Essays on the Economics of Child Labor and Fertility

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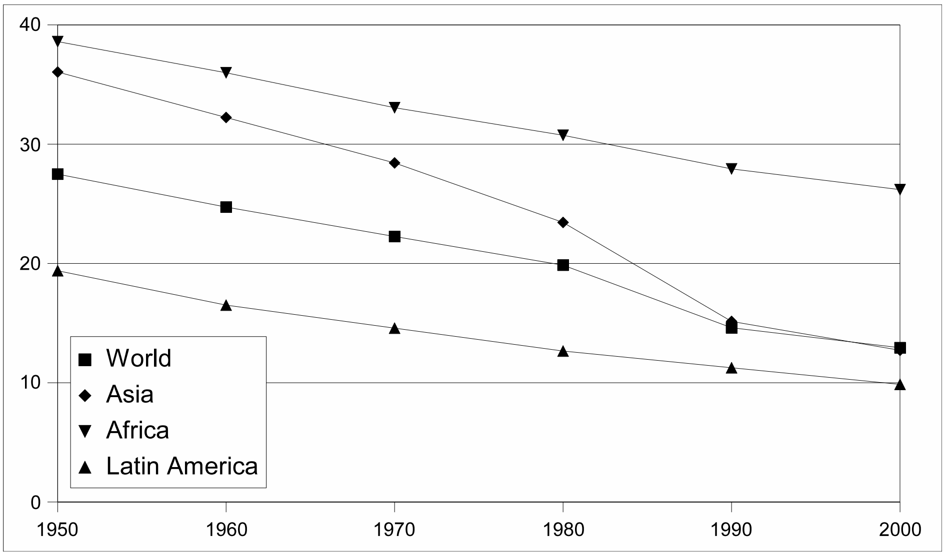
Introduction and Overview

Labor has been a part of childhood for all but short times in the development of humankind. In ancient Greece and Rome, for example, children were often born or sold into slavery. Although they were sometimes protected by laws which prevented excessive abuse, few of them enjoyed education, and many had to work in appalling conditions, e.g. in mines. In Europe in the middle ages, when formal education was a privilege of the rich, virtually all children worked, helping their parents with the fieldwork, tending livestock or doing household chores. Apart from monasteries, where future monks learned how to read and write, the only form of education open to most non-noble boys was an apprenticeship with a craftsman. This started when the child was still very young, by the age of 7, and ended up to 10 years later. Formal education became available in large towns during the 15th century. However, as schooling was neither free nor compulsory, many poor families could not afford to educate their children, who had to work to support their parents. At the time of the industrial revolution, child labor in its worst forms was common in the new factories and mills. As many families moved from the land to town in order to work in the new industries, labor was cheap, and all family members had to work to make a living. Orphans, however, were even worse off, as they had no parents to protect them from abuse, and sometimes had to work without pay. Great Britain was the first country in modern history to pass a child labor act, the Factory Act of 1833, which limited working hours and prohibited children younger than 9 years of age from working at all. Child labor, however, was still a problem in Europe and the United States well into the 20th century.

Figure 1: Economically active children aged 10-14: participation rates. Source: ILO



At present, child labor is insignificant in most industrial countries, with less than one out of 3000 children younger than 14 working in Europe, according to the International Labor Office (ILO). The situation is much worse in other countries and regions, particularly Africa and Asia. About 40% of all children between 10 and 14 years of age were economically active in Eastern Africa in 2000, the number being somewhat lower for the whole continent, where about than one in four children worked. As can be seen from figure 1, the percentage of economically active children is highest in Africa, and about half as high in Asia and Latin America. During the whole period for which data is available, labor market participation rates were falling, on average, by 3 percentage points per decade. While it is clear that working in hazardous conditions for long hours with little pay is undesirable, not all forms of economic activity captured by the numbers in figure 1 are to be condemned. The ILO defines economic activity as ”all market production (paid work) and certain types of non-market production (unpaid work), including production for own use. [...] paid or unpaid, the activity or occupation could be in the formal or informal sector in urban or rural areas”[1]. While this includes ”child labor”, ”children in hazardous work” and ”children in unconditional worst forms of child labor”, it does not include children who perform household chores, which are non-economic activities according to the

[1]Source: Every Child Counts: New Global Estimates on Child Labour, International Labour Office, Geneva, April 2002

ILO's definition, and therefore do not appear in the its statistics. Note that some forms of economic activity, such as light work for a few hours a week, can provide families with the income they need to afford schooling, and can even be part of the education, e.g. in apprenticeships. On the other hand, the non-economic activities which are not captured by the ILO's estimates are often considerable, and can prevent children from attending school.

This distinction is not normally made in the theoretical literature on child labor. Most authors assume that children can either work, and therefore contribute to the household's income, or go to school, thereby foregoing wages. Non-wage work, such as caring for the siblings and households chores, is ignored, as is paid work which is part of the child's education.

The literature on child labor has been reviewed by Basu (1999) and by Brown, Deardorff and Stern (2003). During the 1990s and early 2000s, the literature departed from the analysis of unitary models to consider issues of household bargaining, especially on an intra-household level (Moehling (1995), Basu and Ray (2002)).

More recent literature addresses the possibility of multiple equilibria. Basu and Van (1998), for example, assume that child and adult labor are substitutes, and that parents only send children to work if the household income lies below some exogenous threshold. They consider an economy with inhomogeneous households, which differ with respect to the wage levels for which they send all their children to work, so that the total amount of child labor in an economy will depend on the adult's wage. The authors then show that multiple stable equilibria can exist: in one, all children work and wages are low, and in the other wages are high and children do not work at all. If indeed two such equilibria exist, a governmental ban on child labor, if enforced, will result in the economy switching from the low-wage to the high-wage equilibrium, if the former ruled before the ban.

While the early literature condemned child labor as an evil in itself, the more recent literature focuses on the formation of human capital through education, and on the long-term adverse effects of child labor on the whole economy. Child labor, as opposed to education, is assumed to have negative external effects on the whole society, as children who are not educated will themselves be too poor to educate their own children, thereby impeding economic growth. Baland and Robinson (2000) and Ranjan (1999) show that in the absence of perfect capital markets in which parents can borrow to finance education, child labor will be (inefficiently) high. Both papers focus on bans of child labor as policy interventions, and Baland and Robinson show that, in some cases, a ban can be Pareto improving.

As bans on child labor are often hard to enforce, other, fiscal, measures might be more effective. Bell and Gersbach (2001) use an OLG framework in which individuals live for two periods, where the level of human capital of an adult determines his or her output and income, and human capital is formed through a process involving child-rearing and formal education. Parents are altruistic towards their children, but there are no bequests. Multiple stable steady states are possible in such a setting, such as a poverty trap in which all children work full-time, and human capital is at its minimum at all times. Another steady state is one in which continuous growth is possible. Escaping the poverty trap is possible, even without outside aid, through policy programs consisting of taxes and subsidies. However, all such programs will lead to inequality, at least temporarily.

With few exceptions (notably Rosenzweig and Evenson (1977) and Baland and Robinson (2000)), the literature on child labor has assumed the number of children to be exogenous. However, in his overview article Basu (1999) notes that: "In reality, the number of children is partly volitional, and the decision may well depend on whether children can find work. [...] There is need for such a model [...]." When deciding about schooling, parents must, in general, choose between current consumption, which is financed in part by child labor, and future consumption, financed by an investment in education. When fertility and schooling decisions are both endogenous, there is an additional tradeoff between quantity (i.e. high fertility, but low schooling) and quality (i.e. low fertility and more schooling). The aim of the present dissertation is to analyze household decisions and their long-term effects on the dynamics of an economy, as well as potential policy options, when families decide both about the number of children they intend to raise and level of schooling these are to enjoy. Before turning to an outline of the work before us, however, a brief discussion of the existing literature on fertility, particularly in the context of child labor, is needed.

One of the first articles to address the issue of endogenous fertility and the dynamics of such a model is Raut and Srinivasan (1994). The authors consider an OLG model in which individuals live for three periods. Parents have an incentive to raise many children as a means of financing their old age. However, as raising children is costly, the number of children a family can afford to raise is limited. The dynamics of the system depend on the costs of raising children, and the authors show that steady states with exponential growth of population and constant per capita income are possible. Raut and Srinivasan, however, do not consider child labor and human capital formation.

Becker, Murphy and Tamura (1990) consider a setting with endogenous fertility, where economic growth stems from human capital accumulation. The authors model human capital accumulation through investment of human capital itself, and do not consider child labor. This investment exhibits increasing returns at least for some (low) levels of human capital. The authors show that investing in children is less profitable than investing in human capital if human capital is abundant, and vice versa. Consequently, two stable steady states exist: if the initial level of human capital in a society is low, families will have many children and invest little in their education, whereas families whose initial level of human capital is high will tend to be smaller and their members better-educated.

Dessy (2000) is among the first to analyze the dynamics of a system in which parents decide both about child labor and fertility. He considers an infinitely lived dynasty in which the dynasty head derives utility from the levels of consumption and population sizes in all future periods. Parents need to invest some of their time to raise children, but there are no further child-rearing costs. Education is assumed to be free. If the parents' wage is low, fertility and child labor will be high, as the relative costs of raising children are low. If, however, the parents' wage lies above some threshold, children enjoy full-time schooling. Note that this result resembles the one obtained by Basu and Van (1998). An economy starting from a sufficiently low adult wage will always end up in the child labor trap. The author argues that escaping this trap is possible if there is compulsory schooling and the legislation on education is at least partially enforceable. In this case, both fertility and the incidence of child labor will fall, while the adults' consumption will remain unchanged.

Hazan and Berdugo (2002) focus on the effects of technological progress on the dynamics of child labor, fertility and human capital over time. They assume that parents decide about their children's allocation of time between working and schooling, that the incomes of all household members are pooled, that parents care about the future earnings of their children, and that they need to devote a fraction of their own adulthood to child-rearing. To model technological progress, the authors assume there are two sectors, one a traditional sector which employs raw labor only, and the other a modern sector, employing both raw and efficiency labor, as well as physical capital. The level of technology in the modern sector can be increased through investment. As long as the level of technology in the modern sector is low, fertility and child labor are high. Technological progress decreases the relative wage of child labor and therefore encourages education. Even if the economy is initially in the low steady state it can escape to sustained growth, with rising education and decreasing fertility.

Another strand of literature focuses on the impact of mortality on household decisions regarding children. Chakraborty and Das (2005) consider a three-period OLG setting in which adults can die prematurely at the end of the second period of adult life, before reaching the third. An individuals' likelihood of dying depends on her spending on health. Young parents decide about fertility, child labor and schooling, consumption and investment in health. There are no capital markets, and individuals in the last age group cannot work, so they must rely on transfers from their children to finance old-age consumption. If the parents are sufficiently wealthy (i.e. productive), investment in health can significantly reduce the probability of premature mortality. As a consequence, old-age consumption becomes more important, and investing in education is profitable. Rich parents, therefore, will have few but well-educated children and invest in health. If, on the other hand, parents are poor, they will have more children than rich families, but these children will have to work full-time, rather than go to school. Chakraborty and Das assume that the individual's mortality rate is endogenous, and depends on her spending on health. While this is certainly true with respect to some diseases, such as malaria, where private investment can reduce the likelihood of infection (for instance by buying mosquito nets), there are other diseases where private spending is unlikely to influence the survival probability. One such disease is AIDS. Particularly in developing countries, individuals often do not know how the HIV virus is transmitted, and therefore cannot invest efficiently in protecting themselves from infection. And with medical treatment being very expensive, families in developing countries cannot afford to pay for it when one of their members is HIV positive. Therefore mortality is generally treated as exogenous when analyzing the effects of HIV/AIDS on developing countries, their individuals and economies.

In 2001, AIDS was the fourth-biggest cause of death worldwide, and had lead to a sharp surge in premature adult mortality in some countries. The disease affects mostly young people, who are among the most productive in society. They are responsible for bearing and rearing children, so that their premature death is likely to have a deep impact on the population and economy of the affected country. When the parents are ill, the household's income is reduced and children might have to work more to support their parents and siblings. Similarly, if the survival probability is low, parents could decide to invest less in their children, to reduce fertility or both. As the HIV/AIDS prevalence rates differ substantially among different countries, the economic effects of the disease must be assessed separately for each country. In general, the literature on the economy of HIV/AIDS does not consider the epidemic's effects on human capital. We will review this literature in the third part of the present work, and focus here on articles which analyze human capital in connection with HIV/AIDS.

Bell, Devarajan and Gersbach (2003) consider a 2-period OLG setting, in which either or both of the parents can die prematurely due to HIV/AIDS. Premature death occurs after individuals give birth to their children and before the children reach school-going age; fertility is exogenous. Human capital is formed through formal education at school at the expense of child labor. There are no bequests, and no capital markets. Parents, who are altruistic toward their children, are the sole decision-makers in the economy, and they decide about their own consumption and the level of schooling of their children. As a consequence, the level of human capital attained by a young adult depends on whether her parents survive. The economy has several equilibria, among them a poverty trap where children work full-time and a growth equilibrium if the education technology is sufficiently productive and mortality is sufficiently low. The theoretical model is calibrated to South African data for the years 1960 – 2000. The calibration and its results are used to compute projections of the South African economy until 2080. The authors analyze two different social models: first, the pooling case, where surviving individuals form new couples and all orphans are taken in by relatives. In the second, there are nuclear families, surviving individuals do not form new couples, and full orphans are left to be taken care of by the state. They also analyze different governmental interventions aimed at combating the disease, such as spending on health or school-attendance subsidies. If there had been no HIV/AIDS epidemic, the economy would have grown sustainably, with all children enjoying full-time schooling after 2020. With the outbreak of the epidemic, however, the economy collapses into the poverty trap. Governmental interventions in the health sector combined with support for orphans and poor families can avert this outcome.

Ferreira and Pessoa (2003) consider a continuous-time model with premature mortality due to HIV/AIDS where an individual's decision about schooling depends on her life expectancy. Human capital is formed through formal education. Individuals decide about the point of time when they leave school and enter the labor market, and about consumption. Thus children themselves can decide about their level of schooling, and the parents play no active role in making that decision. In making this assumption, the authors depart from the setting used generally in the child labor literature. They arrive at the conclusion that HIV/AIDS has a strong impact on long-term growth, as individuals reduce formal education when their life expectancy falls. The model is applied to several African countries. The authors show that schooling falls, on average, by half, while income falls by about a quarter in the presence of the HIV/AIDS epidemic.

Corrigan, Glomm and Mendez (2004) consider a 2-generation OLG model where individuals can die prematurely before the start of the last period of life. An individual's level of human capital does not depend on formal education directly, but only on whether her parent survived or not. Child labor, therefore, plays no role. To finance old-age consumption, individuals save some of their labor income while young. They also work while old, and enjoy the returns from their savings. The authors undertake projections of the economies of several countries in Sub-Saharan Africa. They consider several scenarios, depending on the duration and strength of HIV/AIDS mortality shock. Growth rates in all scenarios fall when the epidemic breaks out, and recover if the shock is not permanent, that is, if morality rates subsequently return to their pre-epidemic level. The authors provide an extended version of their initial model in Corrigan, Glomm and Mendez (2005), where individuals live for up to three periods. The process of human capital formation is not only dependent on the parents' survival, but also on the time the child spends pursuing formal education rather than working. In addition to deciding about their own consumption and savings, parents, who are assumed to be altruistic, now also decide about their offsprings' schooling and consumption. If an individual is infected with the

HIV virus, he also decides about spending on medical treatment. Sick individuals do not save for old-age consumption. The effects of the HIV/AIDS epidemic are large, reducing the current income after 10 generations by 5-45% of potential NO-AIDS income, depending of the scenario.

Bell et al. (2004) analyze the long-run effects of the HIV/AIDS epidemic on the Kenyan economy. They employ an OLG model with three generations, where children can either work or go to school, while parents and grandparents work full-time. Parents are altruistic toward their children and decide about consumption and education. All orphans are taken in by relatives, that is, there is pooling. As a consequence, an individual's level of human capital is independent of the health status of his parents but it does depend on schooling and on the parents' expectations about mortality rates, which influence their decisions about education. As in Bell, Devarajan and Gersbach (2003), it is assumed that there are no capital markets. The authors find that GDP in the AIDS case is lower by 40% compared to the NO AIDS case. They identify three major reasons for this effect: the first is a deterioration in the educational system, which was significantly less productive after the 1980s than before. Second, there is a fall in the general productivity in the economy, which took place in the 1990s, and, third, there is the HIV/AIDS epidemic.

As noted previously, the formation of human capital through education and the implications of child labor on the dynamics of a system where fertility and schooling are endogenous have been explored in the literature (Becker, Murphy and Tamura (1990), Dessy (2000), Baland and Robinson (2000), Hazan and Berdugo (2002)). In the area of policy intervention, if taken up at all, these authors discuss regulatory measures – such as bans on child labor or compulsory education (Dessy (2000), Hazan and Berdugo (2002)) – only. Evidence from developing countries, however, suggests that such measures are hard to enforce.

The present dissertation has three aims. The first aim is to construct a setting in which families decide simultaneously about fertility and child labor, and analyze the effects of these decisions on economic growth. In doing so, we will draw upon the framework developed by Raut and Srinivasan (1994) for the analysis of endogenous fertility, and by Bell and Gersbach (2001) for the analysis of child labor. We will develop an OLG model in which parents raise and educate children to finance old-age consumption, while human capital, which is an input in production, is built up through formal education and child rearing. The second aim of the dissertation is to analyze in detail fiscal measures and policy programs aimed at reducing child labor, or fertility, or both. Such measures can be school-attendance subsidies, lump-sum transfers or taxes and subsidies influencing the child-raising costs.

In the first two chapters, we consider a setting in which altruistic parents decide about their own consumption, fertility and the level of schooling their children are to enjoy. We analyze the dynamics of an economy in which growth stems from human capital accumulation and population growth, and find that several steady-states are possible, depending on the underlying educational technology. We also analyze several forms of governmental interventions, such as taxes and subsidies, and construct a policy program which can lead to an escape from the poverty trap. In the first chapter, there is no limiting, fixed factor in production, so that the total population can grow beyond any finite bound. As a consequence, the model presented in the first chapter is extended in the second to include a limiting factor in production, namely, land. In that case, it can be shown that population cannot grow beyond some finite bound in any steady-state process. As in the first chapter, both backward and growth steady-states can exist. In contrast to chapter one, however, alternating steady states (in the sense of fluctuations with a fixed periodicity) are feasible if land ownership plays a role in production. Governmental interventions can lead to sustained economic growth.

In the light of the results of the first two chapters, and the recent literature on human capital and the long-run effects of HIV/AIDS, it seems interesting to analyze the effects of changes in (premature) mortality on household decisions concerning fertility, education and, consequently, economic growth. This is the third aim of the present dissertation, and an application of the framework developed in the first two chapters. In the third chapter, therefore, we extend the basic model, and apply it to Kenya. As we employ the same data as Bell et al. (2004), who assume fertility to be exogenous, the results derived in chapter 3 can usefully be compared to theirs, so that the effects of endogenous fertility decisions can be assessed. We also examine the effects of a governmental program aimed at reducing mortality rates through spending in the health sector, and find it to be highly profitable.

To summarize, we analyze the economic development of societies in which parents decide about not only education but also fertility. The parents' decisions are motivated by both financial and altruistic reasons. We focus on human capital formation and population growth as the determinants of economic development, and find that multiple steady-states can exist. In the first two chapters, the focus of the analysis is on the effects of fiscal interventions on both the household decisions and the dynamic of the system. In the third chapter, we examine the households' reaction to mortality shocks and their long-run economic effects when certain government interventions are financed from sources outside the system.

Chapter 1

Child Labor and Fertility

Abstract

This essay analyzes the economic causes and effects of household decisions concerning fertility, education and child labor when children can supplement family income early in life and must support their parents in old age as adults. Parents, who raise and educate children for both financial and altruistic reasons, will typically choose too little schooling for the economy to grow when all are poor. High child-raising costs or an educational process which is not sufficiently productive are the main reasons for the existence of a poverty trap with a high population growth rate and little or no schooling. Interventions such as taxes and subsidies can lead to sustained long-term economic growth, with full-time schooling and a low population growth rate, even without outside aid, if the child-raising costs are not too high and the educational process is at least moderately productive.

1.1 Introduction

With more than 250 million children working worldwide, the overwhelming majority of them in poor countries, child labor is a major problem. At the same time, fertility is still well above replacement levels, even though it has started to fall from the very high levels that prevailed for most of the 20th century. As population growth rates in most developing countries have started to fall within the last 10 years, perhaps as a delayed response to improved health conditions and an increase in life expectancy, it seems clear that families can influence the number of children they have. This is confirmed by statistics on the use of contraception (contraceptive prevalence), which has increased from 18% in 1990 to 32% in 2000 in the least developed countries, and data on the total fertility rate, which has decreased from 5.9 children in 1990 to 5.4 in the year 2000 in the least developed countries.[1]

Families have several reasons for raising and educating children – altruistic, social and financial. Some religious groups are known to encourage their adherents to have children, and stigmatize families who do not. Parents in highly developed countries do not raise children for financial reasons, as consumption in all stages of adult life is ensured either by their own income or through savings for retirement and social insurance. With access to capital markets being limited in most developing countries, saving for retirement is not a viable option, so that parents have to ensure consumption in old-age by having enough children to support them. Raising children is costly, however, especially when they are very young and cannot earn, and it can even prevent one parent from working. When they become old enough to earn or help in running the family enterprise, educating them involves opportunity costs, at the very least. Therefore, financial reasons will play a major role in fertility decisions.

There is a growing literature on child labor, earlier contributions to which have been surveyed by Basu (1999). When parents decide about their children's education, multiple equilibria can arise, even when fertility is exogenous. Basu and Van (1998) and Swinner-

[1]Source: http://www.childinfo.org/eddb/fertility/index.htm, UNICEF and UN

ton and Rogers (1999) were the first to analyze such a setting. Ranjan (1999) focusses on the connection between capital markets and child labor: without access to credit, parents have to send their children to work. If borrowing were possible, parents could finance the (opportunity) costs of their children's education; for the loans could be paid back through the additional income of well-educated children. Baland and Robinson (2000) were the first to analyze child labor and fertility simultaneously. Inefficiency arises, as parents fail to internalize the negative effects of child labor. The setting in Dessy (2000), who also analyzes both fertility and child labor, also exhibits multiple equilibria: a poverty trap with high population growth and little education, and a steady state with low fertility and high productivity. Hazan and Berdugo (2002) focus on the effects of technological progress, which decreases the relative wage of child labor and therefore encourages education. Even if the economy is initially in the low-level steady state, it can escape to sustained growth, with rising education and decreasing fertility.

This paper will analyze the combined problem of child labor, fertility and provision for old age when the accumulation of human capital can lead to sustained growth. Parents educate children for both altruistic reasons and as a means of financing consumption during retirement. The framework draws on Raut and Srinivasan (1994) and Bell and Gersbach (2001). The former analyze the effects of endogenous fertility on economic growth in the absence of altruism. As capital markets are such that saving in the form of physical capital is possible while borrowing is not, parents have children solely as a means of financing consumption during retirement. Various growth paths are possible: convergence to a steady state, chaotic, and divergent. Bell and Gersbach (2001) examine the interplay between child labor, education and growth when an intergenerational transmission mechanism plays a vital role in the accumulation of human capital. They do not, however, concern themselves with fertility and provision for old age.

The structure of the paper is as follows: Section 1.2 presents the basic model, which is an OLG structure with three generations. Section 1.3 analyzes the different solutions to the household's maximization problem, both interior and at the corner as well as the household decision. Economic growth and possible steady states are examined in section 1.4. Section 1.5 explores governmental interventions, with a focus on financial measures such as taxes and subsidies, and it compares the results derived with those of a model with exogenous fertility. A conclusion is given in section 1.6.

1.2 The model framework

A household consists of three generations: children, parents and grandparents. Each generation is endowed with one unit of time. Children divide their time between working and learning, parents work full-time and grandparents do not work at all. The fraction of childhood assigned to education will be denoted by $e \in [0, 1]$.

For simplicity, assume that only parents with identical labor efficiencies form families (assortative mating). It is assumed that parents raise and educate children in part to increase their own current consumption and to finance their old age. They decide how many children to have (denoted by n) and how well to educate them. Except for the opportunity costs of the children's labor, education is free. Raising children involves direct costs, and well-educated parents normally spend more on their children out of a sense for what is proper and for altruistic reasons. All children are treated identically.

As the grandparents do not work and investment in physical capital is ruled out by assumption, their consumption is financed solely through a grant from their adult children. It is assumed that there is a fixed social norm, under which the young adults must transfer a fixed fraction $\chi \in (0, 1)$ of their income to their parents. Grandparents make no bequests.

Each generation of parents is therefore linked with the two adjoining generations, but there is no direct link between generations more than one period apart. Grandparents cannot influence their children's decisions concerning fertility and education, and these decisions, in turn, do not affect the grandparents' consumption.

Income is generated through the production of a single, non-storable good. Labor – measured in efficiency units – is the only input in production. Let the efficiency of a child be fixed at μ, and denote each parent's endowment of labor in efficiency units by λ. Hence, the total labor supplied by the household in period t, measured in efficiency units, is:

$$L_t = 2\lambda_t + (1 - e_t)\mu n_t. \tag{1}$$

The production function is assumed to exhibit constant returns to scale with respect to labor:

$$y_t = \alpha\, L_t = \alpha\,[2\lambda_t + (1 - e_t)\mu n_t], \quad \alpha > 0, \tag{2}$$

where $\alpha(> 0)$ denotes the output produced with one unit of human capital.

Turning to preferences, the only active decision makers are the parents, whose decisions determine the levels of their consumption in the last two periods of life and the efficiency their offspring will attain in adulthood. Consider a household in period t. The parents' current consumption, C_{1t}, is related to their income $(2\alpha\lambda_t)$, that of the children $(\alpha\mu n_t(1 - e_t))$, the costs of raising the children $(n_t b\lambda_t,\ b > 0)$[2] and the required transfer to the grandparents $(\chi \cdot 2\alpha\lambda_t)$ as follows:

$$C_{1t} = 2\alpha\lambda_t(1 - \chi) + \alpha\mu n_t(1 - e_t) - n_t b\lambda_t \tag{3}$$

Their old-age consumption is given by the number of children, the efficiency each attains in adulthood in period $t + 1$, and the social rule:

$$C_{2t} = n_t(\chi\alpha\lambda_{t+1}) \tag{4}$$

[2]Alternatively, the child-raising costs could be defined as a fraction of the parent's income, $n_t b'(2\alpha\lambda_t)$ where $b' = b/(2\alpha)$. We will return to this specification later on.

We assume that there is no uncertainty in the model in the sense that parents can perfectly foresee all relevant future values of the parameters. To keep matters simple, we will assume that χ does not change over time.

The parents' altruism expresses itself not only through the expenditures on educating and raising children but also in their concern for the children's future income. In contrast to Barro (1974), we do not assume a nested utility function. Instead, parents consider their children's ability to purchase consumption and education for their own children, an ability which is largely determined by the children's efficiency in adulthood. It is further assumed that the utility function is additively separable and has the following form:

$$U(C_{1t}, C_{2t}, \lambda_{t+1}) = ln(C_{1t}) + \beta\, ln(C_{2t}) + \beta_1\, ln(\lambda_{t+1}) \tag{5}$$

The level of fertility n_t does not appear in connection with altruism in order to avoid a tradeoff between quantity, i.e. having many children with little education, and quality, i.e. having few but well-educated children in this context. It is assumed that parents care about the future well-being of each their children, and these are better-off the higher λ_{t+1} is. Note that if child-raising costs were expressed as a fraction of the parents' income, that is, if they took the form $n_t \cdot b'(2\alpha_t\lambda)$, where $b' = b/(2\alpha)$, the labor productivity α would disappear from the household maximization problem as the utility function is logarithmic in form, a consequence which does not seem reasonable.

To summarize thus far: Parents are the sole decision makers in a household. They determine the number of children and their education, and therefore implicitly the amount of child labor and the future efficiency of the children, and the consumption vector (C_{1t}, C_{2t}).

Turning to human capital formation, it is assumed that the efficiency of a grown-up depends on the time she spent at school, the average efficiency of her parents and the productivity of the education process, where it is plausible that these factors are complementary [see Bell and Gersbach (2001)]. If an individual does not spend any time at school, she will attain the minimum level of efficiency $\lambda = 1$. We choose the simplest form:

$$\lambda_{t+1} = ze_t\lambda_t + 1, \tag{6}$$

where $z(> 0)$ can be thought of as the strength of the inter-generational transmission mechanism. Consider a highly developed economy without child labor ($e_t = 1$). In this case, the growth rate of the parents' level of efficiency is given by:

$$g_\lambda \equiv \frac{\lambda_{t+1}}{\lambda_t} - 1 = (z-1) + 1/\lambda_t$$

If $z > 1$, then $g_\lambda > 0\ \forall\lambda$, so that unbounded long-term economic growth is possible. If, on the other hand, $z < 1$, such growth is impossible. In this case, the maximal efficiency an individual can reach is limited, the upper bound for λ being $1/(1-z) > 1$.[3] In the case $z = 1$, long-term economic growth of efficiency beyond any bound is possible, as in the case $z > 1$. The two cases differ only as far as the asymptotic rate of growth is concerned: for $z = 1$, $g_\lambda \to 0$ for very high levels of efficiency.

Using equations (3), (4) and (6), the utility function can be rewritten such that it contains only the decision variables n_t and e_t and the various constants:

$$\begin{aligned} U(C_{1t}, C_{2t}, \lambda_{t+1}) = U(n_t, e_t; \lambda_t) \quad &= \quad ln\Big[2\alpha\lambda_t(1-\chi) + \alpha\mu n_t(1-e_t) - n_t b\lambda_t\Big] \\ &+ \quad \beta ln\Big[\alpha\chi(ze_t\lambda_t + 1)n_t\Big] \\ &+ \quad \beta_1 ln\Big[ze_t\lambda_t + 1\Big] \end{aligned} \tag{7}$$

Note that the utility function $U(n_t, e_t; \lambda_t)$ is not necessarily concave everywhere. The function n_te_t, for example, is neither concave nor convex in (n_t, e_t), as is $ln\big[2\alpha\lambda_t(1-\chi) + \alpha\mu n_t(1-e_t) - n_t b\lambda_t\big]$.

[3]This is not necessarily the highest level of efficiency a society will actually reach, as this upper limit is computed assuming that $e = 1$. If the schooling parents choose at the upper limit of λ is $e < 1$, then the maximal efficiency will be lower, namely, $1/(1-ze) \leq 1/(1-z)$.

1.3 The Household's Optimum

The solution to the households' maximization problem is a tuple $(n_t(\lambda_t), e_t(\lambda_t))$ that maximizes (7) for the given level of efficiency λ_t. For the solution to be relevant economically, it has to fulfill four conditions, with both n_t and e_t being bounded in all periods. The number of children a family can have is bounded above by biological constraints ($n_t \leq n_{max} < \infty$). The total fertility rate of the Hutterites (a religious group in North America) – the highest ever measured historically – was about 9.5 children per woman, suggesting that $n_{max} < 10$. A lower bound might be applicable for social reasons ($0 < n_{min} \leq n_t$).[4] For the classical family with two parents, $n_t = 0$ can never be optimal, as $C_{2t} = 0$ in this case. In order to skirt this problem, the paper will deal with an extended family instead: pooling plays a major role in this case, as in Bell, Devarajan and Gersbach (2003), so that $n_t < 1$ becomes possible. The household's problem, therefore, is to

$$\max_{n_t, e_t} U(n_t, e_t; \lambda_t) \quad \text{s.t} \quad n_t \in [n_{min}, n_{max}] \quad \text{and} \quad e_t \in [0, 1]. \tag{8}$$

1.3.1 The unrestricted solution

The first-order conditions associated with the household's maximization problem (8) above have a unique (unrestricted) solution.[5] Unfortunately, this does not describe a maximum, but rather a saddle point, the utility function being neither convex nor concave in (n_t, e_t), so that the determinant of the hessian of the function is negative for all values of the parameters for the unrestricted solution of the first-order conditions:

$$det(H) = -\frac{\beta_1 \,(1+\beta)^3 \,\mu^2}{4{\lambda_t}^2 \,(\beta_1 + \beta)\,(-1+\chi)^2} < 0$$

If the educational technology were non-linear in schooling, that is, if it took the form $\lambda_{t+1} = z e_t^{\epsilon} \lambda_t + 1$ with $\epsilon < 1$, at least one of the unrestricted solutions would be a maxi-

[4]One can allow n_{min} to be exactly zero if one introduces the assumption that output is storable or that credit contracts can be entered into by members of adjacent generations.

[5]Note that the tuple (n_t, e_t) solving the first-order conditions does not necessarily satisfy the conditions $n_t \in [n_{min}, n_{max}]$ and $e_t \in [0, 1]$

mum.[6] However, if ϵ is less then one, then choosing $e_t = 0$ can never be optimal, as can be seen by deriving the utility function with respect to education and evaluating this derivative for $e_t = 0 : \lim_{e_t \to 0} \partial u / \partial e_t = \infty$. As a consequence, choosing $e_t > 0$ is always optimal, and steady-states with full-time child labor are excluded by construction if $\epsilon < 1$. As one of the aims of the essay is to analyze policy programs which lead to sustained long-term economic growth with full-time schooling, excluding a steady-state of backwardness, that is, one of the main situations in which a policy program is needed, would be a strong limitation. If $\epsilon = 1$ is chosen instead, steady-states with full-time child labor can exist,[7] but steady-states where both schooling and fertility are interior are excluded, as full unrestricted solutions are never optimal. Note, however, that steady-states where either e_t or n_t (but not both) is interior can exist. As a consequence, we choose $\epsilon = 1$ for the remainder of the essay, so as not to exclude steady-states with full-time child labor by construction.

Choosing a Ramsey specification for the subutility function of altruism, that is, $U(\cdot) = ln(C_{1t}) + \beta ln(C_{2t}) + \beta_1(1 - 1/\lambda_{t+1})$, will not change the sign of the determinant of the hessian, so that the unrestricted solution will describe a saddle point in this case too. Unrestricted solutions describing a maximum do exist if one chooses a more general iso-elastic form: $U(\cdot) = ln(C_{1t}) + \beta ln(C_{2t}) + \beta_1(1 - \eta/\lambda_{t+1}^{\eta})$ with $\eta < 1$. However, in this case it is only possible to derive the unrestricted solutions analytically for $\eta = 1/2$, and even then the solutions are very complex, and hard to analyze. We will return to such a specification in the third chapter, where the existence of analytical solutions does not play major role.

As the full unrestricted solution is not relevant in the present case, corner solutions need to be computed. The following sections will present the three corner solutions and analyze the areas where they are relevant economically, that is, where they satisfy the conditions stated above. Both corner solutions with respect to one variable alone are maxima, with

[6] If $\epsilon > 1$, unrestricted solutions are saddle points.

[7] In this case, $\lim_{e_t \to 0} \partial u / \partial e_t < \infty$, so that choosing $e_t = 0$ can be optimal.

their respective second derivatives being negative.

1.3.2 Unrestricted solutions w.r.t. one variable

Whether an unrestricted solution with respect to either fertility or education (but not both) exists and makes sense economically (i.e. $e \in [0,1]$ and $n \in [n_{min}, n_{max}]$) depends on the child-raising costs and the parents' efficiency. A detailed discussion of the solutions is given in the appendix. Unrestricted solutions will be denoted by n^* and e^* respectively, while $\bar{n}$ and $\bar{e}$ denote corner values. Figures 1.1 and 1.2 and tables 1.1 and 1.2 outline the results.

Figure 1.1: The unrestricted solution with respect to fertility for $\bar{e} = 0$, for low and high child-raising costs.

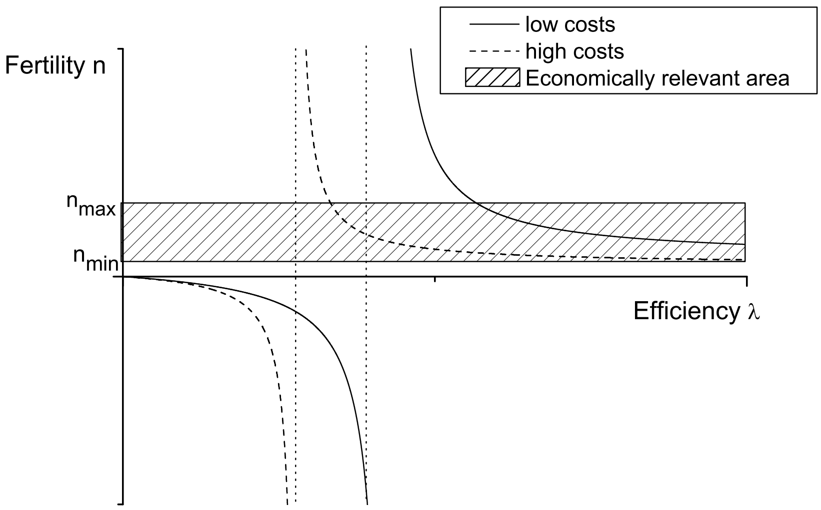


Table 1.1: Unrestricted solution w.r.t fertility for $\bar{e} = 0$ and $\bar{e} = 1$

	$b \in \left[\frac{2\alpha\beta(1-\chi)}{n_{max}(1+\beta)}, \frac{2\alpha\beta(1-\chi)}{n_{min}(1+\beta)}\right]$	$b \notin \left[\frac{2\alpha\beta(1-\chi)}{n_{max}(1+\beta)}, \frac{2\alpha\beta(1-\chi)}{n_{min}(1+\beta)}\right]$
$\bar{e} = 1$	$n^*(\cdot)$ relevant for all λ	$n^*(\cdot)$ is never relevant
$\bar{e} = 0$	$n^*(\cdot)$ is relevant for high levels of efficiency $\lambda > \frac{\alpha\mu n_{max}(1+\beta)}{2\alpha(1-\chi)\beta - n_{max}b(1+\beta)}$	$n^*(\cdot)$ is never relevant

n^* denotes an unrestricted solution w.r.t. fertility.

If the child-raising costs are very low, the unrestricted solution w.r.t n is either negative or too large ($n > n_{max}$) for all levels of efficiency. It is always negative for low levels of λ, and can only be feasible economically if the child-raising costs and the adults' level of

Figure 1.2: Unrestricted solution with respect to education for different costs.

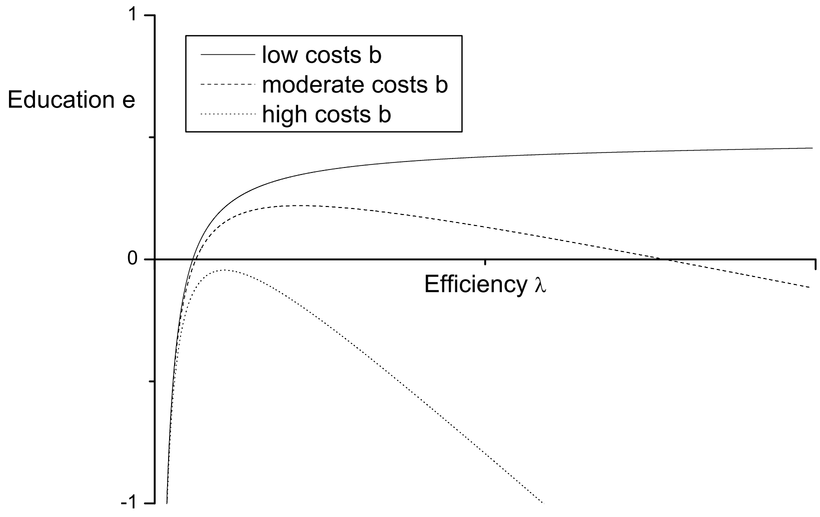


Table 1.2: Unrestricted solution w.r.t education

Child-raising costs	unrestricted solution w.r.t. education
$b < \frac{2\alpha(1-\chi)}{\bar{n}}$	$e^*(\cdot)$ relevant for moderate levels of efficiency
$b > \frac{2\alpha(1-\chi)}{\bar{n}}$	$e^*(\cdot)$ relevant for moderate levels of efficiency $\lambda \in [\lambda_a, \lambda_b]$
$b > \frac{2\alpha(1-\chi)}{\bar{n}} + \frac{\alpha z \bar{n} \mu(\beta+\beta_1)}{4\bar{n}}$	$e^*(\cdot)$ is never relevant

e^* denotes an unrestricted solution w.r.t. education.

human capital are sufficiently large. Depending on the parameters, the unrestricted solution w.r.t. education could exist for low levels of efficiency and be feasible economically. Therefore, parents will choose $e = e^*(\bar{n}, \lambda)$ or $e = 0$ for low levels of efficiency and low costs, where $\bar{n}$ still needs to be determined. For very small and high levels of efficiency, the unrestricted solutions will be economically feasible with respect to neither variable when child-raising costs are low.

For moderate child-raising costs and low levels of efficiency, the unrestricted solution w.r.t. fertility is not feasible. The unrestricted solution $e = e^*(\bar{n}, \lambda)$ will be feasible for low and moderate levels of efficiency, but not available for large λ, where $n^*(\bar{e}, \lambda)$ could be feasible. The values for $\bar{e}$ and $\bar{n}$ as well as the optima in the cases where neither or both of the unrestricted solutions are feasible, still need to be computed.

If the child-raising costs are high, the solution $n^*(\bar{e}, \lambda)$ is available for low levels of efficiency only if $\bar{e} = 1$, while the unrestricted solution w.r.t. education is negative for all levels of efficiency. The optimum for small λ might, therefore, be either a corner solution w.r.t both variables or $n^*(\bar{e} = 1, \lambda)$. For high levels of efficiency, consumption in the first period needs to be financed by child labor, as the child-raising costs are relatively high. Therefore, the solution $n^*(\bar{e} = 0, \lambda)$ will be optimal, if biologically feasible.

1.3.3 The corner solutions w.r.t. both variables

As established above, the unrestricted solutions w.r.t. one variable are not optimal for all levels of efficiency. Therefore, full corner solutions need to be analyzed. In order to find the optimum, it is necessary to compare the utilities generated by any combination of n and e at the boundary of the feasible set. It suffices to consider the following two cases $(n_{min} \neq 0)$:

$$\begin{aligned} (i)\, \triangle U_n &:= U(n_{max}, \bar{e}) - U(n_{min}, \bar{e}) \\ (ii)\, \triangle U_e &:= U(\bar{n}, e = 0) - U(\bar{n}, e = 1) \end{aligned}$$

Table 1.3: $\triangle U_n$ as a function of efficiency and child-raising costs

$b \leq \frac{2\alpha(1+\chi)(n_{max}^{\beta} - n_{min}^{\beta})}{n_{max}^{1+\beta} - n_{min}^{1+\beta}}$	$U(n_{max}, \bar{e}) > U(n_{min}, \bar{e})$ for all levels of efficiency
$b > \frac{2\alpha(1+\chi)(n_{max}^{\beta} - n_{min}^{\beta})}{n_{max}^{1+\beta} - n_{min}^{1+\beta}}$	$U(n_{max}, \bar{e}) > U(n_{min}, \bar{e})$ for sufficiently low levels of efficiency

A detailed analysis of $\triangle U_n$ and $\triangle U_e$ is given in the appendix. It can be shown that if both variables are at the corner and schooling is fixed ($e = \bar{e}$), well-educated parents will be better off choosing the minimum number of children if the child-raising costs are sufficiently large. Otherwise – that is, for low levels of efficiency or low costs – choosing $n = n_{max}$ will be optimal. If n_{min} is sufficiently close to zero, then choosing $n = n_{max}$ will always be optimal if the unrestricted solution w.r.t n is not economically feasible. The results and critical values are derived in the appendix.

Analyzing $\triangle U_e$ for $\bar{n} \neq 0$ yields an inequality involving a transcendental expression. It

Table 1.4: $\triangle U_e$ as a function of efficiency and child-raising costs

$b > \frac{2\alpha(1-\chi)}{\bar{n}}$	$U(e=0,\bar{n}) > U(e=1,\bar{n})$ for all levels of efficiency
$b \leq \frac{2\alpha(1-\chi)}{\bar{n}}$	$U(e=0,\bar{n}) > U(e=1,\bar{n})$ for sufficiently low levels of efficiency

is not possible to express this condition analytically for the critical efficiency. As was the case for $\triangle U_n$, the sign of $\triangle U_e$ depends on the child-raising costs and the parents' level of efficiency. If the unrestricted solution w.r.t education is not feasible economically, choosing $e = 0$ will be optimal for all levels of efficiency if the child-raising costs are large. If, on the other hand, b is small, well-educated parents will be better off choosing full-time schooling, while poor parents need child labor and will choose $e = 0$. Although it is not possible to compute the critical efficiency analytically, it can be shown that it is single-valued.

1.3.4 The household's decision

Having identified and characterized all solutions, it is now possible to outline how the household's decision depends on the parameters that govern its preferences and feasible choices.

If $b < \frac{2\alpha(1-\chi)}{n_{max}}$, and the parents' efficiency is very low, they will choose $n = n_{max}$ [8] and either $e = 0$ or $e = e^*(\bar{n}, \lambda)$. For large levels of efficiency and small costs, neither of the unrestricted solutions will be feasible, as both variables exceed their maximal values. Parents will therefore choose $n = n_{max}$ and $e = 1$.

If $b \in [\frac{2\alpha(1-\chi)}{n_{max}}, \frac{2\alpha(1-\chi)}{n_{min}}]$, and λ is small, the result is the same as above. For moderate levels of efficiency, the unrestricted solution w.r.t. education is feasible, so that parents choose $e = e^*(\bar{n}, \lambda)$ and $n = n_{max}$. For large levels of efficiency, $\triangle U_e$ will be negative while the unrestricted solution w.r.t. education is not feasible. As the unrestricted solution w.r.t. fertility might be feasible, the household's optimum for high levels of efficiency will

[8] With the parents' efficiency being small, the term $z\lambda$ in the educational technology is small while the child-raising costs are relatively large. Therefore, education is not productive, so that full-time schooling cannot be optimal, and parents finance old-age consumption by having numerous children.

be $n = max[n_{min}, n^*(\bar{e}, \lambda)]$ and $e = 1$.

If $b > \frac{2\alpha(1-\chi)}{n_{min}}$, choosing full-time schooling always leads to $C_{1t} \leq 0$. As none of the two unrestricted solutions is feasible for low levels of efficiency when $\triangle U_e < 0$ and $\triangle U_n > 0$, parents will choose $n = n_{max}$ and $e = 0$ for low λ. With increasing levels of efficiency, as the unrestricted solution w.r.t. fertility becomes feasible, parents will choose $n = max[n_{min}, n^*(\bar{e}, \lambda)]$ and $e = 0$.[9] For still higher levels of efficiency, the unrestricted solution w.r.t education is feasible, so that choosing $n = n_{min}$ and $e = e^*(\bar{n}, \lambda)$ will be optimal. For sufficiently high levels of efficiency, none of the unrestricted solutions will be feasible, while $\triangle U_e < 0$ and $\triangle U_n < 0$, so that choosing $e = 0$ and $n = n_{min}$ is optimal. Note that for very high levels of efficiency, even full-time child labor cannot ensure $C_{1t} > 0$, so that this case is of mathematical interest only. Extremely high costs will be analyzed in the context of taxes, as societies cannot carry the burden of such costs for long.

The optimal choice when $\lambda = 1$ is of particular interest, as it determines whether the economy will be trapped at $\lambda = 1$ (if households choose $e(\lambda = 1) = 0$) or not. As the unrestricted solution w.r.t. fertility is generally not feasible for low levels of human capital, and parents choose $n = n_{max}$, we can confine the analysis for $\lambda = 1$ to the unrestricted solution w.r.t. education: $e^*(\lambda = 1)$ will only be positive if the child-raising costs are sufficiently low:

$$b \leq \frac{2\alpha(1-\chi)}{n_{max}} + \alpha\,\mu - \frac{\alpha\mu}{z(\beta+\beta_1)}. \tag{9}$$

Note that the condition is more easily satisfied if z is large, in which case the term on the right hand side of (9) is large, and approaches $\frac{2\alpha(1-\chi)}{n_{max}} + \alpha\mu$ from below. If, on the other hand, z is very low ($z \rightarrow 0$), the last term of the condition will be infinitely large, in which case the condition can never be satisfied. Therefore, parents will generally choose $e(\lambda = 1) > 0$ if raising children is sufficiently cheap while the educational technology is productive, and $e(\lambda = 1) = 0$ otherwise.

[9] As can be seen from figure 1.1, the range of efficiency for which this solution is optimal is very narrow.

1.4 Economic Growth

The next step is to examine the factors influencing economic growth, whereby 'growth' can be measured according to three indices: the parents' efficiency (λ_t), the family's income y_t and the lifetime utility of a generation (U_t). The second index is also the social product of the household, and therefore corresponds to GDP. Finding the growth rate of income or utility is complicated by the fact that efficiency, the number of children and schooling could all change simultaneously within periods, so that $\triangle y_t$ and $\triangle U_t$ might have either sign. Given the fact that it is not possible to find explicit functions for the schooling and fertility for all λ, as critical efficiencies cannot be computed, technical problems arise when trying to compute growth rates for y_t or U_t. Hence, we restrict ourselves to efficiency as a measure of economic growth. A big difference in efficiency will always lead to a big difference in income and utility, and $lim_{\lambda\to\infty}U = lim_{\lambda\to\infty}y = \infty$.[10] Hence, an economic policy aimed at increasing a family's utility or income through an increase in efficiency will be efficacious if increasing λ is at all possible.[11]

1.4.1 The critical level of schooling

The efficiency of a lineage will grow over time only if children spend sufficient time at school. The critical level of schooling depends on the parents' efficiency and the educational technology. It is obvious from (6) that efficiency cannot fall below its minimum $\lambda = 1$, independently of z and e. The critical level of schooling can easily be computed from the following condition for a stationary state in efficiency λ^* :[12]

$$\lambda_{t+1} = \lambda_t \ \ \forall t \quad \Leftrightarrow ze(\lambda_t)\lambda_t + 1 = \lambda_t \ \ \forall t \quad \Leftrightarrow ze\lambda^* + 1 = \lambda^*$$

[10]If $\lambda \to \infty$ is feasible so that $C_{it} > 0$, $i = 1, 2$.

[11]Sustained growth is only possible if $b < \frac{2\alpha(1-\chi)}{n_{min}}$. In this case, parents will choose $n = max[n_{min}, n^*]$ and $e = 1$ when λ is large. Computing income and utility, and then differentiating the results w.r.t. efficiency immediately yields that income grows with efficiency, $\partial y/\partial\lambda = \alpha > 0$ and $\partial U/\partial\lambda = \frac{1+z\lambda(1+\beta+\beta_1)}{\lambda(z\lambda+1)} > 0$. Therefore, growth of efficiency is equivalent to growth of utility and total income, when λ is large, although the growth rates differ.

[12]Note that a stationary state in λ as defined here is not necessarily stable. The issue of stability will be taken up later on.

where λ^* is a stationary value of the system. Therefore, the value of e that induces a stationary state is:

$$e_{crit} := \frac{1}{z} - \frac{1}{z\lambda^*} \tag{10}$$

The higher the initial efficiency, the higher the level of schooling required to attain it. For very high levels of efficiency, the critical level of schooling will approach $1/z$ asymptotically from below. As parents who already are highly efficient choose $e = 1$ automatically if possible,[13] the speed and direction of economic growth depend on z only. For the remainder of the essay, only steady-states with respect to λ will be considered. Steady-states where population is constant will exist only by chance. Note that in all steady-states where λ is constant over time, fertility will be likewise.

If parents choose $e_t > e_{crit}$ for some level of λ_t, the economy will grow. If, however, $e_t < e_{crit}$, the level of human capital will fall with time.

1.4.2 Low and moderate levels of efficiency

If parents are not well educated, they will decide to have as may children as possible ($n = n_{max}$) and to send their children out to work. The exact amount of schooling depends on b and z, with $\partial e/\partial z > 0$ and $\partial e/\partial b < 0$, and can be computed using equation (16) in the appendix. For low costs and $\lambda = 1$, the critical level of schooling is zero, while parents choose $e_t = 0$ if raising children is not cheap. As a consequence, a low-level, stable stationary state with respect to λ with full-time child labor can exist if b does not satisfy condition (9). If, however, b satisfies condition (9), a stationary state with $\lambda = 1$ and $e = 0$ will not exist. For slightly higher levels of efficiency, the existence of stationary states depends on b and z. Parents will usually choose $e > e_{crit}$ if z is sufficiently high and the child-raising costs are sufficiently low. For larger costs, $e^*(\bar{n}, \lambda)$ and e_{crit} will intersect at most twice. A mathematical analysis of potential points of intersection is given in the appendix.

[13] Choosing $e = 1$ is always possible if $b < 2\alpha(1 - \chi)/n_{min}$.

A point of intersection of the two functions is a stationary state with respect to efficiency, provided parents choose the unrestricted solution w.r.t education. It is obvious, therefore, that not all points of intersection (which are computed in the appendix) will yield a steady state if the parents prefer choosing the unrestricted solution w.r.t. fertility rather than the unrestricted solution w.r.t. education for the relevant level of efficiency. As it is not possible to derive the household's choice analytically, the stationary states will also have to be computed numerically. Calculations[14] show that there is only a very limited range of parameters in which two steady-states exist. In most cases, there will be either no steady state (for very low costs) or a single steady-state (for moderate costs) if the parents' efficiency is not too large.

Before turning to the analysis of high levels of efficiency, the stability of the potential stationary states with respect to λ where $e = e^*(\bar{n}, \lambda)$ needs to be analyzed. As $lim_{\lambda \to 0}(e - e_{crit}) = \infty$, the level of schooling parents choose will be higher than the critical level of schooling for levels of efficiency lower than the first steady-state and lower for higher levels of efficiency. As both functions are continuous for low and moderate levels of efficiency, it follows that the first point of intersection will yield a stable steady state, while the second steady state, if it exists, will be unstable.

Recall from section 1.3.4 that parents always choose $e(\lambda = 1) = 0$ if the child-raising costs exceed the value in (9). As the unrestricted solution w.r.t. education is continuous in λ, it is likely that $e = 0$ will still be the optimal choice even if the parents' human capital is slightly higher than $\lambda = 1$, in which case the steady-state with $\lambda_t = 1,\ e_t = 0\ \forall t$, and $n_t = n_{max}\ \forall t$ is stable.

[14]For these calculation, we choose some fixed values for $\alpha, \mu, \chi, \beta, \beta_1$ and n_{min} and n_{max}, and compute the dynamics of the system for different values of z and b. One set of values for the parameters and the ensuing household decisions and phase diagrams can be seen in the appendix.

1.4.3 High levels of efficiency

If the parents' efficiency is large, households will always choose full-time education if they can afford it.[15] Whether the economy will grow, stagnate or contract then depends on z alone. The possible outcomes have been presented in section 1.2. Long-term economic growth is not possible if $z < 1$; in this case, a high-level stationary state will be reached asymptotically if parents always choose $e = 1$.

1.4.4 Conclusion: Economic growth and steady states

As stated earlier, the economic prospects of a society depend on the child-raising costs, the productivity of the education function, z, the social parameter χ and the initial state of the economy, that is, the parents' efficiency in the first period of the analysis.

If $b < \frac{2\alpha(1-\chi)}{n_{max}}$, parents usually choose $n = n_{max}$ and either $e = 0$ or $e = e^*(\bar{n}, \lambda)$ for low and moderate levels of efficiency. If $z < 1$ there will be a single steady-state in λ for low or moderate levels of efficiency. If b does not satisfy condition (9), the stationary state will be at $\lambda = 1$ with $e = 0$, and at some higher level of λ otherwise. If $z = 1$, there will be at least one steady-state, and up to two for moderate costs. If $z > 1$, the parents will always choose $e > e_{crit}$ if b is very small. For high levels of efficiency, parents choose $e = 1$. The economy will grow if $z \geq 1$, and will contract otherwise.

If $\frac{2\alpha(1-\chi)}{n_{max}} \leq b \leq \frac{2\alpha(1-\chi)}{n_{min}}$, the result for low and moderate levels of efficiency will be the same as above, and there will be at least a single stationary state. This steady-state is likely to be at $\lambda = 1$ if b does not satisfy condition (9). For a limited range of parameters, up to three steady states will exist. For high levels of efficiency parents will choose $e = 1$ and $n = max[n_{min}, n^*(\bar{e}, \lambda)]$. The economy will always grow if $z \geq 1$.

As noted above, the case $b > \frac{2\alpha(1-\chi)}{n_{min}}$ is implausible and only of interest in the context of taxes. It is reported here for the sake of completeness. If the parents' efficiency is small,

[15] Parent can always afford $e = 1$ for high levels of efficiency if $b < \frac{2\alpha(1-\chi)}{n_{min}}$.

choosing $e = 0$ and $n = n_{max}$ will be optimal, as long as the unrestricted solution w.r.t. fertility is not feasible.[16] Again, a steady state could exist for $\lambda = 1$. If the efficiency of the parents is large parents will have to reduce schooling as soon as the unrestricted solution w.r.t fertility falls below n_{min}. As C_{1t} will be negative for high levels of efficiency for any schooling and any $n \geq n_{min}$, an economy where $b > 2\alpha(1-\chi)/n_{min}$ can never reach high levels of efficiency. A steady-state will exist for $\lambda \geq 1$. For $z > 1$, there will be up to two steady-states, with the second one always being unstable and the first one stable.

Table 1.5: Conclusion: Steady-States

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$b < \frac{2\alpha(1-\chi)}{n_{max}}$

- At least a single, low level stationary state if $z \leq 1$ either at $\lambda_t = 1$ and $e_t = 0\ \forall t > \bar{t}$, or at $\lambda_t = 1/(1 - z\bar{e})$ and $e_t = \bar{e}\ \forall t > \bar{t}$.
- A growth or high-level stationary state with $e_t = 1 \forall t > \bar{t}$ and $g_\lambda > 0\ \forall t > \bar{t}$ if $z > 1$ and $g_\lambda \geq 0\ \forall t > \bar{t}$ if $z = 1$.
- If b and z satisfy condition (24) there will be no low stationary steady-state for $z > 1$.

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$b \in [\frac{2\alpha(1-\chi)}{n_{max}}, \frac{2\alpha(1-\chi)}{n_{min}}]$

- At least a single, low level stationary state if $z \leq 1$ either at $\lambda_t = 1$ and $e_t = 0\ \forall t > \bar{t}$, or at $\lambda_t = 1/(1 - z\bar{e})$ and $e_t = \bar{e}\ \forall t > \bar{t}$.
- A growth or high-level stationary state with $e_t = 1 \forall t > \bar{t}$ and $g_\lambda > 0 \forall t > \bar{t}$ if $z > 1$ and $g_\lambda \geq 0\ \forall t > \bar{t}$ if $z = 1$.
- As b and z can never satisfy (24) if $b \in [\frac{2\alpha(1-\chi)}{n_{max}}, \frac{2\alpha(1-\chi)}{n_{min}}]$.

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$b > \frac{2\alpha(1-\chi)}{n_{min}}$

- At least a single, low level stationary state if $z \leq 1$ either at $\lambda_t = 1$ and $e_t = 0\ \forall t > \bar{t}$, or at $\lambda_t = 1/(1 - z\bar{e})$ and $e_t = \bar{e}\ \forall t > \bar{t}$.
- A growth or high-level stationary state with $e_t = 1 \forall t > \bar{t}$ cannot exist, as C_{1t} is negative if $e_t = 1$ for all but low levels of efficiency.

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[16]The unrestricted solution w.r.t. education always yields a negative e for very large costs.

An economy will therefore grow forever if z is large while the child-raising costs are very low.[17] In all other cases, stable steady states with low, stationary values of λ, that is, 'poverty traps', will exist. If the initial level of efficiency is sufficiently high, sustained growth in λ is possible for all but very high levels of child-raising costs if $z \geq 1$. If $z < 1$, unbounded growth is not possible for any b and any initial level of efficiency.

Before turning to the analysis of governmental interventions, however, it is of interest to find parameters such that the economy grows by itself. As this can only be the case if the child-raising costs are sufficiently low and $z \geq 1$, the household's decision can be easily modeled: for low levels of efficiency, parents will choose the unrestricted solution w.r.t. education. With the child-raising costs being low, parents will always choose $n = n_{max}$. For high levels of efficiency, $e = 1$ and $n = min[n_{max}, \frac{2\alpha(1-\chi)\beta}{(1+\beta)b}]$ will be optimal. As z is large, it follows immediately that the economy will grow if the initial level of efficiency is sufficiently large. If the level of efficiency in the first period is low, but one can show that the unrestricted solution w.r.t. education always yields $e > e_{crit}$ it follows that the economy will grow independently of its initial state. The upper bound for the costs such that the economy always grows, i.e. $e > e_{crit}\ \forall\lambda$ has been computed in the appendix: if b does not satisfy condition (24), then $e > e_{crit}\ \forall\lambda$, and therefore the economy will always grow for $z \geq 1$. For economies with extremely productive education functions, sustained growth will be possible for a very broad range of costs. On the other hand, if the education function is not very productive, long-term growth will only be possible for very low costs. The paths of education and fertility for different costs, efficiencies and z are depicted in figures 1.6a to 1.6i at the end of the essay. Table 1.5 gives an overview over all possible steady-states.

[17]That is, the unrestricted solution w.r.t. education is feasible for $\lambda = 1$.

1.5 Governmental Intervention

Assuming an economy starts with a low initial level of efficiency, three outcomes are possible: first, if $z \geq 1$ and b is sufficiently low, the economy will eventually attain steady-state growth. Second, if $b > \frac{2\alpha(1-\chi)}{n_{min}}$, the economy will reach a low-level stationary state, and no growth steady-state exists. Third, there is the case where, starting from a low level of λ the economy reaches a low-level stationary state, but a growth steady-state exists. In that case, the economy will be 'stuck' after some periods in a low-level, stable poverty trap, from which it cannot escape without outside intervention. It is assumed that the government's major goal is to induce sustained economic growth, and that it tries to do so by measures designed to promote higher levels of efficiency. This is only possible in the last of the three cases named above. As sustained economic growth is only possible if $z > 1$ and $b < 2\alpha(1-\chi)/n_{min}$, the analysis will be confined to the cases where these conditions are satisfied: there will always exist some level of efficiency, λ' say, such that for all $\lambda \geq \lambda'$, parents will choose $e \geq e_{crit}$. As soon as the government manages to increase the parents' efficiency to just above λ', it will have accomplished its aim. If this is not possible in one period on account of $z\lambda_t + 1 < \lambda'$, intervention will have to stretch over more than one period. Without the introduction of a social welfare function encompassing more than one generation, it is not possible to describe the optimal path to sustained growth in detail. For reasons of simplicity, it is assumed that the government will try to induce parents to choose full-time schooling as long as $z\lambda_t + 1 < \lambda'$ and to choose at least the necessary schooling such that $ze\lambda_t + 1 = \lambda'$ in the last period of intervention. This simplification makes the analysis relatively tractable, and the policy program that emerges from it is plausibly a 'good' one, in the sense that the goal is sensible.

In the present setting, the government can implement both fiscal and regulatory measures: the classical policy is to reduce child labor and increase school attendance through the introduction and enforcement of compulsory education. Another regulatory measure would be to limit the number of children a family may have, China being the most prominent example. There is a broad range of fiscal measures, all of which can be divided into two groups, namely, taxes and subsidies. Governmental measures will usually include a combination of instruments from both classes, with taxes being used to finance subsidies.

1.5.1 Regulatory measures

The impact of a prescribed level of either schooling or fertility has already been analyzed, implicitly, in the sections before. If n or e is fixed, parents will choose the remaining variable using equations (16) and (18), respectively in the Appendix. If the resulting variables are outside their natural ranges, the corner solutions w.r.t. both variables, as discussed in section 1.3.3, will yield the optimum. This class of measures will not be analyzed further in this context: historical evidence shows that compulsory schooling is hard to enforce when the family would experience a heavy loss in income and there are often not enough schools or teachers. Limiting n by decree would also lead to an increase in schooling, but this measure would be very unpopular in most countries, and therefore hard to implement, so we will not consider it here.

1.5.2 Fiscal Measures

Fiscal measures are designed to induce families voluntarily to choose the schooling and/or fertility the government wishes. It is clear that this class of policies is not without its own problems: taxes are sometimes hard to collect, they can lead to social unrest, especially if they impose a heavy burden or if they are perceived to be unfair, and subsidies can fail to reach the needy through corruption and mismanagement. Some of the measures analyzed in the following are not free of these problems, but it will be assumed that these difficulties can be solved. It will also be assumed that the subsidies paid to some families are financed through the taxation of other families or through grants from international organizations: a household is either taxed or subsidized, but not both. Therefore the issue of financing the subsidies and the use of the revenues from taxation, respectively, will be irrelevant for the analysis of the individual household's decision. The following three measures will be dealt with in detail:

- Lump-sum transfers T^f, where $T^f > 0$ denotes a lump-sum tax, and $T^f < 0$ a lump-sum subsidy.
- Taxation/subsidization of the expenditures on raising children, $T^b = \tau bn\lambda$, where both $T^b > 0$ (tax) and $T^b < 0$ (subsidy) are possible.
- Subsidization of school attendance, $S^e = sne$, where $s > 0$. School fees ($s < 0$) will be ignored.

For simplicity, it will be assumed that only parents pay taxes and receive subsidies, while the grandparents' income remains untouched. It is possible to rewrite consumption in the first period of adult life in the following form, which allows the analysis of each of the measures enumerated above:

$$C_{1t} = 2\alpha\lambda_t(1-\chi) + \alpha n_t\mu(1-e_t) - (1+\tau)n_t b\lambda + sn_t e_t - T^f, \tag{11}$$

where at most one of the variables τ, s and T^f is non-zero. Note that none of these taxes alters the obligation to pay $\alpha\lambda_t\chi$ to the grandparents.[18] The impact of these different fiscal measures on the household's decision will be examined in section 1.5.2. As the total revenues which can be raised through each measure and the total costs of subsidizing a household play an important role in the setup of a program, the different measures will also be compared with respect to their benefits and costs in the following sections. An integrated analysis of a program comprising both taxes and subsidies will be developed in sections 1.5.3 and 1.5.4.

Lump-sum transfers

Note first that, the adults' level of efficiency being fixed, a lump-sum transfer T^f will be equivalent to taxing/subsidizing the parents' income ($\tau \cdot (2\alpha\lambda)$), with $\tau = T^f/(2\alpha\lambda)$. Therefore, the following analysis of a lump-sum transfer will also hold for a tax on the

[18] The normal procedure in analyzing such problems is to write down the lifetime budget constraint, and then to appeal to the normalcy of goods in consumption to obtain comparative static results. This is not possible in the present cases, as output is not storable, no capital markets exist and there are two separate budget constraints which cannot be combined.

adults' income.

If a family transfers a fixed amount of money T^f to the government, or receives such a transfer, it will in general change its desired level of fertility or the children's education or both. As we have seen, if parents are poor (λ low), they will typically choose $e < 1$ and $n = n_{max}$. If the budget set is sufficiently enlarged through a lump-sum subsidy, parents will increase the children's education, as a further increase in n is not possible. The lump-sum subsidy in the first period is partially transferred to the second period through investment in additional education, and lifetime consumption is smoothed somewhat.

If parents are rich, they choose $n \leq n_{max}$ and $e = 1$. Since a lump-sum subsidy cannot increase education in this case, parents will 'invest' part of it in raising more children. The higher the parents' efficiency, the lower will be the impact of the transfer. If parents chose $n = n_{max}$ and $e = 1$ before receiving the transfer, all subsidies will be fully consumed in the first period of adult life.

The impact of a lump-sum tax will be similar: poor parents will rather increase child labor than reduce fertility, as the returns from education are limited if the parents' efficiency is low. Again, this leads to a smoothing of income over the life cycle. If the parents are rich, educating children is highly productive, so that rich parents will leave $e(= 1)$ unchanged and reduce fertility, unless raising children is extremely cheap ($b \approx 0$). In that case, parents will maintain fertility and decrease e instead. The impact of the transfer decreases as the parents' efficiency increases. If the tax is so large that a further reduction of fertility is not possible, parents will have to reduce education to finance their tax obligations. The maximal tax a rich family can pay depends on n_{min} and b as follows.

If $b < \alpha\mu/\lambda$ (or $\lambda < \alpha\mu/b$), parents will reduce schooling rather than fertility, until reaching $e_t = 0$.

$$T^f_{max} = 2\alpha\lambda(1-\chi) + n_{max}(\alpha\mu - b\lambda) > 2\alpha\lambda(1-\chi).$$

Raising higher taxes from such poor families is not possible, as they will start to reduce fertility in that case, and $2\alpha\lambda(1-\chi) + n_{max}(\alpha\mu - b\lambda) > 2\alpha\lambda(1-\chi) + n(\alpha\mu - b\lambda)$ if $n < n_{max}$ and $b < \alpha\mu/\lambda$.

For $b > \alpha\mu/\lambda$, parents will always reduce fertility, if possible, before reducing schooling. If $n_{min} > 0$, large taxes will force families to reduce schooling, as child labor will be used to finance the tax and to ensure $C_{1t} > 0$. The maximal tax they can pay in this case is

$$T^f_{max} = 2\alpha\lambda(1-\chi) + n_{min}(\alpha\mu - b\lambda) < 2\alpha\lambda(1-\chi).$$

If fertility is given exogenously (n_{ex}), the total tax such families can pay is

$$T_{ex} = 2\alpha\lambda(1-\chi) + n_{ex}(\alpha\mu - b\lambda).$$

As the condition $b > \alpha\mu/\lambda$ is always satisfied for sufficiently large levels of efficiency, it follows that $T^f_{max} \geq T_{ex}$ if $n_{ex} \geq n_{min}$ for high levels of efficiency. Therefore, the total tax revenues in the case where fertility is exogenous will be no higher than the revenues in the case where fertility is endogenous if λ is high. In the case with endogenous fertility and n_{min} sufficiently close to zero, large taxes will only lead to a strong reduction of fertility, while education remains unchanged, being highly productive: $T^f_{max} = 2\alpha\lambda(1-\chi)$. Similarly, if λ is low ($\lambda < \alpha\mu/b$ or $b < \alpha\mu/\lambda$), T_{ex} will be lower than T^f_{max} if $n_{max} > n_{ex}$.

If the parents' efficiency is sufficiently high, they will be able to pay any lump-sum tax. Ignoring the case where $b < \alpha\mu/\lambda$, as it is irrelevant for high levels of efficiency, a society without constraints on n_{min} will be able to afford higher taxes than a society where n_{min} is appreciably different from zero. The 'price' such a society pays is still very high: rich families could virtually die out when confronted with extremely high taxes. Although a community with $n_{min} > 0$ will pay less in taxes, there is still a danger in requiring rich families to pay the maximum they can afford: as they will finance their tax payments by sending their children to work, it is future generations who will bear the burden of the measure. As education can fall to $e = 0$, efficiency in the next period can be as low as $\lambda = 1$, leading the economy in the poverty trap. It is highly probable that families confronted with such taxes will try to avoid paying them independently of n_{min}. Therefore, these cases are mainly of mathematical interest.

Taxes and subsidies on child-raising costs

A tax on raising children can be interpreted as an increase of the expenditures on raising children b. Its impact on the household's decision can be derived by analyzing (16) for $n = \bar{n}$, (18) for $e = \bar{e}$ and (19) and (22) for the full corner solutions, while taking into account that switching between solutions is possible. As in the previous case, it is necessary to differentiate between poor and rich families, and low and high taxes/subsidies.

Poor parents – who usually choose $n = n_{max}$ and either $e_t = 0$ or the unrestricted solution with respect to education – will reduce schooling and leave fertility unchanged, if possible, as $\partial e^*(\cdot)/\partial b < 0$. Rich parents – who choose the corner solution with respect to education – will reduce fertility ($\partial n^*(\cdot)/\partial b < 0$). If the child-raising costs are larger, the unrestricted solution w.r.t. fertility will become feasible for lower levels of efficiency than in the case where b is low, as depicted in figure 1.1. Therefore, even a small tax on the expenditures on raising children can change the solution dramatically for some λ_t. In this case, parents confronted with low costs will choose $e \leq 1$ and $n = n_{max}$ while parents confronted with a tax will reduce fertility and select full-time schooling $e = 1$ and $n < n_{max}$.

When confronted with a very high tax on the expenditures on raising children, even those poor parents who would have chosen $n = n_{max}$ and $e = 0$ will reduce the number of children. In order to maintain consumption in the last period of life, they will also increase schooling, to $e = 1$ if possible. Therefore, a very high tax on child-raising expenditures will lead to full-time schooling, while reducing consumption in all periods of life and population growth significantly. As in the above cases, it is very improbable that such a tax can be enforced.

If the tax is not too high and parents do not reduce fertility, the maximal revenue to be gained from such a measure will be $\tau b n_{max} \lambda$, where τ is the tax rate defined in (11). In this case, the total tax revenue will be identical to the case where a lump-sum tax was raised, as all variables have identical values for both taxes. The critical level of τ so that parents do not reduce fertility – that is, $U(e = 0, n = n_{max}; \tau) > U(e = 1, n = n^*; \tau)$, which means that the unrestricted solution w.r.t. n becomes feasible – cannot be computed analytically. As soon as τ exceeds this level, n will decrease with $e = 1$, and the total tax revenue will approach $\frac{\beta\,(2\alpha\lambda(1-\chi))}{1+\beta} < 2\alpha\lambda(1-\chi)$ asymptotically as τ grows if n_{min} is sufficiently close to zero.[19] If $n_{min} > 0$, it is not possible to impose an unlimited tax on the child-raising costs, so that τ is limited. The maximal tax revenue will be $2\alpha\lambda(1-\chi) + n_{min}(\alpha\mu - b\lambda)$. It depends on parameters whether the total tax revenue in the case where n is reduced is higher or lower than in the case where e is reduced. In any case, if the parents are poor the total tax revenue raised through a lump-sum tax will be at least as high as the revenue from a tax on the child-raising costs.

Rich parents confronted with a tax will reduce fertility first. If n_{min} is very low ($n_{min} \approx 0$), families will respond to any increase in b by reducing n while leaving $e = 1$ unchanged. The maximal tax revenue is $\frac{\beta\,(2\alpha\lambda(1-\chi))}{1+\beta} < 2\alpha\lambda(1-\chi)$, as in the case where parents are poor. If, on the other hand, $n_{min} > 0$, parents will reduce e if the child-raising costs exceed some level. The result for fertility, education and total tax revenue and its consequences

[19] The term is computed as follows:

$$\lim_{\tau\to\infty} \tau b\lambda n = \lim_{\tau\to\infty} \tau b\lambda \frac{2\alpha\lambda(1-\chi)}{((1+\tau)b)\lambda} \frac{\beta}{1+\beta} = \frac{\beta}{1+\beta} 2\alpha\lambda(1-\chi) \lim_{t\to\infty} \frac{\tau}{1+\tau} = \frac{\beta}{1+\beta} 2\alpha\lambda(1-\chi)$$

for future generations will be the same as in the case where a lump-sum tax was raised; the comparison with the case where fertility is exogenous will also yield similar results.

As soon as parents decide to reduce fertility in order to finance the tax, total government revenues will be reduced. Therefore, this form of taxing is not very efficient as a means of raising public revenue.

If raising children is sufficiently subsidized ($\tau < 0$), poor parents[20] will choose $e = 1$. Therefore, such a subsidy will yield the result desired by the government. The amount of subsidy needed to raise e to the level e' can easily be computed using (16) for the case where no switching between solutions takes place:

$$\tau = \frac{2\alpha(1-\chi)}{nb} - \frac{\alpha n\mu\{1 + z\lambda[e' + (e'-1)(\beta+\beta_1)]\}}{z\,\bar{n}\,\lambda^2\,(\beta+\beta_1)b} - 1$$

School-attendance subsidies

The subsidy to promote education takes the form of a fixed cash transfer $s \geq 0$ for each unit of time each child spends at school. Therefore the total subsidy a family receives will be $S^e = sne$. School fees (i.e. $s < 0$) will be ruled out.

As in the previous section, the families receiving this subsidy are not taxed in any way. Therefore, (4) will not change, while (3) can be rewritten:

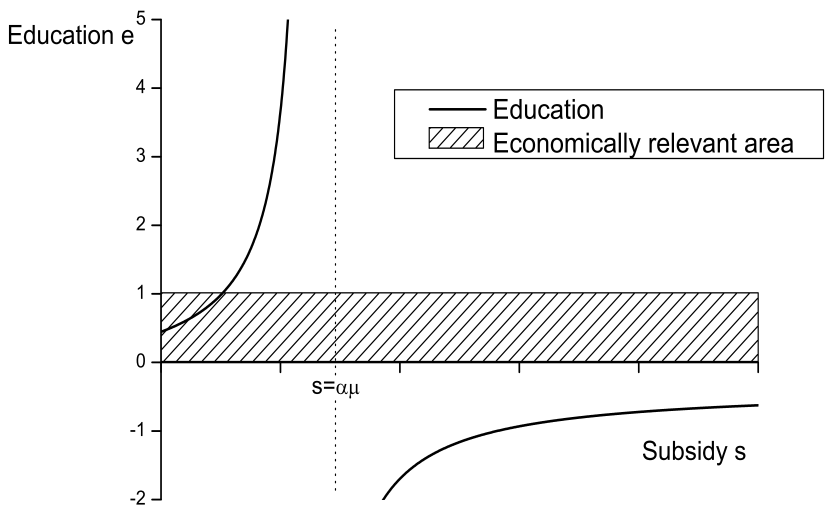
$$\begin{aligned} C_{1t} &= 2\alpha\lambda(1-\chi) + \alpha n\mu(1-e) - nb\lambda + sne \\ &= 2\alpha\lambda(1-\chi) - nb\lambda + \alpha n\mu + ne(s - \alpha\mu) \end{aligned}$$

If the attendance-subsidy per child exceeds $\alpha\mu$, the opportunity costs of education will be negative, so that families will never choose $e < 1$. Trivially, full-time education can easily

[20]Poor parents usually choose $e_t = 0$ or the unrestricted solution w.r.t education and $n = n_{max}$. If the subsidy does not trigger switching between solutions, and if it is sufficiently large, they will choose $n = n_{max}$ and $e = 1$. If the subsidy triggers switching between solutions, parents will choose the unrestricted solution w.r.t fertility and $e = 1$.

be achieved if the government, in effect, makes good for the income-loss families would otherwise experience. It remains to be seen whether there is a smaller subsidy ($s < \alpha\mu$) such that parents still choose $e = 1$, and whether this way of inducing an increase in e is more or less costly than other measures.

Figure 1.3: unrestricted solution with respect to education with subsidies (for $\alpha\mu = 1/4$)



Analyzing first the unrestricted solution w.r.t. education ($n = \bar{n}$), the new level of schooling depends on the subsidy as follows:

$$e(\bar{n}, \lambda, s) = \frac{z\lambda(\beta + \beta_1)[2\alpha\lambda(1 - \chi) + \bar{n}\alpha\mu - b\bar{n}\lambda] + n(s - \alpha\mu)}{\bar{n}z\lambda(1 + \beta + \beta_1)(\alpha\mu - s)} \tag{12}$$

Obviously, e can take any value between zero and infinity for $s \leq \alpha\mu$, as can be seen by computing $\lim_{s \to \alpha\mu-} e(\bar{n}, \lambda, s) = \infty$ if $b < \frac{2\alpha(1-\chi)}{\bar{n}} + \frac{\alpha\mu}{\lambda}$.[21] As the level of schooling the government wants to achieve is $e' \leq 1 (\ll \infty)$, it is obvious that there exists some subsidy $s < \alpha\mu$ that will suffice to induce parents to choose e' in this case (see figure 1.3). Therefore, even if the schooling parents choose for some (low) efficiency is zero, there will exist some subsidy $s < \alpha\mu$ such that parents prefer $e = e'$. For high levels of efficiency,

[21]This condition for the costs states that consumption in the first period of adult life is nonnegative if children work full-time. If the condition is not satisfied, parents cannot survive in the first period without outside help even for $e = 0$. In reality, therefore, one can assume this condition to be fulfilled for some n for all societies.

three scenarios are possible. If $b \leq \frac{2\alpha(1-\chi)}{n_{min}}$ and $z \geq 1$, parents will choose $e > e_{crit}$, and no subsidy is needed. If $b \leq \frac{2\alpha(1-\chi)}{n_{min}}$ and $z < 1$, long-term growth is not possible, so that this case will not be analyzed. If $b > \frac{2\alpha(1-\chi)}{n_{min}}$, very high levels of efficiency are not relevant economically, as parents cannot maintain $C_{1t} > 0$, even if their offspring work full-time. The value of z is irrelevant in this case.

Comparing subsidies: the unrestricted solution w.r.t. education

In this section, the total subsidies needed to induce parents to choose some value of $e' \leq 1$ will be compared. The focus will be on the corner solution w.r.t. fertility, as this case can be analyzed more easily and as the low-level steady state of most economies lies in this area. Therefore, regime-switching will be left out of the analysis at this stage, so that fertility is fixed ($n = \bar{n}$). The problems arising when regime-switching is taken into account – that is, where fertility is not fixed – are addressed at the end of the section.

It is easy to show that subsidizing the expenditures on raising children and a flat transfer will cost the same. The total subsidy needed to raise the level of schooling to e' in both cases is:[22]

$$
\begin{aligned}
S_1 &= \frac{\alpha n\mu(1+ze'\lambda) - z\lambda(\beta+\beta_1)\{\alpha n\mu(1-e') + \lambda[2\alpha(1-\chi) - nb]\}}{z\lambda(\beta+\beta_1)} \\
&= \frac{K_1}{z\lambda(\beta+\beta_1)} \qquad (13)
\end{aligned}
$$

In the case where the government subsidizes school-attendance directly, the total subsidy required to raise the level of schooling from e to e' is:

$$
S_2 = \frac{K_2}{z\lambda(\beta+\beta_1) + z\lambda + 1/e_1} \qquad (14)
$$

[22]This is computed as follows: find the subsidy τ needed to achieve the level of schooling e' for the unrestricted solution. Then compute $\tau bn\lambda$. In an analogous way, one can directly compute the cash transfer T^f needed to raise the level of schooling from some level e to e'. It can then be shown that $K_1 > 0$ for $e' > e^*(\bar{n}, \lambda)$, that is, a positive subsidy is needed to raise the level of schooling.

The numerators of both equations are identical ($K_1 = K_2 > 0$), but the denominators differ. As $z\lambda(\beta + \beta_1) + z\lambda + 1/e' > z\lambda(\beta + \beta_1)$, $\forall \lambda, e' > 0$, it follows immediately that $S_2 < S_1$ for all λ and $e' > e$. That is, the total subsidy required to induce parents to choose some level of schooling $e' > e$ will be lower if education is subsidized directly. This exemplifies the principle of targeting: as a school-attendance subsidy directly attacks the distortion arising from externalities from education, it will be the most efficient way to increase the level of schooling. Comparing the influence on consumption in the first period of a school-attendance subsidy s and a subsidy on the costs of raising children τ confirms this result: $\partial C_{1t}/\partial s < 0$ and $\partial C_{1t}/\partial \tau > 0$. A subsidy on expenditures on raising children will increase C_{1t}, whereas a subsidy on school-attendance will reduce it.

If regime-switching[23] becomes relevant when a tax or subsidy is introduced, no analytic results are possible. The exact subsidy needed in each case depends on the parents' initial efficiency, the child-raising costs and the education productivity factor z, and can only be computed numerically. Therefore, it is not possible to state in advance which subsidy will be cheaper. The interventions calculated above in equations (13) and (14) constitute an upper limit for the total subsidy needed in the case where corner solutions w.r.t. education are optimal after the intervention, and the total transfer required can be much lower in some cases.[24]

1.5.3 Policy Programs

Subsidizing education for a finite number of periods will always lead to parents eventually choosing $e > e_{crit}$ forever after, if $z > 1$ and $b < \frac{2\alpha(1-\chi)}{n_{min}}$, that is, if sustainable growth is possible. The total resources needed depend on everything in the system but

[23] In the present case, regime-switching caused by a subsidy means that families change not only e_t when receiving a subsidy, but also fertility. For example, they switch from choosing the unrestricted solution w.r.t. education and $n = n_{max}$ before the subsidy was introduced, to choosing $e_t = 1$ and the unrestricted solution w.r.t. fertility thereafter. In there is no regime-switching, parents choose unrestricted solution w.r.t. education and n_{max} before and after the subsidy is introduced.

[24] Particularly in the immediate vicinity of the point of discontinuity w.r.t. education and n (see figures 1.6a to 1.6i), where the unrestricted solution w.r.t. fertility becomes optimal, the total subsidy will be very low.

an upper limit thereon can be computed as follows: Assuming that the parents' initial level of efficiency is $\lambda_0 < \lambda'$,[25] let P periods be needed to reach λ' under the following program: parents receive subsidies such that they choose $e = 1$ during the first $P - 1$ periods. In the last period, P, they are induced to choose the level of schooling e_P such that $ze_P\lambda_P + 1 = \lambda'$. Without discounting, the total amount needed, measured in units of output, is $S = S_1 + S_2 + ... + S_P$, where S_p denotes the total subsidy needed in period p. As subsidizing school attendance is most efficient as long as no regime-switching regarding fertility takes place, one can compute S_p using equation (14) and update the parents' efficiency in any period p as follows: $\lambda_p = z\lambda_{p-1} + 1$ for all $p < P$ and $\lambda_P = ze_P\lambda_{P-1} + 1 = \lambda'$. Backwards induction then yields the minimum number of periods needed to reach λ'. Given the restriction that only the unrestricted solution w.r.t. education is analyzed, the number of children born will be constant over all periods.

If it is possible to finance the measure from abroad, for example, through loans to be paid back no earlier than after P periods, or in some other way which does not involve taxation during the periods in which the subsidy is paid, it is possible for the whole society to escape the poverty trap simultaneously and in finite time. If the measure has to be financed through current taxes, however, then whether a successful program can be set up will depend on the system's parameters.

The simplest program, in which no subsidies whatsoever are required, is one where poor families – who would otherwise choose $e \leq e_{crit}$ and $n = n_{max}$ – have to pay a very high tax on the expenditures incurred in raising children. Confronted with this measure, parents will reduce the number of children and increase schooling to $e = 1$. As in the case of a subsidy fully financed from abroad, it is possible to raise the efficiency of the whole society simultaneously to λ' or above. Such a program would not be easy to implement, however, as it would reduce the consumption and utility of at least one generation dramatically. If the taxes raised were used to finance school-attendance subsidies, this measure would

[25] Recall that λ' is chosen so that parents choose $e \geq e_{crit}$ for all $\lambda \geq \lambda'$.

involve a change in relative prices. Families experiencing such a measure might change their decision to reduce fertility and increase education.

In the present setting, there are no other programs such that the whole society can escape the poverty trap simultaneously. If subsidies need to be financed currently, and if this financing is not ensured by means other than taxes, part of the population will have to pay for the subsidies while the rest will enjoy them. Therefore, inequality will arise after the first period in which the measure is introduced. If the process is continued, the efficiency levels of the families so subsidized will exceed λ' after a finite number of periods. This group can now be taxed in some measure, and the revenue obtained can be used to subsidize the poor families. If the tax schedule is chosen such that the efficiency of no succeeding 'rich' generation falls below λ', and if enough revenue is raised to finance a subsidy for the poor which enables the latter to reach λ' after some time, the program will lead to sustainable growth for the entire society after a finite number of periods. The inequality that arises through such a program – due to fertility differences between the groups and due to differences in the level of efficiency – will be discussed in the following section.

1.5.4 Inequality

Consider a society of homogeneous adults, who live in extended families. Assume that in the first period the entire economy is in the poverty trap, which is the only low-level stationary state of the economy. All adults' level of efficiency is low, and households typically choose the corner solution w.r.t fertility and $e < 1$. Let z and b be such that unbounded growth of efficiency is possible, and let $e \geq e_{crit}$ for all $\lambda \geq \lambda'$. As already discussed in previous sections, taxes and subsidies which lead to switching between solutions are hard to analyze. For simplicity, therefore, we will assume that no switching takes place, that is, fertility will be fixed in the first stage of the program. In this case, lump-sum taxes will yield at least the same revenue as all other taxes discussed so far, and school-attendance subsidies will increase schooling more cheaply and efficiently than any other subsidy. In the first part of the program, therefore, a lump-sum tax should be levied on some part of the population and the education of the children of another part should be subsidized. The assumptions of assortative mating and extended families will hold for both groups of individuals, taxed or subsidized. As a consequence there will be no link between the two groups except for fiscal policy and the enforcement of taxation.

Assuming that the subsidy is sufficient to raise the recipient families' level of efficiency just above λ', it follows that these will subsequently choose $e > e_{crit}$, and, after a sufficient number of periods, $e = 1$. The new situation is therefore one where two groups exist: 'rich' families, who usually choose $e = 1$ and $n^r \in [n_{min}, n_{max}]$ and 'poor' families, who usually choose $e < 1$ and $n^p = n_{max}$. Given the assumption of assortative mating, the inequality will be persistent, whereby the 'rich' families' level of efficiency will rise and 'poor' families will be in the poverty trap. Typically, the population growth rates of the two groups will be different, with $n^r \leq n^p$, so that the relative number of rich families will decline over time. The government might now contemplate taxing the rich families and using those taxes to subsidize the poor households.

Whether rich families will ever be able to provide a tax base sufficient to subsidize all poor families depends on several factors. First, the efficiencies of the two groups: the more efficient they are, the more taxes rich parents can pay, and the smaller the subsidy needed by poor families to reach λ'. Second, the relative size of the two groups: if there are only few 'rich' and many 'poor' families, it will not be possible to raise sufficient revenue for all the 'poor'. Third, the minimum number of children a family can have. As discussed in section 1.5.2, if n_{min} is large, current taxable capacity is lower than in the case where n_{min} is very low.

The following simplified example with n_{min} sufficiently close to zero will illustrate the underlying problems. Assume that all poor families are in the poverty trap, that this is the only stable stationary state, that sustainable economic growth is possible for $\lambda \geq \lambda'$

and that the total subsidy 'poor' families need in a given period is S. Rich families – whose initial level of efficiency is $\lambda_0^r \geq \lambda'$ – can pay at most $\lambda_0^r(2\alpha(1-\chi)-n^r b)-\frac{\alpha n^r \mu}{\beta+\beta_1}\left(1+\frac{1}{z\lambda_0^r}\right)$ each, while still choosing $e_0 = 1$. Note that, in general, $n^r = n_{min}$ after the taxes are levied.[26] In the first period, let there be r rich families, so that total tax revenue will be $T(\lambda^r) = r \cdot \left(\lambda_0^r(2\alpha(1-\chi)-n^r b)-\frac{\alpha n^r \mu}{\beta+\beta_1}\left(1+\frac{1}{z\lambda_0^r}\right)\right)$. If the total tax revenue in the first period is not sufficient to pay for the subsidies, the inequality $T(\lambda_0^r) < S$ will hold. The government has two options: it can either tax the rich families and subsidize a fraction of poor families, or it can simply wait until the tax base is sufficiently large to subsidize all poor families simultaneously. We will first take up the case where the government decides to wait. In the following period, with fertility rates n^r for the rich and $n^p = n_{max}$ for the poor, sufficient revenue can be raised if and only if:

$$\frac{n^r}{n_{max}} \cdot T(z\lambda_0^r + 1) \geq S.$$

Obviously, the relative population growth rate $n^r/n_{max} < 1$ and the productivity of the education function z influence whether 'waiting' will ever lead to the government being able to raise sufficient revenue to finance a subsidy for all the poor. After t periods, the difference between revenues and spending $\delta(t)$ will be:

$$\delta(t) \equiv \left(\lambda_t^r(2\alpha(1-\chi)-n^r b)-\frac{\alpha n^r \mu}{\beta+\beta_1}\left(1+\frac{1}{z\lambda_t^r}\right)\right)(n^r)^t - S n_{max}^t, \tag{15}$$

where $\lambda_t^r = \lambda_0^r z^t + (z^t-1)/(z-1)$. Depending on z and the fertility rates of the two groups, the tax revenues after P periods might suffice to finance the subsidies for the 'poor', that is $\delta(t=P) \geq 0$. $\delta(t)$ is a continuous function in t, with $\delta(t=0) < 0$ by assumption. If $zn^r > n_{max}$, it can be shown that $\lim_{t\to\infty}\delta(t) = \infty$ that is, there will be some period $P < \infty$ such that $\delta(P) \geq 0$. Therefore, if $zn^r > n_{max}$ and the government waits for P periods the tax revenues will suffice to finance a subsidy for all poor families. In the following period, $P+1$, all families will be outside the poverty trap, and the level

[26] Recall that families in the high-level stationary state normally reduce fertility before reducing schooling.

of each adults' human capital will be at least λ'.

The government could reduce this period of waiting by subsidizing a fraction of the poor families in the first period, where the proportion of poor families subsidized will be $T(\lambda_0^r)/S$. As rich parents choose $n^r = n_{min}$, the taxable capacity of this group is reduced in the following period. However, the families which have received a subsidy in the previous period can also pay taxes if it is possible to find some tax such that $ze^{tax}\lambda' + 1 \geq \lambda'$. Note that this is always possible as long as the level of human capital attained by the previously subsidized families lies above λ'. The subsidy required by all remaining poor families is also smaller, and amounts to $Sn_{max}(1 - T/S)$. The period of time required to subsidize all poor families depends on the parameters.

1.6 Conclusion

In a society where parents decide freely how many children to have and how well to educate them (as opposed to setting them to work), and also have some measure of altruism towards their children, the child-raising costs, the social norms that govern the provision of support in old age and the productivity of the underlying educational process all have a vital influence. One possibility is that the economy will be trapped in a low-level, stable stationary state – or poverty trap – in which adults' labor efficiency and lifetime utility are low, and child labor is the rule. Fertility will usually be at its exogenously given upper limit, so that, while consumption and income per family are constant, the total population grows exponentially. Only if child-raising costs are sufficiently low and the educational process is highly productive can such a state be avoided.

Escape from this poverty trap is theoretically always possible if the educational process is sufficiently productive ($z > 1$) and child-raising costs are not extremely high. If the government is sufficiently strongly constrained in raising taxes, however – for example, by the ease with which taxes can be evaded or by the upper limits on taxes imposed by minimal consumption needs – it might not be possible to devise a policy such that the whole society can escape the poverty trap.

When parents can decide only about the extent of schooling, fertility being given exogenously, the range of child-raising costs such that sustainable economic growth is possible becomes much narrower; for parents have a more limited menu of choices of how to react to taxes. As they can only reduce current consumption, or education, or both, the imposition of taxes will usually lead to a stronger reduction in overall schooling than if fertility is endogenous. Therefore, the maximal amount of tax rich families can pay without falling back into the poverty trap is lower when fertility is exogenous. On the other hand, the danger of extinction of the lineage is not relevant in such a setting. Poor parents could theoretically be induced to choose full-time schooling through high taxes on the expenditures on raising children, a step they could afford by reducing their fertility. If fertility is fixed, however, the measure cannot have this effect. Therefore, in a setting where schooling alone is analyzed, the simultaneous escape of the whole society from the poverty trap is not possible without outside help.

Incorporating fertility decisions increases the household's flexibility in maximizing its utility and reacting to the introduction of taxes and subsidies. These fiscal measures are at least as efficient in raising revenue or increasing schooling as in the case where fertility is fixed exogenously. In the case of policy programs, under which rich families in a high-level stationary state or growth steady-state pay taxes to subsidize poor families in the poverty trap, the range of values for z (the productivity of the educational production function) for which waiting is profitable is much wider if fertility is exogenous. However, with tax revenues being generally higher and subsidies required being somewhat lower when fertility is endogenous, the number of periods required to lift all families out of the poverty trap might be lower, even if the relative number of poor families increases.

1.7 Appendix

A.1 The corner solution with respect to fertility

If the number of children is fixed ($n_t = \bar{n}$), parents will choose the following level of schooling (the time index t has been suppressed):

$$e^*(\bar{n}, \lambda) = \frac{z\lambda(\beta + \beta_1)[2\alpha\lambda(1-\chi) + \bar{n}\alpha\mu - \bar{n}b\lambda] - \alpha\bar{n}\mu}{\alpha\bar{n}\mu z\lambda(1+\beta+\beta_1)} \tag{16}$$

In order to investigate the properties of $e^*(\cdot)$, we begin by disregarding the restriction $e^* \in [0, 1]$. The shape of the function $e^*(\cdot)$ depends on two parameters: the child-raising costs, b, and the parents' efficiency, λ. Analyzing the function at the borders of its domain reveals that poorly educated parents cannot afford to send their offspring to school, the same being true for the case where raising children is very expensive.

$$\lim_{\lambda \to 0} e^*(\bar{n}, \lambda) = -\infty, \qquad \lim_{b \to \infty} e^*(\bar{n}, \lambda) = -\infty,$$

$$\lim_{\lambda \to \infty} e^*(\bar{n}, \lambda) = sign[2\alpha(1-\chi) - b\bar{n}]\infty$$

Consumption in the first period of adult life can be rewritten so the impact of λ and b becomes clear:

$$C_{1t} = \lambda_t[2\alpha(1-\chi) - b\bar{n}] + \alpha\mu(1-e)\bar{n}$$

If the child-raising costs are sufficiently small ($b < \frac{2\alpha(1-\chi)}{\bar{n}}$), the term in the square brackets will be positive and $\lambda_t[2\alpha(1-\chi) - b\bar{n}]$ will grow without bound for high levels of efficiency. The term describing income from child labor will then be negligible in comparison: as a consequence, parents will be able to afford to send their offspring to school if the child-raising costs are low. If, on the other hand, b is large, the term in squared brackets will be negative, and child labor will be crucial to financing C_{1t}. For very high levels of efficiency, parents will not be able to maintain a nonnegative level of consumption in the first period: a negative school-time would be needed, if that were possible.

With $2\alpha(1-\chi)\lambda_t$ being the residual income of a family after payment of the transfer to the grandparents, the condition $2\alpha(1-\chi) - b\bar{n} \geq 0$, has a natural interpretation. If the condition is satisfied, parents do not have to resort to child labor in order to enjoy positive consumption in the first period of adult life. If, on the other hand, $b > \frac{2\alpha(1-\chi)}{\bar{n}}$, some child labor will be optimal.

Apart from the behavior of the function $e^*(\cdot)$ for extreme values of efficiency and child-raising costs, the schooling chosen is characterized by its zeroes and maxima, which are of major importance for the economic interpretation and relevance of the closed-form solution in (16). As the numerator is a quadratic function of λ, one expects to find up to two zeroes:

$$\lambda_{1,2} = -\frac{1}{2}\frac{\alpha z\bar{n}\mu(\beta+\beta_1) \pm \sqrt{\alpha z\bar{n}\mu(\beta+\beta_1)[\alpha z\bar{n}\mu(\beta+\beta_1) + 8\alpha(1-\chi) - 4bn]}}{z(\beta+\beta_1)[2\alpha(1-\chi) - b\bar{n}]} \tag{17}$$

For sufficiently small costs ($b < \frac{2\alpha(1-\chi)}{\bar{n}}$) only one of these will be positive and the function will have no extrema. For larger costs ($b > \frac{2\alpha(1-\chi)}{\bar{n}}$), there will be two zeroes and a maximum. As the factor under the square root falls with increasing costs, and becomes negative when they are sufficiently large ($b > \frac{2\alpha(1-\chi)}{\bar{n}} + \frac{\alpha z\bar{n}\mu(\beta+\beta_1)}{4\bar{n}}$), the function $e^*(\cdot)$ will be negative for all levels of efficiency if b exceeds this limit. With increasing costs, the first zero of the function will move towards larger levels of efficiency: the larger the costs, the less parents can afford to send their children to school, given that the number of children they have is fixed ($e(b) > e(b') \;\; \forall\, b < b'$).

The shape of $e^*(\bar{n}, \lambda)$ has been plotted in figure 1.2 for different costs. For the purposes of depiction, the intervals for b that have been established above will be defined as 'low', 'moderate' and 'high'. If the function has no maximum and grows without bound for high levels of efficiency, it will be economically relevant for some levels of efficiency $\lambda \in [max(1, \lambda_a), min(1, \lambda_b)]$ where $e(\lambda_a) = 0$ and $e(\lambda_b) = 1$. If the function has a maximum, the ranges can be defined using the zeros of the function, as computed in (17). [27]

The impact of changes in $\bar{n}$ on $e^*(\bar{n}, \lambda)$ remains to be analyzed. With $\partial e^*/\partial \bar{n} < 0\, \forall \lambda, \bar{n}$, it follows immediately that $e^*(n_{min}) > e^*(n_{max})$: the fewer children a family has, the better it will educate them. As parents do not have to spend so much money just raising children, they can afford to educate them better. On the other hand, additional education is necessary in order to finance and maintain consumption in the last period of life. Therefore, if a family has fewer children for any reason, schooling will increase. A reduced number of children increases C_{1t} if the level of schooling remains unchanged, while C_{2t} will fall. Therefore, a shift in consumption between the two periods is needed, and as education is the only variable in which C_{1t} is decreasing and C_{2t} is increasing, schooling will be increased – if that is possible.[28]

A.2 The corner solution with respect to education

In this case, too, the solution depends on the parents' level of efficiency and the child-raising costs as well as the fixed level of schooling.

$$n^*(\bar{e}, \lambda) = \frac{-2\alpha\beta\lambda(1-\chi)}{(1+\beta)[\alpha\mu(1-\bar{e}) - b\lambda]} \tag{18}$$

For very small λ, the function will be zero or negative for all parameters, while the shape of the function for moderate and high levels of efficiency will depend on $\bar{e}$ and the child-raising costs b:

$$\lim_{\lambda \to 0} n^*(\bar{e}, \lambda) = 0, \qquad \lim_{\lambda \to \infty} n^*(\bar{e}, \lambda) = \frac{2\beta\,\alpha\,(1+\chi)}{b\,(1+\beta)} > 0$$

[27] As it can be shown that the maximum of the function lies below 1 for all parameters, the case where the function is relevant in two separate sections can be ignored.

$$e_{max} = \frac{\beta + \beta_1}{1 + \beta + \beta_1}\left[1 - \frac{4\alpha(1-\chi) - 2nb}{\sqrt{z\,(\beta+\beta_1)\,(2\alpha(1-\chi) - bn)\,n\mu\,\alpha}}\right] < 1$$

[28] These considerations are only valid – at this point of the analysis – if the unrestricted solution w.r.t. education is the household's optimum. The intuitive argument will be valid for all solutions.

If $e = 1$, the function $n^*(\cdot)$ will be independent of λ ($n = \frac{2\alpha\beta(1-\chi)}{(1+\beta)b} > 0$), and decreasing in b. For the value of n to be economically relevant in this case, the child-raising costs have to satisfy the condition

$$b \in \left[\frac{2\alpha\beta(1-\chi)}{n_{max}(1+\beta)}, \frac{2\alpha\beta(1-\chi)}{n_{min}(1+\beta)}\right].$$

If $e = 0$, n will be negative for low levels of efficiency up to the point of discontinuity and positive thereafter. In general, the critical efficiency (point of discontinuity) depends on the chosen schooling and the child-raising costs: $\lambda = \frac{\alpha\mu(1-\bar{e})}{b}$. For extremely large λ, n approaches a positive value asymptotically from above, since $\partial n/\partial\lambda \leq 0 \ \ \forall\lambda$:

$$\lim_{\lambda\to\infty} n^*(\bar{e}, \lambda) = \frac{2\alpha\beta(1-\chi)}{b(1+\beta)} > 0$$

For $n^*(\bar{e}, \lambda)$ to be economically feasible for any λ, the child-raising costs have to fulfill the same condition as in the case $e = 1$. An exogenous increase in schooling will lead to a decrease in the number of children a couple decide to have: $\partial n/\partial\bar{e} < 0 \, \forall \, \lambda, b$. The intuitive argument is the same as in the case where education was interior: An increase in schooling will lead to a reduction in family income in the first period, and hence in C_{1t}, if all other variables remain unchanged, while C_{2t} will rise. Therefore, parents will try to shift consumption between periods by reducing fertility.

A.3 The corner solutions w.r.t. both variables

As stated in section 1.3.3, the unrestricted solutions w.r.t. either fertility or education are not feasible for all parameters, and full corner solutions must be analyzed. We will first take up $\triangle U_n$ and then $\triangle U_e$ in the following two sections.

$\triangle U_n$

The following condition for $\triangle U_n > 0$ can be derived:

$$\triangle U_n := U(n_{max}, \bar{e}) - U(n_{min}, \bar{e}) > 0$$

$$\Leftrightarrow$$

$$ln\Big[\frac{2\alpha(1-\chi)\lambda + \alpha(1-\bar{e})\mu n_{max} - b\lambda n_{max}}{2\alpha(1-\chi)\lambda + \alpha(1-\bar{e})\mu n_{min} - b\lambda n_{min}}\Big] + $$
$$+\beta ln\Big[\frac{\alpha(z\lambda\bar{e}+1)\chi n_{max}}{\alpha(z\lambda\bar{e}+1)\chi n_{min}}\Big] + \beta_1 ln\Big[\frac{(z\lambda\bar{e}+1)}{(z\lambda\bar{e}+1)}\Big] > 0$$

$$\Leftrightarrow$$

$$\frac{2\alpha(1-\chi)\lambda + \alpha(1-\bar{e})\mu n_{max} - b\lambda n_{max}}{2\alpha(1-\chi)\lambda + \alpha(1-\bar{e})\mu n_{min} - b\lambda n_{min}} > \Big(\frac{n_{min}}{n_{max}}\Big)^{\beta}$$

A rearrangement yields: $\triangle U_n > 0$ if and only if:

$$\lambda[2\alpha(1-\chi)(n_{max}^{\beta} - n_{min}^{\beta}) - b(n_{max}^{1+\beta} - n_{min}^{1+\beta})] > -\alpha\mu(1-\bar{e})(n_{max}^{1+\beta} - n_{min}^{1+\beta}) \tag{19}$$

As before, b and λ determine whether the condition above is satisfied or not.

If

$$b \leq \frac{2\alpha(1-\chi)(n_{max}^{\beta} - n_{min}^{\beta})}{n_{max}^{1+\beta} - n_{min}^{1+\beta}}, \tag{20}$$

the left-hand side of (19) will be nonnegative. Since $n_{min} < n_{max}$ and $1+\beta > 1$, the term on the right-hand side of (19) will be negative or zero. Therefore, $\triangle U_n$ will be positive, so that $U(n_{max}, \bar{e}) > U(n_{min}, \bar{e})$ for all levels of efficiency if b satisfies condition (20) . If the child-raising costs are low, choosing the maximal number of children (n_{max}) will be optimal for all families when e is fixed.

If

$$b > \frac{2\alpha(1-\chi)(n_{max}^{\beta} - n_{min}^{\beta})}{n_{max}^{1+\beta} - n_{min}^{1+\beta}},$$

both sides of condition (19) will be negative, and $U(n_{max}, \bar{e}) > U(n_{min}, \bar{e})$ only if

$$\lambda < \frac{\alpha\mu(1-\bar{e})(n_{max}^{1+\beta} - n_{min}^{1+\beta})}{2\alpha(1-\chi)(n_{max}^{\beta} - n_{min}^{\beta}) - b(n_{max}^{1+\beta} - n_{min}^{1+\beta})}.$$

Choosing the maximal number of children, then, will only be optimal for poor parents, while well-educated households will prefer having few children if the costs of raising them are large. The lower the child-raising costs and the lower the level of schooling, the higher is the critical level of efficiency.

$\triangle U_e$

To analyze $\triangle U_e$, consider, first, the following simplification:

$$\triangle U_e > 0 \Leftrightarrow \tag{21}$$

$$ln\Big[\frac{2\alpha(1-\chi)\lambda + \alpha\bar{n}\mu - \bar{n}b\lambda}{2\alpha(1-\chi)\lambda - \bar{n}b\lambda}\Big] + \beta ln\Big[\frac{\alpha\chi\bar{n}}{\alpha\chi\bar{n}(z\lambda+1)}\Big] + \beta_1 ln\Big[\frac{1}{z\lambda+1}\Big] > 0 \Leftrightarrow$$

$$\frac{2\alpha(1-\chi)\lambda + \alpha\bar{n}\mu - \bar{n}b\lambda}{2\alpha(1-\chi)\lambda - \bar{n}b\lambda} > \Big(\frac{1}{z\lambda+1}\Big)^{-(\beta+\beta_1)} \tag{22}$$

Analyzing the behavior of $\triangle U_e$ for very small and very high levels of efficiency yields:

$$\lim_{\lambda\to 0} \triangle U_e = \infty \quad \text{and} \quad \lim_{\lambda\to\infty} \triangle U_e = -\infty$$

Therefore $\triangle U_e$ has at least one zero or point of discontinuity. For $b \geq \frac{2\alpha(1-\chi)}{\bar{n}}$, the function will have a single point of discontinuity for positive levels of efficiency, and none otherwise. As all 'goods' are necessary in consumption, and as the condition $b \geq \frac{2\alpha(1-\chi)}{\bar{n}}$ yields $C_{1t}(e=1) \leq 0$, it follows that $e_t = 0$ whenever $b \geq \frac{2\alpha(1-\chi)}{\bar{n}}$. That is to say, the children must then be put to work full-time in order to finance their parents' consumption in the first period of adult life.

If, on the other hand, $b < \frac{2\alpha(1-\chi)}{\bar{n}}$, parents can afford to educate their children, both $e = 0$ and $e = 1$ are possible optima, and the function $\triangle U_e$ has no point of discontinuity. The derivative $\partial\triangle U_e/\partial\lambda$ being always negative for small costs and as $\triangle U_e$ takes all values between $-\infty$ and ∞, it follows that the function $\triangle U_e$ will have a single zero in the interval $0 \leq \lambda < \infty$.

By analyzing the two cases $\beta + \beta_1 > 1$ and $\beta + \beta_1 < 1$ separately, it is possible to approximate the critical levels of efficiency for $\triangle U_e > 0$ and $\triangle U_e < 0$. If $\beta + \beta_1 < 1$ a simplification [29] of the system yields:

$$\triangle U_e > 0 \Leftarrow$$
$$\frac{2\alpha(1-\chi)\lambda + \alpha\bar{n}\mu - \bar{n}b\lambda}{2\alpha(1-\chi)\lambda - \bar{n}b\lambda} > z\lambda + 1$$

This inequality can easily be solved for the critical level of efficiency:

$$\triangle U_e > 0 \;\Leftrightarrow\; \lambda \in \Big(-\sqrt{\frac{\alpha\bar{n}\mu}{z[2\alpha(1-\chi) - \bar{n}b]}}, \sqrt{\frac{\alpha\bar{n}\mu}{z[2\alpha(1-\chi) - \bar{n}b]}}\Big)$$

The result in the case $\beta + \beta_1 > 1$ will be identical. In both cases, the condition for the critical level of efficiency derived above is necessary but not sufficient. It should be noted that for extremely large child-raising costs, even full-time child labor cannot ensure non-negative C_{1t}, with the critical value for the costs depending on efficiency and the number of children: $b < \frac{2\alpha(1-\chi)}{\bar{n}} + \frac{\alpha\mu}{\lambda} \underbrace{\longrightarrow}_{\lambda\to\infty} \frac{2\alpha(1-\chi)}{\bar{n}}$.

A.4 Potential Steady States: the unrestricted solution w.r.t. e

There are two possible efficiencies such that $e^*(\bar{n}, \lambda) = e_{crit}$:

$$\begin{aligned}\lambda = \; & -\frac{1}{2}\frac{\alpha n\mu[(\beta+\beta_1)(z-1)-1]}{z(\beta+\beta_1)(2\alpha(1-\chi)-nb)} \\ & \pm \frac{\sqrt{\alpha n\mu\Big[\Big(\alpha n\mu[(\beta+\beta_1)(z-1)-1]^2\Big) - 4z(\beta+\beta_1)^2(2\alpha(1-\chi)-nb)\Big]}}{2z(\beta+\beta_1)(2\alpha(1-\chi)-nb)}\end{aligned} \tag{23}$$

[29] The simplification used is: $\frac{2\alpha(1-\chi)\lambda+\alpha\bar{n}\mu-\bar{n}b\lambda}{2\alpha(1-\chi)\lambda-\bar{n}b\lambda} > (z\lambda+1)^1 > (z\lambda+1)^{(\beta+\beta_1)}$

For the term under the square root to be positive, the costs of child-rearing must satisfy the condition:

$$b \geq \frac{2\alpha(1-\chi)}{n} - \frac{\alpha\mu[(\beta+\beta_1)(z-1)-1]^2}{4z(\beta+\beta_1)^2} \tag{24}$$

If (24) is not satisfied, there will be no λ so that $e = e_{crit}$ for the unrestricted solution w.r.t education.[30] If, on the contrary, (24) is satisfied, the two functions will have up to two points of intersection, depending on the exact size of b, z and $(\beta + \beta_1)$. If, further, the total weight of the future arguments of utility is sufficiently smaller than that of C_{1t}, that is, $(\beta + \beta_1) < 1$, and if the productivity of education (z) is not too large, then the term $[(\beta + \beta_1)(z - 1) - 1]$ will be negative. In this case, and if $\frac{2\alpha(1-\chi)}{n} - \frac{\alpha\mu[(\beta+\beta_1)(z-1)-1]^2}{z(\beta+\beta_1)^2} \leq b \leq \frac{2\alpha(1-\chi)}{n}$, there will be two points of intersection.[31] If, however, $b > \frac{2\alpha(1-\chi)}{n}$, the root in (23) being larger than the term before it, only one of the signs will yield $\lambda > 0$, so that the two functions will cross only once.

[30] Both values in (23) will be complex numbers.

[31] The term under the square root in (23) will be positive but smaller than the term before it, so that both signs will yield a positive efficiency.

Figures: The Household's Decisions and Phase Diagrams

The following parameters were used to plot figures 1.6a to 1.6i:

$$\alpha = 1, \quad \mu = 0.5, \quad \chi = 0.4$$

$$\beta = 2/3, \quad \beta_1 = 1/5$$

$$n_{min} = 0.2, \quad n_{max} = 10$$

With these parameters, we get:

$$\frac{2\alpha(1-\chi)}{n_{max}} = 0.12$$

and

$$\frac{2\alpha(1-\chi)}{n_{max}} = 6.$$

As a consequence, the following values were chosen for b and z:

$b < \frac{2\alpha(1-\chi)}{n_{max}}$	$\rightarrow$	$b = 0.1$	$z < 1 \rightarrow z = 0.5$
$b \in [\frac{2\alpha(1-\chi)}{n_{max}}, \frac{2A(1-\chi)}{n_{min}}]$	$\rightarrow$	$b = 0.14$	$z = 1 \rightarrow z = 1$
$b > \frac{2\alpha(1-\chi)}{n_{min}}$	$\rightarrow$	$b = 6.01$	$z > 1 \rightarrow z = 2$

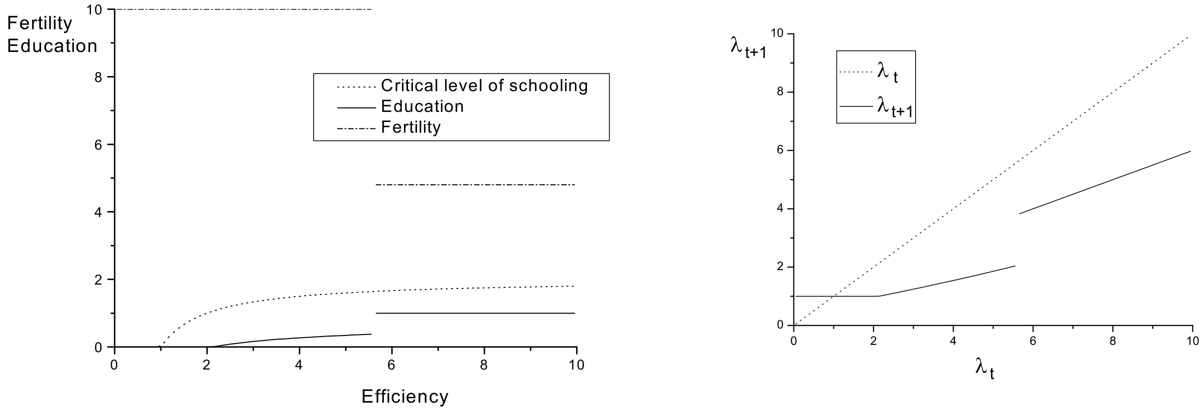


Figure 1.6a: The Household's Decision and Phase Diagram: $z < 1$ and $b < \frac{2\alpha(1-\chi)}{n_{max}}$

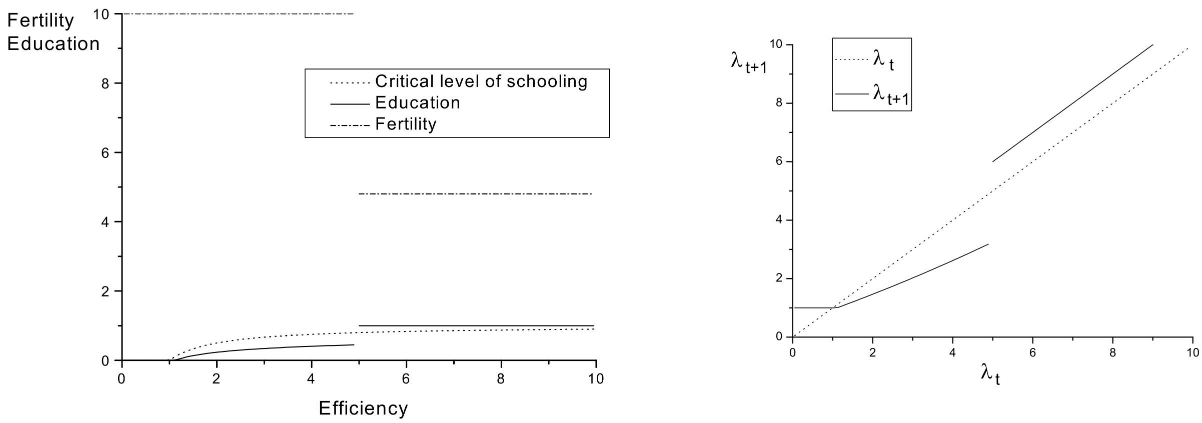


Figure 1.6b: The Household's Decision and Phase Diagram: $z = 1$ and $b < \frac{2\alpha(1-\chi)}{n_{max}}$

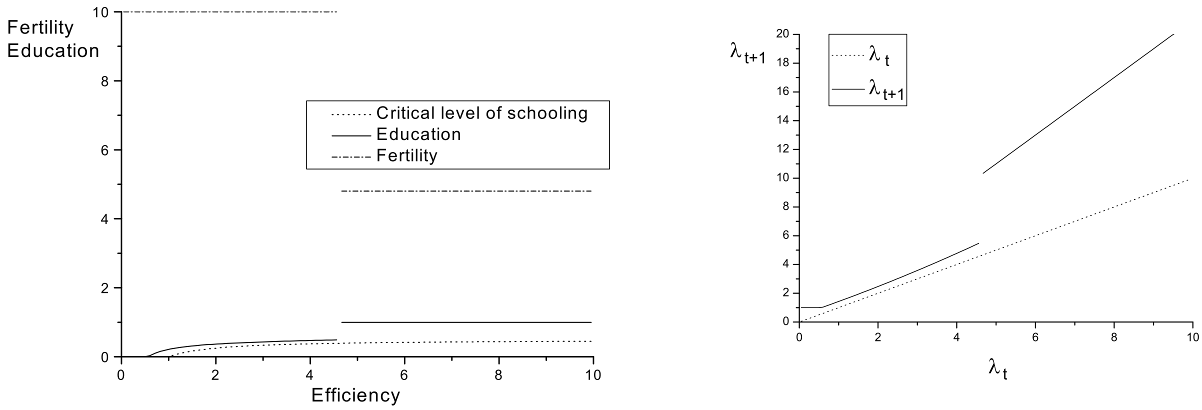


Figure 1.6c: The Household's Decision and Phase Diagram: $z > 1$ and $b < \frac{2\alpha(1-\chi)}{n_{max}}$

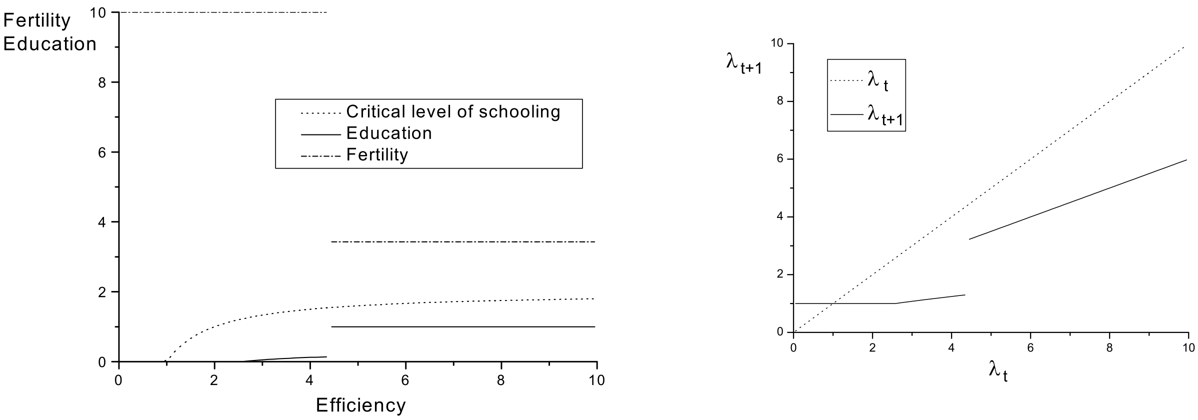


Figure 1.6d: The Household's Decision and Phase Diagram: $z < 1$ and $b \in [\frac{2\alpha(1-\chi)}{n_{max}}, \frac{2\alpha(1-\chi)}{n_{min}}]$

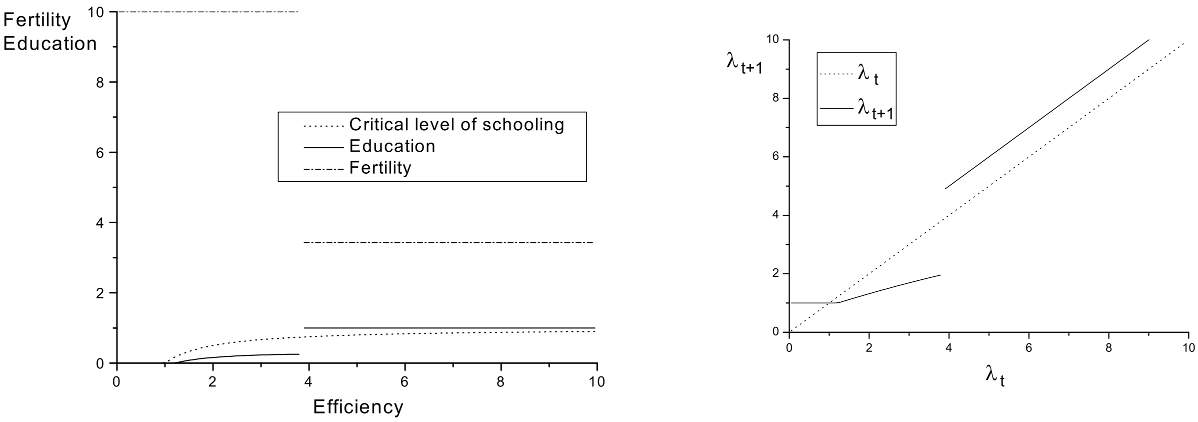


Figure 1.6e: The Household's Decision and Phase Diagram: $z = 1$ and $b \in [\frac{2\alpha(1-\chi)}{n_{max}}, \frac{2\alpha(1-\chi)}{n_{min}}]$

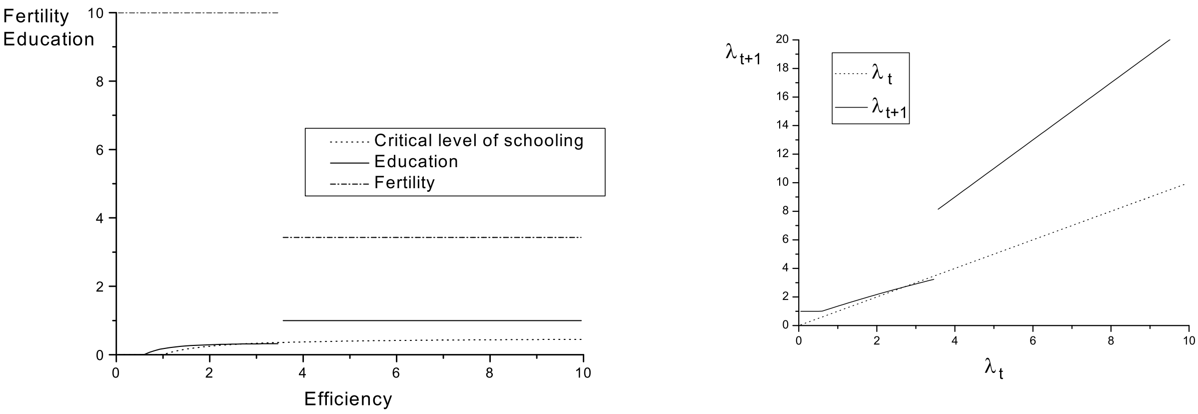


Figure 1.6f: The Household's Decision and Phase Diagram: $z > 1$ and $b \in [\frac{2\alpha(1-\chi)}{n_{max}}, \frac{2\alpha(1-\chi)}{n_{min}}]$

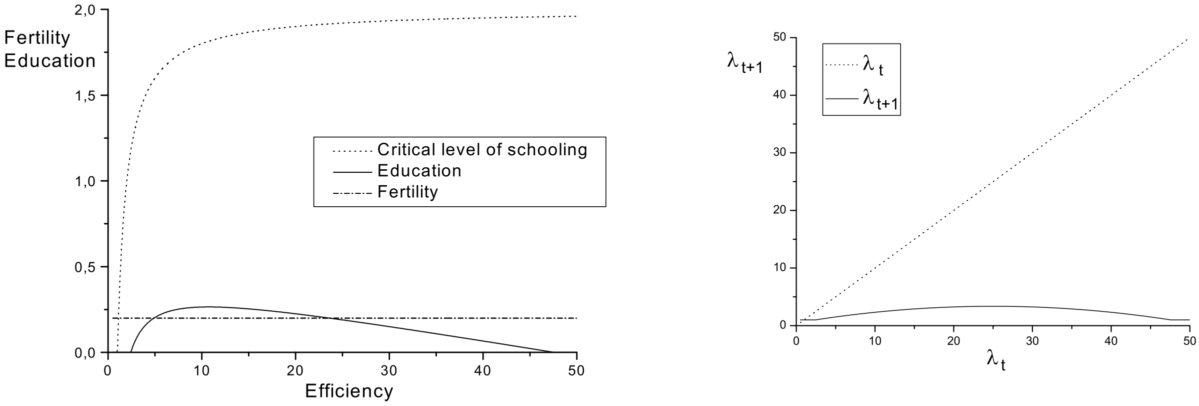


Figure 1.6g: The Household's Decision and Phase Diagram for $z < 1$ and $b > \frac{2\alpha(1-\chi)}{n_{min}}$

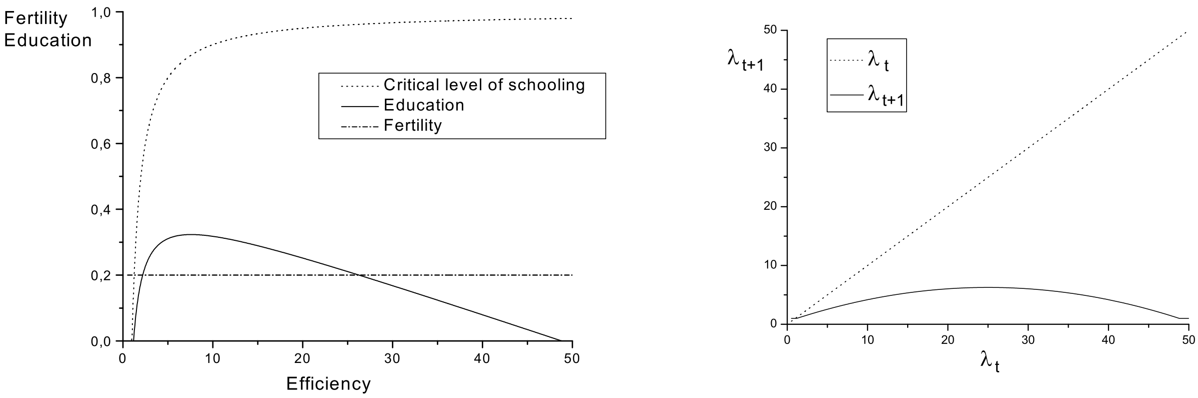


Figure 1.6h: The Household's Decision and Phase Diagram: $z = 1$ and $b > \frac{2\alpha(1-\chi)}{n_{min}}$

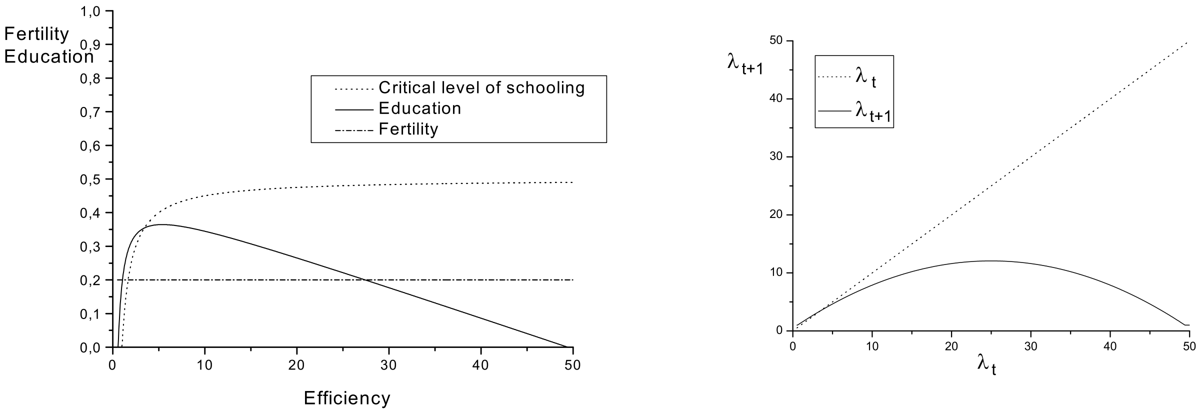


Figure 1.6i: The Household's Decision and Phase Diagram: $z > 1$ and $b > \frac{2\alpha(1-\chi)}{n_{min}}$

Chapter 2

Child Labor, Fertility and Land: Economic growth in developing countries

Abstract

We analyze household decisions in a setting in which altruistic parents decide about fertility, education and child labor. Children can contribute to a household's income while young, and, on becoming young adults, must finance their parents in old age. Apart from labor, the fixed factor land is the only input in production. Multiple steady states can exist, such as a poverty trap with full-time child labor, a growth steady state in which all children enjoy full-time schooling and cyclical steady states. In all non-cyclical steady states, population will be stationary, but output can grow. Sustained long-term economic growth can be attained through programs of fiscal interventions, even without outside aid, if the educational technology is sufficiently productive.

2.1 Introduction

The aim of this essay is to examine household decisions on fertility and child labor, and their consequences for economic growth in the presence of a fixed factor. While economic factors like wealth, employment and wages influence household decisions, other aspects like health, social status, altruism or the households' valuation of time also need to be taken into account. Wealthy parents – be the wealth in the form of assets like land or a high level of education – can usually afford to have more children, or to educate their offspring better, or both. Depending on their environment, they might decide to increase their social status by having many children, as anecdotal evidence from Burkina Faso reveals. A high level of altruism, on the other hand, will more likely induce wealthy parents to have few but well-educated children.

In the context of the paper, it is assumed that parents raise children for both financial and altruistic reasons. Not being able to work when old, parents need transfers from their offspring to finance their consumption in the last period of life. Similar levels of income can be reached by having many uneducated or a few well-educated children when saving in the form of physical capital is not possible. Raising and educating children, however, involves costs, both directly, in the form of clothes, food and medical care, and indirectly, as parents spend time caring for their children instead of working. Even if education is free, sending a child to school instead of out to work involves opportunity costs through lost income. Altruism expresses itself in several different ways: parents who care for their children may be willing to reduce their own current and future consumption in order to spend more time with their children and to finance their education, thereby increasing the children's well-being in the future.

In chapter one, we have shown that well-educated parents will typically choose to have few children, whereby the number depends on the costs incurred in raising them, and that all children will enjoy full-time schooling. For a sufficiently productive education function, the adults' level of efficiency will grow, yielding unbounded growth in both utility and production over time. For all but high child-raising costs, population can also grow beyond any bound. A poverty trap, which will exist in all economies for all but highly productive education functions, will also yield high population growth rates, combined with stagnant levels of efficiency, output and utility. As high population growth rates are not sustainable for very long periods, these results seem unrealistic, and suggest that there is some factor limiting population growth when the total population size is sufficiently high. Including land as an input in the production function is a natural way to limit growth.

The structure of the essay is as follows: Section 2.2 outlines the basic model. The household's optima are discussed in section 2.3, treating interior and corner solutions separately. All potential growth paths are then presented in section 2.4, while section 2.5 focuses on the main steady states of the economy and economic growth. Section 2.6 discusses governmental measures such as taxes and subsidies. A concluding discussion of the main results is given in section 2.7.

2.2 The Model

As in chapter 1, it is assumed that both parents are identical with respect to their labor efficiency, and that they raise and educate children in order to increase their own current consumption and to finance their old age. Education is free except for the opportunity costs of child labor. All children are treated identically.

In the first period, $t = 1$, each household is endowed with a single unit of land, owned solely by the parents. When the parents are old (grandparents), ownership of the land is transferred in equal parts to each of their grown-up children. Therefore, assuming that a family has n_t children in period t, the land owned by a household in period t, H_t, will have the following structure over time:

Period	Number of land-owners (parents)	Land (per family)
$t = 1$	$N = 2$	$H = 1$
$t = 2$	$N = n_1$	$H = \frac{2}{n_1}$
$t = 3$	$N = \frac{n_1}{2} n_2$	$H = \frac{2^2}{n_1 n_2}$
$\vdots$	$\vdots$	$\vdots$
$t = T$	$N = \frac{n_1 n_2 \ldots n_{T-1}}{2^{T-2}}$	$H = \frac{2^{T-1}}{n_1 n_2 \ldots n_{T-1}}$

The behavior of N_t and H_t are governed by the following difference equations:

$$\begin{aligned} N_{t+1} &= \frac{N_t}{2} n_t \\ H_{t+1} &= \frac{2}{N_{t+1}} = \frac{2}{N_t}\frac{2}{n_t} = H_t \frac{2}{n_t} \end{aligned}$$

If $n_t > 2\ \forall t > t'$, total population size will grow beyond any finite bound, and total land ownership will asymptotically approach zero. For $n_t < 2$, the population size will decrease, and if $n_t = 2$, then $N_{t+1} = N_t$.

Income is generated through the production of a single non-storable good. Labor – measured in efficiency units – and land are the only inputs in production. Let the efficiency of a child be fixed at μ, and denote the adults' level of efficiency in period t by λ_t. As each family consists of 2 identical parents and n_t children, the total labor it supplies to production in period t is:

$$L_t = 2\lambda_t + n_t(1 - e_t)\mu. \tag{1}$$

The production function is assumed to exhibit constant returns to scale with respect to labor, and decreasing returns to scale with respect to land. It is also assumed to be Cobb-Douglas in form:

$$y_t = \alpha\, L_t\, H_t^{\gamma}, \quad \gamma < 1,\ \alpha > 0$$

or, from (1), and $H_t \cdot N_t = 2$,

$$y_t = 2\alpha\, \lambda_t \left(\frac{2}{N_t}\right)^{\gamma} + (1 - e_t)\mu n_t \alpha \left(\frac{2}{N_t}\right)^{\gamma}. \tag{2}$$

The above assumptions ensure that it is possible to distinguish between parents' and children's production and income. The assumption is necessary, as grandparents receive a fraction of the parents' income only, which can be easily determined only if the production function exhibits constant returns to scale with respect to labor. Note that the production function permits unbounded growth of output per familiy. The factor α can be interpreted as a parameter describing the productivity of the technology. It is assumed to be the same for all households in the same region.

The only active decision makers in the present setting are the parents, and the decisions they make determine the level of consumption in the last two periods of their life, as well as the level of efficiency attained by their children when these reach adulthood. As in the first chapter, we will now also assume that parents have perfect foresight about all relevant parameters in the future, and that α, γ and χ do not change over time.

The parents' consumption in any period t is given by their income $(2\alpha\lambda_t H_t^\gamma)$ and the income of the children $(\alpha\mu n_t(1-e_t)H_t^\gamma)$ less the costs incurred in raising children $(n_t b\lambda_t)$ and the required transfer to the grandparents $(\chi \cdot 2\alpha\lambda_t H_t^\gamma)$:

$$C_{1t} = 2\alpha\lambda_t H_t^\gamma(1-\chi) + \alpha\mu n_t(1-e_t)H_t^\gamma - n_t b\lambda_t \tag{3}$$

Given the social rule expressed by χ, their old-age consumption is determined by the number of their children and the level of efficiency each child reaches in adulthood:

$$C_{2t} = \chi \cdot \alpha n_t \lambda_{t+1} H_{t+1}^\gamma \tag{4}$$

The parents' altruism expresses itself not only through expenditures on raising children, but also in the parents' concern for their children's future well-being. However, in contrast to the first chapter, their well-being does not depend solely on the level of human capital they attain on reaching adulthood, but also on the amount of land they will own.

Therefore, we adapt the utility function employed in chapter 1, and get:

$$U(C_{1t}, C_{2t}, \lambda_{t+1}) = ln(C_{1t}) + \beta\, ln(C_{2t}) + \beta_1\, ln(\lambda_{t+1} \cdot H^{\gamma}_{t+1}) \tag{5}$$

The educational technology is identical to the one employed in the first chapter:

$$\lambda_{t+1} = ze_t\lambda_t + 1, \tag{6}$$

where $z > 0$ can be interpreted as the strength of the intergenerational transmission mechanism. Recall that the growth rate of the adults' level of efficiency in a highly developed economy without child labor ($e_t = 1\ \forall t$) is then given by:

$$g_\lambda \equiv \frac{\lambda_{t+1}}{\lambda_1} - 1 = (z-1) + 1/\lambda_t$$

If $z < 1$ unbounded growth is not possible, as $g_\lambda < 0\ \forall\, \lambda > \frac{1}{1-z}$ and therefore $\lim_{\lambda\to\infty} g_\lambda < 0$. For $z > 1$, the growth rate will always be positive for sufficiently large levels of efficiencies. If $z = 1$, the economy will also grow for sufficiently large efficiencies, but the growth rate will asymptotically approach zero as $\lambda \to \infty$.

Using (3), (4) and (6), the utility function can be rewritten such that it contains only the decision variables n_t and e_t and the model's parameters:

$$\begin{aligned} U(e_t, n_t; \lambda_t; N_t) &= \ln\Big[2\alpha\lambda_t(1-\chi)\Big(\frac{2}{N_t}\Big)^{\gamma} + \alpha\mu n_t(1-e_t)\Big(\frac{2}{N_t}\Big)^{\gamma} - n_t b\lambda_t\Big] \\ &+ \beta\ \ln\Big[\alpha\chi(ze_t\lambda_t + 1)n_t\Big(\frac{2}{N_t}\Big)^{\gamma}\Big(\frac{2}{n_t}\Big)^{\gamma}\Big] \\ &+ \beta_1 \ln\Big[(ze_t\,\lambda_t + 1)\Big(\frac{2}{N_t}\Big)^{\gamma}\Big(\frac{2}{n_t}\Big)^{\gamma}\Big] \end{aligned} \tag{7}$$

As in chapter 1, the utility function is neither convex nor concave in (n_t, e_t).

2.3 The Household's Optimum

Parents maximise their utility choosing fertility and education. For the solution to be relevant economically, it has to fulfill four conditions regarding e_t and n_t, which are both bounded: $n \in [n_{min}, n_{max}]$ and $e \in [0, 1]$. It is further assumed that n_{min} is sufficiently low, so that the condition $C_{1t}(n_{min}) > 0$ is satisfied for all levels of efficiency and population sizes, given the values of α, χ, μ, b and γ. To make sure that a stationary population size is possible in principle, we choose $n_{min} < 2$ and $n_{max} > 2$. One can therefore formulate the household's maximization problem as follows:

$$\max_{n_t, e_t} \; U(n_t, e_t) \quad \text{s.t.} \;\; n_{min} \le n_t \le n_{max} \quad \text{and} \quad 0 \le e_t \le 1 \tag{8}$$

2.3.1 The unrestricted solution

The maximisation problem (8) has a unique unrestricted solution. However, as the determinant of the hessian is negative for all parameters, as in chapter 1, the solution does not describe a maximum but rather a saddle point:

$$det(Hessian) = -\frac{\left[1 + \beta(1-\gamma) - \beta_1\gamma\right]^3 \mu^2 \left(\beta_1 + \beta\,\gamma + \beta_1\gamma\right)}{4\lambda^2 \left(\chi - 1\right)^2 \left(\beta + \beta_1\right)} \;<\; 0.$$

As argued in chapter one, choosing a non-linear educational function ($\lambda_{t+1} = ze_t^{\epsilon}\lambda_t + 1$ with $\epsilon < 1$ instead of $\epsilon = 1$) or a Ramsey subutility function for altruism would either exclude poverty traps by construction or yield complex and hard to analyze solutions. As a consequence, we defer these two specifications to the third chapter, where no analytical solutions are required and numerical solutions suffice.

Therefore, the full unrestricted solution being irrelevant, corner solutions have to be analyzed. A total of eight possible corner solutions can be found in three classes, as outlined in table 2.1. In the following sections, the household optima will be denoted by n^0 and e^0, while unrestricted solutions – that is, the levels of fertility and education solving the respective FOC – will be denoted by n^* and e^*. The following sections will present the three classes of solutions, and also discuss the areas where they are relevant economically.

Table 2.1: Solutions to the household maximization problem

Class 1: unrestricted solutions w.r.t Education	**Class 2:** unrestricted solutions w.r.t Fertility	**Class 3**: Full Corner Solutions
$n = n_{min},\ e = e^* \in [0, 1]$	$n = n^* \in [n_{min}, n_{max}],\ e = 0$	$n = n_{min},\ e = 0$
$n = n_{max},\ e = e^* \in [0, 1]$	$n = n^* \in [n_{min}, n_{max}],\ e = 1$	$n = n_{min},\ e = 1$
		$n = n_{max},\ e = 0$
		$n = n_{max},\ e = 1$

2.3.2 The unrestricted solution w.r.t. education

If the number of children a family can have is fixed at $n_t = n\ \forall t$, solving the FOC with respect to education yields:[1]

$$e^*(n, \lambda, N) = \frac{z\lambda(\beta + \beta_1)\left[2\alpha\lambda(1 - \chi) + \alpha n\mu - nb\lambda\left(\frac{N}{2}\right)^{\gamma}\right] - \alpha n\mu}{\alpha n\mu z\lambda(1 + \beta + \beta_1)} \tag{9}$$

As the parents' level of efficiency is bounded below ($\lambda \geq 1$), the function $e^*(\cdot)$ has no points of discontinuity. Its behavior in the limit is as follows:

$$\begin{aligned}
\lim_{\lambda \to 0} e^*(\lambda, n, N) &= -\infty \\
\lim_{\lambda \to \infty} e^*(\lambda, n, N) &= \text{signum}\left[2\alpha(1 - \chi)(2/N)^{\gamma} - nb\right] \cdot \infty \\
\lim_{N \to \infty} e^*(\lambda, n, N) &= -\infty
\end{aligned}$$

Therefore, the unrestricted solution w.r.t education will violate the definition of e_t as lying in the interval $[0, 1]$ for some levels of efficiency and population sizes.

The higher the total population size, the lower will be the level of schooling:

$$\frac{\partial e^*(\cdot)}{\partial N} = -\frac{\gamma\lambda b(\beta + \beta_1)}{\alpha N\mu(1 + \beta + \beta_1)(\frac{2}{N})^{\gamma}} < 0 \tag{10}$$

[1] As the dynamics of the system are not discussed in the present sections, the time subscripts are omitted

All families being equal, a large total population is equivalent to the parents' having but little land. Therefore, the less land a family has, the less will its children be educated if fertility is fixed.

The effect of the parents' level of efficiency on the unrestricted solution w.r.t. schooling depends on the total population size: for large populations, where families have only little land, the child-raising costs will be large as compared to the parents' income. In this case, child labor will be needed to finance C_{1t}, and the more so the higher λ is. That is, the level of schooling e^* will decrease with higher levels of efficiency if parents have only little land. If, on the other hand, parents have sufficient land,[2] the level of schooling e^* a child enjoys increases with the parents' level of efficiency:

$$\frac{\partial e^*(\cdot)}{\partial \lambda} = \frac{z\lambda^2\,(\beta+\beta_1)\left[2\,\alpha(1-\chi) - nb\left(\frac{N}{2}\right)^{\gamma}\right] + n\mu\,\alpha}{\alpha n\mu\, z\lambda^2\,(1+\beta+\beta_1)}.$$

2.3.3 The unrestricted solution w.r.t fertility

For a fixed level of schooling, solving the FOC with respect to fertility yields:

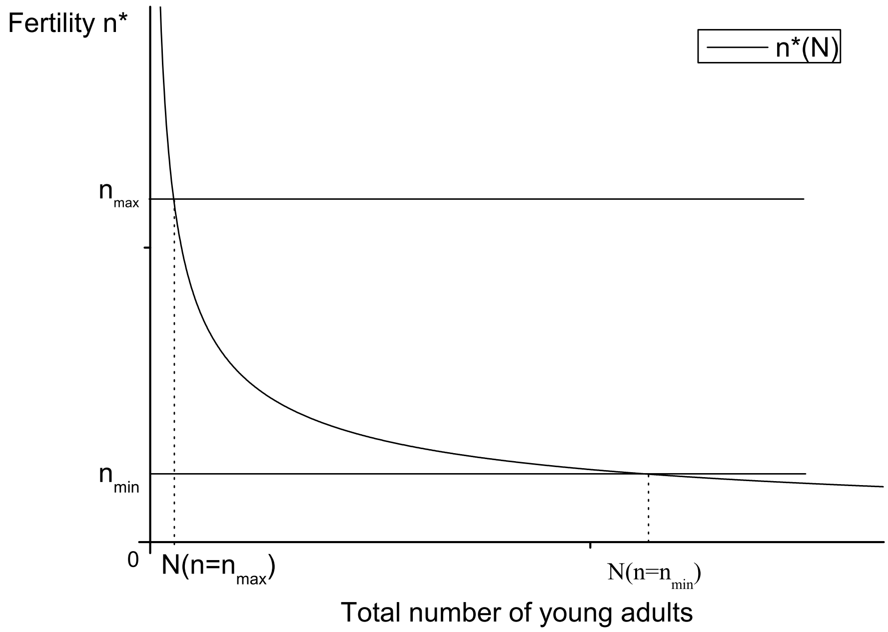
$$n^*(\lambda, e, N) = -\frac{2\alpha\lambda(\frac{2}{N})^{\gamma}(\beta(1-\gamma)-\beta_1\gamma)(1-\chi)}{[1+\beta(1-\gamma)-\beta_1\gamma][\alpha\mu(\frac{2}{N})^{\gamma}(1-\bar{e})-b\lambda]} \tag{11}$$

$$\frac{\partial n^*(\cdot)}{\partial N} = -\frac{\lambda^2\alpha(\frac{2}{N})^{\gamma+1}(\beta(1-\gamma)-\beta_1\gamma)(1-\chi)\gamma b}{[1+\beta(1-\gamma)-\beta_1\gamma][\alpha\mu(\frac{2}{N})^{\gamma}(1-e)-b\lambda]^2} < 0\ \forall\lambda, N, e=\{0,1\}.$$

If $e=1$, the unrestricted solution w.r.t. fertility will be independent of the parents' level of efficiency. It will only be positive if $\beta(1-\gamma)-\beta_1\gamma$ is positive. Similarly, the the level of fertility n^* will decrease with population size if $\beta(1-\gamma)-\beta_1\gamma>0$, and increase otherwise. By choosing $\beta(1-\gamma)-\beta_1\gamma<0$, unrestricted solutions w.r.t. fertility would be excluded when $e=1$, while fertility would be increasing in population size for all other values of $\bar{e}$ and λ. As one would normally expect fertility to be lower when the total population is

[2] $H > \left(\frac{z\lambda^2 nb(\beta+\beta_1)}{2\alpha z\lambda^2(\beta+\beta_1)(1-\chi)+\alpha n\mu}\right)^{1/\gamma}$ respectively $N < 2\left(\frac{2\alpha(1-\chi)}{nb} + \frac{\alpha\mu}{z\lambda^2 b(\beta+\beta_1)}\right)^{1/\gamma}$

Figure 2.1: Unrestricted solution w.r.t. fertility for a fixed level of efficiency as a function of N



higher and families have little land, and as we do not want to exclude unrestricted solutions w.r.t. n when parents choose full-time schooling, we make the following assumption for the remainder of the essay:

Assumption

Assume that the values of β, β_1 and γ satisfy: $\beta(1-\gamma) - \beta_1\gamma > 0$.

Well-educated parents will always have at most as many children as families whose level of efficiency is low, as the costs incurred in raising children ($n\lambda b$) exceed the possible income through child labor ($n\mu\alpha(2/N)^\gamma(1-e)$) when parents are highly efficient:

$$\frac{\partial n^*(\cdot)}{\partial \lambda} = -2\frac{\alpha^2(\frac{2}{N})^{2\gamma}(\beta(1-\gamma)-\beta_1\gamma)(1-\chi)(1-\bar{e})\mu}{[\alpha\mu(1-\bar{e})(\frac{2}{N})^\gamma - b\lambda]^2(1+\beta(1-\gamma)-\beta_1\gamma)} \begin{cases} < 0 & \text{if } \bar{e} < 1 \\ = 0 & \text{if } \bar{e} = 1 \end{cases}$$

If $e < 1$, the function $n^*(e, \lambda, N)$ will be negative for low population sizes and efficiencies $\left(\lambda N^\gamma < \frac{\alpha\mu 2^\gamma(1-e)}{b}\right)$, and positive for large N and λ. It is seen from (11) that n^* is discontinuous for all efficiencies and population sizes fulfilling the condition $\alpha\mu(\frac{2}{N})^\gamma(1-e) = b\lambda$

or $\lambda N^{\gamma} = \frac{\alpha\mu 2^{\gamma}(1-e)}{b}$. In the $(N, \lambda)-$ plane all points of discontinuity lie along a hyperbola. As the derivatives with respect to both λ and N are negative for all parameters if $e < 1$, the function will change sign at the discontinuity. The unrestricted solution w.r.t fertility can only be economically relevant for sufficiently high levels of efficiency or population size if $e < 1$. For $e = 1$, it depends on the population size alone whether $n^*(\cdot)$ is economically relevant or not, as can be seen from figure 2.1.

2.3.4 The corner solutions w.r.t both variables

Neither unrestricted solution is relevant for all efficiencies and population sizes, so that corner solutions w.r.t. both variables need to be analyzed fully to characterize the household's optimum. It is therefore necessary to compare the levels of utility generated by all combinations of n and e at their respective borders. It suffices to analyze:

$$\Delta U(n) \;:=\; U(e, n = n_{max}) - U(e, n = n_{min}) \;\text{ and} \tag{12}$$

$$\Delta U(e) \;:=\; U(e = 0, n) - U(e = 1, n) \tag{13}$$

For $n_{min} \neq 0$, it can be shown that for any given level of efficiency there is a population size N_{un} such that choosing the maximal number of children can be optimal only if and only if the total population size is lower than N_{un}. Therefore, parents will prefer to have as many children as possible if total population size is low, that is, if they have much land. If, on the other hand, land is densely settled, parents will prefer to have the minimum number of children.

The analysis of $\Delta U(e)$ is more complex.

$$\begin{aligned} \Delta U(e) \geq 0 \quad &\Leftrightarrow \\ \frac{\lambda[2\alpha(1-\chi)(\frac{2}{N})^{\gamma} - nb] + \alpha n\mu(\frac{2}{N})^{\gamma}}{\lambda[2\alpha(1-\chi)(\frac{2}{N})^{\gamma} - nb]} \geq (1 + z\lambda)^{(\beta+\beta_1)} & \end{aligned} \tag{14}$$

Choosing full-time child labor will be optimal for all population sizes if the parents' level of efficiency is sufficiently low. For higher levels of efficiency, choosing full-time schooling is optimal for all but high population sizes:[3] if parents are at least moderately efficient, they will choose full-time child labor for their offspring as opposed to full-time schooling only if the family has very little land. The critical value of λ is unique, but cannot be derived analytically.

The household's optimal choice of fertility (denoted by $n_t^0(\lambda_t, N_t)$) and education (denoted by $e_t^0(\lambda_t, N_t)$) cannot be derived analytically for all levels of efficiency and all populations sizes. Both $n_t^0(\lambda_t, N_t)$ and $e_t^0(\lambda_t, N_t)$ will typically have several points of discontinuity, so that the difference equations describing the development of population size and efficiency over time will also be discontinuous in λ_t and N_t:

$$\begin{aligned} \lambda_{t+1} &= z\,\lambda_t\, e^0(\lambda_t, N_t) + 1 \quad \forall t \\ N_{t+1} &= \frac{N_t}{2}\, n^0(\lambda_t, N_t) \quad \forall t \end{aligned} \tag{15}$$

In contrast to the first chapter, where fertility decisions were independent of the households' land holdings, the present system is much more complex. Current decisions do not only influence the future level of human capital, but also land holdings, and these in turn both influence future household decisions on fertility and education. Current fertility decisions therefore have a direct effect on the future levels of both n and e.

2.4 Growth paths: a characterization

Although it is not possible to derive the household's choice and the resulting difference equations for λ_t and N_t analytically, it is possible to analyze some growth paths and ensuing steady states.

---

[3] $N > N_{ue} = 2\left[\frac{2\alpha(1-\chi)}{nb} + \frac{\alpha\mu}{b\lambda}\frac{1}{(z\lambda+1)^{\beta+\beta_1}-1}\right]^{1/\gamma}$

<u>*Definition*</u>

A *non-cyclical steady state* is a growth path in which e_t and n_t are constant $\forall t > t'$. A *cyclical steady state* is a growth path in which e_t and n_t alternate between some fixed values, each with a fixed periodicity, whereby that of fertility, $c_n \geq 0$, does not necessarily coincide with that of education, $c_e \geq 0$.

All possible paths are stated table 2.2. We will analyze whether the paths in table 2.2 can exist in the present setting in sections 2.4.1 to 2.4.5. Note that if e_t is constant, λ_t can be either constant or growing. Similarly, a constant level of fertility implies a constant level of N_t if and only if $n_t = 2\ \forall t$. Table 2.3 gives an overview of all logically possible steady states, both cyclical or non-cyclical, and over the sections in which they are analyzed. Note that path P7 is a subset of all potential paths in P3.

Table 2.2: Cyclical and Non-Cyclical Steady States

	$n_t = const.\ \forall t$	n_t alternates (cycle: c_n)
$e_t = const.\ \forall t$	Non-cyclical Steady state	Cyclical Steady State
e_t alternates (cycle: c_e)	Cyclical Steady-State	Cyclical Steady-State

Table 2.3: Steady States: fertility and education

P1	$n_t = const.$	$n_t < 2\ \forall t$		Section 2.4.1
P2		$n_t > 2\ \forall t$		Section 2.4.1
P3		$n_t = 2\ \forall t$	$e_t = const.$	Section 2.4.2
P4			e_t alternates	Section 2.4.1
P5	$e_t = const.$	$e_t \in \{0, 1\}$ and $\lambda_t = const$		Section 2.4.4
P6		$e_t \in (0, 1)$		Section 2.4.3
P7		$e_t = 1$ and λ_t grows	$n_t = const.$	Section 2.4.2
P8		$e_t = 1$ and λ_t const. or growing	n_t alternates	Section 2.4.5
P9	e_t alternates	n_t alternates		Section 2.4.1

2.4.1 Miscellaneous Paths

<u>Paths P1 and P2</u>

Paths along which N grows without bound cannot exist ($n_t > 2\ \forall t$, path P2 in table 2.3). For if N were to grow without bound, this would yield $\lim_{t\to\infty} C_{1t} < 0$, even if efficiency were growing, as can be seen by rewriting C_{1t} as

$$C_{1t} = [2\alpha(1-\chi)\left(\frac{2}{N_t}\right)^{\gamma} - nb]\lambda_t + \alpha n_t \mu(1-e_t)\left(\frac{2}{N_t}\right)^{\gamma}.$$

For a sufficiently large population, the first and last terms will be negligible compared to the second term, which is negative. Therefore, consumption in the first period would eventually be negative, even if children worked full-time.

The case where population is constantly shrinking with $\lim_{t\to\infty} N_t = 0$, that is, with $n_t^0 < 2\ \forall t$, does not exist either; for parents choose the maximal number of children for sufficiently small population sizes ($N < N_{un}$.)

Since paths P1 and P2 do not exist, then whenever fertility does take a fixed value on some path, it must be $n_t = 2\ \forall t$. Similarly, if fertility alternates with a periodicity c_n, the total population size N must also alternate, as N cannot grow or fall without bound.

Path P4

Consider a path along which population is fixed, that is, $n_t = 2\ \forall t$, while e_t alternates (path P4 in table 2.3). Rewriting equation (11) yields:

$$n^*(\lambda, \bar{e}, N) = -\frac{2\alpha(\frac{2}{N})^{\gamma}(\beta(1-\gamma)-\beta_1\gamma)(1-\chi)}{[1+\beta(1-\gamma)-\beta_1\gamma][\alpha\mu(\frac{2}{N})^{\gamma}\frac{(1-\bar{e})}{\lambda} - b]}$$

Assume that in the first period, we have $n_1 = 2$. As full unrestricted solutions do not exist, e_1 can be either 0 or 1. Assume w.l.o.g. that $e_1 = 0$. As we consider growth paths with alternating levels of schooling, we must have $e_2 = 1$. For the population size to remain unchanged, we must have $n_2 = 2$. This condition can only be satisfied if $\frac{(1-e_1)}{\lambda_1} = \frac{(1-e_2)}{\lambda_2}$, or $1/\lambda_1 = 0$, which cannot be the case. Note that even if the periodicity of schooling and efficiency are higher than 2, there will be some period in which parents will switch from choosing $e = 0$ to $e = 1$. Therefore, the above argument will hold for all paths where population and fertility are constant over time, while schooling and efficiency change. As a consequence, cyclical steady states in which n_t is fixed and e_t alternates do not exist.

Path P9

Another class of possible paths consists of those in which population size and schooling are both alternating with some fixed periodicity. Note that for such paths to be steady-state paths, the initial state of the economy and the parameters relevant for the household decisions must all satisfy certain strict conditions. To illustrate the necessary conditions, consider the following simple example, where both fertility and schooling alternate with a period of 2.

Denoting the initial level of population by N_1, we get $N_2 = (N_1/2)n_1$ and $N_3 = (N_1/4)n_1n_2$. As population must take alternate values if fertility does so, we must have $N_3 = N_1$, which implies $n_1n_2 = 4$. If fertility is always at the corner, this condition can only be satisfied if $n_{min}n_{max} = 4$, which will be the case by pure chance only. If, on the other hand, fertility is not at the corner, but takes interior values, the condition $n_1(\lambda_1, N_1; \cdot) \cdot n_2(\lambda_2, N_2; \cdot) = 4$ will be satisfied for specific values of λ_1 and N_1 only.

If the periodicity is higher than 2, the conditions are stricter still. As a consequence, steady states in which both fertility and schooling are alternating will not be considered further.

2.4.2 Growth paths with a constant total population size

In this section, we will analyze steady states in which $n_t = 2\ \forall t$. We have already shown that paths along which fertility is fixed while schooling alternates (P4 in table 2.3) cannot exist. Therefore, we are left with paths P3 and P7. For a constant level of schooling, and if parents choose $n_t^0 = n^*(\cdot) = 2\ \forall t$, as given by (11) in and around the potential steady state, the total population size will take the stationary value:[4]

$$N_t = N^*(\bar{e}) = 2\left[\frac{\alpha(1-\chi)}{b}\frac{(1-\gamma)\beta-\beta_1\gamma}{[1+\beta(1-\gamma)-\beta_1\gamma]} + \frac{\alpha\mu}{b\lambda_t}(1-\bar{e})\right]^{1/\gamma} \quad \forall t. \tag{16}$$

If $e_t = 1$ and $n_t = n^*(\cdot)\ \forall t$, the stationary value $N^*(\bar{e}=1)$ can be reached asymptotically only.[5] If $e_t = 0$ and $n_t = n^*(\cdot)\ \forall t$, it is not possible to derive N_t as a function of time analytically. However, it is possible to find some sets of parameters and initial conditions for which the stationary population size is reached in finite time.[6]

As the full unrestricted solution is not optimal, only solutions with $e_t^0 = 0\ \forall t$ or $e_t^0 = 1\ \forall t$ are possible if parents choose the unrestricted solution w.r.t. fertility:

- If the level of efficiency is such that parents choose full-time schooling, N^* will be independent of the level of efficiency: $N^*(\bar{e}=1) = 2\left[\frac{\alpha(1-\chi)}{b}\frac{(1-\gamma)\beta-\beta_1\gamma}{[1+\beta(1-\gamma)-\beta_1\gamma]}\right]^{1/\gamma}$. As the total population size is constant, and children do not work, output will grow at the same rate as efficiency, $g_y = g_\lambda = z - 1 + 1/\lambda_t$. Two cases are possible:
  - If $z \geq 1$ and $e_t^0 = 1\ \forall t$, the level of efficiency will grow over time, whereby the growth rate of the level of efficiency and output both approach $z-1$ asymptotically from above. It should be emphasized that $N_t = N^*\ \forall t$ and $g_\lambda = z - 1 > 0$ is a steady-state configuration.
  - If $z < 1$ and $e_t^0 = 1\ \forall t$, efficiency will approach the level $1/(1-z)$ asymptotically.
- If $e^0 = 0$, the adults' level of efficiency will be unity for all z. Therefore, N^* will also be constant: $N^*(\bar{e}=0) = 2\left[\frac{\alpha(1-\chi)}{b}\frac{(1-\gamma)\beta-\beta_1\gamma}{[1+\beta(1-\gamma)-\beta_1\gamma]} + \frac{\alpha\mu}{b}\right]^{1/\gamma}$ $\forall t$. Both efficiency and population size being constant, the total output of the economy will also remain unchanged over time: $y_t = 2(\alpha+\mu)(2/N^*)^\gamma\ \ \forall t$. The configuration $\lambda_t = 1,\ N_t =$

[4]Note that if $e_t = 1 \forall t$, the stationary value will be independent of λ_t. If $e_t = 0\ \forall t$, then $\lambda_t = 1\ \forall t$, so that $N^*(e)$ will be constant.

[5]Rewrite N_t as follows: $N_t = N_0^{(1-\gamma)^t} \cdot (A/2^{1-\gamma})^{(1-(1-\gamma)^t)/\gamma}$, where $A = \frac{2\alpha(\beta(1-\gamma)-\beta_1\gamma)(1-\chi)}{[1+\beta(1-\gamma)-\beta_1\gamma][b]}$ and N_0 is the initial value. One can then show that $\lim_{t\to\infty} N_t = N^*(\bar{e}=1)$, and there is no $T < \infty$ such that $N_T = N^*(\bar{e}=1)$.

[6]For example, if $\gamma = 1/2$ and the initial population size is $N_0 = \frac{2\alpha^2\mu^2((2+\beta-\beta 1)\mu+(\beta+\beta_1)(1-\chi))^2}{b^2(1-\chi)^2(\beta-\beta_1)^2}$, then $n^*(N_0, \bar{e}=0, \lambda=1)\cdot N_0/2 = N^*(\bar{e}=0)$, that is, the stationary value of N is reached within one period.

$N^*(e = 0)$, $e_t = 0$ and $n_t = 2\ \forall t$ describes a non-cyclical stationary state – a poverty trap in the full sense.

Comparing the stationary values of N^* from (16) we obtain:

$$N^*(e = 0, \lambda_t = 1) > N^*(e = 1).$$

If children are educated full time, the total population size in the steady state will be lower than in the case where children work full time.

It is impossible to say which of the two steady states in population size will prevail for which parameters exactly, and whether both are possible for some set of parameters. We will analyze these issues in more detail in sections 2.5.1 and 2.5.2.

2.4.3 Path P6

This section analyzes growth paths with $e_t \in (0, 1)\ \forall t$. As the full unrestricted solution cannot be optimal, this implies that fertility must be at a corner value. Therefore, as $n_{min} \neq 2$ and $n_{max} \neq 2$ by assumption, the total population size cannot be constant in such steady states. As shown in section 2.4.1, population cannot grow or fall without bound, so that N must take alternating values if fertility is not constant. Denote the values N can take by $N', N'', \ldots$.

Two sets of paths are logically possible: first, there are paths along which λ_t grows without bound;[7] and, second, there are paths along which λ_t is constant. In the first case, there will always be some level of human capital, say $\hat{\lambda}$, such that $e^*(\hat{\lambda}, N) \geq 1$ for all values population can take $N \in \{N', N'' \ldots\}$. Consequently, growth paths along which $e_t \in (0, 1)$ while λ_t grows without bound do not exist. Having eliminated this possibility, we consider what happens when λ_t is constant.

[7]Paths along which λ_t falls without bound are not possible, as $\lambda_t \geq 1$ by construction.

Proposition 1

Consider a steady-state growth path with a constant level of schooling $e_t \in (0,1)\ \forall t$ and a constant level of efficiency $\lambda_t = \lambda_{t+1}\ \forall t$. The levels of fertility alternate between the levels n_{min} and n_{max}. For such a growth path to exist, the following conditions must be satisfied:

- The level of schooling is constant in all periods: $e_{t+1} = e_t \in (0,1)\ \forall t$
- $n_{min} n_{max} = 4$
- The total population size alternates, with a period of 2, between $N' = 2\left[\frac{\alpha(1-\chi)\left(\frac{2}{n_{max}}\right)^{\gamma}(n_{max}^2-4)}{2n_{max}b\left(1-\left(\frac{2}{n_{max}}\right)^{\gamma}\right)}\right]^{1/\gamma}$ and $N'' = N' n_{max}/2$.

Note that if the three conditions in Proposition 1 are satisfied, they imply a stationary state in human capital, with $\lambda_t = \lambda_{t+1}\ \forall t$.

Proof :

To prove proposition 1 we will construct such a steady state. This will be done by analyzing the household's decisions in 3 consecutive periods.

The first period

Assume there exists a vector (λ_1, N_1) such that parents choose the level of schooling such that efficiency remains constant over time and $e_1^0 \in (0,1)$. That is,

$$\lambda_2 = z\lambda_1 e_1^0(\lambda_1, N_1, \cdot) + 1 = \lambda_1.$$

W.l.o.g., let $n_1^0(\lambda_1, N_1) = n_{max}$, so that $N_2 = N_1 n_{max}/2 > N_1$, by virtue of the assumption that $n_{max} > 2$.

The second period

For the level of efficiency to remain constant over time ($\lambda_3 = \lambda_2 = \lambda_1$), the level of schooling in the second period must be identical to the level of schooling in the first period: $e_2^0(\lambda_2, N_2) = e_1^0(\lambda_1, N_1)$, with $\lambda_2 = \lambda_1$ and $N_2 = N_1 n_{max}/2$. As $N_1 \neq N_2$, it follows immediately that the level of schooling can only be constant if $n_1 \neq n_2$. With fertility being at the corner, this yields $n_2 = n_{min}$. From (9) we can now derive a condition for n_{min} such that $e_2^0(\lambda_1, N_1 n_{max}/2, n_{min}) = e_1^0(\lambda_1, N_1, n_{max})$ is satisfied:

$$n_{min} = \frac{2\alpha(1-\chi)n_{max}\left(\frac{2}{N_1}\right)^{\gamma}\left(\frac{2}{n_{max}}\right)^{\gamma}}{2\alpha(1-\chi)\left(\frac{2}{N_1}\right)^{\gamma}\left(\frac{2}{n_{max}}\right)^{\gamma} + n_{max}b\left[1-\left(\frac{2}{n_{max}}\right)^{\gamma}\right]}.^{8} \tag{17}$$

In this case, $e_2^0 = e_1^0$, $n_2 = n_{min}$, which yields $\lambda_3 = \lambda_1$ and $N_3 = N_2 n_{min}/2 = N_1 n_{max} n_{min}/4$.

The third period

For the level of efficiency in the fourth period to be equal that in the previous periods ($\lambda_4 = \lambda_3 = \lambda_2 = \lambda_1$), parents must choose $e_3^0(\lambda_3, N_3) = e_1^0(\cdot)$. If parents choose $n_3^0 = n_{min}$ and $e_3^0 \in (0,1)$ in the third period, the level of efficiency will increase, as $e_3^0(\lambda_1, N_3) > e_2^0(\lambda_1, N_2)$ by virtue of equation (10) and $N_3 < N_2$. Again, a stationary state with respect to human capital can only exist if the level of fertility changes, that is, if $n_3 = n_{max}$, and if $e_3^0(\lambda_3 = \lambda_1, N_3) = e_1^0(\lambda_1, N_1)$. Consequently, the level of schooling will remain unchanged iff $N_3 = N_1$, that is, iff

$$n_{max} n_{min} = 4. \tag{18}$$

Therefore, the first and the third period will be identical with respect to all variables.

The last condition in Proposition 1 can be derived using (17) and (18) with $N' = N_1$ and $N'' = N_1 n_{max}/2$:

$$N' = 2\left[\frac{\alpha(1-\chi)\left(\frac{2}{n_{max}}\right)^{\gamma}(n_{max}^2 - 4)}{2n_{max}b\left(1-\left(\frac{2}{n_{max}}\right)^{\gamma}\right)}\right]^{1/\gamma}, \tag{19}$$

while the steady state level of efficiency can be derived using $n_1^0 = n_{max}$, and equations (6), (9) and (19). Note that if $N_1 = N'$, equation (17) reads $n_{min} = 4/n_{max}$, and it

[8] Note that equation (17) will satisfy the condition $n_{min} < 2$ for specific values of N_1 only. We will return to this issue at the end of the proof.

satisfies the condition $n_{min} < 2$, as $n_{max} > 2$ by assumption. ■

Therefore, a steady state with $e_t \in (0, 1)$ can exist only if both conditions (17) and (18) are satisfied simultaneously. Such a steady state is characterized by a constant level of efficiency and schooling and alternating population sizes and fertility levels, whereby $N_{t+2} = N_t$ and $n^0_{t+2} = n^0_t\ \forall t$. As condition (18) will only be satisfied by mere chance, cyclical steady states with a constant level of efficiency, $e \in (0, 1)$ and alternating levels of fertility are knife-edge cases. Consequently, we will not treat them further.

2.4.4 A constant level of efficiency and $e \in \{0, 1\}$

Having demonstrated that steady states with a constant level of efficiency require that $n_{min} \cdot n_{max} = 4$ if $e_t \in (0, 1)$, we now analyze whether steady states in λ are possible if $e^0_t = 0$ or $e^0_t = 1$. We consider these cases in turn.

$\underline{e^0_t = 0}$

Suppose $e^0_1 = 0$. In this case, $\lambda_2 = 1$, and $\lambda_t = 1\ \forall t$ must be the stationary value. Let $S_1 = \{N_1 : e^0_1(\lambda_1 = 1, N_1; \cdot) = 0\}$, which can be derived using (9) and (14) for interior and corner solutions, respectively. If there exists an $N_1 \in S_1$ such that $n^0_1 = 2$, a fully stationary state will have been achieved. Growth paths along which parents choose the unrestricted solution w.r.t. fertility have been discussed in detail in section 2.4.2, and we will now take up corner solutions w.r.t. fertility.

If $n^* \notin [n_{min}, n_{max}]$, parents will choose fertility at a corner. Assume w.l.o.g. that $n^0_1 = n_{min}$. As $n_{min} < 2$, this yields $N_2 < N_1$. In the second period, and as e^0_t is non-decreasing in N_t, there are several possibilities:

(i) The household's optimum is a full corner solution, with $e^0_2 = 0$ and $n^0_2 = n_{min}$. The level of efficiency in the third period will then be equal to the level of efficiency in the first period ($\lambda_3 = \lambda_1 = 1$), while the total population size will decrease: $N_3 < N_2$.

(ii) Due to the smaller population ($N_2 < N_1$), parents now decide to educate their offspring: $e_2^0 > 0$ and $n_2^0 = n_{min}$. It follows that $\lambda_3 > \lambda_2$, which violates the hypothesis that λ is constant.

(iii) Instead of increasing the level of schooling, parents increase the number of children they have: $e_2^0 = 0$ and $n_2^0 = n_{max}$. Thus, $\lambda_3 = \lambda_1$, while the population size rises: $N_3 > N_2$.

(iv) If the change in total population size is sufficiently large, parents might increase both the level of fertility and that of schooling: $e_2^0 > 0$ and $n_2^0 = n_{max}$. Both the total population size and the adults' level of efficiency increase ($N_3 > N_2$ and $\lambda_3 > \lambda_2$), which violates the hypothesis that the level of λ is constant.

Only the first and the third cases can lead to a growth path with $\lambda_t = 1\ \forall t$. In the third case, a growth path with a constant level of efficiency and $e_t^0 = 0\ \forall t$ is feasible only if both conditions

$$N_2 n_{min}/2 \leq N_1 \text{ and } N_1 n_{max}/2 \geq N_2 \tag{20}$$

are satisfied. It can be shown that there exist levels of n_{min} and n_{max} such that the conditions above are satisfied for some (N_1, N_2) if $e_t^0 = 0\ \forall t$. The ensuing growth path is characterized by full-time child labor ($e_t^0 = 0\ \forall t$) and stationary efficiency ($\lambda_t = 1\ \forall t$), with alternating levels of fertility and population size.

In case (i), however, if families choose $n_t^0 = n_{min}$ for a sufficiently large number of periods, total population size will eventually drop below some threshold, and choosing either $e^0 > 0$, or $n^0 = n_{max}$, or the unrestricted solution w.r.t. fertility will be optimal. These constellations are discussed in cases (ii), (iii) and (iv) above, as well as in section 2.4.2.

$\underline{e_t^0 = 1}$

With schooling at the corner, fertility can be either interior, or at a corner value. In this section, we will focus on paths along which $n_t^0 \in \{n_{min}, n_{max}\}\ \forall t$. We have already discussed paths along which fertility is constant in section 2.4.2, while paths along which fertility alternates between values other than n_{min} and n_{max} will be taken up in section 2.4.5.

Proposition 2

There exists no steady state growth path with $e_t^0 = 1\ \forall t$ and $n_t^0 \in \{n_{min}, n_{max}\}\ \forall t$.

Remark:

Proposition 2 treats not only such paths along which λ_t is constant, but also those along which efficiency is growing, i.e., both paths P5 and P7. As the level of efficiency plays no role in determining the optimal level of fertility (and hence of population)[9] when $e_t = 1\ \forall t$, it plays no role whether λ is constant or growing.

Proof :

As can be seen from equation (11), there always exist population sizes N^1 and N^2 ($> N^1$) such that $n^*(e^0 = 1, N^1) = n_{max}$ and $n^*(e^0 = 1, N^2) = n_{min}$. Note that N^1 and N^2 will be independent of λ, as $e^0 = 1$. If the unrestricted solution w.r.t fertility is never optimal for $e_t^0 = 1\ \forall t$, the following conditions must hold:

$$N^2 n_{min}/2 \leq N^1 \text{ and } N^1 n_{max}/2 \geq N^2 \tag{21}$$

It follows from (11) that the first condition can only be satisfied if $n_{max} \leq \left(\frac{2}{n_{min}}\right)^{\gamma} n_{min}$. With $n_{min} < 2$ and $\gamma < 1$ it follows immediately that $n_{max} \leq \left(\frac{2}{n_{min}}\right)^{\gamma} n_{min} < 2$, which cannot be the case, as $n_{max} > 2$ by assumption. Therefore, for $e_t^0 = 1$, choosing the unrestricted solution w.r.t. fertility will always be optimal after a sufficiently large number of periods, which proves Proposition 2. ■

[9] See equations (11) and (16).

2.4.5 Path P8

It remains only to deal with the path along which $e_t = 1\ \forall t$ and fertility takes alternating values. In Proposition 2, it has been shown that such a path cannot exist if n_t alternates between n_{min} and n_{max}. However, paths along which fertility alternates between two (or more) other values might be possible.

To rule out this possibility, consider a path along which fertility alternates between the levels n_1 and n_2, while $e_t = 1\ \forall t$ and λ_t grows. If these levels of fertility have a cycle of 2, the levels of population will have the same cycle,[10] implying $N_{t+2} = N_t \equiv N'\ \forall t$ and $n_t \cdot n_{t+1} = 4\ \forall t$. It can be shown[11] that this latter condition can only be satisfied if $n_t = 2\ \forall t$. For paths with a cycle longer than $c_n = 2$, say, $c_n = x$, it can be shown in a similar way that the condition $N_t = N_{t+x} \equiv N'$ will be independent of x and can only be satisfied if $n_t = 2\ \forall t$. Hence, paths along which fertility takes alternating values while $e_t = 1\ \forall t$ do not exist.

2.5 Conclusion: Paths

Ignoring the chance case where $n_{min}n_{max} = 4$, and full alternating (cyclical) steady states, the following sets of steady states are possible:

- A low-level, full stationary state (backwardness), with $n_t^0 = 2$, $e_t^0 = 0$, $\lambda_t = 1$ and $N_t = N^*(e = 0)\ \forall t$.
- A growth steady state, with a stationary population $n_t^0 = 2$, $e_t^0 = 1$, $N_t = N^*(e = 1)\ \forall t$ and $\lim_{t\to\infty} \lambda_t = \infty$ for $z \geq 1$.
- A high-level, fully stationary state, with $n_t^0 = 2$, $e_t^0 = 1$, $N_t = N^*(e = 1)\ \forall t$ and $\lim_{t\to\infty} \lambda_t = 1/(1 - z)$ for $z < 1$.

[10] Recall that paths along which population grows without bound or falls to zero are not possible.

[11] Define $A = \frac{2\alpha(\beta(1-\gamma)-\beta_1\gamma)(1-\chi)}{[1+\beta(1-\gamma)-\beta_1\gamma][b]}$. From equation (11) and with $e_t = 1\ \forall t$ we get $n_t = A(2/N_t)^\gamma\ \forall t$. Then, the level of N' satisfying $N_{t+2} = N_t \equiv N'\ \forall t$ is $N' = 2(A/2)^{(1/\gamma)}$, and the associated level of fertility is $n = A(2/N')^\gamma = 2$.

- A cyclical steady state of permanent backwardness ($\lambda_t = 1$ and $e_t = 0\ \forall t$), but with alternating population sizes and fertility rates.

If none of the three paths with a constant population size exists, the economy will evolve towards an alternating steady state, whereby the maximum and minimum population sizes and the levels of efficiency reached, as well as the periodicity, depend on the system's parameters. In sections 2.5.1 and 2.5.2 we will analyze the non-cyclical steady states in more detail, determine the set of parameters values under which they will arise and discuss whether it is possible for two or more steady states to exist simultaneously.

2.5.1 The low-level stationary state

For the low-level stationary state to exist, it must be that parents choose the steady state value for education $e^0 = 0$ when $\lambda = 1$ and $N = N^*(e = 0)$. A condition necessary, but not sufficient, for such a path is that parents must prefer full-time child labor to full-time schooling when $N = N^*(e = 0)$.[12] As $\partial n/\partial e < 0$, we have $n^0(e = 1) \in [n_{min}, 2)$:

$$u(e = 0, N^*(e = 0), n^0 = 2, \lambda = 1) > u(e = 1, N^*(e = 0), n^0(e = 1), \lambda = 1),$$

This condition will be satisfied if and only if:

$$z < \left[1 + \frac{\mu[1 + \beta(1-\gamma) - \beta_1\gamma]}{(\beta(1-\gamma) - \beta_1\gamma)(1-\chi)}\right]^{\frac{\beta(1-\gamma)-\beta_1\gamma}{\beta+\beta_1}} - 1 \equiv z_0. \tag{22}$$

Therefore, the low-level stationary state will exist if the education technology is insufficiently productive. The critical value of z, namely z_0, depends on the parameters as follows. If the children's level of efficiency μ is large parents are more likely to send them out to work, for $\partial z_0/\partial\mu > 0$. If the transfer factor χ from the parents to the grandparents is high, adults will need child labor to finance C_{1t}, so that z_0 is high.

The effects of γ, β and β_1 on z_0 cannot be derived analytically. If land plays an im-

[12] Recall that full unrestricted solutions are never optimal.

portant role in production (in the sense that the value of γ is high), numerical simulations show that z_0 will decrease. The intuition for for this is as follows: if γ is high, having many children – who will then own but little land as adults – is not profitable. Parents will therefore prefer full-time schooling of a few children to full-time labor of many even for relatively low values of z when γ is high.

Note that condition (22), while necessary, is not sufficient to ensure the existence of the low-level stationary state, as parents might prefer the unrestricted solution w.r.t. education to the unrestricted solution w.r.t. fertility when $\lambda = 1$ and $N = N^*(e = 0)$, as derived in (16), even if condition (22) is satisfied. In this case, the low-level stationary state will not exist.

2.5.2 The high-level stationary state and growth steady state

A similar necessary condition can be derived for these configurations. However, the level of human capital is not constant over time, but growing, with $\lim_{t\to\infty} \lambda_t = 1/(1-z)$ for the high-level stationary state and $\lim_{t\to\infty} \lambda_t = \infty$ for the growth steady state. Therefore, the condition must be satisfied for all levels of human capital on the path, λ_t :

$$u(e = 1, N^*(e = 1), n^0(e = 1) = 2, \lambda_t) > u(e = 0, N^*(e = 1), n^0, \lambda_t),$$

that is, parents must prefer choosing full-time schooling to full-time child labor. This condition will be satisfied if

$$z > \frac{1}{\lambda_t}\left[1 - \frac{\mu[1 + \beta(1-\gamma) - \beta_1\gamma]}{(\beta(1-\gamma) - \beta_1\gamma)(1-\chi)\lambda_t}\right]^{\frac{-\beta(1-\gamma)+\beta_1\gamma}{\beta+\beta1}} - \frac{1}{\lambda_t} \equiv z_1(\lambda_t). \tag{23}$$

Note that $\lim_{\lambda_t\to\infty} z_1 = 0$, that is, if the parents' level of human capital is high, they will always prefer full-time schooling to full-time child labor. The higher λ_t, the lower will be the level of z_1. As a consequence, z_1 will take its maximal value if λ_t is minimal, that is[13] $z_1(1) \geq \left[1 - \frac{\mu[1+\beta(1-\gamma)-\beta_1\gamma]}{(\beta(1-\gamma)-\beta_1\gamma)(1-\chi)}\right]^{\frac{-\beta(1-\gamma)+\beta_1\gamma}{\beta+\beta 1}} - 1$. Therefore, the growth steady state will exist if $z > 1$ and the initial level of human capital λ_0 is sufficiently high, satisfying $z_1(\lambda_0) \leq z$. The high-level stationary state will exist if $z < 1$ and the implicit condition $z_1(\lambda = 1/(1-z)) \leq z$ is satisfied.

For low z, therefore, the economy will be typically trapped in the low-level stationary state, whereas long-term growth is possible for high z. For medium levels of z, the system can be very unstable, as the optimal levels of schooling and fertility are not smooth functions of N_t and λ_t but have points of discontinuity: small changes in the initial conditions can lead to extreme differences in the long-term population sizes and levels of efficiency. An alternating steady state will always exist if neither of the conditions (22) and (23) are satisfied.

Both the low-level and the high-level stationary states will exist if $z_1(\lambda = 1/(1-z)) < z < z_0$. Similarly, if $1 \leq z < z_0$, both the low-level stationary state and the growth steady state can exist simultaneously. It can be shown numerically that there are sets of parameters for which each of these conditions is satisfied.[14] In these cases, it will depend on the initial levels of λ and N alone which steady state will be reached.

2.5.3 Economic Growth

In the present model, economic growth can be measured with respect to three indices: the adults' level of efficiency, λ_t, the level of family income or output, y_t, and the lifetime utility of a generation, $U(C_{1t}, C_{2t}, \lambda_{t+1})$. However, growth rates for output and lifetime utility can only be derived numerically in most cases. Therefore, we confine our analysis to efficiency as a measure of economic growth. Sufficiently large changes in the adults'

[13]Note that $z_1(1)$ can be both larger and smaller than 1, depending on the parameters.

[14]For example, if $\mu = 1/2$, $\beta_1 = 1/5$, $\beta = 2/3$, $\gamma = 1/4$ and $\chi = 0.4$ we get $z_0 = 0.968$ while the implicit condition $z_1(\lambda = 1/(1-z)) \leq z$ yields $z > 0.671$, and both the low-level and high-level stationary states exist. If $\gamma = 1/5$, we get $z_0 = 1.05$, and both the low-level stationary and the growth steady states exist.

level of efficiency will always lead to large changes in output and utility for a constant population size, and $\lim_{\lambda\to\infty} y_t(N = N^*) = \lim_{\lambda\to\infty} u_t(N = N^*) = \infty$.[15]

For the low-level stationary state, there is no growth: both the level of efficiency and total population size are constant with $\lambda_t = 1$ and $N_t = N^*(e = 0)\ \forall t$.

For the high-level stationary state and growth steady state, parents choose to have two children who enjoy full-time schooling; so that the total population size and land holdings of each family are constant over time ($N_t = N^*(e^0 = 1)$ and $H_t = 2/N^*(e^0 = 1)\ \forall t$). The growth rate of efficiency will be:

$$g_\lambda \equiv (\lambda_{t+1} - \lambda_t)/\lambda_t = (ze_t\lambda_t + 1 - \lambda_t)/\lambda_t = z - 1 + 1/\lambda_t$$

as $e_t = 1\ \forall t$. If $z < 1$, the high-level stationary state is approached asymptotically: $\lim_{t\to\infty} \lambda_t = 1/(1-z)$.

If $z \geq 1$, the adults' level of efficiency will grow at the rate $g_\lambda = z - 1 + 1/\lambda_t\ \forall t$: the growth rate approaches $z - 1$ from above. As $\partial y/\partial\lambda$ and $\partial U/\partial\lambda$ are both positive, per capita GDP and utility will grow when λ grows.

In all other steady states and for all other values of z, the growth rate of λ will eventually be negative or zero. This result holds for all measures of economic growth presented above. Note, however, that the long-term prospects of an economy depend not only on the productivity of the education function, but also on the initial conditions regarding the adults' level of efficiency, population size and land holdings.

Assume, for example, that both a poverty trap and a growth steady state exist. For such a situation to be feasible, it must be that $1 \leq z < z_0$. If the adults' initial level of efficiency λ_0 is high, satisfying $z_1(\lambda_0) < z$, the economy will typically attain the growth

[15] This is valid for all parameters such that $C_{it} > 0\ \forall\lambda_t,\ i = 1, 2$

steady state, so that long-term growth is feasible. If, however, the parents' initial level of efficiency is low, the economy will eventually fall into the poverty trap – if the initial population size is not too high. Long-term growth is feasible even from a low initial level of efficiency, however, if the total population size is very high in the first period. The intuition for this result is that parents are forced to reduce fertility if N is currently high, and therefore to increase their children's level of schooling in order to finance old-age consumption. Therefore, the level of efficiency will grow ($z \geq 1$), while total population size will decrease.

2.5.4 Dynamic Efficiency

It is a well-known result in the literature that household choices in OLG models lead to dynamically inefficient outcomes, particularly if there are no bequests or gifts from one generation to another (Abel (1987)). In the present setting, parents are altruistic toward their children, but bequests are ruled out and capital markets do not exist. Parents can only raise their children's endowment as adults by sending them to school, or by choosing lower fertility, so that each child has more land. The transfers from young to old adults do not stem from children's altruism towards their parents, but rather from a fixed social norm requiring the young to finance their parents' old age. Therefore, it is not to be expected that the steady states derived above will be dynamically efficient.

2.6 Governmental Intervention

In the following sections, the analysis will focus on the case where the only steady states are the low-level stationary state and steady growth. It is assumed that the government's main aim is to induce and maintain growth.

An economy will end up in the stationary state if the initial level of efficiency is low enough, or if the education function is not sufficiently productive. The danger also arises if children have relatively high levels of efficiency, the transfer to the grandparents is generous, or the parameter γ takes a low value, all of which work to increase z_0 as derived in (22). In all these cases, having many children who are put to work full-time is profitable, as the earnings through child labor then make a significant part of the total household income, while the resulting strong increases in the total population of young adults in the next period have but a slight influence on output per family.

Measures enhancing the productivity of the education function might encompass the construction of schools and training teachers, but also setting up adequate, standardized curricula and financing teaching materials. Some of these measures are included in the World Bank Project, "Effective Schooling In Rural Africa", which aims at developing best practices. It turns out, however, that evaluating their impact, both long- and short-term, involves major difficulties. Formulating z as a function of spending on these measures is only possible through extensive data collection and econometric estimation, whereby the results will be valid for specific regions only. A similar argument is also valid for changes in the production technology, as characterized by γ.

Measures designed to influence μ, the children's level of efficiency, and hence the implicit wage for child labor, typically consist of either prohibiting child labor altogether or taxing it – both are unpopular and inefficient measures, as well as hard to enforce. The value of χ, the social rule governing transfers to the old, can be changed if the government can set up capital markets. As Ranjan (1999) has shown, child labor can be reduced if households can save and borrow, which they use to finance either old-age consumption or education.

As measures designed to influence z, γ, μ and χ are hard to analyze, we will focus on direct taxes and subsidies. We will assume that only young adults, the parents, pay taxes and receive subsidies. The government raises revenues by imposing lump-sum taxes (T), which are equivalent to taxing the adults' income (τ) or land holdings (τ_0):

$$C_{1t} = 2\alpha\lambda_t H_t^{\gamma}(1-\chi) + \alpha\mu n_t(1-e_t)H_t^{\gamma} - n_t b\lambda_t - T - \tau_0 \cdot (2\alpha\lambda_t H_t^{\gamma}) - \tau_1 \cdot H_t$$

with $\tau_0 = T/(2\alpha\lambda_t H_t^{\gamma})$ and $\tau_1 = T/H_t$ as the level of land holdings are given for any household. Therefore, the further analysis will focus on lump-sum transfers, whereby a negative lump-sum tax is a subsidy. The only other transfer analyzed in this context will be a direct subsidy for schooling, whereby each child receives a fixed amount $s \geq 0$ for each unit of time she spends at school. While s is a cash transfer in the present model, the subsidy could also take the form of school meals like the Food-for-Education program in rural Bangladesh, free medical service, or other goods or services. As none of these transfers takes place in the last period of an adults' life, (4) remains unchanged while consumption in the first period needs to be rewritten:

$$C_{1t} = 2\alpha\lambda_t(1-\chi)\left(\frac{2}{N_t}\right)^{\gamma} + \alpha\mu n_t(1-e_t)\left(\frac{2}{N_t}\right)^{\gamma} - n_t b\lambda_t - T + n_t s_t e_t, \qquad (24)$$

where at most one of the variables T and s_t is non-zero. Describing the optimal path to permanent and persistent growth is only possible by introducing a social welfare function encompassing several generations. However, as the household decision cannot be derived analytically, and as welfare functions have specific problems like the choice of the temporal discount factor, which strongly influence the optimal outcome, we will refrain from introducing a social welfare function. Instead, assume that the government tries to induce parents into choosing full-time schooling until the economy reaches the high steady state. The government tries to reach its aim through lump-sum transfers and school-attendance subsidies, whereby each family can either be taxed or subsidized, but never both in the same period.

2.6.1 Lump-sum transfers

Assume a family in the low-level stationary state is confronted with a lump-sum tax.[16] Families can react to taxes by changing fertility or education or both. Each household in the low-level stationary state has two children who work full-time, so that a reduction of education as a means of financing the tax is not feasible. Therefore, parents will reduce fertility. As $\lambda_t = 1 \;\forall t$ and $N_t = N^*(e=0)$ from (16) in the low-level stationary state, the total tax a household can pay is limited by the condition $C_{1t} - T_{max} \geq 0$,[17] that is:

$$T_{max} \leq 2\alpha(1-\chi)\Big(\frac{2}{N^*(e=0)}\Big)^{\gamma} + \alpha\mu n_t^0(1-e_t^0)\Big(\frac{2}{N^*(e=0)}\Big)^{\gamma} - n_t^0 b.$$

Poor families confronted with a lump-sum tax choose $e_t^0 = 0$ and $n_t^0 = n_{min}$. Therefore, a household in the poverty trap can pay at most:[18]

$$T_{max} = \frac{b(1-\chi)[2+(2-n_{min})(\beta(1-\gamma)-\beta_1\gamma)]}{(1-\chi)(\beta(1-\gamma)-\beta_1\gamma)+\mu(1+\beta(1-\gamma)-\beta_1\gamma)}. \tag{25}$$

A family in the growth steady state chooses $n^0 = 2 \in (n_{min}, n_{max})$ and full-time schooling for all children. It will reduce fertility before reducing the level of education when confronted with a lump-sum tax if the adult level of efficiency is sufficiently high. Only if the tax is very high will parents reduce e. As was the case for the low-level stationary state, the total tax families can pay is limited by $C_{1t} > 0$ only. Rich families can pay the maximal tax when choosing $n^0 = n_{min}$[19] and $e^0 = 0$:

$$T_{max} = 2\alpha\lambda(1-\chi)\Big(\frac{2}{N^*(e=1)}\Big)^{\gamma} + \alpha n_{min}\mu\Big(\frac{2}{N^*(e=1)}\Big)^{\gamma} - n_{min}b\lambda,$$

with $N^*(e=1)$ from (16). However, after paying this tax once, the level of efficiency of the children when reaching adulthood will be $\lambda = 1$, so that the family will have fallen into the poverty trap. The government might therefore prefer raising a lower tax T^0_{max}, so that

[16] The issues of limits on taxes like tax evasion or minimal consumption will be addressed later.

[17] As $C_{2t} > 0 \;\forall n_t > 0$

[18] Note that T_{max} is positive for all parameters as $n_{min} < 2$ by assumption.

[19] Note that for rich families, the condition $b\lambda > \alpha\mu(2/N^*(e=1))^{\gamma}$ is always satisfied.

children still enjoy full-time schooling. As parents first reduce fertility until $n = n_{min}$ and only then schooling, T^0_{max} must satisfy the condition $e^*(n = n_{min}, \lambda, N^*(e = 1), T^0_{max}) = 1:$

$$T^0_{max} = \frac{2b\lambda}{\beta(1-\gamma)-\beta_1\gamma} - \frac{n_{min}\mu(1+z\lambda)b[1+\beta(1-\gamma)-\beta_1\gamma]}{(\beta(1-\gamma)-\beta_1\gamma)(1-\chi)z\lambda(\beta+\beta_1)} + (2-n_{min})\lambda b \qquad (26)$$

Note that T^0_{max} is positive for sufficiently large levels of efficiency.[20]

2.6.2 School-attendance subsidies

As families in the high-level stationary state already choose full-time schooling, no subsidies are required for rich households. For poor families, which are in the poverty trap, to choose full-time schooling voluntarily, a subsidy is required. Note that if the government reimburses families fully for the loss of child wages, that is, if the subsidy exceeds the maximal income through child labor $\alpha\mu n_t\left(\frac{2}{N^*(e=1)}\right)^\gamma$, all poor families will choose full-time schooling for their offspring. As parents are altruistic towards their children, a lower subsidy, s^0, will suffice to induce full-time education. As switching between solutions can take place as a response to the introduction of a subsidy, finding the required subsidy must be done for both cases separately.

If no switching between solutions takes place, that is, if families choose the unrestricted solution w.r.t. fertility before and after the subsidy is introduced, the minimum subsidy that induces full-time schooling must satisfy the condition

$$u(e = 1, n^*; \lambda, N^*(e = 0), s^0) \geq u(e = 0, n^*; \lambda, N^*(e = 0), s^0),$$

that is,

$$s^0 \geq b\lambda + \left[\mu\alpha\left(\frac{2}{N^*(e=0)}\right)^\gamma - b\lambda\right][1+z\lambda]^{\frac{\beta+\beta_1}{\beta(1-\gamma)-\beta_1\gamma}}$$

[20]For sufficiently low levels of efficiency, parents would require a subsidy to choose full-time schooling, that is, $T^0_{max} < 0$.

This yields the optimal subsidy for families in the low-level stationary state, which induces them into choosing full-time schooling:

$$s^0 \geq b - b\Big[\frac{(1-\chi)((1-\gamma)\beta - \beta_1\gamma)}{(1-\chi)((1-\gamma)\beta - \beta_1\gamma) + \mu[1+\beta(1-\gamma) - \beta_1\gamma]}\Big][1+z]^{\frac{\beta+\beta_1}{\beta(1-\gamma)-\beta_1\gamma}} \tag{27}$$

It can be shown that parents receiving s^0 will always have fewer than two children if $z \geq 1$ and if no switching between solutions takes place. Therefore, the introduction of a school-attendance subsidy will increase the level of education children enjoy while reducing fertility and therefore total population size. For all $z \leq z_0$, s^0 will be positive. For higher levels of productivity, $z > z_0$, parents voluntarily choose full-time schooling for their offspring when $\lambda = 1$ and $N = N^*(e=0)$, so that no subsidy is required. On the contrary, even if school fees were introduced ($s < 0$), parents would still choose full-time schooling if the fees were not too high.

If switching between solutions takes place, the optimal subsidy s^* must satisfy the condition $e^*(s^*; \lambda, N^*(e=0)) = 1$, which yields:

$$s^* = \frac{\alpha\mu\Big(\frac{2}{N^*(e=0)}\Big)^{\gamma}(1+z\lambda) - z\lambda^2(\beta+\beta_1)\Big[2\alpha(1-\chi)\Big(\frac{2}{N^*(e=0)}\Big)^{\gamma}\frac{1}{\bar{n}} - b\Big]}{1+(1+\beta+\beta_1)z\lambda}. \tag{28}$$

Note that the optimal subsidy depends on the level of fertility $\bar{n}$. The larger $\bar{n}$, the higher the subsidy required to induce full-time schooling, as parents need the income from child labor to finance child-raising costs.

Numerical simulations suggest that switching between solutions takes place for large values of z only: in this case, education is sufficiently profitable for parents to reduce fertility to n_{min} and to increase the level of schooling their children enjoy. If, however, z is low, parents will always choose the unrestricted solution w.r.t. fertility and full-time child labor, that is, switching between solutions will not take place. For unproductive education functions parents will therefore effectively forgo the subsidy as long as it is low,[21] but

[21]By choosing $n^0 = n^* = 2$ and $e^0 = 0$ the total subsidy parents receive is $n^0 e^0 s = 0$.

receive the full subsidy for each child if this exceeds s^0.

2.6.3 Governmental Programs

Two types of societies are of interest in the context of governmental intervention: a homogeneous society in which all families are in the poverty trap, and an inhomogeneous society, in which some are enjoying steady-state growth. In both cases we will assume that both the low-level stationary state and the growth steady state exist. The assumption of assortative mating is maintained. In addition, we will assume that there is no link between groups with the exception of fiscal policy.

In the case of a homogeneous society in a poverty trap, the government might tax a fraction of the population and subsidize the rest. The revenue it can raise through a tax can be derived using (25), while the total subsidy required depends on z. If the proportion of the population which is taxed is sufficiently large relative to the proportion receiving a subsidy, some families will escape the poverty trap and eventually reach the growth steady state; so that an inhomogeneous society emerges.

In an inhomogeneous society, some ('rich') families, say r_t in number, are in the growth or high-level stationary state, whereas all other ('poor') families, say p_t in number, are in the low-level stationary state. In this case, the numbers of poor respectively rich families will not change over time, as all households have $n_t^0 = 2\ \forall t$ children. With poor parents sending their offspring to work full-time, the level of efficiency of poor families remains stuck at unity. Rich families, on the other hand, will either get even richer over time, their level of efficiency growing at the rate $z - 1 + 1/\lambda_t \geq 0\ \forall t$ for $z \geq 1$, or, for $z < 1$, their level of human capital will be constant. Therefore the total subsidy poor families need will remain unchanged, while the potential tax revenues increase over time if $z > 1$, due to the growth of the rich adults' level of efficiency, as can be seen from (26) . Even if there are only few rich families, these will eventually be able to finance any arbitrarily large subsidy, if $z > 1$. Note, however, that rich families might become virtually extinct if the tax burden is high, as they typically finance taxes by reducing fertility. If $z < 1$, that is, if no growth steady state exists, the potential tax income does not increase over time, as rich families do not get richer.

Assume, for simplicity, that one single subsidy, say S, is sufficient for all poor families to eventually reach the growth or high-level stationary state, denote the tax each rich family can pay by T as derived in (26) and assume that the subsidy exceeds total tax revenues in the first period, $t = 0$, that is, $S_0 = S > rT_0$. In all following periods, S will remain constant, while the tax revenues will change as the adult's level of human capital grows. Rewriting T_t from equation (26) yields:

$$T_t = X_1\lambda_t - X_2\frac{1}{z\lambda_t} - X_2$$

where $X_1 = \left(\frac{2b}{\beta(1-\gamma)-\beta_1\gamma} + (2-n_{min})b\right)$, $X_2 = \frac{n_{min}\mu b[1+\beta(1-\gamma)-\beta_1\gamma]}{(\beta(1-\gamma)-\beta_1\gamma)(1-\chi)(\beta+\beta_1)}$ and $\lambda_t = \lambda_0 z^t + \frac{z^t-1}{z-1}$. Note that if $z = 1$, T_t will be independent of time. For $z > 1$, $\partial T_t/\partial t > 0$ as $X_1 > 0$, $X_2 > 0$ and $\partial\lambda_t/\partial t > 0$. For tax revenues to exceed expenditures for subsidies, the government must wait for P periods. P is only well-defined if a growth steady state exists, that is, if $z > 1:$

$$P = ln\left(\frac{1}{2}\frac{z(z-1)(X_2+S/r) - 2zX_1 + \sqrt{z(z-1)^2[z(X_2+S/r)^2 + 4X_1X_2]}}{zX_1[\lambda_0(z-1)+1]}\right)\frac{1}{ln(z)}.$$

This period of time will be extended if the government cannot or does not want to impose the maximal tax on rich families, fearing tax evasion or social unrest. Note that inequality will rise during the P periods of 'waiting', falling dramatically in period P. As rich families will typically have a higher level of efficiency, even after the measure, than the households formerly trapped in the low-level stationary state, inequality will again rise.

In the case where land is not an input in production (as in Chapter 1), poor families typically had a higher level of fertility than rich families. Rich families could therefore eventually finance a subsidy for all poor families only if the education function was highly productive ($z \geq \frac{n_{max}b(1+\beta)}{2\alpha(1-\chi)\beta} \gg 1$ as n_{max} is large and for moderate child-raising costs), whereas z need only satisfy the condition $z > 1$ in the present setting, in which fertility is at replacement levels in both groups before the intervention occurs.

2.7 Conclusion

When altruistic parents decide about the number of children to have and the level of schooling these are to enjoy, each family's land holding plays a major role, along with the productivity of the education function and social rules about transfers from young to old adults. If the education function is not sufficiently productive and the initial level of efficiency is low, households will be trapped in a low-level stationary state – the poverty trap – in which the adults' level of efficiency is at its minimum, children work full-time and all families have two children. If, however, the adults are highly efficient, or if the education function is very productive, such a poverty trap can be avoided. Households will send their offspring to school full-time, and the economy will grow with respect to all measures except population, which will eventually become stationary. A permanent escape from the poverty trap is always possible if a growth steady state exists. In a homogeneous society, all families can escape the poverty trap simultaneously only if outside intervention occurs.

Other possible measures – which might be less expensive and more efficient than subsidizing all poor families – encompass reorganizing the education system (that is, trying to increase z), or changes in the technology employed by poor families, thereby increasing adult income. Setting up a functioning financial sector which provides even poor families with access to credits and savings can also reduce child labor, as shown by Ranjan (1999).

Chapter 3

The Long-run Effects of HIV/AIDS in Kenya

Abstract

This essay analyzes the long-run economic effects of HIV/AIDS in Kenya, with emphasis on fertility, education and child labor. Human capital, which is built up through formal education and parental child-rearing, is the only input in production. Two aspects are central to the analysis: First, a mature AIDS epidemic causes massive premature adult mortality, thereby destroying existing human capital and reducing the labor force on a large scale. Second, the transmission of human capital to future generations is weakened, as children are left orphaned and surviving adults are correspondingly burdened. As a consequence, per capita income decreases and communities can less afford to raise and educate children as they did before the outbreak of the disease. The underlying theoretical model, in which it is assumed that parents raise and educate children for both financial and altruistic reasons, is calibrated using data for the period 1920 to 2000. The long-run effects of the disease, which depend heavily on parents' expectations about future mortality rates, are estimated for the years 2000-2040. Both human capital and per capita income grow significantly more slowly after the outbreak of the epidemic, while the incidence of child labor doubles for some periods. The level of fertility falls in the immediate aftermath of the outbreak, but can be significantly higher when the epidemic has reached a mature phase, depending on parents' expectations. Governmental interventions in the health sector in the early phase of the epidemic can strongly mitigate its adverse effects.

3.1 Introduction

Kenya declared the HIV/AIDS epidemic a national disaster in 1999, 15 years after the first HIV/AIDS case had been reported. By that time, more than half a million Kenyans were estimated to have died of the disease, and some 2.5 million adults were infected. Other countries in Sub-Saharan Africa had reacted to the epidemic more than a decade earlier – notably Kenya's neighbor Uganda, which had declared AIDS a national disaster in 1986. Expectations in Kenya in 1999 were grim: the death rate was projected to rise from 560 persons per day in 2000 to 760 by 2005.[1] While governmental interventions following the 1999 declaration seem to have borne some fruit, and the death rate had even been reduced to 300 per day by 2003,[2] the epidemic has not yet been brought under complete control.

Most new infections occur among young people, particularly women aged 15 to 24 and men aged 30 and younger.[3] As 70 per cent of all Kenyan children are born to mothers younger than 30, high HIV/AIDS prevalence rates in this age group will strongly affect the way families raise and educate children. As the parents become ill, family income is reduced, either because they cannot work or because of the stigma towards those infected, who have difficulty finding employment. The high costs of treatment further increase the burden on the household's income. Children raised in families affected by HIV/AIDS often enjoy less parental guidance and care, and their education suffers, as they may have to work to support their parents and siblings. As therapies are often too costly, most victims die within 8-10 years of being infected, leaving their children orphans. By 2003, 37% of all orphans were AIDS orphans, up from an estimated 3% in 1990 and 22% in 1995.[4] Even the education of those children whose parents are not ill suffers, as their teachers are often too sick to conduct their classes. The Kenya Teachers Service Commission reports that deaths among teachers more than tripled between 1995 and 1999, rising from 450 to

[1] Source: *http://www.standwithafrica.org/hiv_aids/reality1.php*

[2] Source: *http://www.aegis.com/news/afp/2003/AF031233.html*

[3] Source: *http://www.unaids.org/EN/Geographical + Area/by + country/kenya.asp*

[4] Sources: 1990 and 1995 figures: 'Children on the Brink',2002, U.N.AIDS, UNICEF and USAID collaboration. 2003 figure: 'Children on the Brink', 2004, U.N.AIDS, UNICEF and USAID collaboration

1500 per year.[5] On average, 1.4% of all teachers are expected to die of HIV/AIDS yearly between 2000 and 2010.[6]

As women of child-bearing age are strongly affected by the disease, and may transmit the virus on giving birth, the total number of children raised by a family is also expected to change. Yet only 30% of all women know that taking anti-retroviral drugs during late pregnancy can reduce the risk of Mother-To-Child-Transmission, according to the 'Kenya Demographic and Health Survey' conducted in 2003. Two different effects are possible: on the one hand, with potential mothers falling ill and dying and higher child mortality due to HIV/AIDS, completed familiy size could fall. On the other hand, families might respond to the rise in mortality by having more children, enough of whom would then survive to take care of their parents when these cannot care for themselves, either due to HIV/AIDS or to old age.

Several studies have projected the impact of the epidemic in Kenya, focusing on economics, the health and education sectors, the development of the population size or specific population groups, like orphans or women. While macroeconomic empirical studies in the 1990s (e.g. Bloom and Mahal (1997)) suggest that the effects of the HIV/AIDS epidemics on the economies of Sub-Saharan African countries are small, more recent work suggests that the effects on specific countries or regions may be very damaging. Bell, Devarajan and Gersbach (2003), for example, simulate the South African economy for the years 1990-2080 and show that the effects of the epidemic will indeed be strong, with a distinct possibility of a collapse of the economy in the absence of government action. Arndt and Lewis (2000) use a shorter simulation period, namely, 1997-2010, and find that both total and per capita GDP are substantially lower in the face of the epidemic, compared to the counterfactual without HIV/AIDS. Bollinger, Stover and Nalu (1999) review several studies of the impact of AIDS in Kenya, and analyze the economic impact of the epidemic on that country's households, agriculture, firms and macroeconomy. They

[5]Source: 'Education and HIV/AIDS: A window of hope', The World Bank, 2002
[6]Source: ibid.

conclude by recommending measures aimed at mitigating the effects of the disease and emphasise the importance of governmental commitment to addressing HIV-related problems, and treating the disease as a ‘national priority’.

Bell et al. (2004) employ a three-generation OLG model to simulate the effects of the disease in Kenya up to 2050. Unlike the South African case, they find that the Kenyan economy is not threatened with a collapse, but GDP in the AIDS case is lower by about 40% in 2040 compared to the counterfactual without AIDS, and population is lower by about a third. Kenya’s long-term problems are attributed to three interrelated factors: first, the ’weakening of the mechanisms through which human capital is transmitted from one generation to the next’; second, a sharp drop in the productivity of human capital in the decade following 1990; and third, the HIV/AIDS epidemic. To analyze the effects of these mechanisms on the Kenyan economy, Bell et al. (2004) focus on household decisions concerning education.

The purpose of the present essay is to extend their analysis by incorporating households’ fertility decisions, as it seems realistic to assume that families react to exogenous shocks to mortality by adjusting not only the level of schooling, but also the number of children they intend to have in the first place. The essay will draw upon Bell et al. (2004), particularly where the model’s structure and the data are concerned.

The essay is structured as follows: Section 3.2 gives a historical overview and discusses the data used. The model is introduced in section 3.3, and its calibration is presented in section 3.4. Several variations of projections of the Kenyan economy until 2040 are discussed in sections 3.5 and 3.6, while the issue of public policy is taken up in section 3.7. The issue of formulating and measuring social welfare in the presence of premature adult mortality is addressed in section 3.8. The main results are stated in section 3.9, with conclusions in section 3.10.

3.2 Historical Overview and Data: 1920-2000

At the beginning of the 20th century, Kenya was a British Protectorate officially called ‘British East Africa’. Inland colonization by white settlers began around 1901, when the railway line connecting Mombasa and Lake Victoria was completed. By 1920, 9000 settlers were living in Kenya, and by 1950 their number had increased nearly 10-fold.[7] Control over nearly all economic activity was concentrated in their hands, and the indigenous population was either employed as workers on the coffee farms, or engaged in mostly low-productivity traditional occupations. Legislation prevented the African population from purchasing and owning property in the highly fertile Kenyan Highlands and from taking part in government. It was only in 1944, that the first African became a member of the Colony’s Legislative Council.[8] These inequalities in economic and political power led to unrest, culminating in the Mau Mau Uprising between 1952 and 1960. While the uprising was suppressed by British troops, some of the Mau Mau’s aims – like land reform – were attained in subsequent years.

In 1963, Kenya gained independence, and Jomo Kenyatta became its first president. He was succeeded by Daniel Arap Moi in 1978, who stayed in power until 2002. After Independence, several reforms were undertaken, especially in the spheres of land policy, the educational system, and the political system. Many of these reforms – particularly those regarding education – can be observed in the time series used in this essay, to which we now turn.

Decadal data on the economies’ output and demography for the years 1950 to 2000 are used, as well as data on the average years of schooling from 1920 onwards. A detailed discussion of the time series and revisions is given in Bell et al. (2004), on which this section draws. For the remainder of the essay, the round years will be used as time points, whereby the variable $t = 1, 2, 3..., 14$ denotes the number of decades elapsed since the year

[7]Source. http://www.lonelyplanet.com/destinations/africa/nairobi/printable.htm
[8]Source: http://www.kenyalogy.com/eng/info/histo12.html

1900, so that $t = 10$ denotes the decade starting in the year 2000.

3.2.1 Population

During the period of interest, five censuses were conducted in Kenya, in the years 1948, 1962, 1969, 1979 and 1989. The United Nations Population Division, the World Bank (in the form of the World Development Indicators, WDI) and the Penn World Tables (PWT) provide secondary data on the Kenyan population and its structure. For the purposes of the calibration, data for the years 1950-1990 are used, with estimates for the nearest round date in the case of the Census figures.

The WDI does not report the total population for 1950, and its data for 1960 onwards are identical to the UN's, as are the PWT data. However, the population in the PWT for 1950 is higher than both the UN and Census data by 3.6% and 7.7% respectively. Bell et al. (2004) reconstruct the early part of the series and reject the PWT's estimates for that year.

Both the UN Population Division and the Censuses provide data on the age distribution. Except for the first and last group, which include infants aged 0-5 and people aged 65 and older, respectively, all 8 age groups span 10 years, their mid-points being 10, 20, 30, 40, 50 and 60. Age-groups will be denoted by the index $a = 0, \ldots, 7$ and the size[9] of an age group in period t will be denoted by N_t^a. As both estimates are problem-ridden, a revised series was derived, in which the implicit age-specific death rates (defined below) are higher for those 35 and older than those implicit in of the original UN data. As a consequence, old cohorts are smaller, while young cohorts are relatively larger than the U.N.'s. Denote the mortality rate in age group i over the period t to $t + j - i$, $(j > i)$ in period t by $q_t^{i,j}$. The age-specific death rate for age group a is defined as the probability that a member of age group a will not survive to become a member of age group $(a + 1)$ 10 years later, and will be denoted by $q_t^{a,a+1}$. Mortality rates can then be computed from

[9] The unit of N_t^a and of all other population data in the essay is 10^3.

Table 3.1: Population Tables

age group/year	1950	1960	1970	1980	1990	2000
0-4	1040	1541	2294	3482	4458	4696
5-14	1606	2317	3371	4951	7182	9006
15-24	1192	1491	2167	3179	4715	6875
25-34	882	1097	1381	2019	2979	4447
35-44	634	784	983	1248	1833	2731
45-54	441	542	677	857	1099	1634
55-64	240	349	432	545	698	911
65+	125	201	238	351	511	864

Source: Bell et al. (2004)

the population tables as follows:

$$q_t^{i,j} = 1 - \frac{N_{t+(j-i)}^j}{N_t^i}.$$

In the remainder of the essay, the probability that a member of age group $a = 2$ reaches age group $a = 4$ will play a major role. It will be denoted by κ_t :

$$\kappa_t \equiv 1 - q_t^{2,4} = \frac{N_{t+2}^4}{N_t^2}. \tag{1}$$

3.2.2 Output

Both the Penn World Tables and the World Development Indicators provide data on aggregate output, with the PWT time series starting in 1950 and the WDI 10 years later. The PWT contains data on per capita GDP in constant purchasing power units with the base year 1995, as well as population data. As discussed in the previous section, the PWT population estimate for 1950 is implausibly high; so that total GDP for that year is derived using the revised estimate instead. The two series do not differ significantly for the following years, and the PWT series is chosen, being the longer of the two. As the purpose of the analysis is to derive long-term effects, short-term shocks to GDP are smoothed by forming 5-year moving averages. As can be seen from table 3.2, the Kenyan economy experienced period of fast growth after Independence, but also a marked slowing down after 1990, with per capita GDP actually falling. For the remainder of the paper, GDP will be denoted by Y_t.

Table 3.2: GDP in Kenya, 1950-2000

Year	1950	1960	1970	1980	1990	2000
GDP (10^7)	436	642	1089	2014	3076	3633
Average annual GDP growth (%)	–	3.9	5.4	6.3	4.3	1.7

Source: Bell et al. (2004)

3.2.3 Educational Attainment

The educational system in Kenya underwent several major changes in the last century. The Department of Education was founded in 1911, but only 3% of the country's African population had enjoyed any formal education by 1925 (Thias and Carnoy, 1972). Primary education was financed and organised by the communities and missionaries, and no common curriculum existed. Until Kenya gained independence in 1963, the African population received mostly technical and vocational training, as recommended by the Fraser Report of 1909. The Ominde Commission, set up in 1964, led to changes in the schooling system which aimed at increasing enrolments in secondary education. Up to 1966, primary and secondary education spanned 8 and 4 years respectively. Primary education was reduced to 7 years after 1966, but was extended again to 8 years in 1985, with all schools using the same curriculum. By 1973, school fees had been abolished for the first 6 years of primary education, following UNESCO's proposals. This led to high enrolment rates, particularly so in 1974 and 1979. As a consequence, the government hired a substantial number of new teachers, many of them poorly trained, thereby possibly reducing the quality – albeit not the quantity – of educational inputs.

Two data sets are used to determine the average years of completed schooling: the Censuses mentioned in section 3.2.1 and reports by the ministry of education. A very detailed description of the method is given in Bell et al. (2004), the results of which are set out in table 3.3. The variable e_t denotes the average years of schooling of the school-age cohort in period t, expressed as a fraction of a 12-year schooling period.

Table 3.3: Educational Attainment

Year	1920	1930	1940	1950	1960	1970	1980	1990
Education e_t	0.047	0.080	0.134	0.201	0.367	0.458	0.520	0.570

Source: Bell et al. (2004)

3.3 The Model

The basic model describes household decisions concerning the bearing and rearing of children in the presence of premature adult mortality, for example, due to HIV/AIDS.

As in Chapters 1 and 2, it is assumed that households consist of three generations, namely, children, parents and grandparents, each of whom is endowed with one unit of time. Children divide their time between working and learning, where the fraction of time assigned to education will be denoted by $e \in [0, 1]$. Surviving parents work full-time, as do grandparents. It is assumed that parents receive the income of the entire family, including that accruing to the grandparents, and then redistribute this income according to some set of social rules, which are described below.

For simplicity, it is assumed that parents (father and mother) are identical with respect to both their levels of human capital and their mortality rates. They raise and educate children in order to increase their own current consumption and to finance their old age. As investment in physical capital is ruled out by assumption, and grandparents work part-time only, they also rely on transfers from their children to finance their consumption. A fixed fraction $\chi \in (0, 1)$ of the family's income is transferred to the grandparents. There are no bequests. It is also assumed that, apart from the opportunity costs of child labor, education is free. In this assumption we depart form Bell et al. (2004), who assume that families spend a fraction of their income on schooling. Raising children, however, is costly, and the better educated parents are, the more they spend on raising their children.

The temporal structure of the model is as follows: When they reach age group $a = 2$, say in period t, young adults form couples and decide on the number of children they intend to have and raise. They also enter into a binding contract on the education these children are to receive when they reach school-going age. If parents have perfect foresight about child mortality and if there is 'replacement fertility', as in Bell, Devarajan and Gersbach (2003), then a decision about fertility is equivalent to deciding about N_t^1, and we will take N_t^1 as the corresponding decision variable for the remainder of the essay. When parents are in age groups $a = 2$ and $a = 3$ their children go to school according to the decision made at birth, as stated in the contract; their level of schooling is e_t, and the level of human capital (measured in efficiency units of labor) they attain upon reaching adulthood is denoted by $\lambda_{t+1}(e_t)$. In period $t+1$, the children start working themselves, and have their own children. When their parents reach age group $a = 4$ in period $t+2$, they start receiving transfers from the younger generations. Therefore, the total number of 'young couples' in period t will be $(N_t^2 + N_t^3)/2$, while the total number of 'old couples' in period $t+2$ will be $(N_{t+2}^4 + N_{t+2}^5 + N_{t+2}^6 + N_{t+2}^7)/2$.

It is assumed that the efficiency of a grown-up depends on the time she spent at school, the average efficiency of her parents and the productivity of the educational process [see Bell et al. (2004)]. If an individual does not spend any time at school, she will attain the basic minimum level of efficiency $\lambda = 1$. It is assumed that adults in the two youngest age groups (i.e. $a = 2$ and $a = 3$) are involved in educating children, and that the educational technology is isoelastic with parameter ϵ, as in Bell et al. (2004). The children which are going to school in period t attain the following level of human capital in period $t+1$:

$$\lambda_{t+1} = 2z_t e_t^{\epsilon} \left(\frac{N_t^2 \lambda_t + N_t^3 \lambda_{t-1}}{N_t^2 + N_t^3} \right) + 1, \qquad z_t, \epsilon > 0 \tag{2}$$

where $z_t (> 0)$ can be thought of as the strength of the mechanism for the inter-generational transmission of knowledge. The growth rate of the adults' level of efficiency in the case of full-time schooling is:

$$g_\lambda \equiv \frac{\lambda_{t+1} - \lambda_t}{\lambda_t} = 2z_t \left(\frac{N_t^2 + N_t^3 \frac{\lambda_{t-1}}{\lambda_t}}{N_t^2 + N_t^3} \right) + \frac{1}{\lambda_t} - 1. \tag{3}$$

Note first that the growth rate will always be positive if previous generations enjoyed no schooling whatsoever, that is,[10] if $\lambda_t = \lambda_{t+1} = 1$. The growth rate g_λ will be positive even for high levels of efficiency if z_t is greater than 0.5. If, however, z_t is lower than 0.5, the growth rate will depend on the level of λ_t : If λ_t is sufficiently close to one, g_λ will be positive, whatever be the level of λ_t, as $1/\lambda_t$ is then sufficiently close to one.[11] If the adults' level of efficiency is growing, however, the term in brackets will be lower than 1, while $1/\lambda_t$ will be falling, so that g_λ will eventually be zero or negative. A steady-state in λ can arise if z_t is stationary, with the steady-state level of efficiency being $\lambda^* = 1/(1-2z_t)$. Note that in a steady-state with respect to λ, population growth plays no role, as the term in brackets in (3) will be 1 in all periods when $\lambda_t = \lambda_{t+1}$. The temporal structure of the population pyramid, however, is important, as it influences the choice of e_t, and therefore also determines whether stationary value of λ can be reached.

Income is generated through the production of a single, non-storable good. Labor – measured in efficiency units – is the only input in production. Let the efficiency of a school-age child be fixed at μ. Assume, further, that those younger than 5 and older than 65 years of age, that is, age groups $a = 0$ and $a = 7$, do not work at all. Therefore, ignoring unemployment, the total labor supply (measured in efficiency units) of an extended family in period t will be:

$$L_t = \underbrace{N_t^2\lambda_t + N_t^3\lambda_{t-1} + N_t^4\lambda_{t-2} + N_t^5\lambda_{t-3} + N_t^6\lambda_{t-4}}_{\text{adult labor}} + \underbrace{N_t^1(1-e_t)\mu}_{\text{child labor}} \tag{4}$$

The production function is assumed to exhibit constant returns to scale with respect to labor (measured in efficiency units). Given the long-term character of the model, assessing the effects of land use in the production function is only possible if long-term data on land use, development and quality is available. While a data series on arable land is available at the FAO, and reaches back to 1960, it does not include any information on the quality

[10] This result is valid not only for the case of full-time schooling, but also for all $e_{t+1} > 0$.

[11] For $\lambda_t = 1$, g_λ is always positive, as $2z_t\left(\frac{N_t^2+N_t^3\frac{\lambda_{t-1}}{\lambda_t}}{N_t^2+N_t^3}\right) + \frac{1}{1} - 1 > 0$

of the land used, and no data are available for the years before 1960, nor are projections for the future. Assessing the quality of the arable land, however, is extremely important in Kenya, as it is highly inhomogeneous: more than 80% of the total land area are semi-arid or arid, and cannot be used for agriculture, while about 12% are humid, and therefore highly suitable for growing crops. Furthermore, the UN Population Division estimates[12] that the rural population, that is, that part of the workforce which needs land to produce output, will stay virtually constant after 1990 (growing by 0.6% per year after 1990, compared to nearly 5% p.a. for the urban population), so that per capita land use in the agricultural sector will also remain unchanged, even if the population grows. Therefore we choose a production function where labor is the only input. With (4), this yields the total output in period t:

$$\begin{aligned} Y_t &\equiv \alpha_t L_t \\ &= \alpha_t (N_t^2 \lambda_t + N_t^3 \lambda_{t-1} + N_t^4 \lambda_{t-2} + N_t^5 \lambda_{t-3} + N_t^6 \lambda_{t-4} + N_t^1 (1 - e_t)\mu), \end{aligned} \tag{5}$$

The factor $\alpha_t > 0$, which denotes the amount of output produced with one unit of efficient labor, describes the general level of economic productivity. It can change over time, for example, as a result of economic policy.

The only active decision-makers in the present setting are the (young) parents, and the decisions they make determine their level of consumption in the last phase of life, as well as the level of efficiency their offspring will attain as adults. For simplicity, assume that the parents' decisions do not influence mortality rates.

Assume that raising infants is free, so that the level of consumption of a family which has infants only is not influenced by their fertility decisions. Therefore, when making fertility decisions in period t, parents, who are in age groups $a = 2, 3$ consider their (expected) level of consumption while their children are going to school (denoted by $c_{1,\,t}$) and later,

[12] Source: Online Database at http://www.un.org/popin/data.html

when they themselves are old, whereby only consumption in the first period of old age is considered for simplicity (denoted by $c_{2,\,t+2}$). The adults also possess altruism, which expresses itself not only through the expenditures on educating and raising children, but also in their concern for the children's future welfare. For simplicity, it is further assumed that the utility function is additively separable, whereby the level of utility of adults who die prematurely is normalized to zero. Following Bell et al. (2004), we choose the form:

$$E_tU(c_{1,\,t}, c_{2,\,t+2}, e_t, N_t^1) = \beta_0 ln(c_{1,\,t}) + \beta_1 \kappa_t ln(c_{2,\,t+2}) + \frac{2N_t^1\kappa_{t+1}}{N_t^2 + N_t^3}\left(1 - \frac{\lambda_{t+1}(e_t)^{-\eta}}{\eta}\right) \quad (6)$$

where $\kappa_{t+k},\ k = 0, 1$ is the parents' subjective estimate thereof at time t.

Note that by choosing

$$\phi = \frac{2N_t^1\kappa_{t+1}}{N_t^2 + N_t^3}\left(1 - \frac{\lambda_{t+1}(e_t)^{-\eta}}{\eta}\right)$$

as the subutility function with respect to altruism and the education production function in (2), equilibria with $\lambda_t = 1$ and $e_t = 0\ \forall t$ (poverty traps) are excluded by construction if $\epsilon < 1$. To see this, differentiate the subutility function with respect to e_{t+1}, and evaluate this derivative for $e_t = 0$:

$$\lim_{e_t \to 0} \frac{\partial \phi}{\partial e_t} = \infty \ \ \forall \epsilon < 1, \ \ \forall \eta.$$

As will be shown in the following section, the model can only be calibrated by choosing $\epsilon < 1$, so that poverty traps are indeed excluded. The step in the calibration needed to determine ϵ is independent of the choice of functional form for preferences, so that the result $\epsilon < 1$ will be valid even if the utility function took a different form. Therefore, as long as the subutility function satisfies the condition $lim_{e_t \to 0} \frac{\partial \phi}{\partial e_t} = \lim_{e_t \to 0} \frac{\partial \phi}{\partial \lambda_{t+2}} \frac{\partial \lambda_{t+2}}{\partial e_t} = lim_{e_t \to 0} \frac{\partial \lambda_{t+1}}{\partial e_t} = \infty$, no poverty trap will exist. Stationary equilibria with respect to λ, however, can still exist if the parents' choice of schooling and fertility satisfies the condition $\lambda = 2z_t e_t(\lambda, N_t^2, N_t^3, \dots)^\epsilon \lambda + 1\ \forall t$. Note, however, that this also implies some kind of equilibrium with respect to population, so that $e_t(\lambda, N_t^2, N_t^3, \dots) = e_{t+1}(\lambda, N_{t+1}^2, N_{t+1}^3, \dots)\ \forall t$, which seems unlikely.

Consider a family that raises N_t^1 children in period t. Each pair of adults in the groups $a = 2,\ 3$ receives the same fraction of the family's total income:

$$\frac{Y_t}{(N_t^2+N_t^3)/2} = \frac{2\alpha_t(N_t^2\lambda_t + N_t^3\lambda_{t-1} + N_t^4\lambda_{t-2} + N_t^5\lambda_{t-3} + N_t^6\lambda_{t-4})}{N_t^2+N_t^3} + \frac{2N_t^1(1-e_t)\mu\alpha_t}{N_t^2+N_t^3}.$$

A fixed fraction $\chi \in [0,1]$ of the total output produced by adults is allocated to the grandparents ($a = 5,\ 6,\ 7$), who consume it. Assume that each child consumes $b\lambda_{t+1}$ units per decade, as in Chapters 1 and 2. Then the consumption of a couple in age groups $a = 2,\ 3$ in period t is:

$$\begin{aligned} c_{1,t} = {} & 2\alpha_t(1-\chi)\frac{(N_t^2\lambda_t + N_t^3\lambda_{t-1} + N_t^4\lambda_{t-2} + N_t^5\lambda_{t-3} + N_t^6\lambda_{t-4})}{N_t^2+N_t^3} \\ & + 2\alpha_t(1-e_t)\mu\frac{N_t^1}{N_t^2+N_t^3} \\ & - 2b\lambda_t\frac{N_t^1}{N_t^2+N_t^3}. \end{aligned} \tag{7}$$

The share χ is divided equally among all old members of the family:

$$c_{2,t+2} = 2\alpha_{t+2}\chi \cdot \frac{(N_{t+2}^2\lambda_{t+2} + N_{t+2}^3\lambda_{t+1} + N_{t+2}^4\lambda_t + N_{t+2}^5\lambda_{t-1} + N_{t+2}^6\lambda_{t-2})}{N_{t+2}^4 + N_{t+2}^5 + N_{t+2}^6 + N_{t+2}^7}. \tag{8}$$

When deciding about N_t^1 and e_t parents can observe all relevant historical and current values of N, λ and e, particularly λ_t and e_t, and the mortality rates in period t. They also form expectations about future mortality rates, and hence expectations about N_{t+k}^a for $a = 2, \ldots, 7$, and $k = 1, 2, 3, 4$.

Assume that parents have perfect foresight about all the $q_t^{i,j}$ in all future periods $t + 1, t+2, \ldots$. They can also observe or deduce the levels of efficiency λ_{t-2}, λ_{t-1} and λ_t associated with age groups $a = 5$ and $a = 6$ in period $t+2$. All the other values needed to determine $c_{2,\,t+2}$, namely, N_{t+2}^2 and N_{t+2}^3 as well as λ_{t+2} are unknown. The level of N_{t+2}^3

can be determined from N_t^1, which is endogenous, using $N_{t+2}^3 = (1 - q_t^{1,3})N_t^1$. Similarly, $N_{t+2}^2 = N_{t+1}^1(1 - q_{t+1}^{1,2})$. However, the level of N_{t+1}^1 is not known to the parents when they make their decisions. Therefore, parents must make conjectures about the future number of school-going children and about e_{t+1}, which will determine λ_{t+2}. Given the complexity of this structure, assume that parents use a simplifying rule: they expect the level of schooling to be stationary:

$$E_t e_{t+1} = e_t, \tag{9}$$

and the number of school-age children raised by couples in $a = 2$ and $a = 3$ in period $t+1$ to be stationary too:

$$\begin{aligned} & E_t\left(\frac{2N_{t+1}^1}{N_{t+1}^2 + N_{t+1}^3}\right) = \frac{2N_t^1}{N_t^2 + N_t^3} \\ \Leftrightarrow \quad & E_t N_{t+1}^1 = N_t^1 \cdot \frac{N_{t+1}^2 + N_{t+1}^3}{N_t^2 + N_t^3} \equiv N_t^1 \theta_t \end{aligned} \tag{10}$$

Note that N_{t+1}^2 appears in θ_t, which is endogenous, as $N_{t+1}^2 = N_t^1(1 - q_t^{1,2})$. For simplicity, however, the ratio θ_t will be computed from the population tables (e.g. table 3.1). In making these assumptions, parents regard both e_{t+1} and N_{t+1}^1 as given: that is, when determining N_t^1 and e_t, they will ignore the influences these have on N_{t+2}^2 and λ_{t+2}, respectively, under the above assumptions about stationarity.

Using (2), (7) and (8), the couple's expected utility can be rewritten as a function of N_t^1 and e_t as well as $E_t e_{t+1}$ and $E_t N_{t+1}^1$ alone. Its optimization problem at time t is then formulated as follows:

$$\max_{e_t, N_t^1} E_t U(e_t, N_t^1; E_t e_{t+1}, E_t N_{t+1}^1, \cdot) \quad \text{subject to} \quad N_{min} \leq N_t^1 \leq N_{max} \quad \text{and} \quad 0 \leq e_t \leq 1 \tag{11}$$

This yields two first-order conditions as functions of $E_t e_{t+1}$ and $E_t N_{t+1}^1$:

$$\frac{\partial E_t U(\cdot; E_t e_{t+1}, E_t N_{t+1}^1)}{\partial N_t^1} \quad \text{and} \quad \frac{\partial E_t U(\cdot; E_t e_{t+1}, E_t N_{t+1}^1)}{\partial e_t}.$$

Together with (9) and (10) these yield, for an interior solution,

$$\left.\frac{\partial E_t U(\cdot)}{\partial N_t^1}\right|_{E_t e_{t+1}=e_t, E_t N_{t+1}^1 = N_t^1 \theta_t} = 0 \quad \text{and} \quad \left.\frac{\partial E_t U(\cdot)}{\partial e_t}\right|_{E_t e_{t+1}=e_t, E_t N_{t+1}^1 = N_t^1 \theta_t} = 0, \tag{12}$$

which, in turn, yield the optimal levels of N_t^1 and e_t.

Note that in the present setup, there is no time-inconsistency regarding the parents' decisions about e_t, as their expectations about mortality rates are correct by assumption if there is no HIV/AIDS shock. That is, when their children reach school-going age, parents have no incentives to depart from the binding contract on e_t which they made at child-birth. If, however, the parents' expectations about mortality rates prove to be incorrect, that is, if there is an unexpected mortality shock like the outbreak of the HIV/AIDS epidemic after the children are born but before they reach school-going age, it is unlikely that the contractual level of e_t is still optimal, from the parents' point of view. Therefore, e_t might be open to renegotiation, an issue which will be addressed in section 3.5.3.

3.4 Calibration

The model presented in the previous section is calibrated to the data from section 3.2. The data on output, population and education can be used to derive the adult's level of efficiency λ_t in each past generation, the children's level of efficiency μ, the factors ϵ, z_t and α_t, which characterize the education and production functions, respectively, the social rules governing transfers to the old and to children, χ and b, and the parameters of the subutility functions, β_0, β_1 and η. This will be done in two steps: first, national aggregates will be used to compute $\epsilon, \mu, z_t, \alpha_t$ and λ_t. Second, the calibration of preferences will yield the values of β_0, β_1, b, χ and η.

Step 1

The first step of the calibration is identical to the one employed in Bell et al. (2004). As noted in section 3.2.3, only 3% of all adult Africans had enjoyed any education by 1925. Therefore, it seems safe to assume that in 1910, before the Department of Education was established, the general population had no education at all and their level of efficiency was $\lambda_1 = 1$, that is, the minimum possible. Following Bell et al. (2004), λ_2 is set to 1.01. Using this as an initial condition and equation (2) yields a set of 8 equations describing the dynamics of the adults' level of efficiency:

$$\lambda_t = 2z_{t-1}e_{t-1}^{\epsilon}\left(\frac{N_{t-1}^2\lambda_{t-1} + N_{t-1}^3\lambda_{t-2}}{N_{t-1}^2 + N_{t-1}^3}\right) + 1, \quad t = 3, \dots, 10 \tag{13}$$

While no data on e_{10} are available yet, it seems reasonable to assume $e_{10} = 0.621$ (see Bell et al. [2004]). The second condition used in the calibration gives the age groups' contribution to GDP, as stated in equation (5):

$$Y_t = \alpha_t(N_t^2\lambda_t + N_t^3\lambda_{t-1} + N_t^4\lambda_{t-2} + N_t^5\lambda_{t-3} + N_t^6\lambda_{t-4} + N_t^1(1-e_t)\mu), \quad t = 5, \dots, 10 \tag{14}$$

The system described by (13) and (14) consists of 14 equations and 24 unknowns: α_t for $t = 5, \dots, 10$, λ_t and z_t for $t = 3, \dots, 10$ as well as μ and ϵ, which are assumed to have stayed constant over time. As the system is underdetermined, solving it is only possible by making assumptions about the values of some of the variables.

Beginning with 1940, adults had enjoyed at least one year of schooling on average (see table 3.3), which suggests that the measures undertaken in the education sector in the early phase of the century had started to bear fruit. Therefore, the first shift in z_t is assumed to have taken place in 1940. The second shift came after Kenya abolished school fees and reformed the educational system, that is, for the decade starting 1980. While per capita GDP grew until 1990, it started to fall thereafter. Hence, we assume that α stayed constant until $t = 9$, and dropped once, in $t = 10$. With these assumptions, the total of 14 variables associated with z_t and α_t is reduced to 5:

$$z_2 = z_3, \quad z_4 = z_5 = z_6 = z_7, \quad z_8 = z_9 = z_{10}$$
$$\alpha_5 = \alpha_6 = \alpha_7 = \alpha_8 = \alpha_9, \quad \alpha_{10}$$

To anchor the system, it is still necessary to choose one more variable. To simplify the calculation, ϵ is chosen exogenously, as the equations are linear in all other parameters. Solutions where $\epsilon < 0.49$ or $\epsilon > 0.62$ are not considered, as they yield either negative values of one of the parameters, or $\mu > 1$, that is, the labor efficiency of a child is higher than that of an adult who did not enjoy any schooling. The exact value of ϵ chosen should reflect the parents' decisions regarding schooling. As these decisions are determined by their preferences, we now turn to their calibration.

Table 3.4: Households' choices of e_t and N_t^1

Year	1920	1930	1940	1950	1960	1970	1980	1990
e_t	0.047	0.080	0.134	0.201	0.367	0.458	0.520	0.570
N_t^1				1606	2317	3371	4951	7182
$2N_t^1/(N_t^2 + N_t^3)$				1.55	1.79	1.90	1.91	1.87

Source: Bell et al. (2004)

Step 2

Table 3.4 presents the households' decisions concerning education and fertility for the years 1950 to 1990. As can be seen, an interior solution existed during the whole period[13] and the two conditions in (12) can be used to recover the preference parameters. For any given value of ϵ and ensuing values of μ, λ_t, z_t and α_t, five parameters need to be determined through a calibration of the preferences, namely, η, χ, b, β_0 and β_1. For computational reasons, solving the system is only possible if one chooses η exogenously, using a grid search method. The other variables are calibrated using (12) for the years 1970 and 1990. A plethora of results exists for the different values of ϵ and η: from all the possible

[13] If the household decisions were corner solutions, this would imply that the social rule on N_{min} required each family to raise at least 1.55 children. As such a high value of N_{min} is rather restrictive, it seems reasonable to assume that 1.55 children per family, as raised in 1950, constitutes an interior solution.

results, one with $\beta_0/\beta_1 \approx 1$ and $c_{1,t=9}/c_{2,t=9} \approx 1$ is chosen. That is, it is assumed that families choose a fairly smooth path of consumption in the two periods of life, apart from the eventuality of premature death, which is captured by the term κ_{t+1} in (6). The results of the calibration are set out in table 3.5. If $\epsilon < 0.49$, the calibration yields $\mu > 1$, that is,

Table 3.5: Calibration Results

Exogenous:			
$\epsilon = 0.5$	$\eta = 1$		
Endogenous:			
$\mu = 0.962$	$\chi = 0.237$	$b = 267.32$	
$\beta_0 = 4.847$	$\beta_1 = 3.776$		
t	λ_t	z_t	α_t
2	1.01	1.3724	–
3	1.5991	1.3724	–
4	2.0442	0.9425	–
5	2.2759	0.9425	585
6	2.8436	0.9425	585
7	3.9729	0.9425	585
8	5.5084	0.4938	585
9	4.4967	0.4938	585
10	4.6451	0.4938	481

a child's level of labor efficiency is higher than that of an adult who enjoyed no schooling whatsoever, which is not realistic. If $\epsilon > 0.51$ or $\eta \notin [0.99, 1.02]$, there are no results with $\beta_0/\beta_1 \approx 1$ and $c_{1,t=9}/c_{2,t=9} \approx 1$. Therefore, the calibration only yields desirable results if $\eta \in [0.99, 1.01]$ and $\epsilon \in [0.49, 0.51]$, and we choose the midpoints $\eta = 1$ and $\epsilon = 0.5$.

The ratio $\beta_0/\beta_1 \approx 0.8$ implies a rate of pure impatience to consume, independently o premature mortality, of about 1.2% per annum. Nearly one fourth of the adults' total income in any given period is transferred to the old ($\chi = 0.237$), a ratio which corresponds roughly to the grandparents' share of the population $((N_t^4 + N_t^5 + N_t^6 + N_t^7)/\sum_{a=0}^{7} N_t^a \approx 0.2$ for $t \leq 9$). A child's level of labor efficiency lies slightly below the level of human capital of an adult who did not receive schooling, $\mu = 0.962$, by way of comparison, the adults' level of efficiency when the epidemic broke out in 1990 was nearly five times higher.

There were two sharp falls in the efficiency associated with the educational technology, one in the 1930s and another in the 1970s, after the Kenyan government had started to reform the school system yet again. The transmission factor z_t fell by nearly 50% during the 1970s, and settled at a value just below 0.5, which implies the existence of a steady state in the adult level of efficiency with full-time schooling and $\lambda^* = 1/(1-2z) \approx 81$. With z_t sharply reduced, the growth rate of λ slows too; indeed, the young adults' level of efficiency falls between 1980 and 1990. The economy experiences another shock over the period 1990 – 2000, as α_t falls by 17%. This shock has two effects: first, labor income is reduced. Second, raising children becomes relatively more expensive. Recalling (7), note that the costs incurred in raising children depend on b, λ_{t+1} and N^1_{t+1} only,[14] and not on α_{t+1}, which was constant during the years used in the calibration of b. As a consequence, the ratio b/α_t was constant too for $t = 7$ and $t = 9$, with $b/\alpha_t = 0.46$. By 2000, however, the ratio rises to $b/\alpha_t = 0.55$ as α falls, so that raising children is now relatively more expensive by about 20%.

To summarize, the calibration is done in two steps: the first determines the parameters of the technologies and historical values of λ, while the second deals with the preference parameters. To derive the parameter values we have imposed two sets of restrictions: first $\mu \leq 1$, $\beta_0/\beta_1 \approx 1$, with $\beta_0 \geq \beta_1$, and $c_{1,t=9}/c_{2,t=9} \approx 1$, which have economic reasons and are independent of the data employed or the country/problem analyzed. The second set of restrictions imposed concerns z_t and α_t. These restrictions are specific to the data set used, and hence to the country/historical experience it reflects. Changing any of the restrictions will lead to significant changes in the calibration results, if any can be derived at all. Koukoumelis (2005) shows that the calibration will, in general, be sensitive to even small measurement errors in Y_5, the 1950 GDP level. He also shows that the degree of sensitivity to errors in Y_5 can be reduced by choosing a different set of restrictions regard-

[14]If the costs incurred in raising children were a function of α, the factor describing labor productivity would drop out of the utility function, as this is logarithmic in form. As a consequence, all decisions on children would be independent of α, which does not seem realistic.

ing z_t. In the present paper, we have kept the original restriction in Bell et al. (2004) in order to ensure that the calibrations are comparable.

3.5 Projections: The Base Case

3.5.1 Preliminaries

Using the results of the calibration presented in the previous section, the household decisions regarding completed fertility and education are determined under several scenarios. First, there is the benchmark case without HIV/AIDS, which is the counterfactual. Second, there is the benchmark case in which the HIV/AIDS epidemic breaks out, and families fully recognize its effects at the very outset in the 1980s. Third, there is the case in which parents fail to take notice of its effects until 1990. In all cases, the projections start with N_9^1 and e_9, that is, the first decisions are made in period $t = 8$ and implemented in period $t = 9$.

Data

To compute the projections, data on mortality rates for 2000 onwards are needed. For both scenarios, revised projections based on those made by the US Bureau of Census are used, following Bell et al. (2004). The population pyramids for both cases are given in tables 3.6 and 3.7. The implicit survival rates κ_t and mortality rates $(1 - \kappa_t)$ are set out in table 3.8. Survival rates for the second age group (age 15 to 24) are lower by up to 30 percentage points if the epidemic breaks out, and mortality rates are at least 1.9 times higher in the case of AIDS. At the peak of the epidemic, mortality rates are more than 250% higher than their respective values in the case without AIDS during four consecutive decades.

N_t^1 is derived endogenously in the present model, in contrast to the projections in tables 3.6 and 3.7. Yet the implicit survival probabilities underlying these tables are used to compute the number of adults in each age group in future periods, and therefore new

Table 3.6: Projections: Population Tables without AIDS

	1990	2000	2010	2020	2030	2040	2050	2060	2070
0-4	4458	4696	4602	4503	4537	4398	4336	4336	4336
5-14	7182	9006	9550	8995	8965	8968	8715	8715	8715
15-24	4715	6875	8666	9236	8744	8759	8807	8601	8644
25-34	2979	4447	6526	8277	8878	8458	8525	8624	8475
35-44	1833	2731	4116	6098	7809	8454	8129	8269	8442
45-54	1099	1634	2463	3755	5628	7290	7982	7761	7982
55-64	698	911	1379	2115	3280	4998	6581	7323	7234
65+	511	864	1294	1969	3131	5266	8395	8395	8395

Source: Bell et al. (2004)

Table 3.7: Projections: Population Tables with AIDS

	1990	2000	2010	2020	2030	2040	2050	2060	2070
0-4	4458	4556	3874	3436	3238	3038	2972	2972	2972
5-14	7182	8612	8416	7197	6578	6258	5971	5971	5971
15-24	4715	6839	8209	8040	6945	6429	6186	5902	5902
25-34	2979	4187	5780	6912	6997	6360	6182	5948	5676
35-44	1833	2410	3052	4136	5266	5873	5878	5713	5498
45-54	1099	1487	1760	2173	3125	4372	5352	5357	5207
55-64	698	906	1149	1361	1746	2666	3956	4843	4847
65+	511	821	1087	1362	1693	2353	3657	3657	3657

Source: Bell et al. (2004)

Table 3.8: Survival rates κ_t and mortality rates $(1-\kappa_t)$ in the benchmark cases

		1990	2000	2010	2020	2030	2040	2050
NO AIDS	κ_t	0.873	0.887	0.901	0.915	0.930	0.944	0.959
AIDS	κ_t	0.647	0.605	0.641	0.730	0.846	0.889	0.889
NO AIDS	$1-\kappa_t$	0.127	0.113	0.099	0.085	0.070	0.056	0.041
AIDS	$1-\kappa_t$	0.353	0.395	0.359	0.270	0.154	0.111	0.111

age pyramids:

$$N_{t+i}^{1+i} = N_t^1 \cdot (1 - q_t^{1,1+i}), \quad i \geq 1 \tag{15}$$

where $q_t^{1,i}$ is the probability that a member of age group 1 in period t will not survive to become a member of age group $1+i$ in period $t+i$. Equation (15) enables the calculation of the diagonals of the population table, namely, values for $N_{t+1}^2, N_{t+2}^3, N_{t+3}^4, \ldots$, starting with N_t^1, which is endogenous. Those values of the age pyramid which cannot be computed using (15), for example N_9^2 or N_{11}^4, are taken from table 3.7.

One last step is necessary before turning to the projections, namely, the determination of N_{min}^1 and N_{max}^1. As the number of school-going children depends on the number of families, one must define N_{min}^1 and N_{max}^1 relative to family size. We choose N_{min}^1 so as to allow stationarity with respect to population size. As some children and adults always die prematurely, even without an AIDS epidemic, the replacement fertility rate will lie somewhat above 1. In the light of these data, N_{min}^1 is set to

$$N_{min}^1 = 1.05 \cdot \frac{N_t^2 + N_t^3}{2} \;\; \forall t$$

in every period t. Variations in N_{min}^1 will be discussed in section 3.6.1. It turns out that the value of N_{max}^1 plays a minor role in all but one variation. Historically, the highest number of children raised ($a = 1$) per family according to the above definition was 1.91 during the 1980s. Therefore, we choose N_{max}^1 slightly higher, so that $2N_t^1/(N_t^2 + N_t^3) \leq 1.95$ or

$$N_{max}^1 = 1.95 \cdot \frac{N_t^2 + N_t^3}{2} \;\; \forall t.$$

This implies that, in a steady state, the total population can double within about 20 years if all families choose $N_t^1 = N_{max}^1$. We also assume that the number of children cannot drop from N_{max}^1 to N_{min}^1 within one period. If the family's optimum yields $2N_t^1/(N_t^2 + N_t^3) > (1.95 + 1.05)/2 = 1.5$ in some period t, then the minimum number of children in the following period, $t+1$, is set to $N_{min}^1 = [1.05/2 + N_t^1/(N_t^2 + N_t^3)] \cdot (N_{t+1}^2 + N_{t+1}^3)/2$ instead of $N^1_{min} = 1.05(N^2_{t+1} + N^3_{t+1})/2$. Only in period $t + 2$ does N^1_{min} take the restricted value of $N^1_{min} = 1.05(N^2_{t+2} + N^3_{t+2})/2$. Consider, for example, $t = 9$, where $2N^1_9/(N^2_9 + N^3_9) = 1.87 > 1.5$. This yields, in period $t = 10 : N^1_{min} = [(1.05 + 1.87)/2] \cdot (N^2_{10} + N^3_{10})/2 = 1.46 \cdot (N^2_{10} + N^3_{10})/2$ instead of $N^1_{min} = 1.05 \cdot (N^2_{10} + N^3_{10})/2$, that is, N^1_{min} is higher by nearly 40%.

Procedure

For each year, the solution to the household maximization problem, that is, the optimal values of N^1_t and e_t, is computed. Both interior solutions, i.e. solutions to (12), and a total of 8 corner solutions are calculated in order to find the optimum, including interior solutions with respect to only one variable, that is, solutions in which N^1_t is at the corner, while e_t solves the corresponding first-order condition in (12), and vice versa. λ_{t+1} and the relevant values of the age pyramid are calculated using (2) and (15), and these values are then used to determine the household optima for the following periods. The projections are limited to 2040 by the need to know $\kappa_{t+1} = N^4_{t+3}/N^2_{t+1}$, as the last value of N^4_{t+3} available is that for 2070.

3.5.2 The Benchmark Cases

The results for the first benchmark case, that is, the scenario without an HIV/AIDS epidemic, are set out in table 3.9. Table 3.10 states the results of the second benchmark case, in which it is assumed that when the HIV/AIDS epidemic breaks out during the 1980s families foresee all its effects ('perfect foresight') and react to them immediately. This assumption does not seem to be very realistic, particularly in the light of African countries' late reaction to HIV/AIDS. Therefore this case will be considered only in this section, as an example of what could have happened if families had perfectly foreseen the epidemic's effects. Like the NO AIDS case, therefore, the AIDS benchmark case is a thought experiment.

The variable y_t denotes GDP per capita, and is computed from the last two columns. Total population, measured in 1000s, is stated in the last column. With $N^1_{min} = 0.525(N^2_t + N^3_t)$,

Table 3.9: Benchmark Case I: NO AIDS

year t	λ_t	N^1_t	$\frac{N^1_t}{(N^2_t+N^3_t)/2}$	e_t	y_t	$Y_t(10^7)$	Pop.
9	4.50	7182	1.87	0.57	1310	3076	23475
10	4.65	8256	1.46	0.65	1263	3792	30023
11	4.67	8211	1.13	0.96	1411	5064	35890
12	5.51	9119	1.17	1.00	1517	6548	43151
13	6.04	11708	1.42	1.00	1533	8177	53329
14	6.72	14002	1.40	1.00	1650	10739	65099

Table 3.10: Benchmark Case I: AIDS, perfect foresight

year t	λ_t	N^1_t	$\frac{N^1_t}{(N^2_t+N^3_t)/2}$	e_t	y_t	$Y_t(10^7)$	Pop.
9	4.50	5684	1.48	0.52	1451	3054	21047
10	4.49	5040	1.05	0.65	1397	3204	22929
11	4.59	4924	1.05	0.85	1453	3430	23616
12	5.14	4593	1.05	1.00	1517	3597	23703
13	5.83	4764	1.12	1.00	1563	3850	24629
14	6.43	5264	1.21	1.00	1680	4452	26501

a value of 1.05 in the fourth column of the tables indicates a corner solution with respect to N^1_t. Similarly, a value of 1.46 for $t = 10$ also denotes a corner solution, as described in section 3.5.1. In the first benchmark case, parents reduce fertility during the 1990s sharply, from 1.87 children per family to 1.46 children a decade later, as the labor productivity parameter α_t drops by 20%, from its 1990 value of 585 to 481 in 2000. As a consequence, raising children is now relatively more expensive, the value of b remaining unchanged, and parents must reduce N^1_t in order to be able to finance child-raising.[15] By 2020, all children start enjoying full-time schooling, and parents find it worthwhile to invest in having more children once again. As a consequence, N^1_{13} and N^1_{14} both lie above N^1_{min}, while $e_{13} = e_{14} = 1.0$.

The HIV/AIDS epidemic does not (measurably) change the adult population structure

[15]Note that this result would not hold if the child-raising costs were defined as $N^1_t b\lambda_t \cdot \alpha_t$ rather than $N^1_t b\lambda_t$.

for 1990, the first year of interest, as can be seen from tables 3.6 and 3.7. It does, however, change the parents' expectations, concerning both future mortality rates and population pyramids. Fewer of them will survive to old age, so that consumption in that phase of life becomes effectively less important than consumption while they are young, as the weight attached to $c_{2,t+2}$, $\beta_1\kappa_t$, falls. The interior solution is $N_9^1 = 3692$ and $e_9 = 0.74$, that is, each family would like to raise only about one child, which would be better-educated than in the NO AIDS case. However, due to the social rule on N_{min}^1, choosing $N_9^1 = 3962 < N_{min}^1 = 5684$ is not an option. As a consequence, parents decide to have the minimum number of children they can have in 1990, that is, $N_9^1 = 5684$, a drop of 21% compared to the NO AIDS case. As raising these children is costly, the offspring have to work, so that parents also reduce the level of schooling these children are to enjoy, from 0.57 in the NO AIDS case, to 0.52. Note that if N_{min}^1 were higher, children would have to work even more; if, however, N_{min}^1 were lower, it could also happen that children would enjoy more schooling in the AIDS case than in the NO AIDS case – this issue will be addressed in section 3.6. Total GDP in 1990 is slightly lower in the NO AIDS case, mostly due to the reduction in the number of school-going children, which is not fully offset by the increase in child labor. After 1990, the level of schooling grows, albeit slowly. Child labor disappears in the both cases by 2020. In the first benchmark case, GDP grows, on average, by 2.5% annually between 1990 and 2040, and by only a third of that rate, (0.8% annually) in the second benchmark case. Total GDP in the NO AIDS case is more than double its AIDS case value for 2040. Population is always lower with the epidemic, by almost 60% in 2040. It grows by 2% annually on average in the NO AIDS case over the period 1990 – 2040, and by 0.5% in the AIDS case. Compared to the US Bureau's projections in tables 3.6 and 3.7, total population in 2040 as predicted by the present model is higher by 15% in the NO AIDS case and lower by 30% in the AIDS case. Note that population and GDP are virtually constant in the AIDS benchmark case after 2010.

Per capita GDP falls between 1990 and 2000 in the NO AIDS benchmark case, due to the drop in α_t in 2000. After this initial reduction of about 3.6%, per capita GDP grows in all decades thereafter; however, the growth rate is very low – only 0.5% annually, on average. In the AIDS case, y_t is higher in 1990 compared to the NO AIDS case, by about 11%. This is due to the fact that parents have fewer children, and therefore the dependent population is lower, while GDP remains virtually unchanged. Per capita GDP falls between 1990 and 2000, and the drop is as large as in the NO AIDS case. y_t grows slowly after 2010, and, by 2020, GDP per capita in the AIDS case is no greater than its level in the NO AIDS case. It is higher, however, in all following periods. This stems from three facts: first, the number of children raised by each family is lower in the AIDS case, which reduces the dependency ratio. Second an adult's level of efficiency and therefore GDP produced per adult are only slightly lower in the AIDS case. Third, children enjoy full-time schooling in both cases and do not contribute to GDP.

3.5.3 Scenarios: Household Behavior when expectations are revised with a delay

In the previous section, it was assumed that parents foresaw the effects of the HIV/AIDS epidemic in the 1980s, and could adapt their decisions for 1990 accordingly. Given the evidence from most African countries, this assumption does not seem realistic. The Kenyan government, in particular, reacted to the epidemic very late, declaring it to be a national disaster only in 1999. It is clear that households had not anticipated the epidemic's effects in the 1980s, more than a decade earlier than their government was prepared to acknowledge its gravity; for the optimal choice of $N_9^1 = 5972$ in the case of perfect foresight departs from the observed value of $N_9^1 = 7182$ for that period. As the population pyramids for the 1990s in the AIDS and NO AIDS cases – as estimated by Bell et al. (2004) – are virtually identical, and as families apparently did not react to the growing epidemic in the 1980s, the number of school-going children, $N_9^{1,NO\ AIDS}$ and $N_9^{1,AIDS}$, will be the same.

Assume therefore that on January, 1st, 1990, parents suddenly realized that an epidemic had broken out. Several scenarios are possible, in which two groups of adults play an active role. The *first group* consists of those individuals whose children are to go to school during the 1990s. This group had decided on N_9^1 and e_9 during the 1980s (when the children were born), and is now confronted with a change in expectations regarding future mortality rates – both their own and their children's. The *second group* consists of those young adults who will decide about N_{10}^1 and e_{10} during the 1990s, and who would normally be the sole decision-makers in that decade. Their decision is influenced by, among other things, the level of N_9^1, which is fixed, and e_9, which might be open to renegotiation among members of the groups $a = 2$ and $a = 3$. Three scenarios will be discussed. The first is the scenario termed 'binding contract', in which it is assumed that the contract about e_9 is binding, and therefore its value remains unchanged. Second, there is the scenario in which the *first group* of adults revise their former decision about e_9, and choose a new level of schooling in the light of changed expectations. Third, there is a scenario in which the *second group* decides not only about N_{10}^1 and e_{10}, but also about e_9. Note that it is also possible that the two groups of individuals negotiate about the level of e_9, whereby the result will lie somewhere between those in the second and third scenarios, depending on the negotiation skills of the two groups and their respective sizes. If there is no HIV/AIDS shock, the results of the benchmark cases and all scenarios but the third will be identical.

Scenario 1: The Binding contract

In this scenario, the level of e_9 remains unchanged: during the 1990s, children enjoy the same level of schooling as if the HIV/AIDS epidemic had not occurred, so that $e_9 = 0.57$. The second group of adults, however, are aware of the force of the epidemic when deciding about N_{10}^1 and e_{10}. The results are set out in table 3.11.

Note that there is no difference between the NO AIDS and the AIDS cases for $t = 9$, due to the binding contract concerning e_9. As N_9^1 is higher in this scenario than in the case of perfect foresight, the number of families which can raise children in 2000 is higher, and therefore total population for that and all following periods is higher. Consequently, total GDP is also higher, and the gap between the two AIDS scenarios widens. GDP per

Table 3.11: Scenario 1: AIDS, e_9 subject to a binding contract

year t	λ_t	N_t^1	$\frac{N_t^1}{(N_t^2+N_t^3)/2}$	e_t	y_t	$Y_t(10^7)$	Pop.
9	4.50	7182	1.87	0.57	1310	3076	23475
10	4.65	8040	1.46	0.45	1273	3686	28944
11	4.05	7058	1.05	0.69	1375	4233	30799
12	4.54	6928	1.05	0.85	1397	4535	32464
13	4.93	7632	1.22	1.00	1384	4903	35423
14	5.69	8040	1.18	1.00	1508	5963	39551

capita, however, is lower in table 3.11 in all years following 2000, albeit the difference is rather small – at most 10% (in 2040). Note that in the case of a binding contract on e_9, the young adults (age group $a = 2$) in 2000 are more numerous and better educated than in the AIDS benchmark case. Individuals who make their decisions about N_{10}^1 and e_{10} during the 1990s are aware of the fact that, by the time they reach old age, their consumption will be provided by the young adults who enjoyed schooling during the 1990s – and these individuals will produce and earn more than their counterparts in the AIDS benchmark case. As a consequence, with their old-age consumption better secured, the decision makers for the 2000s will try to increase their consumption while young. As they cannot further reduce the number of children they intend to raise below N_{min}^1, they will also reduce schooling. For 2000, they choose $e_{10} = 0.45$, which is lower than their choice in the AIDS benchmark case, $e_{10} = 0.65$ (see table 3.10). While e_9 is higher in the 'binding contract' case, the difference between the levels of schooling in the two cases in 2000 is so large, that the young adults' level of efficiency will be lower in all periods following 2000 and per capita GDP is also correspondingly lower. Therefore, from an aggregate point of view (i.e. GDP, population), the economy will be larger if families react to the AIDS epidemic only with a delay, in 2000. Where $\lambda_t$is concerned,, however, an early reaction is better over the long run.

Scenario 2: e_9 revised by the *first group*

Assume that the parents who had their children during the 1980s, and so have N_9^1 school-going children during the 1990s, reconsider their earlier decisions about schooling as they realize the epidemic's impact on mortality rates. In doing so, they find a level of e_9 which solves the first-order condition associated with education in (12), thereby taking N_9^1 as given. Individuals deciding about N_{10}^1 and e_{10} observe the new choice of e_9, and make their decisions accordingly. The results are found in table 3.12.

Table 3.12: Scenario 2: AIDS, e_9 revised by the *first group*

year t	λ_t	N_t^1	$\frac{N_t^1}{(N_t^2+N_t^3)/2}$	e_t	y_t	$Y_t(10^7)$	Pop.
9	4.50	7182	1.87	0.42	1336	3135	23475
10	4.14	8040	1.46	0.45	1217	3521	28944
11	3.82	7058	1.05	0.64	1307	4025	30799
12	4.12	7912	1.20	0.71	1263	4283	33917
13	4.32	9631	1.43	0.76	1210	4762	39355
14	4.64	10293	1.25	0.88	1284	5869	45721

Upon suddenly realizing that both their own and their children's likelihood of surviving to old age is much lower, the adults strongly reduce the level of schooling their children are actually to receive in the 1990s, to $e_9 = 0.42$, compared with $e_9 = 0.52$ if they had realized the full extent of the disease a decade earlier, as in the AIDS benchmark. As children work more, total GDP is higher in 1990 and the level of efficiency of young adults (age group $a = 2$) in 2000 is lower than in all other benchmark cases and scenarios. As each family has already chosen the minimum allowable number of children, it will now choose less schooling for its children as a consequence of the low level of efficiency in all periods after 1990. Per capita GDP in all years but 1990 is the lowest among all cases presented: in 2040, y_t is higher by 22% in the NO AIDS case, by 26% in the case of AIDS with perfect foresight and by 15% in the case of a binding contract about e_9. Total population is higher than in all other AIDS cases: with the adults' level of efficiency being very low, investing in education does not pay off, so that parents increase e_t only slowly after 2020, and increase N_t^1 instead. Compared to the benchmark AIDS case, total population is now higher, as N_9^1, and therefore the number of young adults who can bear and raise children is higher in all future periods. With a greater population, total GDP also lies above the respective values in the other AIDS cases, the difference amounting to 34% compared to the AIDS benchmark case. Note that although the total poplation in 2040 is 83% higher when the first group of adults revises e_9 compared to the binding contract scenario, the GDP values are not as far apart, as the adults' levels of efficiency are correspondingly lower, by nearly 30%. As a consequence, GDP per capita is also much lower, despite the fact that children work more.

Scenario 3: e_9 revised by the *second group*

In the previous scenario, the parents of school-going children revised their choice of e_9 after they suddenly learned about the HIV/AIDS epidemic. Normally, however, the decision-making process in any decade is left to the very young adults in age group $a = 2$. After realizing that the HIV/AIDS epidemic will affect future levels of mortality from January 1st, 1990 onwards, the decision about e_9 is subject to reconsideration. In the third scenario, it is the very young adults who decide about e_9, and not the parents of the children who are about to enjoy e_9. Therefore, the maximization problem in period 9 is revised to read:

$$\max_{e_9, e_{10}, N_{10}^1} \left[\beta_0 ln(c_{1,\,10}) + \beta_1 \kappa_{10} ln(c_{2,\,12}) + \frac{2N_{10}^1 \kappa_{11}}{N_{10}^2 + N_{10}^3} \left(1 - \frac{\lambda_{11}(e_{10})^{-\eta}}{\eta} \right) \right] \tag{16}$$

with

$$\begin{aligned} c_{1,10} = \quad & 2\alpha_{10}(1-\chi)\frac{(N_{10}^2\lambda_{10}(e_9) + N_{10}^3\lambda_9 + N_{10}^4\lambda_8 + N_{10}^5\lambda_7 + N_{10}^6\lambda_6)}{N_{10}^2 + N_{10}^3 + N_{10}^4} \\ + \quad & 2\alpha_{10}(1-e_{10})\mu\frac{N_{10}^1}{N_{10}^2 + N_{10}^3 + N_{10}^4} \\ - \quad & 2b\lambda_{10}(e_9)\frac{N_{10}^1}{N_{10}^2 + N_{10}^3 + N_{10}^4} \end{aligned}$$

$$c_{2,12} = 2\alpha_{12}\chi\frac{(N_{12}^2\lambda_{12}(E_t e_{11}) + N_{12}^3\lambda_{11}(e_{10}) + N_{12}^4\lambda_{10}(e_9) + N_{12}^5\lambda_9 + N_{12}^6\lambda_8)}{N_{12}^5 + N_{12}^6 + N_{12}^7}$$

and

$$\lambda_{10}(e_9) = 2z_9 e_9^{\epsilon}\left(\frac{N_9^2\lambda_9 + N_9^3\lambda_8}{N_9^2 + N_9^3}\right) + 1$$
$$\lambda_{11}(e_{10}) = 2z_{10} e_{10}^{\epsilon}\left(\frac{N_{10}^2\lambda_{10}(e_9) + N_{10}^3\lambda_9}{N_{10}^2 + N_{10}^3}\right) + 1$$

Note that e_9 appears only in $\lambda_{10}(e_9)$, as young adults in age group $a = 2$ do not receive income from child labor and pay the costs of raising the children of school-going age not in period $t = 9$, but rather in period $t = 10$. The results are set out in table 3.13.

Table 3.13: Scenario 3: e_9 revised by the *second group*

year t	λ_t	N_t^1	$\frac{N_t^1}{(N_t^2+N_t^3)/2}$	e_t	y_t	$Y_t(10^7)$	Pop.
9	4.50	7182	1.87	1.00	1236	2902	23475
10	5.83	8040	1.46	0.47	1406	4069	28944
11	4.60	7058	1.05	0.83	1533	4721	30799
12	5.60	6928	1.05	1.00	1613	5238	32464
13	6.05	6591	1.05	1.00	1734	5871	33869
14	6.77	8193	1.30	1.00	1770	6860	38760

It turns out that the first-order condition of the parents' maximization problem with respect to e_9 involves the corner solution $e_9 = 1$, whatever values e_{10} and N_{10}^1 take. Similarly, the first-order condition with respect to N_{10}^1 when $e_9 = 1$ and $e_{10} \in [0, 1]$, involves $N_{10}^1 = N_{min}^1$. The optimal value of e_{10} can then be derived using the remaining first-order condition. It turns out to be scarcely larger than in scenarios 1 and 2.

In the decades following 2000, the level of schooling children enjoy is typically higher than in both the other AIDS scenarios; for the level of adults' efficiency is higher in 2000, so that parents invest more in education. As a related consequence, they also have fewer children, so that the total population is lower than in the first and second scenario. It is, however, higher than in the AIDS benchmark with perfect foresight, as N_9^1 is higher than in that case. The effect of the HIV/AIDS epidemic on λ_t is lowest in the third scenario – indeed, it is almost 1% higher in 2040 than its counterpart in the NO AIDS case. In 2000, per capita GDP is 13% higher than in the NO AIDS case, as children work more, while λ_{10} is also higher. By 2020, children enjoy full-time schooling in both in the third scenario and the NO AIDS benchmark case. However, families in the former have fewer children, while their level of efficiency – and therefore income – is nearly unchanged, so that per capita GDP is somewhat higher.

While it seems likely that adults will revise their decisions when confronted by a shock they did not foresee, it is hardly to be expected that parents who are in their thirties will allow younger adults to decide on their children's education. The third scenario, therefore, will not be discussed in the context of the variations and public spending on health in the subsequent sections.

3.6 Variations: Parameters

To test how sensitive the results derived above are to the assumptions made, some variations in the latter will now be presented. The variations will concern the choice of N^1_{min}, and alternative paths for $\{\alpha_t\}_{t=9,10,...}$ and $\{z_t\}_{t=9,10,...}$. In all variations, it will be assumed that parents perfectly foresee all future changes in parameters, even if they do not immediately foresee the course of the HIV/AIDS epidemic. Three cases will be discussed for each variation; first, a benchmark NO AIDS case; second, the 'binding contract' case; and third, the scenario, in which the parents reconsider their earlier choice of e_9. The case of perfect foresight and the third scenario will not be discussed, as the decision mechanisms and assumptions that underlie them do not seem realistic.

3.6.1 Variation 1: The level of N^1_{min}

The first major assumption concerned the minimum number of children parents can choose to have, which was set by the condition $N^1_{min} = 1.05 \cdot (N^2_t + N^3_t)/2$. As a variation, N^1_{min} is increased slightly, to $N^1_{min} = 1.1(N^2_t + N^3_t)/2$. The results can be found in table 3.14. In the NO AIDS case, parents now raise more children than in the benchmark case in all years but 1990, but they invest less in schooling; so that the level of efficiency of young

Table 3.14: Variation 1: The level of N^1_{min}

NO AIDS							
year t	λ_t	N^1_t	$\frac{N^1_t}{(N^2_t+N^3_t)/2}$	e_t	y_t	$Y_t(10^7)$	Pop.
9	4.50	7182	1.87	0.57	1310	3076	23475
10	4.65	8398	1.48	0.64	1256	3799	30239
11	4.64	8390	1.15	0.95	1402	5089	36292
12	5.46	9270	1.17	1.00	1509	6590	43680
13	6.00	11847	1.41	1.00	1527	8241	53975
14	6.68	14226	1.40	1.00	1640	10818	65984
AIDS, scenario 1: e_9 subject to a binding contract							
year t	λ_t	N^1_t	$\frac{N^1_t}{(N^2_t+N^3_t)/2}$	e_t	y_t	$Y_t(10^7)$	Pop.
9	4.50	7182	1.87	0.57	1310	3076	23475
10	4.65	8178	1.48	0.45	1266	3692	29155
11	4.03	7467	1.10	0.66	1353	4266	31527
12	4.45	7533	1.10	0.80	1366	4626	33859
13	4.75	8375	1.24	0.94	1348	5059	37540
14	5.43	8736	1.18	1.00	1464	6177	42201
AIDS, scenario 2							
year t	λ_t	N^1_t	$\frac{N^1_t}{(N^2_t+N^3_t)/2}$	e_t	y_t	$Y_t(10^7)$	Pop.
9	4.50	7182	1.87	0.42	1336	3135	23475
10	4.14	8178	1.48	0.44	1210	3528	29155
11	3.80	7467	1.10	0.61	1287	4057	31527
12	4.04	8454	1.23	0.68	1240	4369	35219
13	4.20	10538	1.47	0.71	1179	4912	41656
14	4.43	11859	1.33	0.79	1230	6124	49768

adults is somewhat lower in all periods, as is per capita GDP. However, with the parent's level of efficiency being lower by the time children start going to school full-time, that is, by 2020, families choose to have slightly fewer children than in the case where the $N^1_{min}-$ factor is 1.05. Consequently, total population is higher in all periods from 2000 onwards, but the difference is small, reaching 1.3% in 2040. The difference in GDP (per capita and total alike) is even smaller, amounting to less than 1%.

In the first scenario, that is, the case in which e_9 is subject to a binding contract, GDP and population are higher than in the reference case (Table 3.11), as parents have more children from 2000 onwards. They invest less in education, so that e_t, λ_t and per capita GDP lie below their respective reference values. The effects of the epidemic on λ_t and y_t are stronger than in the reference case, as parents can less afford to educate their more numerous children. The effects on GDP, however, are slightly weaker.

In the second scenario, wherein parents reconsider their earlier decisions about e_9 when learning about HIV/AIDS, the new value for e_9 chosen remains unchanged, being independent of N^1_{min}. Compared to the reference case, parents now have more, but worse-educated children (see tables 3.12 and 3.14), and per capita GDP is lower. As in the first scenario, the impact of the epidemic on λ_t and y_t is stronger than in the case where the N^1_{min}–factor is 1.05, while its impact on GDP and population is weaker.

3.6.2 Variation 2: The level of α_t

The labor productivity factor α_t dropped by more than 17% in the decade following 1990, from 585 to 481, and this reduction is the main reason for the households' decision to reduce fertility from 2000 onwards, even in the absence of the disease – see section 3.5.2. Assume that α_t recovers after the sharp drop in the 1990s, to some value between α_{10} and α_9. To be precise, let productivity take the value $\alpha_t = 0.5(\alpha_9 + \alpha_{10}) = 533$ after 2000. As in the previous section, it will be assumed that parents foresee changes in α_t perfectly. The results are set out in table 3.15.

In the NO AIDS case in 2000, parents would like most to have very few children (as raising them is relatively costly when α_t is low) but to educate these children very well, as future pay-offs from schooling are higher, with α_t expected to rise after 2000. However, due to the social rule on N^1_{min} each family in 2000 must raise at least 1.46 children, and therefore cannot afford to educate them above $e_{10} = 0.65$. Thus, the changes in α_t do not lead to an increase in either N^1_{10} or e_{10}, as one might expect. By 2010, however, when α_t has recovered substantially, the relative costs incurred in raising children are lower, and parents have more children than in the case of a low level of α_t for $t > 10$. Note, however, that with α_t higher, the same level of income can be attained for a lower level of efficiency, so that parents choose a lower level of e_t than in the benchmark case. As a consequence, per capita GDP is lower, while population and total GDP are higher (compare tables 3.9 and 3.15).

Table 3.15: Variation 2: Recovery of α_t after 2000

NO AIDS							
year t	λ_t	N_t^1	$\frac{N_t^1}{(N_t^2+N_t^3)/2}$	e_t	y_t	$Y_t(10^7)$	Pop.
9	4.50	7182	1.87	0.57	1310	3076	23475
10	4.65	8256	1.46	0.65	1263	3792	30023
11	4.67	11741	1.62	0.74	1399	5753	41122
12	4.94	16038	1.69	0.77	1423	8104	56946
13	5.18	21203	1.60	0.82	1460	11335	77636
14	5.55	27487	1.54	0.91	1542	16055	104103
AIDS, scenario 1: e_9 subject to a binding contract							
year t	λ_t	N_t^1	$\frac{N_t^1}{(N_t^2+N_t^3)/2}$	e_t	y_t	$Y_t(10^7)$	Pop.
9	4.50	5972	1.87	0.57	1310	3076	23475
10	4.65	8040	1.46	0.45	1273	3686	28944
11	4.05	9548	1.42	0.57	1391	4791	34435
12	4.20	12892	1.66	0.55	1301	5680	43654
13	4.03	19869	1.95	0.50	1172	7210	61509
14	3.87	30042	1.95	0.47	1129	10300	91201
AIDS, scenario 2, e_9 revised by the first group							
year t	λ_t	N_t^1	$\frac{N_t^1}{(N_t^2+N_t^3)/2}$	e_t	y_t	$Y_t(10^7)$	Pop.
9	4.50	7182	1.87	0.42	1336	3135	23475
10	4.14	8040	1.46	0.45	1217	3521	28944
11	3.82	10490	1.56	0.49	1286	4604	35811
12	3.73	16063	1.95	0.43	1138	5604	49239
13	3.45	23617	1.95	0.42	1042	7393	70945
14	3.28	36345	1.95	0.39	989	10647	107687

A similar argument holds for the two AIDS scenarios, in which the effects are even stronger than in the NO AIDS case. In both of these cases, parents are constrained to choose a high level of fertility in 2000, as N_{min}^1 cannot fall to 1.05 within one period. As a consequence, the level of schooling e_{10} is low, and λ_{11} lies well below its NO AIDS value, so that investing in education in all periods after 2010 does not pay as well, particularly in the light of the fact that child-raising costs fall as α_t recovers. By 2030, parents choose the maximal level of fertility, N_{max}, which suggests that the number of children per family in all following periods will remain unchanged and high. As e_t falls after 2010, it is possible that the economy will reach some low equilibrium with respect to λ_t and e_t some time after 2040. Recall, however, that an equilibrium in λ_t with $e_t = 0$ is not possible.

Unfortunately, it is not possible to directly check whether a low-level equilibrium exists, as suggested by the time series $\{e_t, \lambda_t\}_{t \geq 12}$, nor where the equilibrium values lie, as mortality rates and/or population tables after 2070 are not available. In the context of the present model, in which there is no limit to the population Kenya can support, it seems possible to construct a scenario in which the economy moves toward some low equilibrium with respect to e_t, λ_t and y_t, while the total population grows without bound. In reality, however, parents will start to reduce the number of children they have when the total population is large enough. Particularly poor, rural households will not be able to support numerous offspring when land holdings are very small. As a consequence, parents might start investing in education again, thereby leaving the low equilibrium.

3.6.3 Variation 3: The level of z_t

As the adults' choice of e_t is driven, at least in part, by z_t, the last variation presented will address the development of the transmission factor after 2000. Assume that z_t recovers after 1990, and, following Bell et al. (2004), set its value to $z_t = 0.65 \; \forall t \geq 10$. If parents perfectly foresee this change in the productivity of education, they will raise the level of schooling children enjoy during the 1990s, even though z_9 takes a low value; for young adults take into account the education of several future generations when making their decisions, and investing in e_t early is profitable. Therefore, if there is perfect foresight concerning $\{z_t\}$, the results for 1990 will differ from the data. This issue can be solved by assuming either that z_t recovers only later, that is, by 2010, or that parents do not take note of the changes in z_t until they actually take place. To keep matters simple, assume that z_t recovers only late, in 2010, that is: $z_9 = z_{10} = 0.4938$ and $z_{t \geq 11} = 0.65$.

Table 3.16: Variation 3: Recovery of z_t by 2010

NO AIDS							
year t	λ_t	N_t^1	$\frac{N_t^1}{(N_t^2+N_t^3)/2}$	e_t	y_t	$Y_t(10^7)$	Pop.
9	4.50	7182	1.87	0.57	1310	3076	23475
10	4.65	8256	1.46	0.67	1261	3785	30023
11	4.72	8926	1.23	1.00	1372	5070	36949
12	7.09	8516	1.05	1.00	1725	7405	42936
13	8.78	9282	1.12	1.00	1994	9921	49753
14	11.31	10324	1.21	1.00	2382	13650	57311
AIDS, scenario 1: e_9 subject to a binding contract							
year t	λ_t	N_t^1	$\frac{N_t^1}{(N_t^2+N_t^3)/2}$	e_t	y_t	$Y_t(10^7)$	Pop.
9	4.50	7182	1.87	0.57	1310	3076	23475
10	4.65	8040	1.46	0.46	1272	3681	28944
11	4.09	7058	1.05	0.84	1363	4199	30799
12	6.16	6928	1.05	1.00	1548	5025	32464
13	7.69	6591	1.05	1.00	1848	6259	33869
14	10.07	6596	1.05	1.00	2267	8248	36388
AIDS, scenario 2, e_9 revised by the first group							
year t	λ_t	N_t^1	$\frac{N_t^1}{(N_t^2+N_t^3)/2}$	e_t	y_t	$Y_t(10^7)$	Pop.
9	4.50	7182	1.87	0.42	1336	3135	23475
10	4.14	8040	1.46	0.46	1215	3517	28944
11	3.86	7126	1.06	0.77	1293	3996	30898
12	5.54	6962	1.05	1.00	1432	4666	32579
13	7.13	6637	1.05	1.00	1704	5798	34028
14	9.30	6636	1.05	1.00	2086	7630	36570

By construction, the results for 1990 remain unchanged compared to the benchmark cases and scenarios, as z_{11} does not appear in the household's maximization problem when it chooses N_9^1 and e_9. By 2000, however, parents start taking into account future changes in z_t, and increase education accordingly. In the NO AIDS case, child labor is eradicated a decade earlier than in the benchmark case, and the adult level of efficiency in 2040 is higher by 68%, while per capita and total GDP are higher by 44% and 27%, respectively. As λ_t is higher in all periods, raising children is more costly, so that parents prefer to have fewer children than in the case where z_t does not recover.

As expected, the effects of the HIV/AIDS epidemic on λ_t and y_t are slightly weaker than in the case where z_t does not change, as parents invest heavily in education even in the presence of the epidemic. The effects of the disease on population and GDP are generally weaker than in the case where z_t remains unchanged at 0.494 in all cases but the second scenario. Recall from tables 3.9 and 3.12 that, after 2020, parents choose to raise more children per family in the second scenario compared to the NO AIDS case. If z_t improves to 0.65, however, parents always choose $N_t^1 = N_{min}^1$, so that the difference between the population sizes in the NO AIDS case and the second scenario is larger.

Despite the outbreak of the disease, per capita GDP in 2040 when $z_{t\geq 11} = 0.65$ is higher than in the NO AIDS benchmark case (see table 3.9), and the difference amounts to more than 50% if parents perfectly foresee the upcoming epidemic. Child labor is now eradicated by 2020 even in the presence of the epidemic.

3.7 Policy

Governmental intervention has been necessary, since the outbreak of the HIV/AIDS epidemic in Africa in the 1980s, particularly in the health and education sectors, in order to mitigate its adverse effects. Individuals in developing countries do not have the resources to learn unaided about how the disease is transmitted, and cannot afford the treatment if they become infected. With the number of AIDS orphans estimated to have risen by 30% between 2001 and 2003 alone,[16] communities which have taken in orphans in the past are increasingly overwhelmed.

Governmental programs can therefore be classified into three types of measures: preventing the disease from spreading, treating and caring for the ill, and looking after orphans. Preventive measures encompass information campaigns in the media and schools, the dis-

[16]Source: 'Report on the global AIDS epidemic 2004,' U.N.AIDS, 2004, Page 193

tribution of condoms and setting up AIDS test facilities in all affected regions. Before the epidemic breaks out fully, the number of HIV infections typically rises in certain population groups like prostitutes and truck drivers – targeting preventive campaigns at these groups can prove to be highly effective.

After the epidemic breaks out, however, the economy must bear the combined burden of a workforce reduced by illness and death, and the costs of caring for the sick and their families. At this stage, public policies should aim at extending the life of the infected through treatment and preventing them from spreading the disease to the healthy. While information campaigns in the media and schools remain important, measures now also consist of treating HIV-positive pregnant women with drugs to reduce the likelihood of the unborn child also being infected at delivery. With a prevalence rate among Kenyan adults aged 15-49 of 6.7% in 2003,[17] that is, more than 1 million AIDS cases to follow, the medical infrastracture needs to be extended, by building additional hospitals and clinics, as well as training and employing more health personnel.

Preventive measures such as information campaigns and condom distribution are relatively cheap – particularly so before a full outbreak of the epidemic, when it is often sufficient to target focus groups. Marseille, Hofmann and Kahn (2002) estimate the costs of prevention at about 8-12 US$ per case averted. Saving one disability-adjusted-life-year (DALY) through a bundle of measures such as prevention of mother-to-child transmission, supply of condoms for sex workers, control of sexually-transmitted diseases, voluntary counseling and testing as well as blood supply safety, costs an estimated US$12.50. Treatment costs, for example through highly active antiretroviral therapy (HAART) are much higher: in the developed world, they exceed 10,000 US$ per patient per year. With the emergence of generic drugs, which are intended for distribution in developing countries only, drug costs amount to 350 US$ per patient per year. Note, however, that these costs do not include distributing the drugs and payments for medical personnel. As a consequence,

[17] Source: 'Report on the global AIDS epidemic 2004,' U.N.AIDS, 2004, Page 191

Marseille, Hofmann and Kahn estimate the costs of saving a DALY for a cost-effective HAART program to be 395 US$ per patient yearly.

3.7.1 Procedure

Intervention in the health sector will aim at reducing mortality rates, both through preventing AIDS from spreading and by treating those who are already ill. In the initial phase of the epidemic, intervention will consist mainly of cost-effective preventive measures. Treatment will gain importance when preventive measures do not bear fruit any more. Denote public spending on health in period t, normalized per sub-family, by G_t, and mortality rates in the presence of the disease ($D = 1$) by $q_t(D = 1)$. Following Bell et al. (2004), assume the relationship between premature adult mortality in the presence of the disease and public spending on health to be as follows:

$$1 - \kappa_t(G_t; D = 1) \equiv q_t(G_t; D = 1) = d_t + \frac{1}{a_t + c_t e^{-b_t G_t}}, \tag{17}$$

which allows for sufficient curvature (diminishing returns) over a flexible interval. The values of a_t, b_t, c_t and d_t need to be calibrated for each period in which $G_t > 0$. As the present essay uses the same data as Bell et al. (2004), the results derived there remain valid for present purposes.[18] Due to the changes in mortality rates, the population tables

Table 3.17: The parameter values of the function $q_t(G_t; D = 1)$

	2000	2010	2020
a_t	0.9073	0.9863	1.3852
b_t	0.0169	0.0187	0.0266
c_t	0.3124	0.3395	0.4768
d_t	1.2152	1.1128	0.8066

Source: Bell et al. (2004)

need to be recalculated using (17) and table 3.17. To be able to compute the new values of N_t^a, the public spending program needs to be defined and the age-specific mortality rates $q_t^{a,a+1}$, must be formulated as a function of $q_t(G_t; D = 1)$. For reasons of comparability,

[18]Source: Bell et al. (2004), page 47

the same functional form as in Bell et al. (2004) is chosen. Following Bell et al. (2004), G_t, i.e. the level of spending per family, is defined as:

$$G_t \equiv \frac{A_t}{(N_t^2 + N_t^3 + N_t^4)/2}, \tag{18}$$

with A_t being the level of governmental spending on health. Consider the 30-year spending program:

$$\mathbf{A} = (A_{2000}, A_{2010}, A_{2020}) = (50 \cdot 10^6, 100 \cdot 10^6, 100 \cdot 10^6),$$

where $A_{2000} = 50 \cdot 10^6$ means that 50 million dollars are spent on health every year between 2000 and 2009, and A_{2010} and A_{2020} can be interpreted analogously. Assume that the program $\mathbf{A}$ is financed through grants from abroad, which are fully funded by international donors. As the grants do not need to be repaid, levying taxes or reshuffling the governmental budget to finance them is not necessary. It is also assumed that the government has no means to extend the program beyond the donors' grants.

The age-specific mortality rates are then defined as follows:

$$q_t^{a,a+1}(G_t; D = 1) = q_t^{a,a+1} \cdot \left(1 - \frac{q_t(G_t; D = 1)}{q_t(0; D = 1)}\right) \tag{19}$$

As a consequence of the policy program $\mathbf{A}$, the number of children raised by a family and surviving to old age will now change, and the size of G_t is endogenous because as N_t^a is endogenous. Therefore, the population tables need to be recomputed in each period.

Parental decisions are determined in part by their expectations about future mortality rates. In the absence of public spending, parents were assumed to have perfect foresight about mortality rates. To keep matters simple in the case of governmental spending on health, assume parents are more short-sighted: They are aware of the mortality structure if the governmental spending did not take place, and of all future effects of the spending program. However, they do not take into account the effects their own decisions on

N_t^1 will have on future values of G_t. Hence, the subjective estimates of κ_t and κ_{t+1} are computed as follows: The current population tables and mortality rates are taken as a basis to calculate G_t from (18) and the new mortality rates $q_t^{a,a+1}(G_t; D = 1)$, from (17) and (19), which, in turn, are used to compute new values for N_t^a from (15) and hence κ_t and κ_{t+1}. All values of N_t^a which are not influenced by the policy program **A** are left unchanged.

3.7.2 Policy: Results

As the policy program **A** does not affect the NO AIDS case, only the results for the AIDS scenarios are reported. Taking into account the history of policies to deal with AIDS in Kenya, it seems unlikely that households in the 1990s could have foreseen that a program would be implemented in the 2000s. Due to the nature of HIV/AIDS, the effects of such a program on mortality will not make themselves felt immediately, so that it will be assumed that, first, parents during the 1990s do not foresee the program, and second, that even during the 2000s they do not realize its effects, and therefore do not revise their decisions about e_{10}. Young adults in $t = 10$, however, who decide about N_{11}^1 and e_{11}, realize the programs' effects and make their decisions accordingly. Therefore, even if the government intervenes in the health sector, the outcomes for $t = 9$ and $t = 10$ regarding fertility and schooling will remain unchanged. The results are set out in tables 3.25 to 3.28 in the appendix. The direct effects of the policy program on the families' decisions can be observed for 2010, that is, the first year in which they react to changes in mortality. Table 3.18 sets out the adult's level of efficiency in 2010 as well as the qualitative changes in fertility and education: a "+" in the third column states that fertility is higher compared to the case without public spending, a "–" denotes that fertility is lower, and a "0" means that it remains unchanged. The symbols in the last column are interpreted analogously.

As a consequence of the policy program **A**, mortality rates are indeed reduced much, so that more of the parents will survive to old age. At the same time, more of their children will survive to adulthood and be able to finance their parents in old age. Therefore, consumption in the second stage of adulthood becomes more important in all calculations. To finance it, parents can invest either in education, or in fertility, or in both. If λ_t is low, the returns from investing in education will be low, while the ratio of income from child labor to child-raising costs will be relatively high. As a consequence, parents will increase fertility if λ_t is low. This is particularly the case in the second scenario, as can be seen from table 3.18. The effect is strongest when z_t is high, where parents not only increase fertility when λ is low, but also reduce schooling slightly, from $e_{11} = 0.77$ to $e_{11} = 0.76$, in the presence of the program. As this reduction of schooling does not appear in any other variation, it seems to be associated with the high level of z. If mortality rates are high, parents normally do not invest much in education if λ_t is low, except when z_t is high. With mortality rates reduced as a result of the program, investing in fertility becomes more profitable again, for all values of z, and the payoffs from education are lower. Therefore, parents reduce schooling from its very high level in the absence of the policy program to a somewhat lower level in its presence when z is high. The overall effects of the program on λ_t, y_t, Y_t and population will be discussed in detail in section 3.9.

Table 3.18: The effects of the policy program **A** on fertility and education: 2010

	λ_t	$\frac{N_t^1}{(N_t^2+N_t^3)/2}$	e_t
Base Case			
Scenario 1	4.05	0	+
Scenario 2	3.82	+	0
Variation 1 (N^1_{min})			
Scenario 1	4.03	0	+
Scenario 2	3.80	+	+
Variation 2 ($\alpha_{t\geq 10}$)			
Scenario 1	4.05	+	+
Scenario 2	3.82	+	+
Variation 3 ($z_{t\geq 11}$)			
Scenario 1	4.08	+	+
Scenario 2	3.86	+	−

Scenario 1: e_9 subject to a binding contract

Scenario 2: e_9 revised by the first group

3.8 The Effects on Social Welfare

In order to assess the overall effects of the epidemic and the policy program **A**, a social welfare function (SWF) is required. The choice of a SWF, however, poses difficulties of its own. It has to capture and value several effects of the epidemic: first, some individuals die prematurely, and are robbed of the opportunity to enjoy their lives and be productive. Second, some individuals are never born, either because their parents die prematurely or because their parents decide to have fewer children. Third, even those individuals who do not die prematurely often enjoy less schooling than they would have done in the absence of the disease, so that their lifetime income is reduced and their parents' utility from the capacity they attain as adults will fall. There is very little literature on the evolution of welfare when fertility is endogenous. Schweizer (1996) considers an open economy and defines efficiency using the concept of net trade. His welfare measure, however, holds only for economies in a steady state, where population growth is constant over time. As this is not the case in our projections, the measure developed by Schweizer cannot be employed in our framework.

The first step is to decide what unit – monetary or physical – to use. As one of the reasons for introducing a social welfare function is to assess the effects and profitability of the policy program **A**, and as the costs of this program are measured in dollars, we will choose a money-metric utility. In general, changes to an individual's level of utility are valued using the concepts of the compensating or equivalent variation (CV and EV), as these capture all individual-level effects described above and assign them a monetary value. The CV measures how much money, in the form of a lump-sum transfer, is required such that and individual's level of utility in the AIDS case with the lump-sum transfer be identical to her level of utility in the NO AIDS case: $U(NO\ AIDS; \cdot) = U(AIDS, CV > 0; \cdot)$. The EV is the lump-sum an individual is willing to pay in order to avoid the changes in mortality brought about by the HIV/AIDS epidemic and its consequences: $U(NO\ AIDS, EV < 0; \cdot) = U(AIDS; \cdot)$. The CV and EV measure the effects on individuals. By summing the CV and EV over all relevant individuals[19] in some periods, it is possible to assess the overall losses and gains due to the epidemic or an intervention. This aggregation, however, does not include those individuals who are never born, but would have been under other circumstances. The CV and EV can be defined in an analogous way to compute the effects of the policy intervention.

The transfer can be paid or received either in period t, that is, while the children are going to school, or in period $t+2$, that is, when the parents reach old age, or in both periods. Note, first, that even though the transfers are lump-sum in form, parents will change N_t^1 and/or e_t. Consider, for example, a family receiving a lump-sum transfer T_t in period t, and nothing in period $t+2$. As there are no capital markets, individuals cannot smooth consumption by saving, and therefore they will change their choice of fertility, schooling or both.[20] It is not possible to determine e_t and N_t^1 analytically as functions of the transfer, so that the values of the functions $U(AIDS, CV; \cdot)$ and $U(NO\ AIDS, EV; \cdot)$ must be determined numerically. In fact, the CV and EV can only be derived using grid search methods,[21] which are computationally expensive, as they have to be done separately for each period, variation and scenario.

Note, also, that the CV and EV are likely to differ significantly, as the changes in mortality incurred when the epidemic breaks out are large. The difference stems from the fact that it is not only the the levels of consumption which change, but also the weights of the subutility functions. When computing the CV, we consider the AIDS utility function, when computing the EV, the NO AIDS one. These differ, for instance, in the weight

[19] Aggregating the EV^h over all individuals h rests on the assumption that the marginal utility of income is the same for all individuals. This assumption is easily satisfied within a period, as all individuals are identical. However, the marginal utility of income will be identical between two periods by pure chance only.

[20] This argument also holds if parents receive the transfer when they are old, as borrowing is not possible. While it is possible that there will exist a tuple of transfers T_t, T_{t+2} for which fertility and schooling both remain unchanged, it is very unlikely that these transfers will correspond to a CV or EV

[21] The procedure, in the case of the EV, is as follows: First, compute the levels of utility attained in the AIDS case. Second, use the NO AIDS utility function to calculate the optimal levels of fertility and education for each transfer T, and the ensuing level of utility. Third, choose that level of T for which utility in the NO AIDS case with the transfer is equal to the level of utility in the AIDS case without the transfer. Steps two and three must be done separately for every year. The procedure in the case of the CV is similar. Both the EV and CV can be calculated analogously when evaluating the policy program **A**.

attached to old-age consumption, $\beta_1\kappa_t$, which is much lower in the case of the CV. Hence, the CV will generally be much larger than the EV as mortality rates rise, and vice-versa.

The EV and CV being hard to compute, we will now turn to other, simpler, measures of welfare. One such measure is income, summed over all relevant individuals and periods. While income measures, to some extent, the utility which accrues to households through its close connection to consumption, it does not measure the utility that arises from the educational attainment of the individual's children, and it does not fully measure the disutility arising from premature mortality. The contribution of the altruism term to an individual's level of utility is less than 2% in all years, scenarios and variations analyzed; consumption, especially its distribution over time, and the parents' expectations about mortality determine the rest. Therefore, the major drawback of using income as a measure of welfare is that is does not fully capture the losses and gains arising from changes in mortality rates. If these changes are small, income can be used as an easy to compute measure of welfare.

The results, however, require careful interpretation, for they depend on whether per capita or total income (GDP) is used. Total income can rise in two ways: either the number of individuals rises, while income per head is constant, or the income of each individual rises while the number of individuals remains constant. If parents decide to reduce schooling while increasing fertility, per capita income will be lower – suggesting that individuals might be worse off – while total income might rise – suggesting that an economy-wide indicator has improved. If such a tradeoff does not exist, as is the case when the HIV/AIDS epidemic first breaks out,[22] both income per capita and total GDP are arguably acceptable measures of welfare. In the case of the policy program **A**, however, such tradeoffs are frequent, particularly in the second variation, in which per capita GDP rises while total GDP (and population) fall. Therefore, when total GDP is the measure of welfare,

[22]Both per capita and total GDP are higher in the NO AIDS case compared to either AIDS scenario. The single exception is the period $t = 10$, in which per capita GDP in the presence of the disease is somewhat higher than in its absence due to a rise in child labor. As the difference amounts to less than 1% in all cases, we will ignore this problem for the rest of the welfare analysis.

the policy program has negative consequences, as it reduces the size of the population substantially. If per capita GDP is used instead, the program has positive consequences. Which measure, then, is better? When the program is undertaken, mortality rates fall, yet the population size is smaller than in the absence of the program. This is the result of the families' decisions to have fewer but better-educated children. Therefore, from an individual's point of view, the lower total population size is 'optimal' (and, hence, 'good'), suggesting that per capita income is a better measure of welfare than total GDP in the presence of the program. As a consequence, both per capita income and total income can be used to measure the welfare losses incurred when the HIV/AIDS epidemic breaks out. As has been argued above, per capita income seems to be a more appropriate measure of welfare if there is a tradeoff between schooling and fertility, and premature mortality falls, as is the case if the policy intervention is undertaken.

One last aspect needs to be discussed, namely, the treatment of those individuals who are born under one program but not under another. The valuation of such individuals and their lives is the issue of a lively discussion in both philosophy and economics, and there is no commonly accepted method or procedure. Most authors who are concerned with AIDS, for instance, Young (2005), do not take such individuals into account. Note that whatever individual measure of welfare is chosen – CV, EV or per capita income – it has to be summed up over all households/individuals who experience the losses and gains in order to capture the full effects to be analyzed. Consider, for instance, the effects of the HIV/AIDS epidemic. When only those individuals who actually lived at some point of time are considered, the losses are underestimated: if there had been no epidemic, parents would have had more and better-educated children, whose hypothetical income and lives involve losses which need to be taken into account when measuring the effects of HIV/AIDS.

In the present paper, we need to estimate the effects of the HIV/ADS epidemic in the absence of the intervention, when there is no tradeoff, and the effects of the policy program

A, when there is. Two measures of welfare will be employed, both of them based on individual incomes: first, a measure in which unborn individuals are taken into account, and second, one in which they are not. To compute the first measure, we calculate the income, in every period, of every individual alive in that period in the presence of the intervention or shock, and compare it to the total income of every individual in the absence of the intervention/shock. As the income of all individuals in some period is simply the total GDP of that period, the first method compares GDP levels. As argued before, this method is more appropriate when there is no tradeoff between schooling and fertility, that is, when comparing the NO AIDS and AIDS cases. To compute the second measure, we calculate the number of people who have died prematurely, or will do so, because of HIV/AIDS (or, respectively, who would have died if there had been no policy intervention), and assess their lost earnings. This step is somewhat more complicated than computing premature deaths. The simplest way of doing it is to assume that, even if the mortality rates had been different, these individuals would have enjoyed the same education, so that their level of human capital would remain unchanged. In this case, their lost earnings would be $\alpha_t \lambda^a$ for each adult in each decade, where λ^a denotes her level of human capital. It is also possible, however, that the individual would have enjoyed more schooling, and hence a higher level of human capital, if mortality rates had been lower, and vice versa. As it is not possible to compute this hypothetical level of schooling, we will simply assume that e_t takes the same value as in the case without the HIV/AIDS mortality shock or the governmental intervention, as appropriate. Therefore, when computing the (monetary) losses, we will make two assumptions:

(A1) The fact that some potential children will never be born as a consequence of HIV/AIDS will be ignored

(A2) The level of schooling enjoyed by all children will be either

- (A2a) the same level as in the case where the shock/intervention takes place.
- (A2b) the same level as in the case where the shock/intervention does not takes place

For all measures employed, we compute the 1990 net present value (NPV) of all losses and gains, and compare it either to the 1990 NPV of total, NO AIDS GDP, or to the 1990 NPV of the cost of the policy intervention **A**, to assess the effects of the epidemic or the social profitability of the policy program. In all cases, the calculations are limited to the years 1990 to 2040, so that gains and losses which accrue after 2040 are not included.

3.9 Results

The effects of the HIV/AIDS epidemic and the policy program **A** are discussed in sections 3.9.1 and 3.9.2 respectively. We will first present changes in some basic measures of welfare, namely, the adults' level of human capital, λ_t, per capita income, y_t, total GDP, Y_t and population in the last period of the projections ($t = 14$). Second, we will determine the overall effects of the epidemic and the intervention using the procedures described above.

3.9.1 No public spending on health

The effects of the HIV/AIDS epidemic on the Kenyan people and economy are substantial, but they also depend on the way households react. Table 3.19 sets out the levels of several simple measures of welfare as a percentage of their respective NO AIDS levels for the year 2040. While λ and y describe welfare at the individual level, GDP and population are aggregate measures. The effects of the epidemic on λ and per capita GDP are strongest in the case where e_9 is revised in 1990,[23] that is, in the second scenario. The impact of the disease on individuals can be reduced by measures aimed at increasing z_t after 2000, or by reducing N^1_{min}.

In the case of the individual measures of welfare, the first scenario is always better than the second. Recall that e_9 is always higher in the first scenario, as is it revised and therefore reduced in the second, so that the adults' level of human capital is always higher if e_9 is subject to a binding contract. The impact of HIV/AIDS on GDP is a heavy one: it is

[23]Recall that in the second scenario, parents suddenly realize that an epidemic has broken out on January, 1st, 1990, and revise their decision concerning e_9 accordingly.

Table 3.19: The effects of the HIV/AIDS epidemic expressed as a % of the NO AIDS levels in 2040

	λ	GDP per cap. y	GDP Y	Population
Base Case				
Scenario 1	85	91	56	61
Scenario 2	69	78	55	70
Variation 1 (N^1_{min})				
Scenario 1	81	89	57	64
Scenario 2	66	75	57	75
Variation 2 ($\alpha_{t\geq 10}$)				
Scenario 1	70	73	64	88
Scenario 2	59	64	66	103
Variation 3 ($z_{t\geq 11}$)				
Scenario 1	89	95	60	63
Scenario 2	82	88	56	64

Scenario 1: e_9 subject to a binding contract
Scenario 2: e_9 revised by the first group

lower by at least a third in all cases. Increasing N^1_{min} has positive consequences for both GDP and population size, independently of the parents' reaction, or the lack thereof, to the outbreak of the epidemic. Likewise, the recovery of α_t after 2000 has positive effects on the aggregate variables in all cases computed. A recovery of z_t has similar effects on all individual measures of welfare.

In the present setting, the government has several options apart from the policy program **A**; these will be discussed separately in section 3.9.2. One area of intervention concerns N^1_{min}, that is, the minimum number of children a family can raise. After the outbreak of HIV/AIDS, or after the fall in α_t between 1990 and 2000, families would like to reduce the number of children they raise, but it is assumed that they cannot do so due to the social stigma attached to having only very few children and to the lack of available contraceptive measures. Providing condoms and information about contraceptive measures would have two positive effects: first, to reduce N^1_{min} and, second, to reduce the spread of HIV/AIDS. The second area of intervention concerns measures aimed at a recovery of α_t and/or z_t after 2000, for example, through the better conduct of economic policy, investment in schools and the training of teachers. The third option concerns the way households reconsider their choice of e_9 during the 1990s in the absence of perfect foresight. The government might try to influence this decision, for example, by trying to enforce the contract on e_9 (scenario 1) or by endorsing one of the two groups of adults which could revise e_9 (scenarios 2 and 3). It seems very unlikely, however, that the government really can determine which of the mechanisms will be employed.

In general, governments assess the economy's condition by looking at per capita GDP, total GDP, population, or a combination of these factors. If the government's aim is to maximize population (which, in most cases, also maximizes GDP) after the outbreak of the HIV/AIDS epidemic, trying to enforce the decision mechanism employed in scenario 2 is best. Over the long run, however, this will have adverse results for individuals, whose level of efficiency and per capita income will be very low, while child labor will increase.

To increase household welfare, that is the indicators λ_t and y_t, implementing an early information campaign about the effects of the epidemic on mortality rates, so that parents would have more perfect foresight about the disease, is best. It is now arguably too late for such a measure, so that the government can try, instead, to conduct programs aimed at reducing N^1_{min} or improving z_t after 2000. Finally, it can also reduce mortality rates through the public spending program $\mathbf{A}$, the effects of which will be discussed in the following section.

The overall effects of the epidemic are set out in tables 3.20 and 3.21. Table 3.20 gives the 1990 NPV of the overall losses as a fraction of the 1990 NPV of total NO AIDS GDP when unborn individuals are taken into account, for the base case. The variations are reported in the appendix.

Table 3.21 gives the 1990 net present value (NPV) of the loss of income incurred through

Table 3.20: Lost income through premature adult mortality, including unborn individuals

Interest Rate	Scenario 1	Scenario 2
0.0% p.a.	28	31
1.2% p.a.	26	28
5.0% p.a.	18	20

All values expressed as a % of the 1990 NPV of total NO AIDS GDP between 1990 and 2040

Table 3.21: Lost income through premature adult mortality, excluding unborn individuals

Interest Rate	Scenario 1		Scenario 2	
	(A2a)	(A2b)	(A2a)	(A2b)
0.0% p.a.	14	20	13	25
1.2% p.a.	13	19	12	23
5.0% p.a.	9	13	9	16

All values expressed as a % of the 1990 NPV of total NO AIDS GDP between 1990 and 2040

premature adult mortality as a fraction of the NPV of total GDP in the period 1990-2040, when unborn individuals are ignored, using assumptions (A2a) or (A2b). Three different annual rates of interest are considered: first, $r = 0$; second, $r = 1.2\%$, which corresponds to the families' own pure discount rate, as computed from β_0 and β_1; and third, $r = 5\%$, which is the interest rate employed in Bell et. al (2004). As expected, the effects of the epidemic are stronger under assumption (A2b), as the levels of lost income per capita are higher in that case. Whichever scenario is chosen the effects of the epidemic range from 9% to 14% of total GDP under assumption (A2a), and 6 to 8 percentage points more under (A2b) (The effects are about 1.5 times as large when unborn individuals are considered). Even when the incomes of unborn individuals are ignored, therefore, the society suffers significant economic losses when HIV/AIDS breaks out.

3.9.2 Public Spending on Health

The effects of the program **A**, expressed as a % of the 2040 NO AIDS level, are set out in table 3.22. In the base case, all measures of individual and aggregate welfare are somewhat higher if the program is undertaken. The effects of the program in the variations, however, are mixed, and can be negative in several cases, to the discussion of which we

Table 3.22: The effects of the public spending program **A**, expressed as a % of the NO AIDS levels in 2040

	λ	per capita GDP y	GDP Y	Population
Base Case				
Scenario 1	1.3	1.3	3.4	2.8
Scenario 2	2.7	3.2	2.8	0.7
Variation 1 (N_{min})				
Scenario 1	3.3	2.9	3.2	1.4
Scenario 2	4.6	5.5	1.0	-4.0
Variation 2 ($\alpha_{t\geq 10}$)				
Scenario 1	8.3	11.4	-3.7	-16.2
Scenario 2	2.5	2.2	1.0	-2.0
Variation 3 ($z_{t\geq 11}$)				
Scenario 1	0.6	0.3	3.9	3.8
Scenario 2	-0.4	-0.1	4.9	5.7

Scenario 1: e_9 subject to a binding contract
Scenario 2: e_9 revised by the first group

now turn.

If N^1_{min} is higher than in the base case, the policy program **A** has mostly positive effects in all cases but one, namely, total population in the second scenario. This measure is lower by 4% if the program is undertaken; but the cause is benevolent. One would expect population to be higher if mortality rates are lowered, e.g., by spending in the health sector; and total population in the second variation is indeed higher if the program is undertaken, but only up to 2030. Fertility also matters, of course, and rates are higher until 2020. Thereafter, families start investing in education and raise fewer children than in the case without the program **A**. Consequently, total population in 2040 is lower, despite the lower mortality rates.

The policy program **A** has negative effects on some indicators if α_t recovers after 2000. At first, families increase both fertility (as raising children is relatively cheaper) and the level of schooling these children will enjoy, as more of them will survive to finance the parents' old age. Therefore, λ_{12} is higher than without **A**. This has two further effects: child-raising costs are higher than when the program is not undertaken, but education is more profitable. Consequently, fertility is lower, and education is higher, after 2020 in the case where the policy program **A** is undertaken. By 2040, total population is lower under **A**. GDP is also lower if the drop in population is sufficiently large compared to the increase in λ_t (as in scenario 1). Again, the cause of the 'adverse' effect on total population is a benevolent one.

Recall from table 3.15 that the economy seems to be approaching a low-level equilibrium with respect to λ_t in the first and second scenarios. As can be seen from table 3.27 in the Appendix, λ_t grows in the first scenario, but is still falling in the second. Therefore, if e_9 is fixed by a binding contract, the policy program **A** can prevent the economy from reaching a low-level equilibrium, and its effects are arguably strongest in that case.

If z_t recovers after 2000, as in the third variation, the policy program **A** always has positive effects on the aggregate measures of welfare. Surprisingly, however, the effects on individual measures of welfare can be negative, as in the case of the second scenario in variation 3. Recall from section 3.7.2 that parents reduce the level of schooling their children receive when the program is undertaken if the parents' level of human capital is low, and increase e_t otherwise. In second scenario of the third variation, therefore, e_t falls when the program is undertaken, as investing in fertility is more profitable than investing in education. As a consequence, λ_t and y_t are lower. Note, however, that even though the policy program has negative effects on λ_t and y_t, these are small, amounting at most to 0.4% of the respective NO AIDS levels.

To assess the social profitability of the program **A**, we employ the methods presented in section 3.8. Tables 3.23 and 3.24 report the results when unborn individuals are, and are not, considered, respectively. As there are some striking results for the variations in the case where unborn individuals are taken into account, we report those cases here, too.

Assumption (A2) from section 3.8 now reads as follows:

(A2) The level of schooling enjoyed by all children will be either

(A2$\hat{a}$) the same level as in the AIDS case with a policy program.

(A2$\hat{b}$) the same level as in the AIDS case without a policy program.

Note that the levels of schooling and human capital are generally lower in the case of Assumption (A2$\hat{b}$), so that the effects of the policy program **A** will also be lower in that case.

When unborn individuals are taken into account, the policy program seems to be profitable, in general, particularly if the interest rate is low. However, in the case of the second variation, that is, if α_t recovers, the policy intervention seems to have a small negative impact on the economy if the interest rate is low. As the negative effect disappears if the interest rate is higher, and as low interest rates mean that future gains and losses are not discounted much, the losses in question seem to accrue at the end of the period in question. GDP and population both fall in the second variation if the program is undertaken, as parents decide to have fewer but much better educated children. That is, there is a tradeoff between education and fertility, and, consequently, between per capita income (which rises) and total GDP (which falls; see table 3.22). As argued in section 3.8, however, total GDP is not a good measure of welfare if there are such tradeoffs. Similarly, unborn individuals should not be taken into account when parents decide freely to have fewer children and increase their schooling when a policy intervention is undertaken. Therefore, the figures presented in table 3.23 do not constitute good measures of the profitability of the program when fertility rates fall as a consequence of the policy intervention, and the results set out in table 3.24 should be used instead.

In the case where unborn individuals are not taken into account, the discounted gains are at least three times as high as the discounted costs, so that the program is very profitable indeed. Note also that when computing the values in tables 3.23 and 3.24, we did not take into account the program's gains which accrue after 2040. More recent evidence suggests that the costs of saving a DALY through HAART might be lower than those employed in the present essay – in this case, the policy program would be even more profitable, as it saves and prolongs more lives.

Table 3.23: The policy program **A**: Profitability[a], including unborn individuals

Interest Rate	Scenario 1	Scenario 2
0.0% p.a.	8	9
1.2% p.a.	7	8
5.0% p.a.	5	5
Variation 1		
0.0% p.a.	8	6
1.2% p.a.	7	5
5.0% p.a.	4	4
Variation 2		
0.0% p.a.	-0	5
1.2% p.a.	+0	4
5.0% p.a.	1	3
Variation 2		
0.0% p.a.	11	13
1.2% p.a.	9	11
5.0% p.a.	6	7

[a]All entries expressed as multiples of the 1990 NPV of the costs of **A**.

Scenario 1: e_9 subject to a binding contract

Scenario 2: e_9 revised by the first group

Table 3.24: The policy program **A**: Profitability[a], excluding unborn individuals

Interest Rate	Scenario 1		Scenario 2	
	(A2$\hat{a}$)	(A2$\hat{b}$)	(A2$\hat{a}$)	(A2$\hat{b}$)
0.0% p.a.	5	4	5	5
1.2% p.a.	5	4	4	4
5.0% p.a.	3	3	3	3

[a]All entries expressed as multiples of the 1990 NPV of the costs of **A**.

Scenario 1: e_9 subject to a binding contract

Scenario 2: e_9 revised by the first group

3.10 Conclusion

The HIV/AIDS epidemic in Kenya has had, and continues to have, a very damaging impact on the country's economy and population, both at the individual and at the aggregate level. Three factors play a major role in determining the long-term dynamics. First, there is the drop in the inter-generational transmission factor after 1970, which leads to a direct reduction in the formation of human capital and to levels of schooling. Second there is the reduction in labor productivity during the 1990s, as a consequence of which, fertility and total population both fall. Third, there is the increase in mortality rates after the full outbreak of the HIV/AIDS epidemic. The individuals' reaction to the epidemic depends, first, on when they realize its effects, and second on the social mechanisms governing the way exogenous shocks are dealt with. As has been argued in this essay, changes in the inter-generational transmission factor and the labor productivity factor can mitigate or intensify the epidemic's consequences. Similarly, measures aimed at changing N^1_{min} can also form part of an anti-AIDS program.

The policy program called **A**, which is fully financed by foreign grants, has a positive effect on all individual-level welfare measures. In reaction to **A**, parents normally increase the level of schooling their children enjoy, and therefore their future income and per capita GDP. At the aggregate level, however, the program can lead to lower fertility, so that both the total population size and GDP may be lower, too. These effects are strongest if the labor productivity factor recovers after 2000. Combining **A** with measures aimed at influencing N^1_{min}, α_t and/or z_t, can further mitigate the effects of the epidemic. Note, however, that comparing **A** with the other measures discussed in the variations is difficult, as the costs incurred in implementing the changes in the latter are unknown. The gains which accrue if **A** is undertaken outweigh many-fold the costs incurred in financing it, even if the overall returns to this investment that occur after 2040 are left out of account.

3.11 Appendix

Table 3.25: The Policy Program **A**: Base Case

AIDS, Scenario 1: e_9 subject to a binding contract							
year t	λ_t	N_t^1	$\frac{N_t^1}{(N_t^2+N_t^3)/2}$	e_t	y_t	$Y_t(10^7)$	Pop.
9	4.50	7182	1.87	0.57	1310	3076	23475
10	4.65	8040	1.46	0.45	1273	3686	28944
11	4.05	7136	1.05	0.73	1372	4275	31164
12	4.63	7070	1.05	0.88	1409	4719	33494
13	5.03	7962	1.23	1.00	1401	5232	37355
14	5.78	8279	1.18	1.00	1530	6332	41389
AIDS, scenario 2: e_9 revised by the first group							
year t	λ_t	N_t^1	$\frac{N_t^1}{(N_t^2+N_t^3)/2}$	e_t	y_t	$Y_t(10^7)$	Pop.
9	4.50	7182	1.87	0.42	1336	3135	23475
10	4.14	8040	1.46	0.45	1217	3521	28944
11	3.82	7580	1.12	0.64	1283	4083	31812
12	4.13	8230	1.18	0.73	1269	4521	35636
13	4.36	9568	1.32	0.83	1238	5106	41241
14	4.82	9531	1.14	1.00	1337	6169	46154

Table 3.26: The Policy Program **A**: Variation 1: The level of N^1_{min}

AIDS, Scenario 1: e_9 subject to a binding contract							
year t	λ_t	N_t^1	$\frac{N_t^1}{(N_t^2+N_t^3)/2}$	e_t	y_t	$Y_t(10^7)$	Pop.
9	4.50	7182	1.87	0.57	1310	3076	23475
10	4.65	8178	1.48	0.45	1266	3692	29155
11	4.03	7518	1.10	0.70	1352	4306	31854
12	4.54	7670	1.10	0.83	1379	4806	34857
13	4.85	8405	1.21	1.00	1376	5364	38995
14	5.65	8590	1.14	1.00	1511	6520	43149
AIDS, scenario 2: e_9 revised by the first group							
year t	λ_t	N_t^1	$\frac{N_t^1}{(N_t^2+N_t^3)/2}$	e_t	y_t	$Y_t(10^7)$	Pop.
9	4.50	7182	1.87	0.42	1336	3135	23475
10	4.14	8178	1.48	0.44	1210	3528	29155
11	3.80	7711	1.13	0.63	1277	4104	32135
12	4.10	8393	1.19	0.72	1261	4555	36111
13	4.31	9851	1.34	0.81	1226	5152	42013
14	4.74	9812	1.15	0.99	1321	6229	47153

Table 3.27: The Policy Program **A**: Variation 2: Recovery of α_t

AIDS, Scenario 1: e_9 subject to a binding contract							
year t	λ_t	N_t^1	$\frac{N_t^1}{(N_t^2+N_t^3)/2}$	e_t	y_t	$Y_t(10^7)$	Pop.
9	4.50	7182	1.87	0.57	1310	3076	23475
10	4.65	8040	1.46	0.45	1273	3686	28944
11	4.05	9863	1.46	0.58	1379	4845	35146
12	4.25	11887	1.48	0.62	1359	5873	43230
13	4.24	16375	1.64	0.63	1278	7262	56820
14	4.33	20758	1.56	0.67	1305	9699	74304
AIDS, scenario 2: e_9 revised by the first group							
year t	λ_t	N_t^1	$\frac{N_t^1}{(N_t^2+N_t^3)/2}$	e_t	y_t	$Y_t(10^7)$	Pop.
9	4.50	7182	1.87	0.42	1336	3135	23475
10	4.14	8040	1.46	0.45	1217	3521	28944
11	3.82	10628	1.57	0.51	1282	4649	36263
12	3.79	15124	1.80	0.48	1182	5762	48747
13	3.59	23107	1.95	0.44	1073	7576	70635
14	3.42	35107	1.95	0.41	1023	10802	105601

Table 3.28: The Policy Program **A**: Variation 3: Recovery of z_t

AIDS, Scenario 1: e_9 subject to a binding contract							
year t	λ_t	N_t^1	$\frac{N_t^1}{(N_t^2+N_t^3)/2}$	e_t	y_t	$Y_t(10^7)$	Pop.
9	4.50	7182	1.87	0.57	1310	3076	23475
10	4.65	8040	1.46	0.46	1272	3681	28944
11	4.09	7332	1.08	0.86	1350	4244	31450
12	6.23	7168	1.05	1.00	1560	5276	33828
13	7.75	6934	1.05	1.00	1856	6694	36077
14	10.14	6900	1.05	1.00	2275	8774	38558
AIDS, scenario 2: e_9 revised by the first group							
year t	λ_t	N_t^1	$\frac{N_t^1}{(N_t^2+N_t^3)/2}$	e_t	y_t	$Y_t(10^7)$	Pop.
9	4.50	7182	1.87	0.42	1336	3135	23475
10	4.14	8040	1.46	0.46	1215	3517	28944
11	3.86	7797	1.15	0.76	1263	4058	32129
12	5.50	7403	1.05	1.00	1434	4965	34622
13	7.15	7260	1.05	1.00	1697	6308	37173
14	9.26	7177	1.05	1.00	2083	8294	39816

Table 3.29: Lost income through premature adult mortality, including unborn individuals: Variations

Interest Rate	Scenario 1	Scenario 2
Variation 1 (N^1_{min})		
0.0% p.a.	28	30
1.2% p.a.	25	28
5.0% p.a.	17	20
Variation 2 ($\alpha_{t\geq 10}$)		
0.0% p.a.	27	27
1.2% p.a.	25	26
5.0% p.a.	18	19
Variation 3 ($z_{t\geq 11}$)		
0.0% p.a.	28	32
1.2% p.a.	26	30
5.0% p.a.	17	22

All values in % of the 1990 NPV of total NO AIDS GDP between 1990 and 2040

Scenario 1: e_9 subject to a binding contract

Scenario 2: e_9 revised by the first group

Table 3.30: Lost income through premature adult mortality, excluding unborn individuals: Variations

Interest Rate	Scenario 1		Scenario 2	
	(A2a)	(A2b)	(A2a)	(A2b)
Variation 1 (N^1_{min})				
0.0% p.a.	14	21	13	26
1.2% p.a.	13	19	12	24
5.0% p.a.	9	13	9	16
Variation 2 ($\alpha_{t\geq 10}$)				
0.0% p.a.	12	19	11	26
1.2% p.a.	12	18	11	24
5.0% p.a.	9	13	9	17
Variation 3 ($z_{t\geq 10}$)				
0.0% p.a.	10	15	10	19
1.2% p.a.	10	14	10	18
5.0% p.a.	8	11	8	14

All values in % of the 1990 NPV of total NO AIDS GDP between 1990 and 2040

Scenario 1: e_9 subject to a binding contract

Scenario 2: e_9 revised by the first group

Table 3.31: The policy program **A**, excluding unborn individuals: Profitability[a]

Interest Rate	Scenario 1		Scenario 2	
	(A2$\hat{a}$)	(A2$\hat{b}$)	(A2$\hat{a}$)	(A2$\hat{b}$)
Variation 1 (N^1_{min})				
0.0% p.a.	5	4	5	4
1.2% p.a.	5	4	4	4
5.0% p.a.	3	3	3	3
Variation 2 ($\alpha_{t\geq 10}$)				
0.0% p.a.	6	4	6	3
1.2% p.a.	5	4	5	3
5.0% p.a.	4	3	3	3
Variation 3 ($z_{t\geq 10}$)				
0.0% p.a.	6	5	5	6
1.2% p.a.	5	4	5	5
5.0% p.a.	3	3	3	3

[a]All entries expressed as multiples of the 1990 NPV of the costs of the policy program **A**.

Scenario 1: e_9 subject to a binding contract

Scenario 2: e_9 revised by the first group

Chapter 4

Summary and Conclusions

This dissertation has analyzed household decisions regarding fertility, schooling and child labor, and how they affect economic growth. In all three essays, we consider altruistic parents who decide simultaneously about the number of children they will raise and the schooling these children will enjoy. Capital markets being non-existent by assumption, there is a trade-off between consumption in old age, which rises with both the number of children and their schooling, and consumption while young, which is decreasing in schooling and either decreasing or increasing in fertility, depending on the costs of raising children and the productivity of child labor. Aggregate economic growth is driven by population growth and by human capital accumulation. Each individual's level of human capital depends on three factors, namely, the amount of time she spent at school while young, her parents' level of human capital, and the productivity of the educational sector in transforming schooling into human capital. In the first and the third essay, output is produced using a single input, namely, labor in the form of human capital. One drawback of this assumption is that the population growth rate in all steady-states depends on the level of child-raising costs only, and population can grow without bound if raising children is not too expensive. In the second essay, therefore, we assume that two inputs are required in production, namely, labor and the fixed factor land.

In the first two essays, we analyzed potential steady states and the effects of governmental intervention on economic growth. In both essays, we have shown that multiple steady-states can exist, as one might expect from a reading of the recent literature on child labor. A low-level stationary state, or poverty trap, will exist if the educational technology is not very productive and raising children is expensive. Similarly, a high-level stationary state or a growth steady state will exist if the educational technology is sufficiently productive. Poor parents generally choose too little schooling for per capita income to grow, particularly when land plays no role in production or when land is abundant. If land is scarce, even poor parents may choose to have few but well-educated children, as we show in the second essay. Yet appropriate government intervention can induce economic growth, even if the society is stuck in the poverty trap. We have analyzed three instruments, namely, lump-sum transfers, taxes and subsidies on child-raising activities, and school-attendance subsidies. Even without outside help, it is possible to construct policy programs which lead to all households escaping the poverty trap after a finite number of periods if the educational technology is sufficiently productive. In general, however, such programs will also lead to persistent inequality. This result holds independently of the use of land in production. In the second essay, in which land plays a central role, the range of 'sufficiently productive' educational technologies is wider than in the first essay. The intuition is that if land plays a role in producing output, the fertility rate of the those who pay taxes will be equal the fertility rate of those who receive a subsidy in their respective steady states, and both will be at replacement levels, so that the size of the groups will remain constant over time. If, however, land plays no role in production, rich families will generally have fewer children than poor families, and their human capital accumulation – driven by the productivity of the educational technology – must compensate for this effect.

The model developed in the first two essays has been extended and applied in the third, empirical, essay to assess the long-run economic costs of the HIV/AIDS epidemic in Kenya. The central difference between the models in the first two chapters and the model in the third chapter is the possibility of premature adult mortality, either before the children reach school-going age, or before the parents start enjoying old-age consumption. Similarly, some of the children will themselves die as adults after completing their education and before they have started to finance their parents' old age. If premature adult mortality rates rise, our calibration for Kenya yields the result that parents will reduce fertility and also, in general, schooling. As a result, both per capita and total GDP, as well as population and the young adults' level of human capital are significantly lower by 2040, the last year for which we make projections, due to the epidemic. These results confirm the findings in Bell et al. (2004), in which the path of fertility is assumed to be exogenous, but dependent on the presence or absence of the epidemic.

We also analyze the effects of a policy program, which is fully financed by foreign grants, aimed at reducing mortality rates, and show that individuals are always better off if the program is undertaken. Using the individual gains from the policy intervention, it is possible to show that implementing the program is socially profitable. However, it turns out that families first increase fertility somewhat when the policy program is undertaken, and may reduce it later, so that the total population size can be lower when the program is undertaken compared to the case without a governmental intervention. This effect does not appear in Bell et al. (2004), fertility being fixed in that analysis.

To summarize, the present dissertation has contributed to the literature on child labor, fertility and economic growth with the first two essays, while the third essay contributes to the growing literature that assesses the macroeconomic effects and costs of the HIV/AIDS epidemic. The analysis in the first two chapters had two aims. The first aim was to develop a model in which both fertility and schooling decisions are endogenous, and to analyze the effects of these decisions on economic growth. We have drawn upon the frameworks developed in Raut and Srinivasan (1994), which describes the dynamic of economies where fertility decisions are endogenous, and Bell and Gersbach (2001), who analyze child labor. We have shown that multiple steady-states are possible, in accordance with the literature. The second aim was to analyze fiscal measures and policy programs, in contrast to the recent literature on child labor and fertility (Baland and Robinson (2000), Hazan and Berdugo (2002)), which has focused on regulatory interventions. That fertility is endogenous gives households more flexibility in responding to taxes and subsidies. When school-attendance subsidies are introduced, for example, parents react to them by increasing schooling when fertility is exogenous. If fertility is endogenous, however, they might even reduce fertility when such a subsidy is introduced, with an accompanying further increase in education. As a consequence, the subsidy is even more efficient, and policy programs aimed at reducing child labor need less resources if they use school-attendance subsidies than in the case where fertility is exogenous.

In the third essay, we assess the long-run effects of the HIV/AIDS epidemic on Kenya's economy and people. In contrast to the results derived in the early literature on HIV/AIDS (e.g. Bloom and Mahal (1997)), we show that the epidemic's effects are strong, confirming the general results derived in Bell at. al (2004), and other recent contributions, but contradicting Young's (2005) findings for South Africa. We show that families reduce fertility when mortality rates rise, which is a key result in Young's (2005) Solovian framework. Yet, though we agree that frameworks in which fertility is fixed do not capture the full effects of epidemic, we do not arrive at his results that adjustments to fertility outweigh those to schooling - for Kenya, at least.

Future research should strive to construct a sounder theoretical basis for the analysis of mortality in the context of decisions on schooling and fertility, thereby combining the empirical findings of the third chapter with the model developed in the first two. To my knowledge, the only theoretical paper addressing premature adult mortality in the context of child labor and fertility is Chakraborty and Das (2005), who assume that household spending on health can reduce the mortality risks, which is arguably rather restrictive in the case of a communicable disease like HIV/AIDS. On the empirical side, additional research is needed to address the issue of what becomes of orphans. In the third essay, we have assumed that orphans are taken in by surviving couples, and treated like their own children. However, this form of pooling is likely to break down if the number of orphans becomes very large. Evidence from Kenya suggests that many orphans grow up alone, particularly in urban areas. Public interventions outside the health sector, for example, in the educational sector, are also worth analyzing, as are combined programs. For example, Mexico's Oportunidades Program and Brazil's Bolsa Familia Program offer poor households cash transfers conditional on their children's school attendance and visits to a health clinic. Similar programs can be set up in the context of HIV/AIDS, for instance, combining school-attendance subsidies with investment in health infrastructure. Although such complex programs are more challenging to analyze than simple programs targeting only the health sector, they might prove to be significantly more efficient in mitigating the economic effects of the HIV/AIDS epidemic.

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