Essays on Demography-driven Inequality and the Long-run Welfare Consequences of Government Intervention

Inaugural-Dissertation zur Erlangung der Würde eines Doktors der Wirtschaftswissenschaften (Dr. rer. pol.) an der Fakultät für Wirtschafts- und Sozialwissenschaften der Ruprecht-Karls-Universität Heidelberg

vorgelegt von

Jochen Laps

Dezember 2015

To
Michaela
Matilda and Jonathan

Acknowledgments

This dissertation has benefited from the support, encouragement, and constructive comments of a number of people.

An early draft of this dissertation was formed while I was an assistant with the Department of Development Economics at Heidelberg University's South Asia Institute. The institute's working environment brought my latent interest in the fields of development and population economics to life. I thank my supervisor, Prof. em. Clive Bell, Ph.D.. Special thanks to Prof. Dr. Stefan Klonner for his invaluable advice. I also thank Prof. Dr. Zeno Enders and Prof. Dr. Reimut Zohlnhöfer for their immediate willingness to join the dissertation committee.

The present version was written after moving to Berlin, with my young family around. I am deeply grateful to Michaela for all the years, for her patience and trust - and for much more. I thank Matilda and Jonathan, who contributed mostly to my completing the dissertation. Without you, I would never have got this far!

I am grateful to Andreas Kropp for all the fruitful discussions, and to Dr. Christian Almer, Dr. Maik T. Schneider and Dr. Mario S. Mommer for their helpful comments during the last stage of this thesis.

I thank my parents Rosemarie and Jürgen, my brother Marcus and his family for their moral support, for their belief and encouragement.

Berlin, December 2015 Jochen Laps

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Chapter 1

Introduction

The current population age structure in developed countries is to a large extent the result of a secular movement from high fertility and mortality rates towards low, even below replacement level fertility and mortality rates. This so-called demographic transition started in Europe around the year 1800 with decreasing mortality, followed by decreasing fertility around the year 1900. The total fertility rate (TFR) in Germany, for example, dropped from 5.2 at the end of the nineteenth century to 1.4 in 2010.[1] Over the same period, life expectancy at birth among German women increased from 38.5 to 83 years. These numbers are broadly representative of many now developed countries, albeit the demographic transitions were mostly somewhat less pronounced and differed in the exact timing. In the 20$^{\text{th}}$ century, the demographic transition became a global phenomenon. In India, for example, female life expectancy at birth rose from 24.7 years in 1900 to 68 years in 2010, while the TFR dropped from 5.9 in 1950 to 2.4 in 2010.

A central consequence of this secular trend in these vital rates is population ageing and is reflected by the so-called age dependency ratio, defined as the ratio of dependents to the working-age population, a sub-case of which is the old-age dependency ratio.[2] For example, Germany's old-age dependency ratio has increased from 8.5% in 1890 to a currently observed level of about 32%, with a projected level of 58.5% in 2050. Pure scarcity arguments imply that population ageing induces large redistributional effects in the economy. Ludwig et al. (2009) suggest that the 'pure' effect of population ageing on factor prices (abstracting from any government policy) amounts to a 1 percentage point decline in the real rate of worldwide return to physical capital and a 4 percentage point increase in gross wages over the coming decades. Moreover, the shift of the population

[1] The TFR is a cross-section measure, defined as the expected number of newborns per woman throughout her whole fertile phase of life. Detailed historical accounts of vital rates are to be found in Mitchell (1975), Mitchell (1998) and the Institute for Population Research and Social Policy (2000), upon request. Projections are taken from United Nations (2013).

[2] Dependents are usually defined as people younger than 15 and older than 64. A pension system's dependency ratio is typically defined as the ratio of those receiving pension benefits to those accruing pension rights, reflecting old-age dependency.

pyramid towards older age groups that, on average, receive significantly lower income, than the working-aged, increases lifetime income inequality substantially.

This dissertation is concerned with demography-driven inequality which is independent of the demographic transition. To illustrate, consider a generation whose members are fully identical *ex ante* in terms of preferences, the number of children they raise and their probabilities of dying along the life-cycle. Individual fate, however, will sort their heirs into those whose parents survived into old age and those whose parents died prematurely. In a market economy with well-defined and within-family inheritable property rights, uninsured mortality risk generates intragenerational inequality in the transfers received among the members of the next generation, provided that their parents have made private savings to provide for their old age. In short, the demographic environment converts *ex ante* equality into *ex post* inequality, which increases as time passes. This dissertation analyzes the scope for reducing or even eliminating demography-driven inequality.

One prominent alternative that may achieve a reduction in inequality is a Social Security system (see, e.g., Deaton et al., 2002; Ludwig et al., 2009; and Rasner et al., 2013). Chapters 2 and 3 of this dissertation analyze the long-run consequences of Social Security, addressing intragenerational inequality of the kind illustrated by the above example and intergenerational inequality, respectively. The long run is characterized by time-invariant distributions of the key economic variables. The starting point is the link between *ex post* inequality and *ex ante* uncertainty, Social Security is here considered as a tool for insuring risks to lifetime income (see, e.g., Gordon and Varian, 1988; Shiller, 1999; Matsen and Thøgersen, 2004; and Gottardi and Kubler, 2011). The *ex ante* perspective takes into account all insurance possibilities, including the insurance against the state at birth. These scenarios lend themselves to a welfare evaluation on the basis of Rawls's (1973) veil of ignorance, the benchmark for evaluating Social Security systems being the world without it. We do not address the question of whether other institutional arrangements would have occurred without Social Security pensions. Nor do we address the current financial pressures on Social Security systems that arise due to the previous secular trend and the retirement of the baby-boom cohorts. Rather we are concerned with the social long-run costs that would arise without such systems. Particular emphasis is placed on family structure and within-family transfers.

The 'world without it' is also the benchmark against which we evaluate a large, growth-orientated investment programme when there is spatial inequality, *inter alia*, with respect to the vital rates in Chapter 4. Before turning to the differences from - and complementary with - Chapters 2 and 3, we provide an overview of Social Security systems in practice and of the academic debate regarding their long-run consequences.

Social Security pension systems in practice

Social Security pension systems are widespread. Already in the late 1980's, the number of countries that had implemented some kind of old-age Social Security system was 130, albeit with different sizes and coverages (see Sala-I-Martin, 1996). Table 1.1 shows historical levels of Social Security in selected developed countries. Over the past five decades Social Security expenditures grew substantially, with pension expenditures reaching more than 10 per cent of gross domestic product in some countries. Redistribution within the system is not only seen as an essential feature of Social Security, but is also typically an explicit policy objective (Gillion, 2000). Redistribution occurs intragenerationally: for example, from high-income workers to low-income workers or from non-survivors to survivors, and, of course, intergenerationally, from workers to retirees, or from future generations to those suffering current ill-fortune.

Pension systems differ substantially across countries. The chosen systems are typically "multi-pillar", or "multi-tier" implementations that developed over several decades.[3] A clear-cut characterization is therefore not straightforward. We provide a rough taxonomy here: we classify Social Security Pension systems according to the rule which determines the pension benefit payments. A first distinction is made between funded systems and unfunded pay-as-you-go systems. The former invests the pension contributions of a generation at the going interest rate and pays the proceeds plus the assets of those who die prematurely to the survivors of the same generation when old. Chile is a prominent current example of a pension system with substantial reliance on such funding. Countries like Japan, the Netherlands and Norway have had fully funded systems for decades. The German system, the oldest formal Social Security pension system, started as a fully funded disability insurance system in 1889, and was converted to an unfunded pay-as-you-go system only in the aftermath of the Great Depression and World War II.

An unfunded pay-as-you-go system pays the pension benefits of current retirees from the contributions paid by the current generation of workers. Within this class, we further distinguish between defined contribution and defined benefit systems. In the former, contributions into the scheme follow a pre-specified rule, with the pension benefit following as a residual from the balanced budget requirement. In a defined benefit system, the pension formula pre-specifies a targeted benefit level.[4] The benefit formula, in turn, may include several forms of indexation, for example, coupling to consumer prices, real wages or demographic key variables, most prominently to the system's old-age dependency ratio. In fact, unfunded defined benefit pay-as-you-go systems like that

[3]Serious attempts have been made to classify real world pension systems. Two prominent typologies are those employed by the World Bank and the OECD. The World Bank uses a "three-pillar" classification. The first pillar is "a publicly managed system with mandatory participation and the limited goal of reducing poverty among the old. The second pillar is a "privately managed mandatory savings system". Voluntary savings constitute the third pillar (World Bank, 1994). The OECD employs a "two-tiers" classification, the first tier bundling redistribution components "designed to achieve some absolute, minimum standard of living, and the second tier bundling insurance components "designed to achieve some target standard of living" (Queisser and Whitehouse, 2005).

[4]This classification leaves unaddressed the question of whether the system's replacement rate is defined in terms of current or previous wage income, thereby subsuming the alternative taxonomy which splits pension systems into fixed replacement rate systems, and fixed contribution rate systems (see, e.g., Thøgersen, 1998).

Table 1.1: The size of Social Security (% of gross domestic product)

	1960		1970		1980		1990		2000		2010	
	Total	Pensions	Total	Pensions	Total	Pensions	Total	Pensions	Total	Pensions	Total	Pensions
USA	7.2	3.9	10.3	4.9	13.2	6.2	13.6	6.1	14.5	5.9	19.2	6.8
CAN		2.1		2.4	13.7	3.1	18.1	4.2	16.5	4.3	18.6	4.5
GBR		4.0		4.9	16.3	4.2	16.3	4.8	18.4	5.4	22.8	6.1
DEU	15.3	8.4	16.1	8.8	22.1	10.4	21.7	9.0	26.6	11.2	27.3	11.5
FRA	12.4	4.7	15.4	6.7	20.8	9.5	25.1	10.6	28.6	11.8	32.1	13.3
SWE	10.1	3.9	17.12	6.1	27.1	7.1	30.2	7.7	28.4	7.2	28.3	9.6
NLD	10.1	3.0	17.8	5.0	24.8	6.4	25.6	6.7	19.8	5.0	23.5	7.0
JPN		1.2		1.1	10.3	3.0	11.1	4.0	16.3	6.8	22.1	10.2

Pensions: Cash benefits for old age and survivors only. Total: Cash benefits and benefits in kind, including medical care. Administrative expenditures and transfers to other transfer systems are excluded. Source: 1960-1980: Own calculations based on Mannheimer Zentrum für Europäische Sozialforschung (MZES) in cooperation with International Labour Organization (ILO), 'The Cost of Social Security: 1949-1993'. 1980-2010: OECD Social Security Expenditure Database.

Note: Contribution rates are only a weak indicator for the size of the schemes, because some countries finance large parts of their pension benefits by general revenues; also, debt financing is common. The table provides historical data on social security (pension) expenditures as a proxy for the average contributions paid to the pension system.

of the United States is still common. Point systems, which were found, for example, in France and Germany, also fall into this category (Whitehouse and Queisser, 2007). However, the recommendations for reforming the German point system – proposed by the so-called "Rürup commission" in 2003 and adopted by the Bundestag in 2004 – include the indexation of future benefits to a weighted mix of wages and the system's inverse dependency ratio. They converted the existing defined benefit system into a (notional) defined contribution system, in that pension benefits are cut automatically if the system's dependency ratio is high (Barr and Diamond, 2008). Feldstein and Liebman (2002) suggest that, after reforms in the 1990's, the public pay-as-you-go pension systems of countries like Sweden, Italy and Poland should also be classified as (notional) defined contribution systems.

To summarize, the salient features of recent pension policy reforms are the introduction of funded elements into unfunded schemes, the movement from defined benefit systems towards defined contribution systems (for example, by introducing demographic indexation into the pension benefit formula), and the reduction of the general size of the system in order to encourage private savings. In the light of the looming retirement of the baby-boom cohorts, the argument is that these reform packages make Social Security pensions financially sustainable, thereby making future generations better-off. The broader issue is, of course, that an unfunded pay-as-you-go system only functions as long as individuals believe that they will receive an adequate pension payment at old age in exchange for the contribution they made when young. This calculation crucially depends on the next generation's assessment of the risk of not receiving an adequate pension, and so forth.

Social Security pension systems in theory

To take the arguably canonical example, it is commonly perceived that the Great Depression (and not the broader demographic trends) triggered the implementation of unfunded Social Security. Given the U.S. experience, Blinder (1982, pp.60-61) argues that the *ex post* compensation of those generations that suffered most from Great Depression consequences provides an intellectually defensible case for an unfunded public pension system: "The people who retired in the early years of the Social Security system (say, those who reached age 65 in 1940) were 54 or so when the Depression began. For them, the Depression represented a huge and irreparable loss of lifetime income. It is not something they could have been expected to have prepared for, nor subsequently made up for. The Social Security system made huge transfers to these people, who had contributed very little but drew substantial benefits. It thus transferred some of their Depression losses to unborn generations. Was this bad social policy?" The argument also holds for subsequent generations up to the point at which the system is fully matured, albeit in weaker form.

However, the compensation of those suffering ill-fortune is not free: once installed, unfunded Social Security pensions permanently transfer resources from future to current generations. The reduction of net wage income when young and the provision of non-capital income when old tends to decrease private savings, thereby reducing the future national capital stock and, if the economy is dynamically efficient, future resources available for consumption.[5] A full picture therefore includes the (very) long-run and poses a related question: “For whom was this bad/good policy?” A salient feature of deterministic overlapping generations models that address this question is that if egoistic individuals behave rationally, then Social Security pensions are welfare-decreasing in the long run, especially if financed on an (unfunded) pay-as-you-go basis. Most prominently, Auerbach and Kotlikoff (1987) suggest that Social Security is not Pareto-improving.[6] They calculate that the U.S. Social Security pension system causes a reduction of 24% in the steady state capital stock, when compared to the hypothetical outcome without the system.[7] The loss for those generations is equivalent to more than 6% of lifetime resources.

In 1938, President Roosevelt advocated the draft of the Social Security bill as follows: “No one can guarantee this country against the dangers of future depressions, but we can reduce those dangers. ... we can provide the means of mitigating their results” (as cited in Gottardi and Kubler, 2011). Under this *ex ante* perspective, Social Security can be rationalized on efficiency grounds alone, an argument that economists usually feel more comfortable with. Krueger and Kubler (2006) and Gottardi and Kubler (2011), for example, study economies subject to productivity shocks. The authors provide conditions under which the introduction of a marginal unfunded pension system is welfare-improving under the classical Pareto-criterion, generalized to stochastic environments.[8]

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[5]Dynamic efficiency is another term for productive efficiency in a dynamic setup. An economy that is not dynamically efficient is dynamically inefficient in the sense that a reduction of the capital stock today does not reduce resources at any future date and increases it in some futures.

[6]An allocation is called Pareto-optimal if there is no other feasible resource allocation that provides at least an equivalent lifetime utility to every agent, yet even more to some. A Pareto-optimal allocation requires first - on the production side - that the economy provides as much resources available for *aggregate* consumption at every date as possible, given feasibility (i.e. dynamic efficiency), and second, that the largest possible output is allocated optimally across generations. Under certainty, dynamic efficiency implies Pareto-optimality (see De la Croix and Michel, 2002 whose proof builds heavily on Homburg, 1992). Dynamic inefficiency gives rise for potential Pareto-improvements through governmental intervention, indicating that the First Welfare Theorem may fail to hold even in the absence of any traditional market failure.

[7]To employ the proposed taxonomy, the US Social Security system is described as an unfunded defined benefit pension system with benefits replacing 60% of the average indexed monthly earnings.

[8]Two ways of interpreting the term ’some’ in the definition of a Pareto-optimal allocation can be found in the literature. With the overlapping generations structure in mind, a natural optimality benchmark would be one that is achievable in sequentially complete markets. The associated criterion is known as *ex interim* Pareto-optimality (see Chattopadhyay and Gottardi, 1999, Demange and Laroque, 1999 and Demange and Laroque, 2000). Agents are identified by date and state, implying that agents born at the same date but in different states are different agents. However, one might argue that the overlapping generations structure renders all allocations Pareto-suboptimal; for agents are not able to insure themselves against being born into a ’bad’ state of nature. The associated concept is *ex ante* Pareto-optimality under which welfare is evaluated before any uncertainty is revealed. Agents are solely

While the experience of the Great Depression and the ability of Social Security pension systems to mitigate future economic shocks encouraged their implementation, suitably designed systems are also capable of mitigating the economic consequences of a risky demographic environment. For example, Sánchez-Marcos and Sánchez-Martín (2006) calibrate a model with stochastic fertility to the U.S. economy and establish *ex ante* insurance gains from a marginal unfunded Social Security system. They find, however, that these gains are far too small for generating long-run welfare gains once the ensuing reduction in the long-run capital stock is taken into account.

These contributions do not determine a unique optimal allocation. Moreover, most developed countries run Social Security systems on a large scale (see Table 1.1). Chapters 2 and 3 of this dissertation determine the long-run optimal size of alternative Social Security arrangements, which trades off insurance against the crowding out of capital when there is a risky demographic environment. We shortly summarize the chapters. They can be read independently of each other. All references are collected in the final bibliography.

Chapter 2: Fertility

Chapter 2 studies the welfare consequences of unfunded pay-as-you-go Social Security pension systems with (incomplete) demographic indexation when there are perpetual shocks to the workforce size. The latter is of particular importance because it is a key input in the aggregate production process and its relative size is the key variable in the system's budget. Shocks to the workforce arise due to fluctuations in fertility.[9]

Figure 1.1 depicts the historical course of the TFR in selected OECD countries. All of these countries experienced a more or less pronounced baby boom - baby bust scenario in the aftermath of World War II. Much of the debate on pension reforms is about the looming implications of the baby-boomers' retirement to financial sustainability. The fact that there is considerable uncertainty concerning fertility has received less attention. For example, the timing of the baby boom is still puzzling from a theoretical point of view. This can be seen by the *ex post* realized fertility, or completed cohort fertility (CCF). Figure 1.1 contains a limited time series of the CCF, with the horizontal difference between the CCF and TFR curves representing the generational time-span of roughly 30 years. The co-movement of CCF and TFR is striking and challenges the 'catching-up' argument that the baby boom was the result of delayed fertility from the Great Depression and World War II. If this were true, delayed fertility should have had no effect on the CCF. Greenwood et al. (2005) provide additional evidence that "the

identified by the date when they appear on the scene, i.e. by the average of all possible incarnations at birth. Since the *ex ante* criterion takes more insurance possibilities into account, *ex ante* optimality implies *ex interim* optimality but the converse does not hold. The question of which criterion to use, seems to be an ideological one.

[9] In light of current events, it should be noted that shocks to the relative size of the work force can also be explained by stochastic migration flows into the working age population. Under the assumptions that integration succeeds and that these people will stay in the respective target countries when retired, the mechanism and results are similar to those with delayed fertility.

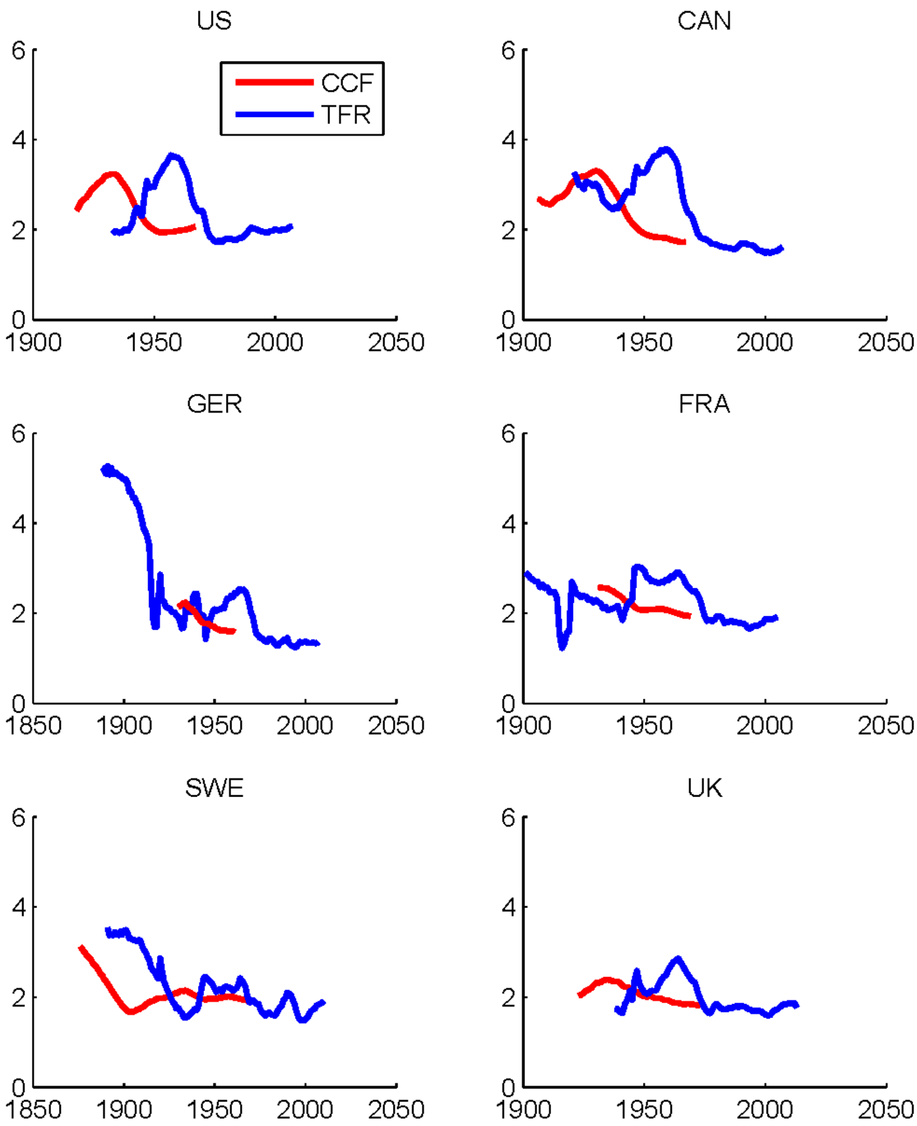
mystery of the baby boom has not been cracked in economics". Instead of providing a further theory of endogenous fertility, we simply treat fertility as an exogenously given stationary process around replacement levels and interpret the baby boom - baby bust events as consecutive shocks to the size of generations.

Empirical evidence in favour of the impact of generational size on factor prices is hard to find and hard to interpret. The reason is that the generational perspective reduces even long historical time series to just a few observations. Further, significant structural breaks, due to for example the two World Wars and the 1918–1920 "Spanish Influenza" pandemic, with its detrimental impact on French vital rates, render standard time series tools inappropriate. The literature on labour economics, however, confirms that the level of lifetime labour income of large cohorts is considerably reduced due to their size (see, e.g., Bloom et al., 1987; and Murphy and Welch, 1992 for the US economy, and Brunello, 2010 for European countries).

Following Sánchez-Marcos and Sánchez-Martín (2006) and Ludwig et al. (2009), Chapter 2 takes a general equilibrium perspective. The economy in each period is populated by two egoistic, overlapping generations, with stochastic total factor productivity and a stochastic number of workers. Two general equilibrium channels that arise in a dynamic setup are explored. First, fertility fluctuations induce movements in factor prices, thereby generating intergenerational heterogeneity among otherwise identical generations that are alive at different points in time. A member of a large generation (a baby boomer) suffers from a relatively low realization of wages during her working life and - since aggregate savings of a large generation tend to be high - a relatively low realization of the real rate of return on her savings when retired. The opposite is true for a member of a baby-bust generation.

Second, the pension system is seen as a means of *sharing* the fertility risk *across generations*, but feeds back in advance on the accumulation of capital by affecting the individual consumption-savings decisions. We argue that the introduction of demographic indexation into the pension benefit formula of an otherwise defined benefit system, pulls the latter into the direction of a defined contribution system, thereby converting the pension claim into an asset with essentially the same risk properties as private savings in physical capital. If fully indexed, Social Security pensions do not contribute at all to the intergenerational sharing of fertility risks. In fact, we establish that the long-run optimal size of an unfunded pay-as-you-go Social Security pension system with full demographic indexation is independent of shocks, brings the economy onto its golden rule growth path, and maximizes a social welfare function which places equal weights on generations that differ in size and the time they appear on the scene. This finding is closely related to Hillebrand (2011), who determines the long-run optimal size of such a system in the presence of stochastic total factor productivity alone. Shocks to the latter, however, pull factor prices in the same direction, thereby limiting or even eliminating the scope for intergenerational risk sharing from the outset. By treating the pension claim as a quasi-asset along the lines of Merton (1983), Thøgersen (1998), Wagener (2003),

Figure 1.1: The baby boom - baby bust in selected countries



Note: The vertical distance between TFR and CCF roughly corresponds with the average age of childbearing, i.e. the time spanned by one generation. If the catching-up argument were correct, then CCF would have been unaffected by the baby boom.
The best indicator for population reproduction is the so-called net reproduction rate, defined as the average number of daughters that a hypothetical cohort of women would have at the end of their reproductive period if they were subject to the fertility and mortality rates of a given period. Since the TFR is so frequently encountered in the literature, we use it here to illustrate the baby boom.
Source: Human Fertility Database. Max Planck Institute for Demographic Research (Germany) and Vienna Institute of Demography (Austria). Available at www.humanfertility.org (data downloaded on 10/18/2015). The UK figure covers England and Wales. The historical depth of the German figure on the cross-sectional TFR is due to the Institute for Population Research and Social Policy (2000), upon request.

and Matsen and Thøgersen (2004), we establish insurance gains from incomplete demographic indexation in general equilibrium when individuals are sufficiently risk averse.
Chapter 3: Mortality
Chapter 3 studies the welfare consequences of funded Social Security pension systems in the presence of otherwise uninsured mortality risk when individuals wish to make transfers to their heirs. As outlined above, in the presence of uninsured mortality risk, within-family transfers depend on realized lifespan and generate intragenerational heterogeneity with respect to transfers of wealth and the lifetime income of future generations. We consider Social Security Pension systems as a means of *insuring* the risks associated with premature mortality among the members of *the same generation.* They provide transfer insurance and insurance of the *ex ante* risk of future generations inheriting a particular amount of transfer wealth, thereby completing a very thin private annuities market (Diamond, 1977, and Pashchenko, 2013). As in Chapter 2, however, the gains from insurance come at the cost of a reduction in the future capital stock, since the provision of annuities reduces intergenerational transfer flows (Auerbach et al., 1995 and Gokhale et al., 2001) and, therefore, the resources from which future generations could save.

The chapter develops an analytical framework wherein both issues are closely linked. While fully funded Social Security crowds out private transfers, we find that, once *ex ante* insurance is taken into account, Social Security is welfare-improving over the long run as long as capital is not too productive and the transfer motive is not too strong. While transfer insurance and crowding out are qualitatively the driving forces behind this result, *ex ante* insurance may tip the scales in qualitative evaluations of the overall welfare effects. Altruists gain far less from Social Security than egoists. These results are at odds with the seminal work of Barro (1974), who finds that government policy intended to change the intergenerational distribution of resources is neutral in welfare terms when individuals are altruistic towards their offspring. If financed on a pay-as-you-go basis, Social Security substitutes for private inter vivos transfers. If fully funded, Social Security substitutes for private savings. Therefore Social Security has no real effects in aggregate. In contrast, the welfare effects in Chapter 3 arise due to the presence of uninsured mortality risk, so that private savings and Social Security pension claims are not real substitutes.

Of course, the notion that Social Security pensions can substantially reduce intragenerational inequality is not new. For example, Deaton et al. (2002) show that inequality rises as time passes, since the effects of random returns and earnings shocks accumulate over time. By substituting for private savings, Social Security pensions are capable of reducing inequality.[10] In Chapter 3, in contrast, uninsured mortality risk, not economic conditions, generates inequality, while private savings and Social Security pension claims

[10] Some authors argue that Social Security pensions may foster inequality (see, e.g., Gokhale and Kotlikoff, 2002). The argument is that annuitization reduces the net worth available for bequests in low-income households more than that in high-income households.

are not full substitutes. Chapter 3 establishes not only the non-neutrality of, but also the existence of long-run welfare improvements from Social Security. The latter is usually found to be at best welfare-neutral in the literature. The reason is that contributions that study the adverse effect of annuitization on transfer flows assume that the assets of those who die prematurely are redistributed in a lump-sum manner to the survivors, thereby ignoring the system's ability to insure the *ex ante* risk of inheriting a particular amount of transfer wealth (see, e.g., Hubbard and Judd, 1987; Kingston and Piggott, 1999; Fehr and Habermann, 2008; and Caliendo et al., 2014).

Both chapters 2 and 3 focus on the *long-run* consequences of Social Security and determine its long-run optimal size in the presence of uncertainties concerning vital rates which are treated as exogenously given. While the chapters are similar in spirit in that they trade off insurance against the crowding out of capital within a general equilibrium framework, there are several conceptional differences. First, chapter 2 determines the long-run optimal size of Social Security pensions assuming that individuals treat the size of the scheme as given when making their consumption-savings decision. While this is common practice in the literature, there is no obvious reason to assume that successive generations are completely unable to revise previous policy. We follow this idea in Chapter 3 and assume that individuals are able to revise the size of a fully funded pension system set by previous generations. Whether upcoming generations are also willing to revise previous policy depends on whether individuals are altruistic towards their children – a point not treated in Chapter 2 where individuals are assumed to save based on perfect selfishness. Second, it is well established that pay-as-you-go pension systems crowd out private savings and thus reduce the future capital stock, compared to the world without such systems (Feldstein, 1974, Auerbach and Kotlikoff, 1987). This standard crowding-out effect, which stems from the provision of non-capital income at old age, is the driving force behind the effect on the capital stock in Chapter 2. This mechanism differs fundamentally from that underlying Chapter 3, which relates to premature mortality. While fully funded systems may even induce individuals to increase their propensities to save from lifetime income, the provision of annuities reduces within-family transfer flows and, therefore, the resources available to future generations. A third difference concerns the nature of risk. The number of newborns constitutes an aggregate risk in the sense that all members a focal generation experience the same shock and this impedes the rise of private arrangements for coping with this kind of risk. Even a mandatory system cannot be expected to fully eliminate fertility risk. Contrastingly, in the case of mortality risk fate sorts members of the same generation into survivors and non-survivors, but the pooling of resources allows them to get rid of idiosyncratic risks. Note the strict separation between fertility and mortality. It is believed that fertility in developed countries is unlikely to be affected by the mortality regime; for most women survive their child-bearing age and child mortality is rather modest.

Chapters 2 and 3 study the insurance and sharing of demographic risks by means of Social Security. By design and construction, they involve a no-growth setup, in the sense that the economies under consideration converge to some kind of a stationary equilibrium. It is interesting, therefore, to analyze the impact of demographic developments in relation to the evaluation of a productive, growth-orientated investment programme.

Chapter 4: Migration[11]

Based on empirical evidence from upland Orissa, a remote and backward region located in the east of India, Chapter 4 attempts to estimate the long-run benefits of India's rural road programme known as Pradhan Mantri Gram Sadak Yojana (PMGSY), the aim of which is an improved integration of rural areas into the mainstream with respect to production and trade, education and health.

In contrast to the settings of Chapters 2 and 3, the demographic transition is still underway in India and life expectancy there is considerably lower, with infant and maternal mortality rates being far from negligible (so that net fertility per family is directly affected by the prevailing mortality regime). The extended family, with its norms of mutual obligations, however, is still alive and well which makes up in part for the lack of a large-scale Social Security system. In contrast to the nuclear family setup of Chapter 3 wherein the pooling of resources only arises through the Social Security system, the pooling of resources within the large extended family eliminates idiosyncratic risk to a substantial extent.

A salient feature is spatial heterogeneity; India's rural and urban areas differ substantially in the demographic environment and economic and educational opportunities. For example, infant mortality rates in 2011 were 55 and 34 (per 1000 live births) in rural and urban India, respectively. At the same time, the urban literacy rate was 85%, approx. 16 percentage points higher than in the rural area.

We employ a two-sector overlapping generations model with human and physical capital formation in the presence of premature mortality. The sectors are connected by movements of both goods and people. As in Bell and Koukoumelis (2009), intersectoral migration is the result of an economic decision in which young adults compare expected utilities in towns and rural areas, a decision which depends on factors including the magnitude of the rural-urban gap in mortality and morbidity. In equilibrium, young individuals in villages are indifferent between migrating to towns and permanently remaining in the rural sector.

Based on the findings of Bell and van Dillen (2014) and Bell and van Dillen (2015), PMGSY is assumed to improve the villagers' net terms of trade both as producers and as consumers, school attendance, and mortality and morbidity rates. By improving those ruling in the rural sector, PMGSY makes human capital less likely to exit. Valuing the benefits of the programme is more complex than in the settings of the previous chapters; for the roads affect decisions concerning the formation and maintenance of human capital, including the quantity and quality of life itself. We estimate that PMGSY generates benefits amounting to approx. 12 per cent of the value of output produced by a surviving member of the first generation. Approx. 18 per cent of this gain accrues in

[11] This chapter was jointly written with Clive Bell.

the spheres of education and health. We find that PMGSY has large long-run effects on welfare through its influence on health and education. From 2020 onwards, improvements in the terms of trade are of secondary importance.

Chapter 2

The Optimal Size of Social Security and the Long-Run Welfare Consequences of Demographic Indexation

2.1 Introduction

Many developed economies finance their social security programs on a pay-as-you-go basis, so that the pensions of current retirees are financed by the contributions of current workers. For quite some time, the pension benefit formulas in a number of countries have been driven by the actual contributions made to the scheme. Feldstein and Liebman (2001) suggest that Germany, Italy and Sweden, for example, run such so-called defined contribution schemes. Demographic indexation of pension benefits reinforces the policy of pre-specified contributions by cutting benefits when the ratio of retirees to workers increases, which implies that the retirees must bear the burden of old-age dependency. The system's replacement rate, defined as the rate at which pension benefits replace wage income, drops, while the system's contribution rate is held constant. In that sense, standard defined contribution systems can be seen as systems with full demographic indexation. Further reform packages intend to force or induce households to establish individual savings accounts as a supplement to, or substitute for, public pensions. However, these are implemented without referring to an optimality benchmark concerning the respective weights of these pension pillars within the overall pension system. Where insurance is concerned, this comes as no surprise because pension claims accumulated within a classic defined contribution system exhibit risk properties similar to private savings. From a risk sharing perspective, the split between the two forms of savings is therefore of secondary importance.

This chapter derives the optimal size of full demographic indexation schemes in the long run and analyzes the welfare effects of introducing a small measure of a defined benefit element. To that end, it develops a two-period overlapping generations economy model that features production, stochastic total factor productivity and a stochastic number of workers, with stochastically stationary productivity and population. The model is set up so that individuals' attitudes towards time determine the capital accumulation process. Further, individuals' attitudes towards risk determine the split of resources among contemporary generations. We find that the long-run optimal size of the full demographic indexation scheme is independent of fertility shocks, brings the economy onto its golden rule growth path, and maximizes a social welfare function that places an equal weight on each generation independently of its size and the time it appears on the scene. This implies that what is commonly referred to as full demographic indexation within a Social Security pension scheme effectively means that no demographic indexation occurs from a social planner's perspective. Introducing an element of defined benefit at the margin entails a crowding out of the long-run capital stock but allows for gains from insuring risks to lifetime resources. The simple example with a unit intertemporal elasticity of substitution in consumption across periods suggests that such a marginal deviation from a full demographic indexation scheme is long-run welfare-improving when the old are sufficiently risk averse, and the actual size of the system exceeds the long-run optimal level. In that case, the economy is dynamically efficient in the sense of Abel et al. (1989), implying that the insurance gains outweigh the negative crowding out effect.

The link between stochastic fertility and the pension scheme design is rarely treated in the literature, although the number of workers is a key variable in both aggregate production and the pension budget. Treating fertility as an exogenously given stochastic process, Ludwig and Reiter (2010) find that members of the German baby-boom generations lose up to five per cent of lifetime consumption under the pension formula prevailing at that time. This result can be interpreted in terms of Easterlin's (1980) informal discussion of the market bias against large generations, which suffer from low wage income when young and low proceeds from savings when old. Bohn (2001) therefore argues that 'pure' defined benefit schemes are more suitable than 'pure' defined contribution schemes to compensate for this market bias[1]. Sánchez-Marcos and Sánchez-Martín (2006) take up the idea and numerically analyze the long-run welfare consequences of the introduction of a marginal defined benefit scheme. Calibrating a four-period OLG model to the U.S. economy, they find substantial welfare losses in the steady state; the crowding out of capital dominates insurance gains over the long-run.

The optimal size of Social Security is usually studied in partial equilibrium models (see, e.g., Thøgersen, 1998; Wagener, 2003; and Matsen and Thøgersen, 2004). Within this strand of literature, only Matsen and Thøgersen (2004) treat fertility risk explicitly

[1]Young (2001) lays out a similar setup and argues that a social planner also has a bias favouring small generations. The argument is simple: the larger the generation, the higher are the social costs of providing utility to each member.

and derive the optimal size of a defined contribution scheme by means of a portfolio choice approach. In the absence of adverse effects on capital accumulation, a case for transfers from the young to the old can be made because the scheme is capable of hedging risks to lifetime income.

This chapter contributes to the above literature in two ways. First, and in contrast to Matsen and Thøgersen (2004), this chapter considers a continuum of pension mixes between the defined contribution system and the defined benefit system, and determines the degree of demographic indexation that minimizes the risk of the consumer's portfolio conditional on the state at birth. Conditional on the state at birth, the incorporation of a defined benefit element in the pension formula is welfare-improving, unless the general size of the system – measured as the replacement rate – is unreasonably high. This result is obtained within a general equilibrium model, wherein factor prices are determined by the relative abundance of capital and labour inputs as well as pension policy parameters, and physical capital is the only asset available in a setting with incomplete markets. Second, and in contrast to Bohn (2001) and Sánchez-Marcos and Sánchez-Martín (2006), the chapter establishes the existence of a long-run stationary equilibrium for an economy which runs with the full demographic indexation system: The equilibrium is characterized by stable distributions of the key variables. The chapter determines the long-run optimal size of Social Security. It is therefore closely related to Hillebrand (2011), who determines the long-run optimal size of a defined contribution scheme in a production economy where the only source of aggregate uncertainty is a stochastic total factor productivity parameter[2]. However, such shocks tend to pull factor incomes in the same direction, thereby limiting or even eliminating the scope for intergenerational risk sharing. The remaining role of the scheme is to bring the economy onto its golden rule growth path, a problem first studied by Phelps (1961).

The remainder of this chapter is structured as follows. Section 2.2 lays out the economic and demographic environment. Section 2.2.3 treats the individual saving decision as exogenously given and derives the variance-minimizing degree of demographic indexation conditional on the state at birth. Section 2.3 endogenizes the savings decision; the associated long-run optimal and socially optimal sizes are derived in Section 2.4. Section 2.5 provides the simulation results. The main conclusions are drawn together in Section 2.6.

2.2 Intergenerational sharing of fertility risk

This section builds a general equilibrium overlapping generations model of a closed economy that features production and stochastic but stationary productivity and population growth. The capital accumulation process and factor prices depend on the relative abundance of factor inputs and the pension parameters.

[2]An early attempt was made by Feldstein (1985) who derives, in a deterministic environment, the optimal size of the scheme when individuals are myopic.

2.2.1 Technology

Let the technology be Cobb-Douglas

$$Y_t = F(\epsilon_t; K_t, N_t) = \epsilon_t K_t^{\alpha} N_t^{1-\alpha}, \ \alpha \in (0,1), \tag{2.1}$$

where Y_t, K_t, N_t, and ϵ_t denote aggregate output, capital stock, work force (equal young population) and total factor productivity, respectively. $\{\epsilon_t\}_{t=0}^{\infty}$ is a sequence of independent and identically distributed random variables with known distribution. In the absence of premature death, shocks to the generational size are conveniently modelled as a stochastic population growth factor $n_t = N_t/N_{t-1} > 0$, where $\{n_t\}_{t=0}^{\infty}$ is a sequence of independent and identically distributed random variables with known distribution. A high realization of n_t represents a baby boom, a low realization a baby bust. Section 2.5 specifies the notion of a 'real' baby bust. The random variables ϵ and n are assumed to be log-normally distributed.

Assumption 1. $\ln \epsilon_t \sim \mathcal{N}(\mu_\epsilon, \sigma_\epsilon^2)$, $\ln n_t \sim \mathcal{N}(\mu_n, \sigma_n^2)$.

'Developed' countries are now at the end of their fertility transition[3], and the question addressed in this chapter concerns fluctuations rather than the previous secular trend of fertility.[4,5] We therefore impose

Assumption 2. *There is replacement fertility on average, i.e.* $En_t = 1$.[6]

Firms make their decision of what levels of inputs to hire after the realization of the shocks. Therefore, they face no uncertainty. Capital depreciates fully within one period, and factors are paid their marginal products:

$$R_t = \alpha \epsilon_t k_t^{\alpha-1} \tag{2.2}$$
$$w_t = (1-\alpha)\epsilon_t k_t^{\alpha}, \tag{2.3}$$

where $k_t = K_t/N_t$ denotes the capital stock per worker. Given K_t, a baby boom causes the capital stock per worker to be low, so that R_t is high and w_t is low. Over the life-cycle, members of a large generation suffer from a relatively low realization of w_t when young and - since aggregate savings of a large generation tend to be high - a relatively low realization of R_{t+1} when old. In that sense, (2.2) and (2.3) restate Easterlin's (1980) informal argument that life is disproportionally good if one belongs to a small generation.

[3]The chapter abstracts from changes in life expectancy and migration.

[4]The economic consequences of the induced population ageing are well documented; see, for example, Roseveare et al. (1996) and Casey et al. (2003) on the general consequences for capital accumulation and growth and the induced financial pressure within social security pension schemes. See also the in-depth treatment within a full-blown (normative) Ramsey model in Cutler et al. (1990).

[5]Greenwood et al. (2005) argue that the mystery of the baby boom following the 1950's has not been cracked within endogenous fertility models.

[6]Still, there is the possibility of extinction, i.e., N_t approaches zero with positive probability.

The Cobb-Douglas technology and Assumption 1 imply that, once the state (k_t, ϵ_t, n_t) has been realized, R_{t+1} is a log-normally distributed random variable.

2.2.2 Demographic indexation of pension benefits

We consider pure pay-as-you-go pension schemes, so that current pension benefits are financed by the contributions of current workers. In particular, there are no pension funds. Demographic indexation can be considered as linking pension benefits to the economy-wide wage bill. Allowing for incomplete demographic indexation, the simplest indexation structure links the pension benefit per retiree partly to the evolution of the wage rate (w_t), and partly to the evolution of the aggregate wage bill ($w_t N_t$),

$$b_{t+1} = \theta \times w_t \times \left(\frac{w_{t+1}}{w_t}\right)^x \times \left(\frac{w_{t+1}N_{t+1}}{w_t N_t}\right)^{1-x}, \ x \in [0,1]. \tag{2.4}$$

Here, θ is the scheme's replacement rate (that is, the rate at which the pension benefit in $t+1$ replaces own previous wage income in t). θ will serve as the measure of the size of the scheme. We assume throughout $\theta > 0$, although we cannot rule out that its long-run optimal level is negative. $1-x$ is the degree of demographic indexation. Demographic indexation allows to display a continuum of pay-as-you-go pension mixes between what is known as a defined benefit scheme ($x = 1$) and a defined contribution scheme ($x = 0$), where an increase in the degree of demographic indexation $(1-x)$ shifts Social Security pensions away from defined benefit towards defined contribution. Both schemes are frequently studied in isolation in the existing literature (see, e.g., Bohn, 2001). Obviously, (2.4) represents a class of pay-as-you-go pension schemes that are equivalent if only fertility was deterministic and constant at replacement level. It follows from (2.4) that $b_{t+1} = \theta \times w_{t+1} \times n_{t+1}^{1-x}$. Under the assumption that the individuals survive both periods of life, n_{t+1}^{-1} is the old-age dependency ratio in period $t+1$. An increase in the latter cuts, *ceteris paribus*, pension benefits in that period whenever $x < 1$.[7] Contributions are proportional to wages, and the pension budget is balanced in each period by assumption, so that the contribution rate in period t is $\tau_t = \tau(\theta, x, n_t) = \theta n_t^{-x}$.

The next section treats the pension claim as a quasi-asset. Denoting the system's implicit rate of return by $R_{t+1}^P = (w_{t+1}/w_t) \times n_{t+1}^{1-x} \times n_t^x - 1$, we may rewrite the pension benefit in (2.4) as

$$b_{t+1} = \tau(n_t) w_t \times R_{t+1}^P. \tag{2.5}$$

[7]For example, the so-called sustainability factor in the German pension system (implemented in 2004) cuts benefits when the old-age dependency ratio increases. The aim was to retain the financial sustainability of the pension system when the baby-boom generations retire. More precisely, it was implemented in order to resolve the conflicting aims of paragraph 154 SGB VI to keep the contribution rate below 22% until 2030 and the replacement rate (related to the artifical 'Eckrentner') above 67%. The cut in benefits is accompanied by measures to force or induce individuals to privately save for old age, the so-called Riester-Rente.

Given the vector (k_t, ϵ_t, n_t), equation (2.1) and Assumption 1, R^P_{t+1} is log-normally distributed.

It is assumed that physical capital and pensions claims are the only assets available. In contrast to Matsen and Thøgersen (2004), we therefore have

Assumption 3. *There is no risk-free asset.*

The combination of Cobb-Douglas technology and full demographic indexation of pension benefits (i.e. $x = 0$) implies that the two sources of old-age income are perfectly, positively correlated. In contrast, if demographic indexation is incomplete (i.e. $x > 0$), then current retirees share the fertility shock of the next generation.

2.2.3 Diversification of lifetime income

This section analyzes the welfare consequences of a deviation from the full demographic indexation scheme, conditional on the state at birth, $(k_t; \epsilon_t, n_t)$. These consequences concern the gains of the diversification of lifetime income; for the pension claim then exhibits different risk properties than private savings in physical capital. Following Matsen and Thøgersen (2004), the pension claim is treated as a quasi-asset. Matsen and Thøgersen (2004) determine the optimal size of a pay-as-you-go pension system with a fixed contribution rate (corresponding to $x = 0$) within a partial equilibrium setup wherein factor prices and population growth are treated as exogenously given. This seems to be at odds with the empirical evidence that the level of labour income of large cohorts is considerably reduced due to their size (see, e.g., Brunello, 2010). In contrast, the current general equilibrium approach makes explicit these interdependencies, through (2.2) and (2.3). As noted above, the combination of Cobb-Douglas technology and full indexation implies perfect correlation between capital income and the pension benefit. As a consequence, the optimal size of the full demographic indexation system remains undetermined, given $(k_t; \epsilon_t, n_t)$. Instead, we establish welfare gains of partial indexation and derive the degree of demographic indexation which minimizes the variance of the overall portfolio conditional on the state at birth.

Each generation consists of many *ex ante* identical purely selfish individuals. When young, each of them inelastically supplies one unit of labour and pays contributions to the scheme. Following Gordon and Varian (1988), Matsen and Thøgersen (2004), and Ball and Mankiw (2007), they are assumed to live for two periods, but only draw felicity from consumption at old age. This simplifying assumption is qualitatively not essential for the intergenerational risk sharing problem, but it admits some analytical results. It is relaxed in Section 2.3. The key insights, namely the existence of diversification gains and the existence and uniqueness of the system's long-run optimal size, carry over to the case when the individual savings decision is endogenous (see Sections 2.4 and 2.5).

The representative member of the young generation therefore privately saves all net wage income. When old, she consumes the proceeds from private savings plus the pension benefit,

$$c_{t+1} = w_t(1-\tau(n_t))R_{t+1} + b_{t+1}. \tag{2.6}$$

Using (2.5), we may rewrite (2.6) as

$$c_{t+1} \quad = \quad w_t \times R^T_{t+1} \tag{2.7}$$

where

$$R^T_{t+1} \quad = \quad R_{t+1} + \tau(n_t) \times \left(R^P_{t+1} - R_{t+1}\right) \tag{2.8}$$

is the effective return of the total portfolio, expressed in terms of a linear combination of the individual asset returns R^P_{t+1} and R_{t+1}.

By virtue of Assumption 1 and the fact that the vector $(k_t; \epsilon_t, n_t)$ and therefore w_t are known, R^P_{t+1} and R_{t+1} are lognormally distributed random variables.[8] While this does not imply that R^T_{t+1} is likewise in general, Campbell and Viceira (2002) show that it is approximately. Taking a second-order Taylor approximation of the logarithm of (2.8) around $\ln R^P_{t+1} - \ln R_{t+1} = 0$ yields the following expression for $r^T_{t+1} \equiv \ln R^T_{t+1}$ (see Appendix 2.A.1):

$$r^T_{t+1} \quad = \quad \tau_t r^P_{t+1} + (1-\tau_t)r_{t+1} + (1/2) \times \tau_t(1-\tau_t)\left(r^P_{t+1} - r_{t+1}\right)^2, \tag{2.9}$$

with mean and variance

$$E_t r^T_{t+1} \quad = \quad \tau_t \mu_{r^P} + (1-\tau_t)\mu_r + (1/2)\tau_t(1-\tau_t)x^2\sigma^2_n \tag{2.10}$$

$$\sigma^2_{r^T} \quad = \quad \tau^2_t \sigma^2_{r^P} + (1-\tau_t)^2\sigma^2_r + 2\tau_t(1-\tau_t)\sigma_{r^P r},. \tag{2.11}$$

where $\mu_r = E_t r_{t+1}$, $\mu_{r^P} = E_t r^P_{t+1}$ and $E_t[(r^P_{t+1} - r_{t+1})^2] = x^2\sigma^2_n$. Finally, $\sigma^2_{r^P} = \sigma^2_\epsilon + ((1-x)-\alpha)^2\sigma^2_n$, $\sigma^2_r = \sigma^2_\epsilon + (1-\alpha)^2\sigma^2_n$ and $\sigma_{r^P r} = \sigma^2_\epsilon + (1-\alpha)(1-\alpha-x)\sigma^2_n$ denote the variances of $r^P_{t+1} \equiv \ln R^P_{t+1}$ and $r_{t+1} \equiv \ln R_{t+1}$ and the associated covariance, respectively, given the information available in period t. As noted by Campbell and Viceira (2002) and Matsen and Thøgersen (2004), the approximation is accurate over short time periods and, fortunately, entails only negligible 'horizon effects' for longer time periods. Also ote that $x = 0$ implies $\sigma^2_{r^P} = \sigma^2_r = \sigma_{r^P r} = \sigma^2_{r^T}$, indicating that the full demographic indexation system does not provide any intergenerational risk sharing for any $\theta \in (0,1)$. The approximation is therefore exact in that case.

The Taylor approximation implies that the stochastic structure of the individual asset returns is inherited by r^T_{t+1}. Still, income and substitution effects depend on the relative size of the generation. Suppose therefore that the current fertility realization is at the replacement level ($n_t = 1$). Then, for any $x \in [0,1]$, the fraction of wage income

[8] Throughout we encounter products of lognormal random variables. If some random variable X is lognormally distributed, then $Y = \ln X$ is normally distributed. It follows that $Y_1 + Y_2 = \ln(X_1 \cdot X_2)$ is normal. Likewise, $Y_1 - Y_2 = \ln(X_1/X_2)$ is normal.

to be saved within the pension system equals θ. This allows a clear-cut analysis of the impact of demographic indexation on the portfolio's riskiness, as measured by $\sigma^2_{r^T}$, given the general size of the system, θ.

By (2.11), $\sigma^2_{r^T}$ depends on the variances of the individual asset returns, $\sigma^2_{r^P}$ and σ_r, and, since these are not independent, on the covariance $\sigma_{r^P r}$. If $n_t = 1$, σ^2_r is unaffected by the degree of demographic indexation. However, the variance of the scheme's implicit rate of return, $\sigma^2_{r^P}$, is a convex function of x with the minimum at $x^*_P = 1 - \alpha \in (0,1)$. The threshold value $x^*_P = 1-\alpha$ separates the system into one which works like a defined contribution ($x < x^*_P$), so that both r^P_{t+1} and r_{t+1} increase with the fertility shock n_{t+1} (albeit the co-movement is dampened by $x > 0$), and one which works like a defined benefits system ($x > x^*_P$), in which case r^P_{t+1} decreases with n_{t+1}. If $x > x^*_P$, then a further increase in x increases the variance of the implicit rate of return.

However, even if x is already large, the variability of the total portfolio may still be reduced by a further increase in x; for the system compensates the market outcome. $\sigma^2_{r^T}$ depends not only on the actual size of the scheme, as measured by θ, but also on the covariance between the asset returns, r^P_{t+1} and r_{t+1}, respectively. An increase in x decreases the said covariance for any $x \in [0,1]$, tending to reduce, *ceteris paribus*, the variance of the total portfolio. Given $n_t = 1$, it is readily verified by differentiating (2.11) that the reduction in $\sigma_{r^P r}$ outweighs the potential increase in $\sigma^2_{r^P}$ as long as $x < x^* = (1-\alpha)/\theta$, where x^* is the variance-minimizing level of x, provided that $\theta > (1-\alpha)$. Note that x^* satisfies, for any $\theta \in (0,1)$, $\partial\sigma^2_{r^T}/\partial\theta = 0$. Therefore, if the objective is to minimize the portfolio's variance, then the generation should opt for the classic defined benefit system ($x = 1$), or for the incomplete demographic indexation system with $x = x^*$ if $\theta > (1-\alpha)$. Given the standard value of the capital share, $\alpha = 0.3$, and social security pension expenditures of around 10 per cent of the gross domestic product (see Table 1 in Chapter 1), $0 < x \leq 1$ almost surely implies a lower portfolio variance, when compared to the full demographic indexation system.

Finally note that the class of pay-as-you-go pension systems considered here is unable to share i.i.d. productivity shocks across generations.

We next show that the risk reduction implies welfare gains of a deviation from full demographic indexation only if individuals are sufficiently risk averse. Let the von Neumann-Morgenstern utility function of a representative member of the generation born in period t be of the CRRA type, with coefficient of relative risk aversion γ. Her expected utility is

$$V_t(\theta, x) = E_t\left[\beta\frac{c^{1-\gamma}_{t+1} - 1}{1-\gamma}\right], \tag{2.12}$$

where (2.12) specializes to $\beta E_t \ln c_{t+1}$ in the limit as $\gamma \to 1$. Maximizing (2.12) is equivalent to maximizing $\ln \frac{\beta}{1-\gamma} E_t c^{1-\gamma}_{t+1}$ (see Campbell and Viceira, 2002). Since $\beta/(1-\gamma)$ is simply a scale factor, the policy problem is equivalent to

$$\max \ln\left[E_t c^{1-\gamma}_{t+1}\right]. \tag{2.13}$$

For later use, note that monotonicity of V_t implies that maximizing V_t, and therefore problem (2.13), is equivalent to maximizing the certainty equivalent $\left[E_t c_{t+1}^{1-\gamma}\right]^{1/(1-\gamma)}$ (i.e. the sure old-age consumption that is necessary to make the individual as well-off as with stochastic old-age consumption). Also note that the certainty equivalent is unaffected by the full demographic indexation pension system ($x = 0$).

Since w_t is treated as given, old-age consumption c_{t+1} is approximately lognormally distributed and so is $c_{t+1}^{1-\gamma}$.[9] Therefore, by virtue of $\ln\left[E\left(c^{1-\gamma}\right)\right] = E\left[\ln\left(c^{1-\gamma}\right)\right] + 1/2 \times var\left(\ln\left(c^{1-\gamma}\right)\right) = (1-\gamma)E\ln c + 1/2 \times (1-\gamma)^2\sigma_c^2$, $\sigma_c^2 = var\left(\ln c\right)$, the problem associated with the objective of maximizing V_t can be written as

$$\max\left[E_t \ln c_{t+1} + 1/2 \times (1-\gamma)\sigma_c^2\right]. \tag{2.14}$$

Since the individual is risk averse, she values a reduction in the variance of c_{t+1}. Yet she dislikes a reduction in risk because it implies a reduction in the expected "pay-off". If $\gamma > 1$ ($\gamma < 1$) the former effect dominates (is dominated by) the latter, and the risk reduction associated with the introduction of $0 < x < x^*$ is, *ceteris paribus*, welfare-improving (welfare-decreasing) conditional on the state at birth. If $\gamma = 1$, then these effects exactly cancel out and the insurance provided by the scheme is welfare-neutral. Since (2.7) implies $\ln c_{t+1} = \ln w_t + \ln R_{t+1}^T$ the policy problem (2.14) finally reads as

$$\max_x\left[E_t r_{t+1}^T + 1/2 \times (1-\gamma)\sigma_{r^T}^2\right], \quad \text{s.t. (2.10) and (2.11).} \tag{2.15}$$

Appendix 2.A.1 differentiates (2.15), given any n_t. If $n_t = 1$, then

$$\begin{aligned}\frac{dV_t}{dx} =& -\theta\left[\mu_n - (1-\theta)x\sigma_n^2\right] \\ & - (1-\gamma)\theta\sigma_n^2\left[\theta(1-\alpha-x) + (1-\theta)(1-\alpha)\right],\end{aligned} \tag{2.16}$$

where the first line captures an income effect associated with the induced change in the portfolios expected return, $E_t r_{t+1}^T$. Given $\theta > 0$ and Assumption 2, $n_t = 1$ implies that $E_t r_{t+1}^T$ increases with x for any $x \in [0,1]$. The second line captures the welfare impact of the induced change in the portfolio's variance, $\sigma_{r^T}^2$. With an eye on Section 2.5, note that the marginal increase in x from $x = 0$, is welfare-improving. It is also readily verified by rearranging that the increase in x from any $x < (1-\alpha)/\theta$ is welfare-improving as long as $\gamma > 1 + \frac{\mu_n - (1-\theta)x\sigma_n^2}{(1-\alpha-x\theta)\sigma_n^2}$. Under Assumption 2, $\mu_n < 0$, implying that the threshold value of γ is unambiguously less than one.

By treating the state (k_t, ϵ_t, n_t) as given, the above focused on one particular general equilibrium channel: fertility fluctuations induce movements in factor prices. While x^* does depend on the actual size of the system, there is no clear guidance on the latter's

[9] $c \sim \mathcal{LN}$ implies $\ln c \sim \mathcal{N}$. Since the normal distribution is closed under linear transformations, $(1-\gamma)\ln c \sim \mathcal{N}$, so that $\exp\left(\ln c^{1-\gamma}\right) \sim \mathcal{LN}$.

optimal level. The next section therefore focuses on a second general equilibrium channel which arises in a dynamic setup: The pension system crowds out private savings and therefore the long-run capital stock for any x. We therefore close this section by defining the optimal size θ of the full demographic indexation system as the solution to

$$\max_{\theta} E_0 \lim_{t\to\infty} V_t(\theta, 0)$$

subject to

$$c_{t+1} = \bar{c}(\theta)\epsilon_{t+1}k_{t+1}^{\alpha}n_{t+1}, \tag{2.17}$$

$$k_{t+1}^{\alpha} = \exp\left\{\alpha \sum_{i=0}^{t} \alpha^i \ln(1-\theta)\right\}, \tag{2.18}$$

where $V_t(\theta, 0)$ is defined in (2.12), $\bar{c}(\theta) = (\alpha + \theta(1-\alpha))$ and (2.18) suppresses all terms that are independent of θ. Appendix 2.A.2 derives the full version of (2.18). The next section shows that k_t^{α} is lognormally distributed, and so is c_{t+1}. Recalling that $x = 0$ implies that the size of the system leaves the variance of old-age consumption unaffected, the optimal size, θ^*, is given by

$$\theta^* = 1 - \frac{\alpha}{1-\alpha}, \tag{2.19}$$

where $\alpha/(1-\alpha)$ is the ratio of factor shares. A formal proof is found in Appendix 2.A.4, which deals with the long-run optimal size when the savings decision is endogenous, but still the young individuals optimally save a constant fraction of output.

Constraint to the particular class of pension systems with $x = 0$, θ^* maximizes the welfare of each generation in the steady state. We therefore use the term 'long-run optimal size'. Note that $\alpha > 1/2$ implies $\theta^* < 0$ so that retirees are compelled to make transfers to their children. In contrast, the long-run optimal size is positive if the resources used to generate the capital stock under laissez-faire, $(1-\alpha)\epsilon_t k_t^{\alpha}$ exceed the contribution of capital to output, $\alpha\epsilon_t k_t^{\alpha}$, or, equivalently, $\alpha < 1/2$. A more detailed discussion of this result is found in Section 2.4, which deals with the more general setup of endogenous savings behaviour. There, we also motivate the long-run perspective by showing that the long-run optimal size is the solution to a social welfare function. Section 2.5 combines insurance of lifetime income and the crowding out capital by studying numerically the long-run welfare effects of introducing an element of defined benefit at the margin.

2.3 The model with endogenous savings

As in the previous section, each generation consists of many *ex ante* identical purely selfish individuals, but the representative individual may now consume in both periods of her life. When young, a member of the generation born in t allocates realized net wage income between consumption c_t^y and saving in physical capital a_t. When old, she receives the proceeds from savings, $a_t R_{t+1}$, plus the pension benefit b_{t+1}.

We employ a simplified version of the non-expected utility function developed in Epstein and Zin (1989) and Weil (1990) (also see Blanchard and Weil, 2001). These recursive preferences resolve the identification problem concerning the intertemporal elasticity of substitution and the coefficient of risk aversion prevailing in expected utility structures. We assume unit elasticity of substitution in consumption across periods, but allow for a general coefficient of relative risk aversion $\gamma > 0$. This is achieved by imposing logarithmic felicity from youth consumption and retaining the felicity function at old age from the previous section. For our purposes, these assumption are appealing. First, if both felicity functions were of the CRRA type, then the long-run equilibrium exists only if $\gamma < 1$, even in the deterministic counterpart of the model. In contrast, the unit intertemporal elasticity of substitution ensures existence and uniqueness of the long-run equilibrium for all $\gamma > 0$, which allows to study the long-run effects of a marginal deviation from the full demographic indexation system under alternative assumptions on the individuals' attitudes towards risk.

Her decision problem is to maximize

$$\mathcal{U}_t = \ln c_t^y + \beta \ln \left[E_t {c_{t+1}^o}^{1-\gamma} \right]^{1/(1-\gamma)}, \tag{2.20}$$

subject to

$$\begin{aligned} c_t^y &= w_t(1-\tau_t) - a_t, & (2.21)\\ c_{t+1}^o &= a_t R_{t+1}^e + b_{t+1}^e, & (2.22)\\ c_t^y &> 0,\ c_{t+1}^o > 0, & (2.23) \end{aligned}$$

where net wage income $w_t(1-\tau_t)$ is treated as given, and R_{t+1}^e and b_{t+1}^e denote subjective expectations concerning the future rate of return on capital and the pension benefit at the time of the consumption savings decision. We retain the specifications of the technology and the pension formula from the previous section, so that R_{t+1} and b_{t+1} follows from (2.2) and (2.4), respectively.

The timing of shocks is relevant. We impose delayed fertility:[10]

Assumption 4. n_{t+1} *is realized after the individuals' consumption savings decision.*

Assumption 4 is implicit in the existing economic literature on stochastic fertility (see, for example Bohn, 2001, Matsen and Thøgersen, 2004 and Sánchez-Marcos and Sánchez-Martín, 2006). The exception is Smith's (1982) early treatment of Pareto-effects from

[10] Delayed fertility probably influences completed fertility, and hence the size, composition, and growth of the population. Since the focus is on intergenerational risk sharing, the timing of fertility is assumed to have no effect on the fertility distribution itself. In particular, the expected number of newborns is unaffected.

unfunded Social Security Pension. We motivate Assumption 4 as follows. Economic uncertainty is one of the main driving forces behind delayed fertility (see e.g. Kreyenfeld et al. (2012) and the references cited therein). In the current setup, the stochastic total factor productivity parameter captures the source of economic uncertainty, and delayed fertility is seen as a response to economic uncertainty.[11] Section 2.5 provides a sensitivity analysis with respect to the timing of fertility.

For general $\gamma > 0$ and $x \in [0, 1]$, the first-order condition with respect to a_t associated with problem (2.20)-(2.23) is

$$\frac{1}{c_t^y} = \beta \frac{E_t \left({c_{t+1}^o}^{-\gamma} R_{t+1} \right)}{E_t {c_{t+1}^o}^{1-\gamma}} \tag{2.24}$$

In the temporary equilibrium, the vector $(a_t, c_t^y, c_t^o, k_{t+1}, w_t, R_t)$ satisfies, for any (θ, x) and all states, the following: (i) factor prices are determined by (2.2) and (2.3); (ii) the scheme is balanced, i.e. intergenerational transfers satisfy (2.4); (iii) given prices and transfers, a_t is the solution to problem (2.20)-(2.23), that is, it solves (2.24) with $R_{t+1}^e = R_{t+1}$ and $b_{t+1}^e = b_{t+1}$;[12] and (iv) the goods market clears. Since contributions to the pure pay-as-you-go pension scheme do not add to the capital stock, the latter evolves according to $K_{t+1} = a_t N_t$. In per worker terms,

$$k_{t+1} = a_t n_{t+1}^{-1}. \tag{2.25}$$

By Walras' Law, (2.25) implies that the goods market clear.

In general equilibrium, the optimal savings decision affects the next period's capital stock and, therefore, the next period's factor prices. Future factor prices, in turn, affect the optimal savings decision. Yet it is well known that the logarithm implies that optimal savings under laissez-faire is independent of the future interest rate, so that the 'feedback' effect is absent. Using equations (2.2), (2.3), (2.21), (2.22), and (2.25) in (2.24), shows that this holds true in the current setup with non-expected utility defined in (2.20) and Social Security, as long as the economy runs with the full demographic indexation system ($x = 0$); for - as noted above - the pension benefit then exhibits the same risk properties than physical capital. The remainder of this section focuses exactly on the case $x = 0$.

[11]Sobotka et al. (2011) conceptionalizes economic uncertainty as an aggregate phenomenon capturing a general uncertainty felt by all people during recession. A prominent example is the productivity slowdown in the 1980's, which dampened both wage and capital income growth. We reject the notion that economic uncertainty is higher in times of recessions.

[12]In contrast to equation (2.4), Abel (2003) assumes that the *present value* of the pension payment received at old age is known at the time of the decision. This entails a strict separability between the evolution of the pension parameters and the evolution of the capital stock even when we deviate from the full demographic indexation scheme, which he calls a *defined contribution* scheme.

2.3.1 Temporary equilibrium with full demographic indexation

We characterize the temporary equilibrium by optimal saving and age-specific consumption as functions of the prevailing state, where the latter is summarized by the current (endogenously determined) capital stock per worker, the exogenous shocks to productivity and fertility, and the pension policy parameters. Since $x = 0$, the aggregate shocks then drop out in (2.24), and the temporary equilibrium allocation is independent of the coefficient of risk aversion and given by the one obtained by imposing logarithmic felicity from old-age consumption ($\gamma = 1$). Solving (2.24) for a_t and plugging the solution back into the budget constraints (2.21) and (2.22) yields the unique temporary equilibrium allocation:

$$
\begin{aligned}
c_t^y(\theta,0) &= \bar{c}^y(\theta,0)\epsilon_t k_t^\alpha, && (2.26)\\
c_{t+1}^o(\theta,0) &= \bar{c}^o(\theta,0)\epsilon_{t+1} n_{t+1} k_{t+1}^\alpha, && (2.27)\\
a_t(\theta,0) &= \bar{a}(\theta,0)\times \epsilon_t k_t^\alpha, && (2.28)
\end{aligned}
$$

where

$$
\begin{aligned}
\bar{c}^y(\theta,0) &= \frac{[\alpha+(1-\alpha)\theta]\,(1-\alpha)(1-\theta)}{\alpha(1+\beta)+(1-\alpha)\theta},\\
\bar{c}^o(\theta,0) &= (\alpha+(1-\alpha)\theta)\,,\\
\bar{a}(\theta,0) &= \frac{\alpha\beta(1-\alpha)(1-\theta)}{\alpha(1+\beta)+(1-\alpha)\theta}.
\end{aligned}
$$

Note the similarity with Section 2.2.3. Since $\tau(\theta,0,n_{t-1}) = \theta$, the individual privately saves a constant fraction of output per worker. While the general environment is stochastic, optimal saving is then consistent with all subjective estimates concerning R_{t+1}^e and b_{t+1}^e. Therefore, the equilibrium allocation (2.26)-(2.28) is consistent with the notion of the so-called self-fulfilling expectations equilibrium, in which the agents' expectations concerning the future interest rate and pension benefit are confirmed *ex post* in all future states, i.e. $R_{t+1}^e = R_{t+1}$ and $b_{t+1}^e = b_{t+1}$ (see Wang, 1993 and Hauenschild, 2002). It should be noted that the self-fulfilling expectations equilibrium is the stochastic analogue to Galor and Ryder's (1989) concept of perfect foresight.

The inspection of $\bar{a}(\theta,0)$ gives the classical result, first obtained by Feldstein (1974), that a pay-as-you-go pension scheme crowds out private savings and hence the future level of capital when compared to the corresponding *laissez-faire* economy. Intuitively, both reduced net wage income when young and non-capital income at old age tend to reduce individual savings, so that for any pair (θ,θ') satisfying $-\alpha/(1-\alpha) < \theta < \theta' < 1$, $a_t(\theta,0) > a_t(\theta',0)$. Therefore, given realizations of the series $\{n_t\}_{t=0}^\infty$ and $\{\epsilon_t\}_{t=0}^\infty$, $k_t(\theta,\cdot) > k_t(\theta',\cdot)\,\forall t$. A continuity argument and the simulations in Section 2.5 suggest that the crowding out result also holds for small deviations from $x = 0$. Corollary 1 below gives a stochastic dominance result for the case $x = 0$.

Let $V_t(\theta, 0)$ denote the associated indirect utility function. Using (2.25) - (2.28), and ignoring terms that are independent of the pension parameters,

$$\begin{aligned} V_t(\theta,0) &= \ln \bar{c}^y(\theta,0) + \beta \ln \bar{c}^o(\theta,0) + \alpha\beta \ln \bar{a}(\theta,0) \\ &\quad + \alpha(1+\alpha\beta)\ln k_t. \end{aligned} \tag{2.29}$$

We next study the evolution of $\ln k_t$.

2.3.2 Long-run equilibrium with full demographic indexation

This section establishes the existence of the long-run equilibrium under full demographic indexation, by showing that the logarithm of the capital stock per worker has stable law. Recalling (2.25) and (2.28), capital accumulation follows the first-order autoregressive process

$$\ln k_{t+1} = \alpha \ln k_t + \ln z(\theta, 0, \epsilon_t, n_{t+1}), \tag{2.30}$$

where $\ln z(\theta, 0, \epsilon_t, n_{t+1}) = \ln \bar{a}(\theta, 0) + \ln \epsilon_t - \ln n_{t+1}$ is a normally distributed random variable with mean $\mu_z = \ln \bar{a}(\theta, 0) + \mu_\epsilon + \mu_n$ and variance $\sigma_z^2 = \sigma_\epsilon^2 + \sigma_n^2$, respectively.

Let $k_\infty(\theta, 0)$ denote the long-run equilibrium capital stock in the presence of the full demographic indexation scheme of size θ. The stochastic structures of the exogenous shocks are fully inherited by $k_\infty(\theta, 0)$; for the exogenous shocks have stable law.[13]

Definition 2.3.1. Stable law

A random variable X has a strictly stable law if

$$a_1 X_1 + a_2 X_2 =^d a_3 X,$$

where X_1 and X_2 are independent copies of X, and the constants $a_1, a_2, a_3 > 0$ satisfy $a_1^\xi + a_2^\xi = a_3^\xi$, $0 < \xi \leq 2$ being the index of the stable law.

Remark. The normal distribution has stable law with index 2.

Proposition 1. *Long-run capital distribution under full demographic indexation*

Suppose the economy runs with a scheme of size $\theta \in (-\alpha/(1-\alpha), 1)$. If $x = 0$, then[14]

$$\ln k_\infty(\theta, 0) =^d \left(\frac{1}{1-\alpha^2}\right)^{1/2} \times \ln z(\theta, 0, \epsilon, n), \tag{2.31}$$

[13] Stable distributions are frequently used in financial economics (see, e.g., Derman et al., 2010 and the references cited therein). A general background on stable distributions is provided, among others, in Whitt (2002). Definition 2.3.1 is taken from Nolan (2016).

[14] $=^d$ means equality in distribution.

where the first and second moments are given by

$$\mu_k = \mu_z/(1-\alpha),$$
$$\sigma_k^2 = \sigma_z^2/(1-\alpha^2).$$

Proof. See Appendix 2.A.3. The condition $\theta \in (-\alpha/(1-\alpha), 1)$ ensures strictly positive consumption levels. □

Since the individual optimally saves a *constant* fraction of output per worker under full indexation, perfect correlation between R^P and R continues to hold from the previous section. Therefore, the crowding out effect stemming from Social Security with full demographic indexation appears as a simple shift of the long-run capital distribution under *laissez-faire*, in which case $\theta = 0$. This notion is formalized by the next corollary to Proposition 1 ($k_\infty(0)$ denotes the long-run equilibrium capital stock under *laissez-faire*).

Corollary 1. *Suppose there is full demographic indexation ($x = 0$). Then*

1. *If $\theta > 0$, then $\ln k_\infty(\theta, 0)$ is first-order stochastically dominated by $\ln k_\infty(0)$.*
2. *If $\theta < 0$, then $\ln k_\infty(0)$ is first-order stochastically dominated by $\ln k_\infty(\theta, 0)$.*

Note that we consider a very general tax-transfer scheme. If $\theta > 0$, then the contributions made by the young generation are transferred in a lump-sum manner as in a pay-as-you-go pension system. As noted in Section 2.2.3, $\theta < 0$ implies that the state transfers a part of the retirees' capital income to the young. In fact, we cannot rule out, *a priori*, that a negative size of the system is optimal over the long run.

2.4 The optimal size of Social Security

While, at first glance, the reduction in the long-run level of the capital-labour ratio due to positive transfers from the young to the old generation seems harmful, it is well known that economies populated by an infinite sequence of overlapping generations with finite lifespans may overaccumulate capital relative to Phelps's (1961) golden rule of capital accumulation. The associated long-run capital stock maximizes aggregate consumption. Noting that $\bar{c}^y(\theta, 0) + \bar{c}^o(\theta, 0) + \bar{a}(\theta, 0) = 1$, the logarithm of aggregate consumption (in per worker terms) can be written as $\ln C_t = \ln(1 - \bar{a}(\theta, 0)) + \ln \epsilon_t + \alpha \ln k_t$.[15] There are two effects stemming from the crowding out of private savings induced by an increase in the size of the scheme. First, at any point in time, the fraction of output dedicated to consumption is increased. Second, output is reduced in the long run. Total differentiation

[15] Since $\ln k_\infty$ is normally distributed, $\ln C_\infty(\theta, 0)$ is also normally distributed with $\mu_C = \alpha\mu_k(\theta, 0) + \mu_\epsilon + \ln(1 - \bar{a}(\theta, 0))$, and $\sigma_C^2 = \alpha\sigma_k^2 + \sigma_\epsilon^2 > \sigma_k^2$ as $\sigma_\epsilon^2 > \sigma_n^2/\alpha$.

with respect to $\bar{a}$ yields the familiar result of deterministic models that the propensity to save that satisfies the golden rule, $\bar{a}_{GR}$, equals the capital share in output, i.e. $\bar{a}_{GR} = \alpha$.

Recalling (2.28), the golden rule can be implemented in the decentralized economy by setting $\theta_{GR} = \beta/(1+\beta) - \alpha/(1-\alpha)$, which is the solution to $\bar{a}(\theta; 0) = \alpha$. This section shows first that θ_{GR} is the long-run optimal size of the scheme in terms of aggregate consumption *and* welfare. Therefore, if the long run is of any interest, then the size of the scheme should be chosen so as to bring the economy onto its golden rule growth path. Secondly, the long run is indeed of interest; for the long-run optimal size implements the allocation chosen by a benevolent planner who cares about undiscounted, average generational welfare.

2.4.1 The long-run optimal size

The long-run optimal size of the scheme is defined as the time- and state-invariant replacement rate under full demographic indexation which maximizes *ex ante* expected indirect utility in the long run. Formally,

Definition 2.4.1. Long-run optimal size

The long-run optimal size of the system is defined as

$$\theta^* = \arg\max \left[V(\theta, 0) \,|\theta \in (-\alpha/(1-\alpha), 1) \right], \tag{2.32}$$

where $V(\theta, 0) = E_0 \lim_{t\to\infty} V_t(\theta, 0)$ and $V_t(\theta, 0)$ is defined in (2.29).

By Proposition 1, the stationary of the exogenous shocks implies that $\ln k_t$ has a time-invariant distribution, given $\theta \in (\alpha/(1-\alpha), 1)$. In particular, $E[\ln k_t] = E[\ln k_{t+1}] = \mu_k$. Using the definition of μ_k, $V(\theta, 0)$ can be written as the function $V : (-\alpha/(1-\alpha), 1) \to \mathbb{R}$

$$V(\theta, 0) = \bar{v} + \ln \bar{c}^y(\theta, 0) + \beta \ln \bar{c}^o(\theta, 0) + \frac{\alpha(1+\beta)}{1-\alpha} \ln \bar{a}(\theta, 0), \tag{2.33}$$

where $\bar{v}$ is a constant which is independent of the pension policy parameters. As noted above, the full demographic indexation scheme does not provide insurance of fertility risk, $\bar{c}^y(\theta, 0), \bar{c}^o(\theta, 0)$ and $\bar{a}(\theta, 0)$ are independent of the coefficient of relative risk aversion, γ, and so is the long-run optimal size of the system. The next proposition establishes the existence and uniqueness of θ^*.

Proposition 2. ***Existence and uniqueness of the long-run optimal size***

The θ^* defined in (2.32) is the unique value

$$\theta^* = \tau^* = \frac{\beta}{1+\beta} - \frac{\alpha}{1-\alpha}, \tag{2.34}$$

and coincides with the golden rule size of the scheme.

Proof. See Appendix 2.A.4. □

Proposition 2 states that the long-run optimal size of the scheme is independent of both productivity and fertility shocks. It extends the result of Hillebrand (2011), who finds that the long-run optimal size of a standard defined contribution scheme is independent of i.i.d. productivity shocks. In particular, θ^* coincides with the long-run optimal size in the deterministic counterpart of the model. Of course, this result is due to the parametrization of the model. The economic intuition is that, due to perfect correlation, both shocks hit capital income and pension benefits alike, so that contributions to the scheme fully substitute for private savings in the form of physical capital. That is to say, full demographic indexation renders the pension claim an asset with the same risk properties as private savings. Since the full demographic indexation scheme does not involve intergenerational risk sharing, the role of the system is restricted to bringing about the golden rule growth path. Moreover, capital accumulation is unaffected by the coefficient of relative risk aversion, implying that θ^* is the long-run optimal size for any $\gamma > 0$.

In contrast to Section 2.2, pure impatience ($\beta < 1$) implies $\beta/(1+\beta) < 1/2$, so that the optimal size may well be negative even if we impose $\alpha < 1/2$. The long-run optimal size is positive if the resources used to generate the capital stock under laissez faire exceed the contribution of capital to output. In the general model with endogenous savings, the former amounts to $\beta(1-\alpha)/(1+\beta)\epsilon_t k_t^\alpha$. Note that this inequality is the sufficient condition for dynamic inefficiency in Abel et al. (1989) (also see Abel, 2003). In passing we note that $\theta^* < 0$ characterizes dynamic efficiency, while $\theta^* > 0$ characterizes dynamic inefficiency of the underlying laissez-faire economy. In the latter case, a reduction in the capital stock, and hence the pay-as-you-go pension scheme with strictly positive transfers from the young to the old generations, is long-run welfare-improving. To get some feeling, set $\beta = 0.8$ and $\alpha = 0.3$. A discussion of these values is found in Section 2.5. These parameters imply a slightly positive long-run optimal replacement rate of $\theta^* = 0.0159$. However, the long-run gains occur exclusively because the crowding out of private savings 'heals' dynamic inefficiency.

The fact that the sizes of real world pension schemes are not long-run optimally chosen (for reasonable parameter values, even $\theta^* < 0$ – see the more detailed discussion in Section 2.5) indicates that other aspects than long-run maximal aggregate consumption are important. In this respect, it should be clear that θ^* can be normatively rationalized only if society does *not effectively* discount future generations' welfare. We show next that zero effective social discounting implies that society does not care about generational size.

2.4.2 The socially optimal size

We rationalize the very long-run perspective of the chapter and the concept of long-run optimality of the previous section by showing that the long-run optimal size defined in

(2.32) implements the socially optimal allocation associated with a normative benchmark. It is the pension policy chosen by a hypothetical social planner who seeks to maximize a class of social welfare functions defined by the sum of an infinite stream of undiscounted generational lifetime-utilities.

Following Young (2001), we consider social welfare functions that differ in the extent to which they account for the (relative) generational size. Let the social planner seek to maximize

$$W_0 = \delta^{-1} N_{-1}^{\eta} \beta \ln c_0^o + E_0 \left[\sum_{t=0}^{\infty} \delta^t N_t^{\eta} U_t \right], \tag{2.35}$$

subject to feasibility,

$$F(\epsilon_t; K_t, N_t) = c_t^y N_t + c_t^o N_{t-1} + K_{t+1}, \tag{2.36}$$

where N_{-1}^{η} is the 'inherited' generation at $t = 0$ and U_t is defined in (2.20). In order to derive analytical solutions, the remainder of this section assumes the logarithmic case, i.e. $\gamma = 1$. The parameter $\delta \in [0, 1]$ discounts time (and needs not to be equal to β), and the parameter $\eta \in [0, 1]$ determines the extent at which the planner seeks to provide utility to large generations.

The effective social discount factor δn_t^{η} involves both time discounting *and* weights on generational size. Therefore, a generation that appears late on the scene may well receive a higher weight in (2.35) than its predecessors as long as it is sufficiently large. We impose $\bar{\delta}(\eta) = \delta E[n^{\eta}] \leq 1$. $\eta = 0$ means that the number of individuals attaining U_t does not matter, and only the representative individual in each generation counts. From this utilitarian perspective, one might argue that the effective discount factor should be equal to $1/En = 1$ (implying $\delta = 1$), because it allows convergence to the golden rule stationary equilibrium discussed above. Note that a zero social discount rate is in the spirit of Ramsey (1928), who argues that discounting future generations' welfare is questionable on moral grounds; for symmetry over all generations at $t = 0$ demands $\delta = 1$. For the current parametrization of the model, the solution to the planning problem is well-defined even when the planner places equal weights on all current and future generations, independent of size. It will turn out that the equal weighting case rationalizes the long-run optimal size of the pay-as-you-go pension system of the previous section. Note that the equal weighting case can be endogenously rationalized on the basis of normative considerations. In contrast, $\eta > 0$ implies non-zero effective social discounting even for $\delta \to 1$, and it is not clear, a priori, what discount rate is appropriate. In fact, convergence requires to exogenously set $\delta < 1/En^{\eta}$, a choice which determines the long-run stationary equilibrium at the outset.

Since the planner respects individual preferences, it is readily verified that the *inter*temporal first-order condition satisfies the individual consumption Euler equation,

$$1/c_t^y = \beta E_t \left(R_{t+1}/c_{t+1}^o \right)$$

Additionally, the *intra*generational first-order condition of problem (2.35)-(2.36) determines the relative consumption of contemporaries,

$$c_t^o = n_t^{1-\eta} \times (\beta/\delta) \times c_t^y.$$

Young (2001) solves the problem in the absence of productivity shocks by calculating the percentage deviation from the age-specific consumption functions chosen by the planner when the realization of n_t is such that $n_t^\eta = En^\eta$. In this case the problem is equivalent to an infinite-horizon planning problem with a constant discount rate. The latter can be solved by standard dynamic programming techniques (see, e.g., Stokey and Lucas, 1989). The following Lemma provides the 'complete' solution, i.e. the socially optimal allocation, when both productivity and fertility shocks are present. Since the productivity shocks are i.i.d., the solution is analogous to Young's (2001).

Lemma 1. ***(Young, 2001) Socially optimal allocation***
The socially optimal allocation associated with (δ, η) *is given by*

$$\begin{aligned} c_t^{y0}(\delta, \eta; n_t) &= \widetilde{c}^y(\delta, \eta; n_t) \times \bar{c}_t^y(\delta), && (2.37)\\ c_t^{o0}(\delta, \eta; n_t) &= n_t^{1-\eta} \times \widetilde{c}^o(\delta, \eta; n_{t-1}) \times \bar{c}_t^o(\delta) && (2.38)\\ k_{t+1}^0(\delta, \eta; n_t) &= \widetilde{k}(\delta, \eta; n_t) \times \bar{k}_{t+1}(\delta), && (2.39)\end{aligned}$$

where

$$\begin{aligned} \bar{c}_t^y(\delta) &= \frac{\delta\,(1-\alpha\delta)}{\delta+\beta}\epsilon_t k_t^\alpha,\\ \bar{c}_t^o(\delta) &= \frac{\beta\,(1-\alpha\delta)}{\delta+\beta}\epsilon_t k_t^\alpha,\\ \bar{k}_{t+1}(\delta) &= \alpha\delta\epsilon_t k_t^\alpha \end{aligned}$$

is the socially optimal allocation associated with constant fertility at replacement level, and

$$\widetilde{c}^y(\cdot; n_t) = \widetilde{c}^o(\cdot; n_t) = \widetilde{k}(\cdot; n_t) = \frac{\delta+\beta}{\delta\,(1+\alpha\beta) + \beta\,(1-\alpha\delta)\,n_t^{-\eta}}.$$

Here, $\widetilde{c}(\cdot)$ is the percentage deviation of the actual consumption allocation in period t from the allocation associated with deterministic replacement fertility that is due to the fertility shock in that period. Note that the log-log structure implies that a fertility shock is distributed equi-proportionally between current youth consumption and capital accumulation.

Proposition 3 derives the socially optimal contribution rate, $\tau_t^0(\delta, \eta, n_t)$. Due to the combination of Cobb-Douglas technology, logarithmic utility and full depreciation, $\tau_t^0(\delta, \eta, n_t)$ is invariant to the capital stock. Therefore, $\tau_t^0(\delta, \eta, n_t)$ can be expressed as the product of its deterministic counterpart and a correction term that reflects fertility fluctuations around replacement level. The latter arises only if $\eta > 0$

Proposition 3. ***Socially optimal scheme***

The socially optimal allocation can be implemented in the decentralized economy by the state-dependent contribution rate

$$\tau_t^0(\delta, \eta, n_t) = \bar{\tau}(\delta) \times \widetilde{\tau}(\delta, \eta, n_t), \tag{2.40}$$

where

$$\bar{\tau}(\delta) = \frac{\beta(1-\alpha\delta)}{(\delta+\beta)(1-\alpha)} - \frac{\alpha}{1-\alpha} \tag{2.41}$$

$$\widetilde{\tau}(\delta, \eta, n_t) = \frac{\widetilde{c}(n_t)\beta(1-\alpha\delta) - \alpha(\delta+\beta)}{\beta(1-\alpha\delta) - \alpha(\delta+\beta)}, \tag{2.42}$$

and $\widetilde{c}(n_t)$ *is defined in Lemma 1.*

Hillebrand, 2011 shows that $\bar{\tau}(\delta)$ is the solution to the planning problem with deterministic and constant fertility at replacement level, with $\lim_{\delta\to 0} \bar{\tau}(\delta) = 1$, and $\lim_{\delta\to 1} \bar{\tau}(\delta) = \frac{\beta}{(1+\beta)} - \frac{\alpha}{1-\alpha}$.

Obviously, $\eta = 0$ implies $\widetilde{c} = \widetilde{\tau} = 1$. Therefore,

Corollary 2. *If* $\eta = 0$*, then* $\lim_{\delta\to 1} \theta^0(\delta) = \theta^*$*.*

Corollary 2 states that the socially optimal size of the scheme chosen by a benevolent planner who does *not* care about generational size can be implemented by the *full* demographic indexation scheme. Intuitively, if the planner does not discount future generations' welfare, then the problem (2.35) - (2.36) is equivalent to maximizing welfare of steady state generations; for an ever increasing number of generations (receiving the same, non-vanishing weight in the social welfare function) is sufficiently 'close' to the stationary equilibrium. Note that the social planner's allocation with $\eta > 0$ cannot be decentralized by the simple demographic indexation system defined in (2.4), because with Social Security payments are linked to wage income rather than lifetime income.

2.5 Long-run welfare effects of partial demographic indexation

This section provides rough calculations on the long-run welfare consequences of introducing defined benefit elements at the margin. To that end, we generate long time-series for productivity and fertility, and derive the time-series of the endogenous variables for two scenarios: In the first, the economy runs with the full demographic indexation scheme; in the second, the economy runs with the pension scheme with a marginal deviation from full indexation. While both economies are faced with the same series of shocks, the associated capital stocks per worker will differ; for the pension design affects individual savings behaviour.

The model parameters are as follows: The coefficient of relative risk aversion is $\gamma = 2$, which is a common value in the macroeconomic literature (see, e.g., Sánchez-Marcos and Sánchez-Martín, 2006). As a simple sensitivity exercise, we also report results for the case $\gamma = 0.5$. We turn to subjective discounting. Auerbach and Kotlikoff (1987), Fullerton and Rogers's (1993) and Sánchez-Marcos and Sánchez-Martín (2006), for example, calibrate β to reproduce either the U.S. capital-output ratio or the gross rate of return on aggregate capital. These works suggest that the annual discount rate lies in 0.1%–0.2%. Since each period spans 40 years, these values imply that β lies in the range $0.65 - 0.92$. Therefore, we continue to assume $\beta = 0.8$.[16]

The work of Abel et al. (1989) suggests that real economies are dynamically efficient. However, this does not mean that these economies were dynamically efficient prior to the implementation of Social Security pensions. In fact, setting the capital's share to the standard value found in the literature, $\alpha = 0.3$, renders the laissez-faire economy dynamically inefficient in the sense that the laissez-faire economy overaccumulates capital relative to the golden rule. The corresponding long-run optimal replacement rate (under full demographic indexation) derived in Proposition 2 is slightly positive, $\theta^* = 0.0159$. The actual size of the system, $\theta = 0.143$, is chosen so as to match social security expenditures of around 10% of gross domestic product (see Table 1 in Chapter 1).[17] As to the degree of demographic indexation, $x = 0.1$ is considered to be marginal. We maintain Assumptions 1 and 2, and impose $\sigma_n^2 = 0.18$ which fits well the empirical values for the growth rate of the population under 25 years in industrialized countries. For example, Sánchez-Marcos and Sánchez-Martín (2006) estimate a variance of 0.164 for the U.S. population.[18] Finally, total factor productivity is simply a scale factor, which does not affect results. For ease of exposition, the associated parameters are chosen so as to generate values of k around 10.

Table 2.1 sets out the average numbers of the time-series. The respective coefficients of variation serve as measures of volatility and are reported in brackets. We start with the results for delayed fertility reported in the upper panel of the table. It is argued above that the deviation from $x = 0$ can increase welfare if γ is sufficiently large. It turns out that $\gamma = 2$ high enough to generate long-run welfare gains. Since the actual size of the scheme is $\theta = 0.143 > 0.016 = \theta^*$, the economy under full demographic indexation

[16]Since the determination of β follows an indirect approach, the appropriate value of β remains debatable. For example, Cooley and Prescott (1995) impose a quarterly psychological discount factor of 0.99, which translates into $\beta = 0.99^{160} = 0.2$, see Chapter 3.

[17]Contribution and replacement rates are only weak indicators of the size of a real world pension system; for some countries finance large parts of the pension benefits by general revenues, and debt financing is common.

[18]In general, the requirement $(1-\theta n^{-1}) > 0$, which ensures positive consumption levels at any state of nature, calls for a truncation of the fertility distribution, which would alter the moments of the underlying distribution and would make necessary adjustments in the mean and variance parameters. While the mean preserving character of the stochastic process $\{k_t\}$ would be unaffected by the truncation, the low value of σ^2 is enough to rule out the violation of the above inequality even for the long time series produced here.

Table 2.1: Allocative and welfare effects from marginal demographic indexation

fertility	γ	x	θ^*	k	w	R	R^P	LTI	c^y	c^o	V
delayed	2	0	0.0159	10.270	19.988	1.203	0.912	19.274	10.708	9.918	4.171
				(0.825)	(0.235)	(0.548)	(0.575)	(0.235)	(0.235)	(0.522)	
		0.1		9.932	19.804	1.225	0.896	18.970	10.582	9.805	4.175
				(0.817)	(0.233)	(0.543)	(0.533)	(0.216)	(0.216)	(0.492)	
	0.5	0		10.270	19.988	1.203	0.912	19.274	10.708	9.918	4.314
				(0.825)	(0.235)	(0.548)	(0.575)	(0.235)	(0.235)	(0.522)	
		0.1		10.093	19.901	1.211	0.964	19.051	10.563	9.853	4.303
				(0.817)	(0.233)	(0.543)	(0.533)	(0.216)	(0.216)	(0.492)	
immediate	2	0	0.0159	10.270	19.988	1.203	0.902	19.274	10.708	9.918	0.011
				(0.825)	(0.235)	(0.548)	(0.548)	(0.235)	(0.235)	(0.522)	
		0.1		9.910	19.844	1.209	0.883	19.004	10.558	9.857	-0.029
				(0.790)	(0.228)	(0.530)	(0.495)	(0.212)	(0.212)	(0.497)	
	0.5	0		10.270	19.988	1.203	0.902	19.274	10.708	9.918	4.065
				(0.825)	(0.235)	(0.548)	(0.548)	(0.235)	(0.235)	(0.522)	
		0.1		9.910	19.844	1.209	0.883	19.005	10.558	9.857	4.026
				(0.790)	(0.228)	(0.530)	(0.495)	(0.212)	(0.212)	(0.497)	

Note: Values in brackets are the coefficients of variation of the respective variable. R^P and LTI are the systems' (gross) implicit rate of return and the present value of lifetime income, respectively. Neither the average allocation nor volatility is affected by the timing of fertility, as $x = 0$.

($x = 0$) is dynamically efficient. The welfare gains of marginal indexation then arise because the (positive) insurance effects dominate the (negative) crowding out effect.

The magnitudes of these effects are as follows: the policy reform crowds out the average capital stock per worker by 3.29 per cent, the coefficient of variation decreases relatively modest by 1.00 per cent. Moreover, even high volatility in the capital stock per worker translates into relatively modest volatility in factor prices. The reform pulls the average factor prices in opposite directions, the wage rate decrease and the return of capital increases. Since benefits are wage related, the system's implicit rate of return (denoted by R^P) is also decreased on average. The volatility of R^P decreases significantly by 7.3 per cent. Moreover, since R and R^P are not perfectly correlated, the pension claim allows a diversification of the individual portfolio. The diversification effect is best seen by the volatility of the present value of lifetime income (LTI), which is reduced substantially by 8.1 per cent.

By construction, volatility is unaffected by γ. Yet if $\gamma = .5$, then the individual values the risk reduction much less so that the insurance effect cannot outweigh the now less pronounced crowding out effect.

The lower panel of Table 2.1 reports the allocative and welfare effects for the case in which fertility is realized prior to the individuals' savings decision. To understand the similarities between the upper and lower panel of the Table, the following corollary to Proposition 2 is useful. It states that the timing of fertility does not affect savings behaviour if the economy runs with the full indexation scheme.

Corollary 3. *Suppose $x = 0$. Then, for all $\gamma > 0$, the timing of fertility has no effect on individuals' optimal savings and the long-run optimal size of the scheme.*

Therefore, neither the average allocation nor volatility is affected by the timing of fertility if $x = 0$. Yet the welfare consequences of the policy reform are sensitive to the timing of fertility: Even if $\gamma = 2$, the reform yields long-run welfare losses for two reasons. First, there is no longer a fertility risk to share across contemporary generations. Second, the crowding out effect of capital which stems from the introduction of $x > 0$ is more pronounced on average when fertility is not delayed. To illustrate, we focus on the equilibrium allocation with incomplete demographic indexation as $\gamma = 1$, in which case (2.20) reduces to the standard time- and state-additive logarithmic expected utility, $\ln c_t^y + \beta E_t \ln c_{t+1}^o$. Using (2.2), (2.3) and (2.25), equation (2.24) reduces to

$$\frac{1}{w_t(1-\theta n_t^{-x}) - a_t} = \beta \int_n \int_\epsilon \frac{R\left(\frac{a_t}{n_{t+1}}; \epsilon_{t+1}\right)}{a_t R\left(\frac{a_t}{n_{t+1}}; \epsilon_{t+1}\right) + \theta w\left(\frac{a_t}{n_{t+1}}; \epsilon_{t+1}\right) n_{t+1}^{1-x}} dG(\epsilon_{t+1}) dH(n_{t+1}), \tag{2.43}$$

where G and H are the time-invariant cumulative distribution functions of the productivity shock and fertility shock, respectively. The solution to (2.43) yields the following consumption-savings allocation in temporary equilibrium:

$$c_t^y(\theta, x; n_t) = \bar{c}^y(\theta, x; n_t) \times \epsilon_t k_t^\alpha \tag{2.44}$$
$$c_{t+1}^o(\theta, x; n_{t+1}) = \bar{c}^o(\theta, x; n_{t+1}) \times \epsilon_{t+1} n_{t+1} k_{t+1}^\alpha \tag{2.45}$$
$$a_{t,}(\theta, x; n_t) = \bar{a}(\theta, x; , n_t) \times \epsilon_t k_t^\alpha \tag{2.46}$$

where

$$\bar{c}^y(\theta, x; n_t) = \frac{(1-\alpha)(1-\tau(n_t; \theta, x))}{1 + \alpha\beta[E_t \bar{c}^o(\theta, x; n_{t+1})^{-1}]}$$
$$\bar{c}^o(\theta, x; n_{t+1}) = (\alpha + (1-\alpha)\tau(\theta, x; n_{t+1}))$$
$$\bar{a}(\theta, x; , n_t) = \frac{\alpha\beta(1-\alpha)(1-\tau(n_t; \theta, x))}{\alpha\beta + [E_t \bar{c}^o(\theta, x; n_{t+1})^{-1}]^{-1}}.$$

Since completed fertility is unaffected by assumption, the actual extent of delay is irrelevant for the determination of optimal saving.

Note that the capital processes with delayed and immediate fertility are defined on the same state space. By virtue of equations (2.28) and (2.46), the effect of delaying fertility on capital accumulation can then be studied by means of the stochastic properties of $\bar{c}^o(\theta, x; n_{t+1})$. Suppose $x > 0$, and define $\widetilde{n} > 0$ as the solution to $(\bar{c}^o(\cdot; \widetilde{n}))^{-1} = E_t(\bar{c}^o(\cdot; n_{t+1}))^{-1}$. Assuming $\theta > 0$, $(\bar{c}^o(\cdot; n))^{-1}$ is concave in n, so $\widetilde{n} < En$.[19] Therefore, given net wage income, i.e. given $(k_t; n_t, \epsilon_t)$, delayed fertility increases savings in period t if there is a 'real' baby bust, i.e. $n_{t+1} < \widetilde{n}$. For realizations $n_{t+1} > \widetilde{n}$, the additional risk due to delayed fertility decreases optimal savings. The imposed parameter value $\alpha = 1/3$ in conjunction with the policy parameters (θ, x) implies that the threshold value $\widetilde{n} = 0.952$ is approximately equal to the median of the lognormal fertility variate, so that the occurrence of a 'real' baby bust is just as likely as 'booms', i.e., fertility realizations above the threshold value $\widetilde{n}$. Due to the concavity of $(\bar{c}^o(\cdot; n))^{-1}$, the propensity to save is higher than with non-delayed fertility. Given the same history of shocks, the next period's capital stock per worker exceeds that with immediate fertility, and so does the wage rate. Finally, since optimal saving is increasing in net wage income and fertility shocks are i.i.d., delaying fertility also increases savings in all subsequent periods and the capital stock per worker is higher *on average*. Similarly, $\theta < 0$ implies convexity of $(\bar{c}^o(\cdot; n))^{-1}$. Table 2.1 confirms the argument for selected values of $\gamma \neq 1$.

The volatility of the equilibrium allocation under full demographic indexation coin-

[19]The second derivative with respect to n is

$$\frac{\partial^2 (\bar{c}^o)^{-1}}{\partial n^2} = \frac{(1-\alpha)\theta x n^{-x-2}}{(\alpha + \theta n^{-x}(1-\alpha))^2} \times \left(\frac{2(1-\alpha)\theta x n^{-x}}{(\alpha + \theta n^{-x}(1-\alpha))} - (1+x) \right),$$

where the term in squared brackets is negative, as $\theta > -\frac{\alpha(x+1)n^x}{(\alpha-1)(x-1)}$, a condition which is satisfied by the assumption that $\theta > 0$.

cides with the one under laissez-faire economy. However, the crowding out of long-run capital due to the prevalence of *any* pay-as-you-go pension scheme is substantial. The average capital stock in the laissez-faire allocation is about 1.6 times higher, at 16.319, than with the reformed scheme. These numbers confirm the results of Sánchez-Marcos and Sánchez-Martín (2006) who find that the crowding out effect of the introduction of a defined benefit scheme far outweighs the induced insurance gains.

2.6 Conclusions

This chapter has studied pay-as-you-go pension systems, wherein the pensions of current retirees are financed by the contributions of current workers. Contributions to such a system do not contribute to capital accumulation, thereby crowding out the long-run capital stock per worker. Against this drawback, a suitably designed scheme provides a way of sharing economy-wide risks across generations, including shocks to the size of generations. For quite some time, pension benefit formulas in many countries have been driven by the actual contributions made to the scheme. In the extreme case, the contribution rate is state- and time-invariant. In the setup of the chapter, such a rule can be implemented by full demographic indexation of pension benefits, which renders the pension claim an asset whose risk properties are the same as private savings in physical capital: an increase in the ratio of retirees to workers cuts pension pension benefits and the system does not provide insurance of lifetime income.

The chapter derives the long-run optimal size of a pay-as-you-go pension scheme with full demographic indexation for an economy that features production, stochastic productivity and stochastic fertility. It shows that the full demographic indexation of pension benefits implements the socially optimal allocation when future generations' welfare is not effectively discounted. The chapter further argues that introducing a small measure of a defined benefit element seem to be appropriate if one is willing to take an intergenerational risk sharing perspective. The simulations suggest long-run welfare gains of this policy if individuals are sufficiently risk averse and the scheme's actual size exceeds the long-run optimal value. In that case, the economy is initially dynamically efficient, implying that the insurance gains will outweigh the negative crowding out of capital. In this connection, the timing of fertility matters. If individuals have their children only after making their consumption-savings decision, then incomplete demographic indexation insures against the risks to lifetime income, and the crowding out of private savings is less pronounced in comparison with non-delayed fertility.

One might argue that the imposed Cobb-Douglas production function is too strong an assumption where distribution is concerned. However, since a pension benefit formula with defined benefit elements breaks the perfect correlation between capital income and benefit payments even with Cobb-Douglas technology, the specification of the presented model provides a useful benchmark for an analysis of the scope of insuring risks to lifetime income. A generalization to the class of constant elasticity of substitution production functions, for example, makes it impossible to sign the involved income and insurance effects without calibrating an economic model more carefully to a particular economy. The latter may be an empirical question for future research.

2.A Appendix to Chapter 2

2.A.1 Derivation of equations (2.9) and (2.16)

Dividing (2.8) by R_{t+1} and taking the logarithm on both sides yields

$$r^T_{t+1} - r_{t+1} = \ln\left[1 + \tau(n_t)\left(\exp\left(r^P_{t+1} - r_{t+1}\right) - 1\right)\right]. \tag{2.A.1}$$

The second-order approximation around $r^P_{t+1} - r_{t+1} = 0$ yields

$$r^T_{t+1} - r_{t+1} = \tau(n_t)\left(r^P_{t+1} - r_{t+1}\right) + (1/2)\tau(n_t)(1-\tau(n_t))\left(r^P_{t+1} - r_{t+1}\right)^2. \tag{2.A.2}$$

Since $(r^P_{t+1} - r_{t+1})^2$ is of the form $(d - x\ln n_{t+1})^2$, with d a constant, the conditional expectation of excess return over the "benchmark asset" physical capital is $E_t(r^P_{t+1} - r_{t+1})^2 = x^2\sigma^2_n$. Rearranging yields equation (2.9) in the main text.

The problem (2.15) yields the following first-order condition with respect to x:

$$\begin{aligned}
\frac{dV_t}{dx} \;=\;& \tau'_{t,x}\mu_{r^P} + \tau_t\mu'_{r^P,x} + \mu'_{r,x} - \left(\tau'_{t,x}\mu_r + \tau_t\mu'_{r,x}\right) \\
&+(1/2)\left[\left(\tau'_{t,x}(1-\tau_t) - \tau_t\tau'_{t,x}\right)x^2 + 2\tau_t(1-\tau_t)x\right]\sigma^2_n \\
&+(1/2)(1-\gamma)\left[2\tau_t\tau'_{t,x}\sigma^2_{r^P} + \tau_t^2\frac{\partial\sigma^2_{r^P}}{\partial x} - 2(1-\tau_t)\tau'_{t,x}\sigma^2_r + (1-\tau_t)^2\frac{\partial\sigma^2_r}{\partial x}\right] \\
&+(1-\gamma)\left(\left(\tau'_{t,x}(1-\tau_t) - \tau_t\tau'_{t,x}\right)\sigma_{r^P r} + \tau_t(1-\tau_t)\frac{\partial\sigma_{r^P r}}{\partial x}\right).
\end{aligned}$$

Using $\partial\sigma^2_{r^P}/\partial x = -2(1-\alpha-x)\sigma^2_n$, $\partial\sigma_{r^P r}/\partial x = -(1-\alpha)\sigma^2_n$, and evaluating at $n_t = 1$ so that $\partial\sigma^2_r/\partial x = 0$, yields (2.16).

2.A.2 Derivation of equation (2.18)

Taking the logarithm on both sides of $k_{t+1} = \left[w_t(1-\theta)\right]/n_{t+1}$, yields

$$\ln k_{t+1} = \ln(1-\alpha) + \ln(1-\theta) + \alpha\ln k_t + \ln\epsilon_t - \ln n_{t+1}.$$

By iteration,

$$\begin{aligned}
\ln k_{t+1} \;=\;& \sum_{i=0}^{t}\alpha^i\ln(1-\alpha) + \sum_{i=0}^{t}\alpha^i\ln(1-\theta) \\
&+\sum_{i=0}^{t}\alpha^i\ln\epsilon_{t-i} - \sum_{i=0}^{t}\alpha^i\ln n_{t+1-i} + \alpha^{t+1}\ln k_0.
\end{aligned}$$

Suppressing all terms that are independent of θ, x and rearranging yields equation (2.18).

2.A.3 Proof of Proposition 1

Since $-\infty \leq \ln\alpha < 0$ and $E\ln n_t < \infty$, there is convergence in distribution, the distribution of $\ln k_\infty$ being the unique solution to the stochastic fixed-point equation $\ln k_\infty =^d \alpha\ln k_\infty + (\ln\bar{a}(\theta,1) + \ln\epsilon - \ln n)$ (Vervaat, 1979). With α constant, it suffices to show, for some constant ξ, that $\xi\ln n =^d \alpha(\xi\ln n) + \ln n_t$. Since $\ln n$ is normally distributed by assumption, it has a stable law with index 2. By virtue of Definition 2.3.1, the solution to $\xi^2 = (\alpha\xi)^2 + 1^2$ confirms the claim. The associated moments follow directly from the discussion in Section 5 of Vervaat (1979).

2.A.4 Proof of Proposition 2

Let $\nu_0 = \frac{1+\alpha\beta}{1-\alpha}$, $\nu_1 = (1+\beta)$, $\nu_2 = -(1+\beta) = -\nu_1$, $\nu_3 = \beta - \frac{\alpha(1+\beta)}{1-\alpha}$, $\mu_1 = \frac{\alpha}{1-\alpha} < \mu_2 = \mu_3 = \frac{\alpha(1+\beta)}{1-\alpha}$.

With full demographic indexation, equation (2.33) can be rewritten in the form

$$V(\theta) = \bar{v} + \nu_0\ln[1-\theta] + \nu_1\ln[\mu_1+\theta] + \nu_2\ln[\mu_2+\theta] + \nu_3\ln[\mu_3+\theta] \tag{2.A.3}$$

where $\theta \in (\underline{\theta}(0)\overline{\theta}(0)) = (-\frac{\alpha}{1-\alpha}, 1)$, and $\bar{v}$ is a constant which is independent of the pension policy parameters. Recall that $\nu_3 <\ (>) 0$ means dynamic (in)efficiency of the hypothetical *laissez-faire* economy. The first and second derivative with respect to θ are

$$\begin{aligned} V'_\theta &= -\nu_0\frac{1}{1-\theta} + \nu_1\frac{1}{\mu_1+\theta} + \nu_2\frac{1}{\mu_2+\theta} + \nu_3\frac{1}{\mu_3+\theta}, \\ V''_{\theta\theta} &= -\nu_0\frac{1}{(1-\theta)^2} - \nu_1\frac{1}{(\mu_1+\theta)^2} - \nu_2\frac{1}{(\mu_2+\theta)^2} - \nu_3\frac{1}{(\mu_3+\theta)^2}. \end{aligned}$$

The limits $\lim_{\theta\to\underline{\theta}(1)} V'_\theta = +\infty$ and $\lim_{\theta\to\overline{\theta}(1)} V'_\theta = -\infty$ imply, by continuity, that there exists at least one critical point. If $V''_{\theta\theta}(\theta^*) < 0$, then any critical point is a maximum. Suppose $\nu_3 \geq 0$. Then,

$$V''_{\theta\theta} < -\left[\nu_1\frac{1}{(\mu_1+\theta)^2} + \nu_2\frac{1}{(\mu_2+\theta)^2}\right] < -(\nu_1+\nu_2)\frac{1}{(\mu_2+\theta)^2} = 0,$$

implying concavity of V for all $\theta \in (\underline{\theta}(1)\overline{\theta}(1))$. Suppose, at the contrary, $\nu_3 < 0$ and let θ_0 denote a solution to $V'_\theta = 0$. Using $V'(\theta_0) = 0$,

$$\begin{aligned} V''_{\theta\theta}(\theta_0) &< -\frac{\nu_0}{(1-\theta_0)^2} - \frac{\nu_1}{(\mu_1+\theta_0)^2} - \frac{\nu_2}{(\mu_2+\theta_0)^2} \\ &\quad -\frac{1}{\mu_3+\theta_0}\left[\frac{\nu_0}{1-\theta_0} - \frac{\nu_1}{\mu_1+\theta_0} - \frac{\nu_2}{\mu_2+\theta_0}\right] \\ &< -\frac{\nu_1}{\mu_1+\theta_0}\left[\frac{1}{\mu_1+\theta_0} - \frac{1}{\mu_3+\theta_0}\right] < 0. \end{aligned}$$

Finally, equation (2.34) is confirmed by directly checking $V_\theta'(\frac{\beta}{1+\beta} - \frac{\alpha}{1-\alpha}) = 0$.

Chapter 3

Fully Funded Social Security Pensions, Lifetime Risk and Income

3.1 Introduction

Social Security pensions are used all around the world. Already in the late 1980's, the number of countries that ran some kind of old–age Social Security programme was 130, albeit with differing sizes and coverages (see Sala-I-Martin, 1996). A common argument against Social Security pensions is that they reduce the national capital stock. Since capital is involved in the aggregate production process and its reduction will lead to a reduced availability of resources in the future, Social Security pensions are seen as welfare-decreasing in the long run, especially if financed on a pay-as-you-go[1] basis (Auerbach and Kotlikoff, 1987). Recent pension policies intend to bring in more funded elements into Social Security pensions. This chapter investigates the welfare consequences of insuring mortality risk by means of fully funded Social Security. For example, the German system, the oldest formal Social Security pension system, started as a fully funded disability system in 1889, and was converted to a pay-as-you-go system only in the aftermath of the Great Depression and World War II. Chile is a prominent current example of a pension system with substantial reliance on funding. Countries like Japan and the Netherlands also have had fully funded pension systems for decades.

In aggregate, fully funded Social Security invests the contributions of a generation at the going interest rate and pays the proceeds plus the assets of those who die prematurely to the survivors of the *same* generation when old. Several empirical studies document a substantial reduction in bequest flows and capital (see, e.g., Auerbach et al.,

[1] If Social Security is financed on a pay-as-you-go basis, then the contributions of current workers finance pension benefits of current retirees. In contrast to the funded scheme studied here, contributions substitute for private savings without adding to the national capital stock.

1995). Two aspects of insurance that arise from premature death are of particular interest when individuals desire to make transfers to their heirs. First, Social Security provides intergenerational transfer insurance by smoothing transfers across states. Second, while death is certain, its timing is not, and those who die prematurely typically do so with some amount of wealth if they have made private savings to provide for their old age. Since the assets of the deceased are naturally transferred within their respective families, intragenerational inequality in wealth arises, so mortality among parents generates the *ex ante* risk of their children inheriting a particular amount of wealth. Due to the scheme's transfer insurance property, future generations face a more concentrated (or even degenerate) wealth distribution. Risk-averse individuals value the reduction in *ex ante* risk.

In this chapter, capital reduction and insurance effects are combined in a simple dynamic overlapping generations model, wherein individuals live for at most two periods, facing a positive probability of dying just before reaching retirement in the second period. Intergenerational transfers occur *within* families, due to both premature death and individuals' desire to make positive inter vivos transfers. While fully funded Social Security leads to a reduction in the long-run capital stock, we find that, once insurance is taken into account, the reduction of capital and long-run welfare gains are not mutually exclusive, whith the reduction in capital being related to premature mortality. The point of departure is selfish behaviour, in the sense that bequest insurance provides no marginal utility, but the scheme promises a higher effective rate of return than the market.

The chapter relates to the literature on the long-run consequences from annuitization by means of private markets (Kingston and Piggott, 1999 and Fehr and Habermann, 2008), and by means of Social Security (Hubbard and Judd, 1987, and most recently Caliendo et al., 2014). The main feature of this strand of the literature is the restriction to the life-cycle savings motive to finance old-age consumption, coupled with a lump-sum redistribution of the assets of the deceased to all members of the next generation. The long-run welfare effect of Social Security pensions may be summarized in terms of two income effects. The first stems from a reduction of bequests, the second from the pension payment at old age. With a zero interest rate, both effects exactly cancel out in the long run so that the introduction of a fully funded Social Security pension system leaves welfare unaffected in the long run. Moreover, if the previous pension policy is revisable, then the pension system provides a higher effective rate of return than the market interest rate: The induced substitution effect is welfare improving in the long run. This chapter contributes to this literature by lifting the restriction to a lump-sum redistribution and studying the pension system's role in reducing the *ex ante* risks to lifetime income. We build on early works on within-family transfers in the absence of a bequest motive of Abel (1985) and Eckstein et al. (1985). To the author's best knowledge, only the appendix of Caliendo et al. (2014) provides a more recent treatment of within-family transfers in the pension context.[2] Our setup allows a generalization of Abel's (1985) model to the case where individuals have an operative bequest motive. In the light of the very small private annuities markets (see, e.g., Pashchenko, 2013) a thorough evaluation of Social Security should account for the system's capability of insuring risks associated with premature mortality. However, previous studies with an operative bequest motive either assume that intragenerational heterogeneity does not arise (see, e.g., Sheshinski and Weiss, 1981; and Lockwood, 2012), or that private markets in annuities are available (see, e.g., Abel, 1986). We contribute to this literature by characterizing the long-run distribution of lifetime income in the presence of uninsured mortality risk and establishing allocative effects of the scheme relative to the world without it. We also quantify the *ex ante* welfare gain associated with the introduction of a fully funded Social Security pension system.

The chapter is structured as follows: Section 3.2 presents the model setup. Section 3.3 establishes the welfare-dominance of Social Security over laissez-faire when individuals are perfectly selfish. The dynamics and long-run equilibria of the altruistic economy with and without Social Security are derived in Section 3.4. Rough calculations in Section 3.5 suggest that Social Security is also welfare-improving in the long run in an economy populated by altruistic individuals as long as capital is not too productive and the desire to make transfers to the heir is not too strong. A decomposition of the overall welfare effect into capital reduction and insurance gains is also provided. The main conclusions are summarized in Section 3.6.

3.2 The model economy

The demographic and economic environment is similar to the one used in Abel (1985), albeit the inclusion of an operative bequest motive requires a slightly more general description of the mortality process. Moreover, we have to distinguish transfers between the living (inter vivos) from transfers that occur after death from the deceased to the living (bequest). We simply use the term transfer when both kinds of transfers are possible. Note that either the one or the other transfer occurs.

3.2.1 The demographic environment

The economy in each period is populated by two overlapping generations whose members live for at most two periods, namely, one young generation of workers and one old generation of retirees. There is a positive probability, q, of dying at the very end of the first period. At time $t = 0$, let there be a continuum of individuals indexed by $j \in [0, 1]$, all of whose parents survived to old age. All lineages stem from individuals

[2]Bell and Gersbach (2013) study the impact of uninsured mortality risk on human capital formation. In their model, transfers are purely inter vivos, but individuals have two parents, which is an additional source of inequality.

born in $t = 0$. Each young individual gives birth (by parthenogenesis) to just one child prior to the realisation of the states premature death and survival. From then onwards, premature mortality sorts individuals into those whose parents survived both periods of life and those whose parents died prematurely. Let the state a member of lineage j is born into at any t be denoted by $I_t^j \in \mathcal{I} := \{0, 1\}$, where 0 and 1 indicate survival and premature death of the individual's parent, respectively. The lineage's particular mortality history up to and including period t is denoted by $\varsigma_t^j \in \mathcal{I}_0 \times \mathcal{I}_1 \times \cdots \times \mathcal{I}_t \equiv \mathcal{I}^t$. By assumption there is no aggregate uncertainty with regard to the number of premature deaths per generation.

3.2.2 The economic environment

There is a single commodity which can be either consumed or saved. Aggregate output, Y, is produced at the very beginning of each period by means of a production technology that is linear in both capital and labour. In per worker terms,

$$Y_t = F(1, K_t) = w + R \times K_t, \tag{3.2.1}$$

where K_t represents the aggregate capital stock at t. Capital has a lifetime of one period, so that K_t equals national savings in $t-1$. Note that (3.2.1) exhibits the property that capital and labour are perfect substitutes, with constant returns to scale. While the latter is standard in the literature, the former is very strong. Note however, that key features of the model, like the optimal size of the scheme, are unaffected by this assumption. Since factors are paid their respective marginal products, saving one unit of the good today yields R units in the next period. We say that capital is unproductive if $R = 1$. Due to linearity of the production function, factor prices are given and deterministic.

There is no private mechanism to pool individual mortality risk. In particular, private annuities markets are assumed to be absent[3]. The proceeds from savings of those who die prematurely are transferred to their respective direct descendants.

3.2.3 Preferences

All individuals have identical tastes. They derive utility from consumption in youth and old age, denoted by c_t^j and c_{t+1}^j, respectively. They also draw utility from inter vivos transfers, $b_{t+1}^{j,0}$, and bequests in case of premature death, $b_{t+1}^{j,1}$, if these motives are operative. The superscripts 0 and 1 refer to the states survival I_t^0 and premature death I_0^1, respectively. Intergenerational transfers stem from own previous savings, a_t^j, and are transferred with interest.

[3]This assumption is also found in Sheshinski and Weiss (1981), Hubbard and Judd (1987) and Caliendo et al. (2014).

Formally, j's preferences are represented by

$$U_t^j = u(c_t^j) + \beta(1-q)\left[u(c_{t+1}^{j,0}) + \nu(b_{t+1}^{j,0})\right] + \beta q \nu\left(b_{t+1}^{j,1}\right) \tag{3.2.2}$$

Pure impatience is captured by $0 < \beta \leq 1$.[4] Effective discounting is a mixture of impatience and mortality. The function ν represents the felicity yielded by giving. The felicity functions $u(\cdot)$ and $\nu(\cdot)$ are assumed to be strictly concave. Felicity from consumption in case of premature death is normalized to zero. Pathological behaviour is ruled out by the assumption $u(x) > \nu(x), \forall x$. The latter condition simply states that an individual who saved x/R units to old age prefers life to death even if the transfer motive is not strong enough to generate positive inter vivos transfers. The transfer motive is operative if $\nu(\cdot) > 0$.

Note that the preferences differ conceptually from 'pure' altruism à la Barro (1974) and Weil (1987) where individuals care about the utility of their offspring, in fact, the well-being of all future generations to come[5]. Here, transfers are motivated by the joy-of-giving, so that the individual draws utility directly from the transfers they make, pleasurable or when premature death afflicts them, but the pleasure does not depend on his children's utility gain (nor on the utility gain of any future generations of the same lineage that appear on the scene after their direct descendants). This kind of bequest motive is used by Abel (1986) and, more recently, Lockwood (2012). While this may look like selfish behaviour at first glance, the preferences defined above may be interpreted as the preferences of an individual who is concerned with the expected initial net wealth position of his offspring, where $\nu(\cdot)$ is the associated utility index.

Also note that the preferences defined in (3.2.2) implicitly impose risk neutrality with respect to lifetime risk[6].

3.2.4 Fully funded Social Security pensions

Under fully funded Social Security Pensions, contributions are proportional to wages and are invested at the going market interest rate and paid as pension benefits to the

[4] Time preference is commonly measured as the marginal rate of substitution between young and old-age consumption along a constant consumption path. With the time-additive preferences used here, β is just built in and invariant to consumption levels.

[5] This imposes a very strong assumption on the individual's capacity to form rational expectations about the decisions of successive generations, and on the compatibility of their expectations with those of their heirs.

[6] In fact, the preferences defined in (3.2.2) are a special case of

$$U_t^j = (1-q)\phi\left[u(c_{t+1}^j) + \nu(b_{t+1}^j)\right] + q\phi\left[\nu(w(1-\tau_t^j) + b_t^j)\right]$$

where the curvature of ϕ capture individual's lifetime risk aversion (see Kihlstrom and Mirman, 1974). Bommier (2013) shows that only an exponential form of ϕ is compatible with time-consistent preferences when individuals live for more than two periods. Since ϕ is assumed linear in this chapter, (3.2.2) defines preferences of an individual who is risk neutral with respect to mortality risk.

surviving members of the same generation one period later. The scheme pools the risks of premature mortality: the pension claims of those who die prematurely are divided among the survivors of the *same* generation. In the happy event that individual j, born in the beginning of period t, survives into old age, she will receive $a_t^j R$ from her privately placed savings a_t^j, and

$$\tau_t^j wR + \tau_t^j \frac{q}{1-q} wR$$

from the pension scheme, where τ_t^j denotes period-t contribution rate for individual j. Note that the second term captures the effect of pooling on old-age income. The latter is

$$\left[a_t^j + \tau_t^j w/(1-q)\right] R. \tag{3.2.3}$$

If she dies prematurely, then her heir will receive only $a_t^j R$ as bequest. Note that private savings and Social Security pensions are *not* perfect substitutes: While saving one unit conventionally yields R units in the next period, whether the individual survives or not, the scheme promises a higher rate of return on contributions, $R/(1-q)$, when the individual survives, and pays nothing when she dies prematurely. The pension claim is a state-dependent, i.e. risky, asset that can be used to equalize bequest streams across states. Also note that the scheme is actuarially fair, so that the expected return (including the death case) of the individual's portfolio is unaffected by the scheme. Since the scheme is fully funded, the contributions to the scheme add to the next period's capital stock, so $K_{t+1} = R \times \int_0^1 \left(a_t^j + \tau_t^j w\right) dj$.

We analyse two cases. First, individuals treat the pension policy as given. Second, each generation can freely decide on the contribution rate. In the latter case, we consider a string of consecutive generations, each member $j \in [0,1]$ of which chooses the contribution rate so as to maximize her expected utility. This assumption is plausible in a two-period setting.

3.3 The egoistic economy

This section studies the case of perfectly selfish individuals. Set $\nu(\cdot) = 0$ so that the proceeds from saving generate no marginal utility when the individual dies prematurely, be they transferred to the next generation or to the survivors of the same generation.[7] Obviously, there will be no intentional bequests. We establish global convergence to the unique long-run equilibrium. Whether the capital stock with the scheme falls short of, or exceeds, the laissez-faire capital stock in the long run depends on the intertemporal elasticity of substitution. Two welfare results are derived: First, a fully funded Social Security scheme with time-invariant fixed contribution rate is welfare-increasing. Second, the time-consistent pension policy, to which all subsequent generations will agree, does

[7] Similar models are developed in Abel (1985) and Eckstein et al. (1985). We use the model for evaluating long-run welfare consequences of the scheme.

even better. We show in Sections 3.4.1 and 3.4.2 that essential properties carry over to the case of altruistic individuals.

3.3.1 Laissez-faire and a fixed contribution rate scheme

As a preliminary to the welfare analysis, we establish existence of a unique stationary equilibrium if the contribution rate is treated as given when individuals make their consumption-savings decisions, i.e. $\tau_t = \tau$. We assume throughout that the fixed contribution rate is small enough to ensure that even the poorest individuals make positive private saving. Laissez-faire serves as the benchmark to evaluate the welfare consequences of Social Security. We use the term 'Laissez-faire' to describe the world without the scheme, so legal and administrative basis of the latter is absent. The laissez-faire economy corresponds to $\tau = 0$.

When young, an egoistic individual j, born in period t, decides on private savings and consumption in youth. When old, if she survives, she simply consumes the proceeds from savings. The maximization problem of individual j in the presence of the fixed contribution rate scheme,

$$\max_{a_t^j} u(w(1-\tau) + b_t^j - a_t^j) + \beta(1-q)u\left(a_t^j R + \frac{\tau w R}{1-q}\right), \tag{3.3.1}$$

yields the Euler equation

$$u'(c_t^j) = \beta(1-q)Ru'(c_{t+1}^j). \tag{3.3.2}$$

Note the presence of mortality in (3.3.2), whose form is such that the scheme with a fixed contribution rate does not affect intertemporal trade-offs. Let u be iso-elastic, with parameter σ, $u(c) = u_0 + \frac{c^{1-\sigma}-1}{1-\sigma}$, $u_0 > 0$. Then, the associated savings function is $a_t^j = \bar{a}_e(w(1-\tau) + b_t^j) - (1-\bar{a}_e)\tau w/(1-q)$, where $\bar{a}_e = \left[1 + (\beta(1-q)R^{1-\sigma})^{-1/\sigma}\right]^{-1}$ is the egoistic individual's propensity to save from the present value of lifetime income. Note that $\tau \leq \tau^0 = \frac{\bar{a}_e(1-q)}{1-q\bar{a}_e}$ ensures non-negative private savings of those individuals who do not receive any bequest. Plugging the optimal decision back into the direct utility function, gives indirect utility of the egoistic individual j, $V_{t,e}^j(\tau)$[8].

Since accidental bequests stem from the parent's savings,

$$a_t^j = \bar{b}_t^j a_{t-1}^j + \bar{w}(\tau), \tag{3.3.3}$$

[8] We use $V_{t,e}^j(\tau)$ as a shortcut for $V_{t,e}^j\left(c_t^j(\tau), c_{t+1}^j(\tau)\right)$, where $c_t^j(\tau)$ and $c_{t+1}^j(\tau)$ denote the optimal decision in the presence of the system with a fixed contribution rate.

where

$$\begin{aligned}\bar{w}(\tau) &= \bar{a}_e \times w(1-\tau) - (1-\bar{a}_e)\tau w/(1-q),\\ \bar{b}_t^j &= 0\chi(I_{t-1}^j = 0) + R\bar{a}_e\chi(I_{t-1}^j = 1),\\ E\bar{b}_t &= R\bar{a}_e q,\end{aligned}$$

and χ is an indicator function so that $\chi(z) = 1$ if z is true, and $\chi = 0$ if z is false. $I_{t-1}^j = 0$ indicates that the individual's parent survived into old age, and makes no intergenerational transfer. $I_{t-1}^j = 1$ indicates that the individual's parent died early and left an accidental bequest to the heir.

Lemma 2 establishes convergence to the unique stationary equilibrium. Part *(ii)* of the Lemma states that the fully funded pension scheme with fixed contribution rate crowds out the long-run aggregate capital stock by reducing the accidental bequests. With an eye on Propositions 4 and 5, note that in the current setup, aggregates coincide with *ex ante* expectations.

Lemma 2. ***Convergence to the unique stationary equilibrium distribution*** *Let $R\bar{a}_e q < 1$ and $\tau \leq \tau^0$. Then*

(i) a_t^j converges to a_∞^j as $t \to \infty$, where the distribution of a_∞^j is the unique solution to[9]

$$a_\infty^j =^d \bar{b}_t^j \times a_\infty^j + \bar{w}(\tau),$$

which is independent of a_0^j.

(ii) the long-run aggregate capital stock

$$Ea_t^j = K_e(\tau) = \bar{w}(\tau)/(1 - R\bar{a}_e q) \qquad (3.3.4)$$

is strictly decreasing in the contribution rate τ.

Proof. (i) By iterating (3.3.3), one obtains

$$a_\infty^j =^d \bar{w}(\tau) + \bar{w}(\tau)\sum_{i=1}^{\infty} \bar{b}_1^j \bar{b}_2^j \cdots \bar{b}_{i-1}^j. \qquad (3.3.5)$$

By Jensen's inequality, $E\bar{b}_t = R\bar{a}_e q < 1$ implies $E\log(\bar{b}_t^j)(\leq \log E\bar{b}_t^j) < 0$, which is a sufficient condition for convergence of the infinite sum (see Vervaat, 1979). (ii) $\bar{w}$ is strictly decreasing in τ. □

In an economy populated by overlapping generations the reduction in the aggregate capital stock is a misleading indicator for the welfare consequences of the scheme. We next show that, once *ex ante* insurance is taken into account, the crowding out of capital

[9] $=^d$ means equality in distribution.

is consistent with welfare improvements. First note that Lemma 2 implies convergences in distribution of all other relevant variables. In particular, lifetime income and therefore consumption in youth, $c_t^j(\tau) = (1-\bar{a}_e)\left(w + b_t^j + \frac{\tau w}{1-q}\right)$ converges in distribution. Since the fraction q of total population dies early and bequests are transferred with interest, i.e. $Eb_t^j = qRK_e(\tau)$, aggregate first-period consumption in the stationary equilibrium is given by

$$Ec_t^j(\tau) \quad = \quad C_e(\tau) = (1-\bar{a}_e)\left(\frac{w}{1-R\bar{a}_e q} + \frac{\tau w q(1-R)}{(1-q)(1-R\bar{a}_e q)}\right), \qquad (3.3.6)$$

where the term in brackets is the *ex ante* expected value of the present value of lifetime income. The latter is unaffected by the scheme whenever capital is unproductive, and so is $Ec_t^j(\tau)$.

In the egoistic economy, different mortality histories across lineages result in different levels of lifetime income. As a consequence, individuals can be unambiguously identified by their lineages' mortality histories. Without ambiguity, therefore, we drop the index j and reinterpret, as Abel (1985) does, the time index t as the number of previous consecutive generations within a particular lineage whose members died prematurely. By the consumption Euler equation (3.3.2), $c_{t+1}^j(\tau)$ is proportional to $c_t^j(\tau)$. It is therefore enough to focus on first-period consumption. In order to prepare for Proposition 4, the following lemma restates a result provided by Abel (1985) in a slightly different form and also recognizes that we work with a geometric distribution.

Lemma 3. ***(Abel, 1985) Geometric long-run distribution***

In the long-run, individuals are geometrically distributed with probability mass function $q^t(1-q)$. First-period consumption of a type-t individual is

$$c_t(\tau) \quad = \quad w(1-\tau) - \bar{w}(\tau)\left(\bar{a}_e R\right)^t + \bar{w}(\tau)(R-1)\sum_{i=0}^{t}\left(\bar{a}_e R\right)^i. \qquad (3.3.7)$$

If $R\bar{a}_e q < 1$, then $\lim_{t\to\infty} c_t(\tau) = \infty$ is compatible with the stationary equilibrium defined in Lemma 2. In that case, there must be both winners and losers in the long run (most likely, earlier). Figure 3.A.1 suggests that this true even when the long-run distributions have bounded supports. Suppose, for example, that capital is unproductive (this case is illustrated in Figure 3.A.1(a)). Solving $c_t(\tau) \geq c_t(0)$ for t shows that welfare of all but those whose parents died prematurely and so left no bequests (the poorest without the scheme) is reduced relative to the laissez-faire, $\tau = 0$; for they experience a negative income effect due to a reduction in bequests that dominates the positive income effect stemming from the scheme. Figure 3.A.1(b) illustrates this result when $R >> 1$. Since type-t individuals occur with frequency $q^t(1-q)$, individuals whose parents died prematurely make up the fraction $(1-q)$ of the total young population. Anticipating the discussion in Section 3.5, let the economically active years start at the age of 20 and let life expectancy at birth be 80 years (i.e. $q = 1/2$). In that case, the scheme makes one half of the young population better off. However, *ex post* inequality translates into *ex ante* uncertainty, and the scheme produces a more concentrated distribution of lifetime income in the stationary equilibrium. Risk-averse individuals value the reduction in *ex ante* risk.

The next proposition avoids distributional aspects by evaluating welfare on the basis of Rawls's (1973) veil of ignorance. It exploits the fact that, under (3.4.7), individuals consume a constant fraction of lifetime income for all $\sigma > 0$ and all $0 < \beta \leq 1$, thereby generalizing Caliendo et al.'s (2014) Proposition 5. Suppose *all* individuals are present behind the veil and are asked whether they prefer to be born into the laissez-faire economy or the economy that runs the fully funded pension scheme. Due to the direct link between an individual's family mortality history and wealth at birth in the egoistic economy, ignorance refers to the *ex ante* risk of being born into a particular lineage.

Proposition 4. *Ex ante welfare-improving Social Security pensions with a fixed contribution rate*

Suppose that capital is unproductive, with $q > 0$, and $\tau \leq \tau^0 = \frac{\bar{a}_e(1-q)}{1-q\bar{a}_e}$. Then, fully funded Social Security Pensions increase ex ante welfare.

Proof. If $R = 1$, then (3.3.6) and (3.3.7) imply $c_t(\tau) = w(1 - \bar{a}_e^{t+1}) + w\tau\left(\frac{1-\bar{a}_e q}{1-q}\bar{a}_e^t - 1\right)$, with $Ec_t(\tau) = C_e(\tau) = \frac{(1-\bar{a}_e)w}{1-R\bar{a}_e q}$. Therefore, $C_e(\tau) > c_0(\tau) > c_0(0)$, and $C_e(\tau) < c_t(\tau) < c_t(0)$, $t > 0$, implying that the scheme induces a mean-preserving reduction in the spread of consumption. The latter, in turn, implies second-order stochastic dominance, i.e. $\int_t V_t(\tau) > \int_t V_t(0)$, which confirms the claim. □

Proposition 4 is important, because it illustrates that the welfare impact of fully funded Social Security pensions is systematically underestimated in the literature. In fact, there are several contributions in the literature on accidental bequests in egoistic economies that find no welfare impact stemming from the scheme if capital is unproductive (see e.g. Hubbard and Judd, 1987, and more recently Caliendo et al., 2014, among others). The underlying assumption in the literature is that accidental bequests are pooled and then transferred anonymously. In the long-run, the income effect from reduced accidental bequests and the income effect stemming from reduced net wage income and additional pension benefit exactly cancel out. Therefore, long-run welfare is unaffected by the scheme when capital is unproductive.

3.3.2 Time-consistent pension policy

This section establishes that the welfare gains from Social Security Pensions are further underestimated relative to those found in 3.3.1; for individuals treated the contribution rate as given when making their decisions. On the other extreme, contemporary generations are able to decide for themselves on the individual contributions made to the pension system. In fact, in the two-period setup presented here, each generation is able to revise previous pension policy without producing intergenerational conflicts. This implies that the pension system essentially acts like privately organised, actuarially fair annuities markets, with a competitive premium taken by the firms which fully and exclusively reflects the prevailing mortality regime.[10]

The maximization problem of individual j is

$$\max_{c_t^j, c_{t+1}^j, \tau_t^j} u(c_t^j) + \beta(1-q)u(c_{t+1}^j) \tag{3.3.8}$$

subject to

$$\begin{aligned} c_t^j &= w(1-\tau_t^j) + b_t^j - a_t^j & (3.3.9)\\ c_{t+1}^j &= a_t^j R + \tau_t^j wR/(1-q) & (3.3.10)\\ \tau_t^j &\in [0,1]. & (3.3.11) \end{aligned}$$

The first-order condition with respect to τ_t^j is

$$u'(c_t^j) = \beta R u'(c_{t+1}^j), \tag{3.3.12}$$

where mortality has dropped out. Since individuals do not value their wealth after death, the scheme promises a higher expected rate of return than the market does, and egoistic individuals do not save privately. Of course, optimality of full annuitization is a well-known result if individuals save due to pure selfishness (see Yaari, 1965). In the egoistic economy, full annuitization eliminates all bequests. Moreover, under the assumption that all are alike at $t = 0$, and receive zero bequests, there is intragenerational agreement on the optimal policy for all $t = 0, 1, 2, \ldots, \infty$. From (3.3.12) we have

Lemma 4. ***Unique time-consistent pension policy***
There exists a unique time-consistent optimal contribution rate:

$$\tau_e^* = \left[1 + (\beta R^{1-\sigma})^{-1/\sigma}/(1-q)\right]^{-1}, \ \forall j, t. \tag{3.3.13}$$

The pension policy (3.3.13) is termed time-consistent: All generations are able, but not willing, to revise previous policies. The time consistent policy is therefore sustainable in the long run. The associated indirect utility is denoted by $V_t(\tau_e^*)$.

For the remainder of this section, let $\beta^{-1} = R = 1$. In that case, Abel (1985) shows that the long-run capital stock of the laissez-faire economy ($\tau = 0$) falls short of the one associated with the time-consistent policy in (3.3.13), i.e. $\tau_e^* w$ if and only if $\sigma < \widetilde{\sigma} \equiv \left[1 - \frac{\ln(1+q(1-q))}{\ln(1-q)}\right]^{-1} < 1$. The reason is that each generation can revise previous policy, so

[10] One can think of a large number of insurance companies which receive the market interest rate on their reserves and earn zero profits under perfect competition.

the increase in effective rate of return affects intertemporal trade-offs (compare (3.3.12) and (3.3.2)). The associated income effect induces the young individual to increase current consumption and to reduce savings. In contrast, the associated substitution effect induces the individual to save more to old age. An intertemporal elasticity of substitution larger than one ($\sigma < 1$) and perfectly selfish individuals imply that the substitution effect dominates the income effect, so the propensity to save is higher with the scheme. Whether this effect compensates for the elimination of accidental bequests depends on whether σ is small enough. The following corollary states that if the fixed contribution rate of Section 3.3.1 is small enough, then the associated capital stock exceeds the one associated with the time-consistent policy defined in (3.3.13).

Corollary 4. *Let $\beta^{-1} = R = 1$, and $\sigma > \widetilde{\sigma}$. Then,*

$$K_e(\tau) > K_e(\tau_e^*) \text{ as } \tau < \widetilde{\tau},$$

where $\widetilde{\tau} = \tau^0 - \frac{1-q}{1+\frac{1}{1-q}} > 0$.

As already shown in Section 3.3.1, the evolution of aggregates is an imperfect indicator of welfare. The next proposition states that the time-consistent policy is *ex ante* welfare-improving and dominates the system with fixed contribution rate for all $\sigma > 0$ and all $\tau < \tau^0$.

Proposition 5. *Dominance of time-consistent Policy*
If $R = 1$, then the time-consistent pension policy defined in Lemma 4 dominates, in welfare terms, the fully funded scheme with fixed contribution rate $\tau < \tau^0$.

Proof. The proof is accomplished by constructing an auxiliary consumption allocation induced by a policy that redistributes the assets of those who die prematurely to the next generation in a lump-sum manner. With this policy, all members of a generation receive identical accidental bequests, and the Euler equation is of the form of (3.3.2). The associated first-period consumption is $c_t^{LS} = (1 - \bar{a}_e)\left(w + b^{LS}\right)$, where $b^{LS} = \frac{qR}{1+(\beta(1-q)R^{1-\sigma})^{-1/\sigma}-qR}w$. First, $R = 1$ implies $Ec_t^{LS} = Ec_t(\tau)$ and therefore $V^{LS} > V(\tau)$; for the lump-sum policy induces a mean-preserving elimination of any spread in consumption. Second, if $R = 1$, then (c_t^{LS}, c_{t+1}^{LS}) and $(c_t(\tau_e^*), c_{t+1}(\tau_e^*))$ are on the same budget line. Therefore, while feasible, the auxiliary allocation is not chosen. By revealed preference, $V(\tau_e^*) > V^{LS} > V(\tau)$. □

The conclusion from Propositions 4 and 5 is that the crowding out of bequests and capital induced by fully funded schemes is consistent with long-run welfare gains whenever capital is not too productive, a claim that follows at once from the strict inequalities above and the continuity of V. Moreover, if each generation is able to revise previous policy, then the scheme even increases long-run capital stock for plausible values of the intertemporal elasticity of substitution. The associated substitution effect is positive in welfare terms. From a generational perspective, the revised contribution rate setup seems plausible; for the scheme should be regarded as acting like annuities markets, although contributions are wage-related. As a final remark, note that neither of the policy regimes is Pareto-improving relative to the laissez-faire. This is obvious from the discussion following Lemma 3 for the case of a fixed contribution rate. Moreover, the time-consistent pension policy yields $c_t(\tau_e^*) = w/(2-q) < w = \lim_{t\to\infty} c_t(0)$.

3.4 The altruistic economy

This section studies the dynamics and long-run behaviour of the altruistic economy with and without Social Security. It uses the apparatus of Section 3.3 to establish convergence to the unique stationary equilibrium in the presence of uninsured mortality risk and an operative bequest motive. The clear-cut welfare results derived for the egoistic economy studied in the previous section are feasible because there are direct relationships, first, between an individual's family mortality history and her expected lifetime income, and second, between expected lifetime income and welfare. Both links break in the altruistic economy. The welfare consequences of the scheme are therefore studied numerically in Section 3.5.

3.4.1 Dynamics under laissez-faire

When young, individual j, born in period t, decides on private savings and consumption in youth. When old, if she survives, she allocates the proceeds from savings between old-age consumption and inter vivos transfers. These choices are determined by the individuals' attitudes towards time, risk and their heirs.

The individual's maximization problem is

$$\max_{a_t^j} U_t^j \tag{3.4.1}$$

subject to

$$\begin{aligned} c_t^j &= w + b_t^j - a_t^j, & (3.4.2)\\ \left[c_{t+1}^{j,0}, b_{t+1}^{j,0}\right] &\equiv \arg\max \left[u(c_{t+1}^j) + \nu(b_{t+1}^j)\right] \text{ s.t. } c_{t+1}^j + b_{t+1}^j = a_t^j R & (3.4.3)\\ b_{t+1}^{j,1} &= a_t^j R & (3.4.4)\\ a_t^j &\geq 0,\ b_{t+1}^{j,0} \geq 0,\ c_{t+1}^{j,0} \geq 0. & (3.4.5) \end{aligned}$$

given the bequest b_t^j.[11] Intergenerational transfers, pleasurable or not, are non-negative; for by assumption, individuals cannot be forced to accept negative bequests (liabilities

[11] By the assumption that bequests received are known prior to the decision, the individual is only concerned with the uncertainty associated with her own life.

left by their parents). For simplicity, both kinds are distributed at the very beginning of each period. Since all individuals born in $t = 0$ have parents who survived into old age, all receive the same bequests at that point in time so that there is no intragenerational heterogeneity at this point. Without loss of generality, let $b_0^j = 0 \, \forall j$. Unlike in the case of egoistic individuals, problem (3.4.1) - (3.4.5) needs to be solved by backwards induction. Assuming an interior solution to (3.4.3), $u'(c) = \nu'(b)$.

The first-order conditions with respect to a_t^j may be written as[12]

$$u'(c_t^j) \quad \geq \quad \beta(1-q)R\nu'(b_{t+1}^{j,0}) + \beta q \nu'(Ra_t^j)R. \tag{3.4.6}$$

Individuals save due to a pure life-cycle motive (i.e. for old-age consumption), and due to an altruistic motive (i.e. for making direct intergenerational transfers). Recalling (3.4.3), both motives are summarized in the first term on the right-hand side of (3.4.6). The last term arises due to the prevalence of uninsured mortality risk, which tends to increase private savings.

Let u and ν be iso-elastic, with common parameter σ:[13]

$$u(c) \quad = \quad u_0 + \frac{c^{1-\sigma} - 1}{1-\sigma}, \tag{3.4.7}$$

$$\nu(b) \quad = \quad \gamma \frac{b^{1-\sigma} - 1}{1-\sigma}. \tag{3.4.8}$$

The parameter $u_0 > 0$ determines the felicity gap between life and death, a condition which is implicit in the above requirement of non-pathological behaviour with respect to the bequest motive (see Section 3.2.3). Also note that $u_0 > 0$ does not affect the inter-temporal consumption allocation. To use terminology carefully, $\sigma > 0$ is, by definition, the elasticity of marginal felicity from consumption and intergenerational transfers, respectively. It is readily verified that $\lim_{\sigma \to 1} u(c) = \ln(c)$. The parameter $0 < \gamma \leq 1$ measures the strength of the bequest motive. For simplicity, we assume that both kinds of transfers receive the same weight γ, albeit one can imagine that inter vivos may be higher valued by the individual. Note that (3.4.8) implies that inter vivos transfers are normal goods.

Since (3.4.7) and (3.4.8) satisfy the lower Inada condition (that is, $\lim_{x \to 0} u'(x) = \infty$, and $\lim_{x \to 0} \nu'(x) = \infty$), inter vivos transfers are always positive if the parents survive

[12] It is readily verified that the second derivative with respect to a_t^j is negative, so the solution is indeed a maximum.

[13] Abel (1986) and Lockwood (2012) employ these functional forms with $u_0 = 0$. Lockwood (2012) extends (3.4.8) to cover inter vivos transfer as luxury goods.

into old age. The consumption-transfer allocation is

$$b_{t+1}^{j,0} = \frac{\gamma^{1/\sigma}}{1+\gamma^{1/\sigma}} R\bar{a}(w+b_t^j) \quad (3.4.9)$$

$$c_{t+1}^{0} = \frac{1}{1+\gamma^{1/\sigma}} R\bar{a}(w+b_t) \quad (3.4.10)$$

Moreover, individuals optimally save a constant fraction of the expected present value of their lifetime incomes, i.e. $a_t^j = \bar{a}(w+b_t^j)$, where

$$\bar{a} = \left\{1 + \left[\beta R^{1-\sigma}\left((1-q)\left(1+\gamma^{1/\sigma}\right)^{\sigma} + q\gamma\right)\right]^{-1/\sigma}\right\}^{-1} < 1 \quad (3.4.11)$$

denotes the propensity to save from lifetime income under laissez-faire, a propensity which is increasing in the strength of the bequest motive. Note that the desire to make transfers to the heir unambiguously increases the said propensity relative to the case of egoistic individuals, i.e. $\bar{a} > \bar{a}_e$.

In the absence of the scheme, the randomness of premature death causes individuals to differ in the bequests they receive at the start of the first period of their lives: b_t^j arises from the mortality history ς_t^j. Since premature death is identically and independently distributed, intragenerational heterogeneity with respect to b_t^j arises and is increasing as time passes. Recalling (3.4.4) and (3.4.9), savings evolve according to the stochastic first-order difference equation with an identically and independently distributed coefficient

$$a_t^j = \bar{b}_t^j a_{t-1}^j + \bar{w}, \quad (3.4.12)$$

with $\bar{w} = \bar{a} \times w$ and the random variable $\bar{b}_t^j = \frac{\gamma^{1/\sigma}}{1+\gamma^{1/\sigma}} R\bar{a}\chi(I_{t-1}^j = 0) + R\bar{a}\chi(I_{t-1}^j = 1)$. $I_{t-1}^j = 0$ indicates that the individual's parent survived into old age, and, in contrast to the previous section, makes an inter vivos transfer. $I_{t-1}^j = 1$ indicates that the individual's parent died early and left an accidental bequest to the heir. Note that $E\bar{b}_t = q \times \frac{\gamma^{1/\sigma}}{1+\gamma^{1/\sigma}} R\bar{a} + (1-q) \times R\bar{a} = R\bar{a}\frac{q+\gamma^{1/\sigma}}{1+\gamma^{1/\sigma}}$.

The stationary equilibrium is given by prices (R, w), an allocation $(c_t^j, c_{t+1}^j, b_{t+1}^j, a_t^j)$, and a time-invariant distribution lifetime income such that, given (R, w), the allocation maximizes expected utility defined in (3.2.2), and individual choices are consistent with the economy-wide resource constraint. Lemma 5 generalizes Lemma 2 by establishing convergence of the altruistic economy to the unique stationary equilibrium.

Lemma 5. ***Convergence in distribution***

(i) Let $R\bar{a}\frac{q+\gamma^{1/\sigma}}{1+\gamma^{1/\sigma}} < 1$. Then, a_t^j converges to a_∞^j as $t \to \infty$, where the distribution of

a^j_∞ is the unique solution to

$$a^j_\infty =^d \bar{b}^j_t \times a^j_\infty + \bar{w},$$

which is independent of a^j_0.

(ii) If $R\bar{a} < 1$, then the long-run capital distribution has bounded support, $a^j \in [\bar{w}/(1-\gamma^{1/\sigma}\bar{a}R/(1+\gamma^{1/\sigma})), \bar{w}/(1-\bar{a}R)]$.

Proof. *By iterating (3.4.12), one obtains*

$$a^j_\infty =^d w + w\sum_{i=1}^{\infty} \bar{b}^j_1\bar{b}^j_2\cdots\bar{b}^j_{i-1} \tag{3.4.13}$$

(i) $E\bar{b}_t = R\bar{a}\frac{q+\gamma^{1/\sigma}}{1+\gamma^{1/\sigma}} < 1$ implies (by Jensen's inequality) $E\log(\bar{b}^j_t)(\leq \log E\bar{b}^j_t) < 0$, which is a sufficient condition for convergence of the infinite sum, (see Vervaat, 1979). (ii) The bounds follow by setting $\bar{b}_t = R\bar{a}, t = 0,1,2,\ldots,\infty$ and $\bar{b}_t = \frac{\gamma^{1/\sigma}}{1+\gamma^{1/\sigma}}R\bar{a},\ \frac{\gamma^{1/\sigma}}{1+\gamma^{1/\sigma}} < 1,\ t = 0,1,2,\ldots,\infty$, respectively. □

The long-run distribution deserves some comment. First, if capital is unproductive (i.e. $R = 1$), then the support is bounded without further assumptions. From (3.4.11), however, the bounds are growing with R whenever $\sigma < 1$. If $R > 1$, then convergence and boundedness require the interest rate to be sufficiently small, albeit $R >> 1$. Intuitively, since bequests are transferred with interest, too large an interest rate makes bequests grow without bounds as time passes, which is at odds with the requirement of a stationary equilibrium. Second, the underlying stochastic difference equation (3.4.12) follows one particular lineage, lineage j, and the associated equilibrium distribution summarizes the possible states into which the members of lineage j can be born. Recall, however, that mortality is identically and independently distributed across both time and individuals, and that all individuals born in $t = 0$ are alike. Therefore, (3.4.13) must hold for all lineages, and the system exhibits asymptotic stationarity: with a continuum of individuals at any point in time and a long enough time horizon, the distribution of lifetime income of members of lineage j across time coincides with the distribution across all lineages at a given point in time.

Proposition 6 states the long-run levels of aggregate lifetime income, capital stock and consumption, respectively.

Proposition 6. ***Long-run Aggregates under Laissez-faire***

Let $R\bar{a}\frac{q+\gamma^{1/\sigma}}{1+\gamma^{1/\sigma}} < 1$. Then, aggregate lifetime income, capital stock and consumption *in the laissez-faire economy converge to the long-run equilibrium values*

$$\omega(0) = \lim_{t\to\infty}\int_0^1 (w+b_t^j)dj = \frac{w}{1-R\bar{a}\frac{q+\gamma^{1/\sigma}}{1+\gamma^{1/\sigma}}} < \infty, \tag{3.4.14}$$

$$K(0) = \lim_{t\to\infty} R\times\int_0^1 \bar{a}(w+b_t^j)dj = \bar{a}R\omega, \tag{3.4.15}$$

$$C(0) = \lim_{t\to\infty}\int_0^1 c_t^j dj + \lim_{t\to\infty}(1-q)\int_0^1 c_{t+1}^j dj = \left((1-\bar{a}) + \frac{R\bar{a}(1-q)}{1+\gamma^{1/\sigma}}\right)\omega \tag{3.4.16}$$

Proof. Follows directly from the discussion in Vervaat (1979). (3.4.16) uses the fact that only the fraction $(1-q)$ of young workers survives into old age. The consumption-transfer allocation is derived in Appendix 3.A.1. □

Figure 3.A.2 displays the long-run distribution of the expected value of lifetime income for alternative values of R. Note that $\bar{a} > \bar{a}_e$ implies that the aggregates exceed their counterparts in the egoistic economy.[14]

3.4.2 Dynamics with the optimal scheme

Following Sheshinski and Weiss (1981), we rely on the assumption that each generation can freely decide on the contribution rate. Recalling (3.2.3), the individual's maximization problem is

$$\max_{a_t^j,\tau_t^j} U_t^j$$

subject to

$$c_t^j = w(1-\tau_t^j) + b_t^j - a_t^j, \tag{3.4.17}$$

$$\left[c_{t+1}^{j,0}, b_{t+1}^{j,0}\right] \equiv \arg\max u(c_{t+1}^j) + \nu(b_{t+1}^j) \text{ s.t. } c_{t+1}^j + b_{t+1}^j = a_t^j R + \frac{\tau_t^j wR}{1-q} \tag{3.4.18}$$

$$b_{t+1}^{j,1} = a_t^j R \tag{3.4.19}$$

$$a_t^j \geq 0,\ b_{t+1}^{j,0} \geq 0,\ c_{t+1}^{j,0},\ \tau_t^j \in [0,1]. \tag{3.4.20}$$

given b_t^j.

The first-order conditions with respect to a_t^j and τ_t^j may be written as

$$u'(c_t^j) \geq \beta(1-q)R\nu'(b_{t+1}^{j,0}) + \beta q\nu'(Ra_t^j)R\,(=\text{ if } a_t^j > 0), \tag{3.4.21}$$

$$u'(c_t^j) = \beta R\nu'(b_{t+1}^{j,0}). \tag{3.4.22}$$

[14] We cannot obtain a similar result for the second moments. Applying the result in Section 5.2.2 of Vervaat (1979), the long-run capital distribution has variance $\frac{\bar{w}^2}{(1-E\bar{b}_t^2)(1-E\bar{b}_t)^2} \times \left(E\bar{b}_t^2 - (E\bar{b}_t)^2\right)$, which can exceed or falls short of its egoistic counterpart.

respectively. Combining (3.4.21) and (3.4.22) gives

$$\nu'(b_{t+1}^{j,0}) = \nu'(Ra_t^j), \tag{3.4.23}$$

so that no accidental bequests occur (i.e. the difference between inter vivos transfers and the bequests in case of premature death is zero). A similar result is derived in Abel (1986) and Lockwood (2012). Fuster (2000) derives this condition for the bequest motive suggested by Barro (1974).[15] By virtue of the assumptions on u and ν, (3.4.22) implies $\tau^* \in (0,1)$. Moreover, the optimal contribution rate that brings about the desired equality of bequests across states in (3.4.23) is unique; for the LHS of (3.4.23) is monotonically decreasing in τ, while the RHS is independent of the contribution rate. Finally, by virtue of the old-age budget constraint, individuals use pension benefits exclusively to finance old-age consumption, while the proceeds from conventional savings are used to finance intergenerational transfers.

Combining (3.4.21) and (3.4.23) yields

$$u'(c_t^j) = \beta R \nu'(Ra_t^j), \tag{3.4.24}$$

which determines the optimal fraction of overall savings to be privately saved. Since individuals are able to revise previous policies, the scheme affects intergenerational trade-offs. In particular, mortality (the only source of uncertainty in the model) vanishes in (3.4.24). The induced temporary allocation is, therefore, the result of the standard income and substitution effects stemming from an increased effective rate of return of the individual portfolio and an insurance effect, since the scheme allows equalization of bequest streams. Recalling the first-period budget constraint, the LHS of (3.4.24) is increasing in a_t^j. Since the RHS is decreasing in a_t^j, the lower Inada condition ensures uniqueness of the optimal decision $(a_t^j, \tau^{j,t})$.

Since all individuals at the beginning of time receive zero bequests, there will be intragenerational agreement on the contribution rate for all $t \geq 0$, so members of the same generation receive the same amount of bequests in the presence of the scheme. In the presence of the scheme, therefore, there is no need to keep the index j.

Using the parametrization (3.4.7) and (3.4.8), the consumption-transfer ratio is independent of lifetime income, and the income expansion paths under full insurance for youth consumption, old-age consumption and both kind of intergenerational transfers are straight lines through the origin. From (3.4.23), $\tau_t w = a_t(1-q)/\gamma^{1/\sigma}$. Using this information in the first-period budget constraint and equation (3.4.24) gives optimal private savings as a function of initial wealth, $a_t = \bar{a}(\tau^*)(w + b_t)$, where

[15] In the pure altruistic case, one applies the envelope theorem to find the impact of the bequest on the maximum utility attainable by her heir born in $t+1$.

$\bar{s}(\tau^*) = \bar{a}(\tau^*)(1 + (1 - q)/\gamma^{1/\sigma})$; or, in closed form,

$$\bar{a}(\tau^*) = \frac{\gamma^{1/\sigma}}{1 - q + \gamma^{1/\sigma} + (\beta R^{1-\sigma})^{-1/\sigma}} \tag{3.4.25}$$

is the time-invariant fraction of lifetime income that is privately saved. The linearity of the model implies that the portfolio decision is independent of the overall savings decision. The sum of annuitized and non-annuitized savings is given by $s_t = \bar{s}(w + b_t^j)$, where

$$\bar{s}(\tau^*) = \left\{1 + \left[\beta R^{1-\sigma}(1 - q + \gamma^{1/\sigma})^{\sigma}\right]^{-1/\sigma}\right\}^{-1} \tag{3.4.26}$$

is the propensity to save from lifetime income. Note that the fraction $\frac{1}{1+\gamma^{1/\sigma}/(1-q)}$ of the individual's portfolio is held as pension claims. We have therefore established

Lemma 6. ***Sequence of optimal contribution rates***

The sequence

$$\tau_t^* \;=\; \frac{\bar{a}_{\tau^*}(1-q)}{\gamma^{1/\sigma}}\frac{w + b_t}{w} = \bar{a}(\tau^*)(1-q)/\gamma^{1/\sigma} \times \sum_{i=0}^{t} (R\bar{a}(\tau^*))^i \,. \tag{3.4.27}$$

converges from below to

$$\lim_{t\to\infty} \tau_t^* = \tau^* = \left[1 + \frac{(\beta R^{1-\sigma})^{-1/\sigma} + \gamma^{1/\sigma}(1 - R)}{1 - q}\right]^{-1} . \tag{3.4.28}$$

Proof. (3.4.27) is obvious from the above discussion. If $R\bar{a}(\tau^*) < 1$, then $\tau_t^* \to \frac{\bar{a}(\tau^*)(1-q)}{\gamma^{1/\sigma}(1-R\bar{a}(\tau^*))}$ as $t \to \infty$. Since the optimal split of the portfolio is independent of lifetime income, i.e. $\frac{\tau^* w}{\omega_t} = \bar{a}(\tau^*)(1-q)/\gamma^{1/\sigma})$, $\tau_{t+1} > \tau_t \,\forall t < \infty$. Finally, using (3.4.25) confirms (3.4.28). □

Note that Lemma 6 accounts for the transfer reduction induced by the scheme. The fact that we can define a sequence of non-zero optimal contribution rates should be contrasted with the seminal work of Barro (1974), who finds that government policy intended to change the intergenerational distribution of resources is neutral in welfare terms, so that the optimal contribution rate remains indeterminate. The present non-neutrality result is not driven by the conceptional difference in bequest motives discussed above. Rather, it is the existence of uninsured mortality risk that breaks the equivalence of Social Security pensions and private savings, a risk that does not appear in Barro (1974).

The essential result of the next corollary is that the sequence of optimal contribution rates in an unproductive economy populated by altruistic individuals converges to the time-consistent contribution rate of its egoistic counterpart.

Corollary 5. $\tau^* \gtreqless \tau_e^* = \frac{1}{1+\beta^{-1/\sigma}/(1-q)}$ *as* $R \gtreqless 1$.

The corollary states that τ^* is independent of the strength of the transfer motive when capital is unproductive. The economic intuition is as follows. Since the proceeds from annuitized savings are used for old-age consumption, the fraction of lifetime income saved in annuities is decreasing in the strength of the transfer motive. This tends to decrease the optimal contribution rate, given lifetime income. In turn, non-annuitized fraction of lifetime income in t increases with γ, and is transferred with interest to the next generation in $t+1$, independently of whether the individual survives into old age. Since contributions are wage-related, this effect tends to increase the optimal contribution rate. If capital is unproductive ($R = 1$), then both effects exactly cancel out in the long run, so τ^* is independent of the strength of the bequest motive. In fact, a comparison of (3.3.13) and (3.4.28) establishes the results that long-run optimal contribution rate coincides with the one obtained in an economy populated with perfectly selfish individuals. Finally, since non-annuitized savings are transferred with interest, the latter (former) effect dominates the former (latter) as $R > 1$ ($R < 1$).

Since the scheme is fully funded, contributions to the scheme add to the economy-wide capital stock in the next period. Let $K(\tau^*)$ denote the long-run aggregate capital stock in the presence of the scheme. We have the following results:

Proposition 7. ***Convergence with the scheme***

If $R\bar{a}(\tau^*) < 1$*, then the economy with the scheme converges to the deterministic steady state with*

$$\omega(\tau^*) = \lim_{t\to\infty}(w + b_t) = \frac{w}{1 - R\bar{a}(\tau^*)}, \qquad (3.4.29)$$

$$K(\tau^*) = \lim_{t\to\infty} K_t = \bar{s}(\tau^*)R\omega(\tau^*), \qquad (3.4.30)$$

$$C(\tau^*) = (1 - \bar{a}(\tau^*))\omega(\tau^*), \qquad (3.4.31)$$

$$(3.4.32)$$

Proof. By iteration, the sequence $K_{t+1} = R(a_t + \tau_t w) = [(1-q)/\gamma^{1/\sigma} + 1]\bar{a}(\tau)R\left(w + b_t^j\right) = \bar{s}w\sum_{i=0}^{t}(R\bar{a})^i + \bar{s}(\bar{a}R)^t$ converges if $R\bar{a}(\tau^*) < 1$. □

In the special case of unproductive capital, aggregate consumption is unaffected by the scheme and is equal to wage income w. If, moreover, $\beta^{-1} = R = \gamma = 1$, then the income expansion paths with respect to consumption and transfers are straight lines through the origin with slopes equal to one, implying a perfectly smooth consumption-transfer profile, $c_t(\tau^*) = c_{t+1}(\tau^*) = b_{t+1}(\tau^*) = a_t(\tau^*) = w/(2-q)$. Since $u = \nu$ in that case, the allocation sequence can be obtained without specifying the felicity function. In stationary equilibrium, $K(\tau^*) = C(\tau^*) = w = C(0) < K(0)$, where $\tau^* = \frac{1-q}{2-q}\ \forall\sigma$. While the scheme contributes to capital accumulation, it unambiguously reduces the long-run capital stock. In general, however, the effect of the scheme on the long-run capital stock is ambiguous *a priori*. To see why, the following lemma will be useful.

Lemma 7. ***Propensities to save***

1. *If $\sigma = 1$, then the propensity to save is unaffected by the scheme, $\bar{s}(\tau^*) = \bar{a}$.*
2. *If $\sigma < 1$, then the propensity to save in the economy with the scheme is higher than in the economy without it, $\bar{s}(\tau^*) > \bar{a}$.*

Proof. The first part follows directly from equations (3.4.11) and (3.4.26). Suppose $\sigma < 1$. Comparing (3.4.11) and (3.4.26) gives $\bar{s}(\tau) > \bar{a}$ if and only if

$$(1-q+\gamma^{1/\sigma})^{\sigma} > (1-q)(1+\gamma^{1/\sigma})^{\sigma} + q\gamma.$$

Rewriting the LHS, we have $(1-q+\gamma^{1/\sigma})^{\sigma} = ((1-q)(1+\gamma^{1/\sigma})+q\gamma^{1/\sigma})^{\sigma} > ((1-q)(1+\gamma^{1/\sigma}))^{\sigma}$, $\forall \sigma > 0$. As to the RHS, $(1-q)(1+\gamma^{1/\sigma})^{\sigma} + q\gamma \leq \left((1-q)(1+\gamma^{1/\sigma})\right)^{\sigma} + q\gamma < \left((1-q)(1+\gamma^{1/\sigma})\right)^{\sigma}$; for $\sigma \leq 1$ by assumption. □

Intuitively, there are two components of the effective return to saving, namely, the expected return of the portfolio and the transfer insurance. Since the scheme is actuarially fair, the expected rate of return of the portfolio (including the premature death case) is simply R and therefore unaffected by the scheme, so the change in the propensity to save must be due to the bequest insurance effect. With bequest insurance, the marginal value of transferring one unit of consumption to the next period increases. In that sense, the bequest insurance appears to be an additional return to saving, and Lemma 7 states that if $\sigma < 1$ the effect of the transfer insurance on savings acts like a substitution effect stemming from an increase in the portfolio's return. The scheme increases the propensity to save from lifetime income, which, in turn, tends to increase the long-run capital stock.

Moreover, while both annuitized and non-annuitized savings add to the capital stock, the scheme reduces the fraction of initial wealth that is privately saved, i.e. $\bar{a}(\tau^*) < \bar{a}$, $t = 0, 1, 2, \ldots$. Only non-annuitized assets are transferred, and the resulting reduction in transfers received at birth tends to reduce the long-run capital stock. Which of these opposing effects dominates is eventually a numerical question.[16]

3.5 Simulations: the cost of (in-)equality

The infinite series $w\left(1+\bar{b}_1+\bar{b}_1\bar{b}_2+\bar{b}_1\bar{b}_2\bar{b}_3+\ldots\right)$ converges quickly, particularly when the $\bar{b}_t$'s are small. It is thus fairly easy to generate approximate samples from the

[16] One might proceed instead by imposing $\sigma \to 1$, in which case the fraction of initial wealth saved for old age (in the form of both annuitized and non-annuitized assets) in the presence of the scheme equals the propensity to save under laissez faire ($\bar{s}(\tau^*) = \bar{a}(0)$). If, moreover, $R = 1$, then the scheme unambiguously decreases the long-run capital stock, i.e. $K(\tau) < K(0)$ for all $0 < \gamma \leq 1$.

distribution of ω_∞ by taking a fairly short truncation. To be more precise, we follow one particular realization of the stochastic time series until $|\omega_t^j - \omega_{t+1}^j| < 10^{-6}$. With $R = 1$, for example, there is convergence within ten periods. A large sample of these stochastic elements is generated.

The parameters of the model economy are determined as follows. In order to maintain the two-period theoretical structure, let the economically active years start at the age of 20, and let one period span 40 years, implying that individuals reach retirement at the age of 60, the upper bound of life being 100 years. It seems tempting to impose $q = 1/2$ as a benchmark as it implies a life expectancy at birth of 80 years. However, $q = 1/2$ also implies that only one half, no less, of all 20 years-old fail to reach retirement. Since mortality rates begin to rise rapidly only after the age of 65 or 70, significantly lower values for q seem more suitable. We opt for the latter perspective and treat $q = 0.3$ as a benchmark.

Let $\sigma = 2$, which corresponds to the standard value of 1/2 for the intertemporal elasticity of substitution frequently found in the literature. By way of sensitivity analysis, Appendix 3.A.3 provides results for $\sigma = .5$. The remaining taste parameters are set so as to match the following targets: First, most of the literature on real business cycles employ a quarterly psychological discount factor of 0.9914 (see, e.g., Cooley and Prescott, 1995). Since premature mortality is an important determinant of subjective discounting in the present setup, let $\beta = 0.35$ which, in conjunction with $q = 0.3$, is equivalent to an effective subjective discount rate of $1/(\beta(1 - q)) - 1$ per period, or 3.58% p.a.[17] Second, the strength of the transfer motive is set so as to match old-age income from public transfers in per cent of their disposable income, $(\tau^* w/(1-q))/(a(\tau^*)+\tau^* w/(1-q)) = 1/(1+\gamma^{1/\sigma})$. Note that this number is independent of time preferences, mortality and the market rate of return. The rough average for OECD countries of about 2/3 serves as the reference, implying $\gamma = 0.25$.

3.5.1 The long-run allocation

Table 3.5.1 reports the average allocations with and without the fully funded Social Security for alternative values of the probability premature death and the real rate of return on physical capital. One can think of the upper and lower panel, respectively, as separate islands, each with its particular q and R. Consider the constellation $(1-q, R) = (0.7, 1)$ in the lower panel, which serves as the benchmark scenario.[18] The remaining parameters yield Social Security expenditures of 29.3% of aggregate wage income, or 19.5% of gross domestic output, which are fairly close to the numbers in developed

[17] β is usually chosen so as to match the market rate of return. In the current setup, however, the latter is exogenously given.

[18] Some data indicate an average annual real rate of return (reported as the lending interest rate adjusted for inflation) for most high income countries roughly between 2.5% and 7.5% p.a. in recent decades; see for example, the World Development Indicator data base, which covers the period 1961-2013. Since we are mainly interested in the scheme's very long-run welfare implications and the data span no more than 1.5 generations, relatively modest real interest rates seem appropriate.

countries when one includes other pillars of Social Security such as health care and the like in the picture. The national savings rate in the long-run equilibrium with the scheme is about 33%. In keeping with the theoretical results, aggregate consumption is unaffected by the scheme, but the national capital stock and wealth are reduced substantially: bequest wealth is 29% lower than under laissez-faire.[19] The magnitudes of both the wealth reduction and the *ex ante* insurance effect increase with R. For example, the coefficient of variation for lifetime income under laissez-faire increases from 0.136 with $R = 1$ to 0.432 with $R = 1.03^{40}$, in which case the equilibrium distribution has unbounded support.

3.5.2 The long-run welfare consequences

Table 3.5.1 also contains the long-run welfare consequences of the scheme. Following Kotlikoff and Spivak (1981) and Fehr and Habermann (2008), the latter are calculated as the percentage increase in initial wealth (which coincides with the expected present value of lifetime income in the current setup) that is necessary to make an individual living in the laissez-faire equilibrium as well off as in the equilibrium with the scheme. For individual $j \in [0,1]$ born in t, this percentage solves $V_j^*(0, \phi^j \times \omega_t^j) = V^*(\tau^*, \omega_t(\tau^*))$, where ϕ^j is derived in the Appendix. Again, Rawls's (1973) veil of ignorance will be used to quantify the gains from insurance against the risk of being born into a particular initial wealth position. Therefore, we 'sum' over all individuals in the laissez-faire equilibrium to derive the long-run welfare consequence before the individual's type is revealed,[20]

$$\phi = \int_0^1 \phi^j dj. \tag{3.5.1}$$

In the linear model, the portfolio decision is independent of the decision on how much to carry over to old age (i.e., the choice of $s_t^j = a_t^j + \tau_t^j w$). Moreover, it can be seen in equations (3.4.25) and (3.4.27) that the individual's problem is scalable in wealth: Both the optimal purchase of pension claims as a fraction of total savings, $\left[1 + \gamma^{1/\sigma}/(1-q)\right]^{-1}$, and the propensity to save are independent of wealth. Since intergenerational transfers arise from previous savings, bequests received (and therefore the expected present value of lifetime income) by any individual j at the beginning of her first period of life are proportional to wage income, where the factor of proportionality is determined by the mortality history of her lineage. Since factor prices are fixed by assumption and unaffected by the scheme, this holds true with and without Social Security pensions, and the population average equivalent variation defined in (3.5.1) is invariant to shifts in the equilibrium wealth distribution. The strength of the bequest

[19]Recall that only the fraction $1-q$ survives into old age, such that aggregate consumption in period t amounts to $c_t^* + (1-q)c_t^*$.

[20]Recall that an individual's family mortality history uniquely determines her wealth at birth if individuals are perfectly selfish. In that case, (3.5.1) 'sums' over all possible states.

Table 3.5.1: Long-run (average) allocations and welfare effects of the optimal scheme, $\gamma = 0.25$, $\sigma = 2$

R	(1-q)	c_t^*	c_{t+1}^*	a_t^*	s_t^*	b_t^*	K/Y	b/Y	SSE	$\phi - 1$	$\phi_1 - 1$	$\phi_2 - 1$	$\phi_3 - 1$
$\tau = 0$													
1.00^{40}	0.5	0.819	0.361	0.542		0.361	0.351	0.234					
	0.6	0.776	0.368	0.553		0.328	0.356	0.211					
	0.7	0.735	0.372	0.559		0.294	0.358	0.188					
1.01^{40}	0.5	0.996	0.536	0.540		0.536	0.299	0.297					
	0.6	0.937	0.543	0.547		0.485	0.302	0.267					
	0.7	0.882	0.545	0.549		0.431	0.302	0.237					
1.02^{40}	0.5	1.267	0.830	0.564		0.832	0.251	0.370					
	0.6	1.179	0.832	0.565		0.744	0.251	0.331					
	0.7	1.094	0.824	0.559		0.653	0.250	0.292					
1.03^{40}	0.5	1.761	1.402	0.645		1.405	0.208	0.453					
	0.6	1.599	1.371	0.631		1.229	0.206	0.402					
	0.7	1.445	1.322	0.608		1.053	0.204	0.353					
$\tau = \tau^*$													
1.00^{40}	0.5	0.772	0.457	0.228	0.457	0.228	0.313	0.157	15.7	0.26	9.00	1.94	-10.64
	0.6	0.738	0.437	0.218	0.480	0.218	0.324	0.147	17.7	0.55	7.61	1.86	-8.92
	0.7	0.707	0.418	0.209	0.502	0.209	0.334	0.139	19.5	0.64	5.93	1.64	-6.93
1.01^{40}	0.5	0.890	0.642	0.216	0.431	0.321	0.263	0.196	13.1	-3.92	7.86	3.56	-15.34
	0.6	0.853	0.616	0.207	0.455	0.308	0.271	0.184	14.8	-2.84	6.70	3.38	-12.91
	0.7	0.819	0.591	0.199	0.477	0.296	0.279	0.173	16.3	-1.90	5.25	2.94	-10.09
1.02^{40}	0.5	1.043	0.917	0.208	0.415	0.459	0.217	0.239	10.8	-9.20	6.84	6.69	-22.75
	0.6	1.002	0.881	0.199	0.439	0.440	0.223	0.224	12.2	-7.14	5.85	6.22	-19.21
	0.7	0.963	0.847	0.192	0.460	0.423	0.228	0.210	13.3	-5.14	4.61	5.33	-15.08
1.03^{40}	0.5	1.201	1.347	0.207	0.413	0.674	0.176	0.287	8.8	-16.50	5.91	13.32	-35.73
	0.6	1.211	1.294	0.198	0.436	0.647	0.180	0.267	9.8	$^{\#}$-13.04	5.08	12.04	-30.16
	0.7	1.164	1.244	0.191	0.458	0.622	0.184	0.250	10.7	$^{\#}$-9.59	4.01	10.03	-23.63

The equivalent variation is reported as a % of the individual's expected present value of lifetime income. SSE: Social Security expenditure (as % of gross domestic output). $^{\#}$: unbounded support of the long-run laissez-faire equilibrium.

motive used to generate Table 3.5.1, $\gamma = 0.25$, suggests an *ex ante* welfare gain of 0.64%. To put this number in perspective, the remainder discusses the relative importance of insurance and the strength of the transfer motive.

Disentangling insurance

In the current setup, the scheme provides only transfer insurance and the induced *ex ante* insurance manifested in a more concentrated long-run wealth distribution.[21] To quantify the relative importance of both forms of insurance, the overall welfare effect is split into effects stemming from the reduction in wealth, transfer insurance, and *ex ante* insurance of the state at birth, respectively. First, the average willingness to pay for transfer insurance is calculated as follows. Suppose that each individual j in the laissez-faire equilibrium enjoys unchanged initial wealth, but also bequest insurance through the possibility of annuitization by means of the scheme. Denote the associated welfare number by $\phi_1 = \int \phi_1^j dj$. Since the initial wealth distribution is unaltered by the scheme, the transfer insurance effect coincides with the short-run welfare effect from the scheme. While the smoothing of transfers across states is beneficial to risk-averse individuals, an increase in the probability of surviving into old age reduces the gains from transfer insurance. At the extreme, $q = 0$ implies that the contribution to the scheme is a perfect substitute for private savings in physical capital (as in Barro (1974)) and such gains vanish. Second, consider the *ex ante* risk of being born into a particular state. The average percentage increase in initial wealth necessary to make the individuals in the laissez-faire equilibrium as well off as they would be if they were faced with the *ex ante* expected initial wealth position, ϕ_2^j, solves $V(\phi_2^j(w + b_t^j)) = V(\int(w + b_t^j)dj)$. If transfers are motivated by the joy of giving, then individuals do not (fully) take into account the scheme's impact on future generations well-being, thereby also generating an externality in the form of wealth reduction. This effect is obtained as the residual $\phi_3 = \phi - \phi_1 - \phi_2$.

The overall welfare gain is decreasing in the market rate of return. For modest real rate of returns, transfer insurance considerably dominates *ex ante* insurance. However, since bequests are transferred with interest, an increase in R increases the spread in the long-run wealth distribution, so that *ex ante* insurance becomes increasingly important. Consider, again, the benchmark constellation $(1-q, R) = (0.7, 1.03^{40})$, in which case the support of the laissez-faire equilibrium distribution is unbounded, and the contribution of *ex ante* insurance to the overall welfare effect is 2.5 times the contribution of transfer insurance. However, the increase in R also enhances the reduction in capital and wealth, which is the far most important contributor to the overall effect if R is high.

By virtue of Lemma 7 the overall propensity to save with the scheme is higher than without it whenever $\sigma < 1$. To illustrate what happens in this range, Table 3.A.2 replicates the allocative and welfare consequences of the scheme for 0.5, which

[21] The model setup is unambiguous at this point: the two period assumption implies that the annuity is only paid once, excluding the insurance of longevity risk (i.e. the risk of running out of resources). Recalling footnote 6, insurance of lifetime risk itself is also excluded.

corresponds to a relatively high intertemporal elasticity of substitution of 2. The increase in long-run capital induced by the scheme now goes along with a long-run reduction in welfare; for the ex ante insurance is significantly reduced (for the unproductive panel, the ratio of *ex ante* insurance bequest insurance gains is 0.062, compared to 0.28 in the case where $\sigma = 2$).

Welfare gains and the strength of the bequest motive

Presuming $R = 1$, Figure 3.A.3 illustrates that the welfare consequences of the scheme depend qualitatively on the strength of the bequest motive. The red line is the net welfare effect as a function of the strength of the bequest motive. While altruists gain far less than egoists from the scheme, the crowding out induced by the scheme is compatible with long-run welfare gains as long as the strength of the transfer motive is low, although the reduction in transfer wealth is quite substantial. However, overall welfare gains turn into losses if the transfer motive is strong enough: while egoists (i.e. $\gamma = 0$) gain on average 3.29% of their expected lifetime income, individuals who value inter vivos transfers and old-age consumption alike (i.e. $\gamma = 1$) suffer from a modest average welfare loss of about -0.1% of expected lifetime income. The driving forces are transfer insurance (in the egoistic economy, a higher effective rate of return) and the crowding out of wealth. Both effects are decreasing in γ. Since $R = 1$, the laissez-faire equilibrium distribution is bounded (see Proposition 6), such that *ex ante* insurance is relatively insensitive to the strength of of the bequest motive. While *ex ante* insurance plays a minor role, it may tip the scales in qualitative evaluations of overall welfare consequences from Social Security pensions.

Long-run welfare consequences: the egoistic economy

Welfare effects crucially depend on whether the transfer motive is operative. Table 3.A.1 provides the welfare numbers for the egoistic economy. Individuals solely save due to perfect selfishness. However, the scheme entails an increase in the effective rate of return, inducing standard income and substitution effects. The latter are bundled in the number ϕ_1 (equivalently, in the first bar for egoistic individuals in Figure 3.A.3). Not surprisingly, ϕ_1 is decreasing in the probability of surviving into old age; for the effective rate of return is decreasing in $(1 - q)$. If capital is unproductive, then, on average, the income effect stemming from the higher return is of the same size in absolute value as the income effect stemming from the elimination of bequests. In the benchmark $1 - q = 0.7$, the remaining substitution effect falls short of the *ex ante* insurance effect: $11.76 - 10.53 = 1.23 < 2.06$. As noted above, egoists gain much more from the scheme than their altruistic counterparts. At first glance, one might argue that the optimal scheme and the associated welfare gains are smaller in size; for the bequest motive reduces the optimal degree of annuitization. By virtue of Corollary 5, however, the long-run optimal contribution rate in the altruistic economy coincides with the one obtained in the egoistic economy if capital is unproductive. Given the insensitiveness of *ex ante* insurance to γ, the increase in the effective rate of return is much more valued by egoists than bequests insurance is valued by altruists.

3.6 Conclusions

The chapter analyzed the consequences of insuring mortality risk by means of standard fully funded Social Security Pensions in two socio-economic setups: an egoistic economy in which individuals save due to a pure life-cycle motive, and an altruistic economy in which individuals save due to both a life-cycle motive and an altruistic motive which reflects the joy of giving. In the former setup, the pension system promises a higher effective return than the market. In the latter setup, the pension system is an instrument to smoothen intergenerational transfers across the states death and survival. Whatever the individuals' attitudes towards their heirs, the pension system reduces intergenerational transfers, resulting in a lower but more concentrated distribution of lifetime income in the long run.

Altruistic behaviour is commonly seen to reduce the fraction of lifetime income to be optimally annuitized. Given equal market interest rate and population growth rate, our model implies that, once the reduction in intergenerational transfer is taken into account, the long-run optimal contribution rate is independent of the strength of the transfer motive, and coincides with the one associated with the egoistic economy. If capital is productive, then the pension system's long-run optimal contribution rate is even higher if individuals are altruistic, reflecting the desire to make transfers to the heir.

The short-run welfare effects are clear-cut. Egoists gain from the higher effective rate of return, while altruists gain from transfer insurance. The long-run welfare consequences are not that obvious a priori. While the pension system reduces intergenerational transfers, it also reduces the mortality-related *ex ante* risk of being born with a particular amount of wealth. The model suggests that, once ex ante insurance is taken into account, fully funded Social Security Pensions cannot be rejected a priori: if capital is not too productive and the bequest motive is not too strong, then the scheme generates long-run welfare gains, because insurance gains outweigh the crowding out of within-family transfers.

3.A Appendix to Chapter 3

3.A.1 The equivalent variation

The consumption-transfer allocation under laissez-faire, namely,

$$\begin{aligned} c_t^{j,0} &= (1-\bar{a})(w+b_t^j) && (3.A.1)\\ b_{t+1}^{j,0} &= \frac{\gamma^{1/\sigma}}{1+\gamma^{1/\sigma}} R\bar{a}(w+b_t^j) && (3.A.2)\\ b_{t+1}^{j,1} &= R\bar{a}(w+b_t^j) && (3.A.3)\\ c_{t+1}^{0} &= \frac{1}{1+\gamma^{1/\sigma}} R\bar{a}(w+b_t) && (3.A.4) \end{aligned}$$

gives the indirect utility of individual j born in period t as

$$V_t^j(0) = \bar{v}(w+b_t^j)^{1-\sigma}, \qquad (3.A.5)$$

where

$$\bar{v} = \left[(1-\bar{a})^{1-\sigma} + \beta\left(\bar{a}R\right)^{1-\sigma}(1-q)\left((1+\gamma^{1/\sigma})^{\sigma} + q/(1-q)\gamma\right)\right]/(1-\sigma), \qquad (3.A.6)$$

which approaches $(1-\bar{a}_e)^{1-\sigma}\left(1+\left(\beta(1-q)R^{1-\sigma}\right)^{1/\sigma}\right)/(1-\sigma)$ as $\gamma \to 0$.

With the scheme, the consumption-transfer allocation

$$\begin{aligned} c_t^{0} &= (1-\bar{s}(\tau^*))(w+b_t) && (3.A.7)\\ b_{t+1}^{0} = b_{t+1}^{s} &= R\bar{a}(\tau^*)(w+b_t) = a_t R && (3.A.8)\\ c_{t+1}^{0} &= \gamma^{-1/\sigma} R\bar{a}(\tau^*)(w+b_t) && (3.A.9) \end{aligned}$$

gives the indirect utility of a member of the generation born in t as

$$V_t(\tau^*) = \bar{v}(\tau^*)(w+b_t)^{1-\sigma}, \qquad (3.A.10)$$

where

$$\bar{v}(\tau^*) = \left[(1-\bar{s}(\tau^*))^{1-\sigma} + \beta(1-q)\left(\bar{a}(\tau^*)R/\gamma^{1/\sigma}\right)^{1-\sigma} + \beta\gamma\left(\bar{a}(\tau^*)R\right)^{1-\sigma}\right]/(1-\sigma), \qquad (3.A.11)$$

which approaches $(1-\tau_e^*)^{1-\sigma}\left(1+\left(\beta R^{1-\sigma}\right)^{1/\sigma}(1-q)\right)/(1-\sigma)$ as $\gamma \to 0$.

Recalling the definition of ϕ^j, the equivalent variation reads

$$\phi_t^j = \left(\frac{\bar{v}(\tau^*)}{\bar{v}}\right)^{\frac{1}{1-\sigma}} \omega(\tau^*)/\omega_t^j \frac{w+b_t^{\tau}}{w+b_t^j}. \qquad (3.A.12)$$

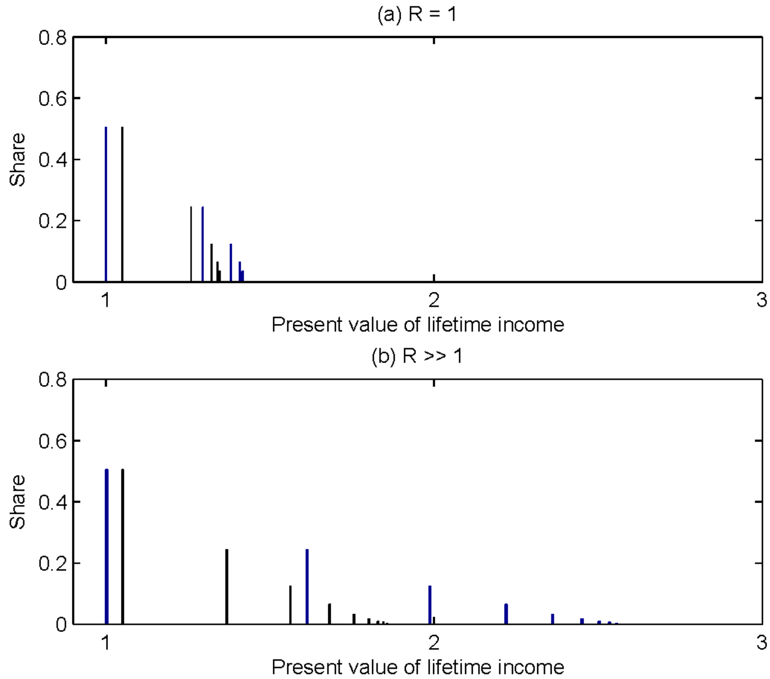
For the egoistic economy, the equivalent variation of an individual whose j previous, consecutive forebears within the lineage died prematurely is given by

$$\phi^j = \left[\frac{1 + \left(\beta(1-q)R^{1-\sigma}\right)^{1/\sigma}}{1 + (1-q)\left(\beta R^{1-\sigma}\right)^{1/\sigma}}\right] \times \left[\frac{1 + (\beta R)^{1/\sigma}}{1 + (1-q)\left(\beta R\right)^{1/\sigma}}\right]^{\frac{1}{1-\sigma}} \times \frac{w}{w + b^j}, \tag{3.A.13}$$

where $w + b^j = w \sum_{i=0}^{j} (\bar{a}R)^i$. Recalling that there are $q^j(1-q)$ type-j individuals in long-run equilibrium, the aggregate welfare effect stemming from the scheme is $\phi = \sum_{j=0}^{\infty} q^j(1-q)\phi^j$, where the double sum converges, provided that the long-run equilibrium exists, i.e. $\bar{a}Rq < 1$.

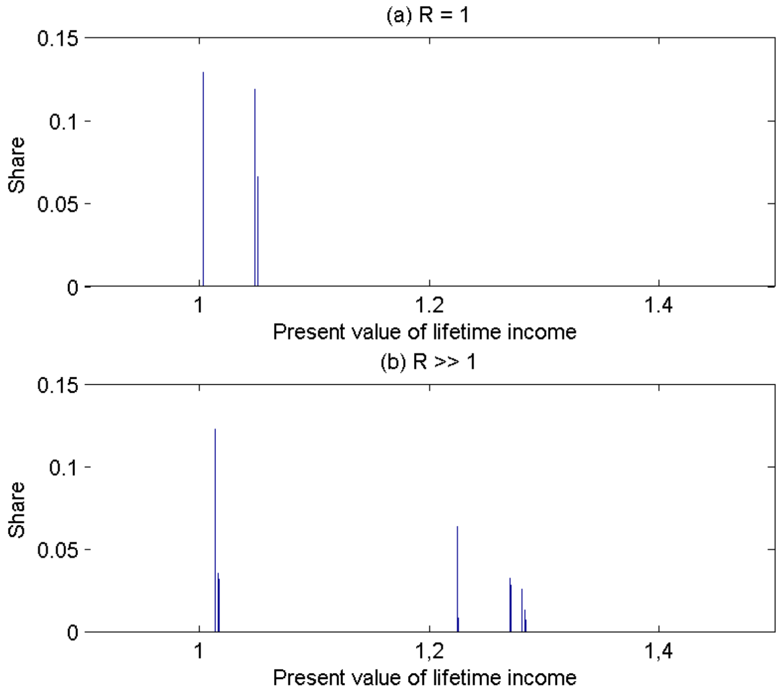
3.A.2 Plots

Figure 3.A.1: Long-run distributions of lifetime income: the egoistic economy



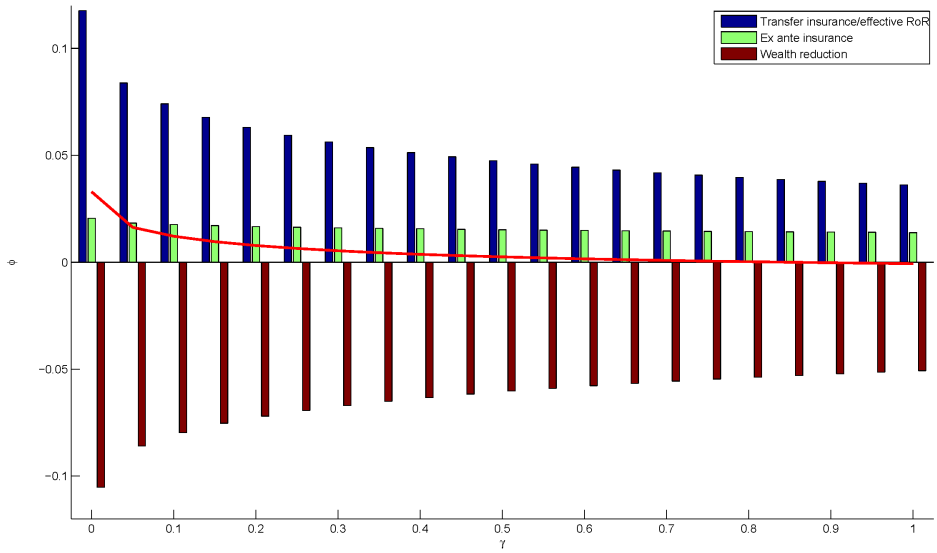
The figure illustrates the distributions of lifetime income in the stationary equilibrium with (black bars) and without the scheme (blue bars). The algorithm used to produce the figure is discussed in Section 3.5. The parameters are $q = 0.5$, $\beta = 0.5$, $\sigma = 2$, $R = 1.02^{40}$, $w = 1$.

Figure 3.A.2: Long-run distributions of lifetime income: the altruistic economy



The figure illustrates the distributions of lifetime income in the stationary equilibrium of the altruistic economy in the absence of the scheme. The algorithm used to produce the figure is discussed in Section 3.5. The parameters are $q = 0.5$, $\beta = 0.5$, $\sigma = 2$, $\gamma = 0.25$, $R = 1.02^{40}$, $w = 1$.

Figure 3.A.3: The population average equivalent variation as a function of the strength of the transfer motive



The figure illustrates the welfare consequences of the optimal scheme for different values of γ when capital is unproductive, i.e. $R = 1$. The red line is the net effect. The bars represent a decomposition of the equivalent variation into bequest insurance, *ex ante* insurance and crowding out of wealth. The first bundle of bars (on the far left) represents the decomposition of the equivalent variation for the egoistic economy (i.e. $\nu(\cdot) = 0$) into (i) income and substitution effects induced by an increase in the effective rate of return, (ii) ex ante insurance and (iii) capital reduction. The parameters use are $R = 1$, $q = 0.3$, $\beta = 0.35$, $\sigma = 2$.

3.A.3 Tables

Table 3.A.1: Long-run welfare effects in the egoistic economy

σ	R	(1-q)	$\phi - 1$	$\phi_1 - 1$	$\phi_2 - 1$	$\phi_3 - 1$
2	1.00^{40}	0.5	4.58	19.81	2.30	-17.53
		0.6	4.04	15.83	2.30	-14.09
		0.7	3.29	11.76	2.06	-10.53
	1.01^{40}	0.5	-1.57	16.82	3.94	-22.33
		0.6	-0.68	13.55	3.89	-18.11
		0.7	0.09	10.14	3.44	-13.66
	1.02^{40}	0.5	-7.60	14.23	6.72	-28.55
		0.6	-5.30	11.54	6.49	-23.32
		0.7	-3.40	8.69	5.64	-17.72
	1.03^{40}	0.5	-13.44	12.02	11.46	-36.91
		0.6	-9.74	9.79	10.77	-30.30
		0.7	-6.57	7.42	9.14	-23.12
0.5	1.00^{40}	0.5	1.48	2.97	0.02	-1.52
		0.6	1.17	2.82	0.04	-1.68
		0.7	0.85	2.43	0.06	-1.64
	1.01^{40}	0.5	1.11	4.36	0.11	-3.36
		0.6	0.60	4.11	0.20	-3.70
		0.7	0.23	3.52	0.30	-3.58
	1.02^{40}	0.5	-0.56	6.33	0.50	-7.39
		0.6	-1.26	5.92	0.89	-8.07
		0.7	-1.47	5.02	1.27	-7.75
	1.03^{40}	0.5	-4.83	9.08	2.32	-16.24
		0.6	-5.29	8.38	3.96	-17.64
		0.7	-4.62	7.02	5.29	-16.92

Table 3.A.2: Long-run (average) allocations and welfare effects of the optimal scheme, $\gamma = 0.25$, $\sigma = 0.5$

R	(1-q)	c_t^*	c_{t+1}^*	a_t^*	s_t^*	b_{t+1}^*	K/Y	b/Y	SSE	$\phi - 1$	$\phi_1 - 1$	$\phi_2 - 1$	$\phi_3 - 1$
$\tau = 0$													
1.00^{40}	0.5	0.977	0.046	0.049		0.026	0.047	0.025					
	0.6	0.965	0.128	0.061		0.026	0.058	0.024					
	0.7	0.950	0.153	0.074		0.024	0.069	0.022					
1.01^{40}	0.5	0.984	0.103	0.074		0.058	0.066	0.052					
	0.6	0.966	0.128	0.091		0.057	0.080	0.051					
	0.7	0.944	0.153	0.109		0.053	0.094	0.046					
1.02^{40}	0.5	1.018	0.235	0.113		0.131	0.090	0.105					
	0.6	0.991	0.288	0.138		0.130	0.106	0.099					
	0.7	0.955	0.341	0.164		0.118	0.120	0.087					
1.03^{40}	0.5	1.134	0.571	0.186		0.320	0.116	0.200					
	0.6	1.087	0.689	0.224		0.312	0.130	0.180					
	0.7	1.021	0.795	0.259		0.280	0.141	0.152					
$\tau = \tau^*$													
1.00^{40}	0.5	0.942	0.115	0.007	0.065	0.007	0.061	0.007	5.42	-0.00	1.78	0.05	-1.84
	0.6	0.932	0.114	0.007	0.076	0.007	0.070	0.007	6.37	-0.10	1.69	0.07	-1.86
	0.7	0.921	0.113	0.007	0.086	0.007	0.079	0.007	7.27	-0.13	1.46	0.09	-1.68
1.01^{40}	0.5	0.921	0.250	0.011	0.095	0.016	0.083	0.014	7.36	-1.25	2.59	0.24	-4.08
	0.6	0.906	0.246	0.010	0.110	0.015	0.094	0.013	8.52	-1.30	2.44	0.34	-4.07
	0.7	0.891	0.242	0.010	0.124	0.015	0.105	0.013	9.61	-1.17	2.09	0.41	-3.68
1.02^{40}	0.5	0.897	0.536	0.015	0.137	0.034	0.105	0.026	9.32	-4.21	3.71	1.11	-9.03
	0.6	0.876	0.523	0.015	0.157	0.033	0.117	0.024	10.56	-3.98	3.47	1.51	-8.96
	0.7	0.856	0.511	0.014	0.176	0.032	0.127	0.023	11.67	-3.34	2.96	1.75	-8.05
1.03^{40}	0.5	0.875	1.140	0.022	0.197	0.071	0.120	0.043	10.65	-10.14	5.23	5.25	-20.63
	0.6	0.845	1.102	0.021	0.224	0.069	0.129	0.040	11.71	-8.88	4.85	6.66	-20.38
	0.7	0.818	1.066	0.020	0.269	0.067	0.137	0.037	12.62	#-6.99	4.08	7.21	-18.28

The equivalent variation is reported as a % of the individual's expected present value of lifetime income. SSE: Social Security expenditure (as % of gross domestic output). #: unbounded support of the long-run laissez-faire equilibrium.

Chapter 4

Estimating the Long-run Benefits of India's Rural Roads Programme: The Movements of Goods and People

4.1 Introduction

Today, 1 billion people worldwide still lack access to an all-weather road (World Bank, 2015). Due to the link between improved infrastructure and rural poverty reduction, especially investments in rural roads are considered by governments and foreign aid donors alike as an effective and efficient way of improving welfare in rural areas. For example, transport lending in 2015 accounts for 21 per cent of the World Bank's total active portfolio. In South Asia, this sector accounts for 16% per cent of the Bank's total lending (World Bank, 2015). The Bank also supports India's rural road programme known as *Pradhan Mantri Gram Sadak Yojana* (PMGSY), one of the largest programmes in the region.[1]

The existing empirical literature indicates that improved road networks indeed can generate diverse benefits in the spheres of production and trade, education and health. For example, Khandker et al. (2009) find that transport costs were substantially reduced,

This chapter was jointly written with Clive Bell.

[1]The programme is large by any measure: it was launched in 2000 and is expected to be completed by 2020. It seeks to provide all-weather road access for every community with a population greater than 1000 by 2003 (all villages with population greater than 500 in 2007). By 2010, about 110 million people had road access, which is equivalent to 70,5000 habitations, or about 47% of the unconnected rural population of India as of the 2001 Census. At that time, the length of new and improved rural road network under the programme reached 274,000 km, and total costs amounted to US$ 14.6 billion. The programme is still underway at the time of this writing, and an estimated further US$ 40 billion will be required to complete the programme by 2020 (World Bank (2010)).

and output and net prices were boosted by rural roads in Bangladesh. Directly applicable is the work of Bell and van Dillen (2014), who estimates that PMGSY reduced unit transport costs by about 5 per cent of the net price received for the commodities marketed. There is evidence that rural roads improve not only the commercial terms of trade, but also education. A related and seemingly established finding is that school construction has positive effects on children's school enrollment and attendance (see, e.g., Duflo, 2001; Burde and Linden, 2013; and Kazianga et al., 2013). Since school construction reduces the effective distance to school in terms of travel time, rural roads can be expected to have similar effects. In fact, Bell and van Dillen (2014) estimate that PMGSY results in substantial savings in travel-time for secondary-school pupils (just over 30 minutes a day) and far fewer days of involuntary absences for all grades. Where health is concerned, Banerjee and Sachdeva (2015) find that PMGSY has promoted preventive health care. The increase in health care usage comes not only from an increase in income and a reduction in travel cost, but also from the increase in the awareness about health care programmes, the improvement in health care supply and the increase in social interactions. Bell and van Dillen (2015) go one step further and show that PMGSY actually affects morbidity itself. They estimate that PMGSY has reduced the average duration of incapacitating illness by 1.4 days per year in a region of upland Orissa.

Valuating these benefits is another matter since the new roads affect not just the decisions of what to produce and consume, but also those having to do with the formation and maintenance of human capital, including life itself. Moreover, while valuating the benefits that arise in connection with more favourable prices of goods, improved educational attainment and lower morbidity involves a common (money) metric, the benefits of reduced mortality do not fit into this convenient scheme of things. Moreover, by virtue of the effects on human capital accumulation, it is very likely that the empirical literature underestimates the long-term effects of PMGSY.

This chapter attempts to estimate the long-run benefits of PMGSY. We develop a computable two-sector overlapping generations model with rural-urban migration, human capital formation and premature mortality, and calibrate it to the available data. Decisions in the spheres of production, consumption and education are made in a particular environment of morbidity and mortality. We estimate that PMGSY generates benefits amounting to approx. twelve per cent of the value of output produced by a surviving member of the first generation. Approx. 18 per cent of this gain accrues in the spheres of education and health. Total benefits more than double for the next generation, and the ratio of commercial to non-commercial benefits falls substantially, to less than one-to-one.

A salient feature of the model is spatial heterogeneity with respect to production and health. We introduce an urban sector and thus allow young adults who grow up in the rural sector to migrate to towns if they wish. Inter-sectoral migration is the result of an economic decision in which young adults compare expected utilities in towns and rural areas. We build on Bell and Koukoumelis's (2009) model of a dual economy. While the urban sector is a simplified version of that in Bell and Koukoumelis (2009), we extend their setup in that the rural and urban sectors differ in the goods they produce. In order to draw a reliable picture of current and future migration patterns with and without rural roads, we need to specify certain demographic features in detail, and to estimate them in relation to the census and other data. In view of the dramatic and rapid demographic transition underway in India, we include secular trends in the demographic environment.[2]

The chapter complements Bell's (2012) analysis of the long-run benefits of PMGSY, which, in turn, extends Bell et al. (2006)'s one-sector model of human capital formation and growth in the presence of premature adult mortality to include two goods, transport costs and morbidity.[3] In Bell (2012), there is just one sector, producing one good that can be 'exported'. The second good is 'imported'. The resulting trade between them necessarily involves transportation. The net prices of both goods are parametrically given, but depend on whether there is an all-weather road. Moreover, the latter reduces mortality rates, pushes family output, school enrollment and attendance, thereby enhancing human capital accumulation.

While the calibration of the model yields taste parameters that are fairly close to those of Bell (2012), our estimates are higher for two reasons. First, we account for a positive effect on the health outcome as detected in Bell and van Dillen (2015). Second, a striking effect of PMGSY is to reduce the scale of migration in all periods under consideration. Improvements in rural living-conditions make human capital less likely to exit the rural sector, which further contributes to the overall welfare gains of PMGSY. Our estimates are still rather conservative; for we employ the short-term results of the above empirical studies. Where morbidity is concerned, for example, timelier and more regular treatment can be expected to have positive long-term effects on health and productivity. Moreover, the costs of pain and suffering during a bout of illness are not taken into account.

The chapter is structured as follows. The model is set out in Section 4.2, the migration equilibrium being defined in Section 4.2.3. The functional forms and parameters used in the simulations follow in Section 4.3, while the estimates of the demographic details are presented in Section 4.3.2. Section 4.4 presents the main results, including the sequences of the key economic variables with and without the road, the equivalent variation and its decomposition. The conclusions are drawn together in Section 4.5.

4.2 The Model

With the model of Bell and Koukoumelis (2009) as a basis, we analyse the introduction of rural roads as the policy intervention. Instead of allowing premature mortality only to

[2]Lee (2003) provides a detailed exposition of the (projected) Indian demographic transition.

[3]Bell (2012) provides a compact summary of the key features in Bell et al. (2006).

occur once at the very end of the phase of life called young adulthood, when individuals are in their late thirties and early forties, we introduce other age-specific mortality, namely, among under-five years olds and among young adults who have just had their children.

We account for the following benefits in the rural sector stemming from all-weather roads: (i) more favorable prices facing the household as producer and consumer, (ii) reduced time for children to go to and from school, and fewer absences of teachers, (iii) lower morbidity and mortality due to timelier medical treatment. While the model has some general equilibrium features, the prices of the two goods in the urban sector are fixed. The roads will therefore affect only the prices faced by producers and consumers in the rural sector, and then only exogenously by the reduction in unit transport costs. The only endogenous prices are the wage rate per efficiency unit of labour and the (implicit) rental of a unit of physical capital. Movements in these prices assist migration to bring about equilibrium in each period. In contrast to the dualistic, labour-surplus economy of Lewis (1954), the marginal productivity of labour in the rural sector is assumed to be positive, and the urban labour market flexible enough to absorb the inflow of migrants at full employment.[4] In the migration equilibrium, young adults in the rural sector are indifferent between migrating to the towns and staying in the rural sector in the current period and all periods to come.

4.2.1 The rural sector

With the important exceptions of migration and a thorough treatment of vital rates along the demographic transition now underway in India, the environment and households' decisions in the rural sector are similar to the setup found in Bell (2012). However, accounting for mortality among under-five years-old and among young adults who just have had their children, requires the formal introduction of a sub-period at the start of each full period.

Demographics

Given the U-shape of age-specific mortality, let death occur at the following possible junctures: during childhood, prior to reaching school age; and during young adulthood, either just after parents have had their children, or at the very end of young adulthood. In accordance with the observed strong age pattern of migration, let all surviving children born in the rural sector (labelled 1) stay there until adulthood, and suppose that only young adults migrate to the urban sector (labelled 2). We use the following notation:

[4] In Lewis (1954), the withdrawal of surplus labour in the rural sector has no effect on the rural output per family. Moreover, the urban wage rate is exogenously given and the supply of labour from the rural sector is perfectly flexible at that rate. Once surplus labour is fully absorbed by the urban sector, the withdrawal of labour is at the cost of lower rural production.

N_{it}^{a}: the number of individuals per family in the age-group $a\,(=1,2,3)$ in period t, and in sector $i\,(=1,2)$,

n_{it}: the number of children born to a representative couple in sector $i\,(=1,2)$ in period t,

q_{it}^{1}: the probability that infants and young children born in period t and in sector $i\,(=1,2)$ will die before reaching school age in that period,

$q_{i,t}^{2}$: the probability that a young adult in sector $i\,(=1,2)$ will die in the early phase of period t,

q_{it}: the probability that a young adult in sector $i\,(=1,2)$ will die at the end of period t, before reaching the third phase of life.[5]

d_{it}^{a}: the fraction of period t that a surviving individual aged $a\,(=1,2,3)$ spends in disability in sector $i\,(=1,2)$,

M_t: the number of (young adult) rural-urban migrants per family in period t.

Figure 4.2.1 depicts the timeline of events. At the very beginning of period t, young adults decide whether to migrate. Only after choosing where to live do they choose like partners[6]. Each couple produces n_{1t} children, out of which the fraction $(1-q_{1t}^{1})$ survives to school age. These young adults die with probability q_{1t}^{2} just after they have had their children. The survivors take in all orphaned children.

All this happens within the subperiod that precedes the decisions of the surviving young adults. Knowing the realized net fertility per family they draw up a consumption and investment plan[7]. At that point in time, the normalised population age structure of the representative rural household comprises the following numbers of individuals:

$$\begin{aligned}\mathbf{N}_{1t} &= (N_{1t}^{1}, N_{1t}^{2}, N_{1t}^{3}) \\ &= \left((1-q_{1t}^{1})n_{1t}, (1-q_{1t}^{2})\cdot 2, \frac{(1-q_{1t-1}^{2})(1-q_{1t-1})\cdot 2\cdot N_{1t-1}^{f}}{N_{1t}^{f}}\right),\end{aligned} \tag{4.2.1}$$

where the normalisation relates to a representative couple at the start of period t and N_{1t}^{f} is the total number of rural households in period t. Denote by $\mathcal{N}_{1t}^{2}$ the total number of young adults in the rural sector after the migrants have departed. Then $N_{1t}^{f}=\mathcal{N}_{1t}^{2}/2$, and it evolves over time according to

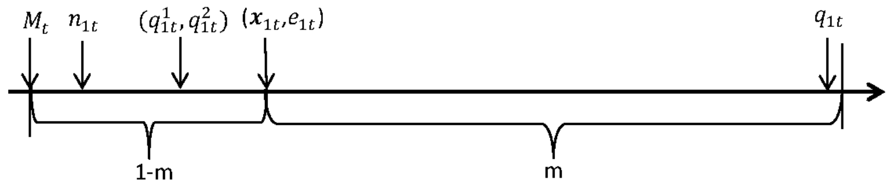
$$N_{1t+1}^{f} = ((1-q_{1t}^{1})n_{1t} - M_{t+1})N_{1t}^{f}/2,$$

[5]In the present model structure, these statistics correspond to ${}_5q_0$, ${}_5q_{20}$ and ${}_{15}q_{25}$, respectively, where ${}_nq_x$ denotes the probability that an individual will die before the age of $x+n$, conditional on surviving until the age of x.

[6]We assume, for simplicity, that there are as many women as men within in each age group, though we are aware of the alarming disproportions in sex ratios in some Indian regions.

[7]Realized net fertility per family is directly affected by the prevailing mortality regime, first, because q_{1t}^{1} affects the number children in school age, and second, q_{1t}^{2} determines the number of orphaned children taken in by a surviving young couple.

Figure 4.2.1: Events and decisions in period t



Note: $(1-m)$ denotes the length of the subperiod prior to the surviving young adults' decision on consumption and investment in human capital. All the old die at the end of each period.

Human capital and output in the rural sector

We use the notation introduced in Bell (2012):

λ^a_{it}: the human capital possessed by an adult in age-group $a\,(=2,3)$ in period t, and in sector $i\,(=1,2)$,

γ: the human capital of a school-age child,

e_{it}: the proportion of their school-age years actually spent in school by the cohort of children $(a=1)$ in period t, and in sector $i\,(=1,2)$.

x_{jit}: the consumption of good $j\,(=1,2)$ by each young adult in period t, and in sector $i\,(=1,2)$,

β: the proportion of a young adult's consumption received by each child,

ρ: the proportion of a young adult's consumption received by each old adult,

σ_{it}: the direct costs per child of each unit of full-time schooling in sector $i\,(=1,2)$,

Human capital is formed through a process that involves the adults' human capital and the educational technology. The human capital attained by a child on becoming an adult in period $t+1$ depends, in general, on the numbers and human capital of the adults, the level of schooling that child received, and the number of siblings of school-going age, who were presumably competing for the adults' attention, care and support – all in the previous period. Formally,

$$\lambda^2_{1t+1} = \Phi(e_{1t}, \boldsymbol{\lambda}_{1t}, \mathbf{N}_{1t}), \tag{4.2.2}$$

where $\boldsymbol{\lambda}_{1t} = (\lambda^2_{1t}, \lambda^3_{1t})$ and Φ is increasing in all its arguments, except for N^1_{1t}.

Simplifying with the application in mind, we ignore competition among siblings and let Φ be multiplicatively separable in: (i) the educational technology; and (ii) the average level of the adults' human capital. This implies that formal education and the adults' human capital are complements in producing the children's human capital. Parents in rural India have most of their children when they are in their twenties, and in their thirties, they are busy rearing them to adulthood. Normalizing the structure to a representative couple within the extended family, these assumptions yield the following specialization of (4.2.2):

$$\lambda_{1t+1}^2 = f_t(e_{1t}) \cdot \frac{N_{1t}^2 \lambda_{1t}^2 + N_t^3 \lambda_{1t}^3}{N_{1t}^2 + N_{1t}^3} + 1. \tag{4.2.3}$$

The function $f_t(\cdot)$ represents the educational technology, whose efficiency may vary with time, and is assumed to be continuous and increasing $\forall e_{1t} \in [0, 1)$, with $f_t(0) = 0$. The assumption $f_t(0) = 0$ implies that a child who receives no schooling will attain only some basic level of human capital, which, without loss of generality, may be normalized to unity – hence the '1' on the RHS of (4.2.3). Let there be no depreciation of human capital. Out-migration impinges on the process of human capital formation because an increased proportion of old adults decreases the average level of the adults' human capital.

The rural household produces a single consumption good (good 1) by means of labor alone, measured in efficiency units, under constant returns to scale. A natural normalization is that a healthy adult who possesses human capital in the amount λ_{1t}^a is endowed with λ_{1t}^a efficiency units of labor, which he or she is assumed to supply completely inelastically.

All output produced in the interval preceding the decision point is simply consumed; for under-fives are not at school (there is no investment in their human capital). Since they do not work, the output in question is

$$y_{1t}^m = (1 - m) \cdot \alpha_{1t} \left[(1 - d_{1t}^2) 2\lambda_{1t}^2 + (1 - d_{1t}^3) N_{1t}^3 \lambda_{1t}^3 \right],$$

where α_{1t} denotes the rural output produced by one unit of human capital input in period t.

In contrast, school-age children do contribute to the family output. However, they, too, suffer ailments, which reduce the effective time left for schooling and work. Each child supplies $(1 - d_{1t}^1 - (1 + \tau)e_{1t})\gamma$ efficiency units of labor when it spends e_{1t} units of time in school and, unavoidably, τe_{1t} units of time travelling to and from school, whereby $\gamma \in (0, 1)$, i.e., a full-time working child is less productive than an uneducated adult. When the decision about (x_{it}, e_{it}) must be made, i.e., after the realization of the rates q_{1t}^1, q_{1t}^2 and n_{1t}, considerably less than a full period remains, with consequences for, *inter alia*, the level of full income. The household produces

$$y_{1t} = m \cdot \alpha_{1t} \left[N_{1t}^1 (1 - d_{1t}^1 - (1 + \tau)e_{1t})\gamma + (1 - d_{1t}^2) N_{1t}^2 \lambda_{1t}^2 + (1 - d_{1t}^3) N_{1t}^3 \lambda_{1t}^3 \right] \tag{4.2.4}$$

units of good 1 in the remaining interval of period t. Aggregate rural output is denoted by $Y_{1t} = (y^m_{1t} + y_{1t})N^f_{1t}$.

Prior to the decision about (x_{it}, e_{it}) the old adults are assumed to consume the entire amount of their contribution to income in the subperiod. Children under five make only limited claims on consumption, the latter being subsumed under young adult consumption. Only after the children go to school do the sharing rules of the 'common pot' apply. Following Bell (2012), we normalize the budget identity at the decision point by the number of (surviving) young adults. The said identity may then be written as:

$$P_{1t}(\beta,\rho)\cdot \mathbf{p}_{1t}\cdot \mathbf{x}_{1t} + Q_{1t}(\alpha_{1t},\gamma,\sigma_{1t},\tau)\cdot e_{1t} \equiv p_{11t}\alpha_{1t}\cdot(\Lambda_{1t} + m\cdot N^1_{1t}(1-d^1_{1t})\gamma)/N^2_{1t}, \quad (4.2.5)$$

where the RHS of (4.2.5) is the level of (normalized) full income in the time t - subperiod of length m, and

$$\Lambda_{1t} \equiv m\cdot\left(N^2_{1t}(1-d^2_{1t})\lambda^2_{1t} + N^3_{1t}(1-d^3_{1t})\lambda^3_{1t}\right) \quad (4.2.6)$$

is the adults' aggregate supply of efficiency units of labor. While the rural household faces the price vector $\mathbf{p}_{1t} = (p_{11t}, p_{21t})$ for the two consumption goods, the term

$$P_{1t}(\beta,\rho) \equiv 1 + (\rho N^3_{1t} + \beta N^1_{1t})/N^2_{1t} \quad (4.2.7)$$

expresses the effect of the family's demographic structure on the 'price' of the consumption bundle $\mathbf{x}_{1t}$ relative to education[8]. Analogously, the 'price' of a unit of full education ($e_{1t} = 1$) is

$$Q_{1t} \equiv (m\cdot p_{11t}\alpha_{1t}(1+\tau)\gamma + \sigma_{1t})N^1_{1t}/N^2_{1t}\,. \quad (4.2.8)$$

Preferences and decisions

The extended family's decisions are motivated by egoistic and altruistic reasons, and by mutual norms and obligations. The young adults' preferences are defined over the levels of consumption in young adulthood and old age, $\mathbf{x}_{1t}$ and $\rho\mathbf{x}_{1t+1}$, respectively, and the human capital attained by their school-age children on attaining full adulthood (λ^2_{1t+1}), which they may value in both phases of their own lives. Investment in the children's education therefore produces two kinds of pay-offs, one altruistic, as expressed by the value directly placed on λ^2_{1t+1}, and the other selfish, inasmuch as an increase in λ^2_{1t+1} will also lead to an increase in $\rho\mathbf{x}_{1t+1}$ under the said social rules.

The young couple draws up a plan for current consumption and investment in the children's education based on the family's resources and expectations about its members' state of health and other relevant variables in the coming and future periods. Given assortative mating, the pair will agree wholly on what is to be done. Appealing to the law of large numbers in order to rid the system of any uncertainty about the realized levels of morbidity in period t,[9] so that (adjusted) full income in that period is non-

[8]Note that (4.2.7) is unaffected by the introduction of the subperiod.

[9]The incidence of morbidity is uncertain at the level of the individual. However, if the extended family is large enough, its realized levels of morbidity will differ little from the population rates.

stochastic, let the preferences be represented as follows:

$$E_tU_1 = b_1u(\mathbf{x}_{1t}) + b_2(1-q_{1t})E_t[u(\mathbf{x}^3_{1t+1})] + E_t[(1-q^2_{1t+1})(1-q_{1t+1})](1-q^1_{1t})n_{1t}\phi(\lambda^2_{1t+1}). \tag{4.2.9}$$

The functions u and ϕ are assumed to be strictly concave. Observe that parents are assumed to be altruistic only towards their own children. The altruistic motive makes itself felt only when they themselves are young and actually make the sacrifices, whereby λ^2_{1t+1} is non-stochastic by virtue of e_{1t} being non-stochastic. As altruism disappears in old age, the elderly make no bequests.

The individuals who make this decision, having survived thus far, will survive into old age with probability $(1-q_{1t})$ and consume the bundle $\mathbf{x}^3_{1t+1} = (1-m)\mathbf{x}(\mathbf{p}_{1t+1}, \alpha_{1t+1}\lambda^3_{1t+1}) + m\rho\mathbf{x}_{1t+1}$, where $\mathbf{x}(\mathbf{p}_{1t+1}, \alpha_{1t+1}\lambda^3_{1t+1})$ is the demand vector for consumer goods at prices $\mathbf{p}_{1t+1}$ and income $\alpha_{1t+1}\lambda^3_{1t+1}$. Should the individuals die, the corresponding von Neumann-Morgenstern utility index u is normalized to zero. Conditional on surviving into old age at $t+1$, $\rho\mathbf{x}_{1t+1}$, is a random variable viewed at time t, for its level depends on a whole variety of future economic and demographic developments. The other hazard is that the children will die as young adults in period $t+1$, each with probability q^2_{1t+1}. The lottery in question, therefore, involves the random number of individuals who survived through full young adulthood in $t+1$. The contribution of this lottery to expected utility is the last term in (4.2.9).

For simplicity, let there be perfect foresight about everything – with the vital exception of individual fates, i.e., whether an individual making a decision will die prematurely. Under this assumption, $\rho\mathbf{x}_{1t+1}$ becomes non-stochastic, conditional on surviving into old age at time $t+1$. After the migrants have departed, a young adult's decision problem therefore is compactly written as:

$$\begin{aligned} \max_{(\mathbf{x}_{1t},\, e_{1t}|\{\mathbf{Z}_{t+t'}\}_{t'=0}^{t'=\infty})} \quad & E_tU_1 = b_1u(\mathbf{x}_{1t}) + b_2(1-q_{1t})u({\mathbf{x}^3}^0_{1t+1}) \\ & + (1-q^1_{1t})(1-q^2_{1t+1})(1-q_{1t+1})n_{1t}\phi(\lambda^2_{1t+1}(e_{1t})) \\ \text{s.t. } & \mathbf{x}_{1t} \geq \mathbf{0},\, e_{1t} \in [0, (1-d^1_{1t})/(1+\tau)], (4.2.3), (4.2.5), \end{aligned} \tag{4.2.10}$$

where the rural endowment and environment at the time of decision, as described by the vector

$$\mathbf{Z}_t \equiv (\mathbf{N}_{1t}, M_t, n_{1t}, q^1_{1t}, q^2_{1t}, q_{1t}, \mathbf{d}_{1t}; \boldsymbol{\lambda}_{1t}, P_{1t}, Q_{1t}, \mathbf{p}_{1t}, \tau, \alpha_t). \tag{4.2.11}$$

are assumed to be known and treated as given, and the superscript '0' denotes the unerringly forecasted optimal choices of the next generation in $t+1$, namely $(\mathbf{x}^0_{1t+1}, e^0_{1t+1})$.

Let E_tU_1 at the optimum be denoted by $V_{1t}(\mathbf{p}_{1t}, q_{1t}, q_{1t+1}, n_{1t})$.

4.2.2 The urban sector

The urban sector is a simplified version of that in Bell and Koukoumelis (2009), with a constant savings rate out of profits and full education of urban children. The townies' preferences are analogous to their rural counterparts', as in (4.2.10).

Demographics

The number of urban households evolves according to

$$N_{2t+1}^{f}=\frac{N_{2t}^{f}(1-q_{2t}^{1})n_{2t}+M_{t+1}N_{1t}^{f}}{2}.$$

Newly arrived migrants are assumed to choose partners in an existing network, to experience the urban morbidity and mortality profiles, and to adopt urban fertility behaviour. Each couple produces n_{2t} children. The family age structure is summarized by the vector

$$\mathbf{N}_{2t}=\left((1-q_{2t}^{1})n_{2t},(1-q_{2t}^{2})\cdot 2,\frac{2(1-q_{2t-1})N_{2t-1}^{2}}{(1-q_{2t-1}^{1})n_{2t-1}(1+\xi_t)}\right), \tag{4.2.12}$$

where $\xi_t=M_tN_{1t-1}^{f}/(n_{2t-1}(1-q_{2t-1}^{1})N_{2t-1}^{f})$ is the number of newly arrived migrants in period t relative to the number of urban young adults who grew up in towns.

Output, physical and human capital in the urban sector

The technology in the urban sector exhibits constant returns to scale in physical capital, K, and efficiency units of labour. Aggregate output may be written as

$$Y_{2t}=F(K_t,L_{2t},t), \tag{4.2.13}$$

where, under the assumptions that workers inelastically supply their human capital endowments to firms and that children and the old are not involved in the production of the urban good,

$$L_{2t}\equiv(1-q_{2t}^{2})\left(\lambda_{1t}^{2}M_tN_{1t-1}^{f}+\lambda_{2t}^{2}n_{2t-1}(1-q_{2t-1}^{1})N_{2t-1}^{f}\right) \tag{4.2.14}$$

denotes the aggregate supply of efficiency units of labour. Recalling the definition of ξ, the supply per family is $\Lambda_{2t}=\frac{N_{2t}^{2}\left(\lambda_{2t}^{2}+\xi_t\lambda_{1t}^{2}\right)}{1+\xi_t}$

Urban children are assumed to spend all of their school-age years in school, except for the time during sickness, so $e_{2t}=(1-2/180)(1-d_{2t}^{1})/(1+\tau_2)$ is treated as exogenously given. Migrants are endowed with human capital λ_{1t}^{2} and make the same contributions to the formation of human capital among the upcoming generation. As in the rural sector, human capital is generated by a combination of formal educational technology and the average level of adults' human capital. Recalling that the fractions of migrants and native townies within a given age group are $1/(1+\xi_t)$ and $\xi_t/(1+\xi_t)$, respectively, that human capital does not depreciate ($\lambda^2_{it-1} = \lambda^3_{it}$), and that the fraction $(1 - q^2_{2t})$ of young adults die before human capital formation within the family takes place, the evolution of human capital is governed by

$$\lambda^2_{2t+1} = \frac{f_t(e_{2t})}{1-q^2_{2t}} \cdot \left(\frac{(\lambda^2_{2t} + \xi_t\lambda^2_{1t})N^2_{2t}}{(1+\xi_t)(N^2_{2t}+N^3_{2t})} + \frac{(\lambda^3_{2t} + \xi_{t-1}\lambda^3_{1t})N^3_{2t}}{(1+\xi_{t-1})(N^2_{2t}+N^3_{2t})} \right) + 1, \tag{4.2.15}$$

Whenever $\lambda^2_{1t} < \lambda^2_{2t}$, migration into towns reduces on the average level of the adults' human capital in that period and therefore dampens the urban human capital formation process. Workers receive wage income according to their respective human capital endowments. There is no urban unemployment, the (competitive) efficiency wage adjusts to the inflow of migrants. Since goods prices in the urban sector are normalized to unity, the wage is given by

$$w_t = \partial F/\partial L_{2t}. \tag{4.2.16}$$

An urban family's wage income in period t is thus $w_t \cdot \Lambda_{2t}$. Note that the efficiency wage moves in the opposite direction to ξ_t (and hence to M_t), given K_t and $N^2_{2t}\lambda^2_{2t}$.

Preferences are the same in both sectors. Since investment in the education of urban children is exogenous, the only remaining task is to determine the urban within-family consumption allocation, given the (normalized) budget identity

$$P_{2t} \cdot \mathbf{p}_{2t} \cdot \mathbf{x}_{2t} + Q_{21} \cdot e_{2t} = w_t \cdot \Lambda_{2t}/N^2_{2t}, \tag{4.2.17}$$

where the RHS is family income normalized to the number of young adults, $\mathbf{p}_{2t} = (1,1)$, $P_{2t} \equiv 1 + (\rho N^3_{2t} + \beta N^1_{2t})/N^2_{2t}$, and $Q_{21} \equiv \sigma_{2t}N^1_{2t}/N^2_{2t}$. Note that family wage income is exhausted by consumption of both goods and investment in education according to the same social norms as in the rural sector.

The aggregate capital stock arises from urban savings in the previous period. As in Lewis (1954), we assume 'classic' savings behaviour, in the sense that a fixed fraction, s, of the accruing profits is reinvested. We simply set $s = 1$. There is 100% depreciation of physical capital within the 20 years spanned by each new generation. A migrant arrives in the city with no savings in physical capital.

4.2.3 Migration equilibrium

The two sectors are connected by rural-urban migration, whose level is endogenously determined. A young adult in a village decides whether to migrate to the towns on the basis of the expected lifetime utilities offered by the two locations. A potential migrant's preferences are defined as in equation (4.2.9), but with with urban mortality and fertility. Denote a migrant's expected utility at the optimum, should he or she move, by

$V_{2t}(\mathbf{p}_{2t}, q_{2t}, q_{2t+1}, n_{2t}, w_{2t}, w_{2t+1})$. Equilibrium is brought about by a level of migration such that young adults who have grown up in villages are indifferent between migrating to the towns and staying in the rural sector. Formally, the migration equilibrium requires $V_{1t} = V_{2t}\,\forall t$, where the equilibrating variables are the number of migrants per family, M_t, and the efficiency wage rates.

4.3 Calibration

Given the degree of underidentification in the model, this chapter follows the standard procedure in the literature on computable general equilibrium models in employing (i) plausible functional forms, if only from common usage in the existing literature, (ii) whatever available econometric estimates for the parameters involved, and (iii) constellations for the remaining parameters such that the simulated model replicates a benchmark year and certain key magnitudes correspond to what is called 'stylized facts'.

4.3.1 Functional forms

Rural technologies and preferences are taken from Bell (2012), albeit some parameter values are adjusted significantly in order to ensure that the calibration algorithm succeeds.

Technologies

Let $f_t(e_{it}) = ze_{it}\,\forall t, i = 1, 2$, where $z > 0$ represents a constant inter-generational transmission factor, which reflects the quality of both child-rearing and the school system.

Let $F(K_t, L_{2t}, t)$ be of the Cobb-Douglas type, with constant returns to scale in physical and human capital, so that

$$Y_{2t} = F(K_t, L_{2t}, t) = \alpha_2 K_t^{\epsilon} L_{2t}^{1-\epsilon}, \;\; \epsilon \in (0, 1).$$

Since all profits are reinvested, the sectoral savings rate equals ϵ. With full depreciation of physical capital within one period $K_{t+1} = K_t \partial F / \partial K_t = \epsilon Y_{2t}$.

Preferences

Recall that preferences are the same in both sectors. We form the Cobb-Douglas aggregate $x_{1it}^{a} \cdot x_{2it}^{1-a}$ $(0 < a < 1, i = 1, 2)$. Applying the logarithm to this index of consumption, we obtain

$$u(\mathbf{x}_{it}) = a \ln x_{1it} + (1-a) \ln x_{2it} \quad \forall\, t,\, i = 1, 2. \tag{4.3.1}$$

Let $\phi(\cdot)$ be of iso-elastic form

$$\phi(\lambda^2_{it+1}) = 1 - (\lambda^2_{it+1})^{-\eta}/\eta\,, \eta > 0,\; i = 1, 2$$

4.3.2 Exogenous parameters

The road programme started in 2000, and since the whole numerical procedure hinges on perfect foresight, this implies that $t = 1$ begins in 2000 and ends at the close of 2019, and so forth for $t = 2, 3, \ldots$.

Demographics: rural sector

In the current setting, each generation corresponds to a span of 20 years, the age at which full adulthood is attained. With $m = 0.75$, the rate q_t therefore corresponds to ${}_{15}q_{25}$, the probability that an individual will die before reaching 40, conditional on reaching 25. We calculate Indian rural mortality rates using SRS-based abridged life tables. The average values in the rural area in the time period 2001-2010 are ${}_{15}q_{25} = 0.044$ and ${}_{25}q_{25} = 0.095$, respectively. Something closer to the latter is suited to our present purposes, to allow for some mortality in the first part of old age. Hence, let $q_{11} = 0.09$. For under-fives, the SRS-based value is $q^1_{11} = 0.083$. Finally, the mortality rate in the early phase of young adulthood is $q^2_{11} = 0.012$. In conjunction with the normalization $N^2_{1t} = 2(1 - q^2_{1t})$, $\forall t$ and the prevailing state of affairs in rural India, let $\mathbf{N}_{11} = (3.254, 1.976, 0.993)$, with $n_{11} = 3.551$ (United Nations, 2013).

If history and international experience are any guides, mortality rates across all age groups are sure to fall over the coming generation, PMGSY or no. Notably, the relevant rates have been fallen considerably and almost linearly in the last decades. It does not seem too much to hope that this trend will continue. We assume, therefore, that mortality rates will fall linearly until 2061 ($t = 4$) (for example, from $q_{11} = 0.09$ to $q_{14} = 0.07$). In contrast, the drop in fertility has started and is assumed to become stationary earlier than mortality. Thus, fertility is also assumed to fall linearly from its current level to replacement fertility in 2041 ($t = 3$); see the upper panel of Table 4.3.1, where we also report the estimated vital rates for $t = 0$.

Where morbidity and disability are concerned, school-age children typically suffer less sickness than their parents, who, in turn, are in better health than their aged parents. The findings from the Orissa survey confirm as much, the average number of days of reported sickness being 5.6 a year. This number covers those individuals who suffered from chronic ailments whenever the morbidity associated with their chronic conditions falls into the category of acute sickness. Following Bell (2012), we use the vector $\mathbf{d}_{11} = (0.02, 0.04, 0.08)$ as an estimate for period 1. Since it is easier to ward off premature death than morbidity, the associated improvement to $\mathbf{d}_{1t} = (0.015, 0.03, 0.06)$, $t \geq 2$ is assumed to be less dramatic than that in mortality.

Finally, we estimate an all-India flow of rural out-migration between 1981 and 2001

Table 4.3.1: The demographic transition with and without the road

Period		0	1	2	3	4
No Road	n_{1t}	4.448	3.551	2.809	2.096	2.056
	q_{1t}^1	0.150	0.083	0.064	0.046	0.027
	q_{1t}^2	0.017	0.012	0.0107	0.009	0.008
	q_{1t}	0.125	0.090	0.083	0.077	0.070
	n_{2t}	3.522	2.758	2.399	2.048	2.024
	q_{2t}^1	0.083	0.046	0.0347	0.0233	0.012
	q_{2t}^2	0.009	0.007	0.006	0.006	0.005
	q_{2t}	0.090	0.070	0.065	0.061	0.056
Road V1	n_{1t}	4.448	3.519	2.790	2.0857	2.050
	q_{1t}^1	0.150	0.0747	0.0579	0.0411	0.0243
	q_{1t}^2	0.017	0.0106	0.0096	0.0084	0.0072
	q_{1t}	0.125	0.081	0.0747	0.0693	0.063
Road V2	n_{1t}	4.448	3.504	2.780	2.081	2.047
	q_{1t}^1	0.150	0.071	0.055	0.039	0.023
	q_{1t}^2	0.017	0.010	0.009	0.0079	0.0068
	q_{1t}	0.125	0.0765	0.0708	0.0652	0.0595

V1: 10% mortality reduction, V2: 15% mortality reduction.

of individuals aged 5-19 in the year 1981. The estimation involves the following steps. First, we use Census data for the initial (1981) and final year (2001), and the mortality rates as reported in the SRS-based life tables to construct a hypothetical rural population pyramid for 2001 that would represent the population age structure if no internal migration had occurred. Second, and in order to correct potential measurement errors concerning the vital rates, the latter is scaled so that the total rural population sums to the actual population number in the final Census year (2001).[10] Third, the difference between the hypothetical and the actual final population in each age class gives the desired estimate for rural-urban migration. Finally, since migration is a flow variable between census dates, and not a stock, the migration *rate* of 12.34% is the average from using the initial population (aged 5-20) and the final population (aged 25-40) as the benchmark population. Our time structure and the scaling to a young couple that survived the early phase of young adulthood gives $M_0 = 0.2439$. The calculation of λ_{22}^2 requires information about ξ_0, which in turn requires N_{1-1}^f and N_{2-1}^f, respectively. Given $\mathbf{N}_{it}$ and the total population by sector, F_{it} say, from Census data, one can readily calculate the numbers of families for each sector and each period:

$$N_{it}^f = F_{it}/(N_{it}^1 + N_{it}^2 + N_{it}^3),\ i = 1, 2.$$

Using $N_{1-1}^f = 54184000$, $N_{2-1}^f = 14461000$, $n_{2-1} = 3.49$, and $q_{2-1}^1 = 0.0829$ gives[11] $\xi_0 = 0.286$. Finally note that the (exogenous) sequences $\{n_{it}\}$, $\{\mathbf{q}_{it}\}$ allow to calculate all future demographic variables for *any* migration stream $\{M_t\}$, $t = 1, 2, \ldots$.

Demographics: urban sector

The abridged life tables give the rates ${}_{15}q_{25} = 0.032$ and ${}_{25}q_{25} = 0.074$, so we choose $q_{21} = 0.07$. Mortality rates in the subperiod are $q_{21}^1 = 0.046$, and $q_{21}^2 = 0.007$, respectively. Similar to their rural counterparts, these rates are assumed to fall linearly along the demographic transition (again see Table 4.3.1). With the normalization $N_{21}^2 = 2(1 - q_{2t}^2)$, the census data give $\mathbf{N}_{21} = (2.631, 1.986, 0.944)$, with $n_{21} = 2.758$ (United Nations, 2013). A diligent search, yielded no quantitative estimates concerning rural-urban differences in morbidity. Since Bell and van Dillen (2014) focus on the effects of PMGSY in a remote and backward region with high poverty and low scores on other social indicators, we assume that townies are still in better health than their rural counterparts in the presence of PMGSY: $\mathbf{d}_{21} = (0.01, 0.02, 0.04)$ and $\mathbf{d}_{2t} = (0.0075, 0.015, 0.0338)$, $t \geq 2$ (see Section 4.3.4).

[10] The procedure abstracts, however, from international migration.

[11] In order to get some historical depth, use the same steps to calculate $\xi_{-1} = 0.3389$.

Human capital

Human capital in both sectors evolves according to (4.2.3) and (4.2.15), given some starting values for period $t = 0$. As to the latter, there is much illiteracy among the old in rural areas, but less among their children, who are today's parents. Literacy rates for the age group 7+ increased from 44.7% in 1991 to 68.9% in 2011. Rising productivity over the past generation also suggests that λ_{11}^2 is substantially larger than λ_{11}^3. Therefore, we impose $\boldsymbol{\lambda}_{11} = (1.9, 1.2)$. The urban literacy rate increased less dramatically than its rural counterpart, from 73.1% to 85.0% over the same period. While the sectoral gap has decreased, urban literacy is still about 16 percentage points higher than in rural areas. This consideration indicates that $\boldsymbol{\lambda}_{21} = (2.2, 1.4)$ is a reasonable guess to accompany $\boldsymbol{\lambda}_{11} = (1.9, 1.2)$.

Prices

For simplicity, set the prices of both goods in the town at unity in all periods. The consequences of this assumption for the economy's growth path are discussed below. A survey of 30 villages in upland Orissa for the year 2009-10 (Bell and van Dillen (2014)) yields the finding that in the absence of an all-weather road, the unit transport costs for paddy, the main crop, were a bit less than 0.1, but those for other commercial crops somewhat higher. Applying the same to fertilisers, seeds and other goods bought in, households in such villages then face the price vector $\mathbf{p}_{1t} = (0.9, 1.1)\ \forall\, t$. As for the trip to school, the great majority of India's villages have a primary one of their own; but only a small minority have a secondary or high school, and in the absence of an all-weather road, the daily round-trip time can be rather long. The averages for primary and secondary school pupils in the Orissa villages lacking such a road were 23 and 76 minutes, respectively. Allowing, say, twelve hours for sleeping, eating and bathing at home, and taking into account the higher opportunity costs of older children's time, let $\tau = 0.08$. The direct costs of state schooling are surely modest: recalling (4.2.8), let σ_{1t} be 0.15 times the opportunity cost factor $m \cdot p_{11t}\alpha_{1t}(1 + \tau)\gamma$, where $\gamma = 0.65$ is taken from Bell (2012).

Preferences

Recall that preferences are the same in both sectors by assumption. Let the social norms demand $\beta = 0.6$ and $\rho = 0.8$. Households are still rather poor, so their taste for good 1 should be at least as strong as that for good 2: accordingly, let $a = 0.5$.

The search algorithm described in Appendix 4.A.2 succeeds only when the sub-utility function ϕ has relatively strong curvature. We choose $\eta = 0.65$, which is substantially higher than that in Bell (2012), namely, 0.1. Note, however, that the 0.65-value is not implausible. In their study of Kenya over the historical period 1950-1990, Bell et al. (2006) found the values of η lie in the range 0.35 – 0.65, with a clustering around 0.5.

4.3.3 Endogenous parameters

The efficiency parameter in the production functions (α_1 and α_2), as well as the efficiency parameter in the educational technology (z), and the inter-temporal taste parameters (b_1 and b_2) are chosen so as to satisfy the following calibration desiderata.

First, the households' current choices of the level of investment in education in both sectors should be in line with what we observe in the present. Children in India's rural areas typically start school at 6 years of age (that is, $m = 0.75$) and complete about 6 years of schooling on average. Hence, with up to 12 years of schooling available, and noting that morbidity reduces the endowment of productive time, the model must be set up so as to yield $(1+\tau)e_{11}^0 = (6/12)(1-d_{11}^1)$. A further adjustment is needed for the number of days lost due to bad weather, especially in the monsoon. The Orissa survey yields an estimated 8.6 and 9.4 days a year, on average, for primary- and secondary-school children, respectively, involuntary absences that were mainly attributable to their teachers' failure to arrive (Bell and van Dillen (2014)). Assuming a school year of 180 days, the required condition becomes $(1 + \tau)e_{11}^0 = (1 - 9/180)(6/12)(1 - d_{11}^1) = (19/40)(1 - d_{11}^1)$, where $\tau = 0.08$ and $d_{11}^1 = 0.02$.

We anchor the sequence to some plausible configuration in the future. Let us suppose, in line with the so-called Millennium Development Goals, that a full education for all is attainable within one complete generation, where the definition of a 'full education' must make a full allowance for the claims on a child's time made by illness, travelling to school and other involuntary absences. That is to say, the model must be set up such that parents in period 2 do choose $e_{12}^0 = \bar{e}_{12} = 0.95(1 - d_{12}^1)/(1 + \tau)$. In anticipating that the next generation's optimal choice involves a full education, the young adults in period 1 do choose $e_{11}^0 = 0.431$.

The parameters (α_1, b_1, b_2) are chosen so as to meet these requirements. We impose pure impatience for consumption, i.e. $b_1 > b_2$. Since premature mortality already appears in connection with preferences over dated consumption, the pure discount rate arguably should not greatly exceed 10 per cent or so per generation of 20 years. By hazarding a guess at α_1, we are then left to find a pair (b_1, b_2), with $b_2 \approx 0.9b_1$, such that the solution to problem (4.2.10) in period 1 indeed involves $e_{11}^0 = (19/20)(6/12)(1-0.02)/1.08$ *and* $e_{12}^0 = 0.866$ is barely attained. We adopt $b_2/b_1 = 0.8$ and $\alpha_1 = 5$ from Bell (2012).

We turn to urban output, beginning with total factor productivity. We set $\alpha_2 = 6.75\,\forall t$ to get an empirically defensible, sectoral aggregate output ratio of one-to-one. While this ratio seems to be too high at first glance (the rural share in gross domestic product was about 23 per cent in the year 2000), it should be noted first that the service sector in towns is not included in the simulation, and second, that estimates of the urban capital-output ratio are available and must be largely respected. There has been much fluctuation in the latter since 1991. A recent estimate for manufacturing in the year 2000 is $K_1/Y_{21} = 4.33$ years (Verma (2012)). Since we assume 100% depreciation, the lifetime of capital goods is one period. In the current setup, one period spans 20 years, so

Verma's estimate translates into 0.2165(= 4.33/20). A capital-output ratio fairly close to this estimate ($0.276 \times 20 = 5.5$ years) is obtained by choosing an initial capital stock per urban family of $k_{11} = 14.732$. Also note that the assumption of zero growth in total factor productivity does not rule out growth in productivity per worker; for individual human capital may well grow without bound when children are fully educated.

Finally, the population vectors and the initial human capital endowments allow for the possibility that $\lambda_{it}^2 < \lambda_{it-1}^2$. In order to rule out such behaviour in human capital formation, we set z sufficiently high, namely $z = 1.3$. The transmission factor is also the candidate for achieving the calibration target $e_{12}^0 = 0.866$ (see Appendix 4.A.2). Table 4.3.2 summarizes the parameter values.

We close this section by noting that the household decision problems in both sectors can be solved for any given migration sequence; for once the next generation's investment in education is correctly anticipated by the current period's young adults, they have all the information needed to determine $\mathbf{p}_1\mathbf{x}^{3^0}_{1t+1}$ from (4.2.5), whereupon $\mathbf{x}^{3^0}_{1t+1}$ follows from (4.3.1). Appendix 4.A.2 provides a compact description of the algorithm employed to generate $b_1^0 = 29.375$, which is used in the simulations. Complications arise because the calibrated value for b_1 depends on the migration equilibrium (see Appendix 4.A.2). The value for b_1^0 so derived is remarkably close to that obtained in Bell (2012).

Table 4.3.2: Calibration: parameters without the road

Preferences	
(β, ρ)	(0.6,0.8)
(b_2/b_1)	0.8
η	0.65
a	0.5
Technology	
γ	0.9
z	1.3
α_1	5
α_2	6.75
ϵ	1/3
Prices	
$\mathbf{p}_1$	(0.9, 1.1)
$\mathbf{p}_2$	(1, 1)
τ_1	0.08
τ_2	0.04
σ	0.15

4.3.4 Life with the road

The improvements induced by the roads in the spheres of goods, education and health are mainly taken from Bell and van Dillen (2014). They estimate that unit transport costs are reduced by about 5 per cent of the net price received for commodities marketed. Khandker *et al.* (2009) obtain a similar estimate for rural roads in Bangladesh, once an allowance is made for greater volumes marketed. The latter also estimate that the farm-gate price of fertilisers declined by 5 per cent. With a new road, therefore, let $\mathbf{p}_{1t} = (0.95, 1.05), \forall t$. Where schooling is concerned, the analysis of the Orissa survey yields substantial savings in travel-time for secondary-school pupils (just over 30 minutes a day) and far fewer days of involuntary absences for all grades (about 2 instead of about 9 for primary schools and 14 for secondary schools). These estimates imply $\tau = 0.04$ and $\bar{e}_{1t} = (1 - 2/180)(1 - d_{1t}^1)/(1 + 0.04) = 0.9508(1 - d_{1t}^1)$, where $\mathbf{d}_{11} = (0.015, 0.03, 0.06)$ is derived from Bell and van Dillen (2015) who estimate that PMGSY reduces the average duration of incapacitating illness by 1.4 days a year. We further impose $d_{2t} = (0.01125, 0.0225, 0.045)$, $t \geq 2$. Let the road be perfectly durable, thereby maintaining these more favourable prices of goods, as well as travel-times to, and involuntary absences from, school indefinitely.

One might expect substantial mortality improvements as the road makes (timely) treatment more likely. Moreover, mortality improvements are likely to be non-neutral in the sense that the reduction in mortality rates is age-specific. No such pattern is found in the sample households (Bell and van Dillen, 2014). However, the respondents in the village focus group interviews claimed that mortality rates had fallen by 25%. In light of these mixed findings, let the provision of the road reduce current and future mortality rates in all age groups by 10% (Variant 1).

4.4 Results

We report results for the PMGSY programme and the counterfactual without it up to and including period $T = 4$. Since the maximization problems in period T require information about the vector Z_{T+1}, we extrapolate the growth rates associated with the sequence $\{M_t\}_{t=1}^{T-1}$ to obtain a first estimate for the value M_{T+1}.

4.4.1 The counterfactual: no road

A long phase of low-to-moderate growth of GDP per capita at an annual rate of 1.3%[12] and high population growth at the annual rate of about 2.3% between the 1950's and 1991 had generated pressure on rural India. In $t = 1$, the rural population is 67% which is close to the official 70% estimate (United Nations, 2014). Viewed in this light, the trajectories of the key economic variables from $t = 1$ (2000-2020) are intuitively plausible

[12] In aggregate, the economy grew at the 'celebrated' Hindu rate of 3.5% p.a.

(see Table 4.4.1). Low incomes and a more hazardous health environment in the rural sector made far strong rural-urban migration in period 1, with a peak immediately following ($t = 2$), when M_2 - further spurred by the drop in rural fertility - will be double that in $t = 1$. The model predicts that mortality-induced migration accounts for 5.63 per cent and 4.02 per cent, respectively, of the total migration streams in those periods[13]. It also indicates a substantial structural change in the aggregate economy at about the year 2040 ($t = 3$). The preceding peak in migration and decreasing marginal productivity in the urban sector make migration to towns increasingly unattractive (the efficiency wage adjusts to in-migration, so $w_2 = 5.061 << w_1$), so migration declines sharply, albeit with a partial recovery in $t = 4$, which reflects the fact that the economy is far away from the steady state in which migration has come to a halt.

In this connection, note that under stationary technological and demographic conditions, equation (4.2.3) becomes

$$\lambda^2_{1t+1} = f(\bar{e}_1)(a_2\lambda^2_{1t} + a_3\lambda^2_{1t-1}) + 1,$$

where $f(\bar{e}_1) = z \cdot 0.866$, $a_2 = N^2/(N^2 + N^3) = 1.984/(1.984 + 1.845) = 0.518$ and $a_3 = N^3/(N^2 + N^3) = 1.845/(1.984 + 1.845) = 0.482$ are constants and $\lambda^2_{1t-1} = \lambda^3_{1t}$. The relevant characteristic root[14] is $.5 \cdot a_2[1 + \sqrt{(}1 + 4a_3/(a_2^2 f(0.866)))]f(0.866)$. Moreover, rural output grows at the same rate as λ^2_1. The value chosen for z, 1.3, implies that the rural sector will grow without bound. In fact, the asymptotic growth rate is $g^*_1 = 8.5\%$ per generation. A more favourable mortality regime in towns implies that urban human capital grows at a higher rate than in the rural sector. Moreover, since all profits are reinvested and there is no TFP growth (in α_2) by assumption, urban output per family will grow at the same rate as λ^2_2, namely at $g^*_2 = 14.4\%$ per generation. As sector 2's growth rate will exceed sector 1's in the asymptotic limit, the rural sector will continue to lose population, and at some point, and as a consequence of assuming fixed goods prices, the demand for good 1 must be satisfied by imports. However, this will not happen in the time span considered.

In 2060 ($t = 4$), a rural young adult is projected to be more than twice as productive as his or her great-grandparents were in young adulthood in period 1. Likewise, the level of real consumption will have doubled by that time. Due to price differences and zero opportunity costs of education, the value of the urban optimal consumption bundle is slightly lower than its rural counterpart. The rural sector is projected to produce 19.1 per cent of total gross domestic output, starting from an initial share of almost one

[13]We constructed an artificial migration equilibrium when the urban mortality rates are the same as their rural counterparts. The migration sequence without an rural-urban mortality gap is $M'_t = \{0.244, 0.444, 0.921, 0.319, 0.487, 0.430\}$. It should be noted that the calibration target $e^0_{11} = 0.431$ is (almost) unaffected by this constellation.

[14]The characteristic roots are derived and set out in Bell (2012) and Bell et al. (2006). In general, the eigenvalues of the linear second-order difference equation $\lambda_{t+1} = \phi_1\lambda_t + \phi_2\lambda_{t-1} + c$, where c is a constant, are $\frac{\phi_1 \pm \sqrt{\phi_1^2 + 4\phi_2}}{2}$ (see, e.g., Hamilton, 1994).

half. The migration flows in equilibrium and the vital rates imply that total population will reach 1.74 billion people in period 4, a number lying comfortably between the high fertility projection and the zero (international) migration projection for the year 2060 provided by the United Nations, namely 1.993 and 1.644 billion people, respectively (United Nations, 2013). The rural population will make up only 31 per cent of the total population at that time.

Table 4.4.1: The sequences of the endogenous variables without the road

Period		0	1	2	3	4
Rural	x_{11t}		3.553	3.637	5.622	7.832
	x_{21t}		2.907	2.975	4.600	6.408
	e^0_{1t}		0.431	0.866	0.866	0.866
	λ^2_{1t}		1.900	1.928	3.158	3.934
	y_{1t}		29.699	33.593	45.380	81.834
	Y_{1t}/Y_t		0.489	0.341	0.274	0.191
Urban	$x_{12t} = x_{22t}$		2.965	3.185	4.788	6.741
	e^0_{2t}		0.941	0.944	0.944	0.944
	λ^2_{2t}		2.200	3.239	3.961	4.894
	k_t		14.732	12.177	21.083	34.590
	y_{2t}		53.215	54.624	86.118	121.977
	k_t/y_{2t}		0.276	0.223	0.245	0.284
	w_t		6.340	5.061	5.579	6.501
	$w_t \times \Lambda_{2t}$		26.075	26.766	42.198	59.769
Demographics	M_t	0.244	0.547	1.065	0.388	0.755
	ξ_t	0.286	0.408	0.694	0.141	0.270
	N^3_{1t}		1.063	1.642	1.619	1.845
	N^3_{2t}		0.793	0.829	1.406	1.471
	N^f_{1t}	63.4	101.0	110.6	123.9	87.1
	N^f_{2t}	25.1	58.9	131.2	173.4	220.2
	F_{1t}		635.7	691.2	694.0	534.0
	F_{2t}		318.6	673.5	935.5	1,202.5
	F_{1t}/F_t		0.67	0.51	0.43	0.31

$b^0_1 = 29.575$. $M_5 = 0.450$. N^f_{it} and $F_{i,t}$ in millions.

4.4.2 PMGSY

The effects of the programme are more easily seen in graphs. The trajectories of the main variables, with and without PMGSY, are depicted in Figure 4.A.1. The numerical details for those with PMGSY are given in Table 4.4.2.

A striking effect of PMGSY is to reduce the scale of migration in all periods under consideration, as it brings about significant improvements in rural living conditions. First, reductions in mortality induce young adults to choose higher investments in education in $t = 1$. The latter is further spurred by an increase in realized net fertility[15] and hence in the weight on altruism in equation (4.2.9). Second, the reductions in involuntary absences and in the time needed to travel to and from school result in a higher natural upper bound on the level of education. In sum, the road promotes human capital formation and output in the rural sector, the latter being directly affected by the involvement of school-age children in the production of the rural good in period 1. While the transmission factor z is sufficiently high to ensure $\lambda_{12}^2 > \lambda_{11}^2$, the value of z in conjunction with the initial population and human capital endowment vectors and the endogenously determined migration flow in the first period implies that human capital formation accelerates only after some time has been passed. As a consequence, the aforementioned growth effect (concerning $\bar{e}_1$) makes itself felt only for $t \geq 2$. Three generations on, in $t = 4$, the levels of real productivity and consumption are 9.35 and 9.16 per cent higher, respectively. By that time, the shares of the rural sector in output and population have been fallen to 42 and 60 per cent, respectively. Table 4.A.1 reports the sequences of the main variables when there is a 15% reduction in mortality (Variant 2). In this more benign environment, the levels of real productivity and consumption in $t = 4$ are about 9.9 and 9.7 per cent higher, respectively.

PMGSY has similar effects in the urban sector, for reduced in-migration enhances physical and human capital accumulation. In $t = 4$, the level of real consumption in the urban sector is about 14.5 per cent (Variant 1) and 15.8 per cent (Variant 2) higher, respectively, than without PMGSY.

4.4.3 The equivalent variation and its decomposition

For the chosen constellation of functional forms and parameter values, how large is the willingness to pay for the more benign economic and health environment yielded by the road? This is obtained by estimating the equivalent variation for rural young adults in period t in the following way (Bell (2012)). One calculates lump-sum payment to be added to the family's (normalized) full income and its allocation subject to the social norms expressed by β and ρ, which leaves them indifferent between having the road and living without it. Since individuals desire to smooth out consumption over the life-

[15]Realized net fertility is positively affected by the road because more women survive childbearing age and more children survive their first years (see Table 4.3.1). As a consequence, total population with the roads in $t = 4$ is slightly higher in the counterfactual.

Table 4.4.2: The sequences of the main variables with the road (Variant 1)

Period		0	1	2	3	4
Rural	x_{11t}		3.544	3.708	5.695	8.161
	x_{21t}		3.207	3.355	5.152	7.384
	e^0_{1t}		0.461	0.940	0.940	0.940
	λ^2_{1t}		1.900	1.998	3.199	4.302
	y_{1t}		29.190	32.287	44.249	71.486
	Y_{1t}/Y_t		0.530	0.440	0.428	0.417
Urban	$x_{12t} = x_{22t}$		3.222	3.559	5.376	7.717
	e^0_{2t}		0.941	0.944	0.944	0.944
	λ^2_{2t}		2.200	3.229	4.032	4.990
	k_t		17.753	14.987	27.001	46.486
	y_{2t}		59.002	61.845	99.696	145.339
	k_t/y_{2t}		0.301	0.242	0.271	0.320
	w_t		6.915	5.520	6.197	7.369
	$w_t \times \Lambda_{2t}$		28.911	30.304	48.851	71.216
Demographics	M_t	0.244	0.266	0.607	0.014	0.285
	ξ_t	0.286	0.208	0.526	0.009	0.094
	N^3_{1t}		0.978	1.373	1.402	1.955
	N^3_{2t}		0.924	0.920	1.590	1.708
	N^f_{1t}	63.4	111.5	147.6	193.0	182.2
	N^f_{2t}	25.1	48.9	98.1	114.6	125.4
	F_{1t}		692.5	883.1	1,039.2	1,082.5
	F_{2t}		270.8	512.4	639.4	714.3
	F_{1t}/F_t		0.72	0.63	0.62	0.60

$b^0_1 = 29.575$. $M_5 = 0.356$. N^f_{it} and F_{it} in millions.

cycle, let there be another payment of equal size in period $t+1$, conditional on the individual surviving into old age. The payment in question is denoted by T_t^2. The total undiscounted benefit accruing to a *surviving* member of the generation born in $t=1$ (i.e. 2×0.887 - see Table 4.4.3) is 12.6 per cent of the output produced over his or her remaining life-cycle (that is, $2 \times 1.017/(0.9 \times 5 \times 1.9 \times ((1-0.04)+(1-0.08)))$. By assumption, roads do not deteriorate, so that the benefits accrue in full in all subsequent periods, causing this measure to increase to 26.9 per cent for those who are young adults in period 2. The equivalent variations associated with the still healthier environment of Variant 2 are 13.2 per cent in period 1 and 27.4 per cent in period 2, respectively. (see Table 4.A.2).

Our estimates are based on the calibrated taste parameter b_1, which is fairly close to the one of Bell's (2012). In Bell (2012), the corresponding numbers are 8.2 per cent and 15.4 per cent, respectively. Our estimates are higher for two reasons. First, we account for a positive effect on the health outcome. Second, improvements in rural living-conditions make human capital more unlikely to exit the rural sector, further contributing to the overall welfare gains of PMGSY.

Table 4.4.3: The EV and its decomposition (Variant 1)

Period		1	2	3
No road	V_{1t}	57.165	65.115	86.591
Road	V_{1t}	60.410	70.548	94.043
	T_t^2	1.017	2.234	3.820
Decompositions	$V_{1t}(\Delta\mathbf{p})$	59.819	67.320	87.716
	$T_t^2(\Delta\mathbf{p})$	0.829 (81.5%)	0.926 (41.5%)	0.571 (15.0%)
	$V_{1t}(\Delta\mathbf{p}^c)$	57.742	68.169	91.903
	$T_t^2(\Delta\mathbf{p}^c)$	0.178 (17.5%)	1.272 (56.9%)	2.700 (70.7%)

Note: Results are reported for the case where the payments affect the migration decision. To obtain the EV for $t>1$, we reset the expected utility *and* migration sequences to their values without the road and without the payment. However, the latter is relatively insensitive to whether it affects migration or not. The contributions do not add to one; for the equivalent variation is not additively separable in its components.

In order to get hold of the individual contributions stemming from the various changes induced by the road, we decompose the overall effect into (i) the effect stemming from the reduction in transport costs alone, the associated equivalent variation being denoted by $T_1^2(\Delta\mathbf{p})$; and (ii) the effect stemming from a combined contribution of the time available for schooling, morbidity and mortality, respectively, the associated equivalent variation being denoted by $T_1^2(\Delta\mathbf{p}^c)$. Due to initially high out-migration benefits in the non-commercial spheres make themselves felt only after some time has been passed, so that the contribution of the reduction in transport costs to the overall benefit is significantly higher than the 44% figure reported in Bell (2012). Over the longer run, there is clear trend for the relative contributions: the value of the reduction in transport costs, $T_1^2(\Delta\mathbf{p})$, is 81.5 per cent of T_1^2 in period 1; it falls to around 41.5 per cent of T_2^2 in period 2, and just 15 per cent in period 3. In the first two periods, these values are fairly close to the differences $T_t^2 - T_t^2(\Delta\mathbf{p}^c)$, indicating that the magnitude of the interaction effects of changes in the terms of trade, education, morbidity and mortality is relatively modest. However, and not surprisingly, uncertainty increases as time passes.

4.5 Conclusions

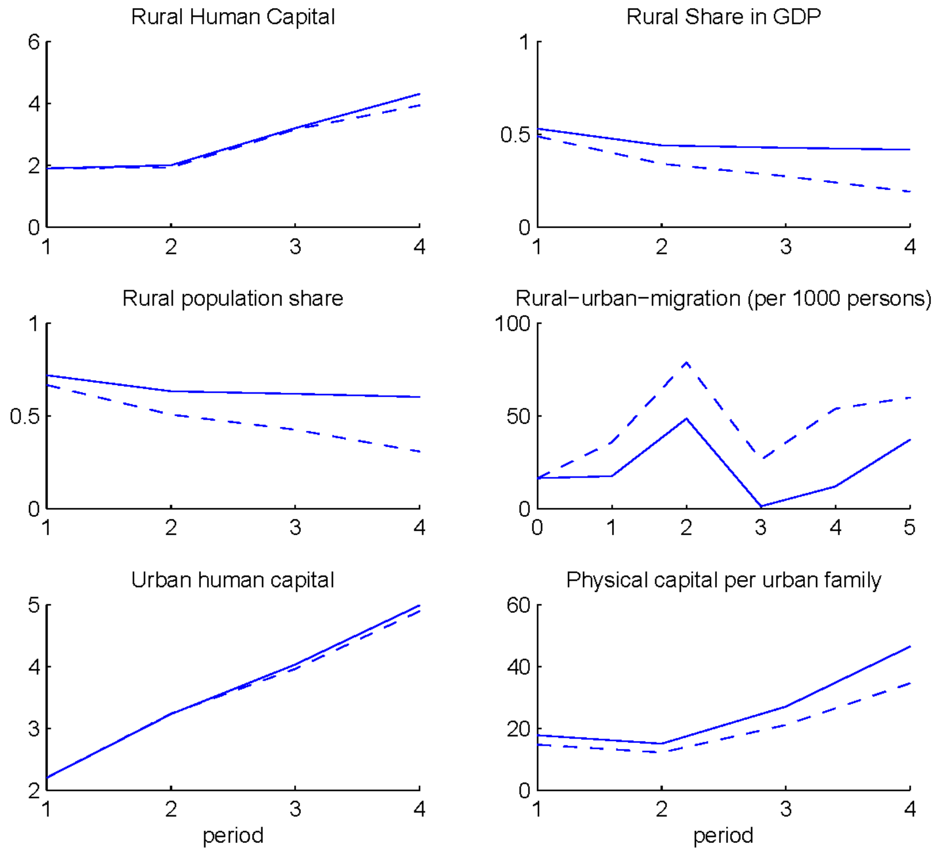
Rural roads enable the movements of goods and people. Yet although a simple dirt track is no barrier to migration to towns in search of employment, rural all-weather roads influence rural-urban migration by improving rural living standards and human capital accumulation. Growth effects are also large. The overall benefits generated by PMGSY are estimated to rise from about 12 per cent of the value of a surviving young adult's output in the absence of a road in the period 2000-20 to about one-fourth thereof in 2020-40. A decomposition of the equivalent variation indicates that improvements in the villager's terms of trades yield only a weak improvement in rural welfare in the long run.

The model indicates that urban dwellers also benefit from rural roads and the ensuing lower in-migration of rural young adults whose human capital endowments typically falls short of the endowments of those who grew up in towns. First, there is less downward pressure on the urban wage rate per efficiency unit of labour. Second, reduced in-migration enhances human capital accumulation in towns. A thorough evaluation of the effects on urban welfare, however, requires a more detailed modelling of the urban sector as was undertaken in this chapter. For example, future research should investigate the urban sector's capability to absorb the inflow of migrants.

4.A Appendix to Chapter 4

4.A.1 Plots and tables

Figure 4.A.1: Selected variables with and without the road



Solid lines: road (Variant 1). Dashed lines: no road.

Table 4.A.1: The sequences of the main variables with the road (Variant 2)

Period		0	1	2	3	4
Rural	x_{11t}		3.5140	3.711	5.728	8.204
	x_{21t}		3.203	3.357	5.183	7.422
	e^0_{1t}		0.463	0.940	0.940	0.940
	λ^2_{1t}		1.900	2.005	3.219	4.323
	y_{1t}		29.139	32.273	44.425	71.809
	Y_{1t}/Y_t		0.532	0.445	0.439	0.431
Urban	$x_{12t} = x_{22t}$		3.235	3.585	5.439	7.807
	e^0_{2t}		0.941	0.944	0.944	0.944
	λ^2_{2t}		2.200	3.228	4.038	4.998
	k_t		17.914	15.195	27.777	47.594
	y_{2t}		59.305	62.381	101.421	147.296
	k_t/y_{2t}		0.302	0.244	0.274	0.323
	w_t		6.943	5.550	6.270	7.447
	$w_t \times \Lambda_{2t}$		29.060	30.567	49.696	72.175
Demographics	M_t	0.244	0.197	0.584	0.016	0.285
	ξ_t	0.286	0.197	0.513	0.011	0.087
	N^3_{1t}		0.974	1.369	1.393	1.949
	N^3_{2t}		0.933	0.928	1.622	1.719
	N^f_{1t}	63.4	111.9	149.5	197.6	188.1
	N^f_{2t}	25.1	48.4	96.4	110.4	120.0
	F_{1t}		694.9	893.7	1,062.7	1,116.3
	F_{2t}		268.8	504.2	619.4	685.0
	F_{1t}/F_t		0.72	0.64	0.63	0.61

$b^0_1 = 29.575$. $M_5 = 0.286$. N^f_{it} and F_{it} in millions.

Table 4.A.2: The EV and its decomposition (Variant 2)

Period		1	2	3
No road	V_{1t}	57.165	65.115	86.591
Road	V_{1t}	60.547	70.849	94.572
	T_t^2	1.061	2.358	4.098
Decompositions	$V_{1t}(\Delta\mathbf{p})$	59.819	67.320	87.716
	$T_t^2(\Delta\mathbf{p})$	0.829 (78.1%)	0.930 (39.4%)	0.570 (13.9%)
	$V_{1t}(\Delta\mathbf{p}^c)$	57.874	68.446	92.195
	$T_t^2(\Delta\mathbf{p}^c)$	0.219 (20.6%)	1.388 (58.9%)	2.847 (69.5%)

4.A.2 Algorithm

Algorithm 1 Calibration and the sequence under perfect foresight

Start with initial guesses: α^g, b_1^g, $(b_1/b_2)^g$ and $\{M_t\}_{t=0}^{T+2}$.
repeat
  repeat
    repeat
      for $t = 1 : T$ **do** ▷ T: # of periods
        (i) Solve the rural household's decision problem under $E_1 e_{12}^0 = \bar{e}_2$,
        (ii) Solve the urban household's decision problem,
        (iii) Calculate the household's next period budget constraint in both sectors
      end for
      Use the resulting $\{M_t^0\}_{t=0}^{T+2}$ to calculate all demographic variables
    until Convergence of $\{M_t^0\}_{t=0}^{T+2}$ ▷ Migration equilibrium
    if $e_{11}^0 \neq 0.431$ **then** ▷ Calibration b_1
      Adjust b_1 (e_{11}^0 is decreasing with b_1)
    end if
  until $e_{11}^0 = 0.431$
  if $e_{1t}^0 \leq \bar{e}_{1t}$, $t \geq 2$ or $e_{1t}^0 >> \bar{e}_{1t}$, $t \geq 2$ **then** ▷ Calibration z
    Adjust z ($e_{1t}^0, \forall t$ is increasing with z).
  end if
until $e_{1t}^0 = \bar{e}_{1t}$, $t \geq 2$ ▷ Sequence

Chapter 5

Summary and Conclusions

A salient feature of the economic outcome is income and wealth inequality (Piketty and Saez, 2014). Yet economic inequality seems to arise almost naturally from demographic developments. For example, there is a seemingly never-ending sequence of generations that differ, amongst others, in size and life expectancy, as was impressively demonstrated by the phenomena of the demographic transition, starting at about 1800, and the baby boom - baby bust scenario in the aftermath of World War II. Chapter 1 summarized the ensuing redistributional effects in the economy. At the individual level, the members of a given generation typically differ with respect to the time when death afflicts them, and the place of birth. The resulting economic heterogeneities are examples of demography-driven intergenerational, intragenerational and spatial inequality.

This dissertation analyzed the scope for reducing demography-driven inequality. Chapters 2 and 3 focused on Social Security systems as a means of reducing inequality. By virtue of the intimate link between *ex post* inequality and *ex ante* uncertainty, Chapter 2 considered Social Security pension systems as a means of sharing risks to lifetime income associated with fluctuations in the size of generations. Chapter 3 considered Social Security pensions systems as a means of insuring risks associated with premature death. These chapters contribute to the theoretical literature on the very long-run economic and welfare effects of Social Security. We found that, once *ex ante* insurance is taken into account, Social Security pension systems are welfare-improving in the long run as long as capital is not too productive, in which case insurance gains dominate the crowding out of resources.

While the steady state comparison is a common tool in the macroeconomic literature, the results ignore the welfare effects on those generations that are living during the transition towards the steady state with the Social Security pension system. Yet a salient feature of Chapter 3, for example, is that if there are long-run welfare gains, then all generations, not only steady state generations, gain from the pension system. In contrast, we found short-run welfare gains even for cases in which the crowding out of resources outweighs the insurance gains in the long run. This finding raises the question of whether the winners can compensate the losers, a question which is not addressed in this dissertation. Future research may employ the lump-sum redistribution mechanism introduced in the deterministic setup of Auerbach and Kotlikoff (1987), which taxes the winners during the early phase of the transition and transfers the proceeds to future generations.

Table 1 in Chapter 1 opens a second door for future research. The staggering increases in medical outlays widens the gap between total and pension specific Social Security expenditures. It is not merely that people are living longer in retirement, but also that treating them is very expensive. Future research should therefore include intergenerational inequality in the quality of life.

The overlapping generations models of Chapters 2 and 3 have been extended in Chapter 4 to assess the benefits of a large-scale investment in rural roads, India's Pradhan Mantri Gram Sadak Yojana (PMGSY). There is a growing number of empirical short-term estimates of its benefits in the spheres of goods, education and health. By employing a calibrated model of a dualistic economy with human capital formation, Chapter 4 provided reasonable estimates on the programme's benefits in the longer run. We showed that the improvements in the villager's terms of trade are of secondary importance in the longer run, not least because the road boosts human capital formation and makes human capital less likely to exit rural areas.

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