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Some Aspects of Inflationary Particle Production

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Einige Aspekte inflationärer Teilchenproduktion

Zusammenfassung

Es werden Teilchenproduktion durch einen variierenden Massenterm und durch die Hintergrundmetrik betrachtet. Wir leiten eine Definition der Teilchenzahl in der kinetischen Theorie her, sowohl für den fermionischen als auch den skalaren Fall, die wir auf die Situation einer flavourmischenden Massenmatrix verallgemeinern. Dies ermöglicht es uns, den Preheatingprozeß mit C und CP Verletzung zu versehen, was zum Szenario der kohärenten Baryogenese führt. Wir stellen Modelle vor, in denen dieser Mechanismus im Zusammenhang mit Hybridinflation und den großen vereinheitlichten Theorien Pati-Salam und SO(10) tätig ist. Es wird gezeigt, daß eine Baryonenasymmetrie im Einklang mit Beobachtungen resultieren kann. Außerdem betrachten wir Fragen der Quantentheorie im gekrümmten Raum. Skalarfelder im expandierenen Universum und im Rindlerraum werden diskutiert. Es stellt sich heraus, daß neben der Teilchendetektionsrate die Lambverschiebung der Energieniveaus ein wichtiger Effekt ist, den Unruhs Detektor in diesen Raumzeiten erfährt.

Some Aspects of Inflationary Particle Production

Abstract

Particle production by a varying mass term and by the background metric are considered. We derive a definition of particle number in kinetic theory for both, fermionic and scalar case, which we generalize to the situation of a flavour-mixing mass matrix. This allows us to endow the process of preheating with C and CP violation, leading to the coherent baryogenesis scenario. We present models where this mechanism is operative in the context of hybrid inflation and the grand unified theories Pati-Salam and SO(10). It is shown that a baryon asymmetry in accordance with observation may result. Moreover, we consider issues of quantum theory in curved space. Scalar fields in the expanding Universe and in Rindler space are discussed. It turns out that besides the particle detection rate, the Lamb shift of energy levels is an important effect experienced by Unruh's detector in these spacetimes.

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Chapter 1

Introduction

The discovery that the world began with the Big Bang immediately brought along as next question where all the energy came from. The answer was given, rather as a by-product of the solution to the flatness, homogeneity and isotropy problems, by inflationary cosmology [1,2]. During inflation, the dominating contribution to the energy density ϱ of the Universe is a *vacuum energy* of negative pressure p, which implies that energy is produced by expansion, opposite to the somewhat more familiar experience that a gas is of positive pressure and has to do work in order to dilate. In particular, when $p = -\rho = \text{const.}$, the Universe expands exponentially fast, corresponding to a *de Sitter* spacetime. The vacuum energy could be provided by a scalar field condensate $\langle\phi\rangle$ with a vacuum expectation value giving rise to a nonvanishing value of the scalar potential, $\varrho = V(\phi) \neq 0$. Almost all of the matter and radiation contained within the present-day Universe then stems from the scalar condensate. At the place of the question of the origin of energy therefore steps the new riddle why the initial state of the Universe was the inflationary vacuum.

According to the standard picture for the end of inflation, $\langle\phi\rangle$ oscillates around a minimum of the potential, where $V(\phi) = 0$, and then perturbatively decays into particles, a process usually named *reheating*. Besides, there is another channel for decay, which is nonperturbative. Couplings of $\langle\phi\rangle$ to other fields induce mass terms for these, and when the condensate is evolving nonadiabatically fast, particle production occurs due to the strongly time-dependent masses [3]. This process of resonant particle production, often referred to as *preheating*, is therefore possibly of great relevance for the history of the early Universe and has been subject of extensive studies [4–8].

No matter how the inflaton energy is transferred, eventually the observed asymmetry between matter and antimatter, or, more precisely, between baryons and antibaryons, ought to arise. Since Sakharov suggested that this asymmetry is not immersed into the Universe as a initial condition but should rather be the result of a dynamical baryogenesis process [9], quite a few of such possible scenarios have been suggested.

They all fulfill the three celebrated Sakharov conditions: first, charge (C) and charge-parity (CP) violation, second, baryon number (B) violation and third, deviation from thermal equilibrium.

While deviation from thermal equilibrium is readily realized at the phase transition which terminates inflation, some important scenarios, *e.g.* electroweak baryogenesis and thermal leptogenesis, assume first equilibration of the Universe and while cooling down by expansion again a departure from equilibrium. The requirement of a sufficient deviation from thermal equilibrium poses important constraints on these mechanisms, which however may also serve to rule them out. It has therefore been suggested that baryogenesis may take place right at the postinflationary phase transition. For example, the condensate may first decay into Majorana neutrinos which subsequently feed baryogenesis by leptogenesis [10].

Here, a novel mechanism for baryogenesis, which does not rely on such a mediating particle but directly yields a charge asymmetry through the nonperturbative decay of the inflaton during preheating, is presented and dubbed *coherent baryogenesis*. The mandatory violation of C and CP is evoked by the presence of a nonsymmetric mixing mass matrix. In chapter 3, we develop an appropriate formalism for resonant particle production in the multiflavour case, which relies on kinetic theory and besides provides a definition of particle number in terms of phase space densities of charges and currents, derived from first principles. As applications, in chapters 4 and 5, we discuss coherent baryogenesis in the context of hybrid inflationary models, where the terminating phase transition goes along with the breaking of the *grand unified theories* (GUTs) Pati-Salam and SO(10), respectively.

For preheating, one considers weakly interacting particles in a quasi-Minkowski background, where *quasi* indicates, that the variation of the background metric is negligible when compared to the variation of the mass term. When eventually the mass term ceases to vary, particle number is well defined. It is very intriguing however, that the expanding background itself can excite the vacuum and thereby produce particles, a process which appears to be similar to preheating, since it can effectively also be described by a varying mass term. A closer glance at the literature reveals however, that besides Parker's seminal work [11], which apparently has been of great influence on the later papers on preheating, there is some disagreement about gravitational particle production – or at least quite different aspects of this phenomenon are advocated.

The probably most popular point of view is that from event horizons, as present *e.g.* for black holes, de Sitter and Rindler spaces, thermal radiation is emitted [12–14]. The usual equipment for *gedankenexperimente* to capture this emanation is an *Unruh detector* [13], an idealized device which, in a certain sense, in fact seems to perceive a thermal particle bath. This aspect of particle production therefore is exponentially suppressed with growing particle energy. Nonetheless it is very often confused with the effect described by Parker, the covariant stress-energy tensor or the generation of cosmic perturbations, which in obvious contrast indicate a power law spectrum.

In this thesis, we therefore consider particle production by the background metric from different angles. In chapter 2, we introduce the theory of a quantum scalar field in a classical expanding background, calculate the stress-energy tensor of the scalar and briefly discuss the occurring divergences, while the results presented in chapter 3 correspond to Parker's definition, when applied to the expanding Universe case. Chapter 6 contains a comprehensive study of the response rate of an Unruh detector in de Sitter space; results for arbitrary dimension and curvature couplings are derived. It is discussed, in what sense all these different spectra can be regarded as thermal. The question whether the detector is in any way sensitive to the power-law behaviour of the scalar stress-energy in the expanding Universe leads us to the investigations of chapter 7, where we calculate the self-energy corrections to the energy levels of an Unruh detector, corresponding to the Lamb shift as familiar from atomic physics. We consider the expanding Universe case as well as a constantly accelerated detector and comment on the applicability of the expressions for the transition amplitudes, which are employed in chapter 6.

We conclude by comparing once more the two main topics of this thesis, particle production from preheating, *i.e.* when endowed with P and CP violation, and particle production induced by the background metric. A consistent picture of both effects and their interrelations shall be given.

Chapter 2

Stress-Energy in the Expanding Universe

Despite strong efforts over many decades, a quantum theory of gravity, which is mathematically consistent and at the same time also describing nature, has not yet been found. Therefore, its formulation is probably the toughest problem in theoretical physics; and being the prerequisite for a unification of gravity with the other known interactions, its outstanding importance is out of question. Interesting results can however already be obtained by treating quantum fields in a classical gravitational background, an approach called quantum theory in curved spacetime [15].

As a basis for our subsequent discussions and to introduce our conventions, we briefly review in the following the basic features of the Friedmann Universe. We give all equations in conformal time, which we find for our purposes to be more suitable than the more commonly used comoving time. Furthermore, we present the theory of a scalar field in an expanding background, in particular its quantization and, as a first aspect of particle production, the calculation of the vacuum expectation value of its stress-energy tensor.

This chapter appears at the beginning of this thesis because in our treatments of kinetic theory and baryogenesis, we take account of the expansion of the Universe. While in those contexts gravitational particle production appears as a side effect only, chapters 6 and 7 are completely devoted to this phenomenon.

2.1 The Friedmann-Lemaître-Robertson-Walker Universe

Let us consider the Einstein-Hilbert action

$$S_{EH} = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left\{ 2\Lambda + R \right\} , \tag{2.1}$$

where Λ is the cosmological constant, or vacuum energy, R the scalar curvature and g the determinant of the metric tensor $g_{\mu\nu}$. Variation with respect to $g^{\mu\nu}$ gives us the left hand side of the Einstein equations,

$$\frac{\delta S_{EH}}{\delta g^{\mu\nu}} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,\{\Lambda g_{\mu\nu} + G_{\mu\nu}\}\ , \tag{2.2}$$

where $G_{\mu\nu}$ denotes the Einstein tensor,

$$G_{\mu\nu} = \frac{1}{2}Rg_{\mu\nu} - R_{\mu\nu}. \tag{2.3}$$

When matter is described by the Lagrangean $\mathcal{L}$, its stress-energy tensor can be derived from the action

$$S = \int d^4x\sqrt{-g}\mathcal{L} \tag{2.4}$$

by the variation

$$\delta S = -\frac{1}{2}\int d^4x\sqrt{-g}T^{\mu\nu}\delta g_{\mu\nu}\ . \tag{2.5}$$

The principle of general coordinate invariance reads

$$\frac{\delta(S_{EH} + S)}{\delta g^{\mu\nu}} = 0 \tag{2.6}$$

and gives us the Einstein equations

$$\frac{1}{8\pi G}\{G_{\mu\nu} + \Lambda g_{\mu\nu}\} = T_{\mu\nu}. \tag{2.7}$$

The Friedmann-Lemaître-Robertson-Walker (FLRW) Universe is a spatial homogeneous spacetime, a feature which becomes manifest, if we choose for the metric tensor the form[1]

$$g^{\mu\nu} = a^2(\eta)\mathrm{diag}(1,-1,-1,-1)\,, \tag{2.8}$$

where a is called the *scale factor* and η *conformal time.*

In the FLRW-Universe, the Einstein tensor (2.3) has the components

$$G_{00} = 3\left(\frac{a''}{a} + \frac{a'^2}{a^2}\right)\,, \tag{2.9}$$

$$G_{ii} = -5\frac{a''}{a} + \frac{a'^2}{a^2}\,, i = 1,2,3\,, \tag{2.10}$$

where the prime denotes derivative *w.r.t.* conformal time, $' \equiv \frac{d}{d\eta}$. Spatial homogeneity also holds for the stress-energy tensor, which we parametrize by

$$T_{\mu\nu} = a^2(\eta)\mathrm{diag}(\varrho, p, p, p)\,, \tag{2.11}$$

where ϱ can be identified with the energy density and p with the pressure.

[1] We restrict ourselves here to the spatially flat case.

The temporal component of the Einstein equations (2.7),

$$\frac{a''}{a} + \frac{a'^2}{a^2} + \frac{1}{3}\Lambda = \frac{8\pi G}{3} a^2 \varrho \,, \tag{2.12}$$

is called the Friedmann equation. Together with the covariant conservation law

$$T^{0\nu}{}_{;\nu} = \varrho' + 3(\varrho + p)\frac{a'}{a} = 0 \,, \tag{2.13}$$

where the semicolon denotes covariant derivative, these constitute a system of two differential equations which may be integrated for given $\varrho(\eta)$ and $p(\eta)$ to give the expansion history in terms of the scale factor $a(\eta)$.

The most important examples are the matter Universe, where

$$p = 0 \quad \text{and} \quad a(\eta) = a_m \eta^2 \,, \tag{2.14}$$

radiation, where

$$p = \frac{1}{3}\varrho \quad \text{and} \quad a(\eta) = a_r \eta \,, \tag{2.15}$$

and de Sitter inflation, where

$$p = -\varrho \quad \text{and} \quad a(\eta) = -\frac{1}{H\eta} \,. \tag{2.16}$$

The respective integration constants are a_m, a_r and H, where the latter quantity happens to be the Hubble rate $H = \frac{a'}{a^2}$.

2.2 Scalar Field in Expanding Background

Let ϕ be a scalar field described by the Lagrangean

$$\sqrt{-g}\mathcal{L} = \sqrt{-g}\left(\frac{1}{2}g_{\mu\nu}\partial^\mu\phi\partial^\nu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{2}\xi R\phi^2\right) \,, \tag{2.17}$$

implying the Euler-Lagrange equation of motion

$$\left[\nabla^2 + m^2 + \xi R\right]\phi(x) = 0 \,, \tag{2.18}$$

where ∇ denotes covariant derivative. In a FLRW-background, this becomes

$$\left[\partial_\eta^2 + 2\frac{a'}{a}\partial_\eta + \left(-\vec{\partial}^2 + a^2 m^2\right) + 6\xi\frac{a''}{a}\right]\phi(x) = 0 \,, \tag{2.19}$$

with $\vec{\partial}$ being the spatial derivative. Upon the substitution $\varphi = a\phi$, the damping term $\propto \partial_\eta\phi$ drops out, *cf.* Eqn. (2.21) below, and the following single mode decomposition of φ then generally holds:

$$\varphi(x) = \int \frac{d^3k}{(2\pi)^3}\left(\mathrm{e}^{i\mathbf{k}\cdot\mathbf{x}}\varphi(\mathbf{k},\eta)a(\mathbf{k}) + \mathrm{e}^{-i\mathbf{k}\cdot\mathbf{x}}\varphi^*(\mathbf{k},\eta)a^\dagger(\mathbf{k})\right) . \tag{2.20}$$

Here $a(\mathbf{k})$ and $a^\dagger(\mathbf{k})$ denote the annihilation and creation operators for the mode with a comoving momentum $\mathbf{k}$, and they are defined by $a^\dagger(\mathbf{k})|0\rangle = |\mathbf{k}\rangle$, $a(\mathbf{k})|\mathbf{k}'\rangle = (2\pi)^3\delta^3(\mathbf{k}-\mathbf{k}')|0\rangle$, where $|0\rangle$ denotes the vacuum state and $|\mathbf{k}\rangle$ the one-particle state with momentum $\mathbf{k}$. The mode functions $\varphi(\mathbf{k},\eta)$ satisfy the equation

$$\Big(\partial_\eta^2 + \left(\mathbf{k}^2 + a^2m^2\right) + (6\xi-1)\frac{a''}{a}\Big)\varphi(\mathbf{k},\eta) = 0\,. \tag{2.21}$$

Throughout this thesis we assume that the modes $\varphi(\mathbf{k}) = \varphi(k)$ ($k \equiv |\mathbf{k}|$) are homogeneous. The field φ obeys the canonical commutation relation,

$$[\varphi(\mathbf{x},\eta), \partial_\eta\varphi(\mathbf{x}',\eta)] = \mathrm{i}\delta^3(\mathbf{x}-\mathbf{x}'), \tag{2.22}$$

which implies the normalization of the mode functions by the Wronskian

$$\varphi^*(\mathbf{k},\eta)\varphi'(\mathbf{k},\eta) - \varphi^{*\prime}(\mathbf{k},\eta)\varphi(\mathbf{k},\eta) = \mathrm{i}\,, \tag{2.23}$$

and for the creation and annihilation operators the commutator

$$[a_{\mathbf{k}}, a^\dagger_{\mathbf{k}'}] = (2\pi)^3\delta^{(3)}(\mathbf{k}-\mathbf{k}')\,. \tag{2.24}$$

2.3 Stress-Energy of a Scalar Field

Here we review the adiabatic expansion of the stress-energy tensor of a scalar field in a FLRW Universe [16, 17]. From the variation (2.5) we find the stress-energy tensor to be

$$\begin{aligned} T^{\mu\nu} &= \partial^\mu\phi\partial^\nu\phi - \frac{1}{2}g^{\mu\nu}\left(g_{\varrho\sigma}\partial^\varrho\phi\partial^\sigma\phi - m^2\phi^2\right) \\ &\quad + \xi G^{\mu\nu}\phi^2 + \xi\left(g^{\mu\nu}g_{\varrho\sigma}\nabla^\varrho\nabla^\sigma - \nabla^\mu\nabla^\nu\right)\phi^2\,. \end{aligned} \tag{2.25}$$

Upon inserting (2.20) into (2.25) and taking expectation value with respect to the vacuum $|0\rangle$, we get for the expectation values of the components of the stress-energy tensor

$$\begin{aligned} \langle 0|T^{00}(x)|0\rangle &= \int\frac{d^3k}{(2\pi)^3}\frac{1}{a^6}\Bigg\{\left[\omega^2 + \frac{1-6\xi}{2}\left(\frac{a'^2}{a^2} - \frac{a''}{a}\right)\right]|\varphi|^2 \\ &\quad - \frac{1-6\xi}{2}\frac{a'}{a}\partial_\eta|\varphi|^2 + \frac{1}{4}\partial_\eta^2|\varphi|^2\Bigg\}, \end{aligned} \tag{2.26}$$

$$\langle 0|T^{0i}(x)|0\rangle = \langle 0|T^{i0}(x)|0\rangle = \int\frac{d^3k}{(2\pi)^3}\frac{k^i}{a^6}\Im(\varphi'\varphi^*)\,, \tag{2.27}$$

$$\begin{aligned} \langle 0|T^{ij}(x)|0\rangle &= \int\frac{d^3k}{(2\pi)^3}\frac{1}{a^6}\Bigg\{\left[k^ik^j + \delta^{ij}\frac{1-6\xi}{2}\left(\frac{a'^2}{a^2} - \frac{a''}{a}\right)\right]|\varphi|^2 \\ &\quad - \delta^{ij}\frac{1-6\xi}{2}\frac{a'}{a}\partial_\eta|\varphi|^2 + \delta^{ij}\left(\frac{1}{4} - \xi\right)\partial_\eta^2|\varphi|^2\Bigg\}, \end{aligned} \tag{2.28}$$

where we defined the single particle energy

$$\omega = \sqrt{\mathbf{k}^2 + a^2 m^2}\,. \tag{2.29}$$

Note that, as a consequence of the isotropy of FLRW spacetimes, $\langle 0|T^{\mu\nu}(x)|0\rangle$ ($\mu \neq \nu$) vanishes. Indeed, the spacetime isotropy implies that φ is a function of the momentum magnitude $k \equiv |\mathbf{k}|$ and η only, such that, when the contributions to the stress-energy tensor of opposite momenta are added, a cancellation occurs.

An exact solution to Eqn. (2.21) can only be found for special $a(\eta)$. We therefore adapt the approach of adiabatic expansion and start with the WKB *ansatz*

$$\varphi(\mathbf{k},\eta) = \alpha(\mathbf{k})\big(2W(\mathbf{k},\eta)\big)^{-\frac{1}{2}}\,\mathrm{e}^{-\mathrm{i}\int^{\eta} d\eta' W(\mathbf{k},\eta')} + \beta(\mathbf{k})\big(2W(\mathbf{k},\eta)\big)^{-\frac{1}{2}}\,\mathrm{e}^{\mathrm{i}\int^{\eta} d\eta' W(\mathbf{k},\eta')}, \tag{2.30}$$

with the normalization condition $|\alpha|^2 - |\beta|^2 = 1$. Then, we find from (2.21)

$$W^2 = \omega^2 - (1-6\xi)\frac{a''}{a} + \frac{3}{4}\frac{W'^2}{W^2} - \frac{1}{2}\frac{W''}{W}\,. \tag{2.31}$$

This equation can be solved iteratively, according to the scheme

$$\begin{aligned} {W^{(0)}}^2 &= \omega^2\,, \\ {W^{(2)}}^2 &= \omega^2 - (1-6\xi)\frac{a''}{a} + \frac{3}{4}\frac{{W^{(0)}}'^2}{{W^{(0)}}^2} - \frac{1}{2}\frac{{W^{(0)}}''}{W^{(0)}}\,, \\ &\cdots \end{aligned} \tag{2.32}$$

We take for the vacuum $|0\rangle$ the purely negative frequency state at infinitely early times, that is $\alpha = 1$ and $\beta = 0$. This choice is well motivated by cosmological inflationary models, and it is a standard choice for studies of de Sitter space [14, 18–23], as well as for general FLRW spacetimes [16, 17]. Up to second adiabatic order, that is to second order in derivatives *w.r.t.* η, the stress-energy (2.26–2.28) is

$$\langle 0|T^{00}_{(2)}(x)|0\rangle = \int \frac{d^3k}{(2\pi)^3}\frac{1}{a^6}\left\{\frac{\omega}{2} + \frac{1-6\xi}{4\omega}\frac{a'^2}{a^2} + \frac{1-6\xi}{4}\frac{a'^2}{a^2}\frac{a^2m^2}{\omega^3} + \frac{1}{16}\frac{a'^2}{a^2}\frac{a^4m^4}{\omega^5}\right\}, \tag{2.33}$$

$$\langle 0|T^{0i}(x)|0\rangle = \langle 0|T^{i0}(x)|0\rangle = 0, \tag{2.34}$$

$$\begin{aligned} \langle 0|T^{ij}_{(2)}(x)|0\rangle = \int \frac{d^3k}{(2\pi)^3}\frac{\delta^{ij}}{a^6}\Bigg\{&\frac{\omega}{6} - \frac{a^2m^2}{6\omega} + \frac{1-6\xi}{12\omega}\left[3\frac{a'^2}{a^2} - 2\frac{a''}{a}\right] \\ &+\frac{1-6\xi}{6}\left[\frac{a'^2}{a^2} - \frac{a''}{a}\right]\frac{a^2m^2}{\omega^3} + \left[\left(\frac{11}{48} - \frac{3}{2}\xi\right)\frac{a'^2}{a^2} - \frac{1}{24}\frac{a''}{a}\right]\frac{a^4m^4}{\omega^5} + \frac{5}{48}\frac{a'^2}{a^2}\frac{a^6m^6}{\omega^7}\Bigg\}, \end{aligned} \tag{2.35}$$

where we made use of $\int d^3k k^i k^j = (\delta^{ij}/3)\int d^3k k^2 = (\delta^{ij}/3)\int d^3k(\omega^2 - a^2m^2)$ and $\Im\big(\varphi'\varphi^*\big) = -1/2$. For both, the zeroth and second adiabatic order contributions separately, the covariant conservation $\nabla_\mu T^{\mu 0}_{(2)} = \partial_\eta T^{00}_{(2)} + 5(a'/a)T^{00}_{(2)} + 3(a'/a)T^{ii}_{(2)} = 0$ holds. When integrated over momentum space, the zeroth order term yields a quartic divergence, corresponding to the vacuum energy, also referred to as cosmological term, and the second order contribution diverges quadratically. In order to deal with the infinity at second adiabatic order, it is suggested either to rescale Newton's constant G by an infinite amount [15, 16, 24], or to simply discard this term [17], which would mean in turn that this contribution is not observable.

Since an ultraviolet regularization should not affect the infrared domain, the latter procedure is in conflict with the amplification of quantum fluctuations at horizon crossing during inflation [25–28], leading to the observed primordial density fluctuations, which is a sound prediction of quantum theory in curved space-times. In that calculation, a subtraction of the second adiabatic order terms is not performed, and would make no sense.

The other alternative, the renormalization of G, seems to disagree with perturbative quantum gravity. Since gravity is a perturbatively nonrenormalizable theory, order by order in loops, new coupling constants for higher order geometric invariant terms are renormalized [29, 30], but not the leading Newton's constant G. In fact, perturbative quantum gravity reproduces at tree level known classical metrics of general relativity and predicts quantum corrections at loop order [30–34] without shifting G by any amount. It is however interesting to note that the adiabatic regularization approach can be useful in identifying divergences occurring at loop order in quantum electrodynamics, since this is a renormalizable theory. In Refs. [35, 36] it is shown that, for pair creation in a stationary electric field, one recovers the familiar logarithmic divergence of the photon vacuum polarization.

In the following, we take the point of view that the second adiabatic order contributions are observable energy and momentum densities, and they should not be removed but need to be regulated by a cutoff. In fact, in chapter 7 we point out that an Unruh detector *can* observe the contributions at second adiabatic order. However, the zeroth order term corresponds to the vacuum energy and shall be subtracted.

Chapter 3

Particle Number in Kinetic Theory

The notion of particles is very intuitive, and at the classical level, in statistical physics, the dynamics is very successfully described by the classical Boltzmann equation for particle densities on phase space. In quantum physics however, the uncertainty principle seems to prohibit the use of phase space densities, and they are replaced by their closest analogues, the Wigner functions [37,38], which are the Fourier transforms of the two point functions *w.r.t.* the relative coordinate, while the center of mass coordinate is kept fixed. Yet, strictly speaking these functions can neither be interpreted as particle numbers nor as probability distributions on phase space, since they may acquire negative values, as already pointed out in Wigner's original work [37]. Attempts have been made to define particle number in relativistic quantum kinetic theory [39], but so far there exists no result which is derived from first principles and that would be applicable to general situations.

In spite of those difficulties, the dynamics of quantum fields and particle numbers in the presence of temporally varying background fields have been extensively studied in the context of early Universe cosmology and are well understood [5, 11, 40]. The particle number operator can be calculated by a Bogolyubov transformation rotating the Fock space to a new basis, which mixes positive and negative frequency solutions or, as an alternative point of view, creation and annihilation operators.

In the analysis presented in this chapter, we apply the particle number definition by Bogolyubov transformations to kinetic theory. We show that the Wigner function, which we take as an expectation value with respect to the ground state of the original basis, provides the necessary information about the rotated basis to calculate the particle number induced by the coupling to time-dependent external fields. We discuss the cases of scalar particles and spin-1/2 fermions and eventually generalize to several mixing flavours, where our formalism proves particularly useful.

3.1 Scalars

3.1.1 Scalar Kinetic Equations

As the first model case, we consider a scalar field with temporally varying mass term in an expanding background, as described in section 2.2.

The fundamental quantity of quantum kinetic theory is the two-point Wightman function, which we here write for the ground state $|0\rangle$. With the rescaling suitable for conformal space-times, it reads

$$\mathrm{i}\bar{G}^{<}(u,v) \equiv a(u)\mathrm{i}G^{<}(u,v)a(v) = \langle 0|\varphi(v)\varphi(u)|0\rangle\,, \tag{3.1}$$

and its Wigner transform is defined as

$$\mathrm{i}\bar{G}^{<}(k,x) = \int d^4r\mathrm{e}^{\mathrm{i}k\cdot r}\mathrm{i}\bar{G}^{<}(x+r/2,x-r/2)\,. \tag{3.2}$$

We note, that under Wigner transformation

$$\begin{aligned}\partial_u \mathrm{i}\bar{G}^{<}(u,v) &= \partial_u \int \frac{d^4k}{(2\pi)^4}\mathrm{e}^{-\mathrm{i}k\cdot(u-v)}\mathrm{i}\bar{G}^{<}\left(k,\frac{u+v}{2}\right) \\ &= \int \frac{d^4k}{(2\pi)^4}\mathrm{e}^{-\mathrm{i}k\cdot r}\left(k\bar{G}^{<}(k,x)+\frac{\mathrm{i}}{2}\partial_x\bar{G}^{<}(k,x)\right)\,,\end{aligned} \tag{3.3}$$

while the transformation of the mass term is done by partial integrations:

$$\begin{aligned}\int d^4k\mathrm{e}^{-\mathrm{i}kr}m(x)e^{-\frac{\mathrm{i}}{2}\overleftarrow{\partial}_x\cdot\partial_k}\mathrm{i}G^{<}(k,x) &= \int d^4k\mathrm{e}^{-\mathrm{i}k\cdot r}m(x+r/2)\mathrm{i}G^{<}(k,x) \\ &= m(u)\mathrm{i}\bar{G}^{<}(u,v)\,,\end{aligned} \tag{3.4}$$

where $\overleftarrow{\partial}$ denotes a partial derivative acting on the left hand side.

The Wigner transform of the Wightman function (3.2) therefore satisfies the Klein-Gordon equation [41]

$$\left[-\mathrm{i}k_0\partial_\eta + \frac{1}{4}\partial_\eta^2 - k^2 + \left(a^2(\eta)m^2(\eta) + (6\xi-1)\frac{a''(\eta)}{a(\eta)}\right)\mathrm{e}^{-\frac{\mathrm{i}}{2}\overleftarrow{\partial}_\eta\partial_{k_0}}\right]\mathrm{i}\bar{G}^{<} = 0\,. \tag{3.5}$$

It is then useful to define the n-th moments of the Wigner function,

$$f_n(\mathbf{k},x) \equiv \int \frac{dk_0}{2\pi}k_0^n\mathrm{i}\bar{G}^{<}(k,x). \tag{3.6}$$

Taking the 1st (0th) moment of the imaginary (real) part of Eqn. (3.5) gives [41]

$$\begin{aligned}f_2' - \frac{1}{2}\left(a^2m^2 + (6\xi-1)\frac{a''}{a}\right)' f_0 = 0\,,\\ \frac{1}{4}f_0'' - f_2 + (\bar{\omega}^2 - \frac{a''}{a})f_0 = 0\,,\end{aligned} \tag{3.7}$$

with ω as defined in Eqn. (2.29) and

$$\bar{\omega}^2(\mathbf{k},\eta) = \omega^2(\mathbf{k},\eta) + 6\xi\frac{a''}{a}. \tag{3.8}$$

Eliminating f_2 from (3.7) yields

$$f_0''' + 4\left(\bar{\omega}^2 - \frac{a''}{a}\right)f_0' + 2\left(\bar{\omega}^2 - \frac{a''}{a}\right)' f_0 = 0. \tag{3.9}$$

This can be integrated once to give

$$\bar{\omega}^2 f_0^2 + \frac{1}{2}f_0{}'' f_0 - \frac{1}{4}f_0'^2 = \frac{1}{4}, \tag{3.10}$$

where the integration constant is obtained by making use of $f_0 = |\varphi|^2$ (*cf.* Eqn. (3.29) below), Eqn. (2.21) and the Wronskian (2.23).

3.1.2 Bogolyubov Transformation

The Hamiltonian density corresponding to the Lagrangean (2.17) reads

$$H(\eta) = \frac{1}{2}\int\frac{d^3k}{(2\pi)^3}\left\{\Omega(\mathbf{k},\eta)(a(\mathbf{k})a^\dagger(\mathbf{k}) + a^\dagger(\mathbf{k})a(\mathbf{k})) + (\Lambda(\mathbf{k},\eta)a(\mathbf{k})a(-\mathbf{k}) + \text{h.c.})\right\}, \tag{3.11}$$

where

$$\Omega(\mathbf{k},\eta) = \left|\varphi'(\mathbf{k},\eta) - (a'/a)\varphi(\mathbf{k},\eta)\right|^2 + \bar{\omega}^2(\mathbf{k},\eta)\left|\varphi(\mathbf{k},\eta)\right|^2, \tag{3.12}$$

$$\Lambda(\mathbf{k},\eta) = \left(\varphi'(\mathbf{k},\eta) - \frac{a'}{a}\varphi(\mathbf{k},\eta)\right)^2 + \bar{\omega}^2(\mathbf{k},\eta)\varphi^2(\mathbf{k},\eta). \tag{3.13}$$

The quantity Ω therefore is the vacuum expectation value of the Hamiltonian. Note that this canonical energy density equals the covariant energy density (2.26) as obtained from the stress-energy tensor only in the minimally coupled case $\xi = 0$.

Consider now the homogeneous Bogolyubov transformation

$$\begin{pmatrix} \hat{a}(\mathbf{k}) \\ \hat{a}^\dagger(-\mathbf{k}) \end{pmatrix} = \begin{pmatrix} \alpha(k) & \beta^*(k) \\ \beta(k) & \alpha^*(k) \end{pmatrix}\begin{pmatrix} a(\mathbf{k}) \\ a^\dagger(-\mathbf{k}) \end{pmatrix}, \tag{3.14}$$

with the norm

$$|\alpha(k)|^2 - |\beta(k)|^2 = 1. \tag{3.15}$$

This transformation therefore rotates the basis of creation and annihilation operators. We call the form of the Hamiltonian (3.11) off-diagonal, because of the presence of the terms $\propto \Lambda$. The particle number definition suggested by Parker [11], is the expectation value of the particle number operator in the basis, where the Hamiltonian is diagonal. In the following, we prove the extremality property of this definition, that the particle number in the diagonal basis is the maximum number for any Bogolyubov transformation.

Upon the rotation (3.14), $\Lambda(\mathbf{k})$ and $\Omega(\mathbf{k})$ transform as

$$\Lambda'(\mathbf{k}) = -2\alpha^*(k)\beta(k)\Omega(\mathbf{k}) + (\alpha^*(k))^2\Lambda(\mathbf{k}) + \beta(k)^2\Lambda^*(\mathbf{k})\,, \tag{3.16}$$

$$\Omega'(\mathbf{k}) = (|\alpha(k)|^2 + |\beta(k)|^2)\Omega(\mathbf{k}) - \alpha^*(k)\beta^*(k)\Lambda(\mathbf{k}) - \alpha(k)\beta(k)\Lambda^*(\mathbf{k})\,. \tag{3.17}$$

In terms of real and imaginary parts, these equations can be recast as

$$\begin{aligned} 2|\alpha(k)||\beta(k)|\Omega(\mathbf{k}) + |\Lambda'(\mathbf{k})|\cos(\phi_\lambda+\phi_\alpha-\phi_\beta) & \\ -(|\alpha(k)|^2+|\beta(k)|^2)|\Lambda(\mathbf{k})|\cos(\phi_\Lambda-\phi_\alpha-\phi_\beta) &= 0, \end{aligned} \tag{3.18}$$

$$|\Lambda'(\mathbf{k})|\sin(\phi_{\Lambda'}+\phi_\alpha-\phi_\beta)-|\Lambda(\mathbf{k})|\sin(\phi_\Lambda-\phi_\alpha-\phi_\beta) = 0, \tag{3.19}$$

$$\Omega'(\mathbf{k})-(|\alpha(k)|^2+|\beta(k)|^2)\Omega(\mathbf{k})+2|\alpha(k)||\beta(k)||\Lambda(\mathbf{k})|\cos(\phi_\Lambda-\phi_\alpha-\phi_\beta) = 0, \tag{3.20}$$

with $|\alpha(k)| = \sqrt{1+|\beta(k)|^2}$, and where we have introduced the phases

$$\Lambda'(\mathbf{k}) = |\Lambda'(\mathbf{k})|\exp(\mathrm{i}\phi_{\Lambda'})\,, \quad \Lambda(\mathbf{k}) = |\Lambda(\mathbf{k})|\exp(\mathrm{i}\phi_\Lambda)\,, \tag{3.21}$$

$$\alpha(k) = |\alpha(k)|\exp(\mathrm{i}\phi_\alpha)\,, \quad \beta(k) = |\beta(k)|\exp(\mathrm{i}\phi_\beta)\,. \tag{3.22}$$

Eqs. (3.18) and (3.20) can be combined to give

$$\cos(\phi_\lambda + \phi_\alpha - \phi_\beta) = \frac{(|\alpha(k)|^2 + |\beta(k)|^2)\Omega(\mathbf{k}) - \Omega'(\mathbf{k})}{2|\alpha(k)|\,|\beta(k)|\,|\Lambda(\mathbf{k})|}\,, \tag{3.23}$$

while (3.20) yields an expression for $\cos(\phi_\Lambda - \phi_\alpha - \phi_\beta)$. Upon squaring Eqn. (3.19) and making use of $\sin^2(\zeta) = 1 - \cos^2(\zeta)$ and the Wronskian (2.23), we find that

$$\Omega(\mathbf{k})^2 - |\Lambda(\mathbf{k})|^2 = \Omega'(\mathbf{k})^2 - |\Lambda'(\mathbf{k})|^2 = \bar{\omega}^2(\mathbf{k}) \tag{3.24}$$

is an invariant of the Bogolyubov transformations (3.14).

Next, we solve (3.20) for $n(\mathbf{k}) \equiv |\beta(k)|^2$ to find

$$n^\pm(\mathbf{k}) = \frac{\Omega(\mathbf{k})\Omega'(\mathbf{k}) \pm \sqrt{|\Lambda(\mathbf{k})|^2x^2(\Omega'(\mathbf{k})^2 - \Omega(\mathbf{k})^2 + |\Lambda(\mathbf{k})|^2x^2)}}{2(\Omega(\mathbf{k})^2 - |\Lambda(\mathbf{k})|^2x^2)} - \frac{1}{2}\,, \tag{3.25}$$

where $x \equiv \cos(\varphi_\Lambda - \varphi_\alpha - \varphi_\beta)$. Upon extremizing this with respect to x^2, one can show that a maximum is formally reached for $x^2_{\mathrm{max}} = \Omega(\mathbf{k})^2/|\Lambda(\mathbf{k})|^2$, which must be greater than one if the Hamiltonian (3.11) is to be diagonalizable. Taking account of $x^2 \leq 1$, one finds that the maximum for $n^\pm(\mathbf{k})$ is reached when $x^2 = 1$, for which

$$n^\pm(\mathbf{k}) = \frac{\Omega(\mathbf{k})\sqrt{\Omega(\mathbf{k})^2 - |\Lambda(\mathbf{k})|^2 + |\Lambda'(\mathbf{k})|^2} \pm |\Lambda(\mathbf{k})|\,|\Lambda'(\mathbf{k})|}{2(\Omega(\mathbf{k})^2 - |\Lambda(\mathbf{k})|^2)} - \frac{1}{2}\,. \tag{3.26}$$

Since $n^-(\mathbf{k}) = 0$ when $|\Lambda'(\mathbf{k})| = |\Lambda(\mathbf{k})|$, the physical branch corresponds to $n(\mathbf{k}) = n^-(\mathbf{k})$. Furthermore, when considered as a function of $|\Lambda'(\mathbf{k})|$, $n(\mathbf{k}) \equiv n^-(\mathbf{k})$ monotonously increases as $|\Lambda'(\mathbf{k})|$ decreases, reaching a maximum when $|\Lambda'(\mathbf{k})| = 0$ (see figure 3.1), for which the particle number is

$$n(\mathbf{k}) = \langle 0|\hat{a}^\dagger(\mathbf{k})\hat{a}(\mathbf{k})|0\rangle = \frac{\Omega(\mathbf{k})}{2\bar{\omega}(\mathbf{k})} - \frac{1}{2}\,. \tag{3.27}$$

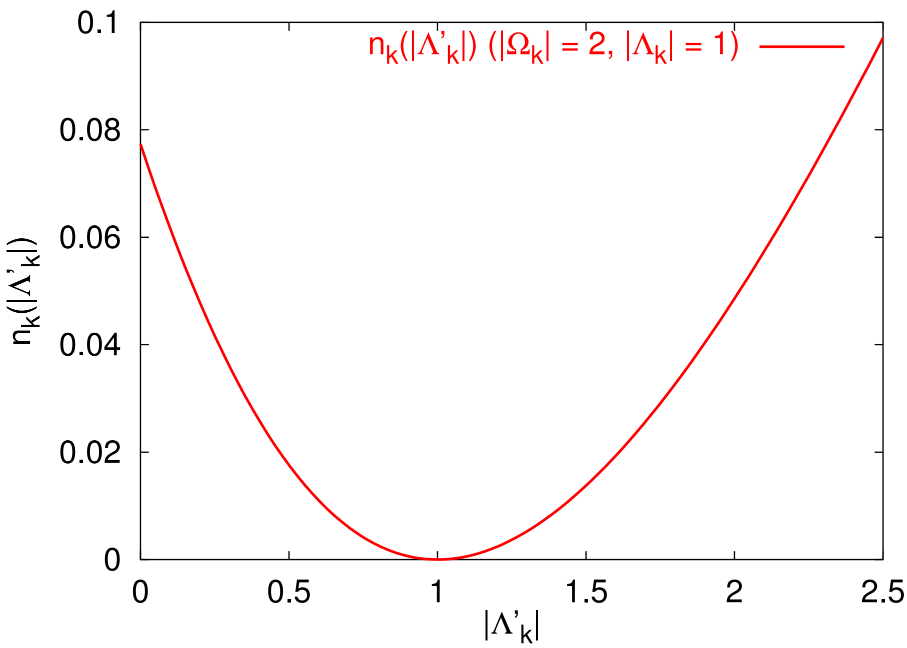

Figure 3.1: Particle number $n(\mathbf{k})$ as a function of $|\Lambda'(\mathbf{k})|$ for $|\Omega(\mathbf{k})| = 2$, $|\Lambda(\mathbf{k})| = 1$. Provided $|\Lambda'(\mathbf{k})| \leq |\Lambda(\mathbf{k})|$, $n(\mathbf{k})$ maximizes at $|\Lambda'(\mathbf{k})| = 0$.

This is the intuitively expected result, since it is the total energy density divided by the individual particle energy, minus the vacuum contribution. It corresponds to the maximum possible expectation value of the particle number operator for any Bogolyubov transformation, and we shall use it as our definition for particle number on phase space.

Moreover, note that in terms of thus transformed creation and annihilation operators $\hat{a}^\dagger(\mathbf{k})$ and $\hat{a}(\mathbf{k})$, the Hamiltonian is diagonal

$$H = \frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\bar{\omega}(\mathbf{k})\left(\hat{a}(\mathbf{k})\hat{a}^\dagger(\mathbf{k}) + \hat{a}^\dagger(\mathbf{k})\hat{a}(\mathbf{k})\right), \tag{3.28}$$

such that our definition agrees with the one advocated, for example, in Refs. [4, 11], *i.e.* when applied to the expanding Universe case it is nothing but the particle number definition suggested by Parker [11].

3.1.3 Particle Number in Scalar Kinetic Theory

It is now a simple matter to calculate the particle number in terms of Wigner functions. Making use of (3.1) and (3.6) we find

$$|\varphi|^2 = f_0, \qquad |\varphi'|^2 = \frac{1}{2}f_0'' + \left(\bar{\omega}_{\mathbf{k}}^2 - \frac{a''}{a}\right) f_0, \tag{3.29}$$

from which it follows

$$\Omega = 2\left(\bar{\omega}^2 f_0 + \frac{1}{4} f_0''\right) - \frac{d}{d\eta}\left(\frac{a'}{a} f_0\right). \tag{3.30}$$

We then insert (3.30) into (3.27) to get

$$n(\mathbf{k}) \;=\; \bar{\omega}_{\mathbf{k}} f_0 + \frac{1}{4\bar{\omega}_{\mathbf{k}}} f_0'' - \frac{1}{2} - \frac{1}{2\bar{\omega}_{\mathbf{k}}}\frac{d}{d\eta}\left(\frac{a'}{a} f_0\right). \tag{3.31}$$

This is our main result for scalars, which is positive, simply because $n(\mathbf{k}) \equiv |\beta(k)|^2 \geq 0$ (see Eqn. (3.27)).

We now apply (3.31) to the Chernikov-Tagirov [18] (Bunch-Davies [21]) vacuum,

$$\varphi(\mathbf{k}, \eta) = \frac{1}{\sqrt{2k}}\Big(1 - \frac{\mathrm{i}}{k\eta}\Big)\mathrm{e}^{-\mathrm{i}k\eta}\,, \tag{3.32}$$

which corresponds to the mode functions of a minimally coupled massless scalar field in de Sitter inflation, $a = -1/H\eta$ (*cf.* Eqn. (2.16)), for which

$$f_0 = (2k)^{-1}\big(1 + 1/(k\eta)^2\big)\,, \tag{3.33}$$

leading to the particle number

$$n(\mathbf{k}) = \frac{1}{4k^2\eta^2} = a^2\Big(\frac{H}{2k}\Big)^2. \tag{3.34}$$

This is to be compared with [42], where, $n(\mathbf{k}) \propto (-k\eta)^{-3}$ $(-k\eta \ll 1)$. We suspect that the origin of the difference is in the approximate methods used in [42].

As a consistency check, we now apply (3.31) to thermal equilibrium, where the Wigner function is (*cf.* Ref. [43])

$$\mathrm{i}G^{<} = 2\pi \mathrm{sign}(k_0)\delta(k^2 - m_\phi^2)\frac{1}{\mathrm{e}^{\beta k_0} - 1}. \tag{3.35}$$

By making use of (3.6) and (3.31) we obtain the standard Bose-Einstein distribution,

$$n(\mathbf{k}) = \frac{1}{\mathrm{e}^{\beta\omega(\mathbf{k})} - 1}\,, \tag{3.36}$$

and therefore have shown that it indeed corresponds to the expectation value of the particle number operator for the canonical ensemble in scalar field theory.

We compare our result to a particle number definition proposed in Refs. [44–46], according to which (expanding space-times are not considered):

$$\Big(\tilde{n}(\mathbf{k}) + \frac{1}{2}\Big)^2 = |\phi(k)|^2\,|\phi'(k)|^2 = f_0\Big(\frac{1}{2} f_0'' + \omega^2(\mathbf{k}) f_0\Big)\,. \tag{3.37}$$

Note that in adiabatic domain, in which $f_0'' \to 0$, Eqs. (3.37) and (3.31) both reduce to $n(\mathbf{k}) \to \omega(\mathbf{k}) f_0 - 1/2$, such that for example in thermal equilibrium of a free scalar theory (3.35), both definitions yield the Bose-Einstein distribution. According to the authors of [44], the definition (3.37) should be applicable to general situations (whenever there is a reasonably accurate quasiparticle picture of the plasma), and it is obtained as a consistency requirement on the energy conservation and quasiparticle current relation, respectively,

$$\frac{\omega^2(\mathbf{k})}{2}|\phi(k)|^2 + \frac{1}{2}|\phi'(k)|^2 = \omega(\mathbf{k})\Big(\frac{1}{2} + \tilde{n}(\mathbf{k})\Big)\,, \quad \omega(\mathbf{k})|\phi(k)|^2 = \frac{1}{2} + \tilde{n}(\mathbf{k})\,. \tag{3.38}$$

The consistency is reached when the kinetic and potential energies are equal, in which case a generalized quasiparticle energy is given by $\omega(\mathbf{k})^2 = |\phi'(k)|^2/|\phi(k)|^2$.

In order to make a nontrivial comparison, consider now a pure state of a scalar theory interacting only weakly with a classical background field (which can be described by a time dependent mass term). The WKB form for the mode functions can be recast as (*cf.* Eqn. (2.30))

$$\begin{aligned} \phi(k) &= \frac{1}{\sqrt{2\epsilon(\mathbf{k})}}\big(\alpha_0 \mathrm{e}^{-\mathrm{i}\int^\eta \epsilon(\mathbf{k})(\eta')d\eta'} + \beta_0 \mathrm{e}^{\mathrm{i}\int^\eta \epsilon(\mathbf{k})(\eta')d\eta'}\big)\,, \\ \phi'(k) &= -\mathrm{i}\sqrt{\frac{\epsilon(\mathbf{k})}{2}}\big(\alpha_0 \mathrm{e}^{-\mathrm{i}\int^\eta \epsilon(\mathbf{k})(\eta')d\eta'} - \beta_0 \mathrm{e}^{\mathrm{i}\int^\eta \epsilon(\mathbf{k})(\eta')d\eta'}\big) - \frac{1}{2}\frac{\epsilon'(\mathbf{k})}{\epsilon(\mathbf{k})}\phi(k)\,, \end{aligned} \tag{3.39}$$

where $\epsilon(\mathbf{k})$ satisfies Eqn. (2.31), $\epsilon^2(\mathbf{k}) = \omega^2(\mathbf{k}) - (1/2)\epsilon''(\mathbf{k})/\epsilon(\mathbf{k}) + (3/4)(\epsilon''(\mathbf{k})/\epsilon(\mathbf{k}))^2$. In a free theory $|\alpha_0|^2 - |\beta_0|^2$ is conserved, and it is usually normalized to *one*. In an interacting theory however, the single particle description breaks down, and consequently $|\alpha_0|^2 - |\beta_0|^2$ is not conserved. For the purpose of this example, we assume that the interactions are weak enough, such that $|\alpha_0|^2 - |\beta_0|^2$ is changing sufficiently slow, and the subsequent discussion applies. In the adiabatic limit $\epsilon(\mathbf{k}) \to \omega(\mathbf{k}) \to$ constant, the particle number (3.27) and (3.31) of the state (3.39) is simply $n^{(0)}(\mathbf{k}) = |\beta_0|^2$.

On the other hand, when applied to the state (3.39), the definition (3.37) yields an oscillating particle number even in adiabatic regime,

$$\Big(\tilde{n}(\mathbf{k}) + \frac{1}{2}\Big)^2 \approx \frac{1}{4} + (1 + |\beta_0|^2)\,|\beta_0|^2 \sin^2(2\epsilon(\mathbf{k})\eta - \chi_\alpha + \chi_\beta)\,, \tag{3.40}$$

where $\alpha_0 = |\alpha_0|e^{\mathrm{i}\chi_\alpha}, \beta_0 = |\beta_0|e^{\mathrm{i}\chi_\beta}$, which is positive and bounded from above by $\tilde{n}(\mathbf{k}) \le |\beta_0|^2 \equiv n^{(0)}(\mathbf{k})$ [1]. Hence, for the state (3.39) our particle number definition (3.31) provides an upper limit for (3.37). This was to be expected, considering that Eqn. (3.31) was derived in section 3.1.2 by an extremization procedure over the Bogolyubov transformations (3.14). We expect that a similar behaviour pertains in other situations.

[1] Note that the definition of the quasiparticle energy $\omega(\mathbf{k}) \equiv |\phi'(k)|/|\phi(k)|$, oscillates even in the adiabatic limit, with the minimum and maximum values given by $\omega_{\min}(\mathbf{k}) = \epsilon(\mathbf{k})(|\alpha_0| - |\beta_0|)/(|\alpha_0| + |\beta_0|)$ and $\omega_{\max}(\mathbf{k}) = \epsilon^2(\mathbf{k})/\omega_{\min}(\mathbf{k})$, respectively, such that $\omega(\mathbf{k}) \neq \epsilon(\mathbf{k})$ in general. This indicates that imposing instantaneous equality of the potential and kinetic energies may not be appropriate in general situations. When particle number is understood as an average over the characteristic oscillation period however, imposing equality of the potential and kinetic energy may lead to a reasonable definition for the particle number density.

3.2 Fermions

Provided that the Dirac field in expanding spacetime is rescaled as

$$a^{3/2}\psi \to \psi \tag{3.41}$$

and the mass as

$$am \to m\,, \tag{3.42}$$

the fermionic Lagrangean reduces to the standard Minkowski form,

$$\sqrt{-g}\mathcal{L}_\psi \to \bar{\psi}\,\mathrm{i}\partial\!\!\!/\,\psi - \bar{\psi}(m_R + \mathrm{i}\gamma^5 m_I)\psi, \tag{3.43}$$

where, for notational simplicity, we omitted the rescaling of the fields and absorbed the scale factor in the mass term. The field ψ then obeys the Dirac equation

$$\left[i\partial\!\!\!/ \; - m_R - \mathrm{i}\gamma^5 m_I\right]\psi = 0\,. \tag{3.44}$$

Note that the complex mass term $m = m_R(\eta) + \mathrm{i}m_I(\eta)$ may induce CP-violation (*cf.* Ref. [47, 48]).

The fermionic Wigner function,

$$\mathrm{i}S^<(k,x) = -\int d^4r \mathrm{e}^{\mathrm{i}k\cdot r}\langle 0|\bar{\psi}(x-r/2)\psi(x+r/2)|0\rangle$$

satisfies the Dirac equation in Wigner representation,

$$\left(k\!\!\!/ \; + \frac{\mathrm{i}}{2}\gamma^0\partial_\eta - (m_R + \mathrm{i}\gamma^5 m_I)e^{-\frac{\mathrm{i}}{2}\overleftarrow{\partial}_\eta\partial_{k_0}}\right)\mathrm{i}S^< = 0, \tag{3.45}$$

where $(\mathrm{i}\gamma^0 S^<)^\dagger = \mathrm{i}\gamma^0 S^<$ is hermitean. Since we are interested in parity and helicity, it is most convenient to work in the chiral or Weyl representation, where

$$\begin{aligned}\gamma^0 &= \mathbb{1}\otimes\sigma_1\,,\\ \gamma^i &= \mathrm{i}\sigma_i\otimes\sigma_2\,,\; i=1,2,3\,,\\ \gamma^5 &= -\mathbb{1}\otimes\sigma_3\,,\end{aligned}$$

and σ_i are the standard Pauli matrices.

Eqn. (3.45) carries of course a Dirac matrix structure, therefore constituting a system of sixteen equations, which can be reduced by noting that due to spatial isotropy the problem is helicity conserving. This means, that the differential operator in Eqn. (3.45), containing the set of Dirac matrices

$$\mathbf{k}\cdot\boldsymbol{\gamma}, \gamma^0, \mathbb{1}, \gamma^5\,, \tag{3.46}$$

commutes with the helicity operator

$$\hat{h} = \gamma^0\hat{\mathbf{k}}\cdot\boldsymbol{\gamma}\gamma^5\,, \quad \hat{\mathbf{k}} = \mathbf{k}/|\mathbf{k}|\,. \tag{3.47}$$

In chiral (Weyl) representation, we have

$$\gamma^0\gamma^i\gamma^5 = \sigma_i \otimes \mathbb{1}\,, \tag{3.48}$$

and define the helicity projector

$$P_h = \frac{1}{2}\left(\mathbb{1} + \hat{h}\right) = \frac{1}{2}\left(\mathbb{1}\otimes\mathbb{1} + \hat{\mathbf{k}}\cdot\boldsymbol{\sigma}\otimes\mathbb{1}\right)\,. \tag{3.49}$$

Therefore, the helicity eigenstates can be expanded as $\mathbb{1}\otimes\sigma^a$, $a = 0,...3$, since these matrices commutes with P_h. Noting that also γ^0 commutes with P_h, we can make the helicity block-diagonal ansatz for the Wigner function (*cf.* Ref. [47, 48])

$$\mathrm{i}S^< = \sum_{h=\pm}\mathrm{i}S^<_h\,, \qquad \mathrm{i}S^<_h = P_h\mathrm{i}S^<\,, \tag{3.50}$$

$$-\mathrm{i}\gamma_0 S^<_h = \frac{1}{4}\big(\mathbb{1} + h\hat{\mathbf{k}}\cdot\boldsymbol{\sigma}\big)\otimes\sigma^a g_{ah}\,. \tag{3.51}$$

Explicitly, Eqn. (3.45) turns into

$$\begin{aligned}\left(\mathrm{i}k_0\mathbb{1}\otimes\mathbb{1} - \mathrm{i}\boldsymbol{\sigma}\otimes\sigma_3\mathbf{k} - \frac{1}{2}\partial_\eta - \mathrm{i}\mathbb{1}\otimes\sigma_1 m_R \mathrm{e}^{-\frac{\mathrm{i}}{2}\overleftarrow{\partial}_\eta\partial_{k_0}} - \mathrm{i}\mathbb{1}\otimes\sigma_2 m_I \mathrm{e}^{-\frac{\mathrm{i}}{2}\overleftarrow{\partial}_\eta\partial_{k_0}}\right)&\\ \times\frac{1}{4}\left(\mathbb{1} + h\hat{\mathbf{k}}\cdot\boldsymbol{\sigma}\right)\otimes\sigma^a g_{ah} = 0\,.&\end{aligned} \tag{3.52}$$

We multiply this equation by

$$\begin{aligned}\mathbb{1} &= \mathbb{1}\otimes\mathbb{1}\,,\\ -h\hat{\mathbf{k}}\cdot\boldsymbol{\gamma}\gamma^5 &= -h\hat{\mathbf{k}}\cdot\boldsymbol{\sigma}\otimes\sigma^1\,,\\ -\mathrm{i}h\hat{\mathbf{k}}\cdot\boldsymbol{\gamma} &= h\hat{\mathbf{k}}\cdot\boldsymbol{\sigma}\otimes\sigma^2\,,\\ -\gamma^5 &= \mathbb{1}\otimes\sigma^3\end{aligned}$$

and take the traces to obtain

$$\begin{aligned}\left(\mathrm{i}k_0 - \frac{1}{2}\partial_\eta\right)g_{0h} - \mathrm{i}h|\mathbf{k}|g_{3h} - \mathrm{i}m_R e^{-\frac{\mathrm{i}}{2}\overleftarrow{\partial}_\eta\partial_{k_0}}g_{1h} - \mathrm{i}m_I e^{-\frac{\mathrm{i}}{2}\overleftarrow{\partial}_\eta\partial_{k_0}}g_{2h} &= 0\,,\\ \left(\mathrm{i}k_0 - \frac{1}{2}\partial_\eta\right)g_{1h} - h|\mathbf{k}|g_{2h} - \mathrm{i}m_R e^{-\frac{\mathrm{i}}{2}\overleftarrow{\partial}_\eta\partial_{k_0}}g_{0h} + m_I e^{-\frac{\mathrm{i}}{2}\overleftarrow{\partial}_\eta\partial_{k_0}}g_{3h} &= 0\,,\\ \left(\mathrm{i}k_0 - \frac{1}{2}\partial_\eta\right)g_{2h} + h|\mathbf{k}|g_{1h} - m_R e^{-\frac{\mathrm{i}}{2}\overleftarrow{\partial}_\eta\partial_{k_0}}g_{3h} - \mathrm{i}m_I e^{-\frac{\mathrm{i}}{2}\overleftarrow{\partial}_\eta\partial_{k_0}}g_{0h} &= 0\,,\\ \left(\mathrm{i}k_0 - \frac{1}{2}\partial_\eta\right)g_{3h} - \mathrm{i}h|\mathbf{k}|g_{0h} + m_R e^{-\frac{\mathrm{i}}{2}\overleftarrow{\partial}_\eta\partial_{k_0}}g_{2h} - m_I e^{-\frac{\mathrm{i}}{2}\overleftarrow{\partial}_\eta\partial_{k_0}}g_{1h} &= 0\,.\end{aligned} \tag{3.53}$$

Taking the real part gives

$$\begin{aligned}-\frac{1}{2}\partial_\eta g_{0h} - m_R\sin\left(\frac{1}{2}\overleftarrow{\partial}_\eta\,\partial_{k_0}\right)g_{1h} - m_I\sin\left(\frac{1}{2}\overleftarrow{\partial}_\eta\,\partial_{k_0}\right)g_{2h} &= 0\,,\\ -\frac{1}{2}\partial_\eta g_{1h} - h|\mathbf{k}|g_{2h} - m_R\sin\left(\frac{1}{2}\overleftarrow{\partial}_\eta\,\partial_{k_0}\right)g_{0h} + m_I\cos\left(\frac{1}{2}\overleftarrow{\partial}_\eta\,\partial_{k_0}\right)g_{3h} &= 0\,,\\ -\frac{1}{2}\partial_\eta g_{2h} + h|\mathbf{k}|g_{1h} - m_R\cos\left(\frac{1}{2}\overleftarrow{\partial}_\eta\,\partial_{k_0}\right)g_{3h} - m_I\sin\left(\frac{1}{2}\overleftarrow{\partial}_\eta\,\partial_{k_0}\right)g_{0h} &= 0\,,\\ -\frac{1}{2}\partial_\eta g_{3h} + m_R\cos\left(\frac{1}{2}\overleftarrow{\partial}_\eta\,\partial_{k_0}\right)g_{2h} - m_I\cos\left(\frac{1}{2}\overleftarrow{\partial}_\eta\,\partial_{k_0}\right)g_{1h} &= 0\,,\end{aligned} \tag{3.54}$$

and the imaginary part

$$
\begin{aligned}
k_0 g_{0h} - h|\mathbf{k}| g_{3h} - m_R \cos\left(\frac{1}{2}\overleftarrow{\partial_\eta}\,\partial_{k_0}\right) g_{1h} - m_I \cos\left(\frac{1}{2}\overleftarrow{\partial_\eta}\,\partial_{k_0}\right) g_{2h} &= 0\,, \\
k_0 g_{1h} - m_R \cos\left(\frac{1}{2}\overleftarrow{\partial_\eta}\,\partial_{k_0}\right) g_{0h} - m_I \sin\left(\frac{1}{2}\overleftarrow{\partial_\eta}\,\partial_{k_0}\right) g_{3h} &= 0\,, \\
k_0 g_{2h} + m_R \sin\left(\frac{1}{2}\overleftarrow{\partial_\eta}\,\partial_{k_0}\right) g_{3h} - m_I \cos\left(\frac{1}{2}\overleftarrow{\partial_\eta}\,\partial_{k_0}\right) g_{0h} &= 0\,, \\
k_0 g_{3h} - h|\mathbf{k}| g_{0h} - m_R \sin\left(\frac{1}{2}\overleftarrow{\partial_\eta}\,\partial_{k_0}\right) g_{2h} + m_I \sin\left(\frac{1}{2}\overleftarrow{\partial_\eta}\,\partial_{k_0}\right) g_{1h} &= 0\,.
\end{aligned}
\tag{3.55}
$$

We take the 0th moments of the kinetic equations (3.54) and the constraint equations (3.55). For the constraints, 0th and 1st moments mix, therefore they are only important when we are interested in the distribution over k_0. From the kinetic equations, writing $f_{ah} \equiv f_{ah}^{(0)}$, we obtain the closed system

$$
\begin{aligned}
f'_{0h} &= 0\,, \\
f'_{1h} + 2h|\mathbf{k}| f_{2h} - 2m_I f_{3h} &= 0\,, \\
f'_{2h} - 2h|\mathbf{k}| f_{1h} + 2m_R f_{3h} &= 0\,, \\
f'_{3h} - 2m_R f_{2h} + 2m_I f_{1h} &= 0\,,
\end{aligned}
\tag{3.56}
$$

where we integrated the terms involving trigonometric functions by parts and discarded boundary terms.

Reinserting the decomposition (3.51), we identify

$$
\begin{aligned}
f_{0h} &\equiv \mathrm{tr}\Big[(\mathbb{1}P_h)\int\frac{dk_0}{2\pi}(-\mathrm{i}\gamma^0 S^<)\Big]\,, \\
f_{1h} &\equiv \mathrm{tr}\Big[(-h\hat{\mathbf{k}}\cdot\boldsymbol{\gamma}\gamma^5 P_h)\int\frac{dk_0}{2\pi}(-\mathrm{i}\gamma^0 S^<)\Big]\,, \\
f_{2h} &\equiv \mathrm{tr}\Big[(-\mathrm{i}h\hat{\mathbf{k}}\cdot\boldsymbol{\gamma} P_h)\int\frac{dk_0}{2\pi}(-\mathrm{i}\gamma^0 S^<)\Big]\,, \\
f_{3h} &\equiv \mathrm{tr}\Big[(-\gamma^5 P_h)\int\frac{dk_0}{2\pi}(-\mathrm{i}\gamma^0 S^<)\Big]\,.
\end{aligned}
\tag{3.57}
$$

Therefore, f_{0h} is the charge density, which is conserved by Noether's theorem, f_{1h} is the scalar density, f_{2h} is the pseudoscalar density and f_{3h} the axial charge density, candidate source for CP-violation.

The moments f_{ah} can be related to the positive and negative frequency mode functions, $u_h(\mathbf{k},\eta)$ and $v_h(\mathbf{k},\eta) = -\mathrm{i}\gamma^2(u_h(\mathbf{k},\eta))^*$, respectively. They form a basis for the Dirac field,

$$
\psi(x) = \sum_h \int \frac{d^3k}{(2\pi)^3} \mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{x}} \big(u_h a_h(\mathbf{k}) + v_h b_h^\dagger(-\mathbf{k})\big)\,, \quad u_h = \xi_h \otimes \begin{pmatrix} L_h \\ R_h \end{pmatrix},
$$

where ξ_h is the helicity two-eigenspinor, $\hat{h}\xi_h = h\xi_h$. The Dirac equation then decomposes into

$$\begin{aligned} \mathrm{i}\partial_\eta L_h - h|\mathbf{k}|L_h &= m_R R_h + \mathrm{i}m_I R_h \\ \mathrm{i}\partial_\eta R_h + h|\mathbf{k}|R_h &= m_R L_h - \mathrm{i}m_I L_h. \end{aligned} \tag{3.58}$$

Note that these equations incorporate CP-violation and thus generalize the analysis of Refs. [5, 7, 40]. Now, from (3.58) one can rederive (3.56) by multiplying with L_h and R_h and employing

$$\begin{aligned} f_{0h} &= |L_h|^2 + |R_h|^2, \qquad f_{3h} = |R_h|^2 - |L_h|^2\,, \\ f_{1h} &= -2\Re(L_h R_h^*), \qquad f_{2h} = 2\Im(L_h^* R_h). \end{aligned}$$

The Hamiltonian density reads

$$\begin{aligned} H = \sum_h \int \frac{d^3k}{(2\pi)^3} \Big\{ &\Omega_h(\mathbf{k})\big(a_h^\dagger(\mathbf{k})a_h(\mathbf{k}) + b_h^\dagger(-\mathbf{k})b_h(-\mathbf{k})\big) \\ &+(\Lambda_h(\mathbf{k})b_h(-\mathbf{k})a_h(\mathbf{k}) + \text{h.c.})\Big\}\,, \end{aligned} \tag{3.59}$$

where

$$\Omega_h(\mathbf{k}) = hk\left(|L_h|^2 - |R_h|^2\right) + mL_h^* R_h + m^* L_h R_h^*\,, \tag{3.60}$$

$$\Lambda_h(\mathbf{k}) = 2kL_h R_h - hm^* L_h^2 + hmR_h^2, \tag{3.61}$$

with $\{\hat{a}_h(\mathbf{k}), \hat{a}^\dagger_{\mathbf{k}'h'}\} = \delta_{h,h'}\delta_{\mathbf{k},\mathbf{k}'}$, $\{\hat{b}_h(\mathbf{k}), \hat{b}^\dagger_{\mathbf{k}'h'}\} = \delta_{h,h'}\delta_{\mathbf{k},\mathbf{k}'}$. We now use the Bogolyubov transformation

$$\begin{pmatrix} \hat{a}_h(\mathbf{k}) \\ \hat{b}_h^\dagger(-\mathbf{k}) \end{pmatrix} = \begin{pmatrix} \alpha_h(k) & \beta_h(k) \\ -\beta_h^*(k) & \alpha_h^*(k) \end{pmatrix} \begin{pmatrix} a_h(\mathbf{k}) \\ b_h(-\mathbf{k})^\dagger \end{pmatrix},$$

to diagonalize the Hamiltonian, where $\alpha_h(k)$ and $\beta_h(k)$ are

$$\frac{1}{2}\left(\left|\frac{\alpha_h(k)}{\beta_h(k)}\right| - \left|\frac{\beta_h(k)}{\alpha_h(k)}\right|\right) = \frac{\Omega_h(\mathbf{k})}{|\Lambda_h(\mathbf{k})|}, \qquad |\alpha_h(k)|^2 + |\beta_h(k)|^2 = 1, \tag{3.62}$$

leading to the particle number density on phase space,

$$n_h(\mathbf{k}) = |\beta_h(k)|^2 = \frac{1}{2} - \frac{\Omega_h(\mathbf{k})}{2\omega(\mathbf{k})}\,, \tag{3.63}$$

where now $\omega(\mathbf{k}) = \sqrt{\mathbf{k}^2 + |m|^2}$.

To construct the initial mode functions in the adiabatic domain, $\eta \to -\infty$, we use the positive frequency solution and its charge conjugate,

$$\psi(\mathbf{k}) \to \begin{pmatrix} \alpha_0 L_h^+ + \beta_0 L_h^- \\ \alpha_0 R_h^+ + \beta_0 R_h^- \end{pmatrix}, \qquad |\alpha_0|^2 + |\beta_0|^2 = 1\,. \tag{3.64}$$

From the Dirac equation under adiabatic conditions it follows

$$
\begin{aligned}
L_h^+ &= \sqrt{\frac{\omega(\mathbf{k})+hk}{2\omega(\mathbf{k})}}, & L_h^- &= -\mathrm{i}\frac{m}{|m|}\sqrt{\frac{\omega(\mathbf{k})-hk}{2\omega(\mathbf{k})}}, \\
R_h^+ &= \frac{m^*}{\sqrt{2\omega(\mathbf{k})(\omega(\mathbf{k})+hk)}}, & R_h^- &= \mathrm{i}\frac{|m|}{\sqrt{2\omega(\mathbf{k})(\omega(\mathbf{k})-hk)}}.
\end{aligned} \tag{3.65}
$$

These mode functions correspond to an initial particle number $n^{(0)}(\mathbf{k}) = |\beta_0|^2$. We now make use of (3.59) to express $\Omega_h(\mathbf{k})$ in terms of the Wigner functions,

$$
\Omega_h(\mathbf{k}) = -(hkf_{3h} + m_R f_{1h} + m_I f_{2h}), \tag{3.66}
$$

which implies our main result for fermions,

$$
n_h(\mathbf{k}) = \frac{1}{2\omega(\mathbf{k})}\left(hkf_{3h} + m_R f_{1h} + m_I f_{2h}\right) + \frac{1}{2}. \tag{3.67}
$$

Note that in the limit $m \to 0$, this expression reduces to the phase space density of axial particles. Moreover, $0 \le n_h(\mathbf{k}) \equiv |\beta_{kh}|^2 = 1 - |\alpha_{kh}|^2 \le 1$ (see Eqs. (3.62-3.63)).

As an application of Eqn. (3.67), we consider particle production at preheating [4,7], where the fermionic mass is generated by an oscillating inflaton condensate. Assuming that the inflaton oscillates as a cosine function results in a fermion production shown in figure 3.2. Observe that, even for a relatively small imaginary (pseudoscalar) mass term, particle production of the opposite helicity states is completely different, implying a nonperturbative enhancement of a CP-violating particle density, $n_{\mathbf{k}+} - n_{\mathbf{k}-}$, which may be of relevance for baryogenesis.

When applied to thermal equilibrium, where (*cf.* Ref. [43])

$$
\mathrm{i}S^< = -(\not{k} + m_R - \mathrm{i}\gamma_5 m_I)\delta(k^2 - |m|^2)\frac{2\pi\mathrm{sign}(k_0)}{\mathrm{e}^{\beta k_0}+1}, \tag{3.68}
$$

we find

$$
\begin{aligned}
f_{0h} &= 1, \\
f_{1h} &= (2m_R/\omega(\mathbf{k}))[\{\exp(\beta\omega(\mathbf{k}))+1\}^{-1} - 1/2], \\
f_{2h} &= (2m_I/\omega(\mathbf{k}))[\{\exp(\beta\omega(\mathbf{k}))+1\}^{-1} - 1/2], \\
f_{3h} &= (2hk/\omega(\mathbf{k}))[\{\exp(\beta\omega(\mathbf{k}))+1\}^{-1} - 1/2],
\end{aligned}
$$

such that Eqn. (3.67) yields the Fermi-Dirac distribution,

$$
n_h(\mathbf{k}) = \frac{1}{\mathrm{e}^{\beta\omega(\mathbf{k})}+1}. \tag{3.69}
$$

After this work appeared as an article, an out-of-equilibrium investigation of the dynamics of chiral fermions coupled to scalars was studied in Ref. [49]. In order to

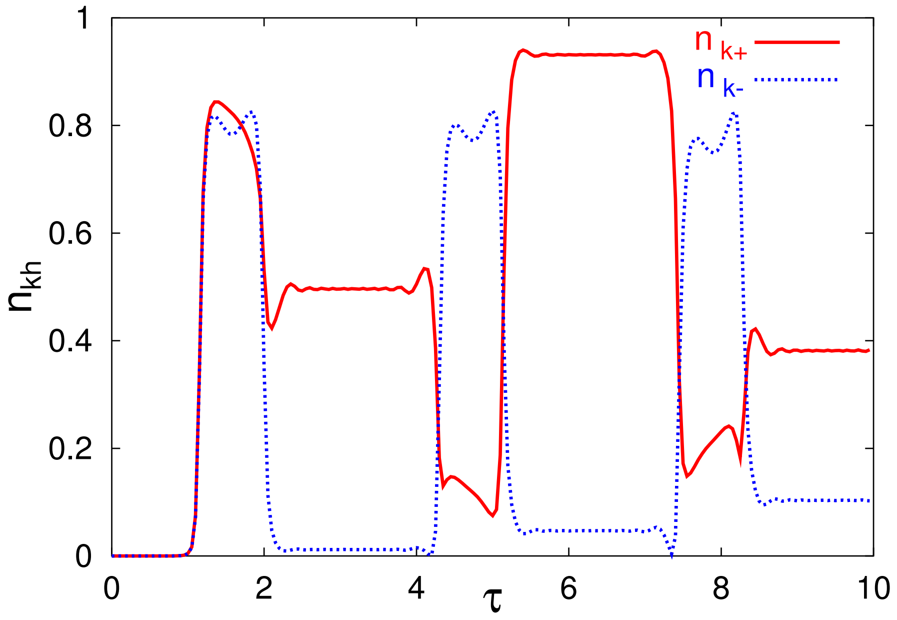

Figure 3.2: The number of produced fermions as a function of time with helicity $h = +$ (*solid*) and $h = -$ (*dotted*), mass $m/\omega_I = 10 + 15\cos(2\tau) - i\sin(2\tau)$, $|\mathbf{k}| = \omega_I$, $\tau = \omega_I \eta$, where ω_I denotes the frequency of the inflaton oscillations.

show that at late times the system thermalizes to the Fermi-Dirac equilibrium, the authors used a particle number definition, which can be in our notation written as

$$\tilde{n}(\mathbf{k}) = \frac{1}{2}\sum_{h=\pm} \tilde{n}_h(\mathbf{k})\,, \qquad \tilde{n}_h(\mathbf{k}) = \frac{1}{2}\left(1 + h f_{3h}\right)\,. \tag{3.70}$$

This definition corresponds to the massless fermion limit $m \to 0$ of our definition (3.67).

3.3 Multiflavour Case

We now generalize the definition of particle number in terms of two-point functions to the case of several species, mixing through a mass matrix. While in the single flavour case always an equal number of particles and antiparticles is produced, we will here encounter the creation of a charge asymmetry when the mass matrix is nonsymmetric. Because of this charge violation, the orthogonality of particle modes with respect to antiparticle modes is not preserved under time evolution, and it is thus impossible to expand the field operators in terms of a time-independent orthogonal basis.

Hence, the use of the basis-independent two-point functions is advantageous. We can either calculate the time evolution of the system in terms of these quantities or measure them, since they correspond to physical charge and current densities. When finally the mass matrix is diagonal and only adiabatically slowly evolving, there exists a well-defined basis, in terms of which the Hamiltonian is diagonal. We use this basis to define the particle number operators and construct their expectation values out of the two-point functions.

3.3.1 Fermions

Since Dirac spinors naturally include particle and antiparticle modes, we first discuss here the fermionic case. We decompose the mass matrix M into a hermitean and an antihermitean part,

$$M_H = \frac{1}{2}(M + M^\dagger), \quad M_A = \frac{1}{2\mathrm{i}}(M - M^\dagger), \tag{3.71}$$

such that the Dirac equation reads

$$\left[\mathrm{i}\partial\!\!\!/ \, - M_H - \mathrm{i}\gamma^5 M^A\right]_{ij} \psi_j = 0. \tag{3.72}$$

One can then attempt to proceed as in the single flavour case and to construct the field operators as

$$\begin{aligned}
\psi_i(x) &= \sum_{h\,j} \int \frac{d^3k}{(2\pi)^3} \frac{\mathrm{e}^{\text{-}\mathrm{i}\mathbf{k}\cdot\mathbf{x}}}{V} \left[U_{h\,ij}(\mathbf{k},\eta) a_{h\,j}(\mathbf{k}) + V_{h\,ij}(\mathbf{k},\eta) b^\dagger_{h\,j}(\text{-}\mathbf{k})\right] , \qquad (3.73)\\
\psi_i^\dagger(x) &= \sum_{h\,j} \int \frac{d^3k}{(2\pi)^3} \frac{\mathrm{e}^{\text{-}\mathrm{i}\mathbf{k}\cdot\mathbf{x}}}{V} \left[a^\dagger_{h\,j}(\mathbf{k}) U^\dagger_{h\,ji}(\mathbf{k},\eta) + b_{h\,j}(\text{-}\mathbf{k}) V^\dagger_{h\,ji}(\mathbf{k},\eta)\right] ,
\end{aligned}$$

with the mode function

$$U_{h\,ij} = \begin{pmatrix} L_{h\,ij} \\ R_{h\,ij} \end{pmatrix} \otimes \xi_h \tag{3.74}$$

and its charge conjugate

$$V_{h\,ij} = -\mathrm{i}\gamma^2 (U_{h\,ij})^* = C U_{h\,ij} C^{-1} = \begin{pmatrix} -hR^*_{h\,ij} \\ hL^*_{h\,ij} \end{pmatrix} \otimes \xi_{-h}. \tag{3.75}$$

This procedure however fails when M is not a symmetric matrix, which can easily be seen by plugging $U_{h\,ij}$ into Eqn. (3.72)

$$\begin{aligned}
\{\mathrm{i}\partial_\eta - h|\mathbf{k}|\}\, \mathbb{1}_{il} L_{h\,ij} &= M^H_{il} R_{h\,lj} + \mathrm{i} M^A_{il} R_{h\,lj} , \qquad (3.76)\\
\{\mathrm{i}\partial_\eta + h|\mathbf{k}|\}\, \mathbb{1}_{il} R_{h\,ij} &= M^H_{il} L_{h\,lj} - \mathrm{i} M^A_{il} L_{h\,lj} ,
\end{aligned}$$

and $V_{h\,ij}$, respectively,

$$\begin{aligned}
\{\mathrm{i}\partial_\eta - h|\mathbf{k}|\}\, \mathbb{1}_{il} L_{h\,ij} &= M^{H*}_{il} R_{h\,lj} + \mathrm{i} M^{A*}_{il} R_{h\,lj} , \qquad (3.77)\\
\{\mathrm{i}\partial_\eta + h|\mathbf{k}|\}\, \mathbb{1}_{il} R_{h\,ij} &= M^{H*}_{il} L_{h\,lj} - \mathrm{i} M^{A*}_{il} L_{h\,lj} ,
\end{aligned}$$

where summation over the repeated index l is implied.

Obviously, when M is not symmetric, Eqs. (3.76) and (3.77) are inconsistent. In particular, for nonsymmetric M, the orthogonality condition

$$U^\dagger_{r\,il} V_{s\,lj} = 0, \tag{3.78}$$

is not preserved at all times, and hence, the expansion of the field operators (3.73) is not suitable. This complication can however lead to the generation of a net charge stored in the produced particles, because the operation of charge conjugation becomes time dependent, an effect which we call coherent baryogenesis [50] and discuss in some detail and with examples in chapters 4 and 5.

The construction of an appropriate Bogolyubov transformation for the case of a symmetric mass matrix, where Eqn. (3.78) holds at all times, is discussed in Ref. [51]. In comparison with the single flavour case this procedure is fairly complicated. For the general nonsymmetric case, we therefore refrain from a computation of a time-dependent orthogonal basis of mode functions, a Bogolyubov transformation and the time evolution of Heisenberg creation and annihilation operators.

It is more convenient to calculate the time evolution of the initial state in terms of two point functions. We straightforwardly generalize the formalism for the single-flavour Wigner functions to the multiflavour case by defining

$$\mathrm{i}S^<_{ij}(k,x) = -\int d^4 r \mathrm{e}^{\mathrm{i}k\cdot r}\langle\bar{\psi}_j(x-r/2)\psi_i(x+r/2)\rangle, \tag{3.79}$$

where a, b are flavour indices. The multiflavour Wigner function obeys the equation of motion

$$\left(\not{k} + \frac{\mathrm{i}}{2}\gamma^0\partial_\eta - (M_H + \mathrm{i}\gamma^5 M_A)\mathrm{e}^{-\frac{\mathrm{i}}{2}\overleftarrow{\partial}_\eta\partial_{k_0}}\right)_{il}\mathrm{i}S^<_{lj} = 0\,. \tag{3.80}$$

As described for the single flavour case in section 3.2, this can be simplified and yields

$$\begin{aligned} f'_{0h} + \mathrm{i}\,[M_H, f_{1h}] + \mathrm{i}\,[M_A, f_{2h}] &= 0 \\ f'_{1h} + 2h|\mathbf{k}|f_{2h} + \mathrm{i}\,[M_H, f_{0h}] - \{M_A, f_{3h}\} &= 0 \\ f'_{2h} - 2h|\mathbf{k}|f_{1h} + \{M_H, f_{3h}\} + \mathrm{i}\,[M_A, f_{0h}] &= 0 \\ f'_{3h} - \{M_H, f_{2h}\} + \{M_A, f_{1h}\} &= 0\,. \end{aligned} \tag{3.81}$$

We can infer from these equations as a necessary condition for the nonconservation of the charge density f_{0h}, that M must not be symmetric, in accordance with our discussion above.

Now assume, that after some time evolution, M has become symmetric and slowly varying. Then, it is possible to expand the field operators as in Eqn. (3.73) and to define the expectation values of the number of particles

$$n^+_{h\,i}(\mathbf{k}) = \langle a^\dagger_{h\,i}(\mathbf{k}) a_{h\,i}(\mathbf{k})\rangle$$

and antiparticles

$$n^-_{h\,i}(\mathbf{k}) = \langle b^\dagger_{h\,i}(\mathbf{k}) b_{h\,i}(\mathbf{k})\rangle.$$

Moreover, we choose this basis such that the Hamilton operator is diagonal and reads

$$\begin{aligned} H = \sum_{h\,ij} \int \frac{d^3k}{(2\pi)^3} \Big(& h|\mathbf{k}| L^\dagger_h L_h + L^\dagger_h \left[M^H + iM^A\right] R_h \\ & -h|\mathbf{k}| R^\dagger_h R_h + R^\dagger_h \left[M^H - iM^A\right] L_h \Big)_{ij} \times \left(a^\dagger_{hi}(\mathbf{k}) a_{hj}(\mathbf{k}) - b_{hi}(\mathbf{k}) b^\dagger_{hj}(\mathbf{k}) \right). \end{aligned} \tag{3.82}$$

We can now also express the functions $f^{ij}_{\mu h}$ employing this basis. Explicitly, they read

$$\begin{aligned} f^{ij}_{0h}(x,\mathbf{k}) &= -\int d^4r\, \mathrm{e}^{ik\cdot r} \langle \bar\psi_{hj}(x-r/2)\gamma^0 \psi_{hi}(x+r/2)\rangle \\ &= \left(L^{il*}_h L^{jl'}_h + R^{il*}_h R^{jl'}_h \right) \times \left\langle a^\dagger_{hl'}(\mathbf{k}) a_{hl}(\mathbf{k}) + b_{hl'}(\mathbf{k}) b^\dagger_{hl}(\mathbf{k}) \right\rangle, \\ f^{ij}_{1h}(x,\mathbf{k}) &= -\int d^4r\, \mathrm{e}^{ik\cdot r} \langle \bar\psi_{hj}(x-r/2) \psi_{hi}(x+r/2)\rangle \\ &= -2\Re \left(L^{il}_h R^{jl'*}_h \right) \times \left\langle a^\dagger_{hl'}(\mathbf{k}) a_{hl}(\mathbf{k}) + b_{hl'}(\mathbf{k}) b^\dagger_{hl}(\mathbf{k}) \right\rangle, \\ f^{ij}_{2h}(x,\mathbf{k}) &= -\int d^4r\, \mathrm{e}^{ik\cdot r} \langle \bar\psi_{hj}(x-r/2)(-\mathrm{i}\gamma^5) \psi_{hi}(x+r/2)\rangle \\ &= 2\Im \left(L^{il*}_h R^{jl'}_h \right) \times \left\langle a^\dagger_{hl'}(\mathbf{k}) a_{hl}(\mathbf{k}) + b_{hl'}(\mathbf{k}) b^\dagger_{hl}(\mathbf{k}) \right\rangle, \\ f^{ij}_{3h}(x,\mathbf{k}) &= -\int d^4r\, \mathrm{e}^{ik\cdot r} \langle \bar\psi_{hj}(x-r/2)\gamma^0\gamma^5 \psi_{hi}(x+r/2)\rangle \\ &= \left(L^{il*}_h L^{jl'}_h - R^{il*}_h R^{jl'}_h \right) \times \left\langle a^\dagger_{hl'}(\mathbf{k}) a_{hl}(\mathbf{k}) + b_{hl'}(\mathbf{k}) b^\dagger_{hl}(\mathbf{k}) \right\rangle. \end{aligned} \tag{3.83}$$

By comparison with the expression (3.82), we obtain

$$\langle H\rangle = -\frac{1}{V}\sum_{h\,i} \int \frac{d^3k}{(2\pi)^3} h|\mathbf{k}| f_{3h\,ii} + M^H_{ii} f_{1h,ii} + M^A_{ii} f_{2h\,ii}. \tag{3.84}$$

We define $\omega_i(\mathbf{k}) = \sqrt{\mathbf{k}^2 + |M_{ii}|^2}$, and since we assumed diagonality of the Hamiltonian, this has to equal

$$\begin{aligned} \langle H\rangle &= \frac{1}{V}\sum_{h\,i} \int \frac{d^3k}{(2\pi)^3} \omega_i(\mathbf{k}) \langle a^\dagger_{h\,i}(\mathbf{k}) a_{h\,i}(\mathbf{k}) - b_{h\,i}(\mathbf{k}) b^\dagger_{h\,i}(\mathbf{k})\rangle \\ &= \frac{1}{V}\sum_{h\,i} \int \frac{d^3k}{(2\pi)^3} \omega_i(\mathbf{k}) \left(n^+_{h\,i}(\mathbf{k}) + n^-_{h\,i}(\mathbf{k}) - 1 \right), \end{aligned} \tag{3.85}$$

while the charge is

$$f_{0h\,ii} = \langle a^\dagger_{h\,i}(\mathbf{k}) a_{h\,i}(\mathbf{k}) + b_{h\,i}(\mathbf{k}) b^\dagger_{h\,i}\rangle = n^+_{h\,i}(\mathbf{k}) - n^-_{h\,i}(\mathbf{k}) + 1. \tag{3.86}$$

We thus find the following generalization of (3.67)

$$n^{+}_{h\,i}(\mathbf{k}) \;=\; \frac{h|\mathbf{k}|f_{3hii}+M^{H}_{ii}f_{1hii}+M^{A}_{ii}f_{2hii}}{2\omega_i(\mathbf{k})}+\frac{1}{2}f_{0hii}\,, \tag{3.87}$$

$$n^{-}_{h\,i}(\mathbf{k}) \;=\; \frac{h|\mathbf{k}|f_{3hii}+M^{H}_{ii}f_{1hii}+M^{A}_{ii}f_{2hii}}{2\omega_i(\mathbf{k})}-\frac{1}{2}f_{0hii}+1, \tag{3.88}$$

which is of course the anticipated result, since the number of particles is just the half of the total particle number (particles plus antiparticles) plus half of the total charge (particles minus antiparticles).

In order to obtain the particle numbers in the mass eigenstates we need to perform a biunitary diagonalization of the mass matrix M, that is a combination of different transformations for the left- and the right-handed Weyl components of the Dirac fermions. We first note, that when the unitary matrices U and V diagonalize $MM^\dagger$ and $M^\dagger M$, respectively, then

$$M_d = UMV^\dagger \tag{3.89}$$

is a diagonal matrix.

Let us define

$$\begin{aligned} X &= P_L\otimes V + P_R\otimes U\,, \\ Y &= P_L\otimes U + P_R\otimes V\,, \end{aligned} \tag{3.90}$$

where

$$P_L = \frac{1-\gamma^5}{2}\,, \qquad P_R = \frac{1+\gamma^5}{2}\,. \tag{3.91}$$

Then the Wigner function (3.79) transforms to the mass diagonal basis as

$$\mathrm{i}S^{<}_{d} = Y\mathrm{i}S^{<}X^\dagger\,. \tag{3.92}$$

For the $f_{\mu h}$, this equation reads

$$\begin{aligned} f_{d\,0h} &= \frac{1}{2}\left[V(f_{0h}+f_{3h})V^\dagger + U(f_{0h}-f_{3h})U^\dagger\right]\,, \\ f_{d\,1h} &= \frac{1}{2}\left[U(f_{1h}+\mathrm{i}f_{2h})V^\dagger + V(f_{1h}-\mathrm{i}f_{2h})U^\dagger\right]\,, \\ f_{d\,2h} &= \frac{1}{2}\left[V(f_{2h}+\mathrm{i}f_{1h})U^\dagger + U(f_{2h}-\mathrm{i}f_{1h})V^\dagger\right]\,, \\ f_{d\,3h} &= \frac{1}{2}\left[V(f_{3h}+f_{0h})V^\dagger + U(f_{3h}-f_{0h})U^\dagger\right]\,. \end{aligned} \tag{3.93}$$

This will be useful, when we want to calculate particle numbers and charges at a time when the mass terms are varying only adiabatically slow, and we transform to the mass-diagonal basis, because flavour oscillations are absent there.

3.3.2 Scalars

Consider now a complex scalar field ϕ_i describing N flavours, which we expand into its hermitean and antihermitean parts as follows,

$$\phi_i = \frac{1}{\sqrt{2}}(\phi_i^1 + \mathrm{i}\phi_i^2), \tag{3.94}$$

such that the multiflavour field operator is

$$\Phi_i = \frac{\varphi_i}{a} = \frac{1}{aV}\sum_{\mathbf{k}} \mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{x}} \times \Big(\varphi_{ij}^1(\mathbf{k},\eta)a_j^1(\mathbf{k}) + \mathrm{i}\varphi_{ij}^2(\mathbf{k},\eta)a_j^2(\mathbf{k}) \tag{3.95}$$
$$+\varphi_{ij}^{1\dagger}(\text{-}\mathbf{k},\eta)a_j^{1\dagger}(\text{-}\mathbf{k}) + \mathrm{i}\varphi_{ij}^{2\dagger}(\text{-}\mathbf{k},\eta)a_j^{2\dagger}(\text{-}\mathbf{k})\Big),$$

where the rescaled fields obey the generalized Klein-Gordon equation

$$\left\{\partial_\eta^2 + \mathbf{k}^2 + M^2 + (1-6\xi)\frac{a''}{a}\right\}_{il} \varphi_{lj}^\alpha = 0\,, \tag{3.96}$$

and M^2 is hermitean. Note that this is independent of whether $\alpha = 1$ or $\alpha = 2$, which is just as in the fermionic case, where the functions U and V both satisfy the Dirac equation. The individual components ϕ_i^1 and ϕ_i^2 are imposed to be hermitean. Therefore, $\sum_j(\varphi_{ij}(\mathbf{k}) + \varphi_{ij}(-\mathbf{k}))$ has to be real, which can in general be satisfied only if M^2 is real or, more precisely, real symmetric. Charge production may however only take place when M^2 is not symmetric.

Let us therefore assume again, that we are in a final state with diagonal and only nonadiabatically varying M. We define

$$\begin{aligned} a(\mathbf{k}) &= \frac{1}{\sqrt{2}}\left[a^1(\mathbf{k}) + \mathrm{i}a^2(\mathbf{k})\right] \quad \text{and} \\ b(\mathbf{k}) &= \frac{1}{\sqrt{2}}\left[a^1(\mathbf{k}) - \mathrm{i}a^2(\mathbf{k})\right]. \end{aligned} \tag{3.97}$$

Then, we find the charge operator to be

$$Q_i(\mathbf{k}) = \langle a_i^\dagger(\mathbf{k})a_i(\mathbf{k}) - b_i^\dagger(\mathbf{k})b_i(\mathbf{k})\rangle \tag{3.98}$$

and the Hamiltonian

$$H = \frac{1}{V}\sum_{\mathbf{k}} \Omega_i(\mathbf{k})\left(a_i^\dagger(\mathbf{k})a_i(\mathbf{k}) + b_i^\dagger(\mathbf{k})b_i(\mathbf{k}) + 1\right), \tag{3.99}$$

where $\Omega_i(\mathbf{k}) = |\varphi_i'(\mathbf{k}) - (a'/a)\varphi_i(\mathbf{k})|^2 + \bar\omega_i^2(\mathbf{k})\,|\varphi_i(\mathbf{k})|^2$, $\bar\omega_i^2(\mathbf{k}) = \mathbf{k}^2 + a^2M_{ii}^2 + 3\xi a''/a$.

We define the multiflavour Wightman function as

$$i\bar G_{ij}^< = \langle\varphi_j^\dagger(u)\varphi_i(v)\rangle \tag{3.100}$$

and adapt straightforwardly the definition of the moments from the single flavour case. These then satisfy the system of equations

$$\begin{aligned} \frac{1}{4}f_0'' - f_2 + \frac{1}{2}\left\{M^2, f_0\right\} + \left(\mathbf{k}^2 - \frac{a''}{a}\right) f_0 &= 0\,, \qquad (3.101)\\ f_1' - \frac{\mathrm{i}}{2}\left[M^2, f_0\right] &= 0\,,\\ f_2' - \frac{\mathrm{i}}{2}\left[M^2, f_1\right] - \frac{1}{4}\left\{(\mathbf{k}^2 + M^2 - a''/a)', f_0\right\} &= 0\,. \end{aligned}$$

We find $Q_i(\mathbf{k}) = f_{1\,ii}(x, \mathbf{k}) + 1$, which is also in accordance with the $U(1)$-Noether charge. Together with the identities (3.29), this leads us to

$$n_i^+(\mathbf{k}) = \omega_i f_{0\,ii} + \frac{f''_{0\,ii}}{4\omega_i} - \frac{1}{2\omega_i}\frac{d}{d\eta}\left(\frac{a'}{a}f_{0\,ii}\right) + \frac{1}{2}f_{1\,ii}\,, \qquad (3.102)$$

$$n_i^-(\mathbf{k}) = \omega_i f_{0\,ii} + \frac{f''_{0\,ii}}{4\omega_i} - \frac{1}{2\omega_i}\frac{d}{d\eta}\left(\frac{a'}{a}f_{0\,ii}\right) - \frac{1}{2}f_{1\,ii} - 1\,, \qquad (3.103)$$

where $n_i^+(\mathbf{k})$ is the number of particles, $n_i^-(\mathbf{k})$ the number of antiparticles, and the same simple interpretation as in the fermionic case applies.

3.4 Remarks

We have derived general expressions for the particle number densities on phase space for single scalars (3.31) and fermions (3.67) in terms of the appropriate Wigner functions. We have then generalized our analysis to the case of mixing scalars (3.102–3.103) and fermions (3.87–3.88). All of these expressions are positive, and moreover, the number of fermions is bounded from above by *unity*, as required by the Pauli principle. In order to incorporate the effect of self-energy into (3.31) and (3.67), one needs to include this correction into the dispersion relation, $\omega = \omega(\mathbf{k}, x) \to \omega + \Sigma_H(\mathbf{k}, x)$, where $\Sigma_H(\mathbf{k}, x) \equiv \int [dk_0/(2\pi)](1/2)[\Sigma^r(k, x) + \Sigma^a(k, x)]$, and Σ^r and Σ^a denote the retarded and advanced self-energies, respectively [52]. When the single particle picture breaks down it is not clear whether a sensible definition of particle number can be constructed.

The kinetic theory definition of the particle number is of course and by construction identical with the definition in terms of Bogolyubov transformations. The number of particles is the total energy of the system divided by the energy of an individual particle. Taking the point of view of kinetic theory proves advantageous when considering the multiflavour case or statistical systems, such as the thermal equilibrium.

While the fermionic particle number definition (3.67) is generally applicable, the scalar one (3.31) fails however when $\bar{\omega}(\mathbf{k})^2 = \mathbf{k}^2 + a^2 m_\phi^2 + 3\xi a'' a < 0$, which can happen at phase transitions. Then $\Omega(\mathbf{k}) < |\Lambda(\mathbf{k})|$ in (3.11), and the Bogolyubov transformation (3.14) does not have a solution. Nevertheless, even in this case, the energy density on phase space $\Omega(\mathbf{k})$ in Eqn. (3.30) is well defined. Another important quantity is $\Lambda^*(\mathbf{k}) = \langle \mathbf{k}, -\mathbf{k}|H|0\rangle$, the transition amplitude for particle pair creation with the momenta $\{\mathbf{k}, -\mathbf{k}\}$; and likewise $\Lambda(\mathbf{k})$ is the transition amplitude for pair annihilation. The appropriate description in this case is in terms of squeezed states. For an account of the inverted harmonic oscillator in terms of squeezed states see *e.g.* Ref. [53].

Our definition of particle number can be used for studies of quantum-to-classical transition, decoherence and entropy calculations of *e.g.* cosmological perturbations [53–55]. Moreover, when suitably normalized, the particle density $n(\mathbf{k})$ can be used to define a density matrix on phase space, $\varrho(\mathbf{k}) = n(\mathbf{k})/\sum_{\mathbf{k}'} n_{\mathbf{k}'}$.

In the derivation of our results, we considered pure quantum states, yet showed explicitly their applicability to thermal states. More generally, our definitions are valid if one requires the density matrix ϱ to satisfy $\langle a(\mathbf{k})a(\mathbf{k})\rangle_\varrho = \langle a^\dagger(\mathbf{k})a^\dagger(\mathbf{k})\rangle_\varrho = 0$. These relations hold *e.g.* for eigenstates of the particle number operator $\hat{N}(\mathbf{k}) \equiv a^\dagger(\mathbf{k})a(\mathbf{k})$, and, as pointed out in Ref. [56], for random phase states, a special case of which is the canonical ensemble. States of this kind can be treated as a linear superposition of the particle number eigenstates which we considered above.

Chapter 4

Coherent Baryogenesis

Baryogenesis scenarios beyond the Standard Model often invoke the out-of-equilibrium decay of heavy particles, a paradigm first suggested by Yoshimura [57], which is also adapted in the very popular and plausible leptogenesis mechanism [58], where superheavy Majorana neutrinos decay in a CP-violating manner. Well known notable exceptions are the mechanisms proposed by Affleck and Dine [59] and by Cohen and Kaplan [60], both being operative in the presence of time-dependent scalar condensates.

Here, we propose a new mechanism for baryogenesis in the context of preheating, which is the coherent production of particles with a strongly time dependent mass term [3–7]. In the early Universe, preheating processes might have occurred during phase transitions, when classical scalar field condensates evolve nonadiabatically fast. Particles coupled to these fields may acquire time-dependent mass terms. If the phase transition comes along with the breaking of the symmetry of a Grand Unified Theory (GUT), then the mass matrix inducing particle production can also mix species with baryonic and leptonic charge. In our discussion of chapter 3, in particular of section 3.3.1, we have seen how under such circumstances fermionic charges can be produced, as described by the multiflavour kinetic equations (3.56). At the end of the phase transition, when the mass matrix becomes constant, the charges get frozen in and there may remain a matter-antimatter asymmetry. Since this mechanism relies on the interplay of coherent particle production and $(B-L)$-violating flavour oscillations, we call it *coherent baryogenesis*. We emphasize the conceptual simplicity of coherent baryogenesis, since it involves the tree-level dynamics of quantum fields only.

In the following, we first demonstrate the mechanism on a simple toy model. Then, we discuss a realistic cosmological scenario, as suggested in Ref. [61], involving inflation and the subsequent breaking of a unified symmetry, namely the Pati-Salam group, down to the Minimal Supersymmetric Standard Model (MSSM), thereby linking various issues of particle physics and cosmology.

4.1 Toy Model

We now consider a two species model, where fundamental SU(2) fermions couple to an adjoint scalar triplet Φ^i, $i = 1, 2, 3$, and to a singlet Φ^0 *via* Yukawa couplings

$$\mathcal{L}_{\text{Yu}} = -y^0 \Phi^0 \bar{\psi}_k \psi_k - y \{\Phi^a \sigma^a\}_{ij} \bar{\psi}_i \psi_j \,. \tag{4.1}$$

The mass matrix is therefore given by

$$M = y^0 \Phi^0 \mathbb{1} + y \Phi^i \sigma^i \,. \tag{4.2}$$

For a discussion of a fermionic mass matrix as the source for electroweak baryogenesis, *cf.* Refs. [47,62–64]. In that case, there is a spatial dependence of the mass term, while for coherent baryogenesis, the mass matrix varies with time.

While we fix the fields associated with the diagonal terms as $\Phi^0 = \mu$ and $\Phi^3 = \mu/2$, with μ being a mass scale, we let Φ^1 and Φ^2 move freely in a harmonic potential, starting from arbitrary initial conditions. Conjugating the fermionic sector of the kinetic equations (3.56) under CP while leaving the scalar condensate invariant, we find that, in spite of M being hermitian, CP is broken for the fermions, because $M \neq M^*$.

It is notable, that in coherent baryogenesis the condensate does not have to carry a charge, while this is necessary for the Affleck-Dine mechanism [59], therefore being conceptually different. This can be seen explicitly from all examples worked out in this thesis, where the scalar condensates ϕ are always real, and therefore the scalar charge vanishes:

$$q_\phi = \mathrm{i} \langle \phi^\dagger \overset{\leftrightarrow}{\partial}_\eta \phi \rangle = 0 \,, \tag{4.3}$$

where η is conformal time.

However, the time dependence of M plays here another important rôle. For $N = 1$, the Dirac equation consists of two first order differential equations due to the two degrees of freedom of the fermionic field. Thus, also $dM/d\eta$ can contribute CP-violation, and in general, both sources from M and $dM/d\eta$ cannot be simultaneously removed by local phase reparametrizations of the fermionic fields, *cf.* figure 3.2 for an example. When $N > 1$, even higher derivatives of M are involved, allowing, in principle, multiple sources of CP-violation. We stress that this is a very different situation from the Standard Model quark mixing Cabibbo-Kobayashi-Maskawa matrix, or lepton mixing Maki-Nakagawa-Sakata matrix, where at least three generations of quarks or leptons are required for one CP-violating phase.

Damping is introduced through a phenomenological decay rate Γ and through the Hubble expansion in a matter dominated universe, *e.g.* the scale factor is $a = a_m \eta^2$, *cf.* Eqn. (2.14). The equation of motion for a scalar $\Phi(\eta)$ is given by

$$\Phi'' + 2\frac{a'}{a}\Phi' + a^2 \frac{dV}{d\Phi} + a\Gamma\Phi' = 0. \tag{4.4}$$

Writing $\omega_\Phi^2{=}d^2V/d\Phi^2$ and setting $\Gamma{=}0$, the solutions are

$$\Phi(\eta) = \frac{c_1}{\eta^3}\cos\left(\frac{1}{3}a_m\omega_\Phi\eta^3\right) + \frac{c_2}{\eta^3}\sin\left(\frac{1}{3}a_m\omega_\Phi\eta^3\right). \tag{4.5}$$

For Φ^1, we employ $c_1 = \mu$, $c_2 = 0$ and $\omega_\Phi = \mu$, for Φ^2, $c_1 = 0$, $c_2 = \mu$ and $\omega_\Phi = 1.5\mu$, and we set $a_m = \mu^2$, $y^0 = y = 1$.

We approximate the effect of damping by multiplying the solutions (4.5) by $A = \exp\left(-\frac{1}{6}a_m\Gamma\eta^3\right)$, where $\Gamma = 0.1\mu \ll \omega_\Phi$.

The equations of motion for the Wigner functions in conformal space-time are then simply obtained by replacing M by aM in (3.56), *cf.* Eqn. (3.42) and Refs. [6, 65]. We illustrate the motion of the mixing contributions to the fermionic mass terms in conformal time in figure 4.1.

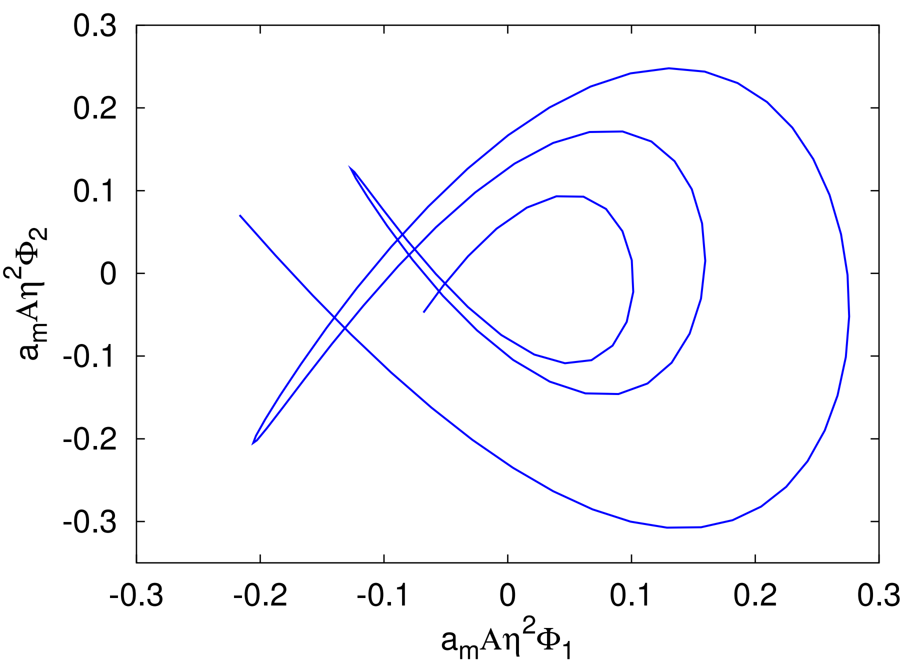

Figure 4.1: Parametric plot of the motion of $a(\eta)A\Phi_1$ and $a(\eta)A\Phi_2$ for $\eta \in [2.3\mu^{-1}, 4\mu^{-1}]$.

Requiring that there are no particles at $\eta{=}2.1\mu^{-1}$, we choose initial conditions in accordance with (3.59) and solve (3.56) numerically, to find fermion number production as displayed in figure 4.2 as a function of the conformal momentum k.

Since we are interested in the produced charge asymmetry and f_{0h} is the zero-component of the vector current, we denote the charge of the species a carried by the mode with momentum k and helicity h by

$$q_{a\,h}(k) = f_{0h\,aa}(k). \tag{4.6}$$

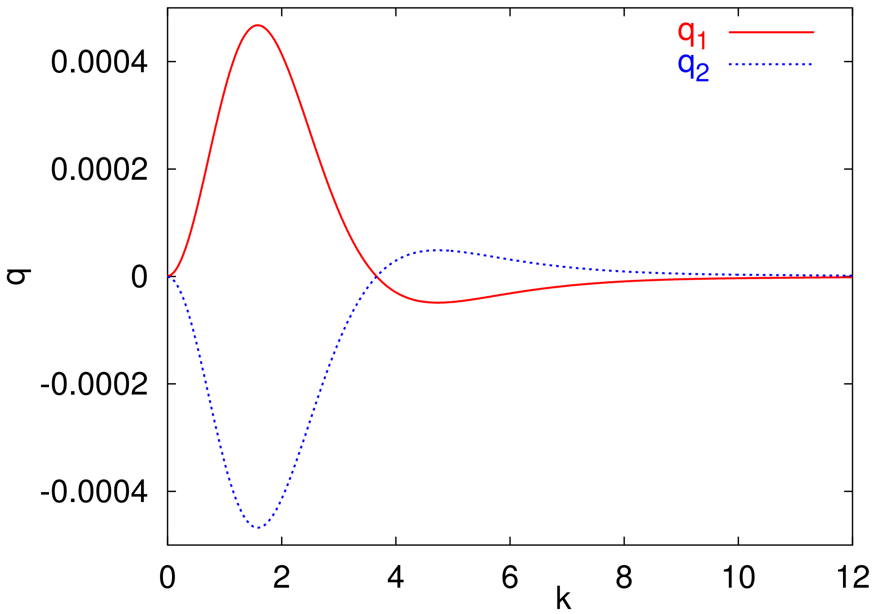

Figure 4.2: The fermion numbers $q_{1+}(k)$ and $q_{2+}(k)$ in the toy model.

We sum over the helicities,

$$q_a = q_{a+} + q_{a-}\,, \tag{4.7}$$

and note, that in the present case $q_{a+} = q_{a-}$, for $M_A = 0$. Integration over k,

$$q_a = \frac{1}{a^3}\int \frac{dk}{2\pi^2} k^2 q_a(k)\,, \tag{4.8}$$

gives the charge densities $q_1 = -q_2 = 2.4\times 10^{-5}(\mu/a)^3$. If q_1 and q_2 were differently charged under B, our toy model would lead to successful baryogenesis. Since there is a global U(1)-symmetry for phase rotations of the fermionic fields, the sum over all produced charges is zero,

$$\sum_a q_{a\pm}(k) = 0\,, \tag{4.9}$$

as can be verified from all examples considered in this thesis.

4.2 Hybrid Inflation in a SUSY Pati-Salam Model

We shall now discuss the implementation of coherent baryogenesis in a more realistic model. In order to generate a baryon asymmetry which survives the sphaleron washout, we require the presence of $(B{-}L)$-violation. This is the case in several GUTs, *e.g.* E(6), a subgroup of which is SO(10), which we shall treat in chapter 5, and in the Pati-Salam group

$$G_{PS} = \mathrm{SU}(4)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R\,, \tag{4.10}$$

which appears as an intermediate stage of breaking of SO(10) down to the Standard Model group

$$G_{SM} = \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y\,. \tag{4.11}$$

For simplicity, here we study an extension of the hybrid inflationary scenario embedded in a supersymmetric (SUSY) Pati-Salam model, which is considered in [61] and does not suffer from the monopole problem, as we explain below. The relevant terms of the superpotential are

$$W \supset \kappa S\left(\bar{H}^c H^c - \mu^2\right) - \beta S\left(\frac{\bar{H}^c H^c}{M_S}\right)^2 + \zeta G H^c H^c + \xi G \bar{H}^c \bar{H}^c\,. \tag{4.12}$$

For a comprehensive SUSY *vademecum*, containing all the facts used here, see Ref. [66].

Under G_{PS}, the fields transform as $H^c = (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$, $\bar{H}^c = (\mathbf{4}, \mathbf{1}, \mathbf{2})$, $S = (\mathbf{1}, \mathbf{1}, \mathbf{1})$ and $G = (\mathbf{6}, \mathbf{1}, \mathbf{1})$ [67]. We adopt the notation

$$H^c = \begin{pmatrix} u^c_{H1} & u^c_{H2} & u^c_{H3} & \nu^c_H \\ d^c_{H1} & d^c_{H2} & d^c_{H3} & e^c_H \end{pmatrix},$$

and likewise for $\bar{H}^c$. With $\mathrm{SU}(4)_C$ broken to the Standard Model, $G = D + \bar{D}$, with $D = \mathbf{3}$ and $\bar{D} = \bar{\mathbf{3}}$ of $\mathrm{SU}(3)_C$. We note that the representation $\mathbf{6}$ of $\mathrm{SU}(4)_C$ is the antisymmetric part of $\mathbf{4}\otimes\mathbf{4}$, and we can therefore represent the tensor G by the matrix

$$G = \begin{pmatrix} 0 & \bar{D}_3 & -\bar{D}_2 & -D1 \\ -\bar{D}_3 & 0 & \bar{D}_1 & -D2 \\ \bar{D}_2 & -\bar{D}_1 & 0 & -D_3 \\ D_1 & D_2 & D_3 & 0 \end{pmatrix}. \tag{4.13}$$

Now, suppose that H^c and $\bar{H}^c$ both obtain a VEV. The scalar potential of a SUSY-gauge theory contains the so-called D-terms, which in the case at hand read

$$\frac{1}{2} g_C^2 \sum_{a=1}^{15} \left(\bar{H}^{c*} T^a \bar{H}^c + H^{c*} T^a H^c\right)^2 + \frac{1}{2} g_R^2 \sum_{a=1}^{3} \left(\bar{H}^{c*} \sigma^a \bar{H}^c + H^{c*} \sigma^a H^c\right)^2\,, \tag{4.14}$$

where the T^a are the 15 generators of $\mathrm{SU}(4)_C$, g_C is the $\mathrm{SU}(4)_C$ gauge coupling constant and g_R is the $\mathrm{SU}(2)_R$ coupling. Since the corresponding components of H^c and $\bar{H}^c$ have opposite charges, vanishing of the D-terms requires $\langle H^c \rangle = \langle \bar{H}^{c*} \rangle$, which we assume throughout this discussion. We furthermore identify the direction of the nonzero VEV with the Standard Model singlet, carrying the quantum numbers of the right-handed neutrino. Therefore, when we consider VEVs of scalar condensates, we introduce the notations $\nu^c_H \equiv H^c$ and $\nu^c_{\bar{H}} \equiv \bar{H}^c$, where $\nu^c_H = \nu^{c*}_{\bar{H}}$.

The other scalar, which will acquire a nonzero VEV is the singlet S, and therefore we consider the scalar potential

$$\begin{aligned} V &= \left|\frac{\delta W}{\delta \nu^c_H}\right|^2 + \left|\frac{\delta W}{\delta \nu^c_{\bar{H}}}\right|^2 + \left|\frac{\delta W}{\delta S}\right|^2 \\ &= 2\left|S\nu^{c*}_H\left(\kappa - 2\beta\frac{|\nu^c_H|^2}{M_S^2}\right)\right|^2 + \left|\kappa\left(|\nu^c_H|^2 - \mu^2\right) - \beta\frac{|\nu^c_H|^4}{M_S^2}\right|^2 . \end{aligned} \tag{4.15}$$

We are not interested in the phases of S and ν^c_H. Ignoring those, we find

$$\begin{aligned} \frac{\partial V}{\partial \nu^c_H} &= 4S^2\nu^c_H\left[\kappa^2 - 8\kappa\beta\frac{{\nu^c_H}^2}{M_S^2} + 12\beta^2\frac{{\nu^c_H}^4}{M_S^4}\right] \\ &\quad + 4\nu^c_H\left[(\kappa^2 - 2\kappa\beta\frac{{\nu^c_H}^2}{M_S^2})({\nu^c_H}^2 - \mu^2) - \beta\frac{{\nu^c_H}^4}{M_S^2}\left(\kappa - 2\beta\frac{{\nu^c_H}^2}{M_S^2}\right)\right] . \end{aligned} \tag{4.16}$$

Imposing $\partial V/\partial \nu^c_H = 0$ leaves us with three candidate extrema for potential minimization, $\nu^c_H = 0$, a case where ν^c_H depends on S and

$$\nu^c_H = M_S\sqrt{\frac{\kappa}{2\beta}}\,, \tag{4.17}$$

where we impose ν^c_H to sit in our patch of the Universe during inflation. Since $V \neq 0$, there is vacuum energy which drives inflation, what also implies that SUSY is broken. We make the choice (4.17), because breaking the symmetry of G_{PS} down to G_{SM} goes along with the creation of magnetic monopoles, which would dominate the energy density of the Universe today, when produced after inflation. By shifting $\langle \nu^c_H \rangle$ away from zero the symmetry is already broken during inflation, and therefore monopoles are diluted. The term $\propto \beta$ in the superpotential (4.12), which is responsible for this displacement, is therefore referred to as shift term. A hybrid inflationary model without shift-term is considered in chapter 5.

We calculate the second derivative of the potential at the inflationary minimum, which is

$$\left.\frac{\partial^2 V}{\partial {\nu^c_H}^2}\right|_{\nu^c_H = M_S\sqrt{\frac{\kappa}{2\beta}}} = 16\kappa^2 S^2 + 4\left(2\kappa^2\mu^2 - \frac{1}{2}\frac{\kappa^3}{\beta}M_S^2\right) . \tag{4.18}$$

Therefore, there is a critical value

$$S_{\text{crit.}} = \frac{1}{2}\sqrt{\frac{1}{2}\frac{\kappa}{\beta}M_S^2 - 2\mu^2}\,, \tag{4.19}$$

such that for $S > S_{\text{crit.}}$ the VEV (4.17) is indeed a minimum of the potential (4.15) and for $S < S_{\text{crit.}}$, it is a local maximum. While at tree level, the potential is flat in S, it acquires a logarithmic slope when including radiative corrections [68].

The hybrid inflationary scenario is then as follows: initially, $S \gg S_{\text{crit.}}$, and ν^c_H has the VEV given in Eqn. (4.17). Logarithmically slow, S rolls down its potential, eventually reaches the value $S_{\mathrm{crit.}}$ and triggers ν_H^c to fall down to a new minimum. This phase transition is usually referred to as the waterfall regime. After the phase transition has ended, the field have attained the VEVs $\langle S \rangle = 0$ and

$$\langle |\nu_H^c| \rangle = \left(\frac{\kappa M_S^2 - \left[\kappa M_S^2 (\kappa M_S^2 - 4\beta\mu^2) \right]^{\frac{1}{2}}}{2\beta} \right)^{\frac{1}{2}} . \tag{4.20}$$

Note that this implies $V = 0$, such that inflation is terminated and SUSY restored.

The reason why one calls this hybrid inflation is that there are two scalar fields involved, and the reason to consider such models is that the inflaton S rolls very slow, because the lifting of the potential is a loop-order effect. This allows for a period of inflation which lasts sufficiently long in order to accord with cosmological observations. One would therefore like to motivate the absence of terms in the superpotential (4.12) which are of higher than linear order in S. This can be achieved by introducing a global $\mathrm{U}(1)_R$ symmetry and assigning the charges $R(H^c) = R(\bar{H}^c) = 0$, $R(S) = R(G) = 1$, such that all superpotential terms have the charge $R(W) = 1$.

We now derive the time-dependent fermionic mass-matrix inducing coherent baryogenesis. CP-violation is provided by a complex phase between the parameters ζ and ξ in the superpotential (4.12). The fields involved are therefore G, H^c and $\bar{H}^c$.

In order to identify the nonzero mass terms, we note

$$\frac{\delta}{\delta \nu_{\bar{H}^c}} \xi G \bar{H}^c \bar{H}^c = 2\xi \sum_{i=1}^{3} d_{\bar{H}j}^c D_j \,, \tag{4.21}$$

$$\frac{\delta}{\delta \nu_{H^c}} \zeta G \bar{H}^c \bar{H}^c = 2\zeta \sum_{i=1}^{3} d_{Hj}^c \bar{D}_j \,, \tag{4.22}$$

where the products are contracted as $G_{\alpha\beta} \bar{H}^{c\alpha} \bar{H}^{c\beta}$ and $\varepsilon^{\alpha\beta\gamma\delta} G_{\alpha\beta} H_\gamma^c H_\delta^c$, respectively.

The fermionic mass terms in the Lagrangean for a chiral supermultiplet are

$$M_{\mathrm{ferm.}} = \frac{1}{2} \left\{ \frac{\delta^2 W}{\delta\phi_i \delta\phi_j} \chi_i \chi_j + \mathrm{h.c.} \right\} , \tag{4.23}$$

where χ_i denotes a Weyl-spinor. For our particular superpotential (4.12), we have

$$M_{\mathrm{ferm.}} = m_d \chi_{d_{Hj}^c} \chi_{d_{\bar{H}j}^c} + 2\zeta \langle \nu_H^c \rangle \chi_{\bar{D}_j} \chi_{d_{Hj}^c} + 2\xi \langle \nu_H^{c*} \rangle \chi_{D_j} \chi_{d_{\bar{H}j}^c} + \mathrm{h.c.} \,, \tag{4.24}$$

where

$$m_d = \langle S \rangle \left(\kappa - 2\beta \langle |\nu_H^c| \rangle^2 / M_S^2 \right) . \tag{4.25}$$

Now, we combine the Weyl spinors to form Dirac fermions

$$\psi_{1j} = \begin{pmatrix} \chi_{d_{\bar{H}j}^c} \\ \bar{\chi}_{\bar{D}_j} \end{pmatrix} \text{ and } \psi_{2j} = \begin{pmatrix} \chi_{D_j} \\ \bar{\chi}_{d_{Hj}^c} \end{pmatrix} , \tag{4.26}$$

with the mass term,

$$\left(\begin{array}{cc}\bar{\psi}_{1j} & \bar{\psi}_{2j}\end{array}\right)\left[\left(\begin{array}{cc}2\Re\left[\langle\nu_H^{c*}\rangle\xi\right] & \frac{1}{2}m_d \\ \frac{1}{2}m_d & 2\Re\left[\langle\nu_H^c\rangle\zeta\right]\end{array}\right)\right.$$
$$\left.+\mathrm{i}\gamma^5\left(\begin{array}{cc}-2\Im\left[\langle\nu_H^{c*}\rangle\xi\right] & -\frac{\mathrm{i}}{2}m_d \\ \frac{\mathrm{i}}{2}m_d & -2\Im\left[\langle\nu_H^c\rangle\zeta\right]\end{array}\right)\right]\left(\begin{array}{c}\psi_{1j} \\ \psi_{2j}\end{array}\right), \tag{4.27}$$

where for Weyl spinors, we have used the notation $\bar{\chi} = \chi^\dagger$, while for Dirac spinors $\bar{\psi} = \psi^\dagger\gamma^0$.

Since not all constants can be made real by field redefinitions, we can allow for a nontrivial phase between ζ and ξ. Note, that the substitution $\zeta \longleftrightarrow \xi^*$ results in the opposite sign of $B-L$ and B and hence, when $\zeta = \xi^*$, there is no generation of $B-L$.

In the presence of the $(B-L)$-violating condensate of $\bar{\nu}_H^c$, the right handed neutrino ν^c acquires a Majorana mass from the nonrenormalizable contribution

$$\gamma\frac{F^c\bar{H}^cF^c\bar{H}^c}{M_S} \tag{4.28}$$

added to the superpotential (4.12), where $F^c\bar{H}^c$ form a gauge singlet, and $F^c = (\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$ are the superfields containing the right handed quarks and leptons,

$$F^c = \left(\begin{array}{cccc}u_1^c & u_2^c & u_3^c & \nu^c \\ d_1^c & d_2^c & d_3^c & e^c\end{array}\right). \tag{4.29}$$

Left handed matter is then contained in $F = (\mathbf{4},\mathbf{2},\mathbf{1})$, and one adds two-Higgs doublet $h = (\mathbf{1},\mathbf{2},\mathbf{2})$. When we make the assignments $R(F) = R(F^c) = \frac{1}{2}$ and $R(h) = 0$, the Standard Model coupling FhF^c is allowed.

The coupling (4.28) gives rise to the Lagrangean terms

$$\gamma(\langle\bar{\nu}_H^c\rangle/M_S)[\psi_{\nu^c}\phi_{d_j^c}\psi_{d^c_{\bar{H}j}} + \psi_{d_j^c}\phi_{\nu^c}\psi_{d^c_{\bar{H}j}}] + c.c.,$$

allowing the decay of the $d^c_{\bar{H}j}$-component of χ_{1j} in (4.26) to $d^{c*}+\nu^{c*}$, where one of the latter particles is fermionic, the other scalar. The Majorana neutrinos ν^c are their own antiparticles; neglecting the small effects induced by possible mixing angles and a CP-violating phase in the Majorana mass matrix familiar from leptogenesis scenarios [6, 58, 69, 70], their decay leaves behind no net charge. Note that in turn, our mechanism does not require any lower constraint on these angles and the phase as in leptogenesis.

The coupling

$$\tilde{\gamma}\frac{F^cH^cF^cH^c}{M_S} \tag{4.30}$$

allows the decay of the H^c-fields through the reaction from the d_H^{c*}-component of χ_{2j} in (4.26) to d^c+u^c, where the contraction is calculated as $\varepsilon^{\alpha\beta\gamma\delta}F^c_\alpha H^c_\beta F^c_\gamma H^c_\delta$. The charges hence get transformed to

$$(B-L) = \frac{1}{3}q_1 - \frac{2}{3}q_2\,. \tag{4.31}$$

$\kappa = 0.007$ $\Gamma_{\mathrm{Ph}} = 0.1\mu$	$\beta = 1$ $M_S = 100\mu$	$\zeta = 0.12\mathrm{i}$ $\xi = 0.12\times\exp\left(\mathrm{i}\times 10^{-3}\right)$	$\mu = 3.9\times 10^{16}\mathrm{GeV}$

Table 4.1: Parameters used in numerical simulation.

This number is promoted by sphaleron processes to [71]

$$B = \frac{10}{31}(B-L) \approx \frac{1}{3}(B-L)\,, \tag{4.32}$$

where we assumed two complex Higgs doublets.

Hence, the final value of $B-L$ arises here due to the transformation of other charges in decay processes. However, we point out, that models are conceivable where coherent particle production directly leads to Standard Model particles, the $(B-L)$-charge of which is conserved in the subsequent history of the Universe.

4.3 Numerical Simulation

Numerically, we simulate the evolution of the scalar fields from the end of inflation until they settle to the supersymmetric vacuum (4.20). The choice of our parameters is listed in table 4.1, for a discussion of their implications for cosmic observations, in particular on the running spectral index of the anisotropies of the CMBR, see Ref. [72]. The phenomenological damping term Γ_{Ph} is chosen such that the scalar fields follow damped trajectories, as suggested by numerical studies of tachyonic preheating [70, 73], which is the exponential production of scalar modes with negative mass squared. Here, this occurs because for ν_H^c the curvature of the potential is negative at onset of the waterfall regime, since $\partial^2 V/\partial {\nu_H^c}^2 > 0$ for $\langle S\rangle < S_{\mathrm{crit.}}$.

Eventually, ν_H^c condenses to the value given by (4.20) and the field S settles to zero, *cf.* figures 4.3 and 4.4. The mass matrix (4.27) then becomes diagonal, such that flavour oscillations are scotched and produced charges frozen in.

Starting with initially zero fermions, we compute the charges $q_{1j}(k)$ and $q_{2j}(k)$ associated with ψ_{1j} and ψ_{2j}, respectively, at the time when the phase transition is completed, by integration of Eqns. (3.56), as shown in figure 4.5. Integration then yields $q_1 = -q_2 = 1.2\times 10^{-44}(\mathrm{GeV})^3$. By Eqn. (4.31) this charge gets transferred into the initial $(B-L)$-density $n^0_{B-L} = q_1 = -q_2 = 1.2\times 10^{46}(\mathrm{GeV})^3$, where the superscript 0 indicates, that this is the density at the end of inflation, before entropy is generated during reheating.

In order to estimate the entropy production, we treat the reheating process as follows [74, 75]. The inflaton fields ν_H^c and S both have the mass

$$m_I = \sqrt{2}\kappa\langle\nu_H^c\rangle\left(1 - 2\frac{\beta}{\kappa}\frac{\mu^2\langle\nu_H^c\rangle^2}{M_S^4}\right)\,, \tag{4.33}$$

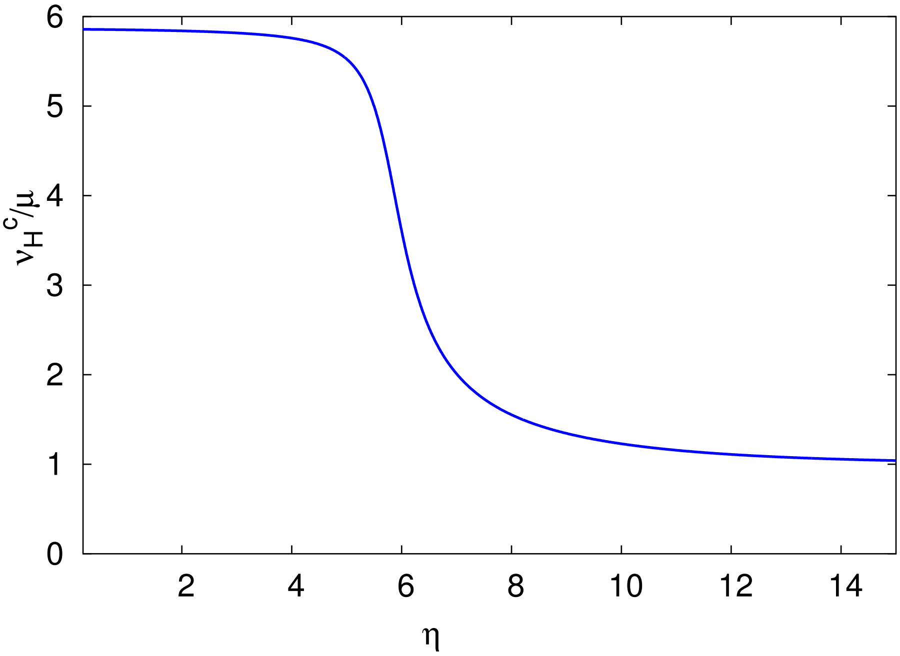

Figure 4.3: Evolution of the field ν_H^c.

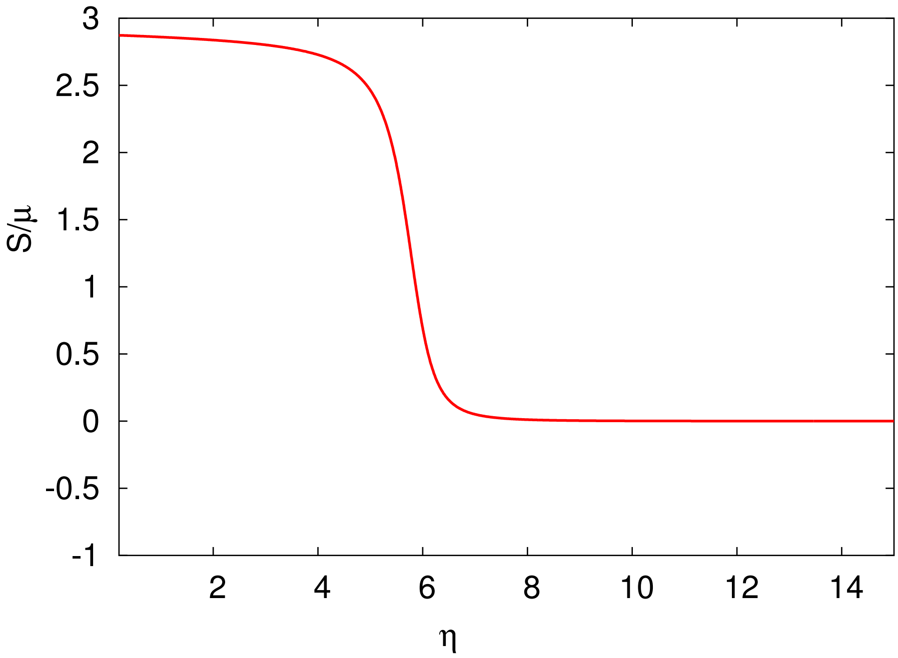

Figure 4.4: Evolution of the field S.

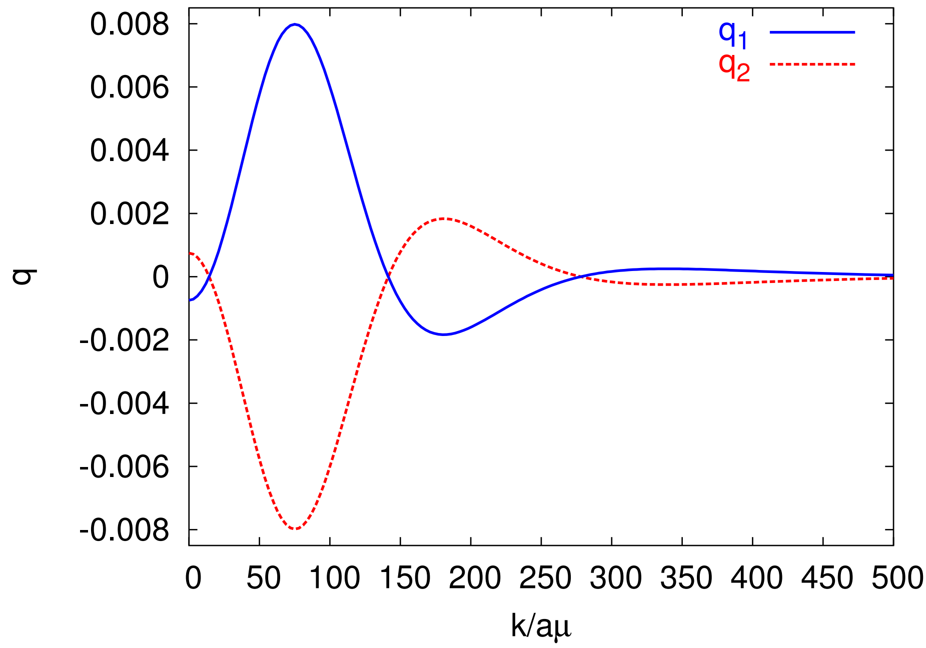

Figure 4.5: The produced charges of the Dirac fermions χ_{1j}, χ_{2j}, summed over both helicities. The scale factor is $a = 60$.

such that for our parameters $m_I \approx 3.9 \times 10^{14}$GeV. This mass term determines the frequency at which the inflaton fields oscillate around their minima, while the Universe expands in a matter-dominated way. The oscillations and the matter dominated epoch are ending at the time, when the inflaton field decays into relativistic matter, behaving as radiation. The coupling (4.28) gives a Majorana mass to the right-handed neutrino ν^c,

$$m_{\nu^c} = \gamma \frac{\langle \nu^c_H \rangle^2}{M_S} \,. \tag{4.34}$$

When we take $\gamma = 10^{-4}$, then $m_{\nu^c} = 3.9 \times 10^{10}$GeV $\ll m_I$, such that the inflaton can decay into relativistic right handed neutrinos at the rate

$$\Gamma = \frac{1}{8\pi} m_I \left(\frac{\gamma \langle \nu^c_H \rangle}{M_S} \right)^2 \,. \tag{4.35}$$

With our parameters, $\Gamma \approx 15$GeV. Since there are three generations of matter, we assume ν^c to be the lightest Majorana neutrino, while the remaining two shall have masses greater than m_I.

The Universe becomes radiation dominated and entropy production stops, when $\Gamma = H$, where H denotes the Hubble expansion rate. The reheat temperature at this time is

$$T_R = 0.55 g_*^{-\frac{1}{4}} \sqrt{\Gamma m_{\rm Pl}} . \tag{4.36}$$

When we take for $g_* = 221.5$, the number of relativistic degrees of freedom in the MSSM and the above results, we find $T_R = 1.9 \times 10^9\text{GeV}$. The entropy density is then given by

$$s = 2\pi^2 g_* T_R^3/45 \tag{4.37}$$

and the Hubble expansion rate by

$$H = 1.66\sqrt{g_*}\frac{T_R^2}{m_{\text{Pl}}}\,. \tag{4.38}$$

During the epoch of coherent oscillations, that is between the end of inflation and the onset of radiation era, the Universe is matter dominated and expands by a factor (*cf.* Eqn. (2.14))

$$\frac{a}{a_0} = \left(\frac{H_0}{H}\right)^{\frac{2}{3}}\,, \tag{4.39}$$

where H_0 is the expansion rate at the end of inflation, given by

$$H_0 = \sqrt{\frac{8\pi}{3}\frac{V}{{m_{\text{Pl}}}^2}}\,. \tag{4.40}$$

Putting everything together, we find[1]

$$\frac{n_B}{s} = \frac{n^0_{B-L}}{s}\left(\frac{a_0}{a}\right)^3 \approx \frac{3}{4}\frac{n^0_{B-L}}{V_0}T_R\,, \tag{4.41}$$

where we have taken account of a factor of three due to three colours, but also a division by three for sphaleron transitions (4.32).

For our model, we find $n_B/s \approx 8 \times 10^{-10}$, such that it is a possible candidate for the explanation of the observed BAU, $n_B/s \approx 8.7 \times 10^{-11}$ [76], since by the choice of a smaller phase between ζ and ξ, the produced asymmetry can be suppressed.

[1]The factor $\frac{3}{4}$ is of course not exact but a numerical accident.

Chapter 5

Coherent Baryogenesis in an SO(10) Framework

Being a product group, the unified gauge group suggested by Pati and Salam [77] gives, strictly speaking, not rise to a Grand Unified Theory. In fact, the number of independent gauge couplings, which is three, is not reduced when compared to the Standard Model, but the appealing feature of regarding lepton number as a fourth colour is introduced, hence placing quarks and leptons of a given chirality into one irreducible gauge multiplet. Yet, there are two different multiplets for the left and right handed matter particles.

On the other hand, the genuine GUT SU(5), as proposed by Georgi and Glashow [78], has just a single gauge coupling constant. The matter fermions have to be grouped however in the two irreducible multiplets $\bar{\mathbf{5}}$ and $\mathbf{10}$ and an additional singlet, if one wants to allow for a right-handed neutrino. Therefore, this arrangement obscures the similarities of the quark and lepton sector, when it comes to weak interactions.

The group SO(10) elegantly combines the virtues of G_{PS} and SU(5) by the simple reason that it contains both as subgroups. In addition, all matter fermions can be accommodated within the single irreducible representation $\mathbf{16}$. Therefore, SO(10) is widely considered to be contained within the ultimate GUT group or even to be that group itself. Consequently, we want to realize a coherent baryogenesis scenario in this framework.

In Appendix A, we discuss some issues concerning the construction of SO(10)-invariants and give reference of our normalizations and conventions. The treatment is less technical but goes into more details than previous literature [79–81], and particular emphasis is put on the construction of charge operators and the assignment of Standard Model particles to the representation $\mathbf{16}$. Many explicit expressions are provided which are useful for the construction of the higgsino mass matrix inducing baryogenesis.

5.1 The Barr-Raby Model

A popular way for breaking SO(10) down to the Standard Model uses a Higgs A in the adjoint representation **45** and another pair of Higgses C and $\bar{C}$ in the spinor representations **16** and $\overline{\mathbf{16}}$. For this purpose, Barr and Raby [80] consider a superpotential of the type

$$\begin{aligned} W \supset\ & \kappa S(C\bar{C} - \mu^2) + \frac{\alpha}{4M_S}\mathrm{tr}A^4 + \frac{1}{2}M_A\mathrm{tr}A^2 + T_1AT_2 + M_T T_2^2 \\ & + \bar{C}'[\zeta\frac{PA}{M_S} + \zeta_Z Z_1]C + \bar{C}[\zeta\frac{PA}{M_S} + \zeta_Z Z_1]C' + M_{C'}C'\bar{C}' \,. \end{aligned} \tag{5.1}$$

The additional fields S, P, Z_1, Z_2 are singlets, T_1 and T_2 **10**-plets of SO(10).

Let us in the following discuss the features of this model and begin with the adjoint sector. The potential is at its minimum, when the condition

$$-F_A^* = \frac{\partial W}{\partial A} = 0 \tag{5.2}$$

is met. When $\langle A \rangle = \mathrm{diag}(a_1, a_2, a_3, a_4, a_5) \otimes \mathrm{i}\sigma_2$, it follows

$$\frac{\alpha}{M_S}a_i^3 + M_A a_i = 0\,. \tag{5.3}$$

This can be solved by either $a_i = 0$, or $a_i = a$, where

$$a = \pm\sqrt{\frac{MM_S}{\alpha}}. \tag{5.4}$$

In order to step towards the Standard Model, it is possible to break SO(10) down to the left-right symmetric group

$$G_{LR} = \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L} \tag{5.5}$$

by the choice of the Dimopoulos-Wilczek (DW) form

$$\langle A \rangle = \begin{pmatrix} a & & & & \\ & a & & & \\ & & a & & \\ & & & 0 & \\ & & & & 0 \end{pmatrix} \otimes \mathrm{i}\sigma_2\,. \tag{5.6}$$

Note, that $\langle A \rangle$ being of DW form is proportional to the $(B-L)$ operator given in Eqn. (A.24).

The two Higgs doublets of the MSSM are contained within T_1 and are identified with the four components which remain massless by the superpotential (5.1), when using the DW-form for $\langle A \rangle$. The additional six degrees of freedom of T_1, two colour triplets, become heavy and hence invisible at low energies. The issue to obtain this bias of masses within the **10**-plet is called the doublet-triplet splitting problem, which is hence solved by the DW-mechanism. The second **10**-plet T_2 becomes necessary since a direct mass-term for the triplet components of T_1 would lead to disastrous rapid proton decay.

The Higgses C and $\bar{C}$ reduce the SO(10) symmetry to SU(5). The absolute values of their VEVs are of course μ, and they point in the SU(5)-singlet direction with the quantum numbers of a right-handed neutrino.

Both sectors, the spinorial and the adjoint, in combination reduce the SO(10)-symmetry to the Standard Model group G_{SM}. However, they need to be linked together in order to get a congruency of the assignment of Standard Model quantum numbers and to remove all pseudo-Goldstone modes from the particle spectrum. The obvious candidate term to add to the superpotential, $\bar{C}AC$, however destabilizes the DW form (5.6), by altering the expression for the F-term (5.3) when the spinors get a nonzero VEV. Barr and Raby therefore suggested to add the additional spinors C' and $\bar{C}'$, which get a zero VEV. The conditions for potential minimization now become

$$-F^*_{\bar{C}'} = \left[\zeta \frac{PA}{M_S} + \zeta_Z Z_1\right] C \,, \tag{5.7}$$

$$-F^*_{C'} = \bar{C}\left[\xi \frac{PA}{M_S} + \xi_Z Z_1\right] . \tag{5.8}$$

When comparing with Eqn. (A.24), we note that in DW-form (5.6), we can identify $\langle A \rangle \equiv \frac{3}{2} a(B-L)$. If we assume that the VEV of P is fixed, then Z_1 and Z_2 settle to

$$Z_1 = -\frac{3}{2}\zeta/\zeta_Z \frac{\langle P \rangle a}{M_S} \,, \tag{5.9}$$

$$Z_2 = -\frac{3}{2}\xi/\xi_Z \frac{\langle P \rangle a}{M_S} \,, \tag{5.10}$$

since C and $\bar{C}$ point in the right-handed neutrino direction, where $B-L=1$. We have hence achieved a link between the spinorial and adjoint sector without changing the form of $-F_A$.

5.2 The Higgsino-Gaugino Mass Matrix

In our model, CP-violation will arise from the phase between ζ and ξ and therefore from couplings of the adjoint to the spinor multiplets. Let us label the multiplets of the Standard Model group (4.11) by K. By inspection of tables A.3 and A.4, we see that **16** and **45** harbour as multiplets with common quantum numbers $K = (\mathbf{3}, \mathbf{2}, \frac{1}{6})$, $K = (\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$ and $K = (\mathbf{1}, \mathbf{1}, 1)$. The corresponding conjugate multiplets in $\overline{\mathbf{16}}$ and **45** are labeled by $\bar{K}$. Furthermore, all these representations contain the singlet $(\mathbf{1}, \mathbf{1}, 0)$.

The spinor pair with 32 degrees of freedom breaks the 45-dimensional SO(10) down to 24-dimensional SU(5). The 21 Goldstone modes come from the multiplets $K =$

$(\mathbf{3},\mathbf{2},\frac{1}{6})$, $K = (\bar{\mathbf{3}},\mathbf{1},-\frac{2}{3})$, $K = (\mathbf{1},\mathbf{1},1)$ plus one linear combination of the singlets $K = (\mathbf{1},\mathbf{1},0)$ within $\mathbf{16}$ and $\overline{\mathbf{16}}$. The 45-dimensional adjoint reduces the SO(10)-symmetry to the 15-dimensional G_{LR}. Because of the DW VEV being proportional to the $(B-L)$ operator, the 30 Goldstone modes can be identified with the multiplets for which $B-L \neq 0$, that are all colour triplets.

Hence, in addition to the Pati-Salam model, we have a mixing of the higgsino modes with the gaugino sector, through the Lagrangean terms

$$\sqrt{2} g \varphi^* T^a \psi \lambda^a + \text{h.c.}\,, \tag{5.11}$$

where T^a is a generator of SO(10), λ^a a gaugino and φ the scalar superpartner of the ψ-fermion. These mass terms occur for both chiral multiplets, A and C when $K = (\mathbf{3},\mathbf{2},\frac{1}{6})$ or $K = (\bar{\mathbf{3}},\mathbf{1},-\frac{2}{3})$, and only for C, when $K = (\mathbf{1},\mathbf{1},1)$.

Let us consider possible mass terms involving only the adjoint Higgs. If we denote the components of either $(\mathbf{3},\mathbf{2},\frac{1}{6})$ or $(\bar{\mathbf{3}},\mathbf{1},-\frac{2}{3})$ by b_K, we have

$$\mathrm{tr} A^4 = 6a^4 - 2a^2 b_K b_{\bar{K}} + \frac{1}{4} b_K^2 b_{\bar{K}}^2\,. \tag{5.12}$$

Hence, the portion $\frac{\alpha}{4M_S}\mathrm{tr}A^4 + \frac{1}{2}M_A \mathrm{tr}A^2$ of the superpotential (5.1) gives for these modes a zero mass term

$$m_K = \frac{1}{2}\left(-\frac{a^2}{M_S} + M_A\right) = 0 \qquad \text{for } K = (\mathbf{3},\mathbf{2},\frac{1}{6}) \text{ and } K = (\bar{\mathbf{3}},\mathbf{1},-\frac{2}{3})\,, \tag{5.13}$$

where we have used the VEV (5.4) for a. This result is expected, since the multiplets in question are Goldstone. In contrast, we find

$$m_K = \frac{1}{2}\frac{a^2}{M_S} \qquad \text{for } K = (\mathbf{1},\mathbf{1},1)\,. \tag{5.14}$$

It remains to discuss the mixing of the adjoints and spinors. ψ_{A_K} and $\psi_{C'_{\bar{K}}}$ get a mass term through

$$\frac{\delta^2 W}{\delta A_K \delta C'_{\bar{K}}} = \xi \frac{\langle \bar{C} \rangle \langle P \rangle}{M_S}\,, \tag{5.15}$$

while for the mixing of the spinors we have

$$\frac{\delta^2 W}{\delta C_K \delta C'_{\bar{K}}} = \xi \alpha_K \frac{a \langle P \rangle}{M_S}\,, \tag{5.16}$$

where $\alpha_K = \frac{3}{2}\left[(B-L)_K - 1\right]$ or explicitly,

$$\alpha_K = \begin{cases} -1 & \text{for} \quad K = (\mathbf{3},\mathbf{2},\frac{1}{6}) \\ -2 & \text{for} \quad K = (\bar{\mathbf{3}},\mathbf{1},-\frac{2}{3}) \\ 0 & \text{for} \quad K = (\mathbf{1},\mathbf{1},1) \end{cases} \,. \tag{5.17}$$

We have used here the VEVs (5.9, 5.10) and again the proportionality of $\langle A \rangle$ to the $(B-L)$ operator.

We are now in the position to write down the higgsino-mass matrix:

$$\begin{pmatrix} \psi_{\lambda_K} & \psi_{A_K} & \psi_{C_K} & \psi_{C'_K} \end{pmatrix} \tag{5.18}$$
$$\times \begin{pmatrix} 0 & \gamma_K\sqrt{2}ga & \sqrt{2}g\langle C\rangle & 0 \\ \gamma_K\sqrt{2}ga & m_K & 0 & \xi\frac{\langle\bar{C}\rangle\langle P\rangle}{M_S} \\ \sqrt{2}g\langle\bar{C}\rangle & 0 & \kappa\langle S\rangle & \alpha_K\xi\frac{a\langle P\rangle}{M_S} \\ 0 & \zeta\frac{\langle\bar{C}\rangle\langle P\rangle}{M_S} & \alpha_K\zeta\frac{a\langle P\rangle}{M_S} & M_{C'} \end{pmatrix} \begin{pmatrix} \psi_{\lambda_{\bar{K}}} \\ \psi_{A_{\bar{K}}} \\ \psi_{C_{\bar{K}}} \\ \psi_{C'_{\bar{K}}} \end{pmatrix} + \text{h.c.}\,,$$

where

$$\gamma_K = \begin{cases} 1 & \text{for} \quad K = (\mathbf{3},\mathbf{2},\frac{1}{6}) \\ 2 & \text{for} \quad K = (\bar{\mathbf{3}},\mathbf{1},-\frac{2}{3}) \\ 0 & \text{for} \quad K = (\mathbf{1},\mathbf{1},1) \end{cases} \quad . \tag{5.19}$$

The mass matrix is nonsymmetric and nonhermitian, therefore being endowed with the prerequisites for coherent baryogenesis.

5.3 Simulation of Coherent Baryogenesis

The superpotential (5.1) we wrote down is of the type suitable for hybrid inflation. We assume that symmetry breaking by the adjoint sector has already taken place before or during inflation and is preserved throughout the subsequent history of the Universe, such that possible monopoles are diluted. We therefore do not consider the dynamics of the field A.

As opposed to the Pati-Salam model considered in chapter 4, hybrid inflation is not shifted. During inflation, the VEVs of C and $\bar{C}$ are zero, and S rolls down a logarithmic slope until reaching the critical value

$$S_C = \mu\,, \tag{5.20}$$

such that the waterfall regime begins, bringing coherent baryogenesis along. We simulate this scenario for the parameter $\kappa = 0.01$ and a damping rate $\Gamma = 5\times 10^{-3}\mu$ and plot the result in figure 5.3.

In order to keep the discussion simple, we do not take the dynamics of the singlet fields Z_1 and Z_2 into account here. In principle, their VEVs only get fixed when C and $\bar{C}$ acquire nonzero VEVs. A possible way to fix Z_1 and Z_2 already during inflation is for example to shift the spinors away from the zero VEV, similar as for the Pati-Salam model. We furthermore come short of giving an assignment of R-charges motivating the absence of higher powers of the inflaton field S in the superpotential and neglect the expansion of the Universe during the phase transition.

These simplifications are of course a step back with respect to our discussion of the Pati-Salam model, but the purpose of this chapter is to point out that coherent

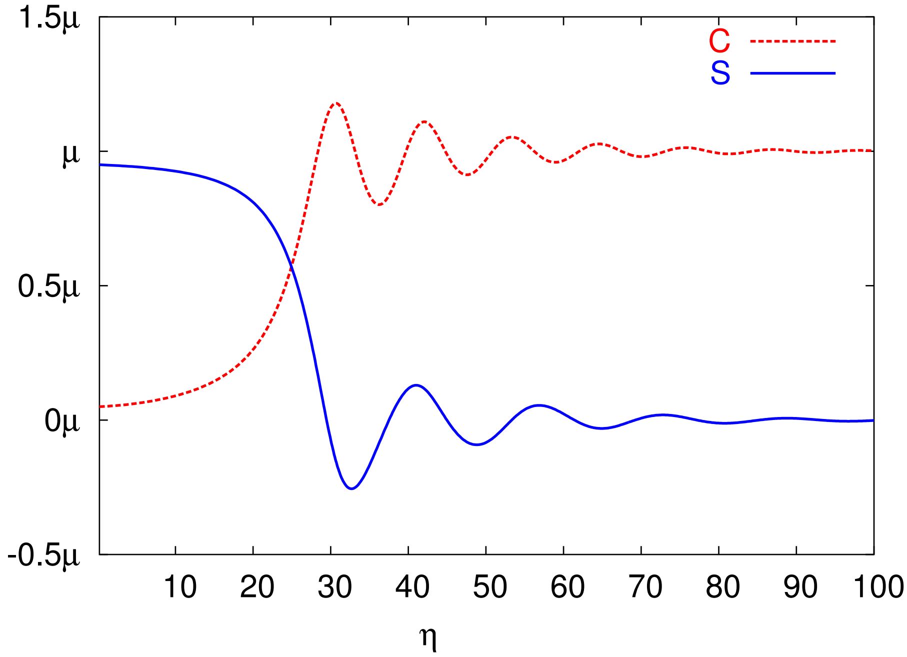

Figure 5.1: Epoch of phase transition in the SO(10)-model

baryogenesis may in principle be operative within SO(10), therefore being a generic mechanism for GUT-baryogenesis, rather than giving an extensive discussion of a plenty of details.

For the remaining parameters, we choose $M_S = 50\mu$, $M_{C'} = 0.01\mu$, $g = 0.2$, $\zeta = -0.8$, $\xi = 0.4 + 0.6\mathrm{i}$, $a = 2.5\mu$ and $\langle P\rangle = 3\mu$.

The charge numbers which are plotted in figures 5.2, 5.3 and 5.4 refer to the final diagonal mass matrix, hence mixing ψ_{λ_K}, ψ_{A_K}, ψ_{C_K} and $\psi_{C'_K}$, and the mass matrix is diagonalized *via* a biunitary transformation (3.92). The produced fermions decay through the ψ_{C_K} and $\psi_{\bar{C}_{\bar{K}}}$ components induced by the couplings

$$\gamma_1 \frac{\bar{C}F\bar{C}F}{M_S}\,, \tag{5.21}$$

$$\gamma_2 \frac{C\Gamma^a F C\Gamma^a F}{M_S}\,, \tag{5.22}$$

$$\gamma_3 \frac{C\Gamma^a\Gamma^b F C\Gamma^a\Gamma^b F}{M_S}\,, \tag{5.23}$$

which are added to the superpotential (5.1), where F are the standard model fermions and the right-handed neutrino, contained in **16**, and Γ denotes the operators defined in (A.6) and (A.7).

We also have to deal with the fact, that for $\alpha_K = 0$, namely for $K = (\mathbf{1}, \mathbf{1}, 1)$, ψ_{C_K} and $\psi_{C'_K}$ do not mix, *cf.* the mass matrix (5.18). Therefore we assume that also the fields C' and $\bar{C}'$ may decay through couplings of the above type, suppressed however by additional powers of $\langle R\rangle/M_S$, where R is some singlet with a VEV.

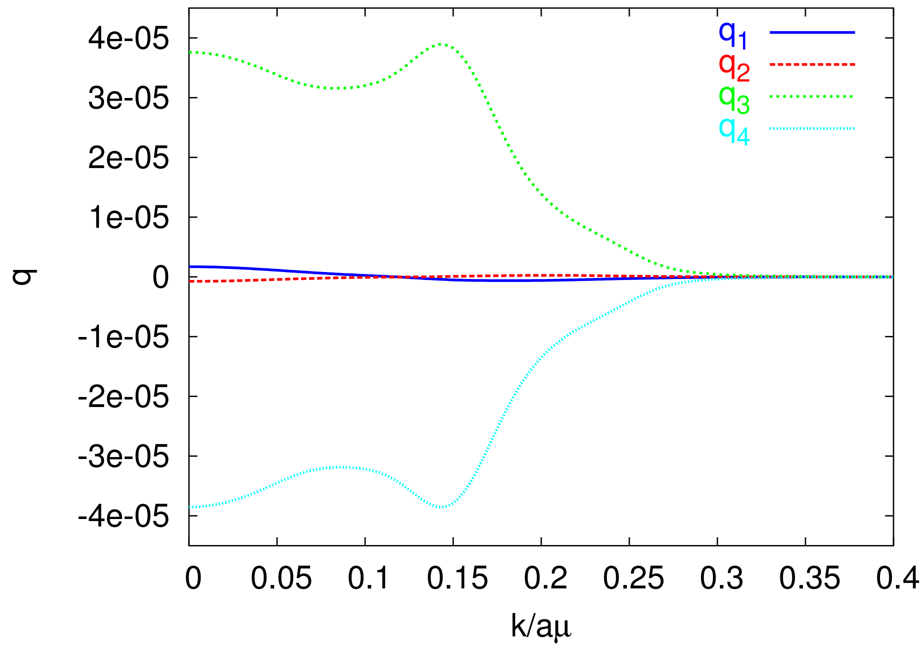

Figure 5.2: The produced charges for the multiplet $(\mathbf{3}, \mathbf{2}, \frac{1}{6})$.

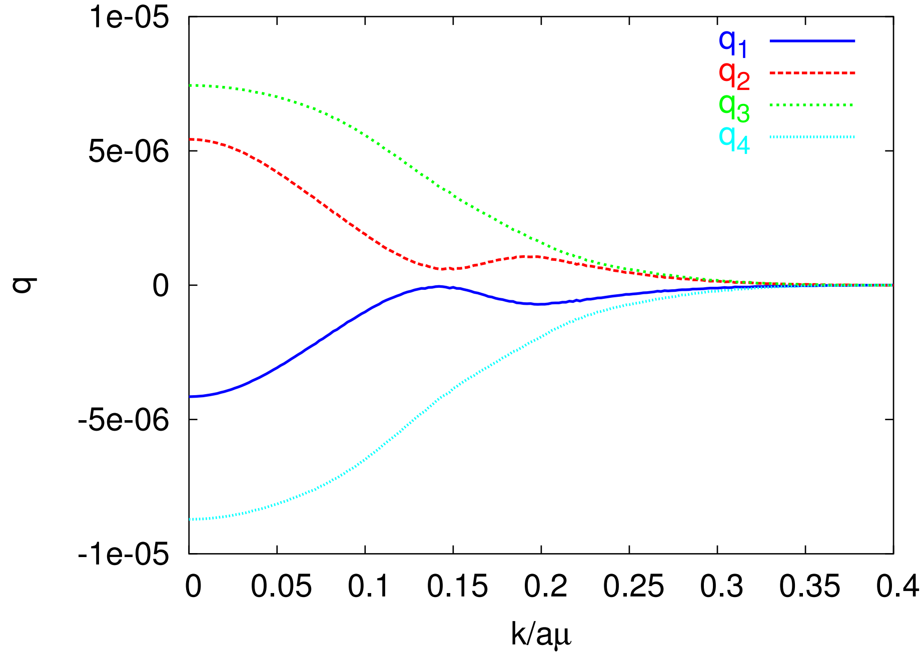

Figure 5.3: The produced charges for the multiplet $(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$.

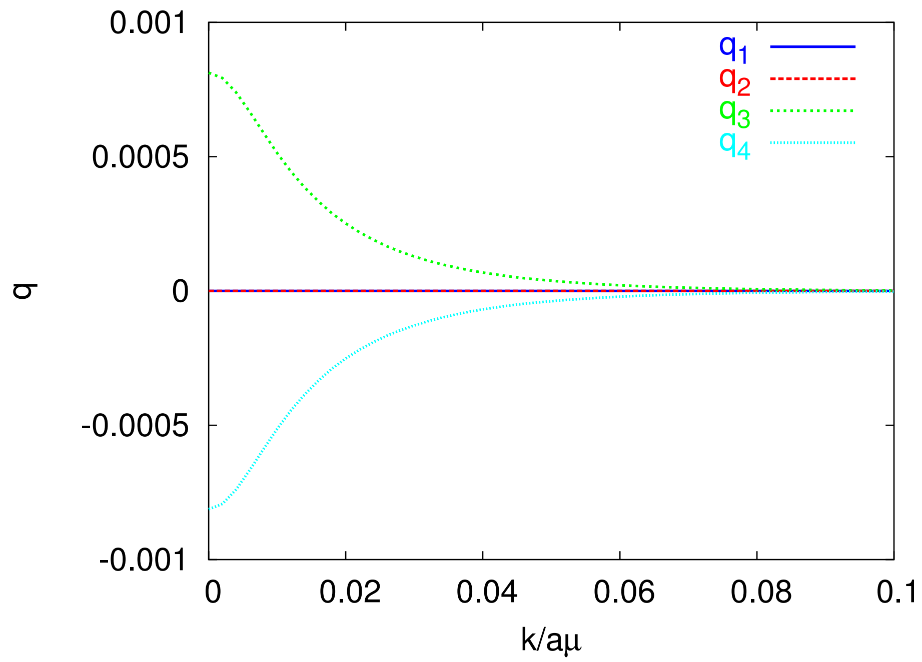

Figure 5.4: The produced charges for the multiplet $(\mathbf{1},\mathbf{1},1)$.

The first coupling (5.21) obviously allows for the decay reaction

$$\psi_{\bar{C}_{\bar{K}}} \longrightarrow F_K^* + \nu^{c*}\,, \tag{5.24}$$

where one of the particles on the right hand side is a scalar, the other one a fermion. Hence, the charges get transformed to

$$(B-L) = -\frac{1}{3}q_{\bar{C}_{\bar{K}}}\,, \quad \bar{K} = (\bar{\mathbf{3}},\mathbf{2},-\frac{1}{6})\,, \tag{5.25}$$

$$(B-L) = \frac{1}{3}q_{\bar{C}_{\bar{K}}}\,, \quad \bar{K} = (\mathbf{3},\mathbf{1},\frac{2}{3})\,, \tag{5.26}$$

$$(B-L) = -q_{\bar{C}_{\bar{K}}}\,, \quad \bar{K} = (\mathbf{1},\mathbf{1},1)\,. \tag{5.27}$$

We can calculate the term (5.22) $\propto \gamma_2$ using the techniques explained in appendix A.2. It is however easier to note that (*cf.* table A.2)

$$(\mathbf{3},\mathbf{2},\frac{1}{6}) \otimes (\bar{\mathbf{3}},\mathbf{1},\frac{1}{3}) \supset (\mathbf{1},\mathbf{2},\frac{1}{2}) \subset \mathbf{10}\,, \tag{5.28}$$

as well as

$$(\mathbf{1},\mathbf{1},0) \otimes (\mathbf{1},\mathbf{2},-\frac{1}{2}) = (\mathbf{1},\mathbf{2},-\frac{1}{2}) \subset \mathbf{10}\,. \tag{5.29}$$

The components of ψ_{C_K} for $K = (\mathbf{3},\mathbf{2},\frac{1}{6})$ therefore decay as

$$\psi_{C_d} \longrightarrow d^{c*} + e^*\,, \tag{5.30}$$

$$\psi_{C_u} \longrightarrow d^{c*} + \nu^*\,, \tag{5.31}$$

where $\psi_{\bar{C}_d}$ denotes the d-quark like higgsino, $\psi_{\bar{C}_u}$ the u-quark like one. The charges hence get transformed to

$$(B-L) = \frac{4}{3} q_{C_K}\,, \text{ for } K = (\mathbf{3}, \mathbf{2}, \frac{1}{6})\,. \tag{5.32}$$

The u^c-quark like higgsino with $K = (\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$ decays through the γ_3-coupling (5.23). We note (*cf.* table A.5)

$$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3}) \otimes (\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3}) = (\mathbf{3}, \mathbf{1}, -\frac{1}{3}) \oplus (\bar{\mathbf{6}}, \mathbf{1}, -\frac{1}{3}) \subset \mathbf{120}\,, \tag{5.33}$$

$$(\mathbf{1}, \mathbf{1}, 0) \otimes (\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3}) = (\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3}) \subset \mathbf{120}\,, \tag{5.34}$$

and therefore have the reaction

$$\psi_{C_{u^c}} \longrightarrow d^{c*} + d^{c*}\,, \tag{5.35}$$

and the charge conversion

$$(B-L) = \frac{2}{3} q_{C_K}\,, \text{ for } K = (\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})\,. \tag{5.36}$$

Finally, the e^c like higgsino $K = (\mathbf{1}, \mathbf{1}, 1)$ turns into matter *via* the γ_2-coupling (5.22), as can be seen by (*cf.* table A.2)

$$(\mathbf{1}, \mathbf{1}, 1) \otimes (\mathbf{1}, \mathbf{2}, -\frac{1}{2}) = (\mathbf{1}, \mathbf{2}, \frac{1}{2}) \subset \mathbf{10}\,, \tag{5.37}$$

$$(\mathbf{1}, \mathbf{1}, 0) \otimes (\mathbf{1}, \mathbf{2}, -\frac{1}{2}) = (\mathbf{1}, \mathbf{2}, -\frac{1}{2}) \subset \mathbf{10}\,. \tag{5.38}$$

Consequently, the decay reaction is

$$\psi_{C'_{e^c}} \longrightarrow e^* + \nu^*\,, \tag{5.39}$$

and the resulting asymmetry

$$(B-L) = 2 q_{C_K}\,, \text{ for } K = (\mathbf{1}, \mathbf{1}, 1)\,. \tag{5.40}$$

The numerical results of the simulation are given in tables 5.1, 5.2 and 5.3. In the second and third column are the squared coefficients when expanding the mass-diagonal basis of Dirac fermions in terms of the original basis of the higgsino mass matrix (5.18). Both bases are related by a biunitary transformation (3.92). The ratio between both columns gives the branching ratios for the decays. In the fourth column are the charge densities, that are the integrals $q_i = \int dk k^2/(2\pi^2) q_i(k)$, *cf.* figures 5.2, 5.3 and 5.4, where the scale factor is $a = 20$. The fourth column contains the $(B-L)$-charges after decay into Standard Model fermions as calculated above, and finally the sum over all decay channels, which we denote by $n^0_{(\mathbf{3},\mathbf{2},\frac{1}{6})}$ and $n^0_{(\bar{\mathbf{3}},\mathbf{1},-\frac{2}{3})}$, respectively.

Table 5.1: Numerical results for the multiplet $(\mathbf{3},\mathbf{2},\frac{1}{6})$.

Charge	contrib. from ψ_{C_K}	contrib. from $\bar\psi_{\bar C_{\bar K}}$	charge produced	final $(B-L)$
q_1	6.8×10^{-2}	6.8×10^{-2}	$-1.4\times10^{-10}\mu^3$	$1.2\times10^{-10}\mu^3$
q_2	6.8×10^{-2}	6.8×10^{-2}	$5.1\times10^{-11}\mu^3$	$4.3\times10^{-11}\mu^3$
q_3	1.0×10^{-1}	7.7×10^{-1}	$5.3\times10^{-9}\mu^3$	$2.4\times10^{-9}\mu^3$
q_4	7.6×10^{-1}	8.2×10^{-2}	$-5.2\times10^{-9}\mu^3$	$-6.4\times10^{-9}\mu^3$
				$-3.9\times10^{-9}\mu^3$

Table 5.2: Numerical results for the multiplet $(\bar{\mathbf{3}},\mathbf{1},-\frac{2}{3})$.

Charge	contrib. from ψ_{C_K}	contrib. from $\bar\psi_{\bar C_{\bar K}}$	charge produced	final $(B-L)$
q_1	8.0×10^{-1}	2.9×10^{-2}	$-2.0\times10^{-10}\mu^3$	$-1.3\times10^{-10}\mu^3$
q_2	3.6×10^{-2}	8.1×10^{-1}	3.2×10^{-10}	$9.1\times10^{-11}\mu^3$
q_3	1.5×10^{-1}	5.7×10^{-4}	$6.7\times10^{-10}\mu^3$	$4.5\times10^{-10}\mu^3$
q_4	5.7×10^{-4}	1.5×10^{-1}	$-7.9\times10^{-10}\mu^3$	$2.6\times10^{-10}\mu^3$
				$6.7\times10^{-10}\mu^3$

Note, that for $K=(\mathbf{1},\mathbf{1},1)$ the mass matrix (5.18) is block-diagonal, such that only the pairs ψ_{λ_K}-ψ_{C_K} and ψ_{A_K}-$\psi_{C'_K}$ are mixed. Only for the second pair, the CP-violating parameters ξ and ζ are relevant and an asymmetry is generated,which vanishes however after the decay into matter.

The total $(B-L)$ number density produced at the phase transition is by the multiplicity of colour and flavour given by

$$n^0_{B-L} = 6n^0_{(\mathbf{3},\mathbf{2},\frac{1}{6})} + 3n^0_{(\bar{\mathbf{3}},\mathbf{1},-\frac{2}{3})} = -2.1\times10^{-8}\mu^3\,. \tag{5.41}$$

The value for the vacuum energy at the end of inflation is $V_0=\kappa^2\mu^4$, and by Eqn. (4.41), we find[1]

$$\frac{n_B}{s} = 0.25\frac{n^0_{B-L}}{V_0}T_R = -1.1\times10^{-10}\,, \tag{5.42}$$

where we have chosen $\mu=10^{15}$GeV and $T_R=2\times10^9$GeV for this example.

The wrong sign could be removed by switching the CP-violating phase, $\zeta\leftrightarrow\xi^*$, or different initial conditions. After all, a universe with more antimatter than matter would be as good as ours. The overall magnitude of the asymmetry is however

[1]Note, that the quantity n^0_{B-L} here already contains the colour and flavour multiplicity, thereby the difference of a factor of *three*.

Table 5.3: Numerical results for the multiplet $(\mathbf{1}, \mathbf{1}, 1)$.

Charge	contrib. from $\psi_{C'_K}$	contrib. from $\bar{\psi}_{\bar{C}'_{\bar{K}}}$	charge produced	final $(B-L)$
q_1	0	0	0	0
q_2	0	0	0	0
q_3	5.0×10^{-1}	5.0×10^{-1}	$3.7 \times 10^{-10}\mu^3$	$5.5 \times 10^{-10}\mu^3$
q_4	5.0×10^{-1}	5.0×10^{-1}	$-3.7 \times 10^{-10}\mu^3$	$-5.5 \times 10^{-10}\mu^3$
				0

in accordance with observation and indicates that supersymmetric SO(10) coherent baryogenesis is a viable cosmological scenario.

Chapter 6

The Unruh Detector

6.1 Nonadiabatic and Adiabatic Particle Production

In the coherent baryogenesis scenario, particle production occurs due to a strongly time-dependent mass term. Initial conditions are however fixed at a time when the mass term is constant or only slowly varying, and the same is true for the time when the final particle number is evaluated. Hence, for both, initial and final time, the solutions to the field equations are asymptotically identical to plane-wave solutions in Minkowski space with constant mass term. The difference between initial and final mode functions is, that there is a mixing of positive and negative frequency solutions, which can be removed by a Bogolyubov transformation.

In the expanding Universe however, we are interested in effects at times when expansion is still going on, that is when the parameter $\mathbf{k}^2 + a^2m^2 + (6\xi - 1)\frac{a''}{a}$ in the Klein-Gordon equation (2.21) is yet varying. We have seen that an approximate solution to the mode functions can be found using the WKB-*ansatz* (2.30) and adiabatically expanding the function $W(\mathbf{k}, \eta)$. When adiabatic expansion is suitable, this means in turn that there is obviously no mixing of positive and negative frequency solutions to the mode functions, in contrast to preheating and coherent baryogenesis.

Therefore, we can refine terminology. We talk of *nonadiabatic* particle production, when we cannot expand $W(\mathbf{k}, \eta)$ by a convergent power series in η, while in the other case particle production is called *adiabatic*.

There is an instructive example due to Ford [82], which serves us here to illustrate this distinction. Consider first a massless minimally coupled scalar field in de Sitter space (2.16), where the solution to the Klein-Gordon equation (2.21) is given by Eqn. (3.32)

$$\varphi^{dS}(\mathbf{k}, \eta) = \frac{1}{\sqrt{2k}}\Big(1 - \frac{\mathrm{i}}{k\eta}\Big)\mathrm{e}^{-\mathrm{i}k\eta}\,.$$

This is a purely negative-frequency solution, leading by the method of Bogolyubov transformation to the particle number (3.34) $n(\mathbf{k}) = (aH)^2/(2k)^2$. The calculation of the energy density $\varrho_E = \int d^3k/(2\pi)^3 kn(\mathbf{k})$ leads to a square divergence, in accordance with our discussion of the stress-energy tensor in chapter 2. Note however, that the parameter β of the Bogolyubov transformation is oscillating as $\propto \mathrm{e}^{-2\mathrm{i}\omega}$, in contrast to the case where positive- and negative frequency modes mix and β is evolving adiabatically slow.

Now consider an instantaneous transition to radiation era (2.15). For the transition, we impose equality of the scale factors and their first derivatives, or the Hubble rates, alternatively. There will however be a kink in the first derivative of the scale factor at the moment of the transition, such that the second derivative is ill-defined and adiabatic expansion breaks down. The mode functions in the radiation Universe are

$$\varphi^R(\mathbf{k}, \eta) = \frac{1}{\sqrt{2k}} \mathrm{e}^{-\mathrm{i}k\eta}, \tag{6.1}$$

also being of purely negative frequency. We can now match the de Sitter mode functions to those in radiation by imposing equality of the amplitudes and the first derivatives at the transition:

$$\begin{aligned} \varphi^{dS} &= \alpha\varphi^R + \beta\varphi^{R*}, \\ \frac{d}{d\eta}\varphi^{dS} &= \frac{d}{d\eta}\left[\alpha\varphi^R + \beta\varphi^{R*}\right]. \end{aligned} \tag{6.2}$$

Then, we find for the matching coefficients

$$\alpha = 1 + \mathrm{i}\frac{H}{k} - \frac{H^2}{2k^2}, \qquad \beta = \frac{H^2}{2k^2}\mathrm{e}^{-2\mathrm{i}k/H} \tag{6.3}$$

and check that the normalization condition $|\alpha|^2 - |\beta|^2 = 1$ is satisfied. This implies a particle number

$$n(\mathbf{k}) = |\beta(\mathbf{k})|^2 = \frac{H^4}{4k^4} \tag{6.4}$$

and therefore a logarithmic divergence in the energy density ϱ_E, which can however be cured by smoothing the transition. We have therefore found an example with positive negative frequency mixing or particle production, which is nonadiabatic by the fact that the matching conditions (6.2) do not allow for adiabatic expansion since the mode function has no second derivative. This result does however not imply, that the contribution from adiabatic particle production vanishes, which is still present.

Hence, there might be substantial differences between nonadiabatic and adiabatic particle production, which therefore also could be differently perceived by an observer. This leads to the question, what an idealized detector would measure, as we shall discuss in the following.

6.2 Unruh Detector and its Response in Flat Spacetime

We consider Unruh's detector [13], a heavy particle with discrete energy levels, which moves along a trajectory $x = x(\tau)$, where τ is its proper time. The Hamiltonian of the detector is given by $H = H_0 + \delta H$, where H_0 is the unperturbed (time independent) Hamiltonian and δH accounts for the interaction with the scalar field ϕ, which we assume to be in a state $|i\rangle$. While we treat the detector by the means of nonrelativistic quantum mechanics, the nature of ϕ as a quantum field is of importance. The situation is therefore very similar to absorption and emission of photons by an atom, a discussion of which can be found in any textbook on quantum mechanics.

Spacetimes of special interest are Minkowski space, de Sitter space, and Rindler space [83]. In all of these cases time translation invariance holds, such that one can expect the detector to equilibrate with the background. We therefore recapitulate the derivation of the detector response in spacetimes endowed with this special symmetry [13, 15, 23, 84, 85].

Let us first define the set of unperturbed eigenstates of the detector by

$$|m^0, \tau\rangle = \mathrm{e}^{-\mathrm{i}H_0\tau}|m^0\rangle = \mathrm{e}^{-\mathrm{i}E_m\tau}|m^0\rangle, \tag{6.5}$$

which can be combined to nondiagonal transition amplitudes through the interaction Hamiltonian

$$\delta H = \hat{h}\phi(x). \tag{6.6}$$

The operator $\hat{h}$ is a quantum mechanical operator determined by the inner structure of the detector, which can be thought of as a bound state, and has the elements $h_{mn} = \langle E_n|\hat{h}|E_m\rangle$, while ϕ is a quantum field operator to be expanded in modes suitable for the given background spacetime.

Initially, at time τ_0, the detector is in the state $|m^0, \tau_0\rangle$, which evolves under the action of the full Hamiltonian H into $|m, \tau\rangle$ at some time τ. We want to determine the amplitude for exciting the detector from $E_m \to E_n$, hence

$$\mathcal{M}_{mn} = \langle n^0, \tau|m, \tau\rangle = \langle n^0|\mathrm{e}^{\mathrm{i}H_0\tau}|m, \tau\rangle = \langle n^0|m, \tau\rangle_I, \tag{6.7}$$

where the interaction state is given to first order in the von Neumann series as

$$|m, \tau\rangle_I = |m^0\rangle - \mathrm{i}\int_{\tau_0}^{\tau} d\tau' \mathrm{e}^{\mathrm{i}H_0\tau'}\delta H(\tau')\mathrm{e}^{-\mathrm{i}H_0\tau'}|m^0\rangle\,, \tag{6.8}$$

and $|n^0\rangle \equiv |n^0, \tau = 0\rangle$, $|m^0\rangle \equiv |m^0, \tau = 0\rangle$. We find

$$\mathcal{M}_{mn} = \delta_{mn} - \mathrm{i}\int_{\tau_0}^{\tau} d\tau' \mathrm{e}^{\mathrm{i}(E_n - E_m)\tau'}\langle n^0|\delta H(\tau')|m^0\rangle\,, \tag{6.9}$$

which, upon inserting (6.6), reads

$$\mathcal{M}_{mn} = \sum_f \langle f|\delta_{mn} - \mathrm{i}\int_{\tau_0}^{\tau} d\tau' \mathrm{e}^{\mathrm{i}(E_n - E_m)\tau'} h_{mn}\phi\left(x(\tau')\right)|i\rangle\,. \tag{6.10}$$

Here the scalar field has undergone a transition from $|i\rangle$ to some element $|f\rangle$ of an orthonormal set of final states, which we summed over. The probability of a transition from $E_m \to E_n$, where $n \neq m$ is hence

$$\mathcal{P}_{mn} = |\mathcal{M}_{mn}|^2 = \sum_f \left| \langle f| \int_{\tau_0}^{\tau} d\tau' \mathrm{e}^{\mathrm{i}(E_n - E_m)\tau'} h_{mn} \phi\left(x(\tau')\right) |i\rangle \right|^2 . \tag{6.11}$$

We sum over the basis f and obtain

$$\mathcal{P}_{mn} = |h_{mn}|^2 \int_{\tau_0}^{\tau} d\tau' \int_{\tau_0}^{\tau} d\tau'' \mathrm{e}^{\mathrm{i}(E_n - E_m)(\tau' - \tau'')} \langle i|\phi\left(x(\tau'')\right) \phi\left(x(\tau')\right) |i\rangle. \tag{6.12}$$

Setting $\tau_0 = 0$, $\Delta E = E_m - E_n$, taking the derivative *w.r.t.* τ and sending $\tau \to \infty$ gives the following result for the response function $\mathcal{F} \equiv \mathcal{P}_{mn}/|h_{mn}|^2$ [85],

$$\frac{d\mathcal{F}(\Delta E)}{d\tau} = \int_{-\infty}^{\infty} d\Delta\tau \mathrm{e}^{\mathrm{i}\Delta E \Delta\tau} \langle i|\phi\left(x(-\Delta\tau/2)\right) \phi\left(x(\Delta\tau/2)\right) |i\rangle , \tag{6.13}$$

where we have assumed that the state of the scalar field respects time translation invariance.

The ten symmetries of Minkowski and de Sitter, three boosts, three rotations, three spatial and one temporal translation, become manifest when the scalar propagator depends just on the geodesic distance between the two points, where it is evaluated. It is therefore useful to rewrite the latter equation as

$$\frac{d\mathcal{F}(\Delta E)}{d\tau} = \int_{-\infty}^{\infty} d\Delta\tau \mathrm{e}^{\mathrm{i}\Delta E \Delta\tau} \mathrm{i}G^{<}\left(x\left(\tau + \frac{\Delta\tau}{2}\right), x\left((\tau - \frac{\Delta\tau}{2}\right)\right) , \tag{6.14}$$

where $\mathrm{i}G^{<}$ is the Wightman function (3.1). From the Wigner function (3.2), one can calculate the response for a detector at rest in Minkowski space,

$$\frac{d\mathcal{F}(\Delta E)}{d\tau} = \int \frac{d^3k}{(2\pi)^3} \mathrm{i}G^{<}(k^0 = \Delta E, \mathbf{k}, x(\tau)) . \tag{6.15}$$

This means that the response function of Unruh's detector is completely insensitive to particle momenta, which is consistent with the assumption that the detector must be very massive, and it absorbs scalar particles of all possible momenta $\mathbf{k}$; likewise, it isotropically emits particles of all momenta.

Now consider Minkowski space filled with particles of the spectrum $|i\rangle \equiv \prod_{\mathbf{k}} \otimes |\nu(|\mathbf{k}|)\rangle$ and mass m, where $\nu(|\mathbf{k}|)$ denotes the particle number per mode, which we assume to be isotropic. We expand the field operator as in Eqn. (2.20), and take $a \equiv 1$, $\eta \equiv \tau$ in flat spacetime.

Making use of the decomposition (2.20), where $\varphi(\mathbf{k}, \tau) = \left(2\omega(|\mathbf{k}|)\right)^{-1/2} \mathrm{e}^{-\mathrm{i}\omega(|\mathbf{k}|)\tau}$, one finds for the response function (6.13)

$$\frac{d\mathcal{F}_{\mathrm{flat}}(\Delta E)}{d\tau} = \frac{k_{\Delta E}}{2\pi} \left[\nu(k_{\Delta E})\vartheta(\Delta E) + \left(\nu(k_{\Delta E}) + 1\right) \vartheta(-\Delta E)\right] , \tag{6.16}$$

with $k_{\Delta E} \equiv \sqrt{(\Delta E)^2 - m^2}$, and the ϑ-function is defined by $\vartheta(x) = 0$ for $x < 0$ and $\vartheta(x) = 1$ for $x \geq 0$. The first term in the square brackets describes particle absorption, induced by the positive frequency part of the scalar field, the second accounts for spontaneous and stimulated emission, due to the negative frequency contribution. This result could of course also be derived by setting $|i\rangle = |0\rangle$, but instead using the Bogolyubov transformed basis of mode functions

$$\varphi(\mathbf{k}, t) = \frac{1}{\sqrt{2\omega(\mathbf{k})}} \left(\alpha(\mathbf{k}) \mathrm{e}^{-\mathrm{i}\omega(\mathbf{k})\tau} + \beta(\mathbf{k}) \mathrm{e}^{\mathrm{i}\omega(\mathbf{k})\tau} \right), \tag{6.17}$$

with $\omega(\mathbf{k}) = \sqrt{\mathbf{k}^2 + m^2}$, $|\beta(\mathbf{k})|^2 = \nu(|\mathbf{k}|)$ and $|\alpha(\mathbf{k})|^2 - |\beta(\mathbf{k})|^2 = 1$. When compared to Eqn. (6.16), an additional term $2\pi\delta(\Delta E) \int d^3k/(2\pi)^3 \, \Re(\alpha\beta^*)/\omega$ arises, which can be imposed to vanish by choosing the phases of the Bogolyubov coefficients such that, upon integration, they average to zero. This example hence illustrates explicitly how a nonzero particle number can be represented by mode mixing.

6.3 Thermal Response

In order to get an insight in how a certain distribution function of a scalar field is perceived by the detector, we consider the response function for the Bose-Einstein distribution. From the Wightman function for a thermally excited scalar field (3.35), the response function (6.15) for d dimensions in flat spacetime is easily calculated,

$$\frac{d\mathcal{F}_{\mathtt{th},d}(\Delta E)}{d\tau} = \frac{2^{2-d}\pi^{\frac{3-d}{2}}}{\Gamma(\frac{d-1}{2})} \mathrm{sign}(\Delta E)\vartheta\left((\Delta E)^2 - m^2\right)\left((\Delta E)^2 - m^2\right)^{\frac{d-3}{2}} \frac{1}{\mathrm{e}^{\beta\Delta E} - 1}, \tag{6.18}$$

and explicitly, for $d = 4$, one finds

$$\frac{d\mathcal{F}_{\mathtt{th},d=4}(\Delta E)}{d\tau} = \frac{\mathrm{sign}(\Delta E)\vartheta\left((\Delta E)^2 - m^2\right)\sqrt{(\Delta E)^2 - m^2}}{2\pi} \frac{1}{\mathrm{e}^{\beta\Delta E} - 1}. \tag{6.19}$$

Hence, the response function of Unruh's detector to a scalar thermal state contains, apart from the Bose-Einstein distribution, an additional factor, which depends on the scalar particle mass and reduces to $\Delta E/(2\pi)$ in the massless limit and $d = 4$.

6.4 Unruh Detector in de Sitter Space

Given the very reasonable result for flat space (6.16), which indicates that Unruh's detector responds to a scalar field similar as an atom to the electromagnetic field does, it is assumed that the response function (6.14) also describes accurately the behaviour in curved spacetimes [13, 15, 23, 84, 85]. In particular, it is commonly accepted, that in de Sitter space, a freely falling observer, corresponding to an Unruh detector, perceives radiation with a thermal spectrum of the de Sitter temperature $T_H = H/(2\pi)$ [14, 15], where H denotes the Hubble parameter. In the following, we want to clarify in what sense this result is universal to scalar fields of different couplings to the de Sitter background and how the detector apprehends the differences.

Let us therefore refine what is meant by the observation of thermal radiation: The detector response function (6.14) is proportional to the Fourier transform of the scalar propagator *w.r.t.* the proper time of the detector, and it describes how many particles are absorbed and emitted per unit time. When being in equilibrium with the de Sitter background, the energy levels of the detector are thermally populated, according to the temperature T_H. As we discuss below, this definition of thermality allows for different response functions.

The fact that in de Sitter space the invariance of the quantum vacuum becomes manifest when the scalar propagator only depends on the proper time separation along a geodesic has led to the practice of defining the de Sitter vacua through this quantity [18, 21, 86–88]. However, in the case of a massless scalar, which is minimally coupled to the curvature, this leads to a problem since the propagator is infrared divergent [87]. We argue that, when regulated by a cutoff, this divergence gives rise to a contribution which is irrelevant to the total detector response.

It is often stated as a simple argument for de Sitter space being thermal that the propagator for scalar fields has in the imaginary direction of proper time τ the periodicity $\tau \to \tau + 2\pi\mathrm{i}/T_H$ [14,22,84], just as for a canonical ensemble at temperature T_H [43]. The rate turns out to depend on the scalar mass and on its coupling to the curvature, as was first shown in Ref. [19], circumventing the use of the scalar propagator.

We first consider the responses to a scalar field ϕ in four-dimensional de Sitter background (2.16), as discussed in chapter 2. The special cases of interest are:

- conformally coupled massless scalar field ($\xi = 1/6$, $m = 0$), for which a conformally rescaled scalar $\varphi \equiv a\Phi$ satisfies the simple differential equation ($\vec{\partial}$ denotes a spatial derivative)
$$(\partial_\eta^2 - \vec{\partial}^2)\varphi(x) = 0\,; \tag{6.20}$$

- nearly minimally coupled light scalar ($|\xi| \ll 1$, $m \ll H$), which obeys
$$\Big(\partial_\eta^2 - \vec{\partial}^2 - \frac{1}{a}\frac{d^2 a}{d\eta^2} + a^2(m^2 + \xi\mathcal{R})\Big)\varphi(x) = 0\,; \tag{6.21}$$

- minimally coupled massless scalar ($\xi = 0$, $m = 0$), which satisfies the following differential equation:
$$\Big(\partial_\eta^2 - \vec{\partial}^2 - \frac{1}{a}\frac{d^2 a}{d\eta^2}\Big)\varphi(x) = 0\,. \tag{6.22}$$

6.4.1 Conformal Vacuum in de Sitter Space

We now calculate the response function (6.13) for a conformally coupled massless scalar field (6.20) ($\xi = 1/6$, $m = 0$), for which the de Sitter invariant Green function during inflation in $d = 4$ reads [15, 21]

$$\mathrm{i}G_{\rm conf}(y) = \frac{H^2}{4\pi^2}\frac{1}{y}\,, \tag{6.23}$$

where y denotes the de Sitter length function

$$y = -\frac{\Delta x^2}{\eta_1\eta_2} \equiv 4\sin^2\Big(\frac{1}{2}H\ell\Big), \tag{6.24}$$

which is related to the geodesic distance ℓ as indicated, and $\Delta x^2 = (\eta_1-\eta_2)^2 - \|\mathbf{x}_1-\mathbf{x}_2\|^2$. The points $x = (\eta_i, \mathbf{x}_i)$ are represented in conformal coordinates, and $\|\cdot\|$ denotes the euclidean norm. For an observer moving along a geodesic, $y = y\big(x(\tau+\Delta\tau/2); x(\tau-\Delta\tau/2)\big) = -4\sinh^2\big(H\Delta\tau/2\big)$.

The response function (6.13) for the conformal vacuum (6.23) can then be written as

$$\frac{d\mathcal{F}_{\rm conf}(\Delta E)}{d\tau} = -\frac{H}{4\pi^2}\int_{-\infty}^{\infty} du\, \mathrm{e}^{\mathrm{i}\Delta E u/H}\frac{1}{4\big[\sinh\big(u/2\big)+\mathrm{i}\varepsilon\big]^2}\,, \tag{6.25}$$

where $u = H\Delta\tau$ and the pole prescription corresponds to that of the Wightman function $\mathrm{i}G^{<}$. This integral can be easily performed by contour integration. The (double) poles (which also correspond to the *zeros* of y) all lie on the imaginary axis, $u_n = H\tau_n = 2\pi\mathrm{i}n$ ($n \,\epsilon\, \mathbb{Z}$). For $E > E_0$ the contour of integration ought to be closed by a large circle above the real axis, such that the integral in (6.25) can be evaluated by summing the residua which lie (strictly) above the real axis, as illustrated in figure 6.1. The result is

$$\begin{aligned}\frac{d\mathcal{F}_{\rm conf}(\Delta E)}{d\tau} &= -\frac{H}{4\pi^2}(2\pi\mathrm{i})\sum_{n=1}^{\infty}\frac{\mathrm{i}\Delta E}{H}\mathrm{e}^{-(2\pi\Delta E/H)n}\\ &= \frac{\Delta E}{2\pi}\frac{1}{\mathrm{e}^{(2\pi/H)\Delta E}-1}\,,\end{aligned} \tag{6.26}$$

which is identical to the response function (6.19) of the thermal Bose-Einstein distribution for massless scalars, confirming thus the well known result [15]. For $\Delta E < 0$, the contour should be closed below the real axis, such that the contributing poles are $n \le 0$, also shown in figure 6.1. The result of integration is again given by Eqn. (6.26).

6.4.2 Nearly Minimally Coupled Light Scalar

The Green function for a massive scalar field minimally coupled to gravity is given by the Chernikov-Tagirov [18] (Bunch-Davies [21]) vacuum

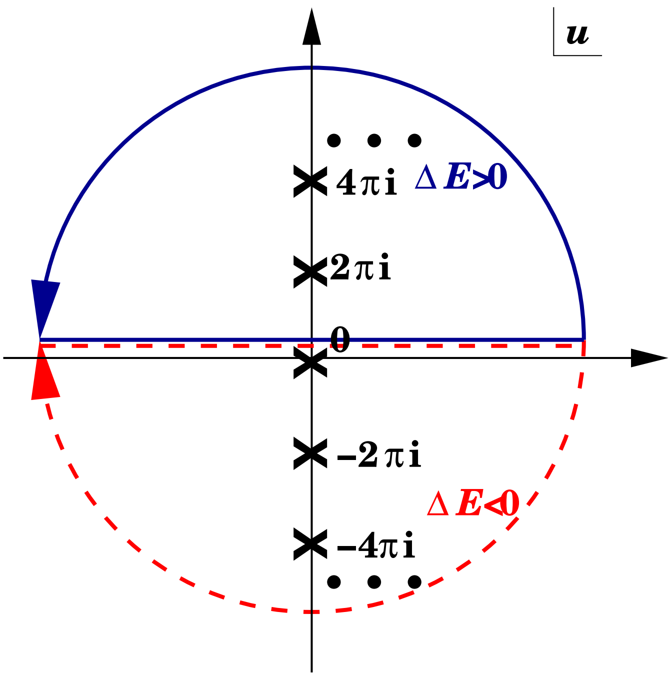

Figure 6.1: The integration contour for the Unruh's detector response function in conformal vacuum. The solid (blue) contour corresponds to $\Delta E > 0$; the dashed (red) contour to $\Delta E < 0$.

$$
\mathrm{i}G(y) = \frac{H^2}{4\pi^2}\Gamma\Big(\frac{3}{2}-\nu\Big)\Gamma\Big(\frac{3}{2}+\nu\Big)\,{}_2F_1\Big(\frac{3}{2}-\nu,\frac{3}{2}+\nu,2;1-\frac{y}{4}\Big)\,, \tag{6.27}
$$

where

$$
\nu = \sqrt{\Big(\frac{3}{2}\Big)^2 - \frac{m^2+12\xi H^2}{H^2}}\,. \tag{6.28}
$$

The uniqueness of $\mathrm{i}G(y)$ follows from the requirement that the lightcone singularity is of the Hadamard form.

When expanded in powers of

$$
\mathtt{s} \;\equiv\; \frac{3}{2}-\nu = \frac{m^2}{3H^2}+4\xi+O\Big(\big[(m^2/H^2)+12\xi\big]^2\Big)\,, \qquad |\mathtt{s}| \ll 1\,, \tag{6.29}
$$

the Green function for the Chernikov-Tagirov vacuum reduces to the following simple form [89]

$$
\mathrm{i}G(y;\mathtt{s}) = \frac{H^2}{4\pi^2}\bigg\{\frac{1}{y}-\frac{1}{2}\log(y)+\frac{1}{2\mathtt{s}}-1+\log(2)+O(\mathtt{s})\bigg\}\,. \tag{6.30}
$$

The nontrivial new integral comes from the term $\mathrm{i}G^{<}_{m=0} \propto \log(y)$, and its contribution to the response function yields the integral

$$
\frac{d\mathcal{F}_{\log}(\Delta E)}{d\tau} = -\frac{H}{8\pi^2}\int_{-\infty}^{\infty} du\, \mathrm{e}^{\mathrm{i}\Delta E u/H}\Big[\log\Big(4\sinh^2(u/2)\Big) - \mathrm{i}\pi\mathrm{sign}(u)\Big]\,, \tag{6.31}
$$

where we broke the logarithm into the real and imaginary contributions, in accordance with the ε-prescription for $\mathrm{i}G^{<}$. The real part of the logarithm can be evaluated by breaking it into positive and negative u and then performing a partial integration (or, alternatively, by expanding the logarithm), while the imaginary part can be integrated trivially:

$$\begin{aligned}\frac{d\mathcal{F}_{\log}(\Delta E)}{d\tau} &= \frac{H^2}{4\pi^2\Delta E}\int_0^\infty du \sin\left(\frac{\Delta E}{H}u\right)\coth\left(\frac{u}{2}\right) - \frac{H^2}{4\pi\Delta E} \\ &= \frac{H^2}{2\pi\Delta E}\frac{1}{\mathrm{e}^{2\pi\Delta E/H}-1}\,,\end{aligned} \tag{6.32}$$

where in the last step we made use of Eqn. (3.981.8) of Ref. [90].

The remaining integrals in the response function (6.13) simply yield δ-function contributions,

$$\frac{d\mathcal{F}_\delta(\Delta E)}{d\tau} = \frac{H^2}{2\pi}\delta(\Delta E)\Big(\frac{1}{2\mathtt{s}} - 1 + \log(2) + O(\mathtt{s})\Big)\,. \tag{6.33}$$

Collecting all terms together, we get the response function for the nearly minimally coupled massive scalar:

$$\frac{d\mathcal{F}_{m\neq 0}(\Delta E)}{d\tau} = \frac{\Delta E}{2\pi}\left(1+\frac{H^2}{\Delta E^2}\right)\frac{1}{\mathrm{e}^{(2\pi/H)\Delta E}-1} + \frac{H^2}{2\pi}\delta(\Delta E)\Big(\frac{1}{2\mathtt{s}} - 1 + \log(2) + O(\mathtt{s})\Big)\,. \tag{6.34}$$

6.4.3 Minimally Coupled Massless Scalar Field

The Green function of a minimally coupled massless scalar field exhibits an infrared divergence [87], and the construction of a finite propagator necessarily breaks de Sitter invariance. For the purpose of calculating loop diagrams and using dimensional regularization, one considers the propagator in d dimensions with appropriate counterterms to cancel the infrared divergence. For a detailed discussion, see Ref. [91]. In four dimensions, one obtains [89, 92]

$$\mathrm{i}G_{m=0}(x_1;x_2) = \frac{H^2}{4\pi^2}\left\{\frac{1}{y} - \frac{1}{2}\log(y) + \frac{1}{2}\log\big(a(\eta_1)a(\eta_2)\big) - \frac{1}{4} + \log(2)\right\}, \tag{6.35}$$

where the term $\propto \log\left(a(\eta_1)a(\eta_2)\right)$ breaks de Sitter invariance.

Yet, this does not imply, that there is no de Sitter-invariant vacuum, as pointed out in Ref. [93], where such an invariant state is explicitly constructed by quantizing the mode with zero momentum separately. However, singling out that mode does not render the propagator finite, as one sees when regulating the propagator with an infrared cutoff k^0, such that one obtains [94]

$$\mathrm{i}G_{m=0,k^0}(x_1;x_2) = \frac{H^2}{4\pi^2}\left\{\frac{1}{y} - \frac{1}{2}\log(y) + \frac{1}{2}\log\big(a(\eta_1)a(\eta_2)\big) - \log(k^0H) - \gamma_E + O(k^0)\right\}, \tag{6.36}$$

where $\gamma_E = 0.577215...$ is Euler's constant. This expression differs from the propagator (6.35), which we shall use for calculating the response, only by a constant.

The response function is easily reconstructed from the results of section 6.4.2:

$$\frac{d\mathcal{F}_{m=0}(\Delta E)}{d\tau} = \frac{\Delta E}{2\pi}\left(1+\frac{H^2}{\Delta E^2}\right)\frac{1}{\mathrm{e}^{2\pi\Delta E/H}-1} + \frac{H^2}{2\pi}\delta(\Delta E)\Big(\log(a) - \frac{1}{4} + \log(2)\Big)\,, \tag{6.37}$$

where $\log(a) = H\tau = N$ is the number of e-folds elapsed since the beginning of inflation, if we set the initial scale factor to be one. This contribution to the response function vanishes for all $\Delta E \neq 0$, such that under the assumption that its energy levels are not degenerate, the detector is insensitive to the breaking of de Sitter invariance by the propagator. Note also, that, when integrated over ΔE around $\Delta E = 0$, the terms $\propto \delta(\Delta E)$ are subdominant, because they only give a finite contribution to the response, provided that the scale factor a and the cutoff k^0 in (6.36) are finite and nonzero, while the remaining terms yield a divergence.

6.4.4 Boundary Terms through Finite-Time Measurements

As we point out in chapter 7, we cannot think of switching Unruh's detector on and off by setting $h_{mn} \neq 0$ and $h_{mn} = 0$ again because this would go along with an infinite shift of the energy levels. However, one can think of fixing the detector to be in the ground state by a measurement. A subsequent second measurement tells us, whether the detector has been excited in the meantime. However, the second observation also influences the detector, the earlier we perform it, the stronger. How much patience does it therefore take to obtain a meaningful result?

At $\tau_0 = 0$, we put the detector in the ground state and at $\tau_f = \tau$, we check out whether it is excited. For this case Eqn. (6.14) generalizes to

$$\frac{d\mathcal{F}(\Delta E)}{d\tau} = \int_{-\tau}^{\tau} d\Delta\tau \mathrm{e}^{\mathrm{i}\Delta E\Delta\tau}\mathrm{i}G^{<}\left(x\left(\tau+\frac{\Delta\tau}{2}\right), x\left(\tau-\frac{\Delta\tau}{2}\right)\right)\,, \tag{6.38}$$

implying that the response function gets modified by the boundary effects. For example, the response function associated with the conformally coupled scalar (6.23) reads

$$\begin{aligned}\frac{d\mathcal{F}_{\mathrm{conf}}(\Delta E, 0, \tau)}{d\tau} &= \frac{\Delta E}{2\pi}\frac{1}{\mathrm{e}^{2\pi\Delta E/H}-1} \\ &+ \frac{H}{2\pi^2}\sum_{n=1}^{\infty} n\mathrm{e}^{-nH\tau}\,\frac{n\cos\big(\Delta E\tau\big) - (\Delta E/H)\sin\big(\Delta E\tau\big)}{\big(\Delta E/H\big)^2 + n^2}\,.\end{aligned} \tag{6.39}$$

Similar, though more technical, analysis can be performed for other contributions from the massless scalar propagator (6.35). Quite generically, boundary effects give rise to oscillatory contributions to the response function of Unruh's detector. For $\Delta E \sim H$, these terms become unimportant when $\tau \gg H^{-1}$. In the ultraviolet, where $\Delta E \gg H$, the oscillatory contributions become subdominant when $\tau \gg \Delta E/H^2$ – much more than a Hubble time.

6.4.5 Dimensions other than Four

So far, we have calculated the response functions from the scalar propagator, which is motivated by the practice of defining vacua in de Sitter space through this quantity. However, there is a method due to Higuchi [19] using a basis of wave functions as starting point, which we generalize here to d dimensions. We define

$$\nu = \sqrt{\left(\frac{d-1}{2}\right)^2 - \frac{m^2+\xi R}{H^2}}\,, \tag{6.40}$$

where the curvature is given by $R = d(d-1)H^2$. The scalar wave equation is (*cf.* Eqn. (6.21)),

$$\left[\partial_\eta^2 + \mathbf{k}^2 - \frac{\nu^2-(1/4)}{\eta^2}\right]\varphi_{\mathbf{k}}(\eta) = 0 \tag{6.41}$$

and has the properly normalized negative frequency solution

$$\varphi_{\mathbf{k}}(\eta) = \frac{1}{2}(-\pi\eta)^{\frac{1}{2}}\mathrm{e}^{\frac{\pi}{2}\Im[\nu]}H_\nu^{(2)*}\left(-|\mathbf{k}|\eta\right)\,. \tag{6.42}$$

The transition probability for the detector in terms of these modes is then

$$\begin{aligned} P(\Delta E) &= \int \frac{d^{d-1}k}{(2\pi)^{d-1}}\int_{-\infty}^{\infty}d\tau\int_{-\infty}^{\infty}d\tau'\frac{1}{a(\tau)a(\tau')}\varphi^*_{\mathbf{k}}(\tau)\varphi_{\mathbf{k}}(\tau')\mathrm{e}^{\mathrm{i}\Delta E(\tau'-\tau)} \\ &= \int \frac{d^{d-1}k}{(2\pi)^{d-1}}\int_{-\infty}^{\infty}d\tau\int_{-\infty}^{\infty}d\tau'\frac{\pi}{4H}\mathrm{e}^{\pi\Im[\nu]-\frac{3}{2}H(\tau+\tau')+\mathrm{i}\Delta E(\tau'-\tau)} \\ &\quad\times H_\nu^{(2)}\Big(\frac{|\mathbf{k}|}{H}\mathrm{e}^{-H\tau}\Big)H_\nu^{(2)*}\Big(\frac{|\mathbf{k}|}{H}\mathrm{e}^{-H\tau'}\Big). \end{aligned} \tag{6.43}$$

From this expression, one obtains for the response function

$$\begin{aligned} \frac{d\mathcal{F}_d(\Delta E)}{d\tau} &= \frac{H^{d-3}\mathrm{e}^{\pi\Delta E/H}}{8\pi^{(d+1)/2}\Gamma\left((d-1)/2\right)} \\ &\quad\times\left|\Gamma\left(\frac{(d-1)/2+\mathrm{i}\Delta E/H+\nu}{2}\right)\Gamma\left(\frac{(d-1)/2+\mathrm{i}\Delta E/H-\nu}{2}\right)\right|^2, \end{aligned} \tag{6.44}$$

which reduces for $d=4$ to Higuchi's result [19]

$$\frac{d\mathcal{F}(\Delta E)}{d\tau} = \frac{H}{4\pi^3}\mathrm{e}^{-\pi\Delta E/H}\left|\Gamma\left(\frac{3/2+\mathrm{i}\Delta E/H+\nu}{2}\right)\Gamma\left(\frac{3/2+\mathrm{i}\Delta E/H-\nu}{2}\right)\right|^2. \tag{6.45}$$

For $\nu = 1/2$ this coincides with our result for the conformal case (6.25), and when expanded in $\mathsf{s} = 3/2-\nu$ with the nearly minimally coupled case (6.34).

It is however also interesting to derive some special responses from the scalar propagator, which is in d dimensions [89]

$$\mathrm{i}G_d(y) = \frac{\Gamma(\frac{d-1}{2}+\nu)\Gamma(\frac{d-1}{2}-\nu)}{(4\pi)^{d/2}\Gamma\left(\frac{d}{2}\right)} H^{d-2}\, {}_2F_1\left(\frac{d-1}{2}+\nu, \frac{d-1}{2}-\nu, \frac{d}{2}; 1-\frac{y}{4}\right) . \tag{6.46}$$

In particular, for $d = 3$, the response function is exactly calculable for arbitrary ν from the propagator, because we can express the hypergeometric function in terms of the geodesic distance ℓ, using Eqn. (9.121.30) of Ref. [90], as

$${}_2F_1\left(1+\nu, 1-\nu, \frac{3}{2}; 1-\frac{y}{4}\right) = \frac{\sin\left[\nu\left(\pi - H\ell\right)\right]}{\nu \sin\left(H\ell\right)}. \tag{6.47}$$

With $\ell \to \mathrm{i}\Delta\tau$, the response function (6.13) is then given by the integral

$$\frac{d\mathcal{F}_3(\Delta E)}{d\tau} = \frac{\Gamma(\nu)\Gamma(1-\nu)}{4\mathrm{i}\pi^2} H \int_{-\infty}^{\infty} d\Delta\tau\, \mathrm{e}^{\mathrm{i}\Delta E \Delta\tau}\, \frac{\sin(\pi\nu - \mathrm{i}\nu H\Delta\tau)}{\sinh(H\Delta\tau)}, \tag{6.48}$$

with the poles of the integrand at $\Delta\tau_n = \pi \mathrm{i} n/H$, $n \,\epsilon\, \mathbb{Z}\backslash\{-1\}$. For odd dimensions, the analytic structure of the propagator is different from the even dimensional case. There are additional poles, but there is no branch cut (except for special values of ν). We perform the integration by closing the contour in the upper complex half plane. According to the ε-prescription for the Wightman function $\mathrm{i}G^{<}$, the poles for $n \geq 0$ contribute, and we obtain

$$\frac{d\mathcal{F}_3(\Delta E)}{d\tau} = \frac{1}{2} \frac{\sinh\left(\pi\frac{\Delta E}{H}\right)}{\cos(\pi\nu) + \cosh\left(\pi\frac{\Delta E}{H}\right)} \frac{1}{\mathrm{e}^{2\pi\Delta E/H} - 1} . \tag{6.49}$$

The above expression applies not only when $m < H$, and ν is real, but also for $m > H$, when ν is imaginary.

Note that for *no* value of the parameter ν the response (6.49) agrees with the thermal response function, which in three dimensions can be read off from Eqn. (6.18),

$$\frac{d\mathcal{F}_{3,\mathtt{th}}}{d\tau} = \frac{1}{2}\mathrm{sign}(\Delta E)\Theta\big((\Delta E)^2 - m^2\big) \frac{1}{\mathrm{e}^{\beta\Delta E} - 1} . \tag{6.50}$$

In particular, for a conformal massless scalar, $\nu = 1/2$, and Eqn. (6.49) yields the following 'fermionic-like' response function,

$$\frac{d\mathcal{F}_{3,\mathtt{conf}}(\Delta E)}{d\tau} = \frac{1}{2} \frac{1}{\mathrm{e}^{2\pi\Delta E/H} + 1} . \tag{6.51}$$

This disagreement with the thermal case is not a special feature of odd dimensions. *E.g.* the conformal Green function in d dimensions

$$\mathrm{i}G_{\mathrm{conf},d} = \frac{\Gamma(\frac{d}{2}-1)}{(4\pi^{d/2})} H^{d-2} y^{1-d/2} \tag{6.52}$$

leads for $d = 6$ to the response

$$\frac{d\mathcal{F}_{6,\mathtt{conf}}(\Delta E)}{d\tau} = \frac{H^3}{12\pi^2}\left(\frac{\Delta E^3}{H^3} + \frac{\Delta E}{H}\right)\frac{1}{e^{2\pi\Delta E/H} + 1}, \tag{6.53}$$

while the flat-space thermal response is

$$\frac{d\mathcal{F}_{6,\mathtt{th}}}{d\tau} = \frac{1}{12\pi^2}\mathrm{sign}(\Delta E)\vartheta\left((\Delta E)^2 - m^2\right)\left(\Delta E^2 - m^2\right)^{3/2}\frac{1}{e^{\beta\Delta E} - 1}. \tag{6.54}$$

Generally, for $d > 4$, the conformal response consists of the Planck factor times a polynomial involving different powers of ΔE, therefore deviating from the thermal response, which involves only a single power of ΔE.

For $d = 2$ conformal and minimally massless coupled case coincide and we find from (6.44)

$$\frac{d\mathcal{F}_{1+1,\mathtt{conf}}(\Delta E)}{d\tau} = \frac{1}{\Delta E}\frac{1}{e^{\beta\Delta E} - 1}, \quad (\Delta E \neq 0), \tag{6.55}$$

in agreement with the flat-space thermal response.

Hence, we have found that an agreement with the thermal response occurs for conformal coupling only in $d = 2$ and $d = 4$, just as for an accelerated observer in flat space [95].

6.5 Detailed Balance, Response Functions and Spectra

Based on the assumption that the principle of detailed balance holds, which states that the absorption rate of the detector $\mathcal{R}_a$ and the emission rate $\mathcal{R}_e$ are equal,

$$\mathcal{R}_a(E_0 \to E) = \mathcal{R}_e(E \to E_0), \qquad (\forall E_0, E), \tag{6.56}$$

and on the fact that the transition probabilities per unit proper time are related by

$$\frac{dP(E_0 \to E)}{d\tau} = e^{-\beta(E-E_0)}\frac{dP(E \to E_0)}{d\tau}, \tag{6.57}$$

or, equivalently, the response function of the detector fulfills,

$$\frac{d\mathcal{F}(\Delta E)}{d\tau} = e^{-\beta\Delta E}\frac{d\mathcal{F}(-\Delta E)}{d\tau} \qquad (\Delta E = E - E_0), \tag{6.58}$$

one can infer that the detector is thermally populated, with the temperature given by $T = 1/\beta$, as follows (for a related discussion see Ref. [84]). Let us rewrite the principle of detailed balance (6.56) as

$$n(E_0)\frac{dP(E_0 \to E)}{d\tau}\big(1 + n(E)\big) = n(E)\frac{dP(E \to E_0)}{d\tau}\big(1 + n(E_0)\big), \tag{6.59}$$

where $n(E)$ and $n(E_0)$ denote the occupation numbers of detector states with energies E and E_0, respectively. Furthermore, we have assumed that there is a Bose enhancement for stimulated excitation of the detector levels. From this, it immediately follows

$$n(E) = \frac{1}{\mathrm{e}^{\beta(E-\mu)} - 1}\,, \tag{6.60}$$

such that the states of the detector are populated according to an equilibrium state at temperature $T = 1/\beta$ and chemical potential μ.

In fact, any response which can be written as

$$\frac{d\mathcal{F}(\Delta E)}{d\tau} = g\,\frac{\Delta E}{2\pi}\frac{1}{\mathrm{e}^{\beta\Delta E} - 1}\,, \tag{6.61}$$

with $g = g(\Delta E)$ being an even function of ΔE, fulfills the relation (6.58). We have shown explicitly for different scalar fields in $d = 4$ (*cf.* Eqns. (6.26), (6.37) and (6.34)), that they are of the form (6.61),

$$\begin{aligned}
g_{\mathrm{conf}} &= 1\,,\\
g_{m=0} &= 1 + \left(\frac{H}{\Delta E}\right)^2 + 2\pi H\delta(\Delta E)\left[H\tau - \frac{1}{4} + \ln(2)\right],\\
g_{m\neq 0} &= 1 + \left(\frac{H}{\Delta E}\right)^2 + 2\pi H\delta(\Delta E)\left[\frac{1}{2\mathtt{s}} - 1 + \ln(2)\right] + O(\mathtt{s})\,,
\end{aligned}$$

with $\beta_H = 1/T_H = 2\pi/H$. Moreover, the more general expressions (6.44), (6.45) and (6.49) also satisfy equation (6.58).

The relation (6.58) can also be viewed as a consequence of the periodicity of the Green function $\mathrm{i}G^{<}$ in imaginary proper time, $\tau \to \tau + 2\pi\mathrm{i}\beta$ [22], which is in turn a consequence of the same periodicity of the metric in Euclidean time[1]. An example where the periodicity of the metric however does not coincide with the Hawking temperature is a quasi-de Sitter space considered in Ref. [96], and hence cannot in general be used as an argument for the thermality of a scalar field.

6.6 Remarks

We found the response functions for different scalar fields to differ strongly in the infrared, where $\Delta E < H$. Moreover, they do not in general coincide with the response to an equilibrium state in flat space. A disagreement with the thermal response does not yet imply that the detector does not equilibrate with the de Sitter background. In fact, the energy levels of the detector are thermally populated. Similar deviations from a

[1] For more general de Sitter invariant states, the so called α-vacua, one can show that (6.58) does not in general hold [22]. However, these states have a different ultraviolet structure than the Chernikov-Tagirov vacuum (6.27), which has the standard Hadamard lightcone singularity, and hence they are most likely unphysical.

Minkowski-space thermal response are also known for accelerated detectors [95,97]. The disagreement of the response functions is attributed to the fact, that for the conformally and the minimally coupled scalar field the density of modes per frequency is different.

However, for fields which are massive or nonconformally coupled to the metric, the infrared enhancement can be seen as a consequence of the amplification of superhorizon modes leading to cosmological density perturbations, an effect which is absent in the conformally coupled massless case. This makes the different fields clearly distinguishable by observables.

By the fact that according to the response function (6.14) the detector equilibrates at temperature T_H and by the conjecture that the de Sitter event horizon is endowed with a temperature [14], suggestions have been made, how the thermality can be understood at a more fundamental level than the detector response. An example for the interpretation of the de Sitter invariant states as thermal can be found in Ref. [19], where scalar field quantization is performed in static coordinates. The mode functions are chosen to vanish beyond the horizon, where the static coordinates exhibit a coordinate singularity. Since the horizon distance is singled out, the mode functions in static coordinates violate spatial homogeneity. An Unruh detector sitting at the coordinate origin of the static vacuum measures no particles, while particles will be captured when the detector is placed at any other site. On the other hand, when the static vacuum is thermally populated, the response function at the origin is the same as for the de Sitter-invariant vacuum, and it is given by (6.45). Note first that a thermally populated state in static coordinates does not correspond to a usual thermal equilibrium state, since spatial homogeneity is broken. In addition, the mode functions in the static and the de Sitter-invariant vacuum have different support. Indeed, the static mode functions vanish beyond the de Sitter horizon, while the de Sitter-invariant mode functions exhibit superhorizon correlations.

Particle production is often attributed to the very existence of an event horizon. Thereby, it will be interesting to investigate the response function for matter or radiation Universes, where no such horizon is present. Since a nonvanishing amplitude for interaction with the vacuum is due to the difference between conformal and proper time, we expect to detect particles anyway. Note also, that strictly speaking, the inflationary Universe has no event horizon since inflation is finite, and therefore it can only be regarded as a quasi de Sitter space.

While we argued in this chapter that the thermal state of the detector does not contain the full information about the quantum field, it is yet remarkable that an observer should be insensitive to the stress-energy tensor, which is clearly nonthermal. In fact, there have been attempts to identify the thermal aspects discussed here with the energy density produced by the de Sitter background [98], but there is apparently no straightforward relation since one spectrum decays exponentially, the other according to a power law, *cf.* chapter 2. In the next chapter however, we point out that there is indeed a way how the detector can observe stress-energy.

Chapter 7

Lamb Shift in Curved Spacetime

While special relativity has done away the theory of the aether, replacing it by the conception that there really is nothing in the vacuum, quantum field theory (QFT) provokes our imagination with the postulation of ubiquitous fluctuating fields, the quantum vacuum. The consequences of its existence appear to be rather abstract: masses and couplings which are supposed to be infinite at a bare level, are rendered finite by corrections from interaction with the vacuum. This curious concept and besides the cosmological constant problem, which is conjured up by the energy density of the fluctuations, are of course only accepted because of the great power of quantum field theory to make accurate predictions, *e.g.* for the anomalous magnetic moment of the electron.

The first experimental result to find an explanation by vacuum fluctuations was the Lamb shift. According to relativistic quantum mechanics, the energy levels $2S_{1/2}$ and $2P_{1/2}$ of hydrogen are degenerate, despite a tiny correction due to the hyperfine structure, insufficient however to account for the actual shift, which was observed by Lamb and Retherford [99]. In 1947, Bethe has shown in a groundbreaking paper [100] that the split is due to interactions of the electron with the vacuum fluctuations of the electromagnetic field, and a finite answer is obtained when subtracting the self-energy corrections for a free electron from those of an electron in the Coulomb potential. This is probably the most illustrative, simple and beautiful example for the effects of the quantum vacuum, which are detected by the hydrogen atom as a probe.

Just like an atom, Unruh's detector is a system with discrete energy levels, which by Bethe's argument also should acquire a Lamb shift correction from the fluctuations of the scalar field ϕ. Since quantum field theory in curved space deals with the distortions of the quantum vacuum induced by the gravitational background, it is perhaps more natural to expect that these become manifest in the Lamb shift rather than in the detection rate of scalar quanta.

Therefore, we calculate in the following the self-energy corrections to the energy levels of an Unruh detector in a spacetime X. At first order in perturbation theory, these are given by [100]

$$\begin{aligned}\delta E_{n\,X} &= \sum_{m\neq n}\int \frac{d^3k}{(2\pi)^3}\frac{\left|\int \frac{d^3k'}{(2\pi)^3}\langle \mathbf{k}', m|\hat{h}a^\dagger(\mathbf{k})\varphi(\mathbf{k},\eta)|0,n\rangle\right|^2}{E_n - E_m - \Omega(\mathbf{k})} \\ &= \sum_{m\neq n}\int \frac{d^3k}{(2\pi)^3}\frac{\left|h_{mn}^2\right| |\varphi(\mathbf{k},\eta)|^2}{E_n - E_m - \Omega(\mathbf{k})}\,,\end{aligned} \tag{7.1}$$

where $\Omega(\mathbf{k})$ is the mode energy (3.12) and the expression is to be evaluated setting the scale factor $a(\eta) = 1$. This shift of energy levels has in flat space a square divergence in the ultraviolet. In Minkowski space, where we define the values of the detector's energy levels E_n, which are finite, the shift $\delta E_{n\,\mathrm{M}}$ is already taken into account. In a curved spacetime C however, the value for the radiative correction differs from the Minkowski space answer; the finite quantity

$$\delta E_n = \delta E_{n\,C} - \delta E_{n\,\mathrm{M}} \tag{7.2}$$

can therefore be observed by comparing the spectra of energy levels in flat and in curved background.

To keep notation simple, we drop the summation over energy levels, corresponding to a two-level detector with spacing $\Delta E \equiv E_n - E_m$ and $|h_{mn}|^2 \equiv h^2$. The sum can simply be reinserted into all subsequent results.

7.1 Lamb Shift in the Expanding Universe

7.1.1 Massless de Sitter Case

As first example, let us consider a minimally coupled massless scalar in de Sitter space because for this situation, exact solutions are available and we do not need to resort to approximation by adiabatic expansion. First, we calculate the shift in Minkowski space,

$$\begin{aligned}\delta E_{\mathrm{M}}^{m=0} &= \int \frac{d^3k}{(2\pi)^3}\frac{1}{2k}\frac{h^2}{\Delta E - k} = \frac{h^2}{4\pi^2}\int_0^\infty dk \frac{k}{\Delta E - k} \\ &= \frac{h^2}{4\pi^2}\int_0^\infty dk\left\{-1 + \frac{\Delta E}{\Delta E - k}\right\} = \frac{h^2}{4\pi^2}\left[-k - \Delta E\log(\Delta E - k)\right]_0^\infty\,,\end{aligned} \tag{7.3}$$

which is to be subtracted.

The de Sitter mode functions are given by Eqn. (3.32), such that we find for their squared amplitude

$$|\varphi(\mathbf{k},\eta)|^2 = \frac{1}{2k} + \frac{1}{2k^3\eta^2} \tag{7.4}$$

and for the mode energy (3.12)

$$\Omega(\mathbf{k},\eta) = k + \frac{1}{2k\eta^2}\,. \tag{7.5}$$

Since we fix $a = 1$, we have by Eqn. (2.16) $\eta = -H^{-1}$. The use of Eqn. (7.1) gives us the unrenormalized Lamb shift in de Sitter space,

$$\begin{aligned}
\delta E_{\mathrm{dS}}^{m=0} &= \int \frac{d^3k}{(2\pi)^3}\left(\frac{1}{2k}+\frac{H^2}{2k^3}\right)\frac{h^2}{\Delta E - \left(k+\frac{H^2}{k}\right)} \\
&= \frac{h^2}{4\pi^2}\int\limits_0^\infty dk\left\{-1+\frac{\Delta E}{\Delta E-\left(k+\frac{H^2}{k}\right)}\right\} \\
&= \frac{h^2}{4\pi^2}\left\{[-k]_0^\infty - \Delta E\int\limits_{-\Delta E/2}^{\infty} dl\frac{l+\Delta E/2}{l^2+H^2-\Delta E^2/4}\right\} \\
&= \frac{h^2}{4\pi^2}\Bigg[-k+\frac{\Delta E^2/4}{\sqrt{\Delta E^2/4-H^2}}\log\left|\frac{k-\Delta E/2+\sqrt{\Delta E^2/4-H^2}}{k-\Delta E/2-\sqrt{\Delta E^2/4-H^2}}\right| \\
&\quad -\frac{\Delta E}{2}\log\left|\frac{(k+\Delta E/2)^2}{\Delta E^2/4-H^2}-1\right|\Bigg]_0^\infty\,.
\end{aligned} \tag{7.6}$$

We evaluate the boundary terms and subtract the flat space result to find for the finite observable shift (7.2)

$$\begin{aligned}
\delta E &= \delta E_{\mathrm{dS}}^{m=0} - \delta E_{\mathrm{M}} \\
&= \frac{h^2}{4\pi^2}\left\{\Delta E\log\left|\frac{H}{\Delta E}\right| - \frac{\Delta E^2}{4\sqrt{\Delta E^2/4-H^2}}\log\left|\frac{\Delta E/2-\sqrt{\Delta E^2/4-H^2}}{\Delta E/2+\sqrt{\Delta E^2/4-H^2}}\right|\right\}.
\end{aligned} \tag{7.7}$$

This expression condenses considerably when expanded in $H/\Delta E$:

$$\delta E = \frac{h^2}{4\pi^2}\frac{H^2}{\Delta E}\left(-\frac{1}{2}-2\log\left|\frac{H}{\Delta E}\right| + O\left(\frac{H}{\Delta E}\right)\right), \tag{7.8}$$

and when we reintroduce the sum to treat the case of more than two energy levels, it reads

$$\delta E = \sum_{m\neq n}\frac{h_{mn}^2}{4\pi^2}\frac{H^2}{E_n-E_m}\left(-\frac{1}{2}-2\log\left|\frac{H}{E_n-E_m}\right| + O\left(\frac{H}{E_n-E_m}\right)\right). \tag{7.9}$$

When compared to the response functions in de Sitter, which decay exponentially in ΔE, this power law behaviour becomes more important in the ultraviolet. Since the mode energy Ω is contributing, we can consider Lamb shift as a way to observe the energy density produced by the de Sitter background.

7.1.2 The General Case

Now, we allow for a general expanding FLRW background given by the scale factor $a(\eta)$, as well as for the scalar field ϕ a curvature coupling ξ and a constant mass m. Adiabatic expansion gives us up to second order

$$|\varphi|^2 = \frac{1}{2\omega} - \frac{1}{4\omega^3}\left\{(6\xi-1)\frac{a''}{a} - \frac{1}{2}\frac{m^2(aa''+a'^2)}{\omega^2} + \frac{5}{4}\frac{m^4a^2a'^2}{\omega^4}\right\} \tag{7.10}$$

and

$$\Omega = \omega + \frac{1}{2\omega}\left(\frac{a'^2}{a^2} + 6\xi\frac{a''}{a}\right) + \frac{1}{2}\frac{a'^2}{a^2}\frac{a^2m^2}{\omega^3} + \frac{1}{8}\frac{a'^2}{a^2}\frac{a^4m^4}{\omega^5}\,, \tag{7.11}$$

$$\begin{aligned}\Lambda = &\left\{\frac{1}{2\omega}\left(\frac{a'^2}{a^2}+\frac{a''}{a}\right) + \frac{1}{4}\left(\frac{a''}{a}+3\frac{a'^2}{a^2}\right)\frac{a^2m^2}{\omega^3} - \frac{1}{2}\frac{a'^2}{a^2}\frac{a^4m^4}{\omega^5}\right. \\ &\left.+\mathrm{i}\frac{a'}{a}\left(1+\frac{1}{2}\frac{a^2m^2}{\omega^2}\right)\right\}\mathrm{e}^{-2\mathrm{i}\int^\eta W(\eta')d\eta'}.\end{aligned} \tag{7.12}$$

We therefore define

$$\Delta_{A^2} = \frac{1}{\omega}\left\{\frac{1-6\xi}{2}\frac{a''}{a} + \frac{1}{4}\frac{m^2(aa''+a'^2)}{\omega^2} - \frac{5}{8}\frac{m^4a^2a'^2}{\omega^4}\right\}\,, \tag{7.13}$$

$$\Delta_\Omega = \frac{1}{2\omega}\left(\frac{a'^2}{a^2} + 6\xi\frac{a''}{a}\right) + \frac{1}{2}\frac{a'^2}{a^2}\frac{a^2m^2}{\omega^3} + \frac{1}{8}\frac{a'^2}{a^2}\frac{a^4m^4}{\omega^5}\,, \tag{7.14}$$

such that $|\varphi|^2 = 1/(2\omega) + \Delta_{A^2}/(2\omega^2)$ and $\Omega = \omega + \Delta_\Omega$.

The Lamb shift in FLRW Universe with respect to flat space is then

$$\begin{aligned}\delta E &= \delta E_{\mathrm{FLRW}} - \delta E_{\mathrm{M}} = h^2\int\frac{d^3k}{(2\pi)^3}\left\{\frac{1}{2\omega}\frac{\Delta_{A^2}/\omega+1}{\Delta E-\omega-\Delta_\Omega} - \frac{1}{2\omega}\frac{h^2}{\Delta E-k}\right\} \\ &\approx \frac{h^2}{4\pi^2}\int_0^\infty dk\left\{\frac{\Delta_{A^2}/\omega^2}{\Delta E-\omega} + \frac{\Delta_\Omega/\omega}{(\Delta E-\omega)^2}\right\} \\ &= \frac{h^2}{4\pi^2}\left\{-\frac{5}{12}\frac{1}{\Delta E}\frac{a''}{a} - \frac{1}{2}\frac{1}{\Delta E}\frac{a'^2}{a^2} + \frac{1-6\xi}{2}\frac{1}{\Delta E}\log\frac{2\Delta E}{m}\frac{a''}{a}\right. \\ &\qquad\left.-\frac{3\pi}{16}\frac{m}{\Delta E^2}\frac{a''}{a} - \frac{3\pi}{32}\frac{m}{\Delta E^2}\frac{a'^2}{a^2} + O\left(\frac{m^2}{\Delta E^3}\right)\right\}\,,\end{aligned} \tag{7.15}$$

where the integrals are given in appendix B. In the limit $m \to 0$, there occurs a logarithmic infrared divergence. This is however an artefact of adiabatic expansion, since the exact expression (7.7) for the massless de Sitter case is infrared finite.

7.2 Lamb Shift in Rindler Space

It was suggested by Unruh [13], that an accelerated observer should perceive particles even in the vacuum, which is due to the fact that quantization in a coordinate system suitable for the observer, referred to as Rindler space[1], is inequivalent to quantization in Minkowski space. Therefore, accelerated observer vacuum and inertial Minkowski vacuum do not coincide [101]. The quantum state in the accelerated system, which is equivalent to the Minkowski vacuum, can be constructed through a Bogolyubov transformation, which corresponds to mode mixing and is known as the Unruh effect [13].

Just as in de Sitter space, the response function of Unruh's detector falls off exponentially [13, 97], therefore resembling to a thermal spectrum. As we have observed for expanding universes, this effect is quantitatively dominated by the Lamb shift of energy levels. In the following, we shall demonstrate that the same holds also true for an accelerated detector.

7.2.1 Scalar Field in Rindler Coordinates

In flat two-dimensional space with the line element

$$ds^2 \equiv g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + dx^2, \quad g_{\mu\nu} = \mathrm{diag}(-1, 1)\,, \tag{7.16}$$

we consider an observer of mass m_{O}, who is constantly accelerated by the force $\mathbf{f}$, for example an ion in a homogeneous electric field. Let us determine his trajectory $y(\tau) = (t(\tau), x(\tau))^T$, where τ is his proper time, defined by $d\tau^2 = -ds^2$.

The Minkowski vector describing the force in the inertial system where the observer is instantaneously at rest is

$$\tilde{f} = \begin{pmatrix} 0 \\ \mathbf{f} \end{pmatrix}. \tag{7.17}$$

When we see the observer moving at the instantaneous velocity v, the force vector f in our coordinate system is obtained from

$$f = \Lambda(-v)\tilde{f} = \mathbf{f} \begin{pmatrix} \frac{v}{\sqrt{1-v^2}} \\ \frac{1}{\sqrt{1-v^2}} \end{pmatrix} = \mathbf{f} \begin{pmatrix} \sinh\psi \\ \cosh\psi \end{pmatrix}, \tag{7.18}$$

where $\Lambda(-v)$ denotes the Lorentz boost transformation, ψ is the rapidity parameter, $\tanh\psi = v$, and the velocity vector is of the standard form

$$u = \frac{dy}{d\tau} = \begin{pmatrix} \frac{1}{\sqrt{1-v^2}} \\ \frac{v}{\sqrt{1-v^2}} \end{pmatrix} = \begin{pmatrix} \cosh\psi \\ \sinh\psi \end{pmatrix}. \tag{7.19}$$

[1] In spite of the caption of this chapter, Rindler space is not a curved spacetime since it is related to Minkowski space by a coordinate transformation. Furthermore, when we talk about gravitational backgrounds, we also include Rindler space. We apologize for this inappropriateness of the terminology used by the author. In the literature, this is common though, since the equivalence principle states that it is impossible for an observer to distinguish between acceleration and the presence of a gravitational field.

With p being his momentum, the observer follows then a trajectory which is solution to the relativistic equation of motion

$$\frac{dp}{d\tau} = m_{\rm O}\frac{d^2y}{d\tau^2} = f\,. \tag{7.20}$$

A solution for $dy/d\tau$ is easily found when setting $\psi = \alpha\tau$ and $\alpha = \mathbf{f}/m_{\rm O}$, and we can interpret the parameter α as a constant proper acceleration

$$\alpha = \left[\left(\frac{d^2y}{d\tau^2}\right)^2\right]^{\frac{1}{2}} = \Big[-\Big(\frac{d^2t}{d\tau^2}\Big)^2 + \Big(\frac{d^2x}{d\tau^2}\Big)^2\Big]^{\frac{1}{2}}. \tag{7.21}$$

A special $y(\tau)$ is given by

$$y(\tau) = \left(\begin{array}{c} \alpha^{-1}\sinh\alpha\tau \\ \alpha^{-1}\cosh\alpha\tau \end{array}\right), \tag{7.22}$$

implying the trajectory

$$x(t) = (t^2 + \alpha^{-2})^{1/2}\,, \tag{7.23}$$

on which we shall consider Unruh's detector in the following.

Since we describe the time evolution of the detector in terms of its proper time τ, we also use τ as the time-variable for canonical quantization of the scalar field, which then manifestly separates into modes which the observer perceives as of positive and of negative frequency, respectively. Let us therefore transform the system to the Rindler coordinates as [83]

$$\begin{aligned} t &= \alpha^{-1}\mathrm{e}^{\xi}\sinh\alpha\tau\,, \\ x &= \alpha^{-1}\mathrm{e}^{\xi}\cosh\alpha\tau\,, \end{aligned} \tag{7.24}$$

such that the metric becomes

$$ds^2 = -\mathrm{e}^{2\xi}d\tau^2 + \alpha^{-2}\mathrm{e}^{2\xi}d\xi^2\,, \tag{7.25}$$

where the detector's site is at $\xi = 0$. The dependence of the metric (7.25) on ξ indicates that Minkowski space appears inhomogeneous to an accelerated observer.

According to the Lagrangean

$$\sqrt{-g}\mathcal{L} = \sqrt{-g}\left(-\frac{1}{2}g_{\mu\nu}\partial^\mu\varphi\partial^\nu\varphi - \frac{1}{2}m^2\varphi^2\right), \tag{7.26}$$

the Klein-Gordon equation for a scalar field with mass m is

$$\Big(-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2} - m^2\Big)\varphi(x,t) = 0\,. \tag{7.27}$$

In the Rindler coordinate system, this transforms to

$$\Big(-\frac{\partial^2}{\partial \tau^2} + \alpha^2\frac{\partial^2}{\partial \xi^2} - \mathrm{e}^{2\xi}m^2\Big)\varphi(\xi,\tau) = 0\,. \tag{7.28}$$

We shall take the scalar field to be in the Rindler vacuum $|0\rangle$, with the field operator

$$\hat{\varphi}(\xi,\tau) = \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} \left\{ c_\lambda \varphi_\lambda(\xi,\tau) + c_\lambda^\dagger \varphi_\lambda^*(\xi,\tau) \right\} , \tag{7.29}$$

where the creation and annihilation operators act as $c_\lambda^\dagger|0\rangle = |\lambda\rangle$ and $c_\lambda|\lambda'\rangle = 2\pi\delta(\lambda - \lambda')|0\rangle$.

Making use of the Jacobian of the coordinate transformation (7.24), $J = \alpha^{-1}\exp(2\xi)$, and the identity $\int dt \int dx \mathcal{H}^\varphi(x,t) = \int d\tau \int d\xi \mathcal{H}^\varphi(\xi,\tau)$, one arrives at the following Hamiltonian

$$\begin{aligned} H^\varphi &= \int_{-\infty}^{\infty} d\xi\, \mathcal{H}^\varphi , \\ \mathcal{H}^\varphi &= \frac{1}{2\alpha}\Big\{\Big(\frac{\partial\hat{\varphi}}{\partial\tau}\Big)^2 + \alpha^2\Big(\frac{\partial\hat{\varphi}}{\partial\xi}\Big)^2 + \mathrm{e}^{2\xi}m^2\hat{\varphi}^2\Big\} . \end{aligned} \tag{7.30}$$

The negative-frequency mode functions in (7.29) can be expressed in terms of Bessel functions (*cf.* Refs. [102] and [103])

$$\varphi_\lambda(\xi,\tau) = \frac{\mathrm{e}^{-\mathrm{i}|\lambda|\tau}}{\sqrt{2|\lambda|}}\,\mathrm{e}^{\frac{\pi}{2}\frac{\lambda}{\alpha}}\Gamma\Big(1+\mathrm{i}\frac{\lambda}{\alpha}\Big) J_{\mathrm{i}\frac{\lambda}{\alpha}}\left(\mathrm{i}\frac{m}{\alpha}e^\xi + \mathrm{sign}(\lambda)\varepsilon\right) . \tag{7.31}$$

Since the Bessel function $J_\nu(z)$ has a branch cut along the imaginary axis, that is for $\arg(z) = \pm\pi/2$, in order to uniquely specify its value, we have introduced in (7.31) an infinitesimal parameter $\varepsilon > 0$. The choice of branch is dictated by the physical requirement that, for $\xi \to -\infty$ (small argument z of $J_\nu(z)$) and for large λ (large ν/z), the modes (7.31) should reduce to plane waves. For the case $m = 0$ the modes will take the form of plane waves for all λ and there will be no additional Lamb shift in two dimensions, when compared to an inertial detector. In higher dimensions however, the momenta perpendicular to the trajectory lead also for the massless case to solutions in terms of Bessel functions.

The normalization of the modes (7.31) was found by considering the surfaces where $U = 0$ and $V > 0$, $V = 0$ and $U < 0$, respectively, where $U = t - x$, $V = t + x$, *cf.* figure 7.1,

$$\varphi_\lambda \overset{\xi\to-\infty}{\longrightarrow} \frac{1}{\sqrt{2|\lambda|}}\left[\theta(\lambda)\Big(\frac{-mU}{2}\Big)^{\mathrm{i}\frac{\lambda}{\alpha}} + \theta(-\lambda)\Big(\frac{mV}{2}\Big)^{\mathrm{i}\frac{\lambda}{\alpha}}\right] . \tag{7.32}$$

In the limit when $m \to 0$, this expression is formally singular. However, multiplying it by a physically unobservable phase, $\exp\left[\mathrm{i}\frac{\lambda}{\alpha}\log\left(\frac{2\mu_0}{m}\right)\right]$, where μ_0 is a constant of mass dimension one, renders Eqn. (7.32) finite. Along the light cones, $U = 0$ and $V = 0$, one can match the modes (7.31) to the Minkowski modes [13, 97],

$$\varphi_k = \frac{1}{\sqrt{2\omega}}\mathrm{e}^{-\mathrm{i}\omega\frac{U+V}{2} - \mathrm{i}k\frac{V-U}{2}} . \tag{7.33}$$

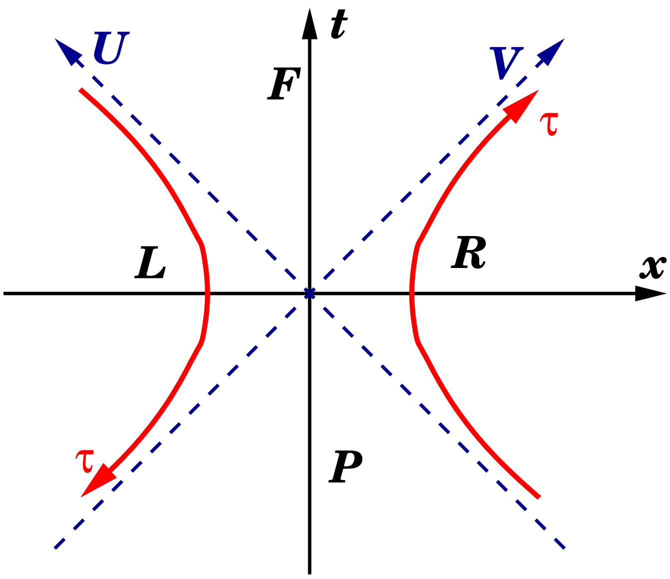

Figure 7.1: Roadmap of Rindler space. The trajectory of the detector is indicated by the arrow in region R.

The mixing is exponentially suppressed however, as can be seen from the Bogolyubov coefficients, $\beta_{\lambda,k} \sim \int dU \varphi_\lambda \big(\mathrm{i}\overleftrightarrow{\partial_V}\big)\varphi_k \propto \mathrm{e}^{-\pi|\lambda|/\alpha}$. Since we are here primarily interested in the ultraviolet domain, where $|\lambda| \gg \alpha$, and where, as we shall see, the radiative corrections fall off as a power law of λ, to recover the leading order behaviour, one does not need to account for the mode mixing. In order not to distract from the main line of argument, we therefore neglect it here.

When the parameter λ becomes large compared to the acceleration α, the modes (7.31) asymptotically reduce to plane waves. The Bessel function can under these circumstances be expanded in powers of α/λ and m/λ, which we do up to second and fourth order, respectively. Since this nice piece of mathematical analysis involves rather lengthy expressions, it is placed in appendix C.

7.2.2 Lamb Shift

In order to calculate Lamb shift, we need the amplitude squared of the expanded mode functions (C.7), which is at the site $\xi = 0$

$$|\varphi_\lambda(\xi = 0, \tau)|^2 = \frac{1}{2\lambda}\frac{1}{1 - \mathrm{e}^{-2\pi|\lambda|/\alpha}}\left(1 + \frac{1}{2}\frac{m^2}{\lambda^2} + \frac{3}{8}\frac{m^4}{\lambda^4} + \dots\right)\left(1 + \frac{1}{2}\frac{\alpha^2 m^2}{\lambda^4} + \dots\right), \tag{7.34}$$

where we made use of $\Gamma(1 + \mathrm{i}a)\Gamma(1 - \mathrm{i}a) = \pi a/\sinh(\pi a)$.

The Hamiltonian (7.30), which is quadratic in the field operators (7.29), can be recast into the following quadratic form in terms of the creation and annihilation operators of the Rindler vacuum,

$$H^\varphi = \frac{1}{2}\int \frac{d\lambda}{2\pi}\{\Omega_\lambda\big(c_\lambda c_\lambda^\dagger + c_\lambda^\dagger c_\lambda\big) + \big(\Lambda_\lambda c_\lambda c_{-\lambda} + \text{h.c.}\big)\}\,, \tag{7.35}$$

where Ω_λ denotes the energy of a Rindler quasiparticle excitation of momentum λ, and Λ_λ is the amplitude for annihilation of a Rindler pair, with the momenta λ and $-\lambda$, respectively.

Here, we are interested in the ultraviolet domain, *i.e.* in the portion of (7.35) where $|\lambda| \gg \alpha$. Thus, it is possible to choose $1 \gg \Delta\xi \gg \alpha/|\lambda|$, which is what we assume in the following. From the analytic behavior of the Rindler modes (7.31), it then follows that in the detector's neighbourhood at $\xi = 0$, the ultraviolet contributions to the Hamiltonian (7.35) are dominated by the local contribution from $\xi \in (-\Delta\xi/2, \Delta\xi/2)$,

$$H^\varphi \approx \int_{-\Delta\xi/2}^{\Delta\xi/2} d\xi\, \mathcal{H}^\varphi\,. \tag{7.36}$$

Then, by using the following approximate relation,

$$\int_{-\Delta\xi/2}^{\Delta\xi/2} d\xi \mathrm{e}^{\mathrm{i}\xi\frac{\lambda\pm\lambda'}{\alpha}\left(1-\frac{1}{2}\frac{m^2}{\lambda^2}-\frac{1}{8}\frac{m^4}{\lambda^4}\right)} \approx 2\pi\alpha\delta(\lambda\pm\lambda')\left(1+\frac{1}{2}\frac{m^2}{\lambda^2}+\frac{3}{8}\frac{m^4}{\lambda^4}\right)\,, \tag{7.37}$$

we find

$$\Omega_\lambda = \left\{(\lambda^2+m^2)|\varphi_\lambda|^2 + \alpha^2|\partial_\xi\varphi_\lambda|^2\right\}\left(1+\frac{1}{2}\frac{m^2}{\lambda^2}+\frac{3}{8}\frac{m^4}{\lambda^4}\right)\,, \tag{7.38}$$

$$\Lambda_\lambda = \left\{(m^2-\lambda^2)|\varphi_\lambda|^2 + \alpha^2|\partial_\xi\varphi_\lambda|^2\right\}\left(1+\frac{1}{2}\frac{m^2}{\lambda^2}+\frac{3}{8}\frac{m^4}{\lambda^4}\right)\mathrm{e}^{-2\mathrm{i}|\lambda|\tau}\,. \tag{7.39}$$

Upon substituting the expanded mode functions (C.7), one obtains after some algebra

$$\Omega_\lambda = \frac{1}{|\lambda|}\left(\lambda^2+m^2+\frac{m^4}{\lambda^2}-\frac{3}{8}\frac{\alpha^2 m^4}{\lambda^4}+\dots\right)\,, \tag{7.40}$$

$$\Lambda_\lambda = -\frac{\alpha^2 m^2}{2|\lambda|^3}\,\mathrm{e}^{-2i|\lambda|\tau}+\dots \tag{7.41}$$

Since we have quantized the Rindler modes employing the energy-like variable $\lambda = \sqrt{k^2+m^2}$, while in flat space we use the momentum k, we note that in order to compare, we need to make the rescaling

$$\Omega_k = \Omega_\lambda\left(1-\frac{\lambda^2}{m^2}\right) = |\lambda| - \frac{3}{8}\frac{\alpha^2 m^4}{|\lambda|^5}+\dots \tag{7.42}$$

We can now assemble the difference between Lamb shift in Rindler space δE_R and flat space δE_M. According to the two-dimensional case of Eqn. (7.1), it is

$$\delta E = \delta E_R - \delta E_M = h^2\int\frac{dk}{2\pi}\left\{\frac{1}{2|\lambda|}\frac{1+\frac{\alpha^2 m^2}{2\lambda^4}}{\Delta E-|\lambda|+\frac{3}{8}\frac{\alpha^2 m^4}{\lambda^5}} - \frac{1}{2|\lambda|}\frac{1}{\Delta E-|\lambda|}\right\} \tag{7.43}$$

$$\approx h^2\int\frac{dk}{2\pi}\left\{\frac{1}{4|\lambda|^5}\frac{\alpha^2 m^2}{\Delta E-|\lambda|} - \frac{3}{16|\lambda|^6}\frac{\alpha^2 m^4}{(\Delta E-|\lambda|)^2}\right\} \approx \frac{h^2}{6\pi}\frac{\alpha^2}{\Delta E m^2}\,,$$

where we have used the integrals (B.19, B.20) and displayed the result up to leading order in $1/\Delta E$. When comparing with the exponentially falling particle number by mode mixing, we see that in the ultraviolet, Unruh effect gets a boost.

7.3 Lamb Shift Versus Response Rate

While the response rate of an Unruh detector falls exponentially with the particle energy, which holds true for de Sitter as well as for Rindler space, we have shown that Lamb shift exhibits a power-law behaviour. According to the discussion of the principle of detailed balance in chapter 6, we can calculate from the response rate the probability to find the detector in an excited state, and also Lamb shift corresponds to the mixing of energy levels. In that sense, both effects are quantitatively comparable, and the Lamb shift is clearly more important in the ultraviolet. Yet, the difference is of course that the response of the detector is a time-dependent while Lamb shift a time-independent effect[2].

The power law behaviour is expected when considering the Hamiltonian or the covariant energy density in curved space, while from this point of view, the exponential decay of the detector response comes out as a surprise. Note, that for the calculation of Lamb shift, we had to use renormalization techniques, though at a rather crude level. Therefore, we have to wonder whether the response function (6.14) for the unrenormalized, bare detector correctly reproduces the response rate for its renormalized, dressed counterpart. In QFT, this question is answered positively by the LSZ reduction formula for scattering amplitudes, the proof of which in particular requires that the external states of the matrix element correspond to well separated wave packets. It is not clear whether this condition can be met for the state being the product of detector in the ground state and curved spacetime vacuum. In particular, our discussion of boundary effects in section 6.4.4 indicates possible problems, since a huge period of interaction with the vacuum and a tremendous coherence is required, while scattering in flat space is a resonance phenomenon on rather short timescales. Unfortunately however, we come short of deciding this question here.

Finally, we want to point out that there is a relation of Lamb shift to work of Tsamis and Woodard [104] on back-reaction in inflation and of Prokopec *et al.* [89, 92, 105, 106] on the generation of primordial electromagnetic fields on cosmic scales, since all these effects are due to self-energy corrections. It is noteworthy, that the expressions for the self-energy of a particle coupled to a minimally coupled scalar in de Sitter background given in these references exhibit no indication of damping, since the analytic structure

[2] Strictly speaking, the Lamb shift in the expanding Universe (7.15) varies as the Hubble rate changes with time. Time-independence means here that we use time-independent perturbation theory.

corresponds to a free stable particle. This is apparently unlike for the case of a particle traveling through a thermal bath in flat space, which experiences scatterings.

Chapter 8

Conclusions

Various aspects of particle production are discussed in this work; and according to the specific settings, we use different techniques. Particle number is expressed in terms of phase space densities which occur in kinetic theory, the covariant stress-energy tensor is calculated and Unruh's detector is employed to probe the particle spectrum. We relate the results to each other and provide explanations whenever there appear to be inconsistencies.

As first presented in Ref. [65], a derivation of particle number in kinetic theory from first principles is given, which comes as close to an interpretation of Wigner functions in terms of number densities on phase space as this is allowed by quantum theory. Because of its independence of a specific Fock-space basis the kinetic theory approach proves very useful when treating the problem of preheating in the multiflavour case, in particular when endowed with C and CP violation. For this situation, it is the only appropriate formalism which appeared in the literature so far. Of course any particle number definition is restricted to the case when interactions are sufficiently weak, such that a quasi-particle picture is applicable. Our work can be generalized to include interactions when calculating the self-energy corrections in the Schwinger-Keldysh closed-time path framework.

The investigations of the multiflavour kinetic equations lead us to the suggestion of a new mechanism for baryogenesis [50, 107]. We name it coherent baryogenesis and assemble it with existing models to realistic scenarios, involving inflation, baryogenesis, reheating and grand unified theories. Coherent baryogenesis therefore serves as an example for the interplay of early Universe cosmology and particle physics, in particular extensions of the Standard Model. Note that for the particular GUTs we consider, Pati-Salam and SO(10), in order to incorporate coherent baryogenesis, we do not need to add additional particle multiplets when compared to the minimal content necessary for hybrid inflation. Furthermore, we use for our numerical simulations parameters which are natural for GUT-scale hybrid inflation, and we obtain for both, Pati-Salam and SO(10) case, baryon asymmetries in accordance with observation. We therefore conclude that coherent baryogenesis should be regarded as a viable scenario for explaining the matter-antimatter asymmetry. When compared to thermal leptogenesis, which requires as only additional ingredients beyond the Standard Model superheavy right-handed Majorana neutrinos and CP-violation in this sector, our coherent baryogenesis scenarios, being examples for GUT baryogenesis, are somewhat more complicated. It is therefore harder to cast bounds on experimentally accessible parameters; however, when the matter-antimatter asymmetry indeed stemmed from GUT baryogenesis, this would open up a window to grand unification and the pattern of symmetry breaking.

As a link to experiment, future investigations should answer the question how initial perturbations in the scalar condensates are transferred to baryon number and whether baryonic isocurvature perturbations are generated. These perturbations have not yet been found by measurements of the cosmic microwave background, but there is already an upper bound for their magnitude. Upcoming experiments should either improve this bound or detect isocurvature perturbations.

When comparing preheating to particle production by the FLRW background, we point out that the former process is nonadiabatic and the latter adiabatic. While in the nonadiabatic case the particle concept causes no problems, the interpretation of results takes greater care in the adiabatic domain. Although often stated otherwise in the literature, we consider the problem of divergences in the stress-energy tensor in curved spacetimes as serious and yet unresolved.

To gain further insight in adiabatic particle production, we give a detailed review of the response functions of Unruh detector in de Sitter space as considered in previous literature and furthermore provide general results for arbitrary mass, curvature coupling and spacetime dimension [85]. We explain the principle of detailed balance, by which all these different results for the response functions may be considered as thermal. However, we also outline where conceptual differences to the usual notion of a thermal state arise in the de Sitter space case. Note that the expression we use for the response function – and thereby a main argument for de Sitter space being thermal – yet needs to be justified by arguments of LSZ type.

We stress the striking bias between the exponentially suppressed detector response on the one side and on the other the covariant stress-energy density and Parker's definition of particle number using the Hamiltonian, which both decay as a power law with particle energy [23]. This motivates us to consider the Lamb shift of the detector's energy levels in the expanding Universe, and in fact, we recover the power law behaviour. Furthermore, we investigate the same effect for a uniformly accelerated observer, which can be conveniently described in the Rindler coordinate system. We provide a novel discussion of the ultraviolet behaviour of the scalar field vacuum in Rindler space, and also find the mode energies and amplitudes to decay with momentum according to power laws. These spectra are then also found to be reflected in the Lamb shift corrections to the detector's energy levels. Thus, the conjectured thermality is not the sole feature to characterize quantum physics in spacetimes with an event horizon, *i.e.* de Sitter and Rindler space. In the ultraviolet, it is quantitatively dominated by background metric-induced self-energy corrections.

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My friendship to Thomas Konstandin goes back to the remote past of our first semesters at university. Since then, we have been going quite parallel paths, not only concerning our studies but also sharing our favourite bars and for a while the yellow house on Allen Street. Since we will eventually move to different towns I express here my best wishes for his future.

Besides the nice view, the members of our institute make it a very pleasant place. Filipe Paccetti, Timo Bergmann, Thomas and I spent quite a few cozy afternoons in our basement office. Furthermore, for sharing leisure activities, common travels or just being gentle persons I would like to mention Gregor Schäfer, Holger Gies, Juliane Behrend, Klaus Klingmüller, Lala Adueva, Ralf Hofmann, Sebastian Diehl, Tania Robens, Tobias Kleinschmidt and Zurab Tavartkiladze.

Finally, my parents and my sister Antje – whom I wish the best for her future plans with Ed – helped me in many ways. It was my family which gave me the possibility to study and I could always find support and understanding. Well, I just lied a little bit about the last point as far as particle physics is concerned.

Appendix A

SO(10) Group Theory

Besides tensors, orthogonal groups may also be represented by spinors, which satisfy a Clifford algebra. In order to construct group-transformation invariants, both types of representations need to be linked together *via* Dirac gamma matrices. For the familiar case of the Poincaré group SO$(3,1)$, it is often convenient to use a specific representation for these matrices, *e.g.* the Weyl-representation, as done in this thesis. In contrast, one has better to circumvent the tedious task of explicitly constructing ten 32×32 gamma matrices for SO(10). Mohapatra and Sakita [79] have therefore devised a very useful technique for performing calculations involving spinors and tensors, employing just abstract commutation and anticommutation relations.

On the other hand, when it comes to symmetry breaking, one has to choose a certain convention, that is a certain basis, how the particles of the Standard Model are assigned to the representation **16** of SO(10). This assignment fixes in turn the definition of the charge operators and hence the quantum numbers of certain entries in vectors and tensors of SO(10).

While the paper by Mohapatra and Sakita [79] does not provide details of tensor representations and symmetry breaking, such a discussion can be found in the comprehensive work by Fukuyama *et al.* [81], where in turn spinors are neglected. In the paper by Barr and Raby [80], which contains the model we consider here, a basis where tensors nicely decompose into blocks of SU(5)-representations is chosen. Unfortunately, the choice of basis and normalizations is not explicitly given, but has to be inferred by the reader.

In the following, we give some detailed account of the construction of SO(10)-singlets, following the conventions of Barr and Raby. Explicit expressions for the charge operators acting on spinors and tensors as well as for the accommodation of the Standard Model particles and the right-handed neutrino in the representation **16** are given, which shall ensure an easier and faster comprehensibility of the Barr and Raby analysis as well as of our calculations.

Table A.1: Quantum numbers of matter

	Q	I_L^3	I_R^3	Y	$B-L$	X
$Q = \begin{pmatrix} u \\ d \end{pmatrix}$	$2/3$ $-1/3$	$1/2$ $-1/2$	0 0	$1/6$ $1/6$	$1/3$ $1/3$	-1 -1
u^c d^c	$-2/3$ $1/3$	0 0	$-1/2$ $1/2$	$-2/3$ $1/3$	$-1/3$ $-1/3$	-1 3
$L = \begin{pmatrix} \nu \\ e \end{pmatrix}$	0 -1	$1/2$ $-1/2$	0 0	$-1/2$ $-1/2$	-1 -1	3 3
ν^c e^c	0 1	0 0	$-1/2$ $1/2$	0 1	1 1	-5 -1

A.1 Charge Assignments and SO(10)-Branching Rules

We denote by Q the electric charge, by Y the weak hypercharge and by I_L^3 the weak isospin. The charges which are not gauge symmetries of the Standard Model are baryon minus lepton number $B-L$ as well as the $\mathrm{SU}(2)_R$-isospin I_R^3 and the charge X. There are linear dependencies among these charges, which are given by

$$\begin{aligned} Q &= I_L^3 + Y\,, \\ B-L &= 2(Y - I_R^3)\,, \\ B-L &= \frac{1}{5}(4Y - X)\,. \end{aligned} \tag{A.1}$$

In table A.1, we give the charge numbers of the Standard Model particles and of the right-handed neutrino.

The way how some representation of a group decomposes in a sum of representations of a subgroup is called a branching rule. In the tables A.2, A.3, A.4 and A.5, we give the branching rules for the representations **10**, **16**, **45** and **120**, which occur in this thesis, as taken from the paper by Fukuyama *et. al.* [81].

A.2 SO($2N$) in an SU(N) Basis

This section contains a review of the paper by Mohapatra and Sakita [79], but adopts the basis conventions of Barr and Raby [80].

Let us introduce five operators χ_i, $(i = 1, ..., N)$ which obey the following anticom-

Table A.2: Decomposition of the representation **10**

$(4, 2_L, 2_R)$	$(3_C, 2_L, 2_R, 1_{B-L})$	$(3_C, 2_L, 1_R, 1_{B-L})$	$(3_C, 2_L, 1_Y)$	$(5, 1_X)$
$(\mathbf{6}, \mathbf{1}, \mathbf{1})$	$(\mathbf{3}, \mathbf{1}, \mathbf{1}; -\frac{2}{3})$ $(\overline{\mathbf{3}}, \mathbf{1}, \mathbf{1}; \frac{2}{3})$	$(\mathbf{3}, \mathbf{1}; 0, -\frac{2}{3})$ $(\overline{\mathbf{3}}, \mathbf{1}; 0, \frac{2}{3})$	$(\mathbf{3}, \mathbf{1}; -\frac{1}{3})$ $(\overline{\mathbf{3}}, \mathbf{1}; \frac{1}{3})$	$(\mathbf{5}, 2)$ $(\overline{\mathbf{5}}, -2)$
$(\mathbf{1}, \mathbf{2}, \mathbf{2})$	$(\mathbf{1}, \mathbf{2}, \mathbf{2}; 0)$	$(\mathbf{1}, \mathbf{2}; \frac{1}{2}, 0)$ $(\mathbf{1}, \mathbf{2}; -\frac{1}{2}, 0)$	$(\mathbf{1}, \mathbf{2}; \frac{1}{2})$ $(\mathbf{1}, \mathbf{2}; -\frac{1}{2})$	$(\mathbf{5}, 2)$ $(\overline{\mathbf{5}}, -2)$

Table A.3: Decomposition of the representation **16**

$(4, 2_L, 2_R)$	$(3_C, 2_L, 2_R, 1_{B-L})$	$(3_C, 2_L, 1_R, 1_{B-L})$	$(3_C, 2_L, 1_Y)$	$(5, 1_X)$
$(\mathbf{4}, \mathbf{2}, \mathbf{1})$	$(\mathbf{3}, \mathbf{2}, \mathbf{1}; \frac{1}{3})$ $(\mathbf{1}, \mathbf{2}, \mathbf{1}; -1)$	$(\mathbf{3}, \mathbf{2}; 0, \frac{1}{3})$ $(\mathbf{1}, \mathbf{2}; 0, -1)$	$(\mathbf{3}, \mathbf{2}; \frac{1}{6})$ $(\mathbf{1}, \mathbf{2}; -\frac{1}{2})$	$(\mathbf{10}, -1)$ $(\overline{\mathbf{5}}, 3)$
$(\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2})$	$(\overline{\mathbf{3}}, \mathbf{1}, \mathbf{2}; -\frac{1}{3})$ $(\mathbf{1}, \mathbf{1}, \mathbf{2}; 1)$	$(\overline{\mathbf{3}}, \mathbf{1}; \frac{1}{2}, -\frac{1}{3})$ $(\overline{\mathbf{3}}, \mathbf{1}; -\frac{1}{2}, -\frac{1}{3})$ $(\mathbf{1}, \mathbf{1}; \frac{1}{2}, 1)$ $(\mathbf{1}, \mathbf{1}; -\frac{1}{2}, 1)$	$(\overline{\mathbf{3}}, \mathbf{1}; \frac{1}{3})$ $(\overline{\mathbf{3}}, \mathbf{1}; -\frac{2}{3})$ $(\mathbf{1}, \mathbf{1}; 1)$ $(\mathbf{1}, \mathbf{1}; 0)$	$(\overline{\mathbf{5}}, 3)$ $(\mathbf{10}, -1)$ $(\mathbf{10}, -1)$ $(\mathbf{1}, -5)$

Table A.4: Decomposition of the representation **45**

$(4, 2_L, 2_R)$	$(3_C, 2_L, 2_R, 1_{B-L})$	$(3_C, 2_L, 1_R, 1_{B-L})$	$(3_C, 2_L, 1_Y)$	$(5, 1_X)$
$(\mathbf{1}, \mathbf{1}, \mathbf{3})$	$(\mathbf{1}, \mathbf{1}, \mathbf{3}; 0)$	$(\mathbf{1}, \mathbf{1}; 1, 0)$ $(\mathbf{1}, \mathbf{1}; 0, 0)$ $(\mathbf{1}, \mathbf{1}; -1, 0)$	$(\mathbf{1}, \mathbf{1}; 1)$ $(\mathbf{1}, \mathbf{1}; 0)$ $(\mathbf{1}, \mathbf{1}; -1)$	$(\mathbf{10}, 4)$ $(\mathbf{1}, 0)$ $(\overline{\mathbf{10}}, -4)$
$(\mathbf{1}, \mathbf{3}, \mathbf{1})$	$(\mathbf{1}, \mathbf{3}, \mathbf{1}; 0)$	$(\mathbf{1}, \mathbf{3}; 0, 0)$	$(\mathbf{1}, \mathbf{3}; 0)$	$(\mathbf{24}, 0)$
$(\mathbf{6}, \mathbf{2}, \mathbf{2})$	$(\mathbf{3}, \mathbf{2}, \mathbf{2}; -\frac{2}{3})$ $(\overline{\mathbf{3}}, \mathbf{2}, \mathbf{2}; \frac{2}{3})$	$(\mathbf{3}, \mathbf{2}; \frac{1}{2}, -\frac{2}{3})$ $(\mathbf{3}, \mathbf{2}; -\frac{1}{2}, -\frac{2}{3})$ $(\overline{\mathbf{3}}, \mathbf{2}; \frac{1}{2}, \frac{2}{3})$ $(\overline{\mathbf{3}}, \mathbf{2}; -\frac{1}{2}, \frac{2}{3})$	$(\mathbf{3}, \mathbf{2}; \frac{1}{6})$ $(\mathbf{3}, \mathbf{2}; -\frac{5}{6})$ $(\overline{\mathbf{3}}, \mathbf{2}; \frac{5}{6})$ $(\overline{\mathbf{3}}, \mathbf{2}; -\frac{1}{6})$	$(\mathbf{10}, 4)$ $(\mathbf{24}, 0)$ $(\mathbf{24}, 0)$ $(\overline{\mathbf{10}}, -4)$
$(\mathbf{15}, \mathbf{1}, \mathbf{1})$	$(\mathbf{1}, \mathbf{1}, \mathbf{1}; 0)$ $(\mathbf{3}, \mathbf{1}, \mathbf{1}; \frac{4}{3})$ $(\overline{\mathbf{3}}, \mathbf{1}, \mathbf{1}; -\frac{4}{3})$ $(\mathbf{8}, \mathbf{1}, \mathbf{1}; 0)$	$(\mathbf{1}, \mathbf{1}; 0, 0)$ $(\mathbf{3}, \mathbf{1}; 0, \frac{4}{3})$ $(\overline{\mathbf{3}}, \mathbf{1}; 0, -\frac{4}{3})$ $(\mathbf{8}, \mathbf{1}; 0, 0)$	$(\mathbf{1}, \mathbf{1}; 0)$ $(\mathbf{3}, \mathbf{1}; \frac{2}{3})$ $(\overline{\mathbf{3}}, \mathbf{1}; -\frac{2}{3})$ $(\mathbf{8}, \mathbf{1}; 0)$	$(\mathbf{24}, 0)$ $(\overline{\mathbf{10}}, -4)$ $(\mathbf{10}, 4)$ $(\mathbf{24}, 0)$

Table A.5: Decomposition of the representation **120**

$(4, 2_L, 2_R)$	$(3_C, 2_L, 2_R, 1_{B-L})$	$(3_C, 2_L, 1_R, 1_{B-L})$	$(3_C, 2_L, 1_Y)$	$(5, 1_X)$
$(\mathbf{1}, \mathbf{2}, \mathbf{2})$	$(\mathbf{1}, \mathbf{2}, \mathbf{2}; 0)$	$(\mathbf{1}, \mathbf{2}; \frac{1}{2}, 0)$	$(\mathbf{1}, \mathbf{2}; \frac{1}{2})$	$(\mathbf{5}, 2)$
		$(\mathbf{1}, \mathbf{2}; -\frac{1}{2}, 0)$	$(\mathbf{1}, \mathbf{2}; -\frac{1}{2})$	$(\overline{\mathbf{5}}, -2)$
$(\mathbf{10}, \mathbf{1}, \mathbf{1})$	$(\mathbf{1}, \mathbf{1}, \mathbf{1}; -2)$	$(\mathbf{1}, \mathbf{1}; 0, -2)$	$(\mathbf{1}, \mathbf{1}; -1)$	$(\overline{\mathbf{10}}, 6)$
	$(\mathbf{3}, \mathbf{1}, \mathbf{1}; -\frac{2}{3})$	$(\mathbf{3}, \mathbf{1}; 0, -\frac{2}{3})$	$(\mathbf{3}, \mathbf{1}; -\frac{1}{3})$	$(\mathbf{5}, 2)$
	$(\mathbf{6}, \mathbf{1}, \mathbf{1}; \frac{2}{3})$	$(\mathbf{6}, \mathbf{1}; 0, \frac{2}{3})$	$(\mathbf{6}, \mathbf{1}; \frac{1}{3})$	$(\overline{\mathbf{45}}, -2)$
$(\overline{\mathbf{10}}, \mathbf{1}, \mathbf{1})$	$(\mathbf{1}, \mathbf{1}, \mathbf{1}; 2)$	$(\mathbf{1}, \mathbf{1}; 0, 2)$	$(\mathbf{1}, \mathbf{1}; 1)$	$(\mathbf{10}, -6)$
	$(\overline{\mathbf{3}}, \mathbf{1}, \mathbf{1}; \frac{2}{3})$	$(\overline{\mathbf{3}}, \mathbf{1}; 0, \frac{2}{3})$	$(\overline{\mathbf{3}}, \mathbf{1}; \frac{1}{3})$	$(\overline{\mathbf{5}}, -2)$
	$(\overline{\mathbf{6}}, \mathbf{1}, \mathbf{1}; -\frac{2}{3})$	$(\overline{\mathbf{6}}, \mathbf{1}; 0, -\frac{2}{3})$	$(\overline{\mathbf{6}}, \mathbf{1}; -\frac{1}{3})$	$(\mathbf{45}, 2)$
$(\mathbf{6}, \mathbf{3}, \mathbf{1})$	$(\mathbf{3}, \mathbf{3}, \mathbf{1}; -\frac{2}{3})$	$(\mathbf{3}, \mathbf{3}; 0, -\frac{2}{3})$	$(\mathbf{3}, \mathbf{3}; -\frac{1}{3})$	$(\mathbf{45}, 2)$
	$(\overline{\mathbf{3}}, \mathbf{3}, \mathbf{1}; \frac{2}{3})$	$(\overline{\mathbf{3}}, \mathbf{3}; 0, \frac{2}{3})$	$(\overline{\mathbf{3}}, \mathbf{3}; \frac{1}{3})$	$(\overline{\mathbf{45}}, -2)$
$(\mathbf{6}, \mathbf{1}, \mathbf{3})$	$(\mathbf{3}, \mathbf{1}, \mathbf{3}; -\frac{2}{3})$	$(\mathbf{3}, \mathbf{1}; 1, -\frac{2}{3})$	$(\mathbf{3}, \mathbf{1}; \frac{2}{3})$	$(\overline{\mathbf{10}}, 6)$
		$(\mathbf{3}, \mathbf{1}; 0, -\frac{2}{3})$	$(\mathbf{3}, \mathbf{1}; -\frac{1}{3})$	$(\mathbf{45}, 2)$
		$(\mathbf{3}, \mathbf{1}; -1, -\frac{2}{3})$	$(\mathbf{3}, \mathbf{1}; -\frac{4}{3})$	$(\overline{\mathbf{45}}, -2)$
	$(\overline{\mathbf{3}}, \mathbf{1}, \mathbf{3}; \frac{2}{3})$	$(\overline{\mathbf{3}}, \mathbf{1}; 1, \frac{2}{3})$	$(\overline{\mathbf{3}}, \mathbf{1}; \frac{4}{3})$	$(\mathbf{45}, 2)$
		$(\overline{\mathbf{3}}, \mathbf{1}; 0, \frac{2}{3})$	$(\overline{\mathbf{3}}, \mathbf{1}; \frac{1}{3})$	$(\overline{\mathbf{45}}, -2)$
		$(\overline{\mathbf{3}}, \mathbf{1}; -1, \frac{2}{3})$	$(\overline{\mathbf{3}}, \mathbf{1}; -\frac{2}{3})$	$(\mathbf{10}, -6)$
$(\mathbf{15}, \mathbf{2}, \mathbf{2})$	$(\mathbf{1}, \mathbf{2}, \mathbf{2}; 0)$	$(\mathbf{1}, \mathbf{2}; \frac{1}{2}, 0)$	$(\mathbf{1}, \mathbf{2}; \frac{1}{2})$	$(\mathbf{45}, 2)$
		$(\mathbf{1}, \mathbf{2}; -\frac{1}{2}, 0)$	$(\mathbf{1}, \mathbf{2}; -\frac{1}{2})$	$(\overline{\mathbf{45}}, -2)$
	$(\mathbf{3}, \mathbf{2}, \mathbf{2}; \frac{4}{3})$	$(\mathbf{3}, \mathbf{2}; \frac{1}{2}, \frac{4}{3})$	$(\mathbf{3}, \mathbf{2}; \frac{7}{6})$	$(\overline{\mathbf{45}}, -2)$
		$(\mathbf{3}, \mathbf{2}; -\frac{1}{2}, \frac{4}{3})$	$(\mathbf{3}, \mathbf{2}; \frac{1}{6})$	$(\mathbf{10}, -6)$
	$(\overline{\mathbf{3}}, \mathbf{2}, \mathbf{2}; -\frac{4}{3})$	$(\overline{\mathbf{3}}, \mathbf{2}; -\frac{1}{2}, -\frac{4}{3})$	$(\overline{\mathbf{3}}, \mathbf{2}; -\frac{7}{6})$	$(\mathbf{45}, 2)$
		$(\overline{\mathbf{3}}, \mathbf{2}; \frac{1}{2}, -\frac{4}{3})$	$(\overline{\mathbf{3}}, \mathbf{2}; -\frac{1}{6})$	$(\overline{\mathbf{10}}, 6)$
	$(\mathbf{8}, \mathbf{2}, \mathbf{2}; 0)$	$(\mathbf{8}, \mathbf{2}; \frac{1}{2}, 0)$	$(\mathbf{8}, \mathbf{2}; \frac{1}{2})$	$(\mathbf{45}, 2)$
		$(\mathbf{8}, \mathbf{2}; -\frac{1}{2}, 0)$	$(\mathbf{8}, \mathbf{2}; -\frac{1}{2})$	$(\overline{\mathbf{45}}, -2)$

mutation relations:

$$\{\chi_i, \chi_j^\dagger\} = \delta_{ij} \,, \tag{A.2}$$

$$\{\chi_i, \chi_j\} = 0 \,. \tag{A.3}$$

The operators defined as

$$T^i{}_j = \chi_i^\dagger \chi_j \tag{A.4}$$

satisfy the SU(N) algebra:

$$[T^i{}_j, T^k{}_l] = \delta^k{}_j T^i{}_l - \delta^i{}_l T^k{}_j \,. \tag{A.5}$$

We now introduce the ten operators

$$\Gamma_j \;=\; -\mathrm{i}(\chi_j - \chi_j^\dagger) \,, \tag{A.6}$$

$$\Gamma_{N+j} \;=\; \chi_j + \chi_j^\dagger \,, \tag{A.7}$$

which obey, by Eqns. (A.2, A.3) the Clifford algebra

$$\{\Gamma_i, \Gamma_j\} = 2\delta_{ij} \,, \tag{A.8}$$

and hence, the algebra of generators of SO($2N$) is given by

$$\Sigma_{ij} = \frac{1}{2\mathrm{i}}[\Gamma_i, \Gamma_j] \,. \tag{A.9}$$

Since the dimension of the spinor representation of SO($2N$) is 2^N, a concrete representation could be constructed for SO(10) in terms of 32×32-matrices, which however shall not be done here.

The spinor states can be constructed by letting the N creation operators $\chi_i^\dagger$ act on the "vacuum" $|0\rangle$, such that the spinor representation is 2^N-dimensional, as it should.

It is well known, that the spinor representation of SO($2N$) is reducible. We therefore define

$$\Gamma_0 = \mathrm{i}^N \prod_{i=1}^{2N} \Gamma_i = \prod_{j=1}^{N} (1 - 2n_j) \,, \tag{A.10}$$

where we have introduced the number operators

$$n_j = \chi_j^\dagger \chi_j \,. \tag{A.11}$$

The chiral projectors $\frac{1}{2}(1 \pm \Gamma_0)$ give therefore rise to the two irreducible 2^{N-1}-dimensional representations containing only even (case "+") or only odd (case "−") numbers of creation operators.

Now let Ψ be an SO($2N$) spinor state. We are interested in calculating products of the form

$$\Psi^T B \Gamma_{i_1} ... \Gamma_{i_M} \Psi \,, \tag{A.12}$$

involving a certain number of gamma matrices. The matrix B is necessary since Ψ^T does not transform as a conjugate spinor when acted upon with an infinitesimal SO(10)-transformation ϵ_{ij}:

$$\begin{aligned} \delta\Psi &= \mathrm{i}\epsilon_{ij}\Sigma_{ij}\Psi\,, \\ \delta\Psi^\dagger &= -\mathrm{i}\epsilon_{\mathrm{ij}}\Psi^\dagger\Sigma_{\mathrm{ij}}\,, \\ \delta\Psi^T &= \mathrm{i}\Psi^T\epsilon_{ij}\Sigma_{ij}^T\,. \end{aligned} \tag{A.13}$$

We require from B the property

$$B^{-1}\Sigma_{ij}^T B = -\Sigma_{ij}\,, \tag{A.14}$$

such that

$$\delta(\Psi^T B) = \mathrm{i}\epsilon_{ij}\Psi^T B B^{-1}\Sigma_{ij}^T B = -\mathrm{i}\epsilon_{ij}(\Psi^T B)\Sigma_{ij}\,, \tag{A.15}$$

i.e. $\Psi^T B$ transforms as a conjugate spinor. The condition (A.14) can be met if

$$B^{-1}\Gamma_i^T B = \pm\Gamma_i\,. \tag{A.16}$$

By choosing the minus-sign in the latter equation, we find

$$B = \prod_{i=1}^{N}\Gamma_i\,, \tag{A.17}$$

because for $i = 1, ..., N$ the Γ_i are represented by antisymmetric matrices, while for $i = N+1, ..., 2N$ by symmetric ones.

We have now reached the point to put the Standard model particles in the spin-**16** representation, which is projected out of the 32-dimensional spinor Ψ by $\frac{1}{2}(1-\Gamma_0)\Psi$. We define

$$\begin{aligned} u_i &= \frac{1}{2}\varepsilon^{ikl45}\chi_k^\dagger\chi_l^\dagger\chi_5^\dagger|0\rangle\,, \\ d_i &= \frac{1}{2}\varepsilon^{ikl45}\chi_k^\dagger\chi_l^\dagger\chi_4^\dagger|0\rangle\,, \\ u_i^c &= \chi_i^\dagger\chi_4^\dagger\chi_5^\dagger|0\rangle\,, \\ d_i^c &= \chi_i^\dagger|0\rangle\,, \\ \nu &= \chi_5^\dagger|0\rangle\,, \\ e &= \chi_4^\dagger|0\rangle\,, \\ \nu^c &= \chi_1^\dagger\chi_2^\dagger\chi_3^\dagger\chi_4^\dagger\chi_5^\dagger|0\rangle\,, \\ e^c &= \chi_1^\dagger\chi_2^\dagger\chi_3^\dagger|0\rangle\,, \end{aligned} \tag{A.18}$$

where $i, k, l = 1, 2, 3$.

The next task is to construct the charge operators. For example, the ladder operators associated with the left isospin take $u \leftrightarrow d$ and $\nu \leftrightarrow e$. They are therefore given by

$$\begin{aligned} I_L^+ &= \chi_5^\dagger \chi_4 \,, \\ I_L^- &= \chi_4^\dagger \chi_5 \,. \end{aligned} \tag{A.19}$$

The weak isospin operator is hence

$$I_L^3 = \frac{1}{2}[I_L^+, I_L^-] = \frac{1}{2}(n_5 - n_4). \tag{A.20}$$

By comparison with the charge numbers in table A.1, we can identify

$$Y = \frac{1}{3}\sum_{i=1}^{3} n_i - \frac{1}{2}\sum_{j=4}^{5} n_j = \frac{1}{12\mathrm{i}}\left([\Gamma_1,\Gamma_6] + [\Gamma_2,\Gamma_7] + [\Gamma_3,\Gamma_8]\right) - \frac{1}{8\mathrm{i}}\left([\Gamma_4,\Gamma_9] + [\Gamma_5,\Gamma_{10}]\right) \,, \tag{A.21}$$

where we have used

$$[\Gamma_{5+j}, \Gamma_j] = -4\mathrm{i} n_j + 2 \,. \tag{A.22}$$

When identifying the indices of the Γ operators with matrix rows and columns, we can explicitly write down Y in tensor representation:

$$Y = \mathrm{diag}\,(1/3, 1/3, 1/3, -1/2, -1/2) \otimes \sigma_2. \tag{A.23}$$

Likewise, we easily find the other charge operators. Putting everything together, we have in spinor and in tensor representation

$$\begin{aligned} Q &= \frac{1}{3}\sum_{i=1}^{3} n_i - n_4 = \mathrm{diag}\,(1/3, 1/3, 1/3, -1, 0) \otimes \mathrm{i}\sigma_2 \,, \\ I_L^3 &= \frac{1}{2}(n_5 - n_4) = \mathrm{diag}\,(0, 0, 0, -1/2, 1/2) \otimes \mathrm{i}\sigma_2 \,, \\ I_R^3 &= \frac{1}{2}(1 - n_4 - n_5) = \mathrm{diag}\,(0, 0, 0, -1/2, -1/2) \otimes \mathrm{i}\sigma_2 \,, \\ B - L &= \frac{2}{3}\sum_{i=1}^{3} n_i - 1 = \mathrm{diag}\,(2/3, 2/3, 2/3, 0, 0) \otimes \mathrm{i}\sigma_2 \,, \\ Y &= \frac{1}{3}\sum_{i=1}^{3} n_i - \frac{1}{2}\sum_{j=4}^{5} n_j = \mathrm{diag}\,(-1/3, -1/3, -1/3, 1/2, 1/2) \otimes \mathrm{i}\sigma_2 \,, \\ X &= -2\sum_{i=1}^{5} n_i + 5 = \mathrm{diag}\,(2, 2, 2, 2, 2) \otimes \mathrm{i}\sigma_2 \,, \end{aligned} \tag{A.24}$$

where we have used the normalization convention (A.1).

The operator representation for the charge operators $\mathcal{Q}$ is suitable for finding the charge eigenvalues q of the spinors through $\mathcal{Q}\Psi = q\Psi$.

Tensors can be constructed from the fundamental 10-dimensional vector Φ_{10} by taking antisymmetric products, such that a rank n tensor is of dimension $10 \cdot 9 \cdot ... \cdot$

$(10-n+1)/n!$. Explicitly, for the vector and the rank two tensor, the charges are given by the eigenvalue equations

$$\begin{aligned} \mathcal{Q}_q \Phi_{10} &= q\Phi_{10} \\ [\mathcal{Q}_q, \Phi_{45}] &= q\Phi_{45}\,, \end{aligned} \tag{A.25}$$

where $\mathcal{Q}$ is acting here by matrix multiplication.

A.3 The Tensor Representations

In order to perform calculations such as $\mathbf{16}.\mathbf{45}.\overline{\mathbf{16}}$, tr$\mathbf{45}^4$ and $\mathbf{16}.\mathbf{10}.\mathbf{16}$, we need to identify the Standard Model multiplets within $\mathbf{10}$ and $\mathbf{45}$, just as we did for the $\mathbf{16}$ in (A.18). We first note, that under SU(5), the fundamental representation of SO(10) decomposes as $\mathbf{10} = \mathbf{5} \oplus \bar{\mathbf{5}}$. Let us denote an element of $\mathbf{5}$ in the representation $\mathbf{10}$ of SO(10) by $\Phi^{\mathbf{5}}_{\mathbf{10}}$, an element of $\bar{\mathbf{5}}$ by $\Phi^{\bar{\mathbf{5}}}_{\mathbf{10}}$. Since they obey

$$X\Phi^{\mathbf{5}}_{\mathbf{10}} = 2\Phi^{\mathbf{5}}_{\mathbf{10}} \quad \text{and} \quad X\Phi^{\bar{\mathbf{5}}}_{\mathbf{10}} = -2\Phi^{\bar{\mathbf{5}}}_{\mathbf{10}}\,, \tag{A.26}$$

they are of the form

$$\Phi^{\mathbf{5}}_{\mathbf{10}} = \frac{1}{\sqrt{2}}\begin{pmatrix} a_1 \\ \vdots \\ a_5 \\ -\mathrm{i}a_1 \\ \vdots \\ -\mathrm{i}a_5 \end{pmatrix} \quad \text{and} \quad \Phi^{\bar{\mathbf{5}}}_{\mathbf{10}} = \frac{1}{\sqrt{2}}\begin{pmatrix} \bar{a}_1 \\ \vdots \\ \bar{a}_5 \\ \mathrm{i}\bar{a}_1 \\ \vdots \\ \mathrm{i}\bar{a}_5 \end{pmatrix}, \tag{A.27}$$

with $\sum_{i=1}^5 |a_i|^2 = 1$ and $\sum_{i=1}^5 |\bar{a}_i|^2 = 1$. To remove this inconvenient mixing of the upper and lower five-blocks, we introduce the unitary transformation

$$U_{\text{BLOCK}} = \frac{1}{\sqrt{2}}\begin{pmatrix} \mathbb{1}_5 & \mathrm{i}\mathbb{1}_5 \\ \mathbb{1}_5 & -\mathrm{i}\mathbb{1}_5 \end{pmatrix}, \tag{A.28}$$

such that

$$U_{\text{BLOCK}}\Phi^{\mathbf{5}}_{\mathbf{10}} = \begin{pmatrix} a_1 \\ \vdots \\ a_5 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad \text{and} \quad U_{\text{BLOCK}}\Phi^{\bar{\mathbf{5}}}_{\mathbf{10}} = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \bar{a}_1 \\ \vdots \\ \bar{a}_5 \end{pmatrix}. \tag{A.29}$$

By this change of basis, the charge operators become diagonal, for example

$$U_{\mathrm{BLOCK}}\mathcal{Q}_X U_{\mathrm{BLOCK}}^{-1} = 2\begin{pmatrix} \mathbb{1}_5 & 0 \\ 0 & -\mathbb{1}_5 \end{pmatrix}. \tag{A.30}$$

We can therfore immediately see how the entries of **45** transform under SU(5), namely

$$U_{\mathrm{BLOCK}}\Phi_{\mathbf{45}}U_{\mathrm{BLOCK}}^{-1} = \left(\begin{array}{c|c} \mathbf{24}\oplus\mathbf{1} & \mathbf{10} \\ \hline \overline{\mathbf{10}} & \mathbf{24}\oplus\mathbf{1} \end{array}\right), \tag{A.31}$$

where the single entries stand for 5×5-blocks and the blocks in the upper left and the lower right are to be related to each other by the factor of minus one. The SU(5)-singlet **1** has here the form of the matrix $1/\sqrt{5}\mathbb{1}_5$. The arrangement of the G_{SM}-multiplets contained in **24** can be schematically written as

$$\begin{array}{|c||c|c|} \hline & 1\quad 2\quad 3 & 4\quad 5 \\ \hline\hline \begin{matrix}1\\2\\3\end{matrix} & (\mathbf{8},\mathbf{1},0)\oplus(\mathbf{1},\mathbf{1},0) & (\mathbf{3},\mathbf{2},-\frac{5}{6}) \\ \hline \begin{matrix}4\\5\end{matrix} & (\bar{\mathbf{3}},\mathbf{2},\frac{5}{6}) & (\mathbf{1},\mathbf{3},0) \\ \hline \end{array}\;; \tag{A.32}$$

and finally **10** decomposes into

$$\begin{array}{|c||c|c|} \hline & 1\quad 2\quad 3 & 4\quad 5 \\ \hline\hline \begin{matrix}1\\2\\3\end{matrix} & (\bar{\mathbf{3}},\mathbf{1},-\frac{2}{3}) & (\mathbf{3},\mathbf{2},\frac{1}{6}) \\ \hline \begin{matrix}4\\5\end{matrix} & -(\mathbf{3},\mathbf{2},\frac{1}{6}) & (\mathbf{1},\mathbf{1},1) \\ \hline \end{array}\;, \tag{A.33}$$

where the matrix is imposed to be antisymmetric, since it is identified with the anti-symmetric part of $\mathbf{5}\otimes\mathbf{5}$ of SU(5).

Appendix B

Integrals for Lamb Shift Calculation

For the calculation of Lamb shift in FLRW-background, we need the following integrals:

$$\begin{aligned} I_1 &= \int \frac{1}{(\Delta E - \omega)\omega^3} \\ &= -\frac{k}{\Delta E \omega} - \frac{m}{\Delta E^2} \arctan \frac{k}{m} + \frac{\sqrt{\Delta E^2 - m^2}}{\Delta E^2} \frac{1}{2} \log \left| \frac{\sqrt{\Delta E^2 - m^2} + k}{\sqrt{\Delta E^2 - m^2} - k} \right| \\ &\quad + \frac{\sqrt{\Delta E^2 - m^2}}{\Delta E^2} \frac{1}{2} \log \left| \frac{\sqrt{\Delta E^2 - m^2}\omega + \Delta E k}{\sqrt{\Delta E^2 - m^2}\omega - \Delta E k} \right| , \end{aligned} \tag{B.1}$$

$$\begin{aligned} I_2 &= \int \frac{1}{(\Delta E - \omega)\omega^5} \\ &= -\frac{1}{2} \frac{k}{\Delta E^2 \omega^2} + \frac{1}{3} \frac{\Delta E^2 k^3 - 3km^2\omega^2}{\Delta E^3 m^2 \omega^3} + \frac{1}{2} \frac{\Delta E^2 - 2m^2}{\Delta E^4 m} \arctan \frac{k}{m} \\ &\quad + \frac{\sqrt{\Delta E^2 - m^2}}{\Delta E^4} \frac{1}{2} \log \left| \frac{\sqrt{\Delta E^2 - m^2} + k}{\sqrt{\Delta E^2 - m^2} - k} \right| \\ &\quad + \frac{\sqrt{\Delta E^2 - m^2}}{\Delta E^4} \frac{1}{2} \log \left| \frac{\sqrt{\Delta E^2 - m^2}\omega + \Delta E k}{\sqrt{\Delta E^2 - m^2}\omega - \Delta E k} \right| , \end{aligned} \tag{B.2}$$

$$\begin{aligned} I_3 &= \int \frac{1}{(\Delta E - \omega)\omega^7} \\ &= -\frac{1}{4} \frac{k}{\Delta E^2 \omega^4} + \frac{1}{15} \frac{5\Delta E^2 k^3 m^2 \omega^2 - 15km^4\omega^4 + \Delta E^4(2k^5 + 5k^3m^2)}{\Delta E^5 m^4 \omega^5} \\ &\quad + \frac{1}{8} \frac{\Delta E^4 + 4m^2(\Delta E^2 - 2m^2)}{\Delta E^6 m^3} \arctan \frac{k}{m} \\ &\quad + \frac{\sqrt{\Delta E^2 - m^2}}{\Delta E^6} \frac{1}{2} \log \left| \frac{\sqrt{\Delta E^2 - m^2} + k}{\sqrt{\Delta E^2 - m^2} - k} \right| \end{aligned} \tag{B.3}$$

$$+\frac{\sqrt{\Delta E^2-m^2}}{\Delta E^6}\frac{1}{2}\log\left|\frac{\sqrt{\Delta E^2-m^2}\omega+\Delta Ek}{\sqrt{\Delta E^2-m^2}\omega-\Delta Ek}\right|,$$

$$\begin{aligned}
J_1 &= \int\frac{1}{(\Delta E-\omega)^2\omega^2} \\
&= \frac{k}{\Delta E^2-\omega^2}+\frac{k\omega}{\Delta E(\Delta E^2-\omega^2)}-\frac{m}{\Delta E^2}\arctan\frac{k}{m} \\
&\quad -\frac{m^2}{\Delta E^2\sqrt{\Delta E^2-m^2}}\frac{1}{2}\log\left|\frac{\sqrt{\Delta E^2-m^2}+k}{\sqrt{\Delta E^2-m^2}-k}\right| \\
&\quad -\frac{m^2}{\Delta E^2\sqrt{\Delta E^2-m^2}}\frac{1}{2}\log\left|\frac{\sqrt{\Delta E^2-m^2}\omega+\Delta Ek}{\sqrt{\Delta E^2-m^2}\omega-\Delta Ek}\right|,
\end{aligned} \tag{B.4}$$

$$\begin{aligned}
J_2 &= \int\frac{1}{(\Delta E-\omega)^2\omega^4} \\
&= -\frac{1}{2}\frac{k(\Delta E^2-3\omega^2)}{\Delta E^2\omega^2(\Delta E^2-\omega^2)}-\frac{k(2\Delta E^2-3\omega^2)}{\Delta E^3\omega(\Delta E^2-\omega^2)}+\frac{1}{2}\frac{\Delta E^2-6m^2}{\Delta E^4 m}\arctan\frac{k}{m} \\
&\quad -\frac{2\Delta E^2-3m^2}{\Delta E^4\sqrt{\Delta E^2-m^2}}\frac{1}{2}\log\left|\frac{\sqrt{\Delta E^2-m^2}+k}{\sqrt{\Delta E^2-m^2}-k}\right| \\
&\quad -\frac{2\Delta E^2-3m^2}{\Delta E^4\sqrt{\Delta E^2-m^2}}\frac{1}{2}\log\left|\frac{\sqrt{\Delta E^2-m^2}\omega+\Delta Ek}{\sqrt{\Delta E^2-m^2}\omega-\Delta Ek}\right|,
\end{aligned} \tag{B.5}$$

$$\begin{aligned}
J_3 &= \int\frac{1}{(\Delta E-\omega)^2\omega^6} \\
&= -\frac{1}{4}\frac{k}{\Delta E^2\omega^4}+\frac{1}{8}\frac{k(\Delta E^2-12m^2)}{\Delta E^4 m^2\omega^2}+\frac{k}{\Delta E^2(\Delta E^2-\omega^2)} \\
&\quad +\frac{1}{3}\frac{k\left[2\Delta E^4k^2+15m^2\omega^4-2\Delta E^2(k^4+7k^2m^2+6m^4)\right]}{\Delta E^5m^2\omega^3(\Delta E^2-\omega^2)} \\
&\quad +\frac{1}{8}\frac{\Delta E^4+4m^2(3\Delta E^2-10m^2)}{\Delta E^6m^3}\arctan\frac{k}{m} \\
&\quad -\frac{4\Delta E^2-5m^2}{\Delta E^6\sqrt{\Delta E^2-m^2}}\frac{1}{2}\log\left|\frac{\sqrt{\Delta E^2-m^2}+k}{\sqrt{\Delta E^2-m^2}-k}\right| \\
&\quad -\frac{4\Delta E^2-5m^2}{\Delta E^6\sqrt{\Delta E^2-m^2}}\frac{1}{2}\log\left|\frac{\sqrt{\Delta E^2-m^2}\omega+\Delta Ek}{\sqrt{\Delta E^2-m^2}\omega-\Delta Ek}\right|.
\end{aligned} \tag{B.6}$$

We evaluate the above integrals at their boundaries and obtain

$$[I_1]_0^\infty = -\frac{1}{\Delta E}-\frac{\pi}{2}\frac{m}{\Delta E^2}+\frac{\sqrt{\Delta E^2-m^2}}{\Delta E^2}\frac{1}{2}\log\left|\frac{\sqrt{\Delta E^2-m^2}+\Delta E}{\sqrt{\Delta E^2-m^2}-\Delta E}\right|, \tag{B.7}$$

$$[I_2]_0^\infty = \frac{1}{3}\frac{\Delta E^2-3m^2}{\Delta E^3m^2}+\frac{\pi}{4}\frac{\Delta E^2-2m^2}{\Delta E^4 m} \tag{B.8}$$

$$+\frac{\sqrt{\Delta E^2 - m^2}}{\Delta E^4}\frac{1}{2}\log\left|\frac{\sqrt{\Delta E^2 - m^2}+\Delta E}{\sqrt{\Delta E^2 - m^2}-\Delta E}\right| ,$$

$$\begin{aligned}[I_3]_0^\infty &= -\frac{1}{\Delta E^5} + \frac{2}{15}\frac{1}{\Delta E m^4} + \frac{\pi}{16}\frac{\Delta E^4 + 4m^2(\Delta E^2 - 2m^2)}{\Delta E^6 m^3} \\ &\quad +\frac{\sqrt{\Delta E^2 - m^2}}{\Delta E^6}\frac{1}{2}\log\left|\frac{\sqrt{\Delta E^2 - m^2}+\Delta E}{\sqrt{\Delta E^2 - m^2}-\Delta E}\right| ,\end{aligned} \tag{B.9}$$

$$[J_1]_0^\infty = -\frac{1}{\Delta E} - \frac{\pi}{2}\frac{m}{\Delta E^2} - \frac{m^2}{\Delta E^2\sqrt{\Delta E^2 - m^2}}\frac{1}{2}\log\left|\frac{\sqrt{\Delta E^2 - m^2}+\Delta E}{\sqrt{\Delta E^2 - m^2}-\Delta E}\right| , \tag{B.10}$$

$$\begin{aligned}[J_2]_0^\infty &= \frac{3}{2}\frac{1}{\Delta E^3} + \frac{\pi}{4}\frac{\Delta E^2 - 6m^2}{\Delta E^4 m} \\ &\quad +\frac{2\Delta E^2 - 3m^2}{\Delta E^4\sqrt{\Delta E^2 - m^2}}\frac{1}{2}\log\left|\frac{\sqrt{\Delta E^2 - m^2}+\Delta E}{\sqrt{\Delta E^2 - m^2}-\Delta E}\right| ,\end{aligned} \tag{B.11}$$

$$\begin{aligned}[J_3]_0^\infty &= -\frac{1}{5}\frac{1}{\Delta E^5} + \frac{2}{3}\frac{1}{\Delta E^3 m^2} + \frac{\pi}{16}\frac{\Delta E^4 + 4m^2(3\Delta E^2 - 10m^2)}{\Delta E^6 m^3} \\ &\quad +\frac{4\Delta E^2 - 5m^2}{\Delta E^6\sqrt{\Delta E^2 - m^2}}\frac{1}{2}\log\left|\frac{\sqrt{\Delta E^2 - m^2}+\Delta E}{\sqrt{\Delta E^2 - m^2}-\Delta E}\right| .\end{aligned} \tag{B.12}$$

When expanded up to second order in $1/\Delta E$, the above expressions read

$$[I_1]_0^\infty \simeq -\frac{1}{\Delta E} - \frac{\pi}{2}\frac{m}{\Delta E^2} + \frac{1}{\Delta E}\log\left|\frac{2\Delta E}{m}\right| , \tag{B.13}$$

$$[I_2]_0^\infty \simeq \frac{1}{3}\frac{1}{\Delta E m^2} + \frac{\pi}{4}\frac{1}{\Delta E^2 m} , \tag{B.14}$$

$$[I_3]_0^\infty \simeq \frac{2}{15}\frac{1}{\Delta E m^4} + \frac{\pi}{16}\frac{1}{\Delta E^2 m^3} , \tag{B.15}$$

$$[J_1]_0^\infty \simeq -\frac{1}{\Delta E} - \frac{\pi}{2}\frac{m}{\Delta E^2} , \tag{B.16}$$

$$[J_2]_0^\infty \simeq \frac{\pi}{4}\frac{1}{\Delta E^2 m} , \tag{B.17}$$

$$[J_3]_0^\infty \simeq \frac{\pi}{16}\frac{1}{\Delta E^2 m^3} . \tag{B.18}$$

The integrals which we need for obtaining the Lamb shift in Rindler space are

$$\begin{aligned}[R_1]_0^\infty &= \int_0^\infty dk \frac{1}{(k^2+m^2)^{5/2}}\frac{1}{\Delta E - \sqrt{k^2+m^2}} \\ &= \frac{2}{3}\frac{1}{\Delta E m^4} + \frac{1}{\Delta E^3 m^2} + \frac{\pi}{4}\frac{\Delta E^2 + 2m^2}{\Delta E^4 m^3} \\ &\quad +\frac{1}{\Delta E^4\sqrt{\Delta E^2 - m^2}}\frac{1}{2}\log\left|\frac{1+\Delta E/\sqrt{\Delta E^2 - m^2}}{1-\Delta E/\sqrt{\Delta E^2 - m^2}}\right| ,\end{aligned} \tag{B.19}$$

$$
\begin{aligned}
[R_2]_0^\infty &= \int_0^\infty dk \frac{1}{(k^2+m^2)^3} \frac{1}{\left(\Delta E - \sqrt{k^2+m^2}\right)^2} \\
&= \frac{4}{3}\frac{1}{\Delta E m^4(\Delta E^2-m^2)} + \frac{8}{3}\frac{1}{\Delta E^3 m^2(\Delta E^2-m^2)} - 5\frac{1}{\Delta E^5(\Delta E^2-m^2)} \\
&\quad + \frac{3\pi}{16}\frac{1}{m^5(\Delta E^2-m^2)} + \frac{9\pi}{16}\frac{1}{\Delta E^2 m^3(\Delta E^2-m^2)} + \frac{7\pi}{4}\frac{1}{\Delta E^4 m(\Delta E^2-m^2)} \\
&\quad - \frac{5\pi}{2}\frac{m}{\Delta E^6(\Delta E^2-m^2)} \\
&\quad + \frac{1}{2}\frac{5m^2-6\Delta E^2}{\Delta E^6(\Delta E^2-m^2)^{3/2}}\frac{1}{2}\log\left|\frac{1+\Delta E/\sqrt{\Delta E^2-m^2}}{1-\Delta E/\sqrt{\Delta E^2-m^2}}\right| .
\end{aligned}
\tag{B.20}
$$

Appendix C

Ultraviolet Behaviour of Rindler Modes

We provide here a systematic expansion of the mode functions (7.31) in Rindler space in the ultraviolet domain, that is where $\lambda \gg \alpha, m$. All terms involving powers up to α^2 and m^4 are displayed. Since $|\lambda|/\alpha \gg 1$, we need an asymptotic expansion of Bessel functions of large order, which is given by the approximation by tangents [108]:

$$
\begin{aligned}
J_{\mathrm{i}\frac{\lambda}{\alpha}}\left(\mathrm{i}\frac{m}{\alpha}\mathrm{e}^{\xi}+\varepsilon\right) \sim \frac{\mathrm{e}^{\mathrm{i}\frac{\lambda}{\alpha}(\tanh\beta-\beta)-\frac{1}{4}\pi\mathrm{i}}}{\left(2\pi\frac{\lambda}{\alpha}\tanh\beta\right)^{1/2}}\Bigg\{1-\mathrm{i}\frac{\alpha}{\lambda}\left(\frac{1}{8}\coth\beta-\frac{5}{24}\coth^3\beta\right) \\
-\frac{\alpha^2}{\lambda^2}\left(\frac{9}{128}\coth^2\beta-\frac{231}{576}\coth^4\beta+\frac{1155}{3456}\coth^6\beta\right)+O\left(\frac{\alpha^3}{\lambda^3}\right)\Bigg\},\ \text{for}\ \lambda>0,
\end{aligned}
\tag{C.1}
$$

where

$$
\cosh\beta = \frac{\lambda}{m}\mathrm{e}^{\xi}+\mathrm{i}\varepsilon, \quad \tanh\beta = \left(1-\frac{m^2}{\lambda^2}\mathrm{e}^{2\xi}\right)^{\frac{1}{2}}.
$$

The expansion (C.1) corresponds to the region 2 of figure 22 in Ref. [108].

When $\lambda < 0$ however, for the argument z of the Bessel function in (7.31) holds $\arg(z) < -\pi/2$, and we cannot make a straightforward use of the approximation by tangents from Ref. [108]. Nevertheless, in this case we can use the following general relations (from [108]),

$$
\begin{aligned}
J_{\mathrm{i}\lambda/\alpha}\left(\mathrm{i}\frac{m}{\alpha}\mathrm{e}^{\xi}-\varepsilon\right) &= \left[J_{-\mathrm{i}\lambda/\alpha}\left(\mathrm{e}^{-\mathrm{i}\pi}\mathrm{i}\frac{m}{\alpha}\mathrm{e}^{\xi}+\mathrm{e}^{-\mathrm{i}\pi}\varepsilon\right)\right]^* \\
&= \mathrm{e}^{-\frac{\pi\lambda}{\alpha}}\left[J_{-\mathrm{i}\lambda/\alpha}\left(\mathrm{i}\frac{m}{\alpha}\mathrm{e}^{\xi}+\varepsilon\right)\right]^*
\end{aligned}
\tag{C.2}
$$

to bring the argument of the Bessel function in (7.31) into the region of validity ($|\arg(z)| < \pi/2$) of the approximation by tangents. Indeed, since the argument fulfills $\arg\big(\mathrm{i}(m/\alpha)\mathrm{e}^{\xi}+\varepsilon\big) < \pi/2$, we can use the approximation by tangents ($\cosh(\beta) = \nu/z$

lies again in the region 2 of figure 22 in Ref. [108])

$$J_{\mathrm{i}\frac{\lambda}{\alpha}}\Big(\mathrm{i}\frac{m}{\alpha}\mathrm{e}^{\xi}-\varepsilon\Big) \sim \mathrm{e}^{-\pi\lambda/\alpha}\frac{\mathrm{e}^{\mathrm{i}\frac{\lambda}{\alpha}(\tanh\beta-\beta)+\frac{1}{4}\pi i}}{\big(-2\pi\frac{\lambda}{\alpha}\tanh\beta\big)^{1/2}}\Bigg\{1-\mathrm{i}\frac{\alpha}{\lambda}\left(\frac{1}{8}\coth\beta-\frac{5}{24}\coth^3\beta\right) \tag{C.3}$$
$$-\frac{\alpha^2}{\lambda^2}\left(\frac{9}{128}\coth^2\beta-\frac{231}{576}\coth^4\beta+\frac{1155}{3456}\coth^6\beta\right)+O\left(\frac{\alpha^3}{\lambda^3}\right)\Bigg\}\ ,\text{for } \lambda<0.$$

The following expressions, which are valid for both for $\lambda < 0$ and $\lambda > 0$, completely specify β,

$$\coth\beta \;=\; \Big(1-\frac{m^2}{\lambda^2}\mathrm{e}^{2\xi}\Big)^{-\frac{1}{2}}\,, \tag{C.4}$$
$$\cosh\beta \;=\; \frac{|\lambda|}{m}\mathrm{e}^{\xi}-\mathrm{i}\varepsilon\mathrm{sign}(\lambda)\,. \tag{C.5}$$

We can now write a general approximation by tangents for the Rindler modes (7.31) (valid for any $|\lambda| \gg \alpha, m$):

$$\varphi_\lambda \sim \frac{\mathrm{e}^{-\mathrm{i}|\lambda|\tau+\frac{\pi}{2}\frac{|\lambda|}{\alpha}}}{\sqrt{2|\lambda|}}\Gamma\Big(1+\mathrm{i}\frac{\lambda}{\alpha}\Big)\frac{\mathrm{e}^{\mathrm{i}\frac{\lambda}{\alpha}(\tanh\beta-\beta)-\mathrm{i}\frac{\pi}{4}\mathrm{sign}(\lambda)}}{\big[2\pi(|\lambda|/\alpha)\tanh\beta\big]^{1/2}}\Bigg\{1-\mathrm{i}\frac{\alpha}{\lambda}\Big(\frac{1}{8}\coth\beta-\frac{5}{24}\coth^3\beta\Big)$$
$$-\frac{\alpha^2}{\lambda^2}\left(\frac{9}{128}\coth^2\beta-\frac{231}{576}\coth^4\beta+\frac{1155}{3456}\coth^6\beta\right)+O\left(\frac{\alpha^3}{\lambda^3}\right)\Bigg\} \tag{C.6}$$

Upon expanding φ_λ in powers of α/λ, $(m/\lambda)^2$ and ξ we get (up to corrections of order $O\big((\alpha/\lambda)^3, \xi^2, (m/\lambda)^6\big)$,

$$\begin{aligned}\varphi_\lambda \simeq\ & \mathrm{e}^{-\mathrm{i}|\lambda|\tau}\frac{\alpha^{1/2}}{(4\pi)^{1/2}|\lambda|}\mathrm{e}^{\frac{\pi}{2}\frac{|\lambda|}{\alpha}-\mathrm{i}\frac{\pi}{4}\mathrm{sign}(\lambda)}\Gamma\Big(1+\mathrm{i}\frac{\lambda}{\alpha}\Big) \\ &\times\exp\left[\mathrm{i}\frac{\lambda}{\alpha}\left(1-\log\left(\frac{2|\lambda|}{m}\right)-\frac{1}{4}\frac{m^2}{\lambda^2}-\frac{1}{8}\frac{m^4}{\lambda^4}\right)\right]\times\exp\left[\mathrm{i}\xi\frac{\lambda}{\alpha}\left(1-\frac{m^2}{2\lambda^2}-\frac{m^4}{8\lambda^4}\right)\right] \\ &\times\left\{1+\frac{1}{4}\frac{m^2}{\lambda^2}+\frac{5}{32}\frac{m^4}{\lambda^4}+\frac{1}{2}\xi\frac{m^2}{\lambda^2}+\frac{5}{8}\xi\frac{m^4}{\lambda^4}\right\} \\ &\times\left\{1+\frac{\mathrm{i}}{12}\frac{\alpha}{\lambda}\left[1+3\frac{m^2}{\lambda^2}+\frac{33}{8}\frac{m^4}{\lambda^4}+6\xi\frac{m^2}{\lambda^2}+\frac{33}{2}\xi\frac{m^4}{\lambda^4}\right]\right. \\ &\quad\left.-\frac{1}{288}\frac{\alpha^2}{\lambda^2}\left[1+78\frac{m^2}{\lambda^2}+\frac{1005}{4}\frac{m^4}{\lambda^4}+156\xi\frac{m^2}{\lambda^2}+1005\xi\frac{m^4}{\lambda^4}\right]\right\}\,.\end{aligned} \tag{C.7}$$

For $\lambda \to \infty$, this reduces to a plane wave solution as it should, since the acceleration parameter α becomes irrelevant.

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