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SEIT 1386

Dissolving an ambiguous partnership

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AWI DISCUSSION PAPER SERIES NO. 733
September 2023

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September 12, 2023

Abstract

Two partners try to dissolve a partnership that owns an asset of ambiguous value, where the value is determined ex post by a draw from an Ellsberg urn. In a within-subject experiment, subjects make decisions in three different bargaining mechanisms: unstructured bargaining, the Texas shoot-out, and a $K+1$ auction. We find that the $K+1$ auction is the most efficient mechanism, which is in line with theory. Free format bargaining yields a surprising number of disagreements, which are not usually observed when the partnership has a certain or risky value.

JEL codes: C91; C72; D74.
Keywords: Bargaining, ambiguity, experiment.

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*We would like to thank Gerrit Bauch, Timo Dimitriadis, Peter Dürsch, Kyle Hyndman, Katharina Momsen and participants at the ESA conference in Santa Barbara and the HeiKaMaX workshop in Heidelberg for comments. We thank Alexander Söket for outstanding research assistance. The University of Heidelberg provided financial support for the experiments.

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1 Introduction

When for whatever reason a partnership has to be dissolved, it is useful to have a mechanism in place that does so efficiently. Some mechanisms have been used in practice for a long time. The ancient "divide & choose" method was already mentioned in the old testament. An application of the divide & choose method to partnership dissolution, where the whole partnership has to end up with one of the partners, is the Texas shoot-out.[1]

In the Texas shoot-out, one of two partners, the proposer, states a price for (half) the partnership. The second partner, the chooser, can then decide whether to sell her half to the proposer at the proposed price or buy the proposer's share at the proposed price. If the proposer knows the value the chooser attaches to the partnership, the partnership will be allocated efficiently to the partner who has the higher willingness to pay for it. While sometimes partners know each other so well that they may have a precise estimate of their partners' respective values, in many other cases there may be substantial uncertainty about them. McAfee (1992) derives the equilibrium in a Bayesian game and shows that the Texas shoot-out loses its efficiency property as proposers with very low or very high values have an incentive to over- or understate their values, respectively.

A mechanism that has better efficiency properties in a Bayesian game is the $K+1$ auction first analyzed by Crampton et al. (1987). In a $K+1$ auction, both partners bid for the (respective) other half of the partnership. The partner with the higher bid wins the partnership and pays a convex combination of the highest and the second highest bid to the other partner. Since the equilibrium bidding functions are strictly increasing in value, the outcome of the $K+1$ auction is ex-post efficient as the partner with the higher value ends up with the partnership.

For some partnerships the assumptions underlying a Bayesian game may be too demanding. For example if the partnership is a start-up that develops a completely new product, it may be difficult to assign probabilities to all possible outcomes. Therefore, in this paper we study those mechanisms experimentally for the case of ambiguity (or Knightian uncertainty). In fact, we let the partnership consist of an asset that can be represented by a standard two-color Ellsberg urn (Ellsberg, 1961). This introduces ambiguity in two ways. First, partners face uncertainty regarding their own valuation for the partnership.

[1] The Texas shoot-out is a divide & choose method because the provider essentially divides total endowment into two lots, one containing the partnership and some amount of money from their joint monetary endowment and the second lot containing the remaining monetary endowment. The responder then chooses one of the lots for herself.

Second, it seems plausible that partners also feel uncertain about their partners' bidding behavior, introducing strategic ambiguity.[2]

An extreme form of ambiguity aversion resulting in maxmin behavior in the $K + 1$ auction and the Texas shoot-out has been studied theoretically by Van Essen and Wooders (2020). Bauch and Riedel (2022) study the Texas shoot-out under ambiguity by allowing partners to have a set of probability distributions over values. Both studies together indicate that the efficiency properties of the Texas shoot-out may actually improve under ambiguity whereas the $K+1$ auction should remain efficient. Our paper would be the first to study this prediction experimentally.

In addition to these well-specified mechanisms, we study whether unstructured bargaining with chat may improve partnership dissolution. The reason for this is that most partnerships are likely not dissolved through well-structured mechanisms so it helps to have unstructured bargaining as a benchmark. Further, given the ambiguity in values, chat may help in allocating the partnership to partners with higher values, as more information about values can be exchanged (as compared to the two structured mechanisms discussed above) through dialog. On the other hand, any messages sent using a chat interface can certainly be seen as cheap-talk.

In a within-subject lab experiment with 220 subjects we find that none of the mechanisms comes close to 100% efficiency. As predicted by some of the theories, the $K + 1$ auction does better than the Texas shoot-out but both mechanisms do not manage to assign the partnership to the partner who values it more in more than 2/3 of cases. However, the overall loss in surplus due to this misallocation is arguably mild (between 15% and 17%) for the two structured mechanisms.

In contrast, the Unstructured bargaining mechanism does rather badly. The partnership is assigned to the partner with the highest value in less than half the cases. The main reason for this is that the bargaining fails to achieve any agreement in a surprising 32% of cases. This is quite unexpected given that recent studies like Hyndman (2021) and Ingersoll and Roomets (2020) have agreement rates of over 90% using a very similar bargaining protocol. We conjecture that the ambiguity of the asset plays a role here.

To the best of our knowledge, bargaining under ambiguity has not been studied much in the literature, either theoretically or experimentally. Salo and Weber (1995), Lo (1998) and Chen et al. (2007) study first or second price auctions with independent private

[2] There may already exist strategic ambiguity even without an ambiguous asset (see e.g. Bauch and Riedel, 2022) but arguably strategic ambiguity increases if the asset is ambiguous.

values under ambiguity about other bidders' valuations. Yu and Chmura (2012) study experimentally the $K+1$ auction for the first price case ($K=0$). For the case where a partnership is initially shared equally, they find that bids are higher under ambiguity than predicted by the Bayesian equilibrium. Huang et al. (2013) study bargaining games under ambiguity between an ambiguity neutral insurer and ambiguity averse clients. Finally, as already mentioned above, Van Essen and Wooders (2020) and Bauch and Riedel (2022) provide the theoretical approaches most suitable for our setting.

Bargaining over a risky asset (rather than an ambiguous one) has been studied somewhat more frequently. White (2008) extends Rubinstein bargaining (1982) to the case when the outcome of the bargaining is risky. Embrey et al. (2021) and Hyndman (2021) study unstructured bargaining when the bargaining outcomes are risky.

Partnership dissolution under standard Bayesian assumptions has been studied by McAfee (1992), Crampton et al. (1987), Moldovanu (2002), Kittsteiner (2003), de Frutos and Kittsteiner (2008), Brooks et al. (2010) and many others. Experimental tests of partnership dissolution include Kittsteiner et al. (2012) and Landeo and Spier (2013), Quin and Zhang (2013) and Brown and Van Essen (2022).

The remainder of the paper is organized as follows. Section 2 details the experimental design. Section 3 establishes the theoretical framework and derives our hypotheses that we later test using our dataset. Section 4 reports the experimental results. Section 5 discusses our results and concludes. The appendix contains supplementary material including the experimental instructions and calculations omitted from the main text.

2 Experimental design

Two partners try to dissolve a partnership that owns an asset of ambiguous value. Both partners initially own 50% of the asset and are each endowed with €10 in cash. The asset will pay out either €20 or €0 depending on the color of a marble drawn from an Ellsberg two-color urn (Ellsberg, 1961). In the experiment, the urn is represented by a bag containing 20 marbles, which are either blue or yellow. The combination of blue and yellow marbles is not known to subjects. They are told that each possible combination from 0 blue marbles (and therefore 20 yellow ones) to 20 blue marbles (and therefore 0 yellow ones) is possible. Subjects will be asked if they would like to choose "blue" or "yellow".[3]

[3] The color choice is made at the very end and applies to any round (including the questionnaire) in which the subject owns the asset.

This should alleviate subjects' potential concern that the experimenters have stacked the odds against them (see e.g. Halevy, 2007, for the same procedure).[4]

At the end of the experiment, if the color of the marble drawn from the bag matches the color choice of the partner who ends up with the asset, this partner will receive €20 for the asset. If the color of the marble drawn from the bag does not match, the partner with the asset will receive €0 for the asset. Participants will also be paid their initial endowment plus or minus any payments agreed upon during the bargaining over the asset.

In a within-subject design, subjects make decisions in three different bargaining mechanisms: unstructured bargaining, the Texas shoot-out, and a $K+1$ auction, where the order of the mechanisms is randomized. After each mechanism, partners are rematched in a perfect stranger design. At the end of the experiment there is a questionnaire. Note that in contrast to most of the auction literature, we do not have induced values. Therefore, there is likely a lot of heterogeneity in values for the ambiguous asset across subjects. Although we try to elicit values, it seems preferable to have a within-subject design so that we can directly compare different bargaining mechanisms.

The details of the three bargaining mechanisms are as follows.

1. Unstructured bargaining: Subjects have 5 minutes to agree on a payment and who should get the asset. If they do not agree, they both receive nothing for the asset (they keep their monetary endowment, though). During the 5 minutes, subjects can chat with their partner by writing in a chat box. They can also, at any time, make proposals about asset allocation and payment. If a proposal is accepted by both partners, bargaining ends. Before submitting proposals, subjects can let the computer calculate the respective payoffs for both partners for any proposal they consider submitting.

2. Texas shoot-out: One subject is randomly chosen to be the proposer who can name a price p. The other subject is the responder who can decide whether to sell her share to the proposer at price p or whether to buy the proposer's share at price p. We implement this with the strategy method. Both subjects name a p in case they are proposer and both name reservation price r in case they are responder. If $p \geq r$, the responder will sell her share at price p. Otherwise, she will buy the proposer's share

[4] Theoretically, this method might offer subjects an hedging opportunity (Oechssler and Roomets, 2014). However, experimental evidence (see Dominiak and Schnedler, 2011, and Oechssler et al. 2019) shows that subjects usually do not use hedging opportunities even if they are offered to them much more saliently.

at price p.

3. $K+1$-price auction: Both subjects name a bid b_i. The one with the higher bid gets the asset and pays the other the price $p = K\min\{b_1, b_2\} + (1-K)\max\{b_1, b_2\}$ for half of the asset, where $K \in [0,1]$. Thus, for $K+1=1$ we would have a first price auction. For $K+1=2$, we would have a second price auction. We pick $K=1/2$ so that the auction becomes a split-the-difference auction (Samuelson, 1985).

To minimize spill-over effects across the bargaining mechanisms, there is no feedback on the outcome of the Texas shoot-out and the $K+1$ auction until the end of the experiment. Nevertheless, order effects can play a role. This is why we used all possible orders and test for order effects.

After the three bargaining rounds, subjects answer an incentivized questionnaire containing two questions designed to elicit their risk and ambiguity attitudes with a BDM mechanism (Becker et al., 1964). In the first question, we ask subjects for their certainty equivalent for a risky asset that pays €15 or €0 depending on the outcome of a fair coin. In the second question, subjects are asked for their value for an ambiguous asset that pays €15 if the color of the marble drawn from the same bag as above is their chosen color and €0 otherwise.[5] To explain the BDM, which is often misunderstood by subjects,[6] we use the very intuitive method suggested by Healy (2017, 2020).[7] According to this method, subjects are asked to imagine answering a long list of questions of the kind "Do you want the asset or [some amount of money]?", where it is very intuitive and transparent for subjects that it is in their interest to answer truthfully if one of these questions is chosen randomly for payment. Then, rather than answering all these questions, subjects are asked for a cut-off level of money beyond which they would prefer the money (see the appendix for the instructions).

At the end of the experiment, one of the bargaining rounds or the questionnaire is randomly selected by rolling a four-sided die and all participants are paid according to the outcome of that bargaining round or questionnaire. In addition, subjects receive a show-up fee of €5. Average pay was €22.51 for about 1 hour.

[5] We chose €15 rather than €20 to make sure subjects would not confuse this question with the ambiguous asset used in the bargaining stages.

[6] See e.g. Cason and Plott (2014).

[7] See Martin and Muñoz-Rodriguez (2022) and Chakraborty and Kendall (2023) for related variants of the BDM and Breitmoser and Schweighofer-Kodritsch (2022) and Brown et al. (2023) for comparisons of different mechanisms.

The experiments took place in July 2022 in the AWI Lab at the University of Heidelberg, mostly with undergraduate students of all fields. In 12 sessions 220 subjects (i.e. 110 bargaining pairs) took part in total. The experiment was computerized using z-Tree (Fischbacher, 2007). At the beginning of each session, subjects were randomly placed in the different cubicles of the lab where they were asked to read the instructions. Instructions on paper for each mechanism were distributed before the respective period, followed by a short comprehension quiz. Once all subjects solved the quiz correctly and there were no more questions, the experiment started. The sessions were conducted in German. English translations of the instructions and the comprehension quiz can be found in the appendix.

3 Theory and Hypotheses

In this section we present intuition for a number of hypotheses which drive the analysis of our experimental results. Our first hypotheses deal with bargaining outcomes, while our latter hypotheses deal with individual choices. In order to motivate these hypotheses, we rely on a number of different theoretical frameworks. These frameworks include conventional non-cooperative equilibria, maxmin behavior, and cooperative solutions. Unfortunately, we do not have a precisely applicable theory for our experiment, as these models do not fully incorporate the ambiguous asset. Instead, both Van Essen and Wooders (2020) and Bauch and Riedel (2022) focus on ambiguity about the valuations of other players. Of course, by making the value of the asset ambiguous in and of itself, we expect that subjects may be more uncertain about other subjects' valuations in our experiment. Bauch and Riedel (2022) show that ambiguity attitudes may result in behavior that links conventional non-cooperative predictions with maxmin predictions (at least for the Texas shoot-out). We then assume that something between these theoretical frameworks would be a reasonable prediction for our situation. Before stating specific hypotheses, we give an overview of each theoretical framework.

3.1 Non-Cooperative Equilibria

Non-cooperative equilibria in the appropriate Bayesian game are useful for analyzing outcomes in the $K+1$ auction and the Texas shoot-out. Solutions for these mechanisms exist in the following forms. For the $K+1$ auction, the symmetric Bayesian Nash equilibrium bidding strategy was derived by Crampton et al. (Prop. 5, 1987). In our case with 2 bidders, the equilibrium bidding functions (for half of the asset) are

$$b(v_i) = \frac{v_i}{2} - \frac{\int_{z=F^{-1}(K)}^{v_i} [F(z) - 1/2]^2 dz}{2[F(v_i) - 1/2]^2}, \tag{1}$$

where v_i is the whole asset value of bidder i, $F(\cdot)$ is the commonly known distribution of values, K is the choice of K in the $K+1$ auction.[8] If we assume for simplicity that values (i.e. certainty equivalents) are distributed via a triangular distribution on $[0, 20]$,[9] we can solve for the bidding functions explicitly (see appendix). While the elicited certainty equivalences are not perfectly distributed according to a triangular distribution (see Figure 3 in the appendix), this assumption certainly seems more appropriate than e.g. a uniform distribution.

For the Texas shoot-out, as shown in McAfee (1992),[10] we first consider the responder's choice. As her choice does not affect the price, the responder has a weakly dominant strategy of setting the reservation price to her true value for half the asset, $r = v_i/2$. For the proposer, facing a responder following this weakly dominant strategy, his expected payoff from offering a price p can be expressed as

$$\pi(p|v_i) = (v_i - p)F(2p) + p(1 - F(2p)). \tag{2}$$

If we again assume that values are distributed triangularly on $[0, 20]$, we can obtain an explicit solution (see appendix).

So far, we have not accounted for the ambiguity regarding the value of the asset. One possibility would be to assume that each partner has a certainty equivalent for the ambiguous asset CE_i^A and that the distribution of these CEs is common knowledge. We could then use the Bayesian game approach simply replacing v_i by CE_i^A. However, this assumption seems too strong. If partners are concerned about ambiguity, it seems more likely that they are also concerned about the *strategic* ambiguity regarding the bidding behavior of their partner that derives indirectly from the ambiguity of the asset's value.

An extreme form of ambiguity aversion is explored in Van Essen and Wooders (2020). They analyze players' bidding behavior under the maxmin rule for both the $K+1$ auction and Texas shoot-out. In the case of the $K+1$ auction, bidders would want to bid half of

[8] Crampton et al. (1987) consider bids for the whole asset and then redistribute the revenues to all bidders. The formulations are equivalent for two bidders.

[9] Of course, in our experiment the asset can only take monetary values of €0 or €20. However, we interpret $F(\cdot)$ here as the distribution of certainty equivalents for the asset.

[10] Referred to therein as the cake-cutting mechanism.

their value so as to ensure they buy at prices below their value and sell at prices above their value, achieving a maxmin of zero. Any bid other than their value can result in selling at a low price or buying at a high price resulting in a payoff of less than zero. For Texas shoot-out responders, reporting one's value as a reservation price r is again weakly dominant and achieves a minimum payoff of zero. For proposers, setting the price p equal to one's value guarantees a payoff of zero, but any other choice opens up the possibility for sales at low prices and purchases at high prices, resulting in payoffs less than zero. So, as shown formally by Van Essen and Wooders (2020) for all players in these mechanisms, half one's value is the maxmin choice, $p(v_i) = r(v_i) = b(v_i) = v_i/2$.

A recent paper by Bauch and Riedel (2022) cleverly incorporates (partial) strategic ambiguity into the Texas shoot-out. Bauch and Riedel (2022) assume that rather than having a single distribution function $F(\cdot)$ as in the standard McAfee (1992) model, partners think that a set of distribution functions is possible. They allow for all distribution functions that are ε-close to the reference distribution according to the Levi-Prohorov metric. Ambiguity averse proposers would then always choose the worst distribution function when deciding on their price p. What is interesting is that depending on their own value, sometimes the worst distribution involves high values of the partner (when they themselves expect to buy the asset) or involves low value of the partner (when they expect to sell the asset). Bauch and Riedel (2022) show that for triangular distribution the bidding function becomes twice kinked. Around the median of the distribution, the bidding function coincides with the maxmin solution of Van Essen and Wooders (2020). For low and for high values it is flatter. Unfortunately, for the $K + 1$ auction we are not aware of a similar analysis. However, we conjecture that strategic ambiguity has a roughly similar effect in the $K + 1$ auction.

We can summarize all theoretical bid functions in Figure 1. All bid functions are increasing and for median values all mechanisms and behavioral assumptions predict very similar bids at about $v_i/2$. The main differences occur for very high and for very low values.

3.2 Hypotheses

In this section we present null and alternative hypotheses that will guide the statistical analysis of our experimental results. For each pair, the hypothesis in bold represents our expectations informed by theory set out earlier in the section. In cases where the alternative is one-sided in nature, we will test using the two-sided alternative instead to be conservative.

Our first hypotheses concern successful asset assignment. We assume that a successful

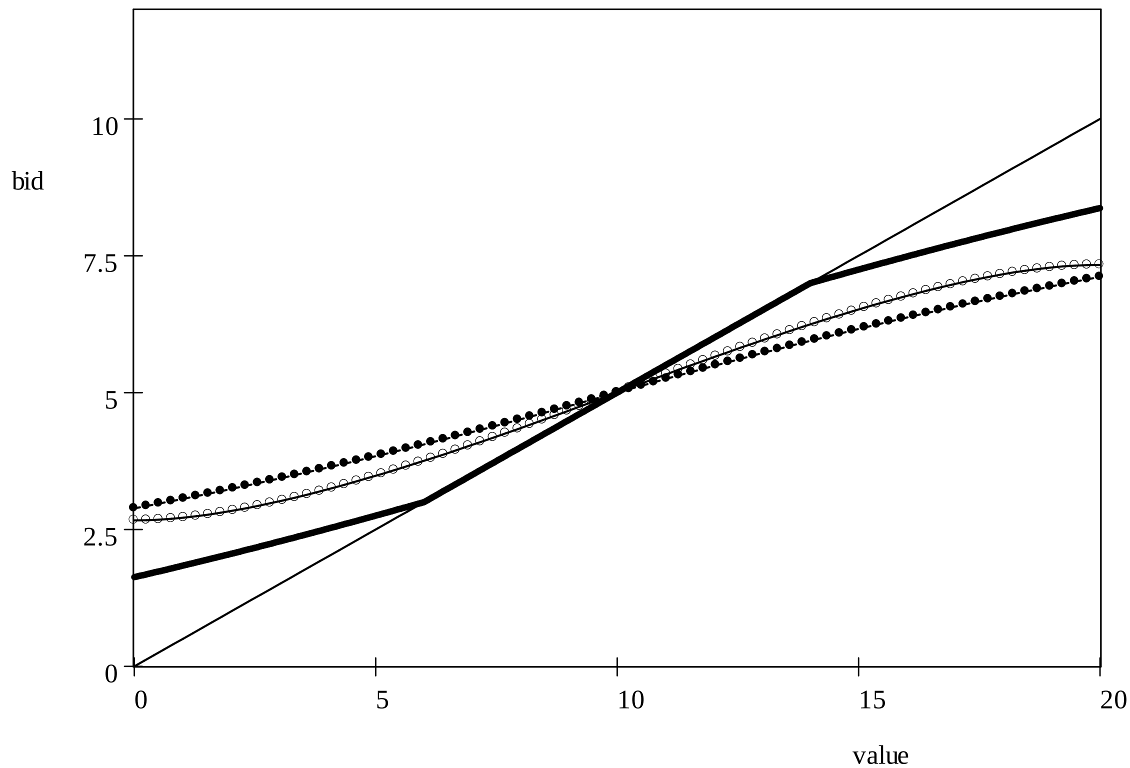

Figure 1: Theoretical bid functions: 1) full circle: McAfee for Texas shoot-out proposer. 2) empty circle: Crampton et al. for K+1 auction. 3) Thin line: Van Essen and Wooders, Texas shoot-out and K+1 auction with maxmin, also Texas shoot-out responder in all cases. 4) Thick line: Bauch and Riedel Texas shoot-out, proposer.

Note: See appendix for the calculations of the bid functions.

bargaining outcome is one where the asset is assigned to the partner with the higher value for the asset. Because the asset is of ambiguous value, we note that each partner's risk and ambiguity attitudes are key to determining their value for the asset. In an attempt to be as inclusive as possible with respect to theoretical models, instead of estimating these attitudes and deriving estimated values for each partner, we instead measure values for an analogous asset in the incentivized questionnaire. By comparing relative values for the analogous asset between partners, we can infer relative values for the original asset in the bargaining session. We then define "successful assignment" as a bargaining outcome where the asset is assigned to the partner who indicated a higher value for the analogous asset in the questionnaire. This brings us to our first hypothesis:

Hypothesis 1 (Null): The probability of successful assignment will be equal across bargaining mechanisms.

Hypothesis 1 (Alternative): The probability of successful assignment in the $K + 1$ auction and unstructured bargaining will be greater than in the Texas shoot-out.

While the null hypothesis is that all mechanisms will perform equally, we plausibly expect some difference here. Theoretically, if subjects follow the same bargaining function, the $K + 1$ auction should be efficient. For Unstructured bargaining our prediction can only be based on cooperative game theory concepts like the Nash bargaining solution (Nash, 1953), which often assume efficiency when there are gains from trade, as there are in our experiment. The Texas shoot-out, on the other hand, is not necessarily efficient, as proposers and responders have different offer and response functions. This allows for misallocation of the asset in equilibrium.

While successful assignment is a good and simple measure for these bargaining mechanisms, a slightly more sophisticated analysis would note that, from an efficiency perspective, it is more important that the bargaining mechanism assigns the asset correctly the further apart the partners' values are. Indeed if the partners have the same value, it does not matter who is assigned the asset. If, on the other hand, one partner had a much higher value for the asset, assigning the asset to the other partner would represent a significant loss in surplus obtained from trade. With this idea in mind, we look also at total surplus obtained per mechanism. This brings us to our second hypothesis:

Hypothesis 2 (Null): Obtained surplus will be equal across bargaining mechanisms.

Hypothesis 2 (Alternative): Obtained surplus in the $K + 1$ auction and unstructured bargaining will be greater than in the Texas shoot-out.

Our null again assumes all bargaining mechanisms are equal. As noted before, the likelihood of success is suspected to differ for the Texas shoot-out. However, misallocation is expected to happen when values are relatively close. So, we expect this represents a somewhat "stronger" test. A difference here would imply a significant welfare loss as a result of misallocation.

With respect to choices, theory (see Figure 1) suggests equal means and medians for $K+1$ bids, Texas proposers, and Texas responders. This brings us to our third hypothesis:

Hypothesis 3 (Null): Mean and median choices for $K+1$ bids, Texas offers, and Texas minimums will be equal.

Hypothesis 3 (Alternative): Mean and median choices will vary across mechanisms.

On the other hand, Figure 1 shows that bids should vary for very low and for very high values.

From a more practical perspective, we are interested in how obtained prices in these mechanisms relate to the values of the two partners. So long as values are not equal between partners, one must have a higher value and one a lower value. Prices in a first price auction would be driven by the higher value, while prices in a second price auction would be driven by the lower value. Particularly for the unstructured bargaining case, it is interesting then to test whether the final price is more related to the higher or lower value. This can give us some insight into whether unstructured bargaining acts more like a first or second price auction. Meanwhile, we expect that in the $K+1$ auction, with $K = 0.5$, the price responds relatively equally to high and low values. A similar argument can be made for the Texas shoot-out as half the time the price is expected to be determined by the high value partner and half the time by the low value partner. This brings us to our fifth hypothesis:

Hypothesis 4 (Null): Prices in each mechanism are affected equally by the higher and lower asset valuations.

Hypothesis 4 (Alternative): For each mechanism, prices are not affected equally by the higher and lower asset valuations.

4 Results

Before getting to our hypotheses, we test for order effects by comparing the bids in the $K+1$ auction and the prices in the Texas shoot-out. Recall that the three mechanisms were played in random order where for each of the 6 possible orders we have between 30 and 40 independent observations. We use pairwise MWU-tests between any two orders and find that none of the differences are significant at the 10% level. Therefore, from now on, we pool all data.

It is also worth looking at the risk and ambiguity attitudes of our subjects based on their responses in the questionnaire at the end of our experiment. Recall that we used a BDM mechanism to measure subjects' certainty equivalent (CE) for a risky and an ambiguous lottery, which pays either €15 or €0. Table 1 shows summary statistics for CEs of the risky and the ambiguous lotteries. The questionnaire responses reveal a subject pool that is both risk and ambiguity averse on average. The average CE for the risky lottery is € 6.78 (compared to the expected value of the lottery of € 7.5). Most subjects are risk averse (54.55%) with about 25% being risk neutral.[11] The average CE for the ambiguous lottery is € 5.80. Subjects are classified as ambiguity averse (as usual in the classical Ellsberg experiment) if they reveal a higher CE for the risky lottery than for the ambiguous lottery. There are about four times as many ambiguity averse subjects (44.09%) as ambiguity loving (11.36%). There were slightly more ambiguity neutral subjects (44.55%) than ambiguity averse ones.

Table 1: Summary of risk and ambiguity attitudes

	Mean	Std Dev	% Averse	% Neutral	% Loving	Obs
CE risky lottery (CE^R)	6.78	2.08	54.55%	25.45%	20.00%	220
CE ambiguous lottery (CE^A)	5.80	2.45	–	–	–	220
Ambiguity Attitude	−0.97	1.97	44.09%	44.55%	11.36%	220

Note: Risk attitude categorized by comparing CE risk to the expected value (€7.50). Ambiguity attitude is the difference between CE ambiguity and CE risk.

4.1 Bargaining outcomes and hypotheses 1 and 2

The main interest of our experiment is in studying which bargaining mechanism performs better in allocating the ambiguous asset to the partner that has a higher willingness to pay for it. Thus, we classify a bargaining outcome as a "successful assignment" if the

[11] We categorize subjects as risk averse (neutral / loving) if their value for the risky lottery is less than (equal to / greater than) the expected value of the lottery. We categorize subjects as ambiguity averse (neutral / loving) if their value for the ambiguous lottery is less than (equal to / greater than) their value for the risky lottery.

asset ends up with the person who reveals a (weakly) higher CE for the ambiguous asset in the questionnaire.[12] For the Texas shoot-out, since we use the strategy method and have data for both roles, the divider and the chooser, we calculate the average of successful assignment across roles. For comparisons of the $K + 1$ auction to the Texas shoot-out we have 220 independent observations (within subjects) for bids and reservations prices, respectively.

For payment purposes we match subjects randomly into pairs. However, for the evaluation of whether the bids and reserve prices lead to efficient outcomes, these random matches are arbitrary since subjects have no knowledge or feedback about their matched partners. Ideally, we would consider the average efficiency across matches with all 219 other subjects. However, given the symmetry (if subject i's bid yields an efficient match against j's, then j'match against i is also efficient) we would double-count observations. It seems intuitive that for efficiency measures we should have 110 independent observations. Thus, what we do is to group subjects randomly into two subgroups and match each subject in subgroup 1 with the 110 members of subgroup 2. This way, there is no double-counting. Since the group assignment is random, test results could depend on a particular group assignment. Therefore, we repeat this process 10,000 times and calculate the p-values for all 10,000 group assignments.

Vovk and Wang (2020) discuss various methods for testing a hypothesis using K different statistical tests, obtaining p-values $p_1, ..., p_K$, and combining them into a single p-value. Apparently, a conservative method would be the Bonferroni method, which would imply taking $K \min\{p_1, ..., p_K\}$. However, with our 10,000 group assignments, the minimal p-values are always below 10^{-13}, which does not strike us as conservative. Rüger (1978, p. 177) suggests using the median of the p-values and proves that 2 times the median is a valid aggregated p-value. In the following we will therefore combine p-values by scaling up their median by a factor of 2 and call this the combined p-value $\bar{p}$.

Table 2 shows the average successful assignment for the different bargaining mechanisms. We will first compare the $K + 1$ auction to the Texas shoot-out and deal with Unstructured bargaining later. While all bargaining mechanisms show an efficiency of well below 100%, the $K+1$ auction is almost 4 percentage points more efficient than the Texas shoot-out. This difference is significant according to a two-sided Wilcoxon sign–ranked test

[12] In the $K + 1$ auction, if two bids are exactly equal, the asset is allocated randomly. In this case, successful assignment takes the expected value of 1/2.

(combined $\bar{p} = 0.016$ from 10,000 group assignments) given our within subject design.[13]

Table 2 also shows the average surplus generated in all matches. The actual surplus equals the CE of the partner who ended up with the asset (if the asset is lost, surplus is zero). The maximum surplus is generated if the asset always goes to the partner who has the higher CE for the ambiguous lottery. The second row in Table 2 shows the average surplus generated by the different mechanisms (for $K+1$ and Texas shoot-out again calculated for each subject in group 1 matched against the 110 subjects in group 2). Again, the $K+1$ auction is more efficient than the Texas shoot-out and this difference is significant according to a Wilcoxon test (combined $\bar{p} = 0.008$ from 10,000 group assignments),[14] however both mechanisms achieve between 85% and 83% of the potential surplus.[15]

For the unstructured bargaining we have to work, of course, with the actual implemented matches as partners interact for 5 minutes. To improve the comparisons to the other treatments, we consider for those the hypothetical matches that would have occurred had partners been matched as in the Unstructured bargaining treatment. For successful assignment we thus have 110 observations per treatment, which are either 0, 1 or 0.5. Wilcoxon sign-rank tests show that successful assignment is significantly lower in Unstructured bargaining than in the $K+1$ auction ($p = 0.012$) or in the Texas shoot-out ($p = 0.020$). Likewise, average surplus in Unstructured bargaining (4.21 which corresponds to 58% of potential surplus) is significantly lower than in the $K+1$ auction and the Texas shoot-out ($p < 0.001$).

Table 2: Efficiency of bargaining mechanisms

	$K+1$ auction	Texas shoot-out	Unstructured bargaining
Successful assignment	64.45%	60.88%	44.55%
	(19.71%)	(13.64%)	(49.93%)
Average surplus	6.13	5.97	4.21
	(1.59)	(1.33)	(3.52)

Note: Values for K+1 auction and Texas shoot-out are based on averages over all potential matches. Standard deviations in parentheses.

[13] In fact, 68.6% of p-values are below 0.05.

[14] 71.5% of p-values are below 0.05.

[15] In the $K+1$ auction and the Texas shoot-out the average potential surplus was 7.22 (averaged over all potential matches between subgroup group 1 and subgroup 2 subjects). In the actual matches for Unstructured bargaining the average potential surplus was 7.14

Result 1 *Efficiency (as measured by successful assignment and surplus) is highest in the* $K+1$ *auction and lowest in Unstructured bargaining.*

The main reason why Unstructured bargaining is performing so badly in terms of efficiency is that many bargaining sessions end without agreement, in which case the asset is lost and the generated surplus is zero. In fact, 31.8% of matches did not agree to a final allocation within the 5 minutes of unstructured bargaining with chat. For the matches that do end with an agreement, Successful assignment and Average surplus are even slightly higher than for the other mechanisms. So the question is, why does unstructured bargaining break down so often here and does this have anything to do with the fact that values of the partnership are ambiguous? One possible reason would be that the time period given for bargaining is simply too short. Recall that subjects had 5 minutes to make a deal. In a similar recent experiment by Hyndman (2021) more than 97% of bargainers were able to strike a deal in unstructured bargaining with chat within 4 minutes. So it seems unlikely that time pressure was the issue. Likewise, the opportunity to chat is unlikely to be the culprit since Hyndman (2021) finds that in unstructured bargaining with chat agreements are more likely than without chat. The software we used to implement the unstructured bargaining protocol was used before in almost the same way in Ingersoll and Roomets (2020), where it also did not result in many bargaining breakdowns (10.3% of subjects in all treatments failed to reach an agreement).

The chat protocol gives some mixed picture on how subjects bargained. While some subjects seem to bargain in good faith and possibly run out of time in the end, others seem to use it like an ultimatum game and lose the asset in a stalemate. For example one pair had the following exchange:

> Subject 1: "I am not willing to bargain with you. Either I get more than €17 or I let the asset expire." (84 sec. left)
>
> Subject 2: "This is not going to work...". (64 sec.)
>
> Subject 1: "Then the asset will expire." (49 sec.)
>
> Subject 2: "Ok, then we let the asset expire." (46 sec.)

And they let the asset go without a deal and any further communication.

Overall, comparing our Unstructured bargaining treatment to Hyndman (2021) and Ingersoll and Roomets (2020), the only major difference seems to be that bargaining is over an ambiguous asset rather than a risky or safe asset. One possible difference between bargaining about an ambiguous asset versus a risky asset is that subjects may (correctly) assume that some subjects have a very low value for the ambiguous asset (see Fig. 3 in the appendix showing the distribution of CE's for an ambiguous asset vs. those for a risky asset). This may induce them to submit very low offers, and make agreements more difficult to achieve. For risky assets, there are fewer of those very low CE's, which might make agreements easier. While we cannot offer any conclusive evidence for why bargaining over ambiguous assets lead to frequent breakdown, we think this is an interesting observation, which should be addressed in future work.

4.2 Subject choices and Hypothesis 3

Table 3 shows summary statistics of subject choices in our main treatments. We find that $K+1$ bids are significantly higher than Texas prices, and that both are significantly lower than Texas reserves (Wilcoxon sign-rank tests, $p < 0.001$). We can therefore reject Hypothesis 3 that mean and median choices for $K+1$ bids, Texas offers, and Texas reserves would be equal.

Table 3: Summary of individual choices

	$K+1$ bids	Texas price p	Texas reserve r
Mean	4.424	3.763	5.057
	(2.051)	(1.593)	(1.985)
25th percentile	3.065	3	4
Median	4.5	3.775	5
75th percentile	5	4.975	6
Subjects	220	220	220

Note: Standard deviations in parentheses.

Looking at Figure 2 and Table 4 allows us to dig deeper into how these choice distributions differ. Figure 2 plots all bids against values, where values (for the whole partnership) are gathered from the CE^A revealed in the questionnaire scaled up by 4/3.[16] For comparison, the theoretical bid functions from Figure 1 and a regression line are also shown.[17]

[16] We use the scaling factor to make CEs comparable. Recall that the asset in the questionnaire would be either €15 or €0, whereas the partnership can pay either €20 or €0.

[17] Although our theoretical predictions are not linear, the curvatures of the predictions are not so extreme as to make a linear estimate unhelpful.

Table 4 reports the corresponding coefficients for the latter.

Our results differ substantially from the theoretical predictions. Concerning intercepts, we would have expected Texas reserves to have an estimated intercept close to zero, but instead it is estimated to be the highest, and statistically different (Wald test, $p < 0.01$) from the intercepts of both, the $K+1$ bids and the Texas prices. With respect to the slopes, we find a similar result. In particular, the lowest estimated slope (with Texas reserves) should theoretically have been the highest. That said, these relationships between slopes are not significantly different from each other at the 5% level according to Wald tests.

Table 4: OLS regressions of individual choices as a function of ambiguity values

	$K+1$ bids	Texas price p	Texas reserve r
Value	0.141***	0.114***	0.066
	(0.041)	(0.032)	(0.041)
Constant	3.333***	2.877***	4.550***
	(0.348)	(0.270)	(0.343)
Observations	220	220	220
Adj. R-squared	0.046	0.051	0.007

Note: *,**,*** - statistically significant at the 10%, 5%, 1% levels, standard deviations in parentheses. Value is calculated by scaling up CE^A by 4/3.

Overall, it seems that subjects deviate substantially from the theory when looking at individual choices. This may also help to explain why we find somewhat low rates of successful assignment across all mechanisms. The largest puzzle presented in this section concerns the Texas reserves regression line implied by Table 4. The intercept is much higher than we would expect and the slope is much lower (and not significantly different from 0).

4.3 Price determination and Hypothesis 4

In Table 5, we explore whether transaction prices are more influenced by the larger or by the smaller of the partners' values. For each partnership, we determine the subject with the higher (max) and lower (min) value derived from the questionnaire. Hypothesis 4 suggests that both values should have a relatively equal effect on final prices. Our regression shows some evidence that the lower value has a stronger influence on final agreed-upon price in the unstructured mechanism. This suggests that the unstructured setting may induce behavior reminiscent of a second price auction.

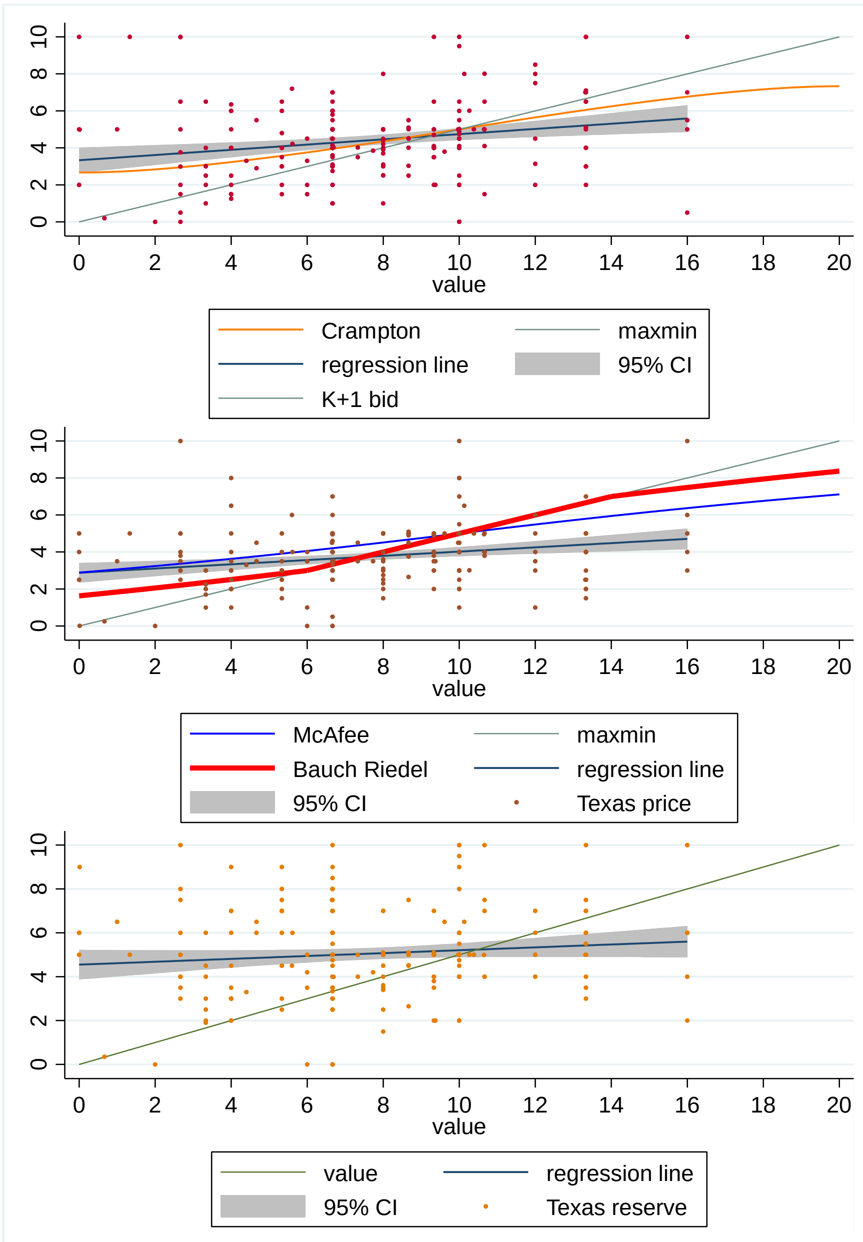

Figure 2: Scatter plot of $K+1$ bids (top graph), Texas prices (middle graph) and Texas reserves (bottom graph).

Note: For comparison theoretical bid functions (cf. Fig. 1) and a regression line with 95% confidence intervals are shown. Values are calculated as $\mathrm{CE}^{A}*4/3$.

Table 5: OLS regressions of transfers as a function of partners' ambiguity values

	$K+1$ Transfers	Texas Transfers	Unstructured Transfers
Max ambig. value	0.060	0.088	0.083
	(0.057)	(0.068)	(0.089)
Min ambig. value	0.046	0.135*	0.244***
	(0.059)	(0.073)	(0.090)
Constant	3.578	2.153	1.999
	(0.491)	(0.593)	(0.749)
Observations	110	110	75
Adj. R-squared	0.013	0.069	0.150

Note: *,**,*** - statistically significant at the 10%, 5%, 1% levels, standard deviations in parentheses. Max and min values are scaled up by 4/3. For Unstructured bargaining only successful agreements are included.

5 Conclusion

Considering the results of our experimental analysis, our conclusions are somewhat mixed. On the one hand, we do see a pattern of bargaining success that aligns roughly with theoretical predictions. The $K+1$ auction was more efficient then the Texas shoot-out, and, when successful, the unstructured bargaining protocol was similarly efficient to the $K+1$ auction and more efficient than the Texas shoot-out. On the other hand, the unstructured bargaining protocol often failed to produce an agreement in time, greatly lowering overall efficiency. Further, individual behavior seems to have been far from behavior predicted by theory.

Behavior in the Texas shoot-out diverged particularly far from theory it seems. Texas reserve prices seem to have been largely unrelated to a measure of value obtained in the incentivized questionnaire. While possible that the questionnaire responses are the issue, we believe that the questionnaire represents a better measure of value due to the relative simplicity of the elicitation mechanism. Understanding that setting a reserve price equal to your value in a strategic scenario is a weakly dominant strategy (and then choosing to play it over something that is weakly dominated) seems more sophisticated than understanding that reporting your value in a BDM-style procedure maximizes your expected payoff. So, if reserve prices were not very responsive to underlying values, we can conclude that much of the inefficiency seen in the Texas shoot-out was a result of reserve price choices. This is basically the opposite of the theoretical justification of inefficiency in the Texas shoot-out (i.e. price offers being less responsive to values while reserve prices are fully responsive).

On average, $K+1$ auction behavior was more in line with theory, but, even though bids were responsive to values on average, noisiness in the bids still caused efficiency loss. This noisiness could be the result of different beliefs about the underlying value distribution, different ideas about the optimal bid function, general confusion about the bargaining mechanism, or other causes. Some of these issues we might expect to be less prevalent in a real partnership dissolution, and so we feel it is safe to assume our results are on the lower end of efficiency expectations if a $K+1$ auction were used in the field.

Our unstructured bargaining results are in a sense disappointing but also intriguing. It would be nice to think that simply sitting down and chatting with a partner about how to move forward would be a sensible solution, but our results suggest that when dealing with ambiguous assets, these types of negotiations are prone to breaking down. We speculate that this may be due to the higher variance of valuations for a risky vs. ambiguous asset.[18] We did not fully anticipate such a result and so more studies will need to be done to better understand this phenomenon, but given the prevalence of negotiations over ambiguous pies,[19] such understanding would be highly valuable.

With respect to the question of partnership dissolution, while we would stop short of claiming the $K+1$ auction as the "best" mechanism for such a dissolution, we do think we have shown a certain value to simplicity in such negotiations. While the Texas shoot-out, as we implemented it, is rather simple, the $K+1$ auction seems simpler. The symmetry of the $K+1$ auction presents a simpler strategic setting than the asymmetric Texas shoot-out. Unstructured bargaining, with its rather indescribable strategy set, is even more complex than both. We believe this difference in relative complexity of the mechanisms played a role in how efficient the mechanisms were at assigning an asset with a value that was already complicated to calculate accurately in the laboratory setting but further studies are required to ascertain this.

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A Appendix (for online publication only)

A.1 Experimental Instructions

Welcome to our experiment! Please read this guide carefully. Stop talking to your neighbors from now on. Please turn off your mobile phone now and leave it off until the end of the experiment. If you have any questions, get in touch. Then we will come to you. All participants received the same instructions. The experiment should last about 60 minutes.

The rounds

You take part in three rounds of bargaining. In each round, you will be randomly assigned a new bargaining partner from this room. You will never be matched twice with the same partner. After the bargaining rounds there will be an additional questionnaire. At the end of the experiment, one of the bargaining rounds or the questionnaire is randomly selected by rolling a four-sided die and all participants are paid according to the outcome of that bargaining round or questionnaire. If the die shows a “1”, the participants are paid according to the first round of bargaining. If the die shows a “2”, the participants are paid according to the second round of bargaining. If the dice shows a “3”, the participants are paid according to the third round of bargaining. If the dice shows a “4”, the participants will be paid according to their answers in the questionnaire.

The bargaining

You bargain with a randomly selected bargaining partner about an asset of unknown value. Both bargaining partners own 50% of the asset at the beginning of each round and also receive €10. The bargaining partners must agree on who should ultimately own 100% of the asset and what payments should be agreed as compensation.

The bargaining rules are different in each round. That is why you get new instructions for each round. Please read them carefully each time and answer a short quiz about the rules.

The asset

You can think of the “asset” as an envelope containing either €0 or €20. This means that the owner of the asset gets paid either €20 or €0 at the end of the experiment. And this payoff is determined in the following way:

On the experimenters’ table we have a bag with 20 marbles, which are either blue or yellow. You are not familiar with the combination of blue and yellow marbles. Any possible combination of 0 blue marbles (and therefore 20 yellow) to 20 blue marbles (and therefore 0 yellow) is possible. At the end of the experiment, you can see the contents of the bag for yourself.

If you are the owner of the asset at the end of the experiment and the corresponding round has been selected for payment, then you can decide whether you want to bet on "Blue" or "Yellow". Then a volunteer draws a marble out of the bag without looking. If the color of the marble drawn from the bag matches your color choice, you will receive €20 for the asset you own. If the color does not match, you get €0 for the asset.

Remember, the bargaining is always about buying or selling 50% of the assets to the other partner and the price for this half. So it would make sense to first consider how much half of the asset is worth to you.

The payoff

All participants will receive a fixed amount of 5€ for participation.

Additionally, if one of the bargaining rounds is selected for payment, you will receive the following:

- If you own the asset this round, you are paid the asset's payoff (as determined by the draw from the bag) in euros.
- All participants receive the starting endowment of 10 € minus or plus the amount agreed during the bargaining.

If the questionnaire is selected for payment, you will also receive the payoffs resulting from your answers as described in the questionnaire, where the payoff for some questions also depends on a drawing from the marble bag.

Round "U" [this sheet with instructions was distributed when Round U started]

In this round you can chat with your partner for 5 minutes about possible deals. For this you will find a chat field on the left half of the screen.

The right half of the screen is divided into two fields: In the upper right corner, both partners can propose deals for consideration. Before making a proposal, you can click "Calculate" to see the resulting possible payoffs for both partners. If you are not happy with the suggestion, you can go back and change it, then click Calculate again. If you are satisfied with the proposal, you can click "Accept the current offer"

A possible deal always consists of an offer to buy or sell **half** of the asset and a payment from the buyer to the seller. Only the latest proposal can be reviewed and accepted. If both partners accept a proposal, the bargaining ends.

Example : You make a proposal to buy half of the asset from the other partner for €1.70. He accepts the suggestion. Then you get his half and pay him €1.70 for it. This means that you have an amount of €8.30 left and you own the whole asset. Your partner has a cash amount of €11.70 but no asset.

If no agreement is reached within the 5 minutes, the asset is lost and both partners only receive their initial endowment of €10. Please note the "**time left**" countdown in seconds on the top right of the screen.

Round "T" [this sheet with instructions was distributed when Round T started]

There are two roles in this round, one of the two bargaining partners is the proposer, the other is the responder. The roles are randomly assigned by the computer.

The proposer proposes a price for **half of the asset**. The responder can then decide if he wants to buy half of the asset from the proposer at that price or if he wants to sell his half of the asset to the proposer at that price.

Since you do not yet know whether you will be the proposer or the responder, you must make a decision for both possibilities:

1. In case you are the proposer, you must suggest the **price** for half of the asset. The responder then decides whether to sell his half to you or buy your half from you at that price. Enter the suggested price at the top of the interface. This amount can be between €0.00 and €10.00.

2. If you are the responder, you must provide a **minimum price**. If the price the proposer proposes is at least as high as your minimum price, then you will sell your half of the asset to the proposer at the price the proposer proposes. If the price the proposer proposes is less than your reserve price, then you will buy half of the asset from the proposer at the price the proposer proposes. Enter the minimum price at the bottom of the screen. It can also be between €0.00 and €10.00.

Example: You enter a price of €3 and a minimum price of €8. Suppose the computer selects you as a responder and your assigned proposer selects a price of €4. Then you will buy half of the asset from the proposer at a price of €4 because your minimum price of €8 is higher than the proposer's price of €4.

Round "K" [this sheet with instructions was distributed when Round K started]

In this round, both partners make a bid **for the other half of the asset**. The partner who submits the higher bid receives half of the asset from the other partner and pays him the average of the two bids.

After you have decided on a bid, you place your bid by computer (without knowing the partner's bid). Bids can range from €0.00 to €10.00. If both bids are equal, one partner will be chosen at random as the winner.

Example : You place a bid of €2.50 and your partner places a bid of €6.00. Since your partner's bid is higher, you sell your half of the asset to your partner for the average of the two bids, i.e. (€2.50 + €6.00) / 2 = €4.25.

Questionnaire [this sheet with instructions was distributed when Round Q started]

In this round, we ask you about your valuation for two other assets. These assets are different from the asset you have been bargaining on so far.

The two assets

Asset A's payoff depends on a coin toss. A volunteer will flip a coin and Asset A pays out €15 on heads and €0 on tails.

Asset B's payoff depends on drawing from the same bag of marbles as before. You choose a color, yellow or blue, and if your chosen color is drawn, Asset B pays out €15, if not, Asset B pays out €0.

To determine your valuation, imagine we were asking you to choose between the asset in question and a fixed amount of money, with the fixed amount of money increasing from €0.00 to €15.00.

Do you want the asset...	...or this amount of money?	
Asset	€0.00	I choose: ?
Asset	€0.01	I choose: ?
Asset	€0.02	I choose: ?
...	...	I choose: ?
...	...	I choose: ?
Asset	€14.99	I choose: ?
Asset	€15.00	I choose: ?

At the end, the computer will randomly choose a row (all are equally likely) and you would then get whatever you chose on that row, either the asset or the money. It should be clear that it is in your best interest to choose what you really prefer.

Since we do not want to bother you with 1500 questions, we will simply ask you up to what fixed amount of money you would want the asset. Again, it is in your best interest to state your true valuation for the asset.

So in the questionnaire we will simply ask you to state your true valuation for asset A and for asset B.

If this round is selected for payment, then you will be paid for both questions, with Asset A's payoff being determined by the volunteer's coin toss and Asset B's being determined by a draw from the bag of marbles.

A.2 Quiz

(the test questions were asked after subject read the instructions for the respective mechanism. They could only proceed, if all questions were answered correctly)

Unstructured bargaining

1) If neither partner makes a proposal, there can be no agreement. The asset is then randomly assigned to one of the partners. Incorrect

2) Suppose you agree on a proposal where your partner gets the asset for a payment of €3.25. Remember your starting endowment of €10. What payoff would you get in the negotiation round?

Texas shoot-out

1) There is a 50% chance that your decision will be applied in your proposer role and a 50% chance that your decision will be applied in your responder role.

2) The suggested price given is €2.50 and the minimum selling price given is €2.00. Remember the starting endowment of 10€. How much money (without asset) will the proposer have after the transfer?

$K + 1$ Auction

1) If you bid €2 and your partner bid €1, who gets the asset?

2) If you bid €2 and your partner bid €1, what would be your cash payoff (without asset) in this negotiation round? Remember your starting endowment.

A.3 Calculations for the triangular distribution

We assume in the following that values (CEs) are distributed according to a triangular distribution (which seems to come close to the empirical distribution of CEs in Figure 3) with density

$$f(v_i) = \begin{cases} \frac{v_i}{100} & \text{if } 0 \leq v_i \leq 10 \\ \frac{(20-v_i)}{100} & \text{if } 10 < v_i \leq 20 \end{cases}$$

and cdf

$$F(v_i) = \begin{cases} \frac{1}{200}v_i^2 & \text{if } 0 \leq v_i \leq 10 \\ \frac{1}{5}v_i - \frac{1}{200}v_i^2 - 1 & \text{if } 10 < v_i \leq 20 \end{cases}.$$

For the $K+1$ auction, equation (1) becomes

$$\begin{aligned} b(v_i) &= \frac{v_i}{2} - \frac{\int_{10}^{v_i} \left[\frac{z^2}{200} - \frac{1}{2}\right]^2 dz}{2\left[\frac{v_i^2}{200} - \frac{1}{2}\right]^2}, \text{ for } v_i < 10 \\ &= \frac{v_i}{2} + \frac{(10 - v_i)\left(90v_i + 3v_i^2 + 800\right)}{30\,(v_i + 10)^2}, \end{aligned} \tag{3}$$

and

$$\begin{aligned} b(v_i) &= \frac{v_i}{2} - \frac{\int_{10}^{v_i} \left[\frac{1}{5}z - \frac{1}{200}z^2 - 1 - \frac{1}{2}\right]^2 dz}{2\left[\frac{1}{5}v_i - \frac{1}{200}v_i^2 - 1 - \frac{1}{2}\right]^2}, \text{ for } v_i \geq 10 \\ &= \frac{v_i}{2} - \frac{(10 - v_i)\left(210v_i - 3v_i^2 - 3800\right)}{30\,(v_i - 30)^2}. \end{aligned} \tag{4}$$

Combining (3) and (4) yields the line with empty circles in Figure (1).

For the Texas shoot-out, equation (2) becomes for $v_i < 10$

$$\pi(p|v_i) = (v_i - p)\frac{(2p)^2}{200} + p\left(1 - \frac{(2p)^2}{200}\right) = p - \frac{1}{25}p^3 + \frac{1}{50}p^2 v_i,$$

which has a maximum at

$$p = \frac{1}{6}v_i + \frac{1}{6}\sqrt{v_i^2 + 300}. \tag{5}$$

For $v_i \geq 10$,

$$\pi(p|v_i) = (v_i - p)\left(\frac{1}{5}2p - \frac{1}{200}(2p)^2 - 1\right) + p\left(1 - \left(\frac{1}{5}2p - \frac{1}{200}(2p)^2 - 1\right)\right),$$

which has a maximum at

$$p = \frac{1}{6}v_i - \frac{1}{6}\sqrt{v_i^2 - 40v_i + 700} + \frac{20}{3}. \tag{6}$$

Combining (5) and (6) yields the line with full circles in Figure (1).

Bauch and Riedel (2022) consider the triangular distribution in their Example 5. For $c = 1/2$ (uncorrelated values) and a noise level of $\varepsilon = 1/5$, they find the bid functions for the Texas shoot-out proposer to be (after adjusting for the different supports)

$$b(v_i) = \begin{cases} \frac{v_i}{6} + \frac{1}{3}\sqrt{100(0.05v_i)^2 + 40(0.05v_i) + 79} - 4/3 & \text{if } 0 \leq v_i \leq 6 \\ \frac{v_i}{2} & \text{if } 6 \leq v_i \leq 14 \\ \frac{v_i}{6} - \frac{1}{3}\sqrt{100(0.05v_i)^2 - 240(0.05v_i) + 219} + 8 & \text{if } 14 < v_i \leq 20 \end{cases}. \tag{7}$$

Bid function (7) is shown as the thick line in Figure (1). The range in the middle where proposers simply bid their maximin payoff depends of course on the noise level ε. The larger ε, the larger this range.

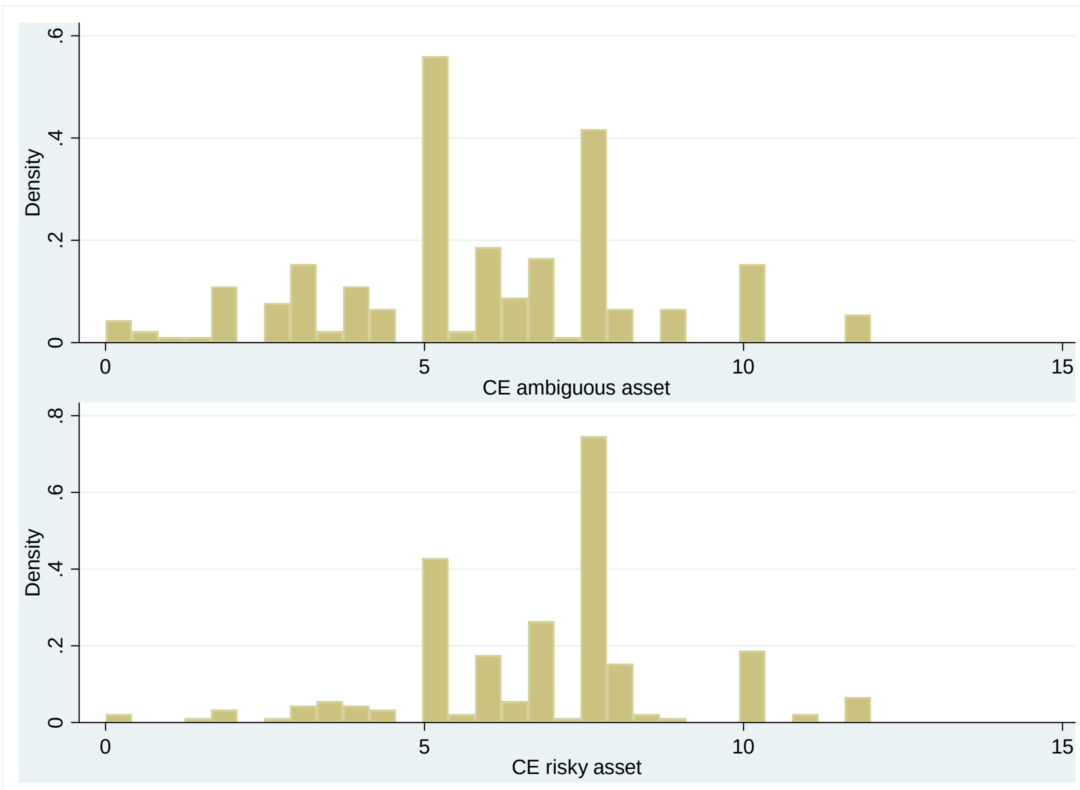

Figure 3: Relative frequencies of CEs for the ambiguous and risky asset

Note: These are the CEs from the questionnaire elicited with BDM for an asset that can either have value of €0 or €15.