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Fundamental Uncertainty**

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Abstract

We propose a new measure of the expected variance risk premium that is based on a forecast of the conditional variance from a GARCH-MIDAS model. We find that the new measure has strong predictive ability for future U.S. aggregate stock market returns and rationalize this result by showing that the new measure effectively isolates fundamental uncertainty as the factor that drives the variance risk premium.

Keywords: Variance risk premium, return predictability, VIX, GARCH-MIDAS, economic uncertainty, vol-of-vol

JEL Classification: C53, C58, E32, G12, G17

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1 Introduction

The findings in Bollerslev et al. (2009, 2012, 2014), Bekaert and Hoerova (2014) and others strongly suggest that the variance risk premium (VRP) predicts medium-term aggregate stock market returns. Economically, the predictive ability of the VRP can be rationalized by its close relation to economic uncertainty and aggregate risk aversion (see Bollerslev et al., 2009, 2011 or Corradi et al., 2013).¹

Formally, the expected VRP is defined as the difference between the ex-ante risk-neutral expectation of future stock market variation and the statistical expectation of the realized variance. While ‘model-free implied volatilities’ can be constructed from option prices, the expected realized variance has to be estimated. The most common approaches are either to assume that the realized variance follows a martingale difference or to estimate a heterogeneous autoregressive model for the realized variance (HAR-RV). We follow a different approach by modeling the conditional variance of daily stock returns as a GARCH-MIDAS process. In this setting, the conditional variance is decomposed into a short-term GARCH component and a long-term component that is driven by macroeconomic explanatory variables. We think of the long-term component as ‘the part’ of the conditional variance of stock market returns that is driven by “uncertainty about the variability of economic prospects” (Bollerslev et al., 2013, p.417).

Our contribution to the literature on the VRP is twofold. First, we suggest a new proxy for the expected VRP that is based on the difference between the option-implied variance and the variance forecast from the GARCH-MIDAS model. We then show that the proposed measure has considerably stronger predictive power for stock returns than conventional measures of the VRP. Second, we rationalize the strong predictive power of our new measure by showing that it effectively isolates the long-term volatility component as the factor that determines the VRP.

¹Using a stylized self-contained general equilibrium model, Bollerslev et al. (2009) show that the equity risk premium can be decomposed into two terms. While the first term describes the classical risk-return trade-off, the second one suggests a positive relation between expected returns and the volatility of consumption growth volatility (vol-of-vol). The predictive ability of the VRP then follows from the observation that the VRP is proportional to the time-varying vol-of-vol.

2 A New Variance Risk Premium Measure

2.1 The GARCH-MIDAS Model

The GARCH-MIDAS model specifies the conditional variance of daily returns as the product of a short-term GARCH component that captures day-to-day fluctuations in volatility and a long-term component that is entirely driven by low-frequency (monthly) macroeconomic variables. The long-term component fluctuates at the monthly frequency only and can be considered as representing economic or fundamental uncertainty. Following Conrad and Loch (2014), we denote daily returns by $r_{i,t}$, where t refers to a certain month and $i = 1, \dots, N^{(t)}$ to the i 'th day within that month. We then assume that

$$r_{i,t} = \mu + \sqrt{g_{i,t}\tau_t}Z_{i,t}, \quad (1)$$

where $Z_{i,t}$ is IID with mean zero and variance one. $g_{i,t}$ and τ_t represent the short- and long-term conditional variances, which are measurable with respect to the information set given at day $i - 1$ of month t . The short-term component follows a mean-reverting asymmetric unit variance GARCH process

$$g_{i,t} = (1 - \alpha - \beta - \gamma/2) + (\alpha + \gamma \cdot \mathbb{1}_{\{r_{i-1,t} - \mu < 0\}}) \frac{(r_{i-1,t} - \mu)^2}{\tau_t} + \beta g_{i-1,t}, \quad (2)$$

with $\alpha > 0$, $\beta > 0$ and $\alpha + \beta + \gamma/2 < 1$. The long-term component is driven by lagged values of an explanatory variable X_t :

$$\log(\tau_t) = m + \theta \sum_{k=1}^K \varphi_k(\omega_1, \omega_2) X_{t-k}, \quad (3)$$

where the behavior of the MIDAS weights $\varphi_k(\omega_1, \omega_2)$ is parsimoniously determined using a flexible Beta weighting scheme. For a more detailed discussion, see Engle et al. (2013) or Conrad and Loch (2014).

At the last day of each month t , we use the GARCH-MIDAS (GM) model to construct out-of-sample forecasts for the realized variance during the following month, RV_{t+1} . Note that next month's long-term volatility, τ_{t+1} , is predetermined with respect to macro realizations up to month t . Then, the realized variance prediction is given by

$$\widehat{RV}_{t+1}^{GM} = \mathbf{E}_t \left[\sum_{i=1}^{N^{(t+1)}} g_{i,t+1} \tau_{t+1} Z_{i,t+1}^2 \right] = \tilde{g}_{t+1} \tau_{t+1}, \quad (4)$$

where $\tilde{g}_{t+1} = \left(N^{(t+1)} + (g_{1,t+1} - 1) \frac{1 - (\alpha + \beta + \gamma/2)^{N^{(t+1)}}}{1 - \alpha - \beta - \gamma/2} \right)$. For a given value of the monthly short-term variance, \tilde{g}_{t+1} , a high (low) value of fundamental uncertainty, τ_{t+1} , will upscale

(downscale) the forecast of the expected monthly realized variance. In this sense, τ_{t+1} is similar to the vol-of-vol factor in the model of Bollerslev et al. (2009).

2.2 Constructing the VRP

We define the monthly expected VRP as $IV_t - \mathbf{E}_t[RV_{t+1}]$, where IV_t is the risk-neutral expected variation during month $t + 1$ and $\mathbf{E}_t[RV_{t+1}]$ is the expected (under the physical measure) realized variation for that period. We build on the approximation of the expected VRP in Bollerslev et al. (2009) and measure IV_t by the end-of-month $t - 1$ value of the squared VIX and, assuming that RV_t follows a martingale difference sequence, replace $\mathbf{E}_t[RV_{t+1}]$ by RV_t . The VRP is thus given by

$$VRP_t = VIX_t^2 - RV_t. \quad (5)$$

This measure is both directly observable and model free. However, as discussed in Bekaert and Hoerova (2014), the assumption that RV_t follows a martingale difference sequence may be inappropriate. As a new measure, we propose to base the expected VRP on the conditional variance forecast from the GARCH-MIDAS model, \widehat{RV}_{t+1}^{GM} . This forecast explicitly takes into account the macroeconomic uncertainty via the long-term component:²

$$VRP_t^{GM} = VIX_t^2 - \widehat{RV}_{t+1}^{GM}. \quad (6)$$

3 Data

We use daily continuously compounded returns, $r_{i,t}$, for the S&P 500 and monthly U.S. macroeconomic data from 1970 to 2011. We include industrial production growth (annualized month-to-month percentage change), the new orders index of the Institute for Supply Management (levels) and the Chicago Fed National Activity Index (NAI).³ Annualized monthly excess returns are calculated as $r_t^{ex} = 12 \cdot (r_t - r_{f,t})$, where $r_t = \sum_{i=1}^{N(t)} r_{i,t}$ and $r_{f,t}$ denotes the one-month T-bill rate. For the 2000 to 2011 period, we employ observations for the ‘new’ VIX and daily realized volatilities, $RV_{i,t}$, based on 5-minute

²In order to predict $\mathbf{E}_t[RV_{t+1}]$, Bekaert and Hoerova (2014) estimate a HAR-RV model. However, in contrast to the GARCH-MIDAS specification, this model does not allow us to explicitly relate the predicted variance to fundamental uncertainty.

³The NAI is a weighted average of 85 monthly national economic indicators. Positive realizations indicate growth above trend, while negative realizations indicate growth below trend. Industrial production and new orders are among the indicators considered.

intra-day returns obtained from the website of the Oxford-Man Institute of Quantitative Finance. Monthly realized variances are constructed as $RV_t = \sum_{i=1}^{N(t)} RV_{i,t}$. Otherwise, all data are obtained from the FRED database at the Federal Reserve Bank of St. Louis.

4 Empirical Results

4.1 VRP Estimation

We estimate the GARCH-MIDAS models for the 1973 to 1999 period. Following Conrad and Loch (2014), we include three MIDAS lag years of the macro variables and use a restricted ($\omega_1 = 1$, i.e. strictly decreasing) Beta weighting scheme. The estimation results presented in Table 1 basically replicate the findings in Conrad and Loch (2014) but for a briefer sample. Specifically, for all variables the estimate of θ is highly significant and negative, thus confirming the counter-cyclical behavior of long-term volatility. Periods of economic growth above trend (e.g. measured by positive NAI realizations) are associated with a decline in long-term volatility, while recession periods coincide with increasing long-term volatility. We use out-of-sample forecasts for τ_{t+1} and \widehat{RV}_{t+1}^{GM} for the 2000 to 2011 period to construct our new measure of the VRP. Table 2 provides summary statistics and Figure 1 depicts the different measures of the VRP over the out-of-sample period.⁴ The table also presents summary statistics for the *ex-post* VRP defined as $VIX_t^2 - RV_{t+1}$. As expected, the VRP is positive on average. Note that the different VRP measures are much less persistent than realized volatility or the VIX squared.

4.2 Return predictability

In this section, we investigate the predictive abilities of the expected VRP measures for future stock market returns. We rely on simple monthly regressions of the form:

$$\frac{1}{h} \sum_{j=1}^h r_{t+j}^{ex} = a_h + b_h Z_t + u_{t,t+h}, \quad (7)$$

⁴Bollerslev et al. (2014) consider the same out-of-sample period, but employ a different risk-free rate in calculating the excess returns and base their RV_t measure on daily squared returns. This explains the slight differences in the summary statistics and the following return predictability regression results.

where $Z_t \in \{VRP_t, VRP_t^{GM}\}$. Following Bollerslev et al. (2014), we use Newey-West robust standard errors.⁵ Table 3 presents the regression results for different horizons h , while Figure 2 shows the estimated b_h coefficients for our VRP measures along with 90% confidence bands based on the critical values simulated in Bollerslev et al. (2014). First, based on these critical values, VRP_t significantly predicts future returns for horizons one to five. In accordance with the theoretical model developed in Bollerslev et al. (2009), the adjusted R^2 initially increases and then decreases with expanding forecast horizon. The maximum R^2 is achieved for $h = 4$ months.⁶ Second, and most importantly, all three proxies for the expected VRP based on the GARCH-MIDAS models have strong predictive power for future returns with significant regression coefficients up to the 6 months horizon. At almost all horizons, the R^2 s from these models are markedly higher than the ones based on VRP_t . In all three cases, the maximum R^2 is achieved at $h = 5$. These findings suggest that our new proxy – which explicitly takes into account the state of the macroeconomy – is a more precise measure for the ex-ante VRP than alternative proxies and, thus, has superior forecasting power for returns. In other words, using \widehat{RV}_{t+1}^{GM} as a measure of the expected variance clearly helps to “isolate the factor that drives the volatility risk premium” (Bollerslev et al., 2009, p.4485).

4.3 The Ex-post VRP and Fundamental Uncertainty

In a final step, we provide an intuitive argument for the successfulness of our new measure in predicting returns. Recall that the variance forecast from the GARCH-MIDAS model can be written as $\widehat{RV}_{t+1}^{GM} = \tilde{g}_{t+1}\tau_{t+1}$, where τ_{t+1} reflects fundamental uncertainty. Then, similarly to Bollerslev et al. (2012), we decompose the squared VIX into the expected conditional variance plus the VRP. In the model of Bollerslev et al. (2012), the VRP can be written as an affine function of fundamental uncertainty. Assuming the same relationship, we obtain:

$$VIX_t^2 = c + \widehat{RV}_{t+1}^{GM} + b^{(\tau)}\tau_{t+1} \quad (8)$$

or $VIX_t^2 - \widehat{RV}_{t+1}^{GM} = c + b^{(\tau)}\tau_{t+1}$ with some constant $b^{(\tau)} > 0$. We test this mechanism by first regressing VIX_t^2 on a constant, \widehat{RV}_{t+1}^{GM} and τ_{t+1} and, second, by regressing the *ex-post*

⁵We choose the same bandwidth in the Bartlett kernel as suggested in their paper. As shown in Bollerslev et al. (2014, p.635), given the low persistence in the VRP (see Table 2), the robust t -statistics “are reasonably well behaved” despite the overlapping nature of the return regressions.

⁶As in Bekaert and Hoerova (2014), we also considered a VRP based on conditional variance forecasts from a HAR-RV model. The corresponding R^2 s are slightly lower.

VRP on a constant, \widehat{RV}_{t+1}^{GM} and τ_{t+1} . Both should be significant in the first regression, but only τ_{t+1} in the second one. Relying on the ex-post VRP in the second regression has the advantage that we do not have to estimate $\mathbf{E}_t[RV_{t+1}]$.

Panel A of Table 4 confirms that VIX_t^2 is positively related to both \widehat{RV}_{t+1}^{GM} and τ_{t+1} . In this regression, the conditional variance forecast can be interpreted as an interaction term: the predicted effect of a change in the long-term component is stronger the higher the forecast for the short-term component is. On the other hand, in the regressions with the ex-post VRP as the dependent variable, only the long-term components are highly significant (see Panel B).⁷ Both regressions support our hypothesis that the long-term volatility components from the GARCH-MIDAS models can be considered as representing the vol-of-vol factor driving the VRP.⁸ The fact that the counter-cyclical long-term component drives the VRP also provides direct evidence for the conclusion of Campbell and Diebold (2009) that expected returns are inversely linked to expected business conditions. However, it should be noted that the R^2 s in the regressions involving the ex-post VRP are quite low. Thus, the VRP is driven by additional factors that are not directly captured by the long-term component, such as aggregate risk aversion and disagreement in beliefs. However, these factors are also likely to behave counter-cyclically and, hence, should comove with τ_{t+1} .

Finally, note that the ex-post VRP corresponds to the payoff from selling a variance swap. Thus, when τ_t is increasing, the expected payoff from selling a variance swap increases as well. Intuitively, in times of high economic uncertainty investors are willing to pay a high premium to ensure against volatility risk.

5 Conclusions

Our results strongly confirm the theoretical insight from the models discussed in Bollerslev et al. (2009, 2012) that fundamental uncertainty (the vol-of-vol) is an important factor driving the VRP. In particular, we show that our new VRP measure, which is based on a volatility component reflecting the ‘state of the macroeconomy’, has considerably higher predictive power for future stock market returns than previously suggested measures.

⁷Additionally including the lagged ex-post VRP does not change our result.

⁸Our findings are in line with Bollerslev et al. (2011) who estimate a time-varying VRP that is driven by macroeconomic state variables and report that, e.g., higher industrial production leads to a decrease in the VRP.

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6 Figures and Tables

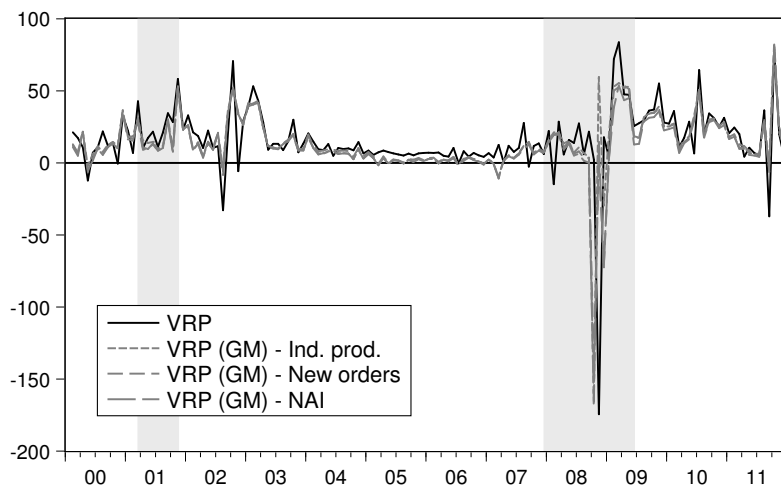


Figure 1: Different measures of the VRP for the January 2000 to December 2011 period. Shaded areas represent NBER recessions.

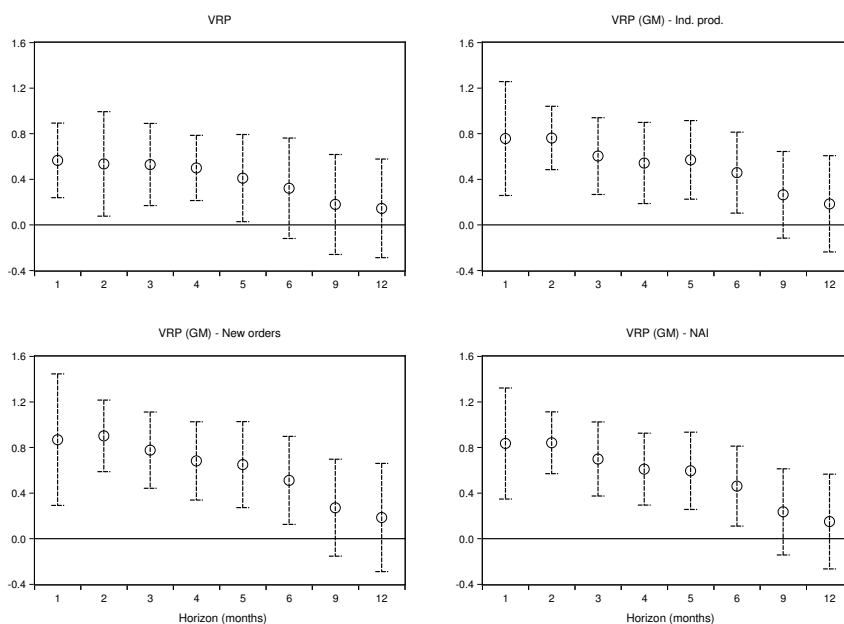


Figure 2: Estimated regression coefficients for the different VRP measures in the return predictability regressions (equation (7)) with 90% confidence bands based on Newey-West standard errors and the simulated critical values from Bollerslev et al. (2014).

Table 1: GARCH-MIDAS model estimation

Variable	μ	α	β	γ	m	θ	ω_2	LLF
Ind. prod.	0.0348*** (0.0098)	0.0253*** (0.0068)	0.9153*** (0.0239)	0.0773** (0.0305)	-0.0003 (0.1647)	-0.0531*** (0.0144)	4.2582*** (1.0124)	-8660.61
New orders	0.0339*** (0.0098)	0.0233*** (0.0069)	0.9176*** (0.0225)	0.0784*** (0.0295)	2.5077*** (0.6514)	-0.0481*** (0.0115)	4.6872** (2.0799)	-8655.37
NAI	0.0343*** (0.0098)	0.0250*** (0.0069)	0.9158*** (0.0230)	0.0782*** (0.0299)	-0.0806 (0.1657)	-0.3503*** (0.0889)	7.2203** (2.9228)	-8658.29

Notes: The table reports estimation results for the GARCH-MIDAS model including 3 MIDAS lag years of a monthly macro variable X , i.e the long-run component is specified as $\log(\tau_t) = m + \theta \cdot \sum_{k=1}^K \varphi_k(\omega_1, \omega_2) X_{t-k}$ with $K = 36$. The three variables require a restricted Beta weighting scheme with $\omega_1 = 1$, see Conrad and Loch (2014) for details. All estimations are based on daily return data from January 1973 to December 1999 and include monthly macroeconomic data beginning in January 1970. LLF is the value of the maximized log-likelihood function. The numbers in parentheses are Bollerslev-Wooldridge robust standard errors. ***, **, * indicate significance at the 1%, 5%, and 10% level.

Table 2: Summary statistics

Variable	Mean	Std. dev.	Skew.	Kurt.	AC(1)
Excess returns	-3.57	57.39	-0.58	3.89	0.15
RV	30.77	48.35	6.01	50.38	0.62
VIX²	46.82	42.35	2.89	14.28	0.81
VRP	16.02	23.89	-3.08	30.61	0.14
VRP^{GM} - Ind. prod.	14.04	21.99	-3.45	34.75	0.13
VRP^{GM} - New orders	13.27	20.78	-2.66	23.92	0.34
VRP^{GM} - NAI	11.96	21.89	-3.63	33.28	0.24
VRP ex-post	16.07	39.83	-5.02	47.65	0.26

Notes: Summary statistics for monthly excess returns and different measures of the VRP, see Section 2.2. Monthly excess returns are constructed using the one-month T-bill rate as the risk-free rate and are in annualized percentage form. Monthly realized volatility (RV) is the sum of daily realized volatilities based on 5-minute intra-day returns. VIX^2 denotes the squared 'new' VIX index in monthly units. The out-of-sample period extends from January 2000 to December 2011 and includes 144 observations.

Table 3: Return predictability regressions

Variance Premium	Horizon	1	2	3	4	5	6	9	12
VRP	Constant	-12.28 (-2.45)	-11.53 (-2.32)	-11.62 (-2.49)	-11.09 (-2.27)	-9.59 (-1.99)	-8.18 (-1.69)	-6.18 (-1.24)	-5.49 (-1.10)
	VRP	0.57 (3.91)	0.54 (3.09)	0.53 (4.42)	0.50 (5.13)	0.41 (3.91)	0.32 (2.78)	0.18 (1.77)	0.15 (1.60)
	adj. R2	4.69	7.60	11.28	12.58	9.60	6.40	2.39	2.02
VRP ^{GM} - Ind. prod.	Constant	-13.52 (-2.15)	-13.33 (-2.32)	-11.35 (-2.16)	-10.47 (-2.00)	-10.79 (-1.99)	-9.28 (-1.76)	-6.90 (-1.38)	-5.67 (-1.17)
	VRP	0.76 (3.43)	0.76 (7.20)	0.60 (5.17)	0.54 (4.81)	0.57 (5.70)	0.46 (4.47)	0.26 (2.50)	0.18 (1.76)
	adj. R2	7.62	13.78	12.66	12.75	16.44	11.78	5.12	3.11
VRP ^{GM} - New orders	Constant	-14.36 (-2.31)	-14.53 (-2.61)	-13.11 (-2.52)	-11.83 (-2.26)	-11.36 (-2.11)	-9.60 (-1.83)	-6.80 (-1.37)	-5.54 (-1.15)
	VRP	0.87 (3.40)	0.90 (7.30)	0.78 (6.20)	0.68 (5.77)	0.65 (6.12)	0.51 (4.88)	0.27 (2.43)	0.19 (1.59)
	adj. R2	9.06	17.39	19.04	18.23	19.10	13.15	4.79	2.72
VRP ^{GM} - NAI	Constant	-12.81 (-2.25)	-12.63 (-2.40)	-11.17 (-2.24)	-10.10 (-2.00)	-9.87 (-1.89)	-8.34 (-1.61)	-6.01 (-1.21)	-4.88 (-1.01)
	VRP	0.84 (3.88)	0.84 (7.91)	0.70 (6.01)	0.61 (5.82)	0.60 (6.35)	0.46 (4.83)	0.24 (2.30)	0.15 (1.43)
	adj. R2	9.33	16.79	17.07	16.17	17.81	11.80	3.86	1.77

Notes: Monthly return predictability regressions $\frac{1}{h} \sum_{j=1}^h r_{t+j}^{ex} = a_h + b_h Z_t + u_{t,t+h}$ with $Z_t \in \{VRP_t, VRP_t^{GM}\}$. In parentheses, we present t -statistics based on Newey-West standard errors, where we adjust the bandwidth in the Bartlett kernel following Bollerslev et al. (2014). The adjusted sample period extends from February 2000 to January 2011 and includes 132 observations. Adjusted R^2 in percentage form.

Table 4: The ex-post VRP and fundamental uncertainty

	c	$b^{(RV)}$	$b^{(\tau)}$	adj. R^2
Panel A: VIX² (depend. Var.)				
Ind. prod.	-18.72 (-1.71)	0.81 (6.36)	41.12 (3.44)	77.05
New orders	-10.59 (-1.35)	0.74 (8.20)	34.70 (3.77)	82.27
NAI	-7.52 (-1.07)	0.71 (6.99)	26.69 (3.74)	80.34
Panel B: Ex-post VRP (depend. Var.)				
Ind. prod.	-28.38 (-2.02)	-0.19 (-0.66)	53.42 (3.31)	5.20
New orders	-22.13 (-2.19)	-0.15 (-0.61)	45.81 (3.33)	6.10
NAI	-18.50 (-2.27)	-0.19 (-0.78)	37.26 (3.68)	6.88

Notes: Regression results for

Panel A: $VIX_t^2 = c + b^{(RV)} \widehat{RV}_{t+1}^{GM} + b^{(\tau)} \tau_t^{GM} + \xi_t$

Panel B: Ex-post $VRP_t = c + b^{(RV)} \widehat{RV}_{t+1}^{GM} + b^{(\tau)} \tau_t^{GM} + \xi_t$
with Ex-post $VRP_t = VIX_t^2 - RV_{t+1}$.

The numbers in parentheses are t -statistics based on Newey-West standard errors. The sample period extends from January 2000 to December 2011. Adjusted R^2 in percentage form.