Knowledge Codification and Endogenous Growth Theory

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Contents

List of Figures

Chapter 1

Introduction

1.1 Purpose of the Thesis

Knowledge codification is central to utilize the non-rivalry of ideas for economic growth. The usual models of economic growth treat knowledge codification as a by-product of R&D-activities and as costless. In contrast to this, one can observe great efforts by private firms for the purposeful codification of knowledge. The present thesis develops a formal model of endogenous growth that incorporates knowledge codification as a means of intergenerational knowledge transfer. It identifies the circumstances under which knowledge codification takes place in the long run and studies its effects on long run economic development.

1.2 Empirical Motivation

The starting point of the present thesis are the following observations.

Firstly, it is widely reported that the codified knowledge base in the world has increased rapidly over the last decades. For example, the European Union research project "Technology and Infrastructures Policy in the Knowledge-Based Economy" (TIPIK) has explicitly concentrated on "the fact that the rapid cumulative expansion of the codified knowledge-base is the salient characteristic of the development of a modern economy based on knowledge." It is argued that "an in-depth understanding of the incentives to codify,[...] [and] of the advantages and drawbacks of the codification of knowledge, is thus becoming essential for analyzing the process of innovation and growth of the economy" (European Commission, 2004, p. 3). Similarly, "OECD analysis is increasingly directed to understanding the dynamics of the knowledge-based economy and its relationship to traditional economics, as reflected in 'new growth theory'. The growing codification of knowledge and its transmission through communications and computer networks has led to the emerging 'information society' " (OECD, 1996, p. 3).

Secondly, codified knowledge is intentionally created, for example, within 'knowledge management' of private firms. Aoshima (2002) reports that Japanese automobile producers extensively use documentation on design know-how, testing results, and problematic and successful cases found in previous development activities as a means to store knowledge about past practices. Another example is Sandia National Laboratories, which conducted videotaped interviews to capture the extensive weapons design and testing expertise of their aging and retired nuclear weapons designers (Sandia National Laboratories, 1996). According to an empirical study by Edler (2003), 85% of the 497 firms from seven economic sectors which answered his questionnaire, stated that they are creating organizational memory by preparing written documentation such as lessons learned, training manuals, good work practice etc. In an online journal article, it is reported that 2.1 billion US $ have been spent in 2000 for knowledge management worldwide (Ball, 2002). At the end of the year 2000, a German consulting company estimated a market volume for "information portals" until 2002 of 6.8 billion Euro in Europe and of 1.61 billion Euro in the German market (Meta Group Deutschland GmbH, 2001).

Thirdly, one major motivation for knowledge codification within firms is to prevent the loss of knowledge of retiring employees. This was indicated by above's example of Sandia National Laboratories conducting videotaped interviews. In Edler's study, 91% of the firms rated "to accelerate and improve the transfer of knowledge to new workers" one or two on a scale from 1 (extremely important) to 6 (not important at all) with regard to the motivations to use knowledge management. This was the most important motivation. The third most often mentioned reason for knowledge management was "to protect your firm or organization from loss of knowledge due to workers' departure" which 82% of responding firms rated one or two.
In fact numerous cases are reported where productive knowledge has been lost or is at risk of being lost. For example, Cowan et al. (2000) argue that "[...] where there are critical bodies of knowledge that are not kept in more-or-less continuous use, inadequate codification and archiving heightens the risks of 'accidental uninvention'." Similarly, "according to MacKenzie and Spinardi (1995), in the nuclear weapons design process specific local and uncodified knowledge was so important that there was a constant appreciable risk that critical elements of the knowledge base would be lost simply through the turnover of scientists and engineers – a risk of technological retrogression, or at best of costly reconstruction of the organization's previous capabilities (competencies)" (Cowan et al., 2000, p. 244). In a recent newspaper article, a spokeswoman of a German high tech firm expressed concerns about a third of the employees in their R&D-department being over 50 years of age and soon becoming eligible for retirement. She claims that especially in research, experience acquired over many years plays an important role as it is very costly if young researchers replicate failures that the old have had before (Astheimer, 2005). DeLong (2004) writes that "the transfer of knowledge to new generations of leadership is a major area of concern for many organizations worrying about their long-term human capital needs." He argues that due to an expected increase in retirements[1], "problems of poor documentation will become increasingly evident as more experienced employees leave behind badly flawed systems for preserving explicit knowledge about operations and the context surrounding important decisions. The implicit or tacit knowledge these

[1]According to press reports, between 2002 and 2008, 75 percent of the U.S. Defense Department's civilian workforce of 675,000 people are expected to retire and the oil and gas production industry expects to lose more than 60 percent of its employees by 2010 (Sandia National Laboratories, 1996; Sapient Corporation, 2003; Farrell, 2002).

veterans used to compensate for idiosyncratic documentation will be gone. What will be left will be a lot of unusable paper and electronic files" (DeLong, 2004, p. 91).

Fourthly, although some information may accrue as a by-product of research and development activities, for example, patent specifications or possibly the product itself, empirical investigations such as Levin (1986) and Mansfield et al. (1981) suggest that this is not sufficient for knowledge to flow freely. Instead substantial real resources are often required to imitate an innovation, even one entirely lacking legal protection. Levin (1986) concludes that public disclosure of a patent claim does not assure eventual diffusion of the knowledge required to make economic use of an innovation. The study of Mansfield et al. (1981) found that in a seventh of the cases imitation cost was no smaller than innovation cost. According to them, this was not due to any superiority of the imitative product over the innovation, but to the innovator's having a technological edge over its rivals in the relevant field. "Often this edge was due to superior 'know-how' - that is, better and more extensive technical information based on highly specialized experience with the development and production of related products and processes. Such know-how is not divulged in patents and is relatively inaccessible (at least for a period of time) to potential imitators" (Mansfield et al., 1981, p. 910). In line with this is Zucker et al. (1998)'s argument that particularly breakthrough inventions "may be better characterized as creating (rivalrous) human capital – intellectual human capital – characterized by natural excludability as opposed to a set of instructions for combining inputs and outputs which can be protected only by intellectual property rights. This natural excludability arises from the complexity or tacitness of the knowledge required to practice the innovation." Based on both extensive interviews and empirical work summarized in Zucker and Darby (1996), they believe that, "at least for the first 10 or 15 years, the innovations which underlie biotechnology are properly analyzed in terms of naturally excludable knowledge held by a small initial group of discoverers, their co-workers, and others who learned the knowledge from working at the bench-science level with those possessing the requisite know-how" (Zucker et al., 1998, p. 291). In another paper they state that "it is misleading to think of scientific breakthroughs as disembodied information which, once discovered, is transmitted by a contagion-like process in which the identities of the people involved are largely irrelevant" (Zucker and Darby, 1996, p. 12709).

The observations suggest that the knowledge transfer between generations is in general imperfect and that purposeful and costly knowledge codification is playing an important role in the transfer of an economy's productive knowledge.

1.3 Theoretical Motivation

By the argument that the use of an item of knowledge – whether it is the Pythagorean theorem, a soft drink recipe or an algorithm to brew coffee – in one application makes its use by someone else no more difficult, new growth theory suggests that knowledge is non-rival. However, although ideas themselves (e.g. the Pythagorean theorem, a soft-drink recipe or an algorithm to brew coffee) are non-rival, they possess direct economic relevance only in so far as they are embodied in either persons or physical objects. We refer to ideas embodied in persons as human capital or knowledge and to those embodied in physical objects as information[2]. Both are rival. For example, an algorithm to brew coffee can be used by many people and coffee machines simultaneously without deteriorating. It is non-rival (and disembodied). However, to really have a cup of coffee, it needs a person or a machine to use this algorithm. Hence, the idea of brewing coffee must be embodied in a person, who devotes her effort to this activity. This precludes the simultaneous use of her human capital by another activity. Similarly, a coffee machine can brew only one kind of coffee at a time at a finite quantity. The fact that ideas that are not embodied in someone or something are not directly relevant economically is immediately obvious for all ideas that have not yet been discovered or have been discovered and forgotten.[3] Consequently long run economic growth is generated by the accumulation of non-rival ideas in people and physical objects. That is, by the accumulation of human capital and information.

A person, however, has only a finite number of years that can be spent acquiring ideas. When this person dies, her human capital is lost. Any non-rival idea that this person has discovered only lives on if it is either embodied in another person or as information. The transformation of the codifiable part of knowledge via some code into information is referred to as knowledge codification.[4] For example, using a natural language to write down an idea in a book, creating a new product or a piece of art. As compared to human capital, information is long lasting if properly maintained and it may be more easily accessible and distributed. In this respect, it may be useful for an economic analysis to refer to an idea as codified, if every individual of a certain group (for example employees of a firm) can access the information independent of others. For instance, there are enough books containing the same idea, such that anyone who is interested in it is able to use a copy, or there are enough servers, such that anyone interested in an idea can download the information via the internet or intranet. In this way, information has the property of a local public good, and knowledge codification can be interpreted as creating non-rival information from rival human capital.

The standard approach in the endogenous growth theory uses the assumption that knowledge codification is an automatic by-product of research and development activities and

[2]Knowing an idea means to understand it. We do not ascribe the capacity to understand to physical objects. For example, a person may be able to learn an idea, that is, understand it by attending to a scientific paper. The paper, however, does not carry knowledge, because reading a scientific paper does not automatically imply that this person can attach meaning to it, that is, knows more than before.

[3]Boldrin and Levine (1999) strongly advocate this line of thinking.

[4]There is a discussion as to what extent knowledge may be codified. On the one hand, it is a function of the code available, on the other, it depends on the idea itself. Highly abstract mathematical ideas seem to allow better codification than the idea "how to ride a bicycle".

can be treated as costless. The (implicit) argument is that when the innovative product is sold at the market, an interested individual or potential competitor could buy and reverse-engineer it. If the innovation is patented, this person could consult the patent specification and would then immediately know the idea embodied in it. However, the empirical observations suggest that the information that accrues as a by-product of research and development is, by itself, not sufficient for the transfer of knowledge between generations. Moreover, large parts of an economy's productive knowledge are not patented or embodied in a product directly sold at the market, such as for instance many process innovations. This gives rise to the observed purposeful and costly knowledge codification.

1.4 Premises

The thesis builds upon the following premises that are derived from empirical evidence.

First, intergenerational knowledge transfer is generally imperfect. In particular, information that may accrue as a by-product of research and development is, by itself, not sufficient to ensure the transfer of knowledge from one generation of employees to the next.

Second, purposeful knowledge codification plays an important role in knowledge management activities of private firms and is largely motivated by transferring knowledge to new employees and by retaining knowledge of retiring employees within the firm.

The line of argument is that the outcome of R&D-activities is human capital and may additionally involve by-product information such as a patent specification and/or a new product. According to empirical observations such as Mansfield et al. (1981) and Zucker et al. (1998), this by-product information of research and development does not convey large parts of the knowledge necessary to productively employ the respective idea and, hence, is not sufficient to ensure the transfer of knowledge between generations. In this way, the idea is not fully codified, which may induce further purposeful codification activities. Focussing on the transfer of knowledge between generations of employees, the analysis will neglect the by-product information and refer to an idea as codified or as information if it is properly codified for intergenerational knowledge transfer and as not codified, else.[5]

1.5 Key Issues

The central questions that are answered in the thesis are the following.

- ▷ How can the idea of intergenerational knowledge transfer by purposeful knowledge codification be formalized within a model of endogenous growth?

[5]The corresponding assumptions and definitions are discussed in greater detail in chapter 2.

- What are the effects of knowledge codification on the economy's long run development in such a model and under what conditions can one observe endogenous knowledge codification?
- Does long run knowledge codification depend on particular assumptions on technological change?
- How does such a model with knowledge codification correspond to standard endogenous growth theory?
- What are potential sources of social inefficiencies in models with knowledge codification?
- Is the suggested model dynamically inefficient?

1.6 The Economic Problem and Modelling Strategy

According to the empirical observations, the underlying economic problem exhibits the following structure.

- The transfer of knowledge between retiring employees and newly hired employees is imperfect.
- Once the retiring employee has left the firm, there is no further transfer of her knowledge possible, except that it has been codified.
- The investment in knowledge codification has to be made before the employee retires and, hence, also before the new employee can utilize it.

In principle the following two situations between retiring employees and young employees of a firm are conceivable. First, the young employees and the retiring employees overlap such that the young, knowing that human capital transfer is imperfect, agree with the retirees on a certain investment in knowledge codification. If the young employees had to pay in advance or there is some other device, that secures the direct exchange, the economic problem basically reduces to a standard problem of human capital transfer by training or education such as e.g. in Jovanovic and Nyarko (1995).

Consider a second situation in which the retiring employees and those newly hired do not have the possibility to negotiate on knowledge codification before the retiring person leaves the firm. For example if the new employee enters the firm after or at the time the retiree leaves. Would further knowledge transfer be feasible in such a situation? Yes, but only via codified sources. Would it happen? Considering only the retiring and new employees, the answer tends to be negative. The reason is that a retiring employee would have to invest in knowledge codification before the young employees enter the firm and hence the investment is sunk when bargaining takes place. This creates a hold-up problem that can not be solved easily. However, within the firm, the problem can at least be attenuated as the profits may be higher in the succeeding period if the newly hired employees had been transferred additional knowledge by knowledge codification. In this way, the firm owners invest in knowledge codification, or respectively, compensate the retiring employee for her codification investment and provide it to the new employees. The firm owner only invests in knowledge codification up to the point to which it maximizes future profits. She does not account for any (equilibrium) effects on the new employees' wage level.

Another aspect of the economic problem with respect to knowledge codification is that the assumed non-rivalry of codified ideas may imply a public good problem. In the first situation it is clear that the codified ideas being local public goods within the firm would create the well known problem of its private provision if the new employees decided decentrally on their investment in knowledge codification. The same problem exists in the second situation with more than one firm owner.
The analysis will focus on the second situation and assume that the firm owners solve the public good problem by a collective decision within the firm. One may also think of a manager acting on behalf of the owners. In this way, the incentive for knowledge codification within the firm stems from the regular profit maximizing motive.

A natural modelling choice that reflects the underlying knowledge transfer problem and allows to address the key issues is a two-sector overlapping generations framework with two-period-lived individuals. The intermediate sector consists of long-lived oligopolistic intermediate firms that carry out in-house R&D-programs. Within these intermediate firms, the amount of knowledge that is transferred from the retiring researchers to those newly hired can be increased by knowledge codification. It is assumed that the capital owners, that is, the members of the old generation are the firm owners and the intermediate firms are managed on their behalf. The young generation will take over the ownership claims at the end of the period and may then be willing to compensate the retiring researchers for knowledge codification. In this way, the codification decision can be depicted as part of the general savings decision of the young generations' utility maximization problem. This decision process drives the main results.

Some central assumptions will play a crucial role for the framework that will be developed.

It is clear that the long run dynamics of the economy centrally depend on the specification of the research process. Two possible variants will be discussed. First, it is assumed that each researcher produces a certain finite amount of ideas in each period. The second specification corresponds to the usual 'standing on the shoulders of giants' assumption of endogenous growth models in which the research productivity per researcher in a period increases linearly in the stock of knowledge. These two specifications have been chosen for several reasons. On the one hand, the 'standing on the shoulders of giants' assumption allows for a comparison to the standard models of endogenous growth. On the other hand, the assumption of a linearly increasing research productivity in the knowledge stock has also been criticized due to the scale effects it involves (see e.g. Jones (1995)) and consequently, it is interesting to study the effects of knowledge codification on long run economic development in other cases. The particular choices highlight the different roles of the information stock for the economy's dynamics. For example, the assumed longevity of the codified knowledge stock plays a more pronounced role in the case with only a constant finite amount of ideas per researcher and may even entail higher long-run output levels for an economy whose researchers are less creative, that is, produce a lower amount of ideas per period.

Rather than assuming a Ramsey household that cares for all generations alike, the model explicitly considers the interaction between the generations without altruism as described above. The Ramsey consumer would face a tradeoff between capital savings, R&D investment and knowledge codification, and knowledge codification should always positively influence economic growth. It will turn out that this may not necessarily be the case in the suggested overlapping generations structure and depends crucially on the assumptions how the codified knowledge stock influences the researchers' productivity.

Of course there are several other areas in which knowledge is codified, for example, the academic sector or the cultural sector, such as in museums and the like. Another aspect is the commercial transfer of knowledge assets, where research firms purposefully codify specific knowledge for transmission to a licensee.[6] The focus on intergenerational knowledge transfer by knowledge codification within firms has been taken up because the knowledge management literature suggests that this is an important problem, yet, it has not been studied in formal models of economic growth.

The thesis is related to the literature on the characteristics of knowledge and particularly to the discussions on the tacitness and codifiability of knowledge such as Cowan et al. (2000) and Dasgupta and David (1994). However, the discourse has been on an entirely verbal basis.

A recent paper of Thoenig and Verdier (2004) is also concerned with the macroeconomic aspects of knowledge management. However, instead of focussing on the incentive to preserve knowledge in intergenerational knowledge transfer, they emphasize the tradeoff that by knowledge codification contractual incompleteness between the firm owner and the employee can be avoided, which, however, implies the risk of information spillovers to competitors.

Further, the thesis is related to the literature on technology adoption models and the literature on endogenous spillovers by own research. It possesses the same underlying motivation, in that blueprints defined as the "by-product" description of a technology are incomplete in conveying what is useful to know about the technology at hand. It

[6] See for example Arora et al. (2001) and The Economist (2005).

needs additional human capital to fully understand and work the blueprints.[7] The focus of building up this human capital in order to apply a new technology for production is discussed in the adoption models. The present work is more related to the models which focus on the human capital necessary to understand the idea embodied in the blueprint (possibly the patent specification) in order to imitate the technology or use the idea for further research. This kind of human capital is commonly referred to as 'absorptive capacity' which is built up by own research.[8] The corresponding strand of the literature usually examines endogenous knowledge spillovers between competitors in a certain market. In such an environment the innovator of a new product is interested in conveying as little information as possible to the competitors. That is, the patent specification or other codified sources of the idea will only include the absolutely necessary descriptions. The focus in the present work is on knowledge transfer between generations of employees within a firm. In such a setting, there may be an incentive for further knowledge codification in order to reduce the "adoption costs" in terms of own research by the newly hired employees.

The proposed formal model corresponds to the standard idea-driven growth models as in Romer (1990),[9] Aghion and Howitt (1992) or Grossman and Helpman (1991). What sets it apart is that it explicitly models a human capital stock and a stock of codified ideas. Ideas can only be used productively if they are learned and, hence, are part of the employee's (intellectual) human capital. The latter can be enhanced by utilizing codified ideas if previous generations have codified them. In this way, the model could be considered as what is sometimes called a hybrid version[10] between the idea driven growth models and the human capital accumulation growth models with the peculiarity of endogenous knowledge spillovers between generations by knowledge codification. Other differences are an oligopolistic intermediate sector with in-house R&D and the overlapping generations framework.

1.7 Outline of the Thesis

The present thesis comprises three parts.

The first chapter of the first part introduces and explains the major assumptions on knowledge and knowledge codification. Thereafter it provides a formalization that will serve as a basis for the dynamics of the knowledge stock of the endogenous growth model. The following chapter specifies a basic model with imperfect knowledge transfer between generations. In order to convey the mechanics of the model as simple as possible, the intermediate sector comprises just one monopolistic firm. It will be shown in

[7] This terminology was taken from Jovanovic (1997).
[8] See Cohen and Levinthal (1989). Griffith et al. (2003) built a model that incorporates 'absorptive capacity' into a model of economic growth.
[9] This will be discussed in the third part of the dissertation.
[10] See for example Klenow (1998).

the second part of the thesis that this is not a necessary assumption for the qualitative results. Chapter four determines the equilibria and the economy's dynamics for both specifications of the research process. The succeeding chapter derives the main results with respect to the economy's codification behavior. It reveals the conditions under which the overlapping generations economy exhibits endogenous knowledge codification and answers when it will do so in the long run. A discussion of the results of the basic model concludes the first part of the dissertation.

The second part elaborates on the robustness of the results obtained from the basic model. As Solow-neutral technological progress has been assumed in the first part, chapter eight asks what other assumptions on technological change would be possible for the results to still hold. Chapter nine identifies the general properties of the model that drive the main results. It is shown in chapter ten that these properties also allow for an interpretation with oligopolistic intermediate sectors, each of which is founded on a separate field of knowledge.

The purpose of chapter eleven in the third part of the dissertation is to elaborate on the differences and similarities of the proposed model structure with standard endogenous growth theory. Romer's model of endogenous technological change has been chosen for a comparison. Chapter twelve features a mostly verbal discussion on aspects of social optimality of the previously studied market allocations. Two perspectives are taken up. First, the relation of the sequential markets equilibrium outcome to a social planner's welfare optimum is discussed, and different potential inefficiencies are identified. The second perspective is characterized by the question as to whether it would be possible to engineer Pareto-improvements to the sequential markets solution.

Final conclusions complete the dissertation.

Part I

A Basic Endogenous Growth Model With Imperfect Knowledge Transfer

Chapter 2

Knowledge and Knowledge Codification

2.1 General Thoughts

Most models of endogenous growth are built around the distinction between technology or abstract knowledge and human capital. The first is non-rival and spills over costlessly whereas the latter is rival and consists of the acquired abilities, skills, and knowledge of individual workers (see for example Romer (2001, p. 133)). Recently a growing number of publications emphasizes a certain degree of complementarity between human capital and technological ideas, such as in the technology adoption literature (Goldin and Katz, 1998; Benhabib and Spiegel, 2005) and the literature on skill biased technological change (Autor et al., 2003).

Particularly, empirical observations such as in Mansfield et al. (1981), Levin (1986) and Zucker et al. (1998), and even more so the case studies of Collins (1974) on the diffusion of the TEA-laser technology and MacKenzie and Spinardi (1995)'s investigation of the ability to produce nuclear weapons suggest a strong complementarity of technological information and personal skills. This has sparked discussions on the economics of R&D and technology transfers which assign "special significance to the tacit elements in technological knowledge, calling attention to the fact that the information contained in patents, blueprints and other codified forms of knowledge often are insufficient for the successful implementation of the technical innovations they purport to describe; much complementary 'know-how' may be required, the acquisition of which, typically, is a costly business" (Dasgupta and David, 1994). This strand of the literature particularly focuses on the tacitness and codifiability of knowledge. 'Tacit knowledge' is usually referred to forms of knowledge that cannot be articulated.[1] Recently, the term 'tacit knowledge' has come to be more widely applied to forms of personal knowledge that remain 'UN-codified' (Cowan et al., 2000). The term 'codified knowledge' is used for knowledge that has been converted into symbols for easy transmission, replication, and storage. With regard to the relationship between codified and tacit knowledge, there are two conflicting positions. The one shares the view that all knowledge can be codified and the fact that there exists tacit knowledge is due to high codification costs for this kind of knowledge. In this way, tacit and codified knowledge are substitutes. The other

[1] The term 'tacit knowledge' goes back to Polanyi (1961), who conceptualized it as the fact that we all are often generally aware of certain objects without being focused on them. They form the context which makes focused perception possible, understandable and productive.

position argues that all forms of codified knowledge require tacit knowledge to be useful. Hence, the two are viewed as essentially complementary. Drawing on this literature, the following paragraphs illustrate the perspective taken here and give definitions that will serve as an operational basis for the subsequent analysis. At this point it is necessary to remark that this does not claim to be a comprehensive treatment of the topic, but tries to capture the central aspects.

Let's first assume that all persons are learning to read and write one common language when young as we are not interested in effects of illiteracy. Further, we assume that there exists an infinite set of disembodied ideas. By the process of thinking or invention, a person P can discover an idea i and henceforth know this idea. Then we say that this idea is part of person P's knowledge, respectively, it is embodied in person P. It would now be possible for P to codify certain parts of idea i. The detailed definitions are introduced and illustrated by the following example.[2]

Consider the idea "solving algebraic equations", in the following also referred to as idea i. A pupil in school does not know this idea and, written on the chalk board by the teacher, sees algebraic equations being solved. Suppose on the board are only mathematical expressions and no further explanation. In this way, the pupil will probably have a hard time to figure out the idea. we will refer to the writings on the chalk board as the **codified component of idea** i. However, there is more that could be articulated. The teacher would be able to explain that the symbol 'x' usually means the same whether it is written in ball point, chalk or print, irrespective of the day of the week, or the temperature of the air. But in another sense the variable x may stand for anything – weight in kilogram, number of people ... – and in general may only mean the same on coincidental and unimportant occasions. Again, sometimes a capital X or an italicized x may have a distinctive meaning. x in the equation $x = 5y$ is the same as x in the equation $5y = x$, but is not the same as x in $x = 5z$, unless $y = z$. But on the other hand, 'x' is being used in the same way in all the equations. All of this could in principle be codified. Hence, the **codifiable component of the idea** comprises more than just the codified part on the chalkboard and shall be defined by that part of an idea that can be articulated or made explicit.[3] With the explanation of the teacher, it is much easier for the scholar to understand the idea. However, the claim is that there is still a tacit part of the idea left that cannot be articulated.[4] That is, although explained comprehensively by the teacher, the young have to exercise mathematics in order to understand it. They oftentimes do not comprehend an idea explained by the teacher immediately but have a "light bulb moment" after attending to it for a while. This last piece of understanding will be referred to as the **tacit component in the narrow sense**, whereas the part that has not been codified is just called the **tacit component of idea** i. When coming across the chalkboard for the first time, the pupil does not understand the idea writ-

[2]The example is in part taken from Collins (1974).
[3]This is obviously a function of the code available.
[4]For example Collins (1974) argues that "all types of knowledge, however pure, consist, in part, of tacit rules which may be impossible to formulate in principle" (Collins, 1974, p. 167).

ten on it, she misses the tacit component, which, in this case comprises the codifiable explanation of the teacher and the light bulb part, that is, the tacit component in the narrow sense. After having understood the idea, for instance, the day after the teacher explained it and after having thought about it on her own, she would be able to attach meaning to the chalkboard writings. Then the idea "solving algebraic equations" is part of the pupil's knowledge. According to this, the codified and the tacit component of an idea are complements. So are the codifiable component and the tacit component in the narrow sense. Knowledge codification is then defined as transforming the tacit but codifiable part of knowledge into information. In this way, the codified component can be interpreted as a substitute to the respective tacit but codifiable part of the idea. Of course besides depending on the code available, the degree to which an idea can be codified depends on its very nature. For example it is often claimed that highly abstract ideas possess larger codifiable components than rather "craft-like" ideas such as "how to ride a bicycle". However, when teaching a child how to ride a bicycle, there are still things to articulate that help in acquiring that skill. If one would like to codify this idea, one could also use videotaping for example. Cowan et al. (2000) argue that very little knowledge is inherently tacit and impossible to codify. Similarly, the assertion here is that ideas of economic relevance do possess codifiable components.

The term knowledge as used in the present work, is defined via the ability to understand an idea. In this way, it is congruent with what is often referred to as intellectual human capital. In the following, we will synonymously use the notion 'human capital'[5] as this thesis is not concerned with other aspects sometimes attributed to human capital such as health. The difference between knowledge or human capital and information is whether the carrier is able to understand and use the respective idea. Hence, in the previous example, the idea "solving algebraic equations" will only be part of the pupil's knowledge, if she is able to solve any algebraic equation, respectively use this concept in thinking about related problems. This is different from learning an algorithm by heart without being able to attach meaning to it. According to the previous definition, an idea only learned by heart would not be part of this person's knowledge because this person misses the tacit counterpart.

With the previous considerations, the seemingly conflicting contentions of standard economic theory and the empirical observations of Levin (1986) and Mansfield et al. (1981) could be resolved as follows. The first time an idea becomes economically relevant is when discovered by an inventor or a research group. Hence, it is initially embodied in its discoverers. According to Zucker et al. (1998), this creates (rival) intellectual human capital. Suppose now, a new product, representing a substantive technological advance, has been invented and is sold at the market. A (potential) competitor can buy the product and reverse engineer it, but, when lacking the respective intellectual human capital,

[5] According to Becker (1975), all activities that influence future monetary and psychic income by increasing the resources in people are called investments in human capital. These investments like schooling, on-the-job training, medical care, migration [...] improve skills, knowledge, or health and thereby raise money or psychic incomes.

may have to expend the same effort to imitate the product as the innovator did to invent it. This is suggested by the observations of Mansfield et al. (1981). In analogy to the previous example, the physical product represents the mathematical expressions on the chalk board. Without any explanation by the teacher, the pupil would have to re-invent the idea. That is, a large part of the technological idea embodied in the new product stays tacit to the competitor. If the competitor does not possess this tacit counterpart, it has to invent the idea itself. This view is supported by empirical evidence concerning endogenous spillovers. For example, Levin (1986) observed that "on average, independent R&D was rated as the most effective means of learning about rival technology." And well known is Cohen and Levinthal (1989)'s argument that if "immediate costs of assimilating technological knowledge" are small, "it is by the virtue of the considerable R&D already conducted by the firms in the vicinity of the 'emission': the firm has already invested in the development of its absorptive capacity in the relevant field" (Cohen and Levinthal, 1989, p. 570). In this way, firms with own R&D would already possess large portions of the tacit counterpart of the new product and are possibly missing only few links. When interpreting the R&D conducting competitor as the pupil of the above example, it would already be acquainted with most of the teacher's explanation. Respectively, coming across the chalk board (the product) it would be much easier to figure out the idea embodied in it. This line of argument suggests that in an industry, where the firms' researchers possess similar levels of intellectual human capital and are researching in the same field, spillover effects would be very high as assumed in large parts of economic literature, for example, in the patent race literature. Hence, patent protection is necessary for rent appropriation. However, spillover effects only through the physical product or patents (the chalkboard writings) are negligible if the innovator has a technological edge, as observed by Mansfield et al. (1981). A similar argument has been put forth by David (1998) for scientific research, claiming that "many 'craft' aspects of scientific practice must be learned in modes of instruction akin to an 'apprenticeship' by being afforded opportunities for first-hand observation of how they are done, leading to trials under the guidance and supervision of experts. Otherwise, something like the original process of acquiring mastery of such knowledge has to be repeated ab initio, guided and encouraged only by the belief that others have found this to be possible" (David, 1998, pp. 125f). With regard to knowledge transfer between generations, it is very plausible, that old employees possess such a 'technological edge' in relation to the new employees. And, referring to the examples of the introduction, within knowledge management, it is the firms' concern to transfer the respective tacit counterparts from one generation to the next in order for the latter not to be forced to re-invent large parts of their predecessors' work.

In this way, the present thesis rests on the assertion that it is an economy's human capital that determines its technological or productive capability. That is, information, such as a book, a machine, or a computer, cannot be productive in the long run without the respective human capital. By this, it is associated to Lucas (1988) who argues that when we talk "about differences in 'technology' across countries we are not talking about knowledge in general, but about the knowledge of particular people, or perhaps particular subcultures of people". Accordingly, the major assumption is that (a) a large amount of an economy's productive knowledge rests in the heads of the working population and is not codified and (b) the information created as a by-product of research and development such as the innovative product itself and patent specifications are not sufficient for the intergenerational knowledge transfer of advanced technological ideas to the younger generation. With respect to (b), the by-product information represents only a small part of the ideas' codifiable component and could, in analogy to the previous example, be interpreted as the chalkboard writings. In summary:

Assumption 2.1 *Suppose an economy's information stock consists only of information created as a direct by-product of research activities, then large parts of an economy's productive knowledge are tacit but codifiable.*

Knowledge codification is defined as the transformation of the codifiable part of knowledge into information such that the idea is easily transferable and accessible. This can take many forms such as using a natural language to write down an idea in a book, but also videotaping or drawing pictures and plans. For the subsequent economic analysis, we will assume that independent of the media used, purposeful knowledge codification codifies the entire codifiable part of the respective idea. Further it is useful to include the dimension of accessibility in a definition of knowledge codification.

Definition 2.1 *An idea is referred to as* ***codified with respect to a group of persons****, if the entire codifiable part of the idea has been codified and the resulting information is accessible to every person in the group independently.*

By this definition, knowledge codification is the link between rival human capital and idealized non-rival information. That is, the resulting information possesses the characteristics of a local public good. If the codifiable part has not been codified entirely, the idea will be referred to as partially codified with respect to a group of persons. This thesis focuses on the knowledge transfer of (substantially) new ideas between generations. It seems then reasonable to argue that there exists a technological edge between the old generation and the young and the by-product information, if existent, plays a negligible role for knowledge transmission. Hence, we will refer to an idea as codified with respect to a group of persons if the above definition applies and as not codified else. In this way, the terms 'information' and 'codified idea' will be used synonymously in the following analysis. It is further assumed that information is long lasting, that is, it does not depreciate, and it is not "consumable".[6]

As a summary, the main assumptions that will serve as a basis for the subsequent analysis are:

[6]Although a book or a hard disk may also wear out, the depreciation rate would probably be very low. To a certain extent, it would be possible to argue that books and computer hardware have positive consumption value. For a first formal analysis of knowledge codification, it is useful to preclude this point for reasons of simplicity.

- ▷ There exists an infinite set of disembodied ideas.
- ▷ The first time an idea becomes economically relevant is when discovered by an inventor. Consequently, it is initially human capital.
- ▷ Each piece of knowledge, that is, each idea embodied in a person, possesses a codifiable part which can be transformed into information.
- ▷ Knowledge codification is a costly activity.
- ▷ Information is long lasting, however, by itself not productive in the long run. It needs the respective human capital to be productively employed.[7]

2.2 A Formal Approach

This section gives a formalization of the notions ‘knowledge’ and ‘information’, which will serve as a basis for the following analysis. Let there be an infinite set of ideas and assign each idea an index $i \in \mathcal{I}$. Consider a person at time t, denoted by P_t. This person’s human capital or knowledge is defined by all ideas that are embodied in her and, hence, can be written as $\mathcal{T}_{P_t} = \{i \in \mathcal{I} \mid \text{i is embodied in } P_t\}$. This is the subset of $\mathcal{I}$ of which the person acquired the respective tacit components over her educative or professional career. The stock of information at time t is the index set of the codified ideas $\mathcal{C}_t = \{i \in \mathcal{I} \mid \text{i is codified}\}$. For illustrative purposes, imagine a bookshelf with each book containing exactly one idea. The book’s title is i indicating the idea it contains. Knowing a person’s human capital, one could immediately tell which books this person can understand, that is $\{i \in \mathcal{T}_{P_t} \cap \mathcal{C}_t\}$, and which ones she would encounter difficulties with $(\mathcal{I} \setminus (\mathcal{T}_{P_t} \cap \mathcal{C}_t))$.

To operationalize this concept of knowledge, it is necessary to specify $\mathcal{I}$. Let’s represent the knowledge stock as an interval in $\mathbb{R}$ for better analytical tractability. In particular, $\mathcal{I} = \mathbb{R}_+$. Further, we order the ideas according to their difficulty. More precisely, if $i < j$ then idea i is easier to comprehend or more basic than idea j. For example, addition is less difficult than solving differential equations. Recalling the bookshelf example, this would mean that the books are ordered, e.g. from left to right, starting with the ones containing the rather basic ideas and becoming successively more difficult. They are indexed continuously beginning at zero. We define a person’s measure of human capital by the index of the most difficult idea she is able to understand. More precisely, $\tau_{P_t} = \sup \mathcal{T}_{P_t}$. The central assumption of the ordering concept is:

[7] It needs human capital to understand and employ in productive processes the information contained in, say, an academic paper.

Assumption (A1) *If i is embodied in person P_t, then $j < i$ is also embodied in person P_t.*

As we defined $\mathcal{I} = \mathbb{R}_+$, τ_{P_t} would be the Lebesgue-measure of $\mathcal{T}_{P_t}$. The definitions imply that an individual with human capital of 5 possesses the set of tacit components $[0, 5]$ and, thus, comprehends the contents of all information indexed by $i \in [0, 5]$, respectively is able to use ideas $i \in [0, 5]$ in production.[8]

Let $C_t := \sup \mathcal{C}_t$ represent the economy's stock of information. The ordering concept may involve:

Assumption (A2) *Before an idea indexed by i can be codified, all ideas with index j, $j < i$ must have been codified. This implies that if i is codified in period t, then $j < i$ is also codified in t.*

With assumption (A2), C_t is identical to the Lebesgue-measure of $\mathcal{C}_t$. The following figure illustrates the ordering concept:

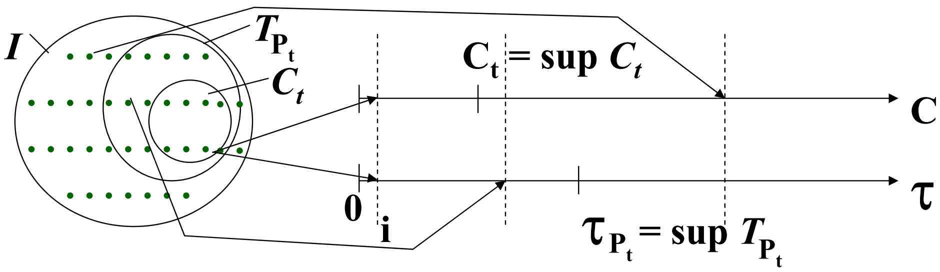

Figure 2.1: Operationalization of the Notions 'Knowledge' and 'Information'

The outer circle depicts the set of ideas $\mathcal{I}$. Human capital of person P_t, $\mathcal{T}_{P_t}$, and information, C_t, are subsets thereof. The arrows indicate possible indices of the respective ideas. For example, idea i is human capital, and it is also codified. In this way, it possesses an index lower than $C_t = \sup \mathcal{C}_t$ and also lower than $\tau_{P_t} = \sup \mathcal{T}_{P_t}$.

[8] Note that other ordering concepts, for example a chronological order, would work as well as long as (A1) holds.

Chapter 3

The Model Specification

Consider an overlapping generations economy similar to the well-known Diamond (1965) model, in which agents live for two periods. Time is infinite in the forward direction and divided into discrete periods indexed by t. There is a continuum of individuals P_t on $[0, 1] =: \mathcal{P}_t$ in each generation. There is no population growth and the size of each generation is normalized to 1. Each individual inelastically supplies one unit of labor when young and consumes its capital savings plus the capital rent when old. Hence, total labor supply in each period t is given by $L_t = 1$. The economy features two sectors. An intermediate goods sector that creates intermediate goods from physical capital and knowledge and a production sector that uses the intermediate product and labor to generate a homogenous physical good that can be used for consumption and investment. The difference to the usual models of endogenous growth is that within the intermediate sector, the individuals can influence the return on capital by investing in knowledge codification.

3.1 The Production Sector

Final-goods production is characterized by a continuum (on [0,1]) of identical firms which produce the homogenous good with the use of labor $L_{A,t}$ and the intermediate good x_t as inputs. Since final-goods firms earn zero profits and own no assets, they can be ignored in the specification of endowments.[1] The firms maximize profits and act competitively in the product and factor markets. For better tractability, the most convenient way to model final production is by means of a representative firm whose production and factor demands represent aggregate values. The aggregate production function is of the constant-returns-to-scale type:[2]

$$F(x_t, L_{A,t}) = x_t^{\alpha} L_{A,t}^{1-\alpha},$$

where $\alpha \in (0, 1)$. Consequently, the representative firm solves the following maximization problem:

$$\max_{x_t, L_{A,t}} \pi_t^{fp} = F(x_t, L_{A,t}) - p_{x,t} x_t - w_{A,t} L_{A,t}.$$

[1] For illustrative purposes one could assume that the final-goods firms are managed and owned by the members of the young generation.

[2] According to Sargent (1979), the preceding assumptions guarantee the existence of such a representative final-product firm.

As it is a constant-returns-to-scale firm, its input demands are defined only after the scale of operation is pinned down. However, the demand for labor and intermediate goods is characterized by the first order conditions. Labor and the intermediate goods are being compensated by their marginal product:

$$\begin{aligned} x_t^d &= \left(\frac{p_{x,t}}{\alpha}\right)^{\frac{1}{\alpha-1}} L_{A,t}, \\ L_{A,t}^d &= \left(\frac{1-\alpha}{w_{A,t}}\right)^{\frac{1}{\alpha}} x_t. \end{aligned}$$

3.2 The Intermediate Goods Sector

In general, the intermediate sector is assumed to consist of long-lived intermediate firms whose ownership is handed down from one generation to the next. More precisely, the capital stock is sold publicly at the end of a period t to the next period's old generation. This process could be interpreted as secondary public offering. Further, the young generation of t may additionally increase the capital stock of $t+1$ by saving more than the amount of capital left from the previous generation. Physical capital left at the disposal of the intermediate firm is interpreted as ownership claims. As mentioned in the introduction, we assume that the firm owners collectively decide on the values of the control variables of the intermediate firm's profit maximization problem. For simplicity, in the basic model the intermediate goods sector is characterized by a single intermediate firm.

3.2.1 The Decision Process

The economic problem with respect to the transfer of knowledge by knowledge codification between two succeeding generations of employees can be translated into the overlapping generations framework by the following three-stage game:

Stage 1: At the end of period t, the new capital owners may invest in knowledge codification before the employees of the intermediate firm, the researchers, retire.[3]

Stage 2: At the beginning of $t+1$, the newly hired researchers are asked to compensate the firm owners for their codification investment. (Under the assumption that the knowledge database or library is excludable[4].)

[3]Note that within the overlapping generations structure, the next period's researchers are not born, yet.

[4]With respect to excludability, two specifications would be possible. On the one hand, a codified idea could be a public good with respect to the new researchers as soon as the the firm owners give access to it. On the other hand each employee could be given exclusive access to certain ideas dependent on her compensatory payment. These specifications have not been explicitly distinguished as it does not change the outcome of the subgame perfect Nash equilibrium.

Stage 3: The capital owners decide on giving access to the firm's information stock.

We assume that allowing access to the knowledge database does not incur costs. It is clear that no newly hired researcher would make a compensatory payment independent of the decisions of her colleagues, as she knows that the capital owners will provide the information for free. By backward induction, the firm owners, knowing that they bear the full codification costs, will only invest in knowledge codification up to the profit-maximizing amount.

One may ask whether the problem within the firm is well depicted by this game structure. For example, why do the firm owners give away the information for free, although they know that the new employees might benefit from it by higher wages? The argument is that as long as the firm owners can only ask for a compensatory payment, the new employees will always deny. Suppose the game were extended to more than three stages, in which after the third stage the new employees are asked for a compensatory payment and the firm owners decide on giving access to the information stock in an alternating sequence. With finitely many stages, there must be a last stage before the production of the intermediate goods has to start. At the last but one stage, the new employees know that if they again deny a compensatory payment, the firm owners would give access to the information for free at the last stage, as otherwise they would forgo a certain amount of profits.

Further, a threat by the firm owners to destroy the information would be empty because they would forgo additional profits and the information stock is not consumable by assumption. The assumption of a non-consumable information stock is not critical as long as the utility derived from consuming it is lower than that from giving access to the information to the new employees. In this case, the new employees again know that the firm owners will not consume the information even if they make no compensatory payment.

In this way, profit maximization of the firm owners proceeds in two stages:

Stage 1: At the end of period t, the firm owners collectively decide on how much to invest in knowledge codification before the researchers retire.

Stage 2: At the beginning of $t+1$, given the amount of ideas that have been codified, the capital owners decide on the amount of intermediate goods to produce and on how many new researchers to hire.

Of course by backward induction, the second-stage maximization problem of $t+1$ is solved given a certain stock of information and then the optimal codification decision is taken. The collective decision process with respect to knowledge codification takes the following form:

- ▷ Every individual capital owner can propose an amount $\zeta_{t,prop}$ to spend on knowledge codification. The costs are split up among the individuals according to their capital shares.
- ▷ Each shareholder votes for or against the proposal and the amount is approved according to the unanimity rule.
- ▷ If the proposal has been accepted, the amount is collected and the retiring researchers are paid to codify their ideas.

For simplicity we assume that this decision process does not incur transaction costs. It would also be possible to assume that a manager acts on behalf of the capital owners.

It is clear that given the capital and information stock in period $t+1$, the second-stage maximization problem is a static profit optimization within the period. However, the codification decision at the first stage cannot be made independent of the households' preferences concerning the distribution of consumption between periods. Hence, the second-stage problem will be solved at the end of this section and the decision on knowledge codification will be elaborated further within the households' utility maximization problem.

3.2.2 Research and Intermediate Goods Production

The intermediate good can be produced according to the following production function: $x_t = G(K_t, \tau_t) = K_t \tau_t$.[5] K_t denotes the measure of the capital stock at time t. The knowledge stock of the intermediate firm which corresponds to the knowledge stock of the economy in period t, τ_t, is defined by the index of the most difficult idea in the union of all sets of tacit components $\tau_t = \sup \cup_{P_t \in \mathcal{P}_t} \mathcal{T}_{P_t}$. It equals the highest measure of human capital in the set $\mathcal{P}_t$ at time t.

Human capital of a person $P_t \in \mathcal{P}_t$ may originate from three sources:[6]

$$\tau_{P_t} = q\tau_{t-1} + \beta \max\{0, C_t - q\tau_{t-1}\} + \varepsilon (q\tau_{t-1})^{\Phi}.$$

First, every individual is exogenously transferred a share q of the economy's knowledge stock of the previous period, e.g. in school or by other educative means.[7] This transfer

[5]Note that this specification implies that a technological improvement increases production of the final product in the same way as an increase in the stock of capital, which is usually referred to as Solow-neutral technological progress. The innovation process could be regarded as both, process or product innovation. The interpretation of the first would be that the higher knowledge stock allows for the production of more homogenous intermediate goods with a certain capital input. On the other hand, one could argue that the capital stock is transformed into higher quality intermediate products, which possess a productivity in final-goods production identical to x_t units of some standard intermediate good.

[6]The dynamics of the knowledge stock have also been inspired by a working paper on technological regress by Aiyar and Dalgaard (2002).

[7]It could also be interpreted as imperfect human capital transfer within the intermediate firm.

is imperfect, $q \in (0,1)$.

As a second source, the members of the new generation who are becoming researchers in the intermediate firm can enhance their human capital by attending to codified ideas if previous generations of researchers have codified their knowledge. Recall the bookshelf example: If the highest index of the books in the shelf C_t is greater than the highest index of the ideas the young generation of t has already learned, $q\tau_{t-1}$, they are interested in reading the books indexed by $(q\tau_{t-1}, C_t]$. In each period the new generation is able to build up additional human capital of a fraction $\beta \in (0,1)$ of those codified ideas.[8] As (A2) is a strong requirement, most part of the analysis will consider situations without (A2). Then, the following assumption with regard to knowledge codification holds instead.

Assumption (A2') *In a period t and for all $i, j \geq q\tau_t$: Before an idea indexed by i is codified, all ideas with index j, $j < i$ have been codified already. This implies that if i is codified in period t, then $j < i$ is also codified in t.*

Finally, a person P_t deciding to do research is able to generate a number $\varepsilon(q\tau_{t-1})^{\Phi}$ of new ideas. The term "new ideas" is supposed to reflect the subjective perspective of person P_t. That is, the ideas generated are new to P_t, but not necessarily to all other persons. As mentioned in the introduction, we intend to examine the cases of $\Phi = 0$, which means that every researcher generates a constant number $\varepsilon \in \mathbb{R}_+$ of new ideas, and of $\Phi = 1$, which reflects the assumption often used in endogenous growth theory that researchers are becoming more productive with an increasing stock of knowledge to draw upon. In the latter case the researchers' productivity increases with the knowledge they received from the previous generation.

The process of knowledge acquisition shows a sequential nature, proceeding from schooling via reading to own research. For analytical simplicity, the first two kinds of knowledge transmission happen at no time at the beginning of each period. Research and development, however, takes time, such that a person has to decide as to whether she is going to work in final-goods production or to do research. Let $L_{R,t}$ denote the number of persons who are hired to do research in period t and let them be arranged on the continuum from 0 to $L_{R,t}$. That is, all persons $P_t \in [0, L_{R,t}]$ decided to become researchers. Since they have been transferred the same amount of human capital from the old generation and generate the same number of new ideas each, they all possess the same level of knowledge. More precisely, $\tau_{P_t^1} = \tau_{P_t^2}$, for all $P_t^1, P_t^2 \in [0, L_{R,t}]$. Analogously, the symmetry of the knowledge level applies to the workers. Until now we have implicitly assumed that each researcher is doing research on her own, which means that they all discover the same ideas. However, when working together in a research group they can exchange their ideas and as a consequence reach a higher level of knowledge altogether. Hence, we assume spillover-effects occur within research groups. A research group is defined as

[8] β reflects how easy ideas can be acquired from reading.

$\mathcal{G}_t \subseteq [0, L_{R,t}]$. Let $L_{\mathcal{G}_t}$ be the Lebesgue-measure of $\mathcal{G}_t$, representing the number of researchers in research group $\mathcal{G}_t$.[9] We further assume that every researcher can participate in one research group only. Since every researcher brings in the same amount of ideas, we define the knowledge level of each person $P_t \in \mathcal{G}_t$ by

$$\tau_{P_t} = q\tau_{t-1} + \beta \max\{0, C_t - q\tau_{t-1}\} + \varepsilon q\tau_{t-1}L_{\mathcal{G}_t}.$$

Consequently, researchers are symmetric with respect to knowledge if they are in the same research group or belong to different research groups of the same size. In detail, if $P_t^1 \in \mathcal{G}_t^1, P_t^2 \in \mathcal{G}_t^2$, then

$$\tau_{P_t^1} \lesseqqgtr \tau_{P_t^2} \Leftrightarrow L_{\mathcal{G}_t^1} \lesseqqgtr L_{\mathcal{G}_t^2}.$$

It immediately follows that

$$\tau_t = \tau_{P_t} \quad \Leftrightarrow \quad (P_t \in \mathcal{G}_t^* \quad \wedge \quad L_{\mathcal{G}_t^*} \geq L_{\mathcal{G}_t}, \forall \mathcal{G}_t).$$

Research groups may be employed by the intermediate firm. As we preclude spillovers between research groups, it follows from the preceding considerations that for the same research expenditures, one would achieve a higher level of knowledge by creating one big research group, instead of many small ones. In this way, the intermediate firm can be interpreted as a single research joint venture owned by the old generation. Having only one research group of size $L_{R,t}$ implies that $\tau_{P_t} = \tau_{R,t}$, $P_t \in [0, L_{R,t}]$, where

$$\tau_{R,t} = q\tau_{t-1} + \beta \max\{0, C_t - q\tau_{t-1}\} + \varepsilon q\tau_{t-1}L_{R,t}.$$

Further there is the symmetry of the workers' knowledge: $\tau_{P_t} = \tau_{A,t}$, $\forall P_t \in \mathcal{P}_t \setminus [0, L_{R,t}]$, where

$$\tau_{A,t} = q\tau_{t-1}.$$

Consequently, the economy's knowledge stock is determined by that of the researchers: $\tau_t = \tau_{R,t}$. It is the maximum level of knowledge that can be productively employed at time t. The dynamics of the economy's knowledge stock can then be written as follows[10]

$$\tau_t = \begin{cases} q\tau_{t-1} + \beta(C_t - q\tau_{t-1}) + \varepsilon q\tau_{t-1}L_{R,t}, & C_t > q\tau_{t-1}; \\ q\tau_{t-1} + \varepsilon q\tau_{t-1}L_{R,t} \quad , & C_t \leq q\tau_{t-1}. \end{cases}$$

[9] Of course, this definition involves a measure theoretic problem. Since single points on the interval $[0, L_{R,t}]$ possess measure 0, there exist research groups $\mathcal{G}_t$ consisting of more than one person with measure 0. We preclude such groups by assumption.

[10] The assumption of only one research group in the economy is not necessary to obtain this difference equation. However, all research groups of the economy must be of equal size.

3.2.3 Second-Stage Profit Maximization

Within period t, the intermediate firm maximizes profits by deciding on the intermediate goods supply and the number of researchers to employ. Hence, it faces the following problem:

$$\max_{x_t, L_{R,t}} \pi_t^{int} = p_{x,t}(x_t)x_t - w_{R,t}L_{R,t}.$$

Taking the information stock and physical capital as given, the problem is one-dimensional in $L_{R,t}$. Using the demand for intermediate products of the final-goods firms, the necessary condition of the intermediate entrepreneurs' optimization problem writes

$$w_{R,t} = \alpha^2 x_t^{\alpha-1} \frac{\partial x_t}{\partial L_{R,t}} L_{A,t}^{1-\alpha}.$$

In this way, the supply of intermediate goods, x_t^s, and the factor demand for researchers, $L_{R,t}^d$, are given by[11]

$$\begin{aligned} x_t^s &= \left(\frac{\alpha^2 \frac{\partial x_t}{\partial L_{R,t}}}{w_{R,t}} \right)^{\frac{1}{1-\alpha}} L_{A,t}, \\ L_{R,t}^d &= \left(\frac{\alpha^2}{w_{R,t}} \right)^{\frac{1}{1-\alpha}} \left(K_t \frac{\partial \tau_t}{\partial L_{R,t}} \right)^{\frac{\alpha}{1-\alpha}} L_{A,t} - \frac{1}{\frac{\partial \tau_t}{\partial L_{R,t}}} (q\tau_{t-1} + \beta \max\{0, C_t - q\tau_{t-1}\}). \end{aligned}$$

Consequently, the profits of the intermediate firm accrue to

$$\pi_t^{int} = \alpha K_t^{\alpha} L_{A,t}^{1-\alpha} \tau_t^{\alpha} (1 - \alpha \tau_t^{-1} \frac{\partial \tau_t}{\partial L_{R,t}} L_{R,t}) =: K_t r_t.$$

For reasons of clarity in exposition, $L_{R,t}^d$ was not inserted into the profit function. The profits are allocated among the shareholders of the intermediate firm proportional to their shares. Hence, the return on capital investment is

$$r_t = \alpha K_t^{\alpha-1} L_{A,t}^{1-\alpha} \tau_t^{\alpha} (1 - \alpha \tau_t^{-1} \frac{\partial \tau_t}{\partial L_{R,t}} L_{R,t}). \tag{3.1}$$

3.3 The Costs of Knowledge Codification

Knowledge codification takes time and physical resources as carriers of information such as paper, CD-ROMs or hard disks. There is also a need for reproduction and storage devices. Additionally knowledge databases have to be maintained and administrated. Costs could even include the creation of new codes, e.g. if the existing one does not

[11] The sufficient condition for a maximum is satisfied due to the strict concavity of the objective function in $L_{R,t}$: $\frac{\partial^2 \pi_t}{\partial L_{R,t}^2} = \alpha^2(\alpha-1)\tau_t^{\alpha-2} \left(\frac{\partial \tau_t}{\partial L_{R,t}} \right)^2 K^{\alpha} L_{A,t}^{1-\alpha} < 0$.

suffice for expression. Although the notion 'knowledge codification' may also comprise the creation of all kinds of artifacts, for illustrative purposes, it will be referred to as the creation of information via some natural language.[12] For simplicity, we assume that every idea can be codified at the same cost of γ_t units of the homogenous good within period t. The codification costs per idea γ_t may change over time, hence the index t in the cost function. However, the marginal costs are supposed to be constant within each period. In this way, the cost function shows a linear form within periods:

$$\Gamma_t(\triangle C_t) = \gamma_t \triangle C_t.$$

Additionally fixed entry costs f may accrue once in the first period of knowledge codification for setting up computer systems, establishing management structures or building archives or libraries.[13]

Two implicit assumptions with respect to the cost function are that, first, the costs of knowledge codification per idea do not depend on the number of new employees to access the information in the next period. According to the definition of information, it may well be that an increasing number of researchers may necessitate further costly copying and distribution activities. However, it is assumed that the cost share of these activities due to variations in the number of new researchers between periods is negligible.

Second, if the new capital owners decided on a positive codification level, those to actually codify the ideas are the researchers of that period as they possess the highest level of knowledge. However, would they codify at the marginal costs γ_t? The reasoning is in the Bertrand-fashion: As long as there is more than one researcher, they will agree to codify at the marginal costs because they are symmetric with respect to knowledge. The situation with only one researcher is mathematically precluded by the representation of the population on the interval $[0, 1]$, because positive research implies a research group of measure greater than zero. For a more general interpretation, it is assumed that when having only one researcher, the firm owners offer a forcing contract that contains a compensation for knowledge codification slightly above the marginal costs. The researcher will accept the contract. This is the case as the firm owners know that the researcher will be better off if she codified at slightly above the marginal costs and hence has no incentive to revise the conditions of the contract, even if the researcher would neglect the contract at first.[14]

3.4 The Problem of the Household

Each individual lives for two periods and maximizes the discounted sum of utilities. There is a constant discount factor $\delta > 0$.[15] The individuals inelastically supply one

[12]For illustrative purposes, one could think of writing a book for the previously described bookshelf.

[13]It will explicitly be mentioned in the analysis when entry costs are considered.

[14]It depends on the number of the stages of the game how often the researcher may neglect the contract. However, she will accept it at the last stage.

[15]This implies a constant rate of time preference $\rho > -1$.

unit of labor when young and may choose as to whether they want to work in final-goods production or in research and development. The budget constraint when young is given by the wage w_t that can be split into consumption today $c_{1,t}$, saving $w_t s_t$ in physical capital and investment in knowledge codification $w_t \varsigma_t$. The physical capital savings plus the real return on capital r_{t+1} equal consumption when old $c_{2,t+1}$[16]. The household privately saves in physical capital which is interpreted as ownership claims on the intermediate firm and then takes part in a collective decision within the firm on knowledge codification. The problem of the household can be depicted by the following three-stage process.[17]

Stage 1: The household decides on how much it would like to save in physical capital and how much to propose for investment in knowledge codification.

Stage 2: The collective decision with respect to knowledge codification within the intermediate firm is taken.

Stage 3: The household may adjust capital savings given the investment in knowledge codification.

It is assumed that the households cannot commit to or be constrained to a certain amount of physical capital saving. In particular, at the third stage, the individual can revise her decision after the amount of knowledge codification has been set. In this way, each young individual makes a decentral decision over her capital savings.[18] As the model is entirely deterministic, the individuals know that they are all symmetric with respect to wage and preferences. Accordingly, at the first stage, she decides whether she would like to make a proposal on the codification investment by solving

$$\max_{s_t, \varsigma_t} U_t(c_{1,t}, c_{2,t+1}) = u(c_{1,t}) + \delta u(c_{2,t+1})$$

subject to

$$c_{1,t} = w_t(1 - s_t - \varsigma_t),$$
$$c_{2,t+1} = (1 + r_{t+1}) s_t w_t.$$

[16]The index indicates whether the person is young (1) or old (2) in period t.

[17]In principle, there are two ways to solve the households' problem. One is a two stage decision process in which, at the first stage, each household decides on how much consumption in total to transfer to the next period, and then at the second stage participate in a collective decision on how much of the consumption forgone to invest in knowledge codification rather than in physical capital. The second way as chosen here is to omit the first stage and directly determine the amount of consumption forgone for knowledge codification and for physical capital.

[18]This is plausible because although by coordination the households could choose the monopoly capital stock, each individual has an incentive to deviate by increasing her saving rate. The coordination in the codification case is possible as by assumption they have to pay the codification costs immediately and hence cannot deviate to lower codification. Of course, every household could privately pay for additional knowledge codification, but none has an incentive to do so.

For notational convenience, let $S_t := s_t w_t$ and $\zeta_t := \varsigma_t w_t$, where $s_t, \varsigma_t \in [0, 1]$. The individual's proposal on the intermediate firm's codification investment would then be $\zeta_{t,prop} = \int_0^1 \zeta_t dP_t$. Further, it is assumed that utility takes the form of a CIES-utility function:

$$u(c) = \frac{c^{1-\theta} - 1}{1 - \theta},$$

where $\theta \in [0.5, 1]$. The agent obtains the utility maximizing pair (s_t, ς_t) from the following necessary conditions:

$$\begin{aligned} u'(c_{1,t}) &= \delta u'(c_{2,t+1})(1 + r_{t+1}), \\ u'(c_{1,t}) &= \delta u'(c_{2,t+1}) \frac{\partial r_{t+1}}{\partial \varsigma_t} s_t. \end{aligned}$$

With the particular functional form of utility, the conditions write

$$\begin{aligned} s_t &= \frac{1 - \varsigma_t}{\delta^{-\frac{1}{\theta}}(1 + r_{t+1})^{\frac{\theta-1}{\theta}} + 1}, \\ \varsigma_t &= 1 - \delta^{-\frac{1}{\theta}} \left(\frac{\partial r_{t+1}}{\partial \varsigma_t} \right)^{-\frac{1}{\theta}} (1 + r_{t+1}) s_t^{\frac{\theta-1}{\theta}} - s_t. \end{aligned}$$

and further transformations yield

$$\begin{aligned} s_t &= \frac{1 + r_{t+1}}{\frac{\partial r_{t+1}}{\partial \varsigma_t}}, \\ \varsigma_t &= 1 - \frac{1 + r_{t+1}}{\frac{\partial r_{t+1}}{\partial \varsigma_t}} \left(\delta^{-\frac{1}{\theta}}(1 + r_{t+1})^{\frac{\theta-1}{\theta}} + 1 \right). \end{aligned}$$

As information cannot be transformed into the consumption good, a negative codification investment suggestion is not possible. Hence, an agent will make a suggestion if her optimal level $\zeta_{t,prop}$ is greater than zero. In particular, if no proposal has been put forth or no proposal has been accepted, the optimization problem is equivalent to the regular maximization problem in an overlapping generation model with production.

Since all individuals are flexible to choose their profession, the model features only one labor market, and wages of the different occupations must be equal in equilibrium. As a consequence, each household faces an identical optimization problem, such that in fact each person makes the same proposal on knowledge codification expenses which, of course, will be accepted. It is thus convenient to work with a representative household as being the young generation characterized by its date of birth t.[19] Each generation faces the first-stage optimization problem and its solution is interpreted as aggregate

[19] In fact, individuals belonging to the same generations may be heterogenous with respect to human capital. Earning the same wage, this heterogeneity does not carry over to the households' utility maximization problem.

savings S_t and investment in knowledge codification $\zeta_t \geq 0$.

Note that the utility function exhibits the following properties:

(c1) $u'(c) > 0, \quad u''(c) < 0$;

(c2) $\lim_{c\to\infty} u'(c) = 0$, $\lim_{c\to 0} u'(c) = \infty$;

(c3) $\frac{\partial s_t}{\partial r_{t+1}} \geq 0$[20] and if $\varsigma_t = 0$, the saving rate is a function of r_{t+1}, only;

(c4) $\frac{u''(c_{2,t+1})}{u'(c_{2,t+1})} + \frac{1}{2c_{2,t+1}} < -\frac{u''(c_{1,t})}{u'(c_{1,t})}\frac{1}{1+r_{t+1}}$.

Given that $r_{t+1} \geq 0$ and $w_t \geq 0$, the Inada conditions (c2) guarantee a positive saving rate s_t. In contrast, the optimal choice for ς_t may be the corner solution $\varsigma_t = 0$. Property (c4) constrains the curvature of the utility function as necessary to guarantee for a unique maximum.[21] With regard to CIES-utility, this condition translates to a lower bound of the constant elasticity parameter[22], $\theta > \frac{1}{2}\frac{1-s_t-\varsigma_t}{1-\varsigma_t}$, and is always satisfied for $\theta \in [0.5, 1]$. Note that this also includes logarithmic utility. More generally, the model allows for other utility functions as long as they satisfy $(c1) - (c4)$.

3.5 Sequence of Events

This section summarizes the time-line of the model's typical events in a regular period t:

(1) At the beginning of period t, the members of the new generation are exogenously transferred a share q of the economy's knowledge stock of the previous period: $\tau_{P_t} = q\tau_{t-1}$, $P_t \in \mathcal{P}_t$.

(2) The intermediate firm hires a number $L_{R,t}$ of researchers. If the researchers of the previous periods have codified their ideas, the new generation of researchers in t would additionally build up human capital of $\beta(C_t - q\tau_{t-1})$.

---

[20] If $\varsigma_t = 0$, the derivative is written as $\frac{\partial s_t}{\partial r_{t+1}} = \delta^{-\frac{1}{\theta}}\frac{1-\theta}{\theta}(1+r_{t+1})^{-\frac{1}{\theta}}\left(\delta^{-\frac{1}{\theta}}(1+r_{t+1})^{\frac{\theta-1}{\theta}}+1\right)^{-2} \geq 0$, $\theta \in (0,1]$. For an interior solution where $\varsigma_t > 0$, $s_t = \frac{\gamma_t(1+r_{t+1})^{\frac{1}{\alpha}}}{w_t\left(\frac{\partial\tau_t}{\partial L_{R,t}}\right)^{\frac{\alpha-1}{\alpha}}\left(\frac{1-\alpha}{\alpha^2}\right)^{\frac{1-\alpha}{\alpha}}(\nu\beta)^{\frac{1}{\alpha}}}$. Consequently, $\frac{\partial s_t}{\partial r_{t+1}} = \frac{\gamma_t\frac{1}{\alpha}(1+r_{t+1})^{\frac{1-\alpha}{\alpha}}}{w_t\left(\frac{\partial\tau_t}{\partial L_{R,t}}\right)^{\frac{\alpha-1}{\alpha}}\left(\frac{1-\alpha}{\alpha^2}\right)^{\frac{1-\alpha}{\alpha}}(\nu\beta)^{\frac{1}{\alpha}}} > 0$.

[21] Appendix 7.1 verifies that there is a unique solution of the necessary conditions which implies a maximum of the objective function.

[22] To be precise, θ would be the constant coefficient of relative risk aversion and its reciprocal value the constant elasticity of substitution between consumption in the two periods.

(3) The intermediate firm produces intermediate good x_t with the capital saved by the now old generation $\mathcal{P}_{t-1}$ and the researchers' human capital. The researchers receive wage $w_{R,t}$. The profits π_t^{int} are split among the capital owners.

(4) The final-product firm rents the intermediate products at price $p_{x,t}$ and hires a number $L_{A,t}$ of workers at a wage $w_{A,t}$ in order to produce the final good Y_t.

(5) At the end of period t, the young generation decides how much of the wage income to consume $c_{1,t}$ and how much to transfer to the next period. On the one hand, the young generation (decentrally) saves in physical capital, which ensures ownership rights of the intermediate firm in $t+1$ involving a return r_{t+1}. On the other hand, it may invest in knowledge codification which is the result of a collective decision made at the shareholder's meeting of the intermediate firm. Knowledge codification increases the knowledge stock of $t+1$ and consequently the rent r_{t+1}.

(6) Dependent on the investment in knowledge codification, ζ_t, the researchers of period t codify an amount $\triangle C_t$ of their ideas.

(7) The old generation sells its capital to the young, consumes its receipts K_t plus the return on capital $r_t K_t = \pi_t^{int}$, and then dies.

As usually assumed in overlapping generations models, there is one physical good that can be consumed. Output that is not consumed can be used as capital in the following period. The literature suggests two ways of interpretation with respect to the capital stock. In both, savings of the young generation in t equal the capital stock of period $t+1$. In the first, the young generation buys the current capital stock from the old with their savings and "net-saves" the difference $S_t - K_t$, which becomes the net increase in capital of the subsequent period. The second approach assumes that capital is "eatable" or consumable, such that the old generation consumes its capital at the end of its life. In this way, the capital stock of the next period is always identical to consumption forgone of the previous period, which has become productive with a time lag.

Both interpretations could apply to our model, however, suggesting a long-lasting intermediate firm, we will go with the first.[23] In detail, we assume that the intermediate firm transforms the capital stock to the intermediate good, which is rent to final-goods production. Over one period the intermediate good depreciates in the sense that it cannot be used in final production unless it is overhauled. But it can still be used as raw capital in the next period.[24]

[23] Adopting the first interpretation and assuming that capital is not consumable may be problematic for finite time horizons and if the capital stock is decreasing. Both problems will not occur in this thesis.

[24] It is well known that overlapping generations economies may exhibit equilibria with asset bubbles. As bubbles are not the focus of the dissertation, we exclude them from the following analysis.

Note that if information were consumable, the firm owners could demand at least the respective consumption value from the next generation of firm owners for those ideas with positive marginal value in the next but one period. The question is how high the amount of consumption from the physical object(s) in which the respective idea is embodied (e.g. books) would be. We argue that it should be lower than γ_t as the costs of knowledge codification do not translate one-to-one into the physical object but also comprise consumable materials like office supplies and the like. It would be possible to introduce the assumption that information is equivalent to e.g. $\varrho\gamma_t$ units of the consumption good in the next period, where $\varrho \in (0, 1)$. However, this would complicate the model without yielding much additional insights. For this reason, the consumption value of information has been set equal to zero.[25]

The following figure sketches the structure of the established overlapping generations model.

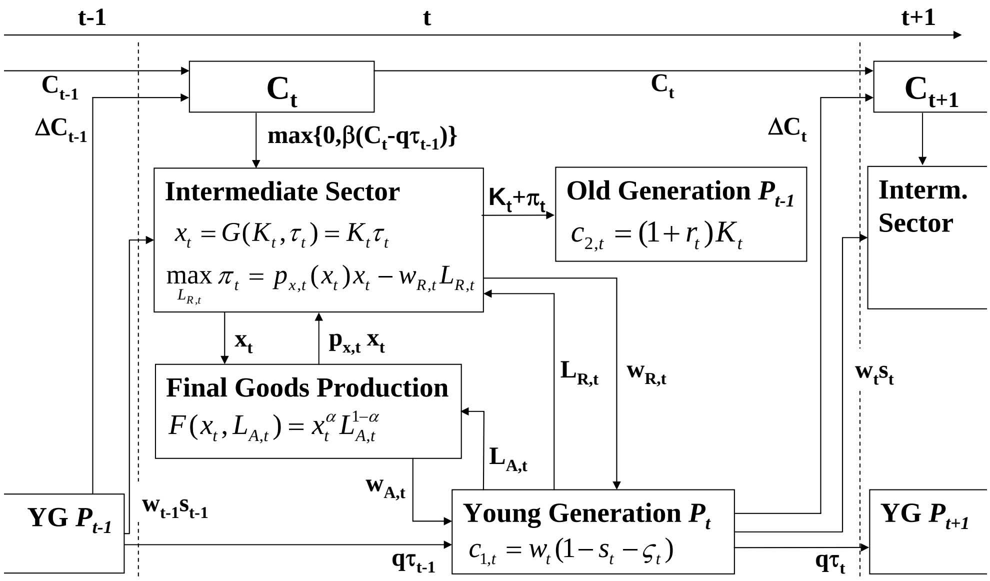

Figure 3.1: The Structure of the Overlapping Generations Model

[25]In this way, the intertemporal externality to future generations from knowledge codification tends to be too high. In particular, with respect to libraries or the hardware of a knowledge database that have been established from fixed overhead costs f in the first period of knowledge codification would be handed down from one generation to the next at its consumption value. This has been neglected for reasons of simplicity.

Chapter 4

Equilibria and General Dynamics

4.1 Sequential Markets Equilibrium

The economy comprises three markets: The labor market, the market for intermediate products and the market for the consumption good. The following analysis will focus on the sequential markets equilibrium, which is defined by the three markets to clear in each period.

Definition 4.1 *Given $K_1, \tau_0 > 0$, $C_1 \geq 0$, and $S_t = K_{t+1}$, $C_{t+1} = f(C_t, \zeta_t)$, a sequential markets equilibrium is allocations $c_{2,1}$, $\{c_{1,t}, c_{2,t+1}, S_t, \zeta_t, L_{A,t}, L_{R,t}, x_t\}_{t=1}^{\infty}$ and prices $\{p_{x,t}, w_t\}_{t=1}^{\infty}$, such that*

(i) they solve the utility maximization problem of the representative household and the profit maximization problems of the representative final-goods firm and of the intermediate firm for all $t \geq 1$ and

(ii) in every period the economy is in temporary equilibrium, that is, for all $t \geq 1$:

(a) (Labor Market)

$$L_t^s = L_{A,t}^d + L_{R,t}^d;$$

(b) (Intermediate Goods Market)

$$x_t^s = x_t^d;$$

(c) (Final Goods Market)

$$Y_t^s = c_{1,t} + c_{2,t} + S_t + \zeta_t.$$

As the supply and demand functions are derived by the respective optimization problems, it is clear that the model satisfies (i). Further, the properties of the final-goods production function imply that the temporary equilibrium of the labor market will exhibit a positive share of workers in this sector due to the Inada conditions. Using the respective demand functions from the first order conditions of the representative final-goods firm and the intermediate firm, the equilibrium condition writes

$$\begin{aligned} L_t^s &= \left(\frac{1-\alpha}{w_t}\right)^{\frac{1}{\alpha}} K_t\tau_t \left(1+\left(\frac{\alpha^2}{w_t}\right)^{\frac{1}{1-\alpha}}\left(K_t\frac{\partial\tau_t}{\partial L_{R,t}}\right)^{\frac{\alpha}{1-\alpha}}\right) \\ &\quad -\frac{1}{\frac{\partial\tau_t}{\partial L_{R,t}}}(q\tau_{t-1}+\beta\max\{0,C_t-q\tau_{t-1}\}). \end{aligned} \tag{4.1}$$

Since $L_t^s - L_{A,t}^d = L_{R,t}^d$, we receive the equilibrium wage

$$w_t = (1-\alpha)^{1-\alpha}\left(\alpha^2 K_t\frac{\partial\tau_t}{\partial L_{R,t}}\right)^{\alpha}.$$

Note that as $\frac{\partial\tau_t}{\partial L_{R,t}} = \varepsilon(q\tau_{t-1})^{\Phi}$, the wage is independent of the amount of knowledge codification of the previous period. In this way, the model characterized by a specification of the production of intermediate goods with Solow-neutral technological progress does not exhibit a hold up problem with respect to knowledge codification. However, knowledge codification may possess positive externalities to the wage levels from the next but one period on. A further discussion on social welfare can be found in chapter 12.

Inserting the equilibrium wage into the labor demand functions and using $L_t^s = 1$ gives

$$L_{A,t} = \frac{1-\alpha}{\alpha^2}\frac{\tau_t}{\frac{d\tau_t}{dL_{R,t}}}.$$

Hence, the equilibrium allocation is

$$\begin{aligned} L_{A,t} &= \min\left\{1, \frac{1-\alpha}{\alpha^2+1-\alpha}\left(1+\frac{q\tau_{t-1}+\beta\max\{0,C_t-q\tau_{t-1}\}}{\frac{\partial\tau_t}{\partial L_{R,t}}}\right)\right\}, \\ L_{R,t} &= \max\left\{0, \frac{\alpha^2}{\alpha^2+1-\alpha}-\frac{1-\alpha}{\alpha^2+1-\alpha}\frac{q\tau_{t-1}+\beta\max\{0,C_t-q\tau_{t-1}\}}{\frac{\partial\tau_t}{\partial L_{R,t}}}\right\}. \end{aligned}$$

The corner solution $(L_{A,t}, L_{R,t}) = (1,0)$[1] would involve a successively decreasing knowledge stock because the ideas of the previous period cannot be recovered fully by direct transfer and knowledge codification. As shown in appendix 7.2, the economy's equilibrium allocation of labor always possesses an interior solution for $\Phi = 0$, that is, with a constant amount of ideas per researcher.[2] In contrast, with $\Phi = 1$ the marginal productivity of the researchers increases with a growing stock of knowledge. In this case, the share of researchers will be positive, whenever the following condition holds:

$$\frac{\alpha^2}{1-\alpha} > \frac{q+\beta(1-q)}{q\varepsilon}. \tag{4.2}$$

[1]Note that there is no other corner solution due to the Inada conditions for final-goods production and a positive value of the knowledge stock in the first period, $\tau_0 > 0$.

[2]If $C_1 < \tau_{st}^{wC}$, which is the interesting case with endogenous codification.

Otherwise, the economy would stay in a low level trap with a declining knowledge stock and declining output as a consequence. Without explicit mention otherwise, we assume that (4.2) is satisfied.

As utility is non-saturated in consumption, one of the market clearing conditions is redundant by Walras law. We leave aside the consumption goods market and concentrate on the intermediate goods market. The equilibrium condition is written as

$$x_t^d = \left(\frac{\alpha}{p_{x,t}}\right)^{\frac{1}{1-\alpha}} L_{A,t} = \left(\frac{\alpha^2 \frac{\partial x_t}{\partial L_{R,t}}}{w_t}\right)^{\frac{1}{1-\alpha}} L_{A,t} = x_t^s.$$

Hence, for all $w_t > 0$, there will be a positive price $p_{x,t}$ that solves the intermediate-goods market equilibrium condition. In particular,

$$p_{x,t} = \frac{w_t}{\alpha K_t \frac{\partial \tau_t}{\partial L_{R,t}}},$$

and with the equilibrium wage from the labor market,

$$p_{x,t} = \alpha^{2\alpha-1} K_t^{\alpha-1} \left(\frac{\partial \tau_t}{\partial L_{R,t}}\right)^{\alpha-1} (1-\alpha)^{1-\alpha}.$$

Let the homogeneous good be the numéraire. The sequential market equilibrium is then characterized by a price vector

$$\{p_{x,t}, w_t\}_{t=1}^{\infty} = \left\{\alpha^{2\alpha-1} K_t^{\alpha-1} \left(\frac{\partial \tau_t}{\partial L_{R,t}}\right)^{\alpha-1} (1-\alpha)^{1-\alpha}, \quad (1-\alpha)^{1-\alpha} \left(\alpha^2 K_t \frac{\partial \tau_t}{\partial L_{R,t}}\right)^{\alpha}\right\}_{t=1}^{\infty}.$$

4.2 Dynamics

This section introduces the general dynamics of the model, which are described by the difference equations of the economy's three stocks: capital, knowledge and information. The development of the capital stock is determined by each period's saving decision, $K_{t+1} = s_t w_t$. Using the equilibrium wage it transforms into

$$K_{t+1} = s_t K_t^{\alpha} (1-\alpha)^{1-\alpha} \alpha^{2\alpha} \left(\frac{\partial \tau_t}{\partial L_{R,t}}\right)^{\alpha}$$

and leads to a growth rate of capital according to

$$g_{K,t} = \frac{K_{t+1} - K_t}{K_t} = s_t K_t^{\alpha-1} (1-\alpha)^{1-\alpha} \alpha^{2\alpha} \left(\frac{\partial \tau_t}{\partial L_{R,t}}\right)^{\alpha} - 1. \tag{4.3}$$

The behavior of the knowledge stock, given the equilibrium number of researchers, can be written as

$$\tau_{t+1} = q\tau_t + \beta(\max\{0, C_{t+1} - q\tau_t\}) + \varepsilon(q\tau_t)^{\Phi} - \tau_{t+1}\frac{1-\alpha}{\alpha^2},$$

which transforms into

$$\tau_{t+1} = \nu(q\tau_t + \beta(\max\{0, C_{t+1} - q\tau_t\}) + \varepsilon(q\tau_t)^{\Phi}),$$

where $\nu = \frac{\alpha^2}{\alpha^2+1-\alpha}$.

From this equation, we obtain the growth rate of knowledge as

$$g_{\tau,t} = \nu\left(q + \beta\frac{\max\{0, C_{t+1} - q\tau_t\}}{\tau_t} + \varepsilon q^{\Phi}\tau_t^{\Phi-1}\right) - 1.$$

Finally the stock of information accumulates according to

$$C_{t+1} = C_t + \triangle C_t, \quad \triangle C_t \geq 0.$$

In the two extreme cases in which the researchers are not codifying at all (indexed by "woC") and the one in which they are codifying every new idea in each period ("wC"), the maximum term in the knowledge stock's difference equation will be zero in the first case and we can set $C_t = \tau_{t-1}$ in the latter. This gives

$$\begin{aligned} g_{\tau,t}^{woC} &= \nu(q + \varepsilon q^{\Phi}\tau_t^{\Phi-1}) - 1, \\ g_{\tau,t}^{wC} &= \nu(q + \beta(1-q) + \varepsilon q^{\Phi}\tau_t^{\Phi-1}) - 1. \end{aligned}$$

The properties of the economy's dynamical system crucially depend on the assumptions concerning the research process, that is, whether $\Phi = 0$ or $\Phi = 1$. As shown in appendix 7.3, $\Phi = 1$ implies long run growth. The economy possesses two kinds of steady states distinguished by whether the economy exhibits positive or zero knowledge codification. Moreover, an economy with zero codification will approach growth at constant rates in the long run. Every steady state exhibits the same relation of the growth rates of capital and knowledge:

$$g_{K,s} = \frac{\alpha}{1-\alpha} g_{\tau,s}. \tag{4.4}$$

For $\Phi = 1$, the growth rates of knowledge with full and zero codification are constant.

$$\begin{aligned} g_{\tau}^{woC} &= \nu q(1+\varepsilon) - 1, \\ g_{\tau}^{wC} &= \nu(q(1+\varepsilon) + \beta(1-q)) - 1. \end{aligned}$$

As we preclude negative codification and codification of ideas that have not yet been discovered, $g_{\tau,t} \in [g_{\tau}^{woC}, g_{\tau}^{wC}]$, $\forall t$.

With regard to output, we can write

$$Y_t = F(K_t, L_{A,t}, \tau_t) = K_t^{\alpha}\tau_t^{\alpha}\left(\frac{1-\alpha}{\alpha^2\varepsilon q}\right)^{1-\alpha}(1 + g_{\tau,t-1})^{1-\alpha}.$$

Log-differentiating and using the steady-state relation of the growth rates of capital and knowledge as given by (4.4) verifies that the steady-state growth rate of output equals that of capital:

$$g_{Y,s} = g_{K,s} = \frac{\alpha}{1-\alpha} g_{\tau,s}.$$

Assuming $\Phi = 0$ implies that the economy approaches a stationary state in the long run. This is also verified in appendix 7.3. Consider an economy with an initial level of information of zero ($C_1 = 0$). By full or zero codification in the following periods, the economy will reach stationary-state levels of knowledge of

$$\begin{aligned} \tau_{st}^{wC} &= \frac{\nu\varepsilon}{1-\nu(q+\beta(1-q))}, \\ \tau_{st}^{woC} &= \frac{\nu\varepsilon}{1-\nu q}. \end{aligned}$$

For initial values $C_1 < \tau_{st}^{wC}$, the system realizes stationary-state knowledge levels of $\tau_{st} \in [\tau_{st}^{woC}, \tau_{st}^{wC}]$. Only in case there is some exogenous source of information such that $C_t > \tau_{st}^{wC}$ from some period t on, the knowledge stock may exceed τ_{st}^{wC}. Focussing on endogenous knowledge codification, only the case $C_1 < \tau_{st}^{wC}$ is of interest for the thesis.

With the saving rate depending positively on the knowledge level in stationary state, the capital stock approaches $K_{st} \in [K_{st}^{woC}, K_{st}^{wC}]$, where

$$K_{st}^{woC,wC} = \left(s_{st}^{woC,wC}\right)^{\frac{1}{1-\alpha}} (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} \varepsilon^{\frac{\alpha}{1-\alpha}}.$$

Clearly then, the economy's stationary-state level of output must be $Y_{st} \in [Y_{st}^{woC}, Y_{st}^{wC}]$, where

$$\begin{aligned} Y_{st}^{woC} &= \left(s_{st}^{woC}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha)\alpha^{\frac{4\alpha-2}{1-\alpha}} \varepsilon^{\frac{\alpha}{1-\alpha}} \frac{\nu}{1-\nu q}, \\ Y_{st}^{wC} &= \left(s_{st}^{wC}\right)^{\frac{\alpha}{1-\alpha}} (1-\alpha)\alpha^{\frac{4\alpha-2}{1-\alpha}} \varepsilon^{\frac{\alpha}{1-\alpha}} \frac{\nu}{1-\nu(q+\beta(1-q))}. \end{aligned}$$

The dynamics show that with codification, the economy would reach a higher stationary-state level of output in the case $\Phi = 0$ as well as a higher steady state growth rate of output for $\Phi = 1$. The magnitude crucially depends on the transfer rate q and the reading capacity β. Hence, knowledge codification may account for a large proportion of an economy's growth rate if other knowledge transfer capabilities are bad, i.e. q is low. When bearing no cost, the researchers would always codify their knowledge to the full extent at the end of the period.

Chapter 5

Codification Behavior of the Economy

The last section of the previous chapter on the economy's dynamics revealed that the economy's output level, respectively its growth rate may substantially increase due to knowledge codification activities. This chapter examines under which conditions knowledge codification will be observed. For this purpose, it will first be necessary to revisit the households' utility maximization problem. Thereafter, the general structure of the problem will be unfolded by three lemmata, which serve as a basis on which the results dependent on the different specifications with respect to the research process are derived.

With the given model specification, the amount of knowledge codification is derived from the representative household's utility maximization problem. As $\zeta_t = \gamma_t \triangle C_t$, γ_t being constant within periods and $S_t = K_{t+1}$, the household's problem can equivalently be solved via the control variables K_{t+1} and $\triangle C_t$. For convenience, we will use this notation in the following. The usual procedure is to calculate the derivative of the rent with respect to the amount of codification in t and solve the first order condition for $\triangle C_t$. This is what will be done, however, $\frac{d\tau_{t+1}}{d\triangle C_t}$ exhibits a discontinuity which necessitates some preliminary considerations.

Recall the difference equation of knowledge given that the economy will be in temporary equilibrium in each period:

$$\tau_{t+1} = \nu q \tau_t + \nu\beta(\max\{0, C_{t+1} - q\tau_t\}) + \nu\varepsilon(q\tau_t)^{\Phi}. \tag{5.1}$$

The maximum term indicates that the codification of one more idea in t contributes to the economy's knowledge stock in t only if the young generation in $t+1$ has not been transferred the respective tacit component. That is, codification in t of ideas that the new researchers of $t+1$ will know without reading the information (because they will have been transferred exogenously), possesses no value in $t+1$. Therefore, codification of an additional idea given the information stock C_t – this idea would be indexed by $C_t + \eta$, $\eta \to 0$ – would enhance the knowledge stock of time $t+1$ by:

$$\frac{d\tau_{t+1}}{d\triangle C_t} = \begin{cases} \nu\beta, & \text{if} \quad C_t \geq q\tau_t; \\ 0\;, & \text{if} \quad C_t < q\tau_t. \end{cases}$$

Accordingly, the ideas to be codified in t can be distinguished by their marginal value in the subsequent period $t+1$. The number of ideas that are of zero marginal value are denoted by $\triangle C_{ie,t} \in [0, \max\{q\tau_t - C_t, 0\}]$ and those with positive marginal value by

$\triangle C_{e,t} \in [0, \max\{0, min\{\tau_t(1-q), \tau_t - C_t\}\}]$. It follows that $C_{t+1} = C_t + \triangle C_{ie,t} + \triangle C_{e,t}$. Consequently, (5.1) can be rewritten as:

$$\tau_{t+1} = \nu q \tau_t + \nu\beta(\triangle C_{e,t} + \max\{0, C_t - q\tau_t\}) + \nu\varepsilon(q\tau_t)^{\Phi}. \tag{5.2}$$

The middle term expresses that the new employees can build up knowledge from reading information codified in the last period $\triangle C_{e,t}$ and, in case it has been transferred a measure of human capital less than what has already been codified in the periods before t, it can additionally utilize "the older books" labeled by indices $i \in [q\tau_t, C_t]$. The latter would imply that all additionally generated information in t is of positive value in the next period, that is, $\triangle C_t = \triangle C_{e,t}$. On the other hand, if $C_t < q\tau_t$, $\triangle C_{ie,t}$ is positive. This implies that the representative agent may also have to pay for information that will not generate any benefit in the next period. Hence, the codification decision is crucially affected by assumption (A2), which may involve an additional "entry cost" for knowledge codification. Without (A2), the representative agent would be able to codify ideas with positive marginal value in the next period only. In this case, C_t cannot be interpreted as the measure of the stock of information. It is just the highest index of all codified ideas. However, with assumption (A2'), $C_{t+1} - q\tau_t$ is well defined as the (Lebesque-)measure of efficient information. In this way, the dynamics of the knowledge stock are also well defined without (A2).

Rather than asking how many ideas in total, that is $\triangle C_t$, the new capital owners are willing to codify, the discontinuity can be eliminated by reformulating the question to: How many ideas of positive marginal value $\triangle C_{e,t}$ are the owners of the intermediate firm willing to codify in period t, given the stock of information C_t, that is, given that they may have to codify some ideas of zero marginal value in $t+1$. Note that this would structurally correspond to the household's problem with (A2') and fixed entry costs f. Altogether, the utility maximization problem then writes:

$$\max_{K_{t+1}, \triangle C_{e,t}} U_t(c_{1,t}, c_{2,t+1}) = u(c_{1,t}) + \delta u(c_{2,t+1})$$

subject to

$$c_{1,t} = w_t - K_{t+1} - \gamma_t \triangle C_t,$$
$$c_{2,t+1} = (1 + r_{t+1})K_{t+1}.$$

The respective first-order conditions yield:

$$u'(c_{1,t}) = \delta u'(c_{2,t+1})(1 + r_{t+1}),$$
$$\gamma_t u'(c_{1,t}) = \delta u'(c_{2,t+1}) \frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} K_{t+1}.$$

Inserting optimal saving in physical capital into the necessary condition with regard to knowledge codification then gives:

$$\gamma_t(1 + r_{t+1}) = \frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} K_{t+1}. \tag{5.3}$$

This equation gives all pairs $(K_{t+1}, \triangle C_{e,t})$ that are candidates of the solution to the representative household's maximization problem. Or, in other words, it defines a function $\triangle C_{e,t}(K_{t+1})^1$, that determines the optimal choice of $\triangle C_{e,t}$ for each optimal K_{t+1}. As negative codification is precluded, the solution to the household's problem must be a point on the graph of this function or involve the corner solution $\triangle C_{e,t} = 0$. Equation (5.3) illuminates the household's tradeoff between saving, that is capital investment, and knowledge codification. On the left hand side is the amount of consumption the young would receive when old, if they used the marginal codification costs for saving in physical capital, which in the optimum must be equal to the marginal benefits of codification. Using the equilibrium allocation of labor, the rent given by (3.1) is written as:

$$\begin{aligned} r_t &= K_t^{\alpha-1} L_{A,t}^{-\alpha} \tau_t^{\alpha} (L_{A,t} - 1 + \alpha) \\ &= K_t^{\alpha-1} \left(\tfrac{1-\alpha}{\alpha^2}\right)^{-\alpha} \left(\tfrac{d\tau_t}{dL_{R,t}}\right)^{\alpha} \left(\tau_t \left(\tfrac{1-\alpha}{\alpha^2}\right) \left(\tfrac{d\tau_t}{dL_{R,t}}\right)^{-1} - 1 + \alpha\right). \end{aligned} \tag{5.4}$$

Calculating the derivative of r_{t+1} with respect to $\triangle C_{e,t}$ and inserting it into the marginal condition (5.3) gives:

$$\gamma_t = \frac{K_{t+1} \left(\tfrac{1-\alpha}{\alpha^2}\right)^{1-\alpha} \left(\tfrac{d\tau_{t+1}}{dL_{R,t+1}}\right)^{\alpha-1} \tfrac{d\tau_{t+1}}{d\triangle C_{e,t}}}{K_{t+1}^{1-\alpha} + \left(\tfrac{1-\alpha}{\alpha^2}\right)^{-\alpha} \left(\tfrac{d\tau_{t+1}}{dL_{R,t+1}}\right)^{\alpha} \left(\tau_{t+1} \left(\tfrac{1-\alpha}{\alpha^2}\right) \left(\tfrac{d\tau_{t+1}}{dL_{R,t+1}}\right)^{-1} - 1 + \alpha\right)}.$$

With the difference equation of the knowledge stock as in (5.2), it is possible to solve for $\triangle C_{e,t}$:

$$\begin{aligned} \triangle C_{e,t} = \frac{K_{t+1}}{\gamma_t} - \frac{K_{t+1}^{1-\alpha}}{\frac{d\tau_{t+1}}{d\triangle C_{e,t}}} \left(\tfrac{1-\alpha}{\alpha^2}\right)^{\alpha-1} \left(\tfrac{d\tau_{t+1}}{dL_{R,t+1}}\right)^{1-\alpha} + \frac{\alpha^2-\nu}{\frac{d\tau_{t+1}}{d\triangle C_{e,t}}} \frac{d\tau_{t+1}}{dL_{R,t+1}} \\ - \frac{\nu q \tau_t}{\frac{d\tau_{t+1}}{d\triangle C_{e,t}}} - \max\{0, C_t - q\tau_t\}. \end{aligned} \tag{5.5}$$

Note that these marginal considerations imply that entry costs are sunk, in the sense that they have to be paid in any case, or there are no entry costs for codification. With assumption (A2), there may be no entry costs because the previous generation has created enough information such that $\triangle C_{ie,t} = 0$. Of course, without entry costs for knowledge codification, the amount of ideas to codify as reflected in equation (5.5) is optimal. With (A2) or (A2') and $f > 0$, it is necessary to verify that the representative agent's life-time utility with codification, given entry costs ($\gamma_t \triangle C_{ie,t}$ and/or f) is higher than her utility when only investing in capital and thence not incurring entry costs. Certainly, assumption (A2) represents an extreme, possibly rather hypothetical case where all previously invented ideas have to be codified before being able to codify an idea i. The reasoning behind it is that if new ideas draw upon previous ideas, the

[1]Using the implicit function theorem, let $M(K_{t+1}, \triangle C_{e,t}) = \frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} K_{t+1} - \gamma_t(1+r_{t+1}) = 0$. We obtain $\frac{\partial M}{\partial \triangle C_{e,t}} = \frac{\partial^2 r_{t+1}}{\partial \triangle C_{e,t}^2} K_{t+1} - \gamma \frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} < 0, \quad \forall K_{t+1} \geq 0$.

codification of the first would have to refer to the latter. However, if those are not codified, it is not obvious whether this can be easily done, for example, if vocabulary of precisely defined and commonly understood terms are lacking. Cowan et al. (2000) refer to this situation as the lack of a 'code-book'. This case however will not be the main focus of the following analysis. Hence, we will proceed with marginal considerations and thereafter address the case with entry costs f that have to be paid once when setting up the structures for knowledge codification. In each period, the case with assumption (A2) can then be interpreted as the previous one with the peculiarity that the entry costs are growing as long as the economy does not codify. Let the case with (A2') and $f = 0$ be the standard situation. This means agents are able to just codify ideas with positive marginal value in $t+1$ without any entry costs. Explicit notion will be made when fading in f or (A2).

It is now interesting to examine the economy's codification behavior over time:

$$\begin{aligned} d\triangle C_{e,t} &= \left(\frac{\partial \triangle C_{e,t}}{\partial \tau_t} + \frac{\partial \triangle C_{e,t}}{\partial K_{t+1}}\frac{\partial K_{t+1}}{\partial \tau_t}\right)d\tau_t + \left(\frac{\partial \triangle C_{e,t}}{\partial C_t} + \frac{\partial \triangle C_{e,t}}{\partial K_{t+1}}\frac{\partial K_{t+1}}{\partial C_t}\right)dC_t + \frac{\partial \triangle C_{e,t}}{\partial K_{t+1}}\frac{\partial K_{t+1}}{\partial K_t}dK_t \\ &= \frac{\partial \triangle C_{e,t}}{\partial \tau_t}d\tau_t + \frac{\partial \triangle C_{e,t}}{\partial C_t}dC_t + \frac{\partial \triangle C_{e,t}}{\partial K_{t+1}}\underbrace{\left(\frac{\partial K_{t+1}}{\partial \tau_t}d\tau_t + \frac{\partial K_{t+1}}{\partial C_t}dC_t + \frac{\partial K_{t+1}}{\partial K_t}dK_t\right)}_{dK_{t+1}}. \end{aligned}$$

Changing stocks of knowledge and information possess a direct and an indirect effect via K_{t+1} on $\triangle C_{e,t}$. For convenience, the indirect effect is subsumed in dK_{t+1} and it is referred to the direct effect when speaking of influences of the knowledge stock or the stock of information on knowledge codification in period t. From (5.5) it is obvious that the direct effect of a marginal increase in C_t is either zero, if the maximum term is zero,[2] or negative. Taking the partial derivative with respect to τ_t yields:

$$\frac{\partial \triangle C_{e,t}}{\partial \tau_t} = \begin{cases} \frac{K_{t+1}^{1-\alpha}}{\nu\beta}\left(\frac{1-\alpha}{\alpha^2}\right)^{\alpha-1}\varepsilon^{1-\alpha}\Phi(1-\alpha)(q\tau_t)^{\Phi(1-\alpha)-1}q + \frac{(\alpha^2-\nu)\varepsilon\Phi q(q\tau_t)^{\Phi-1}-q}{\nu\beta} < 0 & , C_t \leq \tau_t; \\ \frac{K_{t+1}^{1-\alpha}}{\nu\beta}\left(\frac{1-\alpha}{\alpha^2}\right)^{\alpha-1}\varepsilon^{1-\alpha}\Phi(1-\alpha)(q\tau_t)^{\Phi(1-\alpha)-1}q + \frac{(\alpha^2-\nu)\varepsilon\Phi q(q\tau_t)^{\Phi-1}-q}{\nu\beta} + q < 0, & C_t > \tau_t. \end{cases}$$

The direct effect of knowledge growth on codification is negative because it increases the return on physical capital and in this way increases the opportunity costs for knowledge codification. In contrast, an increasing knowledge stock has no direct effect on the codification benefit, because the rent in $t+1$ is linear in τ_{t+1}.[3]
Choosing a marginally higher level of capital saving would influence the representative household's optimal amount of knowledge codification according to:

$$\frac{\partial \triangle C_{e,t}}{\partial K_{t+1}} = \frac{1}{\gamma_t} - \frac{(1-\alpha)K_{t+1}^{-\alpha}}{\frac{d\tau_{t+1}}{d\triangle C_{e,t}}}\left(\frac{1-\alpha}{\alpha^2}\right)^{\alpha-1}\varepsilon^{1-\alpha}(q\tau_t)^{\Phi(1-\alpha)}.$$

[2]Note that in this case the indirect effect via K_{t+1} is also 0. Of course the clause above holds an inaccuracy since if $C_t = q\tau_t$, the maximum term would be zero but the partial derivative of $\triangle C_{e,t}$ with respect to C_t would be negative.

[3]For this reason $\frac{\partial \frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} K_{t+1}}{\partial \tau_t} = 0$. This is due to the model showing Solow neutral technological progress. With other specifications of technological progress an increasing knowledge stock directly decreases the marginal codification benefit.

The partial derivative is negative for small K_{t+1}. More precisely, if

$$\frac{K_{t+1}}{\gamma_t} < \frac{(1-\alpha)K_{t+1}^{1-\alpha}}{\frac{d\tau_{t+1}}{d\triangle C_{e,t}}} \left(\frac{1-\alpha}{\alpha^2}\right)^{\alpha-1} \varepsilon^{1-\alpha}(q\tau_t)^{\Phi(1-\alpha)}.$$

Inserting this condition into (5.5), it is possible to estimate from above the values of $\triangle C_{e,t}$ for which K_{t+1} capital saving exerts a negative influence on the benefits of knowledge codification:

$$\begin{aligned}\triangle C_{e,t} \quad < \quad & -\alpha\frac{K_{t+1}^{1-\alpha}}{\frac{d\tau_{t+1}}{d\triangle C_{e,t}}}\left(\frac{1-\alpha}{\alpha^2}\right)^{\alpha-1}\left(\frac{d\tau_{t+1}}{dL_{R,t+1}}\right)^{1-\alpha} + \frac{\alpha^2-\nu}{\frac{d\tau_{t+1}}{d\triangle C_{e,t}}}\frac{d\tau_{t+1}}{dL_{R,t+1}} \\ & -\frac{\nu q\tau_t}{\frac{d\tau_{t+1}}{d\triangle C_{e,t}}} - \max\{0, C_t - q\tau_t\} < 0.\end{aligned}$$

It shows that a negative partial derivative with respect to K_{t+1} would imply $\triangle C_{e,t} < 0$, which is precluded by assumption. Further, we can formulate:

Lemma 5.1 *There exists a single-valued function $K_{t+1,crit} : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$, which gives the amount of the capital stock in $t+1$, $K_{t+1,crit}(\tau_t, C_t)$, such that $\triangle C_{e,t} = 0$.*

Proof. Consider the derivative of $\triangle C_{e,t}$ with respect to K_{t+1} at $\triangle C_{e,t} = 0$. $\triangle C_{e,t} = 0$ requires

$$\frac{K_{t+1}^{-\alpha}}{\nu\beta}\left(\frac{1-\alpha}{\alpha^2}\right)^{\alpha-1}\varepsilon^{1-\alpha}(q\tau_t)^{\Phi(1-\alpha)} = \frac{1}{\gamma_t} + \frac{\varepsilon(q\tau_t)^{\Phi}(\alpha^2-\nu)-\nu q\tau_t-\nu\beta\max\{0,C_t-q\tau_t\}}{\nu\beta K_{t+1}}$$

and thence[4]

$$\left.\frac{\partial\triangle C_{e,t}}{\partial K_{t+1}}\right|_{\triangle C_{e,t}=0} = \frac{\alpha}{\gamma_t} - (1-\alpha)\frac{\varepsilon(q\tau_t)^{\Phi}(\alpha^2-\nu)-\nu q\tau_t-\nu\beta\max\{0,C_t-q\tau_t\}}{\nu\beta K_{t+1}} > 0.$$

Using the implicit function theorem proves lemma 5.1. □

Lemma 5.2 *Let $\mathcal{U} \subset \mathbb{R}_+^3$ be the set $\{(\tau_t, C_t, K_{t+1}) | \triangle C_{e,t}(\tau_t, C_t, K_{t+1}) \geq 0\}$. $\triangle C_{e,t}$ is a strictly increasing function of K_{t+1} on $\mathcal{U}$.*

Proof. $\triangle C_{e,t} \geq 0$ implies

$$\frac{K_{t+1}^{-\alpha}}{\nu\beta}\left(\frac{1-\alpha}{\alpha^2}\right)^{\alpha-1}\varepsilon^{1-\alpha}(q\tau_t)^{\Phi(1-\alpha)} \leq \frac{1}{\gamma_t} + \frac{\varepsilon(q\tau_t)^{\Phi}(\alpha^2-\nu)-\nu q\tau_t-\nu\beta\max\{0,C_t-q\tau_t\}}{\nu\beta K_{t+1}}$$

and therefore,

$$\left.\frac{\partial\triangle C_{e,t}}{\partial K_{t+1}}\right|_{\triangle C_{e,t}\geq 0} \geq \frac{\alpha}{\gamma_t} - (1-\alpha)\frac{\varepsilon(q\tau_t)^{\Phi}(\alpha^2-\nu)-\nu q\tau_t-\nu\beta\max\{0,C_t-q\tau_t\}}{\nu\beta K_{t+1}} > 0.$$

[4]Note that $\alpha^2 - \nu < 0$ for $\alpha \in (0, 1)$.

From lemma 5.1, we know that $K_{t+1,crit}(\tau_t, C_t)$ is continuous, and since the partial derivative of $\triangle C_{e,t}$ with respect to K_{t+1} can only be negative for $\triangle C_{e,t} < 0$, $\mathcal{U}$ must be a connected subspace of $\mathbb{R}^3_+$. □

The previous lemmata apply to the marginal consideration leaving out possible entry costs of knowledge codification f and/or $\gamma_t \triangle C_{ie,t}$. In order to address the case with entry costs, it is necessary to distinguish the two situations which the agent compares in her decision process. As mentioned previously, from a representative household perspective, the decision on knowledge codification does not depend on whether the fixed entry costs originate from having to create information of zero marginal productivity in the next period due to assumption (A2) or from set-up costs f or both. This is because in the situation with (A2) the household does not take into account that if it did not codify, the next generation would have to pay even higher entry costs. Hence, in the following lemma, total fixed costs are just denoted by $\tilde{f}$, which may originate from both, (A2) and f, that is, in general $\tilde{f}_t = f + \gamma_t \triangle C_{ie,t}$.[5]

For a given budget w_t:

(S1) The representative agent does not pay the entry costs of knowledge codification and, hence, invests in physical capital only. Her optimal choice of physical capital saving in a period t is denoted by $\bar{K}_{t+1}$. The generation of period t will then realize life time utility of

$$u[w_t - \bar{K}_{t+1}] + \delta u[(1 + r_{t+1}(\triangle C_{e,t} = 0))\bar{K}_{t+1}] =: U_t^{woC}.$$

(S2) The household pays entry costs $\tilde{f}_t$ and chooses the optimal pair $(\hat{K}_{t+1}, \triangle \hat{C}_{e,t})$. Lifetime utility in this case can be written as

$$u[w_t - \tilde{f}_t - \hat{K}_{t+1} - \gamma_t \triangle \hat{C}_{e,t}] + \delta u[(1 + r_{t+1}(\triangle \hat{C}_{e,t}))\hat{K}_{t+1}] =: U_t^C,$$

for $w_t \geq \tilde{f}_t$, and let $U_t^C(w_t < \tilde{f}_t) := U_t^C(w_t = 0)$.

Note that the household's optimization problem possesses unique solutions in both situations (see Appendix 7.1).

Lemma 5.3 *For every* $(\tau_t, C_t, \tilde{f}_t) \in \mathbb{R}^3_+$ *exists a unique* $(\hat{K}_{t+1,crit}, \bar{K}_{t+1,crit})$ *such that* $U_t^C = U_t^{woC}$. *For all* $\hat{K}_{t+1} > \hat{K}_{t+1,crit}(\tau_t, C_t, \tilde{f})$, $U_t^C > U_t^{woC}$.

Proof. The proof proceeds as follows. First, we ascertain that for planned capital saving $K_{t+1} > K_{t+1,crit}$[6], the representative agent would be willing to pay entry costs for codification up to a certain amount. In a second step, we identify the uniqueness of a w_t^* for every amount of fixed costs which leaves the agent just indifferent between

[5] Of course, if in a previous period the entry costs f have already been paid, $\tilde{f}_t$ reduces to $\gamma_t \triangle C_{ie,t}$.

[6] $K_{t+1,crit}$ represents the critical value of saving without entry costs as introduced by the previous lemmata.

paying entry costs for codification or investing in capital only. The uniqueness of the solutions $(\hat{K}_{t+1}, \triangle\hat{C}_{e,t}), \bar{K}_{t+1}$ to the household's optimization problem for every w_t yields the lemma's contention.

If $\tilde{f}_t = 0$, lemma 5.1 implies $\hat{K}_{t+1,crit} = \bar{K}_{t+1,crit} = K_{t+1,crit}$. Further, from lemma 5.2 we know that if $\hat{K}_{t+1} > \hat{K}_{t+1,crit}$, $U_t^C > U_t^{woC}$. That is,

$$\begin{aligned} & u[w_t - \hat{K}_{t+1} - \gamma_t \triangle\hat{C}_{e,t}] + \delta u[(1 + r_{t+1}(\triangle\hat{C}_{e,t}))\hat{K}_{t+1}] \\ & \quad > u[w_t - \bar{K}_{t+1}] + \delta u[(1 + r_{t+1}(\triangle C_{e,t} = 0))\bar{K}_{t+1}]. \end{aligned}$$

The representative household's budget is its wage w_t. Fixed costs would reduce the budget the agent can allocate for consumption, physical capital saving and knowledge codification. A reduction of the resources to allocate must reduce utility. The reason is that utility is assumed to strictly increase in consumption. Hence reducing the budget by one unit of the homogeneous good, the agent must abstain from one unit of consumption when young, if she wants to realize the same allocation $(\hat{K}_{t+1}, \triangle\hat{C}_{e,t})$ as before or she chooses another optimal pair $(\hat{K}'_{t+1}, \triangle\hat{C}'_{e,t})$, which must lead to less lifetime utility, because otherwise it is not possible that her previous choice $(\hat{K}_{t+1}, \triangle\hat{C}_{e,t})$ was optimal. The reason is that she could always replicate $(\hat{K}'_{t+1}, \triangle\hat{C}'_{e,t})$ with the higher budget. Hence,

$$\frac{dU_t}{dw_t} > 0.$$

Since being able to choose a pair $(\hat{K}_{t+1}, \triangle\hat{C}_{e,t})$, when $\hat{K}_{t+1} > \hat{K}_{t+1,crit}$, instead of $\bar{K}_{t+1}$ leads to higher lifetime utility, utility is continuous in the budget constraint w_t and $U(w_t = 0) \leq 0$, there must be a unique $w_{net,t}$ such that

$$\begin{aligned} & u[w_{net,t} - \hat{K}_{t+1} - \gamma_t \triangle\hat{C}_{e,t}] + \delta u[(1 + r_{t+1}(\triangle\hat{C}_{e,t}))\hat{K}_{t+1}] \\ & \quad = u[w_t - \bar{K}_{t+1}] + \delta u[(1 + r_{t+1}(\triangle C_{e,t} = 0))\bar{K}_{t+1}]. \end{aligned}$$

Hence, the representative household is willing to pay a maximum entry cost of $w_t - w_{net,t} =: \tilde{f}^{max}$. With this result, we define:

$$\begin{aligned} \triangle U_t(w_t, \tilde{f}_t) \;\; := & \;\; u[w_t - \tilde{f}_t - \hat{K}_{t+1} - \gamma_t \triangle\hat{C}_{e,t}] + \delta u[(1 + r_{t+1}(\triangle\hat{C}_{e,t}))\hat{K}_{t+1}] \\ & \;\; - u[w_t - \bar{K}_{t+1}] - \delta u[(1 + r_{t+1}(\triangle C_{e,t} = 0))\bar{K}_{t+1}] \\ = & \;\; U_t^C(w_t, \tilde{f}_t) - U_t^{woC}(w_t) \end{aligned}$$

and

$$\triangle U_t(w_t, \tilde{f}^{max}) = 0.$$

Let $\Sigma'_t = S_t + \zeta_t$ be the total amount of investment in t without fixed costs.[7] In the situation where the household does not codify $\bar{\Sigma}'_t = S_t = \bar{K}_{t+1}$ and in (S2) $\hat{\Sigma}'_t =$

[7] By this, $\zeta_t = \gamma_t \triangle C_{e,t}$.

$\hat{K}_{t+1} + \gamma_t \triangle \hat{C}_{e,t}$. Due to the strict concavity of utility in consumption, we must have

$$\frac{d\Sigma'_t}{dw_t} > 0.$$

The argument is that the representative household chooses Σ'_t such that $u'(c_{1,t}) = \delta u'(c_{2,t+1}) \frac{dc_{2,t+1}}{d\Sigma'_t}$. Relaxing the budget constraint by one unit would decrease $u'(c_{1,t})$ when keeping Σ'_t constant. If Σ'_t is unchanged, it follows that $u'(c_{1,t}) < \delta u'(c_{2,t+1}) \frac{dc_{2,t+1}}{d\Sigma'_t}$. Hence, equalling out marginal utility, the agent must enhance total investment. We further know from Lemma 5.2 that for $\hat{K}_{t+1} > K_{t+1,crit}$, it is optimal to invest in codification. Hence, when relaxing the budget constraint by one unit, the increase in lifetime utility with knowledge codification must be greater than without. That is, if $\hat{K}_{t+1} > K_{t+1,crit}(\tau_t, C_t)$,

$$\frac{dU_t^C}{dw_t} > \frac{dU_t^{woC}}{dw_t}.$$

It follows that for any level of fixed entry costs to knowledge codification and $\hat{K}_{t+1} > K_{t+1,crit}(\tau_t, C_t)$,

$$\frac{d\triangle U_t(w_t, \tilde{f}_t)}{dw_t} = \frac{dU_t^C(w_t, \tilde{f}_t)}{dw_t} - \frac{dU_t^{woC}(w_t)}{dw_t} > 0.$$

Consequently, $\triangle U_t(w_t, \tilde{f}_t) = 0$ implicitly defines a function $w_t^* : \mathbb{R}_{++} \to \mathbb{R}_{++}$ which gives the wage for every fixed cost level $\tilde{f}_t$ such that the representative household enjoys the same utility in situations (S1) and (S2). As

$$\frac{dw_t^*}{d\tilde{f}_t} = -\frac{\frac{\partial \triangle U_t(w_t, \tilde{f}_t)}{\partial \tilde{f}_t}}{\frac{\partial \triangle U_t(w_t, \tilde{f}_t)}{\partial w_t}} > 0,$$

$w_t^*(\tilde{f}_t)$ is strictly increasing in $\tilde{f}_t$.
In the case with (A2), the knowledge stock τ_t and the stock of information C_t determine entry costs $\gamma_t \triangle C_{ie,t}$. Hence, at $w_t^*(\gamma_t \triangle C_{ie,t})$, $\triangle U_t = 0$. The fact that there is a unique choice of $(\hat{K}_{t+1}, \triangle \hat{C}_{e,t})$ and $\bar{K}_{t+1}$ for every w_t completes the proof.
The claim that $\triangle U_t > 0$ for $\hat{K}_{t+1} > \hat{K}_{t+1,crit}(\tau_t, C_t, \tilde{f}_t)$ follows directly from $\frac{dU_t^C}{dw_t} > \frac{dU_t^{woC}}{dw_t}$, $\frac{d\Sigma'_t}{dw_t} > 0$ and Lemma 5.2. □

Before examining the codification behavior of the economy for $\Phi = 0$ and $\Phi = 1$ separately, the following proposition applies to both cases.

Proposition 5.1 *An overlapping generations economy with $C_1 \leq q\tau_1$, where $\tau_1 > 0$, and an initial stock of capital K_1 close enough to zero will not be codifying at the beginning of its development.*

Proof. In other words, one can always find a K_1 such that for all τ_1 there exists a time interval $I = \{t | 1 \leq t \leq T\}$ in which $\triangle C_{e,t} = 0$. It is sufficient to show that there is no incentive to codify in the first period from a marginal perspective, leaving out entry costs. Lemma 5.1 and Lemma 5.2 imply that if $K_{t+1} < K_{t+1,crit}(\tau_t, C_t)$, the economy realizes the corner solution $\triangle C_{e,t} = 0$ ($\triangle C_{e,t}$ would be negative which is precluded by assumption). It follows from Lemma 5.1 and $\frac{\partial \triangle C_{e,t}}{\partial \tau_t} < 0$ that $K_{t+1,crit}(\tau_t, C_t)$ is an increasing function of τ_t. Hence, for any $\tau_1 \in \mathbb{R}_{++}$, the economy will not be codifying in the first period if $K_2 \leq K_{t+1,crit}(\tau_1, C_1)$. Equation (5.5) implies that $K_{t+1,crit}(\tau_1, C_1) > 0$. Consequently there exists a positive $K_2 \leq K_{t+1,crit}(\tau_1, C_1)$. Since $K_{t+1} = s_t w_t$, s_t bound from above and w_t for given τ_t a continuous function of K_t where $w_t(K_t = 0) = 0$, one can always find an initial value K_1 close enough to zero such that $K_2 \leq K_{t+1,crit}(\tau_1, C_1)$ and hence $\triangle C_{e,1} = 0$. □

The intuition of the proof is the same as for lemma 5.1. That is, the Inada conditions hold for physical capital saving, but not for knowledge codification as the members of the succeeding generation are exogenously transferred a positive share of the previous period's knowledge stock and hence, the marginal benefit of the first idea to be codified is finite.

5.1 Constant Number of Ideas per Researcher and Period

For a detailed discussion on the economy's codification behavior over time, the focus will first be on the case of $\Phi = 0$. For the specification of the research process with a constant number of ideas per researcher and period, the economy will approach a stationary state in the long run. With the previous lemmata, we are now in a position to state the following proposition.

Proposition 5.2 *With constant codification costs γ, an overlapping generations economy that develops over time from initial values K_1, τ_0, C_1 close enough to zero will reach a higher stationary-state level of output than Y_{st}^{woC} if and only if there exists a period t_c in which (S1)-savings satisfy $\bar{K}_{t_c+1} > \bar{K}_{t+1,crit}(\tau_{st}^{woC}, C_{t_c}, \tilde{f}_{t_c})$.*

Proof. "$\Longrightarrow$": From Lemma 5.3 we know that $U_t^C > U_t^{woC}$ if $\hat{K}_{t+1} > \hat{K}_{t+1,crit}(\tau_t, C_t, \tilde{f}_t)$. Consider a period t_c where τ_{t_c} is close enough to τ_{st}^{woC} and $C_{t_c} < q\tau_{st}^{woC}$. If the optimal amount of (S1)-saving in this period is $\bar{K}_{t_c+1} > \bar{K}_{t+1,crit}(\tau_{t_c}, C_{t_c}, \tilde{f}_{t_c})$, the representative household wants to pay for codification of an amount $\triangle C_{e,t_c} > 0$ of ideas with positive marginal value in the next period.[8] Consequently, $C_{t_c+1} > q\tau_{t_c} \simeq q\tau_{st}^{woC}$ and the economy will subsequently move into a stationary-state level of knowledge at

$$\tau_{st} = \frac{\beta C_{st} + \varepsilon}{\frac{1}{\nu} - q(1-\beta)} > \tau_{st}^{woC} \quad , q\tau_{st} < C_{st} \leq \tau_{st}.$$

[8] This payment may also imply entry costs.

Knowledge codification in t_c implies that $\tau_{t+1} > \tau_{t+1}^{woC}$, $\forall t > t_c$. Since $\frac{\partial r_{t+1}}{\partial \tau_{t+1}} > 0$ and from condition (c3) of the utility function $\frac{\partial s_t}{\partial r_{t+1}} \geq 0$, we must have $s_{st} \geq s_{t_c}$. Consequently, $K_{st} \geq \bar{K}_{t_c+1} > \bar{K}_{t+1,crit}(\tau_{st}^{woC}, C_{t_c}, \tilde{f}_{t_c}) \geq K_{st}^{woC}$. As output is an increasing function of knowledge and capital, it immediately follows that $Y_{st} > Y_{st}^{woC}$.

The proposition states that it is sufficient to observe one period t with $\bar{K}_{t+1} > \bar{K}_{t+1,crit}(\tau_{st}^{woC}, C_t, \tilde{f}_t)$ to know that the economy will realize a higher output as a consequence of knowledge codification. However, in such a period t, τ_t may not be close enough to τ_{st}^{woC} to ensure that $\tau_{t+1} > \tau_{st}^{woC}$. The result still holds because as the initial stocks of capital and knowledge K_1, τ_t are close enough to zero, the economy approaches its stationary state from below. Since in this case the knowledge stock increases monotonically, so will the capital stock. Hence, if $\exists t_k$, $\bar{K}_{t_k+1} > \bar{K}_{t+1,crit}(\tau_{st}^{woC}, C_{t_k}, \tilde{f}_{t_k})$ it follows that there must be a period $t_c > t_k$, where $\bar{K}_{t_c+1} > \bar{K}_{t+1,crit}(\tau_{st}^{woC}, C_{t_c}, \tilde{f}_{t_c})$ and τ_{t_c} is close enough to τ_{st}^{woC}.[9]

"$\Longleftarrow$": Suppose the economy realizes stationary-state output $Y_{st} > Y_{st}^{woC}$. With output being an increasing function of knowledge and capital, it follows that $K_{st} \geq K_{st}^{woC}$ and $\tau_{st} > \tau_{st}^{woC}$. By definition, $K_{st} > K_{st}^{woC}$ can only be the case if $\tau_{st} > \tau_{st}^{woC}$. For the latter to hold, the stationary-state level of information must exceed $q\tau_{st}^{woC}$. From Lemma 5.3, $\triangle C_{e,t}$ is only positive if $\bar{K}_{t+1} > \bar{K}_{t+1,crit}(\tau_t, C_t, \tilde{f}_t)$. Hence, for $C_{st} > q\tau_{st}^{woC}$, $\triangle C_{e,t}$ must be positive if τ_t is close enough to τ_{st}^{woC} and consequently there must be a period t_c in which $\bar{K}_{t_c+1} > \bar{K}_{t+1,crit}(\tau_{st}^{woC}, C_{t_c}, \tilde{f}_{t_c})$. □

Note that the assumption that the economy develops from a low initial level of capital K_1 is necessary to make the statement that it is sufficient to observe one period in which $\bar{K}_{t+1} > \bar{K}_{t+1,crit}(\tau_{st}^{woC}, C_t, \tilde{f}_t)$ to infer that the economy will realize a higher level of output in the long run due to knowledge codification activities. The reason is that if, for example, $\bar{K}_2 > \bar{K}_{2,crit}(\tau_{st}^{woC}, C_1, \tilde{f}_1)$, the economy will be codifying in the first period, but, in case of a low initial level of knowledge, it is not clear whether it will still do so when τ_t is close enough to τ_{st}^{woC}. It may be that $K_{st} << K_{t+1,crit}(\tau_{st}^{woC}, C, \tilde{f})$ and the economy will stop codifying before the stock of information C exceeds $q\tau_{st}^{woC}$.[10]

The following discussion explicitly includes the role of the codification costs. It seems reasonable to assume that they fall over time due to technical achievements such as typewriters or computers.

[9] In greater detail, the argument is that with $\tilde{f}_{t_k} \leq f + \gamma_t(q\tau_{st}^{woC} - C_{t_k})$, [the inequality depending on whether f has to be paid or not,] the economy will be codifying in period t_k by the assumption that $\bar{K}_{t_k+1} > \bar{K}_{t+1,crit}(\tau_{st}^{woC}, C_{t_k}, \tilde{f}_{t_k})$. Consequently we must have $C_{t_k+1} > C_{t_k}$ and $\tilde{f}_{t_k+1} = \gamma_t(q\tau_{st}^{woC} - C_{t_k+1}) < \tilde{f}_{t_k}$. As $C_{t_k+1} < q\tau_{st}^{woC}$, otherwise t_k would satisfy the requirements for t_c, C_{t_k+1} does only play a role for the fixed costs not for the marginal considerations as reflected in equation 5.5 and lemma 5.1. By this, $\bar{K}_{t+1,crit}(\tau_{st}^{woC}, C_{t_k+1}, \tilde{f}_{t_k+1}) \leq \bar{K}_{t+1,crit}(\tau_{st}^{woC}, C_{t_k}, \tilde{f}_{t_k})$ and due to $\tau_{t_k+1} > \tau_{t_k}$, $\bar{K}_{t_k+1} > \bar{K}_{t_k}$. Hence, the economy will also codify in $t_k + 1$ and so on at least until there is a period t_c close enough to τ_{st}^{woC} and the above line of the argument applies.

[10] C can be any value smaller $q\tau_{st}^{woC}$ and $\tilde{f} \leq f + \gamma_t(q\tau_{st}^{woC} - C)$.

Proposition 5.3 *For every overlapping generations economy there is a finite maximum entry cost f that will prevent knowledge codification. If $f = 0$, then independent of its codification history, that is, independent of (A2), every economy will start to codify if codification costs approach zero, $\gamma_t \to 0$.*

Proof. The first part of the proposition is a direct consequence of lemma 5.3. For every w_t there is a finite f_t^{max} such that $U_t^C = U_t^{woC}$. Denote by $f^{max} = \sup_t\{f_t^{max}\}$. This supremum must exist as there is a maximum set-up cost, f_{st}^{max}, that the household would be willing to pay in stationary state without codification such that $\triangle U_t(w_{st}^{woC}, f_{st}^{max}) = 0$. Hence, for all $f \geq f^{max}$, there is no period in which the economy would start to codify.

For $f = 0$, in an arbitrary period t, the representative household would like to codify an amount $\triangle C_t$, if and only if

$$\begin{aligned} u[w_t - \hat{K}_{t+1} - \gamma_t \triangle \hat{C}_{e,t} - \gamma_t \triangle C_{ie,t}] + \delta u[(1 + r_{t+1}(\triangle \hat{C}_{e,t}))\hat{K}_{t+1}] \\ \geq u[w_t - \bar{K}_{t+1}] + \delta u[(1 + r_{t+1}(\triangle C_{e,t} = 0))\bar{K}_{t+1}]. \end{aligned}$$

Since $\frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} > 0$ and $\gamma_t \to 0$ it is obvious that

$$\begin{aligned} \lim_{\gamma_t \to 0} u[w_t - \hat{K}_{t+1} - \gamma_t \triangle \hat{C}_{e,t} - \gamma_t \triangle C_{ie,t}] + \delta u[(1 + r_{t+1}(\triangle \hat{C}_{e,t}))\hat{K}_{t+1}] \\ = u[w_t - \bar{K}_{t+1}] + \delta u[(1 + r_{t+1}(\triangle C_{e,t} > 0))\bar{K}_{t+1}] \\ > u[w_t - \bar{K}_{t+1}] + \delta u[(1 + r_{t+1}(\triangle C_{e,t} = 0))\bar{K}_{t+1}]. \end{aligned}$$

□

Instead of codification costs approaching zero, it may be that they drop by a certain amount, e.g. as a consequence of an innovation.[11]

Proposition 5.4 *For overlapping generations economies close enough to stationary state in which $C_{st} < q\tau_{st}^{woC}$ and with equal levels of capital K_{st}^{woC}, the codification costs γ must drop the more the higher the stationary-state level of knowledge in order to induce knowledge codification.*

Proof. The first question is whether economies with identical stationary-state levels of capital K_{st}^{woC} can exhibit different stationary-state values of knowledge. We want to preclude that they possess different aggregate production functions. From section 4.2, we have $\tau_{st}^{woC} = \frac{\nu\varepsilon}{1-\nu q}$ and $K_{st}^{woC} = (s_{st}^{woC})^{\frac{1}{1-\alpha}}(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}\varepsilon^{\frac{\alpha}{1-\alpha}}$. Hence, a higher number of ideas per period ε (creativity) or a higher transfer rate q would enhance τ_{st}^{woC}. However, a higher creativity would directly increase K_{st}^{woC} and q would indirectly increase K_{st}^{woC} via the saving rate s_{st}^{woC}.[12] Differences in the saving rate may result from different

[11] This innovation would be exogenous to the model.

[12] As $\varsigma_t = 0$ in stationary state, the saving rate writes

$$s_{st}^{woC} = \frac{1}{\delta^{-\frac{1}{\theta}}(1 + r_{st}^{woC})^{\frac{\theta-1}{\theta}} + 1}$$

inter-temporal elasticities of utility or different rates of time preference. Let's consider economies with different triples (δ, ε, q). As s_{st}^{woC} is increasing in δ,[13] so is K_{st}^{woC}. Let $K_{st,0}^{woC}$ be characterized by $(\delta_0, \varepsilon_0, q_0)$. The implicit function theorem then implies that there are open neighborhoods $D_0, E_0, Q_0 \subset \mathbb{R}_+$ around δ_0, ε_0 and q_0 respectively and a single valued function $\delta : E_0 \times Q_0 \to D_0$ such that $K_{st,0}^{woC} = constant$. Hence, the model allows for economies achieving equal values of capital, but different levels of knowledge in stationary state.

The representative agent plans to save $K_{t+1} = K_{st}^{woC} < K_{t+1,crit}(\tau_{st}^{woC}, 0)$, given a certain level of codification costs $\hat{\gamma}$. This condition must hold because the economy is assumed to not codify in period t. Denote by $\tilde{\gamma}_t$ the level of codification costs that implies $\triangle C_{e,t} = 0$ as an interior solution for given K_{t+1}, τ_t, C_t. $\tilde{\gamma}_t$ is written as

$$\tilde{\gamma}_t = K_{t+1} \left(\frac{K_{t+1}^{1-\alpha}}{\nu\beta} \left(\frac{1-\alpha}{\alpha^2}\right)^{\alpha-1} \varepsilon^{1-\alpha} - \frac{\alpha^2-\nu}{\nu\beta}\varepsilon + \frac{q\tau_t}{\beta} + \max\{0, C_t - q\tau_t\} \right)^{-1}.$$

Hence, the codification costs would have to drop by more than $\triangle\gamma = \hat{\gamma} - \tilde{\gamma}(K_{st}^{woC}, \tau_{st}^{woC}, C_{st})$ in order to induce codification. Holding K_{st}^{woC} constant as we compare economies with identical stocks of capital, comparative statics with respect to ε and q give

$$\frac{\partial\tilde{\gamma}}{\partial\varepsilon} = -\frac{K_{st}^{woC}\left((1-\alpha)\frac{(K_{st}^{woC})^{1-\alpha}}{\nu\beta}\left(\frac{1-\alpha}{\alpha^2}\right)^{\alpha-1}\varepsilon^{-\alpha} - \frac{\alpha^2-\nu}{\nu\beta} + \frac{\nu q}{(1-\nu q)\beta}\right)}{\left(\frac{(K_{st}^{woC})^{1-\alpha}}{\nu\beta}\left(\frac{1-\alpha}{\alpha^2}\right)^{\alpha-1}\varepsilon^{1-\alpha} - \frac{\alpha^2-\nu}{\nu\beta}\varepsilon + \frac{q\tau_{st}^{woC}}{\beta}\right)^2} < 0,$$

$$\frac{\partial\tilde{\gamma}}{\partial q} = -\frac{K_{st}^{woC}\left(\frac{\nu\varepsilon}{\beta(1-\nu q)^2}\right)}{\left(\frac{(K_{st}^{woC})^{1-\alpha}}{\nu\beta}\left(\frac{1-\alpha}{\alpha^2}\right)^{\alpha-1}\varepsilon^{1-\alpha} - \frac{\alpha^2-\nu}{\nu\beta}\varepsilon + \frac{q\tau_{st}^{woC}}{\beta}\right)^2} < 0.$$

Hence, it is clear that $\triangle\gamma$ increases in q and ε or, equivalently, in τ_{st}^{woC}. □

This result is interesting in the following situation.

Corollary 5.1 *Consider two economies close enough to stationary state with* $K_{1,st}^{woC} = K_{2,st}^{woC}$ *and let one be more sophisticated* $\tau_{1,st}^{woC} > \tau_{2,st}^{woC}$ *due to a higher creativity* $\varepsilon_1 > \varepsilon_2$. *There exist codification costs* γ *and parameter values for* $\alpha, q, \theta, \delta_j, \varepsilon_j$, $j \in \{1, 2\}$ *such that the initially less sophisticated economy 2 will reach a higher output level than economy 1 in the long run.*

Proof. As $Y_{st}^{woC} = (K_{st}^{woC})^\alpha \left(\frac{1-\alpha}{\alpha^2}\right)^{1-\alpha} \varepsilon^\alpha \frac{\nu}{1-\nu q}$, $K_{st}^{1,woC} = K_{st}^{2,woC}$ implies $Y_{st}^{1,woC} > Y_{st}^{2,woC}$ whenever $\varepsilon_1 > \varepsilon_2$. That is, the more sophisticated economy realizes a higher level of output. Proposition 5.4 suggests that there is a γ_j for each economy j at which the

and $r_{st}^{woC} = (s_{st}^{woC})^{-1}(1-\alpha)^{\alpha-1}\alpha^{-2\alpha}\left(\frac{1-\alpha}{\alpha^2}\right)^{-\alpha}\left(\frac{\nu}{1-\nu q}\frac{1-\alpha}{\alpha^2} - 1 + \alpha\right)$. Hence, s_{st}^{woC} does not depend on ε. Using the implicit function theorem gives that s_{st}^{woC} is unique and $\frac{\partial s_{st}^{woC}}{\partial q} > 0$.

[13]The argument again uses the implicit function theorem.

amount of first period consumption they are willing to forgo for knowledge codification is just zero[14] and $\gamma_1 < \gamma_2$. This generates an interval of codification costs $[\gamma_1, \gamma_2)$ for which economy 2 would start to codify, but not economy 1. In this case, economy 2 would approach a new stationary-state level of knowledge at

$$\tau_{2,st} = \frac{\nu\beta C_{2,st} + \nu\varepsilon_2}{1-\nu q(1-\beta)} > \tau_{2,st}^{woC},\ q\tau_{2,st} < C_{2,st} \leq \tau_{2,st}.$$

Note that by the same argument as in the proof of proposition 5.2, $K_{2,st} \geq K_{2,st}^{woC}$. The formerly less sophisticated economy 2 would realize a higher stationary-state output than economy 1 if

$$\left(\frac{K_{2,st}}{K_{1,st}^{woC}}\right)^{\alpha} > \left(\frac{\varepsilon_2}{\varepsilon_1}\right)^{1-\alpha} \frac{\frac{\nu\varepsilon_1}{1-\nu q}}{\frac{\nu\beta C_{2,st}+\nu\varepsilon_2}{1-\nu q(1-\beta)}}. \tag{5.6}$$

The condition only holds if economy 2's stock of information in the long run satisfies

$$C_{2,st} > \frac{1-\nu q(1-\beta)}{\nu\beta}\tau_{1,st}^{woC}\left(\frac{K_{1,st}^{woC}}{K_{2,st}}\right)^{\alpha}\left(\frac{\varepsilon_2}{\varepsilon_1}\right)^{1-\alpha} - \frac{\varepsilon_2}{\beta}.$$

In stationary state $\triangle C_{e2,t}(K_{2,st}, \tau_{2,st}, C_{2,st}) = 0,\ \forall t$. Given $\gamma < \gamma_2$, this implies[15]

$$C_{2,st} = (1-\nu q(1-\beta))\left(\frac{K_{2,st}}{\gamma} - \frac{K_{2,st}^{1-\alpha}}{\nu\beta}\left(\frac{1-\alpha}{\alpha^2}\right)^{\alpha-1}\varepsilon_2^{1-\alpha} + \frac{\alpha^2-\nu}{\nu\beta}\varepsilon_2\right) - \nu\varepsilon_2 q\left(\frac{1-\beta}{\beta}\right).$$

We choose $\gamma = \gamma_1$ such that economy 1's stock of information will stay below or equal to $q\tau_{st}^{woC}$. From the proof of proposition 5.4, the codification costs then write

$$\gamma_1 = K_{1,st}^{woC}\left(\frac{(K_{1,st}^{woC})^{1-\alpha}}{\nu\beta}\left(\frac{1-\alpha}{\alpha^2}\right)^{\alpha-1}\varepsilon_1^{1-\alpha} - \frac{\alpha^2-\nu}{\nu\beta}\varepsilon_1 + \frac{q\tau_{1,st}^{woC}}{\beta}\right)^{-1}.$$

Consequently, economy 2 will approach the following stationary-state level of information:

$$\begin{aligned} C_{2,st} \;=\; & (1-\nu q(1-\beta))\left(\frac{K_{2,st}}{K_{1,st}^{woC}}\left(\frac{(K_{1,st}^{woC})^{1-\alpha}}{\nu\beta}\left(\frac{1-\alpha}{\alpha^2}\right)^{\alpha-1}\varepsilon_1^{1-\alpha} - \frac{\alpha^2-\nu}{\nu\beta}\varepsilon_1 + \frac{q\tau_{1,st}^{woC}}{\beta}\right)\right. \\ & \left. - \frac{K_{2,st}^{1-\alpha}}{\nu\beta}\left(\frac{1-\alpha}{\alpha^2}\right)^{\alpha-1}\varepsilon_2^{1-\alpha} + \frac{\alpha^2-\nu}{\nu\beta}\varepsilon_2\right) - \nu\varepsilon_2 q\left(\frac{1-\beta}{\beta}\right). \end{aligned}$$

Using this expression, the condition for economy 2 to reach a higher stationary-state level of output can be transformed into

$$\begin{aligned} & \left(\frac{1-\alpha}{\alpha^2}\right)^{\alpha-1}\left(\frac{K_{2,st}}{K_{1,st}^{woC}}(K_{1,st}^{woC})^{1-\alpha}\varepsilon_1^{1-\alpha} - K_{2,st}^{1-\alpha}\varepsilon_2^{1-\alpha}\right) - (\alpha^2-\nu)\left(\frac{K_{2,st}}{K_{1,st}^{woC}}\varepsilon_1 - \varepsilon_2\right) \\ & + \tau_{1,st}^{woC}\left(\frac{K_{2,st}}{K_{1,st}^{woC}}\nu q - \left(\frac{K_{1,st}^{woC}}{K_{2,st}}\right)^{\alpha}\left(\frac{\varepsilon_2}{\varepsilon_1}\right)^{1-\alpha}\right) + \nu\varepsilon_2 > 0. \end{aligned}$$

[14]The interior solution is zero.
[15]Equation 5.5 is used.

Inserting $K_{j,st} = s_{j,st}^{\frac{1}{1-\alpha}}(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}\varepsilon_j^{\frac{\alpha}{1-\alpha}}$ yields

$$\alpha^2\left(\left(\frac{s_{2,st}}{s_{1,st}^{woC}}\right)^{\frac{1}{1-\alpha}}\left(\frac{\varepsilon_2}{\varepsilon_1}\right)^{\frac{\alpha}{1-\alpha}}s_{1,st}^{woC}\varepsilon_1 - s_{2,st}\varepsilon_2\right)-(\alpha^2-\nu)\left(\left(\frac{s_{2,st}}{s_{1,st}^{woC}}\right)^{\frac{1}{1-\alpha}}\left(\frac{\varepsilon_2}{\varepsilon_1}\right)^{\frac{\alpha}{1-\alpha}}\varepsilon_1-\varepsilon_2\right)$$
$$+\frac{\nu\varepsilon_1}{1-\nu q}\left(\left(\frac{s_{2,st}}{s_{1,st}^{woC}}\right)^{\frac{1}{1-\alpha}}\left(\frac{\varepsilon_2}{\varepsilon_1}\right)^{\frac{\alpha}{1-\alpha}}\nu q-\left(\frac{s_{2,st}}{s_{1,st}^{woC}}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{\varepsilon_2}{\varepsilon_1}\right)^{\frac{1-2\alpha}{1-\alpha}}\right)+\nu\varepsilon_2 > 0.$$

From this expression, is not clear whether there are reasonable parameter values that satisfy this condition. Let θ be one, that is, assume logarithmic utility. The saving rate in stationary state is then a function of the discount factor, only:

$$s_{j,st} = \frac{1}{\delta_j^{-1}+1}.$$

From $K_{1,st}^{woC} = K_{2,st}^{woC}$ follows that

$$\frac{s_{1,st}}{s_{2,st}} = \frac{\delta_2^{-1}+1}{\delta_1^{-1}+1} = \left(\frac{\varepsilon_2}{\varepsilon_1}\right)^{\alpha}.$$

This simplifies the condition for $Y_{2,st} > Y_{1,st}^{woC}$ to

$$\alpha^2 s_{2,st}\left(\left(\frac{\varepsilon_2}{\varepsilon_1}\right)^{\alpha}\varepsilon_1-\varepsilon_2\right)-(\alpha^2-\nu)(\varepsilon_1-\varepsilon_2)+\frac{\nu\varepsilon_1}{1-\nu q}\left(\nu q-\left(\frac{\varepsilon_2}{\varepsilon_1}\right)^{1-\alpha}\right)+\nu\varepsilon_2 > 0.$$

Choosing $\alpha = 0.5$ yields a quadratic term with respect to ε_j. Assuming a transfer rate of $q = 0.95$ and a discount factor of economy 2 of $\delta_2 = 0.7$, the condition transforms into

$$0.237805\varepsilon_1 - 0.384964\sqrt{\varepsilon_1\varepsilon_2} + 0.147059\varepsilon_2 > 0.$$

It will hold for all $\varepsilon_2 < \varepsilon_1$. Hence, for $\gamma = \gamma_1,\ \theta = 1,\ \alpha = 0.5,\ q = 0.95,\ \varepsilon_2 < \varepsilon_1,\ \delta_2 = 0.7$, and $\delta_1 = \left(\frac{17}{7}\sqrt{\frac{\varepsilon_1}{\varepsilon_2}}-1\right)^{-1}$, economy 2 will realize a higher level of output and, thus, a higher level of each generation's lifetime utility than economy 1 in the long run.[16] □

The intuition of proposition 5.4 is that an economy with a lower rate of time preference is more likely to transfer consumption to the next period. Further less creative researchers leading to a lower stationary-state level of knowledge involves a lower negative influence of the knowledge stock on the incentive for knowledge codification. In this way, it is clear that the individuals of this economy are willing to forgo consumption for knowledge codification at higher costs than less patient individuals of a more sophisticated economy. Interestingly enough, as shown in corollary 5.6, this may lead to a kind of leapfrogging, where an initially less sophisticated economy with lower stationary-state output will reach higher levels of output in the long run than an initially more sophisticated economy if the codification costs stay in the respective interval for a sufficiently long time. If thereafter the codification costs would increase again, the order of output levels would persist due to the assumed longevity of information.

[16]Notice that there are also parameter values for which the condition does not hold.

5.2 Standing on the Shoulders of Giants

The discussion of the economy's codification behavior over time in the case of $\Phi = 1$ will mainly concentrate on whether the overlapping generations economy will be codifying in the long run. "Codifying in the long run" or "codifying from some point in time on" means that there does not exist a period t_0, such that for all $t > t_0, \triangle C_{e,t} = 0$.

Proposition 5.5 *With constant codification costs, an overlapping generations economy will be codifying in the long run if either of the following conditions is satisfied:*

(i) The steady-state growth rate of capital is higher than that of the knowledge stock (or equivalently $\alpha > 0.5$).

(ii) The steady-state growth rate of capital is equal to that of the knowledge stock (or equivalently $\alpha = 0.5$) and

$$\frac{\left(\frac{1-\alpha}{\alpha^2}\right)^{\alpha-1}(\varepsilon q)^{1-\alpha}}{\nu\beta(k_s^{woC})^{\alpha}(1+g_\tau^{woC})^{\frac{\alpha^2}{1-\alpha}}} < \frac{1}{\gamma} + \frac{\varepsilon q(\alpha^2-\nu)-\nu q}{\nu\beta k_s^{woC}(1+g_\tau^{woC})^{\frac{\alpha}{1-\alpha}}}, \tag{5.7}$$

where k_s^{woC} is the steady-state level of $k_t = \frac{K_t}{\tau_t}$ without codification.

Proof. We will prove this result by contradiction. The intuition is the following. No codification implies that the economy approaches steady-state growth. It also requires that $\forall t > t_0$, $(\tau_t, C_t, K_{t+1}) \notin \mathcal{U}$. If (i), steady-state growth causes K_{t+1} to grow more than $K_{t+1,crit}$, $\forall t$. If (ii), the steady-state level of k involves $K_{t+1} > K_{t+1,crit}$. Hence, for (i) and (ii), steady-state growth and no codification are contradictory implying that the economy will be codifying in the long run.

With regard to (i), suppose an economy characterized by $\alpha > 0.5$ will not be codifying in the long run. That is, $\exists t_0$, such that $\forall t > t_0$, $\triangle C_{e,t} = 0$. Consequently, the overlapping generations economy must approach steady-state growth where $g_{K,s} = \frac{\alpha}{1-\alpha} g_{\tau,s}$. $\alpha > 0.5$ implies that $g_{K,s} > g_{\tau,s}$. Further let $\triangle C_{e,t_0} = 0$. No codification in the long run means that it is optimal for each subsequent generation to invest in capital only. Hence, $\forall t > t_0$,

$$\triangle C_{e,t} = \frac{K_{t+1}}{\gamma} - AK_{t+1}^{1-\alpha}\tau_t^{1-\alpha} - B\tau_t - \max\{0, C_t - q\tau_t\} \leq 0, \tag{5.8}$$

where $A = \left(\frac{1-\alpha}{\alpha^2}\right)^{\alpha-1}\frac{(\varepsilon q)^{1-\alpha}}{\nu\beta}$ and $B = \frac{\nu q - (\alpha^2-\nu)\varepsilon q}{\nu\beta} > 0$.

Since by assumption the economy does not codify, the maximum term can be dropped without loss of generality,[17] and the condition can be rewritten as

$$\frac{K_{t+1}}{\gamma} \leq AK_{t+1}^{1-\alpha}\tau_t^{1-\alpha} + B\tau_t.$$

[17] From $\triangle C_{e,t} = 0$ by assumption and τ_t growing at a constant rate, it follows that the maximum term must be 0 from some point in time on. However, since $\max\{0, C_t - q\tau_t\} \leq (1-q)\tau_t$, we could also estimate $B\tau_t + \max\{0, C_t - q\tau_t\}$ from above by $D\tau_t$ with an appropriate $D > 0$ without affecting the results.

For $t \to \infty$ both sides of the equation grow without bound. As the economy is growing with constant rates in the long run, (5.8) will hold for all t if the left hand side is growing at a rate lower than or equal to that of the right hand side. Dividing both sides by τ_t and log-differentiating gives

$$g_{K,t+1} - g_{\tau,t} \leq \frac{AK_{t+1}^{1-\alpha}\tau_t^{-\alpha}((1-\alpha)g_{K,t+1} - \alpha g_{\tau,t})}{AK_{t+1}^{1-\alpha}\tau_t^{-\alpha} + B}.$$

Inserting the steady-state relation of the growth rates of capital and knowledge ($g_{K,s} = \frac{\alpha}{1-\alpha} g_{\tau,s}$) transforms the above inequality into

$$g_{K,s} \leq g_{\tau,s} \quad \text{or} \quad \alpha \leq 1-\alpha.$$

This contradicts the presumption of $g_{K,s} > g_{\tau,s}$ or $\alpha > 0.5$, respectively.

Consider (ii) where $g_{K,s} = g_{\tau,s}$. Again suppose the economy does not codify but satisfies (5.7). Without codification the economy approaches steady-state behavior in the long run, which implies $k_t = \mathit{constant}$. Note that with CIES-utility the steady-state equilibrium without knowledge codification is unique. In accordance with the household's utility maximization, no codification in the long run involves that $\forall t > t_0$, equation (5.8) is satisfied. Inserting $K_{t+1} = k_s^{woC} \tau_t^{\frac{\alpha}{1-\alpha}} (1 + g_\tau^{woC})^{\frac{\alpha}{1-\alpha}}$ yields

$$\frac{\left(\frac{1-\alpha}{\alpha^2}\right)^{\alpha-1} (\varepsilon q)^{1-\alpha}}{\nu\beta(k_s^{woC})^\alpha (1 + g_\tau^{woC})^{\frac{\alpha^2}{1-\alpha}}} \geq \frac{1}{\gamma}\tau_t^{\frac{2\alpha-1}{1-\alpha}} + \frac{\varepsilon q(\alpha^2 - \nu) - \nu q}{\nu\beta k_s^{woC}(1 + g_\tau^{woC})^{\frac{\alpha}{1-\alpha}}}.$$

By assumption $\alpha = 0.5$ and, hence,

$$\frac{\left(\frac{1-\alpha}{\alpha^2}\right)^{\alpha-1} (\varepsilon q)^{1-\alpha}}{\nu\beta(k_s^{woC})^\alpha (1 + g_\tau^{woC})^{\frac{\alpha^2}{1-\alpha}}} \geq \frac{1}{\gamma_t} + \frac{\varepsilon q(\alpha^2 - \nu) - \nu q}{\nu\beta k_s^{woC}(1 + g_\tau^{woC})^{\frac{\alpha}{1-\alpha}}}.$$

This contradicts (5.7). As a consequence, the conditions of proposition 5.5 preclude that there is a t_0 such that $\forall t > t_0$, $\triangle C_{e,t} = 0$. Hence, the economy will codify in the long run. □

It is now assumed that the codification costs decline monotonically over time at a constant rate g_γ. We can then state the following result.

Proposition 5.6 *An overlapping generations economy will be codifying in the long run if the steady-state growth rate of knowledge exceeds that of capital by less than the rate at which the codification costs decline.*

Proof. The proof uses a similar reasoning as that of proposition 5.5. Suppose the economy does not codify in the long run and $g_{\tau,s} - g_{K,s} < -g_\gamma$. Let $\triangle C_{e,t_0} = 0$. Further, (5.8) must hold $\forall t > t_0$. Neglecting the maximum term in (5.8) and log-differentiating yields

$$g_{K,t+1} - g_{\tau,t} - g_\gamma \leq \frac{AK_{t+1}^{1-\alpha}\tau_t^{-\alpha}((1-\alpha)g_{K,t+1} - \alpha g_{\tau,t})}{AK_{t+1}^{1-\alpha}\tau_t^{-\alpha} + B}.$$

Without knowledge codification the economy approaches steady-state behavior. Inserting the relation of the steady-state growth rates, the inequality above can be written as

$$g_{\tau,s} - g_{K,s} \geq -g_\gamma.$$

This contradicts $g_{\tau,s} - g_{K,s} < -g_\gamma$. □

It further follows:

Corollary 5.2 *Every overlapping generations economy will be codifying from some point in time on if the rate at which the codification costs decline is greater than or equal to the steady-state growth rate of the knowledge stock.*

Proof. This is a direct result of the proof of proposition 5.6 as $g_{\tau,s} - g_{K,s} \geq -g_\gamma$ can be transformed into

$$\alpha \leq \frac{1+\frac{g_\gamma}{g_{\tau,s}}}{2+\frac{g_\gamma}{g_{\tau,s}}}, \quad -g_\gamma < 2g_{\tau,s};$$

$$\alpha \geq \frac{1+\frac{g_\gamma}{g_{\tau,s}}}{2+\frac{g_\gamma}{g_{\tau,s}}}, \quad -g_\gamma > 2g_{\tau,s}.$$

The condition $g_{\tau,s} \leq -g_\gamma < 2g_{\tau,s}$ implies that for the economy to not codify in the long run, α must be smaller than or equal to 0. If $-g_\gamma > 2g_{\tau,s}$, α must exceed 1 for zero knowledge codification. This contradicts $\alpha \in (0,1)$, as assumed in section 11.3.1 for all overlapping generations economies. □

Having discussed under what conditions the economy will start to codify sometime, the following proposition focuses on full codification, that is, at the end of each period t, $C_{t+1} = \tau_t$. For simplicity of the argument, let $-g_\gamma < 2g_{\tau,s}$ for the remainder of the paper without loss of generality.

Proposition 5.7 *An overlapping generations economy in steady-state equilibrium that satisfies the condition*

$$\alpha > \frac{1+\frac{g_\gamma}{g_{\tau,s}}}{2+\frac{g_\gamma}{g_{\tau,s}}} \tag{5.9}$$

is codifying fully.

Proof. First, as shown in the proof of proposition 5.6, an overlapping generations economy that satisfies the condition given in the above proposition will start to codify at some point in time. The number of efficient ideas to be codified $\triangle C_{e,t}$ is chosen according to (5.5), which can be written as

$$\frac{\triangle C_{e,t} + \max\{0, C_t - q\tau_t\}}{\tau_t} = \frac{K_{t+1}}{\tau_t \gamma_t} - AK_{t+1}^{1-\alpha}\tau_t^{-\alpha} - B. \tag{5.10}$$

Since the economy is assumed to be in steady state, $\triangle C_{e,t}$ must be greater than 0 (because (5.9) precludes a steady state where $\triangle C_{e,t} = 0$). Hence, above's equality must hold as long as $\triangle C_{e,t}$ possesses an interior solution. Being in steady state, both sides of (5.10) must grow at equal rates. Log-differentiating yields

$$g_{\triangle C_e + \max\{0, C - q\tau\},t} - g_{\tau,t} = \frac{\frac{K_{t+1}}{\tau_t\gamma_t}(g_{K,t+1} - g_{\tau,t} - g_\gamma) - \frac{AK_{t+1}^{1-\alpha}}{\tau_t^{\alpha}}[(1-\alpha)g_{K,t+1} - \alpha g_{\tau,t}]}{\frac{K_{t+1}}{\tau_t\gamma_t} - AK_{t+1}^{1-\alpha}\tau_t^{-\alpha} - B}.$$

Inserting the steady-state relation of the growth rates of knowledge and capital transforms the above equation into

$$g_{\triangle C_e + \max\{0, C - q\tau\},s} - g_{\tau,s} = \frac{\frac{K_{t+1}}{\tau_t\gamma_t}(g_{K,s} - g_{\tau,s} - g_\gamma)}{\frac{K_{t+1}}{\tau_t\gamma_t} - AK_{t+1}^{1-\alpha}\tau_t^{-\alpha} - B}.$$

As the denominator of the right hand side is positive (because $\triangle C_{e,t} > 0$), the entire fraction will be positive whenever $g_{K,s} - g_{\tau,s} - g_\gamma > 0$. The latter can be directly transformed into the condition given in the proposition. The right side being positive implies that the amount of ideas to be codified grows faster than the knowledge stock. This violates the steady-state condition that for $\triangle C_{e,t} > 0$, $g_{\triangle C_e + \max\{0, C - q\tau\},s} = g_{\tau,s}$. The only possibility to satisfy this condition is for $\triangle C_{e,t} + \max\{0, C_t - q\tau_t\}$ to realize its upper bound at $(1-q)\tau_t$ as a corner solution. Consequently, an economy in steady state characterized by $\alpha > \frac{1+\frac{g_\gamma}{g_{\tau,s}}}{2+\frac{g_\gamma}{g_{\tau,s}}}$ must be codifying fully.
Note also that in the case of constant codification costs an economy in steady state is codifying fully if $\alpha > 0.5$, that is, if $g_{K,s} > g_{\tau,s}$. □

It can then be summarized that:

Corollary 5.3 *If $g_{\tau,s} - g_{K,s} \neq g_\gamma$, an overlapping generations economy in steady state exhibits either full or zero knowledge codification in the long run.*

Proof. This corollary is an immediate consequence of propositions 5.6 and 5.7. If $g_{\tau,s} - g_{K,s} > g_\gamma$, full codification directly follows from proposition 5.7.
With regard to $g_{\tau,s} - g_{K,s} < g_\gamma$, the proof of proposition 5.6 implies that this condition is contradictory to $\triangle C_{e,t} > 0$, $\forall t$. Hence, the economy cannot realize steady-state growth with positive codification. From proposition 5.6's proof further follows that if an economy shows steady-state behavior $\forall t > t_0$ and $\triangle C_{e,t_0} = 0$, then $\triangle C_{e,t} = 0$, $\forall t > t_0$.

As a consequence, only steady-state growth without codification is consistent with the condition $g_{\tau,s} - g_{K,s} < g_{\gamma}$. □

This section precluded possible entry costs to knowledge codification until now. The following proposition considers fixed entry costs that have to be paid once at the beginning of knowledge codification as a kind of set-up costs.

Proposition 5.8 *An overlapping generations economy that started to codify without fixed entry costs to knowledge codification will also do so with fixed entry costs.*

Proof. The question to be answered is whether, given the conditions under which an overlapping generations economy without fixed entry costs would be codifying in the long run, there is a period in which the representative household receives a higher life time utility by paying the fixed entry costs to knowledge codification and choosing its optimal amount of physical capital saving and knowledge codification investment than by just investing in physical capital. This is a comparison of the situations (S1) and (S2). (S2) can only yield higher life time utility if there is positive knowledge codification, otherwise the fixed cost payments were just wasted. Hence, the first question is: Would some period's representative agent be willing to codify if she had to pay the fixed entry costs. More precisely, would it be optimal in situation (S2) to choose a positive amount of knowledge codification after having paid the fixed entry costs. If so, lemma 5.3 states that there must be a $w^*(f)$ such that $U_t^C = U_t^{woC}$ and $U_t^C > U_t^{woC}$ for all $w_t > w^*(f)$. As $w^*(f)$ is finite and w_t will grow to infinity in the limit, this second condition will be given from some point in time on if the first one holds.

Consequently, consider an economy in situation (S2) where the representative agent has to pay fixed entry costs f and thereafter decides on the optimal amount of physical capital saving and knowledge codification. The constraints of her utility maximization problem can then be transformed into

$$\begin{aligned} c_{1,t} &= w_{net,t}(1 - s_t - \varsigma_t), \\ c_{2,t+1} &= (1 + r_{t+1}) s_t w_{net,t}, \end{aligned}$$

where $w_{net,t} = w_t - f$. Suppose there will be no knowledge codification for all t. Then the saving rate is a function of r_{t+1}, only. As $g_{\tau,t} = g_{\tau}^{woC} = \mathit{constant}$ and $g_{C,t} = 0$, the economy's dynamics are reflected by

$$K_{t+1} = s_t w_{net,t} = s_t (K_t^{\alpha} \tau_t^{\alpha} F - f),$$

where $F = (1-\alpha)^{1-\alpha} \alpha^{2\alpha} (\varepsilon q)^{\alpha} (1 + g_{\tau}^{woC})^{-\alpha}$. Using $k_t = \frac{K_t}{\tau_t^{\frac{\alpha}{1-\alpha}}}$, it writes

$$k_{t+1} = s_t w_{net,t} = s_t k_t^{\alpha} (1 + g_{\tau}^{woC})^{-\frac{\alpha}{1-\alpha}} F - s_t f \tau_t^{-\frac{\alpha}{1-\alpha}} (1 + g_{\tau}^{woC})^{-\frac{\alpha}{1-\alpha}}.$$

This implies that $f \tau_t^{-\frac{\alpha}{1-\alpha}} (1 + g_{\tau}^{woC})^{-\frac{\alpha}{1-\alpha}}$ is decreasing at a constant rate. As s_t is bound on the interval $[s_{low}, 1)$, we can estimate it from above by 1. Hence, the last term of k_{t+1}

in the above equation becomes arbitrarily small for large enough t. Consequently the difference equation approaches that of an economy without fixed costs in the limit. Being interested in long run behavior, this last term can be neglected for an appropriately large t and the previous propositions apply for positive codification in (S2) in the long run. □

Fixed entry costs that have to be paid once only delay an economy's knowledge codification but do not prevent it. This may be different in the case with assumption (A2). This assumption implies that the codification decision depends on the economy's codification history as previous generations that did not codify accumulated entry costs for later generations.

Proposition 5.9 *An overlapping generations economy that satisfies*

$$\alpha > \frac{1 + \frac{g_\gamma}{g_\tau^{woC}}}{2 + \frac{g_\gamma}{g_\tau^{woC}}}$$

will be codifying in the long run and independent of its codification history if either of the following conditions holds

(i) $\lim_{t\to\infty} \frac{u'(c_{1,t}^{woC})}{u'(c_{1,t}^{woC})-u'(c_{1,t}^{C})} = 0.$

(ii) $\lim_{t\to\infty} \left| \frac{u'(c_{1,t}^{woC})}{u'(c_{1,t}^{woC})-u'(c_{1,t}^{C})} \right| = M < \infty$ *and* $\frac{\alpha}{1-\alpha} F > M \left(\frac{\alpha(1+g_{K,s})^\alpha E}{1+\alpha(k_s^{woC})^{\alpha-1} E(1+g_{K,s})^{\alpha-1}} \right),$

where $E = \left(\frac{1-\alpha}{\alpha^2}\right)^{-\alpha} (\varepsilon q)^\alpha \left(\frac{1-\alpha}{\alpha^2 \varepsilon q}(1+g_\tau^{woC}) - 1 + \alpha \right) > 0$
and $F = (1-\alpha)^{1-\alpha}\alpha^{2\alpha}(\varepsilon q)^\alpha (1+g_\tau^{woC})^{-\alpha}.$

Proof. The proof's reasoning is similar to that of proposition 5.8. No codification in the long run implies that the representative agent must prefer (S1) to (S2) for all $t > t_0$, where t_0 is large enough. In (S1) the overlapping generations economy approaches steady-state behavior without knowledge codification. Again steady-state growth and no knowledge codification are contradictory for the respective values of α if either (i) or (ii) holds. As compared to the proof of proposition 5.8, an additional difficulty is that the codification history enters the representative agents decision problem. In this respect, independence of the codification history means that the economy would start to codify in some period t even if it possessed no information at all, $C_t = 0$. This involves the maximum entry costs in period t of $\gamma_t q \tau_t$. Consequently, if the economy starts to codify under this condition, it must do so with lower entry costs, as well.

The formal argument proceeds as follows. In a first step, we again verify that even if the maximum entry costs had to be paid in (S2), the economy would like to codify eventually. Second, we show that, under the conditions given in the proposition and given the economy would not codify, the hypothetical utility if the representative agent chose (S2) instead of (S1) would satisfy $dU_t^C > dU_t^{woC}$ from some point in time t_1 on. Hence, there must be a t_2, where $U_{t_2}^C > U_{t_2}^{woC}$, contradicting the assumption that the economy does not codify in the long run. The detailed proof is provided in appendix 7.4.

Chapter 6

Conclusions

The first part of thesis has given a set-theoretic formulation of the notions knowledge and information which served as a basis for the economy's knowledge dynamics. Those were generally characterized by imperfect knowledge transfer between generations which could be attenuated by purposeful and costly knowledge codification. The economic problem of knowledge codification occurs within the intermediate firm and has been depicted by a three-stage game, in which the firm owners have to invest in knowledge codification in the first stage before the retiring employees leave and also before the new employees enter the firm. The latter are asked at the second stage to take over a share of the codification costs, which they will deny because they know that at the third stage the firm owners will decide to give access to them for free as a higher level of human capital of the new employees would increase their profits. In this way, the firm owners will only codify up to the profit maximizing amount, not taking into account any positive (equilibrium) effects on the new employees' income.

Chapter 3 developed an overlapping generations framework incorporating this problem of intergenerational knowledge transfer by knowledge codification. We have seen that with a specification of production by Solow-neutral technological progress, the equilibrium wage is independent of the amount of knowledge codification in the previous period, suggesting that there is no hold-up problem between the firm owners and the newly hired employees in this particular setting. However, knowledge codification may possess positive externalities on the wage levels from the next but one period on. The considerations on the dynamics of the established overlapping generations model revealed that knowledge codification may substantially increase the growth rate of an economy's output in the 'standing on the shoulders of giants' specification, respectively, enhances the level of output in the case where each researcher generates a constant amount of ideas in each period. The analysis addressed the question under what conditions these gains in output due to endogenous knowledge codification will be realized in the established overlapping generations framework.

The main results were that in the case of constant ideas per researcher and period, the overlapping generations economy must exceed a critical level of capital in order to be willing to codify. With the economy approaching a stationary state in the long run, it is clear that if the stationary state level of capital is lower than the threshold-level for knowledge codification, the economy will stay on the lower output level without codification. Only a drop in codification costs may then induce the creation of information. Comparing economies with equal stationary state capital stocks, this drop in codification costs must be even stronger the higher is an economy's knowledge stock. In this respect, knowledge codification may cause an initially less sophisticated economy to realize higher utility levels in the long run than an initially more sophisticated one. Independent of the knowledge stock, overlapping generations economies with sufficiently low initial levels of capital will not codify at the beginning of their development. The rather hypothetical case reflected by assumption (A2) that for an idea to be codified the more basic ideas it builds upon have to be codified first, additionally implies a kind of entry cost for knowledge codification. This intensifies the situation for economies developing from low levels of capital. Clearly, every overlapping generations economy will be codifying sometime if codification costs converge to zero. However, the latter only holds true if there are no fixed set-up costs of knowledge codification that have to be paid once in the first period of codification. It is always possible to find a finite level of set-up costs that would prevent an economy from starting to codify.

With regard to the 'standing on the shoulders of giants' specification of the research process, an overlapping generations economy with fixed codification costs will be codifying in the long run if the steady state growth rate of capital is higher than that of the knowledge stock. For equal steady state growth rates of capital and knowledge, the economy will codify if the steady state relation of capital and knowledge is large enough. Allowing for monotonically decreasing codification costs over time, the analysis shows that every overlapping generations economy will be codifying in the long run if the rate at which the codification costs decline is higher than or equal to the steady state growth rate of the knowledge stock. In this case, an overlapping generations economy that shows steady state behavior is codifying fully. Loosely speaking, this means that the economy is codifying every newly discovered idea in the respective period.

These results do not change when additionally considering fixed entry costs to knowledge codification. In contrast to the case with a constant number of ideas per researcher, the entry costs will only delay but not prevent knowledge codification. This may be different in the case with assumption (A2), which implies that the entry costs of knowledge codification increase as long as the economy does not codify. Whether the above conditions for knowledge codification are still sufficient depends on the specific form of the representative household's utility function.

Chapter 7

Appendix

7.1 Sufficient Conditions for the Household's Optimization Problem

A sufficient condition for a unique maximum of the household's optimization problem is that the Hessian matrix at the critical points be negative definite. The Hessian matrix writes

$$H = \begin{pmatrix} u''(c_{1,t})+\delta u''(c_{2,t+1})(1+r_{t+1})^2 & \begin{array}{r}\gamma_t u''(c_{1,t})+\delta u''(c_{2,t+1})\frac{\partial r_{t+1}}{\partial \triangle C_{e,t}}K_{t+1}(1+r_{t+1}) \\ +\delta u'(c_{2,t+1})\frac{\partial r_{t+1}}{\partial \triangle C_{e,t}}\end{array} \\ \begin{array}{r}\gamma_t u''(c_{1,t})+\delta u''(c_{2,t+1})\frac{\partial r_{t+1}}{\partial \triangle C_{e,t}}K_{t+1}(1+r_{t+1}) \\ +\delta u'(c_{2,t+1})\frac{\partial r_{t+1}}{\partial \triangle C_{e,t}}\end{array} & \begin{array}{r}\gamma_t^2 u''(c_{1,t})+\delta u''(c_{2,t+1})\left(\frac{\partial r_{t+1}}{\partial \triangle C_{e,t}}K_{t+1}\right)^2 \\ +\delta u'(c_{2,t+1})\left(\frac{\partial^2 r_{t+1}}{\partial \triangle C_{e,t}^2}K_{t+1}\right)\end{array} \end{pmatrix}.$$

The eigenvalues of H will be negative at the critical points given by

$$\begin{aligned} M_1 &= -u'(c_{1,t}) + \delta u'(c_{2,t+1})(1+r_{t+1}) = 0, \\ M_2 &= -\gamma_t u'(c_{1,t}) + \delta u'(c_{2,t+1})\frac{\partial r_{t+1}}{\partial \triangle C_{e,t}}K_{t+1} = 0, \end{aligned}$$

which together yield

$$M = -\gamma_t(1+r_{t+1}) + \frac{\partial r_{t+1}}{\partial \triangle C_{e,t}}K_{t+1} = 0,$$

if the first principal minor is negative and the second principal minor is positive. The first principal minor is negative due to the concavity of U_t in K_{t+1}:

$$\frac{\partial^2 U_t}{\partial K_{t+1}^2} = u''(c_{1,t}) + \delta u''(c_{2,t+1})(1+r_{t+1})^2 < 0.$$

This implies a unique maximum in (S1), that is, without knowledge codification. The second principal minor will be positive, if and only if

$$\frac{\partial^2 U_t}{\partial K_{t+1}^2}\frac{\partial^2 U_t}{\partial \triangle C_{e,t}^2} - \left(\frac{\partial^2 U_t}{\partial K_{t+1}\partial \triangle C_{e,t}}\right)^2 > 0. \tag{7.1}$$

Using the first order conditions, we obtain

$$\frac{\partial^2 U_t}{\partial \triangle C_{e,t}^2} = \gamma_t^2 \frac{\partial^2 U_t}{\partial K_{t+1}^2} + \delta u'(c_{2,t+1}) \frac{\partial^2 r_{t+1}}{\partial \triangle C_{e,t}^2} K_{t+1}$$

and

$$\frac{\partial^2 U_t}{\partial K_{t+1} \partial \triangle C_{e,t}} = \gamma_t \frac{\partial^2 U_t}{\partial K_{t+1}^2} + \delta u'(c_{2,t+1}) \frac{\partial r_{t+1}}{\partial \triangle C_{e,t}}.$$

Inserting and simplifying transforms (7.1) into

$$\frac{\partial^2 U_t}{\partial K_{t+1}^2} \left(\frac{\partial^2 r_{t+1}}{\partial \triangle C_{e,t}^2} K_{t+1} - 2\gamma_t \frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} \right) > \delta u'(c_{2,t+1}) \left(\frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} \right)^2 .$$

The return on physical capital, r_{t+1}, linearly depends on τ_{t+1} and so does τ_{t+1} on $\triangle C_{e,t}$. Hence, $\frac{\partial^2 r_{t+1}}{\partial \triangle C_{e,t}^2} = 0$ and consequently

$$\frac{\partial^2 U_t}{\partial K_{t+1}^2} \left(-2\gamma_t \frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} \right) > \delta u'(c_{2,t+1}) \left(\frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} \right)^2 .$$

Cancelling $\frac{\partial r_{t+1}}{\partial \triangle C_{e,t}}$ and using M gives

$$-u''(c_{1,t}) - \delta u''(c_{2,t+1})(1 + r_{t+1})^2 > \frac{\delta}{2} u'(c_{2,t+1}) \frac{1 + r_{t+1}}{K_{t+1}}.$$

Further transformations yield

$$\frac{u''(c_{2,t+1})}{u'(c_{2,t+1})} + \frac{1}{2c_{2,t+1}} < -\frac{u''(c_{1,t})}{u'(c_{1,t})} \frac{1}{1 + r_{t+1}}.$$

This is property (c4) of the utility function. Choosing a CIES-utility function, this inequality defines the lower bound of θ by $\theta > \frac{1}{2} \frac{1 - s_t - \varsigma_t}{1 - \varsigma_t}$. This inequality is always satisfied for $\theta \geq 0.5$. □

7.2 Allocation of Labor in Temporary Equilibrium

This section examines under which conditions the equilibrium allocation of labor possesses an inner solution with positive research. The result is that for $\Phi = 0$, the intermediate sector will always employ a positive number of researchers. For $\Phi = 1$ there may be corner solutions for certain parameter constellations.

Generally, the question is under which conditions the following equation possesses a fixed point in (0,1):

$$L_{A,t} = \frac{1 - \alpha}{\alpha^2} \frac{\tau_t}{\frac{\partial \tau_t}{\partial L_{R,t}}}, \tag{7.2}$$

where τ_t is a function of $L_{R,t}(= 1 - L_{A,t})$. Due to the Inada conditions of the aggregate production function, $L_{A,t} > 0$.

First, the case $\Phi = 0$ is considered. To examine whether $L_{A,t} \leq 1$, the intuition is that since the right hand side of equation (7.2) is monotonically increasing in τ_t, there must be a τ_{crit} such that $L_{A,t} = 1$. Consequently, the labor market realizes corner solutions for all periods t where $\tau_t > \tau_{crit}$ and positive research in all periods t where $\tau_t < \tau_{crit}$. Since with $\Phi = 0$ the economy approaches a stationary state, the allocation of labor will possess an interior solution in the long run if $\tau_{st} \leq \tau_{crit}$. As $\tau_{st} = \frac{\nu\beta C_{st}+\nu\varepsilon}{1-\nu q(1-\beta)}$ if $C_{st} > q\tau_{st}^{woC}$, the stationary state of the knowledge stock is increasing in the stationary-state level of information. Without an exogenous source of information, that is, $C_1 \leq \tau_1$ and with $\tau_1 \leq \tau_{st}^{wC}$, the maximum stationary-state level of knowledge the economy can realize is $\tau_{st}^{wC} = \frac{\nu\varepsilon}{1-\nu(q+\beta(1-q)}$. Note that τ_{st}^{wC} implies $C_{st} = \tau_{st}^{wC}$. Hence, if there is positive research with the stock of codified ideas at its maximum, there must also be research with less information. Inserting the maximum knowledge stock into the right hand side of equation (7.2) gives

$$L_{A,st} = \frac{1-\alpha}{\alpha^2}\frac{\nu}{1-\nu(q+\beta(1-q)}.$$

For $L_{A,st}$ to be smaller than 1, we obtain the condition

$$q + \beta(1-q) < 1,$$

which is always satisfied for $q, \beta \in (0,1)$.

For $\Phi = 1$, the considerations proceed along previous lines. That is, we ask what is the highest level of knowledge that can be reached in each period t and then examine the allocation in the labor market. If the respective allocation shows positive research, we can be sure to also have an inner solution for lower levels of the knowledge stock. Here again, the highest knowledge stock in each period t is realized with full knowledge codification. Hence, the fixed point problem takes the following form:

$$L_{A,t} = \frac{1-\alpha}{\alpha^2}\frac{\tau_{t-1}(q+\beta(1-q)+\varepsilon q(1-L_{A,t}))}{q\varepsilon\tau_{t-1}}.$$

Simple transformations yield

$$L_{A,t} = \frac{1-\alpha}{\alpha^2+1-\alpha}\left(\frac{q+\beta(1-q)}{q\varepsilon}+1\right).$$

Consequently, $L_{A,t}$ will only be smaller than 1 if the following condition holds:

$$\frac{\alpha^2}{1-\alpha} > \frac{q+\beta(1-q)}{q\varepsilon}.$$

7.3 Existence of Non-Trivial Steady States

In this section of the appendix, we elaborate whether the dynamical system of the overlapping generations economy possesses non-trivial steady states. To this end, it is necessary to distinguish between the cases where $\Phi = 0$ and $\Phi = 1$. In the first, it is shown that the economy will approach stationary-state behavior in the long run. In the case where $\Phi = 1$, there exist different steady states with positive growth rates. It is verified that the overlapping generations economy must exhibit steady-state behavior in the long run in the case of zero codification. By the attribute “non-trivial”, we intend to preclude $K_1 = 0$ which would imply $K_t = 0$, $\forall t$. In short, we are interested in steady states with positive physical capital and knowledge.

7.3.1 $\Phi = 0$

For the case of $\Phi = 0$, the dynamical system writes

$$\begin{aligned} K_{t+1} &= s_t K_t^{\alpha}(1-\alpha)^{1-\alpha}\alpha^{2\alpha}\varepsilon^{\alpha}, \\ \tau_{t+1} &= \nu q \tau_t + \nu\beta \max\{0, C_{t+1} - q\tau_t\} + \nu\varepsilon, \\ C_{t+1} &= C_t + \triangle C_{ie,t} + \triangle C_{e,t}. \end{aligned}$$

It is now shown that the overlapping generations economy characterized by this system of difference equations will approach a stationary state in the long run. The proof proceeds in two steps. First, it is verified that the dynamical system approaches a fixed point for any constant level of the information stock. The second step shows that the economy’s stock of information will be constant in the long run. Accordingly, the first claim is:

For any fixed level of the information stock C_{st}, the system possesses a fixed point $(C_{st}, \tau_{st}, K_{st}) \in \mathbb{R}_+ \times [\tau_{st}^{woC}, \infty) \times [K_{st}^{woC}, \infty)$.

To verify this statement, choose an arbitrary but fixed $C_{st} \in \mathbb{R}_+$. A consequence of the stock of information being fixed is $\triangle C_{e,t} = 0$. Therefore, the development of the knowledge stock is independent of that of the capital stock. It is characterized by a linear, first order difference equation, where $\frac{d\tau_{t+1}}{d\tau_t} \in (0,1)$. Hence, the knowledge stock will approach a stationary state at

$$\tau_{st} = \begin{cases} \frac{\nu\varepsilon}{1-\nu q}, & C_{st} \le q\tau_{st}^{woC}; \\ \frac{\nu\beta C_{st} + \nu\varepsilon}{1-\nu q(1-\beta)}, & C_{st} > q\tau_{st}^{woC}. \end{cases}$$

As the knowledge stock approaches a stationary point τ_{st}, the capital stock must do so as well, because its dynamics then satisfy the following conditions for a fixed point.
If $K_1 > 0$ and

(a) $\frac{\partial s_t}{\partial r_{t+1}} \ge 0$, $\forall r_{t+1} \ge 0$,

(b) $\lim_{K_t \to 0} \phi'(K_t) > 1$,

(c) $\lim_{K_t \to \infty} \phi'(K_t) = 0$,

the function

$$\phi(K_t) = K_{t+1} = s_t K_t^{\alpha}(1-\alpha)^{1-\alpha}\alpha^{2\alpha}\varepsilon^{\alpha}$$

possesses a non-trivial fixed point.[1]

Condition (a) guarantees the existence of a single valued function $\phi(K_t) > 0, \forall K_t > 0$. The reasoning is as follows. The properties (c2) and (c3) of the utility function imply that $s_t \geq s_{low} > 0$.[2] Hence, for every $K_t > 0$ there exists a $\phi(K_t) > 0$. Uniqueness is given if the derivative of $s_t K_t^{\alpha} Q$ with respect to $\phi(K_t)$ will be smaller than 1.[3] More precisely, if

$$\frac{\partial s_t}{\partial r_{t+1}} \frac{dr_{t+1}}{d\phi(K_t)} K_t^{\alpha} Q < 1,$$

where $Q = (1-\alpha)^{1-\alpha}\alpha^{2\alpha}\varepsilon^{\alpha}$. This condition holds because $\frac{\partial s_t}{\partial r_{t+1}} \geq 0$ and $\frac{dr_{t+1}}{d\phi(K_t)} < 0$[4].

Requirement (b) assures that the function ϕ is steeper than the 45 degree line at the origin and (c) guarantees that there exists a fixed point $\phi(K_t) = K_t$. The derivative of $\phi(K_t)$ can be written as

$$\frac{d\phi(K_t)}{dK_t} = \frac{s_t \alpha K_t^{\alpha-1} Q}{1 - \frac{\partial s_t}{\partial r_{t+1}} \frac{\partial r_{t+1}}{\partial \phi} K_t^{\alpha} Q}. \tag{7.3}$$

Consider first requirement (c). For $K_t \to \infty$ the numerator will approach zero. The second term of the denominator being negative then suffices for (c) to hold. However, it is not immediately clear from equation (7.3) whether (b) holds. A simple argument is the following. As (b) is supposed to guarantee that the graph of $\phi(K_t)$ is above the bisecting line for $K_t \to 0$, the condition can be written as $s_t K_t^{\alpha} Q > K_t$ or $s_t K_t^{\alpha-1} Q > 1$, respectively. Since $s_t \in [s_{low}, 1)$ and $Q > 0$, the condition must hold for all K_t close enough to 0. This completes the proof that the economy will reach a stationary state for fixed C_{st}.[5]

[1] The argument follows Galor and Ryder (1989).

[2] More intuitively: If, as assumed in (c3), the substitution effect prevails when r_{t+1} changes, the lowest the savings rate will become is when $r_{t+1} = 0$. That is, s_{low} solves $u'(w_t(1-s_{low}) - \gamma \triangle C_t) = \delta u'(s_{low} w_t)$. Due to the 'no starvation' requirement or Inada conditions (c2), s_{low} must be greater than 0. Note that (c2) also implies that $s_t < 1$.

[3] The condition is derived from the implicit function theorem.

[4] Since $r_{t+1} = K_{t+1}^{\alpha-1} E$, where $E = \left(\frac{1-\alpha}{\alpha^2}\right)^{-\alpha} \varepsilon^{\alpha} \left(\tau_{st} \frac{1-\alpha}{\alpha^2 \varepsilon} - 1 + \alpha\right) > 0$, the derivative shows the form $\frac{dr_{t+1}}{d\phi(K_t)} = (\alpha-1)\phi^{\alpha-2} E < 0$

[5] Note that as $\tau_{st} \geq \tau_{st}^{woC}$ and $\frac{\partial s_t}{\partial r_{t+1}} \frac{\partial r_{t+1}}{\partial \tau_t} \geq 0$ it follows that $K_{st} \geq K_{st}^{woC} = s_{st}^{woC}(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} \varepsilon^{\frac{\alpha}{1-\alpha}}$.

The next step is to show that the economy's stock of information must be constant in the long run. As a consequence, the overlapping generations economy characterized by $\Phi = 0$ will eventually approach a stationary state.

Recall that the economy's information will not decline by assumption and can only increase in a period t if $\tau_t > C_t$. It is then necessary to distinguish whether the initial stock of information C_1 exceeds τ_{st}^{wC}. If it does, the economy must have inherited information from some exogenous source.
Let's first consider the case $C_1 \leq \tau_{st}^{wC}$. As the highest level of knowledge the economy can realize will then be τ_{st}^{wC}, $(C_t)_{t\in\mathbb{Z}}$ is a monotonically increasing and bounded sequence. Consequently, it must converge to $sup_{t\in\mathbb{Z}}C_t = C_{st} \leq \tau_{st}^{wC}$.
If $C_1 > \tau_{st}^{wC}$, the knowledge stock will approach a stationary-state level at $\tau_{st}(C_{st}) = \frac{\nu\beta C_{st}+\nu\varepsilon}{1-\nu q(1-\beta)} < C_{st}$. Consider $\tau_1 < C_1$. The knowledge stock then monotonically approaches $\tau_{st}(C_1) < C_1$. Hence, $\forall t$, $\tau_t < C_1$, and consequently, $\forall t$, $C_t = C_1 = C_{st}$. Let's turn to $\tau_1 > C_1 > \tau_{st}^{wC}$. An interpretation of this initial situation could be that the economy has previously reached a higher knowledge level due to better knowledge transfer capabilities or a higher creativity. Since $\tau_{st}(C_t) < C_t$, $\forall C_t > \tau_{st}^{wC}$, there must exist a period t_0, where $\tau_t < C_t$, $\forall t > t_0$. Hence, $(C_t)_{t\in\mathbb{Z}}$ is bound from above and $sup_{t\in\mathbb{Z}}C_t = C_{t_0} = C_{st}$. □

7.3.2 $\Phi = 1$

For $\Phi = 1$, the overlapping generations economy is characterized by the following system of difference equations:

$$\begin{aligned} K_{t+1} &= s_t K_t^{\alpha}(1-\alpha)^{1-\alpha}\alpha^{2\alpha}(\varepsilon q)^{\alpha}\tau_{t-1}^{\alpha}, \\ \tau_{t+1} &= \nu q\tau_t + \nu\beta(\triangle C_{e,t} + \max\{0, C_t - q\tau_t\}) + \nu\varepsilon q\tau_t, \\ C_{t+1} &= C_t + \triangle C_{ie,t} + \triangle C_{e,t}. \end{aligned}$$

The respective growth rates are

$$\begin{aligned} g_{K,t} &= s_t K_t^{\alpha-1}(1-\alpha)^{1-\alpha}\alpha^{2\alpha}(\varepsilon q)^{\alpha}\tau_{t-1}^{\alpha} - 1, \\ g_{\tau,t} &= \nu q + \nu\beta\frac{\triangle C_{e,t} + \max\{0, C_t - q\tau_t\}}{\tau_t} + \nu\varepsilon q - 1, \\ g_{C,t} &= \frac{\triangle C_{ie,t} + \triangle C_{e,t}}{C_t}. \end{aligned}$$

The growth rates change from one period to the next according to

$$\begin{aligned} dg_{K,t} &= s_t K_t^{\alpha-1}(1-\alpha)^{1-\alpha}\alpha^{2\alpha}(\varepsilon q)^{\alpha}\tau_{t-1}^{\alpha}(g_{s,t} - (1-\alpha)g_{K,t} + \alpha g_{\tau,t-1}), \\ dg_{\tau,t} &= \nu\beta\frac{\triangle C_{e,t} + \max\{0, C_t - q\tau_t\}}{\tau_t}(g_{\triangle C_e + \max\{0, C - q\tau\},t} - g_{\tau,t}), \\ dg_{C,t} &= \frac{\triangle C_{ie,t} + \triangle C_{e,t}}{C_t}(g_{\triangle C_{ie} + \triangle C_e,t} - g_{C,t}). \end{aligned}$$

Being defined by constant growth rates, steady states imply $(dg_{K,t}, dg_{\tau,t}, dg_{C,t}) = (0,0,0)$, $\forall t$. Consequently, there are two kinds of non-trivial steady states:

(1) Steady state without knowledge codification

If the economy does not codify, $\triangle C_{e,t} + \max\{0, C_t - q\tau_t\} = 0$ and $\triangle C_{ie,t} + \triangle C_{e,t} = 0$. This implies $g_{C,s} = 0$ and $g_{\tau,s} = g_\tau^{woC}$.
As s_t is bound on $[s_{low}, 1)$, the saving rate cannot grow at a constant rate other than 0. Consequently, $g_{K,s} = \frac{\alpha}{1-\alpha} g_{\tau,s}$. Therefore, a steady state without codification is characterized by

$$\begin{aligned} g_{K,s} &= \frac{\alpha}{1-\alpha} g_{\tau,s}, \\ g_{\tau,s} &= g_\tau^{woC} = \nu q(1+\varepsilon) - 1, \\ g_{C,s} &= 0. \end{aligned}$$

(2) Steady state with knowledge codification

Positive knowledge codification implies $\triangle C_{e,t} > 0$. Therefore, the overlapping generations economy can only realize steady-state behavior if $g_{C,s} = g_{\triangle C_{ie} + \triangle C_e, s}$ and $g_{\tau,s} = g_{\triangle C_e + \max\{0, C - q\tau\}, s}$. By the same argument with respect to the saving rate as in the case with zero codification, the steady-state relation of the growth rates of capital and knowledge will be $g_{K,s} = \frac{\alpha}{1-\alpha} g_{\tau,s}$. Consequently, steady states with positive knowledge codification imply

$$\begin{aligned} g_{K,s} &= \frac{\alpha}{1-\alpha} g_{\tau,s}, \\ g_{\tau,s} &= g_{\triangle C_e + \max\{0, C - q\tau\}, s}, \\ g_{C,s} &= g_{\triangle C, s}, \end{aligned}$$

where $\triangle C_t = \triangle C_{ie,t} + \triangle C_{e,t}$. Note that a steady state with full codification is a special case where $g_{\tau,s} = g_\tau^{wC} = g_{\triangle C_e + \max\{0, C - q\tau\}, s} = g_{C,s} = g_{\triangle C, s} = \nu(q(1+\varepsilon) + \beta(1-q)) - 1$.

We will now show that without knowledge codification, the overlapping generations economy will approach a non-trivial steady state.[6]
Let $k_t := \frac{K_t}{\tau_t^{\frac{\alpha}{1-\alpha}}}$. Knowing that in steady state $g_{K,s} = \frac{\alpha}{1-\alpha} g_{\tau,s}$, $k_t = constant$, $\forall t$, is a necessary condition for the economy to be in steady state. Assuming no knowledge

[6] Galor and Ryder (1989) have shown that for any feasible set of well-behaved preferences there exists a production function that satisfies the Inada conditions under which the overlapping generations economy experiences global contraction and the steady-state equilibrium is characterized by the absence of production and consumption.

codification implies $g_{C,t} = 0$ and $g_{\tau,t} = g_\tau^{woC} = constant$. In this case, $k_t = constant$ is also a sufficient condition for steady-state behavior of the economy.

Hence, the economy's dynamics can be summarized by the following first order difference equation:

$$\phi(k_t) = k_{t+1} = \frac{K_{t+1}}{\tau_{t+1}^{\frac{\alpha}{1-\alpha}}} = s_t k_t^\alpha \tilde{Q},$$

where $\tilde{Q} = (1 + g_\tau^{woC})^{-\frac{\alpha(2-\alpha)}{1-\alpha}}(1-\alpha)^{1-\alpha}\alpha^{2\alpha}(\varepsilon q)^\alpha$. To verify that the overlapping generations economy experiences a non-trivial steady-state equilibrium we use the same conditions as in the case of $\Phi = 0$. That is, $k_1 > 0$ and

(a) $\frac{\partial s_t}{\partial r_{t+1}} \geq 0, \forall r_{t+1} \geq 0$,

(b) $\lim_{k_t \to 0} \phi'(k_t) > 1$,

(c) $lim_{k_t \to \infty} \phi'(k_t) = 0$.

Again condition (a) guarantees that for every $k_t > 0$ there exists a unique $\phi(k_t) > 0$. $\phi(k_t) > 0$ follows from $s_t \geq s_{low} > 0$ and uniqueness is given if

$$\frac{\partial s_t}{\partial r_{t+1}} \frac{dr_{t+1}}{d\phi(k_t)} k_t^\alpha \tilde{Q} < 1.$$

As $\frac{\partial s_t}{\partial r_{t+1}} \geq 0$ and $\frac{dr_{t+1}}{d\phi(k_t)} < 0$ this condition is satisfied.[7]

With regard to requirements (b) and (c), the derivative of $\phi(k_t)$ is written as

$$\frac{d\phi(k_t)}{dk_t} = \frac{s_t \alpha k_t^{\alpha-1}\tilde{Q}}{1 - \frac{\partial s_t}{\partial r_{t+1}} \frac{\partial r_{t+1}}{\partial \phi} k_t^\alpha \tilde{Q}}.$$

Requirement (c) holds as the numerator will approach zero for $k_t \to \infty$ and the second term of the denominator is negative. By rewriting condition (b) as $s_t k_t^{\alpha-1}\tilde{Q} > 1$, it becomes clear that the inequality must hold for all k_t close enough to 0 since $s_t \in [s_{low}, 1)$ and $\tilde{Q} > 0$.
Hence, the three conditions for the existence of a non-trivial fixed point are satisfied. Uniqueness and continuity of $\phi(k_t)$ together with requirements (b) and (c) imply that

[7] It is possible to write $r_{t+1} = k_{t+1}^{\alpha-1}\tilde{E}$, where $\tilde{E} = \left(\frac{1-\alpha}{\alpha^2}\right)^{-\alpha}(\varepsilon q)^\alpha (1 + g_\tau^{woC})^{-\alpha}\left(\frac{1-\alpha}{\alpha^2 \varepsilon q}(1 + g_\tau^{woC}) - 1 + \alpha\right) > 0$. Since g_τ^{woC} = constant, the derivative shows the form $\frac{dr_{t+1}}{d\phi(k_t)} = (\alpha - 1)\phi^{\alpha-2}\tilde{E} < 0$.

an economy with zero codification will show steady-state growth in the long run. □

With CIES-utility, the representative household's saving rate without knowledge codification can be written as

$$s_t = \frac{1}{\delta^{-\frac{1}{\theta}}(1 + k_{t+1}^{\alpha-1}\tilde{E})^{\frac{\theta-1}{\theta}} + 1}, \tag{7.4}$$

where $\tilde{E} = \left(\frac{1-\alpha}{\alpha^2}\right)^{-\alpha}(\varepsilon q)^{\alpha}(1+g_\tau^{woC})^{-\alpha}\left(\frac{1-\alpha}{\alpha^2\varepsilon q}(1+g_\tau^{woC}) - 1 + \alpha\right) > 0$. Consequently, a steady state implies

$$\delta^{-\frac{1}{\theta}}(1 + (k_s^{woC})^{\alpha-1}\tilde{E})^{\frac{\theta-1}{\theta}} + 1) = (k_s^{woC})^{\alpha-1}\tilde{Q}. \tag{7.5}$$

This equation possesses a unique solution for k_s^{woC}. Hence, the economy's steady state without knowledge codification is unique.

7.4 Proof of Proposition 5.9

As mentioned in section 5.2 the proof comprises two steps. In a first step, it is necessary to verify that in situation (S2) the representative household invests in a positive level of knowledge codification in the long run. Only in this case a comparison of utilities between situations (S1) and (S2) makes sense. This comparison is subject to the second part of the proof, showing that from some point in time on, utility is growing stronger in (S2) with knowledge codification than in (S1) without knowledge codification. This implies that there must be a period in which the economy will start to codify.

(1) The first part of the proof uses a similar reasoning as the proof of proposition 5.8. Consider an economy in (S2) where the representative agent pays fixed costs $\gamma_t q \tau_t$ and thereafter decides on the optimal amount of physical capital saving and knowledge codification. The constraints of her utility maximization problem can then be transformed into

$$\begin{aligned} c_{1,t} &= w_{net,t}(1 - s_t - \varsigma_t), \\ c_{2,t+1} &= (1 + r_{t+1})s_t w_{net,t}. \end{aligned}$$

Suppose the economy will not be codifying for all t. Then the saving rate is a function of r_{t+1} only. As $g_{\tau,t} = g_\tau^{woC} = constant$ and $g_{C,t} = 0$, the economy's dynamics are reflected by

$$K_{t+1} = s_t w_{net,t} = s_t(K_t^\alpha \tau_t^\alpha F - \gamma_t q \tau_t).$$

Using $k_t = \frac{K_t}{\tau_t^{\frac{\alpha}{1-\alpha}}}$, it writes

$$k_{t+1} = s_t w_{net,t} = s_t k_t^\alpha (1 + g_\tau^{woC})^{-\frac{\alpha}{1-\alpha}} F - s_t \gamma_t q \tau_t^{\frac{1-2\alpha}{1-\alpha}} (1 + g_\tau^{woC})^{-\frac{\alpha}{1-\alpha}}.$$

The condition

$$\alpha > \frac{1 + \frac{g_\gamma}{g_{\tau,s}}}{2 + \frac{g_\gamma}{g_{\tau,s}}}$$

can be transformed into

$$g_\gamma < \frac{2\alpha - 1}{1-\alpha} g_\tau^{woC}.$$

This implies that

$$\gamma_t q \tau_t^{\frac{1-2\alpha}{1-\alpha}} (1 + g_\tau^{woC})^{-\frac{\alpha}{1-\alpha}}$$

is decreasing at a constant rate. We are now in a position to apply the following line of argument again: As s_t is bound on the interval $[s_{low}, 1)$, it can be estimated from above by 1. Hence, the last term of k_{t+1} in the above equation becomes arbitrarily small for large enough t. Consequently the difference equation approaches that of an economy without entry costs in the limit. Being interested in long-term behavior, this last term can be neglected for sufficiently large t and proposition 5.6 implies positive codification in (S2) in the long run.

(2) The representative agent decides to codify if (S2)-utility, U_t^C, is higher than (S1)-utility, U_t^{woC}. Consequently, the question is whether the economy that is currently not codifying (that is with full accumulation of entry costs and $g_{\tau,t} = g_\tau^{woC}$) and with $C_1 = 0$ will reach a period t in which $U_t^C > U_t^{woC}$. This writes

$$u(c_{1,t}^{woC}) + \delta u(c_{2,t+1}^{woC}) < u(c_{1,t}^C) + \delta u(c_{2,t+1}^C).$$

As utility is unbound in consumption, the economy will be codifying if (S2)-utility (the right hand side) is permanently increasing more than (S1)-utility (the left hand side). Assuming that the economy chooses (S1) in every period, we will drop the bar as notational distinction of (S1)-saving, $\bar{K}_{t+1} = K_{t+1}$. However, we still indicate the choice that the representative agent would make in (S2) by $\hat{K}_{t+1}, \triangle\hat{C}_{e,t}$:

$$\begin{aligned} u'(c_{1,t}^{woC})(dw_t - dK_{t+1}) + \delta u'(c_{2,t+1}^{woC})\left(\frac{\partial c_{2,t+1}^{woC}}{\partial K_{t+1}} dK_{t+1} + \frac{\partial c_{2,t+1}^{woC}}{\partial \tau_{t+1}} \frac{\partial \tau_{t+1}}{\partial \tau_t} d\tau_t\right) \\ < \quad u'(c_{1,t}^C)(dw_t - d\hat{K}_{t+1} - \gamma_t d\triangle\hat{C}_{e,t} - \triangle\hat{C}_{e,t} d\gamma_t - \gamma_t d\triangle C_{ie,t} - \triangle C_{ie,t} d\gamma_t) \\ + \delta u'(c_{2,t+1}^C)\left(\frac{\partial c_{2,t+1}^C}{\partial \hat{K}_{t+1}} d\hat{K}_{t+1} + \frac{\partial c_{2,t+1}^C}{\partial \triangle C_{e,t}} d\triangle\hat{C}_{e,t} + \frac{\partial c_{2,t+1}^C}{\partial \tau_{t+1}} \frac{\partial \tau_{t+1}}{\partial \tau_t} d\tau_t\right). \end{aligned}$$

In this way, second period consumption is interpreted as a function of K_{t+1}, τ_t and $\triangle C_{e,t}$. Bringing together all terms with $dK_{t+1}, d\hat{K}_{t+1}, d\triangle\hat{C}_{e,t}$ and using the first order conditions simplifies above's expression to

$$\begin{aligned} u'(c_{1,t}^{woC}) dw_t + \delta u'(c_{2,t+1}^{woC}) \tfrac{\partial c_{2,t+1}^{woC}}{\partial \tau_t} d\tau_t \quad < \quad & u'(c_{1,t}^C)(dw_t - \triangle\hat{C}_{e,t} d\gamma_t - \gamma_t d\triangle C_{ie,t} - \triangle C_{ie,t} d\gamma_t) \\ & + \delta u'(c_{2,t+1}^C) \frac{\partial c_{2,t+1}^C}{\partial \tau_t} d\tau_t. \end{aligned}$$

Using the first order conditions once again yields

$$u'(c_{1,t}^{woC})\left(dw_t + \frac{1}{1+r_{t+1}^{woC}}\frac{\partial c_{2,t+1}^{woC}}{\partial \tau_t}d\tau_t\right) < u'(c_{1,t}^{C})\Bigg(dw_t - \triangle\hat{C}_{e,t}d\gamma_t - \gamma_t d\triangle C_{ie,t} - \triangle C_{ie,t}d\gamma_t + \frac{\gamma_t}{\frac{\partial r_{t+1}^C}{\partial \triangle C_{e,t}}\hat{K}_{t+1}}\frac{\partial c_{2,t+1}^C}{\partial \tau_t}d\tau_t\Bigg).$$

Mathematical transformations give

$$\begin{aligned} dw_t > & -\frac{u'(c_{1,t}^C)}{u'(c_{1,t}^{woC}) - u'(c_{1,t}^C)}(\triangle\hat{C}_{e,t}d\gamma_t + \gamma_t d\triangle C_{ie,t} + \triangle C_{ie,t}d\gamma_t) \\ & + \frac{u'(c_{1,t}^C)}{u'(c_{1,t}^{woC}) - u'(c_{1,t}^C)}\left(\frac{\gamma_t}{\frac{\partial r_{t+1}^C}{\partial \triangle\hat{C}_{e,t}}\hat{K}_{t+1}}\frac{\partial c_{2,t+1}^C}{\partial \tau_t}d\tau_t\right) \\ & - \frac{u'(c_{1,t}^{woC})}{u'(c_{1,t}^{woC}) - u'(c_{1,t}^C)}\left(\frac{1}{1+r_{t+1}^{woC}}\frac{\partial c_{2,t+1}^{woC}}{\partial \tau_t}d\tau_t\right). \end{aligned} \tag{7.6}$$

With respect to (S1), we have $\tau_{t+1} = (1+g_\tau^{woC})\tau_t$. It is then possible to write

$$r_{t+1}^{woC} = K_{t+1}^{\alpha-1}\tau_t^\alpha E,$$

where $E = \left(\frac{1-\alpha}{\alpha^2}\right)^{-\alpha}(\varepsilon q)^\alpha\left(\frac{1-\alpha}{\alpha^2\varepsilon q}(1+g_\tau^{woC}) - 1 + \alpha\right) > 0$. As a consequence,

$$\frac{1}{1+r_{t+1}^{woC}}\frac{\partial c_{2,t+1}^{woC}}{\partial \tau_t} = \frac{\alpha K_{t+1}^{\alpha-1}\tau_t^{\alpha-1}E}{1+K_{t+1}^{\alpha-1}\tau_t^\alpha E}.$$

In (S2), $\tau_{t+1} = (1+g_\tau^{woC})\tau_t + \nu\beta\triangle\hat{C}_{e,t}$.[8] Then

$$c_{2,t+1}^C = \hat{K}_{t+1}(1 + \hat{K}_{t+1}^{\alpha-1}\tau_t^{\alpha-1}G\nu\beta\triangle\hat{C}_{e,t} + (g_\tau^{woC} - \alpha)\hat{K}_{t+1}^{\alpha-1}\tau_t^\alpha G),$$

where $G = \left(\frac{1-\alpha}{\alpha^2}\right)^{1-\alpha}(\varepsilon q)^{\alpha-1}$. It follows that

$$\frac{\partial r_{t+1}^C}{\partial \triangle\hat{C}_{e,t}} = \hat{K}_{t+1}^{\alpha-1}\tau_t^{\alpha-1}G\nu\beta$$

and

$$\frac{\partial c_{2,t+1}^C}{\partial \tau_t} = (\alpha-1)\hat{K}_{t+1}^{\alpha-1}\tau_t^{\alpha-2}G\nu\beta\triangle\hat{C}_{e,t} + \alpha(g_\tau^{woC} - \alpha)\hat{K}_{t+1}^\alpha\tau_t^{\alpha-1}G.$$

[8] Notice that $\max\{0, C_t - q\tau_t\} = 0$ as $C_t = 0$ by assumption.

Inserting this into equation (7.6), gives

$$\begin{aligned}\frac{dw_t}{d\tau_t} \quad > \quad & -\frac{u'(c_{1,t}^{C})}{u'(c_{1,t}^{woC}) - u'(c_{1,t}^{C})} \frac{\triangle\hat{C}_{e,t} d\gamma_t + \gamma_t d\triangle C_{ie,t} + \triangle C_{ie,t} d\gamma_t}{d\tau_t} \\ & + \frac{u'(c_{1,t}^{C})}{(u'(c_{1,t}^{woC}) - u'(c_{1,t}^{C}))}\gamma_t \left((\alpha-1)\frac{\triangle\hat{C}_{e,t}}{\tau_t} + (g_\tau^{woC} - \alpha)\frac{\alpha}{\nu\beta}\right) \\ & - \frac{u'(c_{1,t}^{woC})}{(u'(c_{1,t}^{woC}) - u'(c_{1,t}^{C}))}\left(\frac{\alpha K_{t+1}^{\alpha}\tau_t^{\alpha-1}E}{1+K_{t+1}^{\alpha-1}\tau_t^{\alpha}E}\right).\end{aligned}$$

As $\triangle\hat{C}_{e,t}$ must be smaller than $(1-q)\tau_t$, it can be estimated by $\xi\tau_t$, with an appropriate $\xi \in [0, 1-q]$. Further $w_t = K_t^\alpha \tau_t^\alpha F$ and hence $\frac{dw_t}{d\tau_t} = \frac{K_t^\alpha \tau_t^{\alpha-1} F(\alpha g_{K,t} + \alpha g_\tau^{woC})}{g_\tau^{woC}} > 0$. Cancelling $K_t^\alpha \tau_t^{\alpha-1}$ yields

$$\begin{aligned}\frac{F(\alpha g_{K,t}+\alpha g_\tau^{woC})}{g_\tau^{woC}} \quad > \quad & -\frac{\tau_t^{1-\alpha}\gamma_t}{K_t^\alpha} \frac{u'(c_{1,t}^{C})}{u'(c_{1,t}^{woC}) - u'(c_{1,t}^{C})} \frac{(q g_\tau^{woC} + (q+\xi) g_\gamma)}{g_\tau^{woC}} \\ & + \frac{\tau_t^{1-\alpha}\gamma_t}{K_t^\alpha} \frac{u'(c_{1,t}^{C})}{u'(c_{1,t}^{woC}) - u'(c_{1,t}^{C})} \left((\alpha-1)\xi + (g_\tau^{woC} - \alpha)\frac{\alpha}{\nu\beta}\right) \\ & - \frac{u'(c_{1,t}^{woC})}{u'(c_{1,t}^{woC}) - u'(c_{1,t}^{C})}\left(\frac{\alpha(1+g_{K,t})^\alpha E}{1+\alpha K_t^{\alpha-1}\tau_t^\alpha E(1+g_{K,t})^{\alpha-1}}\right).\end{aligned}$$

Approaching steady-state behavior without knowledge codification in the long run, the limit of the left hand side will be a positive constant, $\frac{\alpha}{1-\alpha}F$. So will the last term of the third summand, because $K_t^{\alpha-1}\tau_t^\alpha = k_t^{\alpha-1}$ is constant in steady state. As a consequence, if

$$\lim_{t\to\infty} \frac{u'(c_{1,t}^{woC})}{u'(c_{1,t}^{woC}) - u'(c_{1,t}^{C})} = 0, \tag{7.7}$$

the last term on the right hand side approaches 0 in the limit. It also implies that $\lim_{t\to\infty} \frac{u'(c_{1,t}^{woC})}{u'(c_{1,t}^{C})} = 0$ and hence

$$\lim_{t\to\infty} \left| \frac{u'(c_{1,t}^{C})}{u'(c_{1,t}^{woC}) - u'(c_{1,t}^{C})} \right| = 1.$$

The condition

$$\alpha > \frac{1 + \frac{g_\gamma}{g_\tau^{woC}}}{2 + \frac{g_\gamma}{g_\tau^{woC}}}$$

guarantees that $\frac{\tau_t^{1-\alpha}\gamma_t}{K_t^\alpha}$ is declining to zero. Consequently, the first two terms will also approach zero in the long run,[9] such that $dU_t^C > dU_t^{woC}$ from some point in time t_1 on.

[9] It should not go without mention that in case the economy reaches the upper bound $\triangle\hat{C}_{e,t} = (1-q)\tau_t$ in (S2), the last term in the second summand will write $\frac{\alpha\hat{K}_{t+1}^{\alpha}\tau_t^{\alpha-1}\hat{E}}{1+\hat{K}_{t+1}^{\alpha-1}\tau_t^{\alpha}\hat{E}}$, where $\hat{E} = \left(\frac{1-\alpha}{\alpha^2}\right)^{-\alpha}(\varepsilon q)^\alpha \left(\frac{1-\alpha}{\alpha^2\varepsilon q}(1+g_\tau^{wC}) - 1 + \alpha\right)$. As this term is positive, the second summand will be negative and consequently the conditions (i) and (ii) still suffice for knowledge codification in the long run.

Hence, there will be a period $t_2 > t_1$, where $U_{t_2}^{C} > U_{t_2}^{woC}$. Or with regard to lemma 5.3, without knowledge codification the economy's savings K_{t+1} will exceed $\bar{K}_{t+1,crit}$ for all $t \geq t_2$. This implies that t_2 denotes the first period of codification.

With regard to (ii), if the limit in (7.7) is greater than zero but finite, it follows that the limit of the relation of marginal utilities in the first two terms must be finite as well. Consequently, with the respective α as given above, only the last term will not approach zero in the long run and the inequality reduces to condition (ii). □

Part II

Notes on the Robustness of the Results

Chapter 8

Robustness with Respect to Assumptions on Technological Change

The question raised in this chapter is whether the results of the basic model depend on the assumption of Solow-neutral technological progress or carry over to other specifications of technological change. For this purpose, we use the model specification as introduced in the first part of the thesis with the only difference that the production function of the intermediate good is altered in a way so as to allow for different weights of the knowledge stock. After deriving the new equilibria and describing the economy's general dynamics, the effects of relaxing the assumption of Solow-neutral technological change on the curvature of the return on capital in the amount of knowledge codification and its consequences for the sufficient conditions of the household's maximization problem are discussed. It will then be shown that by sufficiently restricting the parameter range for the weight of the knowledge stock in intermediate goods production, the more general model exhibits the same structure as the basic one of the first part of the dissertation and the main results apply.

8.1 The New Production Function of Intermediate Goods

Let aggregate production unchanged:

$$F(x_t, L_{A,t}) = x_t^{\alpha} L_{A,t}^{1-\alpha}.$$

The only difference to the basic model is that the production function of the intermediate good is characterized by

$$x_t = K_t \tau_t^{\frac{\psi}{\alpha}}.$$

In this way, ψ reflects the assumptions on technological progress. In particular, $\psi = \alpha$ would represent capital augmenting or Solow-neutral technological progress, $\psi = 1 - \alpha$ labor-augmenting or Harrod-neutral technological progress and $\psi = 1$ Hicks-neutral technological progress. The necessary condition of the intermediate firm's profit maximization problem in its general form can be written as in the basic model with Solow-neutral technological progress:

$$w_{R,t} = \alpha^2 x_t^{\alpha-1} \frac{\partial x_t}{\partial L_{R,t}} L_{A,t}^{1-\alpha}.$$

Considering that $x_t = K_t \tau_t^{\frac{\psi}{\alpha}}$ gives

$$w_{R,t} = \alpha K_t^{\alpha} \psi \tau_t^{\psi-1} \frac{\partial \tau_t}{\partial L_{R,t}} L_{A,t}^{1-\alpha}.$$

For $\psi < 1$, this yields the following demand for researchers and supply of intermediate goods:[1]

$$\begin{aligned} x_t^s &= \left(\frac{\alpha^2 \frac{\partial x_t}{\partial L_{R,t}}}{w_{R,t}} \right)^{\frac{1}{1-\alpha}} L_{A,t}, \\ L_{R,t}^d &= K_t^{\frac{\alpha}{1-\psi}} \left(\frac{\alpha\psi}{w_{R,t}} \right)^{\frac{1}{1-\psi}} \left(\frac{\partial \tau_t}{\partial L_{R,t}} \right)^{\frac{\psi}{1-\psi}} L_{A,t}^{\frac{1-\alpha}{1-\psi}} - \frac{1}{\frac{\partial \tau_t}{\partial L_{R,t}}} (q\tau_{t-1} + \beta \max\{0, C_t - q\tau_{t-1}\}). \end{aligned}$$

In case of Hicks-neutral technological progress ($\psi = 1$), every researcher's contribution to profits is constant, given K_t and $L_{A,t}$. Hence the intermediate entrepreneur would employ as many researchers as possible at wages smaller than $w_{R,t} = \alpha K_t^{\alpha-1} \frac{\partial \tau_t}{\partial L_{R,t}} L_{A,t}^{1-\alpha}$. She would be indifferent with respect to the number of researchers to hire if the equilibrium wage equals $w_{R,t}$ and would hire zero researchers if wages are higher than $w_{R,t}$.

The profits of the intermediate entrepreneur accrue to

$$\pi_t^{int} = \alpha K_t^{\alpha} L_{A,t}^{1-\alpha} \tau_t^{\psi} \left(1 - \psi \tau_t^{-1} \frac{\partial \tau_t}{\partial L_{R,t}} L_{R,t} \right) =: K_t r_t.$$

We did not insert $L_{R,t}$ into the profit function for two reasons: First, for clarity in exposition and second, that it also includes the case of $\psi = 1$. In this generality the return on physical capital investment writes

$$r_t = \alpha K_t^{\alpha-1} L_{A,t}^{1-\alpha} \tau_t^{\psi} \left(1 - \psi \tau_t^{-1} \frac{\partial \tau_t}{\partial L_{R,t}} L_{R,t} \right). \tag{8.1}$$

8.2 Sequential Markets Equilibrium

Again, the focus will be on the sequential markets equilibrium as specified by definition 4.1. In order to receive the equilibrium prices and allocations, the focus will first be on the labor market equilibrium which requires that $L_t^s = L_{A,t}^d + L_{R,t}^d$. Consider the

[1] The second order condition for a maximum is satisfied as $\frac{\partial^2 \pi_t^{int}}{\partial L_{R,t}^2} < 0$. In particular, $\frac{\partial^2 \pi_t^{int}}{\partial L_{R,t}^2} = \alpha^2(\alpha-1)x_t^{\alpha-2} \left(\frac{\partial x_t}{\partial L_{R,t}} \right)^2 L_{A,t}^{1-\alpha} + \alpha^2 x_t^{\alpha-1} \frac{\partial^2 x_t}{\partial L_{R,t}^2} L_{A,t}^{1-\alpha}$. Using $\frac{\partial x_t}{\partial L_{R,t}} = K_t \frac{\psi}{\alpha} \tau_t^{\frac{\psi}{\alpha}-1} \frac{\partial x_t}{\partial L_{R,t}}$ and $\frac{\partial^2 x_t}{\partial L_{R,t}^2} = K_t \frac{\psi}{\alpha} \left(\frac{\psi}{\alpha} - 1 \right) \tau_t^{\frac{\psi}{\alpha}-2} \left(\frac{\partial x_t}{\partial L_{R,t}} \right)^2$, the condition for the second derivative of π_t^{int} with respect to $L_{R,t}$ to be negative simplifies to $\psi < 1$. For $\psi = 1$, the solution to the intermediate firms profit maximization problem is not unique.

case $\psi < 1$. Inserting labor demand of the representative final-goods firm and the intermediate entrepreneur gives

$$\begin{aligned} L_t^s &= K_t^{\frac{\alpha}{1-\psi}} \left(\frac{\psi\alpha}{w_t}\right)^{\frac{1}{1-\psi}} \left(\frac{\partial\tau_t}{\partial L_{R,t}}\right)^{\frac{\psi}{1-\psi}} \left(\left(\frac{1-\alpha}{w_t}\right)^{\frac{1}{\alpha}} K_t \tau_t^{\frac{\psi}{\alpha}}\right)^{\frac{1-\alpha}{1-\psi}} \\ &\quad - \frac{1}{\frac{\partial\tau_t}{\partial L_{R,t}}} \left(q\tau_{t-1} + \beta \max\{0, C_t - q\tau_{t-1}\}\right) + \left(\frac{(1-\alpha)}{w_t}\right)^{\frac{1}{\alpha}} K_t \tau_t^{\frac{\psi}{\alpha}}. \end{aligned}$$

Using $L_t^s = 1$, transformations yield

$$w_t = (1-\alpha)^{1-\alpha} K_t^{\alpha} (\psi\alpha)^{\alpha} \tau_t^{\psi-\alpha} \left(\frac{\partial\tau_t}{\partial L_{R,t}}\right)^{\alpha}$$

and

$$L_{A,t} = \frac{1-\alpha}{\psi\alpha} \frac{\tau_t}{\frac{\partial\tau_t}{\partial L_{R,t}}}.$$

This implies an equilibrium allocation of labor given by

$$\begin{aligned} L_{A,t} &= \min\left\{1, \frac{1-\alpha}{\psi\alpha + 1 - \alpha}\left(1 + \frac{q\tau_{t-1} + \beta\max\{0, C_t - q\tau_{t-1}\}}{\frac{\partial\tau_t}{\partial L_{R,t}}}\right)\right\}, \\ L_{R,t} &= \max\left\{0, \frac{\psi\alpha}{\psi\alpha + 1 - \alpha}\left(1 - \frac{q\tau_{t-1} + \beta\max\{0, C_t - q\tau_{t-1}\}}{\frac{\partial\tau_t}{\partial L_{R,t}}}\right)\right\}. \end{aligned}$$

Consequently, the equilibrium wage calculates to

$$w_t = (1-\alpha)^{1-\alpha} \left(\psi\alpha K_t \tfrac{\partial\tau_t}{\partial L_{R,t}}\right)^{\alpha} \left(\nu\left(q\tau_{t-1} + \beta\max\{0, C_t - q\tau_{t-1}\} + \tfrac{\partial\tau_t}{\partial L_{R,t}}\right)\right)^{\psi-\alpha}, \quad (8.2)$$

where $\nu = \frac{\psi\alpha}{\psi\alpha+1-\alpha}$.

For $\psi = 1$, the intermediate entrepreneur offers wage $w_{R,t}$ that is independent of $L_{R,t}$. In equilibrium, the market either realizes a corner solution at $L_{A,t} = 1$ or the wages in final-goods production and in the research sector are equal. The latter implies

$$w_{R,t} = \alpha K_t^{\alpha} \psi \tau_t^{\psi-1} \frac{\partial\tau_t}{\partial L_{R,t}} L_{A,t}^{1-\alpha} = (1-\alpha) L_{A,t}^{-\alpha} K_t^{\alpha} \tau_t^{\psi} = w_{A,t}.$$

This equation transforms to

$$L_{A,t} = \frac{1-\alpha}{\psi\alpha} \frac{\tau_t}{\frac{\partial\tau_t}{\partial L_{R,t}}}. \quad (8.3)$$

In fact, inserting the respective terms for τ_t and $\frac{\partial\tau_t}{\partial L_{R,t}}$ yields equation (8.2). Hence, the equilibrium wage shows the same form as for $\psi < 1$. In this way, the case $\psi = 1$ does not have to be distinguished any further.

With regard to possible corner solutions, the case $\Phi = 0$ implies that there will be positive research in each period.[2] For $\Phi = 1$, the share of researchers will be positive if the following condition holds:[3]

$$\frac{\psi\alpha}{1-\alpha} > \frac{q + \beta(1-q)}{q\varepsilon}.$$

It is assumed in the following that the above condition is satisfied.

Turning to the intermediate goods market, the market clearing condition implies

$$x_t^d = \left(\frac{\alpha}{p_{x,t}}\right)^{\frac{1}{1-\alpha}} L_{A,t} = \left(\frac{\alpha^2 \frac{\partial x_t}{\partial L_{R,t}}}{w_t}\right)^{\frac{1}{1-\alpha}} L_{A,t} = x_t^s.$$

Using the equilibrium wage, this condition can be written as

$$p_{x,t} = \alpha^\alpha \left(K_t \psi \frac{\partial \tau_t}{\partial L_{R,t}}\right)^{\alpha-1} (1-\alpha)^{1-\alpha} \tau_t^{(\psi-\alpha)\frac{\alpha-1}{\alpha}},$$

and with the equilibrium number of researchers in temporary equilibrium:

$$p_{x,t} = (1-\alpha)^{1-\alpha}\alpha^\alpha \left(K_t \frac{\psi}{1-\alpha}\frac{\partial \tau_t}{\partial L_{R,t}}\right)^{\alpha-1} \left(\nu\left(q\tau_{t-1} + \beta\max\{0, C_t - q\tau_{t-1}\} + \frac{d\tau_t}{dL_{R,t}}\right)\right)^{(\psi-\alpha)\frac{\alpha-1}{\alpha}}.$$

Let the homogeneous consumption good be the numéraire and abbreviate $T_t := q\tau_{t-1} + \beta\max\{0, C_t - q\tau_{t-1}\} + \frac{d\tau_t}{dL_{R,t}}$. In this way, the sequential market equilibrium is characterized by the following price vector:

$$\{p_{x,t}, w_t\}_{t=1}^{\infty} = \left\{1, (1-\alpha)^{1-\alpha}\alpha^\alpha \left(K_t \frac{\psi}{1-\alpha}\frac{\partial \tau_t}{\partial L_{R,t}}\right)^{\alpha-1} (\nu T_t)^{(\psi-\alpha)\frac{\alpha-1}{\alpha}}, (1-\alpha)^{1-\alpha}\left(\psi\alpha K_t \frac{\partial \tau_t}{\partial L_{R,t}}\right)^{\alpha} (\nu T_t)^{\psi-\alpha}\right\}_{t=1}^{\infty}.$$

8.3 Dynamics

With the more general assumptions on technological change the difference equation of the capital stock now writes

$$K_{t+1} = s_t K_t^\alpha (1-\alpha)^{1-\alpha} (\psi\alpha)^\alpha \tau_t^{\psi-\alpha} \left(\frac{\partial \tau_t}{\partial L_{R,t}}\right)^\alpha,$$

and the growth rate of physical capital is

$$g_{K,t} = s_t K_t^{\alpha-1} (1-\alpha)^{1-\alpha} (\psi\alpha)^\alpha \tau_t^{\psi-\alpha} \left(\frac{\partial \tau_t}{\partial L_{R,t}}\right)^\alpha - 1.$$

[2] If $C_1 \leq \tau_1 \leq \tau_{st}^{wC}$.

[3] The proof is identical to the one in the basic model.

Using the equilibrium number of researchers, the knowledge stock transforms to

$$\tau_{t+1} = \nu(q\tau_t + \beta(\max\{0, C_{t+1} - q\tau_t\}) + \varepsilon(q\tau_t)^{\Phi}),$$

where $\nu = \frac{\psi\alpha}{\psi\alpha+1-\alpha}$.
Consequently, knowledge grows at the following rate:

$$g_{\tau,t} = \nu\left(q + \beta\frac{\max\{0, C_{t+1} - q\tau_t\}}{\tau_t} + \varepsilon q^{\Phi}\tau_t^{\Phi-1}\right) - 1.$$

The difference equation of the stock of information remains unchanged:

$$C_{t+1} = C_t + \triangle C_t, \quad \triangle C_t \geq 0.$$

Again the two cases in which the researchers are not codifying at all and that in which they are codifying every new idea in each period are indexed by "woC" and "wC", respectively.[4] For $\Phi = 1$, the economy realizes long run growth. The growth rates of knowledge with full and zero knowledge codification are constant:

$$\begin{aligned} g_{\tau}^{woC} &= \nu q(1+\varepsilon) - 1, \\ g_{\tau}^{wC} &= \nu(q(1+\varepsilon) + \beta(1-q)) - 1. \end{aligned}$$

As negative knowledge codification and codification of ideas that have not yet been discovered are precluded by assumption, $g_{\tau,t} \in [g_{\tau}^{woC}, g_{\tau}^{wC}]$, $\forall t$.

Physical capital grows at the following rate:

$$g_{K,t} = s_t K_t^{\alpha-1}(1-\alpha)^{1-\alpha}(\psi\alpha\varepsilon q)^{\alpha}\tau_t^{\psi}(1+g_{\tau,t-1})^{-\alpha} - 1.$$

Note that another codified idea would increase both, the growth rate of knowledge $g_{\tau,t-1}$ and the knowledge stock τ_t. The former, however, shows a negative exponent. It is hence not clear whether knowledge codification generally increases the growth rate of physical capital. As shown in appendix 8A.2, the system of difference equations possesses two kinds of steady states distinguished by whether the economy exhibits positive or zero knowledge codification. An economy that does not codify at all will approach steady-state behavior in the long run. Both kinds of steady states show the same relation of growth rates of capital and knowledge:

$$g_{K,s} = \frac{\psi}{1-\alpha} g_{\tau,s}.$$

With regard to output, we can write

$$Y_t = F(K_t, L_{A,t}, \tau_t) = K_t^{\alpha}\tau_t^{\psi}\left(\frac{1-\alpha}{\psi\alpha\varepsilon q}\right)^{1-\alpha}(1+g_{\tau,t-1})^{1-\alpha}.$$

[4]Recall that with $C_1 = 0$, the maximum term in the difference equation of the knowledge stock will be zero in the first case and in the latter, $C_t = \tau_{t-1}$ in each period.

As in the basic model of the first part, the steady-state growth rate of output equals that of physical capital:

$$g_{Y,s} = g_{K,s} = \frac{\psi}{1-\alpha} g_{\tau,s}.$$

Note that with the growth rate of output being equal to that of physical capital in steady state, a possible negative effect of knowledge codification on physical capital growth would carry over to the economy's output growth rate.

For $\Phi = 0$ the economy will approach a stationary state in the long run.[5] In the cases of full or zero knowledge codification, the economy would approach the following stationary-state levels of knowledge:

$$\begin{aligned}
\tau_{st}^{wC} &= \frac{\nu\varepsilon}{1-\nu(q+\beta(1-q))}, \\
\tau_{st}^{woC} &= \frac{\nu\varepsilon}{1-\nu q}.
\end{aligned}$$

As in the basic model, the focus will be on endogenous knowledge codification and, hence, on the case $C_1 < \tau_{st}^{wC}$. For initial values $C_1 < \tau_{st}^{wC}$, the system realizes $\tau_{st} \in [\tau_{st}^{woC}, \tau_{st}^{wC}]$.

The economy's physical capital stock in stationary state writes

$$K_{st} = s_{st}^{\frac{1}{1-\alpha}} (1-\alpha)(\psi\alpha\varepsilon)^{\frac{\alpha}{1-\alpha}} \tau_{st}^{\frac{\psi-\alpha}{1-\alpha}}.$$

This leads to a stationary-state level of output according to

$$Y_{st} = s_{st}^{\frac{\alpha}{1-\alpha}} (1-\alpha)(\psi\alpha\varepsilon)^{\frac{2\alpha-1}{1-\alpha}} \tau_{st}^{\frac{\psi-\alpha}{1-\alpha}+1}. \tag{8.4}$$

Here again, the stationary-state knowledge stock's exponent may be negative and, hence, a negative influence of knowledge codification on the economy's output level cannot be precluded. This issue will be further discussed in section 8.4.1 and in chapter 12 on social optimality.

8.4 Knowledge Codification

Having only changed the production function of the intermediate good, the problem of the representative household remains the same. Therefore, an inner solution to the utility maximization problem has to satisfy the equality of the marginal returns to physical capital and knowledge codification:

$$\gamma_t(1 + r_{t+1}) = \frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} K_{t+1}. \tag{8.5}$$

[5] See appendix 8A.2.

When using the equilibrium allocation of labor, the return on physical capital in $t+1$ writes

$$\begin{aligned} r_{t+1} &= K_{t+1}^{\alpha-1} L_{A,t+1}^{-\alpha} \tau_{t+1}^{\psi} (L_{A,t+1} - 1 + \alpha) \\ &= K_{t+1}^{\alpha-1} \tau_{t+1}^{\psi-\alpha} \left(\tfrac{1-\alpha}{\psi\alpha}\right)^{-\alpha} \left(\tfrac{\partial \tau_{t+1}}{\partial L_{R,t+1}}\right)^{\alpha} \left(\tau_{t+1} \left(\tfrac{1-\alpha}{\psi\alpha}\right) \left(\tfrac{\partial \tau_{t+1}}{\partial L_{R,t+1}}\right)^{-1} - 1 + \alpha\right). \end{aligned} \tag{8.6}$$

The particular problem of the case $\psi \neq \alpha$ is that (8.5) cannot be solved for $\triangle C_{e,t}$ explicitly. Therefore, we define

$$M(K_{t+1}, \triangle C_{e,t}, \tau_t, C_t) = \frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} K_{t+1} - \gamma_t (1 + r_{t+1}) = 0.$$

Further let

$$r_{t+1} = K_{t+1}^{\alpha-1} \tau_{t+1}^{\psi} f(L_{A,t+1}),$$

where $f(L_{A,t+1}) = L_{A,t+1}^{-\alpha}(L_{A,t+1} - 1 + \alpha)$, and

$$\frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} K_{t+1} = K_{t+1}^{\alpha} \tau_{t+1}^{\psi-1} g(L_{A,t+1}),$$

where $g(L_{A,t+1}) = \nu\beta L_{A,t+1}^{-\alpha}\left((\psi - \alpha + 1)L_{A,t+1} - (1-\alpha)(\psi - \alpha)\right)$. Appendix 8A.3 verifies that $\frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} > 0$ for $\Phi = 1$ and in the case of $\Phi = 0$, if $\psi \leq \alpha$. For $\psi > \alpha$ and $\Phi = 0$, the derivative may be negative for very small values of the knowledge stock, however must be positive from some point in time on. The following does not account for this special case. When considering $\psi > \alpha$ and $\Phi = 0$ it is assumed that the knowledge stock is large enough.

However, it is possible in the more general setting that the return on physical capital is strictly convex in the amount of knowledge codification. This is subject to the following lemma.

Lemma 8.1

$$\frac{\partial^2 r_{t+1}}{\partial \triangle C_{e,t}^2} = \begin{cases} < 0, & \psi < \alpha; \\ = 0, & \psi = \alpha; \\ > 0, & \psi > \alpha. \end{cases}$$

Proof. Calculating the second derivative gives

$$\begin{aligned} \tfrac{\partial^2 r_{t+1}}{\partial \triangle C_{e,t}^2} &= (\psi - \alpha + 1)(\psi - \alpha) \tau_{t+1}^{\psi-\alpha-1} K_{t+1}^{\alpha-1} \left(\tfrac{1-\alpha}{\psi\alpha}\right)^{1-\alpha} \left(\tfrac{\partial \tau_{t+1}}{\partial L_{R,t+1}}\right)^{\alpha-1} \left(\tfrac{\partial \tau_{t+1}}{\partial \triangle C_{e,t}}\right)^2 \\ &\quad -(1-\alpha)(\psi-\alpha)(\psi-\alpha-1) \tau_{t+1}^{\psi-\alpha-2} K_{t+1}^{\alpha-1} \left(\tfrac{1-\alpha}{\psi\alpha}\right)^{-\alpha} \left(\tfrac{\partial \tau_{t+1}}{\partial L_{R,t+1}}\right)^{\alpha} \left(\tfrac{\partial \tau_{t+1}}{\partial \triangle C_{e,t}}\right)^2. \end{aligned}$$

It is obvious that $\frac{\partial^2 r_{t+1}}{\partial \triangle C_{e,t}^2} = 0$ for $\psi = \alpha$. In the cases where $\psi \neq \alpha$, the second derivative is smaller than 0 if

$$(\psi - \alpha + 1)(\psi - \alpha)\tau_{t+1} < (\psi - \alpha - 1)(\psi - \alpha)\psi\alpha \frac{\partial \tau_{t+1}}{\partial L_{R,t+1}}.$$

The inequality holds for $\psi < \alpha$. In contrast, $\psi > \alpha$ violates this condition and satisfies that for $\frac{\partial^2 r_{t+1}}{\partial \triangle C_{e,t}^2} > 0$. □

The convexity for $\psi > \alpha$ is due to the specification of the amount of newly discovered ideas per researcher to not depend on the ideas acquired by utilizing the information stock but only on those that have been exogenously transferred. In this way, knowledge codification increases the share of workers which increases the equilibrium price of the intermediate good and consequently the profits. This effect is stronger than the negative effect on profits by less research due to a smaller share of researchers. For increasing returns on physical capital in knowledge codification it may even be the case that the marginal benefit of knowledge codification increases stronger than its opportunity costs, which means that $\frac{\partial M}{\partial \triangle C_{e,t}} > 0$. This raises the question as to whether the household's maximization problem still allows for an inner solution. According to appendix 8A.1, it is necessary for the household's second order conditions to hold that $\frac{\partial M}{\partial \triangle C_{e,t}} - \gamma_t \frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} < 0$. The following lemma elaborates on the relation between $\frac{\partial M}{\partial \triangle C_{e,t}}$ and $\frac{\partial M}{\partial \triangle C_{e,t}} - \gamma_t \frac{\partial r_{t+1}}{\partial \triangle C_{e,t}}$ from a long run perspective with constant growth rates.

Lemma 8.2 *If $\psi > \alpha$ and $g_K, g_\tau, g_\gamma > 0$ are constant with $g_\tau + g_\gamma \neq g_K$, then in the long run either*

(i) $\frac{\partial M}{\partial \triangle C_{e,t}} < 0$ *and* $\frac{\partial M}{\partial \triangle C_{e,t}} - \gamma_t \frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} < 0$

or

(ii) $\frac{\partial M}{\partial \triangle C_{e,t}} > 0$ *and* $\frac{\partial M}{\partial \triangle C_{e,t}} - \gamma_t \frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} > 0$.

With constant growth rates, it is possible that $\frac{\partial M}{\partial \triangle C_{e,t}} > 0$ *and* $\frac{\partial M}{\partial \triangle C_{e,t}} - \gamma_t \frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} < 0$, *if and only if* $g_\tau + g_\gamma = g_K$.

Proof. $\frac{\partial M}{\partial \triangle C_{e,t}}$ can be written as

$$\begin{aligned}\frac{\partial M}{\partial \triangle C_{e,t}} &= (\psi-\alpha)(\nu\beta)^2 K_{t+1}^{\alpha}(\tau_{t+1}^{\psi-\alpha-1} D^{1-\alpha}(\psi-\alpha+1) - (1-\alpha)(\psi-\alpha-1)\tau_{t+1}^{\psi-\alpha-2} D^{-\alpha}) \\ &\quad -\gamma_t \nu\beta K_{t+1}^{\alpha-1}((\psi-\alpha+1)\tau_{t+1}^{\psi-\alpha} D^{1-\alpha} - (1-\alpha)\tau_{t+1}^{\psi-\alpha-1}(\psi-\alpha) D^{-\alpha}),\end{aligned}$$

where $D = \frac{L_{A,t+1}}{\tau_{t+1}}$. $\frac{\partial M}{\partial \triangle C_{e,t}} < 0$ implies

$$\begin{aligned}&(\psi-\alpha)(\nu\beta)^2[(\psi-\alpha+1)L_{A,t+1} - (1-\alpha)(\psi-\alpha-1)] \\ &\quad < \frac{\gamma_t \tau_{t+1}}{K_{t+1}} \nu\beta[(\psi-\alpha+1)L_{A,t+1} - (1-\alpha)(\psi-\alpha)].\end{aligned} \tag{8.7}$$

For $\psi > \alpha$ the left hand side must be positive and it is bound from above and below due to $L_{A,t+1}$ being bounded. With respect to the right hand side, $r_{t+1} > 0$ implies $L_{A,t+1} - (1-\alpha) > 0$, which leads to $(\psi-\alpha+1)L_{A,t+1} - (1-\alpha)(\psi-\alpha) > 0$. Constant growth rates imply that $L_{A,t+1}$ is a positive constant and so is the left hand side. The right hand side either approaches 0 or ∞ in the limit, depending on whether $g_\tau + g_\gamma < g_K$ or $g_\tau + g_\gamma > g_K$, respectively.

As $\frac{\partial M}{\partial \triangle C_{e,t}} - \gamma_t \frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} = \frac{\partial^2 r_{t+1}}{\partial \triangle C_{e,t}^2} K_{t+1} - 2\gamma_t \frac{\partial r_{t+1}}{\partial \triangle C_{e,t}}$ only differs from $\frac{\partial M}{\partial \triangle C_{e,t}}$ by subtracting the last summand twice, the necessary condition for the representative household's second order conditions to hold, i.e. $\frac{\partial M}{\partial \triangle C_{e,t}} - \gamma_t \frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} < 0$, requires

$$\begin{aligned}(\psi - \alpha)(\nu\beta)^2[(\psi - \alpha + 1)L_{A,t+1} - (1-\alpha)(\psi - \alpha - 1)] \\ < \frac{\gamma_t \tau_{t+1}}{K_{t+1}} \nu\beta 2[(\psi - \alpha + 1)L_{A,t+1} - (1-\alpha)(\psi - \alpha)].\end{aligned}$$

It is clear that with constant growth rates this inequality shows the same long run behavior as 8.7. This yields the first part of the lemma and it further follows immediately that the signs of $\frac{\partial M}{\partial \triangle C_{e,t}} - \gamma_t \frac{\partial r_{t+1}}{\partial \triangle C_{e,t}}$ and $\frac{\partial M}{\partial \triangle C_{e,t}}$ can only differ if $\frac{\gamma_t \tau_{t+1}}{K_{t+1}} = constant$, which implies $g_\tau + g_\gamma = g_K$. □

It is clear that if $\frac{\partial M}{\partial \triangle C_{e,t}} - \gamma_t \frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} > 0$, the increasing returns to investments in the next period overcompensate for the loss of utility due to less consumption in the first period of the agent's life, which must lead to a corner solution. In this case, the solution derived from the necessary condition $M = 0$ is the minimum of the utility function. If the growth rates satisfy $g_\tau + g_\gamma = g_K$,[6] the marginal benefit of knowledge codification may increase stronger in the amount of codified ideas than its opportunity costs, but the pair $(K_{t+1}, \triangle C_{e,t})$ that yields $M = 0$ could still be a utility maximum. Intuitively, this is the case if the increasing returns are not as pronounced to compensate for the marginal utility lost by forgoing another unit of consumption in the first period. That is, the concavity of the utility function is strong enough to still guarantee an inner solution.

Lemma 8.2 shows that for $g_\tau + g_\gamma \neq g_K$, a steady state implies that either the necessary condition for an inner solution to the household's problem holds and the marginal condition M decreases in $\triangle C_{e,t}$ or on the opposite, M increases in $\triangle C_{e,t}$, but then the household's problem must show a corner solution in the long run.

The following concentrates on the case where $\frac{\partial M}{\partial \triangle C_{e,t}} < 0$. It is shown that the main results of the basic model carry over to this more general setting with $\psi \neq \alpha$. In particular, the results require $\frac{\partial M}{\partial \triangle C_{e,t}} < 0$ and a unique solution to the household's problem.

Assumption (M1) *The household's maximization problem possesses a unique solution in both situations, (S1) and (S2), and* $\frac{\partial M}{\partial \triangle C_{e,t}} < 0$.

From the above discussion, it is clear that there exists a tradeoff between the specification of the utility function and that of technological change if (M1) is to hold. For simplicity, we use $\psi < \alpha$ as this implies that $\frac{\partial^2 r_{t+1}}{\partial \triangle C_{e,t}^2} < 0$, and consequently $\frac{\partial M}{\partial \triangle C_{e,t}} < 0$,

[6] Note that $g_\tau = g_\gamma = g_K = 0$ is also possible.

as well as $\frac{\partial M}{\partial \triangle C_{e,t}} - \gamma_t \frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} < 0$. It further assures unique solutions to the household's utility maximization by using a utility function characterized by the properties (c1)-(c4) as verified in appendix 8A.1. In this way, the basic model specification can be used with $\psi < \alpha$ being the only difference.

Given assumption (M1), it is possible to formulate:

Proposition 8.1 *The lemmata 5.1, 5.2, 5.3 also hold with $\psi \neq \alpha$ and assumption (M1).*

Proof. Lemma 5.1 claims the existence of a function $K_{t+1,crit}(\tau_t, C_t)$ implying $\triangle C_{e,t} = 0$. Consider M at $\triangle C_{e,t} = 0$. The partial derivative with respect to K_{t+1} gives

$$\frac{\partial M(K_{t+1}, 0, \tau_t, C_t)}{\partial K_{t+1}} = \alpha K_{t+1}^{\alpha-1} \tau_{t+1}^{\psi-1} g(L_{A,t+1}) - \gamma_t(\alpha - 1) K_{t+1}^{\alpha-2} \tau_{t+1}^{\psi} f(L_{A,t+1}).$$

$M(K_{t+1}, 0, \tau_t, C_t) = 0$ implies that

$$K_{t+1}^{\alpha} \tau_{t+1}^{\psi-1} g(L_{A,t+1}) = \gamma_t (1 + K_{t+1}^{\alpha-1} \tau_{t+1}^{\psi} f(L_{A,t+1})).$$

Consequently,

$$\left.\frac{\partial M(K_{t+1}, 0, \tau_t, C_t)}{\partial K_{t+1}}\right|_{M(K_{t+1},0,\tau_t,C_t)=0} = \alpha \frac{\gamma_t}{K_{t+1}} + \gamma_t K_{t+1}^{\alpha-2} \tau_{t+1}^{\psi} f(L_{A,t+1}) > 0.$$

The implicit function theorem then establishes lemma 5.1.

Consider now lemma 5.2 which states that $\triangle C_{e,t}$ is an increasing function in K_{t+1} on $\mathcal{U}$. First,

$$\left.\frac{\partial M(K_{t+1}, \triangle C_{e,t}, \tau_t, C_t)}{\partial \triangle C_{e,t}}\right|_{M(K_{t+1},\triangle C_{e,t},\tau_t,C_t)=0} = \frac{\partial^2 r_{t+1}}{\partial \triangle C_{e,t}^2} K_{t+1} - \frac{\left(\frac{\partial r_{t+1}}{\partial \triangle C_{e,t}}\right)^2 K_{t+1}}{1+r_{t+1}} < 0$$

implies that there exists a function $\triangle C_{e,t}(K_{t+1}, \tau_t, C_t)$ such that $M(K_{t+1}, \triangle C_{e,t}, \tau_t, C_t) = 0$. Given τ_t, C_t, the implicit function theorem further gives

$$\frac{\partial \triangle C_{e,t}}{\partial K_{t+1}} = -\frac{\frac{\partial M(K_{t+1}, \triangle C_{e,t}, \tau_t, C_t)}{\partial K_{t+1}}}{\frac{\partial M(K_{t+1}, \triangle C_{e,t}, \tau_t, C_t)}{\partial \triangle C_{e,t}}}.$$

Since

$$\left.\frac{\partial M(K_{t+1}, \triangle C_{e,t}, \tau_t, C_t)}{\partial K_{t+1}}\right|_{M(K_{t+1},\triangle C_{e,t},\tau_t,C_t)=0} = \alpha \frac{\gamma_t}{K_{t+1}} + \gamma_t K_{t+1}^{\alpha-2} \tau_{t+1}^{\psi} f(L_{A,t+1}) > 0,$$

for all $\triangle C_{e,t} \geq 0$, it follows that $\frac{\partial \triangle C_{e,t}}{\partial K_{t+1}} > 0$, which is lemma 5.2's contention.

Lemma 5.3 takes possible entry costs to knowledge codification into account claiming the existence of a unique pair $(\hat{K}_{t+1,crit}, \bar{K}_{t+1,crit})$ for every $(\tau_t, C_t, \tilde{f}_t)$ such that $U_t^C = U_t^{woC}$.[7]

[7]The notation has been introduced in chapter 5 of the first part of the thesis.

As the proof of lemma 5.3 uses lemmata 5.1 and 5.2, the Inada-conditions of the utility function and the uniqueness of the solutions to the households' optimization problem, it also holds in the more general setting. □

As proposition 8.1 implies that the structure of the optimization problem is equal to that in the first part. Hence, a first result is:

Proposition 8.2 *For both, $\Phi = 0$ and $\Phi = 1$, proposition 5.1 holds in the more general setting. That is, an overlapping generations economy with $C_1 \leq q\tau_1$, where $\tau_1 > 0$, and an initial stock of capital K_1 close enough to zero will not be codifying at the beginning of its development.*

Proof. Again it is shown that one can always find a K_1 such that for any $\tau_1 > 0$ there exists a time interval $I = \{t|1 \leq t \leq T\}$ in which $\triangle C_{e,t} = 0$. It is sufficient to show that there is no incentive to codify in the first period from a marginal perspective, leaving out entry costs. For an interior solution $M = 0$. That is,

$$\frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} K_{t+1} = \gamma_t (1 + r_{t+1}),$$

which can be written as

$$K_{t+1}^{\alpha} \tau_{t+1}^{\psi-1} g(L_{A,t+1}) = \gamma_t (1 + K_{t+1}^{\alpha-1} \tau_{t+1}^{\psi} f(L_{A,t+1})).$$

Let $\tau_t > 0$. For $K_{t+1} \rightarrow 0$, the limit of the left hand side is zero and that of the right hand side is infinity. From $\frac{\partial M}{\partial K_{t+1}} > 0$ at the point $M(K_{t+1}, 0, \tau_t, C_t) = 0$ follows $K_{crit,t+1} > 0$. Consequently there exists a positive $K_2 \leq K_{t+1,crit}$. Since $K_{t+1} = s_t w_t$, s_t bound from above and w_t for given τ_t a continuous function of K_t where $w_t(K_t = 0) = 0$, it is always possible to find an initial value K_1 close enough to zero such that $K_2 \leq K_{t+1,crit}(\tau_1, C_1)$ and hence $\triangle C_{e,1} = 0$. □

The following two sections elaborate on what other results carry over to the more general setting with $\psi < \alpha$. At first, the case with a constant number of ideas per researcher and period will be discussed.

8.4.1 Constant Number of Ideas per Researcher and Period: $\Phi = 0$

As shown in appendix 8A.2, the specification of the research process by $\Phi = 0$ leads to a stationary state in the long run. Similar to proposition 5.2, it can be formulated:

Proposition 8.3 *With constant codification costs, γ, an overlapping generations economy that develops over time from initial values K_1, τ_1, C_1 close enough to zero will reach a higher stationary-state level of knowledge than τ_{st}^{woC}, if and only if there exists a period t_c in which (S1)-savings satisfy $\bar{K}_{t_c+1} > \bar{K}_{t+1,crit}(\tau_{st}^{woC}, C_{t_c}, \tilde{f}_t)$ and τ_{t_c} is close enough to τ_{st}^{woC}.*

Proof. The proof uses a similar argument as that of proposition 5.2.
"$\Longrightarrow$": Lemma 5.3 gives that $U_t^C > U_t^{woC}$ if $\hat{K}_{t+1} > \hat{K}_{t+1,crit}(\tau_t, C_t, \tilde{f}_t)$. As $\bar{K}_{t_c+1} > \bar{K}_{t+1,crit}(\tau_{t_c}, C_{t_c}, \tilde{f}_t)$ in a period t_c where τ_{t_c} is close enough to τ_{st}^{woC} and $C_{t_c} < q\tau_{st}^{woC}$ implies that $C_{t_c+1} > q\tau_{t_c} \simeq q\tau_{st}^{woC}$, the economy will subsequently move into a stationary-state level of knowledge at

$$\tau_{st} = \frac{\beta C_{st} + \varepsilon}{\frac{1}{\nu} - q(1-\beta)} > \tau_{st}^{woC},\ q\tau_{st} < C_{st} \leq \tau_{st}.$$

"$\Longleftarrow$": For $\tau_{st} > \tau_{st}^{woC}$ to hold, the stationary-state level of information must exceed $q\tau_{st}^{woC}$. From Lemma 5.3, $\triangle C_{e,t}$ is only positive if $\bar{K}_{t+1} > \bar{K}_{t+1,crit}$. Hence, for $C_{st} > q\tau_{st}^{woC}$, $\triangle C_{e,t}$ must be positive in a period where τ_t is close enough to τ_{st}^{woC}. Consequently there must be a period t_c in which $\bar{K}_{t_c+1} > \bar{K}_{t+1,crit}(\tau_{st}^{woC}, C_{t_c}, \tilde{f}_t)$. □

Notice that a peculiarity of the case $\psi < \alpha$ is that the equilibrium wage w_t declines in τ_t. The reason is the following: The marginal product of an additional idea in final-goods production declines in the knowledge stock. The marginal product of labor in final-goods production increases in the knowledge stock but decreases in the number of workers. As for $\Phi = 0$, each researcher creates a constant amount of ideas in each period, her wage is decreasing in the knowledge stock and hence, a certain share is joining the workforce in final-goods production. For $\psi < 2\alpha - 1$, the decline of the researcher's wage in knowledge and the corresponding shift of labor to final-goods production is so strong that the decline of marginal productivity due to the increasing number of workers is greater than the increase of productivity due to the higher knowledge stock. As a consequence, this would ceteris paribus lead to less capital saving K_{t+1}. On the other hand, the rent r_{t+1} increases in τ_{t+1}. This positively affects the saving rate if $\theta < 1$.[8] Therefore, proposition 8.3 differs from proposition 5.2 of the first part in two respects. First, (S1)-saving in some period t to be greater than $\bar{K}_{t+1,crit}(\tau_{st}^{woC}, C_{t_c}, \tilde{f}_t)$ does not suffice for C_{st} to be greater than $q\tau_{st}^{woC}$. The reason is that capital saving may fall below $\bar{K}_{t+1,crit}(\tau_{st}^{woC}, C_{t_c}, \tilde{f}_t)$ subsequently, and hence, there will be no codification of ideas with indices greater than $q\tau_{st}^{woC}$. Second, a stationary level of information $C_{st} > q\tau_{st}^{woC}$ does not automatically imply higher stationary-state levels of output. The latter is subject to the next proposition.

Proposition 8.4 *If $\alpha > \psi \geq 2\alpha - 1$, a higher stationary-state level of knowledge than τ_{st}^{woC} implies $Y_{st} \geq Y_{st}^{woC}$. In case $\psi < 2\alpha - 1$, knowledge codification may lead to lower output levels in the long run.*

Proof. Recall equation (8.4), describing the stationary-state level of output. It depends positively on the stationary-state level of knowledge if $\psi > 2\alpha - 1$, because the saving rate in stationary state depends positively on the return on physical capital investment, which, in turn, is a positive function of knowledge and the exponent of the stationary-state level of knowledge in (8.4) shows a positive sign. However, this exponent turns

[8] More precisely, if $\frac{\partial s_t}{\partial r_{t+1}} > 0$.

negative for $\psi < 2\alpha - 1$. In this case, the negative effect of an increase of the stationary-state level of knowledge by codification must be offset by a corresponding increase of the saving rate. This is not always the case, as is easily approved for logarithmic utility. Let $\theta = 1$. Then the saving rate in stationary state does not depend on r_{st} and, consequently, neither on τ_{st}. As for $\psi < 2\alpha - 1$ the exponent of τ_{st} is negative, the highest level of output the economy could reach is with τ_{st}^{woC}. □

Proposition 8.4 suggests an inefficiency. It is rational for the firm owners of the intermediate firm to do research and it may also be in line with utility maximization of the young generation to invest in knowledge codification. Both activities enhance the knowledge stock and therefore decrease the equilibrium wage realized by future generations. As shown in the proof of the preceding proposition, the declining wage may affect physical capital savings to an extent that these negative effects overcompensate the positive effects of a higher productivity of the capital stock. This problem will be addressed separately within the discussion on social optimality.

With respect to codification costs, the following analogues to propositions 5.3 and 5.4 of the basic model are verified.

Proposition 8.5 *Even if the knowledge stock negatively effects output in the long run, every overlapping generations economy will start to codify if there is no fixed entry cost to knowledge codification, $f = 0$, and the codification costs approach zero, $\gamma_t \to 0$. For every overlapping generations economy, there is a finite maximum entry cost f that will prevent knowledge codification.*

Proof. The proof is identical to the corresponding one in the first part of the dissertation. Intuitively it is clear that as long as $\frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} > 0$ and $\gamma_t = 0$, the old generation of $t+1$ can only be better off by knowledge codification. They do not take into account any effects on future generations.
With the economy approaching a stationary state in the long run, there must be a maximum entry cost to prevent knowledge codification. □

Proposition 8.6 *For overlapping generations economies close enough to stationary state in which $C_{st} < q\tau_{st}^{woC}$ and with equal levels of capital K_{st}^{woC}, the codification costs γ must drop the more the higher the stationary-state level of knowledge in order to induce knowledge codification.*

Proof. Again, consider economies with identical stationary-state levels of capital K_{st}^{woC} and different stationary-state values of knowledge τ_{st}^{woC}. In particular, the focus will be on economies with different triples (δ, ε, q). The representative agent plans to save $K_{t+1} = K_{st}^{woC} < K_{t+1,crit}(\tau_{st}^{woC}, 0)$, given a certain level of codification costs $\hat{\gamma}$. The level of codification costs that implies $\triangle C_{e,t} = 0$ as an interior solution for given K_{t+1}, τ_t, C_t,

$\tilde{\gamma}_t$, writes

$$\tilde{\gamma}_t = \frac{\left.\frac{\partial r_{t+1}}{\partial \triangle C_{e,t}}\right|_{\triangle C_{e,t}=0} K_{t+1}}{1 + r_{t+1}(\triangle C_{e,t} = 0)}.$$

Focussing on stationary states and letting γ be an argument of M yields $M(\gamma, K_{st}^{woC}, \triangle C_{e,t}, \tau_{st}^{woC}, C_{st})$. As $\frac{\partial M}{\partial \gamma}, \frac{\partial M}{\partial \triangle C_{e,t}} < 0$ and for given K_{st}^{woC}, τ_{st}^{woC}, C_{st} , $M = 0$ implies $\frac{d\triangle C_{e,t}}{d\gamma} < 0$. Hence, the codification costs would have to drop by more than $\triangle\gamma = \hat{\gamma} - \tilde{\gamma}(K_{st}^{woC}, \tau_{st}^{woC}, C_{st})$ in order to induce knowledge codification. Holding K_{st}^{woC} constant as we compare economies with identical stocks of capital, comparative statics with respect to ε and q establish the proposition's contention. More precisely,

$$\tilde{\gamma}_t(K_{st}^{woC}, \tau_{st}^{woC}, C_{st}) = \frac{(K_{st}^{woC})^{\alpha}(\tau_{st}^{woC})^{\psi-\alpha}\varepsilon^{\alpha-1}(\psi-\alpha+1)\left(\frac{1-\alpha}{\psi\alpha}\right)^{1-\alpha} - (1-\alpha)(K_{st}^{woC})^{\alpha}(\tau_{st}^{woC})^{\psi-\alpha-1}\varepsilon^{\alpha}(\psi-\alpha)\left(\frac{1-\alpha}{\psi\alpha}\right)^{-\alpha}}{1+(K_{st}^{woC})^{\alpha-1}(\tau_{st}^{woC})^{\psi-\alpha+1}\varepsilon^{\alpha-1}\left(\frac{1-\alpha}{\psi\alpha}\right)^{1-\alpha} - (1-\alpha)(K_{st}^{woC})^{\alpha-1}(\tau_{st}^{woC})^{\psi-\alpha}\varepsilon^{\alpha}\left(\frac{1-\alpha}{\psi\alpha}\right)^{-\alpha}}.$$

Using $\tau_{st}^{woC} = \frac{\nu\varepsilon}{1-\nu q}$ gives

$$\tilde{\gamma}_t = \frac{\varepsilon^{\psi-1}\overbrace{\left[(K_{st}^{woC})^{\alpha}\left(\frac{\nu}{1-\nu q}\right)^{\psi-\alpha}(\psi-\alpha+1)\left(\frac{1-\alpha}{\psi\alpha}\right)^{1-\alpha} - (1-\alpha)(K_{st}^{woC})^{\alpha}\left(\frac{\nu}{1-\nu q}\right)^{\psi-\alpha-1}(\psi-\alpha)\left(\frac{1-\alpha}{\psi\alpha}\right)^{-\alpha}\right]}^{X>0}}{1+\varepsilon^{\psi}\underbrace{\left[(K_{st}^{woC})^{\alpha-1}\left(\frac{\nu}{1-\nu q}\right)^{\psi-\alpha+1}\left(\frac{1-\alpha}{\psi\alpha}\right)^{1-\alpha} - (1-\alpha)(K_{st}^{woC})^{\alpha-1}\left(\frac{\nu}{1-\nu q}\right)^{\psi-\alpha}\left(\frac{1-\alpha}{\psi\alpha}\right)^{-\alpha}\right]}_{Y>0}}.$$

Consequently,

$$\frac{\partial \tilde{\gamma}}{\partial \varepsilon} = \frac{(\psi-1)\varepsilon^{\psi-2}X(1+\varepsilon^{\psi}Y) - \psi\varepsilon^{2(\psi-1)}XY}{(1+\varepsilon^{\psi}Y)^2} < 0.$$

With regard to the transmission parameter q, we denote:

$$\tilde{X} = (K_{st}^{woC})^{\alpha}(\tau_{st}^{woC})^{\psi-\alpha}\varepsilon^{\alpha-1}(\psi-\alpha+1)\left(\frac{1-\alpha}{\psi\alpha}\right)^{1-\alpha} - (1-\alpha)(K_{st}^{woC})^{\alpha}(\tau_{st}^{woC})^{\psi-\alpha-1}\varepsilon^{\alpha}(\psi-\alpha)\left(\frac{1-\alpha}{\psi\alpha}\right)^{-\alpha} > 0,$$
$$\tilde{Y} = (K_{st}^{woC})^{\alpha-1}(\tau_{st}^{woC})^{\psi-\alpha+1}\varepsilon^{\alpha-1}\left(\frac{1-\alpha}{\psi\alpha}\right)^{1-\alpha} - (1-\alpha)(K_{st}^{woC})^{\alpha-1}(\tau_{st}^{woC})^{\psi-\alpha}\varepsilon^{\alpha}\left(\frac{1-\alpha}{\psi\alpha}\right)^{-\alpha} > 0.$$

Then,

$$\frac{\partial \tilde{\gamma}}{\partial q} = \frac{\frac{\partial \tilde{X}}{\partial q}(1+\tilde{Y}) - \tilde{X}\frac{\partial \tilde{Y}}{\partial q}}{(1+\tilde{Y})^2},$$

where $\frac{\partial \tilde{X}}{\partial q} = \frac{\partial \tilde{X}}{\partial \tau_{st}^{woC}}\frac{\partial \tau_{st}^{woC}}{\partial q}$ and $\frac{\partial \tilde{Y}}{\partial q}$, respectively. From

$$\begin{aligned}
\frac{\partial \tilde{X}}{\partial \tau_{st}^{woC}} &= (K_{st}^{woC})^{\alpha}(\tau_{st}^{woC})^{\psi-\alpha-1}\varepsilon^{\alpha-1}(\psi-\alpha)(\psi-\alpha+1)\left(\frac{1-\alpha}{\psi\alpha}\right)^{1-\alpha} \\
&\quad -(1-\alpha)(K_{st}^{woC})^{\alpha}(\tau_{st}^{woC})^{\psi-\alpha-2}\varepsilon^{\alpha}(\psi-\alpha-1)(\psi-\alpha)\left(\frac{1-\alpha}{\psi\alpha}\right)^{-\alpha} < 0, \\
\frac{\partial \tilde{Y}}{\partial \tau_{st}^{woC}} &= (K_{st}^{woC})^{\alpha-1}(\tau_{st}^{woC})^{\psi-\alpha}(\psi-\alpha+1)\varepsilon^{\alpha-1}\left(\frac{1-\alpha}{\psi\alpha}\right)^{1-\alpha} \\
&\quad -(1-\alpha)(K_{st}^{woC})^{\alpha-1}(\tau_{st}^{woC})^{\psi-\alpha-1}(\psi-\alpha)\varepsilon^{\alpha}\left(\frac{1-\alpha}{\psi\alpha}\right)^{-\alpha} > 0,
\end{aligned}$$

and $\frac{\partial \tau_{st}^{woC}}{\partial q} > 0$, follows that $\frac{\partial \tilde{\gamma}}{\partial q} < 0$. Hence, it is clear that $\triangle\gamma$ increases in q and ε or, equivalently, in τ_{st}^{woC}. □

8.4.2 Standing on the Shoulders of Giants: $\Phi = 1$

As shown in appendix 8A.2, the dynamical system of the economy possesses steady states and the overlapping generations economy will eventually show steady-state growth in the case of zero codification. Again, "Codifying in the long run" or "codifying from some point in time on" means that there does not exist a period t_0, such that for all $t > t_0, \triangle C_{e,t} = 0$. The following proposition is the more general equivalent to proposition 5.5.

Proposition 8.7 *With constant codification costs an overlapping generations economy will be codifying in the long run if either of the following conditions is satisfied:*

(i) The steady-state growth rate of capital is higher than that of the knowledge stock (or equivalently $\psi > 1 - \alpha$).

(ii) The steady-state growth rate of capital is equal to that of the knowledge stock (or equivalently $\psi = 1 - \alpha$) and

$$\gamma + \gamma(k_{s,min}^{woC})^{\alpha-1} f(L_{A,t}) < (k_{s,min}^{woC})^{\alpha} g(L_{A,t}),$$

where $k_{s,min}^{woC}$ is the minimum steady-state level of $k_t = \frac{K_t}{\tau_t}$ without codification.

Proof. The proof proceeds along the lines of the one of proposition 5.5 in the first part. No knowledge codification implies that the economy approaches steady-state growth. It also requires that $\forall t > t_0$, $(\tau_t, C_t, K_{t+1}) \notin \mathcal{U}$. If $\psi > 1 - \alpha$, steady-state growth causes K_{t+1} to grow more than $K_{t+1,crit}$, $\forall t$, and, hence, is contradictory to no knowledge codification in the long run.

Suppose an economy characterized by $\psi > 1 - \alpha$ will not be codifying in the long run. That is, $\exists t_0$, such that $\forall t > t_0$, $\triangle C_{e,t} = 0$. Consequently, the overlapping generations economy must approach steady-state growth where $g_{K,s} = \frac{\psi}{1-\alpha} g_{\tau,s}$. $\psi > 1 - \alpha$ implies that $g_{K,s} > g_{\tau,s}$. From $dM = 0$ follows that

$$d\triangle C_{e,t} = -\frac{\frac{\partial M}{\partial K_{t+1}} dK_{t+1} + \frac{\partial M}{\partial \tau_t} d\tau_t + \frac{\partial M}{\partial C_t} dC_t}{\frac{\partial M}{\partial \triangle C_{e,t}}}.$$

Assuming no knowledge codification, the last term of the numerator becomes zero. Using the following derivatives for the numerator, we obtain

$$\frac{\partial M}{\partial K_{t+1}} = \alpha K_{t+1}^{\alpha-1} \tau_{t+1}^{\psi-1} g(L_{A,t+1}) - \gamma_t(\alpha - 1) K_{t+1}^{\alpha-2} \tau_{t+1}^{\psi} f(L_{A,t+1}),$$

$$\frac{\partial M}{\partial \tau_t} = K_{t+1}^{\alpha}(\psi - 1)\tau_{t+1}^{\psi-2} \frac{\partial \tau_{t+1}}{\partial \tau_t} g(L_{A,t+1}) + K_{t+1}^{\alpha} \tau_{t+1}^{\psi-1} \frac{\partial g(L_{A,t+1})}{\partial \tau_t}$$
$$-\gamma_t K_{t+1}^{\alpha-1} \psi \tau_{t+1}^{\psi-1} \frac{\partial \tau_{t+1}}{\partial \tau_t} f(L_{A,t+1}) - \gamma_t K_{t+1}^{\alpha-1} \tau_{t+1}^{\psi} \frac{\partial f(L_{A,t+1})}{\partial \tau_t} < 0,$$

where the derivatives of $f(L_{A,t+1})$ and $g(L_{A,t+1})$ with respect to τ_t are zero in the case of no knowledge codification.[9] Then, $d\triangle C_{e,t}$ can be written as

$$d\triangle C_{e,t} = -\frac{K_{t+1}^{\alpha}\tau_{t+1}^{\psi-1}g(L_{A,t+1})(\alpha g_{K,t+1}+(\psi-1)g_{\tau,t})-\gamma_t K_{t+1}^{\alpha-1}\tau_{t+1}^{\psi}f(L_{A,t+1})((\alpha-1)g_{K,t+1}+\psi g_{\tau,t})}{\frac{\partial M}{\partial\triangle C_{e,t}}}.$$

Without knowledge codification the economy will approach a steady state where $g_{K,s} = \frac{\psi}{1-\alpha}g_{\tau,s}$. Focussing on steady-state behavior gives

$$d\triangle C_{e,t} = -\frac{K_{t+1}^{\alpha}\tau_{t+1}^{\psi-1}g(L_{A,t+1})(\alpha g_{K,s}+(\psi-1)g_{\tau,s})}{\frac{\partial M}{\partial\triangle C_{e,t}}}. \tag{8.8}$$

Since $\frac{\partial M}{\partial\triangle C_{e,t}} < 0$ for $\psi < \alpha$, the sign of $d\triangle C_{e,t}$ is determined by the numerator. It is positive if $\psi > 1-\alpha$, which implies $g_{K,s} > g_{\tau,s}$. With respect to the long term behavior of (8.8), using

$$\begin{aligned}\frac{\partial M}{\partial\tau_{t+1}} &= K_{t+1}^{\alpha}(\psi-1)\tau_{t+1}^{\psi-2}g(L_{A,t+1}) + K_{t+1}^{\alpha}\tau_{t+1}^{\psi-1}\frac{\partial g(L_{A,t+1})}{\partial\tau_{t+1}}\\ &\quad -\gamma_t K_{t+1}^{\alpha-1}\psi\tau_{t+1}^{\psi-1}f(L_{A,t+1}) - \gamma_t K_{t+1}^{\alpha-1}\tau_{t+1}^{\psi}\frac{\partial f(L_{A,t+1})}{\partial\tau_{t+1}} < 0,\end{aligned}$$

where

$$\begin{aligned}\frac{\partial g(L_{A,t+1})}{\partial\tau_{t+1}} &= (\psi-\alpha+1)\left(\frac{1-\alpha}{\psi\alpha}\right)^{1-\alpha}(1-\alpha)\tau_{t+1}^{-\alpha}\left(\frac{d\tau_{t+1}}{dL_{R,t+1}}\right)^{\alpha-1}+\alpha(1-\alpha)(\psi-\alpha)\left(\frac{1-\alpha}{\psi\alpha}\right)^{-\alpha}\tau_{t+1}^{-\alpha-1}\left(\frac{d\tau_{t+1}}{dL_{R,t+1}}\right)^{\alpha},\\ \frac{\partial f(L_{A,t+1})}{\partial\tau_{t+1}} &= (1-\alpha)\tau_{t+1}^{-\alpha}\left(\frac{1-\alpha}{\psi\alpha}\right)^{1-\alpha}\left(\frac{\partial\tau_{t+1}}{\partial L_{R,t+1}}\right)^{\alpha-1}+\alpha(1-\alpha)\tau_{t+1}^{-\alpha-1}\left(\frac{1-\alpha}{\psi\alpha}\right)^{-\alpha}\left(\frac{\partial\tau_{t+1}}{\partial L_{R,t+1}}\right)^{\alpha}>0,\end{aligned}$$

and dividing (8.8) by $K_{t+1}^{\alpha}\tau_{t+1}^{\psi-1}$, transforms the resulting denominator to

$$\begin{aligned}\frac{\frac{\partial M}{\partial\tau_{t+1}}\frac{\partial\tau_{t+1}}{\partial\triangle C_{e,t}}}{K_{t+1}^{\alpha}\tau_{t+1}^{\psi-1}} &= (\psi-1)\tau_{t+1}^{-1}g(L_{A,t+1})\nu\beta + \frac{\partial g(L_{A,t+1})}{\partial\tau_{t+1}}\nu\beta - \gamma_t K_{t+1}^{-1}\psi f(L_{A,t+1})\nu\beta\\ &\quad -\gamma_t K_{t+1}^{-1}\tau_{t+1}\frac{\partial f(L_{A,t+1})}{\partial\tau_{t+1}}\nu\beta < 0.\end{aligned}$$

Writing τ_{t+1} as $\tau_t(1+g_{\tau,t})$ gives

$$\begin{aligned}\frac{\partial g(L_{A,t+1})}{\partial\tau_{t+1}} &= \tau_t^{-1}\Big[(\psi-\alpha+1)\left(\frac{1-\alpha}{\psi\alpha\varepsilon q}\right)^{1-\alpha}(1-\alpha)(1+g_{\tau,t})^{-\alpha}+\alpha(1-\alpha)(\psi-\alpha)\left(\frac{1-\alpha}{\psi\alpha\varepsilon q}\right)^{-\alpha}(1+g_{\tau,t})^{-\alpha-1}\Big]\\ &\leq \tau_t^{-1}\Big[(\psi-\alpha+1)\left(\frac{1-\alpha}{\psi\alpha\varepsilon q}\right)^{1-\alpha}(1-\alpha)(1+g_{\tau}^{woC})^{-\alpha}+\alpha(1-\alpha)(\psi-\alpha)\left(\frac{1-\alpha}{\psi\alpha\varepsilon q}\right)^{-\alpha}(1+g_{\tau}^{woC})^{-\alpha-1}\Big],\end{aligned}$$

and, thus, it is clear that $\lim_{t\to\infty}\frac{\partial g(L_{A,t+1})}{\partial\tau_{t+1}} = 0$. With regard to the last term of the denominator we consider

$$\tau_{t+1}\frac{\partial f(L_{A,t+1})}{\partial\tau_{t+1}} = (1-\alpha)(1+g_{\tau,t})^{1-\alpha}\left(\frac{1-\alpha}{\psi\alpha\varepsilon q}\right)^{1-\alpha} + \alpha(1-\alpha)(1+g_{\tau,t})^{-\alpha}\left(\frac{1-\alpha}{\psi\alpha\varepsilon q}\right)^{-\alpha}.$$

[9] The reason is that $L_{A,t} = \frac{1-\alpha}{\alpha\psi\varepsilon q}(1+g_{\tau}^{woC})$ in the case of no knowledge codification.

Since g_{τ_t} bound from above and below, so must $\tau_{t+1}\frac{\partial f(L_{A,t+1})}{\partial \tau_{t+1}}$ and, consequently, $\lim_{t\to\infty} \gamma_t K_{t+1}^{-1}\tau_{t+1}\frac{\partial f(L_{A,t+1})}{\partial \tau_{t+1}}\nu\beta = 0$. It then follows that

$$\lim_{t\to\infty} \frac{\frac{\partial M}{\partial \tau_{t+1}}\frac{\partial \tau_{t+1}}{\partial \triangle C_{e,t}}}{K_{t+1}^{\alpha}\tau_{t+1}^{\psi-1}} = 0.$$

Hence,

$$d\triangle C_{e,t} = -\frac{g(L_{A,t+1})(\alpha g_{K,s} + (\psi-1)g_{\tau,s})}{\frac{\partial M}{\partial \tau_{t+1}}\frac{\partial \tau_{t+1}}{\partial \triangle C_{e,t}}K_{t+1}^{-\alpha}\tau_{t+1}^{1-\psi}}$$

will either approach $+\infty$ for $\psi > 1-\alpha$ or $-\infty$ for $\psi < 1-\alpha$. The former implies that there must be a period in which $\triangle C_{e,t}$ is positive, whereas the latter is characterized by the corner solution $\triangle C_{e,t} = 0$. As a consequence $\psi > 1-\alpha$ precludes that there exists a t_0 such that $\forall t > t_0$, $\triangle C_{e,t} = 0$ and, hence, the economy will be codifying in the long run.

Consider (ii). With zero knowledge codification the economy will realize steady-state growth in the long run with an intensive capital stock $k_s^{woC} = constant$. Note that the basic model was characterized by a CIES-utility function. We could, hence, guarantee a unique steady state without knowledge codification. In this part of the thesis, we more generally assume that utility satisfies the properties (c1)-(c4). By this, the steady-state equilibrium is not necessarily unique. Let the different steady-state levels of k without codification be indexed by j and $k_{s,min}^{woC} = \min_j\{k_{s,j}^{woC}\}$. If $\gamma + \gamma(k_{s,min}^{woC})^{\alpha-1}f(L_{A,t}) < (k_{s,min}^{woC})^{\alpha}g(L_{A,t})$, the marginal utility of investing the last unit of the homogenous good transferred to the next period into knowledge codification is higher than its opportunity costs. This implies that the representative agent would be better off by spending the last unit saved in physical capital for knowledge codification instead. In case there are multiple steady states, if (ii) holds for $k_{s,min}^{woC}$, it will also be satisfied for all $k_s^{woC} > k_{s,min}^{woC}$. □

Assuming that codification costs decline monotonically over time at a constant rate g_γ yields the analogon to proposition 5.6.

Proposition 8.8 *An overlapping generations economy will be codifying in the long run if the steady-state growth rate of knowledge exceeds that of capital by less than the rate at which the codification costs decline.*

Proof. We suppose again that the economy will not be codifying in the long run and $g_{\tau,s} - g_{K,s} < -g_\gamma$.
In the case of declining codification costs, M is also a function of γ_t. Then, the total derivative $dM = 0$ implies

$$d\triangle C_{e,t} = -\frac{\frac{\partial M}{\partial K_{t+1}}dK_{t+1} + \frac{\partial M}{\partial \tau_t}d\tau_t + \frac{\partial M}{\partial \gamma_t}d\gamma_t}{\frac{\partial M}{\partial \triangle C_{e,t}}}.$$

Inserting the respective partial derivatives gives

$$d\triangle C_{e,t} = -\frac{K_{t+1}^{\alpha}\tau_{t+1}^{\psi-1}g(L_{A,t+1})(\alpha g_{K,t+1}+(\psi-1)g_{\tau,t})-\gamma_t K_{t+1}^{\alpha-1}\tau_{t+1}^{\psi}f(L_{A,t+1})((\alpha-1)g_{K,t+1}+\psi g_{\tau,t})-\gamma_t(1+r_{t+1})g_{\gamma}}{\frac{\partial M}{\partial\triangle C_{e,t}}}.$$

Using $M = 0$, the preceding expression can be rewritten as

$$d\triangle C_{e,t} = -\frac{K_{t+1}^{\alpha}\tau_{t+1}^{\psi-1}g(L_{A,t+1})(\alpha g_{K,t+1}+(\psi-1)g_{\tau,t}-g_{\gamma})-\gamma_t K_{t+1}^{\alpha-1}\tau_{t+1}^{\psi}f(L_{A,t+1})((\alpha-1)g_{K,t+1}+\psi g_{\tau,t})}{\frac{\partial M}{\partial\triangle C_{e,t}}}.$$

As the economy will eventually show steady-state behavior in the case of zero codification, $d\triangle C_{e,t}$ transforms to

$$d\triangle C_{e,t} = -\frac{K_{t+1}^{\alpha}\tau_{t+1}^{\psi-1}g(L_{A,t+1})(\alpha g_{K,s}+(\psi-1)g_{\tau,s}-g_{\gamma})}{\frac{\partial M}{\partial\triangle C_{e,t}}}. \tag{8.9}$$

By the same line of argument as in the proof of proposition 8.7, $d\triangle C_{e,t}$ approaches $+\infty$ if $\alpha g_{K,s} + (\psi-1)g_{\tau,s} - g_{\gamma} > 0$. Using the relation of the steady-state growth rates of knowledge and capital this condition writes

$$\frac{\psi}{1-\alpha} - 1 > \frac{g_{\gamma}}{g_{\tau,s}} \quad \Leftrightarrow \quad \underbrace{\frac{\psi}{1-\alpha}g_{\tau,s}}_{g_{K,s}} - g_{\tau,s} > g_{\tau,s}.$$

Consequently, there will be a period t where $\triangle C_{e,t} > 0$ if $g_{\tau,s} - g_{K,s} < -g_{\gamma}$ which contradicts steady-state behavior with zero knowledge codification in the long run. □

The respective corollary also holds in the more general setting.

Corollary 8.1 *Every overlapping generations economy will be codifying from some point in time on if the rate at which the codification costs decline will be greater or equal to the steady-state growth rate of the knowledge stock.*

Proof. This is a direct result of the proof of proposition 8.8 as $g_{\tau,s} - g_{K,s} < -g_{\gamma}$ can be transformed to

$$\frac{\psi}{1-\alpha} > \frac{g_{\gamma}}{g_{\tau,s}} + 1.$$

The right hand side will be negative if $g_{\tau,s} < -g_{\gamma}$, whereas the left hand side cannot become smaller than zero for $\alpha \in (0,1)$ and $\psi > 0$. □

Proposition 8.9 *An overlapping generations economy in steady-state equilibrium that satisfies the following condition*

$$g_{\tau,s} - g_{K,s} < -g_{\gamma}$$

is codifying fully.

Proof. First, as shown in the proof of proposition 8.8, an overlapping generations economy that satisfies the condition given in proposition 8.9 will start to codify sometime. Since the economy is assumed to be in steady state, $L_{A,t+1}$ will be constant over time. $dM = 0$ writes

$$\frac{\partial M}{\partial K_{t+1}} dK_{t+1} + \frac{\partial M}{\partial \tau_{t+1}}[d\tau_t(1+g_{\tau,t}) + \tau_t dg_{\tau,t}] + \frac{\partial M}{\partial \gamma_t} d\gamma_t = 0,$$

where

$$dg_{\tau,t} = \nu\beta \tfrac{\triangle C_{e,t}+\max\{0,C_t-q\tau_t\}}{\tau_t}(g_{\triangle C_{e,t}+\max\{0,C_t-q\tau_t\}} - g_{\tau,t}).$$

For notational convenience, we set $\triangle\hat{C}_{e,t} := \triangle C_{e,t} + \max\{0, C_t - q\tau_t\}$. Transforming the above equation such that $\tau_t dg_{\tau,t}$ is on the right hand side, yields

$$-\frac{\frac{\partial M}{\partial K_{t+1}} dK_{t+1} + \frac{\partial M}{\partial \tau_{t+1}}(1+g_{\tau,t})d\tau_t + \frac{\partial M}{\partial \gamma_t} d\gamma_t}{\frac{\partial M}{\partial \tau_{t+1}}} = \nu\beta\triangle\hat{C}_{e,t}(g_{\triangle\hat{C}_{e,t}} - g_{\tau,t}).$$

Note that the right hand side must be zero in steady state. Further transformations give

$$-\frac{\frac{\partial M}{\partial K_{t+1}} K_{t+1} g_{K,t+1} + \frac{\partial M}{\partial \tau_{t+1}}\tau_{t+1} g_{\tau_t} + \gamma_t \frac{\partial M}{\partial \gamma_t} g_\gamma}{\frac{\partial M}{\partial \tau_{t+1}}} = \nu\beta\triangle\hat{C}_{e,t}(g_{\triangle\hat{C}_{e,s}} - g_{\tau,s}).$$

Writing out the derivatives and inserting the relation of the steady-state growth rates of knowledge and physical capital transforms the preceding expression to

$$-\frac{K_{t+1}^{\alpha}\tau_{t+1}^{\psi-1} g(L_{A,t+1})(\alpha g_{K,s} + (\psi-1)g_{\tau,s} - g_\gamma)}{\frac{\partial M}{\partial \triangle\tau_{t+1}}} = \triangle\hat{C}_{e,t}(g_{\triangle\hat{C}_{e,s}} - g_{\tau,s}).$$

As $\frac{\partial M}{\partial \triangle\tau_{t+1}}/(K_{t+1}^{\alpha}\tau_{t+1}^{\psi-1}) < 0$ and approaches zero in the limit, the left hand side is strictly positive for $\alpha g_{K,s} + (\psi-1)g_{\tau,s} - g_\gamma > 0$, which transforms to $g_{\tau,s} - g_{K,s} < -g_\gamma$. This violates the steady-state condition that for $\triangle C_{e,t} > 0$, $g_{\triangle C_e + \max\{0, C - q\tau\},s} = g_{\tau,s}$. The only possibility to satisfy this condition is for $\triangle C_{e,t} + \max\{0, C_t - q\tau_t\}$ to realize its upper bound at $(1-q)\tau_t$ as a corner solution. Consequently, an economy in steady state characterized by $g_{\tau,s} - g_{K,s} < -g_\gamma$ must be codifying fully. □

Consequently, in the more general setting, it is also possible to summarize:

Corollary 8.2 *If $g_{\tau,s} - g_{K,s} \neq g_\gamma$, an overlapping generations economy in steady state exhibits either full or zero codification.*

Proof. Similar to the reasoning of the proof of the respective proposition in the first part, this corollary is an immediate consequence of propositions 8.8 and 8.9. If $g_{\tau,s} - g_{K,s} > g_\gamma$, full codification directly follows from proposition 8.9.
With regard to $g_{\tau,s} - g_{K,s} < g_\gamma$, the proof of proposition 8.8 implies that this condition is contradictory to $\triangle C_{e,t} > 0$, $\forall t$. Hence, the economy cannot realize steady-state

growth with positive codification. From proposition 8.8's proof further follows that if an economy shows steady-state behavior $\forall t > t_0$ and $\triangle C_{e,t_0} = 0$, then $\triangle C_{e,t} = 0$, $\forall t > t_0$. As a consequence, only steady-state growth without codification is consistent with the condition $g_{\tau,s} - g_{K,s} < g_\gamma$. □

It should then also hold that finite fixed entry costs only delay but do not prevent an overlapping generations economy's knowledge codification.

Proposition 8.10 *An overlapping generations economy that started to codify without fixed entry costs to knowledge codification will also do so with fixed entry costs.*

Proof. The proof is the same as in chapter 5.2 with the difference that (S1) now implies the following difference equation of the capital stock:

$$K_{t+1} = s_t w_{net,t} = s_t(K_t^\alpha \tau_t^\psi F - f).$$

Using $k_t = \frac{K_t}{\tau_t^{\frac{\psi}{1-\alpha}}}$, it writes

$$k_{t+1} = s_t w_{net,t} = s_t k_t^\alpha (1 + g_\tau^{woC})^{-\frac{\psi}{1-\alpha}} F - s_t f \tau_t^{-\frac{\psi}{1-\alpha}} (1 + g_\tau^{woC})^{-\frac{\psi}{1-\alpha}}.$$

In this way, the last term of the above equation becomes arbitrarily small for large enough t. Consequently the difference equation approaches that of an economy without fixed costs in the limit and the previous propositions apply for positive codification in (S2) in the long run. □

This section showed that for the specification with $\Phi = 1$, the main results with respect to the economy's codification behavior carry over to the more general setting. However, when relaxing the assumption of Solow-neutral technological progress, the section on the economy's dynamics revealed that knowledge codification not necessarily exerts a positive influence on the economy's growth rate of output. This is due to a shift of labor from research to final-goods production as explained in section 8.4.1.

8.5 Summary

Chapter 8 elaborated on the robustness of the results of the basic model introduced in the first part of the dissertation with respect to assumptions on technological change. For this purpose the production function of the intermediate product has been changed as to allow for different specifications of technological progress. A first finding was that when relaxing the assumption of Solow-neutral technological progress, the return on capital may be strictly convex in the amount of knowledge codification. If the weight assigned to knowledge in intermediate goods production is strong enough this leads to increasing returns in total saving, that is, in the sum of both saving in physical capital and investment in knowledge codification such that the concavity of the households' utility function as specified in the basic model may not be sufficient for an inner solution.

It was shown that this can only be the case for $\psi > \alpha$. Within this parameter range, the situation may occur where the marginal benefit of knowledge codification increases stronger in the amount codification than its opportunity costs, but the households' utility maximization problem still shows a unique inner solution. This is the case if the increasing returns on consumption forgone are not as pronounced and the concavity of the utility function overcompensates for it. It has been verified that this situation is in general inconsistent with steady-state behavior except if the difference between the growth rates of knowledge and capital is equal to the rate at which the codification costs decline. Generally speaking, there is a tradeoff between the specification of technological change and that of the utility function if unique inner solutions are to be assured.

For the remainder of the chapter, this tradeoff has been solved by restricting the weight on knowledge to $\psi < \alpha$ as this guarantees for an inner solution of the households' utility maximization with the specification of utility as in the basic model and the marginal benefit of knowledge codification does not increase stronger in the amount of knowledge codification than its opportunity costs. With these two properties the more general model shows the same structure as the basic model of the first part of the dissertation. It is shown that the main results of the basic model carry over to this more general set-up. The particular difficulty with the proofs in this chapter was that although they generally proceed along the lines of those in the previous part, the first order conditions of the representative household could not be solved explicitly for the optimal amount of knowledge codification.

The results also revealed another peculiarity of the specification $\psi < \alpha$. For ψ small enough, knowledge codification may even exert a negative influence on the stationary-state output level of the economy for the case $\Phi = 0$ and possibly lower growth rates in the case with $\Phi = 1$. This point will be discussed in more detail in section 12 on social optimality.

8A Appendix of Chapter 8

8A.1 Sufficient Conditions for the Household's Optimization Problem

In its general form, the representative household's optimization problem is equivalent to that of the basic model. Hence some steps have been omitted and can be looked up in chapter 7. The difference occurs with respect to the condition 8A.2, as the second derivative of the return on physical capital in knowledge codification is now different from zero.

The Hessian matrix writes

$$H = \begin{pmatrix} u''(c_{1,t})+\delta u''(c_{2,t+1})(1+r_{t+1})^2 & \begin{matrix}\gamma_t u''(c_{1,t})+\delta u''(c_{2,t+1})\frac{\partial r_{t+1}}{\partial \triangle C_{e,t}}K_{t+1}(1+r_{t+1}) \\ +\delta u'(c_{2,t+1})\frac{\partial r_{t+1}}{\partial \triangle C_{e,t}}\end{matrix} \\ \begin{matrix}\gamma_t u''(c_{1,t})+\delta u''(c_{2,t+1})\frac{\partial r_{t+1}}{\partial \triangle C_{e,t}}K_{t+1}(1+r_{t+1}) \\ +\delta u'(c_{2,t+1})\frac{\partial r_{t+1}}{\partial \triangle C_{e,t}}\end{matrix} & \begin{matrix}\gamma_t^2 u''(c_{1,t})+\delta u''(c_{2,t+1})\left(\frac{\partial r_{t+1}}{\partial \triangle C_{e,t}}K_{t+1}\right)^2 \\ +\delta u'(c_{2,t+1})\left(\frac{\partial^2 r_{t+1}}{\partial \triangle C_{e,t}^2}K_{t+1}\right)\end{matrix} \end{pmatrix}.$$

The first principal minor is negative due to the concavity of U_t in K_{t+1}:

$$\frac{\partial^2 U_t}{\partial K_{t+1}^2} = u''(c_{1,t}) + \delta u''(c_{2,t+1})(1+r_{t+1})^2 < 0.$$

This implies the unique maximum in (S1), that is without knowledge codification. The second principal minor will be positive, if and only if

$$\frac{\partial^2 U_t}{\partial K_{t+1}^2}\frac{\partial^2 U_t}{\partial \triangle C_{e,t}^2} - \left(\frac{\partial^2 U_t}{\partial K_{t+1}\partial \triangle C_{e,t}}\right)^2 > 0. \tag{8A.1}$$

Using the first order conditions yields

$$\frac{\partial^2 U_t}{\partial K_{t+1}^2}\left(\frac{\partial^2 r_{t+1}}{\partial \triangle C_{e,t}^2}K_{t+1} - 2\gamma_t\frac{\partial r_{t+1}}{\partial \triangle C_{e,t}}\right) > \delta u'(c_{2,t+1})\left(\frac{\partial r_{t+1}}{\partial \triangle C_{e,t}}\right)^2. \tag{8A.2}$$

According to lemma 8.1, $\psi < \alpha$ implies $\frac{\partial^2 r_{t+1}}{\partial \triangle C_{e,t}^2} < 0$. Hence, the left hand side can be estimated from below by $\frac{\partial^2 r_{t+1}}{\partial \triangle C_{e,t}^2} = 0$ and it is then possible to proceed as with the basic model's specification. Consequently, condition 8A.2 transforms into

$$\frac{u''(c_{2,t+1})}{u'(c_{2,t+1})} + \frac{1}{2c_{2,t+1}} < -\frac{u''(c_{1,t})}{u'(c_{1,t})}\frac{1}{1+r_{t+1}}.$$

This is property (c4) of the utility function and choosing a CIES-utility function, this inequality defines the lower bound of θ. □

8A.2 Existence of Non-Trivial Steady States

In the same way as in the corresponding section of the appendix in the first part of the thesis, it is shown that in the case $\Phi = 0$, the economy will approach a stationary state. For $\Phi = 1$, there exist different steady states with positive growth rates and the overlapping generations economy must exhibit steady-state behavior in the long run in case of zero knowledge codification.

$\Phi = 0$

For $\Phi = 0$, the dynamical system writes:

$$\begin{aligned} K_{t+1} &= s_t K_t^{\alpha}(1-\alpha)^{1-\alpha}(\psi\alpha)^{\alpha}\tau_t^{\psi-\alpha}\varepsilon^{\alpha}, \\ \tau_{t+1} &= \nu q \tau_t + \nu\beta \max\{0, C_{t+1} - q\tau_t\} + \nu\varepsilon, \\ C_{t+1} &= C_t + \triangle C_{ie,t} + \triangle C_{e,t}. \end{aligned}$$

As with the specification of the basic model, the proof that the overlapping generations economy characterized by the above system of difference equations will approach a stationary state in the long run proceeds in two steps. First, it is verified that the dynamical system approaches a fixed point for any constant level of the information stock. The second step shows that the economy's stock of information will be constant in the long run. Accordingly, the first claim is:

For any fixed level of the information stock C_{st}, the dynamical system possesses a fixed point $(C_{st}, \tau_{st}, K_{st}) \in \mathbb{R}_+ \times [\tau_{st}^{woC}, \infty) \times [K_{st}^{woC}, \infty)$.

To verify this statement, choose an arbitrary but fixed $C_{st} \in \mathbb{R}_+$. Consequently, the development of the knowledge stock is independent of that of the capital stock and characterized by a linear, first order difference equation, where $\frac{d\tau_{t+1}}{d\tau_t} \in (0,1)$. Hence, the knowledge stock will approach a stationary state at

$$\tau_{st} = \begin{cases} \frac{\nu\varepsilon}{1-\nu q}, & C_{st} \le q\tau_{st}^{woC}; \\ \frac{\nu\beta C_{st}+\nu\varepsilon}{1-\nu q(1-\beta)}, & C_{st} > q\tau_{st}^{woC}. \end{cases}$$

As the knowledge stock approaches a stationary point τ_{st}, the capital stock must do so as well, because its dynamics then satisfy the following conditions for a fixed point.
If $K_1 > 0$ and

(a) $\frac{\partial s_t}{\partial r_{t+1}} \ge 0, \forall r_{t+1} \ge 0$,

(b) $\lim_{K_t \to 0} \phi'(K_t) > 1$,

(c) $\lim_{K_t \to \infty} \phi'(K_t) = 0$,

the function

$$\phi(K_t) = K_{t+1} = s_t K_t^{\alpha}(1-\alpha)^{1-\alpha}(\psi\alpha)^{\alpha}\tau_{st}^{\psi-\alpha}\varepsilon^{\alpha}$$

possesses a non-trivial fixed point.[10]

[10] The argument follows that of chapter 7 and, hence, that of Galor and Ryder (1989).

Condition (a) guarantees the existence of a single valued function $\phi(K_t) > 0, \forall K_t > 0$. As the properties (c2) and (c3) of the utility function imply that $s_t \geq s_{low} > 0$,[11] there exists a $\phi(K_t) > 0$ for every $K_t > 0$. Uniqueness is given if the derivative of $s_t K_t^\alpha Q$ with respect to $\phi(K_t)$ will be smaller than 1.[12] That is, if

$$\frac{\partial s_t}{\partial r_{t+1}} \frac{dr_{t+1}}{d\phi(K_t)} K_t^\alpha Q < 1,$$

where $Q = (1-\alpha)^{1-\alpha}(\psi\alpha)^\alpha \tau_{st}^{\psi-\alpha}\varepsilon^\alpha$. This condition holds because $\frac{\partial s_t}{\partial r_{t+1}} \geq 0$ and $\frac{dr_{t+1}}{d\phi(K_t)} < 0$[13].

Requirement (b) assures that the function ϕ is steeper than the 45 degree line at the origin and (c) guarantees that there exists a fixed point $\phi(K_t) = K_t$. The derivative of $\phi(K_t)$ writes

$$\frac{d\phi(K_t)}{dK_t} = \frac{s_t \alpha K_t^{\alpha-1} Q}{1 - \frac{\partial s_t}{\partial r_{t+1}} \frac{\partial r_{t+1}}{\partial \phi} K_t^\alpha Q}. \tag{8A.3}$$

Let's first consider requirement (c). For $K_t \to \infty$ the numerator will approach zero. The second term of the denominator being negative then suffices for (c) to hold. However, it is not immediately obvious from equation (8A.3) whether (b) holds. A simple argument is the following. As (b) is supposed to guarantee that the graph of $\phi(K_t)$ is above the bisecting line for $K_t \to 0$, the condition can be written as $s_t K_t^\alpha Q > K_t$ or $s_t K_t^{\alpha-1} Q > 1$, respectively. Since $s_t \in [s_{low}, 1)$ and $Q > 0$, the condition must hold for all K_t close enough to 0.
This completes the proof that the economy will reach a stationary state for fixed C_{st}.

The next step is to show that the economy's stock of information must be constant in the long run. This last step is identical to that in section 7.3.1. □

$\Phi = 1$

For $\Phi = 1$, the overlapping generations economy is characterized by the following system of difference equations:

$$\begin{aligned}
K_{t+1} &= s_t K_t^\alpha (1-\alpha)^{1-\alpha}(\psi\alpha\varepsilon q)^\alpha \tau_t^\psi (1+g_{\tau,t-1})^{-\alpha}, \\
\tau_{t+1} &= \nu q \tau_t + \nu\beta(\triangle C_{e,t} + \max\{0, C_t - q\tau_t\}) + \nu\varepsilon q \tau_t, \\
C_{t+1} &= C_t + \triangle C_{ie,t} + \triangle C_{e,t}.
\end{aligned}$$

The respective growth rates are

$$\begin{aligned}
g_{K,t} &= s_t K_t^{\alpha-1}(1-\alpha)^{1-\alpha}(\psi\alpha\varepsilon q)^\alpha \tau_t^\psi (1+g_{\tau,t-1})^{-\alpha} - 1, \\
g_{\tau,t} &= \nu q + \nu\beta \frac{\triangle C_{e,t} + \max\{0, C_t - q\tau_t\}}{\tau_t} + \nu\varepsilon q - 1, \\
g_{C,t} &= \frac{\triangle C_{ie,t} + \triangle C_{e,t}}{C_t}.
\end{aligned}$$

[11]The reasoning is the same as in section 7.3.1.
[12]The condition is derived from the implicit function theorem.
[13]Since $r_{t+1} = K_{t+1}^{\alpha-1}E$, where $E = \tau_{st}^{\psi-\alpha}\left(\frac{1-\alpha}{\varepsilon\psi\alpha}\right)^{-\alpha}\left(\tau_{st}\frac{1-\alpha}{\varepsilon\psi\alpha} - 1 + \alpha\right) > 0$, the derivative shows the form $\frac{dr_{t+1}}{d\phi(K_t)} = (\alpha-1)\phi^{\alpha-2}E < 0$.

The growth rates change from one period to the next according to

$$\begin{aligned} dg_{K,t} &= s_t K_t^{\alpha-1}(1-\alpha)^{1-\alpha}(\psi\alpha\varepsilon q)^{\alpha}\tau_t^{\psi}(1+g_{\tau,t-1})^{-\alpha}, \\ &\quad (g_{s,t}-(1-\alpha)g_{K,t}+\psi g_{\tau,t}-\alpha(1+g_{\tau,t-1})^{-1}dg_{\tau,t-1}) \\ dg_{\tau,t} &= \nu\beta\frac{\triangle C_{e,t}+\max\{0,C_t-q\tau_t\}}{\tau_t}(g_{\triangle C_e+\max\{0,C-q\tau\},t}-g_{\tau,t}), \\ dg_{C,t} &= \frac{\triangle C_{ie,t}+\triangle C_{e,t}}{C_t}(g_{\triangle C_{ie}+\triangle C_e,t}-g_{C,t}). \end{aligned}$$

Consequently, the following two kinds of non-trivial steady states can be distinguished:

(1) Steady state without knowledge codification

If the economy does not codify, $\triangle C_{e,t}+\max\{0,C_t-q\tau_t\}=0$ and $\triangle C_{ie,t}+\triangle C_{e,t}=0$. This implies $g_{C,s}=0$ and $g_{\tau,s}=g_{\tau}^{woC}$.
As s_t is bound on $[s_{low},1)$, the saving rate cannot grow at a constant rate other than 0. Consequently, $g_{K,s}=\frac{\psi}{1-\alpha}g_{\tau,s}$. Therefore, a steady state without codification is characterized by

$$\begin{aligned} g_{K,s} &= \frac{\psi}{1-\alpha}g_{\tau,s}, \\ g_{\tau,s} &= g_{\tau}^{woC}=\nu q(1+\varepsilon)-1, \\ g_{C,s} &= 0. \end{aligned}$$

(2) Steady state with knowledge codification

Positive codification implies $\triangle C_{e,t}>0$. Therefore, the overlapping generations economy can only realize steady-state behavior if $g_{C,s}=g_{\triangle C_{ie}+\triangle C_e,s}$ and $g_{\tau,s}=g_{\triangle C_e+\max\{0,C-q\tau\},s}$. By the same argument with respect to the saving rate as in the case with zero codification, the relation of the growth rates of capital and knowledge will be $g_{K,s}=\frac{\psi}{1-\alpha}g_{\tau,s}$. Consequently, steady states with positive knowledge codification imply

$$\begin{aligned} g_{K,s} &= \frac{\psi}{1-\alpha}g_{\tau,s}, \\ g_{\tau,s} &= g_{\triangle C_e+\max\{0,C-q\tau\},s}, \\ g_{C,s} &= g_{\triangle C,s}, \end{aligned}$$

where $\triangle C_t=\triangle C_{ie,t}+\triangle C_{e,t}$. Notice that a steady state with full codification is a special case where $g_{\tau,s}=g_{\tau}^{wC}=g_{\triangle C_e+\max\{0,C-q\tau\},s}=g_{C,s}=g_{\triangle C,s}=\nu(q(1+\varepsilon)+\beta(1-q))-1$.

We will now show that without knowledge codification, the overlapping generations economy will approach a non-trivial steady state.

Let $k_t := \frac{K_t}{\tau_t^{\frac{\psi}{1-\alpha}}}$. Without knowledge codification, the economy's dynamics can be summarized by the following first order difference equation:[14]

$$\phi(k_t) = k_{t+1} = \frac{K_{t+1}}{\tau_{t+1}^{\frac{\psi}{1-\alpha}}} = s_t k_t^{\alpha} \tilde{Q},$$

where $\tilde{Q} = (1+g_\tau^{woC})^{-\frac{\psi}{1-\alpha}-\alpha}(1-\alpha)^{1-\alpha}(\psi\alpha\varepsilon q)^{\alpha}$. To verify that the overlapping generations economy experiences a non-trivial steady-state equilibrium we use the same conditions as in the case of $\Phi = 0$. That is, $k_1 > 0$ and

(a) $\frac{\partial s_t}{\partial r_{t+1}} \geq 0$, $\forall r_{t+1} \geq 0$,

(b) $\lim_{k_t \to 0} \phi'(k_t) > 1$,

(c) $\lim_{k_t \to \infty} \phi'(k_t) = 0$.

Again condition (a) guarantees that for every $k_t > 0$ there exists a unique $\phi(k_t) > 0$. $\phi(k_t) > 0$ follows from $s_t \geq s_{low} > 0$ and uniqueness is given if

$$\frac{\partial s_t}{\partial r_{t+1}} \frac{dr_{t+1}}{d\phi(k_t)} k_t^{\alpha} \tilde{Q} < 1.$$

As $\frac{\partial s_t}{\partial r_{t+1}} \geq 0$ and $\frac{dr_{t+1}}{d\phi(k_t)} < 0$ the condition holds.[15]

With regard to requirements (b) and (c), the derivative of $\phi(k_t)$ writes

$$\frac{d\phi(k_t)}{dk_t} = \frac{s_t \alpha k_t^{\alpha-1} \tilde{Q}}{1 - \frac{\partial s_t}{\partial r_{t+1}} \frac{\partial r_{t+1}}{\partial \phi} k_t^{\alpha} \tilde{Q}}.$$

Requirement (c) holds as the numerator will approach zero for $k_t \to \infty$ and the second term of the denominator is negative. By rewriting condition (b) as $s_t k_t^{\alpha-1} \tilde{Q} > 1$, it becomes obvious that the inequality must hold for all k_t close enough to 0 since $s_t \in [s_{low}, 1)$ and $\tilde{Q} > 0$. Hence, the three conditions for the existence of a non-trivial fixed point are satisfied. Uniqueness and continuity of $\phi(k_t)$ together with requirements (b) and (c) imply that an economy with zero codification will show steady-state growth in the long run. □

Note that when assuming CIES-utility the steady state would be unique.[16]

[14] For more details see section 7.3.2.

[15] We can write $r_{t+1} = k_{t+1}^{\alpha-1} \tilde{E}$, where $\tilde{E} = (1-\alpha)^{-\alpha}(\psi\alpha\varepsilon q)^{\alpha}(1+g_\tau^{woC})^{-\alpha}\left(\frac{1-\alpha}{\psi\alpha\varepsilon q}(1+g_\tau^{woC}) - 1 + \alpha\right) > 0$. Since $g_\tau^{woC} =$ constant, the derivative shows the form $\frac{dr_{t+1}}{d\phi(k_t)} = (\alpha-1)\phi^{\alpha-2}\tilde{E} < 0$.

[16] See chapter 7.

8A.3 Influence of Knowledge Codification on the Return on Physical Capital

The return on capital in $t+1$ writes

$$r_{t+1} = K_{t+1}^{\alpha-1}\left(\frac{1-\alpha}{\psi\alpha}\right)^{-\alpha}\tau_{t+1}^{\psi-\alpha}\left(\frac{\partial\tau_{t+1}}{\partial L_{R,t+1}}\right)^{\alpha}\left(\tau_{t+1}\left(\frac{1-\alpha}{\psi\alpha}\right)\left(\frac{\partial\tau_{t+1}}{\partial L_{R,t+1}}\right)^{-1} - 1 + \alpha\right).$$

The derivative with respect to $\triangle C_{t,e}$ uses the chain rule, $\frac{\partial r_{t+1}}{\partial \triangle C_{t,e}} = \frac{\partial r_{t+1}}{\partial \tau_{t+1}}\frac{\partial \tau_{t+1}}{\partial \triangle C_{t,e}}$. As $\frac{\partial \tau_{t+1}}{\partial \triangle C_{t,e}} = \nu\beta = constant$, the sign of the change in the return on physical capital due to a marginal increase in knowledge codification is determined by $\frac{\partial r_{t+1}}{\partial \tau_{t+1}}$.

$$\begin{aligned}\frac{\partial r_{t+1}}{\partial \tau_{t+1}} &= (\psi-\alpha+1)\tau_{t+1}^{\psi-\alpha}K_{t+1}^{\alpha-1}\left(\frac{1-\alpha}{\psi\alpha}\right)^{1-\alpha}\left(\frac{\partial\tau_{t+1}}{\partial L_{R,t+1}}\right)^{\alpha-1} \\ &\quad -(1-\alpha)(\psi-\alpha)\tau_{t+1}^{\psi-\alpha-1}K_{t+1}^{\alpha-1}\left(\frac{1-\alpha}{\psi\alpha}\right)^{-\alpha}\left(\frac{\partial\tau_{t+1}}{\partial L_{R,t+1}}\right)^{\alpha}.\end{aligned}$$

The derivative is positive, if and only if

$$(\psi-\alpha+1)\tau_{t+1} > (\psi-\alpha)\frac{\partial\tau_{t+1}}{\partial L_{R,t+1}}.$$

For $\Phi = 1$, this condition can be written as

$$\frac{\psi\alpha\varepsilon q(\psi-\alpha)}{\psi-\alpha+1} < (1+g_{\tau,t}).$$

Since the growth rate of knowledge must be greater than $g_{\tau}^{woC} = \nu q(1+\varepsilon) - 1$, estimating the right hand side from below gives

$$\frac{(\psi\alpha+1-\alpha)(\psi-\alpha)}{\psi+1-\alpha} - 1 < \varepsilon.$$

As $\alpha \in (0,1)$ and $\psi \in (0,1]$, the fraction $\frac{(\psi\alpha+1-\alpha)(\psi-\alpha)}{\psi+1-\alpha}$ must be smaller than 1. Hence, knowledge codification in t increases the return on physical capital in $t+1$. □

Consider now the case $\Phi = 0$. Knowledge codification exerts a positive effect on the rent if

$$\tau_{t+1} > \frac{(\psi-\alpha)\varepsilon\psi\alpha}{1+\psi-\alpha}.$$

From $\alpha \in (0,1)$ and $\psi \in (0,1]$, it follows that $\frac{\partial r_{t+1}}{\partial \tau_{t+1}} > 0$ if $\psi \le \alpha$. In case $\psi > \alpha$, the right hand side is positive. Hence, for small levels of the knowledge stock, the representative household would decrease its rent by codification. The intuition is that at low levels of knowledge a marginal increase in τ_t causes wage payments of the intermediate entrepreneur to rise more than could be compensated by higher revenues and a decreasing number of researchers. However, every economy will reach levels of knowledge such that $\frac{\partial r_{t+1}}{\partial \triangle C_{t,e}} > 0$. The reason is that every economy approaches a stationary-state level $\tau_{st} \ge \tau_{st}^{woC}$ and

$$\tau_{st}^{woC} > \frac{(\psi-\alpha)\varepsilon\psi\alpha}{1+\psi-\alpha}.$$

To see that this condition holds, we insert $\tau_{st}^{woC} = \frac{\nu\varepsilon}{1-\nu q}$. Mathematical transformations yield

$$\frac{1}{\psi - \alpha} > \alpha\psi(1-q).$$

It is then obvious that the left hand side is greater than 1, whereas the right hand side is smaller than 1. □

Chapter 9

A Generalization

The purpose of this chapter is to give an overview over the model's structural elements that drive the results. The underlying motivation is to elaborate on whether the particular microeconomic structure of the basic model, such as a single monopolistic intermediate firm and one knowledge stock are crucial for the results.

9.1 General Properties of the Model

The argument of this section is that the main propositions of the previous chapter generally hold for overlapping generations economies with microeconomic structures that exhibit the following characteristics.

(U) With respect to the problem of the household, the equilibria allow to work with a representative household whose utility function $U_t(c_{1,t}, c_{2,t+1}) = u(c_{1,t}) + \delta u(c_{2,t+1})$ satisfies:

(c1) $u'(c) > 0, \quad u''(c) < 0;$

(c2) $\lim_{c \to \infty} u'(c) = 0$, $\lim_{c \to 0} u'(c) = \infty;$

(c3') $\frac{\partial s_t}{\partial r_{t+1}} \geq 0$, $\frac{\partial s_t}{\partial w_t} \geq 0$ and $\lim_{k_t \to \infty} \frac{\partial s_t}{\partial w_t} k_t^{\alpha} < \infty$.

(K1) The resulting dynamics of the economy's knowledge stock can be represented by

$$\tau_{t+1} = q\tau_t + \beta \max\{0, C_{t+1} - q\tau_t\} + \varepsilon \left[q\tau_t + \tilde{\Phi}(\beta \max\{0, C_{t+1} - q\tau_t\}) \right]^{\Phi} L_{R,t+1},$$

where $\tilde{\Phi}, \Phi \in \{0, 1\}$.

(A2') is given with respect to knowledge codification. More precisely:

In a period t and for all $i, j \geq q\tau_t$: Before an idea indexed by i is codified, all ideas with index j, $j < i$ have been codified already. This implies that if i is codified in *period t, then $j < i$ is also codified in t.*

(P1) The aggregate production function is of the Cobb-Douglas type

$$F_t(L_{A,t}, K_t, \tau_t) = \xi L_{A,t}^{1-\alpha} K_t^{\alpha} \tau_t^{\psi},$$

where $\xi \in \mathbb{R}_{++}$ is an arbitrary constant.

(P2) There is one labor market and the equilibrium wages of the researchers, $w_{R,t}$, and the workers in final-goods production, $w_{A,t}$, depend (directly or indirectly) on their marginal productivity in final-goods production, such that

$$w_{A,t} = \kappa L_{A,t}^{-\alpha} K_t^{\alpha} \tau_t^{\psi},$$

$$w_{R,t} = \lambda L_{A,t}^{1-\alpha} K_t^{\alpha} \tau_t^{\psi-1} \frac{\partial \tau_t}{\partial L_{R,t+1}},$$

with constants $\lambda, \kappa \in \mathbb{R}_{++}$.

(P3) The return on capital, r_t, exhibits the form

$$r_t = K_t^{\alpha-1} h(\tau_t),$$

where $h(\tau_t) = \tau_t^{\psi} f(L_{A,t}) \geq 0$ and $\frac{\partial h(\tau_t)}{\partial \triangle C_{e,t}} = \tau_t^{\psi-1} g(L_{A,t})$.

Additionally it is necessary that:

(N) The decision maker with respect to knowledge codification possesses positive measure.

The properties have the following implications:

As in the labor market equilibrium $w_{A,t} = w_{R,t} = w_t$, condition (P2) assures an equilibrium allocation of labor according to

$$L_{A,t} = \frac{\kappa}{\lambda} \frac{\tau_t}{\frac{\partial \tau_t}{\partial L_{R,t}}}.$$

This is structurally identical to equation (8.3) giving the equilibrium allocation of labor in the model of the previous chapter. Note that for $\Phi = \tilde{\Phi} = 1$, the shares of workers and researchers are independent of the stock of information as the productivity of both increases in knowledge codification of previous periods by the same factor. Using the equilibrium allocation of labor as derived from (P2) together with (K1) and (A2') then yields the dynamical system

$$\begin{aligned} K_{t+1} &= s_t \kappa^{1-\alpha} \lambda^\alpha K_t^\alpha \tau_t^{\psi-\alpha} \left(\frac{\partial \tau_t}{\partial L_{R,t}} \right)^\alpha, \\ \tau_{t+1} &= \nu q \tau_t + \nu \max\{0, C_{t+1} - q\tau_t\} + \nu\varepsilon \left[q\tau_t + \tilde{\Phi}(\beta \max\{0, C_{t+1} - q\tau_t\}) \right]^\Phi, \\ C_{t+1} &= C_t + \triangle C_t, \end{aligned}$$

where $\nu = \left(1 + \frac{\kappa}{\lambda}\right)^{-1}$. This system of difference equations corresponds to that given in section (8.3). The only difference is that the present specification allows for $\tilde{\Phi} = 1$. It is obvious that for $\Phi = 0$, there is no change. With $(\Phi, \tilde{\Phi}) = (1, 1)$, the equilibrium allocation of labor is independent of knowledge codification. This simplifies the above difference equations to

$$\begin{aligned} K_{t+1} &= s_t \kappa^{1-\alpha} \lambda^\alpha K_t^\alpha \tau_t^{\psi} \left(\frac{\nu(1+\varepsilon)}{\varepsilon} \right)^\alpha, \\ \tau_{t+1} &= \nu[q\tau_t + \beta \max\{0, C_{t+1} - q\tau_t\}](1+\varepsilon), \\ C_{t+1} &= C_t + \triangle C_t. \end{aligned}$$

It is verified in appendix 9A that this dynamical system shows the same long run behavior as the one with $(\Phi, \tilde{\Phi}) = (1, 0)$ and, hence, the nontrivial steady states are characterized by the familiar relation of the growth rates of physical capital and knowledge:

$$g_{K,s} = \frac{\psi}{1-\alpha} g_{\tau,s}.$$

Properties (c1)-(c3') of the utility function further assure that in the case of zero codification (S1), the economy will approach steady-state behavior in the long run. Details are given in appendix 9A. Note that the proof is identical to that in appendix 8A.2, except that the saving rate may now also depend on the representative agent's wage w_t. Barro and Sala-i-Martin (1995) have shown that for a neoclassical growth model to have a steady state, technological progress must be labor-augmenting, which is always satisfied by a Cobb-Douglas production function, hence (P1).[1]

Finally, due to (P3) the first order conditions of the household yield the equality of marginal benefits of knowledge codification and its opportunity costs according to

$$M = K_{t+1}^\alpha \tau_{t+1}^{\psi-1} g(L_{A,t+1}) - \gamma_t (1 - K_{t+1}^{\alpha-1} \tau_{t+1}^{\psi} f(L_{A,t+1})).$$

Further it is important to point out that a necessary condition for knowledge codification is (N). This means e.g. that the economy comprises a finite number of households or the amount of knowledge codification is the outcome of a collective decision of a group of positive measure. In the previous chapters, this group was the shareholders of the

[1] Barro and Sala-i-Martin (1995), p. 54.

intermediate firm. (N) is necessary as a decision maker with zero measure cannot influence the capital rent because her contribution to the stock of information would be nil.

Note that with the properties (c1) - (c2) of the utility function, the household's problem possesses a unique solution in (S1)(but not necessarily in (S2)). Additionally assuming (c3') assures steady-state behavior in the long run. For the case $\Phi = 1$, this is sufficient for the main result.

Proposition 9.1 *An overlapping generations economy characterized by (U), (K1), (A2'),(P1)-(P3) and (N) with constant codification costs will be codifying in the long run if either of the following conditions are satisfied:*

(i) The steady-state growth rate of capital is higher than that of the knowledge stock (or equivalently $\psi > 1 - \alpha$).

(ii) The steady-state growth rate of capital is equal to that of the knowledge stock (or equivalently $\psi = 1 - \alpha$) and

$$\gamma + \gamma(k_{s,min}^{woC})^{\alpha-1} f(L_{A,t}) < (k_{s,min}^{woC})^{\alpha} g(L_{A,t}),$$

where $k_{s,min}^{woC}$ is the minimum steady-state level of $k_t = \frac{K_t}{\tau_t}$ without codification.

For declining codification costs, the economy will be codifying in the long run if the steady-state growth rate of knowledge exceeds that of capital by less than the rate at which the codification costs decline.

Proof. The proof proceeds along the lines of the previous chapters. The difference is that in this general specification, the pair $(K_{t+1}, \triangle C_{e,t})$ that satisfies $M = 0$ may not be a maximum of the utility function. Hence the condition $M = 0$ cannot be used for the argument. However, M still equates the marginal benefit of knowledge codification with its opportunity costs. In this way, the line of argument is that assuming no codification leads to steady-state growth, which implies $M > 0$ for the conditions given in the proposition. However, $M > 0$ indicates that the representative household would be better off when investing in knowledge codification at least the last marginal unit saved in physical capital.
For reasons of simplicity, we will work with

$$\tilde{M} = \gamma_t^{-1} \frac{r_{t+1}}{\triangle C_{e,t}} K_{t+1} - 1 - r_{t+1}.$$

Intuitively, this is just looking at the problem from the other side. In particular, one marginal unit of the homogenous good saved in physical capital yields $1 + r_{t+1}$ of consumption in the next period, whereas spending this marginal unit in knowledge codification enhances consumption in $t + 1$ by $\gamma_t^{-1} \frac{r_{t+1}}{\triangle C_{e,t}} K_{t+1}$. $\tilde{M}$ varies over time according to

$$d\tilde{M} = \frac{\partial \tilde{M}}{\partial K_{t+1}} dK_{t+1} + \frac{\partial \tilde{M}}{\partial \tau_t} d\tau_t + \frac{\partial \tilde{M}}{\partial \gamma_t} d\gamma_t.$$

This equation can be written as

$$d\tilde{M} = \frac{K_{t+1}^{\alpha} \tau_{t+1}^{\psi-1}}{\gamma_t} g(L_{A,t+1})[\alpha g_{K,t+1} - (1-\psi) g_{\tau,t} - g_{\gamma,t}] + \gamma_t K_{t+1}^{\alpha-1} \tau_{t+1}^{\psi} f(L_{A,t+1})[(1-\alpha) g_{K,t+1} - \psi g_{\tau,t}].$$

Using the steady-state growth rates, the total derivative will be positive if

$$\frac{K_{t+1}^{\alpha} \tau_{t+1}^{\psi-1}}{\gamma_t} g(L_{A,t+1}) \left[\frac{\psi - (1-\alpha)}{1-\alpha} g_{\tau,t} - g_{\gamma,t} \right] > 0.$$

Note that in steady state the rent is constant and hence whether $\tilde{M}$ increases or decreases depends on how the marginal codification benefit develops over time. The above equation reveals that for constant codification costs and $\psi > 1 - \alpha$, the term on the left hand side is positive and increases to infinity in the limit. This establishes (i).[2]
With regard to declining codification costs, we rewrite

$$\frac{\psi - (1-\alpha)}{1-\alpha} g_{\tau,t} - g_{\gamma,t} = g_{K,s} - g_{\tau,t} - g_{\gamma,t}.$$

It is then clear that for the difference of the steady-state growth rates of knowledge and capital being smaller than the rate at which the codification costs decline, the codification benefit is strictly increasing over time and hence the agent must face a period with $\tilde{M} > 0$, implying that she would be better off by deviating from zero codification.
(ii) is proven in the same way as for proposition 8.7 and will be omitted at this place, hence. □

Consider additionally:

(M1) The household's utility maximization problem possesses a unique solution in (S1) and (S2) and $\frac{\partial M}{\partial \triangle C_{e,t}} < 0$.

For example, property (M1) would be given for the specification of chapter 8 with $\psi \leq \alpha$ and the utility function additionally satisfying (c4).[3] For $(\Phi, \tilde{\Phi}) = (1, 1)$ and utility satisfying (U) + (c4), (M1) would be given for all $\psi \in (0, 1]$. This is the content of the following lemma.

[2]Note that with $\psi < 1 - \alpha$, the left hand side would approach zero from below.

[3]Note that (c3') is a weaker requirement than (c3). Hence, utility functions satisfying (c3) will also satisfy (c3'). Recall (c4): $\frac{u''(c_{2,t+1})}{u'(c_{2,t+1})} + \frac{1}{2c_{2,t+1}} < -\frac{u''(c_{1,t})}{u'(c_{1,t})} \frac{1}{1+r_{t+1}}$

Lemma 9.1 *For* $(\Phi, \tilde{\Phi}) = (1, 1)$*, the equilibrium allocation of labor in* $t+1$ *does not depend on knowledge codification in* t *and* $\frac{\partial^2 r_{t+1}}{\partial \triangle C_{e,t}^2} \leq 0,\ \forall \psi \in (0, 1]$.

Proof. With $(\Phi, \tilde{\Phi}) = (1, 1)$, the difference equation of the knowledge stock can be written as

$$\tau_{t+1} = \nu(q\tau_t + \beta \max\{0, C_{t+1} - q\tau_t\})(1 + \varepsilon).$$

Consequently, the equilibrium share of workers calculates to

$$L_{A,t+1} = \frac{\kappa}{\lambda} \frac{\nu(1 + \varepsilon)}{\varepsilon}.$$

With the labor shares being independent of the information stock, the second derivative of the return on capital in $t+1$ with respect to the amount of codified ideas in t equals

$$\frac{\partial^2 r_{t+1}}{\partial \triangle C_{e,t}^2} = \psi(\psi - 1)\tau_{t+1}^{\psi-2} K_{t+1}^{\alpha-1} (\nu\beta)^2 f(L_{A,t+1}) < 0.$$

□

As discussed in section 8.4, $\frac{\partial^2 r_{t+1}}{\partial \triangle C_{e,t}^2} < 0$ is sufficient for $\frac{\partial M}{\partial \triangle C_{e,t}} < 0$ and implies that the household's second order condition must be satisfied if the utility function exhibits properties (c1)-(c3') and (c4).
Given (M1), it is then possible to state the following result.

Proposition 9.2 *For an overlapping generations economy satisfying properties* $(U), (K1), (A2'), (P1) - (P3), (N)$*, and* $(M1)$*, the lemmata 5.1-5.3, the propositions 5.1, 8.3, 8.5, 8.6, 8.9, 8.10, and corollaries 8.1 and 8.2 apply.*

Proof. With (M1) additionally assuring the uniqueness of the household's utility maximizer and $\frac{\partial M}{\partial \triangle C_{e,t}} < 0$, the proofs of section 8.4 can be applied. □

9.2 Summary

This chapter expatiated the model's structural elements that drive the results. It has been argued that under more or less standard assumptions with respect to the production sectors and if the dynamics of the knowledge stock can be represented by a difference equation similar to that of the basic specification in the first part, the result that an overlapping generations economy will be codifying in the long run if the steady-state growth rate of capital is higher than that of the knowledge stock, respectively, if the difference between the growth rates of knowledge and capital is smaller than the rate at which the codification costs decline, will generally hold for concave utility functions with one further restriction on the curvature, (c3'), so as to assure for steady-state behavior of the economy without knowledge codification. The reason is that also in the case of increasing returns in consumption forgone in which the sufficient conditions for a maximum of the household's problem are violated, it is favorable to invest in knowledge codification under the respective conditions. However, this situation will then show a corner solution in which the agent does not consume at all in the first period of her life. Further restrictions on utility and/or the parameters of the production functions such that the household's utility maximization problem possesses unique inner solutions yield the structure of the basic model as unfolded by the lemmata 5.1-5.3. Consequently the main results as given in chapter 8 apply. It was further shown that the restriction of the assumptions on technological progress on $\psi < \alpha$ so as to guarantee for an inner solution of the household's problem with the specification of utility in the basic model by (c1)-(c4) is not necessary in the case with $\Phi = 1$ if the productivity of the researchers positively depends on the ideas they have acquired by utilizing the codified sources of knowledge. In this case, more precisely with $(\Phi, \tilde{\Phi}) = (1, 1)$, (c1)-(c4) are sufficient for an inner solution for the entire range $\psi \in (0, 1]$. Intuitively, the productivity of the researchers, when also depending on the knowledge acquired by attending to the information stock, increases in the amount of knowledge codification by the same factor as the productivity of the workers in final-goods production. Thus, the equilibrium allocation of labor is not affected by knowledge codification of previous periods. This eliminates the second channel by which codification activities may increase the return on physical capital, and hence the source of the convexity of the return on physical capital in knowledge codification.

9A Appendix of Chapter 9

9A.1 Equivalence of the Dynamical Systems with $(\Phi, \tilde{\Phi}) = (1, 1)$ and $(\Phi, \tilde{\Phi}) = (1, 0)$

For $(\Phi, \tilde{\Phi}) = (1, 1)$, the overlapping generations economy is characterized by the following system of difference equations:

$$\begin{aligned} K_{t+1} &= s_t \kappa^{1-\alpha} \lambda^{\alpha} K_t^{\alpha} \tau_t^{\psi} \left(\frac{\nu(1+\varepsilon)}{\varepsilon} \right)^{\alpha}, \\ \tau_{t+1} &= \nu[q\tau_t + \beta(\triangle C_{e,t} + \max\{0, C_t - q\tau_t\})](1+\varepsilon), \\ C_{t+1} &= C_t + \triangle C_t. \end{aligned}$$

The respective growth rates are

$$\begin{aligned} g_{K,t} &= s_t \kappa^{1-\alpha} \lambda^{\alpha} K_t^{\alpha-1} \tau_t^{\psi} \left(\frac{\nu(1+\varepsilon)}{\varepsilon} \right)^{\alpha} - 1, \\ g_{\tau,t} &= \nu \left[q + \beta \frac{\triangle C_{e,t} + \max\{0, C_t - q\tau_t\}}{\tau_t} \right] (1+\varepsilon) - 1, \\ g_{C,t} &= \frac{\triangle C_{ie,t} + \triangle C_{e,t}}{C_t}. \end{aligned}$$

The growth rates change from one period to the next according to

$$\begin{aligned} dg_{K,t} &= s_t \kappa^{1-\alpha} \lambda^{\alpha} K_t^{\alpha-1} \tau_t^{\psi} \left(\frac{\nu(1+\varepsilon)}{\varepsilon} \right)^{\alpha} (g_{s,t} - (1-\alpha) g_{K,t} + \psi g_{\tau,t}), \\ dg_{\tau,t} &= \nu\beta(1+\varepsilon) \frac{\triangle C_{e,t} + \max\{0, C_t - q\tau_t\}}{\tau_t} (g_{\triangle C_e + \max\{0, C - q\tau\}, t} - g_{\tau,t}), \\ dg_{C,t} &= \frac{\triangle C_{ie,t} + \triangle C_{e,t}}{C_t} (g_{\triangle C_{ie} + \triangle C_e, t} - g_{C,t}). \end{aligned}$$

As in steady state $(dg_{K,t}, dg_{\tau,t}, dg_{C,t}) = (0, 0, 0)$, it is clear that this system shows the same long run behavior as with $(\Phi, \tilde{\Phi}) = (1, 0)$ and the steady states are characterized by

$$g_{K,s} = \frac{\psi}{1-\alpha} g_{\tau,s}.$$

9A.2 Steady-State Behavior without Knowledge Codification

This section of the appendix shows that (c1)-(c3) suffices for the economy to approach non-trivial steady-state growth without codification. In analogy to section 8A.2, in the situation with zero knowledge codification, that is in (S1), the dynamics of the overlapping generations economy can be described by

$$\phi(k_t) = k_{t+1} = \frac{K_{t+1}}{\tau_{t+1}^{\frac{\psi}{1-\alpha}}} = s_t k_t^{\alpha} \tilde{Q}.$$

Of course, this applies to both cases, $(\Phi, \tilde{\Phi}) = (1, 0)$ and $(\Phi, \tilde{\Phi}) = (1, 1)$.

The concavity of the utility function and the Inada conditions suffice in (S1) for a unique solution of the household's utility maximization problem. The difference to the proof in section 8A.2 is that with utility given by (U), the saving rate may not only depend on the return on capital in the following period (as required by (c3) in the basic model), but also on the representative household's income. However, (c3') is sufficient for a fixed point of $\phi(k_t)$.

To verify that the overlapping generations economy experiences a non-trivial steady-state equilibrium, consider again the following three conditions. That is, $k_1 > 0$ and

(a) $\frac{\partial s_t}{\partial r_{t+1}} \geq 0, \forall r_{t+1} \geq 0,$

(b) $\lim_{k_t \to 0} \phi'(k_t) > 1,$

(c) $\lim_{k_t \to \infty} \phi'(k_t) = 0.$

Condition (a) is satisfied by the same argument as in section 8A.2.

With regard to requirements (b) and (c), the derivative of $\phi(k_t)$ now writes

$$\frac{d\phi(k_t)}{dk_t} = \frac{\alpha k_t^{\alpha-1}\tilde{Q}(s_t + \frac{\partial s_t}{\partial w_t} k_t^{\alpha})}{1 - \frac{\partial s_t}{\partial r_{t+1}} \frac{\partial r_{t+1}}{\partial \phi} k_t^{\alpha} \tilde{Q}}.$$

Requirement (c) holds as with (c3') the numerator will approach zero for $k_t \to \infty$ and the second term of the denominator is negative. Condition (b) is again most easily verified by rewriting it as $s_t k_t^{\alpha-1}\tilde{Q} > 1$, it becomes obvious that the inequality must hold for all k_t close enough to 0 since $s_t \in [s_{low}, 1)$ and $\tilde{Q} > 0$. Hence, the three conditions for the existence of a non-trivial fixed point are satisfied.

The same reasoning applies to the case $\Phi = 0$ in order to show that the capital stock will approach a stationary state for a given knowledge stock. Therewith the proof in section 8A.2 can be applied to show that the overlapping generations economy where the researchers have only a finite amount of ideas per period approaches a stationary state.

Chapter 10

Oligopolistic Intermediate Sectors

The basic model structure comprises one monopolistic intermediate firm. This chapter sketches a simple model structure with oligopolistic competition in several intermediate sectors where each sector draws upon a different field of knowledge. It is shown that under certain symmetry assumptions the resulting equilibrium exhibits the properties specified in the previous chapter. This implies that a monopolistic intermediate firm is not a necessary condition for the results derived from the basic model.

10.1 Outline of the Model Structure

10.1.1 Knowledge and Information

Let there be m different fields of knowledge indexed by $k \in \{1, ..., m\}$ which consist of infinite sets of ideas $\mathcal{I}_k$. Analogously to the definitions in the second chapter, a person P_t's knowledge in the field k is the subset of $\mathcal{I}_k$ which is embodied in P_t: $\mathcal{T}_{P_t,k} = \{i_k \in \mathcal{I}_k | i_k \text{ is embodied in } P_t\}$. And naturally, we define a person's knowledge as $\mathcal{T}_{P_t} = \cup_k \mathcal{T}_{P_t,k}$. Accordingly, the information stock in a specific field of knowledge is $\mathcal{C}_{k,t} = \{i_k \in \mathcal{I}_k | i_k \text{ is codified}\}$ and the entire set of codified ideas is $\mathcal{C}_t = \cup_k \mathcal{C}_{k,t}$. By specifying $\mathcal{I}_k = \mathbb{R}_+$ it is clear that $\mathcal{T}_{P_t}, \mathcal{C}_t \subset \mathbb{R}^k_+$. We assume (A1) and (A2') in each field of knowledge.[1] Then a person's knowledge can be represented by a vector $\tau_{P_t} = (\sup \mathcal{T}_{P_t,k})_{k \in \{1,...,m\}} \in \mathbb{R}^k_+$. Further define $C_t = (\sup \mathcal{C}_{k,t})_{k \in \{1,...,m\}}$.

Let the different fields of knowledge constitute different sectors producing intermediate good x_k , $k \in \{1, ..., m\}$. Suppose each sector is characterized by an oligopolistic structure of n firms indexed by $l \in \{1, ..., n\}$. The oligopolistic structure can be interpreted as resulting from "equivalent innovation" as for example used in Young (1998). The argument is that inventions are oftentimes "paralleled by equivalent but unlike means for reaching the same goal around the same time" (Gilfillan, 1935).[2] This may be the case if ideas cannot be fully monopolized such as when imitation or "inventing around" is possible. Or an increase in the knowledge stock may be interpreted as process innovations which cannot be patented. In contrast to the model of Young (1998), however, the variety in each sector is exogenous and there is only quality improving innovation.

[1] Recall that (A1) requires that if a person knows an idea indexed by i, she must also know all ideas j, with $j < i$.

[2] The quote was taken from Young (1998).

10.1.2 Final Goods Production

The final-goods sector produces the homogeneous consumption good by the following production function:

$$F((x_{k,t})_{k\in\{1,...,m\}}, L_{A,t}) = L_{A,t}^{1-\alpha}\sum_{k=1}^{m} x_{k,t}^{\alpha}.$$

There is a continuum of identical firms on [0,1], and we will again work with a representative final-goods firm whose factor demand and output values are interpreted as aggregate values. Consequently the representative firm solves

$$\max_{x_t, L_{A,t}} \pi_t^{fp} = F((x_{k,t})_{k\in\{1,...,m\}}, L_{A,t}) - \sum_{k=1}^{m} p_{x_k,t}x_{k,t} - w_{A,t}L_{A,t}.$$

Accordingly, the demand for labor and intermediate goods is characterized by

$$\begin{aligned} p_{x_k,t} &= \alpha x_{k,t}^{\alpha-1} L_{A,t}^{1-\alpha}, \quad \forall k \in \{1,...,m\}, \\ w_{A,t} &= (1-\alpha)L_{A,t}^{-\alpha}\sum_{k=1}^{m} x_{k,t}^{\alpha}. \end{aligned}$$

10.1.3 The Intermediate Goods Sectors

As in the basic model, the intermediate firms in the the different intermediate goods sectors are managed by the capital owners, that is the old generation. The shareholders of each firm decide collectively on the investment in knowledge codification by the same process as described in section 3.2.1. Assume that each intermediate firm faces codification costs of γ_t per idea.

Each sector produces an intermediate product x_k according to the production function $G(K_k, \tau_k) = K_k\tau_k^{\frac{\psi}{\alpha}}$. Where K_k denotes the specific capital stock and τ_k represents the knowledge stock in the specific field.[3] The knowledge stock of an intermediate firm is defined as the highest index in the set of all researchers' human capital of firm l with respect to the field of knowledge k. More precisely, $\tau_{k,l,t} := \sup \cup_{P_t\in\mathcal{G}_{k,l,t}} \mathcal{T}_{P_t,k}$, where, in analogy to the basic model, $\mathcal{G}_{k,l,t}$ denotes the research group employed by intermediate firm l in sector k. Accordingly define the knowledge stock of sector k as $\tau_{k,t} := \sup_l \tau_{k,l,t}$ and the knowledge stock of the economy as the vector $\tau_t = (\tau_{k,t})_{k\in\{1,...,m\}}$. Human capital of a person P_t again originates from three sources:

$$\begin{aligned} \tau_{P_t} = {} & q(\tau_{k,t-1})_{k\in\{1,...,m\}} + \beta(0,...,\max\{0, C_{k,l,t} - q\tau_{k,t-1}\}, 0,...,0) \\ & +\varepsilon m(0,...,(q\tau_{k,t-1} + \beta\max\{0, C_{k,l,t} - q\tau_{k,t-1}\})^{\Phi},...,0). \end{aligned} \tag{10.1}$$

[3] The interpretation of K_k as specific capital is not necessary for the results. It can also be interpreted as general capital.

First, every individual exogenously receives a share q of the economy's knowledge stock of the previous period, e.g. by some sort of schooling. Note that the persons are transferred a share of each of the different knowledge stocks τ_k, which then enables them to do research in any sector. Again, the transfer is imperfect: $q \in (0, 1)$.

As a second source, the members of the new generation who are becoming researchers in an intermediate firm can enhance their human capital by attending to codified ideas if the previous generations of researchers in this firm have codified their sector specific knowledge. $C_{k,l,t}$ denotes the highest index of all ideas in firm l's knowledge database. Again (A2') is required to hold with respect to knowledge codification.

A person P_t deciding to do research additionally generates a number $\varepsilon m(q\tau_{k,t-1} + \beta \max\{0, C_{k,l,t} - q\tau_{k,t-1}\})^{\Phi}$ of new ideas in her specific field of research k. It is assumed that the variety m of a person's knowledge positively affects research success and that the number of new ideas generated are a function of a person's entire knowledge in the respective field. This corresponds to a specification with $\tilde{\Phi} = 1$.

The interpretation of equation (10.1) is that the members of the young generation are exogenously transferred a certain general education and will then acquire sector specific knowledge by utilizing the firm's knowledge database and own research. Suppose now a person is hired in sector k by firm l. She is then a member of this firm's research group $\mathcal{G}_{k,l,t}$. In accordance to the basic specification, we assume that there are spillovers within research groups but no spillovers between research groups of different firms within the same sector k or between sectors.[4] Consequently, this person's human capital in the field k then writes

$$\begin{aligned} \tau_{P_t,k} &= q\tau_{k,t-1} + \beta \max\{0, C_{k,l,t} - q\tau_{k,t-1}\} \\ &\quad + \varepsilon m(q\tau_{k,t-1} + \beta \max\{0, C_{k,l,t} - q\tau_{k,t-1})^{\Phi} L_{\mathcal{G}_{k,l,t}}, \end{aligned} \tag{10.2}$$

where $L_{\mathcal{G}_{k,l,t}}$ denotes the size of the research group.[5] This is the $k-th$ projection of P_t's knowledge. All members of a firm's research group are symmetric with respect to knowledge as they are transferred the same ideas exogenously and build up additional human capital from the same stock of codified ideas within the intermediate firm. Equivalently to the basic model there is the symmetry of the workers' knowledge which is determined by the exogenously transferred ideas. Within a period t, an intermediate firm in sector k maximizes

$$\max_{x_{k,l,t}, L_{\mathcal{G}_{k,l,t}}} \pi^{int}_{k,l,t} = p_{x_k,t}(x_{k,t})x_{k,l,t} - w_{R,t}L_{\mathcal{G}_{k,l,t}},$$

where $x_{k,t} = \sum_l x_{k,l,t}$.

[4] Note that spillovers within periods could also be allowed for without qualitative effects on the results with respect to knowledge codification. Conceptionally, they could be interpreted as endogenous spillovers within a generation by own research in the style of Cohen and Levinthal (1989).

[5] More precisely, it is defined as the Lebesgue-measure of $\mathcal{G}_{k,l,t}$.

For a given capital stock $K_{k,l,t}$, the problem is one-dimensional in $L_{\mathcal{G}_{k,l,t}}$ and the factor demand is given by

$$w_{R,t} = p_{x_k,t}(x_{k,t})\frac{\partial x_{k,l,t}}{\partial L_{\mathcal{G}_{k,l,t}}} + \frac{\partial p_{x_k,t}(x_{k,t})}{\partial x_{k,t}}\frac{\partial x_{k,t}}{\partial x_{k,l,t}}\frac{\partial x_{k,l,t}}{\partial L_{\mathcal{G}_{k,l,t}}}x_{k,l,t}.$$

Using the first order condition of the final-goods firm yields[6]

$$w_{R,t} = \alpha x_{k,t}^{\alpha-1}L_{A,t}^{1-\alpha}\frac{\partial x_{k,l,t}}{\partial L_{\mathcal{G}_{k,l,t}}} + \alpha(\alpha-1)x_{k,t}^{\alpha-2}L_{A,t}^{1-\alpha}\frac{\partial x_{k,l,t}}{\partial L_{\mathcal{G}_{k,l,t}}}x_{k,l,t}. \tag{10.3}$$

10.1.4 The Problem of the Household

The problem of the household remains principally the same as in the basic model. That is, each individual decides on the amount of physical capital savings decentrally and with regard to investment in knowledge codification participates in a collective decision within the firm. Let the utility function be specified as in the basic model by (c1)- (c4). As we would like to focus on symmetric equilibria, we make the following additional assumption:

Assumption 10.1

- ▷ *Each individual's physical capital savings are used by only one firm. That is, each individual is capital owner of exactly one firm.*
- ▷ *An individual of the young generation buys the shares of his ancestor and may additionally "net-save" a certain amount of his wage.*

It is assumed that each individual is free to work in final-goods production or as a researcher in any of the intermediate firms. By this, there is only one labor market and in equilibrium, the individuals will be symmetric with respect to wage income when young. Hence, a member P_t of the young generation who invests in firm l of sector k faces the problem:

$$\max_{s_t,\varsigma_t} U_t(c_{1,P_t}, c_{2,P_t}) = u(c_{1,P_t}) + \delta u(c_{2,P_t})$$

subject to

$$c_{1,P_t} = w_t(1 - s_{P_t} - \varsigma_{P_t}),$$
$$c_{2,P_t} = (1 + r_{k,l,t+1})s_{P_t}w_t.$$

we denote the set of capital owners of firm l in sector k in period $t+1$ by $\mathcal{P}_{k,l,t}$. A member of this set then proposes a codification investment

$$\zeta_{k,l,t} = \int_{\mathcal{P}_{k,l,t}} \zeta_{P_t} dP_t,$$

[6] The sufficient condition writes $\alpha(\alpha-1)x_{k,t}^{\alpha-2}\left(\frac{\partial x_{k,l,t}}{\partial L_{\mathcal{G}_{k,l,t}}}\right)^2 L_{A,t}^{1-\alpha}[2+(\alpha-2)\frac{x_{k,l,t}}{x_{k,t}}] < 0$. As $\frac{x_{k,l,t}}{x_{k,t}} \leq 1$, the second order condition for a profit maximum holds.

where $\zeta_{P_t} = \varsigma_{P_t} w_t$. Due to the symmetry of all $P_t \in \mathcal{P}_{k,l,t}$, each individual makes the same proposal (which will then be accepted, of course) derived from the first order conditions:

$$\begin{aligned} u'(c_{1,P_t}) &= \delta u'(c_{2,P_t})(1 + r_{k,l,t+1}), \\ u'(c_{1,P_t}) &= \delta u'(c_{2,P_t}) \frac{\partial r_{k,l,t+1}}{\partial \varsigma_{P_t}} s_{P_t}. \end{aligned}$$

In this way, firm l's capital stock in $t+1$ is equal to

$$S_{k,l,t} = \int_{\mathcal{P}_{k,l,t}} S_{P_t} dP_t,$$

where $S_{P_t} = s_{P_t} w_t$.

10.1.5 Equilibrium

It is assumed that all m sectors are symmetric with respect to the initial knowledge stock $\tau_{k,0} = \bar{\tau}_0$ and capital stocks $K_{k,1} = \bar{K}_1$, and so are all firms: $K_{k,l,1} = \bar{K}_{l,1}$, $\tau_{k,l,0} = \bar{\tau}_0$, $C_{k,l,1} = \bar{C}_1$.

Definition 10.1 *Given* $K_{k,1}, \tau_{k,0} > 0$, $C_{k,l,1} \geq 0, K_{k,l,t+1} = S_{k,l,t}$ *and* $C_{k,l,t+1} = f(C_{k,l,t}, \zeta_{k,l,t})$, *a sequential markets equilibrium is allocations* $(c_{2,P_0})_{P_0 \in \mathcal{P}_0}$, $\{(c_{1,P_t}, c_{2,P_t}, S_{P_t}, \zeta_{P_t})_{P_t \in \mathcal{P}_t}, L_{A,t}, (L_{\mathcal{G}_{k,l,t}})_{\forall(l,k)}, (x_{k,l,t})_{\forall(l,k)}\}_{t=1}^{\infty}$ *and prices* $\{w_t, (p_{x_{k,t}})_{k \in \{1,\ldots,m\}}\}_{t=1}^{\infty}$*, such that*

(i) they solve the utility maximization problem of the households and the profit maximization problems of the representative final-product firm and of the intermediate goods firms for all $t \geq 1$*, and*

(ii) in every period, the economy is in temporary equilibrium, that is, for all $t \geq 1$*:*

(a) (Labor Market)

$$L_t^s = L_{A,t}^d + \sum_{k=1}^{m} \sum_{l=1}^{n} L_{\mathcal{G}_{k,l,t}}^d;$$

(d) (m Intermediate Goods Markets)

$$\sum_{l=1}^{n} x_{k,l,t}^s = x_{k,t}^d;$$

(e) (Final Goods Market)

$$Y_t^s = \int_{\mathcal{P}_t} c_{1,P_t} dP_t + \int_{\mathcal{P}_{t-1}} c_{2,P_{t-1}} dP_{t-1} + \sum_k \sum_l \int_{\mathcal{P}_{k,l,t}} S_{P_t} dP_t + \sum_k \sum_l \int_{\mathcal{P}_{k,l,t}} \zeta_{P_t} dP_t.$$

Before determining the equilibrium prices and allocations, some preliminary considerations are helpful. In the first period, the capital and information stocks of all firms are equal and all members of the young generation are symmetric with respect to human capital before they are hired. Therefore, all intermediate goods markets must show a symmetric equilibrium in the first period. As in all m markets there is the same number n of firms, all research groups will be of the same size and all intermediate firms make the same profit. According to assumption 10.1, the young generation of the next period takes over the firm-specific capital of the old generation and decides on 'net-saving'[7] and knowledge codification. As the members of the young generation are all symmetric with respect to wage income and preferences and each firm is owned by an identical share of the population, it is common knowledge that a symmetric equilibrium will again result in period $t+1$. Consequently, the problems of the individuals are identical and it is convenient to work with a representative household. Note that in the symmetric equilibrium no agent has an incentive to deviate, that is, invest his physical capital in another firm because the return on capital is the same in all intermediate firms. Further, a single individual, possessing measure zero, cannot influence the return on capital. This does not mean that there are no other equilibria, but rather that the symmetric equilibrium exists as long as the individuals cannot effectively coordinate their capital savings decision. In the following, we will only focus on this symmetric equilibrium.

Consider the intermediate goods markets. In a sector k, each firm's reaction function is given by equation (10.3). Due to the symmetry of the firms with respect to physical capital, the resulting equilibrium amount of researchers hired by the firms must also be symmetric. This results in identical supplies of intermediate goods $x_{k,l,t}, \forall(k,l)$. The equilibrium price for the intermediate good x_k can then be written as

$$p_{x_k,t} = \alpha(nx_{k,l,t})^{\alpha-1}L_{A,t}^{1-\alpha}.$$

Turning to the labor market, the equilibrium requires that $w_{A,t} = w_{R,t} = w_t$:

$$(1-\alpha)L_{A,t}^{1-\alpha}\sum_{k=1}^{m} x_{k,t}^{\alpha} = \alpha x_{k,t}^{\alpha-1}L_{A,t}^{1-\alpha}\frac{\partial x_{k,l,t}}{\partial L_{\mathcal{G}_{k,l,t}}} + \alpha(\alpha-1)x_{k,t}^{\alpha-2}L_{A,t}^{1-\alpha}\frac{\partial x_{k,l,t}}{\partial L_{\mathcal{G}_{k,l,t}}}x_{k,l,t}.$$

Due to the above symmetry arguments, it can be transformed to

$$(1-\alpha)L_{A,t}^{-\alpha}m(nx_{k,l,t})^{\alpha} = \alpha(nx_{k,l,t})^{\alpha-1}L_{A,t}^{1-\alpha}\frac{\partial x_{k,l,t}}{\partial L_{\mathcal{G}_{k,l,t}}} + \alpha(\alpha-1)(nx_{k,l,t})^{\alpha-2}L_{A,t}^{1-\alpha}\frac{\partial x_{k,l,t}}{\partial L_{\mathcal{G}_{k,l,t}}}x_{k,l,t}$$

and further to

$$L_{A,t} = \frac{(1-\alpha)mn}{\psi(1-\frac{1-\alpha}{n})}\frac{x_{k,l,t}}{\frac{\partial x_{k,l,t}}{\partial L_{\mathcal{G}_{k,l,t}}}}.$$

[7] Note that the assumption of capital to be eatable would also allow for negative net-saving without problems between generations.

Cancelling $K_{k,l,t}$ yields

$$L_{A,t} = \frac{(1-\alpha)mn}{\psi(1-\frac{1-\alpha}{n})} \frac{\tau_{k,l,t}}{\frac{\partial \tau_{k,l,t}}{\partial L_{\mathcal{G}_{k,l,t}}}}. \tag{10.4}$$

Given the firm's knowledge stock in field k by equation (10.2), the equilibrium amount of labor for the two occupations in the first period writes

$$\begin{aligned} L_{A,t} &= \min\left\{1, \frac{(1-\alpha)}{\psi(1-\frac{1-\alpha}{n})+1-\alpha}\left(\frac{n[q\tau_{k,t-1}+\beta\max\{0, C_{k,l,t}-q\tau_{k,t-1}]}{\varepsilon[q\tau_{k,t-1}+\beta\max\{0, C_{k,l,t}-q\tau_{k,t-1}]^{\Phi}}+1\right)\right\}, \\ L_{\mathcal{G}_{k,l,t}} &= \max\left\{0, \frac{\psi((1-\alpha)^{-1}-n^{-1}) - \frac{n[q\tau_{k,t-1}+\beta\max\{0,C_{k,l,t}-q\tau_{k,t-1}]}{\varepsilon[q\tau_{k,t-1}+\beta\max\{0,C_{k,l,t}-q\tau_{k,t-1}]^{\Phi}}}{\psi((1-\alpha)^{-1}-n^{-1})+1)nm}\right\}. \end{aligned}$$

Using the same line of argument as in appendix 7.2, it is easily verified that the share of researchers will always be positive for $\Phi = 0$ and in the case $\Phi = 1$, positive research in equilibrium requires that the following inequality holds:

$$\psi((1-\alpha)^{-1}-n^{-1}) \geq \frac{n}{\varepsilon}. \tag{10.5}$$

Otherwise, the labor market equilibrium will be characterized by the corner solution $L_{A,t} = 1$ and $L_{\mathcal{G}_{k,l,t}} = 0$, $\forall(l,k)$. Again we assume that condition (10.5) holds in the following.

Using the homogenous consumption good as the numéraire, the equilibrium prices in each period are written as

$$\begin{aligned} w_t &= (1-\alpha)(\psi((1-\alpha)^{-1}-n^{-1})+1)^{\alpha}\left(\frac{n[q\tau_{k,t-1}+\beta\max\{0,C_{k,l,t}-q\tau_{k,t-1}]}{\varepsilon[q\tau_{k,t-1}+\beta\max\{0,C_{k,l,t}-q\tau_{k,t-1}]^{\Phi}}+1\right)^{-\alpha} \\ &\quad mn^{\alpha}K_{k,l,t}^{\alpha}\Bigg(q\tau_{k,t-1}+\beta\max\{0, C_{k,l,t}-q\tau_{k,t-1}\} \\ &\quad +\varepsilon\frac{\psi((1-\alpha)^{-1}-n^{-1})-\frac{n[q\tau_{k,t-1}+\beta\max\{0,C_{k,l,t}-q\tau_{k,t-1}]}{\varepsilon[q\tau_{k,t-1}+\beta\max\{0,C_{k,l,t1}-q\tau_{k,t-1}]^{\Phi}}}{\psi((1-\alpha)^{-1}-n^{-1})+1)n}\Bigg)^{\psi}, \\ p_{x_k,t} &= \alpha n^{\alpha-1}K_{k,l,t}^{\alpha-1}\Bigg(q\tau_{k,t-1}+\beta\max\{0, C_{k,l,t}-q\tau_{k,t-1}\} \\ &\quad +\varepsilon\frac{\psi((1-\alpha)^{-1}-n^{-1})-\frac{n[q\tau_{k,t-1}+\beta\max\{0,C_{k,l,t}-q\tau_{k,t-1}]}{\varepsilon[q\tau_{k,t-1}+\beta\max\{0,C_{k,l,t}-q\tau_{k,t-1}]^{\Phi}}}{\psi((1-\alpha)^{-1}-n^{-1})+1)n}\Bigg)^{(1-\alpha)(1-\frac{\psi}{\alpha})} \\ &\quad (\varepsilon[q\tau_{k,t-1}+\beta\max\{0, C_{k,l,t}-q\tau_{k,t-1}\}]^{\Phi})^{\alpha-1}\left(\frac{(1-\alpha)mn}{\alpha(1-\frac{1-\alpha}{n})}\right)^{1-\alpha}, \quad \forall k. \end{aligned}$$

10.2 Dynamics and Knowledge Codification

The resulting dynamics of the economic system are given by $\tau_{t+1} = (\tau_{k,t+1})_{k\in\{1,\dots,m\}}$, $K_{t+1} = (K_{k,t+1})_{k\in\{1,\dots,m\}}$ and $C_{t+1} = (C_{k,t+1})_{k\in\{1,\dots,m\}}$. Due to the symmetry, the economy's dynamics are completely described by the k-th projection. Hence, the system can be reduced to

$$\begin{aligned} K_{k,l,t+1} &= s_t w_t (nm)^{-1} = s_t(1-\alpha)n^{\alpha-1}L_{A,t}^{-\alpha}K_{k,l,t}^{\alpha}\tau_{k,t}^{\psi}, \\ \tau_{k,t+1} &= q\tau_{k,t}+\beta\max\{0,C_{k,l,t+1}-q\tau_{k,t}\}+\varepsilon m[q\tau_{k,t}+\beta\max\{0,C_{k,l,t+1}-q\tau_{k,t}]^{\Phi}L_{\mathcal{G}_{k,l,t}}, \\ C_{k,l,t+1} &= C_{k,l,t}+\triangle C_{k,l,t}, \end{aligned}$$

which is structurally equivalent to that of the previous chapter. The 'standing on the shoulders of giants' specification would resemble the $(\Phi,\tilde{\Phi})=(1,1)$-case. Furthermore, using the equilibrium values, the return on capital calculates to

$$r_{k,l,t} = K_{k,l,t}^{\alpha-1}\tau_{k,t}^{\psi}\underbrace{n^{\alpha-1}L_{A,t}^{-\alpha}(L_{A,t}-1+\alpha)}_{f(L_{A,t})}.$$

This shows the form required by (P3). The wages can be written as

$$\begin{aligned} w_{R,t} &= \overbrace{\alpha\left(1-\frac{1-\alpha}{n}\right)n^{\alpha-1}\frac{\psi}{\alpha}}^{\lambda} L_{A,t}^{1-\alpha}K_{k,l,t}^{\alpha}\tau_{k,t}^{\psi-1}\frac{\partial\tau_{k,t}}{\partial L_{\mathcal{G}_{k,l,t}}}, \\ w_{A,t} &= \underbrace{(1-\alpha)n^{\alpha}m}_{\kappa} L_{A,t}^{-\alpha}K_{k,l,t}^{\alpha}\tau_{k,t}^{\psi}, \end{aligned}$$

and consequently exhibit the form given in (P2). The production function of final goods simplifies to

$$Y_t = \underbrace{mn^{\alpha}}_{\xi} L_{A,t}^{1-\alpha}K_{k,l,t}^{\alpha}\tau_{k,t}^{\psi},$$

which satisfies (P1). Hence, we can formulate:

Proposition 10.1 *The depicted oligopolistic structure satisfies properties (U), (K1), (A2'), (P1)-(P3). (M1) is satisfied for the case with* $\Phi = 1$.

Proof. (P1)-(P3) is shown above, requirement (K1) follows from the respective equation of the dynamical system as given above and (A2') holds by assumption. (U) results from the fact that all utility functions satisfying (c3) must also satisfy the weaker requirement (c3'). As the specification of ideas per researcher represents the case with $\tilde{\Phi}=1$, lemma 9.1 implies that if additionally $\Phi = 1$, $\frac{\partial^2 r_{t+1}}{\partial\triangle C_{e,t}^2} < 0$, $\forall\psi\in(0,1]$ and hence, $\frac{\partial M}{\partial\triangle C_{e,t}} < 0$. Further, $\frac{\partial^2 r_{t+1}}{\partial\triangle C_{e,t}^2} < 0$ implies that the utility function, as it additionally satisfies (c4), suffices for the second order conditions of the household's maximization problem to hold. Hence, (M1) is given with $\Phi = 1$. In the case with $\Phi = 0$, an additional assumption, for example $\psi \leq \alpha$, would be necessary for (M1). □

Assuming (10.5) to hold precludes atomistic competition and, thus, the oligopolistic firms possess positive measure. In this way, proposition 10.1 implies that the results of the basic model also apply to a setting with more than one knowledge stock and oligopolistic intermediate firms.

The question arises what can be said about the effects of market structure on the allocation of labor in equilibrium and the amount of knowledge codification. Answering these questions requires a thorough analysis which is beyond the scope of this thesis. Hence, we will only give a short sketch of some expected effects assuming $\psi = \alpha$ and $\Phi = 1$.

With regard to the allocation of labor, the intuition would be that the equilibrium share of workers is increasing in the number of firms in each sector. The argument is that with increasing competition, the equilibrium prices of the intermediate goods decrease, respectively, the aggregate supply of the intermediate product in one sector increases, which positively affects the marginal productivity of labor in final-goods production. In contrast to this, research productivity does not increase in the number of firms in a sector. Another reason is that due to a decreasing amount of capital of the single intermediate firms in n in the symmetric equilibrium, the return on an additional idea invented decreases as well. Hence the shift of labor from research to final goods production in n.

The following line of argument shows that the above intuition is correct. The derivative of $L_{A,t}$ with respect to n writes

$$\frac{\partial L_{A,t}}{\partial n} = \frac{(1-\alpha)(2n\alpha(\alpha-1)+n^2+\varepsilon\alpha(1-\alpha))}{\varepsilon(\alpha(1-\alpha)-n)^2}.$$

It will be positive, whenever

$$-\varepsilon\alpha(1-\alpha) < 2n\alpha(\alpha-1)+n^2.$$

Rewriting the right hand side as $n[(1-\alpha)^2+\alpha^2+n-1]$ reveals that it must be positive for $n \geq 1$. As the left hand side is negative for $n \geq 1$, the condition is satisfied. □

Note that such an effect is not present with regard to the number of the economy's sectors m. The reason is that the variety of knowledge is assumed to positively affect research success. Due to the above specification both, the productivity of workers in final-goods production and the research productivity are increasing at the same rate in m. However, the share of researchers per firm is decreasing in a symmetric equilibrium, as the total share is distributed over a higher number of intermediate firms if m is increasing.

With respect to the effect of the market structure on knowledge codification, matters become much more complicated. According to the representative household's first order conditions, the optimal choice of knowledge codification, given a certain amount of physical capital saved, must satisfy

$$\frac{\partial r_{k,l,t+1}}{\partial \triangle C_{k,l,t}} \frac{\partial \triangle C_{k,l,t}}{\partial \zeta_{P_t}} S_{P_t} = 1 + r_{k,l,t+1},$$

where $\gamma_t \triangle C_{k,l,t} = \int_{\mathcal{P}_{k,l,t}} \zeta_{P_t} dP_t = \frac{1}{nm}\zeta_{P_t}$ and $K_{k,l,t} = \int_{\mathcal{P}_{k,l,t}} S_{P_t} dP_t = \frac{1}{nm} S_{P_t}$.
With $\psi = \alpha$, the above condition can be written as

$$K^{\alpha}_{k,l,t} n^{\alpha-1} \bar{\nu}^{1-\alpha} \left(\frac{\partial \tau_{k,t+1}}{\partial L_{\mathcal{G}_{k,l,t}}}\right)^{\alpha-1} \mu\left(1+\frac{\varepsilon}{n}\right) = \gamma_t + \gamma_t K^{\alpha-1}_{k,l,t} n^{\alpha-1} \bar{\nu}^{-\alpha} \left(\frac{\partial \tau_{k,t+1}}{\partial L_{\mathcal{G}_{l_k,t}}}\right)^{\alpha} \left(\bar{\nu}\tau_{k,t+1}\left(\frac{\partial \tau_{k,t+1}}{\partial L_{\mathcal{G}_{k,l,t}}}\right)^{-1} - 1 + \alpha\right), \quad (10.6)$$

where $\bar{\nu} = \frac{(1-\alpha)nm}{\alpha(1-\frac{1-\alpha}{n})}$ and $\mu = (1 + \bar{\nu}\frac{1}{nm})^{-1}$. Again, the left hand side of the above equation represents the increase in consumption when old by an additional unit of the homogenous good spent on knowledge codification, whereas the right hand side depicts the opportunity costs of not having spent it in physical capital. Note that both the return on capital (as the profits are declining) and the codification benefit are declining in n. To see the latter, consider the four terms on the left hand side that depend on n,($n^{\alpha-1}$, $\bar{\nu}^{1-\alpha}$,μ, $\left(1+\frac{\varepsilon}{n}\right)$) and combine $n^{\alpha-1}$ and $\bar{\nu}^{1-\alpha}$ to

$$\left(\frac{(1-\alpha)m}{\alpha(1-\frac{1-\alpha}{n})}\right)^{1-\alpha}.$$

This term is decreasing in n. So is $\mu\left(1+\frac{\varepsilon}{n}\right)$, as we know from the considerations on the equilibrium shares of labor, that the share of researchers $L_{\mathcal{G}_{k,l,t}}$ must be declining in n. This implies that $\tau_{k,t+1}$ is decreasing in n. For $\Phi = 1$ and using (10.4), $\tau_{k,t+1}$ can be rewritten as

$$\tau_{k,t+1} = (q\tau_{k,t} + \beta \max\{0, C_{k,l,t} - q\tau_{k,t}\})\mu\left(1+\frac{\varepsilon}{n}\right),$$

and, hence, $\tau_{k,t+1}$ can only decline in n if $\mu\left(1+\frac{\varepsilon}{n}\right)$ does. With both the return on physical capital and the marginal benefit of knowledge codification declining with lower market concentration, the amount of the homogenous good transferred to the next period will be lower. It may then be expected that both knowledge codification and physical capital investments decline in n, which negatively affects long run economic growth. This indicates that there may also exist a tradeoff between static and dynamic efficiency with respect to the creation of endogenous spillovers by knowledge codification. At this point, a further elaboration must be left for future research.

10.3 Summary

This chapter has outlined a model structure with more than one stock of knowledge, each constituting an intermediate sector with oligopolistic competition. Knowledge of a person has been defined as a vector in a multidimensional knowledge space. Again, it originates from an exogenously transferred share of the economy's knowledge stock of the previous period in every field. When being hired as a researcher by an intermediate firm in a specific sector, this person additionally develops expertise in the respective field of knowledge by attending to the intermediate firm's knowledge database and own research within the firm's research group. It has been shown that under certain symmetry assumptions such a structure exhibits the properties sufficient for the main results as introduced in the previous chapter.

This chapter ended with a brief discussion of effects of market structure in the intermediate sectors on knowledge codification and economic growth, suggesting that there may also exist a tradeoff between static and dynamic efficiency with respect to the creation of endogenous spillovers by knowledge codification. Such an effect, however, needs to be verified by a thorough analysis, which is beyond the scope of this dissertation.

Part III

Relation to Standard Endogenous Growth Theory and Social Optimality

Chapter 11

Relation to Romer's Model of Endogenous Technological Change

This chapter reflects on the relation of the model introduced in the first two parts to standard endogenous growth theory at the example of Romer's model of endogenous technological change (Romer, 1990). This model has been chosen for two reasons. Firstly, it is a seminal contribution to the theory of endogenous growth. In particular many idea- or R&D-driven endogenous growth models, resemble his model structure. Secondly, the engine of growth in Romer (1990) is a growing variety of products, that is, horizontal innovation. This challenges the interpretation of the knowledge dynamics as given in the first part.

The first section of this chapter outlines the main differences between the models introduced in the earlier chapters and the Romer model. Thereafter the question is raised whether Romer's microeconomic structure could be translated into an overlapping generations framework with imperfect knowledge spillovers between generations. A corresponding interpretation of the Romer model is given and problems as well as further necessary assumptions are discussed. The result is that in such a framework, private knowledge codification within intermediate firms could not be explained but only knowledge codification as the outcome of a collective decision, for example, within a state. Allowing for such "social" knowledge codification,[1] the modified Romer structure would satisfy the general properties given in section 9.

11.1 What are the Differences to the Romer Model?

The following points can be identified.

- ▷ Distinction between Knowledge and Human Capital

  In Romer (1990), knowledge is seen as a set of ideas or designs that are idealized as disembodied and non-rival. Once a design has been invented, it can be used for all times. The underlying argument is that by patenting the intermediate good built from the respective design, the idea is fully codified and, hence, can be drawn upon whenever necessary. In contrast, human capital is rivalrous and defined as

[1] The outcome is not necessarily identical to the one chosen by a social planner maximizing welfare of all generations.

years of education or training.[2] Human capital is exogenous in Romer's model and stays at a fixed level.[3]
The model proposed in the present thesis differs in that human capital complements idea-driven growth. More precisely, the researchers need the respective human capital in order to conceive ideas. In this way, the stock of human capital determines which ideas can be used productively. It is argued that ideas embodied in patents or intermediate products are in general not sufficient for the transfer of knowledge between generations if the young generation lacks the requisite human capital. In this way, knowledge may be lost in the transition between generations. It is purposeful and costly knowledge codification that establishes the public good characteristics that Romer attaches to his notion of knowledge.

▷ Horizontal Innovation

In the Romer model growth is driven by innovation that enhances the variety of intermediate goods.
The knowledge concept as introduced in section 2.2, shows a hierarchical structure in that ideas are ordered according to their difficulty. This intuitively suggests an interpretation as vertical innovation, that is, quality improving innovation.

▷ Household is Ramsey Consumer

$$\max_c \int_{t=0}^{\infty} U(c)e^{-\rho t}dt\,, \quad U(c) = \frac{c^{1-\theta}-1}{1-\theta}.$$

This means that the households in the Romer model maximize utility which is the discounted sum of utility over an infinite time horizon instead of only over a lifetime of two periods as in the overlapping generations model of the present dissertation.

▷ Three Production Sectors

Instead of only two production sectors where research is conducted within an intermediate firm, the Romer specification shows three sectors.[4] In principle, an extra research sector in which individual researchers are inventing designs is added. They sell each design to a firm in the intermediate sector, which together with physical capital produce the respective intermediate good. Further, an intermediate firm receives a permanent monopoly on the intermediate good originating from the respective design. By using the intermediate goods and labor, the final goods firms

[2] This is different from the notion of human capital used in models of growth based on unlimited human capital accumulation such as Lucas (1988).

[3] Human capital can be seen as a scaling factor of the researchers' productivity. See also the discussions on the scale effect in growth models.

[4] However, Romer modelled three sectors mainly for analytical convenience.

produce the homogenous consumption good.

▹ No decision maker possesses positive measure

Although the intermediate sector is characterized by monopolistic firms that possesses market power in the market for intermediate goods, a single firm cannot influence aggregate values as it is a point on a continuum of intermediate firms. The latter also holds true with respect to the single final goods firms and the households.

11.2 An Interpretation of the Romer Model

This section raises the following question: Is it possible within the Romer structure to deviate from the assumption of perfect intergenerational knowledge spillovers in the way proposed in the previous parts of the dissertation, and if so, what can be said about the endogenous creation of spillovers by knowledge codification?

The following discussion of this question, considers a version of the Romer model that is characterized by three modifications:

(1) Individuals live for two periods and maximize

$$U_t(c_{1,t}, c_{2,t+1}) = u(c_{1,t}) + \delta u(c_{2,t+1}), \quad \text{where } u(c) = \frac{c^{1-\theta} - 1}{1-\theta};$$

(2) Imperfect knowledge transfer between generations;

(3) Monopoly rights for intermediate goods only for one period.

In principle, the modifications one and three just translate the continuous time structure of the Romer model into an overlapping generations framework where the households do not have altruistic preferences for their successors and the researchers can monopolize their ideas only during their professional life. She neither receives monopoly rents when old nor bequeaths them to their descendants.[5]
The following sections will discuss in more detail how item 2 can be incorporated into the model and which assumptions on the knowledge dynamics would be necessary to show the form of property (K1) with horizontal innovation.[6] It is further elaborated on whether the microeconomic structure of the modified Romer model would satisfy the general properties of the models introduced in this thesis as given in chapter 9? We could ask: Would short lived economic agents codify their knowledge in an overlapping

[5] This corresponds to the model in Judd (1985).

[6] Property (K1) requires that the dynamics of the knowledge stock can be described by $\tau_{t+1} = q\tau_t + \beta \max\{0, C_{t+1} - q\tau_t\} + \varepsilon \left[q\tau_t + \tilde{\Phi}(\beta \max\{0, C_{t+1} - q\tau_t\})\right]^{\Phi} L_{R,t+1}$.

generations economy with the microeconomic structure of the Romer model and imperfect knowledge transfer between generations?

The answer must be "No", since in the structure of the Romer model no decision maker possesses positive measure. Hence, the necessary condition (N) for knowledge codification is violated. No decision maker would invest in knowledge codification as it cannot influence the knowledge stock of the next period. Thus, within the Romer structure there cannot be private knowledge codification within firms as in the model discussed in the previous parts.

In the following model structure, we will allow for social knowledge codification, being the result of a collective decision of the households within a state. This could be interpreted as the state providing public libraries and creating publicly accessible knowledge databases. The corresponding decision process could be modelled in the same way as that within the intermediate firm as described in the specification of the basic model settings in the first part of the dissertation.

11.3 The Modified Romer Model

11.3.1 The Production Sector

Final goods production is characterized by a continuum (on [0,1]) of identical firms producing the homogenous good by using labor $L_{A,t}$ and the intermediate good $x_{i,t}$ as inputs. The firms maximize profits and act competitively in the product and factor markets. Again final goods production is modelled by means of a representative firm whose production and factor demands represent aggregate values. The aggregate production function of the Romer model is given by the following extension of the Cobb-Douglas production function:

$$F((x_{i,t})_{i\leq\tau_t}, L_{A,t}) = L_{A,t}^{1-\alpha} \int_{i=0}^{\tau_t} x_t(i)^\alpha di.$$

The intermediate goods can be distinguished by whether they are competitive or monopolized. As a consequence, the different intermediate goods come at different prices and it will be convenient to write the aggregate production function as

$$F((x_{i,t})_{i\leq\tau_t}, L_{A,t}) = L_{A,t}^{1-\alpha} \left[\int_{i=0}^{\tau_{f,t}} x_{i,t}^\alpha di + \int_{i=\tau_{f,t}}^{\tau_t} x_{i,t}^\alpha di \right],$$

where $[0, \tau_{f,t}]$ is the set of freely available ideas and $(\tau_{f,t}, \tau_t]$ is the set of ideas that are monopolized. The representative firm solves the following maximization problem:

$$\max_{x_{i,t}, L_{A,t}} \pi_t^{fp} = F((x_{i,t})_{i\leq\tau_t}, L_{A,t}) - \int_{i=0}^{\tau_{f,t}} p_{x_i,t} x_{i,t} di - \int_{i=\tau_{f,t}}^{\tau_t} p_{x_i,t} x_{i,t} di - w_{A,t} L_{A,t}.$$

As it is a constant-returns-to-scale firm, its factor demands are defined only after the scale of operation is pinned down. However, the demand for labor and intermediate goods is characterized by the first order conditions. Labor and the intermediate goods are being compensated by their marginal product:

$$
\begin{aligned}
x^d_{i,t} &= \left(\frac{p_{x_i,t}}{\alpha}\right)^{\frac{1}{\alpha-1}} L_{A,t}, \quad i \leq \tau_t, \\
L^d_{A,t} &= \left(\frac{1-\alpha}{w_{A,t}}\right)^{\frac{1}{\alpha}} [\int_{i=0}^{\tau_{f,t}} x^\alpha_{i,t} di + \int_{i=\tau_{f,t}}^{\tau_t} x^\alpha_{i,t} di]^{\frac{1}{\alpha}}.
\end{aligned}
$$

11.3.2 Knowledge Dynamics and the Research Sector

Let the set-theoretic representation of knowledge and information be the same as introduced in the first part of the thesis. The following operationalizes it in a way to explicitly account for horizontal innovations. The knowledge stock is again represented as an interval in $\mathbb{R}$, that is, $\mathcal{I} = \mathbb{R}_+$. This time, however, the ordering concept is characterized by

Assumption (A1') *If i is embodied in Person P_t, then there exists a Person $P'_t \in \mathcal{P}_t$ who knows j, where $j < i$, $\forall i, j \in \mathcal{I}$*

This can be interpreted as chronological order when assuming that within a period, the researchers are discovering the different ideas slightly one after the other. Or if different ideas are invented exactly at the same time, the respective indices are assigned randomly among them. This means that if $i < j$ then idea i has been discovered earlier than idea j. Recalling the bookshelf example, this would imply that the books are ordered, e.g. from left to right, starting with the ones containing the ideas invented early in time. Again, they are indexed continuously beginning at 0.

Note that assumption (A1') does not imply that the Lebesgue measure of a person's human capital is equal to the highest index in the set of ideas, $\lambda(\mathcal{T}_{P_t}) \neq \sup \mathcal{T}_{P_t}$. Let $\tau_{P_t} = \lambda(\mathcal{T}_{P_t})$. In contrast to the hierarchical ordering concept,

$$\lambda : (\mathcal{I}, \mathcal{B}) \to \mathbb{R}_+, \, \tau_{P_t} = \lambda(\mathcal{T}_{P_t}),$$

where $\mathcal{B}$ denotes the Borel-σ-algebra, is no bijection. Let the knowledge stock of an economy in period t be defined as the union of all sets of human capital, $\mathcal{T}_t = \cup_{P_t \in \mathcal{P}_t} \mathcal{T}_{P_t}$. (A1') then implies a bijection

$$\lambda : (\mathcal{I}, \mathcal{B}) \to \mathbb{R}_+, \text{ with } \tau_t = \lambda(\mathcal{T}_t).$$

The unique inverse would be $\lambda^{-1}(\tau_t) = \mathcal{T}_t (= [0, \tau_t])$. In this way, the economy's knowledge stock can be represented in analogy to the first part of the dissertation by $\tau_t = \lambda(\mathcal{T}_t) = \sup \cup_{P_t \in \mathcal{P}_t} \mathcal{T}_{P_t}$.

Again, human capital of a person $P_t \in \mathcal{P}_t$ may originate from three sources and its measure is written as

$$\tau_{P_t} = q\tau_{t-1} + \beta \max\{0, C_t - q\tau_{t-1}\} + \varepsilon(q\tau_{t-1} + \beta \max\{0, C_t - q\tau_{t-1}\})^{\Phi}.$$

First, every individual is exogenously transferred a share q of the economy's knowledge stock of the previous period. This transfer is imperfect $q \in (0, 1)$ and characterized by the following assumption.

Assumption (KT) *In a period t, if an idea indexed by i has been transferred from the previous generation to the young generation of t, so has been idea j, $j < i$.*

In this way, the share q of the economy's knowledge stock that has been discovered earliest is transferred to the next generation in every period.

As a second source, the members of the new generation can enhance their human capital by utilizing codified ideas if previous generations codified their knowledge. With regard to knowledge codification, it is not necessary to modify (A2) or (A2'). As in large parts of the thesis, (A2') is assumed.[7]

Finally, if person P_t decides to do research, she is able to generate a measure $\varepsilon(q\tau_{t-1} + \beta \max\{0, C_t - q\tau_{t-1}\})^{\Phi}$ of new ideas. Denote $\tau_{f,t} := q\tau_{t-1} + \beta \max\{0, C_t - q\tau_{t-1}\}$. As in Romer (1990), we assume that an idea that a researcher discovers is not only new to herself but also to all other persons $P_t \in \mathcal{P}_t$ of the same generation.

Definition 11.1 *A researcher $P_{1,t}$ has invented an idea i that is* ***objectively new with respect to the generation born in period*** *t if at the very moment of its discovery, $i \notin \mathcal{T}_{P_t}, \forall P_t \in \mathcal{P}_t \setminus P_{1,t}$.*

Note, that this is different from the specification of the basic model, which only assumes subjective novelty to the inventor. With respect to the magnitude to which research productivity depends on the existing knowledge stock the Romer specification clearly represents the $\Phi = 1$ case. The assumption with respect to research is then:

Assumption (A3) *In a period t, every researcher invents a measure $\varepsilon\tau_{f,t}$ of objectively new ideas with respect to the generation born in period t.*

In the Romer model, each person decides as to whether she is going to work in final-goods production or to do research. Let $L_{R,t}$ denote the number of persons who decided to do research in period t and let them be arranged on the continuum from 0 to $L_{R,t}$. The knowledge level, that is, the set of ideas embodied in a single person depends on assumptions about spillovers within a generation.

[7](A2') required that in a period t and for all $i, j \geq q\tau_t$, if i is codified in period t, then $j < i$ is also codified in t.

Without spillovers, researchers would be heterogeneous with regard to human capital, as by assumption each researcher invents a measure $\varepsilon(q\tau_{t-1} + \beta \max\{0, C_t - q\tau_{t-1}\})$ of objectively new ideas with respect to her own generation. On the other hand, assuming perfect spillovers within a generation, the researchers (and possibly the workers) would be symmetric with respect to knowledge. The assumptions (KT), (A1'), (A2') and (A3) would allow for both, no spillovers or perfect spillovers within generations, and yield the dynamics of the economy's knowledge stock according to[8]

$$\tau_t = (q\tau_{t-1} + \beta \max\{0, C_t - q\tau_{t-1}\})(1 + \varepsilon L_{R,t}) = \tau_{f,t}(1 + \varepsilon L_{R,t}). \tag{11.1}$$

According to the Romer model, each researcher sells the designs she invented at the price p(i) to an intermediate firm, which then enjoys a monopoly on i. Let $\mathcal{T}_{P_t,inv}$ be the set of ideas discovered by researcher $P_t \in [0, L_{R,t}]$. According to (A3), $\lambda(\mathcal{T}_{P_t,inv}) = \varepsilon(q\tau_{t-1} + \beta \max\{0, C_t - q\tau_{t-1}\})$. Then a researcher's income accrues to

$$w_{R,t} = \int_{\mathcal{T}_{P_t,inv}} p(i)di. \tag{11.2}$$

In the Romer model it is assumed that the intermediate firm receives a patent for the intermediate good produced according to design i. Assuming imperfect knowledge transfer between generations implies that research duplication is possible. And in the present model, (A1') and (A3) imply that those ideas that have not been transferred are re-invented in the next period. Although research duplication is considered in growth models, it is mostly incorporated by a lower marginal productivity of labor engaged in research.[9] With the present specification, the question has to be answered as to whether ideas that are new to a generation but have been patented in a previous period can still yield monopoly rents. If the answer is yes, (11.2) reflects a researcher's income. The following lines of argument would support a positive answer.

- ▷ No spillovers within generations

  Assuming no spillovers between researchers of the same generation, would imply that the respective researcher is the only person whose human capital comprises the re-invented idea and due to the horizontal innovation assumption, every person is researching in different fields such that it is not possible to easily imitate the product once it is on the market. Hence, the re-invented idea earns monopoly profits.

- ▷ Allowing for spillovers within generations

  In the reasoning of the Romer model, the patent assures monopoly rents which otherwise would not accrue as after the intermediate product is launched, every other person could immediately imitate it due to perfect spillovers. This would

[8] Note that with the above assumptions, (KT), (A1'), (A2'), and (A3), the difference equation of the knowledge stock is well defined.

[9] See e.g. Jones (1995).

imply that the re-invented idea cannot earn a profit if it is not possible to receive patent protection again. Allowing for spillovers, the argument must then be that a modified patent application representing the same idea would lead to patent protection. In principle, this would correspond to the reasoning behind "equivalent innovations" as mentioned in section 10.1.1.

On the other hand, it would seem reasonable to assume that the re-invented ideas do not earn profits. For the researchers to still be homogenous with respect to income, it would be necessary that the ratio of research duplication to ideas earning monopoly profits is the same for all researchers. In this case, however, equation 11.2 must be modified to reflect the researchers income. Preliminary calculations show that this may thoroughly complicate the model. Hence, for reasons of simplicity and staying as close to the Romer model as possible with respect to the mathematical structure, we will choose the first line of argument, implying monopoly profits for all ideas that are objectively new with respect to the same generation. This first line of argument is also consistent with the knowledge concept as introduced in chapter 2.[10]

11.3.3 The Intermediate Goods Sector

The intermediate goods sector uses designs/ideas together with forgone output of the last period to produce durables for final goods production. Each intermediate goods firm produces one durable good. As there are two different kinds of ideas in each period, there are two kinds of intermediate goods firms. On the one hand, there are the ideas that have been transferred from the previous period, $t-1$. Those are part of every person's knowledge in period t and, hence, are fully competitive (as their patent protection has expired). On the other hand, the ideas invented by the researchers are new with respect to the generation in t and earn monopoly rents.

(1) The competitive intermediate goods firms produce with a freely available design $i \in [0, \tau_{f,t}]$. They rent capital at the capital market and produce the intermediate good from an idea known to anybody. Hence, they act fully competitive. It is assumed that the production of a unit of the intermediate good takes η units of capital. The profit maximization problem of the competitive intermediate entrepreneur takes the following form:

$$\max_{x_{i,t}} \pi_t^{int} = p_{x_i,t} x_{i,t} - r_t \eta x_{i,t}.$$

The first order condition yields

$$p_{x_i,t} = r_t \eta.$$

[10]Note that the second argument for an idea to still yield monopoly profits using the equivalent innovation principle could also apply to the case without spillovers.

With the inverse demand function of the final goods firms, the competitive entrepreneur's supply of intermediate goods can be written as

$$x_{i,t}^s = \left(\frac{\eta r_t}{\alpha}\right)^{\frac{1}{\alpha-1}} L_{A,t},$$

and consequently the demand for physical capital is

$$K_{i,t}^d = \eta^{\frac{\alpha}{\alpha-1}} \left(\frac{r_t}{\alpha}\right)^{\frac{1}{\alpha-1}} L_{A,t}.$$

(2) The monopolistic intermediate goods firms purchase an idea $i \in (\tau_{f,t}, \tau_t]$ from a researcher in the research sector and rent physical capital to produce the respective intermediate good. On the supply side the intermediate entrepreneur acts as a monopolist. Hence, the optimization problem writes

$$\max_{x_{i,t}} \pi_t^{int} = p_{x_i,t}(x_{i,t})x_{i,t} - r_t \eta x_{i,t} - p(i).$$

Given p(i), this leads to the first order condition

$$p_{x_i,t} + \frac{dp_{x_i,t}}{dx_{i,t}} x_{i,t} = r_t \eta,$$

and consequently to

$$\begin{aligned} x_{i,t}^s &= \left(\frac{\eta r_t}{\alpha^2}\right)^{\frac{1}{\alpha-1}} L_{A,t}, \\ K_{i,t}^d &= \eta^{\frac{\alpha}{\alpha-1}} \left(\frac{r_t}{\alpha^2}\right)^{\frac{1}{\alpha-1}} L_{A,t}. \end{aligned}$$

11.3.4 The Problem of the Household

Although the Romer specification of the household is close to the assumptions of the basic model, there are two main differences. First, the Romer model uses a CIES-utility function with only a non-negativity constraint on the coefficient of constant intertemporal elasticity of substitution, $1/\theta$, whereas the structure of the model presented in the present dissertation needs some further assumptions. Second, the household participates in a collective decision process with respect to knowledge codification.

The particular specification is as follows: There is a continuum of two-period-lived individuals P_t on $[0,1] =: \mathcal{P}_t$ in each generation. There is no population growth and the size of each generation is normalized to 1. The individuals inelastically supply one unit of labor when young and may choose as to whether they want to work in final goods production or in research and development. The budget constraint when young is given by the wage w_t that can be split into consumption today $c_{1,t}$, saving $w_t s_t$ in physical capital and investment in knowledge codification $w_t\varsigma_t$. The savings plus the real return on capital r_{t+1} equal consumption when old $c_{2,t+1}$. With respect to knowledge codification it is assumed that the household's take part in a collective decision within a state. Suppose they have the possibility to decide on a codification tax. This can be conducted by a similar decision process as that within the intermediate firm of the basic model. More precisely:

- ▷ Any individual of the young generation in a period t can propose a tax rate ς_t on labor income in order to finance knowledge codification.
- ▷ Each member of the young generation votes for or against the proposal and the tax will be introduced if it is approved according to the unanimity rule.
- ▷ If the proposal has been accepted, the tax is collected and researchers are paid to codify their ideas.

Assuming for simplicity that this process does not incur transaction costs, it is not necessary to explicitly model a government sector. In this way, each young individual makes a private decision over her physical capital savings and takes part in a collective decision process concerning knowledge codification. A young individual in period t maximizes utility by solving a two-stage optimization problem. At the first stage, she decides whether she would like to make a proposal on the codification tax rate by solving

$$\max_{s_t,\varsigma_t} U_t(c_{1,t}, c_{2,t+1}) = u(c_{1,t}) + \delta u(c_{2,t+1})$$

subject to

$$\begin{aligned} c_{1,t} &= w_t(1 - s_t - \varsigma_t), \\ c_{2,t+1} &= (1 + r_{t+1})s_t w_t. \end{aligned}$$

The household acts competitively with regard to capital investment, taking r_{t+1} as given. However, investing in codification increases r_{t+1}. The agent chooses the pair (s_t, ς_t) that solves the following necessary conditions for a maximum, given by

$$\begin{aligned} s_t &= \frac{1 + r_{t+1}}{\frac{\partial r_{t+1}}{\partial \varsigma_t}}, \\ \varsigma_t &= 1 - \frac{1 + r_{t+1}}{\frac{\partial r_{t+1}}{\partial \varsigma_t}} \left(\delta^{-\frac{1}{\theta}}(1 + r_{t+1})^{\frac{\theta-1}{\theta}} + 1\right). \end{aligned}$$

The agent will make a suggestion if her optimal level $\varsigma_{t,opt}$ is greater zero. She will accept any suggestions $\varsigma_{t,s} < \varsigma_{t,opt}$. Given a tax rate $\varsigma_{t,s}$ has been approved, the agent will then at the second stage maximize utility with respect to capital savings S_t, given $\varsigma_{t,s}$.

Since in the Romer structure the individuals are flexible to choose their profession, the wages of the different occupations must be equal in equilibrium. As a consequence, each household faces an identical optimization problem, such that every person makes the same proposal on the tax rate which, of course, will be accepted. Again, it is thus possible to work with a representative household as being the young generation characterized by its date of birth t. Each generation faces the first-stage optimization problem and its solution is interpreted as aggregate savings S_t and investment in knowledge codification $\zeta_t \geq 0$. As in the basic model, the CIES-utility function exhibits the properties (c1) -(c3) and with $\frac{1}{2}\frac{1-s_t-\varsigma_t}{1-\varsigma_t} < \theta \leq 1$, $\forall t$, it also satisfies (c4).

11.3.5 The Costs of Knowledge Codification

Let the costs of knowledge codification again be characterized by the same cost function as in the basic model,

$$\Gamma_t(\triangle C_t) = \gamma_t \triangle C_t,$$

and possibly fixed entry costs that accrue once in the first period of codification. Note that in the modified Romer structure every researcher invents objectively new ideas. If there are no spillovers, the researchers may have bargaining power with respect to knowledge codification. However, the same line of argument as in section 3.3 applies. That is, the demanding party, in this case the state or government, offers the researcher a forcing contract that contains a compensation for knowledge codification slightly above the marginal costs. Again, the demanding party knows that the researcher will be better off by accepting the contract and, hence, has no incentive to revise the conditions, even if the researcher would neglect the contract in an early stage of a game with finitely many stages. If there were knowledge spillovers within periods, the researchers would codify at the marginal costs according to the Bertrand-reasoning as also put forth in the basic model.

11.3.6 Sequence of Events

The time-line of the model's events in a period t are:

(1) At the beginning of period t, the members of the new generation are exogenously transferred a share q of the knowledge stock of the economy of the previous period: $\mathcal{T}_{P_t} = \{i | i \leq q\tau_{t-1}\}$, $P_t \in \mathcal{P}_t$.

(2) If the researchers of the previous periods codified their ideas, the young generation of t would additionally build up human capital $\{i | q\tau_{t-1} < i \leq \tau_{f,t}\}$.

(3) Each researcher discovers a measure $\varepsilon(q\tau_{t-1} + \beta \max\{0, C_t - q\tau_{t-1}\})$ of new ideas.

(4) The competitive intermediate firms rent capital from the capital market at price r_t and produce intermediate good $x_{i,t}$, $i \leq \tau_{f,t}$. The monopolized intermediate firms additionally purchase a design from a researcher at price $p(i)$ to produce intermediate good $x_{i,t}$, $\tau_{f,t} < i \leq \tau_t$.

(5) The final-product firm rents the intermediate products at prices $p_{x_i,t}$ and hires a number $L_{A,t}$ of workers at a wage $w_{A,t}$ in order to produce the final good Y_t.

(6) At the end of period t, the members of the young generation decide how much of the wage income to consume $c_{1,t}$ and how much to transfer to the next period $t+1$. On the one hand, the young generation may save in physical capital involving a return r_{t+1}. On the other hand, it may invest in knowledge codification, which increases the knowledge stock of $t+1$ and consequently the rent r_{t+1}.

(7) Dependent on the compensation ζ_t, the researchers of period t codify an amount $\triangle C_t$ of their ideas.

(8) The old generation consumes its savings $S_{t-1} = K_t$ plus the return on capital $r_t K_t$ and then dies.

Note, that physical capital is assumed to be putty-putty in Romer (1990), that is, intermediate goods can be converted back into general capital. In this way, an interpretation as in section 3.5 could apply.

11.3.7 Sequential Markets Equilibrium

The economy comprises the following markets: The labor market, the capital market, $\varepsilon\tau_{f,t}L_{R,t}$ markets for designs, τ_t markets for intermediate products and the market for the consumption good. The sequential markets equilibrium is then defined as follows.

Definition 11.2 *Given $K_1, \tau_0 > 0$, $C_1 \geq 0$, and $C_{t+1} = f(C_t, \zeta_t)$, a sequential markets equilibrium is allocations $c_{2,1}$, $\{c_{1,t}, c_{2,t+1}, S_t, \zeta_t, L_{A,t}, L_{R,t}, x_t\}_{t=1}^{\infty}$ and prices $\{w_t, r_t,$ $(p^e_{x_i,t})_{i\leq\tau_t}, (p(i))_{i>\tau_{f,t}}\}_{t=1}^{\infty}$, such that*

(i) they solve the utility maximization problem of the households and the profit maximization problems of the representative final-product firm and of the intermediate firms for all $t \geq 1$, and

(ii) in every period, the economy is in temporary equilibrium, that is, for all $t \geq 1$:

(a) (Labor Market)

$$L^s_t = L^d_{A,t} + L^d_{R,t};$$

(b) (Capital Market)

$$S_{t-1} = K_t;$$

(c) *($\varepsilon\tau_{f,t}L_{R,t}$ Markets for Designs)*

Each new design is purchased by an intermediate entrepreneur;

(d) *(τ_t Intermediate Goods Markets)*

$$x_{i,t}^s = x_{i,t}^d;$$

(e) *(Final Goods Market)*

$$Y_t^s = c_{1,t} + c_{2,t} + S_t + \zeta_t.$$

The intermediate goods markets will be examined first. The representative final goods firm acts fully competitive in the markets, with the same demand for each kind of intermediate good due to the additive separability of the production function. However, supply varies by whether the intermediate entrepreneur possesses a monopoly or acts competitive. In this way, there are $\tau_{f,t}$ competitive markets that clear if

$$x_{i,t}^d = \left(\frac{p_{x_i,t}}{\alpha}\right)^{\frac{1}{\alpha-1}} L_{A,t} = \left(\frac{\eta r_t}{\alpha}\right)^{\frac{1}{\alpha-1}} L_{A,t} = x_{i,t}^s,$$

where $i \leq \tau_{f,t}$. The equilibrium price and equilibrium quantity are then

$$\begin{aligned} p_{x_i,t}^e &= \eta r_t, \\ x_{i,t}^e &= \left(\frac{\eta r_t}{\alpha}\right)^{\frac{1}{\alpha-1}} L_{A,t}. \end{aligned}$$

The competitive intermediate firm's profit is zero. As all competitive entrepreneurs are symmetric, we denote the equilibrium prices and quantities of each intermediate goods by $p_{x_f,t}$ and $x_{f,t}$. The market clearing conditions of the monopolistic intermediate goods markets are

$$x_{i,t}^d = \left(\frac{p_{x_i,t}}{\alpha}\right)^{\frac{1}{\alpha-1}} L_{A,t} = \left(\frac{\eta r_t}{\alpha^2}\right)^{\frac{1}{\alpha-1}} L_{A,t} = x_{i,t}^s,$$

where $\tau_t \geq i > \tau_{f,t}$. And the resulting equilibrium prices and quantities can be written as

$$\begin{aligned} p_{x_i,t}^e &= \frac{\eta r_t}{\alpha}, \\ x_{i,t}^e &= \left(\frac{\eta r_t}{\alpha^2}\right)^{\frac{1}{\alpha-1}} L_{A,t}. \end{aligned}$$

Symmetry also applies to monopolistic intermediate goods firms, such that it is convenient to work with $p_{x_m,t}$ and $x_{m,t}$ as being the equilibrium prices and quantities of each monopolistic intermediate good. A monopolistic intermediate entrepreneur's profit amounts to

$$\pi_t^{int} = p_{x_m,t}x_{m,t} - r_t\eta x_{m,t} - p(i) = (\eta r_t)^{\frac{\alpha}{\alpha-1}}\alpha^{\frac{1+\alpha}{1-\alpha}} L_{A,t}(1-\alpha) - p(i).$$

Next, consider the capital market. The market is competitive on the demand side as well as on the supply side. In equilibrium we have

$$\int_0^{\tau_{f,t}} K_{i,t}^d di + \int_{\tau_{f,t}}^{\tau_t} K_{i,t}^d di = S_{t-1}.$$

Denoting by K_t the total amount of capital available in period t, that is $K_t = S_{t-1}$, and using the respective demand functions as well as the symmetry of intermediate goods firms yields

$$\int_0^{\tau_{f,t}} \eta^{\frac{\alpha}{\alpha-1}} \left(\frac{r_t}{\alpha}\right)^{\frac{1}{\alpha-1}} L_{A,t} di + \int_{\tau_{f,t}}^{\tau_t} \eta^{\frac{\alpha}{\alpha-1}} \left(\frac{r_t}{\alpha^2}\right)^{\frac{1}{\alpha-1}} L_{A,t} di = K_t,$$

and consequently

$$r_t = K_t^{\alpha-1} \eta^\alpha \alpha L_{A,t}^{1-\alpha} [\tau_{f,t} + (\tau_t - \tau_{f,t}) \alpha^{\frac{1}{1-\alpha}}]^{1-\alpha}.$$

With respect to the markets for designs the assumption is that once a design has been produced a large number of potential suppliers of the new intermediate good bid for the right to commercialize it. In this way, the market price for new designs is equal to the profit of the monopolistic entrepreneur:

$$p(i) = (\eta r_t)^{\frac{\alpha}{\alpha-1}} \alpha^{\frac{1+\alpha}{1-\alpha}} L_{A,t}(1-\alpha).$$

Inserting the equilibrium capital rent yields

$$p(i) = K_t^\alpha L_{A,t}^{1-\alpha} \alpha^{\frac{1}{1-\alpha}} \eta^{\alpha \frac{1+\alpha}{\alpha-1}} (1-\alpha) [\tau_{f,t} + (\tau_t - \tau_{f,t}) \alpha^{\frac{1}{1-\alpha}}]^{-\alpha}.$$

Each individual inelastically supplies one unit of labor when young and is free to choose whether to work in final goods production or in research. Therefore, the labor market equilibrium is characterized by

$$L_t = L_{A,t} + L_{R,t}.$$

For convenience, we are first going to determine the share of each occupational group via the condition that wages must be equal in equilibrium. As each researcher produces $\varepsilon \tau_{f,t}$ new ideas, her income amounts to

$$w_{R,t} = \int_{\tau_{P_t,inv}} p(i) di = \varepsilon \tau_{f,t} (\eta r_t)^{\frac{\alpha}{\alpha-1}} \alpha^{\frac{1+\alpha}{1-\alpha}} L_{A,t}(1-\alpha).$$

Using the equilibrium quantities of the intermediate goods markets, the demand for workers is given by

$$w_{A,t} = (1-\alpha) \left(\frac{\eta r_t}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} [\tau_{f,t} + (\tau_t - \tau_{f,t}) \alpha^{\frac{\alpha}{1-\alpha}}].$$

Consequently $w_{R,t} = w_{A,t}$ yields

$$\varepsilon\tau_{f,t}L_{A,t}\alpha^{\frac{1}{1-\alpha}} = [\tau_{f,t} + (\tau_t - \tau_{f,t})\alpha^{\frac{\alpha}{1-\alpha}}].$$

Since $\tau_t - \tau_{f,t} = \varepsilon\tau_{f,t}L_{R,t}$ and due to the inelastic labor supply $L_{R,t} = 1 - L_{A,t}$, we have

$$\begin{aligned} L_{A,t} &= \frac{\alpha^{\frac{\alpha}{\alpha-1}} + \varepsilon}{\varepsilon(1+\alpha)}, \\ L_{R,t} &= \frac{\varepsilon\alpha - \alpha^{\frac{\alpha}{\alpha-1}}}{\varepsilon(1+\alpha)}. \end{aligned}$$

The share of workers is smaller than 1 if $\varepsilon > \alpha^{\frac{1}{\alpha-1}}$. It is assumed that this condition holds.

By Walras law, the final goods market must then be in equilibrium as well. Let the consumption good be the numéraire. The sequential equilibrium is then characterized by the following price vector:

$$\begin{pmatrix} w_t \\ r_t \\ (p^e_{x_i,t})_{i\le\tau_{f,t}} \\ (p^e_{x_i,t})_{i>\tau_{f,t}} \\ (p(i))_{i>\tau_{f,t}} \end{pmatrix} = \begin{pmatrix} K_t^\alpha \tau_{f,t}^{1-\alpha}\alpha^{\frac{1}{1-\alpha}}\eta^{\alpha^{\frac{1+\alpha}{\alpha-1}}}(1-\alpha)^{\frac{\alpha^{\frac{\alpha}{\alpha-1}}+\varepsilon}{1+\alpha}}H^{-\alpha} \\ K_t^{\alpha-1}\tau_{f,t}^{1-\alpha}\eta^\alpha\alpha H^{1-\alpha} \\ K_t^{\alpha-1}\tau_{f,t}^{1-\alpha}\eta^{1+\alpha}\alpha H^{1-\alpha} \\ K_t^{\alpha-1}\tau_{f,t}^{1-\alpha}\eta^{1+\alpha}H^{1-\alpha} \\ K_t^\alpha\tau_{f,t}^{-\alpha}\eta^{\alpha^{\frac{1+\alpha}{\alpha-1}}}\alpha^{\frac{1}{1-\alpha}}(1-\alpha)^{\frac{\alpha^{\frac{\alpha}{\alpha-1}}+\varepsilon}{\varepsilon(1+\alpha)}}H^{-\alpha} \end{pmatrix},$$

where $H = L_{A,t}[1 + \varepsilon L_{R,t}\alpha^{\frac{1}{1-\alpha}}] = \frac{(1+\varepsilon\alpha^{\frac{2-\alpha}{1-\alpha}})(\alpha^{\frac{\alpha}{\alpha-1}}+\varepsilon)}{\varepsilon(1+\alpha)^2}$.

11.3.8 Dynamics and Knowledge Codification

This section shows that using the equilibrium values, the modified Romer structure exhibits the properties of the models introduced in the first two parts of the thesis.

The capital market equilibrium gives

$$r_t = K_t^{\alpha-1}\tau_{f,t}^{1-\alpha}L_{A,t}^{1-\alpha}\eta^\alpha\alpha[1 + \varepsilon L_{R,t}\alpha^{\frac{1}{1-\alpha}}]^{1-\alpha}.$$

With respect to final goods production, inserting the respective equilibrium values yield

$$F((x_{i,t})_{i\le\tau_t}, L_{A,t}) = L_{A,t}^{1-\alpha}[\int_{i=0}^{\tau_{f,t}} x_t(i)^\alpha di + \int_{i=\tau_{f,t}}^{\tau_t} x_t(i)^\alpha di] = L_{A,t}\tau_{f,t}x_{f,t}^\alpha[1 + \varepsilon L_{R,t}\alpha^{\frac{\alpha}{1-\alpha}}],$$

which transforms to

$$F(K_t, L_{A,t}, \tau_t) = K_t^\alpha L_{A,t}^{1-\alpha}\tau_{f,t}^{1-\alpha}\eta^{\alpha^{\frac{1+\alpha}{\alpha-1}}}[1 + \varepsilon L_{R,t}\alpha^{\frac{\alpha}{1-\alpha}}][1 + \varepsilon L_{R,t}\alpha^{\frac{1}{1-\alpha}}]^{-\alpha}.$$

Wages can be written as

$$\begin{aligned} w_{R,t} &= K_t^{\alpha} L_{A,t}^{1-\alpha} \tau_{f,t}^{1-\alpha} \varepsilon \eta^{\alpha \frac{1+\alpha}{\alpha-1}} \alpha^{\frac{\alpha}{1-\alpha}} (1-\alpha)[1+\varepsilon L_{R,t}\alpha^{\frac{1}{1-\alpha}}]^{-\alpha}, \\ w_{A,t} &= K_t^{\alpha} L_{A,t}^{-\alpha} \tau_{f,t}^{1-\alpha} \eta^{\alpha \frac{1+\alpha}{\alpha-1}} (1-\alpha)[1+\varepsilon L_{R,t}\alpha^{\frac{\alpha}{1-\alpha}}][1+\varepsilon L_{R,t}\alpha^{\frac{1}{1-\alpha}}]^{-\alpha}. \end{aligned}$$

Since $L_{R,t}$ is constant over time in the sequential market equilibrium and according to the dynamics with respect to knowledge as given in (11.1), the knowledge stock in period t can be written as

$$\tau_t = \tau_{f,t} \underbrace{\left(1 + \frac{\varepsilon\alpha - \alpha^{\frac{\alpha}{\alpha-1}}}{1+\alpha}\right)}_{\vartheta}.$$

When defining the following constants,

$$\begin{aligned} \xi &:= \eta^{\alpha \frac{1+\alpha}{\alpha-1}} [1+\varepsilon L_{R,t}\alpha^{\frac{\alpha}{1-\alpha}}][1+\varepsilon L_{R,t}\alpha^{\frac{1}{1-\alpha}}]^{-\alpha} \vartheta^{\alpha-1}, \\ \kappa &:= \eta^{\alpha \frac{1+\alpha}{\alpha-1}} (1-\alpha)[1+\varepsilon L_{R,t}\alpha^{\frac{\alpha}{1-\alpha}}][1+\varepsilon L_{R,t}\alpha^{\frac{1}{1-\alpha}}]^{-\alpha} \vartheta^{\alpha-1}, \\ \lambda &:= \eta^{\alpha \frac{1+\alpha}{\alpha-1}} \alpha^{\frac{\alpha}{1-\alpha}} (1-\alpha)[1+\varepsilon L_{R,t}\alpha^{\frac{1}{1-\alpha}}]^{-\alpha} \vartheta^{-\alpha}, \end{aligned}$$

and additionally using $\frac{\partial \tau_t}{\partial L_{R,t}} = \varepsilon \tau_{f,t}$, it follows that the final goods production function, the wages and the return on physical capital show the forms given in properties (P1)-(P3) for the case $\psi = 1-\alpha$.

$$F(K_t, L_{A,t}, \tau_t) = \xi K_t^{\alpha} L_{A,t}^{1-\alpha} \tau_t^{1-\alpha} \Rightarrow \text{(P1)},$$

$$\left.\begin{aligned} w_{A,t} &= \kappa K_t^{\alpha} L_{A,t}^{-\alpha} \tau_t^{1-\alpha} \\ w_{R,t} &= \lambda K_t^{\alpha} L_{A,t}^{1-\alpha} \tau_t^{-\alpha} \tfrac{\partial \tau_t}{\partial L_{R,t}} \end{aligned}\right\} \Rightarrow \text{(P2)},$$

$$r_t = K_t^{\alpha-1} \tau_t^{1-\alpha} f(L_{A,t}) \Rightarrow \text{(P3)},$$

where $f(L_{A,t}) := L_{A,t}^{1-\alpha} \eta^{\alpha} \alpha [1+\varepsilon L_{R,t}\alpha^{\frac{1}{1-\alpha}}]^{1-\alpha}$.
Further, the modified Romer model is characterized by the following system of difference equations:

$$\begin{aligned} K_{t+1} &= s_t K_t^{\alpha} \tau_{f,t}^{1-\alpha} \alpha^{\frac{1}{1-\alpha}} \eta^{\alpha} (1-\alpha) \frac{\alpha^{\frac{\alpha}{\alpha-1}}+\varepsilon}{1+\alpha} H^{-\alpha}, \\ \tau_{t+1} &= (q\tau_t + \beta \max\{0, C_{t+1} - q\tau_t\})(1+\varepsilon L_{R,t}), \\ C_{t+1} &= C_t + \triangle C_t \quad , \quad \triangle C_t \geq 0. \end{aligned}$$

It is easy to see that the difference equation with respect to knowledge exhibits the form given in (K1), resembling the $(\Phi, \tilde{\Phi}) = (1,1)$ case. It can then be formulated:

Proposition 11.1 *The modified microeconomic structure of Romer (1990) satisfies the properties (U), (K1), (P1)-(P3), (M1) and (N).*

Note that the additional assumption of (c4) yields (M1) for the case $(\Phi, \tilde{\Phi}) = (1, 1)$.[11] As discussed earlier, property (N) only holds if a collective decision for example within a state is possible. This is summarized in the following proposition.

Proposition 11.2 *There is no private knowledge codification in the original Romer model.*

It is then clear that with social knowledge codification, it can be stated:

Proposition 11.3 *Since the modified Romer structure represents the case $\psi = 1 - \alpha$, the economy will be codifying in the long run if constant codification costs satisfy*

$$\gamma < \frac{(k_s^{woC})^{\alpha} g(L_{A,t})}{1 + (k_s^{woC})^{\alpha-1} f(L_{A,t})}, \tag{11.3}$$

or the codification costs decline over time.

As an example, assume logarithmic utility. The condition (11.3) then writes

$$\gamma < \frac{\left(\frac{\delta}{1+\delta}\right)^{\frac{\alpha}{1-\alpha}} Q^{\frac{\alpha}{1-\alpha}} \alpha(1-\alpha)\eta^{\alpha} H^{1-\alpha} \vartheta\beta}{1 + \frac{1+\delta}{\delta} Q^{-1} \eta^{\alpha} \alpha H^{1-\alpha}},$$

where $Q = \alpha^{\frac{1}{1-\alpha}} \eta^{\alpha^{\frac{1+\alpha}{\alpha-1}}} (1-\alpha)^{\frac{\alpha^{\frac{\alpha}{\alpha-1}} + \varepsilon}{1+\alpha}} H^{-\alpha} \vartheta q$.

11.4 Summary

This chapter has discussed the relation of the model as introduced in the dissertation and Romer's model of endogenous technological change. The major differences of the Romer model are the assumption of perfect spillovers and the continuous time structure. The question has been elaborated whether the microeconomic structure of the Romer model could be translated into an overlapping generations framework with imperfect knowledge transfer between generations. The answer would be a reserved "yes".

By changing assumption (A1) concerning the ideas' ordering concept and introducing two additional assumptions with respect to knowledge transfer between generations and to research, the difference equation of the knowledge stock as given by (K1) can account for horizontal innovation. Greater difficulties arise with respect to research duplication. The question to be answered is whether an idea that has not been transferred from the previous period and is re-invented, can earn a monopoly profit. Different arguments have been discussed, however, it needs further research as to whether a "no" to this question can combine both, knowledge dynamics according to (K1) and labor market equilibria as induced by the regular Romer structure.

[11] See lemma 9.1.

As a result of this chapter it can be stated that the mathematical structure of the modified Romer model exhibits the general properties of the model introduced in the previous parts. However, (N) is only satisfied if a social decision on knowledge codification is possible as one of the differences between the Romer model and the one of this thesis is that no decision maker possesses positive measure. In this way, the original Romer structure cannot explain the private creation of knowledge spillovers by knowledge codification. Allowing for social knowledge codification, the modified Romer model represents the case of Harrod-neutral technological progress, $\psi = 1 - \alpha$, and the corresponding propositions apply.

Chapter 12

Social Optimality

The discussion on social optimality will be approached from two perspectives. The first is characterized by the question how an allocation chosen by a social planner who maximizes a social welfare function compares to the outcome of the sequential market equilibrium (SME) as discussed in the previous chapters. Rather than providing a full fledged analysis, the purpose is to outline the possible inefficiences and discuss its sources. As the welfare optimum may imply that some generations may be worse off as compared to the SME, the weaker criterion of whether the market outcome is Pareto-optimal will be examined, respectively, whether it is possible to achieve a Pareto-improvement by transfers. This second perspective possesses the advantage that it does not require the specification of a social welfare function.

12.1 The Command Optimum

The command optimum assumes a social planner that may choose all control variables so as to maximize a social welfare function. However, it is not obvious what the relevant social welfare function should be. A thorough discussion on welfare functions is beyond the scope of this chapter and the interested reader is referred to the respective social choice literature.[1] As much of the literature on overlapping generations economies, we will assume that a central planner maximizes the following intertemporal social welfare function:

$$W = \delta u(c_{2,1}) + \sum_{t=1}^{\infty} \delta_s^t [u(c_{1,t}) + \delta u(c_{2,t+1})], \tag{12.1}$$

with a social discount rate $0 < \delta_s < 1$.
The social planner's problem can be stated as

$$\max_{c_{2,1},\{c_{1,t},c_{2,t+1}\}_1^\infty} W = \delta u(c_{2,1}) + \sum_{t=1}^{\infty} \delta_s^t [u(c_{1,t}) + \delta u(c_{2,t+1})]$$

[1] For discussions on social welfare functions see e.g. Ramsey (1928), Harsanyi (1955), Sen (1986), McKenzie (1986), von Weizsäcker (1965).

subject to

$$\begin{aligned} K_{t+1} - K_t &= Y_t - (c_{1,t} + c_{2,t} + \gamma_t \triangle C_t); \\ \tau_t &= q\tau_{t-1} + \beta \max\{0, C_t - q\tau_{t-1}\} \\ &\quad + \varepsilon \left[q\tau_{t-1} + \tilde{\Phi}(\beta \max\{0, C_t - q\tau_{t-1}\}) \right]^{\Phi} L_{R,t}; \\ C_t &= C_{t-1} + \triangle C_{t-1}; \\ L_t &= L_{A,t} + L_{R,t}; \\ \triangle C_{e,t} &\leq (1-q)\tau_t; \\ K_1, \tau_0 &> 0, \quad C_1 \geq 0. \end{aligned}$$

Without loss of generality, let $Y_t = F_t(x_t, L_{A,t}) = F_t(K_t, \tau_t, L_{A,t})$, where $x_t = K_t \tau_t^{\frac{\psi}{\alpha}}$. Further, we preclude parameter values that would violate the problem's concavity without characterizing them further.

Appendix 12A provides the first order conditions for a welfare maximum. With respect to consumption, the social planner would in each period equal out the marginal utility of the young generation and that of the old generation, modified by the relation of the private and the social discount rate. This, of course, must be equal to the next period's return on capital in utility discounted by the social discount rate:

$$u'(c_{1,t}) = \hat{\delta} u'(c_{2,t}) = \delta_s u'(c_{1,t+1}) \left(\frac{\partial F_{t+1}}{\partial K_{t+1}} + 1 \right). \tag{12.2}$$

where $\hat{\delta} = \frac{\delta}{\delta_s}$. This condition is equal to the command optimum in a standard overlapping generations structure in the style of Diamond (1965). In an exogenous growth overlapping generations model this would imply the modified golden rule in steady state.[2] Two issues differ from the market solution as discussed in the first two parts of the dissertation. First, the return on physical capital, $r_{t+1} = \frac{\partial F_{t+1}}{\partial K_{t+1}}$ does not show any distortions due to the market power of intermediate firms. Second, the social optimum would require to allocate consumption such that marginal utilities of the different generations within a period equal out, modified by the relation of the social and private discount factor, rather than the marginal utilities of one generation over two periods, modified by the private discount factor.

According to (12A.23) and similar to the market solution, the marginal future benefit of knowledge codification must be equal to the return on another marginal unit invested in physical capital:

$$\gamma_t u'(c_{1,t+1}) \left(\frac{\partial F_{t+1}}{\partial K_{t+1}} + 1 \right) = \beta u'(c_{1,t+1}) \frac{\partial F_{t+1}}{\partial \tau_{t+1}} + \delta_s \left(\frac{\partial V_{t+2}}{\partial C_{t+2}} + \beta \frac{\partial V_{t+2}}{\partial \tau_{t+1}} \right).$$

However, it is immediately clear that the social planner also accounts for the positive external effect of knowledge codification on later generations, which is given by the last

[2]See for example Blanchard and Fischer (1989).

term on the right hand side. It reveals that the value function of $t+2$ is affected by knowledge codification in period t in two ways: Directly by increasing the information stock available for future generations and indirectly by enhancing the next generations knowledge level. Dividing the above equation by $u'(c_{1,t+1})$ and using

$$\begin{aligned}\frac{\partial F_{t+1}}{\partial K_{t+1}} &= \alpha K_{t+1}^{\alpha-1}\tau_{t+1}^{\psi}L_{A,t+1}^{1-\alpha},\\ \frac{\partial F_{t+1}}{\partial \tau_{t+1}} &= K_{t+1}^{\alpha}\psi\tau_{t+1}^{\psi-1}L_{A,t+1}^{1-\alpha},\end{aligned}$$

yields

$$\gamma_t(1+\alpha K_{t+1}^{\alpha-1}\tau_{t+1}^{\psi}L_{A,t+1}^{1-\alpha}) = \beta K_{t+1}^{\alpha}\psi\tau_{t+1}^{\psi-1}L_{A,t+1}^{1-\alpha} + \frac{\delta_s}{u'(c_{1,t+1})}\left(\frac{\partial V_{t+2}}{\partial C_{t+2}} + \beta\frac{\partial V_{t+2}}{\partial \tau_{t+1}}\right). \quad (12.3)$$

It is then obvious that if $\left(\frac{\partial V_{t+2}}{\partial C_{t+2}} + \beta\frac{\partial V_{t+2}}{\partial \tau_{t+1}}\right) < \infty$ and $\tau_t > 0, \forall t$, the social planner would not codify for small enough K_t. Or put differently, the social planner's optimal program would not include knowledge codification at the beginning of the economy's development given $\tau_1 > 0$ and K_1 small enough. This corresponds to the dynamics of the sequential markets equilibrium. However, the social planner would supposedly begin to codify earlier.

The allocation of labor in the command optimum is given by the following equation:

$$\varepsilon\left[q\tau_{t-1} + \tilde{\Phi}(\beta(C_t - q\tau_{t-1}))\right]^{\Phi}\left[u'(c_{1,t})\frac{\partial F_t}{\partial \tau_t} + \delta_s\left(\delta_s\frac{\partial V_{t+2}}{\partial \tau_{t+1}} + u'(c_{1,t+1})\frac{\partial F_{t+1}}{\partial \tau_{t+1}}\right)\frac{\partial \tau_{t+1}}{\partial \tau_t}\right] = u'(c_{1,t})\frac{\partial F_t}{\partial L_{A,t}}. \quad (12.4)$$

In contrast to the market solution, the social planner also accounts for the research externalities to future generations, leading to a higher share of researchers.

It can be seen from equations (12.2), (12.3) and (12.4), that the sequential market equilibrium may exhibit different inefficiencies.

Proposition 12.1 *As compared to the command optimum, the following potential inefficiencies of sequential market equilibria of model structures satisfying (U), (K1), (A2'), (P1)-(P3), (N) and (M1) can be identified.*

(i) Market imperfection due to (N), which creates market power of at least one decision maker.

(ii) Distributive inefficiency with respect to consumption.

(iii) Socially inefficient investment in knowledge codification.

(iv) *Socially inefficient investment in research and development.*

Possible differences to the social welfare optimum due to a difference between the social and private discount rate have not been explicitly listed, as this is not the focus of this thesis and the social discount rate has no deeper meaning in this context.

With respect to (i), it is clear that the necessary condition for private knowledge codification in the sequential markets equilibrium, (N), implies a welfare loss from distortions due to market power. This constitutes a similar tradeoff as between monopoly rights from patents and innovation. That is, accepting a certain static efficiency loss for a dynamic efficiency gain due to endogenously created spillovers by knowledge codification.

Further, it is clear that due to the concavity of the utility functions, the social planner aims at equalling out each period's consumption levels of old and young.[3] This may not necessarily be given in SME.

In general, the social inefficiencies from knowledge codification originate from the following sources:

(1) The externalities of knowledge codification on future generations from direct access to the information created in t, and from indirectly creating additional exogenous spillovers by enhancing the next periods' knowledge stocks are not taken into account in SME. We will refer to this as the finite horizon effect of knowledge codification.

(2) In the specification with $\Phi = 1$, there may be another source of intertemporal inefficiency with respect to investment in knowledge codification which corresponds to the regular R&D-effect of endogenous growth models. When research productivity is characterized by $\tilde{\Phi} = 1$, this is obvious. For $\tilde{\Phi} = 0$ the effect materializes indirectly by increasing the next period's knowledge stock of which a share q spills over exogenously and hence positively influences the researchers' productivity from the next but one period on.[4]

(3) In the oligopolistic structure the codification costs have been defined as γ_t per idea and firm. The merging of codification activities has not been considered. It seems

[3] Modified by the private discount rate.

[4] The question may arise as to whether there are also social inefficiencies from knowledge codification corresponding to the consumer surplus effect and the business stealing effect of research and development. It seems plausible that such a correspondence exists with respect to the consumer surplus effect as there was no price discrimination in the market for intermediates in the previously discussed models. However, it is not ad hoc clear if there is an equivalent to the regular business stealing effect as the idea that is transferred by knowledge codification must have been invented in the previous period, already. The two effects, that is, the consumer surplus effect and the business stealing effect of knowledge codification should be identified in a full-fledged formal analysis. Here, we will concentrate the following discussion on the R&D-effect of knowledge codification which can be unambiguously verified.

highly plausible that the costs of knowledge codification exhibit subadditivity in the number of accessors. More precisely, having defined the costs of knowledge codification as creating a piece of information and ensuring access to it, the latter seems to cause lower costs than the first. Hence, if for example several firms maintain a knowledge database or library together, it should be cheaper than if they all have one on their own. In this way, the oligopolistic intermediate sectors would engage in inefficiently high knowledge codification efforts. This could be termed the static inefficiency of knowledge codification so as to distinguish it from the finite horizon effect.[5]

Considering research and development, the social inefficiencies may result from:

(1) A finite horizon effect with respect to R&D. The social benefit of an additional idea to future generations is not taken into account. Such an effect also occurs in standard endogenous growth models with finite patent lengths, where the innovator's reward is temporary, but the social gain from the discovery is permanent. This may result from the overlapping generations framework with finite lifetimes without altruism, or in the standard endogenous growth models from finite patent lengths.[6]

(2) The regular externalities of R&D. More precisely, the R&D-effect (if $\Phi = 1$), the consumer surplus effect and the business stealing effect.[7]

(3) In an oligopolistic structure such as that of chapter 10, there will be inefficient research duplication.

To achieve a clear distinction between the first two points in the lists of possible sources of inefficiencies with respect to knowledge codification and research and development, the finite horizon effect refers to the creator of an idea or information to not take into account the social benefit that it entails to future generations' production possibilities. In simple words, by knowledge spillovers this idea can be used by future generations in the production of intermediate goods. In contrast, the R&D-effect only describes the positive externality of the knowledge spillovers on the researchers' productivity. In principle, one may argue that in the overlapping generations model populated by economic agents living for two periods, it is sufficient to distinguish static inefficiencies such as research duplication or the static inefficiency with respect to knowledge codification from intertemporal externalities such as the finite horizon effect and the R&D-effect of

[5]From a static perspective when understanding it as considering only one period, knowledge codification would always be inefficient, of course. This is not what is meant here. It is rather that a certain amount of information is not created at its lowest possible cost.

[6]See e.g. Judd (1985) or Barro and Sala-i-Martin (1995, chapter 6).

[7]Note, that the basic model and the extension in the second part of the thesis do not exhibit negative externalities due to a business stealing effect, since the intermediate sector has been modelled as a monopoly. However, a business stealing effect is not precluded by the general properties given in chapter 9.

knowledge codification, without further differentiating between the latter. This, however, would hamper a discussion on which effects are due to the overlapping generations framework and which would also be present in a standard framework with a Ramsey consumer. Consider the following thought experiment. Let's successively add altruism towards their children to the households' preferences. Although each generation cares directly only about the next, this series of intergenerational links implies that each generation acts as if it cared about the utility of all future generations. As a consequence the effects resulting from the short sightedness of the two period lived individuals in the overlapping generations framework would become successively smaller. Assuming very strong altruism would effectively yield a Ramsey consumer.[8] It seems then reasonable to expect that in this case the social inefficiency due to the distribution of consumption within a period would vanish.

Consider the basic model introduced in the first part of the dissertation and think of the household as a Ramsey-consumer. With the intermediate firm being a monopoly, the first inefficiency from the market imperfection would still prevail and possibly another from a difference between the private and the social discount rate. However, the decisions taken within the intermediate firm should internalize all externalities from knowledge codification and research.

When moving to oligopolistic intermediate sectors with infinitely lived intermediate firms, the R&D-effects of research and of knowledge codification would not be internalized fully anymore and in addition the static inefficiency with respect to knowledge codification and research duplication would arise. However, the finite horizon effect would not occur. Increasing the number of intermediate firms to infinitely many in monopolistic competition, that is, when thinking about the modified Romer model introduced in chapter 11 with a Ramsey consumer, the classical inefficiencies due to market power of the intermediate firms and the R&D-effect with respect research would apply.[9] Due to finite patent lengths, one would also encounter a finite horizon effect with respect to R&D. However, as argued in chapter 11, (N) would be violated leading to inefficiently low, more precisely, zero private knowledge codification. Only social knowledge codification that is allowing for a collective decision of the households, e.g. within a state, would lead to the creation of information. The Ramsey-type households would then fully internalize the intertemporal externalities of knowledge codification.

Taking the model structure of chapter 8 as a reference, the following gives a verbal discussion on the expected major differences of the sequential market solution to the program of the social planner for an economy developing from initial values $K_1, \tau_0 > 0$, but close to zero, and $C_1 = 0$.

[8] See also Barro and Sala-i-Martin (1995, p. 135), Blanchard and Fischer (1989, p. 106).

[9] Note that in the standard Romer model, the consumer surplus effect and the business stealing effect just balance.

Consider first the case with a constant amount of ideas per researcher and period, i.e. $\Phi = 0$.

Note that with $\Phi = 0$, the R&D-effect of innovations is zero. It is clear that in this case, it is not possible to achieve long run growth, as the resources to influence the knowledge stock, that is the number of researchers, are bounded and, thus, would at some point just suffice to regenerate the ideas lost in imperfect intergenerational knowledge transfer. However, the knowledge level in a period positively depends on the information stock which by assumption does not decline. This suggests that as compared to the sequential markets equilibrium, one would observe much more research and knowledge codification in early periods in the command optimum in order to take maximal advantage of the positive externalities of the information stock. On the other hand, in an economy developing from low levels of capital and knowledge, research and knowledge codification are more expensive in terms of utility than in later periods.[10] Nevertheless, it seems reasonable to expect that the social optimum exhibits an information stock $C_{so} > C_{st,sme}$, where C_{so} denotes the socially optimal long run information stock and $C_{st,sme}$ the stationary state information stock of the sequential markets equilibrium.[11] This would imply that due to the finite horizon effect, the sequential markets equilibrium exhibits inefficiently low knowledge codification. Note that this result would be amplified by assumption (A2), because in SME, the generation of period t does not account for an increase in entry costs to knowledge codification for later generations.[12]

If the positive externalities of knowledge codification outweigh the higher costs in terms of utility in early periods, $C_{so} > C_{st,sme}$ must also hold for the parameter values $\psi < 2\alpha - 1$ in the model of chapter 8, although proposition 8.4 states that output may be declining in the knowledge stock, suggesting a negative external effect of knowledge codification on the succeeding generations. However, this effect results from the equilibrium allocation of the homogenous good within a period. The reason was that the decline of the researchers' wage in knowledge and the corresponding shift of labor to final goods production is so strong that the decrease of marginal productivity in final goods production due to the increasing number of workers outweighs the increase of productivity due to the higher knowledge stock. Hence, although total output is ceteris paribus increasing in the knowledge stock, the equilibrium wage is declining in it. Of course, the return on capital shows a corresponding increase. So with the equilibrium wage declining, the saving rate had to show a corresponding increase to prevent a decline in output of the economy. This effect is not due to the market power of intermediate firms but an example of an intertemporal inefficiency created by the equilibrium allocation of the homogenous good. Hence, a first best policy that corrects for the distributional inefficiency would imply higher knowledge codification and research in early periods as discussed in the previous paragraph. However, a second best policy that is constrained

[10] As the marginal utilities of consumption are higher at lower consumption levels.

[11] $C_{st,sme}$ must be smaller than or equal to τ_{st}^{wC} due to the assumption $0 = C_1 < \tau_{st}^{wC}$.

[12] Recall that due to assumption (A2), in order to codify an idea i, all ideas j, $j < i$ have to be codified already.

in reallocating output from old to young within a period, may have the opposite implications with respect to knowledge codification and research and development.

Let's now turn to the 'standing on the shoulders of giants' case, i.e. $\Phi = 1$.

With respect to knowledge codification and research and development, the specification of the research process by $\Phi = 1$ would imply that the SME exhibits inefficiencies resulting from the R&D-effect. With $(\Phi, \tilde{\Phi}) = (1, 1)$, the R&D-effect of knowledge codification is internalized by the capital owners to the extent that the higher research productivity of the next period's researchers affects their return on capital.

Clearly, the positive externalities of knowledge codification on future generations would suggest that the socially optimal level of knowledge codification in each period is weakly higher than in the sequential markets equilibrium. However, on the other hand, the social planner also takes the creation of exogenous spillovers by increasing research and development efforts into account. In this way, there is an additional tradeoff in the transfer of knowledge to future generations between allocating additional resources to research or to knowledge codification. Thus, it is not possible to preclude, that in a certain period t, the command optimum would codify less than in the sequential markets equilibrium. This is more likely for relatively high codification costs, where it may be cheaper to hire another researcher. Note that this tradeoff was excluded in the market solution as the old generation managed research in the intermediate firm and consequently did not take into account any effects of its decision on the next period.

Similar to the model-specification with $\Phi = 0$, the distribution of output within a period may matter for long run growth in the sequential market equilibrium if $\psi < \alpha$. By the same mechanism as described above, the steady state growth rate of output may be declining with a higher growth rate of knowledge. This follows from $g_{Y,s} = g_{K,s}$ and

$$g_{K,s} = s_s k_s^{\alpha-1} (1-\alpha)^{1-\alpha} (\psi \alpha \varepsilon q)^{\alpha} (1 + g_{\tau,s})^{-\alpha} - 1,$$

where $k_s = \frac{K_t}{\tau_t^{\frac{\psi}{1-\alpha}}}$ denotes the intensive capital stock in steady state. Let $\psi = 1 - \alpha$, as otherwise steady state behavior would imply either full or zero knowledge codification. According to the above equation, a higher steady state growth rate of knowledge induced by a higher amount of knowledge codification, would have to entail a correspondingly higher steady state saving rate or lower intensive steady state capital stock to not imply lower economic growth in the long run.

Note, that this effect is due to a shift of labor from research to final goods production resulting from the fact that according to the specification of the research process in chapter 8, the research productivity of the researchers does not increase in knowledge codification, but so does the marginal product in final goods production. Consequently, this effect would not be present in the case $(\Phi, \tilde{\Phi}) = (1, 1)$. Hence, the assumption as to whether knowledge codification of the previous period adds to research productivity or not, may make a major difference to the model's dynamics. And as mentioned previously the first best policy implications with respect to knowledge codification may not carry over to a second best policy that is not able to correct for the respective distributional effect.

To summarize, the positive externalities of knowledge codification on later generations would suggest weakly higher codification efforts in the command optimum as compared to the sequential markets solution. However, such a conclusion would not be unambiguous. As discussed, in the case $\Phi = 1$, the command optimum is characterized by a tradeoff between investing in research and development or knowledge codification to create knowledge spillovers to future periods. If the costs of knowledge codification are high, there may be periods where the social planner prefers to invest another unit in research rather than in knowledge codification; a possibility that is not available to the young generation at the end of a period when taking over the intermediate firm. Their only way to transfer additional knowledge is by knowledge codification and hence, the sequential market equilibrium may show higher investments in the information stock than the command optimum. For the specification $\Phi = 0$, the information stock plays a more pronounced role for long run welfare. With the economy ending up in a stationary state, the size of the information stock crucially determines the long run output level of the economy.[13] In this way, although there is also a tradeoff between creating spillovers to the next generation by knowledge codification or additional research and development, information, by assumption, possesses the characteristic to preserve the idea forever, whereas an additional idea created by further research may be lost in the knowledge transfer between generations. Hence, for $\Phi = 0$ it is without much doubt that the stationary state information stock will be higher in the command optimum than in the sequential markets equilibrium.

Note, however, that the command optimum could well imply that some generations will be worse off as compared to the sequential markets equilibrium. That is, the welfare maximizing time paths of the control variables, may require that early generations heavily invest in knowledge codification in order to maximize the external benefits enjoyed by the later generations. It is important to keep in mind that the inefficiencies as measured by the social welfare function do not exclude that the allocation of the sequential markets equilibrium is Pareto-efficient in the sense that it is not possible to make any one generation better off without worsening another. Hence, the next section elaborates on the question as to whether the allocation of the sequential markets equilibrium allows for Pareto-improvements by transfers between different generations. This is possible if the market outcome is dynamically inefficient. However, there may also exist efficiency gains with respect to knowledge codification.

[13] In the $\Phi = 1$ case, it is rather a means for temporally increasing knowledge transfer (of course with all its long run effects).

12.2 Is the Sequential Markets Equilibrium Dynamically (In-)Efficient?

It is well known that standard overlapping generations economies with production where the steady state growth rate of output exceeds the rate of return on physical capital are dynamically inefficient and redistribution of resources from children to parents would yield a Pareto-improvement to the economy.[14] The intuition is that with the rate of return in $t+1$ to investment in t in steady state being smaller than the steady state growth rate of output, the loss in consumption in t by transferring a marginal unit of the homogenous good to the old generation in t, can be compensated by a transfer of the young generation in $t+1$. As discussed in the previous chapters, without knowledge codification, the model introduced in the present thesis, corresponds very much to the standard Diamond model and it can be easily verified that it will show dynamic inefficiencies for certain parameter values.[15] In this section, we will first ask: Is it possible to make a statement as to whether an economy described by the model structure of chapter 8[16] with positive knowledge codification in steady state is more or less likely to show dynamic inefficiency? Thereafter, another way of achieving a Pareto-improvement by knowledge codification is discussed.

With regard to the first aspect, the focus will be on the case $\Phi = 1$ and $\psi = 1 - \alpha$, as according to the propositions in chapter 8, it is not possible to have steady state behavior other than with full or zero knowledge codification for $\psi \neq 1 - \alpha$. Similar to the discussion above, $g_{Y,s} > r_s$ would imply that it is feasible for a given amount of knowledge codification to transfer at least one marginal unit of the consumption good from the young generation to the old and compensate it by the transfer in the next period.

As verified in 8.3, steady state behavior is characterized by $g_{Y,s} = g_{K,s}$. Using the respective growth rate and the return on capital yields the following condition for the steady state to be dynamically inefficient:

$$
\begin{aligned}
& s_t(1-\alpha)K_t^{\alpha-1}\tau_t^{\psi}L_{A,t}^{-\alpha} - 1 > K_t^{\alpha-1}\tau_t^{\psi}L_{A,t}^{-\alpha}(L_{A,t} - (1-\alpha)) \\
\Leftrightarrow \quad & k_s^{\alpha-1}\left(\tfrac{1+g_{\tau,s}}{\alpha\varepsilon q}\right)^{-\alpha}\left((1-\alpha)(s_s+1) - \tfrac{1+g_{\tau,s}}{\alpha\varepsilon q}\right) > 1. \qquad (12.5)
\end{aligned}
$$

where $k_s = \frac{K_t}{\tau_t}$. The question as to whether an economy that shows steady state behavior with knowledge codification is more or less likely to exhibit dynamic inefficiency, cannot be answered unambiguously. On the one hand, an economy with knowledge codification is characterized by a higher steady state growth rate of output[17] but on the other hand,

[14] In exogenous growth overlapping generations models the subject has been studied by e.g. Diamond (1965), Tirole (1985), Abel et al. (1989). With respect to Romer-type endogenous growth models, see Saint-Paul (1992), Grossman and Yanagawa (1993).

[15] Such parameter values could be easily obtained when assuming logarithmic utility.

[16] The specification with $\psi \leq \alpha$ is used.

[17] The reason is that $g_{Y,s} = g_{K,s} = g_{\tau,s}$ for $\psi = 1 - \alpha$ and the steady state growth rate of knowledge is higher with knowledge codification than without.

also by a higher return on capital. With regard to the above equation, the steady state intensive capital stock k_s must be higher in a steady state with knowledge codification than in one without knowledge codification according to proposition 8.7. Further, the share of researchers will be lower in a steady state with knowledge codification if $\tilde{\Phi} = 0$, that is, in the specification of research as in the basic model.[18] This should lower the steady state growth rate of output. Finally, the steady state saving rate is expected to be higher, due to a higher return on capital. The first two effects would lower the left hand side of (12.5), whereas a higher saving rate increases it. Hence, it depends on the magnitude of the effects whether an economy with knowledge codification in steady state is more likely to be dynamically inefficient than one that is not codifying in steady state. Note, that the specification $\Phi = 0$ is characterized in the long run by $g_{Y,s} = 0$ and $r_s > 0$, indicating that the sequential markets economy does not show a dynamic inefficiency allowing a Pareto-improvement by a reallocation of resources.[19]

With respect to knowledge codification, there may be another possibility to achieve a Pareto-improvement to the sequential markets equilibrium. As the wage of the young generation is a function of the knowledge stock, the overlapping generations economy could gain in efficiency in every period in which the increase of the equilibrium wage by knowledge codification exceeds the present valued costs to the previous generation of codifying another idea. To realize the Pareto-improvement, the generation of t had to codify an additional idea and the subsequent generation would have to compensate them in the next period.

Proposition 12.2 *If*

$$\frac{\partial w_{t+1}}{\partial \tau_{t+1}} \frac{\partial \tau_{t+1}}{\partial \triangle C_{e,t}} > \gamma_t \delta^{-1} - \frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} K_{t+1} = \gamma_t (1 + r_{t+1}) - \frac{\partial r_{t+1}}{\partial \triangle C_{e,t}} K_{t+1} \tag{12.6}$$

and $0 \leq \triangle C_{e,t} < \tau_t - C_t$, *a Pareto-improvement to the sequential markets equilibrium is possible by a higher amount of knowledge codification of the generation born in* t *and a compensating transfer by the descendants in* $t + 1$.

It is clear that if the gain in equilibrium wage payments by another codified idea is greater than or equal to the opportunity costs in consumption of the first period or investment in capital, respectively, minus the gain in profits from codifying an additional idea, the codifying generation can at least be compensated. Taking a closer look at the equilibrium wage,

$$w_t = (1 - \alpha) K_t^{\alpha} \tau_t^{\psi - \alpha} \left(\frac{\kappa}{\lambda} \right)^{-\alpha} \left(\varepsilon (q \tau_{t-1})^{\Phi} \right)^{\alpha},$$

[18] The reason is that for $\tilde{\Phi} = 0$, the productivity in final goods production is increasing in the amount of knowledge codification whereas the researchers' productivity is not.

[19] See e.g. Azariadis (1993), chapter 12.

reveals that for (12.6) to hold, it is necessary that either $(\Phi, \tilde{\Phi}) = (1, 1)$ or $\tilde{\Phi} = 0$ and $\psi > \alpha$.[20] The basic model in the first part of the thesis was characterized by Solow-neutral technological progress, $\psi = \alpha$, and, hence, the equilibrium wage was unaffected by knowledge codification of the previous period. In this way, there is no hold-up problem between the firm owners and the newly hired employees. However, knowledge codification may positively influence the wage levels of the generations from the next but one period on. In the case of $\tilde{\Phi} = 0$ and $\psi < \alpha$, knowledge codification leads to declining wages. Would it be correct to argue that in the latter case a Pareto-improvement could be achieved by giving a transfer in $t + 1$ for the previous generation to not codify?

Even in the case $\Phi = 0$ and $\psi < 2\alpha - 1$ in which the stationary state output is declining in the knowledge stock in sequential markets equilibrium according to proposition 8.4, the answer is not obvious. The reason is, that although the wage of the generation of $t + 1$ increases by less knowledge codification of the previous generation, the return on physical capital in $t + 2$ would ceteris paribus decline. This effect may also carry over to later generations. Hence, it would need a deeper analysis to ensure that no generation would be worse off by such a transfer scheme. With other specifications, such as with $\Phi = 1$, the externalities on future generations due to less knowledge codification in t would be even more pronounced by the R&D-effect of knowledge codification, which would then have additional negative effects on long run growth.

Note, that such an effect was not possible under the conditions $(\Phi, \tilde{\Phi}) = (1, 1)$ or $\tilde{\Phi} = 0$ and $\psi > \alpha$, as with both, wages and the return on physical capital increasing in the knowledge stock, the externalities on future generations of an additional codified idea must be positive. In this case the economic problem with respect to knowledge codification as described in section 3.2.1 constitutes a classical hold-up problem. In the respective three-stage game, the new employees would initially be willing to compensate the firm owners for codifying another idea, however, once the investment is taken, the latter lose in bargaining power. The newly hired employees know that the firm owners benefit from giving access to the information by higher profits and, hence, will not offer a compensatory payment. The above considerations indicate that the solution to the hold-up problem would lead to at least temporarily higher growth rates of output in the case $\Phi = 1$ and imply a weakly higher stationary state level of total consumption for $\Phi = 0$. However, the solution of the hold-up problem possesses a particular difficulty: The generation of t has to take the investment decision at a time where the recipients are not even born. Hence, it is not obvious how to implement a mechanism to solve the commitment problem. In principle the problem shows a similar structure as a pay-as-you-go social security system, where the generations when young transfer consumption to the old, expecting the succeeding generation to do the same.[21] Without altruistic preferences the Pareto-improvement with respect to additional knowledge

[20] With these specifications both, the equilibrium wage and the return on physical capital are increasing in the knowledge stock. Hence, another codified idea will not have negative future externalities.

[21] Note that in contrast to the pay-as-you-go social security system, an infinite time horizon would not be necessary to capture the efficiency gains from knowledge codification.

codification can only be realized with some institutional frame or a constitution that cannot be violated by single generations. In this way, it seems that although such a Pareto-improvement would make no generation worse off, it cannot be enforced more easily than the command optimum.

With respect to research and development the usual policy implications could be applied to Pareto-improve the sequential markets equilibrium. More precisely, at least under the conditions that both, the wages and the return on capital are positive functions of the knowledge stock, the young generation of a period t would be willing to accept an income tax to subsidize research or intermediates, respectively. However, the tax would only be agreed upon, if the corresponding loss in utility in t is lower than the gain from additional knowledge spillovers in $t+1$.

Note, that in the first part of this section, the Pareto-improvement by a transfer of resources from young to old was due to the usual dynamic inefficiency as it may exist in overlapping generations economies with finitely lived agents. The second transfer mechanism leads to a Pareto-improvement by capturing efficiency potentials of the positive externalities of knowledge codification which are not realized in the sequential markets equilibrium.

12.3 Summary

This chapter discussed the welfare aspects of the sequential markets equilibrium with a particular focus on the role played by knowledge codification. It started out with a comparison to the command optimum which is characterized by a social planner maximizing a social welfare function. As prevalent in the literature, social welfare was defined by the discounted sum of each generation's utility. The potential social inefficiencies of the sequential markets equilibrium as compared to the welfare maximum arise from the market imperfection of the intermediate goods market(s), the equilibrium distribution of consumption, inefficient investments in knowledge codification and in research and development. With regard to knowledge codification the inefficiencies originate from a finite horizon effect, meaning that the short lived decision maker does not take into account the positive externalities on future generations from direct access to the respective information and from indirectly increasing exogenous spillovers. In the case of $\Phi = 1$, an additional externality that corresponds to the R&D-effect of the standard endogenous growth models can be identified. Subadditivity of codification costs would bear a static inefficiency in an oligopolistic structure such as that of chapter 10, as the social planner would then consolidate the firms' knowledge codification activities.
Finally, the potential sources of inefficiency with respect to research and development are those well known from the literature such as the R&D-effect, also a finite horizon effect due to the short lifetimes and research duplication in the oligopolistic structure of the intermediate goods markets.

For the case $\Phi = 0$ it is reasonable to expect the command optimum to exhibit a higher level of information in the long run as compared to the stationary state level resulting in the sequential market equilibrium, due to the positive externalities realized by the future generations that are not taken into account in the sequential markets equilibrium. However, this would not generally carry over to a second best policy that is constrained in redistributing consumption from the old generation to the young. The reason is that according to proposition 8.4, knowledge codification may possess a negative effect on long run output for certain parameter values.

Similarly, for a specification of research by $\Phi = 1$, the positive externalities of knowledge codification would suggest weakly higher codification efforts in each period in the command optimum as compared to the sequential markets equilibrium. However, such a conclusion is not unambiguous as the social planner faces a tradeoff with respect to creating spillovers between investing in research and in knowledge codification. This tradeoff has not been present in the sequential market equilibrium as the only way for the new capital owners to transfer additional knowledge to the next period was knowledge codification. In this way, it is not possible to preclude that there are periods of inefficiently high knowledge codification in the sequential markets equilibrium.

The second part of this chapter has asked whether it would be possible to achieve a Pareto-improvement to the sequential markets equilibrium allocation by a transfer scheme between the young and old generation of a period. This is possible if the overlapping generations economy is dynamically inefficient. As the model introduced in the first two parts of the present dissertation shows strong similarities to the Diamond model in case of zero knowledge codification, it is clear that it will be dynamically inefficient for some parameter values. The question has then been raised if an unambiguous statement would be possible as to whether the overlapping generations economy with positive knowledge codification in steady state is more or less likely to show dynamic inefficiency. On the one hand, higher knowledge codification in steady state implies a higher steady state growth rate of output. On the other hand, however, it also involves a higher return on capital. The magnitude of the effects do not show a unique order, such that the above question had to be answered negatively.

Next to Pareto-improvements due to dynamic inefficiencies, it may also be feasible to capture efficiency gains if the young generation of a period t would just codify another idea and is compensated for it by a transfer in the next period. This is always possible if the increase in the equilibrium wage by another piece of information is strong enough. In this situation, the transfer of knowledge by knowledge codification in the sequential markets equilibrium shows the characteristics of a hold-up problem.

12A Appendix of Chapter 12

In order to derive the necessary conditions for a welfare optimum, we rewrite the objective function

$$\begin{aligned} W &= \delta u(c_{2,1}) + \sum_{t=1}^{\infty} \delta_s^t [u(c_{1,t}) + \delta u(c_{2,t+1})] \\ \text{as} \quad W &= \sum_{t=1}^{\infty} \delta_s^t [u(c_{1,t}) + \frac{\delta}{\delta_s} u(c_{2,t})]. \end{aligned}$$

Let $\hat{\delta} := \frac{\delta}{\delta_s}$. The program of the social planner in the command optimum can then be solved via the following Bellman equation:

$$\begin{aligned} V_t(K_t, \tau_{t-1}, C_t) = \; & \max_{cv} [u(c_{1,t}) + \hat{\delta} u(c_{2,t})] + \delta_s V_{t+1}(K_{t+1}, \tau_t, C_{t+1}) \\ & +\lambda_{1,t}(F_t(K_t, L_{A,t}, \tau_t) - c_{1,t} - c_{2,t} - \gamma_t \triangle C_t - K_{t+1} - K_t) \\ & +\lambda_{2,t}(q\tau_{t-1} + \beta \max\{0, C_t - q\tau_{t-1}\} \\ & \qquad +\varepsilon \left[q\tau_{t-1} + \tilde{\Phi}(\beta \max\{0, C_t - q\tau_{t-1}\}) \right]^{\Phi} L_{R,t} - \tau_t) \\ & +\lambda_{3,t}(C_{t-1} + \triangle C_{t-1} - C_t) \\ & +\lambda_{4,t}(L_{A,t} + L_{R,t} - L_t) \\ & +\lambda_{5,t}((1-q)\tau_t - \triangle C_{e,t}), \end{aligned}$$

where $cv := c_{1,t}, c_{2,t}, L_{A,t}, L_{R,t}, \triangle C_{e,t}, K_{t+1}, \tau_t, C_{t+1}$, and initial values $K_1, \tau_0 > 0$, $C_1 \geq 0$. This leads to the following Kuhn-Tucker first order conditions for a global maximum:

$$\frac{\partial V_t}{\partial c_{1,t}} = u'(c_{1,t}) - \lambda_{1,t} = 0; \tag{12A.1}$$

$$\frac{\partial V_t}{\partial c_{2,t}} = \hat{\delta} u'(c_{2,t}) - \lambda_{1,t} = 0; \tag{12A.2}$$

$$\frac{\partial V_t}{\partial \triangle C_{e,t}} = -\gamma_t \lambda_{1,t} + \lambda_{3,t} - \lambda_{5,t} = 0; \tag{12A.3}$$

$$\frac{\partial V_t}{\partial L_{A,t}} = \frac{\partial F_t}{\partial L_{A,t}} \lambda_{1,t} + \lambda_{4,t} = 0; \tag{12A.4}$$

$$\frac{\partial V_t}{\partial L_{R,t}} = \varepsilon \left[q\tau_{t-1} + \tilde{\Phi}(\beta \max\{0, C_t - q\tau_{t-1}\}) \right]^{\Phi} \lambda_{2,t} + \lambda_{4,t} = 0; \tag{12A.5}$$

$$\frac{\partial V_t}{\partial K_{t+1}} = \delta_s \frac{\partial V_{t+1}}{\partial K_{t+1}} - \lambda_{1,t} = 0; \tag{12A.6}$$

$$\frac{\partial V_t}{\partial \tau_t} = \delta_s \frac{\partial V_{t+1}}{\partial \tau_t} + \lambda_{1,t} \frac{\partial F_t}{\partial \tau_t} - \lambda_{2,t} = 0; \tag{12A.7}$$

$$\frac{\partial V_t}{\partial C_{t+1}} = \delta_s \frac{\partial V_{t+1}}{\partial C_{t+1}} - \lambda_{3,t} = 0; \tag{12A.8}$$

$$\frac{\partial V_t}{\partial \lambda_{1,t}} = F_t(K_t, L_{A,t}, \tau_t) - c_{1,t} - c_{2,t} - \gamma_t \triangle C_t - K_{t+1} - K_t = 0; \tag{12A.9}$$

$$\begin{aligned}\frac{\partial V_t}{\partial \lambda_{2,t}} &= q\tau_{t-1} + \beta \max\{0, C_t - q\tau_{t-1}\} \\ &\quad + \varepsilon \left[q\tau_{t-1} + \tilde{\Phi}(\beta \max\{0, C_t - q\tau_{t-1}\}) \right]^{\Phi} L_{R,t} - \tau_t = 0; \qquad (12A.10)\\ \frac{\partial V_t}{\partial \lambda_{3,t}} &= C_{t-1} + \triangle C_t - C_t = 0; \qquad (12A.11)\\ \frac{\partial V_t}{\partial \lambda_{4,t}} &= L_{A,t} + L_{R,t} - L_t = 0; \qquad (12A.12)\\ \frac{\partial V_t}{\partial \lambda_{5,t}} &= (1-q)\tau_t - \triangle C_{e,t} \geq 0 \quad \text{with } \lambda_{5,t}[(1-q)\tau_t - \triangle C_{e,t}] = 0. \qquad (12A.13)\end{aligned}$$

From (12A.1), (12A.2) and (12A.6) follows that with respect to consumption and capital saving, optimality requires

$$u'(c_{1,t}) = \hat{\delta} u'(c_{2,t}) = \delta_s \frac{\partial V_{t+1}}{\partial K_{t+1}} \overset{(12A.15)}{=} \gamma_t^{-1} \delta_s \frac{\partial V_{t+1}}{\partial C_{t+1}}. \qquad (12A.14)$$

Further, with respect to knowledge codification, suppose $\lambda_{5,t} = 0$, that is an inner solution for $\triangle C_{e,t}$. Then we receive from (12A.3), (12A.1) that

$$\gamma_t u'(c_{1,t}) = \delta_s \frac{\partial V_{t+1}}{\partial C_{t+1}}. \qquad (12A.15)$$

Equations (12A.4), (12A.5), (12A.1) together with (12A.7) yield the following optimality condition with respect to the allocation of labor:

$$\varepsilon \left[q\tau_{t-1} + \tilde{\Phi}(\beta \max\{0, C_t - q\tau_{t-1}\}) \right]^{\Phi} \left[\delta_s \frac{\partial V_{t+1}}{\partial \tau_t} + u'(c_{1,t}) \frac{\partial F_t}{\partial \tau_t} \right] = u'(c_{1,t}) \frac{\partial F_t}{\partial L_{A,t}}. \qquad (12A.16)$$

Using the envelope theorem gives

$$\begin{aligned}\frac{\partial V_t}{\partial \tau_{t-1}} &= \lambda_{2,t} \frac{\partial \tau_t}{\partial \tau_{t-1}}, \qquad (12A.17)\\ \frac{\partial V_t}{\partial K_t} &= \lambda_{1,t} \left(\frac{\partial F_t}{\partial K_t} + 1 \right), \qquad (12A.18)\\ \frac{\partial V_t}{\partial C_t} &= \lambda_{3,t} + \lambda_{2,t}\beta, \qquad (12A.19)\end{aligned}$$

and consequently

$$\begin{aligned}\frac{\partial V_{t+1}}{\partial \tau_t} &= \left(\delta_s \frac{\partial V_{t+2}}{\partial \tau_{t+1}} + u'(c_{1,t+1}) \frac{\partial F_{t+1}}{\partial \tau_{t+1}} \right) \frac{\partial \tau_{t+1}}{\partial \tau_t}, \qquad (12A.20)\\ \frac{\partial V_{t+1}}{\partial K_{t+1}} &= u'(c_{1,t+1}) \left(\frac{\partial F_{t+1}}{\partial K_{t+1}} + 1 \right), \qquad (12A.21)\\ \frac{\partial V_{t+1}}{\partial C_{t+1}} &= \delta_s \left(\frac{\partial V_{t+2}}{\partial C_{t+2}} + \beta \frac{\partial V_{t+2}}{\partial \tau_{t+1}} \right) + \beta u'(c_{1,t+1}) \frac{\partial F_{t+1}}{\partial \tau_{t+1}}. \qquad (12A.22)\end{aligned}$$

With respect to the optimal investment in physical capital and knowledge codification, the following equation taken from (12A.14),

$$\delta_s \frac{\partial V_{t+1}}{\partial K_{t+1}} \overset{(12A.15)}{=} \gamma_t^{-1} \delta_s \frac{\partial V_{t+1}}{\partial C_{t+1}}$$

can be rewritten as

$$\gamma_t u'(c_{1,t+1}) \left(\frac{\partial F_{t+1}}{\partial K_{t+1}} + 1 \right) = \beta u'(c_{1,t+1}) \frac{\partial F_{t+1}}{\partial \tau_{t+1}} + \delta_s \left(\frac{\partial V_{t+2}}{\partial C_{t+2}} + \beta \frac{\partial V_{t+2}}{\partial \tau_{t+1}} \right). \qquad (12A.23)$$

Concerning consumption it follows that

$$u'(c_{1,t}) = \delta u'(c_{2,t}) = \delta_s u'(c_{1,t+1}) \left(\frac{\partial F_{t+1}}{\partial K_{t+1}} + 1 \right). \qquad (12A.24)$$

Finally (12A.16) can be transformed to

$$\varepsilon \left[q\tau_{t-1} + \tilde{\Phi}(\beta \max\{0, C_t - q\tau_{t-1}\}) \right]^{\Phi} \left[u'(c_{1,t}) \tfrac{\partial F}{\partial \tau_t} + \delta_s \left(\delta_s \tfrac{\partial V_{t+2}}{\partial \tau_{t+1}} + u'(c_{1,t+1}) \tfrac{\partial F_{t+1}}{\partial \tau_{t+1}} \right) \tfrac{\partial \tau_{t+1}}{\partial \tau_t} \right]$$
$$= u'(c_{1,t}) \frac{\partial F}{\partial L_{A,t}} \qquad (12A.25)$$

Chapter 13

Final Conclusions

The present thesis has developed a formal model of endogenous growth that incorporated endogenous knowledge codification as a means of intergenerational knowledge transfer. The motivation was that in contrast to the usual models of endogenous growth, which treat knowledge codification as a by-product of research and development activities, great efforts of private firms for the purposeful codification of knowledge can be observed.

At first, the underlying economic problem of intergenerational knowledge transfer by knowledge codification has been identified as exhibiting the characteristics of a hold-up problem in which a retiring employee may not be compensated for her codification efforts. This problem can be attenuated within a firm, in which the firm owners, rather than the retiring employees, may have an incentive for knowledge codification as it increases their profits. The knowledge transfer problem has been modelled to occur within long-lived intermediate firms of a two-sector overlapping generations framework. The codification decision was interpreted as a collective decision of the capital owners when they take over the firm. In this way, each generation's utility maximization problem possesses two channels to transfer income to the next period. One is by saving in physical capital and the other is investing in knowledge codification. This tradeoff drives the main results.

Accordingly, a first result derived from the basic model as specified in the first part of the thesis was that an overlapping generations economy developing from low output levels will not be codifying at the beginning. This was due to the Inada conditions applying to physical capital saving, but not to investment in knowledge codification. This result holds true for both specifications with respect to the research process that have been examined, that is, in the case where each researcher produces a constant amount of ideas per period and where the research productivity is increasing linearly in the transferred knowledge stock. The other results differ in the assumptions on research as the latter imply different long-run dynamics. In the specification with a constant number of ideas per researcher and period, the information stock plays a more pronounced role and is a central determinant of the economy's long-run output level. It has been shown that knowledge codification may even cause an initially less sophisticated but more patient overlapping generations economy, that is, an economy with a lower stationary state level of knowledge which is populated by individuals with a lower rate of time preference, to realize a higher stationary output level in the long run than an initially more sophisticated one. This effect occurs when the codification costs were within a certain range for a sufficiently long time. Even if thereafter codification costs would increase again, the different long-run output levels would persist due to the assumed longevity of the information stock.

The longevity assumption plays a minor role in an overlapping generations economy with a 'standing on the shoulders of giants' specification of the research process. In this case, it is the growth rates of capital, knowledge and codification costs that determine the economy's long-run behavior with respect to knowledge codification. A major result was that with constant costs of knowledge codification over time, the overlapping generations economy will be codifying in the long run if the steady state growth rate of capital is higher than that of the knowledge stock. The intuition is that with the capital stock increasing at a higher rate than the knowledge stock, the marginal productivity of knowledge increases relative to that of capital and it will become worthwhile to forgo consumption for preserving ideas by knowledge codification. Allowing for monotonically declining codification costs over time, the analysis showed that the steady state growth rate of knowledge exceeding that of capital by less than the rate at which the codification costs decline is sufficient for long-run knowledge codification.

In these cases, entry costs to knowledge codification that have to be paid once in the first period of codification such as set-up costs for computer networks or libraries and the like, would not prevent but only delay an economy's knowledge codification. This result changes when considering the rather hypothetical assumption that before an idea can be codified, the ideas it builds upon must be codified in advance. In this case, the entry costs are increasing with every period that shows no knowledge codification. It then depends on the particular form of the individuals' utility function whether the overlapping generations economy would still show knowledge codification in the long term.

As one would expect, in the basic model knowledge codification positively influences an economy's output level, respectively, its growth rate. When relaxing the assumption of Solow-neutral technological progress, this may not be the case anymore, as shown in chapter 8 of the second part. The reason is that knowledge codification affects the marginal productivities in research and final goods production differently, which can, under certain parameter values, lower the equilibrium wage. In the case with constant ideas per researcher, this may imply that savings in physical capital decline and with it the economy's total output level. For an overlapping generations economy with a 'standing on the shoulders of giants' specification of the research process it is possible that this effect leads to a declining growth rate of output in knowledge codification. A shift of labor from research to final goods production can be avoided in the 'standing on the shoulders of giants' case by making the assumption that the research productivity also increases in the amount of knowledge codification. In this case, the common intuition of knowledge codification to increase the economy's growth rate would be correct.

Another important finding of the second part of the thesis was that the simplifying assumption of only one intermediate sector that consists of a single intermediate firm is not necessary for the main results. With some symmetry assumptions the general model structure can also account for oligopolistic competition in multiple intermediate sectors, each founded on a different field of knowledge. However, it is necessary that the decision maker with respect to knowledge codification possesses positive measure.

The latter would be violated in Romer's model of endogenous technological change. For this reason, a result of the first chapter of the third part of the thesis was that private knowledge codification cannot be explained within this model. However, a comparison of the model structure introduced in the first two parts of the thesis and the Romer model showed large structural similarity. In particular, when interpreting the Romer model in an overlapping generations framework and assuming imperfect knowledge transfer as well as patent rights only for the professional life of a researcher, it exhibits the general properties of the model introduced in the thesis. The propositions on long-run knowledge codification would apply when allowing for a collective decision on knowledge codification for example within a state.

The last chapter of the third part has discussed the welfare aspects of the sequential markets equilibrium. Two perspectives were taken up. First, potential inefficiencies of the market solution as compared to a command optimum in which a social planner maximizes a social welfare function have been identified. They arise from market imperfections of the intermediate goods market(s), the equilibrium distribution of consumption, inefficient investments in knowledge codification and in research and development.

With regard to knowledge codification, the inefficiencies originate from a finite horizon effect, meaning that the short lived decision maker does not take into account the positive externalities on future generations from direct access to information and from indirectly increasing exogenous spillovers. In the case of the 'standing on the shoulders of giants' specification of the research process, an additional externality that corresponds to the R&D-effect of the standard endogenous growth models can be identified. Subadditivity of codification costs would bear a static inefficiency in an oligopolistic structure such as that of chapter 10, as the social planner would then consolidate the firms' knowledge codification activities.

It has been argued that in the specification with a constant amount of ideas per researcher, the command optimum should show a shift of knowledge codification activities to earlier periods and a higher stock of codified ideas as compared to the sequential market equilibrium in order to fully exploit the positive externatilities of information. Using a 'standing on the shoulders of giants' specification of the research process, an intuitively reasonable conjecture would be that the social planner always prefers weakly higher levels of knowledge codification than in the sequential markets solution. However, this may not be correct as with respect to knowledge spillovers between generations, the social planner faces an additional tradeoff between research and knowledge codification that has not been present in the sequential markets equilibrium as the only way for the new capital owners to transfer additional knowledge to the next period was knowledge codification. For this reason, it could not be precluded that there are periods in which the sequential markets equilibrium shows inefficiently high knowledge codification.

With respect to a second best policy, for example one that only considers measures with respect to knowledge codification, the possibly negative effects of knowledge codification on long-run development as identified in the second part of the thesis have to be kept in mind. It is not possible to generally argue for policy measures implying more knowledge codification, hence.

The command optimum may make certain generations worse off for the benefit of others. This leads to the second perspective with respect to social welfare which asked as to whether it would be possible to achieve a Pareto-improvement by a transfer scheme between the young and old generation of a period. Such an improvement is possible if the overlapping generations economy is dynamically inefficient. In the context of this work, no unambiguous statement could be derived as to whether an economy with positive knowledge codification in steady state is more or less likely to show dynamic inefficiency. However, it may be feasible to capture efficiency gains if the young generation of a period would just codify another idea and is compensated for it by a transfer in the next period. This is possible if the increase in the equilibrium wage by another piece of information is strong enough. This efficiency gain would originate from a (partial) solution to the hold-up problem of knowledge codification as identified at the beginning of the thesis. Within the model's perspective, as the generation to benefit from the additional idea to be codified is not yet born at the time of the actual codification process, it seems that this Pareto-improvement can only be realized with some institutional frame or a constitution that cannot be violated by single generations. In this way, although making no generation worse off, such a transfer scheme cannot be enforced more easily than the command optimum.

In the context of the present thesis, there are several avenues for future research.

With respect to the framework developed in the first two parts of the dissertation, further research should focus on the microeconomic structure of the intermediate sector. For example, it would be desirable to explicitly incorporate an R&D-process whose outcome is a stochastic quality improvement as in the quality ladder models of Aghion and Howitt (1992) or Grossman and Helpman (1991). Knowledge codification of a leading firm could then be motivated by preserving a technologically advanced position in the market.

Next to stochastic idea generation it is plausible to think of the amount of ideas that are lost in the transfer between generations as a random variable. It seems that in a stochastic environment the special characteristics of a long-lived information stock as compared to human capital are more pronounced. In particular, the information stock would not so much be subject to fluctuations as human capital, in this way representing an important pillar of economic development.

A research question could also be how efficiency gains from eliminating the static inefficiency of knowledge codification in an oligopolistic market structure due to the subadditivity of codification costs can be realized. In this respect, a problem may arise in the "real world" from the firms' considering their knowledge stock as a firm secret that they do not want to share with competitors, although they may all possess similar technological levels. Possibly in response to that, the European Commission plans to establish an information clearing house in order to avoid the replication of information that already exists on the internet. The clearing house is supposed to be a broker for information access rather than an all-encompassing archive (European Commission, 2001). However, there are also private initiatives for a common information pool, such as in the oil and gas industry, where the Society of Petroleum Engineers has been founded with the mission to capture and disseminate technical knowledge to its members. The society is sponsored by the oil and gas companies and also wants to respond to large expected numbers of retirements in connection with declining numbers of graduates with a degree in petroleum engineering.[1] It would be interesting to additionally consider such interactions between firms in a model with knowledge codification.

The present thesis has analyzed the intergenerational knowledge transfer within firms by purposeful knowledge codification. Some other areas in which economically relevant knowledge is codified have been mentioned in the introduction, such as the scientific sector. Knowledge codification is also important for knowledge transfer within generations, for example in the context of technology transfer between firms or countries. Furthermore, with respect to biotechnology, there is a vivid discussion on establishing genetic libraries. In these areas, knowledge codification bears different incentives and tradeoffs that may be worthwhile to examine.

In general, the transfer of knowledge between and within generations is at the heart of economic growth theory and knowledge codification plays a central role for it. In this way, the economics of knowledge codification remain an interesting, yet complex, field for future research.

[1] Information available at www.spe.org.

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