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Fundamental and Applied Problems of the String Theory Landscape

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Abstract

In this thesis we study quantum corrections to string-derived effective actions *per se* as well as their implications for phenomenologically relevant setups like the *Large Volume Scenario* (LVS) and the *anti-D3-brane* uplift.

In the first part of this thesis, we improve the understanding of string loop corrections on general Calabi-Yau orientifolds from an effective field theory perspective by proposing a new classification scheme for quantum corrections. Thereby, we discover new features of string loop corrections, like for instance possible logarithmic effects in the Kahler and scalar potential, which are relevant for phenomenological applications like models of inflation.

In the next part of the thesis, we derive a simple and explicit formula, the *LVS parametric tadpole constraint* (PTC), that ensures that the anti-D3-brane uplifted LVS dS vacuum is protected against the most dangerous higher order corrections. The main difficulty appears to be the small uplifting contribution which is necessary due to the exponentially large volume obtained via the LVS. This in turn requires a large negative contribution to the tadpole which is quantified in the PTC. As the negative contribution to the tadpole is limited in weakly coupled string theories, the PTC represents a concrete challenge for the LVS.

The last part of the thesis investigates the impact of α' corrections to the brane-flux annihilation process discovered by Kachru, Pearson, and Verlinde (KPV) on which the anti-D3-brane uplift is based. We find that α' corrections drastically alter the KPV analysis with the result that much more flux in the Klebanov-Strassler throat is required than previously assumed in order to control the leading α' corrections on the NS5-brane. The implication for the LVS with standard anti-D3-brane uplift can again be quantified by the PTC. Incorporating this new bound significantly increases the required negative contribution to the tadpole. In addition, we uncover a new uplifting mechanism not relying on large fluxes and hence deep warped throats, thereby sidestepping the main difficulties related to the PTC.

Zusammenfassung

Diese Dissertation befasst sich mit Quantenkorrekturen zur effektiven, niedrig energetischen Wirkung der Stringtheorie. Dabei werden Quantenkorrekturen *per se* sowie deren Auswirkung auf phenomenologisch relevante Szenarien wie das *Large Volume Scenario* (LVS) und den *anti-D3-Branen Uplift* studiert.

Im ersten Teil werden String-Scheifenkorrekturen aus der effektiven Feldtheorie-Perspektive erforscht. Dabei wird ein neues Schema zur Klassifizierung von Quantenkorrekturen vorgeschlagen, durch welches anschließend bisher unentdeckte Eigenschaften der Schleifenkorrekturen entdeckt werden. Diese beinhalten beispielsweise potentielle logarithmische Korrekturen zum Kähler- oder skalaren Potential. Es zeigt sich, dass diese neuen Effekte einen wichtigen Einfluss auf phenomenologische Anwendungen, wie beispielsweise auf stringtheoretische Modelle der Inflation, haben können.

Im zweiten Teil wird eine einfache und explizite Formel, der sogenannte *LVS parametric tadpole constraint* (PTC), hergeleitet. Diese stellt sicher, dass das de Sitter Vakuum erzeugt mittels des LVS mit anti-D3-Branen Uplift vor den gefährlichsten Quantenkorrekturen geschützt ist. Die größte Schwierigkeit besteht dabei darin, dass der Uplifting-Term aufgrund des exponentiell großen Volumens exponentiell klein sein muss. Dies wiederum erfordert einen großen negativen Beitrag zum Tadpole, welcher durch den PTC quantifiziert wird. Da der negative Beitrag zum Tadpole in schwach gekoppelter Stringtheorie limitiert ist, stellt der PTC eine konkrete Anforderung an konsistente LVS Modelle dar.

Im letzten Teil werden die Auswirkungen von α' Korrekturen auf den von Kachru, Pearson und Verlinde (KPV) entdeckten *Brane-Flux-Annihilationsprozess* untersucht, auf dem der Anti-D3-Branen-Uplift basiert. Es zeigt sich, dass α' Korrekturen die KPV-Analyse drastisch verändern können. Um die führenden α' Korrekturen zu kontrollieren wird viel mehr Fluss im Klebanov-Strassler *Throat* benötigt als bisher angenommen. Dieser neue Effekt wird anschließend in den PTC implementiert. Im Vergleich zum vorherigen Kapitel zeigt sich, dass der erforderliche negative Beitrag zum Tadpole signifikant steigt. Außerdem wird ein neuer Uplifting-Mechanismus vorgeschlagen, der nicht auf langen, gewarpten Throats basiert und damit die Schwierigkeiten, die mit dem PTC einhergehen, umgeht.

Contents

Preface

The research presented in this thesis was carried out at the Institute for Theoretical Physics at the University of Heidelberg from November 2020 until December 2022.

The content of the Chapters 2 to 5 are the result of work [1–4] conducted in collaboration with Arthur Hebecker, Xin Gao, and Gerben Venken. These works are reproduced here with the permission of all coauthors.

My personal role in all of the four papers presented in this thesis is that of the first author. A detailed description concerning my contribution to each of the four papers is provided at the beginning of each chapter where the paper is reproduced. All of the coauthors agreed with these statements.

[1] Xin Gao, Arthur Hebecker, Simon Schreyer, and Gerben Venken. “The LVS parametric tadpole constraint”. *JHEP* 07 (2022) 056. arXiv: 2202.0487 [hep-th]

[2] Xin Gao, Arthur Hebecker, Simon Schreyer, and Gerben Venken. “Loops, local corrections and warping in the LVS and other type IIB models”. *JHEP* 09 (2022) 091. arXiv: 2204.06009 [hep-th]

[3] Arthur Hebecker, Simon Schreyer, and Gerben Venken. “Curvature corrections to KPV: do we need deep throats?”. *JHEP* 10 (2022) 166. arXiv: 2208.02826 [hep-th]

[4] Simon Schreyer, and Gerben Venken. “α' corrections to KPV: An uplifting story”. Under review. arXiv: 2212.07437 [hep-th]

Chapter 1

Introduction

1.1 The current status of high energy physics

The world around us as we understand it today is characterized by four fundamental forces. One of these forces is gravity, described by General Relativity (GR), which treats gravity as the intrinsic curvature of spacetime. The remaining three forces, the strong, weak, and electromagnetic interactions, are described by gauge theories and are unified in what is known as the Standard Model (SM) of particle physics. The SM is the result of decades of mutually stimulating research by experimental and theoretical physicists which culminated in the discovery of the Higgs boson [5, 6]. The striking success of the SM is built on the framework of Quantum Field Theory (QFT). The concept of QFT unifies Quantum Mechanics and Special Relativity and particles are understood as pointlike energy concentrations of some field. In the language of QFT, the SM is a renormalizable effective field theory (EFT) meaning that all its coupling constants have positive or zero mass dimension. In other words, the coupling constants are relevant or marginal, respectively. The SM (extended by massive neutrinos) describes all known elementary particles and their interactions to an astonishing accuracy[1]. Despite this huge accomplishment it is widely believed that the SM can not be the fundamental theory of our world.

The reason being that the SM itself leaves many questions unanswered as, for instance, why the Higgs mass is so small which is related to the electroweak hierarchy problem, how neutrino masses are generated, why there are three fermion generations, or why the (renormalizable) Θ-QCD term is so small. It would also be natural to ask why there are 19 a priori undetermined parameters in the SM.

From a more theoretical perspective, the main reason why the SM can not be the truly, fundamental theory is that it fails to include gravity. Since the gravitational coupling constant, the Newton constant, is irrelevant in EFT language – it is of negative mass dimension – gravity is non-renormalizable and hence can not be completely described perturbatively in QFT (the

[1] After fixing its 19 free parameters by experiments.

framework the SM is based on). Intuitively, this is understood by noting that the gravitational force increases with mass and hence energy. Since in quantum mechanics and therefore also in QFT arbitrarily large energy fluctuations in a sufficiently small region of spacetime are allowed, there is no bound on the strength of gravitational interactions. This is in fact the nature of irrelevant couplings – they are increasingly important with increasing energy. It is the general belief that this tells us that there should be a more fundamental theory at the energy scale where the non-renormalizable theory becomes strongly coupled. This reasoning has proved very useful in the past for Fermi's theory. In that theory the four fermion interaction is non-renormalizable such that the theory becomes invalid at around 100GeV. This led to the prediction of the Glashow-Salam-Weinberg theory of electroweak interactions [7–9].

For gravity the scale of strong coupling is the Planck scale which is in natural units ($\hbar = c = 1$) given by $M_P = (8\pi G_N)^{-1/2} \approx 2.43 \times 10^{18}$GeV. This tells us that as an effective field theorist we expect new physics is guaranteed to become relevant around the Planck scale. The Planck scale is much higher than the energy at currently performed collider experiments which is a few TeV.

On the opposite end of energy scales, in cosmology, the history of the universe is described by the standard model of cosmology called the Lambda-Cold-Dark-Matter model (ΛCDM). It is the simplest model that accomplishes to describe the cosmic microwave background, the large-scale structure in the distribution of galaxies, and the accelerated expansion of the universe that we observe. The ΛCDM model assumes GR and baryonic matter (described by the SM), as well as two additional components namely dark energy denoted by Λ, and cold dark matter (CDM). Dark energy makes up 70% of the energy density today [10] and is given by the cosmological constant. Our universe is accelerated expanding as measured in [11, 12] which can be explained by a *positive* cosmological constant.

But as it was the case for the SM, also ΛCDM can not be the final story. One important reason is that ΛCDM is completely agnostic about the microscopic nature of dark matter and dark energy – it does not explain their origin but just assumes both components to exist.

One very reasonable way of approaching all these unresolved puzzles beyond what is currently known is to study a “theory of everything” that is capable of unifying all four fundamental forces and then study what this theory offers as solutions for our unanswered questions. One possible theory that describe physics even to the Planck scale is *String Theory*. Let us in the next sections explain why string theory is a fascinating theory worth studying and what it can and (so far) can not explain. This presentation will be incomplete and the focus will be on the issues addressed in this thesis.

1.2 String Theory as a candidate for a UV complete theory

The simple basic idea of string theory (see e.g. [13–19] and references therein) is that the fundamental objects of a theory are not point-like as in a QFT, but that the fundamental object is a unique string that extends along a spatial coordinate and time, sweeping out a *worldsheet*. In contrast, point-like objects sweep out a worldline. Different particle species are then found to be different vibration modes of this string. Since strings are one-dimensional objects, there are only two distinct topologies that a string can form: a line (open string) or a circle (closed string). Upon quantizing the string, the massless spin two excitations of the closed string can be identified with the graviton whereas the massless excitations of open strings lead to Yang-Mills gauge bosons. This already hints towards a theory that unifies Yang-Mills theory and gravity. Thinking one step further we observe that since interactions in string theory are understood as the joining and splitting of strings it can be quite intuitively concluded why string theory necessarily requires gravity: The joining and splitting of open strings automatically encapsulate closed strings and hence gravity. This is very remarkably since in a perturbative QFT the unification of Yang-Mills theories with gravity seems to be impossible whereas in string theory describing Yang-Mills theories without gravity is impossible.

Another appealing property of string theory is that it does not have adjustable free parameters besides the string length l_s, or equivalently the string scale $M_s = \alpha'^{-1/2} = 2\pi/l_s$. Rather it is the case that all couplings and masses are set by the dynamics of the theory itself. A particular example is the so-called string coupling constant g_s which arises as the vacuum expectation value (vev) of the dilaton, an excitation mode of the closed string.

It is even more remarkable that all string theories with bosonic and fermionic degrees of freedom turn out to be supersymmetric and only anomaly-free in ten spacetime dimensions. In this sense, (super-)string theory predicts the number of spacetime dimensions. There are five consistent superstring theories, namely heterotic string theory with gauge group $E_8 \times E_8$ or $SO(32)$, type I, type IIA, and type IIB string theory. In fact, all these are related to each other by dualities. They can hence be seen as different limits of one underlying theory, called M-theory. At low energies and small string coupling the five string theories are described by ten dimensional supergravity (sugra) theories[2].

Besides these very desirable features a consistent high-energy theory should have, there is (so far) no direct implication for real world physics. Nevertheless string theory continuously improved our understanding of real world physics. The study of the string worldsheet and its conformal symmetry, for example, led to new insights into 2d conformal field theories (CFTs) that are directly applicable to real world physics like condensed matter theory. Also the *AdS/CFT correspondence* first proposed by Maldacena [20–22] relating strongly coupled QFTs in d dimensions and weakly coupled gravity in $d+1$ dimensions emerged from string theory. Since weakly coupled gravity is well understood, this *gauge/gravity duality* has wide-ranging applications from nuclear physics to condensed matter theory, offering insights into strongly coupled systems. Additionally string theory has also provided new results in mathematics like *mirror symmetry* (see e.g. the lecture notes [23, 24] and references therein), which relates topologically different Calabi-Yau manifolds.

[2]Very intriguingly, there exist only five sugra theories in ten dimensions which are precisely the low-energy limits of the five string theories. This motivates the so called string lamppost principle which postulates that all consistent quantum gravity theories come from string theory such that string theory is the unique theory of quantum gravity.

With all that said, why then do physicists not agree that string theory is *the* UV completion of the physics we observe? The main reason is that by the time of writing up this thesis there is no experimental evidence for string theory. This is on the one hand due to the string scale being incredibly large from the point of view of currently performed collider experiments[3]. On the other hand, one may imagine/hope that there exist emergent phenomena from string theory testable at energies accessible today or in near future. This broadly encapsulates the goal of *string phenomenology*. The main obstacle here is that string theory has a plethora of vacua[4], giving rise to the *string theory landscape* of vacua. The landscape originates from many possible choices of compactifying the ten dimensional theory down to our four observable dimensions. This includes details of the compactification geometry or choices of flux quanta [28, 29]. One interpretation of this is that different solutions of string theory describe different universes, just as different solutions to Einstein's equations describe different galaxies or planetary systems. For the believers of the *multiverse* this may be a necessary property of a fundamental theory whereas for others it is more an obstacle as it radically reduces the predictive power of the theory at low energies. The reason is that a plethora of vacua essentially means that our fundamental 4d physics parameters are not really fundamental but just drawn from a large set of possible choices.

Even more importantly, we observe an accelerated expanding universe. However, completely trustworthy calculations proving that string theory actually allows for accelerated expanding universes are still lacking, despite its vast number of possible vacua/universes and decades of research in this direction. The difficulty of obtaining accelerated expanding universes from string theory will be introduced in detail in Sect. 1.4 and represents also the overarching subject of this thesis.

1.3 Compactification and moduli stabilisation

We have already stated that superstring theories are consistent only in ten spacetime dimensions. This immediately begs the question how this can be in agreement with the four spacetime dimensions we observe. The typical way to achieve four dimensional effective physics is by demanding that six space dimensions are compact and small thereby escaping detection of present experiments. This is a rather ad hoc assumption which, apart from the attempt in [30], lacks a deeper explanation of why exactly six dimensions should be compact. Even though the compact dimensions are not detectable, they have an imprint on the 4d physics: The geometry of the compact space determines the field content of the 4d theory. This intriguing idea of *compactification* goes back to Kaluza and Klein (KK) [31, 32].

[3]As nicely pointed out in [25]: The conceptual shortcoming of string theory not making unique predictions at the TeV scale is as big as the conceptual shortcoming of QCD not making predictions Kopernikus could have verified.

[4]The number of vacua in Calabi-Yau compactifications of IIB string theory is estimated to be of order 10^{500} [26] or $10^{272,000}$ in more general geometries including D-branes [27] which is huge but finite.

In string theory, the ansatz for the background spacetime $\mathcal{M}_{10}$ realizing the idea of Kaluza and Klein is a product structure of the form

$$\mathcal{M}_{10} = \mathcal{M}_{1,3} \times Y \,, \tag{1.1}$$

where $\mathcal{M}_{1,3}$ are the observed non-compact dimensions and Y a six dimensional compact manifold. The associated ansatz for the ten-dimensional metric consistent with four dimensional Poincaré invariance is given by [33–36][5]

$$\mathrm{d}s^2 = \mathrm{e}^{2A(y)}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{e}^{-2A(y)} g_{mn}\mathrm{d}y^m \mathrm{d}y^n \,, \tag{1.2}$$

where $\eta_{\mu\nu}$ with $\mu, \nu \in \{0, \ldots, 3\}$, is the metric on the external spacetime which we have taken to be the four dimensional Minkowski metric. The metric on Y is denoted by g_{mn} with $m, n \in \{4, \ldots, 9\}$ and $\mathrm{e}^{-4A(y)}$ is the *warp factor* depending only on the internal coordinates y^m. The lower dimensional effective theory is obtained by expanding all ten dimensional fields into eigenmodes of the internal manifold Y and then integrate over the compact space. Neglecting the warp factor for the moment, this can be illustrated with a ten dimensional free scalar field $\phi(x, y)$ obeying the 10d Laplace equation $\Delta_{10}\phi = 0$. The ansatz for expanding $\phi(x, y)$ into eigenmodes $\psi_n(y)$ of the Laplace-Beltrami Δ_6 operator on Y reads

$$\phi(x, y) = \sum_{n=0}^{\infty} \phi_n(x)\psi_n(y) \,, \tag{1.3}$$

where $\phi_n(x)$ only depends on the external coordinates. Due to the product structure of the metric the Laplacian splits according to $\Delta_{10}\phi = (\Delta_4 + \Delta_6)\phi = 0$. The internal Laplacian Δ_6 acts only on the eigenmodes ψ_n and yields $\Delta_6\psi_n = m^2_{\mathrm{KK},n}\psi_n$, where $m_{\mathrm{KK},n}$ has dimension mass and is called the KK mass of the n-th eigenmode. From the 4d perspective after integrating out the internal dimensions this looks like a mass term for the $\phi_n(x)$ fields: $(\Delta_4 + m^2_{\mathrm{KK},n})\phi_n(x) = 0$. Since there are infinitely many eigenfunctions on a compact space, an infinite tower of massive scalars called *KK modes* arises. Their masses are quantized (the spectrum of the Laplacian on a compact manifold is discrete) and their scale is given by the inverse of the typical length scale R of the compact manifold: $m_{\mathrm{KK}} \sim 1/R$. The mass scale m_{KK} is also called *KK scale*. If the internal dimensions are sufficiently small, the infinite tower of massive scalars becomes heavy and can be integrated out consistently. In this manner, one obtains an effective theory valid below the KK scale which then includes only the massless zero KK-modes. These zero modes are not only massless but also have no potential and can therefore take arbitrary constant values that all fulfill the vacuum solution. Such fields are called *moduli*. A KK decomposition, as exemplified above for a free scalar field, can be performed for any field of the theory including the metric. Since the moduli arise as zero modes of the internal Laplacian, i.e. $\Delta\phi_0 = 0$, they are in one-to-one correspondence with the harmonic forms on Y and can hence be counted by the dimensions $h^{p,q}$ of the corresponding Dolbeault cohomology classes.

[5] A more general ansatz is presented in [37].

A well trodden path in string theory is to choose Y to be a Calabi-Yau 3-fold or six-torus[6]. A Calabi-Yau k-fold is a manifold admitting a Kahler metric with $SU(k)$ holonomy or, equivalently, a manifold which admits a Kahler metric with vanishing Ricci curvature. One reason for this choice is that these manifolds will retain some supersymmetry even after compactification. Eventually, supersymmetry will then be broken well below the KK scale as we do not observe any supersymmetry at the Large Hadron Collider (LHC).

The moduli of a Calabi-Yau (arising as moduli of the internal metric g_{mn}) are the allowed deformations of the metric that keep the manifold Ricci flat. This leads to $h^{1,1}$ *Kahler moduli* describing the size of 4-cycles of the Calabi-Yau and to $h^{2,1}$ *complex structure moduli* describing the ratio of volumes of 3-cycles of the manifold[7]. For the simple example of a torus, the Kahler modulus is the overall size of the torus whereas the complex structure modulus is the ratio of the sizes of the two 1-cycles of the torus. For generic compactifications on Calabi-Yau manifolds, one typically has $\mathcal{O}(100-1000)$ of moduli.

Moduli stabilisation

The problem of *moduli stabilisation* now arises in the following way. After compactification to 4d and integrating out the massive KK modes, we are left with a theory containing many massless scalars/moduli. If these would remain massless in the full theory, they would mediate long-range fifth forces which are very strongly constrained [40]. Therefore we need to create a potential for all the massless scalars in such a way that in the minimum of the potential all scalars obtain a heavy mass thereby escaping detection. This is known as *moduli stabilisation*.

It is generally not difficult at all to make scalars massive, as they can only be massless if protected by some symmetry. Without supersymmetry, the moduli of a compactification are not protected and hence guaranteed to become massive by some quantum corrections. The problem is that most quantum corrections are not calculable explicitly for a general compactification and thus we can not control the potential for the moduli or their masses.

This is where supersymmetry and a compactification on a Calabi-Yau come to rescue. Compactifying type II string theory on a Calabi-Yau is a controlled framework as it leads to $\mathcal{N}=2$ supersymmetry in four dimensions. In $\mathcal{N}=2$ theories in 4d there is no scalar potential since all scalars are protected by the large amount of supersymmetry and are therefore massless. Accordingly, to obtain a scalar potential we have to break supersymmetry to $\mathcal{N}=1$. To understand how this is usually done we have to introduce the concept of a Dirichlet p-brane and of an orientifold p-plane.

A Dirichlet p-brane, Dp-brane for short, is a $p+1$ dimensional dynamical hypersurface in the $d=10$ dimensional spacetime on which oriented open strings can end [41–43]. Dp-branes have a non-trivial action describing their interaction with open and closed strings. The position of a Dp-brane in the transversal space is given by the vev of $d-p-1$ scalar fields which are

[6]Recent work on compactifications not using Calabi-Yau manifolds can be found in [38, 39].

[7]Note that there also exist moduli coming from all other 10d fields besides the metric. In fact, they pair up with the metric moduli to form $h^{1,1}$ complexified Kahler and $h^{2,1}$ complexified complex structure moduli.

called *position moduli*. Additionally, a Dp-brane breaks half of the supersymmetries since the boundary conditions of the brane relate the left- and right-moving fields at the endpoints of the open string. And since both the left- and right-moving sector preserve some supersymmetry, only a linear combination remains. A crucial fact for phenomenology is that a stack of N Dp-branes carries a $U(N)$ gauge theory on the worldvolumes of the branes. If branes are not on top of each other but intersect, standard model matter can be realized on their intersection locus.

Introducing an arbitrary number of D-branes into the theory generically leads to so called tadpoles which are non-vanishing 1-point functions. As in QFT, they signal an instability of the vacuum. In string theory, a Dp-brane sources a Ramond-Ramond (R-R) $(p+1)$-form field which is part of the closed string spectrum. This linear coupling induces an R-R tadpole. To cancel the R-R tadpole it is necessary to include unoriented strings and therefore Op-planes since O-planes couple with the opposite sign to the R-R $(p+1)$-form fields.

To understand the concept of an orientifold, O-plane for short, we consider first the simpler concept of an orbifold. An orbifold of some manifold X is the quotient space X/G where G is some discrete isometry group of X. This is readily understood for the simple example of the orbifold $S_1/\mathbb{Z}_2$ where the S_1 has radius R and $\mathbb{Z}_2$ acts as a reflection on the coordinate x of the circle: $x \sim -x$. The orbifold is therefore the real line of length πR as illustrated in Fig. 1.1 and has two fixed points[8] at $x = 0, \pi R$ which are invariant under $\mathbb{Z}_2$.

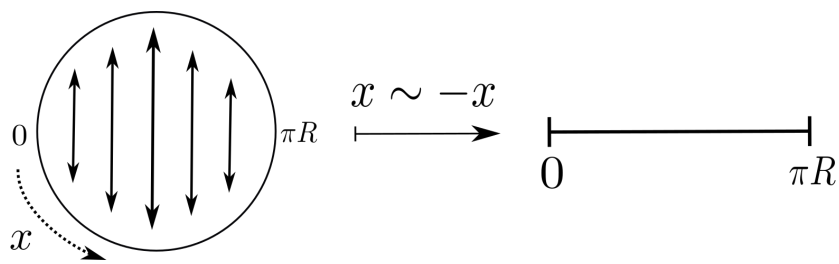


Figure 1.1: Illustration of the orbifold $S_1/\mathbb{Z}_2$ which is an interval.

The orientifold plane (O-plane) now arises when not only a geometric action is modded out (as above for the orbifold) but when additionally worldsheet parity (orientation reversal of the string) is modded out. This combined modding out is called orientifolding and the quotient space is called orientifold. At the fixed points of the orbifold action, the theory is unoriented due to the orientation reversal which projects out half of the states. This indicates the presence of a new object called O-plane located at these fixed points [41]. It can be shown that O-planes have negative tension and since closed strings can end on O-planes, they source various closed string sector fields. Moreover, in string perturbation theory, O-planes are not dynamical and therefore, unlike D-branes, have no degrees of freedom on their worldvolume.

[8] In higher dimensional theories the fixed points are fixed planes. Consider for instance the spacetime $\mathbb{R}^{1,3} \times S_1/\mathbb{Z}_2$ in which the fixed planes are $1+3$ dimensional.

The reason is that open strings can not end on the O-plane due to the orientation reversal of the string. Important for the issue of canceling tadpoles is the fact that Op-planes source the R-R $(p+1)$-form field with a different sign and coefficient compared to Dp-branes. R-R tadpole cancellation is therefore achieved by adding the right number of Dp-branes and Op-planes. In addition, it can be arranged that in theories with D-branes and O-planes the four dimensional effective theory has $\mathcal{N}=1$ supersymmetry. This is exactly what we were after in the context of moduli stabilisation for introducing a scalar potential for the moduli.

In the course of this thesis we will be working in the framework of the low energy supergravity effective action of type IIB string theory exclusively but some results also carry over to other string theories. A motivation to study type IIB string theory in the context of dS vacua will be given in Sect. 1.4. Let us therefore explain moduli stabilisation in more detail in the context of orientifold compactifications of type IIB string theory (for more details see [44]).

In order to stabilize all scalar fields in the effective 4d theory the knowledge of the field content of the 10d string theory is required. The bosonic field content of type IIB string theory contains the following set of fields: the dilaton Φ, the metric $G_{2\,(\mu\nu)}$, the antisymmetric Kalb-Ramond field $B_{2\,[\mu\nu]}$ as well as the Ramond-Ramond form fields C_0, C_2, and C_4. The corresponding action is given in Sect. 2.2.1. Orientifold compactifications of type IIB string theory preserve $\mathcal{N}=1$ supersymmetry in four dimensions if the theory contains either O3/O7-planes or O5/O9-planes. In the following we consider the class of orientifold compactifications with O3/O7-planes since they allow for Klebanov-Strassler throats [45] (to be described below) and a consistent description of the backreaction of fluxes onto the geometry by including a warp factor[9] [36] as in the ansatz (1.2). Further more models with O3/O7-planes allow for rich phenomenology due to intersecting D7-branes.

Since orientifolding projects out part of the spectrum, the cohomology groups split up into even (+) and odd (−) eigenspaces under the orientifold action such that $h^{p,q} = h^{p,q}_{-} + h^{p,q}_{+}$. In the following we assume $h^{1,1}_{-} = 0$ such that no scalar fields from B_2 and C_2 arise in the compactification and further neglect D7- and D3-position moduli. The treatment of position moduli is reviewed for instance in [47] and generalizations to $h^{1,1}_{-} \neq 0$ are discussed in [48]. The number of complex moduli is then given by $h^{2,1}_{-}$ complex structure moduli, the axio-dilaton and $h^{1,1}_{+}$ Kahler moduli[10]. It can be shown that the theory in 4d can be described by $\mathcal{N}=1$ supergravity. Accordingly, the scalar potential V is characterized by a holomorphic function W called superpotential and by a real function K known as the Kahler potential. The scalar potential is then given by

$$V = \mathrm{e}^{K}\left(K^{i\bar{\jmath}}(D_i W)(D_{\bar{\jmath}}\overline{W}) + K^{\alpha\bar{\beta}}(D_\alpha W)(D_{\bar{\beta}}\overline{W}) - 3|W|^2\right)\,, \tag{1.4}$$

[9]Including the warp factor the internal manifold is no longer Calabi-Yau but conformally Calabi-Yau which geometrically ‘comes as close as possible’ to a Calabi-Yau. For the case of O5/O9-planes for instance the backreaction of fluxes is much more drastic and one needs to consider more general manifolds of SU(3) structure. See [46] for more details.

[10]Note that $h^{2,1}_{+}$ counts the number of vector fields arising from the reduction of C_4 and thus does not count scalar fields.

where $D_M W = \partial_M W + (\partial_M K)W$, ∂_M denotes the derivative w.r.t. the M-th modulus, $\alpha, \beta = 1, \cdots, h_-^{2,1}+1$, $i, j = 1, \cdots, h_+^{1,1}$, and M is either a complex structure or Kahler modulus. At leading order in α' and string loops (see below for more details), W and K are given by

$$W = \int_Y \Omega \wedge G_{(3)} \, , \qquad K = -2\ln\mathcal{V} - \ln(-i(\tau-\overline{\tau})) - \ln\left(-i\int_Y \Omega\wedge\overline{\Omega}\right) \, , \tag{1.5}$$

where Ω is the non-vanishing holomorphic three form of the Calabi-Yau, $G_{(3)}$ the 3-form flux, τ the axio-dilaton, and $\mathcal{V}$ the Calabi-Yau volume. The leading order superpotential is called the Gukov-Vafa-Witten superpotential [49, 36]. It is induced by three form fluxes and depends only on the complex structure moduli and the axio-dilaton. The Kahler potential is always a function of all moduli. The dependence on the Kahler moduli in K is precisely such that the first and last term in (1.4) cancel (independently of the choice of W) making type IIB string theory a no-scale model [50]. As a result, all Kahler moduli are in the leading order approximation (1.5) massless.

The complex structure moduli are now stabilized (acquire a mass in the minimum of the scalar potential) by the second term in (1.4) using 3-form fluxes $G_{(3)}$ [11]. The minimum of the scalar potential is obtained by solving the SUSY preserving conditions $D_\alpha W = 0$ [35, 36] for the complex structure moduli leading to a vanishing scalar potential $V = 0$. As we will assume in the following, the complex structure moduli can then be integrated out because they turn out to be much heavier than the Kahler moduli[12]. This can be verified a posteriori after stabilizing Kahler moduli.

Due to the no-scale property, Kahler moduli stabilisation is notoriously difficult. The general strategy to stabilize Kahler moduli is to include a certain set of higher order corrections into W and K, like α' corrections and string loop corrections to K, or non-perturbative corrections to W and K[13]. For this strategy to work out, all corrections should enjoy an expansion in some small parameter. In our case of perturbative string theory this will be the string coupling g_s and the inverse of the Calabi-Yau volume $\mathcal{V}$ for the validity of the string loop expansion and α' expansion, respectively. In this way, there is in principle the possibility to ensure that the neglected corrections do not spoil the stabilisation mechanism because they arise at subleading order in the expansion in g_s and $1/\mathcal{V}$. The difficulty that arises in such settings is usually referred to as the *Dine-Seiberg problem* [59]. It arises when quantum corrections are invoked to stabilize a modulus and the modulus itself is the expansion parameter of quantum corrections as it is the case in string theory where no free parameters

[11] This has recently been questioned in smooth compactifications with large numbers of complex structure moduli [51–56] (see also the discussion of the tadpole problem in [57]).

[12] This is strictly speaking only true for complex structure moduli living in the bulk of the Calabi-Yau. In a throat for example, the masses are exponentially suppressed by the warp factor and are hence much lighter, see e.g. [58].

[13] In principle, all moduli could also be stabilized using only fluxes, D-branes and O-planes if the geometry is not Calabi-Yau but this is for almost all cases excluded by no-go theorems. We will elaborate on different strategies of moduli stabilisation in Sect. 1.4.

except the string length l_s exist. To understand the Dine-Seiberg problem let us for simplicity focus on the one modulus case where the only modulus is $\mathcal{V}$ and the potential V is zero in the weak coupling limit corresponding to $\mathcal{V} \to \infty$. In order to create a local minimum in the quantum corrected scalar potential, (at least) two terms with different sign and different dependence on $\mathcal{V}$ need to be comparable in size. This typically does not happen anymore in the weakly coupled regime of large $\mathcal{V}$ where we trust our expansion. For explicitness, consider the (unrealistic) example of a potential $V = -a/\mathcal{V} + b/\mathcal{V}^2$, where a, b are positive $\mathcal{O}(1)$ constants. The potential has a minimum at $\mathcal{V} = 2b/a \sim \mathcal{O}(1)$ which is not in the weakly coupled regime of large volume where we trust our analysis. Thus, in principle, all higher order corrections eventually become important in the vicinity of the minimum. The Dine-Seiberg problem can for instance be avoided by fine-tuning the numerical prefactors of each correction (in our toy model this corresponds to tuning $a \ll b$)[14].

Kahler moduli stabilisation in IIB string theory

After stabilizing and subsequently integrating out the complex structure moduli we ended up with a flat potential and therefore massless Kahler moduli in the last section. Let us in the following explore possibilities to stabilize Kahler moduli.

The best studied scenarios for Kahler moduli stabilisation in type IIB string theory to 4d on a Calabi-Yau orientifold are *KKLT* [60] and the *Large Volume Scenario* (LVS) [61, 62]. Variants of KKLT and the LVS as well as other proposals of Kahler moduli stabilisation including quantum corrections can be found in [63–73].

As explained above, for Kahler moduli stabilisation in type IIB string theory on Calabi-Yau orientifolds it is necessary to include quantum corrections. The KKLT scenario does so by using non-perturbative corrections to the superpotential whereas in the LVS one uses a combination of non-perturbative corrections to the superpotential and the α' correction of [74, 75] to the Kahler potential. In the main body of the thesis we focus on the LVS since KKLT seems to suffer from the so called *singular bulk problem* [76–78][15].

Let us therefore briefly review the LVS. We will not be concerned with $\mathcal{O}(1)$ constants and refer the reader to a detailed summary of the LVS in Sect. 3.2 or to e.g. [80, 81]. In the minimal LVS setting, the geometry on which we compactify has at least two Kahler moduli, a big 4-cycle $T_b = \tau_b + ic_b$ and a small, blowup cycle $T_s = \tau_s + ic_s$. The real parts of the Kahler moduli describe the sizes of the corresponding cycles and the imaginary parts are axions coming from the KK reduction of the four form C_4. The volume $\mathcal{V}$ of the manifold is given by

[14] In the KKLT for instance the Dine-Seiberg problem is avoided by fine-tuning the value of $W_0 \ll 1$. W_0 denotes the value of the superpotential after stabilizing the complex structure moduli. The value can be tuned by fluxes.

[15] See [79] for a possible resolution.

$\mathcal{V} \sim \tau_b^{3/2} - \tau_s^{3/2}$. The Kahler and superpotential read

$$K = -2\ln\left(\mathcal{V} + \frac{\xi}{g_s^{3/2}}\right)\,, \qquad W = W_0 + A_s e^{-a_s T_s}\,, \tag{1.6}$$

where ξ, A_s, and a_s are model dependent numbers. W_0 is the value of the Gukov-Vafa-Witten superpotential (1.5) after stabilizing complex structure moduli. The exponentially small correction to the superpotential is due to Euclidean D3-instantons or gaugino condensation on D7-branes. We will find below that $\tau_b \gg \tau_s$ such that we neglect the non-perturbative corrections[16] to T_b in W which are generically also present. The correction proportional to ξ in the Kahler potential is due to the α'^3 correction [74, 75]. The scalar potential V for the Kahler moduli follows from plugging in (1.6) into (1.4) and has the form

$$V \sim \frac{g_s\sqrt{\tau_s}\mathrm{e}^{-2a_s\tau_s}}{\mathcal{V}} - \frac{g_s\tau_s|W_0|\mathrm{e}^{-a_s\tau_s}}{\mathcal{V}^2} + \frac{\xi|W_0|^2}{\sqrt{g_s}\mathcal{V}^3}\,, \tag{1.7}$$

where we already minimized the potential w.r.t. the imaginary part of T_s. We can then further minimize (1.7) w.r.t. $\mathcal{V}$ and τ_s and we obtain

$$\mathcal{V} \sim W_0\sqrt{\tau_s}\,\mathrm{e}^{a_s\tau_s}\ \ , \qquad\qquad \tau_s \sim \frac{\xi^{2/3}}{g_s} + \mathcal{O}(1)\,. \tag{1.8}$$

By inserting these values into V, one obtains an AdS vacuum with value

$$V_{\mathrm{AdS}} \sim -\frac{g_s\sqrt{\tau_s}|W_0|^2}{\mathcal{V}^3}\,. \tag{1.9}$$

For small g_s, the volume modulus is therefore stabilized at exponentially large values in string units. Hence, g_s and $\mathcal{V}$ provide good expansion parameters since g_s suppresses higher loop corrections and $\mathcal{V}$ higher order α' corrections. It is moreover crucial for the validity of the 4d effective description that the masses of the moduli should be much lighter than the KK modes which are neglected in the scalar potential. Additionally, as we use the framework of supergravity, the SUSY breaking scale has to be much lower that the KK scale. As the compact dimensions escape detectability of current experiments, the physics of the compact space has to decouple from the low energy 4d physics. For this to be true, the KK length scale $L_{\mathrm{KK}} = 1/m_{\mathrm{KK}}$ should be much smaller than the dS/AdS length scale $L_{\mathrm{dS/AdS}}$ which is the inverse of the Hubble scale. This is commonly referred to as *scale separation*. Note that in recent years the construction of scale-separated AdS vacua was questioned in [82–87].

Higher order corrections

At the example of the LVS above we have seen that moduli stabilisation in type IIB string theory on Calabi-Yau orientifolds requires taking into account a certain set of quantum corrections

[16]Note that in this approximation the imaginary part of T_b will remain massless.

and by that neglecting all other corrections. To be able to do this consistently, we need to know the form of all additional quantum corrections that are neglected in the moduli stabilisation scenario. There are two main difficulties in the business of quantum corrections in string theory. First, they need to be derived as corrections to the 10d theory from string amplitude calculations. In principle this calculation is methodologically straightforward but tedious. Then, to study their implications for moduli stabilisation, they need to be compactified to 4d. This can not be done explicitly for the majority of quantum corrections because of the complicated topology of the internal space. For the case of Casimir-like loop corrections, which will be considered below, the situation is even worse. Their explicit calculation involves infinite sums over KK-modes and therefore knowledge of the eigenfunctions of the compact space.

Besides the danger of quantum corrections, they can in fact also be beneficial. In the LVS for example, string loop corrections are commonly used to stabilize non-blowup Kahler moduli [88, 89]. Furthermore, quantum corrections can be used to realize inflation like in [90–94] or even to establish new de Sitter (dS) uplifting mechanisms like we proposed in [3, 4].

Due to these reasons, higher order corrections to the leading order IIB effective action and thus ultimately to the 4d effective scalar potential together with their implications on phenomenology are a central topic of this thesis. Let us therefore introduce the different kinds of quantum corrections and mention some literature where these are studied.

Quantum corrections to the stringy 10d effective action can in principle be divided into three different types. First, there are higher derivative corrections which are commonly called α' corrections as they are suppressed by the string scale $M_s = \alpha'^{-1/2}$. They can be understood as corrections arising due to strings being extended objects and not point-like. Thus, in the effective action the extendedness of the string is captured by including α' corrections. Second, there are string loop corrections. They are suppressed by the string coupling g_s and originate from higher genus string amplitude calculations. They are the stringy analogue of the standard loop expansion in a QFT. Finally, there are non-perturbative effects due to solitonic objects like Dp-branes which lead for instance to instanton corrections when Dp-branes wrap a $(p+1)$-cycle of the internal manifold. There is also the possibility of gaugino condensates on stacks of D7-branes wrapping 4-cycles in the internal manifold.

In this thesis we will be mainly concerned with perturbative corrections, so let us discuss these in more detail in our setting of type IIB string theory. For the type IIB 10d bulk theory, the leading order α' corrections arise at order α'^3. The purely gravitational corrections are calculated in [74, 75] and corrections including fluxes are only partially known [95–100]. From the M- and F-theory perspective α' corrections are studied in [101–107]. Additionally to the bulk theory, also localized objects like Dp-branes or Op-planes receive α' corrections. They arise at order α'^2 and are investigated in [108–125] and will be crucial for our work [3, 4]. String loop corrections are studied in [126–129, 88, 89, 130–134] and are also the main focus of our work [2].

Additionally to the literature on either purely α' or g_s corrections, in [135–140, 69, 70, 141, 142] the combination of different effects, field redefinitions, and higher derivative supergravity corrections are investigated.

In Part I we will study perturbative quantum corrections with a focus on string loop corrections from the field theory/EFT perspective. It is therefore useful to elaborate on how to understand string loop corrections from the EFT perspective. We assume some knowledge of string loop amplitudes and only discuss their field-theoretic limits.

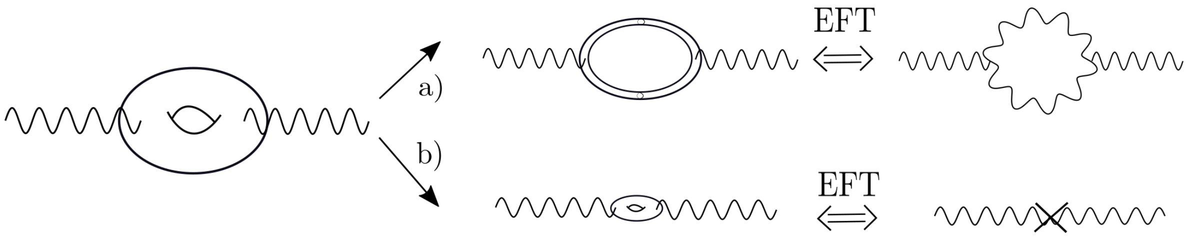


Figure 1.2: A torus diagram with two graviton insertions. Limit a) depicts the limit of the torus diagram where the closed string travels a long distance corresponding to a long and thin torus. In an EFT this corresponds to a loop diagram where the massless closed string excitations run in the loop. Limit b) shows the limit of the torus diagram where the string travels a string-size distance. From the EFT this looks like a local higher dimensional operator (illustrated with the cross between the graviton lines) inserted.

Consider as an example a torus diagram as a string loop correction to the graviton propagator depicted in Fig. 1.2. The external lines correspond to the graviton and the torus corresponds to a closed string running in the loop. To calculate this diagram one has to integrate over all inequivalent tori of different shapes and sizes[17]. This effectively amounts to integrating over different lengths of the loop and of the closed string. The low energy limit of the torus diagram is obtained when the torus is very long and thin. This corresponds to a short closed string[18] traveling in a large loop (see Fig. 1.2 a)) such that the closed string effectively looks point-like and the loop is dominated by the massless states of the closed string. Furthermore, since the closed string travels a large distance (approximately the length scale of the compact space) the diagram is dominated by the KK scale. The string diagram in this limit can then be approximated by the standard loop diagrams of a QFT whose finite contributions are dominated by the KK scale. This works analogously for open string diagrams where the short open string should run in a large loop.

Let us now discuss the limit when both 1-cycles of the torus are string-scale as in Fig. 1.2 b). In this case the closed string travels a string-scale distance. This corresponds to the high-momentum region of field theory loops which might require adding counterterms in form of higher dimensional operators to renormalize the loops (remember that gravity is non-renormalizable). Additionally to the loop-induced counterterms, higher dimensional operators exist also at tree level in the string loop expansion and hence come at leading order in g_s. They

[17] This corresponds to the moduli space of the torus.

[18] Note that the string has a tension set by the inverse string length l_s. Hence it is energetically very expensive to stretch the string.

are known as α' corrections. From the stringy perspective loop induced counterterms are then understood as a mixture of α' and g_s corrections. A well known example is the R^4 term of type IIB string theory [74] whose coefficient has a tree level and 1-loop contribution in g_s.

The classification of the EFT limits will serve as a guiding principle of classifying perturbative corrections to stringy effective actions from the field theory perspective in Part I. We will call loop corrections due to the first limit Fig. 1.2 a) *genuine loop corrections.* From the perspective of the compactified four dimensional theory, they arise from integrating out the tower of KK modes. From 10d, genuine loops can be viewed as fields propagating in the compact space. From the KK reduction above we know that it is precisely the eigenfunctions of the compact space of the respective fields that propagate in the compact space. The distinguishing feature of genuine loops is therefore their non-locality in 10d (or on branes) such that they can not be associated with local operators. This is reminiscent of the Casimir effect where the eigenmodes propagating on the compact space lead to a zero-point energy density. In the effective four dimensional theory, after integrating out the tower of KK modes, genuine loop effects can again be associated to a local operator[19].

Quantum corrections arising through dimensional reduction of classical or loop induced (see Fig. 1.2 b)) higher dimensional operators will be called *local α' corrections* in our classification scheme.

At the end of the day we aim for controlled moduli stabilisation and are hence interested in calculating the effect of quantum corrections on the supergravity scalar potential (1.4). As a result, all quantum corrections have to be recast into corrections to the Kahler potential K and superpotential W. As W is not corrected perturbatively [143, 144], all perturbative corrections enter the Kahler potential. Converting corrections to the 4d effective action into corrections to the Kahler potential can be done by observing that the second derivative of K is the metric on the Kahler moduli space. Hence, in the Einstein frame of the 4d effective theory, the second derivative of the Kahler potential w.r.t. the moduli enters as the prefactor of the kinetic terms of the moduli. Therefore, all terms contributing in 4d Einstein frame to the kinetic terms of the moduli can be interpreted as corrections to K. This can either happen through corrections directly to the moduli kinetic terms, or through corrections to the 4d Einstein-Hilbert term which after Weyl rescaling contribute to the kinetic terms of the moduli. The 4d Einstein-Hilbert term and the kinetic terms of the moduli are corrected directly by loop corrections or by dimensionally reducing higher dimensional operators. As an example of how higher dimensional operators correct the 4d Einstein-Hilbert term consider the 10d R^4 term of type IIB string theory which can be reduced to the 4d Einstein-Hilbert term schematically as

$$\int_{\mathcal{M}_{10}} \mathrm{d}^{10}x\sqrt{-g_{10}}R_{10}^4 \sim \int_{\mathbb{R}^{1,3}} \mathrm{d}^4x\sqrt{-g_4}R_{\text{external}} \int_Y \mathrm{d}^6y\sqrt{-g_6}R_{\text{internal}}^3\,, \tag{1.10}$$

where g is the determinant on the 10d, 6d, or 4d space respectively. In summary, this shows

[19] Analogously to integrating out the gauge bosons in the Glashow-Salam-Weinberg theory to obtain a local four fermion interaction in Fermi's theory.

that genuine loops contribute to the Kahler potential directly through loop corrections and local α' effects indirectly by dimensional reduction. More details and equations can be found in Sect. 2.3.

1.4 de Sitter vacua in String Theory

So far, before giving a rough overview on quantum corrections relevant for type IIB string theory, we have stabilized all complex structure moduli by fluxes and then used some Kahler moduli stabilisation scenario like the LVS to make the remaining moduli heavy. The vacuum energy of the theory then turned out to be at negative value. This seems to be a general feature of moduli stabilisation and is in contradiction with present observations which measure an accelerated expanding universe [11, 12]. An AdS minimum of any moduli stabilisation scenario should hence be *uplifted* to a positive minimum by supplying a source of energy contributing positively to the scalar potential. This contribution should be small enough to obtain a tiny, positive vacuum energy consistent with observations.

As a side remark, let us note that another way of explaining the accelerated expansion of the universe are quintessence models where at present the quintessence field is slowly rolling at positive values. This seems to be as difficult (or even more difficult) to realize in string theory as a dS vacuum, see e.g. [145–149, 81]. We will therefore pursue the aim of constructing dS vacua in this thesis.

Proving that controlled dS minima in string theory exist is notoriously hard. Despite intense work over the last two decades this remains one of the (or even *the*) core problem(s) of string phenomenology since not even a single, rigorous 4d dS vacuum has been found. This has recently cast doubt on the very existence of dS space in quantum gravity [150–153].

Let us overview the current status of 4d dS vacua in string theory at the time of writing this thesis to get a feeling for why dS vacua are so elusive. For this we focus on critical string theory on compact geometric backgrounds (such that the geometry is Riemannian and there are no non-geometric fluxes). Details on other constructions can for instance be found in [150]. In critical, geometric models there are two main strategies on how to (potentially) obtain dS vacua. The first approach is to construct dS vacua at the two derivative level of the string effective supergravity action which are usually referred to as classical dS vacua. The second idea is to include a certain set of quantum corrections beyond the leading order action such as higher derivative, string loops, and non-perturbative corrections. Both strategies can be pursued in either type II or heterotic theories[20].

At first sight, the classical dS solutions seem more appealing since the properties of all ingredients of these setups are well understood. From this perspective, quantum corrections just represent additional components increasing the complexity of the setup. So why then did we advertise the study of these quantum correction above? The reason is a very long history

[20]Type I string theory is S-dual to the $SO(32)$ heterotic string as shown in [154]. Explicit moduli stabilisation in type I has been initiated in [155].

of so called no-go theorems forbidding dS vacua in certain classical solutions.

The no-go theorems are especially strong for heterotic theories. In a series of papers [156–159] it has been shown that heterotic theories at string tree level including α' and non-perturbative stringy α' corrections as well as gaugino condensates do not admit dS vacua.

In type II theories the no-go theorems are not as stringent as in heterotic theories. They only apply to the classical action with Dp-branes and Op-planes included. The current status of no-go theorems in type II theories[21] can be summarized by three (so far not proven) conjectures [174]. First, there are no classical dS solutions with parallel sources. Second, classical dS solutions with intersecting sources are unstable. And third, classical dS solutions can not exist at weak coupling (large volume, small string coupling) with quantized fluxes and a number of orientifolds that is bounded. Note that the no-go theorems do often also forbid slow roll inflation.

As a result classical dS vacua can not be fully excluded and therefore remain an open research field. For some open issues consult [150].

Nevertheless, it seems likely that in order to obtain dS vacua, the degree of complexity of the setup has to be increased. For classical dS solutions this can be done by including manifolds of negative curvature, or anti-branes/NS5-branes [163, 162, 175, 176] as these evade all no-go theorems. Another way of evading the no-go theorems is by incorporating quantum effects as we have already done above when introducing KKLT and LVS. A further motivation for taking into account quantum corrections is that they are typically required to stabilize all moduli. We have seen an example of this above for type IIB orientifolds on Calabi-Yau manifolds where all Kahler moduli remain flat directions at classical level[22]. Other examples of massless moduli at classical level are Kahler axions in IIB and also some complex structure axions in DGKT [161] in the context of IIA string theory.

As there are some no-go theorems excluding heterotic dS solutions including quantum effects, it seems more natural to focus on type II theories. There, the models in IIA have the following drawbacks compared to models in IIB that complicate moduli stabilisation in IIA. First, the backreaction of fluxes in IIA is stronger than in IIB such that the underlying manifold is no longer Calabi-Yau but *half-flat* [178–180]. In type IIB the backreaction of fluxes is taken into account by the warp factor that renders the background manifold conformally Calabi-Yau. Second, the most studied scenarios for establishing dS vacua like KKLT and LVS in IIB feature a very steep AdS minimum since it is obtained by non-perturbative effects in contrast to the DGKT scenario in IIA where all moduli are stabilized perturbatively. The advantage of a steep minimum is that it is in general easier to uplift to a positive value. Third, IIB theories feature the Klebanov-Strassler throat [45] which turns out to be very useful for creating dS uplifts and hierarchies in general as we will review shortly. Note that also the backreation of intersecting O6-planes was for a long time not understood except from the

[21] An incomplete list of references studying classical type II solutions and their for no-go theorems is [160–173, 150, 174] and references therein.

[22] Note that there are also classical solutions of IIB where all moduli are stabilized [173, 177, 165]. In these cases, the compact manifold is not Calabi-Yau.

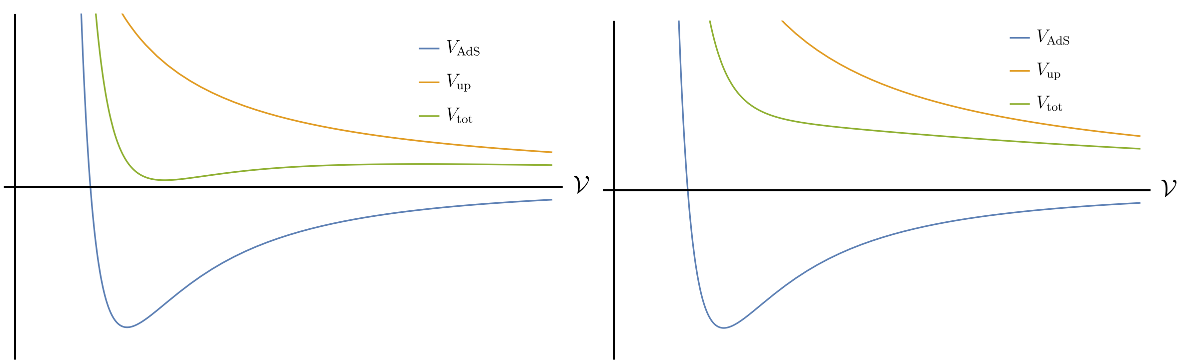
smeared approximation [181]. This has been resolved by now in [182].

Due to these reasons there is much less work on obtaining dS vacua in IIA string theory including quantum effects. There is for instance the possibility of mimicking the LVS in IIA string theory by T-dualizing the corresponding IIB orientifold [183] or the KKLT scenario [175, 176, 184].

Another reason why dS vacua are difficult to construct independently of the type of string theory is that supersymmetry is necessarily broken in a vacuum of a theory arising from a $\mathcal{N} = 1$ supergravity theory as shown for instance in [185]. This makes it more difficult to solve all equations of motion.

This overview should motivate why we focus on the framework of IIB string theory with a particular emphasis on quantum corrections and their implications when investigating dS vacua in string theory in the main body of the thesis.

In order to find dS vacua in IIB string theory one typically starts with a scale separated AdS minimum like the one achieved via the LVS and then adds a source of potential energy to uplift the minimum to a positive value. This can be done in many different ways [60, 186–188, 66, 189, 67, 190–196, 69, 70, 197], see also the reviews [145, 81] for an overview. Additionally, in [3, 4] we also propose a new uplifting mechanism which will be explained in detail in Chapter 4 and 5. The most famous uplifting proposal is the anti-D3-brane uplift [60] which we will review in the following since it is essential for the major part of this thesis [2–4].



(a) (b)

Figure 1.3: Uplifting an AdS minimum of V_{AdS} to dS. In a) a consistent uplift to a dS minimum is depicted. In b) the uplifting contribution V_{up} is too large such that the total potential V_{tot} is a runaway.

As anti-D3-branes have opposite charge compared to D3-branes, they do not satisfy the zero force condition and therefore contribute positively to the potential. This makes them suitable for uplifting. Their contribution to the potential is given by $V_{\text{up}} = 2T_{\text{D3}} \sim 1/\mathcal{V}^2$ in string units with T_{D3} the tension of a D3-brane. This, however, is much larger than the depth of the AdS minimum $|V_{\text{AdS}}|$. For the LVS $V_{\text{AdS}} \sim -1/\mathcal{V}^3$ as given by (1.9). For $V_{\text{AdS}} \ll V_{\text{up}}$

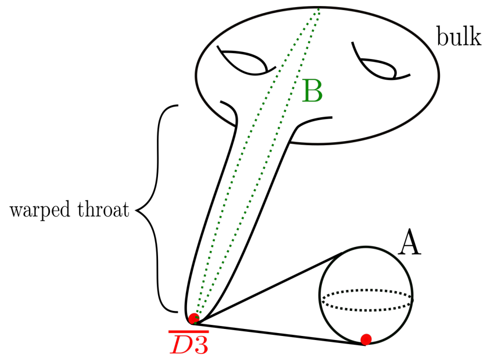


Figure 1.4: An illustration of a KS throat glued into a compact manifold with an anti-D3-brane sitting at the tip. The throat can be described by two 3-cycles A and B.

the total potential $V_{\rm tot} = V_{\rm AdS} + V_{\rm up}$ does not have a metastable minimum anymore – the potential has a runaway behavior driving the volume modulus to infinity which leads to a decompactification of the theory. This is illustrated in Fig. 1.3 b). To obtain a metastable minimum for $V_{\rm tot}$ at positive value the contribution of the anti-brane needs to be diminished. This can be done by placing the anti-brane(s) at the tip of a Klebanov-Strassler (KS) throat [45]. The KS throat provides an exponentially large warp factor which effectively suppresses all quantities of objects living in the throat. In our case this will be the tension of the anti-brane. The feature of creating hierarchies due to exponentially large warping makes the KS throat an extremely popular topology in string compactifications. Such a throat glued into a compact space is illustrated in Fig. 1.4.

The KS throat is six dimensional and consists of a radial direction τ and a five dimensional space called $T^{1,1}$ which can be viewed as an S^2 fibration over S^3. The throat is described by two 3-cycles A and B which are threaded by M units of F_3 flux or K units of H_3 flux, respectively. At the tip ($\tau = 0$), the S^2 of the $T^{1,1}$ pinches off such that the tip is topologically given by an S^3 which coincides with the A-cycle at the tip. The B-cycle extends to infinity or, if glued into a compact geometry, over the entire geometry. The warp factor is generated by the fluxes and is approximately given at the tip of the throat by $\exp(4A(\tau = 0)) \sim \mathcal{V}^{2/3} \exp(-N/(g_s M^2))$ with $N = KM$ the flux in the throat[23]. The size of the positive contribution to the potential from the anti-D3-branes at the tip of a KS throat is then given by $V_{\rm up} = 2T_{\rm D3} \exp(4A(\tau = 0)) \sim \exp(-N/(g_s M^2))/\mathcal{V}^{4/3}$. This uplifting potential can hence be tuned in an almost continuous manner through the amount of fluxes K and M in the throat and the string coupling g_s. For sufficiently large warping $N \gg g_s M^2$, $V_{\rm up} \approx |V_{\rm AdS}|$ so that a small and positive cosmological constant can be achieved.

However, this picture is too simplistic since the anti-branes at the tip can undergo a classical decay into a supersymmetric state which can not be used for uplifting since for a

[23]The volume scaling is due to [198]. See also a derivation in Sect. 3.6.

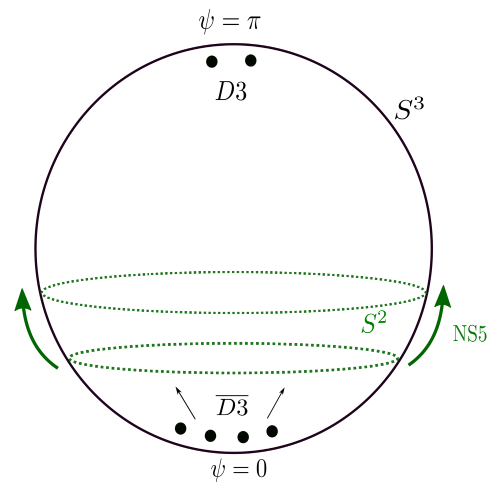


Figure 1.5: Flux-brane annihilation a la KPV. The tip of the throat, topologically an S^3, is depicted. The p anti-D3-branes puff up into an NS5-brane, wrapping an S^2 inside the S^3 (in green). At the north pole the NS5-brane annihilated against flux decaying into a supersymmetric minimum consisting of $M - p$ D3-branes.

supersymmetric state, the potential vanishes. This decay is known as brane-flux annihilation and is described by Kachru, Pearson, and Verlinde (KPV) in [199]. The p anti-D3-branes at the tip can puff up due to the *Myers effect* [200] into a single NS5-brane with p units of worldvolume F_2 flux. In other words, the anti-branes form an energetically favored state, the NS5-brane. The NS5-brane wraps an S^2 inside the S^3 and has a non-trivial potential $V_{\mathrm{KPV}}(\psi)$ where ψ parametrizes the position of the S^2 inside the S^3, see also Fig.1.5. Then, depending on the parameters p and M, the NS5-brane may either provide a metastable supersymmetry breaking vacuum suitable for uplifting or be classically unstable. To be precise, if $p/M < 0.08$ the NS5-brane resides in a metastable minimum at positive energy. If $p/M > 0.08$ the NS5 is classically unstable. In the case of classical instability, the NS5 slips over the equator while its anti-D3-brane charge annihilates against flux, materializing into $M - p$ D3-branes at the north pole of the S^3. This decay leads to a supersymmetric vacuum. If the NS5-brane is metastable, the uplifting potential is given by $V_{\mathrm{up}} = V_{\mathrm{KPV}}(\psi_{\mathrm{min}}) \exp(4A(0)) \sim 2T_{\mathrm{D3}} \exp(4A(0))$ which agrees with the formula given above. The KPV process can also be understood quite intuitively. Roughly, the tension of the NS5-brane pushes the brane towards the south pole of the S^3 whereas the F_3 flux wants to pull the NS5-brane over the equator. Balancing these effects eventually leads to a metastable minimum. Further important formulas and more details are given in Sects. 4.2.1 and 5.2.

Besides this in principle working mechanism, the anti-D3-brane uplift is far from being settled and still remains an open research field in multiple directions.

A much investigated issue is that of backreaction of anti-D3-branes or, equivalently, the puffed-up NS5-brane on the geometry [201–226]. Furthermore, in a series of papers [227, 58, 228–230] a further decay channel of the anti-branes at the tip of the KS throat is studied. This decay channel is called the conifold instability and arises due to a runaway behavior of a light complex structure modulus governing the size of the throat if the bound $g_s M^2 > 46$ is invalidated. This bound is stronger than a similar bound from the KPV decay which reads $g_s M^2 > 12$ (following from $p/M < 0.08$ and $g_s M > 1$ for $p = 1$). Note that the existence of the conifold instability is also not fully established. The instability was recently questioned and shown to be weaker in [231].

It is worth pointing out that the parameter $g_s M^2$ is a phenomenologically very relevant quantity since it is essential for the size of the warp factor $\exp(4A(0)) \sim \exp(-N/(g_s M^2))$. Remember that at the end of the day we aim for a small and positive cosmological constant which, as explained above, requires exponentially large warping for which we need $N \gg g_s M^2$. Thus, strong bounds on the quantity $g_s M^2$ therefore increase the required flux N in the throat and therefore the contribution to the D3-tadpole. This positive contribution to the tadpole needs to be canceled due to *tadpole cancellation* (the net flux on a compact space has to vanish) by a negative contribution coming from O-planes or D7-planes which is limited in compactifications of type IIB string theory.

In the context of the LVS with the assumption that higher order corrections should not spoil the standard LVS construction, we quantified the minimal required negative contribution to the D3-tadpole in [2] which we called the *LVS parametric tadpole constraint* (PTC). As expected from the discussion of the previous paragraph, the quantity $g_s M^2$ is a major player in the PTC.

Another research direction that was opened up in [3, 4] is the effect of higher order α' corrections on the KPV decay channel. In KPV [199], the potential of the NS5-brane remained at leading order in α'. But it can be anticipated that α' corrections will change the leading order behavior substantially. This is most easily understood for curvature corrections but holds also true for all other α' corrections. Since the NS5-brane sits at the tip of the throat, intrinsic and extrinsic curvature corrections will be suppressed by the radius of the S^3 or S^2, respectively. The radii are given by $R_{S^3} \sim \sqrt{g_s M \alpha'}$ and $R_{S^2} \sim \sqrt{g_s M \alpha'} \sin\psi$, respectively. We have seen that the parameter $g_s M^2$ and hence $g_s M$ should be as small as possible for the phenomenological reasons explained above. This is precisely the regime where curvature corrections become important such that their study is mandatory and will increase the KPV bound[24] $g_s M^2 > 12$. In the Chapters 4 and 5 we will calculate that this α' corrected bound for $g_s M^2$ turns out to be by far the most important bound on $g_s M^2$.

All the discussion above concerning the tadpole and the parameter $g_s M^2$ is only relevant in the standard anti-D3-brane uplift where one uses exponentially large warping to warp down the tension of the anti-branes. It would therefore be highly preferable to find an uplifting mechanism not relying on the exponentially large warping. Such a mechanism is proposed in [3, 4] where it was observed that α' corrections overall lower the potential of the NS5-brane

[24]Remember that this uses $g_s M \sim 1$ which is where all α' corrections are equally important!

and therefore the effective tension (the value of the NS5-brane potential at the minimum) of the anti-branes. By tuning the fluxes in the throat and the string coupling thereby balancing tree level and α' corrections, one can achieve a metastable minimum at arbitrarily small energy. As a result, the effective tension of the NS5-brane is itself arbitrarily small and need not be warped down, thereby sidestepping all problems related to exponentially large warping.

1.5 This thesis

This thesis explores higher order corrections to the leading order effective action of type IIB string theory and their implications for phenomenology. As we have reviewed above, this is an important and interesting task for several reasons. First, quantum corrections can in principle endanger moduli stabilisation scenarios like the LVS with and without a dS uplift. Second, if dS minima in string theory exist, they are very likely to be found in the interior of moduli space where higher order corrections potentially become important. Finally, it is possible to take advantage of some of these corrections for approaching long standing problems in string phenomenology like dS vacua or inflation. These questions will be investigated in the main body of the thesis.

Part I mainly investigates higher order corrections to the effective action of type IIB string theory from the field theoretic perspective focusing on string loop corrections. This is motivated by the fact that not much is known about string loop corrections on general Calabi-Yau manifolds. Their form is only conjectured in [129] based on very special but exact string loop calculations on torus orbifolds [127]. The conjecture is known as the Berg-Haack-Pajer (BHP) conjecture. Although only conjectured, string loop effects are often used in phenomenological applications like in moduli stabilisation scenarios or in models of inflation. This can be a dangerous venture, as not only is the form of the loop corrections only conjectured, but it is also not entirely clear when these loop corrections occur. Improving their understanding is therefore very much needed. A step towards this goal is taken in Chapter 2 which reproduces [1]. Therein we provide a classification scheme of higher order corrections suitable for understanding perturbative quantum corrections from the EFT perspective. This resolves an apparent, longstanding inconsistency between the parametric behavior of string loop results and expectations from field theory. Additionally, our results show interesting new features not captured by the BHP conjecture. An example is the possible existence of a logarithmically enhanced loop correction dominant compared to all other loop corrections. This new knowledge is subsequently also applied to fibre inflation.

Part II studies the effects of higher order corrections on the LVS and the anti-D3-brane uplift. In Chapter 3 which reproduces [2], we apply higher supergravity corrections [140] and a correction from warping and α' effects [141] to the LVS with anti-D3-brane uplift. The exponentially large volume of the LVS requires exponentially large warping in order to find a very small but positive cosmological constant. As a consequence, the negative contribution to the D3-tadpole must be very large. We quantify the minimal negative tadpole in order to be in control over the previously mentioned corrections in what we call the *LVS parametric tadpole constraint* (PTC). A large negative contribution to the tadpole severely constraints manifolds suitable for the LVS. We discuss future research directions on how to overcome the PTC.

In the Chapters 4 and 5 we focus on the anti-D3-brane uplift and the underlying KPV decay channel [199]. In recent work [141, 142] the effect of curvature corrections to anti-D3-branes has been investigated. Effectively, the curvature corrections reduce the tension of the anti-brane. This clearly effects the uplifting potential V_{up}. However, as we pointed out in the last section, the picture of the p anti-branes at the tip of the throat is too simplistic as the setup has a classical decay process as observed by KPV [199]. For a consistent uplift potential one has to satisfy the metastability condition $p/M < 0.08$. Hence, also the effect of curvature corrections on this metastability condition of the puffed up NS5-brane should be studied to see whether the curvature corrected potential provides an uplift. In Chapter 4, reproducing [3], we derive curvature corrections for the NS5-brane by S-dualizing known curvature corrections on Dp-branes and investigate the curvature corrected KPV potential. We find that the standard KPV constraint $g_s M^2 > 12$ for a metastable uplift is significantly tightened[25]. As a result, the radius of the S^2 wrapped by the NS5-brane at the tip of the KS throat needs to be much larger than previously assumed requiring much more flux on the A-cycle of the throat. The curvature corrected lower bound on $g_s M^2$ then affects the standard anti-D3-brane uplift in the LVS. This can be quantified using the PTC, with the result that an even larger negative contribution to the D3-tadpole is necessary to control the most dangerous quantum corrections.

Besides the standard anti-D3-brane uplift which suffers from the strong bound on $g_s M^2$ induced by the curvature corrections, we can also take advantage of strong curvature corrections. The reason is that they effectively lower the NS5-brane potential and therefore also the energy at the metastable minimum. As described in Chapter 4, one could hence imagine tuning the value of the potential at the metastable minimum of the NS5 by tuning the size of the curvature corrections. Trusting our incomplete analysis, this tuning can reduce the effective tension to such small values that no exponentially large warping is needed. This mechanism then avoids issues related to the PTC. We also comment in detail on open questions of this construction.

In Chapter 4 we have only calculated curvature corrections to the NS5-brane potential at the tip of the KS throat. However, many more α' corrections to Dp-branes are known. In [4] which is reproduced in Chapter 5, all known α' corrections are S-dualized to obtain the α' corrections to NS5-branes. These hundreds of α' corrections include gauge fields, fluxes, curvature, and mixings thereof. The α'-corrected KPV potential is then studied analogously to Chapter 4. This leads to more precise predictions for the allowed parameter range for a metastable minimum to exist. Also the new uplifting mechanism proposed in 4 can be constructed explicitly.

Finally, in Chapter 6 we combine the results of Part I and II. In Sect. 6.1 we give a detailed self contained summary of the main ideas and results of each publication. Then, in Sect. 6.2

[25] For precise numbers consult the Introduction 4.1 of Chapter 4. However, note that these numbers will change again in Chapter 5 when taking into account all known α' corrections.

we discuss the main results and embed them into the broader picture of string phenomenology. We conclude in Sect. 6.3 with an outlook highlighting potential future directions emerging from or developing results of this thesis.

The following Chapters 2 to 5 are a compilation of papers published by the author together with a number of collaborators. At the time of submission of this thesis, the papers in Chapter 2, 3, and 4 have been published in a peer-reviewed journal. The paper of Chapter 5 is currently under review. The reference to each article is listed in the Preface and at the beginning of each chapter before the publication is reproduced. There, a detailed description of the author's contributions to each multi-authored paper can be found.

Part I

Higher order corrections to type IIB string theory

Chapter 2

Loops, local corrections and warping in the LVS and other type IIB models

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Simon Schreyer is the principal author of this article. The original idea for the project was conceived by Arthur Hebecker and Xin Gao and mainly further developed by Arthur Hebecker and Simon Schreyer. All calculations were performed by Simon Schreyer and all figures except figure 4 were produced by Simon Schreyer. The article was originally written by Simon Schreyer with small exceptions in the Introduction, Conclusion, and some paragraphs in Sect. 3.1. which are originally written by Gerben Venken. Arthur Hebecker and Gerben Venken contributed with corrections and suggestions on the manuscript. Arthur Hebecker, Simon Schreyer, and Gerben Venken were equally involved in the review process.

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Loops, local corrections and warping in the LVS and other type IIB models

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Abstract: To establish metastable de Sitter vacua or even just scale-separated AdS, control over perturbative corrections to the string-derived leading-order 4d lagrangian is crucial. Such corrections can be classified in three types: first, there are genuine loop effects, insensitive to the UV completion of the 10d theory. Second, there are local α' corrections or, equivalently, 10d higher-dimension operators which may or may not be related to loop-effects. Third, warping corrections affect the 4d Kahler potential but are expected not to violate the 4d no-scale structure. With this classification in mind, we attempt to derive the Berg-Haack-Pajer conjecture for Kahler corrections in type-IIB Calabi-Yau orientifolds and extend it to include further terms. This is crucial since the interesting applications of this conjecture are in the context of generic Calabi-Yau geometries rather than in the torus-based models from which the main motivation originally stems. As an important by-product, we resolve a known apparent inconsistency between the parametric behaviour of string loop results and field-theoretic expectations. Our findings lead to some interesting new statements concerning loop effects associated with blowup-cycles, loop corrections in fibre inflation, and possible logarithmic effects in the Kahler and scalar potential.

Keywords: String and Brane Phenomenology, Flux Compactifications, Superstring Vacua

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Contents

1 Introduction

The leading paradigm in the search for realistic vacua in the string theory landscape is to start with type-IIB Calabi-Yau orientifold models with O3/O7-planes, to stabilize complex structure moduli by 3-form flux, and only then to deal with the classically flat Kahler moduli space [1, 2]. Those flat directions may then be stabilized by nonperturbative effects alone [3] or in combination with α' corrections and loop effects [4–6]. In any case, be it as a central ingredient or as a potentially dangerous, subleading effect, perturbative corrections are important in string phenomenology. They affect the scalar potential and hence the prospects of uplifting an initial AdS vacuum to de Sitter — a key step which is still under debate.[1] Clearly, loop and other perturbative effects also impact models of inflation which use Kahler moduli [22, 23].

Motivated by this situation, we devote the present paper to the study of loop corrections to the type-IIB Kahler moduli Kahler potential in the Calabi-Yau context [24–31]. The present level of understanding is not satisfactory: while field-theoretic arguments allow one to make a proposal for such corrections in the simplest Calabi-Yau settings [24, 28], explicit string loop calculations are available only in torus orbifold geometries [25, 26]. It has been conjectured how to generalize the latter to Calabi-Yau models [27, 28], but no derivation for the proposed structure is available. Moreover, there is a seeming inconsistency [28] between field-theoretic and string loop results, which we will resolve in this paper.

In our analysis, we will have the Large Volume Scenario (LVS) [4, 5] in the back of our minds as this is a prototypical example of a model with fluxes where a better understanding of loop and α' corrections is crucial — see [32–44] for recent work on loop and α' corrections in this and related settings. However, our findings are not restricted to the LVS and should be relevant more generally in the type-IIB context.

To explain our approach at a more technical level, let us start by stating the Berg-Haack-Pajer (BHP) conjecture and then describe how, according to our findings, it relates to the three basic types of loop corrections between which we will distinguish. The BHP conjecture proposes two kinds of corrections to the Kahler potential, scaling like

$$\delta K^{KK}_{(g_s)} \sim \sum_a \frac{g_s \mathcal{T}_a(t^i)}{\mathcal{V}}\,, \qquad \delta K^{W}_{(g_s)} \sim \sum_a \frac{1}{\mathcal{I}_a(t^i)\mathcal{V}}\,. \tag{1.1}$$

Here $\mathcal{T}_a$ and $\mathcal{I}_a$ are linear combinations of 2-cycle Kahler moduli t^i, the volume is $\mathcal{V} = \kappa_{ijk} t^i t^j t^k/6$, and we recall that the proper Kahler variables are the complex 4-cycle moduli, with real parts $\tau_i = \partial\mathcal{V}/\partial t^i$. While, as we will see momentarily, our results in part deviate from (1.1), it is nevertheless a good starting point for organizing our discussion.

Next, we clarify the origin and fix terminology for three different kinds of loop corrections.

First, there are *genuine loop corrections* which arise from integrating out the tower of KK modes (4d perspective) or from loops of 10d or brane-localized fields propagating in the compact space (10d perspective). Their distinguishing feature is their non-locality in the

[1] This discussion has recently gained momentum following [7, 8]. For some of the latest additions, see e.g. [9–19]. An important part of the debate is the issue of scale separation [20, 21].

higher-dimensional theory: they can not be associated with local operators in 10d or on a brane. In this sense, they are analogous to the Casimir energy,[2] which arises in geometries with two separated surfaces but can *not* be encoded in a local operator on either surface or in the space between them.

The genuine loop corrections may be thought of as coming from the interacting 4d field theory of moduli and KK modes. In this theory, 3-vertices are suppressed by $1/M_4$. Accordingly, genuine 1-loop effects correct the Kahler moduli kinetic terms as

$$\left(1+\frac{M_{KK}^2}{M_4^2}\right)\frac{1}{\tau^2}\partial_\mu\tau\partial^\mu\tau \qquad \text{or, more generally,} \qquad \left(1+\frac{M_{KK}^2}{M_4^2}\right)K_{ij}\partial_\mu\tau^i\partial^\mu\tau^j\,. \tag{1.2}$$

Here the factor M_{KK}^2 appears on dimensional grounds since, similarly to Casimir energy calculations, no UV mass scales are involved.

It is easy to see that M_{KK}^2/M_4^2 is a homogeneous function of degree -2 in Einstein-frame 4-cycle volumes. Equivalently, we can say that the correction appears at order $\alpha'^4 g_s^2$, such that its scaling agrees with that of the second term of the BHP conjecture (1.1). Our previously mentioned inconsistency is then the apparent absence of the first term in (1.1) in the field-theoretic approach. Moreover, we will argue that the functions $\mathcal{I}_a$ in the Calabi-Yau case are not necessarily linear in 2-cycles. Instead, the additional dependence on ratios of cycles is expected. In section 2, we discuss the genuine loop corrections in detail and also derive (1.2) using Feynman diagrams.

Second, there are *local α' corrections* or, in more precise language, corrections coming from higher-dimension local operators in 10d, on branes and O-planes, or on their intersection loci. We use the adjective 'local' to distinguish them from other effects, such as genuine loop corrections, which also induce 4d EFT operators suppressed by α'. It is important to note that local α' corrections may receive contributions from the high-momentum region of loop integrals. There is in particular no clean separation between local α' corrections which are part of the classical action and the counterterms needed to renormalize the loops. It appears natural to us to collect all corrections which can be associated with higher-dimension local operators, be they fundamental or loop-induced, under the name 'local α' corrections'.

Local α' corrections appear at different order in α' since the underlying higher-dimension operators come with different α' suppression factors. Crucially, such local corrections at order α'^2 can explain the first term in (1.1) and thus resolve the above puzzle. Other local α' corrections contribute to the second term in (1.1), with or without additional g_s factors. This depends on whether the operator in question appears at the string tree level or at higher-loop order.

An important result of our paper, for which we argue in section 3.1.1, is the general expectation that marginal local operators (appearing at order α'^4) introduce logarithmic corrections to the Kahler potential. Examples for this would be, if existent, the R_8^4 operator on a D7-brane/O7-plane and the R_6^3 operator on the intersection locus between D7-branes/O7-planes. In section 3.1, we will deal more generally with loop corrections induced by localized objects and extend the results to multiple Kahler moduli.

[2]As a result, our analysis may be relevant for compactification schemes directly relying on Casimir energy, see e.g. [45].

Correction type	Discussed in section	Induced by	Correction to Kahler potential	Correction to scalar potential
Genuine loops [24]	2 and 3	—	f_{-2}	$\|W_0\|^2 g_s \times h_{-5}$
BBHL+1-loop [48, 49]	2.3	$\frac{M_{10}^2}{g_s^{3/2}}(1+g_s^2)R_{10}^4$	$(g_s^{-1/2}+g_s^{3/2})$ $\times f_{-3/2}$	$\|W_0\|^2(g_s^{-3/2}+g_s^{1/2})$ $\times h_{-9/2}$
Non-intersecting D7/O7 (partly) [38, 39]	3.1.1	$M_{10}^4(1+g_s)R_8^2$	$(0+g_s)\times f_{-1}$	$\|W_0\|^2 g_s^3 \times h_{-5}$
Log-Correction on D7/O7	3.1.1	R_8^4	$\ln(M_{10}g_s^{1/4}L)$ $\times f_{-2}$	$\|W_0\|^2 g_s \ln(M_{10}g_s^{1/4}L)$ $\times h_{-5}$
Intersecting D7/O7 [38–40, 50, 51]	3.1.3	$M_{10}^4(1+g_s)R_6$	$(0+g_s)\times f_{-1}$	$\|W_0\|^2 g_s^3 \times h_{-5}$
Log-Correction on intersecting D7/O7	3.1.3	R_6^3	$\ln(M_{10}g_s^{1/4}L)$ $\times f_{-2}$	$\|W_0\|^2 g_s \ln(M_{10}g_s^{1/4}L)$ $\times h_{-5}$

Table 1. Some of the key corrections discussed in the paper and their effect on the Kahler and scalar potential. The functions $f_{-\lambda}$, $h_{-\lambda}$ are homogeneous of degree $-\lambda$ in 4-cycles and L is the typical length scale of the internal manifold.

Finally, there are *warping corrections* or, more generally, corrections due to the classical backreaction of the background geometry. These can not be cleanly separated from string loop effects since, in the regime where the worldsheet is a long cylinder, the string loop encodes the effects of light 10d fields propagating at tree-level. In our 10d EFT approach, such corrections have to be viewed as classical rather than loop-induced.

As is well known (and reviewed in section 4) warping corrects the Kahler potential by a series of terms $1/\tau^n$, starting at $n = 1$. The complete series does not affect the scalar potential since warping respects the no-scale structure [2]. The $n = 2$ term matches parametrically the second term in (1.1).

In section 6 we work out the explicit form of loop corrections for a blowup modulus (see also [46]) and for fibred geometries. Before concluding in section 8, we devote Section 7 to some further applications where loop corrections can be important: the parametric control of the LVS [18, 19], the control of KKLT with many moduli and small 2- or 4-cycles [14, 15], and the possible presence of dominant log-corrections to the Kahler potential [35–37, 43, 44]. Appendix A contains more details concerning our discussion of warping corrections in section 4, following mainly [47].

Table 1 provides a partial list of the genuine loop and local α' corrections considered in this paper.[3] In particular, concerning the operators on branes and their intersections, we display only the lowest-dimension and marginal operators. It is convenient to express the correction to the Kahler and scalar potential in terms of homogeneous functions of a certain degree in the Kahler moduli since the detailed dependence on ratios of 4-cycle volumes is known only in special cases. We emphasize in particular the corrections induced by an R_8^4 term, potentially present on D7-branes and O7-planes, which has to our knowledge not been

[3]We do not include warping corrections since they do not affect the scalar potential.

considered before. Being marginal and hence probably log-divergent, this operator induces a correction to the scalar potential which is leading compared to the corrections following from the BHP conjecture. It is therefore important for cosmological applications like Fibre Inflation (to be discussed in section 6.2) or moduli stabilization scenarios involving loop effects.

2 Basics of loop corrections — the single modulus case

2.1 Naive power counting

Our goal is a better understanding of the role of loop corrections in type-IIB. Since exact string loop calculations for Calabi-Yau manifolds are not feasible, we will try to develop the parametric estimates based on dimensional analysis as suggested in [24]. We will later on compare our findings with the exact torus-orbifold results of [25] and the corresponding Calabi-Yau form of such corrections conjectured in [27].

Let us start from the bosonic part of the Einstein-frame type-IIB action (see e.g. [52]),

$$S_{\mathrm{EF}} = \frac{1}{2\kappa_{10}^2}\int \mathrm{d}^{10}x\sqrt{-g}\left[R_{10} - \frac{\partial_M\tau\partial^M\overline{\tau}}{2\,(\mathrm{Im}\tau)^2} - \frac{G_{(3)}\cdot\overline{G}_{(3)}}{12\mathrm{Im}\tau} - \frac{\tilde{F}_{(5)}^2}{4\cdot 5!}\right] + S_{\mathrm{CS}}\,, \tag{2.1}$$

where $2\kappa_{10}^2 = (2\pi)^7\alpha'^4$, and S_{CS} the Chern-Simons term. We compactify on a Calabi-Yau orientifold with O3/O7 planes and local tadpole cancellation by D3/D7-branes, without fluxes.[4] The corresponding metric can be written as

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + L(x)^2\tilde{g}_{mn}\mathrm{d}y^m\mathrm{d}y^n, \tag{2.2}$$

with $\mu,\nu\in\{0,\dots,3\}$, $m,n\in\{4,\dots,9\}$ and a Calabi-Yau metric $\tilde{g}_{mn}$ normalized such that the compact space has unit volume. The physical Einstein-frame volume is hence given by $\mathcal{V} = L^6$. The resulting action in 4d Jordan-Brans-Dicke frame (Jordan frame for short) reads

$$S_{\mathrm{JBD}} = \frac{1}{2\kappa_{10}^2}\int \mathrm{d}^4x\sqrt{-g}L^6\left[R_4 + 6(6-1)\frac{(\partial L)^2}{L^2} + \cdots\right], \tag{2.3}$$

where we only display the Einstein-Hilbert and volume Kahler-modulus kinetic terms.

Postponing a more careful, Feynman-diagram-based derivation to section 2.2, we first provide a simple dimensional argument for the parametric behaviour of the genuine loop corrections to (2.3): at one loop, such corrections come from integrating out the tower of all KK modes. The total UV divergence is absorbed in 10d in a renormalization of M_{10}^4. The finite piece knows only about a single dimensionful parameter, the length scale L which governs the KK masses. Hence, on dimensional grounds, the corrections read

$$\Delta S_{\mathrm{JBD}} = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{b_0}{L^2}R_4 + \frac{b_1}{L^4}(\partial L)^2\right), \tag{2.4}$$

[4] In the case of D3-branes, this is only possible in a supergravity toy model, where one can place one fourth of a D3 on each O3-plane to cancel its tadpole. In string theory, one can at best place one D3 on every fourth O3. This implies warping corrections, to be discussed below. In the D7/O7 case, the curvature of the brane and O-plane induces a D3 tadpole, such that our analysis without fluxes is, once again, in most cases only an approximation.

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where b_0, b_1 are $\mathcal{O}(1)$ numerical coefficients depending on the specific Calabi-Yau and, if present, on complex structure moduli.[5] Since we perform our analysis of loop corrections to the Kahler potential in a pure Calabi-Yau orientifold, without fluxes, the complex structure moduli are massless. We treat their vevs as parameters and their fluctuations as light fields, running in the loop just like the Kahler modulus, the 4d graviton and the KK modes. We expect flux-induced modifications of the loop corrections to the Kahler potential to be subleading. To see this, recall that the spacing of KK towers is set by $m_{KK}^2 \sim 1/L^2$. The lowest level of many of the towers is zero, making the corresponding fields moduli. Three-form fluxes thread 3-cycles and hence scale as $1/L^3$. They induce an energy density $G_3^2 \sim 1/L^6$ which depends e.g. on the 10d metric and hence provides as mass correction for the 10d-metric KK tower. One obtains $\delta m_{KK}^2 \sim 1/L^6$, which is clearly subleading. The above is consistent with the well-known fact that complex structure moduli masses scales as $m_{cs} \sim 1/L^3$.

The sum of (2.3) and (2.4) can be translated to 4d Einstein frame. In addition, we trade L for the dimensionless 4-cycle variable $\tau = M_{10}^4 L^4/(2\pi)^4 = L^4/(l_s^4 g_s)$, with $l_s = 2\pi\sqrt{\alpha'}$. This corresponds to common conventions for measuring 4-cycle volumes in type-IIB. The result is

$$(S + \Delta S)_{\rm EF} = \frac{M_4^2}{2}\int {\rm d}^4x\sqrt{-g}\left[R_4 + \left(a_2\frac{(\partial\tau)^2}{\tau^2} + b_2\frac{(\partial\tau)^2}{\tau^4}\right)\right], \tag{2.5}$$

with M_4 the 4d Planck mass, $a_2 = -3/2$, and $b_2 = (114b_0 + b_1)/(32\pi)$ which again depend on complex structure moduli. To make contact with the Kahler potential, we have to interpret (2.5) as a 4d SUSY action, with $\tau = {\rm Re}\,T$ and T the complexified Kahler modulus. Thus,

$$(\mathcal{L} + \Delta\mathcal{L})_{\rm EF} = M_4^2\left(-\frac{3}{(T+\overline{T})^2}\partial T\cdot\partial\overline{T} + \frac{16b_2}{(T+\overline{T})^4}\partial T\cdot\partial\overline{T}\right), \tag{2.6}$$

where we can identify the prefactors as second derivatives of the Kahler potential K and its loop correction δK. Hence $\delta K/M_4^2 = 8b_2/3(T+\overline{T})^2$, which induces a term in the scalar potential of order $\mathcal{V}^{-10/3}$. This matches the winding correction of the BHP conjecture [27], but it does not capture the leading KK correction. We will resolve this issue in section 4 and 5.2.

2.2 Support by Feynman-diagram calculations

We now verify the results of [24] reviewed in section 2.1 using a more explicit Feynman diagram argument. We follow the literature on 'extra dimensions' [55–57], where Kaluza-Klein (KK) expansions are performed in simple geometries, as well as more general and recent studies [58–62]. Of course, we can not be fully explicit in our Calabi-Yau situation.

We start by expanding the metric as $g_{MN} = g_{MN}^{(0)} + \kappa_{10} h_{MN}$. Here $g_{MN}^{(0)}$ denotes the background metric (2.2), but with $L(x)$ replaced by a constant which, by slight abuse of

[5]These coefficients can be large if the number of light fields, including in particular complex structure moduli, is large [53]. It has been suggested in [53] to use such large loop corrections to uplift from AdS to de Sitter. We consider it safer to include the loop effects as corrections to the Kahler potential, and then to study the minima of the resulting supergravity scalar potential. The validity of such an approach, even concerning loops with fields below the SUSY breaking scale, has recently been emphasized in [54].

notation, we call L. In other words, we write $L(x) = L + \delta L(x)$ in (2.2), treating the volume modulus $\delta L(x)$ as part of the metric fluctuation h_{MN}. In order to obtain a 4d action from which Feynman diagrams can be read off, we KK expand all fields in terms of eigenfunctions of their corresponding Laplace operator[6] on the Calabi-Yau. In the following we will focus on the 4d graviton and its massive spin-2 modes as an example but similar terms can be written down for all bulk fields in (2.1) and their KK modes. We first diagonalize the action, eliminating the mixing of graviton modes with scalars and vectors arising from the 10d metric. Focusing on the spin-2 part, the action then reads [55, 58–62]

$$
\begin{aligned}
S = \int \mathrm{d}^4x \Bigg[\sum_a \Bigg(& \frac{1}{2} h^{*\mu\nu,a} \left(\Box + m_a^2\right) h^a_{\mu\nu} - \frac{1}{2} h^{*\mu,a}{}_{\mu} \left(\Box + m_a^2\right) h^{\nu,a}{}_{\nu} \\
& + h^{*\mu\nu,a} \partial_\mu \partial_\nu h^{\lambda,a}{}_{\lambda} - h^{*\mu\nu,a} \partial_\mu \partial_\lambda h^{\lambda,a}{}_{\nu} + c.c. + \dots \Bigg) \\
& + \frac{1}{M_4} \sum_{a_1,a_2,a_3} V_3 \left[h^{a_1}_{\mu\nu}, h^{a_2}_{\rho\sigma}, h^{a_3}_{\lambda\alpha} \right] + \frac{1}{M_4^2} V_4[\dots] + \dots \Bigg],
\end{aligned}
\tag{2.7}
$$

where $h^a_{\mu\nu}$ are the 4d graviton modes with a labeling the eigenfunctions ψ_a of the Laplace-Beltrami operator with eigenvalues m_a^2. In order to obtain (2.7), one has to use the relation

$$
h_{\mu\nu} = \sum_{a \neq 0} h^a_{\mu\nu} \psi_a + \frac{1}{L^3} h^0_{\mu\nu}
\tag{2.8}
$$

between the 4d part of the 10d graviton $h_{\mu\nu}$, the 4d graviton $h^0_{\mu\nu}$, and its massive KK modes $h^a_{\mu\nu}$ [58]. To understand the mass dimensions of our fields, recall that our background metric $g^{(0)}_{MN}$ and its correction $\kappa_{10} h_{MN}$ are dimensionless. Hence $[h_{MN}] = -[\kappa_{10}] = 4$. Correspondingly, the l.h.s. of (2.8) has mass dimension 4, $[\psi_a] = [1/L^3] = 3$, and the fields $h^a_{\mu\nu}$ and $h^0_{\mu\nu}$ have the canonical mass dimension one of 4d bosonic fields. This is consistent with (2.7).

The functionals V_3 and V_4 are sums of cubic and quartic terms in the $h^a_{\mu\nu}$. Each term contains two derivatives. When deducing Feynman rules, V_3 and V_4 will give 3- and 4-vertices. We have for brevity suppressed the arguments of V_4 — they coincide with those of V_3. The ellipsis at the very end of (2.7) stands for higher vertices as well as for analogous quadratic and higher-order action pieces involving all other modes of the KK-expansion — both from the 10d metric and other bulk fields. One can convince oneself that if all those 4d fields are canonically normalized, then the suppression by $1/M_4$ is a universal feature of all 3-vertices. This is a key observation: it implies that all 3-vertex-based loop corrections to the massless 4d graviton or volume modulus propagator (see l.h.s. of figure 1) have the same parametric behavior. This holds independently of the kind of field running in the loop. Other 1-loop contributions come from tadpole diagrams involving a 4-vertex (see r.h.s. of figure 1). The 4-vertices are universally suppressed by $1/M_4^2$, such that all tadpole diagrams have the same parametric behavior as the loops built with two 3-vertices.

[6] By this we mean the Laplace-Beltrami operator for scalar fields, the Laplace-de-Rham operator for p-forms and the Laplace-Lichnerowicz operator for the graviton (in general for symmetric tensors).

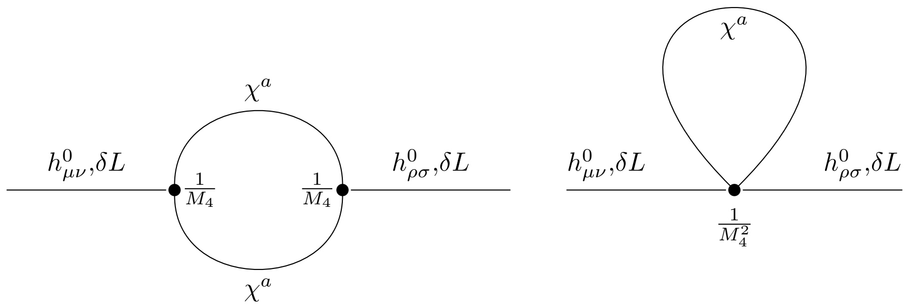

Figure 1. Self-energy diagrams correcting the propagator of the massless 4d graviton and the volume modulus. The 3-vertex is suppressed by $1/M_4$ and the 4-vertex by $1/M_4^2$. The field χ^a symbolizes all fields with their KK towers, including the massless moduli and ghost fields.

Using the KK action of (2.7) and the diagrams in figure 1, one can in principle explicitly compute the 1-loop correction to the propagators of the massless 4d graviton $h^0_{\mu\nu}$ and the volume modulus. Let us first focus on the graviton correction. It can be interpreted as a correction δ_{R_4} to the Ricci scalar term in the Einstein-frame action:

$$(S+\Delta S)_{\mathrm{E}} = \frac{M_4^2}{2}\int \mathrm{d}^4x\sqrt{-g}\,(1+\delta_{R_4})\,R_4 + \cdots\,. \tag{2.9}$$

In dimensional regularization, the loop contribution takes the form

$$\delta^{\varepsilon}_{R_4} = \frac{\mathrm{d}}{\mathrm{d}p^2}\bigg|_{p^2=0}\kappa_4^2\mu^{\varepsilon}\sum_a\int \mathrm{d}^{4-\varepsilon}q\,\frac{f_4\,(p,q,m_a)}{(q^2+m_a^2)\,((p-q)^2+m_a^2)}, \tag{2.10}$$

where $f_4\,(p,q,m_a)$ is of mass dimension 4. For the 3-vertex contribution, this follows from the fact that each 3-vertex comes with two derivatives. For the 4-vertex contribution, one has two derivatives from the vertex and a term cancelling the $(p-q)^2$-expression in the denominator. The sum over KK modes gives (2.10) a maximal degree of divergence which is as strong as in 10d, i.e. octic in cut-off language. The final 4d correction is obtained as

$$\delta_{R_4} = \lim_{\varepsilon\to 0}\left(\delta^{\varepsilon}_{R_4} + \delta^{\varepsilon,\,\mathrm{c.t.}}_{R_4}\right)\,, \tag{2.11}$$

i.e. after adding the counterterm contribution and taking ε to zero. To be specific, we use minimal subtraction, such that $\delta^{\varepsilon,\,\mathrm{c.t.}}_{R_4} = \mathrm{const.}/\varepsilon$.

If the integral in (2.10) were finite, and hence no counterterm were needed, then on dimensional grounds one would find

$$\delta_{R_4} = \frac{\mathcal{O}(1)}{L^2M_4^2}\,. \tag{2.12}$$

This follows because $m_a^2 = f_a/L^2$, with f_a dimensionless numbers encoding the Calabi-Yau geometry. Then, the loop correction in Einstein frame takes the form

$$(S+\Delta S)_{\mathrm{E}} = \frac{M_4^2}{2}\int \mathrm{d}^4x\sqrt{-g}\,(1+\delta_{R_4})\,R_4 = \frac{M_4^2}{2}\int \mathrm{d}^4x\sqrt{-g}\left(1+\frac{\mathcal{O}(1)}{L^2M_4^2}\right)R_4\,, \tag{2.13}$$

in agreement with the first term in (2.4).

Let us now discuss the precise form of (2.11). Note that a non-zero counterterm $\delta_{R_4}^{\varepsilon,\,\text{c.t.}}$ comes with a pole in (2.10) and the latter is necessarily accompanied by a factor $(\mu L)^\varepsilon$. In the limit $\varepsilon \to 0$, a finite term $\sim \ln(\mu L)$ is left. If we take our theory to be defined at the string scale, we may set $\mu = M_s = M_{10} g_s^{1/4}$ and the resulting logarithm would represent a significant enhancement of the $\mathcal{O}(1)$ coefficient in (2.12).

In the following we will argue that such terms do not occur. The reason is that $\delta_{R_4}^{\varepsilon,\,\text{c.t.}}$ vanishes. To understand this, note that compactifying of a theory on a smooth manifold represents an IR modification and does not affect the UV structure. Hence all 4d counterterms derive from 10d counterterms. It is clear that a counterterm proportional to R_{10} in 10d will induce a counterterm proportional to R_4 after compactification. However, this is not the only option. For instance, if the 10d action contains a term of the form[7] R_{10}^5, one way of compactifying this term is schematically as $R_\text{external} \int_{\mathcal{M}_6} d^6y R_\text{internal}^4$. A 10d counterterm $\sim R_{10}^5$, which is completely unrelated to the 10d propagator of h_{MN}, can hence induce a 4d counterterm relevant for the propagator of $h_{\mu\nu}$ and thereby signal a logarithmic enhancement of δ_{R_4}.

To study 4d counterterms we then require some information about higher-order terms in the 10d action. It is known that there are no terms of order R_{10}^2, R_{10}^3 [63], or R_{10}^5 [64] in the IIB supergravity action.

First let us consider the 1-loop correction to R_{10}. By analogy to (2.10), it takes the form

$$\delta_{R_{10}}^\varepsilon = \frac{\mathrm{d}}{\mathrm{d}p^2}\bigg|_{p^2=0} \kappa_{10}^2 \mu^\varepsilon \int \mathrm{d}^{10-\varepsilon} q \frac{f_{10}(p,q)}{q^2(p-q)^2}, \tag{2.14}$$

where f_{10} is again a function of mass dimension 4. The dominant divergence, taking into account the p^2 derivative, is octic. Nevertheless, one could in principle imagine a sub-leading logarithmic divergence and hence a pole being present.[8] However, here this can be excluded on dimensional grounds since no dimensionful parameters like a mass appear and p^2 is set to zero. Thus, $\delta_{R_{10}}^\varepsilon$ vanishes.

This argument can be repeated word by word for the R_{10}^4 term: the dominant 1-loop divergence for its coefficient is quadratic and, since no mass scale is available, there is no sub-leading logarithmic divergence. Hence there is no pole and no non-zero counterterm arises.

Let us make a side remark concerning specifically the R_{10}^4 term (but also relevant more generally): this higher-order term contributes a 4-vertex and can hence also correct the propagator via a diagram of the form figure 1(b). The loop diagram with this R_{10}^4 vertex is, however, suppressed by a factor M_{10}^6 compared to the analogous loop with a 4-vertex from R_{10}. We may hence neglect it.

Note that if the ten-dimensional action would have included a term of order R_{10}^5, such a term could have produced a pole in 10d since it is a marginal operator. This could in

[7] We denote by R_{10}^n any nth power term in the Riemann tensor with all indices contracted.

[8] As a simple example where subleading logarithmic corrections occur, consider the 1-loop correction to the Higgs mass:

$$\sim \frac{\lambda}{2} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{1}{p^2+m^2} = -\frac{\lambda m^2}{32\pi^2}\left(\frac{\Lambda^2}{m^2} - \ln\frac{\Lambda^2}{m^2}\right) + \mathcal{O}(\Lambda^0)\,.$$

In $4-\varepsilon$ dimensions, one would find a pole proportional to m^2.

turn have produced a logarithmic term for the four-dimensional propagator.[9] The absence of such a term in ten dimensions combined with the absence of counterterms in 1-loop diagrams constructed from R_{10} and R_{10}^4 ensures that there are no counterterms in ten dimensions that could induce a logarithmic term in four dimensions.

As (2.4) suggests, there are also direct corrections to the volume modulus kinetic term coming from the loop diagrams in figure 1, with the external legs belonging to the modulus. If the modulus is canonically normalized, the 3- and 4-vertices are again suppressed by $1/M_4$ and $1/M_4^2$. This leads to the same form of loop integrals as for R_4. After returning to the non-canonical field L, one finds

$$M_4^2 \int \mathrm{d}^4x \, \frac{\mathcal{O}(1)}{L^2 M_4^2} \, \frac{(\partial L)^2}{L^2} \,, \tag{2.15}$$

consistently with the correction proportional to b_1 in (2.4).

In summary, the vertices all scale the same way, regardless of the fields correcting the propagator: 3-vertices are suppressed by $1/M_4$ and 4-vertices by $1/M_4^2$. This results in the universal form of the 1-loop correction proposed in (2.5) and [24]. The leading correction to the Kahler potential stemming from genuine loop corrections is therefore proportional to $1/\tau^2$, with τ the 4-cycle volume. The absence of a logarithmic enhancement is a non-trivial consequence of the divergence structure of the 10d effective supergravity theory.

2.3 Local α' corrections from the bulk theory

Our field-theoretic loop analysis of the last subsection required the discussion of 10d higher-curvature terms, which are needed to absorb UV divergences. Specifically, the absence of an R_{10}^5 term prevented the appearance of a logarithmic correction in 4d. In this context, it may be useful to provide a short, more general discussion of such higher-curvature terms and the resulting corrections to the Kahler potential (the local α' corrections). Our line of reasoning will be directly applicable to similar higher-curvature terms localized on branes, O-planes and their intersections, where the implications are less well-established and hence more interesting. From now on we will explicitly keep track of g_s. In a purely low-energy EFT perspective, it can be understood as $g_s \sim \Lambda^4/M_{10}^4$, where $\Lambda = M_s$ is the EFT cutoff.

We focus on the purely gravitational (higher) curvature part of the type-IIB action in the Einstein frame. Suppressing numerical prefactors for brevity, it reads [48]

$$S_{\mathrm{EF}} \sim \int \mathrm{d}^{10}x\sqrt{-g}\left[M_{10}^8 R_{10} + \frac{M_{10}^2}{g_s^{3/2}} R_{10}^4 + M_{10}^2 g_s^{1/2} R_{10}^4 + \mathcal{O}\left(M_{10}^{-2} g_s^{-5/2} R_{10}^6\right)\right]. \tag{2.16}$$

The first two terms appear at string tree-level: the Einstein-Hilbert term and the R_{10}^4 correction. The third term arises at string-theoretic 1-loop order [48] and hence comes with a relative g_s^2 suppression. In our field-theoretic approach, its size is set by the quadratic divergence of the R_{10}^4 term, such that its coefficient can also be understood as Λ^2. In other words, part of this term may be identified as a counterterm of the EFT analysis. Note that the Einstein-Hilbert term does not receive a string-theoretic 1-loop correction [52].

[9] This happens for example in four-dimensional higher derivative gravity with a marginal R^2 term [65].

We will not discuss corrections at order R_{10}^6 since their effects on the Kahler potential are subleading compared to genuine loop effects.

Independently of the genuine loop effects, higher-curvature terms affect the 4d Einstein-Hilbert term obtained after compactification. Specifically, dimensionally reducing the R_{10}^4 term as

$$\left(\frac{M_{10}^2}{g_s^{3/2}} + M_{10}^2 g_s^{1/2}\right) R_{\text{external}} \int \mathrm{d}^6 x R_{\text{internal}}^3 \sim \left(\frac{M_{10}^2}{g_s^{3/2}} + M_{10}^2 g_s^{1/2}\right) R_{\text{external}} \tag{2.17}$$

reproduces the well known string tree-level BBHL correction [48, 49] and its 1-loop counterpart [48]. Comparing to the tree-level term $M_{10}^8 L^6 R_{\text{external}}$, we see that their relative size is $1/(M_{10}^6 L^6 g_s^{3/2})$ and $g_s^{1/2}/(M_{10}^6 L^6)$ respectively. This is also the scaling of the corresponding corrections to the Kahler potential, which arise after Weyl rescaling to 4d Einstein frame.

3 Extending and generalizing the basic analysis

3.1 D-brane and O-plane corrections

We still need to consider additional corrections due to extended objects filling the four external spacetime dimensions. We will focus on D3/D7-branes and O3/O7-planes as they are relevant to phenomenological compactifications such as the LVS, but the same logic applies to other extended objects, like for example even dimensional D-branes/O-planes of type IIA. To study corrections induced by D-branes, we follow the procedure of section 2.2. However, the KK tower of 10d type-IIB bulk fields is now replaced by the analogous tower resulting from the compactification of the worldvolume theory on the brane. For O-planes no such additional tower exists, but the bulk-field KK tower and hence the corresponding loop correction is modified by the orientifold projection. Both for D-branes and O-planes new operators localized on the brane, on intersection cycles, or at the singularity potentially come into play.

Our Dp-branes/Op-planes wrap $p-3$ cycles in the internal dimensions. For the moment, we assume our compact geometry to be governed by a single length scale L. This is then also the typical length scale of these $p-3$ cycles. Further down in this section we will also comment on generalizations to cases with multiple Kahler moduli. Scenarios with hierarchically different cycles will be considered in sections 6.1 and 6.2.

3.1.1 D-branes

The fields of the gauge multiplet living on the brane couple to the graviton and its moduli. Hence, these fields run in loops, such as in figure 1. It is easy to see that, as long as L remains the only relevant length scale, the couplings to graviton and moduli still come with factors $1/M_4$. More precisely, 3-vertices and 4-vertices are again universally suppressed by $1/M_4$ and $1/M_4^2$, respectively. Therefore, the new genuine loop corrections are parametrically the same as those computed in section 2.2.

In addition, we should consider R_{p+1}^n terms on the brane. In analogy to our previous discussion of R_{10}^n terms, brane localised higher-curvature terms can, after dimensional reduction, impact the coefficient of R_4. Again, two distinct effects arise.

First, if the R_{p+1}^n term is marginal, its coefficient at one-loop order can contain a $1/\varepsilon$ counterterm. This means that the one-loop correction from the brane KK-tower can contain a corresponding $1/\varepsilon$ pole. Hence, a logarithm $\ln(M_{10}g_s^{1/4}L)$ can appear in the loop correction to the coefficient of R_4. In terms of our classification proposed in the Introduction, this effect is on the boundary between a localized α' correction and a genuine loop correction. For definiteness, we will count it as part of localized α' corrections. This appears sensible since a logarithmic integral over momentum scales μ in the range between $1/L$ and $\Lambda = M_{10}g_s^{1/4}$ is dominated by scales which satisfy $\mu \gg 1/L$. The effect is hence localized in the sense of not being sensitive to the non-trivial CY geometry with typical length scale L.

Second, if the R_{p+1}^n operator is relevant, the coefficient includes possible power-like divergences, cut off at the string scale (see section 2.3). From the perspective of a loop calculation, such operators generically supply counterterms, which however happen to vanish in dimensional regularization. Thus, for us only the classical part of the coefficient is relevant, providing a localized α' correction.

We will only consider R_{p+1}^n terms up to and including $n = (p+1)/2$, which corresponds to the operator being marginal. Irrelevant operators will not contribute at the same order in $1/L$ as the loop effects we are interested in.

The worldvolume action of a p-brane contains two types of curvature corrections. Firstly, there are curvature corrections to the DBI action [66]. For further work, including curvature-gauge-field terms, see e.g. [67–72]. The curvature corrections start at order R_{p+1}^2. Higher order terms in R_{p+1} have to our knowledge not been computed, but we expect that such terms exist. We will hence include them in our discussion, with the caveat that some of them may turn out to be forbidden. Suppressing again numerical prefactors, the curvature corrections to the DBI action then take the form[10]

$$S_{\mathrm{DBI}} \supset \int \mathrm{d}^{p+1}x \left[M_{10}^{p-3} g_s^{(p-7)/4}(1+g_s)R_{p+1}^2 + M_{10}^{p-5} g_s^{(p-9)/4}(1+g_s)R_{p+1}^3 + \ldots \right] . \quad (3.1)$$

This equation is written in Einstein frame in the sense that the varying part of the dilaton is absorbed into the metric. The g_s scaling follows from the substitution $M_s = \Lambda = M_{10}g_s^{1/4}$. For each of the operators in (3.1), the second, g_s-suppressed term in the round bracket can be interpreted as coming from a power-like divergence cut off at the string scale.

In addition, there are topological curvature terms in the Wess-Zumino (WZ) action. Their form is known, see e.g. [66, 75]. These terms consist of couplings between the bulk Ramond-Ramond fields and even powers of the curvature two-form. Since our analysis is focused on backgrounds without fluxes, we will not consider these terms here. However, even in the absence of background fluxes, D3/O3 loci source F_5 while D7/O7 loci source F_9 (and possibly an induced F_5 field strength). For D7/O7, the induced F_9 RR-field strength can be set to zero by cancelling the D7 tadpole locally. For D3/O3, local D3 tadpole cancellation can not be achieved.

The resulting F_5 field strength is a source for warping to be discussed in section 4. In fact, it is well known since [2] that the leading-order F_5 background is fixed together with

[10] In our understanding, it is expected [66, 73, 74] that there is no R^3 term but the R^4 term is present.

the warp factor. As soon as higher-order corrections like, for example, higher-order terms in the WZ action are included, F_5 effects may become relevant independently of warping, but this would correspond to a superposition of classical backreaction and higher-order α' effects. Hence, this goes beyond the goals of the present paper, where we limit our discussion to each effect separately.

Consider first the impact of counterterms from marginal operators in (3.1) that signal the possibility of a logarithmic term in the R_4 coefficient.

For the D3-branes, the leading R_4^2 term is marginal. There is then potentially a $1/\varepsilon$ counterterm and hence a logarithmic enhancement. However, this cannot possibly impact the R_4 term as the D3-brane is pointlike in the internal dimensions and there is no dimensional reduction to be done that could turn the R_4^2 term into an R_4 term. Moreover, similarly to our discussion in the case of R_{10}^4 in the bulk, a contribution of R_4^2 via the induced vertex will be subleading.

Concerning the D7-branes, a possible R_8^4 term would be a marginal operator. This may give rise to a $1/\varepsilon$ counterterm, leading to a logarithmic enhancement of the form $\ln(M_{10} g_s^{1/4} L)$ in the coefficient of R_4. We note that the logic here is exactly the same as for a potential R_{10}^5 bulk term in section 2.2, which is however known to be absent. The expected appearance of a logarithmic term, related to D7-branes, in our analysis of loop corrections is extremely interesting: if present, it would be dominant compared to genuine loop effects. Moreover, it is conceivable that the numerical coefficient of this logarithm is calculable since it is a universal feature of the UV structure of the 10d theory with D7-branes in flat space.

In the context of logarithmic corrections, let us comment on the log effect at order α'^3 analysed in [35], which may also be used for the construction of novel (A)dS vacua. The relevant correction arises in torus orbifolds from the combination of two effects: first, there is the R^4 term in the bulk (the third term in (2.16)), which is localized at the points of high curvature.[11] Second, there is a backreaction on the R^4 term sourced, in this case, by a D7-brane. The logarithm comes from the codimension-2 behaviour of the relevant Greens function. Crucially, this backreaction is claimed not to involve a further α' suppression, possibly related to the assumption that D7-tadpoles are not cancelled locally, i.e. O7/D7 branes do not come in SO(8) stacks. We note that the log term of [35] appears at the order α'^3, while our previously discussed logarithm arises at the order of genuine 1-loop effects, i.e. α'^4. The correction of [35] can be understood as a combination of a local α' correction, the R^4 part, and a warping effect, the backreaction of the D7 brane on the geometry where the curvature is localized. We will discuss warping in detail in sections 4 and 5.2. A similar correction, based on the interplay of warping and higher-curvature terms, has been recently discussed in [18, 19]. There, the warping does not come from D7-branes and the effect arises at higher order in α'. In the present paper, we do not consider corrections which need warping and higher-curvature terms at the same time. Clearly, such formally higher-order effects can nevertheless be important and should be systematically studied in the future.

[11] We note in passing that this localization as well as the absence of a corresponding tree-level term is a special feature of torus orbifold models.

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Consider now the local α' corrections of branes to the R_4 coupling.

D3-branes are pointlike in the internal dimensions, so no dimensional reduction has to be done. There is then no way for the leading R_4^2 term in (3.1) or any higher term R_4^n to contribute to the coefficient of R_4 in the 4d EFT.

D7-branes wrap 4-cycles with typical length scale L. Thus, R_8^n terms contribute as

$$\begin{aligned} M_{10}^{8-2n} g_s^{(2-n)/2} \left(1+g_s\right) R_{\text{external}} \int_{4-\text{cycle}} \mathrm{d}^4 y \, R_{\text{internal}}^{n-1} \\ \sim M_{10}^{8-2n} g_s^{(2-n)/2} \left(1+g_s\right) L^{4-2(n-1)} R_{\text{external}} \,, \end{aligned} \tag{3.2}$$

with the leading term arising for $n = 2$. The effects of such an R_8^2 term have been studied in [36, 38–40, 76]. The $M_{10}^4 R^2$ term induces a field redefinition and does not correct the Kahler potential [38–40]. However, as displayed in (3.2), a subleading term $M_{10}^4 g_s R^2$ may in general be present and induce a correction to the Kahler potential. Its presence has to our knowledge not yet been confirmed by string amplitude calculations. Such a term would contribute at order $M_{10}^4 L^2 g_s$ to R_4. This would lead to a correction proportional to g_s and of degree -1 in 4-cycles to the Kahler potential, which is dominant compared to BBHL [49]. We note that this matches the KK correction of the BHP conjecture. At the level of the scalar potential, this correction will be subleading compared to BBHL but of the order of genuine loop corrections due to the extended no-scale structure [24, 27, 28].

If an R_8^3 term in (3.1) should exist, it would via (3.2) contribute at the order of BBHL [48, 49]. Since it is not subject to an extended no-scale cancellation, this is dominant compared to loop effects on the level of the scalar potential. Even though we have so far not discussed the case of multiple Kahler moduli, let us briefly note that an R_8^3 would be particularly interesting in this context. Dimensionally reducing the term as above gives

$$\frac{M_{10}^2}{g_s^{1/2}} \int \mathrm{d}^8 x R_8^3 \sim \frac{M_{10}^2}{g_s^{1/2}} \int \mathrm{d}^4 y R_{\text{internal}}^2 \int \mathrm{d}^4 x R_4 \sim \frac{M_{10}^2}{g_s^{1/2}} f(\tau_1, \dots, \tau_n) \int \mathrm{d}^4 x R_4 \,, \tag{3.3}$$

where n labels the Kahler moduli τ_i and $f(\tau_1, \dots, \tau_n)$ is a homogeneous function of degree 0. Crucially, it is possible that f is not just a constant but depends non-trivially on the ratios of 4-cycles. It could hence be the dominant effect lifting the flat directions associated with 'large' 4-cycle ratios, as it is typically required in the LVS.

Finally, the dimensional reduction of an R_8^4 term in (3.2) would result in a correction comparable to one-loop effects, but without logarithmic enhancement.

3.1.2 O-planes

In contrast to D-branes, O-planes do not come with new fields propagating on their world-volume. Thus, no new contributions to the diagrams of figure 1 arise.

The curvature terms of the type of (3.1) also exists for O-planes. At the order α'^2, where the corrections are known, the R^2 curvature term on the O-plane is 2^{p-5} times that on the D-brane. Crucially, D-brane and O-plane curvature correction have the same sign, so they do not cancel against each other. The dimensional reduction of curvature terms on the O-plane worldvolume action then proceeds entirely analogously to the D-brane case.

O-planes have two further effects that are not present for D-branes. First, the orientifold projection removes part of the KK modes. Thus, the KK spectrum relevant for the 4d action (2.7) is modified. The parametric form of the resulting loop correction remains, however, unchanged. Note also that local 10d physics away from the orientifold plane is not affected by the projection, such that the analysis of 10d divergences and counterterms goes through as before.

Second, the orientifolding changes the geometry of the compact space in a UV sensitive manner. Put differently, the O-plane hypersurface represents a singularity within the surrounding, weakly curved 10d geometry. Thus, our logic in section 2.2, which assumed that the UV structure and in particular the counterterms are those of the flat 10d theory, does not apply any more. Instead, the loop calculation involving the orientifold-projected KK spectrum may require counterterms localized at the O-plane. These are the same operators that we discussed above as possible curvature corrections on O-planes and D-branes. Thus, no entirely new effects arise and our previous discussion of loop corrections from the bulk and from D-branes, including the possible log-enhancements, remains valid. Crucially, after an orientifold projection introducing O7-planes a second source for log-enhancements which we discussed in the D7-brane context appears: it is due to the projected spectrum of bulk modes, which may induce a log-divergent R_8^4 term on the O7-plane.

3.1.3 Intersecting D-branes and O-planes

Finally, let us discuss setups where D-branes/O-planes intersect. We focus again on type-IIB orientifolds with D3/D7-branes and O3/O7-planes. In this setting, only D7/O7 intersections are relevant, filling out curves in the internal space. We assume that their 2d geometry is governed by a single length scale L. In total, the intersection manifold is 6-dimensional and potentially supports new operators. We focus on curvature effects, neglecting fluxes and couplings of the branes to higher form fields.

Fields living on the intersection couple to the graviton and its moduli, hence inducing loop correction as in figure 1. The 3- and 4-vertices are again universally suppressed by $1/M_4$ and $1/M_4^2$, respectively. This leads to the same parametric behavior of genuine loop corrections as observed before.

As usual, UV divergences of loop corrections are absorbed in local operators, the most interesting being the marginal operator R^3. If this operator is allowed and the corresponding divergence arises, a logarithmic enhancement in the coefficient of R_4 is induced.

Of the other local curvature operators, the most import one is the Einstein-Hilbert term:

$$S_{\text{int,EH}} \sim M_{10}^4(1+g_s)\int \mathrm{d}^6x\, R_6\,. \tag{3.4}$$

Here we have displayed both the tree level and the string-one-loop contribution. The tree level term does not lead to a correction of the Kahler potential but only to a field redefinition [38–40]. This is supported by scattering analyses in type IIA on intersecting D6-branes/O6-planes[12] [50] and in type IIB with D9/D5-branes [51]. They show that an Einstein-Hilbert term on brane intersections can only be induced at 1-loop level, corresponding to

[12] It would very interesting to study the effect of this term in the context of DGKT [77].

the g_s-suppressed term in (3.4). References [50, 51] discuss the contribution of this term to R_4, which is of order $M_{10}^4 L^2 g_s$:

$$S_{\rm int,EH} \sim M_{10}^4 g_s \int \mathrm{d}^6 x R_6 \sim M_{10}^4 g_s L^2 \int \mathrm{d}^4 x R_4 \,. \tag{3.5}$$

This matches the EFT analysis of [40]. In this analysis, one starts from the string-frame R_8^2 operator on D7/O7. Taking into account the Weyl rescaling to the 10d Einstein frame together with the varying dilaton near D7-branes, one of the Ricci scalars may be replaced by dilaton gradients. One is then left with an integral over the remaining Ricci scalar which is effectively localized on the D7/O7 intersection. This localization is due to the non-trivial dilaton profile which one brane induces in the vicinity of the intersecting brane. The net effect is an R_6 operator on the brane intersection, to be viewed as a local α' correction. The resulting correction to the Kahler potential is proportional to g_s and of degree -1 in 4-cycles and its effect on the scalar potential is subject to the extended no-scale structure. Not much is known about higher order operators on the intersection cycle such as R_6^2 and R_6^3. A comment on this issue can be found in [42]. If terms of the form R_6^2 and R_6^3 on the intersection locus exist, they would induce correction with the volume-scaling of BBHL (but suppressed in g_s) and of genuine loops effects respectively.

Let us briefly comment on the possible R_6^2 term in more detail. Using the metric ansatz (2.2), the R_6^2 term contributes to the 4d Einstein-Hilbert term through the following dimensional reduction:

$$\frac{M_{10}^2}{g_s^{1/2}} \int \mathrm{d}^6 x R_6^2 \sim \frac{M_{10}^2}{g_s^{1/2}} \int \mathrm{d}^2 y \, R_{\rm internal} \int \mathrm{d}^4 x R_4 \sim \frac{M_{10}^2}{g_s^{1/2}} \chi(S) \int \mathrm{d}^4 x R_4 \,. \tag{3.6}$$

Here $\chi(S)$ is the Euler characteristic of the intersection surface S. Comparing this with the tree-level term $M_{10}^8 L^6 R_4$, we see that the relative, parametric suppression of the correction from an R_6^2 term on a 7-brane intersection locus is $1/(L^6 M_{10}^6 g_s^{1/2}) \sim g_s(l_s^6/L^6)$. This is down by a factor g_s compared to BBHL.

3.1.4 Summary

In this section we have studied brane-induced corrections to the four-dimensional Kahler potential. Our goal was to demonstrate that branes do not spoil the analysis of sections 2.1 and 2.2. We have seen that, indeed, brane effects do not alter the power of the volume with which genuine loop corrections scale. However, in the presence of D7-branes/O7-planes, log-enhanced terms may arise. They are expected to be dominant since $M_{10} g_s^{1/4} L \gg 1$ and hence, though to lesser extent, also $\ln(M_{10} g_s^{1/4} L) \gg 1$. The log-enhanced contribution then wins against the $\mathcal{O}(1)$ numerical coefficient b_0 in (2.4). This correction would then be decisive for all moduli stabilization schemes relying on loop corrections. The marginal operators potentially responsible for this effect are of type R_8^4 for 7-branes and of type R_6^3 for their intersections. It would therefore be very important to know whether these terms are really present and to determine their coefficients.

In setups with intersecting D7-branes and/or O7-planes, it has been shown that an Einstein-Hilbert term localized on the intersection curve is induced at 1-loop level. Local

α' corrections coming from this operator then lead to corrections to the Kahler potential proportional to g_s and of degree -1 in 4-cycles. This is fundamentally different from the genuine loop corrections of degree -2 in 4-cycles. Terms of degree -1 in 4-cycles can also be obtained from an $M_{10}^2 g_s R_8^2$ operator on a D7/O7.

In cases with multiple Kahler moduli the corrections considered in this section can be even more interesting since ratios of 4-cycles can potentially appear. These ratios can be large given a hierarchical structure in the Kahler moduli. An explicit example where large ratios appear is discussed in section 6.2.3.

We emphasize once again that for some of the corrections discussed it is not yet clear whether the required term really appears in the DBI action and whether its dimensional reduction works as displayed schematically in (3.2). Moreover, one needs to understand whether the resulting effect can be absorbed in a field redefiniton.[13]

3.2 Multiple Kahler moduli

Most Calabi-Yau manifolds have more than a single Kahler modulus. Moreover, the LVS requires at least two Kahler moduli. It is therefore crucial to extend the analysis above to Calabi-Yaus with multiple Kahler moduli. This is the goal of the present subsection. The fundamental result is the same as in the single-modulus case: the genuine loop correction to the Kahler potential is a homogeneous function of degree -2 in 4-cycle volumes. A logarithmic enhancement is again possible. Readers who are prepared to accept these facts may skip to section 4.

To demonstrate our claims, let us first recall some basics concerning the Kahler moduli sector of type-IIB orientifolds with D3/D7-branes. The tree-level Kahler potential K and volume $\mathcal{V}$ of the internal manifold $\mathcal{M}_6$ read

$$K = -2\ln(\mathcal{V}), \qquad \mathcal{V} = \frac{1}{3!}\int_{\mathcal{M}_6} J\wedge J\wedge J = \frac{1}{3!}\mathcal{K}_{ijk}t^i t^j t^k\,, \tag{3.7}$$

with J the Kahler form and $\mathcal{K}_{ijk}$ the triple intersection numbers. The two-cycle Kahler moduli t^i are related to four-cycle Kahler moduli τ_i as

$$\tau_i = \frac{\partial\mathcal{V}}{\partial t^i} = \frac{1}{2}\mathcal{K}_{ijk}t^j t^k\,. \tag{3.8}$$

The τ_i (t^i) measure the Einstein frame 4-cycle (2-cycle) volume in units of $l_s = 2\pi\sqrt{\alpha'}$. The Kahler potential K has to be interpreted as a function of the complexified 4-cycle moduli T_i. This is achieved by expressing the t^i through the τ_i and the latter as $\tau_i = (T_i + \overline{T}_{\bar{\imath}})/2$.

To argue for the parametric form of loop corrections, we introduce dimensionful Kahler moduli as follows:

$$\tilde{t}^i = \frac{t^i}{M_{10}^2}\,, \qquad \tilde{\tau}_i = \frac{\tau_i}{M_{10}^4}\,, \qquad \tilde{\mathcal{V}} = \frac{\mathcal{V}}{M_{10}^6}\,. \tag{3.9}$$

The dimensionful quantities are characterized by a tilde.

[13] By this we mean that the Kahler manifold as an abstract mathematical object remains unchanged, only the coordinates are modified. In other words, a given point on this manifold might change its interpretation in terms of the volumes of some set of 4-cycles, measured in string units. This implies that, from the perspective of the 4d supergravity model, there is no change (as long as the above 4-cycles do not enter the model in some other way, e.g. through the non-perturbative superpotential). An observer having access to the 'microscopic information' of 4-cycle volumes in string units could discover the correction.

Let us start from the 10D IIB action (2.1) and the metric ansatz (2.2) (but now with multiple Kahler moduli) and dimensionally reduce to four dimensions. This yields the four-dimensional Jordan frame action. At tree level we have[14]

$$S_{\rm JBD} = \frac{1}{2\kappa_{10}^2}\int {\rm d}^4x\sqrt{-g}\,\tilde{\mathcal{V}}\left[R_4 + \tilde{F}_{ij}\partial_\mu\tilde{\tau}^i\partial^\mu\tilde{\tau}^j + \cdots\right]. \tag{3.10}$$

Here we display only the Einstein-Hilbert term and kinetic terms of the dimensionful 4-cycle moduli $\tilde{\tau}^i$, with $\tilde{F}_{ij}$ denoting their prefactors.[15] One-loop corrections come from integrating out the tower of KK modes and from the fluctuations of the moduli themselves. Exactly as in the single-modulus analysis of section 2.1, the UV-scale M_{10} can not appear in the result, except through divergences associated with higher-dimension operators in 10d or on branes (cf. sections 2.2 and 3.1). Thus, on dimensional grounds one expects

$$\Delta S_{\rm JBD} = \int {\rm d}^4x\sqrt{-g}\left(\tilde{a}(\{\tilde{\tau}_k\})R_4 + \tilde{b}_{ij}(\{\tilde{\tau}_k\})\partial_\mu\tilde{\tau}^i\partial^\mu\tilde{\tau}^j\right), \tag{3.11}$$

where $\tilde{a}(\{\tilde{\tau}_k\})$ is a homogeneous function of degree $-1/2$ in the $\tilde{\tau}_k$ (mass dimension 2) and $\tilde{b}_{ij}(\{\tilde{\tau}_k\})$ is of degree $-5/2$ (mass dimension 10).

We now trade all dimensionful quantities for dimensionless ones as it was done in section 2.1 and convert (3.11) to 4d Einstein frame. We can then read off the Kahler metric and its correction:[16]

$$(S+\Delta S)_E = \int {\rm d}^4x\sqrt{-g}\left[\frac{M_4^2}{2}R_4 + \left(\frac{K_{ij}}{4} + f_{ij}(\{\tau_k\})\right)\partial_\mu\tau^i\partial^\mu\tau^j\right]. \tag{3.12}$$

Here $f_{ij}(\{\tau_k\})$ is derived from $\tilde{a}(\{\tilde{\tau}_k\})$ and $\tilde{b}_{ij}(\{\tilde{\tau}_k\})$ as in section 2.1. The functions $f_{ij}(\{\tau_k\})$ are homogeneous of degree -4 in the τ_k. Further, K_{ij} is of degree -2 and so (3.12) shows explicitly that every loop correction is necessarily suppressed by a factor of degree -2 in 4-cycle volumes relative to the leading term. Our simple dimensional analysis is in general insufficient to provide information about the dependence of $f_{ij}(\{\tau_k\})$ on individual 4-cycle volumes. However, we will be able to make progress in specific examples in sections 6.1 and 6.2.

The whole argument goes through the same way using the Feynman diagram approach of section 2.2. After canonically normalizing the moduli fields, each 3-vertex (4-vertex) will again be suppressed by $1/M_4$ ($1/M_4^2$). Moreover, the argument for a possibly log-enhanced correction induced by an R_8^4 term on the D7-brane is still valid. The logarithmic enhancement appears in the coefficient of R_4 and will therefore after Weyl rescaling appear in the coefficients of all kinetic terms of the moduli. This will in turn lead to log-enhanced corrections to the Kahler potential.

[14] From here on we change our index conventions slightly: we use the 4-cycle-moduli as coordinates on the moduli space, hence giving them upper indices.

[15] After Weyl rescaling to the Einstein frame, these prefactors take the form of second derivatives of the tree-level Kahler potential (see e.g. [78] for the corresponding 2-cycle calculation and [49, 79] for the transition to 4-cycles).

[16] Note that derivatives of K with respect to τ^i or T^i differ only by a factor of 2. In the following, we will use the notation $\partial^2 K/(\partial\tau^i\partial\tau^j) \equiv K_{ij}$ and hence $K_{i\bar{j}} = K_{ij}/4$.

4 Warping corrections

In this section we discuss how warping of the type-IIB orientifold geometry [2] affects the 4d moduli action. In our 10d EFT approach below the string scale, warping corrections are simply classical backreaction effects, arising because branes and fluxes deform the CY geometry. In this sense, they are distinct from the loop corrections which are our main subject. However, concerning specifically the Kahler moduli Kahler potential, warping corrections take the form of a series of terms suppressed $1/\tau$, $1/\tau^2$ etc., where τ is a generic 4-cycle variable. This is similar to loop effects, so it is natural to include some discussion of warping in our analysis.

From a stringy perspective, the warping induced by a D-brane can be understood at leading order as a disk diagram with the boundary on the brane. More precisely, the warping far away from the brane corresponds to the regime where this disc is deformed into a long, thin cylinder, ending on the brane on one side and being capped-off by a half-sphere on the other side. Inserting, for example, two 4d graviton vertex operators in the half-sphere region gives the warping correction to the 4d Einstein-Hilbert term. An analogous discussion applies to the warping induced by an O-plane. The only difference is that the long, thin cylinder now ends in a cross-cap on one side and in a half-sphere on the other side.

The proposed association between the disk diagram and warping may at first sight appear unnatural since warping is a gravitational effect, generally associated with closed strings. However, our claim that disk diagrams on D-branes describe the leading warping effect becomes more apparent if one considers as an example a stack of D3-branes in 10d flat space in the holographic limit of [80]. In the holographic limit, the open-string dynamics on the brane clearly corresponds to the closed-string or supergravity dynamics in the $AdS_5 \times S^5$ background. This $AdS_5 \times S^5$ geometry appears precisely due to the warping of the 10d flat space induced by the brane stack, consistently with our discussion above.

One way to get the first subleading order in warping is by having two disconnected disk diagrams. However, at the same order one can also have a long cylinder between two separated branes. This naturally describes the gravitational pull between two spatially separated branes. Now, since we will be interested in comparing 10d EFT loops with string loops, it is clear that the discussion of warping corrections is mandatory.

Although we will not make this concrete, one should also be able to think of the warping corrections from the perspective of Kaluza-Klein fields in a supergravity analysis. One can do so in two different ways.

In the first approach, one starts with the pure CY geometry. The KK mode expansion of 10d metric and fields is performed on the basis of this unwarped background. Introducing sources may lead to warping which, in this language, is equivalent to turning on VEVs of the 4d fields in the KK mode tower.

In the second approach, one first determines the warped geometry and performs the KK modes expansion on this basis. The resulting 4D KK tower is affected by the warping — it is different from the tower in the first approach. The advantage is that now the VEVs of the 4d fields in the tower remain zero. One may also switch between these two perspectives by redefining the 4D KK fields.

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For the analysis in our paper this distinction is not relevant as we only consider loop corrections and warping separately, as independent additive effects. We expect that, for an analysis of the interplay between warping and loop effects it will be crucial to properly account for the background in which one performs the loop analysis. It would be interesting to understand such interplay effects in more detail.

The key information for us, deriving from [2, 47, 81–87], is as follows: warping induced at leading order in α' by fluxes, D3/O3 and curved D7/O7 branes is incorporated in the analysis of GKP [2]. It corrects the Kahler potential K by a series of terms $1/\tau^n$, starting at $n = 1$. However, in total the tree-level no-scale structure of K is not violated, such that no correction to the scalar potential arises. The statements just made follow from classical field theory. More generally, warping corrections do not represent a loop effect from our 10d EFT point of view. Nevertheless, subleading warping corrections do appear as part of a string one-loop calculation. The reader who is willing to simply accept this may move on to the next section.

The claims above may be underpinned by two series of papers which we will briefly discuss in turn. To begin, let us write the metric as

$$\mathrm{d}s^2 = e^{2A(y,\tau)} g_{\mu\nu}(x)\mathrm{d}x^\mu \mathrm{d}x^\nu + e^{-2A(y,\tau)}\tilde{g}_{mn}\mathrm{d}y^m\mathrm{d}y^n \,, \tag{4.1}$$

where we have made it manifest that the warp factor depends on the values τ^i of Kahler moduli governing the unwarped CY geometry. However, as observed by Giddings and Maharana [81], it would be too naive to simply promote the moduli to dynamical 4d fields $\tau^i = \tau^i(x)$ since then the ansatz above does not satisfy the 10d Einstein equations. One has to allow for more general metric fluctuations, parametrized by so-called compensator fields [81]. On this basis, the moduli space metric $K_{I\bar{J}}$ and hence the Kahler potential may be derived. For a deeper understanding, employing in particular the ADM/Hamiltonian formulation, see e.g. [82, 84].

Using the ingredients above, the Kahler potential in the single-modulus case was derived in [83] (see [84] for a generalization allowing for mobile D3-branes):

$$K = -3\ln\left((T+\overline{T}) + 2\frac{V_W^0}{V_{\mathrm{CY}}}\right) . \tag{4.2}$$

Here

$$V_{\mathrm{CY}} = \int \mathrm{d}^6 y\sqrt{\tilde{g}} \qquad \text{and} \qquad V_W^0 = \int \mathrm{d}^6 y\sqrt{\tilde{g}}e^{-4A_0(y)} \tag{4.3}$$

are the CY volume and a fiducial warped-CY volume. The real modulus $\tau = \mathrm{Re}T$ determines the difference between general and fiducial warp factors: $e^{-4A(y,\tau)} = e^{-4A_0(y)} + \tau$. A redefinition, $T \to T' = T + V_W^0/V_{\mathrm{CY}}$ makes it manifest that K is still of no-scale form, as expected for warping corrections [2]. Moreover, a large-volume expansion of (4.2) results in a power series in $1/\tau$. Note that no factor of g_s comes in since we work in the Einstein frame, such that the Poisson equation [2, 81] determining the warp factor e^{-4A_0} contains no string coupling.[17]

[17]We thank Daniel Junghans for correcting an error concerning this important point in an earlier version.

A generalization to the multi-moduli case has been achieved in [47, 87] in a supergravity-based approach, using the earlier work [85, 86]. We briefly state the main ideas and results and give more details in appendix A. The first key idea of [47] is to argue, on the basis of a nonlinearly realized superconformal symmetry of the 4d EFT,[18] that the Kahler potential must have the following implicit form:

$$K = K(a) = -3\ln(V_{\rm CY} a) - 3\ln(4\pi)\,. \tag{4.4}$$

Here a is a universal modulus which, by analogy to what has just been said in the single-modulus case, is defined as $a \equiv e^{-4A} - e^{-4A_0}$. More specifically, one may choose A_0 such that $V_W^0 = 0$. Now the task is to determine the functional dependence of a on the chiral superfields T^i conventionally used to describe the type-IIB Kahler moduli space and on possible further chiral fields, e.g. D3-brane positions Z_I.

The second key idea of [47] is to solve this problem by considering E3 instanton corrections: on the one hand, by holomorphicity the instanton action must be the real part of a chiral superfield. On the other hand, this action is given by the DBI action of the E3-brane in the warped background. It is determined by the warped 4-cycle volume,

$$\frac{1}{2}\int_{D^i} e^{-4A} J_0 \wedge J_0\,, \tag{4.5}$$

with J_0 the unwarped CY Kahler form. Combining these two conditions, one arrives at

$$\mathrm{Re}\, T^i + f^i(Z) + \overline{f}^i(\overline{Z}) = a\mathcal{V}^i + \frac{1}{2}\int_{D^i} e^{-4A_0} J_0 \wedge J_0\,, \tag{4.6}$$

where $\mathcal{V}^i \equiv \frac{1}{2}\int_{D^i} J_0 \wedge J_0$ and $f^i(Z)$ are holomorphic functions of the remaining chiral fields. In the simplest case these are D3-brane positions.

From this, the desired warping-corrected Kahler potential can be derived: one first expands the Kahler form of the unwarped Calabi-Yau as $J_0 = v^i\omega_i$, with ω_i integral harmonic $(1,1)$ forms providing a basis for $H^2(CY,\mathbb{Z})$. The ω_i are chosen to be Poincaré dual to the divisors D^i. The condition on the Kahler form

$$\frac{1}{3!}\int_{\rm CY} J_0 \wedge J_0 \wedge J_0 = V_{\rm CY} \tag{4.7}$$

can then be thought of as a constraint on the v^i, which hence contain only $h^{1,1} - 1$ degrees of freedom. Using (4.6) and (4.7) one can now express a and the v^i in terms of the variables $[\mathrm{Re}\, T^i + f^i(Z) + \overline{f}^i(\overline{Z})]$. Inserting the resulting expression for a in (4.4) gives the warping-corrected Kahler potential. Different choices of the constant $V_{\rm CY}$ correspond to different additive normalizations of the T^i. So far, this is all rather implicit, but it suffices to make our main points. We quote a somewhat more explicit formulation in appendix A. We also note that a more general calculation, including the backreaction of the Kahler moduli to fluxes, appears in [87].

As demonstrated explicitly in [47], the multi-Kahler-moduli Kahler potential just obtained is of no-scale type. For large volumes, (4.4) can be expanded in $\mathcal{V}$ and the leading

[18] Extensive studies of superconformal symmetries can be found in [88].

order correction to the Kahler potential is of degree -1 in 4-cycles. This can be seen as follows: the integral in (4.6) is independent of the volume modulus — it depends only on the ratios of Kahler moduli. This integral is therefore suppressed by $1/a\mathcal{V}^i(v)$ compared to the leading-order term $a\mathcal{V}^i$, which is of degree 1 in 4-cycles. With this, we have collected all the facts stated at the beginning of the present section.

5 Relation to string amplitude calculations

5.1 String loop calculations and the BHP conjecture

In the last sections we have derived field-theoretically how loop corrections on Calabi-Yau geometries scale with the Kahler moduli. Let us now review the string loop results by Berg, Haack and Körs (BHK) in the torus orbifold case [25] and with the conjecture by Berg, Haack and Pajer (BHP) on how this might extend to CYs [27]. We will compare both viewpoints in section 5.2.

String loop calculations on general CYs are currently not feasible. Results are only available for torus orbifolds without flux but with, for example, D3-/D7-branes and O3-/O7-planes. Concretely, the $\mathcal{N}=2$ geometry $T^4/\mathbb{Z}_2\times T^2$ and the $\mathcal{N}=1$ geometries $T^6/(\mathbb{Z}_2\times\mathbb{Z}_2)$ and $T^6/\mathbb{Z}_6'$ were considered in [25]. Subsequently, BHP [27] conjectured how these BHK results might generalize to the CY case. Explicitly, the torus orbifold corrections and their proposed CY generalizations read

$$\delta K_{(g_s)}^{KK} \sim \sum_{i=1}^{3} \frac{\mathcal{E}_i^{KK}(U,\overline{U})}{\mathrm{Re}(S)\tau_i} \qquad \xrightarrow{\mathrm{CY}} \qquad \delta K_{(g_s)}^{KK} \sim \sum_a \frac{\mathcal{C}_a^{KK}(U,\overline{U})\mathcal{T}^a(t^i)}{\mathrm{Re}(S)\mathcal{V}} \tag{5.1}$$

$$\delta K_{(g_s)}^{W} \sim \sum_{i\neq j\neq k=1}^{3} \frac{\mathcal{E}_i^{W}(U,\overline{U})}{\tau_j\tau_k} \qquad \xrightarrow{\mathrm{CY}} \qquad \delta K_{(g_s)}^{W} \sim \sum_a \frac{\mathcal{C}_a^{W}(U,\overline{U})}{\mathcal{I}^a(t^i)\,\mathcal{V}}\,. \tag{5.2}$$

Here $\mathrm{Re}(S)$ is the inverse string coupling and τ_i, t^i are 4-cycle and 2-cycle Kahler moduli respectively. The functions $\mathcal{T}^a$ and $\mathcal{I}^a$ are linear in the t^i. The Calabi-Yau volume is $\mathcal{V}$ and $\mathcal{E}_i^{KK,W}(U,\overline{U})$, $\mathcal{C}_a^{KK,W}(U,\overline{U})$ are functions of the complex structure moduli U. In the Calabi-Yau case, they are unknown. The corrections are presented in the form of two different contributions, $\delta K_{(g_s)}^{KK}$ and $\delta K_{(g_s)}^{W}$, with the indices referring to 'Kaluza-Klein' and 'winding'. These names will be discussed in section 5.2 below.

For a toroidal orbifold, the $\mathcal{I}^a$ are 2-cycles on which D7-brane stacks intersect while the $\mathcal{T}^a$ are 2-cycles transverse to the available D7-brane stacks [27, 28]. For a generic CY it is not obvious whether an unambiguous definition of the latter 'transverse' 2-cycles exists.

In our understanding, the BHP proposal on the r.h.s. of (5.1), (5.2) consists of two steps. First, the scaling in terms of Kahler moduli is assumed not to change in going from torus orbifold to Calabi-Yau. This is rather convincing and in good agreement with the scaling arguments we discussed in previous sections, cf. also [24]. The formulae on the r.h.s. of (5.1), (5.2) would be consistent with this scaling if $\mathcal{T}^a$ and $\mathcal{I}^a$ were replaced by any homogeneous function of the 2-cycle variables of degree 1. The second part of the conjecture then states, non-trivially, that these are not just homogeneous functions but, specifically, linear expressions in the t^i. This linearity does not follow from our derivation in section 3.2, where only the homogeneity of degree 1 is obtained. In particular, extra ratios of 2-cycle volumes may appear. An example suggesting that this indeed happens is provided in section 6.2.

The loop corrections to the Kahler potential induce corrections to the scalar potential. At the perturbative level, the leading such corrections take the form [28, 29]

$$\delta V_{(g_s)}^{\text{1-loop}} = \left(\sum_{i=1}^{h^{1,1}} \frac{(\mathcal{C}_i^{(KK)})^2}{\mathrm{Re}(S)^2} K_{ii}^{\text{tree}} - 2\delta K_{(g_s)}^{W} \right) \frac{W_0^2\, g_s}{\mathcal{V}^2}\,. \tag{5.3}$$

A key role in obtaining this result is played by the 'extended no-scale structure' (ENSS) [24, 28]. This refers to the fact that the leading order contribution from corrections δK to the Kahler potential vanishes if δK is a homogeneous function of degree -1 in 4-cycles. Without the ENSS, one would expect a term linear in $\mathcal{C}_i^{(KK)}$ to be present in (5.3). This term would be dominant since it would scale with the volume as $\mathcal{V}^{-8/3}$. Thanks to the ENSS cancellation, the Kaluza-Klein correction contributes only at second order and the leading loop correction to the potential scales as $\mathcal{V}^{-10/3}$.

5.2 Comparing field-theoretic and (conjectured) string-theoretic loop effects

In this section we compare and match the results of our field-theoretic analysis of corrections to the Kahler potential (Sections 2 and 3) with the expectations from string amplitude calculations (Section 5.1). We will in particular suggest a resolution for a discrepancy between the field-theory analysis of [24] and the string amplitude results [25] (together with the conjecture [27]). This discrepancy was discussed in [28] but has, to the best of our knowledge, so far not been resolved. The discrepancy arises as follows.

From genuine loop effects we obtain corrections to the Kahler potential of degree -2 in 4-cycles. This matches the form of the BHP winding corrections. But the BHP conjecture proposes a leading correction to the Kahler potential, called KK correction by the authors, which is proportional to g_s and of degree -1 in 4-cycles. Thus, it has to be clarified how this correction arises if we take the 10d EFT below the string scale as our starting point. In the remainder of this section, we argue that the EFT counterpart of the BHP KK correction are specific terms of local α' corrections discussed in section 3.1. On the way, we try to develop a better physical understanding of our field theory corrections from a worldsheet perspective and vice versa.

Let us start with the interpretation of genuine loop corrections from a worldsheet perspective. They correspond to those parts of a string 1-loop integral where the worldsheet has, roughly speaking, one long and one short dimension. Pictorially, this means that one has a long and thin torus/Klein bottle in the closed string case or, similarly, a long and thin annulus/Moebius strip in the open string case. Those are the regimes where the string loop integration can be identified with the field theoretic loop integral, i.e. with the propagation of a 10d or brane-localized massless state around a loop. Comparing this interpretation with the BHP conjecture, we find that the two perspectives nevertheless appear to have an imperfection.

For the BHP winding correction to appear, it was argued in [27] that D7-branes or O7-planes need to intersect. Then a short open string connecting the two branes (or brane

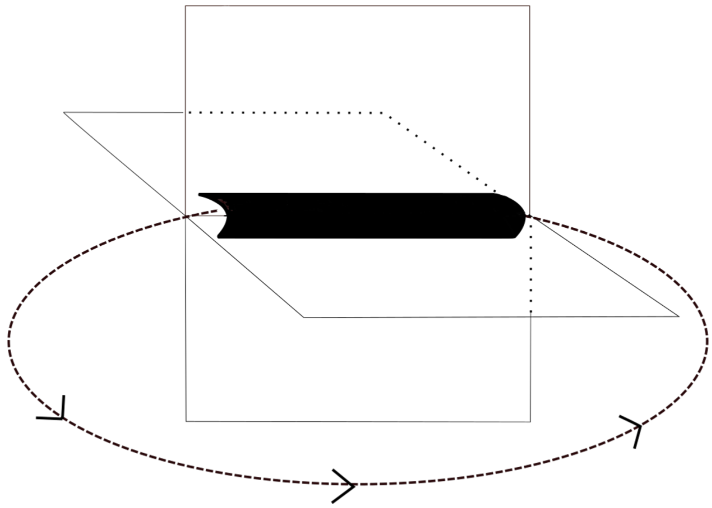

Figure 2. Short open string connecting two localized objects. The ellipse with attached arrows represents that the string is wrapped around the intersection cycle.

and image brane) may propagate in a closed loop along the intersection surface (see figure 2). This corresponds to the thin annulus above or, equivalently, to a field-theoretic loop effect of a massless, intersection-localized state. So far, everything looks perfect. Also the name winding correction is justified if one reinterprets the worldsheet as a closed, winding string which propagates over a short distance from brane to brane. However, as a field theorist one would expect genuine loop corrections to arise more generally — they are not tied to intersecting objects. We have seen examples for this at the beginning of section 3.1.1 where an open string 1-loop effect on a single brane appears to contribute genuine loop correction. Similarly, according to section 2 closed string 1-loop effects in the bulk should also provide a loop correction of the same type and with the same scaling. It is not clear to us why the explicit string loop analysis does not see this more general type of correction producing additional terms of degree -2 in 4-cycles volumes. Conceivably, this is due to the special torus based geometries underlying the calculations.

Next, we discuss local α' corrections. According to our definition, these are classical effects arising from the dimensional reduction of local, higher-dimension operators. However, such operators receive contributions from the high-momentum region of field theory loops. This region corresponds to string 1-loop effects where the worldsheet has a short, string-scale extension in both dimensions. There are two specific examples of this in our context which match the parametric scaling of the BHP KK correction: first, consider the Einstein-Hilbert term on the intersection-2-cycle of two D7-branes or of an D7/O7 pair. This term arises from a short open string stretched from brane to brane near the intersection surface and propagating on an (also short) closed loop. Equivalently, one may think of a short, closed string exchanged between branes. The closed string carries KK momentum and one may hence call this a KK correction, as proposed in BHP. Second, we can consider the operator

$M_{10}^4 g_s R_8^2$ on a D7/O7. This operator can be understood as arising from a 1-loop open string diagram on the D7/O7. Equivalently, it is a short closed string emitted by the brane and absorbed by the same brane after propagating a string-scale distance. From what has just been said, it is clear why the scaling analysis of [24] does not capture these local α' effects: while they *can* be interpreted as loop effect, the relevant scale is the cutoff or string scale. Thus, the in principle correct assumption that finite loop effects are dominated by the KK scale does not apply to the present contribution, which comes from the UV end of the integral.

Finally, we turn to warping effects. As reviewed in section 4, warping effects at subleading order scale as the BHP winding correction. Field-theoretically, warping is a classical backreaction effect and one may think of it as coming from the propagation of massless 10d fields between some source and the point where the geometry is being warped. One of the relevant string diagrams describing this is the tree-level exchange of a closed string between branes, i.e. a long cylindrical worldsheet. Alternatively, this may be viewed as a one-loop diagram, with a long open string propagating in a short loop. Such effects should in principle be part of the analysis performed by BHK/BHP. By contrast, they are clearly not part of field-theoretic loop analyses. As we have discussed in section 4, warping effects do not correct the scalar potential as they are no-scale to all orders in a large-volume expansion [2]. For applications, it would hence be important to split the winding effect in (5.1) according to $\mathcal{C}_i^W = \mathcal{C}_{i,\,\text{gen}}^W + \mathcal{C}_{i,\,\text{warp}}^W$. Then the scalar potential correction of (5.3) would have to include only the genuine loop effect, i.e. only $\mathcal{C}_{i,\,\text{gen}}^W$.

Let us now change perspective and check that we have identified all integration regions of a string 1-loop calculation in our field theoretic approach: an open- or closed-string 1-loop worldsheet which is short in both dimensions corresponds to a local α' effect. The strongest scaling is that of the BHP KK correction. A closed-string 1-loop worldsheet with one long and one short dimension corresponds to genuine loop effects, scaling like the BHP winding correction. An open-string 1-loop worldsheet corresponds either to genuine loop corrections (the case of a long strip) or to warping corrections (the case of a long cylinder). For both cases, the scaling is that of the BHP winding correction.

Finally, a worldsheet with large extension in both dimensions gives an exponentially suppressed contribution, which we can neglect and do not attempt to identify in the field theory perspective. Thus, we appear to have found all relevant regions of the integration over worldsheet geometries in our field-theoretic analysis.

Before closing, let us discuss how loops of a D7-brane gauge theory correct the Kahler modulus kinetic term. This effect has been employed in [28] to argue that genuine loop contributions exist which scale like the BHP KK correction. We will, instead, find that the analysis of this particular effect also supports our earlier conclusion that all genuine loop corrections scale like the BHP winding contribution.

To construct Feynman diagrams for the gauge-theory-derived loop correction to the volume modulus kinetic term, we start from gauge-kinetic term in the DBI action:

$$S_{\text{DBI}} \supset \int \mathrm{d}^4x\sqrt{-g}\,\tau(x) F_{\mu\nu}F^{\mu\nu}\,. \tag{5.4}$$

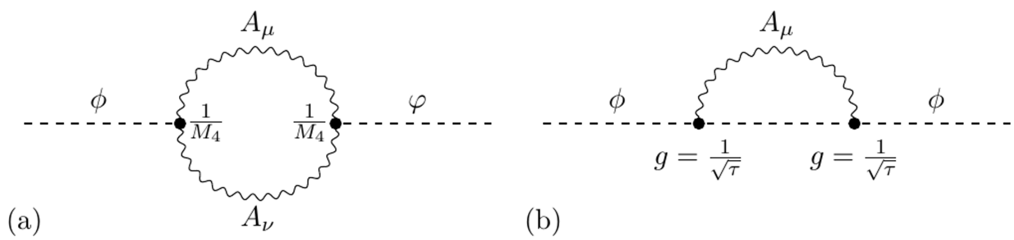

Figure 3. (a) 1-loop diagram for a D7-brane gauge field correcting the kinetic term of a modulus. (b) 1-loop diagram for the wave function renormalization of a charged scalar induced by the gauge boson.

The 4-cycle modulus τ can be expanded around its vev, $\tau(x) = \tau(1 + \varphi(x)/M_4)$. Here we have chosen the fluctuation to be described by a canonically normalized scalar. Redefining $\tilde{A}^\mu = A^\mu\sqrt{\tau}$ and inserting this in the action above, we have

$$S_{\mathrm{DBI}} \supset \int \mathrm{d}^4x\sqrt{-g}\left(\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{1}{M_4}\varphi\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}\right), \tag{5.5}$$

such that the 3-vertex is suppressed by $1/M_4$, as expected. Moreover, the gauge coupling is identified as $g^2 = 1/\tau$. Due to the universal suppression of the 3-vertex by $1/M_4$, we know from section 2 that the loop diagram, depicted in figure 3(a), leads to a correction which scales like BHP winding. A similar calculation has appeared earlier in the unpublished Master Thesis [46].

Instead, the authors of [28] estimate the loop correction by considering the wavefunction renormalization of a scalar field ϕ in ordinary QFT:

$$\int \mathrm{d}^4x\sqrt{-g}\,\frac{1}{2}\partial_\mu\phi\partial^\mu\phi^* \to \int \mathrm{d}^4x\sqrt{-g}\,\frac{1}{2}\left(1 + \frac{g^2}{16\pi^2}\right)\partial_\mu\phi\,\partial^\mu\phi^*\,. \tag{5.6}$$

This suggests a suppression by $g^2 = 1/\tau$ compared to the tree level term, matching the parametric behavior of the KK corrections of BHP. From our point of view this has the following shortcoming: as already noted in [28], the analogy between modulus and charged scalar is not perfect. In the first case, the relevant 3-vertex (cf. figure 3(a)) is $\varphi(\partial A)^2/M_4$. With an effective cutoff M_{KK}, this gives a correction $M_{KK}^2/M_4^2 \sim 1/\tau^2$. In the second case (cf. figure 3(b)), the 3-vertex is $g\phi^*(\partial\phi)A$, with $g \sim 1/\sqrt{\tau}$ and a log-divergent integral. While this gives a correction of order $g^2 \sim 1/\tau$, it is not be applicable to our situation. Moreover, BHP KK corrections have an additional factor g_s, which does not arise in the charged-scalar analogy.

6 Examples and applications

6.1 Blowup modulus: power counting result informed by localization and generic volume scaling

The LVS relies on Calabi-Yau geometries where the volume takes the form

$$\mathcal{V} = f(\tau_1, \ldots, \tau_n) - \beta_1\tau_{s,1}^{3/2} - \cdots - \beta_m\tau_{s,m}^{3/2}\,, \tag{6.1}$$

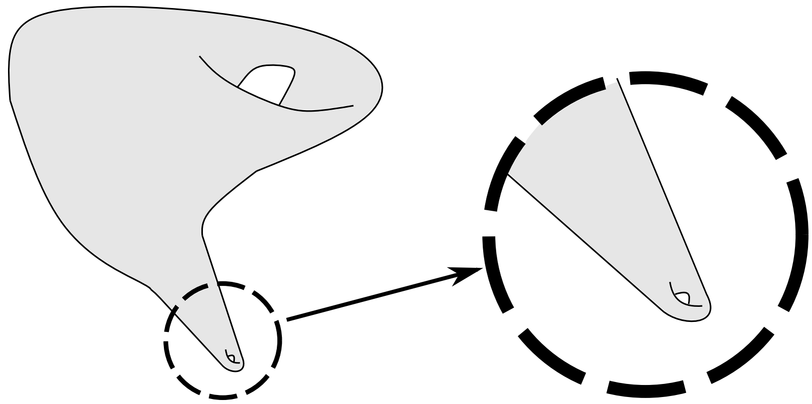

Figure 4. Illustration of a Calabi-Yau manifold with a small blowup cycle. In the vicinity of the blowup, the geometry is assumed to resemble a cone.

with f a homogeneous function of degree $3/2$ in n 'large' 4-cycle moduli τ_i. The m 'small' 4-cycle moduli $\tau_{s,j}$ parametrize blowups and the β_j are numerical constants. In what follows, we will focus on the case of a single blowup, $m = 1$. But our findings generalize straightforwardly to several blowup cycles if these are sufficiently well separated in the full geometry.

The moduli stabilization mechanism of the LVS scenario ensures a hierarchical structure in the vacuum, $\tau_s \ll \mathcal{V}^{2/3}$. We will make the stronger assumption $\tau_s \ll \tau_i$, $\forall i$. One may then expect the geometry to be of the form illustrated in figure 4.

Clearly, it is interesting to know the parametric dependence of loop corrections on the blowup moduli. This is important to be completely certain that loop corrections do not spoil the stabilization scenario in the first place, but it may also be useful for phenomenological applications, e.g. to inflation [22].

In our context, a blowup is a geometric feature which induces a codimension-six singularity once the volume of the relevant 4-cycle (e.g. a $\mathbb{CP}^2$) is taken to zero. A simpler case, useful to build intuition, is the blowup of the singularity of the non-compact geometry $\mathbb{C}_2/\mathbb{Z}_2$. Famously, this is described by the explicitly known Eguchi-Hanson metric [89, 90]. Since we are interested in 3-folds, a better model for us is the Freedman-Gibbons-Pope metric describing the blowup of $\mathbb{C}^3/\mathbb{Z}_3$ [91, 92]. Even closer to our case of interest is the blowup of the related compact geometry $T^6/\mathbb{Z}_3$, for which the Freedman-Gibbons-Pope metric provides an approximation.

To compute loop corrections, we assume $l_s \ll L_{\tau_s} \ll \tilde{\mathcal{V}}^{1/6}$, where $\tilde{\mathcal{V}}$ is the dimensionful Calabi-Yau volume and L_{τ_s} is the typical length scale of the τ_s cycle. An important property of the blowup modulus is that its effect on the geometry is highly localized [93, 94]. Specifically, in the $\mathbb{C}^3/\mathbb{Z}_3$ model the profile of the metric deformation parametrized by the blowup modulus falls off with the sixth power of the distance from the origin [94]. This implies that the integral over the internal geometry which calculates the kinetic term of the modulus is of the type $\int_{\mathcal{M}_6} dy^6/(y^6)^2$ in the region $y \gg L_{\tau_s}$. Thus, the 10d dynamics of the blowup modulus is dominated by the length scale L_{τ_s}. We therefore assume that it is a

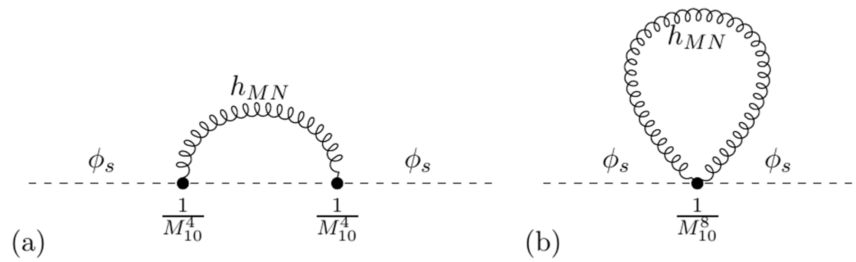

Figure 5. Self-energy (a) and tadpole (b) loop diagram correcting the kinetic term of the scalar field ϕ_s. The scalar is confined to a 4d submanifold whereas the graviton h_{MN} propagates in 10d.

reasonable approximation to treat the blowup modulus as localized in the internal 6d space at a point y_0, which characterizes the locus of the would-be singularity.[19]

Our blowup modulus is thus identified with a localized 4d scalar field, included in the 10d action according to

$$S = \frac{1}{2\kappa_{10}^2}\int \mathrm{d}^{10}x\sqrt{-g}R_{10} + \frac{1}{2}\int \mathrm{d}^{10}x\sqrt{-g}\delta^{(6)}(y-y_0)\partial_\mu\phi_s\partial^\mu\phi_s\,. \tag{6.2}$$

Here the δ-function must be viewed as smeared on the scale L_{τ_s}. It is easy to check that, in the compact case, the relation to the conventional 4d supergravity modulus is $\phi_s/M_4 \sim \tau_s^{3/4}/\sqrt{\mathcal{V}}$. For simplicity, we consider the graviton as the only 10d field, but our following discussion could be repeated including further 10d degrees of freedom. We also disregard higher-dimension operators localized at y_0 which can in principle induce further couplings between ϕ_s and the 10d metric. Their effects will not change our conclusions qualitatively.

The 1-loop diagrams correcting the ϕ_s propagator are shown in figure 5. To work out the corresponding integrals explicitly one has to linearize the metric, $g_{MN} = g^{(0)}_{MN} + \kappa_{10}h_{MN}$, in (6.2). Here the background metric $g^{(0)}_{MN}$ corresponds to the singular geometry, with $\tau_s = 0$. Compared to the loop analysis in section 2.2, a key difference is that the modulus is localized at a special, singular point. The propagator of h_{MN} near this singularity is not known, not even approximately. We may nevertheless make progress by using our assumption that the geometry near the singularity is conical, i.e. in particular scale-free in the limit $\tilde{\mathcal{V}} \to \infty$. Taking this latter limit will be justified a posteriori when we see that the loops are short-distance dominated. Moreover, we need the graviton propagator D_h only with both arguments at the singularity. Thus, on dimensional grounds we have $D_h(x-x') \sim 1/|x-x'|^8$, where x, x' are 4d coordinates and we have suppressed any index structure.

Using also the propagator $D_{\phi_s} \sim 1/|x-x'|^2$ of the scalar ϕ_s, one may now estimate the self-energy diagram of figure 5(a) in position space. This diagram contributes to the correction δ_{ϕ_s} to the kinetic term of the modulus, defined by $\mathcal{L} \supset (1+\delta_{\phi_s})(\partial\phi_s)^2/2$. Using

[19]An alternative approach to derive loop corrections is to sum over the contributions of the KK-tower, taking into account how each mass depends on the blowup cycle. While an estimate for the lowest modes, with wavelength much larger than L_s is possible [46], the challenge of extending such an analysis to modes with wavelength $\sim L_s$ appears daunting to us.

dimensional regularization, we have

$$\begin{aligned}\delta^{\varepsilon}_{\phi_s} &= \frac{\mathrm{d}}{\mathrm{d}p^2}\bigg|_{p^2=0} \kappa_{10}^2 \mu^{-2\varepsilon} \int \mathrm{d}^{4-\varepsilon}(x_1-x_2) f_4(\partial_{x_1},\partial_{x_2}) \mathrm{e}^{-\mathrm{i}(x_1-x_2)p} D_{\phi_s}(x_1-x_2) D_h(x_1-x_2) \\ &\sim \frac{\mathrm{d}}{\mathrm{d}p^2}\bigg|_{p^2=0} \kappa_{10}^2 \mu^{-2\varepsilon} \int \mathrm{d}^{4-\varepsilon}(x_1-x_2) f_4(\partial_{x_1},\partial_{x_2}) \frac{\mathrm{e}^{-\mathrm{i}(x_1-x_2)p}}{|x_1-x_2|^{2-\varepsilon}|x_1-x_2|^{8-\varepsilon}}\,,\end{aligned} \tag{6.3}$$

where we suppressed any index structure and $f_4(\partial_{x_1},\partial_{x_2})$ is a homogeneous function of degree 4 in derivative operators. The integral in (6.3) has an octic UV divergence at $\varepsilon = 0$. In dimensional regularization, this integral vanishes since the only dimensionful parameter, p^2, is set to zero.

A second 1-loop contribution comes from the tadpole diagram figure 5(b). It is proportional to the graviton propagator in the conical geometry, D_h, evaluated at coincident points:

$$D_h(0) \sim \lim_{x\to x'} \frac{1}{|x-x'|^8}\,. \tag{6.4}$$

This is a pure, octic divergence without any intrinsic mass scale, which would again give zero in dimensional regularization.

Thus, all we can learn form a dimensionally (or otherwise) regularized calculation in a low-energy EFT below the scale at which the small cycle is resolved is the following: there is no loop correction to the small-cycle kinetic term coming from the IR. Any possible corrections are dominated by the UV, i.e. by scales at which the small-cycle geometry is resolved. Put differently, there is no interference between the deep IR, encoded in the total volume $\mathcal{V}$ (which we have taken to infinity when talking about a conical geometry) and the small-cycle-scale $L_{\tau_s} \sim \tau_s^{1/4}/M_{10}$.

Nevertheless, UV-dominated quantum corrections do in general exist and can simply be added by hand as a localized operator sitting at the singularity:

$$(\partial\phi_s)^2 \to (\partial\phi_s)^2(1+f(\phi_s))\,. \tag{6.5}$$

In spite of our failure above to write down a loop integral calculating f in the EFT below the scale $1/L_{\tau_s}$, we are able to determine the form of this function. We give two independent arguments.

The first is very intuitive but it requires us to be slightly generous with the concept of a 4d EFT: let us raise our EFT scale above $1/L_{\tau_s}$ but still below M_{10}. In this EFT, one starts seeing 10d supersymmetric cancellations and the loops, still described by figure 5, are cut off at the small-cycle scale: $\Lambda \sim 1/L_{\tau_s}$. The suppression by loop couplings is still governed by $\kappa_{10}^2 \sim 1/M_{10}^8$. Thus,

$$f(\phi_s) \sim \frac{\Lambda^8}{M_{10}^8} \sim \frac{1}{L_{\tau_s}^8 M_{10}^8} \sim \frac{1}{\tau_s^2}\,, \tag{6.6}$$

where $\tau_s \sim (\phi_s/M_{10})^{4/3}$.

The second argument is precise but slightly technical and indirect: for this, we rewrite (6.5) in terms of τ_s, reinstate a finite total volume $\mathcal{V}$, and return to the 4d Einstein frame:

$$(\partial\phi_s)^2(1+f) \to \frac{(\partial\tau_s)^2}{\sqrt{\tau_s}}(1+f) \to \frac{(\partial\tau_s)^2}{\mathcal{V}\sqrt{\tau_s}}(1+f)\,. \tag{6.7}$$

Now we recall that, on the one hand, we have shown that genuine loop corrections to Kahler moduli kinetic terms scale with the power -4 in 4-cycle volumes. On the other hand, from our argument about the decoupling of IR and UV in the conical geometry near the small cycle, we know that f is a function of τ_s only, independent of $\mathcal{V}$. This enforces $f \sim 1/\tau_s^2$, consistently with (6.6).

As result, we have the 4d Einstein frame kinetic Lagrangian (ignoring $\mathcal{O}(1)$ factors)

$$M_4^2 \int \mathrm{d}^4x \frac{1}{\mathcal{V}\sqrt{\tau_s}} \left(1 + f(\phi_s(\tau_s))\right) \partial_\mu \tau_s \, \partial^\mu \tau_s \sim M_4^2 \int \mathrm{d}^4x \frac{1}{\mathcal{V}\sqrt{\tau_s}} \left(1 + \frac{1}{\tau_s^2}\right) \partial_\mu \tau_s \, \partial^\mu \tau_s \,. \tag{6.8}$$

Integrating the 1-loop correction twice with respect to the blowup modulus leads to a correction to the Kahler potential of the form

$$\delta K_{\text{1-loop}} \sim \frac{1}{\mathcal{V}\sqrt{\tau_s}} + \text{subleading terms}\,. \tag{6.9}$$

This matches exactly of the form of the winding corrections of the BHP conjecture.

Note that we only expect blowup corrections of the type just discussed to arise if supersymmetry is broken to 4D $\mathcal{N} = 1$ locally, near the blowup cycle. This is to be contrasted with situations where SUSY is locally $\mathcal{N} = 2$ in 4D language, i.e. all O3/O7-planes and branes are localized elsewhere in the internal geometry. Thus, the above corrections may simply not arise in LVS geometries where the blowup cycle used for moduli stabilization has locally 4D $\mathcal{N} = 2$ SUSY. However, it is also possible that, if the blowup is locally 4d $\mathcal{N} = 2$, nonperturbative corrections to the superpotential of the form $\exp(-a_s\tau_s)$ are absent,[20] making such geometries unsuitable LVS moduli stabilization. If this were the case, then blowup cycles inducing nonperturbative superpotential effects would always come with an appropriate loop correction.

This discussion may be related to the Supersymmetric Genericity Conjecture [97]. According to this conjecture, an allowed correction in a quantum gravity theory can only be systematically zero if the theory at hand descends in some way from a theory with higher supersymmetry.

We finally note that our blowup Kahler potential correction is expected to arise more generally than the similar BHP correction of 'winding-type'. The latter relies on the presence of intersecting D7s. Hence, if our correction always arises in local $\mathcal{N} = 1$ SUSY situations, and if this reduced amount of SUSY is necessary for nonperturbative effects in W to appear, then this should drastically affect blowup inflation [22]. The reason is that this model relies on an exponentially flat potential at $a_s\tau_s \gg 1$, which would then *always* be spoiled by loops, even if no D7s are present in the local geometry.

6.2 Fibred geometries and fibre inflation

As we have seen in sections 3.2 and 6.1, determining the explicit Kahler moduli dependence of loop corrections is complicated if multiple Kahler moduli are present. In this section we

[20] Note that nonperturbative corrections to the superpotential can appear more generally than just on rigid divisors as E3-branes on effective divisors can be 'rigidified' by world-volume flux and provide nonperturbative corrections [95, 96].

will make partial progress in the special case of fibred Calabi-Yau manifolds, governed by two Kahler moduli.

Fibred manifolds are of particular interest in the LVS context, where they are used for example in the Fibre Inflation proposal [23]. Fibre Inflation relies explicitly on the parametric form of loop corrections. We will see that our improved understanding of loop corrections affects Fibre Inflation.

A simple example for a fibred Calabi-Yau is provided by a K3 fibration over a $\mathbb{CP}^1$ base. Let the K3 fibre be governed by the 4-cycle modulus τ_f and the base by the 2-cycle variable t_b. To be useful in the LVS context, a blowup modulus τ_s is also needed. It will however mostly be ignored below since we have already computed the form of loop corrections associated with τ_s in section 6.1. The volume of this fibred Calabi-Yau takes the form

$$\mathcal{V} = t_b\tau_f - \tau_s^{3/2} = \frac{1}{2}\sqrt{\tau_f}\tau_2 - \tau_s^{3/2}\,, \tag{6.10}$$

where we re-expressed the base 2-cycle volume in terms of the 4-cycle modulus $\tau_2 = 2t_b\sqrt{\tau_f}$. The 2-cycle volume t_b (or 4-cycle volume τ_2) can be traded for the total volume $\mathcal{V}$, as is typically done in the LVS analysis. Then the standard LVS potential stabilizes τ_s and $\mathcal{V}$, leaving τ_f a flat direction. In Fibre Inflation, τ_f is identified with the inflaton. Its loop-induced potential governs inflation and also stabilizes the cycle in the post-inflationary vacuum.

Building on the BHP conjecture (cf. (5.1) and (5.2)), it is argued in [23] that loops induce a Kahler potential correction

$$\delta K_{(g_s)} = \frac{g_s C_1^{KK}\sqrt{\tau_f}}{\mathcal{V}} + \frac{g_s C_2^{KK}\sqrt{\tau_2}}{\mathcal{V}} + \frac{C_{12}^W}{\mathcal{V}\sqrt{\tau_f}}\,, \tag{6.11}$$

which in turn, using (5.3), induces a scalar potential for τ_f of the form

$$\delta V_{(g_s)} = \left(\frac{(g_s C_1^{KK})^2}{\tau_f^2} - \frac{2C_{12}^W}{\mathcal{V}\sqrt{\tau_f}} + \frac{(g_s C_2^{KK})^2\tau_f}{2\mathcal{V}^2}\right)\frac{W_0^2\, g_s}{\mathcal{V}^2}\,. \tag{6.12}$$

This assumes a geometry where D7-brane stacks wrap the 4-cycles described by $\tau_2 = 2t_b\sqrt{\tau_f}$ and by τ_f. The first and last term in (6.12) are due to the KK corrections associated with the corresponding transverse cycles. The second term comes from winding corrections related to the intersection 2-cycle with volume $\sqrt{\tau_f}$. Note that the KK corrections are suppressed by g_s^2 compared to the winding correction. For such a potential, inflation corresponds to an initial situation where the fibre is much larger than the base. As the inflaton rolls, this is reversed and eventually the base is much larger than the fibre. During this process, the volume of the Calabi-Yau remains constant.

In the following, we will attempt to derive the explicit form of all loop corrections in the hierarchical regime $\tau_2 \gg \tau_f \gg \tau_s$. We shall see that while all three terms with a Kahler moduli dependence as in (6.12) will indeed appear, our result has two major differences: first, all terms in (6.12) will arise at the same order in g_s. Second, our analysis suggests the presence of additional logarithmic corrections and corrections including ratios of Kahler moduli, some of which could be dominant over the usual fibre-inflation terms. Our discussion will be based on a 2-step compactification process: $10d \xrightarrow{\tau_f} 6d \xrightarrow{t_b} 4d$.

6.2.1 Genuine loop effects in fibred geometries

Let us first consider the 10d to 6d compactification on the fibre 4-cycle and integrate out the heavy KK modes of the fibre. The compact 4d manifold is governed by the single length scale $L_f \sim \tau_f^{1/4}/M_{10}$, to which we can hence apply dimensional analysis. We demand that both the loop correction to the Einstein-Hilbert term and the kinetic term of L_f,

$$\Delta S_{\rm JBD} = \int \mathrm{d}^6 x \sqrt{-g_6}\left[F(L_f) R_6 + G(L_f)(\partial L_f)^2\right] , \tag{6.13}$$

are dimensionless. Since L_f has mass dimension -1, this fixes the scaling of the prefactors to be $F(L_f) \sim 1/L_f^4$ and $G(L_f) \sim 1/L_f^6$. Now we compactify to 4d, Weyl rescale by $M_{10}^8 \tilde{\mathcal{V}} + L_b^2/L_f^4$ with $\tilde{\mathcal{V}} \sim L_b^2 L_f^4$ to arrive at 4d Einstein frame, and express everything in terms of 4-cycle variables:

$$\begin{aligned}
&\frac{R_6}{L_f^4} + \frac{(\partial L_f)^2}{L_f^6} \xrightarrow{\text{compactify}} L_b^2\left(\frac{R_4}{L_f^4} + \frac{(\partial L_f)^2}{L_f^6}\right) \\
&\xrightarrow{\text{Weyl rescale}} \frac{1}{M_{10}^8}\left(\frac{1}{L_b^2 L_f^4}\frac{L_b^2}{L_f^4 L_b^2}(\partial L_b)^2 + \frac{1}{L_b^2 L_f^4}\frac{L_b^2}{L_f^5 L_b}(\partial L_b)(\partial L_f) + \frac{1}{L_b^2 L_f^4}\frac{L_b^2}{L_f^6}(\partial L_f)^2\right) \\
&\qquad \sim \frac{(\partial \tau_2)^2}{\tau_2^2 \tau_f^2} + \frac{(\partial \tau_f)(\partial \tau_2)}{\tau_2 \tau_f^3} + \frac{(\partial \tau_f)^2}{\tau_f^4} ,
\end{aligned} \tag{6.14}$$

where we used that $M_{10} L_b = t_b^{1/2} \sim (\tau_2/\sqrt{\tau_f})^{1/2}$. We implicitly assumed that, in the compactification step from 10d to 6d, supersymmetry has been broken to 6d $\mathcal{N}=1$ by O7-planes/D7-branes wrapping the 4-cycle associated with $\tau_2 = 2t_b\sqrt{\tau_f}$. We expect that the appearance of a non-trivial correction to the kinetic term is then consistent since this corresponds to 4d $\mathcal{N}=2$ SUSY, where corrections to the Kahler potential (or better the prepotential) are allowed.

We now explicitly translate the different terms in the last line of (6.14) in corrections to the Kahler and scalar potentials. The final term gives

$$\delta K_{(\tau_f)} \sim \frac{1}{\tau_f^2} . \tag{6.15}$$

This in turn induces a correction to the scalar potential which is similar to the first term in (6.12), albeit without a g_s^2 suppression, making the correction (6.15) more important.

The first two terms in (6.14) (if not absent due to a magical cancellation) induce a logarithmic correction in the Kahler potential:

$$\delta K_{(\tau_f)} \sim \frac{\ln \tau_2}{\tau_f^2} . \tag{6.16}$$

We immediately recognize a problem: the correction (6.16) induces a further contribution to the kinetic Lagrangian not present in (6.14), a term of the form $\ln(\tau_2)(\partial\tau_f)^2/\tau_f^4$. We will discuss this and related inconsistencies and how they might be resolved in section 6.2.2.

Further genuine loop corrections can arise from D7-brane-localized fields. Concretely, if a D7-brane wraps the fibre, loop corrections to the modulus kinetic term as described by the diagram in figure 3(a) arise. In addition, loop corrections induce a 4d Einstein-Hilbert term on the worldvolume of the brane. The form of the corrections follows from the fact that the KK masses of the gauge fields running in the loop depend only on τ_f and not on t_b (or, equivalently, the volume $\mathcal{V}$). One then finds

$$\frac{R_4}{L_f^2}+\frac{(\partial L_f)^2}{L_f^4}\to\frac{1}{M_{10}^8}\left(\frac{1}{L_b^4L_f^6}(\partial L_b)^2+\frac{1}{L_b^3L_f^7}(\partial L_b)(\partial L_f)+\frac{1}{L_b^2L_f^8}(\partial L_f)^2\right)$$
$$\sim\frac{(\partial\tau_2)^2}{\tau_2^3\tau_f}+\frac{(\partial\tau_f)(\partial\tau_2)}{\tau_2^2\tau_f^2}+\frac{(\partial\tau_f)^2}{\tau_2\tau_f^3}\to\delta K\sim\frac{1}{\mathcal{V}\sqrt{\tau_f}}\,, \tag{6.17}$$

where we also displayed the expression after Weyl rescaling and the resulting effect on the Kahler potential. The corresponding correction to the scalar potential scales like the second term in (6.12).

Genuine loop corrections induced by a D7 wrapping the τ_2-cycle do not lead to parametrically novel corrections to K. As above, these corrections are obtained by gauge fields running in the loop. In particular, they induce a 6d Einstein-Hilbert term on the worldvolume of the brane of the form R_6/L_f^4 which hence matches the correction in (6.13).

A further type of loop corrections arises if one first compactifies on the fibre to 6d and then, using the 6d classical action, considers the quantum effects of the KK modes on the base. In this case the only relevant length scale is L_b and one finds

$$\Delta S_{\rm JBD}=\int \mathrm{d}^4x\sqrt{-g_4}G(L_b)(\partial L_b)^2\,, \tag{6.18}$$

with $G(L_b)\sim 1/L_b^4$ on dimensional grounds. We do not discuss the related loop corrections to the Einstein-Hilbert term since they will not induce parametrically different effects in the Kahler potential.

After Weyl rescaling, (6.18) gives

$$\frac{(\partial L_b)^2}{L_b^4}\to\frac{(\partial L_b)^2}{M_{10}^8L_f^4L_b^6}\sim\frac{(\partial\tau_2)^2}{\tau_2^4}+\frac{(\partial\tau_f)(\partial\tau_2)}{\tau_f\tau_2^3}+\frac{(\partial\tau_f)^2}{\tau_f^2\tau_2^2}\,, \tag{6.19}$$

where we used that $M_{10}L_b=t_b^{1/2}\sim(\tau_2/\sqrt{\tau_f})^{1/2}$. The first term in (6.19) implies a corrections to the Kahler potential of the form

$$\delta K_{(\tau_2)}\sim\frac{1}{\tau_2^2}\sim\frac{\tau_f}{\mathcal{V}^2}\,. \tag{6.20}$$

This reproduces the Kahler moduli dependence of the third term in (6.12). Note that the correction (6.20) is again enhanced by a factor $1/g_s^2$ compared to expectations based on the BHP conjecture.

The last two terms in (6.19) enforce again a logarithmic correction to the Kahler potential,

$$\delta K_{(\tau_2)}\sim\frac{\ln\tau_f}{\tau_2^2}\,, \tag{6.21}$$

which introduces a similar problem as described below (6.16).

6.2.2 Local α' corrections from D7-branes in fibred geometries

Let us now study the possible effects of the R_8^4 operator, focusing first on the case where the D7-brane wraps the 4-cycle parametrized by $\tau_2 = 2t_b\sqrt{\tau_f}$. Let us moreover assume that the coefficient of R_8^4 displays a non-trivial logarithmic running, as is generally expected for marginal operators. Thus, starting with an $\mathcal{O}(1)$ coefficient at the string scale, the coefficient grows as the energy at which we study our EFT decreases to some smaller value $\mu \gtrsim 1/L_f$. If we then compactify the D7-brane theory from 8d to 6d at the scale $1/L_f$, we find an operator $\ln(M_{10}g_s^{1/4}L_f)R_8^4$. Disregarding the small effect of the factor g_s under the log, we may replace this by $\ln(\tau_f)R_8^4$. After compactification to 6d, we obtain a sum of different terms:[21]

$$\int\limits_{\mathbb{R}^{1,5}\times\sqrt{\tau_f}} \ln(\tau_f)R_8^4 \sim \sum_{n=0}^{3} \int\limits_{\mathbb{R}^{1,5}} \ln(\tau_f)R_6^{4-n} \int\limits_{\sqrt{\tau_f}} R_2^n \sim \sum_{n=0}^{3} \int\limits_{\mathbb{R}^{1,5}} \ln(\tau_f)R_6^{4-n}L_f^{2-2n}\,. \tag{6.22}$$

Here the symbol $\sqrt{\tau_f}$ under the integral stands for the integration over the 2-cycle of the fibre wrapped by the brane. The index n runs over all possibilities of how R_8^4 could contribute to the 6d action.[22] Note that we have to allow for all these possible reductions as we do not know how the indices of R_8^4 are contracted. Hence some (or maybe all) of the four terms could turn out to vanish when the detailed structure of the R_8^4 term is specified by a string amplitude calculation.

Before compactifying further down to 4d, we have to run our 6d Lagrangian from the scale $1/L_f$ down to $1/L_b$. In this process, the $n = 1$ term is special because it is a marginal operator in the 6d theory. We expect further logarithmic running, in general with a different prefactor than previously in 8d. Hence, the generic prefactor of the R_6^3 term just above the 4d compactification scale $1/L_b$ reads $\ln(\tau_2^\alpha \tau_f^\beta)$, where α, β are in principle calculable constants. For $n \neq 1$ no further running occurs since the operators are relevant or irrelevant and we also do not expect subleading logarithmic divergences. We have seen this already in section 2.2. Key to this feature was the absence of massive fields in the 1-loop diagrams which also holds in the 6d theory at hand.

After compactification to 4d, we find the following corrections to the coefficient of the Einstein-Hilbert term:

$$\ln\tau_f \sum_{n=0,2,3} \frac{L_b^{2n}}{L_f^{2n}}\frac{L_f^2}{L_b^4} + \ln(\tau_2^\alpha\tau_f^\beta)\frac{1}{L_b^2}\,. \tag{6.23}$$

Focusing, as above, only on the kinetic term of L_b, the Weyl rescaling then produces

[21] To avoid clutter, we keep only the logarithmic piece in the coefficient of R_8^4 in the following terms. In general there are additional constants, such that the reader may always replace $\ln() \to \mathrm{const} + \ln()$.

[22] We do not consider the case $n = 4$ since we know from supersymmetry that a 6d cosmological constant will not be induced.

corrections of the following type:

$$\begin{aligned}&\frac{1}{M_{10}^8}\left(\ln\tau_f\sum_{n=0,2,3}\frac{L_b^{2n}}{L_f^{2n}}\frac{L_f^2}{L_b^4}+\ln(\tau_2^\alpha\tau_f^\beta)\frac{1}{L_b^2}\right)\frac{1}{L_f^4L_b^2}\frac{(\partial L_b)^2}{L_b^2}\\&\sim\left(\ln\tau_f\sum_{n=0,2,3}\frac{\tau_2^n}{\tau_f^n}+\ln(\tau_2^\alpha\tau_f^\beta)\frac{\tau_2}{\tau_f}\right)\left(\frac{\tau_f(\partial\tau_2)^2}{\tau_2^5}+\frac{(\partial\tau_2)(\partial\tau_f)}{\tau_2^4}+\frac{(\partial\tau_f)^2}{\tau_2^3\tau_f}\right).\end{aligned}\tag{6.24}$$

As an illustration, let us restrict attention to the $(\partial\tau_2)^2$ contribution and write out the sum in (6.24) explicitly for this case:

$$\left(\frac{\ln(\tau_2^\alpha\tau_f^\beta)}{\tau_2^4}+\ln(\tau_f)\left(\frac{\tau_f}{\tau_2^5}+\frac{1}{\tau_2^3\tau_f}+\frac{1}{\tau_2^2\tau_f^2}\right)\right)(\partial\tau_2)^2\,.\tag{6.25}$$

Ideally, we want to write down a Kahler potential correction from which all terms in (6.24) and no undesired further terms follow. This can not be achieved. Let us then start our discussion by writing down a correction which induces as many of the terms in (6.24) as possible and no extra terms:

$$\delta K_{(R_8^4)}\sim\frac{1+\ln\tau_f+\ln\tau_2}{\tau_2^2}+\frac{1+\ln\tau_f}{\tau_2\tau_f}+\frac{1+\ln\tau_f}{\tau_f^2}+\frac{\tau_f(1+\ln\tau_f)}{\tau_2^3}\,.\tag{6.26}$$

Here we suppressed numerical factors in each term. We may try to compare this to the structure of corrections deduced from the BHP conjecture in [23] (cf. (6.11) above). At first sight, this is very different since (up to logs) all of our terms are homogeneous of degree -2 in 4-cycles. Thus they are all of 'winding-type' in BHP language. Correspondingly, there are no g_s factors. Interestingly, the difference becomes less dramatic at the level of the scalar potential, cf. (6.12): the first three terms of (6.26) reproduce, at the level of power-like scaling, the structure deduced from the BHP conjecture in (6.12). However, the additional logs could clearly be very important in concrete applications. The last term of (6.26) contradicts the BHP conjecture at an even more elementary level in that it involves an additional 4-cycle ratio: τ_f/τ_2.

Let us now comment on the kinetic terms for which we were not able to write a consistent Kahler potential. This concerns part of the terms in (6.24) as well as the previously emphasized problematic terms in (6.14) and (6.19). First, on the positive side, we note that the previously emphasized problem with the Kahler correction (6.21) associated with (6.19) could now potentially be resolved: the required term appears in (6.26) and its log might thus be explained by the log associated with the R_8^4 operator. However, the problem with the correction (6.16) associated with (6.14) is still there: this Kahler correction induces kinetic terms that do not arise in our analysis. Moreover, problems arise for all those kinetic terms in (6.24) that lead to a product of logarithms in the appropriate correction δK. This happens for example for the last term in (6.25). This kinetic term would require a correction $\delta K\sim\ln(\tau_f)\ln(\tau_2)/\tau_f^2$. This in turn leads to a new kinetic term for τ_f not present in (6.24).

We hence have to draw one of the following conclusions.

First, since in our approach it is not possible to compute numerical prefactors of each correction, the inconsistent kinetic terms could be zero due to some magical cancellation.

This is not implausible since oftentimes more that a single term contributes at a specific order and the required precise compensation could then be a consequence of supersymmetry. The total loop induced correction to the Kahler potential including genuine loop effects is then given by (6.26).

Second, it could be that only the kinetic terms leading to (6.16) and (6.21) vanish due to cancellation. The other inconsistent kinetic terms could vanish due to a particular structure of the R_8^4 operator and its log-divergences. The log-divergence producing the problematic kinetic terms should be absent. Assuming that all inconsistent terms produced by the log enhanced R_8^4 operator vanish due to the structure of the R_8^4 operator, only the $n = 2$ case in (6.24) is left. The total correction to K including genuine loop effects is then given by

$$\delta K \sim \frac{1}{\tau_f^2} + \frac{1 + \ln \tau_f}{\tau_2 \tau_f} + \frac{1}{\tau_2^2} \,. \tag{6.27}$$

It is of course also possible that some of the problematic kinetic terms induced by R_8^4 vanish due to cancellation and some by the structure of R_8^4. The correction δK would then be given by (6.27) extended by some (but not all) of the additional terms in (6.26).

Finally, the inconsistencies could be explained by field redefinitions which can in principle remove certain corrections to K. We have seen an example of this in section 3.1.1.

For a D7-brane wrapping the τ_f-cycle an entirely analogous analysis of corrections coming from an R_8^4 term on the brane worldvolume can be carried out. This does not produce any parametrically novel effects. We finally note that, also in the fibred setting, a correction associated with the small cycle of the type $\delta K \sim 1/(\mathcal{V}\sqrt{\tau_s})$ is expected to arise. This follows in analogy to the discussion of section 6.1.

Let us summarize: we can reproduce the corrections to the scalar potential (6.12) used in [23], but with some important differences and caveats. In our analysis, all three terms appear at the same order in g_s. Thus, the analogue of the hierarchy $(g_s C_1^{KK})^2, (g_s C_2^{KK})^2/2 \ll 2C_{12}^W$, which is required for fibre inflation, can be harder to realize (see [98] for more details on the required hierarchies). Moreover, higher-curvature terms that we expect to be present on D7-branes induce log-enhanced terms of the same structure. Those would be dominant in the Kahler and scalar potential compared to (6.12). Finally, we find a term which, though small in the relevant regime $L_f \ll L_b$, contradicts the BHP conjecture. For a deeper understanding of Fibre Inflation it is hence essential to make sure whether an R_8^4 term on D7-branes exists and to derive its precise structure.

Besides the corrections we have discussed so far, a further, g_s-suppressed effect arises if two brane stacks intersect on the 2-cycle of the fibre with volume $\sim \sqrt{\tau_f}$. It is due to the Einstein-Hilbert term induced at 1-loop order (and hence suppressed by g_s) on the intersection locus (see section 3.1.3). We may then apply (3.5), generalized to the case with multiple Kahler moduli. The resulting correction to the Kahler potential, $\delta K \sim g_s\sqrt{\tau_f}/\mathcal{V}$, reproduces the first term in (6.11) and (6.12), including the right power of g_s.

6.2.3 Loop corrections in the inverse fibration

In the initial stage of Fibre Inflation, we have $\tau_f \gg \tau_2$. This situation is not directly amenable to our earlier analysis as the length scale L_b of the base is smaller than the length scale L_f of the fibre. One would have to first dimensionally reduce on the base to obtain an 8d theory, which could then be further compactified to 4d on the larger fibre-4-cycle. For this, one would need the geometry in this regime to possess some form of 'inverse fibration' structure. In other words, one should be able to reinterpret what was originally the base 2-cycle t_b as being fibred over the 4-cycle τ_f. It is not clear to us whether such an inverse fibration emerges in the case at hand. Clearly, in a toy model where the 6D internal manifold is a product of a 2D and a 4D manifold, the required notion of inverse fibration trivially exists.

Assuming the inverse fibration to exist, one can perform a similar analysis as above in the limit $\tau_f \gg \tau_2$ using $10d \xrightarrow{t_b} 8d \xrightarrow{\tau_f} 4d$ as the 2-step compactification process. In the following, we will only be concerned with the differences compared to the calculation above. The first difference appears in 8d, where we expect no loop corrections to the Kahler potential due to the larger, 8d $\mathcal{N}=1$ and hence 4d $\mathcal{N}=4$, supersymmetry. Next, let us include D7-branes and check whether corrections contradicting the BHP conjecture occur. This is indeed the case and it happens because extra factors involving ratios of cycles appear (cf. the last term in (6.26)). Concretely, such an effect is induced by a D7-brane wrapping τ_2 both through genuine loop corrections and by dimensionally reducing R_8^4 like

$$\int\limits_{\mathbb{R}^{1,5}\times t_b} R_8^4 \sim \int\limits_{\mathbb{R}^{1,5}} R_6 \int\limits_{t_b} R_2^3 \sim \int\limits_{\mathbb{R}^{1,5}} R_6/L_b^4 \sim \int\limits_{\mathbb{R}^{1,3}} R_4\, L_f^2/L_b^4\,. \tag{6.28}$$

This is analogous to the $n=0$ contribution in (6.23). The equivalent genuine loop effect is induced on the 6d worldvolume of the wrapped D7-brane by gauge fields running in the loop. Here we assume that the corresponding KK modes depend only on the length scale of t_b. This induces a localized 6d Einstein-Hilbert term on the brane worldvolume of the form R_6/L_b^4, which after compactification gives the same result as on the r.h.s. of (6.28). Weyl rescaling turns this into a Kahler potential correction $\delta K \sim \tau_f/\tau_2^3$. This correction in now by far more interesting as it is dominant in the regime $\tau_f \gg \tau_2$ and could therefore strongly affect Fibre Inflation. We note that, similar to what has been discussed in section 6.2.2, a log coefficient in front of (6.28) would be inconsistent.

We recall that the last paragraph is to be read with the caveat that the geometry in the regime $\tau_f \gg \tau_2$ has an interpretation as an inverse fibration. This caveat disappears if we leave fibre inflation aside and simply start with a geometry which, by construction, consists of a 2-cycle with volume $t_b \sim \tau_2/\sqrt{\tau_f}$ which fibred over a base with volume τ_f. The analysis of the last paragraph then applies without extra assumptions.

7 Towards applications in LVS and KKLT

Here, we want to collect a number of further observations concerning the role of the loop corrections we studied in concrete phenomenological scenarios.

We first note that in the LVS, loop corrections are commonly used to stabilize non-blowup Kahler moduli for cases where $h^{1,1} > 2$. Doing so rigorously is notoriously difficult as it requires precise knowledge of the Kahler moduli dependence of the loop correction to the scalar potential. This has so far only been achieved for torus orbifold examples [25] and we hope that further work along the lines of the present paper will improve this situation.

We now turn to the question of whether loop corrections are capable of upsetting the LVS. This is in principle the case and hence loop corrections have been rightfully listed as part of the corrections to be dealt with in the recent critical analysis of [18]. We expect that the blowup loop correction $\delta K \sim 1/\mathcal{V}\sqrt{\tau_s}$, which has not been discussed in that analysis, represents one of the leading loop effects.[23] It corrects the scalar potential by terms which are suppressed by $g_s^{3/2}/\sqrt{\tau_s}$ or $1/\tau_s^2$ relative to the leading LVS scalar-potential terms. For establishing an AdS minimum this is not dangerous since g_s (and hence $1/\tau_s$) can be tuned extremely small. But as has been quantified very recently in [19], after including an anti-D3-brane uplift to dS control over the LVS becomes crucially limited by the LVS Parametric Tadpole Constraint: the size of the available negative D3 tadpole limits the size of the volume and hence of the small cycle. Explicitly, one has $a_s\tau_s \lesssim 16\pi N/(9\times(12\cdots 46))$, which very significantly restricts the size of $\tau_s \sim 1/g_s$ [19] and hence our ability to be safe from loop effects. Nevertheless, parametric control is clearly achievable in principle since τ_s is, by the definition of the LVS, a large parameter.

Given the previous comment, it is clearly highly relevant to determine the precise expansion parameter governing this and possibly higher loop corrections. Indeed, recall that in 4d gauge theory the true expansion parameter is $g^2/(16\pi^2)$ rather simply g^2. This line of thinking is known as 'naive dimensional analysis' [99]. In our context, the explicit study of refs. [25, 26][24] suggests that the expansion parameter is $1/(2\pi)^4\tau^2$, with τ being a generic 4-cycle variable. (Relating this to the previous paragraph, we would have $\tau^2 \sim \mathcal{V}\sqrt{\tau_s}$.) To the best of our present understanding, part of this significant suppression by (2π) factors is associated, in field theory language, with the explicit results for sums over KK modes on tori (see e.g. [101], Section II). It would be interesting to understand whether the $1/(2\pi)^4$ factor survives in CY geometries. This is not obvious since the loop factor in a purely 4d approach (following the logic of section 2.2) is $\sim \Lambda^2/(16\pi^2 M_4^2)$. Identifying the cutoff with the KK mass scale, one does of course get the expected parametric behaviour $m_{KK}^2/M_4^2 \sim 1/\tau^2$, but fixing the (2π) factors in this relation appears difficult in a CY geometry.

We have seen that small 4-cycles τ_s induce loop corrections to the Kahler metric which are $1/\tau_s^2$ suppressed compared to the leading terms, see section 6.1. Clearly, such corrections are dangerous when the small-cycle volume becomes $\mathcal{O}(1)$ in Einstein frame. More generally, going beyond the specific case of LVS-type blowup-cycles, one might suspect that even corrections suppressed only by 2-cycle volumes exist. For example, the previously discussed blowup correction came from a term $\delta K \sim 1/\mathcal{V}\sqrt{\tau_s}$, where the 2-cycle volume $\sqrt{\tau_s}$ appears. Now, in a series of papers [15, 102–104], flux compactifications with large $h^{1,1}$ were constructed with the goal to obtain explicit KKLT-type models. While, as already noted in [14], having large $h^{1,1}$ appears to be a promising route to counteract the 'singular-bulk problem', this may force one into a regime of dangerously small 2-cycles. Indeed, in [15] many 2-cycles with Einstein-frame volumes of order unity or smaller arise. While this may

[23] In the LVS, equally important contributions to the scalar potential come from the interplay of KK-type loop corrections with nonperturbative effects, namely from $K^{s\bar{s}}\partial_s W \partial_{\bar{s}} W$, see [18] chapter 3.1.1.

[24] See e.g. eqs. (10), (12) of [26] and eq. (2.6) of [100].

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be harmless in cases where, as the authors argue, the shrinking of the 2-cycle merely leads to a conifold singularity with local $\mathcal{N}=2$ SUSY, it is not clear whether the presence of nearby O-planes with reduced SUSY can always be avoided. If it can not, then our loop corrections with potentially only a 2-cycle Einstein-frame-volume suppression represent a serious concern.

Moreover, the above compactifications have many, typically $\mathcal{O}(100)$, small cycles. One might then be concerned that even if individual small-cycle corrections are controlled, the corrections could add up to become dangerously large if many small 2- or 4-cycles contribute. It would be interesting to better understand the form these small-cycle loop corrections in settings where they are not well-separated.

Let us now change perspective and estimate from a 4d EFT perspective and without the detour via the Kahler metric at which order in the inverse volume $1/\mathcal{V}$ loop corrections to the scalar potential arise. Due to the supersymmetric spectrum, i.e. $\mathrm{Str}\mathcal{M}^0=0$, the quartic divergence $\sim\Lambda^4$ vanishes and we have (disregarding numerical prefactors)

$$V_{\text{1-loop}}\sim V_{\text{tree}}+\Lambda^2\,\mathrm{Str}\mathcal{M}^2+\mathrm{Str}\mathcal{M}^4\,\ln\left(\frac{\Lambda^2}{\mathcal{M}^2}\right)+\ldots\,. \tag{7.1}$$

Here Λ is the cutoff and $\mathcal{M}^2$ the mass matrix. We also assume that SUSY is broken in a flat or approximately flat background (as e.g. in the LVS). This may be quantified by requiring $1/L_{\mathrm{AdS/dS}}\ll m_{3/2}$ and it implies that the supertrace of the mass matrix obeys $\mathrm{Str}\mathcal{M}^2\sim m_{3/2}^2$ (see e.g. [105, 106]). In our case, $\Lambda=m_{KK}\sim M_4/\mathcal{V}^{2/3}$ and $m_{3/2}\sim M_4 g_s^{1/2}W_0/\mathcal{V}$. For sufficiently large volume, the SUSY breaking scale is hence below the KK scale, as required for consistency of our 4d analysis. Using this, the leading correction is given by

$$V_{\text{1-loop}}\sim V_{\text{tree}}+M_4^4\frac{g_sW_0^2}{\mathcal{V}^{10/3}}+\ldots\,. \tag{7.2}$$

This precisely matches the order at which the genuine loop effects correct the scalar potential (cf. table 1). Independently, this represents a very general argument suggesting that the leading loop effect in the scalar potential scales as $\sim\mathcal{V}^{-10/3}$. This appears to clash with the existence of loop corrections $\sim\mathcal{V}^{-8/3}$ which have recently been proposed in the literature [43]. Note also that the Kahler potential term inducing such corrections is $\delta K\sim\ln(\tau)/\tau$, with $\tau\sim\mathcal{V}^{2/3}$. Consistently with what was said before, this does not correspond to a genuine loop but rather to a local α' correction. The log would then be expected if the underlying operator is marginal in 6d, 8d or 10d. However, we have seen that marginal operators always produce Kahler potential corrections which scale as $1/\tau^2$, not $1/\tau$. So we are left with a contradiction.

A possible resolution is as follows: let us start with the $G_4^2R_{11}^3$ term in 11d which according to [36, 42, 107] is the origin of the effect. Let us assume that, using the curvature of the torus fibration of F-theory, this descends to a local operator $\sim(\nabla H_3)^2$ on D7-brane stacks (see e.g. [68]).[25] This would lead to a term $\sim|W_0|^2$ in the 4d scalar potential. The volume

[25] Another option would be terms of the type $R_8(\partial F_2)^2$ computed in [108]. Note that G_4 flux in M-theory descends to H_3, F_3 and F_2 flux in type IIB.

scaling follows by noting that $H_3 \sim 1/\tau^{3/4}$, $\nabla \sim 1/\tau^{1/4}$, $\int_{D7} d^4y \sim \tau$ and that, finally, the Weyl rescaling to the 4d Einstein frame gives a factor $1/\mathcal{V}^2$. In total, one finds $|W_0|^2/\mathcal{V}^{8/3}$.

To summarize, the proposal is that the correction $\sim \ln(\tau)/\tau$ in the Kahler potential comes from a field redefinition and has nothing to do with a marginal operator. It does, however, produce a manifestly physical correction to the scalar potential $\sim |W_0|^2/\mathcal{V}^{8/3}$ which, as we saw earlier, can not be understood as a 4d loop effect. Instead, it comes from a local α' correction, for example a brane-localized 4-derivative term involving flux and hence proportional to $|W_0|^2$. Clearly, this is at the moment only a suggestion and it deserves further study how a possible $\ln(\tau)/\tau$ term and the corresponding scalar-potential effect are to be understood in a 10d SUGRA analysis of type-IIB orientifolds.

We leave it as a challenge for the future to study possible corrections associated with a field redefinition $\tau_s' \equiv \tau_s + \alpha \ln(\mathcal{V})$ [32] in our approach (see also the comment in [19]).

8 Discussion

In this work, we have analysed corrections to the Kahler potential in type-IIB string compactifications on Calabi-Yau orientifolds. We relied on one-loop field theory together with the available information about higher-mass-dimension local operators, both in 10d and on branes or brane intersections. The key corrections are summarized in table 1.

A novel proposal suggested by our analysis is that log-enhanced corrections, dominant w.r.t. established terms, arise on the basis of marginal higher-curvature operators in 8d or in 6d. The operators in question are localized on D7-branes/O7-planes or on their intersection loci respectively. One example is an R_8^4 operator, which is expected to be present on D7/O7s [66], another is the R_6^3 operators on the 6d intersection loci. The correction affects the scalar potential at order $\ln(M_{10} g_s^{1/4} L) \times h_{-5}$, where h_{-5} is a homogeneous function of degree -5 in 4-cycle Kahler moduli and L is a typical length scale of the relevant cycle. Because of its log-enhancement, this correction tends to be dominant and may hence be critical in moduli stabilization schemes relying on loop corrections, such as in Fibre Inflation (cf. section 6.2) or, more generally, in multi-moduli LVS constructions. Moreover, log-enhanced corrections could be dangerous for the LVS per se if an uplift to dS is included. It would hence be essential to confirm the existence of R_8^4/R_6^3 terms localized on 8d/6d from a string amplitude calculation.

One of our declared goals was to derive the Berg-Haack-Pajer conjecture, which we have partially achieved: first, we understand that ‘winding-type’ terms, correcting the Kahler potential at order -2 in 4-cycle variables, come from genuine loop effects and may, as just noted, feature a UV-sensitive log-enhancement associated with local α' corrections. Additional terms with the same volume dependence but suppressed by g_s are expected. We were not able to confirm the more specific suggested form $\sum_a 1/\mathcal{I}_a(t^i)\mathcal{V}$ with $\mathcal{I}_a(t^i)$ linear combinations of 2-cycle variables t^i. On the contrary, fibred examples suggest that a more general Kahler moduli dependence is possible.

Second, we understand the ‘KK-type’ terms as coming from higher-curvature operators. This resolves a discrepancy between the field theory analysis of 1-loop corrections to the Kahler potential in [24] on the one hand and string loop calculations by BHK [25] and the BHP conjecture [27] on the other hand. The local operators, like for example an Einstein-Hilbert term induced at 1-loop level on intersection cycles of D7-branes, induce KK-type Kahler potential corrections which, in turn, may modify the scalar potential.

Returning to the corrections of winding-type, we note that our analysis suggests a different interpretation than what is usually found in the literature: first, part of them is due to warping and, because warping respects the no-scale structure, this part will not affect the scalar potential. Second, the remaining part of the winding-type corrections corresponds to genuine loop effects and should hence be present more generally than proposed by BHP. The reason is that these effects are not tied to intersecting D7-branes but only to the requirement that the relevant tower of KK modes running in the loop displays an $\mathcal{N} = 1$ rather than an $\mathcal{N} = 2$ SUSY spectrum. It is conceivable that the absence of corresponding contributions in [25] is due to the special torus geometries used. We also note that winding-type corrections (and in particular genuine loop effects) contribute to the scalar potential with a $1/g_s^2$ enhancement compared to KK-type corrections since the latter are subject to the extended no-scale structure [24, 27, 28]. Thus, genuine loop effects should always be included in scenarios where KK-type corrections play a role, like in fibre inflation [23] or α' inflation [100]. Another example where this proposed more general occurrence of genuine loop corrections is important is blowup inflation [22]: for this scenario, the loop correction to a blowup cycle τ_s calculated in section 6.1 is dangerous and, in our understanding, it is expected to always be present as long as the geometry is $\mathcal{N} = 1$ locally, near the blowup cycle. We hope that it will be possible to clarify this further, strengthening the proposal of blowup inflation or ruling it out.

Let us close with an optimistic outlook. Naively, one might fear that loop effects will never be explicitly calculable on Calabi-Yaus and will hence always be in the way of fully controlled models. But things could be much better: it is conceivable that, as we argued in this paper, the dominant loop effects in the scalar potential will always come from log-enhanced winding-type corrections. Those are UV sensitive, being tied to certain marginal local operators. If the coefficients of the latter can be determined and the integrals over these operators in the classical Calabi-Yau background can be calculated, one may hope that the dominant effect from loops on moduli stabilization will become accessible.

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A The warped Kahler potential in the multi-moduli case

In section 4 we reviewed some of the results of [47]. A key starting point was the simple expression (4.4) for the Kahler potential in terms of the universal modulus a. Through the identification

$$\operatorname{Re} T^i + f^i(Z) + \overline{f}^i(\overline{Z}) = a\mathcal{V}^i + \frac{1}{2}\int_{D^i} e^{-4A_0} J_0 \wedge J_0\,, \tag{A.1}$$

it is then possible to express a as a function of the other Kahler moduli. In this appendix we add some details concerning how the integral in (A.1) can be evaluated. This material is entirely a review of [47], to which we refer the reader for further information.

First, one observes that the harmonic basis 2-forms may be expressed through so-called $h^{1,1}$ local potentials $\kappa^i(z,\overline{z};v)$:

$$\omega^i = i\partial\overline{\partial}\kappa^i(z,\overline{z};v)\,. \tag{A.2}$$

The $\kappa^i(z,\overline{z};v)$ cannot be defined globally as the ω^i are, by definition, nontrivial in cohomology. This can be remedied by introducing sections ζ^i of the line bundles $\mathcal{O}_{\rm CY}(D^i)$, such that $\zeta^i = 0$ identifies the location of D^i. It can then be shown that the combination

$$\pi\kappa^i - \operatorname{Re}\log\zeta^i \tag{A.3}$$

is globally well-defined.

With this, one can derive that

$$\frac{1}{2}\int_{D^i} e^{-4A_0} J_0 \wedge J_0 = \frac{1}{2\pi l_s^4}\int_{\rm CY}\left(\pi\kappa^i - \operatorname{Re}\log\zeta^i\right) Q_6\,. \tag{A.4}$$

Here Q_6 denotes the D3 charge distribution, which by itself would of course integrate to zero on the compact space. One may isolate from it the mobile D3-brane contribution by writing

$$Q_6 = l_s^4 \sum_{I\,\in\,\{\rm D3s\}} \delta_I^{(6)} + Q_6^{\rm bg}\,, \tag{A.5}$$

where I labels the mobile D3-branes. The background D3 charge distribution $Q_6^{\rm bg}$ includes the contributions from bulk fluxes and O3-planes:

$$Q_6^{\rm bg} \equiv F_3 \wedge H_3 - l_s^4 \sum_{J\,\in\,\{\rm O3s\}} \frac{1}{4}\delta_J^{(6)}\,. \tag{A.6}$$

We could include here the D3 charge of curved D7/O7s but will not do so for notational simplicity. Inserting Q_6 in (A.4) and introducing the functions

$$h^i \equiv \frac{1}{2\pi l_s^4}\int_{\rm CY}\left(\pi\kappa^i - \operatorname{Re}\log\zeta^i\right) Q_6^{\rm bg} \tag{A.7}$$

then leads to the final expression

$$\operatorname{Re} T^i = a\mathcal{V}^i(v) + h^i(v) + \frac{1}{2}\sum_{I\,\in\,\{\rm D3s\}} \kappa^i(Z_I,\overline{Z}_I;v)\,. \tag{A.8}$$

The last term arises since the δ-function part of (A.5) evaluates $\kappa^i(z,\bar{z};v)$ at the positions Z_I, $\overline{Z}_I$ of the mobile D3-branes. The $\mathrm{Re}\log\zeta^i(Z_I)+\mathrm{Re}\log\overline{\zeta}^i(\overline{Z}_I)$ term coming from the mobile D3-branes is of the form $f^i(Z)+\overline{f}^i(\overline{Z})$ and is absorbed in the definition of these functions in (A.1), hence this term does not appear in (A.8). The h^i then encode the data about the geometry, bulk fluxes, and all localized objects except mobile D3-branes.

One should view (A.8) as a system of $h^{1,1}$ equations defining a and v^i in terms of $\mathrm{Re}T^i$, Z_I, and $\overline{Z}_I$. In [47], the expression for a in terms of $\mathrm{Re}T^i$, Z_I, and $\overline{Z}_I$ is explicitly worked out in several examples. For us, the importance of the result (A.8) is that it may, in principle, be used to study the parametric dependence of the warping correction to K on the T^i. One would need to understand in more detail how the last two terms on the r.h.s. of (A.8), which are of degree zero in the Kahler moduli, depend on ratios $\mathrm{Re}T^i/\mathrm{Re}T^j$. This may then be used to disentangle which part of the winding correction comes from warping and which from genuine loop effects. Moreover, a non-trivial check of the BHP conjecture about the form of the winding correction may become possible. We leave these open problems for future work.

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Part II

Implications of higher order corrections for the $\overline{\boldsymbol{D3}}$ uplift and the LVS

Chapter 3

The LVS parametric tadpole constraint

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The LVS parametric tadpole constraint

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Abstract: The large volume scenario (LVS) for de Sitter compactifications of the type IIB string is, at least in principle, well protected from various unknown corrections. The reason is that, by construction, the Calabi-Yau volume is exponentially large. However, as has recently been emphasised, in practice the most explicit models are rather on the border of parametric control. We identify and quantify parametrically what we believe to be the main issue behind this difficulty. Namely, a large volume implies a shallow AdS minimum and hence a small uplift. The latter, if it relies on an anti-D3 in a throat, requires a large negative tadpole. As our main result, we provide a simple and explicit formula for what this tadpole has to be in order to control the most dangerous corrections. The fundamental ingredients are parameters specifying the desired quality of control. We comment on the interplay between our constraint and the tadpole conjecture. We also discuss directions for future work which could lead to LVS constructions satisfying the tadpole constraint with better control, as well as further challenges that may exist for the LVS. Our formula then represents a very concrete challenge for future searches for and the understanding of relevant geometries.

Keywords: Flux Compactifications, Superstring Vacua

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Contents

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1 Introduction

The construction of metastable de Sitter vacua in string theory is challenging and, in spite of many years of work, the focus has remained on two close cousins: the KKLT [1] and the LVS [2] proposals. Recently, fundamental doubts about the very existence of de Sitter space in quantum gravity (see e.g. [3–6]) have triggered intense scrutiny of these models. KKLT has arguably survived attacks related to the stability question of the anti-D3 uplift [7–10] and the gaugino condensate [11–16]. It is too early to judge how it will fare in view of the singular-bulk problem [17], which arises since one is forced to glue a large throat into a fairly small Calabi-Yau [12]. In view of this uncertainty (see [18, 19] for some the recent discussion), it is maybe natural to shift the focus on the LVS, where the singular-bulk problem does, at first sight, not arise [17] because the volume can be made exponentially large.

However, as recently emphasised in [20], the problem does not entirely disappear either. The reason is that at large volume the AdS minimum of the LVS becomes shallow, requiring a small uplift and hence a strongly warped throat. The latter comes with a large positive D3 charge and hence requires a large negative D3 charge of the embedding Calabi-Yau orientifold. This requirement has been emphasised as a potential problem in [21] and argued to be a solvable issue in [22], where the focus was on a concrete model. Most recently, this model has in turn been challenged in [20] on the basis that all the different corrections can not be made sufficiently small at the same time.

Our purpose in this short note is to derive a simple formula quantifying what we believe to be the most serious obstacle to defending the LVS against such skepticism. We call our result the 'LVS parametric tadpole constraint', as it quantifies the amount of negative tadpole which the geometry must possess to be in perturbative control. The desired quality of control is an input to this formula.

Our result will follow from two different corrections to the LVS potential, both with the same parametric behaviour. One is related to warping and hence similar in spirit to the singular-bulk problem, the other to higher F-terms [23–25]. As we will see, this leads to a simple, strong and conceptually clear bound on controlled LVS models with de Sitter uplift, deriving from the requirement to keep warping and higher F-term effects small at the same time. By comparison, the recent analysis of [20] used these conditions together with a number of conditions related to further corrections (see e.g. [26–40]) to collectively constrain the LVS. These analytical constraints were then applied to a specific Calabi-Yau orientifold. The most important of those further corrections are probably certain logarithmic contributions to the Kahler potential, which are not yet unambiguously established. We will comment on this in slightly more detail below. Many of the remaining corrections are interesting but probably not as dangerous. We will discuss them only briefly in what follows.

We note that, in an upcoming paper [41] we will provide a general discussion of loop corrections to the Kahler moduli Kahler potential in type-IIB compactifications [33–37]. We will in particular discuss at which order in α', g_s, and cycle volumes loop corrections can appear and what their potential impact on proposed compactifications, such as the LVS, is.

The rest of the present paper is organized as follows: we list the most important LVS relations in section 2, we derive the proposed key equation for the required minimal tadpole in section 3. Then, in section 4, we discuss how one may try to strengthen the LVS proposal in the future in spite of the presented constraint. We also discuss further challenges and comment on the interplay of our constraint with the tadpole conjecture [21, 42–45].

2 Summary of basic equations

To set the scene, we recall the LVS set-up. One compactifies IIB string theory to 4D on a Calabi-Yau orientifold X with flux and warping. The geometry has at least a big 4-cycle τ_b and a small 4-cycle τ_s. The corresponding Kahler potential reads [2, 46]

$$K = -2\ln\left(\mathcal{V} + \frac{\xi}{2g_s^{3/2}}\right) = -2\ln\left(\tau_b^{3/2} - \kappa_s\tau_s^{3/2} - \frac{\chi\,\zeta(3)}{4(2\pi)^3 g_s^{3/2}}\right). \tag{2.1}$$

Here $\mathcal{V} = \tau_b^{3/2} - \kappa_s\tau_s^{3/2}$ is the Calabi-Yau volume in units of $l_s = 2\pi\sqrt{\alpha'}$ and after a rescaling to the 10d Einstein frame. The constant κ_s is related to the triple intersection number of τ_s and an analogous constant in front of τ_b has been absorbed by rescaling τ_b. The string coupling is g_s and χ is the Euler number of the Calabi-Yau. The superpotential is given by

$$W = W_0 + A_s e^{-a_s T_s}, \tag{2.2}$$

with W_0 due to flux and the nonperturbative correction coming from either ED3-branes ($a_s = 2\pi$) or gaugino condensation on D7-branes (a_s depends on the gauge group, e.g.

$a_s = \pi/3$ for SO(8)). The prefactor A_s is model-dependent. We have absorbed the complex structure Kahler potential in a multiplicative rescaling of W_0 and A_s.

This yields the pure LVS scalar potential (in 4d Planck units)

$$V = \frac{4a_s^2|A_s|^2 g_s\sqrt{\tau_s}\mathrm{e}^{-2a_s\tau_s}}{3\kappa_s\mathcal{V}} - \frac{2a_s|A_s|g_s\tau_s|W_0|\mathrm{e}^{-a_s\tau_s}}{\mathcal{V}^2} + \frac{3\xi|W_0|^2}{8\sqrt{g_s}\mathcal{V}^3}\,, \tag{2.3}$$

which is minimized by

$$\mathcal{V} = \frac{3\kappa_s|W_0|\sqrt{\tau_s}}{4a_s|A_s|}\mathrm{e}^{a_s\tau_s}\,, \qquad \tau_s = \frac{\xi^{2/3}}{(2\kappa_s)^{2/3}g_s} + \mathcal{O}(1)\,, \tag{2.4}$$

leading to an AdS vacuum at

$$V_{\mathrm{AdS}} = -\frac{3\kappa_s g_s\sqrt{\tau_s}|W_0|^2}{8a_s\mathcal{V}^3}\,. \tag{2.5}$$

As noted in the appendix of [47] this value is parametrically suppressed with respect to the individual terms in V by a factor of $1/\tau_s$ or g_s. This has been emphasised and studied in detail in [20] under the name 'non-perturbative no-scale structure'.

One now assumes that a Klebanov-Strassler throat is present and places an anti-D3 brane at its tip. This adds a metastable uplift [1]

$$V_{\mathrm{uplift}} = V_{\overline{\mathrm{D3}}} = 2T_{\mathrm{D3}}e^{4A(0)} \tag{2.6}$$

to the scalar potential. Here T_{D3} is the D3-brane tension in 4D Einstein frame and $e^{A(0)}$ is the warp factor at the tip of the throat. To determine this warp factor, we neglect the difficult issue of backreaction [48, 49] since it is expected to make the uplift smaller. Hence we are being conservative by disregarding it. We rather use the standard Klebanov-Strassler geometry, glued into the conical region of a compact Calabi-Yau. We are careful to keep all $\mathcal{O}(1)$ factors except for those which are unknown since they are related to the specific Calabi-Yau (cf. appendix A for details). Including in particular the corrected volume scaling of [50], we find

$$e^{4A(0)} = \left(\frac{g_s^{3/2}\mathcal{V}}{\mathcal{V}_0}\right)^{2/3} \frac{3\,2^{2/3}\pi}{a_0}\frac{N}{g_s M^2}e^{-\frac{8\pi K}{3g_s M}}. \tag{2.7}$$

Here $a_0 \approx 0.71805$ is a numerical constant related to the explicit Klebanov-Strassler geometry [51], $\mathcal{V}_0$ is the volume of the $T^{1,1}$ where the conifold is glued into the Calabi-Yau, and M is the F_3 and K the H_3 flux on the throat 3-cycles. The contribution of the flux in the throat to the D3-tadpole is $N = KM$. The final form of the uplift potential is then

$$V_{\mathrm{uplift}} = \frac{\left(3^2\,\pi^3\,2^{22/3}\right)^{1/5}}{a_0}\frac{\mathrm{e}^{-\frac{8\pi K}{3g_s M}}}{g_s M^2\mathcal{V}^{4/3}}\,. \tag{2.8}$$

Jumping slightly ahead we note that the weaker volume suppression of the uplift as compared to the depth of the AdS potential in (2.5) does not represent a problem. The matching of the two effects will be ensured by the non-trivial volume scaling of $|W_0|^2$ and of the exponential warping suppression in (2.8).

Before moving on to the derivation of the tadpole constraint, we have to discuss one of the so-called higher F-term corrections. It is important since it bounds the size of W_0 in terms of the Calabi-Yau volume.[1] Concretely, this correction comes from eight derivative terms involving the field strength G_3 and takes the form [20, 25]

$$\delta V_F \sim \frac{W_0^4 g_s^{1/2}}{\mathcal{V}^{11/3}}\,. \tag{2.9}$$

Its ratio to the value of the AdS potential at the minimum (2.5) is required to be small. To quantify this, we multiply this ratio by our control parameter c_{W_0} and set the result equal to unity:

$$1 = c_{W_0}\frac{16a_s}{3(2\kappa_s)^{2/3}\xi^{1/3}}\frac{W_0^2}{\mathcal{V}^{2/3}}\,. \tag{2.10}$$

For $c_{W_0} \gg 1$, higher terms in the superspace derivative expansion are suppressed [23–25].

3 The LVS parametric tadpole constraint

The LVS tadpole constraint is derived in two steps: first, we require the uplift potential (2.8) to be of the size of the potential in the AdS minimum (2.5).[2] This leads to

$$\mathcal{V} = \frac{a_0}{(3^2\pi^3 2^{22/3})^{1/5}}\left(\frac{3(2\kappa_s)^{2/3}\xi^{1/3}}{16a_s}\right)^2\frac{g_s^{1/2}}{c_{W_0}}g_s M^2\, \mathrm{e}^{\frac{8\pi}{3g_sM^2}N}, \tag{3.1}$$

where we replaced $|W_0|$ by (2.10). Setting $\mathcal{V}$ in (2.4) equal to (3.1) gives an implicit equation for $\tau_s = \tau_s(N)$ which can be solved to leading order by comparing the exponentials:[3]

$$a_s\tau_s = \frac{16\pi N}{9g_sM^2} + \ln\left(\mathcal{O}(1)\right). \tag{3.2}$$

In the second step, we bound the volume from below by demanding that we have a well-controlled solution. One argument is to require that the singular-bulk problem is avoided. This imposes [12, 17]

$$1 \gg \frac{N}{\mathcal{V}^{2/3}}\,. \tag{3.3}$$

However, as recently argued in [20], a related and more quantitative bound can be given by studying 10d higher-curvature terms. Indeed, before strong warping leads to the formation of singular regions, it drives curvature corrections large. Specifically, consider a correction to the 4d Einstein-Hilbert term, analogous to BBHL [46], arising from the interplay between the 10d R_{10}^4 term and a varying warp factor [20]:

One uses the warped metric ansatz

$$\mathrm{d}s^2 = \mathrm{e}^{2A(y)}g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + \mathrm{e}^{-2A(y)}\tilde{g}_{mn}\mathrm{d}y^m\mathrm{d}y^n \tag{3.4}$$

[1] The resulting constraint is slightly stronger than the gravitino constraint $m_{3/2} \ll m_{KK}$ needed for a valid low-energy supergravity description [52]. We thank Daniel Junghans for making us aware of this.

[2] The relative fine tuning of the two terms can always be realised by adjusting $|W_0|$, which should not be problematic as long as c_{W_0} has not been chosen too close to unity.

[3] Note that replacing W_0 by the higher F-term constraint changes the exponential factor in (2.4) to $\mathrm{e}^{3a_s\tau_s/2}$.

together with the term

$$\frac{1}{g_s^{3/2}}\int \mathrm{d}^{10}x\sqrt{-G}\varepsilon^{ABM_1\dots M_8}\varepsilon_{ABN_1\dots N_8}R^{N_1N_2}{}_{M_1M_2}\cdots R^{N_7N_8}{}_{M_7M_8} \tag{3.5}$$

from the R_{10}^4 correction to the type-IIB action [46, 53] in 10d Einstein frame. To derive the contribution to the 4d Einstein-Hilbert term, we focus on fluctuations of the type $g_{\mu\nu} = g_{\mu\nu}(x)$. We also disregard terms with derivatives of $A(y)$. This turns (3.5) into

$$\frac{1}{g_s^{3/2}}\int_{\mathcal{M}_{10}} e^{2A(y)}R\wedge R\wedge R\wedge R\wedge e\wedge e\,, \tag{3.6}$$

where R is the Riemann tensor of the unwarped Calabi-Yau compactification with metric $(g_{\mu\nu}, \tilde{g}_{mn})$ and e denotes the vielbein 1-form. Since we are now working with a product geometry, the contribution to the 4d Einstein-Hilbert term can be made explicit as

$$\frac{1}{g_s^{3/2}}\int \mathrm{d}^4xR_4\int_X R_6\wedge R_6\wedge R_6\big(1+(\mathrm{e}^{2A(y)}-1)\big)\approx\frac{1}{g_s^{3/2}}\int d^4xR_4\bigg(\chi(X)+\frac{\chi(X)N}{\mathcal{V}^{2/3}}\bigg)\,. \tag{3.7}$$

Here we assumed that the bulk of the Calabi-Yau is at weak warping, $e^{2A}\simeq 1$. We may then use the fact that, in the 10d string frame, $(e^{-4A}-1)$ is the solution of a Poisson equation with the sources being the throat-localised flux $\sim g_sN$ and the compensating negative tadpole elsewhere in the compact space. The 6d Greens function behaviour $\sim 1/r^4$ then implies that the typical variations of $(e^{-4A}-1)$ and hence of $(e^{2A}-1)$ on the scale of the Calabi-Yau are $\sim g_sN/r^4\sim N/\mathcal{V}^{2/3}$.

We deviate from [20] in that we expect a scaling of the warping correction term with the Euler number $\chi(X)$. The reason is that we assume a slowly varying warp factor [17] and hence do not expect a parametrically strong cancellation between the contributions from different regions.

The correction to the Einstein-Hilbert term then translates into a Kahler potential correction and, analogously to the α' effect in the last term of (2.3), to a correction to the scalar potential:[4]

$$\delta V=\frac{15\,\xi\,N\,|W_0|^2}{8\sqrt{g_s}\,\mathcal{V}^{11/3}}\,\mathcal{O}(1)\,. \tag{3.8}$$

This correction in turn corrects the value of the (A)dS minimum and the stability condition of the dS minimum [20]. We focus on the effect on the value of the potential at the minimum since the stability of the minimum depends crucially on the unknown sign of the correction (3.8). A measure for parametric control is given by comparing the size of the correction to the scalar potential (3.8) and its value at the minimum (2.5). Hence, we are prompted to consider the following improved version of (3.3):

$$1=c_N\frac{10\,a_s\,\xi^{2/3}}{(2\kappa_s)^{2/3}g_s}\frac{N}{\mathcal{V}^{2/3}}\,. \tag{3.9}$$

[4]Another important correction which is of the same form as BBHL was found in [40]. This can be taken into account in our analysis by replacing $\xi\to\xi+\xi'$ where ξ' is computed in [40].

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A large control parameter $c_N \gg 1$ again ensures a parametric suppression of the correction compared to the leading term.[5]

Now, we require the volume in (3.9) to be equal to (3.1) and replace subsequently g_s by τ_s using (2.4). In doing so, we leave the combination $g_s M^2$ untouched since it represents an important parameter characterising the metastability of the throat. This leads to an equation for N which is of the form $we^w = x$, where

$$w = -\frac{16\pi}{21 g_s M^2} N \,, \qquad x = -\frac{3^{3/5}\pi^{9/35}\, a_0^{2/7}}{14\, 2^{2/15}\, 5^{3/7}} \frac{\kappa_s^{2/7}\, \xi^{2/7}}{a_s^{3/7}\, c_{W_0}^{2/7}\, c_N^{3/7}} \frac{1}{(g_s M^2)^{1/7}} \,. \tag{3.10}$$

An equation of the above type is solved by $w = \mathcal{W}_{-1}(x)$, where $\mathcal{W}_{-1}(x)$ is a branch of the Lambert $\mathcal{W}$ function. We find the exact result

$$N = -\frac{21 g_s M^2}{16\pi} \mathcal{W}_{-1}(x) = -\frac{21 g_s M^2}{16\pi} \Big(\ln(-x) + \ln\big(-\ln(-x)\big) + \mathcal{O}(1) \Big) \,, \tag{3.11}$$

where we expanded $\mathcal{W}_{-1}(x)$ around $x = 0$. In order for the D3-tadpole in our compactification to cancel, we must at least have sufficient negative tadpole Q_3 from O3/O7-planes and D7-branes to cancel the flux in the throat

$$-Q_3 > N \,. \tag{3.12}$$

In addition to (3.12), there is another lower bound on $-Q_3$ given by [54][6]

$$-Q_3 \geq 4\pi \frac{g_s W_0^2}{2} \,, \tag{3.13}$$

where the 4π arises since our superpotential W is normalized differently compared to [54]. Expressing W_0 by (2.10) and subsequently $\mathcal{V}$ by (3.9) we find

$$-Q_3 \geq \frac{c_N}{c_{W_0}} \frac{15\pi\xi}{4} N \equiv c_Q N \,. \tag{3.14}$$

Our main result is then that by filling in (3.11) for N in (3.12) and (3.14), we obtain two lower bounds on the D3-tadpole. For any given model and two prescribed values c_N and c_{W_0}, we should check that the strongest of these bounds is satisfied.

This result allows for a more compact formulation if we do not fix c_N and c_{W_0} but merely restrict them such that some minimal quality of control is ensured. Specifically, we demand $c_N \geq c_{N,\min}$ and $c_{W_0} \geq c_{W_0,\min}$, assuming also that the two values $c_{N,\min}$ and $c_{W_0,\min}$ are comparable. This is illustrated in figure 1, where the grey area represents the forbidden regime. The blue line in the plot separates the regimes where one or the other of the two bounds is stronger. It is described by $c_{W_0}/c_N = 15\pi\xi/4$ or, equivalently, $c_Q = 1$. Since $15\pi\xi/4$ is in general significantly larger than unity, the blue line cuts the horizontal rather than the vertical part of the boundary between the grey and white area.

[5]Note that, as described in [20], the dS minimum disappears if the correction (3.8) has an unfavourable sign and is too large. This correspond in our analysis to a value of $c_N < 11 C^{\text{flux}}/(18\xi)$ (see equation (5.8) in [20]). Evaluating this for the explicit model considered there, the dS minimum breaks down for $c_N \simeq 1.35$ (see equation (6.23)).

[6]We thank Erik Plauschinn and the referee for making us aware of this important point.

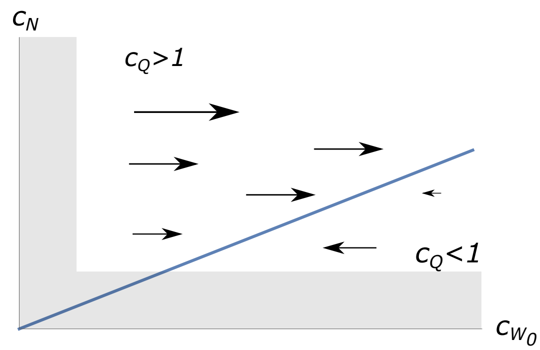

Figure 1. The parameter space of possible values of c_N and c_{W_0}. In the grey area, one of these two control parameters is too small such that corrections are dangerously large. The blue line separates the regimes of validity of the two bounds in (3.12) and (3.14). The arrows specify how c_{W_0} must change to decrease the lower bound $-Q_3^*(c_N, c_{W_0})$ at fixed c_N.

Over the white part of the plot in figure 1, we have a function $-Q_3^*(c_N, c_{W_0})$ given by the r.h. side of (3.12) or (3.14), whichever is largest. For a conservative constraint, we want to know the minimal value of that function. To determine this value, we first consider the area to the right of the blue line, where $c_Q < 1$. It is immediately clear from eqs. (3.10)–(3.12) that, as indicated by the arrows, our function $-Q_3^*(c_N, c_{W_0})$ falls as we move to smaller c_{W_0} at fixed c_N. Hence the minimum is attained on the blue line or to its left. On the contrary, in the region left of the blue line, i.e. for $c_Q > 1$, the value of $-Q_3^*(c_N, c_{W_0})$ generically falls if one moves to larger c_{W_0}. This requires a moment of thought: one first convinces oneself that, in this regime and with fixed c_N, the variation of the minimal tadpole reads $\delta[-Q_3^*(c_N, c_{W_0})] \sim \delta[(a + \ln(c_{W_0}))/c_{W_0}]$, with $a > 0$. This follows from eqs. (3.10), (3.11) and (3.14), using also the leading logarithmic approximation in (3.11). It is now easy to see that, for sufficiently large c_{W_0}, the function $-Q_3^*$ always falls if c_{W_0} grows. The requirement that c_{W_0} is sufficiently large is always satisfied since we are outside the grey band. Thus, the arrows left of the blue line must indeed point to the right, and the minimum is hence on top of the blue line. Finally, restricting attention to points on the blue line, the value of $-Q_3^*$ falls as one moves towards the origin.

As a result, the minimum is at the point where the blue line crosses into the grey area. This is specified by $c_N = c_{N,\mathrm{min}}$ and

$$c^*_{W_0} = c_{N,\mathrm{min}} \frac{15\pi\xi}{4} \,. \tag{3.15}$$

For simplicity, we drop the index 'min' on c_N in what follows. Then, inserting (3.15) together with x into (3.11), we obtain the leading order result:

The LVS parametric tadpole constraint. The D3 tadpole contribution Q_3 of O3/O7-planes and D7-branes must fulfil

$$-Q_3 > N = N_* \left(\frac{1}{3} \ln N_* + \frac{5}{3} \ln c_N + \ln a_s - \frac{2}{3} \ln \kappa_s + 8.2 + \mathcal{O}\big(\ln(\ln)\big) \right), \tag{3.16}$$

where we defined $N_* = 9 g_s M^2/(16\pi)$.

Note that the subleading terms in (3.16) have a significant impact on the final bound one obtains. We only display the leading term to give the reader an intuition of how the bound scales with the parameters. However, to actually compute N we always make use of the exact expression (3.11).

Thus, we require Q_3 to be negative and its absolute value to exceed the throat-flux N since additional 3-form flux is needed to stabilize the complex-structure moduli. Let us discuss the implications of (3.16). The constraint specifies the minimal tadpole needed in LVS constructions to avoid the singular-bulk problem, and to make sure that the corrections induced by a varying warp factor and higher F-terms are parametrically suppressed. The degree of parametric suppression can be chosen at will by specifying c_N. The parameter c_N determines to which extent warping effects are suppressed and fixes, through (3.15), the optimal value of W_0. The approximation of x being close to zero improves with increasing $g_s M^2$ and increasing control parameters. Interestingly, the main expansion parameter N_* coincides with the size of the small cycle τ_s.

An optimistic bound on N is obtained by using the metastability constraint $g_s M^2 \gtrsim 12$ by KPV [55] (to be precise, $M > 12$ combined with $g_s M \gtrsim 1$). Setting $\kappa_s = 1$ and $a_s = \pi/3$ for the moment, as well as using (3.11) together with (3.15), we obtain for the minimal tadpole

$$N \simeq 33.6 \quad \text{for } c_N = 5 \qquad \text{and} \qquad N \simeq 45.8 \quad \text{for } c_N = 100\,. \tag{3.17}$$

This is, however, probably too weak since it ignores backreaction. Indeed, the constraint comes from the process in which the $\overline{\text{D3}}$ polarises into a fluxed NS5, which slips over the equator of the S^3 at the tip of the throat. One expects that, as part of this decay process, the volume of the S^3 is driven to a smaller value, lowering the potential barrier for the decay.[7]

An improvement has been suggested in [48, 49] (see also [57, 58]), where the conifold modulus is treated as a dynamical variable. In [56] this analysis was combined with the polarization of the $\overline{\text{D3}}$ into an NS5-brane. As already discussed in [17, 20], the reliability of these analyses is limited by the assumption that the warp factor behaves as $\exp(4A(0)) \sim |Z|^{4/3}$, also off-shell. This is not obvious because $\exp(4A(0)) \sim \exp(-8\pi K/3g_s M)$ is a priori a fixed number, determined by the flux choice. Put differently, it is not clear why the warp factor should go to zero as the S^3 at the tip of the throat is dynamically driven to zero radius. After all, the fluxes K and M determining $\exp(4A(0))$ à la Klebanov-Strassler are discrete.

Nevertheless, we may take for the moment the value $g_s M^2 \approx 46$ obtained in [48, 49] as a (probably rather conservative) lower bound and insert it together with (3.15) in (3.11). Choosing again $\kappa_s = 1$ and $a_s = \pi/3$, we find

$$N \simeq 132.9 \quad \text{for } c_N = 5 \qquad \text{and} \qquad N \simeq 179.9 \quad \text{for } c_N = 100\,. \tag{3.18}$$

[7] Note that this volume is *not* the same as the conifold modulus Z. Indeed, due to a cancellation between the Z-dependence of the warping and the Z-dependence of the internal metric at the tip of the warped throat, the physical volume of the S^3 at the tip is Z-independent. Correspondingly, the analysis of [56] does not find such a lowering of the potential barrier.

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Combined with the fact that, in the explicit LVS models considered to date the maximal tadpole is ~ 150 [22], we see that the tadpole constraint is neither harmless nor deadly.

The natural way to ensure the constraint (3.16) is satisfied is to search for LVS compactifications with a large D3-tadpole. We will discuss the prospects for finding such examples and stabilizing all moduli in section 4. In addition, one may try to study the LVS in more detail in the regime where the control parameters c_N and c_{W_0} are only moderately large. To convince oneself that the construction remains reliable, one would need to make sure that even higher order corrections of the same type do not blow up. Interestingly, (3.16) increases for small κ_s and is independent of the Euler number. This appears to partly differ from the trend observed in example studies in [22].

4 Discussion of the LVS tadpole constraint

4.1 Interplay with the tadpole problem

Our central result (3.16) provides a quantitative lower bound on the flux-induced D3 tadpole. The tadpole conjecture [21, 42–45] on the other hand states that, for a large number $h^{2,1}$ of complex structure (CS) moduli, the tadpole of the 3-form flux required to stabilize all of them in a non-singular regime grows faster than the negative tadpole of the corresponding orientifold geometry. If the conjecture were true, it would be logical to focus on LVS models with a large tadpole and a small number of moduli, escaping the allegedly dangerous asymptotic regime. But this may not be possible since, according to known examples, a large negative tadpole comes with a complicated topology (as discussed in more detail in section 5 of [17]) and hence many moduli. Indeed, one would expect that a single involution of a certain geometry has many O3 fixed points precisely because the topology is complicated. Similarly, while even a single O7 plane with its D7 branes can produce a large tadpole, this tadpole is tied to its topology. It grows if the 7-brane topology becomes complicated, which is in general associated with the presence of many brane-deformation moduli. These, in turn, require more flux to be stabilized.

It then seems that the allowed LVS vacua are constrained to an interval of permitted Q_3, with (3.16) providing a lower bound and the parametrics of the tadpole conjecture providing an upper bound above which it becomes hard to stabilize all the CS moduli. One might worry that further analyses will demonstrate that this interval shrinks to zero size, casting all LVS de Sitter vacua into the swampland. Indeed, this concern was already raised in [21], though without deriving a quantitative LVS tadpole constraint. However, things need not be all doom and gloom:

We recall an important but perhaps underappreciated condition for the tadpole conjecture to apply, namely that the stabilized geometry is required to be smooth. For example, in the simple $K3 \times K3$ example it is possible to stabilize all CS moduli (including D7 brane moduli in the language of F-theory) if singular geometries are permitted [59–62]. According to the detailed study of [21], developing the method of [61], this fails if one attempts to avoid the gauge enhancement through singularities. Nevertheless, this failure allows for the following more optimistic view on the possible string landscape of de Sitter vacua: the tadpole conjecture says that all vacua in the landscape at large number of CS moduli have singular internal geometries. One may embrace this statement together with the fact that the singularities will in general produce enhanced nonabelian gauge symmetries. The dynamics of those gauge theories together with SUSY breaking might stabilize the new moduli associated to the singularity. If this happens only at low energies, the nonabelian gauge theories make our compactification more interesting phenomenologically. The tadpole conjecture then tells us that the de Sitter vacua that do exist in the landscape are on average much more interesting phenomenologically then one a priori expects.

4.2 Overcoming the LVS Tadpole Constraint and further challenges

With these considerations in mind, let us look at which D3-tadpole has been achieved in IIB string theory and what the prospects for realizing higher tadpoles might be.

In IIB string theory, the D3 tadpole is generally given by

$$N_{\rm D3}+\frac{N_{\rm flux}}{2}+N_{\rm gauge}=\frac{N_{\rm O3}}{4}+\frac{\chi(D_{O7})}{12}+\sum_a N_a\,\frac{\chi(D_a)+\chi(D_a')}{48}\equiv -Q_3\,, \tag{4.1}$$

with $N_{\rm flux}=\frac{1}{(2\pi)^4\alpha'^2}\int H_3\wedge F_3$, $N_{\rm gauge}=-\sum_a\frac{1}{8\pi^2}\int_{D_a}{\rm tr}\,\mathcal{F}_a^2$, where $\mathcal{F}_a$ is the gauge flux turned on on each of the divisors D_a, and N_a the number of D7 branes (their orientifold images) wrapping the divisor D_a (D_a'). Note that a throat with fluxes K and M contributes an amount of $2N=2KM$ to the quantity $N_{\rm flux}$ in (4.1). The reason is that a throat in the orientifold corresponds to two throats in the Calabi-Yau. If the D7 brane tadpole is cancelled locally by placing eight D7 branes on top of the $O7$ plane, (4.1) simplifies to

$$N_{\rm D3}+\frac{N_{\rm flux}}{2}+N_{\rm gauge}=\frac{N_{\rm O3}}{4}+\frac{\chi(D_{O7})}{4}\,. \tag{4.2}$$

Most concrete type-IIB orientifolds considered in the literature are based on Calabi-Yaus with a small number of Kahler moduli $h^{1,1}(X)$. The highest negative tadpole values of explicitly considered models we are aware of are $Q_3=-149$[8] [22] and -104 [63], both in models with $h^{1,1}(X)=2$ and based on reflection involutions. More generally in type-IIB orientifolds with locally cancelled D7 tadpole, the negative contribution to the D3 tadpole can be bounded from above by the Lefschetz fixed point theorem [21, 64, 65]:[9]

$$-Q_3\leq 1+\frac{1}{2}(h^{1,1}+h^{2,1})\,. \tag{4.3}$$

Using the largest Hodge numbers from the Kreuzer-Skarke dataset [66], this gives, $-Q_3\leq 252$.

- *Exploiting non-local D7-tadpole cancellation*: in the models of [22, 63], the D7 tadpole is cancelled locally, i.e. eight D7 branes sit on top of each O7. This restricts the possibilities of getting a large tadpole: indeed, according to [65], canceling the D7 tadpole locally in smooth and favorable Complete Intersection Calabi-Yau threefolds (CICYs) with both reflections and simple divisor exchange involutions gives

[8] This is the model considered in [20].
[9] We thank Jakob Moritz for pointing this out to us.

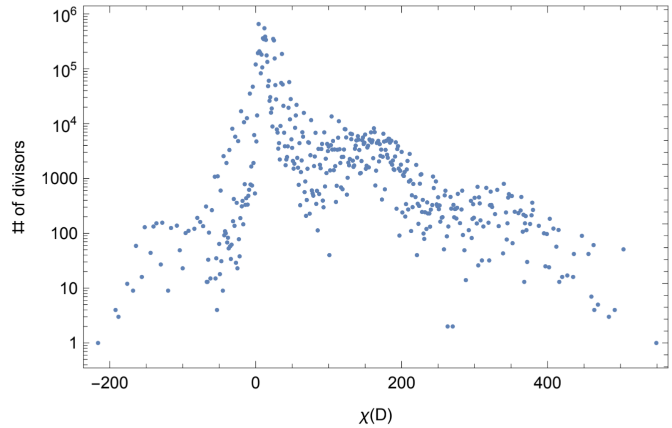

Figure 2. Log distribution of the Euler number of individual divisors $\chi(D)$ in toric orientifold Calabi-Yaus with $h^{1,1}(X) \leq 6$.

even tadpole values in the range $[-36, 4]$. By contrast, non-local D7-tadpole cancellation produced values in the larger range $[-132, -12]$. Moreover, the study of toric orientifold Calabi-Yaus with $h^{1,1} \leq 6$ in [67] gave $Q_3 \in [-30, 0]$. This was based only on the divisor exchange involution and local D7-tadpole cancellation. Models with reflection involution in toric Calabi-Yaus are still under study but we expect that in this class the tadpole will increase significantly. The reason is that, in the examples of [22, 63] with reflection involution the tadpole is already as large as 104 and 149 even for $h^{1,1}(X) = 2$. Thus, one may hope that extending this to the range $h^{1,1}(X) \leq 6$ much larger tadpoles can arise.

- *Using divisors D_a with large Euler number*: (a) from (4.1) one can see another reason for the limited tadpole range arising in the study of [65]. It is the fact that in CICYs the topology of each divisor is relatively simple and the largest Euler number of a divisor is modest: $\chi(D_a)_{\max} = 80$ [65, 68]. By contrast, in the toric setting even with $h^{1,1}(X) \leq 6$, the highest Euler number that gives an integer contribution to the tadpole is $\chi(D_a)_{\max} = 504$ [67]. Even if we only consider a reflection on this divisor and cancel the D7-tadpole locally, it will contribute -126 to the tadpole. A distribution of Euler numbers of divisors in the orientifold Calabi-Yau database of [67] is displayed in figure 2. (b) Another way to enlarge the Euler number of D_a is by considering a D7-brane wrapping a combination of divisors. This relies on detailed model building and represents an interesting project for the future.

- *Large $h^{1,1}(X)$*: increasing $h^{1,1}(X)$ enriches the possibilities for D7-brane model building and hence should make it possible to find larger Euler number for D_a and hence larger $N_{\mathrm{O}3}$. For CICYs, the largest $h^{1,1}$ is just 19 [69] while for toric Calabi-Yau this number increases to $h^{1,1}(X) = 491$ [66]. Although so far no database for orientifolds of toric Calabi-Yaus with higher $h^{1,1}(X)$ exists, progress based on large cluster computation and machine learning has been made [70]. We note that in the search for large gauge groups, a correlation between the rank and $h^{1,1}$ has been observed [63]. The concrete model studied in that paper has only $h^{1,1} = 2$ and a maximal negative tadpole of -92, but the expectation is that models with much larger $h^{1,1}$ and D7 brane numbers going into the hundreds exist. While this raises the hope that very large negative tadpoles can also be achieved by the detailed study of the corresponding geometries, one in general pays the price of large $h^{1,1}$. This means many Kahler moduli, the stabilization of which may in itself be non-trivial in the LVS context, as we discuss below.

- *Increasing* N_{O3}: finally, an obvious way forward is to search for models with more O3 planes. In the particularly tractable setting of CICYs, see e.g. [69, 71], the multiplicities of O3 and O7 planes have recently been studied in [65]. Tadpoles in the range $[-132, -12]$ were found. An interesting aspect uncovered in this context is that the resolution branch of each conifold singularity on an O7 plane produces one extra O3. Although in [65] this did not lead to higher tadpoles, this might change in more general settings, such as toric Calabi-Yaus. Moreover, the existence of this method for increasing the number of O3 planes, possibly in the range of $N_{\mathrm{O3}} \sim 100$, is interesting in itself.

If it should nevertheless prove impossible to construct perturbative type-IIB models with a sufficiently high tadpole to be in control according to (3.16) and with all CS-moduli stabilized in an acceptable manner, another option is to consider proper F-theory compactifications[10] [72, 73]. The immediate advantage is that Calabi-Yau 4-folds with very large Euler number χ_4 are known, leading to negative D3 tadpoles as high as $N = \chi_4/24 = 75852$ [74, 75]. This appears to be sufficient to easily satisfy the constraint (3.16) with excellent control. But the extension of the discussion to F-theory models has its price:

On the one hand, the key parametric relations $\ln \mathcal{V} \sim \tau_s \sim 1/g_s$ demand that we realise a small value of g_s at least in the corner of our compact space where the small cycle is localized. Studying whether this can be implemented in large-tadpole F-theory geometries is an interesting task.

On the one hand, the key parametric relations $\ln \mathcal{V} \sim \tau_s \sim 1/g_s$ demand that we realise a small value of g_s to achieve a large volume. In F-theory, g_s is replaced by an appropriate average of the exponential of the dilaton, $g_s \equiv \langle e^{\phi} \rangle$. Studying whether a small g_s can in this way be implemented in large-tadpole F-theory geometries is an interesting task.

On the other hand, a 4-fold with large Euler number implies, in the smooth case, that the F-theory model has either many Kahler moduli or many complex structure moduli or many 7-brane deformations (cf. section 5 of [17], based on [64, 76, 77]). The latter two options potentially run into trouble with the tadpole conjecture (though this may be acceptable if one allows for singular geometries). The former option, where the number of Kahler moduli is large, potentially has a problem with the LVS moduli stabilization procedure. Indeed, the standard approach to stabilizing non-blowup moduli (other than

[10] The F-theory analogue of the crucial correction (3.5) was derived in [40].

the volume) in the LVS is through loop corrections [36, 37]. The dependence of such corrections on ratios of 4-cycle volumes as well as the complex-structure moduli dependence of the prefactors is not known except for simple torus orbifold geometries [34, 35] (see [41] for an attempt to make progress). This may not be a problem as a matter of principle since, generically, one expects that all Kahler moduli will somehow be stabilized by loop effects. But clearly it *is* a problem if one wants to establish rigorously the existence of at least one explicit LVS de Sitter model. Hence, the way forward may be not to rely on loops but rather demand that all ratios of non-blowup cycles are stabilized by D-term constraints [78–80]. In this case, achieving a purely algebraic and fully explicit construction in the near future appears more plausible.

The correction we have focused on in this note is not the only correction relevant for control in the LVS. As emphasized in [20], further potentially dangerous corrections are associated with logarithmic terms. There are two types:

One of them [26] derives from the field redefinition $\tau_s' \equiv \tau_s + \alpha \log(\mathcal{V})$, where τ_s' rather than τ_s is now the real part of the proper Kahler variable. The Kahler potential then reads

$$K \sim -2 \log \left[\tau_b^{3/2} - \left(\tau_s' - \alpha \log(\mathcal{V})\right)^{3/2} + \ldots \right] + \ldots , \tag{4.4}$$

with the physical small-cycle volume being $(\tau_s' - \alpha \log(\mathcal{V}))$. It is at the moment not entirely settled if and when such a correction appears in string compactifications. A surprising aspect is the implicit claim that, at $\mathcal{V} \to \infty$, the correction grows in importance. For example, the size of non-perturbative effects $\sim \exp(-a_s \tau_s')$ now has a very different relation to the physical volumes of small cycle and Calabi-Yau.

Another type of correction, corresponding to a replacement $\mathcal{V} \to \mathcal{V} + \alpha \log(\mathcal{V})$, has been proposed to appear through the interplay of the R_{10}^4 and D7/O7 branes [27]. An interesting aspect of this proposal is that, while it indeed threatens parametric control of the LVS, it could instead allow for alternative ways to realize a de Sitter minimum.

The correction (3.8) we have focused on in this note and log corrections are not the only potentially relevant corrections for the LVS. As recently highlighted in [20], a host of further corrections are interesting or even potentially dangerous. This requires more research. In this context, in upcoming work [41], we will give a general discussion of loop corrections to the Kahler moduli Kahler potential. While important in many ways, we do not see that these corrections endanger the basic setting of the LVS.

Finally, we emphasize again that the tadpole constraint (3.16) is tied to the $\overline{\text{D3}}$-uplift. Hence, even if this constraint turned out to remain problematic for the LVS, other uplifting mechanisms could work more successfully, especially if they do not require a very large tadpole. One example is the D-term uplift, where the positive energy arises because a D7-brane matter-field is unable to satisfy simultaneously a D-term and an F-term constraint [81, 82]. Implementing this consistently in the stringy geometry leads to the concept of a T-brane uplift [83]. Another suggestion is the Winding uplift [84, 85] (with the idea going back to [86]), which uses the possibility to create a perturbatively flat direction in the complex-structure moduli space by an appropriate flux choice [86]. The non-perturbatively small potential along this flat valley can then arguably be tuned such that a local metastable minimum with non-zero F-term arises. In this setting, the tuning power of the complex-structure landscape could help to realize an exponentially small uplift without a warped throat.

5 Summary

The core result of this short note is (3.16), which bounds the D3 tadpole required in LVS models from below. The bound comes from demanding that warping corrections in the bulk, associated with the Klebanov-Strassler throat housing the anti-D3 brane uplift, are under control. While this constrains the set of geometries suitable for an LVS with a controlled uplift, we so far see no catastrophe that places all LVS vacua in the swampland. Rather, the bound poses a challenge for LVS model-building which one can attempt to overcome by searching for compactifications with a sufficiently large D3-tadpole. While the tadpole conjecture poses an additional challenge in this context, we do not currently see this as impossible to overcome. In type-IIB a tadpole of 149 has been achieved [22], but, as we argued above, much larger tadpoles may be possible. In F-theory, the largest available tadpole of 75852 [74, 75] easily satisfies our bound (3.16) with excellent control. However, then one faces other challenges: one either has to realize a small averaged g_s in F-theory or, alternatively, one has to replace the control parameter $1/g_s$ by a topology which makes the coefficient of the α'^3 correction appropriately large. Moreover, stabilizing all moduli in a controlled manner in geometrically complicated models with high tadpole may introduce new difficulties. Finally, a key quantity determining the bound of the D3 tadpole (3.16) is $g_s M^2$, the minimal value of which is still under debate. In order to see how constraining the bound for the LVS really is, it would be essential to know this value as precisely as possible.

We hope that this note has highlighted a number of directions for future work which can help to clarify whether LVS de Sitter vacua exist.

Acknowledgments

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A Derivation of the $\overline{\text{D3}}$ uplift potential

In this appendix we derive the uplift potential directly from the Klebanov-Strassler (KS) geometry [51], using also [87, 88]. This provides a more precise derivation of the (non-backreacted) uplift potential than we are aware of elsewhere in the literature.

The warp factor in string frame far away from the tip of the throat is given by the following two equivalent expressions. Firstly, it can be parametrized by the flux $N = KM$

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and the radial distance from the tip $r = r_{\max}$ at which the total flux in the throat equals N. At $r_{\max}$, the throat is glued into the compact Calabi-Yau. This yields [87, 88]

$$h_N(r) = \frac{27\pi\alpha'^2}{4r^4}\left(g_s N + \frac{3(g_s M)^2}{2\pi}\ln(r/r_{\max}) + \frac{3}{8\pi}(g_s M)^2\right). \tag{A.1}$$

Secondly, we may use the KS warp factor $h_\varepsilon^{\mathrm{KS}}(r)$ which describes both the UV and the smooth, IR region and which is parametrized by the conifold resolution parameter ε [51, 88]. In the UV, it has the approximate form

$$h_\varepsilon^{\mathrm{KS}}(r) \simeq \frac{81(\alpha' g_s M)^2}{8r^4}\ln\left(\left(\frac{2^{5/3}}{3}\right)^{1/2}\frac{r}{\varepsilon^{2/3}}\right). \tag{A.2}$$

Comparing (A.1) and (A.2) determines $r_{\max}$ as a function of ε to leading order to be

$$r_{\max} = \left(\frac{3}{2^{5/3}}\right)^{1/2}\varepsilon^{2/3}\,\mathrm{e}^{\frac{2\pi K}{3g_s M}}. \tag{A.3}$$

Moreover, from KS, the warp factor at the tip is known. In string frame, it reads

$$h_\varepsilon^{\mathrm{KS}}(0) = \frac{2^{2/3}a_0(g_s M\alpha')^2}{\varepsilon^{8/3}}, \tag{A.4}$$

with the constant $a_0 \approx 0.71805$.

With this in hand, we can calculate the ratio of the warp factors at the tip and far away from the tip at $r = r_{\max}$. This yields the relative warping needed for the uplift potential:

$$\frac{h_\varepsilon^{\mathrm{KS}}(0)}{h_\varepsilon^{\mathrm{KS}}(r = r_{\max})} = \frac{a_0}{3\,2^{2/3}\pi}\frac{g_s M^2}{N}\mathrm{e}^{\frac{8\pi K}{3g_s M}}. \tag{A.5}$$

So far, we neglected the additive, constant contribution $\sim \mathcal{V}^{2/3}$ to the warp factor [89][11] which becomes dominant at sufficiently large volume. Specifically, we replace the KS warp factor according to

$$h_\epsilon^{\mathrm{KS}}(r) \quad\rightarrow\quad (\mathcal{V}_s/\mathcal{V}_{s,0})^{2/3}\,h_\epsilon^{\mathrm{KS}}(r_{\max}) + h_\epsilon^{\mathrm{KS}}(r). \tag{A.6}$$

Here we normalized the constant term by introducing a string-frame, fiducial volume $\mathcal{V}_{s,0}$. Our definition implies that, if $\mathcal{V}_s = \mathcal{V}_{s,0}$, the strong-warping region is set by $r \lesssim r_{\max}$. This is the situation when a strongly warped throat is glued into a weakly warped, compact CY. For larger $\mathcal{V}_s$, a weakly-warped conical region develops above the throat (see e.g. [90]). From all of this, it should be clear that $\mathcal{V}_{s,0}$ must be chosen such that typical length scales in the CY match typical length scales of the $T^{1,1}$ at $r_{\max}$. Hence, for $\mathcal{V}_s \gg \mathcal{V}_{s,0}$, the full, inverse warp factor at the tip is given by

$$\mathrm{e}^{4A(0)} = \left(\frac{\mathcal{V}_s}{\mathcal{V}_{s,0}}\right)^{2/3}\frac{h_\varepsilon^{\mathrm{KS}}(r = r_{\max})}{h_\varepsilon^{\mathrm{KS}}(0)} = \left(\frac{\mathcal{V}_s}{\left(\mathrm{Vol}(T^{1,1})|_{r_{\max}}\right)^{6/5}}\right)^{2/3}\frac{h_\varepsilon^{\mathrm{KS}}(r = r_{\max})}{h_\varepsilon^{\mathrm{KS}}(0)}. \tag{A.7}$$

The volume scaling of this expression matches the corrected volume scaling of [50].

[11] The warp factor is determined by a Poisson equation and can hence be always shifted by a constant.

To obtain the final formula of the uplift potential, we hence need to determine the volume of the $T^{1,1}$ at $r_{\max}$. For this we consider the string frame metric of the conifold far away from the tip

$$\mathrm{d}s_{10}^2 = h_N^{-1/2}(r)\,\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + h_N^{1/2}(r)\big(\mathrm{d}r^2 + r^2\mathrm{d}s_{T^{1,1}}^2\big)\,. \tag{A.8}$$

Using (A.1) and $\mathrm{Vol}\,T^{1,1} = 16\pi^3/27$ from [91], we find

$$\mathcal{V}_{s,0}^{2/3} = \frac{3^{3/5}}{2^{14/5}\,\pi^{3/5}}\left(g_sN + \frac{3}{8\pi}(g_sM)^2\right). \tag{A.9}$$

This can be plugged into (A.7) which then yields the uplift potential

$$V_{\text{uplift}} = V_{\overline{\text{D3}}} = 2T_{\text{D3}}e^{4A(0)} = \frac{(3^2\,\pi^3\,2^{22/3})^{1/5}}{a_0}\,\frac{\mathrm{e}^{-\frac{8\pi K}{3g_sM}}}{g_sM^2\mathcal{V}^{4/3}}\,. \tag{A.10}$$

Here we only used the leading, first term on the r.h. side of (A.9). Note that the g_sM^2 dependence of our the uplift potential agrees with the results of the supergravity-based approach of [48, 57, 58]. At the moment, we do not understand why this agreement occurs in spite of the absence of a fiducial volume $\mathcal{V}_{s,0}$ in these references. In our approach, $\mathcal{V}_{s,0}$ contributes essentially to the final result.

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Chapter 4

Curvature corrections to KPV: do we need deep throats?

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Curvature corrections to KPV: do we need deep throats?

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Abstract: We consider α'^2 curvature corrections to the action of an NS5-brane which plays the key role in the metastability analysis of warped anti-D3-brane uplifts by Kachru, Pearson and Verlinde (KPV). Such corrections can dramatically alter the KPV analysis. We find that for the α'^2-corrections to be sufficiently small to recover essentially the leading-order KPV potential one needs a surprisingly large S^3 radius, corresponding to $g_s M > 20$. In the context of the Large Volume Scenario (LVS) this implies a D3-tadpole of at least $\mathcal{O}(10^3$–$10^4)$. However, large α'^2-corrections do not necessarily spoil the uplift in KPV. Rather, as the curvature corrections lower the tension of the brane, a novel uplifting mechanism suggests itself where the smallness of the uplift is achieved by a tuning of curvature corrections. A key underlying assumption is the existence of a dense discretuum of g_s. This new mechanism does not require a deep warped throat, thereby sidestepping the main difficulty in uplifting KKLT and LVS. However, all of the above has to be treated as a preliminary exploration of possibilities since, at the moment, not all relevant corrections at the order α'^2 are known.

Keywords: String and Brane Phenomenology, Superstring Vacua

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1 Introduction

The set-up of Kachru, Pearson and Verlinde (KPV) [1] is one of the leading proposals for controlled, spontaneous SUSY breaking in string theory. In KPV, a set of anti-D3-branes at the tip of a Klebanov-Strassler throat polarize into a fluxed NS5-brane, which may then decay to restore SUSY or remain metastable[1] at finite radius, ideally in a regime controlled in 10d supergravity.

Our goal is to study α'^2 curvature corrections to the NS5-brane of KPV, which is clearly essential to control metastability. We will show that such corrections can significantly alter the KPV story. To maintain the validity of the leading-order KPV analysis, a strong lower bound on the radius $\sim \sqrt{g_s M \alpha'}$ of the tip of the throat has to be imposed. Here g_s is the string coupling and M counts the units of 3-form flux on the 3-cycle in the throat. If $g_s M$ is small, violating the above bound, α'^2-corrections significantly modify the story of KPV and the possibility of a novel uplifting mechanism suggests itself.

We came to consider α'^2 corrections in the KPV set-up by first thinking about corrections to the anti-brane uplift, as an essential ingredient in the KKLT [4] and Large Volume Scenario (LVS) [5, 6]. We will start by reviewing this simpler anti-D3-brane setup as a motivation and to provide a simplified version of the more complete NS5-brane story which we develop subsequently.

[1] The corresponding tunneling transitions are key in defining the lifetime of the non-SUSY vacuum [2, 3].

In recent years, concerns have been raised about the existence of de Sitter vacua in string theory (see e.g. [7, 8]). In particular, a series of recent papers [9–11] has scrutinized the conditions required to achieve a de Sitter vacuum in the LVS using an anti-D3-brane uplift.

An effect which was not considered in [10] but was crucial to [9, 11] derives from the α'^2 curvature corrections to the action of the anti-D3-brane responsible for the uplift. They lead to the anti-D3-induced potential[2]

$$V_{\overline{D3}} = \mu_3 e^{-\phi}\left[1 - \frac{(4\pi^2\alpha')^2}{384\pi^2} R_{a\alpha b}{}^{\alpha} R^{a}_{\ \beta}{}^{b\beta}\right] = \mu_3 e^{-\phi}\left[1 - \frac{c}{(g_s M)^2}\right], \tag{1.1}$$

where a, b are normal indices and α, β are tangent indices relative to the brane. References [9, 11] demanded that the anti-D3-curvature corrections should be small compared to the leading order DBI action. In our opinion, (1.1) does not necessarily imply such a strong constraint, as we now explain:.[3]

Note first that the curvature correction appears in the 4D scalar potential as a term which has the same dependence on the Kahler moduli as the tree-level tension. This curvature correction then does not alter the basic set-up of the LVS uplift. One can absorb the curvature correction into a redefinition of the anti-D3-brane tension, effectively lowering the tension.

As a result, having a curvature correction of the same order of magnitude as the tree-level tension is not dangerous to the LVS. In fact, it is helpful as the entire purpose of the warped throat is anyway to lower the effective tension of the anti-D3-brane. Reducing the tension through curvature corrections by an $\mathcal{O}(1)$ factor will then slightly reduce the amount of warping needed at the tip of the throat to achieve a de Sitter uplift rather than a runaway.

The curvature correction will only become dangerous when $(g_s M)^2 < c$ since, in this case, the brane gives a negative contribution to the potential and is unable to provide an uplift.

Very intriguingly, if the tension can become negative in this manner, this opens up the prospect for a new uplifting mechanism: we know that for very large $g_s M$ the anti-brane tension is positive and essentially uncorrected by curvature terms. Let us assume that, for small $g_s M$, the anti-brane tension turns negative. Now, given a sufficiently dense discretuum of the parameter $g_s M$, which is likely due to the flux dependence of g_s, it should be possible to realize a positive anti-brane tension which is tuned to be hierarchically smaller than its flat-space value. One may then use this mechanism to tune the uplifting term in the potential to achieve a metastable vacuum rather than a runaway. There is no need to rely on a deep warped throat. This mechanism could be used in KKLT or LVS. Crucially, the

[2] We believe that c should be 3×1.9747 rather than 1.9747 as in [9, 11]. We obtain 1.9747 when setting the first derivative of the warp factor $h(\tau)$ to zero after computing the Riemann tensor. However, we obtain three times this result when setting the first derivative of the warp factor to zero only *after* the scalar curvature-squared expressions have been computed. The difference arises as the scalar expressions contain terms $h'(\tau)/\tau$, which give a finite contribution at the tip, $\tau \to 0$. By contrast, these terms are lost when setting $h'(\tau) = 0$ too early. A more detailed discussion of curvature computations follows in section 2 and appendix A.

[3] Of course, curvature corrections may affect the compactification through effects independent of the modified D3-tension in (1.1). We comment on such effects in section 2.5, but much more work is needed if one really wants to gain confidence in a regime where the curvature is partially high.

strong constraints associated with the large D3-tadpole contribution coming from a deep warped throat are avoided.

One may object to this picture: if the α'^2-corrections are sufficiently large to make the anti-brane tension negative, surely one should also consider e.g. α'^4-corrections and these might contribute positively to the tension. It is then conceivable that, with all corrections summed up, the anti-brane tension is positive for all g_sM. While this may be hard to determine, it is clear that there are two logical possibilities:

First, with all corrections taken into account the anti-D3-brane tension may always remain positive. Corrections shifting the anti-brane tension as in (1.1) are then never a cause for concern.

Second, the anti-brane tension may become negative for some value of g_sM. Corrections like those in (1.1) are then dangerous and one should demand that they remain sufficiently small. However, it is sufficient to require that the corrections are small enough for the anti-brane tension to remain positive, it is not necessary to demand that they are parametrically small as done in [9, 11]. Moreover, in this case an alternate uplifting mechanism emerges which does not rely on the deep warped throat but instead tunes the anti-brane potential to be exponentially small because of curvature corrections.

All this being said, the anti-D3-brane does have decay channels. Their existence threatens the optimistic story just developed. The best-understood decay channel arises because p anti-D3-branes puff up into an NS5-brane with p units of 2-form flux, as described by KPV [1]. This NS5-brane configuration represents a metastable local minimum for $p/M < 0.08$. If, on the contrary, $p/M > 0.08$, the classical decay corresponding to the annihilation of the anti-D3-branes against the background flux in the throat becomes possible. Clearly, to establish or disprove our optimistic anti-D3-story sketched earlier, it is necessary lift the discussion of α'^2 corrections to the technical level of KPV and hence to curvature corrections for the NS5-brane. This is the goal of our paper.

To see more clearly why α'^2 corrections are crucial, let us recall some well-known parametric estimates for KPV and the anti-D3-uplift: the warp factor of the throat is $\sim \exp(-8\pi N/3g_sM^2)$, where N is the contribution of the throat to the total D3 tadpole. Hence, g_sM^2 is a key phenomenological quantity and it may be constrained as follows: first, $p/M < 0.08$ together with $p \geq 1$ imply $M \geq 12$. Second, for supergravity control the radius of the S^3 at the tip of the throat, which scales like $R_{S^3} \sim \sqrt{g_sM}$, should be large. Implementing this through the condition $g_sM \gg 1$ one finds $g_sM^2 \gtrsim 12$. However, this argument is dubious in that for hierarchical supergravity control one must demand $g_sM \gg 1$, not just $g_sM \gtrsim 1$. If one wishes to use a bound of the form $g_sM \gtrsim \mathcal{O}(1)$, one must compute the actual numerical coefficient for this bound and this is where α'^2 corrections become important. Depending on the precise coefficients, the final bound can become either significantly stronger or weaker than $g_sM^2 \gtrsim 12$. Jumping ahead, let us quote some of our results: the conservative requirement of a metastable NS5 configuration in the curvature-corrected analysis suggests $g_sM \gtrsim 20$. It turns out that in this situation the choice $p = 2$ is optimal, which together with $g_s < 1$ and $p/M < 0.08$ implies $M \geq 25$ and $g_sM^2 \gtrsim 500$. As we shall see, the actual impact of α'^2 corrections is more intricate, but the above should suffice to provide a feeling for their importance.

Most optimistically, one may only demand that the barrier preventing the decay of the curvature-corrected NS5 is at positive potential. This leads to $g_s M \gtrsim 4$ and hence, using also $p = 1$, $p/M = 0.08$ and $M = 12$, it implies $g_s M^2 \gtrsim 48$.

While the leading-order KPV analysis suggests $g_s M^2 > 12$, it has been proposed that avoiding a conifold instability for an anti-brane in the Klebanov-Strassler throat requires $g_s M^2 > 46$ [12–16]. Recently, the existence of this instability has been questioned [17]. However, in light of our results it appears that the curvature-corrected KPV stability bound is much stronger than previously thought, possibly dominating any potential conifold instability issue.

Instead of considering the Klebanov-Strassler throat for the uplift one may consider the S-dual set-up [18] where K units of H_3-flux are present on the S^3 cycle in the throat. Now the anti-D3-branes puff up into a D5-brane at the tip.[4] We expect our analysis to go through analogously in this set-up, with the difference that in Klebanov-Strassler the radius of the tip is set by $\sqrt{g_s M}$ while in the S-dual the radius is set by $\sqrt{K}$. We will consider KPV rather than its S-dual as this is the more commonly discussed set-up, but the S-dual has the advantage that we are more certain about curvature corrections.

Finally, it is a key question whether our novel fine-tuned uplifting mechanism will still work, even after lifting the simplified anti-D3 curvature correction analysis to the level of the NS5-brane. A sufficient condition would be that the maximum of the KPV potential is lowered to zero value while remaining in the calculationally controlled regime. Our analysis of expected α'^2 terms does indeed show such a behaviour of the potential barrier. But this is clearly insufficient to claim success. From the perspective the curvature-corrected NS5-brane analysis, the novel fine-tuned uplifing mechanism remains only an intriguing possibility. To establish this proposal, much more work is required.

An essential shortcoming of our analysis is that we have only taken into account the pure curvature corrections, which are known for the D5-brane case and which we have adapted to the NS5-brane. We expect further corrections involving flux which may be equally important and which are expected to change crucial numerical coefficients in our results. While we believe that our parametric results in terms of these numerical coefficients are robust, a complete analysis with all terms has to be performed in future work.

2 Curvature corrections to KPV

2.1 Reviewing KPV

We start by briefly reviewing the set-up of KPV [1]. For this and the computation of the curvature corrections to the NS5-brane, we consider the Klebanov-Strassler throat with metric [19, 20]

$$\mathrm{d}s^2 = h^{-1/2}(\tau)\mathrm{d}x_\mu \mathrm{d}x^\mu + h^{1/2}(\tau)\mathrm{d}s_6^2\,, \tag{2.1}$$

[4] We thank Thomas Van Riet for making us aware of this interesting set-up.

where $\mathrm{d}x_\mu \mathrm{d}x^\mu$ is the metric of 4D Minkowski space and $\mathrm{d}s_6^2$ the metric of the deformed conifold:

$$\mathrm{d}s_6^2=\frac{K(\tau)}{2}\left[\frac{\mathrm{d}\tau^2+(g_5)^2}{3K^3(\tau)}+\cosh^2(\tau/2)((g_3)^2+(g_4)^2)+\sinh^2(\tau/2)((g_1)^2+(g_2)^2)\right]. \quad (2.2)$$

Here, the one-forms $g^{1\ldots5}$ (see e.g. [20]) parametrize the five directions of the $T^{1,1}$, which can be viewed as an S^2 fibration over S^3, and

$$K(\tau)=\frac{(\sinh(2\tau)-2\tau)^{1/3}}{2^{1/3}\sinh(\tau)}\,. \quad (2.3)$$

The warp factor is given by

$$h(\tau)=(g_sM\alpha')\,2^{2/3}I(\tau)\,, \qquad I(\tau)=\int_\tau^\infty \mathrm{d}x\,\frac{x\coth(x)-1}{\sinh^2(x)}\,(\sinh(2x)-2x)^{1/3}\,. \quad (2.4)$$

Here M counts the units of F_3-flux on the A-cycle of the KS throat.

If p anti-D3-branes are placed at the tip of this throat, they can puff up into an NS5-brane with p units of worldvolume flux. This NS5-brane wraps an S^2 inside the S^3 whose metric can be written as

$$R_{S^3}^2\,\mathrm{d}\Omega_3^2=R_{S^3}^2\left(\mathrm{d}\psi^2+\sin^2(\psi)\,\mathrm{d}\Omega_2^2\right)\,, \quad (2.5)$$

where[5] $R_{S^3}=b_0\sqrt{g_sM}$ and $b_0^2=2a_0^{1/2}/6^{1/3}\approx 0.93266$ with $a_0=I(0)\approx 0.71805$.

As realized by KPV [1], this opens up a potential decay channel: the NS5 can slip over the equator, with its anti-D3 charge annihilating against 3-form flux. Eventually, the NS5 turns into $M-p$ D3-branes and supersymmetry is restored. The key quantity determining whether this decay process can take place is p/M: for small enough values of p/M, a meta-stable minimum below the equator exists.

KPV quantify this by deriving the potential $V(\psi)$ of the NS5 at leading order in α' (cf. figure 1):

$$V_{\mathrm{KPV}}(\psi)=\frac{4\pi^2p\mu_5}{g_s}+\frac{4\pi\mu_5M}{g_s}\left(\sqrt{b_0^4\sin^4(\psi)+\left(p\frac{\pi}{M}-\psi+\frac{1}{2}\sin(2\psi)\right)^2}-\psi+\frac{1}{2}\sin(2\psi)\right). \quad (2.6)$$

In the small ψ expansion of (2.6), the meta-stable minimum sits at $\psi_{\mathrm{min}}\approx 2\pi p/(b_0^4M)$. It disappears when $p/M\gtrsim 0.08\equiv(p/M)_*$. At this value, the minimum and the maximum merge to form an inflection point.

The radius of the NS5 at the minimum is given by $R_{\mathrm{NS5}}\approx 2\pi(p/M)\sqrt{g_sM}/b_0^3$. By increasing g_sM at fixed value of p/M, this radius can be made large in string units. Note, however, that this is not the same as making M large while keeping p fixed: in this limit, R_{NS5} approaches the string length such that the solution can not be trusted.

[5] We work in units where $\alpha'=1$ except where we explicitly display α' for clarity.

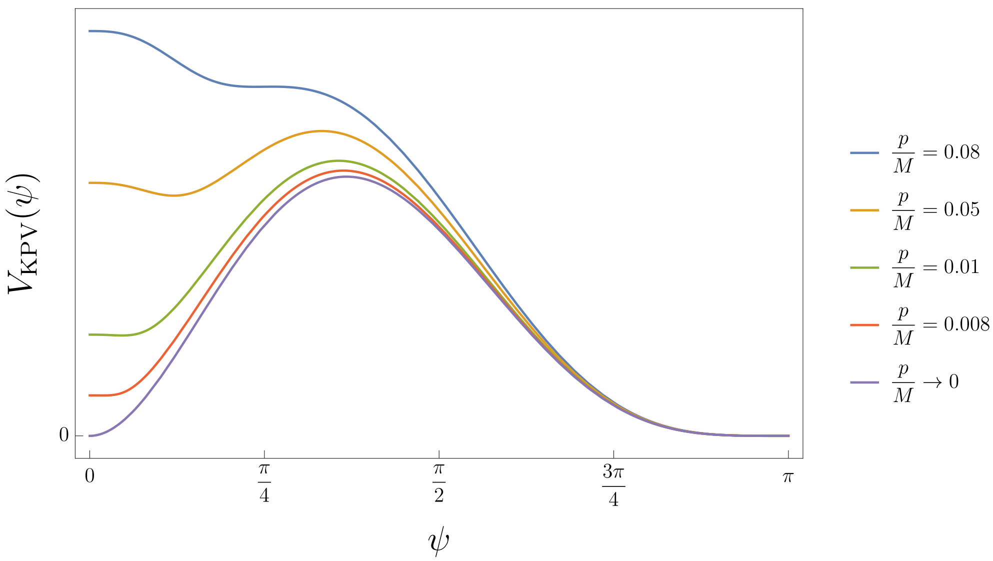

Figure 1. The potential $V_{\mathrm{KPV}}(\psi)$ (suitably normalized) for different values of p/M.

Let us make two comments concerning this regime of small p/M at large S^3 radius $\sim \sqrt{g_s M}$: first, this regime is of great interest since, if one uses the KS throat to uplift an AdS vacuum of type IIB supergravity, the $\overline{\mathrm{D3}}$ tension is warped down by $\exp(-8\pi N/3g_s M^2)$. Here N is the total D3 charge of the throat. Hence, to have large warping without excessive N, one wants M not to become too large. As a result, one wants p to be as small as possible.

Second, this regime is under partial control even though the NS5 radius at the minimum is small. Namely, the maximum of $V_{\mathrm{KPV}}(\psi)$ still corresponds to large NS5 radii and is hence trustworthy. In fact, as can be seen in figure 1, independently of the value of p/M, the position of the maximum is quite stable and only increases slightly as p/M decreases. The minimal value of the maximum $\psi^*_{\max}$ (corresponding also to the inflection point) can be approximated by $\psi^*_{\max} = \psi_{\min}|_{(p/M)_*} \approx 0.58$. For $p/M \to \infty$ the position of the maximum approaches $\psi_{\max} \approx 1.154$. Thus, we can be sure of the maximum and hence of the existence of a minimum to the left of it, even if the minimum itself can not be controlled.

Since our discussion will remain at the probe level, we briefly comment on the regime of validity of the probe approximation: at the very least, the radius R_{back} of the gravitational backreaction of any brane we consider should be small compared to the S^3 radius $\sim \sqrt{g_s M \alpha'}$. The condition for the metric deformation near a k-dimensional brane (a '$(k-1)$-brane') in d dimensions to be small reads $T_k/M_{10}^{d-2} R^{d-k-2} \ll \mathcal{O}(1)$, where T_k is the tension, $1/M_{10}^{d-2}$ the coupling constant, and R the distance to the brane. Solving this for R in the case of p anti-D3-branes gives $R^2_{\mathrm{back}} \sim \sqrt{p g_s}\alpha'$ and hence the condition for backreaction to be small reads

$$\sqrt{p g_s} \ll g_s M \qquad \text{or} \qquad \frac{p}{M} \ll g_s M\,. \tag{2.7}$$

This can be compatible with large p/M. Moreover, one can perform the same analysis for the backreaction of an NS5-brane with p units of worldvolume gauge flux. One reads off from (2.6) that the tension of such a fluxed NS5 at $\psi \sim \mathcal{O}(1)$ and $p \gg M$ is $\sim (p/g_s^2 M)\alpha'^{-3}$. This leads to the same bound (2.7) on the ratio p/M.

2.2 Curvature corrections

The KPV analysis of section 2.1 can be substantially affected by α' corrections if the curvature at the tip of the throat is not very small. As discussed below, such corrections are suppressed by $1/R^4_{S^3} \sim 1/(g_s M)^2$, which for phenomenological reasons can not be taken to zero with arbitrary precision.

Let us start by recalling the curvature corrections to the string-frame DBI action of a Dp-brane [21, 22]. For a constant dilaton, the Dp-brane DBI action with the α'^2 curvature corrections included at leading order in g_s reads

$$\begin{aligned} S_{\text{curv, D}p} = -\frac{\mu_p}{g_s}\int_{\mathcal{M}_{p+1}} \mathrm{d}^{p+1}\xi\,\sqrt{-\det\left(g_{\mu\nu}+\mathcal{F}_{\mu\nu}\right)}\Bigg(1-\frac{(2\pi)^4\alpha'^2}{24\cdot 32\pi^2}\Big[(R_T)_{\alpha\beta\gamma\delta}(R_T)^{\alpha\beta\gamma\delta} \\ -2(R_T)_{\alpha\beta}(R_T)^{\alpha\beta}-(R_N)_{\gamma\delta ab}(R_N)^{\gamma\delta ab}+2\overline{R}_{ab}\overline{R}^{ab}\Big]\Bigg)\,, \end{aligned} \tag{2.8}$$

where μ_p is the brane tension, and $\mathcal{F}_{\mu\nu} = B_{\mu\nu} + 2\pi\alpha' F_{\mu\nu}$, with F the worldvolume field strength. The detailed definitions of the various curvature-squared terms and their indices are given in appendix A.

We could now consider S-dualized Klebanov-Strassler (SDKS) [18], a warped throat geometry in IIB string theory where one has K units of H_3-flux on the S^3 which remains of finite size at the tip of the throat. The p anti-D3-branes can then puff up into a D5-brane wrapping an S^2 on the S^3 at the tip of the throat, entirely analogously to how in KPV the anti-D3-branes puff up into an NS5. We shall instead discuss our results in terms of the KPV set-up in the KS throat. As we will see, we expect the leading curvature corrections to be of the same form and at the same order in g_s compared to the leading term in the DBI action for both the D5-brane and the NS5-brane. We then expect our analysis to be analogous for either the NS5-brane in KPV or the D5-brane in the S-dual set-up, with the difference that in the Klebanov-Strassler throat the tip radius is set by $\sqrt{g_s M}$ while in the S-dual it is set by $\sqrt{K}$.

What are the curvature corrections on the NS5-brane? One may obtain curvature corrections by S-dualizing the D5-brane action from (2.8), i.e. replacing $g_s \to 1/g_s$. This yields, after Weyl rescaling $g_{\mu\nu} \to g_{\mu\nu}/g_s$,

$$\begin{aligned} &S_{\text{curv, NS5 S-duality}} \\ &\quad= -\frac{\mu_5}{g_s^2}\int_{\mathcal{M}_6}\mathrm{d}^6\xi\,\sqrt{-\det\left(g_{\mu\nu}+2\pi g_s F_{2\,\mu\nu}-g_s C_{2\,\mu\nu}\right)}\Bigg(1-g_s^2\frac{(2\pi)^4\alpha'^2}{24\cdot 32\pi^2} \\ &\qquad\times\Big[(R_T)_{\alpha\beta\gamma\delta}(R_T)^{\alpha\beta\gamma\delta}-2(R_T)_{\alpha\beta}(R_T)^{\alpha\beta}-(R_N)_{\gamma\delta ab}(R_N)^{\gamma\delta ab}+2\overline{R}_{ab}\overline{R}^{ab}\Big]\Bigg)\,. \end{aligned} \tag{2.9}$$

We see that these curvature corrections are g_s^2 suppressed compared to the leading-order DBI action, while the D-brane curvature terms were leading order in g_s. One expects such

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g_s^2 corrections for gravitons from loop corrections in the worldvolume field theory on the NS5-brane [23]. By contrast, the curvature corrections on the D5-brane arise at the disk level and are enhanced by a factor $1/g_s$ compared to 1-loop expectations. This leads to a conundrum, as we explain in the two following paragraphs:

Consider a fivebrane (either D5 or NS5) with geometry $\mathbb{R}^{1,3} \times S^2$ and p units of worldvolume flux on the 2-sphere. One can consider this brane either in flat 10D space or in the KS throat with the NS5-brane wrapping an S^2 in the A-cycle or the D5-brane wrapping an S^2 in the B-cycle. The analysis is the same in both scenarios as the curvature of the background becomes negligible relative to that of the S^2 once the latter is taken to be small. Now consider shrinking the S^2 to zero size. The resulting object is a threebrane with the same quantum numbers as p (anti)-D3-branes. We then assume in line with KPV that this object must be a stack of p anti-D3-branes.

Next, consider our fivebrane action as we shrink the S^2 radius R_{S^2} to zero size. We look first at the DBI action at leading order in α'. As $R_{S^2} \to 0$, one has $\int_{S^2} \sqrt{g - B_2} \to 0$ and $\int_{S^2} \sqrt{g - g_s C_2} \to 0$, while one always has $(2\pi/g_s) \int_{S^2} F_2 = 4\pi^2 p/g_s$ for the D5 and $(2\pi/g_s^2) \int_{S^2} g_s F_2 = 4\pi^2 p/g_s$ for the NS5-brane. We then see that for both fivebranes this limit reproduces the leading order action of anti-D3-branes, (2.8). Now consider the α'^2 terms on the fivebranes as one sends $R_{S^2} \to 0$. Clearly for the D5-brane, (2.8), one reproduces the g_s scaling of the R^2 terms for the anti-D3-branes. That the precise index structure for the R^2 terms of the D5 is such that it matches onto the result for a stack of p anti-D3-branes is something we assume for consistency of the theory. We will comment more on this in section 2.6. However, when we perform the $R_{S^2} \to 0$ analysis for the NS5-brane, (2.9), we see that the resulting α'^2 action is g_s^2 suppressed compared to the result required for the anti-D3-branes. Given that by assumption we must reproduce p anti-D3-branes from the NS5-brane when sending $R_{S^2} \to 0$, we conclude that the NS5-brane action must also have R^2 terms at leading order in g_s. Which index structure should this leading term have? Since we asserted that for the D5-brane this index structure should be given by the α'^2 term of (2.8) and the limit $R_{S^2} \to 0$ operates entirely analogously for the NS5 and D5-brane, we propose that in order to obtain the correct anti-D3-brane limit the leading order in g_s curvature correction for the NS5-brane should have the same index structure as that for the D5-brane. We then have

$$\begin{aligned} &S_{\text{curv, NS5}} \\ &\quad = -\frac{\mu_5}{g_s^2} \int_{\mathcal{M}_6} \mathrm{d}^6\xi \sqrt{-\det\left(g_{\mu\nu} + 2\pi g_s F_{2\,\mu\nu} - g_s C_{2\,\mu\nu}\right)} \left(1 - \frac{(2\pi)^4 \alpha'^2}{24 \cdot 32\pi^2}(1 + g_s^2)\right. \\ &\qquad \left. \times \left[(R_T)_{\alpha\beta\gamma\delta}(R_T)^{\alpha\beta\gamma\delta} - 2(R_T)_{\alpha\beta}(R_T)^{\alpha\beta} - (R_N)_{\gamma\delta ab}(R_N)^{\gamma\delta ab} + 2\overline{R}_{ab}\overline{R}^{ab}\right] + \mathcal{O}(g_s)\right). \end{aligned} \tag{2.10}$$

In what follows we shall only consider this NS5-brane action at leading order in g_s. Note that after S-duality this leading order term should contribute to the (unknown) g_s^2 suppressed, 2-loop term for the D5-brane. Such a term is indeed expected to arise for gravitons from field theory loops in an effective field theory on the D5-brane worldvolume [23], analogously to how we also expected such g_s^2 suppressed corrections to arise on the NS5-brane worldvolume. This provides a further argument that such a leading order in g_s curvature term on the NS5-brane should exist. We finally note that a 1-loop R^2 term is also expected to be present on the D5. It would dualize to an $\mathcal{O}(g_s)$ R^2 on the NS5, as displayed at the very end of (2.10) above.

2.3 Curvature-corrected KPV potential and conservative bounds on uplifts

To find the contribution of the NS5-brane to the 4d potential, we evaluate the curvature correction in (2.10) as explained in more detail in appendix A and use the values $I(0) = a_0$, $I'(0) = 0$, and $I''(0) = -2^{2/3}/3^{4/3}$. Keeping only the leading order in g_s, one then has to integrate these terms over the S^2 wrapped by the NS5 which yields (this integral is also calculated in [1])

$$\begin{aligned} V_{\text{curv}}(\psi) &= -\frac{1}{(g_sM)^2}\left(c_1 + 2c_2\cot^2\psi + c_2\cot^4\psi\right)\frac{\mu_5}{g_s^2}\int_{S^2}\sqrt{g+2\pi g_s\mathcal{F}} \\ &= -\frac{1}{(g_sM)^2}\left(c_1 + 2c_2\cot^2\psi + c_2\cot^4\psi\right)\frac{4\pi\mu_5 M}{g_s} \\ &\quad\times\sqrt{b_0^4\sin^4(\psi) + \left(p\frac{\pi}{M} - \psi + \frac{1}{2}\sin(2\psi)\right)^2}\,, \end{aligned} \tag{2.11}$$

where $c_1 \approx 8.825$ and $c_2 \approx 1.891$. The analysis of KPV should now be redone including (2.11), i.e. by considering

$$V_{\text{tot}}(\psi) = V_{\text{KPV}}(\psi) + V_{\text{curv}}(\psi)\,. \tag{2.12}$$

The task is to determine how the KPV bound $g_sM^2 > 12$ (which follows from $p/M \geq 0.08$ together with $g_sM > 1$) is modified.

In the analysis of KPV, the only parameter determining whether a metastable minimum exists is the combination p/M. By including the correction (2.11), there are now two parameters which determine the shape of the potential $V_{\text{tot}}(\psi)$: p/M and g_sM. In addition, $V_{\text{tot}}(\psi)$ contains an overall factor $1/g_s^2$ which sets the overall magnitude of the potential but does not affect its shape.[6]

The potential $V_{\text{tot}}(\psi)$ is plotted for some sample values of p/M and g_sM in figure 2. Besides the fact that the original and corrected KPV potential may or may not have a metastable minimum, the corrected potential has a very different appearance when compared to the original KPV result in figure 1. The most notable difference to KPV is that $V_{\text{tot}}(\psi)$ always has a maximum and always diverges to $-\infty$ as $\psi \to 0, \pi$. These features are due to the extrinsic curvature corrections, which are dominant when the S^2 wrapped by the NS5-brane shrinks to a small sphere as they introduce terms $\sim \cot^2\psi$ and $\sim \cot^4\psi$ in the potential. Not to overrate these features, it is crucial to be aware of the limitations of our analysis.

[6]Interestingly, the corrections to KPV due to conifold dynamics also only depends only on p/M and g_sM [16], suggesting that there may be a convenient way to deal with corrections to KPV due to conifold dynamics and due to curvature simultaneously. We thank Vincent Van Hemelryck for making us aware of this point.

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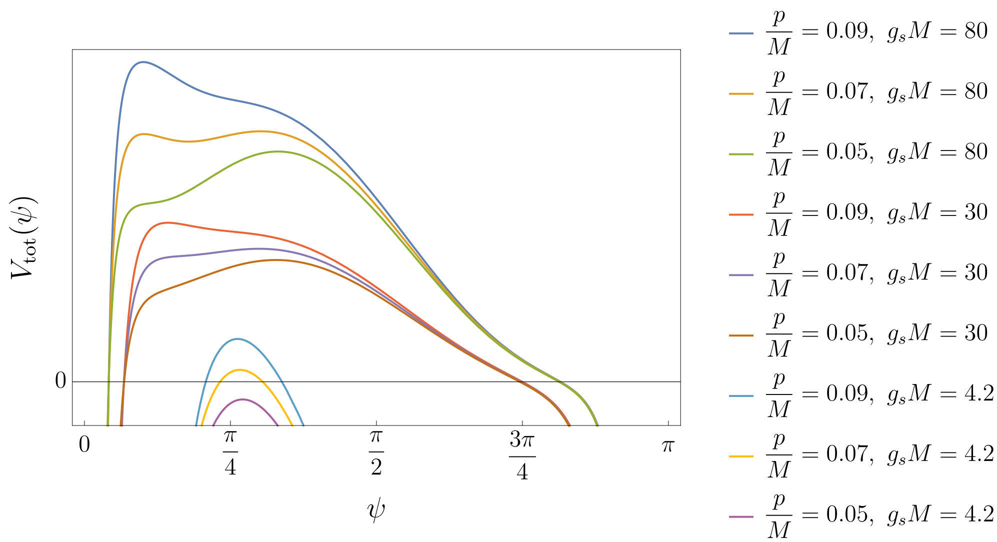


Figure 2. The potential $V_{\text{tot}} = V_{\text{KPV}}(\psi) + V_{\text{curv}}(\psi)$ for different values of p/M and g_sM in units $\mu_5 = 1$. For each set of values of g_sM, g_s is chosen such that the potential is suitably normalized.

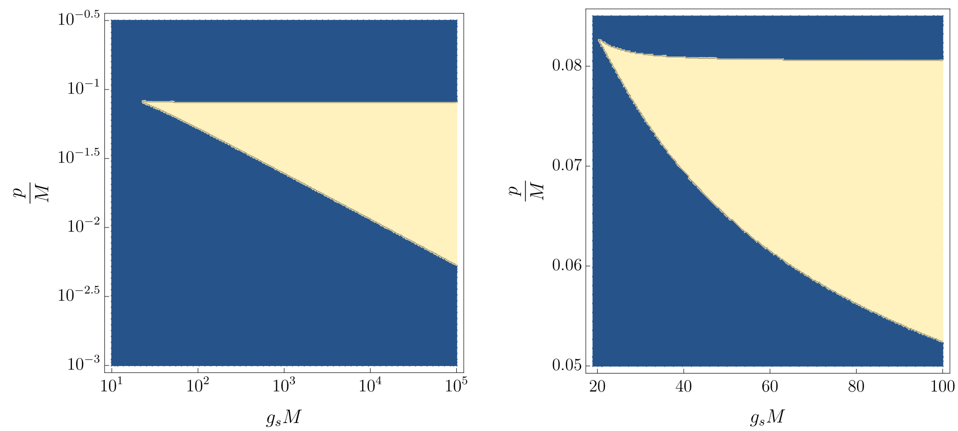
We have only considered α'^2 corrections. As the NS5-brane shrinks, corrections at higher orders in α' become increasingly important. Because of this, we can only trust our analysis in the central region of the potential. Further, even at order α'^2 we have only accounted for those pure curvature corrections which we know, ignoring completely the effect of flux at order α'^2. We expect that these additional corrections prevent any divergent behaviour in the potential. We also assume that, after taking all corrections at all orders in α' into account the potential at $\psi = 0$ matches the (all-orders corrected) potential for p nonabelian anti-D3-branes. Similarly, the potential at $\psi = \pi$ should match the (all-orders corrected) potential for $M - p$ nonabelian D3-branes.[7]

At intermediate values of ψ, the all-orders-corrected potential may or may not feature a local maximum and metastable minimum depending on the parameters. Note that unlike for V_{KPV}, the potential at $\psi = \pi$ need not be zero. This is due to the α' terms in the action of the $M - p$ D3-branes left over in the throat after the p anti-branes have annihilated against flux.

In figure 3 we show for which ranges of p/M and g_sM the potential $V_{\text{tot}}(\psi)$ has a metastable minimum. The tip of this shark fin region is at $p/M = 0.0826$ and $g_sM = 20.363$. The upper horizontal bound is a slightly weakened version of the KPV bound $p/M < 0.08$. This weakening is expected since V_{curv} always possesses a maximum that facilitates the existence of the local KPV minimum at smaller values of ψ. The horizontal line asymptotes to agree with KPV precisely as g_sM grows.

The second, diagonal line bounding the shark fin from below corresponds to the α'^2 curvature corrections being significant enough to destroy the metastable minimum through

[7]When saying 'all orders' we mean including non-perturbative effects.



(a) (b)

Figure 3. The yellow area shows the region in the $(g_sM, p/M)$-parameter space where a meta-stable minimum exists. (a) is a log-log-plot for a wide range of parameters and (b) zooms into the region of smallest allowed values of g_sM.

a runway to $\psi = 0$.[8] Crucially, the loss of a metastable minimum at order α'^2 does in this case not imply that the fully corrected potential has no metastable minimum. It is just that this minimum may be outside the controlled regime. This may also be the reason why the bound on g_sM is very large, $g_sM > 20$. For the metastable minimum at $\psi_{\min} \sim \sqrt{g_sM}p/M$ to appear in the controlled regime, it needs to be pushed to large values of ψ which requires large g_sM. In view of the uplift to dS this is problematic, which can be seen as follows: combined with the perturbativity condition $g_s < 1$, the bound $g_sM > 20$ implies $M > 20$. For $p = 1$, i.e. $p/M > 0.05$, this forces us to even higher g_sM, as can be clearly seen from figure 3. In fact, to stay within the shark fin region, one has to use (at least) $p = 2$, leading to $M = 25$. This appears to be the optimal choice under the above constraints. It implies $g_sM^2 \gtrsim 500$, which requires an enormous tadpole in the LVS as we shall see in section 3.

Note that from this analysis of figure 3, in order for a metastable minimum to exist in the controlled regime where $g_s < 1$ one requires at least $p \geq 2$ anti-D3-branes. Of course additional field strength corrections can alter this result, but assuming this does not occur, the following conclusion is tempting: if $p = 1$ the metastable minimum, if one exists, will be at small or zero ψ where the radius of the NS5-brane is string-scale and all orders in α' corrections are important. This object might be thought of as not really an NS5-brane but the single anti-D3-brane, at most smeared out over its Compton wavelength. A single anti-D3-brane then would not polarize into an NS5-brane. This is in line with the logic when one considers the perspective of the nonabelian DBI action of a stack of anti-D3-branes,

[8] In the interval $g_sM \in [20.363, 100]$, this part of the bound can be approximated by

$$\frac{p}{M}(g_sM) = 0.0313227 + \frac{2.73449}{g_sM} - \frac{72.7164}{(g_sM)^2} + \frac{1182.92}{(g_sM)^3} - \frac{8223.59}{(g_sM)^4} + \mathcal{O}\left((g_sM)^{-5}\right) .$$

where the nonabelianity (and hence the presence of more than one anti-brane) is crucial for the Myers effect [24] to occur.

2.4 Optimistic bounds on uplifts

We have seen that the α'^2-corrected potential (2.12) always has a maximum, but it has a minimum only in the controlled regime of large g_sM. However, for the purpose of constructing de Sitter vacua, the crucial question is whether our potential V_{tot} can provide an uplift. It is not necessary that the corresponding local minimum is in the controlled regime at large ψ. Indeed, let us assume that we have a local minimum at small ψ, capable of uplifting. The radius of the S^2 wrapped by the NS5-brane may then be too small to control the relevant V_{tot} against even higher α' corrections. A necessary but not sufficient condition for such an uplift to exist is the presence of a local maximum with a positive potential energy at larger ψ. We will therefore consider the following very blunt criterion: we demand that V_{tot} has to be positive at its maximum in order to have any chance of providing an uplift. Clearly when the potential is always negative it cannot uplift. As the maximum in the tree level potential V_{KPV} is always at a fairly central value of ψ it seems reasonable that for part of the parameter space this will also be true for V_{tot} and we can neglect even higher order corrections in α' so long as the radius of the tip of the warped throat is not very small in string units. Even then, our criterion can be questioned as we have not accounted for α'^2 flux corrections. Still, while the concrete constraints may change in a more complete analysis, our limited analysis should provide an estimate of the scale at which α'^2 corrections affect KPV.

The region in p/M and g_sM parameter space where the maximum is positive is shown in figure 4. One sees that there are two distinct asymptotic regimes: when $p/M \ll 0.1$ the bound asymptotes towards $g_sM > 5.238$ and for $p/M \gg 0.1$ towards $g_sM > 2.086$. When $p/M \sim 0.1$ there is an interpolating intermediate regime. This clearly suggests that the nature of the maximum of the potential is different in the two asymptotic regimes.

Let us first consider the regime $p/M \ll 0.1$. It is straightforward to evaluate $V_{\mathrm{tot}}(\psi)$ at $p/M = 0$. One can convince oneself that the potential has a maximum at $\psi = \mathcal{O}(1)$, where the NS5-brane radius is comparable to the radius of the A-cycle S^3. The potential drops sharply away from this maximum. Clearly, the maximum is trustworthy in this regime if the A-cycle radius is sufficiently large.

Now let us consider $p/M \gg 0.1$. We can expand $V_{\mathrm{tot}}(\psi)$ at large p/M and find

$$V_{\mathrm{tot}}(\psi) = \frac{4\pi^2}{g_s^3 M}\frac{p}{M}\left(2(g_sM)^2 - c_1 - 2c_2\cot^2(\psi) - c_2\cot^4(\psi)\right) + \mathcal{O}\left(\left(\frac{p}{M}\right)^0\right) . \tag{2.13}$$

We see that the leading p/M term in this expansion provides a table-shape potential which is essentially flat until ψ gets close enough to the north or south pole such that the $\cot^2(\psi)$ term begins to compete with the constant $\sim (g_sM)^2$ term. Near $\psi = 0$, this happens when $\psi \sim 1/g_sM$. As a result, the NS5-brane radius is $R_{\mathrm{NS5}} \sim \sqrt{g_sM}\sin(\psi) \sim 1/\sqrt{g_sM}$. We see that in this region the NS5-brane radius is stringy or sub-stringy in size and hence out of control. The $(p/M)^0$ correction adds a downward slope from $\psi = 0$ to $\psi = \pi$. As a result, at large p/M the potential is essentially flat in the entire controlled regime, with a slight

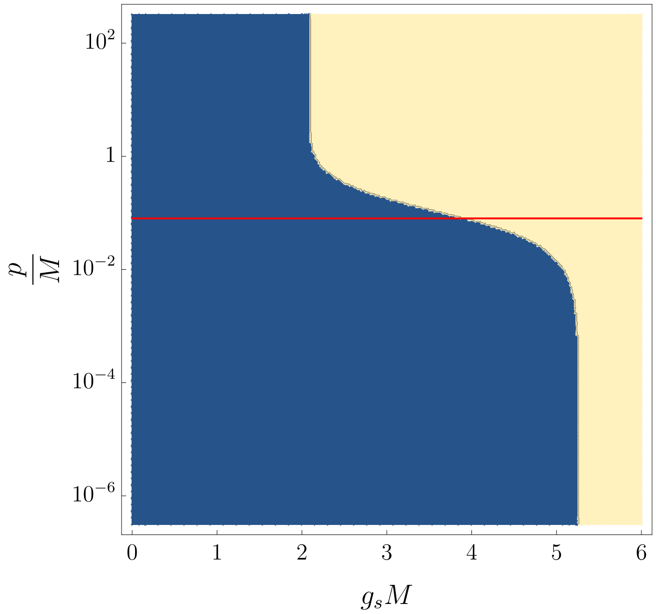


Figure 4. The space of curvature-corrected KPV setups, parameterized by g_sM and p/M. The regions where the maximum of the potential is at positive/negative value are coloured yellow/blue. For $p/M \ll 0.1$ the boundary line asymptotes towards $g_sM \to 5.238$ and for $p/M \gg 0.1$ towards $g_sM \to 2.086$. The red line is at $p/M = 0.08$, approximately above which the maximum is at stringy or substringy radii for the NS5-brane and hence not trustworthy.

downward slope. There is a maximum at the left end of the plateau, near $\psi = 0$, which corresponds to stringy or substringy radii for the NS5-brane and can hence not be trusted.[9]

Somewhere in the intermediate regime $p/M \sim 0.1$ we then lose confidence in our maximum. This transition in where the maximum occurs can already be seen in figure 2: as g_sM falls, the maximum coming from the curvature corrections grows, erasing the KPV local minimum. Moreover, we see that depending on whether we make p/M small or large, this maximum emerges at either larger or smaller ψ. To make this even more apparent, we have plotted the potential for a wider range of p/M in figure 5. Here we chose $g_sM = 15$, a value which is too small for a local minimum to exist.[10] We still clearly see that, at $p/M \approx 0.08$, a transition in the location of the maximum occurs. Once the maximum has shifted to the left, we lose trust in it since it is now very close to the region where the curvature corrections diverge.

We can then also display the regions where a controlled/uncontrolled maximum exists as well as the region with a local minimum of V_{tot} in a single plot, cf. figure 6. We recall that a controlled maximum implies the presence of a local, SUSY-breaking minimum at smaller ψ.

[9]We recall that according to (2.7) throughout this whole paragraph, even at $\psi \sim \mathcal{O}(1)$, the ratio p/M is restricted by $p/M \ll g_sM$.

[10]Note that according to (2.7) the probe approximation is only borderline valid for the blue curve. But as we are only trying to visualize the transition from small to large p/M, we disregard this issue.

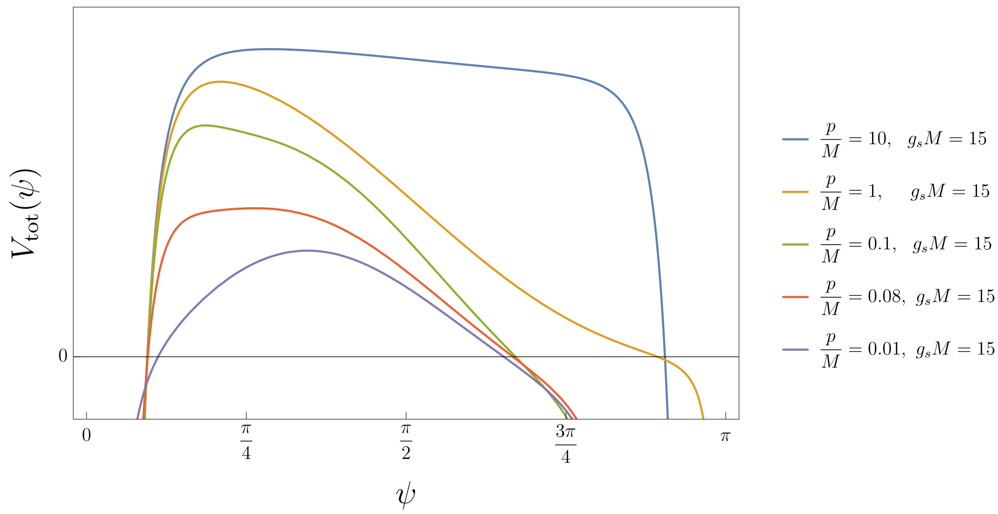


Figure 5. A plot of $V_{\rm tot}(\psi)$ for different p/M showing the transition between the large and small p/M regime. The potentials at different p/M have been rescaled by choosing g_s to make the comparison of their shape clearer.

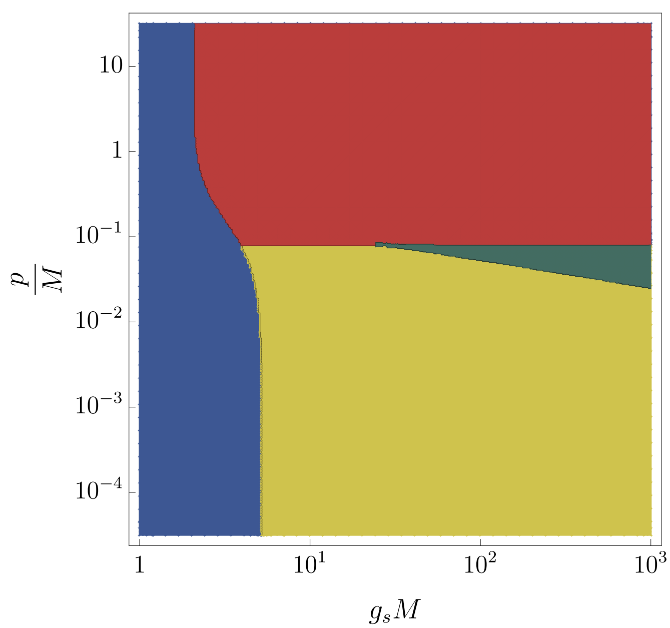


Figure 6. An exclusion plot of the existence of minima and maxima in $V_{\rm tot}(\psi)$. In the blue region the potential is negative everywhere. In the red region, a positive maximum exists, but it is in an uncontrolled regime. In the yellow region a positive maximum exists in a controlled regime. In the green region a metastable uplifting minimum exists.

2.5 Uplifting without deep throats?

When ψ is sufficiently small, $\psi \sim 1/\sqrt{g_s M}$, the NS5-brane radius $R_{\mathrm{NS5}} \sim \sqrt{g_s M}\sin\psi$ becomes of stringy size. Then even higher order α' corrections become important and we can no longer trust our potential $V_{\mathrm{tot}}(\psi)$. Despite this, we can still make some reasonable estimates of the fully corrected potential at small ψ. It is reasonable to assume that at $\psi = 0$ the potential matches the potential for p anti-D3-branes. Up to order α'^2 in curvature corrections this is given by

$$pV_{\overline{D3}} = p\mu_3 \mathrm{e}^{-\phi}\left[1 - \frac{(4\pi^2\alpha')^2}{384\pi^2} R_{\alpha a b}{}^{\alpha} R^{a}_{\ \beta}{}^{b\beta}\right] = p\mu_3 \mathrm{e}^{-\phi}\left[1 - \frac{c}{(g_s M)^2}\right], \tag{2.14}$$

with $c = 5.9241$. If the fully corrected potential has a maximum but no metastable minimum at intermediate ψ where $V_{\mathrm{tot}}(\psi)$ can be trusted and is set to a finite value at $\psi = 0$ given by (2.14) then the fully corrected potential must have a metastable minimum at ψ small (or 0). It then seems a reasonable estimate that the potential at this minimum is given roughly by (2.14) plus small corrections.[11]

It seems that as α'^2 curvature corrections contribute negatively to the potential, they can turn the potential negative. This allows for a novel but speculative alternative uplifting mechanism. Suppose that for some parameters the fully corrected potential has a metastable minimum with a positive vacuum energy, while for different parameters (with increased curvature corrections) there exists a metastable minimum with a negative vacuum energy. Then by tuning g_s it should be possible to obtain a metastable minimum with a positive but hierarchically small vacuum energy by using the tension-lowering effect of curvature corrections, without needing a large throat with hierarchical warping.

To obtain a hierarchically small uplift, one requires a finetuning of g_s. However, due to the flux dependence of g_s and the density of the string theory flux landscape we believe this finetuning to be no worse than e.g. the finetuning in W_0 required to get a cosmological constant of real world scale.

An issue in estimating exactly when this uplifting mechanism occurs is that when in (2.14) the α'^2 corrections are important enough to compete with the leading term, one expects even higher order α' corrections to also be important.

One may take this logic one step further and forget about our warped throat set-up: consider an AdS compactification with small cycles, e.g. the models of [26, 27], and place an anti-D3-brane. The anti-D3-brane will be drawn to the small cycle as this lowers their tension due to curvature corrections. If many models with many such small cycles exist, it may by chance happen that for some of them curvature corrections make the anti-brane tension positive but very small such that it is suitable for an uplift. Of course, all this is very speculative and it takes one to (and possibly beyond) the edge of a controlled regime.

[11]This assumes that the potential rolls down a small amount between $\psi = 0$ and the metastable minimum at small ψ as happens in the original KPV set-up shown in figure 1. While this seems reasonable, this need not be the case as we are ignorant of the effects of additional corrections. It is e.g. possible that at $p = 1$ there is a barrier between $\psi = 0$ and a metastable minimum at small ψ and this from the NS5-brane perspective prevents the anti-D3-brane from puffing up classically into an NS5-brane even though both states have the same quantum numbers and transitions between them should be possible, though now heavily suppressed due to quantum tunnelling, due to the no global symmetries conjecture as discussed in [25].

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Let us try to address some of the most immediate concerns which one might have with the control of compactifications relying on our new uplifting mechanism.

Using this novel uplifting mechanism in the LVS, one may worry that the necessary tuning of g_s will affect the LVS AdS minimum in such a way that $|V_{\mathrm{AdS}}| \approx |V_{\mathrm{uplift}}|$ can not be satisfied at large volume. However, we see from (1.1) that near $g_s \sim \sqrt{c}/M$ the potential $V_{\mathrm{uplift}}(g_s)$ is a growing function, changing sign at $\sqrt{c}/M$. By contrast, $V_{\mathrm{AdS}} \sim -g_s^{1/2}\mathrm{e}^{-3/g_s}$ asymptotes to zero at an exponential rate. This implies that the sum of these functions will always go through zero for some value of g_s^* just slightly above $\sqrt{c}/M$. Thus, the value of g_s^* can be lowered by choosing a larger M, such that a large CY volume appears to be easy to achieve.

Furthermore, one might worry that string-scale curvature at the tip implies large curvature also at the mouth of the throat, affecting the rest of the CY and leading to corrections to the Kahler potential. Parametrically, the radius at the tip is given by $R_{S^3} \sim \sqrt{g_s M}$ and the radius at the mouth by $R_{\mathrm{mouth}} \sim (g_s N + (g_s M)^2)^{1/4}$. Recalling that $N/g_s M^2$ is the exponent governing the warp factor, we have $R_{S^3}^4/R_{\mathrm{mouth}}^4 \approx g_s M^2/N \ll 1$ in the regime of strong warping. At moderate warping, the suppression of the curvature at the mouth is of course limited. Nevertheless, it is parametrically consistent to assume that only the curvature at the tip is string scale. This goes together the general expectation that localized effects in a CY decouple from the bulk.[12]

There is also the potential danger of loop corrections coming from fields propagating in the throat. Since the geometry near the tip is string scale, a 10d EFT analysis is not reliable. The leading closed-string loop diagram contributing to the potential of the anti-D3-brane is a torus with two vertex insertions, one coupling to the anti-D3-brane, the other corresponding to the graviton background. Correspondingly, at the open-string level there is the annulus with a graviton insertion. Compared to the tree level α' corrections, these diagrams are suppressed by g_s^2 and g_s. Hence, in the regime $g_s \ll 1$ and $g_s M \sim \mathcal{O}(1)$, loop corrections are smaller than the curvature corrections computed above.

Another potential concern is that, as the A-cycle shrinks, branes wrapped on it become light. In our context, such branes are relevant if they additionally fill out from zero to four of the non-compact dimensions. In type IIB, only D3- and D5/NS5-branes meet these criteria. They correspond to particles (cf. the discussion in [30]) and domain walls in 4d. Given a string-scale A-cycle, their mass or tension is small — set by the warped-down string scale. But they are in general not mass- or tensionless, so the EFT does not break down. Thus, while they may play an interesting phenomenological role, this does not directly affect the minimum of the potential and does not threaten the moduli stabilization scenario. A key exception are NS5s since the corresponding domain walls mediate the decay of the SUSY-breaking anti-D3 branes. This is, of course, the main subject of the present paper.

Finally, while the effects discussed in the last three paragraphs can all contribute terms to the Kahler potential which mix Kahler and complex-structure moduli, we do not expect this to be deadly for the proposed scenario. First, the warped-down string scale is higher than the Kahler-moduli mass scale, so complex structure moduli may be integrated out first.

[12] This sequestering is studied for the KS throat in [28, 29].

Second, since the KS throat does not contain local 2- or 4-cycles, its geometry measured in string units is insensitive to Kahler deformations of the CY. Thus, we expect α' corrections to the Kahler moduli Kahler metric not to be enhanced by the high curvature present in the throat region.

2.6 Open issues

In this paper we have given only an estimate of how α'^2-corrections affect the KPV uplift. Let us briefly outline what we believe are the main points which must be addressed in a research program to fully understand α'-corrections to KPV.

- We have assumed that the α'^2 curvature corrections for the NS5-brane in KPV are given by the natural generalization from the known curvature terms on the D5-brane. If instead we consider the S-dualized KS set-up [18], we can be certain that we have included all pure curvature corrections in our analysis. It is important to explicitly derive the α'^2 curvature terms for the NS5-brane to be sure that they exist and to compute their exact form. Some discussion of curvature terms on NS5-branes not in the DBI-action but related to anomaly cancellation has appeared in [31–34]. For D-branes, at order α'^2, it is known that besides the pure curvature corrections there also exist pure field strength and field strength-curvature mixing terms in the DBI action and some of these have been computed in [35–39]. One would expect that by S-duality analogous terms exist for the NS5-brane. It would be crucial to derive these and compute their impact on the KPV potential.

- Given the danger posed by the α'^2-corrections, one should convince oneself that at ψ where we claim to trust the α'^2 corrected potential $V_{\mathrm{tot}}(\psi)$, even higher order α' corrections are sufficiently small and can be neglected. Particularly worrying is that, when the number of anti-branes is one or a fixed small number, the leading order analysis of KPV seems to indicate that the metastable minimum is always such that the NS5-brane wraps a string-scale cycle, making it crucial to know the α'-corrections to all orders to know the value of the potential at the metastable minimum (and particularly whether this value is positive). Note that there is some indication that backreaction effects can push the minimum to larger ψ [40].

- Understanding the backreaction of the NS5-brane has played a crucial role in discussions of KPV [40–66]. Our discussion of α'^2 effects has remained at a probe level. It is crucial to understand the interplay between backreaction and higher-derivative effects.

- It would be valuable to understand α'^2-effects from the perspective of a stack of nonabelian anti-D3-branes. This calculation should provide an α'^2 corrected value of the position of the metastable minimum. Of particular interest is the following issue: as we saw, terms related to the extrinsic curvature play a crucial role in the NS5-brane potential. As the anti-D3-branes are pointlike in the internal dimensions, they have no extrinsic curvature. How then do NS5 extrinsic curvature effects manifest from the perspective of a stack of nonabelian anti-D3-branes? Further, even for the intrinsic curvature, the tensor structure of the terms for the NS5-brane does not match that for the anti-D3-branes. Taking into account all α'^2-corrections on the NS5 including pure flux and mixed curvature-flux corrections, we propose the following resolution: for nonabelian brane actions, there is no clear distinction between higher terms in commutators and higher derivative terms [24, 67, 68]. Those higher derivative curvature terms of the abelian NS5-brane which do not match on to higher derivative terms of the anti-D3-brane stack we then propose match higher commutator terms in the nonabelian theory. It should be possible to check this by performing the nonabelian analysis of KPV [1] at higher order in the commutators. As an example where higher commutator terms can be reinterpreted as higher curvature terms, it is known that a stack of flat nonabelian T-branes can be reinterpreted as a curved abelian brane with an R^2 curvature term in the worldvolume action [69]. As usual, it is unclear how or even if a single anti-D3-brane puffs up into an NS5-brane.

- It would be interesting to understand impact of the $1/(g_sM)^2$-curvature corrections on the vacua at $\psi = 0$ and $\psi = \pi$ from the perspective of the holographic dual field theory, especially as g_sM is the 't Hooft coupling in the dual Klebanov-Strassler field theory [19] and the highly curved regime is precisely the weakly coupled holographic field theory regime. An issue here is that with anti-D3-branes present the existence and precise location of the nonsupersymmetric metastable field theory vacuum corresponding to the presence of anti-branes, as proposed in KPV [1], remains to our knowledge poorly understood.[13] In fact, if one would be able to establish the metastable anti-brane vacuum holographically one would be confident that the metastable vacuum indeed exists. The lack of such a holographic field theory understanding of the metastable anti-brane vacuum is thus one of the factors leading to the backreaction discussion [41]. However, our curvature corrections also affect the situation where no anti-D3-branes are present, the situation described from a holographic field theory perspective in Section 2.3 of KPV [72]. In this case, the dual field theory has a baryonic branch, corresponding to a gravity solution with only flux, and a mesonic branch, corresponding to a gravity solution with M D3-branes at the tip of the warped throat [1, 19, 73]. Both branches are supersymmetric, however their moduli spaces are disconnected. There is then a supersymmetric domain wall interpolating between the baryonic and mesonic vacuum, corresponding to the NS5-brane wrapping an S^2 being pulled over the A-cycle S^3 from $\psi = 0$ to $\psi = \pi$ in the gravity picture. According to our curvature-correction computations, when $p = 0$, the NS5-brane potential is everywhere negative as soon as $g_sM < 5.238$ (cf. figure 4). This seems surprising given that for the flux-only (baryonic) supersymmetric vacuum there are no branes present to provide a negative curvature contribution to the potential. It is possible that additional corrections render the potential positive, however it could also be that for the flux-only vacuum α' curvature corrections in the bulk supergravity action render

[13] See e.g. [70–72] for work on the field theory perspective of such a metastable vacuum under the assumption that it exists.

the potential negative e.g. as discussed in [9] the bulk R^4 term of [74] can be corrected by warping and hence lead to terms involving g_sM. It would then be worthwhile to understand this in the holographic gauge theory at $g_sM \ll 1$ where our (uncontrolled) calculation claims that the potential should everywhere be very negative.

We intend to deal with some of these important points in a detailed analysis in future work.

3 Implications for model-building

In this section we want to collect some interesting phenomenological applications of the results of section 2.

So far we have been concerned with the effects of α'^2-curvature corrections to the KPV NS5-brane potential *per se*. The reason we began to study this topic was to better understand the anti-brane uplift in the LVS. With our newfound understanding one can now ask how α'^2 corrections to KPV affect string-phenomenological model building i.e. models such as LVS and KKLT.

Since our results suggest it might be possible to uplift without a deep throat, it is interesting to ask this question not just for the LVS, but also for e.g. KKLT [4]. Since the difficulty of embedding a deep throat in the bulk Calabi-Yau posed the main difficulty in uplifting KKLT [25, 75] (see also [3]) one can ask if our results allow one to resurrect the idea of anti-brane-uplifted KKLT in regimes that previously seemed out of control.

In addition, particles and domain walls in the four external directions have been constructed by wrapping branes on the cycle at the tip of the warped throat [30, 76]. It would be interesting to see how these analyses are affected by curvature corrections of the type considered here.

As each string-phenomenological model comes with many subtleties and additional effects of its own, unrelated to the story of KPV, we will not discuss these in detail here. We will merely state some suggestive results related to the LVS and leave our results on the string-phenomenological impact of α'^2 to an upcoming paper [77].

The natural parameters in which to understand curvature corrections to KPV are g_sM and p/M. If we use KPV in model-building, other parameters will become important. In particular, the individual values of g_s, M, and p will become important. It is straightforward to display the bounds we have derived in terms of these parameters. In figure 7 we show the region where $V_{\rm tot}(\psi)$ has a positive maximum in terms of g_sM as a function of g_s for various p.

Given these bounds, we can now compute for the LVS how they constrain the required negative D3 tadpole $-Q_3$ present in the compactification geometry by using the PTC [10]. We will present a detailed analysis in [77] and merely state the required tadpole for some reasonable parameters here. The additional parameters present in LVS are a_s related to nonperturbative corrections to the superpotential, κ_s related to the triple intersection number of the small divisor, $\xi = \chi\,\zeta(3)/2(2\pi)^3$ where χ is the Euler number of the Calabi-Yau orientifold, and c_N which is a control parameter related to a bulk curvature correction with varying warp factor. c_N must be large for this correction to be small. Not all the

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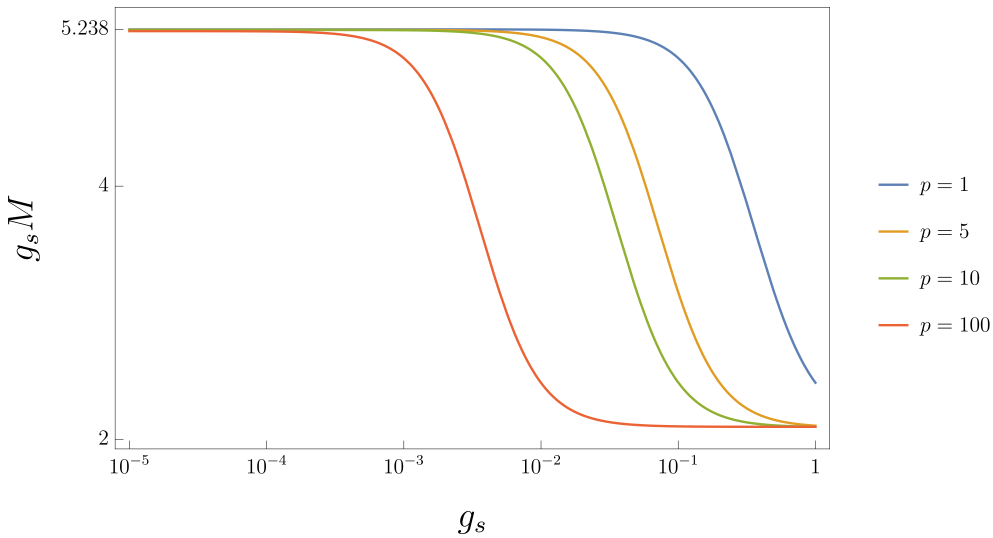

Figure 7. A plot of g_sM over g_s for different values of p. The region above the curves is where V_{tot} has a positive maximum. For $g_s \ll 1$ and $g_s \gg 1$ the curves asymptote again towards $g_sM \to 5.238$ and $g_sM \to 2.086$, respectively. Note that when $p/M > 0.08$ this maximum is not in a controlled regime.

parameters are independent. On way of thinking about this is setting a_s, κ_s, χ, p and c_N will fix g_s. This in turn fixes τ_s, the minimal value of M via section 2, the volume $\mathcal{V}$ and the minimal value of the D3 tadpole N of the warped throat.

Here, we will proceed differently by choosing a value of g_s (and hence a value of M which is in line with figure 6) instead of ξ. We then use the following equations[14] to determine ξ, $\mathcal{V}$ and N:

$$\mathcal{V} = \frac{3\kappa_s^{2/3}\xi^{1/3}|W_0|}{2^{7/3}a_s|A_s|g_s^{1/2}}\mathrm{e}^{a_s\frac{\xi^{2/3}}{(2\kappa_s)^{2/3}g_s}}\,, \quad a_s\frac{\xi^{2/3}}{(2\kappa_s)^{2/3}g_s} = \frac{16\pi N}{9g_sM^2}\,, \tag{3.1}$$

$$N = -\frac{21g_sM^2}{16\pi}\mathcal{W}_{-1}(x)\,, \qquad x = -\frac{3^{11/35}\,a_0^{2/7}}{7\,2^{59/105}\,5^{5/7}\,\pi^{1/35}}\,\frac{\kappa_s^{2/7}}{p^{2/7}\,a_s^{3/7}\,c_N^{5/7}\,(g_sM^2)^{1/7}}\,. \tag{3.2}$$

Here, the first equation in (3.1) is the on-shell solution for the volume modulus in the LVS, the second follows from requiring the uplift to be of the same size as the AdS minimum and (3.2) is called the PTC generalized to generic p.

With this, let us consider some examples where we choose $a_s = 2\pi$ as this minimizes the tadpole compared to the choice $a_s = \pi/3$.

As can be seen from table 1, requiring that a controlled metastable minimum exists via the bound in figure 3 leads to a minimum value for the tadpole at $\mathcal{O}(10^3–10^4)$.

Demanding only that a maximum with a positive potential energy exists leads to much weaker constraints, shown in table 2 where we choose $p = 1$. In the last line we show

[14]The equations are taken from [10], see equ. (2.4), (3.2), (3.11) and (3.15) therein.

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p	a_s	χ	κ_s	c_N	g_sM	p/M	g_s	g_sM^2	$(-Q_3)_{\rm min}$
2	2π	405	0.1	5	20.58	0.0824	0.848	500	1913
3	2π	225	0.1	5	20.4	0.0825	0.561	742	2900
8	2π	56	0.1	5	20.51	0.0824	0.211	1991	8175

Table 1. Minimal value of the required tadpole $(-Q_3)_{\rm min}$ for the minimum of $V_{\rm tot}$ to occur in the controlled regime for different p.

input parameters					$c_N = 5$		$c_N = 100$	
g_sM	p/M	κ_s	g_s	g_sM^2	χ	$(-Q_3)_{\rm min}$	χ	$(-Q_3)_{\rm min}$
5	0.04	0.1	0.2	125	43	455	62	580
10	0.04	0.1	0.4	250	124	921	178	1172
15	0.04	0.1	0.6	375	231	1395	329	1766
20	0.04	0.1	0.8	500	358	1868	510	2365
3.9	0.08	0.1	0.31	48.9	82	175	118	223

Table 2. Minimal value of the required tadpole $(-Q_3)_{\rm min}$ for $a_s = 2\pi$, $p = 1$ and different choices of p/M, c_N and g_sM demanding the maximum of the corrected KPV potential to be positive.

the 'most optimistic' setting, i.e. the point where the blue, red, and yellow region meet in figure 6. This point is at the boundary of the maximum being positive in a somewhat controlled regime. Clearly, all numbers should be viewed as an estimate as flux and possible further curvature corrections at order α'^2 need to be taken into account.

4 Conclusions and outlook

In this paper, we have studied the effects of expected α'^2-curvature corrections to KPV. We have seen that these can radically alter the leading-order KPV story, where the potential has a metastable, positive-energy minimum. However, we have only been able to analyze a subset of corrections — many α'-corrections in the NS5-action remain unknown. The main message then is that in this paper we have opened the Pandora's Box of higher-derivative corrections to KPV, but it is not at all clear at the moment where a complete analysis will lead and whether the final outcome at intermediate g_sM will be anything recognizably like the leading-order story of KPV. The different regimes expected on the basis our (partial) α'^2-corrections to KPV are summarized in figure 6, parametrized by p/M and g_sM.

We have then used the resulting bounds on g_sM from α'-corrections to KPV in the context of the LVS to estimate the minimal negative D3 tadpole. To achieve this, we have employed the Parametric Tadpole Constraint of [10]. To be precise, we have used two different options for how to bound g_sM:

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The conservative option is to demand that $g_s M$ is sufficiently large such that α'^2-corrections to the KPV potential are small, a metastable uplifting minimum exists in the potential, and the leading order story of KPV holds unmodified. How this constrains $g_s M$ can be seen in figure 3. As a result of this constraint, the LVS requires geometries with a negative D3 tadpole of the order of thousands or tens of thousands and at least two anti-D3-branes for the uplift (cf. table 1).

The more optimistic approach is to only demand that a local maximum for the NS5-potential exists at positive vacuum energy, such that the standard KPV decay process is classically forbidden. Clearly, this maximum has to be at sufficiently large radius of the S^2 wrapped by the NS5, such that our computations are controlled against even higher order α'-corrections.

Given the existence of this maximum one then assumes that, on its SUSY-breaking side, a local minimum capable of uplifting also exists. One accepts that, at the minimum, the NS5-brane wraps a string scale S^2, where our computations are not controlled. This constrains $g_s M$ as shown in figure 4 and, in the LVS context, only requires a negative D3 tadpole of the order of hundreds, as summarized in table 2.

In fact, there is even further cause for optimism: in the analysis described as the optimistic approach above, we have assumed that the potential of the α'-corrected minimum is at the same energy scale as the leading-order anti-D3-brane uplift. If, however, the α'^2-corrections significantly alter the NS5-brane potential, it is entirely plausible that the metastable minimum could be at much lower energy. It may even change sign, leading to non-SUSY AdS. This is in fact the situation which a pure anti-D3-analysis (without transition to the NS5) suggests [11]. Now, if one can tune the uplifting potential to extremely small values, one may be able to produce an exponentially suppressed uplift without relying on a deep warped throat. This tuning could be realized using the flux discretuum of g_s, which enters the $1/(g_s M)^2$ prefactor of curvature corrections. If, as a result, one no longer requires a deep throat, the main difficulty in achieving uplifted KKLT or LVS vacua disappears.[15]

Although we have not discussed this in detail, one could perform an analogous analysis in the S-dual set-up to KPV where the anti-D3-branes puff up into a D5-brane at the tip of the throat. It would be interesting to treat this set-up in full detail.

The issue with making the preceding ideas more concrete is that not all α'^2-corrections to KPV are explicitly known. As an outlook for future research, let us summarize again which points we believe need to be understood to obtain a clear, controlled picture of α'-corrected KPV:

- The α'^2-corrected NS5-brane action has to be computed explicitly, both curvature terms and gauge field strength contributions. As we have not included the field strength terms at all, our results may change significantly. This task should be easier in the S-dualized KS set-up as more is known about α'^2 corrections on the D5-brane.

[15] All this is a concrete example of the general philosophy of how one expects the interesting vacua with all moduli stabilized to live in the interior of parameter space rather than at the asymptotics [78]. The more orders in corrections we take into account, the richer the potential will be and the more options there are for model-building. If it is possible to explicitly compute all important α'^2-corrections, we should do so and embrace the added richness this gives us.

- It would be important to check that, at least near the maximum of the corrected potential, the α'^2 effect is trustworthy in the sense that even higher order α'-corrections are small and can be neglected.
- It is crucial to include backreaction effects, especially the notoriously difficult flux backreaction, in our analysis.
- In regimes where the S^2 wrapped by the NS5 is expected to be of stringy size, it would be interesting to consider the anti-D3-brane picture and compute higher-commutator and higher-derivative effects up to a certain order in α' to estimate if and at which angle ψ the minimum exists.
- It would be desirable to develop a deeper understanding in the holographically dual field theory of the vacua on both sides of the domain wall which corresponds to moving the NS5-brane from the south to the north pole. A natural starting point would be to study the case without anti-branes, where both the baryonic and mesonic branch are supersymmetric.

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A Curvature computations

The curvature tensors in (2.8) are given by [21]

$$(R_T)_{\alpha\beta\gamma\delta} = R_{\alpha\beta\gamma\delta} + g_{ab}(\Omega^a_{\alpha\gamma}\Omega^b_{\beta\delta} - \Omega^a_{\alpha\delta}\Omega^b_{\beta\gamma})\,, \tag{A.1}$$

$$(R_N)_{\alpha\beta}{}^{ab} = -R^{ab}{}_{\alpha\beta} + g^{\gamma\delta}(\Omega^a_{\alpha\gamma}\Omega^b_{\beta\delta} - \Omega^b_{\alpha\gamma}\Omega^a_{\beta\delta})\,, \tag{A.2}$$

$$\overline{R}_{ab} = \hat{R}_{ab} + g^{\alpha\alpha'}g^{\beta\beta'}\Omega_{a\,\alpha\beta}\Omega_{b\,\alpha'\beta'}\,. \tag{A.3}$$

Here, $R^\mu_{\nu\rho\sigma}$ is the 10D Riemann tensor, $\mu,\ldots$ indices run over all ten dimensions, $\alpha,\ldots$ indices run tangent to the brane, and $a,\ldots$ indices run normal to the brane. One obtains $\hat{R}_{ab} = R^\alpha{}_{a\alpha b}$ and $(R_T)_{\alpha\beta} = (R_T)^\gamma{}_{\alpha\gamma\beta}$ by contracting only over indices tangent to the brane. The second fundamental form $\Omega^\mu_{\alpha\beta}$ of the brane worldvolume is given by

$$\Omega^\mu_{\alpha\beta} = \partial_\alpha\partial_\beta Y^\mu - (\Gamma_T)^\gamma_{\alpha\beta}\partial_\gamma Y^\mu + \Gamma^\mu_{\nu\rho}\partial_\alpha Y^\nu \partial_\beta Y^\rho\,, \tag{A.4}$$

with $\Gamma^\mu_{\nu\rho}$ the 10D target space connection, $(\Gamma_T)^\gamma_{\alpha\beta}$ the connection on the induced metric on the brane worldvolume and $Y^\mu(\xi^\alpha)$ the coordinate functions describing the brane embedding in the 10D geometry. We defined $\Omega^a_{\alpha\beta} \coloneqq \Omega^\mu_{\alpha\beta}\Big|_{\mu=a}$ such that the normal indices are contracted with g_{ab}.

When a brane is embedded in flat space, the nonzero curvature terms are entirely given by terms related to the second fundamental form. When the brane is totally geodesically embedded, the curvature terms are entirely given by terms related to the 10D Riemann tensor.

As the anti-D3-brane is pointlike in the internal geometry, it is totally geodesically embedded. The NS5-brane however is nongeodesically embedded unless it is wrapping the equator of the S^3 at $\psi = \pi/2$.

As the metric of the Klebanov-Strassler throat, (2.1), is known exactly, it is in principle straightforward to compute all the necessary curvature terms. However, in the natural metric for the deformed conifold, (2.2), the coordinates of the S^2 wrapped by the NS5-brane are not cleanly isolated. To compute the curvature terms we proceed as follows: to obtain the coordinates wrapped by the NS5 we use the spherical parametrization of the deformed conifold at the tip of the throat [79] where the S^3 and the S^2 are parametrized with standard spherical coordinates (ψ, ω, φ) and (θ, ϕ), respectively. The NS5 hence wraps the (ω, φ) plane inside the S^3. The metric is given by

$$\mathrm{d}s_6^2 = \mathcal{F}(\tau)\,\mathrm{Tr}\left(\mathrm{d}W^\dagger \mathrm{d}W\right) + \mathcal{G}(\tau)\left|\mathrm{Tr}\left(W^\dagger \mathrm{d}W\right)\right|^2 , \tag{A.5}$$

where, setting $\varepsilon = 1$ as above,

$$\mathcal{F}(\tau) = \frac{(\sinh(2\tau) - 2\tau)^{1/3}}{2^{4/3}\sinh\tau} , \tag{A.6}$$

$$\mathcal{G}(\tau) = \frac{2 - 3\coth^2\tau + 3\tau\left(\cosh\tau/\sinh^3\tau\right)}{12\,(\cosh\tau\sinh\tau - \tau)^{2/3}} , \tag{A.7}$$

$$W = \left(1 + \frac{\tau^2}{8} + \frac{\tau^4}{384} + \mathcal{O}(\tau^6)\right) L + \left(\frac{\tau}{2} + \frac{\tau^3}{48} + \mathcal{O}(\tau^5)\right) L\hat{L} . \tag{A.8}$$

Here,

$$L = \begin{pmatrix} -\sin\psi\,\sin\omega\,\cos\varphi + \mathrm{i}\sin\psi\,\sin\omega\,\sin\varphi & \cos\psi - \mathrm{i}\sin\psi\,\cos\omega \\ \cos\psi + \mathrm{i}\sin\psi\,\cos\omega & \sin\psi\,\sin\omega\,\cos\varphi + \mathrm{i}\sin\psi\,\sin\omega\,\sin\varphi \end{pmatrix} , \tag{A.9}$$

$$\hat{L} = \begin{pmatrix} -\cos\theta & -\mathrm{e}^{\mathrm{i}\phi}\sin\theta \\ -\mathrm{e}^{-\mathrm{i}\phi}\sin\theta & \cos\theta \end{pmatrix} . \tag{A.10}$$

With this, (2.1) and the warp factor (2.4) it is a straightforward but tedious calculation to obtain the R^2 corrections at the tip. We obtain the following results:

$$(R_T)_{\alpha\beta\gamma\delta}(R_T)^{\alpha\beta\gamma\delta} = \frac{6^{2/3}}{g_s^2 M^2 I(0)} + \frac{6^{2/3}}{g_s^2 M^2 I(0)}\left(\cot^4\psi + 2\cot^2\psi\right) \tag{A.11}$$

$$-2(R_T)_{\alpha\beta}(R_T)^{\alpha\beta} = -2\,\frac{6^{2/3}}{2g_s^2 M^2 I(0)} - 2\frac{6^{2/3}}{2g_s^2 M^2 I(0)}\left(\cot^4\psi + 2\cot^2\psi\right) \tag{A.12}$$

$$-(R_N)_{\gamma\delta ab}(R_N)^{\gamma\delta ab} = -\frac{6^{2/3}}{g_s^2 M^2 I(0)} \tag{A.13}$$

$$\begin{aligned} 2\overline{R}_{ab}\overline{R}^{ab} &= 2\,\frac{3^{2/3}\,(117\,I(0)^2 - 200\,I(0)\,I''(0) + 150\,I''(0)^2)}{25 \times 2^{1/3} g_s^2 M^2 I(0)^3} \\ &\quad + \frac{2 \times 6^{2/3}\cot^2\psi}{g_s^2 M^2 I(0)}\left(2 + \cot^2\psi\right) \end{aligned} \tag{A.14}$$

The terms (A.11) and (A.12) cancel against each other. In (A.14) the term $\sim \cot^2\psi$ stems from the $\overline{R}\Omega^2$ term and the term $\sim \cot^4\psi$ from Ω^4. This also matches the naive expectation of the extrinsic curvature of an S^2 which scales as $1/R_{S^2}$. Moreover, for $\psi = \pi/2$ the extrinsic curvature terms vanish as at the equator the NS5 is geodesically embedded. Inserting (A.11)–(A.14) into (2.10) and integrating over the S^2 gives the α'^2 curvature corrected KPV potential (2.11).

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Chapter 5

α' corrections to KPV: An uplifting story

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Simon Schreyer is the principal author of this article who also conceived the original idea for the project. The calculations were performed by Simon Schreyer with suggestions and corrections by Gerben Venken. All figures were produced by Simon Schreyer. The sections 2, 3, 5, and both Appendices were originally written by Simon Schreyer. The other sections were mainly written by Gerben Venken. Both authors contributed with corrections and suggestions to the manuscript as well as with improvements during the ongoing review process.

α' corrections to KPV: An uplifting story

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Abstract: In earlier work, the effect of α'^2 curvature corrections on the NS5-brane responsible for the decay of anti-D3-branes in the set-up of Kachru, Pearson, and Verlinde (KPV) was considered. We extend this analysis to include all known α'^2 corrections to the action of an abelian fivebrane which involve not just curvature but also gauge fields and flux. We compute the value of these terms at the tip of the Klebanov-Strassler throat to obtain the α'^2 corrected potential for the NS5-brane of KPV. The resulting potential provides a novel uplifting mechanism where one can obtain metastable vacua with an arbitrarily small positive uplifting potential by fine-tuning α' corrections against the tree-level potential. This mechanism works for small warped throats, both in terms of size and contribution to the D3-tadpole, thereby sidestepping the issues associated with a standard deep warped throat uplift which are deadly in KKLT and, as we explicitly check, severely constraining in the Large Volume Scenario.

Contents

1 Introduction

The set-up of Kachru, Pearson, and Verlinde (KPV) [1] provides one of the best-known examples of a proposed metastable spontaneously supersymmetry breaking vacuum in string theory. One considers p anti-D3-branes at the tip of the Klebanov-Strassler (KS) throat [2]. These puff up into a single fluxed NS5-brane which depending on the parameters at the tip either has a metastable supersymmetry breaking vacuum or is classically unstable and pulled over the tip to decay into a supersymmetric vacuum. Our main interest in KPV is as a tool in the anti-D3-brane uplift to construct de Sitter vacua in string theory.

It remains one of the core questions of string phenomenology whether one can achieve controlled de Sitter vacua in string theory [3, 4]. One of the most well-trodden paths towards constructing de Sitter vacua is to start by constructing a scale separated AdS vacuum[1] in IIB with potential V_{AdS}. One then supplies a source of positive potential energy V_{up} to provide an uplift to a de Sitter minimum. This is the route taken by e.g. KKLT [1] and LVS [10, 11]. One requires that $|V_{\text{up}}| \approx |V_{\text{AdS}}|$, in order to obtain a metastable de Sitter vacuum. If $|V_{\text{up}}| \gg |V_{\text{AdS}}|$ the uplifting potential completely overpowers the AdS potential which provided a minimum and one obtains a runaway instead.

An anti-D3-brane, pointlike in the internal dimensions provides a source of positive potential energy but by itself would lead to a runaway. This is where the KS throat comes to the rescue: by placing the anti-D3-brane at the tip of a warped throat, one can exponentially suppress V_{up} through warping. One may then set the warping at the tip of the throat such that V_{up} provides a controlled uplift, as done in KKLT [1] and LVS [10, 11]. The issue with this is that in order to have sufficient warping one must have a sufficiently large throat, both in terms of its size and its contribution to the D3-tadpole. The issue of fitting such a large throat in the compactification geometry seems deadly in KKLT, where it has been dubbed the singular bulk problem [12–14], and seems severely constraining in the LVS [15–18].

Recently, [18] shed new light on this issue by examining how α' corrections affect uplifts with warped throats. Such corrections become important when one considers small warped throats which should be easier to embed in a compactification geometry. In particular, the effect of curvature corrections on the worldvolume of the uplifting antibrane was examined, focussing especially on the impact of these curvature corrections on the KPV decay channel. This analysis opened up many interesting new possibilities. Most exciting was the possibility of a new uplifting mechanism unique to warped throats with a small tip. Curvature corrections correct the potential for a single antibrane as [19, 20, 15]

$$V_{\overline{D3}} = \mu_3 \mathrm{e}^{-\phi}\left[1 - \frac{(4\pi^2\alpha')^2}{384\pi^2} R_{a\alpha b}{}^{\alpha} R^{a}_{\ \beta}{}^{b\beta}\right] = \mu_3 \mathrm{e}^{-\phi}\left[1 - \frac{c}{(g_s M)^2}\right] , \tag{1.1}$$

with $c = 5.924$ and where a, b are normal and α, β tangent indices relative to the brane. Here M is the F_3 flux at the tip of the throat and the radius R_{S^3} of the tip is proportional to $\sqrt{g_s M}$. By fine-tuning g_s one can achieve $c/(g_s M)^2 \approx 1$ and in this way tune $V_{\overline{D3}}$ to an arbitrarily small value. In this way one can achieve an uplift without requiring a large warped throat. We refer to [18] for further discussion.

It is insufficient to analyze this uplift purely in terms of the anti-D3-brane. The antibrane puffs up and to study the metastable vacuum we must consider the KPV set-up. Unfortunately, the analysis of [18] had to remain very incomplete on this point as it only considered curvature corrections. The goal of the present paper is twofold: First, to take into account corrections to the worldvolume theory due to background fluxes which were neglected in [18]. Second, to more fully explore the phenomenological impact of α' corrections on the brane

[1] This first step of constructing a landscape of scale-separated AdS vacua has also been called into question [5–9]. In the remainder of this paper we will proceed under the assumption that a landscape of scale-separated AdS vacua can be obtained in IIB string theory via either KKLT or LVS or some other mechanism, but this assumption is highly nontrivial.

worldvolume in the KPV set-up.

The rest of the paper is structured as follows. In Sect. 2 we very briefly review the set-up of KPV [1] and how the results of [18] affect this set-up.

In Sect. 3 we consider additional corrections from flux, gauge fields and mixed terms involving both of the above as well as curvature. We compute the potential for the NS5-brane with these corrections taken into account. We show that for throats with a large tip the analysis of KPV is recovered, while for small throats the corrections become important and a sensible potential with interesting new features appears.

In Sect. 4 we show that using these new features one can indeed get the new uplifting mechanism by finetuning α' corrections to work, while this was previously only hinted at. As this uplifting mechanism requires balancing α' corrections to the potential to be as large as the tree-level potential one is at the borderline of control. This novel uplifting mechanism allows one to uplift using a small warped throat, requiring a warped throat which contributes as little as $N = 40$ to the D3-tadpole.

In Sect. 5 we instead consider the traditional uplifting mechanism where one uses a large warped throat to exponentially warp down V_{up}. We investigate under what conditions this traditional uplift is controlled for the LVS specifically. Even with the weakest demands on control one finds that the warped throat must have a D3-tadpole $N = 560$. Demanding a bit more control, one rapidly finds that an $\mathcal{O}(10^3)$ tadpole from the warped throat. This severely restricts the possibility of achieving such an uplift with a large warped throat as compactification geometries with sufficient negative D3-tadpole to fit such throats are currently not known in IIB string theory with local D7-tadpole cancellation (but larger tadpoles may be possible for nonlocal cancellation, see [21]).

Our paper focuses on α' corrections to the worldvolume action of an NS5-brane. However, many of the corrections we consider have strictly only been derived for a D5-brane and we have inferred the analogous correction for an NS5-brane. While we believe our method for inferring these corrections is correct, one may object that we strictly lack a derivation of the α' corrections on the NS5-brane. The main reason why we have focused on the KS throat is that this is the set-up most commonly considered for a warped throat uplift in the literature. One may instead consider the S-dual throat with a D5-brane at the tip and use this for the uplift [22]. One can then perform our analysis of α' corrections for this dual set-up now being confident that one knows the α' corrections. We have analyzed our corrections to this S-dual set-up in Sect. 6. Unfortunately the novel uplifting mechanism we propose does not work in this S-dual set-up with the main issue being that the radius of the tip of the throat is set by $\sqrt{K}$, with K the amount of H_3 flux at the tip of the S-dual throat. We then lack a factor of $\sqrt{g_s}$ which allows us to tune the radius of the tip.

We conclude in Sect. 7. The new uplifting mechanism we propose requires far smaller throats than the traditional uplifting mechanism via exponential warping. This allows us to sidestep the issues involved in uplifting KKLT and LVS with a large warped throat. Some questions remain on how controlled our new uplifting mechanism is and we discuss future directions to gain a better understanding of the small throat uplifting mechanism.

The paper contains two appendices. In App. A we gather various technical computations related to α' corrections. In App. B we review the Parametric Tadpole Constraint (PTC) [16] and recast it into a form more useful for applying the bounds on control for the warped throat we obtain to the LVS. We also comment on the discrepancy in the conditions for control in the LVS between [15, 17] and [16].

2 Review of KPV and curvature corrections

In the set-up of KPV [1], p anti-D3-branes are placed at the tip of the Klebanov-Strassler (KS) throat [2]. The KS throat is a deformed conifold with M units of F_3 flux threading the A-cycle of the throat. Topologically, at a distance τ from the tip there is a $T^{1,1}$ geometry, which is an S^2 fibration over an S^3. At the tip $\tau = 0$ the S^2 shrinks to zero size while the S^3 remains of finite radius $R_{S^3} \sim \sqrt{g_s M}$ and coincides with the A-cycle.

The anti-D3-branes at the tip can puff up into an NS5-brane. The NS5 wraps an S^2 inside the S^3. The authors of [1] calculated the 4d potential of this NS5-brane in terms of M, p, and ψ, the angular direction of the S^3 normal to the S^2 wrapped by the NS5, from the tree-level string-frame NS5-brane action

$$S = -\frac{\mu_5}{g_s^2} \int_{\mathcal{M}_6} \mathrm{d}^6\xi \sqrt{-\det\left(g_{\mu\nu} + 2\pi\alpha' g_s F_{2\,\mu\nu} - g_s C_{2\,\mu\nu}\right)} - \mu_5 \int_{\mathcal{M}_6} B_6\,, \tag{2.1}$$

where μ_5 is the brane tension. The resulting potential is of the form

$$V_{\mathrm{KPV}}(\psi) = \frac{4\pi^2 p\mu_5}{g_s} + \frac{4\pi\mu_5 M}{g_s}\left(\sqrt{b_0^4 \sin^4(\psi) + \left(\frac{\pi p}{M} - \psi + \frac{\sin(2\psi)}{2}\right)^2} - \psi + \frac{\sin(2\psi)}{2}\right), \tag{2.2}$$

where $b_0^2 \approx 0.93266$. The potential describes a decay channel for the NS5 into $M - p$ D3-branes. Whether the NS5 is metastable or unstable in this decay channel depends on the value of p/M. During this process, the NS5 slips over the equator while the anti-D3-brane charge of the NS5 annihilates against 3-form flux. The potential is shown in Fig. 1 where one can see that for $p/M > 0.08$ no metastable minimum exists.

The main idea of [18] was to include curvature corrections to the potential of the NS5 since in phenomenologically viable models, the value $g_s M^2$ and hence $R_{S^3} \sim \sqrt{g_s M}$ is typically not too large. Thus, curvature corrections, scaling like $1/R_{S^3}^4$, eventually become important. This is a manifestation of the fact that the validity of the supergravity expansion in α' is on the borderline of control. The curvature corrections evaluated for the NS5 at the tip of the throat lead to a correction of the KPV potential

$$V_{\mathrm{curv}}(\psi) = -\frac{c_1 + c_2 \cot^2\psi(2 + \cot^2\psi)}{(g_s M)^2} \frac{4\pi\mu_5 M}{g_s} \sqrt{b_0^4 \sin^4(\psi) + \left(\frac{\pi p}{M} - \psi + \frac{\sin(2\psi)}{2}\right)^2}\,, \tag{2.3}$$

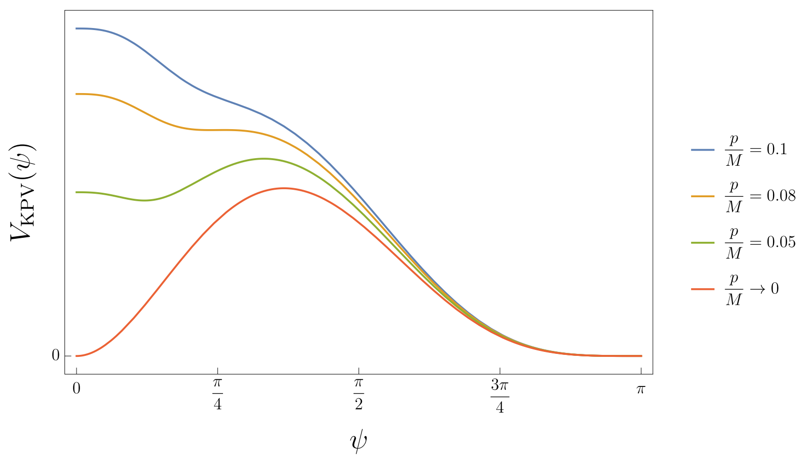


Figure 1. The potential $V_{\mathrm{KPV}}(\psi)$ (suitably normalized) for different values of p/M.

where $c_1 \approx 8.825$ and $c_2 \approx 1.891$. The terms $\sim \cot\psi$ come from the second fundamental form Ω that is non-zero for non-geodesically embedded branes.

As was repeatedly stressed in [18], great care had to be taken in interpreting this corrected potential and its regime of validity. One of the main issues is that only pure curvature α'^2 corrections to the NS5 worldvolume action were included in this potential while at the same order in α' there are also corrections depending on gauge fields, field strengths, and mixing of these with curvature[2]. The main goal of this paper is to account for these additional corrections and we shall see that the resulting potential is much more readily physically interpretable.

3 Flux corrections

As described in Sect. 2, in [18] higher order α' corrections to the Chern-Simons action and corrections to the DBI action that involve fluxes were neglected. In this section, we summarize all flux corrections to D-branes in superstring theory from the papers [23–33] that are non-vanishing for the KS throat, extend them to the NS5-brane and evaluate them at the tip of the KS throat. Many of these corrections have only been derived in flat space without background F_2. We make the following assumptions to extend these corrections to a brane worldvolume with nonzero F_2 in a curved background. We promote all partial derivatives to covariant ones in order for the action to be Lorentz covariant. Moreover, we assume all corrections to hold also for non-zero F_2 and B_2 flux by replacing $\sqrt{-g} \to \sqrt{-(g + 2\pi\alpha'\mathcal{F}_2)}$.

[2] We will generically refer to such corrections as flux corrections, even though they involve not just F_3, H_3, F_5 and H_7 but also C_2 and F_2.

Schematically, the corrections to the DBI action of a Dp-brane important for the KS throat read [27, 28, 32, 33]

$$S_{\mathrm{DBI,D}p} \supset \frac{\mu_p \alpha'^2}{g_s} \int_{\mathcal{M}_{p+1}} \mathrm{d}^{p+1}x \sqrt{-(g+2\pi\alpha'\mathcal{F}_2)} \Bigg[H_3^4 + H_3^2 R + \Omega^4 (2\pi\alpha'\mathcal{F}_2)^2 + (2\pi\alpha'\mathcal{F}_2)\Omega^2 \nabla H_3 \Bigg] , \tag{3.1}$$

where we omitted any index contraction and numerical prefactor, and R symbolically represents either the Riemann or the Ricci tensor. The precise action can be found in App. A.2, in (A.9), (A.14) and (A.17). Note that the first two terms in (3.1) are derived for Op-planes. At order α'^2, up to overall factors, couplings on O-planes can be obtained from couplings on D-branes by orientifold projection [34][3]. From this we conclude that all couplings on O-planes at order α'^2 are also present on D-branes.

3.1 Extending α' corrections to NS5-branes

For $p = 5$, (3.1) can be S-dualized to

$$S_{\mathrm{DBI,NS5}} \supset \mu_5 \alpha'^2 \int_{\mathcal{M}_{p+1}} \mathrm{d}^{p+1}x \sqrt{-(g+2\pi\alpha' g_s\mathcal{F}_2)} \Bigg[(-g_sF_3)^4 + (-g_sF_3)^2 R + \Omega^4 (2\pi\alpha' g_s\mathcal{F}_2)^2 + (2\pi\alpha' g_s\mathcal{F}_2)\Omega^2 \nabla (-g_sF_3) \Bigg] , \tag{3.2}$$

where $2\pi\alpha' g_s \mathcal{F}_2 = 2\pi\alpha' g_s F_2 - g_s C_2$. As observed in [18] for the α'^2 curvature corrections, naively S-dualizing the D-brane action leads to g_s^2 suppressed terms when compared to the leading-order DBI action of the NS5-brane (2.1). This means that tree level terms on D5-branes map to two loop effects on NS5-branes. The comparison of the NS5-brane when shrinking the S^2 to zero size with the anti-D3-brane led [18] to propose that there should also be α'^2 curvature correction at leading order in g_s. S-dualizing this term back to the D5-brane case leads again to (so far unknown) two loop open string terms on the D5-brane.

We will now play the same game for the flux corrections in (3.2) where the story is a little more subtle. To be able to know whether an α' correction appears g_s suppressed compared to the tree level term, the kinetic terms of all fields need to have the same g_s dependence. We normalize all kinetic terms in such a way that they have a g_s^{-2} dependence. This means for instance for F_3 that we write the kinetic term $\int_{10} |F_3|^2 \sim g_s^{-2} \int_{10} |F_{3,\mathrm{norm}}|^2$ with $F_{3,\mathrm{norm}} = g_s F_3$. Similarly, we find $\mathcal{F}_{2,\mathrm{norm}} = g_s \mathcal{F}_2$ as we have already indicated in (3.2). Hence, (3.2) is again g_s^2 – so two loop open string – suppressed compared to the tree level DBI action (2.1). Hence, analogously to the curvature corrections, we propose the following

[3] Note that this is not true for higher order α' corrections, as can be seen from the expansion of anomalous CS couplings.

α'^2 corrections for the NS5-brane at tree level in g_s:

$$S_{\mathrm{DBI,NS5}} \supset \frac{\mu_5 \alpha'^2}{g_s^2}(1+g_s^2) \int\limits_{\mathcal{M}_{p+1}} \mathrm{d}^{p+1}x \sqrt{-(g+2\pi\alpha' g_s \mathcal{F}_2)} \Big[(g_s F_3)^4 + (g_s F_3)^2 R + \Omega^4 (2\pi\alpha' g_s \mathcal{F}_2)^2 + (2\pi\alpha' g_s \mathcal{F}_2)\Omega^2 \nabla(-g_s F_3) \Big] \,. \tag{3.3}$$

The tree level term will after S-dualizing again correspond to so far unknown 2-loop corrections on the D5-brane.

Besides α' corrections to the DBI action, there are also α' corrections to the CS action. The important couplings for the KS throat schematically read [31]

$$S_{\mathrm{CS,D}p} \supset \mu_p \alpha'^2 \int_{\mathcal{M}_{p+1}} \mathrm{d}^{p+1}x \left(-\epsilon_{(6)} \mathcal{F}_2 R \nabla \tilde{F}_5 + \epsilon_{(6)} \mathcal{F}_2 \nabla H_3 \nabla F_7 \right) \,, \tag{3.4}$$

where again we have omitted any index structure. Note that $\epsilon_{(6)}$ denotes the 6d Levi-Civita-symbol, $\tilde{F}_5$ the self dual 5-form flux and F_7 the 7-form flux. The numerical prefactor and index structure are given in (A.19) and (A.21) in the App. A.2. The action (3.4) can be S-dualized for $p=5$ to

$$S_{\mathrm{CS,NS5}} \supset \mu_5 \alpha'^2 \int_{\mathcal{M}_{p+1}} \mathrm{d}^{p+1}x \left(-\epsilon_{(6)} (g_s \mathcal{F}_2) R \nabla (g_s \tilde{F}_5) + \epsilon_{(6)} (g_s \mathcal{F}_2) \nabla(-g_s F_3) \nabla(g_s^2 H_7) \right) \,, \tag{3.5}$$

where we have already properly accounted for factors of g_s to obtain normalized kinetic terms. Let us compare the g_s scaling in (3.5) with the tree level term $\int \mathrm{d}^6 x \mathcal{F}_2 \wedge C_4 = g_s^{-2} \int \mathrm{d}^6 x (g_s \mathcal{F}_2) \wedge (g_s C_4)$ present on the NS5-brane. This shows that the S-dualized action (3.5) appears to be 2-loop suppressed. As before, we propose that the CS couplings exist also at tree level in g_s.

It is interesting that this tree level to 2-loop mapping under S-duality for the D5-/NS5-brane appears to hold for all terms in the α'^2 corrected brane action.

We have accounted for all α^2 corrections to the fivebrane worldvolume action we found in the literature. However, there is to our knowledge no proof that the α'^2 corrections are completely known. Especially corrections to the DBI action involving $F_{(p)}$ flux are not known. Until the fivebrane action at order α'^2 is completely determined, our results must necessarily remain incomplete.

3.2 The flux corrected KPV potential

With the results of App. A.2, the α' corrected KPV potential of all currently known corrections is given by (2.2), (2.3), and (A.23). It is of the form

$$\begin{aligned}
V_{\rm tot} &= V_{\rm KPV} + V_{\rm curv} + V_{\rm flux} \\
&= \frac{4\pi\mu_5 M}{g_s}\sqrt{b_0^4 \sin^4(\psi) + \left(p\frac{\pi}{M} - \psi + \frac{1}{2}\sin(2\psi)\right)^2} \times \frac{1}{(g_s M)^2}\Bigg[c_3 - c_1 \\
&\qquad + (c_4 - 2c_2)\cot^2\psi - c_2\cot^4\psi + \frac{c_5\cot^4\psi}{\sin^4\psi}\left(\frac{\pi p}{M} - \left(\psi - \frac{\sin(2\psi)}{2}\right)\right)^2 \\
&\qquad - \frac{c_6\cot^3\psi}{\sin^2\psi}\left(\frac{\pi p}{M} - \left(\psi - \frac{\sin(2\psi)}{2}\right)\right)\Bigg] \\
&\qquad + \left[\frac{4\pi^2 p\mu_5}{g_s} - \frac{4\pi\mu_5 M}{g_s}\left(\psi - \frac{\sin(2\psi)}{2}\right)\right]\left(1 + \frac{c_7}{(g_s M)^2} + \frac{c_8\cot\psi}{(g_s M)^2\sin\psi}\right),
\end{aligned} \tag{3.6}$$

where the numerical constants c_1 to c_8 are given in Tab. 1.

Table 1. Numerical constants in the α' corrected KPV potential $V_{\rm tot}$.

c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8
8.825	1.891	2.448	8.696	32.61	2.174	14.64	18.65

It was already observed in [18] that the curvature corrected KPV potential appears to have two parameters g_sM and p/M that determine the shape of the potential. Non-trivially, this is still true when the flux corrections are included: All terms precisely line up in terms of g_sM and p/M with an overall $1/g_s^2$ factor. Hence, the α' expansion of the brane action translates into an expansion of the potential in terms of g_sM and p/M. The expansion in p/M is a new effect not observed in [18]. It stems from corrections including F_2. The F_2 corrections in (3.3) are higher order in α' compared to the other corrections. Nevertheless, it still makes sense to include them into the potential since they feature the same g_sM suppression but are additionally suppressed by the second expansion parameter p/M.

The potential $V_{\rm tot}$ is shown in Fig. 2 for $g_sM = 20$ and different values of p/M and in Fig. 3 for $p/M = 0.01$ and varying g_sM. As before, we see that depending on p/M and g_sM there exists a metastable minimum. A new feature is that the value of the potential at the metastable minimum, which we will call $V_{\rm up}$, can be at negative, zero, or positive energy depending on the parameters. This can be summarized in the contour plot in Fig. 4 where (a) is a log-log plot whereas (b) is linear on the g_sM axis. The blue region describes the part of the parameter space where no metastable minimum exists, the yellow region describes the region with a metastable minimum at $V_{\rm up} > 0$, and the minimum is at negative energy in the orange region. As expected for large values of g_sM, the blue region is bounded from below by the KPV bound $p/M = 0.08$.

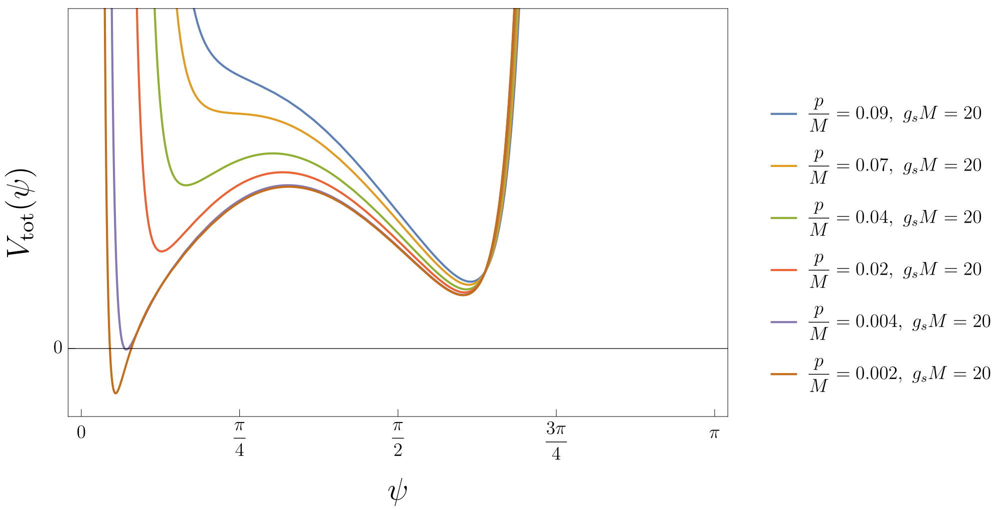


Figure 2. The potential $V_{\text{tot}}(\psi)$ (suitably normalized) for $g_sM = 20$ and different values of p/M. With decreasing values of p/M, the form of the potential transitions from no metastable minimum to a minimum with $V_{\text{up}} < 0$. In between, the vacuum energy is zero or positive. Note that we lose control over the perturbation theory that leads to this potential when either $\psi \gtrsim \pi/2$ or $\sqrt{g_sM}\sin\psi \lesssim 1$.

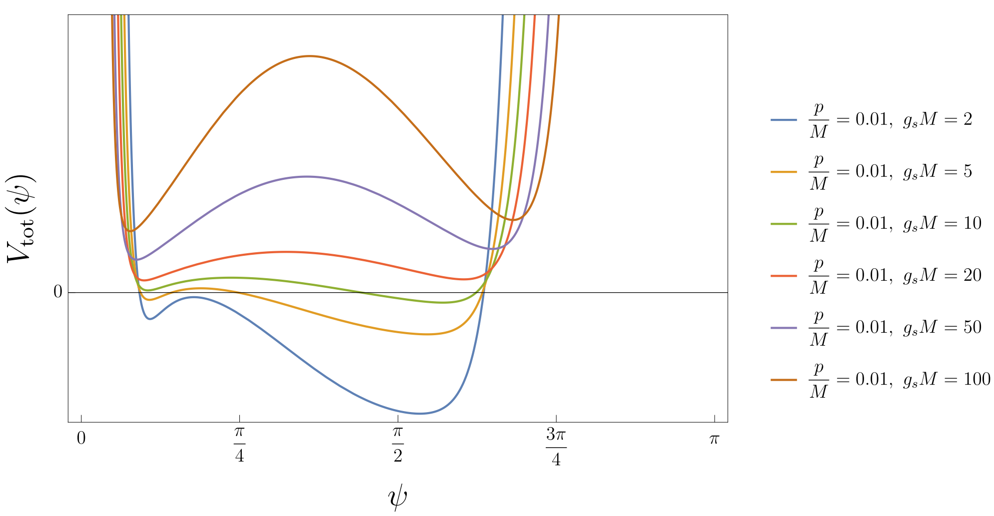
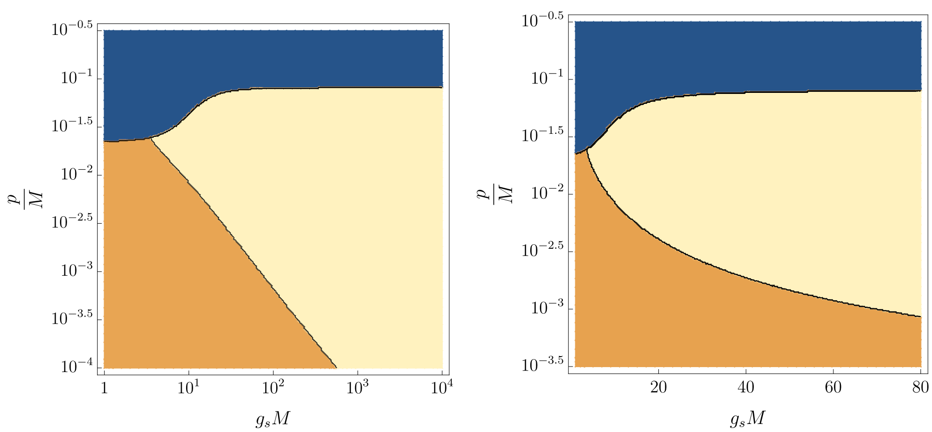


Figure 3. The potential $V_{\text{tot}}(\psi)$ (suitably normalized) for $p/M = 0.01$ and different values of g_sM. Note that we lose control over the perturbation theory that leads to this potential when either $\psi \gtrsim \pi/2$ or $\sqrt{g_sM}\sin\psi \lesssim 1$.

Before discussing the crucial question of in which region of parameter space the potential (3.6) can be trusted, let us comment on the main differences to the curvature corrected potential $V_{\text{KPV}} + V_{\text{curv}}$ calculated in [18]. As the highest order divergence in $1/\psi$ in (3.6) has a positive prefactor coming from the correction $\Omega^4 \mathcal{F}_2^2$, the potential diverges to $+\infty$ when $\psi \to 0, \pi$. This facilitates the existence of a metastable minimum compared to [18] since also for very small values of $g_s M$, a minimum will always exist as long as p/M is sufficiently small. Let us emphasize that in this regime, the radius of the NS5-brane is string size and all higher order α' corrections will become important. We will further elaborate on this below. As already discussed in [18], we assume that summing up all the (unknown) higher order α' corrections, we find a smooth potential without any divergences. We then expect the fully corrected potential to have a finite value at $\psi = 0$. It may be reasonable to expect that the fully corrected value of the potential at $\psi = 0$ will be given by p times the fully corrected potential of an anti-D3-brane.

In the regime of large $g_s M$ both potentials yield similar results.



(a) (b)

Figure 4. The yellow area shows the region in the $(g_s M, p/M)$-parameter space where a metastable minimum at positive energy exists. In the orange region, the minimum is at negative energy. In the blue region there is no metastable minimum, only an instability towards the supersymmetric minimum. In (a) a log-log-plot is shown for a wide range of parameters and (b) zooms into the region of smallest allowed values of $g_s M$.

Let us now turn to the question of when we can trust the approximations that have led to this potential. A first condition is that $g_s M \sin^2 \psi \sim R_{\text{NS5}}^2$ should be sufficiently large. The reason is that higher order α' couplings feature a suppression by R_{NS5}^2 or $R_{S^3}^2$. This applies to curvature and flux couplings. Thus, in order for higher order α' corrections to be negligible one should ensure that $\tilde{R}_{\text{NS5}}^2 = g_s M \sin^2 \psi$ is sufficiently large[4]. Caution is required

[4]Note that we do not take $R_{\text{NS5}}^2 = b_0^4 g_s M \sin^2 \psi$ as the expansion parameter as the powers of b_0 are different

for couplings involving $\mathcal{F}_2$ flux since $\mathcal{F}_2$ scales in the potential as

$$\mathcal{F}_2 \sim \frac{1}{g_s \sin^2\psi}\left(\frac{p}{M} - \psi + \frac{\sin(2\psi)}{2}\right), \tag{3.7}$$

which is only small for moderately small ψ.

This leads to a second condition specific to the terms involving $\mathcal{F}_2$ in the potential (3.6). As can be seen from (3.7), for small ψ, $\mathcal{F}_2$ scales like $\mathcal{F}_2 \sim p/\tilde{R}^2_{NS5}$. But when $\psi \sim \mathcal{O}(1)$, (3.7) implies $\mathcal{F}_2 \sim -(M-p)/M$ which is $\mathcal{O}(1)$ in the regime where $p \ll M$. This is to be expected since with increasing ψ the anti-D3-brane charge of the NS5 is neutralised by flux such that the NS5 describes $M-p$ D3-branes at the north pole. Hence, we lose control over $\mathcal{F}_2$ corrections close to the equator of the S^3 and on the north side of the equator $\psi > \pi/2$. This is not a serious issue since we anyways expect that the NS5 decays into the SUSY minimum at $\psi = \pi$ as soon as the NS5 slips over the equator. Note that it is not guaranteed that the SUSY minimum at $\psi = \pi$ has zero vacuum energy. The SUSY minimum has $M-p$ D3-branes whose α' corrected worldvolume action contributes to the potential as already discussed in [18]. This can lead the SUSY minimum to have negative energy, allowing the metastable minimum to decay even when the metastable minimum has zero or negative energy.

Since in the interesting regime close to the south pole, $\tilde{R}^2_{\text{NS5}}$ is the crucial control parameter, Fig. 5 depicts again the $(g_sM, p/M)$ parameter space but now with contours describing metastable minima of the potential of fixed $\tilde{R}^2_{\text{NS5}}$. Decreasing g_sM also decreases the radius of the NS5. The regime where control over α' corrections is best is the large g_sM and large p/M regime. This again forces highly warped, deep throats as already observed in [15, 35, 17, 18]. We discuss implications of this for phenomenology in Sect. 5.

In the cases where $V_{\text{up}} = 0$, the radius of the NS5 is $\mathcal{O}(1)$ which marks the outermost edge of control as can be seen from the red line in Fig. 5 (b), on which $V_{\text{up}} = 0$ holds. This is precisely the regime needed for the uplifting mechanism without deep throats which we will discuss further in Sect. 4.

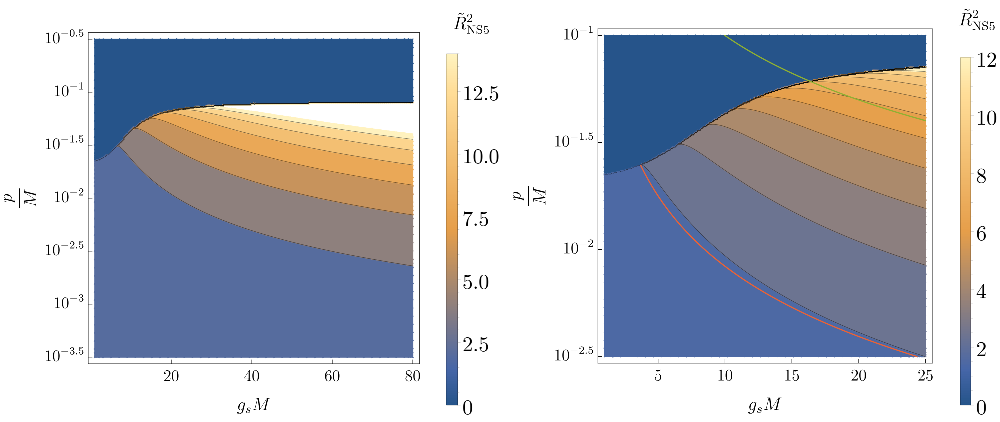
Note that in the regime of large g_sM and p/M one eventually enters the regime where $g_s > 1$. To stay in the weak coupling regime of type IIB string theory, we should ensure that

$$\frac{p}{M} < \frac{p}{g_sM}, \tag{3.8}$$

which drives one to smaller values of R_{NS5}. As also noted in [18], this forbids for $p = 1$ a large region of parameter space. The line $g_s = 1$ for $p = 1$ in the $(g_sM, p/M)$ parameter space is depicted in Fig. 5 (b) in green. Hence, the parameter space above this line is forbidden when choosing $p = 1$. With increasing p, the forbidden region shrinks such that the regime of large g_sM and p/M becomes accessible at weak coupling.

Besides the KPV decay channel, another proposed instability channel for antibranes at the tip of the KPV throat is the conifold transition [36–40] (recently questioned in [41]). To

in some of the on-shell α' corrections.



(a) (b)

Figure 5. A similar contour plot to Fig. 4. The contours indicate lines of constant $\tilde{R}^2_{\rm NS5} = g_sM\sin^2\psi$ in the $(g_sM, p/M)$ parameter space. Note that $\tilde{R}^2_{\rm NS5}$ is proportional to the squared radius of the NS5-brane $R^2_{\rm NS5} = b_0^4\tilde{R}^2_{\rm NS5}$. The dark blue region is again the region where no metastable minimum exists. (a) depicts a wide range of parameters and (b) is a more detailed plot for smaller values of g_sM. In b) the contours are at integer values of $\tilde{R}^2_{\rm NS5}$ starting with $\tilde{R}^2_{\rm NS5} = 1$ on the lowest contour. On the green line in (b), $g_s = 1$ for $p = 1$ and above the green line, $g_s > 1$. The red line indicates the line where the minima of the potential are at zero energy.

avoid a conifold instability for a single antibrane one imposes $\sqrt{g_s}M > 6.8$. Note however that for the smallest values of g_s and M where KPV has a zero vacuum energy metastable vacuum, $\sqrt{g_s}M = 12$, such that after α' corrections requiring the metastable vacuum to have a positive energy is a stronger constraint than that imposed by the conifold instability.

4 Uplifting without a deep throat

We have seen from Fig. 4 that with α'^2 corrections accounted for, there are three types of metastable vacua allowed: those with a negative energy at sufficiently small g_sM and p/M; those with a positive energy at sufficiently large g_sM and p/M; and separating these a line of vacua with zero energy. The line of vacua with zero energy is approximately given by

$$\frac{p}{M} \approx \frac{0.1029}{(g_sM)^{1.0909}}\,, \tag{4.1}$$

and the metastable vacuum with zero energy exists for $g_sM \gtrsim 3.6$. The smallest throat with a zero energy metastable vacuum then has $p/M \approx 0.025$ and $g_sM \approx 3.6$, yielding for $p = 1$ antibranes $M = 40$ and $g_s = 0.09$.

As discussed in the Introduction, one of the main applications of placing an anti-D3-brane at the tip of a warped throats a la KPV is as an uplifting mechanism. The idea is to start from a compactification with a scale-separated AdS vacuum with all moduli stabilized with potential $V_{\rm AdS}$. One then introduces the antibrane as a source of positive potential energy $V_{\rm up}$. One must be able to tune $V_{\rm up} \approx |V_{\rm AdS}|$ which in most cases requires a hierarchically small value $V_{\rm up}$ to avoid a runaway. The existence of the line of metastable vacua with zero energy in the $(g_sM, p/M)$ parameter space now provides a novel uplifting mechanism.

Trusting the potential in the regime where vacua with zero energy exist, one can clearly obtain a metastable vacuum with a hierarchically small positive energy by fine-tuning g_s to be arbitrarily close to the line (4.1) where $V_{\rm up} = 0$. For this to work one must of course have a sufficiently densely spaced in g_s discretuum of vacua in the IIB landscape. This mechanism can be seen in action by tuning close to the $g_sM = 10$ line in Fig. 3.

One may object to this uplifting mechanism on several grounds.

First, one must glue the KS throat into a compact geometry. The resulting geometry will not perfectly match the KS geometry. It seems reasonable that if the modification to the geometry is small the leading effect of this modification will be to alter the coefficients c_i appearing in (3.6) by a small amount. This is not deadly to the uplifting mechanism we propose so long as the corrections to the c_i are sufficiently small that a line of $V_{\rm up} = 0$ continues to exist. However, one may worry that the geometry at the tip is more seriously deformed and the zero energy vacua are lost. One method to achieve a modicum of safety is to remember that the warping at the tip is given by $\exp(-8\pi K/3g_sM)$ and a deep warped throat to some extent decouples the tip of the throat from the bulk geometry (see e.g. [42, 43]). When K is too small there is really no decoupling at all. However, one could demand $8\pi K > 3g_sM$ to ensure there is some amount of decoupling between tip and bulk while at the same time not making K too large to avoid problems such as the singular bulk problem in KKLT or similar issues in LVS that arise when one needs a very deep throat when one attempts to make the uplifting contribution to the potential hierarchically small through only the warping at the tip. In fact, one could combine warping and our uplifting mechanism in a mutually beneficial manner: One achieves part of the hierarchic suppression in $V_{\rm up}$ through warping but stops before one runs into issues with the throat being too big. The remainder of the hierarchy is then achieved through our mechanism; where the required fine-tuning in g_s is now reduced by several orders of magnitude; making the new mechanism in turn easier to implement as one requires a less dense discretuum in g_s.

In fact for the smallest zero vacuum energy throat $g_sM \approx 3.6$ the constraint to have some warping at the tip is that $K > 3g_sM/8\pi \approx 0.43$ which is trivially satisfied and already for $K = 1$ one has a not insubstantial amount of warping at the tip $\exp(-2.33) \approx 10^{-1}$. By choosing $K = 2, 3$ one can already obtain several orders of magnitude of warping while maintaining a reasonable D3-tadpole for the throat.

Second, as can be seen from Fig. 5, the line of zero energy metastable minima occurs when $R_{\rm NS5} \approx 1$. By attempting to fine-tune ourselves arbitrarily close to this line, we come arbitrarily close to the boundary of control and it is highly dubious how reliably our results are as all orders in α' become important. We can only hope that our results give a hint of the behaviour in this regime but clearly the abelian fivebrane picture is not suitable to analyse this regime. Ideally, one would analyse this regime in another picture such as holographically or as a nonabelian D3-brane stack. In fact, $g_s M \ll 1$ is the regime where the holographic picture is perturbatively controlled. If one could establish holographically that there exists a metastable AdS minimum somewhere in the regime $g_s M \ll 1$, $p/M \lesssim 0.025$, then this would qualitatively confirm the picture in Fig. 4 as at large $g_s M$ we know we have controlled metastable vacua with positive energy and by continuity one then expects vacua with zero energy in some intermediate regime.

Third, we have performed our analysis in a probe approximation and have not considered the backreaction of the fivebrane. Backreaction generally was commented on in [18] and we will not rehash those general remarks here. Let us make two specific points. A first point is that [44] have noted that backreaction may result in the metastable minimum being at parametrically larger R_{NS5} than the probe approximation suggests. In this case, the novel uplifting mechanism we propose may occur at better controlled R^2_{NS5} than our results suggest and it would be interesting to investigate this interplay. A second point is specific to KKLT. In recent work [45] have noted that the branes at the tip of the throat source $(0,3)$ three-form flux which in turn can give a mass to gauginos on D7-branes. If in KKLT one stabilized the Kahler moduli using D7-branes, these gauginos are required to condense for moduli stabilization and so must have a mass m_λ below the confinement scale Λ_c. It was shown by [45] that this implies

$$\frac{m_\lambda}{\Lambda_c} \approx 10^2 \frac{1}{M^{1/4}} \frac{p}{M} \frac{1}{(g_s M)^{5/4}} f(2\pi K/g_s M) \ll 1\,, \tag{4.2}$$

with $f(x) = x^{5/4} \exp\left(-19x/9\right)$. The claim is that for small throats this constraint will not be satisfied and so one must demand a sufficiently large throat for the uplift. Note however that our novel uplifting mechanism always satisfies these constraints, even for the smallest throats where the mechanism operates. The function $f(x)$ is bounded from above by 0.14881 and the smallest throats allowed by our mechanism obey $p/M \approx 0.025$ and $g_s M \approx 3.6$ such that the bound becomes

$$\frac{m_\lambda}{\Lambda} \approx 10^{-1} \frac{1}{M^{1/4}} \ll 1\,, \tag{4.3}$$

which is always satisfied by the quantization of M. It then follows that in KPV if the anti-D3-branes polarize into an NS5-brane the loss of control over α' corrections becomes important before the bound (4.2) does.

5 Deep throat phenomenology

In the previous section we discussed a novel uplifting mechanism. In this section we will for the sake of comparison instead consider the traditional mechanism where one also uplifts via p anti-D3-branes at the tip of a warped throat. However, in the traditional mechanism the uplifting condition $|V_{\text{AdS}}| \approx |V_{\text{up}}|$ is achieved solely through a large amount of warping at the tip of the throat and one demands that α' corrections to V_{up} are sufficiently small for control. This leads to the requirement that one has a large warped throat, which is a severely constraining condition on the possibility to achieve an uplift.

For KKLT with large throats there is the singular bulk problem [12–14] which generically appears deadly (see however [46]). Thus we will focus on the LVS where large warping is required in order to control the most dangerous corrections to the LVS scalar potential [15–18]. One such constraint is the Parametric Tadpole Constraint (PTC) [16] which quantifies how large the negative contribution to the D3-tadpole, Q_3, needs to be for control. In App. B we review the PTC in some detail and comment on how dangerous we expect loop corrections to the scalar potential to be to explain why these are not accounted for in the PTC in contrast to [15, 17].

It was already observed in [18] that the fundamental parameters of α' corrected KPV are p/M and g_sM. We have seen from Fig. 5 that increasing g_sM by keeping p/M as large as possible (required that $g_s < 1$) increases the control over α' corrections. The reason being that our parameter of control, the radius of the S^2 wrapped by the NS5-brane, increases. Clearly, if we demand that we have significant control, then we must uplift using a deep warped throat.

Hence g_sM is the main control parameter and it is therefore useful to reformulate the PTC such that it takes $c_e = g_sM$ As an input control parameter. This version of the PTC is derived in App. B.3 with the result that the total negative D3-tadpole of the compactification geometry Q_3 must obey the bound

$$-Q_3 > N_{\{c_N,c_e\}} = \frac{2^{8/3}}{\pi}\frac{\kappa_s^{2/3}\,c_e^2}{\xi^{2/3}\,a_s}\left(\mathcal{W}_{-1}(y)\right)^2\,,\;\; y = -\frac{3^{2/5}\,a_0^{1/4}}{8\;2^{23/40}\,5^{5/8}\,\pi^{1/40}}\,\frac{\kappa_s^{1/6}\,\xi^{1/12}}{p^{1/4}\,a_s^{1/4}\,c_N^{5/8}\,c_e^{1/4}}\,, \tag{5.1}$$

in order for the traditional de Sitter uplift with a large warped throat to be possible. Here κ_s is related to the triple self intersection number of the divisor associated to the LVS blow-up cycle, $\xi = 0.6\chi/(2\pi)^3$ with the Euler number χ of the Calabi-Yau on which is compactified, a_s stems from the nonperturbative corrections to the superpotential responsible for stabilising the Kahler moduli[5], $\mathcal{W}_{-1}(x)$ is the -1 branch of the Lambert $\mathcal{W}$ function, and $a_0 \approx 0.71805$. The control parameter c_N quantifies the control over a correction due to a varying warp factor in the bulk of the Calabi-Yau.

Since the topological quantities (a_s, ξ, κ_s) enter polynomially in (5.1), we can see that they are an essential ingredient to find suitable models where the PTC is weakest but still satisfies the desired amount of control chosen by c_e and c_N. This is also emphasized in [15, 17]. The quantities a_s and ξ should be chosen as large as possible and κ_s as small as possible to minimize $(-Q_3)_{\min}$.

Combining the results of Sect. 3 with the PTC will yield a bound on the minimal negative contribution to the D3-tadpole that respects the constraints coming from the α' corrected KPV potential.

[5] If the nonpertubative corrections are Euclidean D3-branes, one has $a_s = 2\pi$. For gaugino condensation on a stack of D7-branes, a_s depends on the gauge group, for instance $a_s = \pi/3$ for $SO(8)$.

To obtain the minimal tadpole for some examples we proceed as follows. First, we choose some reasonable numbers for a_s and κ_s, namely $a_s = 2\pi$, $\kappa_s = 0.1$. Then, to minimize $(-Q_3)_{\rm min}$, we take p/M as large as possible (such that M is as small as possible for fixed p) for a given value of g_sM. This can be read off from Fig. 5. This will fix p (which we may take as small as possible still compatible with $g_s < 1$) and g_s. These parameters together with some value of c_N will already suffice to determine ξ via (B.9) and then $(-Q_3)_{\rm min}$ using (5.1).

The minimal choice of g_sM depends on the readers notion of control. Each point in the $(g_sM, p/M)$ parameter space corresponds to a specific radius of the NS5 which determines the amount of control over higher order α' corrections. For this reason, we list $(-Q_3)_{\rm min}$ for multiple values of $\tilde{R}_{\rm NS5}$. The results are summarized in Tab. 2.

Table 2. The minimal value of the required negative contribution to the D3-tadpole $(Q_3)_{\rm min}$ for $a_s = 2\pi$ and $\kappa_s = 0.1$ for different values of the control parameter $\tilde{R}^2_{\rm NS5} = g_sM\sin^2\psi$. The choice of g_sM and p/M for a given value of $\tilde{R}^2_{\rm NS5}$ is such that the tadpole is minimal.

input parameters						$c_N = 5$		$c_N = 100$	
$\tilde{R}^2_{\rm NS5}$	p	g_sM	p/M	g_s	g_sM^2	χ	$(-Q_3)_{\rm min}$	χ	$(-Q_3)_{\rm min}$
1	1	3.8	0.025	0.095	152	14	560	20	715
2.2	1	7	0.0323	0.226	217	52	803	75	1015
3	1	9.5	0.0385	0.365	247	108	913	155	1156
5.2	1	13	0.0526	0.684	247	278	913	397	1156
9	2	20.7	0.0667	0.69	310	299	2387	422	3000

Comparing these numbers with [18] we observe that the α' corrections calculated in this work lead to higher values of $(-Q_3)_{\rm min}$. The main reason being that at small values of g_sM, also p/M has to decrease such that a metastable minimum remains. This finally leads to larger values of M (and smaller values of g_s) such that the volume of the Calabi-Yau and the tadpole in the throat increase. In contrast to [18] where the upper bound on p/M at smallish g_sM is always given by the KPV bound $p/M < 0.08$. Interestingly, as can be seen from line four and five in Tab. 2, increasing the control over α' corrections does not necessarily result in a higher tadpole. The reason for this is again that higher values of p/M are allowed when g_sM is increased. Let us also emphasize that the control parameter $\tilde{R}_{\rm NS5}$ increases only slowly at small g_sM. Thus, a slight in- or decrease in control can already have a significant impact on the resulting tadpole.

In total, the results point towards large tadpoles already at small values of g_sM in order to have control over α' corrections. This challenges LVS model-building to find Calabi-Yaus with large tadpole.

Considering the strong constraints on $(-Q_3)_{\rm min}$ in the traditional uplift with large warped throats, it becomes very attractive to study the new way of uplifting discussed in Sect. 4. There, the PTC is not applicable since $V_{\rm up}$ can be tuned exponentially small without large warping. Instead, we saw that this novel uplifting mechanism already works for $M = 40$, $K = 1$ such that for the novel uplifting mechanism one has the much more reasonable constraint $-Q_3 > KM = 40$ for the negative D3-tadpole of the compactification geometry.

6 The S-dual KS set-up

In this paper we have focused on the Klebanov-Strassler throat. One may instead consider an anti-D3 at the tip of the S-dual to the Klebanov-Strassler throat (SDKS) [22]. The advantage to this is that in the SDKS set-up one works with D5-branes for which the α' corrections to the fivebrane worldvolume action we consider are explicitly known, while for the NS5-brane in KS we had to infer the analogous corrections as discussed in Sect. 3.

In the SDKS geometry, there are K units of H_3 flux on the S^3 at the tip of the throat and M units of F_3 flux on the B-cycle. In this S-dual set-up p anti-D3-branes will puff up into a D5-brane wrapping an S^2 at the tip of the throat, rather than an NS5-brane. The radius of the tip is $\sim \sqrt{K}$ in SDKS rather than $\sim \sqrt{g_s M}$ in KS. It is straightforward to compute the α' corrected worldvolume potential for the D5-brane at the tip of the SDKS throat by going through the analysis of Sect. 3 again but now with the SDKS geometry and the action of the D5-brane. One obtains (3.6) but with $g_s M$ substituted by K and p/M substituted by p/K for the potential of the D5-brane. The analysis of Sect. 3 then also goes through for the D5, with these substitutions. For instance, Fig. 4 and Fig. 5 also show the different regimes of the D5-brane after substituting the $g_s M$ axis by a K axis. and the p/M axis by a p/K axis.

The issue now is that K and p are both integer and so we cannot achieve arbitrary points Fig. 4 and Fig. 5. Crucially, to have a metastable vacuum we require at the very least $p/K < 0.08$, which requires $K > 12.5p$. Since then at the very least $K = 13$ it is not possible to achieve a small throat in the SDKS set-up, preventing the use of a small throat uplift.

One may also repeat the analysis of Sect. 5 for a traditional uplift with a large warped throat in the LVS for the SDKS set-up.

To obtain the corresponding formulas, replace $g_s M \to K$ and $K \to g_s M$ everywhere. One finds

$$V_{\text{uplift}} = \frac{\left(3^2\, \pi^3\, 2^{22/3}\right)^{1/5}}{a_0} \frac{g_s}{K^2 \mathcal{V}^{4/3}} \mathrm{e}^{-\frac{8\pi g_s N}{3K^2}} \,. \tag{6.1}$$

The derivation of the PTC leads precisely to (B.11) but with the replacement $c_e \to K$. The same conclusion that for an uplift with a large warped throat one requires a very large negative D3-tadpole in the compactification geometry then also holds when using SDKS to uplift. However, in SDKS unlike in KS there is no hope of achieving an uplift in a small throat using the novel uplifting mechanism.

7 Conclusions and outlook

The main result of this paper is to compute as completely as we were able the α' corrected potential for the NS5-brane at the tip of a warped throat. This is given by V_{tot} in (3.6). The resulting potential matches the results of KPV in the regime where α' corrections are small, but several interesting new features appear when α' corrections become important.

At large g_sM, the condition to have a metastable KPV vacuum is $p/M \lesssim 0.08$ as in KPV. For small g_sM, this bound becomes stronger as can been seen in Fig. 4, getting as strong as $p/M \lesssim 0.025$ in the small g_sM limit.

The α' corrections generically lower the potential of the metastable vacuum compared to the tree-level result. Once the corrections become very strong, the metastable vacuum can even obtain a negative energy as seen in Fig. 4. There exists then a line of metastable vacua with zero energy. This presents the possibility of a novel uplifting mechanism: By keeping the vacuum energy of the metastable minimum positive but tuning g_s to get arbitrarily close to the line of zero-energy vacua one can obtain an arbitrarily small uplifting potential V_{up} in a small throat without needing to rely on warping. This permits an uplift for a throat with a D3-tadpole contribution as low as $N = 40$.

This is in contrast to the traditional anti-D3 uplift using exponential warping. The requirement to have a sufficiently large warped throat for the standard uplifting mechanism is generically impossible to satisfy in KKLT due to the singular bulk problem. In the LVS one requires compactification geometries with $\mathcal{O}(10^3)$ negative D3-tadpole which are difficult to obtain. The fact that these issues are avoided in our new small throat uplifting mechanism makes the new uplifting mechanism we propose in our opinion very promising.

The main issue with the small throat uplift is that it relies on α' corrections to achieve a very small V_{up}. As a result, at the metastable minimum the α' corrections to the potential are of the same order of magnitude as the tree-level potential. By the very nature of our set-up, this uplift then drives us near the boundary of control over the perturbation series of α' corrections as seen in Fig. 5.

One way to deal with this issue is to continue computing higher order α' corrections until one is confident that one has control despite being near (but not past) the boundary where the perturbation series in α' breaks down. In a way this is what we have initiated here. In [18] it was suggested that the new uplifting mechanism might be possible, but this could not explicitly be checked as not taking into account α' flux corrections led to some clearly unphysical behaviour and we had to speculate how this unphysicality would resolve. The explicit computations we have done here confirm the qualitative behaviour guessed at in [18]. Unfortunately, to compute even higher order α' corrections to the fivebrane worldvolume seems a rather grueling task, especially as we have accounted for all α' corrections to the worldvolume whose explicit form we were able to find in the literature. Any further terms one then first has to derive the form of and then compute in the KS throat background.

However, there is also good news when it comes to control. First, the abelian NS5-brane analysis is just one perspective on the metastable KPV vacuum. One may also perform the analysis from the perspective of either a nonabelian stack of anti-D3-branes or a holographic perspective. In fact, the regime $g_sM \ll 1$ where the α' perturbation theory breaks down for the NS5-brane is precisely the regime where the holographic picture is under good perturbative control. If one could holographically establish that there exists a metastable vacuum with negative or zero energy for $g_sM \lesssim 1$, $p/M \lesssim 0.025$ as predicted in Fig. 4, then under the assumption that the potential at the metastable minimum is continuous in g_sM and p/M, it is guaranteed that there exist metastable vacua with zero energy and our new uplifting mechanism works. Second, the issues with control in our analysis seems isolated to the tip of the throat, which in compactifications one can isolate from the rest of the geometry by an intermediate amount of warping. This is in contrast to uplifts with a large warped throat which can strongly affect and destroy the entire compactification geometry as for instance with the singular bulk problem in KKLT [12–14].

We believe that our work shows very encouraging evidence that a de Sitter uplift using α' corrections for anti-D3-branes in small warped throats is possible. This avoids the issues with uplifts using large warped throats. Questions concerning the amount of control in our set-up remain. Many future directions to analyse our set-up are open and we hope to continue this story in the future.

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A Flux corrections to branes and their evaluation at the tip of the throat

In this appendix, we evaluate the α' corrections calculated in [23–33] at the tip of the throat. To do so, we need the KS fluxes close to the tip of the throat. They are given by[6]

$$B_2 \supset \frac{g_sM\alpha'\tau}{6} g^3 \wedge g^4 + \mathcal{O}(\tau^2)\,, \tag{A.1}$$

$$H_3 \supset \frac{g_sM\alpha'}{6} \mathrm{d}\tau \wedge g^3 \wedge g^4 + \frac{g_sM\alpha'\tau}{12} g^5 \wedge \left(g^1 \wedge g^3 + g^2 \wedge g^4\right) + \mathcal{O}(\tau^2)\,, \tag{A.2}$$

$$F_3 \supset \frac{M\alpha'}{2} g^5 \wedge g^3 \wedge g^4 + \frac{M\alpha'\tau}{12} \mathrm{d}\tau \wedge \left(g^1 \wedge g^3 + g^2 \wedge g^4\right) + \mathcal{O}(\tau^2)\,, \tag{A.3}$$

$$\tilde{F}_5 \supset \left(\frac{\tau}{3^{4/3} g_s^3 M^2 a_0^2} + \mathcal{O}(\tau^3)\right) \mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3 \wedge \mathrm{d}\tau + \mathcal{O}(\tau^2)\,, \tag{A.4}$$

$$\begin{aligned} H_7 \supset &- \frac{1}{2^{5/3} a_0 g_s^3 M} \mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3 \wedge g^3 \wedge g^4 \wedge g^5 + \\ &\frac{\tau}{12\, 2^{5/3} a_0 g_s^3 M} \mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3 \wedge \mathrm{d}\tau \wedge \left(g^1 \wedge g^3 + g^2 \wedge g^4\right) + \mathcal{O}(\tau^2)\,, \end{aligned} \tag{A.5}$$

where the one forms $g^{1\ldots5}$ (see for instance [47]) parametrize the $T^{1,1}$.

Before explicitly evaluating α' corrections to the worldvolume action of a fivebrane wrapping an S^2 at the tip of the throat, we list some important properties of the fluxes that will make many corrections vanish.

[6] We only display terms which are relevant to our calculations.

a) There are only a few components of the fluxes that are non-vanishing at the tip. Hence, a wrong number of tangential and/or normal indices will make terms vanish.

b) F_3, C_2 and F_2 are covariantly constant along the S^2. That means that[7] $\nabla_\alpha \mathcal{F}_{2\,\alpha\beta} = 0$ and $\nabla_\alpha F_{3\,\beta\gamma a} = 0$.

c) The covariant derivative of F_3 with respect to a normal index is always zero at the tip except for the term $\nabla_\psi F_{3\,\theta\varphi\psi}$ where (θ, φ) parametrize the non-shrinking S^2 at the tip and ψ the additional direction inside the S^3.

d) If covariant derivatives with respect to a normal index of form fields appear then components linear in τ can be non-vanishing at the tip if the terms in the action have the correct index structure.

A.1 Flux corrections that vanish for the KS set up

Let us start with flux corrections to the DBI action. There are α'^2 corrections involving only covariant derivatives of H_3 [23, 28]

$$S_{\mathrm{D}p,\mathrm{DBI}} \supset \frac{\mu_p}{g_s}\frac{\pi^2\alpha'^2}{48}\int \mathrm{d}^{p+1}\xi\sqrt{-(g+2\pi\alpha'\mathcal{F}_2)}\bigg[-\frac{1}{6}\nabla_\alpha H_{abc}\nabla^\alpha H^{abc} - \frac{1}{3}\nabla_a H_{\alpha\beta\gamma}\nabla^a H^{\alpha\beta\gamma} + \frac{1}{2}\nabla_\alpha H_{\beta\gamma a}\nabla^\alpha H^{\beta\gamma a}\bigg], \tag{A.6}$$

where $2\pi\alpha'\mathcal{F}_{2\,\alpha\beta} = 2\pi\alpha' F_{2\,\alpha\beta} + B_{2\,\alpha\beta}$ for Dp-branes and $2\pi\alpha'\mathcal{F}_{2\,\alpha\beta} \to 2\pi g_s\mathcal{F}_{2\,\alpha\beta} = 2\pi g_s\alpha' F_{2\,\alpha\beta} - g_s C_{2\,\alpha\beta}$ for the NS5-brane. Evaluating (A.6) at the tip of the throat for an NS5-brane (where $H_3 \to -F_3$ and the correct g_s scaling can be inferred from analogous considerations as in Sect. 3.1) gives zero due to property b) and since the term $\nabla_\psi F_{3\,\theta\varphi\psi}$ does not show up (see property c)).

Next, in [29, 31] corrections including $\mathcal{F}_2$ are computed[8]:

$$\begin{aligned} S_{\mathrm{D}p,\mathrm{DBI}} \supset \frac{\pi^2\alpha'^2\mu_p}{12 g_s}\int \mathrm{d}^{p+1}\xi\sqrt{-(g+2\pi\alpha'\mathcal{F})}\bigg[& R_{\beta\delta}\left(\nabla_\alpha\mathcal{F}^{\alpha\beta}\nabla_\gamma\mathcal{F}^{\gamma\delta} - \nabla_\alpha\mathcal{F}_\gamma{}^\delta\nabla^\gamma\mathcal{F}^{\alpha\beta}\right) \\ &+ \frac{1}{2}R_{\beta\delta\gamma\epsilon}\nabla^\gamma\mathcal{F}^{\alpha\beta}\nabla^\epsilon\mathcal{F}_\alpha{}^\delta + \Omega^a{}_\alpha{}^\alpha\nabla_\delta H_\gamma{}^\delta{}_a\nabla_\beta\mathcal{F}^{\beta\gamma} \\ &+ \frac{1}{4}R_\delta{}^\delta\left(\nabla_\alpha\mathcal{F}^{\alpha\beta}\nabla_\gamma\mathcal{F}_\beta{}^\gamma + \nabla_\beta\mathcal{F}_\alpha{}^\gamma\nabla_\gamma\mathcal{F}^{\alpha\beta}\right) \\ &- \Omega^{a\beta\alpha}\left(\nabla_\beta\mathcal{F}_\alpha{}^\gamma\nabla_\delta H_\gamma{}^\delta{}_a + \nabla^\delta\mathcal{F}_\alpha{}^\gamma\nabla_a H_{\beta\gamma\delta} - \frac{1}{2}\nabla^\delta\mathcal{F}_\alpha{}^\gamma\nabla_\gamma H_{\beta\delta a}\right)\bigg], \end{aligned} \tag{A.7}$$

where $R_{\alpha\beta\gamma\delta}$ is the Riemann tensor and $R_{\alpha\beta}$ the Ricci tensor. These terms also vanish since F_2 and C_2 are covariantly constant on the S^2 at the tip (property b)). For the same reason the

[7] We work in conventions where $(\alpha, \beta, \gamma, \cdots)$ are indices tangent to the brane and $(a, b, c, \cdots)$ are indices normal to the brane.

[8] We abbreviate $\mathcal{F}_2$ with $\mathcal{F}$ to not clutter notation.

corrections in [30, 32] vanish at the tip of the throat (except six terms of the form $\mathcal{F}_2\Omega^2\nabla H_3$ in [32] that will be calculated in App. A.2).

Let us move on to corrections of the Chern-Simons (CS) action. The tree level Chern-Simons action reads

$$S_{\mathrm{CS,D}p} = \mu_p \int\limits_{\mathcal{M}_{p+1}} \mathrm{Tr}\left(\mathrm{e}^{2\pi\alpha'\mathcal{F}_2}\right) \wedge \sqrt{\frac{\hat{A}(4\pi^2\alpha' R_T)}{\hat{A}(4\pi^2\alpha' R_N)}} \wedge \bigoplus_q C_q \Bigg|_{p+1} . \tag{A.8}$$

This can be written in a more explicit form by expanding the exponential and the A-roof genus $\hat{A}(R_i)$ of the Riemann curvature 2-form of the normal or tangent bundle and then taking the right C_q form such that the integrand matches a $p+1$ form. For the NS5-brane at the tip of the throat, the only non-vanishing components are $B_6 + \mathcal{F}_2 \wedge C_4$ which are already included in the analysis of [1]. Hence, there are no additional contributions from (A.8) to the scalar potential.

Moreover, there are α' corrections to the CS action which are computed in [24–26, 29, 31]. When carefully taking into account properties a) to d), it turns out that all these terms vanish at the tip of the throat (except one term of the form $\epsilon\mathcal{F}_2 R\nabla\tilde{F}_5$ and two terms of the form $\epsilon\mathcal{F}_2\nabla H_3\nabla F_7$ of [31] which will be computed in App. A.2).

A.2 Non-zero flux corrections

In [27, 28] α'^2 corrections to the O-plane action are calculated. As explained in Sect. 3 these are also present on Dp-branes. The corresponding action reads[9]

$$\begin{aligned} S_{\mathrm{D}p,\mathrm{DBI}} \supset & \frac{\mu_p}{g_s}\frac{\pi^2\alpha'^2}{48}\int \mathrm{d}^{p+1}\xi\sqrt{-(g+2\pi\alpha'\mathcal{F}_2)}\Big[H^{\alpha\beta a}H_{\alpha}{}^{\gamma}{}_{a}(R_T)_{\beta\gamma} - \frac{3}{2}H^{\alpha\beta a}H_{\alpha\beta}{}^{b}\,\overline{R}_{ab} \\ & + \frac{1}{2}H^{abc}H_{ab}{}^{d}\,\overline{R}_{cd} - H^{\alpha\beta a}H^{\gamma\delta}{}_{a}(R_T)_{\alpha\beta\gamma\delta} + H^{\alpha\beta a}H_{a}{}^{bc}(R_N)_{\alpha\beta bc} \\ & - \frac{1}{4}H^{\alpha\beta a}H_{\alpha\beta}{}^{b}H_{a}{}^{cd}H_{bcd} + \frac{1}{4}H^{\alpha\beta a}H_{\alpha\beta}{}^{b}H^{\gamma\delta}{}_{a}H_{\gamma\delta b} \\ & + \frac{1}{8}H^{\alpha\beta a}H_{\alpha}{}^{\gamma b}H_{\beta}{}^{\delta}{}_{b}H_{\gamma\delta a} - \frac{1}{6}H^{\alpha\beta a}H_{\alpha}{}^{\gamma b}H_{\beta\gamma}{}^{c}H_{abc} + \frac{1}{24}H^{abc}H_{a}{}^{de}H_{bd}{}^{f}H_{cef}\Big], \end{aligned} \tag{A.9}$$

where (R_T), (R_N) and $\overline{R}$ are defined as [19]

$$(R_T)_{\alpha\beta\gamma\delta} = R_{\alpha\beta\gamma\delta} + g_{ab}(\Omega^a_{\alpha\gamma}\Omega^b_{\beta\delta} - \Omega^a_{\alpha\delta}\Omega^b_{\beta\gamma})\,, \tag{A.10}$$

$$(R_N)_{\alpha\beta}{}^{ab} = -R^{ab}{}_{\alpha\beta} + g^{\gamma\delta}(\Omega^a_{\alpha\gamma}\Omega^b_{\beta\delta} - \Omega^b_{\alpha\gamma}\Omega^a_{\beta\delta})\,, \tag{A.11}$$

$$\overline{R}_{ab} = \hat{R}_{ab} + g^{\alpha\alpha'}g^{\beta\beta'}\Omega_{a\,\alpha\beta}\Omega_{b\,\alpha'\beta'}\,, \tag{A.12}$$

where $\Omega^{\mu}_{\alpha\beta}$ is the second fundamental form, and $\hat{R}_{ab} = R^{\alpha}{}_{a\alpha b}$.

[9] As before, we assume that the couplings extend to non-geodesically embedded branes and abbreviate H_3 by H.

In the case of the KS throat, we extend (A.9) to NS5-branes leading us schematically to the first two terms of (3.3) from which we can read off the correct g_s scaling. Using the KS flux (A.3) in the spherical parametrization of the deformed conifold at the tip of the throat [48] (see also [18]), we find

$$S_{\mathrm{NS5,DBI}} \supset -\frac{1}{(g_s M)^2}\left(c_3 + c_4 \cot^2\psi\right)\frac{\mu_5}{g_s^2}\int \mathrm{d}^6\xi\sqrt{-(g+2\pi\alpha' g_s \mathcal{F}_2)}\,, \tag{A.13}$$

where $c_3 = \frac{\pi^2}{48}\left(\frac{45}{4I(0)^3} - \frac{12\,6^{1/3}}{I(0)^2}\right) \approx 2.44781$ and $c_4 = \frac{\pi^2}{48}\frac{12\,6^{1/3}}{I(0)^2} \approx 8.69589$ with $I(0) \approx 0.71805$. The coefficient c_3 is smaller than c_4 since the F_3^4 terms compete with the $F_3^2 R$ terms whereas c_4 is only due to $F_3^2\Omega^2$ couplings.

In [33] couplings of the form Ω^4 and $\Omega^4 F_2^2$ are calculated where Ω is the second fundamental form. The couplings are (see equ. (51) in [33])

$$\begin{aligned} S_{\mathrm{D}p,\mathrm{DBI}} \supset -\frac{\mu_p}{g_s}\frac{\pi^2\alpha'^2}{24}\int \mathrm{d}^{p+1}\xi\sqrt{-g}\Big[& 9\Omega^{a}{}_{\alpha}{}^{\varepsilon}\Omega^{b}{}_{\beta}{}^{\eta}\Omega_{b\gamma\varepsilon}\Omega_{a\delta\eta}(2\pi\alpha' F_2^{\alpha\beta})(2\pi\alpha' F_2^{\gamma\delta}) \\ & + 2\Omega_{a\gamma}{}^{\varepsilon}\Omega^{a\gamma\delta}\Omega^{b}{}_{\delta}{}^{\eta}\Omega_{b\varepsilon\eta} - 2\Omega^{b}{}_{\gamma\delta}\Omega^{a\gamma\delta}\Omega_{b\varepsilon\eta}\Omega_{a}{}^{\varepsilon\eta} \\ & + \frac{1}{4}\left(2\Omega_{a\gamma}{}^{\varepsilon}\Omega^{a\gamma\delta}\Omega^{b}{}_{\delta}{}^{\eta}\Omega_{b\varepsilon\eta} - 2\Omega^{b}{}_{\gamma\delta}\Omega^{a\gamma\delta}\Omega_{b\varepsilon\eta}\Omega_{a}{}^{\varepsilon\eta}\right)(2\pi\alpha' F_{2\alpha\beta})(2\pi\alpha' F_2^{\alpha\beta}) \\ & + \underbrace{\cdots}_{\text{12 similar couplings with different contractions}} + \mathcal{O}(\Omega^4 F_2^4)\Big]\,. \end{aligned} \tag{A.14}$$

The terms in the second line are precisely the pure α'^2 curvature terms involving only the second fundamental form that were computed in [19]. We observe that the index structure of the terms in the third line equals the index structure of the second line – the field strength tensors are contracted among themselves. This means that we can identify the second and third line with the first two terms of the expansion of $\sqrt{-(g+2\pi\alpha' F_2)}\Omega^4$ which is already captured by the curvature correction of [19] calculated for the KS throat in [18][10]. Dropping the pure curvature terms, the novel coupling terms in (A.14) read

$$\begin{aligned} S_{\mathrm{D}p,\mathrm{DBI}} \supset -\frac{\mu_p}{g_s}\frac{\pi^2\alpha'^2}{24}(2\pi)^2\int \mathrm{d}^{p+1}\xi\sqrt{-(g+2\pi\alpha'\mathcal{F}_2)}\Big[& 9\Omega^{a}{}_{\alpha}{}^{\varepsilon}\Omega^{b}{}_{\beta}{}^{\eta}\Omega_{b\gamma\varepsilon}\Omega_{a\delta\eta}\mathcal{F}_2^{\alpha\beta}\mathcal{F}_2^{\gamma\delta} \\ & + \underbrace{\cdots}_{\text{12 similar couplings with different contractions}}\Big]\,. \end{aligned} \tag{A.15}$$

Note that we extended the couplings by replacing $\sqrt{-g} \to \sqrt{-(g+2\pi\alpha'\mathcal{F}_2)}$ which means that we assume an infinite tower of (so far not calculated) couplings of the form $\Omega^4\mathcal{F}_2^n$ rearranging in such a form to reproduce $\sqrt{-(g+2\pi\alpha'\mathcal{F}_2)}$. These terms can again be evaluated at the tip of the KS throat. Choosing the worldvolume F_2 flux as in KPV and the g_s scaling as in

[10] Note that this is one reason why we assume that the couplings extend to $2\pi\alpha' F_2 \to 2\pi\alpha'\mathcal{F}_2$.

(3.3), they yield

$$S_{\text{NS5,DBI}} \supset -\frac{\cot^4\psi\, c_5}{(g_sM)^2\sin^4\psi}\left(\frac{\pi p}{M}-\left(\psi-\frac{\sin(2\psi)}{2}\right)\right)^2\frac{\mu_5}{g_s^2}\int \mathrm{d}^6\xi\sqrt{-(g+2\pi\alpha' g_s\mathcal{F}_2)}\,, \quad \text{(A.16)}$$

where we wrote the F_2 flux number p as $p=(p/M)(g_sM)/g_s$ and $c_5=\frac{\pi^2}{6}\frac{45\,3^{1/3}}{4\,2^{2/3}I(0)^2}\approx 32.6096$. The parametrics of this result is understood easily: $\Omega^4 \sim \cot^4\psi/R^4_{S^3}$, $F_2\sim p/R^2_{S^2}$ and $C_2\sim M(\psi-\sin(2\psi)/2)/R^2_{S^2}$ with $R^2_{S^2}=R^2_{S^3}\sin^2\psi\sim(g_sM)\sin^2\psi$.

Next, we evaluate the corrections to the DBI action calculated in [32] at the tip of the throat. As explained above, the only non-vanishing corrections for the KS throat are given by

$$\begin{aligned} S_{\text{D}p,\text{DBI}} \supset -\frac{\pi^2\alpha'^2\mu_p}{96g_s}\int \mathrm{d}^{p+1}\xi\sqrt{-(g+2\pi\alpha'\mathcal{F}_2)}\Big[&5(2\pi\alpha'\mathcal{F}_2^{\alpha\beta})\Omega^{a\gamma}{}_{\alpha}\Omega^{b}{}_{\delta}{}^{\delta}\Big(\nabla_a H_{\beta\gamma b}\\ &-\nabla_b H_{\beta\gamma a}\Big)+(2\pi\alpha'\mathcal{F}_2^{\alpha\beta})\Omega^{a\gamma}{}_{\alpha}\Omega^{b\delta}{}_{\gamma}\left(\nabla_a H_{\beta\delta b}-\nabla_b H_{\beta\delta a}\right)\\ &+(2\pi\alpha'\mathcal{F}_2^{\alpha\beta})\Omega^{a}{}_{\gamma}{}^{\gamma}\Omega^{b}{}_{\delta}{}^{\delta}\nabla_b H_{\alpha\beta a}-(2\pi\alpha'\mathcal{F}_2^{\alpha\beta})\Omega^{b}_{\delta\gamma}\Omega^{a\delta\gamma}\nabla_b H_{\alpha\beta a}\Big]\,. \end{aligned} \quad \text{(A.17)}$$

For the KS throat, the terms in the first two lines pairwise cancel against each other due to property c). The non-vanishing contribution then comes from the last line and yields for the NS5-brane

$$S_{\text{NS5,DBI}} \supset \frac{\cot^3\psi\, c_6}{(g_sM)^2\sin^2\psi}\left(\frac{\pi p}{M}-\left(\psi-\frac{\sin(2\psi)}{2}\right)\right)\frac{\mu_5}{g_s^2}\int \mathrm{d}^6\xi\sqrt{-(g+2\pi\alpha' g_s\mathcal{F}_2)}\,, \quad \text{(A.18)}$$

with $c_6=\frac{6^{1/3}\pi^2}{16I(0)^2}\approx 2.17397$ where the parametrics can be understood from $g_s\nabla F_3\sim g_sM\cot\psi/R^4_{S^3}$.

Additionally to the non-vanishing contributions from the DBI action for the KS throat, we found one non-vanishing α'^2 coupling from the Chern-Simons action that includes $\tilde{F}_5$ and two couplings that involve H_7. Both can be found in [31]. The $\tilde{F}_5$ coupling term on the NS5 is given by

$$S_{\text{NS5,CS}} \supset -\frac{\pi^2\alpha'^2\mu_5}{24g_s^2}\frac{1}{4!}\int \mathrm{d}^6\xi\,\epsilon^{\alpha_0\alpha_1\cdots\alpha_5}(2\pi\alpha' g_s\mathcal{F}_{2\,\alpha_0\alpha_1})\overline{R}^{ab}\nabla_a(g_s\tilde{F}_5)_{b\alpha_2\alpha_3\alpha_4\alpha_5}\,, \quad \text{(A.19)}$$

where $\epsilon^{\alpha_0\alpha_1\ldots\alpha_5}$ is the Levi-Civita symbol on the worldvolume of the NS5-brane. This can be evaluated at the tip yielding

$$\begin{aligned} S_{\text{NS5,CS}} &\supset -\left(-\frac{c_7\left(\psi-\sin(2\psi)/2\right)}{(g_sM)^2\sin^2\psi}+\frac{c_7\pi(p/M)}{(g_sM)^2\sin^2\psi}\right)\frac{\mu_5}{g_s^2}\int \mathrm{d}^6\xi\sqrt{-g_6}\\ &=-\frac{4\pi\mu_5 M}{g_s}\frac{c_7}{(g_sM)^2}\left(\frac{\pi p}{M}-\left(\psi-\frac{\sin(2\psi)}{2}\right)\right)\int \mathrm{d}^4x\sqrt{-g_4} \end{aligned} \quad \text{(A.20)}$$

where $I''(0) = -2^{2/3}/3^{4/3}$ and $c_7 = \frac{\pi^2\, 6^{2/3}(2I(0)-3I''(0))}{18I(0)^{7/2}} \approx 14.6396$.

The H_7 coupling terms read (again already for the NS5 where $F_7 \to H_7$)

$$S_{\mathrm{NS5,CS}} \supset \frac{\pi^2\alpha'^2\mu_5}{48g_s^2}\frac{1}{6!}\int \mathrm{d}^6\xi\epsilon^{\alpha_0\alpha_1\cdots\alpha_5}\Bigg[(2\pi\alpha' g_s\mathcal{F}_2)^{\alpha\beta}\nabla^a(-g_sF_3)_{\alpha\beta}{}^b\nabla_b(g_s^2H_7)_{a\alpha_0\cdots\alpha_5} \\ + (2\pi\alpha' g_s\mathcal{F}_2)^{\alpha\beta}\nabla^a(g_sF_3)_{\alpha\beta}{}^b\nabla_a(g_s^2H_7)_{b\alpha_0\cdots\alpha_5}\Bigg]\,, \tag{A.21}$$

which gives at the tip

$$S_{\mathrm{NS5,CS}} \supset -\frac{4\pi\mu_5 M}{g_s}\frac{c_8\cot\psi}{(g_sM)^2\sin\psi}\left(\frac{\pi p}{M} - \left(\psi - \frac{\sin(2\psi)}{2}\right)\right)\int \mathrm{d}^4x\sqrt{-g_4}\,, \tag{A.22}$$

where $c_8 = 2\frac{\pi^2\, 6^{2/3}}{I(0)^{5/2}} \approx 18.6475$.

Taking all non-vanishing corrections together, the scalar potential following from (A.13), (A.16), (A.18), (A.20) and (A.22) reads

$$\begin{aligned} V_{\mathrm{flux}} =& \frac{4\pi\mu_5 M}{g_s}\sqrt{b_0^4\sin^4(\psi) + \left(p\frac{\pi}{M} - \psi + \frac{1}{2}\sin(2\psi)\right)^2} \times \frac{1}{(g_sM)^2}\Bigg[c_3 \\ &+ c_4\cot^2\psi + \frac{c_5\cot^4\psi}{\sin^4\psi}\left(\frac{\pi p}{M} - \left(\psi - \frac{\sin(2\psi)}{2}\right)\right)^2 \\ &- \frac{c_6\cot^3\psi}{\sin^2\psi}\left(\frac{\pi p}{M} - \left(\psi - \frac{\sin(2\psi)}{2}\right)\right)\Bigg] \\ &+ \left[\frac{4\pi^2 p\mu_5}{g_s} - \frac{4\pi\mu_5 M}{g_s}\left(\psi - \frac{\sin(2\psi)}{2}\right)\right]\left(\frac{c_7}{(g_sM)^2} + \frac{c_8\cot\psi}{(g_sM)^2\sin\psi}\right)\,. \end{aligned} \tag{A.23}$$

B Tadpole constraints in the Large Volume Scenario

In this appendix we first review the PTC [16] and then give a reformulated version of it such that constraints from α' corrected KPV can naturally be implemented. In addition we briefly discuss a loop correction used to constrain the LVS in [15, 17] and explain why we believe it to be less important than discussed in these papers.

B.1 The Parametric Tadpole Constraint

To begin, we quickly review the basic set up of the LVS and the most important ingredients of the PTC. For more details consult [16]. We work in type IIB string theory compactified to 4D on a Calabi-Yau orientifold. In the minimalist case, the Calabi-Yau has two Kahler moduli, a big cycle τ_b and a small cycle τ_s. The volume is given by $\mathcal{V} = \tau_b^{3/2} - \kappa_s\tau_s^{3/2}$ in units of $l_s = 2\pi\sqrt{\alpha'}$ where $\kappa_s = \sqrt{2}/(\sqrt{3}\kappa_{sss})$ and κ_{sss} is the triple self intersection number of the small divisor. The corresponding superpotential is given by

$$W = W_0 + A_s \mathrm{e}^{-a_s T_s}\,, \tag{B.1}$$

where $\mathrm{Re}(T_s) = \tau_s$ and A_s a model-dependent prefactor. W_0 is induced by fluxes and a_s determines whether the nonperturbative correction to W is coming from ED3-branes ($a_s = 2\pi$) or from gaugino condensation on D7-branes (for an $SO(8)$ gauge group, $a_s = \pi/3$). The Kahler potential reads [10, 49]

$$K = -2\ln\left(\mathcal{V} + \frac{\xi}{2g_s^{3/2}}\right) = -2\ln\left(\tau_b^{3/2} - \kappa_s\tau_s^{3/2} - \frac{\chi\,\zeta(3)}{4(2\pi)^3 g_s^{3/2}}\right)\,, \tag{B.2}$$

where g_s is the string coupling, χ the Euler number of the Calabi-Yau and $\zeta(3) \approx 1.2$. The correction proportional to $g_s^{-3/2}$ is due to [49]. At the minimum of the scalar potential

$$\mathcal{V} = \frac{3\kappa_s|W_0|\sqrt{\tau_s}}{4a_s|A_s|}\mathrm{e}^{a_s\tau_s}\,, \qquad \tau_s = \frac{\xi^{2/3}}{(2\kappa_s)^{2/3}g_s} + \mathcal{O}(1)\,. \tag{B.3}$$

For deriving the PTC, higher F-term corrections [50, 15] and a correction coming from the combination of higher curvature corrections and a varying warp factor [15, 16] are taken into account. Demanding the size of the corrections to be small compared to the LVS AdS minimum leads to the definition of the control parameters c_{W_0} and c_N which should be large for parametric control over the higher F-term or varying warp factor correction, respectively, [16]:

$$1 = c_{W_0}\frac{16a_s}{3(2\kappa_s)^{2/3}\xi^{1/3}}\,\frac{W_0^2}{\mathcal{V}^{2/3}}\,, \qquad 1 = c_N\frac{10\,a_s\,\xi^{2/3}}{(2\kappa_s)^{2/3}g_s}\,\frac{N}{\mathcal{V}^{2/3}}\,. \tag{B.4}$$

Additionally to the PTC, there is a bound on Q_3 given by [51]

$$-Q_3 \geq 4\pi\frac{g_s W_0^2}{2}\,, \tag{B.5}$$

which can be used to determine $c^*_{W_0}(c_N)$ such that some minimal quality of control is ensured [16]. This leads to $c^*_{W_0}(c_N) = 15\pi\xi c_N/4$ such that the minimal tadpole required by the PTC reads[11]

$$-Q_3 > N = -\frac{21c_M}{16\pi}\mathcal{W}_{-1}(x)\,, \qquad x = -\frac{3^{11/35}\,a_0^{2/7}}{7\;2^{59/105}\,5^{5/7}\,\pi^{1/35}}\,\frac{\kappa_s^{2/7}}{p^{2/7}\,a_s^{3/7}\,c_N^{5/7}\,c_M^{1/7}}\,, \tag{B.6}$$

where $\mathcal{W}_{-1}(x)$ is the -1 branch of the Lambert $\mathcal{W}$ function and we defined the control parameter $c_M = g_s M^2$.

[11] Note that we generalized the PTC for uplifting with p anti-D3-branes.

B.2 Loop Corrections

In this section we will consider a loop correction to the LVS de Sitter uplift which was not taken into account when deriving the PTC but was considered in [15, 17] and we will discuss the relative importance of this loop correction.

The Kahler potential for the Kahler moduli will generically be loop corrected, see e.g. [52–57]. A detailed study of such loop corrections and their relative importance was recently given in [35].

We will focus only on the most dangerous loop corrections which are loop corrections to the blowup cycle as they are on-shell in the LVS only suppressed by g_s^2 (instead of the volume $g_s^{3/2}\mathcal{V}^{-1/3}$) compared to the leading order terms in the scalar potential [35].

From this it directly follows that loop corrections in the LVS will be small as soon as g_s^2 is sufficiently small. This can be made more precise by formulating this condition in terms of a control parameter. A reasonable way to define the control parameter is to measure the size of the loop correction to the blowup cycle compared to its leading order value (B.3). Using the results of [15] this can for instance be specified for a 'KK-type'[12] loop correction to the Kahler potential of the form [55]

$$\delta K = C_s^{\mathrm{KK}} \frac{g_s\sqrt{\tau_s}}{\mathcal{V}}\,, \tag{B.7}$$

where C_s^{KK} is some unknown prefactor. This leads to a correction[13] to the on-shell value of τ_s in (B.3) of the form $C_s^{\mathrm{KK}} g_s/(3\kappa_s)$. The control parameter is hence defined as

$$1 = c_{\mathrm{loop}}\,\frac{2^{2/3}g_s^2}{3\kappa_s^{1/3}\xi^{2/3}}\,, \tag{B.8}$$

and we demand $c_{\mathrm{loop}} \gg 1$ for control.

This is a much weaker constraint than the related one of [17]. They demand $\lambda_6 = a_s g_s/(3\kappa_s) \ll 1$. This requirement follows from demanding that the size of the loop correction is smaller than one. We believe it is only physically meaningful to compare the loop correction to the leading order term as done in (B.8), and not to compare the size of the loop correction to unity. We therefore propose to use (B.8) instead of λ_6.

Furthermore, as recently discussed in [35], there are settings where loop corrections are assumed to be absent. For example, the loop correction (B.7) is absent if no D7-brane wraps the blowup cycle τ_s. Then, a 'KK-type' loop corrections can not be induced since the $M_{10}^2 g_s R_8^2$ operator on the D7 and an Einstein-Hilbert term on an intersection 2-cycle $\sim \sqrt{\tau_s}$ is absent [35]. Additionally to (B.7) there are 'Winding-type' loop corrections [55] or more generally genuine loop corrections [35] to the blowup cycle. Such corrections appear more generally and are (probably) only absent if there is $\mathcal{N} = 2$ SUSY locally at the blowup cycle. Hence, it should be possible to find models not featuring loop corrections to the small blowup cycle.

[12] We refer the reader to [35] for a classification of the different types of loop corrections.

[13] Note that also winding type loop correction to the blow-up cycle will contribute at the same order in g_s.

As discussed in [35], one may attempt to estimate the numerical prefactor in the expansion parameter of loop corrections and hence their $1/2\pi$ suppression. Such a suppression can play a major role since (B.8) would be weakened significantly such that parametric control over loop corrections could be obtained much earlier than expected. For some toric geometries the expansion parameter is calculated explicitly in [53] and a $1/(2\pi)^4$ suppression is found. For generic Calabi-Yaus naive dimensional analysis [58] in a 4d approach reveals a suppression by $1/16\pi^2$ [35] without evaluating the sum over KK modes which can lead to a further suppression (see e.g. Sect. II in [59]).

As these loop corrections are g_s^2 suppressed and we expect them to be suppressed by a numerical prefactor generically no larger than $1/16\pi^2$, we shall assume they can usually be neglected in our analysis. In cases where $g_s \leq 1$ for some choice of M and one does not want to rely on a small numerical prefactor of loop corrections, it is still possible to increase M and keep g_sM constant to decrease g_s. This is of course at the expense of increasing the D3-tadpole.

B.3 The bound from g_sM

The constraints coming from the α' corrected KPV potential can naturally be thought of as a bound on g_sM since g_sM is the main control parameter of the potential.

We hence aim to rewrite (B.6) such that it takes $c_e \equiv g_sM$ instead of $c_M = g_sM^2$ as an input parameter. As usual, we treat c_e as a control parameter where large c_e increases the control over α' corrections. This can be done using[14]

$$c_e = g_sM = \sqrt{g_s c_M} = \left(\frac{9a_s}{16\pi}\right)^{1/2} \left(\frac{\xi}{2\kappa_s}\right)^{1/3} \frac{c_M}{\sqrt{N}}\,, \tag{B.9}$$

where we used

$$g_s = \frac{9a_s}{16\pi}\left(\frac{\xi}{2\kappa_s}\right)^{2/3}\frac{c_M}{N} \quad \Longleftrightarrow \quad \frac{g_s}{p} = \frac{9a_s}{16\pi}\left(\frac{\xi}{2\kappa_s}\right)^{2/3}\frac{c_e}{N}\frac{M}{p}\,, \tag{B.10}$$

following from a relation $\tau_s(N)$ derived in [16] from comparing the volume in (B.3) with the volume expressed in terms of the uplift potential.

The version of the PTC which takes c_e instead of c_M as input is then derived by using (B.9) in (B.6). We end up with

$$-Q_3 > N_{\{c_N,c_e\}} = \frac{2^{8/3}}{\pi}\frac{\kappa_s^{2/3}\,c_e^2}{\xi^{2/3}\,a_s}\left(\mathcal{W}_{-1}(y)\right)^2\,,\ y = -\frac{3^{2/5}\,a_0^{1/4}}{8\;2^{23/40}\,5^{5/8}\,\pi^{1/40}}\,\frac{\kappa_s^{1/6}\,\xi^{1/12}}{p^{1/4}\,a_s^{1/4}\,c_N^{5/8}\,c_e^{1/4}}\,. \tag{B.11}$$

Let us now compare the two versions of the PTC (B.6) and (B.11). In (B.6) the only parameter that enters non-logarithmically is c_M whereas in (B.11) c_e and the topological quantities enter polynomially. If one considers (B.6) with a given c_M, the value of c_e is

[14] Note that c_e depends on c_N through N logarithmically. With increasing c_N also c_e decreases.

then determined by (B.9) where the topological quantities are important. This can be in contradiction with the constraints on c_e obtained from the α' corrected KPV potential. Taking into account these constraint, it is more useful to work with the PTC (B.11) instead of (B.6).

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Chapter 6

Conclusions

The thesis intends to address (one of) the most urgent and important questions of string phenomenology: Does string theory allow for de Sitter vacua? Even after decades of intense research this question still remains unanswered with many open ends. We hope that the progress of this thesis will eventually help to find a definitive answer to this question and that the methods and insights of this thesis will turn out useful for further research in string phenomenology.

In Sect. 6.1 we summarize in detail the main results of the previous Chapters in order to give an overview of the research conducted in this thesis. Then in Sect. 6.2 those results will be discussed and put into the broader context of string phenomenology. Lastly, in Sect. 6.3, we elaborate on future directions motivated by results of this thesis.

6.1 Summary

In the following we summarize the main findings of the publications associated to this thesis which are reproduced in Part I and II. Therein the corresponding detailed calculations and explanations can be found.

Part I – Higher order corrections to type IIB string theory

The first part of the thesis investigates perturbative higher order α' and g_s corrections to the 10d type IIB supergravity effective action. These corrections are often essential for stabilizing moduli and if they are not, one has to ensure that moduli stabilisation is not spoiled by these corrections.

Chapter 2 – Loops, local corrections and warping in the LVS and other IIB models

The above applies in particular to string loop corrections which are the main subject of study in this Chapter. String loop corrections are frequently used for stabilizing non-blowup Kahler moduli and for inflationary model building in the LVS. But phenomenological applications based on string loop corrections remain on a rather shaky foundation. The reason is that the conditions under which string loop corrections occur and how they scale with the dilaton and the Kahler moduli on general Calabi-Yau orientifolds are only conjectured in a work by Berg, Haack, and Pajer (BHP) [129]. The conjecture is based on explicit string loop calculations on toroidal orbifolds [127] which are very special geometries compared to a Calabi-Yau orientifold.

Trusting the BHP conjecture, it predicts that the Kahler potential (and also the scalar potential according to (1.4)) is corrected by string loops due to two effects. First, there are so-called KK corrections which arise due to the exchange of closed strings carrying KK momentum between D-branes. They induce a correction to the Kahler potential of the form $\delta K_{\mathrm{KK}} \sim g_s \mathcal{T}(t^i)/\mathcal{V}$. The function $\mathcal{T}(t^i)$ is conjectured to be a linear combination of 2-cycle Kahler moduli t^i. The linearity is very special given the complicated topology of a Calabi-Yau. The suppression by the volume $\mathcal{V}$ of the Calabi-Yau is universal due to Weyl rescaling. Second, there are winding corrections when a string winding around the intersection 2-cycle of intersecting D7-branes is exchanged between the branes. The winding correction to the Kahler potential reads $\delta K_{\mathrm{W}} \sim 1/(\mathcal{I}(t^i)\mathcal{V})$ where $\mathcal{I}(t^i)$ is again a linear combination of 2-cycle Kahler moduli.

Due to the strong phenomenological interest in loop corrections it is crucial to deepen the understanding of which correction occurs under which circumstances in the 4d effective potential and how the corrections depend on the Kahler moduli and the dilaton. Only after clarifying these questions, one can reliably use string loop corrections for phenomenological applications without having to rely the BHP conjecture. To make progress in this directions, we take the perspective of an effective field theorist in this Chapter.

To understand string loop corrections and therefore also the BHP conjecture from the field theoretic perspective it is useful to first consider perturbative corrections more generally. As explained in the Introduction, in string theory there are two types of perturbative corrections: α' and string loop (or g_s) corrections. However, for our purposes this classification is not useful. Rather, as one outcome of this Chapter, we propose an EFT-biased classification scheme for perturbative corrections to the 4d string-derived Lagrangian that is suitable for our purposes. We classify the corrections into the following three types: *genuine loop corrections*, *local α' corrections*, and *warping corrections*. The main strategy here is to identify specific limits of string diagrams with objects arising in effective field theory. Some essential connections between string (loop) diagrams and field theory loop diagrams necessary for our classification are discussed in Chapter 1.3.

First, the *genuine loop corrections* are characterized by their non-locality in the higher dimensional theory. Hence, they can not be associated with higher dimensional operators.

From the 4d perspective, they come from integrating out the tower of KK modes. Analogously, in 10d they arise from loops of brane-localized or 10d fields which propagate in the compact space. In this sense, they are reminiscent of the Casimir effect from the perspective of the higher dimensional theory. The second type of corrections, the *local α' corrections* stem from higher-dimensional local operators in 10d, on D-branes or O-planes, and their intersections. These higher dimensional operators can include possible counterterms needed to renormalize genuine loop corrections. Finally there are corrections due to the classical backreaction of the background geometry, which are called *warping corrections*. They can be understood as the backreaction effect of a localized object (like D-branes, fluxes, or O-planes) on a point where the geometry is warped. From the field theory perspective this is due to the propagation of massless 10d fields which is not captured by field theory loops as it is a tree level effect. In contrast, in the language of string diagrams this may be interpreted as a tree-level closed string exchange or, equivalently, a 1-loop open string diagram where for instance a long open string between to branes propagates in a short loop. Hence, warping corrections include contributions from string loop diagrams and should therefore be part of the analysis of [127] and of the BHP conjecture.

Based on this EFT-biased classification scheme we find some important differences to the BHP conjecture (additionally to the warping correction explained above). We find that genuine loop corrections scale like the BHP winding corrections with two crucial differences. First, genuine loop corrections are not tied to models with intersecting D7-branes but appear much more general. These Casimir-like corrections are not directly related to the presence of branes – they are only subject to the SUSY spectrum of the KK tower generating the correction. If the spectrum is $\mathcal{N} = 1$ the genuine loop correction occurs. This difference is important since winding type corrections are often neglected in phenomenological applications without intersecting branes in the literature although they would contribute to the leading loop correction in the 4d scalar potential. An example are winding type corrections to the blow-up cycle which can be dangerous for Kahler moduli inflation. Second, we are only able to confirm that genuine loops correct the Kahler potential at order -4 in 2-cycles. This generalizes the form of winding type corrections of the BHP conjecture to the form $\delta K_{\mathrm{W}} \sim 1/(\tilde{\mathcal{I}}(t^i)\mathcal{V})$ where $\tilde{\mathcal{I}}(t^i)$ is not linear in 2-cycle Kahler moduli anymore. We find evidence for this more general behavior in fibred geometries. Depending on the specific form of $\tilde{\mathcal{I}}(t^i)$ and the size of the 2-cycles, this general scaling can lead to more important corrections. In fibred geometries, we find $\tilde{\mathcal{I}}(t^i) \sim t_1^5/t_2^4$ with t_i 2-cycle Kahler moduli. This leads to a much more important correction to the Kahler potential if $t_2 \gg t_1$.

Including local α' corrections into our discussion of identifying corrections of the BHP conjecture from the EFT perspective we find corrections scaling like the BHP KK type corrections. This resolves the longstanding discrepancy that the BHP KK corrections can not be found from effective field theory reasoning. The higher dimensional operators leading to KK-type corrections are an Einstein-Hilbert term at 1-loop in g_s on the intersection 2-cycle of D7-branes or R^2 operators at 1-loop in g_s on D7-branes. Similarly to the discussion above, we can not reproduce the linearity of $\mathcal{T}(t^i)$ in δK_{KK}. In contrast, we also allow for more general dependencies on 2-cycle Kahler moduli, i.e. we find $\delta K_{\mathrm{KK}} \sim g_s \tilde{\mathcal{T}}(t^i)/\mathcal{V}$ where $\tilde{\mathcal{T}}(t^i)$ is a homogeneous function of degree 1 in 2-cycles.

An additional feature of local α' corrections is that some higher dimensional operators are marginal as for instance a (yet to be proved by string amplitude calculations) R^4 operator on D7-branes. As is well known for marginal operators they might include logarithmically enhanced prefactors induced by counterterms to renormalize the genuine loops. This leads to logarithmically enhanced corrections in the Kahler potential constituting the dominant loop correction. Of course, their existence must ultimately be proven by string amplitude calculations. Evidence for the logarithmically enhanced higher dimensional operator can again be found in fibred geometries. We show that in certain limits, barring possible magical (SUSY) cancellations, the logarithmic corrections are required for consistency of dimensional arguments.

Part II – Implications of higher order corrections for the $\overline{D3}$ uplift and the LVS

The second part of this thesis is devoted to the implications of quantum corrections for phenomenologically relevant settings like the LVS and the anti-D3-brane uplift. In Chapter 3 we study the effect of quantum corrections to the LVS relying on the anti-D3-brane uplift. In Chapter 4 and 5 we study α' corrections to the anti-D3-brane uplift.

The LVS parametric tadpole constraint

In order to provide explicit examples that feature *controlled* dS vacua from string theory, one typically tries to stabilize all moduli by including the leading order corrections into the Kahler and superpotential while neglecting all other corrections. The inconspicuous word *controlled* plays a special role in these constructions because the precise numerical prefactor of many higher order corrections to the effective action of string theories are unknown. Additionally, their compactification on a general Calabi-Yau orientifold can not be done explicitly for most corrections due to the complicated topology. In order to ensure that all these corrections with unknown prefactors neglected in the stabilisation scheme do not render the calculated minimum of the scalar potential unstable, they should be *parametrically* small. If neglected corrections are parametrically small, it is not necessary to rely on a small numerical prefactor.

A prime example potentially realizing controlled dS vacua is the LVS. It is, at least in principle, well protected from higher order corrections due to the exponentially large volume which suppresses higher order corrections. However, combining the LVS with an anti-D3-brane uplift most of the explicit models investigated in the literature are rather at the borderline of (parametric) control. The reason is that for achieving a small dS minimum the uplifting term V_{up} must be comparably large to $|V_{\mathrm{AdS}}|$. Therefore V_{up} needs to be warped down exponentially due to the exponentially large volume $\mathcal{V}$. In equations this reads schematically

$$|V_{\mathrm{AdS}}| \sim \frac{1}{\mathcal{V}^3} \overset{!}{\approx} V_{\mathrm{up}} \sim \frac{\mathrm{e}^{-N/(g_s M^2)}}{\mathcal{V}^{4/3}}\,, \tag{6.1}$$

where N is the flux in the throat. Achieving $|V_{\mathrm{AdS}}| \approx V_{\mathrm{up}}$ requires large flux N contributing positively to the D3-tadpole. Due to tadpole cancellation (vanishing net charge on a compact space), the flux has to be canceled by a large negative contribution to the D3-tadpole. Importantly, in string constructions the negative contribution to the tadpole is limited[1]. This in turn bounds the volume of the Calabi-Yau and thus the expansion parameter of higher order corrections. Accordingly, parametric control in the LVS with anti-D3-brane uplift is limited.

In this Chapter we quantify by a simple formula, called the *Parametric Tadpole Constraint* (PTC), the amount of flux N in the throat necessary to control the most dangerous corrections in the setting described above. The corrections taken into account in the PTC are higher F-terms [140] and a combination of a warping effect and a local α' correction [141]. The input parameters of the PTC are topological quantities of the internal space, the parameter $g_s M^2$, and control parameters specifying the desired control over the respective corrections. The crucial result is that the amount of control increases when increasing the volume of the internal space and therefore the flux in the throat. But since the actual negative contribution to the tadpole is limited in IIB constructions, this represents a very concrete challenge for suitable models for the LVS.

We also discuss additional challenges of the LVS and the interplay of the PTC with the recent tadpole conjecture [51]. Note that in Chapter 4 and 5 we develop the PTC further to include α' corrections to the NS5-brane which will turn out to be crucial for the KPV decay channel on which the anti-D3-brane uplift is based.

Curvature corrections to KPV: do we need deep throats?

In this Chapter, we calculate the effect of curvature corrections on the metastability condition of the NS5-brane at the tip of a Klebanov-Strassler [45] throat. As explained in the Introduction 1, the metastability of the NS5-brane is crucial to realize the anti-D3-brane uplift. So far, the metastability analysis of the NS5-brane has only been carried out to leading order by KPV [199]. The importance of considering curvature corrections in this analysis can be understood as follows. In the last Chapter we have calculated that small values of the parameter $g_s M^2$ and hence also $g_s M$ are phenomenologically preferred since they minimize the necessary flux in the KS throat for uplifting to dS. This in turn minimizes the required negative contribution to the D3-tadpole (which is limited). Therefore, the lowest possible bound on $g_s M$ and $g_s M^2$ is phenomenologically very relevant. But in the regime of small $g_s M$, also curvature corrections

[1]The bound depends on the specific construction. In type IIB string theory with locally canceled D7-tadpole the upper bound is given by $1 + (h^{1,1} + h^{2,1})/2$ which gives 252 for the largest Hodge numbers of the Kreuzer-Skarke database [232]. The bound increases for type IIB with non-local D7-tadpole cancellation and is even higher in F-theory constructions where the largest known negative contribution is 75852 [233, 234].

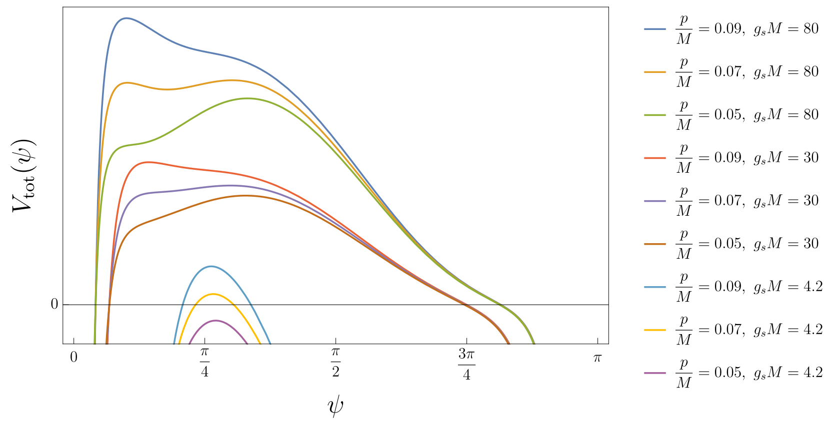

Figure 6.1: Curvature-corrected KPV potential for different values of p/M and g_sM.

become strong: Curvature corrections are suppressed by $R^4_{S^3} \sim (g_sM)^2$ where R_{S^3} is the radius of the S^3 at the tip of the KS throat. Thus, it is mandatory to include curvature corrections in the phenomenologically preferred regime of small g_sM. So far, KPV derived a lower bound of the form $g_sM^2 > 12$, using $g_sM > 1$ such that the radius at the tip is larger than the string length, and $p/M < 0.08$ such that a meta stable minimum exists. The main result of this Chapter is to derive a corresponding bound on g_sM^2 that ensures metastability of the NS5-brane taking curvature corrections into account, thereby making the bound $g_sM > 1$ more precise.

The necessary steps to derive this bound are to first infer the curvature corrections to the NS5-brane by S-dualizing the known curvature corrections to D5-branes and then evaluate them explicitly at the tip of the throat. This is possible since the KS metric is known. The calculation leads to a curvature-corrected potential for the NS5-brane at the tip of the KS throat depicted in Fig. 6.1 for various values of g_sM and p/M. The potential has the unphysical property of diverging to $-\infty$ at $\psi = 0, \pi$ due to extrinsic curvature terms. This is an artifact of our incomplete analysis which neglects α' corrections at the same order in α' that include fluxes and gauge fields and all even higher order α' corrections. As a result, the potential can not be trusted in the regime of strong extrinsic curvature. Once more α' corrections are taken into account the regime where the potential can be trusted will be extended as we show in the next Chapter. Moreover, the potential has two main parameters, namely p/M and g_sM, whose parameter space can be scanned to find the regime where metastable minima exist. The scan is shown in Fig. 6.2 a). From Fig 6.2 a), we find that the minimal value of g_sM where the NS5-brane can be metastable is $g_sM > 20$ (or $g_sM^2 > 500$ for $p = 2$). This drastic change compared to the KPV bound $g_sM^2 > 12$ is caused by the extrinsic curvature corrections which hamper the formation of a metastable minimum.

Applying the curvature-corrected bound on g_sM^2 to the PTC introduced in the last Chapter

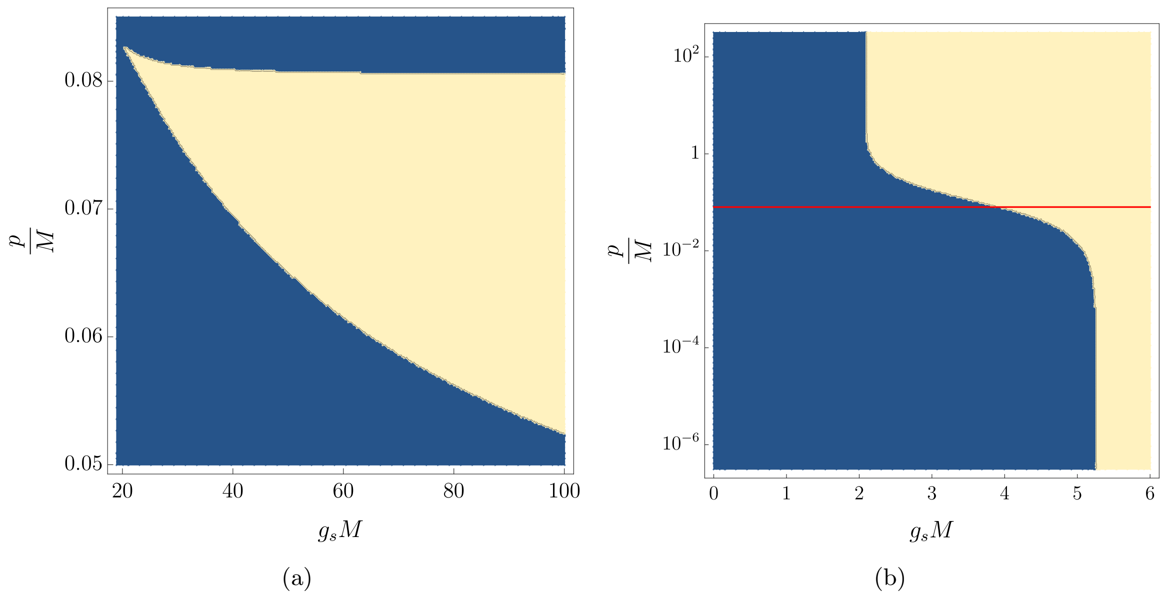

Figure 6.2: Figure a) and b) show the $(g_s M, p/M)$ parameter space. In the yellow region in a) the curvature-corrected potential has a metastable minimum. In the yellow region in b) the maximum of the curvature-corrected potential is at positive energy. Above the red line $p/M = 0.08$ the maximum is at (sub-)stringy radii for the NS5 and therefore not trustworthy.

results in a lower bound on the negative contribution to the D3-tadpole for the LVS with anti-D3-brane uplift of $\mathcal{O}(10^3 - 10^4)$. This is severely constraining since the currently largest explicitly constructed negative tadpole in type IIB string theory is $\mathcal{O}(3000)$ [235].

The bound $g_s M^2 > 500$ derived above is so extreme because of the strong extrinsic curvature corrections at small ψ. Since we assume that this will be resolved as soon as further α' corrections are taken into account it might be possible that our uplifting criterion of a metastable minimum in the curvature-corrected potential is too conservative. It might be that the all orders corrected potential has a metastable minimum in the regime where we do not trust the curvature-corrected potential. Therefore, a more optimistic approach is to demand that the maximum of the curvature-corrected potential should occur at positive values. If a maximum exists at non-zero ψ there will also be a minimum at smaller ψ. This minimum only has a chance of being at positive energy if the maximum has positive energy. This criterion is clearly not a sufficient one but gives at least a most optimistic bound on the parameter $g_s M^2$. The region of parameter space where the maximum occurs at positive values is depicted in Fig. 6.2 b). From there, we can read off the minimal bound on $g_s M$ for the maximum to occur at positive energy. The bound is of the form $g_s M > 4$ (or $g_s M^2 > 48$ for $p = 1$). Even this very optimistic bound which is clearly not sufficient to have a metastable minimum at positive energy seems to outperform the currently strongest bounds on g_sM^2 available in the literature [227]. This emphasizes the importance of taking curvature corrections, and more generally α' corrections into account.

As we have noted already, the above analysis remains preliminary due to multiple reasons. Most importantly, besides the α'^2 curvature corrections there are hundreds of additional α' corrections to the DBI- and CS-action of a Dp-brane which we have neglected so far. This includes corrections involving fluxes, gauge fields, and mixings thereof with curvature terms. This is addressed in Chapter 5. Furthermore, our analysis remains at the probe level such that we do not take into account the issue of backreaction. Finally, the potential we obtain can not be trusted everywhere since in regimes of small radii of the S^2 and S^3 even higher order α' corrections become important.

Motivated by the strong impact of curvature corrections on the metastability of the NS5-brane and the fact that curvature corrections seem to overall lower the KPV potential, we propose a new uplifting mechanism that circumvents all issues related to the D3-tadpole. In this new mechanism one fine-tunes the curvature corrections to the NS5-brane at the tip of the KS throat in such a way that the metastable minimum of the KPV potential is at zero energy. That it is in principle possible to tune the value of the potential at the minimum by curvature corrections can be seen from Fig. 6.1 (the value of the potential at the minimum $\psi = \psi_{\text{min}}$ changes for different values of p/M and g_sM). The advantage then is that the curvature-corrected uplifting term $V_{\text{up}} = V_{\text{curv-corr}}(\psi_{\text{min}})\exp(4A(0))$ can be made arbitrarily small without an exponentially large warp factor $\exp(-4A(0))$ by tuning the parameters g_s and M. The g_s tuning is generically assumed to be possible in the flux landscape which predicts a dense discretuum in g_s.

At this stage, this new mechanism is more a conjecture since the extrinsic curvature corrections become too large to explicitly construct a potential with zero vacuum energy. This will be resolved once all α' corrections are incorporated into the KPV potential. A step towards this aim is taken in the next Chapter.

α' corrections to KPV: An uplifting story

As advertised above, in this Chapter we evaluate all currently known corrections to the DBI- and CS-action of D-branes at the tip of the throat by first S-dualizing them to α' corrections to NS5-branes. All corrections appear at order α'^2 and at tree level in the string coupling g_s. The corrections include hundreds of flux, gauge field, and curvature terms. Incorporating all these terms is crucial as it leads to a more complete and physically interpretable α'^2-corrected potential for the NS5-brane at the tip of a KS throat.

The α'-corrected potential is depicted in Fig. 6.3 for $g_sM = 20$ and different values of p/M. The potential has similar features as the curvature-corrected potential calculated in the last Chapter. It diverges for $\psi = 0, \pi$ and the two crucial parameters are again g_sM and p/M. An important difference is that the α'^2-corrected potential always has a minimum for sufficiently small p/M. This can be shown in a scan over the $(g_sM, p/M)$-parameter space in

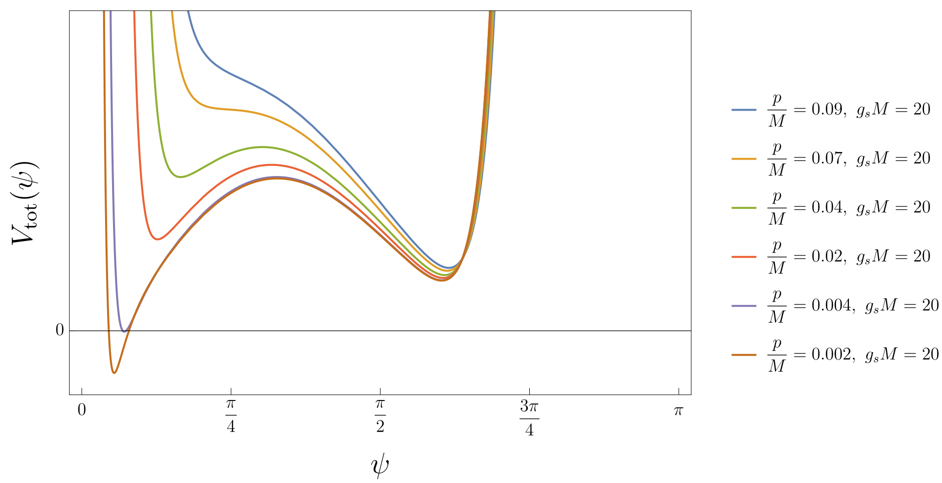

Figure 6.3: α'^2-corrected KPV potential for $g_sM = 20$ and different values of p/M. Depending on the parameters g_sM and p/M, the minimum of the potential can be at negative, zero, or positive energy.

Fig. 6.4. In the yellow/orange region a metastable minimum exists at positive/negative energy, respectively.

Therefore, the α'^2-corrected KPV potential allows for an explicit realization of the new uplifting mechanism proposed in the last Chapter that does not rely on long warped throats. By tuning α' corrections against the tree level KPV potential, metastable minima at zero energy can be constructed (the black line separating the yellow and orange region in Fig. 6.4). The new uplifting mechanism circumvents the control issues related to the PTC. This has to be taken with a grain of salt since the uplifting mechanism is realized in a regime of parameter space at the very boundary of control where also higher order α' corrections become important. The reason is that R_{NS5}, the radius of the S_2 wrapped by the NS5 in the metastable minimum, is of string size. We also discuss future directions how to overcome this control problem. These include treating the anti-D3-branes as a non-abelian brane stack or studying the throat from the holographic perspective.

Additionally, the bounds on the parameters g_sM and g_sM^2 from the last Chapter can be updated by examining the parameter space where the α'-corrected KPV potential exhibits a metastable minimum. From Fig. 6.4 we find as a most optimistic bound $g_sM > 3.6$ (and $g_sM^2 > 144$ for $p = 1$). Note that this point in parameter space occurs again in a regime of bad control since R_{NS5} is again string size. Hence, g_sM and therefore the radius of the S^2 should be increased for control over α' corrections. The implications on the standard anti-D3-brane uplift using long warped throats are, as in the previous Chapter, similarly constraining for the LVS relying on the standard anti-D3-brane uplift. In the Appendices of this Chapter we

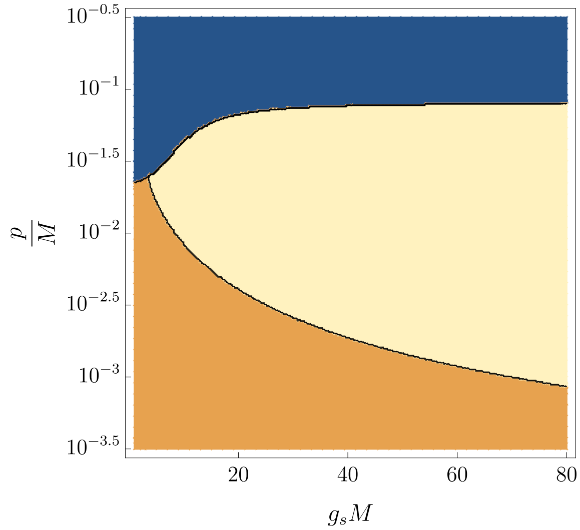

Figure 6.4: Scan for metastable minima in the $(g_sM, p/M)$ parameter space. In the blue region no metastable minimum exists. In the yellow/orange region the minimum occurs at positive/negative energies, respectively. These regions are separated by a black line where the energy at the minimum is zero.

discuss in more detail how to properly incorporate the α' corrected KPV bounds on g_sM into the PTC and furthermore, how string loop corrections relate to the PTC.

6.2 Discussion

The best studied candidate for *the* theory of quantum gravity unifying the standard model of particle physics and gravity is, arguably, string theory. As we have discussed in some detail in the Introduction 1, string theory provides a solid framework for studying physics beyond the standard model of particle physics and the ΛCDM model as the theory has many features we expect our theory of quantum gravity to necessarily have: String theory is capable of unifying GR and gauge theories, realizing the standard model and inflation as well as providing dark matter and axions. A main shortcoming of string theory seems to be the explanation of an accelerated expansion of the universe. Progress in this direction was the main goal of the present thesis. An accelerated expansion of the universe can be explained either through a (meta-)stable dS vacuum (with a longer lifetime than our universe) or through a currently at positive value, slow-rolling Quintessence-like field which eventually will end up in a Minkowski or AdS vacuum. Independently of which explanation the reader may prefer both constructions seem to be similarly difficult to realize within string theory. As reviewed in the Introduction 1, constructing dS vacua in many weakly coupled corners of string theory is forbidden by no-go theorems. Similar conclusions also hold for slow roll in parametrically controlled regimes. This has cast doubt on the very existence of accelerated expanding universes within string theory. This doubt is mainly driven by the swampland program gathering evidence that dS vacua do not exist in asymptotic limits of parameter space.

This led to the general belief that, if dS vacua exist, they will lie in the interior of parameter space. There, the knowledge of the leading order 10D SUGRA effective action is not sufficient to make precise predictions for low-energy physics because quantum corrections have a non-negligible effect. This closes the gap with the research conveyed in this thesis. In Part I we have mainly focused on higher order corrections *per se* whereas in Part II we have investigated the implications of higher order corrections on phenomenologically relevant setups like the LVS and the anti-D3-brane uplift.

The importance of higher order corrections

Due to the web of no-go theorems against dS vacua at classical level and against slow roll in the parametrically controlled regime of string theory, the understanding of quantum corrections seems to be inevitable for connecting string theory with the real world. In the context of moduli stabilisation, quantum corrections are often necessary to stabilize all moduli as it is for instance the case for Kahler moduli in IIB string theory. Therefore, most celebrated scenarios for achieving scale separated AdS vacua including their uplift, namely the KKLT and LVS, are crucially relying on quantum corrections.

One such kind of quantum corrections are string loop corrections which have been the main subject of study in Chapter 2. String loop corrections are frequently used to stabilize non-blowup Kahler moduli and for inflationary model building in the LVS. This is clearly problematic since the scaling of loop corrections with the Kahler moduli is only conjectured for generic Calabi-Yau compactifications based on explicit string loop calculations on orbifolds. Additionally, it is not well understood under which circumstances loop corrections occur. The improved understanding of loop corrections from the EFT perspective provided in Chapter 2 revealed a much more general scaling of loop corrections with Kahler moduli and, possibly, the existence of previously unknown, logarithmically enhanced loop corrections. Furthermore, many corrections to the Kahler potential can now be assigned to specific higher dimensional operators. This improves the understanding of when a specific loop correction occurs. These findings are capable of spoiling/pinning down existing string inflation scenarios like Fibre and Kahler inflation or proposing new models for inflation. Nevertheless, our results can only be viewed as preliminary since the very general EFT arguments that we used in Chapter 2 need to be backed up by string amplitude calculations and/or by Kaluza-Klein reductions on explicit backgrounds together with EFT loop calculations. Both are rather grueling tasks since they require explicit knowledge of the compactification geometry. The EFT loop calculations additionally require knowledge of the eigenfunctions on the compact space. The new findings on loop corrections are only one particular example that shows that there are still many new effects to be uncovered once higher order corrections to the effective action become calculable in a systematic fashion. Another example is the new uplifting mechanism proposed in Chapter 4 and 5 that emerged once α' corrections to the NS5-brane action are taken into account.

Additionally, a profound understanding of quantum corrections is crucial for setting up consistent scenarios for moduli stabilisation. A working scenario relies on a consistent truncation of the Kahler potential and superpotential. At the time of writing it remains unclear whether the truncation underlying the LVS is consistent. The reason being possible logarithmic corrections overpowering the α'^3 BBHL correction [75] crucial for the LVS. These logarithmic corrections could arise in different manners: Through a redefinition of the blow-up cycle [136–138] or a KK exchange between the 10 dimensional R^4 term and distant D7-branes/O7-planes [69, 70][2]. Hence, the proof of existence of these logarithmic corrections is key for designing consistent moduli stabilisation scenarios and therefore also for proving the existence of dS vacua.

Very generally, one might expect that improving the understanding of quantum corrections and therefore the understanding of string theory from the EFT perspective will be very useful in searching for low energy evidences of string theory. It is the author's opinion that the interior of parameter space of string theory, where quantum corrections become important, has many surprises in store for us. We encountered one such surprise, namely a new uplifting mechanism, in Chapter 4 and 5.

Higher order corrections – taming the unknown

A major obstacle in proving the existence of metastable dS vacua in string theory is that the knowledge of quantum corrections is very limited at the time of writing. In particular, this is true for their numerical prefactors. The list of quantum corrections whose numerical prefactors are known is short: the BBHL $\alpha'^3 R^4$ term and its one loop correction [74, 75], an α'^3 correction due to a non-trivial axio-dilaton profile [103] as well as the α'^2 corrections at string tree level to Dp-branes and Op-planes used in Chapter 4 and 5[3].

We have already explained that moduli stabilisation scenarios always rely on a truncated set of corrections, and it is therefore mandatory to verify that all corrections neglected in the scenario do not spoil the previously achieved (A)dS minimum. This is particularly difficult if the numerical prefactor is not known. Depending on their size, a correction can be much more dangerous than one would have naively expected.

Investigating this in the context of the LVS has been the main purpose of Chapter 3 and [141, 142]. At first sight, the LVS seems to be well protected against unknown higher corrections since the volume is exponentially large in $1/g_s$, and g_s can be tuned arbitrarily small. Taking into account the uplift, the LVS rather seems to be at the boundary of control, provided that the negative contribution to the D3-tadpole can be very large. The main lesson to take from there is that due to our limited knowledge of higher order corrections, we need to

[2]The possible log enhanced loop correction we found in [2] is subleading compared to the BBHL correction but log enhanced compared to all other loop corrections.

[3]There has also been progress in determining the prefactor for further α'^3 corrections to the type IIB bulk theory [100].

ensure that unknown corrections are suppressed *parametrically* and not numerically in order to prove the existence of dS vacua. This pushes us towards the weakly coupled corners of the moduli space where we do always find obstacles preventing dS vacua. In the case of the LVS, the limited negative contribution to the D3-tadpole in type IIB string theory is such an obstacle. We quantified this by the *parametric tadpole constraint* in Chapter 3.

But the above discussion is not specific to the LVS. It seems to be a more generic feature that currently available models are at the boundary of control (as a further example consider the new way of uplifting which we proposed in Chapter 4 and 5).

The reason why we have not found controlled dS vacua so far may then be understood as follows. Establishing dS vacua requires us to circumvent no-go theorems by incorporating multiple ingredients like localized objects and quantum corrections into our theory. Doing so we find, at best, dS vacua at the very boundary of control. One reason for this is our limited knowledge of quantum corrections which prevents us from accessing the interior of parameter space where dS solutions are (likely) to be found. Improving our understanding of quantum corrections should therefore be a primary goal of string phenomenology.

Higher order corrections – taking advantage of the known

Following the discussion of the last sections, we should find ourselves in an advantageous position for the anti-D3-brane uplift. In this setup, (most of) the leading order quantum corrections are known explicitly and, further, the metric at the tip of the KS throat is known explicitly. We used this in the Chapter 4 and 5 to quantify by how much α'^2 corrections affect the leading order KPV story and therefore threaten the existence of a metastable minimum necessary for the anti-D3-brane uplift. We found that the standard anti-D3-brane uplift, relying on the exponentially warped down tension of the anti-D3-brane, is only protected *parametrically* against the α' corrections at the cost of even further increasing the flux in the throat. This exacerbates the problem of controllability of dS vacua obtained from the LVS with anti-D3-brane uplift.

Besides quantifying the danger of the leading order quantum corrections, we also observed the emergence of a new uplifting mechanism. The quantum corrections generically lower the overall potential. The new mechanism then relies on balancing quantum corrections against the tree level potential such that the minimum occurs at zero energy. This uplifting mechanism is clearly a surprise from the perspective of leading order physics and requires properly evaluating the quantum corrections on the given background. We should note that also the new mechanism is realized in the interior of moduli space even though it circumvents all problems related to the limited D3-tadpole preventing the LVS from achieving controlled dS vacua. Additionally, the mechanism always requires tuning and relies on the almost continuously tunable value of g_s due to the flux landscape [236].

In summary, evaluating higher order corrections on a given background allows us on the one hand to quantify their danger and hence improve perturbative control. This can be done by either restricting to regimes of parameter space where the quantum corrections can now safely be neglected or by restricting to the parameter space where even higher order quantum corrections are negligible. This second option would again require calculating these higher order effects explicitly. On the other hand, new mechanisms and properties of the theory might emerge that can be used for phenomenological purposes. We assume that this will be a general feature as soon as quantum effects become broadly calculable and explicitly evaluable on given backgrounds.

Increasing perturbative control and the discovery of new effects through quantum corrections are certainly highly desirable and should be motivation enough to further explore the quantum effects of string theory and thus advance into the interior of moduli space.

6.3 Outlook and future directions

The overall goal of this thesis has been set out to make progress in clarifying whether string theory allows for accelerated expanding universes. This remains (one of) the most urgent problems to be solved in order to connect string theory with the real world. We have contributed to this problem by improving the understanding of string loop corrections, we pointed out and quantified challenges of the LVS with anti-D3-brane uplift and the anti-D3-brane uplift itself, and proposed a new uplifting mechanism for obtaining dS vacua. In these final paragraphs, we propose some future directions naturally emerging from results of this thesis.

Based on the results of Chapter 2 it would be important to prove the existence of the R^4 (or R^3) operator on D7-branes (or their intersections) and the related logarithmic correction to the Kahler potential. If existent and calculable, the dominant, logarithmic loop effect would become accessible, constituting an important step towards more controlled models. Additionally, the logarithmic correction together with the improved understanding of loop corrections in general should be thoroughly applied to models of inflation like Kahler and Fibre inflation. Both models are based on the BHP conjecture for loop corrections on Calabi-Yau orientifolds and might be invalidated or pinned down once the new effects are properly accounted for. We initiated this for Fibre inflation in Chapter 2 where we already found substantial differences.

More generally, investigating all kinds of so far mostly conjectured logarithmically enhanced corrections to the Kahler potential would be a crucial task for phenomenology. If they generically exist, some of them would constitute the leading order corrections to the Kahler potential in the Kahler moduli sector and are hence crucial for all moduli stabilisation scenarios relying on perturbative corrections to the Kahler potential.

The main result of Chapter 3 has been that the LVS with standard anti-D3-brane uplift requires much more negative contribution to the D3-tadpole in order to control the most dangerous corrections than one might naively have guessed. It is therefore necessary to study the LVS in IIB on manifolds with non-locally canceled D7-tadpole or, even more difficult, to study the LVS in the framework of F-theory. In both settings many questions remain to be answered, as for instance how dangerous corrections due to a varying dilaton are or how the LVS can in principle work in a F-theoretic context where g_s is not small everywhere in the compact space. Furthermore, it is usually the case for models with large Euler number that they come with a large number of moduli. If these happen to be Kahler moduli, a systematic way of explicitly stabilizing many Kahler moduli is required[4].

Another direction to rescue the LVS with standard anti-D3-brane uplift from large tadpoles is to pin down the numerical coefficients of the most dangerous corrections taken into account in the PTC. If these turn out to be small or harmless due to the right sign, the tadpole would not need to be that high for controlled models[5]. This task is difficult, model dependent, and requires new tools to calculate quantum corrections on a given Calabi-Yau orientifold. Still, we have seen in Chapter 4 and 5 that this calculation can be quite rewarding since new phenomena might emerge. We have explained that to be able to do this requires two conditions. First, the quantum corrections to the 10D SUGRA EFT and/or the worldvolume theory of localized objects need be known. If their numerical prefactor is not known, it must be calculated for instance by string worldsheet calculations. The next step is then to evaluate these quantum corrections on the Calabi-Yau orientifold one wishes to compactify on. A possible tool for doing this in the future might be Machine Learning techniques. There is significant progress [237–240] that shows that neural networks are able to learn the metric of Calabi-Yau manifolds. With the (numerically approximated) metric at hand, the way to study more advanced models explicitly (which we need for finding dS vacua due to no-go theorems) might be paved.

Finally, extending the results of the Chapters 4 and 5 in the following directions would be very interesting future research projects. First, it would be crucial to push our analysis of α'^2 corrections to the KPV decay process [199] to higher orders in α'. This amounts to including α'^4 and also, if they exist, α'^3 corrections to the worldvolume theory of Dp-branes. This leads us back to previous discussions on the crucial importance of deriving higher order quantum corrections to the effective action. With these corrections at hand perturbative control could be improved, the bounds for phenomenology could be made even more precise, and our new uplifting mechanism might be realizable with better control.

Second, the analysis performed in Chapter 4 and 5 remained at a probe level. Hence, including the backreaction effect of the branes onto the geometry is necessary. This might enlarge the radius of the NS5-brane at the minimum of the potential [221] and thereby the regime the control.

Third, the α'-corrected anti-D3-brane uplift should be studied from different perspectives. Investigating the anti-D3-branes as a non-abelian stack of branes will yield a corrected estimate of the position of the metastable minimum. Although very challenging and already intensively studied in the literature, investigating the throat from the holographic perspective could lead to new insights. On the holographic side, the regime of large curvature is the regime of control

[4]Note that one generally assumes all Kahler moduli to be stabilized by loop effects. But to establish controlled dS vacua from string theory this is clearly not sufficient since calculating loop correction explicitly on a general Calabi-Yau is currently not feasible.

[5]To be more precise, the control parameters c_{W_0} and c_N in the PTC could be attenuated or omitted altogether. Nevertheless, the bound on g_sM from the α'-corrected KPV potential still needs to be taken into account enforcing a large negative contribution to the D3 tadpole.

which is precisely the regime where control over the α' expansion is lost. The holographic perspective might enable us to understand the regime of strong curvature and, as a byproduct, it could be a way towards proving our new uplifting mechanism.

In all research conducted in this thesis, and also in the future research directions summarized above, quantum corrections are the main players. They enable us to push the boundaries of control and by this access more of the interior of parameter space. There, dS vacua are (more likely) populating the landscape and new, fascinating stringy phenomena are waiting for us to be discovered.

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