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Abstract

A well–known result from the theory of finitely repeated games states that if the stage game has a unique equilibrium, then there is a unique subgame perfect equilibrium in the finitely repeated game in which the equilibrium of the stage game is being played in every period. Here I show that this result does in general *not* hold anymore if players have social preferences of the form frequently assumed in the recent literature, for example in the inequity aversion models of Fehr and Schmidt (1999) or Bolton and Ockenfels (2000). In fact, repeating the unique stage game equilibrium may not be a subgame perfect equilibrium at all.

Keywords: social preferences, finitely repeated games, inequity aversion, ERC.

JEL-Classifications: C72, C73.

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1 Introduction

A well–known result from the theory of finitely repeated games states that if the stage game has a unique equilibrium, then there is a unique subgame perfect equilibrium in the finitely repeated game in which the equilibrium of the stage game is being played in every period. This result has been much used in applied theory, in particular in industrial organization with the most prominent example being the chain–store paradox of Selten (1980). It is also frequently being invoked in experimental economics when a stage game is played amongst the same set of players for a finite number of periods.

The purpose of this note is to point out that the result described above does in general *not* hold anymore if players have social preferences of the form frequently assumed in the recent literature, for example in the inequity aversion models of Fehr and Schmidt (1999) or Bolton and Ockenfels (2000).[1] In fact, repeating the unique stage game equilibrium may not be a subgame perfect equilibrium at all in some examples. The logic is simple. In the standard case of selfish preferences, payoffs are separable across periods in the sense that the optimal choice in the last period does not depend on anything that has happened in previous periods. For most models of social preferences, this no longer holds. What has happened in previous periods influences the relative payoffs and therefore also the optimal choice in the last period, which makes it impossible to treat the last period as independent from the rest of the game.[2]

2 Examples

Example 1 (Dictator game) As a simple illustration consider the following example of a dictator game with three options for the proposer: $(0, 100)$, $(40, 40)$, $(100, 0)$, where x in (x, y) denotes the the amount of money allocated

[1] See also Engelmann and Strobel (2004) for an extensive discussion.

[2] In other words, the game is now a stochastic game. This insight seems to be some kind of folk wisdom in the literature. It has also been pointed out recently in Hu (2010).

to the dictator and y denotes the amount of money allocated to the other player, the recipient. Choose a model of social preferences and parametrize it such that the proposer ranks $(40, 40) \succ (100, 0) \succeq (0, 100)$. Assume further that preferences are monotone in the sense that $(a, a) \succ (b, b)$ for $a > b$.

Clearly, the unique Nash equilibrium of the stage game is to allocate the money $(40, 40)$. However, in the twice repeated game there are two subgame perfect equilibria (SPE) that yield the sequence of allocations $(100, 0) \rightarrow (0, 100)$ or $(0, 100) \rightarrow (100, 0)$, respectively. Since $(100, 100) \succ (80, 80)$, repeating the unique stage game equilibrium twice is not a SPE of the repeated game.

Example 2 (Trust game) Consider a 5 times repeated trust game. The stage game payoffs of the investor and the trustee are $(\pi_I, \pi_T) = (2, 0)$ if the investor does not invest. If he invests, the trustee can split the pie equally $(4, 4)$ or keep everything for himself $(0, 8)$. Suppose players are inequity averse but not too strongly if it is to their advantage. In particular, suppose that $(2, 0) \succ_I (0, 8)$, $(8, 8) \succ_I (10, 0)$, $(8, 8) \succ_I (a, b)$ if $a < 8 < b$. In all zero–sum choices the trustee likes to have more rather than less: $(M - a, a) \succ_T (M - a', a')$ for all M and $a > a'$.[3] The unique subgame perfect equilibrium of the stage game entails no investment. Yet, all subgame perfect equilibria of the 5 times repeated game entail investment in one of the 5 rounds. Suppose there was no investment in the first 4 rounds. By investing in the last round, the investor can equalize average payoffs to $(8, 8)$ which is preferred by him to $(10, 0)$, which would result from not investing in the last period.

Example 3 (Ultimatum game) Consider a twice repeated ultimatum game, in which the proposer (P) can make offers of $y \in [0, 100]$ to the responder (R). Suppose the responder is known to be inequity averse and his preferences on the total payoff allocations (Π_P, Π_R) are such that $(150, 50) \sim_R (70, 30) \sim_R (0, 0)$.[4] Thus, in a stage game, the responder will reject any offer

[3] These assumptions would be satisfied, for example, in the Fehr and Schmidt (1999) model if $\beta_I > 0.2$ and $\beta_T < 0.5$ in equation (2) below.

[4] This assumption is compatible with Bolton and Ockenfels' (2000) ERC model that

$y < 30$. For a proposer, who is supposed to care only about his own payoff, the unique best reply is to choose $y = 30$. Hence, the SPE of the stage game results in $y = 30$, which is accepted by the responder. However, in the twice repeated game the proposer's optimal action is to offer $y^1 = 30$ in the first period and $y^2 = 20$ in the second. Both are accepted by the responder. But this implies that offering the stage game equilibrium proposal of 30 in both periods is not a SPE.

3 Model and main result

In the following I shall consider a (normal form) stage game $G = \{N, (A_i), (\pi_i)\}$,[5] where $N = 1, ..., n$ is the set of players, A_i is a set of pure actions for player i and $\pi_i(a_i, a_{-i})$ is player i's payoff function given his action and the action profile of all other players $-i$. An $a \in A := \times A_i$ is referred to as an outcome of G. The finitely repeated game $G(T)$ results when G is successively played T times and players are informed about the outcome a^t after each period t.

The crucial assumption is how players evaluate the outcome sequences $(a^t) \in A^T$. We assume that player i evaluates (a^t) according to his total payoff,

$$\Pi_i := \sum_{t=1}^{T} \pi_i(a^t). \qquad (1)$$

When applying the theory to social preferences, there are (at least) two possibilities of how to evaluate payoffs. Given that in finitely repeated games (and in almost all experiments) the payoffs are paid out to players at the end of the game, it seems reasonable that players should evaluate an outcome sequence (a^t) based on the profile of total payoffs of all players, (Π_i).

requires that U_i is strictly increasing in Π_i for given σ_i (see equation (3) below) since $(140, 60) \succ_R (70, 30)$. The assumption would not be compatible with Fehr and Schmidt (1999) preferences.

[5] With slight modifications the analysis can also be applied to sequential stage games with a unique SPE.

Assumption 1 *Social preferences in a finitely repeated game can be represented by a utility function* $U_i(\Pi_i, \Pi_{-i})$.

Likewise, one can assume that U_i depends on average payoffs, $\frac{1}{T}\sum_{t=1}^{T}\pi_i(a^t)$, which does not affect the main results.[6] The crucial feature is that payoffs for each player are first aggregated over periods and then compared across players. An alternative way would be to assume that payoffs are compared period by period without allowing for compensation across periods. This assumption is sometimes made for infinite games (see Duffy and Monoz–Garcia, 2010), however mainly for reasons of tractability. It seems less reasonable for the case of finitely repeated games. After all, why should a player fail to consider the payoffs from different periods as substitutes when in the end all that matters is the total payoff he takes home?

Several prominent social preference models can be applied in accordance with Assumption 1. The inequity aversion model of Fehr and Schmidt (1999) can be specified as

$$U_i(\Pi_i, \Pi_{-i}) = \Pi_i - \alpha_i \frac{1}{n-1}\sum_{j\neq i}\max\left[\Pi_j - \Pi_i, 0\right] - \beta_i \frac{1}{n-1}\sum_{j\neq i}\max\left[\Pi_i - \Pi_j, 0\right], \tag{2}$$

with $0 \leq \beta_i < 1$ and $\beta_i \leq \alpha_i$.

The model of Bolton and Ockenfels (2000) can be written as

$$U_i(\Pi_i, \Pi_{-i}) = V_i\left(\Pi_i, \sigma_i\right), \tag{3}$$

where $\sigma_i := \Pi_i / \sum_j \Pi_j$. Bolton and Ockenfels (2000) assume that V_i is strictly concave in σ_i and assumes a maximum for given Π_i if $\sigma_i = 1/n$. Furthermore, for given σ_i, V_i is strictly increasing in Π_i.

[6] In experiments, sometimes a third way of paying subjects is used, namely paying one randomly chosen period. In this case, one has to make assumptions about how social preferences interact with risk (see Trautmann, 2009 and Fudenberg and Levine, 2011). The current discussion applies only if social preferences are assumed to depend on the expected value of the resulting lottery (as in Trautmann's "process Fehr-Schmidt" model).

Charness and Rabin (2002) assume a payoff function of the form

$$U_i(\Pi_i, \Pi_{-i}) = (1-\gamma)\Pi_i + \gamma \left[\delta \min\{\Pi_1, ..., \Pi_n\} + (1-\delta) \sum_j \Pi_j \right]. \quad (4)$$

Finally, models of linear altruism or spite can be written as

$$U_i(\Pi_i, \Pi_{-i}) = \sum_j \gamma_j \Pi_j. \quad (5)$$

For selfish preferences, the following result is well known (see e.g. Proposition 157.2 in Osborne and Rubinstein, 1994).[7] It is instructive to follow the steps of its proof to see why the result breaks down for most forms of social preferences. Suppose Assumption 1 holds. Suppose further that $U(\cdot,\cdot)$ is a linear mapping and hence

$$\begin{aligned} U_i(\Pi_i, \Pi_{-i}) &= U_i\left(\sum_{t=1}^{T} \pi_i(a^t), \sum_{t=1}^{T} \pi_{-i}(a^t) \right) \\ &= \sum_{t=1}^{T} U_i(\pi_i(a^t), \pi_{-i}(a^t)). \end{aligned} \quad (6)$$

Obviously, (6) is satisfied if U_i is given by (1) or (5) but not if it is given by (2), (3), or (4). This is the reason why for most of the popular social preference models the following proposition does *not* hold.

Proposition 1 *Suppose payoffs in $G(T)$ are evaluated by a utility function $U_i(\Pi_i, \Pi_{-i})$ that satisfies (6). If the strategic game G has a unique Nash equilibrium payoff profile, then for any value of T the action profile chosen after any history in any subgame perfect equilibrium of $G(T)$ is a Nash equilibrium of G.*

If $U_i(\cdot,\cdot)$ is non–linear (e.g. because of the strict concavity with respect to σ_i in the model of Bolton and Ockenfels, 2000, or the max–operators in the

[7] As Osborne and Rubinstein (1994, Definition 137.1) point out, the crucial assumption for the standard result is "weak separability" of preferences.

model of Fehr and Schmidt, 1999), the naive application of the backward–induction argument fails.

Proof. Consider the optimal action in period T. If (6) holds, then

$$\begin{aligned} \arg\max_{a_i^T} U_i(\Pi_i, \Pi_{-i}) &= \arg\max_{a_i^T} \sum_{t=1}^{T} U_i(\pi_i(a^t), \pi_{-i}(a^t)) \\ &= \arg\max_{a_i^T} U_i(\pi_i(a^T), \pi_{-i}(a^T)). \end{aligned}$$

The optimization problem in the second line is the same as the one in the stage game. This implies that players' payoffs in T are independent of the history of play. Thus, in all subgames of $G(T)$ starting in period T, the outcome is a Nash equilibrium of G. Therefore, also in all subgames starting in period $T-1$, the outcome is a Nash equilibrium of G. The result follows then by induction. □

I have emphasized so far the application of the result to partner or fixed matchings in experimental games because it is in this context where Proposition 1 is usually invoked – incorrectly as we see now when social preferences are being considered. However, even with random or stranger matching, problems can occur with the application of social preferences to repeated interaction. To see this consider again Example 1 and let there be four players, two proposers (P1,P2) and two receivers (R1,R2). In the first period, P1 is matched with R1 and P2 with R2. In the second period, P1 is matched with R2 and P2 with R1.

This stranger matching procedure is usually seen as a method that allows to treat each match as an independent observation, essentially as a one–shot game. However, with social preferences this need not be the case anymore. Suppose players evaluate average payoffs according to the ERC model of Bolton and Ockenfels (2000) given in (3). In the one–shot game, both proposers will split (40,40) if they are sufficiently inequity averse. In the twice repeated game however, all SPE are of the form $(100,0) \to (0,100)$ or $(0,100) \to (100,0)$. This is because players have a trade–off between their own payoff and the ratio of their payoff to the sum of all players' payoffs. If the latter is sufficiently important, the claim follows.[8]

Thus, when applying social preference models to finitely repeated games, care has to be taken. It will depend on the specifics of the game whether the T–fold repetition of the unique stage game equilibrium is still a SPE of the repeated game.

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[8] This is true regardless of a proposer's belief about the other proposer's action. In the SPE, $\sigma = 1/4$ and V_i is maximized. But even if a proposer believes that the other proposer is choosing allocations that result in a total payoff of 160 in the other matches, he would choose $(100, 0)$ and $(0, 100)$ in his matches if he is sufficiently inequity averse.

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