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Philipp Saake

born in: Stolberg Rhld., Germany

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Conformal Symmetry and the Hierarchy Problem

1st Referee : Prof. Dr. Dr. h.c. Manfred Lindner
2nd Referee : Prof. Dr. Joerg Jaeckel

Zusammenfassung

In dieser Arbeit stellen wir konform symmetrische Modelle vor, um das Hierarchie und andere Probleme jenseits des Standardmodells in der Hochenergiephysik zu behandeln. Nach einer Einführung in die Konzepte der konformen und der Skaleninvarianz, erläutern wir kurz die quantenmechanische Skalenerzeugung, die konforme Anomalie und das Hierarchieproblem. Wir verwenden diese Konzepte in Form eines Modells, in dem die quantenmechanische Brechung der Skalensymmetrie den einzigen dynamischen Ursprung sowohl für die Planck-Masse als auch für die Skala der elektroschwachen Symmetriebrechung darstellt. Wir stellen fest, dass wir die vorgeschlagenen Lösungen für andere Probleme jenseits des Standardmodells, nämlich die dunkle Materie und die Masse aktiver Neutrinos, sowie die Beschreibung der kosmologischen Inflation in guter Übereinstimmung mit den derzeit stärksten experimentellen Beschränkungen mühelos einbeziehen können. Bei der weiteren Untersuchung des gravitativen Beitrags zur Symmetriebrechung im oben genannten Kontext extrahieren wir den berüchtigten Spin-2-Geist Freiheitsgrad aus dem quadratischen Beitrag des Weyl-Tensors und stellen fest, dass er einen wesentlichen Beitrag nicht nur zur Skalenerzeugung mit einem minimaleren Skalar Sektor, sondern auch zur Aufrechterhaltung der experimentell günstigen Vorhersagen leistet. Schließlich nutzen wir das Skalierungsverhalten von stark gekoppelten konform symmetrischen Theorien, um ein allgemeines Konzept für die UV-Vervollständigung von Little-Higgs-Modellen zu schaffen.

Abstract

In this thesis, we present conformally symmetric models to address the gauge hierarchy problem and other beyond Standard Model (BSM) problems in high-energy physics. After introducing the concepts of conformal, and scale symmetry, we review radiative scale generation, the trace anomaly, and the hierarchy problem. We employ these concepts in the form of a model, where the radiative breaking of scale symmetry denotes the sole dynamical origin for both the Planck mass and the scale of electroweak symmetry breaking. We find that it is natural to include proposed solutions to other problems of BSM physics, namely dark matter, and active neutrino mass, as well as describing cosmological inflation, in good agreement with the currently strongest experimental constraints. Further investigating the gravitational contribution to symmetry breaking in the aforementioned context, we extract the infamous spin-2 ghost degree of freedom from the Weyl tensor squared contribution and find it to induce an essential contribution to not only scale generation with a more minimal scalar sector but also to maintaining experimentally favored predictions. Finally, we leverage the scaling behavior of strongly coupled conformally symmetric theories to provide a general framework of UV completion for Little Higgs models.

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Chapter 1

Introduction

The Standard Model (SM) of particle physics, as well as General Relativity (GR), are both exceptionally successful in describing a wide variety of real-world phenomena utilizing symmetry principles, whether it be internal gauge symmetries or symmetries of the spacetime. Nevertheless, neither manages to provide a complete description of nature and thus apart from the apparent issue of unifying the two fundamentally different theories, both require modification or extension. Building upon the concept of symmetries as the guiding principle of theoretical physics, adding conformal symmetry, which acts directly on the fields and the spacetime, allows for reconciliation or at the least offers an interesting perspective to the (gauge) hierarchy problem and other issues of the SM, GR, and their unification.

The hierarchy problem denotes the complex puzzle concerning resolving the vast disparity between the fundamental scales of the SM, the scale of electroweak (EW) symmetry breaking v_{EW}, and gravity, the Planck scale M_{Pl}. In the absence of other fundamental scales, the SM alone is found to be perturbative and without instabilities, e.g. Landau poles, up to high energies $E \gg M_{\text{Pl}}$ [5–8]. Thus, it alone is free of the hierarchy problem. Nevertheless, the SM requires extension or the inevitable unification with quantum gravity at energies close to M_{Pl}. These beyond the standard model (BSM) physics will (often unavoidably) introduce another elementary scale $\Lambda_{\text{BSM}} \gg v_{\text{EW}}$ much larger than the electroweak scale. Therefore, the heart of the hierarchy problem is found in the Higgs boson's nature as an elementary scalar with mass $m_{\text{h}} \simeq 125\,\text{GeV}$ [9, 10] in the presence of another large fundamental scale. Quantum loop contributions induce a quadratic sensitivity to the heavy scale through radiative corrections to the Higgs mass $m_{\text{h}}^2 \propto \Lambda_{\text{BSM}}^2$, which requires a large amount of unnatural (in the sense of 't Hooft [11]) fine-tuning to reconcile with the experimentally observed Higgs mass. Oftentimes, supersymmetry (SUSY) [12] is invoked to address the hierarchy problem by introducing superpartners that ensure the cancellation of the troublesome quantum corrections. Other approaches feature (large) extra dimensions, specifically addressing the vast discrepancy between v_{EW} and M_{Pl} [13, 14], by utilizing the properties of compactification to describe the SM as an effective 4D theory from a 5D spacetime description of gravity. Others again, question the original "assumption" that the Higgs is a fundamental scalar but rather identify it as a pseudo-Nambu-Goldstone boson (pNGB) of spontaneous symmetry breaking (SSB)[15–17]. However, the hierarchy problem is not yet resolved in a fully satisfying manner, as all proposed solutions come with serious caveats, like a little hierarchy problem [18, 19] or experimental null results for predicted heavy resonances.

Thus, in this thesis, we return to fundamental symmetries as our guiding principle and suggest conformal symmetry, arguably the "maximal" non-supersymmetric group of spacetime symmetries that conserves the light cone and is compatible with the SM gauge groups, to address the hierarchy problem. Conformal symmetry requires not only the absence of dimensionful scales in the (tree-level) Lagrangian, thus demanding the theory to remain unchanged under rescaling of its length or energy scales (scale invariance), but also enforces rotational, boost, and translational invariance, as well as invariance under special conformal transformation. Crucially, for any conformal invariant theory to describe real-world physics at least its scale invariance has to be broken and therefore dimensionful scales are generated dynamically. As a consequence, scale invariance and the breaking thereof play an essential role in understanding the behavior of physical systems across different energy scales and thus the gauge hierarchy problem. Regarding the SM, the logarithmic running of the Higgs mass [20, 21] and the flatness of the Higgs potential at large energies [5, 7] can be understood as hints towards scale invariant extensions. Whereas for general relativity (GR), measurements of the cosmic microwave background (CMB) [22–24] show an approximate scale-invariant power spectrum and a nearly zero value for its tensor-to-scalar ratio, tightly related to the flatness and therefore scale-invariance of the inflationary potential.

From a theoretical point of view, the fact that both the SM and GR each only contain one fundamental scale results in appealing "minimal" conformal symmetric extension. Denoted as the conformal SM (cSM) [25] it was first considered to just omit the mass term of the Higgs potential and thus not rely on tree-level SSB as in the Higgs mechanism to generate masses for gauge bosons, but rather generate masses by radiative spontaneous symmetry breaking (RSSB), i.e. by one-loop quantum corrections spontaneously breaking the symmetry [26]. However, the large mass of the top-quark destabilizes the effective Higgs potential, thus necessitating the inclusion of additional scalars [27] to successfully realize RSSB-generated Higgs mass and potential and address the hierarchy problem [28–30]. Assuming that the conformal symmetry breaking is only anomalous, i.e. spontaneously by quantum corrections, all scales are generated dynamically in relation to the logarithmic renormalization group (RG) running of dimensionless couplings such that quadratic divergences can be removed as remnants of regularization [31–34]. In addition to addressing the hierarchy problem, conformal extensions of the SM naturally allow for the inclusion of proposed solutions to, e.g. dark matter [35, 36] or neutrino masses [37–39].

The scale-invariance of these conformal extensions and the resulting validity up to, in principle, arbitrary energy scales makes them prime candidates for the ultraviolet (UV) completion of an effective theory, consequently ensuring renormalizability, the absence of Landau poles or instabilities, and converging couplings at high energies. Especially, the so-called *Little Higgs* models [40, 41], a class of composite Higgs models, realize the Higgs as pNGB of SSB without quadratic divergences and can be UV completed by strongly coupled conformal dynamics, which we will show in Chapter 4. This opens up another avenue for addressing the gauge hierarchy problem utilizing conformal symmetry. However, when describing theories at large energies close to the Planck scale, gravitational interactions start to play a role in particle physics, such that it becomes inevitable to embed into a theory of gravity. As a consequence, it is important to differentiate between fully conformal (CG) or only scale-invariant gravity, often referred to as quadratic gravity (QG). Whereas in general quantum field theory (QFT), scale-invariance and conformal symmetry can often be used interchangeably, this does not apply anymore when gravity is involved[1].

Quadratic gravity contains all three independent quadratic contractions of the Riemann curvature tensor [42], which include the Weyl tensor squared term, whereas CG is restricted to just the latter. Including these contributions already at tree level is quite natural, as they are inevitably generated by quantum corrections when considering GR to be an effective field theory. Yet, specifically the Weyl tensor squared term induces troublesome contributions to the conformal anomaly (also known as trace anomaly) [43] and massive spin-2 ghost degrees of freedom (DOFs), which potentially threaten unitarity. At the same time, there are promising attempts to resolve the conformal anomaly [44–49], as well as to understand how to properly treat the spin-2 ghost [50–62]. Therefore, both scale-invariant [63–73], and conformal symmetric [74–86] descriptions of quantum gravity still enjoy great popularity and remain an interesting and active field of research. This is in no small part due to the fact that in contrast to the (classical) description of GR, both QG as well as CG are proven to be renormalizable [87, 88] and also due to their versatility in application. Conformal invariance and its anomalous breaking appear, for example, when describing the cosmological constant problem [89–91], Hawking radiation [92] or the information paradox of black holes [93–95], dark energy in the universe [96], and in asymptotic safety [97–101].

In this thesis, we consider scale invariant and conformal symmetric models to address the gauge hierarchy and other problems of BSM physics in the presence of gravity. Therefore, we start in Chapter 2 by closely defining the symmetry concepts and terminologies from scale-invariance to conformal symmetry to Weyl invariance.

[1] More details on this distinction and the relation to the trace anomaly are provided in Sections 2.1 and 2.2, respectively.

We further discuss the related and aforementioned issues of the trace anomaly (Section 2.2) and the gauge hierarchy problem (Section 2.4) and introduce a general description of dynamical scale generation via radiatively induced spontaneous symmetry breaking for (multiple) scalars, in Section 2.3. Subsequently, in Chapter 3 we employ the previously introduced description to dynamically generate the Planck mass, neutrino mass, and the electroweak VEV through RSSB of scale invariance in the model as introduced in [1]. In Section 3.1, we first focus on how one external scalar acquires a non-trivial VEV and thus induces an effective Einstein-Hilbert term that dynamically identifies the Planck mass. Next, in Section 3.2, we showcase how through including the scale-invariant version of the neutrino option, the scalar's VEV is related to the neutrino sector and ultimately to the electroweak VEV via technically natural Yukawa couplings. Thereafter, we introduce the concept of cosmological inflation as a solution to fundamental problems of Big Bang cosmology in Section 3.3 and utilize its constraints by observations of the CMB to match with predictions from our model. We find that indeed our model can achieve inflation in agreement with the most current bounds set by experiments and shortly touch on the possibility of including a dark matter candidate in Section 3.4. In Section 3.5, we switch focus over to investigating the gravitational contributions of the Weyl tensor squared term to RSSB and the inflaton potential, along the lines of [2]. After briefly reviewing the ghost problem, we find that successful symmetry breaking and inflation are already possible with only external scalar due to the graviton's DOFs. At last, in Chapter 4, we explore the aforementioned appealing properties of conformal symmetric dynamics as UV completion for *Little Higgs* models, as shown in [4]. We first review the basic idea of the Higgs as a composite, next we introduce the general framework for UV completing *Little Higgs* models, and show an example before finally concluding with a discussion of our results in Chapter 5.

In this thesis, if not stated explicitly otherwise, we generally employ natural units $c = \hbar = 1$, work in the metric with the signature $(+,-,-,-)$, and when mentioning the Planck mass, we refer to the reduced Planck mass, such that $M_{\mathrm{Pl}} = 1/\sqrt{8\,\pi\,G_{\mathrm{N}}} = 2.435\,32 \times 10^{18}\,\mathrm{GeV}$, with G_{N} denoting the Newtonian constant of gravitation.

Chapter 2

Conformal Symmetry and Scale Invariance

2.1 Scale Invariance, Conformal Transformations, and Weyl Symmetry

In this thesis, we will consider all sorts of scale invariant and conformally symmetric theories to address specific parts of the gauge hierarchy and other problems of beyond standard model (BSM) physics. Therefore, it is important to clarify the terminology and definitions of the symmetries which are going to be the foundation of the subsequently presented results. We start with scale symmetry and then move towards conformal and Weyl symmetry, which both include the former scale invariance, to distinguish the dependencies and directional implications correctly.

Scale invariance or scale symmetry describes the invariance under rescaling of the spacetime coordinates x^μ:

$$x^\mu \to x'^\mu = \Omega(x)\; x^\mu \;, \tag{2.1}$$

where, Ω, often called the scale factor, is a positive and dimensionless function of all x for local and a constant $\Omega(x) = \Omega$ for global transformations. Fields Φ_i transform accordingly to their scaling dimension d_i under dilatations via

$$\Phi_i \to \Phi_i'(x'^\mu) = \Omega^{-d_i}\; \Phi_i(x'^\mu) \;. \tag{2.2}$$

If the quantum field theory is scale symmetric, the scaling dimension is a fixed number $d = d_0$. However, if scale symmetry is broken, the scaling dimension receives a correction $d = d_0 + \gamma$. This correction γ is called the anomalous dimension and depends on the dimensionless couplings of the interaction terms in the Lagrangian. In a scale-invariant theory in $D = 4$ dimensional spacetime, the scaling dimension for scalar, fermion, and vector boson fields are given by $d_\phi = 1,\ d_\psi = 3/2,\ d_A = 1,$

thus they transform as

$$\phi(x^\mu) \to \Omega^{-1} \quad \phi(x'^\mu)\,, \tag{2.3}$$
$$\psi(x^\mu) \to \Omega^{-3/2}\, \psi(x'^\mu)\,, \tag{2.4}$$
$$A_\nu(x^\mu) \to \Omega^{-1} \quad A_\nu(x'^\mu)\,. \tag{2.5}$$

A classical Lagrangian that is invariant under Eqs. (2.1) and (2.3) to (2.5), then preserves the dilatation current D^μ at the classical level. Therefore, in four-dimensional Minkowski space $\mathcal{M}^4$ these can be expressed as

$$D^\mu(x) = x_\nu\, T^{\mu\nu}, \qquad \partial_\mu\, D^\mu = 0\,. \tag{2.6}$$

Now, adding invariance under inhomogeneous Lorentz transformations, also known as Poincaré transformations [102, p.26][103, p.68] and special conformal transformations (SCT) to scale symmetry, we obtain symmetry under the full conformal group.

$$\text{Poincaré}: \qquad x^\mu \to x'^\mu = \Lambda^\mu{}_\nu\, x^\nu + a^\mu\,, \tag{2.7}$$
$$\text{SCT}: \qquad x^\mu \to x'^\mu = \frac{x^\mu + b^\mu\, x^2}{1 + 2\, b \cdot x + b^2\, x^2}\,, \tag{2.8}$$

The infinitesimal form of a general conformal transformation is then given by

$$x^\mu \to x'^\mu = x^\mu + \epsilon \left(a^\mu + b^\mu{}_\nu\, x^\nu + c\, x^\mu + d_\nu \left(\eta^{\mu\nu}\, x^2 - 2\, x^\mu\, x^\nu\right)\right)\,. \tag{2.9}$$

In D spacetime dimensions, the Lie algebra of the conformal group can be expressed by $\frac{1}{2}\,(D+1)\,(D+2)$ generators (we will also refer to them as the group's DOF). In the preferred $D = 4$ dimensions this leads to a total of 15 generators, 1 that generates dilatations, 4 that generate special conformal transformations, and 10 from the Poincaré group's algebra. While the Poincaré symmetry ensures invariance under

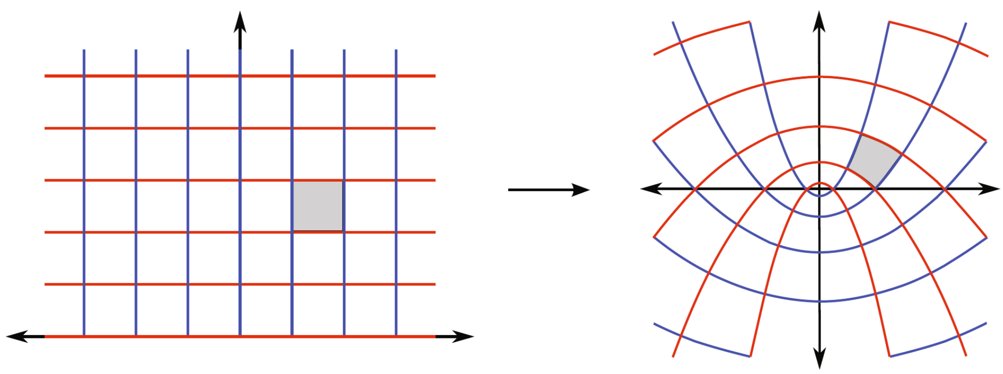


FIGURE 2.1: Conformal transformations in $D = 2$ dimensions, taken from [104].

the full symmetry of special relativity, the invariance under dilatations, i.e. rescaling, leads to conformal transformations preserving angle (or direction) but not length of a vector x^μ. This is nicely visualized in Fig. 2.1, where conformal transformations are displayed for $D = 2$ dimensions.

The conformal group is considered the "maximal" non-supersymmetric group of space-time symmetries that conserves the light-cone $\mathrm{d}s^2 = 0$ and the Yang-Mills EOMs, as argued via the Coleman-Mandula theorem [105] by e.g. [106]. This is especially interesting as symmetry principles have a deep-rooted connection to the laws of nature. For example, promoting Lorentz invariance to be a symmetry of space-time itself denotes the essence of Special Relativity as said by Einstein himself [107]. Furthermore, enforcing that space-time would be invariant under local coordinate transformation then led to the discovery of General Relativity [108]. Given this connection between symmetry principles and fundamental laws of nature, there is hope that demanding conformal invariance might lead to equally fundamental insights in the search for new physics.

Lastly, we introduce the notion of Weyl invariance in the presence of gravity in our theories. Including gravity in our quantum field theory leads to the generalization of (potentially) curved spacetime with the metric being a dynamical DOF. Recalling that conformal transformations are diffeomorphisms that locally transform the coordinates according to Eqs. (2.1) and (2.9), the metric then transforms the following way

$$g_{\mu\nu} \to g'_{\mu\nu}(x') = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} \, g_{\alpha\beta}(x) = \Omega^2(x') \, g_{\mu\nu}(x') \, . \tag{2.10}$$

Here, we see that the metric is invariant under conformal transformations up to $\Omega^2(x')$, which again denotes a (re-)scaling factor that now depends on the transformed coordinates. In contrast, Weyl transformations are not defined as coordinate transformations but rather act directly on the fields, transforming them similarly to Eq. (2.2) as

$$\Phi_i \to \Phi'_i(x^\mu) = \Omega^{w_i}(x) \, \Phi_i(x^\mu) \, . \tag{2.11}$$

On one hand, $\Omega(x)$ again denotes a dimensionless and non-negative parameter of the transformation, that is coordinate-dependent for a local and constant for a global Weyl transformation, much like for conformal transformations. On the other hand, the Weyl weight w differs from the scaling dimension d, particularly for vector bosons. In $D = 4$ spacetime dimensions it is given for the metric, its determinant, scalars, fermions, and vector bosons respectively as:

$$w_g = 2, \quad w_{\det(g)} = 8, \quad w_\phi = -1, \quad w_\psi = -3/2, \quad w_A = 0 \, . \tag{2.12}$$

Thus, Weyl, conformal, and scale symmetry are distinctly different, and there is no equivalence relation between the three concepts of symmetry. Yet, there exists directional implication, where Weyl invariance implies conformal symmetry and conformal symmetry implies scale invariance but not necessarily vice versa. Depending on the situation there are several additional requirements, like e.g. flatness, locality, and unitarity, for these terminologies to be used equivalently. In the following, we will use the term "scale invariance" (or symmetry) to refer to the global case, as it is the guiding principle for Section 2.3 and Chapter 3. In the case of local scale invariance and therefore most often also conformal invariance, we will allude to using the terminology of "conformal" or "Weyl invariance". Since the last part might introduce ambiguity, we will point these out if and when they arise, e.g. when talking about conformal dynamics in Chapter 4. For more information on scale invariance, conformal and Weyl symmetry, as well as Weyl geometry in the context of theories of gravity and their quantization, we gladly refer the reader to these extensive works [3, 61, 62, 109–114]

2.2 Trace Anomaly

As conformal symmetry includes the invariance under scale transformations, and thus forbids the appearance of dimensionful scales and parameters, it is evident that any initial conformal symmetry has to be broken to describe a theory that properly describes real-world physical phenomena. For example, conformal symmetry can be broken explicitly (by e.g. mass terms in the Lagrangian) or spontaneously (via a non-zero VEV, see Section 2.3). To check whether a symmetry of a quantum field theory is preserved one employs the Ward-Takahashi identities [115, 116]. Already in 1974, Capper and Duff found that 1-loop quantum corrections break said Ward-Takahashi identities [43] of the conformal currents, thus rendering conformal symmetry anomalous. Thus, independent from the aforementioned ways of breaking conformal symmetry, it is always already broken anomalously by quantum contributions. The identity-violating terms were found to be proportional to the trace of the improved energy-momentum tensor [64] and thus, the conformal anomaly is also known as the (stress-energy) trace anomaly:

$$T^{\mu}{}_{\mu} = g_{\mu\nu} \frac{\delta S}{\delta g_{\mu\nu}} \neq 0 \,. \tag{2.13}$$

The general form of the (non-local) violating terms depending on the particle content was deduced by Deser, Duff, and Isham in 1976 [117]. For a scale-invariant theory, the anomalous terms can generally be categorized by their origin, either sourced by gravitational curvature, then also called Weyl anomaly W[1], or by local dimension four operators $O^{(4)}$ of the theory's particle content and the running of their couplings

[1] In that respect, we highly recommend the comprehensive and historical review of [118].

through their β-functions

$$T^{\mu}{}_{\mu} = \beta_i \, O_i^{(4)} + W \, , \tag{2.14}$$

First, addressing the gravitational contributions to the anomaly of classical conformal symmetry in Eq. (2.14), i.e. W, they consist of contributions proportional to the Gauss-Bonnet term G, to the dynamics of gravity and in $D = 4$ spacetime dimensions to the square of the Weyl tensor C^2

$$W = a \left(F + \frac{2}{3} \Box R \right) + bG \, , \tag{2.15}$$

$$G = R_{\mu\nu\rho\sigma} \, R^{\mu\nu\rho\sigma} - 4R_{\mu\nu} \, R^{\mu\nu} + R^2 \, , \tag{2.16}$$

$$F = R_{\mu\nu\rho\sigma} \, R^{\mu\nu\rho\sigma} - 2R_{\mu\nu} \, R^{\mu\nu} + \frac{1}{3} R^2 \overset{(D=4)}{=} C_{\mu\nu\rho\sigma} \, C^{\mu\nu\rho\sigma} \equiv C^2 \, . \tag{2.17}$$

While the Gauss-Bonnet term is oftentimes ignored, as it reduces to a topological surface term for $D = 4$ and its counterterms are total divergences and therefore only contribute in non-trivial topologies, the C^2 term is generally non-zero and even if set to zero by assumption, in conformal invariant theories of gravity, it is nevertheless generated by quantum correction. In addition to its role in the anomalous breaking of conformal symmetry, the Weyl tensor also affects other physical observables as we will further discuss in Chapter 3, where it is shown to give corrections to the inflation parameters. Therefore, proper treatment of the Weyl tensor DOFs is crucial when dealing with conformal theories that include gravity.

Now, to the dimension four operators $O_i^{(4)}$ contributions of the theory's particle content in Eq. (2.14). In contrast to W, they describe the anomalous breaking in flat spacetime by quantum corrections. Thus, when $\beta_i \neq 0$, the RG-running of the dimensionless couplings induces a scale dependence, which in turn breaks scale and therefore also conformal invariance at the quantum level. As a consequence, the β function being non-zero is often used synonymously with the anomalous breaking of scale invariance, often overlooking possible contributions from curvature. Since the β functions only introduce logarithmic scale dependence and are not related to the often apparent quadratic divergences, the (only) anomalous breaking of scale invariance is considered a viable perspective to addressing the hierarchy problem [25, 31]. Yet, these arguments only hold for the assumptions of no intermediate scales between IR and UV, no instabilities or poles for the dimensionless couplings between these two scales, and most importantly, that the UV scale is an embedding scale, or scale of validity for QFTs, rather than a second physical mass. Furthermore, as the non-gravitational contribution to the trace anomaly is given by a sum over dimension four operators and their beta functions, it could be possible to adjust the particle content to achieve full cancellation and therefore no anomalous breaking by non-gravitational quantum corrections. If this is possible, this would possibly be only a statement at a specific energy scale as dimensionless couplings would still be running. Here, we leave this interesting topic to be discussed in future works, as the preservation of scale invariance at the quantum level is not our priority, instead, we are interested in the anomalous breaking of scale invariance and the following implications. For an in-depth discussion of quantum scale invariance, we refer to [46, 89, 99, 119–122].

Lastly, we want to note, that renormalization does not break conformal symmetry but (dimensional) regularization does. It seems that a quite straightforward way out is to change the regulator to one that respects the symmetry, yet attempts inevitably introduce non-renormalizable interactions [74, 121, 123–125], while others even state that all regulators must break conformal symmetry [31, 44, 126]. Furthermore, it has been argued that the trace anomaly is a sign of the breaking of global scale invariance, rather than local conformal symmetry [127][2]. It seems that there are still misconceptions and misunderstandings regarding the conformal anomaly, and even after 50 years, it is to this day an unsolved puzzle and an active field in the community. As, in this thesis, we will tackle problems by employing scale invariant and/ or conformal symmetric models, we inevitably come into contact with the conformal anomaly when discussing our results. To avoid a discussion, that is beyond the scope of this work, e.g. in Chapter 3, we will assume that conformal symmetry is only broken by the anomaly and show how one can soften the hierarchy problem in this framework, thus highlighting a possible use case for conformal symmetry and its anomalous breaking. Other examples of the utilization of conformal invariance and its anomalous breaking can be found in the investigation of models of cosmological inflation [1, 2, 86, 128–130] and reheating [131], the explanation of the origin of masses and their generation [1, 110, 128, 130, 132], or addressing the gauge hierarchy problem [76, 133–135]. Furthermore, studying the nature of gravity [65, 69, 74, 83, 85, 136–138], dark matter [137, 139], and possible UV completions for phenomenologically interesting theories [4]. In addition, the conformal anomaly has been argued to be responsible for Hawking radiation [92], to be the reason for the formation of cosmological perturbations [140], to be able to explain dark energy [96], to induce non-Gaussianities in the cosmic microwave background (CMB) [141], as well as playing a large role in the formation of compact stellar objects, that are proposed as alternatives to black holes [142].

2.3 Radiative Breaking of Scale Invariance

As pointed out in the previous section Section 2.2, a sensible scale-invariant theory needs to include a mechanism to generate scales dynamically, i.e. break scale symmetry spontaneously. Hence, before getting into the actual calculations and applications of spontaneously broken scale symmetry later on in Chapters 3 and 4,

[2] The same authors address the conservation of Weyl symmetry at the quantum level by adding a non-local curved spacetime generalization of the dilatation current, thus restoring the conformal Ward identity.

we first introduce the concepts of spontaneous symmetry breaking (SSB) and especially the case where the symmetry breaking contributions are given by (one-loop) quantum corrections, then known as the Coleman-Weinberg mechanism. A critical component is to understand the generation of non-trivial vacuum expectation values (VEVs) of the quantum fields that break scale symmetry. Thus, we start by introducing the quantum effective action, which is comparable to the classical action but takes into account quantum effects of all loop levels. In parallel to the classical action, the field equations of our quantum fields are obtained by functional derivatives of said quantum effective action. Furthermore, the non-derivative terms of the quantum effective action in the local approximation will give the (quantum) effective potential, whereas its minimization gives us information about the nature and generation of the symmetry-breaking VEV. We will only briefly sketch the above-mentioned derivation as it can be found in many more resources, ranging from reviews, e.g. [143–145], to in-detail derivations from mathematical principles, often found in QFT textbooks or lecture notes, see e.g. [146–152]. Roughly following [26, 144, 145], we will highlight the general derivation of the one-loop approximation of the effective potential, i.e. the Coleman-Weinberg potential. As this will need regularization and renormalization, we will furthermore give an expression for the Coleman-Weinberg potential of general particle content in the minimal subtraction scheme ($\overline{\text{MS}}$) and analyze its implications for symmetry breaking in Section 2.3.2. In Section 2.3.3 we will shortly review the Gildener-Weinberg approximation [153], commonly used to systematically minimize the one-loop effective potential in theories with multiple scalar fields.

We begin with the generating functional $Z[J]$ of the Green's functions of a scalar field $\phi(x)$ and its classical action $S[\phi(x)]$ in the presence of a classical source field $J(x)$

$$Z\,[J] = \int \mathcal{D}\phi \;\exp\left(iS[\phi(x)] + i \int \mathrm{d}^4 x \;\phi(x)\; J(x) \right) \,, \tag{2.18}$$

where $\mathcal{D}\phi$ denotes the path integral over ϕ. It is crucial to remark here, that the relation of $Z\,[J]$ being the generating functional to the n-point correlation (or Green's) functions is only due to the properties of canonical quantization and regularization. Hence, Eq. (2.18) is universal for all canonically quantized and regularized (quantum) theories. Given that, $Z\,[J] = \langle 0_{\text{in}} | 0_{\text{out}} \rangle$ also represents the transition amplitude between two vacuum states in the presence of the external field J, factoring out the disconnected Green's functions one is left with the following relation to the generating functional of the connected Green's functions (also known as Schwinger functional) $W[J]$:

$$Z\,[J] = \exp\left(iW[J]\right) \,. \tag{2.19}$$

Now, further following the spirit of reducing towards only the physical information, the quantum effective action $\Gamma[\varphi]$ is the generating functional for the one-particle irreducible (1PI) diagrams. It is then related to $Z[J]$ and $W[J]$ via functional Legendre transformation

$$\Gamma[\varphi] = W[J_\varphi] - \int \mathrm{d}^4x \; \varphi(x) \; J_\varphi(x) \; , \tag{2.20}$$

such that the functional dependency on J is replaced by $\varphi(x)$, the VEV of ϕ. Also called mean or background field, the VEV of ϕ in the presence of the external source J is a classical field given by

$$\varphi_J(x) = \langle \phi(x) \rangle_J = \frac{1}{Z[J]} \frac{\delta Z[J]}{\delta i J(x)} \overset{(2.19)}{=} \frac{\delta W[J]}{\delta i J(x)} \; . \tag{2.21}$$

Assuming that the relation Eq. (2.21) is invertible, it reveals $\varphi_J(x)$ and $J(x)$ to be conjugate objects. Hence, we define $J(x) = J_\varphi(x)$ to be the current that satisfies $\varphi_J(x) = \varphi(x)$ and gain the quantum effective action $\Gamma[\varphi(x)]$ in Eq. (2.20) via aforementioned Legendre transformation. In analogy to the classical action, we derive the equation of motion for $\varphi(x)$ via functional differentiation with respect to the field itself

$$\frac{\delta \Gamma[\varphi]}{\delta \varphi(x)} = -J_\varphi(x) \; , \tag{2.22}$$

thus further affirming the parallel to the classical theory, specifically, that the expectation value of the quantum field is obtained via the stationary principle of the action functional. Here, under the condition of vanishing external sources ($J = 0$) but accounting for all orders of quantum effects. Expanding the effective action $\Gamma[\varphi]$ around $\varphi = 0$ the quantum n-point vertex functions Γ_n (also known as proper vertices) appear as coefficients of the aforementioned expansion. Thus, $\Gamma_n(x_1, \ldots, x_n)$ is obtained via n functional derivatives with respect to $\varphi(x_i)$ for $i = [1, n]$. Furthermore, expressing $\Gamma[\varphi]$ in terms of the Fourier transformation $\tilde{\Gamma}_n$ of the proper vertices and expanding them around vanishing momenta, we can write the quantum effective action of Eq. (2.20) in terms of a derivative expansion

$$\Gamma[\varphi] = \sum_{n=1}^{\infty} \frac{1}{n!} \int \mathrm{d}^4x \; \tilde{\Gamma}_n(0, \ldots, 0) \; \varphi(x)^n + \frac{1}{2} \int \mathrm{d}^4x \; Z(\varphi) \; \partial_\mu \varphi \; \partial^\mu \varphi + \; \ldots \; . \tag{2.23}$$

Here, $Z[\varphi]$ is the mean-field normalization factor and the dots represent all higher orders of derivative terms. We want to remark here, that to get to the expression in Eq. (2.23) one has to assume the mean field $\varphi(x)$ to be only slowly changing in space-time. This is known as the so-called local approximation of the generally non-local effective action $\Gamma[\varphi]$. In analogy to the classical action, the effective potential $V_{\text{eff}}(\varphi)$ is identified as the non-derivative terms of the quantum effective potential in

Eq. (2.23),

$$V_{\text{eff}}(\varphi) = -\sum_{n=1}^{\infty} \frac{1}{n!} \tilde{\Gamma}_n(0,\ldots,0)\varphi(x)^n \,. \tag{2.24}$$

Additionally, if we require, as usually done, the vacuum expectation value to be invariant under translations, i.e. constant in spacetime $\varphi(x) = \varphi_c$, we arrive at

$$\Gamma[\varphi_c] = -V_{\text{eff}}(\varphi_c) \int \mathrm{d}^4 x \,. \tag{2.25}$$

At this point, we have Eq. (2.25) relating the effective Potential $V_{\text{eff}}(\varphi_c)$ of the constant VEV φ_c to the quantum effective action $\Gamma[\varphi_c]$ and Eq. (2.24) that lets us calculate the effective potential through the Fourier transformed proper vertex functions $\tilde{\Gamma}_n$. This, in turn, can be calculated via functional derivatives of the quantum effective action $\Gamma[\varphi_c]$ with respect to φ, on which we will expand further in this section or through representation in Feynman diagrams and proper summation over the corresponding contributions, which is nicely illustrated in e.g. [151]. Yet, in either approach, we have to deal with the external field J or J_φ (after Legendre transformation) and thus we want to remark here, that in this derivation, $J(x)$ is a purely artificial construct with no physical meaning. Thus, by setting $J = 0$ we go from any hypothetical vacuum configuration to the “true” or physical vacuum state. Therefore, the stationary principle of the quantum effective action Eq. (2.22) in the absence of external sources $J = 0$, i.e. for the theory’s physical vacuum state, leads to a stationary condition on the effective potential V_{eff}

$$\left.\frac{\partial V_{\text{eff}}(\varphi)}{\partial \varphi}\right|_{J=0} = 0 \,. \tag{2.26}$$

Here, we want to shortly emphasize the significance of this relation, as this consequently means that analysis of a quantum theory’s physical vacuum structure is equivalent to minimizing the effective potential. Together, Eq. (2.26) and requiring the potential to be bound from below, ensuring stability, give the necessary and sufficient conditions for a minimum of the potential. From here one can carry out all sorts of investigations on, e.g. vacuum decays via analysis of the local and global minima. On the other hand, the problem of actually calculating the effective potential still exists, since both aforementioned methods (functional derivatives and Feynman diagrams) contain infinite sums over contributions that can be grouped by loop-order n, i.e. $\mathcal{O}(\hbar^n)$. Therefore, it is feasible to look at the effective potential in an expansion by order of loops.

2.3.1 The One-loop Effective Potential

Specifically, we will briefly introduce the general derivation of the one-loop contributions to the effective potential as we will discuss their potential for spontaneous breaking of scale symmetry in the rest of this and Chapter 3. Given, the loop-suppression factor that we know from calculations of Feynman diagrams, one might expect that tree-level contributions dominate the mechanism of symmetry breaking. Nevertheless, we will show fairly general examples where this is not the case, and tree-level and one-loop contributions are of the same order of magnitude when it comes to the equations determining the scale of symmetry breaking. Furthermore, as going from tree-level to including one-loop contributions marks taking into account the "leading" order of quantum corrections, we will refrain from including quantum effects of higher loop-order. Including higher-order contributions might increase precision but we believe the fundamental aspects of the findings we will present in Chapters 3 and 4 will not change, hence we stick to the one-loop approximation of the effective potential. The zero-loop order contributions, also known as tree-level are then given by the classical potential $V^{(0)}$, which is obtained in the limit $\hbar \to 0$. The one-loop contributions are then given by the so-called Coleman-Weinberg potential $V^{(1)} = V_{\text{CW}}$, which represents the sum over all 1PI Feynman Diagrams that contain one closed loop and n external legs connected to the mean field ϕ_0. In the following, we will introduce the most important steps and concepts of the calculation on how to derive the general form of the Coleman-Weinberg potential via functional methods from the quantum effective action $\Gamma[\varphi]$, following the detailed and instructive example of [145].

We start with the the usual QFT treatment [26, 145, 154, 155], i.e. we perturbativly expand our quantum field ϕ around a classical, approximately constant, background field ϕ_0

$$\phi \to \phi_0 + \Phi\,. \tag{2.27}$$

Thus, $\Phi(x)$ denotes the small quantum fluctuations around ϕ_0. The classical action, expanded around this field configuration is then given to be

$$\begin{aligned} S\left[\phi_0 + \Phi\right] = S\left[\phi_0\right] &+ \int \mathrm{d}^4x \; \frac{\delta S[\Phi]}{\delta \Phi(x)}\bigg|_{\Phi=\phi_0} \Phi(x) \\ &+ \int \mathrm{d}^4x \int \mathrm{d}^4x' \; \frac{1}{2}\,\Phi(x)\; \frac{\delta^2 S[\Phi]}{\delta\Phi(x)\delta\Phi(x')}\bigg|_{\Phi=\phi_0} \Phi(x') \\ &+ \mathcal{O}(\Phi^3) + \ldots\,, \end{aligned} \tag{2.28}$$

where the functional derivative in the first integrand can be identified as the field's classical EOM and the one in the second integrand precisely gives the definition of the inverse propagator of the classical theory in the constant background ϕ_0

$$\frac{\delta S[\Phi]}{\delta \Phi(x)}\bigg|_{\Phi=\phi_0} = -J(x), \qquad \frac{\delta^2 S[\Phi]}{\delta\Phi(x)\delta\Phi(x')}\bigg|_{\Phi=\phi_0} = iD^{-1}(\phi_0; x, x')\,. \tag{2.29}$$

Seeing, that in $Z[J]$, the action contributes exponentially inside the path integral $\mathcal{D}_\phi$, we find that the second term in above's expansion Eq. (2.28) will give a Gaussian path integral of the form

$$\int \mathcal{D}_\Phi \exp\left(-\Phi\, iD^{-1}\, \Phi\right) = \mathcal{N}\left[\mathrm{Det}\left(iD^{-1}\right)\right]^{-1/2} , \tag{2.30}$$

where $\mathcal{N}$ denotes a spacetime normalization factor. For a detailed derivation of Eq. (2.30), we refer the reader to their preferred QFT textbook, as most of them address Gaussian path integrals, but in the unfortunate case that they do not, see e.g. [146]. Furthermore, note that the EOM in Eq. (2.29) leads to the cancellation of the $\int \mathrm{d}^4x\ \Phi(x)\ J(x)$ term in Eq. (2.18) after plugging in the perturbed classical action $S[\phi_0 + \Phi]$. This only leaves terms that depend on ϕ_0 and the Gaussian path integral Eq. (2.30) to describe $Z[J]$. Now using Eq. (2.19), which is easily inverted to give $W[J] = -i \ln Z[J]$, we can finally calculate the quantum effective action via Eq. (2.20) after Legendre transformation, again trading the functional dependence $J(x)$ with the slowly changing background field φ, according to Eq. (2.21). The effective action $\Gamma[\varphi]$ at one-loop is then given to be

$$\Gamma[\varphi] = S[\varphi] + \frac{i}{2} V \ln \mathrm{Det}\left(iD^{-1}(\varphi)\right) + \mathcal{O}(\hbar^2)\,, \tag{2.31}$$

where $V = \int \mathrm{d}^4x$ denotes the four-dimensional spacetime volume that is consistent with the appearance of the normalization of the Gaussian path integral $\mathcal{N}$ in Eq. (2.30). Further following the derivation above, we can use Eq. (2.25) to identify the one-loop Coleman-Weinberg potential in Eq. (2.31) to be

$$V^{(1)}(\varphi) = V_{\mathrm{CW}}(\varphi) = -\frac{i}{2} \ln \mathrm{Det}\left(iD^{-1}(\varphi)\right) = -\frac{i}{2} \mathrm{Tr}\ \mathrm{Ln}\left(iD^{-1}(\varphi)\right)\,, \tag{2.32}$$

where the capitalized operators Det, Ln and Tr are meant to be acting on the inverse propagator D^{-1} in the functional sense. Here, given the dependence on the inverse propagator in calculating the Coleman-Weinberg potential, one can easily see that it can be quite advantageous to work in a Feynman diagrammatic representation of the theory. Yet, more generally, the dependence on the inverse propagator in Eq. (2.32) is replaced with the functional Hessian of the action with respect to the field content. Thus, once gravity is included as a dynamical DOF, working with the functional Hessian to determine the one-loop contributions becomes favorable. It is necessary to note here, that the full one-loop effective potential V_{eff}, which will determine symmetry breaking and will allow us to investigate the (one-loop) "true" VEV of the theory, also contains the classical potential $V^{(0)}$, which contributes via the classical action $S[\varphi]$ in Eq. (2.31)

$$V_{\mathrm{eff}}(\varphi) = V^{(0)}(\varphi) + V^{(1)}(\varphi)\,. \tag{2.33}$$

To calculate $V^{(1)}$ we first remark, that in most cases the inverse propagator $iD^{-1}(\varphi, p, q)$ (in momentum space) is some kind of diagonal Klein-Gordon operator

$$iD^{-1}(\varphi_i, p, q) = \left[p^2 \delta_{ab} - m^2_{ab}(\varphi_i)\right] \delta^4(p-q) \ , \tag{2.34}$$

here given for multiple scalar fields φ_i, with the field dependent mass (matrix) m^2_{ab} given by the tree-level potential $V^{(0)}$

$$m^2_{ab}(\varphi_i) = \frac{\partial^2 V^{(0)}}{\partial\varphi_a \partial\varphi_b} \ . \tag{2.35}$$

Collecting the pieces from Eqs. (2.32), (2.34) and (2.35), evaluating the trace, performing Wick rotation and dropping field independent terms[3], one finds that the one-loop effective potential contribution most often depends on integrals of the form

$$\int \frac{d^4 p}{(2\pi)^4} \ln\left[p^2 + m^2(\varphi_i)\right] \ , \tag{2.36}$$

this kind of integral is clearly UV divergent and therefore needs regularization and renormalization. Using the well-known $\overline{\text{MS}}$ scheme, a generalized result for the one-loop contribution to the effective potential can be calculated to be [144]

$$V^{(1)}(\varphi_j) = \frac{1}{64\pi^2} \sum_i N_i \, m_i^4(\varphi_j) \left(\ln\left[\frac{m_i^2(\varphi_j)}{\mu^2}\right] - c_i \right) \ , \tag{2.37}$$

where the index i runs over all particles, such that $m_i(\varphi)$ denotes their corresponding field dependent masses, while c_i and N_i are constants that depend on the particle's species, i.e. their spin. These species-dependent constants are due to the appearance of the trace operator in Eq. (2.32), as it directly incorporates the particles' internal degrees of freedom. For real scalars, vector bosons, and Weyl and Dirac fermions N_i and c_i are respectively given by

$$N_i = \{1,\ 3,\ -2,\ -4\}\,, \quad c_i = \left\{\frac{3}{2}, \frac{5}{6}, \frac{3}{2}, \frac{3}{2}\right\} \ . \tag{2.38}$$

While N_i denotes the particle's real number of DOFs and therefore a physical quantity, c_i is "purely" technical and renormalization scheme dependent. Furthermore, the renormalization scale μ is also an unphysical scale and hence, if the particle content of the theory is such that all $c_i = c$ are equal, they can easily be absorbed in the definition of μ. We want to note here, that even though it is only a technical quantity, treating c_i and therefore also μ consistently is important to keep results, like e.g. the scale of symmetry breaking, comparable.

Given the result for the one-loop Coleman-Weinberg potential in Eq. (2.37), one

[3] They would only contribute as infinite constants and thus would be subtracted in the next (necessary) step, i.e. renormalization. Therefore, without loss of generality, they can be neglected.

can somewhat easily investigate toy models for certain properties of radiative symmetry breaking by defining the particle content and its symmetries in the classical Lagrangian, calculating the radiative corrections and determining the (global) VEV and its symmetries. Yet, we want to strongly remark here that, when working with scale invariance or conformal symmetry one always has to be careful with the choice of regulator for the theory's divergences. Using Eq. (2.37) requires dimensional regularization and renormalization via the minimal subtraction scheme. However, the regulator itself already breaks scale invariance (dilatations) explicitly by introducing a scale, even though it is only an unphysical one. This refers back to Section 2.2, conformal symmetry being anomalous, i.e. being broken by quantum corrections already. Thus, working with Eq. (2.37) one ultimately recovers the naive power counting behavior when generating multiple scales together, or in other words; in this framework using $\overline{\mathrm{MS}}$ one does not (easily) find the spontaneously broken, but still non-linearly realized scale symmetry. Unfortunately, this means that if one wants to see whether the non-linearized scale symmetry allows for hierarchically separated scales, one always has to wrestle with the anomaly of scale and conformal symmetry and especially Eq. (2.37) and $\overline{\mathrm{MS}}$ formalism, even though very accessible, appear to be a sub-optimal framework for investigation. There have been several attempts to reconcile this conundrum, yet all of them come with their own concerns, none addressing the problem in a fully satisfying manner. Hence, for this discussion, we refer the interested reader again to Section 2.2 and the therein given literature. Nonetheless, since the main point of the results in Sections 2.3.2 and 2.3.3 and Chapter 3 is separate from this discussion, for the reason of simplicity we use Eq. (2.37) in the $\overline{\mathrm{MS}}$-scheme.

2.3.2 Coleman-Weinberg Mechanism

Having introduced scale and conformal symmetry, and the derivation of the one-loop effective potential to include leading quantum effects, there is still one critical piece missing before we can dive into the specific calculations, namely how (scale) symmetry is broken. As our starting (tree-level) Lagrangian and EOMs are scale-invariant by definition, explicit breaking is not possible, and hence we assume it is broken spontaneously instead. More precisely, the phenomena of spontaneous symmetry breaking (SSB) describes the dynamical process by which a system invariant under certain symmetries ends up in a state that is not invariant under (all) these symmetries. Typically, this state denotes the lowest-energy vacuum solution, the (global) VEV, which then does not respect the symmetries that are present in the EOMs, thus symmetry is broken for perturbations around that VEV even though the symmetry is still upheld by the EOMs and Lagrangian. Since symmetries are one of the most important concepts of physics, their spontaneous breakdown holds incredibly great significance, as it for example, determines the dynamics of phase transitions, the excitations of states, and the appearance of new particles. Thus, it is not surprising that SSB finds application in a variety of different fields of physics.

Early descriptions of SSB can be found in 1950s theoretical condensed matter physics to explain the phenomenon of superconductivity [156–158]. These ideas were taken up quickly by particle physicists and in the early 1960s, especially Nambu [159] and Goldstone [154, 160] developed a generalized description of SSB and with it the according masslessness of bosons, the so-called *Nambu–Goldstone bosons* (NGBs). This led to the exploration of SSB in QFTs via the Goldstone theorem and the subsequent discovery, that it is essential to the generation of the gauge bosons' mass in the Standard Model through the *Englert-Brout-Higgs-Guralnik-Hagen-Kibble mechanism* [161–165], often shortened to just *Higgs mechanism*. The Higgs mechanism utilizes a so-called "Mexican hat" potential at tree level

$$V^{(0)}(h) = \frac{1}{4}\lambda_H h^4 - \frac{1}{2}m^2 h^2 \,, \tag{2.39}$$

with $m^2 > 0$, $\lambda > 0$ and h denoting the neutral component of the Standard Model Higgs doublet H, to generate non-zero VEV for h, thus breaking electroweak symmetry spontaneously and generating all scales of the SM through couplings to this VEV $\langle h \rangle = v_{\text{EW}} \neq 0$. We immediately see, that symmetry breaking of this form is not applicable in scale-invariant theories as Eq. (2.39) vitally depends on the existence of an $m^2 > 0$ term for the generation of a non-zero VEV at tree level, while scale-invariance demands $m^2 = 0$. This is where the Coleman-Weinberg mechanism comes in, describing a theory that is symmetric at tree level but exhibits SSB if quantum contributions are included. Given the widespread application of symmetry breaking and scale-invariance, or conformal symmetry, as principles in physics, there are countless ways in which they are formally introduced. For the Coleman-Weinberg mechanism, we want to follow the original publication [26] and will give a short review via the example of massless scalar QED, as it is easy to comprehend but still raises the important aspects of RSSB for the subsequent chapter. For literature on more extensive treatments we recommend [166–168].

As usual, we start with the classical action of massless scalar QED

$$S_{\text{sQED}} = \int d^4x \, \mathcal{L}_{\text{sQED}} = \int d^4x \left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \left(D_\mu \Phi\right)^\dagger \left(D^\mu \Phi\right) - V^{(0)}(\Phi) \right) \,, \tag{2.40}$$

with the complex scalar $\Phi = \frac{1}{\sqrt{2}}\left(\phi_1 + i\phi_2\right)$ and its tree-level potential $V^{(0)} = \lambda\left(\Phi^\dagger \Phi\right)^2$. Here, λ denotes the dimensionless quartic self-coupling, $D_\mu = \partial_\mu - igA_\mu$ the covariant derivative that couples the complex scalar Φ to the $U(1)$ gauge field A_μ with field-strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and gauge coupling g. Due to the demanded masslessness, Eq. (2.40) exhibits scale symmetry in addition to the evident invariance under $U(1)$ transformations. For our purpose, we will assume the $U(1)$ symmetry to be global, as it is directly relevant to Chapter 3. We want to remark though, that RSSB via the Coleman-Weinberg mechanism allows for a very general description and thus is applicable for global as well as for local symmetries[4] and

[4] For an in-depth discussion of the different implications, see e.g. [147].

even respects gauge invariance in renormalizable theories [169, 170]. The latter is especially important when expanding around the non-trivial minima and investigating the Goldstone modes in specific gauge configurations, which is particularly helpful in discussions of unitarity and propagating (physical) DOFs. Turning back to massless scalar QED, we can calculate the one-loop effective potential using the methods introduced in Section 2.3.1, specifically, Eq. (2.37) is directly applicable and gives

$$V^{(1)}(\varphi;\mu) = \frac{\varphi^4}{64\pi^2}\left(9\lambda^2 \ln\left[\frac{3\lambda\varphi^2}{\mu^2}\right] + \lambda^2 \ln\left[\frac{\lambda\varphi^2}{\mu^2}\right] - 15\lambda^2 + 3g^4 \ln\left[\frac{g^2\varphi^2}{\mu^2}\right] - \frac{5}{2}g^4\right), \tag{2.41}$$

with $\varphi^2 = \varphi_1^2 + \varphi_2^2$ being the gauge invariant combination of the real scalars' ϕ_1, ϕ_2 background fields. Furthermore, separately displaying the dependence on the renormalization scale μ serves as a reminder, that in addition to the explicit dependency in Eq. (2.41), there is also an implicit dependence via the dimensionless couplings $\lambda = \lambda(\mu)$, $g = g(\mu)$ renormalization group (RG) running. Now, to ascertain whether any symmetry is radiatively spontaneously broken, we have to evaluate the stationary condition Eq. (2.26) for $V_{\text{eff}}(\varphi;\mu) = V^{(0)}(\varphi;\mu) + V^{(1)}(\varphi;\mu)$. Given, that physical observables need to be independent of μ, we make the self-consistent choice of $\mu = \langle\varphi\rangle = v \neq 0$. This leads to the criticality equation

$$\lambda = -\frac{1}{16\pi^2}\left(9\lambda^2 \ln[3\lambda] + \lambda^2 \ln[\lambda] - 10\lambda^2 + 3g^4 \ln\left[g^2\right] - g^4\right), \tag{2.42}$$

where we reorganized the term such that the left-hand side represents the tree-level and the right-hand side the one-loop quantum contributions. The first thing we want to point out here is, that for $g(\mu = v) = 0$ RSSB is impossible, since the tree-level contributions of $\mathcal{O}(\lambda)$ would have to be of the same magnitude as (loop-suppressed) quantum corrections of $\mathcal{O}(\lambda^2)$ or $\mathcal{O}(\lambda^2 \ln\lambda)$. In this case, the validity of our perturbative expansion would break down, such that we need a different formalism to investigate whether symmetry breaking occurs. One possibility is to use the renormalization group equations (RGEs) to improve the effective potential. This is already featured in the original publication of Coleman and Weinberg [26], where they find that a single scalar without additional contributions (e.g. gauge) cannot induce RSSB. Secondly, we want to remark that by choosing $\mu = v$, the generally running couplings in Eq. (2.42) are understood to be evaluated at the scale of the VEV, i.e. $\lambda = \lambda(\mu = v) \equiv \lambda_{\text{cr}}$, $g = g(\mu = v) \equiv g_{\text{cr}}$. Now, assuming that $0 < \lambda \ll 1$ to ensure unitarity and perturbativity, we can omit the $\mathcal{O}(\lambda^2)$ terms in Eq. (2.42), as they are subleading. This leaves us with

$$\lambda = -\frac{3g^4}{16\pi^2}\left(\ln\left[g^2\right] - \frac{1}{3}\right), \tag{2.43}$$

which nicely showcases that quantum corrections are of the same order of magnitude as the tree-level contribution $\mathcal{O}(\lambda) \sim \mathcal{O}(g^4)$ and thus can deform the potential sufficiently to allow for a non-trivial (non-zero) vacuum configuration leading to RSSB, all without leaving regime of perturbativity of our expansion. This is visualized for exemplary values in Fig. 2.2. Given our choice for $\mu = v$ above, the relation Eq. (2.43) is not valid for all μ but it rather means that for a given choice of couplings $\lambda_0 = \lambda(\Lambda_{\text{ren}}),\ g_0 = g(\Lambda_{\text{ren}})$ at the scale Λ_{ren}, where the theory is demanded to be renormalizable, there exists a $\mu < \Lambda_{\text{ren}}$ such that Eq. (2.43) is fulfilled. For massless scalar QED, this existence has been proven for (small) perturbative couplings $\lambda,\ g$ in, e.g. [26, 171]. The RG evolution of a running couplings λ_i is given by their β-function

$$\beta_{\lambda_i} = \beta_{\lambda_i}\left(\lambda_i(\mu)\right) = \mu \frac{\partial \lambda_i(\mu)}{\partial \mu} = \frac{\partial \lambda_i(\mu)}{\partial \ln[\mu]} \ . \tag{2.44}$$

With this evolution Eq. (2.44) and the relation between λ and g at the scale $\mu = v$ Eq. (2.43), we can now reparameterize the effective potential Eq. (2.41) to depend on g and v instead of λ and g. This phenomenon is called dimensional transmutation and it is essentially the fact, that the dimensionless couplings' numerical values depend on the arbitrary and dimensionful renormalization scale (or running scale) μ. Knowing now, that a non-zero VEV is indeed generated, we can calculate the masses of the scalar and the gauge boson

$$m_\varphi^2 = \left.\frac{\partial^2 V_{\text{eff}}}{\partial \varphi^2}\right|_{\varphi = v} = \frac{3\, g_{\text{cr}}^4\, v^2}{8\pi^2}, \qquad m_A^2 = g_{\text{cr}}^2\, v^2 \ . \tag{2.45}$$

In summary, we found that under the assumption of perturbative dimensionless couplings and at leading order in quantum corrections, the one-loop gauge contributions lead to successful RSSB for a massless complex scalar. Since higher-order corrections come with greater powers of λ and g (comparable via the scaling in

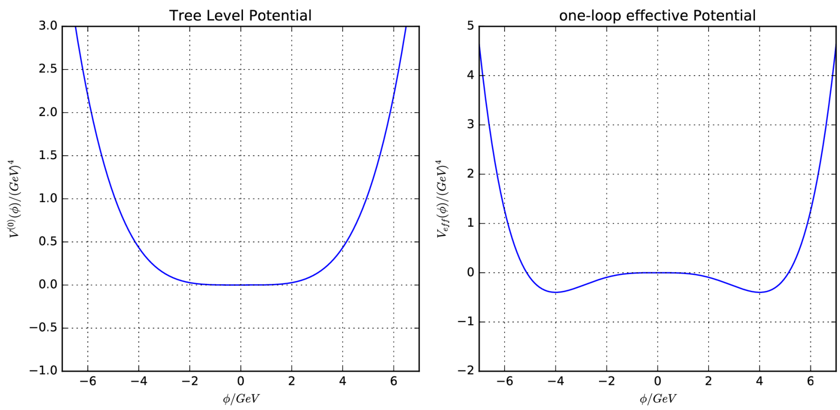


FIGURE 2.2: Example of a scalar QED tree-level and its one-loop effective potential at $\mu = \langle\phi\rangle \neq 0$ and couplings λ, g that satisfy Eq. (2.43).

Eq. (2.43)), they are not expected to fundamentally change the vacuum structure of the theory.

For more general cases with, multiple scalars, multiple gauge groups, etc., it can quickly become much more complex and there are several ways of tackling this complexity, as insights gained from multiscalar toy models are often directly applicable to the dynamical generation of the electroweak scale in scalar extensions of the SM. In the following, we will present a widely used approximation (see e.g. Chapter 3) to massively reduce the complexity of RSSB in models with multiple scalars, the Gildener-Weinberg (GW) approach.

2.3.3 Gildener-Weinberg Approximation

Only a few years after Coleman and E. Weinberg [26] demonstrated that one-loop quantum contributions can cause the spontaneous breakdown of gauge symmetry and induce a non-trivial VEV, Gildener and S. Weinberg [153] developed a systematic approach to minimize the effective one-loop potential for classically scale-invariant theories with multiple scalars. This approach relies on the essential assumption, that there exists a flat direction in the tree-level potential, such that quantum corrections, which are generally smaller than the tree-level contributions, can induce a non-trivial vacuum along that flat direction. Effectively allowing for the reduction of the minimization problem's dimension from N, the number of scalar fields, to one. Since the existence of a tree-level flat direction is the main requirement for RSSB, the condition for criticality depends only implicitly on (one-loop) gauge contributions through RG-running of dimensionless couplings. In contrast, in the full description via Coleman-Weinberg, they are explicitly present in the equations determining criticality, c.f. Eq. (2.42). Thus, if there are non-scalar DOFs present, one needs to carefully check whether the assumptions that go into the Gildener-Weinberg approximation hold for the full effective potential. Given, that in Chapter 3 we employ the most minimal setup to allow for RSSB in the GW-approximation, i.e. two scalars without additional gauge fields or fermions (that take part in the symmetry-breaking), in this work we want to dodge the interesting discussion of symmetry breaking in the full effective potential and instead refer to [172–175] for a more comprehensive treatment. Instead, in the following we will shortly revisit the most important aspects of the GW approach and then apply these to the later utilized two scalar model.

Under the assumption of classical scale invariance, the tree-level potential for N scalars only contains their quartic interactions and can be described by the general form of

$$V^{(0)}(\mathbf{\Phi}) = C_{\text{Norm}}\, \lambda_{ijkl}\, \Phi_i\, \Phi_j\, \Phi_k\, \Phi_l \quad , \tag{2.46}$$

where $\mathbf{\Phi}^\intercal = (\Phi_1, \ldots \Phi_N)$ contains the theory's real scalar DOFs, C_{Norm} denotes a normalization constant and λ_{ijkl} is the general quartic scalar coupling. For reasons of unitarity and boundness of the potential, λ_{ijkl} can only have non-negative individual entries. To ensure, that the tree-level contributions dominate the one-loop effective potential it is assumed that all non-zero entries of λ_{ijkl} are of $\mathcal{O}(g^2)$, with $g \ll 1$ denoting a usual gauge coupling of the theory. As a consequence, it is assumed that at high scales, the theory exhibits only the trivial global minimum, i.e. located at $\mathbf{\Phi} = 0$. Furthermore, after RG-running to some scale Λ_{GW} the dimensionless couplings $\lambda_{ijkl}(\Lambda_{\text{GW}})$ fulfill the GW condition, i.e. the tree-level vanishes along

$$\mathbf{\Phi}_{\text{flat}} = \mathbf{n}\,\phi, \qquad V^{(0)}(\mathbf{\Phi}_{\text{flat}}) = 0\,, \tag{2.47}$$

such that the potential develops a flat direction along the unit vector $\mathbf{n}$, with the radial component ϕ. This leads to an infinite set of degenerate minima along this flat direction. Because of the (nearly) vanishing tree-level contribution along this flat direction, the curvature introduced by quantum corrections is now able to induce a non-degenerate global minimum $\langle\phi\rangle = v_\phi \neq 0$. To find this global minimum, one only has to investigate the one-loop effective potential along the flat direction at the GW scale, which can generally be written as

$$V_{\text{eff}}(\mathbf{\Phi}_{\text{flat}}) = A\,\phi^4 + B\,\phi^4 \ln\left[\frac{\phi^2}{\Lambda_{\text{GW}}^2}\right]\,, \tag{2.48}$$

where the dimensionless A and B are obtained by identification after calculating the full V_{eff} via Eq. (2.32), projection along $\mathbf{\Phi}_{\text{flat}}$ and using scale-invariance to find the scaling of the field-dependent masses to be $m_i^2(\mathbf{\Phi}_{\text{flat}}) = \hat{m}_i^2(\mathbf{n})\,\phi^2$, we thus arrive at

$$A = \frac{1}{64\pi^2}\sum_i N_i\,\hat{m}_i^4(\mathbf{n})\left(\ln\left[\hat{m}_i^2(\mathbf{n})\right] - c_i\right)\,, \tag{2.49}$$

$$B = \frac{1}{64\pi^2}\sum_i N_i\,\hat{m}_i^4(\mathbf{n})\,. \tag{2.50}$$

Deploying the stationary principle Eq. (2.26), we can calculate the value of the (possible) non-trivial minimum along the flat direction depending on the GW scale as

$$\langle\phi\rangle = v_\phi = \Lambda_{\text{GW}}\,\exp\left(-\frac{1}{4} - \frac{A}{B}\right)\,. \tag{2.51}$$

Calculating the second derivative of the effective potential along the flat direction at the above-given value v_ϕ in Eq. (2.51)

$$\left.\frac{\partial^2 V_{\text{eff}}(\mathbf{n}\,\phi)}{\partial\phi^2}\right|_{\phi=v_\phi} = 8\,B\,v_\phi^2 =: m_\phi^2\,, \tag{2.52}$$

verifying that Eq. (2.51) indeed describes a minimum, when assuming $B > 0$. This assumption also has to be verified for a theory's specific particle content but given the quadratic dependence on the field dependence masses in Eq. (2.50) this assumption is rather generally justified. In summary, the existence of the GW scale Λ_{GW} at which the tree-level potential vanishes, allows for RSSB along a flat direction and thus induces one massive scalar (radial) DOF ϕ, the pseudo-Goldstone boson of broken scale invariance, which Gildener and S. Weinberg called the *scalon* [153]. As this approach fundamentally depends on the existence of this flat direction, we want to remark, that the one-dimensional description of $V_{\text{eff}}(\mathbf{\Phi}_{\text{flat}})$ (c.f. Eq. (2.48)) most importantly still is an approximation and that quantum contributions, most notably from non-scalar DOFs, may considerably deform the effective potential and therefore change the orientation of the flat direction or even remove it completely.

Now, for a model only containing scalars, the GW approach is certainly advantageous to investigate possible RSSB based on simple tree-level conditions. For a minimal example, we consider two massless real scalar fields, with their scale-invariant tree-level potential

$$V^{(0)}(\phi_1, \phi_2) = \frac{1}{4}\left(\lambda_1\,\phi_1^4 + \lambda_2\,\phi_2^4 + \lambda_p\,\phi_1^2\,\phi_2^2\right)\,, \tag{2.53}$$

where we, for simplicity, assumed a $\mathbb{Z}_2$ symmetry between ϕ_1 and ϕ_2, excluding terms proportional to e.g. $\phi_1\,\phi_2^3$. To find the GW condition on the couplings, which determines the existence and orientation of a flat direction at tree level we use the stationary equations with respect to ϕ_1, ϕ_2. Combining both equations, we find that the choice of $\lambda_p \lesseqgtr 0$ allows for two distinct sets of solutions:

1. $\lambda_p > 0$: Since we require $\lambda_1, \lambda_2 > 0$ for the potential to be bound from below, with $\lambda_p > 0$, there is no opportunity for cancellations at tree level to achieve a flat direction, i.e. stationary and vanishing tree-level potential. Therefore, the only possible solution is for only one of the two fields to develop a non-zero VEV while this field's quartic coupling vanishes at Λ_{GW}[5]

$$\langle\phi_1\rangle = v_1 \neq 0, \qquad \langle\phi_2\rangle = v_2 = 0\,, \tag{2.54}$$

$$\lambda_1(\Lambda_{\text{GW}}) = 0\,. \tag{2.55}$$

2. $\lambda_p < 0$: We still require $\lambda_1, \lambda_2 > 0$, but given that $\lambda_p < 0$, there can be cancellations at tree-level, such that both the stationary equations as well as the tree-level potential vanish at the GW scale Λ_{GW} for both scalar fields acquiring non-zero VEVs $\langle\phi_1\rangle = v_1 \neq 0$, $\langle\phi_2\rangle = v_2 \neq 0$. The stationary equations then lead to the following GW conditions on the couplings and VEVs

$$\left(\frac{v_2}{v_1}\right)^2 = -\frac{2\lambda_1}{\lambda_p}, \qquad \left(\frac{v_1}{v_2}\right)^2 = -\frac{2\lambda_2}{\lambda_p}\,, \tag{2.56}$$

$$\lambda_p^2 = 4\,\lambda_1\,\lambda_2\,, \tag{2.57}$$

[5] Given the $\mathbb{Z}_2$ exchange symmetry between ϕ_1 and ϕ_2, the GW conditions simply also transform under $1 \leftrightarrow 2$. Therefore, which field acquires a non-zero VEV is a model or parameter choice.

with all the couplings being evaluated at the GW scale $\lambda_i = \lambda_i(\Lambda_{\text{GW}})$ and Eq. (2.57) arises from the combination of the relations in Eq. (2.56). After transforming into polar coordinates $\phi_1 = \phi_r \cos(\theta)$, $\phi_2 = \phi_r \sin(\theta)$, the orientation of the flat direction in two-dimensional field space θ_0 is given by Eq. (2.56) to be

$$\tan\theta_0 = \sqrt{-\frac{2\lambda_1}{\lambda_p}} \,. \tag{2.58}$$

This nicely displays that the orientation of the tree-level flat direction is solely determined by the dimensionless scalar couplings at Λ_{GW}, at the same time fixing the ratio between the VEVs, yet the dimensionful value of the radial VEV $\langle\phi_r\rangle = v_r$ is selected by the one-loop quantum corrections. Including them allows for the calculation of v_r, fully fixing the parameters of the theory depending only on the values of the dimensionless couplings at some scale of renormalization Λ_{ren} via the RG-running and GW conditions on the couplings.

2.4 The (Gauge) Hierarchy Problem

The Standard Model of particle physics is a theory with tremendous success in describing the fundamental interactions of elementary particles. With the discovery of the Higgs boson at the LHC in 2012 [9, 10], all the SM predicted particles have been experimentally confirmed and crucially, the Higgs mechanism has been verified to be responsible for the generation of all masses in the SM through coupling to its one fundamental scale, the electroweak VEV v_{EW}. Despite its success in explaining a huge amount of experimentally observed phenomena, there are various puzzles that the SM cannot explain, e.g. the existence of dark matter and neutrino masses. Another type of problem, that is of great significance, is the SM's unification with the theory of gravity, to ultimately describe phenomena at high energy scales where gravity dominates. To address some or even all of these problems, one often has to introduce another physical scale $\Lambda_{\text{NP}} \gg v_{\text{EW}}$ to incorporate new physics (NP) and extend the SM. This then introduces the SM Higgs' infamous gauge hierarchy problem, where the bare Higgs mass m^2 (c.f. Eq. (2.39)) experiences (one-loop) quantum corrections δm^2 that quadratically depend on some dimensionless coupling λ_i and the scale of new physics $\delta m^2 \propto \lambda_i^2 \Lambda_{\text{NP}}^2$, hence shifting the effective Higgs mass significantly away from the experimentally observed value.

Here we want to remark, that the hierarchy problem is oftentimes falsely interpreted when one does not include a physical high-energy scale but a large scale of renormalization or regularization. For example, regularizing a divergent integral via a high cutoff scale Λ_{UV}, at first glance also leads to large corrections to the Higgs mass. But since the cutoff scale Λ_{UV} is not physical, the large corrections are only a technical artifact from our method of calculation. This inevitably becomes clear, when employing renormalization group methods, as there are no large corrections to the Higgs mass once the proper RG improved mass is used [176]. Hence, the SM by itself does not have a hierarchy problem.

To demonstrate the nature of the hierarchy problem, when extending the SM, we want to utilize an effective field theory (EFT) approach. Imagine, for example, embedding the SM into a theory of quantum gravity, therefore containing the large energy scale $M_{\text{Pl}} \approx 10^{19}$ GeV, we use a toy model with a light scalar φ with mass m to resemble the Higgs boson and heavy scalar Φ with mass M corresponding, in this case, to a gravity DOF with the Planck mass as the UV scale. The Lagrangian of the full (toy-model) theory is then given by

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\, \partial^\mu\varphi - \frac{1}{2}m^2\varphi^2 - \frac{1}{4!}\lambda\varphi^4 + \frac{1}{2}\partial_\mu\Phi\, \partial^\mu\Phi - \frac{1}{2}M^2\Phi^2 - \frac{1}{4}\lambda_p\varphi^2\Phi^2 \,, \tag{2.59}$$

where other quartic terms have been omitted since the above ones are sufficient to demonstrate the nature of the hierarchy problem in the most basic way. Now, running this full theory down to lower energies $\mu \ll M_{\text{Pl}}$, we see that the large mass M will dominate the dynamics of Φ such that we assume it to be constant, leaving us with a low-energy theory with only the light scalar φ as propagating DOF

$$\mathcal{L}_{\text{EFT}} = \frac{1}{2}\partial_\mu\varphi\, \partial^\mu\varphi - \frac{1}{2}m^2_{\text{EFT}}\varphi^2 - \frac{1}{4!}\lambda_{\text{EFT}}\varphi^4 \,. \tag{2.60}$$

This EFT procedure is referred to as integrating out the heavy DOF. To properly incorporate the decoupling behavior of the scalars, one has to match the full theory with the EFT at the scale of integrating out $\bar{\mu}$, often chosen to be at or close to the scale of the large mass $\bar{\mu} \simeq M$. The matching condition for the low-energy effective mass parameter m_{EFT} at $\bar{\mu}$ is given by [176]

$$m^2_{\text{EFT}} = m^2 - \frac{\lambda_p}{32\pi^2}M^2\left(\ln\left[\frac{\bar{\mu}^2}{M^2}\right] + 1\right) \,, \tag{2.61}$$

where all the masses and couplings are understood to be evaluated at the scale $\bar{\mu}$. Here we can nicely revisit that the SM without an additional large physical mass M has no hierarchy problem, as for $\lambda_p = 0$, the large corrections to the (would be) Higgs mass simply vanish. Nevertheless, assuming $\lambda_p \neq 0$, the matching condition Eq. (2.61) shows the hierarchy problem to be a problem of fine-tuning, such that the bare Higgs mass m at $\bar{\mu}$ has to be adjusted greatly to match the large scale M order by order in perturbation theory to allow for extensive cancellations and thus $m^2_{\text{EFT}} \ll M^2$. This adjustment or fine-tuning is considered unnatural in the sense of 't Hooft naturalness [11], where a small parameter is thought to be technically natural if the theory exhibits additional symmetry in the limit said parameter is taken to zero. This is a distinguishing factor for scalar masses from fermion or gauge boson masses, the latter two are protected by chiral or gauge symmetry in their respective massless limits, leading to EFT matching corrections that show no problematic sensitivity to the large mass M. Thus, the fact that all SM particles acquire mass through coupling to the Higgs boson, a fundamental scalar, induces the hierarchy problem when (necessarily) embedded into a UV theory that contains large physical energy scales. These implications were first recognized in 1979 [177] and are still the foundation for attempts to reconcile the SM Higgs boson's hierarchy problem. To get around this fundamental problem, some attempts to solve the hierarchy problem utilize cosmological solutions to dynamically explain the Higgs fine-tuning (see e.g. [178–182]), while others introduce new symmetries or extra dimensions to protect the Higgs mass from large corrections (see e.g. [15–17, 29, 183],[13, 14])[6]. One of the approaches utilizing symmetry protection for the smallness of the Higgs mass is via scale invariance and its anomalous breaking. Under the assumption, that scale invariance is only broken by quantum corrections, i.e. anomaly, and that no intermediate scales are present, the scale generation will be dominated by the RG-running behavior of the dimensionless couplings (their β-funtions), therefore relating the IR to the UV scale only logarithmically [31]. Yet, these two assumptions are not necessarily generally valid, as discussed in Section 2.2, see Eq. (2.13)), curvature can induce a different contribution to the anomaly of scale invariance. Furthermore, the dimensionless couplings can have divergences in the assumed range of validity of the theory, depending on the theory's particle content. But, ensuring that these assumptions hold, in scale-invariant extensions to the SM, one can generate mass that is much smaller than for example the Planck mass and stable under radiative corrections (see e.g. [27, 32–34, 38]). Regardless of this success, the hierarchy problem is not solved, since generating multiple scales that are exponentially separated again introduces a fine-tuning problem in the dimensionless portal couplings of these new scales. To our best knowledge, this problem has not yet been addressed in a fully satisfying manner, yet, among others, scale invariance and therefore also conformal symmetry seem to be an interesting perspective for the continuous investigation of the gauge hierarchy problem.

Thus, in Chapter 3 we will display a classical scale-invariant implementation of the neutrino option to significantly reduce the amount of fine-tuning necessary for the Higgs mass to be small while also generating the Planck mass via SSB, whereas in Chapter 4, we demonstrate that the Higgs boson's mass, as a composite state of strongly coupled conformal UV dynamics, is protected against large (quadratic) corrections.

[6] Recently, the breakdown of the effective quantum field theory has been proposed to induce UV with IR mixing, which in turn is used to address the hierarchy problem [184].

Chapter 3

Unifying Emergence of Scales

Looking to embed the Standard Model into gravity, we present an exemplary model for the dynamical generating of both the Planck mass M_{Pl} and the electroweak VEV v_{EW} via spontaneous breaking of classical scale invariance by additional scalars. Thus, both the SM and gravity have a unified origin for their single respective fundamental mass scale. Given that, $M_{\text{Pl}}/v_{\text{EW}} \sim 10^{-16}$, the generation of scales separated this hierarchically needs further explanation in the light of the gauge hierarchy problem, as introduced in Section 2.4. Utilizing scale invariance and spontaneous breaking thereof is not a new concept in either generating the Planck mass (see e.g. [45, 65–86, 110, 128, 129, 132, 135, 136, 185–187]) or the electroweak VEV (see e.g. [25, 27–29, 32, 33, 35–39, 188–194]). This is well motivated by current experimental observations in particle physics and astrophysics hinting toward (classically) scale-invariant extensions to both the SM as well as Einstein's theory of gravity[1].

Regarding the SM, we consider the absence of Landau poles up to the Planck scale and the resulting perturbativity of the SM Higgs mass parameter m_h as hints for scale invariance being a natural extension to the SM. The fact that m_h only runs logarithmically hints towards a purely anomalous breaking of scale invariance [20, 21], excluding the soft breaking of m_h itself. Finally, the approximate flatness of the Higgs potential at high energies around M_{Pl} can be interpreted as another indication for scale invariance at large scales. This flatness of the scalar potential at high energies ties perfectly into observables of gravity. The measurements by the Planck satellite of the cosmic microwave background (CMB) [22, 23] reveal not only that a slowly rolling scalar field can drive cosmic inflation [196–198], but also that a (super) flat inflationary scalar potential is favorable for complying with the observation that the value for the spectral index n_s of the gravitational fluctuations is close to one, indicating a scale-invariant spectrum and that the tensor to scalar ratio r of the gravitational fluctuations' power-spectrum is nearly equal to zero. The link to the flatness of the potential is easily seen after a local Weyl rescaling from the Jordan frame to the Einstein frame where the tensor-to-scalar ratio r is proportional to the gradient of the potential [199, 200]. This observation makes R^2 inflation [200–204] and Higgs

[1] Here, we refer to Einstein-Hilbert action of gravity, which generates the pure Einstein equations via the stationary principle, see e.g. [195].

inflation [199] models promising candidates. One finds, that in both models the flatness of the scalar potential occurs due to a suppression of the not scale invariant Einstein-Hilbert term $\propto R$ with respect to the scale-invariant terms of R^2 inflation $\propto \gamma R^2$, with $\gamma \sim \mathcal{O}(10^9)$ and Higgs inflation $\propto \beta |H|^2 R$, with $\beta \sim \mathcal{O}(10^4)$. As a result, the Planck CMB measurement strengthens the case for scale-invariant extensions of Einstein gravity not only due to the observation of nearly scale invariance of the scalar perturbation's power spectrum, i.e. $n_s \sim 1$, but also due to the (strong) constraints on r to be small.

In addition, on behalf of scale-invariant extensions, one can argue that the higher derivative terms in the action of local scale symmetric gravity can lead to renormalizability [42, 85, 87, 187]. However, the presence of these higher derivative terms, and therefore also inevitably the Weyl tensor squared term, is well-known to cause violation of unitarity. This problem is directly related to the existence of a spin-2 ghost, therefore often called the ghost problem. An in-depth discussion is beyond the scope of this thesis, yet incredibly important for conformal realizations of gravity. Thus, for now, we refer to e.g. [51, 59, 205, 206] for discussion of the subject and potential ways of addressing it. While these solutions can be compatible with our approach, other potential solutions to the ghost problem depend on the existence of higher curvature terms [207–209], which explicitly break global scale invariance. For Sections 3.1 and 3.2 we assume such terms to be absent and instead follow the guiding principle of global scale invariance and its spontaneous breaking to generate the necessary scales. In Section 3.5 we will focus more on the role of graviton DOFs in spontaneous symmetry breaking and therefore briefly revisit the topic of the ghost problem.

To generate the scale that will be the origin for both M_{Pl} and v_{EW} we employ radiative spontaneous breaking of global scale symmetry via two additional scalars S and σ in the Gildener-Weinberg approximation. For the minimality of extending the SM, we chose two real scalars, since external fermionic DOFs were found to be destabilizing to the non-trivial vacuum structure [27]. As a result of the Gildener-Weinberg-like symmetry breaking (see Section 2.3.3), only the SM singlet S acquires a non-zero VEV $\langle S \rangle \sim \mathcal{O}(10^{16-17})$ GeV, that directly generates the Planck mass of the right size via non-minimal coupling to curvature $\mathcal{L}_{\text{GR}} \supset \beta_S S^2 R$ and thus $M_{\text{Pl}} \simeq \sqrt{\beta_S}\,\langle S \rangle$ (see Section 3.1). Demanding that our quantum effective scalar potential $\tilde{V}_{\text{eff}}(S)$ takes the role of an appropriate slow-roll inflation potential, gives constraints on $\beta_S \sim \mathcal{O}(10^{2-3})$ and therefore leads to the large value of $\langle S \rangle$. To simultaneously have $\langle S \rangle$ as the origin for $v_{\text{EW}} \sim 10^2$ GeV we employ the scale invariant extension of the neutrino option [210] to weaken this hierarchy.

The neutrino option [211] demonstrates a connection between v_{EW} and heavy right-handed neutrinos N_{R} with mass m_{N} via large but finite corrections that may generate $v_{\text{EW}}^2 \simeq \Delta v_{\text{EW}}^2 \propto y_\nu^2 m_{\text{N}}^2 / 4\pi^2$ [212–215]. Here, y_ν denotes the Dirac Yukawa coupling as seen in Section 3.2. We find the "right" combination of the parameters to be $m_{\rm N} \sim 10^7\,\text{GeV}$ and $y_\nu \sim 10^{-4}$ by demanding the right-handed neutrinos also generate proper light active neutrino masses via the usual type-I seesaw mechanism [216–219]. To incorporate these massive right-handed neutrinos in a scale-invariant way, we generate their mass via the Majorana-Yukawa interaction $\mathcal{L}_{\rm N\chi} \supset y_{\rm M}\, S\, N_{\rm R}^T\, C\, N_{\rm R}$, with C denoting the charge conjugation matrix. Thus, the mass of the right-handed neutrinos is given by $m_{\rm N} = y_{\rm M}\, \langle S \rangle$ and therefore lastly show that both the Planck scale $M_{\rm Pl}$ and $v_{\rm EW}$ originate from one scale, the scalar VEV of S. Given the size of $\langle S \rangle$ mentioned above, the Majorana-Yukawa coupling $y_{\rm M}$ has to be $y_{\rm M} \sim \mathcal{O}(10^{-(9-10)})$, which is a technically natural value, because the limit $y_{\rm M} \to 0$ restores an anomaly-free global $U(1)_{\rm B\text{-}L}$ symmetry (more details in Section 3.2). These are not the only problems of suppressed couplings; we do not know of a symmetry that could forbid the scalar portal interaction between the SM Higgs doublet H and the additional scalar S, i.e. $\mathcal{L}_{\rm SM} \supset \lambda_p\, S^2\, |H|^2$. In contrast to the above-discussed interactions, the scalar portal interaction would cause too large corrections to $v_{\rm EW}^2$ for the above-given size of $\langle S \rangle \sim \mathcal{O}(10^{16-17})\,\text{GeV}$. Even though a smallness of λ_p is not natural in the technical sense, we can at least set it up to remain small under quantum corrections. In the absence of $y_{\rm M}$ this is possible due to the multiplicative RG-running of λ_p, which causes the portal-induced radiative corrections to also be proportional to λ_p[2].

As for now, we have touched on how in Sections 3.1 and 3.2 we address some problems of the SM, i.e. hierarchy in the presence of $M_{\rm Pl}$ and small active neutrino masses. Turning towards gravity, we aim to realize an appropriate period of inflation in the early universe, so-called *cosmological inflation*, thus addressing the flatness, horizon, and magnetic monopole problem of the Hot Big Bang cosmology. In Section 3.3 we will further expand on the paradigm of inflation itself and show that we achieve slow-roll (or "new") inflation that is in good agreement with the strongest constraints from CMB measurements [23]. Weyl-transforming our effective scalar potential after symmetry breaking and the identification of $M_{\rm Pl}$ from Jordan to Einstein frame ($g^E_{\mu\nu} = \Omega^2\, g^J_{\mu\nu}$) reveals a second dynamical DOF next to the scalar S, the scalaron $\phi \propto M_{\rm Pl}\, \ln|\Omega^2|$. We find that the potential exhibits a (sufficiently) flat direction and thus can be approximated by an effective one-field model, instead of a more complicated multifield description of inflation. Choosing S to be the field driving inflation, as the so-called *inflaton*, adds a third role to its "responsibilities". The first two are the spontaneous breaking of global scale invariance by developing a non-zero VEV and simultaneously generating $M_{\rm Pl}$ and $v_{\rm EW}$ by coupling to gravity $\mathcal{L}_{\rm GR}$ and the neutrino sector $\mathcal{L}_{\rm N,\chi}$ respectively. This accumulation of roles in S serves as a nice showcase of the unified emergence of energy scales and cosmological inflation. Furthermore, we extensively discuss the validity of this approximation and

[2] There can be a case made for $\lambda_p \ll 1$ being technically natural, in the aforementioned sense, by an enhanced Poincaré symmetry in the limit $\lambda_p \to 0$ [220].

comment on some of the reheating properties that occur after inflation without assuming a specific reheating mechanism, following the reasoning in [12, 221]. Also, in Section 3.4 we introduce a viable candidate for dark matter, the Z_2-odd fermion χ, to account for the observed relic abundance of dark matter [22] via the well-known freeze-in mechanism [222]. This discrete Z_2 symmetry is imposed at the level of the total Lagrangian $\mathcal{L}_{\mathrm{T}}$ and serves two purposes, if unbroken, the stabilization of Z_2-odd fields (here only χ and σ) and the simplification of the scalar potential in $\mathcal{L}_{\mathrm{CW}}$.

Thus, for Sections 3.1 to 3.4 our total Lagrangian $\mathcal{L}_{\mathrm{T}}$ represents the most general function that incorporates the SM and respects its gauge symmetries, exhibits general diffeomorphism invariance[3], a discrete Z_2 symmetry and classical global conformal symmetry. It can be separated by function into four parts:

$$\mathcal{L}_{\mathrm{T}} = \mathcal{L}_{\mathrm{CW}} + \mathcal{L}_{\mathrm{GR}} + \mathcal{L}_{\mathrm{cSM}} + \mathcal{L}_{\mathrm{N},\chi} \ , \tag{3.1}$$

where $\mathcal{L}_{\mathrm{CW}}$ is responsible for the radiative spontaneous breaking of global conformal symmetry, thus generating the origin of all scales in this model, $\mathcal{L}_{\mathrm{GR}}$ is used to identify the dynamically generated M_{Pl} that gives rise to an effective Einstein-Hilbert term and together with $\mathcal{L}_{\mathrm{CW}}$ contains the DOFs that describe inflation. $\mathcal{L}_{\mathrm{SM}}$ contains the scale-invariant SM and the scalar couplings to the "external" scalars S, σ. Lastly, $\mathcal{L}_{\mathrm{N}\chi}$ is responsible for the generation of light active neutrino masses, the radiative generation of the Higgs mass term by corrections from couplings to the right-handed neutrino and contains the fermionic FIMP dark matter candidate χ.

After we have established all the aforementioned concepts and have shown that they indeed allow for a unified emergence of energy scales while simultaneously achieving proper inflation, in Section 3.5 we investigate the contributions from metric (or graviton) DOFs to the scalar potential that governs the spontaneous breaking of conformal invariance. We specifically expand on the remarks regarding the effect of the Weyl tensor squared term as well as on the consequent appearance of the ghost problem. We focus on the scalar and gravitational part of $\mathcal{L}_{\mathrm{T}}$, with the crucial difference that this time only one additional real scalar S is present. It will turn out that the contributions from the graviton will allow for RSSB with only this one additional scalar, such that no additional scalar fields are necessary. Furthermore, we will show that for this case, the predicted parameters of inflation (n_{S}, r) are also well within the current bounds from CMB measurements. The general total action[4] S_{T}^{1S} of global scale symmetric quadratic gravity non-minimally coupled to one real scalar $S(x)$ is then given by

$$S_{\mathrm{T}}^{1\mathrm{S}} = S_{\mathrm{QG}} + S_{\mathrm{S}} \ , \tag{3.2}$$

[3] Even though it is of no significance to our analysis, it is implicitly understood that the vierbein formalism is applied to terms containing minimal fermion-gravitational couplings.

[4] To distinguish the here considered total action from the one in Sections 3.1 to 3.4, we added the superscript "1S", referring to the appearance of only one additional scalar.

where S_{QG} contains the curvature contributions of the squares of the Ricci scalar R^2 and the Weyl tensor C^2, while S_{S} contains the scalar's kinetic term, its quartic self-interaction and the non-minimal coupling to curvature.

3.1 Generating the Planck Mass

Following the concept of scale invariance, to generate the Einstein-Hilbert term and therefore the Planck mass M_{Pl}, one has to add additional scalars, (S, σ) that couple non-minimally to the Ricci scalar R, to the general Lagrangian of (quadratic) scale-invariant gravity

$$\frac{\mathcal{L}_{\text{GR}}}{\sqrt{-g}} = -\frac{1}{2}\left(\beta_S\, S^2 + \beta_\sigma\, \sigma^2 + \beta_{\text{H}}\, H^\dagger H\right) R + \gamma\, R^2 + \kappa\, C_{\mu\nu\rho\sigma}\, C^{\mu\nu\rho\sigma}\,, \tag{3.3}$$

where $C_{\mu\nu\rho\sigma}$ denotes the Weyl tensor and we have dropped the Gauß-Bonnet surface term G (see Eq. (2.16)).

Here, we want to shortly expand on the earlier comment regarding the minimality of extending quadratic gravity with scalars to allow for RSSB of scale invariance à la Coleman-Weinberg [26]. As already argued in [26], a single scalar is not sufficient to allow for RSSB in the region of perturbativity of the theory's dimensionless couplings, thus two real scalars are the next most minimal system to realize RSSB via the Gildener-Weinberg approximation [153] (see Section 2.3.3). Optionally, RSSB could also be allowed for a system containing only one scalar which in turn needs to be charged under a $U(1)$ gauge group, therefore depicting the example in [26] (see Section 2.3.2). In addition to these two well-known options, later in Section 3.5 we will demonstrate that also the dynamical DOFs of the metric can induce contributions that allow for RSSB with only one additional scalar. As opposed to the first two options, RSSB via dynamical DOFs of the metric necessitates a careful treatment of the Weyl tensor squared C^2 terms and therefore also the problems of the Weyl anomaly (see Section 2.2, Eq. (2.15)) and a (possibly) massive spin-2 ghost. Consequently, for simplicity, we choose a system of two real scalars, leave the proper discussion of these problems for later, and first establish the foundation that proper inflation is possible with dynamically generated Planck mass M_{Pl} and v_{EW} from one unified origin.

The Lagrangian $\mathcal{L}_{\text{CW}}$ of these two scalars S, σ is then given by

$$\frac{\mathcal{L}_{\text{CW}}}{\sqrt{-g}} = \frac{1}{2}g^{\mu\nu}\,\partial_\mu S\,\partial_\nu S + \frac{1}{2}g^{\mu\nu}\,\partial_\mu\sigma\,\partial_\nu\sigma - V^{(0)}(S,\sigma)\,, \tag{3.4}$$

where $V^{(0)}(S,\sigma)$ is the two scalars tree-level potential. Since the non-minimal coupling to curvature is contained in $\mathcal{L}_{\text{GR}}$ and the quartic interactions with the SM Higgs doublet H are dealt with in $\mathcal{L}_{\text{N}\chi}$ later in Section 3.2, when v_{EW} is generated via the neutrino option, the most general Z_2 symmetric (S even, σ odd) and scale invariant tree-level potential $V^{(0)}(S,\sigma)$ can be written as

$$V^{(0)}(S,\sigma) = \frac{1}{4}\lambda_{\rm S}\, S^4 + \frac{1}{4}\lambda_\sigma\, \sigma^4 + \frac{1}{4}\lambda_{\rm S\sigma}\, S^2\, \sigma^2\,. \tag{3.5}$$

Up to this point, our setup is symmetric under the exchange of S and σ. Following the Gildener-Weinberg approach, we can choose an orientation for the direction of the tree-level flat direction of the potential (see Section 2.3.3) either along $S = 0$ or $\sigma = 0$. At the same time avoiding the domain wall problem [223] and stabilizing the fermionic dark matter candidate χ, we choose the flat direction along $\sigma = 0$ and $S \neq 0$, thus ensuring the Z_2 symmetry to remain preserved in the presence of the non-trivial VEV[5]. This choice of flat direction is realized when the other GW condition is fulfilled, here for the sake of generality we do not impose $\lambda_S = 0$ exactly but only relative to the other quartic scalar couplings leading to the GW condition for (approximate) flat direction along $S \neq 0, \sigma = 0$:

$$\lambda_{\rm S} \ll \lambda_{\rm S\sigma} \quad \text{and} \quad \lambda_{\rm S} \ll \lambda_\sigma\,. \tag{3.6}$$

On another note, for the utilization of the neutrino option, we already need to assume that the Higgs portal couplings to S and σ respectively, λ_p, $\lambda_{H\sigma}$ in $\mathcal{L}_{\rm cSM}$, and $y_{\rm M}$ in $\mathcal{L}_{{\rm N},\chi}$ are extremely small. Similarly, we assume an approximately vanishing dimensionless coupling $\beta_H \approx 0$, such that the non-minimal coupling interaction between the SM Higgs and the Ricci scalar term in Eq. (3.3) can be neglected. Or more precisely, we assume that $\beta_H R \ll \lambda_p S^2$ during the period of inflation. While none of these assumptions are technically natural (after 't Hooft), at least λ_p, $\lambda_{H\sigma}$, y_M, y_χ, and $y_{{\rm N}\chi}$ exhibit multiplicative RG-behavior, i.e. they are stable under higher order quantum corrections when initially set to zero at tree-level. Thus, their smallness can be perceived as natural in "some" sense despite no restoration or enhancement of symmetry being associated[6].

Ultimately, our assumptions above lead to the SM Higgs playing no part in inflation, as well as in the RSSB of scale invariance. With this, we get exactly the situation described already before, namely that the external fields determine the symmetry breaking while S later on will act as the inflaton. Thus, we can now use the concepts introduced earlier in Sections 2.3.2 and 2.3.3 to calculate the quantum effective potential in $\overline{\rm MS}$ along the flat direction $S \neq 0, \sigma = 0$. In other words, we integrate out the one-loop quantum fluctuations δS and $\delta\sigma$ in the background $S \neq 0$ and $\sigma = 0$. Following Eqs. (2.48) to (2.50), the one-loop $\overline{\rm MS}$ contribution to the effective potential is calculated as

$$V^{(1)}_{\rm eff}(S,R) = \frac{1}{64\pi^2}\left(m_{\rm S}^4 \ln\left[\frac{m_{\rm S}^2}{\mu^2}\right] + m_\sigma^4 \ln\left[\frac{m_\sigma^2}{\mu^2}\right]\right)\,, \tag{3.7}$$

[5] Again, this choice depends only on the previous choice that S is even and σ is odd under the Z_2.
[6] For a discussion of the role of Poincaré symmetry in this, see e.g. [220]

where we have absorbed the constants $c_{S,\sigma} = -3/2$ into the renormalization scale μ and the field dependent masses are given by

$$m_S^2 = 3\lambda_S S^2 + \beta_S R\,, \tag{3.8}$$

$$m_\sigma^2 = \frac{1}{2}\lambda_{S\sigma} S^2 + \beta_\sigma R\,. \tag{3.9}$$

As usual, calculating the one-loop effective potential also gives rise to divergences that are dealt with in the typical way, i.e. they are absorbed in a redefinition of the dimensionless couplings λ_S, γ, and β_S. For the divergences caused by loop diagrams of scalars this method is already demonstrated in the original Coleman-Weinberg paper [26], for divergences due to additional contributions proportional to curvature this method is shown in e.g. [224, 225] and references cited therein. These authors perform a similar calculation and come to results that are in agreement with our computation above. Furthermore, in Eqs. (3.8) and (3.9) the non-minimal couplings should read $(\beta_S - 1/6)$ and $(\beta_\sigma - 1/6)$, where the additional constant contribution is due to the non-flatness of the spacetime [143]. However, for simplicity, we choose to omit these constant contributions of $1/6$, since for proper inflation $\beta_S \gtrsim 10^2$ and we find that physical observables, i.e. parameters of inflation, barely depend on the value of β_σ[7]. The effective potential along the flat direction $V_{\text{eff}}(S, R)$ is then given by

$$\begin{aligned} V_{\text{eff}}(S, R) &= V^{(0)}(S, \sigma = 0) + V_{\text{eff}}^{(1)}(S, R) \\ &= \frac{1}{4}\,\lambda_S\, S^4 + V_{\text{eff}}^{(1)}(S, R)\,, \end{aligned} \tag{3.10}$$

with $V_{\text{eff}}^{(1)}(S, R)$ as shown in Eq. (3.7) with m_S^2 and m_σ^2 respectively from Eqs. (3.8) and (3.9). We find, that σ and λ_σ will not play a further role in symmetry breaking and inflation, as there is no explicit dependence in V_{eff}, and while there is also no tree-level contribution proportional to the couplings $\lambda_{S\sigma}$ and β_σ, these appear in the one-loop contributions via m_σ^2 of Eq. (3.9).

We turn to the calculation of $\langle S \rangle$ to in turn identify the dynamically generated value of the Planck mass M_{Pl} via comparison with the usual Einstein-Hilbert term. Therefore, we separate R from the S dependence in $V_{\text{eff}}(S, R)$. Since we can assume a tiny but non-zero value for the curvature R during inflation, or more specifically $\beta_S R \ll 3\lambda_S S^2$ and $\beta_\sigma R \ll (1/2)\lambda_{S\sigma} S^2$, we can expand Eq. (3.10) in powers of R

$$V_{\text{eff}}(S, R) = V_{\text{CW}}(S) + V_{(1)}(S)\; R + V_{(2)}(S)\; R^2 + O(R^3)\,, \tag{3.11}$$

[7] See Appendix A.1 for a brief numerical evaluation of the dependence of the theories parameters on β_σ.

with

$$V_{\text{CW}}(S) = \frac{1}{4}\,\lambda_{\text{S}}\,S^4 + \frac{S^4}{64\,\pi^2}\left(9\,\lambda_{\text{S}}^2\,\ln\left[\frac{3\,\lambda_{\text{S}}\,S^2}{\mu^2}\right] + \frac{1}{4}\,\lambda_{\text{S}\sigma}^2\,\ln\left[\frac{\lambda_{\text{S}\sigma}\,S^2}{2\,\mu^2}\right]\right)\,, \tag{3.12}$$

$$V_{(1)}(S) = \frac{S^2}{128\,\pi^2}\left(6\,\beta_{\text{S}}\,\lambda_{\text{S}}\left[1+2\ln\left[\frac{3\,\lambda_{\text{S}}\,S^2}{\mu^2}\right]\right] + \beta_\sigma\,\lambda_{\text{S}\sigma}\left[1+2\,\ln\left[\frac{\lambda_{\text{S}\sigma}\,S^2}{2\,\mu^2}\right]\right]\right)\,, \tag{3.13}$$

$$V_{(2)}(S) = \frac{1}{128\,\pi^2}\left(\beta_{\text{S}}^2\left[3+2\,\ln\left[\frac{3\,\lambda_{\text{S}}\,S^2}{\mu^2}\right]\right] + \beta_\sigma^2\left[3+2\,\ln\left[\frac{\lambda_{\text{S}\sigma}\,S^2}{2\,\mu^2}\right]\right]\right)\,. \tag{3.14}$$

Given the smallness of R that we assume during inflation, we obtain a R-independent leading-order results for $\langle S\rangle$ from the zero-order (in R) term $V_{\text{CW}}(S)$ of the full effective potential $V_{\text{eff}}(S,R)$ through analysis of the stationary condition and verification that $\langle S\rangle$ indeed is a minimum of $V_{\text{CW}}(S)$

$$\left.\frac{\partial V_{\text{CW}}(S)}{\partial S}\right|_{S=\langle S\rangle} = 0, \qquad \left.\frac{\partial^2 V_{\text{CW}}(S)}{\partial S^2}\right|_{S=\langle S\rangle} > 0\,, \tag{3.15}$$

We find the analytic expression for $\langle S\rangle$, that $V_{\text{CW}}(\langle S\rangle)$ is non-zero and negative and thus denotes a finite zero-point energy density V_0

$$V_0 := V_{\text{CW}}(\langle S\rangle) = -\frac{\beta_{\lambda_{\text{S}}}}{16}\,\langle S\rangle^4 = -\frac{\beta_{\lambda_{\text{S}}}}{16}\,\mu^4\,\exp\left[-1-\frac{16\,C}{\beta_{\lambda_{\text{S}}}}\right]\,, \tag{3.16}$$

where $\beta_{\lambda_{\text{S}}}$ is the one-loop β-function of λ_{S} in the absence of the Yukawa couplings $y_{\text{M}},\ y_\chi$, and C depends only on the dimensionless scalar couplings $\lambda_{\text{S}},\ \lambda_{\text{S}\sigma}$

$$\beta_{\lambda_{\text{S}}} = \frac{1}{16\,\pi^2}\left(18\,\lambda_{\text{S}}^2 + \frac{1}{2}\lambda_{\text{S}\sigma}^2\right)\,, \tag{3.17}$$

$$C = \frac{1}{4}\lambda_{\text{S}} + \frac{1}{64\,\pi^2}\left(9\,\lambda_{\text{S}}^2\ln\left[3\,\lambda_{\text{S}}\right] + \frac{1}{4}\lambda_{\text{S}\sigma}^2\ln\left[\frac{1}{2}\lambda_{\text{S}\sigma}\right]\right)\,. \tag{3.18}$$

This negative zero-point energy V_0 can be identified as the cosmological constant and is a consequence of the spontaneous breaking of global conformal symmetry and therefore is finite in dimensional regularization[8]. Since this zero-point energy does not agree with the cosmological constant's experimentally observed value, we choose to avoid the cosmological constant problem in this model by subtracting this zero-point energy density, ensuring that $\tilde{V}_{\text{CW}}(\langle S\rangle) = 0$

$$\tilde{V}_{\text{CW}}(S) = V_{\text{CW}} - V_0\,. \tag{3.19}$$

This "solution" comes at the cost of super-soft breaking of scale invariance already at tree-level but since for properly addressing the cosmological constant problem one should take into account gravitational quantum fluctuations, including ones

[8] Note that since there exists no absolute scale, only differences can be measured and thus the zero-point energy cannot be uniquely determined within the framework of quantum field theory in flat spacetime.

with origin in the Weyl tensor squared term C^2 in the tree-level action and therefore (possibly) introducing unitarity violation, we are content with evading this discussion for this thesis and continue with the identification of the dynamically induced Planck mass $M_{\rm Pl}$. As the identification of $M_{\rm Pl}$ follows from comparison with the term of the Einstein-Hilbert action, i.e. $-\frac{1}{2}M_{\rm Pl}^2 R$, we collect the terms proportional to the first power of R in Eq. (3.3) and Eq. (3.11) when evaluated at $S = \langle S \rangle$, compare the pre-factors and find $M_{\rm Pl}$ to be given by

$$M_{\rm Pl} = \langle S \rangle \sqrt{\beta_{\rm S} + \frac{2\, V_{(1)}\left(\langle S \rangle\right)}{\langle S \rangle^2}} \, . \tag{3.20}$$

Taking into account the form of $\langle S \rangle$ as derived from Eq. (3.16)), i.e. $\langle S \rangle = \mu\, f_1\left(\lambda_{\rm S},\, \lambda_{\rm S\sigma}\right)$, the above equation Eq. (3.20) shows that the dynamically generated Planck mass is directly proportional to the renormalization scale μ in the form of

$$M_{\rm Pl} = \mu\, f_2\left(\beta_{\rm S}, \beta\sigma, \lambda_{\rm S},\, \lambda_{\rm S\sigma}\right) \, , \tag{3.21}$$

where f_1, f_2 are dimensionless functions of the theory's dimensionless couplings.

3.2 Standard Model, Neutrinos, and the EW Scale

For now, we have covered the spontaneous symmetry breaking of global conformal symmetry via the two additional scalars and thus the generation of $M_{\rm Pl}$ independent from the SM. In the following, we will discuss the couplings to the SM and the mechanism to generate the electroweak scale via the neutrino option. Since the external scalars in $\mathcal{L}_{\rm CW}$ (see Eq. (3.4)) are singlets under the SM gauge symmetry, there are two scale-invariant coupling terms between the gauge invariant combination $H^\dagger H$ and S, σ. Also, there is a scale-invariant non-minimal coupling of $H^\dagger H$ to R, as shown in $\mathcal{L}_{\rm GR}$ (see Eq. (3.3)), which has great importance in models that utilize inflation driven by the SM Higgs field itself. Here, in our model, we already argued that the contributions of the Higgs to inflation are negligible, due to S being the dominant field during inflation, as well as its heavy VEV $\langle S \rangle \sim \mathcal{O}(10^{16-17})$GeV. Therefore, incorporating the SM into our model is achieved by $\mathcal{L}_{\rm BSM}$ in the total Lagrangian $\mathcal{L}_{\rm T}$ in Eq. (3.1)

$$\mathcal{L}_{\rm BSM} = \mathcal{L}_{\rm cSM} + \mathcal{L}_{\rm N,\chi} \, . \tag{3.22}$$

We separate this part of our model into the one containing the scale-invariant or conformal symmetric SM, $\mathcal{L}_{\rm cSM}$, plus the allowed (quartic) couplings to the additional scalars and into a second one, $\mathcal{L}_{\rm N,\chi}$, which includes the new Majorana fermions $N_{\rm R}$, χ and their couplings to the SM and the external scalar S. First, we discuss the scalars couplings to the SM in $\mathcal{L}_{\text{cSM}}$

$$\frac{\mathcal{L}_{\text{cSM}}}{\sqrt{-g}} = \mathcal{L}_{\text{SM}}|_{m_{\text{H}}=0} - \frac{1}{4}\left(\lambda_p\, S^2 + \lambda_{\text{H}\sigma}\,\sigma^2\right) H^\dagger H\,, \tag{3.23}$$

where $\mathcal{L}_{\text{SM}}|_{m_{\text{H}}=0}$ denotes the conformal or scale-invariant SM Lagrangian, namely the SM with the Higgs's tree-level mass term set to zero (see Eq. (2.39)). The portal couplings to S would induce an effective mass for the SM Higgs proportional to $\langle S\rangle^2 \gg v_{\text{EW}}^2$. As a consequence, $\lambda_p \ll 1$ has to be extremely small so as not to ruin the compatibility with the experimental observed value of v_{EW}. For λ_p to stay small under RG-running, also $\lambda_{\text{H}\sigma}$ is required to be almost vanishing so that it does not induce sizable contributions via quantum loop corrections. Using the portal coupling λ_p, fine-tuned to a specific value to explain the smallness of v_{EW} in relation to M_{Pl}, precisely constitutes the gauge hierarchy problem as discussed in Section 2.4. While not addressing said hierarchy problem in a fully satisfying manner, we use the neutrino option to soften this huge hierarchy.

The neutrino option [211] was proposed as a way of simultaneously generating masses for light active SM neutrinos via a type-I seesaw mechanism [216–219], as well as the Higgs mass via radiative corrections by heavy right-handed Majorana neutrinos N_{R} [212–215]. Including N_{R} in a scale-invariant way (as has been done in e.g. [210, 226, 227]), is achieved by the $\mathcal{L}_{\text{N},\chi}$ contribution in the total Lagrangian $\mathcal{L}_{\text{T}}$ of Eq. (3.1)

$$\begin{aligned}\frac{\mathcal{L}_{\text{N},\chi}}{\sqrt{-g}} = \frac{i}{2}\bar{N}\partial\!\!\!/ N - \frac{1}{2}y_M S N^T C N + \frac{i}{2}\bar{\chi}\partial\!\!\!/\chi - \frac{1}{2}y_\chi S \chi^T C\chi \\ - \left(y_{N\chi}\sigma N^T C\chi \,+\, y_\nu \bar{L}\tilde{H}\,\tfrac{1}{2}(1+\gamma_5)N + \text{h.c.}\right)\,,\end{aligned} \tag{3.24}$$

where, N_{R}, χ are the (three+three) right-handed Majorana neutrinos and C the charge conjugation matrix, while H ($\tilde{H} = i\sigma_2 H^*$) and L denote the SM Higgs and lepton doublets. Furthermore, flavor indices are suppressed throughout this calculation and discussion as we do not address details of the flavor structure. Therefore, we will treat the matrices y_{M}, y_χ, $y_{\text{N}\chi}$ and y_ν as representative real numbers. The large Majorana mass m_{N} is then generated dynamically via the coupling to $\langle S\rangle$ through the second term in Eq. (3.24) to be

$$m_{\text{N}} = y_{\text{M}}\,\langle S\rangle = \frac{y_{\text{M}}\, M_{\text{Pl}}}{\sqrt{\beta_{\text{S}} + \frac{2\,V_{(1)}(\langle S\rangle)}{\langle S\rangle^2}}}\,. \tag{3.25}$$

Considering the parameter space for successful inflation (see Section 3.3), namely $\beta_{\text{S}} \gtrsim 10^2$, we find that $\beta_S \gg 2\,V_{(1)}\,(\langle S\rangle)\,/\,\langle S\rangle^2$ is satisfied, such that we can calculate the leading order estimate for y_{M}

$$y_{\text{M}} \simeq \frac{m_{\text{N}}\,\sqrt{\beta_{\text{S}}}}{M_{\text{Pl}}} \simeq 10^{-10}\times\left(\frac{\beta_S}{10^3}\right)^{1/2}. \tag{3.26}$$

Here we want to shortly note again, that the smallness of $y_{\rm M}$ as shown above is technically natural. Following the arguments of 't Hooft [11], a small $y_{\rm M}$ is considered natural since in the limit $y_{\rm M}, y_{\rm N\chi} \to 0$ the $U(1)_{B-L}$ symmetry is restored. The necessary smallness of $y_{\rm N\chi}$ is in agreement with the requirements for the production of the correct abundance of dark matter as described in Section 3.4. Now, with the large right-handed neutrino mass $m_{\rm N}$, the total neutrino mass matrix M is given by a 2×2 matrix

$$M = \begin{pmatrix} 0 & m \\ m & m_{\rm N} \end{pmatrix} , \tag{3.27}$$

with $m = y_\nu v_{\rm H}$ denoting the off-diagonal mass term of the active neutrino that is generated by the last term in Eq. (3.24) after the spontaneous breaking of the EW symmetry. Since, $v_{\rm EW} = 246\,\text{GeV}$, for (technically) natural values of y_ν, one finds that $m \ll m_{\rm N}$, thus the eigenvalues and determinant of M are then calculated to be

$$M_\pm = \frac{1}{2}\left(m_{\rm N} \pm \sqrt{m_{\rm N}^2 + 4m^2}\right) , \qquad M_+ \approx m_{\rm N} , \qquad M_- \approx -\frac{m^2}{m_{\rm N}} , \tag{3.28}$$

$$\det M = M_+\, M_- = -m^2 . \tag{3.29}$$

The fixed value for the determinant implies that the relation Eq. (3.29) constitutes the eponymous seesaw, an increase in M_+ leads to a decrease in M_- and vice versa. In our case, this results in the generation of the correct active neutrino masses m_ν via the type-I seesaw and $y_\nu \approx 10^{-4}$ as

$$m_\nu \simeq \frac{y_\nu^2\, v_{\rm EW}^2}{m_{\rm N}} \sim 0.1\,\text{eV} . \tag{3.30}$$

At the same time, due to the Feynman diagram shown in Fig. 3.1 (left), heavy right-handed Majorana neutrinos induce a sizable correction to the after EW symmetry breaking effective Higgs mass term $-\mu_{\rm H}^2 H^\dagger H$. The finite part of this contribution is generally renormalization scale μ dependent (here via $m_{\rm N} \sim \langle S \rangle \sim \mu$) and is

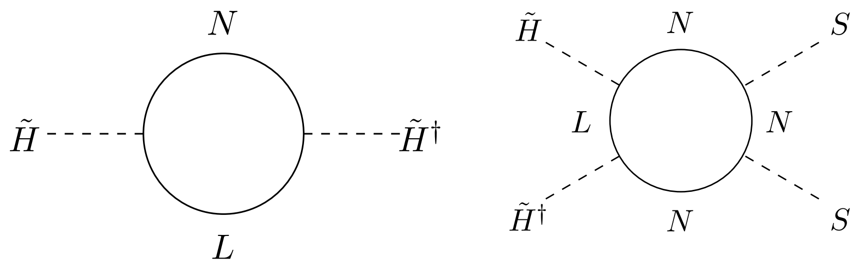


FIGURE 3.1: Neutrino contributions to the Higgs mass term (left) and Higgs portal coupling (right).

calculated to be [212–215]

$$|\Delta\mu_{\mathrm{H}}^2| \sim \frac{y_\nu^2 m_{\mathrm{N}}^2}{4\pi^2} \,. \tag{3.31}$$

Under the main assumption of the neutrino option [211], namely that the radiative correction in Eq. (3.31) constitutes the dominant contribution to the Higgs mass term, we can write $\mu_{\mathrm{H}}^2 \sim \Delta\mu_{\mathrm{H}}^2 \simeq 2(125\,\mathrm{GeV})^2$ and thus find that with $m_{\mathrm{N}} \approx 10^7\,\mathrm{GeV}$, neutrino option successfully generates light active neutrino masses as well as the correct Higgs mass term. While the aforementioned basic assumption of the neutrino option is perfectly in agreement with its scale-invariant implementation, which forces $\Delta\mu_{\mathrm{H}}^2 = 0$ at tree-level, and our previous arguments regarding the smallness of radiative scalar corrections to the Higgs mass parameter, namely λ_p, $\lambda_{\mathrm{H}\sigma} \ll 1$, one has to also consider the radiative fermionic corrections to these scalar couplings via the second diagram in Fig. 3.1

$$\Delta\lambda_p \sim \frac{y_\nu^2\, y_{\mathrm{M}}^2}{16\,\pi^2} \,, \tag{3.32}$$

which is then assumed to provide the dominant contribution to λ_p, for the neutrino option to function properly. Ultimately, we found that the scale-invariant realization of the neutrino option works very well in the here considered model and thus allows for a consecutive identification of the Higgs mass term μ_{H} with larger energy scales, and finally with the Planck scale M_{Pl}, all with the same radiative origin, the non-zero VEV of S.

Lastly, we want to add that the generation of the universe's baryon asymmetry via leptogenesis [228, 229] is compatible with the framework of the neutrino option [230, 231]. Later, in Sections 3.3 and 3.4 we will show that for most of our parameter space and especially the three benchmark points (see Table 3.1), thermal leptogenesis can work successfully under reasonable assumptions.

3.3 Inflation

Inflation or cosmological inflation is a theory that describes an era in the (very) early universe where spacetime expands exponentially. While originally introduced by Guth [232] to resolve the problem of not observed but predicted magnetic monopoles (in grand unified theories), theoretical physicists quickly noticed that it could also address other fundamental issues of standard (hot) Big Bang cosmology. Most notably, inflation is used to explain why the universe is as flat as we observe today (flatness problem), why we observe it as almost perfectly isotropic on large distances (horizon problem), and additionally provides the seed for large-scale structure formation in the universe. Inflation resolves these problems by introducing an early era dominated by dark energy or a cosmological constant with an equation of state $\omega = -1$, thus resulting in the exponential expansion of spacetime, diluting the number density of magnetic monopoles sufficiently to avoid detection, expanding conformal time far enough back to allow for early causal contact that would explain the later observed isotropies and suppressing the curvature energy density exponentially. Unavoidably, there is a huge energy density contained in this early phase of the universe, which has to be converted back to SM DOFs, the so-called *reheating*. This is where the shortcomings of Guth's early model of inflation became apparent. It would not generate the necessary amount of reheating and reintroduce inhomogeneities. This problem was solved when Linde, Albrecht, and Steinhardt introduced the model of *slow-roll* inflation [196–198]. They found that at least one bosonic DOF, namely the inflation field, must be present and thus led to the emergence of a great variety of models with different particle content and origin to realize successful inflation. While there exists already the scalaron as a bosonic DOF in the R^2 to allow for proper slow-roll inflation, the so-called R^2 inflation [201–203], this scalar DOF can not generate the spontaneous breakdown of scale-invariance. Similarly, the Higgs field might be considered a suitable candidate for the inflaton [199], yet when introducing scale-invariance there are again problems with the symmetry breaking. Thus, in models where scales originate from the spontaneous breaking of scale-invariance, one needs at least one additional scalar DOF to incorporate successful slow-roll inflation. But before we get further into the specifics of how our model of Eq. (3.1) can achieve proper inflation, we deem it necessary to introduce the basic notions of slow-roll inflation.

Since inflation denotes an essential component of our current understanding of cosmology, there are multiple excellent reviews, lectures, and presentations introducing the basics of slow-roll inflation. Hereafter, we decided to follow [233] and dive right into our review with the basic Lagrangian of a scalar field (minimally) coupled to gravity

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}\, M_{\text{Pl}}^2\, R + \frac{1}{2}\, g^{\mu\nu}\, \partial_\mu \phi\, \partial_\nu \phi - V(\phi) = \mathcal{L}_{\text{EH}} + \mathcal{L}_\phi \,, \tag{3.33}$$

where ϕ denotes the inflaton, with its potential $V(\phi)$ coupled to the Einstein-Hilbert term of gravity. Furthermore, the metric $g^{\mu\nu}$ is assumed to be the flat Friedmann-Lemaître-Robertson-Walker (FLRW) metric [234–238] so that the line element $\mathrm{d}s^2$ becomes

$$\mathrm{d}s^2 = -\,\mathrm{d}t^2 + a^2(t)\Sigma(r,\vartheta,\theta) \,, \tag{3.34}$$

where $a(t)$ denotes the scale factor that describes the relative size of the spatial slices $\Sigma(r,\vartheta,\theta)$ as time evolves. The evolution of $a(t)$ and therefore also the dynamics of the universe are given by the Einstein equations, which under the assumption of isotropy and homogeneity ($\phi(\mathbf{x},t) = \phi(t)$) lead to the Friedmann equations

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3\, M_{\text{Pl}}^2}\, \rho - \frac{k}{a^2}\,, \qquad \frac{\ddot{a}}{a} = -\frac{1}{6\, M_{\text{Pl}}^2}\, (\rho + 3\, p) \,, \tag{3.35}$$

with H being the Hubble parameter as defined above, a dot denotes a time derivative and k denotes the curvature parameter, that takes the values $k = \{-1,\ 0,\ +1\}$, respectively, for a negatively, flat or positively curved spacelike hypersurface Σ. Furthermore, ρ denotes the matter energy density and p the isotropic pressure as given by the theory's energy-momentum tensor $T_{\mu\nu}$. In the case of Eq. (3.33), $T_{\mu\nu}$ takes the form of a perfect fluid so that

$$\rho_\phi = \frac{1}{2}\dot{\phi} + V(\phi)\,, \qquad p_\phi = \frac{1}{2}\dot{\phi} - V(\phi)\,, \qquad \omega_\phi = \frac{p_\phi}{\rho_\phi} = \frac{\frac{1}{2}\dot{\phi} - V(\phi)}{\frac{1}{2}\dot{\phi} + V(\phi)}\,, \tag{3.36}$$

where ω_ϕ is the inflaton's equation of state parameter. The EOM for ϕ and the Friedmann equations then lead to the following two dynamical equations describing a single scalar particle rolling down a potential with a friction term of $3H\dot{\phi}$

$$\ddot{\phi} + 3H\dot{\phi} + V'(S) = 0\,, \qquad H^2 = \frac{1}{3\,M_{\rm Pl}^2}\left(\frac{1}{2}\dot{\phi} + V(\phi)\right)\,, \tag{3.37}$$

where V' describes a derivative of $V(\phi)$ with respect to ϕ. Inflation addresses the problems of standard cosmology via an era of exponential expansion where a flat scalar potential acts as a cosmology constant $\omega \simeq -1$ and dominates the dynamics of $a(t)$ with subsequent reheating into SM DOFs and thus matter and radiation dominated eras during which the universe exhibits power-law dependent expansion behavior. This flatness of the potential $V(\phi)$ is realized through $\dot{\phi}^2 \ll V(\phi)$. Under this assumption, Eq. (3.36) clearly shows that indeed $\omega_\phi \simeq -1$ and therefore $a(t) \sim e^{Ht}$, with a constant background evolution $H^2 \approx V(\phi)/(3M_{\rm Pl}^2)$. Additionally, imposing that the scalar field only rolls slowly down the potential $\ddot{\phi} \ll 3H\dot{\phi}$ ensures that inflation lasts long enough to solve the cosmological problems. In Fig. 3.2 we show two depictions of appropriate potentials for single-field slow-roll inflation. These slow-roll conditions can be rewritten in terms of the slow-roll parameters $\epsilon,\ |\eta| \ll 1$ with

$$\epsilon(\phi) \equiv \frac{1}{2}M_{\rm Pl}^2\left(\frac{V'}{V}\right)^2\,, \qquad \eta(\phi) \equiv M_{\rm Pl}^2\left(\frac{V''}{V}\right)\,. \tag{3.38}$$

The end of inflation is then quantified by ϵ and $|\eta|$ reaching order unity, such that the duration is expressed by the number of e-folds N_e

$$N_e(\phi) = \ln\left[\frac{a_{\rm end}}{a_{\rm CMB}}\right] = \int_{t_{\rm CMB}}^{t_{\rm end}} H\ {\rm d}t \approx \frac{1}{M_{\rm Pl}^2}\int_{\phi_{\rm end}}^{\phi_{\rm CMB}} \frac{V}{V'}\ {\rm d}\phi\,, \tag{3.39}$$

with the subscripts "CMB" and "end" denoting the quantities at the time of CMB horizon exit and the end of inflation, respectively. It is clear from the above equations for ϵ, η Eq. (3.38) and N_e Eq. (3.39) that the successful realization of inflation depends heavily on the shape and scale of the scalar potential V. Nevertheless, there are many models, also ones containing multiple scalar fields, that generate an appropriate potential. To constrain these models, one utilizes that quantum fluctuations

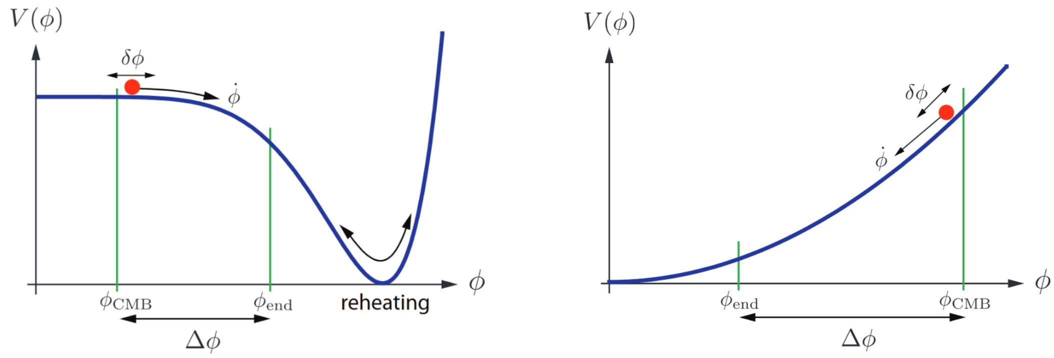


FIGURE 3.2: Examples of a single-field inflation potential where the inflaton ϕ slowly rolls down the flat potential and oscillates around its true minimum, transferring the energy density to SM DOFs. Inflation ends once the kinetic energy has become comparable to the potential energy and starts some time before the CMB horizon decoupling, thus leaving an imprint in the fluctuations observed from the CMB. The distance $\Delta\phi$ travelled between ϕ_{CMB} and ϕ_{end} has a direct influence on the amplitude of inevitably created gravitational waves.

during inflation are blown up by the exponential expansion to macroscopic scale and "frozen-in" after inflation, thus leaving an imprint in later structure formation. This imprint is best observed in the anisotropies of the CMB, which was recently measured by the Planck and BICEP/Keck collaboration [22, 24], and currently sets the strongest limits on inflation [23, 24]. These CMB observables are related to the power spectrum of gravitational scalar and tensor perturbations that originate from the period of inflation. The (for us) relevant observables are the Amplitude A_s and spectral index n_s of the scalar power spectrum, as well as the tensor-to-scalar ratio r. They can be related to the inflationary potential V at the time of CMB horizon exit by

$$A_s = \frac{V_{\text{CMB}}}{24\,\pi^2\,M_{\text{Pl}}^4\,\epsilon_{\text{CMB}}}\,, \qquad n_s = 1 + 2\,\eta_{\text{CMB}} - 6\,\epsilon_{\text{CMB}}\,, \qquad r = 16\,\epsilon_{\text{CMB}}\,, \tag{3.40}$$

where ϵ and η denote the slow-roll parameters as defined in Eq. (3.38). For detailed derivations of the relations in Eq. (3.40) and a more in-depth discussion of inflation, we refer to more elaborate reviews, e.g. [233, 239] and rather continue the investigation of inflation in the context of a unified origin for emerging scales.

3.3.1 Radiatively Induced Effective Action for Inflation

In Sections 3.1 and 3.2 we discussed the emergence of scales with a single origin in the non-zero VEV $\langle S\rangle$, which spontaneously breaks scale invariance through quantum corrections. We have also hinted towards S being the dominant contribution to drive a period of exponential expansion of spacetime, namely inflation. In the following, we want to explicitly investigate these statements and show that indeed inflation that is in agreement with the most current experimental observations is possible in this framework. The assumptions of the previous sections regarding the smallness of the couplings λ_p, $\lambda_{H\sigma}$, $\beta_{\rm H} \ll 1$ result in the Higgs doublet H not playing a role for inflation. On the other hand, the assumptions for a reasonable expansion of $V_{\rm eff}(S,R)$ in R, i.e. $\beta_{\rm S} R < 3\lambda_{\rm S} S^2$ and $\beta_\sigma R < (1/2)\lambda_{\rm S\sigma} S^2$, are confirmed straightforwardly by approximating the scalar curvature during inflation as $R = 12 H_{\rm inf}$, where $H_{\rm inf}$ denotes the Hubble parameter and therefore directly the universe's expansion rate. Accordingly, we collect the terms relevant for inflation of the total Lagrangian Eq. (3.1) in the effective (Jordan frame) inflation Lagrangian $\mathcal{L}^{\rm J}_{\rm eff}$

$$\frac{\mathcal{L}^{\rm J}_{\rm eff}}{\sqrt{-g_{\rm J}}} = -\frac{1}{2}\, M_{\rm Pl}^2 B(S)\; R_{\rm J} + G(S)\; R_{\rm J}^2 + \frac{1}{2}\, g_{\rm J}^{\mu\nu}\; \partial_\mu S\, \partial_\nu S - \tilde{V}_{\rm CW}(S)\;, \tag{3.41}$$

where $M_{\rm Pl}$ takes the dynamical value as calculated in Eq. (3.20), $\tilde{V}_{\rm CW}(S)$ is the "normalized to zero at the minimum" potential of Eq. (3.19) and "J" denotes that all quantities are understood to be dependent on the original metric of Jordan-frame spacetime. Referring to the original factors in front of R and R^2 terms in $\mathcal{L}_{\rm GR}$ (see Eq. (3.3)), i.e. β_i and γ, we have defined the functions $B(S)$ and $G(S)$

$$B(S) = \frac{1}{M_{\rm Pl}^2}\left(\beta_{\rm S}\, S^2 + 2\, V_{(1)}(S)\right)\,, \tag{3.42}$$

$$G(S) = \gamma - V_{(2)}(S)\,. \tag{3.43}$$

These now include the (one-loop) quantum corrections to the original parameters from the effective potential via contributions $V_{(1)}(S)$ and $V_{(2)}(S)$, with their analytical expression found in Eqs. (3.13) and (3.14), respectively. A similar framework with a priori arbitrary functions B, G, and V was considered by [240–244] and studied with purely phenomenological intentions. More closely to our approach is the study of [130], where the authors utilize strong dynamics to break scale invariance and generate an effective potential for inflation. While strong dynamics have their advantages for breaking scale-invariance, we will now continue with our discussion of Coleman-Weinberg type breaking of scale invariance and revisit strong dynamics in Chapter 4. Similarly, we have omitted the Weyl tensor C^2 term under the assumption of a small coupling κ and will further investigate its effects in detail in Section 3.5.

We move on by dealing with the R^2 term, which houses an additional scalar DOF that needs to be accounted for as it contributes significantly to inflation. Even though the slow-roll parameters of inflation are frame-independent [245, 246], they are much easier to calculate in the usual Einstein frame and the dynamical scalar DOF of R^2 is revealed when transforming to the Einstein frame. Therefore, we introduce an auxiliary field ψ of mass dimension two and the appropriate EOMs to make the replacement

$$R_{\rm J}^2 \to 2\, R_{\rm J}\, \psi - \psi^2\,, \tag{3.44}$$

and then go to the Einstein frame through a conformal rescaling of the metric

$$g^E_{\mu\nu} = \Omega^2\, g^J_{\mu\nu}, \qquad \text{with} \quad \Omega^2(S,\psi) = B(S) - \frac{4}{M_{\rm Pl}^2}\, G(S)\, \psi\,. \tag{3.45}$$

While this step here is straightforward, there are reasons for caution when this transformation is taken at the quantum level, namely including quantum fluctuations of the metric. Then, the quantum theories of the individual classical Lagrangians are not necessarily equivalent [247, 248]. This could prove especially problematic for the comparability of our results to other investigations of inflation realized in scale-invariant models, as some of them transform first to Einstein frame (under the assumption of broken scale-invariance) and then calculate the specifics of spontaneous symmetry breaking (see e.g. [137, 249]). However, since they and we treat gravity classically, we will not worry about this in the following discussions. The effective Lagrangian in the Einstein frame is then given by

$$\begin{aligned}\frac{\mathcal{L}^E_{\rm eff}}{\sqrt{-g_{\rm E}}} = &-\frac{1}{2}\, M_{\rm Pl}^2 \left(R_{\rm E} - \frac{3}{2}\, g_{\rm E}^{\mu\nu}\, \partial_\mu \ln\left[\Omega^2\right]\, \partial_\nu \ln\left[\Omega^2\right] \right) \\ &+ \frac{1}{2}\, \Omega^{-2}\, g_{\rm E}^{\mu\nu}\, \partial_\mu S\, \partial_\nu S - V_{\rm E}(S,\psi)\,,\end{aligned} \tag{3.46}$$

with the "E" analog to "J" before, referring to the Einstein frame, $\Omega = \Omega(S,\psi)$ and $V_{\rm E}(S,\psi)$ simply denoting the scalar potential in the Einstein frame, now depending on both S and ψ

$$V_{\rm E}(S,\psi) = M_{\rm Pl}^4\, \frac{\tilde{V}_{\rm CW}(S) + G(S)\psi^2}{\left[\, B(S) M_{\rm Pl}^2 - 4\, G(S)\, \psi \,\right]^2}\,. \tag{3.47}$$

Looking back to Eq. (3.46), we find that due to the second term, ψ is indeed a propagating scalar field in the Einstein frame. We can derive its canonically normalized form ϕ, which is then called the scalaron [250, 251]

$$\phi = \sqrt{\frac{3}{2}}\, M_{\rm Pl}\, \ln\left|\Omega^2\right|\,. \tag{3.48}$$

As a consequence, we can rewrite the effective Einstein-frame Lagrangian in terms of the "proper" fields S, ϕ

$$\frac{\mathcal{L}^E_{\rm eff}}{\sqrt{-g_{\rm E}}} = -\frac{1}{2}\, M_{\rm Pl}^2\, R_{\rm E} + \frac{1}{2}\, g_{\rm E}^{\mu\nu}\, \partial_\mu\phi\, \partial_\nu\phi + \frac{1}{2}\, e^{-\Sigma(\phi)}\, g_{\rm E}^{\mu\nu}\, \partial_\mu S\, \partial_\nu S - V_{\rm E}(S,\phi)\,, \tag{3.49}$$

where the $\Sigma(\phi) = \sqrt{2}\,\phi/\left(\sqrt{3}\,M_{\text{Pl}}\right)$ factor shows that now S is not canonically normalized anymore. When calculating the dynamics of inflation in the one-field approach of slow-roll inflation later, we find that this problem is easily remedied, as we attain a canonically normalized inflaton field through integration over the prefactor of its kinetic term. Furthermore, the Einstein-frame potential of Eq. (3.47), now as of function of S and ϕ can be written as

$$V_{\text{E}}(S,\phi) = e^{-2\,\Sigma(\phi)}\left[\tilde{V}_{\text{CW}}(S) + \frac{M_{\text{Pl}}^4}{16\,G(S)}\left(B(S) - e^{\Sigma(\phi)}\right)^2\right], \tag{3.50}$$

and in principle already denotes a valid two-scalar inflation potential, which we could study employing multifield techniques[9]. Nevertheless, this approach would constitute a great effort and introduce complexity, which seems unnecessary in the light of the potential in Eq. (3.50) exhibiting a valley structure, as e.g. seen in Fig. 3.3, along which the potential is sufficiently flat for slow-roll inflation and thus allows for an effective one-field model description[10].

3.3.2 One-field Slow-roll Inflation Predictions

In the following, we will discuss the validity and the conditions under which the two-field potential in Eq. (3.50) can be approximated by an effective one-scalar potential along a contour either along the S or the ϕ field. The contour plot in Fig. 3.3 shows the existence of said valley structure, along which the potential is suitable for one-field slow-roll inflation. Similarly, in [132] they also found that the classical trajectories with different initial conditions converge to an attractive valley contour. For the validity of the valley, we check the gradient along the contour to be much smaller than the one perpendicular to it, or in other words the eigenvalues of the scalar mass matrix $\partial^2 V(\Phi_i)/(\partial\Phi_i\partial\Phi_j)$ are separated hierarchically. The basic principle to find appropriate contours for different parameter spaces is to look for local minima in the direction of one of the two fields. Comparing both options, one finds, that which one is more suited to describe the potential properly in a one-field manner depends heavily on the parameters chosen (see the appendix in [1] for more details.).

The first contour $\mathcal{C}$ can be found via the analytical observation, that there is exactly one local extremum in the scalaron direction for each value of $S > \langle S \rangle$, thus ensuring flatness along the contour

$$\mathcal{C} = \{S,\, \tilde{\phi}(S)\} \qquad \text{where} \qquad \left.\frac{\partial V_{\text{E}}(S,\phi)}{\partial\phi}\right|_{\phi=\tilde{\phi}(S)} = 0\,, \tag{3.51}$$

9 For an example where multifield techniques are employed to study inflation, see [252].

10 Thus, we will omit the discussion of possible non-Gaussianities of multifield inflation in our analysis. Instead for more detail, we refer to [253, 254].

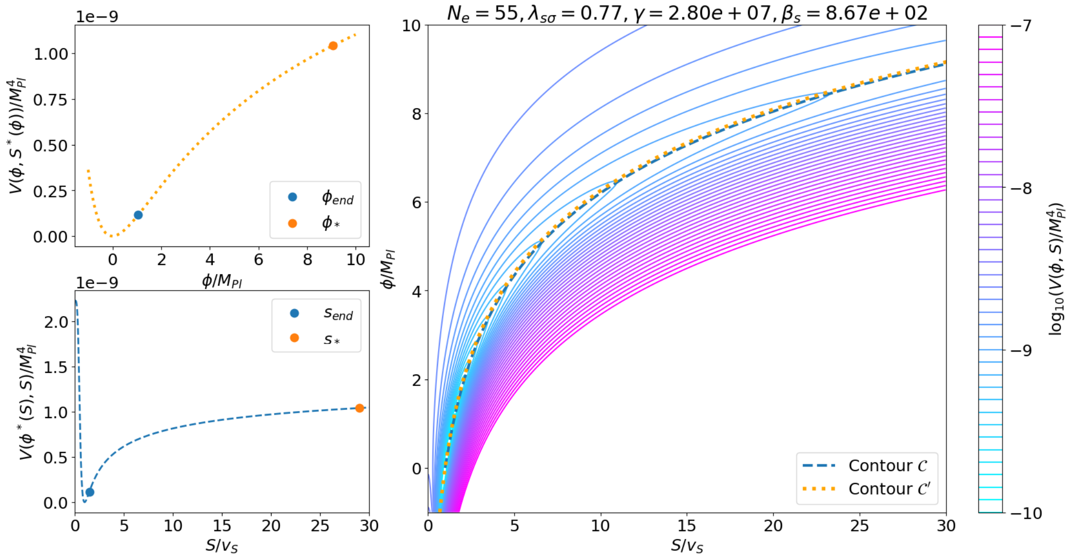


FIGURE 3.3: Scalar potential $V_{\rm E}(S,\phi)$ of Eq. (3.50) (right) for values of benchmark point #3 of Table 3.1. The in Eqs. (3.51) and (3.60) defined contours $\mathcal{C}$ and $\mathcal{C}'$ are shown in the contour plot (right), while the corresponding one-field inflaton potentials are shown on the left. While "end" refers to the end of inflation, "$\star$" denotes the horizon decoupling of the CMB.

where we can calculate the solution to $\tilde{\phi}(S)$ in Eq. (3.51) analytically to be

$$\tilde{\phi}(S) = \sqrt{\frac{3}{2}}\, M_{\rm Pl} \ln\left[B(S) + \frac{16\, G(S)\tilde{V}_{\rm CW}(S)}{B(S)\; M_{\rm Pl}^4} \right] . \tag{3.52}$$

Whether the trajectory of inflation satisfies Eq. (3.51) during inflation is verified by the following condition on the scalaron mass along the contour $\mathcal{C}$

$$\frac{m_\phi^2}{H_{\rm inf}^2} \gg 1\,, \tag{3.53}$$

with $H_{\rm inf}$ again denoting the Hubble parameter during inflation. When Eq. (3.53) above is fulfilled, the large and positive scalaron mass m_ϕ^2 can sufficiently stabilize the contour $\mathcal{C}$ during inflation, thus ensuring that the two-field system moves only along the contour with only insignificant motion away from $\mathcal{C}$. Now, employing the solution for $\tilde{\phi}(S)$ to potential $V_{\rm E}$ of Eq. (3.50), we are left with the one-field inflaton potential $V_{\rm inf}(S)$

$$V_{\rm inf}(S) = V_{\rm E}(S, \tilde{\phi}(S)) = \frac{\tilde{V}_{\rm CW}(S)}{B(S)^2 + 16\, M_{\rm Pl}^{-4}\, G(S)\; \tilde{V}_{\rm CW}(S)} . \tag{3.54}$$

Replacing the scalaron in the rest of the Lagrangian Eq. (3.49) results in a further modification of the kinetic term for S

$$e^{-\Sigma(\tilde{\phi}(S))}\, g_{\mathrm{E}}^{\mu\nu}\, \partial_\mu S\, \partial_\nu S + g_{\mathrm{E}}^{\mu\nu}\, \partial_\mu \tilde{\phi}(S)\, \partial_\nu \tilde{\phi}(S) = F^2(S)\, g_{\mathrm{E}}^{\mu\nu}\, \partial_\mu S\, \partial_\nu S\,, \tag{3.55}$$

where, for simplicity, we have introduced the normalization factor $F^2(S)$

$$F^2(S) = \frac{(1+4\,A(S))\,B(S) + \frac{3}{2}\,M_{\mathrm{Pl}}^2\,[(1+4\,A(S))\,B'(S) + 4\,A'(S)\,B(S)]^2}{[(1+4\,A(S))\,B(S)]^2}\,, \tag{3.56}$$

$$A(S) = \frac{4\,G(S)\tilde{V}_{\mathrm{CW}}(S)}{B(S)^2\,M_{\mathrm{Pl}}^4}\,, \tag{3.57}$$

such that we can write the final effective Lagrangian for inflation as

$$\frac{\mathcal{L}_{\mathrm{eff}}^{E}}{\sqrt{-g_{\mathrm{E}}}} = -\frac{1}{2}\,M_{\mathrm{Pl}}^2\,R + \frac{1}{2}\,F(S)^2\,g_{\mathrm{E}}^{\mu\nu}\,\partial_\mu S\,\partial_\nu S - V_{\mathrm{inf}}(S)\,. \tag{3.58}$$

Here, due to the normalization factor F^2, S is still not a generally canonically normalized field. But we have all the ingredients to obtain the canonically normalized inflaton field $\hat{S}$ through

$$\hat{S}(S) = \int_{\langle S\rangle}^{S} \mathrm{d}x\; F(x)\,. \tag{3.59}$$

With this, we can now calculate the slow-roll parameters and ultimately the CMB observables as predictions of our inflation model.

The other option for a valley contour $\mathcal{C}'$ is obtained in an analog way, namely by locating local minima along the direction of the scalar field S, yielding the solution $\tilde{S}(\phi)$

$$\mathcal{C}' = \{\tilde{S}(\phi),\ \phi\}\,, \quad \text{where} \quad \left.\frac{\partial V_{\mathrm{E}}(S,\phi)}{\partial S}\right|_{S=\tilde{S}(\phi)} = 0\,, \quad V_{\mathrm{inf}}(\phi) = V_{\mathrm{E}}(\tilde{S}(\phi),\phi)\,. \tag{3.60}$$

By going through the steps as discussed above for contour $\mathcal{C}$, one e.g. arrives at the scalaron field normalization $F^2(\phi)$

$$F^2(\phi) = \left[1 + e^{-\Sigma(\phi)}\left(\frac{\partial \tilde{S}(\phi)}{\partial \phi}\right)^2\right]\,, \tag{3.61}$$

as a replacement for $F^2(S)$ in Eq. (3.56). Because of the possible analytic solution $\tilde{\phi}$ of Eq. (3.52) and the limited influence of the choice of contour on the CMB observables, we choose to go forward by mainly using $\mathcal{C}$ to describe our one-field slow-roll inflation. Regardless, in Table 3.1 we also show results for calculations with contour $\mathcal{C}'$ for comparison at the three chosen benchmark points.

The last step before finally calculating the CMB observables A_{s}, n_{s}, and r, and

			Contour $\mathcal{C}$					Contour $\mathcal{C}'$				
#	β_S	γ	n_s	r	A_s	S_{end}/μ	S_{CMB}/μ	n_s	r	A_s	ϕ_{end}/μ	ϕ_{CMB}/μ
1	1.01×10^2	5.24×10^8	0.967	0.004	3.032	0.09	0.11	0.965	0.004	3.088	0.83	4.75
2	5.69×10^2	1.68×10^8	0.972	0.010	3.041	0.11	0.45	0.972	0.010	3.075	2.02	13.46
3	8.67×10^2	2.80×10^7	0.973	0.034	3.038	0.13	2.56	0.973	0.034	3.040	2.74	23.46

TABLE 3.1: Predicted CMB observables from inflation for three benchmark points. For all points, $\lambda_{S\sigma} = 0.77$, $\lambda_S = 0.005$ and $\beta_\sigma = 1$ and thus $\langle S \rangle = 0.088\mu$ have been fixed. We show the results for either one of the two contours, $\mathcal{C}$ Eq. (3.51) or $\mathcal{C}'$ Eq. (3.60) for $N_e = 55$ e-folds.

therefore constraining our model, is to calculate the slow-roll parameters. To compute the slow-roll parameters ϵ and η as in Eq. (3.38), one usually needs to use the canonically normalized inflaton field, however with $\hat{S}$ given by Eq. (3.59), we can instead use S and correct for the non-canonical normalization via $F^2(S)$:

$$\epsilon(S) = \frac{M_{\text{Pl}}^2}{2\,F^2(S)} \left(\frac{V'_{\text{inf}}(S)}{V_{\text{inf}}(S)} \right)^2 , \tag{3.62}$$

$$\eta(S) = \frac{M_{\text{Pl}}^2}{F^2(S)} \left(\frac{V''_{\text{inf}}(S)}{V_{\text{inf}}(S)} - \frac{F'(S)}{F(S)} \frac{V'_{\text{inf}}(S)}{V_{\text{inf}}(S)} \right) . \tag{3.63}$$

This modification also applies to the formula for the number of e-folds N_e of Eq. (3.39), which in this case is then given by

$$N_e = \int_{S_{\text{CMB}}}^{S_{\text{end}}} \frac{F^2(S)}{M_{\text{Pl}}^2} \frac{V_{\text{inf}}(S)}{V'_{\text{inf}}(S)} , \tag{3.64}$$

like in Eq. (3.39), S_{CMB} denotes the value of the scalar S to be evaluated at the time of horizon decoupling of the CMB, and S_{end} refers to S at the time when inflation ends, i.e. when Eqs. (3.62) and (3.63) reach values of order one

$$\max\{\epsilon(S = S_{\text{end}}), |\eta(S = S_{\text{end}})|\} = 1 . \tag{3.65}$$

With this, we have all the ingredients to investigate the dependence of the CMB observables of inflation on our model's parameters, by calculating A_s, n_s, and r via Eq. (3.40) with the aforementioned ϵ of Eq. (3.62), η of Eq. (3.63) and N_e in Eq. (3.64). Given, that ultimately, all the CMB observables of inflation depend on the one-field inflaton potential $V_{\text{inf}}(S)$ Eq. (3.54) and $F^2(S)$ Eq. (3.56), they determine the relevant parameters to be the dimensionless couplings λ_S, $\lambda_{S\sigma}$, β_S, β_σ and γ. The renormalization scale is fixed by the identification of M_{Pl} in Eq. (3.20) and the choice of flat direction Eq. (3.6) results in λ_S and β_σ being almost irrelevant for the CMB observables of inflation (see also Appendix A.1). Therefore, in the following analysis, we set them to the realistic values $\lambda_S = 0.005$ and $\beta_\sigma = 1$. This leaves us with the scalar portal coupling $\lambda_{S\sigma}$, the non-minimal coupling β_S and the R^2 coupling γ as relevant, free parameters of the model. Furthermore, there is some freedom in the number of e-folds between CMB horizon exit and the end of inflation N_e, such that we will use the well-established assumption, $N_e \in [50,\ 60]$ to be a sufficiently long period of inflation to address the problems of hot Big Bang cosmology. Consequently, we can use the tight constraint on A_s from the latest Planck mission data [22, 132]

$$\ln\left[10^{10}A_\mathrm{s}\right] = 3.044 \pm 0.014\,, \tag{3.66}$$

to further reduce our number of free parameters by one. Demanding that points fulfill Eq. (3.66), we find an effective relation between β_s and γ for fixed values of N_e and $\lambda_{\mathrm{s}\sigma}$. Using this we can neatly show the model's parameter dependence in the $n_\mathrm{s} - r$ plane in Fig. 3.4. In addition, demanding Eq. (3.66) results in upper limit on allowed values for $\beta_{\mathrm{s,max}} \sim \mathcal{O}(10^3)$ and $\gamma_{\mathrm{max}} \sim \mathcal{O}(10^9)$, again for fixed values of $\lambda_{\mathrm{s}\sigma}$ and N_e. Comparing our results in Fig. 3.4 with other models for one-field slow-roll inflation we find that our model seems to interpolate between two well-known models of inflation. The lower end (large γ) of our predictions converges to the ones by R^2 (or Starobinsky) inflation [201–203], while the upper end (smaller γ) corresponds to predictions by linear chaotic inflation [185, 255, 256]. Lastly, we find a wide region of parameter space for which the predictions of our model are perfectly in agreement with the constraints on inflation via the CMB observations [22, 23] (see Fig. 3.4), even with the improved constraints from [24] and possible future observations, which are expected to further increase the constraint on r.

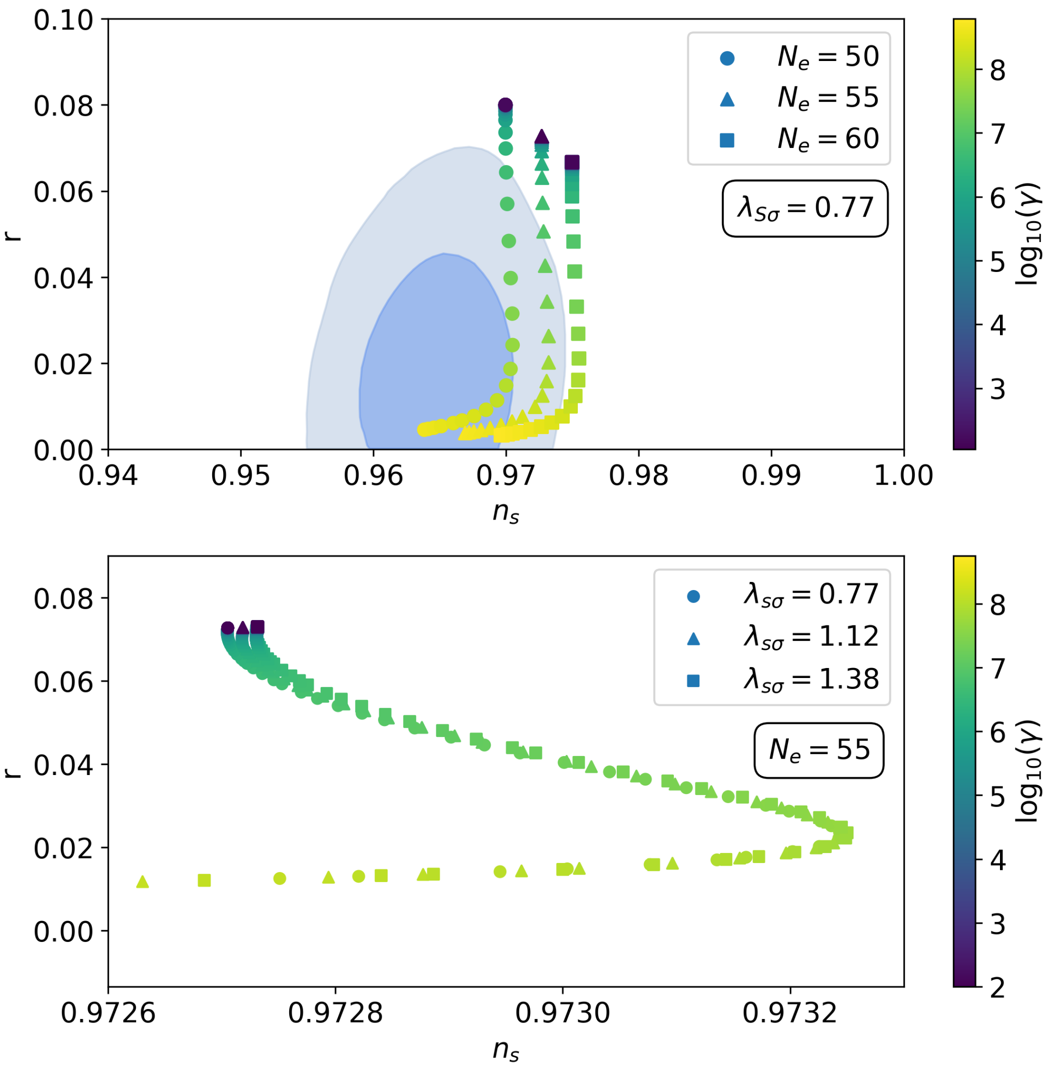


FIGURE 3.4: Predictions for the scalar spectral index n_s and the tensor-to-scalar ratio r with varying number of e-folds N_e (top) and varying $\lambda_{S\sigma}$ (bottom). $\beta_\sigma = 1$ and $\lambda_S = 0.005$ we fixed for all points and only points which satisfy the scalar power spectrum A_s constraint (3.66) and thus fixing β_S w.r.t. γ are displayed. In the top panel, we included the Planck TT,TE,EE+lowE+lensing+BK15 68% and 95% CL regions as displayed in [23].

3.4 Reheating and Dark Matter

At this point, we have discussed the main point of this thesis in the given framework, namely the generation of hierarchical energy scales utilizing conformal symmetry and the spontaneous breaking thereof. Yet, reheating is necessary to convert the inflaton energy density into SM DOFs (like radiation, etc.), and dark matter remains missing in the SM. While we refer to other literature for a more in-depth treatment of these two aspects, we want to give a short sketch of how dark matter can be included in our scale-invariant framework of dynamically generated scales. We can and will do so without specifying a reheating mechanism and without choosing a specific DM model. For more information on reheating we refer to [257, 258], while more details on the present derivation are found in [1] and the references therein.

To include effects of the reheating phase without specifying its mechanism we follow [12, 221] and under general assumptions and model-specific observations derive a relation between the number of e-folds N_e and the reheating temperature T_{RH}, which in turn is directly related to the energy density ρ_{RH} of radiation at the end of the reheating phase by

$$\rho_{\mathrm{RH}} = \frac{\pi^2}{30}\, g_{\mathrm{RH}}\, T_{\mathrm{RH}}^4\,, \tag{3.67}$$

with the subscript "RH" denoting quantities to be evaluated at the end of the reheating phase and thus g_{RH} describing the relativistic DOFs at the end of reheating. This energy density ρ_{RH} and therefore also the reheating temperature T_{RH} can be related to the energy density of the inflaton ρ_{end} at the end of inflation via [12]

$$\ln R_{\mathrm{rad}} = \ln\left[\frac{a_{\mathrm{end}}}{a_{\mathrm{RH}}}\left(\frac{\rho_{\mathrm{end}}}{\rho_{\mathrm{RH}}}\right)^{1/4}\right] = \frac{1-3\,\bar{\omega}}{12(1+\bar{\omega})}\ln\left[\frac{\rho_{\mathrm{RH}}}{\rho_{\mathrm{end}}}\right]\,, \tag{3.68}$$

where a is the scale factor, the subscript "end" refers to the end of inflation and $\bar{\omega}$ denotes the average equation of state parameter during the reheating phase. Through the dependence on a_{end} in Eq. (3.68), we can find the following relation between the number of e-folds N_e and T_{RH}

$$N_e = \ln\left[\frac{a_{\mathrm{end}}}{a_{\mathrm{CMB}}}\right] = C + \frac{1}{4}\ln\left[\left(\frac{3-\epsilon_{\mathrm{end}}}{3-\epsilon_{\mathrm{CMB}}}\right)^{4/3}\left(\frac{V_{\mathrm{inf,CMB}}^2}{M_{\mathrm{Pl}}^4\, V_{\mathrm{inf,end}}^{4/3}}\right)\right] + \frac{1}{3}\ln\left[T_{\mathrm{RH}}\right]\,. \tag{3.69}$$

To obtain the above relation we used the constraints on the current Hubble parameter H_0 and the pivot scale $k_\star$ of [22, 23] and assumed conservation of entropy [259]. Furthermore, we expressed ρ_{end} in terms of the slow-roll parameters, found $\bar{\omega} = 0$ by analyzing the inflaton potentials behavior near its minimum[11] and introduced

[11] For more details on these steps, see [1].

the constant C for simplicity

$$C = \frac{1}{12}\ln\left[\frac{\pi^2}{30}\right] + 66.89 - \ln\left[\frac{k_\star}{H_0}\right] . \tag{3.70}$$

One can see that $T_{\rm RH}$ is the only free parameter on the right-hand side of the above relation, thus assuming that N_e and $\rho_{\rm RH}$ satisfy the constraints from experimental observation [23], we can calculate use Eq. (3.69) to either calculate $T_{\rm RH}$ in terms of N_e or constrain N_e in terms of $T_{\rm RH}$. This will turn out to be of great importance as the reheating temperature $T_{\rm RH}$ plays a vital role in the production of the correct dark matter relic abundance as well as in the generation of the baryon-asymmetry via Leptogenesis. Using Eq. (3.69), in Fig. 3.5 we display the effects of varying N_e on the predictions for CMB parameters of inflation in the $n_s - r$ plane via the corresponding reheating temperature $T_{\rm RH}$. For this, we fix all couplings to the benchmark points 1, 2 or 3 (as seen in Table 3.1) and vary $N_e \in [50, 60]$. We also demand that all points fulfill the constraint Eq. (3.66) and therefore slightly vary β_S accordingly. In the line for points with couplings the same as benchmark 1, the lower bound on β_S only allows for $N_e \in [53.5, 60]$.

With the reheating temperature $T_{\rm RH}$ in hand, we turn towards the production of the right relic abundance of dark matter. We have already introduced χ in the theory's Lagrangian $\mathcal{L}_T$ Eq. (3.1), which is stabilized due to the unbroken Z_2 symmetry and thus constitutes a good candidate for dark matter. Since dark matter can be produced after or during the reheating phase [260–262], we consider both the inflaton S as well as SM (and BSM) particles decaying into χ. Let us note here, that due to the conditions for the flat direction Eq. (3.6), σ is heavier than S and therefore is not

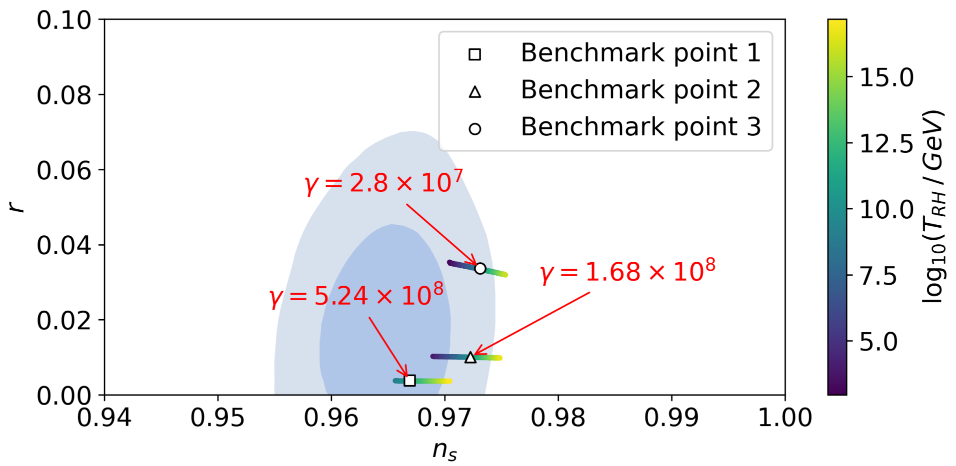


FIGURE 3.5: Inflation predictions in the (n_s, r) plane with varying $N_e \in [50, 60]$ and (slightly) varying β_S to account for the A_s constraint of Eq. (3.66). $T_{\rm RH}$ is shown using its relation to N_e through Eq. (3.69). We have fixed $\beta_S = 0.005$, $\lambda_{S\sigma} = 0.77$ and $\beta_\sigma = 1$ and for each line respectively as displayed. Furthermore, we include the Planck TT,TE,EE+lowE+lensing+BK15 68% and 95% CL regions of [23].

produced during reheating [261]. χ on the other hand can be produced either via inflaton decay $S \to \chi\,\chi$ or via the scattering of right-handed neutrinos $N_\mathrm{R}\,N_\mathrm{R} \to \chi\,\chi$ with the cross-section $\sigma_{\mathrm{N},\chi}$. Upon computation of $\sigma_{\mathrm{N},\chi} \sim y_{\mathrm{N}\chi}^4$ one quickly finds that the process $N_\mathrm{R}\,N_\mathrm{R} \to \chi\,\chi$ is extremely suppressed due to the size of the Yukawa coupling $y_{\mathrm{N}\chi}$, which is required to be small to realize neutrino option (see Section 3.2). Consequently, the process $N_\mathrm{R}\,N_\mathrm{R} \to \chi\,\chi$ is irrelevant for the production of χ [260], such that we focus on the inflaton decay as the production mechanism for χ. The decay width γ_χ for $S \to \chi\,\chi$ is then given by

$$\gamma_\chi = \frac{3\,y_\chi^2\,m_\mathrm{S}}{16\,\pi}\sqrt{1-\frac{4\,m_\chi^2}{m_\mathrm{S}^2}}\;, \tag{3.71}$$

where we assume that S is dominant as the inflaton field compared to ϕ, which if not valid, needs to be included in the decay width above. Furthermore, we want to remark that χ only interacts with the SM particles through the right-handed neutrino N_R, and since $y_{\mathrm{N}\chi} \ll \mathcal{O}(10^{-8})$ this contact is extremely suppressed. Thus, it is sufficient to consider a two-particle system of S and χ where the coupled Boltzmann equations of the number densities n_S, n_χ are given by

$$\frac{\mathrm{d}n_\mathrm{S}}{\mathrm{d}t} = -3\,H\,n_\mathrm{S} - \Gamma_\mathrm{S}\,n_\mathrm{S}\;, \qquad \frac{\mathrm{d}n_\chi}{\mathrm{d}t} = -3\,H\,n_\chi + B_\chi\,\Gamma_\mathrm{S}\,n_\mathrm{S}\;, \tag{3.72}$$

where $B_\chi = \gamma_\chi/\Gamma_\mathrm{S}$ and Γ_S denotes the total decay width of S. We can easily see, that the equation for n_S is not coupled and therefore can be solved on its own [257]. Using this solution to solve for the number density n_χ we can calculate its freeze-in value, namely the value at $t = \infty$, which leads to the relic abundance $\Omega_\chi h^2$ of

$$\Omega_\chi h^2 = \frac{1}{3}\,m_\chi\,B_\chi\,M_\mathrm{Pl}^2\,\frac{\rho_\mathrm{end}}{m_\mathrm{S}}\left(\frac{a_\mathrm{end}}{a_0}\right)^3\left(\frac{H_0}{h}\right)^{-2}\;, \tag{3.73}$$

where $a_0 = 1$ is the current value of the scale factor and $H_0 = h\,2.1332 \times 10^{-42}\,\mathrm{GeV}$ with $h \simeq 0.674$ [22] denotes the present day value of the Hubble parameter. Using the results of the calculation of Eq. (3.69) we can express the DM relic abundance as (see also [261])

$$\Omega_\chi h^2 \simeq 2.04 \times 10^8\,B_\chi\left(\frac{m_\chi}{m_\mathrm{S}}\right)\frac{T_\mathrm{RH}}{1\,\mathrm{GeV}}\;. \tag{3.74}$$

Finally, the branching ratio B_χ can be calculated using Eq. (3.71) and assuming that we can identify the total decay width Γ_S, with the Hubble parameter at the end of reheating $H(a_\mathrm{RH}) = (3\,M_\mathrm{Pl}^2/\rho_\mathrm{RH})^{-1/2}$, so that for the example of benchmark point 2 of Table 3.1 we find

$$m_\mathrm{S} \simeq 4.4 \times 10^{15}\,\mathrm{GeV}\;,\;\langle S\rangle \simeq 1.0 \times 10^{17}\,\mathrm{GeV}\;, \tag{3.75}$$

$$T_\mathrm{RH} \simeq 1.9 \times 10^{10}\,\mathrm{GeV}\;,\;k_\star = 0.002\,\mathrm{Mpc}^{-1}\;, \tag{3.76}$$

and thus an appropriate relic abundance is generated for $y_\chi \simeq 1.4 \times 10^{-11}$

$$\Omega_\chi h^2 \simeq 4.4 \times 10^{31}\, y_\chi^3 \simeq 0.12\,, \qquad m_\chi = y_\chi \, \langle S \rangle \simeq 4.3 \times 10^6\,\mathrm{GeV}\,. \tag{3.77}$$

In Fig. 3.6 we display the possible masses for the dark matter candidate χ that produce the right relic abundance of dark matter, depending on the temperature at the end of reheating $T_{\rm RH}$. Utilizing the relation Eq. (3.69), we change $N_e \in [50, 60]$ and thus considerably vary $T_{\rm RH} \in [6.8 \times 10^3\,\mathrm{GeV}, 4.1 \times 10^{16}\,\mathrm{GeV}]$.

Furthermore, we remember that interaction with S is highly suppressed by $y_{\rm M} \ll 1$ and thus we can assume that the right-handed neutrinos $N_{\rm R}$ are only reheated through interaction with the SM particles. Consequently, the lower bound $T_{\rm RH} \gtrsim 2 \times 10^9\,\mathrm{GeV}$ applies for $m_{\rm N} \gtrsim 2 \times 10^7\,\mathrm{GeV}$, to account for successful thermal leptogenesis [263]. We show this lower bound in Fig. 3.6 as the black dotted line. Lastly, we find that demanding working thermal leptogenesis restricts us to $N_e \gtrsim 54$ for the three benchmark points in Table 3.1. This still leaves plenty of parameter space in which we not only dynamically generate $M_{\rm Pl}$, $\mu_{\rm H}$, $m_{\rm N}$, m_ν, in fact all scales of the SM from the same origin, the spontaneous breaking of scale-invariance by $\langle S \rangle$, which also yields the inflation potential with predictions that are well in agreement with the current observational constraints, but furthermore even allows for the generation of the proper amount of dark matter relic abundance.

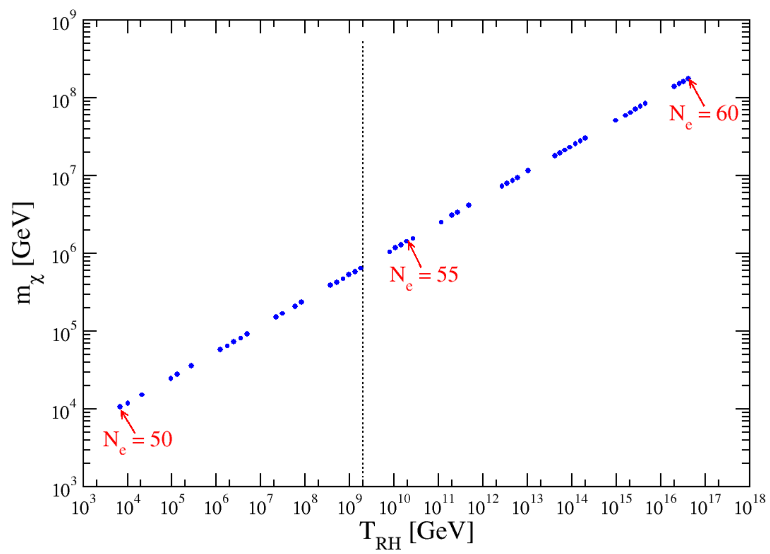


FIGURE 3.6: Dark matter candidate mass m_χ against reheating temperature $T_{\rm RH}$, such that a proper relic abundance is generated. While all the other input parameters are fixed to, β_S was varied around the value of benchmark point 2 (Table 3.1) to vary N_e while respecting Eq. (3.66). The black dotted line shows the lower bound on $T_{\rm RH}$ for a viable thermal leptogenesis for $m_{\rm N} \gtrsim 2 \times 10^7\,\mathrm{GeV}$ [263].

3.5 Gravity as an Extra Scalar

Until now, we have suppressed contributions by the Weyl tensor C^2 term (via $\kappa \approx 0$), therefore, in the following, we will address this shortcoming and focus on the role of curvature in the breaking of scale-invariance and the corresponding inflaton potential. Even though quantum effects inevitably generate these general higher-order contributions of curvature (e.g. C^2), they are often suppressed and rarely included in the action from the start, as they are known to carry not only the massless spin-0 graviton but also propagating massive spin-2 ghost DOFs. These ghosts pose a threat to the unitarity and therefore the probabilistic interpretation of quantum theory and consequently require proper treatment. Nevertheless, the thesis presented here focuses more on the spontaneous breaking of scale invariance in the light of the dynamical generation of scales. So, we will only briefly discuss the so-called *ghost problem*, in which we will explain why we consider globally scale-invariant quadratic gravity not necessarily to be unphysical, despite the existence of the massive spin-2 ghost and rather discuss the ghost's contributions to the radiative breaking of scale-invariance. Thus, we will first give a quick treatment of the aforementioned ghost problem, then expose the gravitational DOFs in a minimal (for RSSB) toy model, further explore the resulting Coleman-Weinberg potential, and show that it can realize successful inflation that is in perfect agreement with the strongest constraints from CMB observations [23, 24].

The Ghost Problem

The *ghost problem* is generally known to be the quantum version of the instability Ostrogradsky already found in 1850 [264], then for classical mechanics. Naturally, quantum and classical instabilities are not always comparable, yet in this case, they are both based on the existence of higher order (> 2) time derivatives at the level of the equations of motion. In the case of scale-invariant or quadratic gravity, these problematic contributions are contained specifically in the C^2 Weyl tensor squared term of the action[12]. Extracting these contributions in terms of propagating fields one finds that there are spin-2 contributions with a relative minus sign on their kinetic term, thus classifying them as ghosts. This relative minus sign leads to negative norm states when canonical quantization is applied to cure the instability. The existence of negative norm states, in turn, breaks unitarity and thus the probabilistic interpretation of QFT. This spin-2 ghost is not to be confused with other often-seen ghosts, like the Boulware-Deser ghost of massive gravity [265] or the Faddeev-Popov ghost [266], often introduced to ensure consistency in the path integration of gauged quantum field theories. The Faddeev-Popov ghost is unphysical and therefore not part of the theory's spectrum, the Boulware-Deser ghost was shown to be vanishing at all orders for massive gravity when parameters were tuned in accordance [267–

[12] This is shown by expanding the metric around its quantum fluctuations, such that these higher derivatives terms can be seen in Eq. (3.82).

269]. In contrast, the spin-2 ghost of quadratic (scale-invariant) gravity whether massive or massless is still an issue as it appears in the theory's asymptotic states threatening unitarity. For a more in-depth discussion of the ghost in scale-invariant or quadratic gravity, we refer to our colleagues [61, 62], who among other things introduce the concept of *conditional unitarity* that offers an interesting perspective on the issue of unitarity in scale-invariant theories with massive spin-2 ghost DOFs.

Generally, there are many interesting suggestions on how to address the ghost problem, i.e. unitarity in higher order derivative theories, some demand different symmetry requirements on the theory's Hamiltonian, e.g. $\mathcal{PT}$-symmetry that results in positive definite inner product [50, 51], others are based on Lee-Wick models [52–54] which employ the Pauli-Villars [270] regularization with a physical mass instead of technical one, ultimately leading to ghost not being created. Further possible resolutions are based on alternative quantization procedures, e.g. the so-called *fakeon* prescription [55, 56], which employs a different quantization for the ghost DOFs, i.e. as "fake" particles (hence the name). These "fake" particles are then ensured to be purely virtual and therefore do not appear in the spectrum of asymptotic physical states. Others, change the notion of probability in quantum mechanics altogether by generalizing the inner product and thus arrive at positive probabilities for processes involving ghosts [57, 58], while yet others propose the ghost to be unstable, consequently decaying into SM particles and therefore to not be part of the spectrum of asymptotic physical states [59, 60].

While this list is by no means a complete recollection of proposed ways to address the ghost problem, none of the existing ones seem to come without substantial caveats, for example, the introduction of (micro) causality violation. Nevertheless, even among the aforementioned there are promising resolutions to the ghost problem, such that the existence of the spin-2 ghost in higher-order gravity should not be a reason for its neglect. Furthermore, we will also find that its contributions can be rather beneficial when it comes to RSSB of scale invariance in the one-loop Coleman-Weinberg potential.

Massive Spin-2 Ghost Inflation

In contrast to $\mathcal{L}_{\mathrm{T}}$ of Sections 3.1 to 3.4, we now do not require κ to be small such that for our minimal model of RSSB we only consider one single additional scalar field $S(x)$ coupled to globally scale-invariant quadratic gravity, as we find that the additional DOFs from the Weyl tensor squared term will allow for the successful breaking of scale invariance. The complete action $S_{\mathrm{T}}^{1\mathrm{S}} = S_{\mathrm{QG}} + S_{\mathrm{S}}$ that respects global scale symmetry and infinitesimal local diffeomorphism invariance can be written as

$$S_{\mathrm{QG}} = \int \mathrm{d}^4x\sqrt{-g}\left(\gamma R^2 - \kappa\, C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\right) \ , \tag{3.78}$$

$$S_{\mathrm{S}} = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{2}\,\nabla_\mu S\,\nabla^\mu S - \frac{1}{2}\beta\, S^2\, R - \frac{1}{4}\lambda\, S^4\right) \ , \tag{3.79}$$

such that $S_{\mathrm{T}}^{1\mathrm{s}}$ contains all individual squared terms of the curvature tensors and scalars except for the omitted contributions from total derivatives like the Gauss-Bonnet term (c.f. Eq. (2.16)). The dimensionless constants γ, κ, β, and λ are taken to be at natural values. To investigate the effects of the gravitational or metric DOFs on breaking of scale-invariance as well as inflation, we separate the dynamical part of the metric by expanding the action in terms of the fluctuations $h_{\mu\nu}(x)$ around flat Minkowski space $\eta_{\mu\nu}$

$$g_{\mu\nu} \to \eta_{\mu\nu} + h_{\mu\nu} \,. \tag{3.80}$$

In the same manner, we rewrite the scalar S as quantum fluctuations $\hat{S}$ around the approximately constant (classical) background field S_{cl}

$$S \to S_{\mathrm{cl}} + \hat{S} \,, \tag{3.81}$$

such that the total action expanded up to second order in the dynamical fields $\hat{S}$ and $h_{\mu\nu}$ using Eqs. (3.80) and (3.81) is calculated to be

$$\begin{aligned} S_{\mathrm{T}}^{(\mathrm{quad})} = \int \mathrm{d}^4x \Big[&\gamma \left(h^{\mu\nu}\partial_\mu\partial_\nu\partial_\rho\partial_\sigma h^{\rho\sigma} + h_\mu{}^\mu \Box^2 h_\nu{}^\nu + 2h_\mu{}^\mu \Box \partial_\nu\partial_\rho h^{\nu\rho} \right) \\ &+ \frac{1}{6}\kappa \left(-3h^{\mu\nu}\Box^2 h_{\mu\nu} - 6h^{\mu\nu}\Box\partial_\nu\partial^\rho h_{\mu\rho} - 2h^{\mu\nu}\partial_\mu\partial_\nu\partial_\rho\partial_\sigma h^{\rho\sigma} \right. \\ &\qquad \left. + h_\mu{}^\mu \Box^2 h_\nu{}^\nu + 2h_\mu{}^\mu \Box\partial_\nu\partial_\rho h^{\nu\rho} \right) \\ &+ \frac{1}{8}\beta\, S_{\mathrm{cl}}^2 \left(h^{\mu\nu}\Box h_{\mu\nu} + 2h^{\mu\nu}\partial_\nu\partial^\rho h_{\mu\rho} - h_\mu{}^\mu \Box h_\nu{}^\nu - 2h_\mu{}^\mu \partial_\nu\partial_\rho h^{\nu\rho} \right) \\ &- \frac{1}{2}\hat{S}\,\beta\, S_{\mathrm{cl}} \left(\partial_\nu\partial_\mu + \Box\eta_{\mu\nu} \right) h^{\mu\nu} + \frac{1}{2}\hat{S}\left(\Box - 3\lambda S_{\mathrm{cl}}^2 \right)\hat{S} \Big] \,, \end{aligned} \tag{3.82}$$

where we use the notation $\Box = -g^{\mu\nu}\partial_\mu\partial_\nu$ to denote the d'Alembert operator, performed partial integration and left out terms $\propto \hat{S}^4$, that would have induced a cosmological constant at tree-level. To further isolate the individual gravitational DOFs, we perform a so-called *York decomposition* [271] to separate the contributions of different spins

$$h_{\mu\nu} = \left(\partial_\mu\partial_\nu - \frac{1}{4}\eta_{\mu\nu}\Box \right) a + \frac{1}{4}\eta_{\mu\nu} h_\alpha{}^\alpha + \partial_\mu V_\nu + \partial_\nu V_\mu + \tilde{h}_{\mu\nu} \,, \tag{3.83}$$

where we find the scalar modes $a(x)$ and $h_\mu{}^\mu$, the transverse ($\partial_\mu V^\mu = 0$) vector modes $V_\mu(x)$ and finally the transverse-traceless ($\partial^\mu \tilde{h}_{\mu\nu} = \tilde{h}_\mu{}^\mu = 0$) tensor modes $\tilde{h}_{\mu\nu}(x)$ [272]. As demonstrated in [273], the two spin-0 DOFs can be combined in the then gauge-invariant[13] scalar quantity $\phi(x)$

$$\phi = h_\mu{}^\mu - \Box a \,, \tag{3.84}$$

13 Here, gauge-invariance is understood to be w.r.t the diffeomorphism $h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$.

which denotes the scalar DOF of the R^2 contribution, the so-called *scalaron*, that also acts as the inflaton in e.g Starobinsky's model of inflation [201]. Applying Eqs. (3.83) and (3.84) to the action in Eq. (3.82) we find that all the quadratic terms of V_μ and a cancel out identically to give $S_{\mathrm{T}}^{(\mathrm{quad})}$ as

$$S_{\mathrm{T}}^{(\mathrm{quad})} = \int \mathrm{d}^4x \left[\frac{9}{16}\gamma\, \phi \left(\Box^2 - m_\phi^2 \Box \right) \phi - \frac{1}{2}\kappa\, \delta_{\mu\nu\rho\sigma}\, \tilde{h}^{\mu\nu} \left(\Box^2 - m_{\mathrm{gh}}^2 \Box \right) \tilde{h}^{\rho\sigma} \right.$$
$$\left. - \hat{S} \left(\frac{3}{4}\beta\, S_{\mathrm{cl}} \Box \right) \phi + \frac{1}{2}\hat{S}\left(\Box - m_S^2\right)\hat{S} \right], \tag{3.85}$$

with $\delta_{\mu\nu\rho\sigma} = \frac{1}{2}(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho})$. In analogy to the general Coleman-Weinberg calculation (see Sections 2.3.1 and 2.3.2 and Eq. (2.37)), we have included the on the classical background field $\hat{S}$ depending masses

$$m_\phi^2 = \frac{\beta}{12\gamma} S_{\mathrm{cl}}^2\,, \qquad m_S^2 = 3\,\lambda\, S_{\mathrm{cl}}^2\,, \qquad m_{\mathrm{gh}}^2 = \frac{\beta}{4\kappa} S_{\mathrm{cl}}^2\,. \tag{3.86}$$

We see, that not only the two scalars S and ϕ but also the spin-2 ghost generate a field-dependent mass term and thus all three are expected to contribute to the quantum effective potential. We use the aforementioned Coleman-Weinberg mechanism to calculate said quantum effective potential at one-loop order by integrating out the fluctuation fields $\hat{S}$, ϕ, and $\tilde{h}_{\mu\nu}$ to determine the VEV of S_{cl}. Following the Coleman-Weinberg procedure, i.e. isolating the terms that are quadratic in the fields $\Phi = (\tilde{h}_{\mu\nu}, \phi, \hat{S})^T$, evaluating the functional Gaussian integral, and dropping the "cl" subscript for S_{cl}, we arrive at the familiar equation for the one-loop contribution $V_{\mathrm{eff}}^{(1)}$ (see Eq. (2.32)) to the effective potential V_{eff}

$$V_{\mathrm{eff}}^{(1)}(S) = -\frac{i}{2} \ln \left[\mathrm{Det} \left(\frac{\delta^2 S_{\mathrm{T}}^{(\mathrm{quad})}}{\delta\Phi\delta\Phi} \right) \right]$$
$$= -\frac{i}{2} \ln \left[\mathrm{Det} M \right] - \frac{i}{2} \mathrm{Tr} \left(\ln \left[\delta_{\mu\nu\rho\sigma} \left(-\Box^2 + m_{\mathrm{gh}}^2 \Box \right) \right] \right), \tag{3.87}$$

where, again, the determinant is understood to be functional. We find, that the scalar contribution to the one-loop effective potential, denoted by the scalar Hessian matrix M (derivatives w.r.t $\phi, \hat{S}$) in the first term, nicely separates from the spin-2 ghost contribution in the second term above, such that we can deal with them individually. First, we will compute the scalar contribution using the standard techniques as outlined in Section 2.3.1, then we will turn to the spin-2 part, which requires some further consideration since the general calculation of the Coleman-Weinberg potential is often shown only for SM-like particles, which are of the spin $s = \left\{0, \frac{1}{2}, 1\right\}$ (see e.g. [144]).

First, regarding the scalar contribution, with the scalar Hessian M given by

$$M = \begin{pmatrix} \frac{9}{8}\gamma\left(\Box^2 - m_\phi^2\Box\right) & -\frac{3}{4}\beta\, S\, \Box \\ -\frac{3}{4}\beta\, S\, \Box & \Box - m_s^2 \end{pmatrix} , \tag{3.88}$$

we can still rewrite the logarithmic determinant in Eq. (3.87) as the functional trace over logarithms but since M, as seen above Eq. (3.88) contains off-diagonal terms, we arrive at the little more complicated expression

$$\ln\left[\mathrm{Det}M\right] = \mathrm{Tr}\left(\ln\left[\Box - m_+^2\right]\right) + \mathrm{Tr}\left(\ln\left[\Box - m_-^2\right]\right) + \cdots , \tag{3.89}$$

where "..." represents constant terms independent S that therefore will not contribute to the effective potential[14]. Furthermore, the two masses $m_\pm^2$ are defined as

$$m_\pm^2 = \frac{1}{2}\left(m_s^2 + (1+6\,\beta)\, m_\phi^2\right) \pm \frac{1}{2}\sqrt{\left(m_s^2 + (1+6\,\beta)\, m_\phi^2\right)^2 - 4\, m_s^2\, m_\phi^2} , \tag{3.90}$$

which we find to agree with the Einstein frame mass eigenstates as calculated in [69]. Using Eq. (3.89), we see that in momentum space, the scalar contributions to $V_{\mathrm{eff}}^{(1)}(S)$ are again of the familiar form of Eq. (2.36). Therefore, we can employ the usual dimensional regularization and renormalization via the $\overline{\mathrm{MS}}$ scheme to calculate the scalar one-loop contribution to the effective potential according to Eq. (2.37) as

$$\begin{aligned} V_s^{(1)}(S) &= -\frac{i}{2}\sum_{j=\pm}\int \frac{\mathrm{d}^4 p}{(2\,\pi)^4}\ln\left[p^2 - m_j^2\right] \\ &= \frac{1}{64\pi^2}\sum_{j=\pm} m_j^4\left(\ln\left[\frac{m_j^2}{\mu^2}\right] - \frac{3}{2}\right) . \end{aligned} \tag{3.91}$$

Here, μ denotes the renormalization scale and divergent terms have been absorbed in the redefinition of the renormalized dimensionless coupling λ[15].

Now, addressing the calculation of the spin-2 contribution works similarly to the CW calculation above and it will turn out that the general $\overline{\mathrm{MS}}$ expression Eq. (2.37) still holds for the massive spin-2 ghost, merely with adjusted $N = 5$ numbers of DOFs and constant $c = 1/10$. Yet, since we believe this not to be obvious, we will showcase the calculation its considerations, and assumptions in the following. Using the well-known properties of the trace and the natural logarithm, we can rewrite the spin-2 contribution of Eq. (3.87) to be

$$\mathrm{Tr}\left(\ln\left[\delta_{\mu\nu\rho\sigma}\left(-\Box^2 + m_{\mathrm{gh}}^2\Box\right)\right]\right) = \mathrm{Tr}\left(\ln\left[\delta_{\mu\nu\rho\sigma}\left(\Box - m_{\mathrm{gh}}^2\right)\right]\right) + \mathrm{Tr}\left(\ln\left[-\Box\right]\right) , \tag{3.92}$$

therefore recovering familiar contribution of $\mathrm{Tr}\ln(\Box - m^2)$ by separating it from the

[14] If not left out here, they would be subtracted when these contributions are renormalized and thus would not contribute either way.

[15] This is the usual procedure in calculating the Coleman-Weinberg potential, see e.g. [26]

"new" $\mathrm{Tr}\ln(-\Box)$ contribution. In addition, we find the "new" contribution to be independent of S and thus we drop it, for the same reason the "..." terms were dropped before. Due to the properties of the logarithm, the relative minus sign is also dropped, thus the overall sign of the spin-2 ghost contribution to the effective potential is the same as for usual particles. We find this to be in agreement with the calculations in [69] of the quartic coupling's β-function. Now, to compute the remaining contribution by the first term in Eq. (3.92) we utilize that $\tilde{h}_{\mu\nu}$ is transverse-traceless in momentum space to arrive at

$$\tilde{h}^{\mu\nu}\,\delta_{\mu\nu\rho\sigma}\,\tilde{h}^{\rho\sigma}=\tilde{h}^{\mu\nu}\,P^{(2)}_{\mu\nu\rho\sigma}\,\tilde{h}^{\rho\sigma}\,, \tag{3.93}$$

with $P^{(2)}_{\mu\nu\rho\sigma}$ being a spin-2 projection operator (see e.g. [274]) of the form

$$P^{(2)}_{\mu\nu\rho\sigma}=\frac{1}{2}\left(\theta_{\mu\rho}\theta_{\nu\sigma}+\theta_{\mu\sigma}\theta_{\nu\rho}\right)-\frac{1}{d-1}\theta_{\mu\nu}\theta_{\rho\sigma}\,, \tag{3.94}$$

$$\theta_{\mu\nu}=\eta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\,. \tag{3.95}$$

Applying the replacement Eq. (3.93) to Eq. (3.92) guarantees that the number of DOF (e.g. N_i in Eq. (2.37)) is counted correctly, which in the case of the massive spin-2 boson comes out to $N=5$. Furthermore, we find that

$$\mathrm{Tr}\left(P^{(2)}_{\mu\nu\rho\sigma}\right)=\delta^{\mu\nu\rho\sigma}P^{(2)}_{\mu\nu\rho\sigma}=\frac{1}{2}(d+1)(d-2)\,, \tag{3.96}$$

and with that proceed to calculate the contribution of the massive spin-2 ghost $\tilde{h}_{\mu\nu}$ to the one-loop effective potential through the usual CW approach to find

$$\begin{aligned}V^{(1)}_{\mathrm{h}}(S)&=-\frac{i}{2}\lim_{d\to4}\left(\mu^{4-d}\int\frac{\mathrm{d}^d p}{(2\,\pi)^d}\,\frac{1}{2}(d+1)(d-2)\ln\left[\frac{p^2-m^2_{\mathrm{gh}}}{p^2}\right]\right)\\&=\frac{5}{64\pi^2}\,m^4_{\mathrm{gh}}\left(\ln\left[\frac{m^2_{\mathrm{gh}}}{\mu^2}\right]-\frac{1}{10}\right)\,,\end{aligned} \tag{3.97}$$

where in the first line we explicitly show the step of dimensional regularization before employing the $\overline{\mathrm{MS}}$ scheme for renormalization.

Now, collecting all the contributions, the quantum effective potential at one-loop level is given by

$$V_{\mathrm{eff}}(S)=\frac{1}{4}\,\lambda\,S^4+V^{(1)}_{\mathrm{s}}(S)+V^{(1)}_{\mathrm{h}}(S)+V_0\,, \tag{3.98}$$

where the first term denotes the tree-level scalar contribution from Eq. (3.79) and, in analogy to before in Section 3.1, specifically Eq. (3.19), we have included the constant contribution V_0 to demand a classically vanishing zero-point energy to avoid problem of the cosmological constant. Here, it is worth noting that our framework ensures that the cosmological constant remains zero in the Einstein frame when canceled in the Jordan frame as in Eq. (3.98). Further following the example of Sections 3.1 and 3.3 we first discuss the spontaneous breaking of scale-invariance by quantum effects in $V_{\text{eff}}(S)$, the resulting dynamically generated M_{Pl} and lastly the corresponding inflation predictions and constraints on this toy model. We will refrain from discussing couplings to the SM and solutions to further problems like dark matter and just remark that (potentially) they can be included in the same way as discussed above in Sections 3.2 and 3.4.

To calculate the VEV of S via Eq. (3.98), we first rewrite the effective potential, factoring out dependencies on S

$$V_{\text{eff}}(S) = V_0 + \left(C_1 + C_2 \ln\left[\frac{S^2}{\mu^2}\right]\right) S^4 \,, \tag{3.99}$$

where C_1 and C_2 depend only on the dimensionless couplings λ, β, γ, and κ of Eqs. (3.78) and (3.79) and can be easily computed from the expressions given for the individual contributions to V_{eff} in Eq. (3.98), i.e. Eqs. (3.91) and (3.97) and are displayed in Appendix A.2. With the effective potential in this form, it is easy to minimize

$$\left.\frac{\partial V_{\text{eff}}(S)}{\partial S}\right|_{S=\langle S\rangle} = 0, \qquad \left.\frac{\partial^2 V_{\text{eff}}(S)}{\partial S^2}\right|_{S=\langle S\rangle} > 0 \,, \tag{3.100}$$

such that we obtain the non-zero value for the VEV of S, that spontaneously breaks global scale-invariance, to be

$$\langle S\rangle = \mu \exp\left(-\frac{1}{4} - \frac{C_1}{2\,C_2}\right) \,, \tag{3.101}$$

and in turn, also determines the value of V_0 through

$$V_{\text{eff}}(\langle S\rangle) \stackrel{!}{=} 0 \qquad \Rightarrow \qquad V_0 = \frac{1}{2}\, C_2\, \langle S\rangle^4 \,. \tag{3.102}$$

Lastly, the dynamically generated Planck mass M_{Pl} is again obtained by identification of the effective Einstein-Hilbert term in the effective action S_{eff}

$$S_{\text{eff}} = \int d^4x \sqrt{-g}\left(\frac{1}{2}\, S\Box S - \frac{1}{2}\,\beta\, S^2\, R + \gamma\, R^2 - \kappa\, C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} - V_{\text{eff}}(S)\right) \,, \tag{3.103}$$

when evaluated at $S = \langle S\rangle$, such that its renormalization scale dependent value is given by

$$M_{\text{Pl}}^2 = \beta\, \langle S\rangle^2 \,. \tag{3.104}$$

At this point, we want to take a breath, shortly conclude, and compare to the previous calculation Section 3.1.

While a lot of arguments and computational steps are similar to Section 3.1 before, there are three points we want to remark on here. First, and possibly most notable, the radiative spontaneous breakdown of global scale-invariance is realized only with one additional scalar due to the additional contributions to the effective potential from the massive spin-2 ghost DOF, that emerges when gravity is not treated classically but quantum mechanically. This specifically, to our knowledge, is a new result, yet maybe not a surprising one given the previous derivation of a more general version of this phenomenon at the level of the RG-equations by [69, 275] that also includes a dynamical solution to the appearance of a cosmological constant. Hereof, and secondly, we again require the cosmological constant to vanish after RSSB already in the Jordan frame, thus ensuring that it stays zero also in the Einstein frame. Hence, we again choose to not address this problem in more detail and rather avoid the discussion. Third and last, one might wonder about the "missing" $\propto R$ contributions to $M_{\rm Pl}$ when comparing Eq. (3.104) with Eq. (3.20). Since we are not treating gravity purely classical anymore, these $\propto R$ contributions from the expansion in powers of R, are implicitly included in the gravitational contributions via m_ϕ^2 and $m_{\rm gh}^2$ of Eq. (3.86), now sourced due to the expansion of the metric $g_{\mu\nu}$ and integrating out its quantum fluctuations.

Now, that we have discussed the spontaneous breaking of scale symmetry in the Jordan frame, we find ourselves in a similar situation to Section 3.3.1. Indeed, our effective potential will again prove to be a proper potential to successfully realize inflation. Since the approach is so similar, we will only briefly go over the intermediate steps and instead refer to the previous treatment for more details. We start with the effective action in Jordan frame of Eq. (3.103), introduce an auxiliary scalar field to remove the R^2 dependency, transform to the Einstein frame through a Weyl rescaling, and again find the auxiliary field to be dynamical as the scalaron. Furthermore, we again find a valley structure in this two-scalar potential, and choose one of two possible flat contours along either field to arrive at an effective one-field inflation potential. Similar to before, the choice of contour has only an insignificant influence on the inflationary predictions, as both contours turn out to be valid for our considered parameter space. Without loss of generality, we then choose to eliminate the scalaron, such that we are left with the final effective one-field Einstein frame action

$$S_{\rm eff}^{\rm E} = \int {\rm d}^4x \sqrt{-g} \left(-\frac{1}{2}\, M_{\rm Pl}^2\, R - \kappa\, C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} + \frac{1}{2}\, F(S)^2\, S\Box S - V_{\rm inf}(S) \right) \,, \tag{3.105}$$

with the modification to the kinetic term of S due to $F^2(S)$ given by

$$F^2(S) = \frac{(1+4\,A)\,B + \frac{3}{2}\,M_{\rm Pl}^2\,[(1+4\,A)\,B' + 4\,A'\,B]^2}{[(1+4\,A)\,B]^2}\,, \tag{3.106}$$

where primes are understood to be derivatives w.r.t. S. The dimensionless shorthand functions A, B are similar to the notation in Section 3.3.1, depending on the dimensionless couplings, and S but now given by

$$A(S) = \frac{4\,\gamma\, V_{\text{eff}}(S)}{B(S)^2 M_{\text{Pl}}^4}\,, \qquad\qquad B(S) = \frac{\beta\, S^2}{M_{\text{Pl}}^2}\,. \tag{3.107}$$

Collecting the results above we can obtain the canonically normalized $\hat{S}$ through the integration shown in Eq. (3.59) and most importantly, the final inflaton potential $V_{\text{inf}}(S)$ can be written as

$$V_{\text{inf}}(S) = \frac{V_{\text{eff}}(S)}{B(S)^2 + 16\, M_{\text{Pl}}^{-4}\,\gamma\, V_{\text{eff}}(S)}\,, \tag{3.108}$$

now only depending on the dimensionless couplings λ, β, γ, κ and the single inflaton field S. V_{inf} allows us to calculate the slow-roll parameters ϵ, η as in Eqs. (3.62) and (3.63) and therefore also the predictions for the CMB observables, the amplitude A_{s} and the spectral tilt of the scalar power spectrum n_{s} as well as the tensor-to-scalar ratio r as shown in Eq. (3.40). We show the predictions for n_{s} and r in Fig. 3.7 for couplings in the parameter space

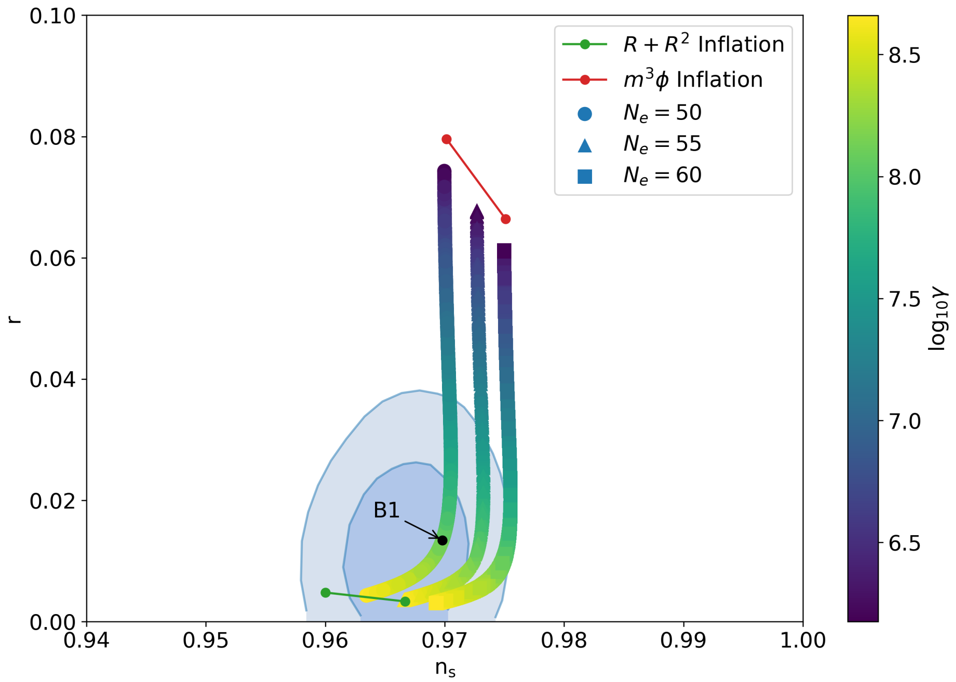
$$\lambda = 0.005\,, \qquad \beta \in [10^3, 10^4]\,, \qquad \gamma \in [10^3, 10^9]\,, \qquad \kappa \in [10^2, 10^{3.25}]\,. \tag{3.109}$$

While μ is fixed by demanding the experimentally observed value for M_{Pl} through Eq. (3.104), the dimensionless couplings of the action Eq. (3.103) are chosen according to Eq. (3.109), to assure that the A_{s} constraint Eq. (3.66) of [23] is fulfilled, all while keeping the theory perturbative. In other words, we ensure that the logarithms (e.g. $\ln[S/\langle S\rangle]$) keep to perturbative values during inflation, thus allowing us to neglect RG-running effects as they are insignificant. In addition to displaying γ (see Fig. 3.7), we also display the coupling of the Weyl tensor squared term κ in Fig. 3.7 to emphasize the C^2 terms contribution. Similar to Section 3.3.2, we find that the predictions for r of our model interpolate between the ones of linear inflation ($m^3\phi$) as an upper and Starobinsky ($R + R^2$) as a lower limit. Both other models' predictions are also displayed in Fig. 3.7, where the circles on each side represent $N_e = 50$ (left) and $N_e = 60$ (right), respectively. All in all, we find a considerably sized parameter space that allows for RSSB of global scale invariance while being in good agreement with the currently strongest constraints on inflation set by the Planck and BICEP/Keck collaborations [23, 24].

Furthermore, the benchmark point ("B1") corresponds to the coupling values of

$$\text{B1}: \ \lambda = 0.005\,, \qquad \beta = 5.62\times 10^2\,, \qquad \gamma = 1.22\times 10^8\,, \qquad \kappa = 837\,, \tag{3.110}$$

and to get a better grasp on contributions of the gravitational DOFs, we compute the to "B1"-values corresponding masses $m_\phi(S)$ and $m_{\text{gh}}(S)$ evaluated at the global



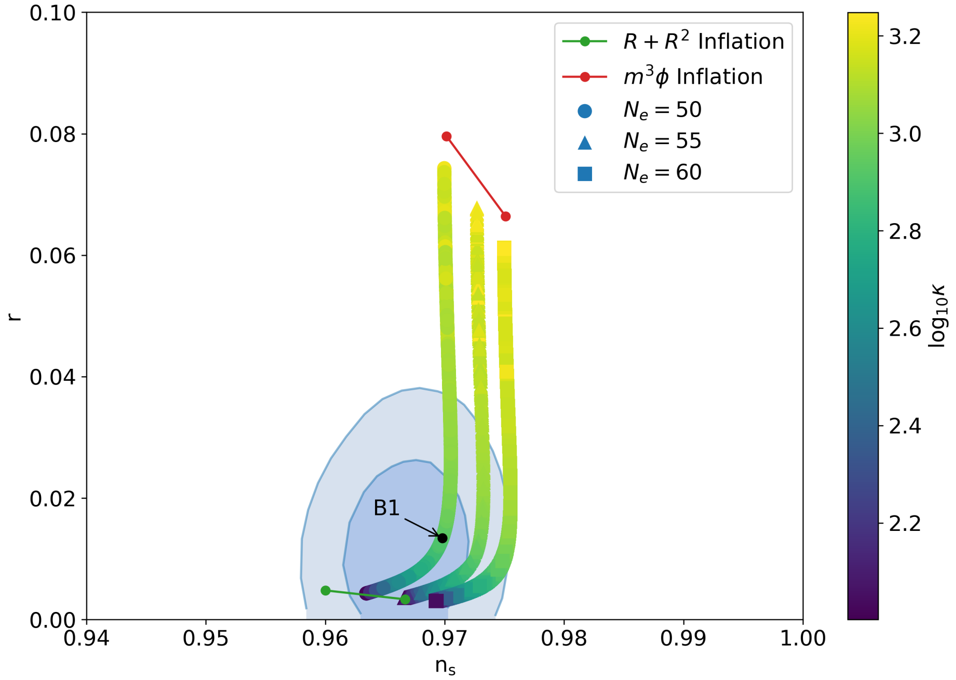


FIGURE 3.7: Predictions for the scalar spectral tilt n_s, the tensor-to-scalar ratio r with varying numbers of e-folds N_e, and γ (top) or κ (bottom) are displayed as color grading. The points shown have fixed $\lambda = 0.005$, while β, γ, and κ are taken randomly from Eq. (3.109), such that Eq. (3.66) is fulfilled. We include the Planck TT,TE,EE+lowE+lensing+BK18+BAO 68% and 95% CL regions from [23, 24]. Additionally, the predictions of the Starobinsky model (green) and linear inflation (red) are included. Corrections to r due to the C^2 term [276–278] have not been included here, though they are accounted for in Fig. A.1.

minimum $\langle S \rangle$ through Eq. (3.86)

$$m_\phi^{\rm B1}(\langle S \rangle) \simeq 6.35 \times 10^{13}\,\text{GeV}\,, \qquad m_{\rm gh}^{\rm B1}(\langle S \rangle) \simeq 4.21 \times 10^{16}\,\text{GeV}\,. \tag{3.111}$$

These exemplary values serve as a good order of magnitude estimate for the entire considered parameters space Eq. (3.109), as they sit in the middle of their range of values, allowing only for deviations of roughly one order of magnitude. Furthermore, since both m_ϕ and $m_{\rm gh}$ display the same dependence on β, yet also dependencies on the remaining curvature couplings $m_\phi \propto \gamma^{-1}$ and $m_{\rm gh} \propto \kappa^{-1}$ (c.f. Eq. (3.86)), we see in Fig. 3.7, that large γ as well as small κ are present for predictions of the experimentally preferred low r region. Thus, we can novelly conclude, that the massive spin-2 ghost contribution, in our scale-invariant model, is not only beneficial but crucial for successfully realizing the spontaneous breaking of scale invariance while maintaining the prediction of a small tensor-to-scalar ratio r. Apart from phenomenological implications, there have been recent works that show that there is also a theoretical requirement for the spin-2 ghost to be (very) heavy in models similar to the one considered here [61].

Lastly, the Weyl tensor contribution in Eq. (3.103) causes additional corrections to the tensor-to-scalar ratio r prediction due to non-trivial tensor perturbations in a de Sitter background [275–278]. Choosing representative field values for S, i.e. $S = S_{\rm CMB}$, we can calculate these corrections using the slow-roll approximation to be

$$r_{\rm corr} = r\left(1 + \frac{2\,H^2}{m_{\rm gh}^2}\right)^{-1} \simeq r\left(1 + \frac{2\,V_{\rm inf}(S_{\rm CMB})}{3\,M_{\rm Pl}^2\,m_{\rm gh}^2(S_{\rm CMB})}\right)^{-1}, \tag{3.112}$$

and find that the corrections are rather small, with a maximal effect of $\approx 10\%$. Thus, the predictions for the same parameter space as before in Eq. (3.109) including the correction factor in Eq. (3.112) are shown Fig. A.1. We still find our model's predictions to be fully compatible with the most stringent constraints from the latest observational data [24]. Consequently, the above conclusion that the inclusion of the C^2 term already at the tree-level can lead to a spin-2 ghost contribution with a large mass and thus allows for the radiative breakdown of scale-invariance with phenomenological favored inflation predictions while dynamically generating the correct $M_{\rm Pl}$, is not altered by these corrections and therefore still stands as the (arguably) most notable outcome of the considerations regarding this model.

Chapter 4

Conformal Dynamics as UV Completion to Little Higgs

So far we have discussed the hierarchy problem of including gravity in the Standard Model only for a fundamental Higgs boson. But what if we question that assumption and consider the Higgs to be non-fundamental, to be composite? At first glance the answer seems simple, we avoid the hierarchy problem (as introduced in Section 2.4) when the Higgs is identified as a bound state of QCD-like strong dynamics with a mass that sits roughly at its confining scale $m_h \sim \Lambda$. Yet, the complete answer, as is often the case, is not so simple and not without significant caveats, as the example of the original technicolor model teaches us.

But first, the basic principle for the non-elementary Higgs to avoid the hierarchy problem is that when probed at energies below the confinement scale Λ, the Higgs will appear as fundamental, thus will exhibit quadratically sensitive corrections and take part in electroweak symmetry breaking as usual, but at energies above the confinement scale, the fact that the bound state has a finite size will be probed as well, hence introducing form factors and thus resulting in quickly converging corrections. Furthermore, one now may wonder whether the confining scale $\Lambda \ll M_{\text{Pl}}$ itself induces another hierarchy problem. This is indeed not the case, as Λ is generated through the dimensional transmutation of QCD-like dynamics, where instead of the dimensionless coupling fulfilling a criticality condition (oftentimes related to smallness) to spontaneously induce a scale as described in Section 2.3, instead the coupling grows large to non-perturbative values at lower energies and thus allows the forming of bound colorless states. This process, also known as confinement, provides naturally exponentially separated scales via the logarithmic RG-running of the dimensionless coupling constants via their β-functions. While the process of scale generation is fairly similar to the considerations of dynamically induced mass scales before, the composite nature of the Higgs here means that it is not present in the particle spectrum at energies above the confinement scale and therefore warrants the embedding into a UV complete theory already above Λ. In contrast, the SM on its own is UV-complete at least up to M_{Pl} when the embedding of quantum gravity becomes necessary. This specifically is where conformal symmetry comes in handy, as one can arrange for only non-Abelian gauge couplings to be fundamental at high energies, such that the theory automatically exhibits the proper fixed points for UV completion.

This chapter is structured as follows, based on the excellent reviews of [15, 17], we first revisit the fundamentals of the Higgs as a non-fundamental scalar in Section 4.1, shortly introducing (historically) important models like technicolor to later arrive at the class of so-called *Little Higgs* models. Thereafter, in Section 4.2 we shortly touch on the general aspects of UV completion and then introduce conformal dynamics as a general framework for the UV completion of Little Higgs models. Finally, we consider the example of UV completing the *Bestest Little Higgs* via strongly coupled conformal dynamics and comment on the corresponding (phenomenological) implications.

4.1 The Higgs as a Non-fundamental Scalar

When describing the electroweak symmetry-breaking, the original composite Higgs models (CHMs) by Georgi and Kaplan [279–285] provide an alternative "middle ground" to the Higgs model, where the Higgs is a fundamental scalar of the SM and simple technicolor [177, 286], which similar to QCD does not contain a scalar at all. They realized that if the Higgs arises as a pseudo-Nambu-Goldstone boson (pNGB) of the global symmetry breaking of strong dynamics, it can be naturally lighter than the other emerging particles. While its composite nature lets them avoid the hierarchy problem of the SM Higgs model, containing the light Higgs boson in the spectrum of physical particles at low energies ensures that electroweak precision tests (EWPT) are satisfied, contrary to technicolor models, which struggle specifically with this aspect. Thus, as we ultimately want to introduce a specific class of CHMs, we think it instructive to first present the fundamental ideas of technicolor models, to then later understand how its issues lead to the formulation of CHMs.

4.1.1 Technicolor

In contrast to the Higgs model, technicolor is fundamentally based on the QCD-like breaking of symmetry. While not QCD itself can account for the breaking of electroweak symmetry, technicolor employs a heavily analog concept. N_{TC} technifermions Q of a new force are charged under the forces strong non-Abelian gauge group G_{TC}, the so-called *technicolor*, they experience spontaneous breaking of their global chiral symmetry down to their vectorial subgroup

$$SU(2)_L \times SU(2)_R \to SU(2)_V \; , \tag{4.1}$$

due to the (assumed) confinement $\langle \bar{Q}Q \rangle$ of this new force rather than the color force of QCD and consequently, generate the required Goldstone bosons. Gauging the electroweak part of the symmetry then gives mass to the SM electroweak gauge bosons. As in QCD, the scale of confinement f_{TC} determines the mass the SM gauge bosons acquire this way. Indeed, one arrives at proper W and Z masses for

$$f_{\text{TC}} = 246\,\text{GeV} \gg f_{\text{QCD}}\,, \tag{4.2}$$

where we want to note, that even though the majority of m_{W}, m_{Z} is due to the technipions, the QCD vacuum already breaks EW symmetry and thus always contributes to the SM gauge boson masses, however negligible compared to other effects from e.g. technicolor or the (composite) Higgs. Regardless, the SM fermions do not directly interact with the condensate $\langle \bar{Q}Q \rangle$ and thus require an additional mechanism for the generation of their mass. The answer was to extend the technicolor gauge group to the larger gauge group G_{ETC} of the so-called *extended technicolor* (ETC) [287, 288]. This larger gauge group is then broken down to G_{TC} at the scale $\Lambda_{\text{ETC}} > \Lambda_{\text{TC}}$, allowing for an interaction between the SM fermions and the technifermions mediated by the massive ETC bosons. Yet, with the solution to this problem, there came substantial caveats, namely the generation of flavor changing neutral currents (FCNCs), which require heavy suppression to not violate the experimental observations. While there were several attempts to reconcile ETC with electroweak precision observations [288–290], we want to shortly remark here that there exists also the framework of walking technicolor [291], which resolves this issue by introducing a conformal fixed point for technicolor in contrast to the usual asymptotically free theory. Regardless, the discovery of the Higgs boson with a mass at 125 GeV in 2012 at the LHC [9, 10][1], and therefore much lighter than the confinement scale, constitute the most vital issue for technicolor models, which eventually lead to them being regarded as unfavorable or even excluded. Now, after reviewing the basic idea of technicolor, in the following, we will discuss how composite Higgs models generate a light Higgs boson in their physical spectrum with a similar but significantly modified flavor structure.

4.1.2 Composite Higgs

Although the flavor structure and its breaking might resemble the ones considered in technicolor, the general class of composite Higgs models exhibit the fundamental difference, that the condensate of the strong confining dynamics does not directly break electroweak symmetry. Therefore, we start with a large global flavor symmetry G, which is spontaneously broken to the smaller subgroup H_{con} due to the confinement of G at the scale f. So far so technicolor, but in contrast, now the electroweak symmetry $G_{\text{EW}} = SU(2)_L \times U(1)_Y$ is unbroken and therefore contained in H_{con}. Furthermore, G is explicitly broken by weakly gauging another one of its subgroups, H_{g} which also contains the SM electroweak symmetry (EW) group G_{EW}, and

[1] It was verified that the observed particle indeed has the properties of the SM Higgs with subsequent measurements (see e.g. [292, 293]). The most recent analysis of experimental data reaches a precision of 0.09% w.r.t the Higgs mass [294].

thus

$$H = H_{\text{con}} \cap H_{\text{g}} \supset SU(2)_L \times U(1)_Y \,, \tag{4.3}$$

ensuring that EW is unbroken by the strong confining dynamics while simultaneously being gauged. Consequently, at the level of preserved EW symmetry, the theory contains $n_{\text{m}} = \dim H_{\text{g}} - \dim H$ massive gauge bosons and $n_{\text{NGB}} = \dim G - \dim H_{\text{con}} - n_{\text{m}}$ massless Nambu-Goldstone boson (NGB). This symmetry-breaking pattern is shown in Fig. 4.1 (left). Under the consideration of Goldstone's theorem [154] we choose G and H_{con} in such a way that the n_{NGB} include a $SU(2)_L$ doublet which we in turn identify with the Higgs doublet

$$G/H_{\text{con}} \supset (1,2)_{1/2} \,. \tag{4.4}$$

At tree level the Higgs doublet does not exhibit a potential and EW symmetry remains unbroken but since we gauged H_{g}, the SM gauge interactions radiatively induce a Higgs potential, thus breaking EW symmetry at the scale v. Consequently, there exists a separation of the confinement scale f and the scale of EW symmetry breaking v, quantified by the parameter ξ

$$\xi = \left(\frac{v}{f}\right)^2 \,, \tag{4.5}$$

which can be interpreted as the angle of vacuum misalignment[2] (see Fig. 4.1 (right)). While this separation allows for a light Higgs doublet from strong confining dynamics and thus avoids the "original" hierarchy problem as discribed in Section 2.4, it introduces new fine-tuning, much smaller than $M_{\text{Pl}}/v_{\text{EW}}$ and thus often referred to as the *little hierarchy* problem.

Since the mechanism of vacuum misalignment is essential for composite Higgs models to separate confinement and electroweak scale we want to further reiterate this unfamiliar concept by shortly discussing the CCWZ formalism introduced by Callan, Coleman, Wess and Zumino [295, 296] to describe the general low-energy effective (or phenomenological) Lagrangian, the corresponding Nambu-Goldstone bosons and heavy resonances for strongly (or weakly) coupled theories with the generic symmetry breaking of $G \to H_{\text{con}}$[3]. One starts by utilizing the Goldstone theorem to describe the NGB $\Pi^{\hat{a}}$ by defining the exponential Goldstone matrix $U(\Pi)$ to express general local fluctuations in G around the vacuum $\vec{V}$ by

$$\vec{\Phi}(x) = U(\Pi)\ \vec{V} = \exp\left[i\,\frac{\sqrt{2}}{f}\,\Pi_{\hat{a}}\hat{T}^{\hat{a}}\right]\vec{V} \,, \tag{4.6}$$

2 The interpretation of ξ as an angle becomes apparent when one chooses $G = SO(3)$, the group of spatial rotations, see [16].

3 Here we chose the generic $H = H_{\text{con}}$ to be in line with the notation used above.

where $\hat{T}^{\hat{a}}$ denotes only the broken generators of the vacuum and we thus have already omitted all contributions from unphysical DOFs, that drop out as they annihilate with the vacuum $\vec{V}$. While G is an exact global symmetry the above NGBs remain exactly massless and thus do not obtain a potential and a non-zero VEV. Even if one manages to construct $\langle \Pi^{\hat{a}} \rangle \neq 0$, it could be removed by global symmetry transformation of G. Now, as already briefly mentioned above, we introduce explicit breaking of G (by gauging H_g) and hence radiatively generate a potential for the NGBs, along with a mass, ultimately making them into pseudo-Nambu-Golstone bosons (pNGBs). Since G is broken explicitly, their VEV is stable $\langle \Pi^{\hat{a}} \rangle = v^{\hat{a}} \neq 0$ against G transformations and therefore becomes physical. The corresponding pNGB potential determines the scale of v and thus is quite dependent on the specific realization of the above formalism. Realizing a large separation, parameterized by $\xi = v^2/f^2 \ll 1$, is possible by constructing the vacuum with an orientation almost along the direction of symmetry preservation. On one hand, the limit of $\xi \to 1$ corresponds to the so-called maximally broken EW symmetry and is technically not much different from technicolor. On the other hand, the limit of $\xi \to 0$, ultimately leads to the decoupling of the strong confining dynamics, leaving only the pNGB Higgs in the low-energy spectrum while at the same time introducing fine-tuning. Since the confinement at f generates the potential, a VEV of a similar order is "natural", whereas a smaller v exhibits the familiar quadratic sensitivity and therefore requires the fine-tuning

$$\delta_\mathrm{ft} = \frac{f^2}{v^2} \,. \tag{4.7}$$

Now considering $v \approx 10^2\,\mathrm{GeV}$, and the experimental null results for new physics up to TeV energies, like the inevitably generated "heavy" resonances with masses of order f, results in fine-tuning issues of general composite Higgs models, also known

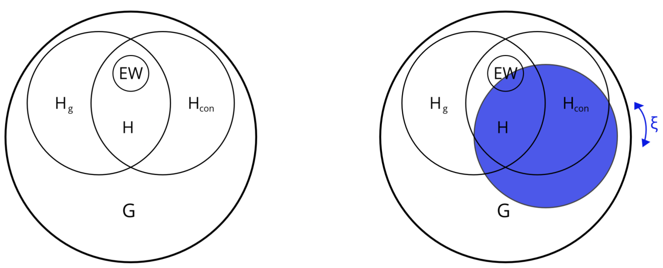


FIGURE 4.1: Symmetry breaking pattern of global symmetry G spontaneously broken to H_con, also explicitly broken to H_g via weakly gauging, Such that $H = H_\mathrm{con} \cap H_\mathrm{g}$ contains the SM gauge group. While the EW symmetry is unbroken at tree-level (left), the one-loop contributions induce a shift in the vacuum and thus breaking of EW symmetry (right).

as the *little hierarchy* problem. The class of composite Higgs models referred to as *Little Higgs* models, additionally addresses specifically this fine-tuning problem and thus we will continue our discussion by introducing the corresponding framework.

4.1.3 Little Higgs

To address this quadratic sensitivity to radiative corrections from the SM gauge bosons, the Little Higgs models follow the symmetry-breaking pattern of general composite models as outlined above but extend the global symmetry G to utilize the so-called *collective symmetry breaking* [40, 297–300] of EW symmetry. In the following, we will discuss this mechanism for removing radiative quadratic sensitive corrections by the example of *The Simplest Little Higgs* [301].

While just altering the gauged content of the theory did not prove sufficient to remove the quadratic sensitive fine-tuning between the confinement scale f and the electroweak VEV v_{EW}, considering a larger global symmetry G as a starting point will allow for multiple non-linear sigma model (nlsm) fields to parameterize the pNGB potential and therefore realize collective breaking. In *The Simplest Little Higgs*, we start out with the strongly interacting sector, invariant under the global product group $G = SU(3) \times SU(3)$. Through confinement at Λ, G is spontaneously broken to its subgroup $H_{\text{con}} = SU(2) \times SU(2)$

$$G \to H_{\text{con}} = SU(3) \times SU(3) \to SU(2) \times SU(2)\,. \tag{4.8}$$

To determine the electroweak symmetry breaking in this framework, we parameterize the NGB potential via the now two nlsm fields Σ_1 and Σ_2[4], which are given by the usual Goldstone matrix $U(\Pi_i)$ (c.f. Eq. (4.6)) of the cosets $SU(3)/SU(2)$, respectively as

$$\Sigma_1 = U\left(\Pi_1\right)\begin{pmatrix}0\\0\\f\end{pmatrix}\,, \qquad \Sigma_2 = U\left(\Pi_2\right)\begin{pmatrix}0\\0\\f\end{pmatrix}\,, \tag{4.9}$$

where we set $f_1 = f_2 = f$ for simplicity in the knowledge that it will not spoil collective symmetry breaking, as we will see in the following. Moreover, G is also broken explicitly to its diagonal (sometimes also called vectorial) subgroup $SU(3)_{\text{D}}$ by only gauging this subgroup

$$G \to H_g = SU(3) \times SU(3) \to SU(3)_{\text{D}}\,, \tag{4.10}$$

[4] Often seen in the context of left-right models, where the degeneracy between either $SU(3)/SU(2)$ coset is usually broken by different couplings to the SM. Nevertheless, this additional assumption is not necessary for the general mechanism of collective breaking and thus we keep our discussion to the general 1, 2 notation, rather than left-right.

such that the leading order nlsm Lagrangian is given by

$$\mathcal{L}_{\text{nlsm}} = |D_\alpha \Sigma_1|^2 + |D_\beta \Sigma_2|^2 \ . \tag{4.11}$$

Here, D_α denotes the $SU(3)_{\text{D}}$ covariant derivative, which does not induce $SU(3)$ breaking spurion contributions in $\mathcal{L}_{\text{nlsm}}$, due to the chosen gauge structure, i.e. that the gauged subgroup of the global G constitutes the full $SU(3)_{\text{D}}$. The absence of these spurion contributions alone ensures that the leading order quadratic divergent terms in the CW-Higgs potential, calculated from Eq. (4.11) are

$$V_{\text{CW}}(H) \propto \text{Tr}\left[\Sigma_1^\dagger \Sigma_1\right] + \text{Tr}\left[\Sigma_2^\dagger \Sigma_2\right] \ , \tag{4.12}$$

and therefore do not depend on the physical Higgs. This allows us to remove these divergent contributions in the process of renormalization. While the quadratic sensitivity is then dealt with, the enlarged global symmetry G needs to provide enough "uneaten" NGB to identify the SM Higgs and thus properly break EW symmetry. Indeed, the breaking $G \to H_{\text{con}}$ results in

$$n_{\text{NGB}} = 2\ \dim\left[SU(3)\right] - 2\dim\ \left[SU(2)\right] = 2\left[\left(3^2-1\right) - \left(2^2-1\right)\right] = 10\ . \tag{4.13}$$

Following the symmetry breaking pattern in Fig. 4.1, and since we only consider the $SU(2)$ part of the EW symmetry and treat $U(1)_Y$ separately, we find $H = H_{\text{con}} \cap H_{\text{g}} = SU(2)$ and thus the number massive gauge bosons n_m to be

$$n_m = \dim\left[H_g\right] - \dim[H] = \left(3^2-1\right) - \left(2^2-1\right) = 5\ . \tag{4.14}$$

This leaves exactly $n_{\text{NGB}} - n_m = 5$ uneaten NGB, four give a complex doublet $H \propto \Pi_1 - \Pi_2$ to be identified with the SM Higgs and the remaining one denotes a real scalar η. Furthermore, in the next step, we will see, that not any enlargement of G is acceptable, but rather one that induces multiple nlsm fields in the Lagrangian, such that the symmetry can be broken collectively, preventing the removed quadratic divergences from reappearing.

Looking for the leading non-quadratic-divergent contributions to the CW-Higgs potential, we turn specifically to the interaction of the nlsm fields Σ_i with the SM gauge bosons W_α in Eq. (4.11), namely

$$\mathcal{L}_{\text{nlsm}} \ni |g\ W_\alpha\ \Sigma_1|^2 + |g\ W_\beta\ \Sigma_2|^2\ , \tag{4.15}$$

where we want to remark that both terms' invariance under the gauged $SU(3)_{\text{D}}$ implies a global $SU(3)$ invariance. In here lies the essence of collective breaking, each term individually in Eq. (4.15) exhibits an extended global $SU(3) \times SU(3)$ symmetry and only the existence of both terms in Eq. (4.15) ensures that the global symmetry is broken to $SU(3)_{\text{D}}$, the symmetry is *collectively* broken. Note here the importance of gauging the diagonal subgroup $SU(3)_{\text{D}}$, as opposed to one of the $SU(3)$, since

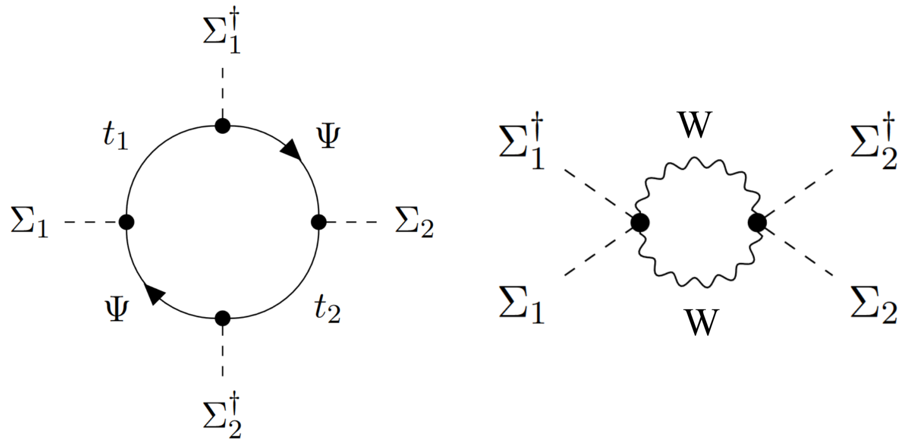


FIGURE 4.2: Leading divergent contributions to V_{CW} from contributions that contain a combination of both $\Sigma_{1,2}$ for the fermion (left) and the gauge (right) sector, respectively.

the latter would lead to a separation of the massive $SU(3)$ gauge bosons and the NGBs in symmetry space such that the NGBs would not obtain a potential and simply remain massless. Indeed when calculating radiative corrections one finds exact cancellations between same-spin particles of the SM gauge contributions that only involve one of the terms in Eq. (4.15) (see e.g. [17]). Consequently, the radiative corrections to the pNGBs' potential, e.g. see Fig. 4.2 (right), only contain contributions that contain both Σ_1 and Σ_2 and therefore (simple power-counting) are only logarithmically divergent, such that the leading order CW-potential contribution is given by

$$V_{\mathrm{CW}}(H) \propto \frac{g^4}{16\,\pi^2} \left|\Sigma_1^\dagger \Sigma_2\right|^2 \ln\left[\frac{\Lambda^2}{\mu^2}\right] , \tag{4.16}$$

which after expanding $\Sigma_{1,2}$ in terms of the doublet H

$$\left|\Sigma_1^\dagger \Sigma_2\right|^2 \sim f^2 - 2\,H^\dagger H + \dots , \tag{4.17}$$

can generate an appropriate SM Higgs mass when f is roughly $\mathcal{O}(\mathrm{TeV})$.

For now, we have only focused on the theory's gauge structure to find the proper symmetry-breaking pattern to successfully induce a naturally light Higgs boson as a pNGB of strong confining dynamics. Nevertheless, the Higgs receives its dominant SM radiative corrections from the fermions, specifically the top quark[5]. To make sure, that these are not quadratically divergent and thus bring back the unwanted quadratic sensitivity, we continue inspired by the results from the gauge sector above, i.e. aiming for same-spin partner cancellations by introducing top partners $(\mathbf{3},\mathbf{1})+(\mathbf{1},\mathbf{3})$ of the global $SU(3)\times SU(3)$. The explicit breaking of G to $SU(3)_{\mathrm{D}}$

5 The above discussed contributions to the Higgs' CW potential should therefore rather read $V_{\mathrm{CW},g}$ to make it clear, these are not the only contributions.

results in its corresponding **3** containing the SM $SU(2)_L$ doublet Q_L and the top partner T_L

$$\Psi = \begin{pmatrix} Q_L \\ T_L \end{pmatrix} = \begin{pmatrix} t_L \\ b_L \\ T_L \end{pmatrix} , \tag{4.18}$$

whereas the **1** includes the two right handed $t_{1,2}$. The top quark and therefore also the dominant contribution to the fermion-nslm Lagrangian is then given by

$$\mathcal{L}_{t,nslm} = \lambda_1 \, \bar{\Psi} \, \Sigma_1 \, t_1 + \lambda_2 \, \bar{\Psi} \, \Sigma_2 \, t_2 \, . \tag{4.19}$$

In the same manner as above for the gauge sector, one finds that a global $SU(3) \times SU(3)$ is collectively broken to a global $SU(3)_{\mathrm{D}}$, thus again inducing cancellation of the quadratic divergent contributions that contain only one of the two terms in Eq. (4.19). Subsequently, the leading divergent contributions come from contributions involving both terms of Eq. (4.19), e.g. see Fig. 4.2 (left). These give the logarithmically divergent contributions of

$$V_{\mathrm{CW},f}(H) \propto \frac{\lambda^4}{16 \, \pi^2} \, \left| \Sigma_1^\dagger \Sigma_2 \right|^2 \, \ln \left[\frac{\Lambda^2}{\mu^2} \right] \, . \tag{4.20}$$

In summary, the above result demonstrates that this framework of collective breaking successfully generates a light SM Higgs boson naturally separated from the confining scale of the strongly coupled extended symmetry G by ensuring that the newly introduced partner particles exactly cancel the quadratic divergent contributions to the radiatively generated Higgs potential. As shown above this cancellation is achieved by introducing appropriate larger global symmetry G, here shown in the form of a product group.

As a promising framework with rich phenomenology, since the first successful *Little Higgs* model [40], there have been numerous realizations of this Little Higgs framework, all based on the same concept of collective symmetry breaking to generate a pNGB Higgs [41, 297, 298, 301–318]. Some employ product groups for G, as seen above for *The Simplest Little Higgs* ($SU(3) \times SU(3)/SU(3)$) [301], others are successful without relying on such, e.g. *The Littlest Higgs* ($SU(5)/SO(5)$) [297], but the general class of Little Higgs models generally face two major problems. First, to match experimental observations of the physical Higgs boson, they are required to include a mechanism to induce a quartic Higgs coupling which often leads to problems with the experimental constraints on ρ parameter[6]. Second, the experimental results for electroweak precision observables and the null results for the inevitably generated heavy resonances, i.e. the newly introduced particles with masse $\mathcal{O}(\mathrm{TeV})$,

[6] The strong experimental constraints on $\rho \sim 1$ are often interpreted to make require a custudial $SU(2)$. This is not a general case, in the absence of a custodial $SU(2)$ one has to merely compute the corrections to ρ and thus show compatibility with the experimental constraints explicitly.

result in required fine-tuning between specifically the top quark and the gauge sector. Especially *The Bestest Little Higgs* [319], based on the $SO(6) \times SO(6)/SO(6)$ coset space, seems to deal with these issues particularly well, such that we will later in Section 4.2.2 use it as an example to demonstrate UV completion with strongly coupled conformal dynamics.

4.2 Conformal Dynamics as UV Completion

While the Higgs not as a fundamental scalar, whether in the form of composite Higgs models or as a Little Higgs as reviewed in the section above Section 4.1, denotes an interesting and promising avenue to address the gauge hierarchy problem, they all are effective (low-energy) theories, only valid up to their cutoff scale $\Lambda \simeq 4\pi f \sim \mathcal{O}(\text{TeV})$. Therefore, they require UV completion, or in other words, embedding into a high energy theory that provides a proper description (often perturbativity) up to at least M_{Pl}, where one runs into the inevitable embedding into quantum gravitational theories. At the same time, the UV completion must provide the appropriate symmetry framework[7] at Λ to realize for example the collective breaking of a Little Higgs model. The UV completion often has to reconcile the perturbativity in $\Lambda < E < M_{\text{Pl}}$ with the non-perturbative breaking (confinement) that generates Λ istself. Furthermore, to avoid spoiling the "rich" phenomenology of the low-energy model one oftentimes requires the UV completion not to introduce new particles with small masses $m_{\text{new}} < f$. Thus, given the popularity and sheer amount of CHM or Little Higgs models, there have been lots of proposed UV completions for a low-energy Higgs that is not a fundamental scalar. Specifically for the framework of Little Higgs models, they can be generally organized into two categories, supersymmetric and non-supersymmetric realizations.

Theories which employ supersymmetry (SUSY) can do so in several different ways and generate the Higgs as a pNGB. Some give supersymmetric low-energy theories [313, 320–327], or employ extra dimensions and their compactification [328]. Others realize the Little Higgs as a composite particle from strong [312, 329] or weakly [330] coupled high energy SUSY theory.

The non-supersymmetric UV completions mostly rely on strongly coupled dynamics in the spirit of technicolor or QCD but with modified symmetry content [312, 329, 331, 332], others employ the AdS/CFT correspondence to generate 4D strongly coupled systems from 5D (so-called) holographic Little Higgs models, e.g. [309, 316, 318] and again others do not rely on strong dynamics but rather on the simple group [317] or a weakly coupled system [314] which itself arises from collective symmetry breaking.

In the following, we combine the two ideas of a non-fundamental Higgs and conformal symmetry, both used to address the gauge hierarchy problem, to propose a

[7] Additional to the global symmetry G and its breaking structure, this also includes all other symmetries required of the low-energy to respect, e.g. scale invariance.

UV completion from conformal strongly interacting dynamics for Little Higgs models. We will demonstrate the general framework in Section 4.2.1 and as an example we review the UV completion of *The Bestest Little Higgs* in Section 4.2.2

4.2.1 General Conformal Framework for Little Higgs Models

Generally, utilizing conformal dynamics to UV complete Little Higgs models, one naturally needs to address the issue of its anomaly, as introduced in Section 2.2. While the anomalous breaking of conformal symmetry can be beneficial to address for example the gauge hierarchy problem itself, here, we entertain the notion of a non-fundamental Higgs to remedy said problem and thus want to avoid contributions from anomalous breaking. To avoid the conformal anomaly altogether is non-trivial but possible by carefully adjusting the symmetry content, specifically the gauge sector, and the representations [333]. This is where the combination with a Higgs as pNGB comes in to ensure that the required UV fixed points (FPs) are present. The emergent nature of the Higgs results in only gauge couplings being fundamental, thus scalar and Yukawa couplings are not present in the UV, and simply by choosing the UV gauge theory to be non-Abelian we ensure that the theory contains the required FPs. For the theory to exit the high-energy conformal phase at Λ, we employ soft (explicit) breaking and find confinement of the strongly coupled dynamics close to Λ, thus generating pNGB related to the spontaneous breaking of chiral symmetry [133, 134, 334] and eventually the physical Higgs via the collective breaking mechanism of Little Higgs models (see Section 4.1.3).

More specifically, to UV complete a Little Higgs model, which relies on the symmetry-breaking coset G/H, we assume the global symmetry G to be the general chiral product symmetry

$$G = SU(N)_L \times SU(N)_R \,, \tag{4.21}$$

where N denotes the number of Dirac fermion flavors. While some Little Higgs are not based on G to be a $SU(N)$ product group (c.f. Section 4.1.3), after introducing the conceptual symmetry-breaking pattern, we will argue that any Little Higgs of a symmetry-breaking coset as above with a weakly gauged subgroup F can be UV complete this way. Now, for G as in Eq. (4.21), we introduce the symmetry of the strongly coupled conformal field theory (CFT) to be $SU(N_c)$, which after exiting the conformal FP at Λ, confines and thus spontaneously breaks G to its diagonal subgroup $H = SU(N)_D$. To ensure conformal dynamics at energies above Λ, we extend the global chiral symmetry by N_m Dirac fermion flavors, such that the theory exhibits the global

$$SU(N_f)_L \times SU(N_f)_R \,, \tag{4.22}$$

with $N_f = N + N_m$. This puts us in the so-called conformal window of non-supersymmetric gauge theories [335, 336]

$$\frac{7}{2} \lesssim \frac{N_f}{N_c} \lesssim \frac{11}{2} \,, \tag{4.23}$$

by adjusting N_m depending on N and N_c. Consequently, this guarantees that at $E > \Lambda$ our theory undergoes a phase of strongly coupled conformal dynamics. The N Dirac fermions are considered to be massless and in the following denoted by ψ_i with $i = \{1, \ldots, N\}$, whereas the N_m flavors carry mass M and therefore constitute the massive Dirac fermions χ_j with $j = \{1, \ldots, N_m\}$. Both ψ_i and χ_j are so-called technifermions in the fundamental representation of the gauge $SU(N_c)$, such that we also include $\hat{\psi}_i$ and $\hat{\chi}_j$ in the antifundamental or conjugate representation. Given the massive nature of χ_j, the UV Lagrangian contains a term of the form

$$\mathcal{L}_{\text{UV}} \supset -M \, \hat{\chi} \, \chi \,, \tag{4.24}$$

describing the aforementioned (soft) explicit breaking of conformal symmetry. For a scaling dimension $d \leq 4$ of the bilinear operator $\hat{\chi} \, \chi$ constitutes a sufficient deformation of the conformal dynamics and thus the theory exists the conformal FP by softly breaking conformal symmetry at

$$\Lambda \equiv M^{1/(4-d)} \,. \tag{4.25}$$

The theory then enters the confining phase at $E < \Lambda$ with now only N flavors and N_c colors. Since we assume our CFT to be strongly coupled the now QCD-like theory directly confines, allowing us to identify $\Lambda \simeq \Lambda_{\text{con}}$, which need not be the case when not considering strongly coupled CFT. Said confinement means a condensate of technifermions ψ (and $\hat{\psi}$) is formed

$$\langle \hat{\psi}^\alpha \psi_\alpha \rangle \sim \frac{\Lambda^d}{16\pi^2} \delta_{ij} \,, \tag{4.26}$$

with α being the $SU(N_c)$ color and i, j the fermion flavor index $i, j = \{1, \ldots, N\}$. The above condensate breaks the flavor symmetry G to its diagonal subgroup H

$$SU(N)_L \times SU(N)_R \to SU(N)_D \,, \tag{4.27}$$

and therefore generate $n_{\text{NGB}} = \left(N^2 - 1\right)$ Nambu-Goldstone bosons in the process. Since ψ and χ belong to the same symmetry before breaking of conformal symmetry, their bilinear operators $\hat{\psi}\psi$ and $\hat{\chi}\chi$ exhibit the same scaling behavior. This scaling behavior is bounded from below by requiring unitarity to $d \geq 1$, where the exact $d = 1$ would correspond to a (free) elementary or fundamental scalar field and thence in our case $d > 1$. At this point, we have introduced a UV completion via strongly coupled conformal dynamics which generates a symmetry structure that allows for the successful implementation of Little Higgs models. Yet, there are two open questions to discuss, which are essential for the above-mentioned framework. These are, firstly, the actual value of the scaling dimension d, which is determined by the CFT and has far-reaching consequences for the validity of addressing not only the gauge hierarchy problem but also the flavor puzzle. Secondly, the generality of the framework, namely that one can UV complete all known Little Higgs models in this way, regardless of whether they are based on chiral product symmetries as assumed above.

First, regarding the scaling dimension d, constructing the physical Higgs to be a composite of the confinement of the technifermions ψ, the Higgs operator $\mathcal{O}_{\rm H} \sim \hat{\psi}\psi$ itself also has scaling dimension $d > 1$ and thus the lowest gauge-invariant Higgs operator $\mathcal{O}_{\rm H}^\dagger \mathcal{O}_{\rm H}$ with scaling dimension Δ would reintroduce quadratic divergencies and thus the gauge hierarchy problem in the limit of $d \to 1$, since it would become weakly coupled and therefore exhibit

$$\lim_{d\to 1} \Delta = 2\,d\,. \tag{4.28}$$

To avoid spoiling the removal of quadratic divergencies by the Little Higgs collective breaking, we require the gauge-invariant Higgs operator $\mathcal{O}_{\rm H}^\dagger \mathcal{O}_{\rm H}$ to be irrelevant, such that it scales like $\Delta > 4$ [133]. The calculation for general CFTs in the limit $d \to 1$ by [337, 338] arrives at

$$\Delta \lesssim 2d + \mathcal{O}(\sqrt{d-1})\,, \tag{4.29}$$

implying that at least $d \gtrsim 1.5$ to ensure $\Delta \gtrsim 4$. Since the results of [337] do not distinguish between scalar operators that differ only by internal symmetries, the result above in Eq. (4.29) should be taken with a grain of salt. While this gives us a (rough) constraint of d from below, to constrain d from above we first look to models with similar symmetry-breaking patterns, namely technicolor models exhibit $d = 3$ [177, 286], whereas in walking technicolor d is only constrained to $d \gtrsim 2$ [339–344]. As this only gives indications to properly constrain d from above, we turn to use the freedom the CFT construction allows for d to address a common problem of strongly coupled UV completions for Little Higgs models, namely the problem of fine-tuning between flavor and electroweak dynamics. Following the arguments in [133], by the appropriate generation of $d \gtrsim 1$ from the CFT, the flavor dynamics decouple from the electroweak scale up to the flavor scale Λ_t through

$$\Lambda_t \equiv \Lambda \left(\frac{4\,\pi\,v_{\rm EW}}{m_t}\right)^{1/(d-1)}\,, \tag{4.30}$$

where Λ denotes the conformal (and therefore also the confining) scale, $v_{\rm EW}$ is the electroweak VEV and m_t denotes the top-quark mass. We see that d close to one leads to a larger decoupling, thus softening the (Little Higgs) flavor puzzle. Consequently, to address both the gauge hierarchy problem and the flavor puzzle, we have to combine the requirements on d of Eq. (4.29) and Eq. (4.30), such that in the end we require d to be large enough to guarantee the gauge-invariant scalar operator's $\mathcal{O}_{\mathrm{H}}^\dagger \mathcal{O}_{\mathrm{H}}$ irrelevance and close enough to one to effectively decouple the flavor dynamics.

Second, regarding the generality of the presented UV completion given that above we assumed some specific (chiral product) group structure for G, we will show how to extend any general Little Higgs to fit the UV completion via conformal dynamics as introduced above, without altering the low-energy physics. The main idea is based on the observations of [329, 331, 345] and also [332], that Little Higgs models that rely on the coset G/H with a weakly coupled gauge subgroup $F \subset G$ (theory-A) exhibit the same low-energy physics as a two-site nonlinear sigma model with a global product symmetry $G \times G$ (or G^2), which is spontaneously broken to subgroup G and gauging the subgroup $F \times H$ in the limit of large H gauge coupling (theory-B). The latter case resembles the one introduced above, where the bifundamental (techni-) fermion's $SU(N_c)$ or $Sp(2N_c)$ confining strong dynamics UV complete the QCD-like breaking G^2/G, which in turn results in a two-site nlsm, as seen in the Moose diagram in Fig. 4.3. A quick and first check that both theory-A and theory-B indeed give the same low-energy physics is to compare the number of uneaten pNGBs $N_{\mathrm{pNGB}}^{\mathrm{A,B}}$, respectively

$$N_{\mathrm{pNGB}}^{\mathrm{A}} = (\dim[G] - \dim[H]) - \dim[F] \,, \tag{4.31}$$
$$N_{\mathrm{pNGB}}^{\mathrm{B}} = \dim[G] - (\dim[H] + \dim[F]) \,, \tag{4.32}$$

where the first term denotes the broken generators and the second one the by the gauge bosons eaten DOFs. We immediately find that both scenarios generate the same number of pNGB. Furthermore, the limit of large H gauge coupling in theory-B results in correspondingly heavy gauge bosons, which can be integrated out similar to Hidden Local Symmetry [346]. In addition to the number of pNGB bosons, this ensures that both theories exhibit the same light gauge boson dynamics of $F \subset G$. With this we can continue the UV completion like before for theory-B, while still attaining the low-energy physics of theory-A. This way we effectively UV complete theory-A through theory-B. Here we need to remark, that our argument is based on the Moose in Fig. 4.3 and therefore demands the G^2/G breaking to be of

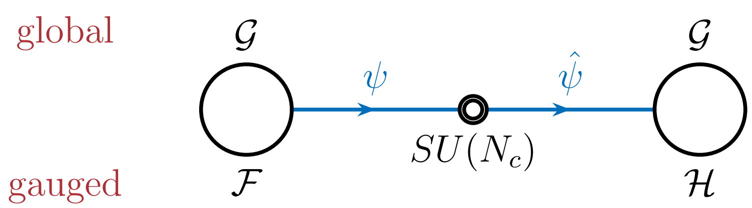


FIGURE 4.3: Moose diagram depicting the symmetry breaking pattern of strongly coupled Little Higgs UV completion.

QCD-like nature, i.e. strongly coupled confining dynamics. Consequently, G needs to be a $SU(N)$ flavor symmetry for N fermions, which in turn are in the fundamental representation of confining $SU(N_c)$ or $Sp(2N_c)$ symmetry. This of course restricts our arguments general validity. Nevertheless, Little Higgs models based on $G = SO(N+1)$ with $N \in 2\mathbb{Z}$ can be expressed in terms of $Sp(N)$, which can then be enlarged to $SU(N)$. To preserve the proper number of pNGB, one can explicitly break the $SU(N)/Sp(N)$ orthogonal symmetry, such that at low energies, the theory exhibits $Sp(N) \sim SO(N+1)$ symmetry. We will shortly discuss the example of $SO(5)^2/SO(5)$ based Little Higgs, i.e. *Minimal Moose Little Higgs* [306], when discussing the exemplary UV completion of *The Bestest Little Higgs* below in Section 4.2.2. Little Higgs models with $SO(N)$ and $N = 2\mathbb{Z}$ can be UV completed in an analog way by utilizing their isomorphic (or larger) SU groups.

With this we have introduced a general mechanism to UV complete all currently known Little Higgs models by employing strongly coupled conformal dynamics, thus simultaneously addressing the flavor puzzle and gauge hierarchy problem while preserving the low-energy physics of Little Higgs models by utilizing the "duality" of symmetry patterns à la theory-A-B.

4.2.2 Conformal Bestest Little Higgs

Having introduced the general concept before, here we want to showcase the UV completion and corresponding low-energy phenomenology for a class of Little Higgs models based on the global symmetry breaking pattern of G^2/G with $G = SU(4)$ (c.f. theory-B in Section 4.2.1) and therefore

$$SU(4)_L \times SU(4)_R \to SU(4)_D \,, \tag{4.33}$$

where $SU(4)_D$ denotes the diagonal (or vectorial) subgroup, whereas the subscripts "L, R" refer to the analogy of the product symmetry to chiral symmetry. Since the above coset is isomorphic to the $SO(6)^2/SO(6)$ coset of *The Bestest Little Higgs* [319] we colloquially refer to the here discussed example as *Conformal Bestest Little Higgs*, even though this symmetry breaking pattern could also refer to other Little Higgs models, depending on the low-energy assumptions. We will return to this point after introducing the general UV completing dynamics when discussing the low-energy implications.

Following the procedure presented in Section 4.2.1, we now choose our confining gauge symmetry, which generates the symmetry breaking pattern of Eq. (4.33), to be $SU(N_c)$ with $N_c = 3$, and therefore add the technifermions ψ with mass M and the massless χ charged under said gauge group. Their corresponding quantum numbers under $SU(N_c)$ and under the $SU(3)_C \times SU(2)_L \times U(1)_Y$ SM gauge group are shown in Table 4.1. We split ψ into the parts which are fundamentals of the electroweak gauge group $SU(2)_L \subset SU(4)_L$, namely $\tilde{\psi}$, and the custodial group $SU(2)'_L \subset SU(4)_L$, denoted by ψ'. Likewise, the conjugate fields $\hat{\psi}$ are charged under subgroups of $SU(4)_R$ and the $U(1)_Y$ hypercharge constitutes the diagonal generators of the custodial $SU(2)$. Furthermore, we assume $N_m = 8$ flavors of χ such that the theory is well within the conformal window (c.f. Eq. (4.23)) and simultaneously believed to give a strongly coupled confining $SU(3)$ gauge symmetry near the conformal FP, in other words after exiting the conformal phase at the scale Λ. The mass term of χ (c.f. Eq. (4.24)) is again responsible for the (soft) explicit breaking of conformal symmetry, such that Λ again is determined by $\Lambda = M^{1/(4-d)}$ (c.f. Eq. (4.25)). After integrating out the heavy χ DOFs, in the effective theory for $E \lesssim \Lambda$ we are only left with the massless ψ and $\hat{\psi}$ which spontaneously break the apparent $SU(4)_L \times SU(4)_R$ by forming the condensate $\langle \hat{\psi}\psi \rangle$. Given the transformation behavior of ψ and $\hat{\psi}$, i.e. as **4** and **4** of $SU(4)_L$ and $SU(4)_R$, respectively, the condensate's transformation behavior follows as

$$\langle \hat{\psi}\psi \rangle = (\mathbf{4}, \bar{\mathbf{4}})_{SU(4)_L \times SU(4)_R} , \tag{4.34}$$

and therefore spontaneously breaks the global flavor $SU(4)_L \times SU(4)_R \to SU(4)_D$ to its diagonal subgroup. The resulting $n_{\text{NGB}} = \dim [SU(4)] = 15$, NGBs transform as the adjoint of $SU(4)_D$, such that we can align the vacuum along the unbroken $SU(4)_D$

$$\langle \hat{\psi}\psi \rangle = \frac{\Lambda^d}{16\,\pi^2} \begin{pmatrix} \mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix} , \tag{4.35}$$

and therefore ensure that EW symmetry is preserved in the presence of $\langle \hat{\psi}\psi \rangle$. Here, Λ denotes the scale of condensation, which can be identified with the conformal symmetry breaking scale due to the strongly coupled conformal dynamics and the orientation of the condensate again depends on the assumption of zero-mass for ψ. Furthermore, d the scaling dimension of the condensate is equal to the scaling dimension of $\hat{\chi}\chi$ due to both ψ and χ being part of the same conformal dynamics (c.f. Section 4.2.1). At this point, we have introduced strongly coupled confining gauge symmetries which exhibit conformal dynamics above the soft conformal breaking scale Λ, while below Λ they generate the appropriate global chiral symmetry which is spontaneously broken to its diagonal subgroup by a condensate that simultaneously preserves EW symmetry to be broken by radiative contributions,

	$SU(N_c)$	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$\tilde{\psi} \equiv \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$	$\square$	**1**	$\square$	0
$\psi' \equiv \begin{pmatrix} \psi_3 \\ \psi_4 \end{pmatrix}$	$\square$	**1**	**1** **1**	$-\frac{1}{2}$ $+\frac{1}{2}$
$\chi \times N_m$	$\square$	**1**	**1**	0

TABLE 4.1: Quantum numbers for the technifermions $\psi = (\tilde{\psi}, \psi')$ and χ under the confining gauge group $SU(N_c)$ and the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$.

therefore allowing to successfully incorporate the collective breaking of Little Higgs models.

But before getting into the specifics of the collective breaking and the low-energy phenomenology, we want to briefly comment on the aforementioned possibility that the here-discussed UV completion can perhaps lead to multiple different Little Higgs models depending on some low-energy assumptions. In particular the $SU(4)^2$ flavor symmetry can also UV complete the *Minimal Moose Little Higgs* [306]. This model relies on the $SO(5)^2/SO(5)$ symmetry-breaking coset. As before mentioned in end of Section 4.2.1, we can utilize that $SO(5)^2/SO(5)$ is isomorphic to $Sp(4)^2/Sp(4)$, and explicitly break "our" $SU(4)^2$ to $Sp(4)^2$ by introducing the non-renormalizable contribution

$$\mathcal{L} \supset \frac{c\, m^2}{\Lambda^{2d-2}} \mathrm{Tr}\left[(\hat{\psi}\psi)\, J\, (\hat{\psi}\psi)^T J \right] , \tag{4.36}$$

with $c \sim 4\pi$ denoting a strong coupling, $m \sim \Lambda$ is a mass scale and J a matrix that explicitly breaks the orthogonal directions of $SU(4)$ while preserving only $Sp(4)$ transformations

$$J \equiv \frac{1}{2}\begin{pmatrix} i\sigma^2 & 0 \\ 0 & i\sigma^2 \end{pmatrix} , \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} . \tag{4.37}$$

This explicit breaking renders 5 of the 15 NGB massive with $m \sim \Lambda$, such that we can integrate them out and are left with the remaining $Sp(4)_L \times Sp(4)_R$ symmetry, spontaneously broken to $Sp(4)_D$, leaving us with the desired $Sp(4)^2/Sp(4)$ coset and the corresponding 10 NGB (c.f. [332]). The low-energy theory as described in the *Minimal Moose Little Higgs* [306] can then be recovered by repeating the aforementioned $Sp(4)^2/Sp(4)$ symmetry breaking pattern. With this, we end our excursion to generating other Little Higgs models and return to the theory above containing the full $SU(4)_L \times SU(4)_R$ symmetry.

Now focusing on the effects of the chiral symmetry breaking $SU(4)_L \times SU(4)_R \to SU(4)_D$, we will first discuss the generated 15 NGBs and the subsequent identification of the Higgs, its mass and its quartic self-coupling. These NGBs transform under the custodial $SO(4) \simeq SU(2)_L \times SU(2)_R \subset SU(4)_D$ as two triplets, two doublets and one singlet

$$\mathbf{15}_{SU(4)_V} = (2,2) + (2,2) + (3,1) + (1,3) + (1,1) . \tag{4.38}$$

Following the usual Little Higgs notation we can write down the NG-matrix $U = \exp\left[i\, 2\Pi/f\right]$ with

$$\Pi = \frac{1}{2}\begin{pmatrix} \sigma^a \Delta_1^a + \eta/\sqrt{2} & -i\Phi_{\mathrm{H}} \\ i\Phi_{\mathrm{H}}^\dagger & \sigma^a \Delta_2^a - \eta/\sqrt{2} \end{pmatrix} , \tag{4.39}$$

where the $\Phi_{\rm H}$ denotes a bi-doublet containing the Higgs doublets H_i and $\sigma^a \Delta^a_{1,2}$ are the triplets, respectively given by

$$\Phi_{\rm H} \equiv \left(\widetilde{H}_1 + i\widetilde{H}_2, \quad H_1 + iH_2 \right) , \tag{4.40}$$

$$\sigma^a \Delta^a = \begin{pmatrix} \Delta^0 & \sqrt{2}\Delta^+ \\ \sqrt{2}\Delta^- & -\Delta^0 \end{pmatrix} , \tag{4.41}$$

with $\widetilde{H}_i \equiv i\sigma_2 H_i^*$, where σ_i (with subscript) denote the Pauli matrices. Both Higgs doublets $H_{1,2}$ developing a non-zero VEV $\langle H_{1,2} \rangle = v_{1,2} \neq 0$ determines the EW symmetry breaking vacuum configuration. Consequently, we can rewrite the custodial symmetry respecting $\Phi_{\rm H}$ VEV in terms of the radial Higgs VEV $v^2 \equiv v_1^2 + v_2^2$ and the Higgs doublets VEVs' angular component $\tan\beta \equiv v_2/v_1$ as

$$\langle \Phi_{\rm H} \rangle = v\, e^{i\,\beta}\, \mathbb{1} \ . \tag{4.42}$$

Then we can parameterize the misalignment between v and VEV of global symmetry breaking $f \equiv \Lambda/(4\pi)$ by the misalignment angle $\sin\theta \equiv v/f$, which can be computed via a rotation with Ω_0 of the NG-matrix U to determine the pNGB-matrix Σ

$$\Sigma = \Omega_0 \cdot U \cdot \Omega_0 \ , \tag{4.43}$$

where the rotation matrix Ω_0 is given by

$$\Omega_0 = \begin{pmatrix} \cos\frac{\theta}{2}\,\mathbb{1} & e^{i\beta}\sin\frac{\theta}{2}\,\mathbb{1} \\ -e^{-i\beta}\sin\frac{\theta}{2}\,\mathbb{1} & \cos\frac{\theta}{2}\,\mathbb{1} \end{pmatrix} . \tag{4.44}$$

The new EW symmetry-breaking vacuum $\langle\Sigma\rangle$ is then consequently calculated to be

$$\langle\Sigma\rangle \equiv \Sigma_0 = \Omega_0 \cdot \Omega_0 \equiv \begin{pmatrix} \cos\theta\,\mathbb{1} & e^{i\beta}\sin\theta\,\mathbb{1} \\ -e^{-i\beta}\sin\theta\,\mathbb{1} & \cos\theta\,\mathbb{1} \end{pmatrix} , \tag{4.45}$$

and therefore in the phase of broken EW symmetry relates the pNGB-matrix Σ to the chiral condensate $(\hat{\psi}\psi)$ through

$$(\hat{\psi}\psi) = \frac{\Lambda^d}{16\,\pi^2}\Sigma \ . \tag{4.46}$$

The factor of $(16\pi^2)$ is from (naive) dimensional analysis (NDA) of a strongly coupled theory. Having defined the proper EW breaking vacuum, we turn to one of the essential aspects of Little Higgs models, the collective breaking, which generates a mass and quartic potential without quadratic divergences (see Section 4.1.3). In analogy to the collective breaking in the *Bestest Little Higgs* [319], we introduce two operators $J_{1,2}$, which project into separate vacuum configurations

$$J_1 = \frac{1}{2}\begin{pmatrix} i\sigma^2 & 0 \\ 0 & i\sigma^2 \end{pmatrix}, \qquad J_2 = \frac{1}{2}\begin{pmatrix} i\sigma^2 & 0 \\ 0 & -i\sigma^2 \end{pmatrix}, \tag{4.47}$$

with the result that the collective quartic potential can be written as

$$\mathcal{L} \supset -\frac{1}{4}\lambda_{12}\, f^4 \left|\mathrm{Tr}\left[\Sigma\, J_1\, \Sigma^T J_2\right]\right|^2 - \frac{1}{4}\lambda_{21}\, f^4 \left|\mathrm{Tr}\left[\Sigma\, J_2\, \Sigma^T J_1\right]\right|^2, \tag{4.48}$$

where λ_{ij} as usual denote dimensionless coupling parameters. Due to the nature of the projectors $J_{1,2}$, the two terms in Eq. (4.48) break the global chiral symmetry $SU(4)_L \times SU(4)_R$ into distinctly different vacuum configurations. In fact, the first term breaks $SU(4)_L \times SU(4)_R$ to $Sp(4)_{L1} \times Sp(4)_{R2}$, while the second term breaks it to $Sp(4)_{L2} \times Sp(4)_{R1}$. Consequently, at tree level, all pNGB bosons except η are protected by the unbroken symmetries, specifically the $Sp(4)_{L1} \times Sp(4)_{R2}$ protects both Higgs doublets from attaining a potential. Since η obtains the mass

$$m_\eta^2 = (\lambda_{12} + \lambda_{21})\, f^2\,, \tag{4.49}$$

we can integrate it out. As a consequence, we obtain a quartic Higgs potential term that is proportional to the product of both dimensionless couplings $\lambda_{12}\ \lambda_{21}$, which is required by the collective breaking of Little Higgs models. At one loop level, radiative corrections from the potential in Eq. (4.48) generate a Higgs mass that is only logarithmically dependent on the confinement scale Λ and also proportional to the product of both dimensionless couplings $\lambda_{12}\ \lambda_{21}$

$$\frac{\lambda_{12}\lambda_{21}}{16\pi^2} f^2\ \ln\left[\frac{\Lambda^2}{\mu^2}\right]\left(H_1^\dagger H_1 + H_2^\dagger H_2\right)\,, \tag{4.50}$$

where μ is understood to be of order m_η (c.f.Eq. (4.49)), such that the finite corrections to the potential are minimal. Having demonstrated that collective breaking successfully generates the Higgs mass with the desired logarithmic dependency on Λ, we refer to *The Bestest Little Higgs* [319] for more details on the phenomenology concerning the Higgs sector, since one can apply their procedure similarly to our theory, and turn to this theory's gauge sector.

We generate collective breaking in the gauge sector by gauging the subgroups $SU(2) \subset SO(4) \subset SU(4)$ of $SU(4)_{L,R}$, respectively, thus resulting in the symmetry breaking pattern as displayed in Fig. 4.4. Thus, we calculate the gauge boson mass terms from the EW breaking pNGB matrix Σ

$$\mathcal{L} \supset \frac{1}{2} f^2 \mathrm{Tr}\left[(D_\mu\Sigma)^\dagger (D^\mu\Sigma)\right]\,, \tag{4.51}$$

with the gauge covariant derivative given as

$$D_\mu \Sigma \equiv \partial_\mu \Sigma + i\, g_L\, \mathcal{A}_L^a T^a\, \Sigma - i\, g_R\, \mathcal{A}_R^a T^a\, \Sigma\,, \tag{4.52}$$

where $\mathcal{A}_{A,B}$ denote to $SU(2)_{LA,B}$ corresponding gauge bosons and $g_{\rm EW}^2 = g_L^2\, g_R^2/(g_L^2 + g_R^2)$. Collecting the above, we can expand Eq. (4.51) in terms of quadratic gauge boson contributions

$$\begin{aligned}\mathcal{L} \supset\, & \frac{1}{2} g_{\rm EW}^2\, f^2 \sin^2\theta\, W_\mu W^\mu + \frac{1}{4\cos^2\theta_W}\, g_{\rm EW}^2\, f^2 \sin^2\theta\, Z_\mu Z^\mu \\ & + \frac{1}{2}\left(g_L^2 + g_R^2\right) f^2 \cos^2\theta\, W'_\mu W'^\mu \\ & + \frac{1}{4}\left(g_L^2 + g_R^2\right) f^2 \cos^2\theta\, Z'_\mu Z'^\mu + \cdots\,,\end{aligned} \tag{4.53}$$

where, for simplicity, we have omitted the $U(1)_Y$ gauge bosons, as they only insignificantly contribute to the little hierarchy problem. furthermore, interaction terms of the Goldstone bosons and the gauge bosons are contained in ellipses above. Now, it is straightforward to read of the gauge boson masses from Eq. (4.53), such that we obtain the same mass for the two heavy gauge boson partners W', Z'

$$m_{W'}^2 = m_{Z'}^2 = \frac{1}{2}\left(g_L^2 + g_R^2\right) f^2 \cos^2\theta\,, \tag{4.54}$$

while the SM gauge bosons acquire the masses

$$m_W^2 = m_Z^2 \cos^2\theta_W = \frac{1}{2} g_{\rm EW}^2 v^2 + \mathcal{O}(v^4/f^2)\,, \tag{4.55}$$

with θ_W as the Weinberg angle. To affirm, that the above calculated massive gauge bosons' contributions do not spoil the absence of quadratic Λ dependencies for the Higgs mass (and other pNGBs), we can now calculate the one-loop contributions from the gauge bosons to $H_{1,2}$ but also Δ_1 to be

$$\mathcal{L} \supset \frac{9\, g_{\rm EW}^2\, m_{W'}^2}{64\,\pi^2}\, \ln\left[\frac{\Lambda^2}{m_{W'}^2}\right] \left(|H_1|^2 + |H_2|^2 + \frac{8}{3}\, |\Delta_1|^2\right)\,, \tag{4.56}$$

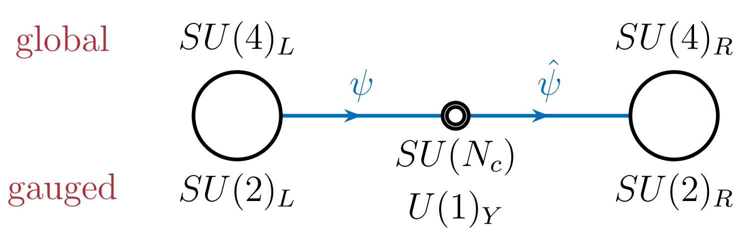


FIGURE 4.4: Moose diagram depicting the symmetry breaking pattern of strongly coupled conformal dynamics UV completing *The Bestest Little Higgs*.

where we find only logarithmic dependence on Λ itself and otherwise quadratic dependence on $m_{W'}$, which might be heavy in comparison to the SM gauge bosons but can still be much smaller than Λ. Thus, so far, neither the Higgs nor the gauge sector spoil the collective breaking as the main aspect of Little Higgs. Before verifying with the fermion sector, we want to shortly note, that most low-energy specific phenomenological considerations, e.g. gauge partners (W', Z') with masses larger than f, can be implemented straightforwardly. For more details, we refer to [4].

We only required the Higgs scalar to be composite to avoid the gauge hierarchy problem and thus the SM fermions are elementary and gain their masses from Yukawa interaction with the composite Higgs operator $\mathcal{O}_H \sim \hat{\psi}_3 \tilde{\psi}$

$$\mathcal{L} \supset \frac{\lambda_t}{\Lambda_t^{d-1}} Q_L t_R \mathcal{O}_H, \tag{4.57}$$

with d again denoting the scaling dimension of $\mathcal{O}_H$ and Λ_t is the scale at which the top coupling grows non-perturbative or strong $\lambda_t \sim 16\pi^2$. Once conformal symmetry is explicitly broken, the strong dynamics confine at $\Lambda \equiv 4\pi f$, generating a VEV for $\langle \mathcal{O}_H \rangle \sim \sin\theta\ \Lambda^d/(16\pi^2)$, which then in turn induces the top-quark mass of the size

$$m_t \equiv y_t\, v_{\text{EW}} \sim \frac{\lambda_t}{16\,\pi^2} \left(\frac{\Lambda}{\Lambda_t}\right)^{d-1} \Lambda\ \sin\theta\ , \tag{4.58}$$

where $v_{\text{EW}} = f \sin\theta$ is the EW VEV in terms of the misalignment angle θ and f. Alternatively, one could generate the top-quark mass via the framework of partial compositeness, by mixing composite fermion operators with the top-quark [347, 348]. While partial compositeness is often used in composite Higgs models, the fundamental nature of our SM fermions marks the top-quark mass generation like above as preferred. As seen in Eq. (4.30), the flavor scale Λ_t naturally decouples from EW breaking for scaling dimensions $1 < d < 2$, resulting in up to three orders of magnitude larger Λ_t than the Λ. This is especially helpful in avoiding experimental flavor constraints when there is no protective UV flavor symmetry. At this point we turn to the verification that no quadratic dependencies on Λ reemerge for the SM Higgs due to couplings to the fermions. Sadly, this is not as straightforward as in the two sectors before, since already the top-quark operator in Eq. (4.57) alongside other fermionic operators, explicitly break the SU(4) symmetry protecting the pNGBs' and therefore also the Higgs' mass. To amend this issue we can introduce additional doublets to complete the original $SU(2)_L$ doublets to a fundamental of $SU(4)_L$, thus removing the harmful contributions to the Higgs and pNGB masses. While this approach works for each quark generation and the charged leptons individually, we demonstrate it in the following at the example of the SM $SU(2)_L$ up/down-type (u/d) quark doublet $Q_L = (u_L, d_L)^{\mathrm{T}}$. To complete the fundamental representation of $SU(4)_L$ we introduce an additional quark doublet $Q'_L = (u'_L, d'_L)^{\mathrm{T}}$ that transforms under $SU(2)'_L$ and carries the same SM $U(1)_Y$ hypercharge as Q_L.

Consequently, $\Psi_L = (Q_L, Q'_L)^{\mathsf{T}}$ denotes a fundamental of $SU(4)_L$. We proceed similarly for $SU(4)_R$, to complete its fundamental representation Ψ_R we introduce the vector-like pair $Q'_R = (U'_R, D'_R)^{\mathsf{T}}$, which similar to the SM $SU(2)_L$ singlet fields U_R, D_R, carry only $U(1)_Y$ hypercharge quantum numbers. The $SU(4)_R$ fundamental is then given by $\Psi_R = (0, 0, \lambda_u U_R, \lambda_d D_R)^{\mathsf{T}}$, such that the quark-Yukawa interactions of the $SU(4)_{L,R}$ fundamentals are encoded in the Lagrangian as

$$\begin{aligned}\mathcal{L} \supset & \frac{1}{\Lambda_t^{d-1}} \bar{\Psi}_L (\hat{\psi}\psi) \Psi_R + \frac{\lambda'}{4\pi} \Lambda \, \bar{Q}'_L Q'_R \\ = & \frac{\Lambda}{(4\pi)^2} \left(\frac{\Lambda}{\Lambda_t}\right)^{d-1} \bar{\Psi}_L \Sigma \Psi_R + \lambda' f \, \bar{Q}'_L Q'_R \, .\end{aligned} \tag{4.59}$$

Here, from NDA estimates we find that the strong couplings $\lambda_{u,d}$ of order $16\pi^2$, and Σ exhibits the from $\langle \hat{\psi}\psi \rangle$ familiar transformation behavior of Eq. (4.34), i.e. as a $(\mathbf{4}, \bar{\mathbf{4}})$ of $SU(4)_L \times SU(4)_R$. The apparent $SU(4)$ invariance of the first term above ensures that all quadratic divergences from quark contributions cancel. Therefore, proceeding like above for the other generations of quarks and charged leptons, one arrives at a valid Little Higgs model without quadratic divergences in the Higgs mass contributions from the Higgs itself, gauge bosons and ultimately also the fermion sector. Furthermore, this construction to avoid fermionic quadratic divergencies in the Higgs mass contributions, leads to the generation of a linear combination of U_R and U'_R that acquires a mass of order f for $\lambda' \sim 1$, next to the usual $SU(2)_L$ singlet up/down-type quark u_R, d_R. The top-quark mass is still generated according to Eq. (4.58) with the top-Yukawa coupling given by

$$y_t = \frac{\lambda_t}{4\pi} \left(\frac{\Lambda}{\Lambda_t}\right)^{d-1} \, . \tag{4.60}$$

At last we want to comment on the low-energy physics of the presented Little Higgs model. Since, it closely resembles the phenomenology of *The Bestest Little Higgs* model [319, 349], we have been brief with discussing actual computational steps and rather focused on present the essential aspects that showcase the viability and effects of strongly coupled conformal dynamics as UV completion to Little Higgs models. To make the comparison easier, we want to clarify, that in our context, H_1 is identified to be the SM Higgs doublet, whereas other pNGBs, i.e. H_2, Δ_1 and η and the heavy gauge boson partners W', Z' acquire masses of $\mathcal{O}(\text{TeV})$, whereas the fermionic top partners have masses of $\mathcal{O}(f)$. The above listed DOFs, constitute scalar, fermionic, and gauge boson states with appropriate masses to be promising targets for the upcoming LHC runs [349]. While these targets are not unique to our framework, the emergence of broad resonance and continuum states from conformal dynamics around the scale of Λ are more distinctive predictions. For $f \sim 1\,\text{TeV}$, equal to $\approx 10\%$ of fine-tuning for the little hierarchy, these characteristic signatures would emerge at roughly 10 TeV and thus prove promising targets for future colliders like the FCC [350, 351].

Chapter 5

Conclusion

In this thesis, we present conformally symmetric models to address the gauge hierarchy problem and other BSM problems in high-energy physics. Since the root of the hierarchy problem lies in the existence of the mass term for the Higgs scalar and its corrections from much larger scales of new physics or eventually gravity, in addition to the gauge symmetries of the SM and general diffeomorphism invariance of gravity, we employ global scale invariance to protect a dynamically generated Higgs mass from fatal corrections. Furthermore, we utilize the theory's resulting scaling behavior to UV complete a class of models, which realize the Higgs not as a fundamental but as a composite field in an extended gauge structure to protect its pNGB mass from quadratic corrections. Motivated by observational hints from SM RG-running, the CMB, and the fact that both the SM and GR each only exhibit one fundamental scale, we propose the radiative breaking of scale symmetry as the sole origin for their dynamic generation. While we find that it is natural to include proposed solutions to other BSM problems, employing conformal symmetry comes with the caveats of its anomaly and propagating a massive spin-2 ghost DOF, threatening the consistency of gauge theory and unitarity. However, there are already multiple promising attempts for the resolution of these issues, such that we rather focus on their beneficial properties. The anomalous breaking of scale invariance due to dimension four operators is believed to protect dynamically generated masses from quadratic divergences while we found the "problematic" massive spin-2 ghost DOFs of the metric to induce substantial contributions to RSSB, replacing an otherwise necessary additional external scalar. Thus allowing for a "more minimal" dynamical generation of the Planck mass and corresponding inflation, in good agreement with the currently strongest experimental constraints from CMB observations. This leaves us assured, that conformal symmetry as the fundamental principle is a natural way to address the embedding of the SM into gravity and the resolution of the consequent puzzle of hierarchically separated scales.

Therefore, after introducing conformal symmetry and the radiative spontaneous breaking of scale invariance in Chapter 2, we construct a model that based on classical scale invariance, dynamically generates all scales with a unified origin in Chapter 3. Thus, we extend the scalar sector by two real scalars S, σ, with one of them attaining a non-zero VEV, namely $\langle S \rangle \neq 0 \sim \mathcal{O}(10^{17}\,\text{GeV})$, in the one-loop effective potential, described by the Gildener-Weinberg approximation. In Section 3.1, we show that the non-minimal coupling to scale-invariant quadratic gravity allows us to identify the dynamically generated Planck mass $M_{\text{Pl}} \sim \sqrt{\beta_{\text{S}}}\,\langle S \rangle$ in terms of the non-trivial VEV, which in turn is solely dependent on dimensionless couplings of the Lagrangian and the technical renormalization scale. For the coupling of these external scalars to the cSM, namely the SM with zero tree-level Higgs mass term, we assume the quartic portal couplings to be almost vanishing to not introduce corrections to the Higgs mass directly proportional to $\langle S \rangle$. Instead, in Section 3.2 we introduce the heavy right-handed Majorana neutrinos N_{R} with mass $m_{\text{N}} \sim \mathcal{O}(10^{7}\,\text{GeV})$, which generate small masses for the active neutrinos via the type-I seesaw mechanism and an appropriate Higgs mass through finite radiative corrections $|\mu_{\text{H}}^2| \sim |\Delta\mu_{\text{H}}^2| \sim y_\nu^2\; m_{\text{N}}^2/(4\pi^2)$ as the dominant contribution. Given that the large Majorana mass of N_{R} is dynamically generated through the Yukawa interaction with S, i.e. $m_{\text{N}} = y_{\text{M}}\,\langle S \rangle$, this constitutes a scale-invariant implementation of the neutrino option, which softens the gauge hierarchy problem by relating the effective Higgs mass to the large scalar VEV $\langle S \rangle$ through small but technically natural Yukawa couplings $y_\nu \sim \mathcal{O}(10^{-4})$, $y_{\text{M}} \sim \mathcal{O}(10^{-10})$ as $\mu_{\text{H}}^2 \sim y_\nu^2 y_{\text{M}}^2\,\langle S \rangle^2/(4\pi^2)$. Furthermore, we can introduce a Z_2 odd Majorana neutrino χ, which we later in Section 3.4 find to be a viable FIMP dark matter candidate when its mass through coupling to S is roughly $m_\chi \in \left[10^4\,\text{GeV}, 10^8\,\text{GeV}\right]$. Furthermore, we notice that with the heavy right-handed Majorana neutrino as above and $m_\chi \gtrsim 10^6\,\text{GeV}$, the baryon asymmetry of the universe (BAU) can be successfully generated in the framework of leptogenesis. In summary, we have an additional scalar sector of two real scalars, with one emerging non-trivial VEV at one-loop level $\langle S \rangle$, which induces the Planck mass as the fundamental scale of gravity through non-minimal coupling to curvature, as well as the effective Higgs mass term of the SM via interaction with a BSM fermion sector. The coupling to the fermion sector not only softens the hierarchy problem by utilizing small yet technically natural values of Yukawa couplings, but also addresses the BSM issues of small active neutrino mass, dark matter, and BAU.

For our scale-invariant model of the unified emergence of fundamental scales in the SM and GR to be a proper description of gravity, it has to provide a mechanism for cosmological inflation. Given the formulation of our model in the Jordan frame, in Section 3.3 we collect the relevant contributing DOFs from the same quantum effective action used before, where due to the aforementioned assumptions the Higgs doublet itself does not play a role. After extracting the *scalaron* DOF ϕ from the R^2 term and Weyl transformation to the Einstein frame, we are left with an effective two-field potential for inflation $V_{\text{E}}(S,\phi)$ exhibiting the valley structure depicted in Fig. 3.3. Making sure that the trajectory along this valley is sufficiently stabilized by the scalaron mass m_ϕ^2, we can, in good approximation, describe inflation as the scalar field S slowly rolling down the effective inflation potential $V_{\text{inf}}(S)$. We calculate the corresponding slow-roll parameters and thus generate predictions for the CMB observables A_s, n_s and r depending on the dimensionless quartic scalar couplings λ_s, $\lambda_{\text{s}\sigma}$, the non-minimal couplings β_s, β_σ and the coupling γ of the R^2 contribution. The choice of flat direction when employing the GW approximation leads to little relevance of λ_s and β_σ for the CMB observable predictions, so we set them to be constant. Furthermore, the stringent constraint from CMB observation on the amplitude of the scalar power spectrum A_s allows us to further reduce the number of free parameters and show our predictions for the scalar spectral tilt n_s and the tensor-to-scalar ratio r in terms of γ for the range of sufficiently long inflationary period, denoted by the number of e-folds $N_e \in [50, 60]$ in Fig. 3.4. We find, that the predictions by our scale-invariant model of unified emergence of SM and gravity scales are in good agreement with the currently strongest constraints provided by the Planck and BICEP/Keck collaborations [22–24].

Further investigating the contributions of quadratic and conformal gravity to the dynamical generation of scales, we devote Section 3.5 to extracting the metrics propagating DOFs, calculating their contributions to the Coleman-Weinberg potential to quantify their contribution in the radiative spontaneous breaking of scale invariance and consequently the realization of inflation in this context. Therefore, we construct a scale-invariant realization of one external scalar S, non-minimally coupled to gravity with the Weyl tensor squared term C^2 already present at tree level. Often, especially this contribution is suppressed as it contains not only a propagating spin-0 but also a propagating (massive) spin-2 ghost DOF. The latter is troublesome as it threatens the theory's unitarity by introducing negative norm states due to its kinetic term coming with a relative minus sign. However, these contributions, even when not included from the start are inevitably generated by quantum effects. Thus we rather decide to include them from the beginning and utilize the promising works [50–62] to develop a proper treatment of these ghost DOFs, so that a theory with a massive spin-2 ghost can still be deemed viable as a QFT, providing unitarity. We simply focus on the dynamical generation of the Planck mass and the resulting inflaton potential, as one can include the scale generation for the SM in the same way as outlined above in Sections 3.1 and 3.2. After expanding the action around quantum fluctuations of the metric, we utilize the *York decomposition* to separate metric contributions by spin. Collecting the contributions quadratic in fields, we find contributions to the CW one-loop effective potential generated from the classical scalar S, the gauge-invariant metric spin-0 DOF ϕ and the massive spin-2 ghost $\tilde{h}_{\mu\nu}$. Due to the nature of contributions to the CW potential, namely $\operatorname{Tr}\ln[\ldots]$, the spin-2 ghost's relative minus sign drops out as a constant term. Hence, the overall sign of the spin-2 ghost contribution to the one-loop effective potential is the same as for non-ghost particles. With the CW potential $V_\text{eff}(S)$ in hand we find that indeed S develops a non-trivial VEV through spontaneous symmetry breaking by quantum corrections, without the need for two external scalar DOFs, as the non-classical treatment of gravitational DOFs provides additional contributions. Now, employing the machinery developed in

Section 3.3 to calculate this model's inflationary CMB predictions, as seen in Fig. 3.7, which again are well in agreement with the strongest modern constraints. Also, similarly to before in Section 3.3, we find our predictions for r to interpolate between the ones from the two well-known models of inflation, with linear inflation ($m^3\phi$) as the less experimentally favored upper and Starobinsky inflation ($R + R^2$) as the lower limit. In sum, in a scale-invariant realization of dynamically generated Planck mass, the massive spin-2 ghost contribution is not only beneficial but crucial for successfully realizing the spontaneous breaking of scale invariance with only one additional scalar, while maintaining the prediction of a small tensor-to-scalar ratio r. However phenomenologically advantageous, its massive presence comes with the so-called *ghost problem*, which in turn evokes a better understanding of QFT in the presence of quantum gravity. Yet, with a mass of roughly $m_{\text{gh}}^2 \sim \mathcal{O}(10^{16}\,\text{GeV})$, our spin-2 ghost qualifies for the recently introduced concept of *conditional unitarity* [61, 62], possibly helping to resolve its threat to unitarity and therefore our fundamental interpretation of QFT.

In Chapter 4 we switch gears and take advantage of a conformally symmetric theory's scaling behavior to provide UV completion by strongly coupled conformal dynamics for *Little Higgs* models, which realize the light physical Higgs boson as a composite state, namely as the pNGB of spontaneously broken global symmetry, free of quadratic divergencies. Utilizing the observation of [329, 331, 332, 345], that Little Higgs models based on the global symmetry breaking coset G/H with a weakly gauged $F \subset G$ (theory-A), yield the same low-energy physics as models based on the breaking of a product group to its diagonal G^2/G with gauged $F \times H$, where H exhibits strong gauge coupling (theory-B), in Section 4.2.1 we propose a generalized way to UV complete Little Higgs models through UV completing a model of type B with the symmetry structure as displayed in Fig. 4.3. Suppose, we want to UV complete a type-B Little Higgs model with G^2 as the chiral global $SU(N)_L \times SU(N)_R$ spontaneously broken to its diagonal subgroup $SU(N)_D$ with N Dirac fermion flavors ψ_i. To ensure the necessary FPs for the couplings at high energies we introduce the strongly coupled confining CFT symmetry $SU(N_c)$ (or $Sp(2N_c)$) and also extend the global chiral symmetry by N_m massive Dirac fermion flavors χ, leaving us with the global $SU(N_f)_L \times SU(N_f)_R$, where $N_f = N + N_m$. The proper adjustment of N_m and therefore N_f/N_c ensures the theory exhibits conformal dynamics at high energies by placing it in the so-called *conformal window* (c.f. Eq. (4.23)), while the massive nature of χ as a fundamental of $SU(N_c)$ softly breaks conformal symmetry at the scale $\Lambda = M^{1/(4-d)}$, thus providing an exit from the conformal FP towards lower energies $E \lesssim \Lambda$. Since we assume our CFT to be strongly coupled the theory almost directly enters a QCD-like confining phase with N flavors and N_c colors, forming the condensate $\langle \hat{\psi}\psi \rangle \sim \Lambda^d$, which breaks the chiral symmetry to its diagonal subgroup while preserving EW symmetry. At this point, we have generated a symmetry structure that allows for the successful implementation of Little Higgs models by introducing strongly coupled conformal dynamics as an embedding UV complete theory.

In Section 4.2.2 we demonstrate, close to the example of *The Bestest Little Higgs*, that this form of UV completion does not spoil the collective breaking of EW symmetry as the main feature of Little Higgs models to remove quadratic divergences of the SM Higgs doublet. Apart from providing a systematic way to secure FPs for the couplings thus extending the theory's validity up to arbitrary scales, strongly coupled dynamics allow for a non-trivial scaling dimension d of the bilinear operator $\hat{\psi}\psi$, such that the flavor dynamics fully decouple from the EW symmetry breaking dynamics. Lastly, we find the non-SM Higgs pNGB, heavy gauge boson partners, and fermionic top partners, constitute promising but generic scalar, fermionic, and gauge boson targets for upcoming LHC runs [349]. Whereas, the strongly coupled conformal dynamics provide also promising and more distinctive targets for future colliders like the FCC [350, 351] in the form of broad resonances and continuum states that emerge at around $\mathcal{O}(10\,\text{TeV})$.

Finally, we want to remark on several topics related to this work that need further exploration and therefore denote possible future avenues of research. First, as conformal symmetry or scale invariance is applied as the guiding principle to investigate the embedding of the SM intro gravity or as UV completion to arbitrary scales (eventually M_{Pl}), the conformal anomaly naturally plays an important role in the realization of such models. While its anomalous breaking of scale invariance can protect the effective Higgs mass from quadratic divergences [31], it also inevitably introduces a finite zero-point energy. Such that, when considering the dynamical generation of both v_{EW} and M_{Pl}, this finite zero-point energy induces an also finite cosmological constant (CC). Similar to a vanishing conformal anomaly for precisely adjusted gauge symmetries and representations [333], the contributions calculated in Sections 3.1 and 3.5 are assumed to precisely cancel with contributions from other sectors to give an approximately vanishing value, matching the observations [352–354]. We implement this assumption by subtracting V_0, such the value of the finite zero-point energy is exactly zero. However, this denotes only a rudimentary solution and naturally requires further investigation of the precise contributions from other sectors and the corresponding cancellations. Especially, in a globally scale-invariant theory where the origin of scales for both the SM and gravity is unified, the connection of both the conformal anomaly and the CC problem becomes evident. Furthermore, when considering conformal gravity, a non-zero conformal anomaly can lead to a fundamental inconsistency in the theory by violating gauge symmetry [147], yet some propose this problem to be considered void if gravity and matter are treated on equal footing and renormalized together [85, 355]. Thus, it is apparent that the subject of the conformal anomaly and with that its origin, i.e. breaking of scale or conformal invariance by the theory's regulator, requires further investigation to form a deeper understanding of the general applicability of conformal symmetry to address outstanding fundamental issues of the (B)SM and gravitational physics.

Other than from fundamental revelations in understanding QFT in the light of conformal symmetric theories, the here-considered models for the dynamical generation of both Planck and EW scale seem only to be constrained by their predictions for the inflation CMB observables. Additional BSM degrees of freedom are too heavy to be probed by current and near-future experiments, and even if observations could determine the nature of neutrinos, Majorana or Dirac, this would not confirm or forbid the inclusion of (scale-invariant) neutrino option to generate the Higgs and active neutrino mass. Thus, stronger constraints on inflation by future CMB measurements as well as exploring the remaining space of inflation CMB observables, like non-Gaussian primordial fluctuations, could help constrain our models. While these non-Gaussianities are heavily suppressed in single-field slow-roll inflation, they can contribute substantially in multifield inflation models [253, 254]. Hence, relaxing the assumption of effective single-field inflation and calculating the occurring non-Gaussianities via, e.g. [356], could potentially further constrain the parameters space of the model discussed in Chapter 3. Especially in the case, when the spin-2 ghost of gravity is present, as in Section 3.5, since it is found to possibly amplify the non-Gaussianities [277]. With the future experimental projects the likes of LiteBird [357], Euclid [358] and LSST [359], to constrain their existence and eventually measure their magnitude, this topic proves to be an interesting direction for future research.

All in all, we hope to supply a new perspective and some deeper insight into how conformal symmetry as a guiding principle can address the gauge hierarchy problem either via generating all (fundamental) mass scales dynamically and also by leveraging its scaling behavior to provide UV completion for successful low-energy effective models à la *Little Higgs*.

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Appendix A

Appendix

A.1 Dependence on the non-minimal couplings to gravity

While the in Section 3.1 given argument for omitting the constant $-1/6$ contribution to β_S, i.e. $\beta_S \gtrsim 10^2$, seems sufficient, here want to shortly elaborate on the fact that even omitting both contributions, also to β_σ does not have a measurable effect and is thus justified. The latter is not as obvious, since for the benchmark values in Table 3.1 we use $\beta_\sigma = 1$. Thus, here we show the results for the benchmark points as seen in Table 3.1 while adjusting for the missing factor by substituting

$$\beta_\sigma \to \beta_\sigma - 1/6 \;\Rightarrow \beta_\sigma = 5/6\,. \tag{A.1}$$

Focusing on the induced difference compared to the original benchmark calculations, in the following we display the relative difference δp_i for each resulting parameter p_i given by

$$\delta p_i = \frac{\left|p_i^{\text{corr}} - p_i\right|}{p_i}\,, \tag{A.2}$$

where p_i^{corr} denotes the "corrected" result with $\beta_\sigma = 5/6$. As seen in Appendix A.1 (the table below), we find that all relative differences are at most $\mathcal{O}(10^{-5})$ and thus we can conclude that omitting the constant $-1/6$ contributions to the non-minimal gravitational couplings β_S, β_σ does not alter the results of our investigation significantly.

	Contour $\mathcal{C}$					Contour $\mathcal{C}'$				
#	δn_s	δr	δA_s	$\delta S_{\text{end}}/\mu$	$\delta S_*/\mu$	δn_s	δr	δA_s	$\delta \phi_{\text{end}}/\mu$	$\delta \phi_*/\mu$
1	2×10^{-8}	2×10^{-6}	9×10^{-7}	6×10^{-7}	4×10^{-6}	3×10^{-8}	1×10^{-6}	4×10^{-7}	1×10^{-5}	1×10^{-5}
2	1×10^{-8}	3×10^{-6}	1×10^{-6}	6×10^{-7}	4×10^{-6}	9×10^{-9}	4×10^{-6}	1×10^{-6}	2×10^{-6}	3×10^{-6}
3	3×10^{-9}	5×10^{-6}	1×10^{-6}	4×10^{-7}	6×10^{-6}	3×10^{-9}	5×10^{-6}	1×10^{-6}	2×10^{-6}	3×10^{-6}

TABLE A.1: Relative differences between the parameters of the benchmark points in Table 3.1 and the adjusted ones $\beta_\sigma = 5/6$. As our focus is the order of magnitude of the corrections, we just display only the first significant digit.

A.2 Explicit Expressions for the Effective Potential

Given the scale-invariant nature of the proposed model in Eqs. (3.78) and (3.79), we can factor out S dependencies in the contributions to the one-loop effective potential $V_{\text{eff}}(S)$ Eq. (3.98) to rewrite it in the form of Eq. (3.99):

$$V_{\text{eff}}(S) = V_0 + \left(C_1 + C_2 \ln \left[\frac{S^2}{\mu^2} \right] \right) S^4 \,. \tag{A.3}$$

Starting by factoring out S contributions from the field-dependent mass contributions m_{s}^2, m_ϕ^2 and m_{gh}^2, as given in Eq. (3.86), we can express them in terms of dimensionless contributions $\hat{m}_i^2 = m_i^2/S^2$ with

$$\hat{m}_\phi^2 = \frac{\beta}{12\gamma} \,, \qquad \hat{m}_{\text{s}}^2 = 3\,\lambda \,, \qquad \hat{m}_{\text{gh}}^2 = \frac{\beta}{4\kappa} \,, \tag{A.4}$$

such that eventually also $m_\pm^2$ are rewritten in terms of $\hat{m}_\pm^2 = m_\pm^2/S^2$, where $\hat{m}_\pm^2$ is given by

$$\hat{m}_\pm^2 = \frac{1}{2}\left(\hat{m}_{\text{s}}^2 + (1+6\,\beta)\,\hat{m}_\phi^2\right) \pm \frac{1}{2}\sqrt{\left(\hat{m}_{\text{s}}^2 + (1+6\,\beta)\,\hat{m}_\phi^2\right)^2 - 4\,\hat{m}_{\text{s}}^2\,\hat{m}_\phi^2} \,. \tag{A.5}$$

Therefore, continuing this procedure for the one-loop contributions $V_{\text{s}}^{(1)}$ and $V_{\text{h}}^{(1)}$,

$$V_{\text{s}}^{(1)}(S) = \frac{1}{64\pi^2}\, S^4 \sum_{j=\pm} \hat{m}_j^4 \left(\ln\left[\frac{S^2}{\mu^2}\right] + \ln\left[\hat{m}_j^2\right] - \frac{3}{2} \right) \,, \tag{A.6}$$

$$V_{\text{h}}^{(1)}(S) = \frac{5}{64\pi^2}\, S^4\, \hat{m}_{\text{gh}}^4 \left(\ln\left[\frac{S^2}{\mu^2}\right] + \ln\left[\hat{m}_{\text{gh}}^2\right] - \frac{1}{10} \right) \,, \tag{A.7}$$

such that we can ultimately bring the full one-loop effective potential $V_{\text{eff}}(S)$ in the aforementioned form of Eq. (3.99), with C_1 and C_2 given by

$$\begin{aligned} C_1 = \frac{\lambda}{4} + \frac{1}{64\pi^2} \Bigg[& \hat{m}_+^4 \left(\ln\left[\hat{m}_+^2\right] - \frac{3}{2} \right) + \hat{m}_-^4 \left(\ln\left[\hat{m}_-^2\right] - \frac{3}{2} \right) \\ & + 5\,\hat{m}_{\text{gh}}^4 \left(\ln\left[\hat{m}_{\text{gh}}^2\right] - \frac{1}{10} \right) \Bigg] \,, \end{aligned} \tag{A.8}$$

$$C_2 = \frac{1}{64\pi^2}\left[\hat{m}_+^4 + \hat{m}_-^4 + 5\,\hat{m}_{\text{gh}}^4\right] \,. \tag{A.9}$$

A.3 Tensor-to-scalar Ratio Corrections Induced by the Weyl Tensor Squared

Here we display the predictions for the same parameter space as before in Eq. (3.109) including the correction factor of Eq. (3.112), induced by the appearance of the Weyl tensor squared term in Eq. (3.78). It is apparent that the model's predictions are still fully compatible with the strongest and most current constraints from the latest CMB observational data [24].

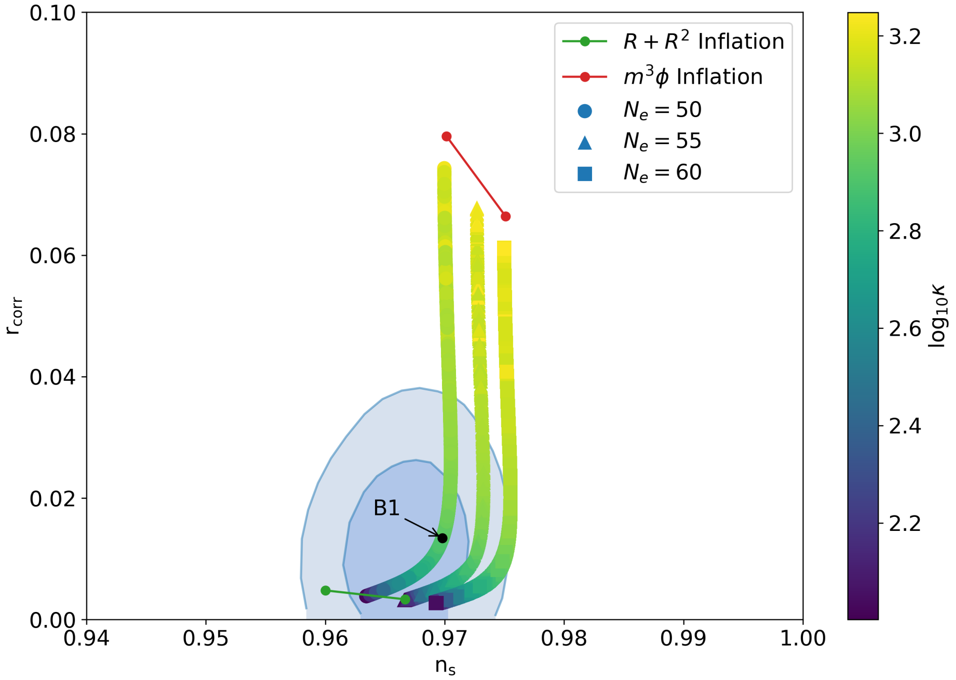

FIGURE A.1: Predictions for the scalar spectral tilt n_s and tensor-to-scalar ratio r_{corr} for three different N_e. This includes corrections due to the C^2 term given in Eq. (3.112). The data is displayed in the same way as the lower plot of Fig. 3.7.

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Disclaimer

This thesis is heavily based on three papers that have been published in peer-reviewed journals. They contain original research performed by the author in collaboration with others. In particular:

- The results of Chapter 3, specifically Sections 3.1 to 3.4, are based on [1] in collaboration with Jisuke Kubo, Jeffrey Kuntz, Manfred Lindner, Jonas Rezacek, and Andreas Trautner.
- The results of Chapter 3, specifically Section 3.5, are based on [2] in collaboration with Jisuke Kubo, Jeffrey Kuntz, and Jonas Rezacek.
- The results of Chapter 4 are based on [4] in collaboration with Aqeel Ahmed and Manfred Lindner.

Publications

[1] Jisuke Kubo et al. "Unified Emergence of Energy Scales and Cosmic Inflation". In: *J. High Energy Phys.* 2021.8 (Dec. 2020), p. 16. DOI: 10.1007/JHEP08(2021)016. arXiv: 2012.09706.

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