Financial Volatility, Dynamic Correlations, and Macroeconomic Fundamentals

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Karin Loch

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Contents

List of Tables

List of Figures

Introduction

Financial volatility, dynamic correlations, and macroeconomic fundamentals

After more than 25 years of research on volatility, the central unsolved problem is the relation between the state of the economy and aggregate financial volatility.

Engle and Rangel (2008, p. 1187)

Some general remarks on financial volatility

Volatility lies at the core of financial risk and plays a crucial role in many financial applications, such as the pricing of financial derivatives, portfolio selection, and risk management. It is closely tracked by private and institutional investors, central bankers, financial regulators, and policy makers. For instance, an estimate of the volatility of an asset is a crucial input for determining capital requirements that are imposed on banks by the so-called Basel accords. Furthermore, policy institutions around the world constantly monitor risk developments across different asset classes, comprising equity, government bond, corporate bond, commoditiy, and FX markets.[1]

[1]Both the ECB and the IMF report *heat maps* with red, yellow, and green colors indicating high, medium, and low volatility estimates across different markets in the bi-annual Financial Stability Review and Global Financial Stability Report report, respectively. In addition, in its September 2015 meeting, the Federal Open Market Committee - the principal decision-making body within the Federal Reserve System - explicitly took into account the (increasing) level of

During the last couple of years, volatility *indices*, most notably the Volatility Index (VIX) published by the Chicago Board Options Exchange (CBOE), which is a measure of the implied volatility of S&P 500 index options, have drawn considerable attention. The VIX represents a measure of the market's expectation of stock market volatility over the next 30-days period. Since it can be viewed as representing investors' sentiments, it is often referred to as the *fear index*. More recently, volatility itself has been considered as an asset class and the number of financial instruments based on volatility indices has increased dramatically.[2]

The accurate modeling of time variation in *co-volatilities* or correlations between single assets as well as (international) asset markets has become just as important. Models of dynamic correlations are applied both on a small scale (e.g., in portfolio allocation) and on a large scale (e.g., for systemic risk measures).

This thesis contributes to the volatility literature by investing several relevant aspects of both financial volatility as well as dynamic correlations and the determination of their macroeconomic fundamentals in the framework of GARCH-MIDAS models - a particular class of volatility models from the *ARCH universe*.

The ARCH model and its extensions

The ARCH (*AutoRegressive Conditional Heteroskedasticity*) model was first introduced by Robert F. Engle in 1982. The economic literature at that time considered conditional heteroskedasticity in the cross-section, but did not regard it as a time-series phenomenon. Twenty years later, Engle was awarded the Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel (2003, shared with Clive Granger) in recognition of his work on "*methods of analyzing economic time series with time-varying volatility (ARCH)*".

Though its first application in Engle (1982) was on UK inflation, the great success of the ARCH model and its *Generalized* version, the GARCH model suggested by Bollerslev (1986), lies in applications to equity and exchange markets. These models have been so popular since they are able to capture the main stylized facts of financial return series, which are characterized by fat tails and volatility clustering. The tendency of financial volatility to cluster had already been observed by Mandelbrot

stock market volatility in its monetary policy decision-making process.

[2] The financial industry has already passed on to a higher, i.e. fourth, moment of asset returns and provides options on the *VVIX*, see "*Double the fun with the VIX on the VIX*" on www.cboe.com.

(1963, p. 418) who noted that "*large changes tend to be followed by large changes - of either sign - and small changes tend to be followed by small changes*".

Ever since its introduction, the number of GARCH model extensions has literally exploded, as has the range of its applications. Yet, the simple GARCH(1,1) remains the benchmark volatility model against which any model extension has to compete.[3] An (almost) exhaustive listing of GARCH model extensions and variants is provided in the ARCH Glossary of Bollerslev (2008).[4] Extensions and applications of the GARCH model are presented in several survey chapters of the handbooks by Andersen et al. (2009) and Bauwens et al. (2012).

On the economic sources of financial volatility

The analysis of the economic fundamentals of financial volatility goes back to Officer (1976), who considered the *variability of the market factor* of the New York Stock Exchange, and Schwert (1989) who raised the question "*Why does stock market volatility vary over time?*". In his analysis, Schwert (1989) considers monthly stock return data spanning from 1857 to 1987 and its relation to real and nominal macroeconomic volatility, the level of economic activity, as well as financial leverage. The counter-cyclical behavior of stock market volatility has generally been acknowledged ever since, i.e. volatility is found to be high during recession and crisis periods and low during economic expansions. However, the link has often seemed unreasonably weak.

In general, fluctuations in asset prices can be rationalized economically by relating them to fluctuations in the arrival and content of news. According to basic financial theory, the price of an asset should reflect the expected present value of its future income flows. This has been formalized in the framework of Campbell (1991) and Campbell and Shiller (1988). Changes in the asset price are then due

[3] In an extensive forecast comparison study of volatility models on exchange rates and stock returns, Hansen and Lunde (2005) ask "*Does anything beat a GARCH(1,1)?*" They find no evidence that the model is outperformed by more sophisticated models for exchange rates. In case of stock returns, it is outperformed only by asymmetric models that account for the leverage effect, such as the GJR-GARCH model (Glosten et al., 1993).

[4] "*The alphabet soup of volatility models continually amazes*" even its inventor (Engle 2002, p. 426). A contest on what other anacronyms ARCH might stand for, spured the following (not to be taken seriously) list of alternatives:
Anything Really Can Happen; Another Risk Can't Hurt; Another Really Cute Hunch; All Reality Comes Here; Applied Research Can Help; Another Rather Crazy Hypothesis; Almost Right Conjected Heuristic; And Robert Can Hit; All Risks Compensate Highly...culminating in the *YAARCH*, the Yet Another ARCH model proposed by the economist Figlewski.

Excerpt from: `http://englenobel.blogs.com/`

to changes in the expectations of investors about these future income streams and these expectations are modified with new information flowing in. Consequently, the volatility of an asset changes over time, since both the content and the arrival rate of news fluctuates over time. The same news may have a different impact on asset prices depending on the general state of the macroeconomy. Furthermore, both the horizon and the persistence of its impact may vary over time.

The last financial crisis and the subsequent great recession have revealed the need for a better understanding of the interaction between risks in financial markets and general economic conditions.[5] This has put the analysis of such macro finance links back on the research agenda, see, e.g., Asgharian et al. (2013), Campbell and Diebold (2009), Christiansen et al. (2012), Corradi et al. (2013), Dorion (2013), Engle et al. (2013), Engle and Rangel (2008), and Paye (2012).

In particular, the crisis has revealed fatal consequences of short-sighted risk management and has spured new research on long-term financial risks and systemic risk measures. In his paper "*How to forecast a crisis*", Engle concludes that "*the crisis was predictable using familiar time series models, but only at short horizons*" (Engle, 2010, p. 1). Furthermore, he stresses that long-term financial risk measures need to incorporate the "*risk that risk will change*", Engle (2009). The new focus on the long-term evolvement of financial risks and their link to economic fundamentals naturally extends the time span of the financial and economic series considered. When analyzing long time series spanning several years or decades, structural breaks become an important issue. Hence, a large part of the literature on GARCH model extensions has focused on developing more flexible models, allowing in particular for changing parameters. There are various alternative approaches to do so, but we will focus on one particular class of GARCH models in this dissertation.

The quote from Engel and Rangel (2008, p. 1187) preceding this introduction resumes as follows: "*The number of models that have been developed to predict volatility based on time series information is astronomical, but the models that incorporate economic variables are hard to find*". In their paper, Engle and Rangel (2008) propose a new GARCH model, called spline-GARCH, that fills this gap and is closely related to the GARCH-MIDAS component model, which will be the starting point of this dissertation.

[5]The financial crisis of 2007-08 is considered by many economists to have been the worst financial crisis since the Great Depression of the 1930s and has seen unprecedented high levels of financial volatility. These were only surpassed by the (short-lived) 1987 stock market crash. For instance, the VIX reached an intraday high of 89.53 on October 24, 2008, whereas its average value since 1990 has been just below 20.

The GARCH-MIDAS and DCC-MIDAS component models

The GARCH-MIDAS model introduced in Engle et al. (2013) is a two component volatility specification in the spirit of the models in Ding and Granger (1996), Engle and Lee (1999), and more recently Bauwens and Storti (2009) as well as Amado and Teräsvirta (2013, 2014, 2015). The model separates short-run fluctuations in volatility from slowly evolving long-term developments. The underlying motivation of these component models is the observation that "*volatility is not just volatility*" (Engle et al., 2013, p. 776), but consists of different components, which ought to be modeled separately. By introducing a time-varying unconditional variance, both the spline-GARCH and the GARCH-MIDAS models relax the assumption that volatility mean reverts to a constant level, which generally underlies many GARCH models and the early component models. In both models, the short-term component is specified as a unit variance GARCH process, which represents day-to-day clustering of volatility, evolving around a long-term trend component. The two models differ only in the specification of the long-term component.

The spline-GARCH model allows the unconditional variance to change smoothly as a function of time (similar to the model of Amado and Taräsvirta, 2012) via a non-parametric exponential quadratic spline. In this model, both volatility components are modeled at the same (high) frequency and the long-term component can only be linked to (low frequency) explanatory macroeconomic variables in a two-step approach. Engle and Rangel (2008) first transform the estimated daily long-term component to a lower frequency, which is then regressed on a set of explanatory variables. In a cross-sectional study comprising equity markets across 50 countries, they show that the long-term component behaves counter-cyclically.

The GARCH-MIDAS model combines the non-stationary volatility model of the spline-GARCH with the *MIxed Frequency DAta Sampling* (MIDAS) approach introduced in Ghysels et al. (2005) and allows to directly link macro economic variables of lower frequency to the long-term volatility component. The central feature of the MIDAS regression (or MIDAS filter) approach is a flexible weighting function that allows to combine data of high and low frequency in a very simple and parsimonious way. The GARCH-MIDAS model applies a beta weighting scheme to link (daily) high frequency financial return data to (monthly / quarterly) low-frequency macroeconomic variables, but there also exist other parsimonious weighting schemes, see Ghysels et al. (2007) for more details. In this model, forecasts of volatility (in particular at longer horizons) are mainly determined by the secular component, since forecasts of the short-run volatility component converge to unity. In their empirical analysis, Engle et al. (2013) consider an extended version of the Schwert (1987) data set (spanning from 1890 to 2010) and link the long-term component to the level and variance of industrial production growth and inflation.

Asymptotic results for the general GARCH-MIDAS model are not yet available, but Wang and Ghysels (2015) establish the asymptotic normality of the quasi-maximum likelihood estimator for a GARCH-MIDAS model including rolling windows of realized volatility as explanatory variable. Conrad and Schienle (2015) present a misspecification test based on the Lagrange multiplier principle and derive its asymptotic properties for testing the null hypothesis that the variable included in the long-term component has no explanatory power.

A multivariate version of the component model is presented in Colacito et al. (2011), who extend the Engle (2002) DCC model by introducing a short- and a long-term in the correlation specification in a similar way as in the GARCH-MIDAS model. In contrast to the latter, the original DCC-MIDAS specification by Colacito et al. (2011), relates the secular correlation component to lags of realized volatilities only.

This dissertation will focus on empirical applications of the GARCH- and DCC-MIDAS component models. It should be noted however, that these models remain reduced-form models, which are not directly linked to any structural model of the macroeconomy. Still, the new models present a feasible approach to relate financial volatility to macroeconomic fundamentals and have been widely applied during recent years, see Asgharian et al. (2013, 2015), Boffelli and Urga (2014), Dorion (2013), Opschoor et al. (2014), amongst others.

Outline of the thesis

This dissertation consists of four research articles that deal with different aspects of the modeling of financial volatility and dynamic correlations. They all focus on the U.S. stock market and its link to macroeconomic fundamentals by applying MIDAS techniques. The contributions of the articles are of theoretical, methodological, and empirical nature. Each chapter is self-contained and can be read independently.

Chapter 1 and 2 consider GARCH-MIDAS component models and the relationship between long-term financial volatility and the stance of the macroeconomy. Both chapters are joint work with my first supervisor Christian Conrad and have been published in the *Journal of Applied Econometrics* and in *Economics Letters*. Chapter 3 is single-authored and presents a new GARCH model that links time-varying volatility persistence to explanatory variables. Finally, Chapter 4 applies the multivariate DCC-MIDAS model to returns on the stock and the oil market and analyzes their relation to macroeconomic fundamentals. It is written jointly with Christian Conrad and my former colleague Daniel Rittler and has been published in the *Journal of Empirical Finance*.[6]

In the following, I will outline the main results and contributions of each chapter.

Chapter 1: Anticipating Long-Term Stock Market Volatility

In Chapter 1, we revisit the link between long-term financial volatility and the general macroeconomic environment using GARCH-MIDAS component models. We focus particularly on the lead-lag relationship between macroeconomic variables and volatility and on the role expectations concerning current and future macroeconomic developments play in predicting volatility.

We present an extensive analysis of the U.S. stock market for the 1969 to 2011 period and consider a variety of measures of economic activity, inflation rates, and interest rates, and combine first release data with expectations from the Survey of Professional Forecasters (SPF). We consider various specifications of the GARCH-MIDAS model. We either let the long-term volatility component be determined by a weighted average of lagged values of the explanatory variable, corresponding to a *one-sided* filter, or we combine lagged and future realizations or expectations thereof, corresponding to a *two-sided* filter. Applying the MIDAS techniques allows

[6]A previous version of this chapter has been part of the Ph.D. dissertation "The Carbon Market, Oil, and the Macroeconomy" (2012) of Daniel Rittler at the *Fakultät für Wirtschafts- und Sozialwissenschaften der Ruprecht-Karls-Universität Heidelberg*.

us to directly combine daily stock return data with quarterly macroeconomic data.

This chapter extends the empirical analysis in Engle et al. (2013) in several important ways and our main results may be summarized as follows. First of all, we confirm the counter-cyclical behavior of long-term financial volatility for a broad set of macro variables. Secondly, we identify several *leading* variables with respect to stock market volatility. For variables such as housing starts growth and the term spread, the optimal weighting scheme in the MIDAS filter is not strictly decreasing, but rather hump-shaped. These two variables perform best in terms of variance ratios (VRs), which measure the fraction of the variation in expected volatility that can be explained by the respective variable. In particular, the term spread specification indicates increasing financial risks well ahead of the recent financial crisis and clearly *anticipates* the build-up of financial risks. Our finding that the term spread and housing starts are leading with respect to long-term volatility is economically plausible, since these two variables are generally considered as leading indicators for the business cycle (see, e.g., Estrella and Mishkin, 1998, and Kydland et al. 2012). On the other hand, unemployment rate changes, industrial production growth, and real GDP growth are found to be coincident/lagging with respect to stock market volatility, since their optimal weighting scheme is a strictly decreasing one. Yet, we demonstrate that the performance for these variables can be improved by using a *feasible* two-sided filter, that is one which combines lagged realizations with expectations of future realizations. In theses cases, switching from one-sided to two-sided filters, leads to sizable increases in the VRs. For the GARCH-MIDAS model that includes past realized volatilities as explanatory variable, we find that the long-term volatility component is mainly dominated by the 1987 stock market crash, and to a lesser extend by the last financial crisis. It hardly varies though during the other recession periods. We also consider model specifications, which include both realized volatilities and a macro variable in the long-term component. Even if we control for realized volatility, we still find significant effects of the macro variables on long-term volatility and the variables mentioned previously are still leading with respect to long-term volatility. This demonstrates that many of the variables considered contain information on stock market risks beyond that contained in past realized volatilities.

In an out-of-sample forecasting exercise, we find further evidence for the predictive power of macroeconomic variables. For most variables, we find significant improvements in forecasting performance over the benchmark model that includes lagged realized volatilities and the gains are particularly large for longer forecasting horizons.

The chapter concludes with some extensions and robustness checks. In particular, we analyze how long-term volatility is related to the uncertainty about macroeconomic fundamentals. Schwert (1989) and later Engle and Rangel (2008) and Engle et al. (2013) measure the volatility of a macro variable by the squared residual of a simple autoregressive model with seasonal dummies. We show that survey based *ex-ante* uncertainty measures are more appropriate than such *ex-post* regression based volatility proxies. As an ex-ante measure, we consider the disagreement among the SPF forecasters as implied by the interquartile range of the individual point forecasts. We find that higher dispersion in expectations increases long-term volatility and more importantly that the *ex-ante* uncertainty measures are more informative than the *ex-post* measures of macro volatility, although the VRs of the disagreement specifications are generally lower than the ones of the level specifications. Finally, we argue that our results complement recent research on the determinants of the secular component of financial volatility in Christiansen et al. (2012) and Paye (2012). These papers consider predictive regressions, where current realized volatility (or a transformation thereof) is regressed on its lagged value(s) and the lag(s) of an explanatory variable. They typically find only weak evidence on the relevance of macro variables in predicting volatility. We find much more promising evidence when considering a slightly different version of the predictive regression, which includes the long-term volatility component obtained from a GARCH-MIDAS model with a macro variable instead of a single lag of the variable. We conclude that these long-term volatility components parsimoniously summarize the information on the lead-lag-structure between a specific macro variable and financial volatility

Chapter 2: The Variance Risk Premium and Fundamental Uncertainty

The insights from Chapter 1 on long-term financial volatility and its macroeconomic determinants are applied to the modeling of the variance risk premium (VRP). The chapter builds on recent findings in Bollerslev et al. (2009, 2012, 2014), Bekaert and Hoerova (2014), and others that strongly suggest that the VRP predicts medium-term aggregate stock market returns. This can be rationalized by the close relation of the VRP to economic uncertainty and aggregate risk aversion. Bollerslev et al. (2009) present a stylized self-contained general equilibrium model and argue that expected returns are positively related to the volatility of consumption growth volatility (*vol-of-vol* or *fundamental uncertainty*). The expected VRP is defined as the difference between the ex-ante risk-neutral expectation of future stock market variation and the statistical expectation of the realized variance. The risk-neutral expected variation is well approximated by the (squared) VIX, a model-free option implied variance measure, but the expected realized variance has to be estimated. However, a typical approach is to simply assume that the realized variance follows a martingale sequence.

The contribution of Chapter 2 is twofold. First, we propose a new measure of the VRP that is based on variance forecasts from the GARCH-MIDAS component model. This new proxy explicitly takes into account macroeconomic uncertainty via the long-run volatility component. In our empirical analysis, we consider the same financial return data set as in Chapter 1, but focus on monthly macroeconomic variables. We construct the VRP based on out-of-sample variance forecasts of the GARCH-MIDAS models. We consider monthly return predictability regressions and show that the new VRP measure has considerably stronger predictive power for stock returns than conventional measures of the VRP. Second, we argue that the strong predictive power stems from the fact that the long-term volatility component effectively isolates the fundamental uncertainty factor that drives the VRP.

Chapter 3: Time-Varying Volatility Persistence in a GARCH-MIDAS Framework

In Chapter 3, we take a different perspective on financial volatility modeling by considering time variation in volatility *persistence*. We suggest a new GARCH model with time-varying persistence (TVP) that is governed by an explanatory variable. We motivate the new model by showing that persistence in squared financial returns, as measured by the speed of decay of their autocorrelation function, varies over time and is high (low) during periods of high (low) realized volatility and weak (strong) business conditions.

In the standard (stationary) GARCH(1,1) model, volatility persistence is determined by the ARCH and GARCH parameters and thus remains constant over time. Estimations of the model on (long) financial return series typically indicate high persistence, i.e. the sum of the two parameters is found to be close to one. However, as already argued by Diebold (1986) and Lamoureux and Lastrapes (1990), and then formalized by Hillebrand (2005), volatility persistence will be over estimated in case there are structural breaks in the model parameters that are not accounted for. This has motivated a large body of literature on GARCH models with time-varying parameters. We add to this literature and extend the asymmetric GJR-GARCH model in Glosten et al. (1993) by introducing time variation in persistence through smooth transitions in the GARCH coefficient. The novelty of the model lies in letting the transition between different persistence regimes be governed by an explanatory variable in the spirit of the GARCH-MIDAS model. We refer to the new model as the *TVP-GARCH-MIDAS* model. It nests the standard GJR-GARCH model in case the variable has no explanatory power. In standard smooth transition type GARCH models, the transition has typically been governed by the lagged squared shock (see Hagerud, 1997, González-Rivera, 1998, Lundbergh and Teräsvirta, 1998, and Anderson et al., 1999) or the lagged conditional variance (Lanne and Saikkonen, 2005). Applying MIDAS techniques now allows us to link the transition to the *history* of an explanatory variable, i.e. to the weighted average of potentially many of its lagged values, in a parsimonious way. This approach yields a reasonably smooth measure of time-varying persistence of volatility.

After introducing the TVP-GARCH-MIDAS model, we derive a misspecification test based on the Lagrange multiplier principle, which has the advantage that it requires estimation of the model under the null only. We examine its finite sample size and power properties in a Monte-Carlo simulation study. The empirical size of the test is found to be close to the nominal size for normally distributed errors. In order to investigate power properties of the test, we consider two different TVP-GARCH-MIDAS model specifications, which include (smoothed versions) of realized volatility and the VIX. We find high power in case of reasonably smooth and pronounced time variation in persistence.

The second part of Chapter 3 presents an empirical application of the new model to the U.S. stock market and considers an extended version of the data set from Chapters 1 and 2. As explanatory variables, we include daily realized volatilities and the ADS business conditions index. The model estimations confirm the intuition from our motivation and we find increasing (decreasing) persistence for high (low) realized volatilities and weak (strong) business conditions. The model with realized volatility generates stronger time variation over a greater range of persistence compared to the model including the ADS. However, both models imply a lower persistence than the GJR-GARCH model on average. In an out-of-sample forecast evaluation, we provide evidence that the new TVP-GARCH-MIDAS model with realized volatility yields significant gains in forecasting performance over the GJR-GARCH model across horizons varying from one day to one quarter. In particular, we demonstrate that the model with realized volatility captures the average level of persistence of volatility particularly well during periods of very low and high volatility.

Chapter 4: On the Macroeconomic Determinants of Long-Term Volatilities and Correlations in U.S. Stock and Crude Oil Markets

Chapter 4 considers a multivariate extension of the GARCH-MIDAS model and its application to dynamic correlation between returns on the U.S. stock market and the oil market.

Based on the presumption that exogenous oil supply shocks are causal for recessions and periods of low growth (see the seminal articles by Hamilton, 1983, 1985, 2003), several empirical studies have analyzed the relationship between oil prices and stock market returns, yielding however conflicting evidence, see among others, Jones and Kaul (1996), Wei (2003), Nandha and Faff (2008), Miller and Ratti (2009). Results from regressing stock returns on oil price changes may be misleading though, as argued in Kilian and Park (2009), due to reverse causality from the U.S. economy to the oil price. Additionally, they argue that the sign of a stock price response to changes in oil price depends on the type of the underlying shock and may change over time. Indeed, Filis et al. (2011) confirm for several oil-exporting and oil-importing countries that the oil-stock correlations vary over time.

The main contribution of this chapter is the identification of a counter-cyclical relation between the long-term correlation component and macroeconomic conditions, which is driven by the same variables that also anticipate changes in both financial and oil long-term volatility. In particular, we provide first evidence on the link between macroeconomic conditions and the *daily* oil-stock correlation.

We first present a modification of the DCC-MIDAS component model introduced in Colacito et al. (2011) that allows to directly incorporate information on the macroeconomic development in the long-term correlation component. Our empirical analysis covers the 1993 to 2011 period and combines daily stock returns and oil return data with monthly macroeconomic data. More precisely, we consider variables measuring the current stance of the economy, such as industrial production growth, non-farm payrolls growth, and changes in the unemployment rate, as well as two forward looking indicators, the leading index and the national activity index. To begin with, we look separately at the macroeconomic determinants of the long-term volatility components for the stock and the oil market. We confirm the counter-cyclical behavior of the financial volatility component, which we analyzed in depth in Chapter 1. More importantly, we also find convincing evidence for a similar counter-cyclical relationship for the oil market. Previous studies such as Barsky and Kilian (2004) and Kilian (2008, 2009) have established reverse causality from the U.S. economy to the oil price and our findings now extend these results to the *volatility* of oil prices. Interestingly, both long-term volatility components respond to the same macroeconomic information.

In a next step, we show that changes in the long-term oil-stock correlation can be anticipated by the same macroeconomic variables that drive the long-term volatilities and find that the oil-stock correlation varies in a counter-cyclical way as well. The model estimates imply a positive long-term correlation component during recessions (or the beginning of expansions with growth still below trend), whereas the correlation decreases or turns negative during periods of strong growth above trend. This can be rationalized economically as follows. During recession periods, a simultaneous drop in oil and stock prices will induce a positive correlation. At the beginning of an economic recovery, increasing oil prices will at first not have a negative effect on the stock market. As argued in Kilian and Park (2009), there will typically be positive short-run effects of an unexpected increase in global demand on oil and stock prices. However, the negative effect of increasing oil prices will dominate in the long-run and in the course of an expansion, the oil-stock correlation will decrease again or even turn negative.

The counter-cyclical nature of the long-term correlation component implies that the sign of the oil-stock correlation critically depends on the state of the macroeconomy. This reinforces the argument made in Kilian and Park (2009) that simple regressions of stock returns on oil price changes may be very misleading.

In our model, the evolution of the long-term correlation component is purely driven by macroeconomic variables, which represent U.S. (or at least to some extent global) aggregate demand. As a consequence, deviations of the short-term component from the long-run trend must be related to other factors the affect the stock and/or oil market. In case of the oil market, typical factors would be either oil-specific (i.e. precautionary or speculative) demand shocks or supply shocks. Indeed, temporary deviations can be related to particular oil-related events, such as the Venezuelan oil supply crisis, the second Iraq war, and the Libyan crisis and political turmoil in North Africa. Since most of these deviations occur for relatively short periods, we consider this being further evidence that the oil-stock correlation is largely determined by U.S. economic activity and global aggregate demand, in line with Hamilton (2008), Kilian (2009), and Kilian and Murphy (2014).

The chapter concludes with some evidence on the potential benefits of accounting for time variation in the oil-stock correlation in a portfolio application. Our results suggest that including macro fundamentals in the conditional correlation specification in the DCC-MIDAS model leads to significantly lower portfolio variances compared to the standard DCC model.

1

Anticipating Long-Term Stock Market Volatility

We investigate the relationship between long-term U.S. stock market risks and the macroeconomic environment using a two component GARCH-MIDAS model. Our results show that macroeconomic variables are important determinants of the secular component of stock market volatility. Among the various macro variables in our dataset the term spread, housing starts, corporate profits, and the unemployment rate have the highest predictive ability for long-term stock market volatility. While the term spread and housing starts are leading variables with respect to stock market volatility, for industrial production and the unemployment rate expectations data from the Survey of Professional Forecasters regarding the future development are most informative.

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1.1 Introduction

Although the question as to whether measures of economic activity actually anticipate changes in stock market volatility dates back to Officer (1973) and Schwert (1989), the last financial crisis has put this issue back into the spotlight. Recent examples are Asgharian et al. (2013), Campbell and Diebold (2009), Christiansen et al. (2012), Corradi et al. (2013), Dorion (2013), and Paye (2012). This paper complements the recent literature by employing the GARCH-MIDAS framework suggested in Engle et al. (2013), which enables us to directly identify the effect of the macroeconomic environment on the secular component of stock market volatility. Our contribution is twofold. First, we provide a detailed exploration of the lead-lag relationship between macroeconomic variables and volatility and, second, we analyze the role of expectations concerning current and future macro developments in predicting volatility. We shall see that both issues are key to enhancing our understanding of the link between macroeconomic conditions and financial volatility.

The GARCH-MIDAS model is a two component volatility specification in the spirit of Ding and Granger (1996), Engle and Lee (1999), and more recently Bauwens and Storti (2009) and Amado and Teräsvirta (2013 and 2014), and separates short-run fluctuations in volatility from long-term developments. Similarly, as in the Engle and Rangel (2008) Spline-GARCH model, the short-term component is specified as a unit GARCH process evolving around a long-term trend component that reflects macroeconomic conditions. In comparison to the Spline-GARCH model, the GARCH-MIDAS has the advantage that it allows us to directly incorporate information on the macroeconomic environment into the long-term component. Using a flexible Beta weighting scheme, long-term volatility of daily stock returns is expressed as a weighted average of either lagged (one-sided filter) or lagged and future (two-sided filter) values of lower frequency macroeconomic variables. While most of the literature on volatility modeling exclusively focuses on the GARCH component, within the GARCH-MIDAS framework the log GARCH component can be thought of as the residual of a regression of the log conditional variance on macroeconomic explanatory variables (see Engle et al., 2013, p.781). Within this new framework, we identify specific economic variables that anticipate changes in long-term volatility.

Our analysis covers U.S. data for the 1969 to 2011 period and provides a detailed analysis of the lead-lag relationship between macroeconomic variables and stock market volatility. For this purpose, we consider a variety of measures of economic activity, as well as inflation and interest rate developments. Furthermore, in order to obtain a realistic picture of the macroeconomic variables' ability to anticipate changes in stock market volatility in real-time, we employ first release instead of revised data. Finally, combining first release data with expectations from the Survey of Professional Forecasters (SPF) allows us to estimate *feasible* two-sided filters.

Our main results can be summarized as follows. First, we reconfirm the countercyclical behavior of stock market volatility – this was first observed in Schwert (1989) – for a broad set of macro variables.

Second, we identify several *leading variables* with respect to stock market volatility. That is, the optimal (one-sided) weighting schemes for these variables do not decay from the beginning but are rather hump-shaped. Among these leading variables, the term spread and housing starts perform best in terms of variance ratios, which measure the fraction of the variation in expected quarterly volatility that can be attributed to the respective macro variable. In particular, the long-term component based on the term spread increases before all the recessions in our sample. The term spread specification clearly indicates increasing stock market risks well in advance of the recent financial crisis. The close relationship between the term spread, housing starts and stock market volatility is not surprising given that both variables are commonly considered as leading indicators for the business cycle (see, e.g. Estrella and Mishkin, 1998, and Kydland et al., 2012).

Third, we find that the performance of some variables, whose weights strictly decrease in the one-sided filter, can be improved by using a feasible two-sided filter – one which combines first release data with SPF expectations about the future. These variables can be described as coincident/lagging with respect to stock market volatility. Specifically, for industrial production, the unemployment rate, and real GDP, the feasible two-sided filters are preferred to their one-sided counterparts. The results for these variables suggest that – besides the current state of the macroeconomy – expectations about future macroeconomic conditions are important drivers of stock market volatility. This interpretation is very much in line with Campbell and Diebold's (2009, p.275) conclusion that expected business conditions forecast future volatility because they are "linked to perceived systematic risk and expected returns".

Fourth, an out-of-sample forecast evaluation provides further evidence for the predictive power of macroeconomic variables. That is, for most macro variables we find significant improvements in forecasting performance over the benchmark model that includes lagged realized volatilities. Modeling the long-term component as a function of the macroeconomic environment pays off particularly when it comes to long-term forecasting.

Finally, it is important to highlight that our results complement the recent findings in Asgharian et al. (2013) and Dorion (2013). The former study focuses on the predictive ability of GARCH-MIDAS models that are based on the first two principal components of several macro variables and the latter applies the GARCH-MIDAS framework in the context of option pricing. While our main focus lies on the macro variable specific lead-lag-structure and the potential gains from using unrestricted one- and feasible two-sided filters, both Asgharian et al. (2013) and Dorion (2013) exclusively employ one-sided filters with restricted weighting schemes.

The remainder of this article is organized as follows. Section 1.2 introduces the GARCH-MIDAS component model. The data and empirical results are presented in Sections 1.3 and 1.4. Finally, Section 1.5 concludes the article. Various additional tables and figures are available in an online Supplementary Appendix.

1.2 The GARCH-MIDAS model

The present value models of Campbell (1991) and Campbell and Shiller (1988) illustrate that unexpected returns can be associated with news that leads to revisions in the discounted sum of future expected dividends and returns. Specifically, the same news can have a small or large impact on unexpected returns depending on whether it affects expectations over short or long horizons. The volatility component models considered in this article capture this idea by relating the size of the new's impact to variables that describe the state of the macroeconomy and, hence, carry information about expected future cash flows. For example, Engle and Rangel (2008) assume that daily unexpected returns can be described by a two component volatility model, i.e.

$$r_i - \mathbf{E}\left[r_i|\mathcal{F}_{i-1}\right] = \sqrt{g_i\tau_i}Z_i, \tag{1.1}$$

where r_i are daily log returns, $\mathcal{F}_i$ is the information set available at day i, $Z_i \overset{iid}{\sim} (0,1)$, g_i is a unit GARCH process and τ_i is an exponential spline function. While the short-term volatility component g_i represents the well-known day-to-day clustering of volatility, the smooth long-term component τ_i reflects the state of the macroeconomy. Hence, Eq. (1.1) illustrates how the same piece of news can have strong or weak effects on unexpected returns depending on the level of τ_i.

The drawback of the Spline-GARCH is that it is not straightforward to incorporate information on the macroeconomy into the long-term component, since the macro variables are typically observed at a lower frequency than the daily returns. Engle and Rangel (2008) opt for a two-step strategy. In a first step, they estimate the model at a daily frequency and then aggregate τ_i to a yearly frequency. In a second step, the aggregated long-term component is regressed on a set of macroeconomic variables. For a panel of nearly 50 countries, Engle and Rangel (2008) show that τ_i behaves counter-cyclically, i.e. it is high during recessions and low during boom phases.

Since we intend to directly model the effects of the macro variables on long-term volatility, we rely on the GARCH-MIDAS model proposed in Engle et al. (2013). This approach allows us to combine daily return data with a long-term volatility component that is entirely driven by the evolution of low-frequency macro variables. We employ a variant of the model which assumes that the long-term component changes at the same frequency that the macro variables are observed. In the following, we use the notation

$$r_{i,t} - \mathbf{E}\left[r_{i,t}|\mathcal{F}_{i-1,t}\right] = \sqrt{g_{i,t}\tau_t} Z_{i,t}, \tag{1.2}$$

where $t = 1, \ldots, T$ denotes a particular period, e.g. a quarter, and $i = 1, \ldots, N^{(t)}$ the days within that period. Daily expected returns are assumed to be constant, i.e. we set $\mathbf{E}\left[r_{i,t}|\mathcal{F}_{i-1,t}\right] = \mu$ for all i and t. The short-term component follows a mean-reverting asymmetric unit GARCH process

$$g_{i,t} = (1 - \alpha - \beta - \gamma/2) + \left(\alpha + \gamma \cdot \mathbb{1}_{\{r_{i-1,t}-\mu<0\}}\right) \frac{\left(r_{i-1,t} - \mu\right)^2}{\tau_t} + \beta g_{i-1,t}, \tag{1.3}$$

with $\alpha > 0$, $\beta > 0$ and $\alpha + \beta + \gamma/2 < 1$. That is, the choice of the constant in Eq. (1.3) ensures that $\mathbf{E}\left[g_{i,t}\right] = 1$.

Following Engle et al. (2013), we consider two alternative versions of the long-term component. In the basic version, long-term volatility is modeled as the weighted average of the *lagged* values of an explanatory variable X_t. We will refer to this version as a *one-sided filter*. Alternatively, the extended version specifies long-term volatility as the weighted average of *past*, *present*, and *future* values of the explanatory variable. This specification corresponds to a *two-sided filter* and will be discussed in Section 1.4.2. In both cases, we opt for modeling $\log(\tau_t)$ rather than τ_t itself which ensures the positivity of the long-term component. We refer to models with macroeconomic explanatory variables as GARCH-MIDAS-X. Our benchmark specification employs quarterly realized volatility as an explanatory variable and is labeled as

GARCH-MIDAS-RV.

The one-sided version of the long-term component is given by[1]

$$\log(\tau_t) = m + \theta \sum_{k=1}^{K} \varphi_k(\omega_1, \omega_2) X_{t-k}, \tag{1.4}$$

with Beta weighting scheme

$$\varphi_k(\omega_1, \omega_2) = \frac{(k/(K+1))^{\omega_1 - 1} \cdot (1 - k/(K+1))^{\omega_2 - 1}}{\sum_{j=1}^{K} (j/(K+1))^{\omega_1 - 1} \cdot (1 - j/(K+1))^{\omega_2 - 1}}. \tag{1.5}$$

Even for a large K, Eq. (1.5) parsimoniously specifies the weights φ_k which are completely determined by the two parameters ω_1 and ω_2. By construction, $\varphi_k \geq 0$ for $k = 1, \ldots, K$ and $\sum_{k=1}^{K} \varphi_k = 1$. For $\omega_1 = \omega_2 = 1$, the weights are equal, i.e., $\varphi_k = 1/K$ for all k. The restriction $\omega_1 = 1$, $\omega_2 > 1$ guarantees a decaying pattern, i.e., the maximum weight is at the first lag. The rate of decay is then determined by ω_2, whereby large values of ω_2 generate a rapidly decaying pattern and small values generate a slowly decaying one. In contrast, the unrestricted scheme can generate hump-shaped or convex weights.[2] Whether an unrestricted weighting scheme leads to a significant improvement relative to the restricted one can be assessed by means of a likelihood ratio test (LRT). The maximum number K of lags to be included is chosen through information criteria. Note that if we restrict θ to zero, the long-run component remains constant. As a consequence, the GARCH-MIDAS-X specification nests the asymmetric GARCH(1,1) process with unconditional variance equal to $\exp(m)$.

Finally, we consider one-period-ahead volatility forecasts. Since at the beginning of period t the long-term volatility τ_t is predetermined with respect to $\mathcal{F}_{N^{(t-1)},t-1}$, the volatility forecast for a specific day i within period t is given by

$$\mathbf{E}\left[g_{i,t}\tau_t Z_{i,t}^2 | \mathcal{F}_{N^{(t-1)},t-1}\right] = \tau_t \mathbf{E}\left[g_{i,t} | \mathcal{F}_{N^{(t-1)},t-1}\right]. \tag{1.6}$$

Since $\mathbf{E}\left[g_{i,t}|\mathcal{F}_{N^{(t-1)},t-1}\right] = 1 + (\alpha + \beta + \gamma/2)^{i-1}(g_{1,t} - 1)$ converges to unity, i.e. to the unconditional variance of $g_{i,t}$, the forecast approaches the long-term component

[1] In order to differentiate between the long-term components of the GARCH-MIDAS-X and GARCH-MIDAS-RV models we also use the notation τ_t^X and τ_t^{RV}.

[2] For a more detailed discussion of the Beta weighting scheme see Ghysels et al. (2005).

for i large. The volatility forecast for period t is then given by

$$\mathbf{E}\left[\sum_{i=1}^{N^{(t)}} g_{i,t}\tau_t Z_{i,t}^2 \middle| \mathcal{F}_{N^{(t-1)},t-1}\right] = \tau_t\left(N^{(t)} + (g_{1,t}-1)\frac{1-(\alpha+\beta+\gamma/2)^{N^{(t)}}}{1-\alpha-\beta-\gamma/2}\right). \quad (1.7)$$

Clearly, if $g_{1,t}$ is equal to its unconditional expectation, the period t forecast would be $\tau_t N^{(t)}$, which resembles the square-root-of-time rule. For a more than one-period-ahead prediction, one needs to forecast the long-term component itself. We will come back to this issue in Section 1.4.3.

We estimate the model parameters via quasi-maximum likelihood. The asymptotic normality of the quasi-maximum likelihood estimator for a 'rolling window' version of the GARCH-MIDAS-RV has been established in Wang and Ghysels (2015). To the best of our knowledge, asymptotic results for the general GARCH-MIDAS-X model are not yet available. However, we performed a Monte Carlo analysis that suggests standard asymptotic inference is also valid for this specification.

1.3 Data

In the empirical analysis, we focus on the S&P 500 and U.S. macroeconomic data for the 1969 to 2011 period. We consider daily stock returns and combine (first release) macroeconomic data with the corresponding SPF expectations. The survey data are obtained from the database at the Federal Reserve Bank of Philadelphia, while all other data are obtained from the FRED database at the Federal Reserve Bank of St. Louis.

Since the SPF data are only available at a quarterly frequency and we intend to employ filters that combine first release with expectations data, we consider all variables at this frequency. That is, for data that are available at a monthly or daily frequency, we take quarterly averages of the levels. For completeness, in Section 1.4.4 we provide estimation results based on monthly macro data and show that the choice of frequency for the macro variables does not affect our main results.

Stock market data: We consider continuously compounded daily S&P 500 stock return data, $r_{i,t}$, from January 2nd 1969 to December 30th 2011. Quarterly realized volatility is calculated as $RV_t = \sum_{i=1}^{N^{(t)}} r_{i,t}^2$.

Macroeconomic data: Data revisions can be substantial for macroeconomic variables. Thus, employing revised instead of first release data can be misleading when it comes to forecast evaluation (see, for example, Stark, 2010). To obtain a realistic evaluation of the various GARCH-MIDAS-X models, we try to match the information that is used in our econometric specification with the one that was available to market participants. That is, for those macro variables that undergo revisions, we employ the advance estimates as published by the Bureau of Economic Analysis.[3]

We employ the following macroeconomic variables: real GDP, industrial production, the unemployment rate, housing starts, nominal corporate profits after tax, real personal consumption, the Chicago Fed national activity index (NAI), the new orders index of the Institute for Supply Management, and the University of Michigan consumer sentiment index. We include the NAI and the new orders index in levels and take the first difference of the respective level for the unemployment rate and the consumer sentiment index. For all other variables, we calculate annualized quarter-over-quarter percentage changes as $100 \cdot ((X_t/X_{t-1})^4 - 1)$. Inflation is measured as the annualized quarter-over-quarter percentage change in the GDP (chain-type) deflator. Finally, to account for interest rate developments, we calculate the term spread as the difference between the 10-year Treasury bond yield and the 3-month T-bill rate.

When dealing with two-sided filters in Section 1.4.2, we also employ forecasts of the macro variables that are based on the following AR(4) model

$$X_t = \sum_{i=1}^{4} \delta_i D_{it} + \sum_{i=1}^{4} \phi_i X_{t-i} + \xi_t, \tag{1.8}$$

where D_{it} are seasonal dummies. This regression is used in Schwert (1989) and Engle et al. (2013) to measure the volatility of a macro variable X_t by the squared residual $\hat{\xi}_t^2$.

Summary statistics for all variables can be found in Table 1.1, while the corresponding times series are plotted in Fig. 1.1. According to standard unit roots tests all series can be considered as being stationary.

Expectations data: We employ expectations data only for those variables that were included in the SPF dataset during our full sample period (see the last column of Table 1.1). The survey is conducted after the release of the Bureau of Economic Analysis's advance report, i.e., survey participants know the first release data for the *previous* quarter when they make their predictions. For each variable, we consider

[3] Nevertheless, our evaluation is not fully in real-time. This is because for some variables – such as real GDP – the advance estimate of X_{t-1} is published in quarter t and therefore not included in the information set $\mathcal{F}_{N^{(t-1)},t-1}$.

the median expectation for the quarter in which the survey is conducted, denoted by $X_{t|t}^{SPF}$, and forecasts up to four-quarters-ahead, denoted by $X_{t+k|t}^{SPF}$, $k = 1, \ldots, 4$.[4]

1.4 Empirical results

In Section 1.4.1 we first present estimation results for various one-sided GARCH-MIDAS specifications and then turn to two-sided models in Section 1.4.2. The out-of-sample forecast performance of the one- and two-sided models is investigated in Section 1.4.3. To complete our model comparisons, Section 1.4.4 provides several extensions and robustness analyses.

1.4.1 One-sided filters

For the time being, we consider one-sided filters. In Section 1.4.1, we first confirm the counter-cyclical behavior of long-term volatility for a broad set of macro variables. We then focus on the lead-lag-structure between the macro variables and stock market volatility and identify variables that require flexible unrestricted filters and, hence, lead long-term volatility. In Section 1.4.1, we analyze the question whether macro variables still contain predictive information on long-term volatility once one controls for lagged realized volatility.

The lead-lag-structure between macro variables and volatility

Estimation results for the parameters of the long-term volatility component of the various one-sided GARCH-MIDAS-X models are summarized in Table 1.2. An extended version of the table containing all parameter estimates can be found in the Appendix. For each macro variable, the first/second line presents the estimates for the restricted/unrestricted weighting scheme. We choose $K = 12$ for all variables which corresponds to three MIDAS lag years.[5] To ensure comparability across the one- and two-sided models (see Section 1.4.2), as well as models based on macroeconomic uncertainty measures (see Section 1.4.4), all models are estimated based on daily return data for the 1973Q1 to 2010Q4 period and quarterly macro data from

[4]There are a few missing observations of the four-quarters-ahead forecasts at the beginning of the sample. Analogously to Eq. (1.8), we estimate $X_{t+4|t}^{SPF} = \sum_{i=1}^{4} \delta_i D_{it} + \sum_{i=1}^{4} \phi_i X_{t+4-i|t}^{SPF} + \xi_t$ using the available data and replace the missing observations by the predictions $\hat{X}_{t+4|t}^{SPF}$.

[5]As long as the selected K is large enough, we find the estimation results to be robust with respect to the specific choice of the maximum number of lags included.

1970Q1 onwards.[6] The table also reports the estimates for the GARCH-MIDAS-RV as well as the one-component GARCH(1,1) model.

First, we note that the estimates of the GARCH parameters $(\mu, \alpha, \beta, \gamma)$ are significant at the 1% level in all cases (see the Appendix). The estimates of α and β take the typical values and, consistent with the leverage effect, the estimate of the parameter γ is found to be positive.

Next, we have a closer look at the estimates of the long-run component τ_t. For all variables except the GDP deflator the estimated θ is highly significant and has the expected sign. For example, for real GDP, the estimated θ is negative, meaning that an increase in the growth rate is associated with a decline in long-term volatility. Conversely, the positive θ for the unemployment rate indicates that a rise in unemployment is associated with higher long-term volatility. That is to say, in all cases the sign of the scale parameter confirms the counter-cyclical property of long-run volatility as observed in Engle and Rangel (2008) and Engle et al. (2013).

In Fig. 1.2 we plot the estimated restricted and unrestricted weighting schemes for the different macro variables. For six out of the eleven macro variables, e.g. the unemployment rate or the NAI, both schemes are declining from the beginning with almost identical shapes. As one would expect, for these variables a LRT (see Table 1.2) does not reject the constraint ($\omega_1 = 1$) imposed by the restricted scheme. In sharp contrast, for housing starts, the GDP deflator, consumer sentiments, real consumption, and the term spread the unrestricted schemes are hump-shaped and clearly different from the restricted ones. For these variables, the restricted scheme appears to be clearly misspecified. For example, for the term spread the unrestricted filter takes its maximum weight at a lag of five quarters, while the restricted scheme is characterized by an almost linear decay. Only in case of the GDP deflator, we find an extreme and somewhat unreasonable weighting scheme, putting almost all weight on the fifth lag. In line with these considerations, the LRT (see Table 1.2) rejects the constraint that $\omega_1 = 1$ for these five variables.[7] Accordingly, we classify all variables that are characterized by hump-shaped weights as leading with respect to long-term volatility. Finally, note that the optimal weighting scheme for the GARCH-MIDAS-RV model is the restricted one.

[6] The first year of macro data (1969Q1-Q4) is used to construct ex-post macro volatility measures based on the AR(4) model in Eq. (1.8).

[7] Although the LRT rejects the restricted weighting scheme for the GDP deflator, the estimate of θ is only marginally significant in the unrestricted filter. That is, the GDP deflator hardly explains any time variation in the conditional variance of the S&P 500 returns. Hence, all subsequent results with respect to the GDP deflator should be taken with a grain of salt.

The finding that some variables are leading with respect to stock market volatility while others are not is economically plausible. Variables such as industrial production (the unemployment rate) are typically considered as coincident (lagging) indicators for the business cycle. For these variables the most recent observations appear to matter most for predicting the counter-cyclical long-term volatility. In contrast, the term spread or housing starts are usually considered as leading indicators.[8] For example, Estrella and Hardouvelis (1991), Estrella and Mishkin (1998) and Ang et al. (2006), among others, provide evidence that the term spread is a powerful predictor of future economic activity and recessions. The predictive ability of the term spread is typically explained by the term spread's relation to investors expectations about future economic activity, demand for credit and monetary policy (see, e.g., Estrella and Trubin, 2006). Similarly, Leamer (2007) and Kydland et al. (2012) show that housing starts lead real GDP. According to Kydland et al. (2012), the leading property of housing starts can be rationalized by the empirical observation of low interest rates for mortgages that precede economic upturns. Our results suggest that variables which lead the business cycle are also leading with respect to financial volatility and, therefore, require unrestricted weighting schemes.

Fig. 1.3 shows the quarterly aggregated long-term component, $\sqrt{N^{(t)}\tau_t^X}$, and the quarterly conditional volatility, $\sqrt{\tau_t^X g_t^X}$ with $g_t^X = \sum_{i=1}^{N(t)} g_{i,t}^X$, for all GARCH-MIDAS-X models along with the realized volatility, $\sqrt{RV_t}$. The figure clearly shows the negative relation between $\sqrt{RV_t}$ and economic activity. The long-term components of all macro variables, except the GDP deflator, mirror this counter-cyclical pattern of stock market volatility.[9] Nevertheless, there are also distinct differences. While the long-term component of the term spread typically increases in advance of a recession, the long-term components of other variables, e.g. real GDP, seem to increase during recessions. Finally, the long-run volatility component of the GARCH-MIDAS-RV model is dominated by the 1987 stock market crash and the recent financial crisis. It hardly increases during the other recession periods.

Next, we compare the fit of the various models by means of the Bayesian information criterion (BIC). According to the BIC, the GARCH-MIDAS-X models based on housing starts, corporate profits, the NAI, new orders, and the term spread are

[8]Our classification of leading vs. coincident/lagging variables is in line with the fact that the yield spread, housing permits, and also consumer expectations are included in the Conference Board's *leading* economic index for the US, while industrial production is included in the *coincident* index.

[9]In line with the only weakly significant estimate of θ for the GDP deflator, the corresponding long-term component is rather flat.

preferred to the GARCH-MIDAS-RV (and to the nested GARCH specification).

From an economic point of view, it is important to know how much of the variation in the expected quarterly variance of a specific GARCH-MIDAS-X model can be attributed to the variation in the corresponding macro variable (see Engle et al., 2013, p.794). In order to answer this question, we provide the value of a variance ratio (VR) statistic for each model. In general, we let the VR be defined as the fraction of the sample variance of the log of total quarterly conditional volatility, $\widehat{\mathbf{Var}}(\log(\tau_t^X g_t^X))$, that can be explained by the sample variance of the log long-term component, $\widehat{\mathbf{Var}}(\log(\tau_t^X))$. For easier comparison across the various GARCH-MIDAS-X models, we report

$$\mathrm{VR}(X) = \frac{\widehat{\mathbf{Var}}(\log(\tau_t^X))}{\widehat{\mathbf{Var}}(\log(\tau_t^{RV} g_t^{RV}))}, \tag{1.9}$$

which relates the sample variance of the log of the long-term component of a specific GARCH-MIDAS-X model to the sample variance of the log of the total expected variance of the *baseline* GARCH-MIDAS-RV model with restricted filter.[10] It is important to note that a small VR does not necessarily imply a poor model fit, since a low $\widehat{\mathbf{Var}}(\log(\tau_t^X))$ can also be an indication of smooth movements in the underlying macro variable. However, in the extreme case, where $\widehat{\mathbf{Var}}(\log(\tau_t^X)) \approx 0$, the long-term component is constant and the GARCH-MIDAS-X reduces to the simple GARCH model. Since τ_t^X dominates the multi-day/period-ahead volatility forecast (see Eq. (1.7)), it is clear that only GARCH-MIDAS-X specifications with high VRs have the potential to outperform the simple GARCH model.

As Table 1.2 shows, the model based on housing starts (unrestricted weighting scheme) achieves the highest VR. Roughly 22% of the variation in expected quarterly volatility is explained by housing starts. The specifications including new orders and the term spread (unrestricted weighting scheme) rank second and third. Most importantly, the models based on these variables achieve higher VRs than the benchmark GARCH-MIDAS-RV model. Interestingly, these are also the three models with the lowest BIC. As expected, the VR for the model based on the GDP deflator is by far the lowest.

[10] Although using $\widehat{\mathbf{Var}}(\log(\tau_t^{RV} g_t^{RV}))$ instead of $\widehat{\mathbf{Var}}(\log(\tau_t^X g_t^X))$ in the denominator does simplify the comparison across models, we verified that it does not affect the ranking of models.

Combining macro information with realized volatility

The previous results suggest that macro variables carry information about long-term volatility that is complementary to that contained in realized volatility. To formally investigate whether the various macroeconomic variables have additional explanatory power over realized volatility, we modify the long-term component by including RV_t and a macro variable, X_t, at the same time:

$$\log(\tau_t) = m + \theta^{RV} \sum_{k=1}^{K} \varphi_k(\omega_1^{RV}, \omega_2^{RV}) RV_{t-k} + \theta^X \sum_{k=1}^{K} \varphi_k(\omega_1^X, \omega_2^X) X_{t-k}. \qquad (1.10)$$

We refer to this model as GARCH-MIDAS-RV-X. It nests both the GARCH-MIDAS-RV and the GARCH-MIDAS-X model. In line with our previous findings we set $\omega_1^{RV} = 1$, but estimate both restricted and unrestricted weighting schemes for the macro variables.

Estimation results of the relevant parameters are presented in Table 1.3. The scaling parameter associated with realized volatility, θ^{RV}, is positive and significant at the 1% level across all models. Most importantly, the parameter associated with the macro variables, θ^X, is significant for all variables (at least in one of two specifications) except for the unemployment rate and real consumption. Once again, the LRT rejects the restricted weighting scheme for housing starts, the GDP deflator, the consumer sentiment index, and the term spread. That is, even if we control for realized volatility, these variables are still leading with respect to long-term volatility. In summary, the results in Table 1.3 clearly demonstrate that most of the macroeconomic variables considered in our analysis expose information on stock market risk beyond that contained in past realized volatility. The results also reconfirm that the optimal weighting schemes differ from one macro variable to the other.[11]

1.4.2 Two-sided filters

Engle et al. (2013) have suggested that the performance of the GARCH-MIDAS-X model can be further improved by employing a two-sided filter of the type:

$$\log(\tau_t) = m + \theta \sum_{k=-K_{lead}}^{K_{lag}} \varphi_k(\omega_1, \omega_2) X_{t-k}. \qquad (1.11)$$

[11] Figures of the weighting schemes and the corresponding long-term components are shown in the Appendix.

Two-sided filters reflect the idea that current stock market volatility depends not only on the past, but also the future (expected) state of the macroeconomy. However, since the future values of the macro variables are unknown in $t-1$, the two-sided specification in Eq. (1.11) is *infeasible* in real-time. Instead, we consider a *feasible* version of the two-sided filter by replacing X_{t-k}, $k = -K_{lead}, \ldots, 0$, with forecasts that are based on information available in $t-1$. We consider both time series forecasts (TSF) that are constructed from the model given in Eq. (1.8) as well as the median forecasts from the SPF. For example, using the SPF forecasts the feasible two-sided filter is given by

$$\log(\tau_t) = m + \theta \sum_{k=1}^{K_{lag}} \varphi_k(\omega_1, \omega_2) X_{t-k} + \theta \sum_{k=-K_{lead}}^{0} \varphi_k(\omega_1, \omega_2) X^{SPF}_{t-k|t-1}. \tag{1.12}$$

Since the maximum forecast horizon of the SPF predictions is four-quarters-ahead, we choose $K_{lead} = 3$. As before, we set $K_{lag} = 12$. Finally, we consider a two-sided filter that is entirely based on SPF data (feasible two-sided SPF + SPF lags) and given by

$$\log(\tau_t) = m + \theta \sum_{k=1}^{K_{lag}} \varphi_k(\omega_1, \omega_2) X^{SPF}_{t-k|t-k} + \theta \sum_{k=-K_{lead}}^{0} \varphi_k(\omega_1, \omega_2) X^{SPF}_{t-k|t-1}. \tag{1.13}$$

Overall, we compare five specifications: the one-sided filter, the infeasible two-sided filter, the feasible two-sided TSF and SPF filters, and the entirely SPF based filter. For those macro variables for which SPF expectations are available for the full sample period, Table 1.4 presents the BICs as well as the VR statistics for the five specifications.[12] A comparison in terms of the BIC reveals that for all six macro variables the preferred two-sided specification achieves a BIC that is at least as low as the BIC of the one-sided model. While the differences between the models are typically small in terms of BICs, they become much more pronounced when looking at the VRs and, again, are in favor of the two-sided models. Most importantly, in all cases one of the feasible SPF based two-sided filters outperforms the one-sided specification.[13] This result is remarkable, because it illustrates the benefits of the feasible two-sided filters, which combine information on the current (and past) state

[12] Detailed estimation results can be found in the Appendix.

[13] In case of real GDP the entirely SPF based specification even outperforms the infeasible one. This finding is plausible since first release data for real GDP are often substantially revised subsequently (see, e.g, Croushore, 2011) and, hence, stock market volatility might be more closely related to expectations data than to first release data.

of the economy with expectations about future macroeconomic conditions. The improvements in the VRs may be rationalized by the argument that the survey expectations are closely related to expectations on future dividends and returns and, therefore, drive long-term volatility. This interpretation supports Campbell and Diebold's (2009) conclusion that expectations data predict future expected returns because they carry information about future volatility and perceived time-varying risk. Finally, for all but one variable the TSF based specification performs worst amongst the two-sided models. Given Stark's (2010, p.2) finding that the SPF "projections generally outperform the benchmark projections of univariate autoregressive time-series models", this result is not surprising.

In order to better understand why for some variables the VRs more than double when two- instead one-sided filters (e.g. the unemployment rate) are employed, it is insightful to compare the estimated weighting schemes for the one-sided, infeasible two-sided, and preferred feasible (in terms of the VR) specifications. Figure 1.4 shows that for all variables for which the VRs are greatly improved when using a two-sided filter, the optimal one-sided filter was the restricted one. That is, for all variables which are coincident (lagging), i.e. the recent (future) observations are most important, switching from one- to two-sided filters leads to sizable increases in the VRs. Clearly, for those variables the availability of the SPF expectations is most valuable. In sharp contrast, for the leading variables housing starts and GDP deflator the optimal one-sided filter is the unrestricted one and, hence, two-sided filters apply very little weight to future values. Also, note that for both variables the weights of the feasible two-sided filters are almost identical with those of the infeasible ones. This explains why we observe much smaller differences in the VRs of the one- and two-sided models for these two variables.

1.4.3 Forecast evaluation

Next, we analyze the out-of-sample forecast performance of the various GARCH-MIDAS-X models. We focus on the one-sided specifications from Section 1.4.1 as well as the best (in terms of the VR statistic) feasible SPF based two-sided models from Section 1.4.2.

As discussed in Section 1.2, forecasts of the $g_{i,t}$ component can be obtained iteratively for any horizon given subsample parameter estimates. The one-quarter-ahead forecast of the long-term component, $\hat{\tau}_{t|t-1}$, is directly given by Eq. (1.4) or (1.12/1.13). For longer horizons, we assume that the long-term component remains at the level of the one-step prediction, i.e. we set $\hat{\tau}_{t+s|t-1} = \hat{\tau}_{t|t-1}$ for $s > 0$. Daily volatility forecasts are calculated as the product of the GARCH and the long-term component forecasts. The quarterly forecasts are given by the sum of the daily forecasts over the respective quarter.

We divide the full sample into an estimation period (based on daily return data from 1973Q1 to 1998Q4 and quarterly macroeconomic data from 1970Q1 on) and an out-of-sample period (1999Q1 - 2010Q4). We then evaluate one- up to four-quarters-ahead volatility forecasts over the 2000Q1 - 2010Q4 period by comparing the models' predicted volatilities with a series of realized volatilities based on 5-minute intra-day returns.[14,15] For each model and forecast horizon we present the parameter estimates of a Mincer-Zarnowitz (MZ) regression as well as the corresponding R^2. In order to compare the forecast performance of a specific GARCH-MIDAS-X relative to the benchmark GARCH-MIDAS-RV we report the ratio of the corresponding mean square errors (MSE). A ratio below one implies an improvement upon the benchmark model. Finally, we test for equal (unconditional) predictive ability over the benchmark model by means of the Giacomini and White (2006) test.

Table 1.5 presents the evaluation of the quarterly volatility forecasts. At the one-quarter-ahead horizon, the results are quite similar across different models. In general, estimates of the constant and slope parameter in the MZ regressions are not significantly different from zero and one. All models except the one for the term spread yield lower MSEs than the RV model, yet not significantly so. However, with increasing forecast horizons the differences among the model specifications become more evident.

At the 2-, 3- and 4-quarters horizons, many GARCH-MIDAS-X models significantly outperform the benchmark model. Interestingly, for industrial production and the unemployment rate, only the two-sided specifications do so. This finding squares with our results in Section 1.4.2, where we found the strongest improvements in the VRs for these two variables when using two- instead of one-sided filters. For housing starts and corporate profits the one- and two-sided filters basically have the same forecasting ability. Once again, this is in line with Section 1.4.2 where we found only modest improvements in the VRs for both variables. The models based on the NAI as well as consumer sentiment significantly outperform the RV based model at the 2-, 3- and 4-quarters-ahead horizons, while the models based on the

[14] Since the first four-quarters-ahead volatility forecast is constructed in 1999Q1 (i.e. in the first out-of-sample period) for 2000Q1, the evaluation period begins in 2000Q1.

[15] The intra-day returns are available at the website of the Oxford-Man Institute of Quantitative Finance.

term spread and new orders do so for 3- and 4-quarters-ahead only.

Why does the performance of most GARCH-MIDAS-X models relative to the RV model improve when increasing the forecast horizon? This question can be addressed by investigating the behavior of the daily volatility forecasts of the various models at different horizons. Exemplary, we focus on the h = 1-, 126-, and 252-days-ahead volatility forecasts. Although, in practical applications one would rarely be interested in the 126- or 252-days-ahead forecasts in isolation (but rather in forecasts of volatility over certain periods), these forecasts illustrate how the dominance of the short- versus the long-term component varies with the forecast horizon.

We evaluate the daily forecasts for a rolling window of 500 observations (i.e. approximately two years of data) again using the Giacomini and White (2006) test. For each macro variable, Figure 1.5 presents the evolution of the corresponding t-statistics for the one- and two-sided models at the three forecast horizons. Each data point refers to a test statistic for a sample *ending* at that point in time. Shaded areas refer to samples that include observations from recession periods. Since the out-of-sample period begins in 2000, we depict the t-statistics from 2002 onwards and include 'recession shadings' until November 2003 (for the March - November 2001 recession) and from December 2007 until the end of the sample (for the December 2007 - June 2009 recession).

First, at least from mid-2004 onwards, for almost all macro variables the t-statistics tend to increase with the forecast horizon under consideration. For example, for the term spread the line which corresponds to $h = 252$ is almost permanently above the one corresponding to $h = 1$. For the 1-day-ahead forecasts the Giacomini and White (2006) test typically neither favors the GARCH-MIDAS-X nor the RV model, i.e. the test statistic is either insignificant or significant but with varying sign over the different sample periods. Since these forecasts are largely determined by the short-term component (which is very similar for both specifications), this result is not surprising. In stark contrast, for $h = 252$ the macro variable based specifications often significantly outperform the RV model. Since predictions of the $g_{i,t}$ component converge to one with an increasing forecast horizon, the long-term component dominates the daily volatility forecast for $h = 252$. Hence, our finding suggests that the long-term components of the GARCH-MIDAS-X models are better than the long-term component of the RV model in anticipating the future level of volatility and, thereby, confirms our interpretation of Table 1.5.

Second, Figure 1.5 illustrates the benefits of the feasible two-sided filters when applied to coincident/lagging variables. At each forecast horizon, the lines which represent the t-statistics for the two-sided filters are typically above the ones for the one-sided filters, i.e. the SPF based feasible two-sided filters improve upon the purely backward-looking one-sided filters. This effect appears to be stronger at longer forecast horizons and, as expected, is most pronounced for industrial production, the unemployment rate and, to some extent, real GDP. On the contrary, for the leading variables, i.e. housing starts and the GDP deflator, the one- and two-sided filters lead to basically the same t-statistics for a given forecast horizon.

Third, even if the focus lies on long-term forecasting ($h = 126$ and $h = 252$) the relative forecasting performance of the GARCH-MIDAS-X models varies considerably over time. The GARCH-MIDAS-X models clearly outperform the RV model in between both recession periods. Also, with the onset of the financial crisis their forecast performance improves relative to the one of the RV model. From mid 2008 onwards, basically all GARCH-MIDAS-X specifications significantly outperform the RV model. The latter finding is in agreement with Paye (2012) and Dorion (2013), who conclude that macroeconomic variables are of greater importance around recessions. In contrast, at the beginning of the evaluation period and during a short episode preceding the recent financial crisis the macro based specifications are at best at par with the RV model.

We also evaluated the forecast performance of the GARCH-MIDAS-RV-X models considered in Section 1.4.1. Although these models generally achieved higher VRs (see Table 1.3) than the corresponding GARCH-MIDAS-X and GARCH-MIDAS-RV models, their forecast performance does not significantly differ from the one of the benchmark RV model. While this finding might be surprising at first sight, a visual inspection of the corresponding long-term volatility components suggests a simple explanation. During periods of high volatility such as the 1987 stock market crash or the recent financial crisis the long-term component of the combined RV-X specification essentially behaves like the one of the RV model. Since the long-term components dominate the predictions of the GARCH-MIDAS model over longer horizons, it is not surprising that we do not find significant differences in the forecast performance of the RV-X and the RV model for our evaluation sample. For details see the Appendix.

Finally, although the forecasting results of the GARCH-MIDAS-X model are encouraging in terms of statistical significance, a more direct approach might be more informative concerning the potential economic gains from using the two component specification. Dorion (2013) provides a first application to long-term option pricing and finds evidence that accounting for business conditions reduces option-pricing errors. Since this is beyond the scope of the current paper, we look forward to evaluate the performance of GARCH-MIDAS-X based volatility predictions in long-term risk management or portfolio choice settings in future work.

1.4.4 Extensions and robustness

In this section we extend our previous results by including the first two principal components of the macro variables and measures of macroeconomic uncertainty as explanatory variables. We then take a fresh look at predictive regressions. Finally, we provide some robustness checks with respect to the sample period and frequency of the macro variables. All tables and figures related to this section can be found in the online Supplementary Appendix.

Principal components: Instead of estimating a separate GARCH-MIDAS-X model for each macro variable, we use the first two principal components of the macro variables as explanatory variables. This approach has for instance been adopted in Asgharian et al. (2013). The first principal component is (contemporaneously) highly correlated with the NAI, real GDP, and new orders. For the second principal component, we find the highest correlation with the term spread. The estimates of the θ coefficients are significant in all principal component based specifications. Consistent with our previous finding that the term spread is a leading variable, the results suggest an unrestricted weighting scheme for the second component, but not for the first one. The forecast evaluation results confirm our conclusion that the relative performance of the macro models over the RV model enhances with increasing forecasting horizon. Still, we neither find in-sample (in terms of BICs or VRs) nor out-of-sample (in terms of MSE ratios) improvements of the principal component based models over the *best* one- and two-sided specifications from Sections 1.4.1 and 1.4.2.

Ex-ante survey disagreement vs. ex-post volatility: Engle and Rangel (2008) and Engle et al. (2013) have empirically investigated whether there is a link between long-term volatility and the uncertainty about macroeconomic fundamentals.[16] Following Schwert (1989), they proxy the uncertainty associated with a macro variable with the squared residual from the regression given by Eq. (1.8). We conjecture that survey based *ex-ante* uncertainty measures are more appropriate than such *ex-post* regression based volatility proxies (see also David and Veronesi, 2013,

[16] For economic models that rationalize a link between stock market volatility and macroeconomic uncertainty see Veronesi (1999) and Bollerslev et al. (2009), among others.

and Arnold and Vrugt, 2008). As an ex-ante measure we employ the disagreement among the SPF forecasters as described by the interquartile range of the individual point forecasts. To analyze whether the potential effect of disagreement varies with the forecast horizon, we employ the disagreement concerning forecasts of the current quarter as well as up to four-quarters-ahead.

We estimate the GARCH-MIDAS-X models including both types of uncertainty measures. While $\hat{\theta}$ is insignificant in all specifications based on the ex-post volatility measure, it is significant for various specifications based on the ex-ante survey disagreement. In particular, we find significant effects regarding the future development of the unemployment rate. As one would expect, the estimated coefficients suggest that higher dispersion in expectations increases stock market volatility.[17] For all variables, the BIC favors one of the disagreement based specifications. Our results suggest that *ex-ante* uncertainty measures are more informative of long-term risks than *ex-post* measures of volatility. Nevertheless, the VRs of the disagreement based specifications are generally lower than the ones for the variables in levels.

Predictive regressions: Our findings can be viewed as being complementary to recent research on the determinants of the secular component of financial volatility such as Christiansen et al. (2012) and Paye (2012). For example, Christiansen et al. (2012) focus on predictive regressions of the type

$$\log(\sqrt{RV_t}) = c + \rho \log(\sqrt{RV_{t-1}}) + \theta X_{t-1} + \zeta_t, \tag{1.14}$$

and find that – controlling for $\log(\sqrt{RV_{t-1}})$ – financial variables appear to be more important predictors of volatility than macroeconomic variables.

The parameter estimates of Eq. (1.14) for the different macro variables in our dataset imply that none of the variables has explanatory power for realized volatility. Similarly, the R^2s of the models that are augmented with macro variables are only marginally higher than the R^2 of the AR(1) benchmark model. Next, we estimate a version of Eq. (1.14) in which X_{t-1} is replaced by $\log(\sqrt{N^{(t)}\hat{\tau}_t^X})$, i.e. by the (log of the scaled) estimated long-term component of the respective GARCH-MIDAS-X model. We find that most long-term components have significant explanatory power for log realized volatility, some even at the 1% level. In addition, we now observe sizable increases in the R^2s compared to the pure AR(1) model. The largest increase is found for the term spread. Thus, by including $\hat{\tau}_t^X$ – which parsimoniously

[17]The only exception is corporate profits for which we obtain a counterintuitive negative sign in some specifications.

summarizes the information about the lead-lag-structure between a specific macro variable and financial volatility – we find more promising evidence for the relevance of the macro variables than from the original predictive regressions.

1973Q1 - 2007Q2 subsample: A potential objection against our findings might be that some of them could be driven by the recent financial crisis. Reestimating all models for a sample that ends in 2007Q2 provides convincing evidence that this is not the case. First, the estimated θ coefficients keep their sign and significance, i.e. the evidence in favor of the counter-cyclical behavior of long-term volatility is reconfirmed. Second, housing starts, the GDP deflator, consumer sentiment, real consumption, and the term spread are again identified as leading variables. Also, the combination of the macro variables with realized volatility does not alter our conclusions.

Monthly data: As mentioned in Section 1.3, some of our macro variables are available at the monthly frequency. Over the full sample period, these variables are industrial production, the unemployment rate, housing starts, the NAI, new orders, and the term spread. In order to analyze the robustness of our results with respect to the frequency of the macro variables, we estimate the GARCH-MIDAS-X and GARCH-MIDAS-RV-X models based on monthly data. The estimation results are again in line with our previous findings in Section 1.4.1. In particular, for all six variables the shape of the weighting scheme is robust with respect to the choice of the frequency.

1.5 Conclusion

This paper revisits the link between long-term financial volatility and the macroeconomic environment using the GARCH-MIDAS component model. In general, our results strongly confirm that long-term financial volatility behaves counter-cyclically. Our particular focus is on the lead-lag-structure between the macro variables and long-term volatility. First, we identify leading variables such as the term spread and housing starts for which the optimal one-sided filters are unrestricted ones. Second, for real GDP, industrial production, and the unemployment rate the most timely information is highly valuable and SPF based feasible two-sided filters considerably improve upon their one-sided counterparts. Hence, our findings highlight the potential role of expectations data in the modeling of stock market volatility. The empirical evidence suggests that long-term volatility is mainly driven by information related to the current state of the economy as well as to expectations regarding future macroeconomic conditions. In addition, we find convincing in- as well as out-of-sample evidence that macro variables contain information that is complementary to that included in lagged realized volatilities.

1.6 Tables and figures

1.6.1 Tables

Table 1.1: Descriptive statistics for stock market and macro data

Variable	Obs	Min	Max	Mean	SD	Skew.	Kurt.	AC(1)	SPF
Stock market data									
S&P 500 daily returns	10852	-22.90	10.96	0.02	1.09	-1.02	28.61	0.01	
S&P 500 quarterly RV	172	11.61	1143.40	74.75	121.92	6.40	51.11	0.38	
Macro data									
Δ real GDP	172	-10.37	11.16	2.44	3.20	-0.97	5.82	0.49	✓
Δ Ind. prod.	172	-29.03	21.16	2.15	6.66	-1.06	6.59	0.54	✓
Δ Unemp.	172	-0.97	1.77	0.03	0.38	1.32	6.62	0.50	✓
Δ Housing	172	-69.03	236.05	5.89	43.69	1.79	10.11	0.12	✓
Δ Corp. prof.	172	-70.81	180.27	12.33	29.69	1.51	10.05	0.13	✓
Δ GDP deflator	172	-0.33	13.69	3.73	2.64	1.21	4.21	0.83	✓
NAI	172	-3.41	1.92	-0.02	0.89	-1.41	6.24	0.73	✗
New orders	172	27.27	71.90	54.74	7.75	-0.75	4.03	0.74	✗
Δ Cons. sent.	172	-14.70	16.27	-0.16	5.37	0.11	3.61	-0.08	✗
Δ real cons.	172	-11.93	10.19	2.95	2.97	-1.25	7.43	0.08	✗
Term spread	172	-1.43	3.80	1.66	1.29	-0.42	2.29	0.88	✗

Notes: The dataset covers the sample from 1969Q1 to 2011Q4. The reported statistics include the number of observations (Obs), the minimum (Min) and maximum (Max), the mean, standard deviation (SD), Skewness (Skew.), Kurtosis (Kurt.), and the first order autocorrelation coefficient (AC(1)). The last column (SPF) shows whether the respective variable is available in the SPF dataset over the full sample period. In case of the unemployment rate and the consumer sentiment index Δ refers to the first difference of the respective levels. For all other variables, Δ refers to annualized quarter-over-quarter percentage changes as in $\Delta X_t = 100 \cdot ((X_t/X_{t-1})^4 - 1)$. The term spread is calculated as the difference between the 10-year Treasury bond yield and the 3-month T-bill rate. All macroeconomic variables included in the SPF dataset are obtained from the Federal Reserve Bank of Philadelphia. The remaining macro and stock market data are obtained from the Federal Reserve Bank of St. Louis.

Table 1.2: One-sided GARCH-MIDAS-X specifications

Variable	m	θ	ω_1	ω_2	LLF	BIC	VR(X)
Δ real GDP	0.1884	-0.0803^{***}	1	4.6508^{***}	−12789.02	2.6733	6.55
	(0.1562)	(0.0251)		(1.1950)			
	0.1921	-0.0823^{***}	1.4895	5.8531^{*}	−12788.93	2.6742	6.90
	(0.1558)	(0.0263)	(1.5532)	(3.0656)	[0.6641]		
Δ Ind. prod.	0.0769	-0.0434^{***}	1	4.5453^{***}	−12788.47	2.6732	7.57
	(0.1375)	(0.0133)		(1.2437)			
	0.0767	-0.0438^{***}	1.6441	6.3975^{**}	−12788.25	2.6741	8.02
	(0.1370)	(0.0129)	(1.3154)	(3.2453)	[0.5064]		
Δ Unemp.	−0.0317	0.5689^{***}	1	6.4943^{***}	−12789.96	2.6735	6.02
	(0.1365)	(0.1865)		(2.1923)			
	−0.0320	0.5751^{***}	1.7441	9.4221^{*}	−12789.82	2.6744	6.30
	(0.1369)	(0.1890)	(1.3159)	(5.1010)	[0.5991]		
Δ Housing	0.0736	-0.0159^{***}	1	1.8226^{***}	−12782.79	2.6720	14.39
	(0.1383)	(0.0049)		(0.2867)			
	0.0651	-0.0173^{***}	2.8071^{**}	4.8430^{*}	−12777.06	2.6718	21.85
	(0.1359)	(0.0047)	(1.4305)	(2.4845)	[0.0007]		
Δ Corp. prof.	0.2249	-0.0187^{***}	1	2.5114^{**}	−12783.30	2.6721	12.69
	(0.1550)	(0.0053)		(1.0048)			
	0.2284	-0.0191^{***}	1.1783^{**}	2.8187^{**}	−12783.20	2.6730	13.24
	(0.1538)	(0.0057)	(0.4851)	(1.3902)	[0.6469]		
Δ GDP deflator	−0.1017	0.0269	1	3.5702^{***}	−12795.21	2.6746	0.99
	(0.1887)	(0.0259)		(0.9506)			
	−0.1385	0.0357^{*}	114.1107^{***}	197.1066^{***}	−12793.47	2.6752	2.10
	(0.1751)	(0.0194)	(9.3931)	(4.5289)	[0.0618]		
NAI	−0.0305	-0.3085^{***}	1	7.7696^{**}	−12783.99	2.6722	12.84
	(0.1315)	(0.0728)		(3.0232)			
	−0.0305	-0.3081^{***}	1.1506	8.4179^{**}	−12783.98	2.6732	12.86
	(0.1315)	(0.0723)	(1.0892)	(3.9454)	[0.8890]		
New orders	2.6787^{***}	-0.0496^{***}	1	4.2905^{***}	−12776.96	2.6708	17.85
	(0.5637)	(0.0101)		(1.5680)			
	2.6904^{***}	-0.0498^{***}	0.9392	4.1023^{***}	−12776.96	2.6717	17.85
	(0.5563)	(0.0099)	(0.6626)	(1.5572)	[0.9776]		
Δ Cons. sent.	−0.0175	-0.1141^{***}	1	1.7135^{***}	−12789.08	2.6733	6.55
	(0.1361)	(0.0368)		(0.2502)			
	−0.0193	-0.1335^{***}	2.4732^{***}	4.0434^{**}	−12783.04	2.6730	12.63
	(0.1382)	(0.0338)	(0.6649)	(1.6654)	[0.0005]		
Δ real cons.	0.2952	-0.0992^{**}	1	3.6582^{***}	−12791.11	2.6737	5.04
	(0.2041)	(0.0422)		(1.1263)			
	0.3406^{*}	-0.1161^{***}	2.6571	8.5625^{*}	−12789.50	2.6743	7.70
	(0.1998)	(0.0413)	(1.9264)	(4.8275)	[0.0731]		
Term spread	0.4155^{***}	-0.2723^{***}	1	1.6276^{***}	−12779.46	2.6713	14.32
	(0.1482)	(0.0554)		(0.5485)			
	0.3658^{***}	-0.2443^{***}	4.2018^{*}	6.2756^{**}	−12777.47	2.6718	15.94
	(0.1388)	(0.0458)	(2.4167)	(3.0765)	[0.0463]		
RV	-0.2761^{**}	0.0033^{***}	1	3.7869	−12785.29	2.6725	12.96
	(0.1203)	(0.0009)		(6.7929)			
	-0.2956^{**}	0.0036^{***}	0.5746	2.3646	−12784.90	2.6734	13.74
	(0.1167)	(0.0010)	(0.6140)	(2.3155)	[0.3800]		
GARCH(1,1)	0.0049	-	-	-	−12796.04	2.6728	-
	(0.1553)						

Notes: The table reports estimation results for the one-sided GARCH-MIDAS-X models including 3 MIDAS lag years of a quarterly macro variable X, i.e, the long-run component is specified as

$$\log(\tau_t^X) = m + \theta \cdot \sum_{k=1}^{K} \varphi_k(\omega_1, \omega_2) X_{t-k},$$

with $K = 12$. All estimations are based on daily return data from 1973Q1 to 2010Q4 and include quarterly macroeconomic data from 1970Q1 on. The numbers in parentheses are Bollerslev-Wooldridge robust standard errors. ***, **, * indicate significance at the 1%, 5%, and 10% level. We estimate each model with a restricted ($\omega_1 = 1$) and an unrestricted weighting scheme. LLF is the value of the maximized log-likelihood function. The numbers in brackets are p-values from a likelihood ratio test $2(L_{UR} - L_R)$, where L_{UR} and L_R refer to the likelihood of the GARCH-MIDAS-X models with unrestricted and restricted weights, respectively. BIC is the Bayesian information criterion and VR(X) denotes the variance ratio statistic, see Eq. (1.9). An extended version of the table containing all parameter estimates can be found in the Appendix.

Table 1.3: One-sided GARCH-MIDAS-RV-X specifications

Variable	m	θ^{RV}	w_2^{RV}	θ^{X}	w_1^{X}	w_2^{X}	LLF	BIC	VR(X)
RV + Δ real GDP	-0.1238	$0.0030^{\star\star}$	2.9016	$-0.0550^{\star\star}$	1	$6.6800^{\star}$	-12780.01	2.6733	16.17
	(0.1393)	(0.0013)	(5.5773)	(0.0255)		(3.9329)			
	$0.0267^{\star\star\star}$	$0.0031^{\star\star}$	2.8773	-0.0515	0.1952	4.5847	-12779.90	2.6743	16.20
	(0.0086)	(0.0013)	(4.5726)	(0.0341)	(2.9720)	(4.4324)	[0.6365]		
RV + Δ Ind. prod.	-0.1706	$0.0025^{\star\star\star}$	6.3608	$-0.0293^{\star\star}$	1	$5.5697^{\star\star}$	-12779.90	2.6733	19.60
	(0.1261)	(0.0008)	(7.1541)	(0.0144)		(2.3289)			
	-0.1661	$0.0025^{\star\star\star}$	6.7958	$-0.0302^{\star\star}$	1.5168	7.0924	-12779.84	2.6742	19.95
	(0.1251)	(0.0008)	(6.6331)	(0.0149)	(1.9851)	(4.4797)	[0.7188]		
RV + Δ Unemp.	$-0.2415^{\star\star}$	$0.0025^{\star\star\star}$	6.1959	0.3403	1	8.5731	-12782.21	2.6738	17.43
	(0.1101)	(0.0008)	(6.4287)	(0.2195)		(5.8779)			
	$-0.2409^{\star\star}$	$0.0025^{\star\star\star}$	6.2585	0.3433	1.2331	$9.5429^{\star}$	-12782.21	2.6747	17.47
	(0.1116)	(0.0009)	(6.8029)	(0.2498)	(2.2599)	(5.6405)	[0.9642]		
RV + Δ Housing	-0.1379	$0.0021^{\star\star\star}$	$7.4497^{\star}$	$-0.0115^{\star\star}$	1	$2.0037^{\star\star\star}$	-12776.34	2.6726	23.50
	(0.1270)	(0.0008)	(4.4271)	(0.0049)		(0.4000)			
	-0.1144	$0.0018^{\star\star}$	$9.1967^{\star\star}$	$-0.0130^{\star\star\star}$	2.9770	5.4547	-12772.31	2.6727	27.34
	(0.1214)	(0.0008)	(4.0207)	(0.0049)	(1.8858)	(3.5738)	[0.0045]		
RV + Δ Corp. prof.	-0.1497	$0.0050^{\star\star\star}$	$2.7708^{\star}$	$-0.0228^{\star\star\star}$	1	$2.6169^{\star\star\star}$	-12758.03	2.6687	37.01
	(0.1068)	(0.0011)	(1.4246)	(0.0047)		(0.5637)			
	-0.1522	$0.0050^{\star\star\star}$	$2.6809^{\star\star}$	$-0.0230^{\star\star\star}$	$1.3462^{\star\star\star}$	$3.3148^{\star\star\star}$	-12757.38	2.6696	37.55
	(0.1075)	(0.0011)	(1.3553)	(0.0048)	(0.3607)	(1.0789)	[0.2559]		
RV + Δ GDP def.	$-0.4420^{\star\star}$	$0.0036^{\star\star\star}$	3.3946	0.0367	1	$3.9340^{\star\star\star}$	-12782.97	2.6739	13.88
	(0.1749)	(0.0011)	(5.3203)	(0.0233)		(1.1784)			
	$-0.4490^{\star\star}$	$0.0035^{\star\star\star}$	3.4273	$0.0388^{\star}$	$125.6962^{\star\star\star}$	$226.7650^{\star\star\star}$	-12781.53	2.6746	14.24
	(0.1793)	(0.0012)	(6.7086)	(0.0226)	(8.9971)	(9.6880)	[0.0893]		
RV + NAI	$-0.2224^{\star}$	$0.0023^{\star\star}$	6.5075	$-0.2006^{\star\star}$	1	12.5022	-12777.44	2.6728	22.16
	(0.1145)	(0.0009)	(7.5933)	(0.0846)		(10.0116)			
	$-0.2236^{\star}$	$0.0023^{\star\star}$	6.4544	$-0.2008^{\star\star}$	0.1002	$7.7109^{\star}$	-12777.40	2.6737	22.20
	(0.1184)	(0.0010)	(8.0210)	(0.0950)	(2.6180)	(3.9621)	[0.7785]		
RV + New orders	$1.8224^{\star\star\star}$	$0.0028^{\star\star}$	2.5878	$-0.0381^{\star\star\star}$	1	$5.8904^{\star}$	-12768.49	2.6709	25.34
	(0.6193)	(0.0011)	(3.0411)	(0.0112)		(3.3223)			
	$1.8749^{\star\star\star}$	$0.0028^{\star\star}$	2.6587	$-0.0391^{\star\star\star}$	0.5671	$4.1934^{\star}$	-12768.39	2.6719	25.62
	(0.6035)	(0.0011)	(3.2878)	(0.0109)	(0.7733)	(2.1694)	[0.6496]		
RV + Δ Cons. sent.	$-0.2842^{\star\star}$	$0.0032^{\star\star\star}$	3.0341	$-0.0918^{\star\star\star}$	1	$2.2077^{\star\star\star}$	-12778.05	2.6729	19.90
	(0.1233)	(0.0012)	(4.6700)	(0.0316)		(0.5215)			
	$-0.3047^{\star\star}$	$0.0035^{\star\star\star}$	2.1847	$-0.1010^{\star\star\star}$	$3.0780^{\star\star\star}$	$7.6379^{\star}$	-12772.21	2.6727	24.15
	(0.1332)	(0.0012)	(1.4167)	(0.0320)	(0.7868)	(4.0328)	[0.0006]		
RV + Δ real cons.	-0.1012	$0.0030^{\star\star\star}$	3.0498	-0.0497	1	$5.7145^{\star\star}$	-12783.22	2.6740	13.86
	(0.1637)	(0.0011)	(5.4317)	(0.0343)		(2.3934)			
	-0.0586	$0.0028^{\star\star\star}$	2.9261	-0.0611	2.6300	$11.7666^{\star}$	-12782.73	2.6748	14.35
	(0.1898)	(0.0010)	(5.4733)	(0.0462)	(2.1412)	(6.2251)	[0.3240]		
RV + Term spread	0.0663	$0.0040^{\star\star}$	3.1613	$-0.2625^{\star\star\star}$	1	$2.2372^{\star\star\star}$	-12760.40	2.6692	30.91
	(0.1185)	(0.0017)	(4.7966)	(0.0585)		(0.8248)			
	0.0207	$0.0041^{\star\star\star}$	2.8889	$-0.2425^{\star\star\star}$	$5.1551^{\star\star}$	$9.8622^{\star\star}$	-12757.05	2.6695	33.02
	(0.1098)	(0.0013)	(2.8781)	(0.0481)	(2.2550)	(4.0127)	[0.0096]		
RV	$-0.2761^{\star\star}$	$0.0033^{\star\star\star}$	3.7869	-	-	-	-12785.29	2.6725	12.96
	(0.1203)	(0.0009)	(6.7929)						

Notes: The table reports estimation results for the one-sided GARCH-MIDAS-RV-X models including 3 MIDAS lag years of quarterly realized volatility and a macro variable X. We include a restricted weighting scheme for the RV variable and both restricted and unrestricted weights for the macro variable, i.e, the long-run component is specified as

$$\log(\tau_t) = m + \theta^{RV} \cdot \sum_{k=1}^{K} \varphi_k(1, \omega_2^{RV}) RV_{t-k} \; + \; \theta^{X} \cdot \sum_{k=1}^{K} \varphi_k(\omega_1^{X}, \omega_2^{X}) X_{t-k},$$

with $K = 12$. Otherwise, see notes of Table 1.2.

Table 1.4: One- and two-sided GARCH-MIDAS-X specifications

Variable	Specification	BIC	VR(X)
Δ real GDP	1-sided	2.6733	6.55
	2-sided: infeasible	2.6733	12.88
	2-sided: feasible TSF	2.6739	7.93
	2-sided: feasible SPF (*)	2.6736	9.76
	2-sided: feasible SPF + SPF lags (*)	2.6724	14.08
Δ Ind. prod.	1-sided	2.6732	7.57
	2-sided: infeasible	2.6725	18.19
	2-sided: feasible TSF	2.6738	9.45
	2-sided: feasible SPF	2.6726	15.26
	2-sided: feasible SPF + SPF lags	2.6730	12.10
Δ Unemp.	1-sided	2.6735	6.02
	2-sided: infeasible	2.6725	20.18
	2-sided: feasible TSF	2.6744	6.39
	2-sided: feasible SPF	2.6725	15.62
	2-sided: feasible SPF + SPF lags	2.6728	12.71
Δ Housing	1-sided (ur)	2.6718	21.85
	2-sided: infeasible	2.6718	22.15
	2-sided: feasible TSF	2.6717	22.30
	2-sided: feasible SPF	2.6717	22.81
	2-sided: feasible SPF + SPF lags	2.6731	10.93
Δ Corp. prof.	1-sided	2.6721	12.69
	2-sided: infeasible	2.6727	16.23
	2-sided: feasible TSF (*)	2.6733	12.50
	2-sided: feasible SPF	2.6727	15.46
	2-sided: feasible SPF + SPF lags	2.6721	15.96
Δ GDP deflator	1-sided (ur)	2.6752	2.10
	2-sided: infeasible	2.6752	2.08
	2-sided: feasible TSF	2.6752	1.96
	2-sided: feasible SPF	2.6752	2.11
	2-sided: feasible SPF + SPF lags	2.6753	2.10

Notes: The table gives an overview over estimation results for the one- and two-sided GARCH-MIDAS-X specifications,

$$\log(\tau_t) = m + \theta \sum_{k=1}^{K_{lag}} \varphi_k(\omega_1,\omega_2)X_{t-k},$$

$$\log(\tau_t) = m + \theta \sum_{k=1}^{K_{lag}} \varphi_k(\omega_1,\omega_2)X_{t-k} + \theta \sum_{k=-K_{lead}}^{0} \varphi_k(\omega_1,\omega_2)\tilde{X}_{t-k},$$

with $K_{lag} = 12$ and $K_{lead} = 3$. All estimations are based on daily return data from 1973Q1 to 2010Q4 and include quarterly macro data from 1970Q1 on. We include a restricted weighting scheme ($\omega_1 = 1$) in the one-sided filter for all variables except for housing starts and the GDP deflator. The infeasible two-sided filter includes leads of the realized macro variable, i.e. $\tilde{X}_{t-k} = X_{t-k}$, whereas feasible filters are based on time series (TSF) or survey forecasts (SPF), i.e. $\tilde{X}_{t-k} = X^{TSF}_{t-k|t-1}$ or $\tilde{X}_{t-k} = X^{SPF}_{t-k|t-1}$. Finally we consider a specification which is entirely based on SPF data, see Eq. (1.13). For all specifications we report the BIC and the variance ratio, see Eq. (1.9). Detailed estimation results can be found in the Appendix.
(*) Due to convergence problems for $K_{lead} = 3$, we choose $K_{lead} = 2$ for these specifications.

Table 1.5: Quarterly RV Forecasting

Forecast horizon	1-quarter-ahead MZ-Regression			MSE	2-quarters-ahead MZ-Regression			MSE	3-quarters-ahead MZ-Regression			MSE	4-quarters-ahead MZ-Regression			MSE
Variable	c	ϕ	R^2	**ratio**	c	ϕ	R^2	**ratio**	c	ϕ	R^2	**ratio**	c	ϕ	R^2	**ratio**
Δ real GDP: 1s	−17.61 (13.95)	1.29 (0.19)	77.04	0.73 [0.29]	10.54 (28.45)	1.27 (0.42)	7.16	0.84 [0.17]	6.83 (70.88)	1.46 (1.32)	2.05	0.85 [0.16]	18.30 (84.73)	1.30 (1.53)	0.85	0.86 [0.15]
Δ real GDP: 2s	−13.29 (14.40)	1.27 (0.21)	75.06	0.78 [0.34]	21.95 (31.86)	1.14 (0.56)	6.18	0.86 [0.18]	29.23 (58.59)	1.11 (1.16)	2.35	0.85 [0.14]	50.25 (52.92)	0.75 (0.90)	0.84	0.86 [0.16]
Δ Ind. prod.: 1s	−18.51 (14.46)	1.26 (0.20)	76.25	0.74 [0.27]	7.82 (30.17)	1.24 (0.45)	7.38	0.83 [0.13]	31.41 (64.20)	0.96 (0.95)	1.21	0.84 [0.13]	55.34 (78.88)	0.59 (1.07)	0.29	0.85 [0.12]
Δ Ind. prod.: 2s	−8.65 (14.73)	1.04 (0.19)	72.28	0.76 [0.20]	40.31 (28.42)	0.66 (0.38)	5.38	**0.84** [0.08]	63.91 (41.56)	0.38 (0.48)	1.00	**0.84** [0.04]	74.44 (43.79)	0.23 (0.39)	0.34	**0.85** [0.04]
Δ Unemp: 1s	−20.00 (12.70)	1.29 (0.17)	79.00	0.68 [0.21]	1.10 (30.08)	1.35 (0.46)	8.50	0.82 [0.12]	23.18 (66.95)	1.11 (1.03)	1.59	0.84 [0.12]	45.82 (69.82)	0.76 (1.03)	0.50	0.85 [0.12]
Δ Unemp.: 2s	−7.50 (17.70)	1.04 (0.26)	63.75	0.99 [0.93]	47.77 (30.29)	0.55 (0.41)	4.51	**0.86** [0.06]	70.40 (38.78)	0.28 (0.39)	0.83	**0.86** [0.02]	75.70 (37.71)	0.21 (0.31)	0.44	**0.87** [0.02]
Δ Housing: 1s (ur)	−34.54 (15.93)	1.40 (0.19)	78.33	0.76 [0.32]	−31.72 (44.22)	1.63 (0.74)	15.06	**0.76** [0.09]	−40.77 (82.40)	1.84 (1.38)	11.26	**0.75** [0.07]	−30.37 (82.48)	1.73 (1.42)	9.04	**0.76** [0.06]
Δ Housing: 2s	−34.45 (15.78)	1.38 (0.19)	78.62	0.74 [0.28]	−30.31 (42.66)	1.58 (0.70)	15.36	**0.76** [0.10]	−42.82 (81.87)	1.82 (1.33)	12.25	**0.74** [0.08]	−38.20 (86.43)	1.79 (1.45)	10.77	**0.75** [0.07]
Δ Corp. prof.: 1s	−10.74 (14.34)	1.25 (0.21)	72.82	0.83 [0.56]	3.98 (20.79)	1.40 (0.39)	14.95	**0.79** [0.10]	−34.72 (51.54)	2.23 (1.12)	15.61	**0.78** [0.05]	−50.13 (61.29)	2.56 (1.37)	14.75	**0.79** [0.04]
Δ Corp. prof.: 2s	−12.17 (12.89)	1.50 (0.20)	78.74	0.90 [0.76]	−5.30 (20.87)	1.97 (0.60)	20.05	0.82 [0.13]	−10.42 (33.59)	2.20 (0.98)	16.31	**0.82** [0.07]	−8.33 (38.72)	2.14 (1.14)	14.05	**0.82** [0.05]
Δ GDP deflator: 1s (ur)	−20.45 (13.12)	1.41 (0.18)	79.36	0.76 [0.39]	−0.48 (26.75)	1.58 (0.32)	8.16	0.86 [0.23]	−22.78 (76.74)	2.21 (1.47)	1.67	0.88 [0.26]	−209.42 (324.34)	6.06 (6.74)	1.66	0.88 [0.25]
Δ GDP deflator: 2s	−21.42 (14.45)	1.40 (0.20)	77.51	0.80 [0.48]	0.86 (26.62)	1.53 (0.34)	8.36	0.85 [0.20]	−67.36 (102.23)	3.03 (2.08)	3.74	0.87 [0.22]	−130.96 (221.47)	4.42 (4.48)	1.10	0.88 [0.24]
NAI: 1s	−11.69 (11.73)	1.11 (0.14)	79.60	0.58 [0.18]	20.74 (27.08)	0.96 (0.39)	8.17	**0.81** [0.10]	48.97 (45.77)	0.62 (0.65)	1.43	**0.83** [0.08]	69.43 (50.55)	0.33 (0.58)	0.30	**0.84** [0.08]
New orders: 1s	−12.41 (13.33)	1.20 (0.17)	77.24	0.68 [0.24]	16.13 (25.60)	1.14 (0.41)	8.96	0.82 [0.11]	30.97 (50.48)	1.00 (0.82)	2.52	**0.83** [0.10]	42.29 (60.69)	0.83 (0.88)	1.22	**0.84** [0.10]
Δ Cons. sent.: 1s (ur)	−16.33 (11.58)	1.20 (0.12)	81.83	0.56 [0.21]	−6.80 (23.53)	1.38 (0.39)	12.99	**0.78** [0.10]	19.70 (46.54)	1.07 (0.68)	2.93	**0.81** [0.08]	79.72 (78.45)	0.17 (0.90)	0.05	**0.84** [0.10]
Δ real cons.: 1s (ur)	−15.57 (13.10)	1.24 (0.17)	78.43	0.67 [0.23]	15.94 (25.77)	1.15 (0.38)	7.61	0.83 [0.16]	45.86 (44.09)	0.77 (0.81)	0.94	0.85 [0.18]	81.17 (41.06)	0.18 (0.66)	0.03	0.87 [0.19]
Term spread: 1s (ur)	−52.98 (20.20)	2.00 (0.31)	78.05	1.19 [0.71]	−77.45 (51.24)	2.92 (1.16)	15.26	0.85 [0.15]	−115.20 (72.13)	3.67 (1.60)	19.02	**0.80** [0.06]	−161.57 (80.35)	4.47 (1.69)	27.49	**0.77** [0.04]
RV	−6.76 (15.75)	0.92 (0.20)	64.99	1	82.18 (36.48)	0.10 (0.19)	0.20	1	107.83 (46.38)	−0.20 (0.25)	0.67	1	112.28 (48.19)	−0.26 (0.29)	1.06	1
GARCH(1,1)	−20.96 (13.49)	1.30 (0.18)	78.88	0.69 [0.26]	7.43 (26.05)	1.25 (0.26)	8.11	0.83 [0.15]	5.00 (63.79)	1.43 (0.98)	1.44	0.84 [0.15]	26.33 (175.36)	1.12 (2.93)	0.12	0.86 [0.15]

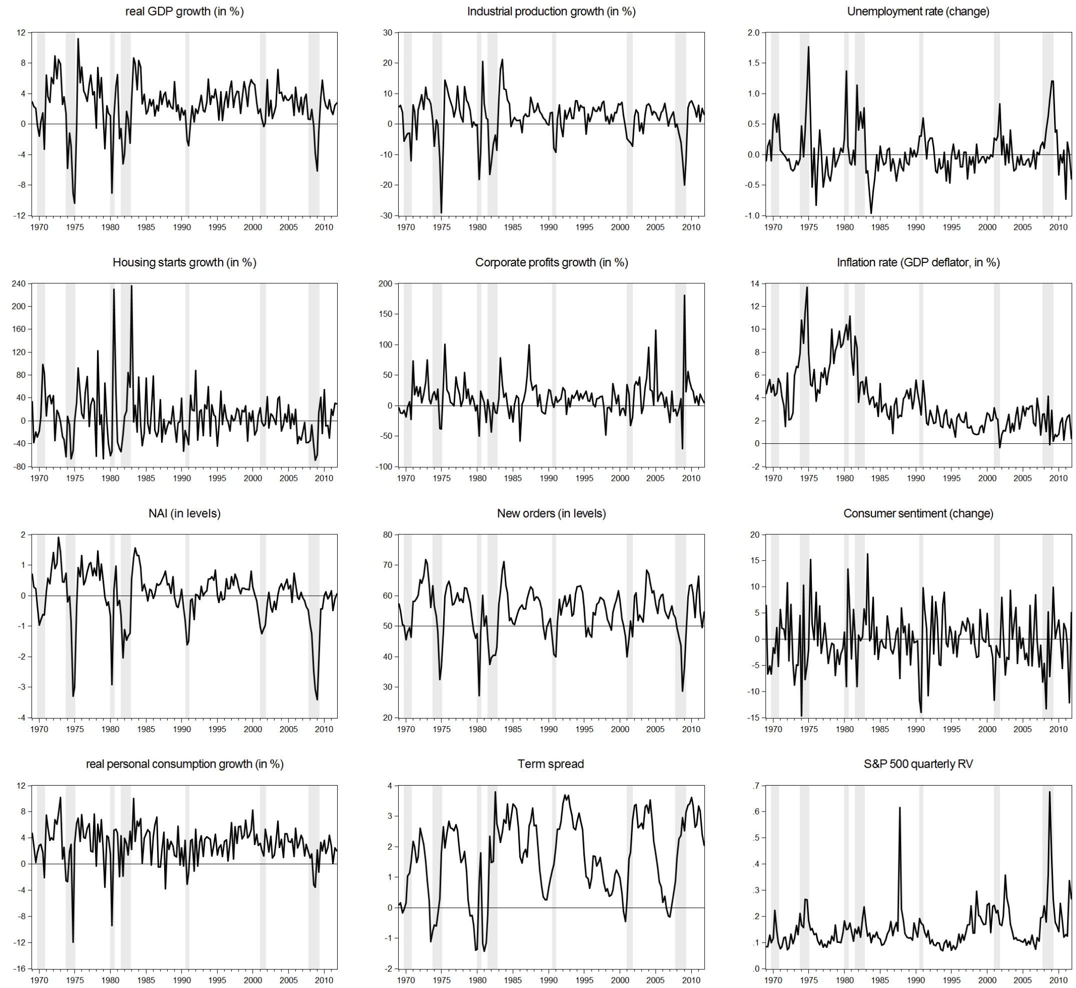
Notes: We estimate all one-sided GARCH-MIDAS-X models as well as the feasible two-sided specifications with the highest variance ratio (see Table 1.4) based on daily return data from 1973Q1 to 1998Q4 and quarterly macroeconomic data from 1970Q1 on. We then compare quarterly volatility forecasts over the 2000Q1-2010Q4 period for varying forecast horizons, $h = 1, 2, 3, 4$, corresponding to one-quarter- up to four-quarters-ahead forecasts. This out-of-sample period consists of 44 quarterly observations. For each horizon we evaluate the forecasts via a Mincer-Zarnowitz-Regression

$$RV_t = c + \phi \cdot \widehat{RV}_{t|t-h} + \xi_t, \qquad \text{with } \widehat{RV}_{t|t-h} = \sum_{i=1}^{N^{(t)}} \hat{g}_{i,t|t-h} \cdot \hat{\tau}_{t|t-h}, \qquad \text{for } h = 1, \ldots, 4,$$

using robust standard errors. RV_t denotes the quarterly realized variance, calculated as the sum of daily realized variances based on 5-minute intra-day returns over one quarter. We report parameter estimates for c and ϕ with standard errors in parentheses, as well as the R^2 percentage value. Finally, we compute MSE-ratios relative to the GARCH-MIDAS-RV model. In brackets we report p-values from the respective Giacomini-White-Test. Bold MSE ratios indicate cases in which the respective p-value is less or equal than 10%.

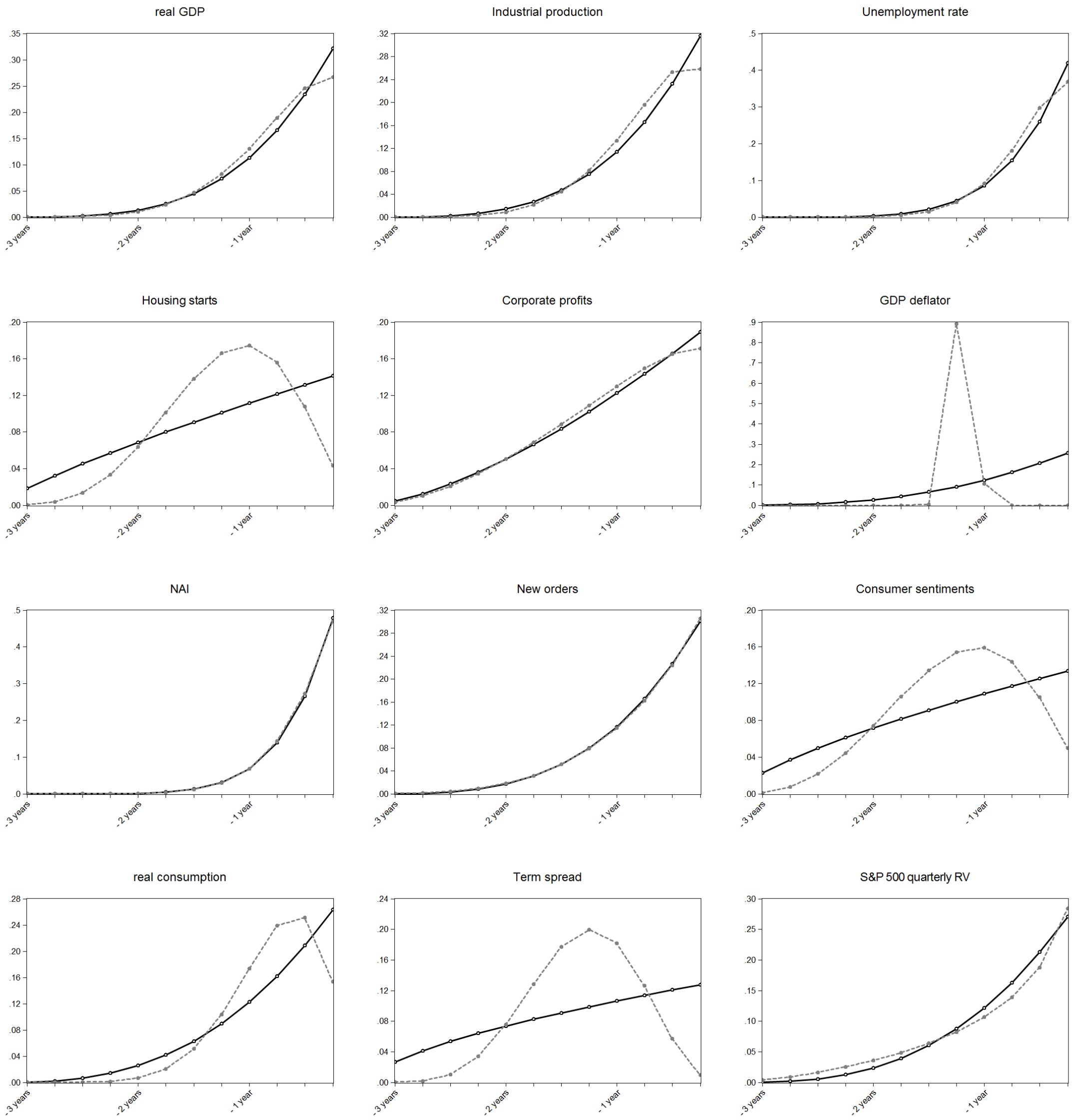
1.6.2 Figures

Figure 1.1: Quarterly macroeconomic data and realized volatility



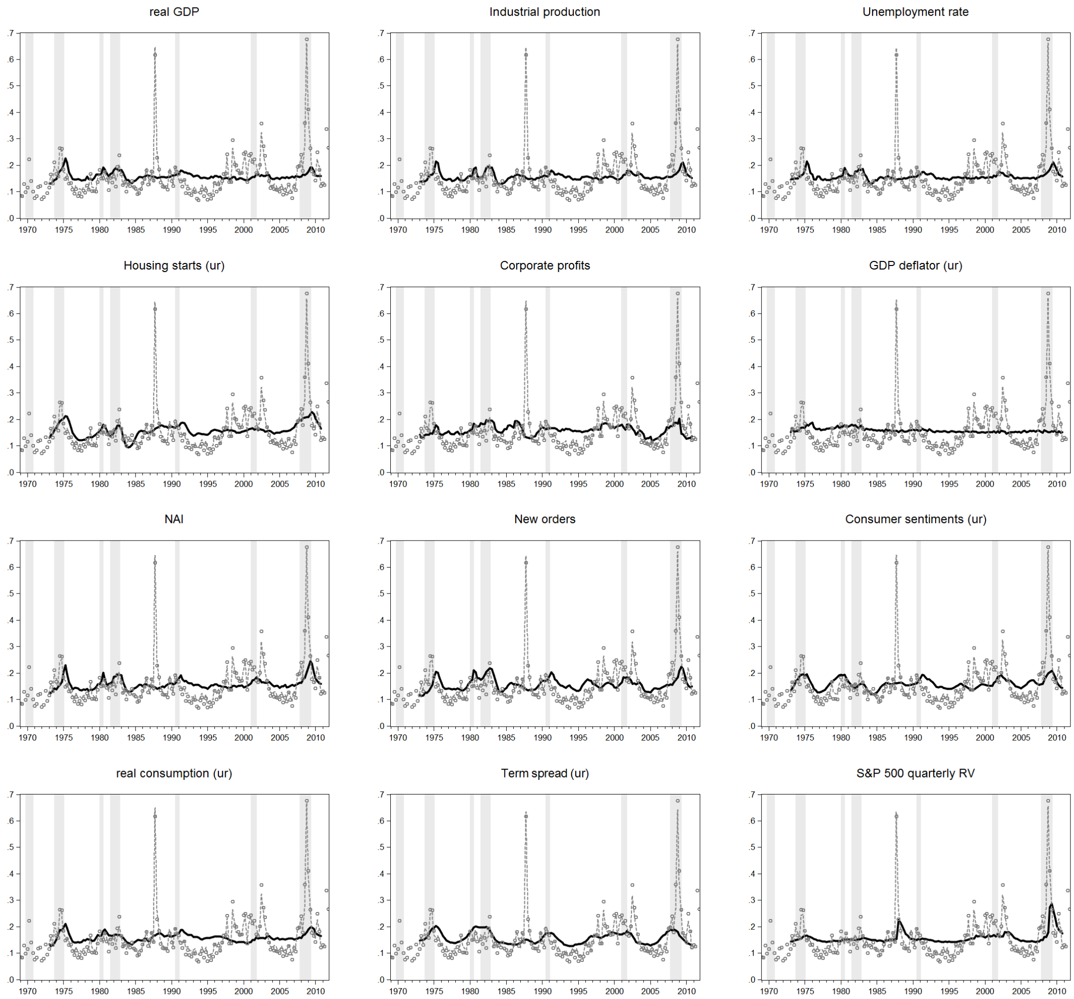
Notes: Quarterly macroeconomic data and realized volatility for the 1969Q1 to 2011Q4 period. We plot quarterly realized volatility $\sqrt{RV_t}$ in annualized terms. Shaded areas represent NBER recession periods. Otherwise, see Table 1.1 for definition and descriptive statistics of the variables.

Figure 1.2: Weighting schemes for the one-sided GARCH-MIDAS-X models



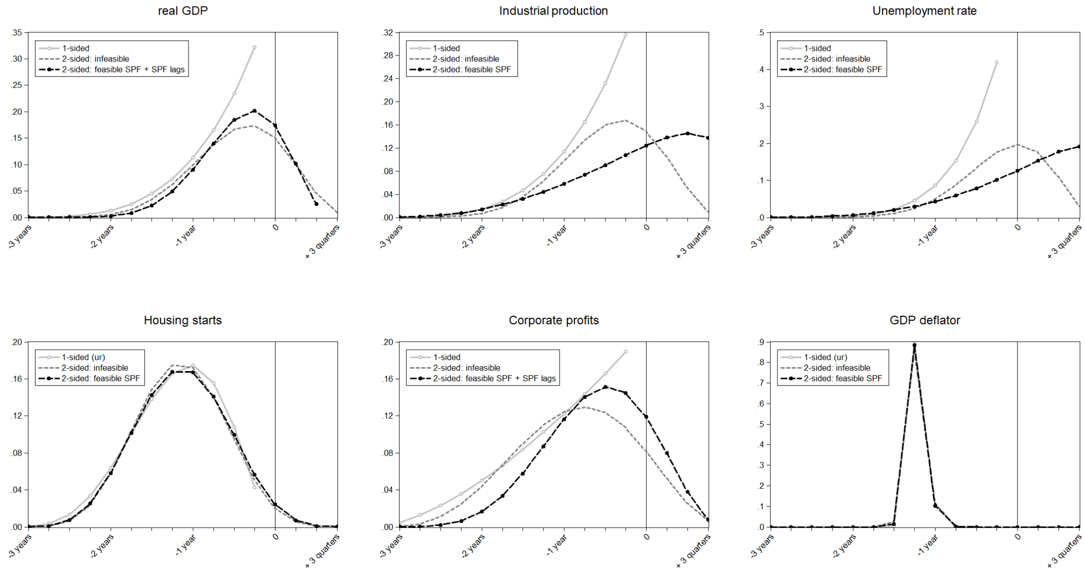
Notes: The figures show the restricted (solid black line) and unrestricted weighting (dashed grey line) schemes, see Section 1.4.1 and Table 1.2.

Figure 1.3: Volatility components for the one-sided GARCH-MIDAS-X models



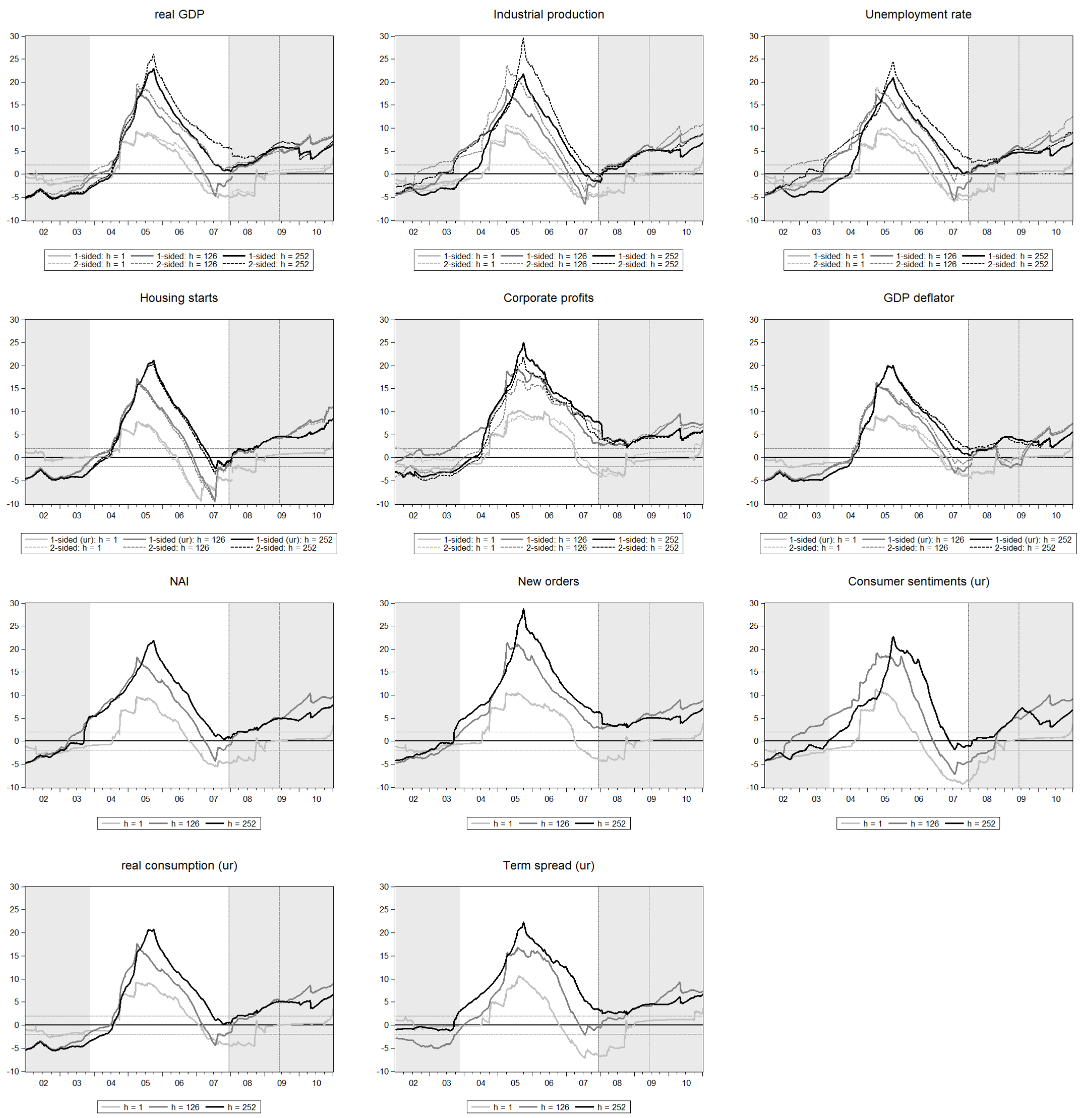
Notes: The figures show quarterly aggregated conditional volatilities $\sqrt{\tau_t^X g_t^X}$ (dashed grey line) and long-run volatility components $\sqrt{N^{(t)}\tau_t^X}$ (solid black line) from all one-sided GARCH-MIDAS-X models (1970Q1 - 2010Q4). Circles correspond to quarterly realized volatility $\sqrt{RV_t}$. For leading variables, the long-term component is based on the unrestricted weighting scheme (ur), see Section 1.4.1 and Table 1.2. Shaded areas represent NBER recession periods. Annualized scale.

Figure 1.4: Comparison of weighting schemes



Notes: The figure shows the weighting schemes for each one-sided (solid light grey line), infeasible two-sided (dashed dark grey line) and feasible two-sided with the highest variance ratio (dashed black line) GARCH-MIDAS-X specification, see Section 1.4.2 and Table 1.4.

Figure 1.5: Out-of-sample forecasting evaluation



Notes: For each GARCH-MIDAS-X model we present the evolution over the out-of-sample period of the t-statistic of a Giacomini-White-Test with the GARCH-MIDAS-RV model being the benchmark model. We evaluate daily volatility forecasts over three different horizons, namely for $h = 1, 126, 252$, corresponding to one-day-ahead, two-quarters-ahead, and four-quarters-ahead forecasts. For each horizon we calculate the respective t-statistic over a rolling window with fixed sample size of 500 days, corresponding to two years of data. Each point of the lines refers to a t-statistic for a sample period ending at that point in time. Thus, the first observations refer to the 2000-2002 sample, whereas the last observations correspond to the 2008-2010 sample. Shaded areas refer to samples that include observations from recession periods. The vertical dashed lines mark the beginning and end of the 2007-2009 recession.

1.7 Appendix

This Appendix presents full estimation results for the one- and two-sided GARCH-MIDAS-X specifications in Sections 1.4.1 and 1.4.2 (Tables A.1 and A.2), a comparison of the weighting schemes and the long-term volatility components for the GARCH-MIDAS-RV, the GARCH-MIDAS-X, and the combined GARCH-MIDAS-RV-X models from Section 1.4.1 (Figures B.1 and B.2), the forecast evaluation of the GARCH-MIDAS-RV-X models in Section 1.4.3 (Table A.3), as well as all results from Section 1.4.4 (Tables A.4 - A.11 and Figure B.3).

A Tables

Table A.1: One-sided GARCH-MIDAS-X specifications

Variable	μ	α	β	γ	m	θ	ω_1	ω_2	LLF	BIC	VR(X)
Δ real GDP	0.0267*** (0.0086)	0.0182*** (0.0050)	0.9188*** (0.0147)	0.0925*** (0.0217)	0.1884 (0.1562)	−0.0803*** (0.0251)	1	4.6508*** (1.1950)	−12789.02	2.6733	6.55
	0.0267*** (0.0086)	0.0182*** (0.0050)	0.9188*** (0.0147)	0.0925*** (0.0217)	0.1921 (0.1558)	−0.0823*** (0.0263)	1.4895 (1.5532)	5.8531* (3.0656)	−12788.93 [0.6641]	2.6742	6.90
Δ Ind. prod.	0.0266*** (0.0086)	0.0174*** (0.0051)	0.9180*** (0.0151)	0.0933*** (0.0219)	0.0769 (0.1375)	−0.0434*** (0.0133)	1	4.5453*** (1.2437)	−12788.47	2.6732	7.57
	0.0267*** (0.0086)	0.0173*** (0.0051)	0.9182*** (0.0152)	0.0932*** (0.0219)	0.0767 (0.1370)	−0.0438*** (0.0129)	1.6441 (1.3154)	6.3975** (3.2453)	−12788.25 [0.5064]	2.6741	8.02
Δ Unemp.	0.0272*** (0.0086)	0.0185*** (0.0050)	0.9177*** (0.0150)	0.0920*** (0.0218)	−0.0317 (0.1365)	0.5689*** (0.1865)	1	6.4943*** (2.1923)	−12789.96	2.6735	6.02
	0.0272*** (0.0086)	0.0184*** (0.0050)	0.9179*** (0.0150)	0.0919*** (0.0218)	−0.0320 (0.1369)	0.5751*** (0.1890)	1.7441 (1.3159)	9.4221* (5.1010)	−12789.82 [0.5991]	2.6744	6.30
Δ Housing	0.0269*** (0.0086)	0.0205*** (0.0050)	0.9141*** (0.0154)	0.0932*** (0.0220)	0.0736 (0.1383)	−0.0159*** (0.0049)	1	1.8226*** (0.2867)	−12782.79	2.6720	14.39
	0.0271*** (0.0086)	0.0184*** (0.0049)	0.9162*** (0.0152)	0.0932*** (0.0216)	0.0651 (0.1359)	−0.0173*** (0.0047)	2.8071** (1.4305)	4.8430* (2.4845)	−12777.06 [0.0007]	2.6718	21.85
Δ Corp. prof.	0.0263*** (0.0086)	0.0174*** (0.0049)	0.9188*** (0.0148)	0.0936*** (0.0214)	0.2249 (0.1550)	−0.0187*** (0.0053)	1	2.5114** (1.0048)	−12783.30	2.6721	12.69
	0.0263*** (0.0086)	0.0172*** (0.0051)	0.9191*** (0.0150)	0.0934*** (0.0215)	0.2284 (0.1538)	−0.0191*** (0.0057)	1.1783** (0.4851)	2.8187** (1.3902)	−12783.20 [0.6469]	2.6730	13.24
Δ GDP deflator	0.0273*** (0.0086)	0.0196*** (0.0049)	0.9211*** (0.0143)	0.0888*** (0.0213)	−0.1017 (0.1887)	0.0269 (0.0259)	1	3.5702*** (0.9506)	−12795.21	2.6746	0.99
	0.0274*** (0.0085)	0.0189*** (0.0049)	0.9220*** (0.0143)	0.0888*** (0.0211)	−0.1385 (0.1751)	0.0357* (0.0194)	114.1107*** (9.3931)	197.1066*** (4.5289)	−12793.47 [0.0618]	2.6752	2.10
NAI	0.0264*** (0.0086)	0.0168*** (0.0052)	0.9171*** (0.0151)	0.0946*** (0.0219)	−0.0305 (0.1315)	−0.3085*** (0.0728)	1	7.7696** (3.0232)	−12783.99	2.6722	12.84
	0.0264*** (0.0086)	0.0168*** (0.0052)	0.9171*** (0.0151)	0.0946*** (0.0219)	−0.0305 (0.1315)	−0.3081*** (0.0723)	1.1506 (1.0892)	8.4179** (3.9454)	−12783.98 [0.8890]	2.6732	12.86
New orders	0.0258*** (0.0086)	0.0147*** (0.0053)	0.9174*** (0.0149)	0.0975*** (0.0217)	2.6787*** (0.5637)	−0.0496*** (0.0101)	1	4.2905*** (1.5680)	−12776.96	2.6708	17.85
	0.0258*** (0.0086)	0.0147*** (0.0053)	0.9173*** (0.0149)	0.0975*** (0.0218)	2.6904*** (0.5563)	−0.0498*** (0.0099)	0.9392 (0.6626)	4.1023*** (1.5572)	−12776.96 [0.9776]	2.6717	17.85
Δ Cons. sent.	0.0270*** (0.0086)	0.0194*** (0.0050)	0.9158*** (0.0154)	0.0935*** (0.0222)	−0.0175 (0.1361)	−0.1141*** (0.0368)	1	1.7135*** (0.2502)	−12789.08	2.6733	6.55
	0.0275*** (0.0085)	0.0193*** (0.0050)	0.9172*** (0.0151)	0.0918*** (0.0218)	−0.0193 (0.1382)	−0.1335*** (0.0338)	2.4732*** (0.6649)	4.0434** (1.6654)	−12783.04 [0.0005]	2.6730	12.63
Δ real cons.	0.0265*** (0.0086)	0.0191*** (0.0049)	0.9179*** (0.0146)	0.0932*** (0.0219)	0.2952 (0.2041)	−0.0992** (0.0422)	1	3.6582*** (1.1263)	−12791.11	2.6737	5.04
	0.0270*** (0.0085)	0.0195*** (0.0049)	0.9188*** (0.0146)	0.0911*** (0.0219)	0.3406* (0.1998)	−0.1161*** (0.0413)	2.6571 (1.9264)	8.5625* (4.8275)	−12789.50 [0.0731]	2.6743	7.70
Term spread	0.0276*** (0.0085)	0.0192*** (0.0050)	0.9144*** (0.0162)	0.0919*** (0.0226)	0.4155*** (0.1482)	−0.2723*** (0.0554)	1	1.6276*** (0.5485)	−12779.46	2.6713	14.32
	0.0272*** (0.0085)	0.0182*** (0.0050)	0.9144*** (0.0164)	0.0930*** (0.0229)	0.3658*** (0.1388)	−0.2443*** (0.0458)	4.2018* (2.4167)	6.2756** (3.0765)	−12777.47 [0.0463]	2.6718	15.94
RV	0.0272*** (0.0085)	0.0203*** (0.0049)	0.9053*** (0.0259)	0.1012*** (0.0308)	−0.2761** (0.1203)	0.0033*** (0.0009)	1	3.7869 (6.7929)	−12785.29	2.6725	12.96
	0.0271*** (0.0085)	0.0205*** (0.0049)	0.9040*** (0.0194)	0.1021*** (0.0260)	−0.2956** (0.1167)	0.0036*** (0.0010)	0.5746 (0.6140)	2.3646 (2.3155)	−12784.90 [0.3800]	2.6734	13.74
GARCH(1,1)	0.0275*** (0.0086)	0.0200*** (0.0048)	0.9215*** (0.0142)	0.0878*** (0.0211)	0.0049 (0.1553)	-	-	-	−12796.04	2.6728	-

Notes: The table reports estimation results for the one-sided GARCH-MIDAS-X models including 3 MIDAS lag years of a quarterly macro variable X, i.e, the long-run component is specified as

$$\log(\tau_t^X) = m + \theta \cdot \sum_{k=1}^{K} \varphi_k(\omega_1, \omega_2) X_{t-k},$$

with $K = 12$. All estimations are based on daily return data from 1973Q1 to 2010Q4 and include quarterly macroeconomic data from 1970Q1 on. The numbers in parentheses are Bollerslev-Wooldridge robust standard errors. ***, **, * indicate significance at the 1%, 5%, and 10% level. We estimate each model with a restricted ($\omega_1 = 1$) and an unrestricted weighting scheme. LLF is the value of the maximized log-likelihood function. The numbers in brackets are p-values from a likelihood ratio test $2(L_{UR} - L_R)$, where L_{UR} and L_R refer to the likelihood of the GARCH-MIDAS-X models with unrestricted and restricted weights, respectively. BIC is the Bayesian information criterion and VR(X) denotes the variance ratio statistic, see Eq. (1.9).

Table A.2: One- and two-sided GARCH-MIDAS-X specifications

Variable	K_{lag}	K_{lead}	m	θ	ω_1	ω_2	LLF	BIC	VR(X)
Δ real GDP	12	-	0.1884 (0.1562)	-0.0803^{***} (0.0251)	1	4.6508^{***} (1.1950)	-12789.02	2.6733	6.55
Δ real GDP	12	3	0.3088^{*} (0.1598)	-0.1327^{***} (0.0336)	8.1523^{***} (2.3815)	4.2075^{***} (1.1460)	-12784.50	2.6733	12.88
Δ real GDP	12	$3^{(TSF)}$	0.2454 (0.1620)	-0.1044^{***} (0.0289)	8.5411^{***} (2.8274)	4.7078^{*} (2.7246)	-12787.47	2.6739	7.93
Δ real GDP (*)	12	$2^{(SPF)}$	0.3684^{*} (0.1973)	-0.1475^{***} (0.0443)	5.6729^{***} (1.6418)	2.1630^{**} (0.9840)	-12785.73	2.6736	9.76
Δ real GDP (*)	$12^{(SPF)}$	$2^{(SPF)}$	0.3714^{**} (0.1749)	-0.1658^{***} (0.0389)	9.3371^{**} (4.3608)	3.8444^{**} (1.7972)	-12780.21	2.6724	14.08
Δ Ind. prod.	12	-	0.0769 (0.1375)	-0.0434^{***} (0.0133)	1	4.5453^{***} (1.2437)	-12788.47	2.6732	7.57
Δ Ind. prod.	12	3	0.1522 (0.1288)	-0.0825^{***} (0.0188)	7.6085^{***} (2.2601)	3.9072^{***} (0.9991)	-12780.76	2.6725	18.19
Δ Ind. prod.	12	$3^{(TSF)}$	0.0975 (0.1372)	-0.0544^{***} (0.0144)	10.7031^{***} (3.4036)	6.2838^{**} (2.6072)	-12786.64	2.6738	9.45
Δ Ind. prod.	12	$3^{(SPF)}$	0.4512^{*} (0.2676)	-0.1578^{***} (0.0611)	3.6772^{***} (1.3667)	1.3264 (0.8758)	-12781.27	2.6726	15.26
Δ Ind. prod.	$12^{(SPF)}$	$3^{(SPF)}$	0.3950 (0.3425)	-0.1418^{*} (0.0811)	4.2192 (3.5359)	1.8766 (2.2438)	-12783.03	2.6730	12.10
Δ Unemp.	12	-	-0.0317 (0.1365)	0.5689^{***} (0.1865)	1	6.4943^{***} (2.1923)	-12789.96	2.6735	6.02
Δ Unemp.	12	3	-0.0596 (0.1242)	1.2381^{***} (0.2763)	9.7239^{**} (4.3270)	3.6440^{**} (1.4595)	-12780.53	2.6725	20.18
Δ Unemp.	12	$3^{(TSF)}$	-0.0356 (0.1363)	0.6837^{***} (0.1999)	13.5712^{***} (4.7068)	6.8646^{**} (3.4311)	-12789.68	2.6744	6.39
Δ Unemp.	12	$3^{(SPF)}$	-0.0132 (0.1298)	2.7466^{***} (0.7783)	4.3472^{***} (1.3590)	1.1997^{**} (0.5743)	-12780.62	2.6725	15.62
Δ Unemp.	$12^{(SPF)}$	$3^{(SPF)}$	-0.0629 (0.1358)	2.3487^{***} (0.7288)	4.8550^{***} (1.8725)	1.5961^{*} (0.8928)	-12782.30	2.6728	12.71
Δ Housing	12	-	0.0651 (0.1359)	-0.0173^{***} (0.0047)	2.8071^{**} (1.4305)	4.8430^{*} (2.4845)	-12777.06	2.6718	21.85
Δ Housing	12	3	0.0648 (0.1344)	-0.0175^{***} (0.0046)	7.2945^{***} (2.6925)	7.4350^{***} (2.8725)	-12777.19	2.6718	22.15
Δ Housing	12	$3^{(TSF)}$	0.0659 (0.1349)	-0.0177^{***} (0.0046)	7.1303^{**} (3.1926)	7.1905^{**} (3.3751)	-12776.97	2.6717	22.30
Δ Housing	12	$3^{(SPF)}$	0.0693 (0.1367)	-0.0184^{***} (0.0055)	6.8567^{*} (3.5110)	6.8895^{*} (3.6895)	-12776.81	2.6717	22.82
Δ Housing	$12^{(SPF)}$	$3^{(SPF)}$	-0.1033 (0.1294)	-0.0210^{***} (0.0065)	3.6753 (2.9977)	5.2542 (3.4620)	-12783.73	2.6731	10.93
Δ Corp. prof.	12	-	0.2249 (0.1550)	-0.0187^{***} (0.0053)	1	2.5114^{**} (1.0048)	-12783.30	2.6721	12.69
Δ Corp. prof.	12	3	0.2849^{*} (0.1552)	-0.0237^{***} (0.0062)	4.4428 (3.5278)	3.4230^{**} (1.7327)	-12781.71	2.6727	16.23
Δ Corp. prof. (*)	12	$2^{(TSF)}$	0.2595 (0.1622)	-0.0217^{***} (0.0059)	3.7175^{*} (2.0642)	2.6406^{**} (1.2702)	-12784.31	2.6733	12.50
Δ Corp. prof.	12	$3^{(SPF)}$	0.3130^{*} (0.1660)	-0.0289^{***} (0.0076)	3.4713^{*} (1.9386)	2.3592^{**} (0.9464)	-12781.60	2.6727	15.46
Δ Corp. prof.	$12^{(SPF)}$	$3^{(SPF)}$	0.2318 (0.1581)	-0.0440^{***} (0.0120)	6.2270^{**} (2.5411)	3.7382^{**} (1.5118)	-12778.57	2.6721	15.96
Δ GDP deflator	12	-	-0.1385 (0.1751)	0.0357^{*} (0.0194)	114.1107^{***} (9.3931)	197.1066^{***} (4.5289)	-12793.47	2.6752	2.10
Δ GDP deflator	12	3	-0.1394 (0.1754)	0.0358^{*} (0.0192)	185.7127^{***} (4.8179)	202.5811^{***} (4.3104)	-12793.48	2.6752	2.08
Δ GDP deflator	12	$3^{(TSF)}$	-0.1318 (0.1746)	0.0341^{*} (0.0184)	344.4171^{***} (2.5678)	403.7092^{***} (1.7643)	-12793.48	2.6752	1.96
Δ GDP deflator	12	$3^{(SPF)}$	-0.1390 (0.1753)	0.0358^{*} (0.0193)	218.7094^{***} (6.6379)	236.9913^{***} (6.5734)	-12793.47	2.6752	2.11
Δ GDP deflator	$12^{(SPF)}$	$3^{(SPF)}$	-0.1642 (0.1838)	0.0408 (0.0255)	399.3586^{***} (3.9616)	388.1800^{***} (3.8229)	-12794.12	2.6753	2.10
GARCH(1,1)	-	-	0.0049 (0.1553)	-	-	-	-12796.04	2.6728	-

Notes: The table compares estimation results for the one- and two-sided GARCH-MIDAS-X specifications,

$$\log(\tau_t) = m + \theta \sum_{k=1}^{K_{lag}} \varphi_k(\omega_1, \omega_2) X_{t-k},$$

$$\log(\tau_t) = m + \theta \sum_{k=1}^{K_{lag}} \varphi_k(\omega_1, \omega_2) X_{t-k} + \theta \sum_{k=-K_{lead}}^{0} \varphi_k(\omega_1, \omega_2) \tilde{X}_{t-k},$$

with $K_{lag} = 12$ and $K_{lead} = 3$. All estimations are based on daily return data from 1973Q1 to 2010Q4 and quarterly macroeconomic data from 1970Q1 on. We include a restricted weighting scheme ($\omega_1 = 1$) in the one-sided filter for all variables except for housing starts and the GDP deflator. The infeasible two-sided filter includes leads of the realized macro variable, i.e. $\tilde{X}_{t-k} = X_{t-k}$, whereas feasible filters are based on time series (TSF) or survey forecasts (SPF), i.e. $\tilde{X}_{t-k} = X^{TSF}_{t-k|t-1}$ or $\tilde{X}_{t-k} = X^{SPF}_{t-k|t-1}$. Finally, we consider a specification which is entirely based on SPF data, see Eq. (1.13). Otherwise, see the notes of Table A.1.
(*) Due to convergence problems for $K_{lead} = 3$, we choose $K_{lead} = 2$ for these specifications.

Table A.3: Quarterly RV Forecasting - GARCH-MIDAS-RV-X models

Forecast horizon	1-quarter-ahead				2-quarters-ahead				3-quarters-ahead				4-quarters-ahead			
	MZ-Regression			MSE	MZ-Regression			MSE	MZ-Regression			MSE	MZ-Regression			MSE
Variable	c	ϕ	R^2	ratio	c	ϕ	R^2	ratio	c	ϕ	R^2	ratio	c	ϕ	R^2	ratio
RV + Δ real GDP	0.51 (17.00)	0.86 (0.23)	58.64	1.20 [0.25]	86.93 (34.30)	0.05 (0.15)	0.06	1.08 [0.25]	101.39 (39.57)	−0.12 (0.16)	0.38	1.07 [0.32]	103.11 (40.85)	−0.15 (0.19)	0.53	1.06 [0.30]
RV + Δ Ind. prod.	7.87 (17.87)	0.74 (0.25)	51.70	1.56 [0.24]	88.62 (33.43)	0.03 (0.11)	0.03	1.24 [0.27]	99.75 (37.91)	−0.09 (0.13)	0.37	1.22 [0.28]	100.33 (38.67)	−0.10 (0.15)	0.42	1.21 [0.26]
RV + Δ Unemp.	2.55 (17.63)	0.81 (0.24)	56.29	1.33 [0.27]	86.63 (33.61)	0.05 (0.13)	0.09	1.16 [0.28]	99.79 (38.89)	−0.09 (0.14)	0.33	1.16 [0.29]	101.04 (39.31)	−0.11 (0.16)	0.44	1.15 [0.26]
RV + Δ Housing (ur)	−29.83 (18.36)	1.32 (0.25)	71.64	0.89 [0.58]	59.37 (35.70)	0.42 (0.38)	1.38	0.88 [0.11]	78.58 (38.36)	0.17 (0.38)	0.20	**0.86** [0.06]	90.27 (40.31)	0.01 (0.29)	0.00	**0.87** [0.06]
RV + Δ Corp. prof.	37.56 (12.65)	0.46 (0.13)	39.41	3.22 [0.31]	79.82 (28.80)	0.11 (0.08)	1.21	1.52 [0.35]	89.46 (31.34)	0.02 (0.09)	0.03	1.54 [0.37]	92.12 (32.27)	−0.01 (0.08)	0.01	1.56 [0.36]
RV + Δ GDP deflator (ur)	−6.31 (15.46)	1.02 (0.22)	65.70	0.94 [0.52]	83.78 (36.51)	0.10 (0.21)	0.14	0.98 [0.59]	106.65 (44.96)	−0.21 (0.26)	0.59	0.97 [0.49]	109.67 (46.33)	−0.26 (0.30)	0.85	0.97 [0.47]
RV + NAI	13.44 (15.78)	0.65 (0.20)	53.47	1.82 [0.28]	85.69 (30.62)	0.05 (0.09)	0.19	1.41 [0.29]	97.01 (34.77)	−0.06 (0.09)	0.22	1.41 [0.29]	98.27 (35.93)	−0.07 (0.10)	0.33	1.40 [0.27]
RV + new orders	21.74 (13.96)	0.60 (0.20)	46.59	2.10 [0.25]	86.76 (30.91)	0.04 (0.09)	0.14	1.44 [0.28]	96.15 (34.48)	−0.05 (0.09)	0.19	1.43 [0.28]	97.05 (35.49)	−0.06 (0.10)	0.26	1.42 [0.27]
RV + Δ Cons. sent. (ur)	34.36 (12.94)	0.45 (0.14)	34.53	3.43 [0.25]	83.75 (28.60)	0.07 (0.07)	0.44	1.66 [0.30]	94.90 (31.37)	−0.03 (0.06)	0.12	1.67 [0.30]	97.70 (32.61)	−0.06 (0.07)	0.35	1.66 [0.29]
RV + Δ real cons.(ur)	11.27 (13.29)	0.72 (0.17)	60.73	1.37 [0.33]	82.62 (30.74)	0.09 (0.12)	0.35	1.14 [0.29]	99.90 (36.33)	−0.10 (0.12)	0.36	1.14 [0.29]	102.00 (37.65)	−0.13 (0.15)	0.55	1.14 [0.27]
RV + Term spread (ur)	34.97 (12.42)	0.51 (0.16)	37.49	2.68 [0.28]	85.28 (29.69)	0.06 (0.07)	0.34	1.59 [0.31]	93.20 (31.26)	−0.02 (0.07)	0.04	1.61 [0.33]	94.45 (31.78)	−0.03 (0.07)	0.11	1.59 [0.34]
RV	−6.76 (15.75)	0.92 (0.20)	64.99	1	82.18 (36.48)	0.10 (0.19)	0.20	1	107.83 (46.38)	−0.20 (0.25)	0.67	1	112.28 (48.19)	−0.26 (0.29)	1.06	1
GARCH(1,1)	−20.96 (13.49)	1.30 (0.18)	78.88	0.69 [0.26]	7.43 (26.05)	1.25 (0.26)	8.11	0.83 [0.15]	5.00 (63.79)	1.43 (0.98)	1.44	0.84 [0.15]	26.33 (175.36)	1.12 (2.93)	0.12	0.86 [0.15]

Notes: We estimate all one-sided GARCH-MIDAS-RV-X models based on daily return data from 1973Q1 to 1998Q4 and quarterly macroeconomic data from 1970Q1 on. We then compare quarterly volatility forecasts over the 2000Q1-2010Q4 period for varying forecast horizons, $h = 1, 2, 3, 4$, corresponding to one-quarter- up to four-quarters-ahead forecasts. This out-of-sample period consists of 44 quarterly observations. For each horizon we evaluate the forecasts via a Mincer-Zarnowitz-Regression

$$RV_t = c + \phi \cdot \widehat{RV}_{t|t-h} + \xi_t, \quad \text{with } \widehat{RV}_{t|t-h} = \sum_{i=1}^{N^{(t)}} \hat{g}_{i,t|t-h} \cdot \hat{\tau}_{t|t-h}, \quad \text{for } h = 1, \ldots, 4,$$

using robust standard errors. RV_t denotes the quarterly realized variance, calculated as the sum of daily realized variances based on 5-minute intra-day returns over one quarter. We report parameter estimates for c and ϕ with standard errors in parentheses, as well as the R^2 percentage value. Finally, we compute MSE-ratios relative to the GARCH-MIDAS-RV model. A ratio smaller than one indicates an improvement over this benchmark model. In brackets we report p-values from the respective Giacomini-White-Test. Bold MSE ratios indicate cases in which the respective p-value is less or equal than 10%.

Table A.4: Principal components - correlation structure

Correlation with	Δ real GDP	Δ Ind. prod.	Δ Unemp.	Δ Housing	Δ Corp. prof.	GDP deflator	NAI	New orders	Δ Cons. sent.	Δ real cons.	Term spread
PC 1	0.91	0.88	-0.85	0.31	0.42	-0.28	0.92	0.89	0.35	0.68	0.24
PC 2	-0.03	-0.27	0.27	0.37	0.24	-0.56	-0.25	-0.06	0.39	0.10	0.81

Notes: We compute the first two principal components of our macroeconomic dataset (1969Q1-2011Q4) and present their correlations with each of the macro variable. The first principal component accounts for 45 percent and the second one for 14 percent of the variation in our variables.

Table A.5: Principal components - one-sided GARCH-MIDAS-X specifications

Variable	m	θ^{RV}	w_2^{RV}	θ^X	w_1^X	w_2^X	LLF	BIC	VR(X)
PC 1	-0.0229	-	-	-0.1516^{***}	1	4.5971^{***}	-12782.05	2.6718	13.19
	(0.1304)			(0.0322)		(1.1558)			
	-0.0236	-	-	-0.1504^{***}	1.5591	6.3095^{**}	-12781.84	2.6728	13.54
	(0.1309)			(0.0309)	(0.9572)	(2.8480)	[0.5195]		
RV + PC 1	-0.2263^{**}	0.0024^{***}	6.2793	-0.1104^{***}	1	5.4296^{**}	-12774.52	2.6722	23.36
	(0.1097)	(0.0008)	(8.7899)	(0.0350)		(2.2506)			
	-0.2239^{**}	0.0023^{***}	6.6836	-0.1110^{***}	1.3897	6.6348^{*}	-12774.47	2.6731	23.59
	(0.1082)	(0.0008)	(7.6584)	(0.0345)	(1.3868)	(3.3924)	[0.7511]		
PC 2	-0.0311	-	-	-0.2698^{***}	1	1.1920^{***}	-12784.34	2.6723	9.72
	(0.1354)			(0.0585)		(0.3445)			
	-0.0371	-	-	-0.2563^{***}	4.4983	4.8846	-12781.07	2.6726	12.78
	(0.1313)			(0.0545)	(5.1288)	(5.6169)	[0.0106]		
RV + PC 2	-0.4086^{***}	0.0046^{***}	2.5985	-0.2974^{***}	1	1.6937^{***}	-12764.32	2.6701	25.75
	(0.1285)	(0.0014)	(2.0335)	(0.0623)		(0.4617)			
	-0.4235^{***}	0.0048^{***}	2.1655^{*}	-0.2824^{***}	4.2344^{*}	6.5692^{**}	-12760.36	2.6702	28.13
	(0.1300)	(0.0014)	(1.1671)	(0.0553)	(2.2723)	(3.1515)	[0.0049]		
GARCH(1,1)	0.0049	-	-	-	-	-	-12796.04	2.6728	-
	(0.1553)								

Notes: The one-sided GARCH-MIDAS-X and GARCH-MIDAS-RV-X models are estimated inlcuding the first two principal components of all macro variables. The estimations are based on daily return data from 1973Q1 to 2010Q4 and include quarterly macroeconomic data from 1970Q1 on. Otherwise, see the notes of Table A.1.

Table A.6: Principal components - quarterly RV forecasting

Forecast horizon	**1-q ahead**				**2-q ahead**				**3-q ahead**				**4-q ahead**			
	MZ-Regression			**MSE**	**MZ-Regression**			**MSE**	**MZ-Regression**			**MSE**	**MZ-Regression**			**MSE**
Variable	c	β	R^2	**ratio**	c	β	R^2	**ratio**	c	β	R^2	**ratio**	c	β	R^2	**ratio**
PC 1	−19.12 (13.04)	1.30 (0.17)	79.18	0.69 [0.25]	−0.38 (30.18)	1.41 (0.53)	9.45	0.82 [0.13]	8.07 (64.71)	1.39 (1.18)	3.25	0.83 [0.11]	21.74 (66.28)	1.18 (1.16)	1.78	**0.84** [0.10]
RV + PC 1	7.69 (16.70)	0.77 (0.23)	53.82	1.44 [0.25]	85.49 (31.89)	0.06 (0.12)	0.16	1.20 [0.30]	96.89 (36.09)	−0.06 (0.12)	0.17	1.19 [0.32]	98.18 (37.12)	−0.08 (0.13)	0.26	1.19 [0.30]
PC 2 (ur)	−34.53 (14.54)	1.83 (0.22)	81.02	1.05 [0.90]	−85.55 (39.37)	3.45 (0.98)	19.79	0.86 [0.20]	−156.97 (85.68)	5.08 (2.05)	21.70	0.83 [0.12]	−206.41 (93.47)	6.08 (2.21)	26.09	**0.83** [0.08]
RV + PC 2 (ur)	46.57 (12.99)	0.41 (0.12)	35.99	3.81 [0.31]	85.27 (28.30)	0.07 (0.05)	0.50	1.75 [0.31]	92.72 (29.51)	−0.02 (0.05)	0.04	1.78 [0.32]	94.31 (30.00)	−0.04 (0.05)	0.15	1.78 [0.33]

Notes: We estimate the one-sided GARCH-MIDAS-X models including the first two principal components based on daily return data from 1973Q1 to 1998Q4 and quarterly macroeconomic data from 1970Q1 on and compare quarterly volatility forecasts over the 2000Q1-2010Q4 period for varying forecast horizons. Otherwise, see the notes of Table A.3.

Table A.7: One-sided GARCH-MIDAS-X specifications - including uncertainty measures

Variable	m	θ	ω_2	LLF	BIC	VR(X)
Δ real GDP - vola	−0.0738	0.0362	4.5674**	−12795.58	2.6747	0.42
	(0.1961)	(0.0494)	(1.8726)			
Δ real GDP - disp(t)	−0.1399	0.0967	5.7121***	−12794.88	2.6745	1.04
	(0.1827)	(0.0733)	(1.6940)			
Δ real GDP - disp(t+1)	−0.2502	0.1654**	12.7999	−12791.49	2.6738	3.36
	(0.1701)	(0.0654)	(9.1566)			
Δ real GDP - disp(t+2)	−0.3228*	0.2166***	7.3951**	−12788.69	2.6732	5.36
	(0.1733)	(0.0677)	(3.0943)			
Δ real GDP - disp(t+3)	−0.2629	0.1860**	7.6622*	−12791.45	2.6738	3.40
	(0.1854)	(0.0771)	(4.4139)			
Δ real GDP - disp(t+4)	−0.1485	0.1034	3.1003***	−12794.67	2.6745	1.21
	(0.1923)	(0.0794)	(0.9693)			
Δ Ind. prod. - vola	−0.0093	0.0036	84.8578***	−12795.93	2.6747	0.05
	(0.1569)	(0.0095)	(0.0863)			
Δ Ind. prod. - disp(t)	−0.1023	0.0303	93.2056***	−12794.37	2.6744	0.77
	(0.1830)	(0.0273)	(0.0401)			
Δ Ind. prod. - disp(t+1)	−0.2087	0.0679**	24.4823***	−12791.30	2.6738	2.89
	(0.1567)	(0.0271)	(8.7081)			
Δ Ind. prod. - disp(t+2)	−0.1649	0.0557	28.3161***	−12791.72	2.6739	2.62
	(0.1757)	(0.0361)	(9.1895)			
Δ Ind. prod. - disp(t+3)	−0.1998	0.0700*	3.8946***	−12793.32	2.6742	2.12
	(0.1887)	(0.0393)	(1.1607)			
Δ Ind. prod. - disp(t+4)	−0.3326	0.1194*	3.3750**	−12790.27	2.6736	4.51
	(0.2305)	(0.0663)	(1.3762)			
Unemp. - vola	−0.0152	0.0837	96.8312***	−12795.85	2.6747	0.08
	(0.1642)	(0.1781)	(0.1480)			
Unemp. - disp(t)	−0.1347	0.8437	7.2600**	−12794.79	2.6745	0.98
	(0.1863)	(0.6182)	(3.3421)			
Unemp. - disp(t+1)	−0.4200*	1.6465**	6.5758***	−12790.10	2.6735	4.46
	(0.2357)	(0.7093)	(1.9051)			
Unemp. - disp(t+2)	−0.4969**	1.3964***	4.7422***	−12788.99	2.6733	5.74
	(0.2085)	(0.4236)	(1.0817)			
Unemp. - disp(t+3)	−0.5178***	1.1970***	7.8313*	−12783.92	2.6722	7.66
	(0.1934)	(0.2764)	(4.5198)			
Unemp. - disp(t+4)	−0.4967***	0.9941***	6.3622***	−12785.04	2.6725	7.02
	(0.1837)	(0.2264)	(2.4066)			
Δ Housing - vola	0.1560	−0.0051	1.0770	−12795.17	2.6746	1.11
	(0.2642)	(0.0067)	(0.9358)			
Δ Housing - disp(t)	−0.0398	0.0020	8.1061**	−12795.96	2.6747	0.8
	(0.2105)	(0.0065)	(3.9674)			
Δ Housing - disp(t+1)	−0.1768	0.0083	8.7074	−12793.60	2.6742	2.80
	(0.1910)	(0.0051)	(7.8799)			
Δ Housing - disp(t+2)	−0.2482*	0.0124***	138.7290***	−12787.84	2.6730	6.22
	(0.1499)	(0.0036)	(1.3050)			
Δ Housing - disp(t+3)	−0.2673	0.0144***	8.2040**	−12790.53	2.6736	5.31
	(0.1695)	(0.0051)	(3.6190)			
Δ Housing - disp(t+4)	−0.1581	0.0096	9.6938	−12794.35	2.6744	1.69
	(0.1984)	(0.0077)	(8.7115)			
Δ Corp. prof. - vola	0.0375	−0.0016	117.7321***	−12795.28	2.6746	0.30
	(0.1642)	(0.0015)	(0.0776)			
Δ Corp. prof. - disp(t)	0.6851**	−0.0437**	1.0528***	−12788.08	2.6731	8.21
	(0.3137)	(0.0180)	(0.3920)			
Δ Corp. prof. - disp(t+1)	−0.2247	0.0167	7.2212**	−12794.60	2.6745	1.41
	(0.2479)	(0.0152)	(3.0518)			
Δ Corp. prof. - disp(t+2)	−0.0040	0.0007	12.0173*	−12796.02	2.6748	0.01
	(0.1639)	(0.0037)	(6.9746)			
Δ Corp. prof. - disp(t+3)	0.0264	−0.0018	82.5353***	−12795.81	2.6747	0.09
	(0.1592)	(0.0028)	(0.0420)			
Δ Corp. prof. - disp(t+4)	−0.4596	0.0422*	3.6093***	−12792.20	2.6740	3.94
	(0.2935)	(0.0238)	(1.3483)			
Δ GDP deflator - vola	−0.0705	0.0739	4.4783**	−12795.60	2.6747	0.49
	(0.1728)	(0.0952)	(2.2468)			
Δ GDP deflator - disp(t)	−0.1677	0.1605	109.9414***	−12793.95	2.6743	1.27
	(0.1765)	(0.1024)	(0.1224)			
Δ GDP deflator - disp(t+1)	−0.1909	0.2033	6.0606**	−12795.05	2.6745	1.13
	(0.2614)	(0.2039)	(2.6030)			
Δ GDP deflator - disp(t+2)	−0.3210	0.3259*	3.7749**	−12793.13	2.6741	3.36
	(0.2409)	(0.1850)	(1.7386)			
Δ GDP deflator - disp(t+3)	−0.0217	0.0273	59.1361***	−12795.99	2.6747	0.03
	(0.1959)	(0.1298)	(0.0653)			
Δ GDP deflator - disp(t+4)	−0.2581	0.2525*	13.1151*	−12793.19	2.6742	2.26
	(0.2127)	(0.1520)	(7.3896)			
GARCH(1,1)	0.0049	-	-	−12796.04	2.6728	-
	(0.1553)					

Notes: The table reports estimation results for the one-sided GARCH-MIDAS-X models including 3 MIDAS lag years of a macro uncertainty measure with a restricted weighting scheme, i.e, the long-run component is specified as

$$\log(\tau_t^X) = m + \theta \cdot \sum_{k=1}^{K} \varphi_k(1, \omega_2) X_{t-k},$$

with $K = 12$. Measures of macroeconomic uncertainty are either based on proxies for macro volatilities, see Eq. (1.8), or on cross-sectional measures of forecast dispersion from the SPF. The latter are available for the current quarter (disp(t)) and up to four-quarters-ahead (disp(t+4)). For the unemployment rate, the uncertainty measures refer to the level of the variable. Otherwise, see the notes of Table A.1.

Table A.8: Predictive regressions

Variable (Model)	ΔR^2	c	ρ	θ
Panel A				
Δ real GDP	0.06	0.6443***	0.6766***	−0.0030
		(0.1305)	(0.0621)	(0.0075)
Δ Ind. prod.	0.13	0.6503***	0.6723***	−0.0022
		(0.1277)	(0.0625)	(0.0036)
Δ Unemp.	0.25	0.6634***	0.6624***	0.0553
		(0.1282)	(0.0645)	(0.0653)
Δ Housing	0.38	0.6443***	0.6742***	−0.0006
		(0.1205)	(0.0600)	(0.0005)
Δ Corp. prof.	0.01	0.6205***	0.6842***	0.0001
		(0.1198)	(0.0594)	(0.0008)
Δ GDP deflator	0.17	0.6479***	0.6824***	−0.0059
		(0.1244)	(0.0594)	(0.0086)
NAI	0.77	0.7113***	0.6380***	−0.0426
		(0.1326)	(0.0667)	(0.0288)
New orders	0.16	0.7685***	0.6684***	−0.0021
		(0.2501)	(0.0638)	(0.0032)
Δ Cons. sent.	0.21	0.6092***	0.6911***	0.0034
		(0.1201)	(0.0600)	(0.0044)
Δ real cons.	0.10	0.5920***	0.6931***	0.0043
		(0.1318)	(0.0617)	(0.008)
Term spread	0.78	0.6701***	0.6823***	−0.0265
		(0.1224)	(0.0590)	(0.0177)
Panel B				
Δ real GDP: 2s	1.05	−0.0207	0.6344***	0.3594*
		(0.3892)	(0.0654)	(0.2072)
Δ Ind.prod.: 2s	2.40	−0.3219	0.5981***	0.5397***
		(0.3737)	(0.0664)	(0.2029)
Δ Unemp.: 2s	3.36	−0.5031	0.5657***	0.6585***
		(0.3728)	(0.0685)	(0.2073)
Δ Housing: 2s	2.57	−0.1446	0.5980***	0.4538***
		(0.3017)	(0.0658)	(0.1647)
Δ Corp. prof.: 2s	2.17	−0.2359	0.6176***	0.4805***
		(0.3595)	(0.0639)	(0.1903)
Δ GDP deflator: 2s	0.03	0.9118	0.6823***	−0.1380
		(1.0084)	(0.0597)	(0.4781)
NAI: 1s	0.57	0.1126	0.6432***	0.2862
		(0.4161)	(0.0672)	(0.2239)
New orders: 1s	0.31	0.3173	0.6607***	0.1707
		(0.3457)	(0.0642)	(0.1815)
Δ Cons. sent.: 1s (ur)	1.66	−0.2520	0.6196***	0.4837**
		(0.4146)	(0.0654)	(0.2199)
Δ real cons.: 1s (ur)	0.02	0.5183	0.6801***	0.0541
		(0.5284)	(0.0624)	(0.2667)
Term spread: 1s (ur)	4.18	−0.5739	0.5914***	0.6707***
		(0.3539)	(0.0626)	(0.1877)

Notes: We estimate an AR(1) model for $\log(\sqrt{RV_t})$ and two types of predictive regressions

$$\log\left(\sqrt{RV_t}\right) = c + \rho \log\left(\sqrt{RV_{t-1}}\right) + \theta X_{t-1} + \zeta_t \qquad \text{(Panel A)}$$

$$\log\left(\sqrt{RV_t}\right) = c + \rho \log\left(\sqrt{RV_{t-1}}\right) + \theta \log\left(\sqrt{N^{(t)}\hat{\tau}_t^X}\right) + \zeta_t \qquad \text{(Panel B)}$$

where the regression is either augmented by the first lag of a macro variable X_t or by the quarterly aggregated long-term component $N^{(t)}\hat{\tau}_t^X$ from the respective GARCH-MIDAS-X model. We either include the long-term component from the feasible two-sided specification with the highest variance ratio, see Section 1.4.2 and Table 1.4 or Table A.2, or from the one-sided specification for variables which are not included in the SPF dataset. For leading variables we include an unrestricted (ur) weighting scheme in the one-sided specification, see Section 1.4.1 and Table 1.2.
Robust standard errors are presented in parentheses and ***,**,* indicate significance at the 1%, 5%, and 10% level. ΔR^2 the increase in the percentage R^2 for the predictive regressions relative to a baseline AR(1) model for $log(\sqrt{RV_t})$. The percentage R^2 value for the latter is 47.06. We consider the 1973Q1 - 2010Q4 sample.

Table A.9: One-sided GARCH-MIDAS-X specifications - subsample until 2007Q2

Variable	μ	α	β	γ	m	θ	ω_1	ω_2	LLF	BIC	VR(X)
Δ real GDP	0.0279***	0.0206***	0.9212***	0.0830***	0.1291	−0.0784***	1	4.9503***	−11228.44	2.5862	8.28
	(0.0088)	(0.0053)	(0.0176)	(0.0247)	(0.1680)	(0.0256)		(1.3855)			
	0.0279***	0.0205***	0.9212***	0.0830***	0.1333	−0.0804***	1.4419	6.1071*	−11228.37	2.5872	8.71
	(0.0088)	(0.0053)	(0.0177)	(0.0247)	(0.1686)	(0.0279)	(1.6310)	(3.4171)	[0.7090]		
Δ Ind. prod.	0.0279***	0.0197***	0.9204***	0.0841***	0.0275	−0.0440***	1	4.7537***	−11227.73	2.5860	9.20
	(0.0088)	(0.0054)	(0.0182)	(0.0249)	(0.1483)	(0.0145)		(1.3649)			
	0.0279***	0.0195***	0.9206***	0.0841***	0.0292	−0.0451***	1.6951	6.8203*	−11227.51	2.5870	10.04
	(0.0088)	(0.0054)	(0.0184)	(0.0250)	(0.1477)	(0.0146)	(1.4509)	(3.7626)	[0.5075]		
Δ Unemp.	0.0285***	0.0207***	0.9200***	0.0830***	−0.0798	0.5949***	1	6.7929***	−11229.13	2.5863	6.96
	(0.0088)	(0.0053)	(0.0182)	(0.0250)	(0.1455)	(0.2146)		(2.4410)			
	0.0285***	0.0206***	0.9201***	0.0829***	−0.0799	0.6067***	1.6474	9.3800	−11229.04	2.5874	7.33
	(0.0088)	(0.0053)	(0.0182)	(0.0249)	(0.1456)	(0.2272)	(1.4843)	(5.7780)	[0.6630]		
Δ Housing	0.0279***	0.0226***	0.9173***	0.0841***	0.0467	−0.0158***	1	1.7973***	−11223.93	2.5851	13.80
	(0.0088)	(0.0053)	(0.0183)	(0.0250)	(0.1612)	(0.0061)		(0.3130)			
	0.0280***	0.0204***	0.9192***	0.0848***	0.0531	−0.0181***	2.7513**	4.6860*	−11218.68	2.5850	24.31
	(0.0088)	(0.0052)	(0.0178)	(0.0244)	(0.1592)	(0.0060)	(1.3976)	(2.5419)	[0.0012]		
Δ Corp. prof.	0.0276***	0.0189***	0.9195***	0.0870***	0.1713	−0.0209***	1	2.7734**	−11220.70	2.5844	19.80
	(0.0088)	(0.0054)	(0.0174)	(0.0241)	(0.1596)	(0.0055)		(1.1714)			
	0.0276***	0.0189***	0.9196***	0.0869***	0.1723	−0.0210***	1.0682**	2.9064*	−11220.69	2.5854	19.99
	(0.0088)	(0.0055)	(0.0177)	(0.0243)	(0.1588)	(0.0058)	(0.5240)	(1.5435)	[0.8775]		
Δ GDP deflator	0.0286***	0.0218***	0.9226***	0.0802***	−0.1971	0.0329	1	3.6621***	−11233.92	2.5874	1.98
	(0.0088)	(0.0052)	(0.0175)	(0.0245)	(0.1969)	(0.0265)		(0.9866)			
	0.0286***	0.0211***	0.9236***	0.0801***	−0.2355	0.0417*	43.3837***	71.5576***	−11232.39	2.5881	3.62
	(0.0088)	(0.0052)	(0.0175)	(0.0245)	(0.1877)	(0.0216)	(10.0414)	(4.7563)	[0.0807]		
NAI	0.0275***	0.0191***	0.9201***	0.0853***	−0.0630	−0.3262***	1	7.3653***	−11223.99	2.5851	13.28
	(0.0088)	(0.0055)	(0.0178)	(0.0247)	(0.1443)	(0.0905)		(2.6667)			
	0.0276***	0.0191***	0.9202***	0.0851***	−0.0630	−0.3270***	1.6149	9.9014**	−11223.90	2.5862	13.56
	(0.0088)	(0.0054)	(0.0179)	(0.0247)	(0.1446)	(0.0908)	(1.2684)	(4.9752)	[0.6796]		
New orders	0.0270***	0.0171***	0.9197***	0.0884***	2.6249***	−0.0495***	1	4.5362***	−11216.69	2.5835	22.31
	(0.0088)	(0.0055)	(0.0177)	(0.0246)	(0.6065)	(0.0108)		(1.7595)			
	0.0270***	0.0171***	0.9197***	0.0883***	2.6143***	−0.0493***	1.0614	4.7447**	−11216.69	2.5845	22.33
	(0.0088)	(0.0055)	(0.0177)	(0.0247)	(0.5977)	(0.0106)	(0.7734)	(2.3070)	[0.9604]		
Δ Cons. sent.	0.0282***	0.0217***	0.9179***	0.0846***	−0.0714	−0.1057***	1	1.7461***	−11229.34	2.5864	6.93
	(0.0088)	(0.0053)	(0.0186)	(0.0255)	(0.1456)	(0.0381)		(0.2664)			
	0.0286***	0.0215***	0.9197***	0.0828***	−0.0712	−0.1291***	2.8365***	4.9146**	−11222.52	2.5859	15.58
	(0.0088)	(0.0053)	(0.0181)	(0.0247)	(0.1489)	(0.0360)	(0.7517)	(2.1287)	[0.0002]		
Δ real cons.	0.0277***	0.0213***	0.9202***	0.0843***	0.2219	−0.0916**	1	3.6243***	−11231.29	2.5868	5.03
	(0.0088)	(0.0052)	(0.0177)	(0.0252)	(0.2313)	(0.0460)		(1.1946)			
	0.0284***	0.0218***	0.9219***	0.0811***	0.2881	−0.1140**	3.1662	9.8547	−11229.28	2.5874	8.97
	(0.0088)	(0.0052)	(0.0177)	(0.0254)	(0.2273)	(0.0454)	(2.8122)	(6.7650)	[0.0450]		
Term spread	0.0290***	0.0215***	0.9144***	0.0839***	0.3454**	−0.2744***	1	1.7635***	−11218.60	2.5839	19.17
	(0.0088)	(0.0055)	(0.0205)	(0.0264)	(0.1507)	(0.0552)		(0.5996)			
	0.0286***	0.0205***	0.9142***	0.0850***	0.3011**	−0.2492***	3.8873*	6.1866*	−11216.83	2.5845	20.94
	(0.0088)	(0.0054)	(0.0209)	(0.0268)	(0.1417)	(0.0464)	(2.1953)	(3.1754)	[0.0599]		
RV	0.0285***	0.0228***	0.9023***	0.0957***	−0.4617***	0.0055***	1	2.8690	−11218.93	2.5840	16.67
	(0.0087)	(0.0052)	(0.0213)	(0.0282)	(0.1495)	(0.0019)		(2.5327)			
	0.0285***	0.0229***	0.9007***	0.0968***	−0.4705***	0.0056***	0.7622*	2.3559	−11218.69	2.5850	17.10
	(0.0087)	(0.0052)	(0.0213)	(0.0282)	(0.1425)	(0.0018)	(0.4033)	(1.8402)	[0.4924]		
GARCH(1,1)	0.0287***	0.0221***	0.9237***	0.0789***	−0.0589	-	-	-	−11235.09	2.5856	-
	(0.0088)	(0.0052)	(0.0172)	(0.0241)	(0.1644)						

Notes: The table reports estimation results for the one-sided GARCH-MIDAS-X models including 3 MIDAS lag years of a quarterly macro variable X, i.e, the long-run component is specified as

$$\log(\tau_t) = m + \theta \cdot \sum_{k=1}^{K} \varphi_k(\omega_1, \omega_2) X_{t-k},$$

where $K = 12$. All estimations are based on daily return data from 1973Q1 to 2007Q2 and quarterly macroeconomic data from 1970Q1 on. Otherwise, see the notes of Table A.1.

TABLE A.10: ONE-SIDED GARCH-MIDAS-RV-X SPECIFICATIONS - SUBSAMPLE UNTIL 2007Q2

Variable	m	θ^{RV}	w_2^{RV}	θ^X	w_1^X	w_2^X	LLF	BIC	VR(X)
Δ real GDP	−0.3139**	0.0055***	2.6642	−0.0608***	1	6.3969***	−11211.91	2.5845	23.52
	(0.1535)	(0.0019)	(2.2918)	(0.0207)		(2.3516)			
	−0.3137*	0.0055***	2.6647	−0.0609***	1.0262	6.4750	−11211.91	2.5855	23.53
	(0.1608)	(0.0020)	(2.3031)	(0.0235)	(2.0301)	(4.3228)	[0.9475]		
Δ Ind. prod.	−0.3725**	0.0052***	3.1892	−0.0357***	1	5.6799***	−11210.28	2.5841	26.28
	(0.1488)	(0.0019)	(3.2753)	(0.0108)		(1.7047)			
	−0.3703**	0.0052***	3.2496	−0.0365***	1.8906	8.6561**	−11210.01	2.5851	26.95
	(0.1501)	(0.0020)	(3.5074)	(0.0109)	(1.5384)	(4.1049)	[0.4592]		
Δ Unemp.	−0.4538***	0.0052***	3.1159	0.4637***	1	7.9774**	−11212.94	2.5847	23.01
	(0.1458)	(0.0020)	(3.3439)	(0.1691)		(3.5296)			
	−0.4534***	0.0052***	3.1236	0.4683***	1.4047	9.6628**	−11212.91	2.5857	23.12
	(0.1462)	(0.0020)	(3.3885)	(0.1812)	(1.4611)	(4.2697)	[0.8185]		
Δ Housing	−0.3372**	0.0046**	3.0866	−0.0098**	1	2.2684***	−11211.56	2.5844	23.82
	(0.1627)	(0.0021)	(3.9622)	(0.0042)		(0.6237)			
	−0.3190**	0.0044**	2.9963	−0.0108**	3.0497	6.3012	−11208.84	2.5848	27.16
	(0.1582)	(0.0021)	(4.2936)	(0.0048)	(2.3699)	(4.6103)	[0.0197]		
Δ Corp. prof.	−0.3022**	0.0067***	3.0202*	−0.0224***	1	2.4239***	−11192.69	2.5800	43.26
	(0.1195)	(0.0016)	(1.6400)	(0.0043)		(0.4825)			
	−0.3196***	0.0069***	3.1179**	−0.0224***	1.7154***	3.6168***	−11191.07	2.5807	45.41
	(0.1147)	(0.0015)	(1.5348)	(0.0041)	(0.5075)	(0.8618)	[0.0721]		
Δ GDP deflator	−0.6143***	0.0056***	2.9068	0.0350*	1	3.8383***	−11216.66	2.5856	17.97
	(0.1812)	(0.0020)	(2.6008)	(0.0200)		(1.2025)			
	−0.6265***	0.0056***	2.9483	0.0378**	116.2248***	213.1336***	−11215.33	2.5863	18.60
	(0.1807)	(0.0020)	(2.7746)	(0.0172)	(11.0846)	(1.7851)	[0.1024]		
NAI	−0.4254***	0.0049**	3.5068	−0.2471***	1	9.1103**	−11208.41	2.5837	27.71
	(0.1462)	(0.0019)	(4.2894)	(0.0693)		(4.2918)			
	−0.4252***	0.0049**	3.5102	−0.2474***	1.2616	10.2807**	−11208.40	2.5847	27.76
	(0.1463)	(0.0019)	(4.2932)	(0.0704)	(1.3772)	(4.2830)	[0.9067]		
New orders	1.8133***	0.0054***	3.1312	−0.0419***	1	5.0161**	−11197.60	2.5812	37.50
	(0.4939)	(0.0016)	(2.5365)	(0.0090)		(2.0022)			
	1.8323***	0.0054***	3.1306	−0.0422***	0.8649	4.5393**	−11197.59	2.5822	37.52
	(0.4841)	(0.0016)	(2.5255)	(0.0089)	(0.7714)	(2.0090)	[0.8642]		
Δ Cons. sent.	−0.4648***	0.0054***	2.5953	−0.0867***	1	2.2106***	−11212.08	2.5845	24.41
	(0.1423)	(0.0018)	(1.8713)	(0.0291)		(0.4158)			
	−0.4736***	0.0056***	2.3139	−0.1016***	3.0757***	6.7877***	−11206.00	2.5841	29.92
	(0.1476)	(0.0018)	(1.4462)	(0.0297)	(0.6866)	(2.4311)	[0.0005]		
Δ real cons.	−0.2996*	0.0053***	2.5858	−0.0475	1	5.6873**	−11216.93	2.5856	18.06
	(0.1763)	(0.0019)	(2.0351)	(0.0317)		(2.2586)			
	−0.2530	0.0052***	2.5707	−0.0601	3.0595	12.9027*	−11216.26	2.5865	18.85
	(0.1767)	(0.0018)	(2.1233)	(0.0391)	(2.8506)	(7.6446)	[0.2454]		
Term spread	−0.1144	0.0057***	3.0076	−0.2442***	1	2.3460***	−11195.72	2.5807	38.34
	(0.1450)	(0.0020)	(2.5423)	(0.0453)		(0.8676)			
	−0.1548	0.0058***	2.8084	−0.2250***	4.8994	9.9784	−11193.06	2.5812	40.37
	(0.1349)	(0.0018)	(1.8745)	(0.0411)	(3.1349)	(7.2785)	[0.0211]		
RV	−0.4617***	0.0055***	2.8690	-	-	-	−11218.93	2.5840	16.67
	(0.1495)	(0.0019)	(2.5327)						

Notes: The table reports estimation results for the one-sided GARCH-MIDAS-RV-X models including 3 MIDAS lag years of quarterly realized volatility and a macro variable X. We include a restricted weighting scheme for the RV variable and both restricted and unrestricted weights for the macro variable, i.e, the long-run component is specified as

$$\log(\tau_t) = m + \theta^{RV} \cdot \sum_{k=1}^{K} \varphi_k(1, \omega_2^{RV}) RV_{t-k} \; + \; \theta^X \cdot \sum_{k=1}^{K} \varphi_k(\omega_1^X, \omega_2^X) X_{t-k},$$

with $K = 12$. All estimations are based on daily return data from 1973Q1 to 2007Q2 and quarterly macroeconomic data from 1970Q1 on. The numbers in brackets are p-values from a likelihood ratio test $2(L_{UR} - L_R)$, where L_{UR} is the likelihood of the GARCH-MIDAS-X specification including unrestricted weights and L_R is the likelihood of the respective specification including restricted weights. Otherwise, see the notes of Table A.1.

Table A.11: One-sided GARCH-MIDAS-(RV)-X specifications - including monthly macro data

Variable	m	θ^{RV}	w_2^{RV}	θ^X	w_1^X	w_2^X	LLF	BIC	VR(X)
Δ Ind. prod.	0.1034	-	-	$-0.0642^{\star\star\star}$	1	$4.0276^{\star\star\star}$	-12780.84	2.6716	12.74
	(0.1318)			(0.0135)		(0.8366)			
	0.1017	-	-	$-0.0641^{\star\star\star}$	$1.3351^{\star\star}$	$5.1695^{\star\star\star}$	-12780.51	2.6725	13.34
	(0.1323)			(0.0131)	(0.5561)	(1.6379)	[0.4179]		
RV + Δ Ind. prod.	-0.1116	$0.0072^{\star}$	$1.6830^{\star\star}$	$-0.0519^{\star\star\star}$	1	$4.9934^{\star\star\star}$	-12776.70	2.6726	17.37
	(0.1432)	(0.0039)	(0.7674)	(0.0134)		(1.4924)			
	-0.1086	$0.0071^{\star}$	$1.6606^{\star\star}$	$-0.0521^{\star\star\star}$	$1.1122^{\star\star}$	$5.4174^{\star\star\star}$	-12776.67	2.6736	17.37
	(0.1463)	(0.0041)	(0.7516)	(0.0138)	(0.5427)	(1.7527)	[0.8127]		
Δ Unemp.	-0.1093	-	-	$0.0110^{\star\star\star}$	1	$6.8834^{\star}$	-12779.93	2.6714	12.20
	(0.1339)			(0.0027)		(3.9504)			
	-0.1098	-	-	$0.0111^{\star\star\star}$	0.5118	$3.9937^{\star\star\star}$	-12778.83	2.6721	11.80
	(0.1316)			(0.0023)	(0.3261)	(1.0882)	[0.1373]		
RV + Δ Unemp.	$-0.3007^{\star\star}$	$0.0084^{\star\star}$	$1.8560^{\star\star}$	$0.0073^{\star\star\star}$	1	$15.6081^{\star}$	-12774.88	2.6723	17.17
	(0.1237)	(0.0034)	(0.8650)	(0.0020)		(8.9620)			
	$-0.2915^{\star\star}$	$0.0076^{\star\star}$	$1.7947^{\star\star}$	$0.0084^{\star\star\star}$	0.3671	$5.3028^{\star\star}$	-12774.34	2.6731	17.09
	(0.1262)	(0.0037)	(0.8251)	(0.0022)	(0.3834)	(2.4831)	[0.3000]		
Δ Housing	0.0427	-	-	-0.0004	1	1.7196	-12795.69	2.6747	0.31
	(0.1787)			(0.0007)		(1.1973)			
	0.0968	-	-	$-0.0009^{\star}$	$13.0877^{\star\star\star}$	$21.2448^{\star\star\star}$	-12792.49	2.6750	4.14
	(0.1701)			(0.0006)	(4.3119)	(4.9438)	[0.0114]		
RV + Δ Housing	-0.2149	$0.0089^{\star\star}$	$1.8619^{\star\star}$	-0.0002	1	2.2841	-12790.58	2.6755	6.68
	(0.1453)	(0.0036)	(0.8480)	(0.0005)		(1.6697)			
	-0.1559	$0.0085^{\star\star}$	$1.8788^{\star\star}$	$-0.0007^{\star}$	$13.2505^{\star\star\star}$	$22.9374^{\star\star\star}$	-12787.76	2.6759	9.50
	(0.1489)	(0.0036)	(0.8257)	(0.0004)	(3.7321)	(5.5934)	[0.0176]		
NAI	-0.0351	-	-	$-0.3796^{\star\star\star}$	1	$7.4250^{\star\star\star}$	-12778.86	2.6712	16.98
	(0.1284)			(0.0693)		(2.5243)			
	-0.0351	-	-	$-0.3823^{\star\star\star}$	0.7809	$6.2046^{\star\star}$	-12778.78	2.6721	16.96
	(0.1281)			(0.0707)	(0.5930)	(2.7065)	[0.6970]		
RV + NAI	-0.1897	0.0059	$1.6853^{\star\star}$	$-0.3022^{\star\star\star}$	1	10.9656	-12776.47	2.6726	19.53
	(0.1393)	(0.0043)	(0.7996)	(0.0813)		(7.0142)			
	-0.1905	0.0059	$1.7228^{\star\star}$	$-0.3050^{\star\star\star}$	0.5587	$7.4063^{\star}$	-12776.30	2.6735	19.57
	(0.1366)	(0.0042)	(0.8441)	(0.0801)	(0.7623)	(4.3574)	[0.5638]		
New orders	$3.0083^{\star\star\star}$	-	-	$-0.0556^{\star\star\star}$	1	$3.8584^{\star\star\star}$	-12774.22	2.6702	19.10
	(0.5606)			(0.0101)		(1.3016)			
	$3.0148^{\star\star\star}$	-	-	$-0.0558^{\star\star\star}$	$0.9821^{\star\star}$	$3.7946^{\star\star}$	-12774.22	2.6712	19.10
	(0.5701)			(0.0101)	(0.4979)	(1.4942)	[1.0000]		
RV + New orders	$2.3471^{\star\star\star}$	$0.0078^{\star\star}$	$1.6271^{\star\star}$	$-0.0473^{\star\star\star}$	1	$4.8753^{\star\star}$	-12769.06	2.6710	23.96
	(0.6044)	(0.0040)	(0.6348)	(0.0103)		(2.3150)			
	$2.4130^{\star\star\star}$	$0.0080^{\star\star}$	$1.6815^{\star\star}$	$-0.0486^{\star\star\star}$	0.7072	$3.6609^{\star\star}$	-12768.88	2.6720	24.38
	(0.5768)	(0.0039)	(0.6766)	(0.0099)	(0.5159)	(1.4962)	[0.5521]		
Term spread	$0.4233^{\star\star\star}$	-	-	$-0.2769^{\star\star\star}$	1	$1.4503^{\star\star\star}$	-12779.29	2.6713	12.93
	(0.1478)			(0.0555)		(0.4300)			
	$0.3653^{\star\star\star}$	-	-	$-0.2446^{\star\star\star}$	4.8830	6.5897	-12776.91	2.6717	14.80
	(0.1378)			(0.0451)	(5.2942)	(5.2859)	[0.0290]		
RV + Term spread	0.0927	$0.0122^{\star\star\star}$	$1.9639^{\star}$	$-0.2755^{\star\star\star}$	1	$1.8675^{\star\star\star}$	-12766.91	2.6706	24.20
	(0.1239)	(0.0034)	(1.0265)	(0.0485)		(0.5257)			
	0.0389	$0.0124^{\star\star\star}$	$1.9383^{\star\star}$	$-0.2502^{\star\star\star}$	5.1274	$8.3131^{\star\star}$	-12763.09	2.6708	27.33
	(0.1133)	(0.0035)	(0.9208)	(0.0408)	(3.8610)	(4.2296)	[0.0057]		
RV	$-0.2405^{\star}$	$0.0091^{\star\star}$	$1.8671^{\star\star}$	-	-	-	-12790.75	2.6737	6.44
	(0.1378)	(0.0037)	(0.8359)						
GARCH(1,1)	0.0049	-	-	-	-	-	-12796.04	2.6728	-
	(0.1553)								

Notes: The table reports estimation results for the one-sided GARCH-MIDAS-X models, i.e, the long-run component is specified as

$$\log(\tau_t^X) = m + \theta^X \cdot \sum_{k=1}^{K} \varphi_k(\omega_1^X, \omega_2^X) X_{t-k},$$

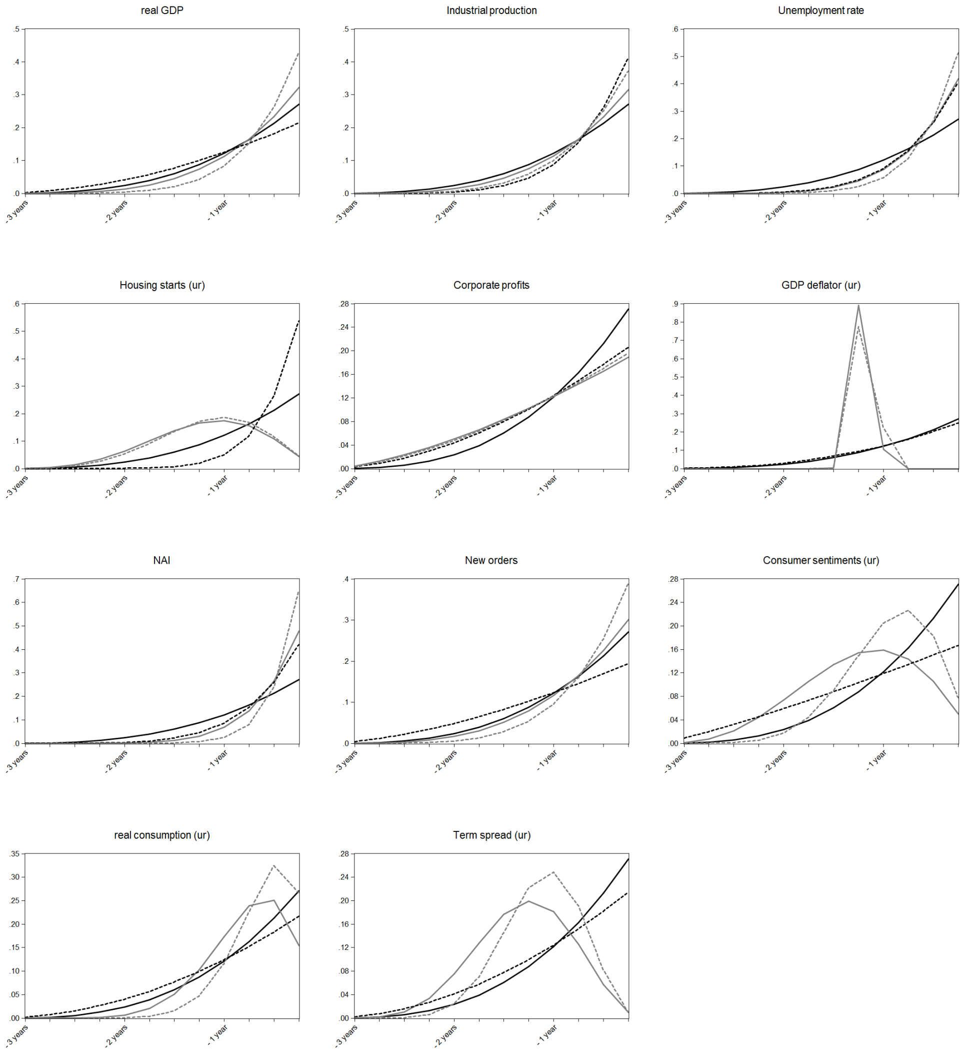
and the one-sided GARCH-MIDAS-RV-X models with a restricted weighting scheme for RV, i.e, the long-run component is specified as

$$\log(\tau_t^X) = m + \theta^{RV} \cdot \sum_{k=1}^{K} \varphi_k(1, \omega_2^{RV}) RV_{t-k} \ + \ \theta^X \cdot \sum_{k=1}^{K} \varphi_k(\omega_1^X, \omega_2^X) X_{t-k},$$

where in both cases we include 3 MIDAS lag years of monthly data, i.e. K = 36. All model estimations are based on daily return data from January 1973 to December 2010 and monthly macroeconomic data from January 1973 on. Otherwise, see the notes of Table A.1.

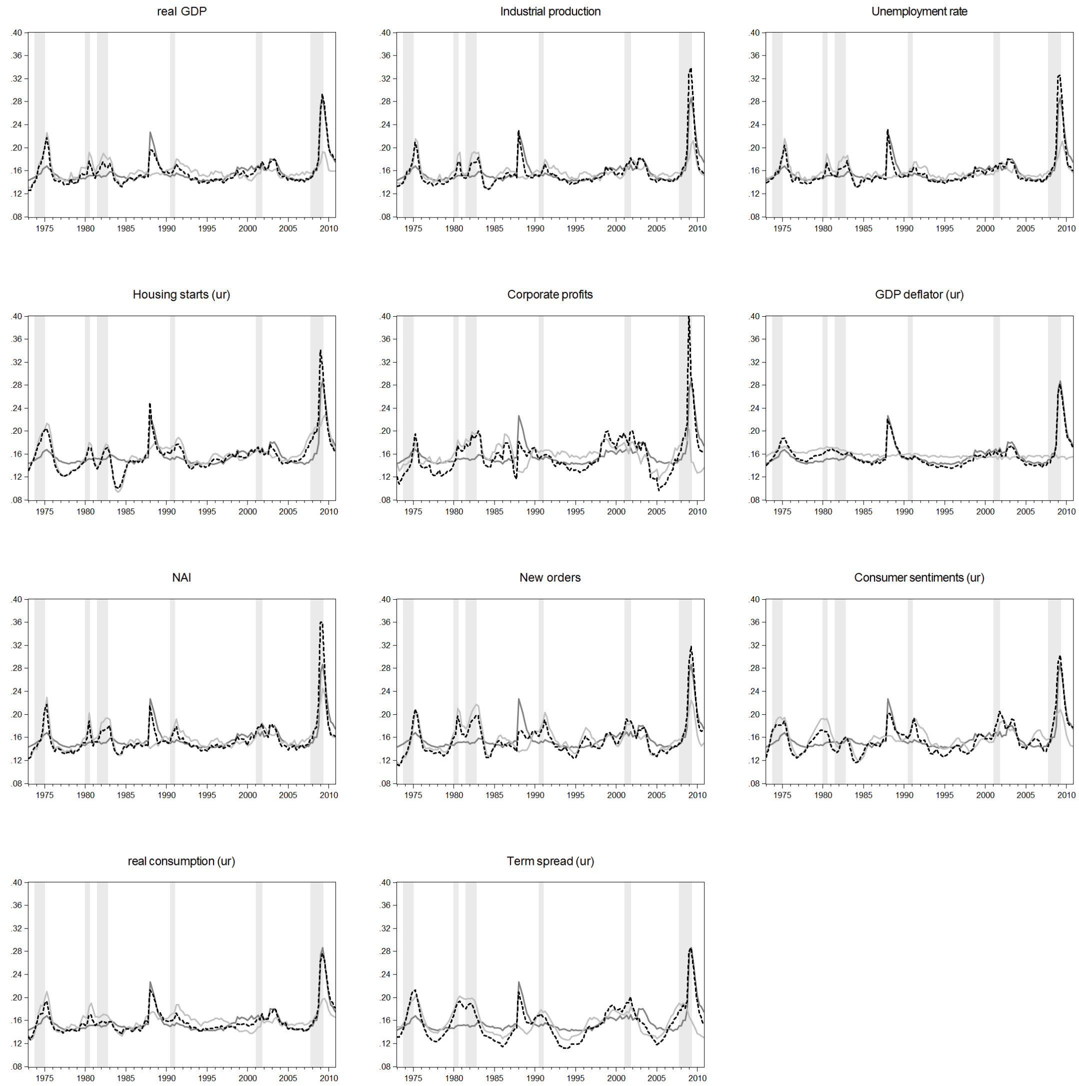
B Figures

Figure B.1: Weighting schemes for GARCH-MIDAS-RV, GARCH-MIDAS-X and GARCH-MIDAS-RV-X mdoels



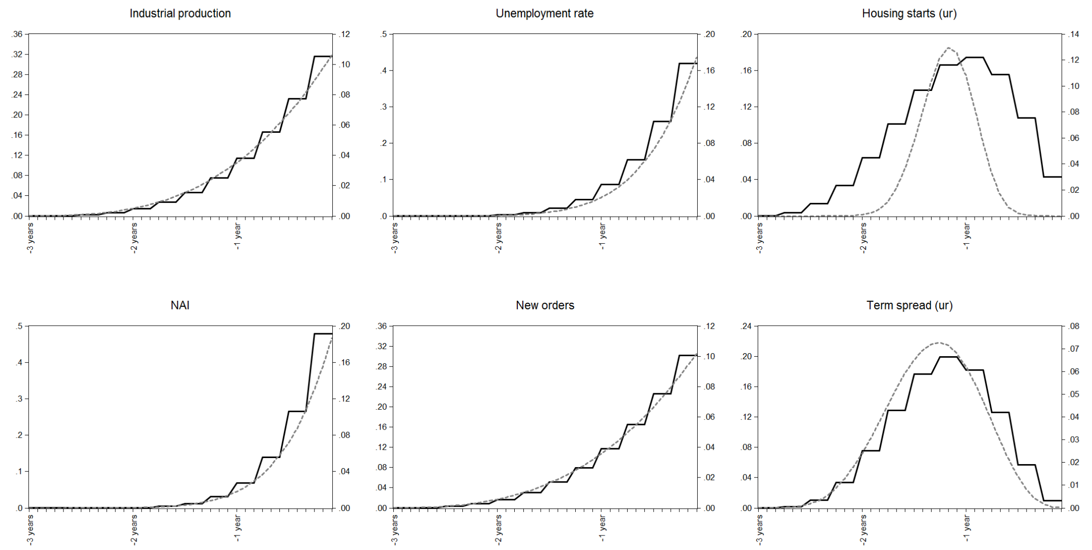
Notes: The figures show the weighting schemes for the GARCH-MIDAS-RV (solid black line), GARCH-MIDAS-X (solid grey line), as well as for the GARCH-MIDAS-RV-X models (RV: dashed black line, X: dashed grey line). We include unrestricted weights for leading variables, see Table 1.3 and Section 4.1.2.

Figure B.2: Long-run volatility components for GARCH-MIDAS-RV, GARCH-MIDAs-X and GARCH-MIDAS-RV-X mdoels



Notes: The figures show the quarterly aggregated long-run volatility components $\sqrt{N^{(t)}\tau_t}$ for the GARCH-MIDAS-RV (solid grey line), GARCH-MIDAS-X (solid light grey line), as well as for the GARCH-MIDAS-RV-X models (dashed black line), see Table 1.3 and Section 4.1.2.

Figure B.3: Weighting schemes for monthly and quarterly GARCH-MIDAS-X models



Notes: The figures show the quarterly (solid black line, left scale) and monthly (dashed grey line, right scale) weighting schemes for all one-sided GARCH-MIDAS-X models for which monthly data is available, see Table A.11. Within each quarter, we keep the quarterly weights constant.

2

The Variance Risk Premium and Fundamental Uncertainty

We propose a new measure of the expected variance risk premium that is based on a forecast of the conditional variance from a GARCH-MIDAS model. We find that the new measure has strong predictive ability for future U.S. aggregate stock market returns and rationalize this result by showing that the new measure effectively isolates fundamental uncertainty as the factor that drives the variance risk premium.

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2.1 Introduction

The findings in Bollerslev et al. (2009, 2012, 2014), Bekaert and Hoerova (2014) and others strongly suggest that the variance risk premium (VRP) predicts medium-term aggregate stock market returns. Economically, the predictive ability of the VRP can be rationalized by its close relation to economic uncertainty and aggregate risk aversion (see Bollerslev et al., 2009, 2011 or Corradi et al., 2013).[1]

Formally, the expected VRP is defined as the difference between the ex-ante risk-neutral expectation of future stock market variation and the statistical expectation of the realized variance. While 'model-free implied volatilities' can be constructed from option prices, the expected realized variance has to be estimated. The most common approaches are either to assume that the realized variance follows a martingale or to estimate a heterogeneous autoregressive model for the realized variance (HAR-RV).

We follow a different approach by modeling the conditional variance of daily stock returns as a GARCH-MIDAS process. In this setting, the conditional variance is decomposed into a short-term GARCH component and a long-term component that is driven by macroeconomic explanatory variables. As discussed in Conrad and Loch (2014), we think of the long-term component as 'the part' of the conditional variance of stock market returns that is driven by "uncertainty about the variability of economic prospects" (Bollerslev et al., 2013, p.417).

Our contribution to the literature on the VRP is twofold. First, we suggest a new proxy for the expected VRP that is based on the difference between the option-implied variance and the variance forecast from the GARCH-MIDAS model. We then show that the proposed measure has considerably stronger predictive power for stock returns than conventional measures of the VRP. Second, we rationalize the strong predictive power of our new measure by showing that it effectively isolates the long-term volatility component as the factor that determines the VRP.

[1] Using a stylized self-contained general equilibrium model, Bollerslev et al. (2009) show that the equity risk premium can be decomposed into two terms. While the first term describes the classical risk-return trade-off, the second one suggests a positive relation between expected returns and the volatility of consumption growth volatility (vol-of-vol). The predictive ability of the VRP then follows from the observation that the VRP is proportional to the time-varying vol-of-vol.

2.2 A new variance risk premium measure

2.2.1 The GARCH-MIDAS model

The GARCH-MIDAS model specifies the conditional variance of daily returns as the product of a short-term GARCH component that captures day-to-day fluctuations in volatility and a long-term component that is entirely driven by low-frequency (monthly) macroeconomic variables. The long-term component fluctuates at the monthly frequency only and can be considered as representing economic or fundamental uncertainty. Following Conrad and Loch (2014), we denote daily returns by $r_{i,t}$, where t refers to a certain month and $i = 1, \ldots, N^{(t)}$ to the i'th day within that month. We then assume that

$$r_{i,t} = \mu + \sqrt{g_{i,t}\tau_t} Z_{i,t}, \tag{2.1}$$

where $Z_{i,t}$ is IID with mean zero and variance one. $g_{i,t}$ and τ_t represent the short- and long-term conditional variances, which are measurable with respect to the information set given at day $i-1$ of month t. The short-term component follows a mean-reverting asymmetric unit variance GARCH process

$$g_{i,t} = (1 - \alpha - \beta - \gamma/2) + \left(\alpha + \gamma \cdot \mathbb{1}_{\{r_{i-1,t} - \mu < 0\}}\right) \frac{(r_{i-1,t} - \mu)^2}{\tau_t} + \beta g_{i-1,t}, \tag{2.2}$$

with $\alpha > 0$, $\beta > 0$ and $\alpha + \beta + \gamma/2 < 1$. The long-term component is driven by lagged values of an explanatory variable X_t:

$$\log(\tau_t) = m + \theta \sum_{k=1}^{K} \varphi_k(\omega_1, \omega_2) X_{t-k}, \tag{2.3}$$

where the behavior of the MIDAS weights $\varphi_k(\omega_1, \omega_2)$ is parsimoniously determined using a flexible Beta weighting scheme. For a more detailed discussion, see Engle et al. (2013) or Conrad and Loch (2014).

At the last day of each month t, we use the GARCH-MIDAS (GM) model to construct out-of-sample forecasts for the realized variance during the following month, RV_{t+1}. Note that next month's long-term volatility, τ_{t+1}, is predetermined with respect to macro realizations up to month t. Then, the realized variance prediction is given by

$$\widehat{RV}_{t+1}^{GM} = \mathbf{E}_t \left[\sum_{i=1}^{N^{(t+1)}} g_{i,t+1} \tau_{t+1} Z_{i,t+1}^2 \right] = \tilde{g}_{t+1} \tau_{t+1}, \tag{2.4}$$

where $\tilde{g}_{t+1} = \left(N^{(t+1)} + (g_{1,t+1} - 1) \frac{1-(\alpha+\beta+\gamma/2)^{N^{(t+1)}}}{1-\alpha-\beta-\gamma/2} \right)$. For a given value of the monthly short-term variance, $\tilde{g}_{t+1}$, a high (low) value of fundamental uncertainty, τ_{t+1}, will upscale (downscale) the forecast of the expected monthly realized variance. In this sense, τ_{t+1} is similar to the vol-of-vol factor in the model of Bollerslev et al. (2009).

2.2.2 Constructing the VRP

We define the monthly expected VRP as $IV_t - \mathbf{E}_t[RV_{t+1}]$, where IV_t is the risk-neutral expected variation during month $t+1$ and $\mathbf{E}_t[RV_{t+1}]$ is the expected (under the physical measure) realized variation for that period. We build on the approximation of the expected VRP in Bollerslev et al. (2009) and measure IV_t by the end-of-month t value of the squared VIX and, assuming that RV_t follows a martingale sequence, replace $\mathbf{E}_t[RV_{t+1}]$ by RV_t. The VRP is thus given by

$$VRP_t = VIX_t^2 - RV_t. \tag{2.5}$$

This measure is both directly observable and model-free. However, as discussed in Bekaert and Hoerova (2014), the assumption that RV_t follows a martingale sequence may be inappropriate. As a new measure, we propose to base the expected VRP on the conditional variance forecast from the GARCH-MIDAS model, $\widehat{RV}_{t+1}^{GM}$. This forecast explicitly takes into account the macroeconomic uncertainty via the long-term component:

$$VRP_t^{GM} = VIX_t^2 - \widehat{RV}_{t+1}^{GM}. \tag{2.6}$$

2.3 Data

We use daily continuously compounded returns, $r_{i,t}$, for the S&P 500 and monthly U.S. macroeconomic data from 1970 to 2011. We include industrial production growth (annualized month-to-month percentage change), the new orders index of the Institute for Supply Management (levels) and the Chicago Fed National Activity

Index (NAI).[2] Annualized monthly excess returns are calculated as $r_t^{ex} = 12 \cdot (r_t - r_{f,t})$, where $r_t = \sum_{i=1}^{N(t)} r_{i,t}$ and $r_{f,t}$ denotes the one-month T-bill rate. For the 2000 to 2011 period, we employ observations for the 'new' VIX and daily realized volatilities, $RV_{i,t}$, based on 5-minute intra-day returns obtained from the website of the Oxford-Man Institute of Quantitative Finance. Monthly realized variances are constructed as $RV_t = \sum_{i=1}^{N(t)} RV_{i,t}$. Otherwise, all data are obtained from the FRED database at the Federal Reserve Bank of St. Louis.

2.4 Empirical results

2.4.1 VRP estimation

We estimate the GARCH-MIDAS models for the 1973 to 1999 period. Following Conrad and Loch (2014), we include three MIDAS lag years of the macro variables and use a restricted ($\omega_1 = 1$, i.e. strictly decreasing) Beta weighting scheme. The estimation results presented in Table 2.1 basically replicate the findings in Conrad and Loch (2014) but for a briefer sample. Specifically, for all variables the estimate of θ is highly significant and negative, thus confirming the counter-cyclical behavior of long-term volatility. Periods of economic growth above trend (e.g. measured by positive NAI realizations) are associated with a decline in long-term volatility, while recession periods coincide with increasing long-term volatility. We use out-of-sample forecasts for τ_{t+1} and $\widehat{RV}_{t+1}^{GM}$ for the 2000 to 2011 period to construct our new measure of the VRP. Table 2.2 provides summary statistics and Figure 2.1 depicts the different measures of the VRP over the out-of-sample period.[3] The table also presents summary statistics for the *ex-post* VRP defined as $VIX_t^2 - RV_{t+1}$. As expected, the VRP is positive on average. Note that the different VRP measures are much less persistent than realized volatility or the VIX squared.

[2] The NAI is a weighted average of 85 monthly national economic indicators. Positive realizations indicate growth above trend, while negative realizations indicate growth below trend. Industrial production and new orders are among the indicators considered.

[3] Bollerslev et al. (2014) consider the same out-of-sample period, but employ a different risk-free rate in calculating the excess returns and base their RV_t measure on daily squared returns. This explains the slight differences in the summary statistics and the following return predictability regression results.

2.4.2 Return predictability

In this section, we investigate the predictive abilities of the expected VRP measures for future stock market returns. We rely on simple monthly regressions of the form:

$$\frac{1}{h}\sum_{j=1}^{h} r^{ex}_{t+j} = a_h + b_h Z_t + u_{t,t+h}, \tag{2.7}$$

where $Z_t \in \{VRP_t, VRP_t^{GM}\}$. Following Bollerslev et al. (2014), we use Newey-West robust standard errors.[4] Table 2.3 presents the regression results for different horizons h, while Figure 2.2 shows the estimated b_h coefficients for our VRP measures along with 90% confidence bands based on the critical values simulated in Bollerslev et al. (2014). First, based on these critical values, VRP_t significantly predicts future returns for horizons one to five. In accordance with the theoretical model developed in Bollerslev et al. (2009), the adjusted R^2 initially increases and then decreases with expanding forecast horizon. The maximum R^2 is achieved for $h = 4$ months.[5]

Second, and most importantly, all three proxies for the expected VRP based on the GARCH-MIDAS models have strong predictive power for future returns with significant regression coefficients up to the 6 months horizon. At almost all horizons, the R^2s from these models are markedly higher than the ones based on VRP_t. In all three cases, the maximum R^2 is achieved at $h = 5$. These findings suggest that our new proxy – which explicitly takes into account the state of the macroeconomy – is a more precise measure for the ex-ante VRP than alternative proxies and, thus, has superior forecasting power for returns. In other words, using $\widehat{RV}^{GM}_{t+1}$ as a measure of the expected variance clearly helps to "isolate the factor that drives the volatility risk premium" (Bollerslev et al., 2009, p.4485).

2.4.3 The ex-post VRP and fundamental uncertainty

In a final step, we provide an intuitive argument for the successfulness of our new measure in predicting returns. Recall that the variance forecast from the GARCH-MIDAS model can be written as $\widehat{RV}^{GM}_{t+1} = \tilde{g}_{t+1}\tau_{t+1}$, where τ_{t+1} reflects fundamental uncertainty. Then, similarly to Bollerslev et al. (2012), we decompose the squared

[4] We choose the same bandwidth in the Bartlett kernel as suggested in their paper. As shown in Bollerslev et al. (2014, p.635), given the low persistence in the VRP (see Table 2.2), the robust t-statistics "are reasonably well behaved" despite the overlapping nature of the return regressions.

[5] As in Bekaert and Hoerova (2014), we also considered a VRP based on conditional variance forecasts from a HAR-RV model. The corresponding R^2s are slightly lower.

VIX into the expected conditional variance plus the VRP. In the model of Bollerslev et al. (2012), the VRP can be written as an affine function of fundamental uncertainty. Assuming the same relationship, we obtain:

$$VIX_t^2 = c + \widehat{RV}_{t+1}^{GM} + b^{(\tau)}\tau_{t+1} \tag{2.8}$$

or $VIX_t^2 - \widehat{RV}_{t+1}^{GM} = c + b^{(\tau)}\tau_{t+1}$ with some constant $b^{(\tau)} > 0$. We test this mechanism by first regressing VIX_t^2 on a constant, $\widehat{RV}_{t+1}^{GM}$ and τ_{t+1} and, second, by regressing the *ex-post* VRP on a constant, $\widehat{RV}_{t+1}^{GM}$ and τ_{t+1}. Both should be significant in the first regression, but only τ_{t+1} in the second one. Relying on the ex-post VRP in the second regression has the advantage that we do not have to estimate $\mathbf{E}_t[RV_{t+1}]$.

Panel A of Table 2.4 confirms that VIX_t^2 is positively related to both $\widehat{RV}_{t+1}^{GM}$ and τ_{t+1}. In this regression, the conditional variance forecast can be interpreted as an interaction term: the predicted effect of a change in the long-term component is stronger the higher the forecast for the short-term component is. On the other hand, in the regressions with the ex-post VRP as the dependent variable, only the long-term components are highly significant (see Panel B).[6] Both regressions support our hypothesis that the long-term volatility components from the GARCH-MIDAS models can be considered as representing the vol-of-vol factor driving the VRP.[7] The fact that the counter-cyclical long-term component drives the VRP also provides direct evidence for the conclusion of Campbell and Diebold (2009) that expected returns are inversely linked to expected business conditions. However, it should be noted that the R^2s in the regressions involving the ex-post VRP are quite low. Thus, the VRP is driven by additional factors that are not directly captured by the long-term component, such as aggregate risk aversion and disagreement in beliefs. However, these factors are also likely to behave counter-cyclically and, hence, should comove with τ_{t+1}.

Finally, note that the ex-post VRP corresponds to the payoff from selling a variance swap. Thus, when τ_t is increasing, the expected payoff from selling a variance swap increases as well. Intuitively, in times of high economic uncertainty investors are willing to pay a high premium to ensure against volatility risk.

[6] Additionally including the lagged ex-post VRP does not change our result.

[7] Our findings are in line with Bollerslev et al. (2011) who estimate a time-varying VRP that is driven by macroeconomic state variables and report that, e.g., higher industrial production leads to a decrease in the VRP.

2.5 Conclusion

Our results strongly confirm the theoretical insight from the models discussed in Bollerslev et al. (2009, 2012) that fundamental uncertainty (the vol-of-vol) is an important factor driving the VRP. In particular, we show that our new VRP measure, which is based on a volatility component reflecting the 'state of the macroeconomy', has considerably higher predictive power for future stock market returns than previously suggested measures.

2.6 Tables and figures

2.6.1 Tables

Table 2.1: GARCH-MIDAS-X model estimation

Variable	μ	α	β	γ	m	θ	ω_2	LLF
Ind. prod.	0.0348*** (0.0098)	0.0253*** (0.0068)	0.9153*** (0.0239)	0.0773** (0.0305)	−0.0003 (0.1647)	−0.0531*** (0.0144)	4.2582*** (1.0124)	−8660.61
New orders	0.0339*** (0.0098)	0.0233*** (0.0069)	0.9176*** (0.0225)	0.0784*** (0.0295)	2.5077*** (0.6514)	−0.0481*** (0.0115)	4.6872** (2.0799)	−8655.37
NAI	0.0343*** (0.0098)	0.0250*** (0.0069)	0.9158*** (0.0230)	0.0782*** (0.0299)	−0.0806 (0.1657)	−0.3503*** (0.0889)	7.2203** (2.9228)	−8658.29

Notes: The table reports estimation results for the GARCH-MIDAS-X model including 3 MIDAS lag years of a monthly macro variable X, i.e the long-run component is specified as $\log(\tau_t) = m + \theta \cdot \sum_{k=1}^{K} \varphi_k(\omega_1, \omega_2) X_{t-k}$ with $K = 36$. The three variables require a restricted Beta weighting scheme with $\omega_1 = 1$, see Conrad and Loch (2014) for details. All estimations are based on daily return data from January 1973 to December 1999 and include monthly macroeconomic data beginning in January 1970. LLF is the value of the maximized log-likelihood function. The numbers in parentheses are Bollerslev-Wooldridge robust standard errors. *** indicates significance at the 1% level. ** indicates significance at the 5% level.

Table 2.2: Summary statistics

Variable	Mean	Std. dev.	Skew.	Kurt.	AC(1)
Excess returns	−3.57	57.39	−0.58	3.89	0.15
RV	30.77	48.35	6.01	50.38	0.62
VIX^2	46.82	42.35	2.89	14.28	0.81
VRP	16.02	23.89	−3.08	30.61	0.14
VRP^{GM}- Ind. prod.	14.04	21.99	−3.45	34.75	0.13
VRP^{GM}- New orders	13.27	20.78	−2.66	23.92	0.34
VRP^{GM}- NAI	11.96	21.89	−3.63	33.28	0.24
VRP ex-post	16.07	39.83	−5.02	47.65	0.26

Notes: Summary statistics for monthly excess returns and different measures of the VRP, see Section 2.2.2. AC(1) denotes the first-order autocorrelation coefficient. Monthly excess returns are constructed using the one-month T-bill rate as the risk-free rate and are in annualized percentage form. Monthly realized volatility (RV) is the sum of daily realized volatilities based on 5-minute intra-day returns. VIX^2 denotes the squared 'new' VIX index in monthly units. The out-of-sample period extends from January 2000 to December 2011 and includes 144 observations.

Table 2.3: Return predictability regressions

Variance Premium	Horizon	1	2	3	4	5	6	9	12
VRP	Constant	-12.28	-11.53	-11.62	-11.09	-9.59	-8.18	-6.18	-5.49
		(-2.45)	(-2.32)	(-2.49)	(-2.27)	(-1.99)	(-1.69)	(-1.24)	(-1.10)
	VRP	0.57	0.54	0.53	0.50	0.41	0.32	0.18	0.15
		(3.91)	(3.09)	(4.42)	(5.13)	(3.91)	(2.78)	(1.77)	(1.60)
	adj. R2	4.69	7.60	11.28	**12.58**	9.60	6.40	2.39	2.02
VRP^{GM}- Ind. prod.	Constant	-13.52	-13.33	-11.35	-10.47	-10.79	-9.28	-6.90	-5.67
		(-2.15)	(-2.32)	(-2.16)	(-2.00)	(-1.99)	(-1.76)	(-1.38)	(-1.17)
	VRP	0.76	0.76	0.60	0.54	0.57	0.46	0.26	0.18
		(3.43)	(7.20)	(5.17)	(4.81)	(5.70)	(4.47)	(2.50)	(1.76)
	adj. R2	7.62	13.78	12.66	12.75	**16.44**	11.78	5.12	3.11
VRP^{GM}- New orders	Constant	-14.36	-14.53	-13.11	-11.83	-11.36	-9.60	-6.80	-5.54
		(-2.31)	(-2.61)	(-2.52)	(-2.26)	(-2.11)	(-1.83)	(-1.37)	(-1.15)
	VRP	0.87	0.90	0.78	0.68	0.65	0.51	0.27	0.19
		(3.40)	(7.30)	(6.20)	(5.77)	(6.12)	(4.88)	(2.43)	(1.59)
	adj. R2	9.06	17.39	19.04	18.23	**19.10**	13.15	4.79	2.72
VRP^{GM}- NAI	Constant	-12.81	-12.63	-11.17	-10.10	-9.87	-8.34	-6.01	-4.88
		(-2.25)	(-2.40)	(-2.24)	(-2.00)	(-1.89)	(-1.61)	(-1.21)	(-1.01)
	VRP	0.84	0.84	0.70	0.61	0.60	0.46	0.24	0.15
		(3.88)	(7.91)	(6.01)	(5.82)	(6.35)	(4.83)	(2.30)	(1.43)
	adj. R2	9.33	16.79	17.07	16.17	**17.81**	11.80	3.86	1.77

Notes: Monthly return predictability regressions $\frac{1}{h}\sum_{j=1}^{h} r^{ex}_{t+j} = a_h + b_h Z_t + u_{t,t+h}$ with $Z_t \in \{VRP_t, VRP_t^{GM}\}$. In parentheses, we present t-statistics based on Newey-West standard errors, where we adjust the bandwidth in the Bartlett kernel following Bollerslev et al. (2014). The adjusted sample period extends from February 2000 to January 2011 and includes 132 observations. Adjusted R^2 in percentage form. For each VRP measure, the bold number indicates the forecast horizon with the highest R^2.

Table 2.4: The ex-post VRP and fundamental uncertainty

	c	$b^{(RV)}$	$b^{(\tau)}$	adj. R^2
Panel A: VIX^2 (depend. Var.)				
Ind. prod.	-18.72	0.81	41.12	77.05
	(-1.71)	(6.36)	(3.44)	
New orders	-10.59	0.74	34.70	82.27
	(-1.35)	(8.20)	(3.77)	
NAI	-7.52	0.71	26.69	80.34
	(-1.07)	(6.99)	(3.74)	
Panel B: Ex-post VRP (depend. Var.)				
Ind. prod.	-28.38	-0.19	53.42	5.20
	(-2.02)	(-0.66)	(3.31)	
New orders	-22.13	-0.15	45.81	6.10
	(-2.19)	(-0.61)	(3.33)	
NAI	-18.50	-0.19	37.26	6.88
	(-2.27)	(-0.78)	(3.68)	

Notes: Regression results for

Panel A: $VIX_t^2 = c + b^{(RV)}\, \widehat{RV}^{GM}_{t+1} + b^{(\tau)}\, \tau^{GM}_{t+1} + \xi_t$

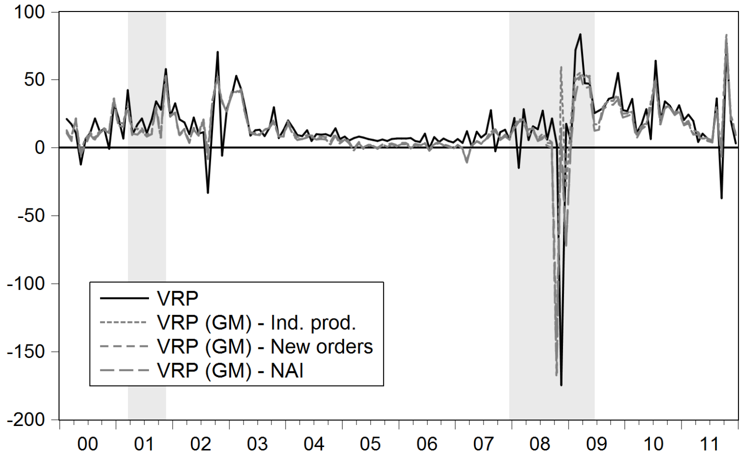
Panel B: Ex-post $\text{VRP}_t = c + b^{(RV)}\, \widehat{RV}^{GM}_{t+1} + b^{(\tau)}\, \tau^{GM}_{t+1} + \xi_t$

with Ex-post $\text{VRP}_t = VIX_t^2 - RV_{t+1}$.

The numbers in parentheses are t-statistics based on Newey-West standard errors. The sample period extends from January 2000 to December 2011. Adjusted R^2 in percentage form.

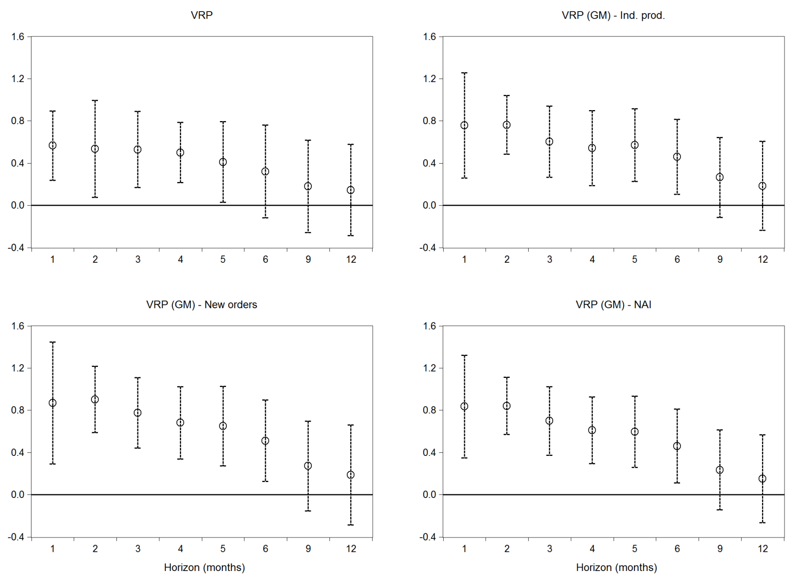
2.6.2 Figures

Figure 2.1: Variance risk premium measures



Notes: Different measures of the VRP for the January 2000 to December 2011 period. Shaded areas represent NBER recessions.

Figure 2.2: Return predictability regression coefficient estimates



Notes: Estimated regression coefficients for the different VRP measures in the return predictability regressions (Eq. (2.7)) with 90% confidence bands based on Newey-West standard errors and the simulated critical values from Bollerslev et al. (2014).

3

Time-Varying Volatility Persistence in a GARCH-MIDAS Framework

This paper presents a new volatility model with time-varying volatility persistence (TVP) that is governed by the dynamics of an explanatory variable. We extend the GJR-GARCH model by introducing a time-varying GARCH coefficient that is linked to the variable in a parsimonious way using MIDAS techniques. We refer to the model as the TVP-GARCH-MIDAS model. It nests the GJR-GARCH under the null that the variable has no explanatory power. We present a misspecification test based on the Lagrange multiplier principle and study its finite sample properties in a Monte-Carlo simulation. In an empirical application to the U.S. stock market, we show that volatility persistence is positively related to realized volatility and that it varies across the business cycle in a counter cyclical way. Finally, forecasting gains of the new model are assessed in a direct forecasting comparison.

3.1 Introduction

This paper adds to the literature on volatility persistence and suggests a new GARCH type model allowing for time variation in volatility persistence that is governed by the dynamics of an explanatory variable.

First, we extend the GJR-GARCH model by introducing smooth transitions for the GARCH parameter that is attached to the lagged conditional variance. The novelty of our model lies in linking the transition between different persistence regimes to the history of an explanatory variable in a parsimonious way. This is done by employing the MIxed Frequency DAta Sampling (MIDAS) framework introduced by Ghysels et al. (2005), in a spirit similar to the GARCH-MIDAS component models in Engle et al. (2013). Our new model generates time-varying persistence (TVP) without actually requiring time-varying parameters. Instead, the time variation is determined via the dynamics of the explanatory variable. Therefore, in contrast to genuine time-varying parameter models, the model is still straightforward to estimate by quasi maximum likelihood methods. The new model is called TVP-GARCH-MIDAS and nests the GJR-GARCH under the null that the variable has no explanatory power for time-varying persistence.

Second, we provide a misspecification test based on the Lagrange multiplier principle along the lines of Lundbergh and Teräsvirta (2002), Halunga and Orme (2009), Conrad and Schienle (2015), and Amado and Teräsvirta (2015). Our test is closely related to the latter misspecification framework of testing the standard GRJ-GARCH against general alternatives with time varying parameters. We find good finite samples size and power properties in a Monte-Carlo simulation study.

Finally, we consider an empirical application to the S&P 500 and let the time-varying persistence be determined by realized volatility dynamics and general macroeconomic conditions captured by the *ADS* business indicator introduced in Aruoba et al. (2009). Our results strongly suggest that volatility persistence varies over time, in line with Karanasos et al. (2014), who present a unified framework for time-varying AR-GARCH models and find strong evidence for time-varying persistence across different European stock markets. Moreover, we show that persistence is high (low) during high (low) volatility regimes and weak business conditions.[1] In a direct

[1]Our findings relate to the literature on GARCH component models introduced in Engle and Rangel (2008) and Engle et al. (2013) that link long-term financial volatility components to the business cycle, see also applications in Asgharian et al. (2013), Conrad and Loch (2014, 2015), Dorion (2013), and Opschoor et al. (2014). Interestingly, we provide strong evidence for volatility persistence being counter cyclical as well.

forecasting evaluation, we find that the model with realized volatility significantly outperforms the benchmark GRJ-GARCH model across horizons from one day up to one quarter.

The new model is motivated by the stylized fact that financial conditional volatility is highly persistent. Estimations of the standard (stationary) GARCH model on financial return series spanning several years or decades typically indicate high persistence, i.e. the sum of the two parameters is found to be close to one (this has been referred to as the IGARCH effect, see also Engle and Bollerslev, 1986). However, as already argued by Diebold (1986) and Lamoureux and Lastrapes (1990), volatility persistence may be overstated due to neglected deterministic shifts in the unconditional variance (see also Mikosch and Stărică, 2004). More general, Hillebrand (2005) formalizes why parameter regime changes in GARCH models that are not accounted for will cause the sum of the estimated parameters to be close to one - a phenomenon he labels "spurious almost-integration". Remarkably, the effect is independent of the estimation method and the statistical properties of the parameter changes. Also, it generalizes to higher-order GARCH models.[2]

Our new model adds to the literature on GARCH models that explicitly take into account structural breaks by introducing time-varying parameters. We extend the popular GJR-GARCH model, which allows for asymmetric volatility response to shocks. The latter can be viewed as the simplest form of a regime-switching volatility model, with two regimes and switches based on the sign of past innovations. In contrast, the smooth transition (ST) GARCH models, proposed by Hagerud (1997), González-Rivera (1998), and Anderson et al. (1999), impose smooth changes between the regimes. In these models, smooth transition is typically attached to the ARCH coefficient with a lesser focus on variation in the GARCH coefficient and the transitions are determined by lagged shocks. Alternatively, Lanne and Saikkonen (2005) use lagged conditional variances as a transition variable. These models have in common that they include only a single lag of the transition variable, whereas the MIDAS approach allows us to include potentially many lags in a parsimonious way. Alternative models with time-varying parameters are Markov-switching GARCH models (e.g., see Hamilton and Susmel, 1994, and more recently Marcucci, 2005), where changes in the regime are governed by an unobservable (or hidden) variable. Instead, Regnard and Zakoïan (2009) present a GARCH model extension that allows

[2] Some of the early volatility component models, such as in Engle and Lee (1999), Ding and Granger (1996), and Bauwens and Storti (2009) can be re-written as higher-order GARCH models and may therefore also be prone to spurious almost-integration.

for regime switches in the parameters to depend on realizations of an exogenous variable.[3]

The organization of the remainder of this paper is as follows. The new TVP-GARCH-MIDAS model is introduced in Section. We present a corresponding misspecification test in Section 3.3 and discuss its size and power properties. The model is applied to U.S. stock market data in Section 3.4 and its forecasting properties are analyzed. Finally, Section 3.5 concludes.

3.2 Time-varying volatility persistence in a GARCH-MIDAS framework

We first motivate the new GARCH model by some stylized facts on time-varying volatility persistence in Section 3.2.1. The TVP-GARCH-MIDAS model is then introduced as an extension to the GJR-GARCH model in Section 3.2.2. In Section 3.3.2, we present a misspecification test based on the Lagrange multiplier principle and study its size and power properties in a simulation study.

3.2.1 Motivation

In order to motivate the new GARCH model, we provide empirical evidence on time varying volatility persistence of financial returns. Figure 3.1 shows the autocorrelation function (ACF) up to lag 100 for daily returns and squared returns on the S&P 500 across four decades (1970-2010). For each subsample, we add the ACF that is implied by the subsample parameter estimates of a GARCH(1,1) model. In the standard Bollerslev (1986) GARCH(1,1) model, volatility persistence is constant over time. It is determined by the model parameters and yields an exponentially decaying ACF of the squared returns.[4] Consistent with financial returns being un-

[3]Though temperature is one of the few variables that can be considered as effectively being exogenous, applications to financial time series may be limited. An application of a GARCH(1,1) model with temperature-dependent coefficients to gas price volatility can be found in Regnard and Zakoïan (2011).

[4]More precisely, in the GARCH(1,1) model specifying the conditional variance of returns $r_t = \mu + \sqrt{h_t}Z_t$ as $h_t = \omega + \alpha(r_{t-1} - \mu)^2 + \beta h_{t-1}$ with parameters α, β such that $\mathbf{E}[r_t^4] < \infty$, the ACF of squared returns $\rho(j)$ at lag j is determined by

$$\rho_j = (\alpha + \beta)^{j-1}\rho_1, \quad j > 1, \qquad \text{with} \qquad \rho_1 = \frac{\alpha(1 - \alpha\beta - \beta^2)}{1 - 2\alpha\beta - \beta^2}.$$

Note that $\rho(j)$ is increasing in both parameters α and β.

predictable, we see hardly any significant autocorrelation for the returns across the subsamples, but find significant autocorrelations up to high lags for the squared returns. However, the figure illustrates a variation over time in the speed of decay of the ACF of the squared returns. The lowest autocorrelation is found for the 1980-1989 subsample, where it becomes insignificant beyond lag 55. Across the other subsamples, the autocorrelation remains significant up to lag 100, with most persistent autocorrelation during the last decade 2000-2009, which included the collapse of the dot-cum bubble as well as the financial crisis and the subsequent Great Recession. In general, the varying autocorrelation structure is reasonably well captured by the subsample GARCH model estimates.[5]

Can the time varying autocorrelation of squared returns be linked to some explanatory variable? We address this question by looking at the autocorrelation across two regimes that are determined by realized volatility on the one hand and by the ADS business conditions index on the other hand. The ADS is a daily macroeconomic indicator with average value zero and positive (negative) values indicating better-(worse-)than-average conditions, see also Section 3.4.1. We calculate autocorrelations conditional on being in a high vs. low volatility regime, and a positive vs. negative ADS regime. To be more precise, we distinguish between above and below mean realized volatility regimes, where we consider a 22-days rolling window realized volatility, $RV_t^{(22)} = 1/22 \sum_{i=0}^{21} r_{t-j}^2$. A similar rolling window version is calculated for the ADS. Adopting the formula in Regnard and Zakoïan (2011), the autocorrelation at lag j for the squared returns conditional on the negative ADS regimes is calculated as

$$\hat{\rho}(j) = \frac{\sum_{t=1}^{T} \left((r_t^2 - \hat{m}_{(-)}) \cdot \mathbb{1}_{\{ADS_t<0\}} \right) \left(r_{t-j}^2 - \hat{m} \right)}{\sqrt{\sum_{t=1}^{T} (r_t^2 - \hat{m}_{(-)})^2 \cdot \mathbb{1}_{\{ADS_t<0\}}} \sqrt{\sum_{t=1}^{T} (r_{t-j}^2 - \hat{m})^2}}, \tag{3.1}$$

where

$$\hat{m}_{(-)} = \sum_{t=1}^{T} r_t^2 \cdot \mathbb{1}_{\{ADS_t<0\}} \Big/ \sum_{t=1}^{T} \mathbb{1}_{\{ADS_t<0\}}, \qquad \hat{m} = \sum_{t=1}^{T} r_{t-j}^2 \Big/ T, \tag{3.2}$$

and analogously for positive ADS regimes and the RV regimes. A similar formula can be adopted for the ACF of the returns.

[5]The persistence implied by the GARCH model can be quantified by the sum of the parameter estimates $\hat{\alpha} + \hat{\beta}$, which varies between 0.954 (1980-1989) and 0.993 (2000-2009) across the subsamples.

Figures 3.2 and 3.3 show the rolling window versions of the ADS and of mean adjusted realized volatility as well as the ACF up to lag 100 for the daily returns and squared returns across the ADS and volatility regimes. Both figures reveal a similar pattern. We find persistent autocorrelation in the squared returns up to high lags during negative ADS regimes, but only weak autocorrelation during positive regimes. Note that negative ADS regimes generally coincide with NBER recession periods. Similarly, there is significant autocorrelation up to high lags during periods with high realized volatility, but virtually no significant autocorrelation at higher lags during low volatility regimes.

Our empirical findings imply that volatility persistence increases during turbulent times and recessions, whereas it decreases during periods of low financial volatility and economic growth. In our new model, volatility persistence will be governed by a time varying GARCH coefficient that is linked to an explanatory variable. These empirical ACFs now suggest an increasing (decreasing) coefficient during negative (positive) ADS regimes. Similarly, we expect the GARCH coefficient to be positively related to the level of realized volatility.

3.2.2 The TVP-GARCH-MIDAS model

We present a new specification of the conditional variance process that extends the GJR-GARCH(1,1) model introduced by Glosten et al. (1993). The GJR-GARCH is one of the most popular modifications of the standard Bollerslev (1986) GARCH model and allows for an asymmetric response of the conditional variance to past shocks, thus accommodating for the so-called leverage effect.[6] Let h_t denote the conditional variance process of (de-meaned) financial returns ε_t. In the GJR-GARCH model, it is specified as

$$h_t = \omega + (\alpha_1 + \alpha_2 \mathbb{1}_{\{\varepsilon_{t-1}<0\}})\varepsilon_{t-1}^2 + \beta_1 h_{t-1}, \tag{3.3}$$

with $\omega > 0, \alpha_1 > 0$, $\beta_1 \geq 0$, and stationarity condition $\alpha_1 + 1/2\alpha_2 + \beta_1 < 1$. The asymmetry parameter α_2 is typically found to be positive, implying that negative shocks have a greater impact on volatility than positive ones.

We extend this model by including a time varying GARCH coefficient that de-

[6] In the empirical analysis in Engle and Ng (1993) this model outperforms various alternative asymmetric specifications. Hansen and Lunde (2005) conclude from an extensive forecast comparison of volatility models that, in case of stock returns, only GARCH models that can accommodate for the leverage effect *beat* the forecasting performance of the simple GARCH(1,1) model.

pends on an explanatory variable $\tilde{x}$. The new conditional variance equation will be given as

$$h_t = \omega + (\alpha_1 + \alpha_2 \mathbb{1}_{\{\varepsilon_{t-1}<0\}})\varepsilon_{t-1}^2 + \beta_1 h_{t-1} + \beta_2 F(\gamma, \tilde{x}_{t-1}) h_{t-1}, \tag{3.4}$$

with $\omega > 0, \alpha_1 > 0,\ \beta_1, \beta_2 \geq 0,\ \alpha_1 + 1/2\alpha_2 + \beta_1 + \beta_2 < 1$, and

$$F(\gamma, \tilde{x}_{t-1}) = \left(1 + \exp(\gamma \tilde{x}_{t-1})\right)^{-1}. \tag{3.5}$$

The function F: $\mathbf{R} \to [0,1]$ is strictly monotonically decreasing with asymptotes $F(\gamma, -\infty) = 1$ and $F(\gamma, \infty) = 0$ in case of a positive γ. It governs the transition between two persistence regimes, a low persistence regime with GARCH coefficient β_1 and a high persistence regime with $\beta_1 + \beta_2$.[7] The parameter γ governs the smoothness of the transition. Additionally, in this new model framework its sign determines whether the explanatory variable is positively or negatively related to the time variation in persistence. The transition function is illustrated for different values of γ in Figure 3.4.

The type of logistic transition function in Eq. (3.5) has been widely used in various non-linear GARCH model extensions of Eq. (3.3) with time varying parameters. In different specifications of a Smooth Transition (ST) GARCH model presented in Hagerud (1997), González-Rivera (1998), Lundbergh and Teräsvirta (1998), and Anderson et al. (1999), it governs transitions in the intercept ω or the (G)ARCH parameters. Typically, the transition is governed by a lag of the shock ε_{t-d}, for some $d > 0$. Alternatively, Lanne and Saikkonen (2005) use the lagged conditional variance as a transition variable, combined with the cumulative distribution function of a gamma distribution as the transition function. They argue that since the innovation is a martingale difference sequence, using ε_{t-1}^2 in the transition function will imply unreasonably frequent changes in regimes whenever large (small) values are followed by small (large) values. More recently, Amado and Teräsvirta (2013, 2014) propose an alternative time varying parameter GARCH model, where the variable triggering the transition for the parameters is the index of time.

In the new variance specification in Eq. (3.4), we opt for introducing smooth transitions for the GARCH parameter attached to the lagged conditional variance, instead of for the ARCH parameter attached to the lagged squared shock. In doing

[7] In case of a negative γ, the transition function is strictly monotonically *increasing*. In case of a negative γ and a strictly positive explanatory variable $\tilde{x}$, the function F yields transitions between $\beta_1 + 1/2\beta_2$ and $\beta_1 + \beta_2$.

so, we have several reasons in mind. If we view the model equation as a forecasting model, then the one step ahead volatility forecast is determined as a weighted average of the long-run forecast, the lagged squared shock, and lagged conditional variance. Yet, estimates of the ARCH parameters are typically found to be small or even insignificant (in particular for the GJR-GARCH specification), implying that the forecast is mainly determined by the lagged conditional variance and its weight, the GARCH parameter. Alternatively, in the ARCH(∞) representation of a GARCH model, the rate at which the effect of the squared innovations on the conditional variance vanishes is primarily governed by the GARCH parameter. Besides, Hillebrand (2005) argues that the effect of "spurious almost-integration" is greater for parameter changes in the GARCH parameter than in the ARCH parameter. Finally, we note that time variation in the GARCH coefficient will induce variation in the *relative* weight associated with the lagged squared shock. In our empirical application in Section 3.4, we will, however, also consider a time varying ARCH coefficient.[8]

The novelty of our model lies in linking the transition function to the *history* of an explanatory variable x in a parsimonious way. We do so by employing the MIxed Frequency DAta Sampling (MIDAS) framework introduced by Ghysels et al. (2005). In a similar vein as in the GARCH-MIDAS model in Engle et al. (2012), we construct $\tilde{x}$ in Eq. (3.4) from a variable x via a MIDAS weighting scheme as in

$$\tilde{x}_{t-1} = \boldsymbol{\Phi}'\boldsymbol{x}_{t-1} = \sum_{k=1}^{K} \varphi_k(\vartheta_1, \vartheta_2) x_{t-k}, \tag{3.6}$$

with $\boldsymbol{\Phi} = (\varphi_1, \ldots, \varphi_K)'$, $\boldsymbol{x}_{t-1} = (x_{t-1}, \ldots, x_{t-K})'$ and beta weights

$$\varphi_k(\vartheta_1, \vartheta_2) = \frac{(k/(K+1))^{\vartheta_1 - 1} (1 - k/(K+1))^{\vartheta_2 - 1}}{\sum_{j=1}^{K} (j/(K+1))^{\vartheta_1 - 1} (1 - j/(K+1))^{\vartheta_2 - 1}}. \tag{3.7}$$

The transition is thus determined by a weighted average of K past realizations of x. With only two parameters, the function in Eq. (3.6) allows for flexible weighting

[8] Bollerslev et al. (2015) take a different approach on time varying volatility persistence modeling and propose an extension to the HAR model of Corsi (2009), where the parameters of the model vary with the (estimated) degree of measurement error. They argue that daily RV provides a stronger (weaker) signal for the next day's volatility when the variance of the measurement error is small (large). Their arguments carry over for time varying parameters in the GARCH(1,1) to the extent that the GARCH parameter should increase (decrease) when squared returns are large (low), since the precision of the squared shocks as a measure of daily realized volatility generally decreases (increases) when the level of volatility is high (low).

schemes, in particular hump-shaped or convex schemes, of potentially many lags of x. The restriction $\vartheta_2 = 1$ yields a strictly decreasing weighting scheme

$$\varphi_k(\vartheta) = \frac{(1 - k/(K+1))^{\vartheta - 1}}{\sum_{j=1}^{K} (1 - j/(K+1))^{\vartheta - 1}}, \tag{3.8}$$

where the rate of decay is increasing in ϑ.

Finally, the equations in (3.4)-(3.7) define the new time varying persistence (TVP) GARCH-MIDAS model.[9] We highlight, that in case the variable x has no explanatory power for time varying volatility persistence, i.e. $\gamma = 0$, our new model only nests the GJR-GARCH in Eq. (3.3) for the shifted transition function with $\tilde{F}(\gamma, \tilde{x}_{t-1}) = F(\gamma, \tilde{x}_{t-1}) - 1/2$ with $\tilde{F}(0) = 0$.[10] However, the specification of the transition function in Eq. (3.5) will be more convenient for our empirical analysis in Section 3.4.

3.3 Misspecification test

In this section, we present a misspecification test for testing the standard GJR-GARCH model against the new model extension presented in the previous section. The test will be based on the Lagrange multiplier (LM) principle, which has emerged as the leading testing principle in the GARCH misspecification testing literature.

Misspecification tests for the ST-GARCH model are discussed in Hagerud (1997), González-Rivera (1998), and Anderson et al. (1999). Lundbergh and Teräsvirta (2002) as well as Halunga and Orme (2009) present a unified framework for a number

[9] The model proposed and applied in this paper does not combine mixed data frequencies - note that all model equations rely only on the daily time index t. Rather, we make use of the flexible weighting scheme of the MIDAS approach to smooth the explanatory variable in a parsimonious way and to link time-varying volatility persistence to regimes of say realized volatility or business conditions (as motivated in Secion 3.2.1). This contrasts to the GARCH-MIDAS model introduced in Engle et al. (2013) and applied for instance in Conrad and Loch (2014, 2015), where daily return data is combined with monthly and quarterly macroeconomic data. However, the TVP-GARCH-MIDAS model could likewise be applied to mixed frequency, though the misspecification test presented in Section 3.3 would need to be modified appropriately.

[10] Using $\tilde{F}$ in Eq. (3.4) instead of F, yields

$$h_t = \omega + (\alpha_1 + \alpha_2 \mathbb{1}_{\{\varepsilon_{t-1}<0\}})\varepsilon_{t-1}^2 + \beta_1 h_{t-1} + \beta_2 \tilde{F}(\gamma, \tilde{x}_{t-1}) h_{t-1},$$

which nests Eq. (3.3) for $\gamma = 0$ and can be re-parameterized as

$$h_t = \omega + (\alpha_1 + \alpha_2 \mathbb{1}_{\{\varepsilon_{t-1}<0\}})\varepsilon_{t-1}^2 + \tilde{\beta}_1 h_{t-1} + \beta_2 F(\gamma, \tilde{x}_{t-1}) h_{t-1},$$

with $\tilde{\beta}_1 = \beta_1 - 1/2\beta_2$.

of misspecification tests for (parametric) GARCH models. More recently, Conrad and Schienle (2015) develop a misspecification test for the GARCH-MIDAS model and derive its asymptotic theory. The derivation of the LM test statistic in this section will follow along the lines of these papers and we adapt a similar notation as in Conrad and Schienle (2015).

The model presented in the Section 3.2 shares the common characteristic with nonlinear models that it is not identified if the true model is the nested standard GJR-GARCH model. This problem can be circumvented by a linear approximation of the transition function, which will be done in the next section. We then present the LM test statistic in Section 3.3.2 and discuss its size and power properties in a simulation study in Section 3.3.3.

3.3.1 Linearizing the model

In the following, we consider the shifted version of the transition function

$$\tilde{F}(\gamma, \boldsymbol{\Phi}'\boldsymbol{x}_{t-1}) = (1 + \exp(\gamma \boldsymbol{\Phi}'\boldsymbol{x}_{t-1}))^{-1} - \frac{1}{2},$$

so that the model nests the standard GJR-GARCH model in Eq. (3.3) if the variable x has no explanatory power. Note, that the alternative model in Eq. (3.4) is not identified under the null hypothesis. It nests the GJR-GARCH if $\gamma = 0$, but then β_2 and $\boldsymbol{\Phi}$ (or $(\vartheta_1, \vartheta_2)$) are nuisance parameters.

Following Luukkonen et al. (1988), Hagerud (1997), and Lundbergh and Teräsvirta (2002), we first linearize the transition function by means of a Taylor expansion in order to break the nonlinear dependence on the parameter γ. The first order Taylor expansion around $\gamma_0 = 0$ is given by

$$\begin{aligned}
\tilde{F}(\gamma, \boldsymbol{\Phi}'\boldsymbol{x}_{t-1}) &= \tilde{F}(\gamma_0, \boldsymbol{\Phi}'\boldsymbol{x}_{t-1}) + (\gamma - \gamma_0) \left.\frac{\partial \tilde{F}(\gamma, \boldsymbol{\Phi}'\boldsymbol{x}_{t-1})}{\partial \gamma}\right|_{\gamma=\gamma_0} + R_1(\gamma, \boldsymbol{\Phi}'\boldsymbol{x}_{t-1}) \\
&= \tilde{F}(\gamma_0, \boldsymbol{\Phi}'\boldsymbol{x}_{t-1}) - (\gamma - \gamma_0) \left(1 + \exp(\gamma_0 \boldsymbol{\Phi}'\boldsymbol{x}_{t-1})\right)^{-2} \exp(\gamma_0 \boldsymbol{\Phi}'\boldsymbol{x}_{t-1}) \boldsymbol{\Phi}'\boldsymbol{x}_{t-1} \\
&\quad + R_1(\gamma, \boldsymbol{\Phi}'\boldsymbol{x}_{t-1}) \\
&= -\frac{1}{4}\gamma \boldsymbol{\Phi}'\boldsymbol{x}_{t-1} + R_1(\gamma, \boldsymbol{\Phi}'\boldsymbol{x}_{t-1}),
\end{aligned}$$

where $R_1(\gamma, \boldsymbol{\Phi}'\boldsymbol{x}_{t-1})$ is the remainder term. This yields the following linearized and re-parameterized version of our model

$$h_t = \omega + (\alpha_1 + \alpha_2 \mathbb{1}_{\{\varepsilon_{t-1}<0\}})\varepsilon_{t-1}^2 + \beta_1 h_{t-1} + \boldsymbol{\pi}'\boldsymbol{x}_{t-1}h_{t-1} + R_1^*, \tag{3.9}$$

with $\boldsymbol{\pi}' = -\beta_2 \frac{1}{4}\gamma\boldsymbol{\Phi}'$ and the remainder term $R_1^* = \beta_2 R_1(\gamma, \boldsymbol{\Phi}'\boldsymbol{x}_{t-1})h_{t-1}$. The model now reduces to the standard GJR-model if $\boldsymbol{\pi} = \mathbf{0}$. The linearized model has the following ARCH(∞) representation

$$h_t = \sum_{i=0}^{\infty}\left[\prod_{j=1}^{i}(\beta_1 + \boldsymbol{\pi}'\boldsymbol{x}_{t-j} + R_1^*)\right]\left(\omega + (\alpha_1 + \alpha_2 \mathbb{1}_{\{\varepsilon_{t-1-i}<0\}})\varepsilon_{t-1-i}^2\right). \tag{3.10}$$

Since under $H_0 : \gamma = 0$, we have $R_1^* = 0$, the stochastic remainder will not affect the distributional properties of the test statistic under the null hypothesis. The representation above then reduces to

$$h_{t|\boldsymbol{\pi}=\mathbf{0}} = \sum_{i=0}^{\infty}\beta_1^i\left(\omega + (\alpha_1 + \alpha_2 \mathbb{1}_{\{\varepsilon_{t-1-i}<0\}})\varepsilon_{t-1-i}^2\right).$$

3.3.2 The LM test statistic

We derive a Lagrange multiplier (LM) test for testing the null hypothesis that the variable has no explanatory power in the linearized model in Eq. (3.9), i.e. we test the hypothesis $H_0 : \boldsymbol{\pi} = \mathbf{0}$ against $H_1 : \boldsymbol{\pi} \neq \mathbf{0}$.[11] The LM testing principle has the advantage that it requires estimation of the model under the null only. It does not rely on the asymptotic properties of the new model, whose derivations will not be considered in this paper. In the following, let $\boldsymbol{\theta} = (\boldsymbol{\eta}, \boldsymbol{\pi})$ denote the parameter vector of the model with $\boldsymbol{\eta} = (\omega, \alpha_1, \alpha_2, \beta_1)'$, whereas $\boldsymbol{\eta}_0$ denotes the true GARCH parameters under the null. $h_{0,t} = h_{t|\boldsymbol{\pi}=\mathbf{0}}(\boldsymbol{\eta})$ refers to the conditional variance model under the null and we specify $h_{0,t}(\boldsymbol{\eta}_0)$ in case of the true GARCH parameters under the null. The observed log-returns are given by $\varepsilon_t = \sqrt{h_{0,t}(\boldsymbol{\eta}_0)}Z_t$, where Z_t is independent and identically distributed (i.i.d.) with mean zero, variance equal to one and finite fourth moment.

The conditional quasi log-likelihood function for observation t is given as

$$l_t(\boldsymbol{\theta}) = -\frac{1}{2}\left(\ln(h_t) + \frac{\varepsilon_t^2}{h_t}\right),$$

[11] Strictly speaking, the LM test we derive applies to testing the null of a GJR-GARCH against the class of all non-linear models that yield the same first-order Taylor approximation as our model alternative.

and the score vector evaluated under the null for observation t is given as

$$\boldsymbol{d}_{0,t}(\boldsymbol{\eta}) = \frac{\partial l_t}{\partial \boldsymbol{\theta}}\bigg|_{\boldsymbol{\pi}=\boldsymbol{0}} = \frac{1}{2}\left[\frac{\varepsilon_t^2}{h_{0,t}} - 1\right]\frac{1}{h_{0,t}}\frac{\partial h_t}{\partial \boldsymbol{\theta}}\bigg|_{\boldsymbol{\pi}=\boldsymbol{0}}.$$

Explicit expressions for these partial derivatives are derived from the representation in Eq. (3.10) as

$$\frac{\partial h_t}{\partial \boldsymbol{\eta}}\bigg|_{\boldsymbol{\pi}=\boldsymbol{0}} = \sum_{i=0}^{\infty}\beta_1^i\left(1,\ \varepsilon_{t-1-i}^2,\ \mathbb{1}_{\{\varepsilon_{t-1-i}<0\}}\varepsilon_{t-1-i}^2,\ h_{0,t-1-i}\right)',$$

and

$$\frac{\partial h_t}{\partial \boldsymbol{\pi}}\bigg|_{\boldsymbol{\pi}=\boldsymbol{0}} = \sum_{i=0}^{\infty}\beta_1^i\boldsymbol{x}_{t-1-i}h_{0,t-1-i}.$$

The average score vector under the null is then obtained as

$$\boldsymbol{D}_0(\boldsymbol{\eta}) = \frac{1}{2T}\sum_{t=1}^{T}\left[\frac{\varepsilon_t^2}{h_{0,t}} - 1\right]\begin{pmatrix}\boldsymbol{y}_t\\ \boldsymbol{r}_t\end{pmatrix}, \tag{3.11}$$

with $\boldsymbol{y}_t = 1/h_{0,t}\sum_{i=0}^{\infty}\beta_1^i\left(1,\ \varepsilon_{t-1-i}^2,\ \mathbb{1}_{\{\varepsilon_{t-1-i}<0\}}\varepsilon_{t-1-i}^2,\ h_{0,t-1-i}\right)'$ and $\boldsymbol{r}_t = 1/h_{0,t}\sum_{i=0}^{\infty}\beta_1^i\boldsymbol{x}_{t-1-i}h_{0,t-1-i}$.

The variance of the score vector under the null and evaluated at the true GARCH parameters is given by

$$\boldsymbol{V} = E\left(\boldsymbol{d}_{0,t}(\boldsymbol{\eta}_0)\boldsymbol{d}_{0,t}(\boldsymbol{\eta}_0)'\right) = E\left(\frac{1}{4}\left(\frac{\varepsilon_t^2}{h_{0,t}(\boldsymbol{\eta}_0)} - 1\right)^2\begin{pmatrix}\boldsymbol{y}_{0,t}\boldsymbol{y}_{0,t}' & \boldsymbol{y}_{0,t}\boldsymbol{r}_{0,t}'\\ \boldsymbol{r}_{0,t}\boldsymbol{y}_{0,t}' & \boldsymbol{r}_{0,t}\boldsymbol{r}_{0,t}'\end{pmatrix}\right) \tag{3.12}$$

$$= \frac{\kappa}{4}\begin{pmatrix}E(\boldsymbol{y}_{0,t}\boldsymbol{y}_{0,t}') & E(\boldsymbol{y}_{0,t}\boldsymbol{r}_{0,t}')\\ E(\boldsymbol{r}_{0,t}\boldsymbol{y}_{0,t}') & E(\boldsymbol{r}_{0,t}\boldsymbol{r}_{0,t}')\end{pmatrix}, \tag{3.13}$$

with $\boldsymbol{y}_{0,t} = \boldsymbol{y}_t(\boldsymbol{\eta}_0), \boldsymbol{r}_{0,t} = \boldsymbol{r}_t(\boldsymbol{\eta}_0)$ and $\kappa = E((\varepsilon_t^2/h_{0,t}(\boldsymbol{\eta}_0) - 1)^2)$.

The LM test statistic is based on the *observed* average score vector, which is an approximation of Eq. (3.11) with truncated versions of $\boldsymbol{y}_t$ and $\boldsymbol{r}_t$, evaluated at the quasi-maximum likelihood estimator ($QMLE$) $\hat{\boldsymbol{\eta}}$ of $\boldsymbol{\eta_0}$ under the null. Its asymptotic distribution will not be shown in this paper, but it should be derived along similar lines as in the proofs in Lundbergh and Teräsvirta (2002), Halunga and Orme (2009), and in Conrad and Schienle (2015), where the arguments in the derivation rely on the results for the $QMLE$ for pure GARCH models in Francq and Zakoïan (2004). Having used a similar notation as in Conrad and Schienle (2015), the formula for the score in Eq. (3.11) differs from theirs only in the specification of the (test) variables in the vector $\boldsymbol{r}_t$ (besides including the additional derivative with respect to the asymmetry parameter α_2). The asymptotic distribution of the test statistic can generally be derived in three steps. First, one shows asymptotic normality of the average score evaluated at the true parameters $\boldsymbol{\eta}_0$. In a next step, the score is related to the "lower part" of the score evaluated at the $QMLE$ in order to derive asymptotic normality of the latter. Finally, it is necessary to show that the observed average score evaluated at the $QMLE$ has the same asymptotic distribution as the unobserved one.

Adopting Theorem 3 in Conrad and Schienle (2015), we obtain the following LM test statistic

$$LM = \frac{1}{4T}\left(\sum_{t=1}^{T}\left[\frac{\varepsilon_t^2}{\hat{h}_t} - 1\right]\hat{\boldsymbol{r}}_t\right)' \hat{\boldsymbol{\Sigma}}^{-1}\left(\sum_{t=1}^{T}\left[\frac{\varepsilon_t^2}{\hat{h}_t} - 1\right]\hat{\boldsymbol{r}}_t\right), \tag{3.14}$$

with parameter estimates from the model under the null $\hat{\boldsymbol{\eta}} = (\hat{\omega}, \hat{\alpha}_1, \hat{\alpha}_2, \hat{\beta}_1)'$, the estimated variance process under the null $\hat{h}_t = \hat{\omega} + (\hat{\alpha}_1 + \hat{\alpha}_2 \mathbb{1}_{\{(\varepsilon_{t-1}<0\}})\varepsilon_{t-1}^2 + \hat{\beta}_1 \hat{h}_{t-1}$, $\hat{\boldsymbol{r}}_t = 1/\hat{h}_t \sum_{i=0}^{t-1} \hat{\beta}_1^i \boldsymbol{x}_{t-1-i}\hat{h}_{t-1-i}$, and $\boldsymbol{x}_{t-1} = (x_{t-1}, \dots x_{t-K})$.

A consistent estimator of the asymptotic variance of the relevant part of the score at the QML estimates is given by

$$\hat{\boldsymbol{\Sigma}} = \frac{\hat{\kappa}}{4T}\left(\sum_{t=1}^{T}\hat{\boldsymbol{r}}_t\hat{\boldsymbol{r}}_t' - \sum_{t=1}^{T}\hat{\boldsymbol{r}}_t\hat{\boldsymbol{y}}_t'\left(\sum_{t=1}^{T}\hat{\boldsymbol{y}}_t\hat{\boldsymbol{y}}_t'\right)^{-1}\sum_{t=1}^{T}\hat{\boldsymbol{y}}_t\hat{\boldsymbol{r}}_t'\right), \tag{3.15}$$

with $\hat{\kappa} = 1/T\sum_{t=1}^{T}(\varepsilon_t^2/\hat{h}_t - 1)^2$. Note, that the inverse of the asymptotic variance $\boldsymbol{\Sigma}$ of the relevant part of the score is given by the "(2,2) element" of $\boldsymbol{V}^{-1}$. The test statistic is asymptotically χ^2 distributed with K degrees of freedom, where K corresponds to the number of lags of the explanatory variable included in the transition function, see Eq. (3.6).[12] Note that in the original non-linear model

[12]Alternatively, the test can be carried out in the so-called TR^2 form (Engle, 1982) based on the auxiliary regression

$$\left(\frac{\varepsilon_t^2}{\hat{h}_t} - 1\right) = \hat{\boldsymbol{y}}_t'\boldsymbol{c}_1 + \hat{\boldsymbol{r}}_t'\boldsymbol{c}_2 + u_t.$$

The LM test statistic in Eq. (3.14) and Eq. (3.15) can be re-written as T times the uncentered R^2 of this regression,

$$LM = T\frac{SSR_0 - SSR_1}{SSR_0},$$

where SSR_1 is the sum of squared residuals from the regression and SSR_0 is the sum of squared specification in Eq. (3.4), the null hypothesis that the variable x has no explanatory power for time varying persistence corresponded to $\gamma = 0$. The linearization of the transition function in Section 3.3.1 leads to a degree of freedom K in the asymptotic χ^2 distribution, since the null is now that $\Pi_1 = \Pi_2 = \cdots = \Pi_k = 0$.

Finally, we compare the test in Eq. (3.14) and Eq. (3.15) to the misspecification tests proposed in Amado and Teräsvirta (2015), who consider testing the GJR-GARCH with constant parameters against a general form of time-varying parameters.[13] Adopting their notation to our setup, results in a decomposition of the conditional variance $h_t = h_{0,t} + g_t$, where g_t introduces non-stationarity and

$$h_{0,t} = \omega + (\alpha_1 + \alpha_2 \mathbb{1}_{\{\varepsilon_{t-1}<0\}})\varepsilon_{t-1}^2 + \beta_1 h_{0,t-1} \tag{3.16}$$

$$g_t = \beta_2 F(\gamma, \tilde{x}_{t-1}) h_{0,t-1}. \tag{3.17}$$

However, note that this does not yield the same model under the alternative as in our model in Eq. (3.4). The above specification is not "recursive in nature" as pointed out by Halunga and Orme (2009), meaning that the functions $h_{0,t}$ and g_t include lags of $h_{0,t-1}$, whereas in our alternative they should include the lag of the volatility process h_{t-1}. The Amado and Teräsvirta (2015) LM test statistic for the linearized model will have the same form, but with $\tilde{\boldsymbol{r}}_t = 1/\hat{h}_t \boldsymbol{x}_{t-1} \hat{h}_{t-1}$. The lack of the recursive nature under the alternative may lead to a decrease in power, as discussed in Halunga and Orme (2009) and Conrad and Schienle (2015). In the following simulation study on the size and power properties, we shall therefore compare our LM test to the Amado and Teräsvirta (2015) test version.

3.3.3 Simulation study: power and size properties

In this section, finite sample properties of the proposed LM test are examined in a Monte-Carlo experiment. We simulate return series with $T = 1250$ observations for $M = 1000$ Monte-Carlo replications. Throughout the simulations, the innovation Z_t is assumed to be either standard normally distributed or (standardized) t-distributed with seven degrees of freedom. We will calculate both the LM test statistic in Eq. (3.14) and Eq. (3.15) and the Amado and Teräsvirta (2015) test version, which we denote by LM_{AT}.

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residuals under the null $H_0 : \boldsymbol{c}_2 = \boldsymbol{0}$, i.e. $SSR_0 = \sum_{t=1}^{T} \left(\varepsilon_t^2/\hat{h}_t - 1\right)^2$.

[13] They consider both an additive and a multiplicative misspecification of the conditional variance.

Size properties We first consider the size properties of the LM test statistic. We simulate the model under the null of a GJR-GARCH(1,1) model, i.e. the data generating process (DGP) for the conditional variance is given by

$$h_t = \omega + \left(\alpha_1 + \alpha_2 \mathbb{1}_{\{\varepsilon_{t-1}<0\}}\right) \varepsilon_{t-1}^2 + \beta_1 h_{t-1}.$$

We set the parameter values to $\alpha_1 = 0.06, \alpha_2 = 0.05$, and consider three different values for β_1, reflecting different degrees of persistence (Low: L, Intermediate: I, High: H)

$$\beta_1^L = 0.82, \qquad \beta_1^I = 0.87, \qquad \beta_1^H = 0.91.$$

The persistence of the simulated model ($\alpha_1 + \alpha_2/2 + \beta_1$) thus varies between 0.90, 0.95, and 0.99. The parameter ω is adjusted accordingly for the unconditional variance to be equal to one. The LM tests are based on different rolling windows of realized volatility,

$$x_t = RV_t^{(N)} = \frac{1}{N}\sum_{j=0}^{N-1} \varepsilon_{t-j}^2, \qquad N = 1, 22, 65,$$

and we set $K = 1$. We report the empirical size for both LM tests in Table 3.1. The empirical size is close to the nominal size for both test versions when Z_t is normally distributed. The size tends to decrease for t-distributed innovations, in line with similar LM test evaluations in Conrad and Schienle (2015). The size properties are robust to increasing K to $K = 22$ as well as in case the DGP includes a constant μ in the mean equation and the LM tests are applied to de-meaned returns, see the simulation results in the Appendix.[14]

Power properties In order to consider a realistic model under the alternative, we include actual data in the TVP-GARCH-MIDAS model

$$h_t = \omega + \left(\alpha_1 + \alpha_2 \mathbb{1}_{\{\varepsilon_{t-1}<0\}}\right) \varepsilon_{t-1}^2 + \left(\beta_1 + \beta_2 \left(F(\gamma, \boldsymbol{\Phi}'\boldsymbol{x}_{t-1}) - \frac{1}{2}\right)\right) h_{t-1}$$

$$\boldsymbol{\Phi}'\boldsymbol{x}_{t-1} = \sum_{k=1}^{K^*} \varphi_k(\vartheta) x_{t-k},$$

[14] The *true* MIDAS lag length is of course unknown in empirical applications. In case of misspecification testing in the GARCH-MIDAS model framework, Conrad and Schienle (2015) argue that their LM test is not suited for selecting the true lag order of the model. Their argument applies just as well to our TVP-GARCH-MIDAS model.

with $\omega = 0.10, \alpha_1 = 0.05, \alpha_2 = 0.06, \beta_1 = 0.82, \beta_2 = 0.18$. We include daily rolling windows of realized volatility, $RV_t^{(N)}$, and of the VIX, $VIX_t^{(N)}$, for $N = 1, 22, 65$, over the 2010-2014 sample.[15] We consider two different smooth transitions in the time-varying GARCH coefficient for each variable, with $\gamma \in \{-1, -10\}$ for realized volatility and $\gamma \in \{-1.5, -4\}$ for the VIX. We denote the true MIDAS lag length by K^* and set $K^* = 1$ or $K^* = 22$ with $\vartheta = 3$. We choose $K = 1$ in the test statistics.

The daily rolling window versions of the variables over the 2010-2014 period as well as the time-varying GARCH coefficients implied by the two model specifications are shown in Figures 3.5 and 3.6. All variables except the daily realized volatility $RV_t^{(1)}$ are highly persistent. Increasing N in the rolling window versions of the variables has a smoothing effect on them. Increasing the absolute value of the transition parameter γ has two effects: it steepens the transition between the persistence regimes and attenuates the range of the time-varying GARCH coefficient if the explanatory variable x is bounded from zero. The latter effect is particularly evident in case of $N = 65$ and should make it more difficult to detect time variation in persistence in that case. We present the results of the Monte-Carlo simulations for normally distributed innovations and realized volatility in Table 3.2 and in Table 3.3 for the VIX. For each specification, we also report the standard deviation of the time-varying GARCH coefficient.

Let us first consider the specifications including realized volatilities with $\gamma = -1$ and $K^* = 1$, i.e. the upper left part of Table 3.2. Our LM test has very good power for the specifications with $N = 22$ and $N = 65$, with rejection rates of 86% and 93% at the 10% nominal level. The rates are always greater than for the LM_{AT} test version, though they are similar in magnitude. Increasing K^* does not have a big effect on the time variation in the GARCH coefficient (note that its standard deviation does not change by much) and consequently, the rejection rates are very similar for $K^* = 22$. For $N = 1$ and $K^* = 1$ however, our test has difficulties in detecting the time-varying persistence and we see substantially lower rejection rates (33% at the 10% nominal level), though now the difference to LM_{AT} is much more pronounced. Recall that the two test versions differ only in the specification of the testing variable $\hat{\boldsymbol{r}}_t$. Our test includes $\hat{\boldsymbol{r}}_t = 1/\hat{h}_t \sum_{i=0}^{t-1} \hat{\beta}_1^i \boldsymbol{x}_{t-1-i} \hat{h}_{t-1-i}$, whereas the LM_{AT} version includes only the first summand $1/\hat{h}_t \boldsymbol{x}_{t-1} \hat{h}_{t-1}$. For the very erratic

[15] More precisely, we take squared returns as a realized volatility measure and define VIX_t as $1/365$ times the squared VIX index. We standardize all variables by dividing with their standard deviation. The sample is chosen so that it includes $T = 1250$ observations.

$RV_t^{(1)}$ variable, the additional smoothing via the summation in our test variable seems to be beneficial in terms of power. Smoothing the time-varying GARCH coefficient by increasing K^* slightly increases the power of our test.

For the model specification with $\gamma = -10$, it becomes more difficult to detect time variation in the persistence for the rolling windows with $N = 22$ and $N = 65$. As already suggested in Figure 3.5, the standard deviation of the time-varying GARCH coefficient decreases, which leads to lower rejection rates. In sharp contrast, our test now has good power for the $N = 1$ specification and yields particularly higher rejection rates than the LM_{AT} version, namely 62%, resp. 91%, compared to 31%, resp. 35%, at the 10% nominal level for $K^* = 1$, resp. $K^* = 22$.

Results for the specifications including the VIX are reported in Table 3.3. The VIX is generally smoother and less erratic than realized volatility, resulting in overall higher power of the tests. For the DGP with $\gamma = -1.5$, our test yields rejection rates well above 95% at the 10% nominal level across all specifications. Again, our test performs particularly better than the LM_{AT} version for the rolling windows with $N = 1$. Choosing $\gamma = -4$, decreases the standard deviation of the time-varying GARCH coefficient, which leads to lower rejection rates. Interestingly, differences in rejection rates for the different rolling windows are much less pronounced than for realized volatility, since already the rolling windows of the VIX are relatively smooth (see Figure 3.6).

Including standardized t-distributed errors instead of normally distributed ones tends to decrease the power of the tests, though our main conclusions are still valid. The simulation results are presented in the Appendix.

In summary, the power of our LM test is high for reasonably smooth and large time variation in persistence and its power is particularly higher than the LM_{AT} test version for erratic explanatory variables.

3.4 Empirical analysis

This section presents an empirical application of the new TVP-GARCH-MIDAS model to stock returns on the S&P 500. The dataset is briefly discussed in Section 3.4.1 and estimation results are presented in Section 3.4.2. In Section 3.4.3, the forecasting performance of the new model is compared to the GJR-GARCH model in a direct forecasting evaluation.

3.4.1 Data

Our empirical analysis focuses on the S&P 500 and general U.S. macroeconomic conditions for the 1969 to 2014 period. We consider continuously compounded daily S&P 500 stock return data, r_t, from January 2nd 1969 to December 31st 2014. We calculate daily realized volatility, $RV_t^{(1)} = r_t^2$, and a 22-days rolling window version thereof, $RV_t^{(22)} = 1/22 \sum_{j=0}^{21} r_{t-j}^2$. In order to account for general macroeconomic conditions, we consider the Aruoba-Diebold-Scotti (ADS) business conditions index, which is introduced in Aruoba et al. (2009) and is provided by the Federal Reserve Bank of Philadelphia. The index tracks real business conditions at a daily frequency. It is based on six economic indicators: weekly initial jobless claims, monthly payroll employment, industrial production, personal income less transfer payments, real manufacturing and trade sales, and quarterly real GDP. Its average value is zero, positive values indicate better-than-average conditions, and negative values worse-than-average conditions. We calculate the 22-days rolling window version of the ADS as $ADS_t^{(22)} = 1/22 \sum_{j=0}^{21} ADS_{t-j}$. Summary statistics of the daily data are presented in Table 3.4 and the variables are depicted in Figure 3.7. Note that compared to the ADS, realized volatility is heavily skewed with an excessive kurtosis. Finally, we also consider a modified version of the ADS that focuses only on its negative values, $neg\ ADS_t^{(22)} = \min\{0, ADS_t^{(22)}\}$, see also Dorion (2013). Throughout the empirical analysis, we consider standardized versions of the variables divided by their standard deviation.

3.4.2 Estimation results

We estimate the TVP-GARCH-MIDAS model defined in the equations (3.4)-(3.7) with the daily 22-days rolling window versions of realized volatility, ADS, and $neg\ ADS$ using quasi-maximum likelihood methods. We include one MIDAS lag year in the MIDAS filter, i.e. we set $K = 252$ in Eq. (3.6).[16] The estimation results are presented in Table 3.5 along with the benchmark GJR-GARCH(1,1) model estimates.

In terms of likelihood criteria, only the model with $RV_t^{(22)}$ yields a lower Bayesian information criterion than the benchmark GJR-GARCH. Accordingly, estimates of

[16] We find $K = 252$ to be sufficiently large for our application. As demonstrated in Engle et al. (2013) and Conrad and Loch (2014), the beta weighting function seems to be robust to the maximum number of lags K included in the MIDAS filter, as long as it is chosen large enough. See also Footnote 14 on the MIDAS lag length choice.

the model extension parameters are highly significant for the *RV* model, whereas only the β_2 parameter is found to be significant at the 5% level for the *neg ADS* variable. However, due to the identification issue under the null discussed in Section 3.3.1, the (in)significance of the model extension parameters have to be taken with a pinch of salt. We therefore add the *LM* test statistic presented in Eq. (3.14) and Eq. (3.15) for testing the null hypothesis that the variable x has no explanatory power for time variation in the GARCH coefficient. The test statistic is significant at the 1% level for both *ADS* variables and lies slightly above the 10% significance level for the *RV* variable.[17] The estimated (restricted) MIDAS weighting schemes are plotted in Figure 3.8.[18] The schemes roughly imply vanishing weights for lags beyond half a year.

Next, we have a closer look at the time variation in persistence that is implied by the model estimations. The estimated time-varying GARCH coefficients, $\hat{\beta}_t = \hat{\beta}_1 + \hat{\beta}_2 F(\hat{\gamma}, \hat{\boldsymbol{\Phi}}'\boldsymbol{x}_{t-1})$, are shown in Figure 3.9 and some descriptive statistics are summarized in Table 3.6. First, the signs of the transition parameter γ confirm our intuition from Section 3.2.1. A negative γ for *RV* implies that the time-varying GARCH coefficient is positively related to realized volatility, i.e. we see high (low) persistence during high (low) volatility regimes. Correspondingly, positive γ estimates for the *ADS* variables imply increasing (decreasing) persistence for weak (strong) business conditions. The *RV* model implies a greater time variation in the GARCH coefficient than the *ADS* models. For *RV*, the coefficient lies in the range of $[0.78, 0.90]$, whereas it lies in the range $[0.91, 0.93]$ for the *ADS* models. Accordingly, we see a higher standard deviation of the GARCH coefficient in the *RV* model.[19] Both versions of the *ADS* variable yield a similar time variation in persistence, though the *neg ADS* version yields a slightly smoother variation. However, note that both models imply a lower persistence than the GJR-GARCH model on average.

For the *ADS* variables, particularly for the negative *ADS*, we see essentially two persistence regimes in Figure 3.9 that roughly correspond to recession and expansion periods with not much variation in between. For realized volatility on the other

[17]The financial crisis period seems to have distorting effects on the *LM* test for the realized volatility model. The test statistic for the subsample ending 2007 is calculated as 4.06 and is significant at the 5% level.

[18]For all three variables, including an unrestricted scheme in Eq. (3.7) instead yielded no significant improvements in terms of the likelihood (as measured by means of a likelihood ratio test).

[19]Note that the range of the time-varying GARCH coefficient implied by the model estimates $\hat{\beta}_1, \hat{\beta}_2$ differs for the strictly positive *RV*/*neg ADS*, and the *ADS*, since in the first case, the transition function is restricted to the $[0.5, 1]$ interval, see also Footnote 7.

hand, there is more variation in the GARCH coefficient during expansion periods. This suggests that there are other factors than the U.S. business conditions affecting volatility persistence, which are reflected in realized volatility but not in the ADS. For instance, monetary policy is an important driver of realized volatility. The term spread (which is not included in the ADS) has strong predictive power for realized volatility, as shown in Paye (2012), and is a leading indicator for financial volatility, as argued in Conrad and Loch (2014).

We illustrate the underlying transitions between high and low persistence regimes implied by the explantory variables in Figure 3.10. The figure plots estimates of the transition function $F(\hat{\gamma}, \hat{\mathbf{\Phi}}'\boldsymbol{x}_{t-1})$ and the time-varying GARCH coefficient $\hat{\beta}_t$, which corresponds to a linear transformation of the transition function, against the weighted average of the respective explanatory variable x, $\tilde{x}_t = \hat{\mathbf{\Phi}}'\boldsymbol{x}_{t-1} = \sum_{k=1}^{252} \hat{\varphi}_k x_{t-k}$. To illustrate how the distribution of the explanatory variables relates to the time variation in persistence, the figure includes histograms of $\bar{x}_t$ and of the transition / GARCH coefficient. The skewness of realized volatility is evident and in line with the descriptive statistics in Table 3.4. Combined with some large outliers, this translates into a very steep transition function for realized volatility, which results in the GARCH coefficient being almost flat during very high volatility regimes, such as the financial crises 2008/09. On the contrary, the distribution of the ADS is symmetric around zero with few outliers and its translation function is smoother.

Finally, we find similar results if we let the explanatory variables govern time variation in the ARCH coefficient. The corresponding estimates of the time-varying ARCH coefficient as well as a summary of their descriptive statistics can be found in the Appendix. However, we note that these model estimates imply a higher level of average volatility persistence, compared to the specification with a time-varying GARCH coefficient. This finding is perfectly in line with the argument by Hillebrand (2005) that the effect of overestimating volatility persistence is stronger if changes in the GARCH parameter are not accounted for (see also the discussion on page 80). This reconfirms the specific choice of our model specification in Eq. (3.4).

3.4.3 Forecasting evaluation

Our forecasting analysis is based on model estimations up to the end of 1999 and an out-of-sample evaluation period from 2000 to 2010. Subsample model estimations as well as descriptive statistics for the estimated time-varying persistence are presented in the Appendix. Note that the LM test statistic is now found to be significant at the 1% level for the $RV_t^{(22)}$ variable. Also, the average estimated persistence from the TVP-GARCH model estimations are found to be lower for the subsample, which excludes the financial crisis, than for the full sample.

In the direct forecasting evaluation, we use the mean squared error (MSE) and the quasi-likelihood ($QLIKE$) loss functions, since Patton (2011) showed that they are both robust in the sense that they yield the same ranking of two volatility forecasts when using an observed (unbiased) volatility proxy instead of the unobserved true volatility. The two loss functions differ in an important way: the MSE is a symmetric loss function, whereas the $QLIKE$ depends on the relative forecast error and penalizes more heavily volatility forecasts that underestimate volatility. Moreover, Brownlees et al. (2011) show that the MSE has a bias that is proportional to the true volatility, whereas the bias of $QLIKE$ is independent of the volatility level. Let RV_{t+l} denote the realized volatility proxy that is based on 5-minutes intra-day returns and let $\hat{h}_{t+l|t}$ denote the l-step ahead volatility forecast.[20] For observation t, the two loss functions are then given by

$$MSE_t = \left(RV_{t+l} - \hat{h}_{t+l|t}\right)^2,$$
$$QLIKE_t = \frac{RV_{t+l}}{\hat{h}_{t+l|t}} - \log\left(\frac{RV_{t+l}}{\hat{h}_{t+l|t}}\right) - 1.$$

Note that at the beginning of period $t+1$, the time-varying GARCH coefficient $\beta_{t+1} = \beta_1 + \beta_2 F(\gamma, \boldsymbol{\Phi}'\boldsymbol{x}_t)$ is predetermined with respect to $\mathcal{F}_t$, since the transition function includes lags of x beyond period t. Thus, the one-step ahead volatility prediction $\hat{h}_{t+1|t}$ from the TVP-GARCH-MIDAS model is simply h_{t+1}, just as in the GJR-GARCH model. In computing volatility forecasts from the TVP-GARCH-MIDAS model beyond horizon $l = 1$, we make the simplifying assumption that $E[\beta_{t+l}|\mathcal{F}_t] = \beta_{t+1}$ for $l > 1$, that is we keep β_{t+1} fixed. Volatility forecasts are then obtained iteratively in a similar way as in the GJR-GARCH model.

The results of a forecast evaluation based on the $QLIKE$ for daily forecasts horizons $l = 1, 10, 22, 65$ are presented in Table 3.7. Based on a Diebold-Mariano test, we find significant improvements over forecasts from the GJR-GARCH benchmark model for the TVP-GARCH-MIDAS model including the RV across all horizons. On the other hand, the models including the ADS yield no significant improvements over the benchmark model. We find similar results for the mean squared error loss

[20]We rely on intra-day returns from the Oxford-Man-Institute of Quantitative Finance.

function. In Figure 3.11, we show the R^2s obtained from Mincer-Zarnowitz regressions across horizons $l = 1, \ldots, 65$, that is from regressing realized volatility for period $t+l$ on a constant and the respective l-step ahead volatility forecast given t. The R^2 obtained from the RV forecasts constantly lies above the other ones across the horizons. On the other hand, the R^2 from the benchmark model is not distinguishable from the ADS models. In summary, we find strong evidence that the model with time-varying persistence determined by $RV_t^{(22)}$ significantly improves volatility forecasts over the GJR-GARCH model.

Finally, we compare the volatility forecasts across volatility regimes in order to get a sense when the adjustment in volatility persistence pays off the most. We follow the approach in Lanne and Saikkonen (2005) and split realized volatility into three categories. Then, based on each realized volatility observation, we calculate forecasts implied by the different model estimations at horizons from 1 to 65 days and take the average of the forecasts for a given horizon within each category. The average realized volatility as well as the volatility forecasts at each horizon are depicted in Figure 3.12. The forecasts are based on the initial realized volatility value RV_0 with $RV_0 < 0.6$, $0.6 \leq RV_0 < 4.3$, and $RV_0 \geq 4.3$. The thresholds correspond to the 50% and 95% quantile of realized volatility and most of the observations falling into the last category coincide with the financial crisis period. Note that we plug in RV_0 as a starting value for all models and then iterate the forecasts based on the respective persistence estimates. This exercise does therefore not evaluate the actual volatility (point) forecasts, but rather compares the *persistence evolvement* that is implied by the models.

We find that on average, forecasts from the TVP-GARCH-MIDAS model with $RV_t^{(22)}$ capture very well the actual rate of persistence of realized volatility for the first (low initial RV_0) and the last (high initial RV_0) category. The two models with the ADS variables yield similar forecast persistence, though the negative ADS improves over the standard ADS variable in the high initial realized volatility regime.

In the low volatility regime, the TVP-GARCH-MIDAS models imply a lower volatility persistence than the GJR-GARCH, but the level implied by the ADS is still to high compared to the actual persistence of realized volatility in this regime. Similarly, the ADS overestimates persistence in the high volatility regime.[21]

[21] In line with the full sample estimations, the TVP-GARCH MIDAS model with realized volatility yields a greater time variation in persistence. The *minimum* persistence value for the model is 0.7788, compared to (0.9829) 0.9777 for the (negative) ADS. The *maximum* persistence value for the model with realized volatility is 0.9545, compared to (0.9922) 0.9972 for the (negative) ADS. The estimated persistence implied by the GJR-GARCH model equals 0.9881. The full descriptive

The "spurious almost-integration" effect for the GJR-GARCH(1,1) is evident in the low realized volatility category, where the rate of persistence is too high. For the intermediate initial realized volatility category, none of the models is able to capture the right level of persistence, but note that the differences to the actual evolvement of realized volatility are small compared to the high volatility regime.

3.5 Conclusions

We suggest a new GARCH volatility model that links time-varying persistence (TVP) to an explanatory variable using MIDAS techniques. The new model nests the GJR-GARCH in case the variable has no explanatory power and we present a misspecification test based on the Lagrange multiplier principle. We find good sample size and power properties in a Monte-Carlo simulation study. In an empirical application to the U.S. stock market, we provide evidence that volatility persistence is counter cyclical and high (low) during periods of weak (strong) business conditions and high (low) realized volatilities. However, the model including realized volatility is able to capture more time variation in the persistence than the model including business conditions and is preferable from a forecasting point of view. It therefore seems natural to extend our empirical analysis to including alternative (high-frequency) realized volatility measures as explanatory variables. Also, our framework allows to include macroeconomic variables of lower frequency, though the misspecification test would have to be adjusted accordingly.

statistics are presented in the Appendix.

3.6 Tables and figures

3.6.1 Tables

Table 3.1: Empirical size properties of the LM-tests

		$Z_t \sim \mathcal{N}(0,1)$			$Z_t \sim t(7)$		
		L	I	H	L	I	H
$x_t = RV_t^{(1)}$							
	1%	0.95	0.62	0.51	0.43	0.20	0.30
LM	5%	3.18	3.62	3.23	2.67	2.05	1.93
	10%	8.05	7.54	6.36	5.97	5.63	5.07
	1%	0.85	0.72	0.81	0.64	0.61	0.30
LM_{AT}	5%	4.03	3.20	3.13	3.62	4.09	3.65
	10%	9.32	8.26	7.58	8.74	8.15	7.09
$x_t = RV_t^{(22)}$							
	1%	0.74	1.14	0.61	0.64	0.61	0.20
LM	5%	4.77	4.86	4.24	5.01	4.40	3.14
	10%	10.06	9.61	8.48	9.59	9.21	6.18
	1%	0.42	0.62	0.51	0.32	0.41	0.20
LM_{AT}	5%	3.18	3.41	3.54	3.52	2.66	3.14
	10%	8.16	6.51	7.07	6.61	5.94	5.88
$x_t = RV_t^{(65)}$							
	1%	1.69	1.76	1.21	0.53	0.92	1.22
LM	5%	6.04	5.27	7.07	5.12	6.02	6.18
	10%	12.08	12.40	12.63	9.28	10.44	11.14
	1%	1.17	1.14	1.11	0.53	0.41	0.41
LM_{AT}	5%	4.98	5.17	5.05	3.20	3.38	3.75
	10%	10.70	10.85	11.21	9.59	8.90	8.41

Notes: Rejection rates in percent at the 1%, 5%, and 10% nominal level. The data generating process is a GJR-GARCH(1,1) process

$$\varepsilon_t = \sqrt{h_t} Z_t$$
$$h_t = \omega + \left(\alpha_1 + \alpha_2 \mathbb{1}_{\{\varepsilon_{t-1}<0\}}\right) \varepsilon_{t-1}^2 + \beta_1 h_{t-1},$$

with parameter values set to $\alpha_1 = 0.05$, $\alpha_2 = 0.06$, and persistence regimes L, I, and H with $\beta_1^L = 0.82$, $\beta_1^I = 0.87$, and $\beta_1^H = 0.91$. The persistence of the simulated model $(\alpha_1 + \alpha_2/2 + \beta_1)$ thus varies between $0.90, 0.95$, and 0.99. We set ω accordingly, so that the unconditional variance equals one. The shocks Z_t are simulated from a standard normal distribution or a t-distribution with seven degrees of freedom. The LM-tests are based on $x_t = RV_t^{(N)} = \frac{1}{N}\sum_{j=0}^{N-1} \varepsilon_{t-j}^2$, with $N = 1, 22, 65$, and we set $K = 1$ in the test statistics. LM refers to the test statistic in Section 3.3.2, see Eq. (3.14) and Eq. (3.15), whereas LM_{AT} refers to the Amado and Teräsvirta (2015) test version, see the discussion at the end of Section 3.3.2.

Table 3.2: Empirical power properties of the LM-tests with RV

N		$K^*=1$: 1	$K^*=1$: 22	$K^*=1$: 65	$K^*=22$: 1	$K^*=22$: 22	$K^*=22$: 65
$x_t = RV_t^{(N)}$	$\gamma=-1$						
SD of β_t		0.013	0.017	0.017	0.009	0.016	0.017
LM	1%	9.3	61.6	72.7	12.7	57.9	72.3
	5%	24.6	78.1	88.2	28.5	75.4	87.2
	10%	33.0	85.6	93.3	36.7	84.5	92.9
LM_{AT}	1%	4.2	50.9	68.3	3.1	43.2	64.7
	5%	12.6	71.7	85.7	9.9	63.0	84.2
	10%	20.0	80.4	92.0	16.3	74.2	90.3
$x_t = RV_t^{(N)}$	$\gamma=-10$						
SD of β_t		0.032	0.018	0.008	0.025	0.016	0.008
LM	1%	32.7	36.6	8.7	65.1	30.2	8.4
	5%	51.0	55.9	22.0	84.3	51.4	21.3
	10%	61.5	66.1	33.3	90.7	60.6	32.4
LM_{AT}	1%	10.2	23.9	6.4	11.6	16.1	5.9
	5%	22.1	44.6	20.0	24.5	35.2	18.2
	10%	31.3	55.5	29.5	34.5	45.8	28.6

Notes: Rejection rates at 1%, 5%, and 10% nominal level. The data generating process is the TVP-GARCH-MIDAS(1,1)

$$\varepsilon_t = \sqrt{h_t} Z_t$$
$$h_t = \omega + \left(\alpha_1 + \alpha_2 \mathbb{1}_{\{\varepsilon_{t-1}<0\}}\right)\varepsilon_{t-1}^2 + \left(\beta_1 + \beta_2\left(F(\gamma, \boldsymbol{\Phi}'\boldsymbol{x}_{t-1}) - \frac{1}{2}\right)\right) h_{t-1}$$
$$\boldsymbol{\Phi}'\boldsymbol{x}_{t-1} = \sum_{k=1}^{K^*} \varphi_k(\vartheta) x_{t-k},$$

with $\omega = 0.1$, $\alpha_1 = 0.05$, $\alpha_2 = 0.06$, $\beta_1 = 0.82$, $\beta_2 = 0.18$, $\vartheta = 3$, and $\gamma = -1$ or $\gamma = -10$. x_t is taken as rolling window versions of realized volatility, $RV_t^{(N)}$ with $N = 1, 22, 65$, over the 2010-2014 sample. K^* denotes the true MIDAS lag order in the DGP. We also report the standard deviation (SD) of the time-varying GARCH coefficient in the DGP, $\beta_t = \beta_1 + \beta_2\left(F(\gamma, \boldsymbol{\Phi}'\boldsymbol{x}_{t-1}) - \frac{1}{2}\right)$.
All test statistics are based on $K = 1$. LM refers to the test statistic in Section 3.3.2, see Eq. (3.14) and Eq. (3.15), whereas LM_{AT} refers to the Amado and Teräsvirta (2015) test version, see the discussion at the end of Section 3.3.2. The shocks Z_t are simulated from a standard normal distribution.

Table 3.3: Empirical power properties of the LM-tests with the VIX

		$K^*=1$			$K^*=22$		
N		1	22	65	1	22	65
$x_t = VIX_t^{(N)}$	$\gamma=-1.5$						
SD of β_t		0.019	0.018	0.018	0.019	0.018	0.018
	1%	79.8	80.0	78.9	78.5	78.5	79.0
LM	5%	93.0	93.1	92.6	92.5	92.3	93.0
	10%	96.3	97.2	96.4	96.3	96.8	96.3
	1%	61.4	72.8	72.1	53.8	66.7	68.3
LM_{AT}	5%	83.3	89.5	90.0	78.4	86.7	88.6
	10%	90.9	95.4	95.6	87.6	92.7	94.6
$x_t = VIX_t^{(N)}$	$\gamma=-4$						
SD of β_t		0.013	0.012	0.010	0.013	0.011	0.010
	1%	33.4	26.4	20.5	29.8	24.5	20.1
LM	5%	56.8	49.6	43.1	53.0	48.6	42.8
	10%	68.5	61.6	55.9	64.6	60.2	55.3
	1%	19.7	18.2	14.2	15.2	15.9	13.7
LM_{AT}	5%	42.1	42.8	36.7	35.4	39.5	35.1
	10%	54.6	54.6	49.5	47.7	51.5	48.3

Notes: Rejection rates at 1%, 5%, and 10% nominal level. The data generating process is the TVP-GARCH-MIDAS(1,1)

$$\varepsilon_t = \sqrt{h_t} Z_t$$

$$h_t = \omega + \left(\alpha_1 + \alpha_2 \mathbb{1}_{\{\varepsilon_{t-1}<0\}}\right)\varepsilon_{t-1}^2 + \left(\beta_1 + \beta_2\left(F(\gamma, \boldsymbol{\Phi}'\boldsymbol{x}_{t-1}) - \frac{1}{2}\right)\right)h_{t-1}$$

$$\boldsymbol{\Phi}'\boldsymbol{x}_{t-1} = \sum_{k=1}^{K^*}\varphi_k(\vartheta)x_{t-k},$$

with $\omega = 0.1$, $\alpha_1 = 0.05$, $\alpha_2 = 0.06$, $\beta_1 = 0.82$, $\beta_2 = 0.18$, $\vartheta = 3$, and $\gamma = -1.5$ or $\gamma = -4$. x_t is taken as rolling window versions of the VIX, $VIX_t^{(N)}$ with $N = 1, 22, 65$, over the 2010-2014 sample. K^* denotes the true MIDAS lag order in the DGP. We also report the standard deviation (SD) of the time-varying GARCH coefficient in the DGP, $\beta_t = \beta_1 + \beta_2\left(F(\gamma, \boldsymbol{\Phi}'\boldsymbol{x}_{t-1}) - \frac{1}{2}\right)$.
All test statistics are based on $K = 1$. LM refers to the test statistic in Section 3.3.2, see Eq. (3.14) and Eq. (3.15), whereas LM_{AT} refers to the Amado and Teräsvirta (2015) test version, see the discussion at the end of Section 3.3.2. The shocks Z_t are simulated from a standard normal distribution.

TABLE 3.4: DESCRIPTIVE STATISTICS OF THE DATA

Variable	Min	Max	Mean	SD	Skew.	Kurt.
S&P 500 returns	-22.90	10.96	0.03	1.07	-1.03	29.09
Realized volatility	0.00	524.40	1.14	6.02	59.80	4977.19
	(0.00)	(87.13)	(0.19)	(1.00)		
$RV_t^{(22)}$	0.06	37.82	1.14	2.35	9.77	123.06
	(0.03)	(16.12)	(0.49)	(1.00)		
ADS_t	-4.42	2.78	-0.09	0.87	-1.23	6.76
	(-5.07)	(3.18)	(-0.10)	(1.00)		
$ADS_t^{(22)}$	-4.30	2.62	-0.09	0.86	-1.26	6.74
	(-5.00)	(3.04)	(-0.10)	(1.00)		

Notes: The reported statistics include the minimum (Min) and maximum (Max), mean, standard deviation (SD), Skewness (Skew.), and Kurtosis (Kurt.). Daily realized volatility is the squared daily return and its 22-days rolling window version is calculated as $RV_t^{(22)} = 1/22 \sum_{j=0}^{21} r_{t-j}^2$, and analogously for the ADS Business Conditions Index. For each variable, the second row displays statistics in parentheses for a standardized version, where the variable is divided by its standard deviation. The full sample includes 11576 daily observations from January 2, 1969 to December 31, 2014.

TABLE 3.5: TVP-GARCH-MIDAS MODEL ESTIMATIONS

Variable	μ	ω	α_1	α_2	β_1	β_2	γ	ϑ	LLF	BIC	LM
$RV_t^{(22)}$	0.0315*** (0.0075)	0.0471*** (0.0110)	0.0093* (0.0052)	0.1359*** (0.0227)	0.5715*** (0.0990)	0.3231*** (0.0877)	−7.7443*** (1.3997)	5.9039*** (1.9796)	−14609.41	2.5956	2.24 [0.13]
$ADS_t^{(22)}$	0.0293*** (0.0076)	0.0175*** (0.0045)	0.0166*** (0.0048)	0.1050*** (0.0203)	0.8947*** (0.0241)	0.0334 (0.0249)	0.8289 (0.9110)	4.5535 (7.3905)	−14654.95	2.6036	8.22 [0.00]
neg $ADS_t^{(22)}$	0.0294*** (0.0076)	0.0182*** (0.0047)	0.0160*** (0.0048)	0.1069*** (0.0208)	0.8879*** (0.0224)	0.0372** (0.0159)	1.1296 (0.8617)	6.5354 (8.4871)	−14654.34	2.6035	8.95 [0.00]
GJR-GARCH(1,1)	0.0291*** (0.0076)	0.0147*** (0.0036)	0.0190*** (0.0047)	0.0996*** (0.0189)	0.9164*** (0.0128)	-	-	-	−14665.58	2.6030	-

Notes: The table reports estimation results for TVP-GARCH-MIDAS models with a time-varying GARCH coefficient based on one MIDAS lag year of a daily macro variable x, i.e the conditional variance process is given by

$$h_t = \omega + \left(\alpha_1 + \alpha_2 \mathbb{1}_{\{(r_{t-1}-\mu<0\}}\right)(r_{t-1}-\mu)^2 + \left(\beta_1 + \beta_2 F(\gamma, \boldsymbol{\Phi}'\boldsymbol{x}_{t-1})\right) h_{t-1}$$

$$F(\gamma, \boldsymbol{\Phi}'\boldsymbol{x}_{t-1}) = \frac{1}{1+\exp(\gamma \boldsymbol{\Phi}'\boldsymbol{x}_{t-1})},$$

$$\boldsymbol{\Phi}'\boldsymbol{x}_{t-1} = \sum_{k=1}^{K} \varphi_k x_{t-k},$$

with restricted beta weights $\varphi_k = \varphi_k(1,\vartheta)$, $k = 1,\ldots,K = 252$. All estimations are based on daily return data from March 1970 to December 2014 and explanatory variable data from March 1969 on. The numbers in parentheses are Bollerslev-Wooldridge robust standard errors. ***, **, * indicate significance at the 1%, 5%, and 10% level. LLF is the value of the maximized log-likelihood function. BIC is the Bayesian information criterion. LM refers to the test statistic in Section 3.3.2, see Eq. (3.14) and Eq. (3.15), for testing the null hypothesis that the variable x has no explanatory power. In brackets, we present the corresponding p-values from the limiting $\chi^2(1)$ distribution.

Table 3.6: Descriptive statistics of time-varying persistence

Variable	Min	Max	Mean	SD
Panel A: Time-varying GARCH coefficient				
$RV_t^{(22)}$	0.7792	0.8946	0.8592	0.0312
$ADS_t^{(22)}$	0.9017	0.9267	0.9118	0.0050
neg $ADS_t^{(22)}$	0.9065	0.9250	0.9106	0.0052
Panel B: Time-varying persistence				
$RV_t^{(22)}$	0.8565	0.9718	0.9364	0.0312
$ADS_t^{(22)}$	0.9708	0.9958	0.9810	0.0050
neg $ADS_t^{(22)}$	0.9760	0.9945	0.9801	0.0052

Notes: The table reports descriptive statistics of the time-varying GARCH coefficients, $\hat{\beta}_t = \hat{\beta}_1 + \hat{\beta}_2 F(\hat{\gamma}, \hat{\boldsymbol{\Phi}}'\boldsymbol{x}_{t-1})$, from the TVP-GARCH-MIDAS model estimations in Table 3.5 and the corresponding time-varying persistence, calculated as $\hat{\alpha}_1 + \frac{1}{2}\hat{\alpha}_2 + \hat{\beta}_t$. The estimated GARCH coefficient, resp. persistence, from the GJR-GARCH model is 0.9164, resp. 0.9852.

Table 3.7: Daily forecast evaluation

Forecast horizon	$l = 1$	$l = 10$	$l = 22$	$l = 65$
Model		$QLIKE$ loss ratio		
GJR-GARCH(1,1)	0.23	0.34	0.42	0.59
	-	-	-	-
$RV_t^{(22)}$	0.96	0.97	0.93	0.88
	[0.00]	[0.06]	[0.00]	[0.00]
$ADS_t^{(22)}$	1.01	1.01	1.00	0.96
	[0.00]	[0.18]	[0.76]	[0.17]
neg $ADS_t^{(22)}$	1.00	1.00	0.99	0.93
	[0.90]	[0.97]	[0.41]	[0.11]

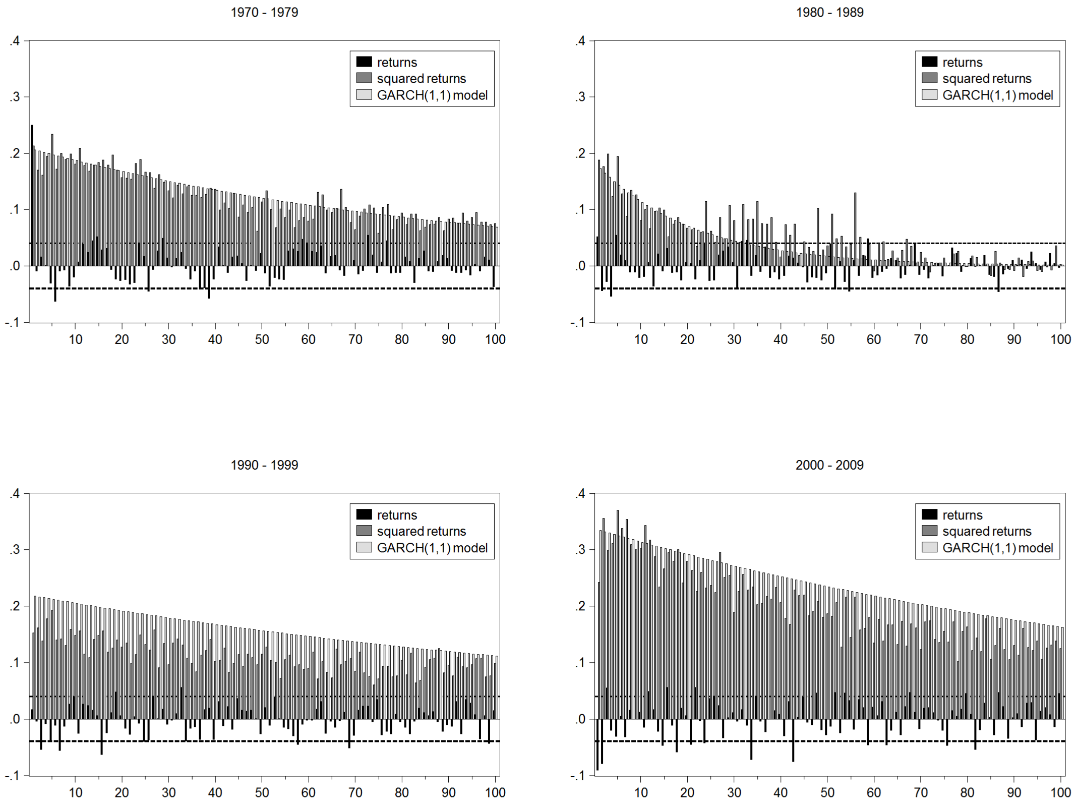
Notes: The forecast evaluation is based on TVP-GARCH-MIDAS model estimations on the subsample from March 1970 to December 1999, see the estimations results in the Appendix. We evaluate daily volatility forecasts for varying forecast horizons $l = 1, 10, 22, 65$ over the January 2000 to December 2014 out-of-sample period with 3744 observations. Volatility forecasts are evaluated using the QLIKE loss function,

$$QLIKE = \frac{RV_{t+l}}{\hat{h}_{t+l|t}} - \log\left(\frac{RV_{t+l}}{\hat{h}_{t+l|t}}\right) - 1,$$

with the $l-$step ahead volatility forecast $\hat{h}_{t+l|t}$ and the RV proxy RV_{t+l} that is based on 5-min intra-day returns. We present mean $QLIKE$ losses for the benchmark GJR-GARCH(1,1) model and mean $QLIKE$ loss ratios relative to the benchmark for the TVP-GARCH-MIDAS models. A ratio below one implies an improvement over the benchmark model. In brackets, we present p-values based on a Giacomini-White test on equal predictive ability.

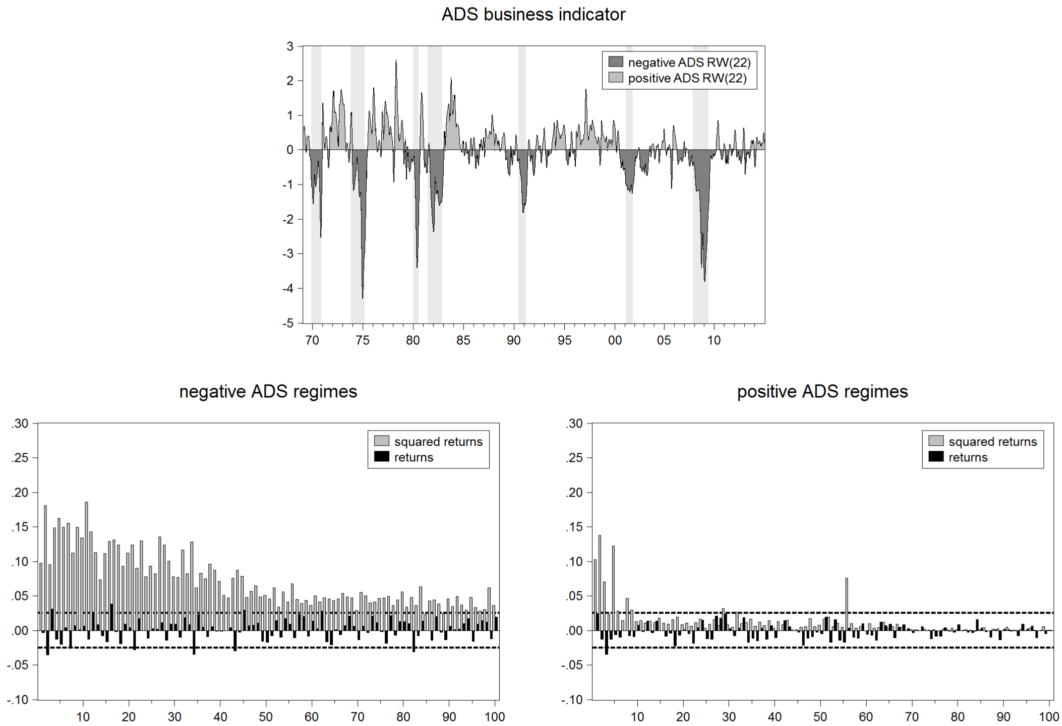
3.6.2 Figures

Figure 3.1: Empirical autocorrelation functions across subsamples



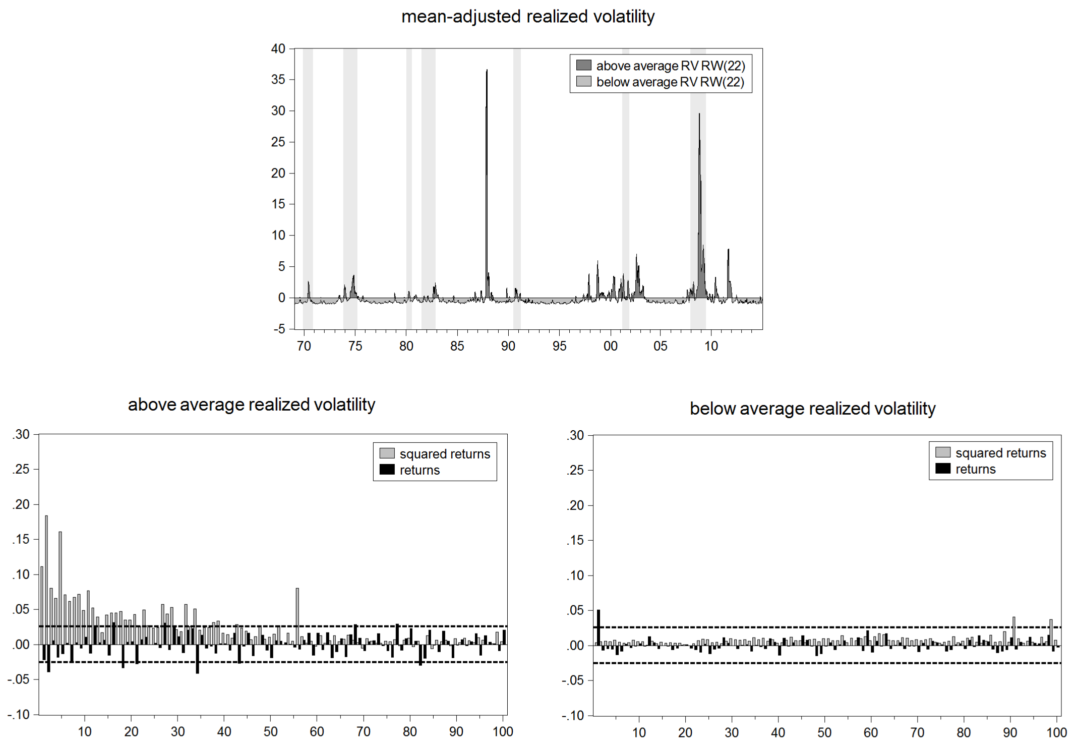
Notes: Empirical autocorrelation functions up to lag 100 for daily returns and squared returns across four decades from 1970 to 2009. The dashed lines are 95% confidence bands for independent white noise, calculated as $\pm 1.96/\sqrt{T}$. For each decade, a simple GARCH(1,1) model is estimated and the model implied autocorrelation function is added to the figure.

Figure 3.2: Empirical autocorrelation functions across *ADS* regimes



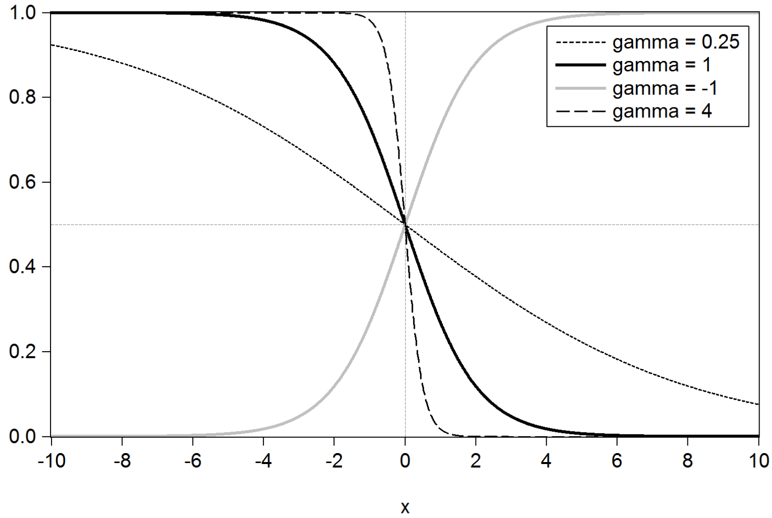
Notes: The top row shows the daily 22-days rolling window of the *ADS* business indicator, denoted by ADS RW(22), over the full 1969-2014 sample. Shaded areas represent NBER recession periods. There are 5732 negative and 5823 positive values of the ADS RW(22). The bottom row shows the empirical autocorrelation functions (ACF) up to lag 100 for daily returns and squared returns across negative and positive 22-days rolling window ADS regimes over the full sample. The dashed lines are 95% confidence bands for independent white noise, calculated as $\pm 1.96\sqrt{T}$. See Eq. (3.1) for the calculation of the ACF across the two regimes.

FIGURE 3.3: EMPIRICAL AUTOCORRELATION FUNCTIONS ACROSS RV REGIMES



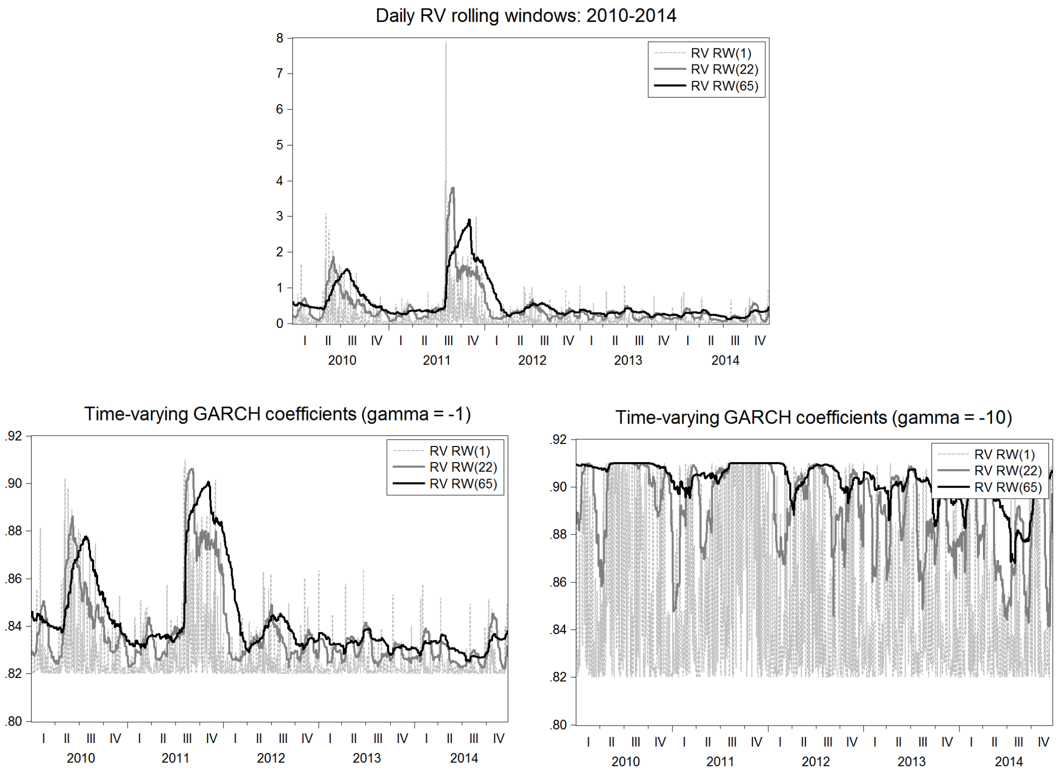
Notes: The top row shows the daily 22-days rolling window of mean-adjusted realized volatility, denoted by RV RW (22), over the full 1969-2014 sample. Shaded areas represent NBER recession periods. The mean value of RV RW(22) is 1.14 and there are 2812 values above and 8743 values below the mean. The bottom row shows the empirical autocorrelation functions (ACF) up to lag 100 for daily returns and squared returns across above and below average realized volatility rolling window (with 22 lags) regimes over the full sample. The dashed lines are 95% confidence bands for independent white noise, calculated as $\pm 1.96\sqrt{T}$. See Eq. (3.1) for the calculation of the ACF across the two regimes.

FIGURE 3.4: TRANSITION FUNCTION



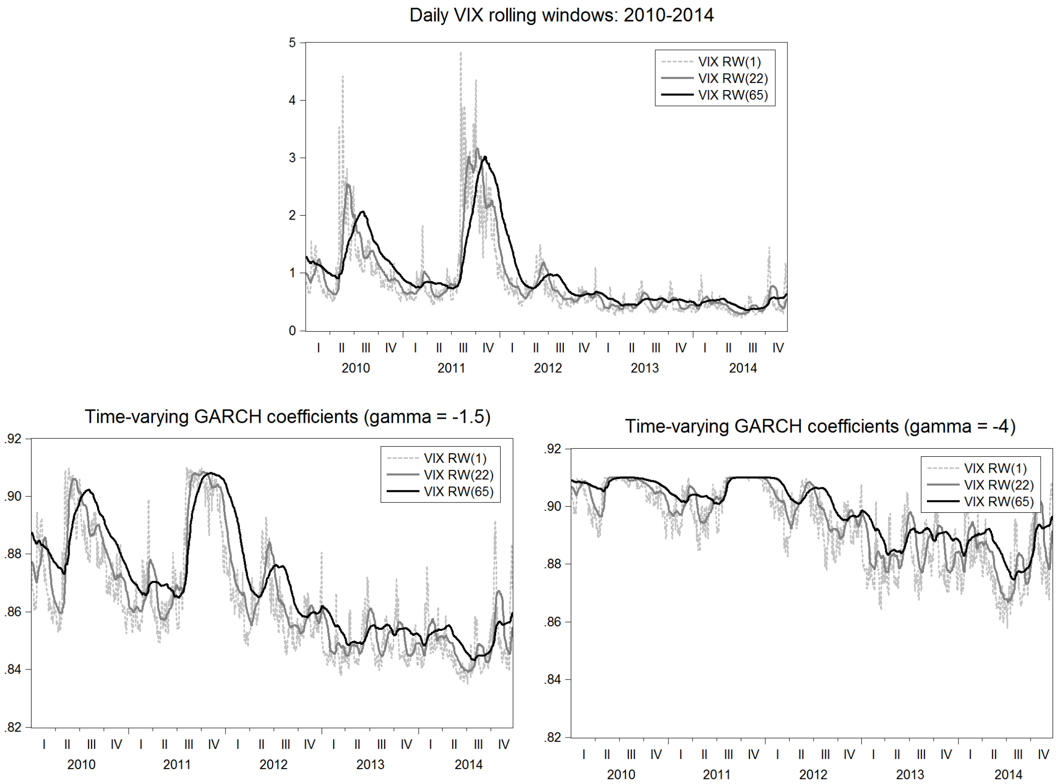
Notes: Transition function $F(\gamma, \tilde{x}) = (1 + \exp(\gamma\tilde{x}))^{-1}$ with $\gamma \in \{0.25, 1, 4\}$ and $\gamma = -1$.

Figure 3.5: Empirical power properties of the LM-tests with RV



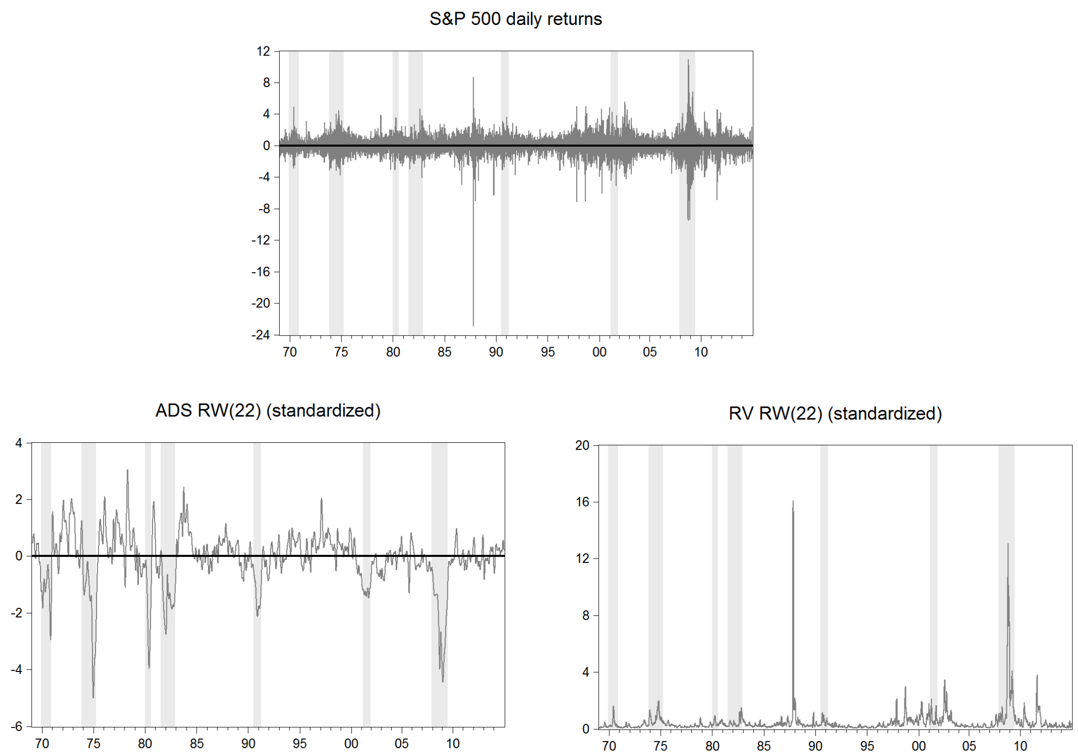
Notes: The top row figure shows daily rolling window versions of realized volatility, denoted by RV RW(N), with $N = 1, 22, 65$, over the 2010-2014 period. The bottom row shows the time-varying GARCH coefficient implied by the two TVP-GARCH-MIDAS model specifications considered in the power simulation for the LM test, see Section 3.3.3 and Table 3.2. The two specifications only differ in the value for the γ parameter, which governs the transitions between the persistence regimes.

Figure 3.6: Empirical power properties of the LM-tests with the VIX



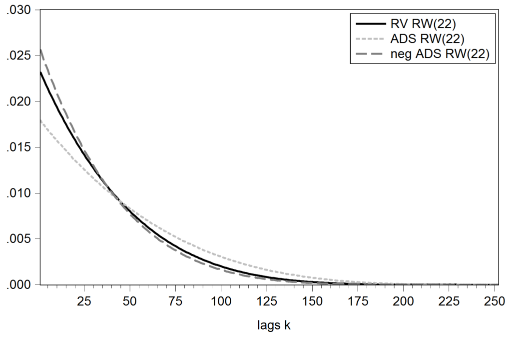
Notes: The top row figure shows daily rolling window versions of the VIX, denoted by VIX RW(N), with $N = 1, 22, 65$, over the 2010-2014 period. The bottom row shows the time-varying GARCH coefficient implied by the two TVP-GARCH-MIDAS model specifications considered in the power simulation for the LM test, see Section 3.3.3 and Table 3.3. The two specifications only differ in the value for the γ parameter, which governs the transitions between the persistence regimes.

Figure 3.7: Financial and macroeconomic data



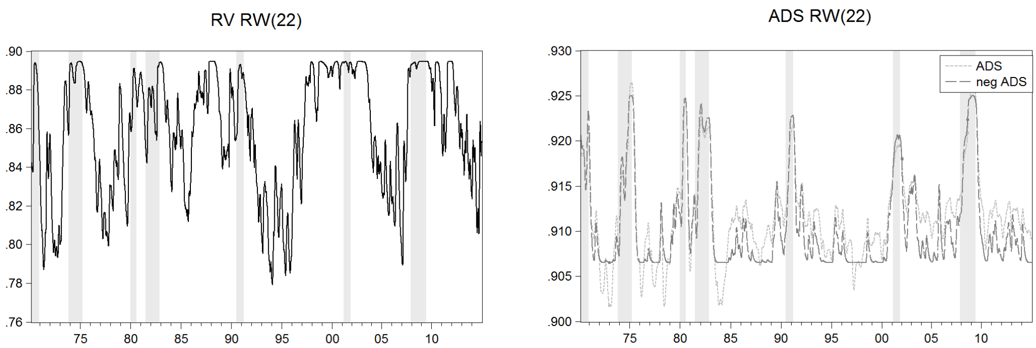
Notes: Daily S&P 500 log returns (top row), 22-days rolling window of the daily *ADS* business conditions index (bottom row, left) and 22-days rolling window of daily realized volatility (bottom row, right). The two variables are standardized through division by their standard deviation. Shaded areas represent NBER recession periods.

FIGURE 3.8: BETA WEIGHTING SCHEMES



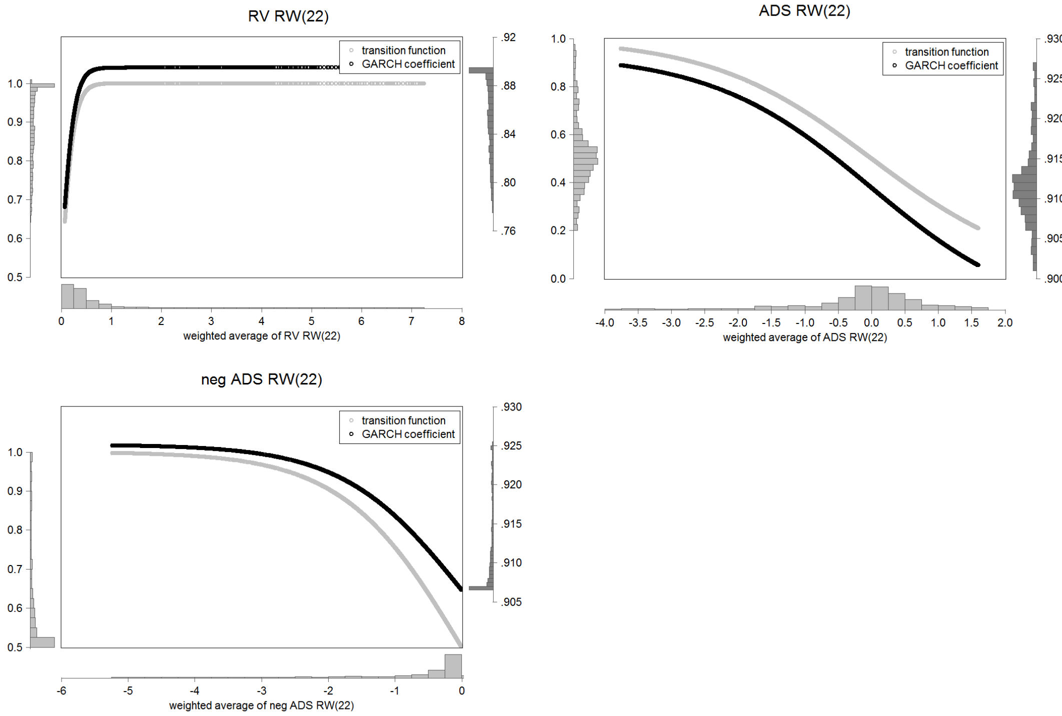
Notes: Beta weighting schemes from the TVP-GARCH-MIDAS model estimations (see Table 3.5) with one MIDAS lag year of daily data, i.e. with $K = 252$ in Eq. (3.8).

FIGURE 3.9: TIME-VARYING GARCH COEFFICIENTS



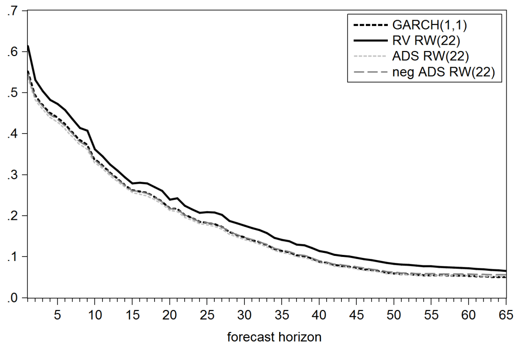
Notes: Time-varying GARCH coefficients calculated as $\hat{\beta}_t = \hat{\beta}_1 + \hat{\beta}_2 F(\hat{\gamma}, \hat{\boldsymbol{\Phi}}' \boldsymbol{x}_{t-1})$ based on the TVP-GARCH-MIDAS model estimations in Table 3.5. Shaded areas represent NBER recession periods.

Figure 3.10: Transition functions and time-varying GARCH coefficients



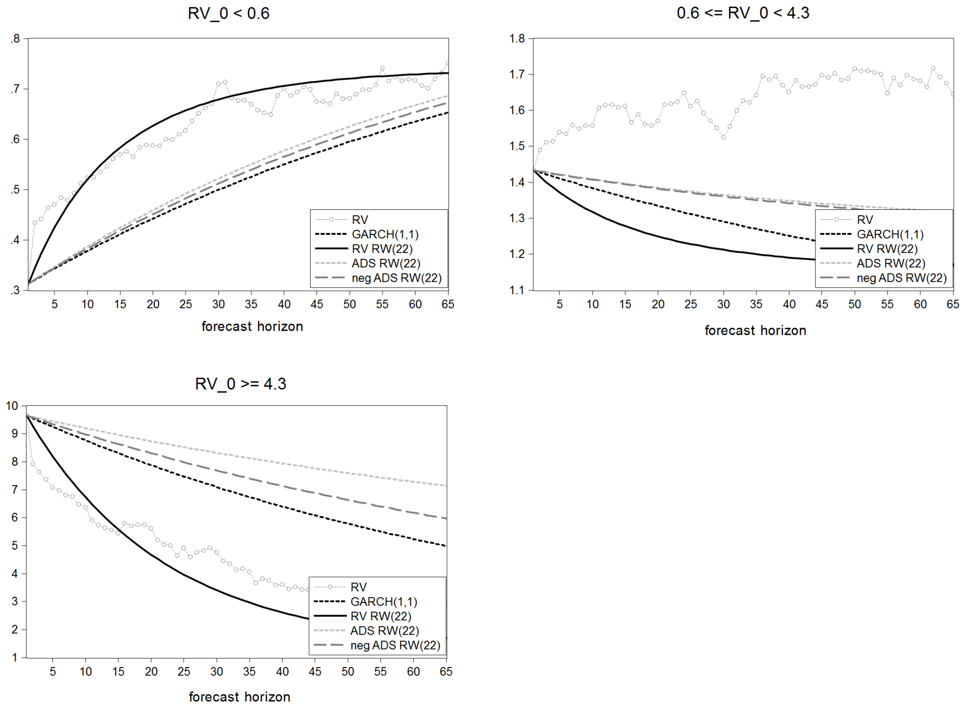
Notes: Transition functions (gray, left axis), $F(\hat{\gamma}, \hat{\boldsymbol{\Phi}}'\boldsymbol{x}_{t-1}) = 1/(\exp(\hat{\gamma}\hat{\boldsymbol{\Phi}}'\boldsymbol{x}_{t-1}))$, and time-varying GARCH coefficients (black, right axis), $\hat{\beta}_t = \hat{\beta}_1 + \hat{\beta}_2 F(\hat{\gamma}, \hat{\boldsymbol{\Phi}}'\boldsymbol{x}_{t-1})$, from the TVP-GARCH-MIDAS model estimations in Table 3.5. The functions are plotted against the weighted average of the respective explanatory variable that is implied by the model estimation, $\hat{\boldsymbol{\Phi}}'\boldsymbol{x}_{t-1} = \sum_{k=1}^{252} \hat{\varphi}_k x_{t-k}$. The axes also include histograms to illustrate the distribution of the (weighted) explanatory variable (bottom), the transition function (left), and the GARCH coefficient (right).

Figure 3.11: Mincer-Zarnowitz regression R^2s



Notes: R^2s from a Mincer-Zarnowitz regression across varying forecast horizons, i.e. the R^2 values from regressing the l-step ahead realized volatility on a constant and the l-step ahead volatility forecast for $l = 1, \ldots, 65$.

Figure 3.12: Average volatility forecast over different volatility regimes



Notes: The figure shows averages of out-of-sample volatility forecasts of the GJR-GARCH(1,1) and the TVP-GARCH-MIDAS model specifications at horizons $l = 1, \ldots, 65$, over different values of realized volatility. The forecasts are based on initial realized volatility values RV_0 with $RV_0 < 0.6$, $0.6 \leq RV_0 < 4.3$, and $RV_0 \geq 4.3$. See also the subsample estimation results presented in the Appendix.

3.7 Appendix

A Tables

Table A.1: Empirical size properties of the LM-tests with $K = 22$

		$Z_t \sim \mathcal{N}(0,1)$			$Z_t \sim t(7)$		
		L	I	H	L	I	H
$x_t = RV_t^{(1)}$							
	1%	0.74	0.83	0.81	1.81	1.52	1.74
LM	5%	4.13	4.75	4.65	4.05	3.55	4.20
	10%	9.11	9.81	8.99	6.93	6.79	6.55
	1%	0.64	0.52	0.51	1.49	1.32	1.74
LM_{AT}	5%	3.18	3.51	4.24	3.62	3.65	3.79
	10%	9.11	9.40	8.59	7.36	5.88	7.06
$x_t = RV_t^{(22)}$							
	1%	1.27	1.34	1.41	1.07	1.22	1.23
LM	5%	4.13	3.72	4.24	5.22	4.15	5.12
	10%	7.10	7.95	8.38	8.42	7.50	7.88
	1%	1.38	1.24	1.31	0.96	0.91	1.02
LM_{AT}	5%	4.13	3.93	3.84	4.48	3.44	4.50
	10%	7.10	7.95	6.77	8.00	7.19	7.98
$x_t = RV_t^{(65)}$							
	1%	1.38	1.14	1.01	2.56	2.74	2.56
LM	5%	5.19	5.17	4.65	6.18	5.78	6.35
	10%	9.75	10.33	10.61	9.28	8.41	9.42
	1%	1.27	1.24	1.11	1.92	1.93	2.05
LM_{AT}	5%	5.19	5.17	4.75	5.97	4.76	5.73
	10%	9.75	9.71	8.99	8.32	8.31	8.70

Notes: Rejection rates in percent at the 1%, 5%, and 10% nominal level. The data generating process is a GJR-GARCH(1,1) process

$$\varepsilon_t = \sqrt{h_t} Z_t$$
$$h_t = \omega + \left(\alpha_1 + \alpha_2 \mathbb{1}_{\{\varepsilon_{t-1}<0\}}\right) \varepsilon_{t-1}^2 + \beta_1 h_{t-1},$$

with parameter values set to $\alpha_1 = 0.05$, $\alpha_2 = 0.06$ and persistence regimes L, I, and H with $\beta_1^L = 0.82$, $\beta_1^I = 0.87$, and $\beta_1^H = 0.91$. The persistence of the simulated model $(\alpha_1 + \alpha_2/2 + \beta_1)$ thus varies between $0.90, 0.95$, and 0.99. We set ω accordingly, so that the unconditional variance equals one. The shocks Z_t are simulated from a standard normal distribution or a t-distribution with seven degrees of freedom. The LM-tests are based on $x_t = RV_t^{(N)} = \frac{1}{N}\sum_{j=0}^{N-1} \varepsilon_{t-j}^2$, with $N = 1, 22, 65$, and we set $K = 22$ in the test statistics. LM refers to the test statistic in Eq. (3.14) and Eq. (3.15), whereas LM_{AT} refers to the Amado and Teräsvirta (2015) test version, see the discussion at the end of Section 3.3.2.

Table A.2: Empirical size properties of the LM-tests with μ

		$Z_t \sim \mathcal{N}(0,1)$			$Z_t \sim t(7)$		
		L	I	H	L	I	H
$x_t = RV_t^{(1)}$							
	1%	0.95	0.93	0.71	0.42	0.20	0.41
LM	5%	3.48	3.81	3.55	3.07	3.68	1.93
	10%	7.70	8.45	6.69	6.03	5.73	4.78
	1%	0.63	0.52	0.81	0.74	0.61	0.30
LM_{AT}	5%	4.01	3.61	3.65	3.60	3.68	4.78
	10%	9.07	8.04	7.91	8.99	7.98	7.52
$x_t = RV_t^{(22)}$							
	1%	0.63	0.82	0.61	0.74	0.61	0.30
LM	5%	4.96	4.64	4.16	4.97	4.40	2.85
	10%	9.49	9.79	9.03	9.21	9.41	6.71
	1%	0.53	0.72	0.61	0.32	0.41	0.20
LM_{AT}	5%	3.06	3.09	3.65	3.07	2.56	2.44
	10%	7.70	6.60	7.10	6.24	5.73	6.71
$x_t = RV_t^{(65)}$							
	1%	1.48	1.55	1.41	0.53	0.92	1.32
LM	5%	5.59	5.67	6.19	5.08	4.81	5.18
	10%	12.34	12.47	11.87	9.42	10.84	11.38
	1%	1.16	1.24	0.91	0.53	0.31	0.30
LM_{AT}	5%	4.96	5.15	4.46	3.07	3.37	3.96
	10%	10.65	10.82	11.46	9.10	8.69	11.38

Notes: Rejection rates in percent at the 1%, 5%, and 10% nominal level. The data generating process is a GJR-GARCH(1,1) process

$$\begin{aligned} r_t &= \mu + \sqrt{h_t} Z_t \\ h_t &= \omega + \left(\alpha_1 + \alpha_2 \mathbb{1}_{\{r_{t-1}-\mu<0\}}\right)(r_{t-1}-\mu)^2 + \beta_1 h_{t-1}, \end{aligned}$$

with parameter values set to $\alpha_1 = 0.05$, $\alpha_2 = 0.06$ and persistence regimes L, I, and H with $\beta_1^L = 0.82$, $\beta_1^I = 0.87$, and $\beta_1^H = 0.91$. The persistence of the simulated model $(\alpha_1 + \alpha_2/2 + \beta_1)$ thus varies between $0.90, 0.95$, and 0.99. We set ω accordingly, so that the unconditional variance equals one. The shocks Z_t are simulated from a standard normal distribution or a t-distribution with seven degrees of freedom. The LM-tests are based on $x_t = RV_t^{(N)} = \frac{1}{N}\sum_{j=0}^{N-1} r_{t-j}^2$, with $N = 1, 22, 65$, and are applied to de-meaned returns. We set $K = 22$ in the test statistics. LM refers to the test statistic in Eq. (3.14) and Eq. (3.15), whereas LM_{AT} refers to the Amado and Teräsvirta (2015) test version, see the discussion at the end of Section 3.3.2.

TABLE A.3: EMPIRICAL POWER PROPERTIES OF THE LM-TESTS WITH RV AND $t(7)$ INNOVATIONS

		$K^* = 1$			$K^* = 22$		
N		1	22	65	1	22	65
$x_t = RV_t^{(N)}$	$\gamma = -1$						
SD of β_t		0.013	0.017	0.017	0.009	0.016	0.017
	1%	7.5	35.9	45.2	8.8	34.0	44.5
LM	5%	15.8	56.2	66.4	17.3	54.1	65.6
	10%	21.0	66.4	77.1	24.3	64.7	76.0
	1%	5.0	28.0	40.5	3.8	23.1	37.6
LM_{AT}	5%	10.3	47.7	63.1	9.2	41.3	60.0
	10%	13.8	59.8	74.1	12.4	53.1	71.4
$x_t = RV_t^{(N)}$	$\gamma = -10$						
SD of β_t		0.032	0.018	0.008	0.025	0.016	0.008
	1%	18.0	19.1	6.4	38.0	16.3	6.1
LM	5%	31.1	36.1	15.4	60.9	31.7	14.8
	10%	40.5	46.9	23.5	70.8	41.5	22.4
	1%	8.1	11.7	4.4	9.1	9.2	4.1
LM_{AT}	5%	14.4	27.6	13.0	16.9	21.0	12.0
	10%	19.7	36.9	20.9	22.4	29.8	19.9

Notes: Rejection rates at 1%, 5%, and 10% nominal level. The data generating process is the TVP-GARCH-MIDAS(1,1)

$$\begin{aligned}
\varepsilon_t &= \sqrt{h_t} Z_t \\
h_t &= \omega + \left(\alpha_1 + \alpha_2 \mathbb{1}_{\{\varepsilon_{t-1}<0\}}\right) \varepsilon_{t-1}^2 + \left(\beta_1 + \beta_2 \left(F(\gamma, \boldsymbol{\Phi}'\boldsymbol{x}_{t-1}) - \frac{1}{2}\right)\right) h_{t-1} \\
\boldsymbol{\Phi}'\boldsymbol{x}_{t-1} &= \sum_{k=1}^{K^*} \varphi_k(\vartheta) x_{t-k},
\end{aligned}$$

with $\omega = 0.1,\ \alpha_1 = 0.05,\ \alpha_2 = 0.06,\ \beta_1 = 0.82,\ \beta_2 = 0.18,\ \vartheta = 3$, and $\gamma = -1$ or $\gamma = -10$. x_t is taken as rolling window versions of realized volatility, $RV_t^{(N)}$ with $N = 1, 22, 65$, over the 2010-2014 sample. K^* denotes the true MIDAS lag order in the DGP. We also report the standard deviation (SD) of the time-varying GARCH coefficient, $\beta_t = \beta_1 + \beta_2 \left(F(\gamma, \boldsymbol{\Phi}'\boldsymbol{x}_{t-1}) - \frac{1}{2}\right)$.
All test statistics are based on $K = 1$. LM refers to the test statistic in Eq. (3.14) and Eq. (3.15), whereas LM_{AT} refers to the Amado and Teräsvirta (2015) test version, see the discussion at the end of Section 3.3.2. The shocks Z_t are simulated from a (standardized) t-distribution with seven degrees of freedom.

TABLE A.4: EMPIRICAL POWER PROPERTIES OF THE LM-TESTS WITH THE VIX AND $t(7)$ INNOVATIONS

		$K^*=1$			$K^*=22$		
N		1	22	65	1	22	65
$x_t = VIX_t^{(N)}$	$\gamma=-1.5$						
SD of β_t		0.019	0.018	0.018	0.019	0.018	0.018
	1%	49.4	52.3	49.7	79.9	49.7	13.3
LM	5%	72.6	74.0	74.0	93.0	72.0	27.0
	10%	82.1	82.5	82.7	96.3	81.5	37.8
	1%	36.2	43.4	41.9	68.3	38.4	8.7
LM_{AT}	5%	57.6	67.1	66.6	88.6	62.3	22.4
	10%	69.8	78.2	78.4	94.6	74.1	32.0
$x_t = VIX_t^{(N)}$	$\gamma=-4$						
SD of β_t		0.013	0.012	0.010	0.013	0.011	0.010
	1%	17.3	14.9	13.5	15.6	13.9	48.9
LM	5%	35.2	30.7	27.5	32.8	29.9	73.3
	10%	47.5	42.1	38.1	45.1	40.6	82.2
	1%	12.7	10.1	8.8	10.3	9.5	40.2
LM_{AT}	5%	28.3	24.1	23.2	23.6	22.3	65.0
	10%	36.6	36.8	32.9	33.4	33.9	77.2

Notes: Rejection rates at 1%, 5%, and 10% nominal level. The data generating process is the TVP-GARCH-MIDAS(1,1)

$$\varepsilon_t = \sqrt{h_t} Z_t$$
$$h_t = \omega + \left(\alpha_1 + \alpha_2 \mathbb{1}_{\{\varepsilon_{t-1}<0\}}\right)\varepsilon_{t-1}^2 + \left(\beta_1 + \beta_2\left(F(\gamma, \boldsymbol{\Phi}'\boldsymbol{x}_{t-1}) - \frac{1}{2}\right)\right)h_{t-1}$$
$$\boldsymbol{\Phi}'\boldsymbol{x}_{t-1} = \sum_{k=1}^{K^*}\varphi_k(\vartheta)x_{t-k},$$

with $\omega = 0.1$, $\alpha_1 = 0.05$, $\alpha_2 = 0.06$, $\beta_1 = 0.82$, $\beta_2 = 0.18$, $\vartheta = 3$, and $\gamma = -1.5$ or $\gamma = -4$. x_t is taken as rolling window versions of the VIX, $VIX_t^{(N)}$ with $N = 1, 22, 65$, over the 2010-2014 sample. K^* denotes the true MIDAS lag order in the DGP. We also report the standard deviation (SD) of the time-varying GARCH coefficient, $\beta_t = \beta_1 + \beta_2\left(F(\gamma, \boldsymbol{\Phi}'\boldsymbol{x}_{t-1}) - \frac{1}{2}\right)$.
All test statistics are based on $K = 1$. LM refers to the test statistic in Eq. (3.14) and Eq. (3.15), whereas LM_{AT} refers to the Amado and Teräsvirta (2015) test version, see the discussion at the end of Section 3.3.2. The shocks Z_t are simulated from a (standardized) t-distribution with seven degrees of freedom.

TABLE A.5: DESCRIPTIVE STATISTICS OF TIME-VARYING (ARCH) PERSISTENCE

Variable	Min	Max	Mean	SD
Panel A: Time varying ARCH coefficient				
$RV_t^{(22)}$	0.0178	0.0294	0.0241	0.0034
$ADS_t^{(22)}$	0.0076	0.0294	0.0164	0.0043
neg $ADS_t^{(22)}$	0.0148	0.0285	0.0177	0.0038
Panel B: Time varying persistence				
$RV_t^{(22)}$	0.9747	0.9863	0.9810	0.0034
$ADS_t^{(22)}$	0.9736	0.9954	0.9824	0.0043
neg $ADS_t^{(22)}$	0.9796	0.9933	0.9824	0.0038

Notes: The table reports descriptive statistics of the time-varying ARCH coefficients, $\hat{\alpha}_{1t} = \hat{\alpha}_{12} + \hat{\alpha}_{12} F(\hat{\gamma}, \hat{\boldsymbol{\Phi}}' \boldsymbol{x}_{t-1})$, from an alternative TVP-GARCH-MIDAS model specification and the corresponding time varying persistence, calculated as $\hat{\alpha}_{1t} + \frac{1}{2}\hat{\alpha}_2 + \hat{\beta}$. The estimated ARCH coefficient, resp. persistence, from the GJR-GARCH model is 0.0190, resp. 0.9852.

Table A.6: TVP-GARCH-MIDAS model estimations: 1970-1999 subsample

Variable	μ	ω	α_1	α_2	β_1	β_2	γ	ϑ	LLF	BIC	LM
$RV_t^{(22)}$	0.0384^{***} (0.0089)	0.0605^{*} (0.0316)	0.0177^{**} (0.0074)	0.1137^{***} (0.0407)	0.3494 (0.3007)	0.5306^{**} (0.2641)	-10.2975^{***} (2.2260)	12.0251^{**} (5.7484)	-9318.06	2.4804	3.19 [0.07]
$ADS_t^{(22)}$	0.0352^{***} (0.0091)	0.0128^{***} (0.0047)	0.0245^{***} (0.0063)	0.0753^{***} (0.0249)	0.9121^{***} (0.0295)	0.0235 (0.0284)	1.0481 (1.8902)	5.0391 (5.2026)	-9348.49	2.4885	3.30 [0.07]
neg $ADS_t^{(22)}$	0.0357^{***} (0.0091)	0.0121^{**} (0.0047)	0.0258^{***} (0.0065)	0.0729^{***} (0.0246)	0.9113^{***} (0.0227)	0.0186^{*} (0.0110)	5.2014 (8.6258)	9.1201 (9.8978)	-9350.24	2.4890	2.72 [0.10]
GARCH(1,1)	0.0357^{***} (0.0091)	0.0113^{***} (0.0041)	0.0274^{***} (0.0063)	0.0707^{***} (0.0240)	0.9253^{***} (0.0175)	-	-	-	-9355.46	2.4868	-

Notes: The table reports estimation results for TVP-GARCH-MIDAS models with a time-varying GARCH coefficient based on one MIDAS lag year of a daily macro variable x, i.e the conditional variance process is given by

$$h_t = \omega + \left(\alpha_1 + \alpha_2 \mathbb{1}_{\{(r_{t-1}-\mu<0\}}\right)(r_{t-1}-\mu)^2 + (\beta_1 + \beta_2 F(\gamma, \boldsymbol{\Phi}'\boldsymbol{x}_{t-1}))\, h_{t-1},$$

$$F(\gamma, \boldsymbol{\Phi}'\boldsymbol{x}_{t-1}) = \frac{1}{1+\exp(\gamma \boldsymbol{\Phi}'\boldsymbol{x}_{t-1})},$$

$$\boldsymbol{\Phi}'\boldsymbol{x}_{t-1} = \sum_{k=1}^{K} \varphi_k x_{t-k},$$

with restricted beta weights $\varphi_k = \varphi_k(1,\vartheta)$, $k = 1,\ldots,K = 252$. All estimations are based on daily return data from March 1970 to December 2014 and explanatory variable data from March 1969 on. The numbers in parentheses are Bollerslev-Wooldridge robust standard errors. ***, **, * indicate significance at the 1%, 5%, and 10% level. LLF is the value of the maximized log-likelihood function. BIC is the Bayesian information criterion. LM refers to the test statistic in Section 3.3.2, see Eq. (3.14) and Eq. (3.15), for testing the null hypothesis that the variable x has no explanatory power for time variation in the GARCH coefficient. In brackets, we present the corresponding p-values from the limiting $\chi^2(1)$ distribution.

Table A.7: Descriptive statistics of time-varying persistence: 1970-1999 subsample

Variable	Min	Max	Mean	SD
Panel A: Time-varying GARCH coefficient				
$RV_t^{(22)}$	0.7042	0.8800	0.8337	0.0457
$ADS_t^{(22)}$	0.9155	0.9351	0.9234	0.0044
neg $ADS_t^{(22)}$	0.9206	0.9299	0.9243	0.0038
Panel B: Time-varying persistence				
$RV_t^{(22)}$	0.7788	0.9545	0.9082	0.0457
$ADS_t^{(22)}$	0.9777	0.9972	0.9856	0.0044
neg $ADS_t^{(22)}$	0.9829	0.9922	0.9866	0.0038

Notes: The table reports descriptive statistics of the time-varying GARCH coefficients, $\hat{\beta}_t = \hat{\beta}_1 + \hat{\beta}_2 F(\hat{\gamma}, \hat{\boldsymbol{\Phi}}'\boldsymbol{x}_{t-1})$, from the TVP-GARCH-MIDAS model estimations over the 1970-1999 subsample and the corresponding time-varying persistence, calculated as $\hat{\alpha}_1 + \frac{1}{2}\hat{\alpha}_2 + \hat{\beta}_t$. The estimated GARCH coefficient, resp. persistence, from the GJR-GARCH model over the 1970-1999 subsample period is 0.9253, resp. 0.9881.

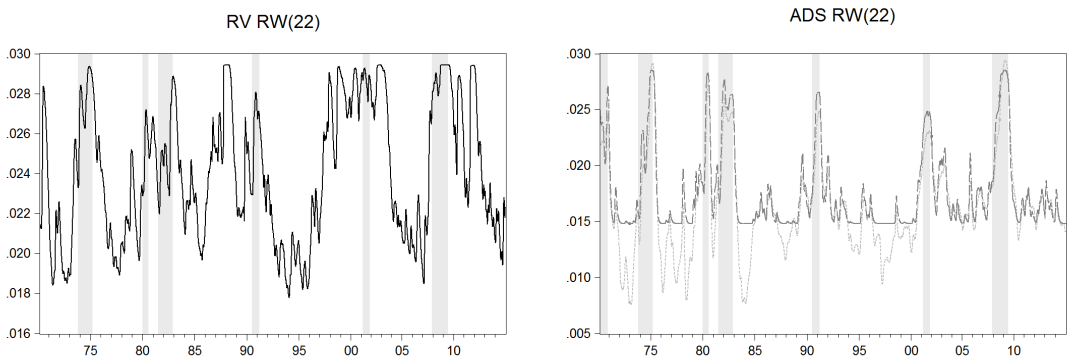
Table A.8: Descriptive statistics of time-varying persistence: 2000-2014 out-of-sample period

Variable	Min	Max	Mean	SD
Panel A: Time-varying GARCH coefficient				
$RV_t^{(22)}$	0.7205	0.8800	0.8487	0.0380
$ADS_t^{(22)}$	0.9198	0.9351	0.9256	0.0033
neg $ADS_t^{(22)}$	0.9206	0.9299	0.9259	0.0034
Panel B: Time-varying persistence				
$RV_t^{(22)}$	0.7951	0.9545	0.9232	0.0380
$ADS_t^{(22)}$	0.9819	0.9973	0.9877	0.0033
neg $ADS_t^{(22)}$	0.9829	0.9922	0.9881	0.0034

Notes: The table reports descriptive statistics of the time-varying GARCH coefficients, $\hat{\beta}_t = \hat{\beta}_1 + \hat{\beta}_2 F(\hat{\gamma}, \hat{\boldsymbol{\Phi}}'\boldsymbol{x}_{t-1})$, from the TVP-GARCH-MIDAS model over the 2000-2014 out-of-sample period based on in-sample parameter estimates from the 1970-1999 model estimation and the corresponding time-varying persistence, calculated as $\hat{\alpha}_1 + \frac{1}{2}\hat{\alpha}_2 + \hat{\beta}_t$. The estimated GARCH coefficient, resp. persistence, from the GJR-GARCH model over the 1970-1999 subsample period is 0.9253, resp. 0.9881.

B Figures

Figure B.1: Time-varying ARCH coefficients



Notes: Time-varying ARCH coefficients, $\hat{\alpha}_{1t} = \hat{\alpha}_{12} + \hat{\alpha}_{12} F(\hat{\gamma}, \hat{\boldsymbol{\Phi}}' \boldsymbol{x}_{t-1})$, from an alternative TVP-GARCH-MIDAS model specification. Shaded areas represent NBER recession periods.

4

On the Macroeconomic Determinants of Long-Term Volatilities and Correlations in U.S. Stock and Crude Oil Markets

Using a modified DCC-MIDAS specification, we endogenize the long-term correlation between crude oil and stock price returns with respect to the stance of the U.S. macroeconomy. We find that variables which contain information on current and future economic activity are helpful predictors for changes in the oil-stock correlation. For the period 1993-2011 there is strong evidence for a counter cyclical behavior of the long-term correlation. For prolonged periods with strong growth above trend our model predicts a negative long-term correlation, while before and during recessions the sign changes and remains positive throughout the economic recovery.

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4.1 Introduction

In this article, we revisit the oil-stock market relationship by analyzing the macroeconomic determinants of the long-term correlation between daily U.S. stock market and crude oil price returns. Recently, Kilian and Park (2009) have shown that on average 22% of the variation in U.S. stock returns in the period 1975–2006 can be explained by oil price shocks. However, whether an oil price shock drives oil and stock prices in the same or in opposite directions crucially depends on the type of the underlying shock. While oil price increases due to precautionary demand have a negative effect on stock prices, demand driven oil price shocks lead to increasing stock prices. Based on these insights, Kilian and Park (2009) argue that the time-varying sign in rolling oil-stock correlations reflects changes in the relative importance of different demand and supply shocks in the oil market.

While Kilian and Park (2009) investigate the oil-stock relationship using monthly data, our purpose is to analyze the correlation between oil and stock returns at a daily frequency. More specifically, we use a novel MIxed Data Sampling (MIDAS) approach to link the smooth component of daily return correlations to changes in monthly U.S. macroeconomic variables. While there is a growing literature on the endogeneity of monthly or quarterly oil prices with respect to U.S. and global macroeconomic conditions (Barsky and Kilian, 2004; Kilian, 2008, 2009), our contribution is to provide first evidence on the link between U.S. economic activity and the daily oil-stock correlation.[1]

Our econometric specification is based on the Dynamic Conditional Correlation MIDAS (DCC-MIDAS) model proposed in Colacito et al. (2011). The DCC-MIDAS combines the Engle (2002) DCC specification with the GARCH-MIDAS framework of Engle et al. (2013). The latter framework extends the simple GARCH specification by modeling volatility as consisting of a short-term and a long-term component. Most importantly, the long-term component is specified as a function of the macroeconomic environment. In the original DCC specification with correlation targeting each quasi-correlation follows a 'GARCH type' process, which is mean-reverting to the unconditional correlation of the volatility-adjusted residuals. The basic idea of Colacito et al. (2011) is to replace this unconditional correlation with a slowly time-varying long-term component. The quasi-correlation then fluctuates around this long-run trend. Hence, the new specification can be considered as a two-component

[1] In the following, we refer to the correlation between oil and stock returns simply as the oil-stock correlation.

model for the dynamic correlations. Colacito et al. (2011) model the long-term component as a weighted sum of the lagged monthly realized correlations between the volatility-adjusted residuals.

Using the GARCH-MIDAS framework, we first analyze whether the long-term oil market volatility is related to the U.S. macroeconomy and whether oil and stock volatility respond to the same macroeconomic information. We then extend the DCC-MIDAS model by directly incorporating monthly macroeconomic explanatory variables X into an appropriately modified long-term correlation component. We refer to this new specification as the DCC-MIDAS-X model.

Our results can be summarized as follows. First, we find that the movements in long-term oil market volatility can be well predicted by various measures of U.S. macroeconomic activity. Our empirical results provide convincing evidence for a counter cyclical relationship between oil market volatility and variables which either describe the current stance of the economy, e.g. industrial production, or provide forward looking information about the future state of the economy, e.g. the leading index for the U.S. Current and expected increases (decreases) in economic activity clearly anticipate downswings (upswings) in long-term oil volatility. While the notion that there is reverse causality from macroeconomic variables to the level of the oil price (see, e.g., Barsky and Kilian, 2004; Kilian, 2008, 2009) is now widely accepted, our result adds a new dimension by establishing a link between U.S. macroeconomic variables and the volatility of oil price returns. Interestingly, we also find that long-term oil and stock market volatility respond to the same macroeconomic information.

Second, our empirical results show that changes in the long-term oil-stock correlation can be anticipated by the same macroeconomic factors that affect the long-term volatilities. We provide strong evidence for a counter cyclical behavior of the long-term oil-stock correlation. Phases with positive long-term oil-stock correlations correspond to values of the macroeconomic factors which either indicate recessions or the beginning of expansions with growth still below or at trend. On the other hand, a negative long-run correlation emerges when the macroeconomic variables signal strong growth above trend. Clearly, the positive correlation during recessions is driven by the simultaneous drop in oil and stock prices. The economic recovery during the early phase of an expansion then leads to increasing oil prices due to higher demand as well as to rising stock prices because of the improved outlook for corporate cash flows. The combination of these two effects causes the long-run oil-stock correlation to remain positive. This interpretation squares with the findings in Kilian and Park (2009) regarding the positive short-run effect on oil and stock prices of an unexpected increase in global demand. Finally, during boom phases with strong growth above trend both the further increasing oil prices as well as the expectation of rising interest rates should have a depressing effect on the stock market. Hence, for these periods our model predicts a decreasing or negative long-term correlation.

Third, the long-term correlation component can be interpreted as the predicted or expected correlation given a certain state of the economy. Since the macroeconomic variables that drive the long-term component represent aggregate demand, the deviations of the short-term from the long-term component should be driven by other factors related to the stock and/or the oil market. Typical examples for the oil market would be either oil specific, i.e. precautionary, demand shocks or supply shocks. However, the fact that various measures of macroeconomic activity lead to a convincing and coherent fit of the long-term correlation suggests that aggregate demand is the most important factor for the oil-stock relationship. This interpretation is very much in line with the view that – in contrast to the 1970s when supply shocks were likely to be predominant – oil prices have been mainly driven by high global aggregate demand since the mid-1990s (see Hamilton, 2008; Kilian, 2009; Kilian and Murphy, 2014).[2]

Fourth, the fact that the sign of the oil-stock correlation critically depends on the state of the economy reinforces Kilian and Park's (2009) argument that simple regressions of stock returns on oil price changes can be very misleading. This point may well explain the conflicting empirical evidence on the oil-stock relationship in Jones and Kaul (1996), Wei (2003), Nandha and Faff (2008), Miller and Ratti (2009) and others.

Fifth, we show that the volatility and correlation predictions from the various DCC-MIDAS-X specifications significantly outperform the ones from the simple DCC model. Hence, the explicit modeling of the long-term correlation component may be very beneficial for portfolio choice, hedging decisions or risk management.

The remainder of the article is organized as follows. Section 4.2 reviews the related literature, while Section 4.3 discusses the GARCH-MIDAS and DCC-MIDAS models. The data and empirical results are presented in Sections 4.4 and 4.5. In Section 4.6 we evaluate the forecasting performance of the different models and

[2]Although we focus on economic activity measures for the U.S. only, while the oil price is driven by global demand, our approach may still be informative to the extent that changes in U.S. real activity are correlated with changes in global real activity.

Section 4.7 concludes the article.

4.2 Related literature

Our analysis is based on two strands of literature. The first one is concerned with the modeling of long-term movements in volatilities and correlations, the second one with the relationship between oil and stock prices and macroeconomic conditions.

The idea of having short- and long-term component models of volatilities dates back to Ding and Granger (1996), Engle and Lee (1999), and more recently Davidson (2004) and Conrad (2010). In their specifications, both components follow 'GARCH-type' processes but with different degrees of persistence. While these specifications allow one to separate the two volatility components, the unconditional variance is still assumed to be constant over time. Engle and Rangel (2008) and Engle et al. (2013) relax this assumption and propose specifications in which the long-term component can be considered a time-varying unconditional variance. While in the Engle and Rangel (2008) Spline-GARCH model both components fluctuate at the same frequency, in Engle et al. (2013) it is assumed that the long-term component evolves at a lower frequency than the short-term component. Using the MIDAS framework of Ghysels et al. (2005, 2007), they directly relate the long-term component to the evolution of macroeconomic time series such as industrial production or inflation. In line with the earlier findings in Schwert (1989), the GARCH-MIDAS model provides strong evidence for a counter cyclical behavior of financial volatility. Recently, Conrad and Loch (2014) extended the analysis of Engle et al. (2013) by using a broader set of macroeconomic variables and expectations data from the Survey of Professional Forecasters. The DCC-MIDAS model proposed in Colacito et al. (2011) simply extends the two-component idea from volatilities to correlations. However, instead of relating the long-term correlation directly to its potential macroeconomic sources, Colacito et al. (2011) only consider lagged realized correlations as explanatory variables.

Since the seminal articles of Hamilton (1983, 1985, 2003) exogenous oil supply shocks were suspected to be causal for recessions and periods of low economic growth. Based on this presumption, several empirical studies have analyzed the relationship between oil prices and stock market returns. While Jones and Kaul (1996) or Nandha and Faff (2008) indeed find that oil price increases negatively affect stock prices, Huang et al. (1996) and Wei (2003) cannot establish a significant relationship. Recently, Miller and Ratti (2009) provide evidence for a time-varying relationship.

For the period after 1999 they even report a positive connection. Hence, the empirical evidence is far from being uncontroversial. Kilian and Park (2009) provide two explanations for the conflicting results. First, there is convincing evidence for reverse causality from the U.S. economy to the oil price (see also Kilian, 2009, and Alquist et al., 2013). Thus, stock and oil price changes may be induced by the same macroeconomic factors and, hence, regressions of stock returns on oil price changes can be misleading due to endogeneity. Second, Kilian and Park (2009) argue that the sign of the effect of an oil price increase on the stock market depends on the type of the underlying shock and, hence, may change over time. While shocks due to an unanticipated economic expansion may have a positive impact, shocks related to precautionary demand, for example, are likely to have a negative impact. For several oil-exporting and oil-importing countries Filis et al. (2011) confirm that the oil-stock correlation is indeed time-varying. Although they informally relate phases of positive or negative correlations to demand and supply shocks, their simple DCC model does not explicitly incorporate information on the state of the economy.

4.3 The DCC-MIDAS model

In this section, we develop the econometric framework to analyze the impact of macroeconomic variables on long-term volatility and correlations. We consider the bivariate vector of asset returns $\mathbf{r}_t = (r_{1,t}, r_{2,t})'$, where $r_{1,t}$ refers to the stock and $r_{2,t}$ to the oil returns, and denote by $\mathcal{F}_{t-1} = \sigma(\mathbf{r}_{t-1}, \mathbf{r}_{t-2}, \ldots)$ the σ-field generated by the information available through time $t-1$. Returns are defined as $r_{i,t} = 100 \cdot (\log(P_{i,t}) - \log(P_{i,t-1}))$, where $P_{i,t}$ denotes the price at time t. Let $\mathbf{E}[\mathbf{r}_t|\mathcal{F}_{t-1}] = \boldsymbol{\mu}_t = (\mu_{1,t}, \mu_{2,t})'$ and define the vector of residuals $\mathbf{r}_t - \boldsymbol{\mu}_t = \boldsymbol{\varepsilon}_t = (\varepsilon_{1,t}, \varepsilon_{2,t})'$. The residuals have mean zero by definition and we denote their conditional covariance matrix by $\mathbf{H}_t = \mathbf{Var}[\boldsymbol{\varepsilon}_t|\mathcal{F}_{t-1}]$. Following Engle (2002), we decompose the conditional covariance matrix into $\mathbf{H}_t = \mathbf{D}_t\mathbf{R}_t\mathbf{D}_t$ where

$$\mathbf{R}_t = \begin{pmatrix} 1 & \rho_{12,t} \\ \rho_{12,t} & 1 \end{pmatrix} \quad \text{and} \quad \mathbf{D}_t = \begin{pmatrix} h_{1,t}^{1/2} & 0 \\ 0 & h_{2,t}^{1/2} \end{pmatrix}. \tag{4.1}$$

Finally, we define the standardized residuals $\boldsymbol{\eta}_t = (\eta_{1,t}, \eta_{2,t})'$ as $\boldsymbol{\eta}_t = \mathbf{D}_t^{-1}\boldsymbol{\varepsilon}_t$. Note that $\mathbf{Var}[\boldsymbol{\eta}_t|\mathcal{F}_{t-1}] = \mathbf{R}_t$. The DCC framework allows us to separately model the conditional variances and the conditional correlations.

4.3.1 Conditional variances

To capture the impact of macroeconomic variables on return volatility, we adopt the GARCH-MIDAS framework of Engle et al. (2013). We assume a multiplicative component model for each conditional variance, i.e. we specify $h_{i,t} = g_{i,t} m_{i,\tau}$, where $g_{i,t}$ is the short-run and $m_{i,\tau}$ the long-run component. While the transitory volatility component changes at the daily frequency t, the long-run component changes at the monthly frequency τ only. We denote $N^{(\tau)}$ as the number of days within month τ. Specifically, we assume that the short-run volatility component follows a mean-reverting unit GARCH(1,1) process

$$g_{i,t} = (1 - \alpha_i - \beta_i) + \alpha_i \frac{(r_{i,t-1} - \mu_{i,t-1})^2}{m_{i,\tau}} + \beta_i g_{i,t-1}, \tag{4.2}$$

with $\alpha_i > 0$, $\beta_i \geq 0$, and $\alpha_i + \beta_i < 1$. The long-term component is modeled as a slowly varying function of an exogenous variable X_τ using the MIDAS specification

$$\log(m_{i,\tau}) = m_i + \theta_i \sum_{k=1}^{K_v} \varphi_k(\omega_i) X_{\tau-k}, \tag{4.3}$$

where the log transformation guarantees the non-negativity of the conditional variances when the exogenous variables can take negative values. X_τ will be a monthly macroeconomic variable.[3]

For the weighting scheme, we follow Engle et al. (2013) and adopt a restricted beta weighting scheme where the weights are computed according to[4]

$$\varphi_k(\omega_i) = \frac{(1 - k/K_v)^{\omega_i - 1}}{\sum_{l=1}^{K_v}(1 - l/K_v)^{\omega_i - 1}}, \qquad k = 1, ..., K_v. \tag{4.4}$$

For all $\omega_i > 1$, the weighting scheme guarantees a decaying pattern, where the rate

[3]Note that we keep the long-run component constant over each calendar month. Obviously, announcements dates for different macro variables vary across the month. Since our focus is not on (short-term) announcements effects, but on modeling the long-term volatility component, we simplify the model implementation by synchronizing all macro variables with the calendar months.

[4]The generalized beta weighting scheme with two parameters

$$\varphi_k(\omega_{i,1}, \omega_{i,2}) = \frac{(1 - k/K_v)^{\omega_{i,1}-1} \cdot (k/K_v)^{\omega_{i,2}-1}}{\sum_{l=1}^{K_v}(1 - l/K_v)^{\omega_{i,1}-1} \cdot (k/K_v)^{\omega_{i,2}-1}}, \qquad k = 1, ..., K_v$$

allows for more flexible, in particular hump-shaped, weights. In line with the results in Conrad and Loch (2014), we find that none of the variables included in our analysis requires such an unrestricted weighting scheme in the MIDAS filter. We therefore restrict the weights to be strictly decreasing in all subsequent specifications.

of decay is determined by ω_i. Large (small) values of ω_i generate a rapidly (slowly) decaying pattern. By construction, the $\varphi_k(\omega_i)$ are nonnegative and sum to one.

In the following, we will refer to the component model with explanatory variables as GARCH-MIDAS-X. Finally, note that when $\theta_i = 0$ the long-run component is simply a constant and, hence, $h_{i,t}$ follows a stationary GARCH(1,1) process with constant unconditional variance.

4.3.2 Conditional correlations

The DCC-MIDAS specification proposed by Colacito et al. (2011) provides a natural extension of the GARCH-MIDAS model to dynamic correlations. We follow Engle (2002) and specify the matrix process $\mathbf{Q}_t = [q_{ij,t}]_{i,j=1,2}$ with GARCH(1,1) dynamics as

$$\mathbf{Q}_t = (1-a-b)\bar{\mathbf{R}}_t + a\boldsymbol{\eta}_{t-1}\boldsymbol{\eta}'_{t-1} + b\mathbf{Q}_{t-1}, \tag{4.5}$$

with $a > 0$, $b \geq 0$, and $a + b < 1$, and a positive definite $\mathbf{Q}_0$. In the Engle (2002) DCC model with correlation targeting the matrix $\bar{\mathbf{R}}_t$ does not depend on time and equals the empirical correlation matrix of $\boldsymbol{\eta}_t$. The process $\mathbf{Q}_t$ can be thought of as an approximation to the true conditional correlation matrix. The $\mathbf{Q}_t$ are therefore sometimes referred to as *quasi*-correlations. Note that the initial condition $\mathbf{Q}_0$ positive definite and the parameter constraints ensure that all $\mathbf{Q}_t$ are positive definite. However, the process does not generally produce valid correlation matrices. The actual conditional correlation matrix is obtained by rescaling as $\mathbf{R}_t = diag\{\mathbf{Q}_t\}^{-1/2}\, \mathbf{Q}_t\, diag\{\mathbf{Q}_t\}^{-1/2}$.

The DCC-MIDAS framework proposed by Colacito et al. (2011) introduces long-term correlations $\bar{\rho}_{12,\tau}$ as the off-diagonal elements in the now time varying matrix $\bar{\mathbf{R}}_t$. As in the GARCH-MIDAS equation the long-term correlation component does not vary at the daily frequency t but at the lower frequency τ. That is, the short-run quasi-correlations fluctuate around the time-varying long-run correlations:

$$q_{12,t} = \bar{\rho}_{12,\tau} + a(\eta_{1,t-1}\eta_{2,t-1} - \bar{\rho}_{12,\tau}) + b(q_{12,t-1} - \bar{\rho}_{12,\tau}). \tag{4.6}$$

Colacito et al. (2011) assume that $\bar{\rho}_{12,\tau}$ can be expressed as a weighted average of the K_c past realized correlations RC_τ:

$$\bar{\rho}_{12,\tau} = \sum_{k=1}^{K_c} \varphi_k(\omega_{12}) RC_{\tau-k}, \tag{4.7}$$

with

$$RC_\tau = \frac{\sum_{t=N_{\tau-1}+1}^{N_\tau} \eta_{1,t}\eta_{2,t}}{\sqrt{\sum_{t=N_{\tau-1}+1}^{N_\tau} \eta_{1,t}^2 \sum_{t=N_{\tau-1}+1}^{N_\tau} \eta_{2,t}^2}}, \tag{4.8}$$

where $N_\tau = \sum_{i=1}^{\tau} N^{(i)}$ and $N_0 = 0$. The weights are again given by Eq. (4.4) with ω_i and K_v replaced by ω_{12} and K_c, respectively. Since the weights $\varphi_k(\omega_{12})$ sum up to one and the RC_τ are correlations, the long-run correlation will itself lie within the $[-1,\ +1]$ interval.

We extend the DCC-MIDAS model by directly incorporating information on the macroeconomic development in the long-run component. Similarly as in the GARCH MIDAS setting – where the specification for $m_{i,\tau}$ has to ensure the non-negativity of the long-term volatility – our specification has to ensure that the long-run correlation lies within the $[-1,\ +1]$ interval although the explanatory variables do not. We follow Christodoulakis and Satchell (2002) and use the Fisher-z transformation of the correlation coefficient, i.e. we assume that

$$\bar{\rho}_{12,\tau} = \frac{\exp(2z_{12,\tau}) - 1}{\exp(2z_{12,\tau}) + 1}, \tag{4.9}$$

with

$$z_{12,\tau} = m_{12} + \theta_{12} \sum_{k=1}^{K_c} \varphi_k(\omega_{12}) X_{\tau-k}, \tag{4.10}$$

where X_τ denotes either a macroeconomic variable or a realized correlation. Note that in our non-linear specification, from θ we can only infer the sign but not directly the marginal effect of a macroeconomic variable on the long-term correlation.

Finally, in the DCC-MIDAS model - as in the standard DCC model - the conditional correlations are obtained by rescaling, i.e. $\rho_{12,t} = q_{12,t}/\sqrt{q_{11,t}q_{22,t}}$. In the subsequent analysis we refer to the specifications with either macroeconomic explanatory variables or the realized correlations as DCC-MIDAS-X or DCC-MIDAS-RC models, respectively.

4.3.3 Estimation

Following Engle (2002) and Colacito et al. (2011) we estimate the model parameters via quasi-maximum likelihood. For asymptotic results on the CCC and DCC models we refer to Ling and McAleer (2003), Engle et al. (2008), Francq and Zakoïan (2012), and Aielli (2013). Asymptotic results for the DCC-MIDAS models are not yet available, but see Wang and Ghysels (2015) for a discussion of the univariate

GARCH-MIDAS-RV model. We adopt the Engle (2002) and Colacito et al. (2011) two-step estimation procedure which is feasible because the log quasi-likelihood function to be maximized

$$\mathcal{L} = -\sum_{t=1}^{T}\left(2\log(2\pi) + 2\log(|\mathbf{D}_t|) + \boldsymbol{\varepsilon}_t'\mathbf{D}_t^{-2}\boldsymbol{\varepsilon}_t\right) - \sum_{t=1}^{T}\left(\log(|\mathbf{R}_t|) + \boldsymbol{\eta}_t'\mathbf{R}_t^{-1}\boldsymbol{\eta}_t - \boldsymbol{\eta}_t'\boldsymbol{\eta}_t\right) \tag{4.11}$$

can be separated into two parts. The first sum in Eq. (4.11) contains the data and the variance parameters while the second sum depends on the volatility-adjusted residuals and the correlation parameters. Hence, in the first step we estimate the GARCH-MIDAS parameters individually for each return series and use the estimated volatility-adjusted residuals in the second step to obtain the correlation parameters. This way, we can analyze separately the macroeconomic determinants of the long-term volatilities and the long-term correlation component.

4.4 Data

We combine daily U.S. stock market and crude oil price data with monthly observations on the macroeconomic variables. While the stock series was obtained from the Kenneth R. French data library, the oil prices and the macroeconomic data are taken from the FRED database at the Federal Reserve Bank of St. Louis. Our data covers the period from January 1993 to November 2011.

4.4.1 Oil and stock market data

For the stock series, we employ the daily returns on the CRSP value-weighted portfolio, which is based on all NYSE, AMEX and NASDAQ stocks and can be considered the best available proxy for 'the stock market'. As in Kilian and Vega (2011), oil price returns are constructed from the daily spot price for West Texas Intermediate (WTI) crude oil for delivery in Cushing, Oklahoma. The data source is the U.S. Energy Information Administration.

Panel A of Table 4.1 provides summary statistics for the two return series.[5] While the sample mean of the returns is positive for both markets, the table provides first

[5] As alternative measures for the stock market we also considered the S&P 500 as well as the DJIA. Similarly, we employed the Brent instead of the WTI crude oil price. All the subsequent results were robust to these changes in the variables.

evidence for stronger fluctuations in oil returns than in stock market returns. The annualized unconditional standard deviation of the oil price returns is 39.21% and, hence, considerably higher than the 19.53% of the CRSP returns. Finally, the unconditional correlation between oil and stock returns is 0.14.

4.4.2 Macroeconomic data

We divide the monthly macroeconomic data into two categories: those which measure the current stance of the economy and forward looking indicators. The first category contains the following variables: industrial production (IP), nonfarm payrolls (NFP), and the unemployment rate (UR). The forward looking indicators are the national activity index (NAI)[6] and the leading index (LI)[7] for the U.S. They are supposed to reflect the role of market participants' expectations concerning the future economic development.

For the variables IP and NFP we compute month-to-month growth rates according to $100 \cdot [\ln(X_\tau) - \ln(X_{\tau-1})]$, while in case of UR we use month-to-month changes. The NAI and LI are included in levels. Panel B of Table 4.1 provides the summary statistics for the macroeconomic data and Figure 4.1 shows the dynamics of the macroeconomic variables. Note that by construction the GARCH- and DCC-MIDAS models require additional lags of the explanatory variables at the beginning of the sample. Since we shall include three MIDAS lag years in the filter, we report descriptive statistics and figures for the macroeconomic variables for the period from January 1990 to November 2011. All data are obtained from the FRED database at the Federal Reserve Bank of St. Louis.

4.5 Empirical results

We first present the estimation results for the GARCH-MIDAS models that relate the long-term volatilities to the macroeconomic environment. Thereafter, the DCC-

[6]The NAI is a standardized weighted average of 85 monthly indicators of national economic activity including figures that represent (i) production and income, (ii) employment, unemployment, and hours, (iii) personal consumption and housing, and (iv) sales, orders and inventories. The NAI is computed and published by the Federal Reserve Bank of Chicago. Positive realizations indicate growth above trend, while negative realizations indicate growth below trend. The variables IP, NFP, and UR are among the indicators used for the computation of the NAI.

[7]The LI predicts the six-month growth rate of the US coincident index based on variables that lead the economy including housing permits, unemployment insurance claims, delivery times from the ISM manufacturing survey, and the term spread. The LI is published by the Federal Reserve Bank of Philadelphia.

MIDAS specifications that focus on the long-run correlations are discussed.

4.5.1 Determinants of long-term volatilities

Tables 4.2 and 4.3 present the estimates for the various stock and oil GARCH-MIDAS models. In addition to the models which include the macroeconomic variables, we consider the stationary GARCH(1,1) with constant unconditional variance as our benchmark specification. Since the serial correlation in daily stock and oil returns is negligible, we choose $\mu_{i,t} = \mu_i$ in both conditional means. To ensure comparability across all specifications, we choose $K_v = 36$ for both markets. However, all results are robust to moderate changes in K_v. We compare the fit of the different models by means of the Akaike and Bayesian information criteria (AIC and BIC).[8]

The constant μ_i is significant in all stock return models, but insignificant in the oil return specifications. In all cases the estimated α_i and β_i parameters are highly significant. Interestingly, while the α_i (β_i) parameters are estimated to be slightly higher (lower) in the stock than in the oil market, the sum $\alpha_i + \beta_i$ is almost identical in both markets and always less than one. That is, in all specifications the short-run volatility component is mean-reverting to the long-run trend. Next, we discuss the estimated long-term volatility components individually for the two markets.

Since the macroeconomic determinants of long-term stock market volatility have been investigated in Engle et al. (2013) and Conrad and Loch (2014) already, we only briefly summarize our findings which are very much in line with theirs. Table 4.2 shows that each macroeconomic variable has a significant effect on long-term stock market volatility. For IP, NFP, NAI, and LI the estimated coefficient $\hat{\theta}_1$ is negative and highly significant, while it is positive and highly significant in case of UR. Since the sign of θ_1 measures whether an increase of the respective variable leads to an upswing or downswing in long-run volatility, the estimates imply that higher (lower) levels of economic activity lead to a reduction (rise) in long-term stock market volatility. All GARCH-MIDAS-X models are preferred over the benchmark GARCH(1,1) by the AIC, but not by the BIC. The best model according to the AIC is the one including LI.

In short, our results reconfirm the observation that long-term stock market volatility behaves counter cyclically. The analysis of the macroeconomic drivers of stock market volatility dates back to Officer (1973) and Schwert (1989) who first revealed

[8] Note that all GARCH-MIDAS-X models include the same number of parameters and, hence, the AIC and BIC will lead to the same ranking. However, the benchmark GARCH(1,1) model includes two parameters less.

this counter cyclical link. Since then, the literature has put forward different economic arguments to explain the channels through which the economic environment relates to stock market volatility. The present value models of Campbell (1991) and Campbell and Shiller (1988) relate unexpected returns to news that induce revisions in the discounted sum of future expected dividends and returns. Specifically, the same news may have different impact on unexpected returns depending on the state of the macroeconomy. As a consequence, counter cyclical stock market volatility arises due to variations in future expected cash flows and future discount rates. For alternative theoretical approaches see for instance Veronesi (1999), and more recently Bansal and Yaron (2004), and Mele (2007).

In Table 4.3 we turn to the analysis of the macroeconomic determinants of the long-term oil return volatility. The estimates of θ_2 suggest that long-term oil return volatility is closely linked to each of the macroeconomic variables describing the current stance of the economy as well as the future economic outlook. In particular, the results imply that downturns in U.S. economic activity, i.e. decreases in IP, NFP, NAI, and LI and increases in UR lead to higher levels of long-term oil return volatility. While Kilian (2008, 2009), Kilian and Murphy (2014) and Alquist et al. (2013) have provided ample evidence for the notion that changes in economic activity predict oil prices, our finding that U.S. economic activity also precedes changes in long-term oil return volatility adds a new insight. Given the positive relation between aggregate demand shocks and the level of the oil price which was established in the previous literature, our finding of a counter cyclical behavior of long-term oil return volatility is very much in line with the observation in stock markets that volatility is low during phases of increasing prices but high during phases of decreasing prices. That is, good news on the macroeconomy is also good news for the oil market, i.e. increases the oil price and at the same time reduces oil return volatility.

Lastly, all GARCH-MIDAS-X models achieve a better fit than the GARCH(1,1) both in terms of the AIC and the BIC. The best model according to the information criteria is the one based on the LI.

Figure 4.2 shows the GARCH-MIDAS-LI estimates of the annualized monthly long-term volatility components for the two markets. While the level of oil return volatility is about twice as high as the one of the stock returns, the evolution of the two components is very similar across markets. It is now straightforward to compare the marginal effects of this variable on the two long-term volatility components. In general, the effect of a one standard deviation increase in X_t in the current month on long-term volatility s-months-ahead can be calculated by $\exp\left(\hat{\theta}_i \cdot \varphi_s(\hat{\omega}_i) \cdot \mathrm{SD}(X_t)\right) - 1$. Thus, a one standard deviation increase in LI this month, i.e. an increase by 0.98, leads to a 7.66% decrease in long-term stock market volatility and a 14.03% decrease in long-term oil market volatility next month. These sizes of the marginal effects imply that they are not only highly significant statistically, but also economically. The observation that the macroeconomic environment affects long-term oil and stock volatility in a very similar manner is very interesting. Our finding suggests that the long-term second moment of oil price returns shares a common component with that of stock returns which reflects the state of the U.S. business cycle.

4.5.2 Determinants of long-term correlations

In this section, we analyze the macroeconomic determinants of the long-term oil-stock correlation. We consider two benchmark specifications. The first natural benchmark is the Engle (2002) DCC model. The second benchmark is the Colacito et al. (2011) specification that uses backward-looking monthly realized correlations as explanatory variables.[9] For these two benchmark models we employ the standardized residuals from the simple GARCH(1,1) models.

Alternatively, we estimate DCC models based on standardized residuals from the GARCH-MIDAS-X models from Section 4.5.1. In the most general DCC-MIDAS-X specifications we replace the realized correlations with key macroeconomic figures. For these models the volatility-adjusted residuals are obtained either from the simple GARCH(1,1) models or from the corresponding GARCH-MIDAS-X models. For each macro variable, we thus compare three different model specifications - one DCC and two DCC-MIDAS-X specifications. As in the case of the long-term volatilities, we include three MIDAS lag years of macroeconomic data, i.e. we choose $K_c = 36$.

Table 4.4 presents the estimation results. Clearly, in all specifications the estimated parameters a and b are highly significant and sum up to a value of less than one. That is, the quasi-correlations are mean-reverting either to the unconditional correlation in the DCC case or to the long-term correlation in the various DCC-MIDAS-X specifications. The estimates of θ_{12} indicate that all macroeconomic variables significantly affect the long-run oil-stock correlation. In line with our analysis in Section 4.5.1, we find negative θ_{12} coefficients on IP, NFP, NAI, and LI,

[9] We calculate monthly realized correlations over the full 1990-2011 sample based on the standardized residuals from GARCH(1,1) models for oil and stock returns over this sample.

while the coefficient on UR is positive. The estimates imply that a contraction of macroeconomic activity leads to an increase of the long-term correlation.

All DCC-MIDAS-X specifications are superior relative to the benchmark DCC according to the AIC. In addition, the specifications based on GARCH-MIDAS-X residuals are also superior according to the BIC. Hence, there is convincing evidence in favor of the component models, which allow for time-varying long-term volatilities and correlations. The model ranking in terms of information criteria is consistent across the different macro variables. According to both the AIC and BIC, the DCC-MIDAS-X based on simple GARCH(1,1) residuals performs worst. The DCC-MIDAS-X models based on the respective GARCH-MIDAS-X residuals perform best in terms of the AIC, whereas the more parsimonious DCC models based on the GARCH-MIDAS-X residuals perform best in terms of the BIC. This model comparison emphasizes the importance of an adequate volatility specification for the statistical fit of the conditional covariance matrix. Confirming our results from the previous section, the best performing models are the ones including LI in the volatility specification, where the DCC (DCC-MIDAS) with GARCH-MIDAS residuals achieves the lowest BIC (AIC).

Interestingly, our second benchmark model, the DCC-MIDAS-RC, performs worst in terms of both information criteria. The fact that the DCC-MIDAS-X models are preferred to the DCC-MIDAS-RC, suggests that the various macroeconomic variables carry information on the evolution of the long-term correlation beyond that included in past realized correlations. Next, we explain how the forward looking properties of the macroeconomic variables which gauge future economic activity as well as inflationary pressures (and thereby future monetary policy) are particularly relevant for anticipating changes in the oil-stock correlation.[10]

Figure 4.3 shows the estimated dynamics of the short- and long-run correlations based on the DCC-MIDAS-LI specification together with a rolling-window of yearly realized correlations. First, although the unconditional correlation between stock and oil returns was found to be 0.14, the figure shows that there is substantial time-variation in the realized correlations with prolonged periods of positive or negative correlations. While the short-run correlation closely follows the behavior of the realized correlations, the long-run correlation evolves much more smoothly. Both the realized correlations as well as the short-run correlations follow this long-run

[10] In the following, we only consider the DCC-MIDAS-X specifications based on the volatility-adjusted residuals from the respective GARCH-MIDAS-X models. However, the subsequent results also hold for the other DCC-MIDAS-X specifications in a similar way.

trend component.

To provide an economic interpretation of the cyclical pattern in the evolution of the correlation dynamics we refer to Figure 4.4, which depicts the long-term component along with the LI. First, the figure clearly shows an inverse relationship between the LI and the long-term oil-stock correlation, which was already evident from the negative θ_{12} estimate in Table 4.4. That is, the oil-stock correlation is increasing (decreasing) when the LI is declining (rising).

Our empirical evidence for a counter cyclical oil-stock correlation is perfectly in line with the recent evidence in Kilian (2009) in favor of a positive oil-growth relation. Kilian and Park (2009) argue that in an early phase of an expansion increasing oil prices may not have negative effects on the stock market. This is because in the short-run the positive effect of higher economic activity on expected future cash flows dominates and, hence, the oil-stock correlation will be positive. However, in the long-run the negative effect of increasing oil prices on corporate cash flows will dominate and, therefore, the oil-stock correlation will decrease or even turn negative.

The long-term correlation in Figure 4.4 very much supports these views. Before and during both recessions bad news on the LI is associated with sharply decreasing stock and oil prices and, therefore, a positive oil-stock correlation. The fact that the correlation turns positive and increases well before both recessions is remarkable and suggests that the long-term oil-stock correlation may itself be used as an early recession indicator. During the recovery phases in 2002-2003 and 2010-2011 the improvement in the LI leads to increasing oil prices and, at the same time, to upward revisions concerning firms' expected dividends and cash flows. In these periods the oil-stock correlation remains positive, but smoothly decreases. The same rationale also applies to the first year of our sample, which falls into the recovery period after the recession of 1990/91. Finally, during the years 1994-1999 and 2004-2006 the LI signals strong growth for a protracted period, which again should positively affect oil prices. However, the (expected) oil price increases now dampen the outlook for future corporate cash flows, i.e. during these periods the good news on the macroeconomy – through the indirect effect via increasing oil prices – turns into bad news for the stock market. Alternatively, the negative effect might also work via interest rates. When the economy is already close to full employment, good news on the LI could signal higher future interest rates and, hence, be bad news for the stock market. During these strong boom phases the negative effect dominates and leads to a decreasing or negative long-run oil-stock correlation.

Since the evolution of the long-term correlation is purely driven by variables which represent U.S. aggregate demand, deviations of the short-term component from the long-run trend must be related to other factors which affect stock and/or oil returns. Typical oil related factors would be oil supply shocks or oil-market specific demand shocks such as precautionary demand or speculative demand shocks. Specifically, the temporary deviation in 2002/03 (see Figure 4.3) can be explained as a combination of the Venezuelan oil supply crisis and precautionary demand provoked by the second Iraq war (see Kilian and Murphy, 2014). Similarly, the drop in the short-term component in 2011/02-2011/04 can be related to the Libyan crisis and political turmoil in North Africa.[11] Another example would be the positive correlation signaled by the short-term component as well as the realized correlations around 1998/99. Following the Asian and Russian financial crises, this positive short-term correlation can be explained by a simultaneous decline in oil and stock prices. Nevertheless, the fact that these deviations occur only for relatively short periods suggests that the oil-stock correlation can be largely explained by U.S. economic activity for most of the time.

A particularly important conclusion that can be drawn from the time-varying oil-stock correlation is that regressions of stock returns on oil price changes are likely to be misleading, since the result will depend on the state of the economy. This insight may explain the controversial empirical findings on the oil-stock relationship and agrees with the arguments put forward in Kilian and Park (2009).

Next, we discuss the MIDAS lag structure and its implications more closely. Recall that the higher ω_{12} the more weight will be given to the more recent observations of the macro variable and, hence, the faster the weights will decline to zero. Table 4.4 reveals that the lowest ω_{12} is estimated for IP and the highest for NFP. Since the DCC-MIDAS-LI model produced the best fit for the correlations, in Figure 4.5 we plot the corresponding weighting function. For comparison, we also display the weighting functions for the GARCH-MIDAS-LI models for the stock and oil market. The figure shows that the weighting function of the correlation model is nearly linear while the weighting functions of the volatility specifications are rapidly declining.[12] [13]

[11] On February 22nd 2011, for instance, oil returns spiked up by 8%, whereas stock market returns went down by 2%.

[12] As a robustness check, we also estimated models including a weighting scheme with two parameters, hereby relaxing the assumption of strictly decreasing weights. However, including an unrestricted weighting scheme did not lead to significant improvements in the value of the maximized log likelihood and the resulting weighting schemes were still strictly decreasing.

[13] Similar results are obtained for the other macroeconomic variables but omitted for reasons of brevity.

Finally, we calculate the marginal effect of a one standard deviation change in a macro variable X_t on the long-run correlation component in the next month. Due to the non-linear Fisher-z transformation in Eq. (4.9), the marginal effect has to be calculated conditional on the history of the explanatory variable.[14] For instance, if we keep all lags of LI fixed at its sample mean of 0.99, then this yields a long-term correlation of 0.088 and a one standard deviation increase in LI this month results in a decrease to 0.072 next month, i.e. a decrease by 18.12%. Thus, the predicted marginal effect on the long-term correlation component is also highly economically relevant.

In the previous considerations we mainly focused on the DCC-MIDAS-LI specification to explain the dynamic behavior of the slowly-moving long-run correlation component. However, Table 4.4 clearly reveals that the fit of the DCC-MIDAS-X specifications with IP, NFP, UR, and NAI are only slightly inferior. Figure 4.6 displays the estimated long-run correlations from the corresponding specifications. The figure illustrates nicely that the long-term components of all specifications follow the same pattern and, hence, further support our argument that the long-term oil-stock correlation is counter cyclical. Note that the spike in the long-term correlation component predicted by IP for October 2005 can be traced back to a significant contraction in industrial production one month earlier. This is not reflected to such a strong extent in the other macroeconomic figures (compare Figure 4.1).

4.6 Model evaluation and hedging performance

Although the main focus of our analysis lies on the macroeconomic determinants of the long-term oil price return volatility as well as the long-term oil-stock correlation, our findings might also have important implications for portfolio choice, hedging decisions or risk management. Therefore, we now have a closer look at the forecasting performance of the different models for the entire conditional covariance matrix $\mathbf{H}_t$. Since a full-fledged out-of-sample analysis is beyond the scope of the current paper, we focus on in-sample results. Following Laurent et al. (2012, 2013) we apply two robust loss functions, i.e. loss functions that deliver the same ordering whether the evaluation the evaluation is based on the true conditional covariance matrix or an

[14] We calculate the percentage change in the long-term correlation component following a one standard deviation increase in X - conditional on all lags of X being fixed at its sample mean $\bar{X}$. More precisely, we compare $\bar{\rho}_{12,\tau}$ in Eq. (4.9) based on the estimates of $z_{12,\tau}$ in Eq. (4.10) evaluated at
$X_{t-1} = \cdots = X_{t-K_c} = \bar{X}$ and at $X_{t-1} = \bar{X} + \mathrm{SD}(X_t), X_{t-2} = \cdots = X_{t-K_c} = \bar{X}$.

unbiased proxy of it.

The first loss function is the Euclidean distance which equally weights the variances and covariances:

$$L_t^E = (r_{1,t}^2 - \hat{h}_{1,t})^2 + (r_{2,t}^2 - \hat{h}_{2,t})^2 + (r_{1,t}r_{2,t} - \hat{h}_{12,t})^2$$

The second one is based on the Frobenius distance and double counts the loss associated with the conditional covariance:

$$L_t^F = (r_{1,t}^2 - \hat{h}_{1,t})^2 + (r_{2,t}^2 - \hat{h}_{2,t})^2 + 2(r_{1,t}r_{2,t} - \hat{h}_{12,t})^2$$

In Table 4.5, we report for each model the average value of the two loss functions. In addition, for each DCC-MIDAS-X model we test whether the average loss is significantly different from the average loss of the DCC benchmark model. Panel A presents results for the full sample, while Panel B covers the subsample of the financial crisis in the years 2007-2009. In case of a positive difference, forecasts from the DCC-MIDAS-X model are superior to those from the benchmark model.

For the full sample, the differences in both loss functions are significant for all DCC-MIDAS-X models except the one based on IP. To the contrary, the DCC-MIDAS-RC model does not lead to a significant improvement over the simple DCC. Unsurprisingly, during the financial crisis period the average losses more than double in comparison to the full sample. During this period we only find a significant improvement over the DCC for the model based on the LI when considering the Frobenius distance. This somewhat disappointing outcome may be due to the fact that during the crisis the forecast quality of all models deteriorated dramatically and it became increasingly difficult to distinguish between them. Another potential explanation could be that during the crisis the quality of our proxies, i.e. the squared returns and the product of daily oil and stock returns, for the true conditional volatilities and covariances has declined.

As an alternative approach to evaluate the forecast performance without the necessity to rely on proxies of the unobserved volatilities and correlations, we consider the problem of hedging a long position of one dollar in the stock market by a short position of $\beta_{12,t}$ dollars in the oil market. The optimal hedge portfolio is given by (see Kroner and Sultan, 1993):

$$r_t^{PF} = r_{1,t} - \beta_{12,t} \cdot r_{2,t}, \qquad \text{with} \qquad \beta_{12,t} = \frac{\hat{h}_{12,t}}{\hat{h}_{2,t}}.$$

We then compare the average portfolio variance based on the volatility and covariance forecasts from the DCC-MIDAS-X models with those from the DCC model. The results in Table 4.5 suggest that the DCC-MIDAS-X models lead to significantly lower portfolio variances compared to the DCC in both the full sample as well as the crisis subsample. Although, the forecasting results are very promising for potential financial applications, a first natural avenue for future research would be to confirm our in-sample findings in a more detailed out-of-sample analysis.

4.7 Conclusion

We investigate the effect of changes in the U.S. macroeconomic environment on the long-term volatilities and correlations in crude oil and U.S. stock price returns. First, our results show that the long-term volatilities in both markets share a common component that reflects the state of the U.S. business cycle. Second, we extend the two-component DCC-MIDAS model of Colacito et al. (2011) by allowing the slowly-moving long-term correlation component to be determined endogenously by the variation of key macroeconomic figures. We show that changes in macroeconomic variables, which reflect the current stance of the economy as well as the future economic outlook, can anticipate counter cyclical fluctuations in the long-term correlation. More specifically, our model predicts a negative correlation during prolonged periods of strong economic expansions, while a positive correlation is observed during recessions and recoveries.

Our results provide further evidence for the argument put forward in Barsky and Kilian (2002, 2004) and Kilian (2008, 2009), among others, that oil price changes should not be considered exogenous with respect to U.S. and global macroeconomic conditions. However, while previous studies focused on a relationship in levels, our analysis shows that there is also feedback from the level of the macro variables to the second moment of the oil price. In addition, our MIDAS approach allows us to establish a link between low frequency data on U.S. economic activity and high frequency oil-stock return correlations, whereas previous evidence in Kilian and Park (2009) was based low frequency data.

4.8 Tables and figures

4.8.1 Tables

Table 4.1: Descriptive statistics

Variable	Obs	Min	Max	Mean	SD*	Skew.	Kurt.
Panel A: Daily return data (Jan 4, 1993 - Nov 30, 2011)							
Oil (WTI)	4743	-17.09	16.41	0.03	39.21	-0.19	7.73
CRSP	4743	-8.96	11.35	0.04	19.53	-0.11	10.66
Panel B: Monthly macro data (Jan 1990 - Nov 2011)							
Current stance of the economy							
IP	263	-4.30	2.10	0.16	0.67	-1.72	11.52
NFP	263	-0.62	0.41	0.07	0.18	-1.16	5.17
UR	263	-0.50	0.50	0.01	0.16	0.39	3.88
Future economic outlook							
NAI	263	-4.55	1.52	-0.17	0.86	-1.82	8.48
LI	263	-3.03	2.42	0.99	0.98	-1.67	6.69

Notes: The reported statistics include the number of observations (Obs), the minimum (Min) and maximum (Max), the mean, standard deviation (SD), Skewness (Skew.), and Kurtosis (Kurt.). For the variables IP and NFP we compute month-to-month growth rates according to $100 \cdot [\ln(X_\tau) - \ln(X_{\tau-1})]$, while in case of UR we use month-to-month changes. The NAI and LI are included in levels. We calculate continuously compounded oil returns based on the WTI crude oil spot price. The CRSP return data is obtained from Kenneth R. French data library, whereas oil prices and macro economic data are obtained from the Federal Reserve Bank of St. Louis.
*The standard deviations are annualized for the daily return series.

TABLE 4.2: GARCH-MIDAS-X MODEL ESTIMATION: STOCK MARKET

Variable	μ_1	α_1	β_1	m_1	θ_1	ω_1	LLF	BIC	AIC
Benchmark model									
GARCH(1,1)	0.0660*** (0.0121)	0.0810*** (0.0118)	0.9109*** (0.0129)	0.2467 (0.2533)	-	-	-6592.08	**2.7868**	2.7814
Current stance of the economy									
IP	0.0672*** (0.0121)	0.0827*** (0.0121)	0.9070*** (0.0135)	0.4005* (0.2283)	−0.9588** (0.4245)	3.2001* (1.8916)	-6589.33	2.7893	2.7811
NFP	0.0677*** (0.0120)	0.0863*** (0.0128)	0.9003*** (0.0147)	0.4052** (0.1989)	−2.3472*** (0.5559)	10.1923 (6.7740)	-6586.41	2.7880	2.7799
UR	0.0675*** (0.0120)	0.0838*** (0.0119)	0.9034*** (0.0136)	0.1642 (0.1954)	4.2234*** (0.9428)	5.5672* (3.0725)	-6585.59	2.7877	2.7795
Future economic outlook									
NAI	0.0674*** (0.0121)	0.0848*** (0.0124)	0.9028*** (0.0144)	0.1374 (0.2073)	−0.5545*** (0.1401)	6.1885 (4.9129)	-6587.38	2.7884	2.7803
LI	0.0672*** (0.0120)	0.0851*** (0.0125)	0.9011*** (0.0146)	0.7015*** (0.2157)	−0.4850*** (0.1165)	6.1449 (6.0315)	-6585.34	2.7876	**2.7794**

Notes: The table reports estimation results for the GARCH-MIDAS-X models including 3 MIDAS lag years of a monthly macro variable X. The estimations are based on daily stock market return data from January 1993 to November 2011 and include macroeconomic data from January 1990 on. The numbers in parentheses are Bollerslev-Wooldridge (1992) robust standard errors. ***, **, * indicate significance at the 1%, 5%, and 10% level. LLF is the value of the maximized likelihood function, BIC is the Bayesian information criterion and AIC is the Akaike information criterion. The bold numbers indicate the models with the lowest values of the information criteria.

TABLE 4.3: GARCH-MIDAS-X MODEL ESTIMATION: OIL MARKET

Variable	μ_2	α_2	β_2	m_2	θ_2	ω_2	LLF	BIC	AIC
Benchmark model									
GARCH(1,1)	0.0363	0.0518***	0.9523***	−0.0074	-	-	-10620.82	4.4857	4.4802
	(0.0394)	(0.0155)	(0.0124)	(0.5920)					
Current stance of the economy									
IP	0.0491	0.0583***	0.9224***	1.8464***	−0.5121**	7.5161***	-10586.33	4.4747	4.4665
	(0.0331)	(0.0160)	(0.0227)	(0.1360)	(0.2135)	(2.3741)			
NFP	0.0477	0.0601***	0.9214***	1.8629***	−1.0675**	15.9548***	-10587.08	4.4750	4.4668
	(0.0331)	(0.0159)	(0.0213)	(0.1473)	(0.4418)	(5.0328)			
UR	0.0489	0.0575***	0.9232***	1.7372***	2.0857***	12.6712***	-10584.25	4.4738	4.4656
	(0.0327)	(0.0145)	(0.0202)	(0.1268)	(0.6705)	(4.2630)			
Future economic outlook									
NAI	0.0486	0.0574***	0.9244***	1.7227***	−0.2952***	14.9278***	-10585.90	4.4745	4.4663
	(0.0331)	(0.0158)	(0.0218)	(0.1364)	(0.1105)	(5.2959)			
LI	0.0465	0.0558***	0.9253***	2.0755***	−0.3111***	21.5657**	-10582.68	**4.4732**	**4.4650**
	(0.0330)	(0.0161)	(0.0226)	(0.1632)	(0.0915)	(8.9818)			

Notes: The table reports estimation results for the GARCH-MIDAS-X models including 3 MIDAS lag years of a monthly macro variable X. The estimations are based on daily oil market return data from January 1993 to November 2011 and include macroeconomic data from January 1990 on. Otherwise, see Notes of Table 4.2.

Table 4.4: DCC and DCC-MIDAS-X model estimation

Model	Variable	a	b	m_{12}	θ_{12}	ω_{12}	LLF	BIC	AIC
Benchmark models									
DCC with GARCH(1,1) residuals	-	0.0203*** (0.0067)	0.9751*** (0.0095)	-	-	-	-4763.60	2.0123	2.0095
DCC-MIDAS with GARCH(1,1) residuals	RC	0.0231*** (0.0056)	0.9559*** (0.0104)	0.0303 (0.0338)	0.7986*** (0.3057)	5.8805* (3.5219)	-4760.65	2.0164	2.0096
Current stance of the economy									
DCC with GARCH-MIDAS residuals	IP	0.0193*** (0.0071)	0.9770*** (0.0098)	-	-	-	-4685.95	1.9795	1.9768
DCC-MIDAS with GARCH(1,1) residuals	IP	0.0195*** (0.0057)	0.9693*** (0.0098)	0.1921*** (0.0546)	-0.6573^{***} (0.1787)	1.8602* (1.0832)	-4755.54	2.0142	2.0074
DCC-MIDAS with GARCH-MIDAS residuals	IP	0.0188*** (0.0060)	0.9712*** (0.0102)	0.2038*** (0.0582)	-0.6931^{***} (0.1885)	1.7758* (0.9689)	-4678.15	1.9816	1.9748
DCC with GARCH-MIDAS residuals	NFP	0.0190*** (0.0072)	0.9780*** (0.0097)	-	-	-	-4679.51	1.9768	1.9741
DCC-MIDAS with GARCH(1,1) residuals	NFP	0.0196*** (0.0058)	0.9685*** (0.0104)	0.1869*** (0.0512)	-1.2418^{***} (0.2866)	4.6308 (3.2031)	-4755.21	2.0141	2.0073
DCC-MIDAS with GARCH-MIDAS residuals	NFP	0.0190*** (0.0063)	0.9708*** (0.0111)	0.2136*** (0.0571)	-1.4176^{***} (0.3316)	4.0474 (2.7107)	-4670.72	1.9784	1.9716
DCC with GARCH-MIDAS residuals	UR	0.0206*** (0.0071)	0.9756*** (0.0098)	-	-	-	-4680.03	1.9770	1.9743
DCC-MIDAS with GARCH(1,1) residuals	UR	0.0204*** (0.0054)	0.9624*** (0.0103)	0.0489 (0.0341)	2.8018*** (0.6571)	1.9959* (1.0738)	-4753.43	2.0133	2.0065
DCC-MIDAS with GARCH-MIDAS residuals	UR	0.0214*** (0.0056)	0.9620*** (0.0103)	0.0505 (0.0360)	3.0976*** (0.6679)	1.9275** (0.8945)	-4668.60	1.9776	1.9707
Future economic outlook									
DCC with GARCH-MIDAS residuals	NAI	0.0192*** (0.0071)	0.9775*** (0.0096)	-	-	-	-4680.65	1.9773	1.9746
DCC-MIDAS with GARCH(1,1) residuals	NAI	0.0193*** (0.0053)	0.9640*** (0.0099)	0.0316 (0.0343)	-0.3356^{***} (0.0714)	2.3380* (1.3299)	-4752.06	2.0127	2.0059
DCC-MIDAS with GARCH-MIDAS residuals	NAI	0.0190*** (0.0055)	0.9661*** (0.0102)	0.0361 (0.0369)	-0.3647^{***} (0.0754)	2.1842* (1.1314)	-4669.00	1.9777	1.9709
DCC with GARCH-MIDAS residuals	LI	0.0193*** (0.0072)	0.9774*** (0.0099)	-	-	-	-4672.44	**1.9738**	1.9711
DCC-MIDAS with GARCH(1,1) residuals	LI	0.0194*** (0.0056)	0.9682*** (0.0099)	0.3241*** (0.0761)	-0.2383^{***} (0.0583)	2.4852 (1.8322)	-4754.78	2.0139	2.0071
DCC-MIDAS with GARCH-MIDAS residuals	LI	0.0189*** (0.0060)	0.9702*** (0.0103)	0.3615*** (0.0810)	-0.2647^{***} (0.0628)	2.3472 (1.5240)	-4663.08	1.9752	**1.9684**

Notes: The table reports estimation results for the different DCC and DCC-MIDAS-X specifications including 3 MIDAS lag years of a monthly macro variable X. The DCC and DCC-MIDAS-X models are based on standardized residuals either from the GARCH(1,1) benchmark models or the GARCH-MIDAS-X models from Tables 4.2 and 4.3. The estimations are based on daily standardized residuals from January 1993 to November 2011 and include macroeconomic data from January 1990 on. Otherwise, see notes of Table 4.2.

Table 4.5: Model evaluation

Variable	Euclidean distance loss	Euclidean distance difference	Frobenius distance loss	Frobenius distance difference	Hedge portfolio variance	Hedge portfolio difference
Panel A: Full sample (Jan 1993 - Nov 2011)						
Benchmark models						
DCC	266.511	-	289.199	-	1.415	-
DCC-RC	266.508	0.003 (0.060)	289.194	0.005 (0.060)	1.414	0.001 (0.446)
Current stance of the economy						
IP	264.604	1.907 (1.158)	287.063	2.136 (1.311)	1.399	0.016* (1.896)
NFP	263.328	3.183** (2.03)	285.744	3.455** (2.181)	1.398	0.017* (1.892)
UR	263.078	3.433** (2.152)	285.394	3.805** (2.344)	1.394	0.021* (1.876)
Future economic outlook						
NAI	263.738	2.772** (2.008)	286.073	3.126** (2.223)	1.395	0.020* (1.919)
LI	262.477	4.034*** (2.895)	284.801	4.398*** (3.054)	1.396	0.019** (1.982)
Panel B: Financial crisis (Jan 2007 - Dec 2009)						
Benchmark models						
DCC	629.845	-	721.984	-	3.113	-
DCC-RC	629.890	−0.045 (−0.178)	722.074	−0.090 (−0.178)	3.118	−0.006 (−0.634)
Current stance of the economy						
IP	629.624	0.221 (0.023)	720.357	1.628 (0.175)	3.012	0.100** (2.151)
NFP	621.292	8.553 (0.967)	711.732	10.252 (1.153)	3.004	0.109** (2.193)
UR	622.677	7.168 (0.824)	712.467	9.517 (1.080)	2.979	0.134** (2.196)
Future economic outlook						
NAI	624.536	5.309 (0.715)	714.491	7.493 (0.995)	2.987	0.126** (2.153)
LI	618.037	11.808 (1.592)	707.883	14.101** (1.846)	2.997	0.116** (2.240)

Notes: For each DCC-MIDAS model we report the average of the Euclidean and Frobenius loss functions:

$$L_t^E = (r_{1,t}^2 - \hat{h}_{1,t})^2 + (r_{2,t}^2 - \hat{h}_{2,t})^2 + (r_{1,t}r_{2,t} - \hat{h}_{12,t})^2,$$
$$L_t^F = (r_{1,t}^2 - \hat{h}_{1,t})^2 + (r_{2,t}^2 - \hat{h}_{2,t})^2 + 2(r_{1,t}r_{2,t} - \hat{h}_{12,t})^2,$$

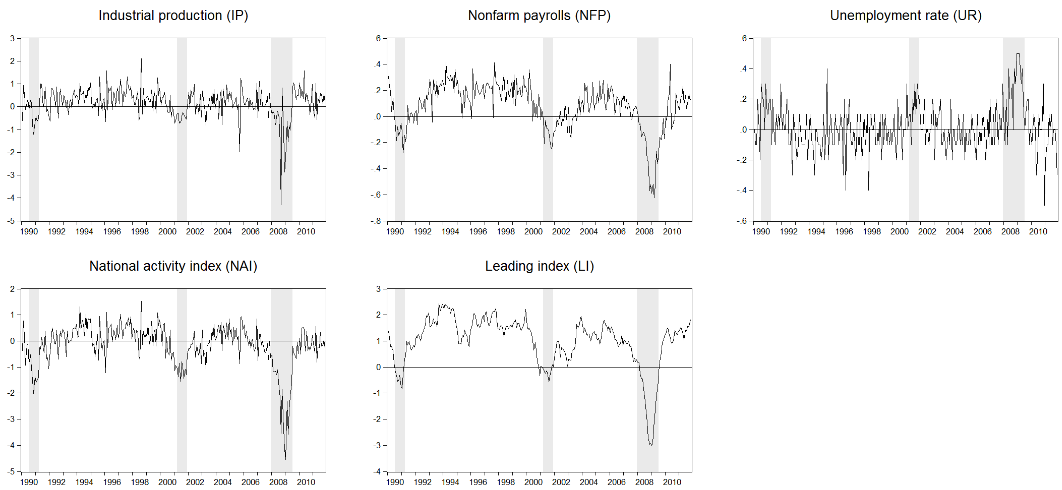
and the average difference relative to the benchmark DCC model along with values of the corresponding t-statistic. For each DCC-MIDAS model we calculate the optimal hedge portfolio

$$r_t^{PF} = r_{1,t} - \beta_{12,t} \cdot r_{2,t}, \qquad \text{with} \qquad \beta_{12,t} = \frac{\hat{h}_{12,t}}{\hat{h}_{2,t}},$$

and report its average variance. The average variance for the portfolio consisting only of stock returns amounts to 1.507 for the full sample and to 3.438 for the subsample. We calculate the average difference of each variance relative to the DCC model and the corresponding t-statistic. ***, **, * indicate significance at the 1 %, 5 %, and 10 % level.

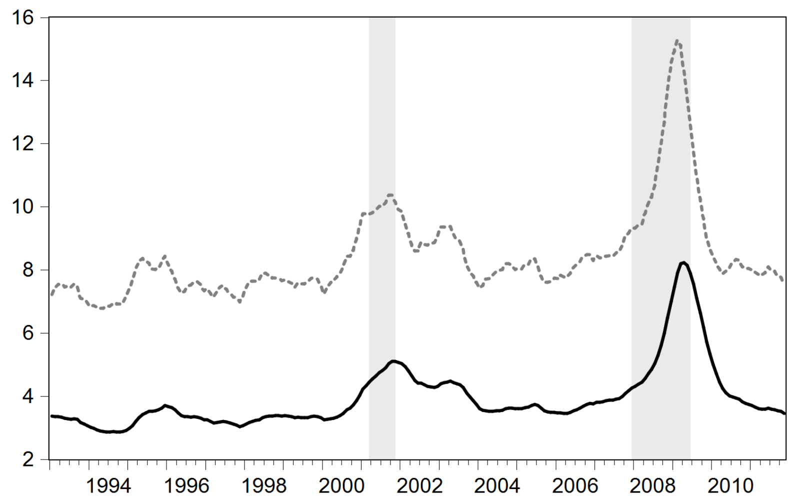
4.8.2 Figures

Figure 4.1: Monthly macroeconomic data



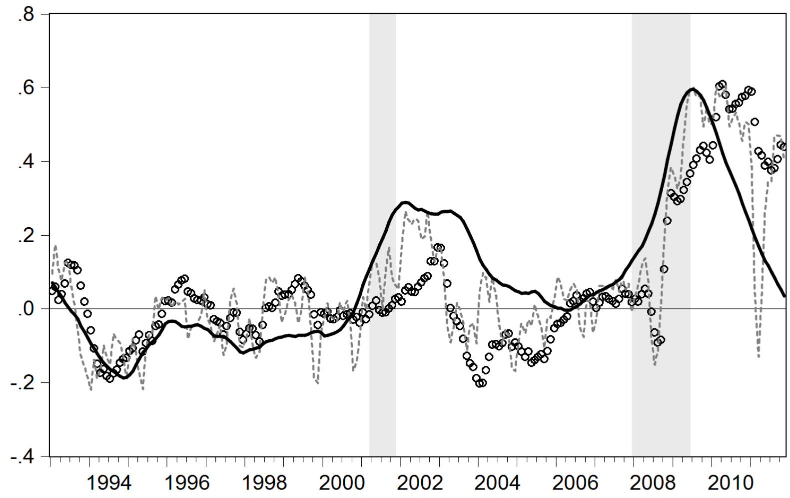
Notes: The figure shows the development of the macroeconomic explanatory variables. Shaded areas represent NBER recession periods.

Figure 4.2: Long-term volatility components for the stock and oil market



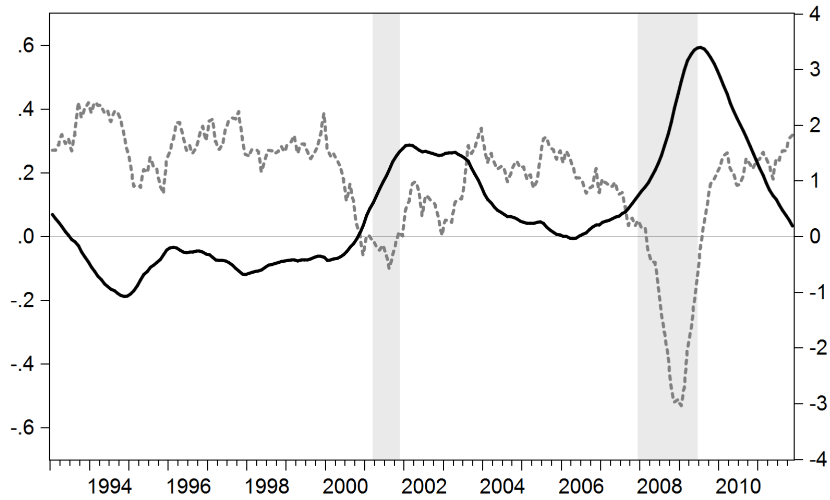
Notes: The figure shows the annualized monthly long-term volatility components (standard deviations) obtained from the GARCH-MIDAS-LI specification. The bold line refers to the stock market, the dashed line to the oil market. Shaded areas represent NBER recession periods.

Figure 4.3: Oil-stock correlation components



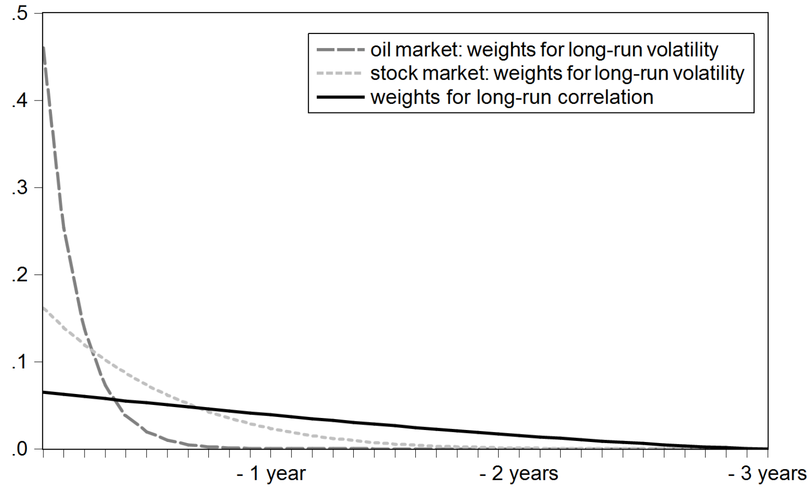
Notes: The figure shows the DCC-MIDAS-LI estimates of the conditional oil-stock correlation (dashed line) and its long-term component (bold black line). The circles correspond to one-year rolling window realized correlations. Each series is shown at a monthly frequency, where monthly realizations are obtained by computing monthly averages. Shaded areas represent NBER recession periods.

Figure 4.4: Long-term correlation component with the leading index



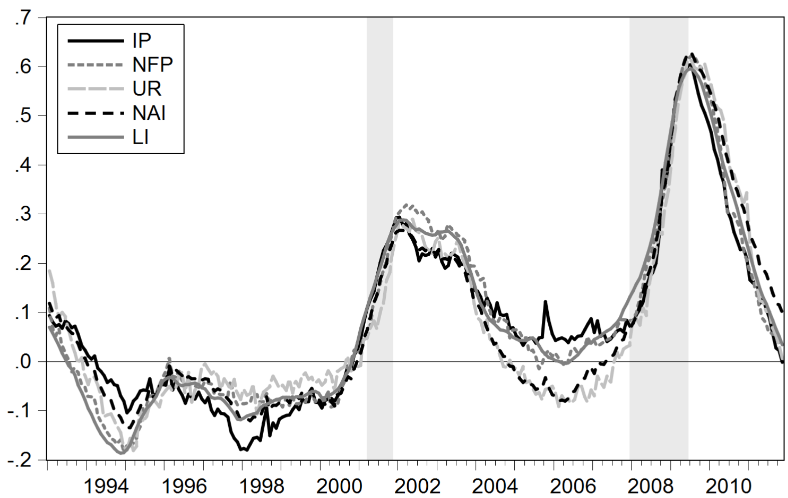
Notes: The bold black line (left scale) represents the DCC-MIDAS-LI estimate of the long-term oil-stock correlation. The dashed line (right scale) corresponds to the LI. Shaded areas represent NBER recession periods.

Figure 4.5: Weighting schemes for the long-term volatility components



Notes: The figure shows the estimated weighting functions for the long-term volatilities based on the GARCH-MIDAS-LI and for the long-term correlation based on the DCC-MIDAS-LI. While the bold black line refers to the long-term correlation, the light-gray and the dark-gray dashed lines refer to the long-term volatilities of CRSP and of oil price returns, respectively.

Figure 4.6: Long-term correlation components



Notes: The figure shows the DCC-MIDAS-X estimates of the long-term oil-stock correlations for all macroeconomic variables. Shaded areas represent NBER recession periods.

References

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