

SEPARABLE C^* -ALGEBRAS AND WEAK* FIXED POINT PROPERTY

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Abstract. We show that the spectrum \widehat{A} of a separable C^* -algebra A is discrete if and only if A^* , the Banach space dual of A , has the weak* fixed point property. We prove further that these properties are equivalent among others to the uniform weak* Kadec-Klee property of A^* and to the coincidence of the weak* topology with the norm topology on the pure states of A . If one assumes the set-theoretic diamond axiom, then the separability is necessary.

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1. INTRODUCTION

It is a well-known theorem in harmonic analysis that a locally compact group G is compact if and only if its dual \widehat{G} is discrete. This dual is just the spectrum of the full C^* -algebra $C^*(G)$ of G . (The spectrum of a C^* -algebra being the unitary equivalence classes of the irreducible $*$ -representations endowed with the inverse image of the Jacobson topology on the set of primitive ideals.) There is a bunch of properties of the weak* topology for the Fourier–Stieltjes algebra $B(G)$ of G , which are equivalent to the compactness of the group, see [7]. Some of them, which can be formulated in purely C^* -algebraic terms, are the topic of this note.

Let E be a Banach space and K be a non-empty bounded closed convex subset. K has the *fixed point property* if any non-expansive map $T : K \rightarrow K$ (i.e. $\|Tx - Ty\| \leq \|x - y\|$ for all $x, y \in K$) has a fixed point. We say that E has the *weak fixed point property* if every weakly compact convex subset of E has the fixed point property. If E is a dual Banach space, we consider the *weak* fixed point property* of E , i.e. the property that every weak* compact convex subset of E has the fixed point property. Since in a dual Banach space convex weakly compact sets are weak* compact, the weak* fixed point property of E implies the weak fixed point property.

As in [7] we shall consider the case of a left reversible semigroup \mathcal{S} acting by non-expansive mappings separately continuously on a non-empty weak* compact convex set $K \subset E$. We say that E has the *weak* fixed point property for left reversible semigroups* if under these conditions there always is a common fixed point in K .

One of the main results of [7] is that a locally compact group G is compact if and only if $B(G)$ has the weak* fixed point property for non-expansive maps, equivalently for left reversible semigroups.

We shall prove that a separable C^* -algebra has a discrete spectrum if and only if its Banach space dual has the weak* fixed point property. We consider separable C^* -algebras only, because a separable C^* -algebra with one-point spectrum is known to be isomorphic to the algebra of compact operators on some Hilbert space [27]. The converse, namely that the C^* -algebra of the compact operators has up to unitary equivalence only one irreducible representation, was proved by Naïmark [23]. His question [24] whether these are the only C^* -algebras with a one-point spectrum became known as Naïmark's problem. Assuming the set-theoretic diamond axiom, independent of ZFC (Zermelo Frankel set theory with the axiom of choice), Akemann and Weaver [1] answered this in the negative. We shall prove that their C^* -algebra does not have the weak fixed point property. This shows that the separability assumption in our theorem is essential.

Section 2 contains our main theorem and its proof. In Section 3 we consider the uniform weak* Kadec-Klee property (see Definition 3.1) of the Banach space dual of the C^* -algebras in question. For the trace class operators this property holds true as proved by Lennard [20]. As he points out, the weak* fixed point property can be obtained, via weak* normal structure of non-empty weakly* compact sets, by an application of UKK*, as shown by van Dulst and Sims [6]. For corresponding results with the weak topology we refer to the articles by Kirk [12] and Lim [21].

2. WEAK* FIXED POINT PROPERTY

In this section A shall be a separable C^* -algebra, unless stated otherwise. We denote by $\pi' \simeq \pi$ the unitary equivalence of $*$ -representations π' and π . By abuse of notation we denote by π also its equivalence class.

The following proposition is based on a theorem of Anderson [3], which itself refines a lemma of Glimm (see [9], Lemma 9, and [8], Theorem 2).

PROPOSITION 2.1. *Let A be a separable C^* -algebra. Let $\pi' \not\simeq \pi \in \widehat{A}$ with $\pi' \in \overline{\{\pi\}}$ be given and assume that φ is a state of A associated with π' . Then there is an orthonormal sequence (ξ_n) in H_π with $(\pi(\cdot)\xi_n | \xi_n) \rightarrow \varphi$ weakly*.*

Proof. By assumption, $\ker \pi' \supset \ker \pi$, so there is a representation π° of $\pi(A)$ such that $\pi' = \pi^\circ \circ \pi$. We may therefore assume that π is the identical representation. We denote by $\mathcal{K}(H)$ the C^* -algebra of compact operators on the Hilbert space H .

(i) Suppose $\varphi|_{\mathcal{K}(H_\pi) \cap A} \neq \emptyset$. Then $\pi' = \pi_\varphi$ does not annihilate $\mathcal{K}(H_\pi) \cap A$. By Corollary 4.1.10 in [5], $\mathcal{K}(H_\pi) \subset A$ and it is a two-sided ideal. This corollary does not cover our case completely but we follow its proof. π' is faithful on $\mathcal{K}(H_\pi)$ and $\pi'|_{\mathcal{K}(H_\pi)}$ is an irreducible representation by [5], 2.11.3. Therefore, it is equivalent to the identical representation of $\mathcal{K}(H_\pi)$. Now π' is equivalent to the identical representation of A , by [5], 2.10.4 (i). This contradicts the assumption, so this case cannot happen.

(ii) If $\varphi|_{\mathcal{K}(H_\pi) \cap A} = \emptyset$, then by the Theorem in [3] there is an orthonormal sequence (ξ_n) in H_π with $(\pi(\cdot)\xi_n|\xi_n) \rightarrow \varphi$ weakly*. ■

LEMMA 2.1. *Let M be a von Neumann algebra. If its predual M_* has the weak fixed point property, then M is of type I. Moreover, M is atomic.*

PROOF. The argument follows the proof of Theorem 4.1 in [26]. We denote by \mathcal{R} the hyperfinite factor of type II₁ and by $\tau_{\mathcal{R}}$ its canonical finite trace (see, e.g., [28]). In [22] it is proved that its predual $L^1(\mathcal{R}, \tau_{\mathcal{R}})$ embeds isometrically into the predual of any von Neumann algebra not of type I. As $L^1([0, 1], dx)$ embeds isometrically into $L^1(\mathcal{R}, \tau_{\mathcal{R}})$ ([15], Lemma 3.1), we conclude from Alspach's theorem [2] that the weak fixed point property of M_* forces M to be a type I von Neumann algebra. So M has a normal semifinite faithful trace (see [5], A35). Now, Proposition 3.4 in [15] implies that M is an atomic von Neumann algebra. ■

Lemma 2.1 and Proposition 3.4 of [15] provide the converse to Lemma 3.1 in [19], and thus answer a question of Lau [14], Problem 1:

COROLLARY 2.1. *Let M be a von Neumann algebra. Then M_* has the weak fixed point property if and only if it has the Radon–Nikodym property.*

REMARK 2.1. *If now A is a C*-algebra whose Banach space dual A^* has the weak fixed point property, then, by Lemma 2.1, A^{**} is a type I von Neumann algebra and we know from [25], 6.8.8, that A is a type I C*-algebra. Especially, its spectrum, which coincides with the space of its primitive ideals in this case, is a T_0 -topological space. A fortiori, this also holds if A^* has the weak* fixed point property.*

PROPOSITION 2.2. *Assume that A is separable. If A^* has the weak* fixed point property, then points in \widehat{A} are closed.*

PROOF. If $\{\pi\}$ is non-closed in \widehat{A} , then there is $\pi' \not\in \pi$ contained in $\overline{\{\pi\}}$. By Proposition 2.1, if φ is a (pure) state associated with π' , then there exists an orthonormal sequence (ξ_n) in H_π such that $\varphi_n := (\pi(\cdot)\xi_n|\xi_n) \rightarrow \varphi$ weakly*. Now we proceed as in [7]. Set $\varphi_0 = \varphi$; then the set

$$C = \left\{ \sum_0^\infty \alpha_i \varphi_i : 0 \leq \alpha_i \leq 1, \sum_0^\infty \alpha_i = 1 \right\}$$

is convex weak* compact. Now we show that the coefficients of every $f = \sum_0^\infty \alpha_i \varphi_i \in C$ are uniquely determined.

Since A^* is assumed to have the weak* fixed point property, the universal enveloping von Neumann algebra A^{**} of A is atomic. By [7], Appendix A, the universal representation of A decomposes into a direct sum of irreducible representations. Hence we may apply Lemma 4.2¹ of [7] to see that the support P_0 of φ_0 (in the universal enveloping von Neumann algebra) is orthogonal to the support of every other φ_i . So, denoting the ultraweak extensions to A^{**} of $f \in C$ and φ_i , $i \geq 0$, by the same symbols again, we have $f(P_0) = \alpha_0 \varphi_0(P_0) = \alpha_0$. It remains to pick out the remaining α_i from the sum $\sum_1^\infty \alpha_i \varphi_i$. Since π is irreducible, its ultraweak extension to A^{**} has $B(H_\pi)$ as its range. Thus we can evaluate the sum $\sum_1^\infty \alpha_i \varphi_i$ at P_n , the one-dimensional projection onto $\mathbb{C} \cdot \xi_n$, which yields exactly α_n .

Now we may define $T : C \rightarrow C$ by

$$T\left(\sum_0^\infty \alpha_i \varphi_i\right) = \sum_0^\infty \alpha_i \varphi_{i+1}.$$

To show that this map is distance preserving it suffices to see that $\|\sum_0^\infty \beta_i \varphi_i\| = \sum_0^\infty |\beta_i|$ for real summable β_i . Clearly, $\|\sum_0^\infty \beta_i \varphi_i\| = |\beta_0| + \|\sum_1^\infty \beta_i \varphi_i\|$, since the support of φ_0 is orthogonal to the support of any φ_i , $i \geq 1$. Since $B(H_\pi) = A^{**}/\ker(\pi)$ isometrically, the norm $\|\sum_1^\infty \beta_i \varphi_i\|$ can be calculated in $B(H_\pi)$.

The element $Q = \sum_1^\infty \text{sign}(\beta_i) P_i \in B(H_\pi)$ has norm one and $\sum_1^\infty \beta_i \varphi_i(Q) = \sum_1^\infty |\beta_i|$. So $\|\sum_0^\infty \beta_i \varphi_i\| \geq \sum_0^\infty |\beta_i|$. In fact, equality holds, since the reverse inequality is plain. Hence T is distance preserving. The definition of T is such that the only possible fixed point would be zero. But $0 \notin C$ and we arrive at a contradiction. ■

THEOREM 2.1. *For a separable C^* -algebra the following are equivalent:*

- (i) *The spectrum \widehat{A} is discrete.*
- (ii) *A^* has the weak* fixed point property.*
- (iii) *A^* has the weak* fixed point property for left reversible semigroups.*

PROOF. We assume that A^* has the weak* fixed point property. If \widehat{A} is not discrete, then there is some point $\pi_0 \in \widehat{A}$ which is in the closure of some set $M \subset \widehat{A}$ not containing π_0 . Because of the last proposition M must be infinite. So, since A is separable, for any state φ_0 associated with π_0 there is a sequence of states (φ_n) associated with pairwise non-equivalent representations π_n with $\varphi_n \rightarrow \varphi_0$ weakly*. By [7], Lemma 4.2, the support projections in the universal representation of A are mutually orthogonal. As in the proof of Proposition 2.2 the set

¹From the context there, one sees that there it is assumed that every *-representation of the C^* -algebra in question decomposes into a direct Hilbert sum of irreducible representations.

$C = \{ \sum_0^\infty \alpha_i \varphi_i : \alpha_i \geq 0, \sum \alpha_i = 1 \}$ is convex and weak* compact. The map $T : C \rightarrow C$ defined like there is well defined and isometric because of the orthogonality of the supports, and it has no fixed point in C . See also [7], Theorem 4.5. So \hat{A} must be discrete.

Conversely, if A has a discrete spectrum, then the Jacobson topology on its set of primitive ideals is also discrete. By [5], 10.10.6 (a), A is a c_0 -direct sum of C^* -algebras with one-point spectrum. Since A is separable, all these algebras are also separable (the sum is on a countable index set of course), and hence each of them is isomorphic to an algebra of compact operators on some Hilbert space (of at most countable dimension), see [5], 4.7.3. By [26], Corollary 3.7, A^* has the weak* fixed point property for left reversible semigroups. By specialisation, the weak* fixed point property for single non-expansive mappings follows. ■

REMARK 2.2. *If one enriches the ZFC set theory with the diamond axiom, then there is a non-separable C^* -algebra with discrete spectrum, whose Banach space dual does not have the weak (and a fortiori not the weak*) fixed point property.*

Proof. The C^* -algebra A constructed by Akemann and Weaver [1] is not a type I C^* -algebra, but it has a one-point spectrum. It follows that A^{**} is not a type I von Neumann algebra (see [25], 6.8.8). By Lemma 2.1 we obtain our assertion. ■

3. UNIFORM WEAK* KADEC-KLEE

Let $K \subset E$ be a closed convex bounded subset of a Banach space E . A point $x \in K$ is a *diametral* point if $\sup\{\|x - y\| : y \in K\} = \text{diam}(K)$. The set K is said to have *normal structure* if every convex non-trivial (i.e. containing at least two different points) subset $H \subset K$ contains a non-diametral point of H .

A Banach space has *weak normal structure* if every convex weakly compact subset has normal structure, and similarly a dual Banach space has *weak* normal structure* if every convex weakly* compact subset has normal structure.

A dual Banach space E is said to have the *weak* Kadec-Klee property* (KK*) if weak* and norm convergence coincide on sequences of its unit sphere.

DEFINITION 3.1. A dual Banach space E is said to have the *uniform weak* Kadec-Klee property* (UKK*) if for $\epsilon > 0$ there is $0 < \delta < 1$ such that for any subset C of its closed unit ball containing an infinite sequence $(x_i)_{i \in \mathbb{N}}$ with separation $\text{sep}((x_i)_i) := \inf\{\|x_i - x_j\| : i \neq j\} > \epsilon$, there is an x in the weak*-closure of C with $\|x\| < \delta$.

For a discussion of these and similar properties we refer the interested reader to [17]. The following proposition is known, but we could not find a valid reference. So, for the reader's convenience, we give a proof.

PROPOSITION 3.1. *Let E be a dual Banach space.*

(i) *The uniform weak* Kadec-Klee property implies the weak* Kadec-Klee property.*

(ii) *If E is the dual of a separable Banach space E_* and has the uniform weak* Kadec-Klee property, then the weak* topology and the norm topology coincide on the unit sphere of E .*

PROOF. To prove (i) assume that $\|x_n\| = 1$, $x_n \rightarrow x$ weakly* and $\|x\| = 1$. If $\{x_n : n \in \mathbb{N}\}$ is relatively norm compact then the only norm accumulation point has to be x , since the norm topology is finer than the weak* topology, and a subsequence has to converge in norm to x . Hence we assume that $\{x_n : n \in \mathbb{N}\}$ is not relatively compact in the norm topology and shall derive a contradiction. Because $\{x_n : n \in \mathbb{N}\}$ is not totally bounded, by induction, we obtain a subsequence $(x_{n_k})_k$ with $\text{sep}((x_{n_k})_k) > 0$. By the UKK* property there is $\delta < 1$ and a weak* accumulation point y of $(x_n)_n$ with $\|y\| < \delta < 1$. Since every weak* neighbourhood of y contains infinitely many x_n , it follows that $y = x$. This contradicts $\|x\| = 1$.

Now (ii) follows since in this case the weak* topology on the unit sphere of E is metrisable. ■

THEOREM 3.1. *For a separable C^* -algebra A the following are equivalent:*

- (i) *The spectrum \widehat{A} is discrete.*
- (ii) *The Banach space dual A^* has the UKK* property.*
- (iii) *On the unit sphere of A^* the weak* and the norm topology coincide.*
- (iv) *On the set of states $\mathcal{S}(A)$ of A the weak* and the norm topology coincide.*
- (v) *On the set of pure states $\mathcal{P}(A)$ of A the weak* and the norm topology coincide.*
- (vi) *A^* has weak* normal structure.*
- (vii) *A^* has the weak* fixed point property for non-expansive mappings.*
- (viii) *A^* has the weak* fixed point property for left reversible semigroups.*

REMARK 3.1. *The C^* -algebras fulfilling the equivalent conditions of the theorem are just the separable dual C^* -algebras (see [28], p. 157, for the definition). This follows from the fact that separable dual C^* -algebras are characterised by the property that their spectrum is discrete (see [5], 9.5.3 and 10.10.6; see also [13], p. 706).*

PROOF OF THEOREM 3.1. Assume (i). Then, as in the proof of Theorem 2.1, A^* is a countable l^1 -direct sum of trace class operators in canonical duality to the corresponding c_0 -direct sum of compact operators. Moreover, considering A^* as block diagonal trace class operators on the Hilbert space direct sum of the

underlying Hilbert spaces gives an isometric embedding of A^* in the trace class operators on this direct sum Hilbert space. The image is closed in the weak* topology and we obtain the UKK* property of A^* from the UKK* property of the trace class operators [20].

Now (ii) implies (iii) by Proposition 3.1 above. Clearly, (iii) implies (iv) and the latter implies (v) by restriction. So the first part of our proof will be finished by proving the implication (v) \Rightarrow (i). We adapt the proof of Lemma 3.7 in [4] to our context. For $\varphi \in \mathcal{S}(A)$ denote by π_φ the representation of A obtained from the GNS construction. Here extreme points yield irreducible representations and conversely a representative of any element of \widehat{A} can be obtained in this way. Moreover, if $\mathcal{P}(A)$ is endowed with the weak* topology then the mapping $q : \varphi \rightarrow \pi_\varphi$ is open ([5], Theorem 3.4.11). By [11], Corollary 10.3.8, for $\varphi, \psi \in \mathcal{P}(A)$ the representations π_φ and π_ψ are equivalent if $\|\varphi - \psi\| < 2$ (see also [10], Corollary 9). Hence, assuming (v), we see that the (norm open) set $\{\psi \in \mathcal{P}(A) : \|\varphi - \psi\| < 2\}$ is a weak* open neighbourhood of φ in $\mathcal{P}(A)$. Its image under q is open but just reduces to the point π_φ . This shows that points in \widehat{A} are open.

Now (ii) \Rightarrow (vi) is proved in [17], (vi) \Rightarrow (vii) is proved in [21], [12] (see also [6]). The implication (vii) \Rightarrow (i) holds true by Theorem 2.1. From this theorem we have also (viii) \Leftrightarrow (i). ■

REMARK 3.2. *Each of the following conditions implies (i)–(viii) above and, if A is the group C^* -algebra of a locally compact group G , is equivalent to them. (See [7], Section 5, for the definitions involved.)*

- (ix) A^* has the limsup property.
- (x) A^* has the asymptotic centre property.

Under the assumption of separability it is shown in [18], Theorem 4.1, that the limsup property implies the asymptotic centre property. From this in turn the weak* fixed point property for left reversible semigroups follows ([18], Theorem 4.2). Without the separability these implications hold equally true, see [7].

REMARK 3.3. *It is proved in [16], Theorem 5, that the limsup property, which is equivalent to Lim's condition considered there, is not fulfilled in the space of trace class operators of an infinite-dimensional Hilbert space H . So for $A = \mathcal{K}(H)$ the limsup property for A^* is not satisfied, and hence not equivalent to (i)–(viii) above. It seems unlikely that the asymptotic centre property holds true in this case.*

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