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Ruben Hans Thies Küspert

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Testing Higher Order Corrections to String Compactifications through Kinetic Mixing and Inflation

Referees: Prof. Dr. Arthur Hebecker
Prof. Dr. Razvan Gurau

Abstract

This thesis investigates phenomenological implications of flux compactifications in type IIB string theory. In particular, we focus on the phenomena of gauge kinetic mixing and cosmic inflation in large volume scenarios in type IIB.

In the first part we discuss kinetic mixing and study settings with $U(1)$s from sequestered D-brane sectors focusing on mixing of D3-D3 and D7-D7-branes. Strikingly, kinetic mixing is absent due to a non-trivial cancellation in rather generic scenarios.

Specifically for D3-branes, precise calculations of string diagrams have previously demonstrated a cancellation on toroidal geometries. Also, field theoretic 10d supergravity approaches have shown that a cancellation remains at leading order between the contributions of B_2 and C_2 in the case of D3-brane mixing, extending the result from special geometries to realistic Calabi-Yau settings. We take the latter approach and furthermore consider non-zero values for C_0 and include sub-leading terms of the D3-brane action and affirm that an exact cancellation persists in this generalised setting. Ultimately, we demonstrate that this cancellation is tied to the $\mathrm{SL}(2,\mathbb{R})$ self-duality of type IIB. Finally, allowing for $SL(2,\mathbb{R})$-breaking 3-form fluxes, kinetic mixing between D3-branes arises at a volume-suppressed level.

In the case of D7-D7-brane mixing, we consider stacks of D7-branes where the non-abelian gauge theory is broken by internal flux of the gauge theory. In this case we find that, in addition to B_2 and C_2 contributions, non-vanishing kinetic mixing is induced by C_4. Yet again, the B_2 and C_2 contributions cancel if no 3-form fluxes are present. We derive explicit formulas for kinetic mixing in both cases and perform a phenomenological analysis of our D3-D3 scenario and find that parametrically small values of kinetic mixing can be realised.

In the second part we discuss cosmic inflation in string theory. We consider a scenario of type IIB where all moduli are stabilised realising a large volume and a Minkowski minimum. The volume and an additional small blow-up modulus are fixed by non-perturbative effects, while the other Kähler moduli are stabilised via loop corrections. Within this framework, we investigate different regimes of the scalar potential which are suitable for slow roll and show that flat plateaus exist quite generally. We apply our observation to a concrete and simple model where we use an additional blowup mode as the inflaton. In this model the respective inflationary potential becomes flat for large field values. We ensure that our model aligns with the observed normalization of scalar perturbations and generates an adequate number of e-foldings. As a consequence, the volume cannot be stabilised at excessively high values and the inflaton starts rolling at largish values, thus introducing a control issue for the stabilisation procedure. Nevertheless, we demonstrate that our model remains in a controlled regime. Encouragingly, a thorough analysis indicates that our model meets all the necessary phenomenological criteria.

Zusammenfassung

In dieser Dissertation werden phänomenologische Auswirkungen von Flusskompaktifizierungen in der Typ IIB Stringtheorie untersucht. Insbesondere liegt der Fokus auf den Phänomenen des kinetischen Mischens von Eichtheorien (“gauge kinetic mixing”) und der kosmischen Inflation im “Large Volume Scenario” von Typ IIB.

Im ersten Teil dieser Arbeit diskutieren wir kinetisches Mischen und untersuchen hierfür Modelle mit $U(1)$-Eichtheorien in räumlich getrennten D-Branen Sektoren. Im Speziellen betrachten wir kinetisches Mischen zwischen D3-D3 und D7-D7-Branen. Bemerkenswerterweise zeigt sich, dass kinetisches Mischen aufgrund einer nicht-trivialen Annullierung in vielen generischen Fällen verschwindet. Im Fall von D3-Branen haben exakte Berechnungen von Stringdiagrammen auf Torus Geometrien eine Annullierung aufgezeigt. Auch feldtheoretische Analysen in 10d-Supergravitation zeigen für diesen Fall, dass eine Annullierung in führender Ordnung zwischen den Beiträgen von B_2 und C_2 vorliegt. Der feldtheoretische Zugang erlaubt stringdiagrammatische Ergebnisse von speziellen Geometrien auch auf komplexe Calabi-Yau-Modelle zu erweitern, weshalb sich diese Arbeit auf die letztere Herangehensweise stützt. Im Folgenden werden wir die Analyse zu führender Ordnung erweitern indem wir hierfür nicht verschwindende Werte für C_0 als auch Selbstkopplungsterme auf der D3-Brane in die Analyse mit einschließen. Interessanterweise bleibt eine exakte Annullierung fortbestehen und es kann gezeigt werden, dass diese Annullierung von der $SL(2,\mathbb{R})$-Selbstdualität von Typ IIB erzwungen wird. Letztlich können wir zeigen, dass $SL(2,\mathbb{R})$-brechende 3-Form Flüsse ein nicht verschwindendes Resultat für kinetisches Mischen zwischen D3-Branen, auf einem volumenunterdrückten Niveau, zulassen.

Im zweiten Fall analysieren wir kinetisches Mischen zwischen D7-D7-Branen, wofür wir Stapel (“stacks”) von D7-Branen untersuchen. Die nicht-abelsche Eichtheorie wird durch einen internen Fluss der Eichtheorie gebrochen. In diesem Fall wird zusätzlich zu den B_2- und C_2-Beiträgen ein nicht verschwindendes Mischen durch C_4 induziert. Wir finden erneut, dass sich die B_2- und C_2-Beiträge aufheben, wenn keine 3-Form Flüsse vorhanden sind. Für beide genannten Fälle leiten wir explizite Formeln für das resultierende kinetische Mischen her und führen eine phänomenologische Analyse des D3-D3 Szenarios durch. Basierend auf dieser Analyse folgern wir, dass parametrisch kleine Werte von kinetischem Mischen realisiert werden können.

Der zweite Teil dieser Arbeit behandelt kosmische Inflation in der Stringtheorie. Hier betrachten wir ein Szenario von Typ IIB, in dem alle Moduli stabilisiert sind und ein großes Volumen sowie ein Minkowski-Minium realisiert werden. Der Volumenmodulus und ein zusätzlicher kleiner Blowup-Modulus werden durch nicht-perturbative Effekte stabilisiert, während anderen Kähler -Moduli durch Quantenkorrekturen stabilisiert werden. In diesem Kontext untersuchen wir verschiedene Regime des Skalarenpotentials und zeigen, dass im Allgemeinen flache Plateaus existieren, welche für Slow Roll Szenarien von Inflation geeignet sind. Diese Beobachtung wenden wir auf ein konkretes und einfaches Modell an, in welchem wir einen zusätzlichen Blowup-Modulus als Inflaton verwenden. In diesem Modell wird das entsprechende Inflationspotential für große Feldwerte des Inflatons flach. Zusätzlich erzwingen wir, dass unser Modell mit der beobachteten Normierung der skalaren primordialen Fluktuationen übereinstimmt und eine angemessene Anzahl an “e-folds” erzeugt. Infolgedessen kann das Volumen nur bei kleinen Werten stabilisiert werden und das Inflaton beginnt bei großen Feldwerten zu rollen. Beide Einschränkungen stellen ein Kontrollproblem für den Stabilisierungsmechanismus dar. Dennoch können wir zeigen, dass unser Modell in einem kontrollierten Bereich verbleibt. Zudem zeigt eine ausführliche Analyse, dass unser Modell alle notwendigen phänomenologischen Kriterien erfüllen kann

Preface

The research presented in this thesis was carried out at the Institute for Theoretical Physics at the University of Heidelberg from November 2020 until January 2024. The content of parts II and III were worked out during this period.

Part II contains the work done in collaboration with my supervisors Arthur Hebecker and Jörg Jäckel which led to the publication [1]. Notably, the content of the chapters 5 and 7 were not part of [1], but were only presented at conferences or in this thesis.

The results of part III are based on work in progress [2] in collaboration with Arthur Hebecker, Michele Cicoli, Sukṙti Bansal and Luca Brunelli. Importantly, the original research that forms the basis of section 10.2.2 and chapter 12 was carried out by Luca Brunelli supervised by Michele Cicoli. For comprehensive coverage and clarity, we have merely reformulated these chapters in our own words.

[1] A. Hebecker, J. Jaeckel, and R. Kuespert. *Small Kinetic Mixing in String Theory.* arXiv: 2311.10817 [hep-th]

[2] S. Bansal, L. Brunelli, M. Cicoli, A. Hebecker, and R. Kuespert. *Loop Blowup Inflation.* in preparation

Contents

Notation, Conventions & Abbreviations

In this dissertation, we will work in natural units where $c = k_B = \hbar = 1$. Hence, the (reduced) Planck mass is given by $M_\mathrm{P} = \sqrt{1/8\pi G} \approx 2.4 \times 10^{18}$ GeV which we may set to one for brevity. We choose the metric signature to be $(-1, 1, \ldots, 1)$ in arbitrary dimensions.
The totally antisymmetric two index tensor may be defined by $\epsilon^{12} = \epsilon_{21} = 1$ and $\epsilon_{12} = \epsilon^{21} = -1$. Complex conjugation is denoted by $\overline{a + ib} = a - ib$.

Tab. 1: List of abbreviations

Abbreviation	Meaning
UV	ultraviolet
IR	Infrared
vev	vacuum expectation value
EFT	Effective field theory
SM	Standard model
CMB	Cosmic Microwave Background
ΛCDM	Lambda-Cold-Dark-Matter
DM	Dark matter
DS	Dark sector
HS	Hidden sector
KM	Kinetic mixing
MM	Magnetic mixing
MCP	Millicharged particle
GR	General relativity
SUSY	supersymmetry
sugra	supergravity
EH	Einstein-Hilbert
RR	Ramon-Ramon
KK	Kaluza-Klein
DBI	Dirac-Born-Infeld
CS action	Chern-Simons action
CS moduli	Complex structure moduli
GKP	Giddings-Kachru-Polchinski
KKLT	Kachru-Kallosh-Linde-Trivedi
LVS	Large volume scenario

1. Introduction

1.1. Challenges of High Energy Physics

"High energy theoretical physics is in trouble."[1] *"[...] [It] can be said that with a wealth of phenomenological and theoretical ideas abounding in particle physics, we have yet to come up with a satisfactorily simple, general, and accurate explanation of all these phenomena."*[2]

These quotes seem to describe the current status of high energy physics, since many open questions remain unsolved and a mystery: the quantum theory of gravity, the origin of dark matter and dark energy, the hierarchy problem of the standard model, the strong CP problem, the masses of neutrinos, the H_0 tension– just to present a strongly biased selection of current problems in (theoretical) high energy physics.

However, the above quotes were taken from an article published in 1977 [3] criticising the state of particle physics at that time. Nowadays, arguably no one would describe the 30 years before and also after 1977 as a period of crisis in fundamental physics. Quite certainly, the whole 20th century can be considered as a period of many fundamental breakthroughs in physics. The historical theoretical achievements start with the advent of special and general relativity in the 1910s and quantum theory in the 1920s leading to the development of relativistic quantum field theory in the 1950s providing *the* frame work to describe all fundamental interactions of nature except gravity in *the standard model of particle physics (SM)*. These theoretical developments were most importantly inspired and accompanied by the discovery of nucleons and fundamental particles like quarks, neutrinos and force carrying bosons until recently when the Higgs boson was discovered in 2012 verifying the final building block of the SM. Simultaneously, seminal observations in cosmology reshaped our understanding of the cosmos: the expansion of the universe in 1929, the discovery of the Cosmic Microwave Background Radiation (CMB) in 1964 (see fig. 1.1), accumulating evidence for the existence of dark matter (DM) as well as the discovery of the accelerated expansion of the universe in 1998. All of these led to the construction of the standard model of cosmology called *the Lambda-Cold-Dark-Matter model (ΛCDM model)*. This list is far from complete and we only highlighted the point of view of high energy physics and cosmology! In summary, one can safely say that the progress in physics during the 20th century fundamentally changed our understanding at the smallest and largest accessible scales.

Besides all these successes, the open questions we have touched upon remain, leaving us in the apparent "trouble" to which [3] might hypothetically refer. However, we think the state of physics is quite the opposite of "in trouble", *"physics is [very, if not] too successful"*[3] in describing nature, which cannot be stressed enough. For all practical purposes, we do have an accurate description which allows to make concrete predictions. It is only in the most extreme regimes of nature that gaps in our current understanding are revealed. So perhaps we should be more optimistic. Our current situation may just be similar to that of 1977 and the upcoming 30 years reveal many new and fundamental discoveries if we only persist in our efforts to uncover them.

[1] see p.104 in [3]
[2] see p.89 in [3]
[3] [4] at 0:10:19

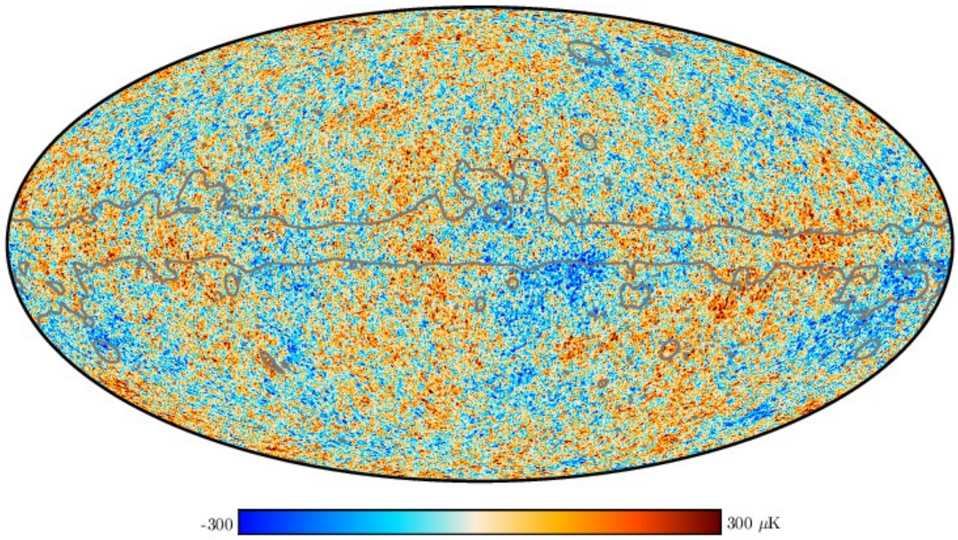


Fig. 1.1: CMB map of temperature fluctuations around mean temperature, taken from fig. 6 of [8].

This thesis is intended to contribute to several of the above issues, on which we will elaborate in the following. From our point of view the most important issues are the lack of a consistent quantum theory of gravity and of a microscopic explanation of dark matter. The unification of gravity and quantum theory into a *theory of everything* is essential for a thorough understand of fundamental physics. However, we will postpone the introduction to our chosen approach, *string theory*, until section 1.2. Before that, we want to discuss dark matter and cosmic inflation. Both subjects address separate problems in physics. If string theory is to be the theory candidate to account for all physical phenomena it must contain a resolution to the dark matter problem and inflation. Therefore, the question we want to answer in this thesis is whether string theory can account for both issues, and if so, what are the characteristic predictions of string theory in both cases.

Dark Matter

Let us begin by discussing dark matter representing a pivotal component of our universe which involves several aspects of astrophysics and cosmology. Among the various pieces of evidence supporting the existence of dark matter the probably strongest one can be extracted from the measurement of the CMB and its power spectrum [5–8], see fig. 1.1 and fig. 1.2. The spectrum shows characteristic *acoustic peaks* which are related to oscillations in the primordial plasma due to the interplay of gravitational contraction and radiative rarefaction. The composition of gravitating matter can be deduced from the relative heights of the acoustic peaks. The results show that an additional only gravitationally interacting matter component is necessary to explain the CMB anisotropies [8]. Moreover, the consistency of large scale structure formation [9–12] requires the presence of dark matter to allow inhomogeneities in the matter distribution to grow over time. Normal matter is repelled by radiative pressure and the increased gravitating pull due to dark matter allows to accumulate normal matter in consistency with the observed large scale structure formation. This is furthermore supported by the effect of matter on light from distant galaxies or stars which is distorted due to gravitational lensing. Applying this method allows to

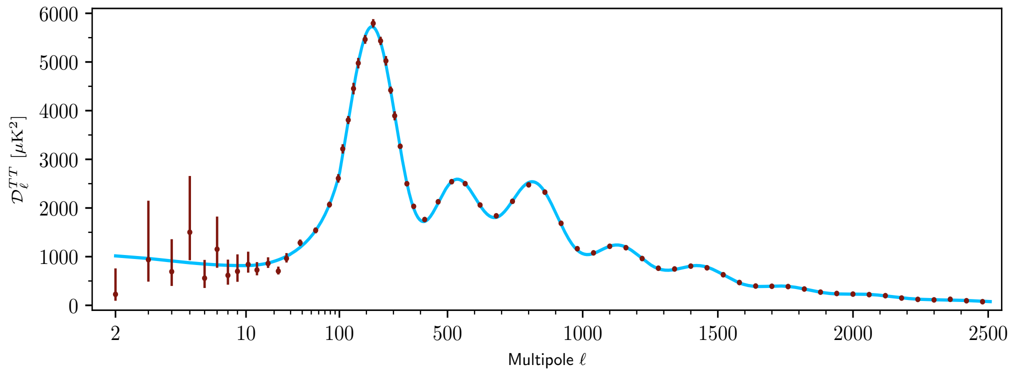


Fig. 1.2: CMB power spectrum, taken from fig. 9 of [8].

reconstruct the distribution of gravitating matter e.g. in galaxy clusters [13, 14] which is only consistent with direct observations if dark matter is taken into account. Several different sources of evidence have been gathered and form a coherent picture of the universe in which ~ 25% of the energy budget can be attributed to dark matter, 5% to normal matter described by the SM, and 70% to dark energy driving the current accelerated expansion of the universe [15–17].

Despite all this evidence for the existence of dark matter, further knowledge about dark matter remains quite limited. Current observational and experimental constraints can fix the abundance [8], its non-relativistic character and stability on cosmic time scales [18–21] as well as the apparent absence of interactions except of gravity [22–25]. However the question whether dark matter can be attributed to an unknown particle and the existence of additional non-gravitational interactions remain unsolved. To tackle these questions one may assume that besides the visible sector described by the SM there exists an additional *"dark or hidden sector"* containing a candidate particle to account for dark matter. A very feeble interaction between the visible and hidden sector can be introduced through so called *"portals"*. A portal is defined by an operator which respects all SM symmetries but incorporates a new mediating particle next to SM fields. In total, there are three renormalizable portals extending the SM: the scalar or Higgs portal [26–29], the fermion or lepton portal [30–32] and the vector or dark photon portal [33, 34] (see [35–39] for reviews and further references). Regarding this thesis, we will exclusively focus on the last of these and discuss the vector portal or synonymously *gauge kinetic mixing*. In this scenario, in addition to the SM gauge group, a further *dark or hidden $U(1)$* gauge group is included. As we will explain in more detail in chapter 2, generically[4] the two U(1)s will mix via their kinetic terms in this situation

$$\mathcal{L} \supset -\frac{\chi}{2} F_{\mu\nu}^{\text{visible}} F^{\mu\nu}_{\text{hidden}} \,, \tag{1.1}$$

where $F_{\mu\nu}$ refer to the respective field strength tensors of the U(1) gauge fields and the kinetic mixing parameter χ controls the strength of the interaction. Thus, as soon as the hidden sector contains an additional U(1) gauge group, a non-trivial interaction between a hidden and a visible sector has to be induced. The hidden U(1) now mediates between the visible and hidden sector, which is assumed to contain dark matter. However, the dark photon itself could be massive and represent a viable dark matter candidate. Studying kinetic mixing and the possible implications such a mixing term can leave is therefore very interesting. Consequently, many experimental searches are already constraining the physical implications of an additional hidden photon, see [39] and references therein.

[4] See footnote 9 on page 17 for examples where kinetic mixing is absent.

Cosmic Inflation

The second topic of our interest concerns *cosmic inflation* [40–43] which is part of the ΛCDM and *Big Bang model* [44–46] describing the thermal history of the universe. In the Big Bang model, the universe was initially filled by fundamental particles in an extremely hot and dense state, the "Big Bang singularity". This initial state expanded and cooled such that first light nuclei could be formed in the process of *Big Bang nucleosynthesis* [47–49] creating a hot plasma of protons, electrons, neutrinos, and photons. The primordial plasma cooled down over 380 000 years until electrons and protons could combine into neutral hydrogen forming the first atoms. After this *recombination* into atoms photons could freely propagate in space without scattering off free electrons or protons. This free stream of photons created the first "snapshot" of the universe which we can see in the CMB. However, observations of the CMB reveal several problems which require further explanation. Here, we want to highlight two pivotal issues: First the *horizon problem* [50, 51] which refers to the almost perfect homogeneity and isotropy of the CMB. The mean temperature through out the whole CMB is given by $2.72548 \pm 0.00057\text{K}$ which underlines the remarkably small fluctuations of only 0.02% [8]. The high degree of homogeneity and isotropy can only be explained if all patches of the CMB could have been in causal contact at some point in the past. Curiously, the time from the initial Big Bang to recombination does not suffice to bring all patches into causal contact, thus posing a problem for cosmology. In addition, the universe is astonishingly flat. This is rather unexpected since the contribution of matter and radiation declines faster then the contribution of spacial curvature. However, today's universe is dominated by matter and not curvature which implies that the initial curvature contribution has to be fine tuned to an extremely small value to evolve into today's contribution. This poses a fine tuning problem and gives rise to the *flatness problem* of cosmology [52, 53].

The horizon and flatness problem can be solved simultaneously if the universe experienced a period of exponential expansion right after the Big Bang which we call *cosmic inflation*. This rapid phase of expansion causes all scales to get stretched out thus driving the universe to a uniform and isotropic state. Intuitively, this explains why the universe appears flat, homogeneous and isotropic. However, the CMB shows that small anisotropies remain. Most naturally these anisotropies are related to quantum fluctuations from before inflation which got stretched to cosmological scales. They seed the large scale structure of the universe. During the period of inflation the universe is dominated by the dynamics of a slowly evolving scalar field which we refer to as the *inflaton*. As we will see in sect. 2.2, the slow evolution of the inflaton leads to an exponential expansion driving the universe to the desired state. The dynamics of the inflaton is governed by its potential which needs to contain a flat region in field space to imply the slow evolution of the inflaton. Eventually, the inflaton rolls to a minimum and starts to coherently oscillate until it decays into SM particles. This "reheates" the universe to the initial hot and dense state which then evolves as a plasma until recombination and follows the rest of the cosmological history until today.

1.2. String Theory

The precise origin of dark matter and inflation are still unresolved and require further experimental as well as theoretical advances in *physics beyond the standard model*. We have already mentioned that, in addition to these issues, another crucial challenge is posed by finding a consis-

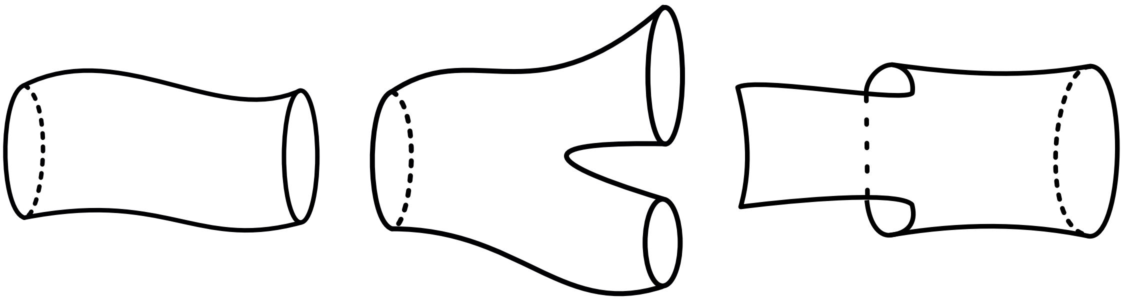


Fig. 1.3: Examples of string diagrams: On the left a closed string propagating, in the middle a closed string splitting into two closed strings and on the right an open string forming a closed string.

tent quantum theory for gravity. Classically, gravity is described exceptionally well by Einstein's theory of general relativity (GR). However, the standard procedure to quantize GR canonically or via path integral methods does not lead to a satisfactory resolution, since it has been shown that the theory is perturbatively non-renormalizable [54–57]. As a result, GR becomes strongly coupled at least at the Planck scale M_{P} defined by

$$M_{\mathrm{P}} = (8\pi G)^{-1/2} \approx 2.4 \times 10^{18}\ \mathrm{GeV}\ , \tag{1.2}$$

where G denotes Newtons constant and we have set $c = \hbar = 1$. It appears reasonable to replace GR with a suitable UV completion as we approach energy scales of this magnitude. However, due to the immense energy that is necessary to probe these scales there has been only little guidance by experiments. The search for a consistent quantum gravity theory has been going on for a long time (see [58–62] for several general reviews about quantum gravity and references therein) and several approaches have emerged (see [63–67] for an incomplete set of reviews about different quantum gravity approaches).

In the course of this thesis we will adopt the position that the correct UV completion of gravity is given by string theory (see [68–77] and references therein). The main idea behind string theory is radical but simple: All point-like particles are replaced by *one-dimensional strings*. Instead of a particle worldline a string will trace out a two-dimensional *worldsheet* embedded into a, so far, arbitrary d-dimensional target space which is identified with spacetime. Topologically one can differentiate between open strings equivalent to a line and closed string equivalent to a circle. As we will see in a moment, both strings behave quite differently and give rise to different phenomena. The interactions of strings are given by splitting and joining, e.g. a closed string can split into two closed strings. Also an open string can split into two or join its ends and form a closed string, see fig. 1.3. The whole dynamics of the string is governed by the Nambu-Goto [78, 79] or the Polyakov action [80–82] which are classically equivalent. Remarkably the only free parameter in string theory is given by the tension of the string T_F, historically expressed in terms of the parameter α', i.e. $T_F = (2\pi\alpha')^{-1}$. All other physical parameters are derived quantities within string theory. More importantly however, string theory is well behaved in the UV since no divergences arise due to the non-local nature of string interactions, see e.g. [83]. Further, the string can also be supersymmetrised to naturally include fermions. This is captured in a suitable supersymmetrised version of the Nambu-Goto or Polyakov action which represents the starting point for all further investigations, see [68–77].

One then proceeds to quantize the string and obtains a spectrum of tachyonic, massless and

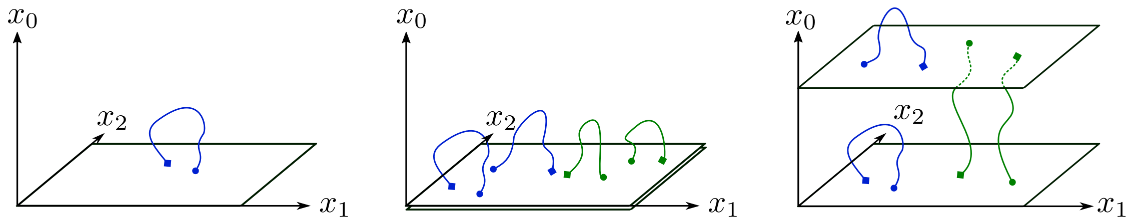


Fig. 1.4: Illustration of D-branes. On the left a single D-brane with one open string attached to the D-brane surface, in the middle a stack of two D-branes with two strings, on the right two parallel branes with strings on the branes as well as between the D-branes. The orientation of the strings is indicated by the different endpoints of the strings.

massive string modes. The masses m_{st} of the massive modes are proportional to $m_{st} \sim \alpha'^{-1/2}$ and mark the scale at which the extended nature of the string becomes apparent. The mass of the lightest string modes is considered to be (very) large such that it is sensible to consider the effective field theory (EFT) of only the massless and tachyonic string modes. However, the presence of tachyons is problematic and threatens the stability of the theory. Yet, this problem can be dealt with by imposing the GSO projection on the string spectrum [84]. Applying the GSO projection reveals that there are *only five consistent superstring theories* containing no tachyons: type I, type IIA and IIB, as well as the heterotic superstring with gauge group $\mathrm{E}_8 \times \mathrm{E}_8$ or $\mathrm{SO}(32)$[5]. In addition to being tachyon-free, the theories must also be free of anomalies. The explicit quantization procedure shows that this is only guaranteed if the string propagates in a *critical number of dimensions*. In the case of the superstring, we require the presence of *a total of ten dimensions*, which can be interpreted as a derivation or prediction of string theory.

By studying the scattering amplitudes of the massless string modes one can obtain an effective field theory formulated in 10d which captures the interactions of these modes at small string coupling g_s. This effective perspective of string theory will also be our main approach to studying phenomenological implications of the theory. Remarkably, one finds that *gravity is an unavoidable part of every string theory*, and thus string theory naturally incorporates gravity in a consistent UV theory. Logically, the 10d EFTs are supergravity theories where gravity is coupled to a set of additional fields identified with massless modes of the string. For the remainder of this thesis we will focus exclusively on the 10d EFT of type IIB string theory due to a variety of technical advantages we will discuss in more detail in chapter 3. The bosonic field content of type IIB is given by the 10d metric $G_{\mu\nu}$, the dilaton scalar ϕ which controls the string coupling, $g_s = \exp \phi$, as well as four different p-forms B_2, C_0, C_2 and C_4 while the fermionic field content is fixed by local supersymmetry[6]. All these fields arise from the closed string and can freely propagate in the 10d bulk. The respective effective action is given in chapter 3.

In addition to the closed string also open strings are present in type IIB. By including open strings one automatically introduces a set of non-perturbative objects called *D(irichlet)-branes* [85] (see [86–88] for reviews) which represent the fundamental objects charged under the C_p forms in string theory. D-branes are defined as objects on which the fundamental string can end and the brane position in 10d is fixed by the Dirichlet boundary conditions of the open string endpoints, see fig. 1.4 for an illustration. Thus, D-branes extend along even more dimensions than

[5]Famously, all five theories are related by a web of dualities. This motivated the idea that all of these string theories are actually certain limits of a 11 dimensional theory called M-theory. For large values of α' the low energy limit of M-theory is given by 11d supergravity.

[6]Note that we do not work in the democratic formulation of type IIB which additionally includes the dual p-forms.

the fundamental string and consequently trace out a *worldvolume* instead. However, due to their non-perturbative nature D-branes are often treated as non-dynamical but couple to the open and closed strings. Most importantly, the open string gives rise to a $\mathrm{U}(1)$ gauge theory confined to the worldvolume of the D-brane and represents the only source for gauge theories of type IIB. Crucially, an arbitrary number N of D-branes can be coincident which enhances the gauge group to $U(N)$ [89, 90]. The effective action for the interactions of D-branes with the string modes is of Born-Infeld type and will be discussed in chapter 3. Furthermore, D-branes can also be parallel but separated or intersect which results in charged states localised at the intersection locus of the branes. These states originate from open strings extending between two different intersecting branes. This illustrates how model building in type IIB is approached. One can consider an almost arbitrary setup of D-branes and choose the appropriate intersections to realise a realistic matter content. Further details will be discussed in part I.

The crucial final step to concrete phenomenological applications is the concept of *Kaluza-Klein compactification* [91, 92] and *moduli stabilisation* first proposed in the context of string theory in [93]. Obviously string theory faces a serious issue since it is necessary to introduce additional six dimensions for consistency. To make contact with the perceived 4d physics one considers the additional six dimensions to be compact and small. This results in the product ansatz for the 10d spacetime manifold $\mathcal{M}^{10}$

$$\mathcal{M}^{10} = \mathcal{M}^{1,3} \times \mathcal{X}^{6} \, , \tag{1.3}$$

where $\mathcal{M}^{1,3}$ refers to the four non-compact dimensions and $\mathcal{X}^6$ refers to the 6d internal compact manifold. The respective metric ansatz preserving 4d Poincare invariance is given by [94–97]

$$\mathrm{d}s^2 = e^{2A(y)} g_{4\,\mu\nu}(x) \mathrm{d}x^\mu \mathrm{d}x^\nu + e^{-2A(y)} g_{6\,mn}(y) \mathrm{d}y^m \mathrm{d}y^n \, , \tag{1.4}$$

where $g_{4\,\mu\nu}(x)$ denotes the 4d metric depending on the 4d coordinates x^μ ($\mu = 0,1,2,3$) and $g_{6\,mn}(y)$ denotes the 6d metric depending on the 6d coordinates y^m ($m = 4,5,\ldots,9$). Additionally, $A(y)$ denotes the *warp factor* which only depends on the 6d coordinates. Since the internal dimensions are compact one can expand the 10d fields Φ into *Kaluza-Klein (KK)-modes* $\phi^n(x)$ and eigenfunctions $Y^n(y)$ of the internal Laplace operator Δ_6

$$\Phi(x,y) = \sum_{n=0}^{\infty} \phi^n(x) Y^n(y) \, . \tag{1.5}$$

Using this ansatz for all 10d fields Φ and integrating over $\mathcal{X}^6$ one obtains a dual 4d theory with a tower of infinitely many fields $\phi^n(x)$ representing one higher dimensional field. Due to the product structure (1.4) one can decompose the 10d Laplacian $\Box_{10} = \Box_4 + \Delta_6$ where we neglected the warp factor for simplicity. In the simplest case the equation of motion of Φ is given by

$$\Box_{10} \Phi(x,y) = 0 \, . \tag{1.6}$$

Using the ansatz (1.5) we obtain the respective 4d equation of motion for every KK mode $\phi^n(x)$

$$\Box_4 \phi^n(x) + (M^n_{\mathrm{KK}})^2 \phi^n(x) = 0 \, , \tag{1.7}$$

where M^n_{KK} denotes the eigenvalue of $Y^n(y)$, i.e. $\Delta_6 Y^n(y) = (M^n_{\mathrm{KK}})^2 Y^n(y)$. In the 4d perspective, the eigenvalues M^n_{KK} determine the mass of the respective KK mode which is related to the typical length scale R of $\mathcal{X}^6$ by $M_{\mathrm{KK}} \sim 1/R$. Crucially for $n = 0$ the respective KK mode is massless and one refers to $M_{\mathrm{KK}} \sim 1/R$ as the *KK scale* at which the internal dimensions become non-negligible.

The idea of compactification is based on the simple observation from above. Similar to the 10d EFT of the string one can now consider the internal dimensions to be very small which results in a high KK scale. It then becomes feasible to turn to an effective 4d description where all massive KK modes have been integrated out. Applying this procedure, one obtains a 4d theory containing only the massless KK modes which explains how a 4d description can be obtained from string theory.

The exact number of 4d massless fields is determined by the number of harmonic functions of the internal Laplacian Δ_6 and is hence tied to the cohomology of $\mathcal{X}^6$. Generically this gives rise to a plethora of massless fields which may be in contradiction with experiments and observations. Most problematic are massless scalars, called *moduli*, because these induce long range fifth forces that are constrained considerably [98]. This is a serious issue since for example the reduction of the internal metric of a complex[7] manifold $\mathcal{X}^6$ gives rise to $h^{1,1}$ *Kähler moduli*, parametrising the size of 2 and 4-cycles of $\mathcal{X}^6$, and $h^{2,1}$ *complex structure moduli* determining the ratio of 3-cycles of $\mathcal{X}^6$ (see e.g. the review [99]). The Betti numbers $h^{1,1}$ and $h^{2,1}$ can easily be $\mathcal{O}(100)$, making the matter even more pressing. This problem can be resolved by the procedure of *moduli stabilisation* which implements a potential for moduli to make these massive and thus evade fifth force constraints. Unfortunately it is very tricky to consistently stabilise all moduli in a technically controlled way such that trustworthy 4d descriptions can be realised. Due to this challenge one exploits supersymmetry which restricts the theory such that very precise predictions for the potential can be made. For this reason one resides to a special class of internal manifolds called *Calabi-Yau 3-folds (CY)* of three complex dimensions (six real dimensions). Very importantly, CY 3-folds are Ricci flat and have $\mathrm{SU}(3)$ holonomy and thus (only) preserve $\mathcal{N} = 2$ supersymmetry upon compactification to 4d. Still, $\mathcal{N} = 2$ supersymmetry is too restrictive and does not allow for a potential for the moduli. The amount of supersymmetry can be reduced further by imposing geometric symmetries and identifying all points which are mapped to each other via the symmetry action. Additionally, the profile of all fields have to be symmetric under the symmetry action. This procedure is equivalent to the concept of *orbifolds*. In the stringy context the symmetry action is accompanied by the reversal of worldsheet parity Ω_p which promotes the orbifold to an *orientifold*. The inclusion of Ω_p breaks supersymmetry to $\mathcal{N} = 1$ in 4d and introduces *O(rientifold)-planes* at the fixpoints of the orientifold action. Due to the action of Ω_p the O-planes get charged under the C_p forms [85] and represent another source for p-forms. Crucially, $\mathcal{N} = 1$ supersymmetry in 4d allows for a potential V for the moduli, the form of which is also dictated by supersymmetry. To obtain a non-trivial potential for all moduli requires to use an interplay of corrections to the 10d EFT [100–102]. Additionally, the presence of non-trivial field configurations of the p-form field strengths are necessary [97] as a first starting point in the details of moduli stabilisation. This gives a rough outline of key concepts and their relation to construct consistent 4d compactifications of string theory. However many, especially technical, details have been relegated to part I. Eventually, this procedure gives rise to consistent 4d descriptions of string theory which can be further investigated for their phenomenological implications.

We want to mention that still many open issues in the context of string compactifications persist. Generally, the potential obtained in the manner outlined above yields only *Anti de Sitter* (AdS) vacuum solutions which is in tension with the observation of an accelerated expansion of the universe. In the standard procedure, one adds a positive contribution to the potential which *uplifts* the AdS minimum to either a Minkowski minimum or a (at least meta stable) *de Sitter* minimum with a small but positive cosmological constant [100, 103–121]. Doing so accounts

[7] We will see in moment why the internal manifold is usually assumed to be complex.

for the observed accelerated expansion. The stability of the uplift mechanisms is currently the subject of much debate [122–137] and further analysis seems of importance. Even claims about the non-existence of dS solutions in string theory have been raised [138–141]. In addition it is unclear whether the KK scale and the dS or AdS scale can be parametrically separated in vacua of string theory [142–148] (see [149] for a recent review). Such vacua are called *scale separated* and the separation is crucial for the 4d low energy theory to be consistent. Some scale separated vacua have famously been constructed e.g. in type IIA [150]. However, their consistency is still under debate, see [151–156] and references therein.

Furthermore, the exact stabilisation mechanisms lead to many different minima in the potential[8] giving rise to the *landscape of string theory vacua* of type IIB [97, 158–162]. This marks a turning point in type IIB string theory, where instead of looking for *the one* compactification, we transition to a statistical approach, scanning the landscape vacua for their properties see e.g. [121, 163–170].

1.3. This Thesis

This thesis assumes the open problems we briefly touched upon at the end of sect. 1.2 will be positively resolved. We will thus focus on phenomenological implications of string theory. For this purpose we will focus on type IIB string theory for several reasons we will discuss in chapter 3. The subjects we will concentrate on are inspired by physics beyond the standard model and cosmology and address current and pressing issues in high-energy physics. First, we will discuss gauge kinetic mixing within the context of string theory and produce results relevant for future model building in string theory. Second, we will turn to cosmic inflation and discuss a general setting of stringy inflation models which yield consistent phenomenological predictions. In this thesis, both phenomena, kinetic mixing and inflation, will crucially rely on higher order corrections to string theory. It is solely due to these corrections that kinetic mixing is induced and a non-trivial potential for inflation is obtained. Therefore, these corrections to string theory have direct physical implications and allow to test them precisely.

To begin our discussion about this matter, we start in part I with a thorough introduction to the theory of kinetic mixing and cosmic inflation in chapter 2. There, we will highlight the basic principles that are relevant for the application within string theory. In chapter 3 we turn to all the details we skipped in the introduction to string theory in sect. 1.2 and discuss the general actions of type IIB and D-branes. We will also give the basic formulae relevant in the context of compactification and moduli stabilisation in type IIB. This will lay the foundation for the stringy applications that will follow.

The second part II of this thesis, will cover kinetic mixing in type IIB string theory. The goal of this part is to explain the prerequisites that lead to a kinetic mixing term in the compactified 4d theory and extract concrete results that can be used for further investigations. First in chapter 4, we will discuss the open string loop diagram which is responsible for the mixing of gauge theories in string theory. Step by step, we will explain how the full string loop diagram can be broken down and applied to phenomenological scenarios which require complicated internal geometries. This will give rise to an effective 10d treatment where kinetic mixing is reinterpreted as an exchange of 10d bulk fields between Dp-branes which naturally carry the gauge theories. The explicit kinetic mixing term becomes apparent after the compactification to 4d and integrating

[8]Estimates including F-theory vacua go up to 10^{27200} [157].

out the massive mediating bulk fields. Opposing to the worldsheet perspective this field theoretic approach will be our method of choice throughout this part.

In addition, we need to obtain parametrically small values for the kinetic mixing parameter in order to satisfy phenomenological constraints. We will argue that there are *four main methods* to achieve this goal: Small gauge couplings, embedding of the U(1) in a non-abelian gauge group, sequestering in higher dimensions and cancellation or exclusion due to a global or gauged symmetry. In our applications in type IIB all of these options can be realised and often several of these effects come into play simultaneously. We devote chapter 5 to discuss this in detail. In particular, we will highlight that the engineering of small gauge couplings is accompanied by several shortcomings and is thus unfavoured. Alternatively and most favourably, we will use sequestering within the internal dimensions to suppress kinetic mixing between spatially separated D-branes.

We will apply this understanding of sequestering to kinetic mixing between D3-D3 and D7-D7 branes in chapter 6 and 7 respectively. However, it has been known for a long time, that several contributions to kinetic mixing can exactly cancel in the case of Dp-Dp-branes at leading order. Explicitly for the mixing between two D3-branes this exact cancellation occurs between the contributions of the bulk fields B_2 and C_2. Importantly, these bulk fields are related by the $\mathrm{SL}(2,\mathbb{R})$ symmetry of type IIB string theory. We extend this leading order analysis and find an exact cancellation for D3-branes using the global $\mathrm{SL}(2,\mathbb{R})$ symmetry. This represents a very specific case where a symmetry is responsible for an exact cancellation. In sect. 6.4, we then introduce 3-form fluxes to break $\mathrm{SL}(2,\mathbb{R})$ and lift the exact zero result for kinetic mixing. Using our findings, we derive some first phenomenological implications for the case of D3-brane kinetic mixing at the end of sect. 6. Most notably we find that in this setting the kinetic mixing parameter χ is suppressed in the volume $\mathcal{V}$ of the internal dimensions

$$\chi \sim \mathcal{V}^{-4/3} \,. \tag{1.8}$$

Here, the volume $\mathcal{V}$ is measured in string units and relying on the large volume scenario [101,102], the volume can be stabilised at exponentially large values [101,102] resulting in tiny values for χ.

In chapter 7, we apply the idea of sequestering also to scenarios involving D7-branes wrapped on 4-cycles in the Calabi-Yau. Specifically, we consider stacks of D7-branes with a non-abelian gauge group which is broken by turning on suitable internal flux of the gauge theory. As in the D3-brane case, our analysis shows that a cancellation between the contributions of B_2 and C_2 occurs, which is again tied to the $\mathrm{SL}(2,\mathbb{R})$ symmetry. This is rather surprising since D7-branes break $\mathrm{SL}(2,\mathbb{R})$. Yet, the parts of the action which are relevant for kinetic mixing feature the same $\mathrm{SL}(2,\mathbb{R})$ structure as in the D3-brane case which leads to the cancellation of the B_2 and C_2 contributions. However, we will find a non-zero contribution mediated by C_4. At the end of chapter 7 we will give an explicit formula yielding the exact kinetic mixing term.

In part III we discuss a specific class of string-derived inflation models which feature a flat plateau in the scalar potential. While it is not hard to obtain explicit slow roll potentials in effective quantum field theory [171–174], realising them in 4d EFTs derived from string theory is challenging [175–179]. Technically, the problem is due to the fact that stringy 4d EFTs are strongly constrained as we have indicated in the introduction. However, in chapter 8, we will argue that flat potentials arise rather naturally in the Kähler moduli sector of type-IIB flux compactifications, given that the volume can be stabilised at a sufficiently large value and an appropriate uplift to an almost-Minkowski vacuum can be realised.

For this purpose we use the framework of the *large volume scenario* (LVS) [101, 102] where the volume is stabilised at exponentially large values. Due to the particular no-scale structure of type-IIB flux compactifications, the naively dominant $1/\mathcal{V}^2$ terms for the Kähler moduli in the scalar potential V cancel [97]. In the LVS, the volume modulus and one additional blowup mode are stabilised by terms of order $\mathcal{V}^{-3}$ which we shall denote by $\hat{V}/\mathcal{V}^3$. The remaining Kähler moduli τ_i are left as flat directions at this order in $1/\mathcal{V}$. Eventually, the remaining Kähler moduli τ_i are stabilised by loop corrections of order $\mathcal{V}^{-10/3}$ [180–186]. Thus, the leading term in V for the τ_i is suppressed in comparison to the leading order term of the potential $\hat{V}/\mathcal{V}^3$. We will demonstrate in chapter 8 that the potential for the Kähler moduli τ_i can be expressed using a generic function f, which is solely dependent on the ratios $\tau_i/\mathcal{V}^{2/3}$. Further f is suppressed by a small coefficient c_{loop} such that the potential V schematically takes the form

$$V = \frac{\hat{V}}{\mathcal{V}^3}\left(1 + \frac{c_{\text{loop}}}{\mathcal{V}^{1/3}}\, f\left(\tau_i/\mathcal{V}^{2/3}\right)\right) . \tag{1.9}$$

Crucially, due the specific form of the metric on moduli space the fields $\tau_i/\mathcal{V}^{2/3}$ are canonically normalised. One may choose the inflaton to parametrise a generic trajectory in the moduli space spanned by the canonical fields $\tau_i/\mathcal{V}^{2/3}$. The potential for ϕ will unavoidably be flat due to the suppressing prefactor $c_{\text{loop}}/\mathcal{V}^{1/3}$. Consequently, also the slow roll parameters ϵ and η, which are proportional to the first and second derivatives of V wrt. the inflaton ϕ, are parametrically small because of the same reason.

We will then proceed and consider the simplest realisation of this observation in chapter 9, where we will choose a blowup mode to represent the inflaton. The blowup mode will be subject to non-perturbative *and* loop corrections in this setting. The non-perturbative corrections determine the global minimum of the inflaton. In contrast, loop corrections dominate the potential at large field values where the potential becomes flat. In chapter 9, we study the simple special case with a single additional Kähler modulus, $\tau_i \equiv \tau_\phi$ which, as explained above, has to be of blowup type. In this setting the form of the dominant loop correction in the regime $\tau_s \ll \tau_\phi \ll \mathcal{V}^{2/3}$ is actually known from an explicit analysis in [186], consistently with the extrapolation from the torus-orientifold case by the Berg-Haack-Pajer conjecture [183]. Then, approaching the regime of $\tau_\phi \lesssim \mathcal{V}^{2/3}$ from the side of small τ_ϕ, we may hope to maintain control of the inflationary potential while also achieving realistic phenomenology. This allows for a very explicit case study. The setting may be viewed as deriving from blowup inflation in a regime where the (naively fatal) loop corrections are taken into account and inflation is saved at the price of moving to much larger values of τ_ϕ. In chapter 10, we derive the inflationary predictions of the simplest realisation as described above. There, we additionally address questions of parametric and numerical control and further investigate stringy restrictions on parameters of the Calabi-Yau geometry. We then devote chapter 11 to the study of two other regimes of the inflationary potential: First, we consider the regime $\tau_\phi \sim \mathcal{V}^{2/3}$, where the functional form of the loop corrections becomes more complicated. Second, we quantify how small loop corrections would have to become to make a transition to blowup inflation. Finally, a detailed phenomenological assessment of the simple scenario from chapter 10, including reheating, dark radiation constraints and an estimate of inflationary parameters is given in chapter 12

We then summarise the results of part II and III in chapter 13 and give a brief outlook for possible future research directions. For a better readability and to separate detailed discussions from the main body of the thesis we relegated several topics to the appendix. In app. A we will discuss the precise $\mathrm{SL}(2,\mathbb{R})$ formulation of type IIB and the proper incorporation of D3-branes respecting the $\mathrm{SL}(2,\mathbb{R})$ structure. A "manifestly" self-dual formulation of the D3-brane action is spelled out in app. B. Several technical details, toy models, and derivations about kinetic mixing can be found in the appendices C, D and E. In app. F we discuss the solution to the Laplace equation in a form which is also applicable to differential forms. Technically, this approach is necessary to integrate out the bulk fields mediating kinetic mixing as we will discuss in part II.

Part I.

Basic Theory and Phenomenology

2. Kinetic Mixing and Cosmic Inflation

In this chapter, we will introduce the important elements of kinetic mixing and cosmic inflation from the viewpoint of field theory. During this discussion, our focus will be on those elements that are essential for understanding kinetic mixing and inflation within the framework of string theory. Furthermore, we discuss the phenomenological implications of both subjects and introduce the experimental and observational constraints. Regarding the discussion about kinetic mixing, we have added section 2.1.3 which details three approaches to achieve small values for the kinetic mixing parameter. These approaches represent relevant suppression mechanisms for our stringy scenarios in part II.

2.1. Kinetic Mixing

2.1.1. General Theory

The term kinetic mixing, as the name suggests, refers to a mixing term of different fields ϕ^a and ϕ^b via their kinetic terms, i.e.

$$\mathcal{L} \supset -\frac{\chi_{ab}}{2}\,\partial_\mu\phi^a\partial^\mu\phi^b\ , \tag{2.1}$$

where a, b shall label only two different fields for simplicity. However, in the literature kinetic mixing commonly refers specifically to the mixing of $\mathrm{U}(1)$ gauge bosons [33, 34], even though a mixing as in (2.1) covers a broader context. In this thesis, we want to proceed in the same fashion and use the term *(gauge) kinetic mixing (KM)* to refer specifically to the following term in a Lagrangian $\mathcal{L}$

$$\mathcal{L} \supset -\frac{\chi_{ab}}{2}F^a_{\mu\nu}F^{b\,\mu\nu}\ , \tag{2.2}$$

where χ_{ab} is called the *kinetic mixing parameter* and $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu$ denotes the field strength tensor of the $\mathrm{U}(1)$ gauge bosons A^a_μ. Besides KM there is a second type of mixing term for $\mathrm{U}(1)$ gauge bosons

$$\mathcal{L} \supset -\frac{\tilde{\chi}_{ab}}{2}F^a_{\mu\nu}\tilde{F}^{b\,\mu\nu}\ , \tag{2.3}$$

where $\tilde{F}^a_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{a\,\rho\sigma}$ is the (magnetic) dual field strength tensor. Consequently (2.3) is referred to as *magnetic mixing (MM)* [188, 189] and $\tilde{\chi}_{ab}$ is called the *magnetic mixing parameter*. In the absence of magnetic monopoles, (2.3) can be rewritten as a total derivative and hence leaves no phenomenological imprint. For this reason we will primarily focus on KM and omit MM in the following discussion. The coupling (2.2) (and (2.3)) represents a gauge invariant operator and hence should be included in a generic theory whenever multiple $\mathrm{U}(1)$ gauge groups are present[9]. Including the usual kinetic terms, the most general and renormalizable Lagrangian reads

$$\mathcal{L} \supset -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a - \frac{1}{4}F^b_{\mu\nu}F^{\mu\nu}_b - \frac{\chi_{ab}}{2}F^a_{\mu\nu}F^{b\,\mu\nu}\ . \tag{2.4}$$

[9] A kinetic mixing and magnetic mixing term could be excluded by symmetry. Specifically, a charge conjugation symmetry $\mathcal{C}$ acting differently on the gauge bosons A^a_μ [190–192], i.e. $\mathcal{C}[A^a] = A^a$ and $\mathcal{C}[A^b] = -A^b$ would obviously exclude mixing terms. We will not consider this in the following.

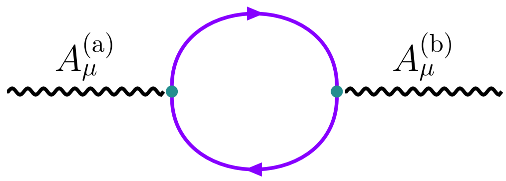


Fig. 2.1: Loop diagram yielding KM. The bicharged particle is depicted in purple.

A KM term (2.2) will generally arise from integrating out heavy particles Φ which are charged under $\mathrm{U}(1)_a \times \mathrm{U}(1)_b$. The relevant loop diagram is shown in fig. 2.1. Evaluating the loop diagram yields the following estimate for KM as an EFT operator [33, 34]

$$\chi_{ab} \sim c_{\text{loop}} g_a g_b \ln\left(\frac{\Lambda^2}{m_\Phi^2}\right) , \tag{2.5}$$

where Λ is the EFT cutoff and g_a and g_b denote the gauge coupling of the respective $\mathrm{U}(1)$. Note that χ_{ab} naturally obtains a suppression by the loop factor c_{loop}. Bicharged particles of this kind are conjectured to exist by the Completeness Hypothesis [193, 194], which postulates that the whole charge lattice of $\mathrm{U}(1)_a \times \mathrm{U}(1)_b$ is populated. Thus bicharged particles should always be present and we have to expect a KM term in the low energy EFT.

To make contact with the rich phenomenology of KM we need to include two further ingredients. First, the gauge bosons could be massive either due to a Higgs [195, 196] or a Stückelberg mechanism [197, 198]. Hence we further include mass and mass mixing terms of the form

$$\mathcal{L} \supset -\frac{m_a^2}{2} A_\mu^a A^{a\,\mu} - \frac{m_b^2}{2} A_\mu^b A^{b\,\mu} - m_a m_b A_\mu^a A^{b\,\mu} , \tag{2.6}$$

and second, we include charged states j_a^μ and j_b^μ, charged only under the respective $\mathrm{U}(1)$

$$\mathcal{L} \supset g_a A_\mu^a \, j_a^\mu + g_b A_\mu^b \, j_b^\mu . \tag{2.7}$$

All bicharged states are considered to be heavy and have been integrated out to yield the kinetic mixing term of (2.2) representing the only interaction operator between the two sectors.

From equations (2.4) and (2.6), it is clear that the gauge bosons are not in a canonical field basis. By applying a linear transformation in the field space of A_μ^a and A_μ^b

$$\begin{pmatrix} A^a \\ A^b \end{pmatrix} = T \begin{pmatrix} A^{\mathrm{A}} \\ A^{\mathrm{B}} \end{pmatrix} , \tag{2.8}$$

we can obtain a canonical kinetic term, i.e. eliminating the kinetic mixing and the mass mixing terms as well as ensuring proper normalization. *This yields four conditions which will unambiguously fix T.* However, in general the transformation T will introduce mixed couplings to the states

$$\mathcal{L} \supset j_b^\mu \, g_a A_\mu^a + j_a^\mu \, g_b A_\mu^b . \tag{2.9}$$

As we will see momentarily, these induced couplings will be proportional to χ_{ab} and yield the physical implication of kinetic mixing: *Transforming to the canonical field basis generically couples the states to both $U(1)$ gauge groups.* However, in the phenomenological settings we will encounter we may set one or both mass parameters m_a and, or m_b to zero. If *one mass parameter remains*, T is still completely fixed. However, *if both masses are set to zero* we only fix three of the four free parameters in T. Thus, we may use the last remaining freedom in T to further eliminate one of the coupling terms in (2.9), such that only one state obtains an additional charge. Now, if both U(1)s are massless and one U(1) does not couple to a state initially we can completely eradicate the presence of the kinetic mixing term. In this setting, kinetic mixing is unphysical and cannot be measured. To summarise: We require at least one charged state to be able to measure physical quantities. In addition, the other U(1) must be massive or must couple to charged states for kinetic mixing to be physical.

All of the above scenarios can be conveniently unified into a single explicit form of the transformation T [39]

$$\begin{pmatrix} A^a \\ A^b \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & -\frac{\chi_{ab}}{\sqrt{1-\chi_{ab}^2}} \\ 0 & \frac{1}{\sqrt{1-\chi_{ab}^2}} \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}}_{=T} \begin{pmatrix} A^{\mathrm{A}} \\ A^{\mathrm{B}} \end{pmatrix} . \tag{2.10}$$

The first matrix on the rhs. of (2.10) will diagonalize the kinetic terms in (2.4). In the case of massless gauge bosons, the parameter θ in the second matrix represents the remaining free parameter of T and can be chosen to eliminate one further term in the coupling structure to the charge states. If at least one gauge boson is massive, the angle θ is fixed such that one additionally obtains a diagonal mass term. The angle θ is then given by

$$\delta = \frac{m_a}{m_b} , \quad \sin\theta = -\frac{\delta\sqrt{1-\chi_{ab}^2}}{\sqrt{1-2\delta\chi_{ab}+\delta^2}} , \quad \cos\theta = \frac{1-\delta\chi_{ab}}{\sqrt{1-2\delta\chi_{ab}+\delta^2}} . \tag{2.11}$$

Without loss of generality, this notation is only adequate for taking m_a to zero[10]. Performing the field transformation (2.10) leaves us with the following Lagrangian

$$\begin{aligned} \mathcal{L} \supset & -\frac{1}{4}F^{\mathrm{A}}_{\mu\nu}F^{\mathrm{A}\,\mu\nu} - \frac{1}{4}F^{\mathrm{B}}_{\mu\nu}F^{\mathrm{B}\,\mu\nu} - \frac{m_a^2+m_b^2-2m_am_b\chi_{ab}}{2(1-\chi_{ab}^2)}A^{\mathrm{B}}_\mu A^{\mathrm{B}\,\mu} \\ & + \left[\frac{g_b\sin\theta}{\sqrt{1-\chi_{ab}^2}}\, j_b^\mu + g_a\left(\cos\theta - \frac{\chi_{ab}\sin\theta}{\sqrt{1-\chi_{ab}^2}}\right) j_a^\mu\right] A^{\mathrm{A}}_\mu \\ & + \left[\frac{g_b\cos\theta}{\sqrt{1-\chi_{ab}^2}}\, j_b^\mu - g_a\left(\sin\theta + \frac{\chi_{ab}\cos\theta}{\sqrt{1-\chi_{ab}^2}}\right) j_a^\mu\right] A^{\mathrm{B}}_\mu . \end{aligned} \tag{2.12}$$

The kinetic mixing term now becomes physical since it yields new coupling terms of the gauge bosons to the states j_a and j_b. All the different scenarios discussed above can be extracted from (2.12).

2.1.2. Phenomenology of Kinetic Mixing

To turn to realistic scenarios, we want one of the U(1) gauge groups to be given by the SM hypercharge $\mathrm{U}(1)_Y$ or the electro-magnetic $\mathrm{U}(1)_{\mathrm{em}}$. For readers interested in the distinctions between these choices, we refer to [39] and the references included therein. We will refer to whatever

[10] Of course one can swap the labels a and b in (2.10) and (2.11).

choice of the SM U(1) as the visible U(1) denoted by $\mathrm{U}(1)_a$ and choose $\mathrm{U}(1)_b$ to represent the dark or hidden U(1). Importantly, we want $\mathrm{U}(1)_a$ to be massless. So in the following we choose

$$m_a = 0 \,. \tag{2.13}$$

Massive Hidden Photon

We keep the mass m_b for the hidden photon. Since $\mathrm{U}(1)_b$ is massive, the angle θ is fixed,

$$\delta = 0 \quad \Rightarrow \quad \sin\theta = 0 \,, \quad \cos\theta = 1 \,, \tag{2.14}$$

and (2.12) simplifies to

$$\begin{aligned} \mathcal{L} \supset & -\frac{1}{4} F^{\mathrm{A}}_{\mu\nu} F^{\mathrm{A}\,\mu\nu} - \frac{1}{4} F^{\mathrm{B}}_{\mu\nu} F^{\mathrm{B}\,\mu\nu} - \frac{m_b^2}{2(1-\chi_{ab}^2)} A^{\mathrm{B}}_\mu A^{\mathrm{B}\,\mu} \\ & + \left[\frac{g_b}{\sqrt{1-\chi_{ab}^2}}\, j_b^\mu - g_a \frac{\chi_{ab}}{\sqrt{1-\chi_{ab}^2}}\, j_a^\mu \right] A^{\mathrm{B}}_\mu + g_a j_a^\mu A^{\mathrm{A}}_\mu \,. \end{aligned} \tag{2.15}$$

We see that the SM states j_a obtained a, so called, *millicharge* Q under $\mathrm{U}(1)_b$

$$Q = \frac{g_a}{g_b} \frac{\chi_{ab}}{\sqrt{1-\chi_{ab}^2}} \,, \tag{2.16}$$

since χ_{ab} is considered to be small. In this scenario, we could also set j_b to zero and kinetic mixing would be observable due to the mass m_b. Since the hidden photon is massive it could play the role of dark matter itself [199–203], otherwise further fields need to be included to serve that purpose.

The phenomenology of the massive hidden photon is determined by the coupling to the SM

$$\mathcal{L} \supset \frac{g_a\, \chi_{ab}}{\sqrt{1-\chi_{ab}^2}}\, j_a^\mu A^{\mathrm{B}}_\mu \simeq g_a\, \chi_{ab}\, j_a^\mu A^{\mathrm{B}}_\mu + \mathcal{O}(\chi_{ab}^2) \,, \tag{2.17}$$

and the parameter space is given by χ_{ab} and m_b. The hidden photon A^{B}_μ interacts directly with the SM and can be produced in collider experiments, decays into SM particles or can be detected directly. The set of constraints on the parameters χ_{ab} and m_b are shown in the left panel of fig. 2.2. We distinguish between direct constraints relying only on the existence of the hidden photon (dark gray region) and constraints assuming the hidden photon is dark matter (solid line). The data is combined and adapted from the collections [39,204–206] where also the original references can be found. The DM constraint is extended to masses larger than one MeV accounting for decays into electron-positron pairs and requiring that the lifetime is greater than the age of the universe. This is analogous to what was done for photons, e.g., in [200] and using decay formulae from [207, 208]. The light gray region is to be covered by future experiments, see refs. in [204].

Massless Hidden Photon

The case of a massless hidden photon we set both masses to zero and the parameter θ can be chosen freely and the millicharged particle is chosen to couple either to hidden or visible photon. The former case can be obtained from (2.15) where θ was fixed due to $m_b \neq 0$. We can take the limit $m_b \to 0$ and obtain a massless hidden photon where the SM states obtain a charge under $\mathrm{U}(1)_b$.

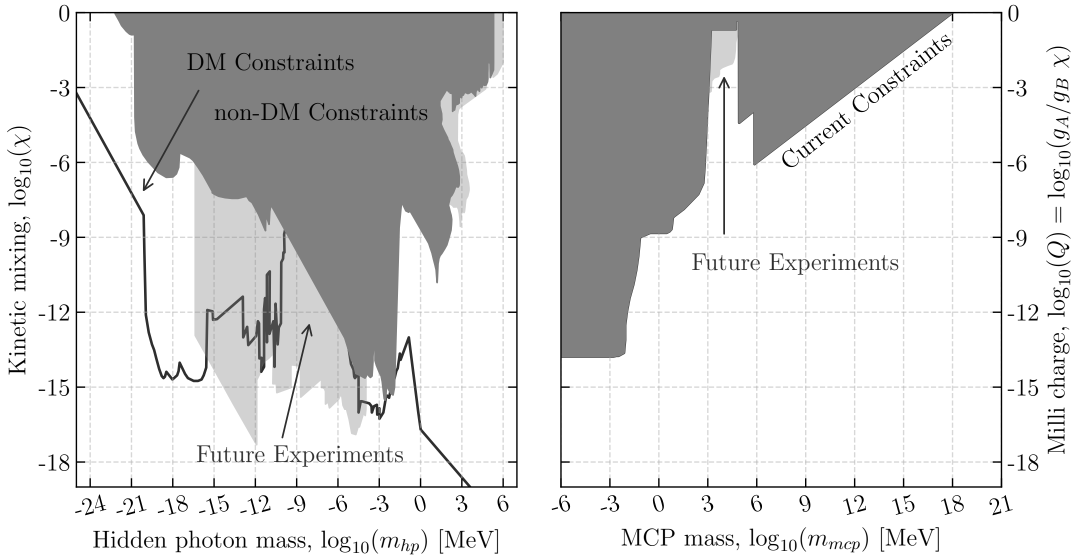


Fig. 2.2: Current constraints on KM in case of a massive hidden photon (left) and a massless hidden photon with millicharged particles (MCP) (right); adapted mainly from [39, 204–206, 209, 210]. We refer to the text for details and further references.

In contrast to this case, if both U(1)s are massless we may choose

$$\sin\theta = -\chi_{ab}\ , \quad \cos\theta = \sqrt{1-\chi_{ab}^2}\ , \tag{2.18}$$

which *cannot* be obtained as a limit of the massive hidden photon. The Lagrangian (2.12) now reads

$$\mathcal{L} \supset -\frac{1}{4}F^{\mathrm{A}}_{\mu\nu}F^{\mathrm{A}\,\mu\nu} - \frac{1}{4}F^{\mathrm{B}}_{\mu\nu}F^{\mathrm{B}\,\mu\nu} + \left[\frac{g_a}{\sqrt{1-\chi_{ab}^2}}\ j_a^\mu - g_b\frac{\chi_{ab}}{\sqrt{1-\chi_{ab}^2}}\ j_b^\mu\right]A^{\mathrm{A}}_\mu + g_b j_b^\mu A^{\mathrm{B}}_\mu\ . \tag{2.19}$$

Now we see that instead the hidden states j_b obtained a millicharge Q under $\mathrm{U}(1)_a$

$$Q = \frac{g_b}{g_a}\frac{\chi_{ab}}{\sqrt{1-\chi_{ab}^2}}\ . \tag{2.20}$$

It is convention to use the Lagrangian (2.19) to define the phenomenology of the massless hidden photon. The hidden photon does not interact directly with the SM but indirect interactions can be encoded in higher-dimensional operators suppressed by the EFT-scale. These operators for example alter the electric and magnetic dipole moment [39] and can be constrained in this way. The precise form of these operators depends on the explicit UV model that is implemented to give rise to kinetic mixing. More prominently, the phenomenology is govern by the direct interaction of the SM with the obtained *millicharged particles* (MCP). In this case the parameter space is spanned by the millicharge Q of (2.20) and the mass m_{mcp} of the MCP. Constraints arise from astrophysics as MCP influence the evolution of stars and supernovae or change the cosmological evolution where MCP are considered to represent dark matter. In addition, MCP can distort the CMB and precision test of QED allow to further constrain the parameter space of MCP. The right panel of fig. 2.2 shows the constraints on MCP (see again [39, 209, 210] and refs.

therein). The gauge couplings are assumed to be equal, such that $Q = \chi$. As can be seen in the figure, a region of large millicharge Q is excluded by CMB limits [211] and the overabundance of DM [212] (adapted to the case where $e = g_a = g_b$). Again, the dark gray region refers to current constraints and the light gray region to future experiments. For a more general and detailed discussion of the constraints see e.g. [39, 204, 213, 214].

2.1.3. Generically Small Kinetic Mixing

From the constraints shown in the plots of fig. 2.2 we can see that χ has to be very small. This is usually accounted for by assuming that kinetic mixing arises only from loop diagrams like fig. 2.1 which yields the typical estimate (2.5)

$$\chi_{ab} \sim g_a g_b \, c_{\text{loop}} \,, \tag{2.21}$$

where we assume the logarithmic terms to be $\mathcal{O}(1)$. The kinetic mixing parameter χ is thus only suppressed by a loop factor. Typically one finds $c_{\text{loop}} \sim 1/16\pi^2$ which however is not sufficient to match the constraints. One could consider that kinetic mixing is only generated at a higher loop level which seams unplausible in light of the completeness conjecture [193, 194]. Instead one may tune the hidden gauge coupling g_b to a small value since g_b could in principle be regarded as a free parameter. We devote chapter 5.1 to explain in detail why this approach has significant limitations. To anticipate this briefly, the argument is based on the magnetic version of the weak gravity conjecture [215], which states that the cutoff Λ of a gauge theory coupled to gravity has to be lower or equal to the gauge coupling g times the 4d Planck mass M_{Pl}

$$\Lambda \lesssim g M_{\text{Pl}} =: \Lambda_{\text{WGC}} \,. \tag{2.22}$$

Therefore, the cutoff Λ decreases if we reduce any gauge coupling g to a small value. We refer to chapter 5.1 for further details. Instead, we want to highlight other scenarios which realize small values for χ *without* relying on small gauge couplings. Hence, these other approaches can be interpreted as a precise cancellation of the logarithmic terms in (2.5).

Charge conjugation symmetry

One way of circumventing small gauge couplings is by excluding the kinetic mixing term due to symmetry. As an example one could consider a hidden charge conjugation symmetry $\mathcal{C}_b$ [11] only acting in the hidden sector as

$$\mathcal{C}_b(A_b) = -A_b \,, \tag{2.23}$$

but leaving the Lagrangian of the visible and hidden sector invariant

$$\mathcal{C}_b(\mathcal{L}_a) = \mathcal{L}_a \,, \quad \mathcal{C}_b(\mathcal{L}_b) = \mathcal{L}_b \,. \tag{2.24}$$

A kinetic mixing term would hence be odd under $\mathcal{C}_b$ and would be forbidden [190–192]. This can also be seen from the loop diagramm in fig. 2.1 which considers bicharged particles running in the loop. Because of the completeness conjecture, we have to include a spectrum of states

[11] Thinking about multiple $U(1)$s in one sector, there can only be discrete global symmetries acting on the $U(1)$s. This can simply be inferred from thinking about the $U(1)$ as compact Lie groups associated to a manifold. Several (n) $U(1)$s in general form a n-torus T^n which only allows for discrete symmetries mapping the T^n to itself. In the simplest case with one $U(1)$ this reduces to the charge conjugation $\mathcal{C}_b$ discussed in the text.

$\mathcal{L}_{completeness}$ with all possible charges under both gauge theories. This spectrum has to satisfy the symmetry $\mathcal{C}_b$

$$\mathcal{C}_b(\mathcal{L}_{completeness}) = \mathcal{L}_{completeness} \ , \tag{2.25}$$

i.e. for every charged state (q_a, q_b) of mass m there has to be another state $(q_a, -q_b)$ also of mass m. This results in a cancellation of kinetic mixing to all orders in perturbation theory, since the contributions of all pairs (q_a, q_b) and $(q_a, -q_b)$ yield

$$\chi_{ab} = c_{\text{loop}}\ g_a\ g_b\ q_a\ q_b \ln\left(\frac{m^2}{m^2}\right) = 0 \ . \tag{2.26}$$

It is well established that global symmetries, like $\mathcal{C}_b$, should either broken or gauged in quantum gravity [194, 216]. In case the symmetry is broken by quantum gravity effects, it is unavoidable to at least have a tiny amount of kinetic mixing. If one is able to quantify the symmetry breaking effect, one can also estimate the minimal amount of kinetic mixing. A possible source for such symmetry breaking effects could be due to wormholes [217, 218], leading to a mass splitting Δm of the otherwise symmetric states. This results in a kinetic mixing contribution

$$\chi_{ab} = c_{\text{loop}}\ g_a\ g_b\ q_a\ q_b \ln\left(\frac{(m+\Delta m)^2}{m^2}\right) \approx 2c_{\text{loop}}\ g_a\ g_b\ q_a\ q_b\ \frac{\Delta m}{m} \ . \tag{2.27}$$

The mass splitting generated by wormholes is suppressed by $\Delta m/m \sim \exp(-M_{\text{Pl}}^2/\Lambda^2)$, hence yielding small kinetic mixing depending on the cutoff, e.g. for $M_{\text{Pl}}/\Lambda = 5$ we find $\chi_{ab} \sim 10^{-11}$. In the other case of a gauged symmetry, a kinetic mixing term would be truly forbidden in the Lagrangian [190].

Non-abelian Gauge Symmetry

One can also consider one or both $U(1)$s to be embedded in non-abelian gauge groups. Famously, kinetic mixing between non-abelian gauge groups is forbidden due to gauge invariance. Kinetic mixing is then only possible upon breaking the non-abelian gauge groups into subgroups containing $U(1)$s at some scale Λ_{SB} [192, 219, 220]. Due to gauge invariance, the relevant terms in the Lagrangian yielding kinetic mixing need to contain $\text{tr}(\Phi_a G_a)$ with a scalar Φ_a in the adjoint of the group labeled by a and G_a referring to the non-abelian field strength. Embedding both $U(1)$s in non-abelian groups, the relevant term for kinetic mixing reads

$$\mathcal{L} \supset \frac{1}{\Lambda^2}\,\text{tr}(\Phi_a G_a)\,\text{tr}(\Phi_b G_b) \ , \tag{2.28}$$

which is suppressed by a scale Λ [192] which we set to $\Lambda = M_{\text{Pl}}$. When the scalars Φ obtain a vev $\langle\Phi\rangle \sim \Lambda_{\text{SB}}$, the gauge groups are assumed to break to a subgroup containing the $U(1)$s we want to use for kinetic mixing. Referring to the stringy settings, we assume that the symmetry breaking scales Λ_{SB} of both sectors are the same. The value of χ_{ab} is then suppressed by the ratio of the scales

$$\chi_{ab} \sim \left(\frac{\Lambda_{\text{SB}}}{M_{\text{Pl}}}\right)^2 \sim 10^{-12}\left(\frac{\Lambda_{\text{SB}}}{10^{13}\ \text{GeV}}\right)^2 \ , \tag{2.29}$$

or

$$\chi_{ab} \sim \frac{\Lambda_{\text{SB}}}{M_{\text{Pl}}} \ , \tag{2.30}$$

if only one of the two $U(1)$s is embedded in a non-abelian gauge group.

Sequestering

The third mechanism takes up the concept of extra dimensions which are naturally present in string theory. When n extra dimensions are available, one can imagine localizing the visible and hidden sectors on branes, rather than having all particles propagate in $(4 + n)$-dim. spacetime. The branes should at least fill our 4-dim. spacetime dimensions and we then obtain our 4-dim. perspective by compactifying the n extra dimensions. Since the sectors are localized, we can separate the sectors geometrically (sequestering) in the q extra dimensions transversal to the branes. The fields of each sector are confined to their branes and cannot propagate transversely in the bulk. Thus, an interaction between the sectors can only be achieved by all the fields which are able to propagate in the bulk from one sector to the other. The interaction naturally scales with the propagator G of the bulk fields [221], which can either be powerlike for massless fields

$$G[y^i] \sim \frac{1}{|y\Lambda|^{q-2}} , \tag{2.31}$$

or exponentially suppressed for massive fields with mass m

$$G[y^i] \sim \frac{\exp(-m\,|y|)}{|y\Lambda|^{q-2}} , \tag{2.32}$$

where y^i refer to the transversal coordinates and Λ to the UV-cutoff. The kinetic mixing parameter χ would scale in the same manner

$$\chi \sim G[y^i] . \tag{2.33}$$

The idea of sequestering will be crucial for our stringy scenarios in part II, where we use D-branes in type IIB string theory to carry the visible and hidden sectors. In a large volume Calabi-Yau the branes can be widely separated, thus yielding suppressed values for χ.

2.2. Cosmic Inflation

2.2.1. Cosmology

In this chapter we want to give a lightning review about cosmology and slow roll inflation. The goal is not to give a comprehensive introduction but merely an overview in order to set the basic equations into context. A detailed introduction can be found for instance in [178, 222–224] which represent the main sources for this section.

At large scales the universe looks homogeneous and isotropic but evolves with time. This is reflected in the spacetime metric which is of *Friedmann-Lemaître-Robertson-Walker* type defined by the line element

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2\left(\frac{\mathrm{d}r^2}{1-kr^2} + r^2\mathrm{d}\Omega^2\right) , \tag{2.34}$$

where we defined $\mathrm{d}\Omega^2 = \mathrm{d}\theta^2 + \sin^2(\theta)\mathrm{d}\phi^2$. Homogeneity and isotropy fix the spacial slices to be maximally symmetric. Therefore, the spatial slices correspond either to the the flat space $\mathbb{R}^3$ ($k = 0$), the sphere S^3 ($k = 1$) or the hyperboloid H^3 ($k = -1$), which can be parametrized conveniently with the *curvature parameter* k. The time dependence of the metric is encoded into the *scale factor* $a(t)$ which is set to unity when evaluated today. The dynamics of the metric (2.34) is govern by Einstein equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = M_{\mathrm{Pl}}^{-2}T_{\mu\nu} , \tag{2.35}$$

where $R_{\mu\nu}$ refers to the Ricci tensor, $R = R^{\mu}{}_{\mu}$ to the Ricci scalar and $T_{\mu\nu}$ denotes the energy-momentum tensor of matter. By assumption, we are considering large scales and are thus averaging out all anisotropies and inhomogeneities. Hence we only need to consider a coarse grained energy momentum tensor. Homogeneity and isotropy eventually enforce that the coarse grained energy momentum tensor $T^{(\mathrm{i})}_{\mu\nu}$ is that of a *perfect fluid.* For a comoving observer $T^{(\mathrm{i})}_{\mu\nu}$ is thus given by

$$T^{(\mathrm{i})}_{\mu\nu} = \mathrm{diag}\left(\rho^{(\mathrm{i})},\ p^{(\mathrm{i})},\ p^{(\mathrm{i})},\ p^{(\mathrm{i})}\right) , \tag{2.36}$$

where $\rho^{(\mathrm{i})}$ and $p^{(\mathrm{i})}$ refer to the *energy density* and to the *pressure* of the fluid respectively and we label different matter components of the universe by (i). Using (2.34) and (2.36) in (2.35) we obtain the *Friedmann equations*

$$H^2 = \sum_{\mathrm{i}}\left(\frac{\rho^{(\mathrm{i})}}{3M_{\mathrm{Pl}}^2}\right) - \frac{k}{a^2} , \tag{2.37}$$

$$\frac{\ddot{a}}{a} = -\sum_{\mathrm{i}}\frac{\rho^{(\mathrm{i})} + 3p^{(\mathrm{i})}}{6M_{\mathrm{Pl}}^2} , \tag{2.38}$$

where we defined the *Hubble parameter* $H = \dot{a}/a$. It is a crucial experimental evidence that our universe is very flat to a good approximation, such that curvature accounts for only about 0.2% of the total energy budget [8]. Thus, we can safely assume that $k = 0$. The energy momentum tensor $T^{(\mathrm{i})}_{\mu\nu}$ is covariantly conserved, i.e. $\nabla^{\mu}T^{(\mathrm{i})}_{\mu\nu} = 0$ which is equivalent to the *continuity equation* of the perfect fluid

$$\dot{\rho}^{(\mathrm{i})} + 3H(\rho^{(\mathrm{i})} + p^{(\mathrm{i})}) = 0 . \tag{2.39}$$

The equations (2.37) and (2.39) imply (2.38) such that we can omit (2.38). To simplify further one defines the equation of state with the *equation of state parameter* $\omega^{(\mathrm{i})}$

$$p^{(\mathrm{i})} = \omega^{(\mathrm{i})}\rho^{(\mathrm{i})} \,. \tag{2.40}$$

For important components one finds

$$\omega^{(\mathrm{i})} = \begin{cases} 0\,, & \text{matter} \\ 1/3\,, & \text{radiation} \\ -1\,, & \text{vacuum energy} \end{cases} . \tag{2.41}$$

Using the equation of state (2.40) in (2.39) we can solve for $\rho^{(\mathrm{i})}$ in terms of a and find

$$\rho^{(\mathrm{i})} = \rho_0^{(\mathrm{i})} a^{-3(1+\omega)} \propto \begin{cases} a^{-3}\,, & \text{matter} \\ a^{-4}\,, & \text{radiation} \\ a^{0}\,, & \text{vacuum energy} \end{cases} , \tag{2.42}$$

where $\rho_0^{(\mathrm{i})}$ is an integration constant. Using (2.42) in (2.37) on can explicitly solve for $a(t)$ if the universe is dominated by a single component. One finds

$$a(t) \propto \begin{cases} t^{2/3(1+\omega^{(\mathrm{i})})}\,, & \omega^{(\mathrm{i})} \neq -1\,, \quad \begin{matrix} t^{2/3}\,, & \text{,matter} \\ t^{1/2}\,, & \text{radiation} \end{matrix} \\ e^{Ht}\,, & \omega^{(\mathrm{i})} = -1\,, \quad \text{vacuum energy} \,. \end{cases} \tag{2.43}$$

The number of efolds N by which the scale factor increases between $t_1 < t_2$ is given by

$$N = \int_{a(t_1)}^{a(t_2)} \mathrm{d}(\ln a) = \ln\left(\frac{a(t_2)}{a(t_1)}\right) . \tag{2.44}$$

If the evolution of the universe is dominated by a single component for a given period of time one can use (2.42) and (2.37) (with $k = 0$) to obtain the useful relation for N we use later in chapter 12

$$N = \frac{2}{3(1+\omega)} \ln\left(\frac{H(t_1)}{H(t_2)}\right) . \tag{2.45}$$

2.2.2. Slow Roll Inflation

As we have mentioned in the introduction, standard cosmology is faced by two pivotal problems: the horizon [50, 51] and the flatness problem [52, 53]. Both problems can be solved if the universe went through a phase of accelerated expansion right after the big bang [40–43] which is referred to as *inflation*. Interestingly, such a phase of accelerated expansion can be generated if the universe is dominated by a scalar field ϕ, the inflaton, slowly evolving in a potential V.

The energy momentum tensor $T_{\mu\nu}^{(\phi)}$ for a scalar field ϕ in a potential $V(\phi)$ is given by

$$T_{\mu\nu}^{(\phi)} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}\partial_\sigma\phi\partial^\sigma\phi + V(\phi)\right) . \tag{2.46}$$

Using the metric (2.34) and restricting to a homogeneous field $\phi = \phi(t)$ one can identify $T_{\mu\nu}^{(\phi)}$ of

(2.46) with (2.36) and extract

$$\rho^{(\phi)} = \frac{1}{2}\dot{\phi}^2 + V(\phi) \ , \quad p^{(\phi)} = \frac{1}{2}\dot{\phi}^2 - V(\phi) \ . \tag{2.47}$$

The continuity equation (2.39) is then equivalent to the equation of motion of ϕ

$$\ddot{\phi} + 3H\dot{\phi} = -V'(\phi) \ , \tag{2.48}$$

where $'$ denotes the derivative wrt. ϕ. An accelerated expansion corresponds to $\ddot{a} > 0$ which can be stated equivalently as (since $a > 0$)

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 > 0 \quad \Leftrightarrow \quad \varepsilon \equiv -\frac{\dot{H}}{H^2} < 1 \tag{2.49}$$

Taking the time derivative of (2.37) ($k = 0$) and using (2.47) one can express the condition (2.49) in terms of ϕ

$$-\frac{\dot{H}}{H^2} = 3\frac{\frac{1}{2}\dot{\phi}^2}{\rho^{(\phi)}} \ll 1 \ , \tag{2.50}$$

where we now demand that the lhs. is much smaller than 1. Therefore, accelerated expansion occurs if the kinetic energy of the inflaton gives only a small contribution to the total energy density $\rho^{(\phi)}$, i.e. $\rho^{(\phi)} \simeq V(\phi)$. A prolonged period of acceleration requires that the acceleration of ϕ is small as well. This is ensured by demanding that dimensionless acceleration per Hubble time is small

$$-\frac{\ddot{\phi}}{H\dot{\phi}} \ll 1 \ . \tag{2.51}$$

Under the conditions (2.50) and (2.51) we can simplify (2.37) and (2.48) to

$$H^2 \simeq \frac{V}{3M_{\rm Pl}} \ , \qquad 3H\dot{\phi} \simeq -V' \ . \tag{2.52}$$

This allows to reformulate the conditions for prolonged accelerated expansion (2.50) and (2.51) in terms of the *slow roll parameters* of the potential V [225]

$$\epsilon = \frac{M_{\rm Pl}^2}{2}\left(\frac{V'}{V}\right)^2 \ , \qquad \eta = M_{\rm Pl}^2\frac{V''}{V} \ . \tag{2.53}$$

Demanding

$$\epsilon \ll 1 \ , \qquad \eta \ll 1 \ , \tag{2.54}$$

ensures that (2.50) and (2.51) are satisfied and inflation ends if (2.54) are violated. The number of efolds gathered in this period are given by (2.44). In terms of ϕ and V this yields[12]

$$N(\phi_i) = \int_{\phi_*}^{\phi_i} \frac{1}{M_{\rm Pl}^2}\frac{V(\phi)}{V'(\phi)}\mathrm{d}\phi \ , \tag{2.55}$$

where ϕ_i denotes the field value where the inflaton starts rolling and ϕ_* denotes the field value where (2.54) are violated and inflation ends.

[12] Here we used $\mathrm{d}(\ln a) = H\mathrm{d}t = H/\dot{\phi}\ \mathrm{d}\phi$ and (2.52) to replace $H/\dot{\phi} = 1/\sqrt{2M_{\rm Pl}\epsilon}$.

2.2.3. Inflationary Phenomenology

Besides resolving the horizon and flatness problem, inflation also offers a unique set of predictions. We have already mentioned in the introduction that inflation stretches small fluctuations to cosmic scales. As a result, small primordial fluctuations appear as the anisotropies of the CMB. The theory of inflation now allows to exactly predict the power spectrum of the temperature fluctuations. We closely follow the presentation of [178] for the following section.

We consider small perturbations of the fields relevant in the primordial universe

$$g_{\mu\nu} = g^0_{\mu\nu} + \delta g_{\mu\nu}\,, \qquad \phi = \phi^0 + \delta\phi\,, \tag{2.56}$$

where we expanded around a background configuration denoted by ^{0}. All but two degrees of freedom in the perturbations δg and $\delta\phi$ correspond to gauge redundancies and can be eliminated by appropriately fixing the gauge. Especially the fluctuations $\delta\phi$ correspond to the Goldstone mode of broken time translations [226] which gets absorbed in the metric degrees of freedom. In unitary gauge all physical perturbations are encoded into scalar curvature perturbations $\mathcal{S}$ and tensor perturbations h_{ij} of the metric

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2 e^{2\mathcal{S}(t,x)}\left(\delta_{ij} + 2h_{ij}\right)\mathrm{d}x^i\mathrm{d}x^j\,. \tag{2.57}$$

By measuring the temperature fluctuations in the CMB on can determine the dimensionless power spectrum of the scalar Δ_s^2 which is given by[13]

$$\Delta_s^2(k) = \frac{1}{8\pi^2}\frac{H^4}{M_{\mathrm{Pl}}^2|\dot{H}|}\,. \tag{2.58}$$

Deviations from a scale invariant spectrum are expressed via the *spectral tilt* n_s defined by[14]

$$n_s - 1 = \frac{\mathrm{d}\ln\Delta_s^2(k)}{\mathrm{d}\ln k} = -2\varepsilon - \frac{\dot{\varepsilon}}{\varepsilon H} \tag{2.59}$$

The tensor modes h_{ij} produce gravitational waves with the corresponding power spectrum Δ_h^2

$$\Delta_h^2(k) = \frac{2}{\pi^2}\frac{H^2}{M_{\mathrm{Pl}}^2}\,. \tag{2.60}$$

The spectrum Δ_h^2 is usually constrained by determining the *tensor-to-scalar ratio* r

$$r = \frac{\Delta_h^2}{\Delta_s^2}\,. \tag{2.61}$$

For slow roll inflation all these physical observable can be related to the slow roll parameters ϵ , η and the potential V of the inflaton. The power spectra can expressed as

$$\Delta_s^2 = \frac{1}{24\pi^2 M_{\mathrm{Pl}}^4}\frac{V}{\epsilon}\,, \qquad \Delta_h^2 = \frac{2}{3\pi^2 M_{\mathrm{Pl}}^4}V\,. \tag{2.62}$$

[13]For slow roll inflation the speed of sound is $c_s = 1$. We will hence not introduce this parameter for the sake of simplicity.

[14]Here we again omit any contributions due to the speed of sound.

This yields for the spectral index and the tensor-to-scalar ratio r

$$n_s - 1 = 2\eta - 6\epsilon \,, \qquad r = 16\epsilon \,. \tag{2.63}$$

The Planck collaboration constrained all of the above parameters [8]. For the scalar power spectrum Planck refers to

$$\Delta_s^2 = A_s \left(\frac{k}{k_\star}\right)^{n_s - 1} \,, \tag{2.64}$$

where k_* denotes the pivot scale set to $k_* = 0.005\mathrm{Mpc}^{-1}$. The amplitude A_s was measured to be [8]

$$A_s \times 10^9 = 2.105 \pm 0.030 \,. \tag{2.65}$$

Further, the measured value of n_s was found to be [8]

$$n_s = 0.9665 \pm 0.0038 \qquad (68\%\,\mathrm{CL}) \,, \tag{2.66}$$

and the tensor to scalar ratio is constrained to

$$r < 0.06 \qquad (95\%\,\mathrm{CL}) \,. \tag{2.67}$$

These results fix all of the parameters we want to match in our stringy inflation model in part III.

3. String Phenomenology

String theory is one of the most studied candidate theories for a consistent quantum theory of gravity. Yet, string theory could neither be verified nor falsified due to the lack of testable predictions. The field of string phenomenology aims to remedy this situation by deriving physical implication and concrete predictions from string theory to be applied in particle physics and cosmology. In this chapter, we will highlight key elements of string phenomenology that are essential to arrive at the effective 4d description of string theory. Of course, this one chapter cannot provide a comprehensive introduction to this broad topic but a detailed presentation can be found in [74, 227–229] or focusing specifically on 4d flux compactifications of string theory [99, 230–234]. We will start in chapter 3.1 with the 10d effective theory of type IIB supergravity which represents the low energy limit of type IIB string theory. This will be the staring point form which we will derive all the results of part II and part III. In addition we will discuss the effective theory of Dp-branes which are an essential part of type IIB and necessary to arrive at realistic 4d models. Upon establishing the higher-dimensional theory, we will turn to the concept of compactification in chapter 3.2 which is crucial for an effective transition from a 10d to a 4d theory. As a result of the compactification, many massless scalar fields (moduli) arise in the effective 4d theory. If not protected by symmetries, these moduli generally become massive due to quantum corrections once supersymmetry is broken. However, this is very model-dependent and it is often not possible to calculate the quantum corrections explicitly. Instead, one would like to implement a concrete mechanism to give mass to the moduli under very general conditions and with technical control. This will be the topic of chapter 3.3 in which we will discuss moduli stabilization in type IIB string theory.

3.1. Low-Energy Effective Actions

3.1.1. Type IIB Supergravity

At low energies, the interaction of the massless string modes of type IIB string theory can be described by a 10d effective field theory where we expand the theory in powers of $\sqrt{\alpha'}$. To leading order in α', we find that all interactions of the massless closed string modes are described by the action of type IIB supergravity. The bosonic field content is given by the fields from the Neveu-Schwarz-Neveu-Schwarz (NSNS) sector: The 10d string frame metric G, the dilaton ϕ and the 2-form B_2. In addition to the NSNS fields we have to include the fields from the Ramond-Ramond (RR) sector: The 0-form C_0, the 2-form C_2 as well as the 4-form C_4. In string frame the bosonic part of the action reads [235–238]

$$\begin{aligned} S_{IIB} = \frac{1}{2\kappa_{10}^2} \int_{\mathcal{M}} \mathrm{d}^{10}x \sqrt{-G}\, e^{-2\phi} \left(R[G] - \frac{1}{2}|H_3|^2 + 4G^{MN} \partial_M \phi \partial_N \phi \right) \\ - \frac{1}{4\kappa_{10}^2} \left[\int_{\mathcal{M}} \mathrm{d}^{10}x \sqrt{-G} \left(|F_1|^2 + |\tilde{F}_3|^2 + \frac{1}{2}|\tilde{F}_5|^2 \right) + \int_{\mathcal{M}} C_4 \wedge H_3 \wedge F_3 \right], \end{aligned} \tag{3.1}$$

where R refers to the 10d Ricci scalar and x^M denote the 10d coordinates of the manifold $\mathcal{M}$ where $M, N = 0, 1, \dots, 9$. Further, we defined $2\kappa_{10}^2 = (2\pi)^7 \alpha'^4$ as well as

$$|F_p|^2 = \frac{1}{p!} F_{\mu_1 \cdots \mu_p} F^{\mu_1 \cdots \mu_p} \; . \tag{3.2}$$

The field strengths of the p-forms are given by

$$\begin{aligned} F_1 &= \mathrm{d}C_0 \; , \quad F_3 = \mathrm{d}C_2 \; , \quad H_3 = \mathrm{d}B_2 \; , \quad F_5 = \mathrm{d}C_4 \; , \\ \tilde{F}_3 &= F_3 - C_0 \wedge H_3 \; , \quad \tilde{F}_5 = F_5 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3 \; , \end{aligned} \tag{3.3}$$

and the action (3.1) is invariant under the following gauge transformations

$$\delta B_2 = \mathrm{d}\lambda_1 \; , \quad \delta C_2 = \mathrm{d}\Lambda_1 \; , \quad \delta C_4 = \mathrm{d}\Lambda_3 + \frac{1}{2}\Lambda_1 \wedge H_3 - \frac{1}{2}\lambda_1 \wedge F_3 \; . \tag{3.4}$$

Strictly speaking, type IIB supergravity is only defined by a set of equations of motion which cannot be derived from an action. The action (3.1) is thus sometimes referred to as a ‘pseudo’ action, because the variation of the action (3.1) yields the correct equations of motion only when the self-duality condition of C_4

$$\tilde{F}_5 = \star_{10} \tilde{F}_5 \; , \tag{3.5}$$

is imposed in addition to the equations of motion.

We stated the action in string frame where the dilaton ϕ couples to the Ricci scalar. Yet, we can transform the action (3.1) to Einstein frame by performing the Weyl rescaling

$$G_{E,MN} = e^{-\phi/2} G_{MN} \; , \tag{3.6}$$

where G_E denotes the Einstein frame metric. At this point we only want to note that the action (3.1) is further invariant under the global $\mathrm{SL}(2,\mathbb{R})$ transformations

$$\Lambda^i{}_j = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \; , \quad \det\Lambda = 1 \tag{3.7}$$

$$\tau = C_0 + i e^{-\phi} \; , \quad \tau' = \frac{a\tau + b}{c\tau + d} \; , \tag{3.8}$$

$$\begin{pmatrix} C_2 \\ B_2 \end{pmatrix}' = \Lambda \begin{pmatrix} C_2 \\ B_2 \end{pmatrix} \; . \tag{3.9}$$

The 4-form C_4 and the Einstein-frame metric G_E do not transform and a manifestly $\mathrm{SL}(2,\mathbb{R})$ -invariant action can be stated in Einstein frame, see eq. (6.1). Given the significance of the $\mathrm{SL}(2,\mathbb{R})$ properties of type IIB for our application to kinetic mixing we will discuss this subject in detail in chapter 6.1 of part II.

3.1.2. Dp-Brane Effective Action

In addition to the string, type IIB string theory contains non-perturbative objects called Dp-branes which extend along the timelike and p spatial dimensions. D-branes can be understood as objects on which open strings can end but are also independent objects of string theory. However, due to their non-perturbative nature D-branes are often treated as non-dynamical but couple to the open and closed strings. Most importantly, the open string gives rise to a $\mathrm{U}(1)$ gauge theory confined to the worldvolume of the D-brane and represents the only source for realistic gauge theories in type IIB. Similar to the 10d bulk action one can derive a low energy effective action describing the interaction of D-branes and massless modes of the open and closed string. The low energy action of a Dp-brane is of *Dirac-Born-Infeld (DBI) type* and is given by [85,89,239–251] (we follow the conventions of [246] and the subsequent discussion is based on the given references.)

$$S_{Dp-brane} = S_{DBI} + S_{CS} \,, \tag{3.10}$$

$$S_{DBI} = -T_p \int\limits_{D_p} \mathrm{d}^{p+1}\xi e^{-\phi} \sqrt{-\det\left(G_{ab} - B_{ab} + F_{ab}\right)} \,, \tag{3.11}$$

$$S_{CS} = \pm\mu_p \int\limits_{D_p} \exp\left(F_2 - B_2\right) \wedge \sum_q \hat{C}_q \wedge \sqrt{\frac{\hat{A}(4\pi^2\alpha' R_T)}{\hat{A}(4\pi^2\alpha' R_N)}} \,, \tag{3.12}$$

where we slightly abuse notation and refer to the Dp-brane worldvolume by D_p. Further, ξ^a refer to the intrinsic worldvolume coordinates $(a, b = 0, 1, \ldots, p)$ and the embedding of the brane in the 10d space is encoded in the functions $x^M(\xi)$. The metric on the brane worldvolume G_{ab} is given by the pullback φ^* of the 10d bulk metric G_{MN}

$$G_{ab} \equiv \varphi^*[G]_{ab} = \frac{\partial x^M(\xi)}{\partial \xi^a} \frac{\partial x^N(\xi)}{\partial \xi^b} G_{MN} \,. \tag{3.13}$$

Similarly B_{ab} and $\hat{C}_q$ refer to the respective pullback of the p-forms and F_{ab} denotes the field strength[15] of the $\mathrm{U}(1)$ gauge theory from the open string excitations localized on the brane. Note, that if the forms $\hat{C}_q$ in the brane action are to be identified with the RR-forms in the bulk action (3.1) we have to impose $\hat{C}_4 \equiv C_4 + 1/2\, B_2 \wedge C_2$ whereas the other RR-forms can be identified, i.e. $\hat{C}_2 \equiv C_2$ and $\hat{C}_0 = C_0$. We devoted app. A to explain this technicality but want to highlight that this is crucial for our applications in part II. Furthermore, the integral in (3.12) picks out the right RR-forms $\hat{C}_q$ in the formal sum $\sum_q \hat{C}_q$, such that the whole expression under the integral gives a $(p+1)$-form. For completeness we included the last term in (3.12) which describes the couplings of the brane to curvature. The subscripts T, N on the curvature forms R distinguish between the curvature form of the tangent (T) and normal (N) bundle with respect to the brane and $\hat{A}$ refers to the A-roof genus. For the rest of this thesis we will omit all curvature contributions.

To preserve the 4d Poincare symmetry, the branes must fill all or none of the four non-compact dimensions. In *static gauge* we choose to align the brane coordinates ξ^a with the first $(p+1)$ 10d coordinates x^M, i.e. $x^a = \xi^a$. The remaining coordinates orthogonal to the brane x^i $(i, j = p+1, \ldots, 9)$ encode the fluctuations of the brane around a fixed position $\langle x \rangle^i$ in 10d space. The fluctuations are parametrised by scalar fields Φ^i defined by

$$x^i \equiv \langle x \rangle^i + 2\pi\alpha' \Phi^i(\xi) \,. \tag{3.14}$$

Thus, we see that all bulk fields couple to the fields Φ^i via their pullback, e.g. in (3.13). We will assume that the pullback implicitly includes all terms dependent on Φ^i, and will only explicitly display these terms when they are important to the context. The tension of the brane T_p and

[15] Note, that we absorbed a factor $2\pi\alpha'$ into the definition of F_{ab} when comparing to the original literature.

charge under RR-forms μ_p are given by

$$1/T_p = (2\pi)^p \alpha'^{(p+1)/2} , \tag{3.15}$$

$$\mu_p = T_p , \tag{3.16}$$

where $(+)$ in (3.12) is used for D-brane and $(-)$ is used for anti-D-branes.

The action S_{DBI} (3.10) can be expanded in powers of derivatives which corresponds to an expansion in F_{ab}. At the two-derivative level we find

$$S_{DBI} = -T_p \int_{D_p} \mathrm{d}^{p+1}\xi e^{-\phi} \sqrt{-\det\left(G_{ab}\right)} \left[1 + \frac{1}{4}\left(F_{ab}F^{ab} + F_{ab}B^{ab} + B_{ab}B^{ab}\right) + \cdots \right] , \tag{3.17}$$

which explicitly demonstrates that a D-brane introduces a gauge field to the low energy effective action. We have already mentioned that the U(1) gauge group of a single D-brane gets enhanced to U(N) if N D-branes are coincident. For our purposes, the generalized non-abelian D-brane action is only required in the study of kinetic mixing between D7-branes. To maintain coherence in our discussion, we have relegated the introduction of the non-abelian D-brane action to chapter 7 However we want to refer to the excellent review [252] about the non-abelian D-brane action where also further references can be found.

3.2. String Compactifications

3.2.1. Kaluza-Klein Compactification

We have already briefly introduced the concept of Kaluza-Klein compactification in the introduction but skimmed over many technical details. In this section we want to return to this subject and elaborate on these details and highlight key insights which will be important for the rest of this thesis. A comprehensive review can be found in [253, 254].

For this reason we turn to a concrete example where we compactify from 5d to 4d where the fifth dimension is given by an S^1, i.e. $\mathcal{M}^5 = \mathcal{M}^{1,3} \times S^1$. We split the 5d coordinates x^m $(m, n = 0, 1, \ldots, 4)$ into 4d coordinates x^μ $(\mu, \nu = 0, 1, 2, 3)$ on $\mathcal{M}^{1,3}$ and the fifth coordinate $x^4 \equiv y$ on S^1. The coordinate y is periodic and we identify $y \simeq y + 2\pi R$ where R corresponds to the radius of the S^1. Let us assume that $\mathcal{M}^5$ is flat such that we can write the Laplacian as $\Box_5 = \partial_\mu\partial^\mu + \partial_y\partial_y$. Analogous to the introduction, we can decompose every 5d field into eigenfunctions $Y^n(y)$ of the internal Laplace operator $\partial_y\partial_y$. In the case of S^1, the eigenfunctions are given by

$$Y^n(y) = e^{\frac{iny}{R}} , \quad n \in \mathbb{Z} , \tag{3.18}$$

where the corresponding eigenvalues read $\lambda_n = -n^2/R^2$. If we again take a 5d scalar field Φ as an example, we decompose Φ as $\Phi(x, y) = \sum_n \phi^n(x) Y^n(y)$. Then, the equation of motion of Φ reads

$$\Box_5 \Phi(x, y) = \sum_{n=-\infty}^{\infty} \left(\partial_\mu\partial^\mu \phi^n(x) - \frac{n^2}{R^2}\phi^n(x) \right) e^{\frac{iny}{R}} = 0 , \tag{3.19}$$

and we explicitly see that a infinite tower of KK-states with masses proportional to $1/R$ arises. Crucially, we can identify a single massless mode for $n = 0$, which will be the only field present in a 4d EFT below the KK-scale $M_{\mathrm{KK}} = 1/R$. As the internal dimensions become small, we can consider an effective 4d action containing only the massless modes from the KK-decomposition.

Similarly, we can repeat the same procedure for the 5d metric G and the 5d Einstein Hilbert action

$$S = \frac{M_{Pl,5}^3}{2} \int \mathrm{d}^4x\mathrm{d}y\sqrt{-G^0}\, R_{(5)}[G^0]\,. \tag{3.20}$$

We are interested in the low energy EFT and hence consider only the zero mode of the metric $G_{mn}^0(x)$. The 4d field content can be read of the metric components G_{mn}^0. Upon compactification the components $G_{\mu\nu}^0$ are associated to the 4d metric $g_{\mu\nu}$, $G_{4\mu}^0$ is associated to a 4d vector A_μ and G_{44}^0 is associated to a 4d scalar ρ. The zero mode of the metric G_{mn}^0 can be conveniently written as [255]

$$G_{mn}^0(x) = \begin{pmatrix} g_{\mu\nu}(x) + e^{\rho(x)}A_\mu(x)A_\nu(x) & e^{\rho(x)}A_\nu(x) \\ e^{\rho(x)}A_\mu(x) & e^{\rho(x)} \end{pmatrix}\,, \tag{3.21}$$

which we plug into the 5d Einstein Hilbert action (3.20) and find [255]

$$S = \frac{2\pi R\, M_{Pl,5}^3}{2} \int \mathrm{d}^4x\sqrt{-g}e^{\rho/2}\left(R_{(4)}[g] - \frac{1}{4}e^\rho F_{\mu\nu}F^{\mu\nu} - \frac{1}{6}\partial_\mu\rho\partial^\mu\rho\right)\,. \tag{3.22}$$

Here we defined $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and obtained the factor $2\pi R$ from the y integration. A few remarks are now in order. First of all, the metric (3.21) is only invariant under 4d diffeomorphisms combined with the reparametrisation $y \to y + \lambda(x)$. Otherwise, the metric components would acquire a dependence with respect to y. However, the reparametrisation $y \to y + \lambda(x)$ implies that A_μ transforms analogously to a gauge transformation $A_\mu \to A_\mu + \partial_\mu\lambda$ such that A_μ genuinely represents a gauge field from the 4d perspective. Furthermore, we find that the scalar ρ has no potential and thus represents a geometric modulus which in principal can take any constant value in the vacuum.

Supposing that the 5d space is flat in the vacuum implies that $\langle\rho\rangle = 0$, $\langle A_\mu\rangle = 0$, and $\langle g_{\mu\nu}\rangle = \eta_{\mu\nu}$. However, $\langle\rho\rangle = 0$ is a priori not fixed and a change in ρ corresponds to a deformation of the S^1. We can see this explicitly from considering the physical volume $\mathrm{vol}(S^1)$

$$\mathrm{vol}(S^1) \equiv 2\pi\mathcal{R} = \int_0^{2\pi R} \sqrt{G_{44}^0} = 2\pi R e^{\rho/2}\,, \tag{3.23}$$

where $\mathcal{R} \equiv Re^{\rho/2}$ represents the physical radius which is determined by ρ. In fact, in our example ρ determines all coupling parameters in the vacuum of the theory. In the case of the gauge coupling and gravitational coupling this is evident from (3.22). One can perform the Weyl rescaling $g_{\mu\nu} \to e^{-\rho/2}g_{\mu\nu}$ to transform to Einstein frame. Yet, the gravitational coupling is still determined by ρ since the 4d theory is unavoidably a scalar-tensor theory of gravity [255].

These particularities arise quite generally in string compactifications. The higher dimensional fields introduce a set of 4d fields with different spin, e.g. from the 5d metric we obtained the 4d metric, a vector, and a scalar. Thus, depending on the higher dimensional field content, we can generate a rich spectrum of 4d fields. Furthermore, the geometry of the internal dimensions will be determined by geometric moduli which correspond to dynamical scalars in 4d. In addition, these moduli control the coupling parameters of the compactified theory. For a consistent compactification these moduli need to be stabilised. Otherwise, the internal dimensions could become singular or decompactify such that our 4d EFT containing only the massless KK-modes breaks down. We thus require a technically controlled mechanism[16] to introduce a scalar poten-

[16]If not protected by symmetries, these moduli generally become massive due to quantum corrections. However, this is very model-dependent and it is often not possible to calculate the quantum corrections explicitly. Instead, one

tial in 4d which stabilises the moduli.

3.2.2. Calabi-Yau and Calabi-Yau Orientifold Compactifications

We now turn to the compactification of the 10d type IIB action (3.1) where we have to compactify in total six internal dimensions. The following discussion on this topic is drawn from [69, 71, 73, 229] and references therein. However, we do not want to choose just any 6d manifold but follow the well trodden path in string phenomenology and consider compactifications on compact Calabi-Yau manifolds (CY). Calabi-Yau manifolds come with a list useful properties which drastically simplify the construction of realistic 4d vacua.

First of all, Calabi-Yau manifolds are in general complex n-dimensional *Kähler manifolds* with vanishing first Chern class. Yaus theorem [256] implies that any compact n-dimensional Kähler manifold with vanishing first Chern class admits a unique *Ricci flat* Kähler metric with SU(n) holonomy. Thus CY 3-folds solve the 10d Einstein equations in vacuum because they are Ricci flat. In addition, due to the SU(3) holonomy compactifications on CY 3-folds preserve only eight of the 32 supercharges of type IIB such that the 4d theory has $\mathcal{N} = 2$ supersymmetry. We will see in a moment that $\mathcal{N} = 2$ can be broken to $\mathcal{N} = 1$ SUSY by orientifolding the CY. With $\mathcal{N} = 1$ SUSY, the 4d theory is still strongly constrained, but allows for a scalar potential of the moduli. What makes this particularly elegant is that $\mathcal{N} = 1$ SUSY dictates the structure of the scalar potential V enabling us to deduce precise expressions for V with minimal input from the higher-dimensional theory.

Furthermore, we have precise knowledge about the number of metric moduli that arise from a CY compactification. Yaus theorem implies that all geometric moduli are in one to one correspondence to the harmonic forms of the CY. Hence, the total number of moduli can be counted using the Hodge numbers $h^{p,q} = \dim_{\mathbb{R}} H^{p,q}(\mathcal{X})$ of the CY 3-fold $\mathcal{X}$. Crucially, SU(3) holonomy restricts the Hodge numbers of $\mathcal{X}$ to

$$\begin{gathered} h^{3,3} = h^{0,0} = 1 \,, \quad h^{3,0} = h^{0,3} = 1 \\ h^{1,1} = h^{2,2} \,, \quad h^{2,1} = h^{1,2} \,, \end{gathered} \tag{3.24}$$

while all other $h^{p,q}$ vanish. Therefore, it is a priori clear how many moduli will appear in the 4d theory.

The rich structure of CY manifolds allows to distinguish between two categories of metric moduli. To delve into this topic, it is essential to first revisit another crucial aspect of Calabi-Yau geometries: As a consequence of Yaus theorem, there exists a unique Ricci flat metric denoted as $g_{i\bar{j}}$, accompanied by its corresponding Kähler 2-form J as defined by

$$J = i g_{m\bar{n}} \, \mathrm{d}z^m \wedge \mathrm{d}\bar{z}^{\bar{n}} \,, \tag{3.25}$$

where z^n and $\bar{z}^{\bar{n}}$ refer to the complex coordinates on the CY. Additionally, there is a unique (up to complex rescaling) holomorphic 3-form Ω

$$\Omega = \frac{1}{3!}\Omega_{mno}(z) \, \mathrm{d}z^m \wedge \mathrm{d}z^n \wedge \mathrm{d}z^o \,, \tag{3.26}$$

and its antiholomorphic counterpart $\overline{\Omega}(\bar{z})$ which is implied by $h^{3,0} = h^{0,3} = 1$. A modulus of the

would like to implement a concrete mechanism to give mass to the moduli under very general conditions and with technical control.

CY corresponds to a deformation of the metric δg

$$g_{m\overline{n}}\mathrm{d}z^m \wedge \mathrm{d}\overline{z}^{\overline{n}} \to g_{m\overline{n}}\mathrm{d}z^m \wedge \mathrm{d}\overline{z}^{\overline{n}} + \delta g_{m\overline{n}}\mathrm{d}z^m \wedge \mathrm{d}\overline{z}^{\overline{n}} + \delta g_{mn}\mathrm{d}z^m \wedge \mathrm{d}z^n + \text{h.c.} , \tag{3.27}$$

which preserves Ricci flatness $R[g+\delta g] = 0$. To maintain Ricci flatness, the Kähler class and the complex structure of the CY have to change accordingly which is accompanied by deformations of the Kähler form J and the holomorphic 3-form Ω.

The metric deformations δg_{mn} violate the hermiticity of the initial metric $g_{m\overline{n}}$. The only possibility to account for this deformation and to return to a hermitian Kähler metric is to adjust the complex structure of the CY. Hence, the moduli associated to δg_{mn} are called *complex structure (CS) moduli* U^I which can be obtained from expanding the harmonic (2,0)-form δg_{mn} in terms of the harmonic (1,2)-forms $\chi_I \in H^{1,2}$ $(I, J = 1, \ldots, h^{1,2})$ and the unique holomorphic 3-form Ω [257]

$$\delta g_{mn} = \frac{i}{||\Omega||^2}\overline{U}^I\, (\chi_I)_{m\overline{p}\,\overline{q}}\, \Omega_{nrs}\, g^{r\overline{p}}\, g^{s\overline{q}} , \tag{3.28}$$

where we defined $||\Omega||^2 = \frac{1}{6}\Omega_{mno}\overline{\Omega}_{\overline{pqr}}\, g^{m\overline{p}}\, g^{n\overline{q}} g^{o\overline{r}}$. The parameters U^I represent the complex structure moduli which determine the ratios in the volume of certain 3-cycles Σ_3 of the CY. Before stating explicit formulas for the 3-cycle volumes we first chose a symplectic basis of harmonic 3-forms α_κ dual to the B-3-cycles $\Sigma^B_{3,\kappa}$ and β^λ dual to the A-3-cycles $\Sigma^\kappa_{3,A}$ $(\kappa, \lambda = 0, 1, \ldots, h^{1,2})$ such that the following relations hold

$$\int\limits_{\mathcal{X}} \alpha_\kappa \wedge \beta^\lambda = \delta^\lambda_\kappa = -\int\limits_{\mathcal{X}} \beta^\lambda \wedge \alpha_\kappa , \quad \int \alpha_\kappa \wedge \alpha_\lambda = 0 = \int \beta^\kappa \wedge \beta^\lambda . \tag{3.29}$$

With the help of these A and B-3-cycles we can determine the complex valued A-periods of Ω

$$X^\kappa = \int\limits_{\Sigma^\kappa_{3,A}} \Omega . \tag{3.30}$$

This provides exactly one period to many for matching the periods to the CS moduli. However, Ω is only defined up to a complex prefactor such that we instead identify

$$U^I \equiv \frac{X^I}{X^0} . \tag{3.31}$$

Given that the A-periods already encompass all the CS moduli implies that the B-periods $\mathcal{F}_\lambda$

$$\mathcal{F}_\lambda = \int\limits_{\Sigma^B_{3,\lambda}} \Omega , \tag{3.32}$$

can be expressed as functions of the A-periods X^κ. The precise form of the $\mathcal{F}_\lambda$ is determined by the Picard–Fuchs equations which rely only on topological properties of the CY [258, 259]. The specific details will not be relevant for the rest of this thesis.

On the other hand, the metric deformations $\delta g_{m\overline{n}}$ are linked to harmonic deformations of the Kähler form δJ [257]. We can expand J in the basis of harmonic (1,1)-forms $\omega_i \in H^{1,1}$ $(i, j = 1, \ldots, h^{1,1})$

$$J = i g_{m\overline{n}}\mathrm{d}z^m \wedge \mathrm{d}\overline{z}^{\overline{n}} = t^i \omega_i , \tag{3.33}$$

such that we explicitly see that metric deformations of the kind $\delta g_{m\overline{n}}$ correspond to a change of the *Kähler moduli* $t^i(x)$. The real valued Kähler moduli t^i determine the volume of 2-cycles Σ_i^2

$$\mathrm{vol}(\Sigma_i^2) = \int_{\Sigma_i^2} J = t^i \ , \tag{3.34}$$

and the volume τ_i of 4-cycles Σ_4^i of the CY

$$\mathrm{vol}(\Sigma_4^i) = \frac{1}{2} \int_{\Sigma_4^i} J \wedge J = \frac{1}{2} \kappa_{ijk} t^j t^k \equiv \tau_i \ . \tag{3.35}$$

Here we introduced the triple intersection numbers κ_{ijk}

$$\kappa_{ijk} = \int_{\mathcal{X}} \omega_i \wedge \omega_j \wedge \omega_k \ , \tag{3.36}$$

which further allow to express the CY volume $\mathcal{V}$ in 4d Einstein frame in terms of t^i

$$\mathcal{V} = \frac{1}{6} \int J \wedge J \wedge J = \frac{1}{6} \kappa_{ijk} t^i t^j t^k \ . \tag{3.37}$$

Given that the metric is positive definite implies that t^i, τ_i and $\mathcal{V}$ are positive. Note, that the above volumes are defined in 4d Einstein frame and are measured in string units where we set $\sqrt{2\pi\alpha'} = 1$. Since the 2-cycles and 4-cycles are related by duality, we will often use the 4-cycle Kähler moduli τ_i instead of the 2-cycle moduli t^i. This represents the usual convention in the literature and the relation between τ_i and t^i can be made explicit through the useful formula $\tau_i = \partial\mathcal{V}/\partial t^i$.

In addition to the metric moduli, there are other massless fields that arise from the KK-decomposition of the 10d fields. Particularly, in the case of p-form fields such as B_2, C_2, and C_4, we encounter non-trivial field configurations in the internal space. These configurations can be represented by the following decomposition into harmonic forms

$$B_2 = b^i(x)\omega_i \ , \quad C_2 = c^i(x)\omega_i \ , \quad C_4 = \theta_i(x)\tilde{\omega}^i \ , \tag{3.38}$$

where we introduced the harmonic 4-forms $\tilde{\omega}^i \in H^{2,2}$ which are dual to ω_i. The 4d scalar fields $b^i(x)$, $c^i(x)$, and $\theta_i(x)$ exhibit a shift symmetry and thus represent 4d axions. Similarly, we obtain the complex *axio-dilaton* τ from the KK-reduction of the C_0-form and the dilaton ϕ

$$\tau = C_0 + ie^{-\phi} \ , \tag{3.39}$$

which yields an additional 4d scalar field. Note that we exclusively focused on the associated 4d scalar fields in (3.38). Performing a full KK-decomposition, additional fields, such as vectors resulting from the expansion of C_4 into harmonic 3-forms would arise in 4d. However, for the discussion of the moduli fields, we will not delve into further details of this topic but refer to the references [260, 261] where this is spelled out explicitly. What holds significance for us is the *complexification* of the Kähler moduli τ_i by coupling these to the C_4 axions θ_i

$$T_i = \tau_i + i\theta_i \ , \tag{3.40}$$

to form supermultiplets of the 4d supergravity theory.

As we have already mentioned, a compactification on a CY yields a $\mathcal{N} = 2$ 4d theory which does not admit a scalar potential for the moduli fields nor chiral fermions. One can obtain a 4d theory with only $\mathcal{N} = 1$ SUSY by orientifolding the CY which we use for the compactification. A CY orientifold [262–266] is obtained from a given CY by modding out a symmetry group which involves the string worldsheet parity operator Ω_p. To achieve $\mathcal{N} = 1$ SUSY in CY-orientifold compactifications of type IIB, the symmetry group for the orientifold projection is taken to be $S\sigma\Omega_p$. Here σ denotes an isometric and holomorphic involution of the 10d space that acts exclusively on the internal space $\mathcal{X}$, while S contains additional symmetries which collectively ensure that $S\sigma\Omega_p$ remains a symmetry of the string theory. In the cases at hand, S will be given by the identity or by the operator $(-1)^{F_L}$ where F_L refers to the left-moving fermion number. The fixed point locus of σ is referred to as *Orientifold-p-plane* (Op-plane) which necessarily fills out the non-compact dimensions since these are invariant under σ. O-planes are fixed in space and represent non-dynamical extended objects which have negative tension identical to D-branes.[17] Furthermore, as a consequence of worldsheet parity Ω_p Op-planes obtain a charged under the RR-$(p+1)$-forms of type IIB which is opposite to the charge of Dp-branes. Note, that this represents an important property because including O-planes in the compactification allows to satisfy the *tadpole cancellation condition* which arises from integrating the Bianchi identity of $\tilde{F}_5$. We will discuss this in more detail in the following section about moduli stabilisation. Since σ is required to be an isometric and holomorphic involution of $\mathcal{X}$ it satisfies the following conditions

$$\sigma\, J = J\,, \quad \sigma^2\, \Omega = \Omega\,. \tag{3.41}$$

This allows for two different projection operators $(S\sigma\Omega_p)^{(i)}$

$$(S\sigma\Omega_p)^{(1)} = \sigma\, \Omega_p\,, \qquad \text{with} \quad \sigma\, \Omega = \Omega\,, \tag{3.42}$$

$$(S\sigma\Omega_p)^{(2)} = (-1)^{F_L}\sigma\, \Omega_p\,, \qquad \text{with} \quad \sigma\, \Omega = -\Omega\,, \tag{3.43}$$

where modding out by $(S\sigma\Omega_p)^{(1)}$ or $(S\sigma\Omega_p)^{(2)}$ allows only for O5/O9-planes or for O3/O7-planes respectively. In the subsequent sections, our focus will be exclusively on O3/O7 orientifolds, since these permit D3 and D7-branes in the compactification. These branes play a critical role in type IIB model building because models with D3/D7-branes allow for incorporating chiral matter fields which are required for realistic constructions of the SM.

Importantly, only KK-modes of the 10d fields which are symmetric under the orientifold projection, in our case $(-1)^{F_L}\sigma\, \Omega_p$, remain in the 4d spectrum. The transformation behaviour of the 10d fields under $(-1)^{F_L}\Omega_p$ has been classified and reads (see e.g. [261])

$$\text{even:} \quad \phi\,,\ g\,,\ C_0\,, C_4 \qquad \text{odd:} \quad B_2\,, C_2\,, \tag{3.44}$$

which implies that invariant states have to satisfy

$$\begin{gathered}\sigma\, \phi = \phi\,, \quad \sigma\, g = g\,, \quad \sigma\, C_0 = C_0\,, \quad \sigma\, C_4 = C_4\,, \\ \sigma\, B_2 = -B_2\,, \quad \sigma\, C_2 = -C_2\,. \end{gathered} \tag{3.45}$$

Note, that this requirement eliminates the constant zero-mode of B_2 and C_2 which will be important in part II . Furthermore, σ is a holomorphic involution which allows to further decompose

[17] However, see [262] for a discussion about a generalisation of this statement.

the cohomology groups $H^{p,q}$ of the CY into even (+) and odd (-) subspaces under the action of σ

$$H^{p,q} = H_+^{p,q} \oplus H_-^{p,q} \,. \tag{3.46}$$

We are thus in the position to explicitly state the moduli spectrum

$$\begin{aligned}
&h_+^{1,1} \quad \text{complexified Kähler moduli } T_i = \tau_i + i\theta_i \,, \\
&h_-^{1,2} \quad \text{complex structure moduli } U^I \,, \\
&1 \qquad \text{axio-dilaton } \tau = C_0 + ie^{-\phi} \,, \\
&h_-^{1,1} \quad \text{2-form axions } G^I = c^I - \tau b^I \,.
\end{aligned} \tag{3.47}$$

Recall, that besides these scalars additional fields arise from the KK-decomposition of the 10d fields. Ultimately, all 4d KK-modes will arrange into supermultiplets of the $\mathcal{N} = 1$ theory [260, 261].

3.3. Moduli Stabilisation

The 4d theory of $\mathcal{N} = 1$ supergravity is entirely defined by the holomorphic superpotential W, the real Kähler potential K and the holomorphic gauge kinetic function f if gauge theories are present in the theory. Crucially, all of these quantities depend only on the chiral superfields present in the 4d theory. We have already discussed the issues that arise from massless scalar fields in the compactified theory, and we emphasized the need for a systematic approach to stabilize the moduli. Now, with the supergravity description at our disposal, we can precisely calculate the scalar potential V using W and K which can be derived from the 10d theory. By minimizing the scalar potential, we aim to ensure that all moduli acquire a non-trivial vev and, consequently, become stabilised. This procedure is now controlled in the sense that we have a precise understanding about the form of the potential, and any further corrections from higher orders in α', string loops, or non-perturbative corrections must conform to the supergravity language.

3.3.1. Flux Compactifications and Complex Structure Moduli Stabilisation

We begin our discussion about moduli stabilisation by determining the Kähler potentail K for the moduli of a CY compactification. Analysing the moduli spaces of CY manifolds shows that the Kähler potential K for the moduli is given by [73, 97, 257]

$$K = K_K + K_{CS} + K_\tau \,, \tag{3.48}$$

$$K_K\left(T_i, \overline{T}_i\right) = -2\ln(\mathcal{V}) \,, \quad K_{CS}\left(U^I, \overline{U}^I\right) = -\ln\left[-i\int_{\mathcal{X}} \Omega \wedge \overline{\Omega}\right], \quad K_\tau(\tau, \overline{\tau}) = -\ln\left[i\left(\overline{\tau} - \tau\right)\right]. \tag{3.49}$$

Here, we assumed $h_-^{1,1} = 0$ such that there are no B_2 and C_2 axions. For $h_-^{1,1} \neq 0$, the T_i supermultiplets would also include contributions from the B_2 and C_2 axions [260]. We do not want to concern ourselves with this complication as the general conclusions drawn in the following are not affected by this. Yet, a generalisation with $h^{1,1}_{-} \neq 0$ can be found e.g. in [267]. The superpotential W is found to be the Gukov-Vafa-Witten superpotential [268]

$$W\left(\tau, U^I\right) = \int_{\mathcal{X}} \Omega \wedge G_3 \,, \tag{3.50}$$

where G_3 is defined as $G_3 = F_3 - \tau H_3$ such that W depends only on the CS moduli U^I and the axio-dilaton τ.

The F-term scalar potential V hence reads

$$V = e^K \left[K^{i\bar{j}} (D_i W)(\overline{D}_{\bar{j}} \overline{W}) + K^{I\bar{J}} (D_I W)(\overline{D}_{\bar{J}} \overline{W}) + K^{\tau\bar{\tau}} (D_\tau W)(\overline{D}_{\bar{\tau}} \overline{W}) - 3|W|^2 \right] \,, \tag{3.51}$$

where we set $M_{\text{Pl}} = 1$. Furthermore, we defined

$$K_\mu = \frac{\partial K}{\partial \varphi^\mu} \,, \qquad K_{\mu\bar{\nu}} = \frac{\partial^2 K}{\partial \varphi^\mu \partial \overline{\varphi}^{\bar{\nu}}} \,, \qquad K^{\mu\bar{\nu}} = \left(K_{\mu\bar{\nu}}\right)^{-1} \,, \qquad D_\mu = \partial_\mu + K_\mu \,, \tag{3.52}$$

where φ^μ may represent any of the above moduli fields T_i, U^I or τ. Due to the special form of the Kähler moduli Kähler potential $K_K = -\ln \mathcal{V}^2$, where $\mathcal{V}^2$ is homogeneous function of degree 3 in T_i, we find the important result

$$K^{i\bar{j}} K_i K_{\bar{j}} = 3 \,. \tag{3.53}$$

In combination with $D_i W = K_i W$ we find that the Kähler moduli contribution in (3.51) cancels against the last term of (3.51) such that we are left with

$$V = e^K \left[K^{I\bar{J}} (D_I W)(\overline{D}_{\bar{J}} \overline{W}) + K^{\tau\bar{\tau}} (D_\tau W)(\overline{D}_{\bar{\tau}} \overline{W}) \right] \,, \tag{3.54}$$

which is positive definite. Models of this type are referred to as *no-scale models* [269]. This indicates that only the CS moduli and the axio-dilaton will be stabilised and we need to invoke further effects which may lift the flatness of the potential wrt. to the Kähler moduli. We will turn to the stabilisation of the Kähler moduli in section 3.3.2.

Before we do so we want to discuss in more detail how the CS moduli and the axio-dilaton are stabilised. The form of the superpotential (3.50) demonstrates that internal G_3-flux is necessary to generate a non-vanishing W. However, introducing non-trivial background fluxes deforms the CY geometry since we no longer require a vacuum solution to the 10d Einstein equations. The seminal work of Giddings, Kachru, and Polchinski [97] found general solutions to the equations of motion including *imaginary self-dual (ISD) G_3-fluxes*, i.e. $\star_6 G_3 = i G_3$. In this case the metric is deformed to be conformal to a CY metric for the internal dimensions and the metric takes the form

$$\mathrm{d}s^2 = e^{2A(y)} g_{4\,\mu\nu}(x) \mathrm{d}x^\mu \mathrm{d}x^\nu + e^{-2A(y)} g_{6\,mn}(y) \mathrm{d}y^m \mathrm{d}y^n \,, \tag{3.55}$$

where the solution for $g_{6\,mn}$ is still given by the unique metric of a CY and $A(y)$ denotes the *warp factor* which is non-trivial in the presences of fluxes or localised sources. Hence, based on the work of [97] there exist concrete realisations of flux compactifications where the internal geometry is a (conformal) CY which yields the scalar potential (3.51) we stated above. With this ansatz it is now possible to stabilize the CS moduli and the axio-dilaton by introducing fluxes.

The 3-form fluxes obey the following quantisation conditions

$$\frac{1}{2\pi\alpha'}\int_{\Sigma_3} F_3 \in 2\pi\mathbb{Z}\,, \qquad \frac{1}{2\pi\alpha'}\int_{\Sigma_3} H_3 \in 2\pi\mathbb{Z}\,, \tag{3.56}$$

when integrated over an integral 3-cycle. We can hence *freely choose* the integral coefficients of the fluxes f_λ, f^κ, h_λ, h^κ and decompose F_3 and H_3 in the basis of harmonic 3-forms α_κ and β^λ

$$F_3 = (2\pi)^2\alpha'\left(f^\kappa\alpha_\kappa - f_\lambda\beta^\lambda\right)\,, \qquad H_3 = (2\pi)^2\alpha'\left(h^\kappa\alpha_\kappa - h_\lambda\beta^\lambda\right)\,. \tag{3.57}$$

The potential (3.54) is minimised when the F-term conditions

$$D_I W = 0\,, \qquad D_\tau W = 0\,, \tag{3.58}$$

are satisfied. The exact form of (3.58) is determined by the fluxes (3.57) and yields $(h^{1,2}+1)$ equations which determine the stabilized values of the CS-moduli and the axio-dilaton. With an appropriate choice of fluxes these stabilised values can be tuned [158,162]. In the minimum SUSY will be broken if $W \neq 0$ since the F-term condition of the Kähler moduli, $D_i W = K_i W = 0$, cannot be fulfilled generically. The SUSY breaking scale which is associated to the gravitino mass $e^K W$ is not fixed since the Kähler moduli are not stabilised, explaining the term no-scale model.

Albeit these nice features, the fluxes cannot be chosen without constraints but have to satisfy the tadpole condition which arises from integrating the Bianchi identity of $\tilde{F}_5$ over the compact space

$$0 = -\int_{\mathcal{X}} H_3 \wedge F_3\,. \tag{3.59}$$

Note, the right side of (3.59) including the minus sign is positive definite since the ISD condition $\star_6 G_3 = iG_3$ implies $F_3 = C_0 H_3 - e^{-\phi}\star_6 H_3$. Hence, in this form the tadpole condition cannot be satisfied in the presence of 3-form fluxes. Fortunately, the tadpole gets modified in the presence of N_{D3} D3-branes and N_{O3} O3-planes since C_4 is sourced by both objects

$$\frac{1}{2}N_{O3} = N_{D3} - \frac{1}{2\kappa_{10}^2 T_3}\int_{\mathcal{X}} H_3 \wedge F_3\,. \tag{3.60}$$

The tadpole (3.60) can now be satisfied in the presence of flux but requires the introduction of O-planes, which yields another reason why orientifolding is required in flux compactifications. Moreover, in the presence of D7-branes higher curvature corrections to the D7-brane action introduce further tadpole contributions [97]. The associated details will not be relevant in the following. Besides this constraint the amount of possible flux choices is vast which gives rise to the 10^{500} possible vacua of type IIB flux compactifications [158, 162] or even 10^{272000} vacua in F-theory constructions [157].

The key takeaway from the CS-moduli stabilization process is that it is possible to achieve consistent stabilization of these moduli using fluxes. Furthermore, the specific values of these moduli, along with the superpotential W in the vacuum, can be adjusted through a suitable choice of fluxes. The use of O-planes is necessary both for phenomenological reasons and for the consistency of the theory. However, it is important to note that the Kähler moduli still represent flat directions in the scalar potential and thus necessitate additional mechanisms to achieve their stabilization.

3.3.2. Kähler Moduli Stabilisation and the Large Volume Scenario

In this section we want to discuss the large volume scenario (LVS) [101, 102] which provides a consistent stabilisation procedure for the remaining Kähler moduli. As we emphasized in section 3.3.1 we require additional ingredients to stabilize Kähler moduli. The sources of these can only come from corrections in α', string loop corrections, or non-perturbative corrections since we are considering a 10d theory which inherently corresponds to an EFT. As we have chosen to compactify on CY-orientifolds which yields $\mathcal{N} = 1$ SUSY all corrections must conform to the supergravity language. The superpotential W is protected from perturbative corrections and receives only non-perturbative corrections. The Kähler potential on the other hand is subject to perturbative corrections as well as non-perturbative corrections. This situation can be summarized by the following equations

$$W = W_{\text{tree}} + \delta W_{\text{non-pert.}}\,, \qquad K = K_0 + \delta K_{\text{pert.}} + \delta K_{\text{non-pert.}}\,, \tag{3.61}$$

where W_{tree} and K_{tree} refer to the respective potentials (3.50) and (3.49) of section 3.3.1.

We will now proceed under the assumption $\delta K_{\text{non-pert.}} \ll \delta K_{\text{pert.}}$ such that the non-perturbative corrections to the Kähler potential can be neglected. Under this assumption, we find that (3.61) induces the scalar potential V of the form [102, 234]

$$V = V_{\text{tree}} + V_{\delta K} + V_{\delta W} + \dots \tag{3.62}$$

where

$$V_{\text{tree}} \sim W_{\text{tree}}^2\,, \qquad V_{\delta K} \sim W_{\text{tree}}^2 \delta K_{\text{pert.}}\,, \qquad V_{\delta W} \sim \delta W_{\text{non.-pert.}}^2 + W_{\text{tree}} \delta W_{\text{non.-pert.}} \tag{3.63}$$

and we neglected higher-order corrections. Furthermore, we assume that all CS-moduli and the axio-dilaton have been stabilized by fluxes and including the above corrections yields only subleading corrections to the flux stabilisation mechanism. We thus use

$$\langle W_{\text{tree}} \rangle = W_0\,, \qquad \langle K_{CS} + K_\tau \rangle = \mathcal{K}_{CS} + \ln\left(\frac{g_s}{2}\right)\,, \tag{3.64}$$

where W_0, $\mathcal{K}_{CS}$, and g_s are constants determined by the fluxes. As a result of fixing these moduli, the tree-level scalar potential V_{tree} in (3.62) vanishes due to the no-scale property. In order to stabilise the remaining Kähler moduli we thus require that at least two terms from $V_{\delta K}$ and $V_{\delta W}$ compete such that a minimum for the Kähler moduli is generated. However, the expansion in (3.61) is genuinely a expansion in the string loop coupling g_s and α' where the latter is equivalent to an expansion in $1/\mathcal{V}$. This gives rise to the *Dine-Seiberg problem* [270]. The main reason for this is that we stabilise a modulus with corrections from an expansion whose expansion parameter is the modulus itself. In general, this will tend to stabilise the modulus at $\mathcal{O}(1)$ values and the perturbative expansion breaks down. To circumvent the Dine-Seiberg problem we need to invoke a hierarchy in the expansion coefficients which allows to stabilise the modulus at appropriate values such that the expansion itself remains under control.

There are two famous proposals how this can be achieved: the KKLT proposal [100] and the large volume scenario [101, 102]. We will focus on the latter of these but briefly comment on the KKLT proposal. In KKLT, the perturbative corrections $\delta K_{\text{pert.}}$ are considered to be subleading and a minimum of the potential is generated by the two terms of $V_{\delta W}$ in (3.63). To establish a minimum in the potential KKLT requires exponentially small values for W_0 such that W_0 becomes comparable to the non-perturbative corrections $\delta W_{\text{non.-pert.}}$, i.e. $W_0 \sim \delta W_{\text{non.-pert.}}$. In this case, the terms of $V_{\delta W}$ indeed give rise to minimum where the Kähler modulus is stabilised in regime of computational control. In addition, due to the exponentially small value for W_0 SUSY can be restored in the minimum.

In the LVS, we choose non-perturbative corrections in the superpotential and perturbative corrections to the Kähler potential to generate a minimum at generic values of W_0. The LVS requires at least two Kähler moduli: a "big" 4-cycle τ_b and a "small" blowup 4-cycle τ_s and $\mathcal{V}$ to be given by

$$\mathcal{V} = \tau_b^{3/2} - \lambda_s \tau_s^{3/2} \,, \tag{3.65}$$

where λ_s is an numerical coefficient determined by CY data. Recall, that we are in 4d Einstein frame and we have set $\sqrt{2\pi\alpha'} = 1$ such that all volumes are dimensionless and measured in string units. This form of the volume $\mathcal{V}$ represents the simplest example of the LVS which can be generalised to include more Kähler moduli. We will comment on this below, but refer to [234] for a detailed discussion. We are interested in solutions with $\mathcal{V} \gg 1$ and $\tau_b \gg \tau_s \gg 1$ such that we may use $\mathcal{V} \sim \tau_b^{3/2}$. The Kähler potential K, including the leading α' correction [101,271], reads

$$K = \mathcal{K}_{CS} + \ln\left(\frac{g_s}{2}\right) - 2\ln\left(\mathcal{V} + \frac{\xi}{2g_s^{3/2}}\right) , \tag{3.66}$$

where, we defined

$$\xi = -\frac{\zeta(3)\,\chi}{2\,(2\pi)^3} \,, \tag{3.67}$$

with χ referring to the CY Euler number. Including non-perturbative corrections to the superpotential we find W to be given by

$$W = W_0 + A_s e^{-a_s T_s} \,, \tag{3.68}$$

where the prefactors A_s are $\mathcal{O}(1)$ numbers depending on the CS moduli. The non-perturbative corrections are induced by instanton effects on the corresponding 4-cycle. These can for instance be generated by D3-branes wrapping the entire 4-cycle (E3-branes) where we find $a_s = 2\pi$ [272–282]. Alternatively, one can wrap a stack of N D7-branes on the respective 4-cycle. It can be argued that the D7-brane stack gauge theory will undergo confinement and introduce a gaugino condensate which generates a non-perturbative correction to W with $a_s = 2\pi/N$ [273, 276, 283–287]. Furthermore, since we assume $\tau_b \gg \tau_s$ we neglected non-perturbative corrections for τ_b. In the large volume limit $\mathcal{V} \gg 1$, the superpotential (3.68) and Kähler potential (3.66) yield the LVS scalar potential V (where the axions θ_i have already been stabilized) [101, 102]

$$V(\mathcal{V}, \tau_s) = \hat{V}\left(\mathcal{A}_s \frac{\sqrt{\tau_s} e^{-2a_s\tau_s}}{\mathcal{V}} - \mathcal{B}_s \frac{\tau_s e^{-a_s\tau_s}}{\mathcal{V}^2} + \frac{3}{4}\frac{\xi}{g_s^{3/2}\mathcal{V}^3}\right) , \tag{3.69}$$

where we included the correct normalization factor [102, 288] and defined

$$\hat{V} = \left(\frac{g_s e^{\mathcal{K}_{CS}}}{8\pi}\right) W_0^2 \,, \quad \mathcal{A}_s = \frac{8(a_s A_s)^2}{3W_0^2 \lambda_i} \,, \quad \mathcal{B}_s = 4\frac{a_s|A_s|}{W_0} \,, \tag{3.70}$$

Minimising the potential (3.69) wrt. τ_s and $\mathcal{V}$ we find the following vacuum solutions $\langle\tau_s\rangle$ and $\langle\mathcal{V}\rangle$

in the limit $a_s \tau_s \gg 1$

$$\langle \tau_s \rangle = \frac{1}{g_s} \left(\frac{\xi}{2\lambda_s} \right)^{2/3} , \quad \langle \mathcal{V} \rangle = \frac{3\lambda_s |W_0| \sqrt{\langle \tau_s \rangle} e^{a_s \langle \tau_s \rangle}}{4 a_s A_s} . \tag{3.71}$$

For the stabilisation mechanism to be consistent we require $\langle \tau_s \rangle > 1$ such that the corresponding cycle is larger than string size. To satisfy this constraint we either need to choose a CY with large Euler number χ or require a small string coupling g_s. The latter can be ensured through a suitable choice of fluxes [162]. Importantly, this result indicate that $\langle \mathcal{V} \rangle \approx e^{1/g_s}$, leading to an exponentially large volume in string units. Furthermore, this assures control over corrections from higher string loops suppressed by g_s and α' corrections suppressed in $1/\mathcal{V}$.

Inserting the values (3.71) for $\mathcal{V}$ and τ_s into the potential (3.69) reveals that one obtains an AdS minimum with

$$V(\langle \mathcal{V} \rangle, \langle \tau_s \rangle) = -\frac{3\lambda_s g_s \sqrt{\langle \tau_s \rangle} |W_0|^2}{8 a_s \langle \mathcal{V} \rangle^3} . \tag{3.72}$$

The AdS minimum may be uplifted to a Minkowski minimum by adding a positive term,

$$V_{\mathrm{up}}(\mathcal{V}) = \frac{\hat{V} \mathcal{D}}{\mathcal{V}^\gamma} , \tag{3.73}$$

to the potential in (3.69), such that $(V + V_{\mathrm{up}})\big|_{\mathrm{minimum}} = 0$. The coefficients $\mathcal{D}$ and γ depend on the specific uplifting mechanism and have to chosen appropriately, see e.g. [289] for a discussion. Famously, an uplifting potential can be generated by placing p anti-D3-branes in a strongly warped region [100]. The effects of warping are necessary to suppress the large positive anti-D3-brane contribution to the potential. Note, that the anti-D3-brane uplift has been challenged in this context [132–136]. However, several other uplifting mechanisms haven been proposed (see e.g. [103, 108, 110, 113–116, 120, 121, 136]).

Loop Corrections

We now want to briefly comment on generalisations of the LVS for including more Kähler moduli based on the discussion in [234]. In general, the form of the volume $\mathcal{V}$ can be extended to include an a priori unfixed number of blow up modes τ_a

$$\mathcal{V} = \tau_b^{3/2} - \sum_a \lambda_a \tau_a^{3/2} . \tag{3.74}$$

Models of this type are referred to as Swiss-cheese models because the blowup cycles can be imagined as small holes of the CY. In Swiss-cheese models, all blowup modes can be stabilised by non-perturbative effects analogously to τ_s as in (3.71). Yet, a deviation from a Swiss-cheese form of the volume in the small cycles, e.g. $\mathcal{V} \supset -(\tau_1 + \alpha \tau_2)^{3/2}$, will require more ingredients for the stabilisation of all moduli [234]. Regarding the "big" cycles, the standard LVS procedure only stabilises one of these as we have demonstrated above. In general, $\tau_b^{3/2}$ can be replaced by a function $\tilde{\mathcal{V}}$ of degree 3 in the 2-cycle moduli t^i and only one linear combination, $\mathcal{V}$, is stabilised in the LVS. All other moduli in $\tilde{\mathcal{V}}$ remain as flat directions in the LVS potential. Eventually, the remaining Kähler moduli are stabilised by loop corrections to the Kähler potential which come to dominate the potential for the Kähler moduli when the cycle shrink to small volume. This behaviour will be important for our discussion about "loop blowup inflation" in part III where we exploit the loop corrections to drive inflation. Thus the rest of this section will fix the notation we will use in part III.

The Kähler potential (3.66) is subject to string loop corrections [181]

$$\delta K_{\text{pert.}} = \delta K^{KK}_{\text{pert.}} + \delta K^{W}_{\text{pert.}}\,, \tag{3.75}$$

which are classified into Kaluza-Klein (KK) and winding (W) type corrections.[18] Based on explicit torus computations, Berg-Haack-Pajer (BHP) [183, 184] *conjecture* that the corrections on a Calabi-Yau have the following form

$$\delta K^{KK}_{\text{pert.}} \sim \sum_a C^{KK}_a \, \frac{g_s \mathcal{T}^a(t^i)}{\mathcal{V}}\,, \tag{3.76}$$

$$\delta K^{W}_{\text{pert.}} \sim \sum_a C^{W}_a \, \frac{1}{\mathcal{I}^a(t^i)\mathcal{V}}\,, \tag{3.77}$$

where the coefficients C^{KK}_a and C^W_a are unknown functions depending on the CS moduli but are expected to be suppressed by π factors (cf. the last paragraph on p. 41 of [186]). The functions $\mathcal{T}^a$ and $\mathcal{I}^a$ are homogeneous functions of degree 1 in the 2-cycle volumes t^i. Because $\delta K^{KK}_{\text{pert.}}$ is of degree -2 in the 2-cycle volumes, the correction in the scalar potential $\delta V^{KK}_{\text{pert.}}$ has an 'extended no-scale structure' [180, 182–184]. In the end, the leading corrections to the scalar potential $\delta V_{\text{pert.}}$ due to loop effects read [184, 185]

$$\delta V_{\text{pert.},t^i} = \left(\frac{g_s\, e^{\mathcal{K}_{CS}}}{8\pi}\right) \frac{W_0^2}{\mathcal{V}^2} \left((g_s C^{KK}_i)^2 K^{\text{tree}}_{ii} - 2\delta K^{W}_{\text{pert.}} \right)\,, \tag{3.78}$$

where we recall the tree-level Kähler potential, $K_{\text{tree}} = -2\ln\mathcal{V}$. Using a form of the volume $\mathcal{V}$ as in (3.74), we find to leading order

$$\delta V_{\text{pert.},\tau_i} \sim \left(\frac{g_s\, e^{\mathcal{K}_{CS}}}{8\pi}\right) \frac{W_0^2}{\mathcal{V}^3} \frac{c_{\text{loop}}}{\mathcal{V}^{1/3}} \left(\sqrt{\frac{\tau_b}{\tau_i}} + \mathcal{O}(\tau_i/\tau_b)\right)\,, \quad c_{\text{loop}} \sim \begin{cases} C^W_i \\ (g_s C^{KK}_i)^2 \end{cases}\,, \tag{3.79}$$

where we introduced the factor c_{loop} to remain agnostic about the origin of the loop corrections. Further, the expression in the brackets arises due to the unknown functions $\mathcal{T}^i$ and $\mathcal{I}^i$ which lead to corrections by ratios of 4-cycle volumes. We may capture this effect by replacing in (3.79)

$$\left(\sqrt{\frac{\tau_b}{\tau_i}} + \mathcal{O}(\tau_i/\tau_b)\right) \to f\left(\sqrt{\frac{\tau_j}{\tau_i}}\right)\,, \tag{3.80}$$

where in principle other ratios with τ_j can occur.

This should demonstrate that loop corrections induce a scalar potential at order $\mathcal{V}^{-10/3}$ which will stabilise the remaining Kähler moduli. Moreover, an explicit incorporation and application of loop corrections in the LVS has been demonstrated e.g. in [290–294].

[18] The label *KK- or winding-type* correction is historical and does not specify the origin of the correction but merely the *form* of the correction [186].

Part II.

Small Kinetic Mixing in String Theory

4. Introduction to Part II

In this part of the thesis, we want to discuss kinetic mixing in string theory where we will focus in particular on the relevant settings in type IIB string theory. To begin, in this chapter we explain the prerequisites that lead to a kinetic mixing term in the compactified 4d theory. First in sect. 4.1, we will discuss the open string loop diagram which is responsible for the mixing of gauge theories in string theory. Unfortunately, the full string loop analysis is technically challenging and therefore limited to toroidal background geometries. In the generic case one would have to evaluate the string loop diagram on Calabi-Yau geometries which, at the moment of writing this thesis, is impossible since no explicit Calabi-Yau metric is known. Alternatively, one turns to the effective 10d supergravity description of e.g. type IIB where kinetic mixing is reinterpreted as an exchange of 10d bulk fields between Dp-branes which naturally carry the gauge theories. This approach is less rigorous but applicable to more complicated situations where an exact string loop analysis is not feasible. The explicit kinetic mixing term becomes apparent after the compactification to 4d and integrating out the mediating fields. The 10d and 4d perspectives will be our method of choice throughout this thesis.

As we have explained in sect. 2.1, we require charged states such that kinetic mixing is physical. In sect. 4.2 we will therefore discuss how charged states arise in the stringy context of Dp-branes. In addition, we will advocate a minimal model which involves stacks of Dp-branes to include charged states. This section will introduce the last set of tools necessary to discuss kinetic mixing from the string theory perspective.

Besides these formal aspects, the experimental constraints for the value of the kinetic mixing parameter χ need to be satisfied. This requires us to find a way to appropriately suppress kinetic mixing. Four options where laid out in sect. 2.1.3: Small gauge couplings, embedding of the $\mathrm{U}(1)$ in a non-abelian gauge group, sequestering in higher dimensions and cancellation due to a global symmetry. In our applications in type IIB all of these options can be realized and often several of these effects come into play simultaneously.

In sect. 5.1 we will review the possibility of suppressing χ with small gauge couplings and argue for shortcomings and limitations of this approach. Ultimately, this will be an incentive to look at other approaches without the use of small values for the gauge couplings.

Further, if one or both $\mathrm{U}(1)$ gauge groups are embedded in a non-abelian gauge theory requires to spontaneously break the non-abelian gauge theory such that kinetic mixing becomes possible. This naturally leads to a suppression of kinetic mixing, see sect. 2.1.3. We already highlighted that sophisticated models involve stacks of Dp-branes which carry non-abelian gauge theories. Unless charges states are not realized otherwise, e.g. by Dp-branes at singular points of the Calabi-Yau, demands that this suppression from breaking the non-abelian gauge group occurs.

We then present the main method of choice in type IIB string theory, which relies on spatially separating the gauge theories in the 6d internal dimensions of a large-volume Calabi-Yau. A suppression of kinetic mixing will then arise simply due to the propagation of fields over a large distance in the internal dimensions. In sect. 5.2 we will exploit this effect and discuss a scenario where even an exponentially small value for kinetic mixing could potentially be realised.

Finally, it has also been known for a long time, that the simple suppression from sequestering can become even much stronger as several contributions to kinetic mixing can exactly cancel. In the Dp-Dp-brane case, an exact cancellation leads to vanishing mixing at leading order. Specifically for the mixing between two $D3$ branes, which will be our main focus in sect. 6, this exact cancellation occurs between the contributions of the bulk fields B_2 and C_2. Importantly, these bulk fields are related by the $\mathrm{SL}(2,\mathbb{R})$ symmetry of type IIB string theory. In sect. 6 we extend this leading order cancellation for D3-branes to all orders using the global $\mathrm{SL}(2,\mathbb{R})$ symmetry. This represents a very specific case where a symmetry is responsible for an exact cancellation. We then introduce 3-form fluxes to break $\mathrm{SL}(2,\mathbb{R})$ and lift the exact zero result for kinetic mixing. Using our findings, we derive some first phenomenological implications for the case of D3-brane kinetic mixing at the end of sect. 6.

In sect. 7, we apply the idea of sequestering also to scenarios involving D7-branes wrapped on 4-cycles in the Calabi-Yau. Specifically we consider stacks of D7-branes with a non-abelian gauge group which is broken by turning on suitable internal flux of the gauge theory. Also in this case, our analysis shows that a cancellation between the contributions of B_2 and C_2 occurs, which is again tied to the $\mathrm{SL}(2,\mathbb{R})$ symmetry. However, we will find a non-zero contribution mediated by C_4. Additionally, we argue that in this setting small kinetic mixing can be achieved by choosing a suitable geometry analogous to sect. 5.2.

4.1. Kinetic Mixing in String Theory

As we discussed in sect. 2, kinetic mixing has to be extremely small to avoid experimental detection and be of phenomenological interest. Typical values are as low as $\chi_{\mathrm{AB}} \sim 10^{-15}$. This has to be contrasted with the field-theoretic expectation that kinetic mixing is a 1-loop effect, cf. fig. 2.1. By the Completeness Conjecture [193, 194] or, more quantitatively [295, 296] by the Weak Gravity Conjecture [215], heavy states Φ charged under $U(1)_{(\mathrm{A})}$ and $U(1)_{(\mathrm{B})}$ should always be present. This leads to the estimate (2.5) for χ_{AB}, which we repeat here for convenience

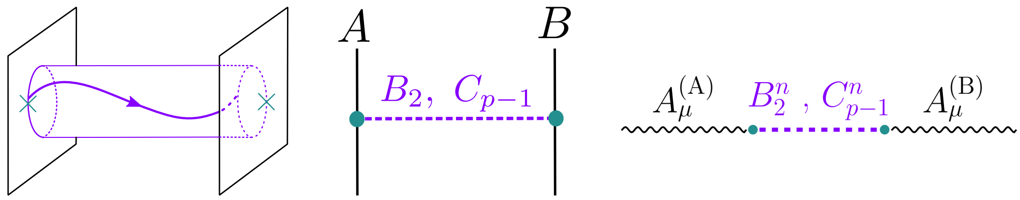
$$\chi_{\mathrm{AB}} \sim c_{\mathrm{loop}}\ g_{\mathrm{A}} g_{\mathrm{B}}\ \ln\left(\frac{\Lambda^2}{m_\Phi^2}\right). \tag{4.1}$$

As before, Λ refers to the EFT cutoff scale and $c_{\mathrm{loop}} \sim 1/16\pi^2$ denotes the loop suppression factor. Clearly, the suppression by c_{loop} is insufficient for phenomenological purposes. One needs either a tiny hidden gauge coupling, e.g. g_{B}, or an overwhelmingly precise cancellation between the loop effects of different charged states Φ.

String theory has the potential to produce such a precise cancellation as we will discuss in the following sections. This may be viewed as providing 'loopholes' in the generic field theory prediction stated above [296]. We will argue that some of the most natural and well-studied settings allow for such loopholes and predict surprisingly small mixing parameters χ_{AB}.

To set the scene, we want to focus exclusively on models in type IIB string theory based on orientifold compactifications with O3/O7 planes [97, 100–102], since these allow for sophisticated incorporations of the standard model [74, 230, 297–314]. However, for further references covering other scenarios of string theory model building see [315–333]. Therefore, we will only consider gauge theories living on D3-branes or D7-branes wrapped on 4-cycles.

It is crucial to identify the appropriate string diagram that produces the mixing of gauge theories in order to proceed with the discussion of kinetic mixing in string theory. Analogously to field theory, kinetic mixing in type IIB string theory is generated by states running in a loop, which are charged under the $\mathrm{U}(1)$ gauge groups of different Dp-branes. These states are given by an open string stretched between the Dp-branes which runs in a loop and forms a cylinder [190, 334], c.f. fig. 4.1a. This cylinder diagram has been computed for toroidal models



(a) (b) (c)

Fig. 4.1: Different perspectives on KM: (a) shows the string loop diagram; (b) is its reinterpretation as exchange of closed string fields in 10d supergravity; (c) symbolizes the corresponding 4d point-of-view, where the closed-string effect is encoded in a tower of KK modes (labeled by n) which mix with the gauge fields on the two branes.

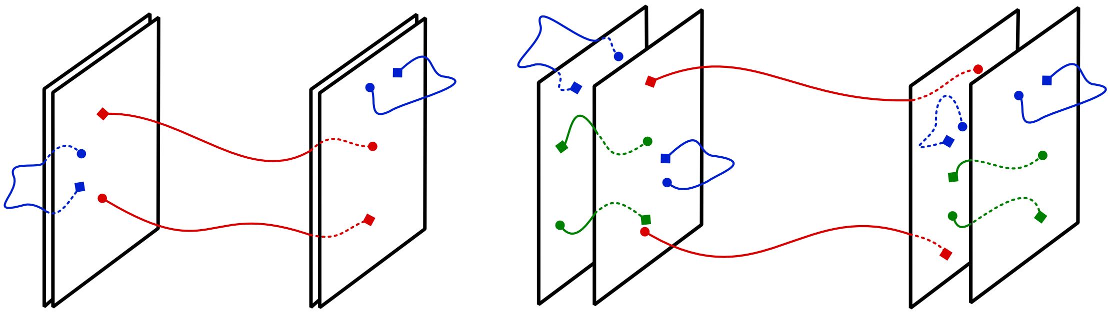
in [190, 334], but it is unclear to which extent these results apply to (orientifolded) Calabi-Yau geometries. However, at large brane separation, it is advantageous to appeal to open-closed string duality and reinterpret the cylinder as the tree-level exchange of massless 10d fields between the branes [209, 221, 335, 336], see fig. 4.1b. Ultimately, this exchange of 10d fields will induce kinetic mixing of the gauge theories living on the branes. Applying this perspective allows to compute kinetic mixing from the 10d supergravity EFT of type IIB string theory, also on more general geometrical backgrounds like orientifolded Calabi-Yau manifolds. Notably the relevant fields mediating kinetic mixing are the Kalb-Ramond field B_2 and the Ramond-Ramond fields C_{p-1}. This can be seen from the following parts of the Dp-brane action where both B_2 and C_{p-1} couple linearly to the gauge theory field strength tensor F_2

$$S_{Dp} \supset T_p \int_{Dp} -\frac{1}{2} e^{-\phi} F_2 \wedge \star F_2 + e^{-\phi} F_2 \wedge \star B_2 + C_{p-1} \wedge F_2 \,, \tag{4.2}$$

where $T_p = 2\pi(2\pi\sqrt{\alpha'})^{-(p+1)}$ denotes the brane tension and ϕ refers to the dilaton. Thus we may interpret F_2 as a source J_2 for B_2 and C_{p-1} and compute kinetic mixing as the interaction of two spatially separated sources. However, the explicit kinetic mixing term in the action can only arise after the compactification to 4d, since the kinetic mixing term would be non-local otherwise. Thus, in 4d, kinetic mixing becomes apparent upon integrating out the tower of massive Kaluza Klein (KK) states of the mediating bulk fields. For this procedure to be consistent, all fields that are integrated out must be massive. Thus we need to ensure that the zero modes of the fields B_2 and C_{p-1} are either projected out by the orientifold or decouple in the mediation process. In the following scenarios we will see that this is indeed the case. Further references to ideas in the context of kinetic mixing with extra dimensions, also outside of string theory, can be found in [337–345].

4.2. Charged States and Minimal Setup

Scenarios only involving single branes are not phenomenologically interesting since they contain no light charged states in their spectrum. These are however necessary to observe kinetic mixing. In the string picture, charged states arise from strings beginning on one brane but ending on a different brane. For example a string beginning on brane A and ending on brane B would be charged positively under $\mathrm{U}(1)_A$ but negatively under $\mathrm{U}(1)_B$. A string starting and ending on the same brane is not charged but gives rise to the gauge bosons confined to the Dp-brane surface. Only those strings that are stuck to an orientifold plane give rise to single charged states due to the orientation reversal of the string at the orientifold plane [74, 75]. Hence, the only charged states in a single Dp-brane setup are the states from the stretched and heavy strings between the branes, which give rise to kinetic mixing in the first place.



(a) (b)

Fig. 4.2: Sketch of the proposed minimal setup which should be considered for stringy realizations of kinetic mixing. First, we have two stacks of coincident branes in (a) where we do not get kinetic mixing due to the non-abelian gauge theory on the branes. Hence in a second step the gauge theory has to be broken to a subgroup containing a U(1) which can be mixed with. This is indicated in (b) by e.g. separating the branes. The charged states necessary to measure kinetic mixing arise from the strings between the branes in each stack coloured in green. Kinetic mixing is mediated by states associated to the strings stretched between the two stacks in red. The blue strings represent the gauge bosons on each brane.

Thus, a minimal setup should consist of two sectors including light and charged states, e.g. brane stacks or branes at singularities. Both sectors are geometrically separated in the internal manifold to realize the visible and hidden sector.

For the simplest setup we consider brane stacks which would contain two Dp-branes carrying a U(2) gauge symmetry. The light charged states are now present due to the strings beginning and ending on the different branes in each stack. The first step to arrange for kinetic mixing is to break U$(2) \to$ U$(1) \times$ U$(1)'$ in both stacks to obtain U(1)s for kinetic mixing. The light states in each sector are however charged under a particular linear combination of U(1) and U(1)', which we call the relative U$(1)^{(r)}$. Thus, kinetic mixing only between the two relative U$(1)^{(r)}$ is observable. Still, in the string picture, kinetic mixing will be induced by the strings stretched between the two brane stacks, see fig. 4.2. As an additional advantage, this automatically allows to suppress kinetic mixing in the sense of sect. 2.1.3, since the U(1)s are embedded in non-abelian gauge groups. A detailed analysis of the above idea can be found in app. D where we consider an unspecified visible sector and assume the hidden sector to be given by a stack of D3-branes, which are separated by a small distance.

In the rest of the thesis, we will focus on scenarios involving either single branes or stacks of branes. However, charges states can also be realised by locating Dp-branes at singular points in the Calabi-Yau [230, 298, 304, 306, 311]. A model involving D-branes at singularities including also a discussion of kinetic mixing can be found in [346].

5. Obtaining Small Kinetic Mixing in String Theory

5.1. Small Gauge Coupling

We now want to take the parametric estimate of (4.1) seriously and consider scenarios where the logarithmic factor is $\mathcal{O}(1)$, as one would naturally expect. Hence, we may simplify (4.1) to

$$\chi_{\mathrm{AB}} \sim \frac{g_{\mathrm{A}} g_{\mathrm{B}}}{16\pi^2} \,. \tag{5.1}$$

Thus, as discussed above in principle small values for kinetic mixing can be achieved by having tiny gauge couplings, see e.g. [347, 348].

Inspired by the weak gravity conjecture [215], we want to argue for a drawback of engineering small kinetic mixing in this way. The argument is based on the magnetic version of the weak gravity conjecture, which states that the cutoff Λ of a gauge theory coupled to gravity has to be lower or equal to the gauge coupling g times the 4d Planck mass M_{Pl}

$$\Lambda \lesssim g M_{\mathrm{Pl}} =: \Lambda_{\mathrm{WGC}} \,. \tag{5.2}$$

Therefore, the cutoff Λ decreases if we reduce any gauge coupling g to a small value. Similar arguments where made in [295] where the authors restricted to stringy models with D5/D9 setups only. The setup with D3/D7-branes was instead considered in [347] but without discussing the issues related to the weak gravity conjecture and the reducing cutoff.

The weak gravity conjecture does not specify the cutoff Λ and in practice the relevant cutoff of the theory is typically much lower then Λ_{WGC}. For example, if one chooses $\Lambda = 10\mathrm{TeV}$ as the upper scale of the experiments at LHC, we obtain a naive lower bound on kinetic mixing. Choosing the hidden gauge coupling to be g_{B} and using the smallest possible value for $g_{\mathrm{B}} \gtrsim \Lambda/M_{\mathrm{Pl}}$ due to the weak gravity conjecture (5.2) yields for χ_{AB}

$$\chi_{\mathrm{AB}} \gtrsim 2.6 \times 10^{-18} \left(\frac{g_{\mathrm{A}}}{0.1}\right)\left(\frac{\Lambda}{1\mathrm{TeV}}\right) \,. \tag{5.3}$$

On the other hand, since the two sectors are supposedly well hidden from each other, the minimal cutoff Λ in the hidden sector can in principle be as low as the hidden photon mass m_x if the hidden photon is massive. This would reduce the bound on χ_{AB} even further in the relevant parameter regions, see fig. 5.1. These naive lower bounds are of course very weak due to the small cutoffs used in (5.2).

Considering stringy settings the relevant cutoff has to be chosen more carefully. In string theories compactified to 4d, a crucial scale is given by the compactification scale M_{KK}, where the internal dimensions become visible and the 4d perspective breaks down. Therefore, the relevant cutoff should at least be M_{KK}, and we replace the cutoff Λ on the left-hand side of (5.2) by M_{KK}.

To manifest the expectation of a decreasing cutoff, we consider type IIB string theory including a $U(1)$ gauge theory of a Dp-brane. With a constant dilaton background, $e^{\phi} = g_s$, the 4d gauge coupling g_p and 4d Planck mass M_{Pl} are derived by dimensional reduction of the 10d theories defined on $\mathcal{M} = \mathcal{M}^{1,3} \times \mathcal{X}^6$, where for $p > 3$ the Dp-brane wraps a $(p-3)$-cycle Σ_{p-3} in $\mathcal{X}^6$. For this section we follow the notation of [102] and express lengths and volumes in string units $l_{\text{s}} = 2\pi\sqrt{\alpha'} = M_{\text{s}}^{-1}$. The 4d Planck mass M_{Pl} and 4d gauge coupling g_p are given by

$$M_{\text{Pl}}^2 = \frac{4\pi}{g_{\text{s}}^2}\mathcal{V}_s\, M_{\text{s}}^2 \,, \tag{5.4}$$

$$g_3^2 = 2\pi g_{\text{s}} \,, \quad \text{and} \quad g_p^2 = \frac{2\pi g_{\text{s}}}{\text{vol}(\Sigma_{p-3})} \quad \text{for } p > 3 \,, \tag{5.5}$$

where $\mathcal{V}_s$ denotes the dimensionless string frame volume of $\mathcal{X}^6$. We immediately see that the gauge coupling for D3-branes is essentially fixed and can only vary with g_{s}. For this reason D3-branes do not lead to small gauge couplings. The other branes wrap a cycle in $\mathcal{X}^6$. We may assume a simple form of the volume with different radii

$$\mathcal{V}_s = (2\pi R_{\parallel})^{p-3}(2\pi R_{\perp})^{9-p} \,, \tag{5.6}$$

where $R_{\perp}$ is perpendicular and $R_{\parallel}$ parallel to the Dp-brane This form of the volume yields for the compactification scale M_{KK}

$$M_{\text{KK}} = \min\left(1/R_{\parallel}\,,\ 1/R_{\perp}\right) M_{\text{s}} \,. \tag{5.7}$$

From (5.5) we see, that a reduction of the gauge coupling is achieved by increasing the size of the wrapped cycle, i.e. increasing $R_{\parallel}$. At the same time, this could lead to a decreasing compactification scale M_{KK}, which fits well with the expectation from the weak gravity conjecture.

Using (5.4), (5.5) and (5.7), we consider the ratio

$$\frac{M_{\text{KK}}}{\Lambda_{\text{WGC}}} \sim \frac{g_{\text{s}}^{1/2}\min\left(1/R_{\parallel}\,,\ 1/R_{\perp}\right)}{R_{\perp}^{(9-p)/2}} \,, \tag{5.8}$$

up to $\mathcal{O}(1)$ factors. We see that the largest ratio is achieved for the extreme case $R_{\perp} \sim 1$. Expressing the ratio in terms of the gauge coupling g_p while setting $R_{\perp} \sim 1$, one obtains

$$\frac{M_{\text{KK}}}{\Lambda_{\text{WGC}}} \sim \frac{g_{\text{s}}^{1/2}}{R_{\parallel}} \sim g_{\text{s}}^{(p-5)/(2p-6)}\, g_p^{2/(p-3)} \,. \tag{5.9}$$

In type IIB, supersymmetric brane setups are restricted to either contain D3/D7- or D5/D9-branes. Since the gauge coupling is suppressed by the volume o the wrapped cycle, we assume the hidden $U(1)$ to reside on D7 or D9 branes and specify (5.9) for these cases

$$\text{D7:} \quad M_{\text{KK}} \sim g_{\text{s}}^{1/4} g_7^{3/2} M_{\text{Pl}} \,, \tag{5.10}$$

$$\text{D9:} \quad M_{\text{KK}} \sim g_{\text{s}}^{1/3} g_9^{4/3} M_{\text{Pl}} \,. \tag{5.11}$$

From (5.9) we see that the weak gravity conjecture, even in this extreme case with $R_{\perp} \sim 1$, is generically satisfied and (5.10) and (5.11) affirm that M_{KK} decreases as we go to smaller gauge couplings g_p. We also see that using D7-branes has a stronger impact on M_{KK} when going to small gauge coupling.

It should also be noted here, that instead of D-brane $U(1)$s there are also $U(1)$s arising from the compactification of the metric which satisfy the weak gravity conjecture. For these metric $U(1)$s to appear it is necessary to have non-trivial 1-cycles in the internal geometry $\mathcal{X}^6$. Since in string theory we usually compactify on Calabi-Yau manifolds that do not contain non-trivial 1-cycles, we will not consider this type of $U(1)$s any further.

In order to relate these results to phenomenology we want to assess the results from different perspectives. The current bounds on kinetic mixing, c.f. fig. 2.2, can be translated to the hidden gauge coupling using (5.1)

$$g_{\mathrm{B}} = \chi_{\mathrm{AB}} \frac{16\pi^2}{g_{\mathrm{A}}} \approx 1.6 \times 10^{-12} \left(\frac{\chi_{\mathrm{AB}}}{10^{-15}} \right) \left(\frac{0.1}{g_{\mathrm{A}}} \right) . \tag{5.12}$$

Using this necessarily tiny hidden gauge coupling in (5.10) and (5.11), assuming $g_s = 0.1$ for perturbative control, yields the respective M_{KK} scale. The results for M_{KK} and the related radius R are given in tab. 5.1, where we fixed $\chi_{\mathrm{AB}} \sim 10^{-15}$ to explain the XENON1T excess [349]. The

Tab. 5.1: Results for the typical length R and associated M_{KK} scale needed to satisfy $\chi_{\mathrm{AB}} \sim 10^{-15}$.

$\chi_{\mathrm{AB}} \sim 10^{-15}$	D9	D7
M_{KK} $[M_{\mathrm{Pl}}]$	8.5×10^{-17}	1.1×10^{-18}
R [m]	9.6×10^{-19}	7.4×10^{-17}

necessary size R of the extra dimensions is still smaller then the experimental bounds listed in tab. 5.2, hence these scenarios are not ruled out experimentally, which was also the conclusion in [295]. The bounds on extra dimensions from tab. 5.2 can also be translated into lower bound on χ_{AB} if one solves (5.10) for the hidden gauge coupling [19]

$$\chi_{\mathrm{AB}} \gtrsim 3.6 \times 10^{-18} \left(\frac{g_A}{0.1} \right) \left(\frac{M_{\mathrm{KK}}}{2.4 \times 10^{-22}} \right)^{2/3} \left(\frac{0.1}{g_s} \right)^{1/6} . \tag{5.13}$$

Tab. 5.2: Experimental bounds on the size R of n extra dimensions and the respective M_{KK} scale. The bounds are taken from [350,351]. The D$(3+n)$-branes extend in d extra dimensions, hence the limits for $n = 6$ apply for the hidden sector on D9-branes and the limits for $n = 4$ apply for D7-branes

	Neutron Star Heating [350]		CMS [351]	
	$n = 6$	$n = 4$	$n = 6$	$n = 4$
M_{KK} $[M_{\mathrm{Pl}}]$	1.9×10^{-21}	2.4×10^{-22}	2.9×10^{-20}	1.3×10^{-22}
R [m]	4.4×10^{-14}	3.4×10^{-13}	2.9×10^{-15}	6.1×10^{-13}

On the other hand if these scenarios are to be implemented in type IIB, one should focus on D3/D7-branes and the LVS for getting large volumes. In the LVS, we face again constraints on

[19] The bound on χ_{AB} using D9-branes would also be $\sim 10^{-18}$.

the maximal size of the internal geometry that can be stabilized consistently. For the consistency of the LVS one should monitor $M_{\rm KK}$ and the mass $m_{\mathcal{V}}$ of the volume modulus, since the volume modulus is the lightest modulus and couples like gravity to all matter fields after Weyl rescaling to 4d Einstein frame. The $M_{\rm KK}$ scale is related to the volume in string units $\mathcal{V}_s$ by [102]

$$M_{\rm KK} \sim \frac{g_{\rm s}}{\mathcal{V}_s^{2/3}} M_{\rm Pl} \,, \tag{5.14}$$

and the mass $m_{\mathcal{V}}$ of the volume modulus is given by [102]

$$m_{\mathcal{V}} \sim \frac{g_{\rm s}^2 W_0}{\mathcal{V}_s^{3/2}} M_{\rm Pl} \,. \tag{5.15}$$

The mass of the volume modulus should obey $m_{\mathcal{V}} \gtrsim 10^{-30} M_{\rm Pl}$ to evade fifth force constraints [98]. This bound on $m_{\mathcal{V}}$ can be used set a lower bound on $M_{\rm KK}$ using (5.14) and (5.15)

$$M_{\rm KK} \gtrsim 3.6 \times 10^{-14} \left(\frac{g_{\rm s}}{0.1}\right)^{1/9} \left(\frac{W_0}{1}\right)^{-4/9} \left(\frac{m_{\mathcal{V}}}{M_{\rm Pl}} \times \frac{1}{10^{-30}}\right)^{4/9} M_{\rm Pl} \,. \tag{5.16}$$

The bound (5.16) is not in accordance with the values listed in tab. 5.1 which are necessary to achieve $\chi_{\rm AB} \sim 10^{-15}$. The required $M_{\rm KK}$ scales from tab. 5.1 are too small, i.e. the radii R are too big. Hence, we can already see that we can obtain a minimum value for $\chi_{\rm AB}$. Focusing on phenomenological more established D3/D7 setups, we can compare (5.10) with (5.16) and replace the hidden gauge coupling g_7 using (5.12) to transfer the bound on $m_{\mathcal{V}}$ also to $\chi_{\rm AB}$

$$\chi_{\rm AB} \gtrsim 10^{-12} \left(\frac{g_{\rm s}}{0.1}\right)^{-5/54} \left(\frac{W_0}{1}\right)^{-8/27} \left(\frac{m_{\mathcal{V}}}{M_{\rm Pl}} \times \frac{1}{10^{-30}}\right)^{8/27} \left(\frac{g_{\rm A}}{0.1}\right) \,. \tag{5.17}$$

This leads us to the conclusion that tuning the gauge couplings in (5.1) to be tiny, values of $\chi_{\rm AB} < 10^{-12}$ are not achievable in the LVS, since the volume modulus would be too light for the required volume to suppress the hidden gauge coupling.

The situation gets even worse because tighter constraints on $m_{\mathcal{V}}$ may be derived from cosmology. To avoid changing the element abundances by energy injection during Big Bang nucleosynthesis (BBN) the volume modulus should decay well before this time [352–354]. In addition, the modulus decays should not deposit a significant amount of energy into its own, ultra-light axion in order to avoid an excess of dark radiation [267, 293, 355–365]. Both problems can be circumvented if the volume modulus can decay efficiently into SM Higges [365]. This, in turn, requires the mass $m_{\mathcal{V}}$ to be large enough, $m_{\mathcal{V}} \gtrsim 2m_H$, where m_H refers to the Higgs mass. Using this bound implies for $\chi_{\rm AB}$

$$\chi_{\rm AB} \gtrsim 1.4 \times 10^{-8} \left(\frac{g_{\rm s}}{0.1}\right)^{-5/54} \left(\frac{W_0}{1}\right)^{-8/27} \left(\frac{m_{\mathcal{V}}}{M_{\rm Pl}} \times \frac{M_{\rm Pl}}{2m_H}\right)^{8/27} \left(\frac{g_{\rm A}}{0.1}\right) \,. \tag{5.18}$$

The different lower bounds on $\chi_{\rm AB}$ are plotted in fig. 5.1. The naive bounds on $\chi_{\rm AB}$ (5.3) using the weak gravity conjecture exclude all values of $\chi_{\rm AB} \lesssim 10^{-20}$. These bounds arise already from a field theoretic perspective and do not imply any string theoretic input. However, assuming that the gauge theories live on D-branes, the respective gauge coupling can be tuned to small values by increasing the wrapped volume of the D-brane. The smallest value of $\chi_{\rm AB}$ achievable this way, (5.13), excludes $\chi_{\rm AB} \lesssim 10^{-18}$ while respecting the experimental bounds on extra dimensions from tab. 5.2. Considering specifically type IIB string theory in the LVS constrains the maximal volume

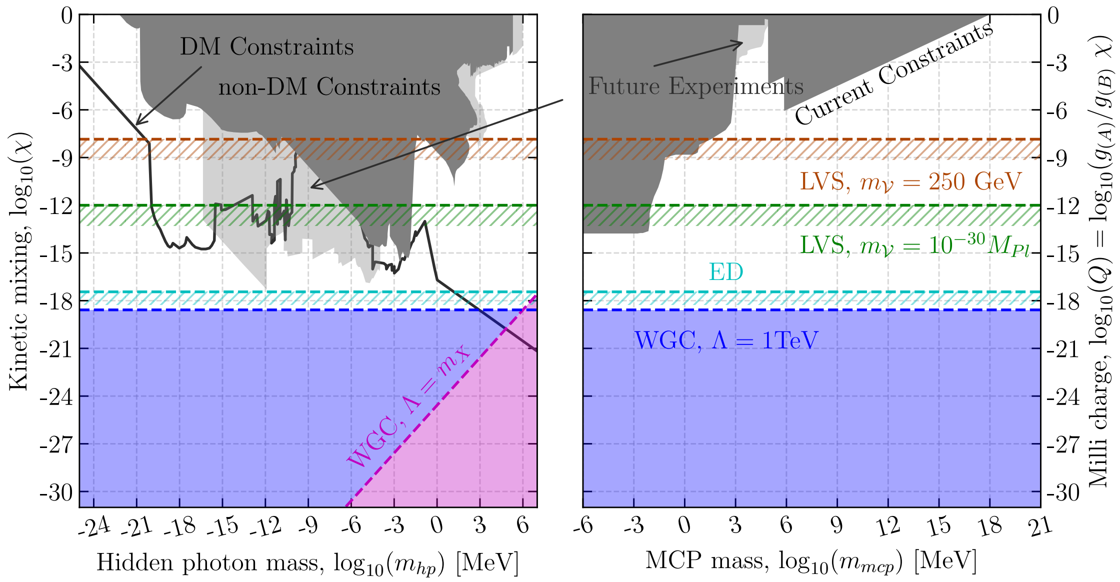


Fig. 5.1: We adapted the plot from [204] where the experimentally excluded regions are given in gray. For detailed references on the excluded regions we refer back to fig. 2.2 and the text below fig. 2.2. The lower bounds due to the WGC (5.3) are given in magenta and blue. Considering type IIB with D-branes gives a bound on χ_{AB} (5.13) due to the maximal size of extra dimensions (ED) given by experiments, plotted in cyan. Specifying to the LVS pushes the bound even further since the maximal allowed volume is constrained by the volume modulus mass, see (5.17) plotted in green and (5.18) in orange.

even further, due to constraints of the volume modulus mass, see (5.17) and (5.18). Respecting these constraints excludes values of $\chi_{AB} \lesssim 10^{-12}$ and $\chi_{AB} \lesssim 1.4 \times 10^{-8}$.

Hence it should be clear, that it is necessary to find other approaches yielding small values for χ_{AB} without reducing the hidden gauge coupling. Possible solutions are discussed in the following chapters.

5.2. Sequestering and Fibred Geometry

As we have just discussed in sect. 5.1, realizing small kinetic mixing by tuning the gauge couplings to tiny values is limited and runs into phenomenological constraints. To explain small kinetic mixing we therefore require other approaches while having $\mathcal{O}(1)$ gauge couplings. As explained in sect. 4.1, from the 10d perspective, an obvious solution is to spatially separate the D-branes over a wide distance in the internal space. Analogous to sect. 2.1.3 we hence expect small kinetic mixing due to a suppression of the propagator of the mediating bulk fields. The suppression will even be exponential if the mediating fields are massive. Naively in 10d the mediating fields are B_2 and C_{p-1} as already mentioned. These fields are of course massless. Still it is possible to arrange for mediation solely by massive fields. This can be realised in a fibred Calabi-Yau where we have a 2D base and a 4d fibre (e.g. $\mathbb{CP}^1$ or T^2 as a 2d base, and K3 or T^4 as 4d fibre, see [290] for more details on the geometry) or with reversed fibre and base, see fig. 5.2 for a sketch. Choosing the length scale of the fibre l_F to be smaller then the length scale of the base l_B allows to compactify the fibre in a first step. This yields a theory in 6d or 8d (depending on the dimension of the fibre) where we keep all KK modes. The branes under consideration can then be arranged to be geometrically separated in the lower dimensional theory (in 6d or 8d) and the mediating fields from the lower dimensional perspective are given by the KK modes. If the zero modes can be projected out by orientifolding, the mediation of kinetic mixing is now due to massive fields only. Hence we expect kinetic mixing to scale (as explained in sect. 2.1.3)

$$\chi \sim e^{-m\,|y^{(1)}-y^{(2)}|}\,, \tag{5.19}$$

where m refers to the mass of the propagating mode and $y^{(i)}$ refer to the different positions of the branes in 6d (8d).

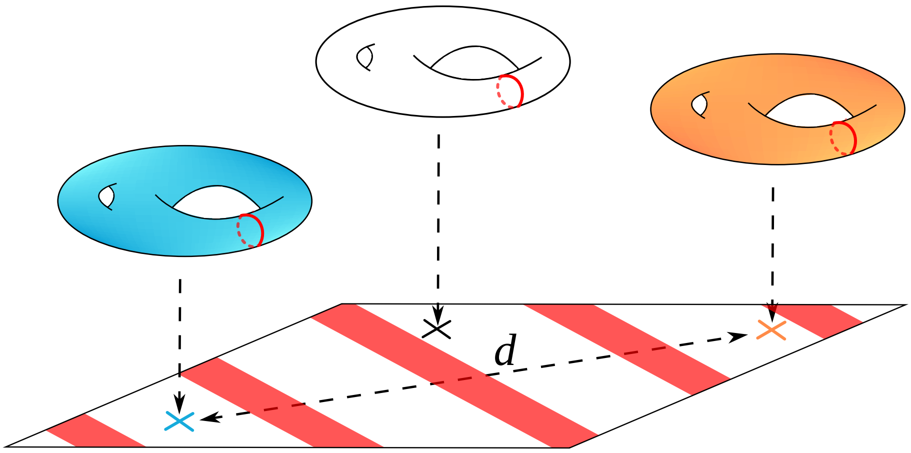


Fig. 5.2: Sketch of a fibred geometry allowing for an exponential suppression of kinetic mixing. D7-branes (blue and orange) wrap the 4d fibre and are separated over a distance d in the 2D base. For consistency we include a O7-plane (red) filling the 2D base and wrapping a 2-cycle in the fibre.

Specifying this idea to O3/O7 orientifolds, consider D7-branes wrapped on the 4d fibre. Compactifying to 6d leaves the D7-branes as points in the 2D base which we separate over a large distance $d = |y^{(1)} - y^{(2)}|$. The KK modes in 6d have a mass of $\sim 1/l_{\mathrm{F}}$, which becomes relevant when propagating from one brane stack to the other, since the base can be significantly larger then the fibre. Including an O7 plane wrapping the base and a 2 cycle of the fibre for consistency, may project out all massless KK modes of of the mediating fields, e.g. of B_2 and C_2, see fig. 5.2 for a sketch. This way we can expect to achieve exponentially suppressed kinetic mixing of the order

$$\chi \sim \exp\left(-\frac{l_{\mathrm{B}}}{l_{\mathrm{F}}}\right)\,, \tag{5.20}$$

where we inserted as the maximal distance $|y^{(1)} - y^{(2)}| = l_{\mathrm{B}}$ between the branes. We will give further details of a sophisticated scenario of this setup in sect. 7.

Similarly, D3-stacks would be represented as points already in the full internal geometry and hence the same reasoning from above should apply to stacks of D3-branes. As was already noted, in the case of D3-branes an exact cancellation between the mediating fields occurs due to the underlying $\mathrm{SL}(2,\mathbb{R})$ structure. Introducing $\mathrm{SL}(2,\mathbb{R})$ breaking fluxes in sect. 6 will yield kinetic mixing which is suppressed by the above sequestering effect and the diluteness of the fluxes.

Unfortunately, this idea comes with a caveat which we can not circumvent. Since in this stepwise compactification, one ends up in a SUGRA theory with dimension d between $4 < d < 10$, there is always either a massless 2-form or vector contained in the gravition multiplet. Thus, there is always a light field present which can couple to F_2 and mediate kinetic mixing. Consequently, exponential suppression, as we have just proposed, is only achievable upon breaking SUSY in the higher dimensional theory, at the cost of losing computational control over the compactification. Further investigations are necessary to clarify this point.

6. D3-Brane Kinetic Mixing

Our focus in this chapter is on kinetic mixing of D3-brane gauge theories. In this case, the relevant fields mediating kinetic mixing are the Kalb-Ramond field B_2 and the Ramond-Ramond field C_2. Crucially, in the phenomenologically most interesting case of O3/O7 orientifold models, B_2/C_2 have no KK zero modes such that integrating out the KK tower gives local 4d mixing terms as in (2.4). The goal of this chapter is to better understand the mysterious but phenomenologically very important cancellation between the B_2 and C_2 kinetic mixing contributions. We go beyond a leading-order analysis, aiming at either the proof of an all-orders zero result or at the identification of the leading, non-zero mixing effect. We start in sect. 6.1 by introducing our notation and presenting the key formulae for an $\mathrm{SL}(2,\mathbb{R})$-covariant treatment of the D3-brane action. In sect. 6.2, we rederive the leading-order cancellation from [221] in a way suitable for the following generalizations. Next, in sect. 6.3, we extend this analysis to include two key subleading effects: A non-zero background value of C_0 as well as the self-couplings and mixing of B_2 and C_2 in the D3-brane action. On the basis of a calculation which crucially relies on the $\mathrm{SL}(2,\mathbb{R})$ structure, we find that the exact cancellation persists. An essential prerequisite for this non-trivial result is the use of the correct and complete D3-brane action. In particular, a key B_2-C_2 coupling term on D3-branes, which surprisingly is missing in standard textbooks [71, 73, 75], must be included to find the zero result. We devote App. A to explaining the origin of this term in detail. In sect. 6.4, we include H_3/F_3 fluxes, breaking $\mathrm{SL}(2,\mathbb{R})$ and inducing different masses for B_2 and C_2 as well as a mixing with C_4. This generically destroys the cancellation and leads to our desired non-zero mixing result, which is however suppressed in the large volume limit by both sequestering and by the diluteness of the 3-form flux. Since our calculation implements the $\mathrm{SL}(2,\mathbb{R})$ structure and hence electric-magnetic duality on D3-branes, we also obtain an explicit expression for the magnetic mixing. This complements magnetic mixing results in the non-sequestered strong-coupling regime discussed in [366, 367]. We use our findings to derive some first phenomenological implications in sect. 6.5.

6.1. $\mathrm{SL}(2,\mathbb{R})$ Structure of type IIB and D3-Branes

We consider type IIB supergravity with D3-branes and start by clarifying its $\mathrm{SL}(2,\mathbb{R})$ symmetry. The 10d bulk action reads [237, 238][20]

$$
\begin{aligned}
S_{\mathrm{IIB}} = \frac{1}{2\kappa_{10}^2}\int_{\mathcal{M}^{10}} \mathrm{d}^{10}x\sqrt{-G_E}\left(R_E - \frac{\partial_N\bar\tau\partial^N\tau}{2(\mathrm{Im}\,\tau)^2}\right) \\
+ \frac{1}{2\kappa_{10}^2}\int_{\mathcal{M}^{10}}\left(-\frac{\hat M_{ij}}{2}F_3^i\wedge\star_{10}F_3^j - \frac{1}{4}\tilde F_5\wedge\star_{10}\tilde F_5 - \frac{\epsilon_{ij}}{4}C_4\wedge F_3^i\wedge F_3^j\right),
\end{aligned}
\tag{6.1}
$$

[20] We chose the conventions $\epsilon^{12} = \epsilon_{21} = 1$ and $\epsilon_{12} = \epsilon^{21} = -1$.

with N a 10d index and $2\kappa_{10}^2 = (2\pi)^7\alpha'^4$. The indices i, j label the two 3-form field strengths

$$F_3^i = \mathrm{d}C_2^i = \begin{pmatrix} \mathrm{d}C_2 \\ \mathrm{d}B_2 \end{pmatrix} . \tag{6.2}$$

We also define

$$\tau = C_0 + ie^{-\phi} , \tag{6.3}$$

$$\hat{M}_{ij} = \frac{1}{\mathrm{Im}\,\tau} \begin{pmatrix} 1 & -\mathrm{Re}\,\tau \\ -\mathrm{Re}\,\tau & |\tau|^2 \end{pmatrix} , \tag{6.4}$$

$$\tilde{F}_5 = \mathrm{d}C_4 + \frac{\epsilon_{ij}}{2} C_2^i \wedge F_3^j . \tag{6.5}$$

The action (6.1) is invariant under the gauge transformations

$$\delta C_2^i = \mathrm{d}\Lambda_1^i , \tag{6.6}$$

$$\delta C_4 = \mathrm{d}\Lambda_3 - \frac{\epsilon_{ij}}{2} \Lambda_1^i \wedge F_3^j \tag{6.7}$$

as well as under the global $\mathrm{SL}(2,\mathbb{R})$ transformations

$$\tau' = \frac{a\tau + b}{c\tau + d} , \qquad \Lambda^i{}_j = \begin{pmatrix} a & b \\ c & d \end{pmatrix} , \tag{6.8}$$

$$C_2'^i = \Lambda^i{}_j C_2^j , \qquad \hat{M}' = (\Lambda^{-1})^T \hat{M} \Lambda^{-1} . \tag{6.9}$$

The form C_4 and the Einstein-frame metric G_E do not transform.

To quadratic order in the gauge field strength F_2, the Einstein-frame action for a D3-brane reads [85, 89, 239–251]

$$S_{D3} = S_{\mathrm{DBI}} + S_{\mathrm{WZ}} , \tag{6.10}$$

$$S_{\mathrm{DBI}} = -T_3 \int_{D3} \frac{e^{-\phi}}{2} (F_2 - B_2) \wedge \star_4 (F_2 - B_2) , \tag{6.11}$$

$$S_{\mathrm{WZ}} = T_3 \int_{D3} C_4 + \frac{1}{2} B_2 \wedge C_2 + C_2 \wedge (F_2 - B_2) + \frac{C_0}{2} (F_2 - B_2) \wedge (F_2 - B_2) , \tag{6.12}$$

with the brane tension given by $T_p = 2\pi\, (2\pi\sqrt{\alpha'})^{-(p+1)}$. While, for our purposes, this action is simply dictated by string theory, we want to highlight a recent discussion on generalizations of similar actions in field theory in the context of axion physics [368, 369].

We emphasise that it will be crucial for us to work with the correct and complete D3-brane action. As noted in the introduction, the second term on the r.h.s. of (6.12) is missing in several standard textbooks [71, 73, 75]. However, this B_2-C_2 coupling is essential for gauge invariance and $\mathrm{SL}(2,\mathbb{R})$ self-duality of the D3-brane. It can be found by going back to the original literature [76, 237, 238, 244, 246, 370–374]. We comment on this term in more detail in app. A.

The brane action (6.10) is invariant under the gauge transformations (6.6) and (6.7) together with the transformation of the gauge field strength

$$\delta F_2 = d\Lambda_1^{(2)} \ . \tag{6.13}$$

To study $\mathrm{SL}(2,\mathbb{R})$ transformation properties, we need to specify the transformation of the gauge field strength F_2. Similarly to B_2, the field strength F_2 transforms as part of a doublet which it forms together with its dual field strength G_2 [246, 371],

$$\begin{pmatrix} G_2' \\ F_2' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} G_2 \\ F_2 \end{pmatrix} \ . \tag{6.14}$$

Formally, the dual field strength G_2 is defined by [375, 376][21]

$$\delta \mathcal{L} = \delta F_2 \wedge G_2 \ , \tag{6.15}$$

which for the D3-brane explicitly yields

$$G_2 = -e^{-\phi} \star_4 (F_2 - B_2) + C_0(F_2 - B_2) + C_2 \ . \tag{6.16}$$

Contrary to widespread beliefs, the D3-brane action is not $\mathrm{SL}(2,\mathbb{R})$ invariant. This can be seen by performing an infinitesimal $\mathrm{SL}(2,\mathbb{R})$ transformation defined by

$$\delta \Lambda^i{}_j = \begin{pmatrix} \alpha & \beta \\ \gamma & -\alpha \end{pmatrix} \ . \tag{6.17}$$

It produces the following change of the D3-brane action,

$$\delta_{SL} S_{D3} = \int_{D3} \frac{\beta}{2} F_2 \wedge F_2 + \frac{\gamma}{2} G_2 \wedge G_2 \ , \tag{6.18}$$

where the second term is not a total derivative. This is in agreement with the general treatment of self-duality in [375, 376]: Although the action is not invariant, the equations of motion of bulk and brane fields are invariant under $\mathrm{SL}(2,\mathbb{R})$ transformations, i.e. the theory is $\mathrm{SL}(2,\mathbb{R})$ self-dual in the spirit of [375, 376]. On the other hand, when restricting ourselves to $\mathrm{SL}(2,\mathbb{Z})$ transformations generated by

$$\mathcal{I} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \ , \quad \mathcal{T} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \ , \tag{6.19}$$

the action is in fact invariant under $\mathcal{T}$ and mapped to its dual theory by $\mathcal{I}$. This mapping is a specific instance [246, 371] of the general concept [375, 376] of mapping to a dual theory. It produces a dual Lagrangian $\mathcal{L}_{dual} = \mathcal{L}_{dual}(G_2, \tilde{g})$, where $\tilde{g}$ is the dual coupling. In our case, the theory is said to be self-dual because the relation

$$\mathcal{L}_{dual}(G_2, \tilde{g}) \equiv \mathcal{L}(F_2 = G_2, g = \tilde{g}) \tag{6.20}$$

holds.

Although $\mathrm{SL}(2,\mathbb{R})$ is not a symmetry, it is convenient to rewrite the D3-brane action (6.10)

[21] We use a different sign convention for G_2 compared to [375, 376].

using $\mathrm{SL}(2,\mathbb{R})$ indices, in a manner similar to (6.1)[22]. At quadratic order we have

$$S_{D3} = T_3 \int_{D3} C_4 + \frac{1}{2} J_{(1)} \wedge J_{(2)} - \frac{1}{2} C_2^i \wedge \star_4 \hat{m}_{ij} C_2^j + C_2^i \wedge \star_4 J_i \,, \tag{6.21}$$

where we defined

$$J_{(1)} = - \star_4 F_2 \,, \quad J_{(2)} = g_s^{-1} F_2 + C_0 \star_4 F_2 \,, \tag{6.22}$$

$$\hat{m}_{ij} = \begin{pmatrix} 0 & -\frac{1}{2}\star_4 \\ -\frac{1}{2}\star_4 & g_s^{-1} + C_0 \star_4 \end{pmatrix} . \tag{6.23}$$

Note that J_i and $\hat{m}_{ij}$ do not transform as a vector and tensor under the full $\mathrm{SL}(2,\mathbb{R})$, but only under the (Borel) subgroup $\mathfrak{b}$ generated by

$$\delta \Lambda^i{}_j = \begin{pmatrix} \alpha & \beta \\ 0 & -\alpha \end{pmatrix} . \tag{6.24}$$

This is a proper, 2-parameter symmetry group of S_{D3}, cf. (6.18).

6.2. Leading Order Cancellation

To begin we repeat the calculation of [221], showing that kinetic mixing vanishes in the D3 scenario at leading order and without fluxes. We consider a generic CY-orientifold with O3/O7-planes, thereby projecting out the massless modes of B_2 and C_2. This allows us to integrate out these fields by solving their equation of motion.

Following [221], we set $C_0 = C_4 = 0$ and consider constant $e^\phi = g_s$. In doing so, the matrix $\hat{M}$ from (6.4) becomes diagonal. This implies that there is no mixing between C_2 and B_2 in the bulk action (6.1). Moreover, we neglect $\hat{m}$, i.e. all self couplings of B_2 and C_2 on the D3-branes. This reduces the relevant parts in the actions (6.1) and (6.21) to

$$S = \frac{1}{2\kappa_{10}^2} \int_{\mathcal{M}^{10}} -\frac{\hat{M}_{ij}}{2} F_3^i \wedge \star_{10} F_3^j + T_3 \int_{\mathcal{M}^{10}} C_2^i \wedge \left[\star_4 J_i \wedge \delta_6(A) + \star_4 J_i \wedge \delta_6(B) \right] . \tag{6.25}$$

Here $\delta_6(A/B) = \delta_6(y - y_{A/B})$ are 6-form delta functions localized at the positions of brane A and brane B in the compact space parameterized by y. Furthermore, the sources J_i simplify to

$$J_{(1)} = - \star_4 F_2 \,, \quad J_{(2)} = g_s^{-1} F_2 \,. \tag{6.26}$$

The pairing of J_i with the delta distributions fixes whether we have to use $F_2^{(A)}$ or $F_2^{(B)}$ in this definition. In order to not clutter the notation, we therefore did not give J_i labels A/B and will also drop these labels on F_2 for now. We will restore them whenever we think it is useful for clarity. The equations of motion for C_2^i following from (6.25) now read

$$\mathrm{d}^\dagger \mathrm{d} C_2^i = 2\kappa_{10}^2 T_3 \left((\hat{M}^{-1})^{ij} J_j \, \delta(A) + (\hat{M}^{-1})^{ij} J_j \, \delta(B) \right) , \tag{6.27}$$

[22] However, [375] state a general formula for a $\mathrm{SL}(2,\mathbb{R})$ self-dual action, if the Lagrangian depends at most quadratically on F_2. Indeed, in app. B we show that the D3-brane action, up to second order in F_2, can be cast into the specific form dictated by [375].

with scalar distributions for the brane positions $\delta(A/B) = \star_6 \delta_6(A/B)$. For the sake of clarity, we will absorb the factor $2\kappa_{10}^2 T_3$ into the six-dimensional δ-function and Laplace operator, respectively:

$$2\kappa_{10}^2 T_3 \; \delta(y - y_{A/B}) =: \underline{\delta}(y - y_{A/B}) \;, \quad 2\kappa_{10}^2 T_3 \; \Delta_6^{-1} =: \underline{\Delta}_6^{-1} \;, \tag{6.28}$$

This defines underlined quantities. Again, we will reintroduce the factor $2\kappa_{10}^2 T_3$ only when helpful. It is obvious from the above equation of motion that only B_2 and C_2 fields with 4d indices are sourced. Thus we treat the 2-forms C_2^i as scalars on the 6d internal manifold $\mathcal{X}^6$. Fixing the gauge by

$$\mathrm{d}^\dagger C_2^i = 0 \;, \tag{6.29}$$

simplifies

$$\Box_{10} \equiv \mathrm{d}^\dagger \mathrm{d} + \mathrm{d}\, \mathrm{d}^\dagger = \mathrm{d}^\dagger \mathrm{d} \;, \tag{6.30}$$

when acting on C_2^i. Since the length scales on which our sources vary in 4d are much larger than the size of the compact space, we may neglect 4d derivatives w.r.t. 6d derivatives. This implies

$$\Box_{10} C_2^i = \Delta_6 C_2^i \;. \tag{6.31}$$

Under this assumption, the solutions to (6.27) are simply given in terms of the Green's function[23] Δ_6^{-1} on the internal manifold $\mathcal{X}^6$

$$C_2^i(x,y) = -\underline{\Delta}_6^{-1}(y, y_A)(\hat{M}^{-1})^{ij} J_j(x) - \underline{\Delta}_6^{-1}(y, y_B)(\hat{M}^{-1})^{ij} J_j(x) \;, \tag{6.32}$$

where x and y refer to 4d and 6d coordinates respectively. Replacing C_2^i in (6.25) with this solution, i.e. integrating out C_2^i, we obtain an effective action which contains the mixing terms

$$S \supset \frac{T_3}{2} \int\limits_{\mathcal{M}^{1,3}} \underline{\Delta}_6^{-1}(y_A, y_B) \left[J_i^{(A)} \wedge \star_4 \, (\hat{M}^{-1})^{ij} J_j^{(B)} + J_i^{(B)} \wedge \star_4 \, (\hat{M}^{-1})^{ij} J_j^{(A)} \right] \;. \tag{6.33}$$

Here we reinstated the brane labels and made use of the symmetry of the Green's function, which can always be imposed on compact Riemannian manifolds. Now one observes that

$$(\hat{M}^{-1})^{ij} J_j(x) = - \star_4 \epsilon^{ij} J_j \;, \tag{6.34}$$

as can be checked explicitly. This relation implies that the square bracket in (6.33) takes the form

$$\epsilon^{ij} \left(J_i^{(A)} \wedge J_j^{(B)} + J_i^{(B)} \wedge J_j^{(A)} \right) = 0 \,, \tag{6.35}$$

vanishing by (anti-)symmetry of the two factors. Thus, the findings of [221] on tori and of [190, 334] on Calabi-Yaus are reproduced. However, this result hinges on the simplifications made at the beginning of this section by setting $C_0 = 0$ and disregarding brane-localized mixing terms between B_2 and C_2. When we drop these simplifications in the next section, it will turn out to be useful that we formulated our analysis in such a way that the crucial cancellation follows from the $\mathrm{SL}(2,\mathbb{R})$ index structure. We note for completeness that one may equivalently ascribe the zero result to a cancellation between the effects of B_2 and C_2 exchange between the branes, cf. fig. 6.1. The exchange of B_2 gives a term proportional to $F^{(A)}_{\mu\nu} F^{\mu\nu}_{(B)}$, while the exchange of C_2 gives $\tilde{F}^{(A)}_{\mu\nu} \tilde{F}^{\mu\nu}_{(B)} = -F^{(A)}_{\mu\nu} F^{\mu\nu}_{(B)}$.

[23]Note, that the total charge on the orientifold vanishes and thus allows to define a Green's function in the "upstaris" geometry before orientifolding.

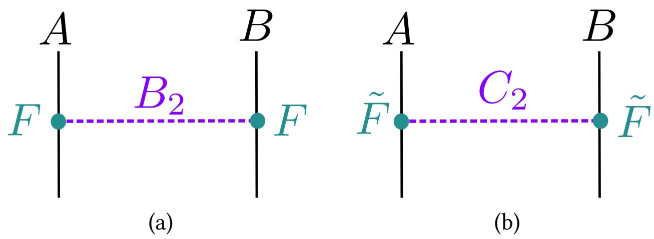


Fig. 6.1: Leading-order diagrams relevant for KM: B_2 exchange (a) and C_2 exchange (b).

6.3. General Cancellation without Fluxes

In flux compactifications, C_0 is generically stabilized at a non-zero $\mathcal{O}(1)$ value [97]. This will introduce mixing of B_2 and C_2 through the matrix $\hat{M}$ – a leading-order effect which has to be taken into account. Moreover, there exist brane-localized mixing terms between B_2 and C_2, encoded in the matrix $\hat{m}$. As we will discuss in app. C, they lead to UV-divergent diagrams, such that these mixing terms are a priori important.

Therefore, we will now extend the analysis of sect. 6.2 (and hence of [221]) by including these generalizations. All we have to do is to repeat the calculation in sect. 6.2 with the complete expression for $\hat{M}$ and with the full D3-brane action,

$$S_{D3} = T_3 \int_{D3} C_2^i \wedge \star_4 J_i - \frac{1}{2} C_2^i \wedge \star_4\, \hat{m}_{ij} C_2^j \,. \tag{6.36}$$

Here the sources J_i are now given by (6.22), including C_0, and we recall that the self-coupling matrix $\hat{m}_{ij}$ can be found in (6.23). Applying these generalizations, the equations of motion for C_2^i read[24]

$$\hat{D}_{ij}(\mathrm{d}^\dagger \mathrm{d})\, C_2^j = J_i\, \underline{\delta}(A) + J_i\, \underline{\delta}(B) \tag{6.37}$$

where

$$\hat{D}_{ij}(\mathrm{d}^\dagger \mathrm{d}) \equiv \left[\hat{M}_{ij}\, \mathrm{d}^\dagger \mathrm{d} + \hat{m}_{ij}\, \underline{\delta}(A) + \hat{m}_{ij}\, \underline{\delta}(B)\right] , \tag{6.38}$$

and we absorbed a factor of $2\kappa_{10}^2 T_3$ in the δ-functions, cf. (6.28). To integrate out C_2^i we fix the gauge and neglect 4d fluctuations analogously to sect. 6.2.

Thus we are left with the problem of inverting the operator $\hat{D}(\Delta_6)$ which we solve using a series expansion in $\hat{m}$:

$$\hat{D}^{-1}(\Delta_6) = \sum_{k=0}^{\infty} \hat{M}^{-1} \Delta_6^{-1} \left(\left[\hat{m}\, \underline{\delta}(A) + \hat{m}\, \underline{\delta}(B)\right] \hat{M}^{-1} \Delta_6^{-1} \right)^k \,. \tag{6.39}$$

This allows us to integrate out C_2^i. One may think of the effects we calculate in terms of diagrams describing C_2^i-exchange between branes, as depicted in fig. 6.2. The resulting action is analogous

[24] We will see in sect. 6.4.1 that when carefully accounting for, so far neglected, C_4 and $\tilde{F}_5$ terms, the equation of motion for C_2^i acquires an extra contribution. This can be seen from (6.50) when recalling $\hat{M}_{ij}(\mathrm{d}^\dagger \mathrm{d})\, C_2^j = \star_{10} \mathrm{d} \left(\hat{M}_{ij} \star_{10} F_3^j \right)$. The extra term may be easily accounted for by replacing $\hat{m}$ with $\tilde{m}$, given by (6.52). The key features of $\hat{m}$ and hence the result of this section remain unchanged.

to (6.33) and reads

$$S \supset \frac{T_3}{2} \int_{\mathcal{M}^{1,3}} \underline{J}_i^{(A)} \wedge \star_4 \, (\hat{D}^{-1})^{ij} \, J_j^{(B)} + \underline{J}_i^{(B)} \wedge \star_4 \, (\hat{D}^{-1})^{ij} \, J_j^{(A)} \, . \tag{6.40}$$

There are two important observations to be made,

$$(\hat{M}^{-1})^{ij} J_j(x) = - \star_4 \epsilon^{ij} J_j \equiv - \star_4 J^i \, , \tag{6.41}$$

$$\hat{m}_{ij} \star_4 J^j(x) = -\frac{1}{2} \epsilon_{ij} J^j \equiv -\frac{1}{2} J_i \, , \tag{6.42}$$

which can be checked by explicit calculation. The first simply generalizes (6.34) to $C_0 \neq 0$. The second is an analogous relation for $\hat{m}$. We have also introduced the natural notation $J^i \equiv \epsilon^{ij} J_j$ [20].

As a result, the application of $\hat{D}^{-1}$ to the source now gives[25]

$$(\hat{D}^{-1})^{ij} \, J_j = - \sum_{k=0}^{\infty} \Delta_6^{-1} \left(\frac{\Delta_6^{-1}}{2} \, \underline{\delta}(A) + \frac{\Delta_6^{-1}}{2} \, \underline{\delta}(B) \right)^k \star_4 J^i \, . \tag{6.43}$$

When we now evaluate (6.40) using this result, we obtain an infinite sum of terms each of which is proportional to

$$\epsilon^{ij} \left(J_i^{(A)} \wedge J_j^{(B)} + J_i^{(B)} \wedge J_j^{(A)} \right) = 0 \, . \tag{6.44}$$

We conclude that kinetic mixing between D3-branes vanishes in full generality.

The structure of the cancellations suggests that the underlying reason is the very peculiar $\mathrm{SL}(2,\mathbb{R})$ structure of D3-branes in type IIB. To be more precise, even though $\mathrm{SL}(2,\mathbb{R})$ is not a symmetry of the action, the subgroup $\mathfrak{b}$ is a symmetry and the sources J_i are doublets under it. Moreover, it is easy to check explicitly that, just like in the case of $\mathrm{SL}(2,\mathbb{R})$, the matrix ϵ^{ij} is the only rank-2 invariant tensor of the group $\mathfrak{b}$. At the lowest non-trivial order contributing to the kinetic mixing we have to build an invariant from the two $\mathfrak{b}$-vectors, J_i. Contraction with ϵ^{ij} then provides the unique $\mathfrak{b}$-invariant (and $\mathrm{SL}(2,\mathbb{R})$-invariant) way to combine the two sources. Therefore, the $\mathfrak{b}$ symmetry together with the obvious symmetry under exchange of A/B ensures that our final result will be proportional to the l.h.s. of (6.44) and hence vanish. The calculation above can be regarded as an explicit confirmation.

6.4. No Cancellation including Fluxes

6.4.1. Deriving relevant Equation of Motion

In a final step we now include a general internal background flux $\bar{F}_3^i$ for the field strength F_3^i, breaking the $\mathrm{SL}(2,\mathbb{R})$ symmetry. Having $\bar{F}_3^i$ fluxes at our disposal, more terms quadratic in the sources J_i can be constructed. Due to Lorentz invariance, at least two fluxes $\bar{F}_3^i$ have to be used, with $J_{i\,\mu\nu} J_j^{\mu\nu} \bar{F}^i_{abc} \bar{F}^{j\,abc}$ being the minimal option. Otherwise Lorentz indices would be left open. Indeed, our explicit result (6.81) has this form.

To begin we note that, after compactification, the $\tilde{F}_5 \wedge \star_{10} \tilde{F}_5$ term in the action (6.1) induces masses for C_2^i [221]. In the analysis of [221], the relevant mass terms explicitly contain the background gauge fields $\bar{C}_2^i$. We find it problematic that the latter are not gauge invariant and

[25] As already remarked in footnote 24 $\hat{m}$ should actually be replaced by $\tilde{m}$. This changes (6.42) into (6.53). While the pre-factor is now 1 instead of $1/2$, the decisive property of lowering the index i on J^i persists.

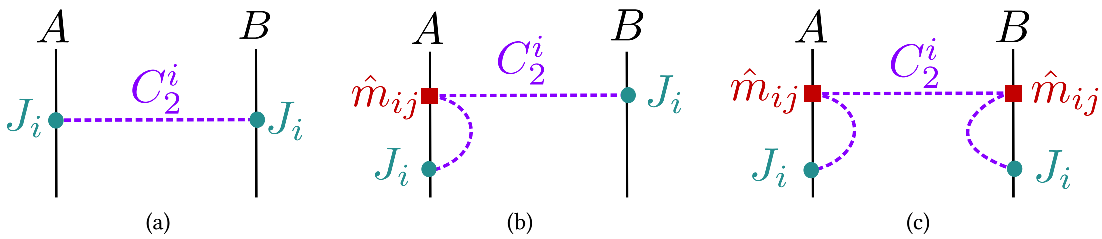


Fig. 6.2: Due to the new coupling $\hat{m}_{ij}$ on each D3-brane, there are now more diagrams contributing to KM. Figures (a), (b) and (c) provide example diagrams in perturbation theory at different orders. As we show, all contributions vanish individually to all orders in perturbation theory.

can not be globally defined on the compact space. We were not able to find a gauge-invariant rewriting at the level of the action. The equations of motion however are fully gauge invariant and thus allow for a consistent analysis. To obtain the equations of motion we recall the relevant parts of the bulk action

$$S_{IIB} \supset \frac{1}{2\kappa_{10}^2} \int_{\mathcal{M}} \left(-\frac{\hat{M}_{ij}}{2} F_3^i \wedge \star F_3^j - \frac{1}{4} \tilde{F}_5 \wedge \star \tilde{F}_5 - \frac{\epsilon_{ij}}{4} C_4 \wedge F_3^i \wedge F_3^j \right) , \tag{6.45}$$

$$\tilde{F}_5 = \mathrm{d}C_4 + \frac{\epsilon_{ij}}{2} C_2^i \wedge F_3^j \tag{6.46}$$

and the D3-brane action

$$S_{D3} = \int_{\mathcal{M}} T_3 \left(C_4 + \frac{1}{2} J_{(1)} \wedge J_{(2)} - \frac{1}{2} C_2^i \wedge \star_4 \hat{m}_{ij} C_2^j + C_2^i \wedge J_i \right) \wedge \delta_6(y - y_{D3}) \, . \tag{6.47}$$

It is convenient to first consider the equation of motion for C_4. Since a D3-brane acts as an electric and magnetic source for C_4, the correct equation of motion is obtained by varying the action w.r.t. C_4 while considering only half of the D3-brane contribution.[26] This gives

$$\mathrm{d} \star_{10} \tilde{F}_5 = \frac{\epsilon_{ij}}{2} F_3^i \wedge F_3^j - \underline{\delta}_6(y - y_{D3}) \, , \tag{6.48}$$

which as usual is further constrained by imposing self-duality, $\tilde{F}_5 = \star_{10}\tilde{F}_5$. Again, we employ our short-hand notation introduced in (6.28).

Varying the action w.r.t. C_2^i yields the equation of motion for C_2^i [27],

$$\begin{aligned} \mathrm{d}\left(\hat{M}_{ij} \star_{10} F_3^j\right) = \frac{\epsilon_{ij}}{2} \left(F_3^j \wedge \star_{10} \tilde{F}_5 + F_3^j \wedge \mathrm{d}C_4\right) + \frac{\epsilon_{ij}}{4} C_2^j \wedge \mathrm{d} \star_{10} \tilde{F}_5 \\ - \left(\star_4 J_i - \star_4 \hat{m}_{ij} C_2^j\right) \wedge \underline{\delta}_6 \, . \end{aligned} \tag{6.49}$$

This equation can now be rewritten by using the definition (6.46) and the equation of motion for

[26] Cf. footnote 6 of [97].

[27] Here we used the identities $\hat{m}_{ij} = \hat{m}_{ji}$ and $A_2^i \wedge \star_4 \hat{m}_{ij} B_2^j = B_2^i \wedge \star_4 \hat{m}_{ij} A_2^j$ for arbitrary A_2^i and B_2^j.

C_4 (6.48):

$$\mathrm{d}\left(\hat{M}_{ij}\star_{10}F_3^j\right)=\epsilon_{ij}F_3^j\wedge\tilde{F}_5-\left(\star_4J_i-\star_4\hat{m}_{ij}C_2^j+\frac{\epsilon_{ij}}{2}C_2^j\right)\wedge\underline{\delta}_6\,. \tag{6.50}$$

One may check explicitly that the two terms containing C_2^j on the r.h. side of (6.50) combine with the brane field F_2 contained in J_i to form a gauge invariant expression:

$$\star_4J_i[F_2]-\star_4\hat{m}_{ij}C_2^j+\frac{\epsilon_{ij}}{2}C_2^j=\star_4J_i[F_2-B_2]\,. \tag{6.51}$$

Thus, our equation of motion is gauge invariant. To simplify it further, we define a new self-coupling matrix $\tilde{m}$ by

$$\star_4\tilde{m}_{ij}\equiv\star_4\hat{m}_{ij}-\frac{\epsilon_{ij}}{2}=\star_4\begin{pmatrix}0 & -\star_4\\ 0 & g_s^{-1}+C_0\star_4\end{pmatrix}. \tag{6.52}$$

Note that the second equality holds only when this matrix is applied to a 2-form. As a key feature, our new $\tilde{m}$ still possesses a property analogous to (6.42):

$$\tilde{m}_{ij}\star_4J^j(x)=-J_i\,. \tag{6.53}$$

This differs from (6.42) only by a missing prefactor 1/2. With this, we can give the final form of the equation of motion which we will use:

$$\mathrm{d}^\dagger\left(\hat{M}_{ij}F_3^j\right)=\epsilon_{ij}\star_{10}\left(F_3^j\wedge\tilde{F}_5\right)+\left(J_i-\tilde{m}_{ij}C_2^j\right)\underline{\delta}(y-y_{D3})\,. \tag{6.54}$$

Now we turn on background fluxes $\bar{F}_3{}^i$, $\bar{\tilde{F}}_5$ for the 3- and 5-form fields strengths. To find the leading effects of these background fields we substitute $F_3^i\to\bar{F}_3{}^i+F_3^i$ and $\tilde{F}_5\to\bar{\tilde{F}}_5+\tilde{F}_5$ in (6.54) and keep only terms linear in the field fluctuations F_3^i and $\tilde{F}_5$ (see also [377, 378]):

$$\begin{aligned}(\hat{M}^{-1})^{ij}J_j\,\underline{\delta}(D3)=&\left(\mathrm{d}^\dagger\mathrm{d}C_2^i+(\hat{M}^{-1})^{ij}\tilde{m}_{jk}C_2^k\,\underline{\delta}(D3)\right)\\&-(\hat{M}^{-1})^{ij}\epsilon_{jk}\star_{10}\left(\bar{F}_3^k\wedge\tilde{F}_5+F_3^k\wedge\bar{\tilde{F}}_5\right)\,.\end{aligned} \tag{6.55}$$

An analogous procedure applied to the C_4 equation of motion (6.48) gives

$$0=\mathrm{d}^\dagger\tilde{F}_5-\epsilon_{ij}\star_{10}\left(\bar{F}_3^i\wedge F_3^j\right)\,, \tag{6.56}$$

where the background $\bar{\tilde{F}}_5$ has to satisfy

$$\mathrm{d}\star_{10}\bar{\tilde{F}}_5=\frac{\epsilon_{ij}}{2}\bar{F}_3^i\wedge\bar{F}_3^j-\underline{\delta}_6(y-y_{D3})\,. \tag{6.57}$$

The delta distribution on the right hand side represents just one D3-brane but, of course, we have to imagine this being replaced by the full set of localised sources, including in particular O3-planes. Theoretically it is now straightforward to derive KM by simultaneously solving the equations of motion for C_2^i (6.55) and C_4 (6.56) in the background of fluxes and localised sources.

To make progress towards an explicit result, we will now argue that, in the large volume limit, it is consistent to set $\bar{\tilde{F}}_5$ to zero. To do so, we first discuss separately the effects coming from two distinct regions: (A) the near-D3 regions with their strongly-peaked $\bar{\tilde{F}}_5$ profile and (B) the generic bulk region, where $\bar{\tilde{F}}_5$ represents a dilute flux background, suppressed at large volume.

As will be discussed in app. C.2, the $\bar{\tilde{F}}_5$ effects from region (A) can, together with further effects related to $\tilde{m}_{ij}$, be absorbed in a renormalization of the brane action. Specifically, this will lead to an effective brane coupling to C_2^i, which we expect to deviate from the leading-order result at most by an $\mathcal{O}(1)$ factor. In fact, it will become clear that we may expect any such effects to be suppressed by g_s if, in addition to being at large volume, we assume $g_s \ll 1$. This is sufficient for our purposes and we disregard $\bar{\tilde{F}}_5$ effects from region (A) for now.

In region (B), both $\bar{\tilde{F}}_5$ and $\bar{F}_3^i$ are dilute and we may consider an expansion in these backgrounds. At zeroth order in $\bar{F}_3^i$ and all orders in $\bar{\tilde{F}}_5$, no kinetic mixing arises by an $\mathrm{SL}(2,\mathbb{R})$ argument similar to sec. 6.3. This is explained in app. C.1. At linear order in $\bar{F}_3^i$ and all orders in $\bar{\tilde{F}}_5$, no kinetic mixing arises. This will become clear when we discuss the corresponding diagrams below. Finally, going to quadratic order in $\bar{F}_3^i$, we will find a nonzero result already at zeroth order in $\bar{\tilde{F}}_5$. Any terms of quadratic order in $\bar{F}_3^i$ involving also $\bar{\tilde{F}}_5$ will then be subleading and we disregard them.

Thus, we now proceed setting $\bar{\tilde{F}}_5 = 0$. The only flux effect is then the mixing between C_2^i and C_4, which arises from the terms $\sim \bar{F}_3^i$ in (6.55) and (6.56). To solve (6.55) and (6.56) together for C_2^i and C_4, it proves useful to split the forms and derivatives in 4d and 6d parts:

$$A_p = \sum_{q=0}^{p} A_{(p-q,q)} \, , \tag{6.58}$$

$$\mathrm{d} = \mathrm{d}^{(4)} + \mathrm{d}^{(6)} \, , \tag{6.59}$$

$$\mathrm{d}^\dagger = \mathrm{d}^{(4)\dagger} + \mathrm{d}^{(6)\dagger} \, . \tag{6.60}$$

Here e.g. $A_{(2,1)}$ corresponds to a 2-form in 4d and a 1-form in 6d. Since the brane source $J_{i,(2,0)}$ is a pure 4d 2-form, we are only interested in the $(2,0)$ component of (6.55). Assuming product form of the 10d metric we have[28]

$$\mathrm{d}^\dagger \mathrm{d} C_2^i = (\mathrm{d}^{(4)\dagger}\mathrm{d}^{(4)} + \mathrm{d}^{(6)\dagger}\mathrm{d}^{(6)}) C_2^i \, . \tag{6.61}$$

It follows that the operator $\mathrm{d}^\dagger\mathrm{d}$ does not mix modes of e.g. C_2^i with different 4d/6d form degrees. Hence, given that we also assume $\bar{\tilde{F}}_5 = 0$, there is no mixing between such different C_2^i modes in (6.55). The (2,0) component of (6.55) thus reads

$$\begin{aligned}(\hat{M}^{-1})^{ij} J_{j,(2,0)} \, \underline{\delta}(D3) = \left(\delta_k^i \, \mathrm{d}^\dagger \mathrm{d} + (\hat{M}^{-1})^{ij} \tilde{m}_{jk} \, \underline{\delta}(D3)\right) C^k_{(2,0)} \\ - (\hat{M}^{-1})^{ij} \epsilon_{jk} \star_{10} \left(\bar{F}^k_{(0,3)} \wedge \tilde{F}_{(2,3)}\right) \, ,\end{aligned} \tag{6.62}$$

where we also used the fact that our 3-form flux is internal: $\bar{F}_3^i = \bar{F}^i_{(0,3)}$. All other modes of C_2^i decouple and (6.62) is the only equation we need to consider further.

The only C_4 modes that couple in (6.62) are $C_{(2,2)}$ and $C_{(1,3)}$ which can be seen by considering the definition of $\tilde{F}_{(2,3)}$,

$$\tilde{F}_{(2,3)} = \mathrm{d}C_4\Big|_{(2,3)} = \mathrm{d}^{(4)} C_{(1,3)} + \mathrm{d}^{(6)} C_{(2,2)} \, . \tag{6.63}$$

Further simplifications arise if, as before, we neglect 4d derivatives of fluctuating fields relative to 6d derivatives. This implies

$$\tilde{F}_{(2,3)} \approx \mathrm{d}^{(6)} C_{(2,2)} \, , \tag{6.64}$$

[28] This can easily be deduced from $\mathrm{d}^\dagger \mathrm{d} A_{\mu_1 \ldots \mu_p} = -(p+1)\nabla^\alpha \nabla_{[\alpha} A_{\mu_1\ldots\mu_p]}$, see e.g. App. A in [379].

$$F^j_{(2,1)} \approx \mathrm{d}^{(6)} C^j_{(2,0)} \,, \tag{6.65}$$

which means that we can forget about $C_{(1,3)}$ since it decouples from $C^j_{(2,0)}$ in (6.62) [29]. The equations of motion for the remaining $C_{(2,2)}$ mode follows from (6.56) and reads

$$0 = \mathrm{d}^\dagger \mathrm{d} C_{(2,2)} - \epsilon_{ij} \star_{10} \left(\bar{F}^i_{(0,3)} \wedge F^j_{(2,1)} \right) \,. \tag{6.66}$$

We thus find a closed set of two equations of motion, (6.62) and (6.66), which we re-write in matrix notation as

$$\begin{pmatrix} - \star_4 J^i_{(2,0)} \underline{\delta}(D3) \\ 0 \end{pmatrix} = \hat{D}^i{}_k \begin{pmatrix} C^k_{(2,0)} \\ C_{(2,2)} \end{pmatrix} \,, \tag{6.67}$$

where we defined

$$\hat{D}^i{}_k = \begin{pmatrix} \delta^i_k \, \Delta_6 + (\hat{M}^{-1})^{ij} \tilde{m}_{jk} \, \underline{\delta}(D3) & (\hat{M}^{-1})^{ij} P_j[\,\cdot\,] \\ P_k[\,\cdot\,] & \Delta_6 \end{pmatrix} \,, \tag{6.68}$$

$$P_j[\,\cdot\,] = \epsilon_{jk} \star_{10} \left(\bar{F}^k_{(0,3)} \wedge \mathrm{d}^{(6)}[\,\cdot\,] \right) \,. \tag{6.69}$$

Here we also replaced $\mathrm{d}^\dagger \mathrm{d} \to \Delta_6$, which is justified if we impose the gauges [377, 378]

$$\mathrm{d}^{(4)\dagger} C^i_2 = 0 = \mathrm{d}^{(6)\dagger} C^i_2 \,, \tag{6.70}$$

$$\mathrm{d}^{(4)\dagger} C_4 = 0 = \mathrm{d}^{(6)\dagger} C_4 \,, \tag{6.71}$$

and if we neglect 4d derivatives.

6.4.2. Leading Order Result

We now want to solve (6.67) in order to integrate out $(C^k_{(2,0)},\, C_{(2,2)})^{\mathrm{T}}$ to obtain a leading order result for KM[30]. Again we need to invert $\hat{D}^i{}_k$ which we do by expanding in $\tilde{m}$ and P_j. For this purpose we decompose $\hat{D}^i{}_k$ according to

$$\hat{D}^i{}_k = (\hat{D}_{(0)})^i{}_k + \delta \hat{D}^i{}_k \,, \quad (\hat{D}_{(0)})^i{}_k = \begin{pmatrix} \delta^i_k \, \Delta_6 & 0 \\ 0 & \Delta_6 \end{pmatrix} \,, \tag{6.72}$$

$$\delta \hat{D}^i{}_k = \begin{pmatrix} (\hat{M}^{-1})^{ij} \tilde{m}_{jk} \, \underline{\delta}(D3) & (\hat{M}^{-1})^{ij} P_j[\,\cdot\,] \\ P_k[\,\cdot\,] & 0 \end{pmatrix} \,. \tag{6.73}$$

Considering only $(\hat{D}_{(0)})^i{}_k$, the analysis is equivalent to sect. 6.2. Including $\delta \hat{D}^i{}_k$ but keeping only the term $\sim \tilde{m}$ in its definition, (6.73), is equivalent to sect. 6.3. Both results were identically zero. Non-zero contributions arise once we include the P_k terms from (6.73). These are proportional

[29] Note that, in general, only the combinations $\mathrm{d}^{(4)} C_{(1,3)} + \mathrm{d}^{(6)} C_{(2,2)}$ and $\mathrm{d}^{(4)} C^j_{(1,1)} + \mathrm{d}^{(6)} C^j_{(2,0)}$ are gauge invariant. However, fixing the gauge by (6.70) and (6.71) we can omit both $\mathrm{d}^{(4)} C^j_{(1,1)}$ and $\mathrm{d}^{(4)} C_{(1,3)}$ since $\mathrm{d}^{(6)} C^j_{(2,0)}$ and $\mathrm{d}^{(6)} C_{(2,2)}$ are invariant under residual gauge transformations left after the gauge choice (6.70), (6.71). Technically this becomes apparent after performing a Hodge decomposition of the form fields [379].

[30] Note that the zero-mode of $C_{(2,2)}$ is *not* projected out by orientifolding. So one may be concerned that $C_{(2,2)}$ can not be integrated out. However, in the key equations (6.67) only 6d derivatives of this field appear. Hence, the zero mode decouples and $C_{(2,2)}$ can be integrated out together with $C^k_{(2,0)}$.

to $\bar{F}^i_{(0,3)}$ and, thinking in terms of diagrams, we may associate them with 3-vertices involving the flux background and two propagating fields C^i_2, $C_{(2,2)}$. This is illustrated in fig. 6.3.

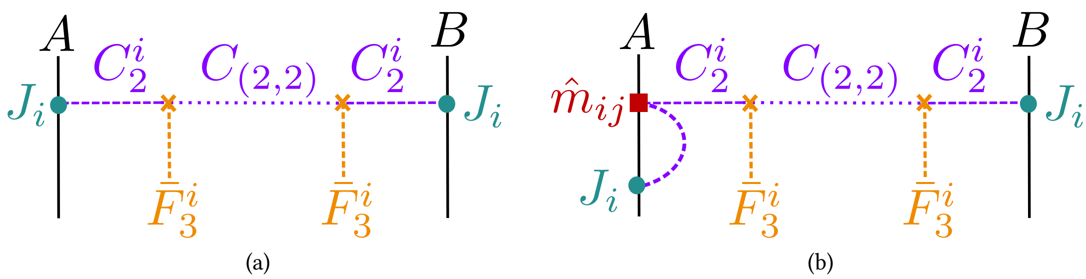


Fig. 6.3: Including the background for $\bar{F}^i_3$ introduces a new coupling term in the bulk. Thus, there are now more diagrams potentially contributing to KM. Diagram (a) corresponds to the leading-order effect and (b) shows an example of a further diagram, involving $\hat{m}$ and contributing to KM at higher order in g_s (cf. app. C.2).

Inverting $\hat{D}$ as a power series in $\delta\hat{D}^k{}_l$ we find

$$(\hat{D}^{-1})^i{}_j = (\hat{D}^{-1}_{(0)})^i{}_j + (\hat{D}^{-1}_{(0)})^i{}_k\, \delta\hat{D}^k{}_l\, (\hat{D}^{-1}_{(0)})^l{}_j + \cdots\,, \tag{6.74}$$

where the first-order term $\hat{D}^{-1}_{(1)}$ explicitly reads

$$\begin{aligned}(\hat{D}^{-1}_{(1)})^i{}_j =&(\hat{D}^{-1}_{(0)})^i{}_k\, \delta\hat{D}^k{}_l\, (\hat{D}^{-1}_{(0)})^l{}_j \\ =&\begin{pmatrix} \Delta_6^{-1}(\hat{M}^{-1})^{ik}\tilde{m}_{kj}[\underline{\delta}(A)+\underline{\delta}(B)]\Delta_6^{-1} & \Delta_6^{-1}(\hat{M}^{-1})^{ik}P_k\circ\Delta_6^{-1} \\ \Delta_6^{-1}P_j\circ\Delta_6^{-1} & 0\end{pmatrix}.\end{aligned} \tag{6.75}$$

Integrating out the vector $(C^k_{(2,0)},\, C_{(2,2)})^{\mathrm{T}}$ at this order contributes to the KM action as

$$S \supset \frac{T_3}{2}\int_{\mathcal{M}^{1,3}} \left(J^{(\mathrm{A})}_{i,(2,0)},\, 0\right)\wedge\star_4\left[(\hat{D}^{-1}_{(1)})^i{}_j\begin{pmatrix} -\star_4 J^{(\mathrm{B})\,j}_{(2,0)} \\ 0\end{pmatrix}\right] + (A\leftrightarrow B)\,. \tag{6.76}$$

Due to the 0 in the lower component of the source vector, only the upper-left entry of $\hat{D}^{-1}_{(1)}$ is relevant. We know, however, that this contribution vanishes by the arguments of sect. 6.3. Hence we have to go to the second order term $\hat{D}^{-1}_{(2)}$, where we again focus only on the top-left entry:

$$\begin{aligned}(\hat{D}^{-1}_{(2)})^i{}_j\Big|_{\text{top-left}} &= (\hat{D}^{-1}_{(0)})^i{}_k\, \delta\hat{D}^k{}_l\, (\hat{D}^{-1}_{(0)})^l{}_r\, \delta\hat{D}^r{}_s\, (\hat{D}^{-1}_{(0)})^s{}_j\Big|_{\text{top-left}} \\ &= \Delta_6^{-1}(\hat{M}^{-1})^{ik}\tilde{m}_{kl}[\underline{\delta}(A)+\underline{\delta}(B)]\Delta_6^{-1}(\hat{M}^{-1})^{lr}\tilde{m}_{rj}[\underline{\delta}(A)+\underline{\delta}(B)]\Delta_6^{-1} \\ &\quad + \Delta_6^{-1}(\hat{M}^{-1})^{ik}P_k\Delta_6^{-1}P_j\Delta_6^{-1}\,.\end{aligned} \tag{6.77}$$

Once again, the KM contribution from the first term in (6.77) cancels by the analysis of sect. 6.3. However, the second term in (6.77) provides a non-zero contribution. This turns out to be our leading-order result. Similarly to (6.76), we may write the corresponding action term as

$$S \supset \frac{1}{2} \int_{\mathcal{M}^{1,3}} \left(J^{(\mathrm{A})}_{i,(2,0)} \wedge \star_4 J^{(\mathrm{B})}_{j,(2,0)}\, K^{ji}(y_A, y_B) + J^{(\mathrm{B})}_{i,(2,0)} \wedge \star_4 J^{(\mathrm{A})}_{j,(2,0)}\, K^{ji}(y_B, y_A) \right) , \tag{6.78}$$

where K^{ji} is given by[31]

$$K^{ji}(y_A, y_B) = 2\pi \int_{y',y''} \left(\frac{1}{4!} G_6(y_A, y')\, \bar{F}^j(y')_{[abc}\, \partial^{(y')}_{d]} \partial^d_{(y'')} \left[G_6(y', y'') \right] \bar{F}^i(y'')^{abc}\, G_6(y'', y_B) \right) . \tag{6.79}$$

Here we introduced the scalar Green's functions G_6, to be distinguished from the general Green's function Δ_6^{-1} which acts on forms and is itself form-valued, cf. [380, 381]. This feature is particularly relevant in connection with $C_{(2,2)}$. The symbolic manipulations above are not affected by this technicality. Here the labels a, b, c, d refer to 6d indices and we used the identity $2\kappa_{10}^2 T_3^2 = 2\pi$. Crucially, the argument from sects. 6.2 and 6.3 for the vanishing of KM does not apply since the tensor K^{ij} used to contract $J^{(A)}_{i,(2,0)}$ and $J^{(B)}_{j,(2,0)}$ is, in contrast to ϵ^{ij}, symmetric rather than antisymmetric:

$$K^{ji}(y_A, y_B) = K^{ij}(y_B, y_A) . \tag{6.80}$$

We can then simplify (6.78) to obtain the final result

$$S \supset \int_{\mathcal{M}^{1,3}} J^{(\mathrm{A})}_{i,(2,0)} \wedge \star_4 J^{(\mathrm{B})}_{j,(2,0)}\, K^{ji}(y_A, y_B) . \tag{6.81}$$

6.4.3. Open Issues with the Supergravity Embedding

From (6.79), (6.81) we can already infer that our result will depend on the distance between the two D3-branes, which may be problematic as will become clear momentarily. More specifically, as will be worked out in detail in the next section, K^{ji} scales as $1/\mathcal{V}^{4/3}$ with $\mathcal{V}$ the Calabi-Yau volume in 10d Planck units. The difficulties arise because our 4d EFT is a genuine $\mathcal{N} = 1$ supergravity theory. Further, it has already been stated that KM arises as a loop correction to the gauge kinetic function f_{AB} [346, 347, 382]. In supergravity, the gauge kinetic function is holomorphic. Thus, the volume dependence noted above implies that f_{AB} is a holomorphic function of the Kahler moduli. However, the shift symmetry of the Kahler moduli excludes any holomorphic Kahler moduli depence of f_{AB} which is not linear or exponential [346, 347, 382]. This clashes with (6.81) and implies the question how our result can be understood from a 4d supergravity perspective.

In fact, our computation in the present paper was not loop-based but relied on the equivalent approach of integrating out 10d p-form fields. In 4d language, this corresponds to integrating out an infinite tower of heavy KK modes. Equivalently, one may say that we are integrating out massive strings stretched between the branes. Such a procedure of integrating out massive fields in supergravity is potentially problematic, as has been pointed out in [383–387]. It is in particular plausible that it induces higher derivative operators in the 4d effective theory, similar to those discussed in [388–404]. Additionally, to obtain our non-zero result it is crucial to include 3-form fluxes to break $\mathrm{SL}(2, \mathbb{R})$. Including such fluxes generically breaks SUSY spontaneously. As a result, higher-derivative corrections to 4d supergravity,[32] such as those in eqs. (3.19) – (3.21)

[31] We have integrated by parts to make the symmetry of $K^{ji}(y_A, y_B)$ more apparent.

[32] By this we mean both terms of the type $\int d^4\theta\, W^\alpha W_\alpha f(\Phi, \overline{\Phi})$, which induce higher-derivatives in the on-shell

of [347] or (3.23) of [398], can affect the gauge-kinetic function, inducing a shift-symmetric and non-holomorphic Kahler moduli dependence. However, this logic also implies that our KM result (6.81) must vanish if only SUSY-preserving fluxes are present. While this might well be the case, it is unfortunately not obvious from our result. More work is necessary to clarify this point.

6.5. Implications for Phenomenology

Before estimating the magnitude of KM on the basis of (6.81), we have to mention a caveat: Strictly speaking, the D3-brane model we investigated is not phenomenologically interesting because it involves no light states charged under the two gauge groups. Hence, one can perform a field redefinition such that any KM visible at low energies disappears. However, even in this toy model the KM we calculated is in principle a well-defined physical observable. Indeed, the model contains heavy charged states in the form of strings stretched between our two D3-branes and the corresponding mirror D3′-branes. The latter must be present due to our use of an O3/O7 orientifold. Allowing for any number and type of such states, one fills out a complete, two-dimensional integer charge lattice. If one works in a gauge field basis defined by this integer lattice, the KM term is fixed in an unambiguous way. For example, one could obtain a static interaction potential between two heavy states one charged under $\mathrm{U}(1)_{(A)}$ the other under $\mathrm{U}(1)_{(B)}$. In this case the interaction potential would be proportional to the KM. In this sense we claim that KM in our model is physical since charged states, even though heavy, are present. However, we note that this is slightly problematic from the effective field theory point of view since these heavy states are not clearly much lighter than the states we have integrated out to obtain the kinetic mixing. Clearly, a better model would contain light charged states, which could be realized by considering branes at singularities or intersecting branes. For the singularity case, a string loop calculation has been performed in a particular class of torus orbifold models [346], but we would need an appropriate 10d supergravity analysis. We leave this to future work. A third way of including light states will be discussed at the end of this section since it will benefit from formulae we will derive momentarily.

Thus, let us continue with the analysis of our example of single D3-branes. We will set $2\pi\sqrt{\alpha'} = 1$ from now on. It will be convenient to introduce a length-scale-type variable R associated with the volume $\mathcal{V}$, measured in 10d Einstein frame. We use the torus-motivated definition

$$\mathcal{V} = (2\pi R)^6 \,, \tag{6.82}$$

but we will think of R more generally as of a typical length scale of our Calabi-Yau. For a parametric estimate of KM from (6.81), we need to characterize the magnitude of fluxes and Green's functions. The 3-form flux $\bar{F}^i_{(0,3)}$ satisfies the standard quantization condition when integrated over a 3-cycle Σ_3 [97]:

$$\int_{\Sigma_3} \bar{F}^i_{(0,3)} = n^i \in \mathbb{Z} \,. \tag{6.83}$$

Using $\mathrm{vol}(\Sigma_3) \simeq \mathcal{V}^{1/2}$ this implies

$$\bar{F}^i_{mno} \sim n^i \, \frac{1}{\mathcal{V}^{1/2}} \,. \tag{6.84}$$

In the regime $y \ll \mathcal{V}^{1/6}$, the Green's function on the Calabi-Yau $\Delta_6^{-1}(y)$ can be estimated on the

action, as well as terms involving higher SUSY derivatives.

basis of its flat-6d counterpart (see [405] for further discussion)

$$\Delta_6^{-1}(y) \simeq -\frac{1}{4\pi^3\, y^4}\,, \quad \partial_{(y)}\partial_{(y)}\Delta_6^{-1}(y) \simeq -\frac{5}{\pi^3 y^6}\,. \tag{6.85}$$

Using (6.84) and (6.85) we may now estimate K^{ji} from its definition in (6.79). In doing so, we will *not* implement the imaginary self-duality (ISD) condition [97]

$$i(F_3 - \tau H_3) = \star_6(F_3 - \tau H_3) \qquad \Leftrightarrow \qquad F_3 = -g_s \star_6 H_3 + C_0 H_3\,. \tag{6.86}$$

As a result, we will also not be able to keep track of the g_s dependence introduced by the ISD condition. This would require keeping track of the g_s dependence of the relative size of 3-cycles, which appears due to relations like (for $C_0 = 0$)

$$\int_{\Sigma_3} F_3 = \int_{\Sigma_3} (-g_s) \star_6 H_3 = n \in \mathbb{Z}\,. \tag{6.87}$$

Controlling the metric at this level of precision goes beyond our goals in this paper.

For our following simple estimates, we set $n^1 = n^2 = 1$ and disregard the non-trivial profiles of the different fluxes on the Calabi-Yau. The behaviour of the Green's functions now comes into play when we try to estimate the integrals defining K^{ji} in (6.79). Crucially, one finds that these integrals are IR dominated, i.e. the integrand does not diverge at $y' \to 0$ and/or $y'' \to 0$. We may then estimate K^{ji} by inserting the maximal distance $y = \pi R = \mathcal{V}^{1/6}/2$ for y' and y'' into the Green's functions (6.85), though this flat-space formula is at this point of course at best correct at the $\mathcal{O}(1)$ level. Replacing the integration by multiplication with the volume of the integration domain, one finds

$$K^{ji} \sim -\frac{2\pi}{4!}\frac{2^{10}}{\pi^9}\frac{5}{\mathcal{V}^{4/3}}\,. \tag{6.88}$$

We may be slightly more precise by ascribing different (though always $\mathcal{O}(1)$) numbers to the RR and NS fluxes: $\bar{F}^{(1)} \sim f$ and $\bar{F}^{(2)} \sim h$. This gives

$$K^{ji} \sim -\frac{2\pi}{4!}\frac{2^{10}}{\pi^9}\frac{5}{\mathcal{V}^{4/3}}\begin{pmatrix} f^2 & fh \\ fh & h^2 \end{pmatrix}^{ji}\,. \tag{6.89}$$

Note that f transforms as a pseudoscalar, which follows from the CP properties of F_3.

We now turn to the product $J_i \wedge \star J_j$ in (6.81). By the definition of J_i in (6.22) this will introduce g_s and C_0 factors. A final subtlety arises because KM is defined with canonically normalised gauge field strengths $\mathbb{F}_2$, cf. (2.4). By contrast, the field strength in our stringy analysis, F_2 from (6.11), is normalised by the coupling to the open string. The relation between the two reads

$$\mathbb{F}_2 = g_s^{-1/2}\sqrt{T_3}\,F_2\,. \tag{6.90}$$

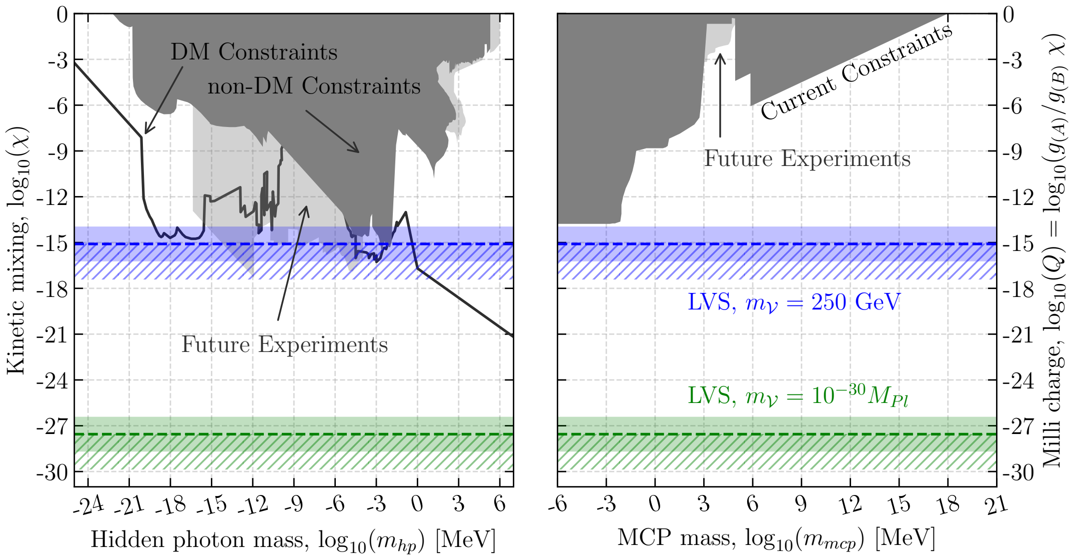


Fig. 6.4: Current constraints on KM in case of a massive hidden photon (left) and a massless hidden photon with millicharged particles (MCP) (right) are given in gray. For detailed references on the excluded regions we refer back to fig. 2.2 and the text below fig. 2.2. The colored delimiters show the indicative lower values obtained for D3-D3 brane setups with fluxes from (6.97) (green) and (6.100) (blue).

Considering all the above details, the result for the parametric scaling of (6.81) is

$$S \supset \sim \int_{\mathcal{M}^{1,3}} \frac{2^{11}\,5}{4!\pi^9\,\mathcal{V}^{4/3}} \Bigg\{ \mathbb{F}_2^{(A)} \wedge \star_4 \mathbb{F}_2^{(B)} \quad \left[f^2 g_s + 2fh\, g_s C_0 + h^2\,(g_s^{-1} - g_s {C_0}^2) \right] \\ + \mathbb{F}_2^{(A)} \wedge \mathbb{F}_2^{(B)} \quad \left[2fh - 2h^2\, C_0 \right] \Bigg\} . \tag{6.91}$$

Comparing to (2.4), we find the following estimates for the kinetic mixing parameter χ_{AB} and the magnetic mixing (MM) parameter $\tilde{\chi}_{AB}$:

$$\chi_{AB} \sim -\frac{2^{10}}{4!\pi^9} \frac{5 g_s^{-1}}{\mathcal{V}^{4/3}} , \tag{6.92}$$

$$\tilde{\chi}_{AB} \sim -\frac{2^{11}}{4!\pi^9} \frac{5(f - C_0)}{\mathcal{V}^{4/3}} . \tag{6.93}$$

Here we also included the factor $1/2$ from the definition (2.4) and we set $h = 1$. We left f explicit such that the CP properties become apparent. The main suppression of KM and MM will be due to the volume factor. We caution the reader that, while we tried to keep track of factors of π in our estimates, the prefactor is nevertheless uncertain at the level of one or two orders of magnitude. This can be seen e.g. by using a 6-torus Green's function [405–408] instead of a flat approximation (6.85), which introduces several π factors due to the six internal dimensions. We indicate this uncertainty in fig. 6.4 with the color shaded band around the bounds.

In order to obtain a parametric estimate of the smallest possible values for KM and MM, we now consider the implementation of our model in the large volume scenario (LVS) [101, 102]. In the LVS, we can constrain the maximal size of the internal geometry $\mathcal{V}$ that can be stabilized consistently. The main constraint comes from the volume modulus, since the volume modulus is the lightest modulus and couples like gravity to all matter fields after Weyl rescaling to 4d Einstein frame. In the LVS, the mass $m_{\mathcal{V}}$ in units of the 4d Planck mass $M_{\rm Pl}$ is given by [102]

$$m_{\mathcal{V}} \sim \frac{W_0}{\sqrt{4\pi}\, g_s^{1/4} \mathcal{V}^{3/2}} M_{\rm Pl}\,. \tag{6.94}$$

Thus we can rewrite (6.92) and (6.93) in terms of $m_{\mathcal{V}}/M_{\rm Pl}$:

$$|\chi_{\rm AB}| \sim\; 5\frac{2^{10}}{4!\pi^9}(4\pi)^{4/8}\left(\frac{m_{\mathcal{V}}}{M_{\rm Pl}}\right)^{8/9} g_s^{-7/9}\, W_0^{-8/9}\,, \tag{6.95}$$

$$|\tilde{\chi}_{\rm AB}| \sim\; 5\frac{2^{11}}{4!\pi^9}(4\pi)^{4/8}\left(\frac{m_{\mathcal{V}}}{M_{\rm Pl}}\right)^{8/9} g_s^{2/9}\, W_0^{-8/9}(f - C_0)\,. \tag{6.96}$$

A very conservative constraint for the volume modulus mass follows by demanding that fifth force limits [98, 409] are respected, which implies $m_{\mathcal{V}} \gtrsim 10^{-30} M_{\rm Pl}$. We can use this constraint on $m_{\mathcal{V}}$ to give a lower bound on $\chi_{\rm AB}$ and $\tilde{\chi}_{\rm AB}$

$$|\chi_{\rm AB}| \gtrsim\; 2.8\times 10^{-28}\left(\frac{m_{\mathcal{V}}}{M_{\rm Pl}}\frac{1}{10^{-30}}\right)^{8/9}\left(\frac{g_s}{0.1}\right)^{-7/9}\left(\frac{W_0}{1}\right)^{-8/9}\,, \tag{6.97}$$

$$|\tilde{\chi}_{\rm AB}| \gtrsim\; 5.7\times 10^{-29}\left(\frac{m_{\mathcal{V}}}{M_{\rm Pl}}\frac{1}{10^{-30}}\right)^{8/9}\left(\frac{g_s}{0.1}\right)^{2/9}\left(\frac{W_0}{1}\right)^{-8/9}\frac{f - C_0}{1}\,. \tag{6.98}$$

Note that W_0 is constrained in terms of the D3-brane tadpole $Q_3 \sim \mathcal{O}(100)$ [162]:

$$W_0 \lesssim \sqrt{\frac{|Q_3|}{g_s}}\,. \tag{6.99}$$

In the LVS, we further have the relation $g_s \sim 1/\ln\mathcal{V}$, which excludes large or extremely small values of g_s, given that we insist on a theoretically and phenomenologically reasonable value for $\mathcal{V}$. We can hence not use the in principle tunable parameters g_s and W_0 to reduce the lower bounds (6.97) and (6.98) significantly. As can be seen in fig. 6.4, the bound (6.97) is well below the experimentally excluded region.

Tighter constraints on $m_{\mathcal{V}}$ may be derived from cosmology. To avoid changing the element abundances by energy injection during BBN the modulus should decay well before this time [352–354]. In addition, the modulus decays should not deposit a significant amount of energy into its own, ultra-light axion in order to avoid an excess of dark radiation [267, 293, 355–365, 410]. Both problems can be circumvented if the volume modulus can decay efficiently into SM Higgses [365]. This, in turn, requires the mass $m_{\mathcal{V}}$ to be large enough, $m_{\mathcal{V}} \gtrsim 2m_H$, where m_H refers to the Higgs mass. Using this bound yields

$$|\chi_{\rm AB}| \gtrsim\; 8.1\times 10^{-16}\left(\frac{m_{\mathcal{V}}}{M_{\rm Pl}}\frac{M_{\rm Pl}}{2m_H}\right)^{8/9}\left(\frac{g_s}{0.1}\right)^{-7/9}\left(\frac{W_0}{1}\right)^{-8/9}\,, \tag{6.100}$$

$$|\tilde{\chi}_{\mathrm{AB}}| \gtrsim\ 1.6\times 10^{-16}\left(\frac{m_{\mathcal{V}}}{M_{\mathrm{Pl}}}\frac{M_{\mathrm{Pl}}}{2m_H}\right)^{8/9}\left(\frac{g_{\mathrm{s}}}{0.1}\right)^{2/9}\left(\frac{W_0}{1}\right)^{-8/9}\left(\frac{f-C_0}{1}\right). \tag{6.101}$$

Notably, values of the order of the bound (6.100) are now being probed by experiments and observations, see fig. 6.4.

We emphasize again that, in generic flux compactifications where the complex structure moduli are stabilized along the line of [97], both $\bar{F}_{(0,3)}$ and $\bar{H}_{(0,3)}$ are turned on. One then expects both kinetic and magnetic mixing to be present. At least at the level of our simple single-D3-brane toy model, both kinetic and magnetic mixing vanish exactly if no 3-form fluxes are turned on.

From eqs. (6.97) and (6.98) we infer that quite small values for KM are achievable without any tuning of the relevant gauge couplings. In particular, the generic estimate that $\mathcal{O}(1)$ gauge couplings imply $\chi\sim\mathcal{O}(1)$ can be easily avoided.[33]

We now want to return to the third possibility of including charged light states mentioned at the beginning of this section: We may replace our two D3-branes by two stacks of D3-branes, with the branes in each of them separated by a small distance d. Light charged states now arise from the strings stretched between the branes in each stack. For concreteness, let each stack consist of two D3-branes. The light states are charged under the "relative" $\mathrm{U}(1)^{(\mathrm{r})}$ which originates from breaking the brane stack gauge group according to $\mathrm{U}(1)^{(\mathrm{o})}\times\mathrm{SU}(2)\to\mathrm{U}(1)^{(\mathrm{o})}\times\mathrm{U}(1)^{(\mathrm{r})}$. The additional "overall" $\mathrm{U}(1)^{(\mathrm{o})}$ has no light charged states. The two relative U(1) gauge groups of the two stacks will then mix due to C_2^i-exchange, as analysed in detail in the bulk of this paper.

Due to the breaking of a non-abelian gauge group, we expect a further suppression factor to come into play. This is most easily seen from a field theory perspective. To make our point, we focus on the even simpler case where only one of the two relevant gauge groups is non-abelian, e.g. $\mathrm{U}(1)_{\mathrm{A}}$ and $\mathrm{SU}(2)_{\mathrm{B}}$. Before $\mathrm{SU}(2)_{\mathrm{B}}$ is broken, KM is clearly impossible because of the non-abelian structure. The leading operator governing KM must involve the $\mathrm{SU}(2)_{\mathrm{B}}$-breaking VEV and takes the form [189, 192, 219, 220]

$$\mathcal{L}\supset\frac{\chi_{\mathrm{AB},0}}{\Lambda}F_{\mathrm{A}}\,\mathrm{tr}(\Phi_{\mathrm{B}}F_{\mathrm{B}})\,. \tag{6.102}$$

Here $\chi_{\mathrm{AB},0}$ is a parameter specifying any a-priori suppression of the interaction between the two gauge groups, as it arises in our context because of sequestering within the large CY volume. Moreover, Φ_{B} is an adjoint scalar and Λ the UV cutoff scale. Thus, after breaking $\mathrm{SU}(2)_{\mathrm{B}}$ the KM mixing between $\mathrm{U}(1)_{\mathrm{A}}$ and the surviving U(1) from $\mathrm{SU}(2)_{\mathrm{B}}$ is governed by

$$\chi_{\mathrm{AB}}\sim\chi_{\mathrm{AB},0}\frac{\langle\Phi_{\mathrm{B}}\rangle}{\Lambda}\,. \tag{6.103}$$

Our key point here is the additional suppression by $\langle\Phi_{\mathrm{B}}\rangle/\Lambda$.

In an analogous stringy setup, with one single D3 brane and a U(2)-stack with adjoint breaking, we find

$$\chi_{\mathrm{AB}}\sim\chi_{\mathrm{AB},0}\,\frac{g_{\mathrm{s}}^{1/4}}{\mathcal{V}^{1/6}}\,\frac{\langle\Phi\rangle}{M_{\mathrm{s}}}\,, \tag{6.104}$$

with $\langle\Phi\rangle\sim d\,M_{\mathrm{s}}$ and d the brane-separation in the U(2) stack in string units. The derivation is given in app. D.

In the formula above, $\chi_{\mathrm{AB},0}$ is the KM parameter as we derived it for two single D3 branes at a large distance. We have also extracted a factor implementing a suppression (for $d\ll 1$) by the

[33] We note an apparent tension between our values and bounds on KM inferred from positivity constraints on gravitational scattering amplitudes argued for in [411] in an explicit Standard Model context. It would be interesting to match the two settings in detail and try to understand and resolve any possible discrepancy.

adjoint VEV, as expected on EFT grounds. Interestingly, our stringy realization displays a further suppression factor $g_s^{1/4}/\mathcal{V}^{1/6}$, corresponding to one power of the inverse CY radius in string units. Intuitively, this can be explained by the fact that C_2^i now couples to the relative $\mathrm{U}(1)^{(r)}$ of the stack, i.e. $\mathrm{U}(1)^{(1)} - \mathrm{U}(1)^{(2)}$. Thus, one is dealing with a dipole coupling in comparison to the monopole-type coupling we were discussing before. Even though these considerations contribute positively to our goal of small KM, we emphasize that this model can not realise chiral matter and hence can not be made fully realistic. A SM sector requires more involved constructions, e.g. branes at singularities or intersecting branes.

7. D7-Brane Kinetic Mixing

In this chapter we discuss kinetic mixing between D7-branes. Specifically, we want to realize kinetic mixing involving D7-brane stacks to respect the considerations that led to the minimal setup of sect. 4.2. This implies that we have to incorporate a mechanism to break the non-abelian gauge theory on the D7-stack. Our method of choice uses internal gauge fluxes of the brane gauge theory. In light of these additional requirements, we will derive the relevant couplings for kinetic mixing in sect. 7.1. In sect. 7.2 we will give first preliminary results on the expectations of kinetic mixing. In this process, we will rediscover the special cancellation structure of B_2 and C_2 as in the case for D3-branes. However, an additional mediation through C_4 will remain and we can expect a non-zero result for kinetic mixing. Additionally, the following model can be used with a fibred geometry to possibly achieve exponentially suppressed kinetic mixing, as in sect. 5.2.

7.1. Mediating Couplings

We restrict ourselves to considering models with O3/O7-planes in the LVS of type IIB. Since we want to consider stacks of branes, the relevant fields for mediating kinetic mixing between D7-branes have to be identified from the non-abelian action of Dp-brane stacks which is given by [90, 252, 412] [34]

$$S_{\mathrm{NDBI}} = -T_p \int\limits_{Dp} \mathrm{d}^{p+1}\xi \;\; \mathrm{STr}\left(e^{-\phi}\sqrt{\mathrm{det}Q^i{}_j}\sqrt{-\mathrm{det}\left(\varphi^* \left[P\right]_{ab} + F_{ab}\right)}\right) \,, \tag{7.1}$$

$$S_{\mathrm{NCS}} = T_p \int\limits_{Dp} \mathrm{STr}\left(\varphi^* \left[e^{i\lambda \;\mathrm{i}_\Phi \mathrm{i}_\Phi} \sum_q \hat{C}_q \wedge e^{-B_2}\right] \wedge e^{F_2}\right) \,, \tag{7.2}$$

where further definitions follow below. Here STr denotes the maximally symmetric trace over all objects carrying gauge indices [412, 413] and the determinant only acts w.r.t. Lorentz indices. Note, that the above action is incomplete [414] and only holds to the fourth order in F_2 [412, 415]. Nevertheless, this form of the action will be sufficient for our application to kinetic mixing but we refer to [252] for further discussion and references on this subject. Additionally, note that we use $\hat{C}_4 = C_4 + 1/2\, B_2 \wedge C_2$ when expanding the sum in (7.2), cf. app. A. In the following we consider the brane stack action in static gauge such that we can choose to identify $\xi^a = x^a$ with $a, b = 0, \dots, p$, where x^M denotes the 10d coordinates with $M, N = 0, \dots, 9$. Further, we fix the position of the brane stack in transverse space to be $x^i = 0$ for $i, j, k = p + 1, \dots, 9$. Fluctuations around the position $x^i = 0$ are conveniently encoded in scalar fields Φ^i defined by

$$x^i = 2\pi\alpha'\Phi^i \equiv \lambda\Phi^i \,, \tag{7.3}$$

[34] Note, that we use a different notation from [90, 252, 412]. Specifically, we flip the sign of B_2 in comparison to [90, 252, 412] and absorb a factor λ into F_2 such that (7.1) and (7.2) reproduce the D3-brane action of (6.10) to (6.12). Crucially, we also take the extra $B_2 \wedge C_2$ term in to account, which arises from the precise definition of C_4, cf. app. A.

which transform in the adjoint representation of the non-abelian gauge theory. The action (7.1) and (7.2) has to be further supplemented by the following definitions:[34]

non-abelian field strength: $$F_{ab} = \partial_a A_b - \partial_b A_a + i\lambda^{-1}\left[A_a, A_b\right] \,, \tag{7.4}$$

covariant derivative: $$D_a\Phi^i = \frac{\partial\Phi^i}{\partial\xi^a} + i\left[A_a, \Phi^i\right] \,, \tag{7.5}$$

$$E_{MN} = g_{MN} - B_{MN} \,, \tag{7.6}$$

$$Q^i{}_j = \delta^i{}_j + i\lambda\left[\Phi^i, \Phi^k\right] E_{kj} \,, \tag{7.7}$$

$$P_{MN} = E_{MN} + E_{Mi}(Q^{-1} - 1)^{ij} E_{jN}, \tag{7.8}$$

non-abelian pullback: $$\begin{aligned} \varphi^*\left[P\right]_{ab} =& P_{MN} D_a x^M D_b x^N \\ =& P_{ab} + \lambda P_{ai} D_b \Phi^i + \lambda P_{ib} D_a \Phi^i + \lambda^2 P_{ij} D_a \Phi^i D_b \Phi^j \,, \end{aligned} \tag{7.9}$$

non-abelian interior Product: $$\mathrm{i}_\Phi \mathrm{i}_\Phi C_{(n)} = \frac{1}{2(n-2)!}\left[\Phi^i, \Phi^j\right] C_{jiM_3 \ldots M_n} \mathrm{d}x^{M_3} \wedge \cdots \wedge \mathrm{d}x^{M_n} \,. \tag{7.10}$$

Another crucial fact to keep in mind is that all bulk fields are functionals of the adjoint scalars Φ^i [413, 416, 417] since they depend on all 10d coordinates x^M. Thus, the bulk fields in the brane action have to be understood as a non-abelian Taylor expansion around a background, e.g. the expansion for the metric g reads

$$\begin{aligned} g_{ab} &= \exp[\lambda\Phi^i \partial_{x^i}] g^0_{ab}(\xi^a, x^i)|_{x^i=0} \\ &= g^0_{ab}(\xi^a, 0) + \lambda\Phi^i\, \partial_i \left[g^0_{ab}(\xi^a, x^i)\right]_{x^i=0} + \frac{\lambda^2}{2}\Phi^i\Phi^j\, \partial_i\partial_j\left[g^0_{ab}(\xi^a, x^i)\right]_{x^i=0} + \ldots \,, \end{aligned} \tag{7.11}$$

where g^0_{ab} refers to the background metric. This implies that bulk fields also contribute in the trace $\mathrm{STr}(\ldots)$. In the following, the expansion (7.11) is implicitly understood and only explicitly applied when necessary. In the case of D3-branes stacks, the non-abelian expansion will turn out to be crucial. In app. E, we will explicitly derive the relevant couplings for kinetic mixing in the case of D3-brane stacks.

To obtain kinetic mixing in the setting with D7-brane stacks, we first have to break the non-abelian gauge group of the stack. As proposed in sect. 4.2 this can be achieved by separating the branes in the stack. However, in the case of D7-branes, there are more possibilities to break the gauge group then just separating the branes. For example, we can either turn on Wilson lines or break the group by turning on gauge flux of the field strength F_2. Let us first consider the last approach and comment on the other mechanisms at the end of this section.

We focus on breaking the gauge group by turning on non-abelian gauge flux $\langle F_2 \rangle$ of the field strength tensor F_2, e.g. along the SM hypercharge generator embedded in a $SU(5)$ GUT model. The gauge group of the stack is thus broken to the subgroup which commutes with $\langle F_2 \rangle$. The field strength flux $\langle F_2 \rangle$ is thereby threaded around a 2-cycle Σ_2 of the divisor D_4, where D_4 is wrapped by the D7-branes. As suggested in [307–310], the 2-cycle Σ_2 of the divisor D_4 has to be trivial in the internal Calabi-Yau $\mathcal{X}^6$ to ensure that the resulting $U(1)$ of the broken gauge group remains massless and hence can be subject to kinetic mixing. In other words, $\langle F_2 \rangle$ has to lie in the kernel of the pushforward $\varphi_* :\ H^2(D_4) \to H^2(\mathcal{X}^6)$ [308, 418]. Imposing this constraint, we decompose the cohomology group $H^2(D_4)$ as

$$H^2(D_4) \cong {}^{\mathcal{X}^6}H^2(D_4) \oplus \tilde{H}^2(D_4) \,, \tag{7.12}$$

where ${}^{\mathcal{X}^6}H^2(D_4)$ denotes those elements inherited via the pullback φ^* of $H^2(\mathcal{X}^6)$ and $\tilde{H}^2(D_4)$ denotes those elements in the kernel of the pushforward φ_* [308, 418]. Thus, we only consider

$$\langle F_2 \rangle \in \tilde{H}^2(D_4) \,, \tag{7.13}$$

where $\langle F_2 \rangle$ obeys the following quantisation condition $\int_{\Sigma_2} \langle F_2 \rangle = (2\pi)^2 \alpha' n$ with $n \in \mathbb{Z}$ the amount of flux on the 2-cycle Σ_2.[35]

We now proceed in extracting the relavant kinetic mixing terms by fixing the scalars $\Phi^i = 0$ and the dilaton $e^{\phi} = g_s$. Doing so reduces (7.7) - (7.9) to

$$Q^i{}_j = \delta^i{}_j \,, \tag{7.14}$$

$$P_{MN} = E_{MN} \,, \tag{7.15}$$

$$\varphi^* \left[P\right]_{ab} = \varphi^* \left[g - B\right]_{ab} = g_{ab} - B_{ab} \,. \tag{7.16}$$

Further, the calibration condition for the D7-branes wrapping the divisor D_4 in the presence of internal B_2 and gauge flux $\langle F_2 \rangle$ has to be satisfied [419, 420]. For fixed scalars Φ^i the calibration condition reads

$$\int_{D_4} \mathrm{d}^4\xi \sqrt{\det \varphi^*[g - B] + \langle F \rangle} = \frac{1}{2} \int_{D_4} \varphi^* J \wedge \varphi^* J - (\langle F_2 \rangle - \varphi^* B) \wedge (\langle F_2 \rangle - \varphi^* B) \,, \tag{7.17}$$

where J refers to the Kähler form of the Calabi-Yau $\mathcal{X}^6$. The relevant terms for mediating kinetic mixing in (7.1) and (7.2) have to contain at least two field strength tensors F_2. At least one field strength for the internal gauge flux and only one field strength with indices in 4d. This is necessary since a single field strength tensor would be eliminated by the trace in (7.1) and (7.2). The field strength with 4d indices should then couple to another field, e.g. B_2 or C_2, to mediate kinetic mixing. In parallel to [347], we find the following terms suitable for mediating kinetic mixing

$$S_{\text{NDBI}} \supset -T_7 \;\mathrm{STr}\left[\left\{\int_{D_4} \frac{1}{2} \langle F_2 \rangle \wedge \langle F_2 \rangle\right\}\left\{\int_{\mathcal{M}^{1,3}} F_2 \wedge \star_4 B_2\right\}\right] \,, \tag{7.18}$$

$$S_{\text{NCS}} \supset T_7 \;\mathrm{STr}\left[\int_{\mathcal{M}^{1,3} \times D_4} \left(\frac{1}{2} C_4 \wedge F_2 \wedge F_2 + C_2 \wedge \frac{(F_2)^3}{3!}\right)\right] . \tag{7.19}$$

Thus, the fields that can mediate kinetic mixing with D7-branes are B_2, C_2, and additionally C_4. In the following, we will discuss how these terms give rise to kinetic mixing.

Finally we want to comment on the other possible mechanisms to break the gauge group of the brane stacks which could be used in principle. Turning on Wilson lines requires 1-cycles which are not present in the internal Calabi-Yau. In principle one could have a non-trivial 1-cycle in the divisor wrapped by the D7-branes which would rule out del Pezzo surfaces as wrapped

[35] Here we follow the convention of [97, 233] and used that F_2 has mass dimension zero equivalent to B_2. This can be seen from the gauge invariant construction $F_2 - B_2$ in the D-brane action.

divisors. This 1-cycle then again has to be trivial in the Calabi-Yau. Scenarios of this kind may be interesting for future work.

7.2. Kinetic Mixing Expectations

We are now in the position to discuss the possible contributions to kinetic mixing between two D7-brane stacks. As discussed above, we use internal gauge flux $\langle F_2 \rangle$ to break the gauge group to a sub-group containing a $\mathrm{U}(1)$ subject to kinetic mixing. In this setting, the possible terms that may mediate kinetic mixing are given in (7.18) and (7.19).

Notice, that the two terms containing B_2 in (7.18) and C_2 in (7.19) again appear to cancel, similar to sect. 6.2 and 6.3. Taking two field strengths F_2 in the C_2 term of (7.19) to be internal fluxes $\langle F_2 \rangle$, introduces a factor 3. Hence, the B_2 and C_2 contributions again cancel equivalently to the leading order cancellation found in sect. 6.2. Even more intriguing, once we take the other terms in (7.1) and (7.2) into account, we find *almost* the exact coupling structure of B_2 and C_2 as in the case of D3-branes in sect. 6.1 and 6.3. Indeed, focusing on the non-vanishing terms under Str, setting $\Phi^i = 0$, and only using $\langle F_2 \rangle$ with indices in 6d we find

$$S_{\mathrm{NDBI}} \supset -T_7 \,\mathrm{Str} \left\{ \left(\int_{D_4} \frac{-1}{2} \langle F_2 \rangle \wedge \langle F_2 \rangle \right) \int_{\mathcal{M}^{1,3}} \frac{e^{\phi}}{2} (F_2 - B_2) \wedge \star_4 (F_2 - B_2) \right\} , \tag{7.20}$$

$$S_{\mathrm{NCS}} \supset -T_7 \,\mathrm{Str} \left\{ \left(\int_{D_4} \frac{-1}{2} \langle F_2 \rangle \wedge \langle F_2 \rangle \right) \int_{\mathcal{M}^{1,3}} C_4 + \frac{1}{2} B_2 \wedge C_2 + C_2 \wedge (F_2 - B_2) + \frac{C_0}{2} (F_2 - B_2)^2 \right\} . \tag{7.21}$$

Except of the different overall factor this looks almost identical to the action of a D3-brane (6.10)-(6.12). Note however that the calibration condition (7.17) introduced a minus sign in the $\int_{D4} \langle F_2 \rangle \wedge \langle F_2 \rangle$ term which does *not* appear in the reduction of the CS-action[36]. We artificially introduced the minus sign in (7.21) with the effect of changing the charge from positive to negative. We do this only because the action written in this way is *(only) technically* identical to the action of an anti-D3-brane for which $S_{\mathrm{CS,\,anti\text{-}D3}} = -S_{\mathrm{CS,\,D3}}$. The same reasoning that led to the cancellation of kinetic mixing between D3-branes in sect. 6 also holds for kinetic mixing between anti-D3-branes. This implies that at this level of rigour all kinetic mixing contributions from B_2 and C_2 cancel. It would be interesting to check if non-zero kinetic mixing contributions from B_2 and C_2 arise if, similar to sect. 6.4, 3-form fluxes are introduced. Further, additional contributions may arise if we do not fix $\Phi^i = 0$, which could also be studied in the future.

The last remaining term of (7.18) and (7.19) contains a coupling to C_4. For the purpose of compactification we decompose C_4 analogous to (6.58) wrt. the indices pointing to the external or internal directions

$$C_4 = E_0 \wedge I_4 + E_1 \wedge I_3 + E_2 \wedge I_2 + E_3 \wedge I_1 + E_4 \wedge I_0 \,, \tag{7.22}$$

where E_p denotes a 4d p-form and I_q denotes a 6d q-form.[37] Using the decomposition (7.22) we

[36] At least at leading order, i.e. ignoring the self coupling terms, the sign has no influence on the cancellation as the coupling appears quadratically in the amplitude. However, we want to mention that the origin of the minus sign might be the varying convention in the literature, cf. eq. (A.5) of [347] and [419–421].

[37] Note, that (7.22) many forms in the decomposition are related by duality, e.g. $E_1 \sim E_3$, and only a subset is necessary to consider. Also, the 6d forms are not assumed to be harmonic but instead represent general 6d q-forms.

can see that the coupling to C_4 can lead to a mediation of kinetic mixing if we use $C_4 = E_2 \wedge I_2$. One of the field strengths F_2 is then also taken to be internal gauge flux and we thus consider the following coupling term

$$S_{\text{NCS}} \supset T_7 \ \text{STr}\left[\left\{\int\limits_{D_4} \langle F_2 \rangle \wedge \varphi^*[I_2]\right\}\left\{\int\limits_{\mathcal{M}^{1,3}} E_2 \wedge F_2\right\}\right] . \tag{7.23}$$

Recall that we required $\langle F_2 \rangle \in \tilde{H}^2(D_4)$ to obtain a massless U(1). Further, by a suitable choice of basis in $H^2(D_4)$ it can always be arranged that

$$\int\limits_{D_4} \varphi^*[\omega] \wedge \tilde{f} = 0 , \tag{7.24}$$

holds for $\tilde{f} \in \tilde{H}^2(D_4)$ and $\varphi^*[\omega] \in {}^{\mathcal{X}^6}H^2(D_4)$ (see fn. 7 of [418]). This implies that the following prefactor of (7.23)

$$\int\limits_{D_4} \langle F_2 \rangle \wedge \varphi^*[I_2] = 0 , \tag{7.25}$$

for $I_2 \in {}^{\mathcal{X}^6}H^2(D_4)$ and we find that the zero mode of C_4 decouples from F_2 on the branes. This is crucial since we can assure that only massive C_4-modes are capable of mediating kinetic mixing even though the zero mode of C_4 is not projected out by the orientifold. Due to this observation, it will be consistent to integrate out the mediating KK modes and one can expect a possible exponential suppression of χ_{AB} in a fibered geometry, as discussed in sect. 5.2.

To extract kinetic mixing mediated by C_4 we proceed analogous to sect. 6. Thus, we focus on the relevant parts of the IIB bulk and D7-brane action which read

$$S \supset -\frac{1}{2}\int\limits_{\mathcal{M}_{10}} \frac{1}{4\kappa_{10}^2} \mathrm{d}C_4 \wedge \star_{10} \mathrm{d}C_4 + \frac{1}{4\kappa_{10}^2}\int\limits_{\mathcal{M}_{10}} C_4 \wedge J_6 . \tag{7.26}$$

For convenience we defined the source J_6 from (7.23) which is given by

$$J_6(y) = 4\kappa_{10}^2 \frac{T_7}{2} F_2(y) \wedge F_2(y) \wedge \delta_2(y - y_0) , \tag{7.27}$$

where $\delta_2(y - y_0)$ fixes the position in the normal directions to the D7-brane stack. The equation of motion for C_4 thus takes the form

$$\mathrm{d}^\dagger \mathrm{d}C_4 = \star_{10}^{-1} J_6 . \tag{7.28}$$

As explained in detail in app. F, the solution to (7.28) is given in terms of form valued Greens functions. To extract kinetic mixing, we now choose

$$C_4 = E_2 \wedge I_2 , \tag{7.29}$$

$$J_6 \coloneqq e_2 \wedge i_4 , \tag{7.30}$$

$$e_2 \coloneqq 4\kappa_{10}^2 T_7 \, F_2 , \tag{7.31}$$

The details of this will not matter in the following.

$$i_4 \coloneqq i_2 \wedge \delta_2(y - y_0) \equiv \varphi_* \left[\langle F_2 \rangle\right] \wedge \delta_2(y - y_0) \,, \tag{7.32}$$

where E_2 and e_2 are 2-forms of $\mathcal{M}^{1,3}$ and I_2, i_2 and i_4 are forms on $\mathcal{X}^6$ and φ_* denotes the push-forward from the divisor D_4 to $\mathcal{X}^6$ [38]. Analogous to sect. 6 we will omit any dependence on the 4d coordinates and treat the equation of motion as a Laplace equation on $\mathcal{X}^6$. As in sect. 6 or app. F, we fix the gauge, i.e. $\mathrm{d}^{(6)\dagger} I_2 = 0$, and find for (7.28)

$$E_2 \wedge \Delta_6 I_2 = - \star_4 e_2 \wedge \star_6 i_4 \,. \tag{7.33}$$

Comparing coefficients we can infer

$$E_2 = - \star_4 e_2 \,, \tag{7.34}$$

$$\Delta_6 I_2 = \mathrm{d}^{(6)\dagger} \mathrm{d}^{(6)} \, I_2 = \star_6 i_4 \,. \tag{7.35}$$

Thus, using (F.12) we find

$$I_2(y') = - \int\limits_{\mathcal{X}^6, y} \hat{G}_4(y, y') \wedge \star_6 i_4(y) \,, \tag{7.36}$$

where $\hat{G}_4$ refers to the form valued Greens function defined in (F.9). In the specific instant of (7.36), $\hat{G}_4$ denotes a 2-form wrt. to the coordinates y' and a 4-form wrt. to the coordinates y. One can further simplify this solution since i_4 restricts the integral to be evaluated on the 4-cycle D_4 wrapped by the D7-branes

$$I_2(y') = - \int\limits_{D_4, y} \langle F_2 \rangle (y) \wedge \varphi^* \left[\star_6 \hat{G}_4(y, y')\right] \,, \tag{7.37}$$

where φ^* denotes the pullback onto D_4. Hence, a single source (7.30) yields the following profile for C_4

$$C_4(x, y) = 4\kappa_{10}^2 T_7 \star_4 F_2(x) \wedge \int\limits_{D_4, y'} \langle F_2 \rangle (y') \wedge \varphi^* \left[\star_6 \hat{G}_4(y', y)\right] \,, \tag{7.38}$$

Assuming now two sources $J_6^{(A)}$ and $J_6^{(B)}$ analogously to (7.30), we can find the appropriate solution as a sum of two times (7.38)

$$\frac{C_4(x, y)}{4\kappa_{10}^2 T_7} = \star_4 F_2^{(A)}(x) \wedge \int\limits_{D_4^{(A)}, y'} \left\langle F_2^{(A)} \right\rangle (y') \wedge \varphi^* \left[\star_6 \hat{G}_4(y', y)\right] + (A \leftrightarrow B) \,. \tag{7.39}$$

Using this solution in the action (7.26), we find that C_4 induces the following kinetic mixing term

$$S \supset 4\kappa_{10}^2 (T_7)^2 \left\{ \int\limits_{D_4^{(A)}, y} \left\langle F_2^{(A)} \right\rangle (y) \wedge \varphi^* \left(\int\limits_{D_4^{(B)}, y'} \left\langle F_2^{(B)} \right\rangle (y') \wedge \tilde{\varphi}^* \left[\star_6 \hat{G}_4(y', y)\right] \right) \right\} \int\limits_{\mathcal{M}^{1,3}} F_2^{(A)}(x) \wedge \star_4 F_2^{(B)}(x)$$
$$+ (A \leftrightarrow B) \,, \tag{7.40}$$

where $\tilde{\varphi}^*$ denotes the pullback to $D_4^{(B)}$. One can think of this integral as a generalization of Biot-Savart's law from classical electrodynamics where we compute the force between two electric

[38] Note, that the factor of 1/2 in e_2 vanishes as there are two choices of F_2 to be internal gauge flux $\langle F_2 \rangle$.

currents. From this result, we infer for $\chi_{AB}/g_A g_B$

$$\frac{\chi_{AB}}{g_A g_B} \sim 4\kappa_{10}^2 (T_7)^2 \left\{ \int\limits_{D_4^{(A)},y} \left\langle F_2^{(A)} \right\rangle (y) \wedge \varphi^* \left(\int\limits_{D_4^{(B)},y'} \left\langle F_2^{(B)} \right\rangle (y') \wedge \tilde{\varphi}^* \left[\star_6 \hat{G}_4(y',y) \right] \right) \right\} + (A \leftrightarrow B) \quad (7.41)$$

Note that it is still necessary to transform to a canonical basis where in addition F_2 obtains mass dimension two, since we for example absorbed a factor $\lambda = 2\pi\alpha'$ into F_2 when defining the action (7.1). In addition, recall that the gauge coupling of a D7-brane depends on the volume of the wrapped 4-cycle D_4, see e.g. (5.5). We can thus eliminate the $(g_A g_B)^{-1}$ factor in (7.41) by identifying the canonically normalized 4d field strength $\mathbb{F}_2$

$$F_2 \to 2\pi\alpha' \sqrt{\frac{2\pi g_s}{\mathrm{vol}(D_4)}}\, \mathbb{F}_2 \,, \tag{7.42}$$

where $\mathrm{vol}(D_4)$ is the volume of the cycle measured in string units. Plugging this into (7.41) yields

$$\chi_{AB} \sim \frac{2g_s}{\sqrt{\mathrm{vol}(D_4^{(A)})\mathrm{vol}(D_4^{(B)})}} \left\{ \int\limits_{D_4^{(A)},y} \frac{\left\langle F_2^{(A)} \right\rangle (y)}{(2\pi)^2\alpha'} \wedge \varphi^* \left(\int\limits_{D_4^{(B)},y'} \frac{\left\langle F_2^{(B)} \right\rangle (y')}{(2\pi)^2\alpha'} \wedge \tilde{\varphi}^* \left[\star_6 \hat{G}_4(y',y) \right] \right) + (A \leftrightarrow B) \right\} , \tag{7.43}$$

where we further simplified the result by using $2\kappa_{10}^2\, T_7 = 1$. Recall, that $\langle F_2 \rangle$ obeys the following quantisation condition $\int_{\Sigma_2} \langle F_2 \rangle = (2\pi)^2 \alpha' n$ with $n \in \mathbb{Z}$ the amount of flux on the 2-cycle Σ_2.

The formula (7.43) needs further investigation with regard to proper phenomenological application, but in principle shows that non-vanishing kinetic mixing can be generated between D7-brane stacks. The result (7.43) is quadratic in the symmetry breaking vevs $\left\langle F_2^{(A)} \right\rangle$ and $\left\langle F_2^{(B)} \right\rangle$ just as predicted in sect. 2.1.3. Also the effects from sequestering are apparent from the integral over the form valued Green's function $\hat{G}_4$. In addition, we see that we can suppress χ_{AB} even further by tuning the gauge couplings to smaller values which corresponds to increasing the volume $\mathrm{vol}(D_4)$ that is wrapped by the respective D7-branes. Thus again, we see that several effects mentioned in sect. 4 can occur simultaneously when discussing kinetic mixing in stringy scenarios.

In flat space, we may simplify (7.43) further by using the flat space solution (F.9) for $\hat{G}_4$ where the scalar kernel G_6 is given by (6.85). Inserting (F.9) into (7.43) yields[39]

$$\chi_{AB} \sim \frac{2g_s}{\sqrt{\mathrm{vol}(D_4^{(A)})\mathrm{vol}(D_4^{(B)})}} \left(\int\limits_{y,y'} \frac{\langle F^{(A)} \rangle_{ab} (y)}{(2\pi)^2\alpha'} \frac{\langle F^{(B)} \rangle^{ab} (y')}{(2\pi)^2\alpha'}\, G_6(y',y) \right) , \tag{7.44}$$

where the integrals contain the respective measures and y and y' refer to the coordinates of the 4-cycles $D_4^{(A)}$ and $D_4^{(B)}$ respectively.

For a very simple parametric estimate we may use the torus-motivated definition for the volume $\mathcal{V}$ (6.82). First, consider the case where the torus is isotropic which implies the following scaling of the gauge fluxes $\left\langle F_2^{(A)} \right\rangle \sim n\mathcal{V}^{-1/3}$, $\left\langle F_2^{(B)} \right\rangle \sim m\mathcal{V}^{-1/3}$ and of the 4-cycle volumes $\mathrm{vol}(D_4) \sim \mathcal{V}^{2/3}$. Further, the Greens function scales like $G_6 \sim \mathcal{V}^{-2/3}$ such that we find

$$\chi_{AB} \sim \frac{g_s\, n\, m}{\mathcal{V}^{2/3}} \,, \tag{7.45}$$

[39]Here, we incorporated the $(A \leftrightarrow B)$ term into the whole expression and several factors of 2 cancel in the computation such that the prefactor $2g_s$ is not changed.

where we omitted all $\mathcal{O}(1)$ factors. We may interpret this estimate in different ways. One can think of the suppression to originate from the symmetry breaking fluxes $\langle F_2^{(A)} \rangle$ and $\langle F_2^{(B)} \rangle$ which are required to break the non-abelian gauge theories and identify this result with (2.29). Alternatively, one can think of the suppression to originate from small gauge couplings since generically $\chi_{AB} \sim g_A g_B$ with $g_{A,B} \sim \mathcal{V}^{-1/3}$ in our specific case of D7-branes. However, a suppression due to sequestering is not present in this scenario.

Yet, one can consider an anisotropic geometry where we chose a different radius R_{D7} for the wrapped 4-cycles. In this case, a sequestering suppression can arise if $R_{D7} \ll \mathcal{V}^{1/6}$ and we separate the 4-cycles in a large volume geometry. For this case, the fluxes scale with $\langle F_2 \rangle \sim n/R_{D7}^2$ and the 4-cycles volumes with $\mathrm{vol}(D_4) \sim R_{D7}^4$. Still, the Green's function yields a suppression wrt. to the volume $G_6 \sim \mathcal{V}^{-2/3}$. With these estimates we again find

$$\chi_{AB} \sim \frac{g_s\, n\, m}{\mathcal{V}^{2/3}} \,, \tag{7.46}$$

where all dependencies on R_{D7} cancelled. The suppression in this case is only attributed to the sequestering of the D7-brane 4-cycles. We want to emphasise that the estimates (7.45) and (7.46) are based on strong simplifications and only invoke scaling arguments. In general, the integrals of (7.43) are highly non-trivial and more familiar to the coupling of e.g. dipoles which would induce a further suppression effect. An exact evaluation of the above formula is necessary to extract a more robust expectation for the suppression.

Part III.

String Loop Inflation

8. Introduction to Part III – Slow Roll Inflation in type IIB

Slow roll inflation requires a scalar potential with a sufficiently flat regime, i.e. a region of field space where slow roll parameters are small. We want to argue that this arises naturally using the Kähler moduli sector of type-IIB flux compactifications, given that the volume can be stabilised at a sufficiently large value and an appropriate uplift to an almost-Minkowski vacuum can be realised.

We recall that, due to the no-scale structure of type-IIB flux compactifications, the naively dominant $1/\mathcal{V}^2$ terms in the scalar potential cancel [97]. As a result, the leading-order Kähler moduli scalar potential V scales like

$$V \sim \frac{|W_0|^2}{\mathcal{V}^3}\,, \tag{8.1}$$

where W_0 is the constant superpotential generated by fluxes and $\mathcal{V}$ is the volume modulus. More specifically, we assume that we are in the regime of validity of the Large Volume Scenario (LVS) [101, 102]: This implies that the total Calabi-Yau volume takes the form $\mathcal{V} = \tilde{\mathcal{V}}(\tau_I) - \tau_s^{3/2}$, with 4-cycle Kähler moduli $\{\tau_I\} = \{\tau_0, \dots, \tau_n\}$ and τ_s. We may eliminate one of the 4-cycle variables in favour of the total volume $\mathcal{V}$ such that the scalar potential takes the form $V = V(\mathcal{V}, \tau_i, \tau_s)$, where now $\{\tau_i\} = \{\tau_1, \dots, \tau_n\}$. We will always be in the regime $\tau_s^{3/2} \ll \mathcal{V}$, referring to $\mathcal{V}$ as the volume, to τ_s as the small-cycle or blowup modulus, and to the $\{\tau_i\}$ as the 'additional Kähler moduli.' A key result of the LVS proposal is that, under rather general assumptions, τ_s and $\mathcal{V}$ get stabilised while, at leading order, the potential for the τ_i, $i = 1, \dots, n$, remains exactly flat: $V(\mathcal{V}, \tau_i, \tau_s) = V(\mathcal{V}, \tau_s)$.

Loop corrections to the Kähler potential induce additional, sub-leading terms in the scalar potential [180–186]. Together with the previously noted leading-order flatness, this allows for the scenario of fibre inflation [290–294], where a particular geometric structure and certain assumptions about the loop corrections induce a realistic inflationary potential. Alternatively, the leading-order flatness of the additional moduli τ_i may be violated non-perturbatively. The resulting scenario is known as blowup-inflation [187], and in this case loop corrections represent a potential problem for the slow-roll requirement, as noted in [290]. Further variants of inflation using the additional Kähler moduli and their flatness were discussed in [399,422–424] and reviews of this broader setting, which one may call 'Kähler moduli inflation', may be found in [177, 178].

Our present proposal builds on the following key observation: very generically, and without any particular assumption about the functional form of loop corrections, the LVS setting allows for inflation in a regime where $\tau_i/\mathcal{V}^{2/3} \sim \mathcal{O}(1)$. To see this, let us disregard the stabilised modulus τ_s, treat the volume as a fixed parameter, $\mathcal{V} \gg 1$, and write the loop-corrected potential as

$$V \sim \frac{|W_0|^2}{\mathcal{V}^3}\left[\mathcal{O}(1) - \frac{c_{\text{loop}}}{\mathcal{V}^{1/3}} f\left(\tau_i/\mathcal{V}^{2/3}\right)\right]. \tag{8.2}$$

Here f is a generic function of the variables $\tau_i/\mathcal{V}^{2/3}$, which we choose to parametrize the field space of the additional Kähler moduli. Furthermore, c_{loop} is a numerical factor to be discussed in more detail below. Consider a generic trajectory on this field space and define ϕ to be a canonically normalised field (our potential inflaton) parametrising this trajectory. Restricting attention to this field ϕ and recalling that the metric $\partial^2 K/(\partial\tau_i\partial\tau_j)$ on Kähler moduli space is homogeneous of degree -2 in the τ_i, one easily shows that

$$\epsilon \equiv \frac{1}{2}\left(\frac{1}{V(\phi)}\frac{dV(\phi)}{d\phi}\right)^2 \sim \left(\frac{c_{\rm loop}}{\mathcal{V}^{1/3}}\frac{\mathrm{d}f}{\mathrm{d}\phi}\right)^2 \quad , \qquad \eta \equiv \frac{1}{V(\phi)}\frac{\mathrm{d}^2V(\phi)}{\mathrm{d}\phi^2} \sim \frac{c_{\rm loop}}{\mathcal{V}^{1/3}}\frac{\mathrm{d}^2 f}{\mathrm{d}\phi^2}\,. \tag{8.3}$$

Here, by abuse of notation, f denotes the function obtained from $f(\tau_i/\mathcal{V}^{2/3})$ by restriction to the single variable ϕ defined above. We then generically have $f'' \sim f' \sim \mathcal{O}(1)$ and hence slow-roll inflation, parametrically guaranteed by $\mathcal{V} \gg 1$.

Crucially, the τ_i-independent term $|W_0|^2/\mathcal{V}^3 \times \mathcal{O}(1)$ in (8.2) arises because we assume that, at the end of inflation, one of the additional Kähler moduli τ_i settles in a non-perturbatively generated minimum. This minimum is analogous to the minimum which stabilises the small-cycle modulus τ_s in the LVS scenario. Hence, we need at least one of the τ_i (or an appropriate combination thereof) to be of blowup type. The depth of this minimum is $\sim |W_0|^2/\mathcal{V}^3$ which is larger than the loop-induced potential $\sim |W_0|^2/\mathcal{V}^{10/3}$. We are thus dealing with a relatively flat inflationary plateau. Its profile is determined by loops and its boundary by the non-perturbative stabilisation of one or several blowup cycles among the τ_i. Together, this suggests the name *'Loop Blowup Inflation'*. We note that this setting has been pointed out previously at the end of section 4.2.1 of [177]. Yet, this model of inflation has never been analysed nor confronted with phenomenological constraints.

In chapter 9, we study the simple special case with a single additional Kähler modulus, $\tau_i \equiv \tau_\phi$ which, as explained above, has to be of blowup type. In this setting the form of the dominant loop correction in the regime $\tau_s \ll \tau_\phi \ll \mathcal{V}^{2/3}$ is actually known from an explicit analysis in [186], consistently with the extrapolation from the torus-orientifold case by the Berg-Haack-Pajer conjecture [183]. Then, approaching the regime of $\tau_\phi \lesssim \mathcal{V}^{2/3}$ from the side of small τ_ϕ, we may hope to maintain control of the inflationary potential while also achieving realistic phenomenology. This allows for a very explicit case study. The setting may be viewed as deriving from blowup inflation in a regime where the (naively fatal) loop corrections are taken into account and inflation is saved at the price of moving to much larger values of τ_ϕ.

In chapter 10, we derive the inflationary predictions of the simplest realisation of Loop Blowup Inflation, as described above. There, we additionally address questions of parametric and numerical control and further investigate stringy restrictions on parameters of the Calabi-Yau geometry.

We then devote chapter 11 to the study of other regimes of the inflationary potential: First, we consider the regime $\tau_\phi \sim \mathcal{V}^{2/3}$, where the functional form of the loop corrections becomes more complicated. Second, we quantify how small loop corrections would have to become to make a transition to blowup inflation.

Finally, a detailed phenomenological assessment of the simple scenario from chapter 10, including reheating, dark radiation constraints and an estimate of inflationary parameters is given in chapter 12

9. The Simplest Realisation of Loop Blowup Inflation

Our goal is to implement the central idea outlined in chapter 8 using a concrete and simple example. For this, we choose the volume to have the form

$$\mathcal{V} \;=\; \tilde{\mathcal{V}}(\tau_b, \tau_\phi) - \lambda_s \tau_s^{3/2} \;=\; \tau_b^{3/2} - \lambda_\phi \tau_\phi^{3/2} - \lambda_s \tau_s^{3/2} \,. \tag{9.1}$$

In other words, we assume that in addition to the big and small cycles τ_b and τ_s of the LVS [101, 102], there is just one further 4-cycle and that the latter is of blowup type.

Let us discuss our setup and notation in more detail: In the above, τ_i are the real parts of the Kähler moduli

$$T_i = \tau_i + i\,\theta_i \,, \quad i \in \{b,\, s,\, \phi\} \,, \tag{9.2}$$

with θ_i their axionic partners. The constants λ_s and λ_ϕ represent ratios of triple intersection numbers. The Kähler potential K, including the leading α' correction [101, 271], reads

$$K = \mathcal{K}_{cs} - 2\log(\mathcal{V} + \hat{\xi}/2) \,, \tag{9.3}$$

with $\mathcal{K}_{cs}$ depending only on complex structure moduli and axio-dilaton. Since these are stabilised by fluxes [97], $\mathcal{K}_{cs}$ can be treated as a constant. Furthermore, we have $\hat{\xi} = -\zeta(3)\,\chi\big/2\,(2\pi)^3\, g_s^{3/2}$, where χ denotes the Calabi-Yau Euler number. The superpotential is given by

$$W = W_0 + A_s e^{-a_s T_s} + A_\phi e^{-a_\phi T_\phi} \,, \tag{9.4}$$

with W_0 the flux-induced constant and the non-perturbative corrections being induced by E3-branes ($a_{s,\phi} = 2\pi$) or gaugino condensation ($a_{s,\phi} = 2\pi/N$). The prefactors $A_{s,\phi}$ are $\mathcal{O}(1)$ numbers depending on the complex structure moduli.

The super- and Kähler potential give rise to the F-term scalar potential for the Kähler moduli

$$V(\mathcal{V}, \tau_s, \tau_\phi) = V_{\rm LVS}(\mathcal{V}, \tau_s) + \hat{V}\left[\mathcal{A}_\phi \frac{\sqrt{\tau_\phi}\, e^{-2a_\phi \tau_\phi}}{\mathcal{V}} - \mathcal{B}_\phi \frac{\tau_\phi\, e^{-a_\phi \tau_\phi}}{\mathcal{V}^2}\right] , \tag{9.5}$$

where $V_{\rm LVS}$ is the scalar potential of the underlying 2-moduli LVS model

$$V_{\rm LVS}(\mathcal{V}, \tau_s) = \hat{V}\left[\mathcal{A}_s \frac{\sqrt{\tau_s}\, e^{-2a_s \tau_s}}{\mathcal{V}} - \mathcal{B}_s \frac{\tau_s\, e^{-a_s \tau_s}}{\mathcal{V}^2} + \frac{3\hat{\xi}}{4\mathcal{V}^3}\right] . \tag{9.6}$$

Here we introduced the abbreviations

$$\hat{V} = \left(\frac{g_s e^{K_{cs}}}{8\pi}\right) W_0^2 \,, \quad \mathcal{A}_i = \frac{8(a_i A_i)^2}{3W_0^2 \lambda_i} \,, \quad \mathcal{B}_i = 4\frac{a_i |A_i|}{W_0} \,, \tag{9.7}$$

with $i = s, \phi$ labelling the blowup cycles. Famously, the potential (9.6) has an AdS minimum at

$\tau_s \sim (\hat{\xi}/2\lambda_s)^{2/3}$ and $\mathcal{V} \sim \exp(a_s\tau_s)$. In the full potential (9.5), the additional τ_ϕ-dependent term stabilises τ_ϕ such that $a_\phi\tau_\phi \sim \ln\mathcal{V}$, analogously to τ_s. Moreover, if we assume [187, 289]

$$\lambda_\phi a_\phi^{-3/2} \ll \lambda_s a_s^{-3/2}\,, \tag{9.8}$$

then the presence of τ_ϕ and its stabilisation do not affect the values of $\mathcal{V}$ and τ_s derived from (9.6). This remains true even during inflation, when τ_ϕ is displaced from its late-time AdS minimum.

The AdS minimum may be uplifted to a Minkowski minimum by adding a positive term

$$V_{\rm up}(\mathcal{V}) = \frac{\hat{V}\mathcal{D}}{\mathcal{V}^2}\,, \tag{9.9}$$

to the potential in (9.5), such that $(V + V_{\rm up})\big|_{\rm minimum} = 0$.[40] We note that, while the feasibility of the famous anti-D3-brane uplift [100, 425] has been challenged in this context [132–136], we are here simply assuming that *some* form of viable uplift for the LVS can be realised (see e.g. [103, 108, 110, 113–116, 120, 121]).

Note that we have arranged the expression for the potential in (9.5) such that one can clearly distinguish the standard LVS scalar potential $V_{\rm LVS}$, independent of the additional modulus τ_ϕ, and the non-perturbative corrections giving τ_ϕ a non-trivial potential. Crucially, these stabilise τ_ϕ at a relatively small value. If no further terms were added, τ_ϕ could be the inflaton of blowup-inflation [187]. In this case, an inflationary plateau appears in the region where τ_ϕ is large enough for the exponential terms to become negligible. Our proposal is different: We will include loop corrections, making the potential for τ_ϕ less flat but, in the regime where τ_ϕ comes close to $\mathcal{V}^{2/3}$, still suitable for slow roll inflation. In fact, we will argue that this is an unavoidable outcome. In other words, blowup-inflation *necessarily* turns into a variant of what we would like to call 'Loop Blowup Inflation'.

Let us be more explicit by specifying the leading loop correction to the potential, as it arises from a loop effect in the Kähler potential K

$$\delta V_{\rm loop} \sim -\frac{\hat{V}}{\mathcal{V}^3}\,\frac{c_{\rm loop}}{\mathcal{V}^{1/3}}\,f\left(\tau_\phi/\mathcal{V}^{2/3}\right)\,. \tag{9.10}$$

Here f is a generic function of $\tau_\phi/\mathcal{V}^{2/3}$. The full potential V hence reads

$$V(\mathcal{V},\tau_s,\tau_\phi) = V_{\rm LVS}(\mathcal{V},\tau_s) + V_{\rm up}(\mathcal{V}) + \hat{V}\left[\,\mathcal{A}_\phi\,\frac{\sqrt{\tau_\phi}\;e^{-2a_\phi\tau_\phi}}{\mathcal{V}} - \mathcal{B}_\phi\,\frac{\tau_\phi\;e^{-a_\phi\tau_\phi}}{\mathcal{V}^2}\right] + \delta V_{\rm loop}\,. \tag{9.11}$$

Two key points have to be made concerning this potential: One concerning our claim that a τ_ϕ-dependent correction $\delta V_{\rm loop}$ is unavoidable and one, closely related, concerning the form of this correction and the one of the function f in (9.10).

We start with the claim that such a correction is unavoidable. Indeed, to realise the minimum which stabilises τ_ϕ after inflation, we have to insist that W receives a non-perturbative correction $\sim \exp(-a_\phi T_\phi)$, cf. (9.4). Requiring this non-perturbative correction implies the presence of an O-plane in the vicinity of the blowup-cycle τ_ϕ in order to break SUSY locally to $\mathcal{N} = 1$.[41] This, in turn implies the presence of the claimed loop effect.

[40] For a precise determination of the constant $\mathcal{D}$ see eq. (4.7) of [289].

[41] Note that fluxes can break SUSY to $\mathcal{N} = 1$ as well. However fluxes cannot introduce non-perturbative corrections, because fluxes dilute with the volume $\mathcal{V} = {\rm Re}T_b$. Hence, the corrections would be of the form $\mathcal{A}(\mathcal{V})\exp(-a_\phi\tau_\phi)$ and spoil the holomorphicity of W in T_b. A familiar argument can be found in sect. 3.2 of [187].

To see this, recall that it has been argued in EFT language that corrections suppressed by $\mathcal{V}^{10/3}$ arise from 10d field-theory loops in $\mathcal{N} = 1$ CY orientifold models [180,185]. At the same time, this has been derived in a very impressive, explicit string-loop calculation, which is however necessarily restricted to torus-based geometries [181]. A generalization to the CY case was suggested in [182]. This was developed and partially debated in [186]. We provided more details on this in section 3.3.2. Not to loose focus, we state here only that, to the best of our knowledge, one-loop closed-string effects unavoidably induce a correction of the type given in (9.10) as soon as the relevant geometry breaks SUSY to $\mathcal{N} = 1$.

More precisely, in the regime where a blowup cycle τ_ϕ is smaller than any other nearby cycle, one can argue in effective field theory for a loop correction depending only on τ_ϕ and, through Weyl rescaling of the 4d metric, on $\mathcal{V}$. As discussed in more detail in [186], this leads to

$$f \sim \frac{\mathcal{V}^{1/3}}{\sqrt{\tau_\phi}} \qquad \text{and hence} \qquad \delta V_{\text{loop}} \simeq -\frac{\hat{V}}{\mathcal{V}^3}\,\frac{c_{\text{loop}}}{\sqrt{\tau_\phi}}\,. \tag{9.12}$$

Here any unknown $\mathcal{O}(1)$ factors in f have been absorbed in c_{loop}. We also note that this numerical factor, which does not involve g_s, is expected to be small in (higher-dimensional) analogy to the familiar loop suppression factor $1/(16\pi^2)$ of 4d field theory (see [186] for an estimate).

As explained above, we may choose CY data such that our potential inflaton τ_ϕ can roll while $\mathcal{V}$ and τ_s remain stabilised (up to small shifts) [187, 289]. We may then work with a potential depending on τ_ϕ only

$$V(\tau_\phi) = V_0\left[1 + \mathcal{A}_\phi\frac{\mathcal{V}^2}{\beta}\,\sqrt{\tau_\phi}\,e^{-2a_\phi\tau_\phi} - \mathcal{B}_\phi\frac{\mathcal{V}}{\beta}\,\tau_\phi\,e^{-a_\phi\tau_\phi} - \frac{c_{\text{loop}}}{\beta\sqrt{\tau_\phi}}\right]. \tag{9.13}$$

Here, we defined

$$V_0 = \left[V_{\text{LVS}}(\mathcal{V},\tau_s) + V_{\text{up}}(\mathcal{V})\right]\Big|_{\text{minimum}} = \frac{\hat{V}\beta}{\mathcal{V}^3} \tag{9.14}$$

and [289]

$$\beta \simeq \frac{3}{2}a_\phi^{-3/2}\lambda_\phi\,(\ln\mathcal{V})^{3/2}\,. \tag{9.15}$$

The constant β encodes the proper adjustment of the uplifting term, ensuring that V_0 precisely compensates the negative value arising from the two exponential terms in (9.13) after minimization in τ_ϕ. Obviously, the resulting value of β is corrected due to the presence of the c_{loop} term, but this is not important at our level of precision.

If we displace τ_ϕ within the regime $\tau_s \ll \tau_\phi \ll \tau_b$ we can neglect the two exponential terms in (9.13). The canonically normalised field corresponding to τ_ϕ is

$$\phi = \sqrt{\frac{4\lambda_\phi}{3\mathcal{V}}}\,\tau_\phi^{3/4}\,. \tag{9.16}$$

In terms of this field, the potential then takes the form

$$V(\phi) = V_0\left(1 - \frac{c_{\text{loop}}}{\mathcal{V}^{1/3}}\,\frac{1}{\beta}\,\left(\frac{4\lambda_\phi}{3}\right)^{1/3}\,\phi^{-2/3}\right). \tag{9.17}$$

This characterizes the slow-roll regime in our simplest scenario, to be analysed below.

10. Loop Blowup Inflation and Slow Roll

To begin, we present in fig. 10.1 a plot of our full inflationary potential (9.13) for different values of c_{loop}. The orange curve corresponds to $c_{\text{loop}} = 0$ and is adjusted such that the minimum is at zero energy. The blue and green curves have positive and negative c_{loop} respectively. Obviously, when applying either of them to cosmology, the constant term must be adjusted such that its minimum (rather than that of the orange curve) is Minkowski. Note that we used extreme values for c_{loop} in fig. 10.1 to make the loop effect more visible.

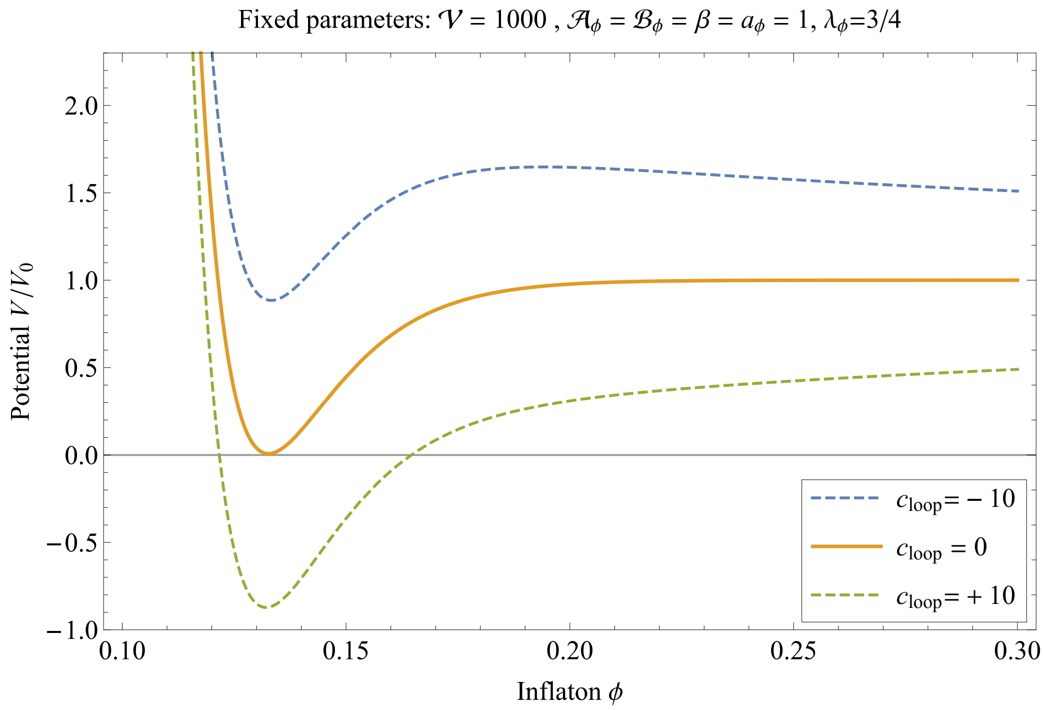


Fig. 10.1: Plot of the potential (9.13) for $c_{\text{loop}} = \pm 10$ and $c_{\text{loop}} = 0$.

As is well-known and also visible in the plot, the pure blowup case with $c_{\text{loop}} = 0$ has a slow-roll region which starts relatively close to the minimum. The reason is that the potential approaches a constant exponentially fast. As we argued above and will quantify later, the loop correction in general destroys this slow-roll region. In case $c_{\text{loop}} > 0$, slow roll can then be regained at much larger ϕ. For $c_{\text{loop}} < 0$, this is impossible.

In the rest of this chapter, we assume that the approximate potential of (9.17) can be used in the inflationary regime. This implies in particular that $\phi \sim \tau_\phi/\mathcal{V}^{2/3}$ is small enough such that the leading-order term in the expansion of f in ϕ is sufficient. Yet, we want to emphasise that this is merely one regime in which slow roll can be realised. In chapter 11, we study two additional regimes of slow roll: The regime where sub-leading terms in the small-ϕ expansion of f are relevant and the regime where loop corrections become negligible due to a small value of c_{loop}.

10.1. Inflation in the Simplest Realisation

In this subsection, we assume that the approximate potential (9.17) is sufficient to describe the observable part of slow-roll inflation. This implies that the exponentially suppressed terms are negligible, which will always hold as long as c_{loop} is not too small. It also relies on the hierarchy

$$\tau_\phi \ll \mathcal{V}^{2/3} \qquad \text{or equivalently} \qquad \phi \ll 1\,, \tag{10.1}$$

and the requirement that this hierarchy is strong enough during the observable ~ 50 e-foldings constrains c_{loop} from above. For convenience, let us rewrite the potential (9.17) in the following form

$$V = V_0 \left(1 - c_{\text{loop}} \frac{b}{\phi^{2/3}}\right), \tag{10.2}$$

where we now defined the constant

$$b = \frac{1}{\beta}\left(\frac{4\lambda_\phi}{3\mathcal{V}}\right)^{1/3} \equiv \frac{\sigma_\phi}{\beta \mathcal{V}^{1/3}}\ . \tag{10.3}$$

The slow roll parameters following from the potential (10.2) read

$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 \simeq \frac{2}{9} b^2 c_{\text{loop}}^2 \phi^{-10/3}\ , \tag{10.4}$$

$$\eta = \frac{V''}{V} \simeq -\frac{10}{9} b\, c_{\text{loop}} \phi^{-8/3}\ . \tag{10.5}$$

We can see that for small values of the factor $(b\, c_{\text{loop}})$ a slow roll regime can be realised. With the parameters ϵ and η, we can determine the spectral index n_s, tensor-to-scalar ratio r and the number of e-folds N_e

$$n_s - 1 = 2\,\eta - 6\,\epsilon \simeq -\frac{20}{9}\, b\, c_{\text{loop}}\, \phi^{-8/3}\ , \tag{10.6}$$

$$r = 16\,\epsilon = \frac{32}{9}\, b^2\, c_{\text{loop}}^2\, \phi^{-10/3}\ , \tag{10.7}$$

$$N_e(\phi_*) = \int_{\phi_E}^{\phi_*} \frac{V}{V'}\, \mathrm{d}\phi \simeq \frac{9}{16}\frac{\phi_*^{8/3}}{b\, c_{\text{loop}}}\ , \tag{10.8}$$

where ϕ_E and ϕ_* denote the values of the inflaton field at the end of inflation and at the scale of horizon exit respectively. In (10.8) we used $\phi_E \ll \phi_*$. This shall be the case study for the following section 10.2 , where we aim to match cosmological constraints. Specifically, we need to assure that we obtain the right number of e-folds N_e and match the amplitude of primordial density fluctuations A_s. These requirements will fix ϕ_* and $\mathcal{V}$ in terms of N_e and $\hat{A}_s$. The hope is now that the large parameters N_e and the inverse spectrum normalisation $\hat{A}_s^{-1}$ are sufficient to make the volume $\mathcal{V}$ large enough to realise a controlled LVS model. At the same time, the condition $\phi_\star \ll 1$ has to be maintained. This will be studied in the next section taking all the other parameters into account.

10.2. Control & Constraints

In this section, we implement phenomenological constraints to our loop blowup inflation model, focusing on the simplest realisation. The inflationary parameters specific to this scenario have been obtained in section 10.1. Our primary objective is to assess whether the inflaton cycle ϕ remains within the blowup regime, $\phi \ll 1$. In section 10.2.1, we derive parametric formulas for ϕ and $\mathcal{V}$, enabling us to evaluate the feasibility of achieving this goal. Following this, section 10.2.2 will demonstrate, using specific numerical values for the key parameters, that our model indeed remains in the controlled regime. Moreover, in section 10.2.3 we discuss the constraint imposed on the volume $\mathcal{V}$ which ultimately challenges the stability of the anti-D3-brane uplift mechanism within our specific model.

10.2.1. Parametric Control

First, we will discuss the parametric control that can be achieved in our model while respecting phenomenological constraints. Our first constraint is to match the amplitude of primordial density fluctuations A_s. The spectrum of primordial density perturbations Δ_s^2 was defined in (2.64)

$$\Delta_s^2 = A_s \left(\frac{k}{k_\star}\right)^{n_s-1} , \tag{10.9}$$

and the amplitude A_s was measured by Planck [8]

$$A_s \times 10^9 = 2.105 \pm 0.030 . \tag{10.10}$$

The scalar density perturbations Δ_s^2 can also be expressed in terms of the inflation potential V

$$\Delta_s^2 = \frac{1}{24\pi^2}\frac{V}{\epsilon}\bigg|_{\phi=\phi(k)} . \tag{10.11}$$

Evaluating (10.11) at the scale of horizon exit, $\phi_* = \phi(k_*)$, is thus equivalent to

$$\frac{V^3}{V'^2}\bigg|_{\phi=\phi_\star} = \hat{A}_s , \tag{10.12}$$

where we introduced

$$\hat{A}_s = 12\pi^2 A_s \simeq 2.5 \times 10^{-7} . \tag{10.13}$$

Using the potential (10.2) and the approximation $1 - c_{\text{loop}} b/\phi_\star^{2/3} \simeq 1$ in (10.12) yields

$$\frac{9V_0}{4c_{\text{loop}}^2}\frac{\phi_*^{10/3}}{b^2} = \hat{A}_s . \tag{10.14}$$

Recall, that V_0 and b contain the volume $\mathcal{V}$, cf. (9.14) and (10.3). Thus, we interpret (10.14) as a relation between the initial value of the inflation ϕ_* and the volume $\mathcal{V}$.

In a second step, the required number of e-folds N_e is determined by the post-inflationary history specific to every inflation model. Yet, for slow roll inflation models N_e will always be in the range $40 \leq N_e \leq 60$. The particular analysis for our model will be performed in sect. 12.1 where we will find $N_e \sim 50$. Thus, we can essentially treat N_e as a fixed constant and hence obtain a second constraint from (10.8) which determines N_e.

The two relations, (10.8) and (10.14), can be solved solve for ϕ_* and $\mathcal{V}$. Using (10.8) we solve for $\mathcal{V}$ in terms of ϕ_* which gives

$$\mathcal{V} = A\,\phi_*^{-8}\,, \qquad A = \left(\frac{16 N_e \sigma_\phi c_{\rm loop}}{9\beta}\right)^3 . \tag{10.15}$$

Inserting this relation for $\mathcal{V}$ in (10.14) we find the following equation

$$\phi_* = \left(BA^7\right)^{1/66}\,, \qquad B = \left(\frac{4\hat{A}_s \sigma_\phi^2 c_{\rm loop}^2}{9\beta^3 \hat{V}}\right)^3 . \tag{10.16}$$

Thus, we can express ϕ_* and $\mathcal{V}$ in terms of A and B. Inserting the definitions of A and B we find

$$\phi_\star = \left(\frac{2^{17}\pi}{3^8}\right)^{1/11}\left[\frac{\hat{A}_s\, c_{\rm loop}^9\, N_e^7\, \sigma_\phi^9}{N_Q\, \beta^{10}}\right]^{1/22}, \quad \mathcal{V} = \left[\frac{1}{2^4\, 3^2\, \pi^8}\frac{N_e^5\, N_Q^4\, \beta^7}{\hat{A}_s^4\, \sigma_\phi^3\, c_{\rm loop}^3}\right]^{1/11}, \tag{10.17}$$

where we introduced the parameter N_Q

$$N_Q \equiv 4\pi\frac{g_s e^{\mathcal{K}_{cs}} W_0^2}{2}\,, \tag{10.18}$$

which contains all W_0 and g_s dependencies. Note that the quantity N_Q is bounded by the negative tadpole of the orientifold $-Q_3$ [162]

$$N_Q < -Q_3 \sim \mathcal{O}(100)\,, \tag{10.19}$$

and hence can only increase $\mathcal{V}$ and decrease ϕ_* to a certain limit. In type IIB, the maximal known value for $-Q_3 \leq 252$ is estimated using the Lefschetz theorem [426–428] and inserting the largest Hodge numbers obtained from the Kreuzer-Skarke dataset [429]. Parametrically, a small value for ϕ_* and a large volume $\mathcal{V}$ can be achieved due to the small amplitude $\hat{A}_s$ and the possible small factor $c_{\rm loop}$. In addition, $N_Q \sim \mathcal{O}(100)$ can provide further but limited improvement.

10.2.2. Numerical Control

We want to emphasise that the analysis of this section is based on work in collaboration [2] and was carried out by Luca Brunelli under the supervision of Michele Cicoli. To maintain a coherent presentation we include this section which we have reformulated in our own words.

This section turns to a numerical evaluation of the constraints (10.8) and (10.14). For this purpose, we insert typical values for the parameters and evaluate the results for consistency.

First of all, consider the relation for N_e (10.8) where N_e is again treated as a fixed constant. This gives a relation between $\phi_*, \mathcal{V}$ and $c_{\rm loop}$. For simplicity, consider a value of $N_e \sim 50$ (to be midway between $40 \leq N_e \leq 60$), which yields

$$\frac{(\mathcal{V}\,\phi_*^8)^{1/3}}{c_{\rm loop}} \sim 100\,. \tag{10.20}$$

Furthermore, let us spell out the condition (10.14) while neglecting $\mathcal{O}(1)$ constants

$$\left(\frac{g_s\, e^{\mathcal{K}_{CS}}}{8\pi}\right)\frac{W_0^2}{\mathcal{V}^{7/3}}\frac{\phi_*^{10/3}}{c_{\text{loop}}^2} \sim 10^{-7}\,. \tag{10.21}$$

For $g_s \sim 0.1$ and $e^{\mathcal{K}_{CS}} \sim 1$, the prefactor in parenthesis in (10.21) is of order 10^{-2} and using (10.20) in (10.21) gives

$$\frac{W_0^2\,\phi_*^{2/3}}{\mathcal{V}^{8/3}\,c_{\text{loop}}} \sim 10^{-7}\,. \tag{10.22}$$

In order to remain in the blowup regime, we focus on scenarios for which we may assume $\phi_* \sim \mathcal{O}(0.1)$. Under this assumption, (10.22) yields

$$\frac{W_0^2}{\mathcal{V}^{8/3}\,c_{\text{loop}}} \sim 10^{-6}\,, \tag{10.23}$$

which demonstrates that the allowed range of the volume crucially depends on values of c_{loop} and W_0. As was already noted in (10.19), W_0 cannot get arbitrarily large and may be set to $W_0 \sim \mathcal{O}(10)$. In this case, the following relation between $\mathcal{V}$ and c_{loop} holds

$$\frac{1}{\mathcal{V}^{8/3}\,c_{\text{loop}}} \sim 10^{-8}\,. \tag{10.24}$$

The leading loop correction is of winding-type, thus c_{loop} is expected to be suppressed only by π factors and does not involve any g_s factors (cf. the last paragraph on p. 41 of [186] where an estimate gives $c_{\text{loop}} \sim (2\pi)^{-4}$). To make a cautious estimate, assume that c_{loop} is of the order $c_{\text{loop}} \sim \mathcal{O}(10^{-2})$. Using this estimate of c_{loop} in (10.24) yields the following maximal value for the volume $\mathcal{V}$

$$\mathcal{V} \sim 10^4\,, \tag{10.25}$$

which corresponds to a rather small volume wrt. the range of volumes one for example obtains in the standard version of Kähler moduli inflation [187]. Relaxing the assumption on W_0 and allowing for $W_0 \sim \mathcal{O}(1-10)$, gives a suitable range of values for the inflationary volume

$$\mathcal{V} \sim 10^3 - 10^4\,. \tag{10.26}$$

Note that these results were obtained under the assumption $\phi_* \sim \mathcal{O}(0.1)$. As will be demonstrated in sect. 12, this precisely corresponds to the order of magnitude of ϕ_* which will be derived in a full phenomenological assessment. Moreover, the above analysis may be justified a posteriori. Using the following values for the parameters $c_{\text{loop}} \sim 0.01$, $\mathcal{V} \sim 10^4$, $N_e \simeq 50$ in (10.8) one can explicitly solve for the respective value of $\phi_* \simeq 0.3$. Therefore, it can be explicitly shown that the simplest implementation of loop blowup inflation remains in the controlled regime.

10.2.3. 10d Curvature Corrections

Furthermore, we face another consistency constraint on the volume $\mathcal{V}$ if the uplift mechanism relies on warped throats. In the case of anti-D3-uplift, warped throats are present and we have to take a correction into account which arises as a combination of the leading α'^3-correction (which corrects the Kähler potential in (3.66)) and a non-constant warp factor [134, 135]. Control over these corrections can be assured if the control parameter c_N, as defined in [134], satisfies $c_N \gg 1$.

The parameter c_N is thereby defined as

$$c_N = \frac{\mathcal{V}^{2/3}}{N} \frac{(2\,\lambda_s)^{2/3}}{10\, a_s\, \hat{\xi}^{2/3}} \,, \tag{10.27}$$

where N denotes the amount of flux in the warped throat. We can rewrite (10.27) purely in terms of the volume $\mathcal{V}$

$$c_N = \frac{\mathcal{V}^{2/3}}{N} \frac{1}{10} \frac{1}{\ln(\mathcal{V})} \,, \tag{10.28}$$

since τ_s stabilises $\mathcal{V}$ at $\mathcal{V} \sim \exp(a_s \tau_s)$ where in addition $\tau_s \sim \hat{\xi}^{2/3}/(2\lambda_s)^{2/3}$, see (3.71). Solving the above equation approximately by neglecting the $\ln(\mathcal{V})$ contribution yields for $\mathcal{V}$

$$\mathcal{V} = (10 N c_N)^{3/2} \,. \tag{10.29}$$

The requirement of control, i.e. $c_N \gg 1$, hence yields a constraint on $\mathcal{V}$

$$\mathcal{V} \gg (10N)^{3/2} \,. \tag{10.30}$$

The minimal value for N which allows for an anti-D3-brane uplift can be obtained from the parametric tadpole constraint (PTC) [134]. To obtain N_{min} we use the minimal value for $g_s M^2 = 144$ from [132, 136] and applied the PTC [134] which gives $N_{\text{min}} \approx 240$ such that we find the following lower bound on $\mathcal{V}$

$$\mathcal{V} \gg 10^5 \left(\frac{N}{240}\right)^{3/2} \,. \tag{10.31}$$

Note, in the derivation of (10.30) we neglected the logarithmic volume contribution and in the determination of N_{min} we further omitted the subleading terms of the PTC. Including any of these subleading corrections will substantially worsen the bound on $\mathcal{V}$ (10.31) for our application.

Therefore, if we insist on using the anti-D3-brane uplift, our inflation model may run into trouble as our volume will not be large enough, cf. (10.25). However, this may not apply to the other regime of inflation that we discuss in sect. 11.1 where we go to the regime $\phi \sim \mathcal{O}(1)$ including more subleading corrections in the ratios of 4-cycles. Ultimately, the constraint of this section vanishes if we consider alternative uplift mechanisms, like D-term effects [113] or dilaton-dependent non-perturbative effects [110, 114].

11. Further Inflationary Regimes

The potential (9.17) represents only one possible regime of the general potential (9.11) which can realise slow roll inflation. So far, we remained in the blowup regime to maintain control over the leading loop correction which specified the function f in (9.12) . Simultaneously, we assumed that loop corrections are non-negligible and spoil the otherwise exponential flatness of the potential. Thus, two further regimes of the potential exist in which slow roll inflation could be realised.

First, we may leave the blowup regime, i.e. $\phi \sim 1$, and consequently loose knowledge about the explicit functional dependence of the loop corrections. However, as we approach this regime from small values of ϕ, we can expand f around the dominant loop correction, introducing subleading corrections which we incorporate into our analysis. In section 11.1, we will argue that a whole class of inflationary models can arise in this regime.

Second, it is possible that the loop factor c_{loop} has a negligible value, such that we can omit the loop corrections completely. This would lead us back to the original proposal of blowup inflation [187]. In section 11.2, we will determine a critical value for c_{loop} where we argue that this transition happens.

11.1. Beyond the Blowup Regime – Subleading Loop Corrections

In this section, we turn to the first alternative regime of the general potential (9.11) where we move beyond the blowup regime, i.e. $\phi \sim 1$. This implies a departure from the precise functional form of the leading loop correction (9.12). However, we can interpret the leading loop correction as the first term of a expansion of f in (9.10), i.e.

$$f \simeq \frac{\mathcal{V}^{1/3}}{\sqrt{\tau_\phi}} + 1 + \frac{\sqrt{\tau_\phi}}{\mathcal{V}^{1/3}} + \ldots \; . \tag{11.1}$$

These additional terms would change the potential (10.2) to

$$V = V_0 \left(1 - c_{\text{loop}}\, b \left[\frac{1}{\phi^{2/3}} + \mathfrak{a} + \mathfrak{b}\, \phi^{2/3} + \ldots \right] \right) \; , \tag{11.2}$$

where we introduced $\mathfrak{a}$, $\mathfrak{b}$ as arbitrary constants which generically should be $\mathcal{O}(1)$ and recall the definition of ϕ in (9.16). Depending on the signs of $\mathfrak{a}$ and $\mathfrak{b}$ (as well as prefactors from other terms in the expansion) we could find a whole class of models which are applicable to slow roll inflation. The factor $\mathfrak{a}$ only influences the height of the potential which we will neglect in the following since the correction is proportional to $(b c_{\text{loop}})$. Further, we will only include the first relevant correction $\sim \phi^{2/3}$ for the following discussion when approaching the regime of $\phi \sim 1$. The reasons will become clear momentarily.

The slow roll parameters for the potential (11.2) read

$$\epsilon \simeq \frac{1}{2} b^2 c_{\text{loop}}^2 \left[\frac{4}{9}\phi^{-10/3} - \frac{8}{9}\mathfrak{b}\phi^{-6/3} + \frac{4}{9}\mathfrak{b}^2\phi^{-2/3} \right] \; , \tag{11.3}$$

$$\eta \simeq -\, b\, \mathfrak{c}_{\text{loop}} \left[\frac{10}{9} \phi^{-8/3} - \frac{2}{9} \mathfrak{b} \phi^{-4/9} \right] . \tag{11.4}$$

We can see that slow roll is possible due to the small prefactor $(b\, \mathfrak{c}_{\text{loop}})$. Using these results we find for the number of e-foldings N_e

$$N_e(\phi_*) = \int_{\phi_E}^{\phi_*} \frac{3}{2} \frac{\phi^{5/3}}{b\mathfrak{c}_{\text{loop}} \left[1 - \mathfrak{b}\phi^{4/3}\right]} \mathrm{d}\phi \,. \tag{11.5}$$

Again, the exact number of N_e is determined by the post-inflationary history and we treat N_e as a constant such that we can read (11.5) as a constraint on the model which determines ϕ_*. Moreover, (11.5) is dominated by large values of ϕ and we may assume $\phi_* \gg \phi_E$. For $\mathfrak{b} > 0$ we consequently require a smaller value of ϕ_* to yield a predetermined value of N_e compared to the situation where $\mathfrak{b} = 0$. This is encouraging since this indicates that the presence of the $\mathfrak{b}$-correction actually leads to a more robust scenario for inflation and we are not required to go to very large values of ϕ_*.

We now want to proceed with the following logic. Assume that the analysis of the potential (11.2) with $\mathfrak{b} = 0$ requires us to go to largish values of ϕ_* e.g. to match the required N_e. Indeed, we found in sect. 10.2.2 that the required value in the simplest realisation will be of the order $\phi_* \sim 0.1$. Thus, the additional terms from (11.1) slowly become important. Still, we may assume $\mathfrak{b}\phi^{-4/3} \ll 1$ and treat $\mathfrak{b}\phi^{-4/3}$ as a correction. In this situation, we can approximate (11.6)

$$N_e \approx \frac{9}{16} \frac{\phi_*^{8/3}}{b\, \mathfrak{c}_{\text{loop}}} (1 + 2\mathfrak{b}\phi_*^{4/3}) \,, \tag{11.6}$$

which clearly confirms that the correction tends to fix ϕ_* at a smaller value. In addition, we want to analyse the effect on the normalisation of scalar perturbations

$$\hat{A}_s = \frac{9V_0}{4b^2 \mathfrak{c}_{\text{loop}}^2} \phi_*^{10/3} \left(1 + 2\mathfrak{b}\phi^{4/3}\right) \,, \tag{11.7}$$

where we used the the approximation $1 - \mathfrak{c}_{\text{loop}} b(\phi_*^{-2/3} + \mathfrak{b}\phi_*^{2/3}) \simeq 1$ and expanded to leading order in $\mathfrak{b}\phi_*^{4/3}$. Recall that the measured value for $\hat{A}_s$ is given by (10.13).

Analogous to section 10.2.1, we now solve (11.6) and (11.7) for ϕ_* and $\mathcal{V}$ where we have to keep in mind that the volume $\mathcal{V}$ is contained in V_0 and b, see (9.14) and (10.3). We can use (11.6) and solve for $\mathcal{V}$ in terms of ϕ_*

$$\mathcal{V} = A\, \phi_*^{-8} \left(1 + 2\mathfrak{b}\phi_*^{4/3}\right)^{-3} \,, \quad A = \left(\frac{16 N_e \sigma_\phi \mathfrak{c}_{\text{loop}}}{9\beta} \right)^3 \,. \tag{11.8}$$

Using this relation for $\mathcal{V}$ in (11.7) we find the following equation

$$\left(BA^7\right)^{2/99} = \phi_*^{4/3} \left(1 + 2\mathfrak{b}\phi_*^{4/3}\right)^{48/99} \,, \quad B = \left(\frac{4\hat{A}_s \sigma_\phi^2 \mathfrak{c}_{\text{loop}}^2}{9\beta^3 \hat{V}} \right)^3 \,. \tag{11.9}$$

For $\mathfrak{b} = 0$, we find that (11.8) and (11.9) are equivalent to (10.15) and (10.16) respectively. To leading order in $\mathfrak{b}\phi_*^{4/3}$, (11.8) is a quadratic equation for $\phi_*^{4/3}$ which is solved by

$$\phi_*^{4/3} = \frac{33}{64\mathfrak{b}}(-1 \pm 1) \pm \left(BA^7\right)^{2/99} \mp \frac{32}{33}\mathfrak{b}\left(BA^7\right)^{4/99} + \mathcal{O}\left(\mathfrak{b}^2\left(BA^7\right)^{6/99}\right) . \tag{11.10}$$

The physical solution corresponds to the upper sign choice in (11.10) which to leading order reads

$$\phi_* = \left(BA^7\right)^{1/66}\left[1 - \frac{8}{11}\mathfrak{b}\left(BA^7\right)^{2/99}\right] . \tag{11.11}$$

We can now clearly see that the correction with $\mathfrak{b} > 0$ will decrease ϕ_*. Furthermore, we obtain the solution for $\mathcal{V}$ by using (11.11) in (11.8) and again expanding to leading order

$$\mathcal{V} = A^{5/33}B^{-4/33}\left(1 - \frac{2}{9}\mathfrak{b}\left(BA^7\right)^{2/99}\right) . \tag{11.12}$$

Note, the results (11.11) and (11.12) are easily transferable to the results (10.17) by setting $\mathfrak{b} = 0$. Consequently, to match (11.6) and (11.7) the required volume is decreased for $\mathfrak{b} > 0$. Note, that the correction to $\mathcal{V}$ is smaller than to ϕ_* and ϕ_* decreases fast due to the corrections. Therefore, one can claim that the correction in principle allows for a more robust realisation of slow-roll inflation.

At this stage, it should at least be plausible that a variety of different models arise when the corrections in (11.1) are taken into account. We have also shown that the corrections can indeed be helpful, leading to a more robust scenario for inflation if the signs of the prefactors turn out right. It could be interesting to further investigate these scenarios. However, a posteriori this may justify to neglect all sub-corrections from (11.1) as we did in chapter 10.

11.2. Regime of Original "Kahler Moduli Inflation"

In this section, we turn to the second alternative regime of the general potential (9.11) where we consider a negligible value of c_{loop}. We aim to determine a critical value of c_{loop} for which we transition to the original model of blowup inflation (originally called "Kahler Moduli Inflation") [187].

For this purpose, imagine we can treat c_{loop} as a free parameter which we set to zero initially such that the loop corrections are truly absent. In this setting, we implement all our phenomenological constraints on N_e, n_s and the normalisation of scalar perturbations, thereby fixing some of the parameters of the blowup inflation model. Now we increase c_{loop}, insisting that the model is not significantly affected. In particular, we demand that the relative corrections $\delta\eta/\eta$ and $\delta\epsilon/\epsilon$ (and hence the correction to N_e) remain small. This will determine a critical value for c_{loop}. To obtain $\delta\epsilon/\epsilon$ and $\delta\eta/\eta$, we first rewrite the potential (9.13) as

$$V(\phi) = V_0 + V_{\text{bu}}(\phi) + V_{\text{loop}}(\phi) , \tag{11.13}$$

where we defined

$$V_{\text{bu}}(\phi) = -V_0\mathcal{B}_\phi\frac{\mathcal{V}}{\beta}\,\tau_\phi(\phi)\,e^{-a_\phi\tau_\phi(\phi)} , \quad V_{\text{loop}}(\phi) = -V_0\frac{c_{\text{loop}}}{\beta\sqrt{\tau_\phi(\phi)}} , \tag{11.14}$$

and $V_{\text{bu}}(\phi)$ corresponds to the relevant term in the potential which generates slow roll inflation as in [187]. Assuming, as explained, $V'_{\rm loop} \ll V'_{\rm bu}$, we have

$$\epsilon \simeq \frac{1}{2}\left(\frac{V'_{\rm bu}(\phi)+V'_{\rm loop}(\phi)}{V_0}\right)^2 = \frac{1}{2}\left(\frac{V'_{\rm bu}(\phi)}{V_0}\right)^2\left(1+\frac{V'_{\rm loop}(\phi)}{V'_{\rm bu}(\phi)}+\ldots\right)\,, \tag{11.15}$$

$$\eta \simeq \frac{V''_{\rm bu}(\phi)}{V_0}\left(1+\frac{V''_{\rm loop}(\phi)}{V''_{\rm bu}(\phi)}\right)\,. \tag{11.16}$$

Using (11.14), the relative corrections become

$$\frac{\delta\epsilon}{\epsilon} \equiv \frac{V'_{\rm loop}(\phi)}{V'_{\rm bu}(\phi)} = \frac{1}{2}\frac{c_{\rm loop}\tau_\phi^{-3/2}}{\mathcal{B}_\phi\mathcal{V}(a_\phi\tau_\phi-1)e^{-a_\phi\tau_\phi}}\,, \tag{11.17}$$

$$\frac{\delta\eta}{\eta} \equiv \frac{V''_{\rm loop}(\phi)}{V''_{\rm bu}(\phi)} = \frac{5}{8}\frac{c_{\rm loop}\tau_\phi^{-3/2}}{\mathcal{B}_\phi\mathcal{V}\,a_\phi\tau_\phi\;e^{-a_\phi\tau_\phi}}\left[a_\phi\tau_\phi-\frac{9}{4}+(4a_\phi\tau_\phi)^{-1}\right]^{-1}\,. \tag{11.18}$$

The ratio of these corrections at horizon crossing, $\tau_\phi = \tau_{\phi*}$, is

$$\frac{\delta\epsilon/\epsilon}{\delta\eta/\eta} = \frac{4}{5}a_\phi\tau_{\phi*}\,\frac{a_\phi\tau_{\phi*}-\frac{9}{4}+(4a_\phi\tau_{\phi*})^{-1}}{a_\phi\tau_{\phi*}-1} \gg 1\,, \tag{11.19}$$

where the last inequality holds since inflation takes place in a regime where $a_\phi\tau_{\phi*} > a_\phi\langle\tau_\phi\rangle$ and $a_\phi\langle\tau_\phi\rangle \approx \ln\mathcal{V} \gg 1$. Thus, the main correction we need to control is $\delta\epsilon/\epsilon$.

To estimate whether the correction due to loop effects is significant during blowup inflation we need to derive the respective values of $\tau_{\phi*}$ and $\mathcal{V}$ for blowup inflation. In pure blowup inflation, the number of e-folds N_e is given by (cf. (4.35) of [289])

$$N_e = \frac{\kappa_e}{\mathcal{V}^2}\,\frac{e^{a_\phi\tau_{\phi*}}}{(a_\phi\tau_{\phi*})^{3/2}}\,,\quad \kappa_e = \frac{3\beta W_0\lambda_\phi}{16a_\phi^{3/2}A_\phi}\,, \tag{11.20}$$

and the normalisation of scalar perturbation reads (cf. (4.39) of [289])

$$\hat{A}_s = \frac{\kappa_s}{\sqrt{a_\phi\tau_{\phi*}}(a_\phi\tau_{\phi*}-1)^2}\,\frac{e^{2a_\phi\tau_{\phi*}}}{\mathcal{V}^6}\,,\quad \kappa_s = \left(\frac{g_s e^{K_{cs}}}{8\pi}\right)\frac{3\lambda_\phi\beta^3W_0^2}{64a_\phi^{3/2}}\left(\frac{W_0}{A_\phi}\right)^2\,. \tag{11.21}$$

Recall, that the measured value for $\hat{A}_s$ is given by (10.13)

$$\hat{A}_s \simeq 2.5\times 10^{-7}\,. \tag{11.22}$$

We can use (11.20) and (11.21) to solve for $\tau_{\phi*}$ and $\mathcal{V}$. In a first step, we use (11.20) to solve for $\mathcal{V}$ in terms of $\tau_{\phi*}$ which gives

$$\mathcal{V} = \left(\frac{\kappa_e}{N_e}\,\frac{e^{a_\phi\tau_{\phi*}}}{(a_\phi\tau_{\phi*})^{3/2}}\right)^{1/2}\,, \tag{11.23}$$

such that we can eliminate $\mathcal{V}$ in (11.21). Suitably rewriting (11.21) in the limit $a_\phi\tau_{\phi*} \gg 1$ one finds

$$(a_\phi\tau_{\phi*})^{-2}e^{a_\phi\tau_{\phi*}} = \left(\frac{N_e}{\kappa_e}\right)^3\frac{\kappa_s}{\hat{A}_s}\,, \tag{11.24}$$

which is approximately solved by

$$a_\phi \tau_{\phi *} \simeq \ln \left[\left(\frac{N_e}{\kappa_e} \right)^3 \frac{\kappa_s}{\hat{A}_s} \right] + \mathcal{O}(1) \, . \tag{11.25}$$

Then, using (11.23) in (11.17) and replacing the exponential dependence with (11.24) yields

$$\frac{\delta \epsilon}{\epsilon} = \frac{1}{2} \frac{c_{\text{loop}}}{\mathcal{B}_\phi} \frac{N_e^2}{\kappa_e^2} \sqrt{\frac{\kappa_s}{\hat{A}_s}} \left(a_\phi \tau_{\phi *} \right)^{-3/4} \, , \tag{11.26}$$

where $a_\phi \tau_{\phi *}$ is the solution to (11.24). However, approximating $a_\phi \tau_{\phi *}$ with (11.25) and explicitly reinstating the factors in κ_e and κ_s gives

$$\frac{\delta \epsilon}{\epsilon} \simeq c_{\text{loop}} \sqrt{\frac{2}{27\pi} \frac{e^{K_{cs}} g_s}{\hat{A}_s \beta}} \frac{N_e^2 W_0 a_\phi^{5/4}}{\lambda_\phi^{3/2}} \left(\ln \left[\frac{8 a_\phi^3 A_\phi e^{K_{cs}} g_s N_e^3 W_0}{9\pi \hat{A}_s \lambda_\phi^2} \right] \right)^{-3/4} \, . \tag{11.27}$$

We can evaluate (11.27) for typical values of the underlying parameters ($a_\phi = 2\pi$, $g_s = 0.1$, $N_e = 50$, $\beta = W_0 = e^{K_{cs}} = \lambda_\phi = A_\phi = 1$) which yields

$$\delta \epsilon / \epsilon \simeq 1.9 \times 10^5 \, c_{\text{loop}} \, . \tag{11.28}$$

Thus, to be safe against considerable deviations in phenomenology we should demand

$$c_{\text{loop}} \ll 5.3 \times 10^{-6} \, . \tag{11.29}$$

However, small values of g_s and in particular of W_0 allow for larger values of c_{loop}. The coefficient c_{loop} is expected to be small due to a suppression by factors of 2π. A careful analysis [186] estimates $c_{\text{loop}} \sim (2\pi)^{-4} \sim 10^{-4}$. Indeed, this value could suffice to neglect the loop corrections, if g_s and W_0 are appropriately small. However, the smallness of g_s is limited by the fact the volume is exponentially large in $1/g_s$. Moreover, tuning W_0 to a small value goes together with making the volume small. This is in turn highly problematic because of warping corrections, as discussed in detail in [134, 135] and also in sect. 10.2.3. Thus, we expect that one can in fact never be in a controlled regime of pure blowup inflation.

12. Phenomenological Assessment

We want to emphasise that the analysis of this chapter is based on work in collaboration [2] and was carried out by Luca Brunelli under the supervision of Michele Cicoli. To maintain a coherent presentation we include this chapter which we have reformulated in our own words.

In this chapter we will include the full phenomenological assessment of the simplest realisation of slow roll inflation which we introduced in chapter 9. To arrive at the final results we analyse our model analogous to [289]. First, we break down the *post-inflationary history* of our model in sect. 12.1. This is necessary because the specific model at hand deviates from the standard cosmological evolution by introducing several periods of moduli domination between the end of inflation and reheating. However, we need to ensure that the energy density at horizon exit ρ_* evolves into the energy density we observe today ρ_0. Thus, taking into account the specific post-inflationary history, we can assess the exact number of e-foldings N_e between horizon exit and the end of inflation which is required for the matching of energy densities [289, 430]. Once N_e is fixed we can determine the field value of the inflaton ϕ_* at the start of inflation. We then use ϕ_* to compute the remaining phenomenological parameters in section 12.2. Eventually, we find that our prediction for the spectral index n_s is too high compared to Planck data [8]. However, these Planck results *do not* take into account the effects of *dark radiation*, which would increase the value of n_s, as was shown in [431]. Hence, in sect. 12.3 we address the amount of dark radiation produced in our model. This analysis will deliver an intrinsic explanation for our elevated value of n_s.

12.1. Post-inflationary Dynamics

In order to determine the post-inflationary dynamics we have to specify our model further. First of all we have to include a SM model sector to determine how reheating occurs. We choose to implement the SM on a stack of D7-branes wrapping a 4-cycle τ_{SM} which we stabilise by loop corrections.

In addition, during inflation the volume modulus is displaced from its global minimum. Hence, at the end of inflation the volume modulus starts to coherently oscillate around the minimum and dilute like matter. This introduces a modulus which will affect the post-inflationary history. After Weyl rescaling to 4d Einstein frame the volume modulus couples like gravity to all fields in the 4d theory. In [365] the dominant decay channel for the volume modulus was computed which is given by the loop enhanced Higgs coupling to the SM. The respective decay rate is given by

$$\Gamma_{\mathcal{V}} \sim \frac{c_l^2 \, W_0^3}{(\ln \mathcal{V})^{3/2} \, \mathcal{V}^{5/2}} \, M_{\rm Pl} \,, \tag{12.1}$$

where c_l denotes a loop factor $c_l \sim 1/16\pi^2$.

Regarding the inflaton we have to distinguish between two different scenarios depending on the effect generating the respective non-perturbative corrections to the superpotential. In *scenario I*, we wrap the inflaton 4-cycle τ_ϕ with a stack of hidden D7-branes yielding a non-perturbative correction due to gaugino condensation [273, 276, 283–287]. In this case the inflaton will dominantly decay into the hidden D7-brane gauge bosons with the associated decay rate [432]

$$\Gamma^{(I)}_{\tau_\phi} \sim \frac{W_0^3}{10} \frac{(\ln \mathcal{V})^3}{\mathcal{V}^2} M_{\mathrm{Pl}} \,. \tag{12.2}$$

On the other hand in *scenario II*, we consider the case where the non-perturbative correction is instead generated by E3-branes wrapping τ_ϕ [272–282]. In this case the inflaton can not decay into gauge bosons from hidden D7-branes wrapping τ_ϕ. Instead, the dominant decay channel is given by the decay into SM gauge bosons since τ_{SM} mixes with τ_ϕ via the Kähler metric and thus couples to the SM gauge theory [365]. The decay rate is given by

$$\Gamma^{(II)}_{\tau_\phi} \simeq \frac{3\lambda_\phi N_g W_0^3 (\ln \mathcal{V})^{9/2}}{8\pi a_\phi^{3/2} \mathcal{V}^4} M_{\mathrm{Pl}} \tag{12.3}$$

where $N_g \geq 12$ is the number of SM gauge bosons, depending on the exact realisation of the SM on τ_{SM}.

Independently of scenario I or II, after inflation ends at $t = t_E$ the universe will be dominated by the energy density of the inflaton $\rho_\phi(t_E)$ and not by the energy density of the volume modulus $\rho_\mathcal{V}(t_E)$. The density $\rho_\phi(t_E)$ is approximately given by the scale of inflation [289]

$$\rho_\phi(t_E) \simeq \frac{M_P^4 W_0^2 \beta}{\mathcal{V}^3} \,, \tag{12.4}$$

while the energy density associated to the volume modulus $\rho_\mathcal{V}(t_E)$ is given by [289]

$$\rho_\mathcal{V}(t_E) \simeq \frac{M_P^4 W_0^2 Y^2}{\mathcal{V}^3 \ln \mathcal{V}} \,. \tag{12.5}$$

Here, Y refers to the initial displacement of the volume modulus from its minimum which is given by [289]

$$Y \simeq \sqrt{\frac{2}{3}} R \,(\ln \mathcal{V}_*)^{3/2} \,, \qquad \text{with} \qquad R = \frac{\lambda_\phi a_\phi^{-3/2}}{\lambda_\phi a_\phi^{-3/2} + \lambda_s a_s^{-3/2}} \ll 1 \,, \tag{12.6}$$

where $R \ll 1$ follows from the condition (9.8). The parameter R generally takes values of $R \sim 0.1 - 0.01$ which implies that $Y \sim 0.1$ [289]. The ratio of the densities (12.5) and (12.4) shows

$$\frac{\rho_\mathcal{V}(t_E)}{\rho_\phi(t_E)} \simeq \frac{Y^2}{\beta \ln \mathcal{V}} \equiv \theta^2 \ll 1 \,. \tag{12.7}$$

Thus, at the end of inflation the universe is at first dominated by the inflaton. The following evolution now depends on the exact relations of the decay rates in the specific scenarios.

Scenario I

In scenario I, after inflation ends we enter a period of *matter domination* due to the presence of the inflaton energy density. The inflaton decays into radiation at $t = t_\phi$ well before the volume modulus decays since we have

$$\frac{\Gamma^{(I)}_{\tau_\phi}}{\Gamma_{\mathcal{V}}} \sim \frac{(\ln \mathcal{V})^{9/2}}{c_l^2}\sqrt{\mathcal{V}} \gg 1 \,. \tag{12.8}$$

The number of e-foldings generated during this period is given by (see (2.45))

$$N_\phi \simeq \frac{2}{3}\ln\left(\frac{H(t_E)}{\Gamma_{\tau_\phi}}\right) \,. \tag{12.9}$$

We can use (2.37) (with $k = 0$) which relates $H(t_E) \simeq \sqrt{\rho_\phi(t_E)}M_{\rm Pl}^{-1}$ such that we obtain

$$N_\phi \simeq \frac{2}{3}\ln\left(\frac{10\sqrt{\beta\mathcal{V}}}{W_0^2(\ln \mathcal{V})^3}\right) \,, \tag{12.10}$$

where we used (12.4) in addition.

After t_ϕ the universe is dominated by the radiation produced by the decay of ϕ and enters a period of *radiation domination*. Further, the volume modulus has not decayed yet and dilutes as matter with the expansion of the universe. As radiation dilutes faster than matter we reach a time of *matter-radiation equality* t_{eq} defined by

$$\rho_{\mathcal{V}}(t_{eq}) = \rho_{rad}(t_{eq}) \,. \tag{12.11}$$

We are thus entering a period of *modulus domination*, driven by the energy density of the volume modulus. To determine the e-folds obtained during radiation domination N_{rad} we use

$$N_{rad} = \ln\left(\frac{a(t_{eq})}{a(t_\phi)}\right) \,. \tag{12.12}$$

To proceed, we suitably rewrite (12.11) by using (2.42)

$$\rho_{\mathcal{V}}(t_\phi)\left(\frac{a(t_{eq})}{a(t_\phi)}\right)^{-3} = \rho_{rad}(t_\phi)\left(\frac{a(t_{eq})}{a(t_\phi)}\right)^{-4} \,. \tag{12.13}$$

We then use $\rho_{rad}(t_\phi) = \rho_\phi(t_\phi)$ and obtain

$$\frac{a(t_{eq})}{a(t_\phi)} = \frac{\rho_\phi(t_\phi)}{\rho_{\mathcal{V}}(t_\phi)} = \theta^{-2} \,, \tag{12.14}$$

where we used in the last equality that $\rho_\phi/\rho_{\mathcal{V}} = \theta^2$ and that the ratio remains constant until t_ϕ because both densities redshift as matter. We hence find

$$N_{rad} = -2\ln(\theta) \,. \tag{12.15}$$

Finally, we have to derive the number of e-folds $N_{\mathcal{V}}$ until the volume modulus decays. Analogously to (12.9), $N_{\mathcal{V}}$ is given by

$$N_{\mathcal{V}} \simeq \frac{2}{3}\ln\left(\frac{H(t_{eq})}{\Gamma_{\mathcal{V}}}\right) \,. \tag{12.16}$$

We can determine $H(t_{eq})$ with $H(t_{eq}) \simeq H(t_\phi)e^{-2N_{rad}}$ and using (12.15) which yields

$$H(t_{eq}) = H(t_\phi)\theta^4 \,. \tag{12.17}$$

Then, we replace $H(t_\phi) \simeq H(t_E)e^{-3/2N_\phi} \simeq \sqrt{\rho_\phi(t_E)}M_{\text{Pl}}^{-1}e^{-3/2N_\phi}$ and use (12.4) for $\rho_\phi(t_E)$ and find

$$N_\mathcal{V} \simeq \frac{2}{3}\ln\left(\frac{\theta^4(\ln\mathcal{V})^{9/2}\sqrt{\mathcal{V}}}{10c_l^2}\right) \simeq \frac{2}{3}\ln\left(\frac{(\ln\mathcal{V})^{5/2}\sqrt{\mathcal{V}}Y^4}{10\beta^2 c_l^2}\right) \,, \tag{12.18}$$

where we used (12.7) to replace θ. The volume modulus then also decays into SM particles and hence reheats the universe.

In summary, we find for scenario I: First the universe is dominated by the inflaton which decays into radiation at t_ϕ with respective e-folds (12.10). Thus at t_ϕ the universe evolves into a radiation dominated epoch where the volume modulus is still present. Matter-radiation equality is reached at t_{eq} with respective e-folds (12.15). The universe transverses into a period of modulus domination until the volume modulus decays with respective e-folds (12.18) and reheats the universe.

Scenario II

In scenario II, after inflation ends the universe is again dominated by the oscillations of the inflaton. However, the ratio of the decay rates now yields

$$\frac{\Gamma_{\tau_\phi}^{(II)}}{\Gamma_\mathcal{V}} \simeq \frac{3\lambda_\phi N_g(\ln\mathcal{V})^6}{8\pi c_l^2 a_\phi^{3/2}\mathcal{V}^{3/2}} \,. \tag{12.19}$$

It appears as if the volume modulus now decays before the inflaton due to the suppression by the $\mathcal{V}^{-3/2}$ factor. Inserting explicit parameters necessary for our model to remain in a controlled regime, e.g. $\mathcal{V} \sim 10^4$, shows that this is however not the case since we find[42] $\Gamma_{\tau_\phi}^{(II)}/\Gamma_\mathcal{V} \sim 100$.

The number of e-folds we gather during inflaton domination is analogous to (12.9) given by

$$N_\phi \simeq \frac{2}{3}\ln\left(\frac{8\pi a_\phi^{3/2}\sqrt{\beta}\mathcal{V}^{5/2}}{3\lambda_\phi N_g W_0^2(\ln\mathcal{V})^{9/2}}\right) \,. \tag{12.20}$$

Similar to above, the Hubble parameter at the time of matter-radiation-equality is given by (12.17). We again replace $H(t_\phi) \simeq H(t_E)e^{-3/2N_\phi} \simeq \sqrt{\rho_\phi(t_E)}M_{\text{Pl}}^{-1}e^{-3/2N_\phi}$ and use (12.4) for $\rho_\phi(t_E)$ to find

$$H(t_{eq}) \simeq H(t_E)\frac{3\lambda_\phi N_g W_0^2(\ln\mathcal{V})^{9/2}\theta^4}{8\pi a_\phi^{3/2}\sqrt{\beta}\mathcal{V}^{5/2}} \,. \tag{12.21}$$

Interestingly this now implies that the volume modulus decays before the time of matter-radiation-equality because we have

$$\frac{H(t_{eq})}{\Gamma_\mathcal{V}} \simeq \frac{3\lambda_\phi N_g(\ln\mathcal{V})^4 Y^4}{8\pi\beta^2 a_\phi^{3/2}c_l^2\mathcal{V}^{3/2}} \,, \tag{12.22}$$

which, for the same choice of parameters as before, yields $H(t_{eq})/\Gamma_\mathcal{V} \sim 10^{-2} - 10^{-3}$. Therefore,

[42] We further used the parameters specified in (12.25).

we can conclude that no period of volume domination will occur and we set

$$N_{\mathcal{V}} = 0 \,. \tag{12.23}$$

In summary, we find for scenario II: First the universe is dominated by the inflaton which decays into radiation at t_ϕ with respective e-folds (12.20). Thus, at t_ϕ the universe evolves into a radiation dominated epoch. The volume modulus decays during radiation domination and no matter-radiation equality is reached. In the end, the universe is reheated by the inflaton.

12.2. Inflationary Parameters

The formula determining the required number of e-folds N_e specified to one additional modulus is given by [289, 430]

$$N_e \simeq 57 + \frac{1}{4}\ln r - \frac{1}{4}N_\phi - \frac{1}{4}N_{mod} + \frac{1}{4}\ln\left(\frac{\rho_*}{\rho(t_E)}\right) \,. \tag{12.24}$$

Here, r refers to the tensor to scalar ratio and ρ_* and $\rho(t_E)$ denote the energy densities at horizon exit and at the end of inflation respectively. Furthermore, N_ϕ and N_{mod} refer to the number of e-folds obtained during the epochs dominated by the inflaton and modulus oscillations. Note that the formula (12.24) is specialised to the case where the inflaton and modulus energy densities redshift as matter and all periods of radiation domination do not contribute [289, 430].

We evaluate (12.24) under the simplifying assumptions that $\rho_* \simeq \rho(t_E)$ and fix our parameters to be given by

$$\mathcal{V} \simeq 10^3 - 10^4 \,, \quad a_\phi = 2\pi \,, \quad W_0 \simeq \lambda_\phi \simeq \beta \simeq 1 \,, \quad N_g = 12 \,, \quad c_{\text{loop}} \sim c_l \sim 0.01 \,. \tag{12.25}$$

The tensor to scalar ratio r is determined by (10.7). For the set of parameters (12.25) and for $N_e = 50$, implying $\phi_* \sim 0.3$ due to (10.8), we find

$$r \simeq 4 \times 10^{-5} \,. \tag{12.26}$$

Evaluating (12.24) by using all this information yields

$$N_e \simeq \begin{cases} 53 & \text{in Scenario I} \\ 52 & \text{in Scenario II} \end{cases} \,, \tag{12.27}$$

which implies for ϕ_* using (10.8)

$$\phi_* \simeq \begin{cases} 0.32 & \text{in Scenario I} \\ 0.33 & \text{in Scenario II} \end{cases} \,. \tag{12.28}$$

At this level of precision both scenarios predict a spectral index n_s of

$$n_s \simeq 0.976 \,, \tag{12.29}$$

where we used (10.6). In comparison to the measured value of n_s [8]

$$n_s = 0.9665 \pm 0.0038 \qquad (68\%\,\text{CL}) \,, \tag{12.30}$$

we find that our value of n_s (12.29) is about 3σ above (12.30). In the following we will discuss how this elevated values can be accounted for.

12.3. Dark Radiation

As we just have seen, our value for the spectral index n_s in (12.29) is elevated in comparison to the analysis of the Planck collaboration [8]. However, the analysis of [8] relies on the ΛCDM model and hence assumes the standard cosmological evolution of the universe. Most importantly for us, the analysis does not include *dark radiation* which denotes an energy density of relativistic particles which are not photons or SM neutrinos. Practically, the contribution of dark radiation to the energy density is incorporated into an *effective number of neutrino species* N_{eff}. As a result the total energy density at the CMB temperature ρ can be expressed in terms of the photon energy density ρ_γ and N_{eff}, see e.g. [223, 433]

$$\rho = \rho_\gamma \left(1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{eff}\right) , \tag{12.31}$$

since the neutrino energy density can be shown to be proportional to ρ_γ. Dark radiation can now be quantitatively accounted for by determining the shift ΔN_{eff} of from its expected value determined by the number of SM neutrinos $\bar{N}_{eff} = 3$, i.e. $\Delta N_{eff} = N_{eff} - \bar{N}_{eff}$. Current observations of the CMB and large scale structure formation constrain ΔN_{eff} [8]

$$\Delta N_{eff} \lesssim 0.2 - 0.4 , \tag{12.32}$$

depending on the specific dataset used in the analysis. Important for our purpose is the observation that including dark radiation in the analysis increases the fitted value of n_s for $\Delta N_{eff} > 0$ [431]

$$n_s = 0.983 \pm 0.006 \quad (68\%\,\mathrm{CL}) \quad \text{for} \quad \Delta N_{eff} = 0.39 . \tag{12.33}$$

Also in string compactifications dark radiation can be generated where moduli axions represent the most significant candidate for dark radiation [267, 293, 355–365, 410]. This suggests that the higher values of n_s observed in our model could be attributed to the presence of dark radiation. Notably, the axionic contribution to ΔN_{eff} can be determined from the branching ratio B of a modulus φ to all available axions a_i [355, 356, 410, 434]

$$\Delta N_{eff} \simeq 6.1 \left(\frac{11}{g_*^4 g_{*,S}^{-3}}\right)^{1/3} B(\varphi \to a_i\, a_i) , \tag{12.34}$$

where g_* and $g_{*,S}$ refer to the effective number of degrees of freedom in the energy density and in the entropy density respectively. For our application, we can set $g_* = g_{*,S} = 106.75$ as we consider early times in the cosmological history where all degrees of freedom are relativistic, see e.g. [223]. It remains to derive the branching ratios B which are different in scenario I and scenario II.

Scenario I

The universe in scenario I is reheated by the decay of the volume modulus which is consequently also responsible for the production of dark radiation. The most dominant source for dark radiation is given by decay the volume modulus into its own axion $a_{\mathcal{V}}$, where the respective decay rate is given by [355]

$$\Gamma_{\mathcal{V}\to a_{\mathcal{V}}\,a_{\mathcal{V}}} = \frac{1}{48\pi}\frac{m_{\mathcal{V}}^3}{M_P^2}\ . \tag{12.35}$$

We have to compare the decay rate (12.35) with the overall dominant decay channel of the volume modulus into SM higgses h given in (12.1)

$$\frac{\Gamma_{\mathcal{V}\to a_{\mathcal{V}}\,a_{\mathcal{V}}}}{\Gamma_{\mathcal{V}\to hh}} \simeq \frac{1}{48\pi(c_l\mathcal{V})^2} \ll 1\ . \tag{12.36}$$

This suggests that the branching ratio in scenario I will be negligible. Consequently, we deduce that there is no substantial production of dark radiation, allowing us to approximate $\Delta N_{eff}^{(I)} \simeq 0$ for scenario I. Additionally, this indicates that the increased value of n_s cannot be accounted for by dark radiation.

Scenario II

The situation in scenario II is different since in this case the inflaton is responsible for reheating. Crucially, the inflaton ϕ does not decay into its own axion a_ϕ because their masses are approximately equal and the decay is thus kinematically forbidden [365]. However, the inflaton can decay into other axions as well as into their associated 4-cycle partner moduli. In sect. 6.1.4 of [365] it was explicitly shown that the decay rates of the inflaton into other 4-cycle moduli and into the axionic partners agree. Fundamentally, this is dictated by supersymmetry. The explicit expressions for the decay rates into the volume axion $a_{\mathcal{V}}$ and the SM axion a_{SM} are [365]

$$\Gamma_{\phi\to a_{\mathcal{V}}\,a_{\mathcal{V}}} \simeq \frac{3\lambda_\phi W_0^3 a_\phi^3 \tau_\phi^{9/2}}{64\pi\mathcal{V}^4} M_P \equiv \Gamma\ , \tag{12.37}$$

$$\Gamma_{\phi\to a_{SM}\,a_{SM}} \simeq \frac{3\lambda_\phi W_0^3 a_\phi^3 \tau_\phi^{9/2}}{16\pi\mathcal{V}^4} M_P = 4\,\Gamma\ , \tag{12.38}$$

where we identified $\Gamma_{\phi\to a_{\mathcal{V}}\,a_{\mathcal{V}}} \equiv \Gamma$. To calculate the branching ratio B, we use the fact that the overall predominant decay process of the inflaton ϕ in scenario II is given by the decay into SM gauge bosons. The respective decay rate $\Gamma_{\phi\to A_{SM}\,A_{SM}}$ has already been given in (12.3) and allows us to state the useful relation

$$\Gamma_{\phi\to A_{SM}\,A_{SM}} = 8\,N_g\,\Gamma\ . \tag{12.39}$$

Using (12.37), (12.38) and (12.39) we find for B

$$B \simeq \frac{\Gamma + 4\Gamma}{8N_g\Gamma} = \frac{5}{8N_g}\ . \tag{12.40}$$

We now use this result with $N_g = 12$ in (12.34) which yields

$$\Delta N_{eff}^{(II)} \simeq 0.14\ , \tag{12.41}$$

where we used $g_* = g_{*,S} = 106.75$ as explained below (12.34). Therefore, we find a substantial amount of dark radiation in scenario II which could account for the result of the elevated value of n_s in (12.29).

13. Conclusions & Outlook

Conclusion on Part II

Dark or hidden photons featuring a small but non-vanishing mixing with the ordinary photon can be probed at a level of mixing angles of sometimes better than $\sim 10^{-15}$. This makes them an ideal tool to probe sequestered or hidden sectors that are often present in string theory. Therefore, understanding how very small values of kinetic mixing can arise in relevant string setups is of significant phenomenological interest. In addition, it is of theoretical interest in the sense that it tests our ability to realize an extremely small coefficient for an operator which should naively be present at the $\mathcal{O}(1)$ level.

In part II we have analysed kinetic mixing between the gauge groups of two D3 branes, chapt. 6, and two stacks of D7-branes, chapt. 7. In the D3-brane case, kinetic mixing occurs due to the propagation of B_2 and C_2 through the bulk of the relevant Calabi-Yau orientifold. Earlier investigations [221, 334, 347] found a leading-order cancellation between the B_2 and the C_2 contribution both in a string theory and in a 10d supergravity calculation. However, an additional coupling arises if a non-zero value for C_0 is taken into account. Moreover, there exists a coupling between B_2 and C_2 localized on the D3-branes. As one of our key results, we demonstrate that an exact cancellation persists after taking these two effects into account. For this it was essential to include a term in the D3-brane action which, while known in principle, is missing in standard textbooks, cf. app. A. Using the complete action for the D3-brane, we can further tie the generalised cancellation to the $\mathrm{SL}(2,\mathbb{R})$ symmetry of type IIB supergravity, which in particular acts as a self duality group on the gauge theory living on the D3-brane.

We then extended our discussion by allowing for non-vanishing 3-form fluxes, which break $\mathrm{SL}(2,\mathbb{R})$ spontaneously. In this case non-vanishing kinetic mixing is present. Moreover, given that $\mathrm{SL}(2,\mathbb{R})$ is the self duality group of the D3-brane gauge theory, it is not surprising that both kinetic and magnetic mixing arise. Any mixing obtained in this setting is small due to two effects: First, independently of any cancellation the B_2 and C_2 propagation leads to a suppression of the mixing by the Calabi-Yau volume. One may call this a sequestering effect. On top of that, the necessary presence of fluxes and the dilution of the latter when the Calabi-Yau grows large leads to a further volume suppression. We provided explicit, $\mathrm{SL}(2,\mathbb{R})$-covariant formulae for kinetic and magnetic mixing. Explicitly, both kinetic and magnetic mixing are suppressed by $\mathcal{V}^{-4/3}$, where $\mathcal{V}$ is the Calabi-Yau volume. Specifically in the Large Volume scenario [101, 102], the parameter $\mathcal{V}$ is linked to the volume modulus mass $m_{\mathcal{V}} \sim \mathcal{V}^{-3/2} M_{\mathrm{Pl}}$, which is subject to experimental constraints. Based on this, we derived lower bounds on kinetic mixing. While it is intriguing that the resulting values of the mixing parameter fall in a range interesting for future probes, we have to recall that our bounds are only indicative because our single-D3-brane model is not realistic.

We then applied our gained insights to a scenario involving stacks of D7-branes. Here we focused on the situation where gauge flux breaks the D7-stack gauge theory to a subgroup containing a $\mathrm{U}(1)$ factor. In this case kinetic mixing is mediated by B_2, C_2 and C_4 propagation in the bulk. Again we also include a non-zero value for C_0 and take the brane-localised self-coupling terms of B_2 and C_2 into account. Analogous to the D3-brane scenario, we find that all contributions of B_2 and C_2 cancel which is again tied to the $\mathrm{SL}(2,\mathbb{R})$ structure of the theory. Yet, a non-vanishing kinetic mixing contribution from C_4 exists and we provide an explicit formula for the kinetic mixing parameter χ. Interestingly, C_4 does not induce a magnetic mixing term. Non-vanishing 3-form fluxes are therefore only required to achieve non-zero magnetic mixing induced by B_2 and C_2 mediation. The flux induced contributions to kinetic mixing are negligible in comparison to the leading C_4 contribution due to the diluteness of the 3-form fluxes.

Clearly, a more detailed phenomenological discussion should be based on realistic models, containing both light charged states and chiral matter. This requires the extension of our analysis of D3-brane kinetic mixing to $U(1)$ gauge groups on branes at singularities or fractional branes. String loop calculations including D3-branes at singularities have been performed e.g. in [346]. However, a supergravity analysis is impossible at the moment since the exact coupling structure of B_2 and C_2 on D3-branes at singularities or fractional D3-branes is not known. Therefore, it requires to first investigate the exact interactions these branes are subject to.

In app. D, we explored an alternative approach to realise light charged states by using D3-brane stacks instead of single D3-branes. Each sector is given by at least two D3-branes which are separated over a short distance but exceeding the string length. In this setting, light charged states arise from the strings that extend between the branes within each sector. However, in this analysis the charged states are still relatively heavy. Reducing the separation to sub-stringy distances would reduce the mass of the states but would simultaneously require a full non-abelian description of the brane stack. We have investigated the action of a non-abelian D3-brane stack for coupling terms which can cause kinetic mixing in app. E. Although the coupling terms reveal that kinetic mixing is still mediated by B_2 and C_2, we find that the structure of the couplings shows notable differences when compared to the above case where non-abelian effects were not necessary to consider. Determining the precise description for transitioning between these two regimes remains an unresolved challenge which we expect to be necessary for a resolution of this discrepancy. In addition, we required the presence of 3-form fluxes to find a non-zero result of kinetic mixing in the D3-brane case and expect the same requirement in the case of D3-brane stacks. Yet, it is well known that p-form fluxes have a polarisation effect on stacks of branes [90]. In a flux background, separating and moving the branes corresponds to flat directions in the potential of the brane position scalars. However, it is energetically more favoured for the branes to arrange into a specific geometry determined by the flux. This could feature a non-trivial interplay of fluxes and kinetic mixing of branes which may be important at this level of precision.

As we highlighted in sect. 6.4.3, of greatest importance is the question of how our 10d EFT analysis may be embedded into a proper 4d formulation of supergravity. It is already known that kinetic mixing in supergravity arises from a one-loop correction to the gauge kinetic function. The gauge kinetic function is holomorphic in all fields and our volume depended result implies that it should be a function of the Kähler moduli. Crucially the Kähler moduli enjoy a shift symmetry which, in combination with the holomorphicity of the gauge kinetic function, restricts any Kähler moduli dependence of the gauge kinetic function to be linear or exponential. This is clearly in tension with the result we obtained. We suggested a possible resolution of this discrepancy in sect. 6.4.3 which is based on higher derivative corrections to the 4d theory and SUSY breaking. Higher derivative operators are not required to be holomorphic and could introduce a non-holomorphic contribution to the gauge kinetic function upon SUSY breaking. However, this logic also implies that our kinetic mixing result must vanish if only SUSY-preserving fluxes are present. Yet, the precise resolution of the discrepancy remains unclear and requires more research. A first step for a better understanding would be to evaluate the derived formula in an explicit and consistent model where the internal geometry is known. In this case, one could explicitly check if our result vanishes if only SUSY-preserving fluxes are present.

In the context of D7-brane mixing, we require a detailed phenomenological analysis of our findings. Furthermore, even though D7-branes break $SL(2,\mathbb{R})$, we observed a cancellation between B_2 and C_2 contributions analogous to the case of D3-branes. One can understand this cancellation on a technical level, however it is unclear why this $SL(2,\mathbb{R})$ structure survives in the context of kinetic mixing. Further, we expect that 3-form fluxes also induce non-zero kinetic mixing from B_2 and C_2 and it would be interesting to work out the exact from of these contributions. Note, that the kinetic mixing contributions induced by C_2 and C_4 are of opposite sign compared to the contributions from B_2. Thus, it is conceivable that a reduction of kinetic mixing can be engineered if the 3-form fluxes are chosen appropriately, such that the B_2 contribution partially cancels against the C_2 and C_4 contributions. Again, an explicit evaluation on concrete geometries would improve the understanding of these scenarios.

Due to time constraints, we could not study any kinetic mixing effects between D3- and D7-branes which remains as an additional open question. Lastly, we did not introduce a mass term for the dark photon which is pivotal in particle phenomenology. A mass term will certainly impose further constraints on the models and might change the results we obtained for kinetic mixing. All of the above comments and remarks represent interesting research directions which we have to leave for future work.

Conclusion on Part III

Part III is based on the observation that the loop-corrected potential of the Kähler moduli generically features flat plateaus when a large volume of the internal Calabi-Yau is realised. In the LVS, the volume $\mathcal{V}$ and a small cycle τ_s are consistently stabilised while yielding an exponentially large value for the volume $\mathcal{V}$. Any additional Kähler moduli τ_i correspond to flat directions as a consequence of the no-scale cancellation in the potential. Non-perturbative corrections and loop corrections spoil the no-scale structure and induce a potential for the remaining Kähler moduli which is suppressed in $\mathcal{V}$ in comparison to the leading order terms. In chapter 8, we demonstrated that this potential becomes flat as one approaches the regime where $\tau_i \sim \mathcal{V}^{2/3}$, a regime where V is dominated by loop corrections. This flatness property allows to implement slow roll inflation where the inflaton parametrizes a generic trajectory in the moduli space of the remaining Kähler moduli τ_i.

Inspired by this insight, we considered a simple realisation of the above in chapter 9. In this simple setting we introduced a single additional blowup 4-cycle τ_ϕ to represent the inflaton. We argued that the blowup τ_ϕ is subject to non-perturbative corrections and *inevitably* loop corrections. The former determine the minimum of τ_ϕ and the loop corrections dominate the potential at large field values where the potential becomes flat. If τ_ϕ remains in a blowup regime the form of the leading loop correction is explicitly known, allowing for a thorough analysis of this model. In sect. 10, we analysed the blowup regime and included only the leading loop correction to determine the inflationary parameters. We then investigated whether this regime allows to realise slow roll inflation in a parametrically and numerically controlled manner. Indeed, we found that this is the case. However, to match phenomenological constraints, we require a small volume, $\mathcal{V} \sim 10^4$, and inflation to start at rather large values of the inflaton. Despite the fact that inflation takes places a in regime of large fields values, the inflaton cycle τ_ϕ remains within the blowup regime ensuring that the model is well-controlled.

In chapter 11, we discussed two other regimes of the inflationary potential which can realise slow roll inflation. First, we discussed the effect of additional sub-leading corrections as we leave the blowup regime. We found that these corrections potentially realise a whole class of inflationary models at large values of the inflaton. Furthermore, the corrections can actually lead to a phenomenologically more robust slow roll potential in the simplest setting we considered. Second, we derived a critical value for the prefactor of the loop corrections, $c_{\text{loop}} < 5.3 \times 10^{-6}$, for which loop corrections can safely be ignored. In this case, we return to the old proposal of blowup inflation [187].

In chapter 12, this theoretical evaluation is followed by the detailed phenomenological assessment of the simplest realisation from chapter 10 which we included for a coherent presentation of this work. The particular implementation of the model leads to various inflationary histories, having mostly similar implications for phenomenological parameters. Notably, the spectral index n_s derived from our model is too high in comparison to Planck data [8]. This issue can be addressed by incorporating dark radiation. Generally, the inclusion of dark radiation in the analysis results in an increased value for n_s [431]. Fortunately, our model can produce dark radiation adequately which provides an explanation for the elevated value of n_s.

Our analysis shows that slow roll inflation can indeed be realised quite generically in the Kähler moduli sector of type IIB flux compactifications. We focused on the simplest and controlled model in this framework which yields promising inflationary predictions that are in agreement with observational data. However, we did not investigate the effects of sub-leading corrections to the potential which could realise more inflationary scenarios. To explore these effects concretely would necessitate explicit calculations of these corrections, a task that is notably complex and challenging.

Appendix

A. Precise formulation of the type IIB and Dp-brane action

The low energy limit of type-IIB string theory is $\mathcal{N} = 2$ type-IIB supergravity. It is defined by a set of covariant equations of motions [235,435]. Since the theory contains a 4-from with self-dual field strength, these equations do not follow from a manifestly Lorentz-invariant action [436]. Famously, this issue can be avoided by imposing self-duality as a constraint after varying the action S_{IIB}, which we repeat here for better readability [237, 238],

$$\begin{aligned} S_{\text{IIB}} = \frac{1}{2\kappa_{10}^2} \int_{\mathcal{M}} \mathrm{d}^{10}x \sqrt{-G_E} \left(R_E - \frac{\partial_M \bar{\tau} \partial^M \tau}{2(\operatorname{Im} \tau)^2} \right) \\ + \frac{1}{2\kappa_{10}^2} \int_{\mathcal{M}} \left(-\frac{\hat{M}_{ij}}{2} F_3^i \wedge \star F_3^j - \frac{1}{4} \tilde{F}_5 \wedge \star \tilde{F}_5 - \frac{\epsilon_{ij}}{4} C_4 \wedge F_3^i \wedge F_3^j \right) . \end{aligned} \tag{A.1}$$

We also restate the field strength

$$\tilde{F}_5 = \mathrm{d}C_4 - \frac{1}{2} C_2 \wedge \mathrm{d}B_2 + \frac{1}{2} B_2 \wedge \mathrm{d}C_2 , \tag{A.2}$$

referring the reader back to sect. 6.1 for the other definitions.

An additional information, which is apparently often overlooked or implicitly understood, is that when writing down the $\mathrm{SL}(2,\mathbb{R})$-invariant action (A.1) one has performed a field redefinition of the original stringy fields, associated to the massless modes of the quantized string [76, 237, 238]. In the following, we will specify these stringy fields by using a hat: "$\hat{}$". The field redefinitions include the transformation of the string frame metric $\hat{G}$ to the Einstein frame metric G_E as well as of the stringy 4-form gauge potential $\hat{C}_4$,[43]

$$C_4 = \hat{C}_4 - \frac{1}{2} \hat{B}_2 \wedge \hat{C}_2 . \tag{A.3}$$

Since the other fields are not transformed and in particular $\hat{B}_2 = B_2$ and $\hat{C}_2 = C_2$, we drop the hat for those fields. We note that $\hat{C}_4$ is not $\mathrm{SL}(2,\mathbb{R})$ invariant,

$$\hat{C}_4' = \hat{C}_4 + \frac{1}{2} \begin{pmatrix} C_2 \, , \; B_2 \end{pmatrix} \wedge \begin{pmatrix} ac & cb \\ cb & bd \end{pmatrix} \begin{pmatrix} C_2 \\ B_2 \end{pmatrix} , \tag{A.4}$$

which explains why C_4 is often used even when working in the string frame.

Particularly relevant for us is the gauge-invariant field strength $\tilde{F}_5$ expressed in terms of the stringy field $\hat{C}_4$ [246, 438–440] ,

$$\tilde{F}_5 = \mathrm{d}\hat{C}_4 - C_2 \wedge \mathrm{d}B_2 , \tag{A.5}$$

[43] A detailed discussion can be found in [76] for the bosons and in [437] for the fermions.

which makes it clear that $\hat{C}_4$ is gauged only by C_2,

$$\delta\hat{C}_4 = \mathrm{d}\Lambda_3 + \Lambda_1^{(1)} \wedge \mathrm{d}B_2 \,. \tag{A.6}$$

This observation is important since we need a consistent description of the IIB bulk theory together with Dp-branes. The Einstein-frame Dp-brane action reads [85, 89, 239–251]

$$S_{\mathrm{D}p\text{-brane}} = S_{\text{DBI}} + S_{\text{WZ}} \,, \tag{A.7}$$

$$S_{\text{DBI}} = -T_p \int\limits_{\mathcal{W}_{p+1}} \mathrm{d}^{p+1}\xi \sqrt{-\det\left(G^E_{ab} - e^{-\phi/2}B_{ab} + e^{-\phi/2}F_{ab}\right)} \,, \tag{A.8}$$

$$S_{\text{WZ}} = \pm\mu_p \int\limits_{\mathcal{W}_{p+1}} \exp\left(F_2 - B_2\right) \wedge \sum_q \hat{C}_q \wedge \sqrt{\frac{\hat{A}(4\pi^2\alpha' R_T)}{\hat{A}(4\pi^2\alpha' R_N)}} \,, \tag{A.9}$$

where we in particular follow the conventions of [246]. We see that S_{WZ} is built using the stringy RR-fields $\hat{C}_q$. The corresponding gauge-invariant field strengths are[44]

$$\tilde{F}_{q+1} = \mathrm{d}\hat{C}_q - \mathrm{d}B_2 \wedge \hat{C}_{q-2} \,, \tag{A.10}$$

with the $\hat{C}_q$ gauge-transforming as

$$\delta\hat{C}_q = \mathrm{d}\Lambda_{q-1} - \mathrm{d}B_2 \wedge \Lambda_{q-3} \,. \tag{A.11}$$

The brane action (A.7) is gauge-invariant under (6.6) and (A.6). The combination $F_2 - B_2$ is gauge invariant due to (6.13). Furthermore, the action (A.7) is $\mathrm{SL}(2,\mathbb{R})$ self-dual as defined by (6.8), (6.9), (6.14) and (A.4).

Most importantly, this means that when using the standard type-IIB bulk action as in (A.1) together with a brane action, then $\hat{C}_4$ in the latter has to be replaced by C_4 using (A.3). Thus, *an extra term* $+\frac{1}{2}B_2 \wedge C_2$ *appears for every* C_4. As explained, this is necessary for gauge invariance of any brane and for $\mathrm{SL}(2,\mathbb{R})$ self-duality of the D3.

The explanations above have no claim to originality and follow from carefully reading the literature [76, 244, 370–374]. We checked in detail the self-consistency of our notation.[45]

[44] Note that $\tilde{F}_3 \neq F_3 \equiv dC_2$.

[45] A different convention one occasionally encounters uses the combination $F_2 + B_2$ instead of $F_2 - B_2$. Such a sign flip for B_2 shows up in (A.10), (A.11) and the $\mathrm{SL}(2,\mathbb{R})$ transformations (6.9). Through the expressions for $\tilde{F}_3$ and $\tilde{F}_5$ it then also affects the bulk action.

B. Manifestly SL$(2,\mathbb{R})$ self-dual D3-brane action

It is common knowledge that the D3-brane in type IIB string theory is mapped to itself under SL$(2,\mathbb{R})$ transformations of type IIB [371, 441]. However, SL$(2,\mathbb{R})$ also acts on the gauge theory on the D3-brane which is evident from the coupling $F_2 - B_2$ that is enforced by B_2 gauge invariance. Hence analogously to B_2, the field strength F_2 transforms as part of a doublet where its doublet counterpart is given by the magnetic dual field strength G_2

$$\begin{pmatrix} G_2' \\ F_2' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} G_2 \\ F_2 \end{pmatrix} . \tag{B.1}$$

The general magnetic dual field strength G_2 is defined by varying the action S_{D3} w.r.t. F_2 [375] [46]

$$\delta_{F_2} S_{D3} = \int_{D3} \delta F_2 \wedge G_2 , \tag{B.2}$$

which in general is different from the known relation in electrodynamics $G_2 = \star F_2$ where the magnetic duality matches Hodge duality of F_2. Thus, to say that the D3-brane maps to itself more precisely means that the D3-brane is self-dual under SL$(2,\mathbb{R})$.

From the SL$(2,\mathbb{R})$ transformation we can see that the gauge theory on the D3-brane is continuously "rotated" into its (magnetic) dual gauge theory. The term rotations is borrowed from the seminal work [375], which showed that the usually discrete duality transformation can be continuous. Such "duality rotations" leave the equations of motion invariant, but non-trivially transform the Lagrangian 4-form L_4

$$\delta L_4 = \frac{1}{2} \delta(F_2 \wedge G_2) , \tag{B.3}$$

such that the action is not invariant. In light of the non-invariance of the Lagrangian, self-duality in the context of the continuous duality rotations of the D3-brane must therefore actually mean that the equations of motion are invariant, in contrast to the usual definition of self-duality $\mathcal{L}_{dual}(G_2, \tilde{g}) \equiv \mathcal{L}(F_2 = G_2, g = \tilde{g})$, cf. (6.20). The invariance of the D3-brane equations of motion is precisely what has been shown in [371, 441].

The change in the Lagrangian (B.3) allows to consistently reconstruct the underlying Lagrangian [375] if one assumes that L_4 contains F_2 only quadratically such that G_2 only linearly

[46] Note that we defined G_2 with a different sign when comparing to the literature [375].

depends on F_2 [47]. In this case, the Lagrangian is given by

$$\begin{aligned} L_4 = \frac{1}{2}F_2 \wedge K(\psi)F_2 + F_2 \wedge I_2 - F_2 \wedge K(\psi)H_2 \\ + \frac{1}{2}\left(K(\psi)H_2 - I_2\right) \wedge H_2 + L_{4,inv}(\psi) \end{aligned} \tag{B.4}$$

where $L_{4,inv}(\psi)$ is an invariant Lagrangian of additional fields ψ, e.g. the axiodilaton τ which also non-trivially transform under duality rotations. The function K depends also on the fields ψ and in addition can contain the hodge star $\star$. The two additional 2-forms I_2 and H_2 can also be coupled to the gauge theory if they form a doublet under duality rotations

$$\begin{pmatrix} I_2' \\ H_2' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} I_2 \\ H_2 \end{pmatrix} . \tag{B.5}$$

From (B.4) we find for G_2

$$G_2 = K\left(F_2 - H_2\right) + I_2 . \tag{B.6}$$

With G_2 one can rewrite (B.4)

$$L_4 = \frac{1}{2}F_2 \wedge G_2 + \frac{1}{2}\left(F_2 \wedge I_2 - G_2 \wedge H_2\right) + L_{4,inv}(\psi) \tag{B.7}$$

which will yield the correct non-invariance of the action (B.3). In this sense, if one finds K and, in the case of D3-branes, also includes suitable I_2 and H_2 one can find a manifestly self-dual action up to quadratic order in F_2.

Specifically for the D3-brane we have the action quadratic in F_2

$$S_{DBI} = -T_3 \int_{D3} \frac{e^{-\phi}}{2}\left(F_2 - B_2\right) \wedge \star_4 \left(F_2 - B_2\right) , \tag{B.8}$$

$$S_{WZ} = T_3 \int_{D3} C_4 + \frac{1}{2}B_2 \wedge C_2 + C_2 \wedge \left(F_2 - B_2\right) + \frac{C_0}{2}\left(F_2 - B_2\right) \wedge \left(F_2 - B_2\right) . \tag{B.9}$$

Hence, we find for G_2

$$G_2 = (-e^{-\phi} \star + C_0)(F_2 - B_2) + C_2 . \tag{B.10}$$

Following the temptation, indeed we can identify

$$K = -e^{-\phi} \star + C_0 , \tag{B.11}$$

$$I_2 = C_2 , \tag{B.12}$$

$$H_2 = B_2 , \tag{B.13}$$

$$L_{inv} = C_4 + (\text{IIB bulk}) , \tag{B.14}$$

where (C_2, B_2) correctly transforms as a doublet. Using these identifications one can rewrite the D3-brane action to match (B.4), which in this sense is manifestly self-dual. Again we want to stress, that it is crucial to have the $B_2 \wedge C_2$ term from app. A present in the D3-brane action.

[47] A generalization to a fully general Lagrangian is given in [376] which is, however, not important for our purposes.

We want to point out that in the quantum theory $\mathrm{SL}(2,\mathbb{R})$ is broken to $\mathrm{SL}(2,\mathbb{Z})$ which is generated by the two transformations

$$\mathcal{I} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} , \quad \mathcal{T} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} . \tag{B.15}$$

The transformation $\mathcal{T}$ leaves the D3-brane action invariant and so does $\mathcal{I}$ in the sense that we only have to replace the fields with the transformed fields, i.e. $\mathcal{I}[\mathcal{L}(\phi)] = \mathcal{L}(\mathcal{I}[\phi])$.

C. Higher-order Diagrammatics

As explained in sect. 6.4.1, our result for KM in (6.81) holds only at leading order. The presence of $\hat{m}$ and $\tilde{\bar{F}}_5$ induces a whole set of additional diagrams which must be taken into account. We can organize them in three different classes: Diagrams which cancel among themselves, diagrams which induce a renormalisation of brane couplings, and diagrams which give volume-suppressed contributions and can hence be neglected in a controlled approximation.

C.1. Cancelling Diagrams

First, we consider the class of diagrams containing any number of $\tilde{m}$ and $\tilde{\bar{F}}_5$ vertices but *no* 3-form flux effects, i.e. no vertex involving $\bar{F}_3^i$. Examples of such diagrams are displayed in fig. C.1. We claim that this set of diagrams gives zero or, put differently, allowing for an $\tilde{F}_5$-flux does not affect the zero result of sec. 6.3.

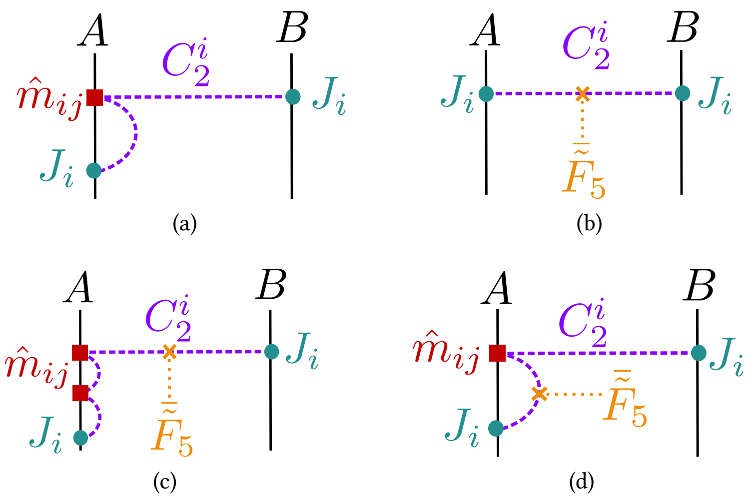


Fig. C.1: Examples of divergent diagrams which will eventually cancel.

Ultimately, this cancellation is a consequence of the underlying SL$(2,\mathbb{R})$ structure of the theory. The key point is that the $\tilde{F}_5$-flux does not introduce any new tensors, beyond the ubiquitous ϵ^{ij}, which carry SL$(2,\mathbb{R})$ indices to contract the sources J_i. To see this in more detail, consider the following part of the action:

$$S \supset \frac{1}{2\kappa_{10}^2} \int_{\mathcal{M}^{10}} \left(-\frac{\hat{M}_{ij}}{2} F_3^i \wedge \star_{10} F_3^j - \frac{1}{2} \tilde{\bar{F}}_5 \wedge \star_{10} \left[\epsilon_{ij} C_2^i \wedge F_3^j \right] \right) . \tag{C.1}$$

Here the first term is the kinetic term for C_2^i and the second term is the vertex which couples C_2^i to the background $\tilde{\bar{F}}_5$. The effect of this vertex is equivalent to that of the $\tilde{\bar{F}}_5$ contribution to the equations of motion, as displayed in (6.55). Besides the coupling to $\bar{\tilde{F}}_5$, we have to include the self-coupling of C_2^i on the brane, which is denoted by $\tilde{m}$ and was discussed in detail in sect. 6.4. Crucially, in (C.1) we see that the vertex with $\bar{\tilde{F}}_5$ uses only the tensor ϵ_{ij} to contract $\mathrm{SL}(2,\mathbb{R})$ indices. Hence, we can conclude that in any diagram of the type displayed in fig. C.1 the indices of the two sources $J_i^{(\mathrm{A,B})}$ are eventually contracted using the matrices $\left(\hat{M}^{-1}\right)^{ij}$, $\tilde{m}_{ij}$ or ϵ_{ij} or arbitrary combinations thereof. Therefore, the contribution of any of these diagrams $\mathcal{A}(\mathrm{C}.1)$ can be schematically written as

$$\mathcal{A}(\mathrm{C}.1) \;\sim\; J_i^{(\mathrm{A})} \wedge \left(f\left[\hat{M}^{-1},\, \tilde{m},\, \epsilon\right]\right)^{ij} J_j^{(\mathrm{B})} + (A \leftrightarrow B)\;, \tag{C.2}$$

where f stands for a monomial built from an arbitrary number of entries $\hat{M}^{-1}$, $\tilde{m}$ and ϵ, in arbitrary order. From sect. 6.3 and sect. 6.4 we know that applying $\left(\hat{M}^{-1}\right)^{ij}$ or $\tilde{m}_{ij}$ to a source J is essentially equivalent to raising or lowering the index with ϵ. Thus, at the end of any calculation the final result will be proportional to the contraction of the two sources using ϵ^{ij}. It will thus be proportional to (6.44), which vanishes and leads us to the conclusion that, in the absence of 3-form flux,

$$\mathcal{A}(\mathrm{C}.1) \sim 0\,. \tag{C.3}$$

Hence, we see that the cancellation persists also if $\bar{\tilde{F}}_5$-flux is included.

C.2. Coupling Renormalization

If $\bar{F}_3^i$ fluxes are present, the cancellations discussed in sects. 6.3, 6.4 and app. C.1 fail because of the non-trivial $\mathrm{SL}(2,\mathbb{R})$ index structure provided by the 3-form doublet. Moreover, some of the higher-order diagrams with $\tilde{m}$ and $\bar{\tilde{F}}_5$ vertices are divergent, making a careful analysis of the corresponding effects necessary.

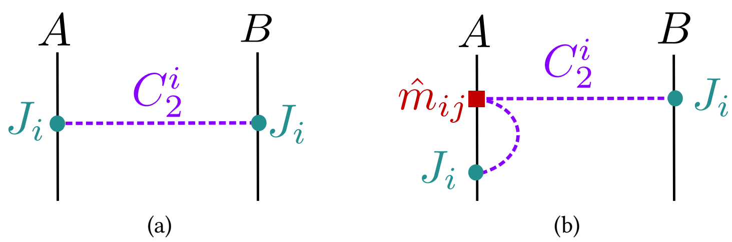


Fig. C.2: Example diagrams used in the text to study the 10d to 6d transition. (Recall that in the end diagrams of this type do not contribute.)

For instance, consider the diagram in fig. C.2a. While the diagram gives zero due to the cancellation explained in (6.33)-(6.35), we may for pedagogical reasons consider just one of the two equal and opposite contributions:

$$\mathcal{A}(\mathrm{C.2a}) \equiv 2\kappa_{10}^2 T_3^2 \int\limits_{x,x',y,y'} \delta(y-y_{\mathrm{A}})\delta(y-y_{\mathrm{B}}) J_i^{(\mathrm{A})}(x) J^{(\mathrm{B})\,i}(x')\; G_{10}(x-x';y-y')\,. \tag{C.4}$$

Here G_{10} denotes the 10d propagator and $\int_{x,x',y,y'}$ stands for the 4d integrations over x, x' and the 6d integrations over y, y'. We have removed the $\mathrm{SL}(2,\mathbb{R})$ index structure of the propagator using

(6.34). Also, we suppress the 4d index contractions between the sources J not to clutter notation. As explained in sect. 6.2, we may neglect any non-trivial dependence of the sources J on x, x'. We hence replace $J_i^{(A)}(x) \to J_i^{(A)}$ and $J^{(B)\,i}(x') \to J^{(B)\,i}$ in the following. The 10d propagator may be written as

$$G_{10}(x - x'; y - y') = \sum\!\!\!\!\!\!\int_{k_4,k_6} \frac{\exp\left[-ik_4(x - x')\right]\, \chi(k_6, y)\overline{\chi}(k_6, y')}{k_4^2 + k_6^2} \,. \tag{C.5}$$

Here $\sum\!\!\!\!\!\!\int_{k_4,k_6}$ stands for the integration over the 4d-momentum k_4 and the sum of the discrete 6d-momenta k_6. We have denoted the eigenfunctions of the 6d scalar Laplacian with eigenvalue k_6^2 by $\chi(k_6, y)$. Inserting (C.5) in (C.4) and performing the y, y' and x' integrations gives

$$\mathcal{A}(\text{C.2a}) = 2\pi \sum\!\!\!\!\!\!\int_{x,k_4,k_6} J_i^{(A)} J^{(B)\,i}\, \frac{\exp\left[-ik_4 x\right]\, \chi(k_6, y_A)\overline{\chi}(k_6, y_B)}{k_4^2 + k_6^2}\, \delta(k_4)\,, \tag{C.6}$$

where we further used $2\kappa_{10}^2 T_3^2 = 2\pi$. Integration over k_4 yields an expression as in (6.33), before the cancellation:

$$\mathcal{A}(\text{C.2a}) = 2\pi \sum\!\!\!\!\!\!\int_{x,k_6} J_i^{(A)} J^{(B)\,i}\, \frac{\chi(k_6, y_A)\overline{\chi}(k_6, y_B)}{k_6^2} = 2\pi \int_x J_i^{(A)} J^{(B)\,i}\, G_6(y_A - y_B)\,. \tag{C.7}$$

The point of this simple exercise was to make it completely clear how scale separation between KK scale and typical source profile scales make the analysis 6-dimensional.

It is now straightforward to repeat the simple calculation above for the diagram of fig. C.2b. This leads to a divergence. Indeed, using the same procedure as above we find

$$\begin{aligned}\mathcal{A}(\text{C.2b}) &\equiv 4\pi\kappa_{10}^2 T_3 \int_{x,x',x''} J^{(A)\,i}(x)\, \hat{m}_{ij}\, J^{(B)\,j}(x'') G_{10}(x - x'; y_A - y_A) G_{10}(x' - x''; y_A - y_B) \\ &= 2\pi \int_x J^{(A)\,i}\, \hat{m}_{ij}\, J^{(B)\,j}\, G_6(y_A - y_B)\, (2\kappa_{10}^2 T_3)\, G_6(y_A - y_A)\,,\end{aligned} \tag{C.8}$$

where we again used (6.34) to remove $SL(2, \mathbb{R})$ indices in the propagator. In comparison to (C.7), the extra factor $2\kappa_{10}^2 T_3\, G_6(y_A - y_A)$ comes from the additional $\hat{m}$ vertex and the second propagator. Clearly, $G_6(y_A - y_A)$ is divergent. Technically, this arises because, as we demonstrated, 4d and 6d integrations may be separated. Then, in spite of the fact that the $\hat{m}$ vertex and the source are separated along the D3 brane, one ends up with a 6d propagator evaluated at zero distance.

At the level of our EFT analysis, the best we can do is to cut off the divergence at the physical string scale $M_s \sim g_s^{1/4}/\sqrt{\alpha'}$. Here the factor $g_s^{1/4}$ is present because we work in the 10d Einstein frame, cf. (6.1). Thus, the relative size of the contributions of the two diagrams considered is

$$\mathcal{A}(\text{C.2b})\Big/\mathcal{A}(\text{C.2a}) \sim 2\kappa_{10}^2 T_3\, G_6(y_A - y_A) \sim 2\kappa_{10}^2 T_3\, (M_s)^4 \sim g_s\,. \tag{C.9}$$

We see that, at small g_s, the corrections introduced by including diagrams that are of higher order in $\hat{m}$ are suppressed. Even in the regime where g_s is not small, these corrections are $\mathcal{O}(1)$ rather than truly divergent.

Of course we know that the two example diagrams we considered are part of a larger set of diagrams, to all orders in $\hat{m}$, which give exactly zero. However, once we include $\bar{F}_3^i$ fluxes, as shown in the diagrams in figs. C.3a and C.3b, we find non-zero contributions to KM. Our analysis above still applies, we still cut off the divergences by M_s as just explained, and get a formal series of non-zero terms corresponding to more and more $\hat{m}$ insertions. As suggested by (C.9), this is at the same time a power series in g_s and may hence be viewed as a set of perturbative string corrections.

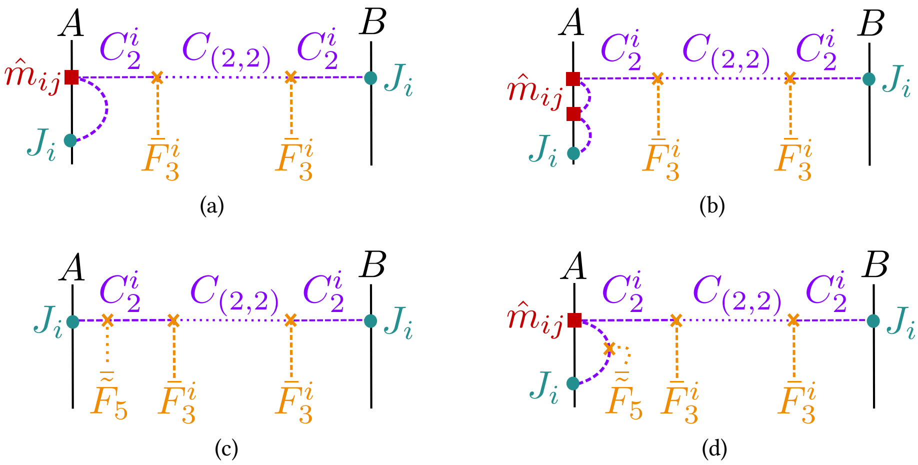


Fig. C.3: Examples of divergent diagrams which will renormalize the brane theory.

Besides $\hat{m}$, there is a second source for divergences: Close to the D3 branes, the profile of $\bar{\tilde{F}}_5$ diverges as y^{-5} according to Gauss' law. Specifically in the diagram of fig. C.3c the integral over the position of the $\bar{\tilde{F}}_5$ vertex is divergent. This divergence arises because both the value of $\bar{\tilde{F}}_5$ and the propagator between J_i and the $\bar{\tilde{F}}_5$ vertex blow up as the vertex approaches the brane. Cutting the integral off at a shortest distance $\sim 1/M_s$ yields an effect with the same prefactor relative to the leading-order term as in (C.9).

More such divergent diagrams exist. In particular, there are also mixed divergences from diagrams involving both $\hat{m}$ and $\bar{\tilde{F}}_5$ insertions, see fig. C.3d. Crucially, all those divergent diagrams have one common feature: Their divergence is proportional to the leading order coupling of the brane source J_i to the bulk field C_2^i. Thus, up to finite terms, their total effect can be absorbed in a renormalization of the brane action, more specifically of the coupling to C_2^i. This is illustrated in fig. C.4. The finite contributions from some of these divergent diagrams may have a structure which is distinct from the leading-order J_i–C_2^i–coupling and can hence not be absorbed in a renormalization. For example, this is the case for the integration region in diagram C.3c for which the $\bar{\tilde{F}}_5$ vertex is distant from the brane. However, such contributions are parametrically suppressed, in this case by the diluteness of the 5-form-flux away from the brane.

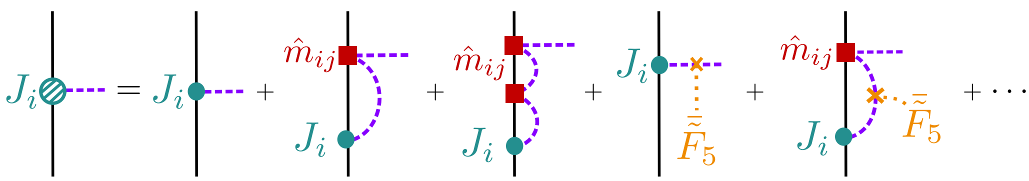


Fig. C.4: Series of diagrams renormalizing the brane coupling between J_i and C_2^i.

In summary, the divergent diagrams correct our general leading order result (6.81) only by terms which are g_s-suppressed in the perturbative regime. Even if $g_s \sim \mathcal{O}(1)$, the corrections are expected to correspond to a renormalization of the brane coupling by an $\mathcal{O}(1)$ factor. This is illustrated in fig. C.5, implying in particular that the crucial parametric dependencies of KM on 3-form flux and volume are not affected.

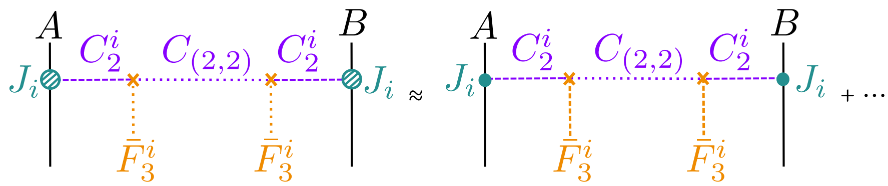
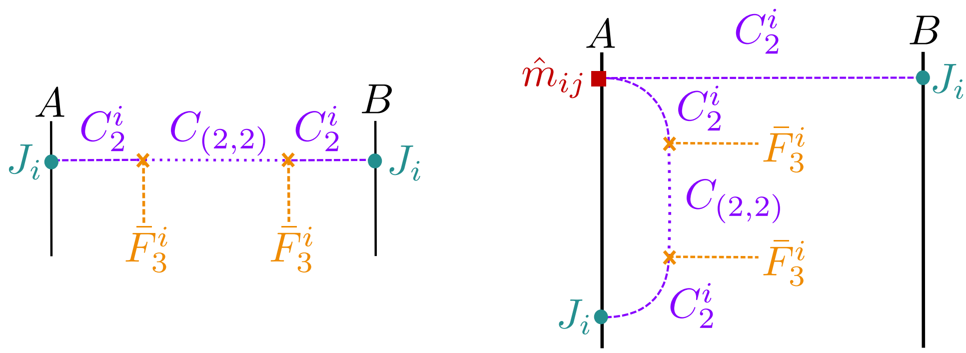


Fig. C.5: Our leading order approximation captures a more general KM result, including a renormalized brane coupling, at the $\mathcal{O}(1)$ level.

C.3. Volume suppressed Diagrams

In addition, there is a third class of diagrams, with the simplest example depicted in fig. C.6b. These diagrams are characterized by the bulk field propagator directly connecting the two branes, with the flux vertices being inserted elsewhere. We will show that such diagrams are volume suppressed.



(a) (b)

Fig. C.6: Example diagram yielding the leading contributions (a) in comparison to the suppressed KM contribution (b).

Let us first reconsider the leading contribution from fig. C.6a. Reformulating our corresponding result, (6.81) with (6.79), in a diagrammatic language, we have

$$\mathcal{A}(\text{C.6a}) \sim \int_{y,y'} G_6(y_A - y)\bar{F}^i_{mno}\partial^2 G_6(y-y')\bar{F}^{i\,mno} G_6(y'-y_B)\,. \tag{C.10}$$

Here we suppressed the y/y' dependence of the 3-form fluxes as well as the sources, the index contractions and constant prefactors. Using the behaviour of G_6 at small distances, cf. (6.85), one can convince oneself that there are no UV divergences and the integral in (C.10) is hence IR dominated. The scaling may then be determined by collecting the factors $(R^6)^2$ from the integrations, $1/(R^4)^3$ from the propagators and $1/R^2$ from the derivatives. Using $R \sim \mathcal{V}^{1/6}$ this implies[48]

$$\mathcal{A}(\text{C.6a}) \sim (\bar{F}^i_{mno})^2\, \mathcal{V}^{-1/3}\, \alpha'^{-1}\,. \tag{C.11}$$

Note that $R \sim \mathcal{V}^{-1/6}$ measures distances in Planck units in the 10d Einstein frame and we kept the units of length coming from the integral in (C.10) explicit in form of the factor $\alpha'^{-1} \sim M^2_{\text{Pl},10}$

We proceed analogously with the diagram in fig. C.6b, finding

$$\mathcal{A}(\text{C.6b}) \sim (2\kappa^2_{10} T_3)\, G_6(y_A - y_B) \int_{y,y'} G_6(y_A - y) \bar{F}^i_{mno} \partial^2 G_6(y - y') \bar{F}^{i\, mno} G_6(y' - y_A)\,. \tag{C.12}$$

While we again suppressed sources and index contractions, we explicitly displayed the relative prefactor $2\kappa^2_{10} T_3$ distinguishing $\mathcal{A}(\text{C.6b})$ from $\mathcal{A}(\text{C.6a})$. Here $2\kappa^2_{10}$ comes from the extra propagator and T_3 from the $\hat{m}$ vertex. The integrations in (C.12) are quadratically divergent in the region $y, y' \to y_A$. As before, we cut off this UV divergence at the physical string scale $M_\text{s} \sim g_\text{s}^{1/4}/\sqrt{\alpha'}$, finding

$$\mathcal{A}(\text{C.6b}) \sim (2\kappa^2_{10} T_3)\, G_6(y_\text{A} - y_\text{B})\, (\bar{F}^i_{mno})^2\, M^2_\text{s}\,. \tag{C.13}$$

We use the estimate $G_6(y_\text{A} - y_\text{B}) \sim |y_\text{A} - y_\text{B}|^{-4} \sim 1/(\sqrt{\alpha'} R)^4$ which yields

$$\mathcal{A}(\text{C.6b}) \sim (\bar{F}^i_{mno})^2\, g_\text{s}^{1/2}\, \mathcal{V}^{-2/3}\, \alpha'^{-1} \tag{C.14}$$

and hence

$$\mathcal{A}(\text{C.6b}) \Big/ \mathcal{A}(\text{C.6a}) \sim g_\text{s}^{1/2} \mathcal{V}^{-1/3}\,. \tag{C.15}$$

This volume suppression extends to all diagrams in which the two $\bar{F}^i_3$ insertions appear in a line connecting one of the branes to itself, as in fig. C.6b. More 3-form flux insertions only make the volume suppression worse. Thus, the diagram in fig. C.6a does indeed represent our leading effect.

[48] Using (6.84) for the volume scaling of the fluxes one confirms the scaling (6.92) from sect. 6.5.

D. D3-brane stack toy model

Consider the following toy model for a more realistic setting of KM. We choose our hidden sector to consist of a stack of two D3-branes and consider the SM to be localised far away from the hidden D3-brane stack, see fig. D.1. We do not specify how the SM is realised, but consider light

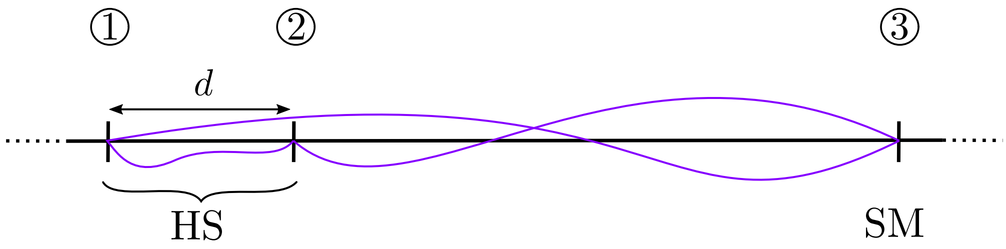


Fig. D.1: Sketch for our toy model. We indicated the strings stretched between the branes in purple.

charged states to be present. Further, we assume that the SM U(1) will kinetically mix with other D3-brane U(1)s and the mixing shall be induced by B_2 and C_2 exchanges as we presented in our paper.

The first step to arrange for KM is to break the gauge group of the stack $\mathrm{U}(2) \to \mathrm{U}(1)^{(1)} \times \mathrm{U}(1)^{(2)}$, e.g. by separating the branes in the stack by a distance $d = \langle y^i \rangle$, where y^i denote the internal coordinates. The (adjoint) coordinates y^i are related to adjoint scalar fields Φ^i specifying the position of the branes [252]

$$\frac{2\pi y^i}{l_s} = \frac{\Phi^i}{M_s} , \tag{D.1}$$

where we reinstated dimensional factors. Analogously, ϕ^i obtains a vev $\langle \Phi^i \rangle$ from the separation in the stack, which will break the U(2) gauge theory. For simplicity we will only consider a separation in one direction and hence suppress the coordinate index, i.e. $\Phi^i \to \Phi$.

The strings between the branes, characterized by Chan-Paton labels (12) and (21), yield states $j_{(12)}$ and $j_{(21)}$ with charges $(1, -1)$ and $(-1, 1)$ under $\mathrm{U}(1)^{(1)} \times \mathrm{U}(1)^{(2)}$ respectively. Further, the mass m_{cs} of the charged states is related to the separation by

$$m_{cs} \sim \langle \Phi \rangle , \tag{D.2}$$

and hence will be small if $d < l_s$. In addition, due to heavy modes of the strings between the different sectors, KM between all U(1)s will be induced.

Hence, the 4d EFT Lagrangian schematically would read

$$\begin{aligned}\mathcal{L}_{4d} =& -\frac{1}{4}\left(F^{(11)}_{\mu\nu}F^{\mu\nu}_{(11)} + F^{(22)}_{\mu\nu}F^{\mu\nu}_{(22)} + F^{(33)}_{\mu\nu}F^{\mu\nu}_{(33)}\right) \\ & -\frac{1}{2}\left(\chi_{(12)}F^{(11)}_{\mu\nu}F^{\mu\nu}_{(22)} + \chi_{(13)}F^{(11)}_{\mu\nu}F^{\mu\nu}_{(33)} + \chi_{(23)}F^{(22)}_{\mu\nu}F^{\mu\nu}_{(33)}\right) \\ & + g\, j^{\mu}_{(12)}\left(A^{(11)}_{\mu} - A^{(22)}_{\mu}\right) + g\, j^{\mu}_{(21)}\left(-A^{(11)}_{\mu} + A^{(22)}_{\mu}\right) + g_{\mathrm{SM}}\, j^{\mu}_{\mathrm{SM}} A^{(33)}_{\mu}\,,\end{aligned} \tag{D.3}$$

where $A^{(11)}_{\mu}$, $A^{(22)}_{\mu}$ and $A^{(33)}_{\mu}$ refer to the gauge fields on the branes, 1 and 2, and the SM sector, 3. The states $j^{\mu}_{(12)}$ and $j^{\mu}_{(21)}$ are charged only under the combination $A^{(11)}_{\mu} - A^{(22)}_{\mu}$ thus only $\mathrm{U}(1)^{(1)}$–$\mathrm{U}(1)^{(2)}$ and $\mathrm{U}(1)^{(3)}$ have light charged states. Hence the mixing terms with $\mathrm{U}(1)^{(1)}$+$\mathrm{U}(1)^{(2)}$ can be eliminated by a suitable field redefinition [39, 189]. The final Lagrangian written in a field basis with canonical kinetic terms reads

$$\begin{aligned}\mathcal{L}_{4d} =& -\frac{1}{4}\left(F^{(\mathbb{1})}_{\mu\nu}F^{\mu\nu}_{(\mathbb{1})} + F^{(\sigma_3)}_{\mu\nu}F^{\mu\nu}_{(\sigma_3)} + F'^{(33)}_{\mu\nu}F'^{\mu\nu}_{(33)}\right) \\ & + \frac{2g}{\sqrt{1+2\chi_{(12)}}}\left(j^{\mu}_{(12)} - j^{\mu}_{(21)}\right)A^{(\sigma_3)}_{\mu} \\ & + \frac{g_{\mathrm{SM}}}{\sqrt{(1-\chi'^2)(1-\chi^2)}}\left[j^{\mu}_{\mathrm{SM}} + Q\left(j^{\mu}_{(12)} - j^{\mu}_{(21)}\right)\right]A'^{(33)}_{\mu}\,,\end{aligned} \tag{D.4}$$

supplemented with the definitions

$$\chi' = \frac{\chi_{(13)} + \chi_{(23)}}{\sqrt{1-2\chi_{(12)}}}\,, \qquad \chi = \frac{\chi_{(13)} - \chi_{(23)}}{\sqrt{(1+2\chi_{(12)})(1-\chi'^2)}}\,, \tag{D.5}$$

$$Q = \frac{-2g}{g_{\mathrm{SM}}}\,\chi\sqrt{\frac{1-\chi'^2}{1+2\chi_{(12)}}}\,, \qquad A'^{(33)}_{\mu} = \sqrt{1-\chi^2}\sqrt{1-\chi'^2}A^{(33)}_{\mu}\,, \tag{D.6}$$

$$A^{(\mathbb{1})}_{\mu} = \frac{A^{(11)}_{\mu} + A^{(22)}_{\mu}}{2} + \chi' A^{(33)}_{\mu}\,, \qquad A^{(\sigma_3)}_{\mu} = \frac{A^{(11)}_{\mu} - A^{(22)}_{\mu}}{2} + \chi\sqrt{1-\chi'^2}A^{(33)}_{\mu}\,. \tag{D.7}$$

We see that in addition to the SM current $\sim j^{\mu}_{\mathrm{SM}}$ we have obtained a current, $\sim \left(j^{\mu}_{(12)} - j^{\mu}_{(12)}\right)$, made from millicharged particles with millicharge Q. The mixing parameter $\chi_{(12)}$ has to be small, since the branes 1 and 2 are not widely separated and the non-abelian structure from the stack suppresses KM. We hence assume $\chi_{(12)} \ll 1$ and expand Q around $\chi_{(12)} \approx 0$

$$Q = \frac{-2g}{g_{\mathrm{SM}}}\left(\chi_{(13)} - \chi_{(23)}\right) + \mathcal{O}(\chi^2_{(12)})\,. \tag{D.8}$$

Considering the results for KM in (6.79) and (6.81), one can think of KM as a function of the D3-brane positions y_A, i.e. $\chi_{(AB)} = \chi_{(AB)}(y_A, y_B)$. Hence we can relate the millicharge Q of our toy model to the separation $d = y_2 - y_1$

$$
\begin{aligned}
Q &\simeq \frac{-2g}{g_{\rm SM}} \left(\chi_{(13)}(y_1, y_3) - \chi_{(23)}(y_1 + d, y_3) \right) \\
&\simeq \frac{2g}{g_{\rm SM}}\, d\, \frac{\mathrm{d}\chi_{(23)}(s, y_3)}{\mathrm{d}s}\bigg|_{s=y_1} \,,
\end{aligned}
\tag{D.9}
$$

or respectively to χ using (2.16)

$$
\chi \sim -d\, \frac{\mathrm{d}\chi_{(23)}(s, y_3)}{\mathrm{d}s}\bigg|_{s=y_1} \,. \tag{D.10}
$$

We can give an estimate of (D.10) by applying the derivative $\mathrm{d}/\mathrm{d}s$ to K^{ij} from (6.79). Using the same procedure of sect. 6.5 we used to estimate the scaling of K^{ij} (6.88), we find that also the derivative of K^{ij} (6.79) is IR dominated. Thus, ignoring all numerical factors of 2 and π to focus on the parametric scaling we find

$$
\frac{\mathrm{d}\chi_{(23)}(s, y_3)}{\mathrm{d}s}\bigg|_{s=y_1} \sim \frac{1}{\mathcal{V}^{4/3}}\, \frac{1}{\mathcal{V}^{1/6}} \,. \tag{D.11}
$$

Using (D.1) and expressing l_s in 10d Planck finally yields for (D.10)

$$
\chi \sim -\frac{1}{\mathcal{V}^{4/3}}\, \frac{g_{\rm s}^{1/4}}{\mathcal{V}^{1/6}}\, \frac{\langle \Phi \rangle}{M_{\rm s}} \,. \tag{D.12}
$$

The first factor, $\mathcal{V}^{-4/3}$, can be identified with the original suppression we found due to the large separation, cf. (6.88). The second term, $g_{\rm s}^{1/4}\mathcal{V}^{-1/6}$, can be explained by the fact that C_2^i now effectively couples to a dipole instead of a monopole, since we are mixing with $A_\mu^{(\sigma_3)}$ of the relative $\mathrm{U}(1)$ of the stack. The third term, $\langle \Phi \rangle\, M_{\rm s}^{-1}$, corresponds to the extra suppression factor we expected due to the breaking of the non-abelian gauge group.

E. D3-brane Stack Mediating Couplings

In this section we identify the fields mediating kinetic mixing between stacks of D3-branes where we break the non-abelian gauge group by separating the branes in the stack. The separation is controlled by a diagonal vev of the position scalars $\langle\Phi\rangle^i$ in the transverse directions. Due to a diagonal $\langle\Phi\rangle^i$, the commutator vanishes $[\langle\Phi\rangle^i, \langle\Phi\rangle^j] = 0$ and (7.7) to (7.10) reduce to[34]

$$Q^i{}_j = \delta^i{}_j \,, \tag{E.1}$$

$$P_{MN} = E_{MN} = g_{MN} - B_{MN} \,, \tag{E.2}$$

$$\varphi^* [P]_{ab} = \varphi^* [g]_{ab} - \varphi^* [B]_{ab} \,, \tag{E.3}$$

$$\varphi^* [g]_{ab} = g_{ab} - g_{ij} \left[A_a, \langle\Phi\rangle^i\right] \left[A_b, \langle\Phi\rangle^j\right] \,, \tag{E.4}$$

$$\begin{aligned} \varphi^* [B]_{ab} = {} & B_{ab} + i \left(B_{ai} \left[A_b, \langle\Phi\rangle^i\right] - B_{bi} \left[A_a, \langle\Phi\rangle^i\right]\right) \\ & - B_{ij} \left[A_a, \langle\Phi\rangle^i\right] \left[A_b, \langle\Phi\rangle^j\right] \,, \end{aligned} \tag{E.5}$$

$$\mathrm{i}_\Phi \mathrm{i}_\Phi C_{(n)} = 0 \,. \tag{E.6}$$

With these simplifications and using the expansion

$$\sqrt{\det(1 + M)} = 1 + \frac{1}{2} \operatorname{tr} M + \frac{1}{8} \left[(\operatorname{tr} M)^2 - 2 \operatorname{tr} M^2\right] + \mathcal{O}(M^3) \tag{E.7}$$

one finds for (7.1)

$$S_{\text{NDBI}} = -\frac{T_3}{g_s} \int_{D3} \mathrm{d}^{3+1}\xi \sqrt{-\det g} \;\operatorname{STr}\left(1 + \frac{1}{2} \operatorname{tr} M + \frac{1}{8} \left[(\operatorname{tr} M)^2 - 2 \operatorname{tr} M^2\right]\right) \,, \tag{E.8}$$

where we fixed $e^\phi = g_s$, the trace tr acts on Lorentz indices and M is given by

$$\begin{aligned} M_{ab} = {} & F_{ab} - g_{ij} \left[A_a, \langle\Phi\rangle^i\right] \left[A_b, \langle\Phi\rangle^j\right] \\ & - B_{ab} - i \left(B_{ai} \left[A_b, \langle\Phi\rangle^i\right] - B_{bi} \left[A_a, \langle\Phi\rangle^i\right]\right) + B_{ij} \left[A_a, \langle\Phi\rangle^i\right] \left[A_b, \langle\Phi\rangle^j\right] \,. \end{aligned} \tag{E.9}$$

Analogously one obtains for (7.2)

$$S_{\text{NCS}} = T_3 \int_{D3} \operatorname{STr}\left(\varphi^* [C_4 + 1/2\, B_2 \wedge C_2] + \varphi^* [C_2] \wedge (F_2 - \varphi^* [B_2]) + \frac{\varphi^* [C_0]}{2} (F_2 - \varphi^* [B_2])^2\right) \,. \tag{E.10}$$

The terms mediating kinetic mixing have to be linear in $F_2 = dA_1$ or A_1, contain any number of $\langle\Phi\rangle^i$ and should contain only one other bulk field to mediate the mixing. Considering first (E.8), we see from

$$\begin{aligned}\operatorname{tr} M = &-g_{ij}\left[A_a, \langle\Phi\rangle^i\right]\left[A^a, \langle\Phi\rangle^j\right] - i\left(B_{ai}\left[A^a, \langle\Phi\rangle^i\right] - B_{ai}\left[A^a, \langle\Phi\rangle^i\right]\right) \\ &+ B_{ij}\left[A_a, \langle\Phi\rangle^i\right]\left[A^a, \langle\Phi\rangle^j\right] \ ,\end{aligned} \tag{E.11}$$

that only

$$i\left(B_{ai}\left[A^a, \langle\Phi\rangle^i\right] - B_{ai}\left[A^a, \langle\Phi\rangle^i\right]\right) = 0 \tag{E.12}$$

fullfills this requirement but vanishes. The terms in $(\operatorname{tr} M)^2$ contain at least two A_1 and are thus also not suitable. On the other hand, from $\mathrm{STr}(\operatorname{tr} M^2)$ one obtains the term

$$2\ \mathrm{STr}\left(\operatorname{tr} F^{ab} B_{bc}\right) = -2\ \mathrm{STr}\left(F^{ab} B_{ab}\right) = -2\ \mathrm{STr}\left[F^{ab}\left(\sum_{n=1} \frac{\lambda^n}{n!} \langle\Phi\rangle^{i_1} \dots \langle\Phi\rangle^{i_n} \partial_{i_1} \dots \partial_{i_n} [B^0_{ab}]_{x^i=0}\right)\right]. \tag{E.13}$$

Considering now (E.10), the only term in question reads

$$\begin{aligned}\mathrm{STr}\left(\varphi^*[C_2] \wedge F_2\right) = -\frac{1}{4}\varepsilon^{abcd}\ \mathrm{STr}\Big(\Big\{&C_{ab} + i\lambda\left(C_{ai}\left[A_b, \langle\Phi\rangle^i\right] - C_{bi}\left[A_a, \langle\Phi\rangle^i\right]\right) \\ &- \lambda^2 C_{ij}\left[A_a, \langle\Phi\rangle^i\right]\left[A_b, \langle\Phi\rangle^j\right]\Big\} F_{cd}\Big) \star_4 1 \ ,\end{aligned} \tag{E.14}$$

where only

$$-\frac{1}{4}\varepsilon^{abcd}\ \mathrm{STr}\left[F_{cd}\left(\sum_{n=1} \frac{\lambda^n}{n!} \langle\Phi\rangle^{i_1} \dots \langle\Phi\rangle^{i_n} \partial_{i_1} \dots \partial_{i_n} [C^0_{ab}]_{x^i=0}\right)\right] \star_4 1 \ , \tag{E.15}$$

is suitable.

From this we again see that only B_2 and C_2 couple linearly to the field strength F_2 in a similar way as in sect. 6 and we thus expect a cancellation unless 3-from fluxes are included. We refer to app. D for the results of this case where we however not work with the explicit Φ dependencies.

F. Solution to the p-form Laplace equation

To fix the notation used in this thesis, we spell out how the Laplace equation for differential forms can be solved. We follow the presentation of [380].

Consider the following action on an m-dimensional manifold $\mathcal{X}$

$$S = -\frac{1}{2} \int_{\mathcal{X}} F_{p+1} \wedge \star_m F_{p+1} + \int_{\mathcal{X}} C_p \wedge J_{m-p} \, , \tag{F.1}$$

where $F_{p+1} = \mathrm{d}C_p$ and $\star_m$ denotes the Hodge star operator associated to the metric on $\mathcal{X}$. The equation of motion for C_p is thus given by

$$\star_m \mathrm{d}^\dagger \mathrm{d} C_p = J_{m-p} \, . \tag{F.2}$$

Fixing the gauge for C_p by

$$\mathrm{d}^\dagger C_p = 0 \, , \tag{F.3}$$

we can rewrite $\mathrm{d}^\dagger \mathrm{d} C_p = (\mathrm{d}^\dagger \mathrm{d} + \mathrm{d}\mathrm{d}^\dagger) C_p = \Delta C_p$. This yields for the equation of motion (F.2)

$$\Delta C_p = \star_m^{-1} J_{m-p} \, . \tag{F.4}$$

A solution to (F.4) can be given in terms of a Greens function $\hat{G}_{m-p}$, which is defined as a p-form $\otimes$ $(m-p)$-form on $\mathcal{X}$ and defined by

$$\Delta_y \hat{G}_{m-p}(y, y') = -\delta_{m-p}(y - y') \, . \tag{F.5}$$

Here Δ_y acts wrt. to the coordinates y. The delta distribution δ_{m-p} in (F.5) is also defined as a p-form $\times$ $(m-p)$-form on $\mathcal{X}$

$$\delta_{m-p}(y - y') \coloneqq \mathrm{d}y'_{i_1} \wedge \cdots \wedge \mathrm{d}y'_{i_p} \otimes \star_m (\mathrm{d}y^{i_1} \wedge \cdots \wedge \mathrm{d}y^{i_p}) \frac{(-)^{p(m-p)}}{p!} \delta(y^1 - y'^1) \ldots \delta(y^m - y'^m) \, , \tag{F.6}$$

such that an integral of a p-form A_p over $\mathcal{X}$ yields

$$\int_{\mathcal{X}, y} \delta_{m-p}(y - y') \wedge A_p(y) = A_p(y') \, . \tag{F.7}$$

In general, (F.5) is a complicated differential equation which can be demonstrated by expressing the lhs. of (F.5) in components (see App. A of [379], we set $m - p = q$ for readability)

$$\left(\Delta \hat{G}_q\right)_{i_1 \ldots i_q} = -D^j D_j \hat{G}_{i_1 \ldots i_q} + \sum_{n=1}^{q} R^j{}_{i_n} \hat{G}_{i_1 \ldots i_{n-1} \; j \; i_{n-1} \ldots i_q} - 2 \sum_{m,n=1\, , \, m<n}^{q} R^{j_1}{}_{i_m}{}^{j_2}{}_{i_n} \hat{G}_{i_1 \ldots i_{m-1} \; j_1 \; i_{m-1} \ldots i_{n-1} \; j_2 \; i_{n-1} \ldots i_q} \, , \tag{F.8}$$

where D_j denotes the covariant derivative. Using the above definitions in flat space where both the Ricci tensors as well as the Riemann tensor vanish, one can express $\hat{G}_{m-p}$ as

$$\hat{G}_{m-p}(y, y') = \frac{(-)^{p(m-p)}}{p!} G_m(y, y') \, \mathrm{d}y'_{i_1} \wedge \cdots \wedge \mathrm{d}y'_{i_p} \otimes \star_m(\mathrm{d}y^{i_1} \wedge \cdots \wedge \mathrm{d}y^{i_p}) \,, \tag{F.9}$$

with the scalar Greens function G_m which satisfies

$$g^{ij} D_{(y),i} \frac{\partial}{\partial y^j} G_m(y, y') = -\delta(y^1 - y'^1) \ldots \delta(y^m - y'^m) \,. \tag{F.10}$$

Importantly, with (F.5) and (F.7) one can express any p-form A_p on $\mathcal{X}$ through the following integral [380] [49]

$$\begin{aligned} A_p(y') = (-)^{m-p} \int\limits_{\mathcal{X},y} &\left[\mathrm{d}\hat{G}_{m-p}(y, y') \wedge \mathrm{d}^\dagger A_p - \mathrm{d}^\dagger \hat{G}_{m-p}(y, y') \wedge \mathrm{d}A_p \right] \\ + \int\limits_{\partial\mathcal{X},y} &\left[(-)^{m-p+s} \star_m \mathrm{d}\hat{G}_{m-p}(y, y') \wedge \star_m A_p - \mathrm{d}^\dagger \hat{G}_{m-p}(y, y') \wedge A_p \right] \,. \end{aligned} \tag{F.11}$$

Specifying this to C_p from above with equation of motion (F.2) and gauge choice (F.3), one finds using (F.11)

$$C_p(y') = -\int\limits_{\mathcal{X},y} \hat{G}_{m-p}(y, y') \wedge \star_m^{-1} J_{m-p}(y) \tag{F.12}$$

where we used the identity

$$\int \mathrm{d}^\dagger A_q \wedge B_{m-q+1} = \int (-)^q A_q \wedge \mathrm{d}^\dagger B_{m-q+1} + (-)^{q+s} \mathrm{d}\big(\star A_q \wedge \star B_{m-q+1} \big) \,, \tag{F.13}$$

and omitted all boundary terms.

[49] $s = 1$ for a Lorentzian signature and $s = 0$ for Euclidean signature of $\mathcal{X}^6$.

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