
&\sim \tau_{\mathrm{b}}^{-9/2}\delta\phi_{\mathrm{b}}\delta a_{\mathrm{I}}\delta a_{\mathrm{L}}\,. && \text{(A.151)}
\end{aligned}$$

- Decay $\delta a_{\mathrm{I}} \to \delta\phi_{\mathrm{L}}\delta a_{\mathrm{b}}$:
  The coupling terms read

$$\mathcal{L}^{(a_{\mathrm{I}}\to\phi_{\mathrm{L}}a_{\mathrm{b}})}_{\text{int,kin},(\theta)} = -\frac{1}{2^{3/2}}K_{mnp}P_{m\mathrm{L}}Q_{n\mathrm{I}}Q_{p\mathrm{b}}m^2_{\theta_{\mathrm{I}}}\delta\phi_{\mathrm{L}}\delta a_{\mathrm{I}}\delta a_{\mathrm{b}}. \tag{A.152}$$

Here the index p is again forced to take on the value 'b' and we obtain

$$\mathcal{L}_{\text{int,kin},(\theta)}^{(a_\text{I}\to\phi_\text{L}a_\text{b})} = -\frac{1}{2^{3/2}}K_{mnp}P_{m\text{L}}Q_{n\text{I}}Q_{p\text{b}}m_{\theta_\text{I}}^2\delta\phi_\text{L}\delta a_\text{I}\delta a_\text{b} \tag{A.153}$$

$$\sim \tau_\text{b}^{-9/2}\delta\phi_\text{L}\delta a_\text{I}\delta a_\text{b}\,. \tag{A.154}$$

- Decay $\delta a_\text{I} \to \delta\phi_\text{L}\delta a_\text{L}$:
  For this decay we have

$$\mathcal{L}_{\text{int,kin},(\theta)}^{(a_\text{I}\to\phi_\text{L}a_\text{L})} = -\frac{1}{2^{3/2}}K_{mnp}P_{m\text{L}}Q_{n\text{I}}Q_{p\text{L}}m_{\theta_\text{I}}^2\delta\phi_\text{L}\delta a_\text{I}\delta a_\text{L}. \tag{A.155}$$

The index p must take on the value 'L' and the coupling terms are given by

$$\mathcal{L}_{\text{int,kin},(\theta)}^{(a_\text{I}\to\phi_\text{L}a_\text{L})} = -\frac{1}{2^{3/2}}K_{mnp}P_{m\text{L}}Q_{n\text{I}}Q_{\text{LL}}m_{\theta_\text{I}}^2\delta\phi_\text{L}\delta a_\text{I}\delta a_\text{L} \tag{A.156}$$

$$\approx -\frac{1}{2^{3/2}}\left(K_{\text{LbL}}P_{\text{LL}}Q_{\text{bI}} + K_{LIL}P_{\text{LL}}Q_{\text{II}} + K_{\text{LLL}}P_{\text{LL}}Q_{\text{LI}}\right)Q_{\text{LL}}m_{\theta_\text{I}}^2 \times \delta\phi_\text{L}\delta a_\text{I}\delta a_\text{L} \tag{A.157}$$

$$\approx \frac{2\sqrt{3\gamma_\text{I}}|W_0|^2\mathfrak{a}_\text{I}^2\tau_\text{I}^{11/4}}{\tau_\text{b}^{15/4}}\delta\phi_\text{L}\delta a_\text{I}\delta a_\text{L}\,. \tag{A.158}$$

A.4.5. Decay rates

To obtain the corresponding decay rates, we use the standard formula

$$\Gamma = \frac{1}{S}\int\frac{|\mathcal{M}|^2}{2E}d\text{LIPS}, \tag{A.159}$$

where S is the symmetry factor, E is the energy of the decaying particle, $|\mathcal{M}|^2$ is the matrix element squared and dLIPS is an element of Lorentz invariant phase space. The decays we consider can be grouped into two categories, either with two identical decay products or with two different ones. The corresponding interaction terms are schematically of the form

$$\mathcal{L}_\text{A} \supset g_\text{A}\varphi_\text{A}\psi_\text{A}^2, \quad \mathcal{L}_\text{B} \supset g_\text{B}\varphi_\text{B}\psi_\text{B}\chi_\text{B}, \tag{A.160}$$

where we assume that the decaying particle φ is much heavier than the decay products ψ and χ, i.e. $m_{\varphi_\text{A}} \gg 2m_{\psi_\text{A}}$ and $m_{\varphi_\text{B}} \gg m_{\psi_\text{B}} + m_{\chi_\text{B}}$. A crucial difference between the two categories lies in their respective symmetry factors and matrix elements. For category A, we have $S = 2$ and $|\mathcal{M}|^2 = 4g_\text{A}^2$ whereas for category B, we have $S = 1$ and $|\mathcal{M}|^2 = g_\text{B}^2$. This results in the following decay rates for the two categories,

$$\Gamma_{\varphi_\mathrm{A}\to\psi_\mathrm{A}\psi_\mathrm{A}} = \frac{g_\mathrm{A}^2}{8\pi m_{\varphi_\mathrm{A}}}, \quad \Gamma_{\varphi_\mathrm{B}\to\psi_\mathrm{B}\chi_\mathrm{B}} = \frac{g_\mathrm{B}^2}{16\pi m_{\varphi_\mathrm{B}}}. \tag{A.161}$$

By reading off the respective couplings g from the trilinear coupling terms above, we can easily obtain the corresponding decay rates.

The relevant decays of the inflaton fall into category A. For the kinetic decay into the volume modulus we have

$$|\mathcal{M}_1|^2 = \frac{3\gamma_\mathrm{I}|W_0|^4\mathfrak{a}_\mathrm{I}^4\tau_\mathrm{I}^{11/2}}{\tau_\mathrm{b}^{15/2}}, \quad m_{\tau_\mathrm{I}}^2 = \frac{4|W_0|^2\mathfrak{a}_\mathrm{I}^2\tau_\mathrm{I}^2}{\tau_\mathrm{b}^3}, \tag{A.162}$$

and thus obtain

$$\Gamma_1 \equiv \Gamma^\mathrm{kin}_{\phi_\mathrm{I}\to\phi_\mathrm{b}\phi_\mathrm{b}} \approx \frac{3\gamma_\mathrm{I}|W_0|^3\mathfrak{a}_\mathrm{I}^3\tau_\mathrm{I}^{9/2}}{64\pi\mathcal{V}^4}. \tag{A.163}$$

Analogously, for the potential decay into the loop modulus, the matrix element squared is given by

$$|\mathcal{M}_2|^2 = \left(-\frac{\sqrt{3\gamma_\mathrm{I}}\tau_\mathrm{I}^{3/4}\left[-3|W_0|^2\gamma_\mathrm{L}\tilde{\mu}^4\tau_\mathrm{L}^2 + W_0^2\left(-4\mu_1\tilde{\mu}^4 + \mu_2(\mu_3^2 - 4\mu_3\sqrt{\tau_\mathrm{L}} + 4\tau_\mathrm{L})\tau_\mathrm{L}\right)\right]}{\gamma_\mathrm{L}\tilde{\mu}^4\tau_\mathrm{L}^2\tau_\mathrm{b}^{15/4}}\right)^2 \tag{A.164}$$

and the decay rate by

$$\Gamma_2 \equiv \Gamma^\mathrm{pot}_{\phi_\mathrm{I}\to\phi_\mathrm{L}\phi_\mathrm{L}} = \frac{3\gamma_\mathrm{I}\sqrt{\tau_\mathrm{I}}\left[-3|W_0|^2\gamma_\mathrm{L}\tilde{\mu}^4\tau_\mathrm{L}^2 + W_0^2\left(-4\mu_1\tilde{\mu}^4 + \mu_2(\mu_3^2 - 4\mu_3\sqrt{\tau_\mathrm{L}} + 4\tau_\mathrm{L})\tau_\mathrm{L}\right)\right]^2}{64\pi\gamma_\mathrm{L}^2|W_0|\tilde{\mu}^8\mathfrak{a}_\mathrm{I}\tau_\mathrm{L}^4\mathcal{V}^4}. \tag{A.165}$$

Note that $\Gamma_1/\Gamma_2 \sim \mathfrak{a}_\mathrm{I}^4\tau_\mathrm{I}^4 \sim (\ln\mathcal{V})^4 \gg 1$. Comparing the coupling functions, all other decay rates of inflaton decays can be related to Γ_1 and Γ_2 as given in Tab. 4.

Likewise, the decay rates of the inflaton axion fall into category B and can also be related to Γ_1 as given in Tab. 4.

Bibliography

[1] Manuel Wittner et al. “Transient weak gravity in scalar-tensor theories”. In: *JCAP* 07 (2020), p. 019. DOI: 10.1088/1475-7516/2020/07/019. arXiv: 2003.08950 [gr-qc].

[2] Arthur Hebecker, Torben Skrzypek, and Manuel Wittner. “The F-term Problem and other Challenges of Stringy Quintessence”. In: *JHEP* 11 (2019), p. 134. DOI: 10.1007/JHEP11(2019)134. arXiv: 1909.08625 [hep-th].

[3] Arthur Hebecker, Joerg Jaeckel, and Manuel Wittner. “Axions in String Theory and the Hydra of Dark Radiation”. In: (Mar. 2022). arXiv: 2203.08833 [hep-th].

[4] N. Aghanim et al. “Planck 2018 results. VI. Cosmological parameters”. In: *Astron. Astrophys.* 641 (2020). [Erratum: Astron.Astrophys. 652, C4 (2021)], A6. DOI: 10.1051/0004-6361/201833910. arXiv: 1807.06209 [astro-ph.CO].

[5] Oliver F. Piattella. *Lecture Notes in Cosmology*. UNITEXT for Physics. Cham: Springer, 2018. DOI: 10.1007/978-3-319-95570-4. arXiv: 1803.00070 [astro-ph.CO].

[6] Steven Weinberg. “Anthropic Bound on the Cosmological Constant”. In: *Phys. Rev. Lett.* 59 (1987), p. 2607. DOI: 10.1103/PhysRevLett.59.2607.

[7] Steven Weinberg. “The Cosmological Constant Problem”. In: *Rev. Mod. Phys.* 61 (1989). Ed. by Jong-Ping Hsu and D. Fine, pp. 1–23. DOI: 10.1103/RevModPhys.61.1.

[8] Ivaylo Zlatev, Li-Min Wang, and Paul J. Steinhardt. “Quintessence, cosmic coincidence, and the cosmological constant”. In: *Phys. Rev. Lett.* 82 (1999), pp. 896–899. DOI: 10.1103/PhysRevLett.82.896. arXiv: astro-ph/9807002.

[9] H. E. S. Velten, R. F. vom Marttens, and W. Zimdahl. “Aspects of the cosmological “coincidence problem””. In: *Eur. Phys. J. C* 74.11 (2014), p. 3160. DOI: 10.1140/epjc/s10052-014-3160-4. arXiv: 1410.2509 [astro-ph.CO].

[10] Catherine Heymans et al. "CFHTLenS: the Canada–France–Hawaii Telescope Lensing Survey". In: *Monthly Notices of the Royal Astronomical Society* 427.1 (Oct. 2012), pp. 146–166. ISSN: 1365-2966. DOI: 10.1111/j.1365-2966.2012.21952.x. URL: http://dx.doi.org/10.1111/j.1365-2966.2012.21952.x.

[11] H. Hildebrandt et al. "KiDS-450: cosmological parameter constraints from tomographic weak gravitational lensing". In: *Monthly Notices of the Royal Astronomical Society* 465.2 (Nov. 2016), pp. 1454–1498. ISSN: 1365-2966. DOI: 10.1093/mnras/stw2805. URL: http://dx.doi.org/10.1093/mnras/stw2805.

[12] Adam G. Riess et al. "New Parallaxes of Galactic Cepheids from Spatially Scanning theHubble Space Telescope: Implications for the Hubble Constant". In: *The Astrophysical Journal* 855.2 (Mar. 2018), p. 136. ISSN: 1538-4357. DOI: 10.3847/1538-4357/aaadb7. URL: http://dx.doi.org/10.3847/1538-4357/aaadb7.

[13] Adam G. Riess et al. "Large Magellanic Cloud Cepheid Standards Provide a 1% Foundation for the Determination of the Hubble Constant and Stronger Evidence for Physics beyond ΛCDM". In: *Astrophys. J.* 876.1 (2019), p. 85. DOI: 10.3847/1538-4357/ab1422. arXiv: 1903.07603 [astro-ph.CO].

[14] Adam G. Riess et al. "A Comprehensive Measurement of the Local Value of the Hubble Constant with 1 km/s/Mpc Uncertainty from the Hubble Space Telescope and the SH0ES Team". In: (Dec. 2021). arXiv: 2112.04510 [astro-ph.CO].

[15] Eleonora Di Valentino et al. "Snowmass2021 - Letter of interest cosmology intertwined II: The hubble constant tension". In: *Astropart. Phys.* 131 (2021), p. 102605. DOI: 10.1016/j.astropartphys.2021.102605. arXiv: 2008.11284 [astro-ph.CO].

[16] Elcio Abdalla et al. "Cosmology intertwined: A review of the particle physics, astrophysics, and cosmology associated with the cosmological tensions and anomalies". In: *JHEAp* 34 (2022), pp. 49–211. DOI: 10.1016/j.jheap.2022.04.002. arXiv: 2203.06142 [astro-ph.CO].

[17] Eleonora Di Valentino et al. "Cosmology intertwined III: $f\sigma_8$ and S_8". In: *Astropart. Phys.* 131 (2021), p. 102604. DOI: 10.1016/j.astropartphys.2021.102604. arXiv: 2008.11285 [astro-ph.CO].

[18] Steven Weinberg. *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*. New York: John Wiley and Sons, 1972. ISBN: 978-0-471-92567-5, 978-0-471-92567-5.

[19] Robert M. Wald. *General Relativity*. Chicago, USA: Chicago Univ. Pr., 1984. DOI: 10.7208/chicago/9780226870373.001.0001.

[20] Steven Weinberg. *Cosmology*. Oxford University Press, 2008. ISBN: 978-0-19-852682-7.

[21] Eleonora Di Valentino et al. "Snowmass2021 - Letter of interest cosmology intertwined IV: The age of the universe and its curvature". In: *Astropart. Phys.* 131 (2021), p. 102607. DOI: 10.1016/j.astropartphys.2021.102607. arXiv: 2008.11286 [astro-ph.CO].

[22] Eleonora Di Valentino, Alessandro Melchiorri, and Joseph Silk. "Planck evidence for a closed Universe and a possible crisis for cosmology". In: *Nature Astron.* 4.2 (2019), pp. 196–203. DOI: 10.1038/s41550-019-0906-9. arXiv: 1911.02087 [astro-ph.CO].

[23] Will Handley. "Curvature tension: evidence for a closed universe". In: *Phys. Rev. D* 103.4 (2021), p. L041301. DOI: 10.1103/PhysRevD.103.L041301. arXiv: 1908.09139 [astro-ph.CO].

[24] M. J. Reid, D. W. Pesce, and A. G. Riess. "An Improved Distance to NGC 4258 and its Implications for the Hubble Constant". In: *Astrophys. J. Lett.* 886.2 (2019), p. L27. DOI: 10.3847/2041-8213/ab552d. arXiv: 1908.05625 [astro-ph.GA].

[25] Kenneth C. Wong et al. "H0LiCOW – XIII. A 2.4 per cent measurement of H0 from lensed quasars: 5.3σ tension between early- and late-Universe probes". In: *Mon. Not. Roy. Astron. Soc.* 498.1 (2020), pp. 1420–1439. DOI: 10.1093/mnras/stz3094. arXiv: 1907.04869 [astro-ph.CO].

[26] Luca Amendola. "Coupled quintessence". In: *Phys. Rev. D* 62 (2000), p. 043511. DOI: 10.1103/PhysRevD.62.043511. arXiv: astro-ph/9908023.

[27] Adrià Gómez-Valent, Valeria Pettorino, and Luca Amendola. "Update on coupled dark energy and the H_0 tension". In: *Phys. Rev. D* 101.12 (2020), p. 123513. DOI: 10.1103/PhysRevD.101.123513. arXiv: 2004.00610 [astro-ph.CO].

[28] Viatcheslav F. Mukhanov, H. A. Feldman, and Robert H. Brandenberger. "Theory of cosmological perturbations. Part 1. Classical perturbations. Part 2. Quantum theory of perturbations. Part 3. Extensions". In: *Phys. Rept.* 215 (1992), pp. 203–333. DOI: 10.1016/0370-1573(92)90044-Z.

[29] Luca Amendola and Shinji Tsujikawa. *Dark Energy: Theory and Observations*. Cambridge University Press, Jan. 2015. ISBN: 978-1-107-45398-2.

[30] Catherine Heymans et al. "KiDS-1000 Cosmology: Multi-probe weak gravitational lensing and spectroscopic galaxy clustering constraints". In: *Astron. Astrophys.* 646 (2021), A140. DOI: 10.1051/0004-6361/202039063. arXiv: 2007.15632 [astro-ph.CO].

[31] Tsutomu Kobayashi. "Horndeski theory and beyond: a review". In: *Rept. Prog. Phys.* 82.8 (2019), p. 086901. DOI: 10.1088/1361-6633/ab2429. arXiv: 1901.07183 [gr-qc].

[32] D. Lovelock. "The Einstein tensor and its generalizations". In: *J. Math. Phys.* 12 (1971), pp. 498–501. DOI: 10.1063/1.1665613.

[33] D. Lovelock. "The four-dimensionality of space and the einstein tensor". In: *J. Math. Phys.* 13 (1972), pp. 874–876. DOI: 10.1063/1.1666069.

[34] Bharat Ratra and P. J. E. Peebles. "Cosmological Consequences of a Rolling Homogeneous Scalar Field". In: *Phys. Rev. D* 37 (1988), p. 3406. DOI: 10.1103/PhysRevD.37.3406.

[35] C. Wetterich. "Cosmology and the Fate of Dilatation Symmetry". In: *Nucl. Phys. B* 302 (1988), pp. 668–696. DOI: 10.1016/0550-3213(88)90193-9. arXiv: 1711.03844 [hep-th].

[36] Eleonora Di Valentino et al. "In the realm of the Hubble tension—a review of solutions". In: *Class. Quant. Grav.* 38.15 (2021), p. 153001. DOI: 10.1088/1361-6382/ac086d. arXiv: 2103.01183 [astro-ph.CO].

[37] M. Ostrogradsky. "Mémoires sur les équations différentielles, relatives au problème des isopérimètres". In: *Mem. Acad. St. Petersbourg* 6.4 (1850), pp. 385–517.

[38] Richard P. Woodard. "Avoiding dark energy with 1/r modifications of gravity". In: *Lect. Notes Phys.* 720 (2007). Ed. by Lefteris Papantonopoulos, pp. 403–433. DOI: 10.1007/978-3-540-71013-4_14. arXiv: astro-ph/0601672.

[39] Hayato Motohashi and Teruaki Suyama. "Third order equations of motion and the Ostrogradsky instability". In: *Phys. Rev. D* 91.8 (2015), p. 085009. DOI: 10.1103/PhysRevD.91.085009. arXiv: 1411.3721 [physics.class-ph].

[40] Mark Trodden. "Theoretical Aspects of Cosmic Acceleration". In: *PoS* DSU2015 (2016), p. 005. DOI: 10.22323/1.268.0005. arXiv: 1604.08899 [astro-ph.CO].

[41] Gregory Walter Horndeski. "Second-order scalar-tensor field equations in a four-dimensional space". In: *Int. J. Theor. Phys.* 10 (1974), pp. 363–384. DOI: 10.1007/BF01807638.

[42] C. Deffayet, S. Deser, and G. Esposito-Farese. “Generalized Galileons: All scalar models whose curved background extensions maintain second-order field equations and stress-tensors”. In: *Phys. Rev. D* 80 (2009), p. 064015. DOI: 10.1103/PhysRevD.80.064015. arXiv: 0906.1967 [gr-qc].

[43] C. Deffayet, Gilles Esposito-Farese, and A. Vikman. “Covariant Galileon”. In: *Phys. Rev. D* 79 (2009), p. 084003. DOI: 10.1103/PhysRevD.79.084003. arXiv: 0901.1314 [hep-th].

[44] C. Deffayet et al. “From k-essence to generalised Galileons”. In: *Phys. Rev. D* 84 (2011), p. 064039. DOI: 10.1103/PhysRevD.84.064039. arXiv: 1103.3260 [hep-th].

[45] Miguel Zumalacárregui and Juan García-Bellido. “Transforming gravity: from derivative couplings to matter to second-order scalar-tensor theories beyond the Horndeski Lagrangian”. In: *Phys. Rev. D* 89 (2014), p. 064046. DOI: 10.1103/PhysRevD.89.064046. arXiv: 1308.4685 [gr-qc].

[46] Jérôme Gleyzes et al. “Healthy theories beyond Horndeski”. In: *Phys. Rev. Lett.* 114.21 (2015), p. 211101. DOI: 10.1103/PhysRevLett.114.211101. arXiv: 1404.6495 [hep-th].

[47] Jérôme Gleyzes et al. “Exploring gravitational theories beyond Horndeski”. In: *JCAP* 02 (2015), p. 018. DOI: 10.1088/1475-7516/2015/02/018. arXiv: 1408.1952 [astro-ph.CO].

[48] David Langlois and Karim Noui. “Degenerate higher derivative theories beyond Horndeski: evading the Ostrogradski instability”. In: *JCAP* 02 (2016), p. 034. DOI: 10.1088/1475-7516/2016/02/034. arXiv: 1510.06930 [gr-qc].

[49] Jibril Ben Achour et al. “Degenerate higher order scalar-tensor theories beyond Horndeski up to cubic order”. In: *JHEP* 12 (2016), p. 100. DOI: 10.1007/JHEP12(2016)100. arXiv: 1608.08135 [hep-th].

[50] David Langlois. “Dark energy and modified gravity in degenerate higher-order scalar–tensor (DHOST) theories: A review”. In: *Int. J. Mod. Phys. D* 28.05 (2019), p. 1942006. DOI: 10.1142/S0218271819420069. arXiv: 1811.06271 [gr-qc].

[51] Luca Amendola et al. “Measuring gravity at cosmological scales”. In: *Universe* 6.2 (2020), p. 20. DOI: 10.3390/universe6020020. arXiv: 1902.06978 [astro-ph.CO].

[52] Emilio Bellini and Ignacy Sawicki. “Maximal freedom at minimum cost: linear large-scale structure in general modifications of gravity”. In: *JCAP* 07 (2014), p. 050. DOI: 10.1088/1475-7516/2014/07/050. arXiv: 1404.3713 [astro-ph.CO].

[53] B. P. Abbott et al. “Gravitational Waves and Gamma-rays from a Binary Neutron Star Merger: GW170817 and GRB 170817A”. In: *Astrophys. J. Lett.* 848.2 (2017), p. L13. DOI: 10.3847/2041-8213/aa920c. arXiv: 1710.05834 [astro-ph.HE].

[54] Jose María Ezquiaga and Miguel Zumalacárregui. “Dark Energy After GW170817: Dead Ends and the Road Ahead”. In: *Phys. Rev. Lett.* 119.25 (2017), p. 251304. DOI: 10.1103/PhysRevLett.119.251304. arXiv: 1710.05901 [astro-ph.CO].

[55] Dario Bettoni and Stefano Liberati. “Disformal invariance of second order scalar-tensor theories: Framing the Horndeski action”. In: *Phys. Rev. D* 88 (2013), p. 084020. DOI: 10.1103/PhysRevD.88.084020. arXiv: 1306.6724 [gr-qc].

[56] Christof Wetterich. “The Cosmon model for an asymptotically vanishing time dependent cosmological ’constant’”. In: *Astron. Astrophys.* 301 (1995), pp. 321–328. arXiv: hep-th/9408025.

[57] Miguel Zumalacarregui, Tomi S. Koivisto, and David F. Mota. “DBI Galileons in the Einstein Frame: Local Gravity and Cosmology”. In: *Phys. Rev. D* 87 (2013), p. 083010. DOI: 10.1103/PhysRevD.87.083010. arXiv: 1210.8016 [astro-ph.CO].

[58] Jurgen Mifsud and Carsten Van De Bruck. “Probing the imprints of generalized interacting dark energy on the growth of perturbations”. In: *JCAP* 11 (2017), p. 001. DOI: 10.1088/1475-7516/2017/11/001. arXiv: 1707.07667 [astro-ph.CO].

[59] Luca Amendola et al. “Fate of Large-Scale Structure in Modified Gravity After GW170817 and GRB170817A”. In: *Phys. Rev. Lett.* 120.13 (2018), p. 131101. DOI: 10.1103/PhysRevLett.120.131101. arXiv: 1711.04825 [astro-ph.CO].

[60] Luca Amendola. “Phantom energy mediates a long-range repulsive force”. In: *Phys. Rev. Lett.* 93 (2004), p. 181102. DOI: 10.1103/PhysRevLett.93.181102. arXiv: hep-th/0409224.

[61] Antonio De Felice, Tsutomu Kobayashi, and Shinji Tsujikawa. “Effective gravitational couplings for cosmological perturbations in the most general scalar-tensor theories with second-order field equations”. In: *Phys. Lett. B* 706 (2011), pp. 123–133. DOI: 10.1016/j.physletb.2011.11.028. arXiv: 1108.4242 [gr-qc].

[62] Niall MacCrann et al. “Cosmic Discordance: Are Planck CMB and CFHTLenS weak lensing measurements out of tune?” In: *Mon. Not. Roy. Astron. Soc.* 451.3 (2015), pp. 2877–2888. DOI: 10.1093/mnras/stv1154. arXiv: 1408.4742 [astro-ph.CO].

[63] P. A. R. Ade et al. “Planck 2015 results. XXIV. Cosmology from Sunyaev-Zeldovich cluster counts”. In: *Astron. Astrophys.* 594 (2016), A24. DOI: `10.1051/0004-6361/201525833`. arXiv: `1502.01597 [astro-ph.CO]`.

[64] Shahab Joudaki et al. “CFHTLenS revisited: assessing concordance with Planck including astrophysical systematics”. In: *Mon. Not. Roy. Astron. Soc.* 465.2 (2017), pp. 2033–2052. DOI: `10.1093/mnras/stw2665`. arXiv: `1601.05786 [astro-ph.CO]`.

[65] T. M. C. Abbott et al. “Dark Energy Survey year 1 results: Cosmological constraints from galaxy clustering and weak lensing”. In: *Phys. Rev. D* 98.4 (2018), p. 043526. DOI: `10.1103/PhysRevD.98.043526`. arXiv: `1708.01530 [astro-ph.CO]`.

[66] C. van de Bruck and J. Morrice. “Disformal couplings and the dark sector of the universe”. In: *JCAP* 04 (2015), p. 036. DOI: `10.1088/1475-7516/2015/04/036`. arXiv: `1501.03073 [gr-qc]`.

[67] C. Wetterich. “Cosmologies With Variable Newton’s ’Constant’”. In: *Nucl. Phys. B* 302 (1988), pp. 645–667. DOI: `10.1016/0550-3213(88)90192-7`.

[68] J. Polchinski. *String theory. Vol. 1: An introduction to the bosonic string*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, Dec. 2007. ISBN: 978-0-511-25227-3, 978-0-521-67227-6, 978-0-521-63303-1. DOI: `10.1017/CBO9780511816079`.

[69] J. Polchinski. *String theory. Vol. 2: Superstring theory and beyond*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, Dec. 2007. ISBN: 978-0-511-25228-0, 978-0-521-63304-8, 978-0-521-67228-3. DOI: `10.1017/CBO9780511618123`.

[70] B. Zwiebach. *A first course in string theory*. Cambridge University Press, July 2006. ISBN: 978-0-521-83143-7, 978-0-511-20757-0.

[71] K. Becker, M. Becker, and J. H. Schwarz. *String theory and M-theory: A modern introduction*. Cambridge University Press, Dec. 2006. ISBN: 978-0-511-25486-4, 978-0-521-86069-7, 978-0-511-81608-6. DOI: `10.1017/CBO9780511816086`.

[72] Michael B. Green, John H. Schwarz, and Edward Witten. *Superstring Theory Vol. 1: 25th Anniversary Edition*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, Nov. 2012. ISBN: 978-1-139-53477-2, 978-1-107-02911-8. DOI: `10.1017/CBO9781139248563`.

[73] Michael B. Green, John H. Schwarz, and Edward Witten. *Superstring Theory Vol. 2: 25th Anniversary Edition*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, Nov. 2012. ISBN: 978-1-139-53478-9, 978-1-107-02913-2. DOI: 10.1017/CBO9781139248570.

[74] Ralph Blumenhagen, Dieter Lüst, and Stefan Theisen. *Basic concepts of string theory*. Theoretical and Mathematical Physics. Heidelberg, Germany: Springer, 2013. ISBN: 978-3-642-29496-9. DOI: 10.1007/978-3-642-29497-6.

[75] Peter C. West. *Introduction to supersymmetry and supergravity*. World Scientific Publishing, 1986. ISBN: 978-9971-5-0028-3.

[76] J. Wess and J. Bagger. *Supersymmetry and supergravity*. Princeton, NJ, USA: Princeton University Press, 1992. ISBN: 978-0-691-02530-8.

[77] Daniel Z. Freedman and Antoine Van Proeyen. *Supergravity*. Cambridge, UK: Cambridge Univ. Press, May 2012. ISBN: 978-1-139-36806-3, 978-0-521-19401-3.

[78] Eran Palti. "The Swampland: Introduction and Review". In: *Fortsch. Phys.* 67.6 (2019), p. 1900037. DOI: 10.1002/prop.201900037. arXiv: 1903.06239 [hep-th].

[79] Arthur Hebecker. "Lectures on Naturalness, String Landscape and Multiverse". In: (Aug. 2020). arXiv: 2008.10625 [hep-th].

[80] Timo Weigand. *Introduction to String Theory*. https://www.thphys.uni-heidelberg.de/courses/weigand/Strings11-12.pdf. Lecture notes from 2011, accessed on 2021-08-23.

[81] Luis E. Ibanez and Angel M. Uranga. *String theory and particle physics: An introduction to string phenomenology*. Cambridge University Press, Feb. 2012. ISBN: 978-0-521-51752-2, 978-1-139-22742-1.

[82] Sidney R. Coleman and J. Mandula. "All Possible Symmetries of the S Matrix". In: *Phys. Rev.* 159 (1967). Ed. by A. Zichichi, pp. 1251–1256. DOI: 10.1103/PhysRev.159.1251.

[83] Rudolf Haag, Jan T. Lopuszanski, and Martin Sohnius. "All Possible Generators of Supersymmetries of the s Matrix". In: *Nucl. Phys. B* 88 (1975), p. 257. DOI: 10.1016/0550-3213(75)90279-5.

[84] Peter Svrcek and Edward Witten. "Axions In String Theory". In: *JHEP* 06 (2006), p. 051. DOI: 10.1088/1126-6708/2006/06/051. arXiv: hep-th/0605206.

[85] D. J. Kapner et al. “Tests of the gravitational inverse-square law below the dark-energy length scale”. In: *Phys. Rev. Lett.* 98 (2007), p. 021101. DOI: 10.1103/PhysRevLett.98.021101. arXiv: hep-ph/0611184.

[86] M. Nakahara. *Geometry, topology and physics*. Taylor & Francis, 2003. ISBN: 978-0-7503-0606-5.

[87] Theodor. Kaluza. “Zum Unitätsproblem der Physik”. In: *Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys.)* 1921 (1921), pp. 966–972. DOI: 10.1142/S0218271818700017. arXiv: 1803.08616 [physics.hist-ph].

[88] Oskar. Klein. “The Atomicity of Electricity as a Quantum Theory Law”. In: *Nature* 118 (1926), p. 516. DOI: 10.1038/118516a0.

[89] Oskar Klein. “Quantum Theory and Five-Dimensional Theory of Relativity. (In German and English)”. In: *Z. Phys.* 37 (1926). Ed. by J. C. Taylor, pp. 895–906. DOI: 10.1007/BF01397481.

[90] Katrin Becker et al. “Supersymmetry breaking and alpha-prime corrections to flux induced potentials”. In: *JHEP* 06 (2002), p. 060. DOI: 10.1088/1126-6708/2002/06/060. arXiv: hep-th/0204254.

[91] Thomas W. Grimm. “The Effective action of type II Calabi-Yau orientifolds”. In: *Fortsch. Phys.* 53 (2005), pp. 1179–1271. DOI: 10.1002/prop.200510253. arXiv: hep-th/0507153.

[92] Xin Gao et al. “Loops, Local Corrections and Warping in the LVS and other Type IIB Models”. In: (Apr. 2022). arXiv: 2204.06009 [hep-th].

[93] Frederik Denef. “Les Houches Lectures on Constructing String Vacua”. In: *Les Houches* 87 (2008). Ed. by Costas Bachas et al., pp. 483–610. arXiv: 0803.1194 [hep-th].

[94] Alexander Giryavets. “New attractors and area codes”. In: *JHEP* 03 (2006), p. 020. DOI: 10.1088/1126-6708/2006/03/020. arXiv: hep-th/0511215.

[95] Sergei Gukov, Cumrun Vafa, and Edward Witten. “CFT’s from Calabi-Yau four folds”. In: *Nucl. Phys. B* 584 (2000). [Erratum: Nucl.Phys.B 608, 477–478 (2001)], pp. 69–108. DOI: 10.1016/S0550-3213(00)00373-4. arXiv: hep-th/9906070.

[96] Steven B. Giddings, Shamit Kachru, and Joseph Polchinski. “Hierarchies from fluxes in string compactifications”. In: *Phys. Rev. D* 66 (2002), p. 106006. DOI: 10.1103/PhysRevD.66.106006. arXiv: hep-th/0105097.

[97] E. Cremmer et al. “Naturally Vanishing Cosmological Constant in N=1 Supergravity”. In: *Phys. Lett. B* 133 (1983), p. 61. DOI: 10.1016/0370-2693(83)90106-5.

[98] Shamit Kachru et al. "De Sitter vacua in string theory". In: *Phys. Rev. D* 68 (2003), p. 046005. DOI: 10.1103/PhysRevD.68.046005. arXiv: hep-th/0301240.

[99] Vijay Balasubramanian et al. "Systematics of moduli stabilisation in Calabi-Yau flux compactifications". In: *JHEP* 03 (2005), p. 007. DOI: 10.1088/1126-6708/2005/03/007. arXiv: hep-th/0502058.

[100] Joseph P. Conlon, Fernando Quevedo, and Kerim Suruliz. "Large-volume flux compactifications: Moduli spectrum and D3/D7 soft supersymmetry breaking". In: *JHEP* 08 (2005), p. 007. DOI: 10.1088/1126-6708/2005/08/007. arXiv: hep-th/0505076.

[101] Arthur Hebecker et al. "Dark Radiation predictions from general Large Volume Scenarios". In: *JHEP* 09 (2014), p. 140. DOI: 10.1007/JHEP09(2014)140. arXiv: 1403.6810 [hep-ph].

[102] Marcus Berg, Michael Haack, and Boris Kors. "String loop corrections to Kahler potentials in orientifolds". In: *JHEP* 11 (2005), p. 030. DOI: 10.1088/1126-6708/2005/11/030. arXiv: hep-th/0508043.

[103] Michele Cicoli, Joseph P. Conlon, and Fernando Quevedo. "Systematics of String Loop Corrections in Type IIB Calabi-Yau Flux Compactifications". In: *JHEP* 01 (2008), p. 052. DOI: 10.1088/1126-6708/2008/01/052. arXiv: 0708.1873 [hep-th].

[104] Michele Cicoli, Joseph P. Conlon, and Fernando Quevedo. "General Analysis of LARGE Volume Scenarios with String Loop Moduli Stabilisation". In: *JHEP* 10 (2008), p. 105. DOI: 10.1088/1126-6708/2008/10/105. arXiv: 0805.1029 [hep-th].

[105] Igor R. Klebanov and Matthew J. Strassler. "Supergravity and a confining gauge theory: Duality cascades and chi SB resolution of naked singularities". In: *JHEP* 08 (2000), p. 052. DOI: 10.1088/1126-6708/2000/08/052. arXiv: hep-th/0007191.

[106] C. P. Burgess, R. Kallosh, and F. Quevedo. "De Sitter string vacua from supersymmetric D terms". In: *JHEP* 10 (2003), p. 056. DOI: 10.1088/1126-6708/2003/10/056. arXiv: hep-th/0309187.

[107] D. Cremades et al. "Moduli stabilisation and de Sitter string vacua from magnetised D7 branes". In: *JHEP* 05 (2007), p. 100. DOI: 10.1088/1126-6708/2007/05/100. arXiv: hep-th/0701154.

[108] Daniel Junghans. "LVS de Sitter Vacua are probably in the Swampland". In: (Jan. 2022). arXiv: 2201.03572 [hep-th].

[109] Xin Gao et al. "The LVS Parametric Tadpole Constraint". In: (Feb. 2022). arXiv: 2202.04087 [hep-th].

[110] Andrei D. Linde. *Particle physics and inflationary cosmology*. Vol. 5. Harwood Acad. Publ., 1990. ISBN: 978-3-7186-0490-6. arXiv: hep-th/0503203.

[111] Shinji Tsujikawa. "Introductory review of cosmic inflation". In: *2nd Tah Poe School on Cosmology: Modern Cosmology*. Apr. 2003. arXiv: hep-ph/0304257.

[112] George Lazarides. "Basics of inflationary cosmology". In: *J. Phys. Conf. Ser.* 53 (2006). Ed. by K. Anagnostopoulos et al., pp. 528–550. DOI: 10.1088/1742-6596/53/1/033. arXiv: hep-ph/0607032.

[113] Andrei D. Linde. "Inflationary Cosmology". In: *Lect. Notes Phys.* 738 (2008), pp. 1–54. DOI: 10.1007/978-3-540-74353-8_1. arXiv: 0705.0164 [hep-th].

[114] Kaloian D. Lozanov. "Lectures on Reheating after Inflation". In: (July 2019). arXiv: 1907.04402 [astro-ph.CO].

[115] Y. Akrami et al. "Planck 2018 results. X. Constraints on inflation". In: *Astron. Astrophys.* 641 (2020), A10. DOI: 10.1051/0004-6361/201833887. arXiv: 1807.06211 [astro-ph.CO].

[116] Michele Cicoli, Joseph P. Conlon, and Fernando Quevedo. "Dark radiation in LARGE volume models". In: *Phys. Rev. D* 87.4 (2013), p. 043520. DOI: 10.1103/PhysRevD.87.043520. arXiv: 1208.3562 [hep-ph].

[117] Tetsutaro Higaki and Fuminobu Takahashi. "Dark Radiation and Dark Matter in Large Volume Compactifications". In: *JHEP* 11 (2012), p. 125. DOI: 10.1007/JHEP11(2012)125. arXiv: 1208.3563 [hep-ph].

[118] Joerg Jaeckel and Wen Yin. "Shining ALP Dark Radiation". In: (Oct. 2021). arXiv: 2110.03692 [hep-ph].

[119] Kiwoon Choi, Eung Jin Chun, and Jihn E. Kim. "Cosmological implications of radiatively generated axion scale". In: *Phys. Lett. B* 403 (1997), pp. 209–217. DOI: 10.1016/S0370-2693(97)00465-6. arXiv: hep-ph/9608222.

[120] Joseph P. Conlon and M. C. David Marsh. "The Cosmophenomenology of Axionic Dark Radiation". In: *JHEP* 10 (2013), p. 214. DOI: 10.1007/JHEP10(2013)214. arXiv: 1304.1804 [hep-ph].

[121] Joseph P. Conlon and Fernando Quevedo. "Kahler moduli inflation". In: *JHEP* 01 (2006), p. 146. DOI: 10.1088/1126-6708/2006/01/146. arXiv: hep-th/0509012.

[122] M. Cicoli, C. P. Burgess, and F. Quevedo. “Fibre Inflation: Observable Gravity Waves from IIB String Compactifications”. In: *JCAP* 03 (2009), p. 013. DOI: 10.1088/1475-7516/2009/03/013. arXiv: 0808.0691 [hep-th].

[123] M. Cicoli and F. Quevedo. “String moduli inflation: An overview”. In: *Class. Quant. Grav.* 28 (2011), p. 204001. DOI: 10.1088/0264-9381/28/20/204001. arXiv: 1108.2659 [hep-th].

[124] C. P. Burgess, M. Cicoli, and F. Quevedo. “String Inflation After Planck 2013”. In: *JCAP* 11 (2013), p. 003. DOI: 10.1088/1475-7516/2013/11/003. arXiv: 1306.3512 [hep-th].

[125] Daniel Baumann and Liam McAllister. *Inflation and String Theory*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, May 2015. ISBN: 978-1-107-08969-3, 978-1-316-23718-2. DOI: 10.1017/CBO9781316105733. arXiv: 1404.2601 [hep-th].

[126] Neil Barnaby et al. “Preheating After Modular Inflation”. In: *JCAP* 12 (2009), p. 021. DOI: 10.1088/1475-7516/2009/12/021. arXiv: 0909.0503 [hep-th].

[127] Michele Cicoli et al. “Moduli Vacuum Misalignment and Precise Predictions in String Inflation”. In: *JCAP* 08 (2016), p. 006. DOI: 10.1088/1475-7516/2016/08/006. arXiv: 1604.08512 [hep-th].

[128] Michele Cicoli and Anupam Mazumdar. “Reheating for Closed String Inflation”. In: *JCAP* 09 (2010), p. 025. DOI: 10.1088/1475-7516/2010/09/025. arXiv: 1005.5076 [hep-th].

[129] C. P. Burgess et al. “Robust Inflation from Fibrous Strings”. In: *JCAP* 05 (2016), p. 032. DOI: 10.1088/1475-7516/2016/05/032. arXiv: 1603.06789 [hep-th].

[130] Michele Cicoli and Gabriel A. Piovano. “Reheating and Dark Radiation after Fibre Inflation”. In: *JCAP* 02 (2019), p. 048. DOI: 10.1088/1475-7516/2019/02/048. arXiv: 1809.01159 [hep-th].

[131] Michele Cicoli, Veronica Guidetti, and Francisco G. Pedro. “Geometrical Destabilisation of Ultra-Light Axions in String Inflation”. In: *JCAP* 05 (2019), p. 046. DOI: 10.1088/1475-7516/2019/05/046. arXiv: 1903.01497 [hep-th].

[132] Michele Cicoli and Eleonora Di Valentino. “Fitting string inflation to real cosmological data: The fiber inflation case”. In: *Phys. Rev. D* 102.4 (2020), p. 043521. DOI: 10.1103/PhysRevD.102.043521. arXiv: 2004.01210 [astro-ph.CO].

[133] Luca Visinelli and Paolo Gondolo. “Dark Matter Axions Revisited”. In: *Phys. Rev. D* 80 (2009), p. 035024. DOI: 10.1103/PhysRevD.80.035024. arXiv: 0903.4377 [astro-ph.CO].

[134] Gonzalo Alonso-Álvarez and Joerg Jaeckel. “Exploring axionlike particles beyond the canonical setup”. In: *Phys. Rev. D* 98.2 (2018), p. 023539. DOI: 10.1103/PhysRevD.98.023539. arXiv: 1712.07500 [hep-ph].

[135] Igor Garcia Irastorza. “An introduction to axions and their detection”. In: *SciPost Phys. Lect. Notes* 45 (2022), p. 1. DOI: 10.21468/SciPostPhysLectNotes.45. arXiv: 2109.07376 [hep-ph].

[136] Joseph P. Conlon. “The QCD axion and moduli stabilisation”. In: *JHEP* 05 (2006), p. 078. DOI: 10.1088/1126-6708/2006/05/078. arXiv: hep-th/0602233.

[137] Michele Cicoli, Mark Goodsell, and Andreas Ringwald. “The type IIB string axiverse and its low-energy phenomenology”. In: *JHEP* 10 (2012), p. 146. DOI: 10.1007/JHEP10(2012)146. arXiv: 1206.0819 [hep-th].

[138] Igor G. Irastorza and Javier Redondo. “New experimental approaches in the search for axion-like particles”. In: *Prog. Part. Nucl. Phys.* 102 (2018), pp. 89–159. DOI: 10.1016/j.ppnp.2018.05.003. arXiv: 1801.08127 [hep-ph].

[139] Luca Di Luzio et al. “The landscape of QCD axion models”. In: *Phys. Rept.* 870 (2020), pp. 1–117. DOI: 10.1016/j.physrep.2020.06.002. arXiv: 2003.01100 [hep-ph].

[140] C. Abel et al. “Measurement of the Permanent Electric Dipole Moment of the Neutron”. In: *Phys. Rev. Lett.* 124.8 (2020), p. 081803. DOI: 10.1103/PhysRevLett.124.081803. arXiv: 2001.11966 [hep-ex].

[141] R. D. Peccei and Helen R. Quinn. “CP Conservation in the Presence of Instantons”. In: *Phys. Rev. Lett.* 38 (1977), pp. 1440–1443. DOI: 10.1103/PhysRevLett.38.1440.

[142] R. D. Peccei and Helen R. Quinn. “Constraints Imposed by CP Conservation in the Presence of Instantons”. In: *Phys. Rev. D* 16 (1977), pp. 1791–1797. DOI: 10.1103/PhysRevD.16.1791.

[143] Giovanni Grilli di Cortona et al. “The QCD axion, precisely”. In: *JHEP* 01 (2016), p. 034. DOI: 10.1007/JHEP01(2016)034. arXiv: 1511.02867 [hep-ph].

[144] Georg G. Raffelt. “Astrophysical axion bounds”. In: *Lect. Notes Phys.* 741 (2008). Ed. by Markus Kuster, Georg Raffelt, and Berta Beltran, pp. 51–71. DOI: 10.1007/978-3-540-73518-2_3. arXiv: hep-ph/0611350.

[145] Luca Visinelli and Paolo Gondolo. “Axion cold dark matter in non-standard cosmologies”. In: *Phys. Rev. D* 81 (2010), p. 063508. DOI: 10.1103/PhysRevD.81.063508. arXiv: 0912.0015 [astro-ph.CO].

[146] Paola Arias et al. “New opportunities for axion dark matter searches in nonstandard cosmological models”. In: (July 2021). arXiv: 2107.13588 [hep-ph].

[147] T. Banks, M. Dine, and M. Graesser. “Supersymmetry, axions and cosmology”. In: *Phys. Rev. D* 68 (2003), p. 075011. DOI: 10.1103/PhysRevD.68.075011. arXiv: hep-ph/0210256.

[148] Ann E. Nelson and Huangyu Xiao. “Axion Cosmology with Early Matter Domination”. In: *Phys. Rev. D* 98.6 (2018), p. 063516. DOI: 10.1103/PhysRevD.98.063516. arXiv: 1807.07176 [astro-ph.CO].

[149] Mark P Hertzberg, Max Tegmark, and Frank Wilczek. “Axion Cosmology and the Energy Scale of Inflation”. In: *Phys. Rev. D* 78 (2008), p. 083507. DOI: 10.1103/PhysRevD.78.083507. arXiv: 0807.1726 [astro-ph].

[150] L. Visinelli and P. Gondolo. “Axion cold dark matter in view of BICEP2 results”. In: *Phys. Rev. Lett.* 113 (2014), p. 011802. DOI: 10.1103/PhysRevLett.113.011802. arXiv: 1403.4594 [hep-ph].

[151] Steven Weinberg. “A New Light Boson?” In: *Phys. Rev. Lett.* 40 (1978), pp. 223–226. DOI: 10.1103/PhysRevLett.40.223.

[152] Frank Wilczek. “Problem of Strong P and T Invariance in the Presence of Instantons”. In: *Phys. Rev. Lett.* 40 (1978), pp. 279–282. DOI: 10.1103/PhysRevLett.40.279.

[153] Joseph P. Conlon. “Seeing the Invisible Axion in the Sparticle Spectrum”. In: *Phys. Rev. Lett.* 97 (2006), p. 261802. DOI: 10.1103/PhysRevLett.97.261802. arXiv: hep-ph/0607138.

[154] Asimina Arvanitaki et al. “String Axiverse”. In: *Phys. Rev. D* 81 (2010), p. 123530. DOI: 10.1103/PhysRevD.81.123530. arXiv: 0905.4720 [hep-th].

[155] David J. E. Marsh. “Axion Cosmology”. In: *Phys. Rept.* 643 (2016), pp. 1–79. DOI: 10.1016/j.physrep.2016.06.005. arXiv: 1510.07633 [astro-ph.CO].

[156] Bobby Samir Acharya and Chakrit Pongkitivanichkul. “The Axiverse induced Dark Radiation Problem”. In: *JHEP* 04 (2016), p. 009. DOI: 10.1007/JHEP04(2016)009. arXiv: 1512.07907 [hep-ph].

[157] Luca Visinelli and Sunny Vagnozzi. “Cosmological window onto the string axiverse and the supersymmetry breaking scale”. In: *Phys. Rev. D* 99.6 (2019), p. 063517. DOI: 10.1103/PhysRevD.99.063517. arXiv: 1809.06382 [hep-ph].

[158] Igor Broeckel et al. “Moduli stabilisation and the statistics of axion physics in the landscape”. In: *JHEP* 08 (2021). [Addendum: JHEP 01, 191 (2022)], p. 059. DOI: 10.1007/JHEP01(2022)191. arXiv: 2105.02889 [hep-th].

[159] Mehmet Demirtas et al. “PQ Axiverse”. In: (Dec. 2021). arXiv: 2112.04503 [hep-th].

[160] Stephen Angus. “Dark Radiation in Anisotropic LARGE Volume Compactifications”. In: *JHEP* 10 (2014), p. 184. DOI: 10.1007/JHEP10(2014)184. arXiv: 1403.6473 [hep-ph].

[161] Michele Cicoli and Francesco Muia. “General Analysis of Dark Radiation in Sequestered String Models”. In: *JHEP* 12 (2015), p. 152. DOI: 10.1007/JHEP12(2015)152. arXiv: 1511.05447 [hep-th].

[162] Bobby S. Acharya et al. “Cosmology in the presence of multiple light moduli”. In: *JCAP* 11 (2019), p. 035. DOI: 10.1088/1475-7516/2019/11/035. arXiv: 1906.03025 [hep-th].

[163] Joerg Jaeckel and Wen Yin. “Using the spectrum of dark radiation as a probe of reheating”. In: *Phys. Rev. D* 103.11 (2021), p. 115019. DOI: 10.1103/PhysRevD.103.115019. arXiv: 2102.00006 [hep-ph].

[164] Stephen Angus, Kang-Sin Choi, and Chang Sub Shin. “Aligned natural inflation in the Large Volume Scenario”. In: *JHEP* 10 (2021), p. 248. DOI: 10.1007/JHEP10(2021)248. arXiv: 2106.09853 [hep-th].

[165] Kwang Sik Jeong and Wan Il Park. “Axion-philic cosmological moduli”. In: (July 2021). arXiv: 2107.13383 [hep-ph].

[166] Andrew R. Frey, Ratul Mahanta, and Anshuman Maharana. “Dark Radiation and the Hagedorn Phase”. In: (Aug. 2021). arXiv: 2108.03317 [hep-th].

[167] Michele Cicoli, Andreas Schachner, and Pramod Shukla. “Systematics of type IIB moduli stabilisation with odd axions”. In: (Sept. 2021). arXiv: 2109.14624 [hep-th].

[168] Soo-Jong Rey. “The Axion Dynamics in Wormhole Background”. In: *Phys. Rev. D* 39 (1989), p. 3185. DOI: 10.1103/PhysRevD.39.3185.

[169] Marc Kamionkowski and John March-Russell. “Planck scale physics and the Peccei-Quinn mechanism”. In: *Phys. Lett. B* 282 (1992), pp. 137–141. DOI: 10.1016/0370-2693(92)90492-M. arXiv: hep-th/9202003.

[170] Richard Holman et al. "Solutions to the strong CP problem in a world with gravity". In: *Phys. Lett. B* 282 (1992), pp. 132–136. DOI: 10.1016/0370-2693(92)90491-L. arXiv: hep-ph/9203206.

[171] Renata Kallosh et al. "Gravity and global symmetries". In: *Phys. Rev. D* 52 (1995), pp. 912–935. DOI: 10.1103/PhysRevD.52.912. arXiv: hep-th/9502069.

[172] Tom Banks et al. "On the possibility of large axion decay constants". In: *JCAP* 06 (2003), p. 001. DOI: 10.1088/1475-7516/2003/06/001. arXiv: hep-th/0303252.

[173] Rodrigo Alonso and Alfredo Urbano. "Wormholes and masses for Goldstone bosons". In: *JHEP* 02 (2019), p. 136. DOI: 10.1007/JHEP02(2019)136. arXiv: 1706.07415 [hep-ph].

[174] Arthur Hebecker, Thomas Mikhail, and Pablo Soler. "Euclidean wormholes, baby universes, and their impact on particle physics and cosmology". In: *Front. Astron. Space Sci.* 5 (2018), p. 35. DOI: 10.3389/fspas.2018.00035. arXiv: 1807.00824 [hep-th].

[175] J. F. G. Cascales et al. "Realistic D-brane models on warped throats: Fluxes, hierarchies and moduli stabilization". In: *JHEP* 02 (2004), p. 031. DOI: 10.1088/1126-6708/2004/02/031. arXiv: hep-th/0312051.

[176] Juan F. G. Cascales, Fouad Saad, and Angel M. Uranga. "Holographic dual of the standard model on the throat". In: *JHEP* 11 (2005), p. 047. DOI: 10.1088/1126-6708/2005/11/047. arXiv: hep-th/0503079.

[177] Sebastian Franco, Amihay Hanany, and Angel M. Uranga. "Multi-flux warped throats and cascading gauge theories". In: *JHEP* 09 (2005), p. 028. DOI: 10.1088/1126-6708/2005/09/028. arXiv: hep-th/0502113.

[178] Keshav Dasgupta, Hassan Firouzjahi, and Rhiannon Gwyn. "On The Warped Heterotic Axion". In: *JHEP* 06 (2008), p. 056. DOI: 10.1088/1126-6708/2008/06/056. arXiv: 0803.3828 [hep-th].

[179] Evgeny I. Buchbinder, Andrei Constantin, and Andre Lukas. "Heterotic QCD axion". In: *Phys. Rev. D* 91.4 (2015), p. 046010. DOI: 10.1103/PhysRevD.91.046010. arXiv: 1412.8696 [hep-th].

[180] Sang Hui Im, Hans Peter Nilles, and Marek Olechowski. "Axion clockworks from heterotic M-theory: the QCD-axion and its ultra-light companion". In: *JHEP* 10 (2019), p. 159. DOI: 10.1007/JHEP10(2019)159. arXiv: 1906.11851 [hep-th].

[181] David Berenstein and Erik Perkins. “Open string axions and the flavor problem”. In: *Phys. Rev. D* 86 (2012), p. 026005. DOI: 10.1103/PhysRevD.86.026005. arXiv: 1202.2073 [hep-th].

[182] Gabriele Honecker and Wieland Staessens. “On axionic dark matter in Type IIA string theory”. In: *Fortsch. Phys.* 62 (2014), pp. 115–151. DOI: 10.1002/prop.201300036. arXiv: 1312.4517 [hep-th].

[183] Gabriele Honecker and Wieland Staessens. “Discrete Abelian gauge symmetries and axions”. In: *J. Phys. Conf. Ser.* 631.1 (2015). Ed. by Nick E. Mavromatos et al., p. 012080. DOI: 10.1088/1742-6596/631/1/012080. arXiv: 1502.00985 [hep-th].

[184] Michele Cicoli et al. “Fuzzy Dark Matter Candidates from String Theory”. In: (Oct. 2021). arXiv: 2110.02964 [hep-th].

[185] Edward W. Kolb and Michael S. Turner. *The Early Universe*. Vol. 69. 1990. ISBN: 978-0-201-62674-2.

[186] Pierre Sikivie. “Axion Cosmology”. In: *Lect. Notes Phys.* 741 (2008). Ed. by Markus Kuster, Georg Raffelt, and Berta Beltran, pp. 19–50. DOI: 10.1007/978-3-540-73518-2_2. arXiv: astro-ph/0610440.

[187] C. J. Hogan and M. J. Rees. “AXION MINICLUSTERS”. In: *Phys. Lett. B* 205 (1988), pp. 228–230. DOI: 10.1016/0370-2693(88)91655-3.

[188] Edward W. Kolb and Igor I. Tkachev. “Axion miniclusters and Bose stars”. In: *Phys. Rev. Lett.* 71 (1993), pp. 3051–3054. DOI: 10.1103/PhysRevLett.71.3051. arXiv: hep-ph/9303313.

[189] Edward W. Kolb and Igor I. Tkachev. “Nonlinear axion dynamics and formation of cosmological pseudosolitons”. In: *Phys. Rev. D* 49 (1994), pp. 5040–5051. DOI: 10.1103/PhysRevD.49.5040. arXiv: astro-ph/9311037.

[190] Luca Visinelli and Javier Redondo. “Axion Miniclusters in Modified Cosmological Histories”. In: *Phys. Rev. D* 101.2 (2020), p. 023008. DOI: 10.1103/PhysRevD.101.023008. arXiv: 1808.01879 [astro-ph.CO].

[191] A. Vilenkin and A. E. Everett. “Cosmic Strings and Domain Walls in Models with Goldstone and PseudoGoldstone Bosons”. In: *Phys. Rev. Lett.* 48 (1982), pp. 1867–1870. DOI: 10.1103/PhysRevLett.48.1867.

[192] C. Hagmann, Sanghyeon Chang, and P. Sikivie. “Axion radiation from strings”. In: *Phys. Rev. D* 63 (2001), p. 125018. DOI: 10.1103/PhysRevD.63.125018. arXiv: hep-ph/0012361.

[193] Masahiro Kawasaki, Ken’ichi Saikawa, and Toyokazu Sekiguchi. “Axion dark matter from topological defects”. In: *Phys. Rev. D* 91.6 (2015), p. 065014. DOI: 10.1103/PhysRevD.91.065014. arXiv: 1412.0789 [hep-ph].

[194] Andreas Ringwald and Ken'ichi Saikawa. "Axion dark matter in the post-inflationary Peccei-Quinn symmetry breaking scenario". In: *Phys. Rev. D* 93.8 (2016). [Addendum: Phys.Rev.D 94, 049908 (2016)], p. 085031. DOI: 10.1103/PhysRevD.93.085031. arXiv: 1512.06436 [hep-ph].

[195] John March-Russell and Hannah Tillim. "Axiverse Strings". In: (Sept. 2021). arXiv: 2109.14637 [hep-th].

[196] John Preskill, Mark B. Wise, and Frank Wilczek. "Cosmology of the Invisible Axion". In: *Phys. Lett.* 120B (1983), pp. 127–132. DOI: 10.1016/0370-2693(83)90637-8.

[197] L.F. Abbott and P. Sikivie. "A Cosmological Bound on the Invisible Axion". In: *Phys. Lett. B* 120 (1983), pp. 133–136. DOI: 10.1016/0370-2693(83)90638-X.

[198] Michael Dine and Willy Fischler. "The Not So Harmless Axion". In: *Phys. Lett.* 120B (1983), pp. 137–141. DOI: 10.1016/0370-2693(83)90639-1.

[199] M. Kawasaki, Kazunori Kohri, and Naoshi Sugiyama. "Cosmological constraints on late time entropy production". In: *Phys. Rev. Lett.* 82 (1999), p. 4168. DOI: 10.1103/PhysRevLett.82.4168. arXiv: astro-ph/9811437 [astro-ph].

[200] M. Kawasaki, Kazunori Kohri, and Naoshi Sugiyama. "MeV scale reheating temperature and thermalization of neutrino background". In: *Phys. Rev.* D62 (2000), p. 023506. DOI: 10.1103/PhysRevD.62.023506. arXiv: astro-ph/0002127 [astro-ph].

[201] Steen Hannestad. "What is the lowest possible reheating temperature?" In: *Phys. Rev.* D70 (2004), p. 043506. DOI: 10.1103/PhysRevD.70.043506. arXiv: astro-ph/0403291 [astro-ph].

[202] Kazuhide Ichikawa, Masahiro Kawasaki, and Fuminobu Takahashi. "Constraint on the Effective Number of Neutrino Species from the WMAP and SDSS LRG Power Spectra". In: *JCAP* 0705 (2007), p. 007. DOI: 10.1088/1475-7516/2007/05/007. arXiv: astro-ph/0611784 [astro-ph].

[203] Francesco De Bernardis, Luca Pagano, and Alessandro Melchiorri. "New constraints on the reheating temperature of the universe after WMAP-5". In: *Astropart. Phys.* 30 (2008), pp. 192–195. DOI: 10.1016/j.astropartphys.2008.09.005.

[204] P. F. de Salas et al. "Bounds on very low reheating scenarios after Planck". In: *Phys. Rev.* D92.12 (2015), p. 123534. DOI: 10.1103/PhysRevD.92.123534. arXiv: 1511.00672 [astro-ph.CO].

[205] Masahiro Kawasaki et al. “Revisiting Big-Bang Nucleosynthesis Constraints on Long-Lived Decaying Particles”. In: *Phys. Rev.* D97.2 (2018), p. 023502. DOI: 10.1103/PhysRevD.97.023502. arXiv: 1709.01211 [hep-ph].

[206] Marco Hufnagel, Kai Schmidt-Hoberg, and Sebastian Wild. “BBN constraints on MeV-scale dark sectors. Part II. Electromagnetic decays”. In: *JCAP* 11 (2018), p. 032. DOI: 10.1088/1475-7516/2018/11/032. arXiv: 1808.09324 [hep-ph].

[207] Lindsay Forestell, David E. Morrissey, and Graham White. “Limits from BBN on Light Electromagnetic Decays”. In: *JHEP* 01 (2019), p. 074. DOI: 10.1007/JHEP01(2019)074. arXiv: 1809.01179 [hep-ph].

[208] Takuya Hasegawa et al. “MeV-scale reheating temperature and thermalization of oscillating neutrinos by radiative and hadronic decays of massive particles”. In: *JCAP* 1912 (2019), p. 012. DOI: 10.1088/1475-7516/2019/12/012. arXiv: 1908.10189 [hep-ph].

[209] Masahiro Kawasaki et al. “Big-bang nucleosynthesis with sub-GeV massive decaying particles”. In: *JCAP* 12 (2020), p. 048. DOI: 10.1088/1475-7516/2020/12/048. arXiv: 2006.14803 [hep-ph].

[210] Paul Frederik Depta, Marco Hufnagel, and Kai Schmidt-Hoberg. “Updated BBN constraints on electromagnetic decays of MeV-scale particles”. In: (Nov. 2020). arXiv: 2011.06519 [hep-ph].

[211] A. Brignole, Luis E. Ibanez, and C. Munoz. “Soft supersymmetry breaking terms from supergravity and superstring models”. In: *Adv. Ser. Direct. High Energy Phys.* 18 (1998), pp. 125–148. DOI: 10.1142/9789812839657_0003. arXiv: hep-ph/9707209.

[212] Arthur Hebecker, Alexander K. Knochel, and Timo Weigand. “A Shift Symmetry in the Higgs Sector: Experimental Hints and Stringy Realizations”. In: *JHEP* 06 (2012), p. 093. DOI: 10.1007/JHEP06(2012)093. arXiv: 1204.2551 [hep-th].

[213] Arthur Hebecker, Alexander K. Knochel, and Timo Weigand. “The Higgs mass from a String-Theoretic Perspective”. In: *Nucl. Phys. B* 874 (2013), pp. 1–35. DOI: 10.1016/j.nuclphysb.2013.05.004. arXiv: 1304.2767 [hep-th].

[214] Ignatios Antoniadis et al. “Effective mu term in superstring theory”. In: *Nucl. Phys. B* 432 (1994), pp. 187–204. DOI: 10.1016/0550-3213(94)90599-1. arXiv: hep-th/9405024.

[215] Stephen P. Martin. “A Supersymmetry primer”. In: *Adv. Ser. Direct. High Energy Phys.* 18 (1998). Ed. by Gordon L. Kane, pp. 1–98. DOI: 10.1142/9789812839657_0001. arXiv: hep-ph/9709356.

[216] Joseph P. Conlon and Fernando Quevedo. “Gaugino and Scalar Masses in the Landscape”. In: *JHEP* 06 (2006), p. 029. DOI: 10.1088/1126-6708/2006/06/029. arXiv: hep-th/0605141.

[217] Sven Krippendorf and Fernando Quevedo. “Metastable SUSY Breaking, de Sitter Moduli Stabilisation and Kahler Moduli Inflation”. In: *JHEP* 11 (2009), p. 039. DOI: 10.1088/1126-6708/2009/11/039. arXiv: 0901.0683 [hep-th].

[218] Gero von Gersdorff and Arthur Hebecker. “Kahler corrections for the volume modulus of flux compactifications”. In: *Phys. Lett. B* 624 (2005), pp. 270–274. DOI: 10.1016/j.physletb.2005.08.024. arXiv: hep-th/0507131.

[219] Marcus Berg, Michael Haack, and Enrico Pajer. “Jumping Through Loops: On Soft Terms from Large Volume Compactifications”. In: *JHEP* 09 (2007), p. 031. DOI: 10.1088/1126-6708/2007/09/031. arXiv: 0704.0737 [hep-th].

[220] J. Richard Bond et al. “Roulette inflation with Kahler moduli and their axions”. In: *Phys. Rev. D* 75 (2007), p. 123511. DOI: 10.1103/PhysRevD.75.123511. arXiv: hep-th/0612197.

[221] J. Beacham et al. “Physics Beyond Colliders at CERN: Beyond the Standard Model Working Group Report”. In: *J. Phys. G* 47.1 (2020), p. 010501. DOI: 10.1088/1361-6471/ab4cd2. arXiv: 1901.09966 [hep-ex].

[222] Prateek Agrawal et al. “Feebly-interacting particles: FIPs 2020 workshop report”. In: *Eur. Phys. J. C* 81.11 (2021), p. 1015. DOI: 10.1140/epjc/s10052-021-09703-7. arXiv: 2102.12143 [hep-ph].

[223] Andrei D. Linde. “Hybrid inflation”. In: *Phys. Rev. D* 49 (1994), pp. 748–754. DOI: 10.1103/PhysRevD.49.748. arXiv: astro-ph/9307002.

[224] P. Binetruy and G. R. Dvali. “D term inflation”. In: *Phys. Lett. B* 388 (1996), pp. 241–246. DOI: 10.1016/S0370-2693(96)01083-0. arXiv: hep-ph/9606342.

[225] Edi Halyo. “Hybrid inflation from supergravity D terms”. In: *Phys. Lett. B* 387 (1996), pp. 43–47. DOI: 10.1016/0370-2693(96)01001-5. arXiv: hep-ph/9606423.

[226] Andrei D. Linde and Antonio Riotto. “Hybrid inflation in supergravity”. In: *Phys. Rev. D* 56 (1997), R1841–R1844. DOI: 10.1103/PhysRevD.56.R1841. arXiv: hep-ph/9703209.

[227] Keshav Dasgupta et al. "D3 / D7 inflationary model and M theory". In: *Phys. Rev. D* 65 (2002), p. 126002. DOI: 10.1103/PhysRevD.65.126002. arXiv: hep-th/0203019.

[228] Maximilian Arends et al. "D7-Brane Moduli Space in Axion Monodromy and Fluxbrane Inflation". In: *Fortsch. Phys.* 62 (2014), pp. 647–702. DOI: 10.1002/prop.201400045. arXiv: 1405.0283 [hep-th].

[229] Federico Carta et al. "Harmonic hybrid inflation". In: *JHEP* 12 (2020), p. 161. DOI: 10.1007/JHEP12(2020)161. arXiv: 2007.04322 [hep-th].

[230] Ignatios Antoniadis, Osmin Lacombe, and George K. Leontaris. "Hybrid inflation and waterfall field in string theory from D7-branes". In: *JHEP* 01 (2022), p. 011. DOI: 10.1007/JHEP01(2022)011. arXiv: 2109.03243 [hep-th].

[231] Iosif Bena, Mariana Grana, and Nick Halmagyi. "On the Existence of Metastable Vacua in Klebanov-Strassler". In: *JHEP* 09 (2010), p. 087. DOI: 10.1007/JHEP09(2010)087. arXiv: 0912.3519 [hep-th].

[232] Jock McOrist and Savdeep Sethi. "M-theory and Type IIA Flux Compactifications". In: *JHEP* 12 (2012), p. 122. DOI: 10.1007/JHEP12(2012)122. arXiv: 1208.0261 [hep-th].

[233] Keshav Dasgupta et al. "de Sitter Vacua in Type IIB String Theory: Classical Solutions and Quantum Corrections". In: *JHEP* 07 (2014), p. 054. DOI: 10.1007/JHEP07(2014)054. arXiv: 1402.5112 [hep-th].

[234] Iosif Bena et al. "Giant Tachyons in the Landscape". In: *JHEP* 02 (2015), p. 146. DOI: 10.1007/JHEP02(2015)146. arXiv: 1410.7776 [hep-th].

[235] Callum Quigley. "Gaugino Condensation and the Cosmological Constant". In: *JHEP* 06 (2015), p. 104. DOI: 10.1007/JHEP06(2015)104. arXiv: 1504.00652 [hep-th].

[236] Diego Cohen-Maldonado et al. "Observations on fluxes near anti-branes". In: *JHEP* 01 (2016), p. 126. DOI: 10.1007/JHEP01(2016)126. arXiv: 1507.01022 [hep-th].

[237] Daniel Junghans and Marco Zagermann. "A Universal Tachyon in Nearly No-scale de Sitter Compactifications". In: *JHEP* 07 (2018), p. 078. DOI: 10.1007/JHEP07(2018)078. arXiv: 1612.06847 [hep-th].

[238] Jakob Moritz, Ander Retolaza, and Alexander Westphal. "Toward de Sitter space from ten dimensions". In: *Phys. Rev. D* 97.4 (2018), p. 046010. DOI: 10.1103/PhysRevD.97.046010. arXiv: 1707.08678 [hep-th].

[239] Savdeep Sethi. "Supersymmetry Breaking by Fluxes". In: *JHEP* 10 (2018), p. 022. DOI: 10.1007/JHEP10(2018)022. arXiv: 1709.03554 [hep-th].

[240] Ulf H. Danielsson and Thomas Van Riet. “What if string theory has no de Sitter vacua?” In: *Int. J. Mod. Phys. D* 27.12 (2018), p. 1830007. DOI: 10.1142/S0218271818300070. arXiv: 1804.01120 [hep-th].

[241] Jakob Moritz and Thomas Van Riet. “Racing through the swampland: de Sitter uplift vs weak gravity”. In: *JHEP* 09 (2018), p. 099. DOI: 10.1007/JHEP09(2018)099. arXiv: 1805.00944 [hep-th].

[242] Michele Cicoli et al. “De Sitter vs Quintessence in String Theory”. In: *Fortsch. Phys.* 67.1-2 (2019), p. 1800079. DOI: 10.1002/prop.201800079. arXiv: 1808.08967 [hep-th].

[243] Shamit Kachru and Sandip P. Trivedi. “A comment on effective field theories of flux vacua”. In: *Fortsch. Phys.* 67.1-2 (2019), p. 1800086. DOI: 10.1002/prop.201800086. arXiv: 1808.08971 [hep-th].

[244] Renata Kallosh and Timm Wrase. “dS Supergravity from 10d”. In: *Fortsch. Phys.* 67.1-2 (2019), p. 1800071. DOI: 10.1002/prop.201800071. arXiv: 1808.09427 [hep-th].

[245] Iosif Bena et al. “Uplifting Runaways”. In: *Fortsch. Phys.* 67.1-2 (2019), p. 1800100. DOI: 10.1002/prop.201800100. arXiv: 1809.06861 [hep-th].

[246] Renata Kallosh et al. “4D models of de Sitter uplift”. In: *Phys. Rev. D* 99.4 (2019), p. 046006. DOI: 10.1103/PhysRevD.99.046006. arXiv: 1809.09018 [hep-th].

[247] Arthur Hebecker and Timm Wrase. “The Asymptotic dS Swampland Conjecture - a Simplified Derivation and a Potential Loophole”. In: *Fortsch. Phys.* 67.1-2 (2019), p. 1800097. DOI: 10.1002/prop.201800097. arXiv: 1810.08182 [hep-th].

[248] F. F. Gautason, V. Van Hemelryck, and T. Van Riet. “The Tension between 10D Supergravity and dS Uplifts”. In: *Fortsch. Phys.* 67.1-2 (2019), p. 1800091. DOI: 10.1002/prop.201800091. arXiv: 1810.08518 [hep-th].

[249] Jonathan J. Heckman et al. “F-theory and Dark Energy”. In: *Fortsch. Phys.* 67.10 (2019), p. 1900057. DOI: 10.1002/prop.201900057. arXiv: 1811.01959 [hep-th].

[250] Daniel Junghans. “Weakly Coupled de Sitter Vacua with Fluxes and the Swampland”. In: *JHEP* 03 (2019), p. 150. DOI: 10.1007/JHEP03(2019)150. arXiv: 1811.06990 [hep-th].

[251] Jay Armas et al. “Meta-stable non-extremal anti-branes”. In: *Phys. Rev. Lett.* 122.18 (2019), p. 181601. DOI: 10.1103/PhysRevLett.122.181601. arXiv: 1812.01067 [hep-th].

[252] F. F. Gautason et al. "A 10d view on the KKLT AdS vacuum and uplifting". In: *JHEP* 06 (2020), p. 074. DOI: 10.1007/JHEP06(2020)074. arXiv: 1902.01415 [hep-th].

[253] Ralph Blumenhagen, Daniel Kläwer, and Lorenz Schlechter. "Swampland Variations on a Theme by KKLT". In: *JHEP* 05 (2019), p. 152. DOI: 10.1007/JHEP05(2019)152. arXiv: 1902.07724 [hep-th].

[254] Iosif Bena et al. "Kähler moduli stabilization from ten dimensions". In: *JHEP* 10 (2019), p. 200. DOI: 10.1007/JHEP10(2019)200. arXiv: 1908.01785 [hep-th].

[255] Keshav Dasgupta et al. "de Sitter vacua in the string landscape". In: *Nucl. Phys. B* 969 (2021), p. 115463. DOI: 10.1016/j.nuclphysb.2021.115463. arXiv: 1908.05288 [hep-th].

[256] Yuta Hamada et al. "On brane gaugino condensates in 10d". In: *JHEP* 04 (2019), p. 008. DOI: 10.1007/JHEP04(2019)008. arXiv: 1812.06097 [hep-th].

[257] Renata Kallosh. "Gaugino Condensation and Geometry of the Perfect Square". In: *Phys. Rev. D* 99.6 (2019), p. 066003. DOI: 10.1103/PhysRevD.99.066003. arXiv: 1901.02023 [hep-th].

[258] Yuta Hamada et al. "Understanding KKLT from a 10d perspective". In: *JHEP* 06 (2019), p. 019. DOI: 10.1007/JHEP06(2019)019. arXiv: 1902.01410 [hep-th].

[259] Federico Carta, Jakob Moritz, and Alexander Westphal. "Gaugino condensation and small uplifts in KKLT". In: *JHEP* 08 (2019), p. 141. DOI: 10.1007/JHEP08(2019)141. arXiv: 1902.01412 [hep-th].

[260] Shamit Kachru et al. "de Sitter vacua from ten dimensions". In: *JHEP* 12 (2021), p. 111. DOI: 10.1007/JHEP12(2021)111. arXiv: 1908.04788 [hep-th].

[261] Xin Gao, Arthur Hebecker, and Daniel Junghans. "Control issues of KKLT". In: *Fortsch. Phys.* 68 (2020), p. 2000089. DOI: 10.1002/prop.202000089. arXiv: 2009.03914 [hep-th].

[262] Georges Obied et al. "De Sitter Space and the Swampland". In: (June 2018). arXiv: 1806.08362 [hep-th].

[263] Sumit K. Garg and Chethan Krishnan. "Bounds on Slow Roll and the de Sitter Swampland". In: *JHEP* 11 (2019), p. 075. DOI: 10.1007/JHEP11(2019)075. arXiv: 1807.05193 [hep-th].

[264] Hirosi Ooguri et al. "Distance and de Sitter Conjectures on the Swampland". In: *Phys. Lett. B* 788 (2019), pp. 180–184. DOI: 10.1016/j.physletb.2018.11.018. arXiv: 1810.05506 [hep-th].

[265] Prateek Agrawal et al. "On the Cosmological Implications of the String Swampland". In: *Phys. Lett. B* 784 (2018), pp. 271–276. DOI: 10.1016/j.physletb.2018.07.040. arXiv: 1806.09718 [hep-th].

[266] Lavinia Heisenberg et al. "Dark Energy in the Swampland". In: *Phys. Rev. D* 98.12 (2018), p. 123502. DOI: 10.1103/PhysRevD.98.123502. arXiv: 1808.02877 [astro-ph.CO].

[267] Yashar Akrami et al. "The Landscape, the Swampland and the Era of Precision Cosmology". In: *Fortsch. Phys.* 67.1-2 (2019), p. 1800075. DOI: 10.1002/prop.201800075. arXiv: 1808.09440 [hep-th].

[268] Marco Raveri, Wayne Hu, and Savdeep Sethi. "Swampland Conjectures and Late-Time Cosmology". In: *Phys. Rev. D* 99.8 (2019), p. 083518. DOI: 10.1103/PhysRevD.99.083518. arXiv: 1812.10448 [hep-th].

[269] P. J. E. Peebles and Bharat Ratra. "Cosmology with a Time Variable Cosmological Constant". In: *Astrophys. J. Lett.* 325 (1988), p. L17. DOI: 10.1086/185100.

[270] R. R. Caldwell, Rahul Dave, and Paul J. Steinhardt. "Cosmological imprint of an energy component with general equation of state". In: *Phys. Rev. Lett.* 80 (1998), pp. 1582–1585. DOI: 10.1103/PhysRevLett.80.1582. arXiv: astro-ph/9708069.

[271] Simeon Hellerman, Nemanja Kaloper, and Leonard Susskind. "String theory and quintessence". In: *JHEP* 06 (2001), p. 003. DOI: 10.1088/1126-6708/2001/06/003. arXiv: hep-th/0104180.

[272] Chien-I Chiang and Hitoshi Murayama. "Building Supergravity Quintessence Model". In: (Aug. 2018). arXiv: 1808.02279 [hep-th].

[273] M. C. David Marsh. "The Swampland, Quintessence and the Vacuum Energy". In: *Phys. Lett. B* 789 (2019), pp. 639–642. DOI: 10.1016/j.physletb.2018.11.001. arXiv: 1809.00726 [hep-th].

[274] Chengcheng Han, Shi Pi, and Misao Sasaki. "Quintessence Saves Higgs Instability". In: *Phys. Lett. B* 791 (2019), pp. 314–318. DOI: 10.1016/j.physletb.2019.02.037. arXiv: 1809.05507 [hep-ph].

[275] Bobby Samir Acharya, Anshuman Maharana, and Francesco Muia. "Hidden Sectors in String Theory: Kinetic Mixings, Fifth Forces and Quintessence". In: *JHEP* 03 (2019), p. 048. DOI: 10.1007/JHEP03(2019)048. arXiv: 1811.10633 [hep-th].

[276] Mark P. Hertzberg, McCullen Sandora, and Mark Trodden. "Quantum Fine-Tuning in Stringy Quintessence Models". In: *Phys. Lett. B* 797 (2019), p. 134878. DOI: 10.1016/j.physletb.2019.134878. arXiv: 1812.03184 [hep-th].

[277] Carsten van de Bruck and Cameron C. Thomas. "Dark Energy, the Swampland and the Equivalence Principle". In: *Phys. Rev. D* 100.2 (2019), p. 023515. DOI: 10.1103/PhysRevD.100.023515. arXiv: 1904.07082 [hep-th].

[278] Iason Baldes, Debtosh Chowdhury, and Michel H. G. Tytgat. "Forays into the dark side of the swamp". In: *Phys. Rev. D* 100.9 (2019), p. 095009. DOI: 10.1103/PhysRevD.100.095009. arXiv: 1907.06663 [hep-ph].

[279] Michele Cicoli, Francisco G. Pedro, and Gianmassimo Tasinato. "Natural Quintessence in String Theory". In: *JCAP* 07 (2012), p. 044. DOI: 10.1088/1475-7516/2012/07/044. arXiv: 1203.6655 [hep-th].

[280] Yessenia Olguin-Trejo et al. "Runaway Quintessence, Out of the Swampland". In: *JCAP* 01 (2019), p. 031. DOI: 10.1088/1475-7516/2019/01/031. arXiv: 1810.08634 [hep-th].

[281] Maxim Emelin and Radu Tatar. "Axion Hilltops, Kahler Modulus Quintessence and the Swampland Criteria". In: *Int. J. Mod. Phys. A* 34.28 (2019), p. 1950164. DOI: 10.1142/S0217751X19501641. arXiv: 1811.07378 [hep-th].

[282] Yasunori Nomura, T. Watari, and T. Yanagida. "Quintessence axion potential induced by electroweak instanton effects". In: *Phys. Lett. B* 484 (2000), pp. 103–111. DOI: 10.1016/S0370-2693(00)00605-5. arXiv: hep-ph/0004182.

[283] Peter Svrcek. "Cosmological Constant and Axions in String Theory". In: (July 2006). arXiv: hep-th/0607086.

[284] Sudhakar Panda, Yoske Sumitomo, and Sandip P. Trivedi. "Axions as Quintessence in String Theory". In: *Phys. Rev. D* 83 (2011), p. 083506. DOI: 10.1103/PhysRevD.83.083506. arXiv: 1011.5877 [hep-th].

[285] Guido D'Amico, Nemanja Kaloper, and Albion Lawrence. "Strongly Coupled Quintessence". In: *Phys. Rev. D* 100.10 (2019), p. 103504. DOI: 10.1103/PhysRevD.100.103504. arXiv: 1809.05109 [hep-th].

[286] Masahito Ibe, Masahito Yamazaki, and Tsutomu T. Yanagida. "Quintessence Axion Revisited in Light of Swampland Conjectures". In: *Class. Quant. Grav.* 36.23 (2019), p. 235020. DOI: 10.1088/1361-6382/ab5197. arXiv: 1811.04664 [hep-th].

[287] M. Cicoli, C. P. Burgess, and F. Quevedo. "Anisotropic Modulus Stabilisation: Strings at LHC Scales with Micron-sized Extra Dimensions". In: *JHEP* 10 (2011), p. 119. DOI: 10.1007/JHEP10(2011)119. arXiv: 1105.2107 [hep-th].

[288] Marcus Berg, Michael Haack, and Boris Kors. "On volume stabilization by quantum corrections". In: *Phys. Rev. Lett.* 96 (2006), p. 021601. DOI: 10.1103/PhysRevLett.96.021601. arXiv: hep-th/0508171.

[289] Shinji Tsujikawa. "Quintessence: A Review". In: *Class. Quant. Grav.* 30 (2013), p. 214003. DOI: 10.1088/0264-9381/30/21/214003. arXiv: 1304.1961 [gr-qc].

[290] Thibault Damour and John F. Donoghue. "Equivalence Principle Violations and Couplings of a Light Dilaton". In: *Phys. Rev. D* 82 (2010), p. 084033. DOI: 10.1103/PhysRevD.82.084033. arXiv: 1007.2792 [gr-qc].

[291] Michele Cicoli. "String Loop Moduli Stabilisation and Cosmology in IIB Flux Compactifications". In: *Fortsch. Phys.* 58 (2010), pp. 115–338. DOI: 10.1002/prop.200900096. arXiv: 0907.0665 [hep-th].

[292] Sidney R. Coleman and Erick J. Weinberg. "Radiative Corrections as the Origin of Spontaneous Symmetry Breaking". In: *Phys. Rev. D* 7 (1973), pp. 1888–1910. DOI: 10.1103/PhysRevD.7.1888.

[293] Sergio Ferrara, Costas Kounnas, and Fabio Zwirner. "Mass formulae and natural hierarchy in string effective supergravities". In: *Nucl. Phys. B* 429 (1994). [Erratum: Nucl.Phys.B 433, 255–255 (1995)], pp. 589–625. DOI: 10.1016/0550-3213(94)00494-Y. arXiv: hep-th/9405188.

[294] A. Brignole, Luis E. Ibanez, and C. Munoz. "Soft supersymmetry breaking terms from supergravity and superstring models". In: *Adv. Ser. Direct. High Energy Phys.* 18 (1998), pp. 125–148. DOI: 10.1142/9789812839657_0003. arXiv: hep-ph/9707209.

[295] Savas Dimopoulos, Kiel Howe, and John March-Russell. "Maximally Natural Supersymmetry". In: *Phys. Rev. Lett.* 113 (2014), p. 111802. DOI: 10.1103/PhysRevLett.113.111802. arXiv: 1404.7554 [hep-ph].

[296] Savas Dimopoulos et al. "Auto-Concealment of Supersymmetry in Extra Dimensions". In: *JHEP* 06 (2015), p. 041. DOI: 10.1007/JHEP06(2015)041. arXiv: 1412.0805 [hep-ph].

[297] Isabel Garcia Garcia, Kiel Howe, and John March-Russell. "Natural Scherk-Schwarz Theories of the Weak Scale". In: *JHEP* 12 (2015), p. 005. DOI: 10.1007/JHEP12(2015)005. arXiv: 1510.07045 [hep-ph].

[298] Andrea Brignole, Ferruccio Feruglio, and Fabio Zwirner. "Signals of a superlight gravitino at e^+e^- colliders when the other superparticles are heavy". In: *Nucl. Phys. B* 516 (1998). [Erratum: Nucl.Phys.B 555, 653–655 (1999)], pp. 13–28. DOI: 10.1016/S0550-3213(97)00825-0. arXiv: hep-ph/9711516.

[299] Markus A. Luty and Eduardo Ponton. "Effective Lagrangians and light gravitino phenomenology". In: *Phys. Rev. D* 57 (1998), pp. 4167–4173. DOI: 10.1103/PhysRevD.57.4167. arXiv: hep-ph/9706268.

[300] G. Abbiendi et al. "Photonic events with missing energy in e+ e- collisions at S**(1/2) = 189-GeV". In: *Eur. Phys. J. C* 18 (2000), pp. 253–272. DOI: 10.1007/s100520000522. arXiv: hep-ex/0005002.

[301] A. Heister et al. "Single photon and multiphoton production in e^+e^- collisions at $\sqrt{s}$ up to 209-GeV". In: *Eur. Phys. J. C* 28 (2003), pp. 1–13. DOI: 10.1140/epjc/s2002-01129-7.

[302] P. Achard et al. "Single photon and multiphoton events with missing energy in e^+e^- collisions at LEP". In: *Phys. Lett. B* 587 (2004), pp. 16–32. DOI: 10.1016/j.physletb.2004.01.010. arXiv: hep-ex/0402002.

[303] J. Abdallah et al. "Photon events with missing energy in e+ e- collisions at s**(1/2) = 130-GeV to 209-GeV". In: *Eur. Phys. J. C* 38 (2005), pp. 395–411. DOI: 10.1140/epjc/s2004-02051-8. arXiv: hep-ex/0406019.

[304] I. Antoniadis et al. "Nonlinear supersymmetry and goldstino couplings to the MSSM". In: *Theor. Math. Phys.* 170 (2012), pp. 26–38. DOI: 10.1007/s11232-012-0004-y.

[305] Kentarou Mawatari and Bettina Oexl. "Monophoton signals in light gravitino production at e^+e^- colliders". In: *Eur. Phys. J. C* 74.6 (2014), p. 2909. DOI: 10.1140/epjc/s10052-014-2909-0. arXiv: 1402.3223 [hep-ph].

[306] Andrea Brignole et al. "Signals of a superlight gravitino at hadron colliders when the other superparticles are heavy". In: *Nucl. Phys. B* 526 (1998). [Erratum: Nucl.Phys.B 582, 759–761 (2000)], pp. 136–152. DOI: 10.1016/S0550-3213(98)00254-5. arXiv: hep-ph/9801329.

[307] D. Acosta et al. "Limits on Extra Dimensions and New Particle Production in the Exclusive Photon and Missing Energy Signature in $p\bar{p}$ Collisions at $\sqrt{s} = 1.8$ TeV". In: *Phys. Rev. Lett.* 89 (2002), p. 281801. DOI: 10.1103/PhysRevLett.89.281801. arXiv: hep-ex/0205057.

[308] Michael Klasen and Guillaume Pignol. "New Results for Light Gravitinos at Hadron Colliders: Tevatron Limits and LHC Perspectives". In: *Phys. Rev. D* 75 (2007), p. 115003. DOI: 10.1103/PhysRevD.75.115003. arXiv: hep-ph/0610160.

[309] P. de Aquino et al. “Light Gravitino Production in Association with Gluinos at the LHC”. In: *JHEP* 10 (2012), p. 008. DOI: 10.1007/JHEP10(2012)008. arXiv: 1206.7098 [hep-ph].

[310] Georges Aad et al. “Search for new phenomena in final states with an energetic jet and large missing transverse momentum in pp collisions at $\sqrt{s}$ =8 TeV with the ATLAS detector”. In: *Eur. Phys. J. C* 75.7 (2015). [Erratum: Eur.Phys.J.C 75, 408 (2015)], p. 299. DOI: 10.1140/epjc/s10052-015-3517-3. arXiv: 1502.01518 [hep-ex].

[311] R. Casalbuoni et al. “A GRAVITINO - GOLDSTINO HIGH-ENERGY EQUIVALENCE THEOREM”. In: *Phys. Lett. B* 215 (1988), pp. 313–316. DOI: 10.1016/0370-2693(88)91439-6.

[312] R. Casalbuoni et al. “High-Energy Equivalence Theorem in Spontaneously Broken Supergravity”. In: *Phys. Rev. D* 39 (1989), p. 2281. DOI: 10.1103/PhysRevD.39.2281.

[313] “Search for New Phenomena in Monojet plus Missing Transverse Momentum Final States using 10fb-1 of pp Collisions at sqrts=8 TeV with the ATLAS detector at the LHC”. In: (Nov. 2012).

[314] Fabio Maltoni et al. “Signals of a superlight gravitino at the LHC”. In: *JHEP* 04 (2015), p. 021. DOI: 10.1007/JHEP04(2015)021. arXiv: 1502.01637 [hep-ph].

[315] C. P. Burgess. “Supersymmetric large extra dimensions”. In: *39th Rencontres de Moriond on Electroweak Interactions and Unified Theories*. June 2004, pp. 109–114. arXiv: hep-ph/0406214.

[316] Hans-Peter Nilles, Antonios Papazoglou, and Gianmassimo Tasinato. “Self-tuning and its footprints”. In: *Nucl. Phys. B* 677 (2004), pp. 405–429. DOI: 10.1016/j.nuclphysb.2003.11.003. arXiv: hep-th/0309042.

[317] C. P. Burgess. “Supersymmetric large extra dimensions and the cosmological constant problem”. In: *Can. J. Phys.* 84 (2006), p. 463. DOI: 10.1139/P06-031. arXiv: hep-th/0510123.

[318] C. P. Burgess and L. van Nierop. “Large Dimensions and Small Curvatures from Supersymmetric Brane Back-reaction”. In: *JHEP* 04 (2011), p. 078. DOI: 10.1007/JHEP04(2011)078. arXiv: 1101.0152 [hep-th].

[319] Jonathan J. Heckman and Cumrun Vafa. “Fine Tuning, Sequestering, and the Swampland”. In: *Phys. Lett. B* 798 (2019), p. 135004. DOI: 10.1016/j.physletb.2019.135004. arXiv: 1905.06342 [hep-th].

[320] Nima Arkani-Hamed et al. “The String landscape, black holes and gravity as the weakest force”. In: *JHEP* 06 (2007), p. 060. DOI: 10.1088/1126-6708/2007/06/060. arXiv: hep-th/0601001.

[321] Anton de la Fuente, Prashant Saraswat, and Raman Sundrum. “Natural Inflation and Quantum Gravity”. In: *Phys. Rev. Lett.* 114.15 (2015), p. 151303. DOI: 10.1103/PhysRevLett.114.151303. arXiv: 1412.3457 [hep-th].

[322] Arthur Hebecker and Pablo Soler. “The Weak Gravity Conjecture and the Axionic Black Hole Paradox”. In: *JHEP* 09 (2017), p. 036. DOI: 10.1007/JHEP09(2017)036. arXiv: 1702.06130 [hep-th].

[323] Arthur Hebecker and Philipp Henkenjohann. “Gauge and gravitational instantons: From 3-forms and fermions to Weak Gravity and flat axion potentials”. In: *JHEP* 09 (2019), p. 038. DOI: 10.1007/JHEP09(2019)038. arXiv: 1906.07728 [hep-th].

[324] Sylvain Fichet and Prashant Saraswat. “Approximate Symmetries and Gravity”. In: *JHEP* 01 (2020), p. 088. DOI: 10.1007/JHEP01(2020)088. arXiv: 1909.02002 [hep-th].

[325] Edward Hardy and Susha Parameswaran. “Thermal Dark Energy”. In: *Phys. Rev. D* 101.2 (2020), p. 023503. DOI: 10.1103/PhysRevD.101.023503. arXiv: 1907.10141 [hep-th].

[326] A. Pourtsidou, C. Skordis, and E. J. Copeland. “Models of dark matter coupled to dark energy”. In: *Phys. Rev. D* 88.8 (2013), p. 083505. DOI: 10.1103/PhysRevD.88.083505. arXiv: 1307.0458 [astro-ph.CO].

[327] Christian G. Boehmer, Nicola Tamanini, and Matthew Wright. “Interacting quintessence from a variational approach Part I: algebraic couplings”. In: *Phys. Rev. D* 91.12 (2015), p. 123002. DOI: 10.1103/PhysRevD.91.123002. arXiv: 1501.06540 [gr-qc].

[328] Christian G. Boehmer, Nicola Tamanini, and Matthew Wright. “Interacting quintessence from a variational approach Part II: derivative couplings”. In: *Phys. Rev. D* 91.12 (2015), p. 123003. DOI: 10.1103/PhysRevD.91.123003. arXiv: 1502.04030 [gr-qc].

[329] Tomi S. Koivisto, Emmanuel N. Saridakis, and Nicola Tamanini. “Scalar-Fluid theories: cosmological perturbations and large-scale structure”. In: *JCAP* 09 (2015), p. 047. DOI: 10.1088/1475-7516/2015/09/047. arXiv: 1505.07556 [astro-ph.CO].

[330] C. Skordis, A. Pourtsidou, and E. J. Copeland. “Parametrized post-Friedmannian framework for interacting dark energy theories”. In: *Phys. Rev. D* 91.8 (2015), p. 083537. DOI: 10.1103/PhysRevD.91.083537. arXiv: 1502.07297 [astro-ph.CO].

[331] Ryotaro Kase and Shinji Tsujikawa. "Scalar-Field Dark Energy Nonminimally and Kinetically Coupled to Dark Matter". In: *Phys. Rev. D* 101.6 (2020), p. 063511. DOI: 10.1103/PhysRevD.101.063511. arXiv: 1910.02699 [gr-qc].

[332] Ryotaro Kase and Shinji Tsujikawa. "Weak cosmic growth in coupled dark energy with a Lagrangian formulation". In: *Phys. Lett. B* 804 (2020), p. 135400. DOI: 10.1016/j.physletb.2020.135400. arXiv: 1911.02179 [gr-qc].

[333] Luca Amendola and Shinji Tsujikawa. "Scaling solutions and weak gravity in dark energy with energy and momentum couplings". In: *JCAP* 06 (2020), p. 020. DOI: 10.1088/1475-7516/2020/06/020. arXiv: 2003.02686 [gr-qc].

[334] Ryotaro Kase and Shinji Tsujikawa. "General formulation of cosmological perturbations in scalar-tensor dark energy coupled to dark matter". In: *JCAP* 11 (2020), p. 032. DOI: 10.1088/1475-7516/2020/11/032. arXiv: 2005.13809 [gr-qc].

[335] Alkistis Pourtsidou and Thomas Tram. "Reconciling CMB and structure growth measurements with dark energy interactions". In: *Phys. Rev. D* 94.4 (2016), p. 043518. DOI: 10.1103/PhysRevD.94.043518. arXiv: 1604.04222 [astro-ph.CO].

[336] Finlay Noble Chamings et al. "Understanding the suppression of structure formation from dark matter-dark energy momentum coupling". In: *Phys. Rev. D* 101.4 (2020), p. 043531. DOI: 10.1103/PhysRevD.101.043531. arXiv: 1912.09858 [astro-ph.CO].

[337] Jose Beltrán Jiménez et al. "Velocity-dependent interacting dark energy and dark matter with a Lagrangian description of perfect fluids". In: *JCAP* 03 (2021), p. 085. DOI: 10.1088/1475-7516/2021/03/085. arXiv: 2012.12204 [astro-ph.CO].

[338] Jose Beltrán Jiménez et al. "Probing elastic interactions in the dark sector and the role of S8". In: *Phys. Rev. D* 104.10 (2021), p. 103503. DOI: 10.1103/PhysRevD.104.103503. arXiv: 2106.11222 [astro-ph.CO].

[339] Roy Maartens et al. "Overview of Cosmology with the SKA". In: *PoS* AASKA14 (2015). Ed. by Tyler L. Bourke et al., p. 016. DOI: 10.22323/1.215.0016. arXiv: 1501.04076 [astro-ph.CO].

[340] R. Laureijs et al. "Euclid Definition Study Report". In: (Oct. 2011). arXiv: 1110.3193 [astro-ph.CO].

[341] Luca Amendola et al. "Cosmology and fundamental physics with the Euclid satellite". In: *Living Rev. Rel.* 16 (2013), p. 6. DOI: 10.12942/lrr-2013-6. arXiv: 1206.1225 [astro-ph.CO].

[342] Luca Amendola et al. "Cosmology and fundamental physics with the Euclid satellite". In: *Living Rev. Rel.* 21.1 (2018), p. 2. DOI: 10.1007/s41114-017-0010-3. arXiv: 1606.00180 [astro-ph.CO].

[343] P. Sikivie. "Experimental Tests of the Invisible Axion". In: *Phys. Rev. Lett.* 51 (1983). Ed. by M. A. Srednicki. [Erratum: Phys.Rev.Lett. 52, 695 (1984)], pp. 1415–1417. DOI: 10.1103/PhysRevLett.51.1415.

[344] Robin Bähre et al. "Any light particle search II —Technical Design Report". In: *JINST* 8 (2013), T09001. DOI: 10.1088/1748-0221/8/09/T09001. arXiv: 1302.5647 [physics.ins-det].

[345] S. J. Asztalos et al. "A SQUID-based microwave cavity search for dark-matter axions". In: *Phys. Rev. Lett.* 104 (2010), p. 041301. DOI: 10.1103/PhysRevLett.104.041301. arXiv: 0910.5914 [astro-ph.CO].

[346] N. Du et al. "A Search for Invisible Axion Dark Matter with the Axion Dark Matter Experiment". In: *Phys. Rev. Lett.* 120.15 (2018), p. 151301. DOI: 10.1103/PhysRevLett.120.151301. arXiv: 1804.05750 [hep-ex].

[347] T. Braine et al. "Extended Search for the Invisible Axion with the Axion Dark Matter Experiment". In: *Phys. Rev. Lett.* 124.10 (2020), p. 101303. DOI: 10.1103/PhysRevLett.124.101303. arXiv: 1910.08638 [hep-ex].

[348] E. Armengaud et al. "Conceptual Design of the International Axion Observatory (IAXO)". In: *JINST* 9 (2014), T05002. DOI: 10.1088/1748-0221/9/05/T05002. arXiv: 1401.3233 [physics.ins-det].

[349] E. Armengaud et al. "Physics potential of the International Axion Observatory (IAXO)". In: *JCAP* 06 (2019), p. 047. DOI: 10.1088/1475-7516/2019/06/047. arXiv: 1904.09155 [hep-ph].

[350] E. Aprile et al. "Excess electronic recoil events in XENON1T". In: *Phys. Rev. D* 102.7 (2020), p. 072004. DOI: 10.1103/PhysRevD.102.072004. arXiv: 2006.09721 [hep-ex].

[351] Luca Di Luzio et al. "Solar axions cannot explain the XENON1T excess". In: *Phys. Rev. Lett.* 125.13 (2020), p. 131804. DOI: 10.1103/PhysRevLett.125.131804. arXiv: 2006.12487 [hep-ph].

[352] Michele Cicoli et al. "Quintessence and the Swampland: The parametrically controlled regime of moduli space". In: *Fortsch. Phys.* 70.40 (2022). DOI: 10.1002/prop.202200009. arXiv: 2112.10779 [hep-th].

[353] Michele Cicoli et al. "Quintessence and the Swampland: The numerically controlled regime of moduli space". In: *Fortsch. Phys.* 70.4 (2022). DOI: 10.1002/prop.202200008. arXiv: 2112.10783 [hep-th].