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Cross-dynastic Intergenerational Altruism

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Abstract

Decisions with long-term consequences require comparing utility derived from present consumption to future welfare. But can we infer socially relevant intertemporal preferences from saving behavior? I allow for a decomposition of the present generation's preference for the next generation into its dynastic and cross-dynastic counterparts, in the form of welfare weights on the next generation in the own dynasty and other dynasties. Welfare weights on other dynasties can be motivated by a concern for sustainability, or if descendants may move or marry outside the dynasty. With such cross-dynastic intergenerational altruism, savings for one's own descendants benefit present members of other dynasties, giving rise to preference externalities. I find that socially relevant intertemporal preferences may not be inferred from saving behavior if there is cross-dynastic intergenerational altruism. I also show that the external effect of present saving decreases over time. This means that intertemporal preferences inferred from saving behavior are time-inconsistent, unless cross-dynastic intergenerational altruism is accounted for.

Keywords: Intergenerational altruism, social discounting, time-inconsistency, declining discount rates, generalized consumption Euler equations, interdependent utility, isolation paradox.
JEL Classification: D64, D71, H43, Q01, Q54.

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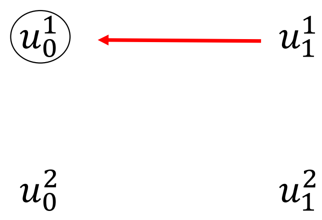
1 Introduction

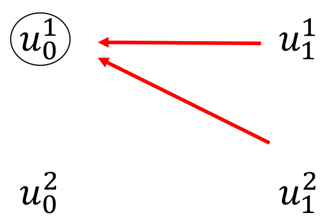
Managing resources requires trading off the interests of different generations. Climate policy, for example, must balance the mitigation costs incurred by the present generation against the benefits from a stable climate that accrue to future generations (Kolstad et al., 2014). As optimal resource management depends on the weights assigned to each generation (Stern, 2007; Nordhaus, 2007; Drupp et al., 2018), determining the trade-off between different generations has been described as "one of the most critical problems of all of economics" (Weitzman, 2001: 260).

Determining this trade-off depends on the sacrifice that the present generation is willing to make for future generations (Goulder and Williams, 2012; Kelleher, 2017). Economists usually impute the social welfare function from returns in the market on either corporate capital, equities or bonds, depending on project maturity and risk profile (Arrow et al., 1995; Gollier, 2012). Hence, the discount rate is implied by saving behavior. However, the traditional models consider altruism only for own descendants. I find that calibration saving behavior might not reveal the sacrifice that the present generation is willing to make if there is altruism for the descendants of others. I also show that the discount rate consistent with saving behavior is decreasing in the time horizon, and only over time approaches the socially desirable level.

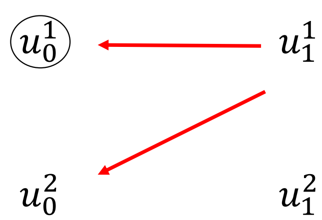
Altruism for the descendants of others can be motivated by a concern for sustainability, or if own descendants may move or marry (Bernheim and Bagwell, 1988; Laitner, 1991). With such altruism, the welfare of the present generation depends on their own utility and the welfare of the next generation across dynasties (parallel families or social groups). This paper extends Sen (1961, 1967) and Marglin's (1963) two-period model of intergenerational altruism to a stationary infinite horizon setting. This allows for a novel decomposition of intergenerational altruism into its dynastic and cross-dynastic counterparts. Dynastic intergenerational altruism gives the own dynasty welfare weights (Barro, 1974), while cross-dynastic intergenerational altruism gives the welfare weights on all other dynasties (Figures 1a and 1b illustrate the preference of generation 0 in dynasty 1 when there are two dynasties).



(a) Dynastic intergenerational altruism.



(b) Cross-dynastic intragenerational altruism.



(c) Consequence of saving for own descendants.

Figure 1: Welfare implication of incremental utility backward in time (direction of arrows). Subscript refers to generation, superscript to dynasty.

The network of altruistic links implies an infinite chain of concerns.

I investigate whether altruism for the next generation is reflected in the market by considering a traditional game of saving for own immediate descendants (Shapley, 1953). The game is a tractable model in which cross-dynastic intergenerational altruism can be studied analytically. Crucially, savings for one's own descendants benefit the present members of the other dynasties when they have cross-dynastic altruism (see Figure 1c), giving rise to preference externalities.

The analysis shows the existence of a stationary Markov-perfect equilibrium in linear strategies with a saving rate that is inefficiently low. I also establish that a unique subgame-perfect equilibrium in the finite horizon version of the game exists. Furthermore, the equilibrium strategies used in these finite horizon games go to the linear strategy when the time horizon goes to infinity. The equilibrium saving rate in this equilibrium is increasing in intergenerational altruism, both within and between dynasties. For constant total intergenerational altruism, it is decreasing in the number of dynasties. Assuming that the altruistic weight on each of the other dynasties goes to zero in the limiting case, when the number of dynasties goes to infinity, the saving rate reduces to the Brock-Prescott-Mehra saving rate (Brock, 1979, 1982; Prescott and Mehra, 1980), the rate without cross-dynastic intergenerational altruism. This means that cross-dynastic intergenerational altruism is not affecting the equilibrium saving rate in the limit when the number of dynasties goes to infinity.

In contrast, dynasties choose the efficient saving rate if they cooperate by acting as if there is only one dynasty. Such a coalition would capture any generation's altruism for the next generation. I find that the efficient saving rate is increasing in intergenerational altruism, both within and between dynasties. The wedge between the efficient and equilibrium saving rates measures the externality problem. I find that this wedge is positive if there is cross-dynastic intergenerational altruism. I also establish that this result is qualitatively robust even with intragenerational altruism, as long as the weight on the utility of the present generation as compared to the weight on the utility of the next generation is higher for the own dynasty than the other dynasties.

The wedge between the efficient and equilibrium saving rates can also be derived from the discount functions. I find that the external effect of present saving becomes less important over time, and vanishes only in the limit. Cross-dynastic intergenerational altruism thus leads to different discount functions in equilibrium and under efficiency. In general, the discount rates converge only in the limit, as time goes to infinity. This means that a dynasty's discount rate is smaller for long-term projects, leading to a time-inconsistency problem unless the dynasties cooperate.

Accounting for cross-dynastic intergenerational altruism beyond what is reflected by saving behavior translates into an increase in the relative weight on future generations. Nordhaus (2008) offers an influential market-based calibration. Respecting the distribution that would arise following the preference of the present generation (thereby retaining Nordhaus' setting but abstracting away from crowding out of saving), I illustrate that cross-dynastic intergenerational altruism of 10% and 20% beyond the level of intergenerational altruism inferred from saving behavior imply utility discount rates of 1.2% and 0.9%, as compared to the Nordhaus rate of 1.5%. The immediate implication for policy guidelines is thus that discount rates inferred from saving behavior should be lowered. The extent of this adjustment depends on the degree of cross-dynastic intergenerational altruism. Even if cross-dynastic intergenerational altruism cannot be inferred from saving behavior, it therefore plays an important normative role.

1.1 Contribution to the literature

Beyond showing that altruism for the next generation may not be reflected in the market, I contribute to the literature on present-biasedness. Cross-dynastic intergenerational altruism implies time-inconsistency (for early contributions on time-inconsistency, Strotz, 1955-1956; Phelps and Pollak, 1968). Time-inconsistency has influenced the study of discounting (Weitzman, 2001; see Arrow et al., 2013 and Groom and Hepburn, 2017), as well as procrastination, intoxication, and addiction (Asheim, 1997). My study on cross-dynastic intergenerational altruism makes two contributions. First, cross-dynastic intergenerational altruism serves as a microfoundation for declining discount rates in equilibrium (relating to Phelps and Pollak, 1968; Sáez-Marti and Weibull, 2005; Galperti and Strulovici, 2017), since the external effect of present saving weakens over time. In the cited papers, time-inconsistency follows from intergenerational altruism being sensitive beyond the next generation of the same dynasty. In my paper, time-inconsistency is due to altruism for the next generation as such. I further establish that cross-dynastic intergenerational altruism implies constant discount rates under efficiency. The equilibrium discount rate converges only to the lower efficient discount rate as time goes to infinity. This follows because the external effect of present saving vanishes only in the limit. Second, the preference formulation permits the saving rates to be derived from generalized consumption Euler equations (Hiraguchi, 2014; Iverson and Karp, 2018; Laibson, 1998), but for a distinct reason: Hiraguchi (2014) and Iverson and Karp (2018) assume declining discount rates when deriving the saving rate. Here, the relation is an outcome of the game of saving.

I contribute to the literature on dynamic interdependent utility and saving behavior. Cross-dynastic intergenerational altruism implies a system of benevolent utility functions (Pearce, 1983; Bergstrom, 1999). Recent additions to the literature focus on static interdependent utility (Bourlès et al., 2017), as well as dynamic interdependent utility without considering saving behavior (Millner, 2019). My paper clarifies consequences in terms of saving. I thus relate most closely to a classical result on dynamic interdependent utility and saving behavior by Sen (1961, 1967) and Marglin (1963). They study a two-period model in which present members of dynasties are altruistic toward own descendants and descendants in other dynasties. As in my paper, they also find that equilibrium saving rate is inefficiently low. Sen (1961) names it the "isolation paradox" because each dynasty would agree collectively to save more, although no dynasty is willing to do so in "isolation". My study on cross-dynastic intergenerational altruism makes two contributions beyond this earlier work. First, through my analysis I reinvestigate conditions for the "isolation paradox" to arise. Sen's (1967) condition is that the relative weight on the utility of the present and next generations is strictly larger for the own dynasty than the other dynasties. I establish that the condition is the same in a stationary infinite horizon setting. The intuition is that if there is a discrepancy between the relative discounting of the first two generations then there is also a discrepancy in any two generations. Second, accounting for cross-dynastic intergenerational altruism also exposes a limitation to, and extends, Sen's (1967) formulation of the "isolation paradox". In Sen's two-period model, cross-dynastic intergenerational altruism cannot effect the decision of how much to save. This is not the case in the stationary infinite horizon model of this paper, except in the limit case, when the number of dynasties goes to infinity.

I also relate to three other literatures. First, the preference formulation offers a new interpretation of discount functions (building on Bernheim and Bagwell, 1988; Laitner, 1991; Zhang, 1994; Myles, 1997). Cross-dynastic intergenerational altruism can be interpreted as the relative probability of immediate descendants ending up in other dynasties, for example through mating. Since the external effect of present saving becomes less important over time and vanishes only in the limit, the weight on each dynasty converges to a uniform distribution. Second, the existence of cross-dynastic intergenerational altruism is consistent with findings from surveys of intergenerational time preferences (Cropper et al., 1991, 1992, 1994; Johanneson and Johansson, 1997; Frederick, 2003) and experiments (Chermak and Krause, 2002; Fischer et al., 2004; Hauser et al., 2014; Molina et al., 2018; see Fehr-Duda and Fehr, 2016 for a perspective). There is also a strong empirical support for a smaller weight on the other dynasties in this generation than the own dynasty (Bernhard et al., 2006 and references therein for evidence on "parochial" altruism; see Schelling, 1995 for a perspective). This is my conceptual basis for claiming that the large discount rates implied from saving behavior do not respect the present generation's preference for the next generation. Third, there is an analogy between cross-dynastic intergenerational altruism towards the other dynasties and back again from the other dynasties, and the indirect concern caused by technological spillovers from one dynasty to another. Such linkages through technology may also lead to time-inconsistency (Harstad, 2019).

The paper proceeds as follows. Section 2 provides an informal motivating example, clarifying how the preference externalities are generated. Section 3 presents the model. Section 4 derives the main results, in the context of a wedge between the equilibrium and efficient saving rates. Section 5 establishes how the main results relate to time-inconsistency, and explains the contributions of the paper. Section 6 establishes how the main results relate to interdependent utility, and explains the contributions of the paper. Section 7 concludes with a numerical exercise, illustrating the policy implications. Appendix A contains additional proofs. Appendix B provides an interpretation of the model if descendants may move or marry someone from other dynasties.

2 Motivating example

Structure the problem by defining $\alpha \in (0,1)$ as any generation's altruism for the next generation. Generation 0 thus assigns weights 1 to itself and α to the next generation, so that $W_0 = (1-\alpha)u_0 + \alpha W_1$, where W, u and subscript refer to welfare, utility and generation. But any future generation t will do so in turn: $W_t = (1-\alpha)u_t + \alpha W_{t+1}$. This leads to the following relative weights on $(u_0, u_1, u_2, \dots)$ from the perspective of generation 0: $1-\alpha,\ (1-\alpha)\cdot\alpha,\ (1-\alpha)\cdot\alpha^2,\ \dots$, which is proportional to (and in line with Samuelson, 1937):

$$1,\ \alpha,\ \alpha^2,\ \dots. \tag{1}$$

This preference is stationary, so that time-consistency follows from time-invariance. The question then, is whether the preference for the future is reflected by saving behavior.

To illustrate the consequences of a particular network, suppose that there are only two dynasties. Assume, for simplicity, that altruism is only intergenerational, so that there is no altruism for contemporaries in the other dynasty. (I show in Section 6 under which condition the main results hold

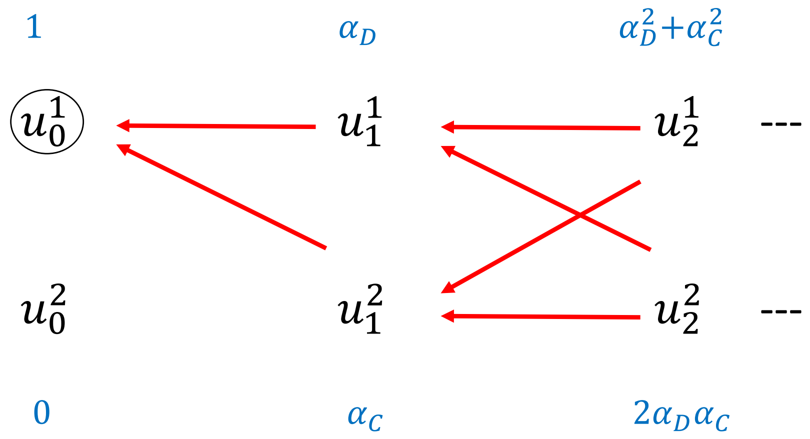


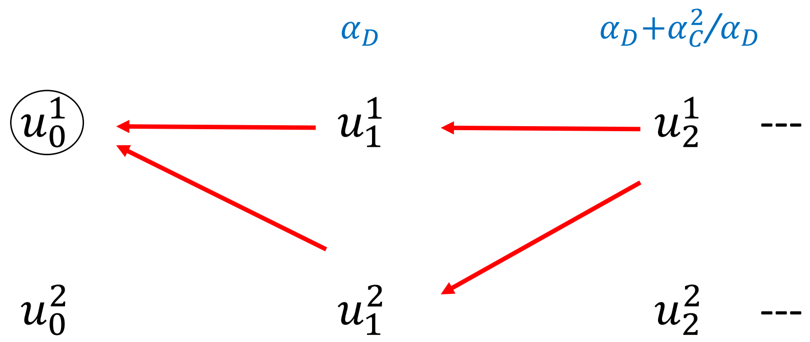
Figure 2: Resulting discount functions with two dynasties (sequences). Welfare implication of incremental utility backward in time (direction of arrows). Subscript refers to generation, superscript to dynasty.

even if there is altruism for contemporaries in the other dynasties.) Consequently, the present generation of any dynasty assigns weights 1 to itself and α to the next generation in both dynasties. The weight on the next generation consists of two parts, defining the network. Write $\alpha \equiv \alpha_D + \alpha_C$, where α_D and α_C are dynastic and cross-dynastic intergenerational altruism. As extreme cases, any dynasty might care only for own descendants: $\alpha_D = \alpha$ (Barro, 1974), or equally for all descendants: $\alpha_D = \alpha_C = \alpha/2$. It is natural to assume that $\alpha_D \geq \alpha_C \geq 0$ (e.g., Myles, 1997), that is a dynasty cares weakly more for its own descendants.

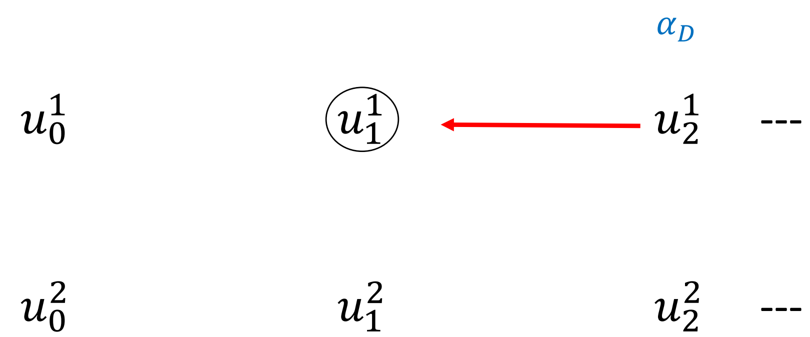
Consider the network implied by $\alpha_C > 0$. Figure 2 illustrates the preference of generation 0 in dynasty 1, with positive weights on future generations also in the other dynasty. These weights follow from accounting for the total number of dynastic and cross-dynastic altruistic links forward in time. The weights assigned by generation 0 in dynasty 1 to dynasties 1 and 2 can be written

$$\begin{aligned} &1,\ \alpha_D,\ \alpha_D^2 + \alpha_C^2,\ \ldots \\ &0,\ \alpha_C,\ 2\alpha_D\alpha_C,\ \ldots \end{aligned} \tag{2}$$

To see this, consider $\alpha_D^2+\alpha_C^2$ in Figure 2. This follows because generation 0 in



(a) Instantaneous discount factors for dynasty 1 (sequence).



(b) Instantaneous discount factors for dynasty 1 (sequence).

Figure 3: Welfare implication of incremental utility backward in time (direction of arrows). Subscript refers to generation, superscript to dynasty.

dynasty 1 cares dynastically for generation 1 which, again, cares dynastically, and because generation 0 in dynasty 1 cares cross-dynastically for generation 1 which, again, cares cross-dynastically. The preference of generation 0 in the other dynasty is the mirror image of (2).

Interpret a game of saving for own immediate descendants (Shapley, 1953) as the relevant market. In a stationary Markov-perfect equilibrium with linear strategies, generation 0 in dynasty 1 considers only the first sequence of (2) as cross-dynastic transfers are not allowed. This gives rise to preference externalities because savings for one's own descendants benefit the present members of the other dynasty. Furthermore, the within-dynasty instantaneous discount factors, $\alpha_D,\ \alpha_D + \alpha_C^2/\alpha_D,\ \dots$ follow from α_D and $(\alpha_D^2 + \alpha_C^2)/\alpha_D$, and are illustrated in Figure 3a. I have that $\alpha_D + \alpha_C = \alpha \geq \alpha_D + \alpha_C^2/\alpha_D > \alpha_D$. This means that that the external effect of present saving on the other dynasty becomes less important over time, leading to non-stationary preferences (as illustrated by comparing Figure 3b, where instead the preference of generation 1 in dynasty 1 is considered, to Figure 3a), thus being time-inconsistent if preferences are time-invariant (Strotz, 1955-1956; Halevy, 2015).

In contrast, the efficient saving captures any generation's altruism for the next generation. Recall that sequences 1 and 2 (from (2)) are the weights of generation 0 in dynasty 1, and that the mirror image gives the weights of dynasty 2. Assuming no side-transfers, it will later be verified that efficiency in the game of saving implies equal relative importance of 1/2 on each of the two dynasties, so that they act as if there were only one dynasty. For dynasty 1, this gives

$$
\begin{aligned}
&1/2 \cdot 1,\ 1/2 \cdot \alpha_D,\ 1/2 \cdot (\alpha_D^2 + \alpha_C^2),\ \dots \\
&1/2 \cdot 0,\ 1/2 \cdot \alpha_C,\ 1/2 \cdot 2\alpha_D\alpha_C,\ \dots
\end{aligned}
$$

And symmetrically, for dynasty 2. Adding up, efficiency thus recovers sequence (1), the preference for the next generation.

3 Model

Society is divided into $N \geq 2$ equally populated dynasties. Index these by $i = 1, 2, \ldots$. Time $t \geq 0$ is discrete and countably infinite. Write $\mathbb{N} = \{1, 2, \ldots\}$ and $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$ for the natural numbers without and with 0. Generations, also indexed by t, are non-overlapping, and live for one period only.

A consumption stream ${}_0c^i = (c_0^i, c_1^i, \ldots) \geq 0$ is feasible given an initial level of wealth $x_0^i \geq 0$ if there exists a wealth stream ${}_0x^i = (x_0^i, x_1^i, \ldots) \geq 0$ such that

$$\begin{aligned} x_t^i &= c_t^i + k_t^i \text{ for all } t \in \mathbb{N}_0, \text{ and} \\ x_t^i &= Ak_{t-1}^i \text{ for all } t \in \mathbb{N}. \end{aligned} \tag{3}$$

The present generation in dynasty i has wealth x_t^i. The action taken by each dynasty i is to save $k_t^i \geq 0$ for own immediate descendants. The residual, c_t^i, is consumed. Hence, no cross-dynastic transfers are possible. Wealth is determined by the saving of the previous generation in the same dynasty, k_{t-1}^i, multiplied by a gross productivity parameter, $A \geq 1$. Such a technology is referred to as the AK model (Romer, 1986), and is a tractable model in which cross-dynastic intergenerational altruism can be studied analytically.

Denote by

$$X_\tau(x_0^i) = \{{}_0x^i : x_0^i = x^i \text{ and } 0 \leq x_t^i \leq Ax_{t-1}^i \text{ for all } t \in \{1, 2, \ldots, \tau\}\} \tag{4}$$

the set of feasible wealth streams until time $\tau \in \mathbb{N}$. Write $X(x_0^i) = X_\infty(x_0^i)$. Hence, $X(x_0^i)$ denotes the set of feasible wealth streams. Furthermore, define $x_t = (x_t^1, x_t^2, \ldots, x_t^N)$ as the distribution of wealth at time $t \in \mathbb{N}_0$.

Define

$$\boldsymbol{c}({}_0x^i) = (x_0^i - x_1^i/A,\ x_1^i - x_2^i/A,\ \ldots)$$

as the consumption stream that is associated with ${}_0x^i$, and denote by

$$C(x_0^i) = \{{}_0c^i :\ \text{there is } {}_0x^i \in X(x_0^i) \text{ s.t. } {}_0c^i = \boldsymbol{c}({}_0x^i)\}$$

the set of feasible consumption streams.

Map consumption $c_t^i \geq 0$ into utility by the utility function $u : \mathbb{R}_+ \to \mathbb{R} \cup \{-\infty\}$ defined by:

$$u(c_t^i) = \begin{cases} \ln c_t^i \text{ if } c_t^i > 0, \\ -\infty \text{ if } c_t^i = 0. \end{cases}$$

The present generation in dynasty i has a logarithmic utility function, which justifies the first equality in (3). Write $\boldsymbol{u}({}_0c^i) = (u(c_0^i), u(c_1^i), \dots)$ and denote by

$$U(x_0^i) = \{{}_0u^i : \text{there is } {}_0c^i \in C(x_0^i) \text{ s.t. } {}_0u^i = \boldsymbol{u}({}_0c^i)\}$$

the set of feasible utility streams. Write $\mathcal{U} = \bigcup_{x^i \in \mathbb{R}_+} U(x_0^i)$.

The present generation of dynasty i cares about immediate descendants in all dynasties. I allow for a decomposition of intergenerational altruism into its dynastic, α_D, and cross-dynastic, α_C, counterparts. The following assumption on the network of altruistic links will be useful:

Assumption 1 *Altruism parameters have the following restrictions:* $1 > \alpha_D + \alpha_C > 0$ *and* $\alpha_D \geq \alpha_C/(N-1) \geq 0$.

The restrictions embody the extreme cases: $\alpha_D > \alpha_C = 0$ (Barro, 1974) and $\alpha_D = \alpha_C/(N-1) > 0$ (e.g., Myles, 1997), weight only on own immediate descendants and equal weight on the immediate descendants of all.

In particular, the preference of each dynasty is represented by the welfare function W^i. Denote by W^{-i} the vector of welfare in other dynasties. Assume that there exists an aggregator function $V : (\mathbb{R} \cup \{-\infty\})^{N+1} \to \mathbb{R} \cup \{-\infty\}$ defined by:

$$V(u^i, W^i, W^{-i}) = (1 - \alpha_D - \alpha_C)u^i + \alpha_D W^i + \frac{\alpha_C}{N-1} \sum_{j \neq i} W^j, \qquad (5)$$

where u^i is the utility of the present generation in dynasty i, W^i is the welfare of the immediate descendants of the same dynasty, and W^j is the welfare of the immediate descendants of another dynasty. Assume furthermore that $V = -\infty$ if $u^i = -\infty$.

The aggregator function, V, implicitly defines the welfare function. It assumes that intergenerational altruism is constant, non-paternalistic and sensitive only for the next generation, in the sense that the welfare of the present generation in dynasty i is derived from own utility and the welfare of immediate descendants in the different dynasties (following Ray, 1987, for the within-dynasty case).

I show in Section 6 that the main results hold qualitatively even with *intra*generational altruism as long as the weight on the utility of the present generation as compared to the weight on the utility of the next generation is higher for the own dynasty than the other dynasties. To ease exposition, I abstract from these complications when deriving the main results.

3.1 Equilibrium concept

The strategic setting is how to best respond to the present saving of other dynasties and the future saving of all dynasties. I defined in (4) the set of feasible wealth streams until time τ for a single dynasty. Write the set of histories as $h_\tau(x_0) = X_\tau(x_0^1) \times \cdots \times X_\tau(x_0^i) \times \cdots \times X_\tau(x_0^N)$, as initial wealth can vary. This gives $h_0(x_0) = x_0$. When deciding how much to save, the dynasties see the entire history, h_τ. Write the union of histories as $\mathcal{H}(x_0) = \bigcup_{\tau \in \mathbb{N}_0} h_\tau(x_0)$. Map the union of histories into present saving by a strategy $k^{i,\sigma} : \mathcal{H}(x_0) \to \mathbb{R}_+$. Unimprovability is defined such that no strategy that differs from it after only one history can increase welfare. The strategy is a subgame-perfect equilibrium (SPE) if and only it, for any i and for any history h_τ, is unimprovable. This follows because the game is continuous at infinity.

I restrict attention to Markovian strategies (Maskin and Tirole, 2001). Under this restriction, there exists a unique equilibrium in linear strategies. This equilibrium is the limit of the unique unrestricted equilibrium of a finite horizon game when the horizon goes to infinity. Define a Markovian strategy $k^{i,\mu} : \mathbb{R}_+^N \to \mathbb{R}_+$ as a function from present wealth x_t to present saving, where x_t contains all payoff-relevant information at time t (the last entry into h_τ). The strategy is also stationary because it is independent of calendar time.

Denote by x and x_{+1} the present and next period wealth levels. Write optimal behavior in the form of a value function from dynamic programming. In particular, a value function $U^i : \mathbb{R}_+^N \to \mathbb{R} \cup \{-\infty\}$ defined over wealth levels satisfies

$$\begin{aligned} U^i(x) &= \max_{k^i \in [0,x^i]} V(u^i, U_{+1}^i, U_{+1}^{-i}) \\ &= \max_{k^i \in [0,x^i]} \Big\{ (1-\alpha_D-\alpha_C)u^i + \alpha_D U_{+1}^i + \frac{\alpha_C}{N-1} \sum_{j \neq i} U_{+1}^j \Big\}, \end{aligned} \tag{6}$$

where $u^i = u(x^i - k^i)$ is defined as the utility of the present generation in dynasty i, $U^i = U^i(A(x^i - k^i),\ x_{+1}^{-i})$ the induced welfare of the immediate descendants of the same dynasty, and $U^j = U^j(x_{+1}^{-i},\ A(x^i - k^i))$ the induced welfare of the immediate descendants of another dynasty.

A Markovian strategy is unimprovable if it satisfies $k^{i,\mu}(x) = \text{argmax}_{k^i} U^i(x)$ for all i and wealth x. As above, it is an SPE if and only if it is unimprovable. A stationary Markovian strategy profile that is an SPE is a stationary Markov-perfect equilibrium (MPE).

4 Main results

4.1 Equilibrium

I now report the main results. I have the following theorem:

Theorem 1 *Under Assumption 1, there exists a stationary MPE where all dynasties use the linear strategy:*

$$k^{i,\mu}(x_t) = s x_t^i \tag{7}$$

for all i and x_t, where the constant saving rate, s, is given by

$$s = \alpha_D + \frac{\alpha_C^2}{(N-1)(1-\alpha_D-\alpha_C)+\alpha_C}. \tag{8}$$

This is the limit of the unique finite time horizon SPE when time goes to *infinity.*

Proof. The proof of existence is an application of the unimprovability property. Assume that all generations in dynasties $j \neq i$ and all future generations in dynasty i use the linear strategy (7). This gives a marginal propensity to consume of $1 - s$.

Write:

$$\begin{aligned} y &\equiv U^i(x_t), \\ z^j &\equiv U^j(x_t), \\ u &\equiv \ln((1-s)x_t^i), \\ v^j &\equiv \ln((1-s)x_t^j). \end{aligned}$$

By observing that the gross growth rate is sA, it follows from (6):

$$\begin{aligned} y &= (1-\alpha_D-\alpha_C)u + \alpha_D\left(y+\ln(sA)\right) + \frac{\alpha_C}{N-1}\sum_{\ell}\left(z^\ell+\ln(sA)\right), \\ z^j &= (1-\alpha_D-\alpha_C)v^j + \alpha_D\left(z^j+\ln(sA)\right) \\ &\quad + \frac{\alpha_C}{N-1}\Big((y+\ln(sA)) + \sum_{\ell\neq j}\left(z^\ell+\ln(sA)\right) \Big), \end{aligned}$$

for all j. Solving the set of these equations, yield:

$$\begin{aligned} y &= \frac{((N-1)(1-\alpha_D)-(N-2)\alpha_C)u + \alpha_C\sum_\ell v^\ell}{(N-1)(1-\alpha_D)+\alpha_C} \\ &\quad + \frac{\alpha_D+\alpha_C}{1-\alpha_D-\alpha_C}\ln(sA), \end{aligned} \tag{9}$$

$$\begin{aligned} z^j &= \frac{((N-1)(1-\alpha_D)-(N-2)\alpha_C)v^j + \alpha_C(u+\sum_{\ell\neq j} v^\ell)}{(N-1)(1-\alpha_D)+\alpha_C} \\ &\quad + \frac{\alpha_D+\alpha_C}{1-\alpha_D-\alpha_C}\ln(sA), \end{aligned} \tag{10}$$

for all j.

Insert for (9) to (10) in (6). The problem is to show that $k_t^i = sx_t^i$

maximizes

$$(1-\alpha_D-\alpha_C)\ln(x_t^i-k_t^i)+\alpha_D\frac{((N-1)(1-\alpha_D)-(N-2)\alpha_C)\ln((1-s)Ak_t^i)}{(N-1)(1-\alpha_D)+\alpha_C}$$
$$+\alpha_C\frac{\alpha_C\ln((1-s)Ak_t^i)}{(N-1)(1-\alpha_D)+\alpha_C}.$$

The first derivative is:

$$-\frac{1-\alpha_D-\alpha_C}{x_t^i-k_t^i}+\frac{\alpha_D((N-1)(1-\alpha_D)-(N-2)\alpha_C)+\alpha_C^2}{(N-1)(1-\alpha_D)+\alpha_C}\frac{1}{k_t^i},$$

which yields the first-order condition:

$$\frac{1-\alpha_D-\alpha_C}{x_t^i-k_t^i}=\frac{\alpha_D((N-1)(1-\alpha_D)-(N-2)\alpha_C)+\alpha_C^2}{(N-1)(1-\alpha_D)+\alpha_C}\frac{1}{k_t^i},$$

Therefore:

$$\frac{k_t^i}{x_t^i}=\alpha_D+\frac{\alpha_C^2}{(N-1)(1-\alpha_D-\alpha_C)+\alpha_C}=s,$$

which gives $k_t^i=sx_t^i$.

The second derivative is:

$$-\frac{1-\alpha_D-\alpha_C}{(x_t^i-k_t^i)^2}-\frac{\alpha_D((N-1)(1-\alpha_D)-(N-2)\alpha_C)+\alpha_C^2}{(N-1)(1-\alpha_D)+\alpha_C}\frac{1}{(k_t^i)^2},$$

and is strictly negative for $k_t^i\in(0,x_t^i)$. This verifies that the problem is concave. $k_t^i=sx_t^i$ therefore maximizes the problem. There is no profitable deviation for the present generation in dynasty i when all generations of dynasties j and all future generations in dynasty i use the linear strategy (7). ■

It is verified that there exists a stationary MPE where all dynasties use a linear strategy. Hence, the saving of one dynasty is independent of the wealth levels of other dynasties.

Proving that (7) is the limit of the unique finite time horizon SPE when time goes to infinity requires additional notation introduced in Section 5. A proof is delegated to Appendix A. It is shown that there exists a unique SPE in the finite horizon game for any horizon. The equilibrium strategies used in these finite horizon games go to the linear strategy with s given by (8) when the horizon goes to infinity.

I obtain the following corollary, describing the properties of the equilibrium saving rate, s:

Corollary 1 *Under Assumption 1, the equilibrium saving rate, s, has the following properties:*

(i) $s = \alpha_D$ *if* $\alpha_C = 0$.

(ii) s *is increasing in* α_D.

(iii) s *is increasing in* α_C.

(iv) s *is decreasing in* N *if* $\alpha_C > 0$.

(v) $s \to \alpha_D$ *if* $N \to \infty$.

This follows from expression (8), and is proved in Appendix A.

Without cross-dynastic intergenerational altruism, the saving rate reduces to the Brock-Prescott-Mehra rate of α_D (Brock, 1979, 1982; Prescott and Mehra, 1980). This follows because the cross-dynastic intergenerational altruism of the descendants in the other dynasties in the next generation has (almost) no concern for dynasty i. The saving rate is decreasing in the number of dynasties, as it increases the externality problem. Since the altruistic weight on other dynasties goes to zero in the limiting case, when the number of dynasties goes to infinity, the saving rate reduces to the Brock-Prescott-Mehra saving rate. This means that cross-dynastic intergenerational altruism is not affecting the equilibrium saving rate when the number of dynasties is infinitely large.

4.2 Efficiency

Recall that the equilibrium saving rate is inefficient due to the preference externalities. Interpret the efficient saving rate as the saving rate that would emerge if all dynasties bargain over how much to save for immediate descendants, under the assumption of cooperation also in the future (and by the assumptions on technology, also no side-transfers). This means that the present representatives of all dynasties come together with the aim of realizing a Pareto optimal trajectory for the present generation, with their preferences also including the preference for the future.

This is a normative setting similar to the cooperative solution to regionally integrated assessment models of the climate and economy (e.g., Nordhaus and Yang, 1996), with the exception that these models do not account for cross-dynastic intergenerational altruism (see Milgrom, 1993 and Hausman, 2011 for perspectives on preference satisfaction in behavioral welfare analysis). More precisely, I derive a stationary saving rate that, if used also in the future, gives a Pareto optimal trajectory for the present generation. By application of Negishi's (1960) theorem, the efficient saving rate is shown to be equal to the equilibrium saving rate if the dynasties act as if there is only one dynasty.

I have the following theorem:

Theorem 2 *Under Assumption 1, saving according to*

$$k_t^i = s^* x_t^i \tag{11}$$

for all i and x_t, where the constant saving rate, s^*, is given by

$$s^* = \alpha_D + \alpha_C, \tag{12}$$

implies a Pareto optimal trajectory for the present generation, given that the rule is used in the future.

Proof. Define the Negishi weights by $\phi^i \geq 0$ for all i and $\sum_i \phi^i = 1$. Cooperation in all periods implements the maximum of

$$\sum_i \phi^i W^i({}_t u) \tag{13}$$

subject to $0 \leq \sum_i c_{t+\tau}^i \leq \sum_i x_{t+\tau}^i$ for all $\tau \in \mathbb{N}_0$. Negishi's (1960) theorem states that all Pareto optimal allocations (for the present generation, in the case of Theorem 2) can be obtained by varying the vector of ϕ^i's. The following proof is an application of this result.

Replace s by s^* (from expression (12)) in y, z^j, u and v^j from the proof of Theorem 1. Denote the new expressions by y^*, z^{*j}, u^* and v^{*j}, and insert these in (13). The problem is to show that $k_t^i = s^* x_t^i$ for all i maximizes

$$\sum_i \phi^i \Bigg[(1-\alpha_D-\alpha_C)\ln(x_t^i - k_t^i) \\ + \alpha_D \Big(\frac{((N-1)(1-\alpha_D)-(N-2)\alpha_C)\ln((1-s^*)Ak_t^i)}{(N-1)(1-\alpha_D)+\alpha_C} \\ + \frac{\alpha_C \sum_\ell \ln((1-s^*)Ak_t^\ell)}{(N-1)(1-\alpha_D)+\alpha_C} \Big) \\ + \frac{\alpha_C}{N-1}\sum_{j\neq i} \Big(\frac{((N-1)(1-\alpha_D)-(N-2)\alpha_C)\ln((1-s^*)Ak_t^j)}{(N-1)(1-\alpha_D)+\alpha_C} \\ + \frac{\alpha_C(\ln((1-s^*)Ak_t^i) + \sum_{\ell\neq j}\ln((1-s^*)Ak_t^\ell))}{(N-1)(1-\alpha_D)+\alpha_C} \Big) \Bigg].$$

The first derivative with respect to k_t^i is:

$$-\phi^i \frac{1-\alpha_D-\alpha_C}{x_t^i - k_t^i} + \Big(\phi^i \frac{\alpha_D((N-1)(1-\alpha_D)-(N-2)\alpha_C)+\alpha_C^2}{(N-1)(1-\alpha_D)+\alpha_C} \\ + \sum_{j\neq i}\phi^j \frac{\alpha_D\alpha_C + \frac{\alpha_C}{N-1}(((N-1)(1-\alpha_D)-(N-2)\alpha_C)+(N-2)\alpha_C)}{(N-1)(1-\alpha_D)+\alpha_C} \Big) \frac{1}{k_t^i},$$

which yields the first-order condition:

$$\frac{1-\alpha_D-\alpha_C}{x_t^i - k_t^i} = \frac{\alpha_D+\alpha_C}{k_t^i},$$

when $\phi^i = 1/N$ for all i. To see this, note that the terms multiplied by $1/k_t^i$ can be written:

$$(\alpha_D+\alpha_C)\frac{(N-1)(1-\alpha_D)+\alpha_C}{(N-1)(1-\alpha_D)+\alpha_C}.$$

Therefore:

$$\frac{k_t^i}{x_t^i} = \alpha_D + \alpha_C = s^*,$$

which gives $k_t^i = s^* x_t^i$.

The second derivative with respect to k_t^i is:

$$-\frac{1-\alpha_D-\alpha_C}{(x_t^i - k_t^i)^2} - \frac{\alpha_D+\alpha_C}{(k_t^i)^2},$$

and is strictly negative for $k_t^i \in (0, x_t^i)$. All cross-derivatives equal 0, implying that the problem is concave. $k_t^i = s^* x_t^i$ therefore maximizes the problem. The k_t^j's follow by symmetry. ■

I obtain the following corollary, describing the properties of the efficient saving rate, s^*:

Corollary 2 *Under Assumption 1, the efficient saving rate, s^*, has the following properties:*

(i) $s^* = \alpha_D$ *if* $\alpha_C = 0$.

(ii) s^* *is increasing in* α_D.

(iii) s^* *is increasing in* α_C.

This follows from expression (12).

Define by

$$s^* - s = \alpha_C - \frac{\alpha_C^2}{(N-1)(1-\alpha_D-\alpha_C)+\alpha_C} \tag{14}$$

the wedge between the efficient and equilibrium saving rates. I obtain the following corollary, describing the wedge, $s^* - s$:

Corollary 3 *Under Assumption 1, the equilibrium saving rate, s, is inefficient if $\alpha_C > 0$.*

This follows from expression (14), and is proved in Appendix A.

The efficient saving rate, s^*, is increasing in intergenerational altruism. It reduces to the Brock-Prescott-Mehra rate of α_D without cross-dynastic intergenerational altruism. With cross-dynastic intergenerational altruism, the efficient saving rate, s^*, is always larger than the equilibrium saving rate, s. It follows from Corollaries 1 and 2 that this wedge increases to α_C in the limit case, when the number dynasties goes to infinity.

The present generation's preference for future generations is reflected by s^*, and can only be inferred from saving behavior when there is no cross-dynastic intergenerational altruism, so that $s^* - s = 0$. Efficient saving may therefore translate into an increase in the relative weight on all future generations when accounting for cross-dynastic intergenerational altruism. As illustrated later, the policy implication could be a lowering of discount rates inferred from saving behavior in the market, even if cross-dynastic intergenerational altruism is small.

5 Time-inconsistency

I now establish how the main results relate to time-inconsistency and present-biasedness (for early contributions, Strotz, 1955-1956; Phelps and Pollak, 1968), which has influenced the study of discounting (Weitzman, 2001; see Arrow et al., 2013 and Groom and Hepburn, 2017), as well as procrastination, intoxication, and addiction (Asheim, 1997). It will prove useful to first derive a non-recursive formulation of the welfare function W^i.

I have the following theorem:

Theorem 3 *Under Assumption 1, welfare can be written non-recursively:*

$$W^i({}_tu) = (1 - \alpha_D - \alpha_C)\Big(\sum_{\tau=0}^{\infty} \Delta_\tau u^i_{t+\tau} + \sum_{j\neq i}\sum_{\tau=0}^{\infty} \Gamma_\tau u^j_{t+\tau}\Big), \tag{15}$$

with discount functions

$$\Delta_\tau = \frac{1}{N}\big((\alpha_D + \alpha_C)^\tau + (N-1)(\alpha_D - \frac{\alpha_C}{N-1})^\tau\big), \tag{16}$$

$$\Gamma_\tau = \frac{1}{N}\big((\alpha_D + \alpha_C)^\tau - (\alpha_D - \frac{\alpha_C}{N-1})^\tau\big). \tag{17}$$

Proof. The welfare function (15) follows by repeated substitution of W^i and W^j's into V from (5), keeping in mind that $V = -\infty$ if $u^i = -\infty$. Discount functions (16) and (17) are proven by induction.

The base case: Discount functions (16) and (17) hold for $\tau = 0$ since $\Delta_0 = 1$ and $\Gamma_0 = 0$.

The step case: Suppose that discount functions (16) and (17) hold for $\tau - 1$. Then,

$$\begin{aligned}
\Delta_\tau &= \alpha_D \Delta_{\tau-1} + \frac{\alpha_C}{N-1}(N-1)\Gamma_{\tau-1} \\
&= \alpha_D \Delta_{\tau-1} + \alpha_C \Gamma_{\tau-1} \\
&= \frac{1}{N}\big(\alpha_D(\alpha_D + \alpha_C)^{\tau-1} + \alpha_D(N-1)(\alpha_D - \frac{\alpha_C}{N-1})^{\tau-1} \\
&\qquad + \alpha_C(\alpha_D + \alpha_C)^{\tau-1} - \alpha_C(\alpha_D - \frac{\alpha_C}{N-1})^{\tau-1}\big) \\
&= \frac{1}{N}\big((\alpha_D + \alpha_C)^\tau + (N-1)(\alpha_D - \frac{\alpha_C}{N-1})^\tau\big),
\end{aligned}$$

by inserting for $\Delta_{\tau-1}$ and $\Gamma_{\tau-1}$. And,

$$\begin{aligned}
\Gamma_\tau &= \frac{\alpha_C}{N-1}\Delta_{\tau-1} + (\alpha_D + \frac{(N-2)\alpha_C}{N-1})\Gamma_{\tau-1} \\
&= \frac{1}{N}\big(\frac{\alpha_C}{N-1}(\alpha_D + \alpha_C)^{\tau-1} + \frac{\alpha_C}{N-1}(N-1)(\alpha_D - \frac{\alpha_C}{N-1})^{\tau-1} \\
&\qquad + (\alpha_D + \frac{(N-2)\alpha_C}{N-1})(\alpha_D + \alpha_C)^{\tau-1} - (\alpha_D + \frac{(N-2)\alpha_C}{N-1})(\alpha_D - \frac{\alpha_C}{N-1})^{\tau-1}\big) \\
&= \frac{1}{N}\big((\alpha_D + \alpha_C)^\tau - (\alpha_D - \frac{\alpha_C}{N-1})^\tau\big),
\end{aligned}$$

by inserting for $\Delta_{\tau-1}$ and $\Gamma_{\tau-1}$. This proves that discount functions (16) and (17) hold for all $\tau \in \mathbb{N}_0$.

It follows from (16) and (17) that $\Delta_\tau + (N-1)\Gamma_\tau = (\alpha_D + \alpha_C)^\tau$. This ensures that W^i is well-defined on $\mathcal{U}^N$. ■

The discount functions (16) and (17) give the weights the present generation of dynasty i puts on the utility of generation τ in the same dynasty and each of the other dynasties. They imply the following weights on the first two generations:

$$\Delta_0 = 1, \quad \Delta_1 = \alpha_D,$$
$$\Gamma_0 = 0, \quad \Gamma_1 = \frac{\alpha_C}{N-1}.$$

Figure 2 illustrates these weights for $N = 2$. More generally, I have that $\Delta_\tau \geq \Gamma_\tau$ for all $\tau \in \mathbb{N}$.

The following observation will be helpful in interpreting the term structure of the discount rates. The total weight on all other dynasties, $(N-1)\Gamma_{\tau+1}$, is of importance to the construction of $\Delta_{\tau+2}$ (in the proof of Theorem 3). It also offers a new perspective on the limiting case of Theorem 1, when the number of dynasties is finite, but goes infinity. Using expressions (16) and (17), the link between Δ_τ and $\Delta_{\tau+2}$ via $(N-1)\Gamma_{\tau+1}$ can be written

$$\frac{\alpha_C}{N-1}(N-1)\frac{\alpha_C}{N-1} = \frac{\alpha_C^2}{N-1} \to 0 \quad \text{as} \quad N \to \infty. \tag{18}$$

The intuition is that although dynasty i gives weight α_C on the other dynasties, the other dynasties give weight $\alpha_C/(N-1)$ to dynasty i. This weight goes to zero as the number of dynasties goes to infinity (see also Asheim and Nesje, 2016).

5.1 Declining discount rates

To see that cross-dynastic intergenerational altruism implies declining discount rates *in equilibrium*, consider the non-recursive formulation of the welfare function (15). Importantly, note that since the behavior of one dynasty does not depend on the utilities of other dynasties (from Theorem 1), only the first summation is relevant for time-inconsistency.

Assume for the moment that $\alpha_C = 0$. Then, $\Delta_\tau = \alpha_D^\tau$ and $\Gamma_\tau = 0$ for all $\tau \in \mathbb{N}_0$. Inserting in (15) gives the dynastic intergenerational altruism welfare function:

$$(1 - \alpha_D) \sum_{\tau=0}^{\infty} \alpha_D^\tau u_{t+\tau}^i.$$

Since $\Delta_\tau / \Delta_{\tau-1} = \alpha_D$ for all $\tau \in \mathbb{N}$, all generations weight within-dynasty utility similarly. This implies a geometric discount function (i.e., constant discount rates). Hence, the preference of each dynasty is time-consistent.

This is no longer the case with cross-dynastic intergenerational altruism. I have the following propositions, which generalizes the claim related to Figure 3a in Section 2:

Proposition 1 *Under Assumption 1, the preference of each dynasty is non-stationary, and thus time-inconsistent, if* $\alpha_C > 0$.

This follows from expressions (16) and (17), and is proved in Appendix A.

Proposition 2 *Under Assumption 1,*

(i) $\Delta_\tau / \Delta_{\tau-1}$ *converges to* $\alpha_D + \alpha_C$ *only in the limit, as time goes to infinity, if* $\alpha_D > \alpha_C/(N-1) > 0$.

(ii) $\Delta_\tau / \Delta_{\tau-1}$ *converges to* $\alpha_D + \alpha_C$ *immediately if* $\alpha_D = \alpha_C/(N-1)$.

This follows from the proof of Proposition 1.

There are two cases: If $\alpha_D > \alpha_C/(N-1)$, then $\Delta_\tau / \Delta_{\tau-1}$ is increasing from α_D and converges only in the limit to $\alpha_D + \alpha_C$, so that all generations weight within-dynasty utility differently. This is a discount function with declining discount rates. If $\alpha_D = \alpha_C/(N-1)$, $\Delta_\tau / \Delta_{\tau-1}$ is increasing from α_D and converges immediately to $\alpha_D + \alpha_C$, so that only subsequent generations weight within-dynasty utility differently. This implies a "quasi-hyperbolic" discount function. In both cases, the preference of each dynasty is time-inconsistent.

This observation differs from Phelps and Pollak (1968) and Sáez-Marti and Weibull (2005), and more recently Galperti and Strulovici (2017), since time-inconsistency in these papers follows from intergenerational altruism being sensitive beyond the next generation of the same dynasty. Here, time-inconsistency is due to altruism for the next generation as such. In line with expression (18), the weight each other dynasty gives to a dynasty goes to zero as the number of dynasties goes to infinity. (Consult the proofs of Theorem 3 and Proposition 1.) This leads to geometric discounting of the own dynasty only in the limit.

In contrast, cross-dynastic intergenerational altruism implies constant discount rates *under efficiency* (Theorem 2). This can be seen from the discount functions (16) and (17), where $(\Delta_\tau+(N-1)\Gamma_\tau)/(\Delta_{\tau-1}+(N-1)\Gamma_{\tau-1}) = \alpha_D + \alpha_C$ for all $\tau \in \mathbb{N}$. From the discussion above, it is clear that $\Delta_\tau/\Delta_{\tau-1}$ is increases from α_D and approaches $\alpha_D + \alpha_C$.

An intuition for why efficient and equilibrium discounting agree in the limit if $\alpha_D > \alpha_C/(N-1)$ can be obtained from the discount functions as time goes to infinity. In a version of the model in Appendix B, I find that the external effect of present saving becomes less important over time, and vanishes only in the limit. This establish that a dynasty's discount rate is smaller for the long term. More precisely, I establish that $\lim_{\tau\to\infty} \Delta_\tau/(\Delta_\tau + (N-1)\Gamma_\tau) = 1/N$. Hence, one dynasty's present value of a gain at time t converges to $1/N$ of the social value of this benefit, when t approaches infinity.

5.2 Generalized consumption Euler equations

An alternative starting point for deriving the stationary saving rate is Laibson (1998). Laibson studies the extent of undersaving by a "quasi-hyperbolic" discounter that is sophisticated, in the sense that he takes into account that his preference is time-inconsistent. Krusell et al. (2002) integrate Laibson's insight into standard discrete-time macroeconomic models. (For continuous-time formulations, see, e.g., Karp, 2007 and Ekeland and Lazrak, 2010.)

Hiraguchi (2014) and Iverson and Karp (2018), that are closer to my contribution, generalize Krusell et al. (2002) to an economy exhibiting declining discount rates.

The equilibrium saving rate, s from expression (8), can be derived from the generalized consumption Euler equation of Hiraguchi (2014) and Iverson and Karp (2018):

$$s = \frac{\sum_{\tau=1}^{\infty} \Delta_\tau}{\sum_{\tau=0}^{\infty} \Delta_\tau}, \tag{19}$$

but for a distinct reason. I have the following proposition:

Proposition 3 *Under Assumption 1, the equilibrium saving rate, s from expression (8), follows from the Hiraguchi-Iverson-Karp solution for s (19).*

This follows from expression (16), and is proved in Appendix A.

Hiraguchi (2014) and Iverson and Karp (2018) assume declining discount rates when deriving the saving rate. Here, the relation is an outcome of the game of saving. This follows since the behavior of one dynasty does not depend on the utilities of other dynasties (from Theorem 1). It is as if society consists of N parallel dynasties with declining discount rates according to expression (16).

6 Interdependent utility

I now establish how the main results relate to interdependent utility (Pearce, 1983; Bergstrom, 1999), focusing on a classical result on dynamic interdependent utility and saving behavior (Sen, 1961, 1967; Marglin, 1963). It will prove useful to first illustrate the qualitative robustness of the main results by considering two formulations of *intra*generational altruism replacing the aggregator function, V, from expression (5).

6.1 Paternalistic intragenerational altruism

Suppose that the present generation of dynasty i cares also about the utility of contemporaries in the other dynasties. I allow for a decomposition of intragenerational altruism into its dynastic, α_A, and cross-dynastic, α_B, counterparts. As argued in the Introduction, there is strong empirical support for $\alpha_A > \alpha_B/(N-1)$.

The following additional assumption on the network of altruistic links will be useful:

Assumption 2 *Altruism parameters have the following restrictions:* $\alpha_A + \alpha_B = 1$ *and* $\alpha_A \geq \alpha_B/(N-1) \geq 0$.

The restrictions embody the extreme cases: $\alpha_A > \alpha_B = 0$ (Section 3) and $\alpha_D = \alpha_B/(N-1) > 0$, weight only on own dynasty contemporaries and equal weight on all contemporaries.

The preference of each dynasty is represented by the welfare function W^i. Denote by u^{-i} and W^{-i} the vectors of utility and welfare in other dynasties. Assume that there exists an aggregator function $V : (\mathbb{R} \cup \{-\infty\})^{2N} \to \mathbb{R} \cup \{-\infty\}$ defined by:

$$\begin{aligned} V(u^i, u^{-i}, W^i, W^{-i}) =& (1-\alpha_D-\alpha_C)\big(\alpha_A u^i + \frac{\alpha_B}{N-1}\sum_{j\neq i} u^j\big) \\ &+ \alpha_D W^i + \frac{\alpha_C}{N-1}\sum_{j\neq i} W^j, \end{aligned} \tag{20}$$

where u^i is the utility of the present generation in dynasty i, u^j is the utility of the present generation of another dynasty, W^i is the welfare of the immediate descendants of the same dynasty, and W^j is the welfare of the immediate descendants of another dynasty. Assume furthermore that $V = -\infty$ if $u^i = -\infty$ or, if $\alpha_B > 0$, $u^j = -\infty$.

I have the following proposition:

Proposition 4 *Under Assumptions 1 and 2, welfare can be written non-*

recursively:

$$W^i({}_tu) = (1 - \alpha_D - \alpha_C)\Big(\sum_{\tau=0}^{\infty} \Delta_\tau u^i_{t+\tau} + \sum_{j\neq i}\sum_{\tau=0}^{\infty} \Gamma_\tau u^j_{t+\tau}\Big), \tag{21}$$

with discount functions

$$\begin{aligned}\Delta_\tau = \frac{1}{N}\Big(&\alpha_A\big((\alpha_D + \alpha_C)^\tau + (N-1)(\alpha_D - \frac{\alpha_C}{N-1})^\tau\big) &(22)\\ &+ \alpha_B\big((\alpha_D + \alpha_C)^\tau - (\alpha_D - \frac{\alpha_C}{N-1})^\tau\big)\Big),\\ \Gamma_\tau = \frac{1}{N}\Big(&\frac{\alpha_B}{N-1}\big((\alpha_D + \alpha_C)^\tau + (N-1)(\alpha_D - \frac{\alpha_C}{N-1})^\tau\big) &(23)\\ &+ (\alpha_A + \frac{(N-2)\alpha_B}{N-1})\big((\alpha_D + \alpha_C)^\tau - (\alpha_D - \frac{\alpha_C}{N-1})^\tau\big)\Big).\end{aligned}$$

This follows from an application of the proof of Theorem 3, and is proved in Appendix A.

The discount functions (22) and (23) give the weights the present generation of dynasty i puts on the utility of generation τ in the same dynasty and each of the other dynasties. They imply the following weights on the first two generations:

$$\begin{aligned}&\Delta_0 = \alpha_A, &&\Delta_1 = \alpha_D\alpha_A + \alpha_C\frac{\alpha_B}{N-1},\\ &\Gamma_0 = \frac{\alpha_B}{N-1}, &&\Gamma_1 = \frac{\alpha_C}{N-1}\alpha_A + \Big(\alpha_D + \frac{(N-2)\alpha_C}{N-1}\Big)\frac{\alpha_B}{N-1}.\end{aligned}$$

Figures 4a and 4b illustrate the weights for $\alpha_A = 1$ (identical to Figure 2) and α_A according to Assumption 2 for $N = 2$. To see this, consider α_A and α_B in Figure 4b. This follows directly from Assumption 2 as the weights the present generation of dynasty i put on itself and contemporaries in the other dynasty. The weight on the next generation in the same dynasty is $\alpha_D\alpha_A + \alpha_C\alpha_B$, and follows because dynasty i cares dynastically and cross-dynastically. By Assumption 2, the dynastic link is weighted by the share put on the own dynasty utility and the cross-dynastic link by the share put on the other dynasty utility. The weight $\alpha_D\alpha_B + \alpha_C\alpha_A$ follows by symmetry. More generally, I have that $\Delta_\tau \geq \Gamma_\tau$ for all $\tau \in \mathbb{N}_0$.

I have the following propositions, generalizing Propositions 1 and 2:

Proposition 5 *Under Assumptions 1 and 2, the preference of each dynasty is time-inconsistent, and thus time-inconsistent, if* $\alpha_A > \alpha_B/(N-1)$ *and* $\alpha_C > 0$.

This follows from expressions (22) and (23), and is proved in Appendix A.

Proposition 6 *Under Assumptions 1 and 2,*

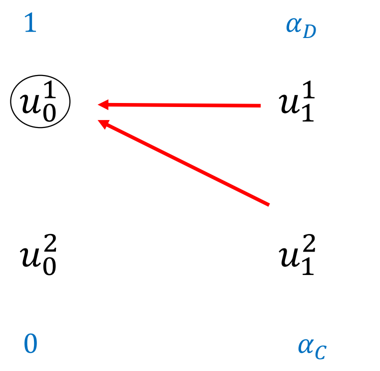
(i) $\Delta_\tau/\Delta_{\tau-1}$ *converges to* $\alpha_D+\alpha_C$ *only in the limit, as time goes to infinity, if* $\alpha_A > \alpha_B/(N-1)$ *and* $\alpha_D > \alpha_C/(N-1) > 0$.

(ii) $\Delta_\tau/\Delta_{\tau-1}$ *converges to* $\alpha_D + \alpha_C$ *immediately if* $\alpha_A > \alpha_B/(N-1)$ *and* $\alpha_D = \alpha_C/(N-1)$.

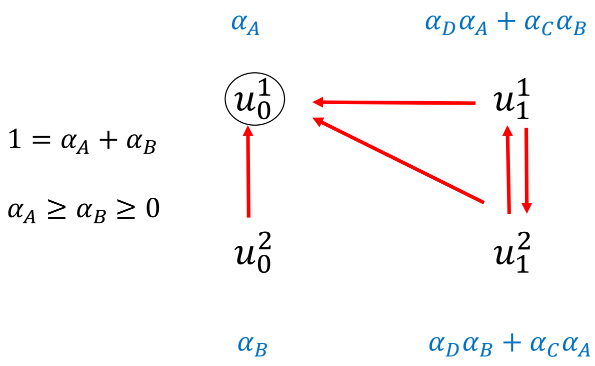
This follows from the proof of Proposition 5.

Assuming $\alpha_A > \alpha_B/(N-1)$, there are two cases: If $\alpha_D > \alpha_C/(N-1)$, then $\Delta_\tau/\Delta_{\tau-1}$ is increasing from $\alpha_D + \alpha_C\alpha_B/((N-1)\alpha_A)$ and converges only in the limit to $\alpha_D + \alpha_C$, so that all generations weight within-dynasty utility differently. This is a discount function with declining discount rates. If $\alpha_D = \alpha_C/(N-1)$, $\Delta_\tau/\Delta_{\tau-1}$ is increasing from $\alpha_D + \alpha_C\alpha_B/((N-1)\alpha_A)$ and converges immediately to $\alpha_D + \alpha_C$, so that only subsequent generations weight within-dynasty utility differently. This implies a "quasi-hyperbolic" discount function. In both cases, the preference of each dynasty is time-inconsistent.

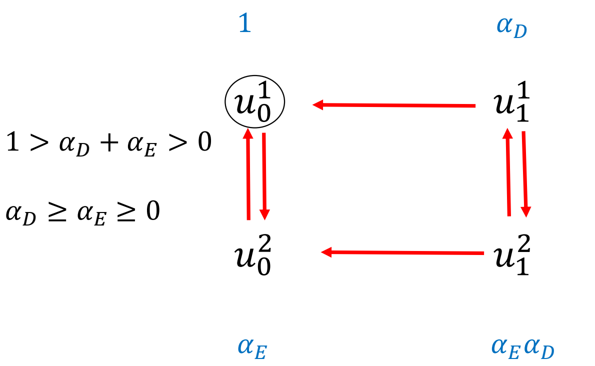
I obtain the following corollary, describing the equilibrium and efficient saving rates:



(a) Non-paternalistic cross-dynastic intergenerational altruism.



(b) Paternalistic cross-dynastic intragenerational altruism.



(c) Non-paternalistic cross-dynastic intragenerational altruism.

Figure 4: Resulting discount functions with two dynasties (sequences) for alternative preference formulations. Welfare implication of incremental utility backward in time (direction of arrows). Subscript refers to generation, superscript to dynasty.

Corollary 4 *Under Assumptions 1 and 2, the equilibrium and efficient saving rates can be written:*

$$s = \alpha_D + \alpha_C \frac{\alpha_A \alpha_C + \alpha_B(1 - \alpha_D)}{\alpha_A((N-1)(1 - \alpha_D - \alpha_C) + \alpha_C) + \alpha_B \alpha_C}, \tag{24}$$

$$s^* = \alpha_D + \alpha_C. \tag{25}$$

This follows from expressions (22) and (23), and is proved in Appendix A.

It then follows from Assumptions 1 and 2 that $s^* - s \geq 0$. Furthermore, $s^* - s > 0$ if $\alpha_A > \alpha_B/(N-1)$ and $\alpha_C > 0$. This means that all results of the main text hold qualitatively even with paternalistic cross-dynastic intragenerational altruism as long as the weight on the other dynasties in this generation is smaller than the weight on the own dynasty in this generation.

Somewhat surprisingly for the stationary infinite horizon setting, it reduces to the following condition for $s^* > s$:

$$\frac{\Delta_0}{\Delta_1} > \frac{\Gamma_0}{\Gamma_1},$$

that the relative weight on the utility of the present and next generations is strictly larger for the own dynasty than for the other dynasty. Equilibrium saving is thus inefficient because of the discrepancy between the dynastic and cross-dynastic discount functions, but only in the first two generations. The intuition follows from Assumptions 1 and 2. If there is a discrepancy between the relative discounting of the first two generations then there is also a discrepancy in any two generations.

6.2 Non-paternalistic intragenerational altruism

Suppose that the present generation of dynasty i now cares cross-dynastically only for the other dynasties in the present generation rather than the next generation. While there is less support for such preferences, it illustrates the limit of the analysis.

The following alternative assumption on the network of altruistic links will be useful:

Assumption 3 *Altruism parameters have the following restrictions:* $1 > \alpha_D + \alpha_E > 0$ *and* $\alpha_D \geq \alpha_E/(N-1) \geq 0$.

The restrictions embody the extreme cases: $\alpha_D > \alpha_E = 0$ (Barro, 1974) and $\alpha_D = \alpha_C/(N-1) > 0$, weight only on own immediate descendants and equal weight on immediate descendants in the own dynasty and contemporaries in the other dynasties.

The preference of each dynasty is represented by the welfare function W^i. Denote by W^{-i} the vector of welfare in other dynasties. Assume that there exists an aggregator function $V : (\mathbb{R} \cup \{-\infty\})^{N+1} \to \mathbb{R} \cup \{-\infty\}$ defined by:

$$V(u^i, W^i, W^{-i}) = (1 - \alpha_D - \alpha_E)u^i + \alpha_D W^i + \frac{\alpha_E}{N-1} \sum_{j \neq i} W^j, \tag{26}$$

where u^i is the utility of the present generation in dynasty i, W^i is the welfare of the immediate descendants of the same dynasty, and W^j is the welfare of the present generation of another dynasty. Assume furthermore that $V = -\infty$ if $u^i = -\infty$.

I have the following proposition:

Proposition 7 *Under Assumption 3, welfare can be written non-recursively:*

$$W^i({}_t u) = (1 - \alpha_D - \alpha_E)\Big(\sum_{\tau=0}^{\infty} \Delta_\tau u^i_{t+\tau} + \sum_{j \neq i} \sum_{\tau=0}^{\infty} \Gamma_\tau u^j_{t+\tau} \Big), \tag{27}$$

with discount functions

$$\Delta_\tau = \alpha_D^\tau, \tag{28}$$

$$\Gamma_\tau = \frac{\alpha_E}{N-1} \alpha_D^\tau, \tag{29}$$

when Δ_0 *is normalized to* 1.

This follows from an application of the proof of Theorem 3, and is proved in Appendix A.

The discount functions (28) and (29) give the weights the present generation of dynasty i puts on the utility of generation τ in the same dynasty and each of the other dynasties. They imply the following weights on the first two generations:

$$\Delta_0 = 1, \qquad \Delta_1 = \alpha_D,$$
$$\Gamma_0 = \frac{\alpha_E}{N-1}, \quad \Gamma_1 = \frac{\alpha_E}{N-1}\alpha_D.$$

Figure 4c illustrates these weights for $N = 2$. To see this, consider the weight the present generation of dynasty i puts on itself. Since cross-dynastic intragenerational altruism is reciprocal, this weight is $(1 + \alpha_E + \alpha_E^2 + \dots) = 1/(1 - \alpha_E)$. Contemporaries in the other dynasty are additionally weighted cross-dynastically, $\alpha_E(1 + \alpha_E + \alpha_E^2 + \dots) = \alpha_E/(1 - \alpha_E)$. Both dynasties care dynastically about the next next generation, so that the resulting weights are $\alpha_D/(1 - \alpha_E)$ for dynasty i and $\alpha_E\alpha_D/(1 - \alpha_E)$ for dynasty j. Multiply through by $1 - \alpha_E$ to ensure $\Delta_0 = 1$. More generally, I have that $\Delta_\tau > \Gamma_\tau$ for all $\tau \in \mathbb{N}_0$.

I obtain the following corollary, describing the equilibrium and efficient saving rates:

Corollary 5 *Under Assumption 3, the equilibrium and efficient saving rates can be written:*

$$s = \alpha_D, \tag{30}$$
$$s^* = \alpha_D. \tag{31}$$

This follows from expressions (28) and (29).

Hence, $s^* - s = 0$ for all α_E. This means that cross-dynastic intragenerational altruism alone is not sufficient for deriving the main results. Cross-dynastic altruism needs to be sensitive to the welfare of future generations.

This reduces to the following condition, implying $s^* = s$:

$$\frac{\Delta_0}{\Delta_1} = \frac{\Gamma_0}{\Gamma_1},$$

that the relative weights on the utility of the present and next generations are equal for the own dynasty and the other dynasties. Equilibrium saving is efficient because of the similarity between the dynastic and cross-dynastic discount functions.

6.3 The "isolation paradox"

Sen (1961, 1967) and Marglin (1963) developed a model of dynamic interdependent utility and saving (see Robson and Szentes, 2014 for a recent addition to this literature). In the terminology of this paper, they study a two-period model in which present members of dynasties are altruistic toward own descendants and descendants in other dynasties. Each dynasty decide how much to save for own immediate descendants. As in the present paper, the equilibrium saving rate is inefficiently low.

Sen (1961) names it the "isolation paradox" because each dynasty would agree collectively to save more, although no dynasty is willing to do so in "isolation" (borrowing Newbery's 1990 explanation). Attempting to solve this problem, Sen (1967) considers a bargain between all dynasties, aiming at realizing a Pareto optimal trajectory for the present generation. The efficient saving rate, s^*, can be interpreted as the saving rate that would emerge if all the dynasties bargain over how much to save for immediate descendants. Thus, the interpretation resembles that of the "isolation paradox" literature.

In Sen's two-period model of within-dynasty saving (see also Lind, 1964), the equilibrium saving rate, s, is inefficient if the relative weight on the utility of the present and next generations is strictly larger for the own dynasty than for the other dynasties. Using my notation, that is

$$\frac{\Delta_0}{\Delta_1} > \frac{\Gamma_0}{\Gamma_1}. \tag{32}$$

It follows from the discussion above that this condition is equal to the condition for $s^* > s$. Sen's (1967) condition thereby generalizes to a stationary infinite horizon setting:

Remark 1 *Under Assumptions 1 and 2, or Assumption 3, the condition for the "isolation paradox" to arise in Sen's two-period model, given by expression (32), is equal in the stationary infinite horizon model.*

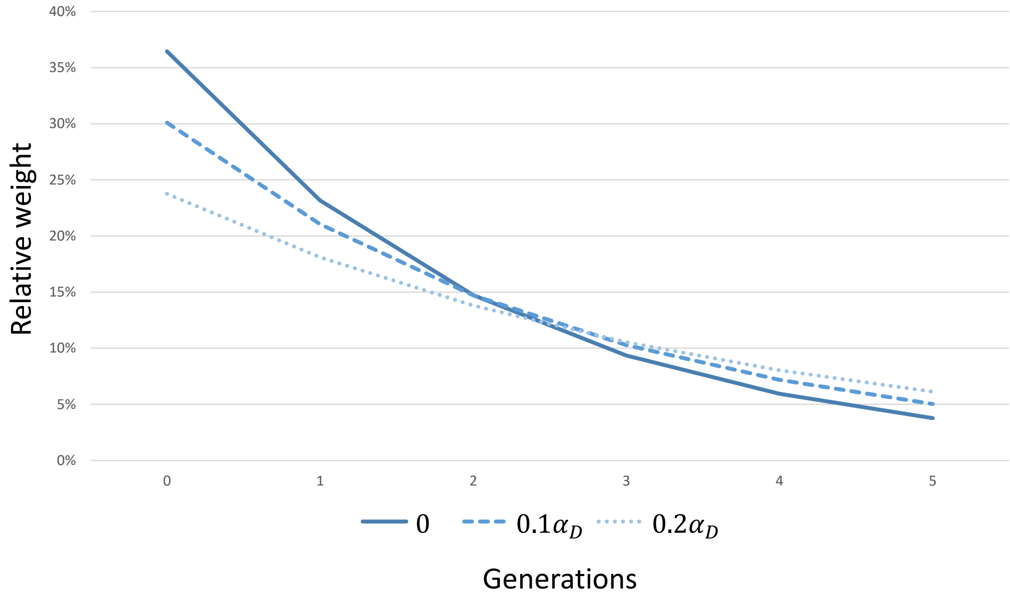
Hence, only the utility weights in the first two generations are relevant for determining whether equilibrium saving is inefficient. The intuition follows from Assumptions 1 and 2, or Assumption 3. If there is a discrepancy between the relative discounting of the first two generations then there is also a discrepancy in any two generations.

Accounting for cross-dynastic intergenerational altruism also exposes a limitation to, and extends, Sen's (1967) "isolation paradox". In Sen's two-period model, cross-dynastic intergenerational altruism cannot effect the decision of how much to save. This is not the case in the model of this paper, except in the limit case, when the number of dynasties goes to infinity:

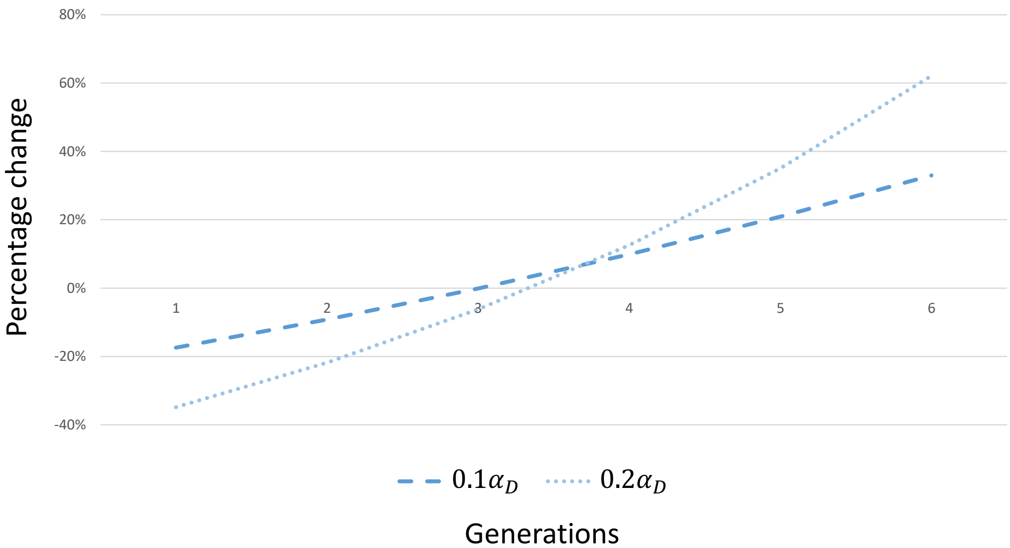
Remark 2 *Under Assumption 1, α_C affects the decision of how much to save, except in the limit as $N \to \infty$. In Sen's two-period model this is not the case for any N.*

7 Concluding remarks

In this paper, I ask whether the trade-off between present utility and future welfare can be inferred from saving behavior. Answering this question, I study a setting with cross-dynastic intergenerational altruism. Cross-dynastic intergenerational altruism is the welfare weight on the next generation in other dynasties. It can be motivated by a concern for sustainability, or if descendants move or marry outside the dynasty. Crucially, savings for one's own descendants benefit present members of other dynasties. This gives rise to preference externalities because the other dynasties also care cross-dynastically. I show that intergenerational altruism may not be inferred from



(a) Resulting relative weights on each generation.



(b) Percentage change in the relative weights.

Figure 5: Implications for the relative weights by changes in the wedge between the efficient and equilibrium saving, $s^* - s$. Assume that a generation is 30 years and that $N \to \infty$. From Nordhaus (2008): $\alpha_D = 0.985^{30}$. The wedge, which is a measure of preference externalities, is given by $s^* - s \to \alpha_C$. Consider cases $\alpha_C = 0$, $0.1\alpha_D$, and $0.2\alpha_D$.

saving behavior as long as the relative weight on the utility of the present and next generations is strictly larger for the own dynasty than the other dynasties. I also find that the external effect of present saving decreases over time. This implies that the utility discount rate consistent with saving behavior is decreasing. In general, this discount rate converges to the efficient level only in the limit, as time goes to infinity.

Yet, the utility discount rate in public guidelines is typically informed by saving behavior (OECD, 2018). To illustrate the consequence of the adoption of such discount rates, assume that a generation is 30 years. Assume furthermore that the number of dynasties goes to infinity, $N \to \infty$. Nordhaus (2008) offers an influential-market based calibration. According to Nordhaus, the *relative* weight on future generations can then be expressed as $s \to \alpha_D = 0.985^{30} \approx 64\%$. The main results gave the following wedge between the efficient and equilibrium saving rates: $s^* - s \to \alpha_C$. This measures the preference externalities due to cross-dynastic intergenerational altruism.

Figure 5a exemplifies the shift of relative weights forward in time by accounting for cross-dynastic intergenerational altruism ($s^* - s \to \alpha_C$, with $\alpha_C = 0$, $0.1\alpha_D$ and $0.2\alpha_D$, respectively), thereby correcting the externality problem. From the restriction that α_C is less than or equal to $(N-1)\alpha_D$, it is clear that I consider very low α_C among those that satisfy this restriction. Accounting for cross-dynastic intergenerational altruism implies relative weights on future generations of 64%, 70%, and 76%, leading to discount rates below the rate inferred from saving behavior (1.2% and 0.9%, as compared to the Nordhaus rate of 1.5%). Figure 5b illustrates the percentage change in these weights as compared to the Nordhaus calibration, clarifying that even accounting for limited levels of cross-dynastic intergenerational altruism is important. The weight on future generations increase by 10% and 20%, respectively. The immediate implication for policy guidelines is that discount rates implied from saving behavior should be lowered.

The analysis has clarified the conceptual basis for the above claim in a model of within-dynasty saving. I argued in the Introduction that the condition on preference parameters for preference externalities to emerge are likely to hold in practice. But, not all transfers to future generations are in the form of dynastic saving. One might additionally consider transfers to the immediate descendants of all dynasties, and whether such transfers can crowd out transfers to own immediate descendants.

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Appendix A

This section contains additional proofs of results.

Proof of Theorem 1 – Uniqueness

The following proves that (7) is the unique MPE in the finite horizon game.

Let the remaining horizon be H. Write

$$U^i((h_{H+1}, x_H)) = \max_{k^i} F(x, k^i, H), \tag{33}$$

$$k^i((h_{H+1}, x_H)) = \frac{\sum_{\tau=1}^{H} \Delta_\tau}{\sum_{\tau=0}^{H} \Delta_\tau} x^i = \operatorname*{argmax}_{k^i} F(x, k^i, H) := s_H x^i, \tag{34}$$

where, based on a finite horizon version of (15):

$$F(x, k^i, H) = (1 - \alpha_D - \alpha_C)\Big(\ln(x^i - k^i) + \sum_{\tau=1}^{H} \Delta_\tau \ln(k^i) + C_H \Big), \tag{35}$$

with constant

$$C_H = \sum_{j \neq i} \sum_{\tau=1}^{H} \Gamma_\tau \ln(k^j) + \sum_{\tau=1}^{H} (\alpha_D + \alpha_C)^\tau \ln\big((1 - s_{H-\tau}) \prod_{\ell=1}^{\tau-1} s_{H-\ell}\, A^\tau\big),$$

depending on the present saving of other dynasties and growth terms implied by future play. The value function (33) and strategy (34) are proven by induction.

The base case: Expressions (33) and (34) hold for $H = 0$ due to the convention $\sum_{\tau=1}^{0} \Delta_\tau = 0$.

The step case: The problem for dynasty i with remaining horizon H is to maximize (35) with respect to k^i. The first derivative is:

$$-\frac{1}{x^i - k^i} + \frac{\sum_{\tau=1}^{H} \Delta_\tau}{k^i},$$

which yields the first-order condition:

$$\frac{1}{x^i - k^i} = \frac{\sum_{\tau=1}^{H} \Delta_\tau}{k^i}.$$

Therefore:

$$k^i = \frac{\sum_{\tau=1}^{H} \Delta_\tau}{\sum_{\tau=0}^{H} \Delta_\tau} x^i,$$

which gives $k^i = s_H x^i$. The second derivative is:

$$-\frac{1}{(x^i - k^i)^2} - \frac{\sum_{\tau=1}^{H} \Delta_\tau}{(k^i)^2},$$

and strictly negative for $k^i \in (0, x^i)$. This verifies that the problem is concave. The solution $k^i = s_H x^i$ satisfies the strategy (34), and also the value function (33) due to the independence of the k^j's.

The above establishes uniqueness in a finite horizon game. From expression (19), it is clear that

$$\lim_{H \to \infty} s_H x^i = s x^i.$$

Hence, it is shown that there exists a unique SPE in the finite horizon game for any horizon. The equilibrium strategies used in these finite horizon games go to the linear strategy with s given by (8) when the horizon goes to infinity.

Proof of Corollary 1

Statements are proven one-by-one:

(i) follows by inserting for $\alpha_C = 0$ in expression (8).

(ii) follows by taking the first derivative of s with respect to α_D:

$$1 + \frac{\alpha_C^2 (N-1)}{\big((N-1)(1 - \alpha_D - \alpha_C) + \alpha_C\big)^2} > 0,$$

since $1 > \alpha_D + \alpha_C$.

(iii) follows by taking the first derivative of s with respect to α_C:

$$\frac{2\alpha_C}{(N-1)(1-\alpha_D-\alpha_C)+\alpha_C}+\frac{\alpha_C^2(N-2)}{\big((N-1)(1-\alpha_D-\alpha_C)+\alpha_C\big)^2}>0.$$

(iv) follows by taking the first derivative of s with respect to N:

$$-\alpha_C^2\frac{(1-\alpha_D-\alpha_C)}{\big((N-1)(1-\alpha_D-\alpha_C)+\alpha_C\big)^2}<0.$$

(v) follows by taking the following limit:

$$\lim_{N\to\infty}\alpha_D+\frac{\alpha_C^2}{(N-1)(1-\alpha_D-\alpha_C)+\alpha_C}=\alpha_D+0=\alpha_D.$$

This completes the proof.

Proof of Corollary 3

Assume $\alpha_C>0$. Compare the equilibrium saving rate, s from (8), with the efficient saving rate, s^*:

$$\alpha_C>\frac{\alpha_C^2}{(N-1)(1-\alpha_D-\alpha_C)+\alpha_C},$$

since $(N-1)(1-\alpha_D-\alpha_C)>0$. This verifies that the equilibrium saving rate is inefficiently low for all $N>1$.

Proof of Proposition 1

Write the relative utility weight of two subsequent generations

$$\frac{\Delta_\tau}{\Delta_{\tau-1}}=\frac{\alpha_D\Delta_{\tau-1}+\alpha_C\Gamma_{\tau-1}}{\Delta_{\tau-1}}=\alpha_D+\alpha_C\frac{\Gamma_{\tau-1}}{\Delta_{\tau-1}}, \tag{36}$$

by inserting from (16). Combine expressions (16) and (17),

$$\frac{\Gamma_{\tau-1}}{\Delta_{\tau-1}} = \frac{(\alpha_D+\alpha_C)^{\tau-1} - (\alpha_D - \frac{\alpha_C}{N-1})^{\tau-1}}{(\alpha_D+\alpha_C)^{\tau-1} + (N-1)(\alpha_D - \frac{\alpha_C}{N-1})^{\tau-1}}. \tag{37}$$

There are two cases:

Case 1: Assume $\alpha_D > \alpha_C/(N-1)$. The fraction $\Delta_\tau/\Delta_{\tau-1}$ in (40) is increasing from α_D and converges only in the limit to $\alpha_D+\alpha_C$. This follows directly from (41): $\Gamma_0/\Delta_0 = 0$, Γ_τ/Δ_τ is increasing in τ (since the nominator is increasing in τ, and the denominator is decreasing), and $\lim_{\tau\to\infty}\Gamma_\tau/\Delta_\tau = 1$. This means that all generations weight within-dynasty utility differently. Hence, the preference of each dynasty is time-inconsistent

Case 2: Assume $\alpha_D = \alpha_C/(N-1)$. The fraction $\Delta_\tau/\Delta_{\tau-1}$ in (40) is increasing from α_D and converges immediately to $\alpha_D+\alpha_C$. This follows directly from (41): $\Gamma_0/\Delta_0 = 0$ and $\Gamma_\tau/\Delta_\tau = 1$ for all $\tau \in \mathbb{N}$. This means that only subsequent generations weight within-dynasty utility differently. Hence, the preference of each dynasty is time-inconsistent.

Proof of Proposition 3

Define the geometric series

$$\sum_{\tau=0}^{\infty}(\alpha_D+\alpha_C)^\tau = \frac{1}{1-\alpha_D-\alpha_C}, \tag{38}$$

$$(N-1)\sum_{\tau=0}^{\infty}(\alpha_D - \frac{\alpha_C}{N-1})^\tau = \frac{N-1}{1-\alpha_D+\frac{\alpha_C}{N-1}}. \tag{39}$$

Hence, by (16), it follows from (38) and (39) that

$$\sum_{\tau=0}^{\infty}\Delta_\tau = \frac{1}{N}\Big(\frac{1}{1-\alpha_D-\alpha_C} + \frac{N-1}{1-\alpha_D+\frac{\alpha_C}{N-1}}\Big),$$

which, by rewriting (19), implies

$$s = \frac{\sum_{\tau=0}^{\infty}\Delta_\tau - 1}{\sum_{\tau=0}^{\infty}\Delta_\tau} = \alpha_D + \frac{\alpha_C^2}{(N-1)(1-\alpha_D-\alpha_C)+\alpha_C}.$$

This is identical to expression (8), the equilibrium saving rate.

Proof of Proposition 4

The welfare function (21) follows by repeated substitution of W^i and W^j's into V from (20), keeping in mind that $V = -\infty$ if $u^i = -\infty$ or, if $\alpha_B > 0$, $u^j = -\infty$. Discount functions (22) and (23) are proven by induction.

The base case: Discount functions (22) and (23) hold for $\tau = 0$ since $\Delta_0 = \alpha_A$ and $\Gamma_0 = \alpha_B/(N-1)$.

The step case: Suppose that discount functions (22) and (23) hold for $\tau - 1$. Then,

$$\begin{aligned}
\Delta_\tau &= \alpha_D \Delta_{\tau-1} + \alpha_C \Gamma_{\tau-1} \\
&= \frac{1}{N}\Big(\alpha_A\big((\alpha_D + \alpha_C)^\tau + (N-1)(\alpha_D - \frac{\alpha_C}{N-1})^\tau\big) \\
&\qquad + \alpha_B\big((\alpha_D + \alpha_C)^\tau - (\alpha_D - \frac{\alpha_C}{N-1})^\tau\big)\Big),
\end{aligned}$$

by inserting for $\Delta_{\tau-1}$ and $\Gamma_{\tau-1}$ (and noting the similarity to the Theorem 3). And,

$$\begin{aligned}
\Gamma_\tau &= \frac{\alpha_C}{N-1}\Delta_{\tau-1} + \Big(\alpha_D + \frac{(N-2)\alpha_C}{N-1}\Big)\Gamma_{\tau-1} \\
&= \frac{1}{N}\Big(\frac{\alpha_B}{N-1}\big((\alpha_D + \alpha_C)^\tau + (N-1)(\alpha_D - \frac{\alpha_C}{N-1})^\tau\big) \\
&\qquad + (\alpha_A + \frac{(N-2)\alpha_B}{N-1})\big((\alpha_D + \alpha_C)^\tau - (\alpha_D - \frac{\alpha_C}{N-1})^\tau\big)\Big).
\end{aligned}$$

by inserting for $\Delta_{\tau-1}$ and $\Gamma_{\tau-1}$ (and noting the similarity to Theorem 3). This proves that discount functions (22) and (23) hold for all $\tau \in \mathbb{N}_0$.

It follows from (22) and (23) that $\Delta_\tau + \Gamma_\tau = (\alpha_D + \alpha_C)^\tau$. This ensures that W^i is well-defined on $\mathcal{U}^N$.

Proof of Proposition 5

Write the relative utility weight of two subsequent generations

$$\frac{\Delta_\tau}{\Delta_{\tau-1}} = \frac{\alpha_D \Delta_{\tau-1} + \alpha_C \Gamma_{\tau-1}}{\Delta_{\tau-1}} = \alpha_D + \alpha_C \frac{\Gamma_{\tau-1}}{\Delta_{\tau-1}}, \tag{40}$$

by inserting from (22). Combine expressions (22) and (23),

$$\frac{\Gamma_{\tau-1}}{\Delta_{\tau-1}} = \frac{\frac{\alpha_B}{N-1} f + \left(\alpha_A + \frac{(N-2)\alpha_B}{N-1}\right) g}{\alpha_A f + \alpha_B g}, \tag{41}$$

where

$$f \equiv (\alpha_D + \alpha_C)^{\tau-1} - (\alpha_D - \frac{\alpha_C}{N-1})^{\tau-1},$$
$$g \equiv (\alpha_D + \alpha_C)^{\tau-1} + (N-1)(\alpha_D - \frac{\alpha_C}{N-1})^{\tau-1}.$$

Assuming $\alpha_A > \alpha_B/(N-1)$, there are two cases:

Case 1: Assume $\alpha_D > \alpha_C/(N-1)$. The fraction $\Delta_\tau/\Delta_{\tau-1}$ in (40) is increasing from $\alpha_D + \alpha_C \alpha_B/((N-1)\alpha_A)$ and converges only in the limit to $\alpha_D + \alpha_C$. This follows directly from (41): $\Gamma_0/\Delta_0 = \alpha_B/(N-1)\alpha_A$, Γ_τ/Δ_τ is increasing in τ (since the nominator is increasing in τ, and the denominator is decreasing), and $\lim_{\tau\to\infty} \Gamma_\tau/\Delta_\tau = 1$. This means that all generations weight within-dynasty utility differently. Hence, the preference of each dynasty is time-inconsistent

Case 2: Assume $\alpha_D = \alpha_C/(N-1)$. The fraction $\Delta_\tau/\Delta_{\tau-1}$ in (40) is increasing from $\alpha_D + \alpha_C \alpha_B/((N-1)\alpha_A)$ and converges immediately to $\alpha_D + \alpha_C$. This follows directly from (41): $\Gamma_0/\Delta_0 = \alpha_B/(N-1)\alpha_A$ and $\Gamma_\tau/\Delta_\tau = 1$ for all $\tau \in \mathbb{N}$. This means that only subsequent generations weight within-dynasty utility differently. Hence, the preference of each dynasty is time-inconsistent.

Proof of Corollary 4

For discount function (22), it follows from (38) and (39) that

$$\sum_{\tau=0}^{\infty} \Delta_\tau = \frac{1}{N}\Big(\frac{1}{1-\alpha_D-\alpha_C} + \Big(\alpha_A - \frac{\alpha_B}{N-1}\Big)\frac{N-1}{1-\alpha_D+\frac{\alpha_C}{N-1}}\Big),$$

which, by the Hiraguchi-Iverson-Karp solution, implies

$$s = \frac{\sum_{\tau=0}^{\infty}\Delta_\tau - \alpha_A}{\sum_{\tau=0}^{\infty}\Delta_\tau} = \alpha_D + \alpha_C \frac{\alpha_A\alpha_C + \alpha_B(1-\alpha_D)}{\alpha_A((N-1)(1-\alpha_D-\alpha_C)+\alpha_C)+\alpha_B\alpha_C}.$$

This is identical to expression (24), the equilibrium saving rate. The efficient saving rate (25) follows immediately from (22) and (23).

Proof of Proposition 7

The welfare function (27) follows by repeated substitution of W^i and W^j's into V from (26), keeping in mind that $V = -\infty$ if $u^i = -\infty$. Discount functions (28) and (29) are proven by induction.

The base case: Discount functions (28) and (29) hold for $\tau = 0$ since $\Delta_0 = 1$ and $\Gamma_0 = \alpha_E$, under the condition that Δ_0 is normalized to 1.

The step case: Suppose that discount functions (28) and (29) hold for $\tau - 1$. Then,

$$\Delta_\tau = \alpha_D \Delta_{\tau-1} = \alpha_D^\tau,$$

by inserting for $\Delta_{\tau-1}$. And,

$$\Gamma_\tau = \frac{\alpha_E}{N-1}\alpha_D\Delta_{\tau-1} = \frac{\alpha_E}{N-1}\alpha_D^\tau,$$

by inserting for $\Delta_{\tau-1}$. This proves that discount functions (28) and (29) hold for all $\tau \in \mathbb{N}_0$.

It follows from (28) and (29) that $\Delta_\tau + \Gamma_\tau = (1+\alpha_E)\alpha_D^\tau$. This ensures that W^i is well-defined on $\mathcal{U}^N$.

Appendix B

This section provides interpretations of the model if descendants can move or marry someone from other dynasties.

The "dynastic family"

In response to Barro's (1974) formulation of intergenerational altruism, Bernheim and Bagwell (1988) consider the case in which each generation consists of a large number of individuals, and that links between dynasties imply that individuals belong to different dynasties. A limitation of their analysis is that these links are hypothesized and not modeled. Laitner (1991) and Zhang (1994) formulate links between two dynasties through marital connections, but focus on cross-sectional neutrality of policies and assortative mating, respectively. Myles (1997) state a more general preference, but is silent about its implications for the discount function.

I give a new interpretation of the discount function. Define for now α_C as the relative probability of immediate descendants ending up in the other dynasties, for example through mating. (Consult Proposition 8 in the next subsection for a statistical interpretation of the discount functions.) Then, discount functions (16) and (17) are Markov chains assigning the relative probabilities that descendants end up in different dynasties:

Remark 3 *Under Assumption 1, the fraction $\Delta_\tau/(\Delta_\tau + (N-1)\Gamma_\tau)$ assigns the probability that the descendants of the present generation of a dynasty are in the same dynasty τ generations from now.*

Note that

$$\frac{\Delta_\tau}{\Delta_\tau + (N-1)\Gamma_\tau} = \frac{1}{N}\frac{(\alpha_D + \alpha_C)^\tau + (N-1)(\alpha_D - \frac{\alpha_C}{N-1})^\tau}{(\alpha_D + \alpha_C)^\tau},$$

by inserting from expressions (16) and (17). Observe that $\lim_{\tau\to\infty} \Delta_\tau/(\Delta_\tau + (N-1)\Gamma_\tau) = 1/N$, implying convergence to a uniform distribution if $\alpha_D > \alpha_C/(N-1)$. In fact, the uniform distribution follows since the external effect of present saving becomes less important over time, and vanishes only in the limit.

Statistical interpretation

Consider discount functions (16) and (17) for $N = 2$. I have the following proposition:

Proposition 8 *Assume $N = 2$. Under Assumption 1, discount functions (16) and (17) can be written:*

$$\Delta_\tau = \sum_{\substack{q \ even \\ 0 \leq q \leq \tau}} \binom{\tau}{\tau - q} \alpha_D^{\tau-q} \alpha_C^q, \tag{42}$$

$$\Gamma_\tau = \sum_{\substack{q \ odd \\ 0 \leq q \leq \tau}} \binom{\tau}{\tau - q} \alpha_D^{\tau-q} \alpha_C^q.$$

Proof. The right-hand side of (42) can be simplified. Do the following rescaling of parameters: $\tilde{\alpha_D} = \alpha_D/(\alpha_D + \alpha_C)$ and $\tilde{\alpha_C} = \alpha_C/(\alpha_D + \alpha_C)$. Since $\tilde{\alpha_D} + \tilde{\alpha_C} = 1$, I can work with sums of binomial distributions. Write the sum over q even and q odd distributions as:

$$\begin{aligned} \sum_{q=0}^{\tau} \binom{\tau}{\tau - q} \alpha_D^{\tau-q} \alpha_C^q &= (\alpha_D + \alpha_C) \sum_{q=0}^{\tau} \binom{\tau}{\tau - q} \tilde{\alpha_D}^{\tau-q} \tilde{\alpha_C}^q \\ &= (\alpha_D + \alpha_C)^\tau, \end{aligned} \tag{43}$$

where the last line follow since the summation is now the total cumulative probability distribution of a binomial distribution, and is equal to 1. The difference between q even and q odd distributions can be expressed as:

$$\sum_{\substack{q \text{ even} \\ 0 \leq q \leq \tau}} \binom{\tau}{\tau - q} \alpha_D^{\tau-q} \alpha_C^q - \sum_{\substack{q \text{ odd} \\ 0 \leq q \leq \tau}} \binom{\tau}{\tau - q} \alpha_D^{\tau-q} \alpha_C^q$$

$$= \sum_{\substack{q \text{ even} \\ 0 \leq q \leq \tau}} (-1)^q \binom{\tau}{\tau - q} \alpha_D^{\tau-q} \alpha_C^q + \sum_{\substack{q \text{ odd} \\ 0 \leq q \leq \tau}} (-1)^q \binom{\tau}{\tau - q} \alpha_D^{\tau-q} \alpha_C^q$$

$$= \sum_{q=0}^{\tau} \binom{\tau}{\tau - q} \alpha_D^{\tau-q} (-\alpha_C^q) = (\alpha_D - \alpha_C)^\tau, \tag{44}$$

using the definitions of $\tilde{\alpha_D}$ and $\tilde{\alpha_C}$.

Using the insights from expressions (43) and (44), expression (42) can be written:

$$\Delta_\tau = \frac{1}{2}\Big(\underbrace{(\alpha_D + \alpha_C)^\tau}_{q \text{ even} + q \text{ odd}} + \underbrace{(\alpha_D - \alpha_C)^\tau}_{q \text{ even - } q \text{ odd}} \Big),$$

which is identical to (16) for $N = 2$.

For completeness, define Γ_τ as:

$$\begin{aligned}
\Gamma_\tau &= (\alpha_D + \alpha_C)^\tau - \Delta_\tau \\
&= (\alpha_D + \alpha_C)^\tau - \frac{1}{2}\big((\alpha_D + \alpha_C)^\tau + (\alpha_D - \alpha_C)^\tau\big) \\
&= \frac{1}{2}\big((\alpha_D + \alpha_C)^\tau - (\alpha_D - \alpha_C)^\tau\big),
\end{aligned}$$

which is identical to (17) for $N = 2$. ■

From the point of view of the present generation of dynasty i, even time periods allow more cross-dynastic altruistic intergenerational links forward in time, as compared to the preceding odd time period. This asymmetry is clear from extending Figure 2 forward in time. The expression within the summation in (42) resembles a binomial distribution, with the exception that $\alpha_D + \alpha_C < 1$.

References

Arrow, K., Cline, W.R., Maler, K.-G., Munasinghe, M., Squitieri, R., and Stiglitz, J.E. (1995), Intertemporal equity, discounting, and economic efficiency. In *Climate Change 1995: Economic and Social Dimensions of Climate Change. Contribution of Working Group III to the Second Assessment Report of the Intergovernmental Panel on Climate Change* [J.P. Bruce, H. Lee, and E.F. Haites (eds.)], pp. 129–144, Cambridge: Cambridge University Press.

Arrow, K., Cropper, M.L., Gollier, C., Groom, B., Heal, G.M., Newell, R.G., Nordhaus, W.D., Pindyck, R.S., Pindyck, W.A., Portney, P.R., Sterner, T., Tol, R.S.J., and Weitzman, M.L. (2013), Determining benefits and costs for future generations. *Science* **341**, 349–350.

Asheim, G.B. (1997), Individual and collective time-consistency. *Review of Economic Studies* **64**(3), 427–443.

Asheim, G.B., and Nesje, F. (2016), Destructive intergenerational altruism. *Journal of the Association of Environmental and Resource Economists* **3**(4), 957–984.

Barro, R.J. (1974), Are government bonds net wealth? *Journal of Political Economy* **82**(6), 1095–1117.

Bergstrom, T.C. (1999), Systems of benevolent utility functions. *Journal of Public Economic Theory* **1**(1), 71–100.

Bernhard, H., Fischbacher, U., and Fehr, E. (2006), Parochial altruism in humans. *Nature* **442**, 912–915.

Bernheim, B.D, and Bagwell, K. (1988), Is everything neutral? *Journal of Political Economy* **96**(2), 308–338.

Brock, W.A. (1979), An integration of stochastic growth theory and the theory of finance, part 1: The growth model. In *General Equilibrium, Growth and Trade* [J. Green and J.A. Scheinkman (ed.)], pp. 165–192, New York: Academic Press.

Brock, W.A. (1982), Asset prices in a production economy. In *The Economics of Information and Uncertainty* [J.J. McCall (ed.)], pp. 1–46, Chicago: University of Chicago Press.

Bourlès, R., Bramoullé, Y., and Perez-Richet, E. (2017), Altruism in networks. *Econometrica* **85**(2), 675–689.

Chermak, J.M., and Krause, K. (2003), Individual response, information, and intergenerational common pool problems. *Journal of Environmental Economics and Management* **43**(1), 47–70.

Cropper, M., Aydede, S., and Portney, P. (1991), Discounting human lives. *American Journal of Agricultural Economics* **73**(5), 1410–1415.

Cropper, M., Aydede, S., and Portney, P. (1992), Rates of time preference for saving lives. *American Economic Review* **82**(2), 469–472.

Cropper, M., Aydede, S., and Portney, P. (1994), Preferences for life saving programs: How the public discounts time and age. *Journal of Risk and Uncertainty* **8**(3), 243–265.

Drupp, M.A., Freeman, M.C., Groom, B., and Nesje, F. (2018), Discounting disentangled. *American Economic Journal: Economic Policy* **10**(4), 109–134.

Ekeland, I., and Lazrak, A. (2010), The golden rule when preferences are time inconsistent. *Mathematics and Financial Economics* **4**(1), 29–55.

Fehr-Duda, H., and Fehr, E. (2016), Game human nature. *Nature* **530**, 413–415.

Fischer, M.-A., Irlenbusch, B., and Sadrieh, A. (2003), An intergenerational common pool resource experiment. *Journal of Environmental Economics and Management* **48**(2), 811–836.

Frederick, S. (2003), Measuring intergenerational time preference: Are future lives valued less? *Journal of Risk and Uncertainty* **26**(1), 39–53.

Galperti, S., and Strulovici, B. (2017), A theory of intergenerational altruism. *Econometrica* **85**(4), 1175–1218.

Gollier, C. (2012), *Pricing the Planet's Future: The Economics of Discounting in an Uncertain World*, Princeton and Oxford: Princeton University Press.

Goulder, L.H., and Williams III, R.C. (2012), The choice of discount rate for climate change policy evaluation. *Climatic Change Economics* **3**(4), 1–18.

Groom, B., and Hepburn, C. (2017), Looking back at social discounting policy: The influence of papers, presentations, political preconditions, and personalities. *Review of Environmental Economics and Policy* **11**(2), 336–356.

Harstad, B. (2019), Technology and time inconsistency. *Journal of Political Economy*, forthcoming.

Hauser, O.P., Rand, D.G., Peysakhovich, A., and Nowak, M.A. (2018), Cooperating with the future. *Nature* **511**, 220–223.

Hausman, D.M (2012), Why satisfy preferences? Mimeo, Max Planck Institute.

Halevy, Y. (2015), Time consistency: Stationarity and time invariance. *Econometrica* **83**(1), 335–352.

Hiraguchi, R. (2014), On the neoclassical growth model with non-constant discounting. *Economics Letters* **125**(2), 175–178.

Iverson, T., and Karp, L. (2018), Carbon taxes and climate commitment with non-constant time preference. Mimeo, Colorado State University.

Johannesson, M., and Johansson, P.-O. (1997), Saving lives in the present versus saving lives in the future: Is there a framing effect? *Journal of Risk and Uncertainty* **15**(2), 167–176.

Karp, L. (2007), Non-constant discounting in continuous time. *Journal of Economic Theory* **132**, 557–568.

Kelleher, J.P. (2017), Descriptive versus prescriptive discounting in climate change policy analysis. *Georgetown Journal of Law and Public Policy* **15**, 957–977.

Kolstad, C., Urama, K., Broome, J., Bruvoll, A., Cariño Olvera, M., Fullerton, D., Gollier, G., Hanemann, W.M., Hassan, R., Jotzo, F., Khan, M.R., Meyer, L., and Mundaca, L. (2014), Social, economic and ethical concepts and methods. In *Mitigation of Climate Change. Contribution of Working Group III to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change* [O. Edenhofer, R. Pichs-Madruga, Y. Sokona, E. Farahani, S. Kadner, K. Seyboth, A. Adler, I. Baum, S. Brunner, P. Eickemeier, B. Kriemann, J. Savolainen, S. Schlömer, C. von Stechow, T. Zwickel, and J.C. Minx (eds.)], pp. 207–282, Cambridge and New York: Cambridge University Press.

Krusell, P., Kuruşçu, B., and Smith, A. (2002), Equilibrium welfare and government policy with quasi-geometric discounting. *Journal of Economic Theory* **105**(1), 42–72.

Laibson, D. (1998), Hyperbolic discounting functions, undersaving and savings policy. National Bureau of Economic Research Working Paper, No. 5635.

Laitner, J. (1991), Modeling marital connections among family lines. *Journal of Political Economy* **99**(6), 1123–1141.

Lind, R.C. (1964), The social rate of discount and the optimal rate of investment: Further comment. *Quarterly Journal of Economics* **78**(2), 336–345.

Marglin, S.A. (1963), The social rate of discount and the optimal rate of investment. *Quarterly Journal of Economics* **77**(1), 95–111.

Maskin, E., and Tirole, J. (2001), Markov perfect equilibrium, I: Observable actions. *Journal of Economic Theory* **100**, 191–219.

Milgrom, P. (1993), Is sympathy an economic value? Philosophy, economics and the contingent valuation method. In *Contingent Valuation: A Critical Assessment* [J.A. Hausman (ed.)], pp. 417–435, Amsterdam: Elsevier.

Millner, A. (2019), Non-dogmatic social discounting. *American Economic Review*, forthcoming.

Molina, J.A., Ferrer, A., Giménez-Nadal, J.I., Gracia-Lázaro, C., Moreno Y., and Sánchez, A. (2018), Intergenerational cooperation within the household: A public good game with three generations. *Review of Economics of the Household* **17**(2), 535–552.

Myles, G.D. (1997), Depreciation and intergenerational altruism in the private provision of public goods. *European Journal of Political Economy* **13**(4), 725–738.

Negishi, T. (1960), Welfare economics and existence of an equilibrium for a competitive economy. *Metroeconomica* **12**(1-2), 92–97.

Newbery, D.M. (1990), The isolation paradox and the discount rate for benefit-cost analysis: A comment. *Quarterly Journal of Economics* **105**(1), 235–238.

Nordhaus, W.D. (1994), *Managing the Global Commons: The Economics of Climate Change*, Cambridge, MA: MIT Press.

Nordhaus, W.D. (2007), A review of the Stern Review on the Economics of Climate Change. *Journal of Economic Literature* **45**(3), 686–702.

Nordhaus, W.D. (2008), *A Question of Balance: Weighting the Options on Global Warming Policies*, New Haven: Yale University Press.

Nordhaus, W.D., and Yang, Z. (1996), A regional dynamic general-equilibrium model of alternative climate-change strategies. *American Economic Review* **86**, 741–765.

OECD (2018), *Cost-Benefit Analysis and the Environment: Further Developments and Policy Use*, Paris: OECD Publishing.

Pearce, D. (1983), Nonpaternalistic sympathy and the inefficiency of consistent intertemporal plans. Mimeo, Princeton University.

Phelps, E.S., and Pollak, R.A. (1968), On second-best national saving and game-equilibrium growth. *Review of Economic Studies* **35**(2), 185–199.

Prescott, E.C., and Mehra, R. (1980), Recursive competitive equilibrium: The case of homogeneous households. *Econometrica* **48**(6), 1365–1379.

Ray, D. (1987), Nonpaternalistic intergenerational altruism. *Journal of Economic Theory* **41**(1), 112–132.

Robson, A.J., and Szentes, B. (2014), A biological theory of social discounting. *American Economic Review* **104**(11), 3481–3497.

Romer, P.M. (1986), Increasing returns and long-run growth. *Journal of Political Economy* **94**(5), 1002–1037.

Sáez-Marti, M., and Weibull, J.W. (2005), Discounting and altruism to future decision-makers. *Journal of Economic Theory* **122**(2), 254–266.

Samuelson, P. (1937), A note on measurement of utility. *Review of Economic Studies* **4**(2), 155–161.

Schelling, T.C. (1995), Intergenerational discounting. *Energy Policy* **34**(4-5), 395–401.

Sen, A.K. (1961), On optimising the rate of saving. *Economic Journal* **71**(283), 479–496.

Sen, A.K. (1967), Isolation, assurance and the social rate of discount. *Quarterly Journal of Economics* **81**(1), 112–124.

Shapley, L.S. (1953), Stochastic games. *Proceedings of the National Academy of Sciences* **39**(10), 1095–1100.

Stern, N. (2007) *The Economics of Climate Change: The Stern Review*, Cambridge and New York: Cambridge University Press.

Strotz, R.H. (1955-1956), Myopia and inconsistency in dynamic utility maximization. *Review of Economic Studies* **23**(3), 165–180.

Weitzman, M.L. (2001), Gamma discounting. *American Economic Review* **91**(1), 260–271.

Zhang, J. (1994), Bequest as a public good within marriages: A note. *Journal of Political Economy* **102**(1), 187–193.