HEIDELBERG UNIVERSITY
DEPARTMENT OF ECONOMICS

UNIVERSITÄT HEIDELBERG
ZUKUNFT
SEIT 1386

Commitment and Conflict in Unanimity Bargaining

Topi Miettinen

Christoph Vanberg

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Topi Miettinen and Christoph Vanberg*

Abstract

We theoretically investigate how the application of unanimity rule can lead to inefficient delay in collective decision making. We do so in the context of a distributive multilateral bargaining model featuring strategic pre-commitment. Prior to each bargaining round, players can declare a minimum share that they must receive in return for their vote. Such declarations become binding with an exogenously given probability. We characterize the set of stationary subgame perfect equilibria under all q-majority rules. Our results suggest that unanimity rule is uniquely inefficient. All other rules, including all-but-one, are fully efficient.
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"The unanimity rule has meant that some key proposals for growth, competitiveness and tax fairness in the Single Market have been blocked for years." (European Commission press release, Jan 15th 2019).

A fundamental constitutional choice facing organizations such as the European Union (EU) concerns the voting rules to be used in decision making bodies, e.g. the Council of Ministers. Many organizations, including the EU, require the unanimous consent of all members at least for certain types of decisions. This rule has the important advantage that it ensures that all decisions reached constitute Pareto improvements over the status quo. However, politicians and expert observers alike have often complained that unanimity rule is responsible for inefficient delays and even gridlock. Prominent EU officials, including former and current Commission presidents Jean-Claude Juncker and Ursula von der Leyen, have therefore proposed the expanded use of qualified majority rule (QMR). The call for such a reform seems to have gained momentum as the number of member states has grown. Other international organizations, including the World Trade Organization (WTO), The United Nations Framework Convention on Climate Change (UNFCCC), and the North Atlantic Treaty Organization (NATO), face similar challenges.[1]

Despite this public debate, there is little game-theoretic literature formally investigating the effect of alternative decision rules on inefficient delay. The goal of the present paper is to develop such a formalization. We therefore consider a setting where immediate agreement would be efficient. In addition to unanimity rule, we investigate the potential efficiency properties of qualified majority rules. Our model builds on the seminal Baron and Ferejohn (1989) model of legislative bargaining, to which it adds a capacity to precommit. In particular, we add a commitment stage at the beginning of each round of negotiations. At this stage, each player can attempt to commit to rejecting, in the subsequent bargaining stage, any proposal where she receives less than a self-imposed threshold. Between the commitment and bargaining stages, any such attempted commitment fails with an exogenously given probability.[2] The commitment status is assumed common knowledge at the

*Miettinen: Helsinki GSE & Hanken School of Economics, Arkadiankatu 7, P.O. Box 479, Fi-00101 Helsinki, Finland; topi.miettinen@hanken.fi. Vanberg: Department of Economics, University of Heidelberg, Heidelberg, Germany; vanberg@uni-hd.de. Thanks to Volker Britz, Colin Campbell, Josh Ederington, Tore Ellingsen, Hülya Eraslan, Guillaume Frechette, Faruk Gul, Bård Harstad, Emin Karagözoglu, Duozhe Li, Brian Rogers, Ariel Rubinstein, Tomas Sjöström, Jonathan Weinstein, Alexander Wolitzky, three anonymous reviewers, and the seminar audiences at BEET 2019, SING 2019, EEA-ESEM 2021, EPCS 2022, Nordic Theory Group Online Seminar, BI Norwegian Business School, Helsinki GSE, Princeton, Rutgers, University of Kentucky, University of Michigan, University of Miami, and Washington University St. Louis for comments and feed-back. Financial support of the Fulbright Finland Foundation, the Norwegian Research Council (250506), the Yrjö Jahnsson Foundation (20187090), and the German Science Foundation (314978473) is gratefully acknowledged.

[1] The WTO was not able to meet the December 2002 deadline imposed in paragraph 6 of the Doha declaration on the TRIPS agreement since a single member prevented consensus (Ehlermann and Ehring, 2005). UNFCCC has not been able to reach a comprehensive and binding agreement on how to limit carbondioxide emissions (Pizer, 2006; Nordhaus, 2006; Stern, 2008; Gollier and Tirole, 2017). A recent example of aggressive commitment tactics is the public posturing of the Turkish president in connection with the NATO membership bids by Finland and Sweden. Expert obervers interpreted this as an attempt to extract concessions from the U.S. and other NATO members (https://www.bloomberg.com/news/articles/2022-05-17/what-turkey-wants-from-sweden-and-finland-in-nato-expansion-spat). Similar negotiation tactics have been employed by the Hungarian prime minister Viktor Orbàn both in EU and NATO contexts.

[2] We adopt the simplest such model where each player's probability of failing is independent of the failures of the commitment attempts of other players and, moreover, each individual commitment attempt has an equal chance of failing. Also, for the sake of simplicity, we assume that commitments need to be re-established at the commitment stage of each round.

bargaining stage. Thus, we pursue a rational, complete information explanation for conflict.[3] Once commitment attempts have been made and their success determined, one of the players is randomly drawn to make a proposal to which all others respond by either accepting or rejecting. Agreement arises if, according to an exogenously given consent rule, sufficiently many players accept.

We find that, under unanimity rule, an inefficient stationary subgame perfect equilibrium (SSPE) with delay always exists. This equilibrium involves all players making maximally agressive commitments, essentially betting on the event that all other players' commitments fail. When all players adopt such a tough stance, the only way to increase the chance of agreement would be for a player to reduce her commitment to her continuation value, leading to a strictly lower payoff. Perhaps more surprisingly, we find that when the commitment success probability is small enough, every SSPE is inefficient. The reason is that, when all players adopt moderate (i.e. mutually compatible) bargaining positions, at least one player will have an incentive to adopt a more aggressive stance in an attempt to capture the surplus that becomes available whenever one or more commitments fail. Moreover, the commitment success probability required for an efficient equilibrium to exist is increasing in the number of players. As a consequence, delay and inefficiency become more severe as the number of players grows.

In contrast, there can be no delay or inefficiency in any SSPE under any less-than-unanimity rule. Giving up unanimity and instead requiring all but one party to agree is enough to restore full efficiency in our model. The mechanism underlying this result is that proposers exclude the most "expensive" players when building minimum winning coalitions, inducing Bertrand-style competition in commitments.[4]

Our model highlights the role of aggressive precommitment as a mechanism contributing to inefficient delay in unanimity decision making. In seminal work, Buchanan and Tullock (1965) posited a fundamental tradeoff between what they called the "external costs" and the "decision costs" associated with a given q-majority rule. The larger is the required majority, the smaller is the chance that a given individual will be harmed by a collective decision. Conversely, Buchanan and Tullock conjectured that more demanding majority requirements will be associated with greater expected "decision costs" in the form of delay and possibly gridlock. Our modeling approach provides a simple game-theoretic micro-foundation for the latter conjecture in a purely distributive bargaining model. By design, we do not investigate complexities generated by externalities or incomplete information. By focusing on the distributive element, our model offers a tractable and transparent framework to understand the effects of (i) commitment, (ii) decision rule, and (iii) number of parties involved on what Buchanan and Tullock coined the decision costs. It thereby contributes to a deeper understanding of such inefficiencies and how they might be reduced by moving to qualified majority rule.

The idea that being committed to an aggressive bargaining position can be advantageous within negotiations can be traced back to Schelling (1956), who discussed several means by which such pre-commitment could be achieved. One example he provides is that national representatives taking part in an international negotiation often "create a bargaining position by public statements, statements calculated to arouse a public opinion that permits no concessions to be made."[5] Another example involves sending delegates who have limited and observable mandates to agree only on certain terms.[6] In both cases, exogenous events may later undermine the commitment attempt. For example, public announcements of a bargaining position will usually be associated with some form of justification which may subsequently be undermined if new information becomes available, giving the bargainers an "excuse" to reconsider the position.[7]

Crawford (1982) formalizes some of Schelling's arguments in a bilateral bargaining framework with both strategic ex-ante pre-commitment and ex-post revoking of commitments.[8] He shows that with sufficiently low commitment success probability, both players make aggressive pre-commitments in the unique equilibrium, which is inefficient since the commitments are mutually incompatible. Ellingsen and Miettinen (2008) show that impasse may be considerably more likely and inefficiency more severe if there is a small cost of commitment. Ellingsen and Miettinen (2014) analyze stochastic commitments in a bilateral dynamic infinite horizon setting. Our paper is the first to apply the stochastic commitment approach in the multilateral case and to compare the

[3]Although inspired by (Schelling, 1960), our model thus falls outside the canonical explanations of rational conflict (Fearon, 1995; Jackson and Morelli, 2011). Another such model is Wolitzky (2023) who shows that imperfectly observed offers or claims can unavoidably result in inefficient conflict with positive probability.

[4]The theoretical principle that "expensive" players are excluded from winning coalitions receives empirical support in the experiment by Miller, Montero and Vanberg (2018).

[5]Putnam (1988) suggests informally that "one effective way to [commit] to a position (...) is to rally support from one's constituents" in order to induce audience costs (1988, p.450). A recent bilateral model which captures some of Putnam's intuition is provided by Basak and Deb (2020) where concession costs depend on political support, which in turn depends on the stochastic realization of an underlying state of nature.

[6]This idea has formalized, among others, by Jones (1989) and Harstad (2010).

[7]In climate negotiations, the basis for the bargaining position or mandate could be that measures to combat climate change in each country should be proportional to per capita net emissions. Suppose that between the date when the commitment position goes public and the following COP meeting starts, new scientific evidence is published that net emissions per capita in the respective country are higher than expected. Then the negotiating party will have more room to maneuver and the commitment fails.

[8]For various modeling approaches, see Crawford (1982); Muthoo (1996); Myerson (1991); Fearon (1994, 1997); Levenotoğlu and Tarar (2005); Jackson and Morelli (2007); Güth, Ritzberger and Van Damme (2004) for instance. Subtle differences in assumptions about the way commitment is built and how it may be lost lead to differences in predicted outcomes. See Muthoo (1999) and Miettinen (2022) for reviews.

performance of alternative decision rules.

A closely related modeling approach is based on the idea that players adopt a tough bargaining stance in an attempt to mimic an obstinate behavioral type. Among bilateral bargaining models using this reputational framework (e.g. Myerson, 1991; Abreu and Gul, 2000; Compte and Jehiel, 2002; Kambe, 1999; Fanning and Wolitzky, 2022), some (but not all) predict inefficient delay.[9] Notice that such models effectively assume unanimity rule. While these models have generated interesting insights and have been widely applied, they quickly become intractable when extended to n-player multilateral settings.[10] An exception is Ma (2023), who analyzes the three-player case with majority rule. He shows that there exists an equilibrium with efficient outcomes and that player's equilibrium payoff in that equilibrium is non-monotone in the probability of obstinacy.

Naturally, our paper also relates to the literature on multilateral bargaining in the tradition of Baron and Ferejohn (1989). Complete information models of this type typically predict immediate agreement in settings where delay is inefficient. This is true irrespective of the decision rule or the number of players (Banks and Duggan, 2000; Eraslan and Evdokimov, 2019). Some authors have analyzed settings in which delay can be efficient, in which case majority rule may lead to inefficient early agreement such that unanimity becomes the more efficient rule (Merlo and Wilson, 1995, 1998; Eraslan and Merlo, 2002).

Finally, our result concerning the sharp contrast between unanimity and less-than-unanimity rules bears some relation to results obtained in the information aggregation literature. Beginning with non-strategic analysis of Condorcet (1785), a number of authors have highlighted the effectiveness of majority rule in aggregating information when interests are aligned. Feddersen and Pesendorfer (1997, 1998) show that, with strategic voting, information aggregates efficiently for all rules other than unanimity. In these models, the proposal being considered is exogenously given. Bond and Eraslan (2010) consider a model in which the group is voting on an endogenous offer made by a proposer. There, unanimity rule can be strategically advantageous for responders, and may even be Pareto superior to majority rule.

The paper is structured as follows. Section 1 presents the model. Section 2 analyzes the simplest multilateral case of three players. Section 3 presents the general analysis and the main results. Section 4 concludes.

1 Model

The bargaining game involves $n \geq 3$ players and takes place in discrete time with infinite horizon. In each period $t \in \{1, 2, ...\}$, actions are taken in two stages – the commitment stage and the bargaining stage. At the commitment stage, players can attempt to make short-lived commitments which last at most the current period.[11] That is, each player i chooses a commitment attempt $x_i \in [0, 1]$, where $x_i = 0$ (or indeed any value below a player's continuation payoff) can be interpreted as a choice not to commit. In between the two stages, each player's commitment attempt may fail independently with probability $1 - \rho$. With a nod to Schelling's original contributions, we will say that a player whose commitment attempt fails has a loophole. The probability that a commitment attempt is successful is ρ. The realization of the attempt, the commitment status, is denoted by s_i and equals x_i with probabiliy ρ and 0 with probability $1 - \rho$.

At the bargaining stage, each player becomes the proposer with probability $1/n$. With probability $(n-1)/n$, a player becomes a responder. The proposer proposes a deal $d = (d_1, ..., d_N)$, with $\sum_{i=1}^{N} d_i \leq 1$, where we refer to d_i as the offer made to player i. Each player then votes to accept or reject. Yet, any player i with commitment status $s_i > d_i$ will automatically reject the proposal. The proposed deal is implemented if at least q players (including the proposer) vote to accept. If not, a new period begins with the commitment stage. If a deal d is implemented in period t, player i's payoff equals $\delta^{t-1} d_i$. Players are impatient, with (common) discount factor $\delta \in (0, 1)$.

Our equilibrium concept is Stationary Subgame Perfect Equilibrium (SSPE). A stationary strategy for player i, denoted ψ_i, specifies a commitment attempt x_i chosen at the commitment stage in any period, a proposal strategy for any realized commitment status profile $s = (s_1, ..., s_n)$, and an accept/reject action given any commitment status profile s, proposer j, and proposed deal d (the latter being relevant only at player i nodes where $s_i \leq d_i$). An SSPE is a collection of stationary strategies $\psi^* = (\psi_1^*, ..., \psi_n^*)$ which induce a Nash Equilibrium after every history. As is common in the literature on Baron-Ferejohn bargaining, we assume that players do not use weakly dominated strategies at the voting stage. See Appendix A.1 for the proof of the

[9] Myerson (1991) introduces obstinate types in a reputational model of bargaining and illustrates how delay occurs to screen the true type. In Kambe (1999), agreement may be immediate even if negotiators are obstinate with a positive probability. In Abreu and Gul (2000), delay occurs in the continuous time limit when players are obstinate with a strictly positive probability. See Fanning and Wolitzky (2022) for a recent review.

[10] In a multilateral bargaining model with obstinate types, each player would have to form beliefs about the obstinacy of each of the other players, and the actions specified by equilibrium strategies would depend on these beliefs and affect how they are updated at various histories. Thus, the dimensionality of the model grows exponentially with the number of players.

[11] In this respect, our model differs from Ellingsen and Miettinen (2014), which involves persistent commitments in a bilateral context. Limiting persistence to at most one period is necessary to make the model tractable for larger n, as otherwise the dimensionality of the state space on which even stationary strategies would be defined grows exponentially.

following Lemma.

Lemma 1. *Let $(x_1^*, ..., x_n^*)$ be the vector of commitment attempts that is part of an SSPE strategy profile ψ^*. And let $(v_1^*, ..., v_n^*)$ be the vector of expected utilities associated with that equilibrium. Then, the equilibrium strategies in the bargaining stage, given any commitment status profile $s = (s_1, ..., s_n)$, satisfy the following conditions:*

1. *Player i votes to accept iff she is offered at least $d_i \geq \hat{x}_i(s_i) \equiv max\{\delta v_i^*, s_i\}$.*
2. *Let C_i be (one of) the "cheapest coalition(s)" consisting of $q-1$ responders other than i. If $\hat{x}_i(s_i) \leq 1 - \sum_{j \in C_i} \hat{x}_j(s_j)$, then when player i proposes, he offers $\hat{x}_j(s_j)$ to each member j of C_i, the residual to himself, and votes to accept. (If there are multiple "cheapest" coalitions, he may randomize between them.) Otherwise, he makes a proposal that fails.*

Define $\pi_i(x|\psi^*)$ as the expected utility that would be achieved by player i if, at the initial commitment stage, (1) players attempted committing to $x = (x_1, ..., x_n)$ and (2) all players followed the equilibrium strategy ψ^* starting at the immediately ensuing bargaining stage.[12] *Then the commitment attempts x_i^* satisfy the following:*

$$x_i^* \in \arg\max \pi_i(x_i, x_{-i}^*|\psi^*)$$

Finally, the equilibrium expected utilites v_i^* satisfy

$$v_i^* = \pi_i(x^*|\psi^*).$$

2 Symmetric equilibria in the three player case

In this section, we illustrate our main results within a simple three-player model, focusing on symmetric equilibria in which all players make the same commitment attempt x^*. A more general analysis for n players and asymmetric equilibria is presented in Section 3. We characterize equilibria under majority and unanimity rule.

Subsection 2.1 considers majority rule and shows that all symmetric equilibria are efficient. Subsection 2.2 considers unanimity rule. There, commitment attempts can conceivably be of three types: An efficient commitment is compatible in the sense that a deal can be reached even if no player has a loophole. A moderately aggressive commitment is such that a deal requires at least one player to have a loophole. Finally, the most aggressive commitment permits a deal only if at least two players have a loophole (i.e. at most one player is committed). We show that under unanimity rule, an inefficient symmetric equilibrium involving the most aggressive commitments always exists. Finally, subsection 2.3 shows that an equilibrium with efficient commitments exists under unanimity rule if and only if the probability that commitments stick (ρ) is large enough.

2.1 Efficiency of majority rule

Consider the case of three players and majority rule, i.e. $q = 2$. It is easy to see that a commitment profile in which no player attempts to commit constitutes an equilibrium. In such an equilibrium, agreement is immediate and the common expected equilibrium payoff is $v^* = 1/3$. Due to stationarity, the common continuation value is δv^*. Any player who deviates to a commitment above δv^* would be excluded from a deal when his commitment sticks and he is responder. If he is proposer, the deviation will either have no effect, or prevent him from making a deal that yields him $(1 - \delta v^*) > \delta v^*$. Any deviation to a commitment below δv^* has no effect. Thus, no deviations from the no-commitment profile are profitable.

Indeed, this is the (essentially) unique symmetric equilibrium. To see this, suppose first that there exists a symmetric equilibrium with inefficient delay. Then, the inefficiency implies that $\delta v^* < 1/3$, and delay implies that $x^* > 1/2$ (otherwise, a deal including two players can be reached even when all players are committed). On the other hand, x^* must be such that it allows for the formation of a coalition consisting of one committed and one uncommitted player, i.e. $x^* \leq 1 - \delta v^*$ (otherwise, a player is never included in a deal when committed). It follows that a deal occurs if and only if at least one player has loophole. One such event is when only player 1 has a loophole and he is chosen to propose. Then, either player 2 or player 3 is included in that deal with probability less than one. Since $x^* > \delta v^*$, this player could deviate to a slightly less aggressive commitment attempt $y = x^* - \epsilon$. For ϵ small enough, this would guarantee that he is included for sure in the event described (and other events), and since ϵ can be arbitrarily small, this deviation is otherwise costless. (As proposer, the deviation does not affect his payoff, as he obtains the residual from agreement in all cases.) Therefore the deviator's payoff increases, a contradiction. This establishes that no symmetric equilibrium with inefficient delay exists.

[12] This payoff is pinned down by the conditions already outlined for the bargaining stage and by the additional condition that, in case no deal is made, player i's expected utility is given by δv_i^*. (The formal details are cumbersome to express explicitly but will be clearly developed in the subsequent analysis.)

Next, suppose there exists an efficient symmetric equilibrium with commitment $x^* > \delta v^*$. Agreement occurs in all events. One such event is when player 1 is proposer and all players are committed. Then again, either player 2 or player 3 is included with probability less than one in this event. By the same argument as above, this player can increase his payoff by deviating to $y = x^* - \epsilon$ for ϵ sufficiently small. A contradiction. It follows that any symmetric equilibrium involves $x^* \leq \delta v^*$, equivalent to not committing, and thus outcome-equivalent to the standard Baron-Ferejohn result.

Proposition 1. *In the three-player game under majority rule, there exists a unique symmetric equilibrium involving no commitments and immediate agreement. The agreement allocates $\delta/3$ to one responder, zero to the other, and $1 - \delta/3$ to the proposer.*

As in the standard Baron Ferejohn game, majority rule leads to asymmetries in the final allocation: One responder is excluded, the included responder receives her continuation value (less than 1/3), and the proposer's share is always greater than 2/3 of the pie. When the discount factor approaches zero (equivalently, a one period game), the outcome is maximally unequal with the proposer receiving the entire pie.

2.2 Aggressive equilibrium under unanimity rule

Next, consider unanimity rule, i.e. $q = n = 3$. Suppose that a symmetric stationary equilibrium with aggressive commitments exists, and denote the common expected equilibrium payoff by v^*. Due to stationarity, each player's continuation value is δv^*. The most aggressive conceivable commitment is targeted to allow for agreement only in the event that exactly one player is committed. The largest commitment compatible with agreement in that event is given by

$$x^* = 1 - 2\delta v^*. \tag{1}$$

This commitment leaves the uncommitted players indifferent between a deal and continuation: they each receive the continuation value δv^*, and the player who is the only one to succeed with her commitment receives the residual.

If all players commit to x^*, three things can happen with positive probability in a given round. If exactly one player is committed, an agreement will be reached in which the committed player receives x^* and the others receive δv^*. This outcome is unaffected by the identity of the proposer. If no player is committed, a deal is reached in which both responders receive δv^* and the proposer receives $1 - 2\delta v^* = x^*$. Note that, in the latter case, the proposer's payoff can be written as $\delta v^* + (x^* - \delta v^*)$. This reflects the fact that, in the event of an "extra" loophole (beyond the minimum number required for agreement), the proposer secures an additional "chunk" of size $(x^* - \delta v^*)$. We will encounter this type of logic again in the more general analysis. Finally, if two or more players are committed, no agreement will be reached, and the game moves to round $t + 1$.

Thus, the equilibrium involves a positive probability of inefficient delay. Concretely, the probability of agreement in a given round is $(1 - \rho)^3 + 3\rho(1 - \rho)^2 = (1 - \rho)^2(1 + 2\rho) \equiv \xi$. Conditional on agreement, the sum of payments is one. Therefore, the expected discounted sum of utilities is $\sum_i v^* = \sum_{t=1}^{\infty} \xi(1 - \xi)^{t-1}\delta^{t-1} = \frac{\xi}{1-\delta(1-\xi)} < 1$, and so the expected equilibrium payoff for each player is given by

$$v^* = \frac{1}{3} \frac{\xi}{1 - \delta(1 - \xi)}$$

Note that v^* is strictly less than 1/3, reflecting the inefficiency. It is straightforward to verify that equilibrium payoffs increase, and commitments become less aggressive, if players are more patient (δ increases) or if they are less likely to succeed with their commitments (ρ decreases).

The following proposition establishes that this equilibrium always exists in the three-player game under unanimity rule.

Proposition 2. *In the three-player game under unanimity rule, there always exists a symmetric SSPE with aggressive commitments and delay.*

To prove the proposition, let us verify that the commitment profile (1) constitutes part of an equilibrium. By construction, if all players attempt commitment $x^* = 1 - 2\delta v^*$, there can be no beneficial deviation at the bargaining stage. Consider then deviations at the commitment stage.

Note that such deviations affect the deviator's payoff only in case he is committed in the ensuing bargaining stage. Thus we must determine its payoff consequences in that event. An upward deviation to $y > x^*$ cannot pay off because no agreement can occur when such a commitment sticks, resulting in a payoff of δv^* instead of a positive probability of receiving x^* (and a guaranteed payoff of δv^* otherwise). A downward deviation can be beneficial only if it increases the chance of a deal. Thus, it would have to allow for a deal in the event that another player's commitment sticks: $y \leq 1 - x^* - \delta v^* = \delta v^*$. But this deviation would guarantee a payoff of

δv^* instead of a positive probability to earn x^*. Thus, no deviation (up or down) can be beneficial. It follows that under unanimity rule, the most aggressive commitment attempt always constitutes an equlibrium.

Notice that the final outcome in the aggressive equilibrium may be highly asymmetric: All players except for the one whose commitment sticks (or the proposer) receive their continuation value. When δ tends to zero, a single player secures the entire pie - as under majority rule. In addition, the inefficiency of the equilibrium implies that continuation values may be close to zero even if δ is large, especially if the probability of a loophole is small. Thus, in addition to the process being inefficent, the final outcome can be more unequal than under majority rule.

2.3 Efficient equilibrium under unanimity rule

Suppose that an efficient symmetric equilibrium exists. As above, denote the (common) expected equilibrium payoff by v^*. (Clearly, $v^* = 1/3$.) Then it can be argued that $x^* = 1/3$, i.e. the commitment is targeted to permit a deal including 3 committed players. Given such commitments, agreement is immediate and so the equilibrium is efficient. The following proposition establishes that this equilibrium exists if and only if ρ is large enough.

Proposition 3. *In the three-player game under unanimity rule, an efficient symmetric equilibrium exists if and only if $\rho \geq \sqrt{\frac{2}{5}}$. In this equilibrium, all players commit to $x^* = 1/3$ and achieve an expected payoff $v^* = 1/3$.*

To understand why the efficient symmetric equilibrium may fail to exist, note that whenever a player has a loophole, the total price of the cheapest coalition is reduced by $(x^* - \delta v^*)$. When k players have a loophole, the proposer can therefore allocate to herself a share that exceeds her own price by $k(x^* - \delta v^*)$. This implies a potential incentive to deviate at the commitment stage. Specifically, a player may want to increase his commitment in an attempt to appropriate, in the event that he is responder and another player (or two) has a loophole, the extra (one or two) "chunks" of size $(x^* - \delta v^*)$ that otherwise accrue to the proposer.

Consider then an upward deviation tailored such that (at least) $k \in \{1, 2\}$ loopholes are needed for a deal to be made. Such a deviation can affect the deviator's payoff only when his own commitment sticks. We therefore analyze its payoff consequences in that event. The best deviation aiming at a single loophole ($k = 1$) is $y = x^* + (x^* - \delta v^*)$, seeking to capture the single "chunk" that becomes available when one other player has a loophole. Then, agreement will no longer occur if both others' commitments stick. Thus, with probability ρ^2 the committed deviator loses $(x^* - \delta v^*)$. The only other condition under which the deviator's payoff changes is when he is responder and at least one other player (proposer or responder) has a loophole. This occurs with probability $\frac{2}{3}(1 - \rho^2)$. In these cases, he gains one "chunk" $(x - \delta v^*)$ which otherwise would have gone to the proposer. Summing up, conditional on his commitment sticking, the change in payoff induced by the "one-chunk" deviation is given by $\frac{2}{3}(1 - \rho^2)(x^* - \delta v^*) - \rho^2(x^* - \delta v^*)$. This is positive iff $\rho < \sqrt{\frac{2}{5}}$. It can be shown that a "two-chunk" deviation aiming at $k = 2$ loopholes can be profitable only if the "one-chunk" deviation is profitable.[13] This will also apply in our more general analysis below.

In the efficient symmetric equilibrium, the pie is shared equally whenever both responder commitments succeed. Each responder loophole results in the reallocation of one "chunk" worth $(x^* - \delta v^*) = (1 - \delta)/3$ to the proposer. For δ close to 1, these chunks are close to zero and the final allocation is equal irrespective of realized commitment statuses. When δ tends to zero, each chunk equals one third of the pie, and so the possible outcomes range from perfect equality to the proposer grabbing the entire pie, as he does under majority rule. Overall, the final outcome is more equal than in the unique efficient equilibrium under majority rule. Therefore, when $\rho \geq \sqrt{\frac{2}{5}}$ both efficiency and equality favor unanimity rule in the case of three players, provided that coordination on the efficient equilibrium can be guaranteed. As we will see, the relevant threshold value for ρ increases rapidly once we consider larger n.

3 General analysis

We now turn to a more general analysis of the n-player game. Subsection 3.1 shows that for any $q-$majority rule with $q < n$, the (essentially) unique equilibrium involves no commitments above the common continuation value and immediate agreement (Theorem 1). Thus, any non-unanimous decision rule is fully efficient.

We then turn to unanimity rule. In Subsection 3.2, we begin by constructing symmetric commitment profiles, each of which is tailored to achieve agreement only when a certain number of commitment attempts fail. In

[13] The best deviation aiming at two loopholes is $y = x^* + 2(x^* - \delta v^*)$. Then, agreement will occur only when both other players have a loophole. Thus, with probability $1 - (1 - \rho)^2$ the committed deviator loses $(x^* - \delta v^*)$. The only other condition under which the deviation affects his payoff is when he is responder and both others have loopholes. This occurs with probability $\frac{2}{3}(1 - \rho)^2$. In these cases, he gains $2(x - \delta v^*)$. Summing up, conditional on his commitment sticking, the change in the deviator's expected payoff is given by $\frac{2}{3}(1 - \rho)^2 2(x^* - \delta v^*) - \left(1 - (1 - \rho)^2\right)(x^* - \delta v^*)$. This is positive iff $\rho < 1 - \sqrt{\frac{3}{7}} < \sqrt{\frac{2}{5}}$.

subsection 3.3, we show that the most aggressive symmetric profile, requiring all but one of the commitments to fail, always constitutes part of an SSPE (Theorem 2).

In subsection 3.4, we show that the efficient symmetric profile, which permits agreement even when all commitments succeed, constitutes an equilibrium if and only if ρ is sufficiently large (Lemma 2). Building on this, we prove the more general result, that all SSPE (symmetric or not) are inefficient if and only if ρ is sufficiently small (Theorem 3).

Given that these results are developed in a discrete time framework, it is of interest to ask whether the results would extend to a continuous time framework with frequent negotiations. In subsection 3.5, we analyze such a version of our model and show that both an efficient equilibrium and inefficient equilibria exist when the length of the time period separating consecutive negotiation rounds tends to zero.

3.1 Effiency of q-majority rules

When majority rule is used, players who commit aggressively may be left out of winning coalitions when successful. It is therefore intuitive to expect Bertrand-like competition in commitments, possibly leading to full efficiency. Although the formal details turn out to be more complicated (see Appendix A.2 for a complete proof), this intuition is helpful to understand the following result.

Theorem 1. *Under any q-majority rule with $q < n$, there exists an (essentially) unique SSPE in which no player attempts a commitment exceeding his continuation value and agreement is immediate. Expected payoffs are $v_i^* = \frac{1}{n}$ for all i, all players are included with the same probability as responders, and each included responder is paid $\frac{\delta}{n}$.*

The equilibrium is unique in the sense that in any equilibrium, $x_i^* \leq \delta v_i^*$ for each i and all players are included with the same probability as responders. Multiplicity remains only in the sense that each player can choose any commitment attempt satisfying $x_i^* \leq \delta v_i^*$, and players could randomize differently over coalition partners when proposing - so long as all players are included with the same probability from an ex ante perspective.

The intuition for this result is as follows. Those who attempt to commit too aggressively will be left out of any winning coalition when successful. This implies that their continuation payoff must be lower than that of players whose commitment attempts permit inclusion to some winning coalition with positive probability. Therefore, players with aggressive commitments have room to deviate down to a less aggressive commitment, such that in fact everyone must be included with positive probability. If all responders commit in a way such that they are included with positive probability, at least one will have an incentive to "undercut" another's commitment in order to be included more often. This triggers a Bertrand-like competition in commitments. In equilibrium, no player's commitment can exceed their continuation value. Thus, no aggressive commitments are made. The outcome is efficient and coincides with that of the Baron and Ferejohn (1989) model.

The analysis starting from the next subsection shows that, contrary to supermajority decision making, commitment strategies are used in any equilibrium, and inefficiency due to delay is often unavoidable under unanimity.

3.2 Symmetric commitment profiles under unanimity rule

We begin our analysis of unanimity rule by considering symmetric commitment profiles as equilibrium candidates. As can be anticipated from the three player example in Section 2, any symmetric SSPE will have the property that there is some number $\ell \in \{0, ..., n-1\}$ such that at least ℓ players must have a loophole for an agreement to be reached. We therefore denote the common commitment attempt x_ℓ^S. It must be such that, in the event that exactly ℓ loopholes arise, the ℓ uncommitted players are exactly indifferent between agreement and continuation when the other $(n-\ell)$ players are paid x_ℓ^S. (Otherwise, any player can benefit by slightly increasing his commitment attempt without affecting the probability of agreement.) That is, $(n-\ell)x_\ell^S + \ell\delta v_\ell^S = 1$. Furthermore, the expected payoff in a symmetric equilibrium, v_ℓ^S, is fully determined by assuming that all players attempt committing to x_ℓ^S in every period.

These observations together constitute a set of necessary conditions for a symmetric commitment profile to be part of an SSPE. We therefore begin by deriving, for any given $\ell \in \{0, ..., n-1\}$, candidate commitments x_ℓ^S and associated expected payoffs v_ℓ^S that satisfy these conditions. Rearranging the condition for indifference of uncommitted players, we obtain the candidate

$$x_\ell^S = \frac{1 - \ell\delta v_\ell^S}{n - \ell}, \tag{2}$$

If all players commit to x_ℓ^S, agreement will be reached whenever at least ℓ players have a loophole. It will be useful to establish the following definition.

Definition 1. *The probability that at least k out of m players have a loophole is denoted*

$$\eta(k,m) = \sum_{l=k}^{m} f(l,m),$$

where $f(l,m) = \begin{pmatrix} m \\ l \end{pmatrix} (1-\rho)^l \rho^{m-l}$ *is the pdf of a binomial probability distribution with loophole probability* $(1-\rho)$.

Assuming that all players consistently commit to x_ℓ^S in every period, the probability that agreement is reached at period t is $\eta(\ell,n)(1-\eta(\ell,n))^{(t-1)}$. Thus, the expected total payoff would be $\eta(\ell,n)\sum_{t=1}^{\infty}[(1-\eta(\ell,n))\delta]^{(t-1)}$, and so the expected payoff associated with this candidate profile is given by

$$v_\ell^S = \frac{1}{n}\frac{\eta(\ell,n)}{1-\delta(1-\eta(\ell,n))}. \tag{3}$$

Note that v_ℓ^S is decreasing in ℓ, with $v_0^S = \frac{1}{n}$ and $v_\ell^S < 1/n$ for all $\ell > 0$. Thus, if x_ℓ^S with $\ell > 0$ constitutes a symmetric SSPE commitment profile, the corresponding equilibrium is inefficient, and the inefficiency is larger the more loopholes are required for a deal to be made. At the same time, x_ℓ^S increases with ℓ and thus, conditional on succeeding with one's own commitment, the earned share when the deal arises is larger the higher is ℓ. Thus, commitment profiles with higher ℓ generate longer conflict duration, greater inefficiency and greater asymmetries in the shares that the parties receive conditional on reaching an agreement. The symmetric commitment profile candidates can therefore be naturally ordered from the least aggressive and efficient ($\ell = 0$) to the most aggressive and inefficient ($\ell = n-1$). Our subsequent analysis will focus on these polar cases.[14]

As stated at the beginning of this subsection, the candidate profiles were constructed to satisfy the necessary condition that the uncommitted players be made indifferent in the event that exactly ℓ players have a loophole. This implies that no player wishes to deviate at the commitment stage in a way that does not affect the probability of agreement. To verify that a given candidate profile indeed constitutes an SSPE, we must additionally verify that no player wishes to engage in larger deviations which would affect the probability of agreement. In the following subsections, we investigate the incentives for such deviations from the most aggresive ($\ell = n-1$) and the least aggressive and efficient ($\ell = 0$) candidate profile, respectively. We will conclude that deviations from the most aggressive profile are never profitable, and that deviations from the efficient profile will be profitable if and only if ρ is small enough. Finally, we will show that the latter condition is necessary and sufficient for the existence of any efficient equilibrium, both symmetric and asymmetric.

3.3 Existence of inefficient equilibrium under unanimity rule

Consider the most aggressive symmetric commitment profile, in which all players attempt commitment to

$$x_{n-1}^S = 1-(n-1)\delta v_{n-1}^S.$$

This profile is such that each player aims to extract the entire surplus from agreement (beyond the sum of continuation values) in the case where all $\ell = n-1$ other players have a loophole. If this profile is part of a symmetric SSPE, the associated expected payoff equals

$$v_{n-1}^S = \frac{1}{n}\frac{\eta(n-1,n)}{1-\delta(1-\eta(n-1,n))},$$

where the probability that an agreement arises in each period, $\eta(n-1,n) = n\rho(1-\rho)^{n-1} + (1-\rho)^n$ is small if n and ρ are large. In that case, the expected delay at this profile is long, thereby severely undermining efficiency.

Agreement occurs only in two cases: (i) only one of the commitment attempts succeeds (probability $n\rho(1-\rho)^{n-1}$) or (ii) none of the commitment attempts succeed (probability $(1-\rho)^n$). In both cases, $n-1$ players will get exactly the continuation value, and the residual, $1-(n-1)\delta v_{n-1}^S$, is secured either by the only committed player or (in case ii) by the proposer.

In order to check whether this is an equilibrium, we must verify that no player wishes to deviate at the commitment stage. To understand the effects of deviations, it will be useful to note that deviating to any $y \neq x_{n-1}^S$ affects the deviator's payoff only if her commitment sticks. Therefore, the analysis that follows focuses exclusively on the payoffs achieved in that event.

Following any upward deviation to $y > x_{n-1}^S$, no deal is made even in the most favorable instance where all players except the deviator have a loophole such that their price equals the continuation value. So a more aggressive commitment cannot be profitable. Consider then a downward deviation to $y < x_{n-1}^S$. Since the

[14] In a working paper version, we also discuss "intermediate" equilibria (Miettinen and Vanberg, 2023).

payoff achieved conditional on commitment success and agreement would then be lower than if the player did not deviate, such a deviation can only be beneficial if the probability of an agreement is increased. Therefore, a profitable deviation must have the property that a deal will be reached in cases where the deviator's own commitment attempt as well as $k \geq 1$ others succeed. The largest commitment that will be met when (at most) $k = 1$ additional player's commitment sticks is such that $(n-2)$ uncommitted players are left with the common continuation value δv_{n-1}^S after giving x_{n-1}^S to one succesful player and y to the deviator, i.e.

$$(n-2)\delta v_{n-1}^S + x_{n-1}^S + y = 1$$

But substituting $x_{n-1}^S = 1 - (n-1)\delta v_{n-1}^S$ we see that this boils down to

$$y = \delta v_{n-1}^S.$$

To understand this, note that in the symmetric aggressive equilibrium, a deal requires that each committed player be paid one "chunk" $(x_{n-1}^S - \delta v_{n-1}^S)$ more than if they had a loophole. A deviation which increases the probability of a deal is one which increases the number of committed players who can be paid. To increase this number by one, the deviation must concede exactly one chunk to an additional committed player. Conceding one chunk amounts to reducing one's commitment from x_{n-1}^S to δv_{n-1}^S.[15]

Note that a deviation to this y will not affect the deviator's payoff in case he is drawn to propose: As committed proposer, he would now form an agreement in case one responder is committed, where before he would have made a proposal that fails. Either way, his payoff equals the continuation value. But in all instances where he is committed and a responder, his payoff drops to the continuation value δv_{n-1}^S, whereas if he stays with the equilibrium commitment, he will get x_{n-1}^S in case all of the other players have a loophole. It follows that this deviation reduces his expected payoff.

It's clear that the argument can be extended to say that for all $k > 1$, there is also no profitable commitment that would be met if the deviating player's and k additional commitments stick, i.e. deviations of the type

$$y = 1 - kx_{n-1}^S - (n-1-k)\delta v_{n-1}^S,$$

since a fortiori these commitments would be strictly smaller than the continuation value. We can conclude that there always exists an SSPE in which all players attempt the most aggressive symmetric commitment x_{n-1}^S. This is enough to establish the following theorem.

Theorem 2. *Under unanimity rule, an inefficient symmetric SSPE always exists.*

Before moving on, we comment briefly on the comparative statics of this most aggressive equilibrium. The comparative statics with respect to ρ is obvious: delay increases in ρ. The number of loopholes at a given round follows the binomial distribution.[16] The number of required loopholes, $n-1$, increases with the number of players n. It is thus intuitive that the duration of the conflict increases with the number of players. As a consequence, the equilibrium payoff, v_{n-1}^S, and the continuation value, δv_{n-1}^S, which is allocated to each flexible responder when the deal is done, decreases with n. Therefore, the fraction of the pie that the deal allocates to the unique successful player, x_{n-1}^S, as well as the difference of the final shares, $x_{n-1}^S - \delta v_{n-1}^S$, increases as the number of players increases.[17]

3.4 Existence of efficient equilibria under unanimity rule

Now consider the opposite extreme, a symmetric commitment profile requiring no loopholes, i.e. $\ell = 0$. If all players commit to this profile, agreement is immediate and thus $v_0^S = 1/n$ and the optimal commitment characterized in equation (2) yields

$$x_0^S = \frac{1}{n}. \tag{4}$$

Clearly, a deviation to a less aggressive commitment $y < x_0^S$ cannot be beneficial because it would not increase the probability of a deal. Thus, consider a deviation to a more aggressive commitment $y > x_0^S$. Any

[15]The same logic applies to "intermediate" inefficient equilibria, which require fewer than $n-1$ loopholes for agreement to be reached. As a consequence, all inefficient equilibria have the property that a deviation which increases the probability of agreement would require stepping down to the continuation value. For an explicit discussion of "intermediate" equilibria, see our working paper (Miettinen and Vanberg, 2023).

[16]This approaches the normal distribution with mean $n(1-\rho)$ and standard deviation $\sqrt{n(1-\rho)\rho}$ as n tends to infinity.

[17]The probability of agreement in a given round is $(1-\rho)^n + n\rho(1-\rho)^{n-1} = (1-\rho)^{n-1}(1+(n-1)\rho) \equiv \xi(n)$. Conditional on agreement, the sum of payments is one. Therefore, the expected discounted sum of utilities is $\sum_i v^* = \sum_{t=1}^{\infty} \xi(n)(1-\xi(n))^{t-1}\delta^{t-1} = \frac{\xi(n)}{1-\delta(1-\xi(n))} < 1$, and so the expected equilibrium payoff for each player is given by$v^* = \frac{1}{n}\frac{\xi(n)}{1-\delta(1-\xi(n))}$.Then
$x_{n-1}^S - \delta v_{n-1}^S = 1 - n\delta v_{n-1}^S = 1 - \delta\frac{\xi(n)}{1-\delta(1-\xi(n))}$where the factor $\frac{\xi(n)}{1-\delta(1-\xi(n))}$ is decreasing in n. Thus, the chunk size is increasing in n and approaches 1 as n tends to infinity.

such deviation will have the property that it permits agreement when the deviator is committed and at least $k \geq 1$ of the other players have a loophole. The most aggressive commitment that permits agreement given exactly k loopholes is

$$y_k = x_0^S + k\left(x_0^S - \delta v_0^S\right). \tag{5}$$

Intuitively, a player deviating to y_k is increasing his demand by k "chunks" of size $\left(x_0^S - \delta v_0^S\right)$, which he is hoping to capture from the proposer in all events where he is a committed responder and at least k other players (proposer or responder) have a loophole.

Let us consider the payoff consequences of such a deviation conditional on the deviator's commitment succeeding. In all cases where fewer than k of the other $n-1$ players have a loophole, agreement will fail and the deviating player will lose $\left(x_0^S - \delta v_0^S\right)$. This is true irrespective of who is chosen to propose, and occurs with probability $1 - \eta(k, n-1)$. In all cases where at least k of the other $n-1$ players have a loophole, the deviation does not affect the deviator's payoff if he proposes, as he would secure the extra "chunks" that become available anyway. If, on the other hand, he is a responder in such events, he gains $k\left(x_0^S - \delta v_0^S\right)$. This occurs with probability $\eta(k, n-1)\frac{n-1}{n}$. Therefore the deviation does not strictly increase his payoff if and only if

$$\eta(k, n-1)\frac{n-1}{n}k\left(x_0^S - \delta v_0^S\right) \leq \left(1 - \eta(k, n-1)\right)\left(x_0^S - \delta v_0^S\right),$$

which boils down to

$$\left(1 + k\frac{n-1}{n}\right)\eta(k, n-1) \leq 1. \tag{6}$$

It follows that an efficient symmetric equilibrium exists if and only if this condition is satisfied for every conceivable deviation, $k \in \{1, ..., n-1\}$. Lemma 4 in the Appendix establishes that if (6) is true for $k = 1$, then it is true for all k. Thus, the efficient symmetric equilibrium exists if and only if $\eta(1, n-1) \leq \frac{n}{2n-1}$, or equivalently $\rho \geq \left(\frac{n-1}{2n-1}\right)^{\frac{1}{n-1}}$. We have thus established the following result regarding the existence of efficient symmetric equilibria.[18]

Lemma 2. *An efficient symmetric SSPE requiring no loopholes for agreement to be reached exists iff* $\rho \geq \left(\frac{n-1}{2n-1}\right)^{\frac{1}{n-1}}$.

Note that Lemma 2 refers only to symmetric SSPE. A natural question to ask is whether efficient asymmetric equilibria can exist even when the condition of the Lemma is violated. The answer is no, as can be established by extending part of the logic underlying the Lemma as follows. (For details, see the proof of Theorem 3 in Appendix A.4). Suppose that an efficient equilibrium exists in which players are not identically committed. Denote by $x^* = (x_1^*, .., x_n^*)$ and $v^* = (v_1^*, .., v_n^*)$ the (asymmetric) equilibrium commitment and payoff profile, respectively. Order players according to the size of the "chunk" $(x_i^* - \delta v_i^*)$ that becomes available when their own commitment fails. Then consider a deviation by player 1, with the smallest such "chunk", in which he increases his commitment attempt by exactly $(x_1^* - \delta v_1^*)$. As in the case of a symmetric equilibrium, player 1 will then gain this extra "chunk" whenever he is a committed responder and at least one other player has a loophole. Conversely, he will lose that same "chunk" in all other events. Thus, the condition under which a "one chunk" deviation does not pay off for player 1 is the same as condition 6 derived above. Therefore, that condition constitutes a necessary condition for the existence of an efficient equilibrium (symmetric or not). And since Lemma 2 provides a sufficient condition, the following more general result follows. (See Appendix A.4 for a proof.)

Theorem 3. *Under unanimity rule, all SSPE are inefficient iff* $\rho < \left(\frac{n-1}{2n-1}\right)^{\frac{1}{n-1}}$.

As n increases, the probability that at least one loophole arises also increases, so that any efficient equilibrium will eventually be destabilized. The condition in the theorem can therefore be interpreted to implicitly define the maximal n for which an efficient equilibrium exists, i.e. $\hat{n}$ such that $\rho < \left(\frac{\hat{n}-1}{2\hat{n}-1}\right)^{\frac{1}{\hat{n}-1}}$. This is displayed in Figure 3.4. For $\rho < \frac{1}{3}$, $\hat{n} < 2$, so an efficient equilibrium does not exist for any $n > 1$.[19] However, as ρ approaches zero, commitments become less relevant. In the limit, the model reduces to the standard Baron-Ferejohn game, and all equilibria are efficient.[20] Remark also that $\hat{n}$ rapidly falls as we move down from ρ close to one. For example, $\hat{n}$ is small (less than 10) even for $\rho = 0.9$.[21]

[18] Notice that the condition for the existence of the efficient equilibrium does not depend on δ, as would be typical for non-stationary equilibria supported by trigger strategies.

[19] Although we did not explicitly include the case $n = 2$ in our analysis, it is easy to verify that an efficient equilibrium involving

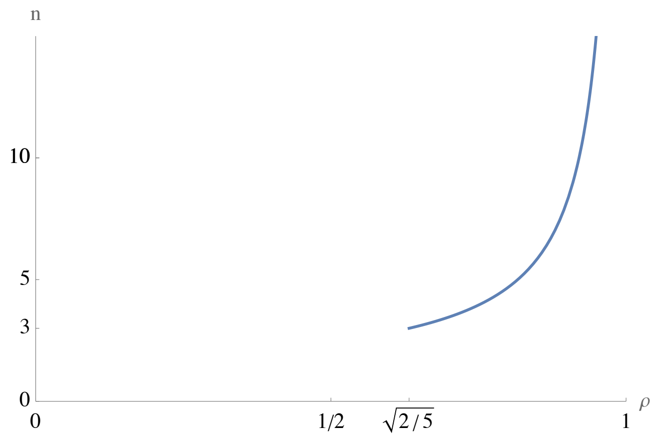

Figure 1: Maximum n such that an efficient equilibrium exists

3.5 Frequent negotiations under unanimity rule

So far we have considered the effects of three institutional features, namely the decision rule being applied, the number of negotiating parties, and the likelihood of commitment success. Let us now analyse another parameter, namely how frequently negotiation rounds take place. Unlike in many face-to-face bilateral negotiations, it is by no means obvious that in international multilateral negotiations, proposals could be made and voted on very frequently.[22]

In such institutionally rich settings, it is not entirely obvious how to think about the frequency of negotations. A commonly used approach in the bargaining literature is to consider a continuous time formulation where discounting over two consecutive negotiation rounds is parametrized by the time gap between the rounds, t, and given by the factor $\delta = exp(-rt)$. Here r is the discount rate reflecting the cost associated with the passage of a naturally given time interval such as a year, and t is the delay between negotiation rounds expressed as a fraction of the natural time interval. For example, if negotiation rounds take place once a year and the yearly interest rate equals 3%, then $\delta = exp(-0.03) \approx 0.97$; if negotiation rounds occur once in every six months $\delta = exp(-0.03/2) \approx 0.985$. A typical question addressed in this setting is what happens when all institutional frictions constraining frequency of rounds are lifted and the rounds rather follow each other in an almost continuous sequence. Formally, what happens when t approaches zero?

In order to formulate that limit, we must first take a stand on what happens to the process of commitments and the exogenous, but stochastic, arrival of loopholes. A straigthforward generalization of the model presented in the previous sections would assume that each player's individual loophole arrivals follow an i.i.d. memoryless Poisson process with an arrival rate of λ. Then within a time period of length t, the probability that no loophole arises for this player, i.e. that his commitment sticks, is given by

$$\rho(t) = exp(-\lambda t).$$

Not surprisingly, the probability of commitment success tends to one as t tends to zero.[23] Equation (3), giving the expected payoff associated with a symmetric equilibrium candidate in the discrete time formulation of the model, can be adjusted to the continuous time formulation, as a function of n, ℓ, λ, r and t, as follows

$$v_\ell^S = \frac{1}{n} \cdot \frac{\eta(\ell, n, t)}{1 - exp(-rt)(1 - \eta(\ell, n, t))},$$

where $\eta(\ell, n, t)$ denotes the probability of at least ℓ loopholes arriving in a round of length t, obtained by substituting $\rho(t)$ for ρ in Definition 1. Note that for any $\ell > 0$, $\eta(\ell, n, t)$ tends to zero as t tends to zero. That

commitments $x^* = 1/2$ exists iff $\rho \geq 1/3$.

[20]The expected payoff in any equilibrium (including the most inefficient) approaches $1/n$ as ρ approaches zero.

[21]One might ask whether the non-existence of efficient equilibria is driven by the assumption that commitment attempts are chosen simultaneously. If instead players moved sequentially in the commitment stage, would efficient equilibria always exist? The answer is no. For $n = 3$, it can be shown that an efficient equilibrium of the sequential move version exists under unanimity rule only if ρ is sufficiently large. See also Chung and Wood (2019) for a similar comparison of simultaneous and sequential commitment in the two-player case based on Ellingsen and Miettinen (2008).

[22]In the European Union, a summit where all heads of state gather together takes place once every six months (June and December), the Doha round of the WTO has had nine comprehensive meetings since the start of the round in 2001 (and are by and large inconclusive by the time of writing this manuscript). In climate change negotiations, general meetings (Conference of the Parties, COP) take place once a year (the 28th COP was organized in Dubai in December 2023 and ended without any conclusive agreement on measures or timeline on how to reach the targets set in Paris 2015).

[23]Despite its simplicity and tractability, this approach implicitly assumes, perhaps unrealistically, that as t tends to zero commitments can be reformulated at an increasing pace between bargaining rounds. In more realistic formulations, one might decouple the process of re-establishing commitments from the frequency of negotiation rounds, but we leave that for future research.

is, the probability of obtaining any positive number of loopholes in a given round tends to zero in the limit. Mirroring our prior analysis, let us now consider the efficient ($\ell = 0$) and the most aggressive ($\ell = n-1$) symmetric commitment profiles as equilibrium candidates and ask if they exist in the limit as t tends to zero and, if so, what the associated equilibrium payoffs are.

The efficient profile (requiring no loopholes for agreement) is associated with an expected payoff of $v_0^S = \frac{1}{n}$ for any t, as in the discrete time framework. With strictly positive delay between bargaining rounds, Theorem 3 says that an efficient equilibrium does not exist when $\rho(t)$ is sufficiently small. This is because a deviation to a more aggressive commitment can be profitable whenever a loophole is sufficiently likely to arrive before the ensuing bargaining stage. As the time between the negotiation rounds gets shorter, $\rho(t)$ increases, loopholes are less likely to arrive, an so an aggressive deviation is less likely to succeed. In the limit, $\rho(t)$ tends to one and the deviation cannot succeed, so that an efficient equilibrium always exists.

Next, consider the most aggressive symmetric profile, requiring $(n-1)$ loopholes for agreement. As shown in the discrete time analysis, this profile constitutes an equilibrium for all values of $\rho(t)$ and thus for all t (Theorem 2). The probability of agreement in a given round is given by $\eta(n-1, n, t) = n\rho(t)(1-\rho(t))^{n-1} + (1-\rho(t))^n$, and the expected equilibrium payoff is given by

$$v_{n-1}^S = \frac{1}{n} \cdot \frac{\eta(n-1, n, t)}{1 - exp(-rt)(1 - \eta(n-1, n, t))}.$$

Note that v_{n-1}^S tends to zero as t tends to zero, as can be shown by applying L'Hopital's rule. The reason is that the expected waiting time before a deal is reached, given the aggressive profile, tends to infinity as t tends to zero. This establishes the following Proposition.

Proposition 4. *When the length of the time period t tends to zero, both the efficient and the most aggressive equilibrium exist under unanimity rule. The expected payoffs associated with these equilibria satisfy $v_0^S = 1/n$ and $\lim_{t\to 0} v_{n-1}^S = 0$.*

Notice moreover that the commitments in the most aggressive equilibrium satisfy

$$lim_{t\to 0} x_{n-1}^S = lim_{t\to 0} \left[x_{n-1}^S - exp(-rt)v_{n-1}^S\right] = 1$$

and an agreement requires $n-1$ loopholes to arise. Yet in the limit, multiple loopholes never arrive in the same round.[24] Thus, each player's expected payoff is zero whichever commitment the player chooses. The maximally aggressive commitment, zero payoff, and each player's indifference between own stratgies in equilibrium are all reminiscient of the inefficient equilibrium of the Nash demand game. The co-existence of efficient and inefficient equilibria is another feature shared with the Nash demand game. Indeed in the limit when t tends to zero, inefficiency is the result of coordination failure in our game. This is in contrast to less frequent negotiations, in which the efficient equilibrium does not exist, and thus inefficiency is unavoidable in equilibrium, and not due to coordination failure.

Finally, we will make some remarks on the symmetric commitment profile requiring just one loophole, with associated payoff

$$v_1^S = \frac{1}{n} \cdot \frac{\eta(1, n)}{1 - exp(-rt)(1 - \eta(1, n))} = \frac{1}{n} \cdot \frac{1 - (exp(-\lambda t))^n}{1 - exp(-rt)(exp(-\lambda t))^n}.$$

This profile is of particular interest since the expected time of first arrival of a single loophole is $1/\lambda$, which is finite. This suggests that the profile leads to some loss of surplus but still strictly positive payoffs $lim_{t\to 0} v_0^S = 1/n > lim_{t\to 0} v_1^S > lim_{t\to 0} v_{\ell>1}^S = 0$. It can indeed be shown that, in the limit, the symmetric commitment profile x_1^S is part of an equilibrium and, thus, there is an equilibrium requiring one loophole for an agreement to arise.[25] Applying l'Hôspital's rule yields the limit payoff

$$lim_{t\to 0} v_1^S = \frac{1}{n} \cdot \frac{\lambda n}{r + \lambda n} \tag{7}$$

which is indeed strictly between 0 and $1/n$. Equation (7) reveals that, when offers are generated very frequently, the efficiency losses in the inefficient one-loophole equilibrium increase in the discount rate r and decrease in the loophole arrival rate λ and the number of players n.[26]

[24] This reflects the property of Poisson processes according to which the probability of multiple loopholes arriving at the same instant in continuous time is zero, so that the probability of $n-1$ loopholes arriving within any round in the future approaches zero as t tends to zero. Indeed, the same reasoning implies that all symmetric commitment profiles for $\ell > 1$ constitute equilibrium commitment profiles in the limit, and all such profiles are associated with the same expected payoff, namely zero.

[25] Intuitively, an upward deviation requiring two (or more) loopholes will never be conceded to, as two (or more) simultaneous loopholes never arrive. As before, a downward deviation would require stepping down to the continuation value. See also section 4.5. of our working paper version (Miettinen and Vanberg, 2023).

[26] Notice also that, because the arrival probability of the first loophole is independent of the length of the time period, the expected length of conflict and the equilibrium payoff v_1^S are independent of the length of the time period, too.

4 Conclusion

International and supranational organizations, including the North Atlantic Treaty Organization, the World Trade Organization, and the European Union, typically require unanimous consent to make decisions in important policy areas. This requirement has the advantage of guaranteeing that all decisions are mutually beneficial. However, many experts and practioners have criticized unanimity rule on the grounds that it often leads to excessive delay and gridlock. Many EU practitioners have proposed expanding the use of QMR to sensitive areas such as foreign policy or taxation.

In this paper, we have presented a tractable model of multilateral bargaining that predicts inefficient delay under unanimity decision making. The mechanism our model highlights is that unanimity rule creates incentives for players to commit to a tough bargaining stance prior to negotiating. Our analysis suggests that the associated inefficiencies grow more severe when the number of players is large. We also show that any less-than-unanimity rule circumvents this problem in the context of our model. Our analysis also suggests that inefficiencies might be reduced by increasing the frequency of negotiations.

Our model abstracts from potentially important details of the applied contexts mentioned. Nevertheless, the substantive predictions fit the empirical patterns in the EU and WTO, and NATO contexts. In all of these international organizations, the number of members is large and has increased prior to the observed impasses: the WTO Doha round failed after the enlargement of the organization in the late 1990's and early this Millennium, and EU decision making in sensitive areas has stalled ever since the enlargement of 2004.

Our analysis would seem to lend support to proposals to shift to the use of qualified majority rule, as such a change may lead to a sharp rise in efficiency. An important caveat to this interpretation is that our model assumes that delay or failure to agree are indeed inefficient. In reality, the expanded use of QMR could also increase the risk of decisions that impose external costs on non-consenting members, an aspect that is absent from our model. Thus, there exists a trade-off between the expected "external costs" and anticipated "decision costs," to borrow terminology introduced by Buchanan and Tullock (1965). By design, our analysis restricts attention to the latter aspect.

Perhaps due to the anticipation of such external costs, the proposal to expand the use of QMR has faced significant resistance from a number of member states. This is well documented among others by Koenig (2020), who conducts a confidential expert survey with diplomatic sources and concludes that only six member states support the expanded use of QMR (as currently defined). Given this resistance, it is perhaps interesting to emphasize that our model suggests that even a move to an all-but-one rule - merely preventing a single member from blocking agreement - may help to dramatically improve the EU's decision making capacity. It seems reasonable to assume that the "external costs" to be expected from such an all-but-one rule will be substantially lower than those to be expected under the currently established - and much less demanding - QMR.[27] It stands to reason that a proposal to introduce a novel, highly demanding, QMR may be more acceptable to many member states than the extension of the currently established procedures.[28]

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[27] There are currently two qualified majority rules in use: "when the Council votes on a proposal by the Commission or the EU's High Representative for Foreign Affairs and Security Policy, a QM is reached if [...] 55% of EU countries vote in favour - i.e. 16 out of 28 [and these countries represent] at least 65 % of the total EU population. When the Council votes on a proposal not made by the Commission or the High Representative, a decision is adopted if [it is supported by] 72 % of EU [countries representing] at least 65 % of the EU population." (https://eur-lex.europa.eu/summary/glossary/qualified_majority.html, accessed 12.6.2020)

[28] Another recommendation which warrants further investigation is to organize bargaining rounds more frequently. However, this policy conclusion hinges on the implicit assumption that commitment positions can be re-established at the beginning of the following round independently of how soon the next round arrives. This implies that the frequency reduces the short-term advantage of committed parties over the uncommitted ones, thereby undermining the incentive to deviate to a more aggressive commitment.

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A Appendix

A.1 Proof of Lemma 1

Part 1 If $d_i > \delta v_i$, this follows from the elimination of weakly dominated strategies. Given this, it is without loss of generality to assume that responders vote "yes" when $d_i = \delta v_i = \hat{x}_i(s_i)$. To see this, suppose there exists an equilibrium in which some player i votes "no" on a proposal d (made by some proposer j given some commitment status profile s) in which i is being offered exactly $\hat{x}_i(s_i) = \delta v_i$. (This proposal need not actually be made along the equilibrium path.) If i is not pivotal, there exists an outcome-equivalent equilibrium in which he votes "yes" in this event. Suppose that i is pivotal. That is, the proposal d fails in equilibrium but it would pass if i were to vote "yes". Suppose that the proposal actually made by j at profile s is $y \neq d$ and it passes. If j (the proposer) weakly prefers passing y over passing d, there exists an outcome-equivalent equilibrium in which i would vote "yes" on d (so it would pass) but j does not propose it. If j strictly prefers passing d over passing y, then she could make a proposal arbitrarily close to d (increasing d_i slightly so that i strictly prefers to vote "yes") and improve over proposing y (recall that i is pivotal), a contradiction. Finally, suppose that the proposal made in this event fails (it could be d itself). Then the cheapest available coalition of $q-1$ players other than j, given s, must cost at least $1 - \hat{x}_j(s_j)$. (Otherwise j would strictly prefer to make a proposal that passes.) If the cheapest coalition costs strictly more than $1 - \hat{x}_j(s_j)$, there is an outcome-equivalent equilibrium in which i votes "yes" on d but j does not propose it (strictly preferring delay). If the cheapest coalition costs exactly $1 - \hat{x}_j(s_j)$, either the proposer or at least one of the responders in the cheapest coalition (say k) must be committed to $x_k > \delta v_k$ (since the sum of continuation values is strictly less than one). Then if player k (it could be j, the proposer) reduces his commitment by an arbitrarily small amount, j will make a deal that includes k and k improves his payoff (relative to the proposal failing). The cost of this deviation is arbitrarily small, and so k's payoff increases, a contradiction.

Part 2 This is immediate if the price of the cheapest coalition is not equal to $1 - \delta v_i$, as in that case the proposer strictly prefers to pass a proposal if his commitment status allows. What remains to be shown is that i must make a proposal that passes in the case where the cheapest coalition costs exactly $1 - \delta v_i$ and the proposer's own "price" is $\hat{x}_i(s_i) = \delta v_i$. To see this, suppose that there exists some event (i.e. a commitment status profile s and a proposer i) such that $\hat{x}_i(s_i) = \delta v_i$ and $\sum_{j \in C_i} \hat{x}_j(s_j) = 1 - \delta v_i$ for the cheapest coalition C_i, but the proposer makes a proposal that fails. Then at least one $k \in C_i$ must be committed to $x_k > \delta v_k$ (since the sum of continuation values is strictly less than one). *Then if k reduces his commitment by an arbitrarily small amount, i will make a deal that includes k and k improves his payoff (relative to the proposal failing). The cost of this deviation is arbitrarily small, and so k's payoff increases, a contradiction.*

Equilibrium commitments The condition $x_i^* \in \arg\max \pi_i(x_i, x_{-i}^* | \psi^*)$ follows from the one-stage deviation principle, which requires that for each i there must not exist a commitment attempt $\tilde{x}_i \neq x_i^*$ with the property that player i could strictly increase his payoff by deviating to $\tilde{x}_i$ in the initial commitment stage and reverting to equilibrium play thereafter. (See Fudenberg and Tirole (1991) Theorem 4.2 and note that our game satisfies their Definition 4.1 due to discounting.)

A.2 Proof of Theorem 1

Lemma 3. *It is without loss of generality to assume the following:*

(a) $x_i \geq \delta v_i$ for all i.

(b) If player j is never included as responder when his commitment sticks, then $x_j = \delta v_j$.

Proof. Part (a) is obvious. Assume it for what follows. For part (b), suppose there exists j who is never included when his commitment sticks, but $x_j > \delta v_j$. We will argue that then there exists an equivalent equilibrium in which $x_j = \delta v_j$ and everything else is the same, in particular, player j is still never included when his commitment sticks. The intuition is that player j would deviate down to a lower commitment if this caused him to be included some of the time. So the fact that he does not do so means that his committing to δv_j makes no difference.

Suppose there is a commitment status profile s and a proposer $k \neq j$ such that (i) j is uncommitted in s, (ii) every set of the cheapest $(q-1)$ responders includes j, and (iii) the cost of these sets (denote it $X = \sum_{i \in C_k} \hat{x}_i(s)$) satisfies $X < 1 - \hat{x}_k(s)$. Then player j could deviate down to commitment attempt $\delta v_j + \epsilon$, and for ϵ sufficiently small, a deal that includes j would be made whenever k is proposer and faced with a commitment profile identical to s except that j is committed. Conditional on being responder, j thus obtains $\delta v_j + \epsilon$ in at least one event where he previously obtained at most δv_j. And since he is never included when his commitment x_j sticks, there are no events in which a downward deviation lowers his payoff as responder. As proposer, he is at least as well off with the smaller commitment, as he will only make a deal if it earns him more than his continuation payoff. A contradiction. Thus, there does not exist a commitment status profile of the type considered.

Suppose that there exists a commitment status profile s and a proposer $k \neq j$ such that (i) j is uncommitted in s, (ii) every set of the cheapest $(q - 1)$ responders includes j, and (iii) the cost of these sets satisfies $X = 1 - \hat{x}_k(s)$. (The difference to the previous case is that k would not make a deal with j if he is committed to more than δv_j.) Then it follows that either the proposer k or at least one responder $i \neq j$ in each of the cheapest coalitions C_k must be committed to $x_i > \delta v_i$. (Otherwise $X = \sum_{i \in C_k} \delta v_i < 1 - \delta v_k = 1 - \hat{x}_k(s)$, contradicting iii.) But then consider a modified event, identical to s except that one of these players has a loophole. Then still j is included in each of the cheapest set(s) of $(q - 1)$ responders, but now its cost satisfies $X < 1 - \hat{x}_k(s)$ and, hence, the argument above applies. A contradiction. Thus, again there does not exist a commitment status profile of the type considered.

Thus, there does not exist a status profile s and a proposer $k \neq j$ such that (i) j is uncommitted in s, (ii) every set of the cheapest $(q - 1)$ responders includes j, and (iii) the cost of these sets satisfies $X \leq 1 - \hat{x}_k(s)$. That is, there is no commitment status profile s such that some player k must make a deal that includes an uncommited j. It follows that j could deviate to commitment attempt δv_j (or smaller), and all proposers could continue to make the same deals (or not make deals) as they do prior to that deviation. Therefore, there exists an equivalent equilibrium strategy profile that is identical except $x_j = \delta v_j$ (or smaller) and equilibrium play is the same in all events. Q.E.D. □

Proof of the Theorem

Proof. Order players such that

$$x_1 \leq x_2 \leq .... \leq x_q \leq ... \leq x_n$$

and assume without loss of generality that $x_i \geq \delta v_i$ and that $x_j = \delta v_j$ for any player who is never included as responder when his commitment sticks. (See Lemma 3.)

Define $H = \{h : x_h > x_q\}$, and assume that H is not empty. Then any $h \in H$ is never included as responder when his commitment sticks. Then by Lemma 3 $\delta v_h = x_h > x_q$, and so in fact h is never included as responder (even with a loophole). Define $L = \{l : x_l \leq x_q\}$, and note that it is not empty. Then $v_h > v_l$ for all $h \in H$ and $l \in L$.

Let $z = \#L$. Define P_L, P_H as the average probability that a member of L and H makes a deal when proposing. It can be argued that $P_L \geq P_H$.[29]

Let $P_F = 1 - \frac{z}{n}P_L - \frac{n-z}{n}P_H$ be the probability that no deal is reached in a given round. Note that all deals include only members of L as responders. Let X_{HL} be the average total payments to responders in L when a member of H makes a deal. Then the average expected payoff among members of H is

$$\bar{v}_h = P_F \delta \bar{v}_h + \frac{1}{n} P_H \left(1 - X_{HL}\right)$$

and the average expected payoff among members of L is

$$\bar{v}_l = P_F \delta \bar{v}_l + \frac{z}{n} P_L \frac{1}{z} + \frac{n-z}{n} P_H \frac{X_{HL}}{z},$$

where the second part of the sum reflects the fact that whenever a member of L makes a deal, the entire pie is shared in some way between the z members of L. But then

$$\left(1 - \delta P_F\right)\left(\bar{v}_l - \bar{v}_h\right) = \frac{P_L - P_H}{n} + P_H \frac{X_{HL}}{z} > 0,$$

contradicting $v_h > v_l$ for all $h \in H$ and $l \in L$. It follows that H is empty, i.e. $x_j = x_q$ for all $j \geq q$.

$$x_1 \leq x_2 \leq .... \leq x_q = x_{q+1} = ... = x_n.$$

Now redefine $H = \{h : \delta v_h = x_q\}$ and $L = \{l : \delta v_l < x_q\}$. (Note that these sets are exhaustive because $\delta v_j \leq x_j \leq x_q$ for all j.) Suppose both sets are nonempty. Again define $z = \#L$, as well as the other notation introduced above. Note that $\hat{x}_h(s) = x_h = \delta v_h = x_q$ for all h and all commitment status profiles s. Also, $v_h > v_l$ for all $h \in H$ and $l \in L$, and the argument in footnote 29 still applies, thus $P_L \geq P_H$.

Suppose $z \geq q$. Then any $h \in H$ is never included as a responder. (To see this, suppose a deal does include some $h \in H$. Then h is paid x_q. Moreover, at least one $l \in L$ is excluded. It follows that l is committed to $x_q > \delta v_l$. But then l can deviate to $x_q - \epsilon > \delta v_l$ and he will be included for ϵ arbitrarily small.) Then the argument above can be repeated to show that the average payoffs satisfy $\bar{v}_l > \bar{v}_h$, a contradiction.

Thus $z \leq q - 1$. Then any deal must include all members of L either as proposer or responder. (To see this, suppose a deal is made that does not include some $l \in L$. Then it includes at most $q - 2$ members of L, and so must include at least one responder $h \in H$, who is paid x_q. It follows that l is committed to x_q. But then l can deviate to $x_q - \epsilon$ and he will be included for ϵ arbitrarily small.) In addition to the notation already introduced, let $X_{LH} = (q - z)x_q$ be the total payment made to responders in H when a member of L makes a deal. Then the average expected payoff among members of H is

$$\bar{v}_h = P_F \delta \bar{v}_h + \frac{n-z}{n} P_H \frac{1 - X_{HL}}{n-z} + \frac{z}{n} P_L \frac{X_{LH}}{n-z}$$

and the average payoff for members of L is

$$\bar{v}_l = P_F \delta \bar{v}_l + \frac{z}{n} P_L \frac{1 - X_{LH}}{z} + \frac{n-z}{n} P_H \frac{X_{HL}}{z},$$

[29]Consider any commitment status profile s such that some $h \in H$ would make a deal if chosen to propose. Let I_h be the set of responders included by h. Then $\hat{x}_h(s) \leq 1 - \sum_{j \in I_h} \hat{x}_j(s)$. Take any $l \in L$. Note that $\hat{x}_l(s) \leq x_l \leq x_q < x_h = \delta v_h = \hat{x}_h(s)$. If $l \notin I_h$, then $\hat{x}_l(s) < \hat{x}_h(s) \leq 1 - \sum_{j \in I_h} \hat{x}_j(s)$, and so l could make a deal with the same coalition I_h (or some other coalition). If $l \in I_h$, then $\hat{x}_l(s) \leq 1 - \sum_{j \in I_h \setminus \{l\}} \hat{x}_j(s) - \hat{x}_h(s)$. Therefore l can make a deal with $(I_h \setminus \{l\}) \cup \{h\}$ (or some other coalition). Thus for any commitment status profile at which a member of H makes a deal, any member of L will also make a deal.

where again each element of the sums reflects the way that the pie is shared among the members of both sets in the event of a deal. Then

$$\begin{aligned}
\bar{v}_L - \bar{v}_h &= P_F\delta\left(\bar{v}_L - \bar{v}_h\right) + \frac{P_L\left(1 - X_{LH}\right)}{n} - \frac{P_H\left(1 - X_{HL}\right)}{n} + \frac{n-z}{nz}P_H X_{HL} - \frac{z}{n\left(n-z\right)}P_L X_{LH} \\
&= P_F\delta\left(\bar{v}_L - \bar{v}_h\right) + \frac{P_L - P_H}{n} + \left(\frac{1}{n} + \frac{n-z}{nz}\right)P_H X_{HL} - \left(\frac{1}{n} + \frac{z}{n\left(n-z\right)}\right)P_L X_{LH} \\
&= P_F\delta\left(\bar{v}_L - \bar{v}_h\right) + \frac{P_L - P_H}{n} + \frac{P_H X_{HL}}{z} - \frac{P_L X_{LH}}{n-z}
\end{aligned}$$

Then using $P_F = \left(1 - \frac{z}{n}P_L - \frac{n-z}{n}P_H\right)$,

$$\bar{v}_L - \bar{v}_h = \left(1 - \frac{z}{n}P_L - \frac{n-z}{n}P_H\right)\delta\left(\bar{v}_L - \bar{v}_h\right) + \frac{P_L - P_H}{n} + \frac{P_H X_{HL}}{z} - \frac{P_L X_{LH}}{n-z},$$

and thus

$$\begin{aligned}
\left(1-\delta\right)\left(\bar{v}_L - \bar{v}_h\right) &= \frac{P_L - P_H}{n} + \frac{P_H X_{HL}}{z} - \frac{P_L X_{LH}}{n-z} - \frac{1}{n}\left(zP_L + \left(n-z\right)P_H\right)\delta\left(\bar{v}_l - \bar{v}_h\right) \\
&= \frac{P_L - P_H}{n}\left(1 - z\delta\left(\bar{v}_l - \bar{v}_h\right)\right) + \frac{P_H X_{HL}}{z} - \frac{P_L X_{LH}}{n-z} - P_H\delta\left(\bar{v}_l - \bar{v}_h\right) \\
&= \frac{P_L - P_H}{n}\left[1 - z\delta\bar{v}_l - \left(n-z\right)\delta\bar{v}_h\right] + P_H\left(\frac{X_{HL}}{z} - \delta\bar{v}_l\right) + P_L\left(\delta\bar{v}_h - \frac{X_{LH}}{n-z}\right)
\end{aligned}$$

Note that all elements of the final sum are positive: The first because the sum of continuation values is less than one, the second because $X_{HL} \geq z\delta\bar{v}_l$, given that any deal includes all members of L and each is paid at least his continuation value. Finally, the last is positive because $X_{LH} = (q-z)\delta\bar{v}_h$ since any deal made by a member of L includes $q-z$ members of H who are each paid exactly their (common) continuation value. Thus $\bar{v}_l \geq \bar{v}_h$, contradicting $v_h > v_l$ for all $h \in H$ and $l \in L$.

It follows that one of the sets, L or H, must be empty. Suppose H is empty, i.e. $\delta v_j < x_q$ for all j. Then by Lemma 3, all $i \geq q$ are included with positive probability when committed to $x_q > \delta v_i$. But any such deal would have to exclude some responder, and so that responder must also be committed to x_q. Then this responder could deviate to $x_q - \epsilon$ for ϵ arbitrarily small and be included. A contradiction.

Thus, the set L is empty and therefore $x_q = \delta v_j$ for all j. Combined with the ordering already established, it follows that

$$\delta v_i = x_i = x_j = \delta v_j$$

for all i, j. Thus, since no player is committed to more than his continuation value, the unique equilibrium is both efficient and symmetric. Q.E.D. □

A.3 Lemma 4

Lemma 4. *If* $\left(1 + k \cdot \frac{n-1}{n}\right) \cdot \eta(k, n-1) \leq 1$ *for* $k = 1$*, then the same holds for all* $k \in \{1, ..., n-1\}$*.*

Proof. Define the statement

$$C(k) \equiv \left[\left(1 + k \cdot \frac{n-1}{n}\right) \cdot \eta(k, n-1) > 1\right]$$

We will show: If there exists $\hat{k} \geq 2$ such that $C(\hat{k})$, then $C(\hat{k}-1)$. We treat separately the case that $\hat{k}-1$ is below or above the expected number of loopholes, $(1-\rho)(n-1)$.

Case 1: Suppose $\hat{k} - 1 < (1-\rho)(n-1)$ A property of the binomial distribution is that the median number of loopholes m satisfies

$$\lfloor(1-\rho)(n-1)\rfloor \leq m \leq \lceil(1-\rho)(n-1)\rceil$$

Therefore in Case 1 we have $\eta(\hat{k}-1, n-1) \geq 1/2$. (Recall that this is the probability of having at least $\hat{k}-1$ loopholes.) It follows that the left hand side (LHS) of statement $C(\hat{k}-1)$ is strictly greater than $\frac{1}{2}\left(1 + (\hat{k}-1) \cdot \frac{n-1}{n}\right)$. This quantity is strictly greater than one if $\hat{k} - 1 > \frac{n}{n-1}$. Thus, $C(\hat{k}-1)$ follows immediately for the subcase where $\frac{n}{n-1} < \hat{k} - 1 < (1-\rho)(n-1)$. Then consider the subcase $\hat{k} - 1 < \frac{n}{n-1}$. Note that $\frac{n}{n-1} \leq \frac{3}{2}$ for all $n > 2$. Thus $\hat{k} - 1 < \frac{n}{n-1}$ implies $\hat{k} \leq \frac{5}{2}$ and so the only conceivable $\hat{k}$ satisfying $\hat{k} \geq 2$ and $\hat{k} - 1 < \frac{n}{n-1}$ is $\hat{k} = 2$. So, assume $\hat{k} = 2$ and suppose $C(2)$:

$$\left(1 + 2 \cdot \frac{n-1}{n}\right) \cdot \eta(2, n-1) > 1.$$

This unpacks as follows:

$$\left(1+2\cdot\frac{n-1}{n}\right)\cdot\left[1-f(0,n-1)-f(1,n-1)\right]>1$$

where, recall, $f(\ell,n-1)$ is the probability of encountering ℓ loopholes out of $n-1$ players. Thus we have

$$\left(1+2\cdot\frac{n-1}{n}\right)\cdot\left[1-\rho^{n-1}-(n-1)\,\rho^{n-2}(1-\rho)\right]>1.$$

This can be rearranged to yield

$$\rho^{n-1}\left(1+(n-1)\,(\frac{1-\rho}{\rho})\right)<\frac{2n-2}{3n-2}$$

Given we are in Case 1, we have $1<(1-\rho)(n-1)$, i.e. $\rho<\frac{n-2}{n-1}$, implying $\frac{1-\rho}{\rho}>\frac{1}{n-2}$, and so we have $\rho^{n-1}\left(1+\left(\frac{n-1}{n-2}\right)\right)<\frac{2n-2}{3n-2}$, equivilantly $\rho^{n-1}<\frac{2n-2}{3n-2}\frac{n-2}{2n-3}$. It can be shown that for $n\geq 2,\frac{2n-2}{3n-2}\frac{n-2}{2n-3}<\frac{n-1}{2n-1}$.[30] Thus we have $\rho^{n-1}<\frac{n-1}{2n-1}$ which implies $C(1)$. To see this, note that

$$\begin{aligned}
C(1) \quad &\equiv \quad \left[\left(1+1\cdot\frac{n-1}{n}\right)\cdot\eta(1,n-1)>1\right]\\
&= \quad \left[\left(1+\frac{n-1}{n}\right)\cdot\left(1-\rho^{n-1}\right)>1\right]\\
&= \quad \left[\left(\frac{n-1}{2n-1}\right)>\rho^{n-1}\right]
\end{aligned}$$

Case 2: Suppose $\hat{k}-1\geq(1-\rho)(n-1)$ Again, suppose $C(\hat{k})$. Since the binomial distribution is discrete log concave, it has the property that $\frac{f(k,m)}{\eta(k,m)}$ is non-decreasing in k (see An (1997) Proposition 10), which implies $\eta\left(k-1,m\right)\geq\frac{f(k-1,m)}{f(k,m)}\eta\left(k,m\right)$. Further, it can be shown that $\frac{f(k-1,m)}{f(k,m)}=\frac{k}{m+1-k}\frac{\rho}{1-\rho}$. Therefore $\eta\left(k-1,m\right)\geq\frac{k}{m+1-k}\frac{\rho}{1-\rho}\eta\left(k,m\right)$. Substituting $m=n-1$ gives us that

$$\eta\left(k-1,n-1\right)\geq\frac{k}{n-k}\frac{\rho}{1-\rho}\eta\left(k,n-1\right)$$

From the definition of Case 2 we have $\frac{\rho}{1-\rho}\geq\frac{n-\hat{k}}{\hat{k}-1}$ and thus

$$\eta\left(\hat{k}-1,n-1\right) \quad \geq \quad \frac{\hat{k}}{\hat{k}-1}\eta\left(k,n-1\right)$$

This give us

$$\begin{aligned}
\left(1+(\hat{k}-1)\cdot\frac{n-1}{n}\right)\cdot\eta(\hat{k}-1,n-1) \quad &\geq \quad \left(1+(\hat{k}-1)\cdot\frac{n-1}{n}\right)\cdot\frac{\hat{k}}{\hat{k}-1}\eta\left(\hat{k},n-1\right)\\
\text{(which by } C(\hat{k})) \quad &> \quad \frac{1+(\hat{k}-1)\cdot\frac{n-1}{n}}{1+\hat{k}\cdot\frac{n-1}{n}}\cdot\frac{\hat{k}}{\hat{k}-1}
\end{aligned}$$

To see that the RHS of this inequality is greater than one, note that

$$\begin{aligned}
&\frac{n-1}{n}\hat{k}<1+\hat{k}\cdot\frac{n-1}{n}\\
\Leftrightarrow\ &-\frac{n-1}{n}\hat{k}>-\left(1+\hat{k}\cdot\frac{n-1}{n}\right)\\
\Leftrightarrow\ &\left(1+\hat{k}\frac{n-1}{n}\right)\hat{k}-\frac{n-1}{n}\hat{k}>\left(1+\hat{k}\cdot\frac{n-1}{n}\right)\hat{k}-\left(1+\hat{k}\cdot\frac{n-1}{n}\right)\\
\Leftrightarrow\ &\left(1+(\hat{k}-1)\cdot\frac{n-1}{n}\right)\hat{k}>\left(1+\hat{k}\cdot\frac{n-1}{n}\right)\left(\hat{k}-1\right)
\end{aligned}$$

It follows that

$$\left(1+(\hat{k}-1)\cdot\frac{n-1}{n}\right)\cdot\eta(\hat{k}-1,n-1)>1,$$

i.e. $C(\hat{k}-1)$.

[30] To see this, note that the difference between the LHS and the RHS is

$$\frac{2n-2}{3n-2}\frac{n-2}{2n-3}-\frac{n-1}{2n-1}=-\frac{(n-1)(2n^2-3n+2)}{(2n-3)(2n-1)(3n-2)}$$

which is negative for $n\geq 3/2$.

Summary We have shown that if there exists $\hat{k} \geq 2$ such that $C(\hat{k})$, then $C(\hat{k}-1)$. It follows that $\sim C(1) \Rightarrow \sim C(k)$ for all $k \in \{2, ..., n-1\}$. Q.E.D.

$\square$

A.4 Proof of Theorem 3

Proof. Suppose there exists an efficient equilibrium under unanimity rule, and let the associated commitment profile be $\{x_i\}_{i=1}^n$. Without loss of generality, assume that $x_i \geq \delta v_i$ for all i. Note that $\sum_{i=1}^n v_i = 1$ and $\sum_{i=1}^n x_i = 1$. (If $\sum_{i=1}^n x_i > 1$, no agreement would be reached when all commitments stick. If $\sum_{i=1}^n x_i < 1$, any player could increase his commitment to $y = 1 - \sum_{j \neq i} x_j$ without affecting the prospects for agreement but improving his payoff when he is a committed responder.) . Also, note that $x_i > \delta v_i$ for all i. To see this, suppose there is i with $x_i = \delta v_i$. Conditional on being responder, player i's payoff is δv_i. (Agreement is certain, he is always included, and his "price" is δv_i irrespective of his commitment status.) As proposer, his payoff is δv_i plus $(x_j - \delta v_j)$ for every j that has a loophole. Let j be the player (other than i) for whom that "chunk" is smallest. Suppose i deviates to $\delta v_i + (x_j - \delta v_j)$. Conditional on the commitment sticking, this deviation prevents agreement only in cases where no other player has a loophole, in which case i's payoff is unchanged at δv_i (irrespective of his proposer status). If at least one other player has a loophole, i will gain the extra chunk if he is responder and still make the same deals as proposer. Thus, the deviation never yields a lower payoff and sometimes yields a greater payoff, so it pays off.

Without loss of generality, order players such that $0 < x_1 - \delta v_1 \leq ... \leq x_n - \delta v_n$. Suppose player 1 deviates to $y_1 = x_1 + (x_1 - \delta v_1)$. Then, conditional on Player 1 being committed (either as proposer or as responder), agreement will occur if and only if at least one of the $n-1$ other players has a loophole. (To see this, suppose any player $k > 1$ has a loophole. Then

$$\begin{aligned}\sum_{j=1}^{n} \hat{x}_j(s) = \sum_{j \neq 1} \hat{x}_j(s) + y_1 &= \sum_{j \neq 1}^{n} \hat{x}_j(s) + x_1 + (x_1 - \delta v_1) \\ &\leq \sum_{j=1}^{n} x_j + (x_1 - \delta v_1) - (x_k - \delta v_k) \leq \sum_{j=1}^{n} x_j = 1,\end{aligned}$$

and so a deal will be reached irrespective of which player proposes.) In that event (i.e. whenever at least one other player has a loophole), Player 1's payoff increases by $(x_1 - \delta v_1)$ if he is responder, and does not change if he is proposer. In all other events (i.e. if no player has a loophole), the deviation will result in a deal not being reached, in which case player 1 would lose $(x_1 - \delta v_1)$ as a result of the deviation (irrespective of whether he is proposer or responder). Thus, the net benefit of the deviation equals $\eta \cdot \frac{n-1}{n}(x_1 - \delta v_1) - (1 - \eta) \cdot (x_1 - \delta v_1)$, where $\eta = \eta(1, n-1)$ is the probability of at least 1 loophole among the $n-1$ players other than 1. Therefore, the deviation pays off if and only if this is strictly positive, equivalently $\eta(1, n-1) > \frac{n}{2n-1}$, or $\rho < \left(\frac{n-1}{2n-1}\right)^{\frac{1}{n-1}}$. It follows that an efficient equilibrium (symmetric or not) does not exist if $\rho < \left(\frac{n-1}{2n-1}\right)^{\frac{1}{n-1}}$. By Lemma 2, an efficient symmetric equilibrium exists iff $\rho \geq \left(\frac{n-1}{2n-1}\right)^{\frac{1}{n-1}}$. Q.E.D. $\square$