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The Measure Problem and Challenges in String Compactifications

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Zusammenfassung

Das Measure Problem ist eine grundlegende theoretische Herausforderung, die beim Studium von Theorien mit einer Vielzahl verschiedener (einer Landschaft) Vakuumzustände auftritt. Die Stringtheorie mit ihrer großen Zahl von vierdimensionalen Lösungen ist ein prominentes Beispiel. Das Measure Problem bezieht sich auf die Frage, wie man das Ergebnis zukünftiger Messungen vorhersagen kann – eine der fundamentalsten Aufgaben der theoretischen Physik. In einer Situation mit einer Landschaft von Vakua ist einfaches Ereigniszählen für Vorhersagen im Allgemeinen unzureichend. Daher ist ein Wahrscheinlichkeitsmaß auf dem Raum der möglichen Messergebnisse erforderlich. Wir schlagen ein solches Wahrscheinlichkeitsmaß vor, das Local Wheeler-DeWitt-Measure, basierend auf grundlegenden Prinzipien der Quantenmechanik und der semi-klassischen Gravitation. Mit diesem Maß sagen wir die Inflationsskala voraus, welche wir in Zukunft messen werden. Darüber hinaus entwickeln wir einen allgemeinen Formalismus zur Untersuchung des Zerfalls der Raumzeit ins Nichts und der Entstehung von Universen aus dem Nichts, unter Einbeziehung von End-of-the-World (ETW)-Branen. Dies ist notwendig, da Messvorhersagen von Vakuumzerfalls- und Vakuum erzeugungsraten abhängen. Wir konstruieren explizit eine ETW-Brane für den am besten verstandenen Teil der String-Landschaft, die in der Lage ist, Raumzeitzerfalls- oder -erzeugungsprozesse zu verursachen. Abschließend schlagen wir einen neuen Mechanismus für die Universumsentstehung aus dem Nichts vor, der auf ETW-Branen in rein off-shell Konfigurationen basiert.

Abstract

The Measure Problem is a fundamental theoretical challenge that arises when studying theories allowing for a landscape of different vacua. String theory, with its large number of four-dimensional solutions, is a prominent example. The Measure Problem refers to the issue of how to predict the outcome of future measurements – one of the most fundamental tasks in theoretical physics. In a landscape scenario, simple event counting is generally insufficient for making predictions. A probability measure on the space of possible measurement outcomes is required. We propose a probability measure, the Local Wheeler-DeWitt Measure, based on fundamental principles of quantum mechanics and semiclassical gravity. We use this measure to predict the scale of inflation to be observed in the future, mainly focusing on the string landscape. Furthermore, we develop a general framework for studying spacetime decays to nothing and the creation of universes from nothing, involving end-of-the-world (ETW) branes. This is necessary because measurement predictions are sensitive to vacuum decay and vacuum creation rates. We explicitly construct an ETW brane for the best-understood part of the string landscape, capable of causing spacetime decay or creation processes. Finally, we propose a new mechanism for universe creation from nothing, utilizing ETW branes in purely off-shell configurations.

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Preface

The research presented in this thesis was carried out at the Institute for Theoretical Physics at Heidelberg University from 2022-2024.

The contents of Chapters 2-5 are the results of [1–4] and are reproduced with permission of my coauthors. My role in the four papers is explained at the beginning of each chapter.

1 Introduction

1.1 Introductory summary of the thesis

String theory rests on the assumption that the fundamental objects of nature are not particles, i.e. objects point-like in space and traveling in time, but rather strings that are extended in one spatial and the time direction. These strings propagate in a spacetime, called the target space, and can vibrate at different frequencies, just as a guitar string. When looking at these strings from a distance, i.e. measuring the strings at low energy, they look like particles, just as every object looks point-like from a large distance away. It is possible to quantize the string in a flat target spacetime and study its quantum behavior. It turns out that the different vibration modes of the string look like different particles to an observer that does not have enough energy available to probe that the strings are actually extended objects.

An important feature of the string spectrum is that there exists a vibration mode that looks to the low-energy observer as a massless spin-two particle. Such a particle would be responsible for the mediation of gravity and is called the graviton. Other vibration modes lead to other massless particles for example having spin-zero or spin-one. As a result, one quickly realized that string theory might be a consistent theory of quantum gravity that can furthermore unify general relativity and ordinary quantum field theory. It is possible to study the interactions of strings with each other, which again at low energies is just as particle interactions. However, due to the strings being extended objects, their interactions are modified at high energies, an important and fundamental property of string theory. It is this property at its essence that allows for a ultra-violet complete quantization of gravity. At low-energies, string-theory describes a universe (the target space) and some field theory living inside it, just like the physics we observe in our universe.

It was realized that string theory can only be self-consistent if the theory is supersymmetric¹ and if the target spacetime is ten-dimensional. String theory defined in this way is (basically) unique to the extent that it has no free parameters. This is a desirable feature for a unified theory of quantum gravity that is unknown to most field theories where coupling constants and mass parameters can usually take arbitrary values.

We certainly only observe four spacetime dimensions and a particle spectrum that is not supersymmetric so that the consistency conditions of string theory appear to be in conflict with observation. However, they do not rule out string theory as our theory of quantum gravity. Six out of the necessary ten spacetime dimensions can be compact and so small that we cannot see them for the very same reason that we do not see that strings are extended objects, namely that we only have access to limited energy. Also, supersymmetry (SUSY) could be spontaneously broken in such a way that the particle spectrum is only supersymmetric above an energy scale larger than what we have available.

¹It was later discovered that there may also exist non-supersymmetric consistent string theories. Presumably, there can only be finitely many of them.

String theory realizes a beautiful connection between physics and geometry since the geometry of the six microscopic dimensions determines the physics in the four macroscopic dimensions we might live in. Since there is a very large (but finite!) number of possible such choices, there is a vast number of different four-dimensional solutions, also called vacua, with the largest estimate being $\mathcal{O}(10^{272\,000})$. This number is by far larger than $\mathcal{O}(10^{80})$, the number of atoms in the observable universe. The space of all possible vacua realized by string theory is known as the *string landscape*. If string theory is our theory of quantum gravity, one of the many solutions must describe our 4-dimensional universe. Due to the large size of the string landscape, it is a very difficult task to find *the* six-dimensional geometry that leads to the four-dimensional universe we observe. We nevertheless assume here that universes of our kind, i.e. expanding universes with a particle content allowing for the formation of structure, are part of the string landscape and it is clear that there will be very many of them.

Expanding universes have the general property that they can decay into other universes by the process of bubble formation. Through a quantum process, a spherical bubble of some other vacuum can form that subsequently expands. One should view such a process as the formation of a new universe. Since, very generically, the expansion rate of such universes is larger than their decay rate, the process of bubble formation will never stop. The result is an *eternally inflating multiverse* of universes that have formed inside other universes. Furthermore, it was proposed that universes are constantly created from nothing. In total, each vacuum will get realized infinitely many times and not only that, but also every physically allowed process will happen infinitely many times. This is often times stated as the phrase

Everything that can happen, will happen, infinitely many times.

In this situation, a fundamental theoretical problem opens up. Imagine a fellow experimental friend to soon perform an experiment measuring a previously unknown quantity. For example, the scale of inflation could be measured. We as theorists should be able to predict the outcome of the measurement. In the multiverse situation there could be two (or in general more) different possibilities A or B for the result. We should be able to state the probabilities p_A, p_B that the measurement results in A or B . The most naive way of doing so would be to count the number of events N_A, N_B in the multiverse where result A and where result B is measured by an observer and then define $p_A/p_B = N_A/N_B$. However, since everything happens infinitely many times, N_A, N_B are both infinite and no unambiguous statement about p_A, p_B can be made. What we need is a probability measure on the space of the outcome of all measurements. Hence, the problem how to make predictions in the multiverse is called the *Measure Problem*.

One may try to regulate the occurring infinities by introducing a cutoff but it turns out that the resulting predictions are highly dependent on the choice of cutoff, making this approach unsatisfying. Rather, it would be desirable to derive a probability measure from first principles.

Our idea to approach the measure problem is to describe the entire multiverse quantum mechanically by a ‘state of the multiverse’ Ψ and use the inner product of the underlying Hilbert space to extract predictions. During my doctoral studies, we developed such an approach, proposed a new measure for the multiverse and made preliminary predictions. Furthermore, the creation of universes from nothing and the decay of universes to nothing is studied in more detail, specifically in the context of string theory.

1.2 Modern high-energy physics

The most important developments in theoretical physics during the 20th century are the discoveries of general relativity (GR) and quantum field theory (QFT). The framework of QFT allows us to explain numerous phenomena at the level of fundamental particles and provides a unification of special relativity and quantum mechanics. The Standard Model of Particle Physics (SM) is a specific QFT that describes the electromagnetic, strong, and weak interactions. It has been tested with remarkable precision, primarily at colliders such as the Large Hadron Collider (LHC). The SM reached a major milestone with the discovery of the Higgs boson [5, 6].

The framework of GR describes gravitational interactions on large scales, explaining the motion of planets, galaxies, the existence of black holes, and gravitational waves. Based on GR, the Standard Model of Cosmology, or Λ -Cold-Dark-Matter (Λ CDM) model, describes the dynamics of our universe. This model postulates that our universe consists of approximately 68% dark energy, 27% cold dark matter, 5% ordinary baryonic matter, and less than 1% radiation [7]. The recent detection of gravitational waves [8, 9] has further validated GR, marking it as one of the most significant successes of modern physics.

However, the Standard Model (SM) and the Λ CDM model, when combined, still fall short of providing a complete description of the universe's physics. Phenomena such as neutrino oscillations [10] and the anomalous magnetic moment of the muon [11] are not accounted for within the SM, while the Λ CDM model does not explain the microscopic origins of dark matter and dark energy. Moreover, there are observations that appear unnatural from the perspectives of the SM and Λ CDM. For instance, the mass of the Higgs boson is measured to be $m_H \simeq 125 \text{ GeV}$, yet arguments from effective field theory (EFT) suggest that it should be much larger, determined by the scale at which the SM ceases to be valid. Similarly, the energy density of dark energy, $\Lambda \simeq 10^{-120} M_P^4$ [12, 13], with $M_P = 1/\sqrt{8\pi G_N} \simeq 2.4 \cdot 10^{18} \text{ GeV}$ the Planck mass, is extraordinarily small. EFT arguments predict it to be of the order of M_P^4 , highlighting a profound discrepancy. Another unresolved puzzle is the strong CP problem, which refers to the observation that the coupling θ of the CP-violating term $\sim \theta \int F \wedge F$, where F is the field strength tensor of QCD, is exceedingly close to zero. This raises questions about why θ is so small and whether there exists an underlying symmetry or mechanism to explain this fine-tuning.

On an even more fundamental level, the theory of General Relativity (GR) can only be valid at low energy densities, and an entirely new theory—a theory of quantum gravity—is required to describe gravity at high energies. The need for a quantum theory of gravity becomes evident when studying the very beginning of the universe or the physics of black holes.

The unification of Quantum Field Theory (QFT) and GR remains one of the most significant research challenges in modern theoretical physics. While it might seem natural to incorporate GR into a field-theoretic framework, this approach proves to be exceptionally challenging in practice, as the resulting field theory is non-renormalizable [14, 15]. String theory is a prominent candidate for unifying GR and QFT. It stands as the most developed framework for the unification of all fundamental forces and has been under investigation for several decades.

In the following, we will introduce string theory as a complete theory of quantum gravity and explore how it addresses some of the hierarchical problems mentioned earlier.

1.3 String Theory

The subject of string theory is covered by many textbooks, some of which are [16–25]. We start by considering a D-dimensional flat Lorentzian *target space*, described by coordinates X^μ , $\mu = 0, \dots, D-1$, and introduce a string, a $1 + 1$ dimensional object, moving within this space. The motion of a string is described by the action

$$S_P = \frac{1}{4\pi\alpha'} \int_{\Sigma} d^2\xi \sqrt{-h} h^{ab}(\xi) \partial_a X^\mu(\xi) \partial_b X^\nu(\xi) \eta_{\mu\nu}, \quad (1.3.1)$$

where Σ is the string worldsheet, the two-dimensional surface traced out by the string in the target spacetime, analogous to the worldline of a particle. The worldsheet is parameterized by coordinates ξ^a , $a = 0, 1$, with $-\infty < \xi^0 < \infty$ and $0 \leq \xi^1 \leq 1$. Here, ξ^0 represents a time-like, and ξ^1 a space-like coordinate on the worldsheet. The metric h_{ab} is the Lorentzian worldsheet metric, and $\eta_{\mu\nu}$ is the flat target space metric. The tension of the string is $1/(2\pi\alpha')$, where α' is a constant with dimensions of length² and is referred to as the Regge slope. The subscript P in Eq. (1.3.1) signifies that this form of the action was first derived by Polyakov [26].

From the perspective of the worldsheet action (1.3.1), the D-many fields X^μ and the metric h_{ab} form a two-dimensional conformal field theory (CFT) including gravity. Remarkably, this theory can be quantized due to the conformal group in 2D being infinite-dimensional. By contrast, gravity in higher-dimensions is not conformal being a reason why quantization as a field theory is difficult. Equation (1.3.1) defines the action of *bosonic string theory*, as the fields X^μ are free bosons on the worldsheet.

For simplicity, let us focus on *closed strings*, i.e., string configurations without boundaries. The fields X^μ then satisfy the periodicity conditions $X^\mu(\xi^0, \xi^1) = X^\mu(\xi^0, \xi^1 + 1)$, describing a string forming a loop like a rubber band. The fields X^μ then obey the following expansion

$$X^\mu(\xi) = X_L^\mu(\xi) + X_R^\mu(\xi), \quad (1.3.2)$$

$$X_L^\mu = \frac{1}{2}x^\mu + \pi\alpha'p^\mu(\xi^0 - \xi^1) + i\sqrt{\frac{\alpha'}{2}} \sum_{n \in \mathbb{Z} - \{0\}} \frac{1}{n} \alpha_n^\mu e^{-2\pi i n(\xi^0 - \xi^1)}, \quad (1.3.3)$$

$$X_R^\mu = \frac{1}{2}x^\mu + \pi\alpha'p^\mu(\xi^0 + \xi^1) + i\sqrt{\frac{\alpha'}{2}} \sum_{n \in \mathbb{Z} - \{0\}} \frac{1}{n} \tilde{\alpha}_n^\mu e^{-2\pi i n(\xi^0 + \xi^1)}, \quad (1.3.4)$$

that is interpret as follows: The string can exhibit various vibration modes that propagate along the string in both directions. These two corresponding sectors are decoupled and are referred to as the left- and right-moving sectors, denoted by subscripts L and R , respectively. The position of the string's center of mass is represented by x^μ , and the corresponding momentum is denoted as p^μ . The vibration modes are characterized by two sets of oscillators, α_n^μ and $\tilde{\alpha}_n^\mu$, which describe the frequencies of oscillation in the left- and right-moving sectors.

Upon quantization, all modes are promoted to operators. The Fock space is constructed by acting with the oscillator operators on a suitably defined vacuum state. The physical Hilbert space is determined by imposing constraints arising from the gauge symmetry of the theory described by (1.3.1).

Quantizing the theory is a nontrivial task and can be approached in various ways. However, all approaches lead to the same conclusions:

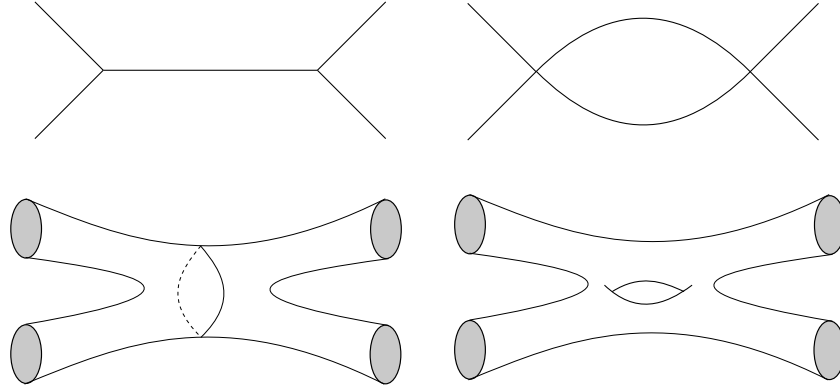


Figure 1.1. Tree-level and one-loop QFT diagrams (at the top) and the corresponding string diagrams (at the bottom) are compared. The string diagrams are Riemann surfaces with boundaries.

- The spectrum of bosonic string theory contains a state that has the target-space interpretation of a *tachyon*, a particle excitation with $m^2 = -\frac{1}{\alpha'} < 0$. This indicates that the theory is unstable. This instability can be resolved by considering superstring theory, and bosonic string theory is best regarded as a toy model.
- The spectrum furthermore includes massless excitations with interpretation:
 - A scalar field ϕ , called the dilaton.
 - An antisymmetric rank-2 tensor $B_{\mu\nu}$, referred to as the Kalb-Ramond field.
 - A symmetric, traceless rank-2 tensor $h_{\mu\nu}$, which corresponds to the graviton and is interpreted as the spacetime metric.
- There exists a tower of massive states with a mass spectrum given by $m^2 = \frac{n}{\alpha'}$ for $n \geq 1$.
- For consistency, the number of spacetime dimensions must be $D = 26$. This is referred to as the *critical dimension* and arises from the requirement that the conformal symmetry be non-anomalous.

Interactions between strings can also be studied. The interaction strength is determined by a string coupling constant g_s that is related to the dilaton. At small g_s , string interactions can be classified using diagrams similar to Feynman diagrams. However, instead of lines that can split and join, the diagrams consist of tubes that merge and split due to the string's extended nature, as illustrated in Fig. 1.1. Each splitting of a string introduces a factor of g_s , causing higher-loop amplitudes to be suppressed. The calculation of string amplitudes requires a deep understanding of the geometry of Riemann surfaces, as each string diagram corresponds to a Riemann surface.

String interactions exhibit the following properties:

- At low energies, $E \ll 1/\sqrt{\alpha'}$, string amplitudes behave like those of particle interactions. However, at high energies, the tower of massive string states becomes relevant, and the amplitudes decrease much faster than QFT amplitudes, making them free from ultraviolet (UV) divergences.

- String interactions exhibit non-local features below an energy scale of order $\sqrt{\alpha'}$, due to the extended nature of the string.

In summary, string interactions are calculable, and the resulting amplitudes are finite. Since the graviton appears in the particle spectrum, string theory provides a framework for well-defined gravitational amplitudes.

Superstrings We have so far discussed the bosonic string but one should consider the *superstring* to get rid of the tachyon and to consider string spectra including fermions. The superstring formalism starts from considering the supersymmetric analogue of (1.3.1)

$$S = \frac{1}{4\pi\alpha'} \int d^2\xi \sqrt{-h} \left(\partial_a X^\mu \partial_b X^\nu h^{ab} \eta_{\mu\nu} + i\alpha' \bar{\psi}_A^\mu \gamma_{AB}^a \partial_a \psi_{\mu B} \right). \quad (1.3.5)$$

Here, the ψ^μ are the superpartners of the X^μ and γ_{AB}^a are the 2D γ -matrices. The quantization procedure is similar as in the bosonic string only that the critical dimension turns out to be $D = 10$ [27,28] and that there are more states arising due to the presence of the ψ^μ that themselves come with oscillator modes. Furthermore, it is possible to perform a projection on the resulting spectrum, the Gliozzi-Scherk-Olive (GSO) projection [29], to a theory that is free of tachyons. On this level, there are two different choices of a GSO projection leading to two different string theories, called type IIA and type IIB.² Their spectra are similar as before: There is a set of massless fields (now bosons and fermions) and a tower of massive states at mass level $m^2 = \frac{n}{\alpha'}$. Similar as before, string interactions can be studied making use of the theory of (super) Riemann surfaces. From now on, we will work with the superstring.

1.4 From String Theory to field theory

We have learned that the (super)string can be quantized, and that its interactions can be calculated. This is analogous to knowing the particle content of a quantum field theory (QFT) and their interactions without having the field-theoretic action that defines the QFT.

A natural question then arises: Can one write down a 10-dimensional action governing the behavior of the string excitations in spacetime? While a field theory describing infinitely many particles is extraordinarily challenging to construct,³ we can make the following observation: The superstring has a set of massless fields, while the first massive states appear at a mass level $m^2 = 1/\alpha'$, creating a large mass gap when α' is small. This allows us to focus on an effective field theory describing only the massless fields, valid at energies below $1/\sqrt{\alpha'}$.⁴

The massless spacetime fields arising from the type IIA and type IIB superstring are:

- The dilaton ϕ , the Kalb-Ramond field B_2 (in the language of differential forms, $B_{\mu\nu}$ is written as a two-form $B_2 = \frac{1}{2} B_{\mu\nu} dx^\mu \wedge dx^\nu$), and the metric $g_{\mu\nu}$, as in the bosonic string.

²With slightly more work, one finds three more tachyon-free superstring theories with critical dimension $D = 10$. They are referred to as heterotic $E_8 \times E_8$, heterotic $SO(32)$ and type I theories. They will not play a major role in the following. Together with the type II theories, they are the only consistent superstring theories.

³This is the subject of string field theory [30–32]

⁴Care must be taken when defining an effective field theory including the first massive modes since gap between two adjacent mass levels is only an $\mathcal{O}(1)$ factor.

- Type IIA contains antisymmetric rank-1 and rank-3 tensor fields C_1 and C_3 .
- Type IIB contains antisymmetric rank-0, rank-2, and rank-4 tensor fields C_0 , C_2 , and C_4 .
- Both theories include fermionic partners to the bosonic fields such that the particle spectrum is supersymmetric.

From the string interactions, effective actions for type IIA and type IIB superstring theories can be derived for the massless sector:

$$S_{IIA} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} e^{-2\phi} \left(\mathcal{R} + 4\partial_\mu \phi \partial^\mu \phi - \frac{1}{2} |H_3|^2 \right) - \frac{1}{4\kappa^2} \int d^{10}x \sqrt{-g} \left(|F_0|^2 + |F_2|^2 + |\tilde{F}_4|^2 \right) - \frac{1}{4\kappa^2} \int B_2 \wedge F_4 \wedge F_4 + \text{fermions}, \quad (1.4.1)$$

$$S_{IIB} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} e^{-2\phi} \left(\mathcal{R} + 4\partial_\mu \phi \partial^\mu \phi - \frac{1}{2} |H_3|^2 \right) - \frac{1}{4\kappa^2} \int d^{10}x \sqrt{-g} \left(|F_1|^2 + |F_3|^2 + |\tilde{F}_5|^2 \right) - \frac{1}{4\kappa^2} \int C_4 \wedge H_3 \wedge F_3 + \text{fermions}, \quad (1.4.2)$$

where $\mathcal{R}$ is the Ricci scalar with respect to the metric g , and we define:

$$H_3 = dB_2, \quad F_k = dC_{k-1}, \quad \tilde{F}_k = F_k + C_{k-3} \wedge H_3. \quad (1.4.3)$$

Here $|F_0|$ should be seen as constant without a dynamical underlying field. The resulting field theories are $\mathcal{N} = 2$ supersymmetric in 10D (32 supercharges), which is a nontrivial discovery as we only started with a supersymmetric action on the 2D worldsheet. The resulting theories are called type IIA and type IIB supergravity (SUGRA). They are the only possible SUGRA theories in D=10 and contain *no free parameters* in the sense that there is not even a coupling constant that can take arbitrary values. All couplings are given by expectation values of scalar fields. In fact, this is a property of string theory in general, as the string coupling constant g_s is determined by the background value of the dilaton

$$g_s = e^{\langle \phi \rangle}. \quad (1.4.4)$$

This is a remarkable property and desirable for a theory of quantum gravity. The fermionic terms, which are not displayed in (1.4.1), (1.4.2), are fully determined by supersymmetry.

If string theory indeed describes the real world, then α' must be so small that $1/\sqrt{\alpha'}$ exceeds the energy scale probed by current experiments, such as those conducted at the LHC. Otherwise, we would have already observed the massive particle tower predicted by string theory. Consequently, the observable physics of our universe should emerge from the effective actions (1.4.1) or (1.4.2).

1.5 Open Strings

In addition to the closed strings discussed above, string theory also allows for open strings, which have two endpoints at $\xi^1 = 0$ and $\xi^1 = 1$. Varying the bosonic action (1.3.1) for strings with boundaries

gives (in a suitable gauge choice for the worldsheet metric):

$$\delta S = -\frac{1}{2\pi\alpha'} \int d\xi^0 \partial_1 X_\mu \delta X^\mu \Big|_{\xi^1=0}^{\xi^1=1}, \quad (1.5.1)$$

which implies that at each endpoint $\xi^1 = 0, 1$, the variation δX^μ must satisfy either Dirichlet (D) or Neumann (N) boundary conditions:

$$\mathbf{D:} \quad \delta X^\mu \Big|_{\xi^1=0,1} = 0, \quad (1.5.2)$$

$$\mathbf{N:} \quad \partial_1 X^\mu \Big|_{\xi^1=0,1} = 0. \quad (1.5.3)$$

An analogous calculation holds for the superstring. The boundary conditions relate the left- and right-moving sectors with each other, such that open strings have only one set of oscillatory modes. The endpoints of an open string can only move in Neumann directions (by definition, Dirichlet conditions fix the endpoints to a certain location), which define a submanifold of the target space. These submanifolds are called *Dirichlet-branes* or D-branes for short. D-branes are dynamical objects with a rich structure, as famously discovered by Polchinski [33] (see [20, 34] and references therein for more details). A Dp-brane is defined as a D-brane extended in p spatial directions and time and its tension is given by

$$T_p = \frac{2\pi}{g_s(2\pi\sqrt{\alpha'})^{p+1}}. \quad (1.5.4)$$

Type IIA string theory only allows for stable Dp-branes with $p \in \{0, 2, 4, 6, 8\}$, while type IIB string theory permits $p \in \{1, 3, 5, 7, 9\}$. This is related to the fact that Dp-branes couple to the anti-symmetric tensor fields C_k as

$$\mu_p \int_{Dp} C_{p+1}, \quad (1.5.5)$$

with $\mu_p = g_s T_p$ the charge of the Dp-branes.⁵ On the worldvolume of a Dp-brane lives a $U(1)$ gauge theory, with the microscopic degrees of freedom given by the open string endpoints. When N Dp-branes coincide, the gauge symmetry is enhanced to $U(N)$. Dp-branes are interesting objects in string theory, coexisting with closed strings.

1.6 Model Building

If we want to connect string theory to the real world, we must explain how $D = 10$ can be consistent with our observation of living in four spacetime dimensions and how the particle physics we observe, i.e., the Standard Model, dark matter, and dark energy, can follow from (one of) the supersymmetric actions (1.4.1), (1.4.2).

⁵The antisymmetric tensor fields C_0, C_1, C_2, C_3 have a dual (magnetic) description in terms of their dual fields C_8, C_7, C_6, C_5 (C_4 is self-dual). Dp branes with $p \geq 4$ couple to those fields.

The first issue is resolved by the concept of *compactification*. It refers to the assumption that some spatial directions are compact and small, a concept that dates back to Kaluza and Klein (KK) [35, 36]. As long as there is not enough energy to probe length scales below the size of the compact extra dimensions, they remain invisible to a low-energy observer. Let us illustrate this point with a simple example, namely a 5D universe with a massless scalar field, where one dimension is a circle. We take the metric to be

$$ds_5^2 = g_{4,\mu\nu} dx^\mu dx^\nu + dy^2, \quad \mu, \nu = 0, \dots, 3, \quad y \in (0, 2\pi R), \quad (1.6.1)$$

such that the x^μ are the coordinates of the large 4D space and y is the coordinate of the circle with radius R . The action of the scalar field is given by

$$S_\phi = - \int d^5x \sqrt{-g_5} \frac{1}{2} (\partial\phi)^2 = \int d^5x \sqrt{-g_5} \frac{1}{2} \phi(x, y) \square_5 \phi(x, y), \quad (1.6.2)$$

with the 5D d'Alembert operator splitting according to

$$\square_5 = \square_4 + \partial_y^2, \quad (1.6.3)$$

where $\square_4$ is the 4D wave operator with respect to $g_{\mu\nu}$. With the general Fourier ansatz

$$\phi(x, y) = \sum_{n=1}^{\infty} \tilde{\phi}_n(x) \sin(ny/R) + \sum_{n=0}^{\infty} \phi_n(x) \cos(ny/R), \quad (1.6.4)$$

the action (1.6.2) becomes

$$S = - \int d^4x \sqrt{-g_4} 2\pi R \left(\frac{1}{2} (\partial\phi_0(x))^2 + \frac{1}{4} \sum_{n=1}^{\infty} \left[(\partial\phi_n(x))^2 + m_n^2 \phi_n^2 + (\partial\tilde{\phi}_n(x))^2 + m_n^2 \tilde{\phi}_n^2 \right] \right), \quad (1.6.5)$$

where $m_n = n/R$. We observe that (1.6.2) turns into a 4D action (1.6.5), containing one massless field ϕ_0 and an entire tower of massive fields $\phi_n, \tilde{\phi}_n, n \geq 1$, with the mass scale set by the size of the compact dimension R . As a result, we conclude that if the circle is small, only the massless mode and not the entire tower of massive particles, signaling the existence of an extra dimension, can be detected by a low-energy observer.

This strategy applies to fields of any kind. Let us illustrate, in the above example, what happens to the metric when considering it as a dynamical field. It can be parametrized in the general form

$$ds_5^2 = (g_{4,\mu\nu} + 2\phi^2 A_\mu A_\nu) dx^\mu dx^\nu + \sqrt{2}\phi^2 A_\mu dx^\mu dy + \phi^2 dy^2, \quad \mu, \nu = 0, \dots, 3, \quad (1.6.6)$$

where the component fields $\phi, A_\mu, g_{\mu\nu}$ can, in principle, depend on x and y . However, we expect a situation similar to the one before, where upon reduction, we find a zero mode, independent of y , and a tower of massive modes. Focusing on the zero mode is achieved by assuming the component fields to depend only on x . The 5D Einstein-Hilbert action then reduces to

$$S = \int d^4x \sqrt{-g_4} \left(\frac{M_P^2}{2} \mathcal{R}_4 - \frac{1}{4} \phi^2 F_{\mu\nu} F^{\mu\nu} + \frac{M_P^2}{3} \frac{(\partial\phi)^2}{\phi^2} \right), \quad (1.6.7)$$

where $F_{\mu\nu}$ is the well-known expression for the field strength tensor built from the vector field A_μ , and the 4D Planck mass M_P relates to the 5D Planck mass M_5 by $M_P^2 = 2\pi R M_5^3$. The existence of a $U(1)$ gauge field in the 4D action is related to the existence of shift symmetry around the S^1 from the 5D perspective. The key lessons to be learned from the above are: Compactification at the massless level reduces the higher-dimensional metric to a lower-dimensional metric plus additional fields. Furthermore, there also exists a tower of massive states in the lower-dimensional theory at a mass scale set by the size of the compact space. This strategy of compactification can analogously be performed for compact spaces of higher dimension, with 6D geometries being relevant for our case of interest. One will again get a set of massless fields in 4D accompanied by a tower of massive modes.

After choosing a geometry for the six-dimensional compact space, one may now use the reduction at the massless level and work with the resulting effective actions. These theories are good effective field theories below the KK-scale, i.e. the scale set by the size of the compact dimensions, which has to itself be below the string scale $1/\sqrt{\alpha'}$, for consistency. One should hence take the actions (1.4.1), (1.4.2) and reduce them on some chosen 6D geometry. However, as type IIA and type IIB string theory are complicated theories, it is very difficult to take any random manifold, apply the KK procedure, and understand the resulting 4D theory. Rather, it proves more practical to choose specific 6D geometries such that the resulting 4D theory has a high degree of symmetry, with the relevant symmetry in our scenario being SUSY. We therefore look for 6D manifolds such that the resulting theory in 4D preserves some supercharges. This requirement is fairly restrictive and leads to the study of *Calabi-Yau* (CY) manifolds.⁶ CY manifolds are Ricci-flat complex Kähler manifolds and precisely lead to lower-dimensional theories preserving SUSY. One calls a complex n -dimensional (i.e. $2n$ real-dimensional) CY manifold a *CY n -fold*. They are a thoroughly studied subjects in physics and mathematics [39–44]. The 6D geometries of interest can be classified as follows:

- T^6 : The six-torus is a compact Riemann-flat (and hence Ricci-flat) manifold and preserves all 32 supercharges.
- $K3 \times T^2$: $K3$ is a complex two-dimensional manifold (a CY 2-fold) that is Ricci-flat but not Riemann-flat. The space $K3 \times T^2$ is compact and breaks half of the 32 supercharges of type IIA/IIB string theory. The $K3$ manifold is, next to T^4 , the only CY 2-fold.
- CY_3 : There exists a large number of proper compact CY 3-folds, i.e., complex three-dimensional Ricci-flat manifolds that are neither T^6 nor $K3 \times T^2$. They break $1/4$ of the 32 supercharges. It is unknown whether or not the number of CY 3-folds is finite, but it is strongly believed to be the case.

Compactifying on a proper CY manifold hence leads to a 4D theory with 8 supercharges, that is $\mathcal{N} = 2$ in 4D. The resulting 4D effective action can be worked out explicitly [45–47]. Since our world is non-supersymmetric, we must spontaneously break the remaining SUSY at a scale larger than that accessible by the LHC to construct models that can describe our real world. There are multiple possibilities how it can be achieved.

Fluxes Recall that type IIA/IIB supergravity contains form fields C_k accompanied by field-strength forms F_{k+1} . These fields can have non-trivial integrals over a $k + 1$ -dimensional submanifold Σ_{k+1} of

⁶Compactifications on non-CY manifolds have been explored e.g. in [37,38].

the compact geometry:

$$\int_{\Sigma_{k+1}} F_{k+1} \neq 0. \quad (1.6.8)$$

This is analogous to how the monopole bundle of a $U(1)$ gauge theory on S^2 obeys

$$\int_{S^2} F_2 \neq 0. \quad (1.6.9)$$

In analogy to the monopole case, the integrals (1.6.8) cannot take arbitrary values but are quantized. Fluxes on submanifolds (called cycles) of the compact space break at least some of the 8 remaining supercharges, and the vast choice of possible fluxes leads to a large variety of 4D models [48]. See [49, 50] for reviews about compactifications with fluxes in string theory.

D-branes Considering D-branes as part of the geometry is another possibility for breaking SUSY, and they furthermore provide a useful tool for building models, as they, as discussed earlier, host gauge theories on their worldvolume. For example, one may consider a Dp-brane with $p \geq 3$ such that it fills out the entire 4D macroscopic space and is wrapped on a $(p - 4)$ -dimensional cycle of the internal space. As a result, the 4D effective physics contains a gauge theory inherited from the theory on the brane. They furthermore also break at least one-half of the 8 supercharges.

Orientifolds A commonly used method, which in fact is almost always necessary for consistency if fluxes or branes are present, is to consider *CY orientifold* compactifications. An *orbifold* is a quotient space Y/G with Y some space (in our case a CY) and G a group that acts on Y including fixed points. An orientifold is defined as the quotient arising from an orbifold action combined with parity symmetry of the worldsheet:

$$\Omega : \xi^1 \rightarrow -\xi^1. \quad (1.6.10)$$

The simplest example is the orientifold of a circle parametrized by a coordinate $y \in (0, 2\pi R)$. The $\mathbb{Z}_2$ -symmetry $y \rightarrow -y$ can be modded out and contains two fixed points at 0 and πR . The resulting space is an interval of length π , which is hence an orbifold of the circle. Together with the action (1.6.10), the theory becomes unoriented at the fixed point locus, where a new object, called an orientifold plane (or O-plane for short), is present [51]. In general, O-planes can come in various dimensions depending on the action of the symmetry group G , and are generally called Op-planes, where p is the number of spatial dimensions. Op-planes are closely related to Dp-branes, and type IIA allows for stable Op-planes for $p = 0, 2, 4, 6, 8$, while type IIB allows for stable Op-planes for $p = 1, 3, 5, 7, 9$. As for D-branes, also O-planes break at least one-half of the remaining supercharges.

1.7 The Landscape

We summarize that the choice of the following ingredients leads to a large family of 4D semi-realistic models from string theory:

- The choice of Calabi-Yau manifold,

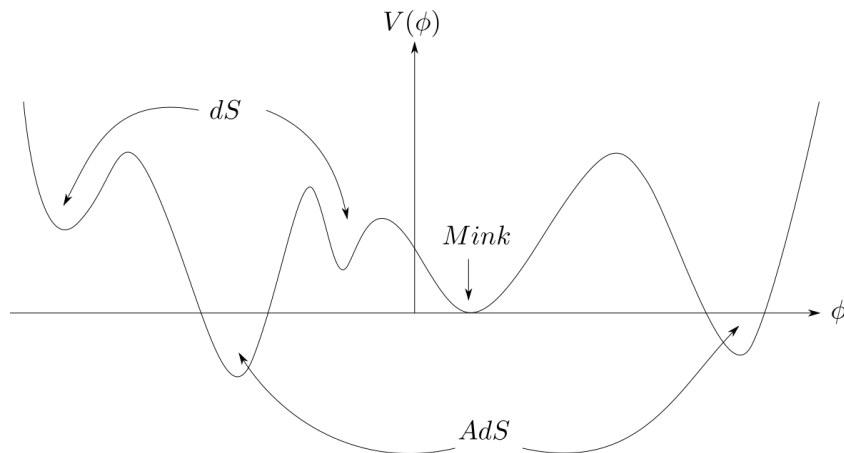


Figure 1.2. An illustration of the string landscape potential. Vacua with positive energy density lead to dS, with vanishing vacuum energy to Minkowski and with negative vacuum energy to AdS spacetimes.

- The choice of orientifolding the CY,
- The choice of fluxes on the cycles of the compact space,
- The choice of D-branes in the geometry.

Choices of this kind lead to a vast number of solutions, with the number of possible solutions being quoted between $\mathcal{O}(10^{500})$ [52] and (using slightly more sophisticated methods) $\mathcal{O}(10^{272,000})$ [53]. All these solutions arise from an (essentially) unique theory in $D = 10$ that does not contain free parameters. The 4D physics varies from solution to solution, i.e., gauge groups, the particle content, and coupling ‘constants’ can differ, making them appear as distinct 4D theories. This space of solutions to string theory is called the *string landscape* [54, 55]. It is often pictured as a scalar potential with very many minima, as illustrated in Fig. 1.2, with each minimum corresponding to a solution which are local vacua of string theory. Importantly, it is believed that the landscape is finite, meaning that there are very many, but only finitely many vacua [56, 57].⁷⁸

The string landscape contains Minkowski solutions, which are necessarily supersymmetric, Anti-de Sitter (AdS) solutions, which are naturally obtained for solutions preserving four supercharges but can equally well be non-supersymmetric, and (potentially) de Sitter (dS) solutions, which can only exist once SUSY is broken completely. The cosmology of our universe today is well described by a positive cosmological constant, i.e. by dS space. However, recently [58–61], it has been questioned whether dS spaces are part of the string landscape. While it is true that explicitly constructing dS solutions proves extraordinarily difficult, there are no clear arguments why it should be impossible.⁹

⁷Potentially, one needs to impose an IR cutoff for this statement to hold.

⁸This is again different from field theories, where a coupling constant can take arbitrary values, allowing for infinitely many different vacua.

⁹The conjecture that string theory does not allow for dS solutions is part of a web of further statements about quantum gravity (mainly string theory) known as the swampland. Its idea goes back to [62, 63] (see [64–66] for reviews) and it

While there exist a number of no-go theorems against dS [67–81], string theory has tools to evade them. The fact that SUSY needs to be broken completely and the Dine-Seiberg problem [82]¹⁰ are reasons why it is hard to find explicit dS solutions. Some people say:

Finding dS solutions in the string landscape could be as hard as showing that a zebra can exist just using QFT.

–Raphael Bousso

We from now on assume that dS solutions are part of the landscape.

We would now like to come back to the heart of theoretical physics: Making predictions for measurements that are going to be performed in the future. We will see that in a landscape situation, making predictions for future measurements becomes an intriguing and difficult problem. To understand the origin of this problem, we need to study the universe dynamics in the presence of multiple vacua.

1.7.1 Comments

In this paragraph, a few more detailed comments about the challenges and possibilities of model building in string theory are given. The reader may jump ahead if reading time is limited.

Due to their applications in string theory, CY manifolds became a target for pure mathematics research, leading to the discovery of mirror symmetry [16, 17, 46, 83–86]. It is a relation between two CY manifolds X and $\hat{X}$ such that the 4D theory arising from compactifying type IIA string theory on X is the same as compactifying type IIB on $\hat{X}$. It can be used as a strong-weak duality, making some seemingly unfeasible calculations possible. As a result, it is a useful tool in building models.

One particular issue in model building with CY manifolds is that they come with *moduli* [45]. These are geometric deformations of the manifold that are unobstructed and appear in the 4D effective action as scalar fields without potential. We have seen an example of a modulus before: The scalar field ϕ , determining the size of the extra dimension in (1.6.6) has no potential in (1.6.7). However, it is desirable that all moduli fields acquire a mass, as massless scalar fields mediate fifth-forces that we do not observe. While part of the moduli, the *vectormultiplets*, can be made massive (one also says stabilized) by fluxes [48],¹¹ the *hypermultiplets* are more difficult to control and one usually has to refer to non-perturbative effects for a stabilization mechanism. There is no general method known for stabilizing all moduli.

The subject of model building is best approached in the context of type IIB string theory, due to the foundational work [89–91]. The leading proposals for realizing a landscape of dS solutions are given by [92–94], building on earlier work [95–97]. Various other proposals for stabilizing moduli and/or realizing de Sitter are [98–113].

Another approach to study CY compactifications is F-theory [114–119]. It is a non-perturbative formulation of type IIB string theory and further deepens the connection between string theory and geometry. It allows for a wide range of model building applications, see e.g. [120–133].

is defined, in opposition to the landscape, as the space of low-energy EFTs that do not allow for a consistent coupling to gravity. Statements are made in the form of conjectures with variable robustness. We refer to the reviews for references to the (by now very many) original articles.

¹⁰The Dine-Seiberg problem refers to the challenge that dS solutions can only exist in strongly coupled regions.

¹¹It was claimed that the moduli stabilization by fluxes is not as straightforward as naively expected [87]. See [88] for a discussion on this issue.

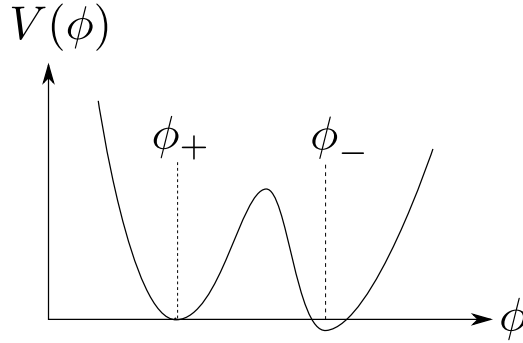


Figure 1.3. A toy-model potential for field-theoretic tunneling. The two minima are located at ϕ_+ and ϕ_- with $V(\phi_+) > V(\phi_-)$. As an overall shift of the potential is irrelevant for field-theoretical physics, we have chosen $V(\phi_+) = 0$. Furthermore, ΔV is much smaller than the height of the potential barrier. As a result, the thin-wall application is applicable.

Finally, the study of string theory led to the discovery of AdS/CFT or gauge/gravity duality [134–136] causing a revolution in a variety of fields such as quantum gravity, cosmology, QFT and even condensed matter theory. It has been used as a tool to understand certain geometries highly relevant for model building purposes [90] and it has been attempted to develop a gauge/gravity duality for dS spaces as well, see e.g. [137–142].

1.8 Vacuum Decay

Given a universe sitting in a vacuum, i.e. a minimum of the landscape potential depicted in Fig. 1.2, it is possible for it to decay into other vacua. This process is the field-theoretic analogue of tunneling in quantum mechanics. In quantum mechanics, the tunneling rate can be obtained by finding a solution to the Euclidean equations of motion that interpolates between two minima of the corresponding potential. This strategy and the following generalization are reviewed e.g. in [143].

Field-theoretic tunneling A similar strategy can be adopted for the field-theoretic case, first developed by Coleman [144, 145]. Let us, for the moment, neglect gravity and consider a scalar field theory with action

$$S = \int d^4x \left(-\frac{1}{2}(\partial\phi)^2 + V(\phi) \right), \quad (1.8.1)$$

where the potential $V(\phi)$ has two minima at ϕ_+ and ϕ_- , with $\Delta V \equiv V(\phi_+) - V(\phi_-) > 0$, as illustrated in Fig. 1.3.

A vacuum transition from the minimum at ϕ_+ to ϕ_- occurs via the quantum formation of a spherical bubble of the new vacuum, which subsequently expands. The transition rate can be calculated by finding an instanton, i.e. a solution to the euclidean equations of motion interpolating between the two vacua. It has been shown that the dominant instanton geometry has $O(4)$ symmetry, such that

the metric and the field profile can be parametrized as

$$ds^2 = dr^2 + r^2 d\Omega_3^2, \quad \phi = \phi(r). \quad (1.8.2)$$

The Euclidean equation of motion is given by

$$\phi'' + \frac{3}{r}\phi' = \frac{dV}{d\phi}, \quad (1.8.3)$$

where a prime denotes a derivative with respect to r . The relevant instanton is a solution to (1.8.3) with boundary conditions $\phi'(r=0) = 0$, $\phi(r \rightarrow \infty) = \phi_+$, and $\phi(r=0)$ in the vicinity of ϕ_- . The on-shell action of this configuration determines the decay rate per time and volume as

$$\Gamma = A \exp(-B), \quad B = S_{\text{instanton}} - S_{\text{vacuum}}, \quad (1.8.4)$$

where A is a dimensionful prefactor that arises at one-loop order, $S_{\text{instanton}}$ is the euclidean action evaluated on the instanton solution and S_{vacuum} is the action of the parent vacuum configuration. As we can shift the action by an overall constant, we choose a shift such that $S_{\text{vacuum}} = 0$ (meaning $V(\phi_+) = 0$). If $B = S_{\text{instanton}} \gg 1$, it gives the dominant contribution to the decay rate.

If ΔV is much smaller than the height of the potential barrier, the *thin-wall* approximation holds. In this case, the profile of $\phi(r)$ is such that it is roughly equal to ϕ_- for $r \lesssim r_{\text{wall}}$ and to ϕ_+ for $r \gtrsim r_{\text{wall}}$ for some parameter r_{wall} determining the location of the *domain wall*. Only around $r = r_{\text{wall}}$ does the field significantly move. In this scenario, one can define the tension of the domain wall as

$$T = \int_{\phi_+}^{\phi_-} d\phi \sqrt{2V(\phi)}, \quad (1.8.5)$$

and one may view the instanton as the $O(4)$ -symmetric configuration where the two vacua are connected by a spherical domain wall of tension T and radius R . The action of the configuration reads

$$S = \int d^4x \left(-\frac{\pi^2}{2} (\Delta V) R^4 + 2\pi^2 T R^3 \right). \quad (1.8.6)$$

The two terms in the action have the following interpretation: The first corresponds to an energy gain when creating a ball of new vacuum, due to the vacuum energy of the daughter vacuum being lower. The second term corresponds to the energy price that needs to be paid for creating a spherical domain wall of tension T . The critical bubble radius, where the energy gain starts outweighing the energy cost, is an extremum of the action (hence it defines a solution to the equations of motion) and is given by

$$R_c = \frac{3T}{\Delta V}. \quad (1.8.7)$$

It requires a quantum fluctuation of size R_c to create a bubble that can go on-shell and so lead to the decay of the vacuum at ϕ_+ . The on-shell action arising from inserting (1.8.7) into (1.8.6) is given by

$$B = S_{\text{instanton}} = \frac{27\pi^2 T^4}{2(\Delta V)^3}. \quad (1.8.8)$$

The action (1.8.6) of course also describes the decay of a vacuum mediated by a fundamental brane with tension T .

Gravitational tunneling In the presence of gravity, gravitational effects can play a significant role in vacuum decay processes. These were first studied in [146], while other related works are given by [147–153]. Pedagogical reviews are given by [24, 143, 154, 155] and more recent work includes [156–159]. We consider a field theory coupled to gravity with the Euclidean action taking the form

$$S = \int_{\mathcal{M}} d^4x \sqrt{g} \left(-\frac{M_P^2}{2} \mathcal{R} + \frac{1}{2} (\partial\phi)^2 + V(\phi) \right) - M_P^2 \int_{\partial\mathcal{M}} d^3x \sqrt{h} \mathcal{K}, \quad (1.8.9)$$

where M_P is the Planck mass, $\mathcal{R}$ is the Ricci scalar with respect to the metric g , h is the induced metric on the boundary $\partial\mathcal{M}$ of the geometry $\mathcal{M}$, and $\mathcal{K}$ is the extrinsic curvature of the boundary. This action is the Einstein-Hilbert action coupled to a scalar field, supplemented by the Gibbons-Hawking-York boundary term.

We again look for $O(4)$ symmetric solutions to the Euclidean equations of motion and parametrize the metric and field profile as

$$ds^2 = dr^2 + \rho(r)^2 d\Omega_3^2 = dr^2 + \rho(r)^2 (d\psi^2 + \sin^2(\psi) d\Omega_2^2), \quad \phi = \phi(r). \quad (1.8.10)$$

The Einstein equation and the equation of motion for ϕ are

$$\rho' = \sqrt{1 + \frac{1}{3M_P^2} \left(\frac{\phi'^2}{2} - V(\phi) \right)}, \quad \phi'' + \frac{3\rho'}{\rho} \phi' = \frac{dV}{d\phi}, \quad (1.8.11)$$

where a prime again denotes a derivative with respect to r . We look for solutions to (1.8.11) with appropriate boundary conditions. These depend on the nature of the decaying space. In the presence of gravity, adding a constant to the Lagrangian is physically relevant such that one must distinguish between

$V(\phi_+) > 0$, Decays of dS: Euclidean dS space is a compact sphere. It is characterized by the solution $\rho(r) = \ell_{dS} \sin(r/\ell_{dS})$, where $\ell_{dS} = \sqrt{3M_P^2/V(\phi_+)}$ is the curvature length of dS. Hence, there is a maximum value $r_{\max} = \pi\ell_{dS}$ of r , with $r \geq 0$. The appropriate boundary conditions are: $\phi'(r=0) = 0$, $\rho'(r=0) = 1$, and $\phi'(r=r_{\max}) = 0$, $\rho'(r=r_{\max}) = 1$. Furthermore, $\phi(r=0)$ is in the vicinity of ϕ_- , and $\phi(r=r_{\max})$ is in the vicinity of ϕ_+ .

$V(\phi_+) = 0$, Decays of Minkowski: Minkowski space is characterized by $\rho(r) = r$. Euclidean Minkowski space corresponds to just $\mathbb{R}^4$. The appropriate boundary conditions are: $\phi'(r=0) = 0$, $\rho'(r=0) = 1$, and $\phi(r \rightarrow \infty) = \phi_+$. Furthermore, $\phi(r=0)$ is in the vicinity of ϕ_- . Decays can only occur to AdS.

$V(\phi_+) < 0$, Decays of AdS: AdS space is characterized by $\rho(r) = \ell_{AdS} \sinh(r/\ell_{AdS})$, where $\ell_{AdS} = \sqrt{-3M_P^2/V(\phi_+)}$ is the curvature length of AdS and $r \geq 0$. Euclidean AdS is a non-compact, infinite-volume space. The appropriate boundary conditions are: $\phi'(r=0) = 0$, $\rho'(r=0) = 1$, and $\phi(r \rightarrow \infty) = \phi_+$. Furthermore, $\phi(r=0)$ is in the vicinity of ϕ_- . Decays can only occur to lower-energy AdS spaces.

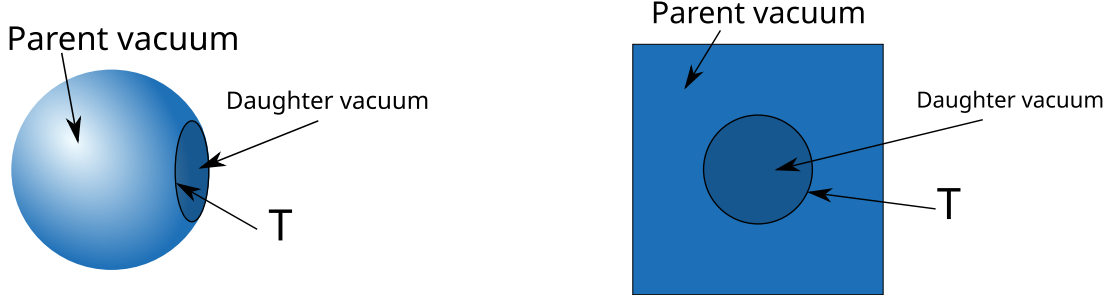


Figure 1.4. The instanton geometry of a decay of dS space (left) and AdS/Minkowski (right) is depicted. In each case, a ball of daughter vacuum is glued along a domain wall of tension T into the parent vacuum configuration. In the case of a dS decay, the instanton is compact whereas for AdS/Minkowski, it is non-compact.

In the thin-wall approximation, one can view the geometry of the instantons as follows: A ball-shaped patch of the new (daughter) vacuum is glued into the parent vacuum along a spherical domain wall of tension T , with T defined as before in (1.8.5). In the case of the decay of a dS spacetime, the instanton geometry is compact; in the other cases, it is non-compact, as illustrated in Fig. 1.4. The critical bubble radius can again be determined by looking for a stationary point of the action. The decay rate per time and volume (1.8.4) can best be expressed in terms of the dimensionless variables

$$x = \frac{3T}{4M_P^2\Delta V}, \quad y = \frac{V(\phi_+) + V(\phi_-)}{\Delta V}, \quad r(x, y) = 2 \frac{1 + xy - \sqrt{1 + 2xy + x^2}}{x^2(y^2 - 1)\sqrt{1 + 2xy + x^2}}, \quad (1.8.12)$$

and is given by

$$\Gamma \propto \exp(-B), \quad B = \frac{27\pi^2 T^4}{2(\Delta V)^3} r(x, y). \quad (1.8.13)$$

This formula was derived for $\Delta V > 0$ and applies to all such decays of dS spaces. It also holds for the decay of Minkowski or AdS spaces, which necessarily decay to AdS, provided that

$$T < 2M_P^2 \left(\frac{1}{\ell_{AdS,-}} - \frac{1}{\ell_{AdS,+}} \right), \quad (1.8.14)$$

where $\ell_{AdS,+}$ is the AdS length of the parent vacuum (taken to be infinity for Minkowski) and $\ell_{AdS,-}$ is the AdS length of the daughter vacuum. This equation has an intuitive interpretation: The volume of a 4-ball of euclidean AdS is, for radii $R > \ell_{AdS}$, given by $\sim R^3 \ell_{AdS}$. As such, it is not guaranteed that the energy gain from creating a lower-energy space outweighs the energy cost of creating a domain wall of tension T , given by $\sim TR^3$. Only if equation (1.8.14) holds, a created bubble can go on-shell and expand. Importantly, the condition (1.8.14) cannot be fulfilled for supersymmetric AdS vacua, as the BPS bound forbids domain walls of small tension. It can generally be shown that supersymmetric spaces are protected from decaying.

Furthermore, dS decays need not fulfill $\Delta V > 0$. Up-tunneling, referring to transitions with $\Delta V < 0$, is possible, and formula (1.8.13) still applies. The expansion of the parent dS space can pull a

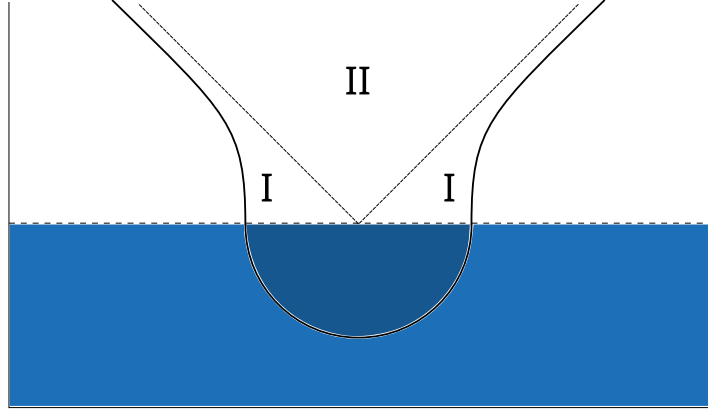


Figure 1.5. The Lorentzian spacetime glued onto the euclidean instanton at $\psi = \pi/2$. The metric (1.8.17) covers region I of the bubble interior whereas the metric (1.8.18) covers region II.

bubble that naively would collapse to larger and larger sizes. In Minkowski and AdS spaces, energy conservation forbids up-tunneling processes. No such symmetry is present for dS spaces, so any transition is allowed. The up-tunneling instanton geometry is the same as for the reverse down-tunneling event, differing only in the vacuum action contributing to (1.8.4). As a result, the up-tunneling rate compares to the down-tunneling process according to

$$\frac{\Gamma_{\text{up}}}{\Gamma_{\text{down}}} \propto \exp(S_{\text{vacuum},+} - S_{\text{vacuum},-}) = \exp(-8\pi^2 M_P^2 (\ell_{dS,-}^2 - \ell_{dS,+}^2)), \quad (1.8.15)$$

showing that up-tunneling is exponentially suppressed. Here, we used the fact that

$$S_{\text{vacuum}} = -8\pi^2 M_P^2 \ell_{dS}^2 \quad (1.8.16)$$

holds for dS spaces with curvature length ℓ_{dS} .

After the creation of a bubble, it expands in the background of the parent space. In the thin-wall limit, the induced metric on the domain wall is that of a 3-dimensional dS space. See Fig. 1.5 for an illustration. The Lorentzian evolution after formation is obtained by analytically continuing the variable ψ in (1.8.10) according to $\psi \rightarrow \pi/2 + it$, resulting in

$$ds^2 = dr^2 + \rho(r)^2 \left(-dt^2 + \cosh(t)^2 d\Omega_2^2 \right). \quad (1.8.17)$$

One can draw this spacetime glued onto the instanton geometry at $\psi = \pi/2$, as illustrated in Fig. 1.5. This metric has the peculiar property of a ‘horizon’ located at $r = 0$, where $\rho(r) = 0$. In fact, these coordinates do not cover the entire interior of the bubble. They only cover region I, as displayed in Fig. 1.5. To obtain the metric for the full interior of the bubble, one must further continue the metric (1.8.17) according to $r \rightarrow i\tau$ and $t \rightarrow i\pi/2 + \chi$, arriving at

$$ds^2 = -d\tau^2 + \tilde{\rho}(\tau)^2 \left(d\chi^2 + \sinh(\chi)^2 d\Omega_2^2 \right), \quad (1.8.18)$$

with $\tilde{\rho}(\tau) = -i\rho(i\tau)$. This metric describes an open FLRW universe with scale factor $\tilde{\rho}$ and covers region II of Fig. 1.5. It is furthermore possible to go to Kruskal-like coordinates that are valid in both regions. In the coordinate x , that is defined implicitly by

$$\rho(r)dx = dr, \quad (1.8.19)$$

the corresponding transformations are given by

$$u = e^{-x} \cosh(t), \quad v = e^{-x} \sinh(t), \quad (1.8.20)$$

and the metric reads

$$ds^2 = -\frac{\rho(x)^2}{u^2 - v^2}(dv^2 - du^2) + \frac{1}{1 - \frac{v^2}{u^2}}\rho(x)^2 d\Omega_2^2. \quad (1.8.21)$$

The horizon is located at $u^2 = v^2$, where $\rho = 0$ also holds.

The key messages to be taken from this section are

- Vacua can decay if they are not protected by supersymmetry.
- Non-SUSY Minkowski (which is not present in the string landscape) and non-SUSY AdS can only down-tunnel due to energy conservation.
- dS spaces can tunnel to any other spacetime as there is no symmetry that forbids it. However, up-tunneling is exponentially suppressed compared to down-tunneling.
- After the formation of the bubble, it expands and the newly created universe is of open Friedmann type.

1.8.1 Witten's bubble of nothing

Witten [160] has discovered that the Kaluza-Klein (KK) geometry $\mathbb{M}_4 \times S^1$ exhibits a peculiar instability. The relevant gravitational instanton is precisely the five-dimensional Schwarzschild black hole geometry, with metric

$$ds^2 = \left(1 - \frac{R_{KK}^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega_3^2 + \left(1 - \frac{R_{KK}^2}{r^2}\right) R_{KK}^2 dy^2, \quad y \sim y + 2\pi, \quad (1.8.22)$$

satisfying the Einstein equations. At $r \gg R_{KK}$, this geometry approaches the KK vacuum $\mathbb{M}_4 \times S^1$ with circle radius R_{KK} . Crucially, the geometry is defined only for $r \geq R_{KK}$ and at $r = R_{KK}$ the internal circle shrinks to zero size (cf. Fig. 1.6). From the four-dimensional point of view, this locus represents the end of the world. As a result, one may interpret this instanton as mediating between ‘nothing’ at $r = R_{KK}$ and the vacuum at $r \rightarrow \infty$. Despite the vanishing radius of the S^1 , the five-dimensional curvature remains small as long as R_{KK} is large in 5d Planck units, ensuring the reliability of a semiclassical analysis. One can analytically continue this instanton just like (1.8.17) and one finds that this hole keeps growing in time, cf. [160].

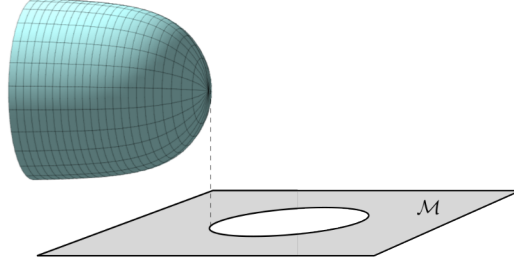


Figure 1.6. Illustration of the bubble of nothing instanton. The four-dimensional space $\mathcal{M}$ has a spherical boundary which coincides with the locus where the internal manifold, shown above $\mathcal{M}$, shrinks to zero size.

The nucleation probability per time and volume of such a bubble can be estimated by (1.8.4) with the relevant action given by

$$S = -\frac{M_5^3}{2} \int \sqrt{g} \mathcal{R}_5 - M_5^3 \int_{\partial \mathcal{M}} \sqrt{h} \mathcal{K}_5, \quad (1.8.23)$$

on the instanton geometry (1.8.22) and in the vacuum. In the above, $\mathcal{R}_5$ and $\mathcal{K}_5$ denote the 5d Ricci- and extrinsic curvature scalar respectively and M_5 is the 5D Planck mass. One explicitly finds [160]

$$B = 2\pi^3 M_5^3 R_{KK}^3 = \pi^2 M_P^2 R_{KK}^3, \quad (1.8.24)$$

where $M_P^2 = M_5^3 2\pi R_{KK}$ is the 4D Planck mass of the effective theory below the KK scale.

Clearly, as string theory has extra dimensions, one might be interested in studying decays to nothing. We will explicitly address this subject in Chapter 3.

1.9 Vacuum creation

One may ask the question about the ultimate beginning of the universe. There exist two leading proposals for how a universe can be created *from nothing*, the Hartle-Hawking [161] and the Linde/Vilenkin [162–166] proposal. Reviews on the subject are given in [167–169]. Both proposals consider the formation of a closed universe, as creating an infinite space through a quantum process appears impossible.

We restrict ourselves to metrics of the form

$$ds^2 = -\frac{N(t)^2}{q(t)} dt^2 + q(t) d\Omega_3^2, \quad (1.9.1)$$

with action

$$S = \int d^4x \sqrt{-g} \frac{M_P^2}{2} (\mathcal{R} - 2\Lambda) + M_P^2 \int d^3x \sqrt{h} \mathcal{K} = 2\pi^2 \int dt \left(-3 \frac{\dot{q}}{4N} + 3(N - q\Lambda) \right), \quad (1.9.2)$$

where $\sqrt{q}$ is the scale factor of the corresponding Friedmann universe and N is the lapse function. Neglecting additional degrees of freedom of the metric is called the *minisuperspace approximation*,

with the minisuperspace being spanned by the variable q , as N is a non-dynamical degree of freedom related to the time-reparametrization invariance of the system. In this section, we take $0 < \Lambda \equiv 3/\ell_{dS}^2$ as otherwise, a spherical universe can never be on-shell.

There are two different methods to obtain the leading-order creation rate of such a spherical universe: The path-integral and the Hamiltonian formalism.

Path-Integral approach Here, one first considers the amplitude for a universe of size q_i to evolve to a universe of size q_f using the path integral

$$A(q_i \rightarrow q_f) = \int_i^f \mathcal{D}g e^{iS[g]}, \quad (1.9.3)$$

where the integral is over all 4-metrics of the kind (1.9.1) interpolating between $q = q_i$ and $q = q_f$. The above path-integral expression simplifies to an ordinary integral upon integrating out q [169]

$$A(q_i \rightarrow q_f) = \sqrt{\frac{3i\pi}{2}} \int_{N_0}^{\infty} \frac{dN}{\sqrt{N}} e^{iS_0}, \quad (1.9.4)$$

with

$$S_0 = 2\pi^2 \left[\frac{\Lambda^2 N^3}{36} + N \left(3 - \frac{1}{2}(q_i + q_f) \right) + \frac{1}{N} \left(-\frac{3}{4}(q_i - q_f)^2 \right) \right]. \quad (1.9.5)$$

Whether the value N_0 in (1.9.4) should be taken as 0 or ∞ is the essential difference between the two proposals for vacuum creation. The remaining integral (1.9.4) can be approximated using a saddle-point approximation. The corresponding saddle points will, in general, be complex. Picard-Lefschetz theory [170] can be used to determine how the original integration contour should be deformed, thereby identifying which saddle points contribute to the amplitude [171–179]. Of course, the original contour, and hence the deformed contour, depend on the choice of N_0 .

One of the original ideas of [161] was to propose that spacetime had a smooth beginning rather than a big-bang singularity. As this is impossible in Lorentzian spacetime, it was proposed that the universe had a smooth Euclidean beginning and then transitioned to Lorentzian through a phase of complex metrics. Another formulation of this condition is that the universe should have no boundary or singularity in the past and hence the proposal [161] is also called the *no-boundary proposal*. The *wavefunction of the universe* $\Psi(q)$, which encodes information about the creation of a universe with scale factor $\sqrt{q}$ from nothing, is then defined as

$$\Psi(q) = \lim_{q_i \rightarrow 0} A(q_i \rightarrow q), \quad (1.9.6)$$

implementing the condition of the universe having ‘no-boundary’ in the past. In Fig. 1.7, a Lorentzian universe having a Euclidean beginning is depicted. In the setting explained here, the Euclidean beginning is reflected by the complex nature of the saddle points that appears when $q_i < \ell_{dS}$ is chosen. Intuition can be drawn from considering an integration contour in the complex N -plane that is initially purely imaginary, corresponding to a growing three-sphere until it reaches a radius of ℓ_{dS} . From there, the integration contour is taken parallel to the real axis, which corresponds to Lorentzian evolution.

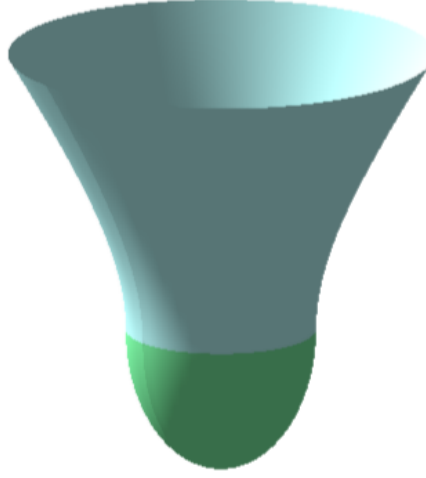


Figure 1.7. Illustration of the Lorentzian geometry (blue) glued onto the Euclidean region (green).

The result is a Lorentzian dS space glued at its ‘waist’ onto half of a 4-sphere (the Euclidean part of the geometry), as depicted in Fig. 1.7, which may be interpreted in total analogy to the vacuum decay processes discussed before. The euclidean part is half of an instanton (the full instanton is just a 4-sphere) onto which the Lorentzian geometry is glued on to.

There are four saddle points for the integral (1.9.4), which, for $q_i = 0$ and $q_f > \ell_{dS}$, are given by

$$N_{\pm,\pm} = \pm \ell_{dS}^2 \left[i \pm \left(\frac{q_f}{\ell_{dS}^2} - 1 \right)^{1/2} \right]. \quad (1.9.7)$$

The Hartle-Hawking proposal takes $N_0 = -\infty$ (with the contour passing below 0), and the corresponding wavefunction, arising from the contribution of all four saddle points, reads (see [169] for more details on the saddle points)

$$\begin{aligned} \Psi_{HaHa}(q) \propto & \exp \left(-4\pi^2 M_P^2 \ell_{dS}^2 \left[\ell_{dS}^2 + i \left(\frac{q}{\ell_{dS}^2} - 1 \right)^{3/2} \right] \right) \\ & + \exp \left(+4\pi^2 M_P^2 \ell_{dS}^2 \left[\ell_{dS}^2 - i \left(\frac{q}{\ell_{dS}^2} - 1 \right)^{3/2} \right] \right) + h.c. \end{aligned} \quad (1.9.8)$$

The Linde/Vilenkin proposal takes $N_0 = 0$, in which case only one saddle point contributes:

$$\Psi_{LV}(q) \propto \exp \left(-4\pi^2 M_P^2 \ell_{dS}^2 \left[\ell_{dS}^2 + i \left(\frac{q}{\ell_{dS}^2} - 1 \right)^{3/2} \right] \right). \quad (1.9.9)$$

We define the ‘creation rate’

$$J = |\Psi(q)|^2 \propto \begin{cases} \exp(+8\pi^2 M_P^2 \ell_{dS}^2) & \text{Hartle-Hawking} \\ \exp(-8\pi^2 M_P^2 \ell_{dS}^2) & \text{Linde/Vilenkin} \end{cases}, \quad (1.9.10)$$

where we have given only the dominant contribution in the Hartle-Hawking case. J cannot be directly interpreted in analogy to (1.8.4) as the creation rate per time and volume, since there is no parent space from which it emerges. Rather, a single J has no physical meaning. Only ratios of the form J_i/J_j , for two different vacua i and j , are physically relevant. We see that the Hartle-Hawking proposal leads to the (maybe counter-intuitive) prediction that universes with large ℓ_{dS} are most likely to be created from nothing whereas the Linde/Vilenkin proposal predicts universes with small rather than large ℓ_{dS} to nucleate. Which of the two proposals is ‘correct’ remains, even forty year into their existence, an unanswered question.

Hamiltonian formalism The action (1.9.2) is time reparametrization invariant. According to the quantization procedure of constrained systems, the generator of this symmetry, which is the Hamiltonian H , must annihilate physical states [180, 181]:

$$H\Psi = 0. \quad (1.9.11)$$

The above equation is known as the Wheeler-DeWitt (WDW) equation and serves as the defining equation for the wavefunction of the universe Ψ in the Hamiltonian formalism. With the metric in the standard FLRW form,

$$ds^2 = -N(t)^2 dt^2 + a(t)^2 d\Omega_3^2, \quad (1.9.12)$$

the Hamiltonian can be derived to be

$$H = -\frac{p_a^2}{24\pi^2 a} - 6\pi^2 a + 2\pi^2 a^3 \Lambda, \quad (1.9.13)$$

where p_a is the canonical momentum conjugate to a . Promoting a and p_a to operators according to $a \rightarrow a$ and $p_a \rightarrow -i\partial_a$ allows us to derive the WDW equation:

$$\left(-\frac{1}{a^p} \partial_a a^p \partial_a + V(a)\right) \Psi(a) = 0, \quad V(a) = a^2 \left(1 - \frac{a^2}{\ell_{dS}^2}\right), \quad (1.9.14)$$

where $p \in \mathbb{N}_0$ is a constant related to an operator ordering ambiguity. Let us focus on $p = 0$, as the wavefunctions in the semiclassical regime are independent of p . In this case, the equation takes the form of a time-independent Schrödinger equation with potential $V(a)$ and an energy eigenvalue of zero. There is a potential barrier between $a = 0$ and $a = \ell_{dS}$, within which the solutions to (1.9.14) will be exponentially increasing or decreasing. For $a > \ell_{dS}$, they are oscillating

$$\Psi_{\pm}(a < \ell_{dS}) \propto \exp\left(\pm \int_a^{\ell_{dS}} da |p(a')|\right), \quad (1.9.15)$$

$$\Psi_{\pm}(a > \ell_{dS}) \propto \exp\left(\pm i \int_{\ell_{dS}}^a da p(a') \mp \frac{i\pi}{4}\right), \quad (1.9.16)$$

with $p(a) = \sqrt{-V(a)}$. In this framework, the difference between the Hartle-Hawking and the Linde/Vilenkin proposals lies in the choice of boundary conditions to the WDW equation. While Linde and Vilenkin interpret the creation of universes at vanishing size as a quantum fluctuation

that must tunnel through a barrier, Hartle and Hawking draw their intuition from a Euclidean path integral aimed at calculating a ground-state wave function. Their respective approaches lead to the choices:

$$\Psi_{HaHa}(a > \ell_{dS}) = \Psi_+(a > \ell_{dS}) + \Psi_-(a > \ell_{dS}), \quad (1.9.17)$$

$$\Psi_{HaHa}(a < \ell_{dS}) = \Psi_-(a < \ell_{dS}),$$

$$\Psi_{LV}(a > \ell_{dS}) = \Psi_-(a > \ell_{dS}),$$

$$\Psi_{LV}(a < \ell_{dS}) = \Psi_+(a < \ell_{dS}) - \frac{i}{2}\Psi_-(a < \ell_{dS}). \quad (1.9.18)$$

The creation probability is argued to follow from integrating the probability current $j = \frac{i}{2}(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*)$ over the expanding branch of the corresponding wavefunction at $a > \ell_{dS}$, yielding (1.9.10).

Comments A few comments are in order: The precise relation between contour choices in the path integral and boundary conditions for the WDW equation is not yet fully clarified. This issue becomes important in the presence of a landscape, where the creation of different universes should be compared with each other. Furthermore, the LV wavefunction is similar to a propagator in the sense that it satisfies

$$H\Psi_{LV}(a) = \delta(a), \quad (1.9.19)$$

with a source at $a = 0$. This reflects the interpretation of the Linde/Vilenkin proposal that small universes can be created from nothing at vanishing cost and then need to tunnel through a potential barrier to go on-shell.

While the Hartle-Hawking wavefunction has the interpretation of a ground-state wavefunction, it is not entirely clear why it should correspond to a *creation from nothing* event.

Moreover, when going beyond the minisuperspace approximation and taking into account fluctuations, it leads to the prediction that the LV proposal predicts unsuppressed fluctuations. While this may seem unsatisfactory, it has been argued that it does not lead to any real inconsistencies [182, 183].

1.9.1 Nontrivial Topology

When adopting the Linde/Vilenkin proposal, one concludes that universes at small size can be created from nothing at very little cost. In the dS model discussed above, these universes need to tunnel through a potential barrier to go on-shell. The potential barrier arises from the fact that the spatial S^3 of dS spaces has positive curvature. It was claimed [184–186], that compact universes of non-positive curvature, i.e. toroidal universes, can be created without exponential suppression. It was speculated that effects such as Casimir energy [184] or the gradient energies of scalar fields [186] determine the most likely properties of the created universe. It could however be that the fluctuation of vacua with higher topology is suppressed by a fundamental reason of quantum gravity.

1.10 The string landscape as a solution to hierarchical questions

The string landscape contains a vast number of solutions, each realizing a different 4D universe. These universes can tunnel into one another and may also be constantly created from nothing. Together, they

form a *multiverse* containing different universes with a wide variety of physical laws. This situation provides a possible explanation for the various hierarchical open questions raised in Sect. 1.2. Namely, the landscape realizes all sorts of universes, some of which, by chance, exhibit ‘unnatural’ relations. A model where universes with unnaturally small cosmological constant are created dynamically was developed in [187]. Following [92, 188] (and other work), it became clear that the scenario [187] might be realized in string theory. Furthermore, it may be that observers like us can only form, or predominantly form, in universes that possess certain hierarchies. For example, one may argue that a large cosmological constant would accelerate the universe too quickly, preventing structure formation. Furthermore, it can be assumed that inflation and subsequent reheating are the only mechanisms capable of creating structure. Such reasoning is referred to as the *anthropic principle* [189–192].¹² Thus, the landscape with its many solutions, together with anthropic arguments, provides an explanation for the existence of hierarchies.

1.11 The Measure Problem

We now return to the task raised at the end of Sect. 1.7, namely making predictions for future measurements in the multiverse. Let us attempt to predict the likelihood of measuring result A rather than B in some future experiment. Assuming complete knowledge of the landscape, we might naively state

$$\frac{p_A}{p_B} = \frac{N_A}{N_B}, \quad (1.11.1)$$

where p_A and p_B denote the probabilities of us observing A or B, and N_A and N_B represent the number of times an observer measures result A or B in the global multiverse.

However, the string landscape naturally leads to the following situation: De Sitter spaces expand exponentially, and their expansion rate generally exceeds their decay rate. As a result, the expansion never stops, while simultaneously, the expanding de Sitter space decays into all other spaces. This process continues indefinitely, and since each decay process produces a new universe, infinitely many universes are created. This situation is illustrated in Fig. 1.8, which becomes fractal-like as time progresses.¹³ This phenomenon is referred to as *eternal inflation*. Additionally, vacua can continuously be created from nothing. Consequently, all possible events will occur infinitely many times, as each universe is realized infinitely often. This situation is typically summarized as follows:

Everything that can happen will happen, infinitely many times.

As a result, both N_A and N_B are infinite, rendering Eq. (1.11.1) meaningless in its current form. One might attempt to introduce a regulator for counting events; however, it turns out that the resulting predictions are highly dependent on the chosen regulator. To make meaningful predictions, it is necessary to define a probability measure over the space of all possible outcomes of future measurements.

¹²It is often times stated that Weinberg predicted the size of the cosmological constant as a result of anthropic arguments in [190]. This is, however, not quite appropriate as he argued that a small nonzero cosmological constant would not affect the formation of life and from there concluded that the cosmological constant is likely to be measured. He could not predict the size of the cosmological constant based on anthropic reasoning.

¹³Colliding bubbles could potentially leave observable imprints [193, 194].

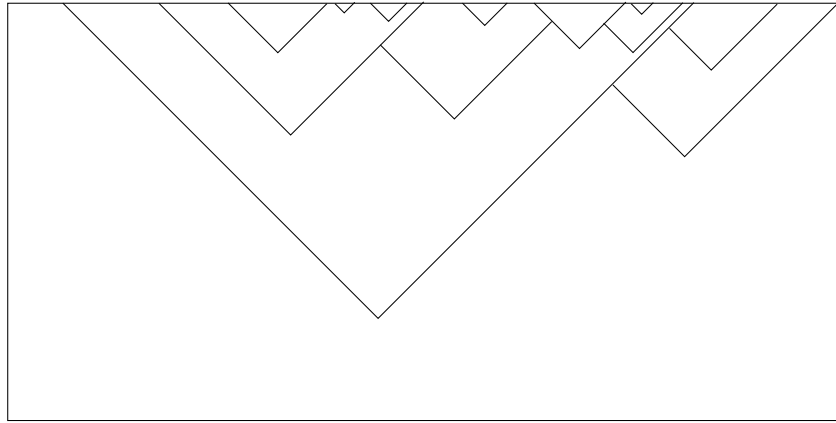


Figure 1.8. An illustration of the eternally inflating multiverse, where bubbles form within bubbles.

This challenge, known as the *Measure Problem*, was first identified in [195] and is comprehensively reviewed in [196–198].

The first proposed measures, developed to address eternally inflating multiverses, were formulated by imposing a late-time cutoff. These measures can be classified into two categories: Global [195, 199–208] and local [209–216] measures. The global measures select a preferred time variable t of the entire spacetime and count events that occur before a given cutoff time t_0 . For each finite t_0 , this method yields finite predictions. In all known cases, taking the limit $t_0 \rightarrow \infty$ is well-behaved. On the other hand, local measures focus on the worldline of an observer and count events occurring within a finite region around it. However, infinities can still arise if the observer’s worldline extends to infinite proper time. To address this issue, it has been suggested to restrict consideration to worldlines of finite proper time. As is reviewed in [197], predictions are highly dependent on the chosen regulator making this approach unsatisfying.

More recent approaches to the measure problem are based on quantum cosmology [217–224], aiming to derive a measure from first principles. Other recent studies on the topic include [225–237].

So far, it remains unclear which definition of a measure is the correct one for making meaningful predictions in the multiverse.

1.12 This Thesis

This thesis primarily focuses on making predictions in the multiverse. Assuming that string theory is our underlying theory of quantum gravity, one quickly arrives at the conclusion that we live in a universe that is part of a multiverse. Even if string theory turns out not to be related to the real world, it is nevertheless likely that a multiverse exists, as the existence of a unique universe of our kind appears highly unlikely. As a result, the measure problem is a general and fundamental problem of theoretical physics.

In Chapter 2, a measure of the multiverse based on *quantum cosmology*, referring to the approach of studying the quantum state of the entire universe Ψ , is developed. The goal is to *derive* the correct measure for the multiverse from fundamental principles. The inner product of the underlying Hilbert

space is used to define probabilities. We adopt a *local* point of view on the multiverse, meaning that Ψ describes regions of the multiverse accessible to a single observer. This is motivated by the complementarity approach [54] to quantum mechanics in black hole spacetimes. The resulting measure is named the *Local Wheeler-DeWitt Measure*. We will be able to relate the theory based on Ψ to semiclassical considerations involving the creation and tunneling rates of vacua. The predictions derived from Ψ then strongly depend on the various rates.

This result triggered a more in-depth study of vacuum decays and vacuum creation in string theory. In Chapter 3, the process of vacuum decay to nothing, which is possible when the underlying theory allows for dynamical spacetime boundaries, also referred to as *end-of-the-world* (ETW) branes, is studied. Orientifold planes can be viewed as examples of ETW branes, and they are generally expected to be abundant in string theory [238]. Also, in Chapter 3, an explicit ETW brane for many vacua of the known string landscape is constructed, and a general formalism for studying decays to nothing is developed. Furthermore, a related framework allows for the study of vacuum creation with boundaries, i.e. the creation of universes of disk topology called a *Bubble of Something*.

In Chapter 4, an entirely new vacuum creation mechanism, referred to as the *boundary proposal*, is developed. It again makes use of the existence of ETW branes, employing them in purely off-shell configurations. The created universe, however, is boundary-free. Such processes are naturally expected to occur in the framework of string theory, and we derive the corresponding creation rates. We find that the boundary proposal can be the dominant mechanism for vacuum creation.

In Chapter 5, the scale of inflation most likely measured by an observer like us living in the string landscape is predicted using the local WDW measure. We identify and overcome some fundamental challenges in using the local WDW measure and reach an interesting set of conclusions. We find that only the Linde/Vilenkin proposal, and not the Hartle-Hawking proposal, leads to predictions in accordance with observations. Furthermore, we find that the prediction for the scale of inflation highly depends on whether or not universes of higher topology can be created from nothing and on the availability of ETW branes with large or small tension. However, we find that in many cases, the scale of inflation is determined by the number density of inflationary plateaus in the landscape as a function of energy.

2 A Local Wheeler-DeWitt Measure for the String Landscape

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Declaration of authorship

The Idea of this project came from Arthur Hebecker, where I carried out all calculations and wrote the entirety of the draft with the exception of a few sentences. Corrections and reformulations of the draft were conducted mainly by Arthur Hebecker and the rest of the collaborators.

High Energy Physics – Theory

A local Wheeler-DeWitt measure for the string landscape

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Abstract

According to the ‘Cosmological Central Dogma’, de Sitter space can be viewed as a quantum mechanical system with a finite number of degrees of freedom, set by the horizon area. We use this assumption together with the Wheeler-DeWitt (WDW) equation to approach the measure problem of eternal inflation. Thus, our goal is to find a time-independent wave function of the universe on a total Hilbert space defined as the direct sum of a variety of subspaces: A finite-dimensional subspace for each de Sitter vacuum and an infinite-dimensional subspace for each terminal Minkowski or AdS vacuum. We argue that, to be consistent with semiclassical intuition, such a solution requires the presence of sources. These are implemented as an inhomogeneous term in the WDW equation, induced by the Hartle-Hawking no-boundary or the Linde/Vilenkin tunneling proposal. Taken together, these steps unambiguously lead to what we would like to think of as a ‘Local WDW measure,’ where ‘local’ refers to the fact that the dS part of the resulting wave function describes a superposition of static patches. The global 3-sphere spatial section of the entire multiverse makes no appearance.

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1. Introduction

In theories allowing for eternal inflation [1–3], the measure problem [4] represents a fundamental challenge on the way to making statistical predictions in a multiverse. The reason is that, in such settings, ‘everything that can happen will happen infinitely many times,’ such that the naive approach of counting becomes insufficient to define a probability measure. Of course, the question about probabilities also arises in fundamental theories with multiple vacua which get populated in some other way. While the measure problem has gained popularity after the advent of the string theory landscape [5–8] (see [8–10] for reviews), it is presumably a general and fundamental question of quantum gravity, possibly independent of its UV completion. Moreover, the measure problem does not disappear if, as recently conjectured, quantum gravity does not allow for metastable de Sitter vacua or for eternal inflation [11–13] (see [14–18] for related work). Indeed, while the traditional landscape of de Sitter vacua would in this case disappear, the measure problem still arises as soon as two solutions with temporal cosmological acceleration, as observed in our universe, exist. While our analysis is phrased in terms of a landscape of (metastable) de Sitter vacua, it will turn out that the resulting local Wheeler-DeWitt measure is also applicable to a multiverse which only contains vacua with a dynamical, decaying dark energy.

The simplest and most studied approach to the measure problem, the introduction of a late-time cutoff, appears to come unavoidably with a certain amount of arbitrariness (see [19,20] for

reviews and [4,21–28] for a selection of specific proposals). We feel that it is more promising to approach the issue in quantum cosmology [29–34], although it is certainly not obvious that an unambiguous answer can be obtained with our present, limited understanding of quantum gravity. For recent work in both directions see e.g. [35–44].

The main paradigm we adopt is that only physics inside the horizon is real and the ever-growing spatial S^3 of global de Sitter (dS) space is irrelevant [32,34,45–47]. This has been called ‘single-observer picture’ since the physics inside the horizon may be viewed as belonging to a single observer, located at the center. While it is advantageous and maybe necessary to put the observer center stage in conventional quantum mechanics, we find this problematic in the present case because one of the questions one wants to ask is precisely about the probability of finding observers, especially observers emerging as a result of an inflationary/reheating period. One then has to introduce some additional, *abstract* observers to count the former. To us, such an additional level of observers appears to be an ad-hoc input, similar to the choice of a cutoff prescription.

To avoid such issues, we prefer to formalize the local approach in an observer-independent way: We base our analysis on the cosmological central dogma [48,49] which states that, from the perspective of the static patch, de Sitter space is a quantum system with a finite-dimensional Hilbert space. In a landscape with several de Sitter vacua, we take this to imply that our total Hilbert space is constructed as the direct sum of such finite-dimensional Hilbert spaces. Each of these subspaces describes a static patch of one of the dS vacua. Semiclassically, transitions between these patches arise since, in any one of them, a bubble of a different dS vacuum can nucleate and grow until the static patch is completely filled out by the new vacuum. In our effective quantum-mechanical model, this is encoded in elements of the Hamiltonian which violate the block-diagonal structure coming with the decomposition of the Hilbert space just described.¹

It has been argued early on [45] that static-patch based approaches generically suffer from a Boltzmann brain problem. This is avoided by including terminal vacua, such that the decay rates of all dS vacua are larger than the production rate of Boltzmann brains, see e.g. [19,32]. We also note that, due to the enormity of the string landscape, finding an anthropically suitable vacuum is computationally complex [50,51], which might embank the spread of the wavefunction through the landscape, even under eternal inflation. This is reminiscent of Anderson localization, which has been discussed in the present context in [31,52,53] and [54].

Our next key ingredient is the Wheeler-DeWitt (WDW) [55] equation, which should clearly be used instead of ordinary quantum-mechanical time evolution. The WDW operator acting on the total Hilbert space still encodes tunnelling between vacua. As a result, generic solutions are wave-functions with non-vanishing projections on all subspaces.

In the WDW approach, the conventional Schrödinger time evolution of quantum subsystems arises through correlations with some semiclassical observable [55–57]. The latter is usually taken to be the scale factor of the global de Sitter 3-sphere. One may try to save this intuitive definition of time in our local approach by imagining some ‘time variable behind the horizon,’ rooted in the huge horizon Hilbert space. We identify a number of conceptual problems related to this perspective and opt for a different, more fundamental approach: Time can arise locally due to correlations between different observables, one of which is semiclassical but does not need

¹ An alternative interpretation of the cosmological central dogma is that the entire Hilbert space of the dS part of the landscape belongs to the highest-entropy metastable dS vacuum with lower-entropy dS spaces emerging as excitations of the former. The dimensions of their Hilbert spaces are exponentially smaller such that, at our level of precision, it is irrelevant whether we view them as subspaces of the Hilbert space of the highest-entropy dS or as extra spaces, to be added in a direct sum.

to be related to the expansion of de Sitter [55,57,58]. The structure of the Hilbert space and the requirement of a time-independent state of the universe are not affected by adopting this local perspective on time.

Due to the Boltzmann brain problem and since the string theory landscape certainly contains supersymmetric Minkowski and anti-de Sitter (AdS) solutions, the framework described above must be enriched by terminal vacua. Such an addition changes the features of a landscape: for example, the probabilities with which dS vacua in the landscape are seen by observers depend on decay rates to terminal vacua [22,59]. A key potential issue for us is that, semiclassically, tunneling from dS to terminal vacua is possible, but not the reverse process. This can only be implemented by a stationary solution Ψ if the corresponding probability current has a source in the dS part of the Hilbert space and ‘runs off to infinity’ in the infinite-dimensional subspaces belonging to the terminal vacua. The source describes the creation of dS vacua out of nothing according to one of the proposals of [60–62]. At the fundamental level, this is implemented by adding an inhomogeneous term χ to the WDW equation:

$$H\Psi = \chi. \quad (1)$$

We will not be able to specify H and χ sufficiently well to solve for Ψ . Instead, we will argue that (1) effectively leads to statements about probability currents and eventually to a simple rate equation for the probability distribution on the vacua of the multiverse. We solve this equation for a simple toy model, illustrating that the distribution is in some cases mainly determined by the amplitude for vacuum creation and the decay rates to terminal vacua. Tunneling between dS vacua could play a subdominant role. A detailed study of possible physics implications is left for future work.

The structure of the paper is as follows: In Sect. 2 we develop a naive model for eternal inflation that is based on the cosmological central dogma and standard quantum mechanical time evolution. We show that semiclassical expectations are consistent with an appropriate generalization of Shnirelman’s theorem. In Sect. 3 we turn to the more appropriate WDW formalism, maintaining the conventional identification of time with the growing scale factor of de Sitter space. We discover that this approach has various conceptional issues, prompting us to adopt a local perspective on time in Sect. 4: Here, no universal time variable is introduced. Instead time emerges independently for every experiment, through correlations with appropriate semiclassical observables. In Sect. 5, we extend the framework just developed by allowing for terminal vacua. Since the wave function of the universe must remain stationary, this entails either tunneling processes from terminals to dS or the presence of sources. Guided by semiclassics, we choose the latter option, i.e. we allow for the creation of dS vacua from nothing. The resulting rate equation for the probability distribution is solved in Sect. 6 for a simple toy model containing two dS and one terminal vacuum. We briefly discuss how one could proceed from here to derive predictions relevant for post-inflationary observers like ourselves but leave a serious study of this subject to future work.

2. The naive, quantum-mechanical perspective

2.1. Density of states

We embrace the notion of de Sitter space as a quantum mechanical system with a finite-dimensional Hilbert space [48,57,63,64]. Based on this, we want to develop a simple toy model for eternal inflation, which is most naively understood as a sequence of tunneling transitions

between several such de Sitter vacua.² The Hilbert space $\mathcal{H}$ of a single dS vacuum is of dimension $N \simeq \exp(S)$, where $S \simeq \pi(RM_P)^2$ denotes the entropy of the horizon. We expect the energy eigenvalues of our dS space to form a very dense discretuum, at least in the energy range that corresponds to what we view as semiclassical de Sitter space. It then makes sense to introduce the continuous function $N(E)$ characterizing the number of states below an energy E asymptotically, in a distributional sense. We also introduce the density of states $\rho(E) = dN(E)/dE$. Since there are only finitely many energy levels, N and $N(E)$ clearly coincide for large enough E . In this notation, the semiclassical dS states lie in a region of high density, presumably, their energies are close to E_* with E_* being defined as the value where $\rho(E)$ is maximal. A small subsystem inside a dS space in a semiclassical state sees a temperature $T \sim 1/R$. The density of states at E_* can then be computed using standard thermodynamic relations:

$$\frac{1}{R} \sim T = \left. \frac{dE(S)}{dS} \right|_{E_*} = \left. \frac{dE(S)}{d \log(N(E))} \right|_{E_*} = N(E_*) \left. \frac{dE(N)}{dN(E)} \right|_{E_*} = \frac{N(E_*)}{\rho(E_*)}. \quad (2)$$

Solving for $\rho(E_*)$ yields

$$\rho(E_*) \sim RN(E_*). \quad (3)$$

We naturally expect that $N(E_*) \sim N$. If we are willing to neglect sub-exponential terms, we may simply write

$$\rho(E_*) \sim N. \quad (4)$$

Note that this fixes the density of states in the energy range $E \sim E_*$ relevant for semiclassical dS, but it does not fix the precise value of E_* . The latter is not relevant in our context.

2.2. Tunneling in the quantum mechanical toy model

Now, we allow for a second dS minimum, such that we can label the two dS vacua as *false* and *true* respectively. Our Hilbert space is then the direct sum of two subspaces of dimensions N_f and N_t , where the index stands for true or false. We assume that $N_f \ll N_t$ and that tunneling transitions can occur. The total Hamiltonian takes the form

$$H = \begin{pmatrix} H_f & \Delta \\ \Delta^\dagger & H_t \end{pmatrix}, \quad (5)$$

where the $(N_f \times N_t)$ -matrix Δ is assumed to have extremely small entries of typical size δ .

We want to determine which δ matches the semiclassical expectation for the decay rate of a generic state in $\mathcal{H}_f$. In the field-theoretic regime, this rate is given by the Coleman-De Luccia (CDL) result [66] $\Gamma_{sc} \simeq R_f^3 \exp(-B) = R_f^3 \exp(-S_B + S_A)$, with S_B the action of the $O(4)$ -symmetric bounce and S_A the action of the false vacuum. The probability for a quantum state being in $\mathcal{H}_f$ then decays in the standard exponential way: $p_t \sim \exp(-\Gamma_{sc} t)$.³

² As will be discussed later on, the Schrödinger equation should in fact be replaced by the Wheeler-DeWitt (WDW) equation. For the moment we assume that (in some approximation) we can instead use a conventional Hamiltonian and a time variable defined e.g. by some physical system (a clock) [48,57,58,65]. A more detailed analysis will be given in Sections 3 and 4.

³ It would be very interesting to also consider de Sitter minima which are connected by paths through flat scalar potentials, such that Hawking-Moss (HM) [67] transitions become relevant. Also, for pairs of vacua that are not connected by an instanton, non-tunneling transitions [68] have to be considered. We leave this subject to future work.

Energy values of false and true vacuum deSitter

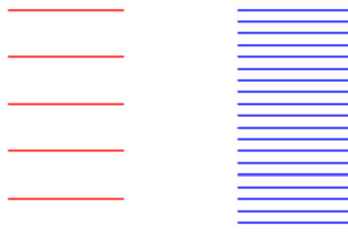


Fig. 1. Energy spectra of the two dS vacua. Most eigenvalues of the false vacuum (left) lie inside the dense region of eigenvalues of the true vacuum (right).

To make progress, we now introduce a key assumption, illustrated in Fig. 1: We assume that the interval in which the energy eigenvalues of H_f are mainly distributed lies inside the high-density region of the H_t spectrum. As a result, the tunneling probability does not depend on which particular state of the false vacuum we are starting from. This agrees with semiclassical expectations since the large number of states comes from the horizon degrees of freedom, and the latter should not influence the semiclassical bubble formation probability of CDL.

Under the assumption above, ρ_t is very large in the relevant energy range and a typical $\mathcal{H}_f$ state decays into a quasi-continuum. The rate in our quantum-mechanical toy model then follows Fermi's Golden Rule:

$$\Gamma_{qm} \sim \delta^2 \rho_t. \quad (6)$$

Demanding $\Gamma_{qm} = \Gamma_{sc}$ allows us to estimate δ (disregarding non-exponential effects) as

$$\delta^2 \sim \exp(-B)/\rho_t. \quad (7)$$

We note that the standard derivation of Fermi's Golden Rule relies on a continuum approximation for the spectrum of the final-state subspace. This limits the validity to a time range below $T_{max} \sim 1/\rho_t$. If we want our false vacuum to actually decay within the time range we control, we need $\Gamma_{qm} \gtrsim 1/T_{max}$, which implies

$$\rho_t \delta \gtrsim 1. \quad (8)$$

The same constraint follows from a static perspective: Namely, we should demand that any eigenstate of the full system mostly points in the $\mathcal{H}_t$ part of the total Hilbert space. If this were not the case, one could prepare the system in an eigenstate pointing mostly in $\mathcal{H}_f$ and forever observe it in the false vacuum. This contradicts the semiclassical expectation that the false vacuum eventually has to decay. A necessary condition for our claim is that perturbation theory in δ , correcting the original eigenstates of H_f , should break down. At leading order, the square of the $\mathcal{H}_t$ -projection of a corrected H_f -eigenvector is estimated by

$$\sum_{E_t} \frac{\delta^2}{(E_f - E_t)^2}. \quad (9)$$

This sum is dominated by the term that contains the eigenvalue E_t^0 which lies closest to E_f . We naturally expect that $E_f - E_t^0 \sim 1/\rho_t$ (for an accidental resonance $E_f = E_t^0$, the cutoff at T_{max} prevents a pole) such that the sum (9) may be approximated by $\delta^2 \rho_t^2$. Requiring this to be of order unity or larger is equivalent to (8).

In our specific case of CDL tunneling, we have $\delta \sim \exp(-B)$ and $\rho_t \sim \exp(S_t)$, such that (8) turns into the condition

$$B \lesssim S_t. \quad (10)$$

In fact, a general argument [6] shows that instantonic tunneling transitions between dS vacua always obey

$$B \leq S_{dS}, \quad (11)$$

with S_{dS} being the entropy of the decaying dS. This is consistent with expectations that dS spaces do not live longer than their recurrence time $t_{rec} \simeq \exp(S_{dS})$ [45,69]. Since $S_f \leq S_t$, it is immediately clear that (11) implies (10). The fact that the relevant bound (10) holds on such general grounds encourages us to think that our basic picture of finite-dimensional Hilbert spaces, linked by decays into a quasi-continuum, is sensible.

We note that certain decay rates may violate our bound, e.g. the non-tunneling transitions of [68]. This does not represent a problem for our approach since there is no reason to expect that dS spaces exist which decay *exclusively* via such extremely slow transitions.

2.3. Ergodicity and distribution of eigenvectors

We expect the overwhelming majority of eigenvectors to deviate only very little from the $\mathcal{H}_t$ subspace of the Hilbert space $\mathcal{H}_t \oplus \mathcal{H}_f$. To quantify this, we consider two general classes of models. First, we draw an analogy to classical mechanics: Our full Hilbert space with a Hamiltonian of the form (5) may be compared to a classical system the phase space of which is the union of two distinct subspaces. Moreover, transitions between these subspaces are suppressed. For example, one may think of a particle in a large container with a peculiar geometry: Namely, it should be the union of two smaller containers connected by a short and narrow throat.

Now, we know that a particle in an irregular, compact geometry in general represents an ergodic system. In other words, integrating over a long period of time the (appropriately averaged) distribution of the particle position on a fixed-energy phase space slice is constant. Moreover, the Shnirelman theorem [70] extends this statement to quantum mechanics. Specifically, it states that for a manifold with ergodic geodesic flow, the probability density associated to an eigenfunction of the Laplacian will generically, in the limit of large eigenvalues, tend towards the uniform distribution in phase space.

In principle, the Shnirelman theorem applies also to our geometry of interest, where the configuration space manifold consists of two well-separated regions connected by a narrow throat. However, in practice one is of course not interested in asymptotically high-energy eigenfunctions but rather hopes that the statement of the theorem holds with sufficient precision above a certain moderate energy threshold. The problem in our case is that this energy threshold presumably grows as the throat becomes more and more narrow. This narrowness is related to the smallness of δ in (5). While it would be interesting to investigate the relevant limiting procedures more carefully, this would in itself not suffice to establish the applicability of Shnirelman's theorem. The reason is that there is another even more obvious obstacle: The Shnirelman theorem, as it stands, only applies to infinite-dimensional Hilbert spaces where the Hamiltonian is an elliptic operator, for example the Laplacian. Our system of interest has by definition a finite Hilbert space to start with. Moreover, we do not know whether it can be derived in some way from a related classical system.

Thus, the best we can do for now is to take the Shnirelman theorem as an inspiration and to proceed from the expectation that an analogous statement holds for our model Hamiltonian (5). Our main motivations are the very large dimensions of the relevant subspaces and the expected random behaviors of the spectra of $H_{f,t}$ as well of the entries of the transition matrix Δ . To formulate our expectation quantitatively, we denote the norm of the projection of a vector on the $\mathcal{H}_{f,t}$ part of the total Hilbert space by $\|\cdot\|_{f,t}$. Now the claim that the probability density associated to an eigenvector tends towards the uniform distribution in phase space translates to the statement that, for a generic normalized eigenvector v of H , one has

$$\|v\|_f^2 \simeq \frac{N_f}{N_f + N_t}, \quad \|v\|_t^2 \simeq \frac{N_t}{N_f + N_t}. \quad (12)$$

Here, the squared norms of the projections of v , i.e. the left-hand sides in the two relations in (12), correspond to the probability distributions associated to v in Shnirelman's sense. The right hand sides represent the relative sizes of the dimensions of the Hilbert subspaces, which corresponds to the claim of equidistribution in phase space volume.

The same result can be obtained using a second, totally different approach, based on different assumptions. To motivate it, recall that depending on the size of δ our system is found in two distinct regimes: For small δ , perturbation theory in δ works and the $N_f + N_t$ eigenvectors split in two subsets – N_f of them lie mostly in $\mathcal{H}_f$, the other N_t lie mostly in $\mathcal{H}_t$. The strong assumption we now make is that, when δ is not very small, the exact eigenvector basis is given by a randomly chosen orthonormal basis in $\mathcal{H}_t \oplus \mathcal{H}_f$. We may specify this proposal by stating that our basis is the eigenbasis of a random hermitian matrix, as analysed in [71,72]. The results of [71,72] concerning the eigenvector distribution can be stated as follows: For a normalized eigenvector v of a random hermitian matrix, the quantity $\|v\|_{f,t}^2$ is, in the large $N_f + N_t$ limit, Gaussian distributed with mean and variance

$$\mu_{f,t} = \frac{N_{f,t}}{N_f + N_t}, \quad \sigma_{f,t}^2 \sim \frac{N_{f,t}}{(N_f + N_t)^2}. \quad (13)$$

In the limit of large N_f and N_t , we see that the distribution of $\|v\|_{f,t}^2$ is sharply peaked around $\mu_{f,t}$. This is in perfect accordance with the proposal (12) motivated using Shnirelman's theorem. Thus, whenever one de Sitter vacuum has an appreciably higher cosmological constant than the other, N_t is exponentially larger than N_f , and hence $\|v\|_f^2$ will be exponentially small for a generic normalized eigenvector v . This conforms with the semiclassical expectation that the false vacuum unavoidably decays.

More specifically, we want to take the point of view of an observer in our landscape of two vacua and ask which vacuum they expect to find themselves in. In our approach, they can make a prediction by taking the solution of the Schrödinger equation describing their universe and projecting on one or the other vacuum. The most general solution is of the form

$$\Psi = \sum_{i=1}^{N_f+N_t} a_i \exp(i\lambda_i t) v_i, \quad (14)$$

where λ_i are the eigenvalues of the full Hamiltonian and v_i are the corresponding normalized eigenvectors. The complex coefficients a_i parameterize the family of solutions. The probability with which an observer sees the false vacuum is then given by

$$R_{qm} = \frac{\int dt \|\Psi\|_f^2}{\int dt \|\Psi\|^2} = \frac{\sum_{i=1}^{N_f+N_t} \|v_i\|_f^2 |a_i|^2}{\sum_{i=1}^{N_f+N_t} |a_i|^2} \simeq \frac{N_f}{N_f + N_t}. \quad (15)$$

Here the last, approximate equality holds only for a generic solution or, more precisely, it is the expectation for an arbitrary but fixed choice of the a_i . Our quantum-mechanical result R_{qm} may be compared with the semiclassical prediction based on following a single observer through the multiverse and integrating the times they spend in each vacuum [46,73]

$$R_{sc} \sim \frac{\exp(-24\pi^2 M_P^2 / \Lambda_f)}{\exp(-24\pi^2 M_P^2 / \Lambda_f) + \exp(-24\pi^2 M_P^2 / \Lambda_t)} \sim \frac{N_f}{N_f + N_t}. \quad (16)$$

This result was derived from rate equations describing an infinite sequence of CDL tunneling events. It agrees with the prediction of our quantum mechanical toy model at the level of exponentially large terms.

As noted in [46], the non-exponential prefactors associated with the tunneling rates violate ergodicity, which in our approach is the basic starting point. At the technical level, this is not an immediate problem for us: On the one hand, we take the quantum mechanical picture more seriously than the semiclassical perspective of following a single observer's worldline. On the other hand, our present work merely attempts to outline an approach that works in principle – we are happy to accept sub-exponential errors. Nevertheless, it would be interesting to pursue this apparent non-ergodicity issue in more detail. For example, our suggestion for a generalized Shnirelman theorem may have sub-exponential errors. It is also conceivable that the tunneling rate has corrections reinstating ergodicity (although we understand that the authors of [46] dismiss this option).

3. Wheeler-DeWitt and ‘time behind the horizon’

This section represents our first attempt to take the crucial step from time-dependent quantum mechanics to the WDW approach. We will work with the usual assumption that time is related to the scale factor of the global dS sphere. This will lead to problems and will be abandoned in Sect. 4 in favor of a more radical approach, where no universal time variable is singled out.

In more detail, an outline for the present section reads as follows: We start by recalling the worldline action of a point-particle moving in target space $\{X^\mu\}$. Worldline reparametrization invariance gives rise to a WDW-type constraint equation, in which target-space time X^0 is singled out by its wrong-sign kinetic term. Hence, also in the worldline view, the argument X^0 of the WDW wave function may naturally be interpreted as ‘time’. Moreover, this toy model serves us to discuss the options of employing the Klein-Gordon or the Hartle-Hawking scalar product [74]. While the former has conceptual advantages, one encounters problems when trying to predict measurements such that we eventually settle on the latter.

We then move on to a further toy model: The mini-superspace description of a homogeneous and isotropic universe. Here, the scale factor appears with a wrong-sign kinetic term and may be taken to play the role of time. This model has been adopted in [31,52,53] to derive a measure on the landscape from a WDW equation. However, since the nucleation of pocket universes breaks spacetime homogeneity, we find such a global mini-superspace approach unsatisfying. We then make an attempt to unify the static-patch-based view of the multiverse with the intuition obtained from the point-particle and mini-superspace discussion. We speculate that the expansion dynamics of the global dS three-sphere is, in the local approach, encoded in the degrees of freedom associated to the dS horizon. We study an effective Hamiltonian appropriate for this kind of model but we quickly encounter various issues, related for example to ambiguities in the structure of the total Hilbert space. As a result, we find it more promising to completely abandon the

scale factor as a dynamical variable and pursue, in Sect. 4, a strictly local approach where time emerges entirely within the horizon, independently of the expansion of global de Sitter space.

The reader not interested in our attempts to develop the approach of [31,52,53] may directly move on to Sect. 4.

3.1. From particle worldlines to mini-superspace and landscape dynamics

We called the discussion of Sect. 2 ‘naive’ for an obvious reason: We are dealing with a time reparameterization invariant theory and the Schrödinger equation is hence not the appropriate tool. Instead, one has to use the Wheeler-DeWitt (WDW) equation [55] (see [75] for a review),

$$H\Psi = 0. \quad (17)$$

While the wave function of the universe Ψ is by definition time-independent, the concept of time naturally arises from correlations between observables.

We find it convenient to start with a particularly simple toy model - a point-particle moving in d -dimensional Minkowski space. Its Polyakov action reads

$$S = \int d\tau \left(\gamma^{-1} \dot{X}^\mu(\tau) \dot{X}^\nu(\tau) \eta_{\mu\nu} - \gamma \left(m^2 + V(X^i(\tau)) \right) \right), \quad (18)$$

where X^μ are the target space coordinates, $\gamma(\tau)$ is the square root of the worldline metric (the einbein), and $V(X^i)$ is a potential in which the particle is trapped. For simplicity, V does not depend on X^0 . From the reparameterization invariance of τ , one derives a constraint which, after quantization, takes the form

$$H\Psi(X^\mu) = 0 \quad \text{with} \quad H = \frac{1}{2}(-\partial_\mu \partial^\mu + m^2 + V(X^i)). \quad (19)$$

From the WDW perspective, the coordinate X^0 is set apart since it enters the equation with a wrong-sign kinetic term. Since V does not depend on X^0 , a product ansatz of the form $\Psi(X^\mu) = \exp(\pm i k X^0) \chi(X^i)$ can be made. This leads to a time-independent Schrödinger equation for χ :

$$(-\partial_i \partial^i + m^2 + V(X^i)) \chi(X^i) = k^2 \chi(X^i). \quad (20)$$

We see that, not only in target space but also in the resulting quantum mechanical system, X^0 plays the role of time. The only special feature is the appearance of positive and negative frequency modes – a consequence of the WDW equation being second rather than first order in ∂_0 .

On the space of wavefunctions $\Psi(X^\mu)$, a natural scalar product is given by

$$(\Psi, \Phi) = \int d^d X \Psi^* \Phi. \quad (21)$$

However, the subspace of solutions to (19) is in addition equipped with an alternative inner product. To see this note that (19) is (a slightly modified version of) the Klein-Gordon equation, implying that the current

$$j^\mu = \frac{i}{2} (\Psi \partial^\mu \Psi^* - \Psi^* \partial^\mu \Psi) \quad (22)$$

is conserved. This allows for the definition of the Klein-Gordon inner product

$$(\Psi, \Phi)_{KG} = \int_{\Sigma} \frac{i}{2} (\Phi \partial_{\mu} \Psi^* - \Psi^* \partial_{\mu} \Phi) dA^{\mu}. \quad (23)$$

Crucially, assuming appropriate boundary conditions, this is independent of the choice of the spacelike codimension-one submanifold Σ over which the current is integrated. However, taking (23) as the Hilbert space inner product of the quantum theory has issues. The first one is the well-known fact that (23) is only positive definite if restricted to positive frequency solutions. The second problem is related to calculating expectation values of operators O . In general, $O\Psi$ does not satisfy (19), implying that a current analogous to (22) but including O ,

$$j^{\mu} = \frac{i}{2} ((O\Psi) \partial^{\mu} \Psi^* - \Psi^* \partial^{\mu} (O\Psi)), \quad (24)$$

is not conserved. As a result, $(\Psi, O\Psi)_{KG}$ depends on the chosen hypersurface Σ . To illustrate this, consider (19) in $1+1$ dimensions with vanishing mass and potential. Then, a simple wave packet,

$$\Psi(X^0, X^1) = \exp(-(X^1 - X^0)^2 + ik_0(X^1 - X^0)), \quad (25)$$

represents a solution. We should be able to predict the most likely value of X^1 when X^0 takes values between X_a^0 and X_b^0 . Put differently, we want to calculate the expectation value of $P_{ab}^0 X^1$, where P_{ab}^0 is the operator projecting the wavefunction to the strip $X^0 \in (X_a^0, X_b^0)$. Using (21), we find the intuitive result

$$(\Psi, P_{ab}^0 X^1 \Psi) = \int_{X_a^0}^{X_b^0} dX^0 \int_{-\infty}^{\infty} dX^1 X^1 \exp(-2(X^1 - X^0)^2) = \frac{\sqrt{2\pi}}{4} (X_b^0 - X_a^0)(X_b^0 + X_a^0). \quad (26)$$

By contrast, the Klein-Gordon inner product with Σ chosen to be the hypersurface $X^0 = \text{const.}$ gives⁴

$$(\Psi, P_{ab}^0 X^1 \Psi)_{KG} = \begin{cases} 0 & \text{if } X^0 \notin (X_a^0, X_b^0) \\ \int dX^1 k_0 X^1 \exp(-2(X^1 - X^0)^2) = \sqrt{\frac{\pi}{2}} k_0 X^0 & \text{if } X^0 \in (X_a^0, X_b^0) \end{cases}. \quad (27)$$

We observe that (27) depends on the chosen value of X^0 , which is expected since $P_{ab}^0 X^1 \Psi$ does not solve (19). Of course, to obtain a result independent of X^0 one may consider averaging (27) over all hypersurfaces $X^0 = \text{const.}$ within the prescribed interval. But this appears ad hoc and goes against the key idea that the Klein-Gordon inner product should be hypersurface-independent.

The problem of $(\Psi, O\Psi)_{KG}$ not being well defined occurs generally for operators that do not commute with H , which is the case for most operators of practical interest. This lends support to the use of (21) rather than (23).

A more relevant model for us is the mini-superspace description of an FLRW spherical geometry with scale factor $a = \exp(\alpha)$, curvature k , as well as a (homogeneous) scalar field ϕ in a potential $V(\phi)$. The appropriately truncated Einstein-Hilbert action reads [55,60]

⁴ For $k_0 \gg 1$, the wave packet (25) contains essentially only positive frequency modes, such that the issue of non-positive-definiteness is absent.

$$S = \int dt \left[-e^{-3\alpha} \dot{\alpha}^2 + k e^\alpha + e^{3\alpha} \dot{\phi}^2 - e^{3\alpha} V(\phi) \right], \quad (28)$$

showing that this time the scale factor plays the special role of the variable with the wrong-sign kinetic term. The resulting WDW operator takes the form

$$H = \frac{1}{2} \left[e^{-3\alpha} \left(\partial_\alpha^2 + p'(\alpha) \partial_\alpha - \partial_\phi^2 \right) + e^{3\alpha} \left(V(\phi) - k e^{-2\alpha} \right) \right], \quad (29)$$

where the function $p(\alpha)$ parameterizes an operator ordering ambiguity [60,76]:

$$e^{-p(\alpha)} \pi_\alpha e^{p(\alpha)} \pi_\alpha \rightarrow -e^{-p(\alpha)} \partial_\alpha \left(e^{p(\alpha)} \partial_\alpha \right). \quad (30)$$

In the late-time, semiclassical regime (i.e. $\alpha \rightarrow \infty$), the WDW equation simplifies to

$$0 = \left[e^{-3\alpha} \left(\partial_\alpha^2 - \partial_\phi^2 \right) + e^{3\alpha} V(\phi) \right] \Psi(\alpha, \phi). \quad (31)$$

In analogy to (21), one may use the scalar product

$$\langle \Psi_1 | \Psi_2 \rangle = \int d\phi \int_{-\infty}^{\infty} d\alpha e^{3\alpha} \Psi_1^*(\alpha, \phi) \Psi_2(\alpha, \phi), \quad (32)$$

where the factor $e^{3\alpha}$ ensures that the WDW operator is self adjoint [60,76].

Alternatively, one could have tried to use the DeWitt inner product [55] on the space of solutions to (31), which is defined in analogy to the Klein-Gordon inner product (23), and has recently proven to be a useful tool in holographic computations [77,78]. As in the Klein-Gordon case, one faces the issue of non-positive-definiteness. Although a probability interpretation can nevertheless be given in special situations, e.g. when working in the WKB approximation where the wavefunction can be decomposed in positive and negative frequency parts, it is not clear how the DeWitt product can be used in more general setups [75,79–81]. Moreover, operators corresponding to observables of interest, such as the counting of galaxies at large α , fail to commute with H . Thus, as in the Klein-Gordon case, making their expectation values well-defined would require an ad-hoc average over hypersurfaces.

Our preferred inner product definition (32), which one may call the ‘Hartle-Hawking inner product’ [74], can be applied before restricting to solutions of (31) and before gauge fixing. One of its main known problems are divergences arising in the integration over the entire parameter space. We give two arguments in its defense: First, even if the integral in (32) is divergent, one may still be able to derive finite relative probabilities. Second, in a theory with a finite-dimensional Hilbert space, which is what we are after eventually, the problem of diverging integrals is presumably entirely absent. Finally, we note that divergent integrals may also arise when using the DeWitt inner product to calculate expectation values, namely in the process of averaging over hypersurfaces.

By comparison with the point particle example above, it now appears very natural to think of α as a time variable [82,83]. One may then encode a (toy-model) landscape in a set of minima of $V(\phi)$ and try to approach the measure problem in the resulting mini-superspace framework. Such an approach has in fact already been pursued in [31,52,53]⁵ and [54]. To be precise, only

⁵ We found the explicit solutions of toy-model WDW equations with concrete scalar-field potentials in [52] more convincing than the general analysis of [31,53]. However, as will be explained in the main text, we have conceptual issues with this framework independently of the technical implementation.

[31,52,53] refer directly to the WDW equation while [54] appears to be closer to what we called ‘naive quantum-mechanical’ approach. Both references appeal to Anderson localization [84] to make statements about the wave function of the universe.

We see conceptual issues with the straightforward use of the mini-superspace model based on α and ϕ advocated in [31,52,53]. The problem is that Coleman-De Luccia instantons and the subsequently expanding bubbles violate global homogeneity of the spatial 3-sphere. But this was a key assumption underlying the model. Put differently, tunneling in the mini-superspace model corresponds to transitions where the whole spatial S^3 changes the vacuum state at once. But this is not what happens according to Coleman-De Luccia and it is inconsistent with causality at late times, when the 3-sphere is much larger than the horizon.

By contrast, Ref. [54] focuses on a single Hubble patch of a dS space and analyzes the scalar-field dynamics resulting from the nonzero temperature of the dS. The starting point is the Langevin equation (derived in this context by Starobinsky [85]). From this, an effective Schrödinger-type equation can be obtained, to which Anderson’s findings are eventually applied. The problem with this approach is that the Langevin equation is only derived in a regime where the slow roll conditions hold, but the later analysis makes statements about the entire landscape (where we do not expect transitions between different minima to occur in the slow roll regime). Still, it may in principle be possible and worthwhile to compare our results with those of [54] in a restricted setting where both are applicable.

3.2. The time variable as part of the horizon degrees of freedom

Let us now try to develop an alternative suggestion for a WDW equation in a landscape situation, avoiding the apparent shortcomings of earlier approaches just discussed. As already noted at the very beginning of Sect. 3, our results will still not be satisfactory and we will eventually turn to an improved, more minimalist point of view in Sect. 4.

Crucially, we can not use a mini-superspace modelling the whole spatial S^3 since we assume that only the spacetime inside the horizon is real. The wave function Ψ of the universe hence describes a superposition of different vacua in a single static patch. In general, the static patch looks homogeneous and isotropic to the observer sitting at the center. This homogeneity is broken if a bubble of a new vacuum nucleates, but it is restored very quickly as the de Sitter expansion continues in the new vacuum. Thus, one may hope that a quantum description similar to the mini-superspace approach exists. To write down an appropriate Hamiltonian, we may take (31) as an inspiration. However, there appears to be no justification for the relative exponential factors between ∂_ϕ^2 and $V(\phi)$ since the local ϕ -field dynamics inside the horizon is not modified in the course of the ongoing exponential expansion. Let us then, in a first attempt, drop the exponential factors altogether and consider the WDW Hamiltonian

$$H = \partial_\alpha^2 + H_\phi, \quad (33)$$

acting on the Hilbert space⁶

$$\mathcal{H}_{dS} = \mathcal{H}_{bulk} \otimes \mathcal{H}_{horizon}, \quad (\text{with } \dim \mathcal{H}_{dS} = \exp(S_{dS})), \quad (34)$$

of the static-patch observer. This tensor product ansatz is inspired, among others, by [32,47]. In the simplest mini-superspace model, $H_\phi = -\partial_\phi^2 + V(\phi)$. More generally, the Hamiltonian

⁶ We view (33) as some form of ‘continuum approximation’ to the true Hamiltonian acting on the finite-dimensional space (34).

H_ϕ should describe the dynamics of all scalar-field and gravitational fluctuations of the static patch. These degrees of freedom are encoded in $\mathcal{H}_{bulk}$, which is hence subject to the action of H_ϕ . It remains at this stage unclear to which extent and how H_ϕ also acts on $\mathcal{H}_{horizon}$. Some form of non-trivial action of H_ϕ on $\mathcal{H}_{horizon}$, which of course contains the majority of degrees of freedom of the entire system [86], is presumably required. The variable α is interpreted as living in $\mathcal{H}_{horizon}$ and encoding a remnant of the semiclassical expansion of the global dS sphere which, of course, we have abandoned. Hence, the operator ∂_α^2 acts only on the space $\mathcal{H}_{horizon}$.

We note that the approximate factorization in bulk and horizon degrees of freedom assumed in (34) relies on the limit of weak gravity, $1/M_P^2 \rightarrow 0$. It is strongly broken if the bulk region contains so much energy that the size of the horizon shrinks significantly. This happens e.g. for black hole states close to the Nariai limit.

Comparing the Hamiltonian (33) with that in (19) and the subsequent analysis, it becomes apparent that α plays the role of a time variable. We may furthermore attempt to identify H_ϕ with the Hamiltonian (5) of our toy-model quantum mechanical description. But such an identification is not perfect since some of the degrees of freedom are already encoded in α .

Unlike in (14), solutions of the WDW equation are now composed of positive and negative frequency parts

$$\Psi(\alpha) = \sum_{i=1}^{N_t+N_f} \left(a_i \exp(i\sqrt{\lambda_i}\alpha) + b_i \exp(-i\sqrt{\lambda_i}\alpha) \right) v_i, \quad a_i, b_i \in \mathbb{C}. \quad (35)$$

We would like to argue that, nevertheless, the crucial calculation (15) remains essentially unchanged:

$$R_{qm} = \frac{\int d\alpha |\Psi|_f^2}{\int d\alpha |\Psi|^2} = \frac{\sum_{i=1}^{N_f+N_t} \|v_i\|_f^2 (|a_i|^2 + |b_i|^2)}{\sum_{i=1}^{N_f+N_t} (|a_i|^2 + |b_i|^2)} \sim \frac{N_f}{N_f + N_t}, \quad (36)$$

concerning the first two equalities this is obvious. However, arguing for the approximate result $N_f/(N_f + N_t)$ after the last step is problematic. Originally, we arrived at this relation by generalizing Shnirelman's theorem and assuming $\|v_i\|_f^2 / \|v_i\|_t^2 \sim N_f/N_t$ for a generic eigenvector v_i in $\mathcal{H}_t \oplus \mathcal{H}_f$. In our present setting we would have to generalize (34) according to

$$\mathcal{H}_{tot} = \mathcal{H}_t \oplus \mathcal{H}_f = \left(\mathcal{H}_{bulk}^t \otimes \mathcal{H}_{horizon}^t \right) \oplus \left(\mathcal{H}_{bulk}^f \otimes \mathcal{H}_{horizon}^f \right) \quad (37)$$

to make this argument.

The difficulty is now that our variable α somehow has to belong to the two horizon Hilbert spaces $\mathcal{H}_{horizon}^t$ and $\mathcal{H}_{horizon}^f$ together. Even worse, in a more realistic case there would be as many such Hilbert spaces as there are dS vacua and our single variable α has to be encoded in all of them at the same time. Even though this sounds maybe surprising, it is certainly conceivable.

Alternatively, we may try to abandon the idea of deriving the last relation in (36) using Shnirelman's theorem. Then, we can speculate that the Hamiltonian H_ϕ only acts on states in $\mathcal{H}_{bulk}$. The action of (33) then factorizes according to the product structure $\mathcal{H}_{bulk} \otimes \mathcal{H}_{horizon}$ of our full Hilbert space. While the Shnirelman theorem now can not be used to argue for the result (36), we may appeal to the more conventional approach of looking for stationary solutions of tunneling rate equations [46,73]. As discussed around (16), this gives the same result: $R_{sc} \sim R_{qm} \sim N_f/(N_f + N_t)$. In this case, the appropriate generalization of (34) would be

$$\mathcal{H}_{tot} = \left(\mathcal{H}_{bulk}^t \oplus \mathcal{H}_{bulk}^f \right) \otimes \mathcal{H}_{horizon}. \quad (38)$$

However, it then remains unclear how exactly to think about $\mathcal{H}_{horizon}$. After all, its dimension depends on the cosmological constant associated with the bulk, the latter in general being very different between true and false vacuum.

Furthermore, we note that following [47] one might also imagine that additional degrees of freedom are present behind the horizon, encoded in a Hilbert space $\mathcal{H}_*$. In this case, (34) has to be replaced by

$$\mathcal{H}_{dS} = \mathcal{H}_{bulk} \otimes \mathcal{H}_{horizon} \otimes \mathcal{H}_*, \quad (39)$$

with⁷ $\dim(\mathcal{H}_*) = \dim(\mathcal{H}_{bulk} \otimes \mathcal{H}_{horizon})$. This may be motivated by related proposals in the black hole context [87–89]. There, it is suggested that the smooth horizon of area A is described by an entanglement subspace of dimension $\exp(A/4l_P^2)$ in a larger space of dimension $\exp(A/4l_P^2) \times \exp(A/4l_P^2)$. The last two factors characterize the exterior (near-horizon) and interior black hole degrees of freedom. The generalization to dS, with the obvious change of meaning of ‘internal’ and ‘external,’ has been discussed in [87,89]. While in this way one might have found, in the form of $\mathcal{H}_*$, a place for the time variable α to live, we feel that this somehow morally clashes with our starting point of giving top priority to the cosmological central dogma. Moreover, while it then appears natural to generalize (39) to the (2-vacuum) landscape according to

$$\mathcal{H}_{tot} = \left(\left(\mathcal{H}_{bulk}^t \otimes \mathcal{H}_{horizon}^t \right) \oplus \left(\mathcal{H}_{bulk}^f \otimes \mathcal{H}_{horizon}^f \right) \right) \otimes \mathcal{H}_*, \quad (40)$$

we are not certain that we have enough motivation for such a proposal.

Let us postpone any further discussion of how to derive the last step in (36) and turn to a different, potentially also problematic aspect of our quantum mechanical analysis. Namely, it may appear to be a radical step to simply drop the exponential factors when going from (31) to (33). We would like to argue that it is possible to proceed in a slightly more general manner. The exponential factors appearing in front of $-\partial_\phi^2$ and $V(\phi)$ in (31) have their origin in the expansion of global space. Since we are only considering the local theory inside a static patch of finite volume that does not change its size, there should be no exponential factors whatsoever multiplying the Hamiltonian H_ϕ . This can be verified by looking at a particle living in a de Sitter space which is localized at a specific point on the spatial S^3 . A straightforward analysis of the equations confirms our intuition that there are no exponential factors appearing in front of the particle Hamiltonian in the resulting WDW equation. Nevertheless, the factor of $e^{-3\alpha}$ multiplying the term ∂_α^2 is still present. Thus, a more appropriate form of the WDW equation taking into account the local dynamics of global expansion is given by

$$0 = \left(e^{-3\alpha} \partial_\alpha^2 + H_\phi \right) \Psi(\alpha). \quad (41)$$

After diagonalization of H_ϕ , equation (41) reduces to a number of Schrödinger-type equations, one for each eigenvalue λ_i of H_ϕ :

$$0 = \left(\partial_\alpha^2 + \lambda_i e^{3\alpha} \right) \psi_i(\alpha). \quad (42)$$

Upon substituting $e^{3\alpha} = x^2$ this turns into Bessel’s differential equation and can hence be solved exactly in terms of Bessel functions. However, we only need the solutions at large α , which

⁷ Reference [47] calls our $\mathcal{H}_{bulk} \otimes \mathcal{H}_{horizon}$ the ‘bulk’ and our $\mathcal{H}_*$ the ‘horizon’ Hilbert space.

can easily be obtained using quantum mechanical intuition. Indeed, (42) describes a particle with zero energy rolling down a potential $V(\alpha) = -\lambda_i \exp(3\alpha)$. This suggests the ansatz $A(\alpha) \exp(\pm i S(\alpha))$ with $S' = \sqrt{-V}$. The prefactor is fixed by noting that the semiclassical probability flux, which is constant along the half-line $\alpha > 0$, may be written as the product of the probability density $\sim |A|^2$ and the velocity v . Hence, $|A|^2 \sim 1/v \sim 1/\sqrt{-V}$. In complete analogy to (14) and (35) we then have

$$\Psi(\alpha) = \sum_{i=1}^{N_f+N_t} e^{-3\alpha/4} \left(a_i \exp\left(\frac{2i}{3} \sqrt{\lambda_i} e^{3\alpha/2}\right) + b_i \exp\left(-\frac{2i}{3} \sqrt{\lambda_i} e^{3\alpha/2}\right) \right) v_i, \quad a_i, b_i \in \mathbb{C}, \quad (43)$$

where the v_i are the eigenvectors of H_ϕ corresponding to λ_i . To demonstrate the claim that the reintroduction of exponential factors in this form leaves physical predictions unchanged, we have to again look at expectation values such as (36). Here, we encounter formal integrals

$$\int_{-\infty}^{\infty} d\alpha e^{3\alpha} \left(e^{-3\alpha/4} \exp\left(\frac{2i}{3} \sqrt{\lambda_i} e^{3\alpha/2}\right) \right) \left(e^{-3\alpha/4} \exp\left(\pm \frac{2i}{3} \sqrt{\lambda_i} e^{3\alpha/2}\right) \right). \quad (44)$$

The factor $e^{3\alpha}$ is part of the definition of the integration measure which is chosen in order for the Hamiltonian (41) to be hermitian, as discussed before. Since the integral (44) is clearly dominated by the region of large α , our restriction to large- α solutions is justified. Changing the integration variable to $x = e^{3\alpha/2}$ shows that the integral may be interpreted in the distributional sense as an oscillatory integral. Due to the Hilbert space being finite-dimensional, this integral should possess some built-in cutoff at large but finite α . This may be modelled by introducing a smooth regulator at $\alpha \rightarrow \infty$. As a result, only terms $\sim |a_i|^2$ and $\sim |b_i|^2$ will survive. This reproduces the result of (36). Thus, the Hamiltonian (41) generates the same predictions as already discussed around (36). We conclude that an appropriate inclusion of exponential factors originally coming from the dS expansion leads to the same phenomenological predictions as the naive calculation (36).

Let us draw preliminary conclusions from this subsection: We tried to turn the scale factor variable α into some form of ‘time behind the horizon’. While this perspective follows rather naturally from trying to merge Wheeler-DeWitt and cosmological central dogma, it also has obvious troubles. In particular, we find it hard to decide whether there should be a separate horizon Hilbert space for each vacuum, or one for all vacua, or whether some compromise must be found. In the first case, which appears to be most consistent with our earlier discussion in Sect. 2, time would have to be encoded by several Hilbert spaces collectively, in a way we do not understand. More generally, since we probably have to give up the eternally expanding spatial sphere of a global de Sitter space, we do not know how the variable α is microscopically related to the degrees of freedom behind the horizon. Is it possible that complexity, which in the black hole case is known to continue growing long after the black hole has settled to a stationary state, has a relation to our desired time variable? Finally, we are forced to accept the puzzling conclusion that time is *not* truly, mathematically non-compact. The reason is that a properly non-compact time could not be encoded in a Hilbert space of finite (no matter how large) dimension. Of course, an effective time variable running over a very large interval is presumably sufficient for all practical purposes.

Given all these difficulties, we find it justified to pursue a more minimalist approach in the next section, abandoning any distinguished classical variable which plays the role of time.

4. The perspective of a ‘WDW purist’

In [55,57] it was explained that, in the WDW context, time is nothing but local correlations between some continuous observable of a semiclassical subsystem and the observables of the quantum subsystem which we are interested in. The resulting time dependence of this quantum subsystem is identical to that prescribed by the time-dependent Schrödinger equation. Similar ideas are discussed in [56,58,65,90,91].

We claim that this perspective straightforwardly applies to our quantum mechanical view of the landscape, for example to the toy model landscape with two vacua and a Hamiltonian (cf. (34))

$$\mathcal{H}_{tot} = \mathcal{H}_t \oplus \mathcal{H}_f = \left(\mathcal{H}_{bulk}^t \otimes \mathcal{H}_{horizon}^t \right) \oplus \left(\mathcal{H}_{bulk}^f \otimes \mathcal{H}_{horizon}^f \right). \quad (45)$$

The condition that the wave function Ψ solves $H\Psi = 0$ ensures that an observer, who is necessarily restricted to measuring in either $\mathcal{H}_{bulk}^t$ or $\mathcal{H}_{bulk}^f$, sees standard quantum mechanical time evolution. There is in particular no need to somehow ‘save’ the scale factor (time) variable a which usually appears in the mini-superspace approach, as we tried to do in Sect. 3.2. The reader to whom this is obvious may skip the rest of Sect. 4.

4.1. Emergent time in its simplest form

For a conventional one-particle Hamiltonian of the form

$$H_0 = \frac{p^2}{2M} + V(x), \quad (46)$$

the familiar WKB solution to the Schrödinger equation $H_0\psi_E = E\psi_E$ reads

$$\psi_E = [2M(E - V(x))]^{-1/4} e^{\pm i \int_x \sqrt{2M(E - V(x'))} dx'}. \quad (47)$$

The WKB method is applicable as long as the potential varies slowly:

$$\left| \frac{V'(x)}{(E - V(x))\sqrt{2M(E - V(x))}} \right| \ll 1. \quad (48)$$

Banks [57] considers a system containing one heavy particle of mass M and a quantum subsystem described by a Hamiltonian H_l (l stands for light degrees of freedom). One then has to solve the WDW equation

$$H\Psi = 0 \quad \text{with} \quad H = H_0 + H_l = \frac{p^2}{2M} + V(x) + H_l. \quad (49)$$

Here the Hamiltonian H_l also contains a coupling between the heavy particle and the quantum system. Moreover, M is assumed to be large enough to apply WKB and to disregard any backreaction of the quantum system on the heavy particle.

One then makes the ansatz

$$\Psi = \sum_E \alpha_E \psi_E(x) \chi_E(x, y), \quad \alpha_E \in \mathbb{C}, \quad (50)$$

where y collectively denotes the positions of the light particles and ψ_E is the WKB solution (47). The WDW equation becomes

$$\begin{aligned}
0 = H\Psi &= \sum_E \alpha_E \left[E\psi_E \chi_E - \frac{1}{2M} \left(2(\partial_x \psi_E)(\partial_x \chi_E) + \psi_E \partial_x^2 \chi_E \right) + \psi_E H_l \chi_E \right] \\
&= \sum_E \alpha_E \left[E\psi_E \chi_E - \frac{1}{2M} \psi_E \left(\pm 2i\sqrt{2M(E - V(x))} \partial_x \chi_E + \partial_x^2 \chi_E \right) + \psi_E H_l \chi_E \right].
\end{aligned} \tag{51}$$

Banks now argue that the term proportional to $\partial_x^2 \chi_E$ can be neglected since it is suppressed by a factor of $1/M$. This does not apply to the term involving $\partial_x \chi_E$ since the factor $\sqrt{2M(E - V)}$ may be large. Defining the parameter t_E through a change of variables,

$$\frac{dt_E(x)}{dx} = \pm \frac{M}{\sqrt{2M(E - V(x))}}, \tag{52}$$

the WDW equation takes the form

$$0 = \sum_E \alpha_E \psi_E (-i\partial_{t_E} + E + H_l) \chi_E. \tag{53}$$

One can now assume that the wave function of the universe is projected on the subspace where the energy of the heavy particle lies in a small interval $(E, E + \delta)$. In practice, this means that (53) is projected to a single, fixed E . Intuitively, one may think of the heavy particle being described by a Gaussian wave packet that moves slowly along the real axis. Its position plays the role of time. Thus, solving WDW implies solving the time-dependent Schrödinger equation

$$i\partial_t \chi_E = (E + H_l) \chi_E \tag{54}$$

for the light degrees of freedom. Here we have replaced t_E by t since the variable E is fixed. The wavefunction $\chi(t(x), y)$ describes the quantum system of interest relative to the position x of the heavy particle. The additive constant E in (54) only contributes a global phase. Time becomes an emergent concept which does not exist outside the validity range of WKB [57].

Clearly, the above is just a toy model. One extension would be to a more complicated semiclassical clock, which is certainly doable but non-essential for us. What we consider more important is to gain confidence that the presence of the whole ‘rest of the world,’ in particular the huge horizon Hilbert space, does not invalidate the logic just presented.

4.2. Including unobserved degrees of freedom

As just explained, we want to repeat the argument of the previous subsection in the presence of an extremely large additional subsystem which, we assume, is very weakly coupled to our clock and our quantum laboratory. Thus, Ψ is a solution to $H\Psi = 0$ with

$$H = H_0 + H_l + H_u, \tag{55}$$

where H_0 and H_l are defined as before. Here H_u (with u for unobserved) governs all the additional degrees of freedom e.g. the universe far away from the laboratory and also the much larger space $\mathcal{H}_{\text{horizon}}$. In the limit where time scales are not excessively long and where the horizon is only weakly coupled to the bulk system, the total Hilbert space completely factorizes in ‘unobserved’ degrees of freedom and the laboratory system. The total Hamiltonian enjoys a corresponding direct-sum structure. In spite of this factorization, the Hamiltonian H_u contributes non-trivially to the WDW equation viewed as a ‘zero-energy condition’. The reason is that the

‘unobserved’ degrees of freedom remain in a superposition of states with different energies. Thus, it may not be entirely obvious whether the previously given argument for how standard time evolution follows from the WDW constraint still goes through. In the following, we want to demonstrate that it does. The reader who finds this sufficiently clear at the intuitive level may jump to Sect. 5.

Introducing eigenfunctions $\phi_\epsilon(z)$ of H_u , i.e. $H_u \phi_\epsilon = \epsilon \phi_\epsilon$ we make the ansatz

$$\Psi(x, y, z) = \sum_{E, \epsilon} \alpha_{E, \epsilon} \psi_E(x) \chi_{E, \epsilon}(x, y) \phi_\epsilon(z). \quad (56)$$

Using the result of the previous subsection, the WDW equation becomes

$$\begin{aligned} 0 = H\Psi &= \sum_{E, \epsilon} \alpha_{E, \epsilon} [\phi_\epsilon H_0(\psi_E \chi_{E, \epsilon}) + \psi_E \phi_\epsilon H_l \chi_{E, \epsilon} + \epsilon \psi_E \chi_{E, \epsilon} \phi_\epsilon] \\ &= \sum_{E, \epsilon} \alpha_{E, \epsilon} \psi_E \phi_\epsilon (-i \partial_{t_E} + E + \epsilon + H_l) \chi_{E, \epsilon}, \end{aligned} \quad (57)$$

assuming, as before, that the energy E of the clock system is known with sufficient precision, we find that the wave function of the universe is

$$\Psi = \sum_{\epsilon} \alpha_{\epsilon} \psi_E \chi_{E, \epsilon} \phi_{\epsilon}. \quad (58)$$

Here the wave functions ϕ_ϵ determine the state of the unobserved part of the universe while, crucially, $\chi_{E, \epsilon}$ are solutions of the time-dependent Schrödinger equation

$$i \partial_t \chi_{E, \epsilon} = (E + \epsilon + H_l) \chi_{E, \epsilon}. \quad (59)$$

At this point, one might be concerned that our wave function of the universe involves a superposition of $\chi_{E, \epsilon} \phi_\epsilon$ with different ϵ , such that the ‘unobserved’ part of the universe may after all affect our measurements. However, this does not happen for the following reason: Note first that solutions of (59) are given by

$$\chi_{E, \epsilon}(t, y) = e^{-i(E+\epsilon)t} \chi(t, y), \quad (60)$$

where $\chi(t, y)$ satisfies the Schrödinger equation for the Hamiltonian H_l :

$$i \partial_t \chi(t, y) = H_l \chi(t, y). \quad (61)$$

It is now easy to see that the phase in (60) is irrelevant for the type of observables we care about. Indeed, such a typical observable is

$$O = O_0 \otimes O_l \otimes \mathbb{1}_u, \quad (62)$$

where $\mathbb{1}_u$ is the identity acting on $\mathcal{H}_u$. In addition, O_l is a generic observable of our quantum system with the light degrees of freedom. Finally O_0 is, without loss of generality, a function of x , which is equivalent to a function $f(t)$ of our time variable t . Recalling (58) and (60), it is now easy to see that

$$\begin{aligned} \langle \Psi | O | \Psi \rangle &= \sum_{\epsilon, \epsilon'} \alpha_{\epsilon'}^* \alpha_{\epsilon} e^{+i\epsilon' t} e^{-i\epsilon t} f(t) \langle \chi(t) | O_l | \chi(t) \rangle \langle \phi_{\epsilon'} | \mathbb{1}_u | \phi_{\epsilon} \rangle \\ &= \left(\sum_{\epsilon} |\alpha_{\epsilon}|^2 \right) f(t) \langle \chi(t) | O_l | \chi(t) \rangle. \end{aligned} \quad (63)$$

We see that, as promised, the unobserved part of our Hilbert space is irrelevant in that it affects only the constant prefactor. Apart from this prefactor, we obtain the desired time-sensitive measurement of O_I on the basis of the solution $\chi(t)$ of our time-dependent Schrödinger equation.

4.3. The complete picture of a universe without time

To summarise, we have recalled and slightly generalized Bank's argument about the emergence of the time-dependent Schrödinger equation. For us, this is a strong motivation to believe that the emergence of time is consistent with (but does not rely on) the presence of a huge, unobserved Hilbert space, especially that 'behind the horizon.' Thus, we do not create any conceptual problems when working exclusively with the WDW equation $H\Psi = 0$, without any scale-factor-related or other time variable.

At this point, one might worry that for a generic Hamiltonian acting on a finite-dimensional Hilbert space, the WDW equation will have no solution. This issue will be resolved in Sect. 5 when more realistic landscapes including terminal vacua are considered. The landscape with two dS vacua discussed until now should only be seen as a toy-model. The solution to the WDW equation is a fundamentally time independent state Ψ that encodes all information about the universe. The probabilities of results of certain observations, labelled by i , are simply proportional to $\|P_i\Psi\|^2$. Here P_i are appropriate projection operators projecting Ψ on subspaces where the result i is guaranteed and $\|\cdot\|$ is the Hilbert space norm of the finite-dimensional space (45). No averaging over time is needed. As an example, the probability that a random observer sees vacuum t or f is now *by definition* given by $\|\Psi\|_{t,f}^2$. We may finally compare this result with the discussion of Sect. 2.3 and, in particular, Eq. (15). From this, we conclude that the WDW approach together with the assumption of quantum ergodicity (cf. (12), as motivated by Shnirelman's theorem) agrees with semiclassical expectations.

5. Terminal vacua

As we have just seen, the model based on (5) and the WDW equation $H\Psi = 0$ leads to predictions consistent with semiclassical physics. This straightforwardly generalizes from our toy-model landscape with two vacua, labelled by t and f for true and false, to a landscape with multiple dS vacua. Labelling the vacua by an index i , we now assume that the full Hilbert space is of the form

$$\mathcal{H} = \bigoplus_i \mathcal{H}_i, \quad (64)$$

with $N_i \equiv \dim(\mathcal{H}_i) = \exp(S_i)$, and with the Hamiltonian being approximately block diagonal, with the off-diagonal blocks inducing tunneling. This is the natural generalization of (5). The Shnirelman theorem and random matrix theory now suggest that

$$\|\Psi\|_i^2 = \frac{N_i}{\sum_j N_j}, \quad (65)$$

where $\|\cdot\|_i$ is the norm of $\mathcal{H}$ restricted to $\mathcal{H}_i$. This again agrees with probabilities derived from rate equations based on semiclassical tunneling [73].

After these preliminaries, we now turn to the main and less straightforward subject of the present section: The inclusion of AdS vacua. We will assume that AdS vacua are terminal, in

other words, we do not consider the possibility that the dynamics of a big crunch induces tunneling processes back to a dS vacuum. One may motivate this by noting the similarity between crunching AdS space and the interior of a black hole near the singularity. For black holes, it is widely believed that they evaporate in a unitary process and hence without transitions to other universes. We then assume the same for AdS crunches.

However, we would not claim that the analogy above is fully convincing. For us, it is merely a working assumption. We do not want to exclude, as a matter of principle, the interesting possibility that the large energy densities of a big crunch induce tunneling to dS, as explored e.g. in [46,92–96]. It will become clear below that such processes can be straightforwardly implemented in the model we are about to construct.

5.1. Terminal vacua as infinite- N limits

One possibility for including a terminal vacuum (AdS or Minkowski) in the WDW framework is to add an infinite-dimensional subspace $\mathcal{H}_{ter}$ and, correspondingly, to extend the Hamiltonian by a block of infinite size. To implement this, we first extend $\mathcal{H}$ by another block of size $N_{ter} \times N_{ter}$ and the corresponding off-diagonal blocks responsible for tunneling. We first assume N_{ter} to be very large but finite: $N_{ter} \gg N_i$ for all i . Inspired by ergodic arguments, one again expects

$$\|\Psi\|_{i,ter}^2 = \frac{N_{i,ter}}{N_{ter} + \sum_j N_j}. \quad (66)$$

This means that ratios of the form $\|\Psi\|_i^2 / \|\Psi\|_j^2$ evaluate to N_i / N_j , as before. If we disregard AdS and Minkowski vacua as anthropically unsuitable,⁸ the dominance of the terminal block, $\|\Psi\|_{ter}^2 \gg \|\Psi\|_i^2$, plays no role. Now we can safely take the limit $N_{ter} \rightarrow \infty$. Clearly, the full state of the multiverse will not be normalizable in this limit, but since we are only interested in ratios between the probabilities for different dS vacua, this is not an issue.

Unfortunately, this suggested treatment of terminal vacua gives rise to a significant concern: We are allowing for a non-zero transition amplitude from dS to the terminal vacuum (or many terminal vacua). At the same time, we are looking either for a stationary solution (in the naive approach) or for a fundamentally time-independent WDW wave function. As we explained, the latter may then be reinterpreted by introducing an emerging semiclassical time variable. Either way, since we are not leaving the framework of conventional quantum mechanics, our stationary solution must be based on a non-zero probability both for tunneling events from dS to terminal vacua *and* for some form of transition from terminal vacua back to dS. For the latter, we have no semiclassical understanding (see, however, [47,97]). One could, of course, simply accept this and postulate that such AdS-to-dS transitions are nevertheless possible, making our approach self-consistent. For example, one may hope that a semiclassical understanding will be achieved in the future or one may assume that such processes exist but do not allow for a semiclassical description. However, we prefer to take a more conservative point of view and look for a different and maybe more natural resolution of the difficulty just described.

⁸ In spite of the big crunch arising in AdS and the infinite, cold and empty future of Minkowski space, this is not an obvious assumption. Observers like ourselves could, of course, nevertheless exist in an initial phase after the last tunneling event. However, this opens up the non-trivial question of how to obtain probabilities specifically for such ‘inflationary’ observers. We will comment on this in Sect. 6.

5.2. The creation of dS vacua and probability currents

As advertised, we now attempt to develop our approach such that terminal vacua are allowed and semiclassical intuition is nevertheless obeyed. To find a time-independent wave function of the universe without tunneling processes from terminal to dS vacua, one then also needs a non-vanishing amplitude for dS space to be created out of nothing. One may think of a 3-sphere being created at zero radius and then growing. The euclidean version of this process is known as the Hartle-Hawking no-boundary proposal [60]. We will be agnostic concerning the value of the corresponding amplitudes, for which different proposals associated with the names of Hartle-Hawking [60], Linde [61] and Vilenkin [62,98] exist. In either case, a source term appears in the WDW equation [99,100], making the wave function of the universe a Green's function to the Hamiltonian.

To illustrate the above, let us start by discussing a toy model entirely unrelated to dS space or cosmology: Consider the time independent Schrödinger equation for a particle on the real line $\mathbb{R} \ni x$ and add an inhomogeneous term, governed by a constant c and localized at the origin:

$$(\partial_x^2 + E)\psi(x) = 2ic\delta(x). \quad (67)$$

One immediately concludes that $\partial_x \psi$ jumps by $2ic$ at $x = 0$ and hence

$$\begin{aligned} \psi(x) &= (c/k) \exp(ikx) \quad \text{for } x > 0, \\ \psi(x) &= (c/k) \exp(-ikx) \quad \text{for } x < 0, \quad \text{with } k = \sqrt{E}, \end{aligned} \quad (68)$$

represents a solution. The familiar quantum mechanical probability current $j(x)$ for this wavefunction is given by

$$\begin{aligned} j(x) &= |c|^2/k \quad \text{for } x > 0, \\ j(x) &= -|c|^2/k \quad \text{for } x < 0. \end{aligned} \quad (69)$$

Because $\nabla \cdot j(x) = 2|c|^2/k \delta(x)$, the solution corresponds to a source at $x = 0$. Similarly, we find a second solution with $k = -\sqrt{E}$ where two waves coming from infinity end in a sink at $x = 0$. Both solutions exist and it is a question of boundary conditions which solutions to use. We see that the source triggers a probability current which can, due to the unboundedness of the real line, flow to infinity and thus realize a stationary solution without the need for a sink or any current coming back from infinity. For a particle localized in an infinitely steep square well, this would be impossible as the energy eigenfunctions are standing waves, leading to a vanishing probability current.

We now want to transfer the above intuition to the multiverse, where the Hartle-Hawking or Linde/Vilenkin mechanism for the creation of dS spaces out of nothing sources the probability flux. As a first step, we need to define a probability current for systems that are more general than a particle in $\mathbb{R}^n$. Such a generalization, using only the existence of a countable Hilbert space basis $\{|n\rangle\}$, is known [101–103]: In this setting, the current is a matrix

$$J_{nm} = -i (H_{nm} \rho_{mn} - \rho_{nm} H_{mn}), \quad (70)$$

where $H_{nm} = \langle n | H | m \rangle$ and $\rho_{nm} = \langle n | \rho | m \rangle$, with ρ being the density matrix. Since we are working with a pure state, $\rho = |\Psi\rangle \langle \Psi|$. The following formulas straightforwardly generalize to more general density matrices. The matrix J is real and antisymmetric and its elements J_{nm} measure the probability flowing from $|m\rangle$ to $|n\rangle$. In an ordinary quantum mechanical system with

standard time evolution, which for the moment serves as a tool to build intuition, J obeys the relation

$$-\partial_t |\langle n | \Psi \rangle|^2 = - \sum_m J_{nm} = \sum_m J_{mn} . \quad (71)$$

The r.h. side of this equation quantifies the rate with which probability flows out of the state $|n\rangle$, similar to the divergence $\nabla \cdot j(x)$ in the continuum case. Equation (71) represents the discrete analogue of the continuity equation. In the continuum limit, the definition (70) coincides with the familiar probability current of point-particle quantum mechanics [102]. We reiterate that we have spelled out the effect of J_{mn} on time evolution in (71) exclusively for better intuition and the rest of this section is devoted to the stationary case.

For stationary solutions to the Schrödinger equation, the l.h. side of (71) is zero and hence the r.h. side also vanishes. This changes if Ψ is a solution to the WDW equation in the presence of a source. However, as long as the Hilbert space remains finite-dimensional, sources always come together with sinks. To show this note that, by antisymmetry, $\sum_n (\sum_m J_{mn}) = 0$. This implies that the divergences of the probability currents at n , summed over all n , give zero. Thus, sources without sinks are impossible. The process of creating dS vacua out of nothing has no inverse, i.e. dS vacua can not decay into ‘nothing’ and hence they do not act as sinks. We immediately conclude that **a landscape containing only dS vacua is inconsistent**. The problem disappears if one includes terminal vacua with their infinite-dimensional Hilbert spaces since now the probability current can run to infinity, cf. our toy-model above. Terminal vacua become a necessary part of the landscape. The full picture is that there is a probability flow from dS to terminal vacua, even though the state of the universe is a solution to the WDW equation and thus static.

It is well known that only dS vacua can be created out of nothing. As a result, the source is a vector from the dS-part of the total Hilbert space. The WDW equation including the source takes the simple form

$$H\Psi = \chi , \quad (72)$$

where χ is a vector from the dS part of $\mathcal{H}$ (i.e. its projection on terminal vacua vanishes). If $\chi \perp \ker(H)$, which we will from now on assume, then solutions to (72) exist and take the general form

$$\Psi = \Psi_0 + \Psi_s . \quad (73)$$

Here the first term is a solution to the homogeneous WDW equation, $\Psi_0 \in \ker(H)$, and the second term denotes a special solution. We would like to fix Ψ by imposing appropriate boundary conditions. As discussed before, the solution to the WDW equation should be such that the probability current runs out to infinity without coming back. This is equivalent to requiring that the vector χ acts purely as a source and not as a sink. Hence, our boundary conditions are that there is no probability current coming from infinity in the Hilbert spaces of the terminal vacua. This is analogous to our toy-model above where we have chosen the solution where the inhomogeneous term acts as a source. One may hope that these conditions are sufficient to fix the state of the universe. It now becomes clear why we do not expect transitions from terminal to dS vacua, even though an off-diagonal matrix elements in the Hamiltonian exists (which is responsible for the familiar down-tunneling process): The Hartle-Hawking or Linde/Vilenkin proposal requires dS vacua to act as sources and not as sinks, leading to the consideration of boundary conditions forbidding a current coming from infinity.

Assuming that Ψ is uniquely determined, the relative probability for a random observer in the multiverse to be in dS vacuum i is then given by

$$p_i \equiv \|\Psi\|_i^2. \quad (74)$$

Since we know neither the precise form of the source vector nor the microscopic Hamiltonian, practical calculations are problematic. In the pure dS landscape without sources (Sect. 2), the Shnirelman theorem was a helpful tool to calculate the probability distribution of vacua and, more importantly, to gain confidence in the quantum-mechanical point of view. In the more complicated setting we are considering at present, where terminal vacua and sources are included, the Shnirelman theorem (at least in the form (65)) is not expected to hold. However we can make progress using semi-classical results. To explain this, let us collect some facts concerning our probability current:

The strength of a source is measured by the divergence of the probability current, as on the r.h. side of (71). Consider now a dS vacuum i with basis $\{|n\rangle\}$ and denote by $\{|\alpha\rangle\}$ a basis of the full Hilbert space. The strength of the source in this dS space then reads

$$J_i \equiv \sum_{\alpha,n} J_{\alpha n}. \quad (75)$$

Consider a second vacuum x (dS or terminal) with basis $\{|r\rangle\}$. The total probability flow from vacuum i to x reads

$$J_{i \rightarrow x} \equiv \sum_{r,n} J_{rn} = 2 \operatorname{Im} \left(\sum_{r,n} \langle r | H | n \rangle \langle n | \Psi \rangle \langle \Psi | r \rangle \right). \quad (76)$$

Here, we encounter projection operators $P_i = \sum_n |n\rangle \langle n|$ and $P_x = \sum_r |r\rangle \langle r|$ on the subspaces belonging to vacua i and x respectively. Using the shorthand notation $|\Psi\rangle_{i,x} \equiv P_{i,x} |\Psi\rangle$, we find

$$J_{i \rightarrow x} = 2 \operatorname{Im} \left({}_x \langle \Psi | H | \Psi \rangle_i \right). \quad (77)$$

This expression is independent of the chosen basis and represents the fact that the total flow between the vacua i and x is governed by the off-diagonal matrix elements of the Hamiltonian.

Consider now the case when x is a dS vacuum. Then, the probability current is linked to the tunneling rates $\Gamma_{i \rightarrow x}$ and $\Gamma_{x \rightarrow i}$ between the two vacua via

$$J_{i \rightarrow x} = p_i \Gamma_{i \rightarrow x} - p_x \Gamma_{x \rightarrow i}. \quad (78)$$

Given (76), we could now determine the decay rates Γ on the r.h. side from the fundamental input data χ and H . The latter is an appropriate generalization of our block-diagonal ansatz for the Hamiltonian in (5).

If x defines a terminal vacuum, the chosen boundary conditions constrain the probability current to the form

$$J_{i \rightarrow x} = p_i \Gamma_{i \rightarrow x}. \quad (79)$$

Again, given H and χ we could in principle compute the rates Γ .

However, as advertised before, we do not understand the fundamental input data well enough and it is more practical to turn the logic around: We can start with our semiclassical knowledge

of (or proposals for) the source strengths J_i and CDL rates Γ [66].⁹ Then the probabilities p_i can be derived without knowledge of Ψ .

Let us start with the J_i , for which the two key competing proposals are ([61,62,104])

$$J_i \propto \begin{cases} \exp(S_i) & \text{no-boundary [104]} \\ \exp(-S_i) & \text{tunneling [61,62]} \end{cases}. \quad (80)$$

Clearly, by the definitions (75) and (77), they satisfy

$$J_i = \sum_x J_{i \rightarrow x}, \quad (81)$$

where the summation is over all vacua of the landscape. Using (78) and (79), one finds the set of equations (one for each dS vacuum i)

$$J_i = \sum_{j \in dS} (p_i \Gamma_{i \rightarrow j} - p_j \Gamma_{j \rightarrow i}) + p_i \sum_{y \in \text{Terminals}} \Gamma_{i \rightarrow y}, \quad (82)$$

where j runs over all dS vacua and y over all terminals. This can be read as matrix equation, $J_i = M_{ij} p_j$, and hence easily solved: $p_i = (M^{-1})_{ij} J_j$.¹⁰ It is interesting to note that the normalization of the J_i does not play a role. A rescaling of the J_i can be absorbed in the p_i , leaving only relative probabilities of the form p_i/p_j as physical predictions. This makes sense also intuitively since we do not expect to be able to derive the precise normalization of the production rates J_i .

We note that the results (72) and (82) are equally meaningful in the absence of a traditional de Sitter landscape. If the underlying quantum gravity theory is such that only solutions with a decaying dark energy are available [11–18], then on the r.h. side of (82) the index j runs only over such relatively short-lived vacua with temporal acceleration. As a result, the equation will be dominated by the source terms and the decays to terminals. Anthropic predictions as discussed in Sect. 6 can nevertheless be made with the same methods. It is also clear how to modify the model when transitions from crunching AdS regions to dS spaces are allowed. One simply includes the corresponding probability current in (82). However, since such events are UV sensitive, the form of the probability current is unknown.

Similar rate equations have appeared before in the discussion of the scale-factor [4,22,25,105] and watcher [46] measure, but in conceptually different frameworks. It is particularly interesting that, if one identifies our p_i with the relative time an eternal observer spends in vacuum i , then an (almost) equivalent rate equation including sources can be found in [46]. There, however, it is assumed that the observer somehow ‘survives’ the crunch in the terminal AdS vacuum and returns to one of the newly-born dS vacua, with a relative probability given by the no-boundary or tunneling proposal. While it is intuitive that this type of dynamics leads to p_i ’s which agree with our result, we feel that conceptually our treatment is rather different: We require no observer and no assumption about any return from a terminal vacuum. This ensures in particular that SUSY-Minkowski vacua, in which there are no further decays and no big crunch (presumably implying no possibility to return), do not require any special treatment in our approach. By contrast, the

⁹ In fact, by identifying the transitions rates Γ on the r.h. side of (78) with CDL rates, we are probably making a (non-exponential) error associated with the precise definition of the volume over which our CDL rate is to be integrated and possibly with further technical details.

¹⁰ The matrix M also appears in [22] where it was shown that all eigenvalues are nonzero and hence the matrix is invertible.

main proposal of [46] is to restrict attention to the measure-zero subset of observer trajectories which never enter a Minkowski vacuum.

We note, however, that one of the different possibilities for defining a measure discussed in [46] is, in our present understanding, equivalent to our result at the technical level: In this approach, it is assumed that the observer trajectories truly end in either eternal Minkowski or crunching AdS, such that ‘returns’ of the observers are replaced by an a priori choice of a certain ensemble of observer trajectories. If this choice agrees with what we implement using the fundamental source of the WDW equation, then predictions should agree.

Finally, a similar rate equation also appears in [22], where the p_i represent the time-averaged fraction of comoving volume occupied by vacuum i and the J_i represent the initial distribution of vacua. The analysis in [22] is based on the conventional picture of the multiverse having infinite volume and containing infinitely many bubbles of each kind. The initial condition dependence of the resulting equation then appears to be counter-intuitive to the authors. The local WDW measure on the other hand naturally does not consider an infinite-volume spacetime geometry and the inhomogeneous version of the WDW equation (72) leads to a constant probability flux responsible for the permanent creation of dS vacua out of nothing. Hence, it is by no means unexpected that the distribution of vacua depends on the creation rates of dS spaces.

Ultimately, it may not seem surprising that we find a result already predicted by one of the many measures based on geometric cutoffs. But in our approach the resulting measure is not just one of many options – we claim it to be the unique outcome of combining quantum mechanics, semiclassical gravity, and the cosmological central dogma.

6. Applications and comments on related approaches

To illustrate our measure proposal, we will now analyze the probability distribution of vacua in a toy model landscape. We will then comment on the more complicated question where post-inflationary observers like ourselves are expected to live.

6.1. A toy-model landscape

Consider two dS vacua labeled ‘1’ and ‘2’ together with a terminal vacuum labeled ‘T.’ Without loss of generality, let $S_1 < S_2$. We are interested in finding p_1, p_2 by solving equation (82). In our case of interest, this equation reads

$$\begin{pmatrix} J_1 \\ J_2 \end{pmatrix} = \begin{pmatrix} \Gamma_{1 \rightarrow 2} + \Gamma_{1 \rightarrow T} & -\Gamma_{2 \rightarrow 1} \\ -\Gamma_{1 \rightarrow 2} & \Gamma_{2 \rightarrow 1} + \Gamma_{2 \rightarrow T} \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}, \quad (83)$$

which is solved by

$$\begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \frac{1}{\det(M)} \begin{pmatrix} \Gamma_{2 \rightarrow 1} + \Gamma_{2 \rightarrow T} & \Gamma_{2 \rightarrow 1} \\ \Gamma_{1 \rightarrow 2} & \Gamma_{1 \rightarrow 2} + \Gamma_{1 \rightarrow T} \end{pmatrix} \begin{pmatrix} J_1 \\ J_2 \end{pmatrix}. \quad (84)$$

Here M denotes the transition rate matrix from (83). Let us make the further simplifying assumption that tunneling to the terminal vacuum is always much faster than tunneling to the other dS vacuum. This is in line with stringy expectations due to the well-known difficulties in constructing dS vacua. The latter are hence expected to be rare in a string landscape consisting of mostly AdS vacua. Transitions between dS vacua are then likely to involve changing the flux by many quanta or even the topology, suppressing the decay rates. We hence approximate the diagonal components of the matrices in (83) and (84) by just $\Gamma_{1 \rightarrow T}$ and $\Gamma_{2 \rightarrow T}$.

Linde/Vilenkin: The tunneling proposal [61,62,98] for the creation of dS vacua suggests $J_i \propto \exp(-S_i)$ and hence

$$\frac{p_1}{p_2} \simeq \frac{\Gamma_{2 \rightarrow T} e^{-S_1} + \Gamma_{2 \rightarrow 1} e^{-S_2}}{\Gamma_{1 \rightarrow 2} e^{-S_1} + \Gamma_{1 \rightarrow T} e^{-S_2}} \simeq \frac{\Gamma_{2 \rightarrow T} e^{-S_1}}{\Gamma_{1 \rightarrow 2} e^{-S_1} + \Gamma_{1 \rightarrow T} e^{-S_2}}. \quad (85)$$

There are now several possibilities: Depending on which of the two large ratios e^{S_2}/e^{S_1} and $\Gamma_{1 \rightarrow T}/\Gamma_{1 \rightarrow 2}$ wins, either the first or the second term in the denominator dominates. If the first term dominates, $p_1 \gg p_2$. If the second term dominates, one has to distinguish two further cases. Now the competition is between the large ratio e^{S_2}/e^{S_1} and the ratio $\Gamma_{1 \rightarrow T}/\Gamma_{2 \rightarrow T}$, which could in principle also be exponentially large. If it is and if it wins over e^{S_2}/e^{S_1} , then $p_2 \gg p_1$. Otherwise we have $p_1 \gg p_2$, as before.

Hartle-Hawking: By contrast, the no-boundary proposal [60] states that $J_i \propto \exp(S_i)$, leading to

$$\frac{p_1}{p_2} \simeq \frac{\Gamma_{2 \rightarrow T} e^{S_1} + \Gamma_{1 \rightarrow 2} e^{S_1 - S_2} e^{S_2}}{\Gamma_{1 \rightarrow 2} e^{S_1} + \Gamma_{1 \rightarrow T} e^{S_2}} \simeq \frac{\Gamma_{2 \rightarrow T} e^{S_1}}{\Gamma_{1 \rightarrow T} e^{S_2}}. \quad (86)$$

The ratio of probabilities is determined by the competition between the large ratio e^{S_2}/e^{S_1} and the ratio $\Gamma_{2 \rightarrow T}/\Gamma_{1 \rightarrow T}$. Since vacuum 1 has the higher energy scale, one would naively expect that it decays more easily, in which case both the entropy factors and the decay rates favor the second vacuum: $p_2 \gg p_1$. However, one can not completely rule out that the tensions of the relevant domain walls lead to $\Gamma_{2 \rightarrow T}/\Gamma_{1 \rightarrow T} \gg e^{S_2}/e^{S_1}$, implying the opposite result.

Preliminary conclusions: In the absence of large hierarchies between the decay rates to the terminal vacuum, the dS space which is more likely to be created out of nothing has a significantly higher probability of being realized in the multiverse. Thus, as naively expected, high-scale vacua or low-scale vacua win depending on whether the Linde/Vilenkin or Hartle-Hawking proposal is correct. However, this expectation can be overturned by large ratios of the decay rates, especially those from the relevant dS to the terminal vacuum. A much more careful analysis, including the realistic case of a large number of dS and terminal vacua and using what is known about decay rates in the string landscape, is clearly required to make progress.

6.2. The most likely vacuum for an anthropic observer

The ultimate goal in analysing the measure problem is, of course, to make phenomenological predictions in an eternally inflating universe. In addition to the famous cosmological constant issue [106] (cf. [107] for a more recent discussion and references), the targets of such analyses include the curvature parameter Ω_k [108–110], the tensor-to-scalar ratio [111,112], and the SUSY breaking scale [113].¹¹

In this subsection, we do not attempt a serious phenomenological study but merely want to discuss the first steps required to go from the distribution of vacua per se to the prediction in which vacuum an anthropic observer is most likely to live. Here by anthropic observers we mean some intelligent beings like us, which developed on the basis of structure formation in

¹¹ See [114] for a recent review emphasizing the alternative perspective of ‘stringy naturalness.’ The latter is based on counting which features occur more frequently in the string landscape.

the universe (as opposed to Boltzmann Brains or ‘abstract observers’ invented to count vacua). It is well-understood that anthropic observers can only develop under special conditions within the set of universes and initial conditions consistent with fundamental theory, and that requiring these conditions leads to strong and robust predictions on observable quantities. The most basic constraint is that the cosmological constant [106] must be small enough to allow for the formation of gravitationally bound structures. Structure formation can only take place following slow-roll inflation when dark matter abundance and strength of primordial fluctuations are in a certain range [115–117]. Eventually, biological evolution must be taken into account. As a result, the conditions on the cosmological parameters above become even more stringent and further cosmological and particle-physics constraints arise [118].

To set the stage, recall that when using the scale factor measure the landscape is populated mainly through the so-called dominant (or master) vacuum [22,59]. This is the dS vacuum with the smallest decay rate. It generally has a small cosmological constant, though not necessarily the smallest one (since decay rates also depend on other parameters). Thus, anthropic observers are most likely to live in vacua to which the tunneling rate from the dominant vacuum is high. More precisely, the vacua in question have to be adjacent to an inflationary plateau and it is the tunneling rate from the dominant vacuum to this plateau that has to be high.

In our case, where the multiverse is characterized by a quantum state Ψ which crucially depends on the possibility of creating dS vacua out of nothing, the situation is different. It is in general not a good approximation to disregard every but the dominant vacuum at late times. The reason is that all vacua are populated by creation out of nothing or by subsequent down-tunneling events. This can easily be more important than the up-tunneling from the dominant vacuum, which is double-exponentially suppressed (by the bounce-action as well as by the large entropy of the dominant vacuum). Thus, the entire landscape will constantly be populated and up-tunneling events may not be particularly important.

On the basis of a stationary state Ψ of the universe, (relative) probabilities for measurements can be obtained by projecting Ψ on the subspace of interest and then taking its norm squared. Without observers and assuming that the measurement simply determines the type of vacuum i , one would find the probabilities p_i of Sects. 5.2 and 6.1. But the projector we are really interested in is on the vacuum i and, on top of that, on the observer in that vacuum. This may be taken into account by an additional weighting factor w_i . The probability O_i that an anthropic observer lives in dS vacuum i is then given by

$$O_i \propto w_i p_i, \quad (87)$$

where the p_i follow from the local WDW measure by solving equation (82). We do not want to enter the non-trivial discussion of how to implement the projection in cases where typical spatial slices of the relevant dS static patch contain multiple anthropic observers. Naively one would expect that this leads to large w_i and hence rewards, for example, vacua with a large horizon size (i.e. small cosmological constant). However, we do not expect hierarchies between the w_i to be exponential.¹² Thus, since we have disregarded exponential effects throughout, we may for the sake of concreteness take $w_i \in \{0, 1\}$, classifying all vacua of the landscape as anthropically suitable or not suitable.

However, as we have argued before, anthropic observers live after a period of inflation, which only occurs shortly after the creation of a pocket universe belonging to vacuum i . Hence, a more

¹² If post-inflationary observers can live in $\mathcal{N} = 2$ Minkowski vacua, this assumption is problematic. Indeed, in this case one of the possible projections on Ψ would be an infinite reheating surface in the relevant vacuum.

appropriate estimate for O_i should be based on the production rate of inflationary universes of vacuum i and not on the abundance of vacuum i in the multiverse. A corresponding ansatz is

$$O_i \propto w_i \left(f_i J_i + \sum_{j \neq i} p_j f_{ji} \Gamma_{j \rightarrow i} \right), \quad (88)$$

where J_i denotes the creation rate of vacuum i from nothing and $\Gamma_{j \rightarrow i}$ the transition rate from vacuum j to vacuum i . In addition, f_i and f_{ji} are the fractions of creation and tunneling events that end specifically in an inflationary state of vacuum i rather than anywhere within the basin of attraction of the minimum of the scalar potential describing vacuum i . Any attempts to derive f_i , f_{ij} in the known part of the string landscape are left to future work. We note that, since the J_i and p_i are defined only up to arbitrary overall rescalings, eq. (88) implies that only ratios of the form O_i/O_j are physically meaningful.

Most naively, one may be tempted to assume that the index i in (88) runs over dS vacua only. However, it takes little thought to see that also Minkowski vacua or AdS spaces with near-zero cosmological constant should be included. Indeed, such vacua may have adjacent inflationary plateaus which can be populated by tunneling or creation out of nothing. Slow roll and reheating can then end in the terminal vacuum and structure formation can occur long before the big crunch (in the AdS case). To summarize, eq. (88) holds also when vacuum i is terminal. It is then understood that $f_i J_i$ stands for the creation rate of a quasi-dS vacuum (i.e. an inflationary state) leading to reheating in vacuum i .

Before closing this section, we would like to discuss an alternative point of view: Slow-roll inflationary plateaus could by definition be promoted to additional landscape vacua. Let us label such vacua by the index ‘ i, inf ’ if the slow-roll ends in the metastable dS or terminal vacuum i . The probability for an anthropic observer to be in vacuum i is then proportional to $w_i p_{i,inf}$, where $p_{i,inf}$ is the probability that the universe is in the ‘vacuum’ defined by the inflationary plateau near minimum i . The $p_{i,inf}$ still follow from (82), where appropriate creation rates of and tunneling rates to vacua of type $p_{i,inf}$ have to be added. Moreover, decay rates of such quasi-dS vacua also have to be introduced. These are dominated by the slow-roll/reheating transition to vacuum i rather than by tunneling.

While we expect that, at the level of the decay rate analysis, this approach of treating inflationary plateaus as additional vacua is equivalent to the previous discussion, it is not obvious how this goes together with the detailed balance assumption in Sect. 2. This issue has been studied in the literature for Hawking-Moss (HM) [67] transitions between adjacent vacua. In this case, detailed balance is obeyed [119] and hence our generalization of Shnirelman’s theorem in the form (65) might apply. However, the real cases of interest are slow-roll rather than HM-plateaus and we have to leave it to future work to determine whether an analogous property holds in this situation.

Ultimately, the clear goal for the future is to develop our brief discussion in Sects. 6.1 and 6.2 to the level needed for deriving statements about anthropic observers in the string landscape.

6.3. Related approaches

Our construction has obvious similarities with [32,47], [22,46] and [53,54]. It is also in part inspired by [34]. In the course of developing our perspective we have, of course, already commented on some of these results. Nevertheless, now that our picture is complete, it may be worthwhile providing a more explicit summary of similarities and differences.

The measure proposal of [32,47] is also based on finding the state of the multiverse living in a Hilbert space which takes a form similar to (89). However, in these references a key role is played by an ‘abstract observer’ moving through the multiverse with the state of the universe describing the regions accessible to the ‘observer.’ By contrast, our approach is observer-independent, as emphasized before. While [32] assumes Schrödinger evolution with a global time variable, [47] refers to the WDW equation. But the selection conditions for the state of the multiverse are different: In our approach, Ψ is a solution to the inhomogeneous equation (1) and it obeys appropriate boundary conditions, such that the state of the universe presumably is uniquely determined. In [47], Ψ must solve the conventional, homogeneous WDW equation and be normalizable. If the WDW operator has multiple normalizable zero modes, there is a residual ambiguity in determining Ψ . This difference is related to the fact that the analysis in [47] does not consider the creation of dS vacua out of nothing. Such a possibility is discussed in [32], but the model built in this reference is not based on the WDW equation. Crucially, in our analysis we are able to make contact between the more abstract WDW or Hilbert space approach and the semiclassical theory in the form of the simple rate equation (82). This should provide a useful starting point for concrete applications.

Moreover, the proposal of [47] relies on special phenomena that were called ‘inverse bubbles.’ These are pocket universes with contracting rather than expanding bubble walls. Such ‘inverse bubbles’ are not well understood semiclassically and one of our main points is the proposal of a stationary solution with terminal vacua which nevertheless does not have to rely on such exotic objects.

In [22,46], rate equations similar to (82) appear. The analysis in these references is, however, based on the semiclassical perspective on the multiverse rather than quantum cosmology and the WDW equation. Hence, as discussed in more detail below (82), the concepts leading to the resulting equation are different in our case.

Reference [34] also combines the creation of universes out of nothing with tunneling transitions in the multiverse to address the measure problem. We are at the moment unable to provide a detailed comparison to our approach since the analysis in [34] is implemented only in a toy model and it is not clear to us how it can be generalized. One aspect that we are struggling with in that approach is to understand how the small Hartle-Hawking 3-spheres are to be included in the very large 3-sphere of the late-time multiverse.

Simultaneous to our paper, reference [44] was uploaded to the arxiv. It discusses the measure problem using Bayesian reasoning as the main ingredient. Although the underlying concepts are rather different compared to the local WDW approach, the resulting predictions appear to be similar.

Finally, also references [53] and [54] have starting points close to our analysis: the WDW equation and a local quantum-mechanical model respectively. But, as we already discussed in some detail at the end of Sect. 3.1, the implementation is rather different. Let us only recall here that, as a key distinction, both references find Anderson localization in the landscape. It is then expected that only vacua around specific centers are populated. By contrast, since in our approach dS vacua can be created out of nothing, the entire landscape is constantly populated such that the probability distribution of vacua is not localized.

7. Conclusions

We have constructed a quantum mechanical model of the multiverse based on the Wheeler-DeWitt approach and the cosmological central dogma. The result is an inhomogeneous version of the WDW equation, the solution Ψ of which lives in the Hilbert space

$$\mathcal{H} = \left(\bigoplus_{i \in dS} \mathcal{H}_i \right) \oplus \left(\bigoplus_{y \in \text{Terminals}} \mathcal{H}_y \right), \quad (89)$$

with $\dim(\mathcal{H}_i) = \exp(S_i)$ and $\dim(\mathcal{H}_y) = \infty$. Predictions for measurements are made by projecting Ψ on the subspace of interest and subsequently taking its norm. Each Hilbert space $\mathcal{H}_i$ in (89) describes a static patch of dS vacuum i , such that the size of the expanding 3-sphere of global dS space is irrelevant. Hence, the resulting measure for the multiverse can be called ‘local Wheeler-DeWitt measure.’ Motivated by semiclassical expectations, we allow for the creation of dS vacua out of nothing according to the no-boundary [60] or tunneling [61,62,98] proposal, and we do not allow for transitions from terminal vacua back to the dS part of the Hilbert space. This forces Ψ to obey specific boundary conditions which imply that the underlying fundamental equation is, as already mentioned, inhomogeneous. The inhomogeneous term encodes the fact that the probability current has sources in the dS-like part of the Hilbert space, cf. (1). In addition, the current runs off to infinity in the terminal part of $\mathcal{H}$.

We note that the WDW approach to landscape dynamics also changes the perspective on the ‘everything happens’ slogan and the spread of the wave function through the landscape. Indeed, in the stationary solution dictated by the inclusion of terminal vacua and the presence of sources, the probability current is ‘eternally flowing’ for consistency, and the Universe does not require additional resources to find the anthropically viable vacua. The ‘static-patch, multiple vacua’ point of view is both observer-independent and consistent with the cosmological version of the ‘central dogma.’

Since the microscopic Hamiltonian is unknown, it is a priori not obvious how to derive predictions from our model of the multiverse. We propose to achieve this by relating the quantum mechanical probability current to semiclassical decay rates, leading to a simple equation for the probability distribution of vacua – eq. (82). This can be taken as the starting point for anthropic predictions, such as the most likely vacuum for anthropic observers. In Sect. 6.1, we have shown that the distribution of vacua appears to depend strongly on the chosen mechanism for creating dS vacua out of nothing. It would hence be crucial to determine on the theory side whether the Hartle-Hawking or the Linde/Vilenkin creation amplitude for dS vacua is the right one to be used in this context. Alternatively, it may become possible to make this decision on the basis of consistency with observations, given of course that our multiverse model is correct.

To summarize, the local WDW measure is a new proposal for making predictions in the multiverse. It has a clear and simple conceptual basis in quantum-cosmology. We expect that, after further reflection and discourse, the proposal will be refined and more details will emerge. Crucially, under certain assumptions to be scrutinized in the future, this proposal leads to explicitly calculable probabilities. Its validity can hence be established by comparing anthropic predictions to observation.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Arthur Hebecker reports financial support was provided by German Research Foundation. Manfred Salmhofer reports financial support was provided by German Research Foundation. Johannes Walcher reports financial support was provided by German Research Foundation.

Data availability

No data was used for the research described in the article.

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3 Cobordism and Bubbles of Anything in the String Landscape

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The idea for this research project was developed by Arthur Hebecker and me. I performed all the calculations and wrote the entirety of the draft which was subsequently edited by Arthur Hebecker and Johannes Walcher.

Cobordism and bubbles of anything in the string landscape

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ABSTRACT: We study bubble of nothing decays and their reverse processes, the creation of vacua through ‘bubbles of something’, in models of the Universe based on string theory. From the four-dimensional perspective, the corresponding gravitational instantons contain an end-of-the-world (ETW) boundary or brane, realized by the internal manifold shrinking to zero size. The existence of such ETW branes is predicted by the Cobordism Conjecture. We develop the 4d EFT description of such boundaries at three levels: first, by generalizing the Witten bubble through an additional defect. Second, by replacing the compact S^1 with a Calabi-Yau orientifold and allowing it to shrink and disappear through a postulated defect. Third, we describe an ETW brane construction for type IIB Calabi-Yau orientifold compactifications with O3/O7 planes through an appropriate additional O5 orientifolding. Our 4d EFT formalism allows us to compute the decay/creation rates for bubbles of anything depending on two parameters: the size of the relevant defect and its tension a.k.a. the induced (generalized) deficit angle.

KEYWORDS: Cosmological models, String and Brane Phenomenology, Superstring Vacua, Flux Compactifications

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1 Introduction

In the presence of multiple vacua, for example in Calabi-Yau compactifications of string theory, the decay of one vacuum into another is in general a common phenomenon. Many such processes admit an effective semi-classical description involving scalar fields in potentials with multiple critical points [1, 2]. Witten [3] discovered that, in addition, for some vacua

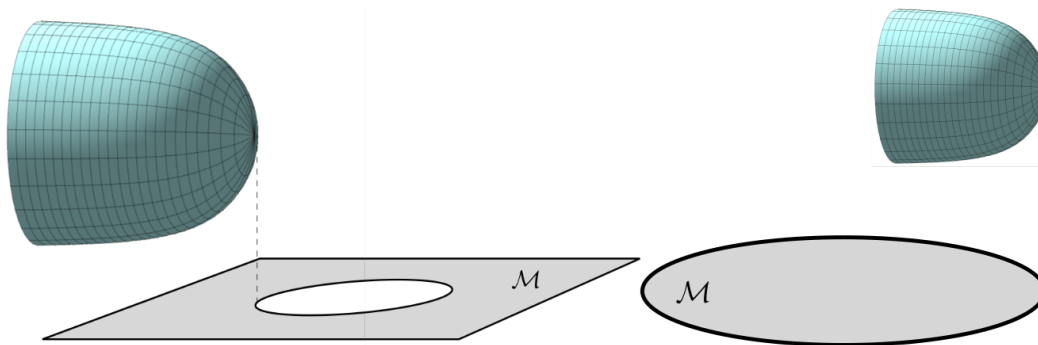


Figure 1. Illustration of the bubble of nothing (left) and the bubble of something (right) instanton. In both cases, the four-dimensional space $\mathcal{M}$ has a spherical boundary which coincides with the locus where the internal manifold, shown above $\mathcal{M}$, shrinks to zero size.

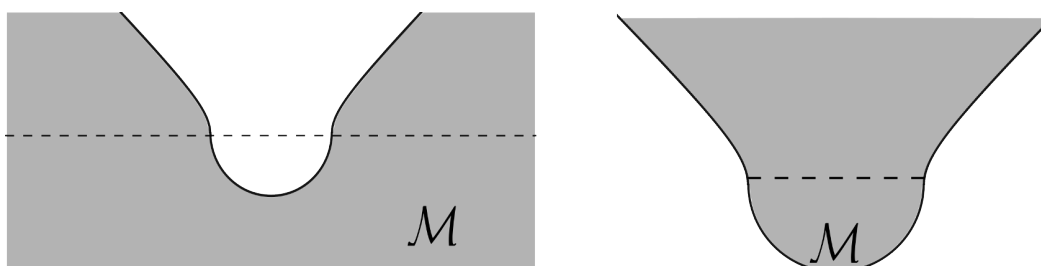


Figure 2. Physical processes corresponding to bubble of nothing (left) and bubble of something (right). The dashed line denotes the locus of analytic continuation from euclidean half instanton to the Lorentzian, expanding bubble. The solid black line is the boundary of 4d space.

arising from compactifications of a higher-dimensional theory, a decay process to ‘nothing’ also exists, whose effective field theory description is much less obvious. Specifically in the case of an S^1 compactification from 5d to 4d, the relevant instanton is characterized by the S^1 shrinking to zero size over the locus of an S^3 , cf. l.h.s. of figure 1. The inside of the S^3 is empty while far away from that region the geometry approaches the vacuum. In the corresponding Lorentzian spacetime, a hole of nothing emerges as a quantum effect and then starts growing, eating up all space, as depicted on the l.h.s. of figure 2. This process and the corresponding instanton is known as a ‘bubble of nothing’. Further studies of such decay processes considered models with stabilized extra dimensions as well as geometries where an originally present supersymmetry is broken by compactification on spheres [4–12] or in string-theoretic toy-models [13–19].

The reverse process of a decay to nothing is vacuum creation from nothing with an instanton of the type illustrated on the r.h.s. of figure 1. The geometry is that of a disk with an S^1 over every point. At the boundary of the disk, the S^1 shrinks to zero size. As the counterpart to the ‘bubble of nothing’, we refer to the creation process as a ‘bubble of something’.¹ Again, these processes are different from the most familiar vacuum creation proposals of [21–23]. Crucially, from the 4d perspective, the instanton contains a boundary and the cosmological constant of the new vacuum need not be positive. Furthermore, the

¹In [20], the term ‘bubbles from nothing’ is used.

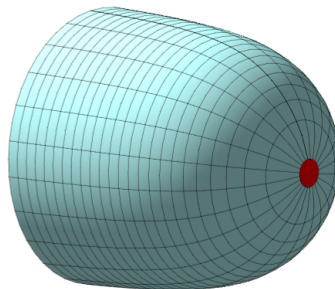


Figure 3. A bordism between S^1 and nothing, including a defect (drawn as a red disk).

created universe is open in the sense that its spatial slices are negatively curved. The idea of a bubble of something was first introduced in [24, 25] and then connected to extra dimensions in [26]. Further research on this topic includes [7, 20, 27–29].

From the four-dimensional perspective, a bubble of nothing looks like an expanding end-of-the-world (ETW) brane with negative tension. The negativity of the tension is enforced by the Israel conditions, which relate the extrinsic curvature of the boundary to the tension of the ETW brane. Ordinary domain walls with negative tension typically suffer from instabilities, since it is energetically favorable to increase their volume. However, it is well known that ETW branes can consistently have negative tension. Examples are provided by O8 planes and the IR-region of the Klebanov-Strassler-throat [30], which in turn can be viewed as a stringy realization [31] of the IR-brane of the Randall-Sundrum model [32]. By contrast, a bubble of something requires a positive-tension ETW brane since, as can be seen by comparing the r.h.s. and l.h.s. of figure 1, the orientation of the boundary and hence the sign of the extrinsic curvature scalar is opposite.

The Witten bubble of nothing can not be naively generalized to supersymmetric configurations since only spinors with anti-periodic boundary conditions on the S^1 can be defined on the cigar geometry of figure 1. The reason is that a circle with periodic boundary conditions for the spinor is not in the trivial class of the spin cobordism group $\Omega_1^{Spin} = \mathbb{Z}_2$. As a consequence, the supersymmetric vacuum does not decay to nothing via the smooth bubble of nothing [3]. More recently, however, it has been argued that this argument for stability is not quite correct in a fundamental theory of quantum gravity. Namely, according to the cobordism conjecture [33] any internal manifold of a string compactification must be cobordant to nothing. In other words, there can not be a topological obstruction to letting spacetime end by the shrinking of the compact space to zero size. Consequently, defects ensuring the triviality of compact spaces in cobordisms are predicted [33–36]. A bordism with defect is illustrated in figure 3. From the 4d point of view, the conjecture implies that all string compactifications possess ETW branes, but it remains a question of dynamics whether bubbles of nothing or something (bubbles of anything for short) arise. A key dynamical ingredient is the tension of the defect. In particular, one expects that in the case of supersymmetric compactifications the relevant defects are always heavy enough to ensure stability. Other applications of the cobordism conjecture can be found in [37–44].

In what follows, we will develop a 4d effective field theory approach to vacuum decay/creation via bubbles of anything, and use it extensively. Let us illustrate this by

discussing the Witten bubble, allowing for a defect as depicted in figure 3. In this case (which of course is just a toy model), the defect has co-dimension 2 in the 5-dimensional spacetime, and its tension is equivalent to a deficit angle θ of the shrinking circle. The relevant 4d Lagrangian consists of the Einstein-Hilbert term and the kinetic term for a scalar ϕ , characterising the radius of the S^1 . In addition, there is a boundary action including the Gibbons-Hawking-York term and the tension of our ETW brane. Our analysis of this setting is related to [10, 11, 45]. It goes beyond these references by allowing for a defect. In addition, we differ in the treatment of the total derivative and boundary terms: we remove the former by integration, obtaining a standard 4d EFT with a completely local description of the ETW brane.

Boundary conditions at the brane fix the scalar field to a certain value, corresponding from the 5-dimensional perspective to a characteristic finite size, η , of the defect. The deficit angle θ is microscopically determined by the tension of the defect. In the 4d EFT, it is identified with the gradient with which ϕ approaches the ETW brane. We will show that η and θ are the key input needed for a simple 4d derivation of the instanton solution corresponding to the Witten bubble with defect and its action, which is related to the decay/creation rate in the usual way.

It can be shown that in the 5d model it is mathematically consistent to view the defect as being point-like, i.e. to perform the limit $\eta \rightarrow 0$. In this case, ϕ runs to infinity at the ETW brane. The original Witten bubble without a defect can now be obtained by setting $\theta = 0$ and performing the above limit $\eta \rightarrow 0$. However, even in this case it is useful to keep η finite since it now plays the role of a regulator making a pure 4d Einstein-frame description possible.

As remarked above, defects of finite size and tension become intrinsic for string theory models such as Calabi-Yau orientifold compactifications with flux. Apart from that, the discussion of the two previous paragraphs carries over almost word by word. The CY is assumed to shrink at the ETW boundary. A defect of size η , which in contrast to the Witten case is now unavoidable, hides the appearing singularity. An appropriately generalized deficit angle θ characterizes the impact of the defect on the nearby geometry, and contributes to the tension of the ETW brane. While instanton solution and action can be given rather explicitly, the limit $\eta \rightarrow 0$ can not be taken. We expect that topology can fluctuate freely at string scale, so $\eta \sim 1$ in string units is plausible, but clearly also much larger ETW defects can exist.

Having thus developed an effective method for computing bubble-of-anything creation/decay rates, we turn to the construction of an ETW brane for type IIB CY orientifold compactifications with D3/D7 branes. This accounts for many vacua of the best understood part of the string landscape [46–49]. The defect corresponding to our proposed ETW brane is an O5/D5 system wrapping a three-cycle of the CY. It can be understood as the mirror dual of a type-IIA orientifold model where, rather obviously, the world can end on an O8/D8 system wrapping the entire mirror manifold and filling three non-compact dimensions. We provide leading order estimates for the transition rates for such vacua and compare the decay rates to traditional decay mechanisms [1, 50–52].

We emphasize that in distinction to the previous discussion based on a rather generic Calabi-Yau cobordism, the orientifold construction does not require the compact space to become small. This illustrates that in controlled situations large defects of size $\eta = R_{KK} \gg 1$ are possible. On the other hand, being supersymmetric, the O5/D5 system will lead to a flat

ETW brane and the associated transition rate will vanish at face value. However, including SUSY breaking and other corrections, it is also possible that our proposed ETW brane is driven to the previously discussed regime of small η .

The paper is organized as follows: in section 2, we set the stage by reviewing the Witten bubble and its 4d description. In section 3, we present the Witten bubble of nothing with defect and characterize it in 4d language. Then we use the 4d intuition gained to analyze more general bubbles of anything, including CY compactifications of string theory. In section 4, we construct an ETW brane for vacua arising from compactifying type IIB string theory on a CY orientifold. We use section 5 to defend the bubble of something against fundamental criticism raised in [7] and to discuss the potential for making our analysis more precise.

2 Bubbles of anything — technical preliminaries

In this section, we first review the Witten bubble of nothing and then discuss the general setup for analyzing bubbles of anything from the four-dimensional perspective.

2.1 The Witten bubble

Witten [3] has discovered that the Kaluza-Klein (KK) geometry $\mathbb{M}_4 \times S^1$ exhibits a peculiar instability. The relevant gravitational instanton is precisely the five-dimensional Schwarzschild black hole geometry, with metric

$$ds^2 = \left(1 - \frac{R_{KK}^2}{\tilde{r}^2}\right)^{-1} d\tilde{r}^2 + \tilde{r}^2 d\Omega_3^2 + \left(1 - \frac{R_{KK}^2}{\tilde{r}^2}\right) R_{KK}^2 dy^2, \quad y \sim y + 2\pi, \quad (2.1)$$

satisfying Einstein's equations. Here, we have chosen to name the radial coordinate $\tilde{r}$ instead of r to reserve the latter for later use. At $\tilde{r} \gg R_{KK}$, this geometry approaches the KK vacuum $\mathbb{M}_4 \times S^1$ with circle radius R_{KK} . Crucially, the geometry is defined only for $\tilde{r} \geq R_{KK}$ and at $\tilde{r} = R_{KK}$ the internal circle shrinks to zero size (cf. the l.h. side of figure 1). From the four-dimensional point of view, this locus represents the end of the world. Despite the vanishing radius of the S^1 , the five-dimensional curvature remains small as long as R_{KK} is large in 5d Planck units, ensuring the reliability of a semiclassical analysis.

The Lorentzian geometry of the bubble of nothing is obtained by an analytic continuation of (2.1): one first writes

$$d\Omega_3^2 = d\psi^2 + \sin^2 \psi d\Omega_2^2 \quad (2.2)$$

and then lets $\psi \rightarrow \frac{\pi}{2} + it$, where $t \in (0, \infty)$. This step is analogous to the analytic continuation employed in Coleman-de Luccia instantons [1]. From the 4d perspective, a spherical hole of nothing with radius R_{KK} nucleates at $t = 0$, starts accelerating and rapidly approaches the speed of light. This Lorentzian geometry corresponds to the region above the dashed line on the l.h. side of figure 2.

The nucleation probability per time and volume of such a bubble can be estimates as

$$\Gamma \sim \exp(-B), \quad B = S_{\text{instanton}} - S_{\text{vacuum}}. \quad (2.3)$$

Here $S_{\text{instanton}}$ and S_{vacuum} are defined by evaluating the Einstein-Hilbert action supplemented by the Gibbons-Hawking-York term,²

$$S = -\frac{1}{2} \int \sqrt{g} \mathcal{R}_5 - \int_{\partial \mathcal{M}} \sqrt{h} \mathcal{K}_5, \quad (2.4)$$

on the instanton geometry (2.1) and in the vacuum. In the above, $\mathcal{R}_5$ and $\mathcal{K}_5$ denote the 5d Ricci- and extrinsic curvature scalar respectively. One explicitly finds [3]

$$B = 2\pi^3 R_{KK}^3. \quad (2.5)$$

2.2 Bubbles of anything from the four-dimensional point of view

We want to find a 4d description of the Witten bubble and then study more general bubbles of nothing as well as bubbles of something from a 4d point of view. Our primary interest will eventually be in bubbles related to Calabi-Yau (CY) compactifications of string theory. We start by considering compactifications from $d = 4 + n$ to 4 dimensions on an n -dimensional compact manifold. To obtain the simplest possible estimate for rates of bubble-of-anything transitions, we restrict attention to the volume modulus. This is also motivated by the fact that in many string models the volume modulus has the lowest mass. We neglect all other fields in the following. As shown in appendix A, the d -dimensional-Einstein-Hilbert action with metric ansatz

$$ds^2 = e^{2\alpha\phi} ds_4^2 + e^{2\beta\phi} ds_n^2 \quad (2.6)$$

reduces to a 4d theory of gravity with Einstein-frame metric ds_4^2 and a canonically normalized volume modulus field ϕ if the constants α, β are chosen as

$$\alpha^2 = \frac{n}{2(n+2)}, \quad \beta = -2\frac{\alpha}{n}. \quad (2.7)$$

Here ϕ is measured in 4d Planck units. In the following, we choose $\alpha < 0$ and, correspondingly, $\beta > 0$. In eq. (2.6), ds_n^2 describes the geometry of the internal manifold with the convention that its volume is fixed to unity in d -dimensional Planck units. For later convenience, we define the quantity R as

$$2\pi R = e^{\beta\phi}, \quad (2.8)$$

which may be thought of as the radius of the internal manifold, also measured in d -dimensional Planck units. Clearly, the case $n = 1$ is appropriate for the Witten bubble.

Motivated by general arguments [53] and similarly to [10, 11], we make an $O(4)$ -symmetric ansatz for the instanton geometry:

$$ds_4^2 = dr^2 + f(r)^2 d\Omega_3^2, \quad \phi = \phi(r). \quad (2.9)$$

²We set the 5d Planck mass $M_5 = 1/8\pi G_5$ to unity. Moreover, our sign convention is such that $\mathcal{K}_5 = \nabla_a n^a$, with $n = n^a \partial_a$ being the outward pointing normal vector of the boundary hypersurface. We adopt this convention throughout the paper.

For completeness, we also record the corresponding d -dimensional metric:

$$ds^2 = e^{2\alpha\phi(r)} \left(dr^2 + f(r)^2 d\Omega_3^2 \right) + e^{2\beta\phi(r)} ds_n^2. \quad (2.10)$$

The KK radius is

$$R_{KK} = \frac{e^{\beta\phi_0}}{2\pi}, \quad (2.11)$$

with ϕ_0 the value of the scalar ϕ when the geometry approaches the vacuum. The equations of motion, i.e. the 4d Einstein equations and the field equation for ϕ take the form

$$\phi'' + 3\frac{f'}{f}\phi' = \frac{dV(\phi)}{d\phi}, \quad (2.12)$$

$$f'^2 = 1 + \frac{f^2}{3} \left(\frac{\phi'^2}{2} - V(\phi) \right), \quad (2.13)$$

where primes denote derivatives with respect to r . A potential $V(\phi)$ can arise for example from internal curvature or fluxes.

A flat-space bubble of nothing instanton is a solution to the field equations where $\phi = \text{const.}$ and $f'(r) \rightarrow 1$ at $r \rightarrow \infty$ and where $\phi \rightarrow -\infty$ at $r = 0$. This implies that we chose r to vanish at the bubble wall, which has radius $f(r = 0)$.

For a bubble of something instanton [26], we may choose $r = 0$ to be at the center of the bubble (see r.h.s. of figure 1), where ϕ takes some finite value and $f = 0$. The boundary is then at some finite $r > 0$, where $\phi \rightarrow -\infty$ and the value of f characterizes the radius of the bubble.

Solutions to (2.12), (2.13) involve three integration constants. Two of them are determined by boundary conditions for ϕ and ϕ' at the bubble wall. The final parameter is then determined by requiring ϕ' to vanish at $r = 0$ in the bubble of something case and at $r \rightarrow \infty$ in the bubble of nothing case.³

We now specialize our discussion to bubble of nothing solutions and focus on internal manifolds without scalar curvature or fluxes, e.g. fluxless CY or torus compactifications, such that the potential vanishes identically. These are merely simplifying assumptions and we discuss their severity in section 5. A more detailed discussion of bubbles of something is given in section 3.5. With the assumptions stated above, we can work with analytic solutions. Equation (2.12) can be integrated to give

$$f^3\phi' = C = \text{const.} \quad (2.14)$$

Substituting the result (2.14) into (2.13) leads to

$$f'^2 = 1 + \frac{C^2}{6f^4}, \quad (2.15)$$

which can be used to obtain [10]⁴

$$\phi(f) = \phi_0 - \frac{\sqrt{6}}{2} \text{arcsinh} \left(\frac{C}{\sqrt{6}f^2} \right). \quad (2.16)$$

³To be more precise, in the bubble of nothing case with vanishing potential, $V(\phi) \equiv 0$, all solutions share the feature that ϕ' vanishes at $r \rightarrow \infty$. In this case, the 3rd integration constant can be used to set the asymptotic value of ϕ , fixing $R_{KK} = \exp(\beta\phi_0)$.

⁴Here we have chosen the positive root of (2.15) such that f increases with growing r .

For $C > 0$, we find that ϕ shrinks when approaching the bubble core. Deriving the profiles $\phi(r)$ and $f(r)$ analytically is more complicated, see [10, 11]. However, it is clear from (2.13) that f is a monotonically growing function of r , such that knowing the profile $\phi(f)$ should be sufficient for practical calculations. In the limit of $f \rightarrow \infty$, ϕ approaches ϕ_0 such that the definition of ϕ_0 in (2.16) is compatible with Equation (2.11). Further, f' approaches unity and spacetime becomes asymptotically Minkowski, as desired. In the limit $f \rightarrow 0$, ϕ approaches $-\infty$ such that the internal S^1 shrinks to zero size. As we will shortly see, C can be fixed by looking at the boundary conditions on ϕ' at the bubble surface. The third integration constant is fixed implicitly by choosing the point where ϕ diverges to be at $r = 0$.

From (2.16) one immediately sees that $f = 0$ at the bubble wall, where $\phi = -\infty$. This means that the bubble has vanishing radius in 4d Planck units. While this may sound confusing, it is simply a consequence of the fact that, at the wall, the compact space shrinks and hence the 4d Planck length diverges. This technical problem of the 4d Einstein frame analysis is overcome when, further down, we regulate the 4d solution by cutting it off when the compact space reaches minimal size η . If appropriate, η may be identified with the size of an ETW defect.

3 Generalized Witten bubbles

We now turn to one of our central points: the development of an entirely four-dimensional description of bubble of nothing/something instantons related to the Witten bubble. In section 3.1, we study the Witten bubble of nothing including a defect at the bubble wall. In section 3.2, we develop a method to effectively compute decay rates for such bubbles, which will also apply to even more general bubble geometries to be discussed below. Section 3.3 discusses a generalization of the Witten bubble with defect to higher dimensions. In section 3.4, bubbles of nothing in 10d Calabi-Yau (CY) compactifications in string theory are analyzed under the assumption that the instanton geometry is similar to the Witten bubble with defect. Finally, in section 3.5, a more detailed discussion of bubbles of something is given.

3.1 Witten bubble with defect

We use the analytic solution from the end of section 2.2, focussing on the case $d = 5$ where (2.7)

$$\alpha = -\sqrt{\frac{1}{6}}, \quad \beta = \sqrt{\frac{2}{3}}. \quad (3.1)$$

The integration constant ϕ_0 is straightforwardly determined by demanding a specific KK radius at $f \rightarrow \infty$, cf. (2.11). The integration constant C remains to be fixed, which is done by analyzing the boundary conditions near the bubble wall. For the geometry (2.10) with $ds_n^2 = dy^2$, $y \sim y + 1$ to be completely regular, one has to demand that the S^1 shrink to zero size without creating a deficit angle. We can however be more general and consider the class of bubbles of nothing where an extra defect is needed to close off the geometry, cf. figure 3. In this case, a deficit angle generically arises. For related discussions of defects see [4, 5, 8, 15, 19].

Since we work semiclassically, when we approach the defect we can only trust the geometry until the internal circle has shrunk to Planck size. If the defect is larger than Planck size, we have to stop even earlier. Let us denote the minimal value of R up to which we trust

our analysis by η , which for the moment we assume to be an $\mathcal{O}(1)$ number. In other words, the geometry stays trustworthy until a minimal $f = f_\eta$, defined by

$$\eta = R(f_\eta). \quad (3.2)$$

Assuming $R_{KK} \gg \eta$ and using (2.8), (2.16), we find

$$f_\eta = \left(\frac{2C}{\sqrt{6}}\right)^{1/2} \left(\frac{\eta}{R_{KK}}\right)^{1/2}. \quad (3.3)$$

As already noted above our defect, being a 3d object in 5d, creates a deficit angle in the geometry. This is entirely analogous to a point mass in three or a cosmic string in four dimensions. We can thus relate the defect tension T_{def} to the deficit angle and hence to the local geometry in the standard way [54, 55],

$$T_{def} = \theta, \quad 1 - \frac{\theta}{2\pi} = \frac{dR}{dx} \Big|_{x=0}. \quad (3.4)$$

Here we use 5d Planck units and x measures the proper distance from the defect. It is related to r by

$$dx = e^{\alpha\phi(r)} dr, \quad (3.5)$$

such that one finds

$$1 - \frac{\theta}{2\pi} = \frac{\beta}{2\pi} \phi' e^{(\beta-\alpha)\phi} \Big|_{x=0} = \frac{\beta}{2\pi} \frac{C}{f^3} e^{(\beta-\alpha)\phi} \Big|_{f=f_\eta} = \sqrt{\frac{4\pi}{3}} \frac{C}{f_\eta^3} \eta^{3/2}. \quad (3.6)$$

Here, we have used the solution (2.14), the definition of η (3.2) and (3.1).

After substituting (3.3) into (3.6), one arrives at

$$C = \sqrt{6}\pi R_{KK}^3 \left(1 - \frac{\theta}{2\pi}\right)^{-2}. \quad (3.7)$$

Interestingly, C does not depend on η which reflects the fact that, even though a defect is present, the limit $\eta \rightarrow 0$ may be taken. In other words, it is mathematically (though maybe not physically) consistent to take the defect to be point-like in the two transverse directions. We will see below that this is no longer true in higher-dimensional models.

We have now fully determined the solutions corresponding to the Witten bubble with defect from the four-dimensional perspective by demanding the three boundary conditions (2.11), (3.2), (3.6). By inserting (3.7) into (2.16), one obtains the instanton solution depending only on the input parameters R_{KK} and θ and where the limit $\eta \rightarrow 0$ has implicitly been taken. The Witten bubble is recovered for the special case $\theta = 0$. The critical bubble radius is

$$\rho_0 = e^{\alpha\phi} f \Big|_{f=f_\eta} = R_{KK}^{-1/2} \left(\frac{1}{\pi\sqrt{6}}\right)^{1/2} \sqrt{C} = \frac{R_{KK}}{1 - \frac{\theta}{2\pi}}. \quad (3.8)$$

We observe that ρ_0 matches the Witten result for $\theta = 0$ and becomes larger when turning on a deficit angle. This agrees with our physics intuition that we expect an additional defect to result in a larger critical bubble and a smaller decay rate.

3.2 The decay rate

We are now ready to calculate decay rates for bubbles of nothing with defects. Although we have the Witten bubble with defect in mind as our primary example, our analysis will turn out to be valid for more general bubbles of nothing, such as the 10d string theory models we are ultimately after. We follow the strategy explained at the end of section 2.1, computing in particular the decay exponent using (2.3). Crucially, we want to work with 4d effective actions for the instanton and the vacuum contribution. Both take the form

$$S = \int_{\mathcal{M}} \sqrt{g} \left(-\frac{1}{2} \mathcal{R}_4 + \frac{1}{2} (\partial\phi)^2 + V(\phi) \right) - \int_{\partial\mathcal{M}} \sqrt{h} (\mathcal{K}_4 - T_4) . \quad (3.9)$$

Here, $\mathcal{R}_4$ is the 4d Ricci scalar, $\mathcal{K}_4$ is the extrinsic curvature, $\partial\mathcal{M}$ is the boundary of the four-dimensional spacetime $\mathcal{M}$ with induced metric h . For both $S_{\text{instanton}}$ and S_{vacuum} , there is a boundary contribution at infinity, without brane tension T_4 . These contributions will cancel. At the center, the vacuum obviously has no boundary while $S_{\text{instanton}}$ receives a contribution with both non-zero $\mathcal{K}_4$ and the tension T_4 of an effective 4d ETW brane sitting at the bubble wall, i.e. at the spherical boundary of $\mathcal{M}$ on the l.h.s. of figure 1. The subscript 4 in T_4 indicates the dimension of space which is bounded by the ETW brane and should not be confused with the tension of a 4-brane in string theory. In eq. (3.9), the 4d Planck mass is set to unity. It is hence important to distinguish between four- and d -dimensional quantities, which are measured in the respective Planck units.

The form of the action (3.9) differs from the one used in [10, 11, 45] by the absence of a term proportional to $\square\phi$ coming from dimensional reduction of $\mathcal{R}_d$. We have replaced this total derivative term by appropriate boundary terms: one at infinity and one at the inner spherical boundary. At the former boundary, the term coming from $\square\phi$ is cancelled by a contribution deriving from the 5d GHY term. The latter boundary is the ETW boundary defined by $f = f_\eta$, where also the ETW brane with tension T_4 sits. Here, the boundary term coming from $\square\phi$ is absorbed in our effective description in terms of $\mathcal{K}_4$ and T_4 . We feel that such a description, based on (3.9) without total derivative terms but viewing the bubble surface as an additional boundary, is more appropriate for our subsequent stringy applications with physical defects at the bubble wall.

In order to compute the on-shell action of the instanton, we first note that the trace of the 4d Einstein equations determines

$$\frac{1}{2} \mathcal{R}_4 = \frac{1}{2} \phi'^2 + 2V(\phi) , \quad (3.10)$$

and hence

$$S_{\text{on-shell}} = -2\pi^2 \int dr f^3 V(\phi) - \int_{\partial\mathcal{M}} \sqrt{h} (\mathcal{K}_4 - T_4) . \quad (3.11)$$

In the absence of a potential, the bulk contribution simply vanishes and the entire contribution comes from boundary terms.

For a Minkowski bubble of nothing geometry, there are two boundaries: one located at asymptotic infinity $f \rightarrow \infty$ and one near the center of the bubble at finite $f = f_\eta$. We calculate the value of the extrinsic curvature scalar for the boundary at infinity using (2.13)

$$\mathcal{K}_4 = 3 \frac{f'}{f} = \frac{3}{f} \sqrt{1 + \frac{C^2}{6f^4}} \simeq \frac{3}{f} + \frac{C^2}{4f^5} , \quad (3.12)$$

where in the last step, we have expanded the previous expression to subleading order in the limit $f \rightarrow \infty$. The contribution from this boundary to the decay exponent B (see (2.3)), can be computed by noting that the vacuum extrinsic curvature scalar is given by

$$\mathcal{K}_{4,\text{vacuum}} = \frac{3}{f}, \quad (3.13)$$

such that the contribution to B , i.e. the difference in the actions, can be calculated

$$B \supset \int_{f \rightarrow \infty} f^3 (-\mathcal{K}_4 + \mathcal{K}_{4,\text{vacuum}}) = 6\pi^2 f^3 \left(-\frac{3}{f} - \frac{C^2}{4f^5} + \frac{3}{f} \right) \Big|_{f \rightarrow \infty} = 0. \quad (3.14)$$

Hence, the entire contribution to B comes from the ETW boundary at the bubble wall. As dictated by the Israel junction conditions, an ETW boundary with non-vanishing extrinsic curvature requires a brane with a certain tension. The (analogue of the) Israel conditions relate the extrinsic curvature $\mathcal{K}_{4,ab}$ to the matter energy momentum tensor S_{ab} located at the end of the world [56, 57]

$$\mathcal{K}_{4,ab} = S_{ab} - \frac{1}{2} h_{ab} h^{cd} S_{cd}. \quad (3.15)$$

An object of pure tension T_4 has an energy momentum tensor $S_{ab} = -T_4 h_{ab}$ and, due to the $O(4)$ symmetry of the problem, we lose no information by taking the trace of (3.15)

$$\mathcal{K}_4 = -\frac{1}{2} h^{ab} S_{ab} = \frac{3}{2} T_4. \quad (3.16)$$

As a consequence of (3.16), the decay exponent takes the form⁵

$$B = 2\pi^2 f_\eta^3 (-\mathcal{K}_4 + T_4) = -2\pi^2 f_\eta^3 \frac{T_4}{2} = -2\pi^2 f_\eta^3 \frac{\mathcal{K}_4}{3}. \quad (3.17)$$

Finally, for the ETW boundary at f_η , the extrinsic curvature scalar takes the value

$$\mathcal{K}_4 = -3 \frac{f'}{f} \Big|_{f=f_\eta} = -\frac{3}{f_\eta} \sqrt{1 + \frac{C^2}{6f_\eta^4}}. \quad (3.18)$$

By comparing with (3.16), we see that a Minkowski bubble of nothing requires a negative tension T_4 , as anticipated before.

We can now compute the decay rate for the Witten bubble with defect. Using (3.16), (3.18), (3.7) in the limit $\eta \ll R_{KK}$, the tension and extrinsic curvature are calculated as

$$T_4 = \frac{2}{3} \mathcal{K}_4 = -\frac{2C}{\sqrt{6}f_\eta^3} = -\left(1 - \frac{\theta}{2\pi}\right) \frac{\eta^{-\frac{3}{2}}}{\sqrt{2\pi}}. \quad (3.19)$$

The decay exponent then follows from substituting (3.3), (3.19) into (3.17)

$$B = \frac{2\pi^3 R_{KK}^3}{\left(1 - \frac{\theta}{2\pi}\right)^2} = \frac{\pi^2 M_P^2 R_{KK}^2}{\left(1 - \frac{\theta}{2\pi}\right)^2}, \quad (3.20)$$

⁵For the bubble of nothing solution, where the action of the vacuum has to be subtracted, we note that the vacuum action does not have a boundary at finite $f = 0$. However, if one considers $f = 0$ to be a boundary of flat space one quickly notices that the extrinsic curvature reads $\mathcal{K}_{4,\text{vacuum}} = -3/f$ and the boundary contribution becomes $6\pi^2 f^3/f|_{f=0} = 0$.

where we have used the definition of the 4d Planck mass $M_P^2 = 2\pi R_{KK}$. We observe that the Witten bubble decay rate (2.5) is recovered for $\theta = 0$. Furthermore, the decay rate decreases for increasing deficit angle (and hence increasing defect tension) and the transition becomes forbidden for $\theta = 2\pi$. By comparing with (3.8), we see that in this limit, the bubble radius grows infinitely large.

Let us collect some further remarks on the above calculation. From (3.19), we see that the Witten bubble solution has, from the four-dimensional perspective, a non-vanishing extrinsic curvature contribution to the action at the bubble surface, which for a low-energy observer signals the presence of a boundary ETW brane. Hence, the description using $\mathcal{K}_4$ and T_4 is justified. In the presence of more complicated defects as discussed below, it is likely that even the d -dimensional theory is described by a boundary action at the locus of the defect, which then naturally descends to a 4d boundary action as in (3.9). Importantly, the action of the form (3.9) also allows us to reliably compute decay rates for more general ETW branes, as will be discussed below.

From (3.19), we see that T_4 diverges in the limit $\eta \rightarrow 0$. This is result from T_4 being measured in units of the 4d Planck mass, which in the limit of $\eta \rightarrow 0$ locally vanishes as $\sim \eta^{1/2}$. Hence, the diverging T_4 is a peculiar looking artefact of our 4d description which however will turn out to be useful in the upcoming sections.

The additional defect was introduced in (3.4) as an object of the 5d theory wrapping the S^3 at fixed f . Its tension in 4d Einstein frame can be obtained by looking at the action

$$S_{def} = -T_{def} \int d^3x \sqrt{g_5|_{P.B.}} = -T_{def} \int d^3x e^{3\alpha\phi} f^3 = -T_{def} e^{-\frac{3}{2}\beta\phi} \int d^3x \sqrt{g_4|_{P.B.}}, \quad (3.21)$$

where we have used (A.9). Hence, the tension in 4d Einstein frame is given by $T_{def}(2\pi R)^{-3/2}$. Remembering that $T_{def} = \theta$, eq. (3.19) becomes intuitive: the defect tension simply adds to the negative tension arising from the shrinking extra dimension and brings the total tension closer to zero, making the decay less likely to occur.

The positive energy theorem [58–60] applies to KK compactifications which asymptote to flat Minkowski space times constant KK manifold. If there exists an asymptotically covariantly constant spinor and the weak energy condition holds,⁶ the ADM mass is non-negative. Furthermore, the mass only vanishes if the asymptotic vacuum situation extends to the complete geometry. Since energy is conserved, this implies that the vacuum is stable. Based on arguments of this type and using the fact that the energy is the square of the supercharges, one expects supersymmetric string vacua to be stable [61]. In our context, we thus expect T_{def} to be large enough to forbid a decay if the relevant vacuum is supersymmetric. Once supersymmetry is broken and the weak energy condition is violated, decays become possible [18].

3.3 Generality of the method developed

Our method for computing the decay rate allows us to go beyond the Witten bubble with defect and to treat more complicated instanton geometries, e.g. with more compact

⁶There are multiple versions of the positive energy theorem with slightly different assumptions. For our purposes it is appropriate to use [59].

dimensions. Using the general metric ansatz (2.10) with appropriate values for α and β (2.7), the equations of motion (2.12), (2.13) continue to hold. If the potential $V(\phi)$ vanishes, also the solution (2.14), (2.16) remains valid. Of course, our ability to let the compact space shrink to zero size in such a way that the higher-dimensional geometry remains smooth is special to S^1 and higher spheres. If the compactification space is generic, the locus where its volume vanishes will represent a singularity. Motivated by the Cobordism Conjecture [33], we may expect that corresponding transitions to nothing are allowed in quantum gravity. Put differently, one may expect that spacetime topology can fluctuate at small scales and that hence our compact space, once it has become small, can disappear. We should then be able to cut out a region with sufficiently small compact space and to replace it by an effective ‘ETW defect’. The relevant region, i.e. the thickness of the ETW defect, may be Planck size or it may be larger. This is to a certain extent a matter of definition of our effective theory. Of course, for our approximation to be valid, the thickness should be much smaller than the critical bubble radius.

This class of instantons, where only the volume of the internal manifold changes until an ETW defect is reached, is conceptually similar to the Witten bubble with defect and might hence be viewed as a generalized Witten bubble. Many known bubble of nothing solutions fall in this category and may hence be described by our general methods, see e.g. [7, 10, 18]. The power of our 4d description and the generality of the ansatz (3.9) allow us to compute the decay rate for a general such instanton, i.e. equations (3.17), (3.18) remain valid.

A natural limit to consider is $\eta \ll R_{KK}$. In this case one finds from (3.17), (3.18) that

$$B = \frac{2\pi^2}{\sqrt{6}} |C|. \quad (3.22)$$

Like for the Witten bubble, the integration constant C depends on R_{KK} and on the boundary conditions at the ETW defect. We will work this out below in examples.

Another natural class of solutions has $\eta \approx R_{KK}$. In this case, $1 \gg C^2/f_\eta^4$, such that $f' \simeq 1$, $\phi' \simeq 0$ on the entire instanton. As a result, the ETW brane cuts out a sphere from flat Minkowski space. Moreover, the ETW defect is maximally thick and essentially identical with the ETW brane. We will see an explicit example in what follows. The general formulas without any restriction on η are shown in appendix B.

We note that, in principle, one could even have exotic bubble-of-nothing or something solutions in which the compact radius grows as one approaches the ETW brane. One may then have $\eta \gg R_{KK}$. Although such boundary conditions are physically unintuitive, the method how to compute the decay rate remains unchanged.

Another interesting observation to be made is that the extrinsic curvature (3.18) and hence the decay rate is symmetric under $C \rightarrow -C$. From (2.16), one observes that this change in sign changes the solution from a shrinking to a growing internal manifold when approaching the bubble surface. The Witten bubble with defect is a solution with shrinking S^1 , i.e. $C > 0$, but due to T -duality, it might not be entirely surprising that an analogous solution with $C < 0$ also exist. Of course, the resulting geometry is somewhat peculiar, because the extra dimension grows large towards the bubble center and it is unclear how the geometry closes. Under T -duality, the codimension-two defect assumed to be present gets mapped to a codimension-one brane wrapping the internal S^1 . This brane can act as an

ETW brane, i.e. fulfilling the Israel junction conditions, such that the geometry can smoothly end. However, for theories in $d > 5$, the T-duality interpretation is not always given. It would be interesting to study the $C \rightarrow -C$ symmetry in more detail.

3.4 Bubbles of nothing in Calabi-Yau compactifications

In this section, we finally discuss bubble of nothing instanton solutions and the corresponding decay rates in string theory models arising from compactification on CY-orientifolds, possibly with fluxes. We will use the intuition from the Witten bubble with defect and the general procedure how to calculate decay rates from above. Fluxes and non-perturbative effects generally present in $\mathcal{N} = 1$ or $\mathcal{N} = 0$ models induce a potential for ϕ which is particularly important at small volumes. We assume that it is a valid approximation to treat the effects of this potential as being part of the effective ETW defect and hence use (2.16) to find the decay rate. This is justified as long as the thickness of the domain wall remains small compared to the critical bubble radius.

Since we now work in $d = 10$, α and β take values according to (2.7):

$$\alpha = -\sqrt{\frac{3}{8}}, \quad \beta = \frac{1}{2\sqrt{6}}. \quad (3.23)$$

Clearly, in the present case, we always require a ‘cobordism transition’, i.e. a region of topology change and hence a non-trivial ETW defect. Thus, we can trust the geometry defined by (2.16) only up to the point where the CY radius has changed to some critical size η . From the condition

$$\eta = R(f_\eta), \quad (3.24)$$

one then finds the relation

$$\frac{C}{f_\eta^2} = \frac{\sqrt{6}}{2} \left[\left(\frac{R_{KK}}{\eta} \right)^4 - \left(\frac{\eta}{R_{KK}} \right)^4 \right]. \quad (3.25)$$

As for the Witten bubble with defect, we expect that C should be fixed by the boundary conditions at the ETW defect. While this ETW defect is an unfamiliar object, it is natural to characterize its effect on the surrounding geometry by a ‘deficit angle’ θ defined in analogy to (3.4):

$$1 - \frac{\theta}{2\pi} = \frac{dR}{dx} \Big|_{x=0} = \frac{\beta}{2\pi} \frac{C}{f^3} e^{(\beta-\alpha)\phi} \Big|_{f=f_\eta} = \frac{1}{4\pi\sqrt{6}} \frac{C}{f_\eta^3} (2\pi\eta)^4. \quad (3.26)$$

For the remainder of this section, we assume $\eta \ll R_{KK}$ and hence $C > 0$, $\theta < 2\pi$, which is the parameter range corresponding to a small ETW defect, as motivated above. Hence, we ignore the second term in (3.25). General equations, including the cases $\eta \approx R_{KK}$ and $\eta > R_{KK}$, are provided in appendix B. Solving (3.25) and (3.26) for C gives

$$C = \frac{\sqrt{6}(2\pi)^6}{32 \left(1 - \frac{\theta}{2\pi}\right)^2} \frac{R_{KK}^{12}}{\eta^4}. \quad (3.27)$$

This fully determines the instanton solution. Unlike the 5d case, C now explicitly depends on η and diverges as $\eta \rightarrow 0$. Thus, the limit of a truly point-like ETW defect can not be taken. To find the decay rate, we go on to compute the 4d extrinsic curvature of the boundary at $f = f_\eta$ (see (3.18)):

$$\mathcal{K}_4 = -6 \left| 1 - \frac{\theta}{2\pi} \right| (2\pi)^{-3} \eta^{-4}. \quad (3.28)$$

Note that $\mathcal{K}_4$ does not depend on R_{KK} , which in appendix B is shown to hold generally in the limit $\eta \ll R_{KK}$. This is consistent with the expectation that the choice of η and θ should completely fix the local geometry near the boundary. By the Israel conditions, the extrinsic curvature also determines the total tension of the ETW brane in the 4d EFT, cf. (3.16).

Next, we compute the critical bubble radius:

$$\rho_0 = e^{\alpha\phi(f_\eta)} f_\eta = \frac{R_{KK}}{4 \left| 1 - \frac{\theta}{2\pi} \right|} \left(\frac{R_{KK}}{\eta} \right)^3. \quad (3.29)$$

It diverges in the limit $\eta \rightarrow 0$, showing again that it is not possible to remove the ETW defect. In comparison to the Witten bubble radius (3.8), ρ_0 is much larger due to the appearance of the factor $(R_{KK}/\eta)^3 \gg 1$. The decay exponent straightforwardly follows from (3.22)

$$B = \frac{(2\pi)^8 R_{KK}^8}{64 \left(1 - \frac{\theta}{2\pi} \right)^2} \left(\frac{R_{KK}}{\eta} \right)^4 = \frac{\pi^2 M_P^2 R_{KK}^2}{16 \left(1 - \frac{\theta}{2\pi} \right)^2} \left(\frac{R_{KK}}{\eta} \right)^4. \quad (3.30)$$

It is again enhanced relative to the Witten bubble case (3.20) by powers of R_{KK}/η .

A peculiar feature of the CY bubble is the fact that when approaching the bubble wall from infinity, the transversal three sphere radius $\rho = e^{\alpha\phi} f$ first shrinks as expected, but at some point starts growing again. This can be seen by looking at the profile $\rho(f)$

$$\rho(f) = R_{KK}^{-3} \left(\frac{C}{\sqrt{6}f^2} + \sqrt{\left(\frac{C}{\sqrt{6}f^2} \right)^2 + 1} \right)^{\frac{3}{4}} f. \quad (3.31)$$

For large values of f , $\rho(f) \approx R_{KK}^{-3} f$ and hence ρ grows with growing f . For small f on the other hand,

$$\rho(f) \approx R_{KK}^{-3} \left(\frac{2C}{\sqrt{6}} \right)^{\frac{3}{4}} \frac{1}{\sqrt{f}}, \quad (3.32)$$

and we see that ρ decreases with growing f . Such a behavior has already been observed in 6d models [8]. In appendix B, it is shown that this peculiar behavior always occurs for $d > 5$.

Clearly, this analysis could have been carried out with $C < 0$ and from (3.22) one can see that the decay rate is symmetric under $C \rightarrow -C$. In contrast to the case of S^1 compactifications, it is not immediately clear how to relate these two types of solutions using T-duality or mirror symmetry. It would be interesting to study this further, presumably invoking Calabi-Yau moduli other than the volume.

3.5 Bubbles of something

The 4d geometry corresponding to the bubble of something instanton is the disc on the r.h.s. of figure 1, with $f = 0$ at the center and $f = f_\eta$ at the boundary. To find the instanton solutions, we again need to consider the field equations (2.12), (2.13) subject to the general boundary condition $\phi'(f = 0) = 0$ as well as the boundary conditions on ϕ and ϕ' at the bubble wall. For instanton solutions in concrete 6d models, see [20].

Given a solution, one can calculate $-S_{\text{instanton}}$. We would now like to compute a creation rate Γ , analogous to the decay rate of (2.3). However, there is no parent vacuum and hence the natural definition of a rate is

$$\Gamma \sim \exp(-B) = \exp(-S_{\text{instanton}}), \quad (3.33)$$

without the subtraction of any parent vacuum action in the exponent. This rate for the creation of a compact universe with boundary is completely analogous to the rate for the creation of a boundary-free, spherical universe suggested by Hartle and Hawking [21]. The competing proposal by Linde and Vilenkin [22, 23] famously differs by a sign in the exponent, but is similar in the aspect that no background action is subtracted.

Furthermore, the meaning of Γ is altered in comparison to decay rates relating different vacua as discussed e.g. in [62–64] (see also [65] for a recent proposal): the rate no longer counts the number of creation events per time and volume since there is no background in which the bubble of something nucleates. Rather, only ratios of the form

$$\Gamma_i/\Gamma_j, \quad (3.34)$$

with i, j labelling vacua of the theory, may be viewed as physical predictions. They specify the relative probabilities of creating vacuum i rather than j from nothing. Such ratios would also be independent of the background action in the case of decay rates.

We first observe that, in general, bubbles of something do not exist for vanishing potential. This follows from (2.14), which shows that the vanishing of f at the center, together with smoothness of ϕ , implies $C = 0$. This special solution corresponds to a flat disc with $\phi = \phi_0$.

However, it is not difficult to show that bubble of something solutions do exist for nontrivial potentials. They can approximate the flat-disc/constant- ϕ solution above as follows: in the central part of the disc, ϕ is essentially constant, sitting near a Minkowski-minimum of the potential. Near the boundary, ϕ starts moving away from that minimum and f starts deviating from $f = r$, such that eventually the boundary conditions associated with the ETW defect are met at $f = f_\eta$. The boundary conditions are given by (3.24) and an equation for the deficit angle similar to (3.26). However, we have to remember that x measures the distance from the ETW brane which, in comparison to (3.5), is now related to r via

$$dx = -e^{\alpha\phi} dr. \quad (3.35)$$

As a result, the second boundary condition becomes

$$1 - \frac{\theta}{2\pi} = \frac{dR}{dx} \Big|_{x=0} = -\frac{\beta}{2\pi} \phi' e^{(\beta-\alpha)\phi} \Big|_{f=f_\eta} = -\frac{1}{4\pi\sqrt{6}} \phi' (2\pi\eta)^4. \quad (3.36)$$

Hence, a solution with deficit angle $\theta < 2\pi$ now corresponds to $\phi' < 0$ at the ETW brane, unlike in bubble of nothing models where $\phi' > 0$ (i.e. $C > 0$) was a necessary condition, cf. (3.26).

The instanton action can be evaluated using our general formula (3.11) for on-shell actions. It receives both bulk and boundary contributions. The boundary term contribution to $S_{\text{instanton}}$ can be computed using

$$\frac{3}{2}T_4 = \mathcal{K}_4 = + \left. \frac{3f'}{f} \right|_{f=f_\eta} = + \frac{3}{f_\eta} \sqrt{1 + \frac{f_\eta^2}{3} \left(\frac{\phi'^2}{2} - V(\phi) \right)}_{f=f_\eta} \geq 0. \quad (3.37)$$

Note that, as anticipated, a bubble of something requires a positive-tension ETW brane.

Given a solution, also the bulk term in (3.11) can in principle be evaluated straightforwardly. However, in this study we do not discuss any details of the potential and we can hence only give explicit results if this term is negligible due to appropriate assumptions.

We can provide a creation rate estimate assuming that ϕ is stabilized such that $V \simeq 0$ for a large part of the instanton. Consequently, ϕ moves significantly only close to the ETW boundary. Using the equations of motion, one can show that the bulk of the instanton is essentially flat space, i.e. $\phi' \simeq 0, f' \simeq 1$. In case of the critical bubble radius being large in comparison to the region where ϕ moves significantly, we may absorb these effects into the ETW brane tension and hence obtain an effective description of a bubble of something instanton where an ETW brane with tension T_4 bounds flat space, similar to [28]. The assumptions stated above are most easily fulfilled for small values of T_4 or a potential stabilizing ϕ strongly. Of course, by absorbing the unknown effects of the potential near the boundary into T_4 , we sacrifice the calculability of T_4 from η and θ .

The extrinsic curvature at $f = f_\eta$, defined as the region where ϕ begins rolling and hence where the ETW brane is located, is given by

$$\mathcal{K}_4 \simeq \frac{3}{f_\eta}. \quad (3.38)$$

The ETW brane tension, now also capturing the effects of the potential and the rolling of ϕ near the ETW boundary, is then related to f_η by (3.16)

$$T_4 = \frac{2}{3}\mathcal{K}_4 = \frac{2}{f_\eta}. \quad (3.39)$$

Disregarding, as discussed, the bulk contribution and reinstating M_P for clarity one finds

$$B = -2\pi^2 f_\eta^3 \frac{M_P^2 \mathcal{K}_4}{3} = -2\pi^2 f_\eta^2 = -8\pi^2 \frac{M_P^6}{T_4^2}. \quad (3.40)$$

The rates governed by this exponent have to be viewed as competing with the famous proposals for vacuum creation [21–23] which result in closed universes and require the initial value of the scalar potential to be positive. By contrast, the bubbles of something governed by (3.40) give rise to open universes with no restriction on the vacuum energy, as long as an ETW brane with sufficient tension exists.

Our Coleman-De Luccia based perspective on (3.40) implies that the quantity in the exponent is eventually positive, such that no tunneling-suppression arises. This is then

analogous to the Hartle-Hawking [21] rather than the Linde/Vilenkin [22, 23] prediction. However, in light of the long-lasting debate concerning these proposals, one should obviously ask whether a Linde-Vilenkin-like exponential suppression by B from (3.40) may also be argued for.

Recently, the creation of universes from nothing according to [21–23] was analyzed based on the Lorentzian path integral [66–70], where it was found that neither proposal in the original form is free of issues. An important role is played by potentially unsuppressed field fluctuations. Clearly, it would be desirable to study the bubble of something from a Lorentzian perspective to validate the creation rate and to analyze the behavior of field fluctuations.

3.6 Comparison to other results

Our results can be tested by comparing with [18], where a bubble of nothing for a T^3 compactification in Einstein-Dilaton-Gauss-Bonnet gravity was constructed. This construction also allows for a string-theoretic embedding. What we referred to as ETW defect can be identified with the ‘inner bubble region’ of [18]. Naively, we would compare their results with our calculations for $d = 7$. However, the presence of the dilaton in [18] modifies the theory such that the resulting equations of motion are analogous to the $d = 5$ theory without a dilaton. Hence, one should compare with equations derived in sections 3.1, 3.2. Using results from [18], the deficit angle can be calculated to be⁷

$$1 - \frac{\theta_{T^3}}{2\pi} = \frac{12\pi\lambda}{(2\pi\eta)^2}, \quad (3.41)$$

with λ denoting the Gauss-Bonnet coupling and $(2\pi\eta)^3$ being the volume of the T^3 at the locus of the ETW defect. The critical bubble radius ρ_{0,T^3} as well as the decay exponent B_{T^3} (assuming $\eta \ll R_{KK}$ where analytical results were obtained in [18]) are

$$\rho_{0,T^3} = \frac{(2\pi\eta)^2 R_{KK}}{12\pi\lambda}, \quad B_{T^3} = \pi^2 M_P^2 \rho_0^2. \quad (3.42)$$

Using (3.41), one finds

$$\rho_{0,T^3} = \frac{R_{KK}}{1 - \frac{\theta_{T^3}}{2\pi}}, \quad B_{T^3} = \frac{\pi^2 M_P^2 R_{KK}^2}{\left(1 - \frac{\theta_{T^3}}{2\pi}\right)^2}, \quad (3.43)$$

in perfect agreement with our previous results (3.8), (3.20).

4 Towards a realization in type IIB string theory

ETW branes for Calabi-Yau or Calabi-Yau orientifold compactifications, as discussed in the previous section, could be responsible for bubble of nothing decays or bubble of something creation processes for many vacua of the string landscape. In the present section, we propose an ETW brane for type IIB compactifications on CY orientifolds with O3 planes. The ETW brane is constructed in a supergravity approximation of string theory and the issue

⁷Here, we use the profile of the volume modulus (which is identical to the dilaton profile) at the surface where the outer bubble-region matches the inner-bubble region. More precisely, we refer to eq. (5.87) in [18].

of Kahler moduli stabilization is not addressed. In this sense, the level of explicitness is similar to that of GKP [71]. We will comment on the relation to the shrinking-CY analysis of section 3.4 at the end of section 4.1.

4.1 The ETW brane

The main idea is most easily explained by starting with type IIA theory on a CY manifold Y_A . The domain wall arises by considering the geometry $\mathbb{R}^4 \times Y_A$, with $\mathbb{R}^4$ parameterized by $(X^0, \dots, X^3)$, and applying a standard orientifold projection $X^3 \rightarrow -X^3$. As a result, one obtains an O8 plane located at $X^3 = 0$ which also plays the role of an ETW brane.⁸

Applying mirror symmetry and recalling its Strominger-Yau-Zaslow (SYZ) interpretation as T-duality of the T^3 fibre of the CY [72], it is clear that we obtain type IIB string theory on $\mathbb{R}^4 \times Y_B$ with a set of O5 planes wrapping a three-cycle of Y_B and filling out three directions of $\mathbb{R}^4$. Clearly, this slightly more complicated ETW brane may also be constructed by directly applying an appropriate orientifold projection to $\mathbb{R}^4 \times Y_B$ on the type IIB side. We place one D5 brane on top of each O5 plane to cancel tadpoles locally.

We can go beyond plain $\mathcal{N} = 2$ in the bulk by combining the orientifold projection above with a holomorphic orientifold projection acting only on Y_B and introducing (for simplicity) O3 planes only. Now, away from the ETW brane, we have a type IIB $\mathcal{N} = 1$ orientifold model of the type underlying the best understood part of the landscape. At the locus of the ETW brane, the simultaneous existence the O5/D5 and O3/D3 systems might break SUSY completely. However, as is argued in appendix C, this configuration preserves two supercharges leading to $\mathcal{N} = 1$ SUSY on the 3d worldvolume of the domain wall.

The above construction can be conveniently illustrated in the following toy-model: consider the space $T^6/\mathbb{Z}_2$, with the T^6 parameterized by three complex coordinates

$$Z^i = U^i + iV^i, \quad U^i \sim U^i + 2\pi, \quad V^i \sim V^i + 2\pi, \quad i \in \{1, 2, 3\}. \quad (4.1)$$

The $\mathbb{Z}_2$ generator acts as $(Z^i \rightarrow -Z^i, (-1)^{F_L}\Omega)$, with F_L the left-moving fermion number and Ω the worldsheet parity. This orientifold action produces 64 O3 planes located at $U^i = 0$ or $U^i = \pi$ and $V^i = 0$ or $V^i = \pi$ for $i = 1, 2, 3$ and filling the non-compact directions. In addition, we introduce an O5 plane generator acting by $(X^3 \rightarrow -X^3, Z^i \rightarrow \bar{Z}^i + i\pi, \Omega)$. The fixed points of this action, which determine the positions of the O5 planes, are given by $V^i = \pm\pi/2$. The full orientifold group is $\mathbb{Z}_4 \times \mathbb{Z}_2$ containing the identity, the two aforementioned generators and their product acting by $X^3 \rightarrow -X^3, Z^i \rightarrow -\bar{Z}^i - i\pi$ without orientation reversal (since $\Omega^2 = 1$). Modding out this product generator does not induce additional singularities since the map $V^i \rightarrow V^i - \pi$ on a torus is fixed-point-free. Table 1 summarizes the generators and table 2 the locations of the O-planes. Note that the O3 and O5 planes do not intersect due to their separation along V^i . We can explicitly check that this configuration does not break SUSY completely. To show this, we first note that Op planes preserve the same supercharges as Dp branes, such that we can equivalently think about the SUSY breaking of D3 and D5 branes. From table 2, we can directly read off that an

⁸In the present section, we use Lorentzian geometry with a time-like ETW brane. This makes it simpler to discuss SUSY and SUSY breaking. We assume that we are allowed to return to Euclidean instanton solutions as usual, by analytic continuation in the 4d EFT.

	X^0	X^1	X^2	X^3	U^1	V^1	U^2	V^2	U^3	V^3	
g_1	X^0	X^1	X^2	X^3	$-U^1$	$-V^1$	$-U^2$	$-V^2$	$-U^3$	$-V^3$	$\Omega(-1)^{F_L}$
g_2	X^0	X^1	X^2	$-X^3$	U^1	$-V^1 + \pi$	U^2	$-V^2 + \pi$	U^3	$-V^3 + \pi$	Ω
$g_1 \cdot g_2$	X^0	X^1	X^2	$-X^3$	$-U^1$	$V^1 - \pi$	$-U^2$	$V^2 - \pi$	$-U^3$	$V^3 - \pi$	$(-1)^{F_L}$

Table 1. Action of the two orientifold generators (of O3 and O5 planes) and of their product.

	X^0	X^1	X^2	X^3	U^1	V^1	U^2	V^2	U^3	V^3
O3	✓	✓	✓	✓	×	×	×	×	×	×
O5	✓	✓	✓	×	✓	×	✓	×	✓	×

Table 2. Summary of dimensions filled by O3/O5 planes (indicated with a ✓).

open string stretching between those branes is defined by four Dirichlet-Neumann boundary conditions which is known to preserve 1/4 of the 32 supercharges.

We expect our $T^6/\mathbb{Z}_2$ toy-model to generalize to proper CY orientifolds with O3 planes, as described at the beginning of this section. In particular, the O5 planes in type IIB are related by mirror symmetry to the O8 plane on the IIA side, which arises from the reflection $X^3 \rightarrow -X^3$. Viewing Y_B in terms of the SYZ fibration $T^3 \rightarrow Y_B \rightarrow B_3$, with B_3 denoting the base, it is clear that the O5 planes are compatible with the symplectic structure of Y_B (i.e. they wrap B_3). We cancel their D5 tadpole locally by placing one D5 brane on top of every O5 plane. One might be concerned that the O5/D5 system induces a D1 tadpole since the Chern-Simons brane action contains a term

$$S_{C.S.} \supset \int_{O5/D5} C_2 \wedge p_1(R). \quad (4.2)$$

However, since the O5/D5 systems wraps a three-cycle of Y_B and the first Pontryagin class $p_1(R)$ is a four-form, the above integral vanishes. Hence, our domain wall does not introduce any uncanceled tadpoles. Following this line of thought, we are lead to refine our initial statements about the O8/D8 system on the mirror-dual IIA side. Indeed, we expect the O8/D8 system to carry a non-trivial gauge bundle. To see this, note that this system wraps the entire CY. The coupling analogous to (4.2),

$$S_{C.S.} \supset \int_{O8/D8} C_5 \wedge p_1(R), \quad (4.3)$$

then induces a D4 charge, which we may cancel by an appropriately wrapped D4-brane. This brane can be taken to be resolved in the D8-stack, inducing a non-trivial gauge bundle and thereby ensuring that the IIA side is tadpole-free as well. For completeness, we recall that the IIA side in addition has an O6/D6 system and the IIB side a mirror-dual O3/D3 system, such that the non-empty side of the ETW-brane is $\mathcal{N} = 1$.

To be sure that the above construction forms an ETW brane of a string theory vacuum, one would need to study its backreaction more properly and address the issue of moduli stabilization. More specifically, corrections to the Kahler moduli Kahler potential and corresponding non-perturbative superpotential effects are not considered in the bulk and

hence Kahler moduli remain unstabilized. The next step would be to include such effects and study their interplay with corresponding effects from the ETW brane. However, we are for the moment satisfied with the construction of an ETW brane at a level of explicitness similar to GKP [71].

Let us comment in more detail on the dynamics of our effective ETW brane. At tree level this brane, viewed as an object of the 4d EFT, has vanishing tension: $T_4 = 0$. This is clear since we use CY geometry and cancel the tadpoles of all O5 planes involved locally. The O3 plane charge can of course only be cancelled globally, which implies non-vanishing F_5 field strength and some warping but does not destabilize the compact space away from the ETW brane or the brane itself. This is also consistent with the fact mentioned above (cf. appendix C for details) that the ETW brane preserves $\mathcal{N} = 1$ SUSY in 3d.

Even in the absence of fluxes, non-perturbative effects will in general induce a non-zero superpotential, leading to an $\mathcal{N} = 1$ AdS vacuum or to a runaway to infinite volume in the bulk. Since our domain wall preserves some supercharges, one might expect that it also receives corresponding corrections, giving it a tension of exactly the right magnitude to keep it flat [73–75]. A flat domain wall with critical tension $T_4 = \pm T_c = \pm 2/\ell_{\text{AdS}}$ allows for a critical bubble of something (positive sign) or a critical bubble of nothing (negative sign) with infinite radius. Thus, neither of the two corresponding tunneling processes can dynamically arise.

We note, however, that the above relies on an analysis performed for domain walls in a 4d theory with smoothly varying superpotential [73–75]. It is not clear to us whether the conclusion concerning the critical tension can really be carried over to our case of a (generically singular) ETW brane. In particular, we feel that we can not firmly exclude the possibility of supersymmetric but non-stationary ETW branes with AdS_3 worldvolume geometry. For instance, it was shown in [76] that 5-dimensional supergravity does admit supersymmetric domain walls with “arbitrary” tension. In 4d, one might expect that the ordinary BPS bound for field theory domain walls will carry over to ETW branes, although this is not immediately compatible with the following holographic argument: find an AdS_3 supergravity theory containing a large- N CFT in the matter sector. This CFT can be holographically interpreted as an additional spatial dimension, implying the existence of an AdS_4 ‘bulk space’. The AdS_3 we started from now plays the role of the boundary of our 4d theory. Being AdS, this boundary is non-stationary and corresponds to a ‘detuned’ ETW brane.

In any event, even if such ‘detuned’ ETW branes should be relevant in our setting, both types of bubble-of-anything processes are still excluded since the tension lies in the range $-T_c \leq T_4 \leq T_c$. Thus, for bubbles of either type to arise, SUSY must be broken in the bulk or by the ETW brane. We will now discuss both options.

First, let us focus on bulk SUSY breaking. This may arise if our $\mathcal{N} = 1$ vacuum is modified by further effects, such as fluxes and/or anti-D3 brane uplifts etc., leading to a metastable SUSY breaking minimum. If fluxes are present, they would first have to be removed by an appropriate domain wall consisting of wrapped D5/NS5 branes, which are extended in parallel to our O5/D5 ETW brane. We will from now on view both together as our effective ETW brane.

The bulk SUSY breaking effects will clearly modify both the vacuum energy and the tension of the domain wall. Most naively, one expects that SUSY breaking increases the

decay probability, i.e. that bubbles of nothing become possible similar to [18]. However, especially if the tension has been positive before SUSY breaking, it is also plausible that a bubble of something arises. We expect the correction to T_4 to be set by a combination of the KK scale and the SUSY-breaking scale.

If, instead of the supersymmetric ETW brane constructed above, we manage to find an ETW brane that does not preserve any supercharges, we expect the tension to be larger such that bubbles of something may become allowed. For example, one may imagine that the D5 branes which are part of our ETW construction are wrapped on cycles that are distinct from the special lagrangian 3-cycles of the O5-planes. Whether this is possible while still cancelling the O5-tadpole and ensuring a sufficiently metastable or long-lived ETW brane is an open question. It appears likely that the small- η ETW defects speculated to exist in section 3 are SUSY-breaking and hence provide an example of the SUSY-breaking ETW branes just discussed. Generally, since non-supersymmetric defects have been predicted by the cobordism conjecture [33, 35, 36], there likely exist ETW branes suitable for bubble of something creation. However, it is a general challenge to explicitly construct metastable objects with large tension and it could be a similarly hard task as finding metastable de Sitter vacua in string theory.⁹

For simplicity, we so far considered CY orientifolds with O3 planes only. However, nothing prevents us from also allowing for O7 planes wrapped on holomorphic cycles on the CY. The resulting 4d vacua still have $\mathcal{N} = 1$ SUSY and, as argued in appendix C, the O5/D5 ETW brane still preserves two supercharges. However, the O7 and the O5 plane now generically intersect.

Note that the T^6 example above also allows for the introduction of O7 planes: consider a compactification of type IIB string theory on $T^2/\mathbb{Z}_2 \times T^4$ containing four O7 planes. Using again the parametrization (4.1) with $\mathbb{Z}_2$ action $(Z^1 \rightarrow -Z^1, (-1)^{F_L}\Omega)$, the O7 planes are located at points where U^1, V^1 take values 0 or π . Introducing again an O5 plane generator compatible with the symplectic structure of the torus, $(X^3 \rightarrow -X^3, Z^i \rightarrow \bar{Z}^i + i\pi, \Omega)$, one sees that the O5 planes are located at $U^i = \pm\pi/2, V^i = \pm\pi/2$. Hence, the O5s and O7s do not intersect in this geometry.

However, it is questionable whether this property can also arise in proper CY compactifications. Here, in analogy to the torus example, the O7 planes are compatible with the holomorphic structure of the CY and the O5 planes with its symplectic structure. The non-trivial fibration of the SYZ-torus over the base B_3 may make it unavoidable for the O5s and O7s to intersect.

Finally, we want to comment on the relation to the possibly more general type of CY-based ETW brane of section 3.4. There, the CY was assumed to shrink to some radius η before the 4d world terminates in a suitable defect. By contrast, in the present section we analysed an ETW brane which terminated the 4d theory without the need of a previous shrinking of the CY. Once SUSY is broken (either in the bulk or on the brane) this solution has corrections and it is conceivable that, when these are included, our present solution is dynamically driven to a solution of the type discussed in section 3.4. Put differently, we may

⁹For example, one expects a 9d non-SUSY positive-tension ETW brane for the 10d Minkowski type-IIB vacuum to exist. Its tension presumably depends on the dilaton, dynamically driving the latter to infinity. It would be interesting to study whether bubble of something processes are possible in settings of this type.

think of the presently discussed ETW brane as of a special case where $\eta \approx R_{KK}$. Corrections may drive this value of η to other, e.g. $\mathcal{O}(1)$, values.

4.2 Creation/decay rates

We now want to estimate the creation/decay rates of bubbles of anything involving the ETW brane constructed above. We assume $\eta \approx R_{KK}$, as discussed before and rely on our calculations for Minkowski vacua from section 3. These remain approximately valid for de Sitter (dS) or AdS models as long as $\ell_{dS/AdS} \gg M_P^2/|T_4|$, which defines the class of vacua we are considering in this section. Clearly, vacua with cosmological constant tuned close to zero most easily fulfill this condition which is why we focus on vacua of this type.

Let us now comment on the scales setting the magnitude of T_4 . The non-vanishing tension is a combination of two effects: the O3/D3 or O7/D7 system present in the bulk leading to $\mathcal{N} = 1$ SUSY in 4d and the additional SUSY breaking effects either in the bulk or on the boundary. Before SUSY breaking, the tension is generically of order $|T_4| \sim T_c \simeq M_P^2/\ell_{AdS}$. We want to express this in terms of the physical KK radius R_{KK}/M_{10} , with M_{10} denoting the 10d Planck mass. Hence we introduce a parameter ϵ measuring the separation of scales between the KK- and the AdS curvature length

$$R_{KK}/M_{10} = \epsilon \ell_{AdS}. \quad (4.4)$$

We are at this point agnostic concerning possible constraints on realizing $\epsilon \ll 1$ [77–79].

Eventually, the tension before SUSY breaking can be expressed as

$$|T_4| \sim \epsilon \frac{M_P^2}{R_{KK}/M_{10}} \sim \epsilon \frac{M_P^3}{R_{KK}^4}. \quad (4.5)$$

We expect that corrections from the uplifting mechanism will not invalidate this estimate. Thus we will continue to work with the ETW brane tension (4.5), with both signs for T_4 being allowed.

Bubbles of something: as discussed before, a tension $T_4 > 0$ suitable for a bubble of something can arise due to SUSY breaking either in the bulk or on the ETW brane. Since we assumed $\eta \approx R_{KK}$, the volume modulus stays approximately constant away from the ETW brane and we may use our estimate (3.40) for the creation rate:

$$B = -\frac{8\pi^2 M_P^6}{T_4^2} \sim -\frac{R_{KK}^8}{\epsilon^2}. \quad (4.6)$$

If the vacuum under consideration is dS, it would be interesting to compare the above result with those of the Hartle-Hawking no-boundary and the Linde/Vilenkin tunneling proposals. As discussed before in section 3.5, it is conceptually unclear how the different creation rates should be compared to each other, but we can nevertheless make a few observations.

The creation exponents for the no-boundary and the tunneling proposals are given by

$$B = \mp 8\pi^2 M_P^2 \ell_{dS}^2, \quad (4.7)$$

with the upper sign corresponding to the Hartle-Hawking and the lower sign to the Linde/Vilenkin process. Clearly, the sign of B in (4.6) matches that of Hartle-Hawking, which is a

straightforward consequence of our euclidean approach. It would be interesting to investigate whether a Linde/Vilenkin-type analysis of the bubble-of-something creation process can be implemented and shown to produce a positive exponent.

Assuming that (4.6) may be directly compared to the Hartle-Hawking version of (4.7), one can use our initial assumption $\ell_{dS} \gg M_P^2/|T_4|$ to conclude that the Hartle-Hawking process is more likely to occur than a bubble-of-something process. It is straightforward to check this remains true after a more general analysis, relaxing the above assumption and including the effects of dS geometry.

Bubbles of nothing: SUSY breaking in the bulk can lead to the existence of a bubble of nothing with $T_4 < 0$. The scales setting the tension are already discussed above. To estimate the decay rate, we approximate the instanton geometry away from the ETW brane by flat space, in analogy to section 3.5. The decay exponent follows from a calculation similar to (3.38)–(3.40), but its sign is reversed due to the extrinsic curvature now being negative:

$$B = \frac{8\pi^2 M_P^6}{T_4^2} \sim \frac{R_{KK}^8}{\epsilon^2}. \quad (4.8)$$

The rate following from (4.8) should be compared to the rates of other decay channels, such as tunneling to decompactification, flux transitions or the Kachru-Pearson-Verlinde (KPV) [50] process. For traditional dS constructions in string theory [47–49], the decay exponents of tunneling to decompactification and flux transitions are, up to a few orders of magnitude,¹⁰ close to [47, 52, 80]

$$B \sim M_P^2 \ell_{dS}^2, \quad (4.9)$$

with ℓ_{dS} being the de Sitter length. The above becomes more exact the closer the dS vacuum energy approaches zero. The value of B in (4.9) is extremely large, of the order of magnitude needed to saturate the de Sitter lifetime. We thus expect bubble of nothing decays to occur faster. This also follows explicitly since, by comparing to (4.8), we immediately see that if $\ell_{dS}^2 T_4^2 > M_P^4$ bubble of nothing decays are faster. But this condition was already assumed to hold at the beginning of the section, such that it is automatically satisfied for all vacua we are considering at the moment.

If the vacuum contains a Klebanov-Strassler throat [30] with $\overline{D3}$ branes as an uplifting mechanism, the KPV decay process [50] might occur. Generically, this decay happens in the field-theoretic regime and the decay exponent is given by [50, 51]

$$B_{KPV} = \frac{27 (0.932)^6 M^6 g_s}{2048\pi (N_{\overline{D3}})^3}. \quad (4.10)$$

Here, M denotes the flux number on the A-cycle, $N_{\overline{D3}}$ the number of anti-D3 branes and g_s is the string coupling. We see that this B is not parametrically large but only enhanced by a largish integer M needed to avoid a classical instability. The KPV estimate $g_s M^2 \gtrsim 12$ is under debate [81–83]. This instability issue endangers the practicality of the KPV uplift, see e.g. [84–87]. In particular, it has been argued that practical applications must consider KPV

¹⁰For example, in KKLT models the tunneling exponent can be suppressed by factors of the volume.

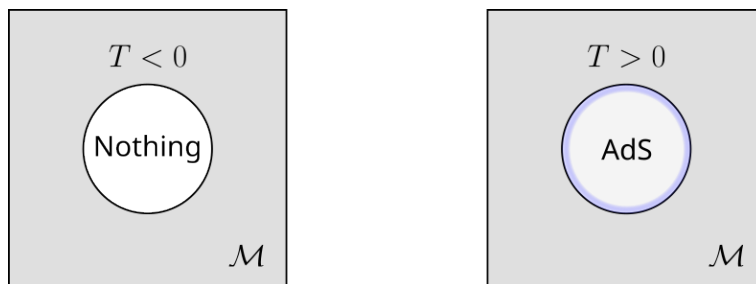


Figure 4. Instantons for a bubble of nothing decay (left) and a decay to AdS (right) of Minkowski space. In the second case, the ball of AdS glued into the flat space $\mathcal{M}$ has most of its contribution to the action localized near the boundary, here illustrated by the blue-shaded region.

with NS5-brane curvature corrections, which change the potential barrier significantly [88, 89]. Remembering that we estimated $|T_4|$ to be parametrically bounded from above by ϵR_{KK}^6 , one may expect our $B \sim R_{KK}^8/\epsilon^2$ to be larger than that of KPV. However, this story is still evolving.

Finally, there remains the possibility of the existence of exotic domain walls with $|\theta| \gg 1$, which can lead to a significant enhancement of the decay rate to nothing.

5 Bubbles of anything — further remarks

In this section, we collect further comments on bubbles of anything and discuss how the creation/decay rates can be computed more precisely in the future.

5.1 Nothing as AdS with infinitely negative vacuum energy

In [7], it was suggested that ‘nothing’ can be modelled as an AdS space in the limit of infinitely negative vacuum energy or, equivalently, infinitely small AdS length ℓ_{AdS} . Indeed, a ball of AdS of radius $R \gg \ell_{\text{AdS}}$ has most of its negative energy localized near the boundary, cf. r.h.s. of figure 4. Hence a brane separating Minkowski or de Sitter space¹¹ and an AdS region (with $\ell_{dS} \gg R \gg \ell_{\text{AdS}}$) may be seen as an effective ETW brane. Its tension receives a correction from the ‘thin’, negative-energy AdS region on one side of the brane. As a result, tunneling between Minkowski/dS and AdS may be a good model for bubble of anything processes. In figure 5, the Penrose diagrams for the decay of Minkowski space through a bubble of nothing and its decay to AdS are shown. The diagrams for the (hypothetical) inverse processes are given in figure 6. Using the analogy between the l.h. and the r.h. sides of figures 4-6 and assuming that up-tunneling from AdS is impossible, one may then claim that the corresponding bubbles of something are forbidden [7]. We want to argue against this conclusion.

To be more precise (cf. appendix D for supporting calculations), consider first tunneling from Minkowski/dS to AdS. As explained above, the instanton is flat space or a large sphere (the euclidean version of the decaying dS), with a small ball of AdS glued in. It is bounded by a brane with positive tension T , cf. figure 4. When evaluating the action of this instanton

¹¹For simplicity, all formulae and the more detailed discussions below are performed only in the $\ell_{dS} \rightarrow \infty$ limit, which is completely sufficient for the qualitative purposes of the present section.

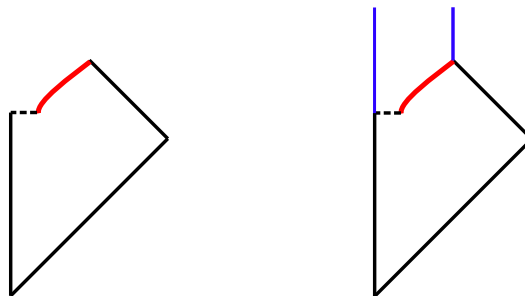


Figure 5. We display the Penrose diagram for a Minkowski bubble of nothing (left) and a Minkowski to AdS decay (right). On the left, the red line denotes the ETW brane. On the right, the blue lines characterize the AdS region and the red line represents the domain wall. In both diagrams, the dashed line denotes the quantum process of the critical bubble emerging. The ETW brane on the left requires a negative tension while the domain wall on the right has positive tension.

configuration, one notices that the AdS on the inner side of the spherical, fundamental brane with tension T contributes an amount $-4\pi^2 R^3/\ell_{\text{AdS}}$, with R being the radius of the bubble. The cubic scaling in R is similar to the contribution from the fundamental brane, which contributes $2\pi^2 T R^3$. This can be understood at the intuitive level by noticing that the AdS has most of its volume localized near the boundary. As a result, the AdS and the brane taken together behave like an ETW brane with total tension $T_{\text{eff}} = T - 2/\ell_{\text{AdS}}$. As shown in appendix D, a nonzero transition rate requires $T_{\text{eff}} < 0$. One then takes the limit $\ell_{\text{AdS}} \rightarrow 0$, $T \rightarrow \infty$ while keeping T_{eff} fixed, ensuring that the critical bubble radius stays finite. Given this renormalization procedure, one finds that the Brown-Dahlen model [7] perfectly describes a decay of dS/Minkowski to nothing.

The suggestion of [7] is that the exponent for up-tunneling from AdS to dS should be calculated using the same instanton, but this time subtracting not a full dS sphere but rather an infinite AdS as background. This clearly leads to a divergence and hence to a vanishing tunneling rate. However, this divergence comes from the infinite AdS volume. This is in conflict with our initial motivation for using AdS as a model for ‘nothing’, which relied on the claim that only a thin slice of AdS contributes to the action. Moreover, this whole approach is not consistent with the standard Coleman-De Luccia procedure [1], where the instanton is constructed by gluing a finite patch of the new vacuum into the background vacuum.

In our opinion, a more appropriate approach to a potential transition $\text{AdS} \rightarrow \text{Minkowski/dS}$ is to construct the instanton following [1]: one glues a small ball of Minkowski or dS into infinite AdS. The subtracted background is also AdS and the divergence from the infinite AdS cancels between instanton and background. Only a thin slice of the background AdS on the inner side of the domain wall of radius R contributes as $+4\pi^2 R^3/\ell_{\text{AdS}}$ to the action. This time the effective ETW brane tension reads $T_{\text{eff}} = T + 2/\ell_{\text{AdS}}$, with the fundamental tension T necessarily being negative. Together with the condition $T_{\text{eff}} > 0$, this ensures the existence of a solution to Einstein’s equations. However, negative-tension branes suffer from instabilities since it is energetically favorable to increase their volume. As a result, it is widely believed that they can not mediate vacuum transitions. This would imply that up-tunneling from AdS is not a good model for bubbles of something. However, we can not completely

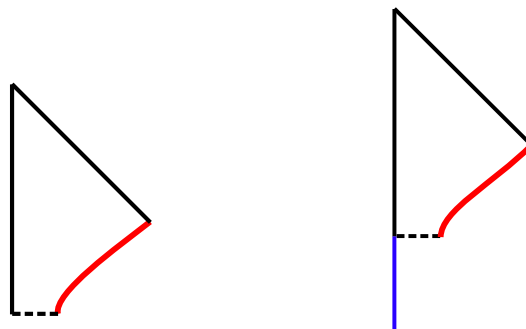


Figure 6. The Penrose diagrams for a Minkowski bubble of something (left) and a hypothetical AdS to Minkowski transition (right) are displayed. The blue region denotes the AdS region and the red lines correspond to the ETW brane (left) and domain wall (right) respectively.

exclude the existence of negative-tension domain walls, especially in the case where one has strongly curved AdS on one of its sides together with a suitably low UV cutoff. If such branes can indeed be constructed, one could complete the analysis above, performing the required ‘renormalization’: taking ℓ_{AdS} to zero, T to $-\infty$ and keeping T_{eff} positive and finite, one would find an AdS-based model for bubbles of something.

Our conclusion is that, at the current level of knowledge, ‘nothing’ can not be universally viewed as AdS in the limit $\ell_{\text{AdS}} \rightarrow 0$: although the model of [7] can describe bubble of nothing decays perfectly, it fails to describe bubble of something creation events. The reason is that in the second case negative-tension domain walls are required and these are in general unstable. While it may be worthwhile to try and overcome this obstacle, the implications for us are in any case clear: either any such efforts fail, then AdS is not a universal model for ‘nothing’. Or they succeed, then bubbles of something can also be recovered from this AdS-based perspective. In both cases we conclude that the Brown-Dahlen approach does *not* exclude bubble-of-something creation events.

A different way of relating deep AdS and ‘nothing’ is discussed in [29].

5.2 Refining the decay/creation rate estimates

Our calculations from sections 3 and 4 provide only leading-order estimates for the decay/creation rates. While we have to leave a more precise treatment to future work, we want to at least comment on the necessary improvements.

5.2.1 Non-trivial potentials

In the present paper, we have focused on compactifications to Minkowski space with vanishing potential for the volume modulus. Clearly, in any realistic compactification, a stabilizing potential must exist. Nevertheless, the general strategy for finding bubble of nothing/something solutions stays similar to what is discussed in sections 2 and 3. However, it now becomes a non-trivial task to solve the equations of motion (2.12), (2.13), subject to the boundary conditions (3.24), (3.26). The calculation of the extrinsic curvature, its relation to the tension T_4 (3.16) and its contribution to the tunneling action are unchanged. In addition to the boundary contribution to B (2.3), there will now also be a bulk term. For both the instanton

and the vacuum solution, this contribution to the action reads (3.11)

$$S_{bulk} = -2\pi^2 \int f^3 V(\phi) dr. \quad (5.1)$$

This has to be evaluated using the profile $\phi(r)$ obtained by solving the equations of motion. For some analytic and numerical estimates in 5d bubble of nothing models see [10–12].

Allowing for non-trivial potentials includes the generalization to situations where the decaying/created vacuum is dS or AdS. The resulting modifications are straightforward. Moreover, if $\ell_{dS/AdS}$ is large compared to the critical bubble radius, we expect only small corrections relative to the results derived in the previous sections. If the considered vacuum is dS, one can not distinguish between a bubble of nothing and a bubble of something instanton. Any solution can be interpreted in both ways, which is a result of euclidean dS space being compact. This is analogous to Coleman-de Luccia instantons for dS $\rightarrow$ dS tunneling where the same instanton can be viewed as an up- or down-tunneling event and only the difference in the vacuum contribution to the decay exponent (2.3) leads to a suppressed up-tunneling rate.

In models where ϕ stays approximately constant away from an ETW brane with tension T_4 , cf. section 3.5, we can directly calculate the dS and AdS decay/creation exponents.

de Sitter. For the dS case, similar calculations in a slightly different setting have appeared in [28]. The ETW brane sits at some radius $f_\eta \leq \ell_{dS}$ of the euclidean dS sphere. The extrinsic curvature and its relation to the tension reads

$$\frac{3}{2}T_4 = M_P^2 \mathcal{K}_4 = \pm \frac{3M_P^2}{f_\eta} \sqrt{1 - \frac{f_\eta^2}{\ell_{dS}^2}}, \quad (5.2)$$

where the positive sign corresponds to a dS patch of size less than a hemisphere and the negative sign to a dS patch of size larger than a hemisphere. The instanton action and thus the creation exponent is given by

$$S_{\text{instanton}} = 4\pi^2 M_P^2 \ell_{dS}^2 \left(\sqrt{\frac{T_4^2 \ell_{dS}^2}{T_4^2 \ell_{dS}^2 + 4M_P^4}} - 1 \right) \quad (5.3)$$

if the tension is positive and

$$S_{\text{instanton}} = -8\pi^2 M_P^2 \ell_{dS}^2 - 4\pi^2 M_P^2 \ell_{dS}^2 \left(\sqrt{\frac{T_4^2 \ell_{dS}^2}{T_4^2 \ell_{dS}^2 + 4M_P^4}} - 1 \right) \quad (5.4)$$

if the tension is negative. To obtain the decay exponent, one needs to subtract the vacuum action $S_{\text{vacuum}} = -8\pi^2 M_P^2 \ell_{dS}^2$ from $S_{\text{instanton}}$.

Let us emphasize a key differences between a Hartle-Hawking [21] or Linde/Vilenkin [22, 23] process and a bubble of something event. The corresponding Penrose diagrams are shown in figure 7. While a Hartle-Hawking or Linde/Vilenkin process nucleates a closed universe (a sphere) with its initial size necessarily being the dS radius, a bubble of something produces a ball bounded by an ETW brane. Hence, the critical bubble radius depends on the brane tension and can in particular be much smaller than the dS radius.

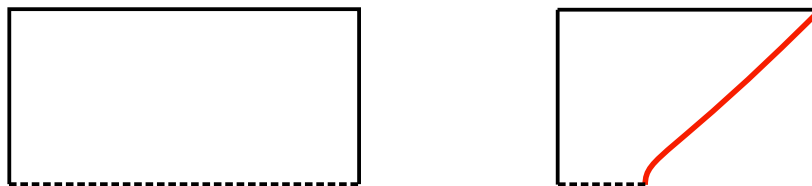


Figure 7. Penrose diagrams of a Hartle-Hawking or Linde/Vilenkin process (left) and of a bubble of something leading to dS (right). The first is simply half a dS space, while the latter contains an ETW brane boundary (red line). The dashed lines indicate the initial universe created from nothing via a quantum process.

Anti de Sitter. Here, similar to Minkowski space, a bubble of nothing instanton is a non-compact patch of AdS with a hole of radius f_η meanwhile a bubble of something instanton is a compact patch of AdS of radius f_η . The former requires a negative ETW brane tension and the latter a positive one. The relation between the extrinsic curvature and the tension reads

$$\frac{3}{2}T_4 = M_P^2 \mathcal{K}_4 = \pm \frac{3M_P^2}{f_\eta} \sqrt{1 + \frac{f_\eta^2}{\ell_{\text{AdS}}^2}}, \quad (5.5)$$

with the upper/lower sign appropriate for a bubble of something/nothing. One immediately sees that $|T_4| > 2M_P^2/\ell_{\text{AdS}}$ is a necessary requirement to have any bubble. The creation/decay exponents are then given by

$$B_{\text{creation}} = -4\pi^2 M_P^2 \ell_{\text{AdS}}^2 \left(\sqrt{\frac{T_4^2 \ell_{\text{AdS}}^2}{T_4^2 \ell_{\text{AdS}}^2 - 4M_P^4}} - 1 \right), \quad (5.6)$$

$$B_{\text{decay}} = 4\pi^2 M_P^2 \ell_{\text{AdS}}^2 \left(\sqrt{\frac{T_4^2 \ell_{\text{AdS}}^2}{T_4^2 \ell_{\text{AdS}}^2 - 4M_P^4}} - 1 \right). \quad (5.7)$$

Using the above results, one can verify that holding fix a tension T_4 , bubbles of nothing/something become more probable if the vacuum energy increases/decreases.

5.2.2 Fluxes

If the CY compactification involves fluxes, one generally requires charged ETW branes. Such ETW branes may be constructed by noting that brane flux transitions are possible [50], realized by wrapped branes forming a domain wall in the non-compact directions. Bubble of nothing decays can then occur as a ‘double process’, where the removal of the flux by a brane is followed by a decay to nothing along the lines of sections 3 and 4. For a decay rate estimate in toy models, see [12]. We may think of our desired charged domain wall as being realized if the flux-removing brane is moved on top of the ETW brane. As already noted in section 3.4, our formalism of section 3 still applies with appropriate values of θ and η , which of course first have to be derived.

5.2.3 Other fields

In an exhaustive treatment of bubbles of anything, all fields, such as moduli fields, gauge fields and the dilaton have to be taken into account. If those fields are only weakly stabilized, they can contribute significantly to the decay rate, as can be seen from [18]. Here, in the

outermost layer, two fields are dynamical, namely the volume modulus and the dilaton. Both fields obtain the same profile such that the equations of motion change to a form that resemble the ones from the Witten bubble of nothing analyzed in section 3.1. In the scenario of [18], the presence of the dilaton increases the decay rate when comparing to the $d = 7$ estimate obtained in appendix B. One can also imagine situations where additional fields lead to a decrease of the decay rate.

5.2.4 Higher-order corrections

The supergravity action obtained as the low-energy limit of string theory receives corrections at higher orders in the α' expansion. Such corrections typically become important at small distance scales which in principle can induce additional effects in the region where the internal manifold has shrunk to small sizes. It would be interesting to investigate the consequences of the higher-order corrections on the instanton geometry and the decay/creation rates.

6 Conclusions

Our main results in the present paper concern the theoretical development and implementation of ‘bubble of anything’ processes in the string theory landscape, making them potentially relevant to cosmology.

First, we formulated a 4d EFT description for Witten’s bubble of nothing based on S^1 compactifications. In doing so, we also allowed for a defect with variable tension, sitting at the point where the S^1 disappears. Crucially, this defect has non-zero size η and induces a deficit angle θ . This deficit angle measures the tension of the defect and contributes to the effective 4d tension T_4 of the boundary. In such a more general setting, both bubble of nothing and bubble of something instantons exist. The latter require a non-trivial potential for the scalar governing the S^1 radius as well as a positive effective ETW brane tension T_4 .

We have demonstrated that the results above may be generalized to Calabi-Yau orientifold compactifications, again with the assumption that the compact manifold shrinks to zero size at the boundary of spacetime. Since the microscopic structure of Calabi-Yau (CY) bordisms to nothing is generally unknown, the introduction of an appropriate defect of size η and with a (suitably generalized) deficit angle θ is in this case mandatory. While it may be natural to assume the defect size to be of higher-dimensional Planck or string scale, it can in principle also be much larger. We provide general formulae for the actions of Calabi-Yau bubbles of anything of this type, which turn out to be parametrically larger than for the Witten bubble. These actions allow for estimates of the decay or creation rates of corresponding 4d vacua.

While we feel that it is well-motivated by the Cobordism Conjecture, the ad-hoc introduction of a defect realizing the bordism of a Calabi-Yau orientifold to nothing is certainly unsatisfactory. We therefore continued our analysis by also proposing a construction of an ETW brane for type IIB Calabi-Yau orientifold models with O3/O7 planes. Our construction employs an O5 orientifolding of the type IIB compactification, involving a reflection in one of the non-compact spatial dimensions. The O5/D5 system wraps the bubble surface and a three cycle of the CY. The resulting ETW brane preserves two supercharges, i.e. $\mathcal{N} = 1$ supersymmetry in 3d. Naively, we then expect such an ETW brane to be either stationary or contracting, allowing neither for a bubble of nothing nor of something. However, once we

include moduli stabilization and SUSY breaking in the bulk, as well as (possibly) additional SUSY breaking on the boundary, this will certainly change. One may generically expect a combination of the KK- and SUSY breaking scale to set the brane tension. In case a bubble of nothing is allowed, one can then compare our decay rates (with decay exponent (4.8)) with the rates of traditional vacuum decay processes, e.g. the decay to decompactification, flux transitions or the Kachru-Pearson-Verlinde (KPV) decay process. It turns out that only the field-theoretic decay of KPV is likely to be faster.

For bubbles of something, the nucleation exponent is given in (5.3), (5.4) and a naive estimate for our setting in (4.6). Bubbles of something may be viewed as an alternative to the Hartle-Hawking no-boundary creation process. The form of the nucleation exponents differs significantly between these two vacuum creation processes: in one case, the key input are the ETW brane tension and 4d vacuum energy, in the other case it is only the 4d vacuum energy. One may ask whether the relative importance of the two creation processes for the landscape can be quantified by directly comparing the respective nucleation exponents.

It would be desirable to analyse the effects of the dilaton, fluxes, vacuum energy and non-perturbative effects in more detail. We also leave a careful study of SUSY-breaking and supersymmetric corrections to the domain wall tension to future work. Other open questions are whether bubbles of something can be interpreted using the Linde-Vilenkin approach to vacuum creation [22, 23] and whether they can be studied in a Lorentzian approach [90].

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A Dimensional reduction

In the following, we present the dimensional reduction from d -dimensions to four dimensions starting from the metric ansatz (2.10)

$$\begin{aligned} ds^2 &= e^{2\alpha\phi(r)} \left(dr^2 + f(r)^2 d\Omega_3^2 \right) + e^{2\beta\phi(r)} ds_n^2 \\ &= e^{2\alpha\phi} \left(ds_4^2 + e^{2(\beta-\alpha)\phi} ds_n^2 \right) = e^{2\alpha\phi} d\tilde{s}^2. \end{aligned} \tag{A.1}$$

We set $M_d = 1$ and normalize ds_n^2 (and hence the compact-space metric) by

$$\int \sqrt{g_n} d^n x = 1, \tag{A.2}$$

such that the 4d Planck mass is unity as well.

Einstein-Hilbert action: when dimensionally reducing the d -dimensional Einstein-Hilbert action, it is convenient to employ the metric $d\tilde{s}^2$ introduced in (A.1). Using the transformation properties of the Ricci scalar under Weyl rescalings, the action takes the form

$$S = -\frac{1}{2} \int \sqrt{g_n} \sqrt{g_4} e^{d\alpha\phi} e^{n(\beta-\alpha)\phi} e^{-2\alpha\phi} \left(\tilde{\mathcal{R}} - 2(d-1)\alpha \tilde{\square}\phi - (d-2)(d-1)\alpha^2 (\partial\phi)^2 \right) \quad (\text{A.3})$$

$$= -\frac{1}{2} \int \sqrt{g_4} e^{[2\alpha+n\beta]\phi} \left(\tilde{\mathcal{R}} - 2(d-1)\alpha \square_4\phi - \left[(d-2)(d-1)\alpha^2 + 2(d-1)n(\beta-\alpha)\alpha \right] (\partial\phi)^2 \right), \quad (\text{A.4})$$

where $\tilde{\square}_4$ is the Laplace operator associated with $d\tilde{s}_4^2$ and $\square_4$ is the Laplace operator associated with ds_4^2 . Further, we have used

$$\tilde{\square}\phi = \frac{1}{f^3 e^{n(\beta-\alpha)\phi}} \partial_r (f^3 e^{n(\beta-\alpha)\phi} \phi') = \frac{1}{f^3} \partial_r (f^3 \phi') + n(\beta-\alpha) (\partial\phi)^2 = \square_4\phi + n(\beta-\alpha) (\partial\phi)^2. \quad (\text{A.5})$$

We proceed by calculating $\tilde{\mathcal{R}}$, the Ricci scalar of the metric $d\tilde{s}^2$. To do this, we first write the metric in the form

$$d\tilde{s}^2 = e^{2(\beta-\alpha)\phi} \left(dz^2 + g(r)^2 d\Omega_3^2 + ds_n^2 \right), \quad (\text{A.6})$$

where we defined $e^{(\beta-\alpha)\phi} dz = dr$ and $g(r) e^{(\beta-\alpha)\phi(r)} = f(r)$. Using twice the transformation properties of the Ricci scalar under conformal transformations one arrives at the result

$$\tilde{\mathcal{R}} = \mathcal{R}_4 - 2n(\beta-\alpha)\square_4\phi - n(n+1)(\beta-\alpha)^2 (\partial\phi)^2 + \mathcal{R}_n e^{-2(\beta-\alpha)\phi}, \quad (\text{A.7})$$

with $\mathcal{R}_4$ denoting the Ricci scalar of the metric ds_4^2 and $\mathcal{R}_n$ the Ricci scalar of the metric ds_n^2 . For completeness, we note that

$$\mathcal{R}_4 = 6 \left(\frac{1}{f^2} - \frac{f'^2}{f^2} - \frac{f''}{f} \right). \quad (\text{A.8})$$

From (A.4) one observes that setting

$$\beta = -2\alpha/n = -2\alpha/(d-4) \quad (\text{A.9})$$

leads to a 4d Einstein frame action (recall that $d = n + 4$)

$$S = \int \sqrt{g_4} \left(-\frac{1}{2} \mathcal{R}_4 + \alpha \square_4\phi + \frac{2(n+2)}{n} \alpha^2 \frac{(\partial\phi)^2}{2} - \frac{\mathcal{R}_n}{2} e^{(\frac{4}{n}+2)\alpha\phi} \right). \quad (\text{A.10})$$

Finally, one can choose α such that the scalar field is canonically normalized:

$$\alpha^2 = \frac{n}{2(n+2)}. \quad (\text{A.11})$$

Gibbons-Hawking-York boundary term: we continue with the reduction of the Gibbons-Hawking-York boundary term from d to four dimensions, assuming that the boundary is a surface defined by $r = \text{const}$. We can directly calculate

$$S_{GHY} = - \int_{\Sigma} \sqrt{h} \mathcal{K}_d = - \int_{\Sigma} \sqrt{g_n} e^{3\alpha\phi} e^{n\beta\phi} f^3 \mathcal{K}_d, \quad (\text{A.12})$$

with h the induced metric on the $(d-1)$ -dimensional boundary. To calculate the d -dimensional extrinsic curvature $\mathcal{K}_d$, we note that the outward pointing normal vector is $n = e^{-\alpha\phi} \partial_r$. We further use $3\alpha + n\beta = \alpha$ from (A.9), resulting in

$$\mathcal{K}_d = \frac{1}{e^{(4\alpha+n\beta)\phi} f^3} \partial_r \left(e^{(3\alpha+n\beta)\phi} f^3 \right) = e^{-\alpha\phi} \left(\alpha\phi' + 3 \frac{f'}{f} \right), \quad (\text{A.13})$$

and hence

$$- \int_{\Sigma} \sqrt{h} \mathcal{K}_d = - \int f^3 \alpha\phi' - \int f^3 \mathcal{K}_4. \quad (\text{A.14})$$

One now observes that the term $\alpha \square_4 \phi$ in (A.10) is a total derivative and contributes to the action as

$$\alpha f^3 \phi' \Big|_{r=0}^{r=\infty}. \quad (\text{A.15})$$

Hence, for Minkowski compactifications, the first term on the r.h.s. of (A.14) precisely cancels the contribution from the $r = \infty$ term of (A.15).

Since, from the 4d perspective, the bubble wall (for bubbles of anything) is another boundary of spacetime, it appears logical to also include a corresponding GHY term in the four-dimensional action. In addition, the bubble wall has an effective 4d tension, such that we supplement our total action with a term

$$- \int_{\partial\Sigma} \sqrt{g_4} (\mathcal{K}_4 - T_4) = -2\pi^3 f^3 (\mathcal{K}_4 - T_4) \quad (\text{A.16})$$

on the boundary corresponding to the bubble wall. At the same time, we remove the term $\alpha \square_4 \phi$ from the bulk action since we take its remaining effect at the bubble wall from (A.15) into account in the new boundary term (A.16).

B Bubble of nothing decays in general dimensions

In this section, we collect the relevant equations to analyze bubble of nothing decays in general dimensions.

For general d , Equations (2.14) and (2.16) are still valid, just the values of α and β change according to (2.7). Demanding that the geometry ends when the extra dimensions reach critical size η results in the defining equation

$$2\pi\eta = 2\pi R(f_\eta) = e^{\beta\phi(f_\eta)} \quad (\text{B.1})$$

for f_η . Equation (B.1) simplifies to

$$\frac{2C}{\sqrt{6}f_\eta^2} = \left(\frac{R_{KK}}{\eta} \right)^{\frac{2}{\sqrt{6}\beta}} - \left(\frac{R_{KK}}{\eta} \right)^{-\frac{2}{\sqrt{6}\beta}}. \quad (\text{B.2})$$

Unless η is extremely close to R_{KK} , we can neglect either the first or the second term in the equation above. In the main text, $\eta/R_{KK} < 1$ was assumed to improve readability of formulae. In the following, we will present the general equations.

Introducing the (analogue of) the deficit angle θ as in (3.26)

$$1 - \frac{\theta}{2\pi} = \frac{dR}{dr} e^{-\alpha\phi} \Big|_{r=0} = \frac{\beta}{2\pi} \frac{C}{f_\eta^3} (2\pi\eta)^{1+\frac{n}{2}} \quad (\text{B.3})$$

and solving for C by using (B.2) results in

$$C = \frac{\beta^2}{(2\pi)^2} \frac{3\sqrt{6}}{4 \left(1 - \frac{\theta}{2\pi}\right)^2} (2\pi\eta)^{n+2} \left[\left(\frac{R_{KK}}{\eta}\right)^{\frac{2}{\sqrt{6}\beta}} - \left(\frac{R_{KK}}{\eta}\right)^{-\frac{2}{\sqrt{6}\beta}} \right]^3. \quad (\text{B.4})$$

The extrinsic curvature in four dimensions can now be calculated to be

$$\mathcal{K}_4 = -\sqrt{6} \frac{2\pi \left|1 - \frac{\theta}{2\pi}\right|}{\beta} (2\pi\eta)^{-1-\frac{n}{2}} \sqrt{\left[\left(\frac{R_{KK}}{\eta}\right)^{\frac{2}{\sqrt{6}\beta}} - \left(\frac{R_{KK}}{\eta}\right)^{-\frac{2}{\sqrt{6}\beta}} \right]^{-2} + \frac{1}{4}}. \quad (\text{B.5})$$

Interestingly, $\mathcal{K}_4$ does not depend on R_{KK} , unless $\eta \approx R_{KK}$. This provides evidence to the claim that bubbles of anything are local quantum effects.

The critical bubble radius ρ_0 is computed to be

$$\rho_0 = \sqrt{\frac{3}{2}} \frac{\beta}{\left|1 - \frac{\theta}{2\pi}\right|} \eta \left| \left(\frac{R_{KK}}{\eta}\right)^{\frac{2}{\sqrt{6}\beta}} - \left(\frac{R_{KK}}{\eta}\right)^{-\frac{2}{\sqrt{6}\beta}} \right|. \quad (\text{B.6})$$

Finally, the decay exponent is given by

$$\begin{aligned} B &= -2\pi^2 f_\eta^3 \frac{\mathcal{K}_4}{3} \\ &= \frac{3}{4} \frac{\beta^2}{\left(1 - \frac{\theta}{2\pi}\right)^2} (2\pi\eta)^{n+2} \sqrt{\left[\left(\frac{R_{KK}}{\eta}\right)^{\frac{2}{\sqrt{6}\beta}} - \left(\frac{R_{KK}}{\eta}\right)^{-\frac{2}{\sqrt{6}\beta}} \right]^4 + \frac{1}{4} \left[\left(\frac{R_{KK}}{\eta}\right)^{\frac{2}{\sqrt{6}\beta}} - \left(\frac{R_{KK}}{\eta}\right)^{-\frac{2}{\sqrt{6}\beta}} \right]^6}. \end{aligned} \quad (\text{B.7})$$

Inspecting the profile of the radius $\rho(f)$ of the transversal S^3

$$\rho(f) = R_{KK}^{-\frac{n}{2}} \left(\frac{C}{\sqrt{6}f^2} + \sqrt{\left(\frac{C}{\sqrt{6}f^2} \right)^2 + 1} \right)^{-\alpha \frac{\sqrt{6}}{2}} f \quad (\text{B.8})$$

shows that for large f

$$\rho(f) \approx R_{KK}^{-\frac{n}{2}} f, \quad (\text{B.9})$$

and hence ρ grows with growing f . On the other hand, if $f_\eta^2 \ll C$, there will be the regime where

$$\rho(f) \approx \begin{cases} R_{KK}^{-\frac{n}{2}} \left(\frac{2C}{\sqrt{6}}\right)^{-\alpha\sqrt{6}/2} f^{1+\alpha\sqrt{6}} & C > 0, \\ R_{KK}^{-\frac{n}{2}} \left(\frac{\sqrt{6}}{2C}\right)^{-\alpha\sqrt{6}/2} f^{1-\alpha\sqrt{6}} & C < 0. \end{cases} \quad (\text{B.10})$$

It now depends on the value of α whether ρ grows or shrinks with growing f . We first note that $\alpha < 0$ for all values of d , such that ρ grows with growing f if $C < 0$. Let us now analyze the case $C > 0$: if, $d = 5$, $\alpha = -\frac{1}{\sqrt{6}}$ and hence ρ stays approximately constant. Since $\alpha < -1/\sqrt{6}$ for $d > 5$, one observes that ρ shrinks with growing f near the bubble wall if $C > 0$. Such a peculiar feature has already been observed in [20].

C Supersymmetry and the ETW brane

In this appendix, we show that a spacetime filling O3/D3 system together with a O5/D5 domain wall wrapped on a special Lagrangian three-cycle of a Calabi-Yau manifold preserves two supercharges. We mostly follow the ideas of [91, 92]. We further argue that this situation also holds for O5/D5, O7/D7 models. Since Op planes preserve the same supercharges as Dp branes, we can equivalently think of D-brane systems only. The two SUSY generators of type II theories will be denoted by $\epsilon, \tilde{\epsilon}$. Then, the condition that a brane preserves supersymmetry is given by

$$\tilde{\epsilon} = \Gamma_{\parallel}^{Dp} \epsilon, \quad \Gamma_{\parallel}^{Dp} = \frac{1}{\sqrt{g|_{Dp}}} \prod_i \Gamma_i, \quad (\text{C.1})$$

where the product is over all gamma matrices in directions parallel to the Dp brane. After compactifying on a Calabi-Yau, the SUSY generators for type IIB string theory can be parametrized as

$$\epsilon = \zeta_+ \otimes \eta_+ + \zeta_- \otimes \eta_- , \quad (\text{C.2})$$

$$\tilde{\epsilon} = \tilde{\zeta}_+ \otimes \eta_+ + \tilde{\zeta}_- \otimes \eta_- . \quad (\text{C.3})$$

Here, $\zeta_+, \tilde{\zeta}_+$ are independent four-dimensional spinors of positive chirality and η_+ is the unique six-dimensional covariantly constant positive-chirality spinor of the Calabi-Yau. Further, $\zeta_- = (\zeta_+)^c, \tilde{\zeta}_- = (\tilde{\zeta}_+)^c, \eta_- = (\eta_+)^c$. The eight real degrees of freedom of $\zeta_+, \tilde{\zeta}_+$ lead to the familiar $\mathcal{N} = 2$ theory in 4d. Using similar arguments as in [92] chapter 9.2 and adopting the same conventions, one finds that the condition

$$\tilde{\epsilon} = \Gamma_{\parallel}^{D3} \epsilon, \quad (\text{C.4})$$

where the D3 brane is spacetime filling, results in the relation

$$\tilde{\zeta}_+ = i\zeta_+ . \quad (\text{C.5})$$

As a result, the preserved supercharges are reduced from eight to four. The second condition

$$\tilde{\epsilon} = \Gamma_{\parallel}^{D5} \epsilon, \quad (\text{C.6})$$

with the D5 brane being extended in the 0, 1, 2 directions of the non-compact space and wrapping a three-cycle S_3 of the CY, implies

$$\zeta_- = \beta\gamma_3\zeta_+, \quad \eta_- = \beta^{-1}\gamma_{\parallel}\eta_+, \quad (\text{C.7})$$

with β a constant to be determined. Here, following [92] chapter 9.2 and 5.6, γ_3 is to be seen as a 4d gamma matrix and $\gamma_{\parallel}$ is the product of gamma matrices along the directions of the three-cycle S_3 . Since $\gamma_{\parallel}$ is unitary, the second condition in (C.7) determines $|\beta| = 1$. Hence, $\beta = e^{i\phi}$. It is now straightforward to see that the first condition of (C.7) always has a solution. For example, in the chiral basis, we have the following representation of γ_3 and a general spinor ζ_+, ζ_-

$$\gamma_3 = i \begin{pmatrix} 0 & -\sigma^3 \\ \sigma^3 & 0 \end{pmatrix}, \quad \zeta_+ = \begin{pmatrix} a \\ b \\ 0 \\ 0 \end{pmatrix}, \quad \zeta_- = \begin{pmatrix} 0 \\ 0 \\ -\bar{b} \\ \bar{a} \end{pmatrix}, \quad a, b \in \mathbb{C}. \quad (\text{C.8})$$

The first condition of (C.7) then becomes

$$\begin{pmatrix} 0 \\ 0 \\ -\bar{b} \\ \bar{a} \end{pmatrix} = e^{i\phi} \begin{pmatrix} 0 \\ 0 \\ ia \\ -ib \end{pmatrix}. \quad (\text{C.9})$$

We see that this system can be solved for any ϕ by choosing $a = ie^{-i\phi}\bar{b}$. This condition further reduces the number of preserved supercharges from four to two, resulting in $\mathcal{N} = 1$ supersymmetry in 3d. Finally, we need to check whether the second condition of (C.7) is fulfilled. Following the logic of [92] chapter 5.6, 9.2 and adopting it to our case of interest, one finds that $\eta_- = e^{-i\phi}\gamma_{\parallel}\eta_+$ is obeyed when the cycle S_3 is calibrated with respect to the form $Re(e^{i\theta}\Omega)$

$$Re(e^{i\phi}\Omega)|_{S_3} = \text{vol}_{S_3}. \quad (\text{C.10})$$

The condition (C.10) precisely states that the three cycle S_3 needs to be special Lagrangian. We conclude that our D3/D5 system preserves $\mathcal{N} = 1$ SUSY in 3d if the D5 brane is wrapped on a special Lagrangian cycle. Since D7 branes wrapped on holomorphic four-cycles of the CY preserve the same supercharges as D3 branes, our results straightforwardly generalize to such D7/D5 models. This can further be seen by again using techniques from [92] chapter 9 and by noting that the condition

$$\tilde{\epsilon} = \Gamma_{\parallel}^{D7}\epsilon, \quad (\text{C.11})$$

with the D7 wrapped on a holomorphic cycle, also leads to (C.5). Then, from the arguments below (C.5), it is clear that two supercharges are preserved if the D5 brane is wrapped on a special Lagrangian cycle of the CY.

D AdS bubble transitions in the limit of small AdS length

In this appendix we discuss whether and how tunneling to/from a deep AdS may be related to tunneling to/from ‘nothing’. This analogy has been suggested and analysed in [7], but we disagree with a key detail which is decisive in our context.

We start by considering thin-wall tunneling instantons for the decay of Minkowski space to AdS as a possible model for bubbles of nothing. We denote the bubble radius by R , the AdS length by ℓ_{AdS} , the matter tension by T and we work in 4d Planck units. The tunneling exponent prior to radius extremization is given by (see e.g. [93])

$$B_{\text{Mink.} \rightarrow \text{AdS}} = 6\pi^2 R^2 + 2\pi^2 T R^3 - 4\pi^2 \ell_{\text{AdS}}^2 \left(\left(1 + \frac{R^2}{\ell_{\text{AdS}}^2} \right)^{3/2} - 1 \right). \quad (\text{D.1})$$

The instanton corresponds to a patch of AdS of radius R glued into Minkowski space. The first term on the r.h.s. of (D.1) represents the extrinsic curvature contribution at the bubble wall on the Minkowski side, the second term is the effect of the wall tension, and the third term combines both the AdS bulk and the AdS-side extrinsic curvature contributions. The extrinsic curvature term at infinity is cancelled by the Minkowski vacuum contribution. In the limit $R \gg \ell_{\text{AdS}}$ the last term in (D.1) reduces to $2\pi^2(-2/\ell_{\text{AdS}})R^3$. As a result, the total effect of the AdS side is to correct the brane tension by $-2/\ell_{\text{AdS}}$. Thus, at $R \gg \ell_{\text{AdS}}$ one has

$$B_{\text{Mink.} \rightarrow \text{AdS}} = 6\pi^2 R^2 + 2\pi^2 T_{\text{eff}} R^3, \quad (\text{D.2})$$

with $T_{\text{eff}} = T - 2/\ell_{\text{AdS}}$. One immediately sees that this is the exponent for a bubble-of-nothing tunneling process in flat space.

The action (D.1) has an extremum only for $T_{\text{eff}} < 0$, in which case the critical bubble radius reads

$$R_{\text{Mink.} \rightarrow \text{AdS}} = \frac{4T}{\frac{4}{\ell_{\text{AdS}}^2} - T^2} = -\frac{4 \left(\frac{T_{\text{eff}} \ell_{\text{AdS}}}{2} + 1 \right)}{\frac{\ell_{\text{AdS}} T_{\text{eff}}^2}{2} + 2T_{\text{eff}}}. \quad (\text{D.3})$$

In the limit $\ell_{\text{AdS}} \rightarrow 0$, one then obtains

$$R_{\text{Mink.} \rightarrow \text{AdS}} = -\frac{2}{T_{\text{eff}}}. \quad (\text{D.4})$$

This could have equivalently been found by extremizing (D.2). We see that tunneling becomes possible if the effective tension is negative. This effective tension may be thought of as a physical quantity by taking the renormalization-type limit $\ell_{\text{AdS}} \rightarrow 0$, $T \rightarrow \infty$ while keeping T_{eff} finite.

For the bubble of something, the corresponding AdS $\rightarrow$ Minkowski tunneling exponent takes the form

$$B_{\text{AdS} \rightarrow \text{Mink.}} = -6\pi^2 R^2 + 2\pi^2 T R^3 + 4\pi^2 \ell_{\text{AdS}}^2 \left(\left(1 + \frac{R^2}{\ell_{\text{AdS}}^2} \right)^{3/2} - 1 \right). \quad (\text{D.5})$$

The instanton geometry is a patch of Minkowski space of radius R glued into AdS, which in contrast to [7] we take to be infinite. Extremizing the expression in (D.5) gives

$$R_{\text{AdS} \rightarrow \text{Mink.}} = \frac{4T}{T^2 - \frac{4}{\ell_{\text{AdS}}^2}} \quad (\text{D.6})$$

for $T < 0$ and no extremum otherwise. Moreover, demanding a positive bubble radius one finds the additional requirement $T_{\text{eff}} \equiv T + 2/\ell_{\text{AdS}} > 0$. In the limit $\ell_{\text{AdS}} \rightarrow 0$, the tunneling exponent (D.5) becomes

$$B_{\text{AdS} \rightarrow \text{Mink.}} = -6\pi^2 R^2 + 2\pi^2 T_{\text{eff}} R^3. \quad (\text{D.7})$$

It is extremized by

$$R_{\text{AdS} \rightarrow \text{Mink.}} = \frac{2}{T_{\text{eff}}}, \quad (\text{D.8})$$

which is also the $\ell_{\text{AdS}} \rightarrow 0$ limit of (D.6) if T_{eff} is kept finite. We see that the limit $\ell_{\text{AdS}} \rightarrow 0$ can only be sensibly taken when the ‘bare tension’ T is negative and diverges. While negative-tension ETW-branes are known to exist, it is not clear to us whether the negative tension brane between AdS and Minkowski required above can be realized in string theory or even makes sense more generally, in an EFT.

Our key conclusion for the present paper is that bubbles of something are not ruled out since, if negative tension branes are forbidden, then up-tunneling from AdS is not a good model for bubbles of something. If, on the contrary, negative-tension branes are allowed, then up-tunneling from AdS reproduces the simpler bubble-of-something results.

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4 The Boundary Proposal

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Declaration of authorship

The idea for this research project was developed by me and Arthur Hebecker. I performed all calculations and wrote the entirety of the draft with the exception of one paragraph in the conclusions. Slight modifications were conducted by Arthur Hebecker.



Letter

The boundary proposal

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ABSTRACT

One of the leading ideas for the beginning of the Universe is the Hartle-Hawking ‘No-Boundary Proposal.’ Since the Cobordism Conjecture claims that any spacetime allows for a dynamical boundary, we suggest that one may equally well consider a ‘Boundary Proposal’. Specifically, the corresponding euclidean instanton is a sphere with two holes around north and south pole cut out. Analogously to the Hartle-Hawking proposal, the sphere is then cut in two at the equator and half of it is dropped. The equator is glued to an expanding Lorentzian de Sitter space, implementing a beginning of the Universe with a spacelike spherical boundary at its earliest moment. This process is in principle on equal footing with the one based on the no-boundary instanton. In fact, if the Linde-Vilenkin sign choice is used, this ‘Boundary’ creation process may even dominate. An intriguing implication arises if tensionless end-of-the-world branes, as familiar from type-IIA or M-theory, are available: Analogously to the Boundary Proposal, one may then be able to create a compact, flat torus universe from nothing, without any exponential suppression or enhancement factors.

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1. Introduction

The idea of vacuum creation from nothing has a long history: Vilenkin, Hartle-Hawking and Linde have proposed that the creation rate of de Sitter universes can be obtained by studying appropriate gravitational instantons [1–5] (see [6,7] for reviews). In fact, the relevant instanton is simply euclidean dS space, i.e. a sphere. Alternatively, one may start from the Lorentzian path integral [8,9]. The relevant instantons then appear in the analytic continuation as those saddle points which give the dominant contribution (see [10,11] for a recent review

and critical discussions). It was furthermore observed that, if end-of-the-world (ETW) branes exist, universes with boundaries may be created from nothing. The corresponding instanton, first introduced in [12,13] and further studied in [14–17], has the topology of a ball with a spherical ETW brane as its boundary. The geometry of the creation process arises by cutting this ball in two and gluing one of its halves to an expanding Lorentzian universe with a cylindrical boundary.

In this work, we propose a new type of creation process in which a boundary-free, spherical, de Sitter universe appears from nothing thanks to an initial, space-like boundary, provided by an ETW brane. The corre-

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sponding instanton may be thought of as sphere with two holes cut out. This results in two spherical boundaries. The size of the holes follows from the ETW-brane tension and the de Sitter cosmological constant. To interpret this instanton as part of the complete geometry, it is cut in two and one half is glued to an expanding, boundary-free spherical universe, precisely as in the Hartle-Hawking case. The resulting spacetime has one boundary, provided by one of the two holes discussed above (for an illustration see Table 1). This boundary may be viewed as an initial surface for the universe. Hence the name ‘Boundary Proposal’, as opposed to ‘No Boundary Proposal’, appears fitting.

Importantly, the instanton action approaches zero in the limit of vanishing ETW brane tension. In this situation, the two holes discussed above eat up almost the whole sphere, such that only a thin strip around the equator is left. As a result, there is no tunneling suppression and our one-boundary creation process dominates over the no-boundary process for the Linde-Vilenkin sign choice in the instanton exponent.

2. A new instanton for the creation of universes

2.1. Instanton geometries and actions

We consider the euclidean action

$$S = -\frac{M_P^2}{2} \int_{\mathcal{M}} d^4x \sqrt{g} (\mathcal{R} - 2\Lambda) - M_P^2 \int_{\partial\mathcal{M}} d^3x \sqrt{h} (\mathcal{K} - T), \quad (1)$$

and take the metric to be of FLRW form:

$$ds^2 = dt^2 + a(t)^2 d\Omega_3^2. \quad (2)$$

Here, $\mathcal{R}$ is the Ricci scalar and $\Lambda = 3/\ell_{dS}^2 > 0$ is the cosmological constant. If a boundary $\partial\mathcal{M}$ with extrinsic curvature $\mathcal{K}$ is present, it is the location of an end-of-the-world (ETW) brane with tension T . Away from the boundary, the classical solution to the resulting Einstein equations is simply dS space. Three possible types of instantons and the corresponding creation processes are illustrated in Table 1.

The first corresponds to the No-Boundary Proposal [1–5], with the instanton being simply a euclidean S^4 . It can be interpreted along the lines of the ‘bounce solutions’ of Coleman and de Luccia [18]: A small S^3 grows, reaches maximal radius at the equator of the S^4 , and shrinks again to zero size. Following the Coleman procedure, one can continue to Lorentzian signature at the turning point. Both the instanton and the resulting creation process are shown in the left column of the table.

In case the underlying theory of quantum gravity allows for ETW branes, further instanton geometries exist. Let us briefly argue that this is a reasonable assumption: First, ETW branes can arise from shrinking extra dimensions [19]. Second, they appear naturally in type IIA string theory in the form of O8 planes. Finally, they are predicted to exist in any quantum gravity theory by the more general Cobordism Conjecture [20]. An early discussion of ETW branes with tension appears in [21]. Recent studies of ETW brane examples include [17,22–32].¹

Once we allow for ETW branes, a further creation process becomes possible as illustrated in the second column of Table 1. This goes back to [12,13], with the interpretation in terms of ETW branes provided by [14,16,33]. Following our recent study of this process in [17], we advocate the name ‘bubbles of something’, since this is the exact counterpart of vacuum decays due to ‘bubbles of nothing’. As seen in the table, the instanton contains a spherical ETW brane, characterized by a tension T . The latter relates to the extrinsic curvature scalar $\mathcal{K}$ as

$$M_P^2 \mathcal{K} = \frac{3}{2} T. \quad (3)$$

¹ We note in particular that, using an explicit regularization of the 4d ETW brane tension, actions can be obtained in a universal way from the 4d EFT [17,32].

In the considered dS model, this determines the critical bubble radius to be²

$$R = \frac{2M_P^2 \ell_{dS}}{\sqrt{T^2 \ell_{dS}^2 + 4M_P^4}}. \quad (4)$$

If $T > 0$, the instanton geometry is a patch of a 4-sphere which is smaller than a hemisphere and bounded by an ETW brane. If $T < 0$, the patch is larger than a hemisphere. As before, the euclidean instanton can be ‘cut in half’ and glued to its Lorentzian analytic continuation, which describes an expanding universe with ETW brane boundary. The instanton action is given by

$$S_{bos} = -4\pi^2 M_P^2 \ell_{dS}^2 \left(1 - \frac{T \ell_{dS}}{\sqrt{T^2 \ell_{dS}^2 + 4M_P^4}} \right). \quad (5)$$

We see that $|S_{bos}|$ becomes small for $T \rightarrow +\infty$ and approaches the no-boundary action $S_{nb} = -8\pi^2 M_P^2 \ell_{dS}^2$ for $T \rightarrow -\infty$. The FLRW universes created in this scenario have negative spatial curvature, similarly to those in creation process of [34].

Now we come to our new proposal, the geometry in the right column of Table 1, relying again on ETW branes. Here, the instanton is a portion of the 4-sphere representing euclidean dS. It is bounded by two ETW branes, each having S^3 geometry and radius R . As before, the extrinsic curvature and the tension are related through Eq. (3) making it necessary for the tension to be negative. The ETW brane radius is again determined by (4). This instanton can then be cut at the equator and glued to standard Lorentzian de Sitter space.

We note that, locally, i.e. focusing on the vicinity of the equator of the sphere and of the waist of the de Sitter hyperboloid, this corresponds to the standard procedure of gluing the euclidean geometry to its analytic continuation. Globally, this is not the case since there is a boundary in the ‘past’ of the euclidean region and no corresponding deviation from a pure de Sitter space in the Lorentzian geometry. One could in principle improve on this by including a field, the profile of which signals the proximity of the ETW brane, and analytically continuing such a more complete instanton solution. However, we believe that our approach is sufficiently precise under the assumption that any such field is stabilized away from the ETW brane, such that the ‘local’ analytic continuation described above can be trusted.

Thus, following the Coleman-de Luccia procedure and analytically continuing the instanton to Lorentzian signature, the nucleated on-shell space does not contain an ETW boundary, cf. the right column of Table 1. The ETW brane we used extends in the euclidean direction and marks the ‘beginning of time’, similarly to what is discussed in [25,35]. Our spacelike ETW brane appears only in the off-shell region. To convince readers who doubt that spacelike ETW branes exist at all, we recall the situation in the more established Witten bubble-of-nothing [19] nucleation process: There, the locus where the S^1 shrinks effectively represents an ETW brane and part of it, in the off-shell region, must be spacelike for the decay to occur.

The instanton action for our new creation process receives contributions from two boundaries and the bulk volume in between. It reads

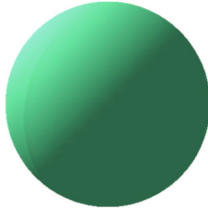
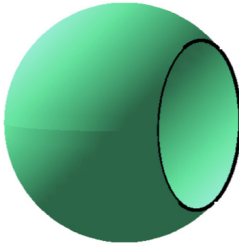
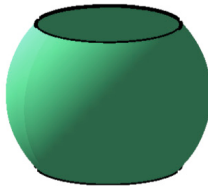
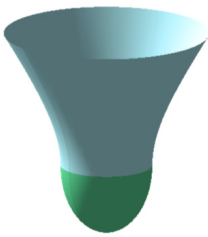
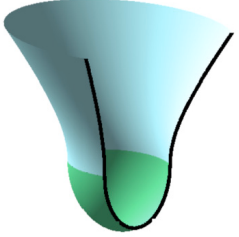
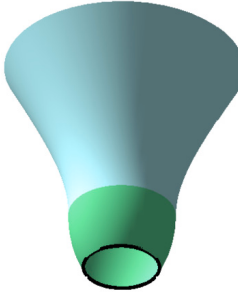
$$S_b = -8\pi^2 M_P^2 \ell_{dS}^2 \sqrt{\frac{T^2 \ell_{dS}^2}{T^2 \ell_{dS}^2 + 4M_P^4}} \quad (6)$$

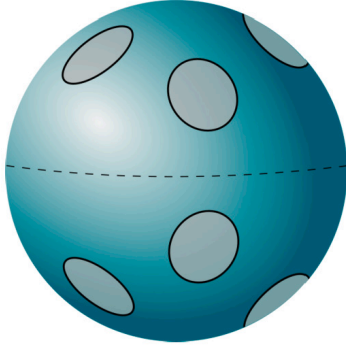
and vanishes if the ETW brane tension approaches zero. This is intuitively clear if one observes from (4) that, in the limit of vanishing

² Note that, if the ETW brane tension is positive, bubbles of something can also lead to the creation of Minkowski or Anti-de Sitter (AdS) universes. In particular, given that one expects the 10d type-IIB theory to possess a non-SUSY and hence presumably positive-tension ETW brane [20], a corresponding ball-shaped universe with eternally expanding spherical boundary can be created.

Table 1

Comparison between the no boundary, the bubble of something and the spacelike ETW brane example. Importantly, if the ETW brane tension is small, the instanton action of the spacelike ETW brane instanton in the last column becomes small.

No-Boundary (nb)	Bubble of Something (bos)	Boundary (b)
		
		
$S = -8\pi^2 M_P^2 \ell_{dS}^2$	$-4\pi^2 M_P^2 \ell_{dS}^2 \left(1 - \frac{T \ell_{dS}}{\sqrt{T^2 \ell_{dS}^2 + 4M_P^4}} \right)$	$-8\pi^2 M_P^2 \ell_{dS}^2 \sqrt{\frac{T^2 \ell_{dS}^2}{T^2 \ell_{dS}^2 + 4M_P^4}}$

**Fig. 1.** Illustration of the ‘Swiss Cheese Instanton’.

tension, the radius of the ETW branes approaches ℓ_{dS} , both branes approach the equator, and hence the instanton volume goes to zero. We provide a path-integral saddle-point derivation of the purely geometric result (6) in Sect. 2.2.

Considering the lower image in the right column of Table 1 and recalling Eq. (4), we see that the limit $T \rightarrow 0_-$ corresponds to the ETW brane approaching the waist of the hyperboloid from below. One may then speculate what happens to this geometry if the tension is further raised, becoming small and positive. The natural expectation is a ‘beginning of the universe’ on a boundary in the on-shell regime. This boundary would have to cut the future dS hyperboloid just above its waist. We would now be dealing with an on-shell, lorentzian version of the ETW brane. The new boundary would relate to our previously discussed euclidean ETW brane in the same way in which the on-shell S-brane [36] (see also [37]) relates to the off-shell euclidean brane of stringy instantons [38]. We do not follow this speculation further, leaving a proper study to future work.

It is clear that the existence of ETW branes allows for further instanton geometries. For example, one could consider spheres with more than two holes cut out, respecting reflection symmetry along the euclidean time axis and with none of the holes intersecting the equator. A corresponding instanton is illustrated in Fig. 1. An appropriate name for such

a geometry would be ‘Swiss Cheese Instanton’. The radius of each hole is again determined by the tension of the ETW brane (4). Each hole reduces the value of $|S_{\text{instanton}}|$. As a result, when adopting the Linde/Vilenkin choice of sign, the Swiss Cheese Instanton with maximal number of holes might turn out to be the most likely creation process for a given de Sitter space.³

2.2. Explicit path integral analysis of the boundary process

Let us further study the boundary process by following [3,18,40,41] and study the euclidean gravitational path integral. It has been proven to be a useful tool for computing vacuum decay rates as well as deriving thermodynamic properties of black holes.

Following e.g. [10,42,43], we consider a minisuperspace setup with metric

$$ds^2 = \frac{N^2}{q(t)} dt^2 + q(t) d\Omega_3^2, \quad (7)$$

a time interval $t \in [0, 1]$ and a gauge where N is constant. Setting $M_P = 1$, the action reads

$$S[q, N] = 2\pi^2 \int_0^1 dt \left(-\frac{3\dot{q}^2}{4N} + N(\Lambda q - 3) \right) + 2\pi^2 q_0^{3/2} \frac{N}{|N|} T. \quad (8)$$

The first term is nothing but the Einstein-Hilbert action evaluated for the metric (7). Since our geometry has two boundaries, defined by $q(0) = q_0$ and $q(1) = q_1$, a standard GHY term is used in the evaluation. The second term in (8) is the contribution of an ETW brane with tension T located at the $q = q_0$ boundary. Its form can be understood as follows: A domain wall with tension T at a fixed physical position makes the following contribution to the action

³ A similar observation relevant for the Hartle-Hawking choice of sign was made in [39]. There, it was argued that an instanton geometry with multiple spheres glued together through small tubular regions is preferred in comparison to the single sphere of the original proposal.

$$\int d^4x \sqrt{g} \delta \left(\frac{N}{\sqrt{q}} (t - t_0) \right) T = \int d^4x N q \delta(t - t_0) \left| \frac{\sqrt{q}}{N} \right| T$$

$$= 2\pi^2 q(t_0)^{3/2} \frac{N}{|N|} T. \quad (9)$$

Setting $t_0 = 0$, the volume of the spherical brane eventually becomes $2\pi^2 q_0^{3/2}$ and (8) follows.

We are interested in the amplitude

$$G[q_1; q_0] = \int_{q(0)=q_0}^{q(1)=q_1} \mathcal{D}q dN e^{-S[q, N]} \quad (10)$$

for an initial three-sphere of radius $q_0^{1/2}$ to evolve to a sphere with radius $q_1^{1/2}$. The wave function of the universe follows after integrating over all possible radii of the ETW brane:

$$\Psi(q_1) = \int dq_0 G[q_1, q_0]. \quad (11)$$

We will evaluate it in the saddle point approximation. The saddle points of $G[q_1, q_0]$ are determined by solutions of the equation of motion and the Hamiltonian constraint:

$$\ddot{q} = -\frac{2}{\ell_{dS}^2} N^2, \quad -\frac{3}{4N^2} \dot{q}^2 + 3 = \frac{3q}{\ell_{dS}^2} + 2T\delta(N)q_0^{3/2}. \quad (12)$$

Using Dirichlet boundary conditions, as specified above, solutions of the former equation read

$$\bar{q}(t) = -\frac{N^2 t^2}{\ell_{dS}^2} + \left(\frac{N^2}{\ell_{dS}^2} + q_1 - q_0 \right) t + q_0. \quad (13)$$

There are four values of N ensuring that $\bar{q}$ is also a solution to the Hamiltonian constraint:

$$N = \pm \ell_{dS}^2 \left[\left(1 - \frac{q_0}{\ell_{dS}^2} \right)^{1/2} \pm \left(1 - \frac{q_1}{\ell_{dS}^2} \right)^{1/2} \right], \quad (14)$$

which for $q_0, q_1 < \ell_{dS}^2$ are real-valued. Following the intuition that a small sphere nucleates and grows, eventually going on-shell, we restrict attention to $q_0 < q_1$. As discussed e.g. in [42,43], in the case without ETW brane, the four solutions define saddle points potentially contributing to the path integral. There are various proposals for which saddle points are relevant. The two solutions with smaller N^2 describe geometries fitting within one hemisphere. By contrast, the solutions with larger N^2 correspond to geometries containing the equator. Hartle and Hawking argue that the saddle point with smaller N^2 and $N > 0$ defines the wavefunction, meanwhile the Linde/Vilenkin proposal corresponds to choosing the saddle point with the same N^2 but $N < 0$. Following these arguments, the amplitude calculated in the saddle point approximation is given by

$$G[q_1; q_0] \propto \exp \left(\pm \left[4\pi^2 \ell_{dS}^2 \left(1 - \frac{q_0}{\ell_{dS}^2} \right)^{3/2} - 2\pi^2 q_0^{3/2} T \right. \right. \\ \left. \left. - 4\pi^2 \ell_{dS}^2 \left(1 - \frac{q_1}{\ell_{dS}^2} \right)^{3/2} \right] \right), \quad (15)$$

with the positive and negative sign corresponding to the Hartle-Hawking and Linde/Vilenkin choice respectively. To find Ψ from (11), we perform the integral over q_0 , again in a saddle point approximation. Extremizing (15) with respect to q_0 yields the condition $T < 0$ and

$$q_0 = \frac{4\ell_{dS}^2}{T^2 \ell_{dS}^2 + 4}, \quad (16)$$

which is precisely the result (4). Ultimately, we find for the wavefunction

$$\Psi(q_1) \propto \exp \left(\pm \left[4\pi^2 M_P^2 \ell_{dS}^2 \sqrt{\frac{T^2 \ell_{dS}^2}{T^2 \ell_{dS}^2 + 4M_P^4}} \right. \right. \\ \left. \left. - 4\pi^2 \ell_{dS}^2 \left(1 - \frac{q_1}{\ell_{dS}^2} \right)^{3/2} \right] \right). \quad (17)$$

As explained in [42,43], one obtains approximate solutions to the WDW equation in the regime $q_1 > \ell_{dS}^2$ using the WKB matching procedure. Specifically, we analytically continue (17) to the on-shell regime and enforce reality in the Hartle-Hawking case or the outgoing-wave condition in the Linde-Vilenkin case:

$$\Psi_{\text{HH}}(q_1) \propto \exp \left(4\pi^2 M_P^2 \ell_{dS}^2 \sqrt{\frac{T^2 \ell_{dS}^2}{T^2 \ell_{dS}^2 + 4M_P^4}} \right) \\ \times \cos \left(4\pi^2 \ell_{dS}^2 \left(\frac{q_1}{\ell_{dS}^2} - 1 \right)^{3/2} - \frac{\pi}{4} \right), \quad (18)$$

$$\Psi_{\text{LV}}(q_1) \propto \exp \left(-4\pi^2 M_P^2 \ell_{dS}^2 \sqrt{\frac{T^2 \ell_{dS}^2}{T^2 \ell_{dS}^2 + 4M_P^4}} \right) \\ - i 4\pi^2 \ell_{dS}^2 \left(\frac{q_1}{\ell_{dS}^2} - 1 \right)^{3/2}. \quad (19)$$

For an analysis of the Swiss-Cheese instanton, one would need to leave the minisuperspace approximation, which is beyond the scope of this note.

2.3. Creation rates

Given the various instanton geometries from Sect. 2.1 or alternatively, the wavefunctions computed in Sect. 2.2, one may compute the vacuum creation rates. For vacuum decays, the instanton action is related to the decay rate Γ via [18]

$$\Gamma \propto \exp(-B), \quad B = S_{\text{instanton}} - S_{\text{vacuum}}, \quad (20)$$

with S_{vacuum} the action of the decaying vacuum. The decay rate is then interpreted as decays per time and volume in the parent vacuum. For the creation from nothing, there does not exist a parent vacuum such that one may define [1–3]

$$\Gamma \propto \exp(-S_{\text{instanton}}), \quad (21)$$

without having the interpretation of creations per time and volume. Rather, physical statements are relative creation rates of the form

$$\Gamma_i / \Gamma_j, \quad (22)$$

for different creation processes i and j . Then, noting that the instanton action is always negative, one obtains

$$\Gamma \propto \exp(+|S_{\text{instanton}}|), \quad (23)$$

with $|S|$ growing for larger ℓ_{dS} (and fixed T) for all discussed instanton geometries. Maybe counter-intuitively, this implies that it is simpler to quantum mechanically nucleate a large rather than small dS sphere. The origin of this behavior can be traced to the negative sign in front of the kinetic term of the conformal mode of gravity.

By contrast, relying on standard tunneling behavior, Linde [4] and Vilenkin [5] suggested that the instanton action should relate to the creation rate according to

$$\Gamma \propto \exp(+S_{\text{instanton}}) = \exp(-|S_{\text{instanton}}|), \quad (24)$$

thus favoring small universes. It remains an open question whether the Hartle-Hawking (23) or Linde/Vilenkin (24) sign choice is the right one in quantum gravity.

Alternatively, one may derive the vacuum creation rates from the wavefunctions (18), (19). Following [42,43], one finds that the creation rates agree with (23) and (24) for the Hartle-Hawking and Linde/Vilenkin wavefunction respectively.

Let us compare the creation rates of the three instanton geometries of Table 1. It is clear that

$$|S_{nb}| > |S_{bos}| \quad \text{and} \quad |S_{nb}| > |S_b| \quad (25)$$

always hold. Hence for the Hartle-Hawking choice of sign the no-boundary process always dominates the creation of de Sitter universes.⁴

The situation is different when adopting the Linde/Vilenkin choice of sign. Here, due to (25), a bubble of something or a boundary instanton are the most likely processes. Furthermore, using negative-tension branes only, it is straightforward to see that $|S_{bos}| > |S_b|$ such that a boundary process wins. This becomes particularly clear for small brane tensions, i.e. $|T|\ell_{dS} \ll M_P^2$. Then the instanton actions take the form

$$|S_{nb}| = 8\pi^2 M_P^2 \ell_{dS}^2, \quad (26)$$

$$|S_{bos}| \simeq 4\pi^2 M_P^2 \ell_{dS}^2, \quad (27)$$

$$|S_b| \simeq 4\pi^2 |T| \ell_{dS}^3 \ll 4\pi^2 M_P^2 \ell_{dS}^2. \quad (28)$$

Hence, for small negative ETW brane tensions, the no-boundary action is of the same order as the bubble of something action while the boundary action is parametrically smaller.

The bubble of something becomes particularly relevant for positive tension ETW branes, when the boundary instanton does not exist: In the regime $T\ell_{dS} \gg M_P^2$ we have

$$|S_{nb}| = 8\pi^2 M_P^2 \ell_{dS}^2 \gg \frac{8\pi^2 M_P^6}{T^2} \simeq |S_{bos}|. \quad (29)$$

The tension of a brane is clearly similar to a 3d vacuum energy. As is well known, it proves difficult to realize string theory vacua with large positive vacuum energy. As a result, one may speculate that ETW branes with large positive tension are also problematic. On the other hand, string-theoretic examples of ETW branes with negative tension exist [19,22,44].⁵ As a consequence, the creation of vacua via the boundary process may turn out to be dominant in the string landscape.

One may study a geometrical model for the ETW brane required to realize our boundary process. The simplest such model is presumably the shrinking S^1 of Witten's bubble of nothing [19]. To implement this, consider a universe with spatial topology $S^n \times S^1$. If the S^1 radius is small and the S^n radius is large, one may interpret the spacelike part of the universe as effectively n -dimensional. In this setting, our boundary instanton corresponds to a creation process in which the S^1 direction shrinks to zero size in the past while the S^n remains finite.

Intriguingly, this process has been studied in [45–47] for the so called Kantowski-Sachs model, where the universe has the topology of $S^2 \times S^1$. Both the Hartle-Hawking no-boundary and the Linde/Vilenkin tunneling initial conditions were analyzed. For us, the tunneling initial conditions are most interesting since this is where we expect that the boundary instanton could dominate. For this case, it was shown [47] that in the limit of a small S^1 , universes with a large S^2 radius are favored to nucleate. One may hence interpret the Picard-Lefschetz-based results of [47] as supporting our suggestion of an (in this case effective) boundary creation process.

⁴ The bubble of something may still be relevant since it also occurs for positive tension ETW branes and allows for the creation of Minkowski and AdS vacua, which are not accessible in the no-boundary process. For example, the ETW brane bounding an empty 10d type IIB string theory universe conjectured to exist in [20] breaks supersymmetry and hence presumably has positive tension. As a result, it may be responsible for the creation of plain type IIB universes.

⁵ The small negative tension of the ETW brane of [22] becomes apparent using the 4d EFT approach as developed in [17].

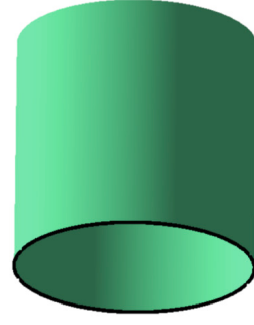


Fig. 2. The spacetime geometry of a toric universe nucleating from nothing.

3. Creation of toric universes with zero cosmological constant

As has been noticed long ago [48–50], a toric universe with positive cosmological constant can classically evolve from zero radius without the need of an off-shell region. A more complicated analysis is required to include quantum effects, such as fluctuations of additional fields or the Casimir energy in the calculation of the creation rate.

Such a creation from nothing process is, of course, also conceivable if the cosmological constant of the torus vanishes. But in this case the created universe would classically not expand and it can only grow due to quantum fluctuations.

One can, however, use a variant of our boundary instanton with an ETW brane with vanishing tension to create a finite-size toric universe with zero cosmological constant from nothing. Unlike dS universes, the so created spacetime could even be supersymmetric, so this could for example apply to M-theory with its familiar ETW brane or to type-IIA string theory with an O8/D8 tensionless boundary. The process is illustrated for the 1+1 dimensional case in Fig. 2, but it works analogously in any number of dimensions.

The instanton, in the classical sense of a bounce solution, would be just $T^{d-1} \times I$, with I an interval of euclidean time of arbitrary length. At the two boundaries, corresponding to the two ends of I , there are two ETW branes with the geometry of T^d . The action clearly vanishes. We can cut this instanton in the middle and glue it to the future half of a static, Lorentzian, flat torus universe. Note that we can choose I arbitrarily small since no off-shell region is really required. There is also no region where the size of the torus is small, such that the issues of corrections at small torus volume do not arise. We admit that it is rather counter-intuitive that this process is in no way suppressed if the torus volume becomes large. However, we were not able to find a hard argument against this conclusion at the present level of precision of our analysis.

Clearly, if we were to wrap our torus by a conventional brane or domain wall rather than by an ETW brane, our conclusion would be completely standard: The instanton would be just a standard brane instanton (see e.g. [51]). Its action would be the product of torus volume and brane tension, going to zero if the tension vanishes. For an ETW brane, the situation is different since its action contains a tension term, just like for a usual brane, but in addition the Gibbons-Hawking-York term, governed by the Planck scale and potentially introducing a wrong-sign kinetic term, as for the gravity conformal mode. Nevertheless, wrapping this brane on a torus gives, in the limit of zero tension, a zero-action instanton.

While we believe that this puzzling result deserves further study, it naively appears that zero-tension ETW branes are able to nucleate (and also terminate) toric universes without any action cost. Torus universes would then undergo a continuous production and destruction cycle.

4. Conclusions

Famously, a de Sitter universe can be created from nothing using the smooth geometry of the No-Boundary Proposal. If the relevant theory of quantum gravity allows for end-of-the world (ETW) branes, two

further options exist: One nucleates a ball of dS space with an ETW-brane boundary from nothing. The other, being the main subject of this paper, nucleates a compact, boundary-free sphere but, in contrast to the No-Boundary Proposal, with a spacelike ETW brane at the beginning of the universe. One may think of this as the ‘Boundary Proposal’.

The Boundary Proposal requires the ETW brane to have negative tension. Explicitly, its instanton action approaches that of the No-Boundary Proposal if the tension goes to minus infinity and it vanishes if the tension approaches zero. If one uses the Linde/Vilenkin (or tunneling) sign choice to compare the rates of creation processes, the Boundary Proposal is dominant if ETW branes with negative or zero tension exist.

A maybe surprising implication of our proposal is that, in the presence of zero-tension ETW branes, torus universes with vanishing cosmological constant can be created from nothing or decay without any exponential suppression or enhancement. Indeed, a flat ETW brane cutting a euclidean torus universe across the time-direction does not contribute to the action. It will be important to understand whether and how this puzzling result changes if fluctuations of the ETW brane are included. Maybe this will require developing a better understanding of the relation between time- and spacelike ETW branes. Moreover, a deeper understanding can hopefully be achieved by investigating carefully various geometrical models for spacelike ETW branes, starting with the simplest case of the effective ETW brane in Witten’s bubble of nothing.

Finally, we recall that the No-Boundary Proposal with Hartle-Hawking sign choice famously has the phenomenological problem of predicting the lowest possible starting point for the inflaton ϕ on its potential plateau. This has very recently been emphasized in [11]. At first sight, one could think that our boundary proposal can help since the inflaton can be forced to take a certain fixed value at the ETW-brane, in the sense of a Dirichlet-boundary condition for ϕ . If this value lies high enough on the plateau, correct predictions could follow. However, for the Hartle-Hawking sign-choice our boundary creation process is sub-leading w.r.t. the no-boundary process, so in total we still get the wrong cosmological prediction. For the Linde-Vilenkin sign choice, our ETW brane proposal can be dominant and can be used to fix the initial value of ϕ .

Very recently, a new creation process has been proposed in [52] which has interesting similarities to our ‘boundary process’. The idea is that the past of our compact universe is a euclidean wormhole leading to euclidean AdS, with the inflaton being the field which interpolates between our inflationary plateau and the AdS minimum defining the euclidean past. One may say that [52] abandons the no-boundary creation process in favor of postulating a spacelike domain wall from euclidean dS to euclidean AdS space in the past.⁶ In our proposal, the place of this spacelike domain wall is, in a sense, taken by a spacelike ETW brane. However, since we still work in the general framework of creation from nothing, the no-boundary process is also there and is more enhanced by its action. In principle, this might also be a concern for the proposal of [52], given in particular that the positive action of euclidean AdS tends to suppress any process in which it appears. Hence, in the case of Hartle-Hawking sign choice, it seems necessary to forbid the creation of small spheres for either the domain wall or ETW brane to dominate the beginning of the universe. It would clearly be important to establish a more detailed relation between No-Boundary, Boundary and further, alternative creation processes.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Bjoern Friedrich reports financial support was provided by German Research Foundation. If there are other authors, they declare that they

have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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5 End-of-the-World Branes and Inflationary Predictions for Rocky and Swampy Landscapes

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Declaration of authorship

The idea for this project was developed by Arthur Hebecker and me and I performed all calculations, wrote with very little exception the entirety of the draft which was edited by the collaborators.

End-of-the-World Branes and Inflationary Predictions for Rocky and Swampy Landscapes

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Abstract

Making cosmological predictions in a multiverse is a fundamental theoretical challenge. Assuming that (quasi-)de Sitter vacua are quantum mechanically described by a finite-dimensional Hilbert space, we develop a detailed framework for making explicit anthropic predictions. A key challenge which we attempt to overcome arises because, almost unavoidably, cosmologies that asymptote to Minkowski space exist. We then apply our framework to predicting the scale of inflation. We find that, even if eternal inflation is allowed, our predictions depend on creation rates of universes from nothing. These, in turn, are highly sensitive to the existence of end-of-the-world branes. The rates for the creation of universes from nothing are the dominant ingredient for ‘Swampy Landscapes’, which may have no metastable de Sitter vacua but only slow-roll solutions. In ‘Rocky Landscapes’, where long-lived de Sitter vacua are abundant, tunneling rates between such vacua represent a further key factor for deriving predictions.

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1 Introduction

For theories exhibiting different vacua, a measure on their set is required to achieve predictivity. If these vacua are continuously populated by eternal inflation, by creation from nothing, or by some other process, this leads to a formidable theoretical challenge known as the measure problem. This was initially studied in the context of eternal inflation [1] and became increasingly

more prominent with the advent of the string landscape [2–6] (see [7, 8] for reviews). The reason is the initial expectation that de Sitter (dS) vacua are abundant and eternal inflation [9–11] is natural. This perspective has changed more recently with the realization that stringy dS solutions are very difficult to realize and the proposal that they may be entirely absent [12–16]. Earlier studies have argued that dS is problematic for exponentially long lifetimes, see e.g. [17–25]. On the other hand, long-lived dS is also taken seriously as a quantum system and studied intensely [26–31]. In our present work, we will be agnostic concerning the abundance and longevity of dS vacua. In other words, we will allow both for ‘swampy’ landscapes (using the terminology of [32]) in which dS is either extremely rare or non-existent, and ‘rocky’ landscapes with abundant dS vacua.

One approach to the measure problem, developed for eternally inflating cosmologies, is to count measurements before some late-time cutoff which regularizes the infinities, see e.g. [1, 33–40]. However, the cutoff dependence makes this somewhat arbitrary, as explained in the reviews [41, 42]. Other approaches appeal to quantum cosmology [43–48] or count measurements along the worldline of a single observer moving through the multiverse [44, 47, 49, 50]. More recent work on the measure problem includes [31, 32, 51–63].

In what follows, we will focus on a measure for the multiverse proposed in [63], which rests on the ‘cosmological central dogma’ (CCD) [26, 64, 65]: The CCD states that de Sitter (dS) space represents a finite quantum mechanical system with Hilbert space dimension $\exp(S_{dS})$, where S_{dS} is the dS entropy. In the following, we assume that the CCD applies to all solutions with a cosmological horizon (quasi-dS solutions), including inflation and quintessence. For simplicity, we will refer to all solutions of this kind simply as ‘dS vacua’. Conceptually, the proposal of [63] is inspired by the quantum mechanical approach of [44, 45, 47], formalizing the latter and trying to develop it to a point where explicit predictions can be derived. The total Hilbert space is taken to be a direct sum, $\mathcal{H} = \bigoplus_{i \in vac} \mathcal{H}_i$, where i labels the vacua of the underlying landscape. This sum contains both finite-dimensional dS as well as infinite-dimensional Minkowski and AdS subspaces. Furthermore, it is assumed that there exists a unique quantum state of the multiverse, $\Psi \in \mathcal{H}$, obtained by solving the Wheeler-DeWitt (WDW) equation with a source term. This source term characterizes the creation of universes from nothing as well as bubble-of-nothing decays. One may think of the proposal [63] as a ‘local WDW measure’, where the term ‘local’ refers to the fact that only a local, horizon-sized patch of each dS vacuum is relevant in the multiverse dynamics. Predictions can be made by constructing projection operators P_α, P_β on subspaces of interest and evaluating ratios

$$\frac{\langle \Psi | P_\alpha | \Psi \rangle}{\langle \Psi | P_\beta | \Psi \rangle}. \quad (1)$$

Note that only relative probabilities of the form (1) can be calculable and physically meaningful. However, not all such ratios are a priori well-defined since, due to the presence of infinite-dimensional $\mathcal{H}_i$, the wave function Ψ is in general not normalizable.

Clearly, the structure of the underlying Hilbert space and the associated observables depend on UV physics, such that a direct construction of Ψ and the relevant projection operators

appears impossible. However, the structure of Ψ can partially be determined semiclassically [63]: The probability distribution of dS vacua in the landscape may be obtained by solving an appropriate rate equation. The required input parameters are various vacuum creation, decay and transition rates. This method does not allow to ascribe probabilities to Minkowski/AdS vacua since the projection of Ψ on such vacua is in general not normalizable.

We note that similar rate equations have appeared before in the context of ‘local measures’, in particular the ‘causal patch measure’ [35]. Such measures are defined by counting events along the worldline of an observer [34, 35, 38, 50, 66]. The rate equations from [34, 35, 50] are closest to our results. Nevertheless, the local WDW measure differs from this class since it is formulated observer-independently, on the basis of quantum mechanical principles. Observers make their appearance only once projections on appropriate Hilbert subspaces are introduced to derive anthropic predictions. Some of the differences between traditional local measures and the local WDW measure will play a key role in this work. The measure proposal of [44, 47], while conceptually related to our approach, also depends on an a priori chosen observer. Additionally, it does not involve the creation of vacua from nothing, which is by contrast of central importance in our analysis.

In this work, we explore the phenomenology and fundamental issues of the local WDW measure. It is clear that the finiteness of predictions of the form (1) is under threat because Ψ is in general not normalizable. However, when considering anthropic predictions, the projection operators in (1) need to involve a projection on observers, which may restore finiteness. Since, arguably, the leading paradigm for creating anthropic observers is a period of inflation followed by reheating and structure formation, we focus on such corners of the landscape.

If observers live in asymptotic dS vacua, i.e. if a (small) cosmological constant is present, we can approximate the observer projection as follows: First, we project on the pure dS-part of the Hilbert space which is finite dimensional and hence leads to well-defined predictions of the form (1). For observers formed after a reheating period, we then project further on the subspace of states describing inflation. We conclude that the probability $p(\text{obs}, i)$ of an observer living in vacuum i is, to leading order, given by

$$p(\text{obs}, i) \propto p_{\text{inf}(i)} , \quad (2)$$

the probability of finding the universe in an inflationary state leading to reheating in vacuum i . This latter probability may be obtained by treating inflationary plateaus as additional, short-lived dS vacua and then deriving their probability from a rate equation.

The situation changes if we consider landscapes with anthropically viable terminal vacua, i.e. AdS or Minkowski. While our method to obtain relative probabilities for the relevant inflationary plateaus still works, a reheating process ending in Minkowski space may now lead to infinitely many observers in a causally connected region.¹ It hence appears that anthropic predictions could become ill-defined, due to both numerator and denominator of (1) being

¹Since cosmological AdS always crunches, the analogous infinite set of observers is not contained in a causally connected patch.

infinite. A similar problem arises for the causal patch measure [41, 67]. A main result of our work is the analysis of several approaches for obtaining well-defined predictions even if observers in Minkowski vacua exist.

One of our suggested solutions is based on the observation that, on the reheating surface, it is not yet visible whether late-time evolution will lead to dS or Minkowski space. Hence, as in the CCD, not all Minkowski observers can be considered independent. In other words, the CCD suggests that there exist gauge redundancies on the reheating surface, such that only part of it can be seen as fundamental. Then, the probability for an observer living in Minkowski space is again estimated by (2), with $\text{inf}(i)$ now denoting an inflationary plateau reheating into a Minkowski vacuum i . Another possibility is to embrace the infinity of observers on a Minkowski-space reheating surface as meaningful. This leads to the prediction that our observed dark energy must be of quintessence type.

Given the vacuum creation and transition rates, we can use the rate equation to derive the relative probabilities for finding the universe on different inflationary plateaus. The required transition rates can be calculated using the Coleman-de Luccia method [68]. We do so specifically for KKL^T- and LVS-type vacua [3, 6], which we thus use as an example of a possible ‘rocky’ landscape. Furthermore, vacua can decay to nothing. This was first demonstrated by Witten [69] and has then been further studied in e.g. [70–75]. Recently [76], decays to nothing in the string landscape were analyzed explicitly.

Our approach to the measure problem is only consistent if, in addition to tunneling transitions, finite-size universes can be created from nothing via a quantum process. Proposals exist for the creation of spherical universes [10, 77–80] (cf. also the new proposal of [81]), universes of disk topology with a boundary [70, 76, 82–85] and closed universes of nontrivial topology [86–88]. There remains an ambiguity concerning the sign with which the instanton action appears in the exponent of the corresponding creation rates (the Hartle-Hawking vs. Linde/Vilenkin sign choice). This is related to different possible definitions of the gravitational path integral, cf. [89] and refs. therein. In a recent paper [31], it was suggested to derive the local density matrix using the Hartle-Hawking proposal. It would be interesting to find a connection of this to the local WDW measure. Phenomenologically, the Hartle-Hawking choice appears to be strongly disfavored as it predicts the shortest anthropically viable period of inflation, in conflict with observation (cf. the recent discussion in [90]).

Since we assume that observers form only near reheating surfaces, the scale of the preceding inflationary period is an important observable. It becomes a natural target for an anthropic prediction because it does not directly impact our existence. This prediction also assumes that, within the underlying landscape, the inflationary scale is largely uncorrelated with anthropically significant parameters, such as certain Standard Model couplings. In the local WDW framework, the prediction turns out to strongly depend on vacuum creation processes. This is particularly apparent for ‘swampy’ landscapes. In this case, tunneling between vacua is subdominant and the probability of finding ourselves in a given vacuum is mainly determined by its probability to nucleate from nothing. In determining the vacuum creation rates, the sign

choice when relating the instanton action to the decay rate, the topology of created universes, and the tensions of ETW branes existing in the landscape play a significant role. Nevertheless, it is possible to establish close links between the gravitational path integral and cosmological predictions. While the Hartle-Hawking sign choice already seems to be ruled out, we argue that the Linde/Vilenkin proposal points to primordial gravitational waves that are detectable by upcoming CMB-experiments.

This article is organized as follows: In Sect. 2, we briefly review the local WDW measure and explain how elementary anthropic predictions can be made. In Sect. 3, we discuss the problem of infinitely many observers living in terminal vacua and provide possible resolutions. In Sect. 4, we discuss the fundamental ingredients needed to predict the most likely scale of inflation to be measured by anthropic observers. In Sect. 5, we explicitly calculate various vacuum decay rates for KKLT and LVS type vacua. In Sect. 6, we combine the results of the previous sections to try and make an explicit prediction for the scale of inflation relevant for anthropic observers in a string multiverse.

2 The local WDW measure and anthropic predictions

2.1 Review of the local WDW measure

It is our goal to implement a measure for the multiverse based on the following assumptions:

- The ‘cosmological central dogma’ (CCD) [26, 65] holds. In other words, de Sitter (dS) space has to be viewed from a static patch perspective and its Hilbert space $\mathcal{H}_{dS}$ has dimension $\dim(\mathcal{H}_{dS}) = \exp(S_{dS})$, where S_{dS} is the dS entropy. The CCD may be interpreted as the statement that different static patches of dS space are gauge-redundant.
- The state vector Ψ or wave function of the multiverse lives in the Hilbert space

$$\mathcal{H} = \bigoplus_{i \in vac} \mathcal{H}_i, \quad (3)$$

with each landscape vacuum i contributing a subspace $\mathcal{H}_i$. These subspaces are finite-dimensional for dS and infinite-dimensional for AdS and Minkowski vacua.

- The wave function of the universe Ψ solves a modified Wheeler- (WDW) equation

$$H\Psi = \chi. \quad (4)$$

Here the source term χ characterizes the creation of universes from nothing as well as the vacuum decay to nothing.

Solving (4) is certainly very difficult since the microscopic structure of the Hamiltonian is unknown. However, as argued in [63], our assumptions stated above imply that the probability

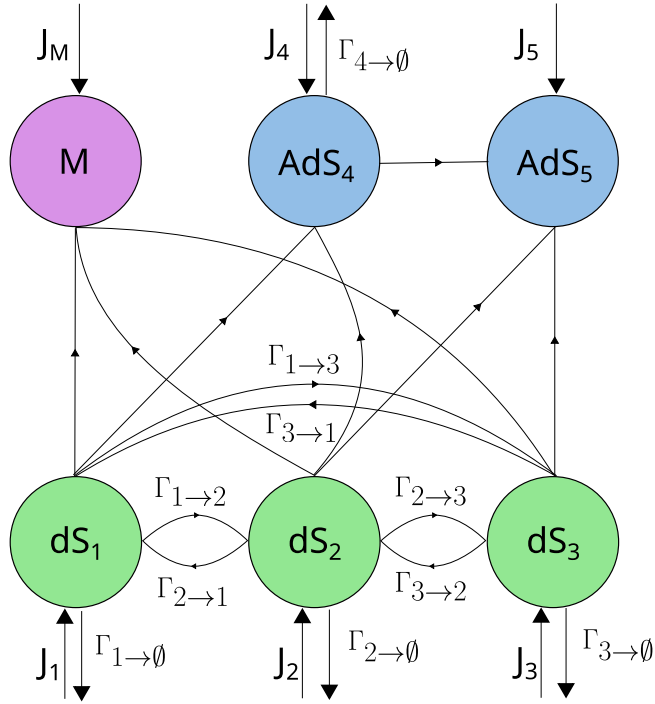


Figure 1: Creation and tunneling for a toy landscape consisting of three dS, one supersymmetric Minkowski (M), one non-supersymmetric and one supersymmetric AdS vacuum (‘AdS₄’ and ‘AdS₅’ respectively). Note that the labels distinguish different vacua and have nothing to do with dimensionality, which is always 4. To avoid overburdening notation, we have not put labels on the arrows for decays to Minkowski and AdS vacua.

distribution corresponding to Ψ can be derived semiclassically for the dS part of the multiverse: Specifically, we denote by

$$p_i = \|P_i \Psi\|^2 \quad (5)$$

the probability for finding a vacuum of type i in the multiverse. Here, P_i is the operator projecting on the subspace $\mathcal{H}_i$. The probabilities p_i obey the semiclassical rate equation

$$J_i = \sum_{j \in dS} (p_i \Gamma_{i \rightarrow j} - p_j \Gamma_{j \rightarrow i}) + p_i \sum_{y \in \text{Terminals}} \Gamma_{i \rightarrow y}, \quad (6)$$

where i, j run over all dS vacua, and $\Gamma_{i \rightarrow j}, \Gamma_{i \rightarrow y}$ and J_i represent the vacuum transition and creation rates, respectively. The index y runs over terminal vacua, i.e. AdS or Minkowski, and ‘nothing’. Similar rate equations have appeared in [34, 50]. The wave function Ψ can be interpreted as characterizing a stationary state with universes constantly being created, decaying and tunneling into each other according to Eq. (6). We have attempted to illustrate this multiverse dynamics in Fig. 1.

With the normalization of Ψ unknown, only ratios of probabilities are meaningful. As already shown in Eq. (1), they take the form

$$\frac{\langle \Psi | P_\alpha | \Psi \rangle}{\langle \Psi | P_\beta | \Psi \rangle}, \quad (7)$$

where P_α, P_β project Ψ on subspaces of interest, e.g. two different vacua. The ratio of their expectation values specifies the relative probability for condition ‘ α ’ being realized rather than ‘ β ’.

Since a direct construction of $\mathcal{H}$ and the projection operators P_α, P_β seems unfeasible, we will focus on predictions which can be obtained from semiclassical considerations by using Eq. (6). While vacuum transition rates $\Gamma_{i \rightarrow j}$ can be calculated using the Coleman-de Luccia method [68], there is a larger uncertainty regarding the creation rates J_i .² Most commonly employed are the Hartle-Hawking no-boundary [82] and the Linde/Vilenkin tunneling [79, 80] proposals, the predictions of which are already drastically different, due to a different sign appearing in the exponent when relating the instanton action to the creation rate. Additionally, a creation via a ‘bubble of something’ has been proposed [82, 83], and a non-trivial topology in the nucleated universe can play an important role [86–88]. Finally, it was speculated that also the creation of universes with a boundary in the off-shell region may be relevant [81].

2.2 Anthropic predictions from the local WDW measure for observers in dS universes

We now address the ‘anthropic questions’ of the most likely vacuum for us to live in and of the inflationary scale we are most likely to observe, following the strategy of [63]. As discussed in the introduction, we shall focus on anthropic observers originating on reheating surfaces after a period of inflation. The probability $p(\text{obs}, i)$ for such an observer to be found in vacuum i is then certainly proportional to $p_{\text{inf}(i)}$, the probability of finding the universe in an inflationary state ‘ $\text{inf}(i)$ ’ leading to reheating in vacuum i . We hence write (cf. Eq. (2))

$$p(\text{obs}, i) = w_i p_{\text{inf}(i)}, \quad (8)$$

where w_i is a weighting factor accounting for various further effects. For example, it is intuitive that a larger number of observers living in i results in a larger probability.³ Eventually, our predictions are ratios

$$\frac{p(\text{obs}, i)}{p(\text{obs}, j)} = \frac{w_i p_{\text{inf}(i)}}{w_j p_{\text{inf}(j)}}, \quad (9)$$

²We shall comment on all these computations in Sect. 4.

³Various anthropic effects, most notably the number density of observers on the reheating surface, may be absorbed in w_i . Furthermore, $p(\text{obs}, i)$ is proportional to the probability of *starting* inflation in $\text{inf}(i)$. This is different from the probability $p_{\text{inf}(i)}$ of *finding* the universe in $\text{inf}(i)$. The latter is enhanced if a given inflationary plateau has a particularly long lifetime before reheating. This difference is a non-exponential effect which may also be absorbed in w_i . We will return to this point in Sect. 6.

for any two anthropically viable vacua i, j (i.e. $w_i \neq 0 \neq w_j$) of the landscape. To calculate the $p_{\text{inf}(i)}$, we treat the inflationary plateaus of any scalar potential as additional dS vacua of the landscape. They will hence appear in the rate equation (6) with a total decay rate dominated by $\Gamma_{\text{inf}(i) \rightarrow i}$ corresponding to inflation ending by reheating in vacuum i . Solving the rate equations, which we will do explicitly in Sect. 4.1, yields the $p_{\text{inf}(i)}$ as functions of all vacuum transition and creation rates.

Let us for the moment assume that observers can only live in universes which are asymptotically dS, i.e. consider the situation where i, j feature a (small) cosmological constant. Since dS spaces are viewed from the perspective of the static patch, which is finite in size, only a finite number of observers will live there. As a result, the ratio w_i/w_j is well defined and not exponentially large or small in the dS radii $\ell_{dS,i}$ and $\ell_{dS,j}$. This is clear since the size of the static patch is $\sim \ell_{dS}^3$. We then approximately have

$$\frac{p(\text{obs}, i)}{p(\text{obs}, j)} \simeq \frac{p_{\text{inf}(i)}}{p_{\text{inf}(j)}}. \quad (10)$$

We conclude that under the assumption that observers can only live in dS vacua, the leading order prediction for the most likely anthropic vacuum is solely determined by the largest value of $p_{\text{inf}(i)}$, which can be computed from (6).

3 Projecting on observers in the presence of terminal vacua

In the previous section, we have seen how to make predictions in case observers can only live in asymptotically dS vacua. In this section, we will discuss the general situation, allowing for observers in terminal vacua. This will lead to difficulties in particular in the case of observers in Minkowski space, to which we refer as the ‘Minkowski-space infinity problem’. Subsect. 3.1 provides an executive summary of the main results of this discussion.

3.1 Summary of approaches to the Minkowski-space infinity-problem

If observers can exist in AdS and Minkowski vacua, the finiteness argument based on the finite dimensionality of dS Hilbert spaces fails. In the case of AdS, this may be argued to be irrelevant since the unavoidable big crunch allows only finite portions of a reheating surface in AdS to be causally connected. For a reheating surface in Minkowski space, no such argument can be made. Moreover, even if no Minkowski-space reheating surfaces with observers exist, the Minkowski-space infinity problem still persists because of bubble collisions: In such collisions, infinite parts of dS-space reheating surfaces can make it into a Minkowski region [91].

We explain this in more detail in Sects. 3.2-3.5 and analyze possible resolutions. Let us here give a brief and systematic summary, following the diagram in Fig. 2:

1. **No Minkowski space:** The issue disappears if Minkowski vacua are in the Swampland.

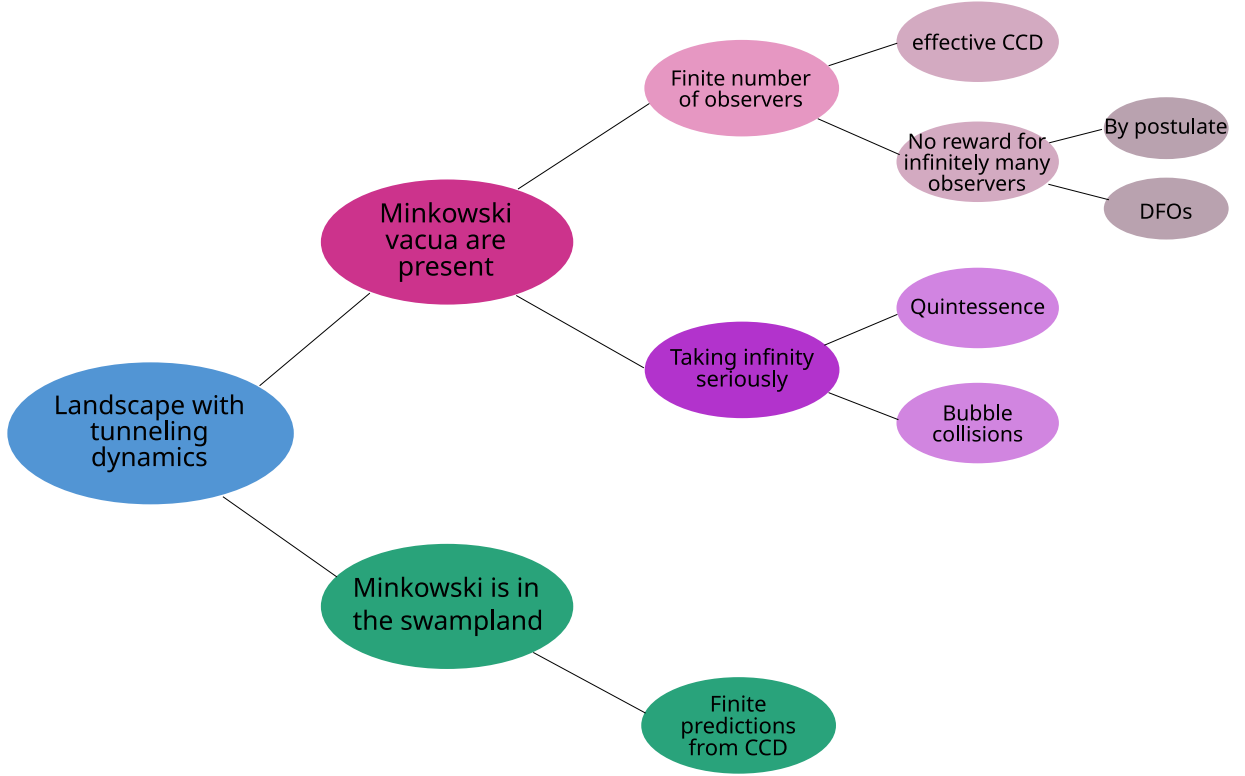


Figure 2: Approaches to the Minkowski-space infinity problem.

2. **Finite number of observers in Minkowski space:** Two lines of argument may avoid observer-infinities in Minkowski space:
 - 2a Effective CCD: The CCD declares that only a finite portion of any spacelike surface in dS is physical. By analogy, we propose that only a finite portion of any reheating surface, including in Minkowski space, contains independent observers.
 - 2b No reward for infinitely many observers: Infinite observer counts may be irrelevant for probabilistic predictions. This may either hold as a postulate [48, 92] or because observers contain finitely many bits: They can then be grouped in a finite set of equivalence classes, which we call ‘dressed finite observers’ or ‘DFOs’.
3. **Taking infinities seriously:** If infinities of observers in Minkowski space do indeed affect predictions, this may happen in two distinct ways (or through a combination thereof):
 - 3a Predicting reheating to Minkowski: If reheating to Minkowski is possible, then anthropic arguments predict Minkowski space in our asymptotic future, suggesting quintessence-like dark energy in our Universe.

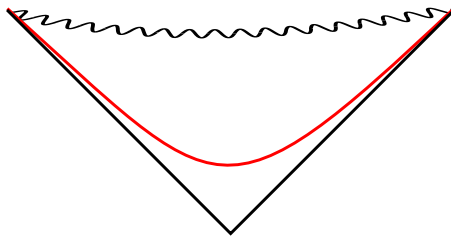


Figure 3: An AdS bubble with crunching surface (wiggly line) and reheating surface (red line) is shown.

- 3b Prediction based on bubble collisions: If there is no direct reheating to Minkowski, then anthropic arguments suggest that we will collide with a Minkowski bubble in the future, introducing a dependence on the dynamics of bubble collisions [41,93,94].

The reader not satisfied with this short version may now read Sects. 3.2-3.5 and, if desired, then return to the present subsection. We note in particular that Sect. 3.5 provides a brief discussion of landscapes without eternal inflation, a subject we did not repeat in this summary.

3.2 Finiteness based on a finite observer Hilbert space

In general, Ψ lives in an infinite-dimensional Hilbert space and may hence not be normalizable. As a result, ratios of the form (7) may be ill-defined. For example, P_α and P_β could be projection operators on AdS vacua. Since the relevant Hilbert subspaces are infinite-dimensional, the numerator and denominator could both be infinite, such that their ratio cannot be quantified. For anthropic predictions, however, we are interested in evaluating (7) for operators P_α, P_β which involve a projection on the subspace $\mathcal{H}_{obs} \subset \mathcal{H}$, describing observers. This projection can restore normalizability, such that the corresponding predictions are well-defined. If anthropic observers live only in dS universes, $\mathcal{H}_{obs}$ is contained in the finite-dimensional dS part of $\mathcal{H}$. The projections of Ψ on $\mathcal{H}_{obs}$ or on subspaces thereof are then normalizable. This is the basic origin of our claim that the ratio w_i/w_j in (9) is finite for i, j denoting dS vacua.

3.3 Finiteness lost

Unfortunately, the assumption that observers only live in dS is hard to justify. Indeed, certain inflationary plateaus may lead to reheating in AdS. While such an inflationary bubble will eventually end in a big crunch singularity, it will contain an infinite reheating surface (cf. Fig. 3). Semiclassically, this surface may contain infinitely many observers. Together with the fact that the relevant part of Ψ now clearly lives in a infinite-dimensional Hilbert space, we have lost our previous, simple argument for finiteness.

At this point, going beyond semiclassics, one may try to argue that the presence of the big crunch singularity somehow limits the number of observers that can be considered independent.

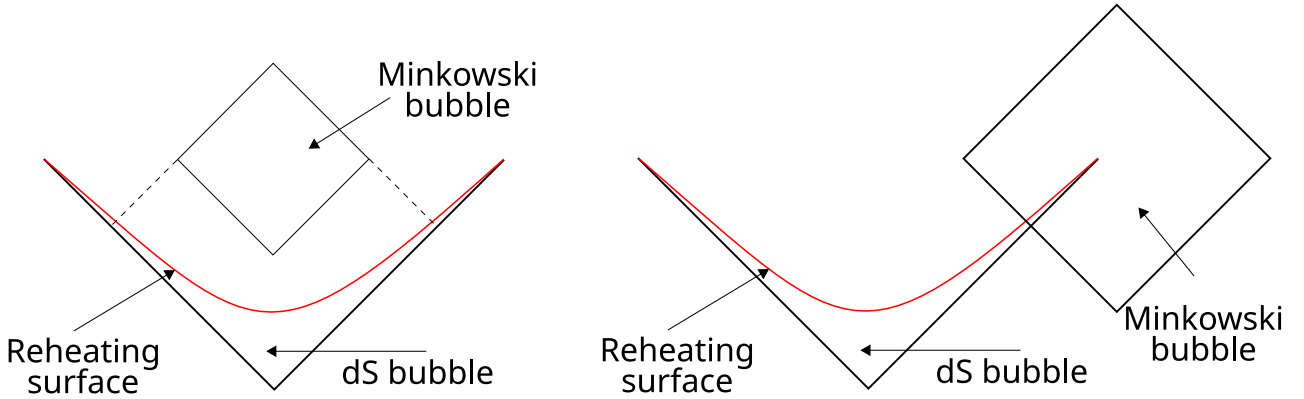


Figure 4: Illustration of a Minkowski bubble inside a dS bubble (left) and a colliding bubble system (right). On the left, the part of the reheating surface that can send signals into the bubble is finite, meanwhile on the right it is infinite.

For example, there are only finitely many observers in any causally connected region of the spacetime displayed in Fig. 3.

However, such a reasoning cannot be applied if reheating occurs in a Minkowski vacuum. To save finiteness, one may appeal to the very reasonable expectation that Minkowski vacua are always $\mathcal{N} = 2$ supersymmetric as otherwise corrections to the scalar potential will induce a non-zero vacuum energy. Since $\mathcal{N} = 2$ SUSY excludes chiral matter, it might become impossible to form observers (see e.g. [95] for a similar argument).

Even if such an argument were valid, we are forced to allow for situations where an inflationary dS bubble with observers on the reheating surface and an additional Minkowski bubble are both present (cf. Fig. 4). A Minkowski bubble that forms completely inside an inflationary dS bubble still does not feature infinitely many observers from the reheating surface since the region of the reheating surface being in causal contact with the Minkowski bubble is only finite in size. The situation is different for a Minkowski bubble colliding with a dS bubble, where the intersection of the reheating surface with the colliding bubble is an infinite 2d surface, cf. [91]. We thus face the unavoidable situation that infinitely many observers (or at least their remnants) enter a Minkowski bubble. Now the CCD can provide no straightforward argument why only a finite region of the reheating surface should be considered.⁴ In other words, it appears as if there are vacua in the landscape contributing to (9) with an infinitely large anthropic weight w .

3.4 Finiteness regained

Effective CCD: In the semiclassical picture of the multiverse, the reheating surfaces in nucleated bubbles are infinitely large. As a result, infinitely many observers will form on a

⁴A similar observation has already been made in [96]

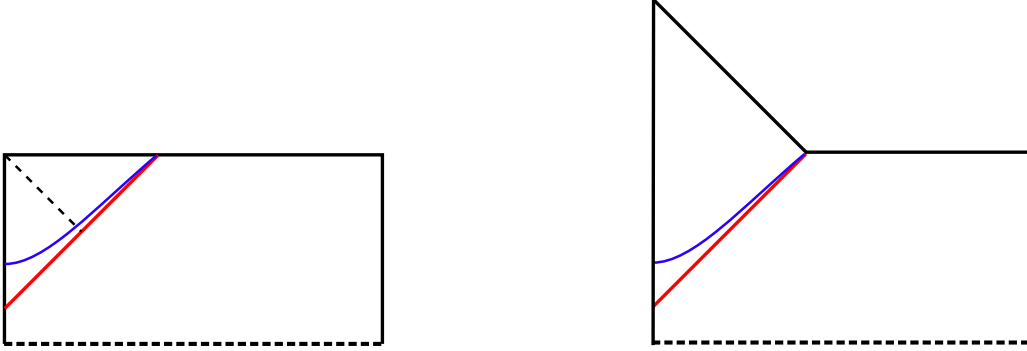


Figure 5: The creation of a spherical universe with a subsequent formation of a dS bubble (left) and Minkowski bubble (right) are displayed. The red line denotes the bubble wall and the blue line illustrates the location of the reheating surface. The diagonal dashed line on the left shows the cosmological horizon for an observer located on the vertical solid black line on the far left.

given reheating surface. If the considered pocket universe is dS, the CCD implies that the infinity is an overcounting of the same information and only finitely many observers should be considered independent: The CCD limits the counting of observers to a single static patch.

At first sight, the situation appears to be qualitatively different when considering a universe with inflation and reheating to Minkowski space – the two cases are compared in Fig. 5. Nevertheless, the observers living on the reheating surfaces experience in both situations a certain energy density which, in case of dS observers, contains a contribution from the cosmological constant. However, if the cosmological constant only provides a negligible contribution to the energy budget, there is no observable difference between the two reheating surfaces in Fig. 5. Such a difference only becomes relevant in the distant future, far away from the reheating surface (where all observers may have died). Thus, given that only finitely many observers are independent in the dS case, it appears counter-intuitive that the subdominant effect of the cosmological constant makes a qualitative difference in the counting of independent observers. This suggests that one should reconsider what the proper quantum gravity way of counting observers on reheating surfaces of Minkowski bubbles is. It should not be taken for granted that the result is infinite. Instead, we should be looking for a way of counting that is similar to the counting of observers living in dS bubbles with a small cosmological constant.

Thus, let us take a fresh look at how many independent observers there are on a reheating surface in Minkowski space: Consider the nucleation of an inflationary bubble, i.e. a bubble in which a scalar is initially rolling along an inflationary plateau. We view this as an approximate dS space with entropy S_{inf} . The initial quantum state after nucleation is contained in an $\exp(S_{\text{inf}})$ -dimensional subspace of $\mathcal{H}$. At late times, the state explores the entire infinite-dimensional Hilbert space associated to Minkowski space and one can imagine that there is a continuous transition letting the dimension of the accessible Hilbert space grow during the cosmological evolution. One may then speculate that, at the moment of reheating, the dimen-

sion of the relevant Hilbert space is set by an entropy $S_{\text{reh}} \sim M_P^2/H_{\text{reh}}^2$, with H_{reh} the Hubble scale on the reheating surface. For simplicity, we do not distinguish between the moment of reheating and the moment of star- and observer formation. We hence identify H_{reh} with the curvature scale relevant for the formation of observers. By analogy to the dS situation, we assume that the Hilbert space of dimension $\exp(S_{\text{reh}})$ describes a region of size $1/H_{\text{reh}}^3$, containing the independent observers.⁵ As a result, ratios of anthropic weighting factors w_i/w_j are finite and not exponentially large or small. Such a reasoning can be viewed as an effective interpretation of the CCD, where passing through a nearly-de Sitter state suffices for making the accessible Hilbert space finite.⁶ Clearly, the same finiteness argument also applies to the case when reheating to AdS is considered.

This argument resonates nicely with the finding [96] that requiring validity of effective field theory combined with the CCD limits the number of independent modes of inflationary origin. If for example our Universe is rolling into a Minkowski vacuum without cosmological horizon, the number of fluctuations originating from inflation that can enter our horizon is in principle unbounded. However, the CCD implies that these modes must be correlated, leaving at most $\mathcal{O}(S_{\text{inf}})$ many independent degrees of freedom, with S_{inf} denoting the entropy of the approximate dS universe during inflation. This situation is fully analogous to radiation coming out of a black hole during the evaporation process which at late times must be correlated with the degrees of freedom already being outside the horizon [98]. The statement of the CCD that $\dim \mathcal{H}_{dS} < \infty$ is then analogous to the claim that the transition of modes into Minkowski space follows similar principles as the process of degrees of freedom leaving the black hole horizon.

Equivalent observers: Independently of the above, there is a further way of reasoning why well-defined predictions can be derived in the presence of Minkowski vacua: One may argue that anthropic observers necessarily correspond to finite subsystems of the total space accessible to the wave function of the universe. In other words, they have a finite-dimensional Hilbert space, $\dim \mathcal{H}_{\text{obs}} < \infty$. By virtue of this definition, any observer can encode only a finite amount of information about the state of the rest of the universe (roughly S_{obs} bits, with the finite-dimensional observer Hilbert space allowing for about $e^{S_{\text{obs}}}$ different inequivalent quantum states which the observer can be in). Therefore, the future-infinite amount of observers on open slices of Minkowski spacetime inside the bubble will decompose into a finite $\mathcal{O}(e^{S_{\text{obs}}})$ set of equivalence classes which we may call ‘dressed finite observers’ (DFOs). In the limit to asymptotic future, each DFO contains infinitely many finite observers in the same observer quantum state.

We can now declare that a ‘real’ observer must be identified with a DFO since the infinitely many observers in that DFO equivalence class perceive themselves and their internal description of the rest of the universe to be exactly the same, and are thus inseparable by any operation

⁵One may think of the CCD as the statement that there exist gauge redundancies along surfaces of constant field value (i.e. on slices of constant FLRW time).

⁶In the context of the census taker and causal diamond measure, it was speculated [45] on the basis of holographic arguments [97] that information in Minkowski bubbles is redundant.

accessible for a finite observer. This yields a finite observer count even in Minkowski space. We note that there is no intrinsic cutoff on the size of the finite-dimensional Hilbert space and thus S_{obs} per se. In this sense, there is a minimal amount of ‘anthropic’ modeling in our choice of DFOs, in that we manifestly limit $\dim \mathcal{H}_{obs} < \infty$ and $S_{obs} < \infty$ to be small in a sense of being parametrically tiny compared to the dS horizon entropies of cosmological constants corresponding to EFT-accessible sub-Planckian vacuum energy densities. This notion thus is guided by the smallness of the entropy of human-like observers relative to the late-time dS entropy of our observable universe.

Furthermore, it was claimed elsewhere that one should not focus on the total number of observers in a given universe [48, 92]. Instead, one should demand that the probability for at least one observer forming is near unity. Given this proposal, an infinity of observers that can exist in a Minkowski bubble poses no threat to deriving anthropic predictions from the local WDW-measure.

No Minkowski space: A particularly drastic possibility for how infinities associated with Minkowski space could be avoided is the following: Our full quantum gravity theory might simply not contain Minkowski vacua. Instead, there could be only (metastable) dS states and AdS vacua. This resonates with the ‘fundamentalist-AdS/CFT’ point of view that the only way to define quantum gravity is via a CFT. Presumably metastable dS or slow-roll states can then only arise as excitations of AdS (cf. also [25, 99, 100]).

Taking infinitely many Minkowski-space observers seriously: By contrast, we can also not exclude the possibility that infinite observer counts in Minkowski space are indeed physical. One can argue for this by including cosmological Minkowski vacua in our WDW approach on the basis of a ‘FRW-slicing’, with each slice being a hyperboloid of constant density. It could then be that no variant of the CCD applies, i.e. there are no gauge redundancies along each surface. This clearly leads to infinities of observers. Alternatively, one could slice the Minkowski space on the r.h. side of Fig. 5 horizontally, such that each slice has only a finite number of observers. Nevertheless, infinities can still arise because these slices continue to intersect the reheating surface all the way into the infinite future. In fact, these intersection loci keep growing. In any case, once we accept these infinities as real, one straightforwardly concludes that our measure prefers universes in which our FRW-cosmology asymptotes to Minkowski space. In other words, the presently observed dark energy should decay, possibly in a quintessence-like manner. However, this conclusion might be premature: Indeed, we might live in a landscape where no observers form on reheating surfaces leading to Minkowski space. This could come about because Minkowski space requires $\mathcal{N} = 2$ SUSY or SUSY-restoration through decompactification. In such a landscape, the preferred vacua would be those which have a large probability of colliding with Minkowski bubbles. This is similar to the predictions obtained from the census taker measure, cf. [41, 93, 94].

3.5 A landscape without eternal inflation

It has been suggested that, while being compatible with slow-roll inflation, quantum gravity does not allow for de Sitter minima and eternal inflation because of a fundamental inconsistency caused by quantum breaking [16, 23–25, 101–103]. This is also a possible implication of the de Sitter conjecture in string theory [12–15]. If so, creation from nothing can only lead to states on an inflationary slow-roll plateau, with further tunneling processes not playing a significant role. If furthermore bubbles of something [70, 76, 82–85] do not exist, the closed topology of the dS universes created according to Hartle-Hawking or Linde/Vilenkin ensures finiteness of the reheating surface and the number of observers. As a result, in such a landscape no CCD-based argument is needed for making anthropic predictions. If, however, bubbles of something can occur, one nevertheless has to deal with infinite reheating surfaces. The arguments of Sect. 3.1 for treating infinities are then relevant even though eternal inflation is not possible. A different way in which infinite reheating surfaces can arise in the absence of metastable de Sitter is through topological inflation [104, 105]. In this context, it is an interesting question to what extent arguments against de Sitter or other swampland criteria also affect topological inflation [16, 106–110].

4 Explicit predictions in the multiverse – basic ingredients

Let us now try to make progress towards explicit prediction in the multiverse using the local WDW measure. We assume that eternal inflation is not excluded and that the finiteness arguments of Sect. 3.4 hold, such that anthropic predictions are not a priori dominated by Minkowski universes. This corresponds to the cases 1. and 2. of Sect. 3.1.

More precisely, we base our further analysis on (10) and it is crucial for us that the transition from (9) to (10) still holds. This is obvious in case 1. of Sect. 3.1. In case 2.a, it holds because ratios $w_i/w_j \propto \ell_{reh,i}^3/\ell_{reh,j}^3$ are not exponentially large. In case 2.b, it holds either by definition or, if we rely on the DFO argument, using the extra assumption that the number of independent DFOs is not exponentially large. In any case, evaluating (10) will now be our main focus.

4.1 Solving the rate equation

Our first goal is to solve (6) for the p_i . Since the normalization of creation rates is unknown, the p_i can only be determined up to a multiplicative constant. This is fine since predictions are of the form p_i/p_j . Equation (6) can be written in matrix form,

$$J_i = M_{ij}p_j, \quad M_{ij} = \delta_{ij}\Gamma_i - \Gamma_{j \rightarrow i}, \quad (11)$$

with Γ_i being the total decay rate of vacuum i . This equation can be solved in the form of the series (see Appendix A for more details):

$$p_i = \sum_{N=0}^{\infty} p_{i,N}, \quad p_{i,0} = \frac{J_i}{\Gamma_i}, \quad p_{i,N} = \frac{1}{\Gamma_i} \sum_{j_1, \dots, j_N} J_{j_1} \frac{\Gamma_{j_1 \rightarrow j_2}}{\Gamma_{j_1}} \frac{\Gamma_{j_2 \rightarrow j_3}}{\Gamma_{j_2}} \dots \frac{\Gamma_{j_N \rightarrow i}}{\Gamma_{j_N}}. \quad (12)$$

We note that the structure of (12) is straightforward to interpret. In the sum over N , the N^{th} term corresponds to N tunneling processes between creation from nothing and vacuum i . Each tunneling process is suppressed by a small number coming from a ‘branching ratio’ of the form $\Gamma_{j_s \rightarrow j_{s+1}}/\Gamma_{j_s} \ll 1$. It becomes apparent that we need to investigate both the creation rates J_i as well as the tunneling rates $\Gamma_{i \rightarrow j}$ in more detail.

4.2 Creation rates

The traditional proposals for the creation of universes from nothing are the Hartle-Hawking no boundary [78] and the Linde/Vilenkin tunneling [79, 80] proposals. Both study the quantum mechanical nucleation of compact spherical universes which eventually go on shell and expand. In both cases, the basic ingredient is an instanton geometry which describes euclidean dS, i.e. a 4-sphere. Its action $\mathcal{S}$ is given by minus the dS entropy, $\mathcal{S} = -S_{dS}$, with

$$S_{dS} = 24\pi^2 M_P^4 / V_{dS} = 8\pi^2 M_P^2 \ell_{dS}^2. \quad (13)$$

Here V_{dS} is the energy density and ℓ_{dS} the dS radius. The proposed rates are

$$J \propto \begin{cases} \exp(S_{dS}) & \text{Hartle-Hawking [82]} \\ \exp(-S_{dS}) & \text{Linde/Vilenkin [79, 80]} \end{cases}. \quad (14)$$

Clearly Hartle-Hawking favor small and Linde/Vilenkin favor large vacuum energies.

A different proposal for creating universes from nothing was introduced in [82, 83] and further developed in [70, 84, 85]. Its key ingredient are end-of-the-world (ETW) branes, representing boundaries of 4d spacetime. The existence of such boundaries allows for the nucleation of universes with the geometry of a ball, bounded by a spherical ETW brane. The prime example of an ETW brane arises in 5d to 4d compactifications, where the S^1 compact space can shrink to zero size at a codimension-one locus over the 4d base. This gives rise to Witten’s famous ‘bubble of nothing’ [69]. The creation process we are discussing may be thought of as the inverse process, i.e. a ‘bubble of something’ [76].⁷ The relevance of this creation process is enhanced by the cobordism conjecture [111], implying that every vacuum of a complete theory of quantum gravity exhibits ETW branes. Further work on ETW branes include [75, 112–114]. An explicit geometry of such ETW branes in the type IIB flux landscape was provided in [76], where first steps towards an analysis of the corresponding creation process in string theory were also made.

⁷In [70], the term ‘bubble from nothing’ was used instead.

As in the no boundary proposal, ball-shaped universes nucleate quantum-mechanically, go on-shell and subsequently expand. By contrast, however, this process also allows for the creation of Minkowski and AdS vacua. The bubble-of-something process produces open Friedmann universes, similar to the interiors of Coleman-de Luccia (CdL) bubbles [68].

Recall that we are only interested in observers on post-inflationary reheating surfaces and that we treat inflationary plateaus as short-lived dS vacua. We may hence restrict our attention to the bubble-of-something creation of dS. In the regime of a thin ETW brane, the relevant euclidean instanton action reads [76]

$$\mathcal{S} = -4\pi^2 M_P^2 \ell_{dS}^2 \left(1 \pm \sqrt{\frac{T_4^2 \ell_{dS}^2}{T_4^2 \ell_{dS}^2 + 4M_P^4}} \right). \quad (15)$$

Here the upper sign is relevant for negative tension, $T_4 < 0$, and the lower sign for $T_4 > 0$. Using (13), it is straightforward to see that $\mathcal{S} > -S_{dS}$, with equality holding in the limit $T_4 \rightarrow -\infty$.⁸ If one uses the ‘Hartle-Hawking sign-choice’, i.e. $J \sim \exp(-\mathcal{S})$, it then follows that the standard no-boundary-process is more likely than the bubble-of-something process. By contrast, adopting the ‘Linde/Vilenkin sign-choice’, i.e. $J \sim \exp(\mathcal{S})$, the bubble-of-something process wins. Specifically, the smallest enhancement arises if $T_4 < 0$ and $T_4^2 \ell_{dS}^2 \gg M_P^4$, in which case we have

$$\mathcal{S} \simeq -S_{dS} + 8\pi^2 \frac{M_P^6}{T_4^2}. \quad (16)$$

The relative enhancement of the bubble-of-something relative to the Linde/Vilenkin process is then by

$$\exp(8\pi^2 M_P^6 / T_4^2), \quad (17)$$

which is enormous whenever the brane tension is sub-Planckian. In all other cases, the enhancement is by an exponent of the order of S_{dS} , i.e. even stronger. Thus, if the Linde/Vilenkin sign choice is the right one and ETW branes exist in the validity range of the EFT, the bubble-of-something process is always completely dominant. The strongest enhancement occurs for $T_4 \ell_{dS} \gg M_P^2$, in which case

$$\mathcal{S} \simeq -8\pi^2 \frac{M_P^6}{T_4^2}. \quad (18)$$

Finally, another possibility to create de Sitter vacua from nothing, again making use of the existence of ETW branes, was suggested in [81]. Analogously to the ‘No Boundary Proposal’, this ‘Boundary Proposal’ is based on an off-shell spherical universe in our past. However, instead

⁸A bubble of something may be thought of as the creation of a universe together with a bubble of nothing. In the limit $T_4 \rightarrow -\infty$, the initial size of the bubble of nothing vanishes. In our understanding, the bubble of something in the limit $T_4 \rightarrow -\infty$ may correspond to the ‘creation from nothingness’ considered in [115].

of starting at zero size, our spatial 3-sphere starts its existence at a spacelike negative-tension ETW brane. The euclidean action for the corresponding instanton reads

$$\mathcal{S} = -8\pi^2 M_P^2 \ell_{dS}^2 \sqrt{\frac{T_4^2 \ell_{dS}^2}{T_4^2 \ell_{dS}^2 + 4M_P^4}}. \quad (19)$$

The boundary instanton again satisfies $\mathcal{S} > -S_{dS}$. It is hence again suppressed relative to the standard no-boundary process in case of the Hartle-Hawking sign choice and enhanced in case of the Linde/Vilenkin sign choice. As before, the smallest possible enhancement occurs for $T_4 < 0$ and $T_4^2 \ell_{dS}^2 \gg M_P^4$, in which case the exponent is

$$\mathcal{S} \simeq -S_{dS} + 16\pi^2 \frac{M_P^6}{T_4^2}. \quad (20)$$

Thus, whenever the EFT is valid, the Boundary Proposal rate completely dominates over the Linde-Vilenkin rate.

The enhancement is the strongest in the limit $|T_4| \ell_{dS} \ll M_P^2$, in which case we find

$$\mathcal{S} = -4\pi^2 |T_4| \ell_{dS}^3. \quad (21)$$

While the exponential suppression completely disappears for $T_4 \ll \ell_{dS}^{-3}$, this has to be taken with some caution: In the formulae above, non-exponential prefactors have remained undetermined, so more research is needed to quantify the outcome of this proposal in the limit of small T_4 .

4.3 Nontrivial topology

The Linde/Vilenkin proposal for the creation of universes from nothing can be viewed as the statement that small spheres can quantum mechanically fluctuate from nothing and then undergo a tunneling process through a potential barrier to become on-shell. One manifestation of this interpretation is the fact that the Linde/Vilenkin wavefunction Ψ_{LV} satisfies an inhomogeneous WDW equation with a localized source for zero-size universes, see e.g. [116].⁹ The potential barrier arises from the positive spatial curvature of the spherical universe.

Now, as observed in [86–88], if spherical universes at vanishing size can fluctuate from nothing, it appears inevitable that compact universes with different topology, e.g. small tori, can also nucleate at little to no cost. Since there is no spatial curvature, there is no potential barrier and hence no tunneling suppression for going on-shell. As a result, the creation of a toroidal or negatively curved compact universe appears to be far more likely than the creation of ordinary de Sitter space with spherical spatial section. Moreover, there is no exponential

⁹By contrast, the Hartle-Hawking wavefunction is real and may be viewed as a standing wave, not requiring any source [116].

preference for a large cosmological constant of the nucleated vacuum.¹⁰ It was speculated that subleading effects such as Casimir energy [86] or the gradient energies of scalar fields [88] determine the most likely properties of the created universe.

Regarding applications to the string landscape, one might expect that also details of the compactification, e.g. the geometry of the internal manifold, affect which vacua are preferred in the creation process. Work in this direction includes [119–123]. However, in the present analysis we adopt a 4d EFT approach and hence do not account for such effects. This amounts to assigning all dS universes which avoid a potential barrier by their nontrivial topology a similar probability for creation from nothing.

4.4 Tunneling rates

In the thin wall limit, tunneling rates are determined by the domain wall tension T , by the true- and false-vacuum energy densities V_t, V_f , and the Planck mass $M_P^2 = 1/(8\pi G)$ [68]. The false-vacuum decay rate Γ is conveniently expressed using the dimensionless variables [68, 124]

$$x = \frac{3T^2}{4M_P^2(V_f - V_t)}, \quad y = \frac{V_f + V_t}{V_f - V_t}, \quad r(x, y) = 2 \frac{1 + xy - \sqrt{1 + 2xy + x^2}}{x^2(y^2 - 1)\sqrt{1 + 2xy + x^2}}. \quad (22)$$

It reads

$$\Gamma \sim \exp(-B), \quad \text{with} \quad B = \frac{27\pi^2 T^4}{2(V_f - V_t)^3} r(x, y). \quad (23)$$

The importance of gravitational effects is measured by x . In the regime $x \ll 1$, $xy \ll 1$, the expression for the decay rate reduces to the field theory result, given by $r(x, y) \rightarrow 1$. If x or xy become $\mathcal{O}(1)$ or larger, gravitational effects are important and are quantified by the function $r(x, y)$. In the limit $x \gg 1$, $x \gg y$, the tunneling exponent B approaches the entropy S_f of the decaying vacuum. Hence, the decay occurs on the recurrence time scale of the corresponding dS space. See App. B for some further details and useful formulae.

5 Tunneling rates in the string landscape

In this section, we study tunneling rates for the most promising dS vacuum candidates in the string landscape: Type IIB flux compactifications on Calabi-Yau orientifolds with Kahler moduli stabilization à la KKLT [3] or LVS [6]. Some of the following results have appeared elsewhere in the literature. Our summary of the various rates can be of general interest in studies of the string landscape.

¹⁰It was argued in [117, 118] that the creation of spherical universes may, in fact, also be free of any tunneling suppression. The reason is that, given the Linde/Vilenkin sign choice, fluctuations are enhanced and these compensate the curvature term in the action which was responsible for the suppression.

We will show that, to the best of our present understanding, the dominant decay process of KKLT/LVS-type vacua is the destruction of the uplift. Tunneling between two dS vacua occurs only via flux transitions and is generically close to being maximally suppressed. While tunneling to decompactification is generically slower than the destruction of the uplift, it still proceeds much faster than tunneling between dS vacua.

5.1 Tunneling to decompactification

This tunneling transition always exists and its final state is 10d Minkowski space. The tension of the domain wall has been estimated for both KKLT and LVS models [125]. For generic vacua, the resulting decay exponent B is given by

$$B_{\text{dec}} = \frac{S_f}{(1 + 1/x)^2} \quad \text{with} \quad x \lesssim 1, \quad (24)$$

and is thus smaller but of a similar order of magnitude as the dS entropy S_f . Details of the analysis are given in Appendix C. If, however, the vacuum energy of the dS space is fine-tuned to be very close to zero, the decay exponent becomes even larger and approaches S_f [3, 125].

5.2 Flux transitions

Flux transitions are mediated by domain walls made from D5/NS5 branes wrapped on CY 3-cycles. They can always occur and their final state can be either a dS or an AdS vacuum. The latter case will not be discussed here since, as will become clear in the next subsection, fast transitions to AdS always exist due to the ‘destruction of the uplift’. Thus, the decay to AdS via flux transitions is not essential.

Generically, both transitions from KKLT to KKLT as well as from LVS to LVS occur in a regime where gravitational effects are large. In both cases, the decay exponent can be approximated as

$$B_{\text{flux}} = S_f - \frac{64\pi^2 M_P^6}{T^2}, \quad (25)$$

with $64\pi^2 M_P^6/T^2 \ll S_f$. This is shown in App. C, which in part reviews the analysis of [126, 127]. It is easy to convince oneself that this result also holds for transitions from KKLT to LVS and vice versa. By comparing with (24), we notice that generic flux transitions are significantly suppressed compared to tunneling to decompactification.

In [128], the backreaction of the D5/NS5-brane domain walls on the volume has been studied. It was found that for generic transitions in the KKLT and LVS landscape, the backreaction is relevant and it has even been questioned whether flux transitions can occur at all. In [129] it was argued that non-instantonic transitions are nevertheless expected as long as there is no symmetry forbidding them. Decay rates for such transitions were estimated in [130] in

toy-landscapes. For situations where also a CdL transition exists, it was found that the non-instantonic decay exponent for down tunneling is slightly larger than the corresponding CdL exponent. In the case of downtunneling-events, Eq. (25) therefore both represents a lower bound on the decay exponent and may be viewed as a reasonable approximation so that Eq. (25) can serve as an order-of-magnitude estimate even if instanton transitions do not exist. However, for up-tunneling it was argued that the decay exponent can be significantly smaller than its CdL analogon.

Finally, non-generic flux transitions featuring a much smaller decay exponent than (25) could exist. Specifically, if the domain wall tension T can be tuned very small, the decay occurs in the field-theoretic regime. In principle, a lower bound on the domain wall tension follows from a BPS argument. Although SUSY is broken in our cases of interest, branes can nevertheless be ‘near BPS’ with respect to the SUSY theory of the CY before fluxes and uplifting. As explained in [131], BPS domain walls cannot arise from wrapping a brane on an arbitrarily small cycle without the loss of geometric control. As a result, typical ‘near BPS’ domain walls will not have a tension small enough to lead to a decay in the field-theoretic regime. However, controlled small-tension domain walls can nevertheless arise due to warping, such as in the presence of Klebanov-Strassler throats. Flux transitions in this region are then similar to the KPV decay [132], with a D5/NS5 wrapped on the A -cycle of a KS throat (cf. Sect. 5.3). Clearly, we also have to avoid producing too many D3 branes in such a warped-down flux transition, such that any pre-existing $\overline{D3}$ uplift in a different throat does not get destroyed. In summery, while they are highly non-generic and probably rare, we have a priori no reason to exclude such transitions altogether.

5.3 The destruction of the uplift

Both KKLT and LVS models realize dS by uplifting an AdS minimum. The most-studied concrete realization is the $\overline{D3}$ uplift, which can decay by the KPV transition [132]. This process, in which the $\overline{D3}$ brane(s) annihilates against flux, generally occurs in the field-theoretic regime with decay exponent [132, 133] (cf. App. C)

$$B_{\text{KPV}} = \frac{27b^6 M^6 g_s}{2048\pi (N_{\overline{D3}})^3}. \quad (26)$$

Here $b \approx 0.932$, M is a flux number, g_s the string coupling and $N_{\overline{D3}}$ the number of decaying $\overline{D3}$ branes.

It is clear that the decay exponent (26) will very generally not be as large as for flux or decompactification transitions. Nevertheless, choosing appropriate values for g_s , M and $N_{\overline{D3}}$ appears to allow for $B_{\text{KPV}} \gg 1$ and hence long-lived dS. However, recent work shows that realizing such a parameter choice is non-trivial – both in the KKLT [134–136] and the LVS [137–142] case: On the one hand, singularity and tadpole constraints limit the value of M . On the other hand, at small M the KPV process is subject to curvature corrections to the

action of the NS5 brane which mediates the decay [140–142]. Thus, if a metastable $\overline{D3}$ -uplift can be achieved at all, we expect settings where B_{KPV} is relatively small (maybe outside the regime where the derivation of (26) is reliable) to be more common. This strengthens our point that KPV transitions occurs much faster than decompactification or generic flux decays. As discussed in the last subsection, a non-generic flux transition could be similarly likely.

Let us briefly comment on the possibility that a KPV decay takes us from dS to dS: In principle, this is possible if there are two uplifts realized in two throats. However, the $\overline{D3}$ decay in one throat produces many D-branes which migrate and are very likely to annihilate all the $\overline{D3}$ s of the other throat. Thus, following [143], we conclude that KPV processes always or almost always end in AdS.

Of course, we should also consider different uplifting proposals, such as the F -term [144] or T-brane [145] uplift (for recent work on the former see [146, 147]). To the best of our understanding, all ideas of this type share the feature of a metastable SUSY-breaking minimum protected by a small barrier, such that a relatively low-energy process can return the model to a non-uplifted AdS phase. We then expect that also the decay of these alternative uplifts proceeds in the field-theory regime and hence much faster than flux or decompactification decays.

5.4 Bubbles of nothing

A bubble of nothing is a vacuum decay process discovered in [69] (cf. [70–75] for a selection of recent work). In the simplest case, the underlying geometry is $\mathbb{M}_4 \times S^1$, i.e. 4d flat space with one internal dimension. The relevant instanton is obtained by letting the internal S^1 shrink to zero size over the locus of an S^3 in the non-compact space. From the 4d perspective, this locus is an ETW brane with negative tension (the 4d EFT description was in particular developed in [76]). In the Lorentzian evolution, this ETW brane expands and eats up all space. According to the cobordism conjecture [111], ETW branes always exist and bubbles of nothing should hence be considered as a potential decay process for all string models with broken SUSY [72].

Specifically for type IIB CY orientifold compactifications with O3/O7 planes, which are most relevant in the present context, an ETW brane was constructed in [76]. It is based on an O5 orientifolding reflecting one of the non-compact dimensions. The preliminary analysis of the tension suggests that the induced decays to nothing occur faster than tunneling to decompactification and generic flux transitions, but are nevertheless unlikely to compete with the destruction of the uplift. It would, however, be important to understand whether other ETW branes with more negative tension and hence higher decay rates exist.

6 Explicit predictions in the (string theory) multiverse

We now return to the study of anthropic predictions in the multiverse. While much of what follows is generic and applies to any underlying landscape, some of the arguments used in this section rely on the string-specific results of Sect. 5. As discussed in Sect. 2.2, $p(\text{obs}, i)$, the

probability of finding an observer in vacuum i , is mainly determined by the value $p_{\text{inf}(i)}$. This is the quantity we will now try to calculate on the basis of (12).

6.1 Creation from nothing or tunneling from a higher-energy dS?

In the solution to the rate equation (12), the term $p_{i,N}$ corresponds to the contribution where i is reached via N tunneling transitions after creation from nothing. Each transition comes with an additional suppression which may be approximately quantified by a factor

$$\frac{\sum_{k \in \text{dS}} \Gamma_{j \rightarrow k}}{\Gamma_j} = \frac{\sum_{k \in \text{dS}} \Gamma_{j \rightarrow k}}{\sum_{k \in \text{dS}} \Gamma_{j \rightarrow k} + \sum_{y \in \text{Terminals}} \Gamma_{j \rightarrow y}}. \quad (27)$$

Here, j denotes a fixed dS vacuum, k runs over all other dS vacua and y runs over all terminal vacua and ‘nothing’. In the KKLT/LVS landscape analyzed in Sect. 5, dS-dS transitions are generally maximally suppressed. By contrast, transitions to decompactification, to nothing, and to AdS are much faster. The fact that dS-dS transition rates are strongly suppressed compared to the total dS decay rate could be a general feature of the string landscape and it would be interesting to explore this further. In addition, from the difficulties of constructing dS vacua in string theory, one may expect that they are vastly outnumbered by AdS vacua. We conclude that $\sum_{k \in \text{dS}} \Gamma_{j \rightarrow k} / \Gamma_j \ll 1$ holds for almost all dS vacua j .¹¹ As a result, the sum (12) converges quickly and we may use the approximation

$$p_i \simeq p_{i,0} + p_{i,1}. \quad (28)$$

In this approximation, vacuum i can either be directly created from nothing (corresponding to the term $p_{i,0}$) or via one intermediate tunneling process (corresponding to $p_{i,1}$).

We now pick an anthropically viable vacuum i and apply (28) to its inflationary plateau which leads to:

$$p_{\text{inf}(i)} \simeq p_{\text{inf}(i),0} + p_{\text{inf}(i),1} = \frac{1}{\Gamma_{\text{inf}(i)}} \left(J_{\text{inf}(i)} + \sum_{o \neq \text{inf}(i)} J_o \frac{\Gamma_{o \rightarrow \text{inf}(i)}}{\Gamma_o} \right). \quad (29)$$

The relative size of the two terms in parenthesis determines whether the direct creation from nothing or tunneling in the background of another dS space is the most likely process to induce reheating in vacuum i . The summation index o in the second term indicates that we sum over all vacua ‘other’ than $\text{inf}(i)$. From (12), we observe that each p_i contains an overall factor $1/\Gamma_i$. The appearance of this factor is intuitively clear: A smaller decay rate makes a vacuum longer-lived and thus increases the probability of *finding* it in the multiverse. However, as we

¹¹A minor exception arises because we treat inflationary plateaus as separate dS vacua. If j stands for such a plateau, the factor (27) is obviously very close to unity in cases where the rolling takes us to a metastable dS minimum. Since inflationary plateaus are presumably rare, we consider it justified to ignore this subtlety.

discussed in Footnote 3, $p(\text{obs}, i)$ should be proportional to the probability of *starting* inflation in $\text{inf}(i)$ (leading to reheating in vacuum i) and should hence be independent of $\Gamma_{\text{inf}(i)}$. An improved estimate of $p(\text{obs}, i)/p(\text{obs}, j)$, with i, j anthropically viable vacua, is thus given by

$$\frac{p(\text{obs}, i)}{p(\text{obs}, j)} \simeq \frac{\Gamma_{\text{inf}(i)} p_{\text{inf}(i)}}{\Gamma_{\text{inf}(j)} p_{\text{inf}(j)}}. \quad (30)$$

Although we expect $\Gamma_{\text{inf}(i)}/\Gamma_{\text{inf}(j)}$ to be a subleading effect and thus (30) to be only a minor refinement of (10), we shall employ the refined ratio of probabilities (30) from here on.

In what follows, it will be convenient to work mostly with the decay exponents B rather than with the decay rates Γ . We will label these exponents in the same way as rates, i.e.

$$\Gamma_i \propto \exp(-B_i), \quad \Gamma_o \propto \exp(-B_o), \quad \Gamma_{o \rightarrow \text{inf}(i)} \propto \exp(-B_{o \rightarrow \text{inf}(i)}), \quad \text{etc.} \quad (31)$$

As before, we denote by S_i the entropy of vacuum i and by $\mathcal{S}_i$ the euclidean action of the instanton responsible for the creation of vacuum i from nothing. In this notation, the inequality

$$0 < B_o < B_{o \rightarrow \text{inf}(i)} < S_o \quad (32)$$

always holds. From the analysis of Sect. 5, we learned that in a KKLT/LVS landscape, generic transitions between dS vacua are close to being maximally suppressed while decays to AdS can proceed via the destruction of the uplift and are hence much faster. In this type of landscape, we have the refined statement

$$0 < B_o \ll B_{o \rightarrow \text{inf}(i)} \simeq S_o. \quad (33)$$

Evidently, in a landscape without eternal inflation, tunneling is a subdominant effect such that inflationary vacua are mainly created directly from nothing with rate $J_{\text{inf}(i)}$.

6.1.1 Hartle-Hawking-type creation

When adopting the sign choice of Hartle and Hawking, i.e. $J \sim \exp(-\mathcal{S})$, Bubble-of-Something and Boundary creation processes are always subdominant and we can focus on the standard creation of a spherical universe from nothing. Since the terms in the sum of $p_{\text{inf}(i),1}$ (29) are all exponentially large or small, we approximate the sum by its largest term. We then have

$$\frac{p_{\text{inf}(i),1}}{p_{\text{inf}(i),0}} = \frac{J_o \Gamma_{o \rightarrow \text{inf}(i)} / \Gamma_o}{J_{\text{inf}(i)}} \simeq \exp(S_o - B_{o \rightarrow \text{inf}(i)} + B_o - S_{\text{inf}(i)}). \quad (34)$$

If $S_{\text{inf}(i)} > S_o$, it is immediately clear that the direct creation of the inflationary state $\text{inf}(i)$ from nothing is more likely than tunneling to that state from vacuum o .

If $S_{\text{inf}(i)} < S_o$, i.e. when uptunneling needs to occur, more thought is required: In this case, we may use the general identity¹² $B_{o \rightarrow \text{inf}(i)} = S_o - S_{\text{inf}(i)} + B_{\text{inf}(i) \rightarrow o}$ to simplify (34) as follows:

$$\frac{p_{\text{inf}(i),1}}{p_{\text{inf}(i),0}} = \frac{J_o \Gamma_{o \rightarrow \text{inf}(i)} / \Gamma_o}{J_{\text{inf}(i)}} \simeq \exp(-B_{\text{inf}(i) \rightarrow o} + B_o). \quad (35)$$

As shown in Sect. 5, the KKLT/LVS landscape generically has $B_{\text{inf}(i) \rightarrow o} \simeq S_{\text{inf}(i)}$. By contrast, B_o is in the field-theoretic regime, determined by the destruction of the uplift (see Sect. 5.3). As discussed below (26), especially when considering the $\overline{D3}$ -brane uplift, B_o cannot be very large. Hence, one generally expects $B_o < B_{\text{inf}(i) \rightarrow o}$. Let us be more specific about this point:

CMB data restricts the tensor to scalar ratio to $r < 0.04$ [151] leading to a bound on the Hubble scale during the inflationary era that occurred in our own past: $H_{\text{inflation}} \lesssim 0.4 \cdot 10^{-4} M_P$. Hence the inflationary plateau of our universe satisfies $S_{\text{inflation}} = 8\pi^2 M_P^2 / H_{\text{inflation}}^2 \gtrsim 5 \cdot 10^{10}$. For consistency of the uplift (see e.g. [134–142]), the flux number M in (26) cannot be larger than $\mathcal{O}(100)$. As a result,¹³ $B_{\text{KPV}} \lesssim 10^8$. Recalling the great difficulties in realizing a metastable uplift (briefly mentioned below (26)), it becomes clear that our assumptions are very generous and that, for inflationary plateaus that are observationally viable, $B_o < B_{\text{inf}(i) \rightarrow o} \sim S_{\text{inf}(i)}$ appears unavoidable. As a result, direct creation from nothing is the dominant way of creating inflationary universes of such type.

However, for inflationary plateaus at very high energies and for vacua o with a relatively stable uplift (possibly not the $\overline{D3}$ -uplift), $B_o > B_{\text{inf}(i) \rightarrow o}$ appears to be generally possible. Moreover, if non-generic flux transitions between dS vacua are relevant, $B_{\text{inf}(i) \rightarrow o} < S_{\text{inf}(i)}$ and thus $B_o > B_{\text{inf}(i) \rightarrow o}$ can also be achieved. Finally, we could be dealing with a landscape which is very different from any KKLT/LVS-based expectations. Also in this case $B_o > B_{\text{inf}(i) \rightarrow o}$ could hold for certain inflationary plateaus and other vacua o . In these cases, the inflationary plateau $\text{inf}(i)$ is more likely to arise through tunneling than through direct creation from nothing à la Hartle and Hawking.

In summary, expectations based on an LVS/KKLT-type landscape imply that for the Hartle-Hawking sign choice inflationary plateaus are directly created from nothing rather than through tunneling events. A key possibility for how this might fail is if low-scale dS vacua are much more long-lived than the known fragile uplifts suggest.

6.1.2 Linde/Vilenkin creation with nontrivial topology

In case the creation of toric or other universes with nontrivial topology is considered, $J_o \simeq J_{\text{inf}(i)}$ holds to leading order for all o . As a result, the direct creation from nothing of vacuum i in an

¹²This can fail if, as argued in [130, 148] (see also [149, 150]), detailed balance does not hold such that uptunneling rates are much larger than naively expected. The relevant transitions are such that, from the outside observer's perspective, the created vacuum is behind a black hole horizon. It is not clear how to treat this in the local WDW framework and we leave this issue to future work.

¹³Note that the warp factor suppressing the uplift is $\exp(8\pi N/3g_s M^2)$. If we very generously require this to only be $\exp(-10)$ and restrict the tadpole by $N < 10^3$, we find $g_s M^2 \lesssim 10^3$. Then (26) gives $B_{\text{KPV}} \lesssim 3 \times 10^8$.

inflationary state is the most likely scenario (recall, however, the caveat mentioned at the end of Sect. 4.3).

6.1.3 Linde/Vilenkin creation with spherical topology

In this approach, the Euclidean action $\mathcal{S}$ of the creation instanton is related to the creation rate by $J \sim \exp(+\mathcal{S})$. If no ETW branes exist, we have $-\mathcal{S} = |\mathcal{S}| = S_{dS}$. Otherwise, a bubble of something or a boundary instanton is the most likely creation process, with $-\mathcal{S} = |\mathcal{S}| < S_{dS}$ given in (15),(19). Let us start with the part of analysis which applies to both cases – with and without ETW branes:

As before, a key quantity of interest is the ratio

$$\frac{p_{\text{inf}(i),1}}{p_{\text{inf}(i),0}} = \frac{J_o \Gamma_{o \rightarrow \text{inf}(i)} / \Gamma_o}{J_{\text{inf}(i)}} \simeq \exp(-|\mathcal{S}_o| - B_{o \rightarrow \text{inf}(i)} + B_o + |\mathcal{S}_{\text{inf}(i)}|). \quad (36)$$

Let us first see under which conditions, for given $\text{inf}(i)$, there exists a vacuum o making this ratio large. Clearly, we have to focus on vacua o with small $|\mathcal{S}_o|$. We may then assume $|\mathcal{S}_o| < |\mathcal{S}_{\text{inf}(i)}|$.¹⁴ Using (32), we can estimate

$$-|\mathcal{S}_o| - B_{o \rightarrow \text{inf}(i)} + B_o + |\mathcal{S}_{\text{inf}(i)}| > -|\mathcal{S}_o| - B_{o \rightarrow \text{inf}(i)} + |\mathcal{S}_{\text{inf}(i)}| > -(|\mathcal{S}_o| + S_o) + |\mathcal{S}_{\text{inf}(i)}|. \quad (37)$$

We conclude that the condition

$$|\mathcal{S}_{\text{inf}(i)}| > S_o + |\mathcal{S}_o| \quad (38)$$

is sufficient to ensure that $p_{\text{inf}(i),1}/p_{\text{inf}(i),0} \gg 1$ and hence that $\text{inf}(i)$ is more likely to arise through tunneling from o than via creation from nothing.

Let us now ask under which conditions $p_{\text{inf}(i),1}/p_{\text{inf}(i),0}$ always remains small. For this, we recall that in the KKLT/LVS landscape (33) holds. This implies

$$-|\mathcal{S}_o| - B_{o \rightarrow \text{inf}(i)} + B_o + |\mathcal{S}_{\text{inf}(i)}| \simeq -|\mathcal{S}_o| - S_o + |\mathcal{S}_{\text{inf}(i)}|. \quad (39)$$

We conclude that $p_{\text{inf}(i),1}/p_{\text{inf}(i),0} \ll 1$ and hence that $\text{inf}(i)$ is most likely to be created directly from nothing if

$$|\mathcal{S}_{\text{inf}(i)}| < S_o + |\mathcal{S}_o| \quad (40)$$

holds for all o .

To be more specific, we have to distinguish different cases:

¹⁴This of course requires that, for all anthropically viable vacua i , vacua o satisfying $|\mathcal{S}_o| < |\mathcal{S}_{\text{inf}(i)}|$ exist. We believe this is a good assumption since most vacua are likely not inflationary plateaus leading to an anthropically viable vacuum.

No (relevant) ETW branes: As discussed around Eq. (15),(19), if there are no branes satisfying $T_4 \ell_{\text{inf}(i)} \gg M_P^2$ or $0 < -T_4 \ell_{\text{inf}(i)} \ll M_P^2$ (or if ETW branes do not exist at all), then $|\mathcal{S}_{\text{inf}(i)}|$ is of order $S_{\text{inf}(i)}$. Moreover, $S_{\text{inf}(i)}$ is expected to be much larger than the smallest S_o in the landscape. Now, using the fact that $0 < |\mathcal{S}_o| < S_o$, we arrive at the conclusion that (38) always holds and thus the creation of $\text{inf}(i)$ in the background of some o is more likely than the direct creation of $\text{inf}(i)$ from nothing.

Relevant ETW branes: If ETW branes with $T_4 \gg M_P^2/\ell_{\text{inf}(i)}$ exist, $|\mathcal{S}_{\text{inf}(i)}|$ is given by the bubble of something result (18)

$$|\mathcal{S}_{\text{inf}(i)}| \simeq 8\pi^2 \frac{M_P^6}{T_4^2}. \quad (41)$$

It is then possible that (40) holds for all vacua o , making the direct creation from nothing most likely. More concretely, if the landscape contains a vacuum with minimal de Sitter radius $\ell_{\min}$ (i.e. with the largest cosmological constant), then condition (40) is always fulfilled if $T_4 \gg M_P^2/\ell_{\min}$. Clearly, an ETW brane tension is similar to a 3d cosmological constant. From what we know about the landscape, one may argue that it is easier to obtain large energy densities in 3d than in 4d. Hence it is quite plausible that (40) does indeed hold for many inflationary plateaus $\text{inf}(i)$, which are hence predominantly created by a bubble-of-something process. Obviously, this holds true if the landscape has no dS vacua.

Alternatively, if ETW branes with $0 < -T_4 \ell_{\text{inf}(i)} \ll M_P^2$ exist, the boundary proposal becomes relevant and the corresponding euclidean action is given by (21):

$$|\mathcal{S}_{\text{inf}(i)}| \simeq 4\pi^2 |T_4| \ell_{\text{inf}(i)}^3. \quad (42)$$

Again, the direct creation from nothing can easily be the dominant process leading to a cosmology which starts on the inflationary plateau $\text{inf}(i)$. In particular, this will be the case if the landscape has no dS vacua and if even positive-tension ETW branes are forbidden by Swampland constraints.

We see that ETW branes can drastically change phenomenological predictions in the multiverse. This is particularly true in the presence of strong Swampland constraints on models with positive energy density.

6.2 Most likely scale of inflation

Let us now try to use (10) and the dependencies of creation and decay rates on the vacuum energies to make an anthropic prediction for the scale of inflation.¹⁵

¹⁵Our work is focused on the energy scale at the beginning of inflation. We shall assume that it is similar to the phenomenologically relevant scale associated with the CMB. Clearly, in principle there are models where these two scales are very different.

6.2.1 Hartle-Hawking-type creation

We have discussed below (34) that most inflationary plateaus, and especially the ones that are still in the observationally viable regime, get dominantly created directly from nothing. Using (30) and (12) to zeroth order, we find

$$p(\text{obs}, i) \propto J_{\text{inf}(i)} \propto \exp(S_{\text{inf}(i)}) = \exp(24\pi^2 M_P^4 / V_{\text{inf}(i)}) . \quad (43)$$

We see that the landscape dynamics gives an exponential bias towards low-scale inflation. The details of tunneling transitions do not play an important role.

This result leads to an immediate tension with observations, as has recently been made explicit in [90]. The exponential preference for a low scale of inflation leads to an overwhelming probability for having the shortest possible period of inflation that is still compatible with the formation of anthropic observers. However, CMB measurements such as the observed spatial flatness of our Universe show inflation lasted longer than anthropically required. Therefore, creation out of nothing in the Hartle-Hawking scenario appears to be incompatible with data.

In the discussion below (34), we have specified what precisely it would take to find inflationary plateaus which are populated through tunneling: At least in the KKLT/LVS landscape, they can only exist at very high energies or if very stable uplifts can be realized. Let us nevertheless try to understand whether such plateaus (if existent) can be anthropically preferred. For this purpose, we assume that $\text{inf}(i)$ is a low-energy plateau with dominant creation directly from nothing and $\text{inf}(j)$ a high-energy plateau with dominant creation in the background of some high entropy dS o . To compare them, we use (30),(12) and obtain

$$\frac{p(\text{obs}, i)}{p(\text{obs}, j)} \simeq \frac{\Gamma_{\text{inf}(i)} p_{\text{inf}(i),0}}{\Gamma_{\text{inf}(j)} p_{\text{inf}(j),1}} = \frac{J_{\text{inf}(i)}}{J_o \Gamma_{o \rightarrow \text{inf}(j)} / \Gamma_o} \simeq \exp(S_{\text{inf}(i)} - S_o + B_{o \rightarrow \text{inf}(j)} - B_o) . \quad (44)$$

Employing the relation $B_{o \rightarrow \text{inf}(j)} = S_o - S_{\text{inf}(j)} + B_{\text{inf}(j) \rightarrow o}$ this simplifies to

$$\frac{p(\text{obs}, i)}{p(\text{obs}, j)} \simeq \exp(S_{\text{inf}(i)} - S_{\text{inf}(j)} + B_{\text{inf}(j) \rightarrow o} - B_o) . \quad (45)$$

At least in the KKLT/LVS landscape, we expect $p(\text{obs}, i)/p(\text{obs}, j) \gg 1$ since $S_{\text{inf}(i)} \gg S_{\text{inf}(j)} + B_o$ is expected for low-energy plateaus $\text{inf}(i)$ (recall the discussion below (35)). As a result, low-scale inflationary regions directly created from nothing are most likely to be realized and the conclusions below (43) still hold.¹⁶

6.2.2 Linde/Vilenkin creation with nontrivial topology:

If the creation of small universes with non-trivial topology is possible, then all (quasi-)dS vacua are equally likely to be created from nothing. This expectation holds at leading order,

¹⁶These conclusions, including the claim that the Hartle-Hawking sign choice is ruled out by observations, could be altered by detailed-balance-violating, fast upward transitions such as [130, 148]. It would be interesting to study how such transition should be treated in the local WDW framework.

i.e. disregarding non-exponential effects, and in the 4d EFT. Accepting these caveats, one may conclude that the distribution of the scale of inflation measured by anthropic observers is equal to the distribution of the number of inflationary plateaus in the landscape as a function of energy. Such a distribution has been estimated for a toy-model of the string landscape [152].¹⁷ It was found that the number frequency is highest for small-field models of plateau inflation, which are favored by recent cosmological measurements [151, 154]. Among the small-field scenarios, those with the largest scale of inflation appear to be preferred [152].

Key challenges for the future are the analysis of the creation with non-trivial topology at sub-exponential level and beyond the 4d EFT, i.e. including effects of the compact space. Moreover, we obviously need the distribution of inflationary plateaus in the actual string landscape.

6.2.3 Linde/Vilenkin type creation with spherical topology:

For the Linde/Vilenkin type creation with spherical topology, we again distinguish between the following cases:

No (relevant) ETW branes: If there are no relevant ETW branes to significantly change the traditional Linde/Vilenkin result, the creation of i in the background of some other vacuum o is the most likely scenario. Hence, observers in the multiverse could measure signatures of bubble collision events [91, 155].

Quantitatively, we find at leading order

$$p(\text{obs}, i) \propto \Gamma_{\text{inf}(i)} p_{\text{inf}(i)} \simeq J_o \frac{\Gamma_{o \rightarrow \text{inf}(i)}}{\Gamma_o} \simeq \exp\left(-\frac{24\pi^2 M_P^4}{V_o}\right) \frac{\Gamma_{o \rightarrow \text{inf}(i)}}{\Gamma_o}. \quad (46)$$

Clearly, it is favorable to nucleate a vacuum with large energy density from nothing. Since Γ_o is dominated by the destruction of the uplift, which happens in the field-theoretic regime, it is independent of the vacuum energy. The tunneling rate $\Gamma_{o \rightarrow \text{inf}(i)}$ becomes a key player in making anthropic predictions. This is in contrast to (43) where solely the energy of the inflationary plateau determines the prediction. Eq. (46) favors those inflationary plateaus which are most closely connected to the highest-energy vacua of the landscape. In the KKLT/LVS landscape of Sect. 5, the generic dS-dS tunneling rate is close to being maximally suppressed, cf. (25). As a result, we do not expect a strong correlation between the energy densities $\text{inf}(i)$ and o in the string landscape. Therefore, the prediction for the scale of inflation may turn out to be similar to that of Sect. 6.2.2, as a result of the creation of universes with non-trivial topology. However, this may change if there are many dS-dS transitions which are not maximally suppressed, such that the energy dependence of the tunneling rate $\Gamma_{o \rightarrow \text{inf}(i)}$ in (46) becomes important. For example, as discussed in Sect. 5.2, non-generic flux transitions may lead

¹⁷Note that [153] proposed to require both sufficient slow-roll inflation and rolling into an anthropically viable state thereafter. The number frequency distribution of inflation together with such a ‘graceful exit’ can differ strongly from what one obtains by just asking for sufficient slow-roll inflation.

to such fast dS-dS transitions. Moreover, dS-dS tunneling rates could naturally be important if the relevant landscape is very different from expectations based on the flux-landscape of type-IIB supergravity.

Relevant ETW branes: If bubble of something creation events are dominant, the prediction becomes

$$p(\text{obs}, i) \simeq \Gamma_{\text{inf}(i)} p_{\text{inf}(i)} \simeq J_{\text{inf}(i)} \simeq \exp\left(-\frac{8\pi^2 M_P^4}{T_4^2}\right). \quad (47)$$

In this case, a new quantity comes in to determine the most probable scale of inflation: It is neither the energy density of the plateau nor the tunneling rates towards $\text{inf}(i)$, but rather it is the tension of the ETW brane which may exist in the quantum-gravity EFT describing the inflationary plateau.

If the boundary proposal is relevant, a similar situation occurs:

$$p(\text{obs}, i) \simeq \Gamma_{\text{inf}(i)} p_{\text{inf}(i)} \simeq J_{\text{inf}(i)} \simeq \exp(-4\pi^2 |T_4| \ell_{\text{inf}(i)}^3). \quad (48)$$

Now those inflationary plateaus are favored which allow for an ETW brane with negative tension and with the smallest values of $|T_4| \ell_{\text{inf}(i)}^3$.

The above results are novel predictions in the multiverse clearly showing that relations between ETW branes and cosmology deserve further studies. It would be especially important to uncover any possible correlations between the values of T_4 and features of the corresponding inflationary plateau, such as its energy scale.

No eternal inflation: If quantum gravity does not allow for eternal inflation, direct creation of an inflationary state from nothing is obviously preferred. If the traditional Linde/Vilenkin proposal is relevant, this favors the largest anthropically viable inflationary scale. If creation proposals including ETW branes are relevant, the same conclusions hold as described above.

7 Conclusions

A measure is needed both for ‘rocky’ landscapes, with many dS vacua and eternal inflation, as well as for ‘swampy’ landscapes, with cosmological expansion mostly or exclusively driven by some form of decaying dark energy. For both scenarios, we have developed and applied the ‘local WDW measure’, resting on two ingredients: First, the assumption that (quasi-)dS vacua have finite-dimensional Hilbert spaces (the Cosmological Central Dogma or CCD). Second, the expectation that in this context the wave function of the universe Ψ follows from solving an inhomogeneous version of the Wheeler-DeWitt (WDW) equation, which accounts for creation from nothing. Predictions are made by projecting Ψ on the subspace of interest.

Concretely, we focused on anthropic observers living near post-inflationary reheating surfaces and developed the local WDW measure to the point where explicit predictions for the scale of inflation become possible. It would be interesting to similarly employ the local WDW measure for the study of alternative cosmological approaches, such as [156, 157]. An obstacle for making predictions is the fact that, in the local WDW framework, Ψ is not normalizable. Hence, predictions of relative probabilities like $\langle \Psi | P_\alpha | \Psi \rangle / \langle \Psi | P_\beta | \Psi \rangle$, with $P_{\alpha, \beta}$ appropriate projectors, may in general be ill-defined. We argued that anthropic predictions can nevertheless be possible because Ψ is always projected on the subspace $\mathcal{H}_{obs}$ in which observers are present. We then investigated whether this projected state is normalizable.

Clearly, if the landscape has finitely many (quasi-)dS vacua, which by assumption all have finite-dimensional Hilbert spaces, then the projected wave functions arising in this context are normalizable. While, by contrast, AdS has an infinite-dimensional Hilbert space, cosmological AdS always crunches. We argue that in the cosmological transition from inflation to AdS and to the crunch only finitely many independent observers can appear. The subspace of $\mathcal{H}_{obs}$ associated with observers in AdS is then nevertheless finite-dimensional. The most problematic case is that of Minkowski space: In a dS-Minkowski transition, an infinite reheating surface with infinitely many observers can form which, moreover, can all be seen from the infinite future. We argued that the CCD nevertheless suggests that only a finite portion of this surface and hence a finite number of the observers can be considered as independent. We provided a set of independent arguments supporting this claim. In addition, we also analyzed how predictions in the multiverse change if one considers the infinitely many observers in Minkowski space as independent. In particular, this leads to the prediction that our observed dark energy should decay. All these arguments are summarized in Sect. 3.1. To clarify whether our suggested counting of observers is correct, the CCD and its application to quasi-dS, i.e. inflationary universes, needs to be studied in more detail. A key problem appears to be the interplay with the infinite-dimensional Hilbert space of asymptotic Minkowski cosmologies.

Assuming the projection of Ψ on $\mathcal{H}_{obs}$ to be normalizable, we then proceeded towards our goal of predicting the scale of inflation. The result turned out to significantly depend on the creation rates of universes from nothing. Such an ‘initial conditions dependence’ is expected for local measures. In ‘swampy’ landscapes, creation from nothing is of overwhelming importance and tunneling transitions are subdominant. For ‘rocky’ landscapes, creation and tunneling rates together determine the predictions.

The creation rates depend strongly on the choice of sign in the relation between instanton action and creation amplitude. When adopting the ‘Hartle-Hawking’ choice, one finds that our universe was created directly from nothing. Moreover, the scale of inflation is predicted to be as low as anthropically possible, in strong tension with observation.

When adopting the ‘Linde/Vilenkin’ choice, different vacuum creation mechanisms need to be considered. For example, it is possible that small compact universes with zero or negative curvature (e.g. 3-tori) are as likely to fluctuate out of nothing as small 3-spheres. It is then a prediction that we live in one of these spatially compact universes with non-trivial topology.

Since there are no obvious exponential suppression effects, the scale of inflation is predicted on the basis of vacuum counting, i.e., it is in the energy-region where most inflationary plateaus of the landscape can be found. While it would be important to study creation with non-trivial topology in more detail, this is challenging since one expects such processes to be UV-sensitive. If the creation of universes with non-trivial topology is suppressed and no relevant ETW branes exist, we predict that our inflationary plateau was populated through a tunneling process from a high-scale dS vacuum. The most likely scale of inflation is again determined by the number density of inflationary plateaus in the landscape. If, by contrast, ETW branes with appropriate tensions exist, we predict that a universe on our inflationary plateau has been created directly from nothing. The scale of inflation is then determined by correlations between this energy scale and the availability and tension of ETW brane involved in this (‘bubble of something’ or ‘boundary’) creation process. ETW branes may hence drastically affect predictions in cosmology. Correlations between their tension and observable features of our (inflationary) vacuum represent a key research target.

Let us finally state some explicit but preliminary connections to observations: The Hartle-Hawking sign choice for creation from nothing appears to be observationally strongly disfavored. However, this could change if fast, detailed-balance-violating up-tunneling transitions can be established. When using the sign choice of Linde and Vilenkin, both direct creation from nothing and tunneling dynamics can be important. Nevertheless, in many cases anthropic predictions can be dominated simply by the available number of vacua of a certain type in the landscape. This includes the particularly important case in which small universes with non-trivial topology can be created from nothing. A detailed study of the distribution of the scale of inflation in the landscape would hence be important (for first steps see [152]).

The tensor-to-scalar ratio r , related to the amplitude of primordial gravitational waves, is an observable directly linked to the energy scale of inflation (see e.g. [158]):

$$V_{\text{inf}(i)}^{1/4} \approx 10^{16} \text{ GeV} \left(\frac{r}{0.01} \right)^{\frac{1}{4}}. \quad (49)$$

So far, no detection has been made and the current bound [151, 154] $r \lesssim 0.04$ restricts the scale of inflation to $V_{\text{inf}(i)}^{1/4} \lesssim 1.4 \cdot 10^{16} \text{ GeV}$. One may then conclude that, while the distribution of inflationary plateaus in the landscape cannot be peaked at energy scales above 10^{16} GeV , a peak at any lower energy value is consistent with present data. This matches with the results of [152], where a toy-model for the plateau distributions in the type-IIB flux landscape was studied, finding a preference for $r \sim 10^{-3}$ (detectable with planned CMB measurements such as CMB-S4 [159] and LiteBIRD [160]).

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A Solving the rate equation

Solving the rate equation (6) amounts to inverting the matrix $M = X + Y$ with

$$X_{ij} = \delta_{ij}\Gamma_i, \quad Y_{ij} = -\Gamma_{j \rightarrow i}. \quad (50)$$

Formally, the inverse is given by

$$(X + Y)^{-1} = X^{-1} - X^{-1}YX^{-1} + X^{-1}YX^{-1}YX^{-1} - \dots \quad (51)$$

To see that this series converges and M is hence indeed invertible (an observation also made in [34]), we write

$$(YX^{-1})_{ij} = -\sum_k \Gamma_{k \rightarrow i} \frac{\delta_{kj}}{\Gamma_j} = -\frac{\Gamma_{j \rightarrow i}}{\Gamma_j}, \quad (52)$$

and observe that

$$\forall i, j : \quad \sum_i |(YX^{-1})_{ij}| < 1. \quad (53)$$

This crucial inequality is a consequence of the sum going only over (approximate) dS solutions, excluding terminal vacua.

Given that the standard matrix norm $\|A\| \equiv \sup_x (\|Ax\|/\|x\|)$ satisfies $\|A\| = \max_j \sum_i |A_{ij}|$, we may now conclude that $\|YX^{-1}\| < 1$. Hence the series (51) converges and (12) solves our rate equation.

B The decay rate in certain limits

For later use, we collect some facts about the tunneling rate (23) and consider certain limits. First, we note that V_f and V_t can be expressed using the dimensionless variables (22) as

$$V_f = \frac{3T^2(y+1)}{8M_{Pl}^2 x}, \quad V_t = \frac{3T^2(y-1)}{8M_{Pl}^2 x}. \quad (54)$$

The entropy of the false vacuum may be written as

$$S_f = \frac{64\pi^2 M_P^6 x}{T^2(y+1)}. \quad (55)$$

Using

$$\frac{27T^6}{(V_f - V_t)^3} = 64M_P^6 x^3 \quad (56)$$

and (55), the decay exponent (23) can be given in the form

$$B = \frac{64M_P^6 x^3}{2T^2} r(x, y) = S_f \frac{x^2(y+1)}{2} r(x, y) = S_f \frac{1+xy - \sqrt{1+2xy+x^2}}{(y-1)\sqrt{1+2xy+x^2}}. \quad (57)$$

Tunneling to Minkowski: The tunneling process to Minkowski space is of interest for example in the decay to decompactification. $V_t = 0$ is equivalent to $y = 1$ for which B becomes

$$B = S_f \frac{x^2}{(1+x)^2} = \frac{S_f}{(1+1/x)^2} = \frac{S_f}{\left(1 + \frac{4M_P^2 V_f}{3T^2}\right)^2}. \quad (58)$$

We see that for large x , $B \approx S_f$ and the lifetime of the dS space approaches its recurrence time.

Large x expansion: For $x \gg y$ and $x \gg 1$, we may characterize B by expanding the function of x and y appearing in (57) in $1/x$:

$$B = S_f \left(1 - \frac{y+1}{x} + \frac{3y(y+1)}{2x^2} + \mathcal{O}\left(\frac{1}{x^3}\right) \right). \quad (59)$$

Using (55) and (22), one finds

$$B = S_f - \frac{64\pi^2 M_P^6}{T^2} + \frac{128\pi^2 M_P^8 (V_t + V_f)}{T^4} + \mathcal{O}(1). \quad (60)$$

We see that the dependence of the true vacuum energy density only enters at third order.

C Tunneling rates for KKLT and LVS type vacua

We recall that, if two vacua are connected by a barrier in a scalar-field potential $V(\phi)$, then the tension of the corresponding domain wall reads

$$T = \int_{\phi_f}^{\phi_t} d\phi \sqrt{2V(\phi)}. \quad (61)$$

Here ϕ_f and ϕ_t are the field values characterizing the false and true vacuum. Parametrically, the tension obeys

$$T \sim \Delta\phi \sqrt{V_{\text{barrier}}} , \quad (62)$$

where $\Delta\phi$ is the width and V_{barrier} the height of the potential barrier.

Decompactification: This tunneling transition generically exists due to the typical runaway behavior of the potential at field-space infinity. Equations (61), (62) still hold as an approximation, with ϕ_t being replaced by the field value at which the runaway potential has fallen to the value of the false vacuum. The width and the height of the barrier and hence the tension T have been estimated in [3, 125]. For KKLT, the barrier height is parametrically given by $\sim M_P^4 g_s |W_0|^2 / \mathcal{V}^2$, where $\mathcal{V}$ is measured in 10d Planck units. The width is [125]

$$\Delta\phi \sim \frac{M_P}{\mathcal{V}^{2/3}} , \quad (63)$$

implying

$$T_{\text{dec, KKLT}} \sim \frac{M_P g_s^{1/2} M_P^2 |W_0|}{\mathcal{V}^{2/3} \mathcal{V}} = \frac{M_P^3 g_s^{1/2} |W_0|}{\mathcal{V}^{5/3}} . \quad (64)$$

Using the definition (22) and assuming that $V_f - V_t = V_f \sim M_P^4 g_s |W_0|^2 / \mathcal{V}^2$, we find

$$x_{\text{dec, KKLT}} \sim \frac{M_P^6 g_s |W_0|^2 \mathcal{V}^2}{\mathcal{V}^{10/3} M_P^6 g_s |W_0|^2} \sim \frac{1}{\mathcal{V}^{4/3}} < 1 . \quad (65)$$

According to (58), the tunneling exponent is given by

$$B = \frac{S_f}{(1 + 1/x_{\text{dec, KKLT}})^2} , \quad (66)$$

and is thus **a few orders of magnitude** smaller than the entropy. Here we also recall that $\mathcal{V} \sim \ln(1/|W_0|)$ and hence, even if W_0 is tuned very small, can never become extremely large. We also note that, if the uplifted cosmological constant is tuned to be very close to zero, such that V_f is parametrically smaller than $g_s |W_0|^2 / \mathcal{V}^2$, the estimate (65) is no longer valid. From the definition of x (22) (or directly from (58)), one can see that a small V_f in comparison to the barrier height leads to large x and hence to the tunneling exponent being very close to the entropy. However, the decay time cannot exceed the recurrence time [3].

For LVS models, the barrier height is $\sim M_P^4 g_s^{1/2} |W_0|^2 / \mathcal{V}^3$ and the width is $\mathcal{O}(1) M_P$. As a result, the tension of the domain wall to decompactification scales as [125]

$$T_{\text{dec, LVS}} \sim \frac{M_P^3 g_s^{1/4} |W_0|}{\mathcal{V}^{3/2}} . \quad (67)$$

Generically, the parameter x and the decay exponent B are then given by

$$x_{\text{dec, LVS}} \sim \mathcal{O}(1), \quad B = \frac{S_f}{(1 + 1/x_{\text{dec, LVS}})^2}. \quad (68)$$

We conclude that the decay exponent is **of the same magnitude** as the entropy. As before, tuning the vacuum energy to be very close to zero results in $x_{\text{dec, LVS}} \gg 1$ and hence in a maximally suppressed decay rate.

Flux transitions: The domain walls relevant for flux transitions in the type IIB landscape are given by D5/NS5 branes wrapping 3-cycles of the Calabi-Yau manifold. For LVS models, this has been discussed in [127]. In units where $2\pi\sqrt{\alpha'} = 1$, the D5 tension is $T_{\text{D5}} = 2\pi/g_s$. Using $M_P^2 = 4\pi\mathcal{V}/g_s^{1/2}$ and wrapping the D5 on a 3-cycle Σ , the domain wall tension reads¹⁸

$$T_{\text{D5, D.W.}} = M_P^3 \frac{2\pi g_s^{1/2}}{(4\pi)^{3/2}\mathcal{V}} \int_{\Sigma} \sqrt{g_{\Sigma}}. \quad (69)$$

If the 3-cycle Σ were one of the special Lagrangian cycles A^a or B_b of a symplectic basis (A^a, B_b) , the last integral would be exactly equal to the absolute value of the corresponding period,

$$z^a = \int_{A^a} \Omega, \quad \mathcal{G}_b = \int_{B_b} \Omega. \quad (70)$$

In general, Σ is of the form

$$\Sigma = \sum_i (r_B^i B_i + s_i^A A^i), \quad r_B^i, s_i^A \in \mathbb{Z}, \quad (71)$$

and the volume of Σ can be given an upper and lower bound¹⁹

$$\left| \sum_i (r_B^i \mathcal{G}_i + s_i^A z^i) \right| \leq \int_{\Sigma} \sqrt{g_{\Sigma}} \leq \sum_i (|r_B^i \mathcal{G}_i| + |s_i^A z^i|). \quad (72)$$

The D5 brane changes the F_3 flux on the cycles A^a and B_b precisely by s_a^A and r_B^b units such that the change in the flux superpotential becomes

$$\Delta W = \sum_i (r_B^i \mathcal{G}_i + s_i^A z^i). \quad (73)$$

¹⁸We use the convention that $M_P \int_{CY_3} \sqrt{g} = 1 = M_P \int_{CY} \Omega \wedge \bar{\Omega}$. Moreover, our volume $\mathcal{V}$ in 10d Planck units is related to the volume in string units by $\tilde{\mathcal{V}} = \mathcal{V} g_s^{3/2}$.

¹⁹Here we differ from [127], where it is stated that the volume of the cycle Σ is always given by the left hand side of (72). This difference does not parametrically affect the subsequent analysis of [127]. The reason is that they focus on the LVS, where $|W_0| \sim \mathcal{O}(1)$ before and after the transition and hence the lower and upper bound in (72) are comparable. However, this difference is crucial for KKLT models, where both values of $|W_0|$ are tiny and hence the lower bound is much smaller than the upper bound.

Hence, the domain wall tension satisfies the inequality

$$T_{\text{D5, D.W.}} = \frac{M_P^3 2\pi g_s^{1/2}}{(4\pi)^{3/2} \mathcal{V}} \int_{\Sigma} \sqrt{g_{\Sigma}} \geq \frac{M_P^3 2\pi g_s^{1/2}}{(4\pi)^{3/2} \mathcal{V}} |\Delta W|. \quad (74)$$

If, by analogy, the domain wall is built using an NS5 brane, the H_3 flux jumps and the superpotential changes by

$$\Delta W = \sum_i S (r_B^i \mathcal{G}_i + s_i^A z^i), \quad (75)$$

with S the axio-dilaton. The corresponding relation for the tension is

$$T_{\text{NS5, D.W.}} = \frac{M_P^3 2\pi}{(4\pi)^{3/2} g_s^{1/2} \mathcal{V}} \int_{\Sigma} \sqrt{g_{\Sigma}} \geq \frac{M_P^3 2\pi}{(4\pi)^{3/2} g_s^{1/2} \mathcal{V}} |\Delta W/S|. \quad (76)$$

We focus on the regime where g_s is small such that $|S| \sim 1/g_s$. As a result, for both the D5 and NS5 case, we have parametrically $T \gtrsim g_s^{1/2} M_P^3 |\Delta W|/\mathcal{V}$. However, this is not too important since generic domain walls are not near-BPS (see e.g. [126]) and hence do not saturate the lower bounds of (74),(76). Instead, a more useful estimate is provided simply by $T \gtrsim g_s^{1/2} M_P^3/\mathcal{V}$.

Let us first focus on LVS models and their dS-dS transitions, with $|W_0|$ generically an $\mathcal{O}(1)$ number and $\mathcal{V} \sim \exp(1/g_s)$ exponentially large. The requirement that g_s remains small in the new vacuum constrains possible flux transitions. The volume can change during the transition and for down-tunneling events it generally increases. In this case, assuming also that $|W_0|$ does not change drastically, we estimate $V_f - V_t \sim V_f \sim M_P^4 g_s^{1/2} |W_0|^2/\mathcal{V}^3$, with $\mathcal{V}$ from now on always denoting the volume of the decaying vacuum. The parameter y is then $\mathcal{O}(1)$ and x may be estimated according to

$$x_{\text{LVS, flux}} \gtrsim \frac{g_s^{1/2} \mathcal{V}}{|W_0|^2} \gg 1. \quad (77)$$

Using (59),(60), we obtain

$$B_{\text{LVS, flux}} = S_f \left(1 - \frac{y+1}{x_{\text{LVS, flux}}} + \mathcal{O}\left(\frac{1}{x_{\text{LVS, flux}}^2}\right) \right) \quad (78)$$

$$= S_f - \frac{64\pi^2 M_P^6}{T^2} + \mathcal{O}(\mathcal{V}) \sim \frac{24\pi^2 \mathcal{V}^3}{g_s^{1/2} |W_0|^2} - \frac{16(4\pi)^3 \mathcal{V}^2}{g_s \left(\int_{\Sigma} \sqrt{g_{\Sigma}}\right)^2} + \mathcal{O}(\mathcal{V}). \quad (79)$$

Here, S_f denotes the entropy of the false vacuum. At leading order,

$$B_{\text{LVS, flux}} \sim S_f. \quad (80)$$

We note that the dependence on the true vacuum energy density V_t only enters at third order (see (60)).

For dS-dS transitions in KKLT models, both $|W_0|$ and $|W_0 + \Delta W|$ must be tuned exponentially small. Thus, only a tiny fraction of flux transitions from a KKLT vacuum end in another KKLT dS state. This amounts to an extremely strong restriction on the allowed flux changes. As before, we have $T \sim M_P^3 g_s^{1/2} / \mathcal{V}$. For down-tunneling events, $|W_0|$ generically decreases during the transition. Using again the approximation $V_f - V_t \sim V_f \sim M_P^4 g_s |W_0|^2 / \mathcal{V}^2$, we find

$$x_{\text{KKLT, flux}} \sim \frac{1}{|W_0|^2} \gg 1, \quad (81)$$

with $|W_0|$ the value of the flux superpotential of the parent vacuum. Hence, by (59),(60) with $y \sim 1$ due to our assumptions,

$$B_{\text{KKLT, flux}} = S_f \left(1 - \frac{y+1}{x_{\text{KKLT, flux}}} + \mathcal{O}\left(\frac{1}{x_{\text{KKLT, flux}}^2}\right) \right) \quad (82)$$

$$= S_f - \frac{64\pi^2 M_P^6}{T^2} \sim \frac{24\pi^2 \mathcal{V}^2}{g_s |W_0|^2} - \frac{16(4\pi)^3 \mathcal{V}^2}{g_s (\int_\Sigma \sqrt{g_\Sigma})^2}. \quad (83)$$

As for LVS models, the dependence on the true vacuum energy density V_t only enters at third order.

KPV decay: For completeness, we review the derivation of the KPV decay rate [132, 133]. The $\overline{D3}$ -brane of the KPV SUSY breaking mechanism can annihilate against the flux of the Klebanov-Strassler throat [132]. This KPV or brane-flux annihilation process leads to the decay of dS models employing the $\overline{D3}$ -brane uplift. The corresponding domain wall tension follows from the action of an NS5 brane wrapping the S^3 at the tip of the throat. In string frame with $2\pi\sqrt{\alpha'} = 1$, it reads

$$T_{\text{KPV}} = \frac{2\pi}{g_s^2} \text{vol}(S^3) h_{\text{tip}}^{-3/4} \tilde{\mathcal{V}}^{1/2} = \frac{(bM)^{3/2} \tilde{\mathcal{V}}^{1/2}}{2g_s^{1/2}} h_{\text{tip}}^{-3/4}. \quad (84)$$

Here $b \approx 0.932$ is a numerical constant, $h_{\text{tip}}^{-1/4} \sim \exp\left(-\frac{2\pi K}{3g_s M}\right)$ is the warp factor at the tip of the throat, M and K are the flux numbers on the A and B -cycle of the throat, and $\tilde{\mathcal{V}}$ is the CY volume measured in string units. The energy density provided by $N_{\overline{D3}}$ anti-branes at the tip of the throat is given by

$$\Delta V = \frac{4\pi N_{\overline{D3}}}{g_s} h_{\text{tip}}^{-1} \tilde{\mathcal{V}}^{2/3}. \quad (85)$$

Using $M_P^2 = 4\pi\tilde{\mathcal{V}}/g_s^2$ and employing (22) we find

$$x_{\text{KPV}} = \frac{3b^3 g_s^2 M^3}{256\pi^2 \tilde{\mathcal{V}}^{2/3} N_{\overline{D3}}} h_{\text{tip}}^{-1/2}. \quad (86)$$

This formula is dominated by the exponentially small warp factor, ensuring that $x_{\text{KPV}} \ll 1$. Since $y_{\text{KPV}} \sim 1$, the KPV tunneling process occurs in the field-theoretic regime and represents the dominant decay channel. As a result, all dS vacua decay predominantly to AdS. This ensures that the assumptions which went into deriving the perturbative solution (12) hold. In the regime of $x_{\text{KPV}} \ll 1$, the KPV tunneling exponent follows from (23) and reads [132, 133]

$$B_{\text{KPV}} = \frac{27b^6 M^6 g_s}{2048\pi (N_{\overline{D3}})^3}. \quad (87)$$

Since 4d gravity plays no role, one expects the decay exponent not to depend on volume or warp factor but only on local physics at the tip of the throat. This is indeed the case.

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6 Summary and Discussion

This thesis contributes to the study of theories of quantum gravity allowing for a landscape of solutions, with string theory being the most prominent example. In such theories, making predictions for future measurements is a notoriously difficult task. We hope that the ideas developed in this thesis will be useful for further research in this field.

In Sect. 6.1, we summarize the results from Chapter 2-5. In Sect. 6.2, the results will be discussed, put into context with other work in this area and an outlook towards the future is given.

6.1 Summary

6.1.1 The local Wheeler-DeWitt Measure

Chapter 2 is concerned with developing a measure for the string landscape from first principles. The main idea is to construct the quantum state Ψ of the entire multiverse, from which predictions can be extracted using the inner product of the underlying Hilbert space.

It has been claimed [239] that de Sitter spaces should be viewed from a static patch perspective and that the corresponding Hilbert space is finite-dimensional, with dimension

$$\dim \mathcal{H}_{dS} = \exp(|S_{dS}|) = \exp(8\pi^2 M_P^2 \ell_{dS}^2), \quad (6.1.1)$$

where $|S_{dS}|$ is referred to as the dS entropy. In analogy to the central dogma of black hole physics, the statement (6.1.1) was named the *Cosmological Central Dogma* (CCD) [240]. The arguments for this proposal are threefold:

The cosmological horizon of dS spaces is similar to black hole horizons. An observer in dS experiences a temperature

$$T_{dS} = \frac{1}{2\pi\ell_{dS}}, \quad (6.1.2)$$

similar an observer outside of a black hole, and it was then conjectured that dS thermodynamics exists in analogy to black hole thermodynamics. From (6.1.2), the dS entropy can be derived in analogy to the black hole entropy. The entropy of dS is then interpreted as the entropy of a black hole, namely by signaling that the underlying Hilbert space should be finite dimensional. Furthermore, it has been argued that consistent quantum mechanics can only be formulated in a causally connected region of spacetime [241]. One can speculate that this principle of *complementarity* also holds for the cosmological horizons of dS spaces, making it necessary to view dS from the static patch perspective.

The second argument in favor of (6.1.1) comes from black holes in dS space. The most effective way for an observer in dS to collect as much entropy as possible is to form a black hole whose entropy is bounded by its horizon area. Since black holes cannot grow larger than the cosmological horizon, there is no way for an observer to collect more entropy than $|S_{dS}|$.

The third argument rests on the observation that Euclidean dS space is a sphere that is compact, unlike Euclidean AdS space, which has a boundary. The compactness of Euclidean dS space suggests that all diffeomorphisms of dS space, including the isometries, should be interpreted as pure gauge transformations. This is in contrast to Euclidean AdS, where isometries act non-trivially on the boundary. Green's functions of the Euclidean theory, analytically continued to Lorentzian spacetime, have the interpretation of being the Green's functions of a static patch [242], and they can be given a Hilbert space interpretation of the form (6.1.1). To obtain the Green's function of global dS, one can use the dS isometries to 'move' the Green's function from one static patch to another. As the isometries are interpreted as gauge transformations, it appears incorrect to introduce separate degrees of freedom for different static patches.

The proposed measure for the multiverse follows from considering the Hilbert space to be of the form

$$\mathcal{H} = \left(\bigoplus_{i \in \text{dS}} \mathcal{H}_i \right) \oplus \left(\bigoplus_{y \in \text{Terminals}} \mathcal{H}_y \right), \quad (6.1.3)$$

and the state of the multiverse Ψ being obtained from solving

$$H\Psi = \chi. \quad (6.1.4)$$

Here, the Hilbert space is chosen as a direct sum over the individual Hilbert spaces of all vacua of the landscape. The dS spaces contribute with a finite-dimensional Hilbert space according to (6.1.1), whereas the terminal vacua (Minkowski and AdS) contribute with an infinite-dimensional Hilbert space. Eq. (6.1.4) is a modified Wheeler-DeWitt equation, where H is the Hamiltonian and χ is a source term responsible for the creation of universes from nothing. The state Ψ is a superposition of states describing different universes from the perspective of a 'single observer' (i.e. from a static-patch perspective).

The source χ needs to be such that it generates a quantum mechanical probability current J flowing into the dS-like part of the Hilbert space. One may interpret Ψ as a stationary state where universes are constantly created from nothing (due to the presence of χ) and simultaneously decay into other spaces according to the dynamics defined by H . Predictions can be made by constructing projection operators P_A, P_B on subspaces of interest and evaluating ratios

$$\frac{\langle \Psi | P_A | \Psi \rangle}{\langle \Psi | P_B | \Psi \rangle}. \quad (6.1.5)$$

It is furthermore shown that the microscopic definition of Ψ can be linked to the semiclassical theory of the multiverse in terms of the following equation:

$$J_i = \sum_{j \in \text{dS}} (p_i \Gamma_{i \rightarrow j} - p_j \Gamma_{j \rightarrow i}) + p_i \sum_{y \in \text{Terminals}} \Gamma_{i \rightarrow y}, \quad (6.1.6)$$

where i, j run over all dS vacua and y runs over terminal vacua, i.e. AdS, Minkowski, and 'nothing'. Here, the p_i are defined as

$$p_i = ||P_i \Psi||^2, \quad (6.1.7)$$

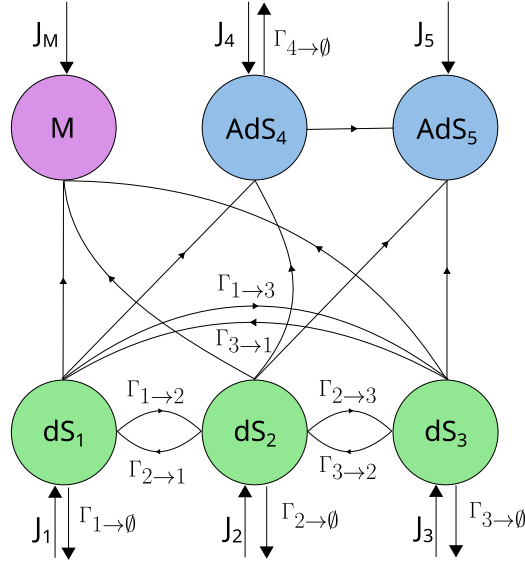


Figure 6.1. Creation and tunneling for a toy landscape consisting of three dS, one supersymmetric Minkowski (M), one non-supersymmetric and one supersymmetric AdS vacuum (‘AdS₄’ and ‘AdS₅’ respectively). Note that the labels distinguish different vacua and have nothing to do with dimensionality, which is always 4. To avoid overburdening notation, we have not put labels on the arrows for decays to Minkowski and AdS vacua.

with P_i being the projection operator projecting onto the Hilbert space of dS space i . The p_i have the interpretation of the probability of dS space i being realized in the multiverse. The J_i are the creation rates, and the $\Gamma_{i \rightarrow j}, \Gamma_{i \rightarrow \emptyset}$ are the decay rates as discussed in Sect. 1.9. An illustration of the multiverse dynamics is given in Fig. 6.1. The rate equation (6.1.6) provides a starting point for making predictions in the multiverse as the creation and decay rates are semiclassically calculable.

6.1.2 Bubbles of Anything

In order to make predictions in the string landscape using the local WDW measure, it is clear from Eq. (6.1.6) that the possible vacuum creation and vacuum transition rates in the string landscape need to be understood. In the best models for realizing dS solutions in string theory, there are four different decay channels that can occur:

- **Tunneling to decompactification:** This transition is from a 4D de Sitter space to a 10D spacetime arising from the volume of the compact space becoming infinite. As reviewed in Sect. 1.6, the volume of the internal CY manifold is a scalar field in the 4D theory that is stabilized at a certain value. It can tunnel through a potential barrier, as discussed in Sect. 1.8, and trigger a decay.
- **Flux transitions:** Fluxes are an integral part of model building in string theory, and a transition can occur by a flux changing its value. The flux can change through the spontaneous nucleation

of a D-brane that represents the domain wall in this context. The final state after a flux transition can be a different dS space or a terminal vacuum.

- **Destruction of the uplift:** All known dS proposals try to realize dS by first constructing an AdS solution and then adding an ‘uplift’ contribution, meaning an object providing a positive energy density that lifts the AdS solution to a dS solution. Naturally, this uplift can decay, and the destruction of the uplift is generally a quickly occurring phenomenon compared to the other decay channels. The final state of this process is an AdS spacetime.
- **Bubbles of nothing:** As discussed in Sect. 1.8.1, vacua can decay to nothing, which could plausibly also commonly happen for vacua of the string landscape [243].

While the first three decay channels are well understood in the string landscape, little work has been done in studying decays to nothing. As a result, we investigated such decays, specifically in the context of string theory, in more detail.

The first observation of Chapter 3 is that the Witten bubble, reviewed in Sect. 1.8.1, from the 4D perspective, looks like the nucleation of a negative-tension brane behind which is nothing. Such branes are called end-of-the-world (ETW) branes and have received extensive interest in recent years following the formulation of the Cobordism Conjecture [238], stating that ETW branes are expected to be ubiquitous in string theory. More precisely, the conjecture states that the internal manifolds $\mathcal{M}_k$, with k denoting their dimension, of a consistent theory of quantum gravity (e.g. string theory) should be cobordant to nothing. In other words, the cobordism group of ‘Quantum Gravity’ must satisfy

$$\forall k : \quad \Omega_k^{QG} = 0. \quad (6.1.8)$$

Here, the structure ‘QG’ that the bordisms have to preserve is not yet fully understood, as the ultimate theory of quantum gravity is not yet known, but it represents the ‘quantum gravity’ structure. Nevertheless, (6.1.8) states that the internal manifolds of string theory should be deformable to nothing, i.e. able to shrink to zero size, just as the internal S^1 in the Witten bubble does. The corresponding geometry is nothing but the cigar geometry, as depicted in Fig. 1.6. Another way of interpreting the Cobordism Conjecture is that it predicts the existence of additional objects, often called *defects*, that allow the internal manifolds to shrink. The simplest example may be the internal manifold being a circle and the theory exhibiting a spinor with periodic boundary conditions upon going around the S^1 . The cigar geometry is inconsistent with the periodic boundary conditions, and one hence requires a spin defect in order to have a bordism to nothing (see Fig. 6.2 for an illustration). As a result, many ETW branes of Witten type (and in fact many others) are expected to exist in string theory.

In Chapter 3, a systematic way of studying decays to nothing, independent of the microscopic realization of the ETW brane, is developed. The analysis is based on the Witten bubble viewed from the 4D perspective. Here, the size of the internal S^1 is given in terms of a scalar field ϕ without potential. The original Witten solution, where the S^1 shrinks to zero size, corresponds to the field ϕ going to infinite distance in field space within a finite distance in spacetime. It was claimed that this feature can be taken as a general definition of an ETW brane from an EFT point of view [244–249]. However, in such a situation, one necessarily loses control of the EFT as the scalar field gains infinite kinetic energy. We proposed that a (4D) defect needs to be introduced to capture the effective

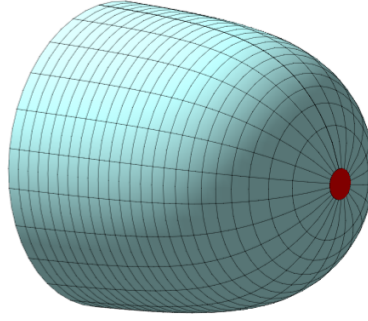


Figure 6.2. A bordism between S^1 and nothing, including a defect (drawn as a red disk).

physics in the region where control is lost. The introduction of this defect is natural from the higher-dimensional point of view, as a (10D) defect predicted by the Cobordism Conjecture is likely to be present, and it is needed from the 4D perspective to maintain control over the EFT. In the simplest case, applicable to Witten-type bubbles, the 4D defect is characterized by its size η and its tension T_4 .

One can then look for an instanton solution to the theory defined by the action

$$S = \int d^4x \sqrt{g} \left(-\frac{\mathcal{R}}{2} + \frac{1}{2}(\partial\phi)^2 \right) - \int d^3x \sqrt{h} (\mathcal{K}_4 - T_4), \quad (6.1.9)$$

where g is the 4D metric, and h is the induced metric on the boundary. Making the $O(4)$ symmetric ansatz

$$ds^2 = dr^2 + f(r)^2 d\Omega_3^2, \quad \phi = \phi(r), \quad (6.1.10)$$

and looking for a solution to the Euclidean equations of motion subject to the condition that the ETW brane with tension T_4 is located at some value $\phi = \phi_\eta$ and enforcing the Israel junction conditions leads to an instanton solution that mediates the decay to nothing. Importantly, a flat space bubble of nothing instanton can only be found if $T_4 < 0$. This however, is nothing exotic for ETW branes [20, 90, 250–252]. The EFT proposal (6.1.9) is widely applicable for many microscopic realizations of ETW branes. In certain cases, it will be necessary to extend the EFT of the ETW brane, e.g. if the ETW brane couples to other fields.

An important observation can now be made: ETW branes with positive tension do not allow for an instanton geometry as depicted in Fig. 1.6. Rather, they allow for a geometry where a finite-sized ball of some spacetime is bounded by a spherical ETW brane, as illustrated in Fig. 6.3. Such an instanton solution has the interpretation of being responsible for the creation of a universe with boundary from nothing, as is illustrated in Fig. 6.4. In contrast to the ‘bubble of nothing’, we named this creation proposal the ‘bubble of something’. Similar constructions are given in [253–256] where also the term ‘bubble from nothing’ has been used. Our EFT proposal (6.1.9) can hence also describe such creation processes. The creation rate for creating a flat universe is given by

$$J \propto \exp\left(\pm \frac{8\pi^2 M_P^6}{T_4^2}\right), \quad (6.1.11)$$

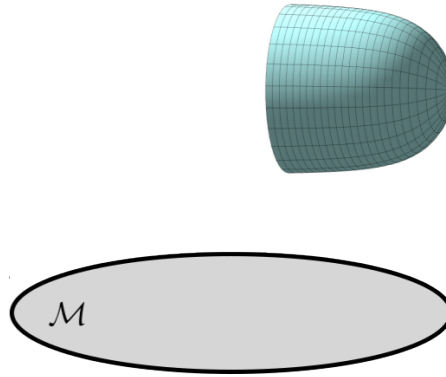


Figure 6.3. Illustration of the bubble of something instanton.

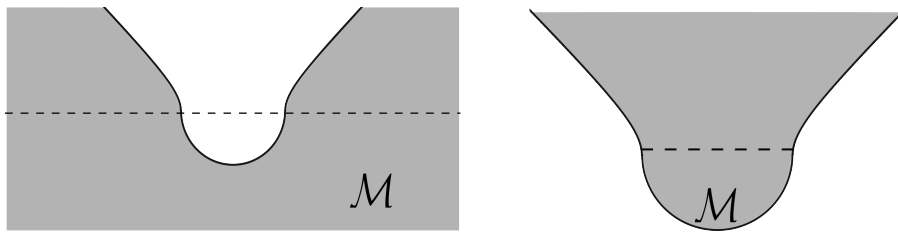


Figure 6.4. Physical processes corresponding to bubble of nothing (left) and bubble of something (right). The dashed line denotes the locus of analytic continuation from the euclidean half instanton to the Lorentzian, expanding bubble. The solid black line is the boundary of 4d space.

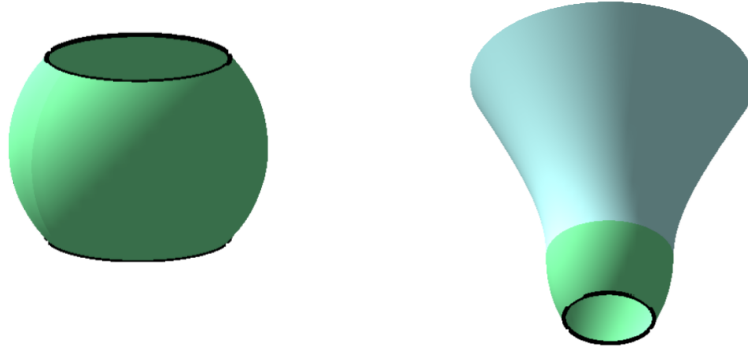


Figure 6.5. The instanton geometry for the Boundary Proposal (left) and the Lorentzian spacetime (right).

where the choice of sign is similar to the one in (1.9.10). Such events create a universe with boundary from nothing and contribute to the total creation rate entering Eq. (6.1.6). Interestingly, also Minkowski or AdS vacua can be created this way, in contrast to the creation proposals discussed in Sect. 1.9.

Finally, we explicitly constructed an ETW brane for the best understood part of the string landscape, namely type IIB compactifications on CY orientifolds. The ETW brane is constructed by making use of an appropriate orientifolding. This brane could be responsible for the decay of universes to nothing or the creation of universes from nothing.

6.1.3 The Boundary Proposal

By looking at Fig. 6.4, one realizes that the ETW brane in a bubble of something geometry is part of both the Euclidean and Lorentzian regions of the bubble of something creation process. In Chapter 4, a new creation mechanism for dS universes utilizing ETW branes with negative tension purely in the off-shell region is proposed. The corresponding instanton geometry is S^4 (i.e. Euclidean dS space) with two S^3 s cut out by ETW branes, as illustrated on the lh. side of Fig. 6.5. Following the standard technique, this instanton geometry can be cut in half, and onto it can be glued the Lorentzian, analytically continued spacetime, resulting in a geometry of the form depicted on the rh. side of Fig. 6.5. As we observe, the created dS space is boundary-free, just as in the creation proposals of Sect. 1.9. However, the Euclidean region differs by the existence of a boundary, which is why we call this process ‘The Boundary Proposal.’

The creation rate, given by the on-shell action, takes the form

$$J \propto \exp \left(\pm 8\pi^2 M_P^2 \ell_{dS}^2 \sqrt{\frac{T^2 \ell_{dS}^2}{T^2 \ell_{dS}^2 + 4M_P^4}} \right), \quad (6.1.12)$$

with ℓ_{dS} the curvature length of the created dS space and $T_4 < 0$ the tension of the ETW brane. We have now identified three different creation mechanisms for dS spaces, which are compared to each other in Table 6.1, where the bubble of something creation rate for dS universes is also displayed.

We observe that if the Hartle-Hawking sign is chosen, i.e. the ‘+’ sign in the exponent of J , then the original no-boundary proposal remains the dominant creation process. However, if the Linde/Vilenkin

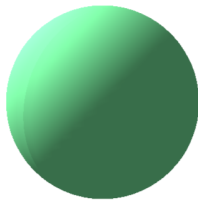
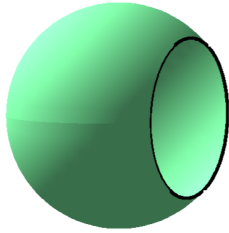
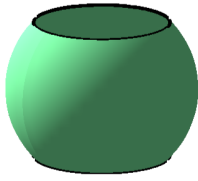
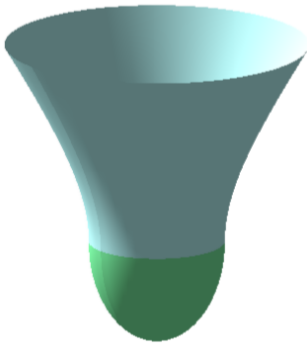
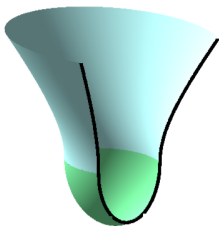
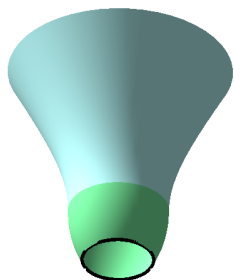
	No-Boundary (nb)	Bubble of Something (bos)	Boundary (b)
			
			
$\log(J) \propto$	$\pm 8\pi^2 M_P^2 \ell_{dS}^2$	$\pm 4\pi^2 M_P^2 \ell_{dS}^2 \left(1 - \frac{T_A \ell_{dS}}{\sqrt{T_4^2 \ell_{dS}^2 + 4M_P^4}} \right)$	$\pm 8\pi^2 M_P^2 \ell_{dS}^2 \sqrt{\frac{T_4^2 \ell_{dS}^2}{T_4^2 \ell_{dS}^2 + 4M_P^4}}$

Table 6.1. Comparison between the no boundary, the bubble of something and the spacelike ETW brane example. Importantly, if the ETW brane tension is small, the instanton action of the spacelike ETW brane instanton in the last column becomes small.

sign is chosen, either the bubble of something or the boundary proposal are the dominant creation processes. The bubble of something becomes especially relevant if $T_4 \gg M_P^2/\ell_{dS}$, in which case the creation rate is given by (6.1.11) with a negative sign. If T_4 is negative and satisfies $|T_4| \ll M_P^2/\ell_{dS}$, the boundary proposal becomes especially dominant with the creation rate

$$J \propto \exp(-4\pi^2|T_4|\ell_{dS}^3). \quad (6.1.13)$$

In general, we have identified an entirely new creation process that can, in certain situations, be the dominant creation mechanism.

6.1.4 Inflationary Predictions for Rocky and Swampy Landscapes

In Chapter 5, it is the idea to use the local WDW measure and the identified creation/decay rates to make a prediction in the multiverse. However, there is a fundamental challenge in the definition of the local WDW measure as Ψ is non-normalizable due to the Hilbert spaces of terminal vacua being infinite-dimensional. In Chapter 5, it is proposed that predictions for future measurements of the form (6.1.5) can nevertheless be well defined as the projection operators P_A, P_B necessarily involve a projection on the Hilbert space $\mathcal{H}_{obs}$ describing states in the multiverse where observers are present. Since, arguably, the leading paradigm for creating anthropic observers is a period of inflation followed by reheating and structure formation, we focus on such corners of the landscape. If observers could only live in dS spaces, predictions from the local WDW measure are automatically well defined due to the CCD (6.1.1). For observers formed after a reheating period, P_A, P_B involve a projection on the subspace within the dS-like part of the Hilbert space describing inflationary states. We conclude that the probability $p(\text{obs}, i)$ of an observer living in vacuum i is, to leading order, given by

$$p(\text{obs}, i) \propto p_{\text{inf}(i)}, \quad (6.1.14)$$

the probability of finding the universe in an inflationary state leading to reheating in vacuum i . This probability may be obtained by treating regions of the landscape where inflation occurs, which we call inflationary plateaus, as additional, short-lived dS vacua and then deriving their probability from the rate equation (6.1.6). However, there is no (strong) argument excluding the existence of observers in Minkowski (or AdS with cosmological constant near zero) universes. A main result of Chapter 5 is the analysis of several approaches for obtaining well-defined predictions even if observers in Minkowski vacua exist. First, we observe that if Ψ describes only finitely many independent observers, the projected state on $\mathcal{H}_{obs}$ is likely to be normalizable. Observing that, on the reheating surface, it is not yet visible whether late-time evolution will lead to dS or Minkowski space, see Fig. 6.6 for an illustration we make the conclusion that, as for dS spaces, not all Minkowski observers can be considered independent. In other words, the CCD suggests that there exist gauge redundancies on the reheating surface, such that only part of it can be seen as fundamental. Then, the probability for an observer living in Minkowski space turns out to be again estimated by (6.1.14), with $\text{inf}(i)$ now denoting an inflationary plateau reheating into a Minkowski vacuum i . In Chapter 5, a detailed discussion on the counting of independent observers is given.

Furthermore, a prediction for the scale of inflation in the landscape is performed. It becomes a natural target for an anthropic prediction because it does not directly impact our existence. This

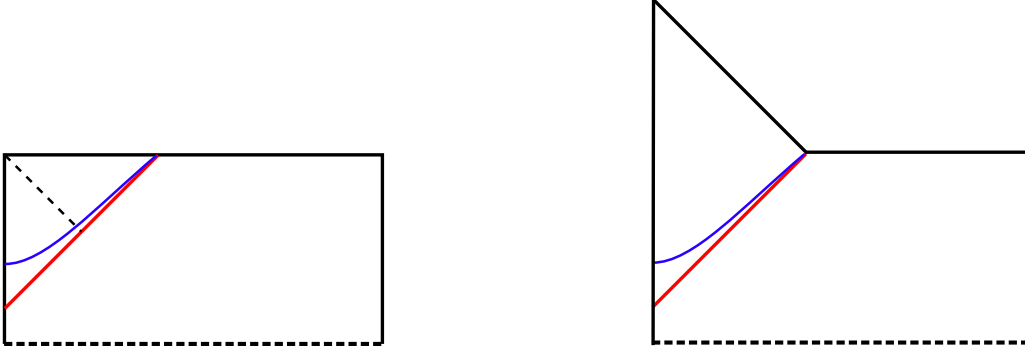


Figure 6.6. The creation of a spherical universe with a subsequent formation of a dS bubble (left) and Minkowski bubble (right) are displayed. The red line denotes the bubble wall and the blue line illustrates the location of the reheating surface. The diagonal dashed line on the left shows the cosmological horizon for an observer located on the vertical solid black line on the far left.

prediction also assumes that, within the underlying landscape, the inflationary scale is largely uncorrelated with anthropically significant parameters, such as certain Standard Model couplings. As mentioned before, the adopted strategy is to treat inflationary plateaus in the landscape as vacua of the theory and compute the $p_{\text{inf}(i)}$ from the rate equation (6.1.6). The solution to (6.1.6) can be given in the form

$$p_i = \sum_{N=0}^{\infty} p_{i,N}, \quad p_{i,0} = \frac{J_i}{\Gamma_i}, \quad p_{i,N} = \frac{1}{\Gamma_i} \sum_{j_1, \dots, j_N} J_{j_1} \frac{\Gamma_{j_1 \rightarrow j_2}}{\Gamma_{j_1}} \frac{\Gamma_{j_2 \rightarrow j_3}}{\Gamma_{j_2}} \dots \frac{\Gamma_{j_N \rightarrow i}}{\Gamma_{j_N}}, \quad (6.1.15)$$

where the J_i are the creation rates, $\Gamma_{i \rightarrow j}$ denote the vacuum transition rates between dS vacua, and Γ_i denotes the total decay rate (to dS and terminal vacua) of dS space i . If the $\Gamma_{i \rightarrow j}$ are much smaller than the Γ_i , which is estimated to be the case for the string landscape, then the series converges quickly, and one may approximate

$$p_i = p_{i,0} + p_{i,1} = \frac{1}{\Gamma_i} \left(J_i + \sum_{j \neq i} J_j \frac{\Gamma_{j \rightarrow i}}{\Gamma_j} \right). \quad (6.1.16)$$

The two terms have an intuitive interpretation: Space i can be directly created from nothing with probability or it can be created by tunneling in the background of some other space j that has been created from nothing. Another observation can furthermore be made. The probability $p(\text{obs}, i)$ of an observer being in vacuum i should be proportional to the probability of *starting* inflation leading to reheating in vacuum i rather than the probability $p_{\text{inf}(i)}$ of *finding* the multiverse in an inflationary state of vacuum i . Implementing this condition results in the refined statement for the probabilities:

$$\frac{p(\text{obs}, i)}{p(\text{obs}, j)} \simeq \frac{\Gamma_{\text{inf}(i)} p_{\text{inf}(i)}}{\Gamma_{\text{inf}(j)} p_{\text{inf}(j)}} \simeq \frac{J_i + \sum_{o \neq i} J_o \frac{\Gamma_{o \rightarrow \text{inf}(i)}}{\Gamma_o}}{J_j + \sum_{o \neq j} J_o \frac{\Gamma_{o \rightarrow \text{inf}(j)}}{\Gamma_o}}. \quad (6.1.17)$$

In Chapter 5, a detailed analysis of the predictions is then performed. We find that when adopting the Hartle-Hawking choice of sign, the direct creation from nothing is the dominant creation process (i.e. $p_{i,0} \gg p_{i,1}$ holds for all i) such that the prediction becomes

$$\frac{p(\text{obs}, i)}{p(\text{obs}, j)} \simeq \frac{J_{\text{inf}(i)}}{J_{\text{inf}(j)}} = \exp\left(8\pi^2 M_P^2 \left(\ell_{\text{inf}(i)}^2 - \ell_{\text{inf}(j)}^2\right)\right), \quad (6.1.18)$$

resulting in the prediction that the scale of inflation is as low as possible. However, this result turns out to be in tension with observation, providing an argument for the Linde/Vilenkin choice of sign being ‘correct’.

When adopting the Linde/Vilenkin sign, i.e. the ‘-’ sign, the analysis depends on details of the landscape:

- **Nontrivial topology:** If universes with non-trivial topology can be created from nothing at little to no cost, as discussed in Sect. 1.9.1, then the direct creation of a toroidal (or generally non-positively curved) compact universe is the dominant creation mechanism. To leading order, all inflationary plateaus are equally likely to be created from nothing, such that the most likely scale of inflation for us to measure is determined by the number density of inflationary plateaus in the landscape as a function of energy.
- **No relevant ETW branes:** If the creation of vacua with nontrivial topology is impossible or, for some reason, strongly suppressed and if there do not exist ETW branes with particularly large (i.e. $T_4 \gg M_P^2 \ell_{\text{inf}(i)}$) or small negative (i.e. $0 < -T_4 \ell_{\text{inf}(i)} \ll M_P^2$) tension, then it can be shown that the most likely creation process of $\text{inf}(i)$ is by the creation of a small- ℓ_{dS} space from nothing and then tunneling to $\text{inf}(i)$. The dS-dS tunneling rates in the best-understood part of the string landscape, which are also discussed in detail in chapter 5, are largely independent of the vacuum energy of the daughter vacuum. As a result, the most likely scale of inflation for us to measure is again determined by the number density of inflationary plateaus in the landscape as a function of energy.
- **Heavy ETW branes:** If there generally exist ETW branes with tension $T_4 \ell_{\text{inf}(i)} \gg M_P^2$, then the most likely creation of $\text{inf}(i)$ is the direct creation from nothing via a bubble of something. Using (6.1.11), the desired prediction becomes

$$\frac{p(\text{obs}, i)}{p(\text{obs}, j)} \simeq \exp\left(-8\pi^2 M_P^6 \left(\frac{1}{T_{4,\text{inf}(i)}^2} - \frac{1}{T_{4,\text{inf}(j)}^2}\right)\right), \quad (6.1.19)$$

resulting in a new landscape prediction: The plateau allowing for the highest-tension ETW brane is most likely to be realized!

- **Light ETW branes:** If there generally exist ETW branes with tension $0 < -T_4 \ell_{\text{inf}(i)} \ll M_P^2$, then the most likely creation of $\text{inf}(i)$ is the direct creation from nothing via a boundary process. Using (6.1.13), the desired prediction becomes

$$\frac{p(\text{obs}, i)}{p(\text{obs}, j)} \simeq \exp\left(-4\pi^2 \left(|T_{4,\text{inf}(i)}| \ell_{\text{inf}(i)} - |T_{4,\text{inf}(j)}| \ell_{\text{inf}(j)}\right)\right), \quad (6.1.20)$$

the prediction that the plateau-ETW brane system with the smallest $|T_{4,\text{inf}(i)}| \ell_{\text{inf}(i)}$ is most likely to be realized. This is again a novel prediction in the landscape.

6.2 Discussion

String theory is (basically) a unique theory of quantum gravity with no free parameters. It fulfills all the criteria for a good theory of everything. Although it admits no free parameters, string theory leads to a large (but finite) landscape of solutions, as discussed in Chapter 1, each realizing a different 4D universe. Since the existence of a large landscape is arguably the only explanation for the observed hierarchies or ‘unnatural’ relations in nature, string theory appears to be the prime candidate for our theory of quantum gravity. The vast size of the landscape is a blessing when trying to explain the existence of hierarchies, but it becomes a curse when searching for solutions that describe our real world. Arguably, the main shortcoming of string theory is that no convincing solution has been found that realizes universes with a positive cosmological constant. It is known that dS solutions cannot be found in the weak coupling regime of string theory, but despite the recent success of the swampland program, there are no hard arguments why dS solutions should be entirely absent. Even if dS vacua can be constructed, finding our world within the string landscape will still be extremely difficult, given its $\mathcal{O}(10^{500} - 10^{272\,000})$ number of vacua. As a result, we are forced to make statistical predictions to test the landscape paradigm. The Measure Problem then becomes a fundamental and crucial issue at the heart of theoretical physics. The goal of this thesis was to make well-defined predictions within the multiverse. While this goal has been achieved with the prediction of the most likely scale of inflation to be observed, many interesting research challenges remain for the future.

Finding dS vacua Some parts of this thesis become futile in case dS vacua are indeed absent from the landscape. The fact that we do not have a convincing model for realizing dS in string theory is discouraging, to say the least. Following the claims in [58, 60, 61], the construction of quintessence models has found renewed interest (see, e.g. [257–259]), as well as the study of non-supersymmetric string theories (see e.g. [260, 261]) and compactifications on non-CY manifolds [37]. However, both the KKLT [92] and LVS [93, 94] constructions, while identified to have issues [262–265], remain the leading candidates for realizing a realistic cosmology. The problems in these models arise from the difficulties of finding an ‘uplift,’ the addition of some positive energy-density source responsible for lifting an AdS solution to a dS solution. Various uplift mechanisms have been proposed, but none of them have so far been proven to be under computational control. Nevertheless, it is far from excluded that uplifting the AdS constructions of KKLT or LVS is impossible. Computational searches over explicit CY geometries have, to a large extent, verified the statistical expectations of analytical estimates. So far, constructing trustworthy dS solutions has not been achieved, but it is not inconceivable that a breakthrough in this direction, making use of advanced computer technology, is around the corner. It would be of tremendous importance to establish the existence of dS in string theory in order to provide the landscape approach to solving ‘hierarchical problems’ a better foundation. A different approach to finding dS solutions is to look for a string worldsheet description for strings propagating in dS space (see, e.g. [266]). While this approach may establish the existence of dS in string theory, it appears very difficult to establish an entire landscape of dS solutions in this way.

Local or global Measures and the Cosmological Central Dogma The various approaches to the measure problem can be categorized into ‘global’ and ‘local’ measures. The global measures define a way of counting events occurring in the global spacetime, i.e. not focused on a finite region of dS. The

local measures, on the other hand, only count events that occur in a local region of the global multiverse. The Local WDW Measure falls into the category of local measures, as it only takes the static patch region of dS as fundamental. The local approach is a consequence of the CCD, which is taken as a fundamental principle. The question of whether the local or the global approach is appropriate is deeply related to fundamental properties of quantum mechanics, such as complementarity [241] or the interpretation of the wavefunction.

The Cosmological Central Dogma is a proposal with wide-ranging implications, and it would be interesting to study it further. This is certainly a difficult task, as its sibling, the black hole information paradox, is known as a notoriously difficult problem in quantum gravity. A considered breakthrough in this problem has been achieved using the AdS/CFT correspondence through the discovery of the ‘Island Formula’ [267–269] and applications thereof. It refers to a concept describing how entanglement entropies in systems of quantum gravity can be computed and has its origin in a Euclidean gravitational path integral. While the corresponding formula provides a prescription for computing an entropy, the microscopic physics behind it remains unknown. Using the Island Formula, the discovery of ‘replica wormholes’ was made, and it was argued [270] that the entanglement entropy between the black hole system and the outside radiation starts decreasing after some time giving further evidence towards the claim that black holes evaporate in a unitary process. However, the microscopic processes that lead to the information escaping from the black hole can not be understood from the Island Formula. Rather, the computation [270] can be seen as further motivation for the idea that information is not lost in black holes. If the replica calculation is correct, the microscopic physics must involve non-local phenomena that are yet to be understood.

Being able to perform some analogous calculation for dS horizons to show that they have similar properties as black hole horizons would be a primary step toward further establishing the Cosmological Central Dogma. However, as AdS/CFT cannot be applied in dS, and since the horizon of a dS space is observer-dependent and not absolute, performing such a calculation proves to be very difficult.

It may be worthwhile to study the CCD in the context of 2D models of gravity, which have proven to be useful toy models of quantum gravity. One of the most studied models is Jackiw-Teitelboim (JT) gravity, where the gravitational path integral can be performed exactly. It is a widely used model for computations involving black hole horizons (and has been used significantly in [270]), and dS solutions of JT gravity have also been investigated [271–273] recently. For example, the first steps toward deriving the Island Formula for dS spaces have been taken in [274]. Studying the CCD in the context of JT gravity could lead to important insights and thus provides an interesting research target for the future.

ETW branes and their properties The ubiquitous existence of ETW branes has played an important role in a large part of this thesis. The cobordism conjecture [238] predicts their existence but does not make any statements about their properties. While we have developed an EFT description for ETW branes of Witten type that can also be applied to a variety of other ETW branes, it certainly does not describe all possible ETW branes. For example, it is possible that many ETW branes are unstable in the sense that the brane itself disappears. One may think of a brane whose energy density is set by a scalar field with a potential that is monotonically decreasing. As such, the energy density and hence the brane itself will disappear over time. Furthermore, ETW branes can be unstable to ‘thickening’. This can be pictured by a scalar field in the spacetime bulk that has a Dirichlet condition

at the spacetime boundary defining the ETW brane. If the fixed value of the scalar at the boundary is on top of a potential maximum, the scalar will develop a profile into the bulk and the ETW brane ‘thickens’. Examples of domain walls of such type are given in [275, 276]. While the majority of known ETW branes in string theory preserve SUSY and are hence stable, it is expected that there exists a large variety of non-SUSY ETW branes whose properties remain largely unknown. It may very well be that our simple EFT proposal needs to be refined in order to describe these branes, and it would be interesting to study them further. Studying the properties of existing ETW branes would be of tremendous importance in order to understand the possible creation and decay mechanisms existing in the landscape.

The study of gravity localized to an ETW brane, first considered in [250, 251], as an alternative to compactification, has received renewed interest (cf. [277, 278]). Studying the properties of ETW branes would be important for pursuing this research direction.

In general, the phrase ‘ETW brane’ is used in two slightly different contexts in the literature. While an ETW brane is often defined as an object sitting at the boundary of spacetime and described by an action involving the Gibbons-Hawking-York boundary term, in the context of ‘dynamical cobordism,’ an ETW brane is said to arise when a scalar field runs to infinite distance in field space over a finite distance in spacetime [244–249]. While we have managed to connect these two ‘definitions’ in the context of Witten-type ETW branes, it remains to be clarified whether they can be connected in every context.

Creation from Nothing The creation rates are a fundamental ingredient in making predictions, and they deserve to be studied further. Most pressingly, the question of whether the Linde/Vilenkin or the Hartle-Hawking proposal (or neither of them) is the correct one in quantum gravity remains unanswered. The Hartle-Hawking wavefunction is real, has the interpretation of a ground-state wavefunction, and appears to be independent of UV properties. However, it remains largely unclear how this wavefunction is related to a *creation from nothing* event. The Linde/Vilenkin wavefunction, on the other hand, has a source for universes nucleating at vanishing size. The precise form of the source is unknown, and it is strongly suggested that it is determined by the microscopic details of our theory of quantum gravity. Although we believe that the dominant contribution to the creation rate comes from the tunneling through a barrier region, a complete understanding of the creation from nothing process is difficult to achieve. To make matters worse, if the Linde/Vilenkin approach is correct, it is suggested that the creation of universes with nontrivial topology is most likely. However, to verify this statement, it is necessary to understand the source of the WDW equation in more detail. In principle, string theory should explain how to create a universe from nothing, but no significant progress has been made in the last forty years. This may be related to the fact that the process by which spacetime emerges from a state of strings is not (yet) known. This question is part of a more fundamental open problem in quantum gravity, namely, the study of topology change. While it is believed that topology change is possible in theories of quantum gravity, developing a method to systematically study such processes would be of tremendous importance.

Furthermore, the Linde/Vilenkin proposal has the problem of predicting unsuppressed metric fluctuations. It was argued long ago [182, 183] that considering fluctuations causes the potential barrier to decrease, leading to an unsuppressed creation from nothing process. However, the effects of fluctuations, and whether or not they are problematic, deserve further study. It would be interesting to

derive the effects of fluctuations on the creation from nothing processes.

The structure of the landscape One of our main results is that the most likely scale of inflation is determined by the number density of inflationary plateaus in the landscape as a function of energy. This prediction may be unsatisfying in the sense that it requires detailed knowledge of the properties of the landscape in order to make a precise prediction that can be tested by experiments. One could have hoped that the Local WDW Measure would lead to an exponential bias toward high- or low-scale inflation, making the precise structure of the landscape a second-order effect.

As we currently have access only to corners of the string landscape that are weakly coupled, a large part of the landscape, where dS solutions are to be expected, remains unknown. As such, making predictions in the multiverse is a tremendously difficult task. Obtaining a better understanding of the landscape is, therefore, an important objective. Beyond finding dS solutions, studying the possible particle contents, gauge groups, and couplings of the string landscape is an important subject.

Current efforts in this context come from F-theoretic constructions of the Standard Model as well as the study of inflation models. The limitation in these contexts arises from the stabilization of moduli, which remains to be understood in a general context.

String theory is oftentimes criticized for not being predictive. While this thesis has made a significant step toward developing a theoretical framework for making statistical predictions in the string landscape, the mathematical understanding of the landscape prevents us from drawing more precise conclusions. I want to stress that this shortcoming stems from the limitations of our calculational ability and not from a fundamental inconsistency in string theory. String theory, as such, with its finite-sized (or at least discrete) landscape, is on a fundamental level more predictive than theories allowing for arbitrary tuning of coupling constants.

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