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How do people cope with an ambiguous situation when it becomes even more ambiguous?

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Abstract

As illustrated by the famous Ellsberg paradox, many subjects prefer to bet on events with known rather than with unknown probabilities, i.e., they are ambiguity averse. In an experiment, we examine subjects' choices when there is an additional source of ambiguity, namely, when they do not know how much money they can win. Using a standard independence assumption, we show that ambiguity averse subjects should continue to strictly prefer the urn with known probabilities. In contrast, our results show that many subjects no longer exhibit such a strict preference. This should have important ramifications for modeling ambiguity aversion.

Keywords: ambiguity aversion, uncertainty, minmax-expected utility

JEL-Classifications: C91, D81.

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1 Introduction

In Ellsberg's famous two–color experiment (1961), subjects can choose between placing bets on the color of a ball drawn from one of two urns. The first urn (urn H) contains a number of colored balls, half of which are known to be black and half of which are known to be red. The second urn (urn U) contains balls of the same colors but in unknown proportions. Subjects who irrespectively of the color strictly prefer betting on the urn where half of the balls are black are classified as ambiguity-averse.

In the classic experiment, ambiguity only concerns the composition of the urns. In reality, however, ambiguity is rarely limited to a specific aspect of a situation. In particular, the gains from winning are often not clear.[1] In this paper, we examine experimentally how ambiguity aversion is affected when ambiguous situations become more ambiguous in the sense that there is also uncertainty about the prizes one can win. As we shall see, this has important consequences for modeling ambiguity aversion.

We extend Ellsberg's two-color experiment by systematically varying the information available about the prize. Subjects decide on an urn (H or U) and a color (black or red). If their color matches that of the ball drawn from the respective urn, subjects receive an envelope that is marked with an equal sign (=). If not, they receive a (different) envelope that is marked with an unequal sign ($\neq$).

[1] For example, lottery stands often do not state the possible prizes when claiming that "Every second ticket wins!"

We consider three situations. In situation O (for open envelope), subjects see the contents of the envelopes. There are 3 euro in the envelope with the $=$ sign and 1 euro in the other envelope. Situation O corresponds to the usual Ellsberg experiment. In addition, we consider the following two variations. In situation S (for sealed envelope), subjects only know that one of the two envelopes contains 3 euro and the other 1 euro but they do not know which amount is in which envelope. In situation R (for random), subjects know that the content of the envelope (3 euro or 1 euro, respectively) will be determined by flipping a fair coin after they have made their choice on which urn to bet.

Since situation O describes the standard Ellsberg experiment, ambiguity averse subjects should strictly prefer to bet on the urn with the known composition of colors. In situation R, one could follow Raiffa (1961) and argue that decision makers face equal odds of winning the 3 euro no matter which urn they choose.[2]

For situation S, one could argue as follows: "Given that I have no way of knowing *what* I win if I win, I should not care *whether* I win." Given this line of reasoning (which is actually entertained by at least two of the authors of this paper), a subject should not care whether he bets on the known or the unknown urn.

Predicting formally how ambiguity averse subjects behave in situation S is more involved. Our starting point is to represent ambiguity aversion using the MaxMin Expected Utility (MEU) approach by Gilboa and Schmeidler

[2] This argument can be formalized in the Anscombe and Aumann (1963) setting.

(1989). Later, we also consider alternative approaches and their predictions like Choquet expected utility (CEU) pioneered by Schmeidler (1989) and the smooth ambiguity model by Klibanoff, Marinacci, and Mukerji (2005).

Suppose now that subjects believe the envelope's content to be independent from the color of the ball drawn, which can be formalized with the notion of independence advanced by Gilboa and Schmeidler (1989). Given this notion and the MEU representation, we show that decision makers who strictly prefer to bet on urn H in situation O would also do so in situation S but be indifferent in situation R.

Our results are only partially in line with these predictions. As usual in such experiments, about $2/3$ of subjects are ambiguity averse in the sense that they prefer to bet on urn H in situation O. However, we find that in situation S, this share drops significantly. The additional ambiguity about the contents of the envelope seems to mitigate the ambiguity about the contents of the urn. Finally, few subjects seem to be indifferent between urns in situation R, although most theories (like SEU or MEU) predict them to be so. Instead, most subjects still strictly prefer to bet on urn H.

Apart from very mild assumptions on the set of priors, our theoretical predictions rely decisively on the assumption that this set is a product set of the set of priors for the urn composition and the envelope's content. Without this assumption, subjects who strictly prefer urn H when the envelope's content is known, may well cease to do so when it is unknown.

Our paper contributes to a growing literature that examines specific predictions of ambiguity models. Superficially, these experiments concern different aspects of ambiguity, for example, its relationship to preferences for randomization (Dominiak and Schnedler 2011) or that to dynamic consistency (Cohen et al., 2000, Dominiak, Duersch, and Lefort, 2012). However, all these predictions have in common that they critically depend on how independence is modeled.

As an example take preferences for randomization. In response to the Ellsberg paradox, Raiffa (1961) advanced the intuitive argument that, by choosing whether to bet on the color Red or the color Black conditional on the outcome of a coin flip, an ambiguity-averse decision maker could transform the ambiguous choice into the preferred unambiguous gamble. Eichberger and Kelsey (1996b) show that such a preference for randomization depends on how the random device is modelled. In the Anscombe-Aumann framework, randomization over acts corresponds to forming a state-wise convex combination of the outcome lotteries. In this case, ambiguity aversion implies a preference for randomization. If one models the random device explicitly as part of the state space, however, then no such implication follows. Indeed, as Klibanoff (2001) shows, one needs specific behavioral assumptions in order to model behavior corresponding to the notion of an independent random device and, in consequence, a clear preference for randomization. Schmeidler's (1989) ambiguity aversion axiom, which underpins various representations of ambiguity averse behavior, directly stipulates preferences for randomization. Dominiak and Schnedler (2011) experimentally test whether ambiguity averse subjects prefer randomization but find no such relationship. Indeed, a considerable share of ambiguity averse subjects dislikes randomization.[3]

With respect to dynamic consistency, Eichberger and Kelsey (1996) show that it is, in general, incompatible with ambiguity aversion in the CEU model. Even if dynamic consistency is restricted to a particular event tree, it implies additive separability of preferences and, hence, conditional independence of beliefs up to the final stage. For the multiple prior model, Sarin and Wakker (1998) show that dynamic consistency of intertemporal choices implies constraints on the set of priors if they are updated pointwise according to Bayes' rule. Epstein and Schneider's (2003) concept of "rectangularity" characterizes the condition on the sets of priors which allow for independence. Hansen et al. (2006) criticize the emphasis given to dynamic consistency and the implied independence in situations of genuine uncertainty. Bade (2008) points out there is a direct association between the updating rule and how independence is defined.

Despite the large number of experiments on ambiguity in general (for a partial survey see Camerer, 1995, or Halevy, 2007) there is to our knowledge only one other study that considers an Ellsberg type experiment in which the size of the prize is also uncertain. Eliaz and Ortoleva (2011) consider a three–color Ellsberg urn, where in some treatments the amount of money won depends on the (unknown) number of balls of a given color. Thus, in

[3]Using a different experimental design Spears (2009) comes to a similar conclusion.

contrast to our setting, the uncertainty with respect to the number of balls of a given color and with respect to the size of the prize money come from the *same* source. Hence, the issue of stochastic independence of different sources of uncertainty can play no role. In fact, due to their design, the probability of winning is perfectly correlated with the size of the prize, depending on the treatment either positively or negatively. Eliaz and Ortoleva (2011) find that most subjects prefer to bet on a color with unknown proportion in the positively correlated situation. In addition, many subjects prefer a gamble with positively correlated ambiguity to gambles without any ambiguity.[4]

The rest of the paper is organized as follows. In the next section we describe the experimental design and procedures. In Section 3, we derive various theoretical hypotheses. Results are analyzed and discussed in Section 4. In Section 5, we discuss alternative models of ambiguity. Finally, we close with a brief discussion of the implications of our findings in Section 6.

2 Design of the experiment

Our experiment encompasses three variations on a standard two-urn Ellsberg setting. There are two urns. Each urn contains 40 balls, which are either black or red. In the first urn (urn H) half of the balls are black and the other half red. The second urn (urn U) contains an unknown proportion of

[4] These observations may be rationalized by the fact that a subjective expected utility maximizer who follows the principle of insufficient reason would actually strictly prefer betting on a color with unknown proportion given positive correlation.

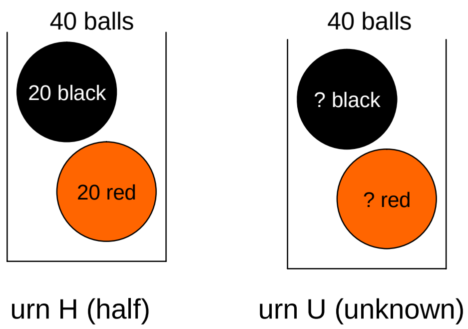

Figure 1: Composition of urns used in the experiment

black and red balls.[5] Subjects can win the money in one of two envelopes, one of which is marked with an equal sign (=) and the other marked with an unequal sign (≠). Subjects have to decide on an urn (U, H, or indifferent) and a color (black, red, or indifferent). Then, a ball is drawn from the chosen urn and if the drawn ball has the chosen color, they receive the money in the envelope marked with = and otherwise that in the envelope marked with ≠ (see Figures 1 and 2 for illustrations). If subjects indicate that they are indifferent, the first option for the respective decision is chosen as payoff relevant.[6]

In our experiment, we consider three different *situations*. In each situation, subjects are informed that one of the envelopes contains 3 euro, while the other contains 1 euro. The knowledge about which envelope contains

[5]In the actual experiment, we used bags and blue and green marbles. For expositional reasons, we employ the more customary urns, balls, and colors in the text.

[6]Note that strictly speaking indifferent subjects have no strict incentive to mark that they are indifferent. However, there is evidence that subjects have an aversion to lying, in particular in situations in which they have no strategic reason for doing so (cf. Gneezy, 2005; Vanberg, 2008; and Hurkens and Kartik, 2009).

the 3 euro differs across the three situations. In situation O (open), subjects know that the envelope with the $=$ sign contains the 3 euro (and the other 1 euro). In situation S (sealed), subjects are informed that whether the envelope with the $=$ sign contains 3 euro or the envelope with the $\neq$ sign has been determined according to some unknown probability. In situation R (randomized), subjects know that the envelope, which contains 3 euro, is determined by throwing a fair coin after the experiment; so that the envelope with the $=$ sign contains the 3 euro with probability one half. Situation O thus represents a classical Ellsberg-two-color urn experiment, while situation S introduces additional ambiguity about the envelopes' contents.

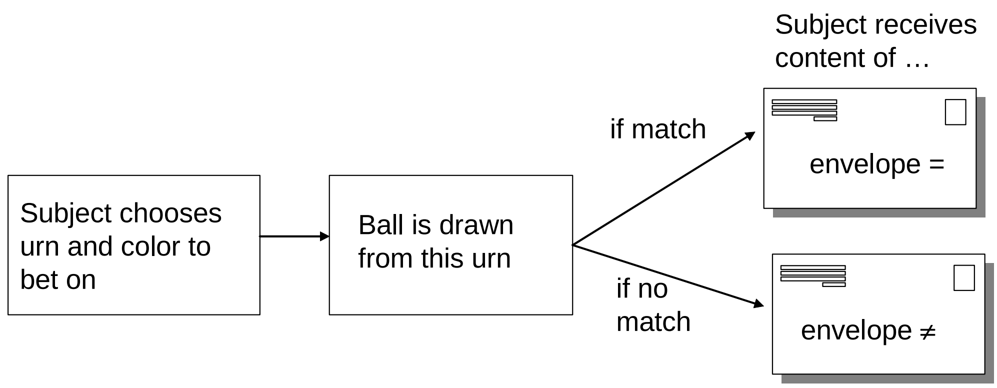

Figure 2: Structure of the experiment

Since we are particularly interested in the question whether ambiguity-averse subjects continue to prefer urn H when the prize is uncertain, we run a "within" subjects treatment, in which subjects have to make decisions in all three situations. In order to control for order effects, we run two subtreatments. In the OS-subtreatment, subjects are first confronted with situation O, and then situation S. In the SO-treatment, the order is reversed. In both treatments, subjects are asked about situation R in the end. Each subject participates in one of the two subtreatments only. Subjects are paid the sum of their payoffs from all three situations in cash at the end of the experiment.

Paying subjects for all three decisions may be problematic if there is a portfolio effect (see e.g. Cox et al. 2011, for an extended discussion). However, the alternative of paying one randomly selected situation is not appropriate if subjects violate the independence axiom of expected utility theory (Holt 1986, Karni and Safra, 1987), which is not a desirable feature if one wants to study non-expected utility theories. To control for portfolio effects, we also conduct a "between" subjects treatment, in which each subject faces just one of the three situations. Subjects in this treatment only face two outcomes (win or lose) so that risk aversion does not matter for behavior.

The urn chosen as default in case that a subject indicates indifference may also affect behavior, e.g., if subjects believe this default to be informative about the urn's composition. In order to check whether the default affects results, we systematically vary the default in treatment "between": About half the subjects have urn U as default, about half have urn H. In treatment "within", the default was urn U. Table 1 summarizes the treatment characteristics.

The experiment was run as a classroom experiment with paper and pencil in July 2010 and October 2011 using bags marked with the letters H and

Table 1: Treatments

name	description	subtreatment	# of subjects
"within"	subjects choose in all three situations in the order O-S-R...	OS	23
	or S-O-R...	SO	25
"between"	subjects choose in just one situation. Default urn in case of indiff. is H...	default H	35
	or U...	default U	36

U that were filled with marbles. Bags were on display during the experiment, so that subjects could be certain that the bags' contents could not be manipulated. Subjects were allowed to verify the bags' contents after the experiment and some did. The participants were 119 undergraduate economics students from the University of Heidelberg (48 in treatment "between" and 71 in treatment "within"). They came from classes in microeconomics and game theory, none of which had covered decision theory in general or the Ellsberg paradox in particular. The instructions (see appendix) were distributed on paper and were read aloud by the experimenter. The experiments lasted between 15 and 30 min. By design, in treatment "within" earnings ranged from 3 to 9 euro. The average payment was 6.42 euro. Since payments in treatment "between" could only be either 1 or 3 euro, we added a show-up fee of 3 euro such that average earnings were 4.87 euro.

3 Theoretical predictions

A state in the experiment is described by a triplet listing the color of the ball (b or r) drawn from the unambiguous urn H, the color of the ball (B or R) drawn from the ambiguous urn U, and the amount in euro (3 or 1) in envelope $=$.[7] Thus, in total there are the eight states, $S = \{s_1, ..., s_8\}$, listed in Table 2. For example, we denote state s_3 by $bR3$ because in this state, ball b was drawn from urn H, ball R was drawn from urn U, and the number of euro in envelope $=$ was 3.

Table 2: States, bets, and probabilities

S		Hb	Hr	Ub	Ur	probabilities
s_1	$bB3$	3	1	3	1	$\pi_1(p,q) = \frac{1}{2}qp$
s_2	$rB3$	1	3	3	1	$\pi_2(p,q) = \frac{1}{2}qp$
s_3	$bR3$	3	1	1	3	$\pi_3(p,q) = \frac{1}{2}(1-q)p$
s_4	$rR3$	1	3	1	3	$\pi_4(p,q) = \frac{1}{2}(1-q)p$
s_5	$bB1$	1	3	1	3	$\pi_5(p,q) = \frac{1}{2}q(1-p)$
s_6	$rB1$	3	1	1	3	$\pi_6(p,q) = \frac{1}{2}q(1-p)$
s_7	$bR1$	1	3	3	1	$\pi_7(p,q) = \frac{1}{2}(1-q)(1-p)$
s_8	$rR1$	3	1	3	1	$\pi_8(p,q) = \frac{1}{2}(1-q)(1-p)$

We denote a bet (or act) on some color c in urn X by Xc. There are four possible bets: $\mathcal{B} = \{Hb, Hr, Ub, Ur\}$. The consequences (or payoffs) associated with these bets $f \in \mathcal{B}$ are also shown in Table 2. We assume that there is a utility function $u(\cdot)$ over consequences. Without loss of generality, we set $u(3) = 1$ and $u(1) = 0$.

[7] The content of envelope $\neq$ follows by implication.

The last column of Table 1 shows the probabilities of the states as they would be derived for a subjective expected utility maximizer who considers

- the draws from the two urns and the filling of the envelopes as independent events, and who assumes that
- the probability of a black ball b drawn from urn H is $r = \frac{1}{2}$, because the composition of urn H was announced to be half black and half red,
- the probability of a black ball B drawn from urn U is q, and
- the probability of envelope = containing 3 euro equals p.

The probability of state s, $\pi_s(p, q)$, then follows by the usual product rule for independent events.

Note that the content of an envelope (3 euro or 1 euro) was either known (in situation O), decided before the experiment started (in situation S), or determined by a fair coin after the experiment (in situation R). Thus, in all three situations one can reasonably assume that the envelopes' content is independent from the colors of the balls drawn from urns U or H. Likewise, the colors of the balls drawn from urns H and U are independent from each other. Whether subjects in the experiment actually consider these events to be independent is another matter that is addressed by our experiment.

In the presence of ambiguity, the notion of "independence" is no longer clear. Depending on how ambiguity is modeled, different concepts of independence arise. Here, we adopt the notion of independence suggested in

Gilboa and Schmeidler (1989, p. 150).[8] Alternative ways to model ambiguity and independence will be discussed in Section 5.

Let P be the set of priors for the probabilities that envelope = contains 3 euro and Q for the probability that B is drawn from U. Let $\pi(p, q) := (\pi_1(p, q), \ldots, \pi_8(p, q))$, denote the probability distribution over states, where $\pi_s(p, q)$ is the product measure for state s as defined in the last column of Table 2.

Assumption 1 *The set of priors $\mathcal{P}$ is the set of Gilboa-Schmeidler-independent product measures,*

$$\mathcal{P} := \text{co}\left\{\pi(p, q) \mid p \in P,\ q \in Q\right\} \subseteq \Delta(S).$$

The set of priors is thus the convex hull of all product measures that can be constructed in the familiar way.[9]

We model ambiguity–averse subjects using the MEU approach by Gilboa Schmeidler (1989). A decision maker whose preferences are described by MEU evaluates a bet $f \in \mathcal{B}$ by

$$MEU(f) = \min_{\pi \in \mathcal{P}} \sum_{s \in S} \pi_s u(f(s)). \tag{1}$$

We assume that an MEU–maximizer has a set of priors that is compatible with the actual compositions of the urns and the content of the envelope.

[8] Bade (2008) provides a discussion of alternative ways for defining independence of sets of priors.

[9] Since payoffs in (1) are linear in probabilities, minimal payoffs are, of course, unaffected by taking the convex hull.

When subjects are informed of the objective probabilities of certain events, then they are assumed to consider those events as unambiguous in the sense of Nehring (1999). For example, if the envelope is known to contain 3 euro, then $P = \{1\}$. Conversely, we assume that an MEU–maximizer has non–degenerate priors in each dimension (P or Q) for which no objective probabilities are known.

As a benchmark we take a subjective expected utility (SEU) maximizer with a unique prior π who evaluates bets in the following way,

$$SEU(f) = \sum_{s \in S} \pi_s u(f(s)).$$

3.1 Situation *O*

In situation O the content of envelope = is known; the probability that it contains 3 euro is $p = 1$. When betting on urn H, there is no ambiguity and both the SEU– and the MEU–maximizer evaluate bets equally. Using the probabilities in Table 2 we obtain

$$SEU(Hb) = SEU(Hr) = MEU(Hb) = MEU(Hr) = \frac{1}{2}.$$

When betting on urn U, SEU–maximizers have subjective beliefs $q \in [0, 1]$. Hence, $SEU(Ub) = q$ and $SEU(Ur) = 1 - q$. Since

$$\max\{q, 1 - q\} \geq \frac{1}{2},$$

with strict inequality for $q \neq \frac{1}{2}$, SEU–maximizers weakly prefer betting on urn U (denoted as $U \succeq H$).

Now, consider an MEU-maximizer. This maximizer strictly prefers urn H to urn U if and only if the set of priors Q on urn U satisfies

$$Q \cap \left[0, \frac{1}{2}\right) \neq \varnothing \text{ and } Q \cap \left(\frac{1}{2}, 1\right] \neq \varnothing. \tag{2}$$

In order to see this, consider the bets Ub and Ur. Bet Ub yields 3 euro in states $bB3, rB3, bR1$, and $rR1$, while Ur yields 3 euro in four different states $(bR3, rR3, bB1, rB1)$. Evaluating both bets, gives:

$$\begin{aligned} MEU(Ub) &= \min_{\pi \in \mathcal{P}} [\pi_1 + \pi_2 + \pi_7 + \pi_8] = \min_{q \in Q} q, \\ MEU(Ur) &= \min_{\pi \in \mathcal{P}} [\pi_3 + \pi_4 + \pi_5 + \pi_6] = \min_{q \in Q} (1 - q). \end{aligned}$$

For a strict preference to bet on urn H, both terms need to be smaller than the value for the bets on urn H, $MEU(Hb) = MEU(Hr) = \frac{1}{2}$. This, however, is the case if and only if condition (2) holds.

Prediction 1 *In situation O, SEU–maximizers weakly prefer betting on urn U. MEU–maximizers strictly prefer betting on urn H if and only if condition (2) holds.*

Situation O describes the classical Ellsberg two–color urn experiment. In line with the customary notion, we classify subjects as *ambiguity-averse* if they choose to bet on urn H rather than on urn U in this situation. Theoretically, we model these subjects as MEU–maximizers for whom condition (2) holds.

3.2 Situation *S*

In situation S, the content of envelope = is not known; the probability that it contains 3 euro may be any $p \in [0, 1]$. Evaluating bet Hb for a SEU maximizer yields

$$SEU(Hb) = SEU(Hr) = \frac{1}{2}.$$

Let us now evaluate bet Hb for a decision maker with MEU preferences. This bet wins in four states, two ($bB3$ and $bR3$) in which the color drawn from H is black and the 3 euro are in envelope = and two ($rB1$ and $rR1$) in which the color is red and the 3 euro are in the other envelope.

$$\begin{aligned} MEU(Hb) &= \min_{\pi \in \mathcal{P}} [\pi_1 + \pi_3 + \pi_6 + \pi_8] \\ &= \min_{p \in P} \left[\frac{1}{2}p + \frac{1}{2}(1-p) \right] = \frac{1}{2}. \end{aligned}$$

Completely analogous, $MEU(Hr) = \frac{1}{2}$. In other words, the probability with which the envelope with the = sign contains 3 euro is irrelevant for evaluating the bets on urn H, Hb and Hr.

When betting on urn U, SEU–maximizers evaluate the bets as

$$\begin{aligned} SEU(Ub) &= qp + (1-p)(1-q), \\ SEU(Ur) &= q(1-p) + (1-q)p. \end{aligned} \tag{3}$$

Obviously,

$$\max\{SEU(Ub), SEU(Ur)\} \geq \frac{1}{2} \text{ for all } p \in [0,1], q \in [0,1]$$

with strict inequality for all $(p,q) \neq (\frac{1}{2}, \frac{1}{2})$. Hence, for SEU–maximizers $U \succeq H$ still holds.

Next, we evaluate the bets on the urn with unknown composition for a MEU–maximizer. For each bet, there are again four winning states and the

MEU-maximizer assigns the following values:

$$
\begin{aligned}
MEU(Ub) &= \min_{\pi \in \mathcal{P}} \left[\pi_1 + \pi_2 + \pi_7 + \pi_8\right] \\
&= \min_{\substack{p \in P \\ q \in Q}} \left[qp + (1-p)(1-q)\right], \\
MEU(Ur) &= \min_{\pi \in \mathcal{P}} \left[\pi_3 + \pi_4 + \pi_5 + \pi_6\right] \\
&= \min_{\substack{p \in P \\ q \in Q}} \left[q(1-p) + (1-q)p\right].
\end{aligned}
\tag{4, 5}
$$

Since P is non–degenerate, it holds that $P \neq \{\frac{1}{2}\}$. For all $p \neq \frac{1}{2}$, $qp + (1 - p)(1 - q)$ and $q(1 - p) + (1 - q)p$ are either strictly increasing or strictly decreasing in q. Thus, the respective minimizers are on a boundary of Q. Condition (2) implies that $\frac{1}{2}$ is in the interior of Q. Evaluating the products at $q = \frac{1}{2}$, we get $qp + (1 - p)(1 - q) = q(1 - p) + (1 - q)p = \frac{1}{2}$. Accordingly, the value at the minimum must be smaller,

$$
\max \{MEU(Ub), MEU(Ur)\} < \frac{1}{2}. \tag{6}
$$

Consequently, MEU–maximizers for whom condition (2) holds strictly prefer to bet on urn H in situation S.

Prediction 2 *In situation* S, *SEU–maximizers weakly prefer betting on urn* U. *MEU–maximizers, for whom condition (2) holds, strictly prefer betting on urn* H.

According to this prediction, subjects who are classified as ambiguity-averse because they choose H in situation O will also choose H in situation S.

In deriving Prediction 2, we used Assumption 1, which implies the independence between the content of the envelope and the color of the ball drawn from urn U. In fact, a much weaker assumption suffices for the prediction. Suppose a MEU–maximizer has correlated priors, e.g., he may believe that there are more black balls in urn U whenever the envelope with the equal sign contains 3 euro. Formally, the MEU–maximizer may consider the event $(b3 \cup r1)$ as more likely than the event $(b1 \cup r3)$. What is sufficient for Prediction 2 to hold is the assumption that whenever a MEU–maximizer has a correlated prior such that $prob\,(b3 \cup r1) > \frac{1}{2} > prob\,(b1 \cup r3)$, there also exists a prior in his set of priors with the inverse inequality.[10] Then, betting on black still yields $MEU(Ub) < \frac{1}{2}$. In summary, as long as the MEU–maximizer is not absolutely certain about the *sign* of the correlation, he strictly prefers to bet on urn H.

3.3 Situation *R*

In situation R, the content of envelope = is determined by a fair coin and hence $P = \left\{\frac{1}{2}\right\}$. Evaluating (3) and (4) at $p = \frac{1}{2}$, it is easy to see that

$$\begin{aligned} SEU(Ub) &= SEU(Ur) = MEU(Ub) = MEU(Ur) = \frac{1}{2}, \\ SEU(Hb) &= SEU(Hr) = MEU(Hb) = MEU(Hr) = \frac{1}{2}. \end{aligned}$$

Hence both SEU– and MEU–maximizers are indifferent between all four bets.

Prediction 3 *In situation* R, *SEU–maximizers and ambiguity–averse indi-*

[10]Note that this condition is automatically satisfied if Assumption 1 and condition (2) hold.

viduals are indifferent between betting on either urn.

We can summarize all three predictions in Table 3.

Table 3: Summary of theoretical predictions

	SEU	**MEU**
Situation O	$U \succsim H$	$H \succ U$
Situation S	$U \succsim H$	$H \succ U$
Situation R	$U \sim H$	$U \sim H$

4 Experimental results

Before coming to our main results, the comparison between behavior when envelopes are sealed and open, we address three preliminary concerns: order, portfolio, and default effects.

We test for order effects by using a variation in our "within" treatment. In sub-treatment SO, subjects were first presented to situation S, while in sub-treatment OS, they first were confronted with situation O. Table 8 in the appendix lists the frequency of urn choices in situation O and S for the two sub-treatments. A χ^2–test shows that the frequencies are not significantly different (p-value 0.51). Accordingly, we pool both sub-treatments in our following analysis. In order to see whether the default affects behavior, we use the variation in the default urn in the "between" treatment. The percentage of subjects choosing urn H was 48.6% when the default was H versus 55.6% when the default was U. A χ^2–test shows that the frequencies are not significantly different (p-value: 0.28). Portfolio effects can be assessed by comparing urn choice distributions in the "within" and "between" treatment (see Table 4). We find no indication for such effects at any conventional level (p-value of χ^2–test: 0.60).

Table 4: Percentage of subjects choosing the urns in the different situations

		urn choices in %		
		urn H	urn U	indifferent
treatment "within"	Situation O	62.5	22.9	14.6
	Situation S	39.6	35.4	25.0
	Situation R	52.1	25.0	22.9
treatment "between"	Situation O	62.5	33.3	4.2
	Situation S	45.5	45.5	9.1
	Situation R	48.0	36.0	16.0

Note: A total of 48 subjects made urn choices for all three situations of treatment "within"; 71 subjects made choices in one of the situations of treatment "between".

4.1 Main results

Table 4 shows the percentage of subjects who chose the various urns in the three different situations. When the content of the envelope is known (situation O), we get the standard result that almost 2/3 of subjects prefer the unambiguous urn *H.* In fact, in both treatments, exactly 62.5% of subjects prefer urn *H.*

Most interestingly, when the content of the envelope is also ambiguous, we get the lowest number of subjects choosing urn *H.* In situation *S*, about as many subjects prefer urn H as prefer urn *U*, with the remaining explicitly stating that they are indifferent. The majority no longer seems to strictly prefer urn H in situation *S.*

Table 5: Probit regression: probability of choosing H in Situations O

	marg. effect	std. error	p-value	pseudo $R^2 = .06$
situation S	$-.225^{***}$	.079	.006	$n = 212$
situation R	$-.095$	.077	.219	$\log L = -138.69$
first situation	.098	.086	.259	
H is default	.039	.112	.726	
between	$-.067$	.116	.563	
female	$.207^{**}$	.086	.018	

Note: Standard errors are clustered by individuals in treatment "within" and by sessions in treatment "between". A constant term is included. $^{***}(^{**})$ significant at the 1% or 5% level, respectively. Three subjects are not included in the regression because their gender information were missing

Table 5 presents the result of a probit regression where the probability of choosing urn H is explained by six dummy variables: two dummies for the situation, with situation O being the default, a dummy that indicates whether the observation stems from the first choice by an individual,[11] a dummy for the default urn being H in case of indifference, a dummy for treatment "between", and a dummy for the subject being female. Reported are percentage changes for an average participant when the respective characteristic is changed.[12] Standard errors are clustered on the individual level in treatment "within" and on the session level in treatment "between". The probability of choosing urn H is reduced by almost 22% in situation S versus situation O and this difference is highly significant. Females are more than

[11] This dummy measures any general tendency to change behavior from the first to the second choice. It is thus related but different from the order effect. The latter cannot simply be assessed with a dummy in this regression because the value of this dummy for subjects in the treatment "between" would be missing (as they only face one situation).

[12] These estimates of the marginal effects for the average participant are very similar to the average marginal effects.

20% more likely to choose urn H, a difference which is significant at the 5% level. All other dummies are not significant. In particular, the regression confirms that there are no default or portfolio effects.

In order to analyze the choice behavior in more detail, Table 6 shows a cross tabulation of choices in situation O versus choices in situation S of all subjects that have decided in both situations (i.e. in treatment "within"). From the 30 subjects that choose urn H in situation O, only 14 stick to this choice in situation S. On the other hand, 13 of the 18 subjects that choose an urn different from H in situation O continue to pick an urn different from H in situation S. These differences are significant according to an exact McNemar test ($p = 0.026$, two–sided). Let us summarize these findings.

Result 1 *Significantly fewer subjects have a strict preference for urn* H *in situation* S *than in situation* O.

This result stands in contrast with Prediction 2, according to which subjects who prefer H in situation O should also do so in situation S.

Table 6: Number of subjects choosing the urns in situations O versus S in treatment "within"

		urn choice in situation O			
		urn H	urn U	indiff	Total
urn choices in situation S	urn H	14	5	0	19
	urn U	11	4	2	17
	indiff	5	2	5	12
	Total	30	11	7	48

Table 7 shows the respective cross tabulation of urn choices in situation O

versus choices in situation R. Of the 30 subjects who prefer H in situation O, 21 continue to prefer H in situation R, while 14 of the 18 subjects who did not prefer H, continue not to prefer H. The inflows and outflows of the two groups are not significantly different according to an exact McNemar test ($p = 0.27$).

Table 7: Number of subjects choosing the urns in situations O vs. R in treatment "within"

		urn choice in situation O			
		urn H	urn U	indiff	Total
urn choices in situation R	urn H	21	4	0	25
	urn U	4	7	1	12
	indiff	5	0	6	11
	Total	30	11	7	48

Note in particular the small number of subjects who claim to be indifferent in situation R, where according to the theory a coin flip should make all subjects indifferent. Yet only 11 of the 48 subjects indicate that they are indifferent. Of the 30 subjects who expressed a preference for urn H in situation O, only 5 are made indifferent by the coin flip in situation R. If we consider all subjects (as in Table 4), only 20.5% of subjects are indifferent in situation R. This confirms (albeit weakly) earlier findings by Dominiak and Schnedler (2011) that ambiguity-averse subjects do not view randomization devices as means to overcome ambiguity.

Result 2 *The preferences for* H *in situations* O *and* R *are not significantly different. The coin flip in situation* R *makes only a few subjects indifferent* *between the two urns.*

5 Alternative models of ambiguity

In this paper, we have focused on the MEU approach to model ambiguity. Two prominent alternatives are Choquet expected utility (CEU) with convex capacities pioneered by Schmeidler (1989) and the smooth ambiguity model by Klibanoff, Marinacci, and Mukerji (2005).

Klibanoff, Marinacci, and Mukerji (2005) assume a two-stage representation where a decision maker with ambiguity in terms of multiple priors over states has beliefs represented by a probability distribution over these priors. The support of the probability distribution over priors describes the set of priors about states. It is not difficult to see that the independence notion of Assumption 1 carries over in a natural way. Maintaining Assumption 1, however, one will obtain the same predictions as in Table 3. To the best of our knowledge, there exists no thorough investigation of notions of independence for the smooth model. Hence, it must remain an open question, whether one can obtain sensible concepts of independence that would support the behavior observed in our experiment.

Interestingly, for the CEU approach of Schmeidler (1989) predictions are not necessarily the same. Independence in this setting could be represented in various ways as product capacities (see Hendon, Jacobsen, Sloth, and Tranæs, 1996). A particular well-known case is the Möbius product (for details see, e.g., Denneberg 1997). However, there is no product capacity that yields results equivalent to assuming Gilboa-Schmeidler-product-independence in the MEU approach as shown by Chateauneuf and Lefort (2008) building on Ghirardato's work (1997) on the independence of capacities.[13] While there are no obvious alternative concepts of independence for MEU, it is not difficult to find some product capacity for which the CEU representation can accommodate the behavior observed in our experiment.

6 Discussion

Our experiment examined the effect of introducing additional ambiguity to the standard two-color Ellsberg experiment. Subjects were classified as ambiguity-averse according to their behavior in a standard Ellsberg experiment. We found that many of these subjects no longer preferred betting on the urn with known probabilities if they did not know the prizes they could win (situation S in our experiment). In other words, fewer subjects preferred to bet on events with known proportions once a second source of ambiguity was in place.

The observed behavior contrasts with the predictions of various theories (MEU, smooth ambiguity) for a decision maker who regards the two sources of ambiguity as independent (in the sense of Assumption 1). In order to describe the observed behavior with these theories, we would have to impose some form of dependence. For example, a decision maker who has chosen to bet on urn H and the color black, could believe with certainty that given a

[13] See also Nehring (1999) who shows that this is true even if one of the marginal capacities is additive.

black ball is drawn, the envelope = contains 1 euro, whereas given a red ball is drawn, he is certain that the very same envelope contains 3 euro. More generally, the decision maker must believe that the probability describing the contents of the envelopes changes depending on the color of the ball drawn.

Alternatively, our findings can be accommodated by representing preferences differently, for example, by using Schmeidler's CEU approach (1989).

In any case, our paper highlights the importance of suitable definitions of independence for ambiguity models which so far may have been not fully appreciated.

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Appendix

Table 8: Urns choices in treatments SO and OS

	Urn choice				
	HH	HN	NH	NN	Total
subtreatment *SO*	8	7	4	6	25
subtreatment *OS*	6	9	1	7	23
Total	14	16	5	13	48

The first letter indicates the urn choice in situation O, the second in situation S. "H" denotes a preference for urn H, while "N" denotes no preference for urn H.

Instructions[14]

Welcome to our experiment. Please read these instructions carefully. Turn off your mobile phone, don't talk to your neighbors, and remain quiet throughout the experiment. If you have any questions, please raise your hand, and someone will come to you.

In this experiment you'll make a number of decisions. Make your decisions carefully since you can earn some money, which will be paid in cash at the end of the experiment. The decisions you are supposed to make differ for all participants somewhat. So, copying from your neighbor(s) makes no sense.

The experimenter has two bags on his table, with each bag containing 40 marbles. Each marble is either blue or green. In **Bag *H*** half of the marbles are green, and the other half are blue. For **Bag *U*** you do not know how many marbles are blue and how many are green. That is, any combination is possible for bag U, from 0 blue marbles (that is, 40 green marbles) to 40 blue marbles (that is, 0 green marbles). After completion of the experiment, you are invited to check the content of bag H and bag U.

In total, we have three situations, each of them is associated with two envelopes containing money. In each situation one of the two envelopes contains 1 euro and the other 3 euros. It depends on the situation which of the two envelopes contains 3 euros.

In each of the three situations you specify

[14] These are the instructions for treatment "within". The instructions for treatment "between" are modified in an obvious way such that subjects only have to make one choice in one of the situations.

- the bag (U or H) from which to draw a marble
- the color (blue or green) of the marble

If the marble being drawn has the color you specified, you will get the amount contained in the envelope labeled =. If the marble being drawn has not the color you specified, you will get the amount contained in the envelope labeled $\neq$.

If you do not care from which bag the ball is drawn or for a particular color, please indicate so. Since it does not matter for you, we will for simplicity take the first bag or first color, respectively.

The marbles will be drawn at the end of the experiment by one of the participants you picked. After each drawing, the marble is put back into the bag.

Situation 1

With a probability unknown to you, it was determined whether the = envelope or the $\neq$ envelope contains 3 euros. That is, you do not know whether the = envelope or the $\neq$ envelope contains 3 euros.

☐ I want the marble to be drawn from bag U

☐ I want the marble to be drawn from bag H

☐ I don't care which bag is selected

I specify the following color: ☐ blue ☐ green ☐ I don't care

Situation 2

The = envelope contains 3 euros and the $\neq$ envelope contains 1 euro.

☐ I want the marble to be drawn from bag U

☐ I want the marble to be drawn from bag H

☐ I don't care which bag is selected

I specify the following color: ☐ blue ☐ green ☐ I don't care

Situation 3

At the end of the experiment a participant will toss a fair coin. If heads wins, 3 euros are put in the $=$ envelope, and 1 euro in the $\neq$ envelope. If tails wins, the money is allocated vice versa.

☐ I want the marble to be drawn from bag U

☐ I want the marble to be drawn from bag H

☐ I don't care which bag is selected

I specify the following color: ☐ blue ☐ green ☐ I don't care