Essays on the Theory of Productive Government Activity and Economic Growth

Inauguraldissertation zur Erlangung der Würde
eines Doktors der Wirtschaftswissenschaften an der
Fakultät für Wirtschafts- und Sozialwissenschaften
der Ruprecht-Karls-Universität Heidelberg

vorgelegt von

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Dezember 2010

Acknowledgments

First and foremost, I would like to thank my supervisor Andreas Irmen for his excellent guidance, ongoing support, and remarkable availability during the writing of this dissertation. I have benefited a lot from his valuable comments and motivating feedback on the content of my research and the way of presenting it. I am grateful for his continuous help and confidence in me.

I also wish to thank Berthold Wigger for his immediate willingness to join the dissertation committee.

Moreover, I would like to express my gratitude to all members of the Chair of Economic Policy who have assisted and accompanied me over the past years. Special thanks go to Thomas Eife who was always ready to discuss my work and provided many helpful comments. I have learned a lot from his suggestions and advice and greatly appreciate his encouraging support at the right time.

Many other colleagues and friends have helped to make my time at the University of Heidelberg both enjoyable and enriching, especially Jianfeng Chen, Nathalie Jorzik, Andrea Leuermann, Christoph Lipponer, Amaranta Melchor del Río and of course all my AWI football friends. Furthermore, I would like to thank my family and friends outside university who always believed in me.

Finally, I am deeply grateful to my husband Heriberto, my parents, and my brother Matthias for everything. Without their love, unconditional support, and patience this dissertation would not have been possible. It is dedicated to them.

Contents

Chapter 1

Introduction

Public expenditure on infrastructure such as roads, ports, or communication systems, public research and education spending as well as the enforcement of the "rule of law" are vital to the production possibilities of firms, and thus to the economic potential of an economy. Easterly and Rebelo (1993) and, more recently, Canning and Pedroni (2004) find empirical evidence for long-run growth effects associated with public investment in infrastructure. Similarly, Knack and Keefer (1995) and Kaufmann and Kraay (2002) establish that the strength of the rule of law has a positive impact on long-run economic growth. This dissertation studies in four essays the theoretical relationship between such forms of productive government activity and long-run economic growth.

Solow (1970) and Arrow and Kurz (1970) already discussed criteria for optimal public investment in the context of the neoclassical growth model. However, in the long run the per capita growth rate in this type of model depends entirely on the exogenous rate of technological progress. Thus, in order to assess the effect of productive government activity on long-run economic growth, this dissertation focuses on endogenous growth models in which variations in fiscal policy parameters may have an effect on long-run growth. First, we give a comprehensive overview of the existing literature. Then, we address some of its shortcomings and hitherto unexplored issues.

Chapter 2 provides a critical survey of the recent theoretical literature that studies the role of productive government expenditure for sustained economic growth. For this purpose, we develop a uniform analytical framework in which the seminal paper in this field, Barro (1990), as well as the ensuing contributions can be discussed and compared. The existing literature incorporates, as we argue, many but not all relevant facets of the link between productive government activity and economic growth. Three of them are addressed in more detail in this dissertation.

First, the existing literature usually models the services derived from productive government activity as an argument in the production function of individual firms. In this way productive government expenditure enhances the productivity in the economy by raising the marginal product of private capital. This appears to be a natural form of modeling for services derived from public infrastructure, i. e., from roads, power and communication networks or the public education system. However, activities that strengthen the rule of law such as police services, courts, the design and enforcement of patent rights, or the stability of laws and institutions are better viewed as affecting the ability of people to retain the rights to their goods or profits from production; thereby shaping their incentives to invest, innovate, and produce.

Second, the main body of the existing literature is rooted in the tradition of investment-based endogenous growth models, in which growth originates with private investment either in physical or human capital. Incorporating productive government activity into idea-based endogenous growth models, i. e., models in which growth arises from technological innovations, allows us to analyze new questions, e. g., related to the effect of public policy on the incentives to invest in innovations and the speed of technological progress.

Third, in the literature presented in the second chapter the share of productive government expenditure is either exogenous or chosen by a benevolent planner while in reality it is the outcome of an election process, and thus reflects fundamental characteristics of the process of collective decision-making and the distribution of preferences and endowments in the population. In other words, this literature does not capture how changes in the distribution of preferences, for instance due to population aging, endogenously affect government activity and economic growth via a democratic voting process.

Chapter 3 addresses the first and the second point by studying the government's ability and willingness to enforce the rule of law and the ensuing consequences for innovation and economic growth in an endogenous growth framework with an expanding set of product varieties. The strength of the rule of law influences economic growth by determining the profit that firms expect from an innovation investment.

Chapters 4 and 5 focus on the third point. Population aging, i. e., the process by which older individuals become a proportionally larger fraction of the total population, is one of the most important demographic phenomena of our time. For instance, in Germany the old-age dependency ratio, which measures the ratio of elderly persons to non-elderly adults, increased from 16 in 1955 to 28 in 2005. Thus, in 2005 100 individuals of working age had to support 28 retired individuals. According to forecasts of the United Nations this ratio will rise to 59 in the year 2050. Population aging is likely to alter the support for different types of government spending, thereby affecting economic growth. A typical concern raised in the public debate is that population aging, by increasing the political weight of the elderly, leads to increased spending on the elderly, e.g., on health and care services. For instance, the former German president Roman Herzog in 2008 issued the warning that Germany risks becoming a "pensioners' democracy" (Rentner-Demokratie). It is feared that this trend crowds out public productive expenditure and increases the overall tax burden, thus slowing down economic growth. To theoretically evaluate this question, Chapters 4 and 5 incorporate heterogeneity in age and preferences as well as a democratic voting process into the analysis.

The following sections contain more detailed summaries of Chapters 2 to 5. The dissertation is organized in such a way that the chapters can be read independently of each other. All references are collected in the bibliography.

Chapter 2: Survey

The second chapter, which is based on Irmen and Kuehnel (2009), establishes a well-defined analytical framework in which we then review the recent theoretical literature that aims at the identification of possible channels through which productive government activity and its financing affect economic growth and welfare. In this framework, the services derived from productive government activity directly enter the production function of individual firms such that there are constant returns to scale from these services and private capital. Our setup allows government activity to be either a flow or a stock variable. In the former case, government spending corresponds to the provision of public services that instantaneously affect the production technology of firms. In the latter case, today's government spending adds to the stock of public capital and affects the future production technology of firms. In both cases, the presence of productive government activity assures sustained growth of per capita variables. Moreover, following most of the literature, the framework is set up in continuous time with many identical infinitely-lived dynasties.

We first focus on the flow model. The benchmark scenario introduces government expenditure as a pure public good into the production function of individual firms. We then extend the analysis to incorporate other relevant aspects presented in the literature that interact with the effect of productive government expenditure on economic growth. They include adjustment costs, congestion effects, utility-enhancing public consumption services, endogenous labor supply both in closed and small open economies. The flow modeling approach is appealing because of its analytical simplicity: we show that an understanding of the mechanics and the core results of each aspect can be gained from the study of the respective Euler equation. We use this property to characterize the determinants of the equilibrium growth rate and to analyze the role of fiscal policy measures on this growth rate. Moreover, for each variant we conduct a welfare analysis and derive the circumstances under which the Pareto-efficient allocation can be implemented.

The main insights of this approach can be summarized as follows. Productive government expenditure affects the steady-state growth rate of consumption through a direct effect on the technology and an indirect effect on investment incentives through its financing. In a closed economy, the direct effect is always strictly positive. The reason for this is that the consumption growth rate positively depends on the rate of return to private capital, which in turn increases in productive government activity. By contrast, in a small open economy, the consumption growth rate is independent of domestic production conditions such that the direct effect is zero. If productive government spending is fully financed by an income tax, then the indirect effect on consumption growth is strictly negative because such a tax reduces the after-tax marginal return on private capital. Thus, in the polar case of full income tax financing, the opposing forces of the direct and indirect effect give rise to a growth-maximizing share of productive government expenditure. In most variants of the flow model, this growth-maximizing share corresponds to the output elasticity of the public input. Generally, the welfare-maximizing, i. e., the

Pareto-efficient, share of productive government expenditure coincides with the growth-maximizing one. However, when government not only spends on productive purposes, but also provides public consumption services this need not be the case. As public consumption services are non-productive but utility-enhancing, their provision may drive a wedge between the goals of growth and welfare maximization. Finally, we show that in each setting appropriate fiscal policy measures can implement the Pareto-efficient allocation. For instance, a strictly positive income tax rate may be used to correct for overaccumulation of private capital due to a negative externality such as congestion.

Then, we turn to the stock approach. Modeling productive government expenditure as a stock variable appears to be more realistic as many public services like public infrastructure are considered to be stocks. However, this approach makes the analysis more complicated because it entails complex transitional dynamics and the steady-state growth rate is no longer determined by the Euler equation alone. Nevertheless, we argue that the analysis of the balanced growth path in the stock case confirms most results that are obtained in the flow case. The most important difference occurs in the welfare analysis. The fact that current public investment only becomes productive tomorrow whereas the cost in terms of foregone consumption is paid today reduces the welfare-maximizing share of government investment. However, the stock approach allows us to address new questions that cannot be raised in a flow context. We make this point with an analysis that introduces an additional productive use of government expenditure, namely the maintenance of public capital.

Finally, the last part of Chapter 2 considers three more fundamental variations of the analytical framework and checks the robustness of the policy implications derived so far. First, in a stochastic version of the flow model the policy implications turn out to be similar in spite of the fact that precautionary savings drive a wedge between the goals of growth and welfare maximization. Second, under increasing returns to scale multiple equilibria exist. This makes it hard to formulate policy recommendations because there is no natural way to select among multiple equilibria. Third, in a non-scale endogenous growth model productive government services cease to have an effect on the steady-state growth rate. The reason for this is that in a non-scale model the steady-state growth rate is entirely determined by the technology of the economy and its consistency with a balanced growth path.

Chapter 3: Rule of Law, Innovation, and Growth

The focus of the third chapter is on the link between a weak rule of law, firms' incentives to engage in innovation investments, and economic growth. For this purpose, the chapter incorporates an imperfect rule of law into an endogenous growth framework where growth results from an expanding set of product varieties à la Grossman and Helpman (1991). The rule of law is imperfect in the sense that producing firms' property rights over profits are not fully secured. One may think of firms being subject to partial expropriation by an organization such as the mafia. The strength of the rule of law is then captured by the fraction of profits protected from expropriation.

In a first step, we take the strength of the rule of law as exogenous. In this scenario, we establish that a weak rule of law, because of the resulting weak innovation incentives, is a major reason why an economy may be caught in a "no-growth trap". Hence, a minimum strength of the rule of law can be thought of as a necessary condition for sustained growth. However, a weak rule of law may be Pareto-improving. This is the case when the equilibrium growth rate exceeds the Pareto-efficient one. Such a situation occurs in the variety expansion growth model when the gains from specialization captured by the CES production function are small (see Bénassy, 1998; de Groot and Nahuis, 1998). Then, a weakening of innovation incentives is indeed preferable. A means to accomplish this is a weaker rule of law which essentially acts on innovation incentives as a "mafia tax".

In a second step, we endogenize the strength of the rule of law by allowing the government to use tax resources to improve its strength. We consider two situations. First, the government can invest produced output in the enforcement of the rule of law. Second, the government has to hire a fraction of the workforce as policemen who then enforce the rule of law. In the first case, government intervention can always shift the economy from a no-growth path onto a path with strictly positive growth. In the second case, the necessity to employ a scarce resource reduces the government's ability to move the economy to a positive growth equilibrium. In both cases, government activity that assures positive equilibrium growth rates may not be optimal from a welfare point of view. This is the case if the costs of fighting the mafia induce an instantaneous welfare loss in terms of foregone current consumption that outweighs the increasing future consumption possibilities that arise along a path with strictly positive growth. Moreover, we find that a government intervention is more likely to be desirable if the economic environment is more prone to growth, e. g., if the economy's research technology is sufficiently productive or its market size large enough.

Chapters 4 and 5: Democratic Voting and the Implications of Population Aging

The purpose of the fourth and fifth chapter is to analyze how population aging endogenously affects taxes, the composition of government spending, and long-run economic growth via a democratic voting process.

The fourth chapter approaches this question by incorporating heterogeneity and a demographic structure directly into a version of the infinitely-lived agent (ILA) endogenous growth framework presented in the second chapter. More specifically, we consider a continuum of infinitely-lived households, each of them comprising not only working young but also economically-dependent elderly members. The age composition of a household is captured by the so-called support ratio, which measures the share of workers among the number of total household members. Households are heterogeneous with respect to their support ratio. Then, population aging corresponds to a shift in the distribution of households such that there are more households with a small support ratio. This shift leaves the overall size of the population unaffected. Besides a productive public input, the government provides a public consumption good that satisfies the preferences of the elderly. Both types of government spending are fully financed via taxes on household income. By majority voting, the households determine the policy mix that will be implemented by the government with full commitment in all periods.

In a first step, we show that for a given time-invariant public policy mix there exists a unique decentralized competitive equilibrium in which all households (independent of their support ratio) accumulate at the same rate. However, the age composition of each household determines its level of aggregate household income and consumption per household member.

In a second step, we endogenize government policy. To derive the political equilibrium, we first establish that each household has a unique most preferred policy mix that does not depend on time. While all households want the same share of output to be invested in productive purposes, they prefer different shares of output to be spent on the public consumption good that benefits their elderly members, and therefore different tax levels. Household preferences can be ranked according to their idiosyncratic support ratio: households with a smaller support ratio prefer higher spending on the elderly. The equilibrium policy mix then coincides with the one preferred by the median voter.

Finally, population aging is predicted (i) to increase public spending on the elderly (as a share of output), (ii) not to affect productive government expenditure (as a share of output), (iii) to raise the overall tax burden, and (iv) to lower the economy's growth rate.

Thus, the fourth chapter shows that the link between population aging, endogenous government spending, and endogenous economic growth can be analyzed in a heterogeneous agent version of the standard infinite-horizon framework used by most of the literature. The qualitative results of this approach are in line with notions voiced in the public debate.

The fifth chapter compares these results to those obtained in a model of overlapping generations in which agents live for two periods and vote each period.

Population aging now corresponds either to an exogenous, permanent decline in fertility or to an exogenous, permanent increase in life expectancy. Both phenomena increase the economy's old-age dependency ratio and change the population size. We focus on the same public spending categories: productive public expenditure and a public consumption good that yields utility to the elderly. The government finances its expenditure by levying a uniform, proportional tax on the income of the young and the old.

As government policy choices are of differing concern to the young and the old, they disagree on which policy mix they prefer to be implemented. The resolution of the resulting political conflict is modeled under the assumption of probabilistic voting. This assures that the policy proposals represent the interests of both groups of society, reflecting the political process in representative democracies more realistically than the median voter model.

Rational, forward-looking agents vote on government policy at the beginning of each period. Since elections take place each period, policy makers cannot commit to future policy choices. Voters therefore have to form expectations about future policies. In particular, the rational young voters are aware that the current policy choices influence the evolution of the economy, and thus next period's policy choices. In this respect, we focus on Markov perfect equilibria, i. e., equilibria in which the policy choices expected for a certain period depend only on the value of the fundamental state variable at that time. Such an equilibrium in which voters consider the economic as well as political repercussions of their policy choices is called a politico-economic equilibrium.

In this setup, population aging has two opposing effects on economic growth. On the one hand, population aging, by increasing the relative weight that the political process attaches to the interests of the old relative to the young, (i) increases public spending on the elderly (as a share of output), (ii) does not affect productive government spending (as a share of output), and (iii) increases the income tax rate. The latter depresses the economy's growth rate of per capita variables. These results are qualitatively the same as in Chapter 4. On the other hand, for a given policy mix population aging accelerates economic growth. This positive growth effect is due to reduced capital dilution if population aging follows a decline in fertility. In contrast, an increase in life expectancy positively impinges on economic growth by increasing the incentives to save. In both scenarios, we find that the second effect dominates the first such that population aging overall increases the economy's long-run growth rate.

Chapter 2

Productive Government Expenditure and Economic Growth - A Survey

2.1 Introduction

Few would dispute that public expenditure on infrastructure such as roads, ports, or communication systems, public research spending as well as the provision of basic education and medical services raises the economic potential of an economy. At least since the influential study of Aschauer (1989) and the following discussion (see de Haan and Romp, 2007, for a recent survey of this empirical literature) it is argued that a rise in productive government activity increases output. Easterly and Rebelo (1993) and, more recently, Canning and Pedroni (2004) find evidence for long-run growth effects associated with public investment in infrastructure. In addition, many case studies highlight the growth-enhancing potential associated with such investments (see, e. g., OECD, 2007).

The purpose of this chapter is to provide a critical survey on the recent theoretical literature that aims at the identification of possible links between productive government activity and long-run economic growth and the assessment of the resulting allocation in terms of welfare. To accomplish this, we have to focus on endogenous growth models where variations in fiscal policy parameters may have an effect on long-run growth.[1] To the best of our knowledge, Barro (1990) is the seminal paper in this field. It introduces government expenditure as a public good into the production function of individual firms. In this way the rate of return to private capital increases which in turn stimulates private investment and growth.

We show that the ensuing literature is able to extend Barro's analysis to incorporate many relevant aspects that interact with the effect of public services on economic growth. They include adjustment costs, congestion effects, utility-enhancing public consumption services, endogenous labor supply both in closed and small open economies. We establish that the mechanics and the core results for each aspect can be gained from the study of the respective Euler equation. We use this approach to characterize the determinants of the equilibrium growth rate and to analyze the role of fiscal policy measures on this growth rate. Moreover, we conduct a welfare analysis and derive the circumstances under which the welfare-maximizing allocation can be implemented.[2]

While Barro (1990) treated productive government expenditure as a flow variable, the paper by Futagami et al. (1993) introduces the provision of productive government services as a stock. At first sight, this approach is more appealing because services like public infrastructure are more realistically described as stocks. However, the advantage in terms of realism has a price in terms of analytical complexity. For instance, this approach usually entails complex transitional dynamics and the steady-state growth rate is no longer determined by the Euler equation alone. Nevertheless, we argue that the analysis of the balanced growth path in the stock case confirms most results that are obtained in the flow case. An important difference occurs in the welfare analysis. The fact that current public investments become only productive tomorrow tends to reduce the welfare-maximizing share of government investment. We show that these findings are robust in a setting where a flow and a stock of public services are provided simultaneously.

[1] For the study of various aspects of public expenditure in the neoclassical growth model, the interested reader is referred to Arrow and Kurz (1970), Aschauer (1988), Barro (1989), Baxter and King (1993), Fisher and Turnovsky (1995), or Fisher and Turnovsky (1998).

[2] Throughout we stick to a continuous-time framework with infinitely-lived dynasties. Moreover, we do not explicitly consider education and human capital formation as a government activity. This is at the heart of, e. g., Glomm and Ravikumar (1992) or, more recently, Gómez (2008). See Zagler and Dürnecker (2003) for a survey of this literature.

However, the stock approach allows to address new questions that cannot be raised in a flow context. We make this point with an analysis that introduces an additional productive use of government expenditure, namely the maintenance of the stock of public capital.

Finally, we turn to more fundamental variations of the analytical framework and ask for the robustness of the policy implications derived so far. For a stochastic setting we conclude that the policy implications are similar in spite of the fact that precautionary savings drive a wedge between the goals of growth and welfare maximization. In contrast, we argue that the knife-edge assumption of constant returns to scale with respect to private and public capital is responsible for many findings. For instance, under increasing returns multiple equilibria are endemic. This complicates the policy implications as the effect of fiscal policy measures is now conditional on expectations. Similarly, if productive government services are provided in a non-scale model, they cease to have an effect on the steady-state growth rate.

In light of these findings, we conclude that future research ought to focus on a deeper understanding of the policy implications that matter in reality. Certainly, a focus on the analysis of productive government services on economic growth in idea-based endogenous growth models is likely to enhance our understanding of the relationship between productive government expenditure and economic growth.

The remainder of this chapter is organized as follows. Section 2.2 sets out the basic analytical framework. In Section 2.3 we deal with the flow model and variants of this approach. Section 2.4 presents variants of the stock approach and compares them to the respective flow cases. Important extensions such as uncertainty, increasing returns and non-scale models are covered in Section 2.5. Section 2.6 concludes. The Appendix derives somewhat more complicated results that appear in Section 2.4.1.

2.2 The Basic Analytical Framework

Consider a closed economy in continuous time with many identical and competitive household-producers and a government. We denote per-household variables by small letters, whereas capital letters represent aggregates. For instance, $k(t)$ is the private capital input of an individual firm at t, and $K(t)$ the economy's aggregate capital stock at t. Henceforth, we suppress the time argument whenever this does not cause confusion. We represent household-producers by the interval $[0, N]$, $N > 1$, such that $K = Nk$. The "number" of household-producers remains constant over time. The economy has one good that can be consumed or invested. At all t, prices are expressed in units of the contemporaneous output of this good.

Each producer has access to the per-period production function

$$y = f(k, g) = Ak^{1-\alpha} g^{\alpha}, \qquad 0 < \alpha < 1, \tag{2.1}$$

where y denotes firm output at t, $A > 0$ the time-invariant total factor productivity, and g the services derived by the firm from productive government activity at t. Private capital, k, has a positive but diminishing marginal product, and for simplicity does not depreciate.[3] The function f has constant returns to scale with respect to both inputs. The possibility of steady-state growth arises since government activity acts as a countervailing force on the diminishing marginal product of private capital. To keep the marginal and the average productivity of private capital constant, in a steady state k and g have to grow at the same rate. To simplify the exposition, we work with the Cobb-Douglas specification.

Household-producers are infinitely-lived and derive utility in each period from private consumption. Their intertemporal utility is

$$u = \int_0^\infty e^{-\rho t} \ln c \, dt, \tag{2.2}$$

where $\rho > 0$ is the instantaneous rate of time preference. For expositional convenience, we stick to a logarithmic per-period utility function. Most of the results presented in what follows readily extend to more general per-period utility functions with a constant

[3]Labor is not mentioned as a separate input in the production function. This is a valid shortcut if we interpret the profit of each firm as the wage income that is earned by labor. More precisely, we may admit to each household-producer an exogenous per-period labor endowment $\bar{l} = 1$ that is inelastically supplied and consider a production function $y = Ak^{1-\alpha} \left(\bar{l} g\right)^{\alpha}$. Marginal cost pricing of labor and a real wage consistent with firms hiring $\bar{l}$ determines the wage income equal to the profit income of firms that produce according to (2.1) without labor. This is an implication of Euler's law for linear-homogeneous functions.

elasticity of intertemporal substitution different from unity. Each household-producer receives net output and determines how much to consume and how much to invest in private capital. Her flow budget constraint is

$$\dot{k} = (1 - \tau_y)\, f\,(k, g) - (1 + \tau_c)c - \tau, \tag{2.3}$$

where τ_y and τ_c denote time-invariant tax rates on income/output and consumption, and τ is a lump-sum tax. When choosing c and k to maximize her utility the individual household-producer takes the level of public services as given and disregards the possible impact of her decision on the amount of public services provided. Then, her intertemporal optimization leads to the Euler condition

$$\gamma_c \;=\; (1 - \tau_y)\frac{\partial f}{\partial k} - \rho, \tag{2.4}$$

i. e., the growth rate of consumption γ_c depends on the difference between the after-tax private marginal return on private capital and the rate of time preference. Throughout, we assume that the economy is sufficiently productive to sustain a strictly positive growth rate γ_c.

We denote G the aggregate amount of productive government activity at t from which individual firms derive the services g. Conceptually, G may be a flow or a stock variable. In the former case, government spending corresponds to the provision of public services that instantaneously affect the production technology of firms. In the latter case, today's government spending adds to the stock of public capital and affects the future production technology of firms. In any case, the government claims resources from household-producers and transforms them one-to-one into a productive input to which firms get access. We assume that the government's budget is balanced in all periods. Let Y and C denote aggregate output and consumption at t and define total tax receipts at t by $T \equiv \tau_y Y + \tau_c C + \tau N$. Then, the budget constraint at t is

$$G = T \quad \text{or} \quad \dot{G} = T \tag{2.5}$$

for the flow and the stock case, respectively. Throughout, we focus on tax-financed expenditure and disregard funding via public debt.

2.3 Productive Government Activity as a Flow

Along a steady-state growth path with all variables growing at a constant rate, government expenditure must be proportionate to the size of the economy. To comply with this requirement, we stipulate for all t that

$$G = \theta_G Y, \tag{2.6}$$

where $\theta_G \in (0,1)$ is a time-invariant constant measuring the fraction of current output that constitutes the current flow of productive government expenditure. If G includes public investment as well as government expenditure on public order and safety, on economic affairs, and on health and education, one finds for a sample of 19 Organization for Economic Cooperation and Development (OECD) countries that the average θ_G over the time period 1995 to 2002 ranges between 10% and 20%.[4]

2.3.1 The Pure Public Good Case

Following Barro (1990), we first consider the case where government services are neither rival nor excludable. In this case, G is a pure public good and $g = G$ such that the production function (2.1) becomes

$$y = AG^{\alpha}k^{1-\alpha}, \qquad 0 < \alpha < 1. \tag{2.7}$$

One may of think of G as government expenditure on basic education, the provision of medical services, or public research spending that increases the productivity of private inputs of all firms in the same manner.

[4]This finding is based on our own computations. We use data collected in UNdata (2008). Public investment corresponds to gross fixed capital formation of general government. Government expenditure on public order and safety, on economic affairs, and on health and education are subcategories of government final consumption expenditure. The sample includes Austria, Belgium, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Japan, Luxembourg, Mexico, the Netherlands, Norway, Portugal, Sweden, the United Kingdom and the United States of America. This selection of OECD countries maximizes the number of countries and the length of the time period for which a full set of comparable annual data on the components of G mentioned above are available. The sample average of θ_G across countries and time is approximately 15%.

Decentralized Equilibrium

Following the reasoning that led to the Euler equation (2.4) we find

$$\gamma_c = (1-\tau_y)(1-\alpha)A\left(\frac{G}{k}\right)^{\alpha} - \rho. \tag{2.8}$$

The ratio of government spending per unit of private capital consistent with condition (2.6), the aggregation $Y = Ny$, and the production function (2.7) is $G/k = (AN\theta_G)^{1/(1-\alpha)}$. Upon substitution of the latter in (2.8) we obtain

$$\gamma_c = (1-\tau_y)(1-\alpha)\left(AN^{\alpha}\theta_G^{\alpha}\right)^{\frac{1}{1-\alpha}} - \rho. \tag{2.9}$$

At this stage, three remarks are in order. First, there are admissible values for τ_y, τ_c, τ, and θ_G that satisfy the budget constraint (2.5). Hence, given τ_c and τ, τ_y and θ_G that appear in (2.9) are not independent. Second, one can show that the economy immediately jumps onto its steady-state path along which all per capita variables grow at rate γ_c. Third, the equilibrium growth rate depends on the "number" of household-producers, i. e., there is a scale effect. The latter occurs since individual firm productivity depends on aggregate spending ($G = \theta_G Y$). Then, with more firms the externality is more pronounced.[5]

An interesting question is how the size of the government and the mode of funding government spending affects the economy's steady-state growth rate. A useful benchmark has $\tau_c = \tau = 0$ such that $\theta_G = \tau_y$. In this case, there is a growth-maximizing expenditure share equal to the output elasticity of government expenditure, $\theta_G^* = \alpha$ (Barro, 1990).[6] Intuitively, it balances two opposing effects. A rise in θ_G increases the private marginal product of private capital and reduces its after-tax value through a necessary increase in the distortionary income tax. At θ_G^*, government expenditure satisfies the so-called *natural condition of productive efficiency*, i. e., the marginal contribution of government

[5] To eliminate the scale effect one may assume that the government service is not excludable but rival such that each producer receives a proportionate share of government services, i. e., $g = G/N$. In this case, the economy's steady-state growth rate is $\gamma_c = (1-\tau_y)(1-\alpha)\left(A\theta_G^{\alpha}\right)^{1/(1-\alpha)} - \rho$ and is independent of N. We shall get back to this case in Section 2.3.3 where we discuss different forms of congestion.

[6] For more general production functions $f(k, g)$ with constant returns to scale in its inputs the growth-maximizing expenditure share remains equal to the respective output elasticity. This elasticity, however, need not be constant but may vary with G and other parameters. This generalization may prevent closed-form solutions (see, e. g., Ott and Turnovsky, 2006, for a discussion)

expenditure to aggregate output is one.[7] The steady-state growth rate may be further increased if a strictly positive consumption or lump-sum tax is levied. In the present context, a consumption tax acts like a lump-sum tax and both may be used to reduce the distortionary income tax. However, there is little reason why the steady-state growth rate should be arbitrarily large because faster economic growth has a cost in terms of foregone consumption. To assess the desirability of a given consumption growth rate we have to compare it to the allocation chosen by an omniscient social planner.

Pareto Efficiency

Contrary to the household-producer, the social planner knows that - given θ_G - the choice of k affects the level of government expenditure G through condition (2.6) and $Y = Ny$. Hence, he perceives the production function of the representative household-producer as

$$y = (AN^{\alpha}\theta_G^{\alpha})^{\frac{1}{1-\alpha}} k. \tag{2.10}$$

The aggregate resource constraint is $N\dot{k} = Ny - G - Nc$. It results as the sum of all individual flow budget constraints (2.3) in conjunction with the government's budget constraint and (2.6). Expressed in per household terms, this is

$$\dot{k} = (1 - \theta_G)y - c. \tag{2.11}$$

Throughout, we shall refer to an allocation as *constrained Pareto-efficient* if the planner takes the share of government expenditure θ_G as a given constant. The unconstrained or, in short, the *Pareto-efficient* allocation is the one obtained when the planner chooses θ_G optimally.

Here, the constrained Pareto-efficient allocation obtains from the maximization of u given by (2.2) with respect to c and k subject to (2.11). The corresponding Euler

[7] To grasp the natural condition of productive efficiency consider a coal mine that uses its coal as an input. Then, what is the output-maximizing amount of the coal input? Intuitively, as long as an additional unit of coal raises output by more than one unit it will be used; if its marginal product is smaller, it will not. The quantity that maximizes output obtains when the marginal product of coal in its production is one. In the present context, we have from equation (2.7) with $Y = Ny = ANG^{\alpha}k^{1-\alpha}$: $dY/dG = \alpha\,(Y/G) = \alpha/\theta_G^{*} = 1$.

condition delivers the steady-state growth rate of all per household variables and is given by

$$\gamma_c^P = (1-\theta_G)\left(AN^{\alpha}\theta_G^{\alpha}\right)^{\frac{1}{1-\alpha}} - \rho. \tag{2.12}$$

The first term on the right-hand side is the social marginal return on private capital. It need not coincide with the after-tax private marginal product that matters in (2.9). The comparison of the equilibrium to the planner's growth rate reveals that

$$\gamma_c = \gamma_c^P \quad \Leftrightarrow \quad (1-\tau_y)(1-\alpha) = 1-\theta_G. \tag{2.13}$$

These growth rates generically differ for two reasons. First, in equilibrium intertemporal prices are distorted due to the income tax. If the government sets $\tau_y = 0$ to eliminate this distortion and finances its expenditure via lump-sum taxes these growth rates may still differ as θ_G need not be equal to α. This reflects the second difference. The planner internalizes the externality associated with the provision of the public good, i. e., when choosing c and k he accounts for condition (2.6).

If we extend the planner's choice set and allow him to determine the size of the government in addition to c and k, one finds that he chooses $\theta_G^P = \alpha$. The Pareto-efficient growth rate is then given by γ_c^P of (2.12) with $\theta_G = \alpha$. As a consequence, the equilibrium and the Pareto-efficient allocation coincide if $\tau_y = 0$ and θ_G is chosen optimally.

2.3.2 Productive Public Expenditure and Adjustment Costs

Often the productive use of new private capital requires adjustment costs. Examples include costs for the installation of equipment or the schooling of employees. Adjustment costs increase the effective costs of private investment and may therefore discourage the accumulation of private capital. Here, we introduce this feature into the pure public good framework of the previous section.

Following Turnovsky (1996a), we assume that productive government expenditure reduces adjustment costs. For instance, due to a better road network the setup costs of a new factory may be lower. We capture this feature with an adjustment cost function per unit of investment given by $\phi(\theta_G)\, i/(2k)$, where i denotes investment per household-producer. A higher share of government activity reduces adjustment costs, though at a declining rate, i. e., $\phi' < 0 < \phi''$. As in Hayashi (1982), we assume that adjustment costs are proportional to the rate of investment per unit of installed capital and not to the absolute level of investment. Accordingly, the investment cost function is

$$\varphi(i, k, \phi(\theta_G)) \equiv \left(1 + \frac{\phi(\theta_G)}{2}\frac{i}{k}\right) i. \tag{2.14}$$

Decentralized Equilibrium

Individual household-producers choose a plan (c, k, i) for each t to maximize u of (2.2) subject to the constraints

$$i = \dot{k} \quad \text{and} \tag{2.15}$$

$$(1-\tau_y)\, AG^{\alpha}k^{1-\alpha} - \tau = (1+\tau_c)c + \varphi(i, k, \phi(\theta_G)), \tag{2.16}$$

where the latter equalizes disposable income to consumption and investment outlays.

The resulting optimality condition with respect to i reveals that the current-value shadow price of installed capital in units of current output is equal to the marginal investment costs, i. e.,

$$q = 1 + \phi(\theta_G)\frac{i}{k}. \tag{2.17}$$

Hence, for an investing firm ($i > 0$) the value of installed capital exceeds unity. With (2.15) it follows that the steady-state growth rate of private capital is

$$\gamma_k = \frac{q-1}{\phi(\theta_G)}. \tag{2.18}$$

The Euler equation is now given by

$$\gamma_c = \frac{(1-\tau_y)(1-\alpha)\left(A\theta_G^{\alpha}N^{\alpha}\right)^{\frac{1}{1-\alpha}}}{q} + \frac{\dot{q}}{q} + \frac{(q-1)^2}{2q\phi(\theta_G)} - \rho. \tag{2.19}$$

The first three terms on the right-hand side represent the rate of return on acquiring a unit of private capital at price q. The first term denotes the after-tax private marginal return on private capital deflated by the cost of capital q. The second term is the rate of capital gain. The third term reflects the marginal reduction in adjustment costs when k increases for given i deflated by q. In the absence of adjustment costs $q = 1$ for all t and (2.19) reduces to (2.9).

In the steady state, per household variables such as c, k, and i grow at the same rate; from (2.17) we also have $\dot{q} = 0$. Using (2.17) and (2.18) in the Euler condition (2.19) delivers the steady-state growth rate implicitly as

$$\frac{\phi(\theta_G)}{2}\gamma_c^2 + [1 + \phi(\theta_G)\rho]\gamma_c = (1-\tau_y)(1-\alpha)\left(A\theta_G^{\alpha}N^{\alpha}\right)^{\frac{1}{1-\alpha}} - \rho. \tag{2.20}$$

Hence, the right-hand side of (2.20) coincides with the equilibrium growth rate of (2.9) where adjustment costs are absent. However, since the left-hand side of (2.20) increases faster than proportionately in γ_c, the resulting steady-state growth rate must be smaller with than without adjustment costs.

Turning to the effect of productive government expenditure on the steady-state growth rate for the benchmark scenario with full income tax funding ($\theta_G = \tau_y$) we find

$$\frac{d\gamma_c}{d\theta_G} = \frac{-\gamma_c\phi'(\theta_G)\left(\frac{\gamma_c}{2} + \rho\right) + \alpha(1-\theta_G)\frac{y}{k\theta_G} - (1-\alpha)\frac{y}{k}}{1 + \phi(\theta_G)(\gamma_c + \rho)}. \tag{2.21}$$

The latter highlights three channels through which government activity affects the growth rate. First, the reduction in adjustment costs ($\phi' < 0$) increases the growth rate. The second and the third channel matter in the same way as in the scenario without adjustment costs: on the one hand, productive government expenditure enhances the productivity of the existing capital stock, on the other hand, the government must balance its budget which brings about a rise in the distortionary income tax rate.

Observe that a growth-maximizing expenditure share $\theta_G^* \in (0,1)$ may exist. It must be strictly greater than in the world without adjustment costs since $d\gamma_c/d\theta_G|_{\theta_G=\alpha} > 0$ (Turnovsky, 1996a). If government expenditure is fully funded by a non-distortionary lump-sum tax the third channel in (2.21) vanishes such that an increase in government spending unambiguously raises the growth rate.

Pareto Efficiency

The social planner internalizes (2.6) and the equilibrium condition $K = Nk$. He maximizes utility (2.2) subject to the resource constraint $(1-\theta_G)y = c + \varphi(i, k, \phi(\theta_G))$ and $i = \dot{k}$, where y is given by (2.10). Following the steps that led to the implicit statement of the equilibrium growth rate in (2.20), we obtain here

$$\frac{\phi(\theta_G)}{2}\left(\gamma_c^P\right)^2 + [1 + \phi(\theta_G)\rho]\gamma_c^P = (1-\theta_G)\left(A\theta_G^{\alpha}N^{\alpha}\right)^{\frac{1}{1-\alpha}} - \rho. \tag{2.22}$$

The latter generalizes (2.12) to the case with adjustment costs. Again the left-hand side is strictly convex in γ_c^P such that the constrained optimal growth rate is smaller under adjustment costs.

If we allow the social planner to determine the size of θ_G optimally, the welfare-maximizing share of government expenditure, θ_G^P, is greater than α, thus exceeding its level without adjustment costs. Intuitively, the possibility to reduce adjustment costs provides an additional incentive for the government to expand its activity relative to the size of the economy.

The comparison of the equilibrium to the constrained optimal growth rate reveals that both rates are the same if the right-hand sides of (2.20) and (2.22) take on the same value. From (2.13) we know that this is the case whenever $(1-\tau_y)(1-\alpha) = 1-\theta_G$. Interestingly, adjustment costs alter the implications of this condition for the optimal tax policy. For instance, if government expenditure is fully financed via lump-sum taxes ($\tau_y = 0$), then the Pareto-efficient growth rate cannot be implemented since $\theta_G^P > \alpha$. The reason is that a higher θ_G^P does not only internalize the externality associated with the pure public good but also reflects the planner's incentive to reduce adjustment costs. Therefore, at θ_G^P the equilibrium incentives to invest are too pronounced relative to the efficient growth rate. Accordingly, a strictly positive income tax $\tau_y^P = \left(\theta_G^P - \alpha\right) / (1-\alpha) > 0$ is needed to support the Pareto-efficient allocation.

2.3.3 Public Goods Subject to Congestion

Often, the services derived from the provision of a public good are subject to congestion. Congestion effects arise if public goods are partially rival, i. e., their use as a productive input by one firm diminishes their usefulness to other firms. Examples include road infrastructure or police and fire protection.

Two forms of congestion can be distinguished, relative and aggregate (absolute) congestion. In the former case, the level of services derived by an individual firm depends on its size relative to the aggregate of firms. We refer to aggregate congestion if the level of services received by the individual firm is decreasing in the aggregate usage. As noted by Eicher and Turnovsky (2000, p. 344) highway usage is an example of the former and police protection an example of the latter:

> "Unless an individual drives his car, he derives no service from a publicly provided highway, and in general the services he derives depend upon his own usage relative to that of others in the economy, as total usage contributes to congestion. Police protection may serve as an example of absolute congestion: in principle, people always enjoy this service, independent of their own actions, though the amount of service they may actually derive varies inversely with aggregate activity and the demands this places on the limited resources devoted to this public service."

To study relative congestion, we use the ratio of individual to aggregate private capital, k/K, to measure the size of an individual firm relative to the economy. Then, the productive services that a firm derives from public expenditure G is

$$g = G\left(\frac{k}{K}\right)^{1-\sigma_G}, \tag{2.23}$$

where $\sigma_G \in [0,1]$ parameterizes the degree of relative congestion associated with the public good G. This specification includes the pure public good case (without congestion) for $\sigma_G = 1$. As σ_G declines, congestion becomes more pronounced. Yet, as long as $\sigma_G \in (0,1)$, the government services derived by a firm of size k increases if G and K grow at the same rate. Barro and Sala-í-Martin (1992) analyze the case where $\sigma_G = 0$. Then, g increases only if G grows faster than K. The latter case is called *proportional congestion* (Turnovsky, 2000b, p. 618). As in equilibrium $K = Nk$, the public good is then rival yet not excludable and the individual firm receives its proportionate share of services $g = G/N$.

One specification the literature uses to capture aggregate congestion is $g = GK^{\sigma_G - 1}$, $\sigma_G \in [0,1]$, i. e., government services are independent of firm size. With this specification, the firms' production function ceases to exhibit constant returns to scale in private and public capital. Therefore, steady-state growth can only arise under additional restrictive conditions. To avoid these complications, we restrict attention to the case of relative congestion with and without excludability.[8] If a public good is excludable, then the government can identify the user and charge an access fee.

[8] See, e. g., Glomm and Ravikumar (1994) for a discrete-time model with absolute congestion. Their

2.3.3.1 Relative Congestion Without Excludability

Under relative congestion, we obtain the production function of the individual firm from (2.1) and (2.23) as

$$y = A\left(\frac{G}{K}\right)^{\alpha}\left(\frac{k}{K}\right)^{-\sigma_G\,\alpha} k. \tag{2.24}$$

Decentralized Equilibrium

Individual firms believe that a rise in k increases their benefit from the provision of public services and disregard the impact of their investment decision on G and K. Applying the reasoning that led to the Euler equation (2.9) and taking into account that $G/k = \left(A\theta_G N^{1-\alpha(1-\sigma_G)}\right)^{1/1-\alpha}$ we find

$$\gamma_c = (1-\tau_y)(1-\sigma_G\alpha)\left(AN^{\sigma_G\alpha}\theta_G^{\alpha}\right)^{\frac{1}{1-\alpha}} - \rho. \tag{2.25}$$

Again, the first term on the right-hand side is the after-tax marginal private return on private capital. An increase in the degree of congestion, i. e., a decline in σ_G, has two effects on γ_c: On the one hand, it augments the output elasticity of private capital, $1-\sigma_G\alpha$. On the other hand, it weakens the scale effect through $N^{\sigma_G\alpha}$. Which effect dominates depends on the number of household-producers and σ_G.

As in the pure public good case, the growth-maximizing share of government expenditure θ_G^* for the benchmark scenario with full income tax financing is equal to α (Turnovsky, 1996c).

Pareto Efficiency

The social planner is aware of the negative externality that the choice of k by an individual firm exerts on the production technology of all other firms via the implied increase in the aggregate capital stock K. He also knows that in a symmetric configuration no firm can gain an advantage from the provision of public services by raising its capital

setup has a one period lag between the collection of taxes and the conversion of these revenues into public services. Hence, methodologically this study belongs to the "stock case" to which we turn in Section 2.4. Ott and Soretz (2007) argue that relative congestion of productive government activity may also be important for the spatial distribution of economic activity.

stock. Since all firms are identical, no firm can increase its size relative to other firms and/or the economy.

In other words, the planner internalizes the equilibrium condition $K = Nk$ in (2.23) which then reduces to $g = GN^{\sigma_G - 1}$. As a consequence, with (2.6) the relevant production function is $y = (AN^{\sigma_G\,\alpha}\theta_G^{\alpha})^{1/(1-\alpha)}\,k$ and the constrained efficient growth rate of consumption becomes

$$\gamma_c^P = (1-\theta_G)\,(AN^{\sigma_G\,\alpha}\theta_G^{\alpha})^{\frac{1}{1-\alpha}} - \rho. \tag{2.26}$$

The comparison of the equilibrium growth rate (2.25) to the constrained optimal growth rate (2.26) gives

$$\gamma_c = \gamma_c^P \quad \Leftrightarrow \quad (1-\tau_y)(1-\sigma_G\,\alpha) = 1-\theta_G. \tag{2.27}$$

With $\sigma_G < 1$ the latter two equations generalize (2.12) and (2.13) to the case of congestion. With congestion, the equilibrium growth rate may again be too high relative to the efficient one. To see this, consider the case where θ_G is chosen optimally, i. e., $\theta_G^P = \alpha$. Then, if government expenditure is entirely financed by lump-sum taxes, i. e., $\tau_y = 0$, it holds that $\gamma_c > \gamma_c^P$. Intuitively, congestion drives a wedge between the private and the social marginal return to private capital and induces an incentive to over-accumulate private capital in the decentralized equilibrium. From (2.27) we derive the income tax rate for an optimally chosen share of government expenditure as

$$\tau_y^P = \frac{\alpha\,(1-\sigma_G)}{1-\sigma_G\alpha}, \quad \text{with} \quad \frac{d\tau_y^P}{d\sigma_G} < 0. \tag{2.28}$$

An income tax rate τ_y^P eliminates this wedge and implements the Pareto-efficient allocation. Clearly, τ_y^P increases the stronger the degree of congestion. In the extreme case of proportional congestion all government expenditure should be financed via income taxes, i. e., $\tau_y^P = \theta_G^P = \alpha$.

2.3.3.2 Relative Congestion With Excludability

Some public services subject to congestion are excludable. This means that a potential user of the service can be identified and charged a user fee. Examples include highways, bridges, universities, or schools.

Ott and Turnovsky (2006) extend the previous setup and introduce a second public service that is excludable. The modified production function of individual firms is

$$y = f(k, g, e) = Ag^{\alpha} e^{\beta} k^{1-\alpha-\beta}, \qquad 0 < \alpha, \beta < 1, \tag{2.29}$$

where e is the benefit derived by the firm from the excludable public service. Just as the non-excludable public service G, the excludable one is subject to relative congestion such that

$$e = E \left(\frac{k}{K} \right)^{1-\sigma_E}; \tag{2.30}$$

here, E is the total amount of the excludable public service supplied by the government, and $\sigma_E \in [0, 1]$ measures the degree of relative congestion. Using (2.30) and (2.23) in (2.29) gives the production function as perceived by the individual firm

$$y = A \left(\frac{G}{K} \right)^{\alpha} \left(\frac{k}{K} \right)^{-\alpha\sigma_G} \left(\frac{E}{K} \right)^{\beta} \left(\frac{k}{K} \right)^{-\beta\sigma_E} k. \tag{2.31}$$

For the government, the key difference between the provision of G and E is that the former must be financed through taxes whereas the latter can be financed through a fee paid by the individual user. Denote p this fee per unit of E. Then, the expression for the balanced government budget (2.5) becomes $G + E = \tau_y Y + \tau_c C + \tau N + pEN$. Similar to condition (2.6) for G, we assume that the provision of E is proportionate to the size of the economy, i. e., $E = \theta_E Y$ for all t.

Decentralized Equilibrium

For the individual household-producer, the new element is that besides c and k, she also determines in each period her demand for the excludable public service. The associated expenditure pE^d must be added to the flow budget constraint (2.3) which modifies to $\dot{k} = (1 - \tau_y) f(k, g, e) - (1 + \tau_c)c - \tau - pE^d$.

Since the choice of E^d does not affect utility directly, it is chosen to maximize the right-hand side of the flow budget constraint. The associated optimality condition equates the marginal after-tax product of the excludable input to its marginal cost, i. e.,

$$(1 - \tau_y) \frac{\partial y}{\partial E^d} = (1 - \tau_y) \beta \frac{y}{E^d} = p. \tag{2.32}$$

The latter delivers the demand of each household-producer, $E^d(p)$. Since E is a public good, in equilibrium we have $E = E^d(p)$. Together with the proportionality constraint $E = \theta_E\, Ny$, we obtain from (2.32) the equilibrium value of p as

$$p = \frac{(1-\tau_y)\beta}{\theta_E N}. \tag{2.33}$$

Intuitively, the equilibrium user fee declines with the total number of users.

Applying the same reasoning that led to the Euler equation (2.25) delivers the equilibrium growth rate

$$\gamma_c = (1-\tau_y)(1-\alpha\sigma_G-\beta\sigma_E)\left(AN^{\sigma_G\,\alpha+\sigma_E\,\beta}\theta_G^{\alpha}\theta_E^{\beta}\right)^{\frac{1}{1-\alpha-\beta}} - \rho, \tag{2.34}$$

which generalizes (2.25) of the non-excludable public input case to $\beta > 0$. Since firms neglect the congestive consequences of their own choice of the private capital input on the aggregate economy, they continue to overestimate the before-tax marginal product of capital.

Finally, consider the growth-maximizing government expenditure shares for the benchmark scenario where the provision of G is fully financed through income taxes, i. e, $\theta_G = \tau_y$. With two public goods and a user fee given by (2.33) the two shares θ_G and θ_E are linked by the government budget such that $\theta_E = \beta\,(1-\theta_G)$. Using this condition, we obtain $\theta_G^* = \alpha$ and $\theta_E^* = \beta\,(1-\alpha)$.

Pareto Efficiency

The social planner internalizes congestion effects, i. e., he considers (2.31) in conjunction with $K = Nk$ and the proportionality conditions $Y = G/\theta_G = E/\theta_E$. Then, $y = \left(AN^{\sigma_G\,\alpha+\sigma_E\,\beta}\theta_G^{\alpha}\theta_E^{\beta}\right)^{1/(1-\alpha-\beta)} k$ and the resource constraint is $\dot{k} = (1-\theta_G-\theta_E)y - c$. The constrained efficient growth rate obtains as

$$\gamma_c^P = (1-\theta_G-\theta_E)\left(AN^{\sigma_G\,\alpha+\sigma_E\,\beta}\theta_G^{\alpha}\theta_E^{\beta}\right)^{\frac{1}{1-\alpha-\beta}} - \rho, \tag{2.35}$$

the first term on the right-hand side denoting the social marginal return on private capital.

If the social planner is allowed to choose θ_G and θ_E optimally, he picks $\theta_G^P = \alpha$ and

$\theta_E^P = \beta$.[9]

The comparison of the equilibrium to the constrained optimal growth rate reveals that

$$\gamma_c = \gamma_c^P \quad \Leftrightarrow \quad (1-\tau_y)(1-\sigma_G\alpha-\sigma_E\beta) = 1-\theta_G-\theta_E. \tag{2.36}$$

The latter has interesting consequences for the budgeting of government services. To see this, consider the case where $\theta_G^P = \alpha$ and $\theta_E^P = \beta$. Then, (2.33) and (2.36) deliver the following pair (τ_y^P, p^P) that implements the efficient allocation

$$\tau_y^P = \frac{(1-\sigma_G)\alpha + (1-\sigma_E)\beta}{1-\sigma_G\alpha-\sigma_E\beta} \quad \text{and} \quad p^P = \frac{1}{N}\left(\frac{1-\alpha-\beta}{1-\sigma_G\alpha-\sigma_E\beta}\right). \tag{2.37}$$

To satisfy the government's budget constraint at (τ_y^P, p^P) a residual lump-sum tax or subsidy may be necessary.

As a benchmark, consider the case where $\sigma_G = \sigma_E = 1$ such that both public services are congestion-free. Then, $\tau_y^P = 0$ and $p^P = 1/N$, i. e., there is no distortion of intertemporal prices and each firm's demand for the excludable public service satisfies the natural efficiency condition $\partial y/\partial E^d = p^P = 1/N$. In this case, the user fee fully finances the provision of E. However, the provision of G must be financed through some lump-sum tax to guarantee a balanced budget (Ott and Turnovsky, 2006).

In the presence of congestion, $\tau_y^P > 0$ is necessary to correct for the congestion externalities. However, as τ_y^P increases the price of the excludable service must fall since its after-tax marginal product declines. Then, the provision of E requires cross-subsidization.

2.3.4 Public Consumption Services

Many publicly provided services matter for an economy because they directly enhance the utility of households without affecting technology. Examples include cultural and recreational public services such as museums, public parks, or public social events like fireworks. To study the role of such public consumption services, we extend the analysis

[9] Again, this result can be linked to the natural condition of productive efficiency. We obtain from equation (2.31) with $K = Nk$ and $Y = Ny$ that $dY/dG = \alpha\,(Y/G) = \alpha/\theta_G^P = 1$ and $dY/dE = \beta\,(Y/E) = \beta/\theta_E^P = 1$. Hence, the marginal product of both government services provided out of current production is one.

of the pure public good case of Section 2.3.1 and add a non-excludable service that enters the utility function. This service is subject to absolute congestion. With these properties, our analysis combines the framework of Barro (1990) and Turnovsky (1996c).[10]

The household's intertemporal utility is now

$$u = \int_0^\infty e^{-\rho t} \left(\ln c + b_h \ln h\right) dt, \tag{2.38}$$

where h is the service the individual household derives from the public consumption good, and $b_h \geq 0$ measures the relative weight of this form of consumption. For simplicity, the per-period utility is separable in c and h.

The public consumption good is subject to aggregate congestion in total output such that the service, h, derived by each household falls short of the aggregate service, H, provided by the government. More precisely, we follow Turnovsky (1996c) and stipulate

$$h = H^{\sigma_H} \left(\frac{H}{Y}\right)^{1-\sigma_H}; \tag{2.39}$$

here, $\sigma_H \in [0, 1]$ measures the degree of aggregate congestion with $\sigma_H = 1$ and $\sigma_H = 0$ capturing the special cases of a pure public good and of proportional congestion, respectively.

On the production side, we maintain the production function of equation (2.7). On the government side, we need to add H as government expenditure such that a balanced budget requires $G + H = \tau_y Y + \tau_c C + \tau N$. As in the previous sections, we tie the size of H to the size of the economy: $H = \theta_H Y$.

Decentralized Equilibrium

The individual household-producer behaves as in Section 2.3.1. Since H is non-excludable, there is no optimization with respect to h. Moreover, when choosing k, she disregards the link between k, aggregate output Y, and h that materializes under congestion.

[10] Cazzavillan (1996) studies the role of a public good that simultaneously affects per-period utility and the production function of the representative household-producer. Under a more general utility function that allows for increasing returns in the consumption externality of public expenditure, he shows that local indeterminacy and endogenous stochastic fluctuations may arise.

As a result, the expression of the consumption growth rate in equilibrium is again given by (2.9).[11] However, if at least some part of H is funded via the distortionary income tax, then, the level of γ_c that satisfies the government's budget constraint is smaller. To see this, consider the benchmark where $G + H = \tau_y Y$. Then, $\tau_y = \theta_G + \theta_H$ such that

$$\gamma_c = (1 - \theta_G - \theta_H)(1 - \alpha)\left(AN^{\alpha}\theta_G^{\alpha}\right)^{\frac{1}{1-\alpha}} - \rho. \tag{2.40}$$

Since the government channels additional resources into non-productive uses, the latter falls short of (2.9) with $\tau_y = \theta_G$. For the same reason, the growth-maximizing expenditure share of the productive government services $\theta_G^* = \alpha(1-\theta_H)$, declines in θ_H. Further, in this case the growth-maximizing share of public consumption services is zero.

Pareto Efficiency

The omniscient planner considers the individual production function as in (2.10). The resource constraint is $\dot{k} = (1 - \theta_G - \theta_H)y - c$. The key new element appears in the per-period utility function. The planner knows that the congestion effect of equation (2.39), the proportionality requirement, $H = \theta_H Y$, and the aggregation $Y = Ny$ imply $h = \theta_H (Ny)^{\sigma_H}$. Hence via (2.10), the choice of k directly affects per-period utility for $\sigma_H > 0$. The resulting constrained efficient steady-state growth rate is

$$\gamma_c^P = (1 - \theta_G - \theta_H)\left(AN^{\alpha}\theta_G^{\alpha}\right)^{\frac{1}{1-\alpha}} - \frac{\rho}{1 + b_h\sigma_H}. \tag{2.41}$$

The first term on the right-hand side is the social marginal return on private capital. The second term is the social rate of time preference. The provision of the non-productive public service reduces this rate. Intuitively, the presence of b_h captures the fact that a higher capital stock tomorrow raises the level of h, and hence tomorrow's utility. This effect is stronger the smaller the congestion effect.[12]

The equilibrium and the planner's growth rate coincide if and only if

$$\gamma_c = \gamma_c^P \Leftrightarrow \left(AN^{\alpha}\theta_G^{\alpha}\right)^{\frac{1}{1-\alpha}}\left[(1-\tau_y)(1-\alpha) - (1 - \theta_G - \theta_H)\right] = \rho\left[1 - \frac{1}{1 + b_h\sigma_H}\right]. \tag{2.42}$$

[11] This result hinges to some extent on the separability of c and h in the per-period utility function. If the marginal utility of c depends on h, then the household's willingness to postpone consumption depends on the growth rate of h. In a steady state with congestion, the latter need not coincide with the steady-state growth rate of all other per-capita variables. We leave a more detailed study of the impact of the interaction between c and h on the steady-state growth rate for future research.

[12] In the special case of proportional congestion, where $\sigma_H = 0$, the effect of k on h disappears because (2.39) in conjunction with $H = \theta_H Y$ implies $h = \theta_H$.

The term in brackets on the left-hand side reflects the possible deviation of the private from the social marginal rate of return on private capital. The gap between these rates depends on the way government finances its expenditure. A novelty compared to the pure public good case of Section 2.3.1 is the deviation of the private and the social rate of time preference that appears on the right-hand side.

We may expand the planner's choice set and allow him to determine the size of θ_G and θ_H. Then, the efficient pair $\left(\theta_G^P, \theta_H^P\right)$ satisfies the following optimality conditions

$$\theta_G^P = \alpha \left[\theta_H^P \left(\sigma_H - 1\right) + 1\right], \tag{2.43}$$

$$\theta_H^P = \frac{b_h \rho}{1 + b_h \sigma_H} \left[A N^\alpha \left(\theta_G^P\right)^\alpha\right]^{\frac{-1}{1-\alpha}}. \tag{2.44}$$

Assume that the pair $\left(\theta_G^P, \theta_H^P\right)$ is unique in $[0,1]^2$ such that equations (2.43) and (2.44) intersect only once as depicted in Figure 2.1.

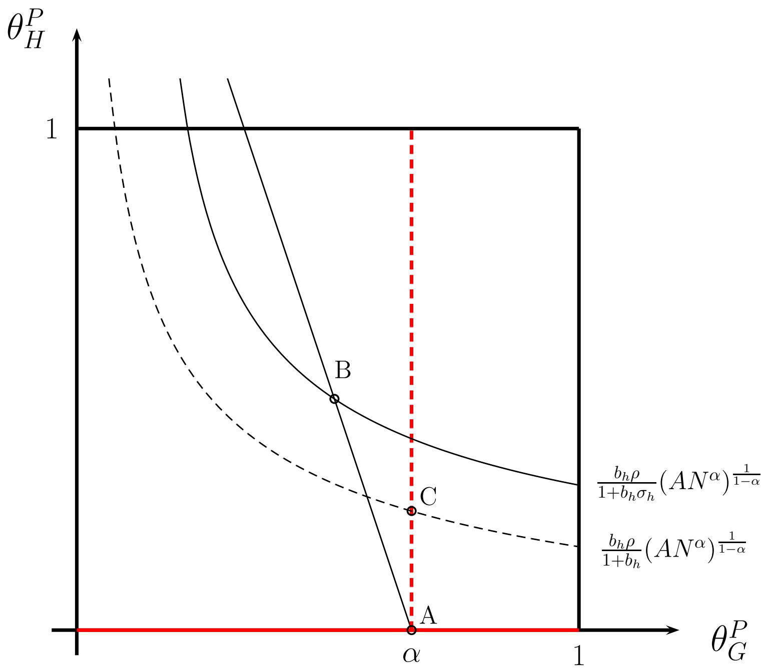

Figure 2.1: Efficient pair $\left(\theta_G^P, \theta_H^P\right)$

Intersection A corresponds to the case where $b_h = 0$ and $\sigma_H \in [0,1]$. Then, there is no utility associated with h. Hence, independently of the degree of congestion, the planner chooses $\theta_H^P = 0$ and $\theta_G^P = \alpha$ and the optimal allocation coincides with the one of Section 2.3.1. Case B has $b_h > 0$ and $\sigma_H \in (0,1)$. Here, $\theta_H^P > 0$ and $0 < \theta_G^P < \alpha$.

Moreover,

$$\frac{d\theta_G^P}{db_h} < 0 \qquad \text{and} \qquad \frac{d\theta_H^P}{db_h} > 0, \tag{2.45}$$

$$\frac{d\theta_G^P}{d\sigma_H} > 0 \qquad \text{and} \qquad \frac{d\theta_H^P}{d\sigma_H} < 0. \tag{2.46}$$

Since $b_h > 0$, the planner is ready to provide public consumption services according to the optimality condition $\partial u/\partial H = \partial u/\partial c$. As a consequence, the relative size of G falls. To grasp the effect of congestion, recall that the planner is aware of the positive effect of θ_G on h in the utility function. This effect is more pronounced the lower the degree of congestion. Hence, a rise in σ_H increases θ_G^P. In the limit $\sigma_H \to 1$, H is a pure public good and $\theta_G^P \to \alpha$ as shown as intersection C in Figure 2.1. In any case, the welfare-maximizing share of public consumption services is positive. Hence, the provision of public consumption services may introduce a wedge between the goals of growth and welfare maximization.[13]

The income tax rate that implements the Pareto-efficient allocation is found to be $\tau_y^P = (1 - \sigma_H)\,\theta_H^P$. Hence, without the congestion externality, i. e., $\sigma_H = 1$, no income tax is needed to implement the Pareto-efficient allocation.

2.3.5 Endogenous Labor Supply

This section incorporates the labor-leisure decision, i. e., individual labor supply becomes endogenous. In this context, a consumption tax as well as a tax on labor income are distortionary since they affect the trade-off between consumption and leisure. Contrary to the analysis of Section 2.3.4, public consumption expenditure turns out to have a positive effect on the equilibrium growth rate. We develop an intuition for this result following the presentation of Turnovsky (2000a).

The representative agent has a per-period time endowment equal to one and allocates the fraction $l \in (0, 1)$ to leisure and $(1 - l)$ to work. The per-period utility function

[13] Park and Philippopoulos (2002) confirm this result in a setting that allows for a different set of second-best optimal policies.

takes the positive utility of leisure into account. More precisely, we stipulate

$$u = \int_0^\infty [\ln c + b_h \ln h + b_l \ln l]\, e^{-\rho t}\, dt, \quad b_h, b_l \geq 0. \tag{2.47}$$

Since the focus is on the role of labor supply, we abstract from congestion effects associated with the provision of public consumption services, i. e., $\sigma_H = 1$ in (2.39) such that $h = H$.

On the production side, we incorporate labor as a productive input and generalize the production function of (2.7) to

$$y = A\,[G\,(1-l)]^{\alpha}\, k^{1-\alpha}, \qquad 0 < \alpha < 1. \tag{2.48}$$

Hence, there are constant returns to scale both with respect to private capital and labor, and with respect to public and private capital. The former implies zero profits in a competitive environment whereas the latter allows for steady-state growth of labor productivity.

On the government side, we split the proportional income tax τ_y into a tax on wage income at rate τ_w and a tax on capital income at rate τ_r. With w and r denoting the real wage and the real rate of return on private capital, the balanced government budget becomes $G + H = \tau_w w(1-l)N + \tau_r rK + \tau_c C + \tau N$.

Decentralized Equilibrium

Household-producers choose a plan (c, l, k) for each t such that (2.47) is maximized subject to the budget constraint $\dot{k} = (1-\tau_w)w(1-l) + (1-\tau_r)rk - (1+\tau_c)c - \tau$.

Following the steps that led to Euler equation (2.9) we obtain

$$\gamma_c = (1-\tau_r)(1-\alpha)\,(AN^{\alpha}\theta_G^{\alpha})^{\frac{1}{1-\alpha}}\,(1-l)^{\frac{\alpha}{1-\alpha}} - \rho. \tag{2.49}$$

Observe that l appears as a determinant of γ_c. Intuitively, since labor and capital are complements in the production function (2.48), more leisure reduces the marginal rate of return on private capital. Moreover, l is a choice variable that needs to be pinned down.

To find a second condition that determines the level of leisure consistent with steady-state growth, consider the product market equilibrium condition that coincides with the economy's resource constraint $\dot{k} = (1 - \theta_G - \theta_H)\, y - c$. Expressing the latter in terms of the growth rate of private capital and using the static optimality condition for the consumption-leisure decision[14] delivers

$$\gamma_k = \left[(1 - \theta_G - \theta_H) - \left(\frac{1 - \tau_w}{1 + \tau_c} \right) \frac{\alpha}{b_l} \left(\frac{l}{1 - l} \right) \right] (A N^\alpha \theta_G^\alpha)^{\frac{1}{1-\alpha}} (1 - l)^{\frac{\alpha}{1-\alpha}}. \quad (2.50)$$

A steady state needs $\gamma_c = \gamma_k$. If this requirement gives rise to a unique and strictly positive steady-state growth rate, then there is a time-invariant level of steady-state leisure depending on the policy variables $\theta_G, \theta_H, \tau_r, \tau_w, \tau_c$.

Turnovsky (2000a) shows that a rise in either public consumption services ($\theta_H \uparrow$) or in public productive services ($\theta_G \uparrow$) financed by a lump-sum tax increases the steady-state growth rate. As to θ_H, this is the result of two opposing forces. For a given labor supply, the growth rate declines since the government claims additional resources. However, households increase their labor supply to make up for this negative income effect. Overall the steady-state growth rate increases due to greater employment. These effects do not materialize when labor supply is inelastic as in Section 2.3.4. In such a setting a lump-sum financed increase in θ_H has no impact on steady-state growth.

The same two forces also operate in the case of an increase in θ_G. In addition, there is a third effect since a higher θ_G raises the equilibrium wage and, hence, the labor supply. As a result, the steady-state growth rate increases further such that $\partial\gamma_c/\partial\theta_G > \partial\gamma_c/\partial\theta_H > 0$.

If the lump-sum tax is accompanied by a consumption tax and/or a tax on wage income the positive link between steady-state consumption growth rate γ_c and θ_i, $i = G, H$, weakens. The reason is the distorted consumption-leisure decision, i. e., the household tries to avoid the additional tax burden and substitutes leisure for labor. Hence, with endogenous labor supply, a consumption tax ceases to be lump-sum and impinges on the economy's growth rate.

[14]The latter condition requires the marginal rate of substitution between leisure and consumption to equal the relative price of both goods, i. e., $b_l c/l = w(1 - \tau_w)/(1 + \tau_c)$. Marginal cost pricing of labor gives $w = \alpha y/(1 - l)$. Using both equations, we can determine the ratio c/y, which is then used to derive (2.50).

Pareto Efficiency

The social planner chooses a plan (c, l, k) for each t to maximize household utility subject to the economy's resource constraint. Compared to the optimization of competitive households, the omniscient planner takes into account that the choice of l and k has an effect on the level of government consumption services. Since $H = \theta_H Y$ appears in the utility function this channel affects the constrained optimal steady-state growth rate of consumption.

Given l, the optimization generates the following expressions for the planner's choice of γ_c^P and γ_k^P:

$$\gamma_c^P = \frac{(1-\theta_G-\theta_H)}{1-b_h\Omega(l)}\left(AN^\alpha\theta_G^\alpha\right)^{\frac{1}{1-\alpha}}(1-l)^{\frac{\alpha}{1-\alpha}}-\rho, \tag{2.51}$$

$$\gamma_k^P = (1-\theta_G-\theta_H)\frac{1-\Omega(l)(1+b_h)}{1-b_h\Omega(l)}\left(AN^\alpha\theta_G^\alpha\right)^{\frac{1}{1-\alpha}}(1-l)^{\frac{\alpha}{1-\alpha}}. \tag{2.52}$$

Here, $\Omega(l) \equiv \alpha l/\left[b_l(1-\alpha)(1-l)\right]$. The presence of $b_h\Omega(l)$ in the Euler equation (2.51) captures the fact that, for a given level of leisure/labor, the presence of utility-enhancing government consumption expenditure increases the benefits from capital investment today, lowering the consumption-output ratio and positively affecting the growth rate of consumption.[15] Further, the additional term $b_h\Omega(l)$ in the numerator of (2.52) reflects the fact that, from the planner's point of view, the marginal disutility of labor is lower since more labor means a higher consumption of H. Via this channel, the consumption-output ratio is lowered and the growth rate of capital is positively affected.

If we add the steady-state requirement $\gamma_c^P = \gamma_k^P$, then (2.51) and (2.52) give an expression for the constrained optimal steady-state growth rate that is similar to (2.41) with $\sigma_H = 1$ and a level of labor supply yet to be determined

$$\gamma_c^P = (1-\theta_G-\theta_H)\left(AN^\alpha\theta_G^\alpha\right)^{\frac{1}{1-\alpha}}(1-l)^{\frac{\alpha}{1-\alpha}}-\frac{\rho}{1+b_h}. \tag{2.53}$$

In addition, (2.51) and (2.52) determine the steady-state labor supply implicitly.[16]

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[15] The latter dominates the negative effect of leisure on the rate of return of private capital for any value of l if $b_h/b_l > 1$. If $0 < b_h/b_l < 1$, this only applies for not too small values of l.

[16] This condition is $\Omega(l)(1+b_h(1+z(l))) = z(l)$, where
$z(l) \equiv \rho\left[(1-\theta_G-\theta_H)\left(A\theta_G^\alpha\right)^{\frac{1}{1-\alpha}}(1-l)^{\frac{\alpha}{1-\alpha}}\right]^{-1}$.

We can use our results to derive conditions under which a fiscal policy mix implements the constrained efficient allocation. This requires

$$\gamma_c = \gamma_c^P \quad \Leftrightarrow \quad (1-\tau_r)(1-\alpha) = \frac{1-\theta_G-\theta_H}{1-b_h\Omega(l)}, \tag{2.54}$$

$$\gamma_k = \gamma_k^P \quad \Leftrightarrow \quad \left(\frac{1-\tau_w}{1+\tau_c}\right)(1-\alpha)\Omega(l) = (1-\theta_G-\theta_H)\frac{\Omega(l)}{1-b_h\Omega(l)}. \tag{2.55}$$

According to the first of these conditions, τ_r has to be set such that the private marginal after-tax return on private capital equals the social rate of return on private capital. The second condition equalizes the consumption-output ratios of the equilibrium and planner's choice. It is then straightforward to see that the desired policy mix must satisfy the condition (Turnovsky, 2000a)

$$(1-\tau_r)\left(\frac{1+\tau_c}{1-\tau_w}\right) = 1. \tag{2.56}$$

Intuitively, the effect of a distortionary tax on capital income can be offset by a compensating distortion of the consumption-leisure trade-off that strengthens labor supply. As long as lump-sum taxation is a feasible option any policy mix satisfying (2.56) is consistent with the government's budget constraint.[17]

If the social planner is allowed to pick θ_G and θ_H optimally, one finds that the optimal choice involves

$$\theta_G^P = \alpha \quad \text{and} \quad \theta_H^P = (1-\alpha)\, b_h\Omega(l), \tag{2.57}$$

i. e., the optimal share of productive government expenditure satisfies the natural condition of productive efficiency, and the optimal share of consumption expenditure is tied to the optimal level of leisure. Interestingly, from (2.54) and (2.55) the implementation of (θ_G^P, θ_H^P) is only possible if $\tau_r = 0$ and $\tau_w = -\tau_c$.

2.3.6 Small Open Economy

Next, we turn to a small open economy with productive government expenditure, where agents are free to accumulate internationally traded bonds in a perfect world capital

[17] Raurich (2003) studies optimal tax policies in the model of Turnovsky (2000a) assuming that neither lump-sum nor consumption taxes are admissible, yet the government's budget must be balanced in all periods.

market. To highlight the role of openness we restrict attention to a pure public good such that the production function of household-producers is given by (2.7). Moreover, we abstract from the presence of public consumption services.

As bonds and private capital are perfect substitutes as stores of value, in equilibrium they must pay the same after-tax rate of return, which is tied to the exogenous world interest rate $\bar{r}$. Hence, with government expenditure a fixed fraction of aggregate output according to (2.6) and an exogenous labor supply, this implies

$$(1-\tau_y)(1-\alpha)\left(A\theta_G^{\alpha}N^{\alpha}\right)^{\frac{1}{1-\alpha}} = \bar{r}(1-\tau_b), \tag{2.58}$$

where τ_b is the tax rate on foreign bond income. Obviously, the pair of tax rates τ_y and τ_b that satisfies this condition cannot be chosen independently. To circumvent this problem, we introduce adjustment costs such that the price of installed capital, q, is variable and adjusts in equilibrium such that these after-tax rates of return are the same for any arbitrarily specified tax rates. The investment cost function is independent of government activity and given by

$$\varphi(i,k) \equiv \left(1+\frac{i}{2k}\right)i, \tag{2.59}$$

which simplifies (2.14) by fixing $\phi = 1$.

First, we study the case of an exogenous labor supply and then incorporate a labor-leisure trade-off. We shall see that the implications for government activity substantially differ in both cases. The exposition is based on Turnovsky (1999a).

2.3.6.1 Exogenous Labor Supply

Decentralized Equilibrium

Denote b the stock of net foreign bonds held by a household-producer at t and recall that k is the stock of capital in her (domestic) firm. Then, her flow budget constraint is given by

$$\dot{b} = \bar{r}(1-\tau_b)b + (1-\tau_y)y - (1+\tau_c)c - \varphi(i,k) - \tau. \tag{2.60}$$

The government budget modifies to $G = \tau_y Y + \tau_c C + \bar{r}\tau_b bN + \tau N$.

The objective of household-producers is to choose a plan (c, i, b, k) that maximizes utility (2.2) subject to (2.60) and $\dot{k} = i$. From the individual's optimality conditions with respect to c and b we obtain the Euler equation as

$$\gamma_c = \bar{r}(1 - \tau_b) - \rho. \tag{2.61}$$

Hence, in a small open economy, the consumption growth rate is independent of domestic production conditions. It only depends on the given world interest rate, the tax rate on foreigns bonds, and on the rate of time preference.

The optimality conditions with respect to k and i deliver

$$q = 1 + \frac{i}{k}, \tag{2.62}$$

$$\bar{r}(1 - \tau_b) = \frac{(1 - \tau_y)(1 - \alpha)\left(A\theta_G^{\alpha} N^{\alpha}\right)^{\frac{1}{1-\alpha}}}{q} + \frac{\dot{q}}{q} + \frac{(q-1)^2}{2q}. \tag{2.63}$$

As we saw in (2.17) the value of installed capital for an investing firm is greater than one. Equation (2.63) is a non-linear differential equation that describes the evolution of q such that the after-tax rate of return on traded bonds is equal to the after-tax rate of return on private domestic capital. The latter comprises the same elements as discussed following equation (2.19). Observe that (2.63) collapses to (2.58) for $q = 1$.

Turning to the steady state, we know from (2.62) that private domestic capital grows at the rate $\gamma_k = q - 1$, where q satisfies (2.63) for $\dot{q} = 0$.[18] Thus, the steady-state growth rate of capital (and output) depends on the domestic production technology as well as on various fiscal policy parameters. In contrast to the closed economy, consumption, capital and output generically grow at different rates, with the difference being reconciled by the accumulation of traded bonds.

As to the role of government activity on steady-state growth one finds that (Turnovsky, 1999a)

$$\frac{d\gamma_k}{d\theta_G} > 0, \quad \frac{d\gamma_c}{d\theta_G} = 0; \quad \left.\frac{d\gamma_k}{d\theta_G}\right|_{\theta_G = \tau_y} \lesseqgtr 0 \quad \Leftrightarrow \quad \alpha \lesseqgtr \theta_G, \quad \left.\frac{d\gamma_c}{d\theta_G}\right|_{\theta_G = \tau_y} = 0. \tag{2.64}$$

[18] The presence of convex adjustment costs may prevent the existence of a balanced growth path; for a discussion see Turnovsky (1996b). Further, the transversality condition of the household-producer's problem requires $\bar{r}(1 - \tau_b) > \gamma_k$ and implies that only the smaller root of (2.63) is consistent with steady-state growth. Moreover, at this root, the right-hand side of (2.63) is negatively sloped.

The first two derivatives describe the effect of a rise in government expenditure financed through an adjustment in lump-sum taxes. The steady-state growth rate of capital increases since a higher θ_G increases the marginal product of capital such that the steady-state value of q in (2.63) increases; hence, $\gamma_k = q - 1$ rises. Due to (2.61), γ_c is independent of θ_G. The second two derivatives consider the benchmark case where government activity is only financed by income taxation, i. e., $\theta_G = \tau_y$. This introduces an offsetting effect on the steady-state value of q since a necessary rise in taxes reduces the after-tax marginal product of capital in (2.63). As in previous sections, there is a growth-maximizing share of government expenditure equal to α at which the price of installed capital is maximized.

The effect of τ_b on steady-state growth is given by

$$\frac{d\gamma_k}{d\tau_b} > 0, \quad \frac{d\gamma_c}{d\tau_b} < 0. \tag{2.65}$$

Intuitively, an increase in the tax on bond income lowers the net rate of return on traded bonds, which requires a lower rate of return on installed capital, hence a higher q according to (2.63). Moreover, a higher τ_b reduces the households' willingness to postpone consumption and γ_c declines.

Pareto Efficiency

The planner maximizes u with respect to c, i, k, and b subject to $\dot{k} = i$ and the resource constraint

$$\dot{b} = (1 - \theta_G)y + \bar{r}b - c - \varphi\,(i, k)\,. \tag{2.66}$$

Accordingly, we obtain the constrained efficient steady-state growth rates of consumption and capital as

$$\gamma_c^P = \bar{r} - \rho \quad \text{and} \quad \gamma_k^P = q^P - 1, \tag{2.67}$$

where q^P is determined by

$$\bar{r} = \frac{(1 - \theta_G)\,(A\theta_G^\alpha N^\alpha)^{\frac{1}{1-\alpha}}}{q^P} + \frac{(q^P - 1)^2}{2q^P}. \tag{2.68}$$

The interpretation of (2.67) and (2.68) mimics the one of (2.61) and (2.63) in the decentralized equilibrium. Due to the presence of τ_b, we have $\gamma_c^P > \gamma_c$. It follows that

$\tau_b = 0$ is necessary to implement the constrained efficient allocation. Then, for the same reasons set out in Footnote 18, we find

$$q \lesseqqgtr q^P \quad \Leftrightarrow \quad \gamma_k \lesseqqgtr \gamma_k^P \quad \Leftrightarrow \quad \frac{\theta_G - \alpha}{1-\alpha} \lesseqqgtr \tau_y. \tag{2.69}$$

Allowing the social planner to additionally determine the optimal size of the government reveals that the growth-maximizing share of government expenditure is also welfare-maximizing, i. e., $\theta_G^P = \alpha$. If $\theta_G \neq \alpha$, we obtain that capital and interest income should be taxed at different rates. This result is driven by the assumption that government expenditure is a fixed fraction of output and thereby independent of interest income.

2.3.6.2 Endogenous Labor Supply

In this section we introduce an endogenous labor supply in the small open economy of the previous section.

Decentralized Equilibrium

The household-producer chooses a plan (c, l, i, b, k) to maximize

$$u = \int_0^\infty [\ln c + b_l \ln l] \, e^{-\rho t} \, dt \tag{2.70}$$

subject to $\dot{k} = i$, the budget constraint $\dot{b} = (1-\tau_w)w(1-l) + (1-\tau_r)rk + \bar{r}(1-\tau_b)b - (1+\tau_c)c - \varphi(i,k) - \tau$, and the production function (2.48). This leads to the conditions for consumption and domestic capital growth (2.61) and (2.62) as well as the following optimality conditions

$$\bar{r}(1-\tau_b) = \frac{(1-\tau_r)(1-\alpha)\left(AN^\alpha\theta_G^\alpha\right)^{\frac{1}{1-\alpha}}(1-l)^{\frac{\alpha}{1-\alpha}}}{q} + \frac{\dot{q}}{q} + \frac{(q-1)^2}{2q}, \tag{2.71}$$

$$\frac{c}{y} = \left(\frac{1-\tau_w}{1+\tau_c}\right)\frac{\alpha}{b_l}\left(\frac{l}{1-l}\right). \tag{2.72}$$

In the steady state $\dot{q} = 0$ such that (2.71) determines the equilibrium price of installed capital given l. Condition (2.72) implies that in a steady state with constant labor supply c and y must grow at the same rate. Moreover, one can show that in a steady state y, k, and G must grow at the same rate. It follows that

$$\gamma_k = q - 1 = \bar{r}(1-\tau_b) - \rho = \gamma_c. \tag{2.73}$$

Equation (2.73) implies that, contrary to the case with exogenous labor supply, in equilibrium capital, output, and consumption grow at the same rate determined by the net interest rate on foreign bonds and the rate of time preference. Hence, with endogenous labor supply the production side is irrelevant for the steady-state growth rates of consumption *and* domestic capital (Turnovsky, 1999a).

From (2.73) it follows that out of the set of fiscal policy variables, only changes in τ_b generate steady-state growth effects. The reason is that (2.73) also pins down q. Therefore, in a steady state changes in θ_G, τ_r, and τ_b lead to adjustments of labor supply such that (2.71) remains valid. One readily verifies that $dl/d\theta_G > 0$, $dl/d\tau_r < 0$, and $dl/d\tau_b > 0$. Moreover, since τ_w and τ_c do not show up in (2.71) it follows that these taxes are essentially lump-sum, i. e., $dl/d\tau_w = dl/d\tau_c = 0$. This is in stark contrast to the results obtained under endogenous labor supply in the closed economy of Section 2.3.5.

Pareto Efficiency

The social planner chooses a plan (c, l, i, b, k) to maximize individual utility (2.70) subject to $\dot{k} = i$ and the resource constraint (2.66) where y is given by (2.48). Following the same procedure as in the decentralized setting we obtain the steady-state conditions

$$\gamma_k^P = q^P - 1 = \bar{r} - \rho = \gamma_c^P \tag{2.74}$$

$$\bar{r} = \frac{(1-\theta_G)\left(AN^{\alpha}\theta_G^{\alpha}\right)^{\frac{1}{1-\alpha}}(1-l)^{\frac{\alpha}{1-\alpha}}}{q^P} + \frac{(q^P-1)^2}{2q^P} \tag{2.75}$$

$$\frac{c}{y} = \left(\frac{1-\theta_G}{1-\alpha}\right)\frac{\alpha}{b_l}\left(\frac{l}{1-l}\right). \tag{2.76}$$

The tax rates that replicate the constrained efficient steady-state path, bring (2.71) - (2.73) in line with (2.74) - (2.76). These are $\tau_b = 0$ and $(1-\tau_r)(1+\tau_c)/(1-\tau_w) = 1$, where the latter is a restatement of condition (2.56) derived for the closed economy. The welfare-maximizing share of government expenditure is equal to α. Moreover, with

$\theta_G = \theta_G^P$, the optimal tax rates can be shown to be $\tau_r = 0$ and $\tau_w = -\tau_c$. This confirms the results of the small open economy with exogenous labor supply and for the closed economy with endogenous labor supply. However, here the choice of θ_G^P does not have a growth effect but assures the static efficiency of the steady state.

2.4 Productive Government Activity as a Stock

The difference between the stock and the flow approach to modeling productive government activity is that $G(t)$ is not provided out of current output but results from past public investments, i. e., $G(t)$ is the aggregate stock of public capital at t.

The first paper that treats productive government activity as a stock in our analytical framework is Futagami et al. (1993). These authors assume that the public capital stock is a pure public good such that $g = G$. Here, we begin our discussion of the stock approach by directly allowing for the congestion of public services. Then, we incorporate two aspects that arise only if we think of productive government activity as a stock.[19]

2.4.1 Public Goods Subject to Congestion

We follow Turnovsky (1997a) and assume that current public investment is a constant fraction of aggregate output denoted by $\theta_G \in (0, 1)$. We abstract from depreciation such that G evolves according to

$$\dot{G} = \theta_G Y. \tag{2.77}$$

The household-producer's production technology continues to be as in (2.1). As a consequence, in the stock case G will only be a constant fraction of Y in the steady state,

[19] The stock modeling approach has incorporated many facets that we will not discuss in detail. For instance, Lau (1995) and Chen (2006) incorporate public consumption expenditure affecting the per-period utility function. See Baier and Glomm (2001) and Raurich-Puigdevall (2000) for stock models with an endogenous labor supply. Turnovsky (1997b) is the reference for a small open economy. This framework is applied by Chatterjee et al. (2003) to analyze the process of developmental assistance through unilateral capital transfers tied to investment in public capital. Gómez (2004) devises a fiscal policy that allows to implement the Pareto-efficient allocation when investments are irreversible. Devarajan et al. (1998) study alternative ways how to provide public capital.

whereas in the flow case this holds for all t in accordance with condition (2.6).

Let the service derived by the individual household-producer g be given by (2.23). As in the flow model of Section 2.3.3.1, the individual household-producer chooses c and k to maximize utility u of (2.2) subject to her flow budget constraint (2.3) and the production function (2.24) which we repeat here for convenience

$$y = A\left(\frac{G}{K}\right)^{\alpha}\left(\frac{k}{K}\right)^{-\sigma_G\,\alpha} k.$$

In her intertemporal optimization the individual household-producer neglects her impact on the aggregate private capital stock K and takes the stock of public capital G as given. Then, the Euler condition obtains as

$$\gamma_c = (1-\tau_y)(1-\sigma_G\alpha)AN^{\alpha(\sigma_G-1)}\left(\frac{G}{k}\right)^{\alpha} - \rho \equiv \gamma_c\left(\frac{G}{k},\tau_y\right), \qquad (2.78)$$

where we use the fact that in equilibrium $K = Nk$.

This growth rate looks similar to the Euler condition in the flow model (see, e. g., equation (2.8) where $\sigma_G = 1$). Again, the first term on the right-hand side of the Euler equation is the private marginal product of private capital. In the flow model, the ratio G/k is determined by exogenous parameters since G is proportionate to Y at all t. Therefore, the growth rate of consumption is time-invariant. Here, this is not the case since the proportionality of G and Y occurs only in the steady state. As a consequence, additional differential equations are needed to fully characterize the dynamical system.

To derive these conditions, we divide the aggregate resource constraint (2.11) by k and the public accumulation equation (2.77) by G. Taking into account that equilibrium production is given by

$$y = AN^{\alpha(\sigma_G-1)}\left(\frac{G}{k}\right)^{\alpha} k, \qquad (2.79)$$

we find two additional differential equations in G and k

$$\gamma_G = \theta_G AN^{\alpha(\sigma_G-1)+1}\left(\frac{G}{k}\right)^{\alpha-1} \equiv \gamma_G\left(\frac{G}{k}\right), \qquad (2.80)$$

$$\gamma_k = (1-\theta_G)\,AN^{\alpha(\sigma_G-1)}\left(\frac{G}{k}\right)^{\alpha} - \frac{c}{k}. \qquad (2.81)$$

The dynamical system of the economy is then described by equations (2.78), (2.80), and (2.81) in conjunction with initial conditions k_0, G_0, and the transversality condition of the household-producer's optimization problem.

Here, we focus on the steady state and its properties. From (2.78) and (2.79), G, k, and y have the same steady-state growth rate. This growth rate and the steady-state ratio, $(G/k)|_{ss}$, can be obtained from (2.78) and (2.80). Figure 2.2 (left) illustrates the loci γ_c and γ_G as functions of G/k.

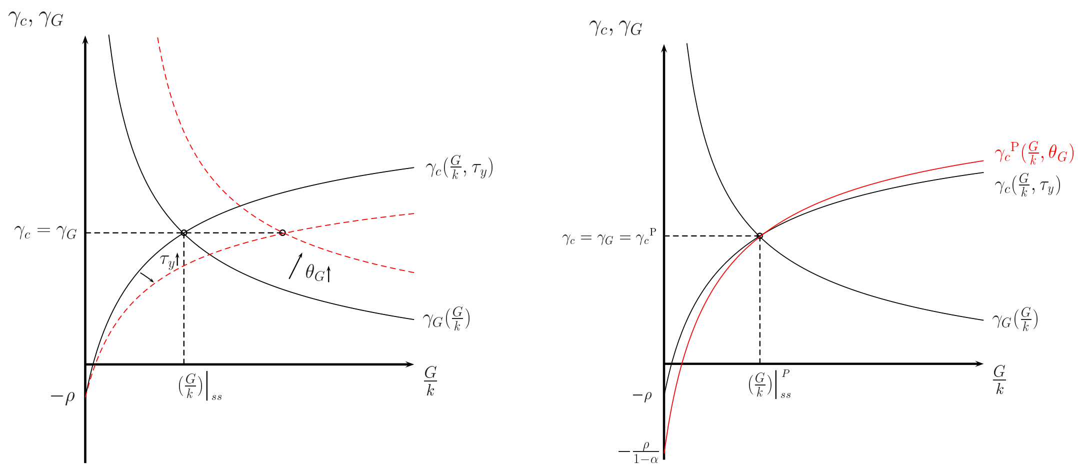

Figure 2.2: Steady-State Growth Rates: Decentralized Equilibrium (left) and the Implementation of the Pareto-Efficient Allocation (right).

Next, we turn to the effect of fiscal policy variables on steady-state growth. A lump-sum financed increase in the share of government investment, θ_G, corresponds to an upward shift of the γ_G-locus in Figure 2.2 (left), which implies a higher steady-state growth rate. If instead of a lump-sum tax a distortionary income tax is used for funding such that $\theta_G = \tau_y$, then in addition the γ_c-locus pivots downwards. The overall effect on the steady-state growth rate depends on the relative strength of both shifts. Analytically, one can show that

$$\frac{d\gamma_c}{d\theta_G} \gtreqless 0 \quad \Leftrightarrow \quad \alpha \gtreqless \theta_G. \tag{2.82}$$

Hence, as in the flow model, the growth-maximizing share of government investment is $\theta_G^* = \tau_y^* = \alpha$ (Futagami et al., 1993).

Pareto Efficiency

In contrast to the individual household-producers the social planner not only chooses c and k but also the public capital stock G to maximize utility (2.2) subject to the aggregate resource constraint (2.11) and the accumulation equation of public capital (2.77) with aggregate production y given by (2.79). This problem delivers the steady-state Euler equation[20]

$$\gamma_c^P = (1-\theta_G)\frac{\partial y}{\partial k} + \theta_G N \frac{\partial y}{\partial G} - \rho. \tag{2.83}$$

The first two terms on the right-hand side have an interpretation as the social return of an additional marginal unit of output. Along the optimal path, the planner allocates the fraction $1-\theta_G$ of this unit to private capital and the fraction θ_G to public capital. This partition is imposed by the public accumulation equation (2.77). The second term corresponds to the benefit of a marginal increase in the provision of public capital associated with θ_G units of current output. Since the planner views G as a pure public good, the marginal increase in aggregate output is N times the marginal increase in individual output.

In light of (2.79) and (2.80), (2.83) can be written as

$$\gamma_c^P = (1-\theta_G)\,(1-\alpha)\,AN^{\alpha(\sigma_G-1)}\left(\frac{G}{k}\right)^{\alpha} + \alpha\gamma_G - \rho. \tag{2.84}$$

In a steady state $\gamma_c^P = \gamma_G$ such that (2.84) becomes

$$\gamma_c^P = (1-\theta_G)\,AN^{\alpha(\sigma_G-1)}\left(\frac{G}{k}\right)^{\alpha} - \frac{\rho}{1-\alpha} \equiv \gamma_c^P\left(\frac{G}{k},\theta_G\right). \tag{2.85}$$

The steady-state ratio $(G/k)|_{ss}^P$ is then determined by the conditions (2.85) and (2.80). See Figure 2.2 (right) for an illustration.

Comparing γ_c of (2.78) to γ_c^P of (2.85) shows that

$$\gamma_c = \gamma_c^P \quad \Leftrightarrow \quad AN^{\alpha(\sigma_G-1)}\left(\frac{G}{k}\right)^{\alpha}[(1-\tau_y)(1-\sigma_G\,\alpha)-(1-\theta_G)] = \frac{-\rho\alpha}{1-\alpha}. \tag{2.86}$$

Hence, an income tax rate τ_y that implements $(G/k)|_{ss}^P$ given θ_G exists. It is lower the lower the degree of congestion (i. e., the larger σ_G) and higher the greater θ_G.

Allowing the planner to choose θ_G optimally delivers

$$\theta_G^P = \alpha - \frac{\rho}{AN^{\sigma_G\,\alpha}\left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha}} < \alpha. \tag{2.87}$$

[20] A detailed derivation of this and other results discussed in this section can be found in the appendix of this chapter.

Interestingly, θ_G^P falls short of the growth-maximizing level $\theta_G^* = \alpha$ (Turnovsky, 1997a). This difference occurs as the advantage of a larger public investment share only materializes tomorrow whereas the cost in terms of foregone consumption is paid today. This intertemporal aspect explains why θ_G^P declines in ρ. Since the benefit of an increase in the stock of public capital accrues to all firms, θ_G^P increases in N. Notice that no intertemporal consideration is present when θ_G^P is determined in the flow model of Section 2.3.3.1. Therefore, in that case the growth-maximizing and the welfare-maximizing expenditure shares coincide, i. e., $\theta_G^* = \theta_G^P = \alpha$.

One can show that an income tax rate equal to

$$\tau_y^P = \frac{\alpha\,(1-\sigma_G)}{1-\sigma_G\,\alpha}, \tag{2.88}$$

implements the Pareto-efficient steady-state allocation involving θ_G^P. The optimal income tax corrects for the congestion externality and recommends the same tax rate as in the flow model of Section 2.3.3.1 (see equation 2.28). The larger the degree of congestion the greater the optimal income tax.[21]

A curious implication arises when the degree of congestion is sufficiently high, i. e., σ_G close to zero. For instance, in the extreme case of proportional congestion, $\sigma_G = 0$, $\tau_y^P = \alpha > \theta_G^P$ such that the government should impose an income tax rate in excess of its current investment costs and refund the excess revenue in form of lump-sum taxes. In the respective flow model (equation 2.28) the optimal income tax is $\tau_y^P = \alpha = \theta_G^P$ so that government expenditure is exactly covered. Thus, in the stock model a larger income tax rate is required in order to offset the incentive to overaccumulate private capital due to congestion.

[21]Marrero and Novales (2005) show that the presence of a significant level of wasteful public expenditure that does neither affect the economy's technology nor preferences is another reason for why a positive income tax leads to faster long-run growth and higher welfare than lump-sum taxes. Turnovsky (1997b) confirms the results of (2.87) and (2.88) for a small open economy with exogenous labor supply and private and public investments subject to adjustment costs.

2.4.2 Maintenance of Public Capital

Due to its use or the passage of time, a fraction of the current stock of public capital depreciates. Maintenance refers to investments that replace depreciated public capital. Conceptually, the incorporation of such replacement investments requires the identification of wear and tear with different parts of the existing public capital stock. Since here this stock comprises homogeneous capital goods, such an identification is not possible. Therefore, we follow the literature, in particular Rioja (2003) and Kalaitzidakis and Kalyvitis (2004), and model replacement investments as an attempt to reduce the instantaneous rate of depreciation of public capital. The new question is then how the economy splits up its expenditure on public capital into "new" public capital goods and in replacement investments, i. e., investments that reduce the rate of depreciation.

Denote G_I the per-period investments in "new" public capital goods and M the level of per-period maintenance investments. Then, the economy's gross investment is $G_I + M$. With D denoting depreciation, the stock of public capital evolves according to $\dot{G} = G_I + M - D$.

As proposed by Kalaitzidakis and Kalyvitis (2004), we model the difference between replacement investments and actual depreciation as

$$D - M \equiv \delta_G\left(\frac{M}{Y}\right)G, \quad \text{with} \quad \delta_G(.) > 0 > \delta_G'(.). \tag{2.89}$$

The idea is that a higher level of maintenance M reduces the level of depreciation whereas a more intense usage measured by Y increases it. With (2.89) the accumulation of public capital is governed by

$$\dot{G} = G_I - \delta_G\left(\frac{M}{Y}\right)G. \tag{2.90}$$

We assume that the government finances its total expenditure, G_I+M, via income taxes such that the government's budget constraint is

$$M + G_I = \tau_y Y. \tag{2.91}$$

Let θ_M and $(1-\theta_M)$ denote the shares of total government expenditure that are allocated to maintenance and "new" capital goods, respectively, i. e.,

$$M = \theta_M \tau_y Y \quad \text{and} \quad G_I = (1-\theta_M)\tau_y Y. \tag{2.92}$$

To simplify, we abstract from congestion effects such that the individual household-producer's production function is (2.7) which we restate here for convenience[22]

$$y = A\left(\frac{G}{k}\right)^{\alpha} k.$$

Decentralized Equilibrium

The individual household-producer chooses c and k to maximize her utility u given by (2.2) subject to her flow budget constraint $\dot{k} = (1-\tau_y)y - c$. The Euler condition is then

$$\gamma_c = (1-\tau_y)(1-\alpha)A\left(\frac{G}{k}\right)^{\alpha} - \rho, \tag{2.93}$$

which corresponds to (2.78) for $\sigma_G = 1$. The growth rates of public and private capital result from the public accumulation equation (2.90) and the individual's resource constraint

$$\gamma_G = (1-\theta_M)\tau_y AN\left(\frac{G}{k}\right)^{\alpha-1} - \delta_G(\theta_M\tau_y), \tag{2.94}$$

$$\gamma_k = (1-\tau_y)\,A\left(\frac{G}{k}\right)^{\alpha} - \frac{c}{k}. \tag{2.95}$$

Then, the dynamical system of the economy is given by (2.93) - (2.95) and initial conditions k_0, G_0, and the transversality condition of the household-producer's optimization problem.

Analogously to the previous section, we obtain the steady-state ratio $(G/k)|_{ss}$ and the common steady-state growth rate for c, G, and k from (2.93) and (2.94). These equations also reveal that no clear cut comparative statics for the steady-state growth rate with respect to θ_M and τ_y are available. However, a steady-state growth-maximizing share of maintenance investments, θ_M^*, can be determined, at least implicitly. The total differential of (2.93) and (2.94) delivers the condition

$$AN\left[\left(\frac{G}{k}\right)^{*}\right]^{\alpha-1} = -\delta_G'\left(\theta_M^*\tau_y\right). \tag{2.96}$$

Intuitively, the optimal allocation of current output to public capital investments satisfies $\partial\dot{G}/\partial G_I = \partial\dot{G}/\partial M$, i. e., the last marginal unit spent on maintenance contributes the

[22] See Dioikitopoulos and Kalyvitis (2008) for an analysis of public capital maintenance and congestion.

same amount to the change in public capital stock as the last marginal unit spent on "new" public capital goods.

Kalaitzidakis and Kalyvitis (2004) show further that the growth-maximizing income tax rate, τ_y^*, evaluated at $\theta_M = \theta_M^*$ is

$$\tau_y^* = \frac{\alpha}{1 - \theta_M^*(1-\alpha)} > \alpha. \tag{2.97}$$

This result contrasts with the finding of the previous sections where the growth-maximizing tax rate was found to equal α. Intuitively, the presence of maintenance adds a productive use to public capital expenditure. To exploit this opportunity, the optimal income tax should be higher than without it. To strengthen this intuition we introduce an explicit functional form such that $\delta_G = (\theta_M \tau_y)^{-\varepsilon}$, $\varepsilon > 0$. Then, τ_y^* of (2.97) becomes

$$\tau_y^* = \frac{\alpha\,(\varepsilon + 1)}{1 + \alpha\varepsilon}. \tag{2.98}$$

In the limit $\varepsilon \to 0$, the effect of maintenance vanishes and the optimal income tax is $\tau_y^* = \alpha$. On the other hand, the effect of maintenance becomes more pronounced the larger ε and $\tau_y^* \to 1$ as $\varepsilon \to \infty$.[23]

Further, it can be shown that the growth-maximizing share of new public capital goods, $(G_I/Y)^* = (1-\theta_M^*)\tau_y^* < \alpha$. With $\delta_G = (\theta_M \tau_y)^{-\varepsilon}$, we find using (2.98) in (2.97) that $\theta_M^* = \varepsilon/\left(\varepsilon + 1\right)$. Then,

$$(G_I/Y)^* = \frac{\alpha}{1 + \alpha\varepsilon} < \alpha. \tag{2.99}$$

Hence, for $\varepsilon = 0$ we are back in the case without maintenance and $(G_I/Y)^* = \alpha$. Moreover, as $\varepsilon \to \infty$ all public expenditure goes to maintenance and $(G_I/Y)^* \to 0$.

[23] Similar to the present setup, Greiner and Hanusch (1998) have a stock model where government expenditure can be allocated to two uses. They are the accumulation of the public capital stock and a subsidy to private capital accumulation. The point of their paper is that a rise in the subsidy rate for private capital investment is not necessarily growth-enhancing because it diverts resources away from productive government spending. Moreover, these authors show that for strictly positive subsidy rates the growth-maximizing income tax rate is strictly greater than α. Hence, the qualitative finding of (2.97) may also be the consequence of a growth policy that strengthens the investment incentives of private firms.

2.4.3 Stock-Flow Model of Public Goods

Thus far, we have considered either the flow or the stock approach to modeling public services. An interesting question taken up by Tsoukis and Miller (2003) and Ghosh and Roy (2004) is whether and how new implications for growth and welfare arise if both approaches appear simultaneously.

Let G_f denote the flow of public services and G_s the stock of public capital. Then, a natural extension of the production function (2.7) is

$$y = \left(G_s^{\beta} G_f^{1-\beta}\right)^{\alpha} k^{1-\alpha}, \quad 0 < \beta < 1. \tag{2.100}$$

We assume that $\dot{G}_s = \theta_{G_s} Y$ and $G_f = \theta_{G_f} Y$. Moreover, total government expenditure is fully financed via a distortionary income tax and continues to be a fixed fraction of output, i. e.,

$$\dot{G}_s + G_f = \tau_y Y = \theta_G Y, \quad \theta_G \equiv \theta_{G_s} + \theta_{G_f}. \tag{2.101}$$

Tsoukis and Miller (2003) show that the growth-maximizing shares are

$$\theta_G^* = \alpha, \quad \theta_{G_s}^* = \alpha\beta, \qquad \theta_{G_f}^* = \alpha(1-\beta). \tag{2.102}$$

Hence, each facet of public expenditure receives a share equal to its respective output elasticity.

The Pareto-efficient allocation mimics the properties of the previous sections. In particular, one finds that the equilibrium shares of total expenditure and of public capital investment are too large relative to their welfare-maximizing level whereas the equilibrium flow share is the welfare-maximizing one, i. e.,

$$\theta_G^P < \theta_G^*, \quad \theta_{G_s}^P < \theta_{G_s}^*, \quad \theta_{G_f}^P = \theta_{G_f}^*. \tag{2.103}$$

Ghosh and Roy (2004) analyze the question how the government by deciding on the ratio of the two types of public spending can at least partially compensate for the non-optimal choices of the private sector.

2.5 Variations on a Theme

2.5.1 Stochastic Environments

Turnovsky (1999c) studies the role of productive government expenditure in a stochastic version of the flow model with congestion as presented in Section 2.3.3.1. He finds that under uncertainty the growth-maximizing level of government expenditure depends on the degree of relative risk aversion. If the latter is strong, then the growth-maximizing expenditure share exceeds the Pareto-efficient one.

On the production side, uncertainty is introduced via a productivity shock, du, that is independent and identically distributed - normal with zero mean and variance $\sigma_u^2 dt > 0$. This shock is proportional to the current mean flow of output. More precisely, the flow of output, dy, produced by the individual household-producer over the small time period $(t, t + dt)$ is $dy = Ag^{\alpha}k^{1-\alpha}[dt + du]$, where g is given by (2.23). Government expenditure comprises a deterministic, productivity-enhancing component, G, and a stochastic component, G'. The total flow of resources claimed by the government over the period dt amounts to $d\bar{G} = Gdt + G'du$. Both types of government expenditure are fixed fractions of the aggregate mean rate of the output flow, i. e., $G = \theta_G NAg^{\alpha}k^{1-\alpha}$ and $G' = \theta_G^{'} NAg^{\alpha}k^{1-\alpha}$. Thus, the fraction θ_G now represents the government's choice of the (deterministic) size of government, while $\theta_G^{'}$ represents the fraction of the aggregate output shock absorbed by the government.

To allow for varying degrees of risk aversion the per-period utility function is now $\left(c^{1-\upsilon} - 1\right) / (1 - \upsilon)$, $\upsilon \geq 1$. Here, υ is the coefficient of relative risk aversion.

This setting delivers a unique stochastic balanced growth path where the mean growth rate depends on the degree of relative risk aversion, the variance of the shock, the shares of government expenditure, and the degree of relative congestion. With $\sigma_u^2 = 0$ and $\upsilon = 1$ this growth rate collapses to the one under certainty as given by (2.25). To interpret the equilibrium under uncertainty we follow Turnovsky (1999c) and consider reasonable degrees of relative risk aversion to be $\upsilon > 1$.

The mean steady-state growth rate increases in the variance of du. Intuitively, a higher variance of the shocks means higher risk. Therefore, more risk-averse agents increase their precautionary savings, which allows for faster growth.

The deterministic growth-maximizing share of government expenditure under full income tax financing, θ_G^*, exceeds α. The reason is that a higher θ_G raises the productivity of private capital and, since the shock is proportional to output, magnifies the volatility of output. As the latter induces more precautionary savings that increase the mean growth rate there is an additional reason to increase θ_G.

The introduction of uncertainty reduces the Pareto-efficient share of deterministic government expenditure below θ_G^*. Intuitively, the planner takes the individual's risk aversion into account and chooses a smaller steady-state growth rate that comes along with lower volatility. The optimal tax structure that implements the Pareto-efficient allocation has to internalize the congestion externality. This is accomplished with a strictly positive income tax. This tax reduces the growth rate of the economy and, hence, the degree of volatility.[24]

2.5.2 Increasing Returns

Thus far, we have assumed that the production function of the individual firm exhibits constant returns to scale with respect to private capital and productive government expenditure at the social level. Constant returns are, among others, responsible for the existence of a balanced growth path and the absence of transitional dynamics in the flow models based on Barro (1990). Intuitively, this assumption is not mandatory. For instance, in developing countries the density of the road network may be so low that twice as much private capital and twice as many roads more than double output.

Conceptually, in the presence of external effects associated with productive government expenditure, the expected return on private capital investments of individual firms de-

[24]Turnovsky (1999b) considers a small open economy under the same uncertainty as above. He shows that the Pareto-efficient share of government expenditure is greater in the open than in the closed economy if and only if the economy is a net creditor. The reason is that some of the risk of domestic productivity shocks is exported and reduces the volatility of domestic income. Hence, for a given degree of risk aversion, the individual is ready to accept a greater volatility caused by a bigger size of the government.

pends on the investment decisions of all other firms. Thus, there is scope for a self-fulfilling prophecy (Krugman, 1991). If all household-producers believe the return on investment to be high, they will invest a lot today. Then, tomorrow aggregate output and, accordingly, government expenditure will be large. The latter raises the return on investment such that the belief of a high rate of return is confirmed in equilibrium.

Abe (1995) and Zhang (2000) incorporate increasing returns at the social level into the flow setup and find multiple equilibria and sophisticated transitional dynamics.[25] For instance, the dynamical system of Abe (1995) delivers a new locally-stable and stationary steady state in addition to an endogenous growth path. Accordingly, the economy may be trapped in a sufficiently small neighborhood of the stationary steady state. Alternatively, a coordinated hike in investment activity may push the economy sufficiently far away from this steady state such that it embarks on an endogenous growth path. The latter may be either due to a self-fulfilling prophecy or to an unpredicted and temporary rise in government activity.[26]

2.5.3 Non-Scale Growth

In previous sections, we have emphasized that the steady-state growth rate depends on the size of the economy measured by the "number" of household-producers - at least as long as the provision of the public good has an element of non-rivalry (see Footnote 5). The larger N, the faster the economy grows. This finding is often referred to as the *scale effect* and has been criticized on both empirical and theoretical grounds (see Jones, 1995).[27] Here, it arises since the level of government expenditure is tied to the size of

[25]Both authors generalize the production function (2.7) to $y = AG^{\alpha}k^{\beta}$ where $\alpha + \beta > 1$. Moreover, they allow for the public good to affect per-period utility. Abe (1995) adopts the research production function of Romer (1986, p. 1019) to model capital accumulation.

[26]Some details necessary to guarantee the success of the suggested government intervention are quite involved. Zhang (2000) reaches similar policy conclusions, e. g., when his interior stationary steady state is an unstable focus.

[27]The scale effect is a feature of the first-generation endogenous growth models of Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992). Here, it results from the specification of the production function for new knowledge. Subsequent idea-based growth models follow Jones (1995) and modify this functional relationship to find qualitatively similar steady-state growth rates as the one derived in equation (2.106). See Jones (1999) for a concise summary of this literature and

the economy measured by aggregate output Ny.

Eicher and Turnovsky (2000) study productive government spending as a flow in a non-scale endogenous growth model in the spirit of Jones (1995). As new elements, their approach incorporates population growth, i. e., a constant growth rate of the "number" of household-producers, $\gamma_N \neq 0$, and a simultaneous treatment of relative and aggregate congestion of public services.[28] The latter is achieved with a modification of equation (2.23). Here, the functional form of productive services derived by an individual firm from public expenditure is

$$g = G\left(\frac{k}{K}\right)^{1-\sigma_R} K^{\sigma_A - 1}, \tag{2.104}$$

where $\sigma_R, \sigma_A \in [0, 1]$ parameterize the degree of relative and aggregate congestion, respectively. Clearly, $\sigma_R = \sigma_A = 1$ is the special case of a pure public good.

In addition, Eicher and Turnovsky (2000) allow for increasing or decreasing returns to scale in the production function of the individual firm such that (2.1) is replaced by $y = Ak^{\beta}g^{\alpha}$ with $\alpha, \beta \in [0, 1]$. Upon combining this production function and equations (2.104) and (2.6) one obtains at the social level

$$y = (A\theta_G^{\alpha})^{\frac{1}{1-\alpha}}\, k^{\frac{\alpha(\sigma_A-1)+\beta}{1-\alpha}} N^{\frac{\alpha(\sigma_R+\sigma_A-1)}{1-\alpha}}. \tag{2.105}$$

The latter is consistent with a balanced growth path involving $\gamma_c = \gamma_k = \gamma_y$ if

$$\gamma_c = \frac{\alpha\,(\sigma_R + \sigma_A - 1)}{1 - \beta - \alpha\sigma_A}\gamma_N \gtreqless 0. \tag{2.106}$$

To fix ideas, assume that the marginal product of capital in (2.105) is strictly positive, i. e., $\alpha(\sigma_A - 1) + \beta > 0$, and let $\gamma_N > 0$. If the denominator of (2.106) is positive, then the marginal product of capital is decreasing in (2.105). As a consequence, y cannot grow as fast as k unless some of the growth of y is due to population growth. Indeed, the numerator is only strictly positive if the output elasticity of labor is positive such that population growth contributes positively to the growth of y. In turn, this is the case if the degrees of congestion are not too pronounced.

The way we find the steady-state growth rate of (2.106) is quite different from previous sections. In fact, here we are not concerned with first-order conditions to determine

Eicher and Turnovsky (1999) for a general treatment.

[28]Pintea and Turnovsky (2006) study the role of relative and aggregate congestion in a two-sector non-scale model with private and "public" firms.

intertemporal prices and, hence, the households' Euler condition. Instead, we require the consistency of equal growth rates of per capita variables with the economy's technology given by (2.105). As a result, the steady-state growth rate is independent of preference parameters like ρ or fiscal policy variables such as θ_G. Consequently, the derivation of a growth-maximizing share of government expenditure θ_G^* as discussed in Section 2.3 becomes irrelevant.

By contrast, a welfare-maximizing share of government expenditure, θ_G^P, can still be determined since the static allocation consistent with steady-state growth need not be efficient. Eicher and Turnovsky (2000) show that $\theta_G^P = \alpha$. Moreover, there is a time-invariant income tax rate that implements the Pareto-efficient allocation $\tau_y^P = \alpha(2 - \sigma_R - \sigma_A)/(\beta + \alpha(1 - \sigma_R))$. Intuitively, τ_y^P internalizes both externalities caused by relative and aggregate congestion. Clearly, τ_y^P decreases in σ_R and σ_A.

2.6 Concluding Remarks

What is the role of productive government expenditure for sustained economic growth? The literature surveyed in this chapter provides a rich set of hints to a full-fledged answer.

First, it establishes an analytical framework in which productive government activity is necessary for balanced growth of per capita variables. Without government activity, we would be back in the neoclassical growth model without technical change and sustained long-run growth. In this framework, government activity can be treated either as a flow or as a stock. In both cases the technology of the economy has the following properties. At the level of individual firms, there are constant returns to scale with respect to private capital, k, and the services derived from productive government activity, g. At the social level, two assumptions imply that the production function of the individual firm becomes, at least asymptotically, linear in k. First, services, g, derived by individual firms are proportional to the level of total government activity, G. Second, the current flow of government expenditure is proportional to the size of the economy. In the flow case, since $G = \theta_G Y$, the linearity in k holds at all t; in the stock case, since $\dot{G} = \theta_G Y$, this linearity holds only in the steady state.

As a consequence, the steady-state properties of the scenarios under scrutiny are similar to those of the AK-model such that the Euler equation determines the steady-state growth rate. We use this property to study and compare the link between productive government activity, economic growth, and welfare in different economic settings.

Second, productive government expenditure affects the steady-state growth rate of consumption through a direct effect on the technology and an indirect effect on investment incentives through the mode of financing. The direct effect is strictly positive except for the small open economy where consumption growth is determined by parameters that are exogenous to the domestic economy. This can be verified from the first column of Table 2.1. It shows the effect of a larger government share, θ_G, on consumption growth under full lump-sum financing. Another polar case has full income tax financing. Such a tax reduces the after-tax marginal return on private capital. Hence, the indirect effect on consumption growth is strictly negative. Column 2 in Table 2.1 reveals that these opposing forces tend to give rise to a growth-maximizing government share. In most settings, this share is equal to the output elasticity of the public input, α. If the government service in addition reduces adjustment costs, then $\theta_G^* > \alpha$; if the government also provides consumption services, then $\theta_G^* < \alpha$.

Third, the welfare-maximizing, i. e., Pareto-efficient, share of government expenditure - Column 3 of Table 2.1 - need not coincide with the growth-maximizing government share. This reflects the trade-off involved in the consumption-savings decision that the planner takes into account: faster growth requires higher investment outlays and reduces consumption today. Most interestingly, here the difference between the flow and the stock variant matters. In the stock case, the benefit from government expenditure today is smaller since it augments output only tomorrow. Therefore, the welfare-maximizing share of government expenditure is smaller.

Fourth, as shown in Column 4 of Table 2.1, appropriate fiscal policy measures can implement the Pareto-efficient allocation. Intuitively, a strictly positive income tax can be used to correct for overaccumulation of private capital due to a negative externality such as congestion.

Arguably, within this well-defined analytical framework further facets of the link between productive government expenditure and sustained economic growth can be studied. One

Table 2.1: Summary of Main Findings

	$\frac{d\gamma_c}{d\theta_G}$	θ_G^*	θ_G^P	τ_y^P
Flow Modeling				
1. Pure Public Good Case	$+$	α	α	0
2. Adjustment Costs	$+$	$> \alpha$	$> \alpha$	> 0
3.1 Public Goods Subject to Congestion Without Excludability	$+$	α	α	$> 0 \,(\text{for}\, \sigma_G \neq 1)$
3.2 Public Goods Subject to Congestion With Excludability	$+$	α	α	$\begin{cases} 0 & \sigma_G = \sigma_E = 1 \\ > 0 & \text{otherwise} \end{cases}$
4. Public Consumption Services	$+$	$\alpha(1-\theta_H)$	$0 < \theta_G^P \leq \alpha$	$(1-\sigma_H)\theta_H^P$
5. Endogenous Labor Supply	$+$	N.A.	α	$\tau_r = 0,\, \tau_w = -\tau_c$
6.1 SOE: Exogenous Labor Supply	0	N.A.	α	$\tau_y^P = \tau_b^P = 0$
6.2 SOE: Endogenous Labor Supply	0	N.A.	α	$\tau_r = 0,\, \tau_w = -\tau_c$
Stock Modeling				
Relative Congestion Without Excludability	$+$	α	$< \alpha$	$\begin{cases} > 0 & \sigma_G \in [0,1) \\ 0 & \sigma_G = 1 \end{cases}$

important aspect for economic growth is the government's ability and willingness to enforce "the rule of law." On the one hand, we can think of private corruption that a strong government may want to combat. This introduces an alternative form to use collected resources in a productive way. An interesting question is then what the optimal degree of corruption depends on if a given amount of tax revenues must be allocated towards competing productive tasks. This goes beyond Mauro (1996) who introduces corruption as a proportional tax on income in the setup of Barro (1990) and finds no distortion in the composition of public spending.[29] On the other hand, the government itself may be weak and corrupt, hence, an impediment to economic growth.[30] One way to incorporate the consequences of inefficient government behavior is to assume that the government cannot transform collected tax revenues one-to-one into, say, productive public infrastructure. Finally, an interesting and related question concerns the determinants of the share of productive government expenditure. While in the models discussed above θ_G was either exogenous or chosen optimally by a planner, in reality this parameter reflects fundamental characteristics of the process of collective decision-making and the distribution of preferences and endowments (see, e. g., Chapters 4 and 5 of this dissertation).

How about the role of productive government expenditure for sustained economic growth once we leave the well-defined analytical framework based on Barro (1990)? Arguably, one weakness of this approach is the knife-edge assumption of constant returns to scale (see, e. g., Solow (1994) for a critique of such assumptions). We have seen in Section 2.5.2 that increasing returns substantially alter the predictions of the growth performance. While the presence of increasing returns is empirically not implausible the policy recommendations of these models are hard to formulate since there is no natural way to select among multiple equilibria. Clearly, more research is needed here.

Some authors argue forcefully against the framework of Barro (1990) because neither the prediction of scale effects nor the dependency of the steady-state growth rate on taxation

[29]The third chapter of this dissertation analyzes the optimal enforcement of the rule of law in the context of an idea-based endogenous growth model. In Chapter 3, government investment into the rule of law determines the fraction of firms' profits that are protected from expropriation by the "mafia" and shapes the incentives to engage in innovation investments. As this investment is costly, the optimal degree of mafia activity is always positive and depends on the economic environment of the economy.

[30]See Acemoglu (2005) for a different notion of weak and strong states and their implications for economic development.

finds empirical support (see, e. g., Peretto, 2003). Indeed, the steady-state growth rate generated by non-scale models tends to be independent of government activity and the size of the economy. However, as we have seen in Section 2.5.3, the steady-state growth rate in the model of Eicher and Turnovsky (2000) is entirely determined by the technology of the economy and its consistency with a balanced growth path. The role of economic agents is then quite passive. Moreover, in cross-country growth regressions the partial correlation between population growth and the growth rate of per-capita GDP is often found to be negative (see, e. g., Barro and Sala-í-Martin, 2004; Kormendi and Meguire, 1985).

In any case, it seems fair to say that the main body of the existing literature on productive government expenditure and economic growth is rooted in the tradition of investment-based endogenous growth models. In view of the strength and weaknesses of this approach it will be desirable in future research to incorporate productive government expenditure into idea-based endogenous growth models. This allows to address new questions, e. g., related to the effect of government activity on the productivity of an economy's research technology. On the other hand, these studies will generate findings that should be compared to those presented in this chapter in order to select robust policy implications. Chapter 3 of this dissertation is a first step in this direction.

2.7 Appendix

The Pareto-Efficient Allocation of Section 2.4.1

Derivation of Equation (2.83)

The present-value Hamiltonian for the social planner's optimization problem is

$$H = \ln c\, e^{-\rho t} + \lambda e^{-\rho t}\left[(1-\theta_G)AN^{\alpha(\sigma_G-1)}G^{\alpha}k^{1-\alpha} - c\right] + \upsilon e^{-\rho t}\theta_G AN^{\alpha(\sigma_G-1)+1}G^{\alpha}k^{1-\alpha}.$$

The optimality conditions with respect to c, k and G, for a given θ_G, then obtain as

$$\frac{1}{c} = \lambda \tag{2.107}$$

$$(1-\alpha)AN^{\alpha(\sigma_G-1)}\left(\frac{G}{k}\right)^{\alpha}[(1-\theta_G)+\mu N\theta_G] = \rho - \frac{\dot{\lambda}}{\lambda} \tag{2.108}$$

$$\frac{\alpha AN^{\alpha(\sigma_G-1)}\left(\frac{G}{k}\right)^{\alpha-1}}{\mu}[(1-\theta_G)+\mu N\theta_G] + \frac{\dot{\mu}}{\mu} = \rho - \frac{\dot{\lambda}}{\lambda}, \tag{2.109}$$

where $\mu \equiv \upsilon/\lambda$ denotes the endogenously determined shadow value of public capital in terms of private capital.

Then, (2.107) to (2.109) deliver the planner's consumption growth rate, γ_c^P, and a differential equation describing the evolution μ

$$\gamma_c^P = (1-\alpha)AN^{\alpha(\sigma_G-1)}\left(\frac{G}{k}\right)^{\alpha}[(1-\theta_G)+\mu N\theta_G] - \rho, \tag{2.110}$$

$$\dot{\mu} = \left[\frac{(1-\alpha)\mu G}{k} - \alpha\right]AN^{\alpha(\sigma_G-1)}\left(\frac{G}{k}\right)^{\alpha-1}[(1-\theta_G)+\mu N\theta_G]. \tag{2.111}$$

The growth rates of private and public capital are given by (2.80) and (2.81). For convenience, we repeat them here

$$\gamma_G = \theta_G AN^{\alpha(\sigma_G-1)+1}\left(\frac{G}{k}\right)^{\alpha-1}, \tag{2.112}$$

$$\gamma_k = (1-\theta_G)\,AN^{\alpha(\sigma_G-1)}\left(\frac{G}{k}\right)^{\alpha} - \frac{c}{k}. \tag{2.113}$$

As the steady-state equilibrium of this economy is one in which consumption, private and public capital all grow at the same rate, it is convenient to express equations (2.110)-(2.113) in terms of the stationary variables $z \equiv G/k$ and $x \equiv c/k$. Then, the following set of differential equations determines the equilibrium dynamics of this economy

$$\frac{\dot{z}}{z} = \theta_G A N^{\alpha(\sigma_G-1)+1} z^{\alpha-1} - (1-\theta_G) A N^{\alpha(\sigma_G-1)} z^{\alpha} + x \tag{2.114}$$

$$\begin{aligned}\frac{\dot{x}}{x} &= [(1-\theta_G) + \mu N \theta_G](1-\alpha) A N^{\alpha(\sigma_G-1)} z^{\alpha} - (1-\theta_G) A N^{\alpha(\sigma_G-1)} z^{\alpha} \\ &+ x - \rho\end{aligned} \tag{2.115}$$

$$\dot{\mu} = [(1-\alpha)\mu z - \alpha] A N^{\alpha(\sigma_G-1)} z^{\alpha-1} [(1-\theta_G) + \mu N \theta_G]. \tag{2.116}$$

Further, the following transversality conditions must hold

$$\lim_{t\to\infty} \lambda k e^{-\rho t} = 0 \quad \text{and} \quad \lim_{t\to\infty} \upsilon G e^{-\rho t} = 0.$$

The steady-state condition $\dot{z} = \dot{x} = \dot{q} = 0$ delivers

$$x - (1-\theta_G) A N^{\alpha(\sigma_G-1)} z^{\alpha} = -\theta_G A N^{\alpha(\sigma_G-1)+1} z^{\alpha-1}, \tag{2.117}$$

$$x - (1-\theta_G) A N^{\alpha(\sigma_G-1)} z^{\alpha} = \rho - [(1-\theta_G) + \mu N \theta_G](1-\alpha) A N^{\alpha(\sigma_G-1)} z^{\alpha}, \tag{2.118}$$

$$[(1-\alpha)\mu z - \alpha]\, A N^{\alpha(\sigma_G-1)} z^{\alpha-1} \,[(1-\theta_G) + \mu N \theta_G] = 0. \tag{2.119}$$

Equation (2.119) implies that the steady-state value of μ is given by

$$\mu = \frac{\alpha}{1-\alpha}\frac{1}{z}. \tag{2.120}$$

Substituting (2.120) into (2.110) delivers

$$\begin{aligned}\gamma_c^P &= A N^{\alpha(\sigma_G-1)} z^{\alpha-1} \,[(1-\theta_G)(1-\alpha) z + \alpha N \theta_G] - \rho \\ &= (1-\theta_G) \underbrace{(1-\alpha) A N^{\alpha(\sigma_G-1)} z^{\alpha}}_{\partial y/\partial k} + \theta_G N \underbrace{\alpha A N^{\alpha(\sigma_G-1)} z^{\alpha-1}}_{\partial y/\partial G} - \rho,\end{aligned} \tag{2.121}$$

such that (2.121) corresponds to equation (2.83) of Section 2.4.1 with y given by (2.79).

Derivation of θ_G^P of Equation (2.87)

Maximizing the Hamiltonian with respect to θ_G, i. e., $\partial H/\partial\theta_G = 0$, delivers

$$\begin{aligned} \lambda A N^{\alpha(\sigma_G-1)} G^{\alpha} k^{1-\alpha} &= \upsilon A N^{\alpha(\sigma_G-1)+1} G^{\alpha} k^{1-\alpha} \\ \mu^P &= \frac{1}{N}. \end{aligned}$$

Hence, for an unconstrained Pareto-optimum $\dot{\mu} = 0$ is required. From (2.120) it follows that $z^P = \alpha N/(1-\alpha)$, and thus $\dot{z} = 0$. Then, from (2.116) and the transversality condition we know that also x must be constant at all times. Equalizing the right-hand sides of (2.117) and (2.118) and substituting z^P and μ^P gives

$$\theta_G A N^{\alpha\sigma_G} \left(\frac{\alpha}{1-\alpha}\right)^{\alpha-1} = (1-\alpha) A N^{\alpha\sigma_G} \left(\frac{\alpha}{1-\alpha}\right)^{\alpha} - \rho \tag{2.122}$$

$$\frac{\rho\alpha}{1-\alpha} = (\alpha-\theta_G) A N^{\alpha\sigma_G} \left(\frac{\alpha}{1-\alpha}\right)^{\alpha}. \tag{2.123}$$

Resubstituting (2.123) into (2.118) we obtain

$$\begin{aligned} x^P &= \rho + (\alpha-\theta_G) A N^{\alpha\sigma_G} \left(\frac{\alpha}{1-\alpha}\right)^{\alpha} \\ &= \rho + \frac{\rho\alpha}{1-\alpha} \\ &= \frac{\rho}{1-\alpha}. \end{aligned}$$

Moreover, solving (2.122) for θ_G delivers

$$\theta_G^P = \alpha - \frac{\rho}{A N^{\alpha\sigma_G} \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha}}, \tag{2.124}$$

which corresponds to (2.87) in the main text.

Derivation of τ_y^P of Equation (2.88)

From (2.110) with $\mu = \mu^P$ we obtain

$$\gamma_c^P = (1-\alpha) A N^{\alpha(\sigma_G-1)} \left(z^P\right)^{\alpha} - \rho. \tag{2.125}$$

Then, comparing γ_c of (2.78) to γ_c^P of (2.125) reveals that

$$\gamma_c = \gamma_c^P \quad \Leftrightarrow \quad (1-\tau_y)(1-\sigma_G\,\alpha) = (1-\alpha).$$

Thus, the income tax rate that implements the Pareto-efficient allocation is given by

$$\tau_y^P = \frac{\alpha\,(1-\sigma_G)}{1-\sigma_G\,\alpha},$$

which corresponds to (2.88).

Chapter 3

Innovation, Growth, and the Optimal Enforcement of the Rule of Law

3.1 Introduction

The legal framework of an economy is often thought of as defining the rules of the game that economic agents play. However, what really matters for the incentives of these agents is the strength of the "rule of law". While precise definitions of this term are hard to come by, they usually involve the notion of an economy's degree of property rights protection, the enforceability of contracts, the likelihood of crime and violence, and the effectiveness of an economy's judiciary (see, e. g., Kaufmann et al., 2007; Weil, 2009, p. 346). The focus of this chapter is on the link between a weak rule of law, the incentives to engage in innovation investments, and endogenous economic growth. On the positive side, we want to know whether a minimum rule of law enforcement is a prerequisite for economic growth. On the normative side, we ask whether a stronger rule of law is desirable and discuss the circumstances under which the government should intervene and use resources to strengthen the enforcement of the rule of law.

This chapter addresses these questions in an endogenous growth framework where growth is the result of an expanding set of product varieties in the sense of Grossman and Helpman (1991). The strength of the rule of law is captured by the fraction of profits taken away by, say, mafia activity. This "mafia tax" (Maddison and Pollicino, 2003) deters innovation investments and reduces economic growth. Thus, the framework of this chapter is consistent with the empirical literature that establishes a positive relationship between the strength of the rule of law and economic growth (see, e. g., Kaufmann and Kraay, 2002; Clague et al., 1999; Knack and Keefer, 1995; Barro, 1996, for a recent survey of the empirical literature see Aron, 2000). It is also in line with recent empirical support for a positive link between the rule of law and entrepreneurship (Estrin et al., 2009) or technological development (Giménez and Sanaú, 2007).

In a first step, we take the strength of the rule of law as exogenous and abstract from government interventions. In this scenario, we establish that a weak rule of law is a major reason why an economy may be caught in a no-growth trap. Hence, a minimum strength of the rule of law can be thought of as a necessary condition for sustained growth. Though, on the normative front, we show that a weaker rule of law may be Pareto-improving. This is the case when the equilibrium growth rate exceeds the Pareto-efficient one.[1] Then, it is indeed preferable to weaken innovation incentives. A means to accomplish this is a weaker rule of law which essentially acts as a "mafia tax" on innovation incentives.

In a second step, we endogenize the strength of the rule of law. First, we assume that the government collects final output via taxes and invests these resources to strengthen the rule of law. We find that such government activity can shift the economy from a no-growth path onto a path with strictly positive growth. Second, we analyze the case where the government has to hire a fraction of the workforce as policemen who then enforce the rule of law. We show that the necessity to employ a scarce resource reduces the government's ability to move the economy to a positive growth equilibrium.

Government activity that triggers positive equilibrium growth rates may not be optimal from a welfare point of view. Indeed, we characterize environments where no growth is better than some growth even if the government is able to intervene. For instance, this is the case if the economy's research technology is sufficiently unproductive or if its

[1] Bénassy (1998) and de Groot and Nahuis (1998) establish this possibility in the variety expansion growth model when the gains from specialization captured by the CES production function are small.

market size is too small.

This chapter relates and contributes to at least two different strands of the literature on property rights and growth. First, it makes a contribution to the literature on predation, economic growth, and governmental enforcement of property rights. This literature includes Economides et al. (2007), Zak (2002), and Dincer and Ellis (2005).[2] These three studies explicitly model individuals' decision how to allocate their time or resources between productive and expropriative activities. Individuals have access to an expropriation technology, the specific design of which determines how many and what type of equilibria exist. In contrast to these papers, we do not model the decision of individuals to exert an expropriative activity or not. Rather, we assume the existence of a mafia that diverts resources from the production to the household sector. Hence, the focus of our analysis is on the consequences of mafia activity rather than on the conditions that may cause a mafia to operate. An advantage of this approach is that the equilibrium we identify for each setting is unique and allows for clear-cut predictions.

Second, this chapter contributes to the literature that studies the relationship between intellectual property rights (IPR), i. e., the danger of imitation and the erosion of monopoly power, growth, and welfare in the framework of the variety expansion growth model. Related studies include Kwan and Lai (2003) and Furukawa (2007). They analyze the social benefits and costs of IPR protection assuming that the government can choose the degree of IPR protection at no cost. By contrast, the focus of this chapter is on the role of property rights over profits. In our model the strength of the rule of law determines the share of monopoly profits that is protected from expropriation. Moreover, we argue that the enforcement of the rule of law through governments is endogenous and costly. Accordingly, the optimal degree of law enforcement equilibrates the advantage of better incentives and faster growth to the disadvantage of foregone consumption used up as an enforcement device.

The remainder of this chapter is organized as follows. Section 3.2 presents a basic analytical setup. Section 3.3 establishes the dynamic general equilibrium and compares it to the Pareto-optimum for an exogenous strength of the rule of law. In Section 3.4 we endogenize the strength of the rule of law. Here we also derive our main results

[2] See, e. g., Grossman and Kim (1996), Tornell (1997) and Lindner and Strulik (2004) for the analysis of predation and growth in environments without a law enforcing government.

on the effect of government intervention on growth and welfare. Section 3.4.1 deals with the case where the rule of law can be strengthened through an investment of final output whereas Section 3.4.2 presents the case where labor is needed to enforce the rule of law. Section 3.5 concludes. Proofs are relegated to Appendix A. Appendix B contains additional material. It establishes the robustness of the qualitative results for a lab-equipment specification of the variety expansion growth model.

3.2 The Basic Setup

We consider a closed economy with four sectors and a mafia. In later sections a government will be added. *Households* work, consume, and save. The *final-good sector* produces a consumption good out of a variety of existing intermediate goods. The *intermediate-good sector* consists of monopolistically competitive firms that manufacture one intermediate good using labor as the only input. The blueprint for the production of each intermediate good is invented in a *research sector*. The rule of law is imperfect in the sense that property rights in the intermediate-good sector are not fully secured. One may think of intermediate-good firms being subject to expropriation by an organization such as the *mafia*.

We interpret the strength of the rule of law as the fraction of profits in the intermediate-good sector that is protected from expropriation and denote it by $q \in [0, 1]$. Mafia income is "laundered" and increases consumption and savings of the household sector.

The Household Sector

There is a continuum of identical households of mass 1. We study their behavior through the lens of a single representative household that supplies the time-invariant aggregate labor endowment L inelastically to the intermediate-good and the research sectors. Her consumption-savings decision maximizes intertemporal utility

$$U = \int_0^\infty \ln c(t)\, e^{-\rho t} dt, \tag{3.1}$$

where $c(t)$ is consumption at date t and ρ the subjective discount rate. Henceforth, we suppress time arguments whenever this does not cause confusion. Household income comprises at each t labor income, returns on assets, Ω, and laundered mafia income, M, such that the household's flow budget constraint is given by

$$\dot{\Omega} = wL + r\Omega + M - p_c c, \quad \text{with} \quad \Omega(0) > 0. \tag{3.2}$$

Here, w denotes the wage rate at t, r the rate of return on assets, and p_c the price of the consumption good. The budget constraint (3.2) captures the fact that in a closed economy total mafia income is laundered, re-introduced into the economy, and used for consumption or saving of the household sector.

The household's maximization of (3.1) is subject to (3.2) and a No-Ponzi condition. Following Grossman and Helpman (1991), we choose consumption expenditure as the numéraire, i. e., $p_c c = 1$ at all t. Then, the Euler condition implies $r = \rho$, and the transversality condition is $\lim_{t\to\infty} e^{-\rho t}\Omega(t) = 0$.

The Final-Good Sector

The final-good firms produce a homogeneous output y out of a variety of exiting intermediates, and then sell it to the household sector for consumption. The production function is

$$y = \left[A^{(\sigma-1)(1-\alpha)} \int_0^A x(j)^\alpha dj \right]^{1/\alpha}, \tag{3.3}$$

where $A \in \mathbb{R}_{++}$ is the "number" of available intermediate goods at t and $x(j)$, $j \in [0, A]$ denotes the quantity of intermediate-good input j used at t. The parameter $\alpha \in (0, 1)$ determines the elasticity of substitution between any pair of intermediates, $\epsilon \equiv 1/(1-\alpha)$. Following Ethier (1982), the term in front of the integral introduces $\sigma > 0$ as a measure of the gains from specialization. As σ increases, these gains become more pronounced, for $\sigma \to 0$ they vanish.

The representative producer of y is competitive and chooses $\{x(j)\}_{j=0}^{A}$ to maximize $p_c y - \int_0^A p(j)x(j)dj$ at all t, where $p(j)$ is the price of input j.

The Intermediate-Good Sector

Each intermediate-good firm $j \in [0, A]$ produces a single intermediate good in a monopolistically competitive environment with demand $x(j) = yp(j)^{-\epsilon}/P$, where $P \equiv [A^{(\sigma-1)(1-\alpha)} \int_0^A p(j')^{1-\epsilon} dj']^{\epsilon/(\epsilon-1)}$. The production function for all varieties is $x(j) = l(j)$, where $l(j)$ is the amount of labor hired by firm j. The price $p(j)$ charged by intermediate-good firm j maximizes his profits $\pi(j) = q\,[p(j) - w]\,yp(j)^{-\epsilon}/P$. Here, $q \in [0, 1]$ denotes the fraction of profits that remains in the hands of intermediate-good producer j after the mafia has made its claims. The weaker the rule of law, i. e., the lower q, the lower are the net profits of intermediate-good producers. Intermediate-good producers regard q as a given constant. The resulting monopoly price satisfies $p(j) = p = w/\alpha$ such that $x(j) = x = yA^{\frac{\sigma}{1-\epsilon}-1}$ and $\pi(j) = \pi = q(1-\alpha)px$.

The Research Sector

Previous to the marketing of an intermediate good it is invented by competitive research firms. The production function of the research sector for new intermediates is

$$\dot{A} = AL_A/a, \tag{3.4}$$

where L_A is the aggregate amount of labor used for research and a is a productivity parameter. Once a new variety is invented, it is sold by auction to the highest bidder who also receives a perpetual patent. Accordingly, the price for such a patent at t is $v(t) = \int_t^\infty \pi(s)e^{-\rho(s-t)}ds$. The profit-maximization problem of the representative research firm is then to choose L_A that maximizes $vAL_A/a - wL_A$. For L_A to be finite the first-order condition is

$$v \leq \frac{wa}{A} \quad \text{with} \quad \text{“} = \text{”}, \quad \text{if} \quad \dot{A} > 0. \tag{3.5}$$

3.3 Exogenous Strength of the Rule of Law

The purpose of this section is to examine the effect of an exogenously given strength of the rule of law on innovation activity and growth. In particular, we show that and how a weak enforcement of the rule of law causes the economy to be trapped in an equilibrium without growth.

3.3.1 Dynamic General Equilibrium

Given q, the Dynamic General Equilibrium (DGE) consists of an allocation $\{c(t), \Omega(t), M(t), y(t), x(j,t), l(j,t), L_x(t), L_A(t), A(t)\}_{t=0}^{t=\infty}$ and a price system $\{r(t), p_c(t), w(t), p(j,t), v(j,t)\}_{t=0}^{t=\infty}$ such that households, final-good, intermediate-good and research firms behave optimally at all t, the market for the final good clears, i. e., $c(t) = y(t)$, there is full employment of labor at all t, i. e., $L_x(t) + L_A(t) = L$, the capital market values firms according to fundamentals and $\Omega(t) = A(t)v(t)$. Moreover, total mafia income at t is equal to the fraction $1-q$ of total gross profits in the intermediate-good sector, i. e., $M(t) = (1-q)A\left(\frac{\pi(t)}{q}\right)$.

The following proposition establishes the existence of a steady-state equilibrium with and without growth.

Proposition 3.1. *The steady-state growth rate of intermediate-good varieties is*

$$g_A^* = max\left\{0, \frac{q(1-\alpha)L - a\alpha\rho}{a\left[(1-\alpha)q + \alpha\right]}\right\} \equiv g_A^*(q). \tag{3.6}$$

The economy immediately jumps to the steady state for any admissible set of initial conditions.

Moreover, the steady-state growth rate of consumption is given by

$$g_c^* \equiv \frac{\dot{c}}{c} = \frac{\sigma}{\epsilon - 1}\, g_A^*. \tag{3.7}$$

Proposition 3.1 reveals that the steady-state growth rates of the economy depend on the exogenously given strength of the rule of law. If $q = 1$, then its enforcement is perfect, and we are back in the world of Grossman and Helpman (1991), where the economy's growth rates depend on $g_A^*(1) = max\{0, (1-\alpha)L/a - \alpha\rho\}$. Whether this rate is strictly positive hinges on the parameters a, ρ, α, and L, which characterize the economic environment. The effect of an imperfectly enforced rule of law on the steady-state allocation depends crucially on whether this set of parameters allows for strictly positive growth or not.

Corollary 3.1. *It holds that*

1. *if* $g_A^*(1) > 0$*, then there is* $q_{min} \in (0,1)$ *such that* $g_A^* > 0$ *and* $g_c^* > 0$ *if and only if* $q > q_{min}$*. If* $q > q_{min}$*, then* $dg_A/dq > 0$*.*

   Moreover,

$$\frac{\partial q_{min}}{\partial \alpha} > 0, \quad \frac{\partial q_{min}}{\partial \rho} > 0, \quad \frac{\partial q_{min}}{\partial a} > 0, \quad \frac{\partial q_{min}}{\partial L} < 0. \tag{3.8}$$

2. *if* $g_A^*(1) = 0$*, then the strength of the rule of law has no growth effects.*

Statement 1 of Corollary 3.1 applies to the case of a strictly positive growth rate under a perfect rule of law. Then, an imperfect enforcement of the rule of law may shift the economy into a no-growth equilibrium. A more intense mafia activity increases the prospect of expropriation. This reduces the value of an innovation and discourages research activity. For the economy to grow at a strictly positive rate, the strength of the rule of law has to surpass a threshold level q_{min}. Thus, we can assert that a certain strength of the rule of law is an underlying prerequisite for sustained growth. If $q > q_{min}$, then raising q speeds up economic growth.

In turn, the threshold level q_{min} also depends on the environment of the economy. The comparative statics of (3.8) reveal that q_{min} increases in α, ρ, and a, but declines in L. Intuitively, the greater the degree of substitutability of intermediate goods, α, the lower are the monopoly profits in the intermediate-good sector. The greater the discount rate, ρ, the lower is the incentive to save and to acquire equity shares issued by research firms. The higher a, the lower is the productivity in the research sector and the smaller is the research output. Finally, the smaller the aggregate labor endowment, L, the less labor is available for research. All these factors have negative effects on the incentives to engage in research. The greater these countervailing forces on the invention of new products, the stronger the rule of law has to be to ensure a strictly positive growth rate. For instance, larger economies with very productive researchers can more easily afford to have a weak rule of law, and will nevertheless grow.

By contrast, if the economy does not admit a strictly positive growth rate under a perfect enforcement of the rule of law, improvements in q have no growth effects. However, even in this case, q affects the equilibrium income distribution.

Corollary 3.2. *For all $q \in [0,1]$, mafia income accounts for the fraction $\mu = (1-q)(1-\alpha)$ of aggregate final-good output while legal income constitutes the fraction $1-\mu$. For any $g_A^* > 0$, the distribution of labor and capital income is independent of q. If $g_A^* = 0$, a rise in q increases the share of capital income in legal household income.*

Corollary 3.2 shows that a rise in q always increases the share of legal income that accrues to households. By contrast, the effect of the rule of law on the distribution of income between labor and capital depends on the level of q. If $q < q_{min}$, then no research activity takes place and the total labor supply is employed in the manufacturing sector. Consequently, the equilibrium wage rate is determined in this sector as $w = \alpha/L$ and is independent of q. Thus, a rise in q has to increase the share of capital income relative to labor income in legal household income. Intuitively, the greater q the greater is the value of the household's assets. By contrast, if $q > q_{min}$, then $g_A^* > 0$ and labor is allocated to the manufacturing as well as the research sector. As in the previous case, a rise in q increases the value of the household's assets. But additionally, a greater patent value boosts the incentives to engage in research and thus the demand for labor in the research sector. Consequently, the equilibrium wage rate as determined by (3.5) rises. This in turn positively affects the household's total labor income. Income from labor and capital rise simultaneously so that the shares of capital and labor income in total legal income remain unaffected by an increase in q.

3.3.2 Welfare Analysis

Consider a social planner who allocates the factors of production and outputs to households and firms. Naturally, this allocation is independent of the rule of law.

Due to the decreasing marginal product of the intermediate goods in the production of the final good, the social planner chooses a symmetric configuration $c = A^{\sigma/(\epsilon-1)} L_x$ at all t. The intertemporal optimization determines the allocation of labor between manufacturing and research. Formally, the planner maximizes U of (3.1), invoking full employment and the production function of the research sector. This problem has previously been solved by Bénassy (1998) and de Groot and Nahuis (1998). In our notation their result appears in the following proposition.

Proposition 3.2. *The Pareto-efficient growth rate of intermediate goods is*

$$g_A^P = \max\left\{0, \frac{L}{a} - \frac{(\epsilon - 1)\rho}{\sigma}\right\}. \tag{3.9}$$

The economy immediately settles at this steady-state growth rate. For all $g_A^P > 0$ holds that $\partial g_A^P / \partial \sigma > 0$.

Corollary 3.3 compares the Pareto-efficient allocation to the equilibrium allocation for all $\sigma > 0$ and $q \in [0, 1]$.

Corollary 3.3. *Let $g_A^*(1) > 0$. Then, there are threshold values $\underline{\sigma}$ and $\bar{\sigma}$ with $0 < \underline{\sigma} < \bar{\sigma} < 1$ such that*

- *if $\sigma > \bar{\sigma}$, then $g_A^P > g_A^*$ for all q.*
- *if $\sigma \in (\underline{\sigma}, \bar{\sigma}]$, then there is a unique $q^P \equiv q^P(\sigma) \in (q_{min}, 1]$ such that $g_A^*(q^P) = g_A^P$. Moreover, $q^P(\sigma)$ is a function satisfying*

$$\frac{dq^P}{d\sigma} > 0, \quad \lim_{\sigma \to \underline{\sigma}} q^P(\sigma) = q_{min}, \quad \text{and} \quad q^P(\bar{\sigma}) = 1.$$

- *if $\sigma \leq \underline{\sigma}$, we have $g_A^P = 0$. It follows $g_A^*(q) = g_A^P$ for all $q \leq q_{min}$.*

Corollary 3.3 applies to economies with a strictly positive equilibrium growth rate under a perfect enforcement of the rule of law. In these economies the Pareto-efficient growth rate may be smaller than the equilibrium growth rate if the gains from specialization are sufficiently small. This is the essence of Bénassy (1998) and de Groot and Nahuis (1998). Corollary 3.3 extends their findings to an economy with an imperfect enforcement of the rule of law.

If $\sigma > \bar{\sigma}$, then the gains from specialization imply too little equilibrium research for any q. By contrast, if $\sigma \leq \bar{\sigma}$, then the level of q decides whether there is too much or too little research in equilibrium. According to Corollary 3.3, if $\sigma \in (\underline{\sigma}, \bar{\sigma}]$, then the equilibrium has too much research when $q = 1$. Thus, there is a unique $q^P \in (q_{min}, 1)$ that implements the Pareto-efficient allocation. Intuitively, since the mafia reduces research incentives Pareto-efficiency can be established. Finally, if σ is so small that $\sigma \leq \underline{\sigma}$, then the Pareto-efficient growth rate is zero and coincides with the equilibrium growth rate for any $q \leq q_{min}$.

In other words, some degree of mafia activity can be socially optimally if $\sigma < \bar{\sigma}$. Intuitively, in this scenario mafia activity acts as a tax on intermediate-good firms, which helps to internalize the externality due to the profit-destruction effect.

3.4 Endogenous Strength of the Rule of Law

From now on the government plays an active role. It uses tax resources to enforce the rule of law. We consider two cases. First, we assume that the government invests produced output in the enforcement of the rule of law. Second, we analyze the case where the government has to hire a fraction of the workforce as policemen.

3.4.1 Final Output as an Investment in the Enforcement of the Rule of Law

Consider a government that levies a tax $\tau \in [0,1]$ on final-good output, y, and uses these resources to invest in the enhancement of the rule of law. Denote G the amount of government investment. Then, a balanced budget in all periods requires

$$G = \tau y. \tag{3.10}$$

The strength of the rule of law will now depend on the share of total government expenditure in final-good output,[3] i. e.,

$$q = F\left(\frac{G}{y}\right) \quad \text{with} \quad F : [0,1] \to [0,1]. \tag{3.11}$$

F is $\mathcal{C}^2$ with $F(0) = q_0 \in (0, q_{min})$, $F(1) = 1$, $F' > 0 > F''$, and $\lim_{G\to 0} F' = \infty$.

This reduced form relationship captures the idea that the government via increased spending relative to the size of the economy can improve the rule of law, though at a declining rate. Naturally, government expenditure is bounded by aggregate output. Without government spending on the rule of law firms keep a fraction q_0 of their profits.

[3] For a steady state to exist government spending has to be proportionate to the size of the economy, at least asymptotically.

The fraction q_0 has an interpretation as the minimal strength of the rule of law that norms of the society guarantee without any governmental enforcement. However, by assumption $q_0 < q_{min}$. Hence, such an economy does not grow. If the government spent total aggregate output on the enforcement of the rule of law, property rights in the intermediate-good sector would be fully secured. Moreover, the function F fulfills an Inada-type condition. Note also that q is a flow variable, i. e., the enforcement level of the rule of law has to be maintained constantly.

3.4.1.1 Dynamic General Equilibrium

The tax on final-good output acts as a tax on consumption. The final good y is manufactured according to (3.3), but final-good producers have to pay the tax τ on their total production. In other words, final-good producers are aware that they will only be able to sell $c = (1-\tau)y$ as private consumption goods to the households. The remaining fraction of output, $G = \tau y$, is claimed by the government and used for the enhancement of the rule of law. In equilibrium, final-good producers pass on the tax to consumers via a higher price p_c.

Given τ, the equilibrium consists of an allocation $\{y(t), c(t), \Omega(t), M(t), G(t), x(j,t), l(j,t), L_x(t), L_A(t), A(t)\}_{t=0}^{t=\infty}$ and a price system $\{r(t), p_c(t), w(t), p(j,t), v(j,t)\}_{t=0}^{t=\infty}$ such that (3.10) and (3.11) hold in addition to the equilibrium conditions of the DGE of Section 3.3.1.

The same steps that led to Proposition 3.1 deliver now the steady-state growth rates as a function of τ:

$$g_A^* = \max\left\{0, \frac{(1-\alpha)F(\tau)L - a\alpha\rho}{a\,[F(\tau)(1-\alpha)+\alpha]}\right\} \equiv g_A^*(F(\tau)) \tag{3.12}$$

and

$$g_c^* = \frac{\sigma}{\epsilon - 1}\, g_A^*. \tag{3.13}$$

Henceforth, we assume that the environment of our economy is such that there would be positive growth if the rule of law were perfect, i. e., $g_A^*(F(1)) > 0$.[4]

[4]For a detailed derivation of Equation (3.12) see the Appendix A.

Analogously to Corollary 3.1, there exists a $\tau_{min} \in (0,1)$ such that for all $\tau > \tau_{min}$ the steady-state growth rates are positive. A strong government that spends sufficient resources on the enforcement of the rule of law can increase the incentives to engage in research such that the economy moves to an equilibrium with strictly positive growth rates.

3.4.1.2 Welfare Analysis

From (3.12) and (3.13) it is clear that $\partial g_c^*/\partial\tau > 0$ for any $g_c^* > 0$ and $\tau \in (0,1)$. However, there is little reason why the tax rate should be arbitrarily large since a higher tax and faster consumption growth have a cost in terms of foregone current consumption. In this section, we study the tax rate the government should choose in order to maximize the welfare of the representative household in equilibrium. By solving the integral of (3.1) using $c(t) = c_0\, e^{g_c^* t}$, household welfare in equilibrium obtains as

$$U = \frac{1}{\rho}\left(\ln c_0 + \frac{g_c^*}{\rho}\right), \tag{3.14}$$

where c_0 denotes the initial level of consumption at $t = 0$. By combining $c = (1-\tau)y$ with the equilibrium conditions $y = A^{\sigma/(\epsilon-1)}Ax$ and $L_x = Ax$ we obtain c_0, for a given initial quantity of intermediates A_0, as

$$c_0 \;=\; (1-\tau)A_0^{\frac{\sigma}{\epsilon-1}}L_x. \tag{3.15}$$

Upon substitution of (3.15) into (3.14) we find[5]

$$U = \begin{cases} \frac{1}{\rho}\ln\left[(1-\tau)A_0^{\frac{\sigma}{\epsilon-1}}L\right] & \text{if } \tau \in [0,\tau_{min}] \\ \frac{1}{\rho}\ln\left[\frac{(1-\tau)\alpha(L+a\rho)A_0^{\frac{\sigma}{\epsilon-1}}}{[F(\tau)(1-\alpha)+\alpha]}\right] + \frac{g_c^*}{\rho^2} & \text{if } \tau \in [\tau_{min},1]. \end{cases} \tag{3.16}$$

[5]For $\tau \in [\tau_{min}, 1]$, one obtains the first term of U in (3.16) by substituting $L_x = \alpha/w$ in (3.15). The steady-state wage rate is determined by condition (3.5) which has to hold with equality in a steady state with positive R&D activity, i. e., $w = vA/a$. The aggregate value of equities, $\Omega = vA$, is constant in the steady state. From $A(s) = A_0\, e^{g_A s}$ and $v(t) = \int_t^\infty \frac{1-\alpha}{A(s)}e^{-\rho(s-t)}ds$ one finds that $vA = F(\tau)(1-\alpha)/(g_A^* + \rho)$ and thus $w = [F(\tau)(1-\alpha)+\alpha]/(L+a\rho)$. Note, that the former also implies that the initial value $A_0 > 0$ determines $v(0)$ such that $\Omega_0 = F(\tau)(1-\alpha)/(g_A^* + \rho)$.

Notice that U is piecewise-defined reflecting the regimes without and with growth.[6]

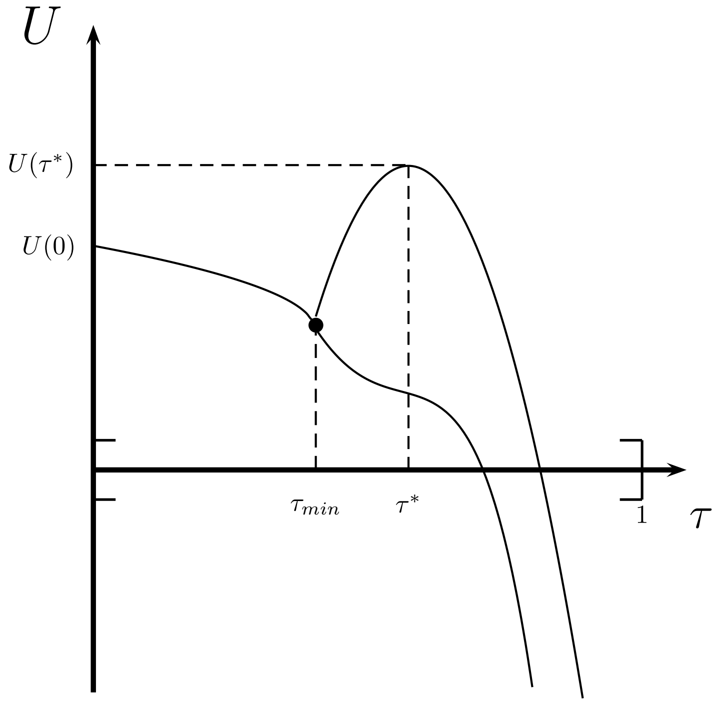

Figure 3.1: Welfare U as a function of $\tau \in [0, 1]$.

For levels of τ smaller than τ_{min}, there is no research. Hence, $L_x = L$, and $g_A^* = g_c^* = 0$. Accordingly, on $[0, \tau_{min}]$, a rise in τ reduces consumption in all periods and welfare declines monotonically in τ (see Figure 3.1). Moreover, a higher τ affects the income distribution by increasing capital income and reducing mafia income (see Corollary 3.2).

For levels of τ greater than τ_{min}, there is research and growth. In this regime a rise in τ has a level effect on current consumption and a growth effect. The level effect is due to the consumption tax and the reallocation of labor from manufacturing to research. The growth effect speeds up $g_A^*(F(\tau))$. If the positive growth effect of a higher tax rate outweighs the negative effects on the level of initial consumption near τ_{min}, then the welfare function is inversely U-shaped on $[\tau_{min}, 1]$. Otherwise, U continues to decline in τ (see Figure 3.1).

The following proposition establishes the welfare-maximizing tax rate.

[6]One readily verifies that U is continuous for all $\tau \in [0, 1]$.

Proposition 3.3. *Let $g_A^*(F(1)) > 0$. Then, it holds that*

1. *on the interval $[0, \tau_{min}]$, U is maximized at $\tau = 0$.*
2. *if $dU/d\tau|_{\tau=\tau_{min}} > 0$, then $\underset{\tau\in[\tau_{min},1]}{\arg\max} U = \tau^* \in (\tau_{min}, 1)$. Otherwise $\tau_{min} = \underset{\tau\in[\tau_{min},1]}{\arg\max} U$.*
3. *if $\tau^* \in (\tau_{min}, 1)$ exists and $U(\tau^*) > U(0)$, then τ^* maximizes U on $[0, 1]$. Otherwise $\tau = 0$ maximizes U on $[0, 1]$. In the non-generic case where $U(0) = U(\tau^*)$, the solution of $\max_{\tau\in[0,1]} U$ is not unique.*

If a positive globally welfare-maximizing tax rate exists, it is strictly smaller than one. Thus, Proposition 3.3 implies that the optimal tax rate will never fully enforce the rule of law which in turn means that there is a positive optimal level of mafia activity. At this point we can draw a comparison to the literature on optimal law enforcement initiated by the seminal paper of Becker (1968). Most of the models in this literature (see, e. g., Garoupa, 1997, for a survey) find that the optimal amount of crime deterrence does not eliminate crime altogether. The principal reason for this is that eradicating crime is costly and has a declining social benefit. In our framework, the enforcement of the rule of law has a social cost in form of tax payments. Moreover, Statement 3 of Proposition 3.3 shows that in terms of welfare no growth can be better than some growth.[7] Observe that $U(\tau^*)$ may not be a global maximum because $U(0) > U(\tau^*)$. Then, it is preferable to set $\tau = 0$. This is the case if the negative static welfare effect of the consumption tax is so large that it is optimal for the government not to levy any taxes, leave the rule of law unchanged and remain in an equilibrium without growth.

Corollary 3.4. *The lower τ_{min} and the greater σ, the more likely it is that τ^* is the global maximizer of U on [0,1]. Moreover, it holds that*

$$\frac{\partial \tau^*}{\partial a} < 0, \quad \frac{\partial \tau^*}{\partial L} > 0, \quad \frac{\partial \tau^*}{\partial \rho} < 0, \quad \frac{\partial \tau^*}{\partial \sigma} > 0, \quad \frac{\partial \tau^*}{\partial \alpha} < 0. \tag{3.17}$$

[7] Arnold and Bauer (2009) draw a similar conclusion in a Grossman-Helpman type variety expansion growth model with erosion of monopoly power due to exogenous imitation and a non-innovative traditional sector. Similarly, Gonzalez (2007) finds in an AK growth model with an exogenous institutional structure of property rights that increases in the security of property rights and growth might not be in the interest of the society.

Corollary 3.4 shows that government intervention that strengthens the rule of law is more likely to be desirable if the economic environment is more prone to growth. The threshold tax rate τ_{min}, which describes the minimum government investment necessary to move an economy onto a positive growth path, is determined by preferences, endowment, and technology as described by the parameters α, a, ρ, and L. As discussed following Corollary 3.1, these parameters determine how pronounced an economy's incentives to engage in research are. The stronger these incentives, the lower is τ_{min}. Moreover, it is straightforward to see from (3.16) that the lower τ_{min} the lower is the instantaneous welfare loss due to reduced current consumption at τ_{min} and the more likely it is that a welfare-maximizing tax rate $\tau^* \in (0, 1)$ exists. The same line of reasoning applies to the condition $U(\tau^*) > U(0)$. Similarly, for sufficiently large values of σ, the gains from specialization are so strong that the welfare gain from additional varieties and increased future consumption possibilities outweighs the static welfare loss.

Finally, the comparative statics of (3.17) reveal that if a positive globally welfare-maximizing tax rate exists, it increases with the productivity in the research sector, the aggregate labor endowment, the propensity to save as well as the gains from specialization, and decreases with the degree of competition in the intermediate-good sector. Hence, we can assert: the more favorable an economy's environment for research activity and thus growth, the greater is its welfare-maximizing tax rate, and thus its welfare-maximizing steady-state consumption growth rate. This may be interpreted as a self-reinforcing feedback.

3.4.2 Policemen as an Investment in the Enforcement of the Rule of Law

Consider a government that levies a lump-sum tax T on households and uses these tax resources to hire a fraction $\delta \in [0, 1]$ of the total workforce as policemen, L_P, to enforce the rule of law. Hence, under a balanced budget we have for all t

$$T = wL_P = w\delta L. \tag{3.18}$$

We stipulate that the strength of the rule of law, q, depends on the share of the policemen in the total workforce, $\delta = L_P/L$, according to

$$q = F(\delta) \quad \text{with} \quad F : [0,1] \to [0,1]. \tag{3.19}$$

F is $\mathcal{C}^2$ with $F(0) = q_0 \in (0, q_{min})$, $F(1) = 1$, $F' > 0 > F''$, and $\lim\limits_{\delta \to 0} F' = \infty$.

3.4.2.1 Dynamic General Equilibrium

The representative household's flow budget constraint is now $\dot{\Omega} = wL + \Omega + M - p_c c - T$. This modification leaves the Euler and the transversality condition of the household's problem unaffected. Since workers are also used as policemen, the labor market equilibrium condition is now $L_x(t) + L_A(t) = (1-\delta)L$. Then, given δ, the equilibrium consists of an allocation $\{y(t), c(t), \Omega(t), M(t), x(j,t), l(j,t), L_x(t), L_A(t), L_P(t), A(t), T(t)\}_{t=0}^{t=\infty}$ and a price system $\{r(t), p_c(t), w(t), p(j,t), v(j,t)\}_{t=0}^{t=\infty}$ such that (3.18), (3.19), the new labor market equilibrium condition and the remaining conditions of the DGE of Section 3.3.1 hold.

Then, following the same steps that led to the steady-state growth rates in Section 3.4.1.1 we obtain the steady-state growth rates as

$$g_A^* = \max\left\{0, \frac{(1-\alpha)(1-\delta)F(\delta)L - a\alpha\rho}{a\left[F(\delta)(1-\alpha)+\alpha\right]}\right\} \equiv g_A^*(\delta) \tag{3.20}$$

and

$$g_c^* = \frac{\sigma}{\epsilon - 1}\, g_A^*. \tag{3.21}$$

In contrast to Section 3.4.1.1, government activity as captured by δ has two opposing effects on the steady-state growth rates of A and c. On the one hand, government spending on the rule of law positively affects the equilibrium growth rates through its effect on q. On the other hand, government activity reduces the labor supply available for research and intermediate-good production to $(1-\delta)L$. The following proposition establishes the growth-maximizing government policy.

Proposition 3.4. *It holds that*

1. *if* $\hat{\delta} = \arg\max\limits_{\delta \in [0,1]} (1-\delta)F(\delta)$ *is such that* $(1-\hat{\delta})F(\hat{\delta}) \leq q_{min}$, *then* $g_A^* = 0$ *for all* $\delta \in [0,1]$.

2. *if* $(1-\hat{\delta})F(\hat{\delta}) > q_{min}$*, then there are* δ_{min} *and* δ_{max} *with* $0 < \delta_{min} < \delta_{max} < 1$ *such that* $g_A^* > 0$ *for all* $\delta \in (\delta_{min}, \delta_{max})$*. In this case, there is a unique* $\delta^* \in (\delta_{min}, \delta_{max})$ *that maximizes* g_A^* *and* g_c^**.*

The first statement of Proposition 3.4 reveals that the government's ability to move the economy to an equilibrium with strictly positive growth rates depends on the environment, in which the economy operates, i. e., q_{min}, and on the effectiveness of the police as specified by the function F.[8] If q_{min} is large and/or the police not very effective, then the reduction of the workforce due to public employment of policemen outweighs its benefits and the steady-state growth rate is zero, independent of the choice of δ. If government intervention can trigger positive growth rates, then according to the second statement of Proposition 3.4, there is a growth-maximizing share of government activity, δ^*, which balances the two opposing effects of government activity.

3.4.2.2 Welfare Analysis

In this section, we derive the welfare-maximizing policy of the government and compare it to the growth-maximizing policy. Following the same steps as in Section 3.4.1.2, we obtain the following piecewise-defined welfare function:

$$U = \begin{cases} \frac{1}{\rho}\ln\left[A_0^{\frac{\sigma}{\epsilon-1}}(1-\delta)L\right] & \text{if } g_A^* = 0 \\ \\ \frac{1}{\rho}\ln\left[\frac{\alpha((1-\delta)L+a\rho)A_0^{\frac{\sigma}{\epsilon-1}}}{[F(\delta)(1-\alpha)+\alpha]}\right] + \frac{g_c^*}{\rho^2} & \text{if } g_A^* > 0. \end{cases} \tag{3.22}$$

The following proposition establishes the share of government employment that maximizes U and compares it to the growth-maximizing share discussed in Proposition 3.4.

Proposition 3.5. *The following statements are true.*

1. *If Statement 1 of Proposition 3.4 holds, then* U *is maximized at* $\delta = 0$*.*

[8]As before, we assume that the environment of our economy is such that there would be positive growth if the rule of law were perfect without any government intervention, i. e., $q_{min} = a\alpha\rho/(1-\alpha)L < 1$.

2. *If Statement 2 of Proposition 3.4 holds, then*

   (a) *on the interval* $[0, \delta_{min}]$, *U is maximized at* $\delta = 0$, *and on the interval* $[\delta_{max}, 1]$, *U is maximized at* δ_{max}.

   (b) *if* $dU/d\tau|_{\delta=\delta_{min}} > 0$, *then* $\arg\max_{\delta\in[\delta_{min},\delta_{max}]} U = \delta^{**} \in (\delta_{min}, \delta_{max})$. *Otherwise* $\delta_{min} = \arg\max_{\delta\in[\delta_{min},\delta_{max}]} U$.

   (c) *if* $\delta^{**} \in (\delta_{min}, \delta_{max})$ *exists and* $U(\delta^{**}) > U(0)$, *then* δ^{**} *maximizes U on* $[0, 1]$. *Otherwise* $\tau = 0$ *maximizes U on* $[0, 1]$.[9]

   (d) *if* $\delta^{**} \in (\delta_{min}, \delta_{max})$ *exists and* $U(\delta^{**}) > U(0)$, *then* $\delta^{**} < \delta^{*}$.

Similarly to Proposition 3.3 , Proposition 3.5 reveals that in terms of welfare no growth can be better than some growth. Moreover, if a positive welfare-maximizing public employment share exists it will be strictly smaller than the growth-maximizing one, i. e., $\delta^{**} < \delta^{*}$. To grasp the intuition for this, consider that

$$\frac{dU}{d\delta} = \frac{1}{\rho^2}\left(\rho\frac{\partial lnc_0}{\partial\delta} + \frac{\partial g_c^*}{\partial\delta}\right). \tag{3.23}$$

The second term, i. e., the consumption growth rate is maximized at δ^*. By contrast, the first term, which corresponds to the static welfare effect, is always negative because a rise in δ reduces the resources available for final-good production. Thus, the public employment share that maximizes U has to be smaller than the one that maximizes g_c^*.

Hence, we conclude that our qualitative results regarding the welfare-maximizing government policy do not depend on whether the government uses final output or part of the labor force to enforce the rule of law.

3.5 Concluding Remarks

This chapter studied the interdependence between innovation, economic growth, and the rule of law in an economy where growth results from an expanding set of product varieties. The strength of the rule of law determines the profit that firms expect from an innovation investment. The results may be summarized as follows.

[9] In the non-generic case where $U(0) = U(\delta^{**})$, the solution of $\max_{\delta\in[0,1]} U$ is not unique.

First, on the positive side, we find that a weak rule of law may be the reason why an economy is caught in a "no-growth trap". In other words, a minimum strength of the rule of law is a prerequisite for sustained growth. Second, on the normative side we establish that a weaker rule of law may be Pareto-improving. This is the case when the equilibrium growth rate exceeds the Pareto-efficient one. Then, the mafia acts like a government charging a tax on monopoly profits, which reduces the incentive to innovate in a desirable way.

Third, when government investment determines the rule of law endogenously, such an investment may shift the economy from a no-growth equilibrium onto a welfare-improving equilibrium with strictly positive growth rates. This is always possible if the government invests final output in the enforcement of the rule of law. By contrast, if policemen are necessary to enforce the rule of law, this possibility arises only if the economic environment is sufficiently favorable to research and/or policemen are sufficiently effective. Finally, even if the government is ready to intervene, the price of fighting the mafia may be too high. In this case, in terms of welfare no growth may be better than some growth. Overall, however, the more favorable the economic environment is towards innovation and growth, the more likely it is that the welfare-maximizing strength of the rule of law requires taxes and government intervention.

3.6 Appendix A

Proof of Proposition 3.1

We start the derivation of the steady-state growth rate by looking at the labor market.[10] The linear production function of intermediates implies for a symmetric configuration that the aggregate labor demand of this sector is $L_x = Ax$. Moreover, constant returns to scale in the production of the final good, clearance of the final-good market ($y = c$), and our normalization imply $1 = Apx$. Thus, $L_x = \alpha/w$. Aggregate labor demand in the research sector obtains from the production function of research as $L_A = ag_A$, where $g_A \equiv \dot{A}/A$. Hence, the labor market is in equilibrium if and only if $g_A = L/a - \alpha/wa$. When employment in research is positive, we need $v = wa/A$ (see equation 3.5). Hence, a necessary condition for positive steady-state growth of A is $v \geq \alpha a/AL$. Defining $V \equiv \Omega^{-1} = 1/Av$ as the inverse of the economy's equity value, we obtain

$$g_A = \max\left\{0, \frac{L}{a} - \alpha V\right\}. \tag{3.24}$$

For the capital market to be in equilibrium, the return that a shareholder can expect must be equal to the return of a riskless loan. As the former is the sum of dividends and capital gains and the latter is equal to ρ, we obtain as a no-arbitrage condition $\rho = (\pi + \dot{v})/v$, where instantaneous net profits with $1 = Apx$ are $\pi = q(1-\alpha)/A$. Then, observing that $\dot{V}/V = -g_A - \dot{v}/v$, we obtain

$$\frac{\dot{V}}{V} = -g_A - \rho + q(1-\alpha)V. \tag{3.25}$$

Equations (3.24) and (3.25) jointly describe the equilibrium paths of V and g_A. Setting $\dot{V} = 0$ in (3.25) and substituting $V = \frac{g_A+\rho}{q(1-\alpha)}$ in (3.24) delivers equation (3.6).

As to the transitional dynamics consider the phase-diagram in the (g_A, V)-plane depicted in Figure 3.2. The kinked curve LL depicts the labor market equilibrium as expressed by equation (3.24) and has to be satisfied at every moment in time. The lines VV_1 and VV_q reflect the combinations of V and g_A that imply $\dot{V} = 0$. While the VV_1-locus

[10] This proof extends the one of Grossman and Helpman (1991, p. 57-62) to an environment with $\sigma \neq 1$ and mafia activity.

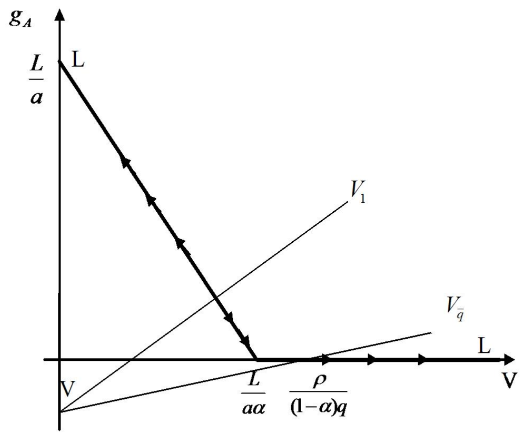

Figure 3.2: Equilibrium Growth Rate with Exogenous Strength of the Rule of Law

represents the case without mafia activity, the VV_q-locus corresponds to a situation with an imperfect rule of law. The intersection of the LL-locus with the VV_1 or the VV_q-locus, respectively, determines the equilibrium growth rate of intermediate goods. Figure 3.2 has been drawn to depict the case in which the rule of law is so weak that there is no growth in equilibrium. Moreover, from the phase diagram one sees that starting the economy outside of the steady state leads either to $V \to \infty$, $g_A = 0$ or $V \to 0$, $g_A \to L/a > 0$. Both cases violate rational expectations.

Consider the first case. As $V \equiv 1/Av \to \infty$ it must be that $v \to 0$ since A cannot decline. However, with $g_A = 0$ and $\pi = q(1-\alpha)/A$ the value of a patent v obtains as

$$v(t) = \int_t^\infty \frac{q(1-\alpha)}{A(t)} e^{-\rho(s-t)} ds = \frac{q(1-\alpha)}{\rho A(t)} > 0,$$

i. e., without innovations the monopoly profits and their present value remain positive. We arrive at a contradiction to $v \to 0$.

The second case has $g_A > 0$ which implies that $A(s) > A(t)$ for all $s > t$, so that

$$v(t) = \int_t^\infty \frac{q(1-\alpha)}{A(s)} e^{-\rho(s-t)} ds < \int_t^\infty \frac{q(1-\alpha)}{A(t)} e^{-\rho(s-t)} ds = \frac{q(1-\alpha)}{\rho A(t)},$$

or $V > \rho/q(1-\alpha)$ which contradicts $V \to 0$.

To obtain the consumption growth rate, consider that for a symmetric configuration

$c = A^{\sigma/(\epsilon-1)}L_x$. Then, $\dot{c}/c = \sigma/(\epsilon-1)g_A + \dot{L}_x/L_x$. As L_A and L_x have to be constant in the steady state g_c^* is given by (3.7). □

Proof of Corollary 3.1

Starting with the first statement, note that $g_A^*(1) > 0$ if and only if $a\rho\alpha/(1-\alpha)L < 1$. Moreover, from (3.6) follows $g_A^*(q) > 0$ if and only if $q > a\rho\alpha/(1-\alpha)L$. Denote $q_{min} \equiv a\rho\alpha/(1-\alpha)L$. Then, $g_A^* > 0$ if and only if $q > q_{min}$, where it is obvious that $q_{min} \in (0,1)$. Moreover, $\forall g_A^* > 0$ it follows from (3.6) that

$$\frac{\partial g_A^*(q)}{\partial q} = \frac{(1-\alpha)\alpha(L+a\rho)}{[q(1-\alpha)+\alpha]^2} > 0.$$

The comparative statics of (3.8) obtain directly from differentiation of q_{min} with respect to α, ρ, a and L.

As to the second statement, note that the second term in brackets of (3.6) positively depends on q. Thus, if $g_A^* = 0$ for a perfect rule of law, i. e., $q = 1$, then it also has to be zero for all $q < 1$. □

Proof of Corollary 3.2

The aggregate value of equities Ω is constant in the steady state. From $A(s) = A_0 e^{g_A^* s}$ and $v(t) = \int_t^\infty \frac{1-\alpha}{A(s)} e^{-\rho(s-t)} ds$ one finds that

$$v(t) \;=\; \frac{q(1-\alpha)}{A_0(g_A^*+\rho)} e^{-g_A^* t} \quad \text{so that} \quad \Omega(t) = A(t)v(t) = \frac{q(1-\alpha)}{g_A^*+\rho}.$$

As $1 = Apx$, the mafia expropriates $(1-q)(1-\alpha)/A$ from each of the A intermediate-good firms. Thus, total mafia income is given by $M = (1-q)(1-\alpha)$. As we have chosen consumption expenditure as the numéraire, $\mu \equiv (1-q)(1-\alpha)$ represents a fraction of aggregate output. Moreover, as legal and mafia income sum up to unity, total legal income corresponds to $1-\mu = q(1-\alpha)+\alpha$.

In a steady state with positive R&D activity ($g_A^* > 0$) condition (3.5) has to hold with equality such that $w = vA/a$. Thus, $wL/(wL + r\Omega) = L/(L + a\rho)$ and $r\Omega/(wL + r\Omega) =$

$a\rho/(L+a\rho)$. Thus, a change in q does not affect the distribution between these two types of income. However, if $g_A^* = 0$, then $L_x = \alpha/w$ with $L_x = L$ implies $wL = \alpha$. Moreover, $r\Omega = q(1-\alpha)$. Then, $wL/(wL+r\Omega) = \alpha/(q(1-\alpha)+\alpha)$ and $r\Omega/(wL+r\Omega) = q(1-\alpha)/(q(1-\alpha)+\alpha)$. Thus, in this case an increase in q raises the share of capital income in legal household income at the cost of a decline in the share of labor income.

□

Proof of Proposition 3.2

The program of the social planner is

$$\max_{g_A, A} \int_0^\infty \ln c\, e^{-\rho t} dt, \quad \text{where} \quad c = A^{\sigma/(\epsilon-1)}\left(L - a g_A\right)$$
$$\text{s.t.} \quad \dot{A} = A g_A.$$

The current-value Hamiltonian of this problem is

$$\mathcal{H} \equiv \sigma/(\epsilon-1)\ln A + \ln\left(L - a g_A\right) + \lambda A g_A.$$

The necessary and sufficient conditions for an optimum are

$$\frac{\partial \mathcal{H}}{\partial g_A} = \frac{-a}{L - a g_A} + \lambda A \leq 0, \quad \text{with "=" if } g_A > 0, \tag{3.26}$$

$$\dot{\lambda} = \rho\lambda - \frac{\sigma}{\epsilon-1}\frac{1}{A} - \lambda g_A, \tag{3.27}$$

$$0 = \lim_{t\to\infty} e^{-\rho t}\lambda A. \tag{3.28}$$

Denote $S \equiv 1/\left(\lambda A\right)$. Then, the first-order conditions (3.26) - (3.28) become

$$g_A = \max\left\{0, \frac{L}{a} - S\right\} \tag{3.29}$$

$$\frac{\dot{S}}{S} = \frac{\sigma}{\epsilon-1}S - \rho \tag{3.30}$$

$$0 = \lim_{t\to\infty}\frac{e^{-\rho t}}{S}. \tag{3.31}$$

In a steady state $\dot{S} = 0$ has to hold. From (3.30) one has $S = (\epsilon - 1)\rho/\sigma$ so that (3.29) becomes (3.9). There are no transitional dynamics. The proof of this mirrors Arnold (1997, p. 132-134) for $\sigma \neq 1$. □

Proof of Corollary 3.3

While the Pareto-efficient growth rate (3.9) is monotonically increasing in σ, the equilibrium growth rate (3.6) is independent of σ and increasing in q. Then, there must exist a $\bar{\sigma}$ such that $\forall \sigma > \bar{\sigma}$ holds $g_A^P > g_A^*(1) > g_A^*(q < 1)$. From (3.9) and (3.6) with $q = 1$ follows

$$g_A^P > g_A^*(1) \quad \Leftrightarrow \quad \sigma > \frac{a\rho}{(1-\alpha)(L+a\rho)}.$$

Denote $\bar{\sigma} \equiv \frac{a\rho}{(1-\alpha)(L+a\rho)}$. Observe that $\bar{\sigma} < 1$ if $g_A^*(1) > 0$ holds (remember that $g_A^*(1) > 0$ implies $q_{min} = a\alpha\rho/(1-\alpha)L < 1$). Thus, the first statement has been proven.

For small σ, g_A^P drops to zero. More specifically, $g_A^P = 0$ for all $\sigma < a\alpha\rho/(1-\alpha)L$. Denote $\underline{\sigma} \equiv \frac{a\alpha\rho}{(1-\alpha)L} = q_{min}$. As $a\alpha\rho/(1-\alpha)L < 1$, one readily verifies that $\underline{\sigma} < \bar{\sigma}$. Thus, in the interval $(\underline{\sigma}, \bar{\sigma}]$, $g_A^P > 0$. Moreover, as $dg_A^*/dq > 0$, $g_A^*(q_{min}) = 0$ and $g_A^*(1) = g_A^P(\bar{\sigma})$, for each $\sigma \in (\underline{\sigma}, \bar{\sigma}]$ must exist a $q^P \in (q_{min}, 1)$ such that $g_A^*(q^P) = g_A^P$. Equalizing (3.9) and (3.6) and solving for q delivers

$$q^P(\sigma) = \frac{\sigma(L+a\rho)(1-\alpha) - a\alpha\rho}{(1-\alpha)a\rho}. \tag{3.32}$$

From (3.32) one may verify that q^P is increasing in σ, $q^P(\underline{\sigma}) = a\alpha\rho/(1-\alpha)L = q_{min}$ and $q^P(\bar{\sigma}) = 1$. Then, the second statement follows directly.

As to the third statement, remember that $g_A^P = 0$ for all $\sigma < \underline{\sigma}$. Thus, for $g_A^* = g_A^P$ to hold g_A^* has to be zero. This is the case for all $q < q_{min}$ (see Corollary 3.1). □

Detailed Derivation of Equation (3.12)

The household optimizes over his consumption expenditure such that his maximization problem remains unchanged.

The final good continues to be produced according to (3.3), but final-good producers have to pay a tax τ on output such that they maximize

$$\max_{\{x(j)\}_{j=0}^{A}} \; p_c(1-\tau)y - \int_0^A p_j x_j dj.$$

The demand function of intermediate-good producers that follows from this optimization problem continues to be given by $x(j) = yp(j)^{-\epsilon}/P$.

For a symmetric configuration, final-good producers make zero profits such that $p_c(1-\tau)y = Apx$. As $p_c(1-\tau)y = p_c c = 1$, we obtain $1 = Apx$.

When maximizing their profits, $\pi(j) = F(\tau)\,[p(j) - w]\,x(j)$, intermediate-good firms treat government expenditure and hence the strength of the rule of law, $q = F(\tau)$, as given. Then, the intermediate-good firms still charge $p = w/\alpha$ and instantaneous profits in the intermediate-good sector obtain as $\pi = F(\tau)(1-\alpha)/A$.

The labor market equilibrium is still described by condition (3.24) and the evolution of the inverse of the economy's legal equity value is governed by

$$\frac{\dot{V}}{V} = -g_A - \rho + F(\tau)(1-\alpha)V. \tag{3.33}$$

Similarly to Section 3.3.1, equations (3.24) and (3.33) jointly describe the equilibrium paths of V and g_A. The steady-state level is obtained from setting $\dot{V} = 0$ in (3.33) and substituting $V = (g_A + \rho)\,/\,[F(\tau)(1-\alpha)]$ in (3.24) yields g_A^* of Equation (3.12).

Proof of Proposition 3.3

We proof each statement of the Proposition separately, starting with Statement 1.

1. On the interval $[0, \tau_{min}]$, U is a monotonically declining function in τ. Thus, on this interval U has its global maximum at $\tau = 0$.

2. On the interval $[\tau_{min}, 1]$, increasing τ has two opposing effects on U. A higher τ negatively impinges on welfare by lowering initial consumption c_0 and positively affects welfare by enabling a higher consumption growth rate g_c^*. For large values

of τ, the former effect dominates the latter and $\lim_{\tau \to 1} U \to -\infty$. For values of τ close to τ_{min} it is not clear a priori which effect dominates. In the following we demonstrate that the maximization of U in $[\tau_{min}, 1]$ has a corner (unique interior) solution if a marginal increase in τ at $\tau = \tau_{min}$ has a negative (positive) effect on utility, i. e., $dU/d\tau|_{\tau=\tau_{min}} < 0$ (> 0).

Let $dU/d\tau|_{\tau=\tau_{min}} < 0$. In this case U is a monotonically declining function in $[\tau_{min}, 1]$ and U is maximized at τ_{min}.

$$\left.\frac{dU}{d\tau}\right|_{\tau=\tau_{min}} < 0 \Leftrightarrow \left|\frac{\partial \ln c_0}{\partial \tau}\right|_{\tau=\tau_{min}} > \left.\frac{1}{\rho}\frac{\partial g_c^*}{\partial \tau}\right|_{\tau=\tau_{min}}$$

$$\Leftrightarrow \frac{a\alpha^2\rho(L+a\rho)}{F'(\tau_{min})(1-\tau_{min})} > \sigma(1-\alpha)^2L^2 - (1-\alpha)a\rho\alpha L, \tag{3.34}$$

where we have substituted $F(\tau_{min}) = \frac{a\alpha\rho}{(1-\alpha)L}$. For future reference, note that $\sigma < F(\tau_{min})$ is a sufficient but not necessary condition for (3.34) to hold.

Inversely,

$$\left.\frac{dU}{d\tau}\right|_{\tau=\tau_{min}} > 0 \Leftrightarrow \left.\frac{1}{\rho}\frac{\partial g_c^*}{\partial \tau}\right|_{\tau=\tau_{min}} > \left|\frac{\partial \ln c_0}{\partial \tau}\right|_{\tau=\tau_{min}}$$

$$\Leftrightarrow \frac{a\alpha^2\rho(L+a\rho)}{F'(\tau_{min})(1-\tau_{min})} < \sigma(1-\alpha)^2L^2 - (1-\alpha)a\rho\alpha L. \tag{3.35}$$

In what follows we show that a unique interior local extremum in $[\tau_{min}, 1]$ exists if (3.35) holds. In $[\tau_{min}, 1]$,

$$\frac{dU}{d\tau} = 0 \Leftrightarrow$$

$$\frac{F'(\tau)(1-\tau)}{[F(\tau)(1-\alpha)+\alpha]^2} = \frac{a\rho}{\sigma(1-\alpha)^2(L+a\rho) - (1-\alpha)a\rho\,[F(\tau)(1-\alpha)+\alpha]} \tag{3.36}$$

Define the left-hand side of (3.36) as $LHS(\tau)$ and the right-hand side of (3.36) as $RHS(\tau)$. A unique solution to (3.36), $\tau^* \in (\tau_{min}, 1)$, exists if $LHS(\tau)$ and $RHS(\tau)$ intersect exactly once in $[\tau_{min}, 1]$. While $LHS(\tau)$ monotonically decreases in τ, $RHS(\tau)$ is a monotonically increasing function in τ. Moreover,

$$LHS(\tau_{min}) > RHS(\tau_{min})$$

$$\text{as} \quad \frac{F'(\tau_{min})(1-\tau_{min})L^2}{\alpha^2(L+a\rho)^2} > \frac{a\rho L^2}{(L+a\rho)\,[\sigma(1-\alpha)^2L^2 - (1-\alpha)a\alpha\rho L]}. \tag{3.37}$$

Condition (3.37) is equivalent to condition (3.35) if the right-hand side of (3.37) is positive which is true for all $\sigma > F(\tau_{min}) = a\alpha\rho/(1-\alpha)L$. The latter has to hold because $\sigma < F(\tau_{min})$ would imply $dU/d\tau|_{\tau=\tau_{min}} < 0$, which is a contradiction. $\sigma > F(\tau_{min})$ also implies that $RHS(1) > 0$. Thus, $LHS(\tau)$ and $RHS(\tau)$ intersect exactly once in $(\tau_{min}, 1)$. As U is a continuous function in $[\tau_{min}, 1]$ with $\lim_{\tau\to 1} U \to -\infty$ and $dU/d\tau|_{\tau=\tau_{min}} > 0$, the unique local extremum at τ^* has to be a global maximum on $[\tau_{min}, 1]$.

3. Statements 1 and 2 determine the welfare-maximizing tax rates on the intervals $[0, \tau_{min}]$ and $[\tau_{min}, 1]$, respectively. To find the globally welfare-maximizing tax rate in $[0, 1]$ we have to compare the welfare associated with these two optimal rates. On the interval $[0, \tau_{min}]$, welfare is always maximized at $\tau = 0$. On the interval $[\tau_{min}, 1]$, welfare is either maximized at τ_{min} or at τ^*. Thus, if τ^* exists, it is the globally welfare-maximizing tax rate on the whole interval $[0, 1]$ if and only if $U(\tau^*) > U(0)$. This is fulfilled if

$$\frac{g_c^*(\tau^*)}{\rho} > -ln(1-\tau^*) - ln\left[\frac{\alpha(L + a\rho)}{L\,(F(\tau^*)(1-\alpha) + \alpha)}\right]. \tag{3.38}$$

Note that the right-hand side of (3.38) is positive as $\tau^* < 1$ and $a\alpha\rho/(1-\alpha)L < F(\tau^*)$. If τ^* exists and $U(\tau^*) < U(0)$, then $\tau = 0$ is globally welfare-maximizing. If $\tau_{min} = \arg\max_{\tau^* \in [\tau_{min}, 1]} U$, $\tau = 0$ is again globally welfare-maximizing as $U(0) > U(\tau_{min})$.

□

Proof of Corollary 3.4

From $F(\tau_{min}) = a\alpha\rho/(1-\alpha)L$ follows

$$\tau_{min} = F^{-1}\left(\frac{a\alpha\rho}{(1-\alpha)L}\right),$$

where F^{-1} is an increasing function. Then,

$$\frac{\partial\tau_{min}}{\partial\alpha} > 0, \quad \frac{\partial\tau_{min}}{\partial a} > 0, \quad \frac{\partial\tau_{min}}{\partial\rho} > 0, \quad \frac{\partial\tau_{min}}{\partial L} < 0.$$

Condition (3.35), which guarantees the existence of an interior welfare maximum on the interval $[\tau_{min}, 1]$, can be rearranged to

$$\frac{a\alpha^2\rho(1 + \frac{a\rho}{L})}{F'(\tau_{min})(1-\tau_{min})} \quad < \quad (1-\alpha)\left[\sigma(1-\alpha)L - a\rho\alpha\right]. \tag{3.39}$$

One readily verifies that the left-hand side of (3.39) increases in α, a, ρ, and decreases in L whereas the right-hand side of (3.39) decreases in α, a, ρ and increases in L and σ. Thus, the smaller τ_{min}, i. e., the smaller α, a, ρ, and the greater L and σ, the more likely it is that condition (3.39) is fulfilled and an interior maximum exists. A similar argument applies for condition (3.38), which assures that $U(\tau^*) > U(0)$. As the right-hand side of (3.38) increases in a, α, ρ, and decreases in L, it is more likely that this condition is satisfied the smaller a, α, ρ, and the greater L.

If $\tau^* \in (\tau_{min}, 1)$ exists, it is determined by condition (3.36) (see proof of Proposition 3.3). Then, the comparative statics results follow from implicit differentiation of (3.36). □

Proof of Proposition 3.4

The equilibrium growth rate (3.20) is equal to zero if

$$F(\delta)(1-\delta) \leq \frac{\alpha\rho a}{(1-\alpha)L} = q_{min}. \tag{3.40}$$

Define the left-hand side of (3.40) as $LHS(\delta)$. $LHS(\delta)$ is greater or equal than zero for all $\delta \in [0,1]$ with $LHS(0) = q_0$ and $LHS(1) = 0$. Moreover, $\partial LHS/\partial\delta = -F + (1-\delta)F'$ is positive for values of δ close to zero and negative for values of δ close to one. Finally, $\partial^2 LHS/\partial\delta^2 < 0$. Thus, there exists a unique $\hat{\delta} \in (0,1)$ such that $\partial LHS/\partial\delta = 0$. If $\hat{\delta} = \arg\max[(1-\delta)F(\delta)]$ is such that $(1-\hat{\delta})F(\hat{\delta}) \leq q_{min}$, then $g_A^* = 0$ for any δ. This proves Statement 1.

For g_A^* to become positive in (3.20) the government has to set δ such that

$$F(\delta)(1-\delta) > \frac{\alpha\rho a}{(1-\alpha)L} = q_{min}. \tag{3.41}$$

If $\hat{\delta} = \arg\max[(1-\delta)F(\delta)]$ is such that $(1-\hat{\delta})F(\hat{\delta}) > q_{min}$, then there exist δ_{min} and δ_{max} with $0 < \delta_{min} < \delta_{max} < 1$ such that $(1-\delta_{min})F(\delta_{min}) = q_{min} = (1-\delta_{max})F(\delta_{max})$. Then, for all $\delta \in (\delta_{min}, \delta_{max})$ holds $g_A^* > 0$. This proves the first part of Statement 2. Moreover, for all $\delta \in (\delta_{min}, \delta_{max})$ g_A^* is a positive function with $\partial^2 g_A^*/\partial\delta^2 < 0$ such that g_A^* has to have a unique maximum. Denote $\delta^* = \underset{\delta\in(\delta_{min},\delta_{max})}{\arg\max}\, g_A^*$. As $g_c^* = \sigma/(\epsilon-1)\, g_A^*$, g_c^* is also maximized at δ^*. This proves the second part of Statement 2. □

Proof of Proposition 3.5

We proof each statement of the Proposition separately, starting with Statement 1.

1. If $g_A^* = 0$, then U is a monotonically declining function in δ. Thus, in this case U has its global maximum at $\delta = 0$.

2. If Statement 2 of Proposition 3.4 holds, then $g_A^* > 0$ for $\delta \in (\delta_{min}, \delta_{max})$ and $g_A^* = 0$ otherwise. Thus,

   (a) on the intervals $[0, \delta_{min}]$ and $[\delta_{max}, 1]$, U is a monotonically declining function in δ. Thus, on these intervals U has its global maximum at $\delta = 0$ and at δ_{max}, respectively.

   (b) At δ_{min}, increasing δ has two opposing effects on U. A higher δ negatively impinges on welfare by lowering initial consumption c_0 and positively affects welfare by enabling a higher consumption growth rate g_c^*. If the latter effect dominates the former, i. e., if $dU/d\tau|_{\delta=\delta_{min}} > 0$, then the maximization of U on $[\delta_{min}, \delta_{max}]$ has a unique interior solution δ^{**}. If this is not the case, then welfare monotonically declines on the interval $[\delta_{min}, \delta_{max}]$ and U is maximized at δ_{min}.

   (c) Statements 2a and 2b determine the welfare-maximizing government activity on the intervals $[0, \delta_{min}]$, $[\delta_{max}, 1]$ and $[\delta_{min}, \delta_{max}]$, respectively. To find the globally welfare-maximizing government share on $[0, 1]$ we have to compare the welfare associated with these optimal shares. First note that $U(0) > U(\delta_{max})$. Moreover, on the interval $[\delta_{min}, \delta_{max}]$, welfare is either maximized at δ_{min} or at δ^{**}. Thus, if δ^{**} exists, it is the globally welfare-maximizing share on the whole interval $[0, 1]$ if and only if $U(\delta^{**}) > U(0)$.

   (d) As $\partial \ln c_0 / \partial \delta < 0$ at any point, the welfare-maximizing share has to be smaller than the growth-maximizing one, i. e., $\delta^{**} < \delta^*$.

□

3.7 Appendix B

Innovation, Growth, and the Optimal Enforcement of the Rule of Law in the Lab-Equipment Specification of the Variety Expansion Growth Model

To study the relationship between innovation, growth, and the optimal enforcement of the rule of law in a variety expansion model with a research sector using the final good as an input several modifications are in order.[11] To justify them, we start with a proof that the lab-equipment specification in the variety expansion model of Grossman and Helpman (1991) that we used in the main text is generically inconsistent with a steady state involving a strictly positive growth rate. Then, we establish the robustness of our qualitative results derived in Section 3.4 in a lab-equipment specification that allows for steady-state growth.

The Model of Section 3.2 with a Lab-Equipment Specification of the Research Technology

In the lab-equipment specification of the variety expansion model the research sector uses final output instead of labor as a productive input, i. e., the research technology is

$$\dot{A} = \frac{z}{a}, \tag{3.42}$$

where z is the final output used in research at t.

Proposition 3.6. *Consider the variety expansion growth model of Section 3.2 and replace the research technology of (3.4) by the one of (3.42). With this modification the variety expansion growth model generically does not admit a steady-state growth path with* $g_A > 0$.

[11] Following Rivera-Batiz and Romer (1991), this specification of the research technology is often referred to as the lab-equipment model.

Proof *Suppose to the contrary that a steady state with $g_A > 0$ exists. Along such a trajectory all variables must grow at a constant rate. Denote $g_x \equiv \dot{x}/x$ the growth rate of any variable x at t. From the research equation (3.42) we have*

$$g_z = g_A. \tag{3.43}$$

Since final output is used as a research input, the market clearing condition for the final good is

$$y = c + z \quad \text{for all } t. \tag{3.44}$$

Differentiating (3.44) with respect to time and dividing the resulting condition by (3.44) gives

$$g_y\left(1+\frac{z}{c}\right) = g_c + g_z\frac{z}{c} \quad \text{for all } t. \tag{3.45}$$

Now, differentiate (3.45) with respect to time and find

$$\dot{g}_y\left(1+\frac{z}{c}\right) + g_y\left(\frac{\dot{z}c-\dot{c}z}{c^2}\right) = \dot{g}_c + \dot{g}_z\frac{z}{c} + g_z\left(\frac{\dot{z}c-\dot{c}z}{c^2}\right) \quad \text{for all } t. \tag{3.46}$$

The steady state requires $\dot{g}_c = \dot{g}_z = \dot{g}_y = 0$, hence, (3.46) collapses to $g_y = g_z$. Combining the latter with (3.43) gives

$$g_y = g_A \quad \text{for all } t. \tag{3.47}$$

The labor market clears if $L = Ax$. Hence, the CES aggregator (3.3) under symmetry implies that $y = A^{\sigma(1-\alpha)/\alpha}L$. Hence,

$$g_y = \sigma\frac{1-\alpha}{\alpha}g_A \quad \text{for all } t. \tag{3.48}$$

Generically, equations (3.47) and (3.48) are inconsistent. Except in the non-generic case where $\sigma = \alpha/(1-\alpha)$, we arrive at a contradiction. □

Endogenous Strength of the Rule of Law in a Lab-Equipment Model

The purpose of this section is to establish the robustness of our qualitative results of Section 3.4 in a lab-equipment specification.

The Basic Setup

As in Section 3.4.1, the government transforms tax resources into expenditure on the rule of law and the strength of the rule of law is endogenously determined by (3.11). Again, the assumption is that the government finances its expenditure by levying a tax $\tau \geq 0$ on consumption c. Then, a balanced budget in all periods requires

$$G = \theta y = \tau c, \tag{3.49}$$

where $\theta = G/y$ measures the share of total government expenditure in final-good output. For convenience, we choose the price of the final good as numéraire.

The representative household maximizes (3.1) subject to her flow budget constraint

$$\dot{\Omega} = r\Omega + wL + M - (1+\tau)c, \tag{3.50}$$

with $\Omega(0) > 0$ and a No-Ponzi game condition. The household's consumption plan then satisfies a standard Euler equation, $g_c = r - \rho$, and the transversality condition is $\lim_{t\to\infty} e^{-\rho t}\lambda(t)\Omega(t) = 0$.

We consider a variant of the lab-equipment model in which research *and* intermediate-good firms use final output instead of labor as a productive input. The aggregate labor endowment L is now employed besides the set of all existing intermediate goods to produce the final-good output y according to

$$y = \frac{1}{\alpha} L^{1-\alpha} \int_0^A x(j)^\alpha dj, \tag{3.51}$$

where the term α in the denominator in the front of the integral is included for notational simplicity. Final-good producers choose L and $\{x(j)\}_{j=0}^{A}$ to maximize $y - wL - \int_0^A p(j)x(j)dj$. The first-order conditions of this maximization problem yield the final-good sector's demand for labor and for intermediate goods, respectively,

$$w = \frac{1-\alpha}{\alpha} L^{-\alpha} \int_0^A x(j)^\alpha dj \tag{3.52}$$

$$x(j) = p(j)^{-\frac{1}{1-\alpha}} L. \tag{3.53}$$

In the intermediate-good sector, each firm produces one unit of its variety at a marginal cost equal to $\mu > 0$ units of the final good. Then, the price $p(j)$ charged by the intermediate-good firm j maximizes his profits

$$\pi(j) = F(\theta)\,(p(j) - \mu)\,p(j)^{-1/(1-\alpha)}L. \tag{3.54}$$

Normalizing $\mu \equiv \alpha$, the resulting monopoly price satisfies $p(j) = p = 1$ such that $x(j) = x = L$, $\pi(j) = \pi = F(\theta)(1-\alpha)L$, and $y = AL/\alpha$. Then, aggregate spending on intermediates denoted by X obtains as $X = A\mu x = \alpha^2 y$.

The research sector invents new intermediates according to (3.42). The price of a patent is

$$v(t) = \int_t^{\infty} \pi\, exp\left(-\int_t^{s} r(s')ds'\right) ds. \tag{3.55}$$

Then, the profit-maximization problem of the research firm yields the first-order condition

$$v \leq a \quad \text{with} \quad \text{“} = \text{”}, \quad \text{if} \quad \dot{A} > 0. \tag{3.56}$$

Dynamic General Equilibrium

Given θ, the equilibrium consists of an allocation $\{c(t), \Omega(t), M(t), y(t), x(j,t), z(t), X(t), A(t), G(t)\}_{t=0}^{t=\infty}$ and a price system $\{r(t), w(t), p(j,t), v(j,t)\}_{t=0}^{t=\infty}$ such that households, final-good, intermediate-good and research firms behave optimally at all t, the capital market values firms according to fundamentals, for all t the value of households' assets is $\Omega(t) = A(t)v(t)$, and total mafia income is $M(t) = (1-F(\theta))A\left(\frac{\pi(t)}{F(\theta)}\right)$. All other markets clear as well such that the economy's resource constraint, $c(t)+X(t)+z(t)+G(t) = y(t)$, holds at all t.

The following proposition establishes the existence of a steady-state equilibrium with strictly positive levels of consumption.

Proposition 3.7. *There exists $\theta_{max} \in (0,1)$ such that for all $\theta < \theta_{max}$ the steady-state growth rate of c, y, and A is*

$$g^* = max\left\{0, \frac{F(\theta)(1-\alpha)L}{a} - \rho\right\} \equiv g^*(F(\theta)) \tag{3.57}$$

and the steady-state level of consumption is strictly positive and given by

$$c = \begin{cases} \frac{1-\alpha^2-\theta}{\alpha} AL & \text{if } g^* = 0 \\ \\ A\left[\frac{1-\alpha^2-\theta-F(\theta)(1-\alpha)\alpha}{\alpha} L + a\rho\right] & \text{if } g^* > 0. \end{cases} \tag{3.58}$$

For $\theta > \theta_{max}$ no steady state with strictly positive consumption levels exists.

The economy immediately jumps to the steady state for any admissible set of initial conditions.

Proof

A steady-state growth path requires that consumption grows at a constant rate g_c^*. This is possible from the Euler equation if and only if the interest rate is constant, i. e., $r(t) = r^*$. Then, it follows from (3.55) that $v = \pi/r^* = F(\theta)(1-\alpha)L/r^*$, i. e., v is constant. Then, the equilibrium condition $\Omega = vA$ implies $\dot{\Omega} = v\dot{A}$. The wage rate is given from equation (3.52) by $w = (1-\alpha)A/\alpha$ and total mafia income by $M = (1-F(\theta))(1-\alpha)A$. Hence, aggregate income, $wL + r\Omega + M$, equals $y - \alpha^2 y$. It follows that the household's budget constraint in equation (3.50) becomes

$$v\dot{A} = (1-\theta)y - X - c, \tag{3.59}$$

where we used $X = \alpha^2 y$ and condition (3.49). From $y = AL/\alpha$ and (3.42) we know that output as well as z have to grow at the same rate as the number of intermediates. Then, one readily verifies from (3.59) that in a steady state also consumption has to grow at this rate. Thus, in a steady state it has to hold that $g^* = g_c^* = g_y^* = g_A^*$. A steady state with a strictly positive growth rate of intermediates requires $v = a$. Thus, substituting for r^* in the Euler equation delivers the steady-state growth rate (3.57). Note that $v\dot{A} = vAg_A$ corresponds to research spending equal to $z = aAg^*$, if $\dot{A} > 0$, and $z = 0$, if $\dot{A} = 0$. Thus, the economy's resource constraint is fulfilled at all t. The steady-state level of consumption obtains from (3.59) as

$$c = \begin{cases} \frac{1-\alpha^2-\theta}{\alpha} AL & \text{if } g^* = 0 \\ \\ A\left[\frac{1-\alpha^2-\theta-F(\theta)(1-\alpha)\alpha}{\alpha} L + a\rho\right] & \text{if } g^* > 0. \end{cases} \tag{3.60}$$

One readily verifies that $c > 0$ for all $\theta < \theta_{max}$, where θ_{max} is such that $\theta_{max} = (1-\alpha)\left[1+\alpha(1-F(\theta_{max}))\right]$. *For values of $\theta > \theta_{max}$ no steady state with positive consumption levels exists.*

It is straightforward to see that there are no transitional dynamics. For a proof see, e. g., Acemoglu (2009, Chapter 13). □

Assume as in the main text that the environment of the economy is such that there would be positive growth if the rule of law were perfect, i. e., $g^*(F(1)) > 0$. Then, analogously to our main model, there exists a $\theta_{min} \in (0, \theta_{max})$ such that the steady-state growth rate is positive for all $\theta > \theta_{min}$. For future reference, note that $F(\theta_{min}) = \frac{a\rho}{(1-\alpha)L}$.

Welfare Analysis

From Proposition 3.7 one immediately verifies that $\partial g^*/\partial\theta > 0$ for any $\theta \in [\theta_{min}, \theta_{max}]$. However, as government expenditure is financed by levying a tax on consumption, an increase in θ has a cost in terms of foregone current consumption. Thus, in the following, we determine the value of $\theta \in [\theta_{min}, \theta_{max}]$ that maximizes the representative household's welfare in the steady state as given by (3.14). Upon substitution of (3.58) for $t = 0$ in (3.14) we obtain the following piecewise-defined welfare function:

$$U = \begin{cases} \frac{1}{\rho}\ln\left[\frac{1-\alpha^2-\theta}{\alpha}A_0 L\right] & \text{if } \theta \in [0, \theta_{min}] \\ \\ \frac{1}{\rho}\ln\left[A_0\left(\frac{1-\alpha^2-\theta-F(\theta)(1-\alpha)\alpha}{\alpha}L + a\rho\right)\right] + \frac{g^*}{\rho^2} & \text{if } \theta \in [\theta_{min}, \theta_{max}]. \end{cases} \tag{3.61}$$

For values of θ smaller than θ_{min}, there is no research and the economy does not grow. Thus, on the interval $[0, \theta_{min}]$, a rise in θ just reduces consumption at all t and welfare declines monotonically in θ. For values of $\theta \in [\theta_{min}, \theta_{max}]$ there is research and growth. A rise in θ then has two effects on welfare. On the one hand, the steady-state growth rate increases; on the other hand, as a fraction of final output is spent on research, less resources are available for current consumption. If the positive growth effect of a higher expenditure share θ outweighs the negative effects on the level of initial consumption near θ_{min}, then the welfare function is inversely U-shaped on $[\theta_{min}, \theta_{max}]$. Otherwise, U continues to decline in θ.

The following proposition establishes the welfare-maximizing government policy.

Proposition 3.8. *It holds that*

1. *on the interval* $[0, \theta_{min}]$, U *is maximized at* $\theta = 0$.
2. *if* $dU/d\theta|_{\theta=\theta_{min}} > 0$, *then* $\underset{\theta \in [\theta_{min}, \theta_{max}]}{\arg\max} U = \theta^* \in (\theta_{min}, \theta_{max})$. *Otherwise* $\theta_{min} = \underset{\theta \in [\theta_{min}, \theta_{max}]}{\arg\max} U$.
3. *if* $\theta^* \in (\theta_{min}, \theta_{max})$ *exists and* $U(\theta^*) > U(0)$, *then* θ^* *maximizes* U *on* $[0, \theta_{max}]$. *Otherwise* $\theta = 0$ *maximizes* U *on* $[0, \theta_{max}]$.[12]

Proof

We proof each statement of the proposition separately, starting with Statement 1.

1. *On the interval* $[0, \theta_{min}]$, U *is a monotonically declining function in* θ. *Thus, on this interval* U *has its global maximum at* $\theta = 0$.
2. *On the interval* $[\theta_{min}, \theta_{max}]$, *increasing* θ *has two opposing effects on* U. *A higher* θ *negatively impinges on welfare by lowering initial consumption* c_0 *and positively affects welfare by enabling a higher steady-state growth rate* g^*. *For large values of* θ, *the former effect dominates the latter and* $\lim_{\theta \to \theta_{max}} U \to -\infty$. *For values of* θ *close to* θ_{min} *it is not clear a priori which effect dominates. In the following we demonstrate that the maximization of* U *in* $[\theta_{min}, \theta_{max}]$ *has a corner (unique interior) solution if a marginal increase in* θ *at* $\theta = \theta_{min}$ *has a negative (positive) effect on utility, i. e.,* $dU/d\theta|_{\theta=\theta_{min}} < 0$ (> 0).

   If $dU/d\theta|_{\theta=\theta_{min}} < 0$, *then* U *is a monotonically declining function on* $[\theta_{min}, \theta_{max}]$ *and* U *is maximized at* θ_{min}. *By contrast, if* $dU/d\theta|_{\theta=\theta_{min}} > 0$, *then holds*

   $$1 + F'(\theta_{min})(1-\alpha)\alpha < \frac{(1-\alpha^2-\theta)\, F'(\theta_{min})(1-\alpha)L}{a\rho}. \tag{3.62}$$

   In what follows we show that a unique interior local extremum on $[\theta_{min}, \theta_{max}]$ *exists if (3.62) holds. On* $[\theta_{min}, \theta_{max}]$, $dU/d\theta = 0 \Leftrightarrow$

   $$\frac{1}{\alpha} = F'(\theta)\left[\frac{c_0}{a\rho A_0} - 1\right]. \tag{3.63}$$

[12] There is a non-generic case where $U(0) = U(\theta^*)$. Then, the solution of $\max_{\theta \in [0, \theta_{max}]} U$ is not unique.

Define the right-hand side of (3.63) as $RHS(\theta)$*.* $RHS(\theta)$ *is a monotonically declining function in* θ*. Moreover,* $RHS(\theta_{max}) < 0$ *and* $RHS(\theta_{min}) > 1/\alpha$ *if condition (3.62) holds. Thus,* $RHS(\theta)$ *intersects exactly once with* $1/\alpha$ *in* $(\theta_{min}, \theta_{max})$*, i. e., there is a unique* $\theta^* \in (\theta_{min}, \theta_{max})$ *such that* $RHS(\theta^*) = 1/\alpha$*. As* U *is a continuous function on* $[\theta_{min}, \theta_{max}]$ *with* $\lim_{\theta \to \theta_{max}} U \to -\infty$ *and* $dU/d\theta|_{\theta=\theta_{min}} > 0$*, the unique local extremum at* θ^* *has to be a global maximum on* $[\theta_{min}, \theta_{max}]$*.*

3. *Statements 1 and 2 determine the welfare-maximizing government policy on the intervals* $[0, \theta_{min}]$ *and* $[\theta_{min}, \theta_{max}]$*, respectively. To find the globally welfare-maximizing government policy on* $[0, \theta_{max}]$ *we have to compare the welfare associated with these two optimal rates. On the interval* $[0, \theta_{min}]$*, welfare is always maximized at* $\theta = 0$*. On the interval* $[\theta_{min}, \theta_{max}]$*, welfare is either maximized at* θ_{min} *or at* θ^**. Thus, if* θ^* *exists, it is the globally welfare-maximizing government policy on the whole interval* $[0, \theta_{max}]$ *if and only if* $U(\theta^*) > U(0)$*.*

□

Equivalently to Proposition 3.3, Proposition 3.8 reveals that no growth might be better than some growth. Moreover, if a positive globally welfare-maximizing share of government expenditure exists, it will never fully enforce the rule of law. Thus, the qualitative results of the main text hold true under the lab-equipment specification of the expanding variety model.

Chapter 4

Population Aging, Endogenous Government Spending, and Economic Growth in a Heterogeneous Infinitely-Lived Agent Framework

4.1 Introduction

It is widely recognized that most economies in the 21^{st} century will experience population aging. This demographic shift involves a change in the relative size of the self-supporting (working) population compared to the number of economically-dependent old. This change is reflected in an economy's old-age support ratio, which measures the workers' share in total adult population. Table 4.1 presents actual data and forecasts of the old-age support ratio for selected countries and regions based on data from the United Nations.[1] Between 2005 and 2050, this ratio is forecasted to decline substantially, e. g.,

[1] Table 4.1 is based on the data that appear as the 'medium variant' prediction in United Nations (2008). The old-age support ratio is the ratio of the population aged $15-64$ over the population aged 15 or over.

in Europe from 81% to 68%.[2]

Table 4.1: Old-Age Support Ratio in Selected Countries and Regions

Year	Europe	Northern America	Japan	South Korea	Brazil	Chile
2005	0.81	0.84	0.77	0.88	0.91	0.89
2050	0.68	0.74	0.57	0.61	0.74	0.74

Population aging is likely to alter the distribution of preferences and the support for different types of government spending, thereby affecting economic growth. A typical concern expressed in the public debate is that falling old-age support ratios and an increasing political weight of the elderly lead to a growing overall tax burden, and thus slow down economic growth. Moreover, it is feared that increased spending for the elderly, e. g., on health and care service, crowds out public investment spending, thus again lowering economic growth. Recent experience suggests that the concerns with respect to public spending may be (at least partly) warranted. For instance, in the United States, the allocation of public spending expenditure is highly skewed towards older members (see, e. g., Iqbal and Turnovsky, 2008) and public spending for the elderly has grown much faster during the second half of the twentieth century than other categories of public expenditure (see Mulligan and Sala-í-Martin, 1999). A similar pattern holds in other OECD countries (see Poterba, 1997). Moreover, from 1960 to the mid-1990's, U.S. federal public spending on infrastructure declined from 5% to 2.5% of GDP. However, over the same time period, total productive government spending on infrastructure, educational institutions, and R&D remained stable at 10% of GDP (CBO, 1998).

This chapter incorporates heterogeneity and a demographic structure into a simple

[2]Note that the total support ratio defined as the ratio between working-age and total population (including children) displays the same negative trend. However, the magnitude of the decline is a bit smaller. This is not surprising as population aging is usually due to a combination of increased life-expectancy and lower fertility rates.

infinite-horizon, endogenous growth model à la Barro (1990). It then shows how this heterogeneous infinitely-lived agent (ILA) framework can be used to analyze how population aging via a democratic voting process endogenously changes the level and composition of government budgets and to derive the ensuing consequences for long-run economic growth.

Specifically, we assume that their is a continuum of infinitely-lived households, each of them comprising not only working young but also economically-dependent elderly members. The age composition of a household is captured by her support ratio, i. e., the ratio of working members to total household members.[3] Households are heterogeneous with respect to their support ratio. Then, population aging corresponds to a shift in the distribution of households such that there are more households with a small support ratio. Moreover, we focus on two public spending categories - productive government expenditure (e. g., on infrastructure, education, or law and order) that increases private production possibilities and public consumption spending that satisfies the preferences of the elderly (e. g., on health and care services). Both types of public spending are fully financed via income taxes. By majority voting, the households determine the policy mix that will be implemented by the government with full commitment. More precisely, voting takes place over two separate income tax rates, which correspond to the share of each spending component in aggregate output.

This framework yields the following results. In a first step, we show that for a given time-invariant public policy mix there exists a unique decentralized competitive equilibrium in which all households (independent of their support ratio) accumulate at the same rate. However, the age composition of each household determines her level of aggregate household income and consumption per household member. Moreover, the economy immediately settles on its balanced growth path. In a second step, we endogenize government policy. To derive the political equilibrium, we first show that each household has a unique most preferred policy mix that indeed does not depend on time. All households want the same share of output to be invested in productive purposes. However, they prefer different shares of output to be spent on the public consumption good that benefits their elderly members. Household preferences can be ranked according to their idiosyncratic support ratio: households with a smaller support ratio prefer higher spend-

[3] Cutler et al. (1990) have introduced such a support ratio in a representative agent, neoclassical growth model.

ing for the elderly. The equilibrium policy mix then coincides with the one preferred by the median voter. It results that the equilibrium policy mix is time-consistent, but not necessarily growth-maximizing. Finally, a comparative static analysis suggests that population aging (i) increases public spending on the elderly (as a share of output), (ii) does not affect productive government expenditure (as a share of output), (iii) raises the total tax burden, and (iv) lowers the growth rate of per capita variables.

Following the seminal paper by Barro (1990), a large literature has discussed the possible links between government expenditure and long-run economic growth. Papers in this strand of the literature usually analyze the growth-and welfare-maximizing size and composition of different types of government expenditure, predominantly in an infinite-horizon, representative agent framework.[4] Thus, in these papers the shares of government expenditure are either exogenous or chosen optimally by a social planner and do not reflect the fundamental characteristics of collective decision-making and the distribution of preferences.[5] Moreover, due to the choice of the infinite-horizon representative agent framework these papers are not concerned with the role of population aging.

By contrast, the standard approach to model population aging is to consider a decline in the population growth rate or an increase in life expectancy in a model with overlapping generations. For instance, Razin et al. (2002), Holtz-Eakin et al. (2004), Bassetto (2008), Gonzalez-Eiras and Niepelt (2008), and Song et al. (2009) study how demographic change endogenously affects fiscal policy via a democratic political process. Hitherto, this literature has not considered a public policy mix that involves productive expenditure as well as spending for a consumption-type public good whose valuation depends on the household's age structure. Moreover, the above mentioned papers do not consider an endogenous economic growth framework such that the link from government expenditure to long-run economic growth is absent. To the best of our knowledge, the only exception is Gonzalez-Eiras and Niepelt (2007) who quantitatively analyze the effect of population aging on public spending for education, public transfers between

[4]See, e.g., Fisher and Turnovsky (1995), Park and Philippopoulos (2002), Kalaitzidakis and Kalyvitis (2004), Ghosh and Gregoriou (2008), or the second chapter of this dissertation for a survey of this literature.

[5]A notable exception is Darby et al. (2004) who examine how random voter turnout, when voters are heterogeneous in their discount rates, affects the actual composition of government expenditure and growth.

workers and retirees, and endogenous productivity growth in a three-period overlapping generations model with human and physical capital accumulation.

The present chapter deliberately follows a different approach to model population aging. The aim is to see whether and how the link between population aging, endogenous government spending, and endogenous economic growth can be analyzed in an infinite-horizon framework directly comparable to the Barro (1990) literature. The qualitative results of this approach are in line with notions voiced in the public debate.

The remainder of this chapter is organized as follows. Section 4.2 describes the economic framework and determines the economic equilibrium, which is the decentralized competitive equilibrium of the economy for an exogenously given, time-invariant government policy. Section 4.3 establishes the political equilibrium. While Section 4.3.1 derives each household's most preferred policy mix, Section 4.3.2 determines the actual policy mix that will be chosen under pure majority rule. Section 4.4 analyzes the implications of population aging. Section 4.5 concludes. Proofs are relegated to Appendix A. Appendix B contains additional material. It establishes the robustness of the qualitative results for a different initial capital distribution and for a utility function with non-separable preferences between private and public consumption.

4.2 The Economic Environment

Consider a closed economy in continuous time that is populated by a continuum of infinitely-lived household-producers of mass 1, i. e., each household is represented by a unique real number i that belongs to the unit interval $[0, 1]$, and a government. Household-producers differ in their composition between working young and economically-dependent elderly members. A household's age structure is captured by her respective support ratio that we denote by $\phi^i = L^i/N^i \in (0, 1]$. Here, N^i represents household i's total "number of members" and L^i her working "members", where we assume that both do not change over time.[6] This seems particularly acceptable in our setup as we are not interested in the demographic evolution of each single household over time but rather

[6]This assumption simplifies aggregation over households and assures the existence of a unique balanced growth path.

in the composition of the population, i. e., in the distribution of households. Household-producers behave competitively and produce one good that can be consumed or invested. At all t, prices are expressed in units of the contemporaneous output of this good. While all household members derive utility from private consumption, the elderly additionally benefit from public spending, e. g., on health care. The government taxes household income to finance the utility-enhancing public good as well as productive government expenditure.

4.2.1 Technology

Household-producer i produces her output $Y^i(t)$ each period t according to

$$Y^i(t) = A\left[K^i(t)\right]^{\alpha}\left[G^i(t)\right]^{1-\alpha}\left[L^i(t)\right]^{1-\alpha}, \quad 0 < \alpha < 1, \tag{4.1}$$

where $A > 0$ denotes the time-invariant total factor productivity, $K^i(t)$ producer i's private capital stock at t and $G^i(t)$ the flow of services derived by firm i from total productive government expenditure at t. Each firm faces positive, but diminishing marginal products in all factors, constant returns to scale in the private factors capital and labor and constant returns to scale in private capital and public productive services. Thus, if G^i rises with K^i the diminishing returns to the accumulation of private capital do not set in. For this reason, the economy will exhibit endogenous steady-state growth. Note also that private capital for simplicity does not depreciate. The economy's total output at t, denoted by $Y(t)$, obtains from aggregation over all firms, i.e., $Y(t) = \int_0^1 Y^i(t)di$. Moreover, equation (4.1) yields household-producer i's output per worker in t as

$$y^i(t) \equiv \frac{Y^i(t)}{L^i} = A\left[k^i(t)\right]^{\alpha}\left[G^i(t)\right]^{1-\alpha}, \tag{4.2}$$

where $k^i(t) \equiv K^i(t)/L^i$ denotes household-producer i's capital stock per worker. We assume that all households have the same initial capital stock per worker, i.e., $k^i(0) = k_0 > 0$.[7]

[7] This assumption implies that households with fewer working members have a lower initial capital holding. Alternatively, one could suppose that the economy starts off with an equal distribution of capital. This assumption does not affect the qualitative results. For details see the first part of Appendix B.

The level of productive services that producer i in t enjoys from aggregate productive government expenditure $G(t)$ is given by

$$G^i(t) = G(t)\,\frac{y^i(t)}{y(t)}, \tag{4.3}$$

where $y(t) \equiv Y(t)/L$ is the economy's aggregate output per worker and $L \equiv \int_0^1 L^i di$ the economy's aggregate labor supply. Equation (4.3) describes a situation of relative congestion (see, e. g., Barro and Sala-í-Martin, 1992, or Turnovsky, 1996c), in which the level of services producer i derives from the public good G at t depends upon her own usage, as represented by her own output per worker, relative to aggregate usage, as represented by economy's aggregate output per worker.[8]

Combining (4.3) and (4.1) producer i's output per worker can be expressed as

$$y^i(t) = A^{\frac{1}{\alpha}} \left(\frac{G(t)}{y(t)}\right)^{\frac{1-\alpha}{\alpha}} k^i(t). \tag{4.4}$$

Equation (4.4) says that each household's production per worker has constant returns to the private input k^i as long as the government maintains a given state of congestion, i. e., as the long as the ratio G/y is constant.

4.2.2 Preferences

Each household i seeks to maximize her overall intertemporal utility which is given by

$$\begin{aligned} U^i(t) &= \int_0^\infty \left[N^i \ln \tilde{c}^i(t) + (N^i - L^i) b \ln H(t)\right] e^{-\rho t}\, dt \\ &= N^i \int_0^\infty \left[\ln \tilde{c}^i(t) + (1-\phi^i) b \ln H(t)\right] e^{-\rho t}\, dt, \end{aligned} \tag{4.5}$$

where $\tilde{c}_i(t)$ denotes private consumption per household member at date t, $H(t)$ aggregate public spending on the elderly at date t, $b > 0$ measures the weight the elderly assign to

[8] Note that this specification of congestion in per worker terms eliminates an undesirable scale effect in the accumulation path of households. If the level of services derived by an individual household-producer depended on her own total output relative to the economy's aggregate output, i. e., if $G^i = GY^i/Y$, then households with more workers would accumulate at a faster rate.

public relative to private consumption goods, and $\rho > 0$ is the constant instantaneous rate of time preference.[9] Note that if all members of household i work, i. e., if $\phi^i = 1$, then (4.5) reduces to the standard utility function $U^i(t) = \int_0^\infty N^i e^{-\rho t} \ln \tilde{c}^i(t)\, dt$. Henceforth, we suppress time arguments whenever this does not cause confusion.

As usual, each household i may use her after-tax income either for consumption or investment in private capital. However, when there are more household members than workers, i. e., when $\phi^i < 1$, then consumption per household member at each t is only a fraction of after-tax output per worker net of investment per worker:

$$\tilde{c}^i = \phi^i \left[(1-\tau) y^i - \dot{k}^i \right], \tag{4.6}$$

where $\tau \geq 0$ denotes a non-discriminatory income tax rate.

4.2.3 Government Policy

The government in each period t taxes household income at rate $\tau = \tau_G + \tau_H$. Revenues collected from household-producers fund productive government expenditure (the component corresponding to τ_G) as well as spending on the elderly (the component corresponding to τ_H). Thus, a balanced government budget requires

$$\tau Y = G + H = \tau_G Y + \tau_H Y. \tag{4.7}$$

Note that $\tau_G = G/Y \in [0,1]$ and $\tau_H = H/Y \in [0,1]$ also represent the ratio of the respective spending component to aggregate output. In Section 4.3, when we turn to the determination of government policy, voting will take over the policy mix (τ_G, τ_H). This policy mix then automatically yields the overall income tax rate τ.

[9] Two remarks are in order. First, note that in the above utility specification the working members do not derive any utility from the public consumption good. This assumption is used to highlight the intergenerational conflict. The key point here is that the old derive considerably greater benefit from spending on health and care services than the young. Second, we have chosen an additively separable utility specification. However, our results do not change if we use a similar utility function with non-separable preferences between private and public consumption. For details see the second part of Appendix B.

4.2.4 Economic Equilibrium

This section derives the economic equilibrium, which is the decentralized competitive equilibrium of our economy for an exogenously given, time-invariant government policy.

The optimization problem for each household-producer i is to choose $\tilde{c}^i(t)$ and $k^i(t)$ to maximize (4.5), subject to (4.6), (4.4), and an initial capital stock per worker $k_0 > 0$. When making her consumption-savings decision each household-producer takes the paths of G, H, y, and τ as given and disregards the possible impact of her investment decision on the amount of public services provided. Then, the intertemporal optimization problem leads to the following Euler condition

$$\frac{\dot{\tilde{c}}^i(t)}{\tilde{c}^i(t)} = (1-\tau)A^{\frac{1}{\alpha}}\left(\frac{G}{y}\right)^{\frac{1-\alpha}{\alpha}} - \rho, \quad \text{for all } i. \tag{4.8}$$

In addition, the equilibrium requires the following transversality condition to be met

$$\lim_{t\to\infty}\left[\lambda^i(t)k^i(t)\right] = 0, \tag{4.9}$$

where λ^i denotes the present-value shadow price of household-producer i's capital stock.

The ratio of productive government spending per unit of the economy's output per worker consistent with condition (4.7) and the fact that $y = Y/L$ is

$$\frac{G}{y} = \tau_G L. \tag{4.10}$$

Upon substitution of (4.10) in (4.8) and taking into account the government's budget constraint (4.7) we obtain

$$\frac{\dot{\tilde{c}}^i(t)}{\tilde{c}^i(t)} = (1-\tau_G-\tau_H)A^{\frac{1}{\alpha}}\left(\tau_G L\right)^{\frac{1-\alpha}{\alpha}} - \rho, \quad \text{for all } i. \tag{4.11}$$

Thus, the per capita consumption growth path chosen by household i is independent of her support ratio ϕ^i. Now assume that τ_G and τ_H remain unchanged over time. (We will show in Section 4.3 that this will be the case in the political equilibrium). Then, we can establish the following proposition, which characterizes the economic equilibrium.

Proposition 4.1. *For given, time-invariant τ_G and τ_H, there exists a unique steady-state growth path along which all variables at the household and the economy-wide level as well as government expenditure grow at the same constant rate*

$$\gamma(\tau_G,\tau_H) \equiv \frac{\dot{\tilde{c}}^i}{\tilde{c}^i} = (1-\tau_G-\tau_H)A^{\frac{1}{\alpha}}\left(\tau_G L\right)^{\frac{1-\alpha}{\alpha}} - \rho. \tag{4.12}$$

The economy immediately jumps to this steady-state growth path for any admissible set of initial conditions.

Proposition 4.1 reveals that all household-producers, independent of their support ratio, accumulate at the same rate. As they all have the same initial capital stock per worker, this in turn implies that all i have the same capital stock per worker and output per worker at all t. However, each household's demographic composition determines her instantaneous level of total income

$$Y^i(t) = y^i(0)L^i e^{\gamma(\cdot)t} = A^{\frac{1}{\alpha}} (\tau_G L)^{\frac{1-\alpha}{\alpha}} k_0 L^i e^{\gamma(\cdot)t} \tag{4.13}$$

[from (4.4) and (4.10)], and of consumption per capita

$$\tilde{c}^i(t) \;=\; \tilde{c}^i(0)e^{\gamma(\cdot)t} = \phi^i \rho k_0 e^{\gamma(\cdot)t}, \tag{4.14}$$

[from (4.6) and (4.12)]. The argument of γ is (τ_G, τ_H) if not mentioned otherwise. Intuitively, the smaller a household's labor force, the smaller is her aggregate income at each t. Similarly, the smaller the share of workers among the total number of household members, i. e., the smaller the support ratio, the lower is the level of consumption per capita at each t.

As all households accumulate at the same rate and the labor supply of each household is constant, it is not surprising that the growth rate $\gamma(\cdot)$ also applies to the economy's aggregate variables, which obtain from aggregation over all households. For instance, the economy's aggregate output is given by

$$Y(t) = \int_0^1 Y^i(t)di = A^{\frac{1}{\alpha}} (\tau_G L)^{\frac{1-\alpha}{\alpha}} k_0 L e^{\gamma(\cdot)t}. \tag{4.15}$$

Finally, as G and H are proportional to aggregate income, they also have to grow at the rate $\gamma(\cdot)$.

The steady-state growth rate depends on the public policy parameters τ_G and τ_H. There is a negative relationship between the government's expenditure ratio for services that benefit the elderly and the steady-state growth rate, i. e., $\partial\gamma(\cdot)/\partial\tau_H < 0$. The reason for this is that each household-producer in her optimization problem disregards that her choice of k^i via aggregate output, Y, affects the aggregate amount of public spending for the elderly, H, and thus the household's overall per-period utility. Thus, τ_H only affects the steady-state growth rate by reducing each household's net income. By contrast, a rise in τ_G has two opposing effects on $\gamma(\cdot)$. A rise in τ_G increases the provision of G, and thus the private marginal product of private capital, but at the same time reduces its after-tax value due to the distortionary tax financing of government expenditure. For a given τ_H, the steady-state growth rate $\gamma(\cdot)$ is maximized at $\tau_G = (1-\alpha)(1-\tau_H)$. Overall, maximum growth is obtained for $\tau_H = 0$ and $\tau_G = 1-\alpha$. Finally, the steady-state growth rate depends on the economy's aggregate labor supply, L, i. e., there is a scale effect. The latter occurs since a greater aggregate labor supply increases aggregate household income, and thus the tax base from which productive government expenditure is financed.

4.3 The Political Equilibrium

So far we have analyzed each household's accumulation path for a given public policy mix (τ_G, τ_H). Now we proceed to endogenize government policy. For this purpose, we first characterize each household's policy preferences and then determine the policy mix that will be implemented by the government under pure majority voting.

4.3.1 Policy Preferences

This section derives the ith household's most preferred policy mix. Each household i considers the economic equilibrium effects of each policy mix and calculates the associated utility level. Household i's most preferred policy mix then is the combination of τ_G and τ_H that delivers the highest utility level. The relevant optimization problem thus is

$$\begin{aligned}
\max_{\tau_G,\tau_H} U^i &= N^i \int_0^\infty \left[\ln \tilde{c}^i(t) + (1-\phi^i) b \ln H(t)\right] e^{-\rho t}\, dt \\
\text{s.t.} & \\
\tilde{c}^i(t) &= \phi^i \rho k_0 e^{\gamma(\tau_G,\tau_H)t} \\
H(t) &= \tau_H A^{\frac{1}{\alpha}} (\tau_G L)^{\frac{1-\alpha}{\alpha}} k_0 L e^{\gamma(\tau_G,\tau_H)t} \\
\gamma(\tau_G,\tau_H) &= (1-\tau_G-\tau_H) A^{\frac{1}{\alpha}} (\tau_G L)^{\frac{1-\alpha}{\alpha}} - \rho,
\end{aligned}$$

where $H(t)$ follows from (4.7) and (4.15). The constraints make clear how the choice of policy affects household i's indirect utility. First, a rise in τ_G has two effects on U^i. On the one hand, a higher τ_G increases utility by raising aggregate production today and thus today's provision of H. On the other hand, a change in τ_G affects U^i by altering the steady-state growth rate. The direction of this effect depends on the size of τ_G compared to its growth-maximizing size $(1-\alpha)(1-\tau_H)$. A greater growth rate is utility-enhancing because it increases future private as well as public consumption possibilities. Second, a rise in τ_H positively affects households' well-being by directly increasing the provision of H but impinges on U^i by reducing the steady-state growth rate.

Henceforth, we assume that the following assumption, which will be motivated below, holds:

Assumption 4.1. $\rho \leq \frac{1+b}{b} A^{\frac{1}{\alpha}} \left[(1-\alpha)L\right]^{\frac{1-\alpha}{\alpha}}$.

Then, the solution to the above optimization problem yields household i's unique most preferred policy mix as[10]

$$\tau_G^i = 1-\alpha \tag{4.16}$$

and

$$\tau_H^i = \frac{(1-\phi^i)b\rho}{\left[1+(1-\phi^i)b\right] A^{\frac{1}{\alpha}} \left[(1-\alpha)L\right]^{\frac{1-\alpha}{\alpha}}}. \tag{4.17}$$

Since time does not enter these expressions, they confirm our conjecture that the actual policy mix, which will be implemented by the government (see Section 4.3.2), will involve constant tax rates. Thus, household behavior based on time-invariant τ_G and τ_H is fully consistent with the actual equilibrium outcome.

Assumption 4.1, which is easily met for a small ρ or a large A, assures that $\tau_H^i \leq 1$ for any ϕ^i. Note that the equilibrium growth rate associated with household i's most preferred policy mix can be negative if τ_H^i is sufficiently large.

According to equation (4.16), the ideal share of productive government expenditure, τ_G^i, for all households corresponds to $1-\alpha$. The latter is the output elasticity of productive expenditure, which is the same for all households. As productive government

[10]For a detailed derivation see Appendix A.

expenditure affects all household-producers in the same way, it is intuitive that the preferred expenditure ratio is independent of the households' demographic composition, i. e., $\partial\tau_G^i/\partial\phi^i = 0$. It is noteworthy that at τ_G^i productive government expenditure satisfies the so-called natural condition of productive efficiency, i. e., the marginal contribution of government expenditure to aggregate output is one (see, e.g., Barro, 1990). In the present context, as aggregate output in equilibrium can be written as $Y = AK^{\alpha}(GL)^{1-\alpha}$, we have $dY/dG = (1-\alpha)(Y/G) = (1-\alpha)/\tau_G^i = 1$.

By contrast, equation (4.17) reveals that household i's preferred spending ratio for services that benefit the elderly, τ_H^i, depends on ϕ^i. Thus, households with different support ratios prefer different tax rates. Since τ_H^i affects the steady-state growth rate, this difference also translates into the preferred growth rate. Assuming that i's most preferred policy mix is the one implemented by the government, one readily establishes that

$$\frac{d\tau_H^i}{d\phi^i} < 0 \quad \text{and} \quad \frac{d\gamma(\tau_G^i, \tau_H^i)}{d\phi^i} > 0. \tag{4.18}$$

Intuitively, households with a greater share of elderly members (i. e., a lower ϕ^i) are willing to pay higher taxes for the provision of public services that benefit these members and to accept lower growth rates of private consumption.

4.3.2 Policy Choice under Majority Voting

Let's turn to the policy mix that will be implemented by the government under pure majority rule. In particular, we will show that the median voter theorem can be applied to this voting problem.

For all household-producers the optimal policy mix involves $\tau_G = 1-\alpha$. Thus, voters only differ in their preferences for τ_H and the voting problem becomes one-dimensional. Moreover, each voter's preferences for τ_H are single-peaked because the indirect utility function U^i is strictly concave in τ_H^i for $\tau_G = 1-\alpha$. In addition, there exists a monotonic relationship between household i's ideal tax rate τ_H^i and her support ratio ϕ^i. Thus, the median voter theorem can be applied to this voting problem and the share of public spending for the elderly that the government implements coincides with the one preferred by the median voter. The following proposition summarizes the political equilibrium, i.e., the actual choice of policy under majority rule, and the resulting economic growth rate.

Proposition 4.2. *The actual policy mix involves*

$$\tau_G^* = 1 - \alpha \quad \textit{and} \quad \tau_H^* = \frac{(1-\phi^m)b\rho}{\left[1+(1-\phi^m)b\right] A^{\frac{1}{\alpha}} \left[(1-\alpha)L\right]^{\frac{1-\alpha}{\alpha}}}, \tag{4.19}$$

where ϕ^m denotes the support ratio of the median household. The corresponding steady-state growth rate of household and economy-wide variables is

$$\gamma^* = \alpha A^{\frac{1}{\alpha}} \left[(1-\alpha)L\right]^{\frac{1-\alpha}{\alpha}} - \frac{(1-\phi^m)b\rho}{1+(1-\phi^m)b} - \rho. \tag{4.20}$$

Implicitly, we have assumed that taxes are voted on and implemented with full commitment at time zero. However, due to the infinite time horizon and exponential discounting, this policy choice is time-consistent (see Laibson, 2003). Thus, it has to coincide with the solution that would be obtained if the government could not commit itself to future policies. Intuitively, as households only differ in their support ratio (which does not affect the accumulation path) and as the identity of the median voter does not change over time, strategic intertemporal voting cannot occur.

Equation (4.19) reveals that $\tau_H^* > 0$ for any $\phi^m < 1$. Thus, as long as the median household is not solely composed of working members, majority voting cannot yield the economy's maximum growth rate (which requires $\tau_H = 0$).

4.4 Implications of Population Aging

How does population aging in this framework affect actual government spending and long-run economic growth? Population aging here corresponds to an (exogenous) change in the distribution of households such that there are more households with a large fraction of elderly members and the median household has a lower support ratio.[11] This process may entail a decline in the total labor force. The implications of population aging are

[11] As the economy in equilibrium immediately settles on its steady-state growth path, i. e., does not feature any transitional dynamics, the present framework does not permit to study a demographic transition.

summarized in the following corollary, which immediately follows from Proposition 4.2 and equation (4.18).

Corollary 4.1.

1. *It holds that*

$$\frac{d\tau_H^*}{d\phi^m} < 0, \quad \frac{d\tau_G^*}{d\phi^m} = 0, \quad \text{and} \quad \frac{d\gamma^*}{d\phi^m} > 0. \tag{4.21}$$

2. *It holds that*

$$\frac{d\tau_H^*}{dL} < 0, \quad \frac{d\tau_G^*}{dL} = 0, \quad \text{and} \quad \frac{d\gamma^*}{dL} > 0. \tag{4.22}$$

The first statement of Corollary 4.1 reveals that a fall in the median voter's support ratio, ϕ^m, involves a higher τ_H^*, an unchanged τ_G^*, and a lower γ^*. Thus, the framework of this chapter predicts that demographic aging increases public spending on the elderly (as a share of output), does not affect productive government expenditure (as a share of output), increases the overall tax burden (because $\tau = \tau_G + \tau_H$), and lowers the economy's growth rate. This is consistent with the concerns raised in the public debate. Statement 2 reveals that the steady-state effects of a fall in L have the same sign, and thus potentially reinforce the effects of a decline in ϕ^m. This result is due to the scale effect of aggregate labor supply in aggregate production.

4.5 Concluding Remarks

This chapter has shown how a simple infinite-horizon, endogenous growth model à la Barro (1990) with households that are heterogeneous in their age composition can be used to analyze how population aging, via a democratic voting process, endogenously affects government spending for productive purposes and for a public consumption good that benefits the elderly, and how a change in the level and composition of the government budget impinges on long-run economic growth.

The results can be summarized as follows. While all households prefer the same share of output to be used for productive public expenditure, the age pattern of each household determines which share of output they want the government to spend on the needs of the elderly. Consequently, the policy mix that will be implemented under majority voting depends on the age distribution of the economy. Population aging is predicted to increase public spending on the elderly (as a share of output) as well as the overall tax burden, and thus to lower the economy's growth rate. However, it does not affect public productive expenditure (as a share of output). Thus, the present framework yields results that are consistent with recent evidence (see Section 4.1).

The next chapter compares these results to those obtained in a comparable model of overlapping generations in which agents live for two periods and vote each period. It can be expected that the results for τ_H will not differ qualitatively. However, the solution under repeated voting then does not necessarily have to coincide with the one under full commitment. This is due to the finite lifetime of individuals in this type of model. Moreover, to achieve tractability in the present framework, the size of the population has been stationary and we have focused on changes in the composition of the population. In an overlapping generations model, if population aging occurs due to a slowdown in population growth, then this does not only affect the composition, but also the size of the population. The latter in turn reduces the usual capital dilution effect and has an opposing, positive effect on the growth rate of per capita variables.

4.6 Appendix A

Proof of Proposition 4.1

Household-producer i's intertemporal optimization problem gives rise to the following present-value Hamiltonian

$$\mathcal{H}^i \equiv N^i \left[\ln \tilde{c}^i + (1-\phi^i) b \ln H\right] e^{-\rho t} + \lambda^i \left[(1-\tau) A^{\frac{1}{\alpha}} \left(\frac{G}{y}\right)^{\frac{1-\alpha}{\alpha}} k^i - \frac{\tilde{c}^i}{\phi^i}.\right], \quad (4.23)$$

where λ^i denotes the present-value shadow price of household-producer i's capital stock. In performing the optimization, the individual household-producer takes τ, y, G, and H as given. Then, the necessary and sufficient optimality conditions are[12]

$$\frac{1}{\tilde{c}^i} e^{-\rho t} = \frac{\lambda^i}{L^i} \quad (4.24)$$

$$\dot{\lambda}^i = -\lambda^i \left[(1-\tau) A^{\frac{1}{\alpha}} \left(\frac{G}{y}\right)^{\frac{1-\alpha}{\alpha}}\right] \quad (4.25)$$

$$0 = \lim_{t\to\infty} \left[\lambda^i k^i\right]. \quad (4.26)$$

As L^i is constant, combining (4.24) and (4.25) and taking into account $G/y = \tau_G L$ and $\tau = \tau_G + \tau_H$ yields

$$\frac{\dot{\tilde{c}}^i}{\tilde{c}^i} = (1-\tau_G-\tau_H) A^{\frac{1}{\alpha}} (\tau_G L)^{\frac{1-\alpha}{\alpha}} - \rho. \quad (4.27)$$

From the flow budget constraint (4.6) we know that

$$\frac{\tilde{c}^i}{\phi^i k^i} = (1-\tau) A^{\frac{1}{\alpha}} (\tau_G L)^{\frac{1-\alpha}{\alpha}} - \frac{\dot{k}^i}{k^i}. \quad (4.28)$$

In a steady state the growth rate of the household's capital stock per worker per definition has to be constant. Therefore, for constant expenditure shares τ_G and τ_H the right-hand side of (4.28) is constant. Consequently, $\tilde{c}^i/(\phi^i k^i)$ is constant and for a constant ϕ^i the

[12]The Hamiltonian H^i is the sum of a concave function of $\tilde{c}^i$ and a linear function of $(k^i, \tilde{c}^i)$. Therefore, it is concave in $(k^i, \tilde{c}^i)$. Moreover, it is strictly concave in $\tilde{c}^i$. Thus, the paths of $\tilde{c}^i$ and k^i implied by (4.24)-(4.26) achieve a unique global maximum.

growth rate of the household's capital stock per worker has to equal the growth rate of consumption per household member. As y^i from (4.4) with $G/y = \tau_G L$ obtains as

$$y^i(t) = A^{\frac{1}{\alpha}} \left(\tau_G L\right)^{\frac{1-\alpha}{\alpha}} k^i(t), \tag{4.29}$$

output per worker in the steady state also grows at the same rate as k^i and $\tilde{c}^i$. Moreover, as all households have the same initial capital-labor share $k^i(0) = k_0$ equation (4.29) implies that all households at all t have the same output and capital per worker such that in equilibrium individual and economy-wide variables per worker coincide, i. e., $k = \int_0^1 k^i di = k^i$ and $y = \int_0^1 y^i di = y^i$. Each household's instantaneous level of aggregate capital, output, and consumption depends on her respective (but constant) labor supply, but is proportional to k^i:

$$\begin{aligned} K^i(t) &= k^i(t) L^i && (4.30)\\ Y^i(t) &= A^{\frac{1}{\alpha}} \left(\tau_G L\right)^{\frac{1-\alpha}{\alpha}} k^i(t) L^i && (4.31)\\ C^i(t) &= \rho k^i(t) L^i. && (4.32) \end{aligned}$$

Thus, these aggregate household variables in a steady state grow at the same rate as k^i. Finally, the economy-wide aggregate variables are given by

$$\begin{aligned} K(t) &= \int_0^1 K^i(t) di = k^i(t) \int_0^1 L^i di = k^i(t) L && (4.33)\\ Y(t) &= \int_0^1 Y^i(t) di = A^{\frac{1}{\alpha}} \left(\tau_G L\right)^{\frac{1-\alpha}{\alpha}} k^i(t) L && (4.34)\\ C(t) &= \int_0^1 C^i(t) di = \rho k^i(t) L. && (4.35) \end{aligned}$$

Hence, for a constant aggregate labor supply (L), K, Y, and C have to grow at the same rate as k^i. Finally, as G and H are proportional to aggregate output, these variables also have to grow at this rate. Thus, we have established the existence of a steady-state growth path at which all variables at household and economy level as well as the level of public services grow the same constant rate

$$\gamma(\tau_G, \tau_h) \equiv \frac{\dot{\tilde{c}}^i}{\tilde{c}^i} = \frac{\dot{k}^i}{k^i}. \tag{4.36}$$

Moreover, using (4.24) and (4.25) to evaluate (4.26) one readily verifies that the transversality condition holds for any parameter constellation.

Finally, it is straightforward to show that the economy immediately jumps onto this steady-state path. The proof of this mirrors the one of a standard AK model. □

Derivation of Household i's Most Preferred Policy Mix

Substituting for $c^i(t)$ and $H(t)$ in household i's utility function (4.5) and solving the integral we obtain the steady-state utility of household i as

$$\begin{aligned} U^i\left(\tau_G, \tau_H\right) &= \frac{N^i}{\rho}\left[\ln(\phi^i \rho k_0) + (1-\phi^i) b \ln\left(\tau_H \left(\tau_G\right)^{\frac{1-\alpha}{\alpha}} k_0 \left(AL\right)^{\frac{1}{\alpha}}\right)\right] \\ &+ \frac{N^i(1+(1-\phi^i)b)\gamma(\cdot)}{\rho^2}. \end{aligned} \tag{4.37}$$

Then, the optimization problem of household i reduces to choosing $\tau_G \in [0,1]$ and $\tau_H \in [0,1]$ to maximize (4.37) with $\gamma(\cdot)$ given by (4.12). To determine the global maximum of U^i in the square $0 \leq \tau_G \leq 1$ and $0 \leq \tau_H \leq 1$ we proceed in two steps. First, we show that there exists a unique local maximum in the interior of the square. Second, we verify that this policy mix represents the global maximum in the square by comparing its implied utility level with the utility obtained at the local extrema on the boundary of the square and at corner points.

1. Derivation of the unique local maximum in the interior of the square:

   The above optimization problem delivers the following pair of necessary first-order conditions for an interior optimum

$$\frac{\partial U^i}{\partial \tau_G} = \frac{(1-\phi^i)b(1-\alpha)}{\alpha \tau_G} + \frac{1+(1-\phi^i)b}{\rho}\frac{\partial \gamma(\cdot)}{\partial \tau_G} = 0 \tag{4.38}$$

$$\frac{\partial U^i}{\partial \tau_H} = \frac{(1-\phi^i)b}{\tau_H} + \frac{1+(1-\phi^i)b}{\rho}\frac{\partial \gamma(\cdot)}{\partial \tau_H} = 0, \tag{4.39}$$

   where

$$\frac{\partial \gamma(\cdot)}{\partial \tau_G} = \frac{A^{\frac{1}{\alpha}}\left(\tau_G L\right)^{\frac{1-\alpha}{\alpha}}}{\alpha \tau_G}\left[(1-\alpha)(1-\tau_H) - \tau_G\right] \tag{4.40}$$

$$\frac{\partial \gamma(\cdot)}{\partial \tau_H} = -A^{\frac{1}{\alpha}}\left(\tau_G L\right)^{\frac{1-\alpha}{\alpha}} < 0. \tag{4.41}$$

   Rewriting conditions (4.38) and (4.39), household i's most preferred expenditure shares τ_G^i and τ_H^i are implicitly determined by

$$\frac{1+(1-\phi^i)b}{(1-\phi^i)b\rho} = -\frac{(1-\alpha)}{\alpha \tau_G^i \left(\frac{\partial \gamma(\cdot)}{\partial \tau_G}\right)\Big|_{\tau_G^i, \tau_H^i}} \tag{4.42}$$

$$\frac{1+(1-\phi^i)b}{(1-\phi^i)b\rho} = -\frac{1}{\tau_H^i \left(\frac{\partial \gamma(\cdot)}{\partial \tau_H}\right)\Big|_{\tau_G^i, \tau_H^i}}, \tag{4.43}$$

respectively. Then, combining (4.42) and (4.43) and taking into account (4.40) and (4.41) yields

$$-\frac{1-\alpha}{(1-\tau_H^i)(1-\alpha)-\tau_G^i} = \frac{1}{\tau_H^i}$$

and thus $\tau_G^i = 1-\alpha$.

Then, substituting $\tau_G^i = 1-\alpha$ and (4.41) in (4.39) and rearranging yields

$$\tau_H^i = \frac{(1-\phi^i)b\rho}{\left[1+(1-\phi^i)b\right] A^{\frac{1}{\alpha}} \left[(1-\alpha)L\right]^{\frac{1-\alpha}{\alpha}}}, \tag{4.44}$$

which is equation (4.17).

The sufficient condition for (τ_G^i, τ_H^i) to be a local maximum is

$$D\left(\tau_G^i, \tau_H^i\right) \equiv \left.\frac{\partial^2 U^i}{\partial \tau_G^2}\right|_{\tau_G^i,\tau_H^i} \times \left.\frac{\partial^2 U^i}{\partial \tau_H^2}\right|_{\tau_G^i,\tau_H^i} - \left(\left.\frac{\partial^2 U^i}{\partial \tau_G \partial \tau_H}\right|_{\tau_G^i,\tau_H^i}\right)^2 > 0, \tag{4.45}$$

where

$$\frac{\partial^2 U^i}{\partial \tau_H^2} = -\frac{N^i}{\rho}\frac{(1-\phi^i)b}{(\tau_H)^2}, \tag{4.46}$$

$$\frac{\partial^2 U^i}{\partial \tau_G \partial \tau_H} = -\frac{N^i}{\rho}\frac{1-\alpha}{\alpha}\frac{1+(1-\phi^i)b}{\rho}\left(AL^{1-\alpha}\right)^{\frac{1}{\alpha}}(\tau_G)^{\frac{1-\alpha}{\alpha}-1}, \tag{4.47}$$

$$\frac{\partial^2 U^i}{\partial \tau_G^2} = \frac{N^i}{\rho}\left[-\frac{(1-\phi^i)b\,(1-\alpha)}{\alpha\,(\tau_G)^2} + \frac{1+(1-\phi^i)b}{\rho}\frac{\partial^2 \gamma}{\partial \tau_G^2}\right] \quad \text{with} \tag{4.48}$$

$$\frac{\partial^2 \gamma}{\partial \tau_G^2} = -\frac{1-\alpha}{\alpha}\left(AL^{1-\alpha}\right)^{\frac{1}{\alpha}}(\tau_G)^{\frac{1-\alpha}{\alpha}-2}\left[\frac{\tau_G}{\alpha} + \left(1-\frac{1-\alpha}{\alpha}\right)(1-\tau_H)\right].$$

Evaluating equations (4.46)-(4.48) at $\tau_G^i = 1-\alpha$ and at τ_H^i as given by (4.44), substituting the resulting expressions in (4.45) and rearranging yields

$$D\left(\tau_G^i, \tau_H^i\right) = \frac{\left[1+(1-\phi^i)b\right]\left(AL^{1-\alpha}\right)^{\frac{3}{\alpha}}(1-\alpha)^{\frac{3(1-\alpha)}{\alpha}}}{(1-\phi^i)b\rho^3} > 0.$$

Thus, the policy mix (τ_G^i, τ_H^i) represents a local maximum in the interior of the square. The utility level associated with this policy mix is

$$\begin{aligned} U^i\left(\tau_G^i, \tau_H^i\right) &= \frac{N^i}{\rho}\left[\ln(\phi^i \rho k_0) + (1-\phi^i)b\ln\left(\frac{(1-\phi^i)b\rho}{1+(1-\phi^i)b}k_0 L\right)\right] \\ &+ \frac{N^i(1+(1-\phi^i)b)}{\rho^2}\alpha\left(A(1-\alpha)^{1-\alpha}L^{1-\alpha}\right)^{\frac{1}{\alpha}} \\ &- \frac{N^i}{\rho}\left[(1-\phi^i)b + (1+(1-\phi^i)b)\right]. \end{aligned} \tag{4.49}$$

2. Comparison to local maxima on the boundary of the square and to corner points:

The boundary of the square consists of 4 parts. On the first two sides with either $\tau_G = 0$ or $\tau_H = 0$ no relative maximum exists as U^i tends to $-\infty$ if one of the tax rates approaches zero. Side 3 is $\tau_G = 1$ and $\tau_H \in [0, 1]$. On this side, we have

$$\begin{aligned} U^i(1, \tau_H) &= \frac{N^i}{\rho}\left[\ln(\phi^i \rho k_0) + (1-\phi^i) b \ln\left(\tau_H k_0 (AL)^{\frac{1}{\alpha}}\right)\right] \\ &+ \frac{N^i(1+(1-\phi^i)b)}{\rho^2}\left(-\tau_H \left(AL^{1-\alpha}\right)^{\frac{1}{\alpha}} - \rho\right) \end{aligned}$$

and $\frac{\partial U^i(1,\tau_H)}{\partial \tau_H} = 0$ delivers the relative extremum

$$\hat{\tau}_H = \frac{(1-\phi^i) b \rho}{[1+(1-\phi^i)b]\left(AL^{1-\alpha}\right)^{\frac{1}{\alpha}}}.$$

Evaluating U^i at this critical point gives

$$\begin{aligned} U^i(1, \hat{\tau}_H) &= \frac{N^i}{\rho}\left[\ln(\phi^i \rho k_0) + (1-\phi^i) b \ln\left(\frac{(1-\phi^i) b \rho}{1+(1-\phi^i)b} k_0 L\right)\right] \\ &- \frac{N^i}{\rho}\left[(1-\phi^i) b + (1+(1-\phi^i)b)\right], \end{aligned}$$

which is strictly smaller than $U^i(\tau_G^i, \tau_H^i)$ given by (4.49).

Side 4 is $\tau_H = 1$ and $\tau_G \in [0, 1]$. On this side, we have

$$\begin{aligned} U^i(\tau_G, 1) &= \frac{N^i}{\rho}\left[\ln(\phi^i \rho k_0) + (1-\phi^i) b \ln\left((\tau_G)^{\frac{1-\alpha}{\alpha}} k_0 (AL)^{\frac{1}{\alpha}}\right)\right] \\ &- \frac{N^i(1+(1-\phi^i)b)\left(\tau_G A L^{1-\alpha}\right)^{\frac{1}{\alpha}}}{\rho^2} \end{aligned}$$

and $\frac{\partial U^i(1,\tau_H)}{\partial \tau_G} = 0$ delivers the relative extremum

$$\bar{\tau}_G = \left[\frac{(1-\phi^i)\rho b (1-\alpha)}{[1+(1-\phi^i)b]\left(AL^{1-\alpha}\right)^{\frac{1}{\alpha}}}\right]^{\alpha}.$$

Evaluating U^i at this critical point then gives

$$\begin{aligned} U^i(\bar{\tau}_G, 1) &= \frac{N^i}{\rho}\left[\ln(\phi^i \rho k_0) - \left[(1-\phi^i) b (1-\alpha) + (1+(1-\phi^i)b)\right]\right] \\ &+ \frac{N^i}{\rho}(1-\phi^i) b \ln\left(\left[\frac{(1-\phi^i)\rho b (1-\alpha)}{[1+(1-\phi^i)b]\left(AL^{1-\alpha}\right)^{\frac{1}{\alpha}}}\right]^{1-\alpha} k_0 (AL)^{\frac{1}{\alpha}}\right), \end{aligned}$$

which under Assumption 1 can be shown to be strictly smaller than (4.49).

The only candidate for a corner solution is $\tau_G = \tau_H = 1$. In this case we obtain

$$\begin{aligned} U^i(1,1) &= \frac{N^i}{\rho}\left[\ln(\phi^i \rho k_0) + (1-\phi^i) b \ln\left(k_0 (AL)^{\frac{1}{\alpha}}\right) - \frac{(1+(1-\phi^i)b)\left(AL^{1-\alpha}\right)^{\frac{1}{\alpha}}}{\rho}\right] \\ &- \frac{N^i}{\rho}(1+(1-\phi^i)b), \end{aligned}$$

which is strictly smaller than (4.49) because

$$\left[1 + \ln\left(\frac{[1+(1-\phi^i)b]\left(AL^{1-\alpha}\right)^{\frac{1}{\alpha}}}{(1-\phi^i)b\rho}\right)\right] < \frac{(1+(1-\phi^i)b)\left(AL^{1-\alpha}\right)^{\frac{1}{\alpha}}}{(1-\phi^i)b\rho}\left[1+\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}\right].$$

Thus, we have shown that all corner points and local extrema on the boundary of the square yield a lower utility than the interior local maximum at (τ_G^i, τ_H^i). Thus, this policy mix is the global maximum in the square. □

4.7 Appendix B

4.7.1 Economic Equilibrium with an Equal Initial Capital Distribution

This section analyzes the economic equilibrium for an equal initial capital distribution, i. e., for $K^i(0) = K_0 > 0$ for all i.

The optimization problem for each household-producer i is

$$\max_{\{\tilde{c}^i(t), k^i(t)\}_{t=0}^{\infty}} \quad U^i \qquad \text{s.t. (4.6), (4.4), and } K^i(0) = K_0 > 0, \\ \text{taking } \tau,\ G,\ H, \text{ and } y \text{ as given} \qquad (4.50)$$

which gives rise to the same necessary and sufficient optimality conditions as in the proof of Proposition 4.1, namely equations (4.24) - (4.26). Thus, it is straightforward to show that (equivalent to Proposition 4.1) along the steady-state growth path for given time-invariant expenditure ratios τ_G and τ_H all variables will grow at the same constant rate $\gamma(\tau_G, \tau_H)$ given by (4.12).

The main difference with an equal initial capital distribution occurs at the level of per-period household variables. With the same initial capital stock all households have the same initial income and produce the same output at all t. To see this, note that

$$Y^i(t) = Y^i(0)e^{\gamma(\cdot)t}, \tag{4.51}$$

$$\text{where} \quad Y^i(0) = A^{\frac{1}{\alpha}}\left(\tau_G L\right)^{\frac{1-\alpha}{\alpha}} K_0. \tag{4.52}$$

The argument of γ is (τ_G, τ_H). Thus, all households independent of the size of their labor force produce the same output but differ in their output per worker

$$y^i(t) = \frac{Y^i(t)}{L^i} = A^{\frac{1}{\alpha}}\left(\tau_G L\right)^{\frac{1-\alpha}{\alpha}} \frac{K_0}{L^i} e^{\gamma(\cdot)t}. \tag{4.53}$$

This is possible because firms in this setting asymmetrically benefit from productive government expenditure at each t

$$G^i = G\frac{y^i}{y} = G\frac{L}{L^i}, \tag{4.54}$$

where we have used that in equilibrium at each t

$$y = \int_0^1 y^i di = \int_0^1 \frac{Y^i}{L^i} di = Y^i \int_0^1 \frac{1}{L^i} di = \frac{Y^i}{L}. \tag{4.55}$$

Intuitively, equation (4.54) implies that the government via the provision of public productive services subsidizes the production of household-producers with a smaller labor force. By contrast, when households have the same initial capital stock per worker as assumed in the main text they all produce the same output per worker at each t but differ in their aggregate output according to the size of their labor force.

4.7.2 Non-Separable Preferences

To gauge the sensitivity of our results, this section considers an alternative specification of the utility function with non-separable preferences between private and public consumption.

In particular, we assume that household i's intertemporal utility is given by

$$U^i(t) = \int_0^\infty \frac{\left(\left(C^i(t)\right)^{\phi^i} H(t)^{1-\phi^i}\right)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt, \tag{4.56}$$

where $C^i(t) = \tilde{c}^i(t)N^i$ denotes household i's aggregate private consumption and σ is the reciprocal of the intertemporal elasticity of substitution for consumption. We assume $1-\sigma < 1$ such that the instantaneous utility function is strictly concave in its arguments. The share of private consumption in household i's utility relative to public consumption is given by the support ratio ϕ^i. The greater the share of elderly members, i. e., the smaller ϕ^i, the more important is the public consumption good for overall household utility. For simplicity, we normalize each households labor supply to unity, i. e., $L^i = 1$, such that in equilibrium $L = 1$. As the size of the household and her labor supply are assumed to be constant, this assumption does not affect our qualitative results, but simply eliminates the scale effect in the steady-state growth rate.

Economic Equilibrium

In this case, the optimization problem for each household i is to choose $\tilde{c}^i(t)$ and $k^i(t)$ to maximize (4.56), subject to (4.6), (4.4), and an initial capital stock per worker $k^i(0) = k_0 > 0$, taking G, H, and $\tau = \tau_G + \tau_H$ as given. The corresponding present-value Hamiltonian is

$$\mathcal{H}^i \equiv \frac{\left((\tilde{c}^i/\phi^i)^{\phi^i} H^{1-\phi^i}\right)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} + \lambda^i \left[(1-\tau_G-\tau_H)A^{\frac{1}{\alpha}}\left(\frac{G}{y}\right)^{\frac{1-\alpha}{\alpha}} k^i - \frac{\tilde{c}^i}{\phi^i}.\right], \quad (4.57)$$

where λ^i denotes the present-value shadow price of household-producer i's capital stock. The necessary and sufficient first-order conditions of this optimization problem yield

$$\left[(1-\sigma)\phi^i - 1\right]\frac{\dot{\tilde{c}}^i}{\tilde{c}^i} + (1-\phi^i)(1-\sigma)\frac{\dot{H}}{H} - \rho = \frac{\dot{\lambda}^i}{\lambda^i}, \quad (4.58)$$

$$\left[(1-\tau_G-\tau_H)A^{\frac{1}{\alpha}}\left(\frac{G}{y}\right)^{\frac{1-\alpha}{\alpha}}\right] = -\frac{\dot{\lambda}^i}{\lambda^i}, \quad (4.59)$$

$$\lim_{t\to\infty}\left[\lambda^i k^i\right] = 0. \quad (4.60)$$

The government's budget constraint (see equation 4.7) implies for a time-invariant τ_H that $\dot{H}/H = \dot{Y}/Y$. Moreover, on a balanced growth path with a stationary population all variables have to grow at the same rate, i. e., $\gamma \equiv \dot{\tilde{c}}^i/\tilde{c}^i = \dot{H}/H$. Taking this into account and combining (4.58) - (4.59) with (4.10) for $L = 1$ yields the steady-state growth rate as

$$\gamma(\tau_G, \tau_H) = \frac{\dot{\tilde{c}}^i}{\tilde{c}^i} = \frac{1}{\sigma}\left[(1-\tau_G-\tau_H)\,A^{\frac{1}{\alpha}}\,(\tau_G)^{\frac{1-\alpha}{\alpha}} - \rho\right], \tag{4.61}$$

which generalizes equation (4.12) to $\sigma \neq 1$. As before, the economy has no transitional dynamics and is always in a position at which all variables at the household and the economy-wide level as well as government expenditure grow at the rate $\gamma(\cdot)$. For utility to be bounded $\rho > \gamma(\cdot)(1-\sigma)$ has to hold.

Given a starting amount of capital, $k^i(0)$, the levels of all variables are again determined. In particular, the initial quantity of consumption is

$$\tilde{c}^i(0) = \phi^i\left[(1-\tau_G-\tau_H)\,A^{\frac{1}{\alpha}}\,(\tau_G)^{\frac{1-\alpha}{\alpha}} - \gamma(\cdot)\right]k^i(0) \tag{4.62}$$

and the initial level of the public consumption good is

$$H\,(0) = \tau_H A^{\frac{1}{\alpha}}\,(\tau_G)^{\frac{1-\alpha}{\alpha}}\,k^i\,(0)\,. \tag{4.63}$$

Also note that equations (4.61) and (4.62) imply that $\tilde{c}^i(0)$ can be written as

$$\tilde{c}^i\,(0) \;=\; \phi^i\,[\rho - \gamma(\cdot)\,(1-\sigma)]\,k^i\,(0)\,. \tag{4.64}$$

Political Equilibrium

In the following we use the above results to determine household i's most preferred policy mix. The relevant optimization problem is

$$\max_{\tau_G,\tau_H} \int_0^\infty \frac{\left(\left(\tilde{c}^i(t)/\phi^i\right)^{\phi^i} H(t)^{1-\phi^i}\right)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t}\,dt \quad \text{s.t.}$$
$$\tilde{c}^i(t) = \tilde{c}^i(0)e^{\gamma(\tau_G,\tau_H)t} \quad \text{and} \quad H(t) = H(0)e^{\gamma(\tau_G,\tau_H)t}.$$

For a constant $\gamma(\cdot)$ the integral in the above equation can be simplified to yield (aside from a constant)

$$U^i(\tau_G, \tau_H) \;=\; \frac{\left(\tilde{c}^i\,(0)\,/\phi^i\right)^{\phi^i(1-\sigma)}\,H\,(0)^{\left(1-\phi^i\right)(1-\sigma)}}{(1-\sigma)\,[\rho - \gamma(\cdot)\,(1-\sigma)]}. \tag{4.65}$$

Then, using equations (4.63) - (4.64) in (4.65) gives i's indirect utility function as

$$U^i = \frac{k^i(0)^{1-\sigma}}{1-\sigma}\left(\rho-\gamma(\cdot)(1-\sigma)\right)^{\phi^i(1-\sigma)-1}\left(\tau_H A^{\frac{1}{\alpha}}(\tau_G)^{\frac{1-\alpha}{\alpha}}\right)^{\left(1-\phi^i\right)(1-\sigma)}. \quad (4.66)$$

Maximizing (4.66) with respect to τ_G and τ_H yields the following pair of first-order conditions

$$\frac{(1-\alpha)\left(1-\phi^i\right)}{\alpha\frac{\partial\gamma(\cdot)}{\partial\tau_G}}\left([\rho-\gamma(\cdot)(1-\sigma)]\right)^{\phi^i(1-\sigma)-1}\left(\tau_H A^{\frac{1}{\alpha}}\right)^{\left(1-\phi^i\right)(1-\sigma)}(\tau_G)^{\frac{(1-\alpha)(1-\sigma)\left(1-\phi^i\right)}{\alpha}-1}$$

$$= -\left(1-\phi^i(1-\sigma)\right)\left([\rho-\gamma(\cdot)(1-\sigma)]\right)^{\phi^i(1-\sigma)-2}\left(\tau_H A^{\frac{1}{\alpha}}(\tau_G)^{\frac{1-\alpha}{\alpha}}\right)^{\left(1-\phi^i\right)(1-\sigma)}, \quad (4.67)$$

and

$$\frac{1-\phi^i}{\frac{\partial\gamma(\cdot)}{\partial\tau_H}}\left([\rho-\gamma(\cdot)(1-\sigma)]\right)^{\phi^i(1-\sigma)-1}(\tau_H)^{\left(1-\phi^i\right)(1-\sigma)-1}\left(A(\tau_G)^{1-\alpha}\right)^{\frac{(1-\sigma)\left(1-\phi^i\right)}{\alpha}}$$

$$= -\left(1-\phi^i(1-\sigma)\right)\left([\rho-\gamma(\cdot)(1-\sigma)]\right)^{\phi^i(1-\sigma)-2}\left(\tau_H A^{\frac{1}{\alpha}}(\tau_G)^{\frac{1-\alpha}{\alpha}}\right)^{\left(1-\phi^i\right)(1-\sigma)}, \quad (4.68)$$

where

$$\frac{\partial\gamma(\cdot)}{\partial\tau_G} = \frac{A^{\frac{1}{\alpha}}(\tau_G)^{\frac{1-\alpha}{\alpha}}}{\sigma\alpha\tau_G}\left[(1-\alpha)(1-\tau_H)-\tau_G\right], \quad (4.69)$$

$$\frac{\partial\gamma(\cdot)}{\partial\tau_H} = -\frac{1}{\sigma}A^{\frac{1}{\alpha}}(\tau_G)^{\frac{1-\alpha}{\alpha}} < 0. \quad (4.70)$$

Combining conditions (4.67) and (4.68) and taking into account equations (4.69) and (4.70) yields

$$-\frac{1-\alpha}{(1-\tau_H^i)(1-\alpha)-\tau_G^i} = \frac{1}{\tau_H^i}$$

and thus $\tau_G^i = 1-\alpha$.

Then, substituting $\tau_G^i = 1-\alpha$ and (4.70) in (4.68) and rearranging delivers

$$\left(1-\phi^i\right)\left[\rho-\gamma(\cdot)(1-\sigma)\right] = \frac{1}{\sigma}A^{\frac{1}{\alpha}}(1-\alpha)^{\frac{1-\alpha}{\alpha}}\left(1-\phi^i(1-\sigma)\right)\tau_H^i. \quad (4.71)$$

Then, using (4.61) in (4.71) yields

$$\tau_H^i = \frac{(1-\phi^i)\left[\rho-\alpha A^{\frac{1}{\alpha}}(1-\alpha)^{\frac{1-\alpha}{\alpha}}\right]}{A^{\frac{1}{\alpha}}(1-\alpha)^{\frac{1-\alpha}{\alpha}}\phi^i\sigma}.$$

As τ_H^i cannot be negative, household i's most preferred spending share is given by

$$\tau_H^i = \max\left\{0, \frac{(1-\phi^i)\left[\rho - \alpha A^{\frac{1}{\alpha}}(1-\alpha)^{\frac{1-\alpha}{\alpha}}\right]}{A^{\frac{1}{\alpha}}(1-\alpha)^{\frac{1-\alpha}{\alpha}}\phi^i\sigma}\right\}. \tag{4.72}$$

Intuitively, if ρ is sufficiently small, i.e., if households care a lot about the future, then i prefers a high growth rate and thus $\tau_H^i = 0$. Henceforth, we assume that $\rho > \alpha A^{\frac{1}{\alpha}}(1-\alpha)^{\frac{1-\alpha}{\alpha}}$.

Equivalently to the main text, the voting problem has become one-dimensional. Moreover, preferences are single-peaked as U^i is strictly concave in τ_H^i for $\tau_G = 1-\alpha$.[13] Thus, the median voter theorem can be applied and the actual policy mix involves

$$\tau_G^* = 1-\alpha \quad \text{and} \quad \tau_H^* = \frac{(1-\phi^m)\left[\rho - \alpha A^{\frac{1}{\alpha}}(1-\alpha)^{\frac{1-\alpha}{\alpha}}\right]}{A^{\frac{1}{\alpha}}(1-\alpha)^{\frac{1-\alpha}{\alpha}}\phi^m\sigma}, \tag{4.73}$$

where ϕ^m denotes the support ratio of the median household. The corresponding steady-state growth rate of household and economy-wide variables is $\gamma^* \equiv \gamma(\tau_G^*, \tau_H^*)$.

Finally, it is straightforward to verify that population aging, i.e., a decline in the median voter's support ratio has the following steady-state effects

$$\frac{d\tau_H^*}{d\phi^m} < 0, \quad \frac{d\tau_G^*}{d\phi^m} = 0, \quad \text{and} \quad \frac{d\gamma^*}{d\phi^m} > 0,$$

thereby confirming the results of Corollary 4.1.

[13] A proof of this is available upon request.

Chapter 5

Population Aging, the Composition of Government Spending, and Economic Growth in the Politico-Economic Equilibrium of a Simple OLG Economy

5.1 Introduction

Population aging, i. e., the shift in the distribution of a country's population towards older ages, is one of the most important demographic phenomena of our time. It will neither be confined to the West nor to industrialized economies. Table 5.1 presents actual data and forecasts of the old-age dependency ratio for selected countries and regions based on data from the United Nations.[1] Roughly speaking, between 2005 and 2050 this ratio is forecasted to double in Europe and Northern America. In Japan, India, Brazil, and Chile its estimated increase is even greater.

[1]Table 5.1 is based on the data that appear as the 'medium variant' prediction in United Nations (2008). The old-age dependency ratio is the ratio of the population aged 65 or over to the population aged $15-64$. The ratio is stated as the number of dependents per 100 persons of working age ($15-64$).

Table 5.1: Old-Age Dependency Ratios in Selected Countries and Regions.

Year	Europe	Northern America	Japan	India	Brazil	Chile
2005	23	19	30	7	9	12
2050	47	36	74	20	36	36

Population aging is likely to alter the distribution of preferences and the support for different types of government spending, thereby affecting economic growth. A typical concern raised in the public debate is that rising old-age dependency ratios lead to growing tax burdens and increased spending on the elderly, e. g., on health and care service, which crowds out public investment spending and has negative effects on economic growth. Recent evidence suggests that the concerns with respect to public spending are partly justified. In the United States, for example, the allocation of expenditure is highly skewed towards older members (see, e. g., Rogers et al., 2000; Iqbal and Turnovsky, 2008) and since 1959 public spending on the elderly has grown much faster than other categories of public expenditure (Mulligan and Sala-í-Martin, 1999). Poterba (1997) finds that a similar pattern holds in other OECD countries. Moreover, U.S. federal public spending on infrastructure declined from 5% to 2.5% of GDP over the time period from 1960 to the mid-1990's. However, total productive government spending on infrastructure, educational institutions, and R&D remained stable at 10% of GDP over the same time period (CBO, 1998).

In this chapter, we introduce a democratic voting process into a simple two-period overlapping generations model with endogenous growth à la Barro (1990) in order to analyze how population aging, i. e., an increase in the old-age dependency ratio, endogenously changes the composition of government spending and long-term economic growth.

We focus on two public spending categories: productive government expenditure (e. g., on infrastructure, education, or law and order) that increases private production possibilities and (unproductive) public consumption spending that only benefits the elderly (e. g., on health and care services or public infrastructure for the elderly). To finance its expenditure the government levies a uniform, proportional tax on labor and capital income. We solve for the politico-economic equilibrium in which government policy is set each period through voting by rational, forward-looking agents. In particular, voters take into account that current policy choices influence the evolution of the economy and future policies.

As government policy choices are of differing concern to the young and the old, they disagree on which policy mix they prefer to be implemented. We model the resolution of the resulting political conflict under the assumption of probabilistic voting. In contrast to the median voter model, the probabilistic voting model assures that policy proposals represent the interests of both groups of society, reflecting the political process in representative democracies more realistically.

Since elections take place each period, policy makers cannot commit to future policy choices. Therefore, voters have to form expectations about future policy outcomes. We focus on Markov perfect equilibria, i. e., equilibria in which the policy choices expected for a certain period depend only on the value of the fundamental state variable at that time.[2]

Under standard functional form assumptions, we are able to determine the politico-economic equilibrium and the implications of population aging in closed form. (This is in contrast to most of the literature that has to resort to numerical methods to characterize the politico-economic equilibrium. When we relax the functional form assumptions, and thus have to use numerical examples, the qualitative results turn out to be robust.)

We find that in the politico-economic equilibrium both types of government expenditure are chosen as constant shares of output and all variables in per capita terms grow at the same constant rate. The equilibrium share of output devoted to productive purposes corresponds to the exogenous output elasticity of productive public expenditure. In other words, it does not depend on preferences or demographic parameters, and thus is not affected by any form of population aging. By contrast, the equilibrium share of

[2] For a discussion of Markov perfect equilibria in the context of endogenous dynamic fiscal policy see, for instance, Krusell et al. (1997).

public spending on the elderly balances the interests of the old who support this type of spending as long as its benefits outweigh its tax costs and those of the young taxpayers who as net contributors oppose this form of public spending.

Voters internalize only those effects of government policy that materialize during their lifetimes; negative consequences borne by subsequent cohorts (due to higher overall taxes and lower capital accumulation) are not taken into account. By contrast, a benevolent planner with "dynastic" welfare weights (i. e., with welfare weights reflecting the households' discount factor and cohort sizes) also considers the effects on future households. Therefore, the share of public consumption spending on the elderly implemented by such a planner tends to be smaller than the corresponding share in the politico-economic equilibrium.

Population aging in our framework occurs either due to a decline in the population growth rate or due to an increase in life expectancy. Both phenomena increase the old-age dependency ratio and the relative weight that the political process attaches to the interests of the old relative to the young voters. The model predicts an increase in the old-age dependency ratio to be associated with (i) higher public consumption spending on the elderly (as a share of output), (ii) unchanged public productive expenditure (as a share of output), and (iii) a higher distortionary income tax rate. The latter has a negative effect on the economy's growth rate of per capita variables.

However, population aging not only affects economic growth indirectly via the composition and financing of government spending, but also has a direct effect on economic growth. For a given policy mix, an increase in the old-age dependency ratio accelerates economic growth. If the increase in the old-age dependency ratio follows a slowdown in the population growth rate this result is due to reduced capital dilution. In the case of a higher life expectancy the positive growth effect results from an increase in precautionary savings. The same channels are at work in any AK-type OLG growth model. In this chapter, we evaluate whether a positive growth bias of population aging persists when an increasing fraction of elderly prefers higher public consumption spending and less economic growth. We find that in both scenarios the direct effect dominates the indirect effect such that population aging overall increases the economy's growth rate of per capita variables.

This chapter relates and contributes to at least two strands of the literature. First, it makes a contribution to the recent politico-economic literature on dynamic fiscal policy where rational, forward-looking agents vote repeatedly on the level and financing of different types of government spending. Recent contributions that analyze how population aging endogenously affects government spending include Bassetto (2008), Gonzalez-Eiras and Niepelt (2008), and Song et al. (2009).[3] Hitherto, this literature has not considered a public policy mix that involves productive government expenditure and public consumption spending that only benefits the elderly. Moreover, the above mentioned papers do not consider an endogenous economic growth framework such that they cannot study the effect of demographic change on long-term economic growth. To the best of our knowledge, the only exception is Gonzalez-Eiras and Niepelt (2007) who quantitatively analyze the effect of population aging on public spending for education, public transfers between workers and retirees, and endogenous productivity growth in a three-period overlapping generations model with human and physical capital accumulation. In their framework, population aging induces a reallocation of public resources from education spending to retirement benefits, which has a negative growth effect. Similar to our results, they also find that reduced capital dilution more than outweighs this effect and that the long-term growth rate overall increases.

Second, this chapter complements the theoretical literature on the causal effect of population aging on long-term economic growth in models with overlapping generations and endogenous economic growth. Most contributions in this strand of the literature find this effect to be positive.[4] It results, for instance, from the following channels: (i) reduced capital dilution due to a slowdown in population growth (see, e. g., Gonzalez-Eiras and Niepelt, 2007), (ii) changes in individual saving behavior because of a longer expected lifetime (see, e. g., Futagami and Nakajima, 2001), (iii) more investment in innovations that increase labor productivity because a smaller labor force makes the input factor labor more expensive (see, e. g., Heer and Irmen, 2008), (iv) more private investments into new technologies as they are more likely to pay off when the individual time horizon

[3]See, e. g., Hassler et al. (2007, 2005) or Krusell and Ríos-Rull (1999) for insights about the politico-economic determination of taxes, transfers, and/or public consumption spending in environments where agents are heterogeneous in human capital and earnings. However, these papers do not consider the role of population aging.

[4]By contrast, Irmen (2009) finds that in the presence of capital-saving technical change population aging does not affect the economy's steady-state growth rate.

expands (see, e. g., Prettner, 2009). In the present chapter, either channel (i) or (ii) is at work. Additionally, a new channel operates in the opposite direction: population aging by shifting political power from the young to the old leads to an increased demand for public consumption spending and a slowdown of economic growth.[5]

The remainder of this chapter is organized as follows. Section 5.2 describes the model and characterizes the allocation conditional on policy. Section 5.3 describes the political decision-making process and establishes the politico-economic equilibrium. The allocation chosen by a Ramsey planner, who cares about all future generations, is studied in Section 5.4. Section 5.5 analyzes how an increase in the old-age dependency ratio affects the composition of government expenditure and economic growth in the politico-economic equilibrium. While Section 5.5.1 considers a decline in the population growth rate, Section 5.5.2 studies the case where the old-age dependency ratio increases because of a higher life expectancy. Section 5.6 discusses and extends the analysis in two directions. First, in Section 5.6.1 the Markov perfect equilibrium of Section 5.3 is compared to two other voting equilibria. Second, numerical examples in Section 5.6.2 suggest that our main findings are robust to the use of two alternative utility functions. Section 5.7 concludes. Proofs are relegated to the Appendix.

5.2 The Economic Environment

Consider an overlapping generations economy in which non-altruistic agents live for two periods: a working period and a retirement period.[6] Individual labor supply when young is inelastic and normalized to one. The size of generation t is denoted by L_t and grows at the exogenous rate $n > (-1)$. The population at any t thus consists of L_t young and L_{t-1}

[5] There are a few papers (see, e. g., Yakita, 2008; Dioikitopoulos, 2009) that examine the effect of population aging on the growth-maximizing composition of government expenditure. However, in these papers policy is not determined endogenously via a political process and thus does not reflect the distribution of preferences.

[6] This can be considered the most conservative scenario. A setup with agents that are altruistic towards future generations would represent an intermediate case between the framework presented in this section and the Ramsey planner of Section 5.4. Thus, it can be expected that altruistic agents would vote for a lower share of public consumption spending and a higher equilibrium growth rate than in the politico-economic equilibrium of Proposition 5.1.

old individuals.[7] Note that n corresponds to the growth rate of the total population and determines the old-age dependency ratio defined as $L_{t-1}/L_t = (1+n)^{-1}$. The economy starts at time 0 with $L_{-1} = 1$.

5.2.1 Preferences

In the economy at each t there is one private good and one public (consumption) good. The private good delivers utility to the agents when young and when old, whereas the public consumption good only benefits the old agents. For concreteness, one may think of this public good involving publicly-provided health and care services or public infrastructure for the elderly.

The preferences of an individual born at t are described by the following log-linear utility function[8]

$$\ln c_t^y + \beta \left(\ln c_{t+1}^o + b \ln \tilde{h}_{t+1} \right), \tag{5.1}$$

where c_t^y and c_{t+1}^o are consumption of the private good of a member of generation t when young and old, respectively, and $\tilde{h}_{t+1}$ is the level of provision of the public good per old agent at $t+1$, i. e., $\tilde{h}_{t+1} \equiv H_{t+1}/L_t$, where H_{t+1} denotes aggregate spending on the public consumption good at $t+1$. The fact that $\tilde{h}_{t+1}$ and not H_{t+1} enters the utility function implies that there is congestion in the public consumption good.[9] Moreover, $\beta \in (0,1)$ denotes the discount factor and $b > 0$ measures the weight an old agent assigns to the public relative to the private consumption good.

[7] In the following sections we focus on a deterministic life time. Only in Section 5.5.2 we reinterpret and extend the setup of Section 5.2 to incorporate an uncertain life time and the concept of life expectancy.

[8] The choice of logarithmic utility guarantees analytical tractability, but does not affect the qualitative findings. We return to this point in Section 5.6.2, where the results from the logarithmic utility case are compared to those of (i) a utility function with a constant intertemporal elasticity of substitution different from unity and (ii) non-separable preferences.

[9] In other words, this type of government activity has the character of a utility-enhancing transfer to the old that is not excludable, but rival. Alternatively, we could assume that public consumption spending enters the utility function as a pure public good. The main difference of this modeling approach concerns the long-term effect of population aging on the level of services derived by each old agent from public consumption spending. See Section 5.5.1, for a more detailed discussion.

5.2.2 Technology

At each t, the private good is produced by competitive firms operating a technology that uses capital K_t procured by the old, labor L_t supplied by the young, as well as a productivity-enhancing input G_t provided by the government. One may think of G as government expenditure on infrastructure, education, or law and order. More specifically, we assume that total output of the private good at t, Y_t, is manufactured according to

$$Y_t = AK_t^{\alpha} \left(g_t L_t\right)^{1-\alpha}, \quad 0 < \alpha < 1, \tag{5.2}$$

where $A > 0$ denotes the time-invariant total factor productivity and $g_t \equiv G_t/L_t$ is the productive public input per worker at t. Thus, there is also congestion in the productive public input.[10] Given the length of the considered period (one generation) it is assumed that capital fully depreciates after each use.

Let $y_t \equiv Y_t/L_t$ and $k_t \equiv K_t/L_t$ denote output and capital per worker, respectively. Then, we obtain output per worker from (5.2) as

$$y_t = Ak_t^{\alpha} g_t^{1-\alpha}. \tag{5.3}$$

The initial capital stock per worker is given by $k_0 > 0$.

Note that the technology displays diminishing returns in private capital, but constant returns to scale in private capital and the productive public input. Thus, if g rises with k the diminishing returns to the accumulation of capital do not set in. For this reason, the economy will exhibit endogenous steady-state growth.

At any time t, the private good can either be consumed, saved as capital for the next period, or be converted one-to-one into units of H_t and G_t. We take the private good produced at each period t as the numeraire.

[10] The congestion specification assures the existence of a politico-economic equilibrium and a balanced growth path. By contrast, if G were a pure public good, then the interest rate in the politico-economic equilibrium would depend on the aggregate labor supply (see Section 2.3.1 of this dissertation for a more detailed discussion), which in our framework grows over time. However, for an endogenous balanced growth path to exist, the equilibrium interest rate has to be constant. The Barro (1990) literature that uses a pure public good specification avoids this problem by assuming a stationary population.

5.2.3 Government Policy

In each period, the government raises tax revenues and uses the proceeds to purchase private consumption goods to be converted into the public consumption good and the public productive input. Specifically, the government at each t levies a proportional tax $\tau_t \in [0,1]$ on labor income of the young and capital income of the old. The government cannot issue age-dependent taxes and the government's budget is assumed to balance in each period t, i. e., $G_t + H_t = \tau_t \left(w_t L_t + R_t K_t\right)$, where w_t is the pre-tax wage rate at time t and R_t is the rental rate of capital at time t.

Then, the government's budget constraint in per worker terms is given by

$$g_t + h_t = \tau_t \left(w_t + R_t k_t\right), \tag{5.4}$$

where $h_t \equiv H_t/L_t = \tilde{h}_t/(1+n)$ is the level of the public consumption good per worker at t. As h_t is proportional to $\tilde{h}_t$ and n is exogenous, we focus - for notational simplicity - on the policy mix (g_t, h_t).

Then, a feasible government policy at t is a vector $(g_t, h_t) \in \mathbb{R}^2_+$ such that (5.4) holds and $\tau_t \in [0,1]$.

5.2.4 Timing

Within each period t the timing of events is as follows: At the beginning of the period, after a new generation of young people has been born, all individuals (the young and the old) democratically elect a political candidate who chooses current policy. When deciding which candidate to support, voters anticipate how each candidate's policy platform would affect subsequent economic and political decisions. Then, firms hire workers and rent capital to produce. The policy vector and the resulting income tax rate together with the wage rate and the rental rate of capital determine the consumption of the old and the disposable income of the young. The young then choose how much to consume and how much to save as capital for the next period. Finally, the old generation dies, while the young generation ages and becomes old in the next period.

In order to solve for the equilibrium we proceed by backward induction. We start in Section 5.2.5 by analyzing the economic choices of households and firms subject to exogenously given (prices and) government policy. We refer to the allocation that results at time t for a given policy mix as the economic equilibrium.[11] Section 5.3 then considers the political determination of policy.

5.2.5 Economic Equilibrium

In an economic equilibrium, each household maximizes her lifetime utility given by (5.1) taking factor prices and the benefits from the public consumption good as given. Each individual that is born at time $t \geq 0$ faces the per-period budget constraints $c_t^y + s_t \leq (1-\tau_t)\, w_t$ and $c_{t+1}^o \leq s_t\,(1-\tau_{t+1})\, R_{t+1}$, where s_t denotes savings at t.

The optimal choices of a member of cohort t are then given by

$$c_t^y = \frac{1}{1+\beta}(1-\tau_t)\, w_t, \tag{5.5}$$

$$c_{t+1}^o = \frac{\beta}{1+\beta}(1-\tau_t)\, w_t\,(1-\tau_{t+1})\, R_{t+1}, \tag{5.6}$$

$$s_t = \frac{\beta}{1+\beta}(1-\tau_t)\, w_t. \tag{5.7}$$

Note that optimal saving of a young agent at t, given by (5.7), does neither depend on the population growth rate nor on future fiscal policy.[12]

Moreover, in an economic equilibrium each firm maximizes its profits taking factor prices and the level of provision of the productive public input as given. Thus, the firms' profit maximization problem determines the rental rate of capital and the pre-tax wage rate as

$$R_t = \alpha \frac{y_t}{k_t} \quad \text{and} \quad w_t = (1-\alpha)\, y_t, \tag{5.8}$$

respectively, where y_t is given by (5.3). Due to constant returns to scale in private inputs the firm sector makes zero profits, i. e., $Y_t = w_t L_t + R_t K_t$. This in turn implies that the

[11] This is the term used in the politico-economic literature, see, e. g., Persson and Tabellini (2000, p.271). The economic equilibrium corresponds to what the growth literature calls a "temporary equilibrium", see, e. g., de la Croix and Michel (2002, p. 16).

[12] The latter independence is a direct consequence of the logarithmic utility and greatly simplifies the analysis. We relax this restriction in the numerical sensitivity analysis of Section 5.6.2.

government budget constraint (5.4) may be written as

$$g_t + h_t = \tau_t y_t. \tag{5.9}$$

For the capital market to clear it has to hold at all t that

$$K_{t+1} = s_t L_t, \tag{5.10}$$

i. e., the aggregate capital stock in period $t+1$ corresponds to aggregate saving in period t.

Combining conditions (5.5) - (5.10), the equilibrium allocation in t can be expressed in terms of government policy and the capital stock per worker

$$c_t^y = \frac{1-\alpha}{1+\beta}\left(Ak_t^\alpha g_t^{1-\alpha} - g_t - h_t\right), \tag{5.11}$$

$$c_t^o = \alpha(1+n)\left(Ak_t^\alpha g_t^{1-\alpha} - g_t - h_t\right), \tag{5.12}$$

$$k_{t+1} = \frac{s_t}{1+n} = \frac{\tilde{B}}{1+n}\left(Ak_t^\alpha g_t^{1-\alpha} - g_t - h_t\right) \equiv \pi^k(g_t, h_t, k_t), \tag{5.13}$$

where $\tilde{B} \equiv \beta(1-\alpha)/(1+\beta)$.[13]

The function $\pi^k(\cdot)$ is the economic equilibrium condition that describes how young agents optimally choose their savings and thus determine the next period's capital stock per worker for given g_t, h_t, and k_t. Equation (5.13) also reveals how the composition and financing of government spending affects capital accumulation. First, the income tax financing of both types of public expenditure has a negative effect on the accumulation of capital (negative terms in brackets). Second, g_t has an additional positive effect on the accumulation of capital by raising the productivity of private capital.

In an economic equilibrium, the indirect utility of a young and an old agent alive at t, respectively, can be expressed as functions of government policy and the capital stock per worker:

$$\begin{aligned} U_t^Y &= \ln c_t^y + \beta\left(\ln c_{t+1}^o + b \ln \tilde{h}_{t+1}\right) = \ln\left(Ak_t^\alpha g_t^{1-\alpha} - g_t - h_t\right) \\ &\quad + \beta \ln\left(Ak_{t+1}^\alpha g_{t+1}^{1-\alpha} - g_{t+1} - h_{t+1}\right) + \beta b \ln h_{t+1} + \text{t.i.p.}, \end{aligned} \tag{5.14}$$

$$U_t^O = \ln c_t^o + b \ln \tilde{h}_t = \ln\left(Ak_t^\alpha g_t^{1-\alpha} - g_t - h_t\right) + b \ln h_t + \text{t.i.p.}, \tag{5.15}$$

s.t. $k_{t+1} = \pi^k(g_t, h_t, k_t)$. Here, t.i.p. denotes terms independent of the policy choice.

[13] Note that the above equilibrium conditions imply that the market for the private good clears at all t, i. e., $L_t c_t^y + L_{t-1} c_t^o + K_{t+1} + G_t + H_t = Y_t$.

5.3 Politico-Economic Equilibrium

In the politico-economic equilibrium, the government policy mix (g_t, h_t) is chosen through voting at the beginning of each period t. Electoral competition is modeled under the assumption of probabilistic voting. As elections take place each period, policy makers cannot commit to future policy choices. Therefore, voters have to form expectations about future policy outcomes. In order to limit the set of potential equilibria, we restrict attention to Markov perfect equilibria, i. e., equilibria in which the policy choices expected for a certain period depend only on the value of the fundamental state variables expected at that time, and not on the past history of policies or artificial state variables sustaining trigger strategy equilibria.[14] In the present setup, the only state variable is the level of the private capital stock per worker; it affects future wages and returns, and therefore income of future voters.[15]

5.3.1 Probabilistic Voting

The political process is represented via a two-candidate probabilistic voting model. In this model agents cast their votes on one of two candidates, who maximize their probability of becoming elected. Voters support a candidate not only for her policy platform, but also for other characteristics like "ideology" that are orthogonal to the fundamental policy dimensions of interest. The evaluation of these features differs across voters and is subject to random aggregate shocks, realized after candidates have chosen their platforms.[16]

In a probabilistic-voting Nash equilibrium, two candidates maximizing their respective vote shares both propose the same policy platform and each of them has a 50 % probabil-

[14] This assumption rules out equilibria that rely on reputation mechanisms, see, e. g., Chari and Kehoe (1990), Kotlikoff et al. (1988), or for a concise discussion Persson and Tabellini (2000, p.314-317), and allows to identify the fundamental forces that shape the policy mix of interest.

[15] Note that the population growth rate n as well as life expectancy in Section 5.5.2, i. e., the variables that determine the old-age dependency ratio, will affect the actual policy choice. However, in their decision-making process all agents treat these variables as exogenous.

[16] For a more detailed discussion of the probabilistic voting model, see Lindbeck and Weibull (1987) or Persson and Tabellini (2000).

ity of winning. The proposed policy platform maximizes a "political objective function" which is a weighted average utility of all voters, with the weights reflecting the group size and the sensitivity of voting behavior to policy changes. Groups that have a low concern for the orthogonal policy dimension have more political influence since they are more likely to alter their support in response to small changes in the proposed platform. In other words, these groups of "swing voters" are more attractive to power-seeking candidates and exert a stronger influence on the equilibrium policy outcome. Formalizing the foregoing discussion, we assume that the "political" aggregation of different preferences is summarized by the following political objective function

$$U_t = (1+n) U_t^Y + \omega U_t^O, \tag{5.16}$$

where U_t^Y and U_t^O are given by (5.14) and (5.15), respectively, $\omega > 0$ represents the per-capita political weight of the old relative to the young, and $(1+n)$ the relative group size of the young compared to the old. Thus, the political objective function (5.16) to be maximized in the political process attaches a positive weight to the welfare of the elderly, even if the median voter is a young agent. This appears to be a realistic implication. In fact, it is often argued that the old are more policy-focused, i. e., care less about ideology and have more swing voters, and thus even exert a stronger political influence per capita than the young (see e. g., Rhodebeck (1993, p.357), Dixit and Londregan (1996, p.1144) or Grossman and Helpman (1998, p.1309)). This case would correspond to an $\omega > 1$.

Using the expressions for U_t^Y and U_t^O, the political objective function obtains as

$$\begin{aligned} U\left(g_t, h_t, k_t, g_{t+1}, h_{t+1}, k_{t+1}\right) &= (1+n+\omega) \ln \left(A k_t^\alpha g_t^{1-\alpha} - g_t - h_t\right) + \omega b \ln h_t \\ &+ (1+n) \beta \ln \left(A k_{t+1}^\alpha g_{t+1}^{1-\alpha} - g_{t+1} - h_{t+1}\right) \\ &+ (1+n) \beta b \ln h_{t+1} \end{aligned}$$

subject to $k_{t+1} = \pi^k(g_t, h_t, k_t)$.

5.3.2 Definition of the Politico-Economic Equilibrium

As mentioned above, we look for Markov perfect equilibria, i. e., for equilibria in which the policy choices are functions only of the level of private capital per worker in the economy. The dynamic aspect of the voting game stems from the fact that current policy affects capital accumulation, and thus income and the strategic position of the currently young in the next period. Agents are assumed to be fully forward-looking. Thus, when voting over today's policy, young agents correctly anticipate how future policy will depend on current policy via the state of the economy.

Definition 5.1. *The Politico-Economic Equilibrium is defined as a pair of functions* $\left\langle \pi^g, \pi^h \right\rangle$*, where* π^g *and* π^h *are public policy rules,* $g_t = \pi^g(k_t)$ *and* $h_t = \pi^h(k_t)$*, such that the following functional equation holds:*

$\left\langle \pi^g(k_t), \pi^h(k_t) \right\rangle = \arg\ \max_{\{g_t, h_t\}} U(g_t, h_t, k_t, g_{t+1}, h_{t+1}, k_{t+1})$*, subject to*

$$\begin{aligned}
k_t & \quad \textit{given}, \\
k_{t+1} &= \pi^k(g_t, h_t, k_t), \\
g_{t+1} &= \pi^g(k_{t+1}) = \pi^g\left(\pi^k(g_t, h_t, k_t)\right), \\
h_{t+1} &= \pi^h(k_{t+1}) = \pi^h\left(\pi^k(g_t, h_t, k_t)\right).
\end{aligned}$$

The equilibrium condition requires the political mechanism in t to choose g_t and h_t to maximize the political objective function U, for a given k_t, taking into account that future government policies, g_{t+1} and h_{t+1}, depend on the current policy mix (g_t, h_t) via the state of economy, k_{t+1}, as described by the economic equilibrium decision rule π^k. Moreover, the above definition of the politico-economic equilibrium has the usual fixed point structure induced by a rational expectations equilibrium: the anticipated policy functions coincide with the optimal ones. In other words, suppose that agents believe future government policy to be set according to $g_{t+1} = \pi^g(k_{t+1})$ and $h_{t+1} = \pi^h(k_{t+1})$. Then, we require that the same functions $g_t = \pi^g(k_t)$ and $h_t = \pi^h(k_t)$ define optimal spending today.

5.3.3 Solving for the Politico-Economic Equilibrium

To solve for the politico-economic equilibrium we need to find two functions π^g and π^h satisfying Definition 5.1. Guided by the fact that government expenditure is financed by a proportional tax on income, we conjecture that π^g and π^h are linear functions in the capital stock. Specifically, we make the following guess for future policy variables: $\pi^g(k_{t+1}) = \eta^g k_{t+1}$ and $\pi^h(k_{t+1}) = \eta^h k_{t+1}$, with some yet undetermined coefficients η^g

and η^h.[17] We derive the equilibrium choice of government policy in period t under this conjecture and show that the spending shares in t are indeed linear in the capital stock, thereby verifying the conjecture.

First of all, note that with this guess the production function (5.3) at $t+1$ becomes linear in the capital stock $y_{t+1} = A(\eta^g)^{1-\alpha} k_{t+1}$ and we can write $Ak_{t+1}^{\alpha} g_{t+1}^{1-\alpha} - g_{t+1} - h_{t+1} = \left(A(\eta^g)^{1-\alpha} - \eta^g - \eta^h\right) k_{t+1}$.

Using these results and omitting terms independent of the policy choice, the program characterizing equilibrium policy choices in period t can be expressed as

$$\max_{\{g_t,h_t\}} \bar{U}(g_t,h_t,k_t) \quad \text{s.t.} \quad k_t \text{ given}, \quad \text{where}$$

$$\bar{U}(g_t,h_t,k_t) \equiv [(1+n)(1+\beta(1+b))+\omega] \ln\left(Ak_t^{\alpha} g_t^{1-\alpha} - g_t - h_t\right) + \omega b \ln h_t. \tag{5.17}$$

After some algebra, the first-order conditions of the program (5.17) with respect to g_t and h_t yield

$$g_t = (1-\alpha)y_t \quad \text{and} \quad h_t = \frac{\alpha\omega b}{(1+n)(1+\beta(1+b))+\omega(1+b)} y_t, \tag{5.18}$$

$$\text{where} \quad y_t = A(\eta^g)^{1-\alpha} k_t. \tag{5.19}$$

Equations (5.18) and (5.19) verify the tentative guess as a fixed point of the functional equation of Definition 5.1 if $\eta^g = (A(1-\alpha))^{1/\alpha}$ and $\eta^h = \alpha\omega b\left[(1+n)(1+\beta(1+b))+\omega(1+b)\right]^{-1} A^{1/\alpha}(1-\alpha)^{(1-\alpha)/\alpha}$, which allow us to establish the following proposition.

Proposition 5.1. *The politico-economic equilibrium is characterized as follows:*

$$\pi^g(k_t) = (1-\alpha)y_t \quad \text{and} \quad \pi^h(k_t) = \tau_h^{\mathcal{P}} y_t,$$
$$\text{with} \quad y_t = A^{\frac{1}{\alpha}}(1-\alpha)^{\frac{1-\alpha}{\alpha}} k_t$$

and $\alpha > \tau_h^{\mathcal{P}} \equiv \alpha\omega b\left[(1+n)(1+\beta(1+b))+\omega(1+b)\right]^{-1} > 0$ *such that* $0 < \tau = 1-\alpha+\tau_h^{\mathcal{P}} < 1$ *for all* t.

[17] Note that the above decision problem is a stationary Markov decision problem because the problem facing voters looks the same (contingent on the state) at each t. Moreover, note that guessing a policy function that does not depend on time per se is not the same as imposing ex-ante that the expenditure has to be a constant fraction of the capital stock. For details on this see Section 5.6.1.1.

Moreover, the equilibrium growth factor of the capital stock per worker, $\gamma_{t+1} \equiv k_{t+1}/k_t$, *is constant and given by*

$$\gamma_{t+1} = \frac{B}{1+n}\left(\alpha - \tau_h^{\mathcal{P}}\right) \equiv \gamma, \tag{5.20}$$

with $B \equiv A^{\frac{1}{\alpha}}(1-\alpha)^{\frac{1-\alpha}{\alpha}}\tilde{B}$. *The economy immediately settles on its steady-state path on which the economy's relevant variables such as per capita consumption, per capita output, the per capita capital stock, government spending as well as wages all grow at the same constant rate* $\gamma - 1$.

According to Proposition 5.1, under rational voting both types of government expenditure are chosen as constant shares of output. The equilibrium share of output devoted to the productive public input corresponds to $1-\alpha$, which is the output elasticity of the productive public input. Thus, productive government expenditure satisfies the so-called natural condition of productive efficiency, i. e., the marginal contribution of government expenditure to aggregate output is one (see, e.g., Barro, 1990). In the present context, as aggregate output is $Y = AK^{\alpha}G^{1-\alpha}$, we have $dY/dG = (1-\alpha)(Y/G) = (1-\alpha)(y/g) = 1$. This also implies that the young and the old prefer the same share of output devoted to productive purposes. In other words, there is no conflict about this type of public spending. The reason for this is that g_t symmetrically affects the labor income of the young and the capital income of the old.

The equilibrium share of output spent on the public consumption good benefiting the old is given by $\tau_h^{\mathcal{P}}$ and depends on preferences, technology, and demographic parameters. Intuitively, it balances the interests of the elderly who support public consumption spending as long as the related benefits outweigh the tax costs and those of the young taxpayers who oppose this form of spending as they are net contributors to the system. This reflects the intergenerational conflict more realistically than what would be observed under simple majority voting. For instance, assume that the median voter is a young agent.[18] Then, if we anticipate that all agents will prefer the same share of productive government spending such that the voting problem becomes one-dimensional, we find that the median voter would set public consumption spending on the old equal to zero. In our probabilistic voting setup, $\tau_h^{\mathcal{P}} = 0$ could only occur if the old had no political

[18] In a two-period OLG model there are always more young people than old as long as $n > 0$.

influence at all (i. e., if $\omega = 0$) or if they did not care about the public consumption good (i. e., if $b = 0$).[19]

The equilibrium income tax rate corresponds to the sum of the two public expenditure shares and turns out to be strictly smaller than one such that the equilibrium policy mix is feasible. Moreover, note that the income tax rate in equilibrium is time-invariant. In other words, it is independent of the economy's endogenous state variable, i. e., the capital stock per worker. Nevertheless, the equilibrium tax rate will be affected by population aging because it depends on demographic parameters.

Finally, Proposition 5.1 reveals that in the politico-economic equilibrium the economy's relevant variables in per capita terms grow at the same constant rate given by $\gamma - 1$. There is no guarantee that this rate is positive for all parameter combinations. However, a positive steady-state growth rate can be assured if we assume that the economy is sufficiently productive, i. e., if A is large enough.

The following corollary verifies that the Markov perfect equilibrium derived above is the limit of a unique finite-horizon equilibrium.[20]

Corollary 5.1. *The equilibrium policy functions $g_t = (1-\alpha)y_t$ and $h_t = \tau_h^{\mathcal{P}} y_t$ of Proposition 5.1 represent the unique equilibrium policy mix in (the limit of) the corresponding finite-horizon economy. In the last period, the policy function for h_t is different, but also unique.*

5.4 The Ramsey Allocation

This section compares the politico-economic equilibrium with the Ramsey allocation chosen by a benevolent planner who has the ability to commit to all its future policy choices at the beginning of time, but is constrained by the same economic equilibrium conditions.

Specifically, we consider the Ramsey solution in the case where the planner's weight on

[19] In this chapter we abstract from these polar cases.

[20] This allows us to rule out potential reputation-like equilibria that can only be supported if the horizon is infinite.

generation $t \geq 0$ is $\beta^{t+1}(1+n)^{t+1}$, i. e., the planner's weights on future generations reflect the discount factor of households as well as the cohort size ("dynastic discounting").[21] The planner's decision problem is therefore to choose the sequence $\{g_t, h_t\}_{t=0}^{\infty}$ in order to maximize

$$W\left(k_0, \{g_t, h_t\}_{t=0}^{\infty}\right) \equiv \beta U_0^O + \sum_{t=0}^{\infty} \left(\beta\left(1+n\right)\right)^{t+1} U_t^Y$$
$$\text{subject to} \quad (5.13) - (5.15) \quad \text{and} \quad k_0 \quad \text{given.} \tag{5.21}$$

In the following we assume that $\beta(1+n) < 1$ such that the planner's objective function W is finite.

The main difference to the program solved by the political candidates is that the Ramsey program (5.21) involves the choice of an entire sequence of policy mixes. Moreover, the Ramsey planner values the welfare of all households, not only of those currently alive and voting.

In order to solve for the Ramsey allocation it is helpful to first establish the following lemma.

Lemma 5.1. *The Ramsey program (5.21) is equivalent to the following recursive program:*

$$V\left(k_t\right) = \max_{\{g_t, h_t, k_{t+1}\}} \left\{T_t\left(g_t, h_t, k_t\right) + \left(1+n\right)\beta V\left(k_{t+1}\right)\right\} \quad \text{for} \quad t \geq 0, \tag{5.22}$$

subject to (5.13), where $T_t\left(g_t, h_t, k_t\right) \equiv \beta\left(2+n\right)\ln\left(Ak_t^{\alpha}g_t^{1-\alpha} - g_t - h_t\right) + \beta b \ln h_t$.

The fact that the planner's problem admits the standard recursive formulation of Lemma 5.1 reveals that its solution is time consistent. Intuitively, the generational weights (in the case of dynastic discounting) are such that the Ramsey plan is dynamically consistent (see, e. g., Heijdra, 2009, p.656-658).

The following proposition summarizes the solution of the Ramsey problem.

Proposition 5.2. *Let* $\beta(1+n) < 1$*. Then, the solution of the Ramsey program (5.21) involves for* $t \geq 0$

$$g_t = \left(1-\alpha\right) y_t \quad \text{and} \quad h_t = \tau_h^{\mathcal{R}} y_t,$$

[21] See, e. g., Gonzalez-Eiras and Niepelt (2008) for a discussion.

where $y_t = A^{1/\alpha}(1-\alpha)^{(1-\alpha)/\alpha} k_t$ *and* $0 < \tau_h^{\mathcal{R}} \equiv \alpha b\,(1-(1+n)\beta)\,(2+n+b)^{-1} < \alpha$. *Moreover,* g_t*,* h_t*,* y_t *and* k_t *grow at the same constant rate determined by (5.13).*

Proposition 5.2 reveals that the Ramsey planner sets the levels of both types of government expenditure proportional to output. He chooses the same share of output, namely $1-\alpha$, to be devoted to the productive public input as in the politico-economic equilibrium. The optimal share of output spent on the public consumption good benefiting the old is given by $\tau_h^{\mathcal{R}}$ and depends again on preferences, technology, and demographic parameters. The following corollary compares $\tau_h^{\mathcal{R}}$ to $\tau_h^{\mathcal{P}}$ of the politico-economic equilibrium.

Corollary 5.2. *It holds that*

$$\tau_h^{\mathcal{R}} \leq \tau_h^{\mathcal{P}} \Leftrightarrow \omega \geq 1-\beta(1+n).$$

Corollary 5.2 shows that the share of public spending on the elderly chosen by the Ramsey planner falls short of the corresponding share in the politico-economic equilibrium whenever ω or $\beta\,(1+n)$ are sufficiently large. For instance, this is the case for any $\omega \geq 1$, i. e., if the old have at least the same per capita political weight as the young.

The intuition for $\tau_h^{\mathcal{R}} < \tau_h^{\mathcal{P}}$ is that voters in their optimization problem only consider the effects of their policy choice that materialize during their lifetimes. Negative consequences borne by subsequent generations due higher taxes and lower capital accumulation are not taken into account. By contrast, the Ramsey planner internalizes the effects of policy on all current *and* future households.

5.5 Implications of Population Aging

5.5.1 Declining Population Growth

This section studies the effect of a permanent decline in the population growth rate n on government policy and economic growth in the politico-economic equilibrium. A decline in n causes a rise in the old-age dependency ratio $(1+n)^{-1}$. Increases in the latter are meant to capture the tendencies shown in Table 5.1. The decline in n materializes at the beginning of the period (see Section 5.2.4) and is then taken into account by all agents alive in that period. Note that it does not affect the results whether or not the decline in n is anticipated by the generation born in the previous period as their savings decision is independent of n (see equation 5.7).

In the politico-economic equilibrium of Proposition 5.1 the economy at all t grows at the constant rate $\gamma - 1$ given by (5.20). Recall that there are no transitional dynamics in the economy. Denote τ_g the fraction of current output devoted to productive public services, i. e., $\tau_g = g_t/y_t = 1 - \alpha$. Then, the results of the comparative static analysis described above can be summarized in the following corollary.

Corollary 5.3.

1. *If $\omega \neq (1+n)$, then it holds that*

$$\frac{d\tau_h^{\mathcal{P}}}{dn} < 0, \quad \frac{d\tau_g}{dn} = 0, \quad \textit{and} \quad \frac{d\tau}{dn} < 0.$$

2. *It holds that*

$$\frac{d\gamma}{dn} < 0.$$

The first statement of Corollary 5.3 reveals that an increase in the old-age dependency ratio (due to a decline in n) raises $\tau_h^{\mathcal{P}}$, i. e., the fraction of output used for the provision of public services that benefit the old. The reason is that a decline in n reduces the share of young agents relative to old agents in the population, and thus their weight in the political objective function (5.16). Intuitively, the old prefer greater spending on the public consumption good than the young. In the non-generic case $\omega = 1 + n$, i. e., when both groups have exactly the same weight in the political objective function, $\tau_h^{\mathcal{P}}$ does not depend on n. The share of output devoted to the productive public input corresponds to $1-\alpha$, and is thus always independent of the population growth rate. Overall, the income tax rate, τ, which is levied on households to finance government expenditure, has to increase in the politico-economic equilibrium. Statement 1 of Corollary 5.3 also implies that population aging increases the share of public consumptive expenditure in total government expenditure, i. e., $\tau_h^{\mathcal{P}}/\tau$, and decreases the share of productive government expenditure in total government expenditure given by $(1 - \alpha)/\tau$.

According to the second statement of Corollary 5.3, an increase in the old-age dependency ratio leads to a higher equilibrium growth rate of per capita variables. This is the result of two opposing forces. On the one hand, reduced labor force growth weakens the effect of capital dilution, i. e., a given amount of capital implies a higher capital stock per worker at each t and a rise in the equilibrium growth rate of per capita variables. Intuitively, a lower n reduces the break-even investment, the amount of investment necessary for k to grow at a constant rate, without affecting saving at any given level of capital.[22] On the other hand, there is a negative, indirect tax effect via $\tau_h^{\mathcal{P}}$. As discussed in the previous paragraph, an increase in the old-age dependency ratio raises spending for the public consumption good, and thus taxes. Since taxes are levied on capital *and* labor income, they reduce the incentive to save and to accumulate capital, and hence have a negative effect on the steady-state growth rate. The point of the second statement of Corollary 5.3 is that the former effect dominates the latter. Therefore, population aging accelerates the economy's growth rate of per capita variables.

Finally, Corollary 5.3 implies that an increase in the old-age dependency ratio in the long run raises the benefits derived by each old agent from aggregate public consumption spending. To see this note that $\tilde{h}_t$ is given by

$$\tilde{h}_t = (1+n)h_t = (1+n)\tau_h^{\mathcal{P}} y_t = (1+n)\tau_h^{\mathcal{P}} y_0 e^{(\gamma-1)t}. \tag{5.23}$$

From the definition of $\tau_h^{\mathcal{P}}$ in Proposition 5.1 one readily verifies that $(1+n)\tau_h^{\mathcal{P}} y_0$ declines if n decreases. Thus, the level of benefits per old initially falls, but then grows at a higher rate (as $d\gamma/dn < 0$) and at some point reaches a higher level than what would have be attainable without a change in n.[23] The initial decline is due to congestion effects; intuitively, the benefits of the public consumption good have to be spread over more old people.

[22] In the context of a conventional neoclassical growth model, Cutler et al. (1990) refer to this channel as the "Solow effect".

[23] By contrast, if aggregate public consumption spending H entered the utility function (5.1) as a pure public good, then an increase in the old-age dependency ratio would lower the level of H in the long run. To see this note that in this case the level of public consumption services would be given by $H_t = \tau_h^{\mathcal{P}} Y_t = \tau_h^{\mathcal{P}} Y_0 e^{g_K t}$, where $g_K = B(\alpha - \tau_h^{\mathcal{P}}) - 1$ corresponds to the growth rate of aggregate variables. As $d\tau_h^{\mathcal{P}}/dn < 0$ and $dg_K/d\tau_h^{\mathcal{P}} < 0$, a decline in n thus implies that H_t initially increases, but then grows at a lower rate. Therefore, population aging here leads to an increase in taxes and at the same time to a decline in the long-term provision of H. The reason for this is that the tax base is lower at all t.

5.5.2 Increasing Life Expectancy

In the previous section, we studied population aging as a decline in the population growth rate. With a slight reinterpretation of the analytical framework, we can also analyze the effect of an increasing life expectancy on government policy and economic growth.

For this purpose, suppose that each individual faces an exogenous probability of dying at the end of its first period of life equal to $(1-v) \in (0,1)$. This implies that the old-age dependency ratio at t becomes vL_{t-1}/L_t and increases in v.

Let $\beta_v \in (0,1)$ denote the pure discount factor, i. e., the discount factor that the individual would apply if he or she were sure to reach the retirement age. Moreover, normalize the utility after death to zero. Then, we may interpret the utility function of (5.1) as the expected utility of a member of generation t with $\beta \equiv \beta_v v$ as the effective discount factor of the agent and with $\tilde{h}_{t+1} \equiv H_{t+1}/vL_t$ as the provision of the public consumption good per surviving old agent.

Against the survival risk individuals may buy annuity assets with which they receive insurance payments if they are alive and nothing if dead in the retirement period. Assuming that the private annuity markets are perfectly competitive, insurance payments are actuarially fair.

Finally, let ω_v denote the pure per capita political weight of the old, i. e., the political power the old would exert if all individuals survived. Then, the political objective function remains given by (5.16) with $\omega \equiv \omega_v v$ as the effective political weight of the old and the economy inherits the properties established in Proposition 5.1 and Corollary 5.1.

A permanent increase in life expectancy due to a permanent rise in the survival probability, v, increases the effective discount factor, β, and the effective political weight of the old, ω. The following proposition summarizes the effects of such an increase on government policy and the equilibrium growth rate.

Proposition 5.3. *Consider an economy that at $t = 1$ experiences a small but permanent increase in its life expectancy, i. e., $\hat{v} > v$ for all generations $t = 1, 2, 3, ..., \infty$. Assume that this change is unexpected, i. e., it is anticipated by all generations $t = 2, 3, ..., \infty$,* *but not by generation* 1*. Denote variables associated with an evolution under* $\hat{v}$ *by a hat such that the politico-economic equilibrium at* t *is characterized by* $\hat{\tau}_{h_t}^{\mathcal{P}}$ *and* $\hat{\gamma}_{t+1}$.

Then, it holds that

$$\hat{\tau}_{h_1}^{\mathcal{P}} = \tau_h^{\mathcal{P}}, \quad \hat{\gamma}_2 = \gamma, \quad \textit{and}$$

$$\hat{\tau}_{h_t}^{\mathcal{P}} = \hat{\tau}_h^{\mathcal{P}} > \tau_h^{\mathcal{P}}, \qquad \hat{\gamma}_{t+1} = \hat{\gamma} > \gamma \quad \textit{for} \quad t = 2, 3, ..., \infty.$$

Proposition 5.3 reveals that an increase in life expectancy increases the share of public consumption spending and the equilibrium growth rate. Intuitively, a higher life expectancy increases savings per next period's worker since the weight on the expected old-age utility increases. This has a positive effect on capital accumulation and dominates the negative tax effect that results from a greater political weight of the old.

However, contrary to the case of a permanent decline in the population growth rate, this effect is delayed by one generation. The reason for this is that the increase in life expectancy is unexpected, i. e., generation 1 makes its consumption and savings plan anticipating an effective discount factor of β instead of $\hat{\beta}$.[24]

Arguably, this is a realistic assumption as expectations of one's own life expectancy are usually myopic, i. e., coincide with the actual life expectancy of the previous generation. Moreover, it is reasonable to assume that the choice of s_1 is made by the young agents before the change in the survival probability is experienced.

Consequently, a permanent increase in life expectancy affects public spending and economic growth in the same direction as a decline in the population growth rate, but with a period delay.

[24]If the increase in life expectancy were anticipated by generation 1, then savings would already increase in $t = 1$. However, the effective political weight of the elderly ($\omega_v v$) and the public policy choice in $t = 1$ are not affected by an anticipated change in life expectancy. In $t = 1$, the young of generation 1 and the old of generation 0, whose size is determined by the initial life expectancy v, vote on government policy. Therefore, the equilibrium growth rate would first jump to a level $\hat{\gamma}_2 > \hat{\gamma}$ and then from $t = 2$ onwards correspond to $\hat{\gamma}$. For a more detailed discussion see the proof of Proposition 5.3 in the Appendix.

5.6 Discussion and Extensions

This section discusses and extends the analysis in two directions. First, Section 5.6.1 compares the politico-economic equilibrium to two other voting equilibria. Second, Section 5.6.2 presents numerical examples for two alternative preference specifications.

5.6.1 Other Voting Equilibria

In this section we compare the politico-economic equilibrium of Section 5.3 to (i) the voting equilibrium that results when voters ex-ante are restricted to choose constant policy paths and (ii) the myopic voting equilibrium.

5.6.1.1 Voting Equilibrium under Commitment to Constant Policy Paths

This section analyzes a voting equilibrium in which taxes and expenditure shares are ex-ante restricted to a constant path. In other words, we assume that the political candidates in period t propose and fully commit to policies that set government expenditure as the same constant fraction of output.

For this purpose, suppose that a feasible government policy is a vector $(\tau_g^c, \tau_h^c) \in [0,1] \times [0,1]$ such that $g_t = \tau_g^c y_t$, $h_t = \tau_h^c y_t$ and $\tau^c = (\tau_g^c + \tau_h^c) \in [0,1]$. Otherwise the economic environment is identical to that of Section 5.2.

Then, following the same steps as in Section 5.2.5 the economic equilibrium in period t, i. e., the allocation conditional on the policy mix (τ_g^c, τ_h^c) and for a given k_t is characterized by

$$c_t^y = \frac{1-\alpha}{1+\beta}\left(1-\tau_g^c-\tau_h^c\right) y_t, \tag{5.24}$$

$$c_t^o = \alpha(1+n)\left(1-\tau_g^c-\tau_h^c\right) y_t, \tag{5.25}$$

$$k_{t+1} = \frac{\tilde{B}}{1+n}\left(1-\tau_g^c-\tau_h^c\right) y_t, \tag{5.26}$$

with $y_t = A^{1/\alpha}\left(\tau_g^c\right)^{(1-\alpha)/\alpha} k_t$.

Using equations (5.24) - (5.26) and dropping terms independent of policy yields the indirect utilities of a young and an old agent at t as

$$U_t^Y \simeq [1+\beta(2+b)]\ln\left(1-\tau_g^c-\tau_h^c\right)+[1+2\beta(1+b)]\frac{1-\alpha}{\alpha}\ln\tau_g^c + \beta b\ln\tau_h^c, \tag{5.27}$$

and

$$U_t^O \simeq \ln\left(1-\tau_g^c-\tau_h^c\right)+(1+b)\frac{1-\alpha}{\alpha}\ln\tau_g^c+b\ln\tau_h^c, \tag{5.28}$$

respectively.

The political candidates in period t then choose (τ_g^c, τ_h^c) to maximize the political objective function (5.16) with U_t^Y and U_t^O given by (5.27) and (5.28). The following proposition establishes the equilibrium policy mix and the resulting economic growth rate.

Proposition 5.4. *The equilibrium policy mix under commitment to constant policy paths is given by*

$$\tau_g^c = 1-\alpha \quad \text{and} \quad \tau_h^c = \frac{((1+n)\beta+\omega)\alpha b}{(1+n)(1+2\beta(1+b))+\omega(1+b)} < \alpha. \tag{5.29}$$

Under this policy mix, the economy's growth factor of all per capita variables, government spending and wages is given by

$$\gamma^c = \frac{B}{1+n}(\alpha-\tau_h^c). \tag{5.30}$$

Proposition 5.4 reveals that policy makers in this voting equilibrium also set the share of output devoted to the productive public input equal to $1-\alpha$.[25] The following corollary concerns public consumption spending on the elderly.

Corollary 5.4. *Comparing the share of output spent on the public consumption good for the elderly, τ_h^c of (5.29), to the corresponding expenditure share in the politico-economic equilibrium of Proposition 5.1 yields*

$$\tau_h^c > \tau_h^{\mathcal{P}}.$$

[25] In other words, they choose the same share as in the politico-economic equilibrium and as a Ramsey planner.

Thus, policy makers that commit to a constant tax path will opt for a higher share of public consumption spending in output than in the politico-economic equilibrium without commitment. Intuitively, when voting on government policy today the current young are aware that they decide about the expenditure share for public good provision that benefits them tomorrow. Hence, they choose a higher share of output to be spent on these services than in the politico-economic equilibrium.

The implications of population aging on the voting equilibrium of Proposition 5.4 are summarized in the following corollary.

Corollary 5.5. *An increase in the old-age dependency ratio - either due to a permanent decline in the population growth rate or due to a permanent, but unexpected increase in life expectancy - does not affect τ_g^c, but increases τ_h^c and γ^c. In the case of an increase in life expectancy the latter effects are delayed by one period.*

Thus, we can conclude that qualitatively the effects of population aging on government spending and economic growth in this voting equilibrium are the same as in the politico-economic equilibrium (see Corollary 5.3 and Proposition 5.3).

5.6.1.2 Myopic Voting Equilibrium

This section derives the equilibrium policy mix when agents vote myopically, i. e., when they ignore the effect of the current political decision on future political outcomes, and then compares it to the politico-economic equilibrium of Section 5.3.

More specifically, in a myopic voting equilibrium agents at t treat future policy variables, i. e., g_{t+1}, h_{t+1}, and τ_{t+1}, as given. However, they are aware that their policy choice today affects tomorrow's capital stock and output per worker. Then, the economic equilibrium at t continues to be characterized by equations (5.11) - (5.13). Moreover, using (5.13) we obtain consumption of an agent that is old in period $t+1$ as

$$\begin{aligned} c_{t+1}^o &= (1+n)\alpha\left(1-\tau_{t+1}\right)Ak_{t+1}^{\alpha}g_{t+1}^{1-\alpha} \\ &= (1+n)^{1-\alpha}\alpha A\tilde{B}^{\alpha}\left(Ak_t^{\alpha}g_t^{1-\alpha}-g_t-h_t\right)^a g_{t+1}^{1-\alpha}\left(1-\tau_{t+1}\right). \end{aligned}$$

Omitting all terms independent of policy and those that involve future policy variables (as they are treated as exogenous), the relevant indirect utilities of a young and an old agent at t are

$$U_t^Y \simeq (1+a\beta)\ln\left(Ak_t^{\alpha}g_t^{1-\alpha}-g_t-h_t\right) \tag{5.31}$$

and

$$U_t^O \simeq \ln\left(Ak_t^{\alpha}g_t^{1-\alpha}-g_t-h_t\right)+b\ln h_t, \tag{5.32}$$

respectively. The political candidates at t then choose (g_t, h_t) to maximize the political objective function (5.16) with U_t^Y and U_t^O given by (5.31) and (5.32). The following proposition provides the equilibrium policy mix and the resulting economic growth rate.

Proposition 5.5. *The equilibrium policy mix under myopic voting for all t is given by*

$$g_t = (1-\alpha)y_t \tag{5.33}$$

and

$$h_t = \tau_h^m y_t, \quad \text{where} \quad \tau_h^m = \frac{\alpha\omega b}{(1+n)(1+\alpha\beta)+\omega(1+b)} < \alpha \tag{5.34}$$

and $y_t = A^{1/\alpha}(1-\alpha)^{(1-\alpha)/\alpha}k_t$.

Under this policy mix, the economy's growth factor of all per capita variables, government spending and wages is given by

$$\gamma^m = \frac{B}{1+n}(\alpha-\tau_h^m). \tag{5.35}$$

Proposition 5.5 reveals that policy makers in this voting equilibrium again choose the same share of output to be devoted to the productive public input, namely $1-\alpha$. With respect to the equilibrium share of public consumption spending, we can establish the following corollary.

Corollary 5.6. *Comparing the share of output spent on the public consumption good for the elderly given by (5.34) to the corresponding expenditure share in the politico-economic equilibrium of Proposition 5.1 yields*

$$\tau_h^m > \tau_h^{\mathcal{P}}.$$

Thus, if agents vote myopically the equilibrium share of government expenditure for the public consumption good exceeds the one of the politico-economic equilibrium. The reason for this is that the young agents at t neglect that the choice of h_t via savings and the accumulation of capital affects tomorrow's provision, h_{t+1}. Therefore, they agree to a too high spending level today.

The implications of population aging on the myopic voting equilibrium of Proposition 5.5 are summarized in the following corollary.

Corollary 5.7. *An increase in the old-age dependency ratio - either due to a permanent decline in the population growth rate or due to a permanent, but unexpected increase in life expectancy - does not affect τ_g^m, but increases τ_h^m and γ^m. In the case of an increase in life expectancy the latter effects are delayed by one period.*

Thus, we can conclude that qualitatively the effects of population aging on government spending and economic growth in a myopic voting equilibrium are the same as in the politico-economic equilibrium (see Corollary 5.3 and Proposition 5.3).

5.6.2 Alternative Utility Functions

This section presents several numerical examples to gauge the sensitivity of the comparative static effects of population aging to the specification of the utility function. In particular, we consider two alternative specifications: one with a constant intertemporal elasticity of substitution and the other with non-separable preferences between private and public consumption when old. Both specifications encompass the benchmark separable, log utility function of (5.1) as a special case. Otherwise the economic framework is as described in Section 5.2.

A necessary price of this sensitivity analysis is that (at some point) we must adopt specific parameters for the model. For this purpose, let a period represent 30 years. Then, set $\beta = 0.55$, implying a 2% annual discount rate. The parameter that measures the weight of public relative to private consumption in the utility function is $b = 0.1$. As there is no strong prior on ω, we simply assume equal political weights on the young and the old ($\omega = 1$).[26] Moreover, a parameter value for the output elasticity of productive

[26]However, we have solved for a range of economies with ω different from unity and holding constant the other parameters. The comparative static results are qualitatively unchanged. Moreover and

government expenditure, $1-\alpha$, is needed. As one period represents 30 years, it seems acceptable to suppose that an estimate of the output elasticity of public capital is a good proxy for $1-\alpha$. Glomm and Ravikumar (1997) review the empirical results related to the output elasticity of public capital and find estimates in the range of zero to 0.39. Therefore, we choose $1-\alpha = 0.2$ as an intermediate value. This implies that the elasticity of output with respect to private capital corresponds to $\alpha = 0.8$. This appears reasonable if we consider that private capital encompasses physical as well as human capital.

We start with the assumption that the population growth rate is 2% annually. This annual rate corresponds to growth of 81% over a model period ($n = 1.02^{30} - 1 \simeq 0.81$). This in turn implies an old-age dependency ratio of $(1+n)^{-1} = 0.55$. Note that in a model where agents live for two periods, it is impossible to match the actual population growth rate and the old-age dependency ratio of a country. The above choice reflects this trade-off, with both the population growth rate and the dependency ratio being somewhat higher than currently in Europe or in the US.[27] Then, we investigate the comparative static effect resulting from a shift in the population growth rate from 2.0% to 1.0%. In other words, n declines to 0.35 and the dependency ratio rises to 0.74. Finally, the productivity parameter A is set such that the annual growth rate of per capita variables is 1.8% for the benchmark utility (5.1) when $n = 0.81$.

5.6.2.1 Constant Intertemporal Elasticity of Substitution Utility Function

This section generalizes the analysis to a more general constant intertemporal elasticity of substitution utility function. Assume that the preferences of an individual born at t are described by

$$\frac{\left(c_t^y\right)^{1-\sigma} - 1}{1-\sigma} + \beta \left[\frac{\left(c_{t+1}^o\right)^{1-\sigma} - 1}{1-\sigma} + b \frac{\left(\tilde{h}_{t+1}\right)^{1-\sigma} - 1}{1-\sigma} \right], \tag{5.36}$$

where $\sigma > 0$ and $1/\sigma$ is the intertemporal elasticity of substitution. This specification includes the benchmark log utility for $\sigma \to 1$.

equivalently to the log utility case, the results suggest that $d\tau_h^{\mathcal{P}}/d\omega > 0$.

[27] Introducing a survival probability $v \neq 1$ as in Section 5.5.2 allows - conditional on n - to calibrate the ratio of retirees to workers.

One aim of this generalization is to analyze whether there is a third channel (besides the two discussed in Section 5.5.1) by which a decline in the population growth rate potentially affects the steady-state growth rate. In a standard two-period OLG model under (5.36) with $b = 0$ and a neoclassical production function $Y_t = K_t^{\alpha} L_t^{1-\alpha}$ an increase in the capital stock per worker (e. g. due to decline in n) lowers the rental rate of capital. If the intertemporal elasticity of substitution is different from unity this in turn affects savings, and thus the accumulation of capital. However, in the present framework the interest rate (independent of the utility specification) turns out to be constant in the politico-economic equilibrium. Hence, this third channel is mute and we will see that the qualitative comparative static results are unchanged.

To see this, we first derive the economic equilibrium at t and then define the politico-economic equilibrium. Finally, numerical results for the equilibrium policy mix are presented. We consider the three cases: $\sigma = 0.5$, $\sigma = 1$, and $\sigma = 2$, with the other parameters as described above.

The Economic Equilibrium

Maximizing the lifetime utility of an individual born at t given by (5.36) subject to her per-period budget constraints, and then taking into account the remaining equilibrium conditions of Section 5.2.5, i. e., equations (5.8) - (5.10), yields the equilibrium allocation at t as

$$c_t^y = \frac{(1-\tau_t)\, w_t}{1+\beta^{\frac{1}{\sigma}}\left[(1-\tau_{t+1})\, R_{t+1}\right]^{\frac{1-\sigma}{\sigma}}} = \frac{(1-\alpha)\,(y_t - g_t - h_t)}{1+\beta^{\frac{1}{\sigma}}\left[\frac{\alpha(y_{t+1}-g_{t+1}-h_{t+1})}{k_{t+1}}\right]^{\frac{1-\sigma}{\sigma}}}, \tag{5.37}$$

$$c_t^o = k_t(1+n)\,(1-\tau_t)\, R_t = \alpha\,(1+n)\,(y_t - g_t - h_t)\,, \tag{5.38}$$

$$k_{t+1} = \frac{(1+n)^{-1}(1-\tau_t)\, w_t}{1+\beta^{-\frac{1}{\sigma}}\left[(1-\tau_{t+1})\, R_{t+1}\right]^{\frac{\sigma-1}{\sigma}}} = \frac{(1+n)^{-1}(1-\alpha)\,(y_t - g_t - h_t)}{1+\beta^{-\frac{1}{\sigma}}\left[\frac{\alpha(y_{t+1}-g_{t+1}-h_{t+1})}{k_{t+1}}\right]^{\frac{\sigma-1}{\sigma}}}. \tag{5.39}$$

The Politico-Economic Equilibrium

In a politico-economic equilibrium the public policy rules $\left\langle \pi^g, \pi^h \right\rangle$ have to maximize the political objective function $U_t = (1+n)\, U_t^Y + \omega U_t^O$ with

$$U_t^Y = \frac{(c_t^y)^{1-\sigma}-1}{1-\sigma} + \beta\frac{\left(c_{t+1}^o\right)^{1-\sigma}-1}{1-\sigma} + \beta b\frac{(h_{t+1}(1+n))^{1-\sigma}-1}{1-\sigma}$$

and

$$U_t^O = \frac{(c_t^o)^{1-\sigma}-1}{1-\sigma} + b\frac{(h_t(1+n))^{1-\sigma}-1}{1-\sigma},$$

subject to (5.37) - (5.39).

Making the same policy guess as in Section 5.3.3, i. e., $\pi^g(k_{t+1}) = \eta^g k_{t+1}$ and $\pi^h(k_{t+1}) = \eta^h k_{t+1}$, the economic equilibrium conditions (5.37) - (5.39) yield

$$\begin{aligned} c_t^y &= Y(y_t - g_t - h_t), \quad Y \equiv \frac{1-\alpha}{1+\beta^{\frac{1}{\sigma}}\left[\alpha\left(A(\eta^g)^{1-\alpha}-\eta^g-\eta^h\right)\right]^{\frac{1-\sigma}{\sigma}}}, \\ c_{t+1}^o &= Z(y_t - g_t - h_t), \quad Z \equiv \frac{(1-\alpha)\alpha\left(A(\eta^g)^{1-\alpha}-\eta^g-\eta^h\right)}{1+\beta^{-\frac{1}{\sigma}}\left[\alpha\left(A(\eta^g)^{1-\alpha}-\eta^g-\eta^h\right)\right]^{\frac{\sigma-1}{\sigma}}}, \\ k_{t+1} &= \frac{X(y_t-g_t-h_t)}{1+n}, \quad X \equiv \frac{1-\alpha}{1+\beta^{-\frac{1}{\sigma}}\left[\alpha\left(A(\eta^g)^{1-\alpha}-\eta^g-\eta^h\right)\right]^{\frac{\sigma-1}{\sigma}}}. \end{aligned}$$

Using the latter conditions and omitting additive constant terms, the political objective function simplifies to

$$\begin{aligned} U_t &= \left((1+n)\left(Y^{1-\sigma}+\beta Z^{1-\sigma}+\beta b\left(\eta^h X\right)^{1-\sigma}\right)+\omega(\alpha(1+n))^{1-\sigma}\right)\frac{(y_t-g_t-h_t)^{1-\sigma}}{1-\sigma} \\ &+ \omega b(1+n)^{1-\sigma}\frac{(h_t)^{1-\sigma}}{1-\sigma}, \end{aligned} \tag{5.40}$$

and the equilibrium policy mix (g_t, h_t) has to maximize (5.40). The first-order conditions of this optimization problem with respect to g_t and h_t are

$$\frac{\left((1+n)\left(Y^{1-\sigma}+\beta Z^{1-\sigma}+\beta b\left(\eta^h X\right)^{1-\sigma}\right)+\omega(\alpha(1+n))^{1-\sigma}\right)\left((1-\alpha)\frac{y_t}{g_t}-1\right)}{(y_t-g_t-h_t)^{\sigma}} = 0$$

and

$$\frac{-\left((1+n)\left(Y^{1-\sigma}+\beta Z^{1-\sigma}+\beta b\left(\eta^h X\right)^{1-\sigma}\right)+\omega(\alpha(1+n))^{1-\sigma}\right)}{(y_t-g_t-h_t)^{\sigma}} + \frac{\omega b(1+n)^{1-\sigma}}{(h_t)^{\sigma}} = 0,$$

respectively.

The former condition is fulfilled if and only if $g_t = (1-\alpha)\, y_t$ which verifies our guess for $\eta^g = A^{1/\alpha}\,(1-\alpha)^{1/\alpha}$. If a political equilibrium exists, i. e., if the guess for h_t can also be verified, then the above result implies that the equilibrium interest rate is constant and given by $R = \alpha A^{1/\alpha}(1-\alpha)^{(1-\alpha)/\alpha}$.[28]

Using $g_t = A^{1/\alpha}\,(1-\alpha)^{1/\alpha}\,k_t$ and the guess $h_t = \eta^h k_t$ in the second first-order condition then yields

$$\eta^h = \frac{\alpha D}{1 + \left(\omega b\,(1+n)^{1-\sigma}\right)^{-\frac{1}{\sigma}} \left((1+n)\left(Y^{1-\sigma} + \beta\left(Z^{1-\sigma} + b\left(\eta^h X\right)^{1-\sigma}\right)\right) + \omega\left(\alpha\,(1+n)\right)^{1-\sigma}\right)^{\frac{1}{\sigma}}} \tag{5.41}$$

where $D \equiv A^{1/\alpha}\,(1-\alpha)^{(1-\alpha)/\alpha} = \left(A\,(\eta^g)^{1-\alpha} - \eta^g\right)/\alpha$. For the guess to be correct condition (5.41) needs to have a unique solution for η^h in the interval $(0, \alpha D)$. As this problem cannot be solved analytically, the following section considers numerical examples for $\sigma = 0.5$ and $\sigma = 2$ and compares them to the benchmark case of $\sigma = 1$.

Numerical Results

In this section, we set $A = 34.5$, implying an annual growth rate of per capita variables of 1.8% if $\sigma = 1$. For both choices of σ exists a unique $\eta^h \in (0, \alpha D)$ that solves (5.41). The results are summarized in Table 5.2. Note that $\tau_h^{\mathcal{P}} \equiv \eta^h / D$ denotes the share of public consumption spending that benefits the elderly in aggregate output.[29]

Table 5.2 suggests that an intertemporal elasticity of substitution different from unity does not alter the qualitative comparative static results of Section 5.5.1, i. e., $d\tau_h^{\mathcal{P}}/dn < 0$ and $d\gamma/dn < 0$. Nevertheless, the equilibrium ratio of public spending on the elderly and the equilibrium growth rate depend on σ. The numerical examples reveal that $d\tau_h^{\mathcal{P}}/d\sigma > 0$ and $d\gamma/d\sigma < 0$. Intuitively, a greater intertemporal elasticity of substitution (i. e., a smaller σ) implies a stronger negative substitution effect of a higher tax rate on

[28]For this reason, the savings decision even in the CIES case with $\sigma \neq 1$ is independent of n and it does not matter whether the change in n is anticipated or not.

[29]All examples were computed using Maple. All files are available upon request.

Table 5.2: Comparative Static Analysis of Demographic Change: CIES Utility

σ		$n = 0.81$	$n = 0.35$
0.5	$\tau_h^{\mathcal{P}}$ (public consumption spending / GDP)	0.0009	0.0011
	annual p.c. growth rate	5.16%	6.19%
1	$\tau_h^{\mathcal{P}}$ (public consumption spending / GDP)	0.0200	0.0245
	annual p.c. growth rate	1.80%	2.78%
2	$\tau_h^{\mathcal{P}}$ (public consumption spending / GDP)	0.0475	0.0621
	annual p.c. growth rate	−2.11%	−1.19%

savings such that households prefer a lower tax rate, which in turn involves a higher growth rate.

5.6.2.2 Non-Separable Preferences

This section generalizes the analysis to non-separable preferences between private and public consumption when old. Assume that the preferences of an individual born at t are given by

$$\ln c_t^y + \beta \ln \left(\left[\frac{1}{1+b} \left(c_{t+1}^o\right)^\rho + \frac{b}{1+b} \left(\tilde{h}_{t+1}\right)^\rho \right]^{\frac{1}{\rho}} \right), \tag{5.42}$$

where $\rho < 1$. This specification encompasses the benchmark separable log utility as $\rho \to 0$.[30] Private and public consumption when old are substitutes if $\rho > 0$ and complements if $\rho < 0$. This generalization has interesting implications: for instance, if agents can substitute private for public health services when old they will be less concerned for public good provision and vote for a lower tax rate. Nevertheless, the qualitative comparative static results with respect to population aging will not be affected by this generalization.

[30]Note that for $\rho \to 0$ (5.42) reduces to $\ln c_t^y + \beta/(1+b)\left(\ln c_{t+1}^o + b \ln \tilde{h}_{t+1}\right)$. This specification only differs from the benchmark utility (5.1) by a constant factor which does not affect the qualitative results.

Equivalently to Section 5.6.2.1, we first determine the economic equilibrium and then analyze the politico-economic equilibrium analytically. To analyze the comparative static effects of a decline in the population growth rate we consider three numerical examples for $\rho = -0.1$, $\rho = 0$, and $\rho = 0.1$, with the other parameters as before.

The Economic Equilibrium

Maximizing the lifetime utility (5.42) with respect to an individual's per-period budget constraints delivers the following implicit characterization of optimal savings at t

$$\beta\left(1-\tau_t\right)w_t = s_t\left[1+\beta+b\left(\frac{\tilde{h}_{t+1}}{s_t\left(1-\tau_{t+1}\right)R_{t+1}}\right)^{\rho}\right]. \tag{5.43}$$

Optimal consumption of a young and an old agent at t then follow from the respective per-period budget constraints.

Taking into account the equilibrium conditions (5.8) - (5.10), equation (5.43) becomes

$$\beta\left(1-\alpha\right)\left(y_t-g_t-h_t\right) = k_{t+1}\left(1+n\right)\left[1+\beta+b\left(\frac{h_{t+1}}{\alpha\left(y_{t+1}-g_{t+1}-h_{t+1}\right)}\right)^{\rho}\right]. \tag{5.44}$$

The Politico-Economic Equilibrium

In a politico-economic equilibrium the public policy rules $\left\langle\pi^g,\pi^h\right\rangle$ have to maximize the political objective function $U_t = \left(1+n\right)U_t^Y + \omega U_t^O$ with the indirect utilities of the young and the old at t (disregarding terms independent of policy) given by

$$U_t^Y \simeq \ln c_t^y + \frac{\beta}{\rho}\ln\left[\left(c_{t+1}^o\right)^{\rho} + b\left(h_{t+1}\right)^{\rho}\left(1+n\right)^{\rho}\right]$$

and

$$U_t^O \simeq \frac{1}{\rho}\ln\left[\left(c_t^o\right)^{\rho} + b\left(h_t\right)^{\rho}\left(1+n\right)^{\rho}\right].$$

With the linear policy guess, $\pi^g\left(k_{t+1}\right) = \eta^g k_{t+1}$ and $\pi^h\left(k_{t+1}\right) = \eta^h k_{t+1}$, condition (5.44) can be written as

$$k_{t+1}\left(1+n\right) = \frac{\beta\left(1-\alpha\right)\left(y_t-g_t-h_t\right)}{1+\beta+b\left(\frac{\eta^h}{\alpha\left(A\left(\eta^g\right)^{1-\alpha}-\eta^g-\eta^h\right)}\right)^{\rho}}. \tag{5.45}$$

Moreover, using $s_t = k_{t+1}(1+n)$ and (5.44) in the per-period budget constraints yields consumption of a young and an old agent at t as

$$c_t^y = X\left(y_t - g_t - h_t\right), \text{ where } X \equiv \frac{(1-\alpha)\left(1+b\left(\frac{\eta^h}{\alpha\left(A(\eta^g)^{1-\alpha}-\eta^g-\eta^h\right)}\right)^\rho\right)}{1+\beta+b\left(\frac{\eta^h}{\alpha\left(A(\eta^g)^{1-\alpha}-\eta^g-\eta^h\right)}\right)^\rho} \quad (5.46)$$

and

$$c_t^o = (1+n)\,\alpha\left(y_t - g_t - h_t\right), \quad (5.47)$$

respectively. Additionally, we obtain the levels of private and public consumption of an old agent at $t+1$ as

$$c_{t+1}^o = Y\left(y_t - g_t - h_t\right), \text{ where } Y \equiv \frac{\beta(1-\alpha)\,\alpha\left(A(\eta^g)^{1-\alpha}-\eta^g-\eta^h\right)}{1+\beta+b\left(\frac{\eta^h}{\alpha\left(A(\eta^g)^{1-\alpha}-\eta^g-\eta^h\right)}\right)^\rho} \quad (5.48)$$

and

$$\tilde{h}_{t+1} = Z\left(y_t - g_t - h_t\right), \text{ where } Z \equiv \frac{\eta^h\beta(1-\alpha)}{1+\beta+b\left(\frac{\eta^h}{\alpha\left(A(\eta^g)^{1-\alpha}-\eta^g-\eta^h\right)}\right)^\rho}, \quad (5.49)$$

respectively. Using conditions (5.46) - (5.49) in the indirect utility functions and omitting terms independent of policy variables, the political objective function simplifies to

$$U_t = (1+n)(1+\beta)\ln\left(y_t - g_t - h_t\right) + \frac{\omega}{\rho}\ln\left[\alpha^\rho\left(y_t - g_t - h_t\right)^\rho + b\left(h_t\right)^\rho\right] \quad (5.50)$$

and the equilibrium policy mix (g_t, h_t) has to maximize (5.50). The first-order conditions of this optimization problem with respect to g_t and h_t are

$$\left[(1-\alpha)\frac{y_t}{g_t} - 1\right]\left[\frac{(1+n)(1+\beta)}{y_t - g_t - h_t} + \frac{\omega\alpha^\rho\left(y_t - g_t - h_t\right)^{\rho-1}}{\alpha^\rho\left(y_t - g_t - h_t\right)^\rho + b\left(h_t\right)^\rho}\right] = 0$$

and

$$\frac{-(1+n)(1+\beta)}{y_t - g_t - h_t} + \omega\frac{-\alpha^\rho\left(y_t - g_t - h_t\right)^{\rho-1} + b\left(h_t\right)^{\rho-1}}{\alpha^\rho\left(y_t - g_t - h_t\right)^\rho + b\left(h_t\right)^\rho} = 0,$$

respectively.

The first condition is fulfilled if and only if $g_t = (1-\alpha)\,y_t$ which verifies our guess for $\eta^g = A^{1/\alpha}(1-\alpha)^{1/\alpha}$. If a political equilibrium exists, i. e., if the guess for h_t can also be verified, then the above result again implies that the equilibrium interest rate is constant and given by $R = \alpha D$, where $D \equiv A^{1/\alpha}(1-\alpha)^{(1-\alpha)/\alpha}$.

Then, using $g_t = A^{1/\alpha}(1-\alpha)^{1/\alpha} k_t$ and the guess $h_t = \tau_h^{\mathcal{P}} y_t$, where $\tau_h^{\mathcal{P}} \equiv \eta^h/D$, in the second first-order condition and rearranging yields

$$(1+n)(1+\beta) = \omega \frac{-1+\alpha^{-\rho} b \left(\tau_h^{\mathcal{P}}\right)^{\rho-1} \left(\alpha-\tau_h^{\mathcal{P}}\right)^{1-\rho}}{1+\alpha^{-\rho} b \left(\tau_h^{\mathcal{P}}\right)^{\rho} \left(\alpha-\tau_h^{\mathcal{P}}\right)^{-\rho}}. \tag{5.51}$$

For the guess to be correct, condition (5.51) needs to have a unique solution $\tau_h^{\mathcal{P}}$ in the interval $(0, \alpha)$. This is the case for not too large values of ρ. For a proof of this see the Appendix.

Numerical Results

To analyze the comparative static effect of a decline in the population growth rate, this section considers numerical examples for $\rho = -0.1$ and $\rho = 0.1$ and compares them to the benchmark case of $\rho = 0$.

In the examples of this section, we set $A = 36.198$ in order to again obtain an annual growth rate of per capita variables of 1.80% if $\rho = 0$. For both choices of ρ, there exists a unique $\tau_h^{\mathcal{P}} \in (0, \alpha)$ that solves (5.51). The results are summarized in Table 5.3.

Table 5.3: Comparative Static Analysis of Demographic Change: Non-Separable Preferences

ρ		$n = 0.81$	$n = 0.35$
-0.1	$\tau_h^{\mathcal{P}}$ (public consumption spending / GDP)	0.0254	0.0308
	annual p.c. growth rate	1.70%	2.68%
0	$\tau_h^{\mathcal{P}}$ (public consumption spending / GDP)	0.0191	0.0235
	annual p.c. growth rate	1.80%	2.78%
0.1	$\tau_h^{\mathcal{P}}$ (public consumption spending / GDP)	0.0134	0.0168
	annual p.c. growth rate	1.89%	2.87%

Table 5.3 suggests that allowing for non-separable preferences between private and public consumption when old does not change the qualitative comparative static results of Section 5.5.1, i. e., $d\tau_h^{\mathcal{P}}/dn < 0$ and $d\gamma/dn < 0$.

Moreover, the numerical examples reveal that the equilibrium ratio of public spending on the elderly declines in ρ, i. e., $d\tau_h^{\mathcal{P}}/d\rho < 0$. Intuitively, a higher degree of substitutability between private and public consumption goods makes the old less concerned for the public consumption good and induces them to vote for a lower spending ratio.

5.7 Concluding Remarks

What is the role of population aging for the composition of government spending and long-term economic growth? This chapter addressed this question in an overlapping generations model in which economic growth is endogenous and agents each period vote on the composition of government spending between productive public expenditure and public consumption spending on the elderly. Population aging corresponds either to a decline in the population growth rate or to an increase in life expectancy. Both phenomena increase the economy's old-age dependency ratio.

The model predicts that population aging, by increasing the relative weight of the old in the political process, leads to an increase in public spending on the elderly (as a share of output), but does not affect the share of public productive expenditure in output. This is in line with recent evidence (see Section 5.1). To finance the additional government spending, the income tax rate has to increase, which in turn has a negative effect on the economy's growth rate of per capita variables. However, the model also suggests that population aging overall accelerates the economy's growth rate. If the increase in the old-age dependency ratio is due to a decline in the population growth rate, then reduced capital dilution is at the source of this acceleration of growth. By contrast, an increase in life expectancy generates higher long-term growth by strengthening the incentives to save.

The present analysis leaves scope for future research. For instance, for analytical tractability this chapter introduced the productive public input as a flow into production. Considering that the length of a model period corresponds to one generation, this appears to be a good benchmark. Alternatively, one could treat the publicly-provided productive input as a stock rather than as a flow, thereby introducing public as well as private capital. In this case the advantages of a larger public investment today only materialize tomorrow whereas the tax costs have to be borne today. Then, the young and the old are no longer symmetrically affected by current public productive spending. Additionally, the stock approach introduces transitional equilibrium dynamics into the analysis. This would allow us to study the effects of the projected demographic transition not only on the steady state but also on the dynamics of transition between steady states. A second suggestion for future research is to disentangle the uniform income tax rate into a separate labor and capital income tax rate. This introduces another dimension of policy choice and a further source of potential conflict between the young and the old. It would be interesting to see whether in this case the public consumption good that benefits the elderly will be entirely financed via capital income taxes and the productive public input via both types of taxes.

5.8 Appendix

Detailed Derivation of Condition (5.18)

The first-order conditions of the program (5.17) with respect to g_t and h_t are

$$\bar{U}_{g_t} = \frac{(1-\alpha)\,y_t - g_t}{g_t\,(y_t - g_t - h_t)}\left[(1+n)\,(1+\beta\,(1+b)) + \omega\right] = 0 \tag{5.52}$$

$$\bar{U}_{h_t} = -\frac{(1+n)\,(1+\beta\,(1+b)) + \omega}{y_t - g_t - h_t} + \frac{\omega b}{h_t} = 0, \tag{5.53}$$

where $\bar{U}_x \equiv \partial\bar{U}/\partial x$. The first condition is fulfilled if and only if $g_t = (1-\alpha)y_t$. Using this in the second condition and rearranging yields

$$\left[(1+n)\,(1+\beta\,(1+b)) + \omega\right] h_t = \omega b\,(\alpha y_t - h_t)\,, \tag{5.54}$$

and thus

$$h_t = \underbrace{\frac{\alpha\omega b}{(1+n)\,(1+\beta\,(1+b)) + \omega\,(1+b)}}_{\equiv \tau_h^{\mathcal{P}}} y_t. \tag{5.55}$$

Thus, the unique interior solution is given by $g_t = (1-\alpha)y_t$ and $h_t = \tau_h^{\mathcal{P}} y_t$ as stated in the main text.

Proof of Proposition 5.1

In addition to what is stated in the text, it remains to be verified that (i) the first-order conditions are sufficient for a global maximum, (ii) the economy's relevant variables grow at the rate $\gamma - 1$.

(i) The unique interior solution derived above is a global maximum if

$$\bar{U}_{g_t g_t} < 0, \bar{U}_{h_t h_t} < 0 \text{ and } \bar{U}_{g_t g_t}\bar{U}_{h_t h_t} - \left(\bar{U}_{g_t h_t}\right)^2 > 0, \quad \text{for any} \quad (g_t, h_t),$$

where $\bar{U}_{xy} \equiv \partial^2\bar{U}/\partial x\partial y$. First, note that

$$\begin{aligned} \bar{U}_{g_t g_t} &= -\left[(1+n)\,(1+\beta\,(1+b)) + \omega\right] \frac{\alpha\,(1-\alpha)\,y_t\,(y_t - g_t - h_t) + \left[(1-\alpha)\,y_t - g_t\right]^2}{g_t^2\,(y_t - g_t - h_t)^2} \\ &< 0 \\ \bar{U}_{h_t h_t} &= \frac{-\left[(1+n)\,(1+\beta\,(1+b)) + \omega\right]}{(y_t - g_t - h_t)^2} - \frac{\omega b}{(h_t)^2} < 0. \end{aligned}$$

Then, $\bar{U}_{g_t g_t}\bar{U}_{h_t h_t}$ can be written as

$$\bar{U}_{g_t g_t}\bar{U}_{h_t h_t} = \left[(1+n)(1+\beta(1+b))+\omega\right]^2 \frac{\left[(1-\alpha)\frac{y_t}{g_t}-1\right]^2}{(y_t-g_t-h_t)^4} + X + Y + Z,$$

where X, Y, and Z are positive constants. Moreover,

$$\bar{U}_{g_t h_t} = -\left[(1+n)(1+\beta(1+b))+\omega\right] \frac{\left[(1-\alpha)\frac{y_t}{g_t}-1\right]}{(y_t-g_t-h_t)^2},$$

and thus

$$\left(\bar{U}_{g_t h_t}\right)^2 = \left[(1+n)(1+\beta(1+b))+\omega\right]^2 \frac{\left[(1-\alpha)\frac{y_t}{g_t}-1\right]^2}{(y_t-g_t-h_t)^4}$$

such that

$$\bar{U}_{g_t g_t}\bar{U}_{h_t h_t} - \left(\bar{U}_{g_t h_t}\right)^2 = X + Y + Z > 0, \quad \text{for any} \quad (g_t, h_t).$$

□

(ii) First, it is straightforward that, as in the standard AK model, there are no transitional dynamics such that the economy immediately jumps onto its steady-state path. Moreover, note that output per worker in equilibrium is linear in the capital stock per worker k, and thus has to grow at the same rate as k, namely at rate $\gamma - 1$.

Then, output per capita at t is given by

$$\frac{Y_t}{L_t + L_{t-1}} = \frac{y_t L_t}{L_t + L_{t-1}} = \frac{y_t}{1 + L_{t-1}/L_t} = \frac{1+n}{2+n} y_t,$$

and is thus proportional to output per worker and has to grow at the same rate. Using (5.11) and (5.12), consumption per capita at t obtains as

$$\begin{aligned}\frac{C_t}{L_t + L_{t-1}} &= \frac{c_t^y L_t + c_t^o L_{t-1}}{L_t + L_{t-1}} = \frac{L_t}{L_t + L_{t-1}}\left(c_t^y + \frac{c_t^o}{1+n}\right) \\ &= \frac{1+n}{2+n}\frac{1+\alpha\beta}{1+\beta}(1-\tau)y_t,\end{aligned}$$

and is also proportional to output per worker. Similar arguments apply to all other relevant variables such as government spending and wages. □

Proof of Corollary 5.1

Assume that the economic environment is identical to that of the previous sections, except in a final period T where there is a generation of newborns that lives only for one period. The consumption of old and young households in this period are given by

$$c_T^y = (1-\tau_T)\, w_T = (1-\alpha)\left(Ak_T^\alpha g_T^{1-\alpha} - g_T - h_T\right), \tag{5.56}$$

$$c_T^o = (1-\tau_T)\, R_T s_{T-1} = \alpha(1+n)\left(Ak_T^\alpha g_T^{1-\alpha} - g_T - h_T\right), \tag{5.57}$$

respectively. The policymaker (voters) then chooses g_T and h_T to maximize the political objective function $U_T = (1+n)\, U_T^Y + \omega U_T^O = (1+n)\ln c_T^y + \omega \ln c_T^o + \omega b \ln h_T$. Omitting terms independent of the policy choices g_T and h_T, the political objective function reduces to

$$U_T \simeq (1+n+\omega)\ln\left(Ak_T^\alpha g_T^{1-\alpha} - g_T - h_T\right) + \omega b \ln h_T. \tag{5.58}$$

The first-order conditions of maximizing (5.58) with respect to g_T and h_T yield

$$g_T = (1-\alpha) y_T \quad \text{and} \quad h_T = \frac{\omega b \alpha}{1+n+\omega(1+b)} y_T, \qquad \text{with} \quad y_T = A(1-\alpha)^{1-\alpha} k_T. \tag{5.59}$$

Now we can proceed by backward induction. In period $T-1$ voters choose g_{T-1} and h_{T-1}, correctly anticipating g_T and h_T , to maximize

$$\begin{aligned} U_{T-1} &= (1+n)\, U_{T-1}^Y + \omega U_{T-1}^O \\ &= (1+n)\ln c_{T-1}^y + \beta(1+n)\left[\ln c_T^o + b\ln h_T\right] + \omega\left[\ln c_{T-1}^o + b \ln h_{T-1}\right] \end{aligned}$$

with c_T^o given by (5.57), c_{T-1}^y follows from (5.11) for $t = T-1$ and c_{T-1}^o from (5.12) for $t = T-1$. Using g_T and h_T of (5.59) as well as k_T of (5.13) for $t = T-1$ and omitting terms independent of policy variables the political objective function at $T-1$ can be written as

$$U_{T-1} \simeq \left[(1+n)(1+\beta(1+b)) + \omega\right] \ln\left(Ak_{T-1}^\alpha g_{T-1}^{1-\alpha} - g_{T-1} - h_{T-1}\right) + \omega b \ln h_{T-1}. \tag{5.60}$$

After some algebra, the first-order conditions of maximizing (5.60) with respect to g_{T-1} and h_{T-1} yield

$$g_{T-1} = (1-\alpha) y_{T-1} \tag{5.61}$$

and

$$h_{T-1} = \frac{\alpha\omega b}{(1+n)(1+\beta(1+b)) + \omega(1+b)} y_{T-1} = \tau_h^{\mathcal{P}} y_{T-1}, \tag{5.62}$$

where $y_{T-1} = A(1-\alpha)^{1-\alpha}k_{T-1}$. The policy functions (5.61) and (5.62) correspond to the equilibrium policy functions of the infinite-horizon economy (see equation 5.18). Proceeding in the same way for all preceding periods one readily verifies that this equilibrium policy mix results for all periods $t = 0, 1, ..., T-1$. □

Proof of Lemma 5.1

First note that the indirect utility of a young agent of generation t given by (5.14) is additively separable in (h_t, g_t, k_t) and $(h_{t+1}, g_{t+1}, k_{t+1})$, i. e.,

$$U_t^Y\left(h_t, g_t, k_t, h_{t+1}, g_{t+1}, k_{t+1}\right) = P_t(g_t, h_t, k_t) + Q_{t+1}(g_{t+1}, h_{t+1}, k_{t+1}),$$

where

$$\begin{aligned} P_t(g_t, h_t, k_t) &\equiv \ln\left(Ak_t^{\alpha}g_t^{1-\alpha} - g_t - h_t\right) \quad \text{and} \\ Q_{t+1}(g_{t+1}, h_{t+1}, k_{t+1}) &\equiv \beta\ln\left(Ak_{t+1}^{\alpha}g_{t+1}^{1-\alpha} - g_{t+1} - h_{t+1}\right) + \beta b\ln h_{t+1}. \end{aligned}$$

Then, the Ramsey planner's objective function in (5.21) can be expressed as

$$\begin{aligned} &\max_{\{g_t,h_t,k_{t+1}\}_{t=0}^{\infty}} W(\cdot) \equiv \max_{\{g_t,h_t,k_{t+1}\}_{t=0}^{\infty}} \left\{\beta U_0^O + \sum_{t=0}^{\infty}\left((1+n)\beta\right)^{t+1}U_t^Y\right\} \\ &= \max_{g_0,h_0,k_1}\left\{\beta U_0^O + \max_{\{g_t,h_t,k_{t+1}\}_{t=1}^{\infty}} \sum_{t=0}^{\infty}\left((1+n)\beta\right)^{t+1}U_t^Y\right\} \\ &= \max_{g_0,h_0,k_1}\left\{\beta U_0^O + \max_{g_1,h_1,k_2}\left\{(1+n)\beta U_0^Y + \max_{\{g_t,h_t,k_{t+1}\}_{t=2}^{\infty}}\sum_{t=1}^{\infty}\left((1+n)\beta\right)^{t+1}U_t^Y\right\}\right\} \\ &= \max_{g_0,h_0,k_1}\left\{\beta U_0^O + \max_{g_1,h_1,k_2}\left\{(1+n)\beta\left[P_0(\cdot)+Q_1(\cdot)\right] + \max_{\{\cdot\}_{t=2}^{\infty}}\sum_{t=1}^{\infty}\left((1+n)\beta\right)^{t+1}U_t^Y\right\}\right\} \\ &= \max_{g_0,h_0.k_1}\left\{\beta\left(U_0^O + (1+n)P_0(\cdot)\right) + \max_{\{\cdot\}_{t=1}^{\infty}}\sum_{t=1}^{\infty}(1+n)^t\beta^t\left[Q_t(\cdot) + (1+n)\beta P_t(\cdot)\right]\right\}, \end{aligned} \tag{5.63}$$

where the argument of $\{\cdot\}$ is g_t, h_t, k_{t+1}.

Now let $T_t\left(g_t, h_t, k_t\right) \equiv Q_t\left(g_t, h_t, k_t\right) + (1+n)\beta P_t\left(g_t, h_t, k_t\right)$ and note that from (5.15) and the definition of P_t follows $\beta U_0^O + (1+n)\beta P_0(\cdot) = \beta Q_0(\cdot) + (1+n)\beta P_0(\cdot) = T_0(\cdot)$

such that (5.63) can be written as

$$\max_{\{g_t,h_t,k_{t+1}\}_{t=0}^{\infty}} W(\cdot) = \max_{g_0,h_0,k_1} \left\{ T_0\left(g_0,h_0,k_0\right) + \max_{\{g_t,h_t,k_{t+1}\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} (1+n)^t \beta^t T_t\left(g_t,h_t,k_t\right) \right\}. \tag{5.64}$$

Defining the value function

$$V\left(k_t\right) \equiv \max_{\{g_{t+s},h_{t+s},k_{t+1+s}\}_{s=0}^{\infty}} \sum_{s=0}^{\infty} (1+n)^s \beta^s T_{t+s}\left(g_{t+s},h_{t+s},k_{t+s}\right),$$

standard recursion on (5.64) yields the functional Bellman equation (5.22). □

Proof of Proposition 5.2

In order to solve the Ramsey problem, we start by guessing that the solution to the functional equation (5.22) takes the form of $V(k) = a_0 + a_1 \ln k$ for all k, where a_0 and a_1 are yet undetermined coefficients. Then, the Bellman equation becomes

$$\begin{aligned} a_0 + a_1 \ln k_t = \quad & \max\nolimits_{\{g_t,h_t,k_{t+1}\}} \quad \left\{(2+n)\beta \ln\left(A k_t^{\alpha} g_t^{1-\alpha} - g_t - h_t\right) + \beta b \ln h_t \right. \\ & \left. +(1+n)\beta a_0 + (1+n)\beta a_1 \ln k_{t+1}\right\} \end{aligned}$$

subject to (5.13). Substituting for k_{t+1}, the Bellman equation reduces to

$$\begin{aligned} a_0 + a_1 \ln k_t = \quad & \max\nolimits_{\{g_t,h_t\}} \quad \left\{\beta\left(1+(1+n)\left(1+a_1\right)\right) \ln\left(A k_t^{\alpha} g_t^{1-\alpha} - g_t - h_t\right) \right. \\ & \left. +\beta b \ln h_t + (1+n)\beta a_0 + (1+n)\beta a_1 \ln \tilde{B}\right\}. \end{aligned} \tag{5.65}$$

After some algebra, the first-order conditions with respect to g_t and h_t yield

$$g_t = (1-\alpha) y_t \quad \text{and} \quad h_t = \frac{b\alpha}{1+b+(1+n)\left(1+a_1\right)} y_t, \tag{5.66}$$

with $y_t = A^{1/\alpha}(1-\alpha)^{(1-\alpha)/\alpha}$. Using this in (5.65) and collecting the terms that multiply $\ln k_t$ results in

$$\begin{aligned} a_0 + a_1 \ln k_t \quad = \quad & \beta\left(1+b+(1+n)\left(1+a_1\right)\right) \ln k_t + (1+n)\beta a_0 + (1+n)\beta a_1 \ln \tilde{B} \\ & +\beta\left(1+(1+n)\left(1+a_1\right)\right) \ln\left[\frac{1+(1+n)\left(1+a_1\right)}{1+b+(1+n)\left(1+a_1\right)} \alpha A^{\frac{1}{\alpha}} (1-\alpha)^{\frac{1-\alpha}{\alpha}}\right] \\ & +\beta b \ln\left[\frac{b\alpha}{\left(1+b+(1+n)\left(1+a_1\right)\right)} A^{\frac{1}{\alpha}} (1-\alpha)^{\frac{1-\alpha}{\alpha}}\right]. \end{aligned}$$

The functional equation holds for all k if and only if $a_1 = \beta\left(1+b+(1+n)\left(1+a_1\right)\right)$. This in turn implies that

$$a_1 \quad = \quad \frac{\beta\left(2+b+n\right)}{1-\beta(1+n)}$$

is required for a solution. This expression can then be used to solve for a_0. Thus, it has been verified that the tentative guess is indeed a solution to the functional equation. Substitution of a_1 in (5.66) then yields $\tau_h^{\mathcal{R}}$ of Proposition 5.2. □

Proof of Corollary 5.2

The result follows directly from comparing $\tau_h^{\mathcal{R}}$ of Proposition 5.2 to $\tau_h^{\mathcal{P}}$ of Proposition 5.1. □

Proof of Corollary 5.3

1. From Proposition 5.1 we have

$$\tau_h^{\mathcal{P}} = \frac{\alpha\omega b}{(1+n)\left[1+\beta\left(1+b\right)\right]+\omega\left(1+b\right)}. \tag{5.67}$$

   Partial derivation of (5.67) with respect to n immediately yields $d\tau_h^{\mathcal{P}}/dn < 0$. Moreover, $\tau_g = 1-\alpha$ such that $d\tau_g/dn = 0$. The comparative static result for τ immediately follows from the definition of τ and from the first two results.

2. Using (5.67) in (5.20) yields the equilibrium growth rate as

$$\gamma = \alpha B \frac{\left[1+\beta\left(1+b\right)\right]+\omega/\left(1+n\right)}{(1+n)\left[1+\beta\left(1+b\right)\right]+\omega\left(1+b\right)}.$$

   Then, partial derivation immediately yields $d\gamma/dn < 0$. □

Proof of Proposition 5.3

In the presence of a perfect annuity market, an individual born at t chooses the plan (c_t^y, c_{t+1}^o, s_t) to maximize her lifetime utility (5.1) subject to $c_t^y + s_t = (1-\tau_t)w_t$ and $c_{t+1}^o = s_t(1-\tau_{t+1})R_{t+1}/v$. Writing the problem like this uses the fact that the assets at $t+1$ of a member of generation t are equal to $s_t + (1-v)s_t/v = s_t/v$. Moreover, it incorporates the results of Yaari (1965) and Sheshinski and Weiss (1981) according to which individuals without a bequest motive want to annuitize all their wealth. The optimal choices of a member of cohort t are given by (5.5), (5.7), and
$c_{t+1}^o = \beta(1-\tau_t)w_t(1-\tau_{t+1})R_{t+1}/v(1+\beta)$ with $\beta \equiv \beta_v v$. Then, one readily verifies that all other equations in Sections 5.2 and 5.3 remain valid.[31]

As an increase in the survival probability v raises the effective discount factor β, equation (5.7) yields $\partial s_t/\partial v > 0$. Thus, for a given government policy, an increase in v raises savings per worker. This, in turn has a positive effect on the growth rate of capital per worker, see equation (5.20). However, we assume that the increase in life expectancy is unexpected for generation 1 such that it makes its plan (c_1^y, s_1, c_2^o) without anticipating the increase of the survival probability from v to $\hat{v}$. Hence, the positive growth effect only materializes from generation 2 onwards.

The second effect of an increase in life expectancy is that the effective weight of the old, $\omega = \omega_v v$, in the political objective function (5.16) increases. However, this effect only becomes effective from period 2 onwards too. In period $t = 1$, the young of generation 1 and the old of generation 0, whose size is determined by the initial life expectancy v, vote on government policy. Thus, in $t = 1$ government policy is unaffected by an increase in the survival probability. This in turn implies that the accumulation rule that determines the capital stock per worker in period 2 is unchanged. Thus, $\hat{\tau}_{h_1}^{\mathcal{P}}$ and $\hat{\gamma}_2$ correspond to $\tau_h^{\mathcal{P}}$ and γ of Proposition 5.1 with $\omega = \omega_v v$.

By contrast, from period 2 onwards the relevant effective discount factor and the effective political weight are $\hat{\beta} \equiv \beta_v \hat{v}$ and $\hat{\omega} \equiv \omega_v \hat{v}$. Then, the politico-economic equilibrium for

[31] The only exception is equation (5.12) that modifies to $c_t^o = \alpha(1+n)\left(Ak_t^\alpha g_t^{1-\alpha} - g_t - h_t\right)/v$.

any $t = 2, 3, ..., \infty$ is characterized by

$$\hat{\tau}_{g_t} = 1 - \alpha \equiv \hat{\tau}_g, \tag{5.68}$$

$$\hat{\tau}^{\mathcal{P}}_{h_t} = \frac{\alpha \omega_v \hat{v} b}{(1+n)\left[1 + \hat{v}\beta_v (1+b)\right] + \omega_v \hat{v} (1+b)} \equiv \hat{\tau}^{\mathcal{P}}_h, \tag{5.69}$$

$$\hat{\gamma}_{t+1} = \frac{\beta_v \hat{v}}{1 + \beta_v \hat{v}} \left(\alpha - \hat{\tau}^{\mathcal{P}}_h\right) X \equiv \hat{\gamma} \tag{5.70}$$

where $X \equiv A^{1/\alpha} (1-\alpha)^{1/\alpha} / (1+n)$.

Partial derivation of (5.69) with respect to $\hat{v}$ gives

$$\begin{aligned} \frac{d\hat{\tau}^{\mathcal{P}}_h}{d\hat{v}} &= \frac{\alpha \omega_v b \left[(1+n)(1 + \hat{v}\beta_v (1+b)) + \omega_v \hat{v} (1+b) - (1+n)\hat{v}\beta_v (1+b) - \omega_v \hat{v} (1+b)\right]}{\left[(1+n)\left[1 + \hat{v}\beta_v (1+b)\right] + \omega_v \hat{v} (1+b)\right]^2} \\ &= \frac{\alpha \omega_v b (1+n)}{\left[(1+n)\left[1 + \hat{\beta}(1+b)\right] + \hat{\omega}(1+b)\right]^2} > 0. \end{aligned} \tag{5.71}$$

Moreover,

$$\frac{\partial \hat{\gamma}}{\partial \hat{v}} = \frac{\beta_v}{\left(1+\hat{\beta}\right)^2} \left(\alpha - \hat{\tau}^{\mathcal{P}}_h\right) X - \frac{\hat{\beta}}{1+\hat{\beta}} \frac{d\hat{\tau}^{\mathcal{P}}_h}{d\hat{v}} X = \frac{X\beta_v}{1+\hat{\beta}} \left(-\hat{v}\frac{d\hat{\tau}^{\mathcal{P}}_h}{d\hat{v}} + \frac{\alpha - \hat{\tau}^{\mathcal{P}}_h}{1+\hat{\beta}}\right). \tag{5.72}$$

Using (5.71) in (5.72) and rearranging yields

$$\begin{aligned} \frac{\partial \hat{\gamma}}{\partial \hat{v}} &= \frac{X\beta_v \alpha}{1+\hat{\beta}} \frac{(1+n)^2 \left(1 + \hat{\beta}(1+b)\right)^2 + \hat{\omega}^2 (1+b) + \hat{\omega}(1+n)\left(2\left(1 + \hat{\beta}(1+b)\right) + \hat{\beta} b^2\right)}{\left[(1+n)\left(1 + \hat{\beta}(1+b)\right) + \hat{\omega}(1+b)\right]^2} \\ &> 0. \end{aligned}$$

Thus, $\hat{\tau}^{\mathcal{P}}_h > \tau^{\mathcal{P}}_h$ and $\hat{\gamma} > \gamma$ for all $t = 2, 3, ..., \infty$. □

Proof of Proposition 5.4

Substituting (5.27) and (5.28) in the political objective function (5.16) yields the program to be solved by the political mechanism as $\max_{\{\tau^c_g, \tau^c_h\}} \bar{U}_t$ with

$$\begin{aligned} \bar{U}_t &= \left[(1+n)(1 + \beta(2+b)) + \omega\right] \ln\left(1 - \tau^c_g - \tau^c_h\right) + \left[(1+n)\beta + \omega\right] b \ln \tau^c_h \\ &+ \left[(1+n)(1 + 2\beta(1+b)) + \omega(1+b)\right](1-\alpha)/\alpha \ln \tau^c_g. \end{aligned}$$

The first-order conditions of the above program with respect to τ_g^c and τ_h^c yield

$$\frac{(1+n)(1+\beta(2+b))+\omega}{1-\tau_g^c-\tau_h^c} = \frac{(1-\alpha)[(1+n)(1+2\beta(1+b))+\omega(1+b)]}{\alpha\tau_g^c} \tag{5.73}$$

and

$$\frac{(1+n)(1+\beta(2+b))+\omega}{1-\tau_g^c-\tau_h^c} = \frac{(1+n)\beta b+\omega b}{\tau_h^c}. \tag{5.74}$$

Combining (5.73) and (5.74) yields

$$\tau_g^c = \frac{(1-\alpha)[(1+n)(1+2\beta(1+b))+\omega(1+b)]}{[(1+n)\beta+\omega]\alpha b}\tau_h^c. \tag{5.75}$$

Substituting (5.75) in (5.74) and solving for τ_h^c yields

$$\tau_h^c = \frac{((1+n)\beta+\omega)\alpha b}{(1+n)(1+2\beta(1+b))+\omega(1+b)} < \alpha,$$

which is τ_h^c of (5.29). Finally, using (5.29) in (5.75) yields $\tau_g^c = 1-\alpha$.

The policy mix of (5.29) is the global maximizer of the political objective function. To see this note that

$$\bar{U}_{\tau_g^c\tau_g^c} = -\frac{(1+n)(1+\beta(2+b))+\omega}{\left(1-\tau_g^c-\tau_h^c\right)^2} - \frac{(1-\alpha)[(1+n)(1+2\beta(1+b))+\omega(1+b)]}{\alpha\left(\tau_g^c\right)^2} < 0$$

$$\bar{U}_{\tau_h^c\tau_h^c} = -\frac{(1+n)(1+\beta(2+b))+\omega}{\left(1-\tau_g^c-\tau_h^c\right)^2} - \frac{[(1+n)\beta+\omega]b}{\left(\tau_h^c\right)^2} < 0$$

$$\left(\bar{U}_{\tau_g^c\tau_h^c}\right)^2 = \frac{[(1+n)(1+\beta(2+b))+\omega]^2}{\left(1-\tau_g^c-\tau_h^c\right)^4}$$

$$\bar{U}_{\tau_g^c\tau_g^c}\bar{U}_{\tau_h^c\tau_h^c} = \frac{[(1+n)(1+\beta(2+b))+\omega]^2}{\left(1-\tau_g^c-\tau_h^c\right)^4} + X + Y + Z,$$

where X, Y, and Z are positive constants. Then,

$$\bar{U}_{\tau_g^c\tau_g^c}\bar{U}_{\tau_h^c\tau_h^c} - \left(\bar{U}_{\tau_g^c\tau_h^c}\right)^2 = X+Y+Z > 0, \quad \text{for any} \quad (\tau_g^c, \tau_h^c).$$

□

Proof of Corollary 5.4

Proof by contradiction. Suppose that $\tau_h^c \leq \tau_h^{\mathcal{P}}$, then

$$
\begin{aligned}
& \frac{((1+n)\beta+\omega)\,\alpha b}{(1+n)\,(1+2\beta\,(1+b))+\omega\,(1+b)} \leq \frac{\alpha b\omega}{(1+n)\,(1+\beta\,(1+b))+\omega\,(1+b)} \\
\Leftrightarrow\ & [(1+n)\beta+\omega]\,(1+n)\,(1+\beta\,(1+b))+(1+n)\beta\omega\,(1+b) \\
& \leq (1+n)\,(1+2\beta\,(1+b))\,\omega \\
\Leftrightarrow\ & (1+n)\,[-\beta\,(1+b)\,\omega+(1+n)\beta\,(1+\beta\,(1+b))+\beta\omega\,(1+b)] \leq 0 \\
\Leftrightarrow\ & (1+n)\beta\,(1+\beta\,(1+b)) \leq 0,
\end{aligned}
$$

which is a contradiction. Thus, it has to hold that $\tau_h^c > \tau_h^{\mathcal{P}}$.

□

Proof of Corollary 5.5

1. Comparative statics for a change in n

   Partial derivation of each of the expenditures shares of (5.29) with respect to n yields

$$
\begin{aligned}
\frac{\partial \tau_g^c}{\partial n} &= 0 \\
\frac{\partial \tau_h^c}{\partial n} &= \frac{-\alpha b\omega\,[1+\beta\,(1+b)]}{[(1+n)\,(1+2\beta\,(1+b))+\omega\,(1+b)]^2} < 0.
\end{aligned}
$$

   Using τ_h^c in (5.30) we obtain the equilibrium growth factor as

$$
\gamma^c = \alpha B \frac{[1+\beta\,(2+b)]+\omega/\,(1+n)}{(1+n)\,[1+2\beta\,(1+b)]+\omega\,(1+b)}.
$$

   Then, partial derivation immediately yields $d\gamma^c/dn < 0$.

2. Comparative statics for an increase in life expectancy

   Consider the reinterpretation of the economic framework as described in Section 5.5.2. Then, τ_h^c and γ^c can be rewritten as

$$\tau_h^c = \frac{((1+n)\beta_v + \omega_v)\,\alpha b}{(1+n)(1/v + 2\beta_v(1+b)) + \omega_v(1+b)}$$

and

$$\gamma^c = \frac{\beta_v v X}{1+\beta_v}\left(\alpha - \tau_h^c\right),$$

where $X \equiv A^{1/\alpha}(1-\alpha)^{1/\alpha}/(1+n)$. Then, a permanent increase in the survival probability v has the following effects on government policy and economic growth:

$$\frac{\partial \tau_g^c}{\partial v} = 0$$

$$\frac{\partial \tau_h^c}{\partial v} > 0$$

$$\frac{\partial \gamma^c}{\partial v} = \alpha\beta_v X \frac{\beta_v(1+n)^2\left[(2+b)(1+2v) + (1+b)(3+b)\omega + 2\beta(1+b)^2\right]}{(1+\hat{\beta})^2\left[\frac{1+n}{v} + 2\beta_v(1+n)(1+b) + \omega(1+b)\right]^2}$$
$$+\frac{(1+n)\left[2\omega_v/v + \beta_v\omega_v + (1+n)/v^2\right] + (1+b)\omega_v^2}{(1+\hat{\beta})^2\left[\frac{1+n}{v} + 2\beta_v(1+n)(1+b) + \omega_v(1+b)\right]^2} > 0.$$

However, as the increase in the life expectancy is unexpected these effects only materialize with a period delay; for an intuition see the proof of Proposition 5.3.

□

Proof of Proposition 5.5

Substituting (5.31) and (5.32) in the political objective function (5.16) yields the program to be solved by the political mechanism as

$$\max_{\{g_t,h_t\}} \bar{U} \quad \text{with} \quad \bar{U} \equiv \left[(1+n)(1+\alpha\beta) + \omega\right]\ln\left(Ak_t^\alpha g_t^{1-\alpha} - g_t - h_t\right) + \omega b \ln h_t.$$

The first-order conditions of the above program with respect to g_t and h_t are

$$\bar{U}_{g_t} = \frac{(1-\alpha)y_t - g_t}{g_t(y_t - g_t - h_t)}\left[(1+n)(1+\alpha\beta) + \omega\right] = 0$$

$$\bar{U}_{h_t} = -\frac{(1+n)(1+\alpha\beta) + \omega}{y_t - g_t - h_t} + \frac{\omega b}{h_t} = 0.$$

The first condition is fulfilled if and only if $g_t = (1-\alpha)y_t$. Using this in the second condition and rearranging immediately yields τ_h^m of (5.34).

The policy mix of Proposition 5.5 is the global maximizer of the political objective function. To see this note that

$$\begin{aligned}\bar{U}_{g_t g_t} &= -\left[(1+n)(1+\alpha\beta)+\omega\right]\frac{\alpha(1-\alpha)y_t(y_t-g_t-h_t)+\left[(1-\alpha)y_t-g_t\right]^2}{g_t^2(y_t-g_t-h_t)^2}<0\\ \bar{U}_{h_t h_t} &= \frac{-\left[(1+n)(1+\alpha\beta)+\omega\right]}{(y_t-g_t-h_t)^2}-\frac{\omega b}{(h_t)^2}<0.\end{aligned}$$

Then, $\bar{U}_{g_t g_t}\bar{U}_{h_t h_t}$ can be written as

$$\bar{U}_{g_t g_t}\bar{U}_{h_t h_t} = \left[(1+n)(1+\alpha\beta)+\omega\right]^2\frac{\left[(1-\alpha)\frac{y_t}{g_t}-1\right]^2}{(y_t-g_t-h_t)^4}+X+Y+Z,$$

where X, Y, and Z are positive constants. Moreover,

$$\bar{U}_{g_t h_t} = -\left[(1+n)(1+\alpha\beta)+\omega\right]\frac{\left[(1-\alpha)\frac{y_t}{g_t}-1\right]}{(y_t-g_t-h_t)^2},$$

and thus

$$\left(\bar{U}_{g_t h_t}\right)^2 = \left[(1+n)(1+\alpha\beta)+\omega\right]^2\frac{\left[(1-\alpha)\frac{y_t}{g_t}-1\right]^2}{(y_t-g_t-h_t)^4}$$

such that

$$\bar{U}_{g_t g_t}\bar{U}_{h_t h_t}-\left(\bar{U}_{g_t h_t}\right)^2 = X+Y+Z>0, \quad \text{for any} \quad (g_t,h_t).$$

□

Proof of Corollary 5.6

Proof by contradiction. Suppose that $\tau_h^m \leq \tau_h^{\mathcal{P}}$, then

$$\frac{\alpha\omega b}{(1+n)(1+\alpha\beta)+\omega(1+b)} \leq \frac{\alpha\omega b}{(1+n)(1+\beta(1+b))+\omega(1+b)}$$

$$\Leftrightarrow \quad 1+b \leq \alpha,$$

which is a contradiction. Thus, it has to hold that $\tau_h^m > \tau_h^{\mathcal{P}}$.

□

Proof of Corollary 5.7

1. Comparative statics for a change in n

   Partial derivation of (5.34) with respect to n immediately yields $d\tau_h^m/dn < 0$. Moreover, $d(g_t/y_t)/dn = 0$. Then, using (5.34) in (5.35) we obtain the equilibrium growth factor as

   $$\gamma^m = \alpha B \frac{(1+\alpha\beta) + \omega/(1+n)}{(1+n)(1+\alpha\beta) + \omega(1+b)}.$$

   Then, partial derivation immediately yields $d\gamma^m/dn < 0$.

2. Comparative statics for an increase in life expectancy

   Consider the reinterpretation of the economic framework as described in Section 5.5.2. Then, τ_h^m and γ^m can be rewritten as

   $$\tau_h^m = \frac{\alpha\omega_v b}{(1+n)(1/v + \alpha\beta_v) + \omega_v(1+b)}$$

   and

   $$\gamma^m = \frac{\beta_v v X}{1+\beta_v}\left(\alpha - \tau_h^m\right),$$

   where $X \equiv A^{1/\alpha}(1-\alpha)^{1/\alpha}/(1+n)$. Then, a permanent increase in the survival probability v has the following effects on government policy and economic growth:

   $$\frac{\partial(g_t/y_t)}{\partial v} = 0$$

   $$\frac{\partial \tau_h^m}{\partial v} > 0$$

   $$\begin{aligned}\frac{\partial \gamma^m}{\partial v} &= \frac{\beta_v X}{1+\hat{\beta}}\left(-v\frac{\partial \tau_h^m}{\partial v} + \alpha - \tau_h^m\right) \\ &= \frac{X\beta_v\alpha}{1+\hat{\beta}} \frac{(1+n)\left(1+\alpha\hat{\beta}\right)\left((1+n)(1+\alpha\hat{\beta}) + \hat{\omega}\right)}{\left[(1+n)\left(1+\alpha\hat{\beta}\right) + \hat{\omega}(1+b)\right]^2} \\ &\quad + \frac{X\beta_v\alpha\hat{\omega}}{1+\hat{\beta}} \frac{(1+b)\hat{\omega} + (1+n)\left(1+(1+b)\alpha\hat{\beta}\right)}{\left[(1+n)\left(1+\alpha\hat{\beta}\right) + \hat{\omega}(1+b)\right]^2} > 0.\end{aligned}$$

   However, as the increase in the life expectancy is unexpected these effects only materialize with a period delay; for an intuition see the proof of Proposition 5.3.

□

Existence proof for numerical example of Section 5.6.2.2

To see that a unique $\tau_H^{\mathcal{P}}$ exists if ρ is sufficiently small, rewrite equation (5.51) as

$$(1+n)(1+\beta)+\omega=\alpha^{-\rho}b\left[\omega\left(\tau_h^{\mathcal{P}}\right)^{\rho-1}\left(\alpha-\tau_h^{\mathcal{P}}\right)^{1-\rho}-(1+n)(1+\beta)\left(\tau_h^{\mathcal{P}}\right)^{\rho}\left(\alpha-\tau_h^{\mathcal{P}}\right)^{-\rho}\right]. \tag{5.76}$$

Denote the right-hand side of (5.76) by $RHS(\tau_h^{\mathcal{P}},\rho)$ and the left-hand side, which does not depend on τ_h, by LHS. One readily verifies that $\partial RHS(\tau_h^{\mathcal{P}},\rho)/\partial\tau_h < 0$ for any $\rho < 1$. Moreover, for a given ρ, $RHS(\tau_h^{\mathcal{P}},\rho) > 0$ when $\tau_h^{\mathcal{P}}$ is sufficiently small (i. e. close to zero) and $RHS(\tau_h^{\mathcal{P}},\rho) < 0$ when $\tau_h^{\mathcal{P}}$ is sufficiently close to α. Therefore, there is a unique value of $\tau_h^{\mathcal{P}} \in (0,\alpha)$ which satisfies (5.76) if and only if $RHS(\tau_h^{\mathcal{P}},\rho)$ for $\tau_h^{\mathcal{P}}$ close to zero is greater than LHS. Now note that for a given $\tau_h^{\mathcal{P}}$

$$\frac{\partial RHS(\tau_h^{\mathcal{P}},\rho)}{\partial\rho}=\frac{b\left(\tau_h^{\mathcal{P}}\right)^{\rho-1}}{\alpha^{\rho}\left(\alpha-\tau_h^{\mathcal{P}}\right)^{\rho}}\ln\left(\frac{\left(\alpha-\tau_h^{\mathcal{P}}\right)\alpha}{\tau_h^{\mathcal{P}}}\right)\left(\tau_h^{\mathcal{P}}(1+\beta)(1+n)-\omega\left(\alpha-\tau_h^{\mathcal{P}}\right)\right).$$

Thus, $lim_{\tau_h^{\mathcal{P}}\to 0}\left(\partial RHS(\tau_h^{\mathcal{P}},\rho)/\partial\rho\right) < 0$ and $lim_{\tau_h^{\mathcal{P}}\to 0}RHS(\tau_h^{\mathcal{P}},\rho)$ is greater the smaller ρ. Therefore, we can conclude that a solution to (5.76) only exists if ρ is not too large.

□

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