Betting on Global Warming

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The Impact of Insurance Instruments on Mitigation of Climate Change

Inaugural-Dissertation zur Erlangung der Würde
eines Doktors der Wirtschaftswissenschaften (Dr. rer. pol.)
an der Wirtschaftswissenschaftlichen Fakultät der
Ruprecht-Karls-Universität Heidelberg

vorgelegt von
Susanne Klimpel
aus Bielefeld

Heidelberg im Oktober 2001

Acknowledgements

Especially, I would like to thank Prof. Till Requate for his support and advice during the writing of this doctoral thesis. His suggestions and critical comments have greatly improved the quality of this thesis. Besides, I would like to thank him for encouraging me to present parts of this thesis at international conferences which also delivered stimulating input.

I would also like to thank Prof. Clive Bell for his very valuable comments and helpful suggestions.

Last but not least, I am grateful to all my colleagues at the *Interdisciplinary Institute for Environmental Economics* as well as the members of the *Graduate Programme in Environmental and Resource Economics* for their helpful comments at all stages of this research. Equally important, I would like to thank them all for providing motivation during our famous "Kvg"-sessions. In particular, I am indebted to Harald Tauchmann for his moral support and patience.

Heidelberg, October 2001 Susanne Klimpel

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Chapter 1

Introduction

Since the 1980's the phenomenon of potential climate change or "global warming" has been one of the predominant topics in environmental economics. *Uncertainty* is one of the main characteristics of climate change since many important scientific relationships are not yet well enough understood. In particular, we cannot be sure how exactly anthropogenic emissions of greenhouse gases will affect future climate. Furthermore, the economic and social impacts of the uncertain future development of the climate are also highly uncertain.

Facing the risks associated with global warming different types of response could be taken. The most prominent ones are mitigation, adaptation and insurance.[1] *Mitigation* means taking measures to prevent damages from occurring or to reduce the probability of their occurrence, i.e. abatement of greenhouse gas emissions. Note that due to human activity, including mitigation activity, climate uncertainty is *endogenous uncertainty. Adaptation* is concerned with minimizing the impact of global warming, given that it occurs. In contrast, the *insurance approach* does not attempt to influence damages in any way, it merely arranges for compensation in case of damage having occurred. The term "insurance" is here used in a broad sense, including all instruments that enable agents to transfer income between different states of the world. Insurance can be advantageous if, for example, the parties involved are affected differently by global warming or if their assessments

[1] Cf. CHICHILNISKY [1994], p. 102.

of climate uncertainty differ. The latter situation will be a central issue in Chapters 5 to 7 below.

Up to date, attention has, quite rightfully, concentrated on mitigation activity. In this context, the *Kyoto Protocol* was an important step towards international cooperation with respect to mitigation of climate change, i.e. abatement of greenhouse gases. However, the process of its ratification and thereafter implementation has been seen to be extremely lengthy and controversial during the past few years. After the election of George W. Bush as President of the United States in January 2001 it became even more doubtful whether the Kyoto Protocol would ever be ratified by a sufficient number of industrialized countries. Subsequently, the administration of President Bush stated that it had *"no interest in carrying out the so-called Kyoto Protocol,"*[2] substantially threatening the process of mitigation within the framework of the Kyoto Protocol.

At present, it seems that this difficulty was overcome at the resumed *Sixth Conference of the Parties to the United Nations Framework Convention on Climate Change* (COP6-2) staged in Bonn in July 2001. Although the United States did not change their position, a sufficient number of industrialized countries expressed their intention to ratify the Kyoto Protocol. However, the compromise agreed on at COP6-2 in Bonn could only be obtained at the cost of a further slackening of the required emission targets, either directly or indirectly, for some of the key countries as, for example, Japan. In addition, the Kyoto process still remains very fragile as long as the ratification process has not been completed.

Therefore, alternative or supplementary concepts must not be neglected. This doctoral thesis introduces, analyses and discusses such a concept. It focuses on the relation between the insurance and the mitigation approach and studies the impact of insurance instruments on mitigation of climate change.[3] The main characteristic of the financial instruments considered below, called *"climate securities,"*[4] is that their payoff depends on the future

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[2]See Meller [2001].

[3]The adaptation approach is not studied here.

[4]Note that the expression "climate securities" stems from the fact that these instruments are defined in analogy to "Arrow securities". Such instruments were first introduced

state of the climate, i.e. on a set of climate-relevant data. Broadly speaking, buying or selling such assets amounts to "betting on the climate state."

Up to date, such assets do not exist in reality, but from the technical point of view they seem to be feasible as explained below in Chapter 2. Note that the use of such financial instruments seems a natural approach with respect to global warming since, on the one hand, uncertainty is one of the main characteristics of climate change and, on the other hand, the main purpose of financial markets is the allocation of uncertainty.

A large part of the analysis below is concerned, in particular, with the specific claim that financial instruments defined on the climate state can reduce free riding behaviour with respect to abatement of greenhouse gases. This claim originates from the IPCC's Second Assessment Report (SAR), specifically Arrow, Palikh and Pillet [1996] which corresponds to Chapter 2 of the contribution by Working Group III (IPCC [1996c]). Since there is no formal analysis of this claim in the literature, Chapters 5 to 7 are concerned with a formal analysis of that claim.

The claim, referred to as *"IPCC claim"* below, was inspired by a risk-bearing approach for so-called *unknown risks*. This approach, presented in more detail in Chapter 3, was developed in Chichilnisky and Heal [1992] and generalized in Klimpel and Requate [2000]. It provides a framework for the allocation of unknown risks by combining two different kinds of financial instruments, *Arrow securities*, on the one hand, and *mutual insurance contracts*, on the other hand.

In Arrow et al. [1996] the unknown risks approach is mainly discussed as an insurance approach. It is pointed out that the existing insurance markets cannot adequately handle risks associated with climate change and that appropriately designed insurance, as proposed by Chichilnisky and Heal, could improve both efficiency and equity. The authors further note that trading in securities whose payoff depends on the future climate amounts *"to betting on particular climate outcomes."*[5] Although at first sight, the idea of countries actually "betting on the future climate state" might sound strange, it is not as unrealistic as it seems. Examples for newly emerged

in Arrow [1953].

[5] Cf. also Pillet [1994].

financial instruments with similar properties are presented and discussed in Chapter 4.

In addition to the insurance point of view Arrow et. al. also claim that[6]

> *"[...] creating risk securities may provide an objective test of the honesty of national positions on the risks of climate change. If, as part of a negotiating ploy to avoid onerous abatement commitments, a country were to argue that climate change does not involve substantial risks, that country would have to be prepared to hold associated high payoff securities. Hence, there would be an economic penalty for misrepresentation of true beliefs. These penalties could offset some of the incentive to free ride on other countries' efforts to reduce greenhouse emission."*

This suggests that financial structures devised for insurance purposes could, in addition, have a positive effect on mitigation activities. In other words, the claim is that the insurance approach to climate risk encourages mitigation. Considering the difficulties experienced in international climate change negotiations throughout the process and, in particular, the difficulty of achieving the Kyoto targets without participation of the United States, this seems a very attractive prospect.

The intuition behind the IPCC claim is roughly the following. Consider a situation with uncertainty about the future climate state. For simplicity assume there are only two possible climate states. Either there is a drastic climate change or things stay roughly as they are. As argued in Chapter 2 there will be a variety of different assessments of climate uncertainty due to the large number of scientific uncertainties. In particular, different probabilities will be attributed to the two possible climate states. Since (costly) abatement of greenhouse gases has the characteristics of a public good, countries may have an incentive to misrepresent their true assessment, i.e. their type, in order to free ride on the other countries' abatement measures. The implicit assumption made at this point is that a country would not publicly admit to be concerned about climate risk and at the same time refuse

[6] Cf. Arrow et al. [1996], p. 72.

to participate in abatement measures. Then public pressure would probably become unbearable for such a country. Therefore it would rather pretend not to be concerned about climate risk in the first place, since its true assessment is not verifiable, and free ride on the other countries' abatement measures.

What is the effect of introducing climate securities in such a situation? On markets for climate securities countries want to behave according to their true type, i.e. security markets can reveal information about the countries' true assessment of the climate risk. This information can be used to generate a *trade-off* between gains from free riding and gains from trade in climate securities for potential free riders. This could be done either by establishing restrictions on security trading or by introducing a penalty[7] for free riding behaviour. Concerning their effect, the two approaches – trading restrictions or free riding penalties – are equivalent, only the interpretation differs. In Chapters 5 to 7 the trading restrictions approach is used since it seems more practical. It involves linking a country's feasible trades on security markets to its abatement behaviour. In particular, each country would be allowed to transact only such trades that are "consistent" with its abatement behaviour and the "type" represented by this behaviour. I.e. these trading restrictions would be designed in such a manner that free riders cannot profit from security markets.

The structure of this doctoral thesis is as follows. Chapter 2 summarizes the development and present status of scientific knowledge with respect to climate change. From this, implications for modelling of climate change within the context explained above are derived. Chapter 3 then introduces the unknown risks approach from which the IPCC claim originated. Chapter 4 presents some recent developments on the financial markets which show that instruments with characteristics similar to those of climate securities have already developed.

The impact of markets for climate securities on abatement behaviour is then investigated within two different modelling frameworks. The first approach focuses on *incomplete information* with respect to countries' assessments of climate uncertainty whereas the second approach concentrates

[7] In reality, such a penalty might be of an informal nature, e.g. damage to a country's reputation or retaliation regarding other issues.

on *endogenity of climate uncertainty.* Uncertainty with respect to the future climate state is common to both models.

The incomplete information approach in Chapters 5, 6 and 7 follows the intuitive argumentation for the IPCC claim above. The main feature of this approach is incomplete information with respect to countries' types which are represented by their probability distribution across the possible climate states. Abatement decisions are modelled discretely. Probabilities for climate states are considered exogenous for a large part of the analysis, although endogenizing probabilities, as done in Chapter 7.1, does not change anything substantial.

The impact of climate securities on abatement activity is studied both for a scenario with and for a scenario without regulation of security markets, in each case in comparison to a benchmark scenario without security markets. Formally, each of the scenarios defines a *Bayesian game.* In order to analyse the impact of climate securities on abatement we compare the conditions under which certain Bayesian equilibria emerge in the different scenarios. However, a shortcoming of this approach is that, due to discreteness of abatement strategies, we cannot say much about the resulting levels of abatement.

Therefore, Chapter 8 adapts a different approach for analysing the effect of insurance markets (in the sense of climate securities) on mitigation activity. The focus of the analysis is on endogenity of the climate state probabilities. Incomplete information is not assumed any longer and the abatement decision is modelled continuously. Endogenity of probabilities is the only link between the abatement decision and the portfolio decision, i.e. trading behaviour on markets for climate securities. In particular, there is no scope for regulation so that only two scenarios are considered, one with climate securities and one without climate securities.

Formally, the setup defines a non-cooperative game for each of the two scenarios. Chapter 8 compares the outcomes of non-cooperative behaviour with and without markets for climate securities with respect to abatement activity and welfare. Finally, Chapter 9 summarizes and critically discusses the analysis.

Chapter 2

Climate change

2.1 Scientific background

2.1.1 Development and organizational structure of international climate change research and policy

Since the beginning of the 1980s there has been increasing concern with respect to the potential threat of human-induced climate change. In response to this concern the *World Meteorological Organization* (WMO) and the *United Nations Environment Programme* (UNEP) established the *Intergovernmental Panel on Climate Change* (IPCC) in 1988.

According to JEPMA AND MUNASINGHE [1998], p. xi, the role of the IPCC is *"to (a) assess scientific information on climate change; (b) evaluate the environmental and socioeconomic effects of climate change; and (c) determine response strategies."* In 1990 the IPCC completed its FIRST ASSESSMENT REPORT (IPCC [1990]) which *"played an important role in establishing the* Intergovernmental Negotiating Committee for a UN Framework Convention on Climate Change *(UNFCCC) by the UN General Assembly. The UNFCCC*[1] *was adopted in 1992 and entered into force in 1994."*[2] The IPCC states that *"[i]ts Second Assessment Report Climate Change 1995 provided key input to the negotiations which lead to the adoption of the Kyoto*

[1] Cf. UNITED NATIONS [1992].
[2] Cf. IPCC [2000].

Protocol[3] *to the UNFCCC in 1997."*[4] Recently, the work summarized in the THIRD ASSESSMENT REPORT (see IPCC [2001a], IPCC [2001b] and IPCC [2001c]) has been published.

2.1.2 Development of scientific knowledge on climate change

Since the creation of the IPCC in 1988 a large amount of research has been carried out. Although there has been a lot of progress, numerous and large uncertainties with respect to many aspects of climate change still remain.

The FIRST ASSESSMENT REPORT (IPCC [1990]) predicted a global warming estimate of about 1°C, but did not find evidence of this effect being human-induced: *"Thus the observed increase could be largely due to this natural variability; alternatively this variability and other human factors could have offset a still larger human-induced global warming."*

The SECOND ASSESSMENT REPORT (SAR) already emphasized that *"[t]he balance of evidence suggests a discernible human influence on global climate,"* although acknowledging *"uncertainties in key factors."* It predicted an increase in global mean temperature in the range of $1°C$ to $3.5°C$ (see IPCC [1990a,b,c]).

The recently published contribution of IPCC Working Group I to the THIRD ASSESSMENT REPORT (TAR) projects an even larger rise in globally averaged surface temperature between $1.4°C$ and $5.8°C$ (cf. IPCC [2001a], "Summary for Policymakers", p. 13). Moreover, it concludes that *"[t]here is new and stronger evidence that most of the warming observed over the last 50 years is attributable to human activities."* With respect to uncertainty it comments *"[a]lthough many of the sources of uncertainty identified in the SAR still remain to some degree, new evidence and improved understanding support an updated conclusion."*

"New evidence" includes the observations of climate-relevant data made

[3]Cf. UNITED NATIONS [1997].

[4]The SECOND ASSESSMENT REPORT (SAR) consists of of the contributions of Working Group I, (IPCC [1996a]), Working Group II, (IPCC [1996b]) and Working Group III, (IPCC [1996c]).

since the previous Report. The THIRD ASSESSMENT REPORT comments on a large number of relevant parameters and events in addition to global-average surface temperature, e.g. the global average temperature of the lowest 8 km of the atmosphere, snow cover, ice extent, global-average sea level, global-ocean heat content, precipitation, cloud cover, the frequency of extremely low temperatures, warm episodes (El Niño), storm intensity and frequency etc.

The contribution of IPCC Working Group II to the TAR (IPCC [2001b]) emphasizes regional diversity of potential climate change. It notes that according to the results of Working Group I *"the warming would vary by region."*[5] Concerning the overall effects for different regions the Working Group II Report concludes:[6]

> *"[...] increases in global mean temperature would produce net economic losses in many developing countries for all magnitudes of warming studied [...]. In contrast, an increase in global mean temperature of up to a few degrees C would produce a mixture of economic gains and losses in developed countries [...], with economic losses for larger temperature increases [...]."*

Therefore, developing countries are likely to be adversely affected by global warming, no matter what the magnitude of the warming is. In contrast, for developed countries there can be gains for smaller magnitudes of warming which then turn into economic losses for larger magnitudes of warming.

With respect to the global effects the Working Group II Report concludes that[7]

> *"[...] world GDP would change by ± a few percent for global mean temperature increases of up to a few degrees C [...] and increasing net losses would result for larger increases in temperature [...]. More people are projected to be harmed then benefited by climate change, even for global mean temperature increases of less than a few degrees C [...]."*

[5] Cf. IPCC [2001b], "Summary for Policymakers", p. 3.
[6] Cf. IPCC [2001b], "Summary for Policymakers", p. 8.
[7] Cf. IPCC [2001b], "Summary for Policymakers", p. 8.

We see that concerning the global effects of climate change the Report differentiates between the effect on world GDP, on the one hand, and the effect with respect to how many individuals will be harmed. For world GDP the sign of the effect is ambiguous for smaller magnitudes of warming and turns negative as the magnitude of warming increases. However, concerning the effect on individuals, the number of individuals harmed will be larger than of individuals benefiting no matter how the magnitude of the warming is, according to the Working Group II Report.

2.1.3 The present status of scientific knowledge on climate change

Comparing the conclusions reached in the three IPCC Assessment Reports, it is evident that potential climate change is a more serious problem than initially expected. The projected increases of globally averaged surface mean temperature have increased substantially since 1990. It has to be acknowledged that scientific knowledge with respect to climate change has improved. In particular, we can now be relatively sure that it is human-induced to some extent. Nevertheless a wide range of uncertainties remains. This current state of affairs was summarized as follows by Robert Watson, chairman of the IPCC, in his *Report to the Sixth Conference of the Parties (COP 6) to the UNFCCC* (cf. WATSON [2000a]) on November 20, 2000:

> *"As you debate the weighty issues associated with effective implementation of the Convention and the Kyoto Protocol let me remind you that the overwhelming majority of scientific experts, whilst recognizing that scientific uncertainties exist, nonetheless believe that human-induced climate change is already occurring and that future change is inevitable."*

In a different presentation,[8] also at COP 6, Mr. Watson said:

> *"The question is not whether climate will change in response to human activities, but rather how much (magnitude), how fast (the rate of change) and where (regional pattern)."*

[8] See WATSON [2000b].

Section 2.2 below examines the implications of this widely accepted[9] point of view for modelling of climate risk.

2.2 Implications for modelling climate risk

This section discusses the implications of the present status of scientific knowledge with respect to climate change for modelling climate risk. First of all, the long list of climate-relevant parameters in Section 2.1.2 shows that the "state of the climate" is a highly multi-dimensional variable. This should be kept in mind with respect to the definition of climate states. From the present status of scientific knowledge, four further characteristics of climate risk emerge, endogenity, subjectivity and incomplete information with respect to climate risk assessments and regional diversity. Each of these issues is discussed in detail in the following.

2.2.1 Endogenity of climate risk

As outlined in Sections 2.1.2 and 2.1.3 above, scientists are increasingly certain that climate change is human-induced. Human activity, in particular emissions of greenhouse gases or, equivalently, abatement of such emissions affects future climate change. Therefore, one important characteristic of climate risk is that it is *endogenous*. This is also emphasized by CHICHILNISKY AND HEAL [1993]. Obviously, the extent of endogenity depends on the time horizon under consideration. In the short run, exogenously given climate risk may be an appropriate approximation. In the long run, however, endogenity of climate risk should be taken into account. For this reason a model which focuses on endogenity of climate risk is presented and analysed in Chapter 8. An extension of the incomplete information model to endogenous probabilities is considered in Section 7.1.

[9] Although the IPCC documents are the most widely accepted documents on the subject of climate change there are divergent opinions.

2.2.2 Subjectivity of climate risk assessments

A second characteristic aspect of climate risk is that the remaining scientific uncertainties lead to *subjective* evaluations of the existing scientific evidence. This is reflected in the careful formulations in documents like the IPCC ASSESSMENT REPORTS or in the speech by the IPCC chairman (WATSON [2000a]), already mentioned in Section 2.1.3 above. A study by GRANGER MORGAN AND KEITH [1995] titled "Subjective Judgements by Climate Experts" conducted with 16 leading US climate scientists concluded:

> *"The results reveal a rich diversity of expert opinion and, aside from climate sensitivity, a greater degree of disagreement than is often conveyed in scientific consensus documents. Research can make valuable contributions, but we interpret our results to mean that overall uncertainty about the geophysics of climate change is not likely to be reduced dramatically in the next few decades."*

Since all scientific knowledge concerning climate change is subject to a certain level of uncertainty there is room for differing interpretations. Moreover, the aggregation of results from different studies leaves more scope for subjective evaluations of climate risk. Subjectivity of assessments of climate risk is catered for in both models studied below.

2.2.3 Incomplete information with respect to climate risk assessments

In particular, the subjective evaluations of the available scientific evidence cannot be verified by third parties. This leads to *incomplete information* with respect to the true assessment of the parties involved concerning the climate risk. Then, misrepresentation of the true evaluation for strategic reasons becomes an option. I.e. countries may conceal their true assessment of climate risk and, for example, profess not to be concerned about adverse effects of climate change in order to avoid mitigation expenses. Due to the nature of the situation it is, of course, difficult to identify such behaviour.

It seems that some of the statements made by US President George W. Bush soon after he came into office in which he expressed an opinion completely contrary to that of the IPCC chairman might be an example for such behaviour. An article from the *New York Times*[10] comments on March 13, 2001: *"Under pressure from conservative Republicans and industry groups, President Bush reversed a campaign pledge today and said his administration would not seek to regulate power plants' emissions of carbon dioxide, a gas that many scientists say is a key contributor to global warming."* In a letter[11] to four Republican senators who where leading in the protest against Bush's original plans, Bush said:

> *"As you know I oppose the Kyoto Protocol because it exempts 80 percent of the world, including major population centers such as China and India, from compliance, and would cause serious harm to the U.S. economy. The Senate's vote, 95-0, shows that there is a clear consensus that the Kyoto Protocol is an unfair and ineffective means of addressing global climate change concerns."*

He continued to argue that according to a recent Energy Department study regulating carbon dioxide emissions would have led to *"significantly higher electricity prices"* which he considered *"important new information"*, warranting a re-evaluation.

> *"At a time when California has already experienced energy shortages, and other Western states are worried about price and availability of energy this summer, we must be very careful not to take actions that could harm consumers. This is especially true given the incomplete state of scientific knowledge of the causes of, and solutions to, global climate change and the lack of commercially available technologies for removing and storing carbon dioxide."*

Note that Mr. Bush's interpretation of the available scientific evidence is completely contrary to that expressed by the IPCC chairman, Robert Watson.

[10] Cf. Jehl [2001].

[11] Cf. Bush [2001].

In particular, Mr. Bush takes scientific uncertainties as an argument against mitigation of climate change and refuses to acknowledge the present status of scientific knowledge that there is already a discernible human influence with respect to global climate change.[12] Compared with the views expressed by the IPCC it seems that Mr. Bush, influenced by strong lobbying activity, deliberately chooses a very different interpretation of the scientific evidence. If he acknowledged considering immediate and substantial mitigation activities to be necessary, the international community would demand from the United States that they take over an appropriate share of that activity. Therefore, Bush may choose to question the scientific evidence for strategic reasons in order to avoid the consequences of acknowledging the scientific evidence as sufficient for immediate action. However, due to subjectivity and incomplete information it is not possible to prove that he is misrepresenting his true evaluation of the global warming problem for strategic purposes.

2.2.4 Regional diversity

According to the IPCC Third Assessment Report[13] global warming is expected to vary by region:

> *"The vulnerability of human populations and natural systems to climate change differs substantially across regions and across populations within regions. Regional differences in baseline climate and expected climate change give rise to different exposures to climate stimuli across regions."*

Therefore, *regional diversity* is a further characteristic of climate change. In principle, many regions can experience both adverse and beneficial effects through global warming. Projected *adverse impacts* include, for example,[14]

- a reduction in potential crop yield in most tropical and sub-tropical regions, and, for strong warming, also in most regions in mid-latitudes,

[12] Cf. Section 2.1.3 above.
[13] Cf. IPCC [2001b], "Summary for Policymakers", p. 14.
[14] Cf. IPCC [2001b], page 4.

- decreased water availability for populations in many scarce water regions,
- an increase in the number of people exposed to vector-borne diseases (e.g. malaria) and water-borne diseases (e.g. cholera) and an increase in heat stress mortality,
- an increase in the risk of flooding for many human settlements from both increased heavy precipitation events and sea-level rise,
- increased energy demand for space cooling due to higher summer temperatures.

Projected *beneficial impacts* include, for example,

- an increase in potential crop yields in some regions ad mid-latitudes for moderate increases in temperature,
- a potential increase in global timber supply from appropriately managed forests,
- increased water availability in some water scarce regions, e.g. in parts of South East Asia,
- reduced winter mortality in mid- and high-latitudes,
- reduced energy demand for space heating due to higher winter temperatures.

It is stressed by the authors of IPCC [2001b] that *"less-developed regions are especially vulnerable because a larger share of their economies are in climate-sensitive sectors and their adaptive capacity is low due to low levels of human, financial, and natural resources, as well as limited institutional and technological capability."*

For most regions the net impact is predicted to be an adverse one, in particular, if more than a moderate warming takes place. However, in contrast to most environmental problems the overall effect of global warming may be beneficial for some regions. Therefore, models of climate risk should allow for varying effects of global warming, and, in particular, the possibility of a few regions actually benefiting from global warming.

Chapter 3

Unknown Risks

The analysis in Chapters 5 to 8 below is concerned with the effect of insurance instruments on mitigation of climate change. In particular, Chapters 5 to 7 investigate whether the *IPCC claim* holds true, i.e. whether securities defined on the climate state, later called *climate securities*, can reduce free-riding behaviour with respect to abatement of greenhouse gases.[1] As explained in the Introduction this claim was inspired by a risk-bearing approach for so-called *unknown risks* which was developed in CHICHILNISKY AND HEAL [1992] and slightly modified in CHICHILNISKY AND HEAL [1998]. This approach provides a framework for the allocation of unknown risks by combining two different kinds of financial instruments, *Arrow securities*,[2] on the one hand, and *mutual insurance contracts*, on the other hand. However, CHICHILNISKY AND HEAL's main result is only shown for identical beliefs which is a major deficiency in the context of unknown risks. In KLIMPEL AND REQUATE [2000] the approach of CHICHILNISKY AND HEAL is generalized to include the case of differing beliefs.

This chapter summarizes the unknown risks approach analysed in CHICHILNISKY AND HEAL and KLIMPEL AND REQUATE in order to illustrate how the use of climate securities as insurance instrument in Chapters 5 to 8 was inspired by that approach. The presentation is based on the more general results in KLIMPEL AND REQUATE [2000].

[1] This was claimed in ARROW, PALIKH AND PILLET [1996] which is part of the IPCC's SECOND ASSESSMENT REPORT.

[2] Such instruments were first introduced in ARROW [1953].

3.1 Introduction to Unknown Risks

In recent years, a number of new risks have emerged, e.g. the risks of global warming, ozone depletion or BSE, and, more recently, the foot and mouth disease. These risks are hard to manage since we know too little about them. In particular, it is not possible to associate objective probabilities with different possible consequences since there is too little data available. As pointed out in CHICHILNISKY AND HEAL [1998] some of these risks, e.g. global warming, are even *unknowable,* i.e. there can be no repetition of such events.

CHICHILNISKY AND HEAL study exchange economies with unknown risks in the following sense. Each household faces the risk of being in one of Σ individual states. The risk is unknown, but each household has a subjective probability distribution across the states of the world. In principle, the framework of Chapter 7 in DEBREU [1959] can be applied, i.e. a complete set of contingent markets would lead to a Pareto efficient allocation. However, CHICHILNISKY AND HEAL emphasize that *"this approach may be impracticable as the number of markets needed with individual risks [...] rises exponentially with the number of agents in the economy."* The reason is that a contingent market approach requires decisions over the complete enumeration of all possible combinations of individuals and states over the whole population of an economy.

Therefore CHICHILNISKY AND HEAL propose an alternative framework[3] which uses a mix of two types of financial instruments, *mutual insurance contracts*, on the one hand, and *Arrow securities*, on the other hand. They argue that agents face two types of uncertainty, collective uncertainty with respect to the overall distribution of the risk, and – given the overall distribution – the remaining individual risk. The overall distribution of the unknown risk can be described by the corresponding *statistical state*. The statistical state specificies the distribution of individuals across the set of individual states, but neglects their identity. Arrow securities deal with the collective aspect of the unknown risk, i.e. which statistical state emerges, whereas mutual insurance contracts insure the individual risk given a certain overall distribu-

[3] Such a framework is also used in CASS, CHICHILNISKY AND WU [1996].

tion, i.e. given a certain statistical state. The decomposition of the unknown individual risk is illustrated in Figure 3.1.

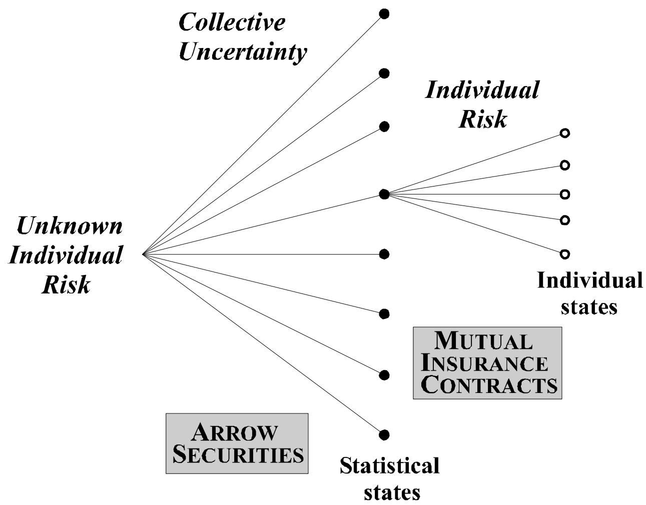

Figure 3.1: *Illustration of the decomposition of the unknown individual risk into collective uncertainty with respect to the statistical state and individual risk given the statistical state. Each component of the unknown individual risk is dealt with by the appropriate financial instrument.*

For identical beliefs Chichilnisky and Heal show that in a competitive contingent equilibrium each agent's equilibrium consumption depends on the statistical state only. Building on this result, they further show that contingent market equilibria – known to be Pareto-efficient – can also be supported in a framework with Arrow Securities and mutual insurance contracts.

Obviously, the assumption of identical beliefs is extremely critical in the context of unknown risks. However, in Klimpel and Requate [2000] the above results are generalized to the case of differing beliefs. Moreover, some flawed results for the case of different beliefs in Chichilnisky and Heal [1992] and Chichilnisky and Heal [1998] are corrected. Therefore, even with differing beliefs contingent market equilibria can be achieved by mutual insurance contracts and Arrow securities.

A similar decomposition is introduced for catastrophic risks in Schlesinger [1999]. Schlesinger decomposes losses into two multiplicative components, one independent among insureds and one highly correlated. Magill

AND SHAFER [1992] also use a related approach to the allocation of collective and individual risks.

3.2 The Model

Consider a pure exchange economy with H consumers and denote the set of consumers by $\tilde{H} := \{1, \dots, H\}$.

States

Three different types of *states,* defined below, will be used in the analysis: *individual states, collective states* and *statistical states.*

Let $\tilde{\Sigma} := \{1, \dots, \Sigma\}$ be the set of *individual states*[4] that can occur for an individual consumer. Initial endowments depend on the individual state only, thus reflecting the individual component of the unknown individual risk. Moreover, it is assumed that this dependance is the same for all households.

The *collective state* of the economy can be described by a list of the individual states of each consumer, i.e. an $H-$tuple $\omega = (\sigma_1, \dots, \sigma_H)$ with $\sigma_h \in \tilde{\Sigma}$ for all $h \in \tilde{H}$. Formally, a collective state ω is a function $\omega : \tilde{H} \to \tilde{\Sigma}$ with $h \mapsto \omega(h)$. Therefore, $\Omega := \{\omega \mid \omega : \tilde{H} \to \tilde{\Sigma}\}$ is the set of collective states. Clearly, $|\Omega| = \Sigma^H$.

The overall distribution of the unknown risk is characterized by how many consumers are in a certain individual state, whereas *who* is in which individual state is not relevant from the collective point of view. Recall that by definition, $\omega(h)$ is the individual state of consumer h in the collective state ω, and define $\tilde{H}^\omega_\sigma := \{h \in \tilde{H} \mid \omega(h) = \sigma\}$ as the set of all consumers who find themselves in the individual state σ when the collective state is ω. Then the sets $(\tilde{H}^\omega_\sigma)_{\sigma \in \tilde{\Sigma}}$ form a partition of $\tilde{H}$, i.e. $\cup_{\sigma \in \tilde{\Sigma}} \tilde{H}^\omega_\sigma = \tilde{H} \quad \forall \omega \in \Omega$. Now define for $\sigma \in \tilde{\Sigma}$

$$r_\sigma(\omega) := \frac{|\tilde{H}^\omega_\sigma|}{H} \tag{3.1}$$

as the proportion of all consumers who are in the individual state σ when the collective state is ω. Clearly, $\sum_{\sigma \in \tilde{\Sigma}} r_\sigma(\omega) = 1$ by (3.1) and the definition of

[4] Without loss of generality this set is assumed to be the same for all consumers.

$\tilde{H}^{\omega}_{\sigma}$ so that $r(\omega) := (r_1(\omega), \ldots, r_\Sigma(\omega)) \in \Delta^\Sigma$ is the distribution[5] of consumers over the individual states in the collective state ω. We call $r(\omega)$ a *statistical state* since it contains the statistical information about how many consumers are in a certain individual state, but does not specify *who* is in which individual state. Let $R := r(\Omega) \subset \Delta^\Sigma$ be the set of statistical states. By use of simple combinatorics it is easy to prove that $|R| = \binom{H+\Sigma-1}{\Sigma-1}$. Clearly, $|R| < |\Omega|$ for $\Sigma \geq 2$.[6]

Given a certain statistical state, there is only individual risk. The unknownness of the individual risk is reflected in the model by consumers having different beliefs about the statistical state. Summing up, the unknown individual risk is represented by *collective uncertainty* about the statistical state and *individual risk* for a given statistical state.

Probability distributions across collective and statistical states

Now turn to the subjective beliefs about the states of the world.[7] Let Π^h denote the subjective probability distribution of consumer h over the set of collective states Ω. Assume that $\Pi^h_\omega := \Pi^h(\omega) > 0$ for all $h \in \tilde{H}$ and $\omega \in \Omega$. The following assumption about the probabilities Π^h, originating in MALINVAUD [1973] and being recalled by CHICHILNISKY AND HEAL, is important for the main results.

Assumption 1 ***(Anonymity Assumption)***
For each consumer $h \in \tilde{H}$, we have

$$\Pi^h_\omega = \Pi^h_{\hat{\omega}} \qquad \forall \omega, \hat{\omega} \in \Omega \quad \textit{with} \quad r(\omega) = r(\hat{\omega}). \tag{3.2}$$

By the Anonymity Assumption any two collective states leading to the same statistical state are considered equally likely by all consumers. Defining $\Omega_r := \{\omega \in \Omega \mid r(\omega) = r\}$ for $r \in R$, the Anonymity Assumption implies[8] that the

[5] Δ denotes the unit interval $[0, 1]$.

[6] Note that $|R|$ is a *polynomial* in H, whereas $|\Omega|$ increases *exponentially* with H. This will be of importance later on when we need financial assets for each state.

[7] For convenience we start with subjective probability distributions over collective states from which we will later derive the distributions over statistical states.

[8] There should be no confusion by using the same symbol for the function $r(\cdot)$ and a statistical state $r \in R$.

subjective probabilities are the same for all states in Ω_r, i.e.

$$\Pi^h_\omega = \Pi^h_{\hat{\omega}} =: \Pi^h_r \qquad \forall \omega, \hat{\omega} \in \Omega_r, \forall h \in \tilde{H}. \tag{3.3}$$

Note that $\dot{\bigcup}_{r \in R} \Omega_r = \Omega$ by definition of Ω_r. The Anonymity Assumption is illustrated in Figure 3.2.

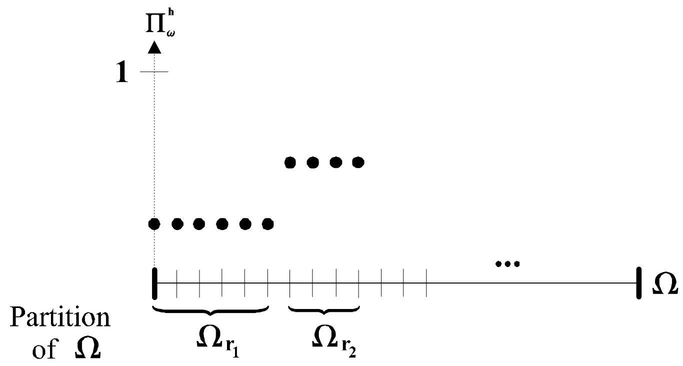

Figure 3.2: *Illustration of the Anonymity Assumption. The probabilities Π^h_ω are constant within each subset Ω_r of the collective state set Ω.*

Concerning the conditional probability $\Pi^h_{\sigma|r}$ of consumer h for being in the individual state σ given the statistical state r we have the following

Lemma 1

$$\Pi^h_{\sigma|r} = r_\sigma \qquad \forall h, \sigma, r. \tag{3.4}$$

The proof is given in Appendix A. By Lemma 1 the conditional probability $\Pi^h_{\sigma|r}$ of consumer h for being in the individual state σ given the statistical state r equals the proportion of consumers in the individual state σ (given the statistical state r).

Consumption sets and preferences

Let $L \in \mathbb{N}$ be the number of commodities. Within a contingent markets framework the commodity space formally becomes $\mathbb{R}^{L\Sigma^H}$, and the consumption set of consumer h is $\mathbb{R}^{L\Sigma^H}_+$. Let $z_h = (z_{h\omega})_{\omega \in \Omega} \in \mathbb{R}^{L\Sigma^H}_+$ denote a consumption vector of consumer h and $z_{h\omega} \in \mathbb{R}^L_+$ the consumption of h in the collective state ω. Further let $e^h_\sigma = e_\sigma \in I\!R^L_{++}$ be the initial endowment of consumer h when she or he is in the individual state $\sigma (h \in \tilde{H}, \sigma \in \tilde{\Sigma})$. Then the initial endowment e^h_ω of consumer h in the collective state ω is given by $e^h_{\omega(h)} \in I\!R^L_{++} (h \in \tilde{H}, \omega \in \Omega)$. Accordingly, $e^h := (e^h_{\omega(h)})_{\omega \in \Omega} \in I\!R^{L\Sigma^H}_{++}$ defines the initial endowment of h over all collective states.

Assumption 2
The preferences of consumer h can be represented by a utility function $U^h : I\!R^{L\Sigma^H} \longrightarrow I\!R$ with

$$U^h(z_h) := \sum_{\omega \in \Omega} \Pi^h_\omega u^h(z_{h\omega}) \tag{3.5}$$

where $u^h : I\!R^L_+ \longrightarrow I\!R$ is twice continuously differentiable, strictly increasing and strictly concave for all $h \in \tilde{H}$.

Thus, u^h is a utility function of the von Neumann-Morgenstern type whereas U^h denotes the expected utility function of consumer h. Assumption 2 implies that the consumer demand correspondences are continuous functions.[9]

3.3 Properties of Contingent Market Equilibrium

Although contingent markets are not an appropriate framework for unknown risks, their well–known properties can be exploited for the main result, presented in Section 3.4 below. Consider a complete system of contingent markets with regard to the set of collective states Ω. The well–known results concerning existence of equilibria and their properties then hold. In particular, contingent market equilibria are Pareto–efficient by the *First Welfare Theorem*. However, the disadvantages of the contingent market approach are also obvious. Assuming the existence of a complete system of contingent markets with respect to the state space Ω is tantamount to considering an

[9]See Aliprantis, Brown and Burkinshaw [1990], Theorem 1.3.8 on page 24.

Arrow–Debreu economy with $L\Sigma^H$ commodities. Since the number of markets rises exponentially with the number of consumers, a complete system of contingent markets is unlikely to be established.

Proposition 1 below, which generalizes Proposition 1 of CHICHILNISKY AND HEAL [1998], shows that – given the assumptions above – there is some redundancy in contingent market equilibrium in the sense that equilibrium prices and allocations are constant for each statistical state, i.e. within certain groups of collective states.

Proposition 1
Let $(p^*, z^*) = ((p^*_\omega)_{\omega\in\Omega}, (z^*_{h\omega})_{\omega\in\Omega, h\in\tilde{H}}) \in I\!R^{L\Sigma^H}_{++} \times I\!R^{L\Sigma^H H}_{+}$ *be a contingent market equilibrium. Write* $z^*_\omega := (z^*_{h\omega})_{h\in\tilde{H}} \in I\!R^{LH}_{+}$ *and* $z^*_h := (z^*_{h\omega})_{\omega\in\Omega} \in I\!R^{L\Sigma^H}_{+}$. *Then*

$$z^*_\omega = z^*_r \qquad \forall\omega \in \Omega_r, \forall r \in R, \tag{3.6}$$

$$\text{and} \qquad p^*_\omega = p^*_r \qquad \forall\omega \in \Omega_r, \forall r \in R. \tag{3.7}$$

The proof is given in Appendix A. In contrast to CHICHILNISKY AND HEAL [1998], identical beliefs are *not* assumed in KLIMPEL AND REQUATE [2000]. This extends the range of application of the model considerably, because if risks are *unknown*, it is extremely unlikely that all consumers have the same, possibly wrong probability beliefs.

3.4 Economies with Securities and Mutual Insurance Contracts

Recall from Section 3.2 that the unknown individual risk has a collective and an individual component. Whereas securities are suitable for collective uncertainty, the adequate way of dealing with individual risk is the use of insurance markets. For this reason CHICHILNISKY AND HEAL introduce the appropriate kind of asset for each component of the unknown individual risk. *Arrow Securities* defined on statistical states enable consumers to insure themselves against the collective uncertainty with respect to the statistical state, whereas the remaining individual risk can be insured by a suitable set of *mutual insurance contracts* contingent on the statistical state.

Assume that there is a complete system of Arrow securities $(\alpha_r)_{r\in R}$ where "α" stands for "**A**rrow Securities". Security α_r is a contract paying one unit of the numéraire–good if the statistical state turns out to be r and nothing otherwise.[10] Assume that security markets are competitive and denote the price of security α_r by $q_r \in I\!R_+$ for $r \in R$. Define the vector of security prices as $q := (q_1, \dots, q_{|R|}) \in I\!R_+^{|R|}$.

Denote by $s_r^h \in I\!R$ the amount of security α_r bought (or sold in case of $s_r^h < 0$, respectively) by consumer h. For $h \in \tilde{H}$ define $s^h := (s_1^h, \dots, s_{|R|}^h) \in I\!R^{|R|}$ and $s := (s^1, \dots, s^H) \in I\!R^{|R|H}$.

Mutual insurance is modelled as follows. For each statistical state r consumer h chooses a vector of insurance transfers $m_r^h := (m_{1r}^h, \dots, m_{\Sigma r}^h) \in I\!R^{\Sigma}$, where $m_{\sigma r}^h \in I\!R$ is the transfer received (or paid) by consumer h if the statistical state is r and his individual state is σ. Each consumer chooses a tuple $m^h := (m_1^h, \dots, m_{|R|}^h)$ of insurance vectors.

Assumption 3 *Consumer h can choose his tuple of insurance vectors from the set*

$$M^h := \left\{ m^h \in I\!R^{\Sigma|R|} | \sum_{\sigma=1}^{\Sigma} \Pi_{\sigma|r}^h m_{\sigma|r}^h = 0 \quad \forall r \in R \right\}. \tag{3.8}$$

By Assumption 3, given a statistical state r, consumer h can choose any insurance vector m_r^h with contingent expectation of zero. This means, the insurance vectors have to be *actuarially fair*.

An *economy with securities and mutual insurance contracts* satisfying the above requirements will be denoted $\mathcal{E}_{SI}$ in the following. The budget set of consumer h in $\mathcal{E}_{SI}$ at spot market prices p and security prices q is given by

$$B_{SI}^h(p, q; e^h) := \left\{ (z_h, s^h, m^h) \in I\!R_+^{L\Sigma^H} \times I\!R^{|R|} \times I\!R^{\Sigma|R|} \;\middle|\; \begin{array}{c} (z_h, s^h, m^h) \quad \text{satisfies} \\ (3.9), (3.10), (3.11) \end{array} \right\},$$

where

$$p_\omega(z_{h\omega} - e_\omega^h) = s_{r(\omega)}^h + m_{\omega(h)\, r(\omega)}^h \quad \forall \omega \in \Omega, \tag{3.9}$$

$$\sum_r q_r s_r^h = 0, \tag{3.10}$$

$$\sum_\sigma \Pi_{\sigma|r}^h m_{\sigma r}^h = 0 \qquad \forall r \in R. \tag{3.11}$$

[10] Formally, α_r is defined as the $r-$th unit vector in $I\!R^{|R|} (r \in R)$.

Equation (3.9) represents the budget constraints of consumer h on the spot markets in the collective states where the right hand side consists of income (or obligations) from the portfolio of securities and the mutual insurance vector. Equation (3.10) ensures that his or her portfolio is self-financing (there are no endowments in securities) whereas (3.11) requires the insurance vectors to be actuarially fair.

Sometimes, only the first component of the tuples contained in the budget set is relevant. Hence define the projection onto this component

$$\hat{B}^h_{SI}(p,q;e^h) := \left\{ z_h \in I\!R^{L\Sigma^H}_+ \;\middle|\; \begin{array}{c} \exists s^h \in I\!R^{|R|}, m^h \in I\!R^{\Sigma|R|} \\ (z_h, s^h, m^h) \in B^h_{SI}(p,q;e^h) \end{array} \right\}.$$

Finally, we formally define the notion of equilibrium in an economy with securities and insurance. Doing this we follow the intuition presented in the papers by Chichilnisky and Heal [1992] and Chichilnisky and Heal [1998] where no formal definition was given.

Definition 1 *An equilibrium in an economy with securities and mutual insurance contracts (equilibrium in $\mathcal{E}_{SI}$) is a tuple $(p^*, q^*, z^*, s^*, m^*) \in I\!R^{L\Sigma^H}_{++} \times I\!R^{|R|}_{++} \times I\!R^{L\Sigma^H H}_{+} \times I\!R^{|R|H} \times I\!R^{|R|\Sigma H}$ satisfying*

$$\begin{aligned} z^*_h &\in \arg\max \left\{ U^h(z_h) | z_h \in \hat{B}^h_{SI}(p^*, q^*; e^h) \right\} && (3.12)\\ \textit{and} \quad (z^*_h, s^{h*}, m^{h*}) &\in B^h_{SI}(p^*, q^*; e^h) \quad \textit{for all } h \in \tilde{H}, && (3.13)\\ \textit{as well as} \quad \sum_{h=1}^{H} (z^*_h - e^h) &= 0 && (3.14)\\ \textit{and} \quad \sum_{h=1}^{H} s^{h*}_r &= 0 \qquad \forall r \in R. && (3.15) \end{aligned}$$

Here, (3.12) requires utility maximization with respect to the budget set $B^h_{SI}(p^*, q^*; e^h)$ at equilibrium prices.[11] Equation (3.14) guarantees market clearing on the spot markets whereas (3.15) ensures market clearing on the security markets.

[11] Note that (3.11) and (3.12) guarantee that in each statistical state the sum of premia and payments are balanced, i.e. $\sum_{h,\sigma} \Pi^h_{\sigma|r} m^{h*}_{\sigma r} = 0 \quad \forall r \in R$. So this need not be required in the definition of equilibrium.

3.5 Supporting Contingent Market Equilibria in Economies with Securities and Mutual Insurance

This section shows that each contingent market equilibrium can be supported as an equilibrium in the corresponding economy with securities and mutual insurance. Since the $\mathcal{E}_{SI}$-equilibrium is defined on a larger space, we have to "extend" the contingent market equilibrium by defining security equilibrium prices, equilibrium portfolios and insurance vectors for consumers such that we get an equilibrium in $\mathcal{E}_{SI}$. In contrast to Chichilnisky and Heal [1998] equal beliefs are not assumed, generalizing their main result considerably.

If (p^*, z^*) is a contingent market equilibrium, then contingent market equilibrium consumption only depends on the statistical state by equation (3.6) in Proposition 1. Therefore write $z_r^{h*} := z_{h\omega}^*$ for $r = r(\omega)$.

Proposition 2
Let $(p^*, z^*) \in \mathbb{R}_{++}^{L\Sigma^H} \times \mathbb{R}_{+}^{L\Sigma^H H}$ be a contingent market equilibrium. Define $q^* \in \mathbb{R}_{++}^{|R|}, s^* \in \mathbb{R}^{|R|H}$ and $m^* \in \mathbb{R}^{\Sigma|R|H}$ by

$$q_r^* := |\Omega_r| \qquad \forall r \in R, \tag{3.16}$$

$$s_r^{h*} := \sum_{\sigma=1}^{\Sigma} \Pi_{\sigma|r}^h p_r^* (z_r^{h*} - e_\sigma) \qquad \forall r \in R, \forall h \in \tilde{H}, \tag{3.17}$$

$$m_{\sigma r}^{h*} := p_r^*(z_r^{h*} - e_\sigma) - s_r^{h*} \qquad \forall r \in R, \forall \sigma \in \tilde{\Sigma}, \forall h \in \tilde{H}. \tag{3.18}$$

Then $(p^*, z^*, q^*, s^*, m^*)$ is an equilibrium in the corresponding economy with securities and mutual insurance contracts.

The proof is given in Appendix A. Proposition 2 is constructive in the sense that it not only proves that contingent market equilibria can be supported in economies with securities and mutual insurance, but also states explicitely *how* portfolios and insurance vectors as well as security prices have to be chosen. The equilibrium allocation of securities and insurance can be interpreted as follows.[12] s_r^{h*} is the expected value of the excess demand of consumer h conditional on being in the statistical state r, so that on

[12] See Chichilnisky and Heal [1992] and Chichilnisky and Heal [1998].

average his budget is balanced in the state r. Of course, the actual value of the excess demand can differ from the expected value. The mutual insurance contracts are designed to make up for these differences.[13]

Proposition 2 allows several important conclusions. First of all, it tells us that it is possible to have Pareto-efficient equilibria in $\mathcal{E}_{SI}$. More precisely, if we have an $\mathcal{E}_{SI}$-equilibrium induced by a contingent equilibrium in the sense of Proposition 2, then the allocation of goods in this equilibrium is Pareto-efficient. Considering this Proposition in connection with the Second Welfare Theorem, we can conclude that every Pareto-efficient allocation can be supported (if necessary after a suitable redistribution of endowments) in $\mathcal{E}_{SI}$. So the analogue to the Second Welfare Theorem holds in economies with securities and mutual insurance, whereas this is not true for the First Welfare Theorem. (From Proposition 2 we can only conclude that a certain kind of equilibrium in $\mathcal{E}_{SI}$ is Pareto-efficient.)

Proposition 2 also implies that there are always at least as many equilibria in $\mathcal{E}_{SI}$ as in the underlying contingent economy. In particular, the existence of contingent equilibrium implies the existence of equilibrium in the corresponding $\mathcal{E}_{SI}$-economy. On the other hand, if the contingent market equilibrium is not unique, the E_{SI}-equilibrium cannot be unique either.

3.6 Summary

The unknown risks approach decomposes the unknown individual risk into a collective and an individual component and introduces the appropriate financial instrument for each of these components. The main result is that with a combination of securities and mutual insurance contracts a contingent market equilibrium can be supported as an equilibrium with respect to the adequate equilibrium concept for such an economy. Therefore, the *analogue to the Second Welfare Theorem* holds for such an economy, i.e. Pareto-efficient allocations can potentially be decentralized.

In principle, one might consider applying the unknown risks approach to

[13] $m^{h*}_{\sigma r}$ is the difference between the actual value of the excess demand if the statistical state is r and the individual state σ and the expected value of the excess demand for the state r.

global warming. That would involve, firstly, defining an appropriate set of *(global) climate states,* corresponding to the statistical states. Classification of the climate states should take into account all relevant parameters, for example those listed in Section 2.1.2 above. Securities would then be defined with respect to these global climate states. Mutual insurance contracts would be written conditional on the occurrence of a certain global climate state, arranging for compensation between differently affected parties.

Whereas securities written on global climate states seem feasible, as argued in Chapter 4 below, a complete set of mutual insurance contracts, contingent on climate states, would be unlikely to be established. Therefore, the approach, although an improvement relative to the contingent markets benchmark, still lacks in practicability.

Recall also that this approach is primarily an insurance approach. It is aimed at achieving an efficient allocation of climate risk, i.e. the unknown individual risk, but does not attempt to mitigate climate risk. Since at the present stage climate policy is rightfully focused on mitigation of climate change this approach may be better suited for future use at a later point in time.

However, thoughts on the application of this approach to climate change prompted the *IPCC claim*[14] that such an insurance approach could also encourage mitigation. If that claim was proved to be true, this would substantially increase the attractivity of the related financial instruments, i.e. primarily climate securities, for present use.

[14] Cf. the Introduction.

Chapter 4

Relevant new developments on financial markets

At present, the financial instruments analysed in Chapter 5 to Chapter 8 below, there called *climate securities,* do not (yet) exist in reality. Nevertheless, recent developments on the international financial markets suggest that "betting on the future climate state" could, in principle, become reality in the near future. During the last ten years several types of new financial instruments have emerged, each of them sharing a number of the characteristics with the climate securities studied below.

This chapter introduces some of these new financial instruments, focusing on weather derivatives and insurance derivatives. Moreover, it compares the characteristics of these instruments to those assumed for climate securities below. Since climate securities are basically a simple kind of "climate derivatives", there are many similarities. Weather derivatives are the instruments that seem most similar to climate securities and are therefore presented first.

4.1 Weather derivatives

4.1.1 Development of markets for weather risk

Weather risk is the uncertainty with respect to profits due to weather volatility. The weather has an impact on a large number of industries. In particular utility and energy companies can face an unusually low demand for energy due to a cool summer (less need for air conditioning) or a mild winter (less need for heating). Other industries, ranging from agriculture and construction, over transportation, tourism and the catering trade to the film industry, are also subject to substantial weather risks. According to the US Department of Energy[1] about 15% of the US GDP is weather dependent. For Europe, estimates yield similar figures.

Therefore it is not surprising that instruments for hedging against weather risk, so called *weather derivatives,* have emerged during the past five years. SANDOR [2000] defines weather derivatives as *"risk management tools based on a variety of metrics such as temperature or precipitation."* These instruments enable firms that could be adversely affected by weather fluctuations to hedge against this risk.

Weather derivatives first developed in 1997 in the United States.[2] The weather derivative market was jump started during the El Niño winter of 1997-98, one of the strongest such events on record. At that point, many companies, faced with the possibility of significant earnings declines because of an unusually mild winter, decided to hedge their seasonal weather risk. Since then the market has grown rapidly. The convergence of capital markets and insurance markets observed in recent years is one of the drivers of the development of the market for weather derivatives. Besides, increasing importance of shareholder value has also driven companies' increasing desire for balance sheet protection against the weather.[3]

In the beginning, weather derivatives were traded on *over-the-counter* (OTC) markets, i.e. each contract was negotiated individually. Since September 1999 *standardized* weather derivative contracts are traded at the *Chicago Mercantile Exchange* (CME). Both standardized instruments and over-the-counter transactions are described in more detail in Section 4.1.2 below. Recently, a number of online exchanges specializing in standardized

[1] Cf. BANHAM [1999].

[2] SANDOR [2000] notes that *"[a]lthough their roots lie in agriculture and catastrophic events (such as hurricanes), weather derivatives achieved a separate identity about three years ago [i.e. 1997]."*

[3] Cf. MUELLER AND GRANDI [2000], p. 1.

weather derivatives have been launched and the *London International Financial Futures and Options Exchange* (LIFFE) also plans to start trading weather derivatives soon.

4.1.2 Description of weather derivatives instruments

Heating Degree Days and Cooling Degree Days

Heating Degree Days (HDDs) and *Cooling Degree Days* (CDDs) are the most typical reference parameters for weather derivatives. HDDs are a measure of a day's coldness. The *Daily Heating Degree Day* (Daily HDD) is the number of degrees by which the day's average temperature[4] is below a base temperature. The base temperature is usually 65 degrees Fahrenheit,[5] but sometimes 75 degrees Fahrenheit in warmer climates. The daily HDD is calculated by subtracting a day's average temperature from 65 degrees Fahrenheit (which corresponds to 18 degrees Celsius), i.e. one defines

$$\text{Daily HDD} \quad := \quad \max \quad (0, 65^\circ \text{ F} - \text{daily average temperature}).$$

For example, an average daily temperature of 40 degrees Fahrenheit results in a daily HDD of 25, whereas for average daily temperatures above 65 degrees F the daily HDD is zero (no heating is required). The *CME HDD Index* is an accumulation of daily HDDs over a calender month. For example, assume the average daily HDDs for a city in the month of November were 25. With 30 days in the month of November, the HDD index would then be 750 (25 daily HDDs x 30 days).

Similarly, a Cooling Degree Day (CDD) measures the warmth of a day relative to the 65° F benchmark temperature and is defined by

$$\text{Daily CDD} \quad := \quad \max \quad (0, \text{daily average temperature} - 65^\circ \text{ F}).$$

[4] The *daily average temperature* is measured as the average between the daily high and daily low.

[5] The choice of 65° F can be explained as follows. The utility industry uses 65° Fahrenheit as a baseline because years ago that was the temperature at which furnaces would be switched on. Now it is used as a benchmark with the assumption being that for each degree below 65°, consumers will use more energy to heat their homes and for each degree above 65°, they will consume more energy to run their air conditioners.

The *CME CDD Index* is defined accordingly.[6] The definition of the Daily HDD and the Daily CDD is illustrated in Figure 4.1.

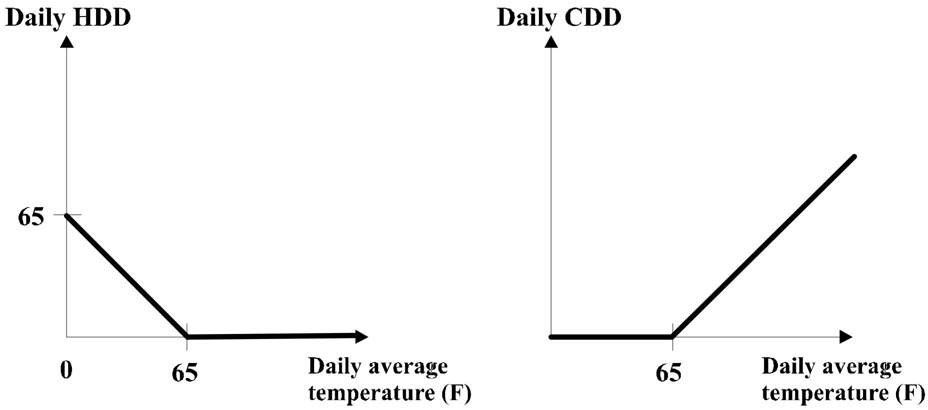

Figure 4.1: *Illustration of the Daily HDD and the Daily CDD.*

Standardized instruments

As mentioned above the *Chicago Mercantile Exchange* (CME) was the first institution to offer standardized weather derivatives. The CME offers futures and options based upon indices of *Heating Degree Days* (HDDs) and *Cooling Degree Days* (CDDs) for selected population centers and energy hubs with significant weather related risks throughout the United States.[7] Currently, these contracts are available for ten cities in the United States.[8]

The *CME HDD/CDD Index futures contracts* are legally binding agreements to buy or sell the value of the HDD/CDD Index at a specified future date. In contrast to other futures contracts which specify physical delivery of the underlying product they are *cash settled*. The HDD/CDD futures have a notional value of $100 times the CME HDD or CDD Index and are quoted in

[6] The indices are calculated by *Earth Satellite Corporation*, an international services firm specialized in the development and application of remote sensing and geographic information technologies. The HDD/CDD data is available, updated daily, on the CME website, cf. CME [2001].

[7] See CME [2001].

[8] Cities are chosen based upon population, the variability in their seasonal temperatures and the activity seen in over-the-counter trade in HDD/CDD derivatives.

HDD/CDD Index points. The minimum *tick size*[9] is one index point. Each index point has a value of 100 dollars.

Continuing the above example, with an HDD index of 750 for November, the nominal value of a futures contract on that city would be $75,000 (750 HDD index x $100). The nominal value of the futures contract for this example is illustrated in Figure 4.2.

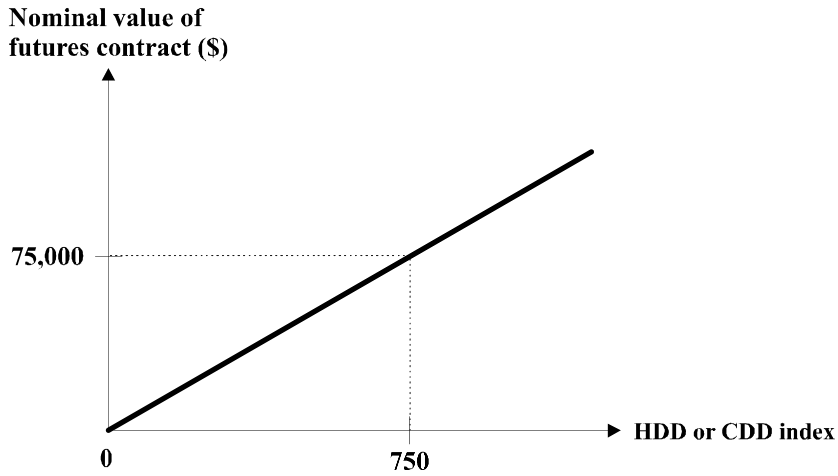

Figure 4.2: *Illustration of the nominal value of a HDD or CDD index futures contract for the case in which each index point is worth 100 dollars. The slope of the value function is determined by the value of the index points, i.e. here it is equal to* 100.

Options on CME HDD/CDD Index futures are defined in the standard manner. For example, the purchase of an HDD call option gives the buyer the right, but not the obligation, to buy one HDD futures contract at a specific price, the strike or exercise price. The options are *European Style options,* which means that they can only be exercised at expiration. The HDD/CDD futures and options are traded on CME's electronic GLOBEX®2 system.

Alternatively, weather derivatives contracts may be given the structure of a call option without referring to an underlying CME Index future. Consider, for example, a call option defined with respect to the HDD as underlying. Assume a strike of 500 with respect to the HDD index and a payout of $100

[9]The minimum tick size is the smallest tradable unit.

per degree day, i.e. if the value of the HDD index at the expiry date is larger than 500 the investor receives \$100 for each index point above the strike value. The resulting payoff structure is depicted in Figure 4.3.

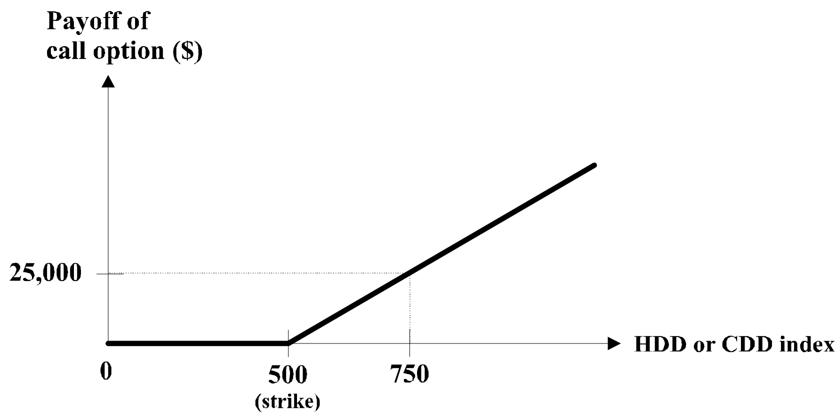

Figure 4.3: *Illustration of the payoff a HDD or CDD call option for the case in which each index point is worth 100 dollars. The strike is set at 500 index points.*

Note that as emphasized by MUELLER AND GRANDI [2000] *"[w]eather derivatives differ from conventional derivatives in that there is no original negotiable underlying or price of an underlying, which normally forms the basis of any derivative. For example, financial derivatives are based on shares, share indices, bonds, exchange rates or currencies – all of which are themselves negotiable objects, unlike the weather."*

Since it is impossible to put a price on the weather, weather derivatives are mainly aimed at hedging the *volumetric risk* arising from weather volatility, for example the risk of declining sales in the energy and power sector. However, depending on the industry under consideration the volumetric change in demand will obviously also influence the price of the good provided. Whereas the price for ice cream would probably not be affected by a decline in demand due to a cool, wet summer, the fuel price reacts sensitively to a greater demand for fuels in cold winters.

Over-the-counter transactions

This section presents two examples for over-the-counter weather derivatives transactions. Although the energy sector is the most active sector with respect to weather derivatives other sectors have also participated in innovative transactions. For example, *Massive Limited*, owner of 26 Pubs and Restaurants in London, completed a transaction in order to protect revenues from its most weather sensitive site, "The White Swan" which is situated in Twickenham beside the Thames, against unusually cool weather. The contract pays out if the number of Fridays and Saturday between April and July, which are above or below a reference temperature, reaches a certain level.[10] For example, the White Swan is protected if the number of Fridays and Saturday in April that are above 14 C $^\circ$, and in May that are above 18 C$^\circ$, are too few. It is also protected if there are too many Fridays and Saturday in June that are below 18 C$^\circ$, and in July that are below 20 C$^\circ$.

The second example is interesting for two reasons. The transaction is one of the first weather transactions in Germany and, moreover, one of the first transactions protecting business revenues against rainfall instead of against extreme temperatures. The policy provided financial protection to German utility *Elektrizitätswerk Dahlenburg AG* against excessive rainfall during summer months. Excessive rainfall makes the utility's revenues decline since many of their customers are farmers who use electric power to pump water during periods of insufficient precipitation. The power supplied to the farmers represents an important part of the company's revenues, thus motivating the transaction. The contract[11] ran from May 1, 2001 to August 31, 2001 and the reference level of rainfall was 70 mm of precipitation per square meter. For each millimeter of rain above the reference level the utility was entitled to receive approx. 2000 DM, whereas the premium for the contract was about 10.000 DM.

[10] Cf. press release by SPEEDWELL WEATHER DERIVATIVES [2000] who structured the transaction.

[11] Cf. NDR [2001], an article on weather derivatives related to a programme on the topic by NDR, a TV station in Northern Germany.

4.1.3 Comparison of weather derivatives and climate securities

This section compares the characteristics of weather derivatives and climate securities with respect to the reference parameters, the state space and payoff structure, the time horizon and the geographical range.

Concerning the reference parameters, the focus in weather derivatives up to date is on temperature-related transactions, although there has been a small number of transactions defined with respect to precipitation or wind. Temperature is certainly one of the most important climatic characteristics, but only one amongst a large number of important climate metrics (cf. Chapter 2.1.2). A description of the climate state requires a **combination** of many relevant factors, also including precipitation, wind etc. Whereas the reference parameters for weather derivatives are usually one-dimensional, a climate index would have to be highly multi-dimensional.

Obviously, the standardized weather derivatives instruments, e.g. HDD or CDD index options and futures, have a different state space and payoff structure by definition than the highly stylized climate securities.[12] Recall that standardized weather derivatives all have a continuous state space whereas the state space for climate securities in the following chapters will be discrete.[13] Moreover, the payoffs for climate securities will be normalized, yielding a structure as depicted in Figure 4.4 for each of the instruments. However, the differences concerning state space and payoff structure could be eliminated by defining climate derivatives analogously to weather derivatives.

Evidently, the time-horizon for weather derivatives is completely different to the time-horizon that would be appropriate for climate securities. Weather derivatives are traded on a monthly, at most a seasonal basis whereas climate securities would have to be defined with respect to long-term developments of the climate-relevant parameters.

Weather derivatives also differ from climate securities with respect to

[12] For over-the-counter weather derivatives this can be different for each individual transaction, therefore a general comparison is not possible.

[13] For simplicity there will be only two possible states.

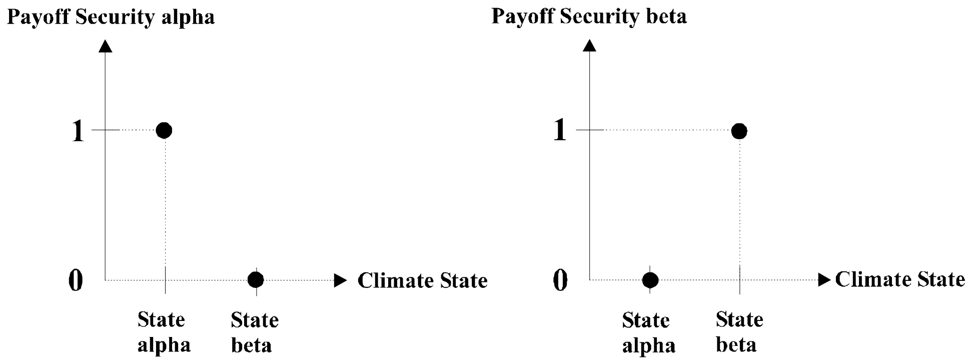

Figure 4.4: *Illustration of the state space and payoff structure of the climate securities analyzed below in Chapters 5 to 8. In analogy to Arrow-Securities the state space is discrete and payoffs are normalized.*

their geograhical range since they refer to one certain location only whereas climate-data would have to be collected on a world-wide basis.

Summarizing, although there are a number of differences between weather derivatives and climate securities the basic structure of the instruments, being defined with respect to the underlying weather or climate index, is very similar.

Furthermore, the rapid increase of the trading volume in weather derivatives during the last few years shows that the technology and the expertise that would be needed in order to define an appropriate climate index is already available. Therefore trading with climate securities or more sophisticated climate derivatives should, in principle, not pose a serious problem in the near future.

The following section discusses a different type of financial instruments, insurance derivatives, which are also defined with respect to parameters that can be considered as climate-relevant data.

4.2 Insurance derivatives

4.2.1 History

Insurance derivatives were first introduced in December 1992 by the *Chicago Board of Trade*. They were directed at insurance and reinsurance companies as means of hedging their underwriting risk. In particular, at that point in time the insurance industry needed additional capital sources to participate in insuring the financial risk posed by natural catastrophes.[14] Initially, both catastrophe insurance futures and options were traded.

The *catastrophe insurance future* (CAT-future) was based on an index determined by the US *Insurance Services Office* (ISO). The index used data from selected insurance companies within the property/casualty insurance industry concerning the amount of claims having occurred within the relevant period.[15] However, O'Brien [1997] remarks that *"CAT-futures failed to generate much interest and trading was halted in 1995."* Vorm Christensen and Schmidli [2000] agree and also explain why CAT-futures were not successful: *"The product never became popular with private investors. The reasons were that the index was announced only once before the settlement date, there was information asymmetry between insurers and investors, and that there was a lack of realistic models."*

Options based on CAT-futures were more successful, but also exhibited some inherent problems. Therefore, ISO catastrophe insurance options were replaced by the so-called *PCS Options* in September 1995. "PCS" stands for *"Property Claims Service"* which is an independent authority. PCS Options are discussed in more detail in the following section. They were traded at the Chicago Board of Trade, but trading was stopped at the end of the year 2000. I.e. PCS options did not gain sufficient acceptance even though a number of improvements, explained in detail below, were implemented relative to ISO options. It seems that the increasing number of securitizations of catastrophe risk, discussed in Section 4.3 below, was a more attractive method to insurers and reinsurers and thus rendered PCS Options superfluous.

[14] See O'Brien [1997], p. 53.

[15] Note that only claims having been *reported* to the insurance companies before a certain date entered the calculation of the index.

4.2.2 PCS Options

Like the HDD/CDD Index Options the PCS Options were also *index options*,[16] based on the *PCS-index.* The PCS-index measured *"the amount of catastrophe claims in the region and period of the contract"*, according to O'Brien [1997]. Each index point represented 100 million dollars in catastrophe losses. The underlying futures contracts themselves were not traded anymore. The options based on the PCS index were defined in a standard way, i.e. the buyer of a call option has the right, but not the obligation, to buy the underlying futures contract at the pre-specified strike price. Since, in principle, the structures are analogous to those of weather derivatives, presented in Section 4.1 above, and PCS options are not traded anymore, graphical illustrations and further details are omitted here.

There were nine different contracts, one covering the entire territory of the United States (National), several regional contracts (East, Southeast, Northeast, Midwest, West) and contracts specifically covering those states with high catastrophe risk (Florida, Texas, California).

The loss period was a quarter for most contracts with the exception of the Western and California contracts which had annual loss periods and the National contract which traded on both a quartely and an annual basis.

An important improvement relative to the ISO options was that the PCS-index was announced daily. Moreover, the development period had been lengthened from three months (for ISO options) to either six or twelve months. Most of the trading involved the longer development period. The predominant trading pattern was the "call spread", i.e. simultaneous buying and selling of of call options with different strike values.[17] Such a call spread enables *layered* covers similar to the layers of non-proportional reinsurance.

The pricing of these options was very complex which may be a further reason why the trading volumes were relatively small. Therefore, a widely accepted pricing approach as, for example, the *Black and Scholes model* for

[16] An *index option* is an option whose underlying security is an index rather than a stock as for stock options. If an index option is exercised, settlement is made by cash payment, since physical delivery is not possible.

[17] Cf. Kielholz and Durrer [1997], p. 7.

stock options,[18] had not emerged during the first years of trading for ISO and PCS options.[19]

4.2.3 Comparison of insurance derivatives and climate securities

This section compares the characteristics of insurance derivatives to those of climate securities with respect to the same criteria as in Section 4.1.3 above.

Note that the reference parameters underlying the PCS options, i.e. the catastrophe damage indices, can also be interpreted as climate-related data in the following sense although the link is not as obvious as with weather data. On the one hand, scientists attribute the higher frequency and larger impact of natural catastrophes during the last decades to global warming, at least to some extent.[20] On the other hand, the PCS indices measure the impact of natural catastrophes for a certain region and time period. They represent an indirect method of measuring catastrophe impacts since only insured damages enter the calculation. But since insured damages will, in general, be highly correlated with total damages the PCS indices represent a good approximation of catastrophe damages. Therefore, the PCS indices are an indirect way of measuring climate-related data, and thus PCS options were financial instruments with payoffs dependant on climate-relevant data. Similarly as for the case of weather derivatives, the reference parameters for insurance derivatives only represent a very small subset of the parameters relevant for the climate state.

Concerning the state space and payoff structure the comparison to climate securities yields the same results as that for weather derivatives, since the

[18] Cf. Black and Scholes [1972].

[19] Geman and Yor [1997] discuss several of the pricing approaches for catastrophe options and suggest a modified approach. Later, Flasse et al. [1999] discuss the applicability of the traditional option pricing approaches to the case of PCS Options and also suggest an alternative approach to valuation of PCS Options.

[20] See, for example, Watson [2000b]: *"Indeed, during the last few years many parts of the world have recently suffered major heat-waves, floods, droughts and extreme weather events leading to significant loss of life and economic costs. While individual extreme weather events cannot be directly linked to human-induced climate change, the frequency and magnitude of these types of events are expected to increase in a warmer world."*

structures of the insurance derivatives strongly ressemble those of weather derivatives.

With respect to the time horizon the comparison also yields similar results to those in Section 4.1.3 above. The loss period for PCS options was either 3 months or one year whereas the adequate period for measuring climate change would have to be substantially longer. Geographically, PCS options covered, at most, the United States, whereas climate securities would have to be defined globally.

Summarizing, we also observe similarities between insurance derivatives and climate securities. In particular the reference parameter underlying the insurance derivatives would also be one of the relevant parameters for climate securities. However, insurance derivatives seem to be less relevant than weather derivatives with respect to the importance of the underlying parameters for measurement of climate development.

4.3 Securitizations

One way of dealing with insurance risks or weather risks is to hedge against these risks by means of insurance derivatives and weather derivatives, respectively, as discussed in Sections 4.1 and 4.2 above. An alternative way of responding to insurance or weather risks is their *securitization,* i.e. transferring these risks to the capital markets. With respect to insurance derivatives it seems that, as noted above in Section 4.2, the market preferred the securitization method.

Basically, securitization is the process of aggregating, re-packaging and re-selling similar risks to capital market investors in order to share these risks among a large number of agents. Some examples for securitizations of catastrophe risks and weather risks are given in the following sections. However, it should be kept in mind that for purposes of comparison with climate securities securitizations are less relevant than derivative instruments since the climate securities analysed in the following chapter can also be interpreted as derivative instruments.[21] The interesting point in this section

[21] Note that the similarity in the expressions "climate securities" and "securitization" is mis-leading here. The expression "climate securities" stems from their formal similarity to

is with respect to which parameters of weather risk or insurance risk the securitizations are defined.

4.3.1 Securitizations of weather risk

Since the market for weather risks has only emerged a few years ago the number of securitizations of weather risk is still limited. The first weather-related securitization that took place was a 50 million dollar weather bond issued in November 1999[22] by Houston-based *Koch Energy Trading*, working with underwriter *Goldman Sachs*.[23] PERIN [1999] explains the details: *"Koch's weather bond is a three-year transaction that transfers risk from a fixed portfolio of 28 weather derivative contracts based on temperatures in 19 US locations. The portfolio is diversified in terms of season, geographical region and direction of temperature deviation."*

Basically, such a weather bond constitutes an exchange of *principal*[24] for periodic coupon payments and the return of the principal. In the case of the Koch weather bond both the payment of the interest **and** the return of the principal of the bond are linked to the occurrence of the insured event, i.e. the performance of the underlying portfolio of weather risks.

Since that first weather-related securitization there have been a number of further transactions, most of them relating to windstorm.[25]

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the well-known concept of "Arrow securities" whereas "securitization" is a technical term from the finance vocabulary.

[22] See SANDOR [2000] and PERIN [1999].

[23] Structuring the deal took a long time and in the end it had to be scaled down from the originally planned 227 million dollars to the 50 million dollars that were finally issued.

[24] The *principal* of a bond is the amount initially invested in the bond by the investors or, equivalently, the amount borrowed from the investors by the issuer. Usually, investors provide the principal for the duration of the bond in return for receiving the principal plus interest payments at the end of the duration. Sometimes, only the interest is at risk, i.e. depends on the underlying risk, sometimes both the interest and the principal are at risk.

[25] Windstorm can be considered as lying on the border between weather risk and catastrophe risk.

4.3.2 Securitizations of catastrophe risk

Securitizations of catastrophe risk, usually by means of a *catastrophe bond* (CAT bond), have been far more prominent than securitizations of weather risk. In principle, the structure of a CAT bond is the same as that of a weather bond. SHEPHERD [1999] gives the following definition: *"A CAT bond constitutes an exchange of principal for periodic coupon payments wherein the payment of the coupon and/or the return of the principal of the bond is linked to the occurrence of an insured event."* The insured event can refer to hurricanes, earthquake or other natural perils as well as combinations of these perils. If the actual catastrophe losses surpass a specified amount, called *trigger*, the bondholders lose some or all payments of interest or principal, depending on the specifications of the contract.[26] In that case, the issuing insurance company can pay claims with the funds that would otherwise have gone to the bondholders.[27]

In 1999 the US *Insurance Services Office* study ISO [1999] already commented that *"[A]t least ten insurers have used catastrophe bonds to obtain protection against losses caused by hurricanes, earthquakes, or other perils. For example, in 1997 a special-purpose reinsurer, Residential Re, sold 477 million dollars in catastrophe bonds and used the proceeds to provide reinsurance protection to the United Services Automobile Association. That same year, another special-purpose reinsurer, SR Earthquake Fund Ltd., issued 137 million dollars in catastrophe bonds and provided reinsurance for California earthquake losses to Swiss Reinsurance Company."*

Similarly, in May 1999 the operator of Tokyo Disneyland, *Oriental Land Company*, issued a bond with the aim of hedging against earthquake risk.[28] It was intended to cover a loss in cash flow due to a decline in visitors following a potential earthquake rather than the direct physical damage, e.g. the collapse

[26] Naturally, the interest payments for CAT bonds where the principal is at risk are higher than those where only the coupon payments have to be forfeited when the trigger point is reached. Often such bonds are split into several tranches to offer investors a broader spectrum of riskiness.

[27] Cf. the study by the *Insurance Services Office,* ISO [1999].

[28] This deal is described in an article from *The Nikkei Weekly* (see EQECAT [1999]), put online by a *EQECAT*, a US company that took charge of the risk assessment for the deal.

of buildings.[29]

4.3.3 Comparison with climate securities

A general comparison of instruments like weather bonds or CAT bonds to climate securities is not feasible since each securitization transaction has its individual characteristics and features. In most cases, weather bonds and CAT bonds each have only one climate-relevant parameter as reference parameter although recently there have been transactions based on two different parameters, e.g. windstorm and earthquake.

Concerning the payoff structure weather bonds and CAT bonds can be more similar to climate securities than derivative instruments are. Broadly speaking, for the parties buying or selling such bonds the state space can be interpreted as consisting of two different states, one where the trigger is not activated and one where the trigger is surpassed.[30] With this interpretation, the state space is similar to that for climate securities.

With respect to the time horizon the comparison again depends on each transaction, but since a number of multi-year transactions have been completed recently the difference in time horizon can, potentially, be smaller than with standardized derivative instruments.

The same applies to the geographical range of the transactions which also varies widely. Initially, transactions were defined with respect to one geographical region only, but in order to improve diversification it has recently become fashionable to include two or more different geographical regions, e.g. Europe and Japan, in one transaction.

Summarizing, on the one hand the individuality of such securitizations opens up the possibility of transaction structures that are relatively similar to

[29] The Oriental Land bond consists of two types of bonds, a *principal-risk* type and a *credit-risk* type. The principal-risk type entitles Oriental Land to receive a percentage of the principal of the bond if a big earthquake occurs within a certain radius, whereas the credit-risk-switch bond provides Oriental Land with the option of borrowing from the special-purpose company if a quake occurs.

[30] Of course, the state in which the trigger is surpassed may contain an interval of "sub-states" if, for example, the contract specifies differentiated payoffs depending on the extent to which the trigger was surpassed.

those of climate securities. On the other hand, climate securities would have to be standardized instruments which is the one aspect that securitizations lack.

4.4 Summary

This section introduced and discussed three different types of innovative financial instruments which exhibit some of the characteristics that will be attributed to climate securities below. Although at present there are no financial instruments exhibiting *all* the characteristics that would be attributed to climate securities the financial instruments discussed in this section share some of the characteristics of climate securities.

In particular, the experience gained from these innovative transactions shows that the expertise that would be required for the introduction of climate securities is already available. Therefore, such instruments seem to be feasible from the technical point of view. The difficult and possibly controversial task due to climate being a highly multi-dimensional variable would be the appropriate definition of the state space for climate securities, i.e. the definition of "climate states."

Chapter 5

Incomplete Information about Types

Recall from Chapter 2 that due to the wide range of remaining scientific uncertainties countries' assessments of climate risk will necessarily be subjective. In particular, such assessments are not verifiable by third parties so that there is *incomplete information* with respect to other countries' assessments of climate risk. This opens up an "easy way out" with respect to abatement of greenhouse gases. A country could profess not to believe that global warming is a serious threat and thus free ride on other countries' abatement activities. The *IPCC claim,* explained in more detail in the Introduction, is that in such a situation markets for climate securities can reduce free riding.

An incomplete information approach for modelling the IPCC claim is presented in this chapter and the following two chapters. This chapter presents the model and the general analysis, whereas Chapter 6 illustrates the results for the case of two countries and also presents some examples. Chapter 7 is concerned with extensions and limits of the approach.

As explained in the Introduction there is no formal analysis of the IPCC claim in the literature. In particular, there is almost no strongly related literature. However, Bac [1996] has a number of features in common with the approach below. Bac studies incentives to free ride on international environmental resources. He considers a two-country infinitely repeated dynamic game of transboundary pollution with incomplete information about valuations. In each constituent game countries first decide whether to participate in abatement or not and, in the affirmative, how much to abate.

BAC investigates the perfect Bayesian equilibria of the resulting game and finds that, depending on the level of discount factor and the prior beliefs, two contrasting types of perfect Bayesian equilibria emerge. For some constellations of discount factors and prior beliefs *"the inclusion of incomplete information has no impact on the pattern of the abatements."* Otherwise the game becomes a *war of attrition.*[1] In contrast to the model presented in Section 5.1 below, BAC focuses on the dynamic aspects of mitigation under incomplete information. Moreover, he does not include insurance instruments in his framework.

5.1 The Basic Model

The key characteristics of the model are uncertainty with respect to the future climate state, on the one hand, and incomplete information with respect to countries' assessments of climate risk, on the other hand. They are introduced in detail in Sections 5.1.1 and 5.1.2 below.

Consider a world with n countries, indexed by $i = 1, ..., n$ and one aggregate consumption good. Countries can only differ with respect to their type, defined below.[2] The time structure is depicted in Figure 5.1. In the first period ($t = 1$) each country decides on its abatement policy with respect to greenhouse gases. At $t = 2$ there is uncertainty about the climate state in the following period. Countries take their portfolio decision on the markets for climate securities if these are available. At $t = 3$ the uncertainty about the climate state is resolved and the resulting payoffs from security trading are realized. W.l.o.g. there is no discounting.

[1] The *war of attrition* was first analysed by MAYNARD SMITH [1974]. See FUDENBERG AND TIROLE [1991], p. 119.

[2] This assumption is necessary only for the results in Sections 5.1.4 and 5.4.

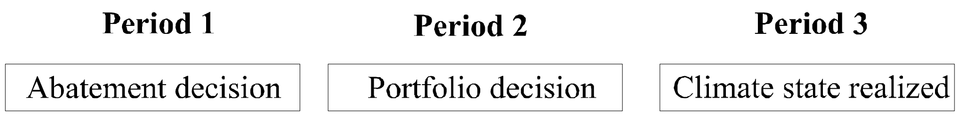

Figure 5.1: *The time structure*

5.1.1 Uncertainty with respect to the climate state

There are two possible climate states that can emerge in $t = 3$, state α (no noticeable climate change) and state β (severe climate change). The countries' probability distributions on α and β are assumed to be *exogenous*,[3] in particular they are independent of abatement policy in $t = 1$. The climate state influences countries' endowments of the aggregate consumption good in $t = 3$. Countries are assumed to be expected utility maximizers using the same von Neumann-Morgenstern utility function u in all periods. Assume $u' > 0$ and $u'' < 0$, i.e. countries are risk-averse.

5.1.2 Incomplete information about countries' types

A country's type θ_i is characterized completely by its *beliefs*, i.e. its subjective probability distribution, on the two climate states.[4] The type θ_i of a country will be identified with the probability it attributes to state α. Types are determined before $t = 1$.[5] Each country knows its own type, but not that of the other countries. Apart from possibly being of a different type, countries are taken to be completely identical.

For simplicity, there are only two types, A and B, i.e. two possible probability distributions on climate states. Type A has the distribution $(\theta^A, 1-\theta^A)$

[3] This assumption will be dropped in Chapter 7.

[4] Recall from Chapter 2 that in the context of climate change there can be no objective probability distribution due to remaining scientific uncertainties.

[5] A common interpretation is that types are determined randomly by *nature* before play starts. This interpretation does not seem plausible in our context. Rather, the type of a country will be the result of the country's individual interpretation and aggregation of the results of a certain group of climate experts. The assumption in the model then is that the result of these processes is the same *as if* nature had chosen the types with the probabilities in the prior distribution.

on climate states and type B the distribution $(\theta^B, 1-\theta^B)$ as illustrated in Table 5.1. Assume $\theta^A > \theta^B$ so that type A puts more weight on state α.

Probability distributions on climate states		
	State $\boldsymbol{\alpha}$	**State $\boldsymbol{\beta}$**
Type A	θ^A	$1-\theta^A$
Type B	$1-\theta^B$	θ^B

Table 5.1: *Probability distributions of the two types on the set of climate states.*

W.l.o.g. we can assume an "independent prior distribution"[6] $P(\Theta)$ with respect to the possible profiles of types $\theta = (\theta_1, ..., \theta_n) \in \Theta$. The prior distribution $P(\Theta)$ can then be specified by giving the individual distribution of types for each country. Denote by p the probability that a country is of type A. The country is then type B with probability $1-p$.

5.1.3 The abatement decision

In $t = 1$ each country has an endowment $\bar{\omega} > 0$ of the aggregate consumption good which can be either consumed or invested in abatement.[7] The abatement decision is modelled discretely, i.e. it is assumed that countries can choose between two different actions, L ("low abatement effort") and H ("high abatement effort"). In contrast to a country's type each country's abatement decision is *observable* to all other countries.

Note that in the scenario with regulated markets for climate securities in Section 5.4 below the abatement decision will be interpreted as a – not necessarily truthful – revelation of the type of a country.

[6] See Myerson [1991], pp. 67.

[7] The good is assumed to be *non-durable* in order to exclude intertemporal saving effects.

Abatement cost

Each country's consumption in $t = 1$ is reduced by the cost of its abatement activity a_i, measured in terms of the aggregate good,

$$\gamma(a_i) := \begin{cases} 0, & \text{if } a_i = L \\ \gamma, & \text{if } a_i = H \end{cases}$$

where $\gamma > 0$. For simplicity, the cost of pursuing the low abatement effort L is normalized to zero.[8] Therefore, the low abatement effort L will often be interpreted and referred to as "no abatement" from now on. The cost of choosing the high abatement level H (or "cost of abatement"), expressed in terms of utility, is

$$c := u(\bar{\omega}) - u(\bar{\omega} - \gamma). \tag{5.1}$$

Since $\gamma(L) = 0$, the **marginal** cost of abatement is also c.

Abatement benefits

Abatement benefits depend on the aggregate abatement effort, represented by the number of countries abating,

$$\tilde{a} := \#\{\, i \,|a_i = H\}.$$

By definition, $0 \leq \tilde{a} \leq n$ holds. Abatement benefits do not materialize until after the realization of the climate state in $t = 3$ where they are included in the state-dependent endowments.

For simplicity, third period endowments for state α are assumed to be independent of the abatement effort, i.e. $\omega^\alpha(\tilde{a}) = \bar{\omega}^\alpha \quad \forall \tilde{a}$. Third period endowments $\omega^\beta(\tilde{a})$ for state β are increasing in aggregate period 1 abatement $\tilde{a}$, i.e.

$$\bar{\omega}^\beta := \omega^\beta(0) < \omega^\beta(1) < ... < \omega^\beta(n-1) < \omega^\beta(n). \tag{5.2}$$

Abatement benefits (in terms of utility) if i countries abate are

$$b_i^{abs} := u(\omega^\beta(i)) - u(\bar{\omega}^\beta) \quad \text{for} \quad i = 0, ...n \tag{5.3}$$

[8] One could equivalently define $\gamma(L) = \gamma_L$ and $\gamma(H) = \gamma_H$ with $0 < \gamma_L < \gamma_H$.

where the index "abs" indicates that these are the **absolute** or total benefits from abatement. Define the **marginal** benefits (in terms of utility)

$$b_i := u(\omega^\beta(i)) - u(\omega^\beta(i-1))$$

for each country if i countries abate instead of $i-1$. Note that

$$b_i^{abs} = \sum_{j \leq i} b_j \qquad \text{and} \qquad b_i = b_i^{abs} - b_{i-1}^{abs} \tag{5.4}$$

hold for $i = 1, ..., n$.

Assumptions on marginal abatement cost and benefits

Assume that *unilateral abatement* is worthwhile for neither of the two types, i.e. (marginal) abatement cost c is larger than the expected marginal benefits from unilateral abatement for both types:

$$c > (1 - \theta^B)b_1 > (1 - \theta^A)b_1. \tag{5.5}$$

In addition, in order to focus on the most interesting constellation, assume that abating is never attractive for type A – no matter how many other countries abate –, i.e.

$$c > (1 - \theta^A)b_i \qquad \text{for} \qquad i = 2, ..., n. \tag{5.6}$$

Note that by assumptions (5.5) and (5.6) the low abatement effort L is a dominant strategy for type A.

In order to exclude the less interesting case of type B also having L as a dominant strategy further assume that there exists an $\hat{\imath} \in \{2, ..., n\}$ with

$$(1 - \theta^B)b_{\hat{\imath}} > c. \tag{5.7}$$

Assumption (5.7) requires the expected marginal benefits of abatement for type B to be larger than marginal abatement cost c given that $\hat{\imath} - 1$ countries abate. In other words, given that $\hat{\imath} - 1$ other countries abate type B has an incentive to join these $\hat{\imath} - 1$ countries and also abate.

Note that assumption (5.7) implies that abatement benefits are "convex" at least for a small range by monotonicity of state β endowments. This effect is reinforced by strict concavity of the utility function. Therefore, assumption (5.7) may be controversial in the sense that it seems to represent a deviation from the standard assumption of concave abatement benefits often used in environmental economics. However, in the context of climate change assumption (5.7) is a plausible assumption. As noted in the recently published contribution of Working Group III to the IPCC Third Assessment Report (IPCC [2001c]) the climate reaction to greenhouse gas concentrations can exhibit discontinuities at certain threshold values.[9] Then, abatement benefits will also exhibit discontinuities for the corresponding levels of aggregate abatement. The "convexity" of abatement benefits at the threshold number $\hat{\imath}$ of countries abating may be interpreted as representing such a discontinuous climate effect. Interpreted in this way, assumption (5.7) is not less plausible than assuming "concavity" of abatement benefits, in particular since concavity may hold for most of the relevant range even with assumption (5.7).

In order to demonstrate that the results do not hinge on assumption (5.7) section 5.5.1 summarizes and discusses the results that would emerge without that assumption. It turns out that the main results only change slightly if assumption (5.7) is abandoned.

Consistency of abatement strategies

Note that a country's observed abatement strategy is interpreted as revelation of its type. If a country chooses the costly abatement strategy H, then this is interpreted as the country being type B, whereas choice of L as abatement strategy is interpreted as the country being type A. In this sense, abatement strategy L is associated with type A and abatement strategy H is associated with type B. If a country chooses the abatement strategy associated with its type, it behaves *type-consistently* or *consistently.* In Section 5.4 regulation is aimed at achieving consistent behaviour in the sense explained above.

[9]See IPCC [2001c], p. 3: *"Developing a response to climate change is characterized by decision-making under uncertainty and risk, including the possibility of non-linear and/or irreversible changes."*

Summary

In summary, each country's abatement decision affects its own utility in $t = 1$ (cost of abatement) and everybody's utility in $t = 3$ (benefits from abatement). Therefore, a country's endowment $w^{\sigma}(\tilde{a})$ in $t = 3$ depends on the climate state $\sigma \in \{\alpha, \beta\}$ and, if the climate state is β, on the aggregate abatement effort $\tilde{a}$.

5.1.4 The portfolio decision

Markets for climate securities

In the second period we consider a complete system of Arrow-Securities defined on climate states, called "climate securities". Assume that there is one financial asset associated with each of the two climate states α and β. The payoffs of the climate securities are summarized in Table 5.2. Security α pays one unit of the consumption good, if and only if the climate state is α. Security β is defined accordingly.[10]

Payoffs for climate securities		
	Security $\boldsymbol{\alpha}$	**Security $\boldsymbol{\beta}$**
State $\boldsymbol{\alpha}$	1	0
State $\boldsymbol{\beta}$	0	1

Table 5.2: *Payoffs for climate securities*

Assume that security markets are competitive and denote the price of security α by q and that of security β by $1 - q$. Further denote by s_i^{α} the amount of security α bought by country i, and by s_i^{β} the amount of security β bought.[11]

Country i's portfolio decision then consists of choosing a pair $(s_i^{\alpha}, s_i^{\beta})$. There is no endowment of climate securities[12] so that a feasible portfolio has

[10] We abstract from any transaction costs arising from the set–up and operation of securities markets.

[11] If, for example, country i is a *seller* of security α, then $s_i^{\alpha} < 0$ holds.

[12] Nor is there any endowment of the consumption good in $t = 2$. Endowment from the first period cannot be used for trade in climate securities, since non-durability was

to satisfy

$$qs_i^\alpha + (1-q)s_i^\beta = 0. \tag{5.8}$$

Assume that short-selling is not possible, i.e. the following two inequalities have to hold:

$$s_i^\alpha \geq -\bar{\omega}^\alpha \quad \text{and} \quad s_i^\beta \geq -\omega^\beta(\tilde{a}) \tag{5.9}$$

where $\tilde{a}$ is determined in the previous period. For a given security price q and given aggregate abatement effort $\tilde{a}$ from the first period countries choose their portfolio (s_i^α, s_i^β) as to maximize expected utility

$$\theta_i u(\bar{\omega}^\alpha + s_i^\alpha) + (1-\theta_i)u(\omega^\beta(\tilde{a}) + s_i^\beta) \tag{5.10}$$

from period 3 subject to equation (5.8) and the inequalities in (5.9). The first order condition for country i (for an interior solution) is

$$-\frac{q}{1-q} = -\frac{\theta_i}{1-\theta_i}\frac{u'(\bar{\omega}^\alpha + s_i^\alpha)}{u'(\omega^\beta(\tilde{a}) - \frac{q}{1-q}s_i^\alpha)}, \tag{5.11}$$

which requires the slope of country i's budget set, $-\frac{q}{1-q}$, to be equal to its *marginal rate of substitution between state α and state β* (the right hand side of (5.11)).

Equation (5.11) yields country i's net demand $s_i^\sigma(\theta_i, q, \tilde{a})$ for each of the securities[13] as a function of the security price q, aggregate abatement $\tilde{a}$ and its own type θ_i for $\sigma = \alpha, \beta$. Since countries are identical apart from the type, so are all the parameters entering the first order condition (5.11) which determines a country's security demand. Therefore, the security demand function $s^\alpha(\theta_i, q, \tilde{a})$ is the same for all countries. But note that security demand still depends on i's type θ_i.

Security market equilibrium

Since security demand functions are the same for all countries, aggregate net demand for the securities depends only on the number of countries of each type eligible for security trading.

assumed (cf. Section 5.1.3).

[13] Given the above assumptions $s_i^\sigma(\theta_i, q, \tilde{a})$ exists and is a singleton.

Consider first the case **without regulation** of security markets. Then all countries can trade if they want to and aggregate security demand depends on the number of countries of each type. Define the number of type A and type B countries for a given type profile θ as

$$n_A(\theta) := \#\{i|\theta_i = \theta^A\} \quad \text{and} \quad n_B(\theta) := \#\{i|\theta_i = \theta^B\} = n - n_A(\theta).$$

The market clearing condition for security α in the absence of regulation is

$$n_A(\theta) \cdot s^\alpha(\theta^A, q, \tilde{a}) + (n - n_A(\theta)) \cdot s^\alpha(\theta^B, q, \tilde{a}) = 0. \tag{5.12}$$

It determines the equilibrium security price vector $q^*(n_A(\theta), \tilde{a})$ which depends on the number of countries of type A, $n_A(\theta)$, and aggregate abatement $\tilde{a}$. Existence of equilibrium is guaranteed by standard conditions, see, for example, MAS-COLELL, WHINSTON AND GREEN [1995], p. 585. Note that via the endowments $\omega^\beta(\tilde{a})$ security market equilibrium in the second period depends on first period abatement $\tilde{a}$.

Due to identical security demand functions there can only be trade in securities in equilibrium if there are countries of both types, i.e. if $0 < n_A(\theta) < n$ holds. Otherwise, the net demand for each of the securities would be zero.[14] In that case each country would consume its endowment in equilibrium and, consequently, there would be no gains from trade.

Now turn to the case **with regulation** of markets for climate securities. With regulation of security markets the number of countries allowed to trade is endogenous, in particular it may be smaller than n. Denote the number of countries of each type allowed to participate in trading by $n_A^R(\theta)$ and $n_B^R(\theta)$ where "R" stands for "Regulation". In general, $n_B^R(\theta)$ will not be equal to[15] $n - n_A^R(\theta)$ and we write $q^*(n_A^R(\theta), n_B^R(\theta), \tilde{a})$ for the equilibrium price.[16] The details of the regulation scheme, including the precise definition of $n_A^R(\theta)$ and $n_B^R(\theta)$ are explained in Chapter 5.4 below.

[14] Cf. the market clearing condition (5.12).

[15] Without regulation, $n_B(\theta) = n - n_A(\theta)$ always holds, simplifying matters.

[16] With regulation the market clearing condition is

$$n_A^R(\theta) \cdot s^\alpha(\theta^A, q, \tilde{a}) + n_B^R(\theta)) \cdot s^\alpha(\theta^B, q, \tilde{a}) = 0.$$

The equilibrium security trades for the two scenarios with securities are denoted

$$\begin{cases} s^{\sigma *}(\theta_i, n_A(\theta); \tilde{a}) := s^{\sigma}(\theta_i, q^*(n_A(\theta), \tilde{a}), \tilde{a}), \text{ without regulation} \\ s^{\sigma *}(\theta_i, n_A(\theta), n_B(\theta); \tilde{a}) := s^{\sigma}(\theta_i, q^*(n_A^R(\theta), n_B^R(\theta), \tilde{a}), \tilde{a}), \text{ with regulation} \end{cases} \tag{5.13}$$

for $\sigma = \alpha, \beta$ and $i = 1, ..., n$.

Characteristics of the equilibrium trading pattern

The equilibrium trading pattern, i.e. who buys and who sells security α in equilibrium, is characterized in Lemma 2 below. This information is exploited for the appropriate design of regulation in the scenario with regulated security markets (cf. Section 5.4).

Implicitly define a function $q^0(\theta_i, \tilde{a})$ by

$$-\frac{q^0(\theta_i, \tilde{a})}{1 - q^0(\theta_i, \tilde{a})} = -\frac{\theta_i}{1 - \theta_i} \frac{u'(\bar{\omega}^{\alpha})}{u'(\omega^{\beta}(\tilde{a}))}. \tag{5.14}$$

In the context of trade theory $q^0(\theta_i, \tilde{a})$ corresponds to the *autarky price*, i.e. $q^0(\theta_i, \tilde{a})$ is the security price for which country i's marginal rate of substitution between state α and state β in the endowment $(\bar{\omega}^{\alpha}, \omega^{\beta}(\tilde{a}))$ equals the slope of the budget set. Consequently, $q^0(\theta_i, \tilde{a})$ is the security price for which country i has no net demand for either security.

Lemma 2
Let θ be a given type profile and $\tilde{a}$ the aggregate abatement effort from the first period. If the type profile θ satisfies $0 < n_A(\theta) < n$,

a) the equilibrium security price $q^*(n_A(\theta), \tilde{a})$ lies between the autarky prices, i.e.

$$q^0(\theta^B, \tilde{a}) < q^*(n_A(\theta), \tilde{a}) < q^0(\theta^A, \tilde{a}), \tag{5.15}$$

and

b) in equilibrium, type A countries buy security α and type B countries sell security α, i.e.

$$s^{\alpha}(\theta^A, q^*(n_A(\theta), \tilde{a})) > 0 \quad \textit{and} \quad s^{\alpha}(\theta^B, q^*(n_A(\theta), \tilde{a})) < 0.$$

The proof is straightforward and given in Appendix B. The intuition behind Lemma 2 is clear: the countries attaching a higher probability to state α, the type A countries, will *buy* security α in equilibrium, i.e. transfer income to state α. Type B countries will *sell* security α, i.e. transfer income to state β. Note that Lemma 2 is formulated for the case without regulation, but holds analogously for the case with regulation.[17]

Knowledge of the equilibrium trading pattern is crucial for any kind of regulatory intervention, because it enables us to anticipate the different types' equilibrium behaviour. In particular, regulation can then be designed in such a manner that free riders are excluded from security trading by preventing them from transacting their desired equilibrium trades.[18]

5.1.5 The general structure of the resulting Bayesian Games

This section summarizes the general structure, common to all three scenarios, derived from the set-up described above in a general Bayesian Game. From this Bayesian game the specific Bayesian game for each of the three scenarios analysed below can be easily derived.

Combining the abatement decision described in Section 5.1.3 and its consequences for security market equilibrium analysed in Section 5.1.4 we can now write total utility for country i as

$$\begin{aligned} U_i(\theta; a) &= u(\bar{\omega} - \gamma(a_i)) + \theta_i u(\bar{\omega}^\alpha + s^{\alpha *}(\theta_i, \theta_{-i}, \tilde{a})) \\ &\quad + (1 - \theta_i) u(\omega^\beta(\tilde{a}) + s^{\beta *}(\theta_i, \theta_{-i}, \tilde{a})), \end{aligned} \tag{5.16}$$

i.e. the sum of utility from the first period, $u(\bar{\omega} - \gamma(a_i))$, and expected utility from the second period. The three different scenarios analysed below only differ in the definition of the equilibrium security trades $s^{\alpha *}(\theta_i, \theta_{-i}, \tilde{a}))$ and $s^{\beta *}(\theta_i, \theta_{-i}, \tilde{a}))$. In the absence of securities the equilibrium security trades are

[17] In that case substitute $n_A^R(\theta)$ for $n_A(\theta)$, $n_A^R(\theta) + n_B^R(\theta)$ for n and $q^*(n_A^R(\theta), n_B^R(\theta), \tilde{a})$ for $q^*(n_A(\theta), \tilde{a})))$.

[18] If, alternatively, punishments for free riding were used instead of regulation, the types of the trading countries could equivalently be deduced from observation of their trading behaviour.

set to zero, with markets for climate securities they are defined according to (5.13), depending on whether there is regulation or not.

Each country's total utility defined in (5.16) depends not only on the aggregate abatement effort $\tilde{a}$, but also – via the equilibrium security trades – on the type profile $\theta = (\theta_1, ..., \theta_n)$.

Since there is incomplete information about types, i.e. the type of the other countries is not known, the utility functions in (5.16) define a *Bayesian Game*. Formally, the Bayesian game consists of the set of players $\{1, ..., n\}$, their action sets $\{L, H\}$, the set of type profiles Θ, the prior distribution $P(\Theta)$ and the utility functions defined in (5.16).

In the Bayesian game, a *strategy* for country i is a function $a_i(\theta_i)$ that gives the country's choice of abatement action for each realization of its type θ_i. In *Bayesian Nash Equilibrium* countries maximize their expected utility with regard to the other countries' types conditional on their own type[19] given the other countries' equilibrium strategies.

In the following we interpret an abatement vector $a = (a_1, ..., a_n)$ as the result of each country's strategy $a_i(\cdot)$ being applied to its type θ_i, i.e. $a = a(\theta) := (a_1(\theta_1), ..., a_n(\theta_n))$. Note that then $\tilde{a}$ and similarly defined variables depend on both the strategies $(a_i(\cdot))_{i=1,...,n}$ and the type profile θ. For simplicity of notation we mostly write $\tilde{a}$ and only make explicit its dependency on strategies and types where it is essential.

5.2 No Security Markets

First of all, a scenario without markets for climate securities is studied as a benchmark scenario to which the two scenarios with markets for climate securities will be compared below.

[19]Due to the assumption of an independent prior distribution a country's probability for a certain type profile of the other countries conditional on its own type is the same as the ex ante probability.

5.2.1 The Bayesian Game

Without markets for climate securities, total utility for country i from (5.16) simplifies to[20]

$$\begin{aligned} & U_i(\theta_i; a_i(\theta_i), \tilde{a}(a(\theta))) && (5.17) \\ = \; & u(\bar{\omega} - \gamma(a_i(\theta_i))) + \theta_i u(\bar{\omega}^\alpha) + (1-\theta_i) u(\omega^\beta(\tilde{a})) \\ \overset{(5.3)}{=} \; & u(\bar{\omega}) - c(a_i(\theta_i)) + [\theta_i u(\bar{\omega}^\alpha) + (1-\theta_i) u(\bar{\omega}^\beta)] + (1-\theta_i) b^{abs}_{\tilde{a}(a(\theta))} \end{aligned}$$

where

$$c(a_i(\theta_i)) := \begin{cases} 0, & \text{if } a_i(\theta_i) = L \\ c, & \text{if } a_i(\theta_i) = H \end{cases}.$$

For the purpose of characterizing the Bayesian equilibria the expression in (5.17) can be simplified further. W.l.o.g. normalize $u(\bar{\omega}) = 0$ for all countries.[21] Moreover, total utility for country i in (5.17) always contains the term in square brackets, $[\theta_i u(\bar{\omega}^\alpha) + (1-\theta_i) u(\bar{\omega}^\beta)]$, if the type is θ_i. Since each of these expressions appears in *all* the payoffs of the respective country for a fixed type they cancel out in the equilibrium analysis below and can be omitted.

Define

$$\tilde{a}_{-i} := \#\{\, j \,|\, j \neq i \text{ and } a_j = H\} \qquad (5.18)$$

as the aggregate abatement effort of all countries $j \neq i$ and define also

$$\tilde{a}_i := \begin{cases} 1, & \text{if } a_i = H \\ 0, & \text{if } a_i = L \end{cases}.$$

Then country i's utility can be represented by a modified utility function

$$\tilde{U}_i(\theta_i; a_i, \tilde{a}_{-i}) = -c(a_i(\theta_i)) + (1-\theta_i) b^{abs}_{\tilde{a}_i + \tilde{a}_{-i}}. \qquad (5.19)$$

[20] Refer to Section 5.1.3 in order to recall the definitions of γ, $b^{abs}_{\tilde{a}}$ etc.

[21] The expression $u(\bar{\omega})$ is a constant and appears in all such expressions.

5.2.2 Bayesian Equilibria

This section characterizes the Bayesian equilibria of the Bayesian game induced by the utility functions in (5.19). The four potential equilibrium strategy profiles[22] are listed in Table 5.3.

Strategy profile $\mathbf{a^x}$	$\mathbf{a_i^x(\boldsymbol{\theta}^A)}$	$\mathbf{a_i^x(\boldsymbol{\theta}^B)}$	**Interpretation**
a^1	L	L	"never abate"
a^2	L	H	"type-consistent","truth-telling"
a^3	H	H	"always abate"
a^4	H	L	"inconsistent"

Table 5.3: *Strategy profiles*

With strategy profile a^1 both types of countries, i.e. *all* countries, always choose L (low or no abatement), whereas with strategy profile a^3 both types of countries always abate. Strategy profile a^2 corresponds to *type-consistent behaviour*, i.e. type A chooses L and type B chooses H. Finally, for strategy profile a^4 this behaviour is reversed so that a^4 corresponds to *inconsistent behaviour.*

The equilibrium conditions are given in Proposition 3 below.

Proposition 3

a) Strategy profile a^1 ("no abatement") is a Bayesian Nash Equilibrium, if and only if

$$(1-\theta^B)b_1 \leq c. \tag{5.20}$$

b) Strategy profile a^2 ("type-consistency") is a Bayesian Nash Equilibrium, if and only if

$$(1-\theta^A)\sum_{k=0}^{n-1} p_k b_{n-k} \leq c \leq (1-\theta^B)\sum_{k=0}^{n-1} p_k b_{n-k} \tag{5.21}$$

[22]The analysis focuses on symmetric strategy profiles since the analysis of asymmetric strategy profiles does not yield any further insight.

where

$$p_k := \binom{n-1}{k} p^k (1-p)^{n-1-k} \qquad \text{for } k = 0, ..., n-1 \tag{5.22}$$

is the binomial coefficient with respect to $n-1$.

c) *Strategy profiles* a^3 *and* a^4 *cannot be Bayesian Nash equilibria for any prior* p.

The details of the proof are given in Appendix B. The intuition for Proposition 3 is as follows. Strategy profile a^1 is a Bayesian equilibrium if deviating from L to H is not worthwhile for any of the two types, given that all other countries stick to L. Equivalently, a^1 is a Bayesian equilibrium if unilateral abatement is not worthwhile for any of the two types, i.e. if $(1-\theta^A)b_1 \leq c$ and $(1-\theta^B)b_1 \leq c$. By assumption (5.5) neither of the types has an incentive for unilateral abatement so that the equilibrium conditions for strategy profile a^1 are satisfied.[23] Since strategy profile a^1 is a *pooling strategy*, i.e. both types choose the same action, the equilibrium conditions do not depend on the prior p. Therefore, a^1 is an equilibrium for all priors p. This ensures existence of (at least one) Bayesian equilibrium for any prior p.

Concerning strategy profile a^2 the equilibrium conditions are more complicated, because a^2 is a *separating strategy profile*, i.e. the different types choose different actions. Therefore, the expected marginal benefits from deviating depend on the type profile θ of which each country knows only its own type. Consequently, expectations with respect to the type profile have to be taken in the equilibrium conditions (5.21) for strategy profile a^2. The binomial coefficient p_k represents the probability that k out of $n-1$ countries are type A. Given that all the countries with the exception of the country under consideration stick to strategy profile a^2, p_k is the probability that k of these $n-1$ countries choose action L (and the remaining $n-1-k$ countries choose H).

In order for strategy profile a^2 to be an equilibrium, type A must not have an incentive to abate, whereas type B has to have an incentive to abate. This

[23] Since the inequality for type B is stronger than that for type A, only the type B inequality appears in Proposition 3.

is required in inequality (5.21) where the expected[24] marginal benefits from abatement for each of the types are set into relation to marginal abatement cost c. Strategy profile a^2 is a Bayesian Nash equilibrium for the set of priors p satisfying condition (5.21).

Strategy profiles a^3 and a^4 are ruled out immediately as equilibria by the fact that L is a dominant strategy for type A by assumptions (5.5) and (5.6).

By Proposition 3 there is always at least one Bayesian Nash equilibrium for a given prior distribution in the Bayesian game without security markets, in some cases there may even be two Bayesian equilibria. The following section analyses the efficiency properties of these equilibria.

5.2.3 Efficiency

In an incomplete information context, the efficiency concept must not depend on the actual realization of the type profile. Depending on whether utilities are evaluated before anybody knows their own type *(ex ante)*, when everybody knows their own type but not that of the others *(interim)* or when all types are known *(ex post)*, Holmstrom and Myerson [1983] define the notions of *ex ante-, interim- and ex post-efficiency.* These concepts are now applied to the equilibrium strategy profiles in the benchmark scenario without securities.

Proposition 4
If strategy profile a^2 is an equilibrium, then it Pareto-dominates strategy profile a^1 from the ex ante and from the interim point of view. From the ex post point of view neither of the strategy profiles a^1 and a^2 Pareto-dominates the other one.

The proof is given in Appendix B. By Proposition 4 the *type-consistent* or *truth-telling* equilibrium strategy profile a^2 is a "better" equilibrium than the *never abate* equilibrium strategy profile a^1 in terms of ex ante- and interim-efficiency. Recall that by Proposition 3 the less desirable equilibrium a^1 rules for a larger set of economies without securities than the preferred equilibrium

[24] Note that expectations are here taken both with respect to the climate state and with respect to the type profile.

a^2. In particular, strategy profile a^1 is an equilibrium for all priors and, moreover, the *only* equilibrium for certain values of p, whereas strategy profile a^2 is never the only equilibrium.

In Sections 5.3 and 5.4 below it is analysed how the equilibrium regions with respect to the priors for the strategy profiles are affected by the introduction of security markets.

5.3 Unregulated security markets

This section deals with the effect of markets for climate securities on abatement when there is no regulatory intervention. Since the approach with regulation may not be feasible or enforceable, it is very important to also consider markets for climate securities in the absence of regulation.

5.3.1 The Bayesian Game with unregulated securities

For the purposes of this section it is convenient to rewrite the utility function in (5.19) in order to separate between the abatement cost and benefits, on the one hand, and the gains from security trading, on the other hand.

Recall from Section 5.1.4 that there will be no trade and therefore no gains from trade if all countries are of the same type, i.e. if $n_A(\theta) = 0$ or if $n_A(\theta) = n$.

If, however, the type profile θ satisfies $0 < n_A(\theta) < n$, there is trade in equilibrium and, in general,[25] $s^{\sigma *}(\theta_i, n_A(\theta); \tilde{a}) \neq 0$. In that case, the *equilibrium gain from trade in climate securities* for country i is defined as

$$\begin{aligned} & g(\theta_i, n_A(\theta); \tilde{a}) \\ := \quad & \theta_i u\left(\bar{\omega}^\alpha + s^{\alpha *}(\theta_i, n_A(\theta); \tilde{a})\right) + (1-\theta_i) u\left(\omega^\beta(\tilde{a}) + s^{\beta *}(\theta_i, n_A(\theta); \tilde{a})\right) \\ & \quad - \left[\theta_i u(\bar{\omega}^\alpha) + (1-\theta_i) u(\omega^\beta(\tilde{a}))\right]. \end{aligned} \tag{5.23}$$

The gain from trade $g(\theta_i, n_A(\theta); \tilde{a})$ depends on the type θ_i, the distribution of types amongst the n countries, represented by $n_A(\theta)$, and the aggregate abatement effort $\tilde{a}$ in the first period. Since trade is voluntary $g(\theta_i, n_A(\theta); \tilde{a})$ is always non-negative.

[25] Unless, by coincidence, the equilibrium security price equals the autarky price.

The utility function for country i with unregulated security markets can be written as

$$U_i^{NR}(\theta_i, n_A(\theta); a_i, \tilde{a}_{-i}) := \begin{cases} -c(a_i(\theta_i)) + (1-\theta_i) b^{abs}_{\tilde{a}_i + \tilde{a}_{-i}} \\ \qquad +g(\theta_i, n_A(\theta); \tilde{a}), \text{ if } 0 < n_A(\theta) < n \\ -c(a_i(\theta_i)) + (1-\theta_i) b^{abs}_{\tilde{a}_i + \tilde{a}_{-i}}, \text{ if } n_A(\theta) = 0, n \end{cases} \tag{5.24}$$

where "NR" stands for "No Regulation" and the same normalization as in Section 5.2.1 has been applied. Comparison with the utility function in the absence of security markets in (5.19) shows two main differences. First of all, with unregulated securities a "gain from trade function" is added to the "basic utility function" in (5.19) whenever trade takes place. Secondly, total utility for country i in (5.24) also depends on the type profiles of the other countries via these gains from trade.

It is important to note that the additive structure of the utility functions in (5.24) relative to those in (5.19) does **not** imply that countries will, in general, be better off with unregulated security markets than in the absence of securities. For a given type profile and **fixed** strategies $(a_i)_i$ countries can only gain from trading in security markets due to participation being voluntary. However, in general, equilibrium strategies and, in particular the equilibrium level of $\tilde{a}$, will be different in the game with unregulated security markets than in the absence of securities. Therefore the additive structure of the utility functions in (5.24) does not permit any conclusions about the overall effect of security markets on countries' equilibrium utility levels.

5.3.2 Bayesian Equilibria

This section characterizes the Bayesian equilibria of the Bayesian game induced by the utility functions in (5.24). The equilibrium conditions for the different strategy profiles naturally resemble those in Proposition 3, however, some additional expressions arising from trade in climate securities appear. These expressions are the *expected additional*[26] *abatement incentives* gener-

[26] "Additional" in the sense of the incentive arising from markets for climate securities and not from abatement benefits.

ated by unregulated security markets. They determine the effect of climate securities on abatement behaviour.

The expected additional abatement incentives appearing in the equilibrium conditions for the "never abate"-strategy profile a^1 are[27]

$$g^A_{uni}(p) := \sum_{k=0}^{n-2} p_k[g(\theta^A, k+1; 1) - g(\theta^A, k+1; 0)] \qquad (5.25)$$

$$\text{and} \quad g^B_{uni}(p) := \sum_{k=1}^{n-1} p_k[g(\theta^B, k; 1) - g(\theta^B, k; 0)] \qquad (5.26)$$

where p_k is again the binomial coefficient defined in Proposition 3 above and the subscript "uni" stands for "unilateral".[28] Recall that p_k, the probability that k out of $n-1$ countries are type A, is a function of the prior p. Therefore the expressions $g^A_{uni}(p)$ and $g^B_{uni}(p)$ are also functions of the prior p. Their interpretation is the following. $g^A_{uni}(p)$ is the difference between the expected gains from trade, when type A abates, and the expected gains from trade, when type A does not abate, given that all other countries stick to strategy profile a^1 and never abate. In different words, $g^A_{uni}(p)$ is type A's *expected additional incentive for unilateral abatement.* Although a^1 is a pooling strategy and the other countries' behaviour does not depend on their type,[29] expectations with respect to the type profile are taken because the security market equilibrium, and therefore the gains from trade, depend on the type profile. The interpretation of $g^B_{uni}(p)$ is analogous.[30]

The expected additional abatement incentives appearing in the equilib-

[27] See Proposition 5 below.

[28] See explanation below.

[29] Note that $\tilde{a}$ is either one or zero and determined entirely by the country under consideration.

[30] Note that in the expected additional abatement incentives the sums run from $k = 0$ to $k = n-2$ if country i is type A and from $k = 1$ to $k = n-1$ if country i is type B. This is due to the fact that for the omitted values of k all countries are of the same type so that there is no trade in equilibrium and, consequently, no gains from trade, either.

rium conditions for the type-consistent strategy profile a^2 are

$$g^A_{con}(p) := \sum_{k=0}^{n-2} p_k[g(\theta^A, k+1; n-k) - g(\theta^A, k+1; n-k-1)]$$

$$\text{and} \quad g^B_{con}(p) := \sum_{k=1}^{n-1} p_k[g(\theta^B, k; n-k) - g(\theta^B, k; n-k-1)]$$

where "con" stands for "consistent". $g^A_{con}(p)$ are the expected additional incentives for type A to abate, given that all other countries behave consistently, i.e. according to strategy profile a^2. Since a^2 is a separating strategy profile the number of other countries abating depends on their type profile[31] so that expectations with respect to the number of type A countries in that type profile, indexed k, are taken in $g^A_{con}(p)$. The interpretation of $g^B_{con}(p)$ is analogous.

Note that the main difference relative to the expressions $g^A_{uni}(p)$ and $g^B_{uni}(p)$ is the underlying assumption about the behaviour of the other countries. For $g^A_{uni}(p)$ and $g^B_{uni}(p)$ the other countries are assumed to behave according to strategy profile a^1, whereas for $g^A_{con}(p)$ and $g^B_{con}(p)$ they are assumed to behave according to strategy profile a^2.

Similarly, the expected additional abatement incentives appearing in the equilibrium conditions for strategy profiles a^3 and a^4 are

$$g^A_{join}(p) := \sum_{k=0}^{n-2} p_k[g(\theta^A, k+1; n) - g(\theta^A, k+1; n-1)]$$

$$\text{and} \quad g^B_{join}(p) := \sum_{k=1}^{n-1} p_k[g(\theta^B, k; n) - g(\theta^B, k; n-1)],$$

$$\text{as well as} \quad g^A_{inc}(p) := \sum_{k=0}^{n-2} p_k[g(\theta^A, k+1; k+1) - g(\theta^A, k+1; k)]$$

$$\text{and} \quad g^B_{inc}(p) := \sum_{k=1}^{n-1} p_k[g(\theta^B, k; k+1) - g(\theta^B, k; k)].$$

$g^A_{join}(p)$ and $g^B_{join}(p)$ are associated with strategy profile a^3, i.e. it is assumed that all other countries choose H, irrelevant of their type. Therefore $g^X_{join}(p)$

[31] The type profile of "the other $n-1$ countries" is denoted θ_{-i} in the proof (cf. Appendix B) where θ_{-i} is defined in the usual way as $\theta_{-i} := (\theta_1, ..., \theta_{i-1}, \theta_{i+1}, ..., \theta_n)$.

is the expected additional abatement incentive for type X assuming that all other countries abate, i.e. the expected additional incentive to *join* the other countries in abatement ($X = A, B$), thus also explaining the subscript "join".

Finally, the expressions $g^A_{inc}(p)$ and $g^B_{inc}(p)$ appear in the context of the inconsistent[32] strategy profile a^4. They resemble the expressions $g^X_{con}(p)$, the difference being that they assume that all other countries behave according to strategy profile a^4.

The most important feature of the expected additional abatement incentives, i.e. the expressions $g^X_{uni}(p), g^X_{con}(p), g^X_{join}(p)$ and $g^X_{inc}(p)$ for $X = A, B$, is that their signs are *ambiguous.* Although the gains from trade $g(\theta^X, n_A(\theta); \tilde{a})$ are always non-negative since participation is voluntary, differences of two such non-negative expressions, appearing in the expected additional abatement incentives, can become smaller than zero. Therefore, a weighted sum of such differences can also become smaller than zero. The examples in Section 6.2 below suggest that negative expected additional abatement incentives are indeed a relevant case. In particular, the expected additional abatement "incentive", when smaller than zero, represents an incentive *not* to abate.

Proposition 5 gives the equilibrium conditions for strategy profiles a^1 to a^4 with unregulated security markets.

Proposition 5

a) *Strategy profile a^1 is a Bayesian equilibrium, if and only if*

$$(1-\theta^X)b_1 + g^X_{uni}(p) \leq c \qquad \textit{for} \quad X = A, B. \tag{5.27}$$

b) *Strategy profile a^2 is a Bayesian equilibrium, if and only if*

$$(1-\theta^A)\sum_{k=0}^{n-1} p_k b_{n-k} + g^A_{con}(p) \;\leq\; c \tag{5.28}$$

$$\textit{and} \qquad (1-\theta^B)\sum_{k=0}^{n-1} p_k b_{n-k} + g^B_{con}(p) \;\geq\; c. \tag{5.29}$$

[32] The subscript "inc" stands for "inconsistent".

c) Strategy profile a^3 is a Bayesian equilibrium, if and only if

$$(1-\theta^X)b_n + g^X_{join}(p) \geq c \qquad \text{for} \quad X = A, B. \tag{5.30}$$

d) Strategy profile a^4 is a Bayesian equilibrium, if and only if

$$(1-\theta^A)\sum_{k=0}^{n-1} p_k b_{k+1} + g^A_{inc}(p) \;\; \geq \;\; c \tag{5.31}$$

$$\text{and} \qquad (1-\theta^B)\sum_{k=0}^{n-1} p_k b_{k+1} + g^B_{inc}(p) \;\; \leq \;\; c. \tag{5.32}$$

The proof is given in Appendix B. Due to the ambiguity of the signs of the expected additional abatement incentives the effect of climate securities on abatement relative to the case without security markets is not easy to determine. This effect is analysed in detail in the following section.

5.3.3 The effect of security markets on the equilibrium conditions

Since the signs of the expected additional abatement incentives generated by unregulated security markets are ambiguous, so are, not surprisingly, the effects on abatement. If the expressions $g^X_z(p)$ with $X = A, B$ and $z = uni, con, join, inc$ are larger than zero, the effect of security markets corresponds to that of an increase in expected marginal abatement benefits, i.e. security markets *encourage abatement.* If, however, the expressions $g^X_z(p)$ are smaller than zero, the effect of security markets corresponds to that of a decrease in expected marginal abatement benefits, i.e. security markets *render abatement less attractive.*

The equilibrium regions for the four strategy profiles are affected differently by, for example, an increase in expected abatement benefits. The resulting effect depends on the direction of the inequalities in the equilibrium conditions, which, in turn, is determined by the strategies.

If, for example, the expressions $g^A_{uni}(p)$ and $g^B_{uni}(p)$ are larger than zero, thus making action H more attractive, naturally strategy profile a^1 with both types choosing L becomes harder to sustain as an equilibrium. The inequalities in (5.27) are both stronger than the corresponding inequalities in the absence of securities, inequality (5.20) for type B and the corresponding inequality for type A. Note, in particular, that with unregulated securities the equilibrium conditions in (5.27) are not independent of the prior p anymore. Therefore, strategy profile a^1 may be an equilibrium only for certain values of p, i.e. emerge for a smaller set of economies than in the absence of security markets.

If, to the contrary, the expressions $g^A_{uni}(p)$ and $g^B_{uni}(p)$ are smaller than zero, the inequalities in (5.27) are less binding than in the absence of climate securities and strategy profile a^1 remains an equilibrium for all priors.

Since strategy profile a^2 is a separating profile, the inequalities for the two types are affected differently by, for example, expected additional abatement incentives $g^X_{con}(p)$ larger than zero. If $g^A_{con}(p)$ is larger than zero, inequality (5.28) becomes stricter than the corresponding part of inequality (5.21). This is the case since action H is rendered more attractive for type A when $g^A_{con}(p)$ is larger than zero, but according to strategy profile a^2 type A chooses L. In contrast, a positive abatement incentive for type B, i.e. $g^B_{con}(p) > 0$, weakens inequality (5.29), since type B chooses H according to strategy profile a^2. If $g^A_{con}(p)$ and $g^B_{con}(p)$ are smaller than zero, the effects are reversed.

Recall from Section 5.2.2 that in the absence of security markets strategy profiles a^3 and a^4 could not be supported as Bayesian equilibria.[33] With unregulated markets for climate securities strategy profiles a^3 and a^4 can be supported as Bayesian equilibria for certain priors p if the expected additional abatement incentives are large enough.

By (5.30) and (5.6) a necessary condition for strategy profile a^3 to be an equilibrium is $g^A_{join}(p) > 0$. Similarly, (5.31) and (5.32) imply that a necessary condition for strategy profile a^4 to be an equilibrium is

$$g^A_{inc}(p) - g^B_{inc}(p) \geq \left[(1-\theta^B) - (1-\theta^A)\right] \sum_{k=0}^{n-1} p_k b_{k+1},$$

i.e. the expected additional benefits for type A have to be sufficiently larger than those for type B.

[33] This was ruled out by L being a dominant strategy for type A.

Due to the complexity of the expected additional abatement incentives no more can be said for the general case. Therefore the effects of unregulated climate securities on abatement behaviour are analysed and graphically illustrated in Section 6.1 for the case of two countries. In particular, the effects on the *equilibrium regions*, i.e. the sets of priors for which each of the strategy profiles is a Bayesian equilibrium, are studied. In Section 6.2 some illustrative examples are presented.

5.3.4 The effect of security markets on aggregate abatement

This section is concerned with an equally important aspect, the *equilibrium level of aggregate abatement* and how it is affected by security markets. Table 5.4 below lists the equilibrium values of aggregate abatement $\tilde{a}$ for each of the equilibrium strategy profiles.

Strategy profile	**Aggregate abatement $\tilde{\mathbf{a}}$**
a^1	0
a^2	$n_B(\theta)$
a^3	n
a^4	$n_A(\theta)$

Table 5.4: *Aggregate abatement levels for the four strategy profiles.*

Note that due to discreteness of the abatement strategies each equilibrium strategy profile leads to the same aggregate abatement level $\tilde{a}$ for all three scenarios investigated. For the two pooling equilibrium strategy profiles a^1 and a^3 the aggregate level of abatement $\tilde{a}$ does not depend on the type profile θ, but for the separating strategy profiles a^2 and a^4 it does. Recall that, depending on the type profile θ, $n_A(\theta)$ and $n_B(\theta)$ can assume values between 0 and n and note that

$$n_A(\theta) > n_B(\theta) \quad \Leftrightarrow \quad n_A(\theta) > \frac{n}{2}.$$

Obviously, the equilibrium level of aggregate abatement will not be affected by security markets if the *same* equilibrium reigns both without and with unregulated security markets. The equilibrium regions with unregulated security markets, however, may differ from the equilibrium regions in the absence of security markets since they depend on the gains from trade expressions. Note that constancy of aggregate abatement levels in this model is due to *discreteness* of the abatement strategy, since there can be no gradual adaption of the equilibrium strategy as, for example, in the model of Chapter 8 below.

Recall from the analysis above that there can be **multiple equilibria**, at least for some values of the prior p, in both scenarios investigated so far. In that case, the equilibrium played with unregulated security markets may differ from the equilibrium played in the absence of security markets. In different words, comparing the situation without securities to that with unregulated securities there may be a "switch" from one equilibrium to another equilibrium.[34] With multiple equilibria the level of aggregate abatement is indeed affected if there is a switch from one equilibrium to another equilibrium when unregulated markets for climate securities are introduced.

Consider first the case that a^1 is the equilibrium played in the absence of security markets. Then, none of the countries abates and the aggregate abatement level is $\tilde{a} = 0$. Naturally, since $\tilde{a} = 0$ is also the lower bound for aggregate abatement, the level of aggregate abatement with unregulated securities cannot decrease any further.

If a^1 is also the equilibrium played with unregulated security markets, aggregate abatement is evidently not affected by unregulated security trading. Besides, Table 5.4 shows that there are two further constellations for which the aggregate abatement level is not affected by unregulated security markets: firstly, if there is a switch to a^2 as equilibrium and all countries are type A, i.e. $n_B(\theta) = 0$, and secondly, if there is a switch to a^4 as equilibrium and if all countries are type B, i.e. $n_A(\theta) = 0$.

However, a switch from equilibrium strategy profile a^1 to a^2 leads to an increase in aggregate abatement from $\tilde{a} = 0$ to $\tilde{a} = n_B(\theta)$ for all type profiles θ with at least one type B country, i.e. with $n_B(\theta) > 0$. A switch from a^1 to a^3 leads to an increase in aggregate abatement from $\tilde{a} = 0$ to $\tilde{a} = n$,

[34] Note that with multiple equilibria we cannot be sure *which* equilibrium will be played unless applying a theory of equilibrium selection which is not attempted here.

independently of the type profile θ. Finally, a switch from a^1 to a^4 leads to an increase in aggregate abatement from $\tilde{a} = 0$ to $\tilde{a} = n_A(\theta)$ when at least one country is type A, i.e. $n_A(\theta) > 0$ holds.

Now turn to the case in which a^2 is the equilibrium played in the absence of security markets. Then the initial level of aggregate abatement $\tilde{a} = n_B(\theta)$ depends on the type profile θ. Trivially, if a^2 also reigns with unregulated securities, aggregate abatement is not affected by security markets.

For "interior" type profiles θ, i.e. type profiles satisfying $n > n_B(\theta) > 0$, aggregate abatement can increase, remain constant or decrease due to a switch in equilibrium as follows. If there happens to be a switch in equilibrium from a^2 (without securities) to a^1 (with unregulated securities), then aggregate abatement *decreases* from $\tilde{a} = n_B(\theta) > 0$ to $\tilde{a} = 0$. If, however, there is a switch from a^2 to a^3, aggregate abatement *increases* to $\tilde{a} = n$. A switch from a^2 to a^4 causes an increase in aggregate abatement if $n_A(\theta) > \frac{n}{2}$, a decrease in aggregate abatement if $n_A(\theta) < \frac{n}{2}$ and no change if $n_A(\theta) = \frac{n}{2}$.

For type profiles with $\tilde{a} = n_B(\theta) = 0$ the above considerations for a^1 as equilibrium strategy profile in the absence of securities apply analogously, for type profiles with $\tilde{a} = n_B(\theta) = n$ the effects are reversed.

The important fact to note is that within the framework of this model unregulated markets for climate securities do not necessarily increase aggregate abatement compared to the situation in the absence of securities. They might lead to less abatement if a^2 is the equilibrium played in the absence of securities, but that will not necessarily be the case. Although in the discrete model[35] analysed here it seems that this phenomenon is driven mainly by the existence of multiple equilibria, the results for the continuous model investigated in Chapter 8 display similar effects.

Obviously, multiplicity of equilibria is not specifically related to the existence of security markets. But the existence of unregulated security markets may increase the number of equilibria, thus generating more "switching possibilities" and a larger range of potential effects on aggregate abatement.

[35]With discrete abatement strategies there is no scope for gradual, small adaptions of the equilibrium amount of abatement as would be the case in a continuous model.

5.4 Regulated Security Markets

This section studies the effect of regulated markets for climate securities on abatement activity. In particular, it is aimed at capturing the intuition of the IPCC claim. Regulation is the means of establishing that link. The following section introduces and discusses the regulation approach employed below.

5.4.1 Regulation

As explained in the Introduction the intuition behind the IPCC claim is to generate a *trade-off* between the gains from free riding and the gains from trade in climate securities. Such a trade-off can be created by linking the abatement decision and the portfolio decision to each other. Crucial for the creation of this trade-off is the fact that, by Lemma 2, the two different types behave differently on markets for climate securities. Therefore these markets can contribute valuable information about countries' true types.[36]

The desired trade-off could equivalently be achieved either by regulation of security trading or by punishing free riders. The feature common to both approaches is that they aim at preventing *inconsistent behaviour,* as defined in Section 5.1.3, either by excluding it directly or, indirectly, by punishing it.

Regulation involves making a country's trading opportunities (its "feasible set" of trades) dependent on its abatement decision. In particular, regulation of trading would be designed to prevent inconsistent behaviour, i.e. countries would only be allowed to transact trades being consistent with their abatement activity. Regulation may be designed to apply to both types, as is the case below, or, for example, only to type B. The latter kind of regulation is discussed in Section 5.5.2 below.

The second approach to generating the desired trade-off is punishing free riders. In that case there would be no regulatory intervention with respect to trade in climate securities. After trade has taken place, countries' types would be derived according to Lemma 2. Then, inconsistent behaviour could be identified, and subsequently sanctioned, by comparing abatement be-

[36]Note, however, that countries' types can only be deduced from their trading behaviour if countries are identical otherwise. See Section 7.2.

haviour, on the one hand, with observed trades and their implications for countries' types, on the other hand. If the punishments for inconsistent behaviour are large enough, then such a scheme will also prevent inconsistent behaviour.[37] Therefore, the desired trade-off could equivalently be generated by means of regulation or by means of punishments for free riders. In the following, we concentrate on a regulative approach w.l.o.g.

The regulation scheme used in Section 5.4.2 below consists of *trading restrictions* for climate securities. Both types of countries are restricted to those security trades that are consistent with their abatement strategy[38] in the first period. For example, consider a country of type B that chose L in $t = 1$, thus "pretending" to be type A. As a type A-country it would want to buy security α and sell security β (cf. Lemma 2). This is exactly what it is allowed to do by the trading restrictions on the security markets. If the country were really type A, then these restrictions would not be binding. But since its true type is B, they are binding, in fact they prevent the country from becoming active on the security markets, because it would want to sell security α and buy security β.

Regulation punishes countries who do not "tell the truth" in the sense that their abatement behaviour does not correspond to their true type, while it does not affect "truth-telling" countries at all. Therefore, only countries who chose the "type-consistent" action[39] in the first period can profit from security trading. In particular, free riders are excluded from trade in securities.[40]

Whether such a regulation scheme is a feasible policy instrument is questionable since its enforcement would be very difficult. It is studied here because it represents the intuition underlying the IPCC claim very well. Note that in the IPCC SECOND ASSESSMENT REPORT the authors, ARROW ET AL. [1996], are less explicit about the nature of the link between abatement decisions and trade in climate securities which may also be of an informal

[37] Naturally, it has to be known beforehand that inconsistent behaviour will be subject to such punishments in order to deter countries from behaving inconsistently.

[38] Cf. Section 5.1.3.

[39] Cf. Section 5.1.3. The "type-consistent" action is L for type A and H for type B.

[40] Note, however, that type A countries which chose H are also excluded from security trading. An alternative regulation scheme which only restricts type B countries is discussed in Section 5.5.2.

nature. Therefore it may be more adequate to interpret the formal regulation scheme used in this section as representing an *informal* kind of link between abatement and portfolio decisions. In particular, it may be interpreted as an approximation of a kind of *issue linkage.*

Since feasibility of such regulation is unlikely, the scenario with regulation should rather be considered as a *second benchmark* besides the scenario without securities rather than a potential policy option.

5.4.2 The Bayesian Game with regulated securities

For the Bayesian Game with regulated security markets the utility functions defined in (5.24) for the case without regulation have to be modified appropriately. The difference is that, with regulation, countries exhibiting inconsistent behaviour receive no gains from trade since they are excluded from trade in climate securities.

For a given type profile θ and strategies $a = (a_i(\cdot))_{i=1,\dots,n}$ define

$$\begin{aligned} &\mathcal{I}^A(\theta, a) := \left\{ i | \ \theta_i = \theta^A \text{ and } a_i(\theta_i) = H \right\} \\ \text{and} \quad &\mathcal{I}^B(\theta, a) := \left\{ i | \ \theta_i = \theta^B \text{ and } a_i(\theta_i) = L \right\} \end{aligned}$$

as the sets of countries of each type behaving inconsistently. The set of all countries behaving inconsistently is their union

$$\mathcal{I}(\theta, a) := \mathcal{I}^A(\theta, a) \ \cup \ \mathcal{I}^B(\theta, a).$$

The set of countries potentially participating in security trading, i.e. exhibiting consistent behaviour, is[41]

$$\mathcal{N}^R(\theta, a) := \{1, ..., n\} \setminus \mathcal{I}(\theta, a)$$

where "R" stands for "Regulation". Write $n^R(\theta, a) := \#(\mathcal{N}^R(\theta, a))$ for the number of countries potentially participating in trade altogether and

$$n^R_X(\theta, a) := \#\{i \in \mathcal{N}^R(\theta, a) | \theta_i = \theta^X\} \qquad X = A, B$$

[41] Due to regulation all countries in $\mathcal{I}(\theta, a)$ cannot transact their desired trades and will remain inactive on security markets.

for the number of type X countries within the set $\mathcal{N}^R(\theta, a)$. For brevity of notation, the arguments (θ, a) are often omitted below.

For countries $i \in \mathcal{N}^R(\theta, a)$ the *equilibrium gains from trade in climate securities subject to regulation* are defined in analogy to (5.23) as

$$\begin{aligned} & g^R(\theta_i, n_A^R, n_B^R; \tilde{a}) \\ := & \theta_i u(\bar{\omega}^\alpha + s^{\alpha *}(\theta_i, n_A^R, n_B^R; \tilde{a})) + (1-\theta_i) u\left(\omega^\beta(\tilde{a}) + s^{\beta *}(\theta_i, n_A^R, n_B^R; \tilde{a})\right) \\ & - \left[\theta_i u(\bar{\omega}^\alpha) + (1-\theta_i) u(\omega^\beta(\tilde{a})\right]. \end{aligned} \quad (5.33)$$

Since participation in trade in climate securities is voluntary, $g^R(\theta_i, n_A^R, n_B^R; \tilde{a})$ is always non-negative. The utility function for country i with regulated security markets can be written[42]

$$U_i^R(\theta_i, n_A^R, n_B^R; a_i, \tilde{a}_{-i}) = \begin{cases} -c(a_i(\theta_i)) + (1-\theta_i) b^{abs}_{\tilde{a}_i + \tilde{a}_{-i}}, & \text{if } i \in \mathcal{I}(\theta, a) \\ -c(a_i(\theta_i)) + (1-\theta_i) b^{abs}_{\tilde{a}_i + \tilde{a}_{-i}}, & \\ \quad \text{if } i \in \mathcal{N}^R(\theta, a) \text{ and } n_A^R(\theta, a) = 0, n^R(\theta, a) & \\ -c(a_i(\theta_i)) + (1-\theta_i) b^{abs}_{\tilde{a}_i + \tilde{a}_{-i}} + g^R(\theta_i, n_A^R, n_B^R; \tilde{a}), & \\ \quad \text{if } i \in \mathcal{N}^R(\theta, a) \text{ and } 0 < n_A^R(\theta, a) < n^R(\theta, a) & \end{cases} . \quad (5.34)$$

Like the utility function with unregulated securities in (5.24) the utility function with regulated securities can be derived from the basic utility function in (5.19) by adding gains from trade expressions if trade takes place. With regulation, trade will only occur if there are countries of both types among the countries potentially participating in security trading, i.e. if $0 < n_A^R(\theta, a) < n^R(\theta, a)$ holds.

As already emphasized in Section 5.3.1 the additive structure of the utility function does not imply that countries will generally be better off with regulated markets for climate securities than with unregulated markets or without securities since the equilibrium value of $\tilde{a}$ will not be the same for the different scenarios.

5.4.3 Bayesian Equilibria

This section characterizes the Bayesian equilibria of the Bayesian Game induced by the utility functions in (5.34). Note that due to regulation trade

[42] The same normalizations as in Sections 5.2 and 5.3 have been applied.

in climate securities takes place for fewer constellations of type profiles and strategies than in the absence of regulation. Consequently, the expressions representing the expected additional abatement incentives are less complicated than in the absence of regulation.

In the equilibrium conditions for strategy profile a^1 only the inequality for type B contains an expected additional abatement incentive stemming from trade in climate securities,

$$r^B_{uni}(p) := \sum_{k=1}^{n-1} p_k g^R(\theta^B, k, 1; 1).$$

It represents type B's expected additional abatement incentive given that all other countries stick to strategy profile a^1, i.e. type B's *additional incentive for unilateral abatement.*[43]

The expressions

$$r^A_{con}(p) := \sum_{k=0}^{n-2} p_k g^R(\theta^A, k+1, n-k-1; n-k-1)$$

$$\text{and} \qquad r^B_{con}(p) := \sum_{k=1}^{n-1} p_k g^R(\theta^B, k, n-k; n-k)$$

are the expected additional abatement incentives for type A and B, assuming that all other countries follow the consistent strategy profile a^2.

In contrast to the case of unregulated security markets the additional abatement incentives contain no *differences* of gains from trade expressions. The reason is that each of the equilibrium conditions compares utility derived from a certain strategy with the utility that could be achieved when deviating from that strategy. Since only either the strategy itself or the deviation can be consistent for a given type, gains from trade only arise on one side of the equilibrium condition.

In contrast to the case without regulation, the expressions $r^B_{uni}(p)$, $r^A_{con}(p)$ and $r^B_{con}(p)$ contain no differences of gains from trade expressions. They are always larger than zero so that there is no ambiguity about their sign. Proposition 6 gives the equilibrium conditions for the Bayesian Game with regulated security markets.

[43]Note that type B can only profit from trading by deviating from strategy profile a^1.

Proposition 6

a) *Strategy profile* a^1 *is a Bayesian equilibrium, if and only if*

$$(1-\theta^A)b_1 \leq c \tag{5.35}$$

$$\text{and} \quad (1-\theta^B)b_1 + r^B_{uni}(p) \leq c \tag{5.36}$$

hold.

b) *Strategy profile* a^2 *is a Bayesian equilibrium, if and only if*

$$(1-\theta^A)\sum_{k=0}^{n-1} p_k b_{n-k} \leq c + r^A_{con}(p) \tag{5.37}$$

$$\text{and} \quad (1-\theta^B)\sum_{k=0}^{n-1} p_k b_{n-k} \geq c - r^B_{con}(p). \tag{5.38}$$

c) *Strategy profiles* a^3 *and* a^4 *cannot be Bayesian equilibria for any prior* p.

The proof is given in Appendix B. Note that for strategy profile a^1 the equilibrium condition for type A, (5.35), is identical to the corresponding equilibrium condition without securities, $(1-\theta^A)b_1 \leq c$. The intuition is the following. For type A countries a^1 is a consistent strategy, whereas type B countries are excluded from trade in climate securities if they follow a^1. Therefore a type A country will have no (type B) trading partner if it follows strategy a^1, and there will be no gains from trade. If, however, type A deviates from a^1 it is itself excluded from trading. Therefore, type A never benefits from security trading, given that the other countries follow strategy a^1.

The equilibrium condition for type B, inequality (5.36), is explained as follows. If type B follows strategy a^1, he is not allowed to trade. However, deviating from L to H he may trade, giving rise to the expression $r^B_{uni}(p)$.

Inequalities (5.37) and (5.38) for strategy profile a^2 are explained similarly. Either sticking to a^2 or deviating from a^2 represents inconsistent behaviour for each of the types so that gains from trade emerge only for exactly one of the two situations compared in the equilibrium conditions.

Since the signs of the additional abatement incentives are positive, the effects of regulated security markets are more straightforward than those for unregulated security markets. They are analysed in detail in the following section.

5.4.4 The effect of security markets on the equilibrium conditions

The crucial condition for strategy profile a^1 is (5.36) since inequality (5.35) is satisfied independently of the prior p by assumption (5.6). Inequality (5.36) is more binding than the corresponding inequality (5.20) in the absence of securities due to $r^B_{uni}(p)$ being larger than zero. However, if the expected additional abatement incentive $r^B_{uni}(p)$ is small, i.e. if

$$r^B_{uni}(p) \leq c - (1 - \theta^B)b_1 \tag{5.39}$$

holds, inequality (5.36) is satisfied for all priors p, and strategy profile a^1 is an equilibrium for all priors p, like in the absence of securities. If the expected gains from trade $r^B_{uni}(p)$ are large enough to make unilateral abatement worthwhile for type B, i.e. (5.39) does not hold, strategy profile a^1 is an equilibrium for those priors satisfying (5.36). Therefore, if the gains from trade expression $r^B_{uni}(p)$ is large enough, strategy profile a^1 is an equilibrium for a smaller set of economies with regulated security markets than in the absence of securities.

Concerning the equilibrium conditions for strategy profile a^2, inequalities (5.37) and (5.38) are both relaxed by the additional abatement incentives $r^A_{con}(p)$ and $r^B_{con}(p)$. Therefore, depending on the size of $r^A_{con}(p)$ and $r^B_{con}(p)$, strategy profile a^2 can be an equilibrium for a larger set of economies. The equilibrium regions with respect to the prior p are analysed in more detail for the case of two countries in Section 6.1 below.

5.4.5 The effect of security markets on aggregate abatement

In principle, a large part of the analysis in Section 5.3.4 applies equally to the case of regulated security markets. Again, the equilibrium level of aggregate abatement will not be affected by security markets if the same equilibrium reigns both without and with regulated security markets. If, however, different equilibria are played in the absence of securities and with regulated securities, the level of aggregate abatement is affected in the same way as described in Section 5.3.4. The main difference is that with regulation there are only two possible equilibrium strategy profiles, a^1 and a^2, and therefore less possibilities for a switch from one equilibrium to another when securities are introduced.

Consider first the case that a^1 is the equilibrium played in the absence of security markets. A switch from equilibrium strategy profile a^1 in the absence of security markets to strategy profile a^2 with regulation of security markets leads to an increase in aggregate abatement from $\tilde{a} = 0$ to $\tilde{a} = n_B(\theta)$ for all type profiles θ with at least one type B country. In contrast, if a^2 is the equilibrium without securities a switch to a^1 with regulated securities decreases aggregate abatement unless there is no type B country.

The conclusions are similar to those in Section 5.3.4. In particular, aggregate abatement is not necessarily higher under a regime of regulated markets for climate securities than in the absence of security markets.

5.5 Variations of the basic model

5.5.1 Dominant strategy equilibrium in the absence of security markets

This section summarizes the results that would emerge without assumption (5.7), i.e. when L is a dominant strategy for type B, and shows that the nature of the results does not change substantially if that assumption is dropped.

Recall that L is a dominant strategy anyway for type A by assumption (5.5). Therefore, in the absence of security markets strategy profile a^1 is an equilibrium in dominant strategies for all priors p. Moreover, strategy profile a^1 is the only Bayesian equilibrium[44] so that, in contrast to the analysis in Section 5.2.2, there can be no multiplicity of equilibria.

[44] Assumption (5.7) can be shown to be a necessary condition in order for strategy profile a^2 to be an equilibrium.

With unregulated security markets, in principle, all strategy profiles can be equilibria.[45] The only noteworthy implication of abandoning assumption (5.7) is that a necessary condition for strategy profile a^2 to be an equilibrium[46] is $g^B_{con}(p) > 0$.

With regulated security markets abandoning assumption (5.7) does not have any tangible implications on the results in Proposition 6 above.

Altogether, the results do not change substantially when assumption (5.7) is abandoned. The main difference lies in the scenario without security markets where assumption (5.7) produces a second equilibrium strategy profile in addition to a^1. Therefore the reference situation is determined by assumption (5.7) to a relatively large extent. However, the effects produced by security markets, either with or without regulation, relative to that reference situation are not changed substantially by assumption (5.7).

5.5.2 An alternative regulation scheme

This section considers an alternative regulation scheme to that used in Section 5.4 above. Recall that in Section 5.4 regulation was tantamount to the prevention of inconsistencies between abatement behaviour, on the one hand, and security trading behaviour, on the other hand.[47] Thus, free riders, i.e. type B countries which did not abate in the first period, were excluded from trade in climate securities in the second period. However, type A countries which had abated in the first period were also excluded from trade in climate securities, i.e. they were punished for abating *too much*.

This property of the regulation scheme applied in Section 5.4 may be controversial for the following reason. At present, in the context of climate change there is no danger of the abatement effort being too high. To the contrary, scientists warn that the targets of the Kyoto process are by far too low. Thus, preventing countries from abating "too much" may be considered the wrong approach. Therefore, an alternative regulation scheme which only

[45] This can be seen by checking the implications of abandoning assumption (5.7) in the equilibrium conditions for unregulated security markets given in Proposition 5 above.

[46] Note, however, that according to the analysis in Chapter 6 below $g^B_{con}(p)$ is often smaller than zero. In that case strategy profile a^2 would be ruled out as an equilibrium.

[47] Cf. Section 5.4.1.

restricts free riders, i.e. type B countries behaving inconsistently, in their trading activities is now studied. Type A countries are not restricted by the regulation.

Formally, if type A countries are not subject to regulation, we have $n_A^R(\theta, a) = n_A(\theta, a)$ for all type profiles θ and for all strategy profiles a. With this modification the utility functions defined in (5.34) still apply.

Bayesian Equilibria

The equilibrium conditions for the four strategy profiles can be calculated in the same way as in Section 5.4. Note that this alternative regulation scheme is less stringent than the regulation scheme studied in Section 5.4. In particular, the alternative regulation scheme is a combination of the two different scenarios with security markets above. For type A countries the situation is similar to that with unregulated securities, whereas for type B countries the situation is similar to that under the regulation scheme in Section 5.4 above. Therefore it is not surprising that two of the expressions in the equilibrium conditions stemming from trade in climate securities, $g_{con}^A(p)$ and $g_{join}^A(p)$, originate in the framework without regulation in Section 5.3 and two of these expressions, $r_{uni}^B(p)$ and $r_{con}^B(p)$, originate from the regulation framework in Section 5.4.

The equilibrium conditions under the alternative regulation scheme are given in Proposition 7.

Proposition 7

a) Strategy profile a^1 is a Bayesian equilibrium, if and only if

$$(1 - \theta^A)b_1 \leq c \tag{5.40}$$

$$\text{and} \quad (1 - \theta^B)b_1 + r_{uni}^B(p) \leq c \tag{5.41}$$

hold.

b) Strategy profile a^2 is a Bayesian equilibrium, if and only if

$$(1 - \theta^A)\sum_{k=0}^{n-1} p_k b_{n-k} + g_{con}^A(p) \leq c \tag{5.42}$$

$$\text{and} \quad (1 - \theta^B)\sum_{k=0}^{n-1} p_k b_{n-k} + r_{con}^B(p) \geq c. \tag{5.43}$$

c) Strategy profile a^3 is a Bayesian equilibrium, if and only if

$$(1-\theta^A)b_n + g^A_{join}(p) \geq c \tag{5.44}$$

$$\text{and} \quad (1-\theta^B)b_n + r^B_{join}(p) \geq c \tag{5.45}$$

where $r^B_{join}(p) := \sum_{k=1}^{n-1} p_k g^R(\theta^B, k, n-k; n)$.

d) Strategy profile a^4 cannot be a Bayesian equilibrium.

The proof is given in Appendix B. Comparison with Proposition 6 shows that the equilibrium conditions and equilibrium regions for strategy profile a^1 are not at all affected by the choice of regulation scheme.[48] For strategy profile a^2 there is only a minor change in one of the equilibrium conditions.[49]

The main impact of the modification of the regulation scheme is observed for strategy profile a^3. Note that now strategy profile a^3 can, in principle, be an equilibrium whereas this was not the case under the more stringent regulation scheme.[50] This is the most striking difference to the other kind of regulation. But strategy profile a^3 cannot be an equilibrium if $g^A_{join}(p) < 0$ holds,[51] since then we have

$$(1-\theta^A)b_n + g^A_{join}(p) < (1-\theta^A)b_n \overset{(5.6)}{\leq} c$$

which contradicts (5.44). In that case the most striking difference between the results of the two different regulation schemes vanishes. The only difference remaining is then inequality (5.42).

However, if the expected additional abatement incentives are smaller than zero as suggested by the examples in Chapter 6 below, the main difference

[48] The equilibrium conditions for strategy profile a^1, (5.40) and (5.41), are identical to (5.35) and (5.36) in Proposition 6.

[49] The equilibrium condition for type A, (5.42), is identical to that for security markets **without** regulation, (5.28), whereas the condition for type B, (5.43), is the same as with the more stringent kind of regulation, (5.38).

[50] The equilibrium condition for type A, (5.44), is again identical to that for unregulated securities in (5.30). Inequality (5.45) is the only equilibrium condition that has not yet appeared in Section 5.4 above.

[51] Recall that the signs of the expected additional abatement incentives $g^A_{con}(p)$ and $g^A_{join}(p)$ are, in principle, ambiguous. However, the evidence from the examples in Section below suggests that $g^A_{join}(p)$ will often be smaller than zero.

concerning strategy profile a^3 vanishes. In particular, studying the case of two countries all the differences mentioned above vanish for negative expected additional abatement incentives, i.e. both types of regulation lead to exactly the same results concerning the equilibrium regions for the different strategy profiles. Therefore, the choice of the regulation scheme is not decisive for the results.

5.6 Summary

This chapter introduced a framework for the formal analysis of the IPCC claim that insurance instruments, represented by climate securities, could reduce free riding with respect to abatement of greenhouse gases. The focus of the three-period static[52] model introduced is on *incomplete information* with respect to countries' assessments of climate risk.

Countries first decide on their abatement strategy and then trade in climate securities if such instruments are available. Due to incomplete information the setup leads to a *Bayesian Game* in abatement strategies. Three different scenarios were studied, a benchmark scenario without climate securities, as well as two variations of the basic setup with markets for climate securities, one without and one with regulatory intervention. For the two scenarios with climate securities the outcome of trading in climate securities, security market equilibrium, is anticipated within the Bayesian game.

A large part of this chapter was concerned with the analysis of the Bayesian equilibria of those three Bayesian games. It turned out that two of the four potential equilibrium strategy profiles can be equilibria for all three scenarios, the "never abate" strategy profile a^1 and the "type-consistent" strategy profile a^2. With respect to their efficiency properties it was shown that strategy profile a^2 ex ante- and interim dominates strategy profile a^1 whenever a^2 is an equilibrium. In other words, strategy profile a^2 is *preferred* to strategy profile a^1.

The determining factors for the effect of climate securities on abatement activity are the so-called *expected additional abatement incentives* generated

[52] The model is "static" because each kind of decision is taken only once.

by security markets. In the absence of regulation, the signs of the expected additional abatement incentives and, consequently, also the effect of trade in climate securities are ambiguous.

If, however, security markets are regulated in such a manner that countries can only transact security trades *consistent* with their abatement activity, the expected additional abatement incentives are always larger than zero. In particular, the equilibrium conditions for the preferred strategy profile a^2 are relaxed, whereas those for the less preferred strategy profile a^1 are tightened. Therefore, under regulation the preferred strategy profile a^2 will be an equilibrium for a larger set of economies, whereas the less preferred strategy profile a^1 will be an equilibrium for a smaller set of economies, i.e. there is a *shift towards the preferred strategy profile.*

A more precise analysis of the effects on the equilibrium regions, i.e. the sets of priors for which the strategy profiles are equilibria, follows in Chapter 6.1 for the case of two countries. The analysis there confirms that there is such a shift with respect to the equilibrium regions.

Moreover, two variations of the basic model with respect to underlying assumptions were studied. For both of these variations it turned out that the nature of the results does not change fundamentally, i.e. the corresponding assumptions that were varied are not decisive for the results.

Concerning the IPCC claim that climate securities can reduce free riding in the context of climate policy the results for regulated securities up to this point seem supportive of that claim. Note that, although we do not actually observe free riding in equilibrium, strategy profile a^1 is an equilibrium, because each of the countries wants to free ride on the other countries' abatement effort. It is the *potential* free riding of the others that stops countries from abating in the first place. Therefore, the equilibrium strategy profile a^1 can be interpreted as a consequence of free riding tendencies. In this sense the results in Chapter 5.4 seem supportive of the IPCC claim.

Recall that the results in this chapter are based on two assumptions that may be subject to criticism. Firstly, the kind of regulation applied is only feasible if countries are identical apart from the type or, as an approximation, very similar. If that is not the case, countries cannot be "separated" according to their types by means of trade in climate securities as in Lemma

2 and regulation is not an option. However the results for unregulated security markets are too ambiguous to be interpreted as supportive of the IPCC claim. For heterogeneous countries it is argued in Chapter 7.2 below that it is virtually impossible to deduce countries' types from their trading behaviour, at least without very specific assumptions, so that regulation is not possible.

Secondly, the assumption of exogenous probabilities for climate states is not satisfactory. Its adequacy depends on the time horizon under consideration. In the short run exogenous probabilities may be a good enough approximation, but in the long run climate state probabilities ought to be endogenous. For the incomplete information approach introduced in this chapter, climate state probabilities can be endogenized without changing the main results. See Chapter 7.1 for the generalization of the approach to endogenous probabilities. A different model, focusing on endogenity of probabilities, is analysed in Chapter 8.

Chapter 6

Illustration of the incomplete information approach for the case of two countries

In Chapter 5 the model and the general results for the incomplete information approach were presented. However, for the general case of n countries the crucial expressions, the *expected additional abatement incentives*, were too complex to allow precise conclusions about the effect of regulated or unregulated climate securities on the equilibrium regions.

Therefore, this chapter considers the special case of only two countries[1] in order to give a more detailed analysis than Chapter 5. First, the equilibrium regions for all three scenarios analysed in Chapter 5 are derived and graphically illustrated in Section 6.1. Then, the equilibrium regions are explicitely calculated for several functional specifications in Section 6.2.

[1] Obviously, the assumption of competitive markets for climate securities is not convincing for the case of two countries, but in order to build up some intuition concerning the underlying mechanisms it is useful.

6.1 The equilibrium regions for the case of two countries

6.1.1 No security markets

Recall from Chapter 5.2.2 that the equilibrium condition (5.20) for strategy profile a^1

$$(1-\theta^B)b_1 \leq c \tag{6.1}$$

holds for all priors p by assumption (5.5). Therefore, the equilibrium region for strategy profile a^1 extends over all of the unit interval $[0,1]$ as depicted in Figure 6.1 below.[2]

For the case of two countries the equilibrium condition from Proposition 3 for strategy profile a^2, (5.21), simplifies to

$$(1-\theta^A)\left[pb_1+(1-p)b_2\right] \leq c \leq (1-\theta^B)\left[pb_1+(1-p)b_2\right]. \tag{6.2}$$

Inequality (6.2) can be interpreted as follows. The expression in brackets, $[pb_1+(1-p)b_2]$, represents the expected[3] marginal benefits from choosing H to one of the countries if state β emerges, given that the other country behaves according to strategy profile a^2. The other country is type A with probability p, in which case it will choose L if it behaves according to strategy profile a^2, and type B with probability $1-p$, in which case it will choose H according to strategy profile a^2. Therefore, if the first country chooses H with probability p it is the only country abating, i.e. abating *unilaterally*. In that case, marginal abatement benefits are equal to b_1. With probability $1-p$ both countries abate, i.e. they abate *jointly*,[4] yielding marginal abatement benefits equal to b_2. Since abatement benefits only arise if state β occurs, they are weighted with the probability $(1-\theta^X)$ attributed to state β by type X $(X=A,B)$.

[2] This observation holds for general values of n, not only for $n=2$.

[3] "Expected" here refers to the unknown type of the other country.

[4] Note that in this framework "joint abatement" merely stands for a situation in which both countries abate, but is **not** the outcome of cooperation. Each country takes its decision independently and if the result happens to be that both countries abate we call it "joint abatement".

We now proceed to derive the equilibrium region, i.e. the set of priors p for which the equilibrium condition is satisfied, for strategy profile a^2 from inequality (6.2). Note that by assumption (5.7)

$$(1-\theta^B)b_2 > c \tag{6.3}$$

has to hold for $n = 2$.[5] The equilibrium region for a^2 is characterized in

Corollary 1

Strategy profile a^2 is a Bayesian equilibrium, if and only if

$$p \leq \frac{(1-\theta^B)b_2 - c}{(1-\theta^B)(b_2 - b_1)} =: p_{2B} < 1 \quad \textit{holds.} \tag{6.4}$$

Proof. The left inequality in (6.2) holds for arbitrary p by assumptions (5.5) and (5.6). The right inequality is equivalent to (6.4) and, moreover, p_{2B} is well-defined, since $0 < p_2 < 1$ follows from (5.5) and (6.3). ■

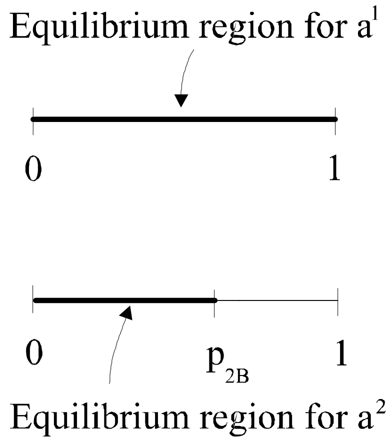

Figure 6.1: *The equilibrium regions for strategy profiles a^1 (top) and a^2 (bottom) with respect to the prior p in the absence of security markets.*

By Corollary 1 strategy profile a^2 is an equilibrium if and only if the probability p of the other country being type A is small enough, i.e. p not larger than p_{2B}. Figure 6.1 depicts the equilibrium regions for strategy profiles a^1 and a^2. For a given prior p there exists at least one Bayesian Nash equilibrium. For priors in the interval $[0, p_{2B}]$ there are two equilibria.

[5] See Chapter 5.5.1 for a discussion of the results without assumption (6.3).

The intuition behind the result is that type B has an incentive for joint abatement by assumption (6.3), but not for unilateral abatement by assumption (5.5). In order for type B to abate, as required in the equilibrium conditions for strategy profile a^2, the weight on the marginal benefits from joint abatement, $1-p$, has to be large enough and that on the marginal benefits from unilateral abatement, p, small enough. In different words, the expected marginal benefits from abatement for type B, $pb_1 + (1-p)b_2$, are decreasing in p so that in order for the expected marginal benefits to exceed marginal cost c, the prior p has to be small enough.

6.1.2 Unregulated security markets

Recall from Chapter 5.3 that there will be no trade if both countries are of the same type. Therefore, whenever trade actually takes place, there must be exactly one country of each type, i.e. $n_A(\theta) = n_B(\theta) = 1$. For this reason we can omit $n_A(\theta)$ as an argument in $g(\theta^X, n_A(\theta); \tilde{a})$ and write

$$g_{\tilde{a}}^X := g(\theta^X, 1; \tilde{a}) \tag{6.5}$$

for the gains from trade for type X when aggregate abatement is $\tilde{a}$.

The expected additional abatement incentives

The expected additional abatement incentives generated by unregulated markets for climate securities, denoted $g_{uni}^X(p)$, $g_{con}^X(p)$, $g_{join}^X(p)$ and $g_{inc}^X(p)$ for $X = A, B$ in Section 5.3, simplify considerably for $n = 2$. For example,

$$\begin{aligned} g_{uni}^A(p) &= \sum_{k=0}^{n-2} p_k[g(\theta^A, k+1; 1) - g(\theta^A, k+1; 0)] \\ &= p_0[g(\theta^A, 1; 1) - g(\theta^A, 1; 0)] \\ &\overset{(6.5)}{=} \binom{1}{0} p^0 (1-p)^1 [g_1^A - g_0^A] \\ &= (1-p)[g_1^A - g_0^A]. \end{aligned}$$

The remaining seven expressions are of a similar structure. In particular, they all consist of a difference of two gains from trade expressions multiplied by either the prior p or by $1-p$.

For the analysis of the equilibrium regions below it is convenient to define

$$\begin{aligned} \bar{g}^X_{uni} &:= g^X_1 - g^X_0 \qquad \text{for} \quad X = A, B \\ \text{and} \quad \bar{g}^X_{join} &:= g^X_2 - g^X_1 \qquad \text{for} \quad X = A, B \end{aligned}$$

without including the multiplicative factor[6] containing the prior p. The bar indicates that the expressions $\bar{g}^X_{uni}$ and $\bar{g}^X_{join}$ do *not* depend on the prior p. $\bar{g}^X_{uni}$ is the difference between the gains from trade for type X when one of the countries abates and when none of the countries abates. In particular, $\bar{g}^X_{uni}$ can be interpreted as the (expected) *additional incentive for unilateral abatement*, i.e. assuming that the other country does not abate. Similarly, $\bar{g}^X_{join}$ can be interpreted as the *additional incentive to join in abatement* for type X, i.e. assuming that the other country abates ($X = A, B$). The signs of $\bar{g}^X_{uni}$ and $\bar{g}^X_{join}$ are equally ambiguous as those of the expressions $g^X_{uni}(p)$, $g^X_{con}(p)$, $g^X_{join}(p)$ and $g^X_{inc}(p)$.

The equilibrium conditions

The equilibrium conditions from Proposition 5 simplify to

a) Strategy profile a^1 is an equilibrium, if and only if

$$\begin{aligned} (1-\theta^A)b_1 + (1-p)\bar{g}^A_{uni} &\leq c \qquad (6.6) \\ \text{and} \qquad (1-\theta^B)b_1 + p\bar{g}^B_{uni} &\leq c. \qquad (6.7) \end{aligned}$$

b) Strategy profile a^2 is an equilibrium, if and only if

$$\begin{aligned} (1-\theta^A)\left[pb_1 + (1-p)b_2\right] + (1-p)\bar{g}^A_{join} &\leq c \qquad (6.8) \\ \text{and} \qquad (1-\theta^B)\left[pb_1 + (1-p)b_2\right] + p\bar{g}^B_{uni} &\geq c. \qquad (6.9) \end{aligned}$$

c) Strategy profile a^3 is an equilibrium, if and only if

$$\begin{aligned} (1-\theta^A)b_2 + (1-p)\bar{g}^A_{join} &\geq c \qquad (6.10) \\ \text{and} \qquad (1-\theta^B)b_2 + p\bar{g}^B_{join} &\geq c. \qquad (6.11) \end{aligned}$$

[6]The weights p or $1-p$ arising from the binomial distribution appear directly in the equilibrium conditions below.

d) Strategy profile a^4 is an equilibrium, if and only if

$$(1-\theta^A)\left[(1-p)b_1+pb_2\right]+(1-p)\bar{g}^A_{uni} \geq c \qquad (6.12)$$

$$\text{and} \qquad (1-\theta^B)\left[(1-p)b_1+pb_2\right]+p\bar{g}^B_{join} \leq c. \qquad (6.13)$$

Whether the equilibrium conditions for each strategy profile become more or less stringent as a result of markets for climate securities depends on the sign and size of the additional abatement incentives $\bar{g}^X_{uni}$ and $\bar{g}^X_{join}$.

The equilibrium regions for strategy profile $\mathbf{a^1}$

In order to determine the equilibrium regions for the four strategy profiles the pairs of inequalities above have to be analysed in more detail. That involves solving each of the inequalities for the prior p and then determining the sets of priors for which each pair of inequalities holds. The results for strategy profile a^1 are summarized in Corollary 2 below.

Corollary 2
The equilibrium regions for strategy profile a^1 with respect to the prior p, dependent on the size of the additional abatement incentives $\bar{g}^A_{uni}$ and $\bar{g}^B_{uni}$, are listed in the following table.

Equilibrium regions for strategy profile $\mathbf{a^1}$				
	$\bar{g}^B_{uni} \leq c-(1-\theta^B)b_1$		$\bar{g}^B_{uni} > c-(1-\theta^B)b_1$	
$\bar{g}^A_{uni} \leq c-(1-\theta^A)b_1$	***a)***	$[0,1]$	***b)***	$[0,p^{NR}_{1B}]$
$\bar{g}^A_{uni} > c-(1-\theta^A)b_1$	***c)***	$[p^{NR}_{1A},1]$	***d)***	$[p^{NR}_{1A},p^{NR}_{1B}]$

$$\textit{where} \quad p^{NR}_{1A} := \frac{1}{\bar{g}^A_{uni}}\left[(1-\theta^A)b_1-c+\bar{g}^A_{uni}\right] \qquad (6.14)$$

$$\textit{and} \quad p^{NR}_{1B} := \frac{1}{\bar{g}^B_{uni}}\left[c-(1-\theta^B)b_1\right]. \qquad (6.15)$$

The proof is given in Appendix C. The table in Corollary 2 has to be interpreted as follows. If, for example, $\bar{g}^A_{uni} \leq c-(1-\theta^A)b_1$ and $\bar{g}^B_{uni} \leq c-(1-\theta^B)b_1$ hold, then case a) arises and strategy profile a^1 is an equilibrium for all priors in the unit interval $[0, 1]$. Figure 6.2 illustrates the equilibrium regions determined in Corollary 2 in comparison to the equilibrium region for the scenario without securities.

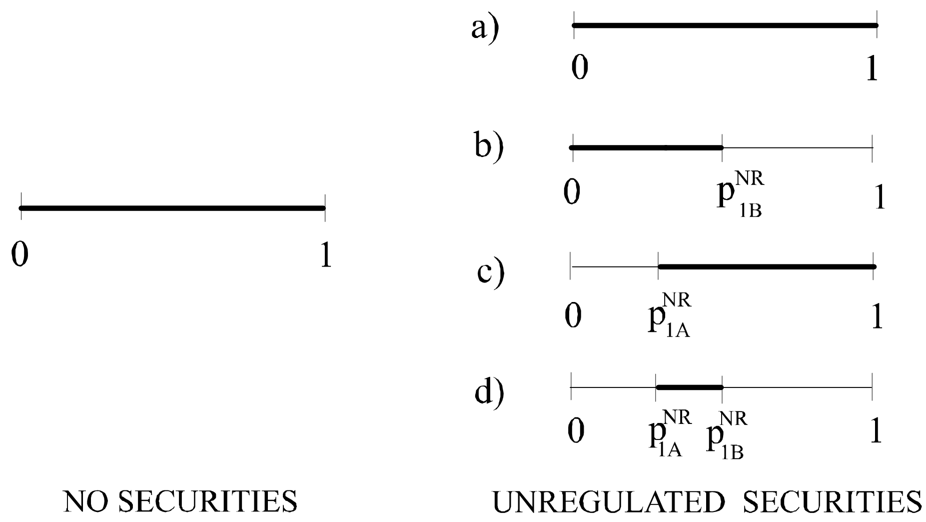

Figure 6.2: *Equilibrium regions for strategy profile* a^1 *in the absence of security markets (left) and with security markets that are not subject to regulation (right). The four intervals marked a) to d) on the right hand side correspond to case a) to d) in Corollary 2.*

Recall from Section 5.2 that in the absence of securities strategy profile a^1 is a Bayesian equilibrium for all priors since neither of the two types has an incentive for unilateral abatement. The intuition for case a) in Corollary 2 is then obvious. If the *additional* incentives for unilateral abatement $\bar{g}^A_{uni}$ and $\bar{g}^B_{uni}$ are both relatively small (or even negative), neither of the types has an incentive for unilateral abatement. Therefore, the equilibrium region for strategy profile a^1 is the complete unit interval as in the absence of securities (cf. case a) in Figure 6.2).

If, for example, $\bar{g}^B_{uni}$ is relatively large, strategy profile a^1 is an equilibrium only for sufficiently small priors (cf. case b) in Figure 6.2). The reason is that in the equilibrium condition for type B, (6.7), the basic incentive for unilateral abatement, $(1-\theta^B)b_1$, is augmented by the additional incentive for unilateral abatement multiplied by the prior, $p\bar{g}^B_{uni}$. For large priors, inequality (6.7) does not hold.

Similarly, if $\bar{g}^A_{uni}$ is relatively large, strategy profile a^1 is an equilibrium only for sufficiently large priors (cf. case c) in Figure 6.2). Note that in the equilibrium condition for type A (6.6) the additional incentive for unilateral abatement $\bar{g}^A_{uni}$ is multiplied by $(1-p)$. Therefore, inequality (6.6) does not hold for small priors.

If the additional incentives for unilateral abatement are large for *both* types (cf. case d) Figure 6.2) the equilibrium region is the *intersection* of the two intervals in b) and c). Note that $p^{NR}_{1A} > p^{NR}_{1B}$ may hold in d). In that case, the interval $[p^{NR}_{1A}, p^{NR}_{1B}]$ is the empty set and there is no prior for which strategy profile a^1 is an equilibrium.

The equilibrium regions for strategy profile $\mathbf{a^2}$

The equilibrium regions for the type-consistent strategy profile a^2 were derived in the same manner as for strategy profile a^1 above and are illustrated in Figure 6.3. Their derivation, omitted here for brevity, and the definitions of p^{NR}_{2A} and p^{NR}_{2B} can be found in Corollary 6 in Appendix C.

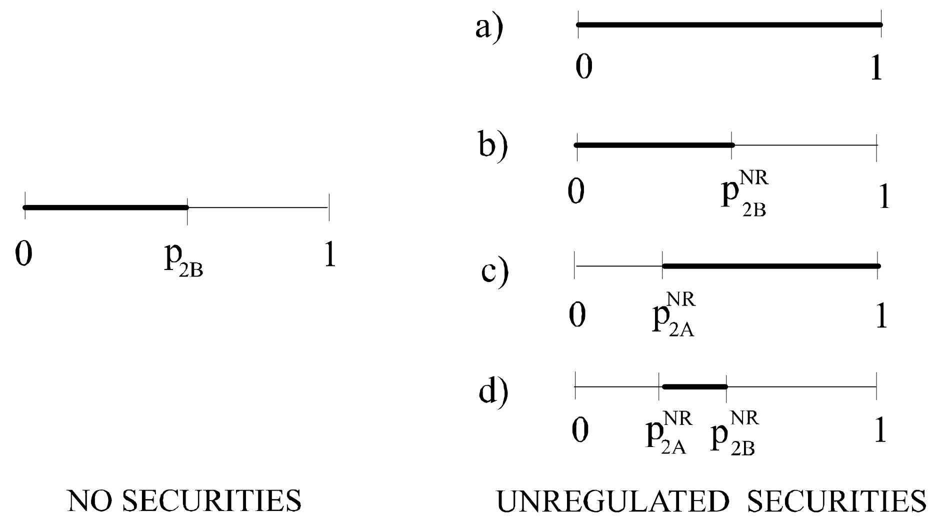

Figure 6.3: *Equilibrium regions for strategy profile a^2 in the absence of security markets (left) and with security markets that are not subject to regulation (right). The four figures on the right hand side, marked a) to d), correspond to cases a) to d) in Corollary 6.*

Figure 6.3 shows that four different cases may arise with respect to the equilibrium region for strategy profile a^2. The structures of the equilibrium regions strongly ressemble those for strategy profile a^1 in Figure 6.2. However, the relevant additional abatement incentives are $\bar{g}^A_{join}$ and $\bar{g}^B_{uni}$ since they appear in the equilibrium conditions.[7] Moreover, the expressions p^{NR}_{2A} and p^{NR}_{2B} are defined differently to p^{NR}_{1A} and p^{NR}_{1B}. Note that the expressions $\bar{g}^A_{join}$ and $\bar{g}^B_{uni}$ enter both the definition of p^{NR}_{2A} and p^{NR}_{2B} and the conditions determining which of the four cases arises.

In general, we cannot say whether $p^{NR}_{2A} \leq p^{NR}_{2B}$ or $p^{NR}_{2A} > p^{NR}_{2B}$ holds. Figure 6.3 is drawn for the case $p^{NR}_{2A} < p^{NR}_{2B}$. If $p^{NR}_{2A} > p^{NR}_{2B}$ holds, the interval in case d) becomes the empty set. Therefore, the possible range for the equilibrium region for strategy profile a^2 stretches from the unit interval to the empty set, depending on the sign and size of the additional abatement incentives $\bar{g}^A_{join}$ and $\bar{g}^B_{uni}$.

For purposes of comparison with the scenario without securities the crucial point is the relation between p^{NR}_{2B} and p_{2B}. Whether p^{NR}_{2B} is smaller or larger than p_{2B} depends on the sign of $\bar{g}^B_{uni}$ as shown in the following Corollary.

Corollary 3 *(Comparison of p_{2B} and p^{NR}_{2B})*
Assume $\bar{g}^B_{uni} < c - (1-\theta^B)b_1$. Then

$$\bar{g}^B_{uni} < 0 \quad \Leftrightarrow \quad p_{2B} > p^{NR}_{2B}$$

$$\text{and} \quad 0 < \bar{g}^B_{uni} < c - (1-\theta^B)b_1 \quad \Leftrightarrow \quad p_{2B} < p^{NR}_{2B}.$$

The proof is given in Appendix C. By Corollary 3 the equilibrium region with unregulated securities in case b) of Figure 6.3 is smaller than that without securities if $\bar{g}^B_{uni}$ is smaller than zero and larger than that without securities if $\bar{g}^B_{uni}$ is larger than zero.[8]

Summarizing, the impact of unregulated markets for climate securities on the equilibrium regions for strategy profile a^2 can vary strongly. If case a) of Figure 6.3 is the relevant case the equilibrium region is the unit interval

[7] Cf. (6.8) and (6.9).

[8] Note that the assumption in Corollary 3 corresponds to that relevant for case b) in Figure 6.3.

with securities whereas it consisted of the interval $[0, p_{2B}]$ in the absence of securities. The other extreme is case d) where the equilibrium region may be the empty set.

The equilibrium regions for strategy profile a^3

The equilibrium regions for strategy profile a^3 are illustrated[9] in Figure 6.4. Recall from Section 5.2.2 that in the absence of securities strategy profile a^3

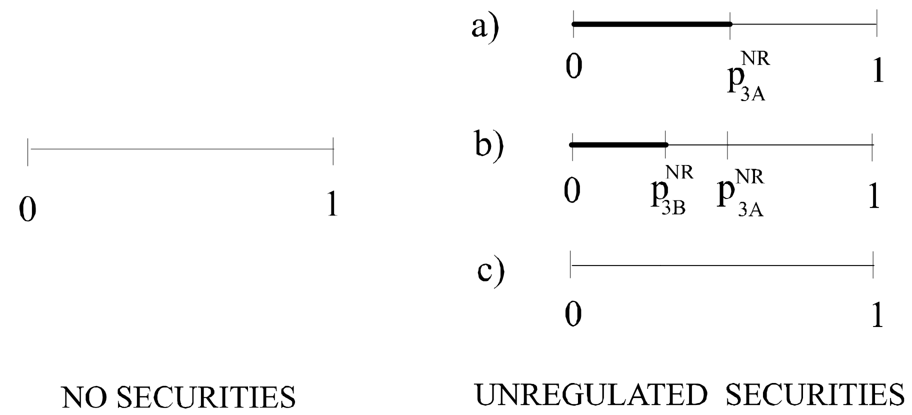

Figure 6.4: *Equilibrium regions for strategy profile a^3 in the absence of security markets (left) and with security markets that are not subject to regulation (right). Note that the equilibrium region in the absence of security markets is the empty set. The three figures on the right hand side, marked a) to c), correspond to cases a) to c) in Corollary 7.*

cannot be an equilibrium.[10]

With unregulated securities, the relevant additional abatement incentives are $\bar{g}^A_{join}$ and $\bar{g}^B_{join}$. Strategy profile a^3 is a Bayesian equilibrium for sufficiently small priors if the additional incentive for joint abatement for type A, $\bar{g}^A_{join}$, is large enough (cf. case a) and b) in Figure 6.4 and Corollary 7). In particular

[9] The formal derivation which is analogous to that for strategy profile a^1 above and the definitions of p^{NR}_{3A} and p^{NR}_{3B} are presented in Corollary 7 in Appendix C.

[10] I.e. the equilibrium region is the empty set. The reason is that the equilibrium conditions for strategy profile a^3 require both types to have an incentive for joint abatement which is excluded by the assumption of type A having L as dominant strategy in (5.5) and (5.6).

$\bar{g}^A_{join}$ has to be larger than zero in order for strategy profile a^3 to be an equilibrium.

The equilibrium regions for strategy profile $\mathbf{a^4}$

Figure 6.5 illustrates the equilibrium regions for strategy profile a^4 with unregulated securities.[11] Recall from Section 5.2 that strategy profile a^4 can-

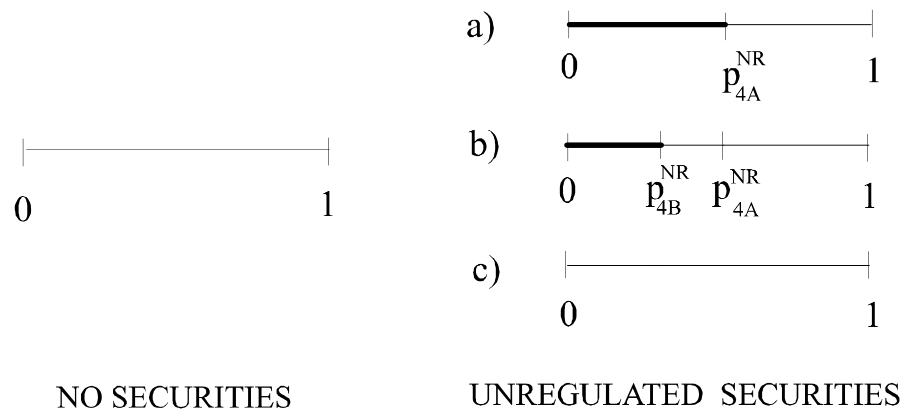

Figure 6.5: *Equilibrium regions for strategy profile a^4 in the absence of security markets (left) and with security markets that are not subject to regulation (right). The three figures on the right hand side, marked a) to c), correspond to cases a) to c) in Corollary 8.*

not be an equilibrium in the absence of securities. With unregulated security markets, the relevant additional abatement incentives are $\bar{g}^A_{uni}$ and $\bar{g}^B_{join}$. Strategy profile a^4 can be an equilibrium for sufficiently small priors if type A's additional incentive for unilateral abatement $\bar{g}^A_{uni}$ is relatively large.

The case of additional abatement disincentives

Recall that the expressions $\bar{g}^X_{uni}$ and $\bar{g}^X_{join}$ $(X = A, B)$ can have a positive or a negative sign. Evidence from the examples in Section 6.2 below suggests that the case of negative signs is indeed relevant.

[11] The details are given in Corollary 8, again in Appendix C.

Therefore consider the special case $\bar{g}^A_{uni} < 0$, $\bar{g}^B_{uni} < 0$ and $\bar{g}^A_{join} < 0$ for a moment.[12] Then the implications of Corollaries 2, 6, 7 and 8 in combination with assumptions (5.5) and (5.6) are clear cut. Strategy profile a^1 is an equilibrium for all priors $p \in [0, 1]$ whereas the equilibrium region for strategy profile a^2 is $[0, p^{NR}_{2B}]$. Strategy profiles a^3 and a^4 cannot be equilibria, i.e. their equilibrium regions are the empty set.

The effect of security markets on aggregate abatement

The analysis of the equilibrium regions above shows that for certain values of the prior p there are multiple equilibria both in the absence of securities and with unregulated securities. Whenever the equilibrium regions for two different strategy profiles overlap, the analysis of Sections 5.3.4 and 5.4.5 above applies.

Recall from Section 6.1.1 that in the absence of security markets both strategy profile a^1 and strategy profile a^2 are equilibria if p lies in the interval $[0, p_{2B}]$, which is the intersection of the equilibrium regions for a^1 and a^2. With unregulated security markets there may be up to four equilibria for some values of p. For example, if case a) is the relevant case in each of the Figures 6.2 to 6.5, all four strategy profiles are equilibria for values of p satisfying $0 \leq p \leq \min\{p^{NR}_{3A}, p^{NR}_{4A}\}$.

Therefore the introduction of securities may be accompanied by a switch from one equilibrium in the absence of these markets to another equilibrium when there are security markets. In particular, as argued in Sections 5.3.4 and 5.4.5, aggregate abatement can increase or decrease with security markets relative to a situation without these markets.

6.1.3 Regulated security markets

With regulated security markets and only two countries trade can only take place if there is exactly one country of each type[13] and if both countries

[12] The expression $\bar{g}^B_{join}$ does not appear in the equilibrium conditions given in Corollaries 2, 6, 7 and 8.

[13] Cf. Section 6.1.2.

behaved type-consistently in the first period.[14] Consequently, whenever trade takes place under regulation $n_A^R(\theta) = n_B^R(\theta) = 1$ holds.

Note that the gains from trade for type θ_i with and without regulation do not differ for a given level of aggregate abatement $\tilde{a}$ if there is the same number of countries of each type trading. Thus, for the case of two countries

$$g^R(\theta_i, 1, 1; \tilde{a}) = g(\theta_i, 1; \tilde{a}) \qquad \forall\, \theta_i, \tilde{a}$$

holds and we can write $g_{\tilde{a}}^X$, defined as abbreviation for $g(\theta_i, 1, \tilde{a})$ in (6.5), for $g^R(\theta^X, 1, 1, \tilde{a})$.

For the case of two countries the equilibrium conditions from Proposition 6 simplify to[15]

a) Strategy profile a^1 is an equilibrium, if and only if

$$\begin{aligned} (1-\theta^A)b_1 &\leq c && (6.16)\\ \text{and} \quad (1-\theta^B)b_1 + pg_1^B &\leq c. && (6.17) \end{aligned}$$

b) Strategy profile a^2 is an equilibrium, if and only if

$$\begin{aligned} (1-\theta^A)[pb_1 + (1-p)b_2] &\leq c + (1-p)g_1^A && (6.18)\\ \text{and} \quad (1-\theta^B)[pb_1 + (1-p)b_2] &\geq c - pg_1^B. && (6.19) \end{aligned}$$

Note that, with regulation, expected additional abatement incentives do not appear in all the equilibrium conditions. This is due to the fact that with regulation there is trade in fewer situations than without regulation.

The equilibrium regions for strategy profile $\mathbf{a^1}$

The equilibrium regions for strategy profile a^1 with regulated security markets are illustrated[16] in Figure 6.6. With respect to strategy profile a^1 the

[14] If only one of the countries behaves consistently and is therefore allowed to trade it has no trading partner. Cf. Section 5.4.

[15] Recall from Proposition 6 that strategy profiles a^3 and a^4 cannot be equilibria with regulated security markets.

[16] They are derived analogously to the equilibrium regions for unregulated securities in Section 6.1.2. The details, in particular the definition of p_{1B}^R, are given in Corollary 9 in Appendix C.

equilibrium condition for type A, (6.16), is not affected by security trading. Therefore, only the gains from trade for type B, g_1^B, have an impact on the results. If g_1^B is relatively small, strategy profile a^1 is an equilibrium for all priors as was the case in the absence of securities (cf. case a) in Figure 6.6). If g_1^B is relatively large, i.e. large enough to create an incentive for unilateral abatement, strategy profile a^1 is only an equilibrium if the weight p on these gains from trade is small enough (cf. part b) in Figure 6.6).

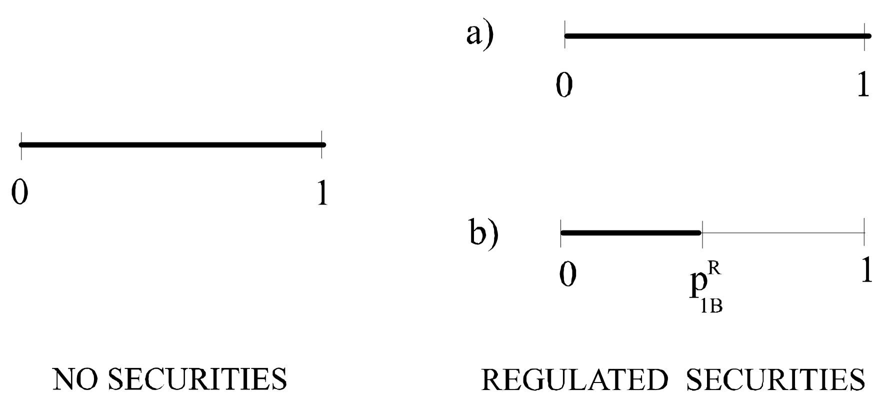

Figure 6.6: *Equilibrium regions for strategy profile a^1 in the absence of security markets (left) and with regulated security markets (right). The two figures on the right hand side, marked a) and b) correspond to parts a) and b) of Corollary 9.*

The equilibrium regions for strategy profile $\mathbf{a^2}$

Figure 6.7 illustrates[17] the equilibrium regions for strategy profile a^2. Recall that in the absence of markets for climate securities the type-consistent strategy profile a^2 is an equilibrium only for priors in the interval $[0, p_{2B}]$. With regulated securities strategy profile a^2 is an equilibrium for all priors if the relevant gains from trade are relatively large for type B (cf. case a) in Figure 6.7). If the relevant gains from trade are relatively small, strategy profile a^2 is only an equilibrium for priors in the interval $[0, p_{2B}^R]$ (cf. case b) in Figure 6.7).

[17] For the derivation see Corollary 9 in Appendix C.

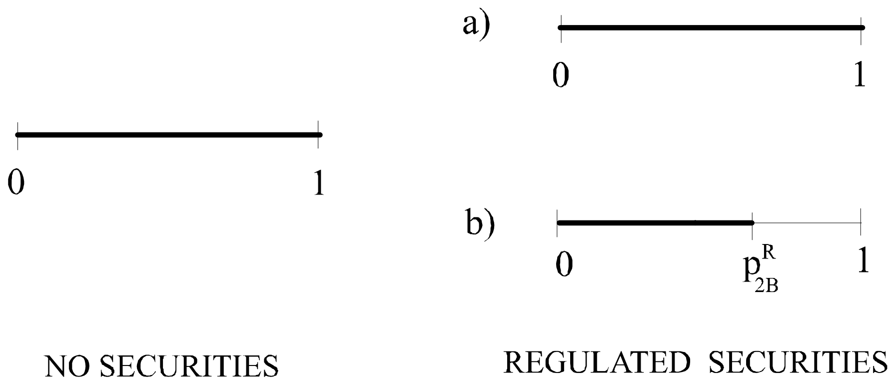

Figure 6.7: *Equilibrium regions for strategy profile a^2 in the absence of security markets (left) and with regulated security markets (right). The two figures on the right hand side marked a) and b) correspond to parts a) and b) of Corollary 9.*

Corollary 4 *(Comparison of p^R_{2B} with p_{2B})*
Assume $g^B_1 < c - (1 - \theta^B)b_1$. Then $p^R_{2B} > p_{2B}$ holds.

The proof is given in Appendix C. Corollary 4 shows that p^R_{2B} is larger than p_{2B} under the assumptions of case b) in Figure 6.7 and Corollary 9. Therefore, with regulated security markets the equilibrium region for strategy profile a^2 is always larger than the equilibrium region for strategy profile a^2 without security markets, no matter which of the two cases depicted in Figure 6.7 arises. In different words, with regulated security markets strategy profile a^2 is an equilibrium for a larger set of economies than without security markets.

The effect of regulated markets for climate securities on aggregate abatement

In Sections 5.3.4 and 5.4.5 it was pointed out that the level of aggregate abatement is only affected by security markets through the existence of multiple equilibria and potential switching from one equilibrium to another. The same applies here.

With regulation, both strategy profile a^1 and strategy profile a^2 are equilibria at least for the interval $[0, \min\{p^R_{1B}, p^R_{2B}\}]$. Depending on which of the cases a) and b) arises for each of the strategy profiles the overlap of the equilibrium regions may be even larger.

Therefore, as described in Section 5.4.5 above, there is potential for switching between equilibria and the level of aggregate abatement can be smaller or larger with regulated securities than in the absence of such markets.

Summary

In contrast to the case of unregulated securities the impact of climate securities on abatement is not ambiguous if trade in climate securities is feasible. The equilibrium region for the "never abate" strategy profile a^1 is either identical to or smaller than that in the absence of security markets. Thus, there is a tendency for the less desirable strategy profile a^1 to be an equilibrium for a smaller set of economies than in the absence of markets for climate securities.

For the preferred type-consistent strategy profile a^2, the equilibrium region is always larger with regulated security markets than without securities. Therefore regulated security markets make the preferred equilibrium "appear" for a larger set of economies, i.e. a wider range of priors than is the case without securities. Put differently, if regulation of markets for climate securities is a feasible way to proceed, then these markets generate a shift from the "never abate" strategy profile a^1 to the "type-consistent" strategy profile a^2.

6.2 Examples

Although the analysis for the case of two countries in Section 6.1 delivered new insight, some open questions remained. This section presents a number of examples in order to improve the intuition with respect to the effects analysed above and illustrate the process of calculating the equilibrium regions.

So far, the only assumption with respect to the von Neumann-Morgenstern utility function $u(\cdot)$ was risk aversion, i.e. $u'(x) > 0$ and $u''(x) < 0$ for all x. Within this section, several different utility functions will be used. They differ, for example, with respect to how the degree of risk aversion changes with either the absolute or the relative level of consumption. Therefore, the following subsection briefly recalls the well-known concepts of *absolute and relative risk aversion.*

6.2.1 Measures of risk aversion

Absolute risk aversion

Risk aversion is strongly related to the curvature of $u(\cdot)$. However, the second derivative $u''(x)$ is not a suitable measure, since it is not invariant to positive linear transformations of the utility function.[18] Therefore the *Arrow-Pratt coefficient of absolute risk aversion*[19] $R_a(x)$ is defined as

$$R_a(x) := -\frac{u''(x)}{u'(x)}.$$

PRATT [1964] argued in favour of *decreasing absolute risk aversion* (DARA). The underlying intuition is that willingness to bear absolute risks increases with wealth, i.e. a rich individual is less risk averse than a poor one. This hypothesis is widely accepted in the literature and also well-supported empirically. The assumption of *constant absolute risk aversion* (CARA) corresponds to the absence of wealth effects with respect to risk aversion and may be an acceptable approximation or simplification when wealth effects are small.

Relative risk aversion

Considering proportional rather than absolute changes in wealth yields

$$R_r(x) := -\frac{x \cdot u''(x)}{u'(x)} = xR_a(x)$$

[18] Cf. MAS-COLELL ET AL. [1995], Chapter 6.

[19] See ARROW [1965] and PRATT [1964]. Note that since the utility function can be recovered up to two integration constants from R_a by integrating twice, the preferences are characterized completely by the Arrow-Pratt coefficient of absolute risk aversion (cf. HIRSHLEIFER AND RILEY [1992] or MAS-COLELL, WHINSTON AND GREEN [1995]).

as *measure of relative risk aversion*.[20]

Which assumption concerning relative risk aversion is most plausible is controversially debated in the literature. Arrow [1965] argued in favour of *increasing relative risk aversion* (IRRA). However, this hypothesis has not found general acceptance. For lack of convincing arguments and evidence in favour of one of the three possible assumptions *constant relative risk aversion* (CRRA) is often assumed.

6.2.2 Exponential utility

As first example consider a utility function of the exponential type defined as

$$u(x) = -\lambda e^{-\mu x} \qquad \text{with} \quad \lambda, \mu > 0. \tag{6.20}$$

It is easily checked that the coefficient of absolute risk aversion is $R_a(x) = \mu$, a constant larger than zero. Therefore this type of utility function exhibits CARA. Furthermore, we get $R_r(x) = x \cdot R_a(x) = \mu x$ which increases in x, implying IRRA.

Typical patterns of equilibrium regions

Recall from Section 6.1 that the structure of the equilibrium regions for unregulated securities depends decisively on the signs of the (expected) additional abatement incentives. These, in turn, depend on differences of gains from trade for different levels of aggregate abatement $\tilde{a}$. The calculations for exponential utility functions strongly suggest[21] that the gains from trade for type X, $g_{\tilde{a}}^X$, are decreasing in aggregate abatement $\tilde{a}$ ($X = A, B$). In that case the additional abatement incentives $\bar{g}_{uni}^X$ and $\bar{g}_{join}^X$would be smaller than zero.[22] The consequences for the equilibrium regions were already analysed in Section 6.1.2.

[20] Note that it corresponds to the *elasticity* of the utility function.

[21] However, analytically, the comparative statics of security market equilibrium do not yield a sign for the partial derivative of the gains from trade for type X, $g_{\tilde{a}}^X$, with respect to aggregate abatement $\tilde{a}$, $\frac{\partial g_{\tilde{a}}^X}{\partial \tilde{a}.}$. This should be a topic of further research.

[22] Recall that $\bar{g}_{uni}^X = g_1^X - g_0^X$ and $\bar{g}_{join}^X = g_2^X - g_1^X$ for $X = A, B$.

For regulated security markets the gains from trade themselves are the relevant expressions. They do not exhibit any special properties so that for regulated securities two different patterns of equilibrium regions can arise.[23]

For the case of the expressions $\bar{g}^X_{uni}$ and $\bar{g}^X_{join}$ being smaller than zero for $X = A, B$ there are only two possible overall patterns of equilibrium regions which are depicted in Table 6.1 below.

	No Securities	**Unregulated Securities**	**Regulated Securities**	
			Pattern 1	Pattern 2
a^1	$[0,1]$	$[0,1]$	$[0,p^R_{1B}]$	$[0,1]$
a^2	$[0,p_{2B}]$	$[0,p^{NR}_{2B}]$	$[0,1]$	$[0,p^R_{2B}]$

Table 6.1: *Possible pattern of equilibrium regions for exponential utility functions when the gains from trade $g^X_{\tilde{a}}$ are decreasing in $\tilde{a}$. Note that by Corollary 3 $p_{2B} > p^R_{2B}$ holds.*

Example 1 below gives a parameter constellation leading to the respective pattern of equilibrium regions for each of the two patterns in Table 6.1.

Example 1 *(Exponential utility)*
Let the utility function be of the exponential type as defined in (6.20) with $\lambda = 100$ and $\mu = \frac{1}{100}$, i.e. $u(x) = -100 \cdot e^{-\frac{x}{100}}$. The parameters concerning first period endowment, abatement cost[24] and third period endowments for state α and β as well as the subjective probability distributions of two different types are

Parameters	$\bar{\omega}$	γ	$\bar{\omega}^\alpha$	$\omega^\beta(0)$	$\omega^\beta(1)$	$\omega^\beta(2)$	θ^A	θ^B
Example 1 a)	*50*	*10*	*50*	*20*	*30*	*42*	$\frac{4}{10}$	$\frac{2}{10}$
Example 1 b)	*50*	*10*	*50*	*20*	*30*	*45*	$\frac{4}{10}$	$\frac{3}{10}$

.

[23] See Corollary 9 in Appendix C.
[24] Recall that γ denotes abatement cost measured in units of the aggregate good.

Calculation of equilibrium regions for Example 1

First of all, marginal abatement cost c and marginal abatement benefits b_1 and b_2, all three measured in utility, can be calculated according to the definitions in Section 5.1.3. Note that for the case of two countries aggregate abatement $\tilde{a}$ can only assume the values $\tilde{a} = 0, 1, 2$. Then, it is easy to verify that the parameters chosen for Example 1 indeed satisfy the assumptions in Section 5.1.

The second step consists of the calculation of security market equilibrium for all possible values of aggregate abatement $\tilde{a}$, i.e. for $\tilde{a} = 0, 1, 2$. Given a fixed level of $\tilde{a}$, each type's demand for security α can be determined as a function of the security price q. The equilibrium price for security α is derived by setting aggregate demand[25] for security α equal to zero.

The third step involves calculating the gains from trade $g_{\tilde{a}}^X$ for each type X and for all values of $\tilde{a}$ as well as the additional abatement incentives. The gains from trade are given in the following table.

Gains from trade				
	Example 1 a)		Example 1 b)	
	Type A (X=A)	Type B (X=B)	Type A (X=A)	Type B (X=B)
g_2^X	1.6066	1.4123	0.3476	0.3330
g_1^X	1.6518	1.4630	0.3639	0.3503
g_0^X	1.6872	1.5034	0.3741	0.3613

Recall that by definition "gains from trade" are non-negative. As pointed out above, the gains from trade are decreasing in aggregate abatement $\tilde{a}$ for both types both in Example 1 a) and in Example 1 b). Therefore, the additional abatement incentives $\bar{g}_{uni}^X$ and $\bar{g}_{join}^X$, given in the table below, are all smaller than zero.

[25]Since there can only be trade if the two countries are of different types, aggregate demand equals the sum of type A demand and type B demand.

Additional abatement incentives				
	Example 1 a)		**Example 1 b)**	
	Type A	Type B	Type A	Type B
$\bar{g}^X_{uni}$	-0.0354	-0.0403	-0.0103	-0.0110
$\bar{g}^X_{join}$	-0.0452	-0.0507	-0.0163	-0.0173

Finally, the equilibrium regions for the strategy profiles can be determined according to Corollary 2 and Corollaries 6 to 8 for unregulated security markets and according to Corollary 9 with regulation. Calculating the equilibrium regions amounts to checking which of the cases in each of these Corollaries applies and then, if necessary, calculating the missing boundary points of the equilibrium regions, e.g. p^{NR}_{2B} or p^{R}_{1B}.

The resulting equilibrium regions for Example 1 a) and Example 1 b) are drawn in Figures 6.8 and 6.9 below.

Figure 6.8: *The equilibrium regions for Example 1 a).*

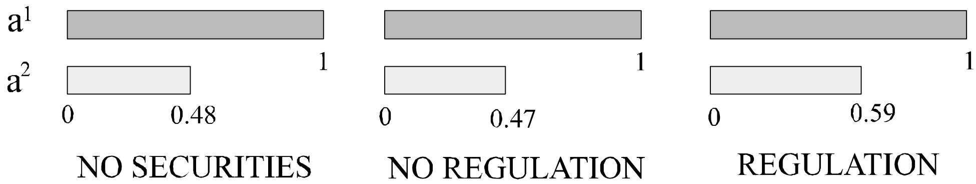

Figure 6.9: *The equilibrium regions for Example 1 b).*

Interpretation of Example 1

Note that Example 1 a) and Example 1 b) only differ with respect to two of the parameters. Firstly, the state β endowment for $\tilde{a} = 2$, $\omega^{\beta}(2)$, is larger in Example 1 b). Secondly, type B's probability for state α, θ^B, is larger in Example 1 a). Although the parameters in Example 1 a) and 1 b) are very similar, they lead to different patterns of equilibrium regions. In Example 1 a) the first of the two patterns in Table 6.1 emerges, whereas Example 1 b) exhibits the second pattern.

Further note that for Example 1 a) the gains from trade[26] are generally much larger than for Example 1 b). Intuitively, this can be explained as follows. Recall that the countries are identical apart from the type. Therefore the difference between the two types concerning their probability distribution across the two climate states is an important motive for trade in climate securities. In Example 1 a) the difference between θ^A and θ^B is larger, leading to larger gains from trade than in Example 1 b).

Since under a regime of regulated markets for climate securities the gains from trade, not the expected additional abatement incentives, determine the equilibrium regions, this also explains why the effect of climate securities under regulation is much larger for Example 1 a). With regulated securities the equilibrium region for the preferred strategy profile a^2 is the unit interval, whereas the equilibrium region for strategy profile a^1 is reduced to the interval $[0, 0.1]$. In Example 1 b) the equilibrium region for strategy profile a^2 is also larger with regulated securities than for the other two scenarios, but the equilibrium region for strategy profile a^1 is still the unit interval.

Note further that without regulatory intervention the equilibrium region for strategy profile a^2 is slightly smaller than in the absence of securities in both Example 1 a) and 1 b). This is due to the expected additional abatement incentives, in particular $\bar{g}^B_{uni}$, being smaller than zero.[27]

6.2.3 Ramsey utility

Now consider the utility function

$$u(x) = -\frac{1}{x} \qquad \text{for} \quad x > 0, \tag{6.21}$$

called "Ramsey utility" here, since this type of utility function originates in Ramsey [1928] and has subsequently been used in the so-called "Ramsey

[26] See table above.

[27] See Corollary 3 in Section 6.1.2.

growth model."[28] The coefficient of absolute risk aversion is $R_a(x) = \frac{2}{x}$, implying that the utility function is characterized by DARA and, since $R_r(x) = x \cdot R_a(x) = 2$, by CRRA.

Typical patterns of equilibrium regions

For the Ramsey utility function the calculations also suggest that the gains from trade $g^X_{\tilde{a}}$ decrease with aggregate abatement $\tilde{a}$ for both types and the expected additional abatement incentives are then smaller than zero.[29] Therefore, it seems that we get the same effects as for exponential utility functions (cf. Section 6.2.2 above). In particular, we expect the same two patterns of equilibrium regions depicted in Table 6.1 to emerge.

Example 2 *(Ramsey utility)*
Let the utility function be as defined in (6.21). The parameters concerning first period endowment, abatement cost and third period endowments for state α and β as well as the subjective probability distributions of the different types are

Parameters	$\bar{\omega}$	γ	$\bar{\omega}^\alpha$	$\omega^\beta(0)$	$\omega^\beta(1)$	$\omega^\beta(2)$	θ^A	θ^B
Example 2 a)	60	20	60	20	25	40	$\frac{5}{6}$	$\frac{1}{3}$
Example 2 b)	60	20	60	20	25	40	$\frac{2}{3}$	$\frac{1}{3}$

The equilibrium regions for Example 2 a) and Example 2 b) are drawn in Figures 6.10 and 6.11 below. Basically, the effects are the same as for the exponential type of utility function in Section 6.2.2 above. Since the Ramsey utility function satisfies DARA, whereas the exponential utility function satisfied CARA the similarity of the results suggests that the difference between DARA and CARA does not have a substantial impact on the structure of the equilibrium regions.

[28] In the Ramsey growth model $u(x) = \frac{x^{1-\lambda}-1}{1-\lambda}$ with $\lambda > 0$ a constant. For $\lambda = 2$ that yields $u(x) = -\frac{1}{x}+1$ which can be transformed into (6.21) by a monotonic transformation.

[29] Again, the comparative statics with respect to aggregate abatement $\tilde{a}$ do not yield any general results.

Figure 6.10: *The equilibrium regions for Example 2 a).*

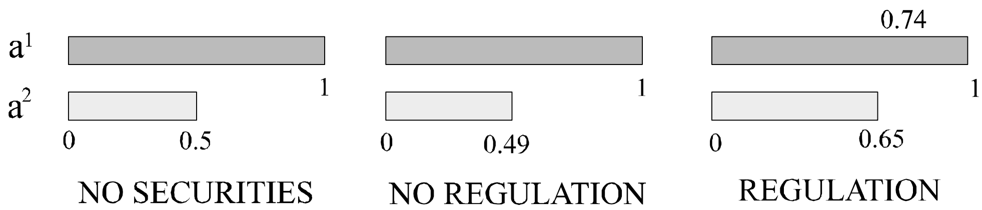

Figure 6.11: *The equilibrium regions for Example 2 b).*

6.2.4 Logarithmic utility

Consider the natural logarithm

$$u(x) = \ln x \tag{6.22}$$

as von Neumann-Morgenstern utility function. Here the coefficient of absolute risk aversion is $R_a(x) = \frac{1}{x}$, i.e. the utility function exhibits DARA. Moreover, $R_r(x) = x \cdot \frac{1}{x} = 1$, implying CRRA. Recall that the Ramsey utility function also exhibited DARA and CRRA. However, the coefficients $R_a(x)$ and $R_r(x)$ themselves are *not* the same for the two functions. In particular, the logarithmic utility function is unit-elastic, whereas the elasticity of the Ramsey utility function was equal to 2.[30] The analysis below suggests that this property has a decisive impact on the results.

Gains from trade

The logarithmic utility function stands out from the other utility functions analysed, because the gains from trade do *not* depend on aggregate abate-

[30] Recall that the coefficient of relative risk aversion also represents the elasticity of the utility function.

ment as shown in the following Proposition.

Proposition 8 *(Gains from trade for logarithmic utility)*
Let the utility function be as defined in (6.22) and let $n = 2$. For $\theta^1 \neq \theta^2$

a) the gains from trade do not depend on aggregate abatement $\tilde{a}$, i.e.

$$g_0^X = g_1^X = g_2^X \quad \textit{for} \quad X = A, B, \qquad \textit{and consequently}$$

b) the additional abatement incentives are zero, i.e.

$$\bar{g}_{uni}^X = 0 \quad \textit{and} \quad \bar{g}_{join}^X = 0 \qquad \textit{for} \quad X = A, B. \tag{6.23}$$

The proof is given in Appendix C. The result in Proposition 8 is surprising in the sense that, in general, security market equilibrium and therefore also the gains from trade depend on state β endowments, in particular, on aggregate abatement $\tilde{a}$. Although security market equilibrium does depend on the endowments in the context of Proposition 8 they cancel out in the gains from trade due to the characteristic properties of the natural logarithm.

Typical patterns of equilibrium regions

Since the expected additional abatement incentives determine the effect of unregulated markets for climate securities on the equilibrium regions, Proposition 8 immediately yields[31]

Corollary 5 *(Equilibrium regions with unregulated securities)*
Let the utility function be logarithmic and $n = 2$. Then the equilibrium regions for unregulated security markets are identical to the equilibrium regions in the absence of security markets, i.e. the unit interval for strategy profile a^1 and the interval $[0, p_{2B}]$ for strategy profile a^2.

With regulation there are no peculiarities compared to the general case. Table 6.2 shows the two possible patterns of equilibrium regions.

Example 3 gives a parameter constellation for each of the two patterns.

[31] Set $\bar{g}_{uni}^X$ and $\bar{g}_{join}^X$ to zero for $X = A, B$ in Corollary 2 and Corollaries 6 to 8.

	No Securities	**Unregulated Securities**	**Regulated Securities**	
			Pattern 1	Pattern 2
a^1	$[0,1]$	$[0,1]$	$[0,p^R_{1B}]$	$[0,1]$
a^2	$[0,p_{2B}]$	$[0,p_{2B}]$	$[0,1]$	$[0,p^R_{2B}]$

Table 6.2: *Possible patterns of equilibrium regions for logarithmic utility functions.*

Example 3 *(Logarithmic utility)*
Let the utility function be as defined in (6.22). The remaining parameters concerning endowments, abatement cost and abatement benefits as well as the subjective probability distributions of the different types are

Parameters	$\bar{\omega}$	γ	$\bar{\omega}^\alpha$	$\omega^\beta(0)$	$\omega^\beta(1)$	$\omega^\beta(2)$	θ^A	θ^B
Example 3 a)	20	8	20	4	6	12	$\frac{3}{4}$	$\frac{1}{8}$
Example 3 b)	20	8	20	4	6	12	$\frac{3}{4}$	$\frac{1}{4}$

.

As shown in Proposition 8 the gains from trade for Example 3, given in the table below, do not depend on aggregate abatement $\tilde{a}$ which can take on the values $\tilde{a} = 0, 1, 2$.

Gains from trade	$g^A_{\tilde{a}}$	$g^B_{\tilde{a}}$
Example 3 a)	0.2	0.23
Example 3 b)[32]	0.13	0.13

The gains from trade are larger for Example 3 a) than for Example 3 b) since the difference between θ^A and θ^B is larger in Example 3 a) than in Example 3 b). The equilibrium regions for Example 3 a) and Example 3 b) are drawn in Figures 6.12 and 6.13 below.

Note that although the parameters for Example 3 a) and b) only differ in type B's probability for state α, θ^B, this difference suffices to yield fundamentally different patterns of equilibrium regions for each of the two parameter sets.

[32] For Example 3 b) the gains from trade are even identical for the two types due to $\theta^A + \theta^B = 1$, but that is not a general property.

Figure 6.12: *The equilibrium regions for Example 3 a).*

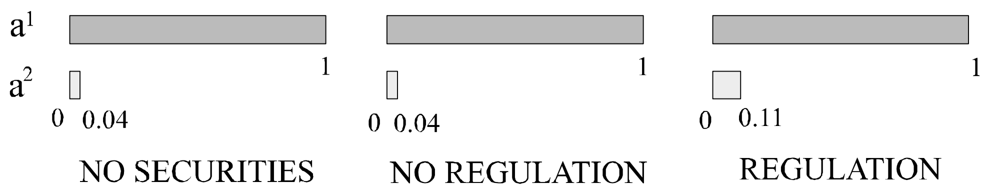

Figure 6.13: *The equilibrium regions for Example 3 b).*

6.2.5 Quadratic Utility

The utility functions considered so far all exhibited either decreasing or constant gains from trade $g_{\tilde{a}}^X$ with respect to aggregate abatement $\tilde{a}$. In order to show that the gains from trade can also be *increasing* in aggregate abatement $\tilde{a}$ consider a quadratic utility function with

$$u(x) = \lambda x - \mu x^2 \qquad \text{where} \qquad \lambda, \mu > 0. \tag{6.24}$$

The use of a quadratic utility function may be controversial since this type of utility function exhibits *increasing absolute risk aversion* (IARA) due to $R_a(x) = x$. Concerning IARA, it was argued in Section 6.2.1 that it is commonly agreed that IARA is not usually observed in reality. On the other hand, one of the key assumptions in the *Capital Asset Pricing Model* (CAPM) is motivated by a quadratic utility function. Specifically, the CAPM assumes that expected utility depends only on the mean and the variance of consumption, an assumption which can be derived from an underlying quadratic utility function.[33]

Irrespective of whether the assumption of a quadratic utility function is considered adequate or not, the interesting point here is that for this type

[33] See LAFFONT [1989], p. 117.

of function the gains from trade have different properties than for all the functions analysed so far.

Evidence from the examples calculated strongly suggests that, in contrast to the results for all the other utility functions above, the gains from trade for one of the types are *increasing* in aggregate abatement $\tilde{a}$, whereas the gains from trade for the other type decrease in $\tilde{a}$. Then, the additional abatement incentives for the type with the increasing gains from trade would be larger than zero, whereas the additional abatement incentives for the other type would be smaller than zero.

Consequently, some of the cases ruled out by negative additional abatement incentives for the other examples above (cf. Corollaries 2, 6, 7 and 8) can arise. The implications for the equilibrium regions are that further patterns become possible in addition to the patterns shown in Table 6.1. Furthermore, for the two patterns in Table 6.1 the inequality $p_{2B}^{NR} > p_{2B}$ might hold by Corollary 3 if the type with the increasing gains from trade were type B.[34]

6.2.6 Summary

Whereas the derivation and graphic illustration of the potential equilibrium regions for the case of two countries in Section 6.1 improved the understanding of the general analysis in Chapter 5 a number of open questions still remained. In particular, with unregulated security markets up to four different cases could arise for each of the four strategy profiles with respect to the equilibrium region.

The examples in this Section showed that the expected additional abatement incentives can be smaller than zero. In particular, they suggest that for utility functions satisfying DARA or CARA the expected additional abatement incentives always tend to be smaller than zero. Therefore, the case of negative additional abatement incentives seems to be the more relevant case. Positive additional abatement incentives emerged only for the controversial case of quadratic utility functions, satisfying IARA, and even then only for one of the two types. The tendency towards negative additional abatement

[34] Then $\bar{g}_{uni}^{B} > 0$ would hold.

incentives rules out all but one of the four possible cases listed in Section 6.1 for unregulated securities.

For security markets subject to regulation the examples prove that in both Figure 6.6 and Figure 6.7 each of the two cases depicted there can arise.

Taking into account that regulation of markets for climate securities is unlikely to be established, the evidence from the examples is "bad news" in the following sense. Without regulation such markets seem to produce abatement disincentives rather than abatement incentives if IARA is not an acceptable assumption on utility functions. Then the preferred strategy profile a^2 is an equilibrium for a smaller set of economies than in the absence of such markets.

Concerning the crucial factors for the results it seems that the difference between DARA and CARA utility functions has less impact than the unit elasticity of the logarithmic utility function. This issue should be a topic of further research.

Chapter 7

Extensions and limits of the incomplete information approach

This chapter supplements the analysis of the incomplete information approach presented in Chapters 5 and 6 with respect to two issues. Firstly, the incomplete information model is extended to the case of endogenous probabilities in Section 7.1. Secondly, Section 7.2 discusses the case of heterogeneous countries.

7.1 Extension to endogenous probabilities

In Chapters 5 and 6 the probability distributions across climate states, which characterized the different types, were assumed to be exogenous. However, as argued in Chapter 2.2, climate risk is clearly an endogenous risk, i.e. human activity, in particular mitigation, affects the future climate. Therefore this section extends the basic model to the case of endogenous climate state probabilities.

7.1.1 Modelling endogenity of climate state probabilities

In the framework of the incomplete information model introduced in Chapter 5 the natural way of endogenizing the climate state probabilities, represented by θ^A and θ^B, is to make them depend on aggregate first period abatement $\tilde{a}$. By assumption, $\tilde{a}$ is observable to all countries involved. Before we can define the endogenous probabilities an additional assumption on the endowments is required.

Endowments

Recall that state β endowment $\omega^\beta(\tilde{a})$ increases in aggregate abatement $\tilde{a}$ by assumption (5.2). Assume in addition that

$$\bar{\omega}^\alpha > \omega^\beta(n), \tag{7.1}$$

i.e. the endowment in state α is larger than that in state β when all n countries abate, which is the largest state β endowment. Therefore endowment in state α is always larger than endowment in state β, no matter what the level of aggregate abatement $\tilde{a}$ is. This is consistent with the interpretation in Section 5.1.1 that state α is the "good" state (no climate change) and state β the "bad" state (severe climate change).[1] The extent of the loss of endowment in the bad state β can be partially mitigated by abatement, but never completely.

Endogenous probabilities

Let $\theta^X_{\tilde{a}}$ (short for $\theta^X(\tilde{a})$) denote the probability attributed to state α by type X when aggregate first period abatement is $\tilde{a}$ ($X = A, B$). The corresponding probability for state β is then $1-\theta^X_{\tilde{a}}$. Intuitively, the probability for the good

[1] Note that the interpretation is compatible with the views expressed in the latest IPCC Report, cf. IPCC [2001b], that the net global effect of climate change is expected to be an adverse one as warming increases (cf. Chapter 2.1.2). Since countries are identical in this model there is no scope for including the regional differences emphasized in Chapter 2.2.

state α should increase with the aggregate abatement effort $\tilde{a}$. Therefore, assume that $\theta_{\tilde{a}}^X$ is increasing in $\tilde{a}$, i.e.

$$\theta_0^X < \theta_1^X < ... < \theta_{n-1}^X < \theta_n^X \qquad X = A, B. \tag{7.2}$$

The natural generalization of the former assumption $\theta^A > \theta^B$ is to assume

$$\theta_{\tilde{a}}^A > \theta_{\tilde{a}}^B \quad \text{for all } \tilde{a}. \tag{7.3}$$

Increments of endogenous probabilities

The increment in the probability attributed to climate state α by type X, if $\tilde{a}$ countries abate instead of $\tilde{a}-1$, is denoted by

$$\Delta_{\tilde{a}}^X := \theta_{\tilde{a}}^X - \theta_{\tilde{a}-1}^X \quad \text{for } X = A, B \quad \text{and } \tilde{a} = 1, ..., n. \tag{7.4}$$

By assumption (7.2) $\Delta_{\tilde{a}}^X > 0$ holds for $X = A, B$ and $\tilde{a} = 1, ..., n$.

Assumptions on marginal abatement cost and marginal benefits

Basically, the assumptions made in Section 5.1 are maintained. However, they have to be adapted to the new framework as follows. Assumption (5.5), by which there was no incentive for unilateral abatement, is replaced by

$$(1 - \theta_0^B)b_1 < c. \tag{7.5}$$

Assumption (7.5) implies that *if* the probability θ_0^B stayed constant, type B would have no incentive for unilateral abatement. The same, i.e. $(1-\theta_0^A)b_1 < c$, holds true for type A by assumption (7.3).

Assumption (5.6), by which strategy L was a dominant strategy for type A, is replaced by

$$(1 - \theta_{\tilde{a}-1}^A)b_{\tilde{a}} < c \qquad \text{for all } \tilde{a} = 1, ..., n, \tag{7.6}$$

which is simply the generalized version of assumption (5.6). Assumption (7.6) is also formulated for the hypothetical case of the probability remaining constant.

Similarly, assumption (5.7), ensuring that L is not a dominant strategy for type B as well, is replaced. Assume that there exists an $\hat{\imath} \in \{2, ..., n\}$ satisfying

$$(1 - \theta_{\hat{\imath}-1}^B)b_{\hat{\imath}} > c. \tag{7.7}$$

Total utility

Without markets for climate securities, total utility for country i can be written as[2]

$$U_i(\theta_i, a_i(\theta_i); \tilde{a}(a(\theta))) = -c(a_i(\theta_i)) + \bar{u}(\theta_i, \tilde{a}) + (1 - \theta^i_{\tilde{a}}) b^{abs}_{\tilde{a}} \tag{7.8}$$

where $c(a_i(\theta_i))$ is defined as in Section 5.2 and where

$$\bar{u}(\theta_i, \tilde{a}) := \theta^i_{\tilde{a}} u(\bar{\omega}^\alpha) + (1 - \theta^i_{\tilde{a}}) u(\bar{\omega}^\beta) \tag{7.9}$$

is the expected utility derived from the minimum endowments $\bar{\omega}^\alpha$ and $\bar{\omega}^\beta$ weighted with probabilities associated with type θ_i and based on an aggregate abatement of $\tilde{a}$.

Note that by the definition of $\Delta^X_{\tilde{a}}$ in (7.4)

$$\bar{u}(\theta^X, \tilde{a}) - \bar{u}(\theta^X, \tilde{a} - 1) = \Delta^X_{\tilde{a}} [u(\bar{\omega}^\alpha) - u(\bar{\omega}^\beta)] \tag{7.10}$$

for $\tilde{a} = 1, ..., n$ and $X = A, B$.

7.1.2 No Security Markets

This section characterizes the Bayesian equilibria of the Bayesian Game induced by the utility functions in (7.8) in the absence of markets for climate securities. In analogy to Proposition 3 the equilibrium conditions for the strategy profiles are given in Proposition 9.

Proposition 9

a) *Strategy profile a^1 ("no abatement") is a Bayesian Nash Equilibrium, if and only if*

$$(1 - \theta^X_0) b_1 + \Delta^X_1 \left[u(\bar{\omega}^\alpha) - u(\omega^\beta(1)) \right] \leq c \quad \textit{for } X = A, B. \tag{7.11}$$

[2] Like in Chapter 5 the normalization $u(\bar{w}) = 0$ has been applied. Note that $\bar{u}(\theta_i, \tilde{a})$ cannot be omitted here as done for simplicity in Chapter 5.2 since this expression now depends on $\tilde{a}$.

b) *Strategy profile* a^2 *("type-consistency") is a Bayesian Nash Equilibrium, if and only if*

$$\begin{aligned} & \sum_{k=0}^{n-1} p_k \left[(1-\theta^A_{n-k-1})b_{n-k} + \Delta^A_{n-k}\left[u(\bar{\omega}^\alpha) - u(\omega^\beta(n-k))\right]\right] \\ \leq \; & c \qquad (7.12) \\ \leq \; & \sum_{k=0}^{n-1} p_k \left[(1-\theta^B_{n-k-1})b_{n-k} + \Delta^B_{n-k}\left[u(\bar{\omega}^\alpha) - u(\omega^\beta(n-k))\right]\right] \end{aligned}$$

where p_k *is again the binomial coefficient with respect to* $n-1$ *defined in (5.22).*

c) *Strategy profile* a^3 *("always abate") is a Bayesian Nash Equilibrium, if and only if*

$$(1-\theta^X_{n-1})b_n + \Delta^X_n\left[u(\bar{\omega}^\alpha) - u(\bar{\omega}^\beta(n))\right] \geq c \quad \text{for } X = A, B. \qquad (7.13)$$

d) *Strategy profile* a^4 *("type-inconsistency") is a Bayesian Nash Equilibrium, if and only if*

$$\begin{aligned} & \sum_{k=0}^{n-1} p_k \left[(1-\theta^B_{k})b_{k+1} + \Delta^B_{k+1}\left[u(\bar{\omega}^\alpha) - u(\omega^\beta(k+1))\right]\right] \\ \leq \; & c \qquad (7.14) \\ \leq \; & \sum_{k=0}^{n-1} p_k \left[(1-\theta^A_{k})b_{k+1} + \Delta^A_{k+1}\left[u(\bar{\omega}^\alpha) - u(\omega^\beta(k+1))\right]\right]. \end{aligned}$$

The proof is given in Appendix D. Naturally, the inequalities for strategy profiles a^1 and a^2 in Proposition 9 strongly ressemble the corresponding inequalities in Proposition 3. The only difference is the second expression in each inequality which arises from the endogenity of probabilities.

First, take a closer look at the equilibrium conditions for strategy profile a^1 in (7.11). The first term in the inequalities, $(1-\theta^X_0)b_1$, is the incentive for unilateral abatement for type X, given that nobody else abates, i.e. using θ^X_0 as probability for state α. This first term represents only the expected benefits arising *directly* from abatement, but does not include the indirect effect arising from the increase in the probability through abatement.

This indirect effect is represented by the second expression in (7.11). $\Delta_1^X[u(\bar{\omega}^\alpha) - u(\omega^\beta(1))]$ is the expected additional utility for type X from unilateral abatement arising *indirectly* through the increment of the probability for state α. Utility from state β enters with a negative sign since an increase in the probability θ_0^X is equivalent to a decrease in the probability $1 - \theta_0^X$.

Note that by assumptions (7.1) and (7.2) the indirect effect is always larger than zero. Therefore it has the same effect as an increase in expected marginal abatement benefits. Since abatement cost is not affected by endogenization of the probabilities the inequalities in (7.11) become stricter compared to the corresponding inequalities for exogenous probabilities.[3] Therefore, explicitely taking into account the positive effect of abatement on the probability of the "good" state strengthens the unilateral abatement incentive for both types, weakening the case for the "never abate" strategy a^1.

Now turn to the equilibrium conditions for strategy profile a^2. Concerning the decomposition into the direct and the indirect effect of abatement the structure of the two inequalities in (7.12) is similar to that of the inequalities in (7.11). The difference is that in (7.12) all other countries are assumed to behave according to strategy profile a^2 so that the number of countries abating depends on the type profile θ and, therefore, expectations have to be taken with respect to the types of the other $n-1$ countries.

Since the indirect effect $\Delta_{n-k}^X[u(\bar{\omega}^\alpha) - u(\omega^\beta(n-k))]$ is always larger than zero for all k, the expected indirect effect $\sum_{k=0}^{n-1} p_k \Delta_{n-k}^X[u(\bar{\omega}^\alpha) - u(\omega^\beta(n-k))]$ is also larger than zero. This makes the first inequality (for type A) in (7.12) stricter than the corresponding inequality for exogenous probabilities in (5.21), but relaxes the second inequality (for type B) in (7.12) relative to (5.21). Again, endogenity of probabilities has the same effect as an increase in abatement benefits.

In contrast to the case with exogenous probabilities, strategy profiles a^3 and a^4 can also, in principle, be equilibria when climate state probabilities are endogenous. Recall that with exogenous probabilities, strategy profile a^3 was ruled out as an equilibrium by assumption (5.6) and for strategy profile a^4 the two equilibrium conditions contradicted each other. If the indirect effect is large enough to compensate the effects originally ruling out a^3 and

[3] Cf. inequality (5.20) in Proposition 3.

a^4, these strategy profiles can be equilibria.

In comparison to the case of exogenous probabilities abatement is rendered more attractive when climate state probabilities are endogenous. This is not surprising, since abatement is now rewarded in two different ways. In addition to the benefits in terms of a higher endowment in state β there is also the increase in probability for state α, the "good" state.

7.1.3 Unregulated security markets

With unregulated markets for climate securities utility for country i can be written as

$$U_i^{NR}(\theta_i, a_i(\theta_i); \tilde{a}(a(\theta))) = \begin{cases} -c(a_i(\theta_i)) + \bar{u}(\theta_i, \tilde{a}) + (1-\theta_{\tilde{a}}^i) b_{\tilde{a}}^{abs} \\ \quad +g(\theta_i, n_A(\theta); \tilde{a}), \text{ if } 0 < n_A(\theta) < n \\ -c(a_i(\theta_i)) + \bar{u}(\theta_i, \tilde{a}) + (1-\theta_{\tilde{a}}^i) b_{\tilde{a}}^{abs}, \\ \quad\quad \text{if } n_A(\theta) = 0, n \end{cases} \tag{7.15}$$

where the gains from trade $g(\theta_i, n_A(\theta); \tilde{a})$ are defined as in (5.23). The utility functions in (7.15) differ from those for exogenous probabilities in (5.24) only by the additional expression $\bar{u}(\theta_i, \tilde{a})$. The equilibrium conditions for the Bayesian Game induced by the utility functions in (7.15) are given in Proposition 10 below.

Proposition 10

a) *Strategy profile a^1 ("no abatement") is a Bayesian Nash Equilibrium, if and only if*

$$(1-\theta_0^X)b_1 + \Delta_1^X[u(\bar{\omega}^\alpha) - u(\omega^\beta(1))] + g_{uni}^X(p) \leq c \tag{7.16}$$

where $g_{uni}^X(p)$ is defined as in Section 5.3.2 for $X = A, B$.

b) *Strategy profile a^2 ("type-consistency") is a Bayesian Nash Equilibrium,* *if and only if*

$$\begin{aligned}
&\sum_{k=0}^{n-1}\left[p_k(1-\theta^A_{n-k-1})b_{n-k}+\Delta^A_{n-k}[u(\bar{\omega}^\alpha)-u(\omega^\beta(n-k))]\right] \\
&\qquad +g^A_{con}(p) \qquad (7.17)\\
\leq\ & c \\
\leq\ & \sum_{k=0}^{n-1}p_k\left[(1-\theta^B_{n-k-1})b_{n-k}+\Delta^B_{n-k}\left[u(\bar{\omega}^\alpha)-u(\omega^\beta(n-k))\right]\right] \\
&\qquad +g^B_{con}(p) \qquad (7.18)
\end{aligned}$$

where p_k is again the binomial coefficient with respect to $n-1$ defined in (5.22) and $g^A_{con}(p)$ and $g^B_{con}(p)$ are defined as in Section 5.3.2.

c) *Strategy profile a^3 ("always abate") is a Bayesian Nash Equilibrium, if and only if*

$$(1-\theta^X_{n-1})b_n+\Delta^X_n[u(\bar{\omega}^\alpha)-u(\bar{\omega}^\beta(n))]+g^X_{join}(p)\geq c \qquad (7.19)$$

for $X=A,B$ where $g^X_{joint}(p)$ is defined as in Section 5.3.2.

d) *Strategy profile a^4 ("type-inconsistency") is a Bayesian Nash Equilibrium, if and only if*

$$\begin{aligned}
&\sum_{k=0}^{n-1}p_k\left[(1-\theta^B_k)b_{k+1}+\Delta^B_{k+1}[u(\bar{\omega}^\alpha)-u(\omega^\beta(k+1))]\right]+g^B_{inc}(p)\\
\leq\ & c \qquad (7.20)\\
\leq\ & \sum_{k=0}^{n-1}p_k\left[(1-\theta^A_k)b_{k+1}+\Delta^A_{k+1}[u(\bar{\omega}^\alpha)-u(\omega^\beta(k+1))]\right]+g^A_{inc}(p)
\end{aligned}$$

where $g^A_{inc}(p)$ and $g^B_{inc}(p)$ are defined as in Section 5.3.2.

The proof is given in Appendix D. The equilibrium conditions in Proposition 10 can be interpreted both in comparison to those in Proposition 9 (equilibrium conditions **in the absence of security markets** with endogenous probabilities) and in comparison to those in Proposition 5 (equilibrium conditions for unregulated securities with **exogenous probabilities**).

In comparison to Proposition 9 each of the equilibrium conditions includes an additional expression, e.g. $g^X_{uni}(p)$, which represents the expected additional abatement incentive generated by security markets. These additional terms in the equilibrium conditions are exactly the same expressions as in the corresponding inequalities with exogenous probabilities. In particular, the sign of the expected additional abatement incentives is ambiguous so that they may deter countries from abating. Apart from the initial situation (characterized in Proposition 9) being a different one when probabilities are endogenous the effect of introducing markets for climate securities with endogenous probabilities is the same as with exogenous probabilities.

In comparison to Proposition 5 endogenous probabilities again give rise to an additional term, e.g. $\Delta^A_1[u(\bar{\omega}^\alpha) - u(\omega^\beta(1))]$, in each of the equilibrium conditions which reflects the indirect effect of abatement on expected utility. As argued above in the analysis of Proposition 9, abatement is more attractive when probabilities are endogenous since it is rewarded two-fold.

Note that given the additive structure of the utility function in (7.15) and the prior analysis in Sections 5.3 and 7.1.2 the results in this section are not at all surprising.

7.1.4 Regulated security markets

Trade in climate securities is now regulated in the same way as in Section 5.4 so that gains from trade are only feasible for countries with a type-consistent abatement decision in the first period.

Recall from Section 5.4.2 that for a given type profile θ and strategy profile a we denote by $\mathcal{I}(\theta, a)$ the set of countries that cannot benefit from security trading. The set of countries potentially participating in security trading is $\mathcal{N}^R(\theta, a)$. However, if the countries belonging to $\mathcal{N}^R(\theta, a)$ are all of the same type, i.e. if $n^R_A(\theta, a)$ equals either zero or $n^R(\theta, a)$, there is no trade in equilibrium. In analogy to (5.34) country i's utility with regulated markets for climate securities can be written as

$$U_i^R(\theta_i, n_A^R, n_B^R; a_i, \tilde{a}_{-i}) = \begin{cases} -c(a_i(\theta_i)) + \bar{u}(\theta_i, \tilde{a}) + (1-\theta_{\tilde{a}}^i)b_{\tilde{a}}^{abs}, \\ \quad \text{if } i \in \mathcal{I}(\theta, a) \\ -c(a_i(\theta_i)) + \bar{u}(\theta_i, \tilde{a}) + (1-\theta_{\tilde{a}}^i)b_{\tilde{a}}^{abs}, \\ \quad \text{if } i \in \mathcal{N}^R(\theta, a) \text{ and } n_A^R(\theta, a) = 0, n^R(\theta, a) \\ -c(a_i(\theta_i)) + \bar{u}(\theta_i, \tilde{a}) + (1-\theta_{\tilde{a}}^i)b_{\tilde{a}}^{abs} \\ \quad +g^R(\theta_i, n_A^R, n_B^R; \tilde{a}), \\ \quad \text{if } i \in \mathcal{N}^R(\theta, a) \text{ and } 0 < n_A^R(\theta, a) < n^R(\theta, a) \end{cases} \tag{7.21}$$

where the gains from trade $g^R(\theta_i, n_A^R, n_B^R; \tilde{a})$ are defined as in (5.33).

The equilibrium conditions for the Bayesian Game induced by the utility functions in (7.21) are given in Proposition 11 below.

Proposition 11

a) Strategy profile a^1 is a Bayesian Nash Equilibrium, if and only if

$$(1-\theta_0^A)b_1 + \Delta_1^A[u(\bar{\omega}^\alpha) - u(\omega^\beta(1))] \leq c. \tag{7.22}$$

$$(1-\theta_0^B)b_1 + \Delta_1^B[u(\bar{\omega}^\alpha) - u(\omega^\beta(1))] + r_{uni}^B(p) \leq c \tag{7.23}$$

where $r_{uni}^B(p)$ is defined as in Section 5.4.

b) Strategy profile a^2 is a Bayesian Nash Equilibrium, if and only if

$$\sum_{k=0}^{n-1}\left[p_k(1-\theta_{n-k-1}^A)b_{n-k} + \Delta_{n-k}^A[u(\bar{\omega}^\alpha) - u(\omega^\beta(n-k))]\right] - r_{con}^A(p) \tag{7.24}$$

$$\leq c$$

$$\leq \sum_{k=0}^{n-1}\left[p_k(1-\theta_{n-k-1}^B)b_{n-k} + \Delta_{n-k}^B[u(\bar{\omega}^\alpha) - u(\omega^\beta(n-k))]\right] + r_{con}^B(p) \tag{7.25}$$

where $r_{con}^A(p)$ and $r_{con}^B(p)$ are defined as in Section 5.4.

c) Strategy profile a^3 is a Bayesian Nash Equilibrium, if and only if

$$(1-\theta_{n-1}^X)b_n + \Delta_n^X[u(\bar{\omega}^\alpha) - u(\bar{\omega}^\beta(n))] + r_{join}^X(p) \geq c \tag{7.26}$$

for $X = A, B$ where

$$r^A_{join}(p) := \sum_{k=0}^{n-2} p_k g^R(\theta^A, 1, n-1-k; n-1)$$
$$\textit{and} \quad r^B_{join}(p) := \sum_{k=1}^{n-1} p_k g^R(\theta^B, 1, n-1-k; n-1).$$

d) Strategy profile a^4 is a Bayesian Nash Equilibrium, if and only if

$$\begin{aligned} & \sum_{k=0}^{n-1} p_k \left[(1-\theta^B_k) b_{k+1} + \Delta^B_{k+1}[u(\bar{\omega}^\alpha) - u(\omega^\beta(k+1))]\right] \\ \leq \; & c \\ \leq \; & \sum_{k=0}^{n-1} p_k \left[(1-\theta^A_k) b_{k+1} + \Delta^A_{k+1}[u(\bar{\omega}^\alpha) - u(\omega^\beta(k+1))]\right] \end{aligned} \tag{7.27}$$

The proof, given in Appendix D, is analogous to that of Propositions 6 and 10 and the structure of the equilibrium conditions ressembles that already observed in Propositions 9 and 10.

Relative to the corresponding equilibrium conditions in the absence of securities (cf. Proposition 9) the same expected gains from trade expressions as in Section 5.4, $r^B_{uni}(p), r^A_{con}(p)$ and $r^B_{con}(p)$, appear in addition for strategy profiles a^1 and a^2. Since the expected gains from trade are always larger than zero, inequality (7.23) becomes stricter than the corresponding inequality in the absence of security markets in (7.11), whereas (7.25) does not change. Both (7.24) and (7.25) are slackened by the gains from trade relative to the inequalities in (7.12).

In contrast to the case of exogenous probabilities with regulated security markets (cf. Proposition 6) strategy profiles a^3 and a^4 can now be equilibria. This is the case since in each of the equilibrium conditions the additional expressions arising from endogenity of probabilities appear.[4] Depending on their size, the equilibrium conditions for strategy profiles a^3 and a^4 may be satisfied.

[4]Inequalities (7.22) to (7.24) become stricter, reducing the incentive to chose strategy L, whereas (7.25) is slackened, increasing the incentive to chose H.

Like in the case of exogenous probabilities regulated security markets generate a tendency towards strategy profile a^2 for which the equilibrium conditions are slackened whereas those for strategy profile a^1 become stricter.

7.1.5 Summary

By comparison of the three scenarios under consideration it is evident that with endogenous probabilities for climate states abatement is rendered more attractive than with exogenous probabilities. Abatement benefits are increased by the indirect effect from abatement $\Delta_{\tilde{a}}^{X}[u(\bar{\omega}^{\alpha}) - u(\omega^{\beta}(\tilde{a}))]$ which is always larger than zero.

This has two different implications. On the one hand, the shift from the "never abate" equilibrium a^1 to the "type-consistent" equilibrium a^2 observed in the model with exogenous probabilities is reinforced when climate state probabilities are endogenous. On the other hand, due to the additional indirect effect of abatement, strategy profiles a^3 and a^4 can, in principle, also be equilibria in the absence of securities and with regulated security markets. This yields a by far larger range of possible constellations even though the sign of the indirect effect is always positive.

7.2 Heterogeneous countries

This section discusses the problems that arise when countries are heterogeneous. Recall that within the framework of the incomplete information model it was assumed that countries can only differ by their type. The type corresponds to their probability distribution[5] across climate states. Countries are *identical* with respect to all other characteristics.

[5]For simplicity, probability distributions are assumed to be *exogenous* again in this section.

7.2.1 Heterogenity with respect to first period parameters

Concerning the first period parameters, e.g. first period endowment ω and abatement cost γ as well as the utility function in the first period, it is not essential for the results that countries be identical. Similar but more complicated results can be derived when countries are heterogeneous with respect to the first period parameters. The assumption of countries being identical with respect to the first period parameters was only included for simplicity of notation and exposition.

7.2.2 Heterogenity with respect to second and third period parameters

With respect to the second and third period parameters it depends on the scenario under consideration whether similar results could be derived for heterogeneous countries. For the benchmark scenario without security markets and for the scenario with unregulated security markets the same analysis as above could, in principle, be applied to a setting with heterogeneous countries.

However, for the scenario with regulation of trade in climate securities analysed in Section 5.4.1 it is crucial that countries only differ with respect to their type. Recall from Section 5.1.4 that then all countries have the same security demand functions. The only argument in the security demand function which can differ from country to country is the type θ_i. Therefore, in security market equilibrium countries will be "separated" according to their types as shown in Lemma 2, i.e. we know which of the types will want to buy and which of the types will want to sell each security in equilibrium. This information is exploited in order to design a regulation scheme that, on the one hand, prevents countries exhibiting type-inconsistent behaviour from transacting their desired trades and, on the other hand, does not restrict countries exhibiting type-consistent behaviour in any way.[6]

If countries are heterogeneous, the decisive property of security market

[6] Cf. Section 5.4.

equilibrium for the design of the regulation scheme in Lemma 2, *separation* of the types, does not hold any longer. In order to see this reconsider country i's first order condition for security demand when, for example, third period endowments are heterogeneous, i.e. $\bar{\omega}_i^\alpha$ and $\omega_i^\beta(\tilde{a})$ differ between the countries:

$$-\frac{q}{1-q} = -\frac{\theta_i}{1-\theta_i}\frac{u'(\bar{\omega}_i^\alpha + s_i^\alpha)}{u'(\omega_i^\beta(\tilde{a}) - \frac{q}{1-q}s_i^\alpha)}. \tag{7.28}$$

Obviously, the security demand function induced by (7.28) then also depends on the endowments $\bar{\omega}_i^\alpha$ and $\omega_i^\beta(\tilde{a})$. In particular, there is no simple criterion by which the equilibrium behaviour of the two different types can be separated. Consequently, it is virtually impossible to design a regulation scheme similar to that introduced in Section 5.4.1, at least without very specific assumptions on the functions involved. If utility functions are also heterogeneous, even more so. Therefore it is important to note that the regulation approach as introduced in Section 5.4.1 only works when countries are similar.

7.2.3 Summary

This section discussed whether similar results would also hold for heterogeneous countries. It turned out that to a large extent the approach would go through for heterogeneous countries. However, the design of the regulation of security trading in the third scenario hinges on countries being identical apart from their type. Thus, if countries are heterogeneous, the relevant scenario is the regime of unregulated security markets in Section 5.3. In particular, the effect of markets for climate securities on abatement behaviour then remains ambiguous and we find no support for the IPCC claim.

Chapter 8

Endogenous probabilities for climate states

This chapter analyses the impact of markets for climate securities on mitigation of climate change when climate state probabilities are endogenous. In Chapter 2.2 it was argued that, clearly, the risks arising from global warming are endogenous risks. They are driven by human activity, to what extent and in what way exactly remains uncertain.[1] In particular, these risks are also influenced by policy measures, e.g. mitigation policies for climate change, since these, in turn, affect human activity.

However, the standard risk allocation framework does not allow for endogenity of risks. In particular, risks are exogenous in the sense that different states of the world are realized with fixed, exogenous probabilities. Since this approach is not appropriate in the context of climate change, the analysis in this chapter focuses on endogenity of probabilities. The following section summarizes the literature on the subject.

8.1 Endogenous probabilities in the literature

In the literature, there are relatively few articles dealing with endogenous uncertainty. They can roughly be divided into two categories, the first one con-

[1] However, the latest IPCC-Report projects a *higher* increase in global mean temperature than the 1995 IPCC-Report (cf. IPCC [1996a]). See Chapter 2.1.

taining generalizations of microeconomic results to endogenous uncertainty, the second one containing applications of catastrophe theory and related fields to global warming or other environmental risks.

Most of the work belonging to the first category is relatively recent work. CHICHILNISKY [1996] extends the analysis of general competitive equilibrium with production to a setting in which the state space and the probabilities of events are endogenously determined as part of the equilibrium. She formulates an equilibrium concept for this setting and proves existence of equilibrium. Moreover, she shows that the equilibrium allocations are efficient in a restricted sense. For this, endogenous uncertainty is decomposed into "scientific uncertainty", on the one hand, and "strict endogenous uncertainty", on the other hand. Scientific uncertainty refers to the uncertain effect of economic activity on the climate, strict endogenous uncertainty is uncertainty about the resulting economic activity itself. Whereas scientific uncertainty can be fully hedged, strict endogenous uncertainty cannot.

In CHICHILNISKY [1996] the vector of aggregate output in the economy determines both the set of possible states and the corresponding probability distributions. This relation is formalized by an implicit function linking aggregate output and probabilities on the variable state space. One of the key assumptions is competitive behaviour with respect to endogenous uncertainty, i.e. each agent considers the set of states and their probability as independent of his own actions. CHICHILNISKY also considers applications of her framework for endogenous uncertainty to climate change. These applications and the same market approach to endogenous risks are also discussed in CHICHILNISKY [1998].

KURZ AND WU [1996] focus on equilibrium theory with endogenous uncertainty in an overlapping generations context. Agents are assumed to have rational beliefs and prices are the state variables themselves. In this setup, KURZ AND WU investigate the role of price-contingent contracts for the allocation of risk.

In a very different context BRIYS, EECKHOUDT AND LOUBERGE [1989] point out that there are several different definitions of the risk premium for endogenous risks in the literature. They show that two of these proposed concepts do not preserve the intuitive properties of the Arrow-Pratt risk premium[2] and suggest an alternative definition.

The classic reference for the second category is CROPPER [1976]. She investigates two types of catastrophes, one leading to a temporary reduction in utility and one which is irreversible. Whether or not a catastrophe occurs depends on the stock of pollution relative to the critical pollution level. The critical pollution level, in turn, is a random variable. In the first context CROPPER analyses the steady state equilibria and approach paths for pollution when effects of pollution are catastrophic. In the second context she studies the optimal rate of extraction of a non-renewable resource when available resources are uncertain.

An intertemporal approach, studying a growing economy with endogenous uncertainty, is also presented in HEAL [1984], [1990]. There, the atmosphere may be in a favourable or in a unfavourable state so that there is a possibility of a future climate change. The transition is stochastic and irreversible, i.e. the unfavourable state is an absorbing state. The probability of transition to the unfavourable state is endogenous and increases with the level of cumulative emissions from the use of fossil fuels. In this setting, HEAL characterizes optimal paths of consumption, capital accumulation and use of fossil fuels and compares them to those that are optimal in the absence of an atmospheric impact.

CLARKE and REED [1994] analyse the impact of an avoidable risk of irreversible environmental catastrophe on the optimal management of a renewable environmental resource. The catastrophic risk is assumed to be a nondecreasing function of stock pollution. In TSUR AND ZEMEL [1996] there is an uncertain critical pollution level which triggers undesirable events associated with the greenhouse effect. In contrast to CROPPER and CLARKE AND REED they assume that the probability distribution also depends on the history of pollution.

TORVANGER [1997] considers a three-generation planning model with uncertain climate change. The probability and scale of future climate change are affected by production activities in terms of accumulated resource use. The optimal resource extraction path is determined by means of stochastic

[2] The definition of the Arrow-Pratt risk premium originates in PRATT [1964] and ARROW [1965].

dynamic programming.

All of the articles concerned with application of catastrophe theory to environmental issues focus on the optimal dynamic path only, i.e. they consider the issues from a social planner's perspective. They do not study a decentralized framework and the equilibrium behaviour within such a framework.

8.2 The Model

In contrast to all the papers mentioned above the analysis in this chapter deals with *strategic interaction under endogenous uncertainty.* In order to analyse the effect of financial instruments on environmental decision-making the model combines a general equilibrium framework with a standard model of strategic abatement behaviour. Unlike CHICHILNISKY [1996], competitive behaviour with respect to endogenous uncertainty is not assumed. In contrast, countries' influence on endogenous probabilities is anticipated and affects their abatement action. This is, in fact, the key element of the approach presented here. The model also differs from that of CHICHILNISKY with respect to endogenity of probabilities being modelled explicitely and the fixed state space. Probabilities for the different states are endogenous and subjective.

The focus here is on the effect of climate securities on abatement, i.e. the outcomes of strategic interaction under endogenous uncertainty without and with climate securities are compared. In contrast to the articles concerned with catastrophes this model is not a dynamic model. Although it consists of two stages, it is of static nature since each type of decision (abatement decision and portfolio decision) is taken only once. Catastrophic outcomes are not explicitely incorporated in the model. However, they can be approximated by adequate choice of the endowments in the two possible climate states.

What welfare effects would one expect from the introduction of markets for climate securities? Introducing such markets removes a market imperfection, namely lack of "insurance" markets. Therefore, at first sight one might expect welfare to improve. However, we know from the *Theory of the Second*

Best[3] that if there is more than one market imperfection, abolishing one of these imperfections does not necessarily improve welfare. I.e. if one of the Paretian conditions is not attainable due to a market imperfection the other Paretian conditions are not necessarily optimal anymore. Therefore we cannot compare the welfare properties of markets with differing inherent market imperfections with one another. In particular, removing one of several market imperfections will not not necessarily enhance welfare. Since the setting to be analysed in this Chapter indeed exhibits several market imperfections we can anticipate that the welfare effects may be ambiguous.

8.2.1 Basics of the Model

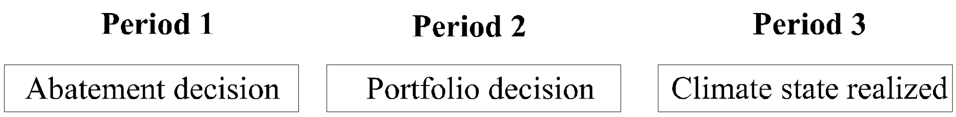

Figure 8.1: *The time structure*

The time structure of the model ressembles that of the incomplete information approach studied in Chapters 5 to 7 and is illustrated in Figure 8.1. In the first period ($t = 1$) each country $i = 1, \ldots, n$ decides on its abatement strategy, i.e. the amount of greenhouse gases it will emit. In the second period ($t = 2$) it chooses its portfolio, i.e. can buy or sell climate securities. Like in Chapter 5 climate securities are financial instruments whose payoff depends on the climate state that will be realized in the subsequent period. In the

[3]The Theory of the Second Best originates from LIPSEY AND LANCASTER [1956] according to whom *"The general theorem for the second best optimum states that if there is introduced into a general equilibrium system a constraint which prevents the attainment of on of the Paretian conditions, the other Paretian conditions, although still attainable, are, in general, no longer desirable. In other words, given that one of the Paretian optimum conditions cannot be fulfilled, then an optimum situation can be achieved only by departing from all the other Paretian conditions."* DAVIS AND WHINSTON [1964] warn that *"... the policy maker must consider the entire functionally interconnected subset of units in order to avoid undesirable consequences."*

third period ($t = 3$) the uncertainty about the climate state is resolved,[4] and the payoffs arising from security trading in period 2 are realized.

The key element of the model is *endogenity* of countries' probability distributions across climate states. Aggregate first period emissions influence the (subjective) probability distributions across climate states used by the countries in the second period. This feature generates a link between the decisions in the two periods.

Consider a world with two goods, a (highly aggregated) consumption good[5] and emissions of greenhouse gases. Note that utility is derived only from the consumption good, whereas emissions do not enter the utility functions directly. In particular, there are no instantaneous damages arising from pollution. Instead, they influence the probabilities of the climate states and, therefore, have an indirect effect on expected utility.[6]

8.2.2 The abatement decision

In contrast to the incomplete information model of Chapter 5 the abatement decision is modelled continuously. Denote by e_i the emissions of country i. The abatement cost function for country i is denoted $C_i(e_i)$. It measures abatement cost in units of the aggregate good and satisfies standard properties. It is assumed non–negative ($C_i \geq 0$), strictly decreasing ($C_i' < 0$) and convex ($C_i'' \geq 0$) on its relevant range. Denote by e_i^{max} the maximal amount of emissions, i.e. the emission level when no abatement effort is made, and assume $C_i(e_i^{\max}) = 0$. An abatement cost function with linear marginal abatement cost is illustrated in Figure 8.2.

Country i's first period endowment of the aggregate good is denoted w_i. Interpret w_i as the total production in country i when emissions are e_i^{max} and no abatement effort is made. Endowment can either be consumed or "invested" in emission reductions, i.e. there is a trade–off between consump-

[4] In reality, the uncertainty will only be partially resolved. This could be modelled by considering increasingly finer partitions as in DEBREU [1959], Chapter 7.

[5] The aggregate good will also be the numeraire.

[6] Emissions would usually be considered a *"public bad"*. In this context, the terminology "bad" is not appropriate in general since in some cases emissions may increase a country's utility.

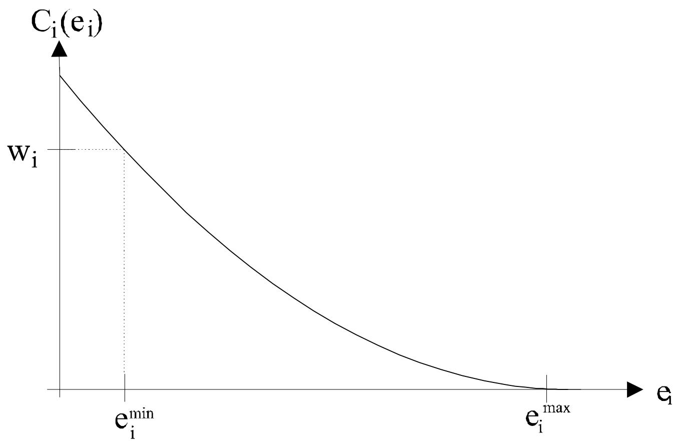

Figure 8.2: *An abatement cost function with linear marginal abatement cost.*

tion and abatement.[7]

If country i emits $e_i < e_i^{\max}$ in the first period, it has to give up the corresponding amount of its first period endowment w_i. Consumption, which equals $w_i - C_i(e_i)$, is restricted to non-negative values by definition.[8] The finite endowment w_i then defines a lower boundary for emissions, with $e_i^{min} = C^{-1}(w_i)$.

If the abatement cost function, in addition to the properties assumed above, has the property that it is (for technical reasons) not possible to reduce emissions completely, i.e. $\lim_{e_i \to 0} C_i(e_i) = \infty$, then $e_i^{min} > 0$ will hold. This is a realistic assumption for most greenhouse gases, e.g. CO_2. For simplicity, abatement cost functions with linear marginal abatement cost – as drawn in Figure 8.2 – will also be used in Section 8.6. In that case $e_i^{min} > 0$ does not hold if first period endowment w_i is large.

Let $u_i(\cdot)$ denote the utility function for country i. The utility function is the same for all periods. Assume $u_i' > 0$ and $u_i'' < 0$, representing risk aversion. Country i's utility from the first period is $u_i(w_i - C_i(e_i))$ which,

[7]Abatement expenditures can be interpreted as an investment in future consumption. Thus, the trade–off is between *sure consumption* in period 1 and *uncertain future consumption* in period 3.

[8]Note that *borrowing*, i.e. consumption levels smaller than zero, is not possible in this framework.

given the assumptions above, is strictly increasing and concave in e_i.

Assume that each country's emissions and, in particular, aggregate emissions $E := \sum_{i=1}^{n} e_i$ can be observed by all countries. The range for aggregate emissions is the interval $[E^{\min}, E^{\max}]$ where $E^{\min} := \sum_i e_i^{\min}$ and $E^{\max} := \sum_i e_i^{\max}$.

8.2.3 The portfolio decision

Uncertainty with respect to the climate state

In the second period there is uncertainty about the climate state that will be realized in the subsequent period. For simplicity, assume like in Chapter 5 that there are only two potential climate states,[9] state α (no noticeable climate change) and state β (severe climate change). Interpret state α as corresponding to a lower global mean temperature than state β, i.e. state β stands for an increase in the global mean temperature ("global warming").

In the third period the climate state realized will determine the endowment of the countries for that period. I.e. whether a country would be harmed by an increase in global mean temperature, e.g. the Mediterranean, or whether it would benefit from global warming, e.g. Siberia or northern Europe, is reflected in the relation between its endowments for the two different states.[10] The underlying interpretation is that climate can influence the production possibilities, especially the agricultural sector, and that this is reflected by the endowment which represents total production.

Denote by $w_{i\sigma}$ the endowment of country i in the third period if the climate state realized is σ ($i = 1, ..., n$ and $\sigma = \alpha, \beta$). Then, for example, country i profits from global warming if $w_{i\alpha} < w_{i\beta}$ and is adversely affected by global warming if $w_{i\alpha} > w_{i\beta}$.

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[9] In reality, of course, there will be a continuum of potential climate states. Classification of climate states would have to refer to a large number of criteria as explained in Chapter 2.1.2.

[10] Cf. IPCC [2001b], p. 15: *"There will be some broadly positive effects on agriculture in northern Europe [...]; productivity will decrease in southern and eastern Europe [...]."*

Markets for climate securities

The assumptions on markets for climate securities are the same as in the incomplete information model of Chapter 5. Again a complete system of climate securities is assumed, i.e. one instrument associated with each climate state $\sigma \in \{\alpha, \beta\}$. Climate securities are again defined in the tradition of Arrow [1953]: security σ pays one unit of the aggregate good, which is also the numeraire, if state σ occurs and nothing otherwise.[11] By means of climate securities countries can transfer income (endowment) from favorable states to unfavorable ones and vice versa.

Security markets are again assumed to be competitive, the price of security α being denoted q. The price of security β is then $1 - q$. Assume further that at the time of decision-making in the first period all countries are informed about the possibility of trade in climate securities in the second period.

Endogenous probabilities on climate states

Each country has a subjective[12] probability distribution on the set of climate states which depends on aggregate emissions in period 1. For $E \in [E^{\min}, E^{\max}]$ denote by $\Pi_i(E)$ country i's subjective probability for state α and by $1 - \Pi_i(E)$ its probability for state β. Intuitively, it is clear that with increasing aggregate emissions countries will put more weight on the state associated with an increase in global mean temperature, i.e. on state β. Therefore assume

$$\Pi_i'(E) < 0 \qquad \text{“for all } E\text{”} \quad \text{and all } i = 1, ..., n, \tag{8.1}$$

where “for all E” has to be interpreted appropriately, as explained in the following paragraph.

Whereas Π_i is defined on the closed interval $[E^{\min}, E^{\max}]$, its derivative Π_i' is only defined on the open interval $(E^{\min}, E^{\max})$. For the endpoints of the interval, $E^{\min}$ and $E^{\max}$, only the derivative from the right or left, respectively, is well-defined. I.e. “$\Pi_i'(E) < 0$ for all E” should be interpreted

[11] The payoff structure shown in Table 5.2 applies. Again, transaction costs of security trading are neglected.

[12] In the context of climate change, there is no justification for objective probabilities.

as "$\Pi_i'(E) < 0$ for all $E \in (E^{\min}, E^{\max})$, $lim_{E \to E^{\min}, E > E^{\min}} \Pi_i'(E) < 0$ and $lim_{E \to E^{\max}, E < E^{\max}} \Pi_i'(E) < 0$." Note that in the following the phrase "for all E" will be used repeatedly in the above interpretation.

It is not obvious what should be assumed about the second derivative $\Pi_i''(E)$ of the endogenous probabilities. This depends on the scientific evidence concerning the impact of mitigation on global warming, and also the aggregation and interpretation of this scientific evidence.[13] The probability function Π_i could, given that we assume continuity,[14] be convex or concave or could even be less well-behaved and have neither of these properties. For simplicity, we will often consider the special case of a linear relationship between aggregate emissions and climate state probabilities, i.e. assume $\Pi_i''(E) = 0$ for the appropriate interval with respect to E.[15]

An important implication of endogenity of probabilities is that countries' preferences on contingent consumption bundles become endogenous, too. Therefore a change in first period aggregate emissions E leads to a change in preferences on contingent consumption. An example for such a change in preferences, assuming maximization of expected utility, is drawn in Figure 8.3.[16]

The decision problem

Write $s_{i\sigma}$ for the amount of security σ bought or sold by country i in period 2. The constraints of country i's decision problem are the same as those in Chapter 5. Firstly, a country's portfolio has to satisfy the "self-financing portfolio constraint"

$$q s_{i\alpha} + (1 - q) s_{i\beta} = 0 \tag{8.2}$$

[13] See Chapter 2.

[14] Note that probability functions exhibiting discontinuities at certain threshold values of E may also be a relevant case in this context.

[15] Cf. assumption (8.1) and the subsequent interpretation which applies analogously.

[16] Note that the (negative) slope of the indifference curves for country i contains the expression $-\frac{\Pi_i(E)}{1-\Pi_i(E)}$, which increases in E by assumption (8.1), as a multiplicative factor. Therefore an increase in E causes the (negative) slope of the indifference curves to become flatter.

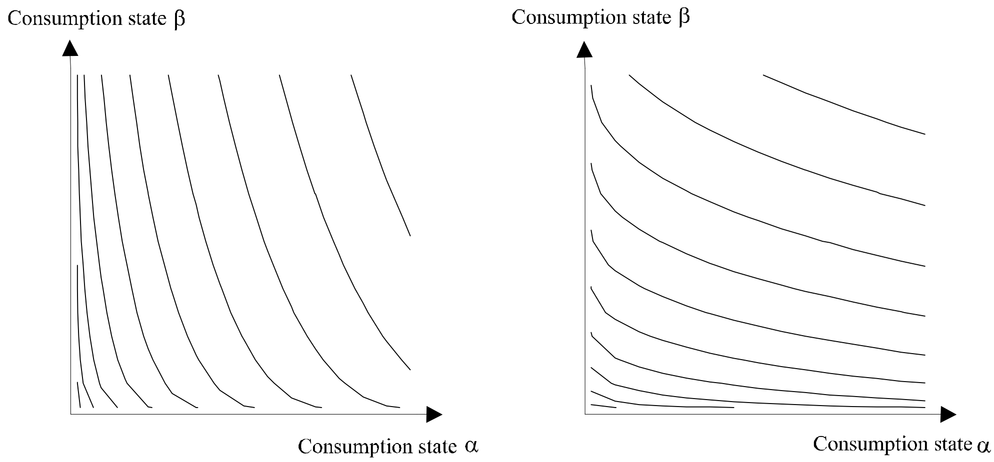

Figure 8.3: *Preferences on contingent consumption bundles for levels of aggregate emissions $\hat{E}$ (to the left) and $\bar{E}$ (to the right) with $\hat{E} < \bar{E}$. A higher level of aggregate emissions E increases the probability for state β and the indifference curves become flatter.*

since there is no endowment in securities. I.e., if a country wants to buy securities of one type, then it has to sell securities of a different type in order to be able to do so. Secondly, short-selling is not possible which is equivalent to restricting third period consumption to non-negative values.

Note that the utility function does not depend on the climate state and that there is no consumption in the second period. Assume that country i maximizes expected utility from period 3

$$v_i(s_{i\alpha}, s_{i\beta}; E) := \Pi_i(E)u_i(w_{i\alpha} + s_{i\alpha}) + (1 - \Pi_i(E))u_i(w_{i\beta} + s_{i\beta})$$

for given security prices $(q, 1 - q)$ and first period emissions E subject to (8.2) and a non-negativity restriction on third period period consumption.[17]

8.2.4 Security market equilibrium

Solving the decision problem described above yields net demand for securities $s_{i\sigma}(E, q)$ as a function of aggregate period 1 emissions E and security prices q.

[17]Third period consumption in state σ is $w_{i\sigma} + s_{i\sigma}$ for $\sigma = \alpha, \beta$.

The equilibrium security price[18] $q^*(E)$ is determined by the market clearing condition[19]

$$\sum_{i=1}^{n} s_{i\alpha}(E, q^*(E)) = 0.$$

Plugging equilibrium security trades for each state, $s^*_{i\sigma}(E) := s_{i\sigma}(E, q^*(E))$, into the expected utility function v_i yields the indirect expected utility function

$$\begin{aligned} v_i^*(E) &:= v_i(s^*_{i\alpha}(E), s^*_{i\beta}(E); E) \\ &= \Pi_i(E) u_i(w_{i\alpha} + s^*_{i\alpha}(E)) + (1 - \Pi_i(E)) u_i(w_{i\beta} + s^*_{i\beta}(E)) \end{aligned} \tag{8.3}$$

which only depends on period 1 aggregate emissions E. For a given outcome of the first period in terms of aggregate emissions E, the indirect expected utility function v_i^* summarizes the outcome of (competitive) decision-making in the second period in terms of expected utility for country i. The asterisk as superscript indicates that these are *equilibrium values* with respect to security trading in the second period.

8.2.5 Total utility

Total utility for country i can then be written as

$$U_i(e_1, \ldots, e_n) := u_i\left(w_i - C_i(e_i)\right) + v_i^*\left(\sum_{i=1}^{n} e_i\right). \tag{8.4}$$

For simplicity, there is no discounting.[20] Due to competivity of security markets there is no strategic interaction in the second period and the contribution to total utility from that period, $v_i^*(E)$, depends only on aggregate first period emissons E. It is assumed that countries *anticipate* the security market equilibrium in the second period when deciding on their emission level in

[18] Given the assumptions above $s_{i\sigma}(E, q)$ exists and is a singleton. Existence of the equilibrium security price is guaranteed by the standard conditions. See, for example Mas-Colell, Whinston and Green [1995], p. 585.

[19] Since Walras' Law holds, the market for security β then also clears.

[20] This could easily be included in the model, but does not yield any new insights.

the first period.[21] Basically, that amounts to *backward induction*, but with competitive instead of strategic interaction in the second period. Note that anticipation of the security market equilibrium makes strong demands on the information structure.[22]

Via the indirect utility functions v_i^* each country's payoff U_i depends, in principle, on the emissions of all other countries besides its own. Thus, the utility functions defined in (8.4) induce a *non-cooperative game*. The analysis in the following sections is concerned with the impact of markets for climate securities on mitigation activity, i.e. on emission levels. For this purpose, we study and compare two non-cooperative games, the game induced by the utility functions in (8.4) and, as benchmark, the game derived from the same model when there are no markets for climate securities.

Note that this comparison only makes sense if it is common knowledge at the beginning of the first period that there will be markets for climate securities in the following period. Only then will countries take into account the effect of their abatement decision on these markets when making their abatement decision. Otherwise, there would be no effect on emissions at all. The anticipation of the effect of emission decisions on the future gains from trade in climate securities is the crucial mechanism in this chapter that links the mitigation and insurance decisions.

[21] Of course, a very basic requirement for anticipation of security market equilibrium is that countries are informed about the trading possibilities they will have in the second period.

[22] Perfect information about all relevant characteristics of all the other countries is required, e.g. concerning endowments, utility functions, and probability functions. In particular, as argued in Chapter 5, the subjective probability functions are critical, since they may be misrepresented.

8.3 The non-cooperative game in the absence of climate securities

8.3.1 Total utility

Formally, the benchmark scenario without climate securities is a special case of the more general framework defined in Section 8.2 above. Countries always consume their third period endowment, i.e. $s^*_{i\sigma}(E) = 0$ for all i, σ and for all $E \in [E^{\min}, E^{\max}]$. The indirect utility function, defined in (8.3), reduces to expected utility from third period endowments and is denoted by

$$x_i(E) := v^*_i(0, 0; E) = \Pi_i(E)u_i(w_{i\alpha}) + (1 - \Pi_i(E))u_i(w_{i\beta}), \tag{8.5}$$

where "x" stands for "e**x**pected utility from endowments." Since $x_i(\cdot)$ does not depend on security market equilibrium it carries no asterisk.

Total utility, substituting x_i for v^*_i in (8.4), reduces to

$$U^N_i(e_1, \ldots, e_n) := u_i\left(w_i - C_i(e_i)\right) + x_i(E) \tag{8.6}$$

where the superscript "N" stands for "**N**o securities." Denote the Nash equilibrium – given there exists a unique Nash equilibrium[23] – of the induced non-cooperative game by $(e^N_1, \ldots, e^N_n)$ and the corresponding equilibrium utility levels by $u^N_i := U^N_i(e^N_1, \ldots, e^N_n)$ for $i = 1, .., n$. Before proceeding to the analysis of non-cooperative equilibrium some basic properties of the derivatives of the indirect utility function x_i are stated.

8.3.2 The effect of aggregate emissions on expected utility from third period endowments

The derivative of x_i with respect to aggregate emissions E is given by

$$x'_i(E) = \underbrace{\Pi'_i(E)}_{<0}[u_i(w_{i\alpha}) - u_i(w_{i\beta})]. \tag{8.7}$$

Recall that the derivative $\Pi'_i(E)$ is well-defined only on the open interval $(E^{\min}, E^{\max})$ whereas for the endpoints of that interval only the right or

[23]Existence and uniqueness of Nash equilibrium are discussed in Section 8.3.3 below.

left hand derivatives are defined.[24] The same then applies to the derivative $x_i'(E)$. Since $\Pi_i'(E) < 0$ "for all E" in the same sense, we immediately get the following result for the sign of x_i' where again "for all E" has to be interpreted accordingly.

Lemma 3
For $i = 1, ..., n$

$$x_i'(E) \quad \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} \quad 0 \qquad \text{“}\textit{for all } E\text{”} \qquad \Longleftrightarrow \qquad w_{i\alpha} \quad \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} \quad w_{i\beta}.$$

By Lemma 3 the sign of $x_i'(E)$ is completely determined by the relationship between country i's endowments $w_{i\alpha}$ and $w_{i\beta}$, i.e. by whether the country would be adversely affected by an increase in global mean temperature or whether it might even benefit from climate change. In particular, the sign of $x_i'(E)$ neither depends on further properties of the probability function $\Pi_i(\cdot)$ (apart from assumption (8.1)), nor on the utility function $u_i(\cdot)$.

Taking the derivative with respect to E once more in (8.7) we see that the second derivative, x_i'', is also determined by the endowment pattern and the properties of the probability function. We have

$$sign(x_i''(E)) = \left\{ \begin{array}{l} \;\; sign(\Pi_i''(E)) \text{ “for all } E\text{”, if } w_{i\alpha} > w_{i\beta} \\ -sign(\Pi_i''(E)) \text{ “for all } E\text{”, if } w_{i\alpha} < w_{i\beta} \end{array} \right. . \tag{8.8}$$

If $w_{i\alpha} > w_{i\beta}$, i.e. country i is adversely affected by global warming, x_i'' takes on the sign of Π_i''. In particular, x_i is concave (convex) if and only if Π_i is concave (convex). If $w_{i\alpha} < w_{i\beta}$, i.e. if country i benefits from global warming, the sign is reversed. Then, x_i is concave (convex) if and only if Π_i is convex (concave).

8.3.3 Non-cooperative behaviour

We now proceed to the analysis of the non-cooperative game induced by the utility functions U_i^N defined in (8.6). Recall that the first term in U_i^N

[24] Cf. assumption (8.1).

is strictly concave in e_i (cf. Section 8.2.2), whereas the properties of the second term can vary depending on the endowment pattern (cf. equation (8.8) above). Therefore U_i^N is not necessarily a concave function and the standard analysis not applicable.

For the partial derivative of U_i^N with respect to country i's emissions e_i we get

$$\frac{\partial U_i^N(e_1, \ldots, e_n)}{\partial e_i} = \underbrace{u_i'(w_i - C_i(e_i))\left[-C_i'(e_i)\right]}_{>0} + x_i'(E). \tag{8.9}$$

The first expression on the right hand side of (8.9) is marginal utility with respect to emissions e_i from the first period. Recall that it is larger than zero by the assumptions above. The sign of $x_i'(E)$ is determined by the relationship between the third period endowments as discussed above in Lemma 3. In particular, the sign of $x_i'(E)$ can be positive or negative.

First order conditions for interior solution

Setting $\frac{\partial U_i^N(e_1,\ldots,e_n)}{\partial e_i} \stackrel{!}{=} 0$ for all i yields the first order conditions for an interior Nash equilibrium

$$\underbrace{u_i'(w_i - C_i(e_i))\, C_i'(e_i)}_{<0} = x_i'(E) \qquad i = 1, ..., n. \tag{8.10}$$

From (8.9) it is clear that for an interior Nash equilibrium $(e_1^{int}, ..., e_n^{int})$ with respect to $e_i \leq e_i^{\max}$ the derivative $x_i'(E^{int})$ has to satisfy $x_i'(E^{int}) < 0$ for all i where $E^{int} := \sum_i e_i^{int}$. In different words, country i will only engage in costly abatement, i.e. choose an interior solution with respect to $e_i \leq e_i^{\max}$, if it is adversely affected by climate change.

Corner solutions and dominant strategies

Denote the aggregate emissions of all countries $j \neq i$ by $E_{-i} := \sum_{j \neq i} e_j$. The range of E_{-i} is $[E_{-i}^{\min}, E_{-i}^{\max}] := [\sum_{j \neq i} e_j^{\min}, \sum_{j \neq i} e_j^{\max}]$. In the following "for all e_i" means "for all $e_i \in [e_i^{\min}, e_i^{\max}]$" and "for all E_{-i}" means "for all $E_{-i} \in [E_{-i}^{\min}, E_{-i}^{\max}]$". As explained above, derivatives for the endpoints of the interval under consideration should be interpreted as right or left hand derivatives only.

Country i has the dominant strategy $e_i = e_i^{\max}$, if and only if

$$\begin{aligned} \frac{\partial U_i^N(e_1,\ldots,e_n)}{\partial e_i} &\geq 0 \quad \text{for all } e_i \\ \Leftrightarrow \quad \underbrace{u_i'\left(w_i - C_i(e_i)\right) C_i'(e_i)}_{<0} &\leq x_i'(e_i + E_{-i}) \quad \text{for all } e_i, E_{-i}. \qquad (8.11) \end{aligned}$$

Since the term on the left hand side of inequality (8.11) is strictly increasing in e_i by the assumptions on u_i and C_i, (8.11) is equivalent to

$$\begin{aligned} u_i'\left(w_i - C_i(e_i^{\max})\right) C_i'(e_i^{\max}) &\leq x_i'(e_i^{\max} + E_{-i}) \quad \text{for all } E_{-i} \\ \Leftrightarrow \quad u_i'(w_i)\, C_i'(e_i^{\max}) &\leq x_i'(e_i^{\max} + E_{-i}) \quad \text{for all } E_{-i}. \qquad (8.12) \end{aligned}$$

In particular, (8.12) holds when $x_i'(E) \geq 0$ for all E. So, not surprisingly, if country i is not affected by climate change ($w_{i\alpha} = w_{i\beta}$) or even profits from it ($w_{i\alpha} < w_{i\beta}$), it will not be prepared to invest in abatement.

By a similar argument, $e_i^N = e_i^{\min}$ is a dominant strategy for country i, if and only if

$$\begin{aligned} \frac{\partial U_i^N(e_1,\ldots,e_n)}{\partial e_i} &\leq 0 \quad \text{for all } e_i \\ \Leftrightarrow \quad u_i'\left(w_i - C_i(e_i^{\min})\right) C_i'(e_i^{\min}) &\geq x_i'(e_i^{\min} + E_{-i}) \quad \text{for all } E_{-i} \\ \Leftrightarrow \quad u_i'(0)\, C_i'(e_i^{\min}) &\geq x_i'(e_i^{\min} + E_{-i}) \quad \text{for all } E_{-i} \qquad (8.13) \end{aligned}$$

Since the left hand side of inequality (8.13) is smaller than zero, this requires the respective country to be adversely affected by climate change ($x_i'(e_i^{\min} + E_{-i}) < 0 \quad$ for all E_{-i}) and, moreover, the adverse effect has to be relatively large.[25]

Existence of Nash equilibrium

Sufficient conditions for the existence of Nash equilibrium are given, for example, in Theorem 1.2 of Fudenberg and Tirole [1991]. Clearly, the strategy spaces $[e_i^{\min}, e_i^{\max}]$ obey the requirements[26] listed there. Continuity

[25] Since the argument of u_i' in (8.13) is zero there may be technical problems checking (8.13) for some utility functions, e.g. for $u_i(\cdot) = \ln(\cdot)$.

[26] The strategy spaces have to be nonempty compact convex subsets of a Euclidean space.

of the utility functions is also obvious.[27] The interesting requisite for existence is quasi-concavity of the utility functions in e_i. Taking the second partial derivative with respect to e_i we get

$$\frac{\partial^2 U_i^N(e_1,\dots,e_n)}{\partial^2 e_i} = \underbrace{u_i''(w_i - C_i(e_i))\left[C_i'(e_i)\right]^2 - u_i'(w_i - C_i(e_i))\,C_i''(e_i)}_{<0} + x_i''(E).$$

The sum of the first two expressions is smaller than zero by the assumptions on u_i and C_i. If $\frac{\partial^2 U_i^N(e_1,\dots,e_n)}{\partial^2 e_i} < 0$ holds, the utility function for country i is concave in e_i. Since concavity implies quasi-concavity the sufficient conditions in the existence theorem would then be satisfied. Therefore, existence of equilibrium is guaranteed, if $x_i''(E)$ is sufficiently small, in particular, if $x_i''(E) \leq 0$ for all E and all i. Note, however, that for the general case existence is *not* guaranteed.

Reaction functions

Implicitly differentiating the first order condition in (8.10) with respect to E_{-i} yields the derivative of the reaction function for country i

$$e_i'(E_{-i}) = -\frac{x_i''(E)}{\underbrace{u_i''(w_i - C_i(e_i))\left[C_i'(e_i)\right]^2 - u_i'(w_i - C_i(e_i))C_i''(e_i)}_{<0} + x_i''(E)}. \tag{8.14}$$

If the sufficient conditions for existence of equilibrium hold, i.e. $x_i''(E)$ is sufficiently small, the denominator of $e_i'(E_{-i})$ is smaller than zero. Then, the sign of $e_i'(E_{-i})$ is determined by that of $x_i''(E)$. For $x_i''(E) < 0$ the reaction function is downward sloping and for $x_i''(E) > 0$ it is upward sloping.[28]

For concave x_i the absolute slope of the reaction function is smaller than one, i.e. $|e_i'(E_{-i})| < 1$ holds for all E_{-i}. If that is the case for all countries, uniqueness is also guaranteed.[29]

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[27] Continuity follows from differentiability, which was implicitly assumed in requiring $u_i' > 0$.

[28] Reaction functions may be non-monotonous if $x_i''(E)$ exhibits a change of sign.

[29] See Tirole [1988], p. 226.

8.3.4 Linear probability functions

Non-cooperative behaviour

Matters simplify considerably for linear probability functions. Since x_i is a linear function of E, its derivative x_i' is a constant, denoted $\bar{x}_i'$ in the following. Then the derivative $\frac{\partial U_i^N(e_1,\dots,e_n)}{\partial e_i}$ does not depend on E_{-i} and there is *no strategic interaction* between the different countries. Consequently, all reaction functions are constants, i.e. the slope of the reaction function in (8.14) is zero. The resulting Nash equilibrium is characterized in Proposition 12.

Proposition 12 *(Non-cooperative equilibrium without security markets)*

Let the probability functions Π_i be linear for all $i = 1, ..., n$. Then there exists a unique Nash equilibrium $(e_1^N, ..., e_n^N)$ in the non-cooperative game induced by (8.6) where

$$e_i^N = \begin{cases} e_i^{\max}, & \text{if } u_i'(w_i)\, C_i'(e_i^{\max}) \leq \bar{x}_i' \\ e_i^{\min}, & \text{if } u_i'(0) C_i'(e_i^{\min}) \geq \bar{x}_i' \\ e_i^{int}, & \text{else} \end{cases} \tag{8.15}$$

and where e_i^{int} solves (8.10) for country i.

The proof follows immediately from the analysis in Section 8.3.3 above. By Proposition 12 there is a unique Nash equilibrium in dominant strategies when probability functions are linear. Note that the standard sufficient conditions for existence are also satisfied, since $\frac{\partial^2 U_i^N(e_1,\dots,e_n)}{\partial^2 e_i} < 0$ holds. Uniqueness trivially follows due to reaction functions being constant functions.

Cooperation

As a further benchmark consider the outcome if there was cooperation between the countries. For simplicity, assume that the sum of countries' utilities is maximized rather than a weighted sum. The first order conditions for cooperative behaviour, assuming an interior allocation,[30] are

$$\underbrace{C_i'(e_i)u_i'(w_i - C_i(e_i))}_{<0} = \sum_{j=1}^{n} \bar{x}_j' \quad \text{for all } i = 1, ..., n. \tag{8.16}$$

Denote the solution to the system of equations in (8.16) – if existent and uniquely determined – by $e_1^C, ..., e_n^C$.

In order to focus on the difference between cooperation and non-cooperation assume for a moment that countries are *identical concerning the first period parameters.* Then the left hand sides of (8.16) are the same for all countries and, consequently, cooperative emission levels are always symmetric.

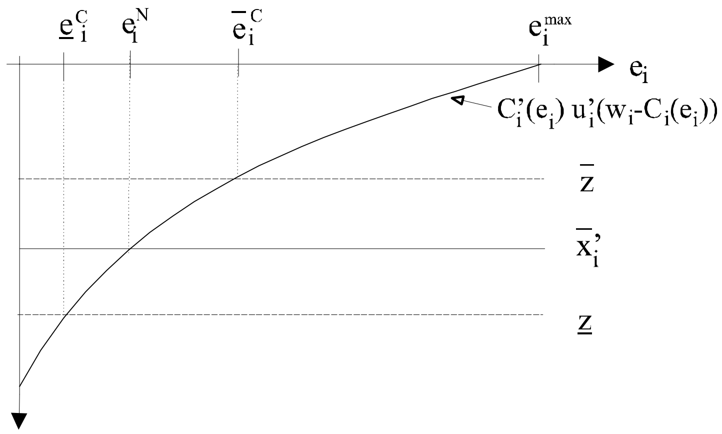

Figure 8.4: *Illustration of the first order conditions for cooperative behaviour. When $\bar{x}_i'$ is larger than $\sum_{j=1}^n \bar{x}_j'$, e.g. $\sum_{j=1}^n \bar{x}_j'$ equal to $\underline{z}$, cooperative emissions $\underline{e}_i^C$ are smaller than non-cooperative emissions e_i^N. When $\bar{x}_j'$ is smaller than $\sum_{j=1}^n \bar{x}_j'$, e.g. $\sum_{j=1}^n \bar{x}_j'$ equal to $\bar{z}$, cooperative emissions $\bar{e}_i^C$ are larger than non-cooperative emissions e_i^N.*

Since the left hand side in the respective first order conditions (8.10) and (8.16) is identical, the crucial point for comparison of country i's emissions in a non-cooperative and a cooperative environment is the relationship between $\bar{x}_i'$ and $\sum_{j=1}^n \bar{x}_j'$. The situation is illustrated in Figure 8.4. Recall that the left

[30] In analogy to (8.11) and (8.13) there will be corner solutions, if $\sum_{j=1}^n \bar{x}_j'$ is either very large or very small so that it does not intersect with the left hand side of (8.16) in the relevant range of e_i.

hand hand side of both types of first order conditions is smaller than zero and increases with e_i, whereas due to linearity of the probability functions both $\bar{x}_i'$ and $\sum_{j=1}^n \bar{x}_j'$ are constants. If $\bar{x}_i' > \sum_{j=1}^n \bar{x}_j'$ holds, cooperative emissions e_i^C for country i are smaller than non-cooperative emissions e_i^N and if $\bar{x}_i' < \sum_{j=1}^n \bar{x}_j'$ holds, cooperative emissions for country i are larger than non-cooperative emissions.

In particular, $\bar{x}_i' > \sum_{j=1}^n \bar{x}_j'$ always holds for all i by Lemma 3 if we have only adversely affected countries. Then we have the typical *Prisoner's Dilemma Situation* in which the non-cooperative abatement effort is too low. However, if there are countries which benefit from global warming, $\bar{x}_i' < \sum_{j=1}^n \bar{x}_j'$ may hold and then non-cooperative emission levels may be higher than the cooperative levels for some countries. This would be a situation not often encountered in environmental economics.

Of course, in general, neither the utility functions nor the abatement cost functions nor the first period endowment can be expected to be identical for all countries. Then, cooperative emission levels are no longer symmetric and the above discussion has to be modified accordingly.

8.4 The non-cooperative game with climate securities

8.4.1 Total utility

The utility functions for the scenario with climate securities were already defined in (8.3). The key term is expected utility v_i^* from the third period. For further analysis it is convenient to decompose v_i^* into expected utility from endowments, x_i, as defined in Section 8.3.3 and an additional term, *expected gains from trade in climate securities*. For $i = 1, ..., n$ define the expected gains from trade as the difference between total expected utility v_i^* and expected utility from endowments x_i,

$$g_i^*(E) := v_i^*(E) - x_i(E). \tag{8.17}$$

The asterisk as superscript again indicates that these are equilibrium[31] values. Since participation in security trading is voluntary, naturally $g_i^*(E) \geq 0$ has to hold for all E and for all i. Plugging in the definitions of v_i^* and x_i we get

$$g_i^*(E) = \Pi_i(E)\gamma_{i\alpha}^*(E) + (1 - \Pi_i(E))\gamma_{i\beta}^*(E)$$
$$\text{where} \quad \gamma_{i\sigma}^*(E) := u_i(w_{i\sigma} + s_{i\sigma}^*(E)) - u_i(w_{i\sigma}) \quad \text{for } \sigma = \alpha, \beta.$$

Whereas $s_{i\alpha}^*(E)$ and $s_{i\beta}^*(E)$ represent the transfer made by country i between the two states expressed in terms of the numeraire, $\gamma_{i\alpha}^*(E)$ and $\gamma_{i\beta}^*(E)$ represent the same transfer expressed in terms of utility. Since $s_{i\alpha}^*(E)$ and $s_{i\beta}^*(E)$ have opposite signs by the self-financing portfolio constraint (8.2), $\gamma_{i\alpha}^*(E)$ and $\gamma_{i\beta}^*(E)$ also have opposite signs.

By (8.4) and the following decomposition

$$v_i^*(E) = x_i(E) + g_i^*(E) \tag{8.18}$$

write total utility with markets for climate securities as

$$U_i^S(e_1, \ldots, e_n) := u_i\left(w_i - C_i(e_i)\right) + x_i(E) + g_i^*(E) \tag{8.19}$$

where the superscript "S" stands for "**S**ecurities." Denote the Nash equilibrium[32] in the non-cooperative game induced by (8.19) by $e_1^S, \ldots, e_n^S$ and the corresponding equilibrium utility levels by $u_i^S := U_i^S(e_1^S, \ldots, e_n^S)$ for $i = 1, ..., n$. Before proceeding to the analysis of the induced non-cooperative game the expected gains from trade $g_i^*(E)$, the key expression in the utility functions, are discussed in detail.

8.4.2 The effect of aggregate emissions on expected third period utility

Using the decomposition of expected utility from the third period in (8.18)

$$(v_i^*)'(E) = x_i'(E) + (g_i^*)'(E).$$

[31] "Equilibrium" here refers to security market equilibrium in the second period.

[32] Given there is a unique Nash equilibrium. Existence and uniqueness are discussed in Section 8.4.3 below.

By Lemma 3 we know that the sign of $x_i'(E)$ depends on the relation between country i's endowments $w_{i\alpha}$ and $w_{i\beta}$. For the derivative of the expected gains from trade $g_i^*(E)$ we get

$$\begin{aligned}(g_i^*)'(E) =& \Pi_i'(E)\gamma_{i\alpha}^*(E) + \Pi_i(E)(\gamma_{i\alpha}^*)'(E) \\ & - \Pi_i'(E))\gamma_{i\beta}^*(E) + (1 - \Pi_i(E))(\gamma_{i\beta}^*)'(E) \\ =& \Pi_i'(E) \left[\gamma_{i\alpha}^*(E) - \gamma_{i\beta}^*(E)\right] + \Pi_i(E)u_i'(w_{i\alpha} + s_{i\alpha}^*(E))(s_{i\alpha}^*)'(E) \\ & + (1 - \Pi_i(E))u_i'(w_{i\beta} + s_{i\beta}^*(E))(s_{i\beta}^*)'(E). \end{aligned} \tag{8.20}$$

The interpretation of the expression derived for $(g_i^*)'(E)$ in (8.20) is as follows. A marginal increase in first period aggregate emissions E has two different effects.[33] The first effect stems from the change in preferences[34] and the resulting change in the evaluation of the given security market equilibrium. For a marginal increase in E, country i's probability for state α increases by $\Pi_i'(E)$ so that utility from state α contributes more to expected utility and utility from state β contributes less. This effect is represented by the term $\Pi_i'(E) \left[\gamma_{i\alpha}^*(E) - \gamma_{i\beta}^*(E)\right]$. If, for example, $\gamma_{i\alpha}^*(E) > 0$, i.e. country i buys security α in equilibrium, then the extra utility $\gamma_{i\alpha}^*(E)$ transferred to state α contributes more to expected utility when E increases. On the other hand, the transfer of utility $\gamma_{i\beta}^*(E) < 0$ away from state β counts less when E increases. Therefore, $\Pi_i'(E) \left[\gamma_{i\alpha}^*(E) - \gamma_{i\beta}^*(E)\right]$ would be the net increase in expected utility for a marginal increase of E if the security market equilibrium did not adjust to the increase in E.

However, the security market equilibrium does adjust to a marginal increase in E. Thus, the second effect arises from this adjustment of security market equilibrium to an increase in E, holding probabilities constant. In particular, equilibrium security trades react by $(s_{i\alpha}^*)'(E)$ and $(s_{i\beta}^*)'(E)$. This in turn affects marginal utility $u_i'(w_{i\alpha} + s_{i\alpha}^*(E))$ in equilibrium $(\sigma = \alpha, \beta)$. These changes in utility for each of the states from the adjustment of security market equilibrium to a marginal increase in E are weighted with the original probabilities $\Pi_i(E)$ and $(1 - \Pi_i(E))$ held constant.

Therefore, the overall effect of a marginal increase in E on the expected gains from trade $g_i^*(E)$ can be interpreted as sum of a "preference effect"

[33] Formally, these two effects stem from the application of the product rule.
[34] See Figure 8.3.

and an "equilibrium adjustment effect". Both effects can have a positive or negative sign for the general case.

Consequently, the sign of $(v_i^*)'(E)$ is also ambiguous in general. It is shown in Section 8.6 that $(v_i^*)'(E)$ often exhibits a change of sign, i.e. $v_i^*(E)$ is not necessarily monotonous, and can take on quite irregular shapes, depending on the functional specifications and parameters.

Due to the complex structure of v_i^* the analysis of the non-cooperative game for the case with securities in the following section is unlikely to yield clear-cut results.

8.4.3 Non-cooperative behaviour

Since the utility functions for the case with security markets in (8.19) only differ from those for the case without security markets (8.6) by the additional gains from trade term $g_i^*(E)$ we can draw from the analysis for the benchmark scenario in Section 8.3.3, merely substituting $x_i'(E) + (g_i^*)'(E)$ for $x_i'(E)$. Obviously, differences in the outcome compared to that of the benchmark scenario will depend on, firstly, the signs of x_i' and $(g_i^*)'$ and, secondly, their relative size. The main question is, do the expected gains from trade reinforce or reduce the basic abatement incentive represented by x_i'?

First order conditions for interior solution

Substituting $x_i'(E) + (g_i^*)'(E)$ for $x_i'(E)$ in (8.10) an interior Nash equilibrium[35] with securities is characterized by the first order conditions

$$\underbrace{C_i'(e_i)u_i'(w_i - C_i(e_i))}_{<0} = \underbrace{x_i'(E) + (g_i^*)'(E)}_{(v_i^*)'(E)} \tag{8.21}$$

for all $i = 1, \ldots, n$.

[35] In analogy to (8.12) and (8.13) there will be corner solutions if $(v_i^*)'(E)$ is relatively large or relatively small so that there is no intersection with marginal utility on the left hand side of (8.21).

Existence of Nash equilibrium

By the same argument as in Section 8.3.3 the standard conditions for existence of Nash equilibrium will be satisfied, if $(v_i^*)''(E)$ is not too large. In particular, a sufficient condition for the existence of Nash equilibrium in the game induced by (8.19) is

$$(v_i^*)''(E) = x_i''(E) + (g_i^*)''(E) < 0 \quad \text{for all } i \text{ and for all } E. \tag{8.22}$$

The examples in Section 8.6 below illustrate that inequality (8.22) is often satisfied, in particular for linear probability functions, since then $g_i^*(E)$ is inversely U-shaped. However, inequality (8.22) does not hold in general. Section 8.6 also presents examples for which (8.22) is violated. Therefore, existence of Nash equilibrium in pure strategies is not always guaranteed in the scenario with climate securities.[36]

Reaction functions

Concerning the reaction functions, the structure of $e_i'(E_{-i})$ naturally parallels that for the benchmark case in (8.14). Given that the sufficient conditions for existence of equilibrium are satisfied, i.e. $\frac{\partial^2 U_i^S(e_1,\dots,e_n)}{\partial^2 e_i} < 0$, by the same argument as in Section 8.3.3, the slope of the reaction function is determined by the sign of $(v_i^*)''(E)$. If $(v_i^*)''(E) < 0$, the reaction function is downward sloping, if $(v_i^*)''(E) > 0$, it is upward sloping. Therefore, the reaction function does not necessarily have to be monotonous.

Note that if the expected gains from trade are concave in E, i.e. $(v_i^*)''(E) < 0$ for all E, it is straightforward to show that the absolute slope of the reaction function is smaller than one. In that case existence and uniqueness of equilibrium are guaranteed.[37]

8.4.4 Linear probability functions

For linear probabilities, the expression $x_i''(E)$ drops out everywhere in the analysis above. If the expected gains from trade are a concave function of E,

[36] Of course, an equilibrium may exist, even if the sufficient conditions do not hold. This is the case for Example 6 in Section 8.6 below.

[37] See Tirole [1988], p. 226.

existence and uniqueness of Nash equilibrium are guaranteed. In contrast to the benchmark scenario there is no dominant strategy equilibrium.

8.5 The effect of markets for climate securities on non-cooperative behaviour

8.5.1 Emission levels

In Sections 8.3.3 and 8.4.3 above the non-cooperative games without and with markets for climate securities were analysed. In order to isolate the effect of markets for climate securities on emission levels, the equilibrium outcomes of these two games have to be compared. So far, the question how the equilibrium outcomes, i.e. the Nash equilibrium emissions e_i^N and e_i^S, compare with each other, still remains unanswered.

However, this question cannot be answered in general. The analytical complexity of the key difference between the two games, the expected gains from trade which were studied in Section 8.4.3, already suggested that there cannot be a general result concerning this comparison. This is indeed the case. It is shown in Section 8.6 below that the effect of markets for climate securities on equilibrium emissions is fundamentally ambiguous. The conclusions that can be drawn are summarized in Proposition 13.

Proposition 13
Equilibrium emissions e_i^S in the non-cooperative game with security markets can be either higher or lower than equilibrium emissions e_i^N in the corresponding game without security markets for all countries i. More precisely, each of the three cases

a) $e_i^S \geq e_i^N$ *for all* i,

b) $e_i^S < e_i^N$ *for all* i,

c) $e_j^S \geq e_j^N$ *for all* $j \in J$, *where* $J \subset \{1, ..., n\}$, *and*
$e_k^S < e_k^N$ *for all* $k \in \{1, ..., n\} \setminus J$

may arise.

Proof. See Examples 4 and 5 in Section 8.6 below. ■

By Proposition 13 markets for climate securities can have a wide range of effects on abatement activity. They can encourage abatement for all countries, but they can also make abatement less attractive for all countries. Even more complicated, they may have different effects on the abatement activity of different countries.

Remark 1 *Note that non-cooperative emissions with markets for climate securities are not necessarily closer to the cooperative emission levels than non-cooperative emissions without markets for climate securities. The effect is equally ambiguous as the effect on emission levels in general.*

This can easily be seen in Example 4 in Section 8.6 below.

8.5.2 Welfare

This section adresses the question how countries' utility levels are affected by markets for climate securities, given that non-cooperative behaviour reigns. Note first of all, that – given there are markets for climate securities – each country can only benefit from these markets since participation is voluntary. The question, however, is do countries benefit from markets for climate securities relative to a situation in which there are no such markets? For this aspect, equilibrium utility levels without and with security markets have to be compared. Taking into account the feedback, i.e. the effect of security markets onto the abatement decision, there may be a negative effect onto a country's utility altogether. This is confirmed by Proposition 14.

Proposition 14
Equilibrium utility levels u_i^S in the non-cooperative game with security markets can be either higher or lower than equilibrium utility levels u_i^N in the corresponding game without security markets for all countries i. In particular, equilibrium utility may be smaller for all countries in the non-cooperative game with securities than in the non-cooperative game without securities.

Proof. See Section 8.6.3 below and, in particular, Example 6. ■

Proposition 14 confirms the ambiguity of the impact of markets for climate securities. In addition to the ambiguity of their effect on equilibrium emission levels there is also ambiguity with respect to the welfare effect. Whereas the direct welfare effect of such markets cannot be negative since participation is voluntary, Proposition 14 shows that the overall effect, transmitted indirectly via the abatement decisions in the first period, can potentially make all countries worse off.[38]

Within the context of the Second Best Theorem the ambiguity of the welfare effects had to be expected. Introducing markets for climate securities removes one of the distortions, absence of insurance markets, i.e. "climate state dependent" markets. However, markets for climate securities do not eliminate the second market imperfection, the external effect of one country's emissions on all other countries' utility. Therefore, it is not surprising that by Proposition 14 welfare will not necessarily improve if only the first market imperfection is removed.

8.6 Examples

This section presents the examples, amongst others, that Propositions 13 and 14 are based on. They prove the ambiguity of the effect of climate securities with respect to emission levels and welfare.

8.6.1 Additional assumptions and functional specifications

All calculations in this section are made for the case of two countries only. Obviously, this is not a satisfactory simplification since for $n = 2$ the assumption of competitive markets for climate securities is not plausible anymore. Nevertheless, in order to prove Propositions 13 and 14 and illustrate some further points the case of two countries is sufficient.[39]

[38] Note that the indirect effect corresponds to a change in first period Nash equilibrium.

[39] Concerning some aspects we might think of the two countries here as representatives of a larger number of countries. Note that security market equilibrium would not change

For simplicity, abatement cost functions of the quadratic type

$$C_i(e_i) := \frac{(a_i - b_i e_i)^2}{2b_i} \qquad \text{with } a_i, b_i > 0 \tag{8.23}$$

are assumed. These abatement functions exhibit linear marginal abatement cost, i.e. $-C_i'(e_i) = a_i - b_i e_i$.

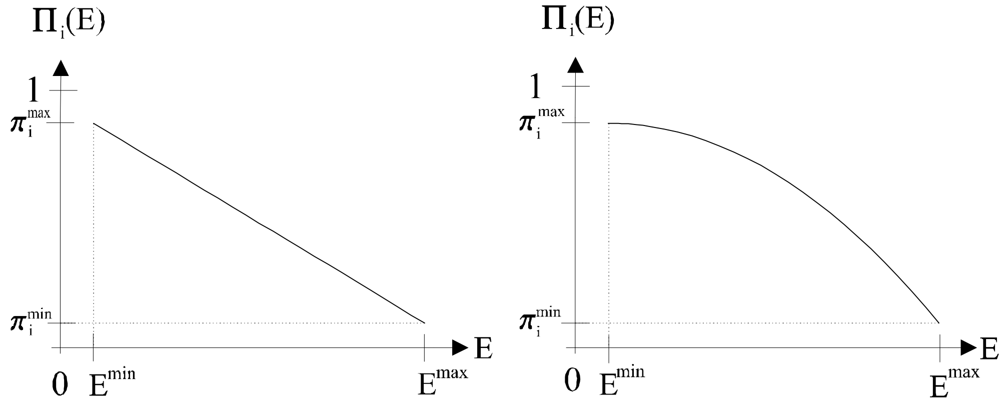

Figure 8.5: *A linear probability function (left) and a concave probability function (right).*

Probability functions of both the linear and the concave type are used. They are illustrated in Figure 8.5. Probability functions have to be defined on the range of the aggregate emissions E which is the interval $[E^{\min}, E^{\max}]$.

A linear probability function is completely characterized by its values on the boundaries of the interval $[E^{\min}, E^{\max}]$. Denote these by $\pi_i^{\max}$ and $\pi_i^{\min}$ where $\pi_i^{\max}$ is the maximum probability attributed to state α by country i (when emissions are $E^{\min}$) and $\pi_i^{\min}$ the minimum probability (when emissions are $E^{\max}$). Assume $1 \geq \pi_i^{\max} > \pi_i^{\min} \geq 0$ for all i. Note that the parameters $\pi_i^{\max}$ and $\pi_i^{\min}$ will in general not be one or zero, respectively. Formally, a linear probability function, as drawn in Figure 8.5 on the left, can be defined by

$$\Pi_i(E) = (E^{\max} - E)\left(\frac{\pi_i^{\max} - \pi_i^{\min}}{E^{\max} - E^{\min}}\right) + \pi_i^{\max} \quad i = 1, ..., n.$$

if we replicated the two-country-economy. Of course, for the strategic component, Nash equilibrium with respect to the abatement decision, replicating would make a difference.

It satisfies $\Pi_i(E^{\min}) = \pi_i^{\max}$, $\Pi_i(E^{\max}) = \pi_i^{\min}$ and $\Pi_i'(E) = -\frac{\pi_i^{\max}-\pi_i^{\min}}{E^{\max}-E^{\min}} < 0$ as well as $\Pi_i''(E) = 0$.

A canonical prototype for a non-linear, in particular concave, probability function is a quadratic polynomial, i.e.

$$\Pi_i(E) = \pi_i^{\max} - (\pi_i^{\max} - \pi_i^{\min}) \frac{\left(E - E^{\min}\right)^2}{(E^{\max} - E^{\min})^2} \qquad i = 1, ..., n. \qquad (8.24)$$

Such a probability function is depicted in Figure 8.5 on the right. It also satisfies $\Pi_i(E^{\min}) = \pi_i^{\max}$, $\Pi_i(E^{\max}) = \pi_i^{\min}$ and is strictly decreasing in E. Moreover, it is concave, i.e. $\Pi_i''(E) < 0$ "for all E."

For linear probability functions, the impact of a marginal increase in aggregate emissions on the probability for state α is the same for all aggregate emission levels. In contrast, for a concave probability function the impact of a marginal increase in aggregate emissions increases for larger values of aggregate emissions.

8.6.2 The effect of security markets on equilibrium emission levels

This section presents the examples which show that the effect of markets for climate securities on non-cooperative emission levels is fundamentally ambiguous, proving Proposition 13. For these examples, Nash emission levels are calculated with and without markets for climate securities. Comparison of the Nash emission levels for both scenarios proves that the equilibrium emissions with securities can be larger or smaller than without securities.

Example 4 considers a setting in which both countries are harmed by global warming, whereas in Example 5 the impact of global warming has different signs for the two countries, i.e. one of the countries is better off in state β and the other country is worse off in state β.

Example 4 *(Both countries adversely affected by global warming)*
Consider identical abatement cost functions with linear marginal abatement cost as defined in (8.23). Let $a_i = 10$ *and* $b_i = 1$ *for* $i = 1, 2$. *Probability functions are linear and identical with* $\pi_i^{\max} = 1$ *and* $\pi_i^{\min} = 0$ *for both* i. *Utility functions are assumed to be of the logarithmic type, i.e.* $u_i(\cdot) = \ln(\cdot)$

for $i = 1, 2$. Let the first period endowment for both countries be $w_i = 100$. Countries are also identical with respect to third period endowment in state α with $w_{1\alpha} = w_{2\alpha} = 10$.

Example 4 describes one of the simplest settings imaginable.[40] Apart from the endowment in state β which remains to be specified countries are completely identical. Nevertheless, the effect of security markets on equilibrium emission levels varies strongly, depending on how the endowments for state β are chosen. Table 8.1 lists the Nash emission levels without and with security markets for three different specifications of state β endowments for Example 4.

Example 4					
State β endowments		**Without securities**		**With securities**	
		Country 1 e_1^N	Country 2 e_2^N	Country 1 e_1^S	Country 2 e_2^S
a)	$w_{1\beta} = 2$ $w_{2\beta} = 1$	3.601	2.089	3.643	2.156
b)	$w_{1\beta} = 9$ $w_{2\beta} = 8$	9.474	8.891	9.467	8.884
c)	$w_{1\beta} = 5$ $w_{2\beta} = 2$	6.721	3.601	6.696	3.625

Table 8.1: *Nash emission levels with and without security markets for Example 4.*

Given the parameters chosen for the abatement cost functions we have $e_i^{\min} = 0$ and $e_i^{\max} = 10$ for $i = 1, 2$. Therefore, the relevant range for aggregate emissions is $[E^{\min}, E^{\max}] = [0, 20]$.

Note that by Proposition 12 a unique Nash equilibrium in dominant strategies exists for the benchmark case without securities in Example 4. For the setting with climate securities $(v_i^*)'' = (g_i^*)''$ holds by linearity of the

[40] Note that the examples are chosen as simply as possible in order to illustrate the effects. The choice of parameters is purely for illustrative purposes, they are not intended to give a realistic representation of reality.

probability functions. Since $(g_i^*)'' < 0$ holds for both countries in Example 4a), 4b) and 4c), existence and uniqueness of Nash equilibrium are guaranteed[41] for all three choices of state β endowments. Section 8.6.4 below gives a more detailed analysis of the indirect expected utility functions v_i^* and their components. In particular, g_1^* and x_1 are illustrated graphically for Example 4a).

The interpretation of the results in Table 8.1 is the following. Since the two countries only differ by their state β endowments, the country with the larger state β endowment (which is always country 1 in part a), b) and c) of Example 8.1) emits more than the other country (country 2). Country 1's abatement incentive is smaller than that of country 2 because for country 1 the difference between its endowments $w_{1\alpha}$ in state α and $w_{1\beta}$ in state β is smaller than for country 2 for all three cases.

The effect of the gap between endowment in the two states[42] is also clearly visible in the equilibrium emission levels. In Example 4a) the gap is large for both countries, leading to a relatively large abatement effort,[43] i.e. relatively low emission levels, for both countries. However, in Example 4b) the gap is rather small for both countries, and consequently abatement in equilibrium is also relatively small.

In Example 4a) equilibrium emissions for both countries are larger with securities than without, i.e. $e_i^S > e_i^N$ for $i = 1, 2$. With climate securities there is less abatement than without these instruments. However, in Example 4b) the effect is reversed. Non-cooperative equilibrium emissions with securities are smaller than without securities, i.e. markets for climate securities lead to more abatement. Formally, $e_i^N > e_i^S$ for $i = 1, 2$. Finally, in Example 4c) the effect has a different sign for each of the two countries. For country 1, equilibrium emissions increase with securities, $e_1^N < e_1^S$, whereas they decrease for country 2, $e_2^N > e_2^S$.

Summarizing the insight gained from Example 4, the effect of markets for climate securities on emission levels is strongly ambiguous. In particular, all three different cases listed in Proposition 13 arise in Example 4. Therefore,

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[41] See Section 8.4.3.

[42] The "gap between endowment in the two states" refers to the difference $w_{i\alpha} - w_{i\beta}$.

[43] Equilibrium abatement by country i is $e_i^{\max} - e_i = 10 - e_i$ for $i = 1, 2$.

Example 4 serves as proof of Proposition 13.

Note that linearity of marginal abatement cost is not essential for the ambiguity of the effect on emissions. The three different effects illustrated in Example 4 can also be generated for abatement cost functions with non-linear marginal abatement cost.

In the same sense, linearity of the probability functions is not essential either for the ambiguity of the effect of climate securities on equilibrium emissions. I.e. an increase or a decrease in emissions for both countries as in Example 4a) and 4b) can also be generated for probability functions of the concave type defined in (8.24).

Recall from Section 8.3.3 that Example 4 corresponds to a typical Prisoner's Dilemma situation since both countries are adversely affected. Therefore it is not surprising that in Example 4a), 4b) and 4c) cooperative emission levels without security markets are smaller than non-cooperative emissions both without and with climate securities, i.e. $e_i^C < e_i^N$ and $e_i^C < e_i^S$ for all i.[44] Since the effect of climate securities on emissions is ambiguous, we can conclude that climate securities do not necessarily bring non-cooperative emissions closer towards the corresponding cooperative emissions. This confirms Remark 1 in Section 8.5.1.

In general, different regions will be affected differently by global warming. In particular, there will be regions for which global warming would be beneficial. Therefore, in Example 5 below one the countries benefits from global warming whereas the other country is adversely affected.

Example 5 *(Only one of the countries adversely affected by global warming) Consider the same functional specifications and parameters as in Example 4.*

The endowments for state β are chosen in such a way in Table 8.2 below that third period endowments satisfy $w_{1\alpha} < w_{1\beta}$ and $w_{2\alpha} > w_{2\beta}$. The resulting Nash emission levels without and with security markets are shown in Table 8.2.

[44] For brevity, cooperative emission levels are not included in Table 8.1. Cooperative emissions are $e_i^C = 0.075$ in Example 4a), $e_i^C = 8.379$ in Example 4b) and $e_i^C = 2.089$ in Example 4c), $i = 1, 2$.

Example 5					
State β endowments		**Without securities**		**With securities**	
		Country 1 e_1^N	Country 2 e_2^N	Country 1 e_1^S	Country 2 e_2^S
a)	$w_{1\beta} = 10.5$ $w_{2\beta} = 5$	10	6.721	10	6.457
b)	$w_{1\beta} = 10.5$ $w_{2\beta} = 2$	10	3.601	9.990	3.336
c)	$w_{1\beta} = 10.5$ $w_{2\beta} = 1$	10	2.089	9.978	2.239

Table 8.2: *Nash emission levels with and without security markets for Example 5.*

Existence and uniqueness of Nash equilibrium are again guaranteed both without and without climate securities by the same argument as in Example 4 above. By Lemma 3, country 1 always choses $e_1^{\max} = 10$ in the benchmark scenario without security markets, since it benefits from global warming and therefore has no abatement incentive. The interesting point is how – and if at all – country 1's abatement behaviour is affected by markets for climate securities.

In Example 5a) the impact of the security markets is not large enough so that country 1 does not abate with security markets, either. Country 2's equilibrium emissions are, however, smaller with climate securities than without, i.e. $e_2^N > e_2^S$. With a larger gap between endowment in state α and β for country 2, as in Example 5b), gains from trade in climate securities become more influential. Therefore, country 1 abates – although only slightly – when there are climate securities. For country 2, again emissions with climate securities are less than without. In Example 5c) the gap between state α and β endowments is even larger for the second country. Again, markets for climate securities can induce country 1 to engage in abatement. Country 1's abatement effort is now higher than in Example 5b). However, for country 2, Nash emissions with security markets are actually higher than without security markets, $e_2^N < e_2^S$.

Therefore, the effect of markets for climate securities on Nash emissions in Example 5 is similarly ambiguous as in Example 4. The only exception is that in Example 5 country 1's equilibrium emissions cannot increase beyond $e_1^{\max} = 10$ with securities since the maximal emission level is already reached without security markets.

8.6.3 Welfare Effects

Whereas Section 8.6.2 above focused on the effect of climate securities on equilibrium emissions, this section is concerned with the effect of climate securities on equilibrium utility levels. Note that the welfare effect is positive, but very small,[45] for all three cases in both Example 4 and Example 5 above. I.e. equilibrium utility levels with securities are larger than equilibrium utility levels without securities for each country, formally $u_i^S > u_i^N$ for all i. However, Example 6 below illustrates that the welfare effects are not necessarily positive.

Example 6 *(Negative welfare effects)*
Consider the same functional specifications and parameters as in Example 4. For endowments in state β assume $w_{1\beta} = 9$ and $w_{2\beta} = 6$.

Example 6				
Nash equilibrium	**Without securities**		**With securities**	
	Country 1	Country 2	Country 1	Country 2
Emission levels	9.47393	7.99766	9.58954	8.14972
Utility levels	6.81433	6.47942	6.81402	6.47774

Table 8.3: *Equilibrium emissions and equilibrium utility with and without security markets for Example 6.*

The equilibrium emissions and the equilibrium utility levels for the non-cooperative games without and with securities are given in Table 8.3. Equilibrium utility is smaller for both countries with securities than without, i.e. $u_1^N > u_1^S$ and $u_2^N > u_2^S$. Given that they behave in a non-cooperative

[45] The exact figures have been omitted for sake of brevity.

way both countries would be better off without markets for climate securities. This proves the ambiguity of the welfare effects of markets for climate securities already emphasized in Proposition 14. Examples of parameter constellations for which the welfare effect is negative for one of the countries and positive for the other country can also be easily found.

Note that the expected gains from trade are convex for both countries, i.e. $(g_i^*)'' > 0$ for $i = 1, 2$. Therefore the standard sufficient conditions for existence of equilibrium with security markets do not hold. Nevertheless, an equilibrium does exist.[46]

In order to build up some intuition concerning the behaviour of the indirect expected utility functions which cannot be determined in general we now study these key elements in more detail.

8.6.4 The indirect utility functions and their decomposition

For the simple setting of Example 4 the structure of the functions $v_i^*(E)$ and, in particular, their two components $x_i(E)$ and $g_i^*(E)$ is not very complex. These two components are depicted in Figure 8.6 for country 1 with the specifications of Example 4a). Due to linearity of the probability function, $x_1(E)$ is also a linear function of E. Since $w_{1\alpha} > w_{1\beta}$ holds for country 1's third period endowments, $x_1(E)$ is strictly decreasing (see Lemma 3).

The shape of $g_1^*(E)$ is more interesting. It is inversely U-shaped and satisfies $g_1^*(E^{\min}) = g_1^*(E^{\max}) = 0$. Consequently, we have $(g_1^*)'(E) > 0$ for smaller values of E and $(g_1^*)'(E) < 0$ for larger E. This explains why comparative statics of the security market equilibrium with respect to E do not (and cannot) yield a sign for $(g_i^*)'(E)$ in the general case. Moreover, $(g_1^*)''(E) < 0$ holds for all E, i.e. the expected gains from trade are a concave function of E.

The properties of $g_1^*(E)$ in Example 4 can be explained as follows. First of all, note that for $E = E^{\min} = 0$ the probability attributed to state α by both countries is one, the probability attributed to state β is zero. Therefore

[46] Existence of equilibrium was checked numerically. Each of the equilibrium strategies in Table 8.3 is indeed the best answer to the other strategy.

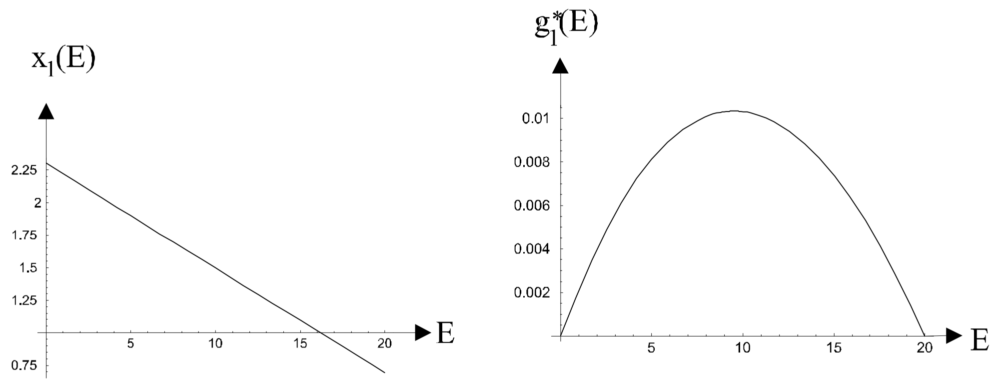

Figure 8.6: *The decomposition of the indirect expected utility function* $v_1^*(E)$ *for Example 4a). Expected utility from endowments* $x_1(E)$ *is drawn on the left and expected gains from trade* $g_1^*(E)$ *on the right. Note that the vertical axes are scaled differently in the two diagrams.*

both countries want to transfer all their state β endowment to state α.[47] Consequently, there is no trade in climate securities and no gains from trade so that $g_i^*(E^{\min}) = 0$ holds for both countries.[48] For $E > E^{\min}$ both countries have interior solutions for their security demand and trade takes place. Since countries are identical up to state β endowment, the country with the larger endowment in state β (country 1) buys security α and sells security β. The gains from trade increase as E becomes larger and then decrease again.

The shape of $v_1^*(E)$ as sum of the two functions $x_1(E)$ and $g_1^*(E)$ depends on which of two components is more dominant. Note that in Figure 8.6 the values for $g_1^*(E)$ are extremely small compared to those for $x_1(E)$. Therefore, the expected gains from trade $g_1^*(E)$ do not have a large impact and $v_1^*(E)$ is almost identical to $x_1(E)$. In particular, $(v_1^*)'(E) < 0$ holds. The second derivative $(v_1^*)''(E)$ is identical to $(g_1^*)''(E) < 0$ by linearity of the probability function. The properties of $v_i^*(E)$ and its two components are similar for country 2 and also for both countries in Examples 4b) and 4c).

The analysis of the indirect expected utility functions for Example 4 suggests that the expected gains from trade do not have a large impact. Recall, however, that according to the equilibrium emission levels in Table 8.1 they

[47]Formally, there is a corner solution for both countries' security demand.

[48]By a similar argument, $g_i^*(E^{\max}) = 0$ is explained.

can have quite a large impact.

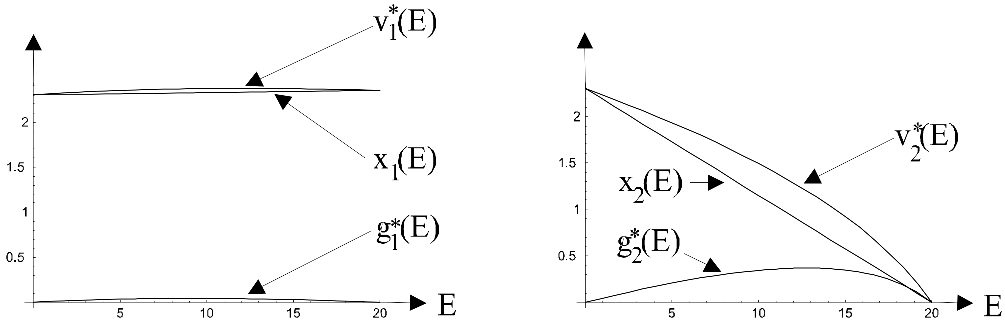

Figure 8.7: *The decomposition of the indirect expected utility functions* $v_i^*(E)$ *for country 1 (left) and country 2 (right) in Example 5c).*

There are also parameter constellations for which $v_i^*(E)$ is dominated by $g_i^*(E)$. In particular, $v_i^*(E)$ then takes on the inverse U-shape property of $g_i^*(E)$ as is the case in Example 5c) for country 1. The situation is illustrated in Figure 8.7 on the left. The expected gains from trade $g_1^*(E)$ and their derivative are relatively large in comparison to the slope of $x_1(E)$ which is almost neglegible. Therefore, $v_1^*(E)$ is also (slightly) inversely U-shaped, and in particular not monotonous, although this is hardly visible in Figure 8.7 and the derivative $(v_1^*)'(E)$ exhibits a change of sign. Again, $(v_1^*)''(E) < 0$ holds.

For country 2, the decomposition of $v_2^*(E)$ is illustrated in Figure 8.7 on the right. Although the gains from trade are also relatively large, the endowment effect in $x_2(E)$ remains dominant and $(v_2^*)'(E) < 0$ holds for all E. However, $(v_2^*)''(E) < 0$ holds once more.

Recall that Figures 8.6 and 8.7 are drawn for situations in which the two countries only differ in their state β endowment. For complexer settings, the expected gains from trade $g_i^*(E)$ also become more complicated and loose the nice properties exhibited in Figure 8.6 and 8.7.

For identical, but flatter probability functions, i.e. with $\pi_i^{\max} < 1$ and $\pi_i^{\min} > 0$, $g_i^*(E)$ is still inversely U-shaped, however the expected gains from trade are not zero on the edge of the interval $[E^{\min}, E^{\max}]$. Since probabilities cannot become one or zero,[49] there is trade even if aggregate emissions are equal to $E^{\min}$ or $E^{\max}$.

With linear but heterogeneous probability functions and all other functional specifications and parameters as in Example 4 the expected gains from trade $g_i^*(E)$ can be of quite an irregular shape. In particular, $(g_i^*)''(E) = (v_i^*)''(E) < 0$ for all E does not hold anymore since $(g_i^*)''(E)$ has a change of sign and existence of Nash equilibrium with securities is not guaranteed.

As final illustration of the structure of the indirect expected utility functions Example 7 looks at a setting with heterogeneous and non-linear probability functions.

Example 7 *(Heterogeneous concave probability functions)*
Consider the same utility functions, abatement cost functions and state α endowments as in Example 4. State β endowments are identical to those in Example 4c). Probability functions are of the concave type defined in (8.24) and the parameters are $\pi_1^{\max} = \pi_2^{\max} = 1$, $\pi_1^{\min} = 0$ and $\pi_2^{\min} = 0.4$.

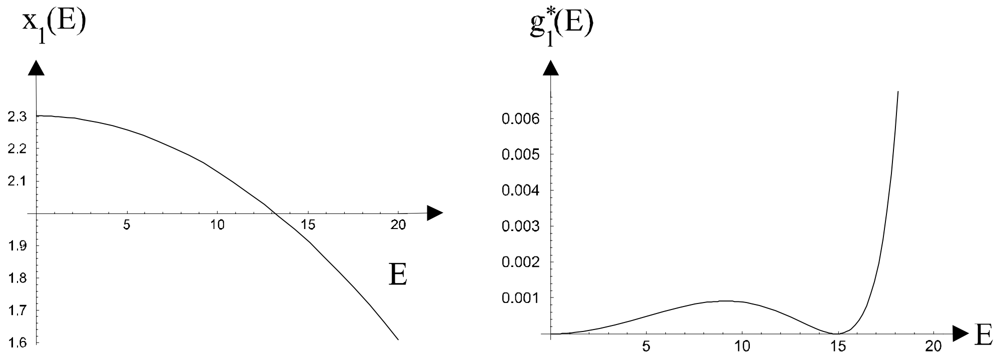

Figure 8.8: *Illustration of Example 7. Expected utility from endowments (left) and expected gains from trade (right) for country 1. Note that the vertical axes in the two diagrams are scaled differently.*

Note that in Example 7 country 2 always has a probability of at least 0.4 for state β. The decomposition of expected utility for country 1, v_1^*, is depicted in Figure 8.8. From the analysis in Section 8.3.2 we know that x_1

[49] This is excluded by assuming $\pi_i^{\max} < 1$ and $\pi_i^{\min} > 0$.

will be decreasing by Lemma 3. Moreover, by (8.8) $(x_1)''(E) < 0$ will hold for all E. This is confirmed by the diagram on the left of Figure 8.8.

The shape of g_1^* is rather irregular. We still observe $g_1^*(E^{\min}) = 0$ since for $E = E^{\min}$ both countries attribute probability one to state α and there is no trade. Since for $E = E_{\max}$ at least country 2 attributes a positive probability to state β, the expected gains from trade are larger than zero for $E = E_{\max}$. In particular, g_1^* attains its global maximum on the interval $[E^{\min}, E^{\max}]$ for $E = E^{\max}$. Within the interval $[E^{\min}, E^{\max}]$ we observe a local maximum and a local minimum. Therefore $(g_1^*)''(E) < 0$ for all E does not hold.

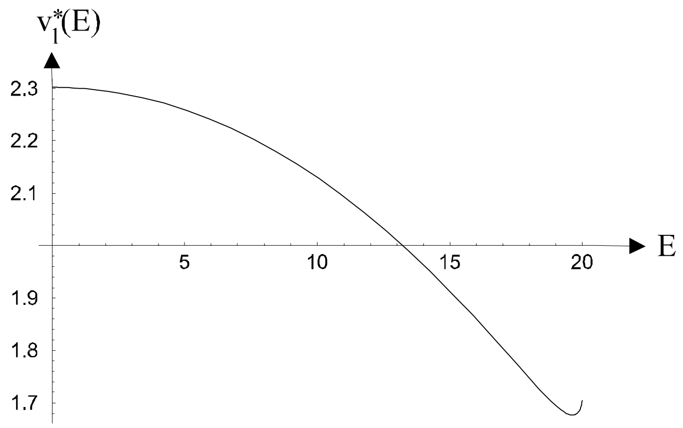

Figure 8.9: *Illustration of Example 7. The indirect expected utility function $v_1^*(E)$ for country 1.*

The indirect expected utility function $v_1^*(E)$ is drawn in Figure 8.9. The dominant component for the larger part of the relevant range for E obviously is x_1 so that v_1^* is decreasing for most values of E. However, for values of E close to $E^{\max}$ indirect expected utility increases again, i.e. in this region g_1^* is dominant. Due to the "kink" in v_1^* its second derivative exhibits a change of sign. In particular, $(v_1^*)''(E) < 0$ for all E does **not** hold[50] and existence of Nash equilibrium with securities is not guaranteed.

The analysis of the structure of the indirect expected utility function which turned out to be highly irregular in this section explains the fundamental ambiguity of the effect of markets for climate securities. In particular,

[50] Note that for non-linear probabilities $(v_i^*)''(E)$ is no longer identical to $(g_i^*)''(E)$.

it illustrates why standard approaches do not go through.

8.7 Summary

This chapter analysed the effect of markets for climate securities on mitigation of climate change when climate state probabilities are endogenous. Countries first decide on emission levels and can then trade with climate securities. In contrast to the model in Chapter 5 the abatement decision is modelled continuously. Aggregate first period emissions influence the probabilities attributed to the possible climate states when taking the portfolio decision. The endogenous probabilities generate a link between the abatement decision and the portfolio decision since countries anticipate the impact on the subsequent portfolio decision when taking their abatement decision.

Formally, the setup induces a non-cooperative game. The Nash equilibrium of this game is compared to the Nash equilibrium of the corresponding game in the absence of security markets.

A key element of the model are the expected gains from trade in climate securities. Since the expected gains from trade exhibit an irregular behaviour, a comparison of the equilibrium outcomes of the two non-cooperative games for the general case does not yield any results. However, such a comparison cannot yield any general results since it is shown that the effect of markets for climate securities is indeed fundamentally ambiguous. This main result holds true both with respect to emission levels and with respect to welfare.

Markets for climate securities can be interpreted as removal of a market imperfection relative to the situation without such instruments. However, lack of insurance markets is not the only distortion since the external effects of one country's emissions on all other countries represent a second distortion. Considered in the context of the Second Best Theorem the result is not surprising since only one of the distortions is removed by markets for climate securities.

Chapter 9

Conclusions

The main objective of the analysis in the previous chapters was to study the impact of insurance approaches to climate change on mitigation of climate change. For this purpose two different models were presented and analysed, each with a different focus. In both models, the insurance approach was represented by means of climate securities, financial instruments defined on climate states with a structure analoguous to that of Arrow Securities.

The first model concentrated on the issue of incomplete information with respect to countries' types. This model was tailored to a second objective, the formal analysis of the IPCC claim that financial structures devised for insurance purposes encourage mitigation or, equivalently, reduce free riding with respect to abatement. Within the framework of this model the effect of climate securities on abatement activity was studied by comparing scenarios with unregulated and regulated markets for climate securities, respectively, to a benchmark scenario without such financial instruments. Since the abatement decision was modelled discretely, the effect of the climate securities was mainly reflected in the changes to the conditions under which certain Bayesian equilibria emerged in the different scenarios.

The main results derived from the incomplete information framework were the following. For the scenario without regulatory intervention with respect to trade in climate securities the effects of climate securities turned out to be ambiguous. This was the case both with respect to the equilibrium regions and the level of aggregate abatement.

For the scenario with regulation of security trading, however, the effect on the equilibrium regions was not ambiguous. The preferred equilibrium appeared for a larger set of economies than for the case without climate securities. In other words, markets for climate securities produced a "shift" with respect to equilibrium regions towards the Pareto-preferred type-consistent equilibrium strategy profile. The effect on aggregate abatement remained indeterminate due to the abatement decision being modelled discretely.

Recall that in terms of the underlying intuition the IPCC claim is best approximated by the scenario with regulation of markets for climate securities, since in the context of the IPCC claim some kind of mechanism ensuring consistent behaviour is assumed implicitly. Furthermore, the less desirable equilibrium strategy profile where nobody abates can be interpreted as consequence of free riding intentions. With this interpretation the results are supportive of the IPCC claim in the sense that we observe a shift away from the "free riding" equilibrium.

However, whereas the effect on the equilibrium regions can be considered as supportive of the IPCC claim, the effect on aggregate abatement is not necessarily supportive. In particular, the level of aggregate abatement may be lower with climate securities than without. In that case, the conclusion that insurance markets encourage mitigation does not seem justified. Nevertheless, it should be emphasized that the ambiguity of the effect on aggregate abatement is due to, firstly, discrete modelling of the abatement decision, and, secondly, multiplicity of equilibria. In particular, the ambiguous effect only arises if "switches" between equilibria occur when changing scenarios. Therefore, less weight should be attributed to the effect on aggregate abatement levels than to the effect on the equilibrium regions. In particular, the conclusion that the results are supportive of the IPCC claim to a certain extent, in the sense explained above, seems justifiable.

A shortcoming of the incomplete information approach is that the analysis focused on similar countries. Although most of the findings could be generalized to the case of heterogeneous countries the design of regulation used for the trading restrictions would not be applicable to heterogeneous countries. This increases the relevance of the scenario without regulatory intervention whereas it renders the scenario with regulation irrelevant when countries are heterogeneous.

It should be emphasized that the scenario with regulation of trade in climate securities functions as a second benchmark rather than intending to represent a realistic description of reality. Obviously, it is unlikely that such a system of regulated markets for trade in climate securities could be established in reality. But this scenario is well-suited as means of capturing the intuition behind the IPCC claim within the model and for this reason it makes sense to include such a scenario in the analysis.

Evidently, the more realistic setting is the scenario with unregulated markets for climate securities. This increases the relevancy of the results for this scenario relative to that of the results for the scenario with regulation. Although instruments exactly corresponding to the climate securities in the two models do not exist at present, it seems from the analysis of similar instruments in Chapter 4 that such instruments could become reality in the near future. In particular, a large part of the required infrastructure seems to be available already. However, since the results for the scenario without regulation were ambiguous they do not permit any conclusions as to whether the creation of markets for climate securities would be desirable or not. In particular, one should keep in mind that such insurance markets will not necessarily have a beneficial effect on mitigation activity.

The second model focused on endogenity of probabilities with respect to climate states within a complete information framework. Endogenity of probabilities is an aspect that has not found much attention in the literature so far. Within that setting the abatement decision was modelled continuously. The analysis compared the outcome of non-cooperative behaviour with and without markets for climate securities, in particular with respect to emission levels and welfare levels.

For the second model the main result was the fundamental ambiguity of the effect of climate securities on abatement activity. It was shown that both the abatement effects and the welfare effects of markets for climate securities are ambiguous. It should be emphasized that the result concerning the welfare effects is not at all surprising when viewed in relation to the Second Best Theorem, since the framework without climate securities exhibits a second distortion besides lack of insurance markets. Again, the lesson to be learnt from the fundamental ambiguity of the results is that it should not be asserted that the creation of new financial markets will improve mitigation activity or welfare.

Combinedly reviewing the results from both models the main conclusion to be drawn from the analysis is that financial instruments like climate securities do not qualify as an instrument of international climate policy. In that case, it could have been proposed to include such instruments in the process of implementation of the Kyoto Protocol. Although, initially, the IPCC claim evoked the impression that such instruments could be an extremely attractive instrument of international climate policy, the ambiguity of their effect rules out this possibility. One might expect further research to resolve this ambiguity, but at least with respect to the welfare effects the Second Best Theorem leaves little hope, unless *all* inherent distortions can be eliminated which is unlikely.

Considering the recent development of a weather derivatives market as described in Chapter 4 it seems that markets for climate derivatives will develop autonomously as soon as payoffs of agents, e.g. countries or firms, depend sufficiently on the development of the climate. However, it is difficult to estimate when that might be the case. At the moment, products related to the flexible mechanisms provided for in the framework of the Kyoto Protocol are in the process of being developed.

Summarizing, one can say that the idea of enhancing the process of mitigation of climate change by means of insurance instruments serving a two-fold purpose is a very elegant proposal from an intuitive point of view. However, the formal analysis showed that the intuition given in the IPCC Report does not necessarily carry through due to the resulting ambiguities, thus substantially reducing the attractivity of the proposed policy instrument. Taking into account the fact that the proposed structure is too unrealistic further diminishes its attractivity. Therefore, the results of the analysis do not allow for a recommendation of instruments like climate securities as (supplementary) policy instrument, e.g. within the framework of the Kyoto process.

Appendix A

Proofs for Chapter 3

Proof of Lemma 1:

Step 1: Making use of elementary methods from combinatorics it can be shown that

$$|\Omega_r| = \frac{H!}{(H \cdot r_1)! \cdot \ldots \cdot (H \cdot r_\Sigma)!} \qquad \forall r \in R. \tag{A.1}$$

Step 2: For $h \in \tilde{H}, r \in R$ and $\sigma \in \tilde{\Sigma}$ define $\Omega^h_{r\sigma} := \{\omega \in \Omega_r \mid \omega(h) = \sigma\}$ as the set of all collective states in Ω_r, in which consumer h is in the individual state σ. This defines a partition of Ω_r for each h and each r.

Arguing similarly as in step 1 proves that

$$|\Omega^h_{r\sigma}| = r_\sigma \cdot |\Omega_r| \qquad \forall h \in \tilde{H}, r \in R, \forall \sigma \in \tilde{\Sigma}. \tag{A.2}$$

Step 3: The conditional probability $\Pi^h_{\sigma|r}$ can now be calculated (in terms of collective states) as follows:

$$\begin{aligned}
\Pi^h_{\sigma|r} &= \Pi^h\{\omega \in \Omega : \omega(h) = \sigma | r(\omega) = r\} \\
&= \frac{\Pi^h\{\omega \in \Omega : \omega(h) = \sigma, r(\omega) = r\}}{\Pi^h\{\omega \in \Omega : r(\omega) = r\}} \\
&= \frac{\sum_{\omega \in \Omega^h_{r\sigma}} \Pi^h_\omega}{\sum_{\omega \in \Omega_r} \Pi^h_\omega} \overset{(3.2)}{=} \frac{\sum_{\omega \in \Omega^h_{r\sigma}} \Pi^h_r}{\sum_{\omega \in \Omega_r} \Pi^h_r} = \frac{\Pi^h_r}{\Pi^h_r} \cdot \frac{|\Omega^h_{r\sigma}|}{|\Omega_r|} \overset{(A.2)}{=} \frac{r_\sigma |\Omega_r|}{|\Omega_r|} \\
&= r_\sigma,
\end{aligned}$$

thus completing the proof.

Proof of Proposition 1:

Lemma 4 *For all $\omega \in \Omega_r$ and for all $r \in R$*

$$\sum_{h=1}^{H} e_\omega^h = H \sum_{\sigma=1}^{\Sigma} r_\sigma \cdot e_\sigma = E_r. \tag{A.3}$$

I.e. total endowments of the economy are the same for all collective states leading to the same statistical state.

The proof is easy and omitted. It uses a number of suitable partitions as well as the definition of the function r_σ.

Step 1: Assume that (3.6) does *not* hold, then

$$\exists h \in \tilde{H}, r \in R, \omega_1, \omega_2 \in \Omega_r \quad \text{with} \quad z^*_{h\omega_1} \neq z^*_{h\omega_2}. \tag{A.4}$$

Starting from the equilibrium allocation z^* we now define a different feasible allocation and show that it is a Pareto improvement on z^*. The First Welfare Theorem then yields the desired contradiction.

Step 1.a: For $h \in \tilde{H}$ and $r \in R$ we will define the conditional expectation Ez^*_{hr} of the $(z^*_{h\omega})_{\omega\in\Omega}$ given r. Let $\Pi^h(\omega|r)$ denote as usual the conditional probability of ω given r for consumer h. In a similar way $\Pi^h(\omega, r)$ is the probability of the collective state being ω and the statistical state being r. By definition of Ω_r we have

$$\begin{aligned} \Pi^h(\omega|r) &= 0 \qquad \forall \omega \notin \Omega_r && \text{(A.5)}\\ \text{and} \qquad \Pi^h(\omega, r) &= \Pi^h_\omega \qquad \forall \omega \in \Omega_r && \text{(A.6)} \end{aligned}$$

for all $h \in \tilde{H}$. Note that every probability distribution Π^h on the set of collective states Ω induces an associated probability distribution $\hat{\Pi}^h$ on R ($h \in \tilde{H}$), defined by $\hat{\Pi}^h_r := \sum_{\omega\in\Omega_r} \Pi^h_\omega$ ($\forall r \in R$). Using the Anonymity Assumption (here implicit in (3.3)) we obtain[1]

$$\hat{\Pi}^h_r \overset{Def.}{=} \sum_{\omega\in\Omega_r} \Pi^h_\omega \overset{(3.3)}{=} \sum_{\omega\in\Omega_r} \Pi^h_r = |\Omega_r| \cdot \Pi^h_r. \tag{A.7}$$

[1]Note that $\hat{\Pi}^h_r \neq \Pi^h_r$ holds whenever $|\Omega_r| > 1$. Since $\Pi^h > 0$ in each component, it follows that $\hat{\Pi}^h_r > 0$ ($\forall r \in R, \forall h \in \tilde{H}$).

$$\begin{aligned} Ez^*_{hr} := \quad & \sum_{\omega\in\Omega} \Pi^h(\omega|r) z^*_{h\omega} \overset{(A.5)}{=} \sum_{\omega\in\Omega_r} \frac{\Pi^h(\omega,r)}{\hat{\Pi}^h(r)} z^*_{h\omega} \\ \overset{(A.6)}{=} \quad & \sum_{\omega\in\Omega_r} \frac{\Pi^h_\omega}{\hat{\Pi}^h_r} z^*_{h\omega} \overset{(3.3)}{=} \sum_{\omega\in\Omega_r} \frac{\Pi^h_r}{\hat{\Pi}^h_r} z^*_{h\omega} \overset{(A.7)}{=} \sum_{\omega\in\Omega_r} \frac{1}{|\Omega_r|} z^*_{h\omega} \end{aligned} \tag{A.8}$$

Define $Ez^*_h := (Ez^*_{h\,r(\omega)})_{\omega\in\Omega} \in I\!R^{L\Sigma H}_+$ and $Ez^* := (Ez^*_1, \ldots, Ez^*_H)$. The bundle $Ez^*_{h\,r(\omega)}$ gives consumer h the conditional expectation of the bundles $(z^*_{h\omega})_{\omega\in\Omega}$ – conditional on $r(\omega)$ – instead of $z^*_{h\omega}$. As Ez^*_{hr} does not depend on Π^h for all $h \in \tilde{H}$ and for all $r \in R$, Ez^* is also independent of the consumers' probability distributions.

Step 1.b: We show that Ez^* is a feasible allocation. Let $r \in R$.

$$\sum_h Ez^*_{hr} \overset{(A.8)}{=} \sum_h \sum_{\omega\in\Omega_r} \frac{1}{|\Omega_r|} z^*_{h\omega} = \sum_{\omega\in\Omega_r} \frac{1}{|\Omega_r|} \sum_h z^*_{h\omega}. \tag{A.9}$$

By assumption, z^* is an equilibrium allocation. Hence, market clearing yields $\sum_h z^*_{h\omega} = \sum_h e^h_\omega$ for all $\omega \in \Omega$. By (A.3) we have $\sum_h e^h_\omega = E_r$ for all $\omega \in \Omega_r$. Hence, $\sum_h z^*_{h\omega} = \sum_h e^h_\omega = E_r$ for all $\omega \in \Omega_r$. Substituting into (A.9) yields (for each $\omega \in \Omega_r$):

$$\sum_h Ez^*_{hr} = \sum_{\omega\in\Omega_r} \frac{1}{|\Omega_r|} E_r = E_r \cdot \frac{1}{|\Omega_r|} \sum_{\omega\in\Omega_r} 1 = E_r = \sum_h e^h_\omega. \tag{A.10}$$

Hence,

$$\begin{aligned} \sum_h Ez^*_h &= \left(\sum_h Ez^*_{hr(\omega_1)}, \ldots, \sum_h Ez^*_{hr(\omega_{\Sigma H})} \right) \\ &= \left(\sum_h e^h_{\omega_1}, \ldots, \sum_h e^h_{\omega_{\Sigma H}} \right) = \sum_h e^h. \end{aligned}$$

This completes the proof of Ez^* being a feasible allocation.

Step 1.c: We now show that Ez^* is a Pareto improvement on z^*.

(i) It is easily shown that for $h = 1, \ldots, H$:

$$[z^*_{h\omega} = z^*_{hr} \qquad \forall \omega \in \Omega_r \qquad \forall r] \iff Ez^*_h = z^*_h. \tag{A.11}$$

(ii) We now consider the following statement:

$$\left[h \quad \text{satisfies} \quad z^*_{h\omega_1} = z^*_{h\omega_2} = z^*_{hr} \quad \forall \omega_1, \omega_2 \in \Omega_r, \forall r\right] \tag{A.12}$$

Let $h \in \tilde{H}$. Then two cases are possible:

Case 1: h satisfies (A.12), which by (i) is equivalent to $Ez_h^* = z_h^*$. But then

$$U^h(Ez_h^*) = U^h(z_h^*).$$

Case 2: h does *not* satisfy (A.12), i. e.

$$\exists r \in R \quad \text{and} \quad \omega_1, \omega_2 \in \Omega_r \quad \text{with} \quad z_{h\omega_1}^* \neq z_{h\omega_2}^*. \tag{A.13}$$

To start with

$$\begin{aligned} U^h(Ez_h^*) &\overset{(3.3),(3.5)}{=} \sum_r \Pi_r^h \sum_{\omega \in \Omega_r} u^h(Ez_{hr}^*) \\ &\overset{(A.8)}{=} \sum_r \Pi_r^h \sum_{\omega \in \Omega_r} u^h\left(\sum_{\omega \in \Omega_r} \frac{1}{|\Omega_r|} z_{h\omega}^*\right). \end{aligned} \tag{A.14}$$

Since $\sum_{\omega \in \Omega_r} \frac{1}{|\Omega_r|} = 1$, the argument of u^h in (A.14) is a convex combination of the $z_{h\omega}^*$, $\omega \in \Omega_r$. In order to distinguish easily between non-trivial and trivial convex combinations we define the sets

$$\begin{aligned} R_=^h &:= \quad \{r \in R \mid z_{h\omega}^* = z_{hr}^* \quad \forall \omega \in \Omega_r\} \\ \text{and} \quad R_{\neq}^h &:= \quad \left\{r \in R \mid \exists \omega_1, \omega_2 \in \Omega_r \quad \text{with} \quad z_{h\omega_1}^* \neq z_{h\omega_2}^*\right\}. \end{aligned}$$

Then $R_=^h \dot{\cup} R_{\neq}^h = R$. For all $r \in R_=^h$ we are dealing with a trivial convex combination, i.e. $\sum_{\omega \in \Omega_r} \frac{1}{|\Omega_r|} z_{h\omega}^* = z_{hr}^*$. Consequently, for all $r \in R_=^h$

$$u^h\left(\sum_{\omega \in \Omega_r} \frac{1}{|\Omega_r|} z_{h\omega}^*\right) = u^h(z_{hr}^*) = \sum_{\omega \in \Omega_r} \frac{1}{|\Omega_r|} u^h(z_{h\omega}^*). \tag{A.15}$$

For $r \in R_{\neq}^h$ we have non-trivial convex combinations. By strict concavity of u^h

$$u^h\left(\sum_{\omega \in \Omega_r} \frac{1}{|\Omega_r|} z_{h\omega}^*\right) > \sum_{\omega \in \Omega_r} \frac{1}{|\Omega_r|} u^h(z_{h\omega}^*) \qquad \forall r \in R_{\neq}^h. \tag{A.16}$$

Since $R^h_{\neq} \neq \emptyset$ by (A.13),

$$\begin{aligned} U^h(Ez_h^*) &\overset{(A.14)}{=} \sum_r \Pi_r^h \sum_{\omega\in\Omega_r} u^h\left(\sum_{\psi\in\Omega_r}\frac{1}{|\Omega_r|}z_{h\psi}^*\right) \\ &\overset{(A.15),(A.16)}{>} \sum_r \Pi_r^h \sum_{\omega\in\Omega_r} \underbrace{\sum_{\psi\in\Omega_r}\frac{1}{|\Omega_r|}u^h(z_{h\psi}^*)}_{indep.\, of\, \omega} \\ &= \sum_r \sum_{\psi\in\Omega_r}\frac{1}{|\Omega_r|}u^h(z_{h\psi}^*)\sum_{\omega\in\Omega_r}\Pi_r^h \\ &\overset{(3.3)}{=} \sum_{\omega\in\Omega}\Pi_\omega^h u^h(z_{h\omega}^*) = U^h(z_h^*). \end{aligned}$$

iii) By (ii) we have $U^h(Ez_h^*) \geq U^h(z_h^*)$ for *all* $h \in \tilde{H}$. Due to assumption (A.4) there exists at least one h that does not satisfy (A.12). For this/these h we even have $U^h(Ez_h^*) > U^h(z_h^*)$. Therefore Ez^* is a Pareto improvement on z^*.

Step 1.d: By the First Welfare Theorem z^* is Pareto-efficient, thus there can not be an allocation that is a Pareto improvement on z^*. This gives us the desired contradiction and (3.6) must hold.

Note that (3.6) also implies that within each statistical state the equilibrium consumption of consumer h does not depend on the individual state, i.e. in contingent equilibrium we have full insurance within each statistical state. As consumers are risk-averse according to Assumption 2 this is not surprising.

Step 2: As (p^*, z^*) is a contingent market equilibrium, we know that consumers maximize utilities. Let $h \in \tilde{H}$. From the first order conditions for utility maximization of consumer h we get

$$\Pi_\omega^h \cdot \operatorname{grad} u^h(z_{h\omega}^*) = \lambda_h p_\omega^* \qquad \forall \omega \in \Omega, \tag{A.17}$$

where λ_h is the Lagrange-multiplier in the maximization problem of consumer h. Using the Anonymity Assumption equation (A.17) yields

$$\begin{aligned} p_{\omega_1}^* &= \frac{\Pi_r^h}{\lambda_h}\cdot \operatorname{grad} u^h(z_{h\omega_1}^*) \\ \text{and} \quad p_{\omega_2}^* &= \frac{\Pi_r^h}{\lambda_h}\cdot \operatorname{grad} u^h(z_{h\omega_2}^*) \overset{(3.6)}{=} \frac{\Pi_r^h}{\lambda_h}\cdot \operatorname{grad} u^h(z_{h\omega_1}^*) = p_{\omega_1}^*, \end{aligned}$$

which proves (3.7). This completes the proof.

Proof of Proposition 2:
According to Definition 1 we have to show that $(p^*, z^*, q^*, s^*, m^*)$ satisfies (3.12) to (3.15).

Step 1: We use the fact that (p^*, z^*) is a contingent market equilibrium in order to derive some useful identities.

Step 1.a: By Proposition 1

$$p^*_\omega = p^*_r \qquad \forall \omega \in \Omega_r, \forall r \tag{A.18}$$

$$\text{and} \qquad z^*_{h\omega} = z^{h*}_r \qquad \forall \omega \in \Omega_r, \forall r. \tag{A.19}$$

Step 1.b: By our assumptions, z^*_h satisfies the budget restraint of consumer h for all $h \in \tilde{H}$. Due to Proposition 1 (equations (A.18) and A.19)) as well as by the definition of $\Omega^h_{r\sigma}$ and (A.2) we can express this equation in terms of r and σ instead of ω.

$$\begin{aligned} p^*(z^*_h - e^h) &\overset{(A.18),(A.19)}{=} \sum_{r \in R} p^*_r \sum_{\omega \in \Omega_r} (z^{h*}_r - e^h_{\omega(h)}) \\ &\overset{(A.2)}{=} \sum_{r \in R} p^*_r \sum_{\sigma=1}^{\Sigma} r_\sigma |\Omega_r| (z^{h*}_r - e^h_\sigma) \end{aligned}$$

Therefore

$$p^*(z^*_h - e^h) = 0 \quad \Longleftrightarrow \quad \sum_{r \in R} p^*_r |\Omega_r| \sum_{\sigma=1}^{\Sigma} \Pi^h_{\sigma|r} (z^{h*}_r - e^h_\sigma) = 0. \tag{A.20}$$

Step 1.c: We deal with the market clearing condition in the same way.

Here – besides Proposition 1 – we use the definition of $\tilde{H}^{\omega}_{\sigma}$ and (3.1), obtaining

$$\begin{array}{rrcll}
 & \sum_h (z^*_h - e^h) & = & 0 & \\
\Longleftrightarrow & \sum_h (z^*_{h\omega} - e^h_{\omega}) & = & 0 & \forall \omega \in \Omega \\
\Longleftrightarrow & \sum_h z^*_{h\omega} - \sum_h e^h_{\omega} & = & 0 & \forall \omega \in \Omega_r, \forall r \\
\overset{(A.3),(A.19)}{\Longleftrightarrow} & \sum_h z^{h*}_r - H \sum_{\sigma} r_{\sigma} e_{\sigma} & = & 0 & \forall r \\
\Longleftrightarrow & \sum_h z^{h*}_r \underbrace{\left(\sum_{\sigma} r_{\sigma} \right)}_{=1} - \sum_h \sum_{\sigma} r_{\sigma} e_{\sigma} & = & 0 & \forall r \\
\Longleftrightarrow & \sum_{h,\sigma} r_{\sigma} (z^{h*}_r - e_{\sigma}) & = & 0 & \forall r \qquad \text{(A.21)}
\end{array}$$

Step 2: First of all we show that (3.13) is satisfied for all $h \in \tilde{H}$, i.e. we show that $(z^*_h, s^{h*}, m^{h*}) \in B^h_{SI}(p^*, q^*; e^h)$ for all $h \in \tilde{H}$.

Let $h \in \tilde{H}$. By definition of $B^h_{SI}(p^*, q^*; e^h)$ we have to show that (z^*_h, s^{h*}, m^{h*}) satisfies equations (3.9), (3.10) and (3.11) w.r.t. p^* and q^*.

Step 2.a: We begin with (3.9). It is straightforward to check that (3.9) is satisfied with respect to p^* by the definition of $m^{h*}_{\sigma r}$ in (3.18) and Proposition 1.

Step 2.b: Now we show that (3.10) holds with respect to q^*.

$$\sum_r q^*_r s^{h*}_r \overset{(3.16),(3.17)}{=} \sum_r |\Omega_r| \sum_{\sigma} p^*_r \Pi^h_{\sigma|r} (z^{h*}_r - e^h_{\sigma}) \overset{(A.20)}{=} 0$$

Step 2.c: It remains to prove that equation (3.11) also holds with respect to p^* for (z_h^*, s^{h*}, m^{h*}). Let $r \in R$. Then

$$\begin{aligned}
& \sum_{\sigma=1}^{\Sigma} \Pi^h_{\sigma|r} m^{h*}_{\sigma r} \\
\overset{(3.17),(3.18)}{=} & \sum_{\sigma=1}^{\Sigma} \Pi^h_{\sigma|r} \left[p^*_r(z^{h*}_r - e^h_\sigma) - \sum_{\tau=1}^{\Sigma} \Pi^h_{\tau|r}(p^*_r(z^{h*}_r - e^h_\tau)) \right] \\
= & \sum_{\sigma=1}^{\Sigma} \Pi^h_{\sigma|r}[p^*_r(z^{h*}_r - e^h_\sigma)] - \sum_{\sigma=1}^{\Sigma} \Pi^h_{\sigma|r} \underbrace{\left[\sum_{\tau=1}^{\Sigma} \Pi^h_{\tau|r}(p^*_r(z^{h*}_r - e^h_\tau)) \right]}_{independ.\, of\, \sigma} \\
= & \sum_{\sigma=1}^{\Sigma} \Pi^h_{\sigma|r}[p^*_r(z^{h*}_r - e^h_\sigma)] - \left[\sum_{\tau=1}^{\Sigma} \Pi^h_{\tau|r}(p^*_r(z^{h*}_r - e^h_\tau)) \right] \underbrace{\sum_{\sigma=1}^{\Sigma} \Pi^h_{\sigma|r}}_{=1} \\
= & \ 0,
\end{aligned}$$

i.e. (3.11) holds w.r.t. p^*. Thus $(z_h^*, s^{h*}, m^{h*}) \in B^h_{SI}(p^*, q^*; e^h)$ for all $h \in \tilde{H}$, i.e. (3.13) holds.

Step 3: Now we show (3.12), i.e.

$$z_h^* \in \text{argmax} \left\{ U^h(z_h) \mid z_h \in \hat{B}^h_{SI}(p^*, q^*; e^h) \right\} \qquad \forall h \in \tilde{H}.$$

Step 3.a: We begin by proving

$$\hat{B}^h_{SI}(p^*, q^*; e^h) \subseteq \bar{B}^h(p^*, e^h) := \{z_h \in I\!R^{L\Sigma H}_+ | p^*(z_h - e^h) = 0\} \quad \forall h \in \tilde{H}. \tag{A.22}$$

$\bar{B}^h(p^*, e^h)$ is the budget set of consumer h at prices p^* in the corresponding contingent market economy.

Let $h \in \tilde{H}$ and $z_h \in \hat{B}^h_{SI}(p^*, q^*; e^h)$. By definition of $\hat{B}^h_{SI}(p^*, q^*; e^h)$ there exist $s^h \in I\!R^{|R|}$ and $m^h \in I\!R^{\Sigma|R|}$, such that $(z_h, s^h, m^h) \in B^h_{SI}(p^*, q^*; e^h)$.

Consequently (z_h, s^h, m^h) satisfies (3.9) w.r.t. p^*. Summing over ω leads to

$$\begin{aligned}
\sum_{\omega\in\Omega} p^*_\omega(z_{h\omega} - e^h_\omega) &= \sum_{\omega\in\Omega} s^h_{r(\omega)} + \sum_{\omega\in\Omega} m^h_{\omega(h):r(\omega)} \\
\Longleftrightarrow \quad p^*(z_h - e^h) &= \underbrace{\sum_r \sum_{\omega\in\Omega_r} s^h_{r(\omega)}}_{=0 \text{ by } (3.16),(3.10)} + \sum_r \sum_{\omega\in\Omega_r} m^h_{\omega(h):r(\omega)} \\
\Longleftrightarrow \quad p^*(z_h - e^h) &= \sum_{r\in R} \sum_\sigma \sum_{\omega\in\Omega^h_{r\sigma}} m^h_{\sigma r} \overset{(A.2)}{=} \sum_{r\in R} |\Omega_r| \underbrace{\sum_\sigma \Pi^h_{\sigma|r} m^h_{\sigma r}}_{=0 \text{ by } (3.11)} \\
&= 0. \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (A.23)
\end{aligned}$$

So $z_h \in \bar{B}^h(p^*, e^h)$ which proves (A.22).

Step 3.b: Let $h \in \tilde{H}$. By assumption we know that

$$z^*_h = argmax \left\{ U^h(z_h) \mid z_h \in \bar{B}^h(p^*, e^h) \right\}$$

holds.[2] Since $\hat{B}^h_{SI}(p^*, q^*; e^h) \subseteq \bar{B}^h(p^*, e^h)$ and $z^*_h \in \hat{B}^h_{SI}(p^*, q^*; e^h)$, as the tuple (z^*_h, a^{h*}, m^{h*}) belongs to $B^h_{SI}(p^*, q^*; e^h)$ by Step 2, we directly get $z^*_h = \text{argmax}\left\{ U^h(z_h) \mid z_h \in \hat{B}^h_{SI}(p^*, q^*; e^h) \right\}$ which proves (3.12).[3]

Step 4: Regarding (3.14) there is nothing to show since $\sum_h (z^*_h - e_h) = 0$ continues to hold by assumption. Only (3.15) remains to be shown. Let $r \in R$. Then

$$\sum_h s^{h*}_r \overset{(3.17)}{=} \sum_h \sum_{\sigma=1}^{\Sigma} \Pi^h_{\sigma|r} p^*_r (z^{h*}_r - e^h_\sigma) \overset{(3.4)}{=} p^*_r \sum_h \sum_\sigma r_\sigma \left(z^{h*}_r - e_\sigma \right) \overset{(A.21)}{=} 0.$$

This completes the proof.

[2] By Assumption 2 there is a unique maximizer.

[3] In this case there is even a unique maximizer. See (3.12).

Appendix B

Proofs for Chapter 5

Proof of Lemma 2:
By the definition of $q^0(\theta_i, \tilde{a})$ in (5.14)

$$s^\alpha(\theta_i, q, \tilde{a}) = 0 \quad \Leftrightarrow \quad q = q^0(\theta_i, \tilde{a})$$

holds[1] for all θ_i and all $\tilde{a}$. Obviously, demand for security α is negative if and only if the security price q is larger than $q^0(\theta_i, \tilde{a})$ and positive if q is smaller than $q^0(\theta_i, \tilde{a})$. Therefore, the sign of the demand for security α is characterized by

$$s^\alpha(\theta_i, q, \tilde{a}) \begin{pmatrix} > \\ = \\ < \end{pmatrix} 0 \qquad \Leftrightarrow \qquad q \begin{pmatrix} < \\ = \\ > \end{pmatrix} q^0(\theta_i, \tilde{a}) \quad \text{for all } \theta_i, \tilde{a}. \tag{B.1}$$

Since

$$\frac{\partial q^0(\theta_i, \tilde{a})}{\partial \theta_i} = \frac{(1 - q^0(\theta_i, \tilde{a}))^2}{(1 - \theta_i)^2} \frac{u'(\bar{\omega}^\alpha)}{u'(\omega^\beta(\tilde{a}))} > 0,$$

we get[2] $q^0(\theta^A, \tilde{a}) > q^0(\theta^B, \tilde{a})$.

If $q \geq q^0(\theta^A, \tilde{a}) > q^0(\theta^B, \tilde{a})$, then by (B.1) we have

$$s^\alpha(\theta^A, q, \tilde{a}) \leq 0 \quad \text{and} \quad s^\alpha(\theta^B, q, \tilde{a}) < 0$$

[1] Note that the right hand side of the first order condition in (5.11) is strictly decreasing in s_i^α.

[2] W.l.o.g. treat θ_i as a continuous variable.

therefore q cannot be an equilibrium security price.

If $q \leq q^0(\theta^B, \tilde{a}) < q^0(\theta^A, \tilde{a})$, then (B.1) implies

$$s^\alpha(\theta^A, q, \tilde{a}) > 0 \quad \text{and} \quad s^\alpha(\theta^B, q, \tilde{a}) \geq 0$$

which cannot be an equilibrium either, proving part a) of Lemma 2. Part b) follows from (5.15) and (B.1), q.e.d.

Proof of Proposition 3:

Recall that, by definition, a strategy profile is a Bayesian equilibrium, if and only if neither type A countries nor type B countries can improve their expected[3] utility by deviating from that strategy profile.

a) We derive the equilibrium conditions for strategy profile a^1 with $a_i^1(\theta^A) = a_i^1(\theta^B) = L$ for $i = 1, ..., n$ as defined in Table 5.3. Define the type profile of all countries with the exception of country i as

$$\theta_{-i} := (\theta_1, ..., \theta_{i-1}, \theta_{i+1}, ..., \theta_n).$$

First, consider a possible deviation of country i when it is type A Given that the remaining $n-1$ countries follow strategy a^1 they always choose L, independently of their type, so that

$$\tilde{a}_{-i}(\theta_{-i}) = 0 \quad \text{for all} \quad \theta_{-i} \tag{B.2}$$

where $\tilde{a}_{-i}$ is defined analogously to $\tilde{a}$. Thus, we do not have to take expectations with respect to the type profile θ_{-i}.

Country i with $\theta_i = \theta^A$ will not deviate from $a_i^1(\theta^A) = L$ to to a strategy $\bar{a}_i^1$ with $\bar{a}_i^1(\theta^A) = H$, if and only if

$$\begin{aligned} \tilde{U}_i(\theta^A; a_i^1(\theta^A), \tilde{a}_{-i}) &\geq \tilde{U}_i(\theta^A; \bar{a}_i^1(\theta^A), \tilde{a}_{-i}) \\ \overset{(B.2)}{\Leftrightarrow} \quad \tilde{U}_i(\theta^A; L, 0) &\geq \tilde{U}_i(\theta^A; H, 0) \\ \overset{(5.19)}{\Leftrightarrow} \quad (1-\theta^A)b_0^{abs} &\geq (1-\theta^A)b_1^{abs} - c \\ \overset{(5.4)}{\Leftrightarrow} \quad c &\geq (1-\theta^A)b_1. \end{aligned} \tag{B.3}$$

[3]Expectations are taken with respect to the other countries' types conditional on a country's own type.

Similarly, country i with $\theta_i = \theta^B$ will not deviate to a strategy $\bar{a}_i^1$ with $\bar{a}_i^1(\theta^B) = H$, if and only if

$$\begin{array}{rcl} \tilde{U}_i(\theta^B; a_i^1(\theta^B), \tilde{a}_{-i}) & \geq & \tilde{U}_i(\theta^B; \bar{a}_i^1(\theta^B), \tilde{a}_{-i}) \\ \overset{(5.19),(B.2)}{\Leftrightarrow} \quad (1-\theta^B)b_0^{abs} & \geq & (1-\theta^B)b_1^{abs} - c \\ \overset{(5.4)}{\Leftrightarrow} \quad c & \geq & (1-\theta^B)b_1. \end{array} \tag{B.4}$$

Since $\theta^A > \theta^B$ implies $(1-\theta^A)b_1 < (1-\theta^B)b_1$, inequality (B.3) is implied by inequality (B.4), proving part a) of Proposition 3**.**

b) Now turn to strategy profile a^2 with $a_i^2(\theta^A) = L$ and $a_i^2(\theta^B) = H$ for $i = 1, ..., n$ as defined in Table 5.3. In contrast to part a) of the proof above, the value of $\tilde{a}_{-i}$ now depends on the type profile θ_{-i}, since strategy profile a^2 is a separating strategy profile. Therefore we have to take expectations with respect to θ_{-i}. Note that, in principle, the expectations are *contingent* on country i's own type θ_i, but due to the independency of the prior distribution the contingent probabilities are equal to the uncontingent probabilities. Therefore we can write $P(\theta_{-i})$ below instead of $P(\theta_{-i}|\theta_i = \theta^X)$ for the probability of θ_{-i} contingent on country i being type X $(X = A, B)$.

For a given type profile θ the number of type A countries amongst the $n-1$ countries $j \neq i$ is

$$n_A^{-i}(\theta_{-i}) := \{j \neq i | \theta_i = \theta^A\}.$$

Assuming that the $n-1$ countries $j \neq i$ stick to strategy a^2, the type A countries amongst these $n-1$ countries, of which there are $n_A^{-i}(\theta_{-i})$, choose L and the type B countries amongst these $n-1$ countries, of which there are $n-1-n_A^{-i}(\theta_{-i})$, choose H. Thus the number of countries abating, i.e. choosing H, amongst these $n-1$ countries is

$$\tilde{a}_{-i}(\theta_{-i}) = n - 1 - n_A^{-i}(\theta_{-i}).$$

Note that $n_A^{-i}(\theta_{-i})$ is binomially distributed with

$$P(n_A^{-i} = k) = \binom{n-1}{k} p^k (1-p)^{n-1-k} =: p_k \qquad k = 0, ..., n-1. \tag{B.5}$$

Country i with $\theta_i = \theta^A$ will not deviate from strategy a_i^2 to a strategy $\bar{a}_i^2$ with $\bar{a}_i^2(\theta^A) = H$, if and only if

$$\begin{aligned}
&\sum_{\theta_{-i}} P(\theta_{-i})\tilde{U}_i(\theta^A; a_i^2(\theta^A), \tilde{a}_{-i}(\theta_{-i})) \geq \sum_{\theta_{-i}} P(\theta_{-i})\tilde{U}_i(\theta^A; \bar{a}_i^2(\theta^A), \tilde{a}_{-i}(\theta_{-i})) \\
\overset{(5.19),(B.5)}{\Leftrightarrow} \quad &\sum_{k=0}^{n-1}(1-\theta^A)p_k b_{n-1-k}^{abs} \geq -c + \sum_{k=0}^{n-1}(1-\theta^A)p_k b_{n-1-k+1}^{abs} \\
\overset{(5.4)}{\Leftrightarrow} \quad &(1-\theta^A)\sum_{k=0}^{n-1} p_k b_{n-k} \leq c. \qquad \text{(B.6)}
\end{aligned}$$

Country i with $\theta_i = \theta^B$ will not deviate to $\bar{a}_i^2$ with $\bar{a}_i^2(\theta^B) = L$, if and only if

$$\begin{aligned}
&\sum_{\theta_{-i}} P(\theta_{-i})\tilde{U}_i(\theta^B; a_i^2(\theta^B), \tilde{a}_{-i}(\theta_{-i})) \\
&\geq \sum_{\theta_{-i}} P(\theta_{-i})\tilde{U}_i(\theta^B; \bar{a}_i^2(\theta^B), \tilde{a}_{-i}(\theta_{-i})) \\
\overset{(5.19),(B.5)}{\Leftrightarrow} \quad &(1-\theta^B)\sum_{k=0}^{n-1} p_k b_{n-k}^{abs} - c \geq (1-\theta^B)\sum_{k=0}^{n-1} p_k b_{n-k-1}^{abs} \\
\overset{(5.4)}{\Leftrightarrow} \quad &(1-\theta^B)\sum_{k=0}^{n-1} p_k b_{n-k} \geq c. \qquad \text{(B.7)}
\end{aligned}$$

Combining inequalities (B.6) and (B.7) yields part b) of Proposition 3.

c) In principle, the equilibrium conditions for strategy profiles a^3 with $a_i^3(\theta^A) = a_i^3(\theta^B) = H$ for all $i = 1, ..., n$ and a^4 with $a_i^4(\theta^A) = H$ and $a_i^4(\theta^B) = L$ for all $i = 1, ..., n$ can be derived analogously.

Assume that all countries $j \neq i$ follow strategy profile a^3. Since strategy profile a^3 is a pooling strategy the value of $\tilde{a}_{-i}$ is independent of θ_{-i} like in a) above, in particular

$$\tilde{a}_{-i}(\theta_{-i}) = n-1 \quad \text{for all} \quad \theta_{-i}. \qquad \text{(B.8)}$$

Country i will not deviate from strategy profile a^3 as type X, if and only if

$$\begin{aligned} \tilde{U}_i(\theta^X; a_i^3(\theta^X), \tilde{a}_{-i}) \geq & \tilde{U}_i(\theta^X; \bar{a}_i^3(\theta^A), \tilde{a}_{-i}) \\ \overset{(5.19),(B.8)}{\Leftrightarrow} \quad (1-\theta^X)b_n^{abs} - c \geq & (1-\theta^X)b_{n-1}^{abs} \\ \overset{(5.4)}{\Leftrightarrow} \quad (1-\theta^X)b_n \geq & c \qquad X = A, B. \end{aligned} \tag{B.9}$$

But (B.9) cannot hold for $X = A$ by assumption (5.6) so that strategy profile a^3 cannot be a Bayesian equilibrium.

Assuming that all other countries stick to strategy profile a^4, a type A country will not deviate from $a_i^4(\theta^A) = H$ to a strategy $\bar{a}_i^4$ with $\bar{a}_i^4(\theta^A) = L$, if and only if

$$\begin{aligned} \sum_{\theta_{-i}} P(\theta_{-i})\tilde{U}_i(\theta^A; a_i^4(\theta^A), \tilde{a}_{-i}(\theta_{-i})) \geq & \sum_{\theta_{-i}} P(\theta_{-i})\tilde{U}_i(\theta^A; \bar{a}_i^4(\theta^A), \tilde{a}_{-i}(\theta_{-i})) \\ \overset{(5.19)}{\Leftrightarrow} \quad -c + \sum_{k=0}^{n-1}(1-\theta^A)p_k b_{k+1}^{abs} \geq & \sum_{k=0}^{n-1}(1-\theta^A)p_k b_k^{abs} \\ \overset{(5.4)}{\Leftrightarrow} \quad (1-\theta^A)\sum_{k=0}^{n-1} p_k b_{k+1} \geq & c. \end{aligned} \tag{B.10}$$

Analogously, type B will not deviate, if and only if

$$\begin{aligned} \sum_{\theta_{-i}} P(\theta_{-i})\tilde{U}_i(\theta^B; a_i^4(\theta^A), \tilde{a}_{-i}(\theta_{-i})) \geq & \sum_{\theta_{-i}} P(\theta_{-i})\tilde{U}_i(\theta^B; \bar{a}_i^4(\theta^A), \tilde{a}_{-i}(\theta_{-i})) \\ \overset{(5.19)}{\Leftrightarrow} \quad \sum_{k=0}^{n-1}(1-\theta^B)p_k b_k^{abs} \geq & -c + \sum_{k=0}^{n-1}(1-\theta^B)p_k b_{k+1}^{abs} \\ \overset{(5.4)}{\Leftrightarrow} \quad (1-\theta^B)\sum_{k=0}^{n-1} p_k b_{k+1} \leq & c. \end{aligned} \tag{B.11}$$

However, (B.10) and (B.11) contradict each other, since $1-\theta^A < 1-\theta^B$. Therefore, strategy profile a^4 cannot be a Bayesian equilibrium, proving part c) of Proposition 3, q.e.d.

Proof of Proposition 4:

Since interim Pareto-dominance implies ex ante Pareto-dominance,[4] it is sufficient to prove that strategy profile a^2 Pareto-dominates strategy profile a^1 from the interim point of view. For this purpose we calculate the interim utilities[5] induced by a^1 and a^2. Type A's interim utility derived from strategy profile a^1 is[6]

$$\begin{aligned} U^{int}(a^1|\theta^A) &= \sum_{\theta_{-i}} P(\theta_{-i}|\theta_i = \theta^A)\tilde{U}_i(\theta^A; a_i^1(\theta^A), \tilde{a}_{-i}(a^1(\theta))) \\ &= \sum_{\theta_{-i}} P(\theta_{-i})\tilde{U}_i(\theta^A; L, 0) = \tilde{U}_i(\theta^A; L, 0) \\ &\overset{(5.19)}{=} (1-\theta^A) \underbrace{b_0^{abs}}_{=0 \text{ by } (5.3)} = 0. \end{aligned}$$

In the same way, interim utility for type B is $U^{int}(a^1|\theta^B) = 0$. For strategy profile a^2 similar calculations yield the interim utilities

$$\begin{aligned} U^{int}(a^2|\theta^A) &= (1-\theta^A)\sum_{k=0}^{n-1} p_k b_{n-1-k}^{abs} \\ \text{and} \quad U^{int}(a^2|\theta^B) &= -c + (1-\theta^B)\sum_{k=0}^{n-1} p_k b_{n-k}^{abs} \end{aligned}$$

where p_k is defined as in (B.5) above. Note that $(1-\theta^A)\sum_{k=0}^{n-1} p_k b_{n-1-k}^{abs} > 0$ by (5.3) and the definition of p_k. Comparing $U^{int}(a^2|\cdot)$ and $U^{int}(a^1|\cdot)$, we then see immediately

$$U^{int}(a^2|\theta^A) = (1-\theta^A)\sum_{k=0}^{n-1} p_k b_{n-1-k}^{abs} > 0 = U^{int}(a^1|\theta^A).$$

If strategy profile a^2 is an equilibrium, the equilibrium condition for type B

[4] See Myerson [1991], p. 485, and Holmstrom and Myerson [1983].

[5] Since countries are identical apart from the type interim utility does not depend on a country's name, only on the type.

[6] By independence of the prior distribution $P(\theta_{-i}|\theta_i = \theta^X) = P(\theta_{-i})$ again holds for $X = A, B$ and all $i = 1, ..., n$.

in (5.21) is equivalent to

$$
\begin{aligned}
(1-\theta^B)\sum_{k=0}^{n-1} p_k[b^{abs}_{n-k} - b^{abs}_{n-k-1}] &\geq c \\
\Rightarrow \quad (1-\theta^B)\sum_{k=0}^{n-1} p_k b^{abs}_{n-k} &\geq c. \qquad \text{(B.12)}
\end{aligned}
$$

Then,

$$U^{int}(a^2|\theta^B) = -c + (1-\theta^B)\sum_{k=0}^{n-1} p_k b^{abs}_{n-k} \overset{(B.12)}{\geq} 0 = U^{int}(a^1|\theta^B),$$

and strategy profile a^2 interim-dominates strategy profile a^1 whenever strategy profile a^2 is an equilibrium.

Concerning ex post-dominance, for type A we have

$$
\begin{aligned}
U^{ex\,post}(a^2|(\theta^A, \theta_{-i})) &= (1-\theta^A) b^{abs}_{n-1-n_A^{-i}} \\
&\geq 0 = U^{ex\,post}(a^1|(\theta^A, \theta_{-i})) \qquad \forall \theta_{-i}, \forall i,
\end{aligned}
$$

on the one hand, and

$$U^{ex\,post}(a^2|(\theta^B, \theta_{-i})) = -c + (1-\theta^B) b^{abs}_{n-n_A^{-i}}$$

with

$$-c + (1-\theta^B) b^{abs}_{n-n_A^{-i}} \overset{(5.4)}{<} -c + (1-\theta^B) b_{n-n_A^{-i}} \overset{(5.5)}{<} 0 = U^{ex\,post}(a^1|(\theta^A, \theta_{-i}))$$

for $n_A^{-i} = n-1$, on the other hand, so that neither of the strategy profiles a^1 and a^2 ex post-dominates the other one, q.e.d.

Proof of Proposition 5:

The proof proceeds in exactly the same way as that of Proposition 3, the only difference being that the utility functions in the absence of security markets, $\tilde{U}_i(\theta_i; a_i, \tilde{a}_{-i})$, are substituted by those for the case of unregulated security markets defined in (5.24), $U_i^{NR}(\theta_i, n_A; a_i, \tilde{a}_{-i})$. Since $n_A^{-i}(\theta_{-i})$ now enters the utility function we have to take expectations in the analysis of the equilibrium conditions for strategy profiles a^1 and a^3 as well.

a) For strategy profile a^1 we get the following equilibrium conditions. Type A will not deviate, if and only if

$$
\begin{aligned}
& \sum_{\theta_{-i}} P(\theta_{-i}) U_i^{NR}(\theta^A, n_A^{-i}(\theta_{-i}) + 1; L, 0) \\
\geq \; & \sum_{\theta_{-i}} P(\theta_{-i}) U_i^{NR}(\theta^A, n_A^{-i}(\theta_{-i}) + 1; H, 0) \\
\overset{(5.24)}{\Leftrightarrow} \; & (1-\theta^A) b_0^{abs} + \sum_{k=0}^{n-2} p_k g(\theta^A, k+1; 0) \\
\geq \; & (1-\theta^A) b_1^{abs} - c + \sum_{k=0}^{n-2} p_k g(\theta^A, k+1; 1) \\
\overset{(5.4)}{\Leftrightarrow} (1-\theta^A) b_1 + & \sum_{k=0}^{n-2} p_k [g(\theta^A, k+1; 1) - g(\theta^A, k+1; 0)] \leq c.
\end{aligned}
$$

Note that the sums only run from $k = 0$ to $k = n-2$ since for $k = n-1$ all countries are of type A so that trade does not take place.[7] Type B will not deviate, if and only if

$$
\begin{aligned}
& \sum_{\theta_{-i}} P(\theta_{-i}) U_i^{NR}(\theta^B, n_A^{-i}(\theta_{-i}); L, 0) \\
\geq \; & \sum_{\theta_{-i}} P(\theta_{-i}) U_i^{NR}(\theta^B, n_A^{-i}(\theta_{-i}); H, 0) \\
\overset{(5.24)}{\Leftrightarrow} \; & (1-\theta^B) b_0^{abs} + \sum_{k=1}^{n-1} p_k g(\theta^B, k; 0) \\
\geq \; & (1-\theta^B) b_1^{abs} - c + \sum_{k=1}^{n-1} p_k g(\theta^B, k; 1) \\
\overset{(5.4)}{\Leftrightarrow} \quad (1-\theta^B) b_1 + & \sum_{k=1}^{n-1} p_k [g(\theta^B, k, ; 1) - g(\theta^B, k; 0)] \leq c.
\end{aligned}
$$

The sum only runs from $k = 1$ to $k = n-1$ since for $k = 0$ all countries are type B and there no trade.

b) Analogously, the equilibrium conditions for strategy profile a^2 can be

[7] Cf. the definition of the utility functions in (5.24).

derived. Type A will not deviate, if and only if

$$
\begin{aligned}
& \sum_{\theta_{-i}} P(\theta_{-i}) U_i^{NR}(\theta^A, n_A^{-i}(\theta_{-i}) + 1; L, \tilde{a}_{-i}(\theta_{-i})) \\
\geq \;& \sum_{\theta_{-i}} P(\theta_{-i}) U_i^{NR}(\theta^A, n_A^{-i}(\theta_{-i}) + 1; H, \tilde{a}_{-i}(\theta_{-i})) \\
\overset{(5.24)}{\Leftrightarrow} \;& \sum_{k=0}^{n-1} p_k (1-\theta^A) b_{n-k-1}^{abs} + \sum_{k=0}^{n-2} p_k g(\theta^A, k+1; n-k-1) \\
\geq \;& \sum_{k=0}^{n-1} p_k (1-\theta^A) b_{n-k}^{abs} + \sum_{k=0}^{n-2} p_k g(\theta^A, k+1; n-k) - c \\
\overset{(5.4)}{\Leftrightarrow} \;& (1-\theta^A) \sum_{k=0}^{n-1} p_k b_{n-k} \\
& + \sum_{k=0}^{n-2} p_k \left[g(\theta^A, k+1; n-k) - g(\theta^A, k+1; n-k-1) \right] \\
\leq \;& c
\end{aligned}
$$

and type B will not deviate, if and only if

$$
\begin{aligned}
& \sum_{\theta_{-i}} P(\theta_{-i}) U_i^{NR}(\theta^B, n_A^{-i}(\theta_{-i}); H, \tilde{a}_{-i}(\theta_{-i})) \\
\geq \;& \sum_{\theta_{-i}} P(\theta_{-i}) U_i^{NR}(\theta^B, n_A^{-i}(\theta_{-i}); L, \tilde{a}_{-i}(\theta_{-i})) \\
\overset{(5.24)}{\Leftrightarrow} \;& \sum_{k=0}^{n-1} p_k (1-\theta^B) b_{n-k}^{abs} + \sum_{k=1}^{n-1} p_k g(\theta^B, k; n-k) - c \\
\geq \;& \sum_{k=0}^{n-1} p_k (1-\theta^B) b_{n-k-1}^{abs} + \sum_{k=1}^{n-1} p_k g(\theta^B, k; n-k-1) \\
\overset{(5.4)}{\Leftrightarrow} \;& (1-\theta^B) \sum_{k=0}^{n-1} p_k b_{n-k} \\
& + \sum_{k=1}^{n-1} p_k \left[g(\theta^B, k; n-k) - g(\theta^B, k; n-k-1) \right] \\
\geq \;& c.
\end{aligned}
$$

c) Similarly, type A will not deviate from strategy profile a^3, if and only if

$$\begin{aligned}
& \sum_{\theta_{-i}} P(\theta_{-i}) U_i^{NR}(\theta^A, n_A^{-i}(\theta_{-i})+1; H, n-1) \\
\geq & \sum_{\theta_{-i}} P(\theta_{-i}) U_i^{NR}(\theta^A, n_A^{-i}(\theta_{-i})+1; L, n-1) \\
\overset{(5.24)}{\Leftrightarrow} \quad & (1-\theta^A) b_n^{abs} - c + \sum_{k=0}^{n-2} p_k g(\theta^A, k+1; n) \\
\geq & (1-\theta^A) b_{n-1}^{abs} + \sum_{k=0}^{n-2} p_k g(\theta^A, k+1; n-1) \\
\overset{(5.4)}{\Leftrightarrow} \quad & (1-\theta^A) b_n \\
& + \sum_{k=0}^{n-2} p_k [g(\theta^A, k+1; n) - g(\theta^A, k+1; n-1)] \\
\geq & c.
\end{aligned}$$

For type B the resulting inequality is very similar to that for type A. The only difference is that the sum of the gains from trade expressions runs from $k = 1$ to $k = n - 1$. Therefore, type B will not deviate, if and only if

$$(1-\theta^B) b_n + \sum_{k=1}^{n-1} p_k [g(\theta^B, k, n) - g(\theta^B, k, n-1)] \geq c.$$

d) Finally, the equilibrium conditions for strategy profile a^4 are derived. Type A will not deviate, if and only if

$$\begin{aligned}
& \sum_{\theta_{-i}} P(\theta_{-i}) U_i^{NR}(\theta^A, n_A^{-i}(\theta_{-i})+1; H, \tilde{a}_{-i}(\theta_{-i})) \\
\geq & \sum_{\theta_{-i}} P(\theta_{-i}) U_i^{NR}(\theta^A, n_A^{-i}(\theta_{-i})+1; L, \tilde{a}_{-i}(\theta_{-i})) \\
\overset{(5.24)}{\Leftrightarrow} \quad & - c + \sum_{k=0}^{n-1} p_k (1-\theta^A) b_{k+1}^{abs} + \sum_{k=0}^{n-2} p_k g(\theta^A, k+1; k+1) \\
\geq & \sum_{k=0}^{n-1} p_k (1-\theta^A) b_k^{abs} + \sum_{k=0}^{n-2} p_k g(\theta^A, k+1; k)
\end{aligned}$$

$$\overset{(5.4)}{\Leftrightarrow} \quad (1-\theta^A)\sum_{k=0}^{n-1} p_k b_{k+1}$$
$$+ \quad \sum_{k=0}^{n-2} p_k \left[g(\theta^A, k+1; k+1) - g(\theta^A, k+1; k)\right] \geq c$$

and type B will not deviate, if and only if

$$\sum_{\theta_{-i}} P(\theta_{-i}) U_i^{NR}(\theta^B, n_A^{-i}(\theta_{-i}); L, \tilde{a}_{-i}(\theta_{-i}))$$
$$\geq \sum_{\theta_{-i}} P(\theta_{-i}) U_i^{NR}(\theta^B, n_A^{-i}(\theta_{-i}); H, \tilde{a}_{-i}(\theta_{-i}))$$
$$\Leftrightarrow \quad \sum_{k=0}^{n-1} p_k (1-\theta^B) b_k^{abs} + \sum_{k=1}^{n-1} p_k g(\theta^B, k; k)$$
$$\geq -c + \sum_{k=0}^{n-1} p_k [(1-\theta^B) b_{k+1}^{abs} + \sum_{k=1}^{n-1} p_k g(\theta^B, k; k+1)$$
$$\Leftrightarrow \quad (1-\theta^B)\sum_{k=0}^{n-1} p_k b_{k+1} +$$
$$\sum_{k=1}^{n-1} p_k \left[g(\theta^B, k; k+1) - g(\theta^B, k; k)\right] \leq c,$$

completing the proof of Proposition 5, q.e.d.

Proof of Proposition 6:

The proof again proceeds like that of Proposition 3, now the utility functions $U_i^R(\theta_i, n_A^R, n_B^R; a_i, \tilde{a}_{-i})$ for the case of regulated security markets, defined in (5.34), are substituted for $\tilde{U}_i(\theta_i, a_i, \tilde{a}_{-i})$.

a) First we derive the equilibrium conditions for strategy profile a^1. Note that strategy profile a^1 is a consistent strategy for type A countries, but not for type B countries.[8] Formally, $\mathcal{I}_B(\theta, a^1) = \{i|\theta_i = \theta^B\}$and $\mathcal{N}^R(\theta, a^1) = \{i|\theta_i = \theta^A\}$ for all type profiles θ so that

$$n_A^R(\theta, a^1) = n^R(\theta, a^1) = n_A(\theta, a^1) \quad \text{and} \quad n_B^R(\theta, a^1) = 0 \quad \text{for all } \theta. \tag{B.13}$$

[8] Cf. Section 5.4.2.

Therefore, if they follow strategy a^1, all type B countries are restricted with respect to security trading and do not receive any gains from trade.

Now consider country i with $\theta_i = \theta^A$. If country i chooses $a_i^1(\theta^A) = L$, it could potentially trade in securities, but there is nobody on the opposite side of the market, since all potential trading partners (the type B countries) are restricted in their trades. Consequently, i cannot benefit from gains from trade. In the definition of $U_i^R(\theta_i, n_A^R, n_B^R; a_i, \tilde{a}_{-i})$ in (5.34) this corresponds to $i \in \mathcal{N}^R(\theta, a^1)$ and $n_A^R(\theta, a^1) = n^R(\theta, a^1)$.

If country i deviated to a strategy $\bar{a}_i^1$ with $\bar{a}_i^1(\theta^A) = H$, it would exhibit inconsistent behaviour itself and would thus not be able to transact its desired trades. This case corresponds to $i \in \mathcal{I}(\theta, \bar{a}^1)$ in the definition of $U_i^R(\theta_i, n_A^R, n_B^R; a_i, \tilde{a}_{-i})$ in (5.34).

Assuming that the other $n-1$ countries play according to strategy profile a^1 we again have $\tilde{a}_{-i}(\theta_{-i}) = 0$ for all θ_{-i}. Therefore, type A will not deviate from strategy a_i^1 to a strategy $\bar{a}_i^1$ with $\bar{a}_i^1(\theta^A) = H$, if and only if

$$\begin{aligned}
& & & U_i^R(\theta^A, n_A^R(\theta, a^1), n_B^R(\theta, a^1); a_i^1(\theta^A), \tilde{a}_{-i}) \\
& & \geq\ & U_i^R(\theta^A, n_A^R(\theta, \bar{a}^1), n_B^R(\theta, \bar{a}^1); \bar{a}_i^1(\theta^A), \tilde{a}_{-i}) \\
& \overset{(B.13)}{\Leftrightarrow} & & U_i^R(\theta^A, n^R(\theta, a^1), 0; L, 0) \\
& & \geq\ & U_i^R(\theta^A, n^R(\theta, a^1) - 1, 0; H, 0) \\
\overset{(5.34)}{\Leftrightarrow} & (1-\theta^A) b_0^{abs} & \geq\ & (1-\theta^A) b_1^{abs} - c \\
\overset{(5.4)}{\Leftrightarrow} & (1-\theta^A) b_1 & \leq\ & c.
\end{aligned}$$

Now consider country i with $\theta_i = \theta^B$. If country i sticks to strategy profile a^1, it behaves inconsistently, therefore there are no gains from trade for country i (this corresponds to $i \in \mathcal{I}(\theta, a^1)$ in the definition of the utility function $U_i^R(\theta_i, n_A^R, n_B^R; a_i, \tilde{a}_{-i})$ in (5.34)). Then country i's payoff does not depend on $n_A^{-i}(\theta_{-i})$ so there are no expectations involved in calculating the payoff.

If country i deviated to $\bar{a}_i^1$ with $\bar{a}_i^1(\theta^B) = H$, it would be the only type B country allowed to trade and none of the type A countries would be restricted, i.e.

$$n_B^R(\theta, \bar{a}^1) = 1 \quad \text{and} \quad n_A^R(\theta, \bar{a}^1) = n_A(\theta). \tag{B.14}$$

Recall from the proof of Proposition 3 that $n_A^{-i}(\theta_{-i})$ is binomially distributed. Moreover, $\tilde{a}_{-i}(\theta_{-i}) = 0$ holds for all θ_{-i}. Therefore, type B will not deviate from strategy a_i^1 to a strategy $\bar{a}_i^1$ with $\bar{a}_i^1(\theta^B) = H$, if and only if

$$\begin{aligned}
& & & U_i^R(\theta^B, n_A^R(\theta, a^1), n_B^R(\theta, a^1); a_i^1(\theta^B), \tilde{a}_{-i}) \\
& & \geq \; & U_i^R(\theta^B, n_A^R(\theta, \bar{a}^1), n_B^R(\theta, \bar{a}^1); \bar{a}_i^1(\theta^B), \tilde{a}_{-i}) \\
& \overset{(B.13),(B.14)}{\Leftrightarrow} & & U_i^R(\theta^B, n_A(\theta), 0; L, 0) \\
& & \geq \; & \sum_{\theta_{-i}} P(\theta_{-i}) U_i^R(\theta^B, n_A(\theta), 1; H, 0) \\
\overset{(5.34)}{\Leftrightarrow} & (1-\theta^B) b_0^{abs} & \geq \; & (1-\theta^B) b_1^{abs} - c + \sum_{k=1}^{n-1} p_k g^R(\theta^B, k, 1; 1) \\
\overset{(5.4)}{\Leftrightarrow} & c & \geq \; & (1-\theta^B) b_1 + \sum_{k=1}^{n-1} p_k g^R(\theta^B, k, 1; 1).
\end{aligned} \tag{B.15}$$

Like in the proof of Proposition 5 above the sum in inequality (B.15) only runs from $k = 1$ to $k = n - 1$ since there is no trade for $k = 0$.

b) Note that strategy profile a^2 is consistent for both types. Therefore, none of the countries is restricted with respect to security trading, i.e.

$$n_A^R(\theta, a^2) = n_A(\theta) \qquad \text{and} \qquad n_B^R(\theta, a^2) = n_B(\theta) \quad \text{for all } \theta. \tag{B.16}$$

First, consider country i with $\theta_i = \theta^A$. If country i sticks to strategy a^2, its security trades are not restricted (this corresponds to $i \in \mathcal{N}^R(\theta, a^2)$ in the definition of the utility function U_i^R in (5.34)). However, if there are only type A countries, i.e. $n_A^{-i}(\theta_{-i}) = n - 1$, there is nobody to trade with.

If country i deviated to strategy $\bar{a}^2$ with $\bar{a}_i^2(\theta^A) = H$, it would exhibit inconsistent behaviour (this corresponds to $i \in \mathcal{I}(\theta, \bar{a}^2)$ in (5.34)) and could not trade.

Again, the expressions $n_A^{-i}(\theta_{-i})$ and $\tilde{a}_{-i}(\theta_{-i})$ are both binomially distributed with $\tilde{a}_{-i}(\theta_{-i}) = n - 1 - n_A^{-i}(\theta_{-i})$. By (B.16) type A will not deviate from strategy a^2, if and only if

$$\begin{aligned}
&\sum_{\theta_{-i}} P(\theta_{-i}) U_i^R(\theta^A, n_A^{-i}(\theta_{-i}) + 1, n_B(\theta); L, \tilde{a}_{-i}(\theta_{-i})) \\
&\geq \sum_{\theta_{-i}} P(\theta_{-i}) U_i^R(\theta^A, n_A^{-i}(\theta_{-i}), n_B(\theta); H, \tilde{a}_{-i}(\theta_{-i})) \\
\overset{(5.34)}{\Leftrightarrow}\ & (1-\theta^A) \sum_{k=0}^{n-1} p_k b_{n-k-1}^{abs} + \sum_{k=0}^{n-2} p_k g^R(\theta^A, k+1, n-k-1; n-k-1) \\
&\geq -c + (1-\theta^A) \sum_{k=0}^{n-1} p_k b_{n-k}^{abs} \\
\Leftrightarrow\ & (1-\theta^A) \sum_{k=0}^{n-1} p_k b_{n-k} \\
&\leq c + \sum_{k=0}^{n-2} p_k g^R(\theta^A, k+1, n-k-1; n-k-1).
\end{aligned}$$

Now consider country i with $\theta_i = \theta^B$. Sticking to strategy profile a^2 is consistent for type B (this corresponds to $i \in \mathcal{N}^R(\theta, a^2)$ in (5.34)). However, if $n_A^{-i}(\theta_{-i}) = 0$, there is no trading partner.

Deviating from strategy a^2 to a strategy $\bar{a}^2$ with $\bar{a}_i^2(\theta^A) = L$ means forfeiting the gains from trade (this corresponds to $i \in \mathcal{I}(\theta, \bar{a}^2)$ in (5.34)).

Therefore type B will not deviate from strategy profile a^2, if and only if

$$\begin{aligned}
&\sum_{\theta_{-i}} P(\theta_{-i}) U_i^R(\theta^B, n_A^{-i}(\theta_{-i}), n_B(\theta); H, \tilde{a}_{-i}(\theta_{-i})) \\
\geq\ &\sum_{\theta_{-i}} P(\theta_{-i}) U_i^R(\theta^B, n_A^{-i}(\theta_{-i}), n_B(\theta) - 1; L, \tilde{a}_{-i}(\theta_{-i})) \\
\overset{(5.34)}{\Leftrightarrow}\quad & -c + (1-\theta^B) \sum_{k=0}^{n-1} p_k b_{n-k}^{abs} + \sum_{k=1}^{n-1} p_k g^R(\theta^B, k, n-k; n-k) \\
\geq\ & (1-\theta^B) \sum_{k=0}^{n-1} p_k b_{n-k-1}^{abs}
\end{aligned}$$

$$\overset{(5.4)}{\Leftrightarrow} \quad (1-\theta^B)\sum_{k=0}^{n-1} p_k b_{n-k} + \sum_{k=1}^{n-1} p_k g^R(\theta^B, k, n-k; n-k) \geq c.$$

c) We now turn to strategy profile a^3 which is a consistent strategy for type B, but not for type A. Therefore, $\mathcal{N}^R(\theta, a^3) = \{i|\theta_i = \theta^B\}$ and $n_A^R(\theta, a^3) = 0$ for all θ.

If country i with $\theta_i = \theta^A$ sticks to $a_i^3(\theta^A) = H$, its trading is restricted ($i \in \mathcal{I}(\theta, a^3)$ in (5.34)).

If it deviates from strategy profile a^3 to strategy $\bar{a}_i^3$ with $\bar{a}_i^3(\theta^A) = L$, it exhibits consistent behaviour ($i \in \mathcal{N}^R(\theta, \bar{a}^3)$ in (5.34)) and $n_A^R(\theta, \bar{a}^3) = 1$ for all θ. However, $n_B^R(\theta, \bar{a}^3) = n_B(\theta)$ depends on the type profile so that expectations with respect to θ_{-i} have to be taken.

Assuming that all other countries stick to strategy profile a^3, we have $\tilde{a}_{-i}(\theta_{-i}) = n-1$ for all θ_{-i}.

Type A will not deviate from strategy a^3, if and only if

$$U_i^R(\theta^A, 0, n_B; H, n-1) \geq \sum_{\theta_{-i}} P(\theta_{-i}) U_i^R(\theta^A, 1, n_B(\theta); L, n-1)$$

$$\overset{(5.34)}{\Leftrightarrow} \quad (1-\theta^A)b_n^{abs} - c \geq (1-\theta^A)b_{n-1}^{abs} + \sum_{k=0}^{n-2} p_k g^R(\theta^A, 1, n-1-k; n-1)$$

$$\overset{(5.4)}{\Leftrightarrow} \quad (1-\theta^A)b_n \geq c + \sum_{k=0}^{n-2} p_k g^R(\theta^A, 1, n-1-k; n-1). \quad \text{(B.17)}$$

However, inequality (B.17) contradicts (5.6) since the sum on the right hand side is always larger than zero. Therefore strategy profile a^3 is ruled out as an equilibrium when security markets are regulated.

d) Finally, consider strategy profile a^4 which is inconsistent for both types. Therefore, a country can only attain gains from trade when deviating from strategy a^4. But then, assuming that all other countries stick to strategy a^4, there is nobody on the opposite side of the market so that there will be no trade. Consequently, the payoffs correspond exactly to those in the absence of security markets (cf. part c) in the proof of Proposition 3) and the resulting equilibrium conditions are the same as those in the absence of security markets, (B.10) and (B.11). Since they contradict each other (see proof of Proposition 3 above), strategy profile a^4 cannot be a Bayesian equilibrium when security markets are regulated, completing the proof of Proposition 6, q.e.d.

Proof of Proposition 7:
Naturally, the proof strongly ressembles that of Proposition 6 above. Recall from Section 5.5.2 that the utility function $U_i^R(\theta_i, n_A^R, n_B^R; a_i, \tilde{a}_{-i})$ defined in (5.34) still applies. The only difference relative to the kind of regulation in Proposition 6 is the definition of n_A^R. With the alternative regulation design

$$n_A^R(\theta, a) = n_A(\theta, a) \quad \text{holds for all } \theta, a, \tag{B.18}$$

since regulation is not directed at type A.

a) First we derive the equilibrium conditions for strategy profile a^1. Note that a^1 is an inconsistent strategy for type B countries, therefore all type B countries are subject to trading restrictions if they follow strategy a^1, i.e. $n_B^R(\theta, a^1) = 0$ holds for all θ.

   Consider country i with $\theta_i = \theta^A$. Since type A countries are not subject to regulation, country i could potentially trade in securities, no matter whether it sticks to strategy a^1 or deviates to $\bar{a}_i^1$ with $\bar{a}_i^1(\theta^A) = H$. However, assuming that all other countries follow strategy a^1 there is no trading partner, since all potential trading partners (the type B countries) are restricted in their trades. Consequently, country i does not receive any gains from trade and the equilibrium condition for type A is identical to the corresponding inequality in the absence of security markets, inequality (B.3) in the proof of Proposition 3 above, which was also the equilibrium condition when both types were subject to regulation.

Now consider country i with $\theta_i = \theta^B$. If country i sticks to strategy a^1, it behaves inconsistently, therefore there are no gains from trade ($i \in \mathcal{I}(\theta, a^1)$ in (5.34)). If country i deviated to $\bar{a}_i^1$ with $\bar{a}_i^1(\theta^B) = H$, it would be the only type B country allowed to trade and none of the type A countries would be restricted, i.e. the properties in (B.14) above apply. Therefore, n_A^R and n_B^R have the same properties as in part a) of the proof of Proposition 6 above for all θ, a. Consequently, the equilibrium condition for type B is identical to (B.15).

Note that both of the equilibrium conditions for strategy profile a^1 under the alternative regulation scheme are identical to the corresponding equilibrium conditions when both types are regulated.

b) Note that strategy a^2 is a consistent strategy for type B. Therefore, there are no restrictions on security trading when type B countries behave according to strategy a^2, i.e.

$$n_B^R(\theta, a^2) = n_B(\theta) \quad \text{for all } \theta. \tag{B.19}$$

First, consider country i with $\theta_i = \theta^A$. Assuming that the other countries follow strategy a^2, country i can trade with all type B countries, no matter whether it sticks to strategy a^2 or deviates. Only, for $n_A^{-i}(\theta_{-i}) = n-1$ there is nobody to trade with. Therefore type A will not deviate from strategy profile a^2, if and only if

$$\begin{aligned}
&\sum_{\theta_{-i}} P(\theta_{-i}) U_i^R(\theta^A, n_A^{-i}(\theta_{-i}) + 1, n_B(\theta); L, \tilde{a}_{-i}(\theta_{-i})) \\
&\geq \sum_{\theta_{-i}} P(\theta_{-i}) U_i^R(\theta^A, n_A^{-i}(\theta_{-i}) + 1, n_B(\theta); H, \tilde{a}_{-i}(\theta_{-i})) \\
\overset{(5.34)}{\Leftrightarrow}\quad &(1-\theta^A) \sum_{k=0}^{n-1} p_k b_{n-k-1}^{abs} + \sum_{k=0}^{n-2} p_k g^R(\theta^A, k+1, n-k-1; n-k-1) \\
&\geq -c + (1-\theta^A) \sum_{k=0}^{n-1} p_k b_{n-k}^{abs} + \sum_{k=0}^{n-2} p_k g^R(\theta^A, k+1, n-k-1; n-k)
\end{aligned}$$

$$\overset{(5.4)}{\Leftrightarrow} \quad (1-\theta^A)\sum_{k=0}^{n-1} p_k b_{n-k} + \sum_{k=0}^{n-2} p_k g^R(\theta^A, k+1, n-k-1; n-k)$$
$$- \sum_{k=0}^{n-2} p_k g^R(\theta^A, k+1, n-k-1; n-k-1)$$
$$\leq c. \tag{B.20}$$

Note that by definition

$$g^R(\theta_i, n_A, n_B^R; \tilde{a}) = g(\theta_i, n_A; \tilde{a}) \quad \text{for all } \theta_i \text{ and all } \tilde{a} \tag{B.21}$$

for all tuples (n_A, n_B^R) with $n_B^R = n - n_A = n_B$, i.e. whenever none of the type B countries is excluded from trading. In particular, by (B.19) this is the case in inequality (B.20) and we can write

$$\begin{aligned} g^R(\theta^A, k+1, n-k-1; n-k) &= g(\theta^A, k+1; n-k) \\ \text{and} \quad g^R(\theta^A, k+1, n-k-1; n-k-1) &= g(\theta^A, k+1; n-k-1) \end{aligned}$$

for all $k = 0, ..., n-2$. Then, by the definition of $g^A_{con}(p)$ in Section 5.3.2,

$$(B.20) \quad \Leftrightarrow \quad (1-\theta^A)\sum_{k=0}^{n-1} p_k b_{n-k} + g^A_{con}(p) \leq c. \tag{B.22}$$

Now consider country i with $\theta_i = \theta^B$. Note that the situation is exactly the same as for type B in part b) of the proof of Proposition 6 above. There, strategy a^2 was consistent for type A countries so that they were not restricted in their security trades. Here, type A countries are not restricted by definition of the regulation scheme. The result for the type B country under consideration is exactly the same, yielding

$$(1-\theta^B)\sum_{k=0}^{n-1} p_k b_{n-k} + \sum_{k=1}^{n-1} p_k g^R(\theta^B, k, n-k; n-k) \geq c$$

as equilibrium condition for type B.

c) We now turn to strategy profile a^3 which is a consistent strategy for type B so that $n_B^R(\theta, a^3) = n_B(\theta)$ for all type profiles θ. Assuming that all countries apart from the country under consideration stick to strategy profile a^3, $\tilde{a}_{-i}(\theta_{-i}) = n-1$ again holds for all θ_{-i}. Country i will not deviate as type A, if and only if

$$\begin{aligned}
& \sum_{\theta_{-i}} P(\theta_{-i}) U_i^R(\theta^A, n_A^{-i}(\theta_{-i})+1, n_B(\theta); H, n-1) \\
& \geq \sum_{\theta_{-i}} P(\theta_{-i}) U_i^R(\theta^A, n_A^{-i}(\theta_{-i})+1, n_B(\theta); L, n-1) \\
\overset{(5.34)}{\Leftrightarrow} \quad & (1-\theta^A) b_n^{abs} - c + \sum_{k=0}^{n-2} p_k g^R(\theta^A, k+1, n-1-k; n) \\
& \geq (1-\theta^A) b_{n-1}^{abs} + \sum_{k=0}^{n-2} p_k g^R(\theta^A, k+1, n-1-k; n-1) \\
\overset{(5.4)}{\Leftrightarrow} \quad & (1-\theta^A) b_n + \sum_{k=0}^{n-2} p_k g^R(\theta^A, k+1, n-1-k; n) \\
& - \sum_{k=0}^{n-2} p_k g^R(\theta^A, k+1, n-1-k; n-1) \geq c
\end{aligned}$$

$$\overset{(B.21)}{\Leftrightarrow} \quad (1-\theta^A) b_n + g^A_{joint}(p) \geq c. \tag{B.23}$$

Now consider country i with $\theta_i = \theta^B$. Deviating to a strategy $\bar{a}^3$ with $\bar{a}_i^3(\theta^B) = L$ corresponds to inconsistent behaviour ($i \in \mathcal{I}(\theta, \bar{a}^3)$ in (5.34)). Type B will not deviate from strategy a^3, if and only if

$$\begin{aligned}
& \sum_{\theta_{-i}} P(\theta_{-i}) U_i^R(\theta^B, n_A^{-i}(\theta_{-i}), n_B^{-i}(\theta_{-i})+1; H, n-1) \\
\geq \; & \sum_{\theta_{-i}} P(\theta_{-i}) U_i^R(\theta^B, n_A^{-i}(\theta_{-i}), n_B^{-i}(\theta); L, n-1) \\
\overset{(5.34)}{\Leftrightarrow} \; & (1-\theta^B) b_n^{abs} - c + \sum_{k=0}^{n-1} p_k g^R(\theta^B, k, n-k; n) \\
\geq \; & (1-\theta^B) b_{n-1}^{abs} \\
\overset{(5.4)}{\Leftrightarrow} \; & (1-\theta^B) b_n + \sum_{k=1}^{n-1} p_k g^R(\theta^B, k, n-k; n) \\
\geq \; & c.
\end{aligned} \tag{B.24}$$

d) Finally, consider strategy profile a^4 which is an inconsistent strategy for type B, i.e. $n_B^R(\theta, a^4) = 0$ for all type profiles θ. First consider country i of type A. It is not restricted in its trading, no matter whether it sticks to strategy a^4 or whether it deviates. However, since all type B countries are restricted in their trading no trade will take place, no matter whether country i sticks to strategy a^4 or whether it deviates. Therefore, the equilibrium condition is identical to the corresponding inequality in the absence of security markets, (B.10) in part c) of the proof of Proposition 3 above,

$$(1-\theta^A)\sum_{k=0}^{n-1} p_k b_{k+1} \geq c. \tag{B.25}$$

Now consider country i with $\theta_i = \theta^B$. If country i sticks to strategy a^4, its trading activities are restricted ($i \in \mathcal{I}(\theta, a^4)$ in (5.34)). If country i deviates to a strategy $\bar{a}_i^4$ with $\bar{a}_i^4(\theta^B) = H$ it can only trade unrestrictedly. Type B will deviate from strategy profile a^4, if and only if

$$\begin{aligned}
&\sum_{\theta_{-i}} P(\theta_{-i}) U_i^R(\theta^B, n_A^{-i}(\theta_{-i}), 0; L, \tilde{a}_{-i}) \\
\geq &\sum_{\theta_{-i}} P(\theta_{-i}) U_i^R(\theta^B, n_A^{-i}(\theta_{-i}), 1; H, \tilde{a}_{-i}) \\
\overset{(5.34)}{\Leftrightarrow} \quad &(1-\theta^B)\sum_{k=0}^{n-1} p_k b_k^{abs} \\
\geq &-c + (1-\theta^B)\sum_{k=0}^{n-1} p_k b_{k+1}^{abs} + \sum_{k=1}^{n-1} p_k g^R(\theta^B, k, 1; k+1) \\
\overset{(5.4)}{\Leftrightarrow} \quad &(1-\theta^B)\sum_{k=0}^{n-1} p_k b_{k+1} + \underbrace{\sum_{k=1}^{n-1} p_k g^R(\theta^B, k, 1; k+1)}_{=:r_{inc}^B} \\
\leq &c.
\end{aligned} \tag{B.26}$$

Recall that $r_{inc}^B \geq 0$ by definition. Therefore $\theta^A > \theta^B$ implies

$$(1-\theta^A)\sum_{k=0}^{n-1} p_k b_{k+1} < (1-\theta^B)\sum_{k=0}^{n-1} p_k b_{k+1} \leq (1-\theta^B)\sum_{k=0}^{n-1} p_k b_{k+1} + r_{inc}^B,$$

contradicting (B.25) and (B.26). Thus, strategy profile a^4 cannot be an equilibrium, completing the proof of Proposition 7.

Appendix C

Detailed Results and Proofs for Chapter 6

Proof of Corollary 2:

1) The equilibrium condition with respect to strategy profile a^1 for type A, (6.6), is equivalent to

$$p\bar{g}^A_{uni} \geq (1-\theta^A)b_1 - c + \bar{g}^A_{uni}. \tag{C.1}$$

If $\bar{g}^A_{uni}$ is small, i.e. $\bar{g}^A_{uni} \leq c-(1-\theta^A)b_1$, then the right hand side of (C.1) is not larger than zero.[1] In the case of $0 \leq \bar{g}^A_{uni} \leq c-(1-\theta^A)b_1$, (C.1) is clearly satisfied for all values of p. For $\bar{g}^A_{uni} < 0 < c-(1-\theta^A)b_1$, (C.1) is equivalent to

$$p \leq \frac{1}{\bar{g}^A_{uni}}\left[(1-\theta^A)b_1 - c + \bar{g}^A_{uni}\right], \tag{C.2}$$

but inequality (C.2) is not binding, since the right hand side of (C.2) can be shown to be larger than 1 due to assumption (5.5). Therefore, if $\bar{g}^A_{uni} \leq c-(1-\theta^A)b_1$, the equilibrium condition for type A, (6.6), holds for all priors p.

If $\bar{g}^A_{uni}$ is large, i.e. $\bar{g}^A_{uni} > c-(1-\theta^A)b_1$, then the right hand side of (C.1) is larger than zero. In particular, $\bar{g}^A_{uni} > 0$ holds and (C.1) is

[1] Recall that $c-(1-\theta^A)b_1 > 0$ by assumption (5.5).

equivalent to

$$p \geq \frac{1}{\bar{g}^A_{uni}} \left[(1-\theta^A)b_1 - c + \bar{g}^A_{uni}\right] = p^{NR}_{1A}$$

with $0 < p^{NR}_{1A} < 1$ under the relevant assumptions.[2]

Summarizing, inequality (6.6) holds

$$\begin{cases} \text{for all priors } p \in [0,1], \text{ if } \bar{g}^A_{uni} \leq c - (1-\theta^A)b_1 \text{ and} \\ \text{for all priors } p \in [p^{NR}_{1A}, 1], \text{ if } \bar{g}^A_{uni} > c - (1-\theta^A)b_1. \end{cases} \tag{C.3}$$

2) The equilibrium condition for type B, (6.7), is equivalent to

$$p\bar{g}^B_{uni} \leq c - (1-\theta^B)b_1. \tag{C.4}$$

Note that the right hand side of inequality (C.4) is larger than zero by assumption (5.5). If $\bar{g}^B_{uni} \leq 0$ holds, then inequality (C.4) is satisfied for all priors p.

If $\bar{g}^B_{uni} > 0$, then inequality (C.4) is equivalent to

$$p \leq \frac{1}{\bar{g}^B_{uni}} \left[c - (1-\theta^B)b_1\right] = p^{NR}_{1B}. \tag{C.5}$$

Note that then $p^{NR}_{1B} > 0$ by assumption (5.5). Since

$$p^{NR}_{1B} < 1 \quad \Leftrightarrow \quad c - (1-\theta^B)b_1 < \bar{g}^B_{uni},$$

inequality (C.5) is only binding, if $\bar{g}^B_{uni} > c - (1-\theta^B)b_1$, else inequality (C.4) holds for all values of p.

Summarizing, inequality (6.7) holds

$$\begin{cases} \text{for all priors } p \in [0,1], \text{ if } \bar{g}^B_{uni} \leq c - (1-\theta^B)b_1 \text{ and} \\ \text{for all priors } p \in [0, p^{NR}_{1B}], \text{ if } \bar{g}^B_{uni} > c - (1-\theta^B)b_1. \end{cases} \tag{C.6}$$

In order for strategy profile a^1 to be an equilibrium both (6.6) and (6.7) have to hold. Therefore the equilibrium region for strategy profile a^1 is the

[2] Since $\bar{g}^A_{uni} > 0$ and $(1-\theta^A)b_1 - c + \bar{g}^A_{uni} > 0$, it is obvious that $p^{NR}_{1A} > 0$ holds. Moreover, $p^{NR}_{1A} < 1$ is equivalent to $(1-\theta^A)b_1 < c$ which holds by assumption (5.5).

intersection of the sets of priors satisfying the equilibrium condition for type A and type B, respectively. Depending on which of the two possible cases holds in (C.3) and (C.6), respectively, the four different equilibrium regions listed in Corollary 2 may arise, q.e.d.

Corollary 6

Equilibrium regions for a^2 ***with unregulated securities***		
	$\bar{g}^B_{uni} \geq c-(1-\theta^B)b_1$	$\bar{g}^B_{uni} < c-(1-\theta^B)b_1$
$\bar{g}^A_{join} \leq c-(1-\theta^A)b_2$	***a)*** $[0,1]$	***b)*** $[0, p^{NR}_{2B}]$
$\bar{g}^A_{join} > c-(1-\theta^A)b_2$	***c)*** $[p^{NR}_{2A}, 1]$	***d)*** $[p^{NR}_{2A}, p^{NR}_{2B}]$

where the parameters are defined as

$$p^{NR}_{2A} := \frac{(1-\theta^A)b_2 - c + \bar{g}^A_{join}}{(1-\theta^A)(b_2-b_1) + \bar{g}^A_{join}} \tag{C.7}$$

$$\textit{and} \quad p^{NR}_{2B} := \frac{(1-\theta^B)b_2 - c}{(1-\theta^B)(b_2-b_1) - \bar{g}^B_{uni}}. \tag{C.8}$$

Proof:

1) The equilibrium condition with respect to strategy profile a^2 for type A, (6.8), is equivalent to

$$p\left[(1-\theta^A)(b_1-b_2) - \bar{g}^A_{join}\right] \leq c-(1-\theta^A)b_2 - \bar{g}^A_{join}. \tag{C.9}$$

If the left hand side of inequality (C.9) is larger than zero, i.e. $\bar{g}^A_{join} < (1-\theta^A)(b_1-b_2)$, inequality (C.9) is equivalent to

$$p \leq \frac{c-(1-\theta^A)b_2 - \bar{g}^A_{join}}{(1-\theta^A)(b_1-b_2) - \bar{g}^A_{join}} = p^{NR}_{2A}. \tag{C.10}$$

Note that

$$(1-\theta^A)(b_1-b_2) = (1-\theta^A)b_1 - (1-\theta^A)b_2 \overset{(5.5)}{<} c-(1-\theta^A)b_2. \tag{C.11}$$

Therefore $\bar{g}^A_{join} < (1-\theta^A)(b_1-b_2)$ implies $\bar{g}^A_{join} < c-(1-\theta^A)b_2$ so that both the numerator and the denominator of p^{NR}_{2A} are larger than zero, implying $p_{2A}^{NR} > 0$. However, $c > (1-\theta^A)b_1$, which holds by assumption (5.5), implies $p_{2A}^{NR} > 1$, so that inequality (C.10) is not binding. Thus, the equilibrium condition (6.8) holds for all priors p.

If $\bar{g}_{join}^A = (1-\theta^A)(b_1-b_2) \overset{(C.11)}{<} c-(1-\theta^A)b_2$, the left hand side of inequality (C.9) is zero, the right hand side larger than zero so that (C.9) also holds for all priors.

If the left hand side of (C.9) is smaller than zero, i.e. $\bar{g}_{join}^A > (1-\theta^A)(b_1-b_2)$, inequality (C.9) is equivalent to $p \geq p_{2A}^{NR}$. However, the inequality $p \geq p_{2A}^{NR}$ is only binding, i.e. $p_{2A}^{NR} > 0$, if $c-(1-\theta^A)b_2 < \bar{g}_{join}^A$ holds. Otherwise p_{2A}^{NR} is smaller than or equal to zero so that $p \geq p_{2A}^{NR}$ is not binding and (C.9) is satisfied for all priors p.

Note that $(1-\theta^A)(b_1-b_2) < 0$ by assumptions (5.5) and (6.3) and $0 < c-(1-\theta^A)b_2$ by assumption (5.6), implying $(1-\theta^A)(b_1-b_2) < c-(1-\theta^A)b_2$. Therefore, we can summarize as follows. The equilibrium condition (6.8) for type A holds

$$\begin{cases} \text{for all } p \in [0,1], \text{ if } \bar{g}_{join}^A \leq c-(1-\theta^A)b_2 \\ \text{for all } p \in [p_{2A}^{NR},1], \text{ if } \bar{g}_{join}^A > c-(1-\theta^A)b_2 \end{cases}. \tag{C.12}$$

2) The equilibrium condition with respect to strategy profile a^2 for type B, (6.9), is equivalent to

$$p\left[(1-\theta^B)(b_1-b_2)+\bar{g}_{uni}^B\right] \geq c-(1-\theta^B)b_2. \tag{C.13}$$

By assumption (6.3) the right hand side of inequality (C.13) is smaller than zero. Therefore, if the left hand side of inequality (C.13) is larger than or equal to zero, i.e. $\bar{g}_{uni}^B \geq (1-\theta^B)(b_2-b_1)$, inequality (C.13) is satisfied for all values of the prior p.

If the left hand side is smaller than zero as well, i.e. $\bar{g}_{uni}^B < (1-\theta^B)(b_2-b_1)$, inequality (C.13) is equivalent to

$$p \leq \frac{c-(1-\theta^B)b_2}{(1-\theta^B)(b_1-b_2)+\bar{g}_{uni}^B} = p_{2B}^{NR}$$

with $p_{2B}^{NR} > 0$. The inequality $p \leq p_{2B}^{NR}$ is binding, if

$$\begin{aligned} p_{2B}^{NR} < 1 \quad &\Leftrightarrow \quad c - (1-\theta^B)b_2 > (1-\theta^B)(b_1 - b_2) + \bar{g}_{uni}^B \\ &\Leftrightarrow \quad \bar{g}_{uni}^B < c - (1-\theta^B)b_1. \end{aligned}$$

Since assumption (6.3) implies

$$c - (1-\theta^B)b_1 < (1-\theta^B)(b_2 - b_1),$$

we can summarize as follows: the equilibrium condition (6.9) for type B holds

$$\begin{cases} \text{for all } p \in [0,1], \text{ if } \bar{g}_{uni}^B \geq c - (1-\theta^B)b_1 \\ \text{for all } p \in [0, p_{2B}^{NR}], \text{ if } \bar{g}_{uni}^B < c - (1-\theta^B)b_1 \end{cases}. \tag{C.14}$$

In analogy to the proof of Corollary 2 the equilibrium conditions for both types, (6.8) and (6.9), have to hold in order for strategy profile a^2 to be an equilibrium. Combining the conditions given in (C.12) and (C.14) yields the equilibrium regions for strategy profile a^2 as the intersection of the sets of priors satisfying each of the equilibrium conditions, q.e.d.

Proof of Corollary 3:

Recall that

$$\begin{aligned} p_{2B} &= \frac{(1-\theta^B)b_2 - c}{(1-\theta^B)(b_2 - b_1)}, \text{ as defined in Corollary 1} \\ \text{and} \quad p_{2B}^{NR} &= \frac{(1-\theta^B)b_2 - c}{(1-\theta^B)(b_2 - b_1) - \bar{g}_{uni}^B}, \text{ as defined in Corollary 6.} \end{aligned}$$

The numerators of p_{2B} and p_{2B}^{NR} are identical. Therefore we get

$$\begin{aligned} \bar{g}_{uni}^B < 0 \quad &\Leftrightarrow \quad (1-\theta^B)(b_2 - b_1) < (1-\theta^B)(b_2 - b_1) \underbrace{-\bar{g}_{uni}^B}_{>0} \\ &\Leftrightarrow \quad p_{2B} > p_{2B}^{NR} \end{aligned}$$

and, for the relevant case[3] of $\bar{g}_{uni}^B < c - (1-\theta^B)b_1$,

$$\begin{aligned} 0 < \bar{g}_{uni}^B \quad &\Leftrightarrow \quad (1-\theta^B)(b_2 - b_1) > (1-\theta^B)(b_2 - b_1) \underbrace{-\bar{g}_{uni}^B}_{<0} \\ &\Leftrightarrow \quad p_{2B} < p_{2B}^{NR}, \end{aligned}$$

[3] Note that p_{2B}^{NR} only appears for $\bar{g}_{uni}^B < c - (1-\theta^B)b_1$. Cf. Corollary 6 above.

q.e.d.

Corollary 7

Equilibrium regions for a^3 *with unregulated securities*		
	$\bar{g}^B_{join} \geq c - (1-\theta^B)b_2$	$\bar{g}^B_{join} < c - (1-\theta^B)b_2$
$\bar{g}^A_{join} > c - (1-\theta^A)b_2$	***a)*** $[0, p^{NR}_{3A}]$	***b)*** $[0, \min\{p^{NR}_{3A}, p^{NR}_{3B}\}]$
$\bar{g}^A_{join} \leq c - (1-\theta^A)b_2$	***c)*** $\emptyset$	***d)*** $\emptyset$

where the parameters are defined as

$$p^{NR}_{3A} := \frac{(1-\theta^A)b_2 - c + \bar{g}^A_{join}}{\bar{g}^A_{join}} \tag{C.15}$$

$$\textit{and} \quad p^{NR}_{3B} := \frac{c - (1-\theta^B)b_2}{\bar{g}^B_{join}}. \tag{C.16}$$

Proof:

1) The equilibrium condition with respect to strategy profile a^3 for type A, (6.10), is equivalent to

$$p\bar{g}^A_{join} \leq (1-\theta^A)b_2 - c + \bar{g}^A_{join}. \tag{C.17}$$

For $\bar{g}^A_{join} > 0$ inequality (C.17) is equivalent to

$$p \leq \frac{(1-\theta^A)b_2 - c + \bar{g}^A_{join}}{\bar{g}^A_{join}} = p^{NR}_{3A}.$$

Note that $p^{NR}_{3A} < 1$ since $(1-\theta^A)b_2 < c$ holds by assumption (5.6). Furthermore

$$\begin{aligned} p^{NR}_{3A} > 0 \quad &\Leftrightarrow \quad (1-\theta^A)b_2 - c + \bar{g}^A_{join} > 0 \\ &\Leftrightarrow \quad \bar{g}^A_{join} > c - (1-\theta^A)b_2. \end{aligned}$$

For $\bar{g}^A_{join} = 0$ there is no prior satisfying (C.17). For $\bar{g}^A_{join} < 0$ inequality (C.17) is equivalent to $p \geq p^{NR}_{3A}$. Then, however, $p^{NR}_{3A} > 1$ holds due to $(1-\theta^A)b_2 < c$ so that there exists no prior satisfying $p \geq p_{3A}^{NR}$ or equivalently (C.17).

Summarizing, since $c-(1-\theta^A)b_2 > 0$, the equilibrium condition (6.10) for type A holds

$$\begin{cases} \text{for all } p \in [0, p_{3A}^{NR}], \text{ if } \bar{g}_{join}^A > c-(1-\theta^A)b_2 \\ \text{for } \textbf{no } p, \text{ if } \bar{g}_{join}^A \geq c-(1-\theta^A)b_2 \end{cases}. \tag{C.18}$$

2) The equilibrium condition with respect to strategy profile a^3 for type B, (6.11), is equivalent to

$$p\bar{g}_{join}^B \geq c-(1-\theta^B)b_2. \tag{C.19}$$

If $\bar{g}_{join}^B \geq 0$, then inequality (C.19) holds for all priors p since the right hand side is smaller than zero by (6.3). If $\bar{g}_{join}^B < 0$, then inequality (C.19) is equivalent to

$$p \leq \frac{c-(1-\theta^B)b_2}{\bar{g}_{join}^B} = p_{3B}^{NR}$$

with $p_{3B}^{NR} > 0$. The inequality $p \leq p_{3B}^{NR}$ is binding for $\bar{g}_{join}^B < c-(1-\theta^B)b_2$ since

$$p_{3B}^{NR} < 1 \quad \Leftrightarrow \quad \bar{g}_{join}^B < c-(1-\theta^B)b_2.$$

Summarizing, the equilibrium condition (6.11) for type B holds

$$\begin{cases} \text{for all } p \in [0,1], \text{ if } \bar{g}_{join}^B \geq c-(1-\theta^B)b_2 \\ \text{for all } p \in [0, p_{3B}^{NR}], \text{ if } \bar{g}_{join}^B < c-(1-\theta^B)b_2 \end{cases}. \tag{C.20}$$

The equilibrium regions in Corollary 7 can again be derived as the intersections of the different sets of priors satisfying the respective equilibrium conditions in (C.18) and (C.20), q.e.d.

Corollary 8

Equilibrium regions for a^4 ***with unregulated securities***		
	$\bar{g}^B_{join} \leq c - (1-\theta^B)b_2$	$\bar{g}^B_{join} > c - (1-\theta^B)b_2$
$\bar{g}^A_{uni} \geq c - (1-\theta^A)b_1$	***a)*** $[0, p^{NR}_{4A}]$	***b)*** $[0, \min\{p^{NR}_{4A}, p^{NR}_{4B}\}]$
$\bar{g}^A_{uni} < c - (1-\theta^A)b_1$	***c)*** $\emptyset$	***d)*** $\emptyset$

where the parameters are defined as

$$p^{NR}_{4A} := \frac{c - (1-\theta^A)b_1 - \bar{g}^A_{uni}}{(1-\theta^A)(b_2-b_1) - \bar{g}^A_{uni}} \tag{C.21}$$

$$\textit{and} \quad p^{NR}_{4B} := \frac{c - (1-\theta^B)b_1}{(1-\theta^B)(b_2-b_1) + \bar{g}^B_{join}}. \tag{C.22}$$

Proof:

1) The equilibrium condition with respect to strategy profile a^4 for type A, (6.12), is equivalent to

$$p\left[(1-\theta^A)(b_2-b_1) - \bar{g}^A_{uni}\right] \geq c - (1-\theta^A)b_1 - \bar{g}^A_{uni}. \tag{C.23}$$

For $\bar{g}^A_{uni} < (1-\theta^A)(b_2-b_1)$ the expression in square brackets on the left hand side of inequality (C.23) is larger than zero and inequality (C.23) is equivalent to

$$p \geq \frac{c - (1-\theta^A)b_1 - \bar{g}^A_{uni}}{(1-\theta^A)(b_2-b_1) - \bar{g}^A_{uni}} = p^{NR}_{4A}.$$

However, in this case p^{NR}_{4A} is not well-defined, since

$$p^{NR}_{4A} > 1 \Leftrightarrow (1-\theta^A)b_2 < c$$

which holds by assumption (5.6). Therefore, there exists no prior p satisfying (C.23).

For $\bar{g}^A_{uni} = (1-\theta^A)(b_2-b_1)$ the left hand side of (C.23 is zero, but the right hand side larger than zero since $(1-\theta^A)(b_2-b_1) < c - (1-\theta^A)b_1$ by assumption (5.5). Then, there is again no prior p satisfying (C.23).

For $\bar{g}^A_{uni} > (1-\theta^A)(b_2-b_1)$, inequality (C.23) is equivalent to $p \leq p^{NR}_{4A}$ and, moreover, $p^{NR}_{4A} < 1$ holds since equivalent to $(1-\theta^A)b_2 < c$ which is assumption (5.6). For $\bar{g}^A_{uni} \geq c-(1-\theta^A)b_1 > (1-\theta^A)(b_2-b_1)$, we also get $p^{NR}_{4A} \geq 0$.

Summarizing, the equilibrium condition (6.12) for type A holds

$$\begin{cases} \text{for all } p \in [0, p^{NR}_{4A}], \text{ if } \bar{g}^A_{uni} \geq c-(1-\theta^A)b_1 \\ \text{for } \mathbf{no}\ p, \text{ if } \bar{g}^A_{uni} < c-(1-\theta^A)b_1 \end{cases}. \tag{C.24}$$

2) The equilibrium condition with respect to strategy profile a^4 for type B, (6.13), is equivalent to

$$p\left[(1-\theta^B)(b_2-b_1)+\bar{g}^B_{join}\right] \leq c-(1-\theta^B)b_1. \tag{C.25}$$

If the bracket on the left hand side of inequality (C.25) is not larger than zero, i.e. if $\bar{g}^B_{join} \leq (1-\theta^B)(b_1-b_2)$, then inequality (C.25) holds for all priors p since the right hand side is larger than zero by assumption (5.5).

If $\bar{g}^B_{join} > (1-\theta^B)(b_1-b_2)$, then inequality (C.25) is equivalent to

$$p \leq \frac{c-(1-\theta^B)b_1}{(1-\theta^B)(b_2-b_1)+\bar{g}^B_{join}} = p^{NR}_{4B}.$$

Since

$$p^{NR}_{4B} < 1 \quad \Leftrightarrow \quad \bar{g}^B_{join} > c-(1-\theta^B)b_2$$

and

$$(1-\theta^B)(b_1-b_2) < c-(1-\theta^B)b_2,$$

inequality (C.25) is not binding for $(1-\theta^B)(b_1-b_2) < \bar{g}^B_{join} \leq c-(1-\theta^B)b_2$. Then (C.25) holds for all priors p. If $\bar{g}^B_{join} > c-(1-\theta^B)b_2$, inequality (C.25) holds for all $p \leq p^{NR}_{4B}$.

Summarizing, the equilibrium condition (6.13) for type A holds

$$\begin{cases} \text{for all } p \in [0,1], \text{ if } \bar{g}^B_{joint} \leq c-(1-\theta^B)b_2 \\ \text{for all } p \in [0, p^{NR}_{4B}], \text{ if } \bar{g}^B_{joint} > c-(1-\theta^B)b_2 \end{cases}. \tag{C.26}$$

Combining the sets of priors in (C.24) and (C.26) yields the equilibrium regions in Corollary 8, q.e.d.

Corollary 9

Equilibrium regions for $\mathbf{a}^1$ ***and*** $\mathbf{a}^2$ ***with regulated securities***		
	Strategy profile a^1	*Strategy profile* a^2
$g_1^B < c - (1-\theta^B)b_1$	$[0,1]$	$[0, p_{2B}^R]$
$g_1^B = c - (1-\theta^B)b_1$	$[0,1]$	$[0,1]$
$g_1^B > c - (1-\theta^B)b_1$	$[0, p_{1B}^R]$	$[0,1]$

where the parameters are defined

$$p_{1B}^R := \frac{1}{g_1^B}\left[c - (1-\theta^B)b_1\right] \tag{C.27}$$

$$\textit{and} \quad p_{2B}^R := \frac{c - (1-\theta^B)b_2}{(1-\theta^B)(b_1-b_2) + g_1^B}. \tag{C.28}$$

Proof:

The equilibrium conditions for strategy profile a^1 and a^2 are analysed separately. First consider the equilibrium conditions for strategy profile a^1. The equilibrium condition for type A, (6.16), is satisfied for all priors p by assumption (5.5). The equilibrium condition for type B, (6.17), is equivalent to[4]

$$p \le \frac{1}{g_1^B}\left[c - (1-\theta^B)b_1\right] = p_{1B}^R.$$

The inequality is binding, i.e. $p_{1B}^R < 1$, for $g_1^B > c - (1-\theta^B)b_1$. If $g_1^B \le c - (1-\theta^B)b_1$ the constraint $p \le p_{1B}^R$ is not binding and (6.17) holds for all priors p. Since the equilibrium condition for type A is satisfied for all priors, strategy profile a^1 is an equilibrium for

$$\begin{cases} \text{all } p \in [0,1], \text{ if } g_1^B \le c - (1-\theta^B)b_1 \\ \text{all } p \in [0, p_{1B}^R], \text{ if } g_1^B > c - (1-\theta^B)b_1 \end{cases}. \tag{C.29}$$

Now consider the equilibrium conditions for strategy profile a^2.

[4]Note that the gains from trade g_1^B are always larger than zero since they are only defined for $\theta_1 \neq \theta_2$.

1) The equilibrium condition for type A, (6.18), is equivalent to

$$p\left[(1-\theta^A)(b_1-b_2)+g_1^A\right] \leq c-(1-\theta^A)b_2+g_1^A. \tag{C.30}$$

If $g_1^A \leq (1-\theta^A)(b_2-b_1)$, the bracket on the left hand side of inequality (C.30) is not larger than zero. However, the right hand side is larger than zero by assumption (5.6) and $g_1^A \geq 0$ so that inequality (C.30) holds for all values of p.

If $g_1^A > (1-\theta^A)(b_2-b_1)$, the bracket on the left hand side of inequality (C.30) is larger than zero so that (C.30) is equivalent to

$$p \leq \frac{c-(1-\theta^A)b_2+g_1^A}{(1-\theta^A)(b_1-b_2)+g_1^A} =: p_{2A}^R. \tag{C.31}$$

However,

$$p_{2A}^R < 1 \quad \Leftrightarrow \quad c < (1-\theta^A)b_1$$

which is excluded by assumption (5.5). Therefore, inequality (C.31) is not binding and the equilibrium condition (6.18) holds for all priors p.

2) The equilibrium condition for type B, (6.19), is equivalent to

$$p\left[(1-\theta^B)(b_1-b_2)+g_1^B\right] \geq c-(1-\theta^B)b_2. \tag{C.32}$$

For $g_1^B \geq (1-\theta^B)(b_2-b_1)$ the bracket on the left hand side of inequality (C.32) is larger than or equal to zero, whereas the right hand side is strictly smaller than zero by assumption (6.3). Thus, inequality (C.32) holds for all priors p.

For $g_1^B < (1-\theta^B)(b_2-b_1)$ inequality (C.32) is equivalent to

$$p \leq \frac{c-(1-\theta^B)b_2}{(1-\theta^B)(b_1-b_2)+g_1^B} = p_{2B}^R.$$

Note that

$$p_{2B}^R < 1 \quad \Leftrightarrow \quad g_1^B < c-(1-\theta^B)b_1 \overset{(6.3)}{<} (1-\theta^B)(b_2-b_1),$$

implying that $p \leq p_{2B}^R$ is not binding for $g_1^B \geq c-(1-\theta^B)b_1$.

Summarizing, the equilibrium condition (6.19) holds for

$$\begin{cases} \text{all } p \in [0,1], \text{ if } g_1^B \geq c - (1-\theta^B)b_1 \\ \text{all } p \in [0, p_{2B}^R], \text{ if } g_1^B < c - (1-\theta^B)b_1 \end{cases}. \tag{C.33}$$

Since the equilibrium condition (6.19) for type A holds for all priors, (C.33) also summarizes the equilibrium regions for strategy profile a^2. Comparison of (C.29) and (C.33) yields Corollary 9, q.e.d.

Proof of Corollary 4:
Recall that as defined in Corollary 1

$$p_{2B} = \frac{(1-\theta^B)b_2 - c}{(1-\theta^B)(b_2 - b_1)},$$

and as defined in Corollary 9

$$\begin{aligned} p_{2B}^R &= \frac{c - (1-\theta^B)b_2}{(1-\theta^B)(b_1 - b_2) + g_1^B}, \\ &= \frac{(1-\theta^B)b_2 - c}{(1-\theta^B)(b_2 - b_1) - g_1^B}. \end{aligned} \tag{C.34}$$

By assumption

$$g_1^B < c - (1-\theta^B)b_1 \overset{(6.3)}{<} (1-\theta^B)(b_2 - b_1),$$

implying $(1-\theta^B)(b_2 - b_1) - g_1^B > 0$ so that the denominator of p_{2B}^R in (C.34) is larger than zero. Then

$$\begin{aligned} p_{2B}^R &> p_{2B} \\ \Leftrightarrow \frac{(1-\theta^B)b_2 - c}{(1-\theta^B)(b_2 - b_1) - g_1^B} &> \frac{(1-\theta^B)b_2 - c}{(1-\theta^B)(b_2 - b_1)} \\ \Leftrightarrow \frac{1}{(1-\theta^B)(b_2 - b_1) - g_1^B} &> \frac{1}{(1-\theta^B)(b_2 - b_1)} \\ \Leftrightarrow (1-\theta^B)(b_2 - b_1) &> (1-\theta^B)(b_2 - b_1) - g_1^B \\ \Leftrightarrow g_1^B &> 0 \end{aligned}$$

which always holds, given that trade actually takes place, q.e.d.

Proof of Proposition 8:

The security demand functions can be derived from (5.11). Since $n = 2$ and $\theta_1 \neq \theta_2$ aggregate demand for a security is the sum of type A and type B demand. Setting aggregate demand for one of the securities equal to zero yields the equilibrium security price[5]

$$q^*(\tilde{a}) = \frac{(\theta^A + \theta^B)\omega^\beta(\tilde{a})}{(2 - \theta^A - \theta^B)\bar{\omega}^\alpha + (\theta^A + \theta^B)\omega^\beta(\tilde{a})}.$$

Inserting $q^*(\tilde{a})$ into the security demand functions yields the equilibrium security demands

$$\begin{aligned} s^{\alpha *}(\theta^A, \tilde{a}) &= \frac{(\theta^A - \theta^B)\bar{\omega}^\alpha}{\theta^A + \theta^B} = -s^{\alpha *}(\theta^B, \tilde{a}) \\ \text{and} \qquad s^{\beta *}(\theta^A, \tilde{a}) &= \frac{(\theta^B - \theta^A)\omega^\beta(\tilde{a})}{2 - \theta^A - \theta^B} - s^{\beta *}(\theta^B, \tilde{a}). \end{aligned} \tag{C.35}$$

For the gains from trade for type X with unregulated securities we then get for $X = A, B$

$$\begin{aligned} g^X_{\tilde{a}} &= \theta^X \ln\left(\bar{\omega}^\alpha + s^{\alpha *}(\theta^X, \tilde{a})\right) + (1 - \theta^X) \ln\left(\omega^\beta(\tilde{a}) + s^{\beta *}(\theta^X, \tilde{a})\right) \\ &\quad - \left[\theta^X \ln(\bar{\omega}^\alpha) + (1 - \theta^X) \ln(\omega^\beta(\tilde{a}))\right] \\ &= \theta^X \ln\left(\frac{\bar{\omega}^\alpha + s^{\alpha *}(\theta^X, \tilde{a})}{\bar{\omega}^\alpha}\right) + (1 - \theta^X) \ln\left(\frac{\omega^\beta(\tilde{a}) + s^{\beta *}(\theta^X, \tilde{a})}{\omega^\beta(\tilde{a})}\right) \\ &\overset{(C.35)}{=} \theta^X \ln\left(\frac{2\theta^X}{\theta^A + \theta^B}\right) + (1 - \theta^X) \ln\left(\frac{2 - 2\theta^X)}{2 - \theta^A - \theta^B}\right). \end{aligned}$$

Since $g^X_{\tilde{a}}$ does not depend on aggregate abatement $\tilde{a}$, part a) of Proposition 8 follows immediately. Part b) then follows by the definition of the additional abatement incentives $\bar{g}^X_{uni}$ and $\bar{g}^X_{join}$, q.e.d.

[5] Recall that under the assumptions of Proposition 8 we always have $n_A(\theta) = n_B(\theta) = 1$ and $n_A(\theta)$ is omitted as argument in the components of security market equilibrium.

Appendix D

Proofs for Chapter 7

Proof of Proposition 9:
Note that by (7.4) we can write

$$1-\theta^X_{\tilde{a}} = 1-\theta^X_{\tilde{a}-1} - \Delta^X_{\tilde{a}}. \tag{D.1}$$

a) Assuming that the remaining $n-1$ countries follow strategy profile a^1, $\tilde{a}_{-i}(\theta_{-i}) = 0$ holds for all θ_{-i}. By the definition of the utility functions in (7.8) a type A country will not deviate from strategy profile a^1, if and only if

$$\begin{aligned}
&\bar{u}(\theta^A,0) + (1-\theta^A_0)b^{abs}_0 \geq -c + \bar{u}(\theta^A,1) + (1-\theta^A_1)b^{abs}_1 \\
\Leftrightarrow\quad & (1-\theta^A_1)b^{abs}_1 - (1-\theta^A_0)b^{abs}_0 + \bar{u}(\theta^A,1) - \bar{u}(\theta^A,0) \leq c \\
\overset{(D.1),(7.10)}{\Leftrightarrow}\quad & [(1-\theta^A_0) - \Delta^A_1]b^{abs}_1 - (1-\theta^A_0)b^{abs}_0 + \Delta^A_1[u(\bar{\omega}^\alpha) - u(\bar{\omega}^\beta)] \leq c \\
\overset{(5.4)}{\Leftrightarrow}\quad & (1-\theta^A_0)b_1 + \Delta^A_1[u(\bar{\omega}^\alpha) - u(\bar{\omega}^\beta) - b^{abs}_1] \leq c \\
\overset{(5.3)}{\Leftrightarrow}\quad & (1-\theta^A_0)b_1 + \Delta^A_1[u(\bar{\omega}^\alpha) - u(\omega^\beta(1))] \leq c.
\end{aligned}$$

By the analogue argument, type B will not deviate from strategy profile a^1, if and only if

$$\begin{aligned}
&\bar{u}(\theta^B,0) + (1-\theta^B_0)b^{abs}_0 \geq -c + \bar{u}(\theta^B,1) + (1-\theta^B_1)b^{abs}_1 \\
\Leftrightarrow\quad & (1-\theta^B_0)b_1 + \Delta^B_1[u(\bar{\omega}^\alpha) - u(\bar{\omega}^\beta(1))] \leq c.
\end{aligned}$$

b) Given that the remaining $n-1$ countries follow strategy profile a^2, they abate if and only if they are type B. Recall, for example from the proof of Proposition 3 above, that then $\tilde{a}_{-i}(\theta_{-i}) = n - 1 - n_A^{-i}(\theta_{-i})$ and that $n_A^{-i}(\theta_{-i})$ is binomially distributed. A type A country will not deviate from strategy profile a^2, if and only if

$$\begin{aligned}
&\sum_{k=0}^{n-1} p_k \left[\bar{u}(\theta^A, n-1-k) + (1-\theta^A_{n-1-k}) b^{abs}_{n-1-k}\right] \\
&\geq -c + \sum_{k=0}^{n-1} p_k \left[\bar{u}(\theta^A, n-k) + (1-\theta^A_{n-k}) b^{abs}_{n-k}\right] \\
\overset{(D.1),(7.10)}{\Leftrightarrow} \quad &\sum_{k=0}^{n-1} p_k \left[(1-\theta^A_{n-1-k}) b_{n-k} + \Delta^A_{n-k} \left(u(\bar{\omega}^\alpha) - u(\omega^\beta(n-k))\right)\right] \\
&\leq c.
\end{aligned}$$

A type B country will not deviate from strategy a^2, if and only if

$$\begin{aligned}
&-c + \sum_{k=0}^{n-1} p_k \left[\bar{u}(\theta^B, n-k) + (1-\theta^B_{n-k}) b^{abs}_{n-k}\right] \\
&\geq \sum_{k=0}^{n-1} p_k \left[\bar{u}(\theta^B, n-1-k) + (1-\theta^B_{n-1-k}) b^{abs}_{n-1-k}\right] \\
\Leftrightarrow \quad &\sum_{k=0}^{n-1} p_k \left[(1-\theta^B_{n-1-k}) b_{n-k} + \Delta^B_{n-k} \left(u(\bar{\omega}^\alpha) - u(\omega^\beta(n-k))\right)\right] \\
&\geq c.
\end{aligned}$$

c) Assuming that the remaining $n-1$ countries stick to strategy a^3 we know that $\tilde{a}_{-i}(\theta_{-i}) = n-1$ holds for all θ_{-i}. A type X country ($X = A, B$) will not deviate from strategy profile a^3, if and only if

$$\begin{aligned}
&(1-\theta^X_n) b^{abs}_n - c + \bar{u}(\theta^X, n) \\
&\geq (1-\theta^X_{n-1}) b^{abs}_{n-1} + \bar{u}(\theta^X, n-1) \\
\Leftrightarrow \quad &(1-\theta^X_{n-1}) b_n + \Delta^X_n \left[u(\bar{\omega}^\alpha) - u(\bar{\omega}^\beta(n))\right] \\
&\geq c.
\end{aligned}$$

d) Assuming that the remaining $n-1$ countries follow strategy profile a^4 they abate if and only if they are type A so that $\tilde{a}_{-i}(\theta_{-i}) = n_A^{-i}(\theta_{-i})$.

A type A country will not deviate from strategy profile a^4, if and only if

$$\begin{aligned} & -c + \sum_{k=0}^{n-1} p_k \left[\bar{u}(\theta^A, k+1) + (1-\theta^A_{k+1}) b^{abs}_{k+1}\right] \\ \geq & \sum_{k=0}^{n-1} p_k \left[\bar{u}(\theta^A, k) + (1-\theta^A_k) b^{abs}_k\right] \\ \Leftrightarrow \quad & \sum_{k=0}^{n-1} p_k \left[(1-\theta^A_k) b_{k+1} + \Delta^A_{k+1}[u(\bar{\omega}^\alpha) - u(\omega^\beta(k+1))]\right] \geq c. \end{aligned}$$

Similarly, a type B country will not deviate, if and only if

$$\begin{aligned} & \sum_{k=0}^{n-1} p_k \left[\bar{u}(\theta^B, k) + (1-\theta^B_k) b^{abs}_k\right] \\ \geq & -c + \sum_{k=0}^{n-1} p_k \left[\bar{u}(\theta^B, k+1) + (1-\theta^B_{k+1}) b^{abs}_{k+1}\right] \\ \Leftrightarrow \quad & \sum_{k=0}^{n-1} p_k \left[(1-\theta^B_k) b_{k+1} + \Delta^B_{k+1}[u(\bar{\omega}^\alpha) - u(\omega^\beta(k+1))]\right] \leq c, \end{aligned}$$

q.e.d.

Proof of Proposition 10:

a) The countries' payoffs, defined in (7.15), now additionally include the gains from trade in climate securities if trade takes place. There is no trade if all countries are of the same type.[1] Assuming that the remaining $n-1$ countries follow strategy profile a^1, a type A country will not deviate from strategy a^1, if and only if

$$\begin{aligned} & \bar{u}(\theta^A, 0) + (1-\theta^A_0) b^{abs}_0 + \sum_{k=0}^{n-2} p_k g(\theta^A, k+1; 0) \\ \geq & -c + \bar{u}(\theta^A, 1) + (1-\theta^A_1) b^{abs}_1 + \sum_{k=0}^{n-2} p_k g(\theta^A, k+1; 1) \end{aligned}$$

[1] For this reason the sum below runs from $k = 0$ to $k = n-2$ instead of running to $k = n-1$.

$$\Leftrightarrow \quad \underbrace{(1-\theta_0^A)b_1 - \Delta_1^A[u(\bar{\omega}^\alpha) - u(\omega^\beta(1))] + \sum_{k=0}^{n-2} p_k \left[g(\theta^A, k+1; 1) - g(\theta^A, k+1; 0)\right]}_{g^A_{uni}(p)} \leq c$$

By a similar argument, type B will not deviate from strategy a^1, if and only if

$$(1-\theta_0^B)b_1 + \Delta_1^B[u(\bar{\omega}^\alpha) - u(\omega^\beta(1))] + g^B_{uni}(p) \leq c.$$

b) In analogy to part b) in the proof of Proposition 9 above a type A country will not deviate from strategy a^2, if and only if

$$\begin{aligned}
&\sum_{k=0}^{n-1} p_k \left[\bar{u}(\theta^A, n-1-k) + (1-\theta^A_{n-1-k})b^{abs}_{n-1-k}\right] \\
&\quad + \sum_{k=0}^{n-2} p_k g(\theta^A, k+1; n-1-k)] \\
&\geq -c + \sum_{k=0}^{n-1} p_k \left[\bar{u}(\theta^A, n-k) + (1-\theta^A_{n-k})b^{abs}_{n-k}\right] \\
&\quad + \sum_{k=0}^{n-2} p_k g(\theta^A, k+1; n-k)] \\
\Leftrightarrow \quad &\sum_{k=0}^{n-1} p_k[(1-\theta^A_{n-1-k})b_{n-k} - \Delta^A_{n-k}[u(\bar{\omega}^\alpha) - u(\omega^\beta(n-k))]] \\
&\quad + g^A_{con}(p) \leq c.
\end{aligned}$$

Similarly, a type B country will not deviate from strategy a^2, if and only if

$$\sum_{k=0}^{n-1} p_k \left[(1-\theta^B_{n-1-k})b_{n-k} - \Delta^B_{n-k}[u(\bar{\omega}^\alpha) - u(\omega^\beta(n-k))]\right] + g^B_{con}(p) \geq c.$$

c) Assuming that the remaining $n-1$ countries follow strategy profile a^3 they all choose H and $\tilde{a}_{-i}(\theta_{-i}) = n-1$ for all θ_{-i}. A type A country will not deviate from strategy a^3, if and only if

$$\begin{aligned}
&\bar{u}(\theta^A, n) + (1-\theta_n^A) b_n^{abs} - c + \sum_{k=0}^{n-2} p_k g(\theta^A, k+1; n) \\
&\geq \bar{u}(\theta^A, n-1) + (1-\theta_{n-1}^A) b_{n-1}^{abs} + \sum_{k=0}^{n-2} p_k g(\theta^A, k+1; n-1) \\
\Leftrightarrow \quad & (1-\theta_{n-1}^A) b_n - \Delta_n^A [u(\bar{\omega}^\alpha) - u(\omega^\beta(n))] \\
& + \underbrace{\sum_{k=0}^{n-2} p_k [g(\theta^A, k+1; n) - g(\theta^A, k+1; n-1)]}_{g_{join}^A(p)} \geq c.
\end{aligned}$$

Similarly, a type B country will not deviate, if and only if

$$\begin{aligned}
&\bar{u}(\theta^B, n) + (1-\theta_n^B) b_n^{abs} - c + \sum_{k=1}^{n-1} p_k g(\theta^B, k; n) \\
&\geq \bar{u}(\theta^B, n-1) + (1-\theta_{n-1}^B) b_{n-1}^{abs} + \sum_{k=1}^{n-1} p_k g(\theta^B, k; n-1) \\
\Leftrightarrow \quad & (1-\theta_{n-1}^B) b_n - \Delta_n^B [u(\bar{\omega}^\alpha) - u(\omega^\beta(n))] \\
& + \underbrace{\sum_{k=1}^{n-1} p_k \left[g(\theta^B, k+1; n) - g(\theta^B, k+1; n-1)\right]}_{g_{join}^B(p)} \geq c.
\end{aligned}$$

d) Type A will not deviate from strategy profile a^4, if and only if

$$\begin{aligned}
& - c + \sum_{k=0}^{n-1} p_k [\bar{u}(\theta^A, k+1) + (1-\theta_{k+1}^A) b_{k+1}^{abs}] \\
& + \sum_{k=0}^{n-2} p_k g(\theta^A, k+1; k+1) \\
& \geq \sum_{k=0}^{n-1} p_k [\bar{u}(\theta^A, k) + (1-\theta_k^A) b_k^{abs}] + \sum_{k=0}^{n-2} p_k g(\theta^A, k+1; k) \\
\Leftrightarrow \quad & \sum_{k=0}^{n-1} p_k [(1-\theta_k^A) b_{k+1} + \Delta_{k+1}^A [u(\bar{\omega}^\alpha) - u(\omega^\beta(k+1))]
\end{aligned}$$

$$+\underbrace{\sum_{k=0}^{n-2} p_k[g(\theta^A, k+1; k+1) - g(\theta^A, k+1; k)]}_{g^A_{inc}(p)} \geq c.$$

Similarly, a type B country will not deviate, if and only if

$$\begin{aligned}
&\sum_{k=0}^{n-1} p_k[\bar{u}(\theta^B, k) + (1-\theta^B_k)b^{abs}_k] + \sum_{k=1}^{n-1} p_k g(\theta^B, k+1, k) \\
\geq &- c + \sum_{k=0}^{n-1} p_k[\bar{u}(\theta^B, k+1) + (1-\theta^B_{k+1})b^{abs}_{k+1}] \\
&\quad + \sum_{k=1}^{n-1} p_k g(\theta^B, k+1, k+1)] \\
\Leftrightarrow \quad &\sum_{k=0}^{n-1} p_k[(1-\theta^B_k)b_{k+1} + \Delta^B_{k+1}[u(\bar{\omega}^\alpha) - u(\omega^\beta(k+1))] \\
&+\underbrace{\sum_{k=1}^{n-1} p_k[g(\theta^B, k+1, k+1) - g(\theta^B, k+1, k)]}_{g^B_{inc}(p)} \leq c,
\end{aligned}$$

q.e.d.

Proof of Proposition 11:

a) In contrast to the proof of Proposition 10 above countries can now only attain gains from trade if they behave consistently. Note that strategy profile a^1 is inconsistent for type B countries, therefore they cannot trade in climate securities.

A type A country is not restricted as long as it sticks to a^1, but there is nobody on the opposite side of the market since type B countries cannot trade. If a type A country deviates, it receives no gains from trade since it then exhibits inconsistent behaviour. Therefore the equilibrium condition for type A is identical to the equilibrium condition in the absence of security markets (7.11).

Note that a type B country can trade in climate securities if it deviates from strategy a^1. Type B will not deviate from strategy a^1, if and only if

$$\begin{aligned} &\bar{u}(\theta^B,0)+(1-\theta_0^B)b_0^{abs} \\ &\geq -c+\bar{u}(\theta^B,1)+(1-\theta_1^B)b_1^{abs}+\sum_{k=1}^{n-1}p_k g^R(\theta^B,k,1;1) \\ \Leftrightarrow \quad &(1-\theta_0^B)b_1-\Delta_1^B[u(\bar{\omega}^\alpha)-u(\omega^\beta(1))]+\sum_{k=1}^{n-1}p_k g^R(\theta^B,k,1;1)\leq c \end{aligned}$$

b) Since strategy profile a^2 is consistent for type A countries, a type A country benefits from gains from trade only when sticking toy a^2. A type A country will not deviate from strategy a^2, if and only if

$$\begin{aligned} &\sum_{k=0}^{n-1}p_k\left[\bar{u}(\theta^A,n-1-k)+(1-\theta_{n-1-k}^A)b_{n-1-k}^{abs}\right] \\ &\quad+\sum_{k=0}^{n-2}p_k g^R(\theta^A,k+1,n-k-1;n-k-1) \\ &\geq -c+\sum_{k=0}^{n-1}p_k[\bar{u}(\theta^A,n-k)+(1-\theta_{n-k}^A)b_{n-k}^{abs}] \\ \Leftrightarrow \quad &\sum_{k=0}^{n-1}p_k[(1-\theta_{n-1-k}^A)b_{n-k}-\Delta_{n-k}^A[u(\bar{\omega}^\alpha)-u(\omega^\beta(n-k))]] \\ &\quad-r_{con}^A(p)\leq c. \end{aligned}$$

Similarly, type B will not deviate, if and only if

$$\sum_{k=0}^{n-1}p_k[(1-\theta_{n-1-k}^B)b_{n-k}-\Delta_{n-k}^B[u(\bar{\omega}^\alpha)-u(\omega^\beta(n-k))]]+r_{con}^B(p)\geq c.$$

c) Assuming that the remaining $n-1$ countries follow strategy profile a^3 they all choose H and $\tilde{a}_{-i}(\theta_{-i})=n-1$ for all θ_{-i}. Note that strategy profile a^3 is inconsistent for type A countries, but consistent for type B countries. Country i with $\theta_i=\theta^A$ will not deviate from strategy a^3, if and only if

$$\begin{aligned}
&\bar{u}(\theta^A, n) + (1-\theta_n^A)b_n^{abs} - c \\
&\geq \bar{u}(\theta^A, n-1) + (1-\theta_{n-1}^A)b_{n-1}^{abs} + \sum_{k=0}^{n-2} p_k g^R(\theta^A, 1, n-1-k; n-1) \\
\Leftrightarrow\quad &(1-\theta_{n-1}^A)b_n - \Delta_n^A[u(\bar{\omega}^\alpha) - u(\omega^\beta(n))] \\
&+\sum_{k=0}^{n-2} p_k g^R(\theta^A, 1, n-1-k; n-1) \;\geq\; c.
\end{aligned}$$

Similarly, a type B country will not deviate, if and only if

$$\begin{aligned}
&\bar{u}(\theta^B, n) + (1-\theta_n^B)b_n^{abs} - c + \sum_{k=1}^{n-1} p_k g^R(\theta^B, 0, n-k; n) \\
&\geq \bar{u}(\theta^B, n-1) + (1-\theta_{n-1}^B)b_{n-1}^{abs} \\
\Leftrightarrow\quad &(1-\theta_{n-1}^B)b_n - \Delta_n^B[u(\bar{\omega}^\alpha) - u(\omega^\beta(n))] \\
&+\sum_{k=1}^{n-1} p_k g^R(\theta^B, 0, n-k; n) \;\geq\; c.
\end{aligned}$$

d) Note that strategy profile a^4 is inconsistent for both types of countries. Both types could, in principle, trade if they deviate from strategy a^4.But then there is nobody on the other side of the market and no trade takes place. Therefore the equilibrium conditions are identical to those in the absence of security markets in (7.14), q.e.d.

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