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Legislative bargaining with private information: A comparison of majority and unanimity rule

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Abstract

We present a three-person, two-period bargaining game with private information. A single proposer is seeking to secure agreement to a proposal under either majority or unanimity rule. Two responders have privately known "breakdown values" which determine their payoff in case of "breakdown". Breakdown occurs with some probability if the first proposal fails and with certainty if the second proposal fails. We characterize Bayesian Equilibria in Sequentially Weakly Undominated Strategies. Our central result is that responders have a signaling incentive to vote "no" on the first proposal under unanimity rule, whereas no such incentive exists under majority rule. The reason is that being perceived as a "high breakdown value type" is advantageous under unanimity rule, but disadvantageous under majority rule. As a consequence, responders are "more expensive" under unanimity rule and disagreement is more likely. These results confirm intuitions that have been stated informally before and in addition yield deeper insights into the underlying incentives and what they imply for optimal behavior in bargaining with private information.

1 Introduction

A fundamental problem in Political Economics is the choice between alternative q-majority rules. As argued informally by, among others, Buchanan and Tullock (1965), unanimity rule ($q = n$) may create incentives for individuals to take a "tough" bargaining stance. That is, bargainers may have an incentive to "pretend" to be opposed to agreement (or more strongly opposed than they actually are), hoping to secure concessions from others. Such incentives are mitigated by the use of less-than-unanimity rules, because bargainers who take an excessively "tough" stance can be excluded from a winning coalition.

Certain aspects of this idea can be formalized using models involving perfect information (see, e.g., Miettinen and Vanberg (2020)). However, the core intuition as well as its real-world counterpart clearly involve *private information* about individual inclinations to support a proposal. In the presence of private information, the actions taken by individual bargainers may act as (costly) *signals* of their privately known preferences. Our aims in this paper are to model the role of private information explicitly, in order to investigate how such *signaling incentives* are affected by the decision rule being used, and to gain a deeper understanding of what they imply about optimal behavior by proposers and responders in bargaining with private information. In particular, our focus will be on the *signaling value* of voting to support or reject a proposal and how it is affected by the decision rule and the particular proposal being considered.

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We analyze a two-period, three-player bargaining game involving a fixed proposer who is trying to secure agreement on a proposal, either under majority ($q = 2$) or unanimity ($q = 3$) rule. The two responders can be of two different types, who differ in their willingness to agree to the proposal. This difference is modeled by endowing each responder with a privately known "breakdown value" (either "high" or "low"), which they receive if no agreement is reached. The substantive interpretation is that a player's "breakdown value" reflects the utility of the "status quo" which will prevail if no agreement is reached. The proposer is endowed with a single monetary unit that she can use to "buy" the responders' votes, and she has (at most) two chances to get her proposal passed. The substantive interpretation is that the monetary unit represents a surplus that will be generated by the proposed change to the "status quo" and that this surplus is transferable between the members of the committee. Breakdown occurs with some probability if the first proposal fails, and for sure if the second proposal fails. (The game is formally defined in Section 3.)

Our main results are as follows. Under both majority and unanimity rule, voting "no" on certain first period proposals constitutes a *signal* that a responder is a "high" type. Under unanimity rule, this signaling effect benefits the responder in the sense that it causes the proposer to make a more favorable offer to him in Period 2. We refer to this as a positive "signaling value" from rejection. Under majority rule, by contrast, the signaling value is generally *negative*, because responders who vote "no" can be excluded from subsequent coalitions (and it is in the interest of the proposer to do so). As a consequence of this difference in signaling incentives, responders are "more expensive" (in a sense that will be made more precise) under unanimity rule. The set of proposals that can pass in Period 1 is significantly larger under majority rule, not just because agreement requires fewer "yes"-votes, but also because responders are more willing to vote "yes" when allocated a given share. Under majority rule, agreement in the first period is more likely in the sense that, for a wide parameter range, the proposer will find it optimal to immediately secure one responder's agreement, while under unanimity rule, there is a large parameter range for which the proposer prefers not to ensure immediate agreement.

Overall, these results lend support to the intuitive argument that unanimity rule induces "tough" bargaining stances and is associated with greater delay. In addition, our analysis provides a deeper understanding of the signaling incentives in multilateral bargaining as well as implications for optimal behavior in the presence of private information. The rest of the paper is organized as follows. Section 2 discusses related literature. Section 3 presents our game and equilibrium concept. Section 4 previews our main findings using examples with specific parameter constellations. Sections 5 and 6 present more general analyses of unanimity and majority rule, respectively. Section 7 compares the results in order to make precise our conclusions that unanimity rule is associated with higher "prices" and more delay. Section 8 concludes and discusses the additional strategic insights generated by the analysis. Proofs are presented in the Appendix.

2 Related Literature

The model and analysis presented in this paper fits broadly into a literature on q-majority rules in multilateral bargaining games with complete information. More specifically, there is a connection to models that involve any sort of heterogeneity with respect to individual players' willingness to agree. For example, consider a standard Baron-Ferejohn bargaining game (see Baron and Ferejohn (1989)) with complete information and heterogeneous discount factors. There, a larger discount factor implies a "tougher" bargaining stance and a higher "price" for a player's vote. It is folk knowledge that this is 'good' under unanimity rule but may be disadvantageous under majority rule. That is, a player's expected payoff is increasing in his discount factor under unanimity rule, but non-monotone (and eventually decreasing) under less-than-unanimity rules. Miller et al. (2018) consider a modified Baron-Ferejohn game with *commonly known* breakdown values and show that a larger breakdown value is advantageous under unanimity rule, but can be bad under majority rule because "expensive" players are excluded from winning coalitions. These results are *suggestive* of the idea that, when the source of heterogeneity is private information, players would like to *signal* that they are "tough" (e.g. have a large discount factor or breakdown value) under unanimity rule, while they would not like to do so under less-than-unanimity rules.

The literature on multilateral bargaining with private information is scarce. Tsai and Yang (2010) investigate a three-player, three-period Baron-Ferejohn game with private information about discount factors. They show that under majority rule, equilibrium might involve delay and oversized coalitions. Though our model differs in several respects, we will have substantively similar results for the case of majority rule. The main differences between our paper and Tsai and Yang (2010) is that we also investigate unanimity rule, and that our focus is on contrasting the signaling incentives under the different rules.[1] Another is that Tsai and Yang (2010) focus on pure strategy equilibria. As we will see, such equilibria may fail to exist in our model under unanimity rule. One of the main technical contributions of our paper will be to characterize mixed strategy equilibria under unanimity rule.

Chen and Eraslan (2014) investigate a three-player majoritarian bargaining game over an ideological and distributive decision with private information about ideological intensities. As in our setting, a single proposer is seeking the support of either one (majority rule) or both (unanimity rule) responders, and she is unsure as to how much each responder must be paid in order to secure his vote. Unlike in our setting, the proposer can make only one proposal, so there is no room for costly signaling by voting "no". Instead, Chen and Eraslan (2014) consider cheap talk communication prior to the proposal stage. They investigate the circumstances under which cheap talk communication can credibly convey information. A central finding is that competition between the two responders may result in less information conveyed in equilibrium and may make the proposer worse off as compared to the case under unanimity rule (and only one responder). (Chen and Eraslan (2013) investigate a similar model in which the responders' ideal points are private information.)

Our paper is also related to the literature on reputational bargaining. Abreu and Gul (2000) investigate a bilateral bargaining game in which players may, with some probability, be of an "obstinate" type that is committed to claim a certain share of the pie. They show that the presence of such types creates incentives for "rational" types to imitate them, i.e. to "act tough". Ma (2021) applies this idea to a multilateral *majoritarian* bargaining game and finds that the threat of exclusion under majority rule removes this incentive to act "tough".

Finally, our paper is also related to the literature in *bilateral* bargaining with private information. Naturally, these models only consider unanimity rule ($q = n = 2$). The paper that is most relevant to our analysis is Fudenberg and Tirole (1983), who investigate a two-person, two-period bargaining game in which a seller is looking to sell an object to a buyer whose valuation is privately known. The seller has two chances to make a proposal (suggested price). Like in our model, (Sequential) equilibrium requires that some buyers *mix* between accepting and rejecting certain offers.

[1] After completing this draft, we learned that Tsai (2009) studied a similar game both under both majority and unanimity rule, though the analysis is less elaborate than in Tsai and Yang (2010). Some of our results are similar to his. For example, he also finds that immediate agreement is more likely under majority rule. As indicated, our model differs in several respects, and our analysis is more elaborate. Tsai's analysis identifies conditions for the existence of "separating" and "pooling" equilibria. We explicitly characterize the set of equilibria and analyze among other things the "signaling value" of voting "yes" or "no" and its implications for the "prices" that must be paid to secure a responder's vote.

3 Bargaining Game and Equilibrium Concept

The bargaining game The game involves three players, a proposer (P, she) and two responders (R1 and R2, he). The game has two periods $t = 1, 2$. Each period consists of a *proposal stage* and a *voting stage*: P first makes a *proposal* (x_1^t, x_2^t), with $x_i^t \geq 0$ and $x_1^t + x_2^t \leq 1$. (In much of the analysis, we will be able to drop the superscript t without causing confusion.) Then R1 and R2 simultaneously *vote* either "yes" (Y) or "no" (N). If q responders vote Y, the proposal *passes*. In this case the payoffs are $\pi_P = 1 - x_1^t - x_2^t$ and $\pi_{Ri} = x_i^t$. If fewer than q responders vote Y, the proposal *fails*. If the Period 1 proposal fails, the game ends in "breakdown" with probability $(1 - \delta)$ and otherwise continues to Period 2. If the Period 2 proposal fails, the game ends in breakdown.

In case of breakdown in either period, P's payoff is $\pi_P = 0$ and Ri's payoff is $\pi_{Ri} = b_i$, where b_i denotes an exogenously given "breakdown value". Responders can be of two types, which we refer to as "high" (H) or "low" (L). The "low" type's breakdown value is denoted $l > 0$, and the "high" type's is $h > l$. For some of what follows, it will be convenient to define $\tau = h - l$ to be the difference in high and low breakdown values. For simplicity, we assume that types are independently drawn and equally likely ex ante.

Throughout the analysis, we assume that agreement is *efficient* even in the "worst case" where both responders are of type H. This requires $h < \frac{1}{2}$ (implying $\tau < \frac{1-2l}{2}$). This assumption implies that P is willing to offer both responders h in Period 2 if she is sufficiently sure that they are of type H. Note that payoffs are not discounted, but delay is costly due to the probability of breakdown.

Equilibrium concept Our equilibrium concept is Bayesian equilibrium in "sequentially" weakly undominated strategies. By "sequentially" we mean that we proceed by backward induction and eliminate weakly dominated strategies *once* in each subgame. (That is, we are not applying *iterative* weak dominance.) Concretely, we first identify Bayesian Equilibria in weakly undominated strategies for arbitrary beliefs in Period 2. Then we take the implied continuation payoffs as given and look for a Bayesian Equilibrium in weakly undominated strategies for Period 1. We will refer to this concept as "Sequentially Weakly Undominated Bayesian Equilibrium" (SWUBE), noting its conceptual similarity to a related concept introduced by Acemoglu et al. (2009) ("Sequentially Weakly Undominated Equilibrium").[2] In the following definition, a "history" is a complete description of all proposals and votes that occur before an information set at which a given player is called upon to take an action.

Definition 1. A strategy profile $(\sigma_{R1}, \sigma_{R2}, \sigma_P)$ and (common) belief system ω is a *Sequentially Weakly Undominated Bayesian Equilibrium (SWUBE)* if the action specified by each σ_i at each history maximizes i's payoff given beliefs at that history and given σ_{-i}, and in addition, the following criteria are met:

1. Beliefs are formed using Bayes rule whenever possible.
2. Weakly dominated strategies are played with probability zero in the *Period 2 voting stage.*
3. Strategies that are weakly dominated in earlier stages (*Period 2 proposal, Period 1 voting, Period 1 proposal*), *given* continuation payoffs implied by equilibrium play in later stages, are played with probability zero.

[2]We will later consider an additional refinement to deal with certain multiplicities in a way that we find plausible: If after certain histories the proposer is indifferent between multiple Period 2 proposals, she makes the proposal that maximizes her Period 1 expected utility. Substantively, this can be motivated by noting that the proposer could declare, ex ante, her *intention* to follow a certain strategy. Such an announcement would be credible if, following any history, the proposer does not strictly prefer to deviate from it.

4 Preview of Results using Examples

To preview some of our main results, we begin by considering examples with specific parameter values. Concretely, let the high type's breakdown value be $h = \frac{4}{10}$ (i.e. slightly less than one half), and consider a setting where the types are "different" ($l = \frac{1}{10}$, i.e. $\tau = \frac{3}{10}$) and one where they are "similar" ($l = \frac{3}{10}$, i.e. $\tau = \frac{1}{10}$). In addition, we will consider three values for δ, corresponding to (i) a one shot game ($\delta = 0$), (ii) a game where Period 2 is reached with certainty ($\delta = 1$), and (iii) a game where Period 2 is reached following failure with probability $\delta = \frac{1}{2}$. For each case, we will outline an informal intuitive analysis and identify what we will call "salient" equilibria. The subsequent general analysis (Sections 5 and 6) will be conducted with more attention to detail.

One shot game $\delta = 0$ If the game ends for sure following Period 1, responders are not concerned about the signaling implications of their vote. Therefore, they simply vote Y if and only if they are offered at least their breakdown value. Given this, the proposer's choice of proposal is a straightforward exercise of comparing the payoffs from candidate proposals that are not obviously dominated.

Under unanimity rule, P could consider offering both responders h (which passes for sure), offering both l (which passes with probability $\frac{1}{4}$), or offering one l and the other h (which passes with probability $\frac{1}{2}$). Then if high and low types are "similar" ($\tau = \frac{1}{10}$), she offers both h, securing immediate agreement. If types are "different" ($\tau = \frac{3}{10}$), she offers one h and the other l, resulting in failure and immediate breakdown with probability $\frac{1}{2}$.

Under majority rule, she considers offering one responder h (passes for sure), one responder l (passes with probability $\frac{1}{2}$), or both l (passes with probability $\frac{3}{4}$). Then if types are "similar" ($\tau = \frac{1}{10}$), she offers h to one responder, securing immediate agreement. If types are "different" ($\tau = \frac{3}{10}$), then under our specific parameter values P would be indifferent between offering one responder h (which would pass for sure) and offering both responders l (which would pass with probability $\frac{3}{4}$).

This example illustrates the intuitive fact that in a one-shot setting ($\delta = 0$), responders' (type specific) "prices" do not depend on the voting rules or on the other responder's behavior. In addition, this example corresponds to what *would* happen if, in a game with $\delta > 0$, Period 2 were reached without any information having been revealed about the responders. Note that agreement is more likely under majority rule than under unanimity rule when responder types are sufficiently different. In the one shot context, this difference is driven entirely by the fact that the proposer needs only one vote, not that the "prices" of votes differ between the rules.

Two period game $\delta = 1$ Suppose that Period 2 is reached with certainty following failure of the Period 1 proposal. Clearly, responders' Period 2 "prices" still coincide with their breakdown values. However in this case their Period 1 "prices" may differ, as they will generally correspond to their expected payoff from entering Period 2, which in turn will depend on the proposal that P will make, which in turn may depend on the way that responders have voted in Period 1.

Under unanimity rule, it is easy to see that the Period 1 "price" of a high type is also h. Low type responders, in contrast, may well vote N if offered their breakdown value, depending on what they expect the proposer to do in Period 2. In general, responders are more likely to vote Y if they expect the other responder to do so. As an example, suppose that responder types are similar ($\tau = \frac{1}{10}$) and that R2 is expected to vote Y irrespective of his type. Then it can be argued that a low type R1 must vote Y if and only if he is offered at least $\hat{z}_L \equiv \frac{l+h}{2} = \frac{3.5}{10}$. (This "price" is exactly equal to R1's expected payoff from entering Period 2 and being perceived as a high type while R2's type remains unknown - see next paragraph.) If instead R2 is expected to vote Y only if he is a low type, it turns out that a low type R1's "price" becomes $h = \frac{4}{10}$, i.e. it is larger.

Similar reasoning can be used to determine R1's Period 1 "price" under other circumstances. Following this, P's optimal Period 1 proposal under unanimity rule can be identified. For the particular examples we are considering, we find that if types are similar ($\tau = \frac{1}{10}$), P will secure immediate agreement by offering $x^1 = (h, h)$ in Period 1. If types differ ($\tau = \frac{3}{10}$), she instead initially offers $x^1 = (h, \hat{z}_L)$ (or symmetrically), where $\hat{z}_L = \frac{3.5}{10}$ is the "price" discussed above. Then R1 votes Y for sure and R2 votes Y only if he is a low type. Since the proposal fails if R2 is a high type, there is a positive probability of delay under unanimity rule. In case of delay (i.e. if R2 votes N), P will propose (l, h) in Period 2, which will pass only if R1 is a low type. (This is why R2 expects to get h with probability $\frac{1}{2}$ following failure, which determines his price - see previous paragraph.)

Under majority rule, there exists a great multiplicity of equilibria. Unlike under unanimity rule, resonders of *both* types generally might vote Y even if they are offered strictly less than their breakdown value in Period 1. The reason is that no responder can guarantee himself his breakdown value by voting N in both periods. In fact, there exist equilibria in which the proposer's first offer is $x^1 = (0, 0)$ and at least one responder votes Y. The reason is that the proposer can credibly threaten to exclude, say, R1 in Period 2 if he votes N. There also exist equilibria where both vote N on this proposal irrespective of their type. However, even in the "worst case" scenario (i.e. in all possible equilibria), proposals like $x^1 = \left(\frac{h}{2}, \frac{h}{2}\right)$ or $x^1 = (h, 0)$ must pass for sure in Period 1 (all proposals where $x_1 + x_2 \geq h$). We would argue that the equilibrium where $x^1 = (0, 0)$ passes is "salient" in the sense that P prefers it and could credibly announce the strategy that supports it if there is any opportunity to communicate.

This example illustrates that, when Period 2 is reached for sure following proposal failure, responders will consider the "signaling value" of their Period 1 vote. Under unanimity rule, voting N carries "positive signaling value" and guarantees that Period 2 will be reached. Conversely, voting Y carries *negative* signaling value and does not guarantee agreement. Therefore a low type responder's "price" is generally higher than his breakdown value. Under majority rule, by contrast, voting N carries negative signaling value (supporting the threat of exclusion), which makes responders "cheap". So "prices" lie below the responder's breakdown value and might even be 0. When responder types are sufficiently different, the probability of delay and breakdown is higher under unanimity rule. In contrast to the case of $\delta = 0$, this inefficiency is now driven by the fact that Period 1 "prices" of votes differ between the rules.

Period 1 breakdown probability $\delta = \frac{1}{2}$ Finally, we consider the intermediate case where failure of the first proposal is followed by immediate breakdown with probability $\frac{1}{2}$. As in the previous case, responders' Period 2 "prices" coincide with their breakdown values, and Period 1 "prices" correspond to expected payoffs after proposal's failure. Again, these expected payoff depends on Period 2 beliefs about both responders and thus on how both responders are expected to vote in Period 1.

Under unanimity rule, high type responders follow the same cutoff strategy as above, while a low type's "price" depends on the other responder's voting behavior. If R2 votes Y irrespective of his type, a low type R1's Period 1 "price" is $\hat{z}_L = \frac{l+h}{2} = \frac{3.5}{10}$ when $\tau = \frac{1}{10}$, and $\tilde{z}_L \equiv \frac{3}{4}l + \frac{1}{4}h = \frac{1.75}{10}$ when $\tau = \frac{3}{10}$. If R2 votes Y only if he is a low type, R1's "price" is h if $\tau = \frac{1}{10}$ and $\frac{l+h}{2} = \frac{2.5}{10}$ if $\tau = \frac{3}{10}$. If R2 votes N for sure, then R1's price is h, i.e. even low types vote N if offered anything less than h. This is because agreement will not be reached anyway, and so all that matters is the negative signaling value from voting Y.[3]

[3]More formally: *If* low types were to vote Y on a proposal that the other responder is sure to reject and which allocates less than h to them, then voting Y perfectly reveals that a responder is low while voting N would perfectly reveal that they are high. Thus voing Y would carry negative signaling value and neither type would want to do so.

As in the case where $\delta = 1$, it turns out that P will want at least one of the responders to vote Y with certainty. If types are "similar" ($\tau = \frac{1}{10}$), P secures immediate agreement by offering $x^1 = (h, h)$ in Period 1. If types are "different" ($\tau = \frac{3}{10}$), she instead initially offers $x^1 = (h, \tilde{z}_L)$ (or symmetrically), where $\tilde{z}_L = \frac{1.75}{10}$ is the "price" discussed above. Then R1 votes Y for sure and R2 votes Y only if he is a low type. If Period 2 is reached, P will offer $x^2 = (l, h)$. Since the Period 1 proposal fails if R2 is a high type and the second proposal fails if R1 is also a high type, there is a positive probability of inefficient delay and breakdown under unanimity rule.

Under majority rule, responders now must vote N if offered less than $\frac{1}{2}$ (breakdown probability) times their breakdown value. In the "salient" equilibrium, R1 votes Y on such an offer if R2 votes N for sure. This is because P can credibly threaten to exclude R1 in Period 2, as the belief after R1 voted N would be $\omega = \left(1, \frac{1}{2}\right)$, making R2 the more attractive partner. So in this case, the high type's Period 1 "price" is $\frac{h}{2}$ and the low type's is $\frac{l}{2}$. If R2 votes Y only if he is a low type, then R1's "price" might be higher because Period 2 would be reached only if both responders have revealed that they are high types, so that the proposer may include R1 in his Period 2 coalition with positive probability. If P were to include both with equal probability in this case (mixing evenly between $(h, 0)$ and $(0, h)$ in Period 2), the low type responder's "price" would be $\frac{1}{2}l + \frac{1}{4}h$, and the high type's "price" would be $\frac{3}{4}h$.

As in the previous example, there are multiple equilibria under majority rule. In the "salient" one, P offers $\left(\frac{h}{2}, 0\right)$ (or $\left(0, \frac{h}{2}\right)$) and R1 (R2) votes Y with certainty. This equilibrium is plausible because P can credibly threaten to exclude R1 (R2) from a possible Period 2 coalition. However, if Period 2 is reached, P will actually be indifferent as to whom to include in a coalition. Therefore a whole range of other equilibria exists. Even so, the proposal $x^1 = (h, 0)$ is an upper bound of offers that must pass for sure in Period 1, i.e. $x^1 = (h, 0)$ passes in the whole range of possible equilibria.

This example illustrates that for intermediate values of δ, the signaling incentives lie in between the ones of the previous cases where we considered $\delta = 0$ and $\delta = 1$, respectively. Responders' under unanimity rule are thus less "expensive" than in the previous $\delta = 1$ case as voting N now comes with a lower signaling value (due to the threat of immediate breakdown). Responders' under majority rule, by contrast, are more "expensive" than in the previous $\delta = 1$ case as the threat of exclusion is lower (thanks to the possibility of immediate breakdown). What still holds true is that responders under majority rule are "cheaper" than under unanimity rule. This again is the main reason why breakdown and delay might only occur under unanimity rule.

Discussion and comparison of rules Despite the multiplicity of equilibria under majority rule, certain differences between the rules can be clearly identified. The first is that the threat of exclusion in Period 2 translates into lower "prices" for votes in Period 1. To see this, Table 1 summarizes the low type responders' "prices" under both voting rules in the examples discussed above. For majority rule, we list the "prices" corresponding to what we have called the "salient" equilibrium which is the "best case" from the proposer's perspective. Another pattern that is visible in the table is that "prices" seem to increase in δ under unanimity and to decrease in δ under majority rule. The reason for these effects is that the signaling concern and the threat of exclusion become more relevant when Period 2 is reached with greater probability. Also, the probability with that the other responder votes Y on a given proposal has a (weakly) negative impact on "prices" under unanimity rule and a (weakly) positive one on "prices" under majority rule. The following sections generalize these findings and discuss them in more detail.

Another important difference between the decision rules is that the "signaling value" of voting N can be *negative* under majority rule. As under unanimity rule, voting N generally constitutes a signal that one is a high type. This is because high type responders are (weakly) more likely to vote N on any given

Table 1: Responders' "Prices"

	$\delta=1$				$\delta=\frac{1}{2}$				$\delta=0$			
$h=\frac{4}{10}$	$l=\frac{1}{10}$		$l=\frac{3}{10}$		$l=\frac{1}{10}$		$l=\frac{3}{10}$		$l=\frac{1}{10}$		$l=\frac{3}{10}$	
	z_L^m	z_L^u	z_L^m	z_L^u	z_L^m	z_L^u	z_L^m	z_L^u	z_L^m	z_L^u	z_L^m	z_L^u
R2 votes Y for sure	0	$\frac{2.5}{10}$	0	$\frac{4}{10}$	?	$\frac{1.75}{10}$	?	$\frac{3.5}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{3}{10}$
R2 votes Y iff low	0	$\frac{4}{10}$	0	$\frac{4}{10}$	$\frac{1.5}{10}$	$\frac{2.5}{10}$	$\frac{2.5}{10}$	$\frac{4}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{3}{10}$
R2 votes N for sure	0	$\frac{4}{10}$	0	$\frac{4}{10}$	$\frac{0.5}{10}$	$\frac{4}{10}$	$\frac{1.5}{10}$	$\frac{4}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{3}{10}$

The table presents the "price" of a low type R1 (denoted z_L^m), given R2's (pure strategy) voting behavior and given a certain voting rule, for each parameter combination of our example above. Under majority rule we assume the "salient" equilibrium (see text).

proposal. Unlike under unanimity rule, however, being perceived as a "high" type is in most cases *not* advantageous in Period 2. Instead, it supports and makes credible P's threat to exclude a responder who voted N in Period 1.[4]

5 Unanimity Rule

We analyze the game by backward induction, beginning with Period 2. Period 2 play will generally depend on the (common) *beliefs* formed following Period 1 play. We begin by solving for Bayesian equilibria in weakly undominated strategies for arbitrary beliefs. (In cases where multiple equilibria exist for given beliefs, Period 2 play could, and as it will turn out *must* in some cases depend "directly" on *which history* led to those beliefs.)

5.1 Period 2 Voting Stage

Lemma 1. *In any SWUBE under unanimity rule, responders vote Y in Period 2 iff they are offered at least their breakdown value.*

Proof. Voting N (Y) when offered *strictly* more (less) is weakly dominated. When offered exactly the breakdown value neither choice is dominated. Assuming that responders vote Y in this case is without loss of generality. □

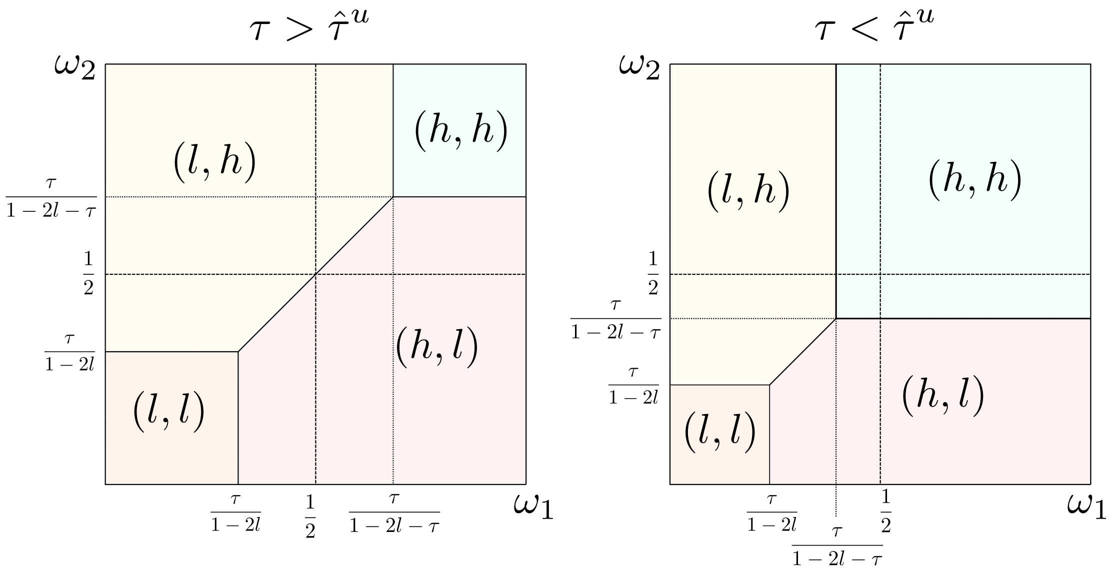
5.2 Period 2 Proposal Stage

Period 2 is reached if the first period proposal failed and breakdown did not occur. Following any such history, we denote the (common) beliefs concerning the probability that responder j is a high type by ω_j. W.l.o.g., let $\omega=(\omega_1,\omega_2)$ be such that $\omega_1\le\omega_2$. It can then be shown that the optimal Period 2 proposal is given by

$$p_2(\omega_1,\omega_2)=\begin{cases}(l,l) & \omega_1\le\omega_2<\frac{\tau}{1-2l}\\ (l,h) & \omega_1<\frac{\tau}{1-2l-\tau}\text{ and }\omega_2>\frac{\tau}{1-2l}\\ (h,h) & \omega_1>\frac{\tau}{1-2l-\tau}\end{cases}$$

[4]Note that in our 3 player model, Period 2 is reached only after *both* responders have voted N, a circumstance that complicates - but does not invalidate - this intuition. We will return to this in our concluding remarks.

Figure 1: Period 2 Proposals (unanimity rule)



The figure depicts the Period 2 proposals that P prefers to make under unanimity rule given arbitrary beliefs about the responders' types. The horizontal axis measures the probability that R1 is a high type, denoted ω_1. The vertical axis measures the belief about R2, denoted ω_2. The left and right panel distinguish the cases where τ is greater or smaller than $\hat{\tau}^u \equiv \frac{1-2l}{3}$, which determines what P would do in case $\omega_1 = \omega_2 = \frac{1}{2}$.

For special values of (ω_1, ω_2), P may be *indifferent* between some of these proposals: If $\omega_1 = \frac{\tau}{1-2l-\tau}$, she will be indifferent between (h,h) and (l,h). If $\omega_2 = \frac{\tau}{1-2l}$, she will be indifferent between (l,h) and (l,l). Finally, if $\omega_1 = \omega_2 \in \left(\frac{\tau}{1-2l}, \frac{\tau}{1-2l-\tau}\right)$, she is indifferent between (l,h) and (h,l). For these values, P could mix between two (or even three) proposals. In case $\omega_1 > \omega_2$, the optimal proposal would be given by the symmetrically modified $p_2(\omega_2, \omega_1)$.

Figure 1 provides a graphical representation of the second round proposal. The horizontal and vertical axes measure beliefs about R1 and R2 respectively, and the regions identify the proposals made for various combinations of beliefs. We define $\hat{\tau}^u \equiv \frac{1-2l}{3}$ as the cutoff value such that in case P has not learned anything about the responders (i.e. $\omega_2 = \left(\frac{1}{2}, \frac{1}{2}\right)$), then P prefers to offer (h,h) for all $\tau < \hat{\tau}^u$ and to offer (l,h) (or (h,l)) for all $\tau > \hat{\tau}^u$.

As noted, Period 2 play and consequently expected utilities will generally be determined by the belief ω induced by the prior history.[5] We will write $v_P(\omega)$ to denote the proposer's expected utility and $v_L(\omega_i, \omega_{-i})$ and $v_H(\omega_i, \omega_{-i})$ to denote the expected utility of a responder of type L or H, given that beliefs about his own and the other responder's type are given by ω_i and ω_{-i}, respectively. Note that $v_H(\omega) = h$ for all ω. Another important fact is that a low type responder is always (weakly) better off if the belief about her type "increases".

Lemma 2. *Let* $(\omega_1, \omega_2) \in [0,1]^2$ *and* $\tilde{\omega}_1 \in (\omega_1, 1]$. *Then in any SWUBE under unanimity rule, Period 2 expected utilities satisfy* $v_L(\tilde{\omega}_1, \omega_2) \geq v_L(\omega_1, \omega_2)$.

Proof. See Appendix. □

[5]If beliefs allow for mixing, an entire *set* of Period 2 expected utilities can conceivably be induced by different mixing probabilities. If multiple histories lead to the same such belief, the expected utilities following each of these histories may differ, because the mixing probabilities could be conditioned the different histories.

5.3 Period 1 Voting Stage

We now characterize equilibrium play following an *arbitrary* Period 1 proposal $x = (x_1, x_2)$. In what follows, we denote the probability that responder j votes Y when his type is T by μ_{jT}. (For ease of notation, we suppress its dependence on the proposal x, which is exogenously given and fixed for this part of the analysis.) In general, a given pair of acceptance probabilities (μ_{jL}, μ_{jH}) will determine equilibrium beliefs about responder j that would be induced by him voting either Y or N. We denote these beliefs by ω_{jY} and ω_{jN}, respectively. (Again, we suppress its dependence on x for the time being.) For example, if $(\mu_{jL}, \mu_{jH}) = (1, 0)$, then $\omega_{jY} = 0$ and $\omega_{jN} = 1$. That is, if responder j votes Y if and only if she is type L, then voting Y or N perfectly reveals her type. More generally, beliefs determined by Bayes rule are given by

$$\omega_{jY} = \frac{\mu_{jH}}{\mu_{jH} + \mu_{jL}}$$

$$\omega_{jN} = \frac{1 - \mu_{jH}}{2 - \mu_{jH} - \mu_{jL}}$$

In the special cases $(\mu_{jL}, \mu_{jH}) = (0, 0)$ or $(\mu_{jL}, \mu_{jH}) = (1, 1)$, beliefs following an "unexpected" (off path) Y or N vote, respectively, are not determined by Bayes rule.

We begin our analysis by establishing the following Lemma, which places bounds on the "price" that must be paid to secure agreement from a responder depending on his type.

Lemma 3. *Let (x_1, x_2) be an arbitrary Period 1 proposal. Then in any SWUBE under unanimity rule,*

1. *if $x_i > h$, then $\mu_{iL} = \mu_{iH} = 1$,*
2. *if $x_i < h$, then $\mu_{iH} = 0$,*
3. *if $x_i < l$, then $\mu_{iL} = \mu_{iH} = 0$.*

Proof. See Appendix. □

An implication of Lemma 3 is that for any proposal, a low type responder is at least as likely to vote Y as is a high type. This, in turn, means that voting Y signals that one is (more likely to be) a low type, and voting N signals that one is (more likely to be) a high type, at least on the equilibrium path.

Corollary 1. *In any SWUBE under unanimity rule: For any Period 1 proposal x, if $\mu_{2L} + \mu_{2H} \neq 0$ (or $x_1 \neq h$, see Lemma 4), then $\mu_{1L} \geq \mu_{1H}$ and therefore*

- *if $\mu_{1L} + \mu_{1H} \neq 0$ (i.e. not $\mu_{1L} = \mu_{1H} = 0$), then $\omega_{1Y} \leq \frac{1}{2}$ and*
- *if $\mu_{1L} + \mu_{1H} < 2$ (i.e. not $\mu_{1L} = \mu_{1H} = 1$), then $\omega_{1N} \geq \frac{1}{2}$.*

Proof. See Appendix. □

Lemma 3 *suggests* that high type responders follow a simple cutoff strategy, rejecting anything below and accepting anything above h. However it does not pin down the response to an offer of exactly h. For simplicity, we make the following additional assumption.[6]

[6] We believe that this assumption is without loss of generality, but a proof eludes us so far. Intuitively, suppose a type H R1 were to reject some offer (h, x_2) with positive probability. By Lemma 3, he must vote Y on $(h + \epsilon, x_2)$ for ϵ arbitrarily small. Suppose that R2's vote is the same on both of these proposals. Then one of two things must be true. Either (a) (h, x_2) *would* be the best proposal that can be made if R1 were to vote Y for sure, or (b) it *would not* be. In case (a) P can get arbitrarily close to the (counterfactual) maximum payoff by offering $(h + \epsilon, x_2)$ for ϵ arbitrarily small, and thus a payoff maximizing proposal does not exist. The only way to resolve this would be to assume that R1 votes Y on (h, x_2) for sure. In case (b), it is without consequence to assume that the high type accepts for sure, since P will not make this proposal anyway. The reason that this argument does not constitute a complete proof is that R2 may vote *differently* on $(h + \epsilon, x_2)$ and (h, x_2).

Assumption 1. If $x_i = h$, then $\mu_{iH} = 1$. (And from this it then follows that $\mu_{iL} = 1$ as well, by Corollary 1.)

Voting by low types offered less than h Lemma 3 and Assumption 1 imply that a responder is sure to vote Y on any proposal that allocates at least h to him. Thus what is left to investigate is how he votes when offered strictly less than h. By Lemma 3, a high type votes N on such proposals. Therefore, the following analysis deals with voting by type L responders. Without loss of generality, we investigate the voting decision of R1, taking R2's voting probabilities as given. By Lemma 3 and Assumption 1, type H agents play a pure strategy and so we restrict attention to $\mu_{2H} \in \{0, 1\}$. As we will see, low type responders may mix between voting Y and N in equilibrium, and so we allow $\mu_{2L} \in [0, 1]$. Given these parameters, the consequences of R1 voting Y or N are the following:

Consequences of voting Y With probability $\frac{\mu_{2H}+\mu_{2L}}{2}$, the proposal passes and responder 1 receives x_1. With probability $(1 - \frac{\mu_{2H}+\mu_{2L}}{2})$, responder 2 votes N and the proposal fails. Then with probability $(1 - \delta)$ responder 1 receives l, and with probability δ, Period 2 is reached and beliefs are given by $(\omega_{1Y}, \omega_{2N})$. Therefore a low type R1's expected utility from voting Y is given by

$$\frac{\mu_{2H} + \mu_{2L}}{2} x_1 + \left(1 - \frac{\mu_{2H} + \mu_{2L}}{2}\right) ((1 - \delta)l + \delta v_L(\omega_{1Y}, \omega_{2N}))$$

Consequences of voting N The proposal fails. With probability $(1 - \delta)$ the responder receives l, and with probability δ, Period 2 is reached. Then with probability $(1 - \frac{\mu_{2H}+\mu_{2L}}{2})$, responder 2 voted N and beliefs are given by $(\omega_{1N}, \omega_{2N})$. With probability $\frac{\mu_{2H}+\mu_{2L}}{2}$, responder 2 voted Y and beliefs are given by $(\omega_{1N}, \omega_{2Y})$. Therefore a low type R1's expected utility from voting N is given by

$$(1 - \delta)l + \delta \left(\frac{\mu_{2H} + \mu_{2L}}{2} v_L(\omega_{1N}, \omega_{2Y}) + \left(1 - \frac{\mu_{2H} + \mu_{2L}}{2}\right) v_L(\omega_{1N}, \omega_{2N}) \right)$$

Voting when the other responder votes *N* for sure Suppose that R2 votes N irrespective of his type ($\mu_{2L} = \mu_{2H} = 0$). Then the proposal is sure to fail. Intuitively, R1 then has no incentive to vote Y, as doing so will not result in agreement, and it would be associated with a harmful signaling effect: If the low type were to vote Y in an equilibrium, this would reveal his type, and so anyone voting Y would be offered l in Period 2, while anyone voting N would be offered h. This intuition can be analytically confirmed to yield the following result.

Lemma 4. *Suppose $x_1 < h$ and $\mu_{2H} = \mu_{2L} = 0$. Then in any SWUBE under unanimity rule, $\mu_{1L} = \mu_{1H} = 0$ and it is without loss of generality to assume that $\omega_{1Y} = \omega_{1N} = \frac{1}{2}$.*

Proof. See Appendix. □

An important implication of Lemma 4 is that for any proposal that allocates less than h to both responders, there exists a continuation equilibrium in which both responders vote N irrespective of their type. Another is that P can learn nothing about either responder by making a proposal that is sure to fail. Thus, any such proposal is equivalent to simply skipping Period 1 and entering Period 2 without updating beliefs. For the purposes of the analysis, we can simply treat *all* proposals of this type as one, and focus the rest of the analysis on the case where R2 votes Y with positive probability.

Voting when the other responder votes Y with positive probability Assume from now on that $\mu_{2H} + \mu_{2L} > 0$. Then R1 *strictly* prefers to vote Y if

$$x_1 > l + \delta\Big(v_L(\omega_{1N}, \omega_{2Y}) - l + (\frac{2}{\mu_{2H} + \mu_{2L}} - 1)\big(v_L(\omega_{1N}, \omega_{2N}) - v_L(\omega_{1Y}, \omega_{2N})\big)\Big)$$

(And he strictly prefers to vote N if the inequality is reversed, and he is indifferent in case of equality.) This condition can be used to investigate the "price" at which R1 is willing to vote Y, which depends on the probability with which R2 is expected to vote Y.

First, consider proposals on which R2 votes Y irrespective of his type in equilibrium. Then R1's vote is pivotal and we can establish the following result, which identifies the "price" of his vote under this circumstance.

Lemma 5. *Suppose $x_1 < h$ and $\mu_{2H} = \mu_{2L} = 1$. Then in any SWUBE under unanimity rule, $\mu_{1H} = 0$ and*

1. If $\tau < \hat{\tau}^u$, $\mu_{1L} = \begin{cases} 0 & x_1 < l + \delta\tau \\ \in [0,1] & x_1 = l + \delta\tau \\ 1 & x_1 > l + \delta\tau \end{cases}$

   and following $v = (N, Y)$, P offers (h, h) in Period 2.

2. If $\tau > \hat{\tau}^u$, $\mu_{1L} = \begin{cases} 0 & x_1 < l + \frac{\delta}{2}\tau \\ \in [0,1] & x_1 = l + \frac{\delta}{2}\tau \\ 1 & x_1 > l + \frac{\delta}{2}\tau \end{cases}$

   and following $v = (N, Y)$, P offers (h, l) in Period 2.[7]

Proof. See Appendix. □

The intuition behind these results is as follows. If $x_1 < h$, R1 must vote N if he is a high type. If he votes Y as a low type ($\mu_{1L} = 1$), then voting N would reveal that he is type H. Thus, x_1 must exceed the expected value of failure *given* that P's belief in Period 2 would be given by $(\omega_1, \omega_2) = \big(1, \frac{1}{2}\big)$. (Since R2 votes Y irrespective of his type, P's belief about him stays at $\frac{1}{2}$.) Looking back at Figure 1, we see that in this case the Period 2 proposal would be (h, h) if $\tau < \hat{\tau}^u$ and (h, l) if $\tau > \hat{\tau}^u$. In the former case, the Period 2 proposal would pass for sure, so a low type R1's expected utility from voting N is $l + \delta\tau$. In the latter case, the second proposal will pass only if R2 is low. Therefore R1's expected utility from voting N is $l + \frac{\delta}{2}\tau$. In each case, x_1 must weakly exceed these values in order for R1 to vote Y.

Next, consider proposals where R2 votes both Y and N with positive probability in equilibrium. By Corollary 1, this implies that $\mu_{2H} = 0$ and $\mu_{2L} > 0$. Then using reasoning similar to that in the preceding paragraph, we can establish the following.

Lemma 6. *Suppose $x_1 < h$, $\mu_{2H} = 0$, and $\mu_{2L} > 0$. Then in any SWUBE under unanimity rule, $\mu_{1H} = 0$ and*

1. *If $\tau < \hat{\tau}^u$, then* $\mu_{1L}(x_1, \mu_{2L}) = \begin{cases} 0 & x_1 < l + \frac{2\delta}{\mu_{2L}}\tau \\ \in [0,1] & x_1 = l + \frac{2\delta}{\mu_{2L}}\tau \\ 1 & x_1 > l + \frac{2\delta}{\mu_{2L}}\tau \end{cases}$

[7] P *must* offer (h, l) even if $\mu_{1L} = 0$ and thus $(\omega_{1N}, \omega_{2Y}) = (\frac{1}{2}, \frac{1}{2})$. While P is then indifferent between (h, l) and (l, h), if she offered (l, h) with positive probability, then R1 would prefer to vote Y for $x_1 < l + \frac{\delta}{2}\tau$, yielding irreconsilable contradictions.

and Period 2 proposals are as follows: After $v = (Y,N)$ P offers (l,h).[8] *Symmetrically for $v = (N,Y)$. (The same will be true for all cases that follow.) Following $v = (N,N)$ P offers (h,h). (Note that if $\mu_{2L} \leq 2\delta$, then $\mu_{1L} = 0$ for all $x_1 < h$.)*

2. *If $\tau > \hat{\tau}^u$ and $\mu_{2L} > \bar{\mu} \equiv \frac{2h+\tau-1}{\tau}$, then*

$$\mu_{1L}(x_1, \mu_{2L}) = \begin{cases} 0 & x_1 < l + \delta\tau \\ \in [0, \bar{\mu}] & x_1 = l + \delta\tau \\ \bar{\mu} & x_1 \in \left(l + \delta\tau, l + \frac{2\delta}{\mu_{2L}}\tau\right) \\ \in [\bar{\mu}, 1] & x_1 = l + \frac{2\delta}{\mu_{2L}}\tau \\ 1 & x_1 > l + \frac{2\delta}{\mu_{2L}}\tau \end{cases}$$

and following $v = (N,N)$ P offers (l,h) if $x_1 < l + \delta\tau$, and (h,h) if $x_1 > l + \frac{2\delta}{\mu_{2L}}\tau$, and for $x_1 \in \left[l + \delta\tau, l + \frac{2\delta}{\mu_{2L}}\tau\right]$, P offers (h,h) with probability $\rho = \frac{\mu_{2L}}{2-\mu_{2L}}\left(\frac{x_1-l}{\delta\tau} - 1\right)$ and (l,h) otherwise.[9]

3. *If $\tau > \hat{\tau}^u$ and $\mu_{2L} \in (0, \bar{\mu})$, then*

$$\mu_{1L}(x_1, \mu_{2L}) = \begin{cases} 0 & x_1 < l + \delta\tau \\ \in [0, \mu_{2L}] & x_1 = l + \delta\tau \\ \mu_{2L} & x_1 \in \left(l + \delta\tau, l + \frac{\delta}{\mu_{2L}}\tau\right) \\ \in [\mu_{2L}, 1] & x_1 = l + \frac{\delta}{\mu_{2L}}\tau \\ 1 & x_1 > l + \frac{\delta}{\mu_{2L}}\tau \end{cases}$$

and following $v = (N,N)$ P offers (l,h) if $x_1 \leq l + \delta\tau$, and (h,l) if $x_1 \geq l + \frac{\delta}{\mu_{2L}}\tau$, and for $x_1 \in \left(l + \delta\tau, l + \frac{\delta}{\mu_{2L}}\tau\right)$, P offers (h,l) with probability $\rho = \frac{\mu_{2L}}{1-\mu_{2L}}\left(\frac{x_1-l}{\delta\tau} - 1\right)$ and (l,h) otherwise.[10]

4. *If $\tau > \hat{\tau}^u$ and $\mu_{2L} = \bar{\mu}$, then*

$$\mu_{1L}(x_1, \bar{\mu}) = \begin{cases} 0 & x_1 < l + \delta\tau \\ \in [0, \bar{\mu}] & x_1 = l + \delta\tau \\ \bar{\mu} & x_1 \in \left(l + \delta\tau, l + \frac{\delta}{\bar{\mu}}\tau\right) \\ \in [\bar{\mu}, 1] & x_1 \in \left[l + \frac{\delta}{\bar{\mu}}\tau, l + \frac{2\delta}{\bar{\mu}}\tau\right] \\ 1 & x_1 > l + \frac{2\delta}{\bar{\mu}}\tau \end{cases}$$

and following $v = (N,N)$ P offers (l,h) if $x_1 \leq l + \delta\tau$, mixes (in any way) between (h,h) and (h,l) if $x_1 > l + \frac{2\delta}{\bar{\mu}}\tau$, and (a) for $x_1 \in \left(l + \delta\tau, l + \frac{\delta}{\bar{\mu}}\tau\right)$, P mixes between (h,l), (l,h), and (h,h) in such a way as to make R1 indifferent, i.e. to induce $v_L(\bar{\omega}, \bar{\omega}) = l + \left(\frac{\bar{\mu}}{2-\bar{\mu}}\right)\left(\frac{x_1-l}{\delta\tau} - 1\right)\tau$. (Several such mixing strategies are possible, for example proposing (h,l) with prob $\rho = \left(\frac{\bar{\mu}}{1-\bar{\mu}}\right)\left(\frac{x_1-l}{\delta\tau} - 1\right)$ and

[8]For $x_1 \leq l + \delta\tau$, P *could* offer (h,h) following an "unexpected" (Y,N) vote if the *off path* belief ω_{1Y} is large enough. However it is without loss of generality to ignore this, as R1 would still vote N. If $x_1 \in \left(l + \delta\tau, l + \frac{2\delta}{\mu_{2L}}\tau\right)$, P *must* offer (l,h) after (Y,N) because otherwise R1 would vote Y, leading to a contradiction. This requires that the off-path belief satisfies $\omega_{1Y} \leq \frac{\tau}{1-2l-\tau}$.

[9]Mixing in this way makes R1 exactly indifferent between voting Y and N. When $\rho \in (0,1)$, this requires that P be indifferent between the two proposals in Period 2. This, in turn, requires that R1 votes Y with probability $\bar{\mu}$.

[10]Mixing in this way makes R1 exactly indifferent between voting Y and N. When $\rho \in (0,1)$, this requires that P be indifferent between the two proposals in Period 2. This, in turn, requires that R1 accepts with exactly the same probability as R2. The difference between this case and the previous one is that if P offers h to R1 in Period 2, he will offer l to R2, and therefore the proposal may fail. This is why the "price" at which R1 votes Y is lower when R2 is less likely to vote Y.

(l,h) otherwise.) And (b) for $x_1 \in \left[l+\frac{\delta}{\bar{\mu}}\tau, l+\frac{2\delta}{\bar{\mu}}\tau\right]$, P offers (h,h) with probability $\rho \leq \bar{\mu}\frac{x_1-l}{\delta\tau}-1$ and (h,l) otherwise, where in case $\mu_1 < 1$ this holds with equality.

Proof. See Appendix. □

An important feature of this result is that if $\tau > \hat{\tau}^u$, there exists a range of offers such that R1 *must mix* between voting Y and voting N. It is worth pausing to explain the intuition behind this result and to outline its implications for the construction of equilibria. The offers for which mixing occurs have the property that a low type R1 would strictly prefer to vote N if doing so constituted a *perfect signal* that he is a high type (inducing $\omega_1 = 1$), but would strictly prefer to vote Y if voting N constituted *no signal* at all (i.e. inducing $\omega_1 = \frac{1}{2}$). Then a low type R1 cannot vote Y for sure because this would mean that voting N is a *perfect signal* and thus he would prefer to vote N. And he cannot vote N for sure because then doing so constitutes *no signal* and hence he would prefer to vote Y. Thus, equilibrium requires that the low type *mixes* between Y and N, inducing what one might call "partial" or "imperfect" signaling value. This mixing by R1 requires that P must *also mix* between Period 2 proposals in such a way as to equalize R1's payoffs from voting Y and voting N. And this mixing by P, in turn, requires that R1 mix between Y and N (in Period 1) in such a way as to induce a *belief* that makes P indifferent between proposals in Period 2. These conditions, together, determine the precise probabilities with which R1 and P must mix, as identified in Lemma 6.

Period 1 voting stage equilibria Having characterized one responder's optimal voting strategy taking the others' voting probabilities as given, we now characterize equilibria of the "continuation game" following an arbitrary Period 1 proposal. These are voting probabilities for both responders such that they are simultaneously consistent with Lemmas 4, 5, and 6. First, consider the simple cases where at least one responder is offered h or more, such that he votes Y for sure.

Proposition 1. *Consider any Period 1 proposal which allocates $x_2 \geq h$ to R2. Then under unanimity rule in any SWUBE, R2 votes Y irrespective of his type. A high type R1 votes Y iff $x_1 \geq h$, and a low type R1 votes Y iff $x_1 \geq l + \delta\tau$. Following an N vote by R1, P offers (h,h) in Period 2. (Symmetrically when $x_1 \geq h$.)*[11]

Proof. These conclusions follow directly from Lemma 5. □

Next consider proposals where both responders are allocated strictly less than h. For all such proposals, high type responders vote N. Thus what needs to be determined are the equilibrium probabilities with which low types vote Y, which we denote by $\mu_L^*(x) = \left(\mu_{1L}^*, \mu_{2L}^*\right) : \mu_{jL}^* = \mu_{jL}(x_j, \mu_{-jL}^*)$, where $\mu_{jL}(x_j, \mu_{-jL}^*)$ refers to the optimal voting strategy, derived above (Lemma 6), when the "other" responder's (low type) acceptance probability was exogenously given.

Proposition 2. *Suppose $\tau < \hat{\tau}^u$ and consider proposals $x = (x_1, x_2)$ with $x_i < h$ for $i = 1, 2$. Then in any SWUBE under unanimity rule, high type responders vote N following such proposals. In addition, (only) the following continuation equilibria exist:*

1. $\mu_L^*(x) = (0,0)$ always exists.[12]

[11] When $x_1 = l + \delta\tau$, R1 *could* mix between Y and N in an equilibrium of the subgame beginning with the voting stage. However such equilibria can be ignored without loss of generality because P could ensure that a low type R1 votes Y by increasing x_1 by an arbitrarily small amount.

[12] If Period 2 is reached, P offers (h,h).

2. $\mu_L^*(x) = (1,1)$ exists iff $\delta \le \frac{1}{2}$ and $x_j \ge l + 2\delta\tau$ for $j = 1$ and 2.[13]

Proof. These conclusions follow from Lemma 5. □

Note that for $\tau < \hat{\tau}^u$ and $\delta > \frac{1}{2}$ (i.e. if types are similar and breakdown is sufficiently unlikely), all equilibria involve failure of any Period 1 proposal which allocates less than h to both responders. Following such failure, the second period proposal would be (h,h). Clearly, any such Period 1 proposal is (sequentially) weakly dominated by offering (h,h) immediately. It follows that under the stated conditions P must allocate at least h to at least one responder.

Proposition 3. *Suppose $\tau > \hat{\tau}^u$ and consider proposals $x = (x_1, x_2)$ with $x_i < h$ for $i = 1,2$. Then in any SWUBE under unanimity rule, high type responders vote N. In addition, the following continuation equilibria exist (see Figure 2):*

1. $\mu_L^*(x_1,x_2) = (0,0)$ *always exists.*[14]
2. $\mu_L^*(x_1,x_2) = (1,1)$ *exists if* $x_j \ge l + 2\delta\tau, j = 1,2.$[15]
3. $\mu_L^*(x_1,x_2) = (1,\bar{\mu})$ *exists if* $x_1 \ge l + \frac{\delta}{\bar{\mu}}\tau$, $x_2 \in [l+\delta\tau, l+2\delta\tau]$, *and* $x_2 \le l + \bar{\mu}(x_1 - l)$.[16]
4. $\mu_L^*(x_1,x_2) = (\bar{\mu},\bar{\mu})$ *exists if* $x_j \ge (2-\bar{\mu})\, x_{-j} - (1-\bar{\mu})\left(l + \frac{2\delta}{\bar{\mu}}\tau\right)$ *and* $x_1 + x_2 \ge 2l + \delta\frac{1+\bar{\mu}}{\bar{\mu}}\tau$.[17]
5. $\mu_L^*(x_1,x_2) = (\mu_1,\mu_2)$ *with* $0 < \mu_1 = \delta\frac{\tau}{x_2 - l} < \mu_2 \le \bar{\mu}$ *exists if* $x_1 = l + \delta\tau$ *and* $x_2 \in \left(l + \frac{\delta}{\bar{\mu}}\tau, h\right)$.[18]
6. $\mu_L^*(x_1,x_2) = (\hat{\mu},\bar{\mu})$ *with* $\hat{\mu} = \frac{2\bar{\mu}(x_1-l)-2\delta\tau}{(x_2-l)+\bar{\mu}(x_1-l)-2\delta\tau} > \bar{\mu}$ *exists if* $x_1 \in \left(l + \frac{\delta}{\bar{\mu}}\tau, l + \frac{2\delta}{\bar{\mu}}\tau\right]$ *and* $x_2 \in \left(l + \bar{\mu}\,(x_1 - l)\,, l + (2-\bar{\mu})\,(x_1 - l) - \frac{2(1-\bar{\mu})}{\bar{\mu}}\delta\tau\right)$.[19]
7. $\mu_L^*(x_1,x_2) = (\mu_1,\mu_2)$ *with* $\mu_i = 2\delta\frac{\tau}{x_{-i} - l} \in (\bar{\mu}, 1)$, $i = 1,2$ *exists if* $x_i \in \left(l + 2\delta\tau, l + \frac{2\delta}{\bar{\mu}}\tau\right)$, $i = 1,2$.[20]
8. $\mu_L^*(x_1,x_2) = (\mu,\mu)$ *with* $\mu = \frac{\delta\tau}{x_1+x_2-2l-\delta\tau} \le \bar{\mu}$ *exists if* $x_i \in [l+\delta\tau, h]$ *and* $x_1 + x_2 \ge 2l + \frac{1+\bar{\mu}}{\bar{\mu}}\delta\tau$.[21]

where $\bar{\mu} \equiv \frac{2h+\tau-1}{\tau}$ *and* $\bar{\omega} = \frac{1}{2-\bar{\mu}}$.

Proof. These conclusions follow from Lemma 6. □

[13] If Period 2 is reached (off path), P offers l to any responder who voted Y and h to any responder who voted N. (Continuation equilibria involving mixing also exist, but can be ignored without loss of generality because P could achieve sure acceptance by increasing either offer by an arbitrarily small amount.)

[14] Following $v = (Y,N)$ (off path) P offers (l,h) and the (off path) belief satisfies $\omega_{1Y} \le \frac{1}{2}$. (Another possible continuation involves $\omega_{1Y} = \frac{1}{2}$ and P mixes in any way that is worse for R1 than what occurs following (N,N). However this can be ignored without loss of generality, because R1 would still vote N.) Symmetrically for $v = (N,Y)$. (The same will be true for all cases that follow.) Following $v = (N,N)$, P mixes in any way between (l,h) and (h,l). Following $v = (Y,Y)$ (off path) multiple equilibria exist (because off path beliefs are undetermined) but which is selected is irrelevant for equilibrium play because neither responder can reach this path by unilaterally deviating.

[15] Following failure, P offers l (h) to any responder who voted Y (N).

[16] Following $v = (N,N)$, P offers (h,h) with probability $\rho = \frac{x_2-l}{\delta\tau} - 1$ and (h,l) otherwise.

[17] Following $v = (N,N)$ P mixes between (h,h), (h,l), and (l,h) such that both responders are indifferent in Period 1. Specifically, she offers (h,h) with probability $\rho = \frac{1}{3-\bar{\mu}}\left(\frac{\bar{\mu}(x_1+x_2-2l)}{\delta\tau} - (1+\bar{\mu})\right)$, (h,l) with probability $\rho_1 = \frac{1}{3-\bar{\mu}}\left(2 - \frac{\bar{\mu}}{\delta\tau}\left(\frac{2-\bar{\mu}}{1-\bar{\mu}}x_2 - \frac{1}{1-\bar{\mu}}x_1 - l\right)\right)$, and symmetrically for (l,h) otherwise. In all cases, the proposer's Period 2 utility following $v = (N,N)$ is $1-2h$.

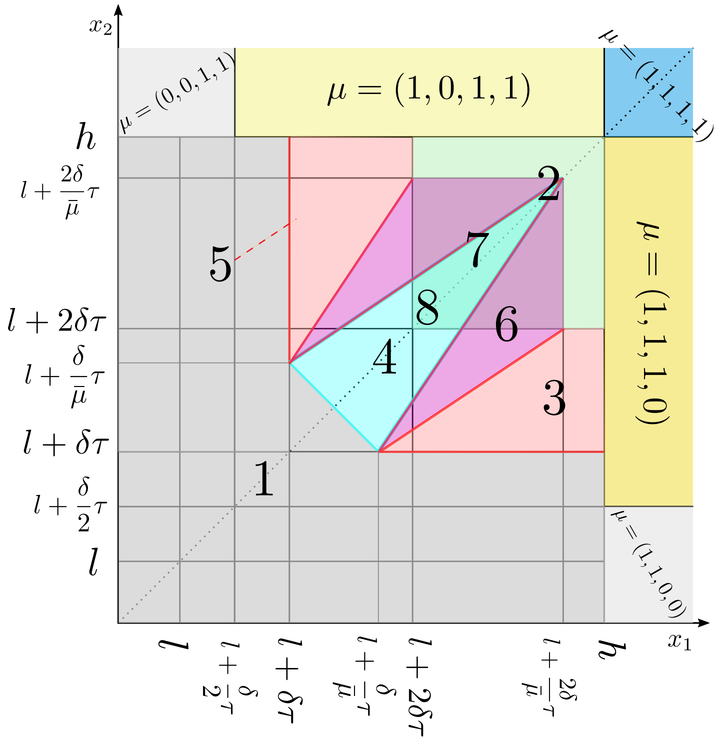
[18] Following $v = (N,N)$, P offers (l,h).

[19] Following $v = (N,N)$, P mixes between (h,h) and (h,l) in such a way as to make both responders indifferent in Period 1. Concretely, she offers (h,h) with probability $\rho = \bar{\mu}\frac{x_1-l}{\delta\tau} - 1$ and (h,l) otherwise.

[20] Following $v = (N,N)$, P offers (h,h).

[21] Following $v = (N,N)$, P mixes between (h,l) and (l,h) so as to make both responders indifferent. Specifically, she offers (h,l) with probability $\rho = \frac{\mu}{1-\mu}\left(\frac{x_1-l}{\delta\tau} - 1\right)$ and (l,h) otherwise.

Figure 2: Equilibria following Stage 1 Proposal (unanimity rule)



The figure depicts the equilibria as described in Proposition 2 following a Stage 1 proposal of (x_1, x_2) under unanimity rule ($l = \frac{1}{10}$, $\tau = \frac{1}{3}$, $\delta = \frac{1}{4}$ so that $\tau > \hat{\tau}^u$). In the whole colored (not grey) area where both responders are offered less than h there exist multiple equilibria (for instance, equilibria as described in 8 exist in this whole area).

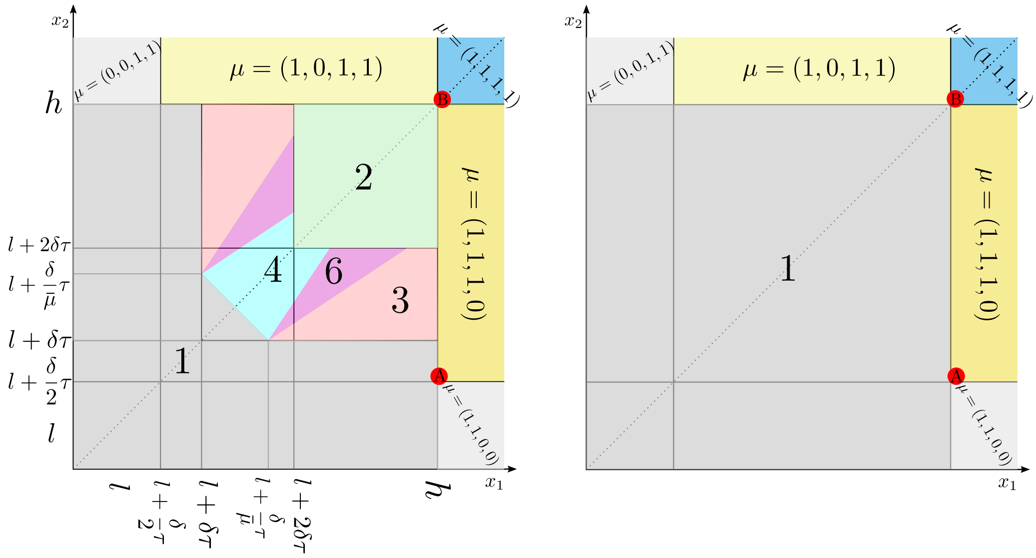
Figure 2 illustrates which equilibria exist for various offers in case $\tau > \hat{\tau}^u$. The figure is qualitatively general under the assumption that $\bar{\mu} > max\left\{2\delta, \frac{1}{2}\right\}$, and $\delta < \frac{1}{2}$ (e.g. $l = \frac{1}{10}$, $\tau = \frac{1}{3}$, $\delta = \frac{1}{4}$, and therefore $\bar{\mu} = 0.6$). In the colored (not grey) area where the proposal allocates less than h to *both* responders, there exist multiple continuation equilibria.

In order to get a feel for the range of equilibria that exist, we characterize the most "optimistic" and most "pessimistic" equilibria in the sense of maximal or minimal probabilities of first round passage. Clearly, the minimum probability of passage is associated with the "sure reject" equilibrium where both responders vote N irrespective of their type, and this equilibrium exists for all proposals allocating less than h to both responders (see RHS of Figure 3). On the other extreme, the largest probability of passage is associated with equilibria of type 2 when they exist (both responders vote Y if low), and in other cases with the equilibria of type 3, 4, or 6 (see LHS of Figure 3).

5.4 Period 1 Proposal Stage

The multiplicity of continuation equilibria following many Period 1 proposals turns out to be inconsequential when we turn to identifying the proposer's optimal Period 1 proposal. This is because even under the most "optimistic" assumptions regarding continuation play after offers allocating less than h to both responders, P will still prefer to make a proposal that allocates h to at least one responder.

Figure 3: “Best” and “worst” equilibria (unanimity rule)



The figure depicts the equilibria as described in Proposition 2 following a Stage 1 proposal of (x_1, x_2) under unanimity rule ($l = \frac{1}{10}$, $\tau = \frac{1}{3}$, $\delta = \frac{1}{4}$ so that $\tau > \hat{\tau}^u$). In the graph on the left hand side, the multiplicity of equilibria is resolved in the most “optimistic” way. In the graph on the right hand side, the multiplicity of equilibria is resolved in the most “pessimistic” way. The red points A and B depict the two relevant options for P’s optimal Period 1 proposal (see Proposition 4).

Proposition 4. *(See Figure 3) In any SWUBE under unanimity rule, the proposer’s Period 1 offer is*

$$p_1 = \begin{cases} (h, h) & \tau < \hat{\tau}^u \\ \left(l + \frac{\delta}{2}\tau, h\right) \;\; or \;\; \left(h, l + \frac{\delta}{2}\tau\right) & \tau > \hat{\tau}^u \end{cases}.$$

Following these offers, both responders vote Y independent of their types if offered h, while only low types vote Y if offered $l + \frac{\delta}{2}\tau$.

Proof. See Appendix. □

For τ small, P buys both responders for sure and there is an immediate agreement. For larger τ, P takes a “risk” and tries to buy one responder “cheap”. Even so, P offers one responder h in order to secure his vote for sure. In addition to increasing the probability of passage from $\frac{1}{4}$ to $\frac{1}{2}$, this also makes the other responder cheaper: A low type responder’s “price” decreases from $\max\{l + 2\delta\tau, h\}$ to $l + \frac{\delta}{2}\tau$ if the other responder’s probability of voting Y jumps from $\frac{1}{2}$ to 1. The intuition is that if the respective responder’s vote becomes pivotal, it eliminates the negative signaling value of voting Y (as voting Y then leads with certainty to an immediate agreement) - making voting Y more attractive.

6 Majority Rule

We now turn to the analysis of majority rule equilibria, proceeding in the same way we did for unanimity rule, i.e. by backward induction, beginning with the Period 2 voting stage. As we will see, majority rule admits a large multiplicity of equilibria. This is due mainly to the fact that the proposer will often be indifferent between the two “minimum winning coalitions” that he can build in Period 2. Depending on how she resolves this indifference, responders’ continuation values and their incentives to vote Y in Period 1 change. We will consider different ways that this multiplicity could be dealt with in order to get a feel for the "range" of equilibria that exist. This will be discussed when we get to the relevant parts of the analysis.

6.1 Period 2 Voting Stage

Lemma 7. *In any SWUBE under majority rule, responders vote Y in Period 2 iff they are offered at least their breakdown value.*

Proof. Voting N (Y) when offered *strictly* more (less) is weakly dominated. When offered exactly the breakdown value neither choice is dominated. Assuming that responders vote Y in this case is without loss of generality. □

6.2 Period 2 Proposal Stage

As under unanimity rule, let $\omega = (\omega_1, \omega_2)$ denote P's belief concerning the probabilities that R1 and R2 are "high" types. Without loss of generality, assume that $\omega_1 \leq \omega_2$. It can then be shown that the optimal Period 2 proposal is given by

$$p_2(\omega_1, \omega_2) = \begin{cases} (h, 0) \text{ or } (0, h) & \omega_1 > \frac{\tau - l}{\omega_2(1-2l)} \\ (l, l) & \omega_1 < \frac{\tau - l}{\omega_2(1-2l)} \end{cases} \text{ for } \omega_2 < \frac{1-l}{1-2l} - \frac{l}{\omega_1(1-2l)}$$

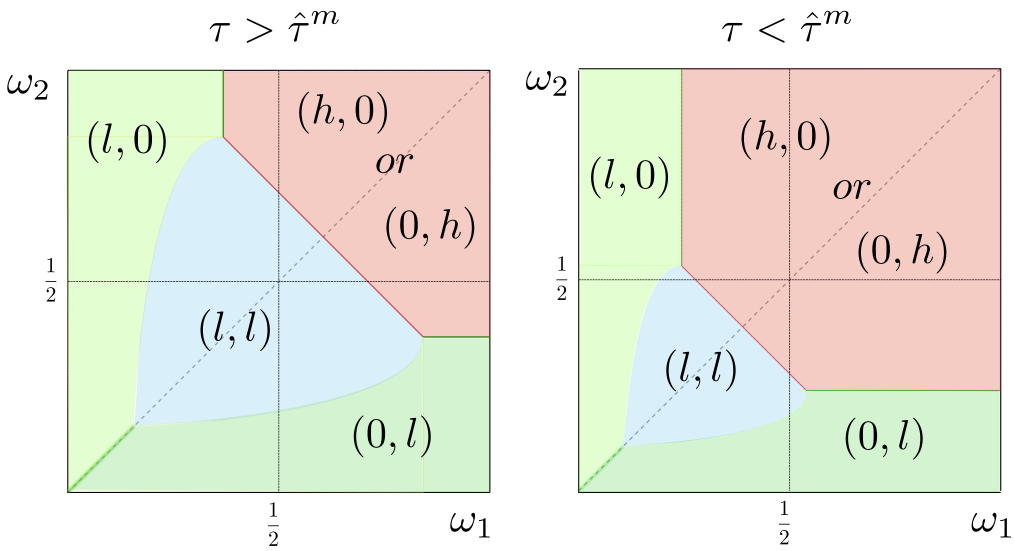
$$p_2(\omega_1, \omega_2) = \begin{cases} (h, 0) \text{ or } (0, h) & \omega_1 > \frac{\tau}{1-l} \\ (l, 0) & \omega_1 < \frac{\tau}{1-l} \end{cases} \text{ for } \omega_2 > \frac{1-l}{1-2l} - \frac{l}{\omega_1(1-2l)}$$

Figure 4 presents the optimal proposals in (ω_1, ω_2)-space.[22] There are several things to notice about these figures. First, holding ω_2 fixed, R1 will often prefer that $\omega_1 = 0$, i.e. that P believes R1 is a low type. This is because P will then offer him l, while for larger ω_1, she may exclude him and make an offer to R2 instead. And since R1 cannot veto such proposals, he may end up with nothing if ω_1 is large. We will see later that this translates into a *negative signaling value* from voting N in Stage 1. (Note also that this applies irrespective of R1's type.) Here, we define $\hat{\tau}^m \equiv \frac{1+2l}{4}$ as the cutoff value such that in case P has not learned anything about the responders (i.e. $\omega_2 = \left(\frac{1}{2}, \frac{1}{2}\right)$), P prefers to offer $(h, 0)$ (or $(0, h)$) for all $\tau < \hat{\tau}^m$ and to offer (l, l) for all $\tau > \hat{\tau}^m$. Second, the multiplicity of equilibria in the red areas of Figure 4 implies an enormous multiplicity of equilibria in the game as a whole: When beliefs about both responders are sufficiently high, P will offer one responder h, and this responder will vote Y for sure. Therefore P is indifferent between $(h, 0)$ and $(0, h)$. Depending on how she mixes between these offers following a given history, R1 and R2's continuation values can be any convex combination of $(h, 0)$ and $(0, h)$. This, in turn, implies that the "prices" that must be paid in order to secure a responder's vote in Stage 1 are not uniquely determined, leading to a "folk theorem"-like multiplicity of equilibria.

Additional Assumptions under Majority Rule In order to make progress, we reduce this multiplicity by introducing additional assumptions as to what P will do in situations where she is indifferent between proposing $(h, 0)$ and $(0, h)$. In addition, we will make an assumption about off-path beliefs following an unexpected N vote, and we will assume that voters use pure strategies whenever possible.

[22] Note that the relationship between beliefs and optimal proposals is qualitatively different depending on whether $\tau < \frac{1+2l}{4}$ or not. Also note that $\tau > \frac{1+2l}{4}$ is possible only if $l < \frac{1}{6}$, as we have already assumed that $\tau < \frac{1-2l}{2}$.

Figure 4: Period 2 Proposals (majority rule)



The figure depicts the Period 2 proposals that P makes given arbitrary beliefs about the responders' types. The horizontal axis measures the probability that R1 is a high type, denoted ω_1. The vertical axis measures the belief about R2, denoted ω_2. The left and right panel distinguish the cases where τ is greater or smaller than $\hat{\tau}^m \equiv \frac{1+2l}{4}$, which determines what P would do in case $\omega_1 = \omega_2 = \frac{1}{2}$.

Thus, for the remainder of the analysis, we will restrict attention to equilibria which satisfy the following conditions, in addition to those stated in Definition 1:

Assumption 2. *The following conditions are assumed to hold:*

1. If $\omega_2 = 1$ and $\omega_1 \in \left(\frac{1}{2}, 1\right)$, $p_2(\omega) = (h, 0)$. That is, P favors the responder whom he believes is less likely to be a high type. This assumption is in the spirit of 'extended proper equilibrium' refinements (see Milgrom and Mollner (2021)): Although both responders *should* vote Y when offered h, only low types *strictly* prefer to do so. Therefore, if we introduced cost-dependent trembles, low types would be less likely to 'accidentally' vote N, and therefore P would prefer to include the responder who is less likely to be a high type.

2. If $\omega_1 = \omega_2$, then P mixes symmetrically if she is indifferent between two proposals.

3. Following an unexpected stage 1 N vote by responder i, off-equilibrium beliefs satisfy $\omega_i = 1$.

4. Responders use pure strategies whenever possible.

6.3 Period 1 Voting Stage

Note that a responder's vote makes a difference only in the event that the other responder votes N. In this case, voting Y yields x_i, while voting N yields $(1-\delta)b_i + \delta v_{T_i}(\omega_{1N}, \omega_{2N})$. Therefore, voting Y is weakly dominated if $x_i \leq (1-\delta)b_i + \delta v_{T_i}(\omega_{1N}, \omega_{2N})$, while voting N is weakly dominated if $x_i \geq (1-\delta)b_i + \delta v_{T_i}(\omega_{1N}, \omega_{2N})$, where $b_i \in \{l, h\}$ is the responder's breakdown value and $T_i \in \{L, H\}$ is his type. The continuation values $v_{T_i}(\omega_{1N}, \omega_{2N})$ are defined in the obvious way.

Lemma 8. *Let (x_1, x_2) be an arbitrary Period 1 proposal. Then in any SWUBE under majority rule satisfying our additional Assumption 2,*

1. *if* $x_i > h$, *then* $\mu_{iL} = \mu_{iH} = 1$,

2. *if* $x_i > (1-\delta)l + \delta h$, *then* $\mu_{iL} = 1$,

3. *if* $x_i < (1-\delta)h$, *then* $\mu_{iH} = 0$,

4. *if* $x_i < (1-\delta)l$, *then* $\mu_{iL} = \mu_{iH} = 0$.

Proof. (1) - (4) all follow directly from the (sequential) elimination of weakly dominated strategies. □

Note that a pattern visible in Lemma 8, which is analogous to Lemmas 3 for unanimity rule, is that low type responders are generally more likely to vote Y than high type responders. In contrast to Lemma 3, Lemma 8 does not yet pin down the relative voting probabilities for proposals that allocate $x_i \in ((1-\delta)h, (1-\delta)l + \delta h)$. However, it is straightforward to verify the following result which is analogous to Corollary 1 and makes a statement about the beliefs that can be formed if Period 2 is reached. (Note that, unlike under unanimity rule, Period 2 is reached only if both responders vote N, so that only the belief ω_{iN} is relevant.)

Corollary 2. *In any SWUBE under majority rule satisfying our additional Assumption 2: For any Period 1 proposal* x, $\mu_{iL} \geq \mu_{iH}$, *and therefore* $\omega_{iN} \geq \frac{1}{2}$.

Proof. See Appendix. □

We now proceed analogously to the analysis of unanimity rule, treating R2's type dependent voting probabilities $\mu_2 = (\mu_{2L}, \mu_{2H})$ as exogenously given and identifying R1's voting probabilities as a function of the amount x_1 being allocated to him. It will turn out that, unlike under unanimity rule, there always exist equilibria in which both responders use pure strategies. We therefore (and recalling $\mu_{1L} \geq \mu_{1H}$) restrict attention to $\mu_i \in \{(0,0), (1,0), (1,1)\}$.

Lemma 9. *In any SWUBE under majority rule satisfying our additional Assumption 2, the following holds.*

1. *Suppose* $\mu_2 = (0,0)$ *and* $\tau < \hat{\tau}^m$. *Then*[23]

$$\mu_{1L} = \begin{cases} 0 & x_1 < (1-\delta)l \\ 0 \text{ or } 1 & x_1 \in \left[(1-\delta)l, (1-\delta)l + \frac{\delta}{2}h\right] \\ 1 & x_1 > (1-\delta)l + \frac{\delta}{2}h \end{cases}$$

and

$$\mu_{1H} = \begin{cases} 0 & x_1 < (1-\delta)h \\ 0 \text{ or } 1 & (1-\delta)h \leq x_1 \leq max\left\{(1-\delta)h, (1-\delta)l + \frac{\delta}{2}h\right\} \\ 1 & otherwise \end{cases}$$

2. *Suppose* $\mu_2 = (0,0)$ *and* $\tau > \hat{\tau}^m$. *Then*

$$\mu_{1L} = \begin{cases} 0 & x_1 < (1-\delta)l \\ 0 \text{ or } 1 & x_1 \in [(1-\delta)l, l] \\ 1 & x_1 > l \end{cases}$$

[23] Let us give you a short intuition of why the high type's voting behavior depends on the low type's breakdown value: If $x_1 > (1-\delta)l + \frac{\delta}{2}h$, then the low type must vote Y in any SWUBE and therefore it cannot be that both types are expected to vote N. Hence, voting N leads to a Period 2 belief of 1 so that a high type R1 MUST vote Y on all offers $x_1 > (1-\delta)h + \frac{\delta}{2}h$ (i.e. also at offers $x_1 > (1-\delta)l + \frac{\delta}{2}h$).

and

$$\mu_{1H} = \begin{cases} 0 & x_1 < (1-\delta)h \\ 0 \text{ or } 1 & (1-\delta)h \le x_1 \le max\{(1-\delta)h, l\} \\ 1 & otherwise. \end{cases}$$

3. *Suppose* $\mu_2 = (1,0)$ *or* $\mu_2 = (1,1)$. *Then*

$$\mu_{1L} = \begin{cases} 0 & x_1 < (1-\delta)l + \frac{\delta}{2}h \\ 0 \text{ or } 1 & x_1 \in \left[(1-\delta)l + \frac{\delta}{2}h, (1-\delta)l + \delta h\right] \\ 1 & x_1 > (1-\delta)l + \delta h \end{cases}$$

and

$$\mu_{1H} = \begin{cases} 0 & x_1 < (1-\delta)h + \frac{\delta}{2}h \\ 0 \text{ or } 1 & (1-\delta)h + \frac{\delta}{2}h \le x_1 \le max\left\{(1-\delta)h + \frac{\delta}{2}h, (1-\delta)l + \delta h\right\} \\ 1 & otherwise. \end{cases}$$

Proof. See Appendix. □

It may be interesting to compare Lemma 9 to Lemmas 5 and (especially) 6. Under unanimity rule, there is a range of offers for which a low type must mix between voting Y and N in equilibrium. The reason is that low types are less willing to vote Y if they are *expected* to do so, as that expectation creates a signaling incentive to vote N. Therefore there exist proposals on which low types would want to vote Y if they were expected to vote N and vice versa. The opposite is true under majority rule. Here, a low type responder is "cheaper" if P expects low types to vote Y than if she expects them to vote N. The reason is that in the former case, voting N causes P to conclude that the responder is a high type, so she excludes him from the Period 2 coalition with higher probability. Thus there exists a range of offers where responders want to vote Y if expected to do this and N if expected to do that, resulting in multiple equilibria.

In addition, notice that R1's "price" is higher if R2 votes Y with positive probability (e.g. $\mu_2 = (1,0)$) than if R2 votes N for sure (e.g. $\mu_2 = (0,0)$). The intuition is that in the former case, P will have learned that R2 is a high type if Period 2 is reached. Therefore she is more likely to include R1 in her Period 2 proposal, making him more expensive in Period 1.

Lemma 9 can be used to establish the following result, which identifies *necessary* conditions on the proposal x for a certain combination of voting probabilities for both responders to be consistent with SWUBE (and our additional Assumption 2).

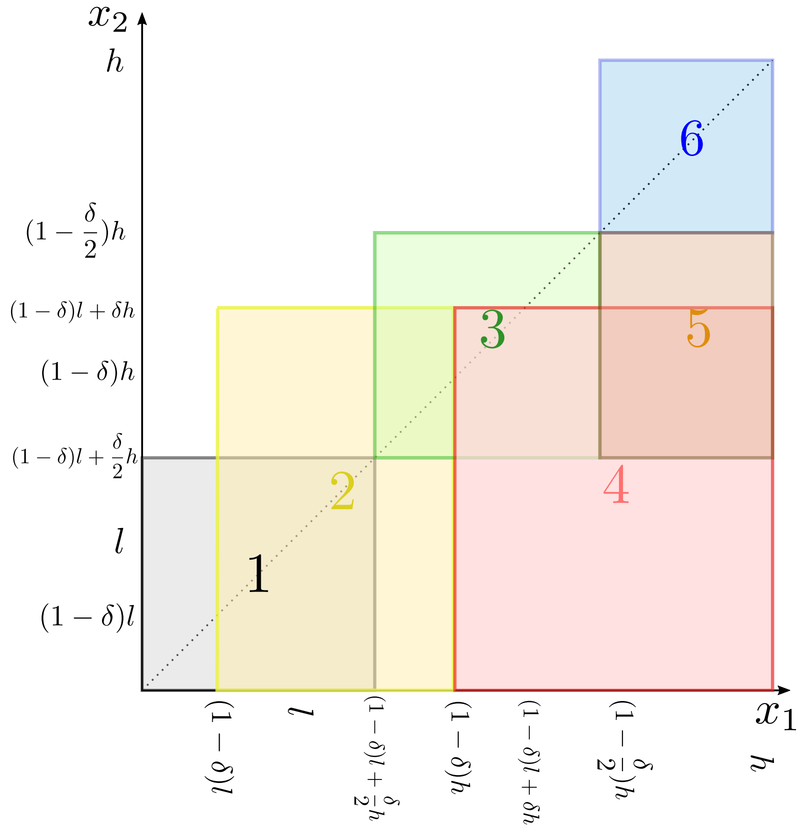
Proposition 5. *(See Figure 5) Consider an arbitrary Period 1 proposal* $x = (x_1, x_2)$. *Suppose that there exists a SWUBE satisfying our additional Assumption 2 in which the acceptance probabilities following this proposal are given by* $\mu = (\mu_{1L}, \mu_{1H}, \mu_{2L}, \mu_{2H})$ *with* $\mu_i = (\mu_{iL}, \mu_{iH}) \in \{(0,0), (1,0), (1,1)\}$. *Then in any such SWUBE under majority rule,*

1. *If* $\mu = (0,0,0,0)$ *then* $x_i \le \begin{cases} (1-\delta)l + \frac{\delta}{2}h & \tau < \hat{\tau}^m \\ l & \tau > \hat{\tau}^m \end{cases}, i = 1, 2.$[24]

2. *If* $\mu = (1,0,0,0)$ *then* $x_1 \in \left[(1-\delta)l, (1-\delta)h\right]$ *and* $x_2 \le (1-\delta)l + \delta h$.

[24]If $\tau = \hat{\tau}^m$, then the proposer can in Period 2 mix between offering (l,l), $(h,0)$, and $(0,h)$ in a symmetric way. So $\mu = (0,0,0,0)$ exists for $x_i \le max\left\{l, (1-\delta)l + \frac{\delta}{2}h\right\}$.

Figure 5: Equilibria following Stage 1 Proposal (majority rule)



The figure depicts the equilibria as described in Proposition 5 following a Stage 1 proposal of (x_1, x_2) under majority rule ($l = \frac{1}{10}$, $\tau = \frac{3}{10}$, and $\delta = \frac{1}{2}$). For some offers there exist multiple equilibria.

3. *If* $\mu = (1, 0, 1, 0)$ *then* $x_i \in \left[(1-\delta)l + \frac{\delta}{2}h, (1-\delta)h + \frac{\delta}{2}h\right]$, $i = 1, 2$.

4. *If* $\mu = (1, 1, 0, 0)$ *then* $x_1 \geq (1-\delta)h$ *and* $x_2 \leq (1-\delta)l + \delta h$.

5. *If* $\mu = (1, 1, 1, 0)$ *then* $x_1 \geq (1-\delta)h + \frac{\delta}{2}h$ *and* $x_2 \in \left[(1-\delta)l + \frac{\delta}{2}h, (1-\delta)h + \frac{\delta}{2}h\right]$.

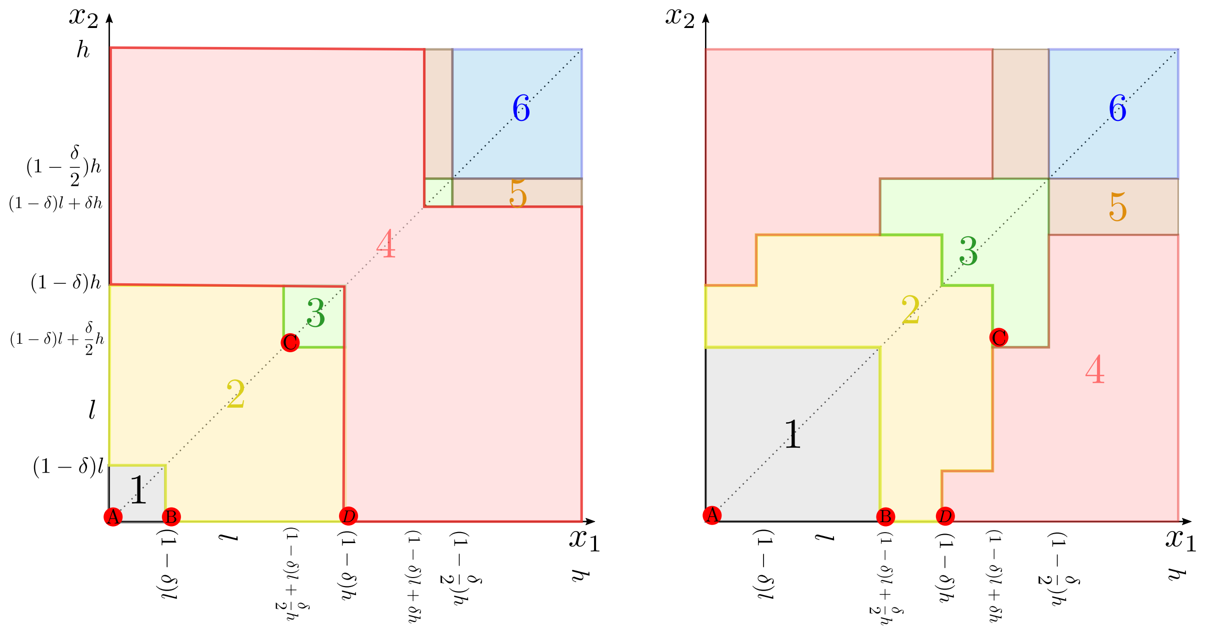
6. *If* $\mu = (1, 1, 1, 1)$ *then* $x_1 \geq (1-\delta)h + \frac{\delta}{2}h$.

Proof. This proposition follows by combining Lemma 9 and Assumption 2. □

Note that the continuation equilibria identified in Proposition 5 are lexicographically ordered according to, first, the probability of passage, and second, the expected number of Y votes. Since only one vote is necessary for passage, equilibria 4, 5, and 6 all involve certain passage but differ in the expected number of Y votes.

Figure 5 illustrates, for one set of parameter conditions ($l = \frac{1}{10}$, $\tau = \frac{3}{10}$, and $\delta = \frac{1}{2}$), which equilibria exist following all possible first round proposals which allocate less than h to both responders. (Any proposal that allocates at least h to one responder must pass for sure.) These areas overlap, meaning that multiplicity remains even with our refining Assumption 2.

Figure 6: “Best” and “worst” equilibria (majority rule)



The figure depicts the equilibria as described in Proposition 5 following a Stage 1 proposal of (x_1, x_2) under majority rule ($l = \frac{1}{10}$, $\tau = \frac{3}{10}$, and $\delta = \frac{1}{2}$). In the graph on the left hand side, the multiplicity of equilibria is resolved in the most “optimistic” way. In the graph on the right hand side, the multiplicity of equilibria is resolved in the most “pessimistic” way. The red points A, B, C, and D depict the four relevant options for P’s optimal Period 1 proposal (see Propositions 6 and 7).

6.4 Period 1 Proposal Stage

P’s optimal Period 1 proposal will depend on how the multiplicities in continuation equilibria are resolved. As under unanimity rule, we can gain an understanding of the “range” of equilibria by characterizing the most “optimistic” and “pessimistic” cases in the sense of maximal or minimal probabilities of first round passage (lexicographically: the largest or smallest expected number of Y votes).

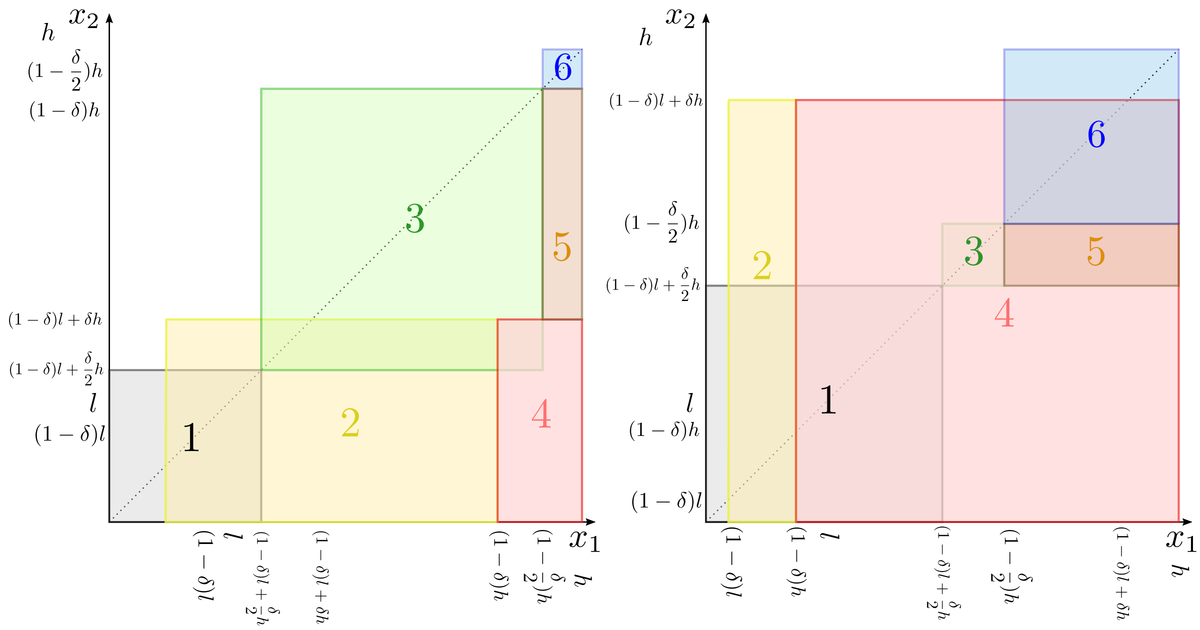
“Optimistic” case The left panel of Figure 6 resolves all multiplicities by picking the continuation equilibria with the largest probability of passage. It is immediately clear that P’s best choice is one of the following four: (A) offer nothing to both responders, (B) offer either responder $(1-\delta)l$ and the other nothing, (C) offer both responders $(1-\delta)l + \frac{\delta}{2}h$, or (D) offer either responder $(1-\delta)h$ and the other nothing. Under the parameter conditions assumed in Figure 6, (A) will be rejected for sure, (B) will pass if the targeted responder is a low type (probability 1/2), (C) will pass if either is a low type (probability 3/4), and (D) passes for sure. (Under some conditions, option C falls out because the point becomes part of area 4.) Comparing these options, gives us the following Proposition.

Proposition 6. *(See Figure 6) In any SWUBE under majority rule satisfying our additional Assumption 2, the proposer’s Period 1 offer is in the “optimistic” case*

$$p_1^* = \begin{cases} ((1-\delta)l + \frac{\delta}{2}h, (1-\delta)l + \frac{\delta}{2}h) & \tau > \hat{\tau}^m \text{ and } \delta < \frac{4h-6l-1}{8h-6l-1} \\ ((1-\delta)h, 0) \text{ or } (0, (1-\delta)h) & \text{otherwise} \end{cases}$$

Only low types vote Y on the first of these proposals (thus it passes if one responder is “low”), while both types vote Y if included on the second proposal (thus it passes for sure).

Figure 7: Equilibria following Stage 1 proposals ($\delta = \frac{1}{6}$ on the LHS and $\delta = \frac{5}{6}$ on the RHS)



The figure depicts the equilibria as described in Proposition 5 following a Stage 1 proposal of (x_1, x_2) under majority rule ($l = \frac{1}{10}$, $\tau = \frac{3}{10}$). The graph on the left hand side uses $\delta = \frac{1}{6}$. The graph on the right hand side uses $\delta = \frac{5}{6}$.

Proof. This proposition follows from comparing P's expected utilities from the options (A)-(D) defined above. □

Note that the proposer's choice and the responder's "prices" depend on the continuation probability δ. To understand this, note that the size of the area in Figure 7 that leads to certain agreement (4, 5, and 6 combined) is increasing in δ. This is because the threat of being excluded in the following period becomes more relevant if the probability of reaching Period 2 following failure is larger. Thus, P can more easily convince responders to vote Y, i.e. an increase in δ makes responders "cheaper". On the other hand, in case breakdown gets more likely (see left panel of Figure 7), buying only the low type responders (as in areas 2 and 3) works for an increasing range of offers. That is because larger breakdown probabilities (small δ) - similar to a larger τ - lead to an increased difference between low and high type responders because the breakdown values become more relevant.

"Pessimistic" case The right panel of Figure 6 resolves all multiplicities by picking the equilibria with the smallest probability of passage. Note that the border between two areas must, like in the "optimistic" case, belong to the one with a larger probability of passage.[25] Again, P's best option will be one of: (A) offer nothing to both responders, (B) offer either responder something a low type must vote Y on (in Figure 6 this is $(1-\delta)l + \frac{\delta}{2}h$) and the other nothing, (C) offer both responders something that only a low type responder votes Y on (above this is for instance $\big((1-\delta)l + \delta h, (1-\delta)l + \frac{\delta}{2}h\big)$), or (D) offer either responder something that both types vote Y on (above this is $(1-\delta)h$) and the other nothing. Note that the offers that are required for these four options (and thus the responders' "prices") depend on the parameter values.

Proposition 7. *(See Figure 6) In any SWUBE under majority rule satisfying our additional Assumption*

[25] Otherwise there would not be an equilibrium as the proposer would want to offer $(x_i + \epsilon, x_j)$ instead of (x_i, x_j) with ϵ being as small as possible.

2, the proposer's Period 1 offer is in the "pessimistic" case (or the symmetric one)

$$p_1^* = \begin{cases} ((1-\delta)h, 0) & \tau < \hat{\tau}^m \text{ and } \delta < \frac{2\tau}{3h-2l} \text{ or} \\ & \tau > \hat{\tau}^m \text{ and } \delta \in \left(\frac{8h-12l-2}{19h-26l-2}, \frac{\tau}{h}\right) \\ \left((1-\delta)l + \frac{\delta}{2}h, 0\right) & \tau < \hat{\tau}^m \text{ and } \delta > \frac{2\tau}{3h-2l} \\ (l, 0) & \tau > \hat{\tau}^m \text{ and } \delta > \frac{\tau}{h} \\ \left((1-\delta)l + \delta h, (1-\delta)l + \frac{\delta}{2}h\right) & \text{otherwise} \end{cases}.$$

Both type of responders vote Y if included in one of the upper three proposals (thus they all pass for sure), while only low types vote Y on the bottom proposal (it passes if at least one of the responders is "low").

Proof. This proposition follows from comparing P's expected utilities from the options (A)-(D) defined above. □

For a vast range of parameter values, buying one responder for sure is P's preferred option. Only for δ sufficiently small and τ sufficiently large, securing immediate agreement with certainty becomes too expensive. This is because if the likelihood of reaching Period 2 is close enough to zero, the threat of exclusion becomes less relevant so that high type responders become so expensive that P prefers to risk breakdown rather than pay both responders the high "price".

7 Unanimity vs. Majority Rule

In the following section, we conduct a comparison of unanimity rule and majority rule. Our goal is to make formally precise the notion that responders are "more expensive" and that immediate agreement is "less likely" or "more difficult to achieve" under unanimity rule than it is under majority rule.

7.1 Responders' Period 1 "Prices"

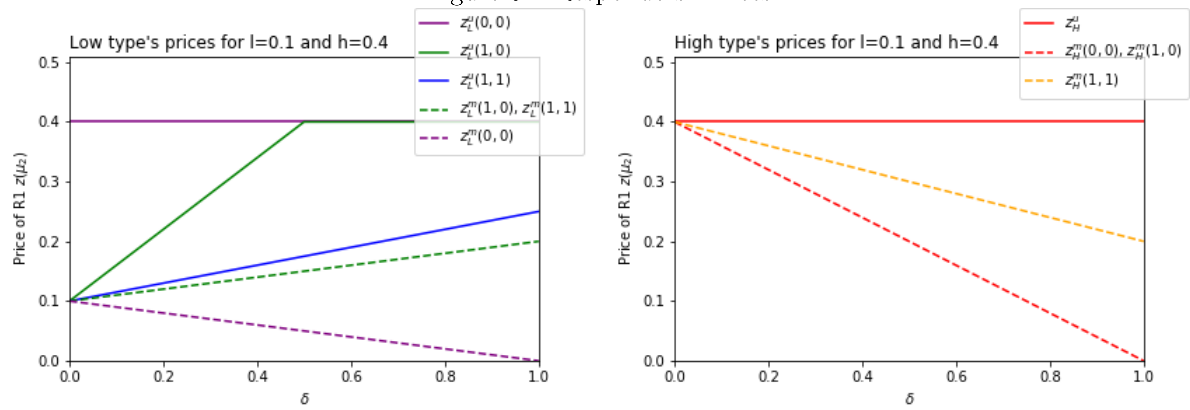
Intuitively, we want to use the word "price" to refer to the amount that a responder must be offered in order to secure his vote. As summarized in Lemmas 4, 5, 6, and 9, this is a function, not just of the voting rule, but also of the probabilities with which the other responder votes Y on the proposal being considered. Taking these probabilities as exogenously given, Lemmas 4, 5, 6, and 9 identify conditions on x_i such that a low or high type responder i would vote Y on a proposal in equilibrium, provided that the other responder votes with the specified probabilities. With reference to the relevant Lemmas, we now explicitly define responder "prices" as follows.

Definition 2. Let μ_2 be fixed. Then the Period 1 price

- $z_L^v(\mu_2)$ of the *low type* of R1 under voting rule $v \in \{m, u\}$ is defined as the smallest value of x_1 such that $\mu_{1L} = 1$ and $\mu_{1H} = 0$ is consistent with the relevant Lemma 4, 5, 6, or 9.
- $z_H^v(\mu_2)$ of the *high type* of R1 under voting rule $v \in \{m, u\}$ is defined as the smallest value of x_1 such that $\mu_{1L} = 1$ and $\mu_{1H} = 1$ is consistent with the relevant Lemma 4, 5, 6, or 9.

Recall that we assumed that responders follow pure strategies under majority rule. Therefore to compare prices between the decision rules, we restrict attention to cases where μ_2 reflects a pure strategy. Also,

Figure 8: Responders' Prices



Responder 1's prices as a function of the continuation probability δ under unanimity rule (solid lines) and majority rule (dashed lines) given R2's voting probabilities. R2 is expected to play a pure strategy. For instance, $z_L^u(1,0)$ is the price of the low type of R1 under unanimity rule given that R2 votes Y if and only if he is of a low type.

recall that low types vote Y whenever high types do. Thus for both voting rules, we will identify R1's prices for the case that R2's voting probabilities are $\mu_2 = (\mu_{2L}, \mu_{2H}) \in \{(0,0),(1,0),(1,1)\}$.

We begin by looking at the Period 1 price of *high type* responders. Under unanimity rule, this is $z_H^u = h$ in any equilibrium and independently of μ_2 (see Lemma 3). Under majority rule, it can take on different values depending on μ_2 and is given by $z_H^m(0,0) = (1-\delta)h$ and $z_H^m(1,0) = z_H^m(1,1) = (1-\frac{\delta}{2})h$ (see Proposition 5). In case that R2 votes N for sure and therefore does not signal that he is of the high type after voting N, R1 will be excluded in a possible second period.[26] On the other hand, if R2 votes Y with positive probability, P, by our symmetry assumption (see Assumption 2), excludes both responders in Period 2 with equal probability. In this sense, we can say that a high type's price is *unambiguously* lower under majority rule than under unanimity rule.

Next, consider a *low type* responder. Under majority rule, the prices of a low type of R1 are given by $z_L^m(0,0) = (1-\delta)l$ and $z_L^m(1,0) = z_L^m(1,1) = (1-\delta)l + \frac{\delta}{2}h$ (see Proposition 5). Under unanimity rule, we have $z_L^u(1,1) = \begin{cases} l + \frac{\delta}{2}\tau & \tau > \hat{\tau}^u \\ l + \delta\tau & \tau < \hat{\tau}^u \end{cases}$ (see Lemma 5) and $z_L^u(1,0) = min\Big\{l + 2\delta\tau, h\Big\}$ (see Lemma 6). If the other responder votes N with certainty, then under unanimity rule, the low type responder can always pool at the high type's behavior with no cost. Therefore, the low type's price must equal that of the high type, i.e. $z_L^u(0,0) = h$ (see Lemma 4). Again, we can conclude that responders' prices are lower under majority rule than under unanimity rule.[27] This finding is illustrated by Figure 8 for a given combination of parameter values ($l = \frac{1}{10}$ and $\tau = \frac{3}{10}$).

Figure 8 also shows how prices vary in the continuation probability δ. Under unanimity rule, prices increase if it becomes more likely that Period 2 is reached. That is because the positive signaling value of voting N becomes increasingly beneficial. Under majority rule, the threat of exclusion makes in most cases responders cheaper if δ is large. An exception is the case where beliefs in Period 2 are such that P is unsure which responder to exclude. At the same time τ must be large enough so that the threat of exclusion is outweighed by the possibility of getting the high share.

[26] Recall that above we have assumed that P's belief after an unexpected (off equilibrium path) N vote of R1 is $\omega_{1N} = 1$.

[27] Note that under unanimity rule (if we drop the assumtion of R1 playing a pure strategy), there are also cutoff values above which the low type responders in equilibrium vote Y with a positive probability smaller than one while below which the responders vote N for sure. These cutoff values are $\begin{cases} l + \frac{\delta}{2}\tau & \mu_{-i} = (1,1) \\ l + \delta\tau & \mu_{-i} = (1,0) \wedge \tau > \hat{\tau}^u \\ l + 2\delta\tau & \mu_{-i} = (1,0) \wedge \tau < \hat{\tau}^u \end{cases}$.

7.2 Delays and Probability of Breakdown

Perhaps the most important measure on which to compare the two rules is the probability of inefficient delay and breakdown. Under unanimity rule, delay can occur if and only if $\tau > \hat{\tau}^u$. Under this condition, P will make one responder an offer that he only accepts if he is a low type (while buying the other "for sure"). Should this proposal fail, P will offer the responder who rejected the high share, while attempting to buy the other "cheap". Therefore delay occurs with probability $\frac{1}{2}$ in Period 1, and if so the Period 2 proposal fails with probability $\frac{1}{2}$. Thus, the probability that no agreement is reached when $\tau > \hat{\tau}^u$ is $\frac{1}{2}\left(1 - \delta + \frac{\delta}{2}\right) = \frac{1}{2} - \frac{\delta}{4}$. Conversely, the probability of *agreement* is $\frac{1}{2} + \frac{\delta}{4}$. For $\tau < \hat{\tau}^u$, agreement in Period 1 is certain.

Under majority rule, delay can occur if and only if $\tau > \hat{\tau}^m$ and $\delta < \frac{4h-6l-1}{8h-6l-1}$ ("optimistic" case) or $\delta < min\left\{\frac{8h-12l-2}{19h-26l-2}, \frac{\tau}{h}\right\}$ ("pessimistic" case). Under these conditions, P will offer both responders something that only low types accept. Thus, delay occurs with probability $\frac{1}{4}$ (if both responders are of the high type). As there will for sure be an agreement if the second period is reached (on equilibrium path), the overall probability of reaching no agreement is $\frac{3}{4} + \frac{\delta}{4}$ under the above conditions. The probability of *agreement* is $\frac{1-\delta}{4}$. For $\tau < \hat{\tau}^m$ or $\delta > \frac{4h-6l-1}{8h-6l-1}$ ("optimistic" case) and for $\tau < \hat{\tau}^m$ or $\delta > min\left\{\frac{8h-12l-2}{19h-26l-2}, \frac{\tau}{h}\right\}$ ("pessimistic" case), agreement in Period 1 is certain.

Comparing these implications, we can conclude that, for a vast range of parameter values, the probability of a delay is (weakly) larger under unanimity rule than under majority rule. The only exception is for $\tau \in (\hat{\tau}^m, \hat{\tau}^u)$ (nonempty if $l < \frac{1}{14} \approx 0.07$) and δ close enough to 0.[28] This result confirms the central part of the informal argument that motivates our analysis, namely that the "tough" bargaining induced by unanimity rule can result in inefficiencies even in a situation where agreement is always efficient.

8 Discussion

The choice between alternative q-majority rules, including unanimity rule, is a central problem in constitutional design. Unanimity rule has the attractive property that decisions taken are guaranteed to be *efficient* in the sense that all members prefer the proposed change relative to the status quo. A potential disadvantage is that agreement might be more difficult to reach under unanimity rule *even if it is efficient.* This is because, even when all members of a group can potentially benefit, each individual member may have an incentive to overstate their opposition, in an attempt to secure a larger share of the surplus that results from agreement. Arguments along these lines have been made informally, among others by Buchanan and Tullock (1965). The goal of the present paper was to provide an explicit formalization of this argument in a multilateral bargaining game with private information.

Our central results confirm the informal argument that unanimity rule creates incentives to "act tough" and thereby makes agreement more difficult to achieve. In general, the prices necessary to secure a responder's vote are higher under unanimity rule than under majority rule, ceteris paribus. The main reason for this is that voting N carries a positive "signaling value" under unanimity rule, while it is

[28] These conditions are extremely restrictive. For instance, if $l = 0.05$, then they are $\tau \in (0.275, 0.3)$ and $\delta \leq 0.0\bar{6}$ (if $\tau \approx 0.3$). To get an intuition for this, consider the extreme case of $\delta = 0$ and $l = 0$. Under both rules, P compares two types of proposals: Those that pass for sure because she offers both responders enough to secure even the high type's vote, and those where she "gambles" by offering at least one responder something they might reject. If $\tau > \frac{1}{4}$, P prefers to take the gamble under majority rule and offer both responders the low type's price $l = 0$, which passes with probability $\frac{3}{4}$ (i.e. unless both are high types). Under unanimity rule, however, a similar gamble is too risky if $\tau < \frac{3}{4}$. P needs both responders to vote Y. Thus, even if she offers the low type's price to only one of the responders (and buys the other for sure), there is only a 50 : 50 chance that the proposal will pass. Therefore P prefers to buy both high types under unanimity rule. Thus for these special parameter conditions, delay only might occur under majority rule.

discouraged through the threat of subsequent exclusion under majority rule. The combination of higher prices and the need to secure more votes implies that the proposer will more often choose to risk delay and breakdown under unanimity rule, resulting in a larger probability of inefficient disagreement. In addition to confirming the informal argument outlined above, the formal analysis yields additional insights into the underlying incentives and the properties of optimal strategies.

For example, the analysis yields interesting comparative statics with respect to the continuation probability δ. Under unanimity rule, the high type's price always corresponds to his breakdown value, since he can veto any other outcome. The low type's price, however, is larger than his breakdown value and increases in δ. This is because acting "tough" is both less costly and more beneficial if the next period is reached with a larger probability following proposal failure. Under majority rule, in contrast, responders' prices are generally lower than their breakdown values and decreasing in δ, and indeed approach zero as δ approaches 1. The intuition is that acting "tough" becomes counterproductive if period 2 is likely to be reached, as the "tough" responder will simply be excluded from the period 2 coalition.[29]

Another insight concerns strategic complementarities in responders' vote choices. Under unanimity rule, a responder is more willing to vote Y (in the sense that his price is lower) the higher the probability that the *other* responder votes Y, in which case his own vote becomes pivotal. The reason is that voting Y carries a *negative signaling value* in case the proposal fails. The more likely it is that the other responder votes Y, the less relevant this becomes.[30] This translates into an incentive for the proposer to offer one responder enough to secure his vote for sure, thereby making the other responder pivotal and consequently "cheap".[31] Interestingly, the opposite is true under majority rule. Here, a responder is *less* willing to agree (his price is higher) the more likely the other responder is to vote Y.[32]

Appendix

Proof of Lemma 2: Note that a low type responder will receive either l or h in Period 2. Thus, the statement can be false only if he receives h with positive probability at (ω_1, ω_2). Then with positive probability $p_2(\omega_1, \omega_2) = (h, x_2)$ for some x_2 so that the proposal passes with positive probability. It can be verified by inspection that then for any $\tilde{\omega}_1 > \omega_1$, the optimal proposal at $(\tilde{\omega}_1, \omega_2)$ is unique and allocates h to R1. Moreover, the offer to R2 is unaffected unless $\omega_1 = \omega_2 \in \left(\frac{\tau}{1-2l}, \frac{\tau}{1-2l-\tau}\right)$ and P mixes between proposing (l, h) and (h, l) at (ω_1, ω_2). (And the latter proposal passes with positive probability.)

[29] An exception can occur if the offer is such that the other responder is likely to accept if he is a low type. Then, in case the other responder votes N, he would be signaling that he is a high type. Then, if both responders vote N, the proposer offers the high share to each with equal probability in Period 2. If $\tau > l$, this possibility of getting the high share (with probability $\frac{1}{2}$) is preferred over getting the low breakdown value. Therefore, in such cases the responders' prices increase in δ, as under unanimity rule.

[30] There is one exception to this pattern: For a small interval of τ, R1's price is lower if R2 mixes with a certain probability μ_{2L} when low than if R2 votes Y for sure when low. The reason is that if R2 votes Y if low, then in Period 2 after (N,N) P will offer the high share to both responders. This makes voting N more attractive compared to the case where P offers the high share only to R1 (and thus the Period 2 proposal only passes if R2 is low) - as in the case with $\mu_{2L} < \bar{\mu}$. For this to hold we need $\mu_{2L} \in (\frac{1}{2}, \bar{\mu})$ and thus $\tau > \frac{2-4l}{5}$ (so that $\bar{\mu} > \frac{1}{2}$).

[31] In a model involving a larger number of players, this may translate into an interesting tradeoff: P will wish to offer a sufficiently large number of responders large enough shares to secure their certain agreement such that the remaining responders are sufficiently convinced that agreement is likely so as to make their votes pivotal, and therefore "cheap".

[32] The reasons underlying this effect are subtle and have to do with the beliefs formed in case the first round proposal fails (see the discussion following Lemma 9). In addition, the precise way this plays out is specific to the three player case. The general insight is that under majority rule, responders are more likely to accept if they believe that they will be excluded from a future coalition. This threat of exclusion, in turn, will be more credible if *other* responders are not induced to signal their types, i.e. if P offers them something such that they vote either Y or N irrespective of their type. It appears potentially interesting to investigate in more detail the implications of this for the design of optimal proposals in larger groups. This is the subject of ongoing follow up research in which we extend the model to $n > 2$ responders.

In this case she will shift to proposing (h, l) with probability 1 at $(\tilde{\omega}_1, \omega_2)$. So in all instances, the probability that responder 1 receives h (rather than l) weakly increases in ω_1.

Proof of Lemma 3: For part (1): Suppose w.l.o.g. that $x_1 > h$. Suppose R1 is type H. If he votes N, the proposal fails and he obtains utility h. If he votes Y, he obtains x_1 in case R2 votes Y as well. Thus, voting N is (sequentially) weakly dominated, and by Definition 1, he votes Y. Suppose R1 is type L, and suppose $\mu_{1L} < 1$ (seeking a contradiction). Then voting N reveals that R1 is of type L, and so $\omega_{1N} = 0$. Hence, the expected utility from voting N is l. Voting Y yields a strictly greater payoff if R2 votes Y, and cannot yield a lower payoff otherwise. Thus, voting N is (sequentially) weakly dominated, a contradiction. So we must have $\mu_{1L} = 1$. Part (2) is trivial as the high type can always secure h by voting N in both periods. For part (3): Suppose $x_1 < l$. By (2), $\mu_{1H} = 0$. Suppose $\mu_{1L} > 0$. Then $\omega_{1Y} = 0$ and $\omega_{1N} > \frac{1}{2}$. Voting Y gives R1 x_1 if R2 votes Y and $v_L(0, \omega_{2N}) = l$ otherwise. Voting N gives R1 $v_L(\omega_{1N}, \omega_2) \geq l > x_1$ (where $\omega_2 \in \{\omega_{2Y}, \omega_{2N}\}$ but either way R1 obtains at least l). Thus, voting Y is (sequentially) weakly dominated.

Proof of Corollary 1: The only part that does not follow immediately from Lemma 3 is that $\mu_{iL} \geq \mu_{iH}$ for $x_1 = h$. To see this, suppose $x_1 = h$ and $\mu_{1H} > \mu_{1L}$ (seeking a contradiction). Then we have $\omega_{1N} < \frac{1}{2} < \omega_{1Y}$. Letting $\mu_2 = \frac{\mu_{2L}+\mu_{2H}}{2}$, the utility from voting N is $(1-\delta)l + \delta\Big(\mu_2 v_L(\omega_{1N}, \omega_{2Y}) + (1 - \mu_2)v_L(\omega_{1N}, \omega_{2N})\Big)$ while the utility from voting Y is $\mu_2\,(h) + (1-\mu_2)\Big((1-\delta)l + \delta v_L(\omega_{1Y}, \omega_{2N})\Big)$. The Y-N difference is $\mu_2\Big(\tau + \delta l - \delta v_L(\omega_{1N}, \omega_{2Y})\Big) + (1-\mu_2)\delta\Big(v_L(\omega_{1Y}, \omega_{2N}) - v_L(\omega_{1N}, \omega_{2N})\Big)$. By Lemma 2, $\omega_{1N} < \frac{1}{2} < \omega_{1Y}$ implies $v_L(\omega_{1Y}, \omega_{2N}) \geq v_L(\omega_{1N}, \omega_{2N})$ and so this difference is strictly positive. (If $\mu_2 = 0$, it could be zero if $v_L(\omega_{1Y}, \omega_{2N}) = v_L(\omega_{1N}, \omega_{2N})$). Therefore, $\mu_{1L} = 1$, contradicting $\mu_{1H} > \mu_{1L}$. Hence, we must have $\mu_{1L} \geq \mu_{1H}$.

Proof of Lemma 4: Suppose $x_1 < h$ and $\mu_{2H} = \mu_{2L} = 0$. By Lemma 3, $\mu_{1H} = 0$. The proposal is sure to fail, and only "signaling motives" play a role. R1 strictly prefers to vote Y if $v_{1L}(\omega_{1Y}, 1/2) > v_{1L}(\omega_{1N}, 1/2)$. If so, then $\mu_{1L} = 1$, $\omega_{1Y} = 0$, and $\omega_{1N} = 1$, yielding an immediate contradiction (using Lemma 2). Suppose he strictly prefers to vote N. Then ω_{1Y} is not determined by Bayes rule and $v_{1L}(\omega_{1Y}, 1/2) < v_{1L}(1/2, 1/2)$. This pattern can always be "supported" by appropriate beliefs, e.g. $\omega_{1Y} = 0$. Suppose he is indifferent. Then if $\mu_{1L} > 0$, $\omega_{1Y} = 0$ and $\omega_{1N} > \frac{1}{2}$, yielding a contradiction. Thus, $\mu_{1L} = 0$ and ω_{1Y} is such that $v_{1L}(\omega_{1Y}, 1/2) = v_{1L}(1/2, 1/2)$, e.g. $\omega_{1Y} = \frac{1}{2}$.

Suppose $\mu_{1H} \in (0, 1)$ and $\mu_{1L} < \mu_{1H}$. Then $\omega_{1N} < \frac{1}{2} < \omega_{1Y}$ and so R1 cannot strictly prefer to vote N. Thus, he must be indifferent. Then $v_{1L}(\omega_{1Y}, 1/2) = v_{1L}(\omega_{1N}, 1/2)$. If $\tau < \hat{\tau}^u$, we have $v_{1L}(\omega_{1Y}, 1/2) = h$, and it follows that $\omega_{1N} > \frac{\tau}{1-2l-\tau}$. That is, μ_{1L} must be large enough such that P offers (h, h) after R1 votes N. And so it is without loss of generality to say that $\mu_{1L} \geq \mu_{1H}$. If $\tau > \hat{\tau}^u$, we have $v_{1L}(\omega_{1N}, 1/2) = l < v_{1L}(\omega_{1Y}, 1/2)$, a contradiction. Thus, we must have $\mu_{1L} \geq \mu_{1H}$.

Suppose $\mu_{1H} \in (0, 1)$ and $\mu_{1L} > \mu_{1H}$. Then $\omega_{1Y} < \frac{1}{2} < \omega_{1N}$, R1 cannot strictly prefer to vote Y and must be indifferent. Then $v_{1L}(\omega_{1Y}, 1/2) = v_{1L}(\omega_{1N}, 1/2)$. If $\tau < \hat{\tau}^u$, we have $v_{1L}(\omega_{1N}, 1/2) = h$, and it follows that $\omega_{1Y} > \frac{\tau}{1-2l-\tau}$. That is, μ_{1L} must be small enough such that P offers (h, h) after R1 votes Y. And so it is without loss of generality to say that $\mu_{1L} \leq \mu_{1H}$. If $\tau > \hat{\tau}^u$, we have $v_{1L}(\omega_{1Y}, 1/2) = l < v_{1L}(\omega_{1N}, 1/2)$, a contradiction. Thus, we must have $\mu_{1L} \leq \mu_{1H}$.

In sum, in all cases it is conceivable that low type responders mix with a different probability than high type responders, but only in such a way as to induce the same Period 2 play as when they mix with the same probability. Thus, P cannot obtain information that would affect her Period 2 choices, i.e. nothing that makes her better off than she is by proposing something on that R1 surely votes N.

The next proof makes use of an additional Lemma.

Lemma 10. *Suppose $x_1 < h$ and $\mu_{2H} = \mu_{2L} = 1$. Then in any SWUBE under unanimity rule, $\mu_{1H} = 0$ and a low type responder 1*

- *strictly prefers to vote Y iff $x_1 > l + \delta\left(v_{1L}(1, \frac{1}{2}) - l\right)$,*
- *strictly prefers to vote N iff $x_1 < l + \delta\left(v_{1L}(\frac{1}{2}, \frac{1}{2}) - l\right)$,*
- *is indifferent between voting Y or N iff $x_1 = l + \delta\left(v_{1L}\left(\frac{1}{2-\mu_{1L}}, \frac{1}{2}\right) - l\right)$.*

Proof. Note that $\omega_{2Y} = \frac{1}{2}$ and R1 is pivotal. Necessity: Suppose a low type R1 *strictly* prefers to vote *Y*. Then $\mu_{1L} = 1$ and so $\omega_{1N} = 1$ and $x_1 > (1-\delta)l + \delta v_{1L}(1, \frac{1}{2}) = l + \delta\left(v_{1L}(1, \frac{1}{2}) - l\right)$. Suppose he *strictly* prefers to vote *N*. Then $\mu_{1L} = 0$ and so $\omega_{1N} = 1/2$ and $x_1 < (1-\delta)l + \delta v_{1L}(\frac{1}{2}, \frac{1}{2}) = l + \delta\left(v_{1L}(\frac{1}{2}, \frac{1}{2}) - l\right)$. Suppose he is *indifferent* between *Y* and *N*. Then $\mu_{1L} \in [0, 1]$, $\omega_{1N} = \frac{1}{2-\mu_{1L}}$, and $x_1 = (1-\delta)l + \delta v_{1L}\left(\frac{1}{2-\mu_{1L}}, \frac{1}{2}\right) = l + \delta\left(v_{1L}\left(\frac{1}{2-\mu_{1L}}, \frac{1}{2}\right) - l\right)$. These (in)equalities constitute *necessary conditions* for the responder to prefer *Y* or *N*, or to be indifferent. Sufficiency: Suppose $x_1 > l + \delta\left(v_{1L}(1, \frac{1}{2}) - l\right)$. By Lemma 2, $v_{1L}(\omega_1, \frac{1}{2}) \leq v_{1L}(1, \frac{1}{2})$ and so the necessary conditions for preferring *N* or being indifferent are violated. Therefore, the responder strictly prefers to vote *Y* and $\mu_{1L} = 1$. Suppose $x_1 < l + \delta\left(v_{1L}(\frac{1}{2}, \frac{1}{2}) - l\right)$. By Lemma 2, $v_{1L}\left(\frac{1}{2-\mu_{1L}}, \frac{1}{2}\right) \geq v_{1L}(\frac{1}{2}, \frac{1}{2})$, and so the necessary conditions for indifference and for preferring *Y* are violated. Therefore, the responder strictly prefers *N* and $\mu_{1L} = 0$. Suppose $x_1 = l + \delta\left(v_{1L}(\frac{1}{2-\mu_{1L}}, \frac{1}{2}) - l\right)$. By Lemma 2, $v_{1L}(1, \frac{1}{2}) \geq v_{1L}\left(\frac{1}{2-\mu_{1L}}, \frac{1}{2}\right) \geq v_{1L}(\frac{1}{2}, \frac{1}{2})$, and so the necessary conditions for strictly preferring *Y* or *N* are violated. Therefore, the responder is indifferent between *Y* and *N*. □

Proof of Lemma 5; For part (1): Suppose $\tau < \hat{\tau}^u$. Then $v_{1L}(\frac{1}{2}, \frac{1}{2}) = v_{1L}(1, \frac{1}{2}) = h$. R1 *strictly* prefers *Y* iff $x_1 > l + \delta\tau$ and he *strictly* prefers *N* iff $x_1 < l + \delta\tau$. Therefore, he must be *indifferent* for $x_1 = l + \delta\tau$. Therefore, $v_{1L}\left(\frac{1}{2-\mu_{1L}}, \frac{1}{2}\right) = h$. This will be satisfied for any $\mu_{1L} \in [0, 1]$ (because P offers (h, h) even at $\omega = (\frac{1}{2}, \frac{1}{2})$). For part (2): Suppose $\tau > \hat{\tau}^u$. Then $v_{1L}(1, \frac{1}{2}) = \frac{l+h}{2}$ and $v_{1L}(\frac{1}{2}, \frac{1}{2}) \in \left[l, \frac{l+h}{2}\right]$, depending on how P mixes between (h, l) and (l, h). Suppose $v_{1L}(\frac{1}{2}, \frac{1}{2}) < \frac{l+h}{2}$. Then for $x_1 \in \left[l + \delta\left(v_{1L}(\frac{1}{2}, \frac{1}{2}) - l\right), l + \delta\frac{\tau}{2}\right]$, both conditions for strict preference are violated. Thus, R1 would have to be indifferent for all offers in this range. Then for each such offer, there must exist μ_{1L} such that $x_1 = l + \delta\left(v_{1L}\left(\frac{1}{2-\mu_{1L}}, \frac{1}{2}\right) - l\right)$. However, for offers strictly inside of the interval, such μ_{1L} do not exist, since $\mu_{1L} = 0$ implies $v_{1L}\left(\frac{1}{2-\mu_{1L}}, \frac{1}{2}\right) = v_{1L}(\frac{1}{2}, \frac{1}{2})$, and any $\mu_{1L} > 0$ implies $v_{1L}\left(\frac{1}{2-\mu_{1L}}, \frac{1}{2}\right) = \frac{l+h}{2}$. Therefore, we must have $v_{1L}(\frac{1}{2}, \frac{1}{2}) = \frac{l+h}{2}$, i.e. P must propose (h, l) with probability 1 at $\omega = (1/2, 1/2)$. It follows that responder 1 strictly prefers *Y* iff $x_1 > l + \delta\frac{\tau}{2}$, *strictly* prefers *N* iff $x_1 < l + \delta\frac{\tau}{2}$, and is *indifferent* for $x_1 = l + \delta\frac{\tau}{2}$. Therefore, $v_{1L}\left(\frac{1}{2-\mu_{1L}}, \frac{1}{2}\right) = l + \delta\frac{\tau}{2}$. This will be satisfied for any $\mu_{1L} \in [0, 1]$ and requires that P offers (h, l) with probability 1 at $\omega = (\frac{1}{2-\mu_{1L}}, 1/2)$, even if $\mu_{1L} = 0$.

Proof of Lemma 6: The following additional result can be used to prove each of the items.

Lemma 11. *Suppose $x_1 < h$, $\mu_{2H} = 0$, and $\mu_{2L} > 0$. Then in any SWUBE under unanimity rule, $\omega_{2Y} = 0$, $\omega_{2N} = \frac{1}{2-\mu_{2L}} > \frac{1}{2}$, $\mu_{1H} = 0$, and a low type responder 1*

1. *strictly prefers to vote Y iff $x_1 > l + \delta\Big(\tau + (\frac{2}{\mu_{2L}} - 1)\left(v_{1L}(1, \omega_{2N}) - l\right)\Big)$,*
2. *strictly prefers to vote N iff $x_1 < l + \delta\Big(\tau + (\frac{2}{\mu_{2L}} - 1)\left(v_{1L}(\frac{1}{2}, \omega_{2N}) - v_{1L}(\omega_{1Y}, \omega_{2N})\right)\Big)$*
   where ω_{1Y} is not determined by Bayes rule,

3. *is indifferent between voting Y or N iff* $x_1 = l + \delta\Big(v_{1L}(\omega_{1N}, 0) - l + \big(\frac{2}{\mu_{2L}} - 1\big)\big(v_{1L}(\omega_{1N}, \omega_{2N}) - v_{1L}(\omega_{1Y}, \omega_{2N})\big)\Big)$
   where $\omega_{1N} = \frac{1}{2-\mu_{1L}} \geq \frac{1}{2}$*, and* $\omega_{1Y} = 0$ *if* $\mu_{1L} > 0$ *(and not determined by Bayes rule otherwise).*

Proof. Recall that the general condition for preferring to vote Y is $x_1 > (1-\delta)l + \delta\Big(v_{1L}(\omega_{1N}, \omega_{2Y}) + \big(\frac{2}{\mu_{2H}+\mu_{2L}} - 1\big)\big(v_{1L}(\omega_{1N}, \omega_{2N}) - v_{1L}(\omega_{1Y}, \omega_{2N})\big)\Big)$. Necessity: Suppose a low type R1 strictly prefers to vote *Y*. Then we have $\mu_{1L} = 1$, $\omega_{1Y} = 0$, $\omega_{1N} = 1$, and $x_1 > l + \delta\left(\tau + \left(\frac{2}{\mu_{2L}} - 1\right)(v_{1L}(1, \omega_{2N}) - l)\right)$. Suppose he strictly prefers to vote N. Then $\mu_{1L} = 0$, $\omega_{1N} = \frac{1}{2}$, and $x_1 < l + \delta\Big(\tau + \big(\frac{2}{\mu_{2L}} - 1\big)\big(v_{1L}(\frac{1}{2}, \omega_{2N}) - v_{1L}(\omega_{1Y}, \omega_{2N})\big)\Big)$. Suppose he is indifferent. Then $\mu_{1L} \in [0,1]$, $\omega_{1N} = \frac{1}{2-\mu_{1L}} \geq \frac{1}{2}$, and $x_1 = l + \delta\Big(v_{1L}(\omega_{1N}, 0) - l + \big(\frac{2}{\mu_{2L}} - 1\big)\big(v_{1L}(\omega_{1N}, \omega_{2N}) - v_{1L}(\omega_{1Y}, \omega_{2N})\big)\Big)$. Sufficiency: Suppose $x_1 > l + \delta\Big(\tau + \big(\frac{2}{\mu_{2L}} - 1\big)\big(v_{1L}(1, \omega_{2N}) - l\big)\Big)$. By Lemma 2, $v_{1L}(1, \omega_{2N}) \geq v_{1L}(\omega_{1N}, \omega_{2N}) \geq v_{1L}(\frac{1}{2}, \omega_{2N})$, and $v_{1L}(\omega_{1Y}, \omega_{2N}) \geq h$. Therefore, the necessary conditions for preferring *N* or indifference are violated and so he strictly prefers *Y*. Suppose $x_1 < l + \delta\Big(\tau + \big(\frac{2}{\mu_{2L}} - 1\big)\big(v_{1L}(\frac{1}{2}, \omega_{2N}) - v_{1L}(\omega_{1Y}, \omega_{2N})\big)\Big)$. Then for the same reasons the necessary conditions for preferring *Y* or indifference are violated, and he strictly prefers *N*. Suppose $x_1 = l + \delta\Big(v_{1L}(\omega_{1N}, 0) - l + \big(\frac{2}{\mu_{2L}} - 1\big)\big(v_{1L}(\omega_{1N}, \omega_{2N}) - v_{1L}(\omega_{1Y}, \omega_{2N})\big)\Big)$. Then for the same reasons the necessary conditions for strictly preferring *N* or *Y* are violated, and he is indifferent. □

Proof of Proposition 4

Proof. If $\tau < \hat{\tau}^u$, continuation equilibria for proposals allocating less than h to both responders are characterized by Proposition . So P has the following options. **(i)** Offering a small share to at least one of the responders so that the Period 1 offer fails for sure. This would lead to a Period 2 belief of $\omega = (\frac{1}{2}, \frac{1}{2})$ and an EU(i) of $\delta(1-2h)$. **(ii)** Buying both responders iff low, i.e. offering $(l + 2\delta\tau, l + 2\delta\tau)$. This option is feasible as long as $\delta < \frac{1}{2}$. Otherwise, P buys at least one responder for sure. **(iii)** Thereby, P might want to offer $(h, l + \delta\tau)$ or **(iv)** (h, h). It can be shown that option **(iv)** dominates all other options. If $\tau > \hat{\tau}^u$, it can be shown that offering $(h, l + \delta\tau)$ dominates all other proposals even if we decide multiplicities in favor for the most optimistic continuation equilibria. □

Proof of Corollary 2:

Proof. For $x_i \leq (1-\delta)h$ and $x_i \geq (1-\delta)l + \delta h$, the underlying corollary follows directly from Lemma 8. For $x_i \in ((1-\delta)h, (1-\delta)l + \delta h)$ (if non-empty), $\mu_{iL} \geq \mu_{iH}$ must still be shown. Suppose for $x_i \in ((1-\delta)h, (1-\delta)l + \delta h)$, $\mu_{iH} > \mu_{iL}$ holds (seeking a contradiction). Then by our assumption that pure strategies are played when ever possible (see Assumption 2), it follows directly that a high type responder must always be (weakly) more willing to vote *N* because the only difference between the types' continuation utility after voting *N* lies in their breakdown value (and this is larger for the high type responder). Given that $\mu_{iL} \geq \mu_{iH}$, we have $\omega_{iN} \geq \frac{1}{2}$ because on equilibrium path beliefs are determined by Bayes' Rule and because of our assumption that after an unexpected *N* vote (off-equilibrium path), we have $\omega_{iN} = 1$. □

Proof of Lemma 9:

Proof. Recall that by Corollary 2 we have $\mu_{1L} \geq \mu_{1H}$. For Part (1): Suppose $\mu_2 = (0,0)$ and $\tau < \hat{\tau}^m$. By Lemma 8, we know that a low type responder must in any SWUBE vote *N* if offered less than $(1-\delta)l$ and similarly a high type must in any SWUBE vote N if offered less than $(1-\delta)h$. In addition, there exists a SWUBE where both types of R1 vote N if offered (weakly) less than $(1-\delta)l + \frac{\delta}{2}h$. That is because P's Period 2 belief would be $\omega_{NN} = (\frac{1}{2}, \frac{1}{2})$, i.e. he would equally mix between offering $(h, 0)$ and $(0, h)$. There also exists a SWUBE where the low type votes Y if offered (weakly) more than $(1-\delta)l$. That is because P's Period 2 belief would then be $\omega_{NN} = (1, \frac{1}{2})$, i.e. he would offer $(0, h)$. By the same argument, there is a SWUBE where the high type votes Y if offered (weakly) more than $(1-\delta)h$. There, the low type must vote Y in any SWUBE if offered more than $(1-\delta)l + \frac{\delta}{2}h$. The intuition is that voting N could never give him more than this because the best he can hope for is the $\frac{1}{2}$ chance of getting h in Period 2 (in case P would expect R1 to vote N for sure). The high type must vote Y in any SWUBE if the low type votes Y (see Corollary 2) *and* if offered more than $(1-\delta)h$ (as in a possible Period 2, he would for sure be excluded from the coalition). The only difference of Part (2) is that we have $\tau > \hat{\tau}^m$ and thus P offers (l, l) at $\omega_{NN} = (\frac{1}{2}, \frac{1}{2})$ (instead of equally mixing between offering $(h, 0)$ and $(0, h)$ as before). Thus, there now exists a SWUBE where both types of R1 vote N if offered less than $(1-\delta)l + \delta l = l$. Everything else is like in Part (1). In Part (3), P's belief about R2's type in a possible second period (i.e. after both have voted N) changes from $\frac{1}{2}$ to 1. Thus, a low (high) type of R1 votes N in any SWUBE if offered less than $(1-\delta)l + \frac{\delta}{2}h$ ($(1-\delta)h + \frac{\delta}{2}h$). Still, there also exists a SWUBE where the low type votes N if offered less than $(1-\delta)l + \delta h$. That is because now P would in Period 2 offer $(h, 0)$ if R1 is expected to vote N for sure (then $\omega_{NN} = (\frac{1}{2}, 1)$). Any offer above $(1-\delta)l + \delta h$ a low type must vote Y on in any SWUBE. A high type R1 must in any SWUBE vote Y if the low type votes Y (see Corollary 2) *and* if at the same time he is offered more than $(1-\delta)h + \frac{\delta}{2}h$. That is because in a possible Period 2, he would just get h with probability $\frac{1}{2}$. □

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