Monetary and Fiscal Policies in Currency Board Systems and Monetary Unions

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Chapter 1

Introduction

1.1 The Theme

In recent years, two monetary regimes have gained great importance, namely currency board arrangements and monetary unions: Currency boards reemerged in the 1990s and the European Monetary Union started in 1999.

Both exchange-rate systems have in common that the participating countries give up monetary independence, but retain fiscal independence. Therefore, fiscal policies remain the only instruments to offset idiosyncratic shocks and to follow country-specific targets. The arising conflict for fiscal authorities is whether to exercise a more aggressive fiscal policy in order to reach the domestic output goals, or to be disciplined to help monetary policy to achieve its objectives.

The main objective to operate a currency board system is to establish credibility and stability during transition periods and to overcome hyperinflations and currency crises by adopting an exchange-rate regime based on a strict rule. The objective of the monetary policy of the European Central Bank (ECB) is to ensure a stable money within the European Monetary Union and to ease market integration and political integration for the thirteen countries that have already abandoned national monetary discretion and joined the Euro-area.

The main theme of this dissertation is whether currency boards achieve their objectives and how fiscal policy will and should operate under both monetary systems, a currency board and a monetary union. The dissertation is divided into two parts. In the first part, we examine:

- the credibility and stability of currency boards, and compare the credibility of currency board arrangements to the credibility of standard fixed exchange-rate systems,

- fiscal policy under currency boards, and analyze whether currency boards induce more fiscal discipline compared to standard fixed exchange-rate systems, and
- how fiscal variables, i.e. the budget deficit and public debts, affect the stability of currency board systems.

In the second part, we consider a heterogeneous monetary union and analyze:

- which target function is socially optimal for individual fiscal authorities and for the common central bank in a microeconomic model,
- the interplay between fiscal and monetary policies, while we allow for non-cooperative and cooperative behavior, as well as for simultaneous and sequential decision-making of fiscal and monetary authorities.

We give a detailed outline of the thesis in the following.

1.2 Structure of the Thesis

1.2.1 Part I: Credibility and Stability of a Currency Board

Functioning of Currency Boards (Chapter 2)

In chapter 2, we characterize a currency board system and highlight the main differences of a currency board and a standard peg. We also sum up the advantages and disadvantages of operating a currency board system and elaborate the prerequisites for success in the intermediate-term.

The Credibility of Currency Boards (Chapter 3)

In chapter 3, which is based on Feuerstein and Grimm (2006b), we compare the credibility of currency boards and standard fixed exchange-rate regimes.[1] Abandoning a currency board requires a time-consuming legislative process and an abolition can, thus, never catch the public by surprise. A currency board, therefore, solves the time inconsistency problem of monetary policy. Policy can, however, react to unexpected shocks only

[1] The thesis is structured cumulatively and, therefore, repetitions cannot be excluded, completely. Especially, in chapter 4 the reader will come across several recapitulations.

with a time lag and, thus, the threat of large shocks makes the abolition necessary. We show in a theoretical model that currency boards are more credible than standard pegs if the time inconsistency problem dominates. In contrast, standard pegs, which can be left at short notice, are more credible if exogenous shocks are highly volatile and constitute the dominant problem.

Fiscal Policy and Stability of Currency Boards (Chapter 4)

Chapter 4 analyzes fiscal policy under a currency board. We give an overview of past and present currency systems and show that modern currency board systems have been stable arrangements associated with low inflation, stable economic growth and fiscal discipline. We claim that this was also true for Argentina in the first years operating the currency board system. In this context, we discuss the possible causes for the Argentinean crisis and the abandonment of the currency board system in 2001. We, furthermore, give a brief survey of several empirical studies that confirm a central finding of our theoretical analysis that currency boards are associated with higher fiscal discipline compared to standard peg regimes.

In the theoretical part we analyze, first, fiscal policy under a currency board system and under a standard fixed exchange-rate system, and show that a currency board induces higher fiscal discipline.

Second, we examine the stability of a currency board system. To relate this to empiricism, modern currency board systems have proven to be rather stable: Hong-Kong has been operating a currency board since 20 years, Estonia and Lithuania for more than ten years, and Bosnia and Bulgaria for almost ten years without a realignment of the central rate.[2] In our theoretical framework, we determine the factors which increase or lessen the stability of a currency board. We also compare the credibility of a currency board and a standard peg with respect to the probability that the particular system is maintained in the future period. We use a similar proceeding as in chapter 3, but incorporate the role of government expenditure and public debts into our analysis.

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[2]Lithuania switched from pegging its currency against the dollar to peg its currency against the Euro in 2002. This was necessary for participating in the (Exchange Rate Mechanism) ERM II which belongs to the Maastricht criteria that have to be fulfilled to adopt the Euro.

1.2.2 Part II: Monetary and Fiscal Policies in a Monetary Union

Part II of the thesis examines strategic interactions of fiscal and monetary policies in a monetary union; it consists of four interdependent chapters and is based on Grimm and Ried (2006).

After the introduction in chapter 5, the basic framework for the policy analysis is derived by using a microeconomic model in chapter 6. For a monetary union which comprises two regions, we determine the region-specific output and inflation equations. Moreover, we derive the region-specific and union-wide welfare function from microeconomic foundations. In particular, we show that fiscal policies should aim at regional inflation and output targets, whereas the common monetary policy should focus on maximizing union-wide welfare.

In chapter 7, we analyze the interplay between fiscal and monetary policies for several scenarios: We distinguish whether monetary and fiscal authorities decide upon their policies simultaneously, or whether one policy moves first. We, furthermore, distinguish between independently acting fiscal and monetary policies, cooperation of fiscal policies and joint cooperation of all policy makers. We analyze the different scenarios for two types of fiscal policies, namely supply-side and demand-side fiscal policies.

The results are obtained by using a simulation approach. We show that the more heterogeneous the monetary union is, the higher are the welfare losses. Heterogeneities also hamper cooperation of all policy makers. Cooperation, however, leads to the lowest welfare losses, if all policy makers agree upon inflation and output targets which are close to or equal the socially optimal levels. We discuss that such an agreement seems to be implausible in a heterogeneous monetary union and show that monetary leadership is the second-best solution for both types of fiscal policies. Chapter 8 concludes the second part of the thesis.

Part I

Credibility and Stability of Currency Board Systems

Chapter 2

Functioning of Currency Boards

This chapter gives an overview of the features and characteristics of currency board systems. First, we show the characteristics of an *orthodox currency board system*, and identify briefly how *modern* currency board systems differ from the orthodox type. Second, we examine the pros and cons of a currency board system. Third, we highlight which prerequisites a country should have for operating a currency board arrangement. Fourth, we give a summary and state under which conditions a currency board system may be an adequate exchange-rate system.

2.1 Features of a Currency Board System

A currency board is a fixed exchange-rate system, where the exchange rate is pegged extremely tight to the anchor currency, i.e. it belongs to the classification of a "hard-peg"-regime.[1] The first currency board was introduced in the British colony Mauritius in 1849. Further British colonies introduced currency board systems in the second half of the 19th century. They reached their peak in the middle of the 20th century. The purpose was that the colonies have an own stable currency, which was fully backed by reserves of the anchor currency, the British Pound Sterling. Thus, the costly transport of Pounds from Britain to its colonies to issue money in the colonies and to replace destroyed notes and coins was no longer necessary to guarantee smooth payment transactions (Williamson,

[1] A hard peg regime is a fixed exchange-rate system with a credible commitment to vary the exchange rate within a very narrow margin or even a "zero-margin" to its anchor currency. Hence, it denies its monetary policy to access to devaluation as a policy instrument. Hard pegs comprise currency board systems, dollarization and monetary unions. See Fatas and Rose (2000) for a detailed discussion of hard peg regimes.

1995). At the same time, there was no room for any kind of independent monetary policy in the British colonies, meaning that the monetary base could only be changed by buying or selling the British Pound.[2] Furthermore, the colonies gained interest rate revenues from the liquid foreign reserve assets that were held in London. After the Second World War, currency boards disappeared as the former colonies became independent and established own central banks.[3]

Currency board systems reemerged at the beginning of the 1990s. The main motives for the countries, which introduced a currency board system were, on the one hand, to (re)gain stability after times of economic turmoil and hyperinflation and, on the other hand, to generate credibility during the transition process (Ho, 2002). A question which arises is, wherefrom a currency board system derives its credibility. To answer this question, we focus on its features in the following.

A clear definition of a currency board system is difficult to give, as the design of the existing currency board systems is quite different among each other. Williamson (1995) defines a currency board *"as a monetary institution that issues base money solely in exchange for foreign assets, specifically the reserve currency* (p.2)". We use a more specific definition and state that a currency board is an *establishment by law*, which covers the following issues:[4]

- *The exchange rate is pegged to the anchor currency without the existence of a certain fluctuation band.*
- *The monetary base is covered with at least 100% of foreign reserves (of the anchor currency).*
- *The currency board guarantees full convertibility.*

In this context, full convertibility means that the currency can be used for transactions with foreign countries without any restrictions. Furthermore, full convertibility contains

[2]Britain, furthermore, favored to introduce currency board systems in her colonies, because this exchange-rate regime gives not much autonomy away.

[3]Among economists there is a controversy whether a currency board "is a central bank or not". We do not want to unroll this question here. In this dissertation, a currency board is classified as a special form of a central bank, by pointing out that modern currency boards have also a limited leeway for making use of monetary policy instruments.

[4]Note that this definition is given by Feuerstein (2000) and is also used in Feuerstein and Grimm (2006b), a joint work to which chapter 3 is mainly referred to.

Figure 2.1: T-Accounts of a Currency Board and a Central Bank

A	Orthodox Currency Board P
Liquid reserve-currency assets	Cash
	(Deposits of Commercial Banks)
	Net worth

A	(Common) Central Bank P
Liquid reserve-currency assets	Cash
Domestic assets	(Deposits of Commercial Banks)
(Government Debts)	Net worth

Source: WILLIAMSON (1995), p. 3.

also the right to exchange the domestic currency for units of the anchor currency at the currency board for the official exchange-rate parity.[5]

We, henceforth, distinguish between an *orthodox currency board system*, which is defined by a strict, conservative interpretation of the rules, and a *modified currency board*, which contains some deviations from the classical blueprint (HO, 2002). All modern currency board systems differ from the orthodox interpretation by allowing some flexibility in certain aspects.[6]

To elaborate the differences of an orthodox currency board system and a common central bank, we compare the T-accounts of both institutions in figure 2.1. A currency board holds liquid reserve-currency assets to cover the issued amount of cash, i.e. the

[5]For a detailed overview of the criterion of convertibility and the criterion of reserve backing see e.g. JANSEN (2000).

[6]In theoretical examinations an orthodox currency board is usually considered. This is also done in chapter 3 and 4 of this dissertation.

notes and coins in circulation. Under an orthodox currency board assets comprise solely liquid reserves. In principal, a currency board can also hold deposits from commercial banks. However, in an orthodox currency board commercial banks hold usually their reserves completely in liquid assets denominated in the reserve currency. The monetary base exclusively depends on buying or selling foreign currency at the fixed rate. Hence, there is no room for independent monetary policy under a currency board, and the money supply is exogenously determined.

In contrast, a common central bank includes also domestic assets which comprise assets from domestic banks and government debts. Money supply under a common central bank is mainly controlled by open market operations, i.e. through buying assets from the commercial banks (with reverse transactions).[7] On the liability-side, we see that commercial banks hold their reserves at the central bank.

To summarize, a common central bank can do discretionary monetary policy, which is also possible to a certain degree when operating a standard peg regime.[8] On the contrary, under an orthodox currency board system no independent monetary policy is possible due to its strict rule-based definition.

Feuerstein (2000) points out that one important role that monetary authorities occupy is to act as a *lender of last resort*. This means that the central bank grant loans on illiquid but solvent banks to prevent a bank run. This is, of course, not possible under an orthodox currency board system.

Therefore, modern currency board systems deviate in their design from the orthodox type to establish some political leeway. Feuerstein (2000) shows that all modern currency boards take a minimum reserve from commercial banks, which leaves some restricted room to use independent monetary policy. Furthermore, a modified currency board can grant loans to solvent but illiquid banks if the 100% reserve-backing of the monetary base is maintained, which means that a limited intervention as lender of last resort is also possible under a currency board. Monetary reserves can also comprise loans granted from the International Monetary Fund (IMF).[9]

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[7] To explain the difference of a common central bank and a currency board system, we give here only a very rough exhibition about the functioning of money supply under a common central bank. For details see European Central Bank (2004).

[8] The term *standard fixed exchange-rate system* comprises all fixed-exchange rate systems which do not belong to the classifications of a hard peg regime. Examples for standard pegs are fixed exchange-rate systems with a certain fluctuation band or crawling pegs.

[9] For details see Feuerstein (2000), p.230, and figure 2 on p.231.

2.2 Pros and Cons of a Currency Board System

The pros and cons of currency board systems are widely discussed in the economic literature. Thereby, currency boards are assessed quite differently. The reason for that may be found in the different experiences with currency board systems. On the one hand, Estonia, Lithuania, and Bulgaria have performed well on their way to the European Union and the European Monetary Union so far. On the other hand, the breakdown of the Argentinean currency board in 2002 raised some negative aftertaste.[10] We begin this section by showing the advantages of operating a currency board system.[11]

2.2.1 Advantages

a) **Convertibility and 100%-Reserve Backing**

The features of full convertibility and the backing of the monetary base by at least 100% reduces strongly the risk of a speculative attack and increases the stability and credibility of the exchange-rate peg: A currency board cannot break down because it is running out of reserves, but only if the economic cost of maintaining it exceeds the costs which arise when the currency board is reneged. We take up this point when analyzing the credibility of a currency board in a theoretical model in chapter 3.

b) **Fiscal Discipline**

A currency board induces fiscal discipline, as the currency board cannot finance government debts: The T-account shown in figure 2.1 implies that the central bank cannot buy government bonds, because the monetary base has to be fully backed by liquid foreign reserves. Furthermore, the strict rule of monetary policy under a currency board implicates that government deficits also cannot be devalued by creating inflation through open market operations and via other channels. Therefore, the government must finance its deficits by tax income and/or from borrowing

[10] We discuss the economic performance of modern currency board systems in chapter 4.

[11] Note that the borderline between the pros and cons differs among economists. Hanke (2002), for example, contradicts most of the negative arguments by using the unworthy notation of *"straw man arguments"*, (p.100). We point out that there exist drawbacks associated with operating a currency board system, but concede that in some cases the arguments can also be interpreted in the opposite direction if certain conditions hold.

abroad. This suggests that fiscal policy is likely to be disciplined, because excessive deficits are costly in real economic terms if financed by high taxes, and because the possibility of borrowing from abroad may be constrained in the case of excessive deficits (Schweikert, 1998).

In chapter 4 of this dissertation, we compare fiscal policies under a currency board system and a standard fixed exchange-rate regime using a theoretical model and confirm that a currency board system leads to more fiscal discipline than a standard peg.

c) **Establishment by Law**

A further important feature of a currency board system is its establishment by law. Brockmann and Keppler (2003) claim that the embodiment of the currency board mechanisms in the law implies that the cost of modifying some features or abandoning the currency board is relatively high compared to a standard peg. Hence, a currency board derives much of its credibility from this feature. Ho (2002) adds that the the written law makes a *"valuable contribution via its role as an information device"* (p.20), which means that the rules of the game become clear and transparent, and this contributes also to the credibility of a currency board system. In our theoretical analysis in the chapters 3 and 4, we go one step ahead and assume that the anchorage of the main currency board mechanisms by law makes a surprising abandonment implausible and solves, thus, the time inconsistency problem of monetary policy. This accounts for a sound environment to achieve low inflation rates and stable economic growth.

d) **Balance of Payments Equilibrating Effect**

A trade balance deficit leads to a reduction of money supply and induces an increase of the interest rates. This leads to a reduction of private demand (consumption and investments) and to falling prices under the fixed exchange rate, which contributes to an increase of exports as domestically produced goods become relatively more competitive. At the same time imports are reduced. Both effects improve the current account.

The mechanism is almost the same as the well-known *adjustment mechanism under the gold-standard*. Williamson (1995) associated this effect with a currency board system. We decided to mention this effect at the end of the list of advantages, as we state that if wage rigidities and price stickiness exist, the adjustment process to a balanced current account can possibly lead through an intermediate-term recession, which may even put some pressure on the currency board.

2.2.2 Disadvantages

a) **Lender of Last Resort**

When acting under an orthodox currency board system, the policy maker cannot act as a lender of last resort. This means that the banking sector is potentially vulnerable to a liquidity crisis: monetary policy cannot grant loans to solvent but illiquid banks to prevent bank runs which may induce high economic costs.

This is the reason for modern currency board systems to implement some political leeway as discussed in the previous section. Therefore, currency boards can also act as a lender of last resort, but only as long as the 100% coverage of the monetary base by foreign reserves is not violated.

Note that the limited possibility to act as a "lender of last resort" has also a silver lining: unfortunate bailouts of insolvent institutions are avoided and moral hazard is restricted (Dornbusch, 2001). Under a currency board a sound banking system is a necessary requirement for intermediate-term success. Furthermore, an open banking sector with international players is helpful, as subsidiary companies from international banks can usually access the liquidity of their mother banks. In this context, however, Caprio et al. (1996) annotated the problem that foreign banks are generally considered as disadvantaged compared to domestic banks in monitoring domestic investments, which may lead to slightly higher interest rates due to an increase of the risk premia.[12]

b) **Asymmetric Shocks**

A further disadvantage of a currency board is that the strict rules do not allow for monetary policy to react to asymmetric shocks. Honohan (1994) argues that asymmetric shocks will affect directly the prices and money demand, as policy makers cannot anticipate and offset economic shocks. If price rigidities and labor market frictions are present, it can take several periods until the shock is completely abated.

[12] A detailed discussion of the lender of last resort function under a currency board, which also includes a case study of Argentina, is found in Caprio et al. (1996).

In contrast, Dornbusch (2001) states that the importance of the nominal exchange rate to offset asymmetric shocks is overdone: *"Most disturbances are temporary rather than permanent"*, (p.239). This may be true for countries like e.g. Hong-Kong where prices and wages are relatively flexible in both directions. However, if price rigidities exist shocks may happen to persist for several periods. We take up this point in our model in chapter 3 and show that the degree of shock persistence, which can be interpreted as a certain degree of price stickiness, has a negative impact on the credibility of a currency board system.

c) **Procyclical Money Supply**

In the previous subsection we have already mentioned that money supply is exogenous under a currency board system. A problem which is likely to appear under a currency board is that the money supply behaves procyclical: In good times money flows in and the interest rate falls, which leads to more private consumption and investments and may contribute to an overheating of the economy. In bad times, the opposite is true: Money flows out, interest rates rise and private demand is crowded out, which makes a recession more severe (Williamson, 1995).

d) **Real Appreciation**

When the inflation rates after fixing the exchange rate under a currency board do not converge quickly and completely to the inflation rates of the anchor currency, the country experiences a real appreciation, which induces a loss of competitiveness. This problem occurs when the currency board is not completely credible (i.e. when the private sector assumes that it is repealed with a certain probability) or when the tax system is inefficient.[13]

e) **Start-up Problem**

Another problem, which is frequently discussed is the start-up problem, i.e. how to solve the question where the foreign reserves come from to issue the own currency and fulfill the 100% reserve-backing criterion. The present currency board arrangements did not have such problems: the former Soviet Republics, Estonia

[13]The inflation rates in the present currency board systems were strongly reduced within a short period after the introduction of the systems. However, a full convergence was reached only after several months. The performance of the present currency board systems is demonstrated in chapter 4.

and Lithuania, received gold transfers from wealth, which was transferred to London and Stockholm before the beginning of the Second World War. The rest of the currency board countries suffered from hyperinflations before the adoption of their currency boards, which reduced the real money supply strongly and increased, thereby, automatically the cover ratio (Feuerstein, 2000).

f) **Seigniorage Problem**

The last problem we highlight is the seigniorage problem: the currency board's 100% backing-rule requires that the monetary base is completely backed by liquid reserve-currency assets. The revenues of these assets are usually lower compared to that of a common central bank, which also holds higher interest-bearing domestic assets and government bonds as reserves.

Dornbusch (2001), however, considers this problem as subordinated. He states that the savings on debt services through lower interest rates due to the credible commitment of the currency board may compensate or even overcompensate the missed earnings from seigniorage.

2.3 Prerequisites for Success under a Currency Board

The discussion of the pros and cons of currency board systems has shown that a currency board can contribute to a gain in credibility by its strict rule-based monetary policy (also under a modified currency board system). This is especially helpful during a transition period where the policy makers do not have any reputation from a tough past. Currency board systems have also proven to be a useful remedy to overcome currency crises. However, the credibility of a currency board does not come per se and there exist many disadvantages from which a currency board country will possibly suffer. In this section, we name and discuss the requirements a country has to fulfill to operate a currency board system successfully in the intermediate-term or in the long-run.

We begin with the *degree of openness* which is an important feature that a currency board country should exhibit. An open economy is characterized by a relatively large relation of tradeable goods to all domestically produced goods. Therefore, it participates with a large fraction of its domestic production in the world market. Hence, exchange-rate uncertainty imposes large costs which can be reduced by operating a fixed exchange-rate system. This is particularly true with respect to currency boards due to their establishment by law and the strict rules. The reduction of exchange-rate uncertainty facilitates a lower price variability.

Furthermore, *labor market stickiness* and *inflation inertia* tend to be lower under an open economy compared to a relatively closed economy. This implicates that the nominal exchange rate is less important as a stabilization tool, meaning that asymmetric shocks have a lower persistence because they can be offset or mitigated more quickly by an adjustment of domestic prices and wages. As in most cases small countries are more open than larger countries, currency board systems are typically found in small, open economies.

To avoid large and frequent asymmetric shocks, the country to which the domestic currency is pegged has to show *similar business cycle movements*. Usually the business cycles of a country move very much synchronous to the business cycles of the country most traded with. Hence, it seems to be logical to peg one's currency to that of the most important trading partner. This idea is referred to MUNDELL's "*theory of optimum currency areas*" (MUNDELL, 1961).[14] For the reasons explained above, this criterion becomes even more important when price and wage rigidities exist.

One main motive of introducing a currency board system is to reduce inflation and interest rates to create a sound economic environment through a (re)gain of credibility. At the initial stage, however, inflation may possibly not converge immediately to the levels of the anchor currency and the domestic currency appreciates in real terms. This makes domestic products relatively more expensive compared to foreign goods and, thus, decreases competitiveness. KOPCKE (1999) proposes to fix the *initial exchange rate* (in price-notation) at a level somewhat *above the estimates of the equilibrium exchange rate* to reduce or totally circumvent this problem. Hence, the currency approaches to its real equilibrium level by the real appreciation during the initial stage.

A further aspect required for success under a currency board system is that fiscal policy has to be willing to accept the importance of *fiscal discipline*. In chapter 4, we show that fiscal policy is more restrictive under a currency board system when its target function contains output, inflation and debts. However, if policy makers focus on other goals, like e.g. the maximization of the re-election probability by an excessive provision of public goods or tax-cuts which are financed by foreign debts, the interest rates will rise and crowd out private investment. Such a behavior *"endangers the sustainability of government*

[14] We argue in chapter 4 that the choice of the US-$ as the anchor currency contributed to the breakdown of the Argentinean currency board in 2002. Argentina was far away from constituting an optimum currency area with the United States

finances, and eventually of the whole currency board arrangement" (STUKENBROCK, 2004, p.49). For guaranteeing long-term success, fiscal policy must accept the rules of the game and must endorse that monetary policy is independent from financing government debts.

Due to the missing or restricted lender of last resort-function, a *sound and liberalized banking sector* with the participation of foreign banks is also an important prerequisite. We have discussed this point already in the previous section.

2.4 Summary

So far, we have discussed the characteristics and the pros and cons of operating a currency board arrangement and we have highlighted the essential prerequisites for a country to run a currency board successfully over an intermediate or longer time-horizon.

We suggest that a currency board can be an eligible exchange-rate system for small, open transition countries to achieve high credibility and, thus, low inflation rates, which is a precondition for sustainable growth. In this context DORNBUSCH and GIAVAZZI (1999) claim that the central and eastern European countries lose nothing in giving up their monetary sovereignty. They even recommended that all accession countries should introduce a Euro-based currency board.

The introduction of a currency board arrangement can also be very effective to stabilize an economy suffering from a currency crisis or hyperinflations. In this context by its strict rule of monetary policy and its establishment by law, a currency board arrangement signals great commitment and helps to restore confidence. The small requirement of skills and staff also increases the attractiveness of currency boards further. This argument is especially applicable for transition countries without much experience in central banking.

Admittedly, a currency board system is no panacea as demonstrated by the prerequisites and by the disadvantages in the previous sections. In general, a successful implementation of the system needs to be combined with reforms and liberalization of labor and goods markets to increase flexibility, with reforms of the tax system, with establishing a sound banking regulation, and with a cooperative and disciplined fiscal policy.

KOPCKE (1999) sees a currency board system as an interim solution: *"Currency Boards represent a start, more than a destination for the design of monetary authorities. They can offer economies a temporary shield for cultivating reputable central banks and financial institutions"*, (p.36). We agree on this point and add that a currency board system seems to be an eligible transitory system. An unsolved question remains: How to make a smooth exit? This would only be possible if a government can credibly signal that it has done its homework and created a sound economic environment, so that there is no incentive to misuse monetary policy for the creation of surprise inflation or to devalue government debts. However, a case in which a country abandoned its currency board during good times does not exist.[15]

A currency board may also be acceptable in the long-run if it fulfills nearly all prerequisites at the same time. This can be seen from Hong-Kong, which have operated a currency board since 1983. The credibility of currency board systems is analyzed theoretically in chapter 3 and chapter 4 and an overview of the performance of modern currency boards is given in chapter 4.

[15]Estonia and Lithuania will adopt the Euro in the near future, but this is just a transformation from an unilateral ultimate fixed exchange-rate system pegged to the Euro to directly adopting the Euro. Therefore, this will not deliver further insights into our question.

Chapter 3

Credibility of Currency Boards

3.1 Introduction

During the crisis-prone decade of the 1990s, currency boards proved to be remarkably robust. Even in the case of the 2002 Argentinean currency board collapse, both the event itself and the durability of the arrangement in the face of such large strains, require an explanation. In this context, the question arises as to what constitutes the difference between a currency board and a standard peg system and under what circumstances does a currency board possess a credibility advantage.[1]

As demonstrated in chapter 2 a currency board is characterized by a fixed exchange rate to a stable anchor currency and full coverage of the monetary base by foreign reserves. It requires a long-term commitment by policy makers and is usually introduced by law (which also specifies the fixed exchange rate). The main advantage of a currency board is the gain in credibility. The monetary base is changed only through buying and selling the anchor currency at the fixed rate. Thus, the trilemma that it is not possible to maintain a fixed exchange rate, free movement of capital and an independent monetary policy at the same time is solved by clearly abstaining from monetary independence. Moreover, the time inconsistency problem of monetary policy is solved, as it is not possible for the monetary authorities to create surprise inflation. Calvo (2000, p.4) strengthens that view by suggesting that in emerging markets when sudden stops are possible, the emphasis should be on credibility, "where the central banker may have to tie himself to the mast or a currency board to command any respect".

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[1]On Argentina's experience with the currency board see De la Torre et. al. (2003), Gurtner (2004) and chapter 4.

The anti-inflationary effects of currency boards have been confirmed empirically. Ghosh et al. (2000) find that countries with a currency board experienced lower inflation compared both with floating regimes and with simple pegs. Other econometric studies that investigate the relevance of exchange rate regimes for economic performance pool currency boards and countries with a shared currency into the group of hard pegs. The result that countries with hard pegs have lower inflation rates than countries with soft pegs or other regimes is found unequivocally (Levy-Yeyati and Sturzenegger, 2001; Ghosh et al., 2003; Bleaney and Francisco, 2005). There is, however, mixed evidence whether soft pegs are also associated with lower inflation than floating regimes and whether the gain in stability comes at the cost of lower growth. While Ghosh et al. (2000) find that countries with a currency board experienced a higher growth, Levy-Yeyati and Sturzenegger (2001) conclude that there is a trade-off between inflation and growth also for hard pegs. In the study of Bleaney and Francisco (2005), the result of lower growth of hard peg countries vanishes when regional dummies are introduced. The reason is that a large share of the hard peg countries is located in Sub-Saharan Africa, the region with the lowest growth. The cited studies do not only vary in the period and the countries covered in the sample, but also in the exact classification of soft pegs and so-called intermediate regimes.

A new strand of the literature considers the transition between different exchange rate regimes Masson (2001), Masson and Ruge-Murcia (2005) and Husain et al. (2005).

Obviously, the gain in credibility of monetary policy relies on the credibility of the currency board itself, which is, of course, not complete. Following the seminal work of Drazen and Masson (1994), the credibility of an exchange rate regime is defined as the probability that it is maintained. A currency board does not break down because it runs out of reserves necessary to intervene on the foreign exchange market, as may be the case in a standard peg system. Nevertheless, it can be abolished if the costs of maintaining it – for example in case of a recession, a debt crisis or problems within the banking sector – exceed its advantages.

Although currency boards have been discussed thoroughly, there is little literature which analyzes the difference between a currency board and a standard fixed exchange-rate regime from a theoretical point of view.[2] One of the few papers which try to model

[2]Spiegel and Valderrama (2003) and Chan and Chen (2003) theoretically discuss specific aspects of currency boards. However, they do not distinguish between a normal peg and a currency board system. Spiegel and Valderrama (2003) characterize a currency board system by assuming that the costs of

this difference is Chang and Velasco (2000). They characterize a currency board by the full coverage of the monetary base, which excludes a run on the central bank's reserves that is possible in a standard peg. As Chang and Velasco consider opportunity costs of holding reserves, but do not model any possible disadvantages of flexible exchange rates, their conclusion that a flexible exchange rate is the optimal regime comes as no surprise. A different approach to capture the difference between a currency board and a standard peg is used by Oliva et al. (2001) who analyze whether monetary authorities can signal their preferences on price stability by choosing between these two exchange rate systems. They emphasize that a currency board constitutes a long term commitment and assume that it can only be abolished in the second period of their two period model, whereas with a standard peg, a realignment in response to a supply shock is possible in either period. In contrast, Irwin (2004) assumes that the policy maker can abolish a currency board without any time lag, and he characterizes a currency board only by its high exit costs. The central result is that "the combination of incomplete information and persistence of unemployment can lead to a build up of pressure on a currency board system to the extent that it does collapse, even where the true devaluation cost is very high".

In this model, we take up the idea of Oliva et al. (2001) that a currency board cannot be abolished overnight, but we model this feature more consequently. A currency board is established by law, and leaving it requires a political process including preceding public discussion. While Oliva et al. allow for a *sudden exit* out of the system in the *second* period, our model captures that it is hardly possible to generate a surprise, as repealing a currency board takes time. In this context, we claim that when Argentina finally left its currency board, this had been largely expected (see also chapter 4). The models also differ in the assumption on the exchange rate regime with respect to abandoning the system. Whereas Oliva et al. assume that the currency will devalue by an *exogenously* given amount after giving up the fixed exchange rate, in our model, the exchange rate will become flexible.

Therefore, in our model the currency board is characterized by the assumption that

abandoning the fixed exchange rate *decrease* over time, being high in the first period, whereas the system is always abolished in the second period. In this framework, they study the effect of dollarized liabilities on monetary policy and compare the results to the case of a free float. Chan and Chen (2003) consider the possibility of increasing the credibility of a currency board by introducing irrevocable commitments to sell foreign currency at the prespecified exchange rate up to its commitment level even if the currency board is abandoned. Such a commitment could be achieved by depositing part of the currency board's foreign reserves with a third party.

it can only be abandoned with a one period delay, in contrast to the case of a standard peg. The currency board can only be left if this was – at least implicitly – announced earlier. Of course, this is not to be understood literally in the sense that in the real world the exact date of the abolition will be announced. The point is, that the breakdown of the currency board will be expected and no surprise inflation can be created. Thus, when expectations on inflation are formed, it is public information whether the currency board will still be in place in the next period. Hence, the currency board completely solves the time inconsistency problem of monetary policy. Nevertheless, announcing the abolition of the currency board may make sense in case of a lasting misalignment.

Pressure to change the exchange rate emerges from asymmetric shocks that require an adjustment of the real exchange rate, i.e. from stochastic shocks on the purchasing power parity (*PPP*) (Berger et al., 2001). These shocks may for example arise from differing business cycles. They may also reflect exchange rate movements between the anchor currency and third countries. Although the Argentinean currency board ultimately collapsed because of unresolved budgetary problems, the sharp devaluation of the Brazilian Real in 1999 and the strength of the US-$ in the years 2000 until mid 2002 contributed to Argentina's difficulties and help to explain the timing of the breakdown.

The *PPP*-shocks are assumed to be autocorrelated, meaning that a shock has lasting effects. After observing the shock of the first period, the policy maker will decide whether to initiate the process of repealing the currency board. If the shock is large, he knows that the misalignment will continue with a high probability in the following period. However, if the currency board is abolished, the time inconsistency problem of monetary policy reemerges. In contrast, in a standard peg regime the policy maker can make use of an escape clause after observing the shock in each period. He can respond to a large shock, but he is also tempted to create surprise inflation. As a result, a currency board arrangement is more credible than a standard-peg regime, if the time inconsistency problem is dominant, whereas the peg is maintained with a higher probability, if the ability to react to future shocks is more important.

This chapter is organized as follows: in the next section we develop our two-period model. In section 3.3 we consider the regime of a floating exchange rate, which will be in operation if the fixed exchange rate is abolished. Section 3.4 analyzes the policy options under a currency board. We derive conditions under which the currency board will be maintained and show in which situations it will be abandoned. In section 3.5, the behavior of the policy maker in a standard fixed exchange-rate system is considered. Section 3.6 compares the credibility of the two fixed exchange-rate regimes, that are introduced in the previous sections. Section 3.7 summarizes the essential results of this chapter.

3.2 Basic Model

We consider a two-period model of a small open economy that has a time inconsistency problem of monetary policy modeled as in Kydland and Prescott (1977) and in Barro and Gordon (1983). In each period t, output y_t is given by a standard *Lucas supply function*

$$y_t = \gamma(\pi_t - \pi_t^e), \quad \gamma > 0. \tag{3.1}$$

Output depends on unanticipated inflation $(\pi_t - \pi_t^e)$, where π_t denotes inflation in period t and π_t^e is the inflation rate expected by the private sector. Expectations are formed rationally. Strictly speaking, y_t denotes the deviation of output from its natural level, i.e. the natural output level is normalized to zero. The inflation rate and the exchange rate are linked by the *stochastic purchasing power parity (PPP)*

$$\pi_t = \pi_t^* + e_t + \phi_t \ , \tag{3.2}$$

where π_t^* denotes foreign inflation, e_t the percentage change of the nominal exchange rate in period t, and ϕ_t is a random shock (Berger et al., 2001).
The shock ϕ_t is autocorrelated[3]

$$\phi_t = \eta\phi_{t-1} + u_t \ , \quad \eta \in (0,1) \ , \tag{3.3}$$

and it is assumed that initially there is no inherited shock, i.e. $\phi_1 = u_1$.

ϕ_t represents an asymmetric shock that changes the equilibrium real exchange rate reflecting for example differing business-cycles or exchange rate movements between the anchor currency and third countries. A positive ϕ_t corresponds to the necessity of a real appreciation, which can either be realized by an inflation rate exceeding foreign inflation or by a falling exchange rate. New shocks u_t are i.i.d. with $\mathrm{E}(u_t) = 0$ and $\mathrm{Var}(u_t) = \sigma_u^2$ for all t. In the sections 3.5 and 3.6, we will assume in addition that u_t is uniformly distributed on the interval $[-A, A]$.

Normalizing the foreign inflation π^* to zero, equation (3.2) can be rewritten as

$$\pi_t = e_t + \phi_t \ . \tag{3.4}$$

[3]Equivalently, we could assume that shocks ϕ_t are uncorrelated, but prices are sticky instead. This would mean that inflation reflects the current shock only partially, leaving some of the required adjustment for future periods. In this sense, the parameter η can be interpreted as a degree of price stickiness.

The monetary authorities' loss in period t is given by the function

$$L_t = (y_t - k)^2 + \theta\pi_t^2 + \delta c \ , \tag{3.5}$$

and depends on the deviation of output from its target level $k > 0$ and the actual inflation rate. The assumption that the policy makers' target output k is above the natural level can be interpreted as capturing distortions on goods and factor markets that lead to too low a natural level. The relative weight on inflation in the loss-function is given by θ. In addition, there are political costs c that arise when the fixed exchange rate is given up under a peg regime or when the currency board arrangement is abandoned. These political costs can either be interpreted as reputation costs or as costs caused by political institutions in society (Lohmann, 1992). δ is a dummy variable, which equals one when leaving the peg or the currency board and equals zero otherwise. Inserting the supply function and the PPP (equation 3.1 and 3.4) into the loss function (3.5) yields[4]

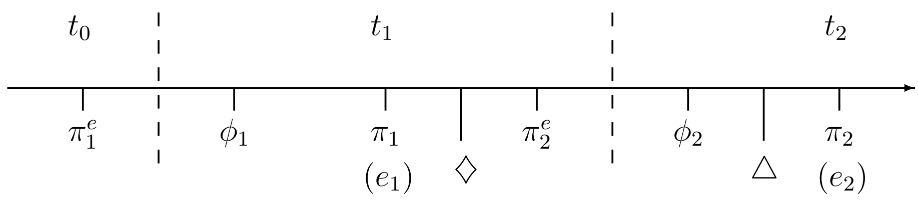
$$\begin{aligned} L_t &= (\gamma(\pi_t - \pi_t^e) - k)^2 + \theta\pi_t^2 + \delta c \\ &= (\gamma(e_t + \phi_t - \pi_t^e) - k)^2 + \theta(e_t + \phi_t)^2 + \delta c \ . \end{aligned} \tag{3.6}$$

The first term shows that the positive k leads to an incentive to create surprise inflation in order to push output above its natural level. According to the assumption of rational expectations, the private sector will take this incentive into account when forming its expectations. Therefore, the policy maker cannot generate a surprise and a time inconsistency problem of monetary policy arises, that is the larger the larger the target output k is. A high weight on inflation in the loss function θ, i.e. a high preference for price stability, mitigates the time inconsistency problem. The effect of these two parameters on the time inconsistency problem will be discussed again in section 3.3, where the results for a free float, i.e. for the case of unrestricted discretion of monetary policy, are derived.

In our model, the essential difference between a currency board and a standard peg lies in the procedure of abolishing the particular system. A currency board, characterized by its establishment by law and the complete renunciation of individual monetary policy,

[4]This type of loss function, i.e. the policy maker's objective to create a certain amount of surprise inflation although she dislikes inflation itself, can also be derived from public finance considerations. In Sachs et al. (1996), the policy maker is tempted to create unanticipated inflation to deflate the real value of outstanding government debt. An alternative interpretation given by Sachs et al. (1996) is that the inflation tax increases with actual inflation but falls with anticipated inflation, again giving rise to aiming at surprise inflation.

Figure 3.1: Sequence of the Model



can only be repealed, if this was announced one period in advance, i.e. before the private sector made its expectations on inflation. In contrast, monetary authorities can leave the fixed exchange rate in a far more flexible way after the shock was observed. The sequence of the model is depicted in figure 3.1. Expectations π_t^e have to be formed before the shock ϕ_t is observed. Actual inflation is determined by the purchasing power parity (equation 3.4) if the exchange rate is fixed, and it is optimally set by the policy maker in response to the shock if the exchange rate is flexible or the fixed exchange rate is abandoned. In our two-period model, the decision whether to repeal the currency board in the second period or not is announced in the first period at time $\diamondsuit$ before the private sector forms expectations π_2^e. In the case of a standard peg, the fixed exchange rate can be abandoned after observing the shock which would actually be possible in the first as well as in the second period. However, we assume that the fixed exchange rate is maintained in the first period and the decision whether to defend it or not is considered at time $\triangle$ in the second period. This could be justified by the fact that a standard peg has some commitment value, too, and cannot be abolished immediately after its adoption. The more important reason for making the assumption is, however, that a meaningful comparison of the credibility of a currency board and a standard peg has to be based on a *single* decision whether to give up the fixed exchange rate or not in both systems. If the currency board can only be abolished at one point of time, but for the standard peg abolition is considered both in period 1 and period 2, a statement that an abolition of the standard peg is more probable will be irrelevant.

3.3 Free Float Regime

In our model the process of repealing a currency board system takes time and has to be announced one period in advance. In this case, the policy maker will set the inflation rate (and thus the exchange rate) in period 2 optimally, and this policy will be taken into account when expectations are formed. This regime amounts to a free float in period 2 that will briefly be analyzed in this section.

Consider the loss in period 2 (equation 3.6)

$$L_2 = (\gamma(\pi_2 - \pi_2^e) - k)^2 + \theta \pi_2^2 \ .$$

The monetary authorities can freely choose inflation. By minimizing L_2, inflation in period 2 equals

$$\pi_2 = \frac{\gamma(\gamma \pi_2^e + k)}{\gamma^2 + \theta} \ . \tag{3.7}$$

As the private sector's expectations are rational, it follows that

$$\pi_2^e = E(\pi_2) = \frac{\gamma^2 \pi_2^e + \gamma k}{\gamma^2 + \theta} \ ,$$

and thus

$$\pi_2^e = \gamma \frac{k}{\theta} \ . \tag{3.8}$$

Using equations (3.1), (3.4) and (3.7) yields the equilibrium values for period t

$$\pi_2^f = \gamma \frac{k}{\theta}, \qquad e_2^f = \gamma \frac{k}{\theta} - \phi_2, \qquad y_2^f = 0 \ . \tag{3.9}$$

The superscript f denotes "free float". With flexible exchange rates, the PPP-shock ϕ_2 is fully absorbed by the change of the exchange rate e_2^f and by the inflation rate π_2^f being independent of ϕ_2. The actual inflation rate is proportional to k, which is the difference between the targeted and the natural output level. Although the policy maker wants to push output above its natural level, output y_2 is not affected. Obviously, the loss would be smaller if actual and expected inflation equaled zero. However, a zero-inflation policy would be time inconsistent, as with $\pi^e = 0$ a higher inflation rate would actually be chosen. Thus in case of a free float, when there is unrestricted discretion of monetary policy, the inflation rate will be inefficiently high, and this problem is the larger, the larger the parameter k. In contrast, a high weight θ on inflation in the policy makers' loss function makes the time inconsistency problem of monetary policy smaller. The two parameters k and θ will therefore be interpreted as determining the size of the time inconsistency problem in the remainder of this analysis. Of course, the size of the time inconsistency problem also depends on γ. But it is natural to focus the interpretation on the parameters occurring in the policy makers' loss function in this context. The aim to increase output above its natural level — reflected in $k > 0$ — is the basic cause for the time inconsistency problem, and θ measures the degree of inflation aversion.

The resulting loss in period 2 is given by

$$L_2^f = \frac{1}{\theta} k^2 (\gamma^2 + \theta) \ . \tag{3.10}$$

Note that L_2^f does not depend on the shock ϕ_2, but only on k, hence $E(L_2^f) = L_2^f$.

3.4 Policy Options under a Currency Board

The policy maker has to announce one period in advance (at time $\diamondsuit$, see figure 3.1) whether to maintain or to abolish the currency board in the next period. Thus, the decision depends on the expected second period loss of the two cases, which are compared to each other in the following analysis.

3.4.1 Currency Board Maintained over Both Periods

First, we consider the case of a currency board regime that is kept over both periods; i.e. the monetary authorities do not announce the abolition of the currency board in period 1, implying that $e_2 = 0$ irrespective of the shock in period 2. Using equations (3.3) and (3.4), the second period's inflation rate is given by

$$\pi_2 = \phi_2 = \eta\phi_1 + u_2 \ , \tag{3.11}$$

meaning that inflation depends only on the shock ϕ_2. Thus the expected inflation π_2^e equals[5]

$$\pi_2^e = E_1(\phi_2) = \eta\phi_1 \ . \tag{3.12}$$

[5]In the following analysis, we use E_1 as the abbreviation for the expectation contingent on available information in the first period I_1, i.e. $E_1 = E(\cdot|I_1)$.

Expected and actual inflation differ by the new shock u_2. Substituting $\eta\phi_1$ for π_2^e in equation (3.6) yields a second period loss of [6]

$$L_2^{cc} = (\gamma(\phi_2 - \eta\phi_1) - k)^2 + \theta\phi_2^2 \ . \tag{3.13}$$

Equilibrium values are given by

$$\begin{aligned} e_2^{cc} &= 0 \\ \pi_2^{cc} &= \phi_2 = \eta\phi_1 + u_2 \ , \\ y_2^{cc} &= \gamma(\pi_2^{cc} - \pi_2^e) = \gamma(\eta\phi_1 + u_2 - \eta\phi_1) = \gamma u_2 \ . \end{aligned} \tag{3.14}$$

As the exchange rate cannot be changed, e_2^{cc} equals zero. Thus, π_2^{cc} is independent of k and the time inconsistency problem of monetary policy is solved at the cost of having no policy option to counteract ϕ_2. Equilibrium output depends on the realization of the unexpected part of ϕ_2, the new shock u_2. The expectation of period 2 loss, contingent on first period information, is given by

$$\begin{aligned} E_1(L_2^{cc}) &= E_1\left((\gamma u_2 - k)^2 + \theta\phi_2^2\right) \\ &= \gamma^2 E_1(u_2^2) + k^2 + \theta E_1(\eta\phi_1 + u_2)^2 \\ &= (\gamma^2 + \theta)\sigma_u^2 + k^2 + \theta\ (\eta\phi_1)^2 \ . \end{aligned} \tag{3.15}$$

The threat of a large expected second period shock, represented by a high variance σ_u^2 and a high inherited shock ϕ_1, leads to a high expected second period loss, as the policy maker has no options to counteract the shock. This effect is reinforced by a large time inconsistency problem of monetary policy, represented by a high level of k.

3.4.2 Currency Board Abolished after the First Period

In this subsection, we consider the case that the government has adopted a currency board regime, but announces its abolition at the end of the first period and introduces a free float system for period 2. Note that e_1 equals zero and the monetary authorities can set π_2 (and therefore e_2) optimally after observing ϕ_2.

The monetary authorities optimize the period 2 social loss according to the flexible exchange rate case. The equilibrium values of e_2^{cf}, y_2^{cf} and π_2^{cf} are identical to those of a

[6] The superscript cc stands for the case in which the currency board is maintained over both periods and cf denotes the situation of abolishing the currency board in the second period.

flexible exchange rate system. Hence, L_2^{cf} and also $E_1(L_2^{cf})$ are given by equation (3.10) plus the political costs c^{cf} of abandoning the currency board, yielding

$$E_1(L_2^{cf}) = L_2^{cf} = \frac{1}{\theta}k^2(\gamma^2 + \theta) + c^{cf} \ . \tag{3.16}$$

3.4.3 Maintaining or Leaving the Currency Board Arrangement

The decision whether to maintain or abandon the currency board takes place in the first period before the private sector forms its inflation expectations for the second period. The policy maker will decide to maintain the currency board, if the expected second period loss of leaving it exceeds the expected second period loss of perpetuating the currency board, i.e. if

$$E_1(L_2^{cf}) - E_1(L_2^{cc}) > 0 \ . \tag{3.17}$$

Using equation (3.15) and (3.16), this condition is equivalent to

$$\phi_1^2 \ < \ \frac{1}{\theta\eta^2}\left(\frac{1}{\theta}k^2\gamma^2 - (\gamma^2+\theta)\sigma_u^2 + c^{cf}\right) \ . \tag{3.18}$$

The inequality shows that the decision whether to keep the currency board after the first period or not depends on the absolute value of the shock realization ϕ_1. The currency board is maintained for small shocks, whereas a large shock ϕ_1 prompts the policy maker to announce its abolition. A high target output k and a low weight θ of inflation in the loss function – both reflecting a large time inconsistency problem of monetary policy – make the inequality more likely to hold.[7] The policy maker will continue the currency board in spite of a large shock requiring an adjustment of the real exchange rate, as an abolition of the currency board will revive a huge inflation bias. This effect is reinforced by the political costs c^{cf} of abolishing the currency board.

In contrast, a high variance σ_u^2 of the *PPP*-shock makes the interval in which the fixed exchange rate is defended smaller, and it is possible that the right hand side of inequality (3.18) becomes negative which would mean that the currency board would be abolished irrespective of the shock (or it would not be a suitable system from the very beginning and would never be introduced). The higher σ_u^2, the more important it is for the policy maker to be able to react to a large possible shock ϕ_2. A high autocorrelation of the shocks ϕ_t,

[7] If the right hand side of equation (3.18) is positive, it depends negatively on θ. If it is negative, the inequality does not hold anyway

represented by a large η, decreases the range of shock realizations for which the currency board is maintained. In this case, the first period shock contains much information about the second period shock, implying that a large first period shock makes the need for large further adjustments in the second period more likely. If η is interpreted as the degree of price stickiness (see footnote 3), a currency board is more likely to be maintained in the case of high price flexibility, whereas a relatively large η increases the probability of announcing its abolition.

3.5 Standard Peg

To ensure an unbiased comparison of a standard peg and a currency board, we only consider the policy makers' decision of the second period (see section 3.2). After observing the shock ϕ_2, monetary authorities decide whether to maintain or to abandon the peg. In this section, the range of realizations of ϕ_2 in which monetary authorities would defend the peg is derived. Multiple equilibria for π_2^e may occur in this case. However, we will derive sufficient conditions for the uniqueness of the equilibrium.

3.5.1 Policy Decisions under a Peg

The second period loss when leaving the peg L_2^{pf} is given by the term[8]

$$L_2^{pf} = \theta \frac{(\gamma \pi_2^e + k)^2}{\gamma^2 + \theta} + c^{pf} \ , \tag{3.19}$$

which equals the second period loss in a free float system (equation 3.10) plus the political costs c^{pf}. If the monetary authorities decide to maintain the peg after observing ϕ_2, L_2^{pp} equals

$$L_2^{pp} = (\gamma(\phi_2 - \pi_2^e) - k)^2 + \theta \phi_2^2 \ . \tag{3.20}$$

The fixed exchange rate is defended if the second period loss in case of leaving the peg exceeds the loss in case of maintaining the peg, i.e. if

$$L_2^{pf} - L_2^{pp} = \left(\theta \frac{(\gamma \pi_2^e + k)^2}{\gamma^2 + \theta} + c^{pf} \right) - (\gamma(\phi_2 - \pi_2^e) - k)^2 - \theta(\phi_2)^2 > 0 \ . \tag{3.21}$$

[8]The superscript *pf* denotes the situation of leaving the peg in the second period and *pp* stands for the case of maintaining the peg in both periods.

Isolating ϕ_2 in the above equation yields the result that the exchange rate remains fixed if and only if

$$\underbrace{\Gamma(\pi_2^e, k) - \sqrt{\frac{c^{pf}}{\gamma^2 + \theta}}}_{\phi_2^l} < \phi_2 < \underbrace{\Gamma(\pi_2^e, k) + \sqrt{\frac{c^{pf}}{\gamma^2 + \theta}}}_{\phi_2^u} , \tag{3.22}$$

where $\Gamma(\pi_2^e, k) = \gamma \dfrac{(\gamma \pi_2^e + k)}{\gamma^2 + \theta}$.

The peg is maintained if the shock ϕ_2 lies in an interval of length $2\sqrt{\frac{c^{pf}}{\gamma^2+\theta}}$, which is increasing in the political costs c^{pf}. Without political costs, this interval vanishes and monetary authorities will always abandon the fixed exchange rate and respond optimally to shocks. If $\phi_2 < \phi_2^l$, the monetary authorities will devalue; if $\phi_2 > \phi_2^u$, they will revalue. Using equation (3.3), the lower and upper boundary of the interval in which the peg is defended can be expressed in terms of the new shock u_2,

$$u_2^l = -\eta \phi_1 + \Gamma(\pi_2^e, k) - \sqrt{\frac{c^{pf}}{\gamma^2 + \theta}} \tag{3.23}$$

$$u_2^u = -\eta \phi_1 + \Gamma(\pi_2^e, k) + \sqrt{\frac{c^{pf}}{\gamma^2 + \theta}} . \tag{3.24}$$

The policy maker devalues, if the new shock u_2 is below u_2^l and revalues the currency if $u_2 > u_2^u$. In the following, it is assumed that the new shock u_t is uniformly distributed with $u_t \sim U[-A, A]$. Note that for certain parameter values, the boundaries u_2^u and u_2^l may lie outside the support of u_2.

It is assumed that political costs c^{pf} are small enough to ensure that $\sqrt{\frac{c^{pf}}{\gamma^2+\theta}} \leq A$, i.e. the (maximum) length of the interval in which the policy maker maintains the fixed exchange rate is smaller than the length of the support of u_2. Thus, independent of the realization of ϕ_1, the probability of abandoning the peg is always positive. The probability of defending the fixed exchange rate is a measure of the credibility of the exchange rate system. In case that the whole interval $[u_2^l, u_2^u]$ is contained in the support $[-A, A]$, the probability of maintaining the peg equals $\frac{1}{A} \cdot \sqrt{\frac{c^{pf}}{\gamma^2+\theta}}$, which is an upper boundary of the credibility of the fixed exchange-rate system in all cases.

We use this upper boundary for the comparison of the credibility of a currency board and a standard peg in section 3.6. This way, the standard peg appears in a favorable light, and the credibility gains of a currency board can only be underestimated. Therefore, when pointing out situations in which a currency board has a credibility advantage, we will remain on the safe side.

3.5.2 Unique and Multiple Equilibria

The focus of this subsection is the position of the interval in which the peg is defended. The center of that interval depends on π_2^e, which is determined by rational expectations, i.e. $\pi_2^e = E_1(\pi_2)$.

The expected value of π_2 is given by

$$\begin{aligned} E_1(\pi_2) &= \mathrm{P}(u_2 < u_2^l)E_1(\pi_2|u_2 < u_2^l) + \mathrm{P}(u_2^l < u_2 < u_2^u)E_1(\pi_2|u_2^l < u_2 < u_2^u) \\ &\quad + \mathrm{P}(u_2 > u_2^u)E_1(\pi_2|u_2 > u_2^u)\ , \end{aligned} \tag{3.25}$$

which is of course a function of the expected inflation π_2^e.

As in Obstfeld (1996), the existence of an equilibrium is ensured, but multiple equilibria may occur when determining π_2^e from equation (3.25).[9] However, multiplicities can be excluded for certain parameter sets (see appendix A). The condition $\theta > \frac{\gamma^2}{2}$, which means that the weight on inflation θ in the policy makers' loss function is high relative to γ, the parameter in the Lucas supply function, is sufficient for a unique equilibrium to exist. Moreover, if there is a solution for equation (3.25) for which $[u_2^u, u_2^l] \subset [-A, A]$ (corresponding to case (iii) in the appendix A), the equilibrium is unique.

In this case

$$\pi_2^e = E_1(\pi_2|u_2^l > -A \wedge u_2^u < A) = \Gamma(\pi_2^e, k), \tag{3.26}$$

implying that[10]

$$\pi_2^e = \frac{\gamma k}{\theta}\ . \tag{3.27}$$

The same expected inflation $\pi_2^e = \frac{\gamma k}{\theta}$ would result, if the interval $[u_2^l, u_2^u]$ lay completely outside the support of $[-A, A]$, meaning that the peg would be abolished in any case (case (i) and (v) in the appendix A). If the interval $[u_2^l, u_2^u]$ lies partly in the support of u_2 and partly outside of the left boundary (case ii), it follows that $\pi_2^e > \frac{\gamma k}{\theta}$, whereas $\pi_2^e < \frac{\gamma k}{\theta}$ if a part of $[u_2^l, u_2^u]$ lies outside $[-A, A]$ on the right hand (case iv). In the latter case it is not excluded that expected inflation π_2^e is negative. However, a non-negative inherited shock

[9] Obstfeld (1996) discusses the possibility of multiple equilibria in the private sector's expectations under fixed exchange-rate regimes. In contrast to our model, the random shock occurs in the Lucas supply curve.

[10] The inflation rate expected by the private sector is the same as in the free float system. This result is a consequence of the assumption of a uniform distribution.

$\eta\phi_1$ continues to ensure that the expected inflation rate is positive. A negative π_2^e may occur if ϕ_1 is sufficiently negative, the persistence parameter η is high, the target output k is small, and case (iv) is the relevant one. In this situation the conditional expectation on the inflation rate given that the peg is defended may be negative (reflecting that on average, a real depreciation is required), and due to the small k, inflation will be low if the peg is abandoned.

If π_2^e is negative, $\Gamma(\pi_2^e, k)$, the center of the interval of period 2's shocks ϕ_2 in which the peg is defended (equation 3.22) may also be negative.[11] Nevertheless, a positive $\Gamma(\pi_2^e, k)$ should be considered as the normal case.

3.6 Comparison of Peg and Currency Board

In the previous sections, we derived the ranges of the *PPP*-shock in which the particular regimes are maintained. In section 3.4 (see equation 3.18), the condition to keep the currency board was derived as

$$\phi_1^2 < \frac{1}{\theta\eta^2}\left(\frac{1}{\theta}k^2\gamma^2 - (\gamma^2+\theta)\sigma_u^2 + c^{cf}\right) .$$

Assuming as in section 3.5, that the new shock u_t is uniformly distributed on $[-A, A]$, the length of the interval is proportional to the probability of maintaining the currency board and can also be interpreted as a measure of credibility. This probability is given by[12]

$$P(\text{maintain CB}) = \frac{1}{A}\sqrt{\frac{1}{\theta\eta^2}\left(\frac{1}{\theta}k^2\gamma^2 - (\gamma^2+\theta)\sigma_u^2 + c^{cf}\right)} . \tag{3.28}$$

From section 3.5 (equation 3.22 and 3.27), we know that the fixed exchange rate is defended in the second period, if

$$\Gamma(\pi_2^e, k) - \sqrt{\frac{c^{pf}}{\gamma^2+\theta}} < \phi_2 < \Gamma(\pi_2^e, k) + \sqrt{\frac{c^{pf}}{\gamma^2+\theta}} ,$$

leading to the upper boundary for the probability of defending the peg given by

$$P(\text{maintain peg}) \leq \frac{1}{A}\sqrt{\frac{c^{pf}}{\gamma^2+\theta}} , \tag{3.29}$$

[11]The case of $\pi_2^e < 0$ and $\Gamma(\pi_2^e, k) < 0$ occurs for example for the parameter values $A = 1,\ \eta = -0.3,\ \phi_1 = 0.9,\ k = 0.05,\ \gamma = 0.3,\ c = 0.5$ and $\theta = 0.6$.

[12]Of course, the probability must be an element of [0,1]. If the expression exceeds one, the probability equals one; if the term under the square root is negative, the probability equals zero. Note that $\sigma_u^2 = A^2$.

where equality holds for $[u_2^l, u_2^u] \subset [-A, A]$.

A comparison of the intervals in which the particular exchange rate system is maintained shows that the interval is symmetric around zero in the case of a currency board but shifted by $\Gamma(\pi_2^e, k) = \frac{\gamma^2\pi_2^e + \gamma k}{\gamma^2 + \theta}$ in the case of a peg. When $\Gamma(\pi_2^e, k)$ is positive, which can be considered as the normal case,[13] this shift amounts to an inflation bias under a standard peg that does not exist in a currency board system. A standard peg will rather be abolished in case of a negative ϕ_2 requiring a real depreciation than in case of $\phi_2 > 0$ which leads to a positive inflation when the exchange rate remains fixed.

Moreover, the credibility of the peg hinges on the political costs c^{pf} of abandoning it, as without these costs, the probability of maintaining the peg shrinks to zero (equation 3.29). In contrast, the credibility of a currency board system is not exclusively based on the political cost c^{cf} (equation 3.28). A large target output k (or a low degree of inflation aversion θ), reflecting a large time inconsistency problem that would lead to a high future inflation in case of leaving the currency board system, may prevent the policy maker from announcing its abolition even if $c^{cf} = 0$. Conversely, the probability of maintaining the currency board may be zero in spite of positive political costs c^{cf} of abolishing it if σ_u^2 is large and the expression under the square root in equation (3.28) becomes negative. In this case, the ability to offset shocks promptly is more important for the policy makers than avoiding the inflation that results from the time inconsistency problem.

In addition, the credibility of the currency board depends negatively on η, the parameter representing the autocorrelation of the *PPP*-shocks as the decision of abolishing the currency board is based on the expectation on the second period shock $E_1(\phi_2) = \eta\phi_1$. In contrast, the credibility of a standard peg does not depend on η, as the decision whether to abolish the peg is made after observing ϕ_2 and does not depend on the degree of shock persistence.

For further comparison of the credibility of the two exchange rate regimes, it is assumed that the political costs are equal in both regimes[14] which means that $c = c^{cf} = c^{pf}$.

A currency board system is more credible if $P(\text{maintain CB}) > P(\text{maintain peg})$,

[13]See the discussion at the end of section 3.5.2. In particular, $\Gamma(\pi_2^e, k)$ is always positive if there is no inherited shock or if ϕ_1 is positive. The case $\Gamma(\pi_2^e, k) < 0$ may occur only if ϕ_1 is sufficiently negative.

[14]Intuitively, the political costs of giving up the fixed exchange rate are higher under a currency board than under a standard peg as assumed by Irwin 2004. Higher political costs would give the currency board an additional credibility advantage.

which is the case if

$$\frac{1}{A}\sqrt{\frac{1}{\eta^2\theta}\left(\frac{1}{\theta}(\gamma k)^2-(\gamma^2+\theta)\sigma_u^2+c\right)}>\frac{1}{A}\sqrt{\frac{c}{\gamma^2+\theta}}\ , \tag{3.30}$$

i.e., if the length of the interval of maintenance is larger in the case of a currency board than under a fixed exchange-rate regime.[15] The expression can be rewritten as

$$\frac{\gamma^2k^2}{\theta}-\sigma_u^2(\gamma^2+\theta)+c\left(1-\frac{\theta\eta^2}{\gamma^2+\theta}\right)>0\ . \tag{3.31}$$

This inequality shows that a currency board is more credible than a standard peg regime when the time inconsistency problem of monetary policy is large (as represented by a high k or a low θ). In the case of a currency board arrangement, the monetary authorities are tied by law to keep the fixed parity, when its abolition was not announced in the previous period. Hence, it is not possible to create surprise inflation. Thus, the time inconsistency problem of monetary policy is solved, which is not the case in a standard peg system. High political costs c also increase the credibility of a currency board relative to a standard peg, as $1-\frac{\theta\eta^2}{\gamma^2+\theta}>0$.

The peg regime achieves a credibility advantage vis-a-vis a currency board, when σ_u^2 becomes so high that the ability to react to shocks is more relevant than solving the time inconsistency problem of monetary policy – higher shock variances lead to a higher probability that large shocks may hit the economy, and hence it can be important to be able to react immediately to the shock by choosing an optimal e_2 (and thus π_2), instead of having to keep a misalignment over one period.

3.7 Conclusion

In this chapter we have addressed the issue of whether a currency board arrangement is indeed more credible than a standard peg system, and what exactly may make it more credible. The essential feature of a currency board captured in our model is its longer-term nature. The currency board can only be abolished if this has been announced one period in advance – reflecting the fact that a currency board can only be abandoned after a time-consuming political process. As a result, it is not possible to create surprise inflation, and the time-inconsistency problem of monetary policy is solved completely. In

[15] As discussed at the end of section 3.5.2, we use the upper boundary of P(maintain peg) for the comparison of the two regimes. Therefore the credibility of the standard peg appears in a favorable light.

contrast, a standard peg does not solve the time inconsistency problem, because of the permanently existing escape clause from the fixed exchange rate. The policy maker can abandon the peg overnight, and he is only deterred from doing so by the political costs of exiting the exchange rate system.

The comparison of both exchange rate regimes in section 3.6 shows that the currency board is more credible – in the sense of having a higher probability of being maintained – if the time inconsistency problem is dominant in the economy considered. The threat of high future inflation will prevent the policy maker from starting the process of abolishing the currency board unless there is a large persisting misalignment. In contrast, the currency board is more likely to be abandoned than a standard peg if shocks with a high volatility constitute the dominant problem, i.e. if the flexibility to be able to react immediately to future shocks is of paramount importance. In summary, its capability of solving the time inconsistency problem makes the currency board credible, but only as long as this advantage is not outweighed by the need for stabilization of shocks occurring with a high volatility.

Chapter 4

Fiscal Policy and Stability of Currency Boards

4.1 Introduction

Despite the successful reappearance of currency board regimes since the beginning of the 1990s, only few of researchers have addressed to model the differences between a currency board regime and a standard fixed exchange-rate systems: Chang and Velasco (2000), Oliva et al. (2001), Irwin (2004) and Feuerstein and Grimm (2006b).

All these papers have in common that the effects of fiscal policy and the role of public debts are neglected or are captured by realizations of stochastic shocks. However, although hardly analyzed in theoretical literature, the economic performance of currency board countries, especially those in Europe and Hong-Kong, but also Argentina during the period of 1991 to 1995, was good and accompanied by a sound fiscal policy and a sustainable development of public debts. This has also been subject to many econometric studies, like for example Ghosh et al. (2000), Fatas and Rose (2001), Sun (2003) and Grigonyte (2003) to mention only a few of them.

In this chapter, the focus will be laid on the stability and credibility of currency board systems by additionally taking fiscal policy into account. The central questions of our analysis are (i) whether and under what circumstances a currency board guarantees more fiscal discipline and a more sustainable growth of debts compared to a standard peg, and (ii) which factors contribute to higher or less stability of a currency board system.

The chapter is structured as follows. First, related empirical literature and the few existing theoretical papers on that topic are summarized. Second, we look at the economic performance of recent and present currency board systems by also taking fiscal aspects into account, i.e. the budget deficit and the development of debts. Within this section, we discuss also the main reasons for the abandonment of the Argentinean Convertibility Plan and the default of sovereign debts. We state that, despite the breakdown of the Argentinean Convertibility Plan, a currency board system can be a proper exchange rate system to (re)gain credibility and stability after a financial turmoil or during a transition process. This point of view is also supported by the theoretical results of this chapter. Third, we develop a two-period model, which integrates fiscal policy and public debts. Fourth, we show that optimal debt levels and the optimal amount of government expenditure are lower under a currency board compared to a standard fixed exchange-rate regime, when assuming at the same time that both exchange rate systems are maintained.[1] Hence, we state that a currency board increases the discipline of fiscal policy. Fifth, we examine the stability of a currency board system by using two numerical scenarios. We define the stability of a currency board as the difference between the expected policy losses occurring in the cases that the currency board is maintained and that it is abandoned. Then, we examine which factors drive the stability of a currency board system and which factors contribute to less stability. Sixth, we finish our theoretical analysis by comparing a currency board system and a standard peg regime. In this context, we use again the concept of the "credibility of an exchange rate system" which denotes the probability that the current exchange rate system is maintained in the next period. We derive the conditions under which a currency board gains a higher credibility compared to a standard peg and vice versa. Seventh, we conclude and highlight the main results of our analysis and give a brief outlook on further research.

4.2 Empirical and Theoretical Background

In the existing literature and in economic discussions, the effect of the introduction of a fixed exchange-rate regime on fiscal policy is discussed differently.

On the one hand, there is a broad view that fiscal policy is the only remaining stabilization instrument when large asymmetric shocks occur in a fixed exchange-rate regime. This may suggest that fiscal policy tends to be more expansionary in times of a recession.

[1] Maintaining the standard fixed exchange-rate system means in this context that the exchange rate remains pegged to its initial level and no realignment takes place, i.e. both the currency board and the standard peg survive the first period and are not replaced in period 2.

On the other hand, a country operating a fixed exchange-rate system aims at a gain in credibility and stability to achieve sustainable levels of the inflation rate and, thereby, to create a sound environment for economic growth. Unsound fiscal policy and high debts, however, can be interpreted by the private sector as an increasing risk of leaving the exchange rate peg in the future. As policy makers realize that a collapse of the exchange rate would induce high economic and political costs, they have an incentive to exercise a more restrictive fiscal policy. This argument becomes even stronger in case of a hard peg-regime like a monetary union or a currency board, as the political costs of repealing such a system are higher compared to a realignment under a soft peg. This is demonstrated by FATAS and ROSE (2001) and it is also an essential assumption in the theoretical analysis of currency board arrangements by IRWIN (2004).

In the following, we give an overview of the theoretical models and empirical findings, analyzing the impact of a fixed exchange-rate regime – and especially for the case of a currency board arrangement – on fiscal policy. We begin with the work of TORNELL and VELASCO (1995a, 1995b, 1995c and 1998). In the theoretical parts of their models the effect of a fixed exchange-rate regime on fiscal policy is not uniquely determined, which is against conventional wisdom. They distinguish in their model between two systems:
(i) money-based stabilization, where the central bank sets the money growth rate to some constant and the exchange rate is obtained endogenously, and
(ii) exchange-rate stabilization, where the nominal devaluation rate is treated as a constant (and could also be set equal to zero e.g. for the currency board case) and the money supply is determined endogenously.

They show that fiscal discipline is in both systems influenced by the fiscal authority's intertemporal discount factor and by the level of the interest rate. Their main findings are that money-based programs induce more fiscal discipline if the fiscal policy makers are *impatient*, i.e. if the policy maker puts a greater weight on present utility than on future utility. In contrast, exchange-rate based programs lead to a higher discipline of fiscal policy, if the fiscal authorities are *patient*.

The intuition behind is that the choice of each system can be considered as a certain rule to distribute the burden of the inflation tax intertemporally. Operating under a peg system, the real exchange rate is determined by the central bank. Thus, fiscal authorities can execute higher expenditures financed by an increasing level of debt or a reduction of foreign reserves in the short-run. Resulting from the fact that indicators like e.g. foreign reserves are not transparent enough under a fixed exchange-rate system to reveal significant fiscal imbalances, especially for the case of transition and developing countries, the punishment of an unsound fiscal policy may be delayed to some future point in time where the situation becomes "more unsustainable" and the peg collapses. In contrast, under a flexible exchange rate system an unsound fiscal imbalance is recognized by the private sector in the short run, which leads to an expectation of higher future money growth and thus to an immediate rise in the inflation rate.

These results are confirmed empirically by several studies of TORNELL and VELASCO (1995a, 1995b, 1995c and 1998). However, the authors do not distinguish between soft and hard fixed-exchange rate regimes and focus on Sub-Saharan countries during the 1980s and Latin American Countries from 1960 to 1994, a time horizon where currency board systems played no major role in the geographical regions.[2]

In a similar examination, HAMANN (2001) compares *exchange-rate based stabilization programs* to the broader class of *other stabilization programs*. The data used comprises 143 countries over the years 1960-1997 and includes stabilization via the adoption of a hard peg in Ecuador (dollarization) and Argentina (currency board). The author draws a conclusion which is in line with the results of TORNELL et al.: The key argument in favor of an exchange-rate based stabilization, to gain fiscal discipline by reducing the inflation bias, is not confirmed in the data. However, HAMANN does not distinguish between currency boards and other fixed exchange-rate systems.

In contrast, in their empirical analysis on this topic FATAS and ROSE (2001) split up countries with fixed exchange-rate regimes classified as hard pegs into countries belonging to a currency area, dollarized countries and countries operating a currency board. They show that belonging to a currency area – as for example to the European Monetary Union – or dollarization are not automatically accompanied by a greater fiscal discipline. For a monetary union, the results depend on the number of participating countries. Though, they find that currency board countries are associated with a more restrained fiscal policy compared to the rest of the countries considered in their sample, also relative to countries operating under other fixed exchange-rate regimes. Furthermore, they show that the budget in currency board countries is shifted to components providing more social insurance like transfers, subsidies and social security taxes, which contribute to a sound domestic environment, too.

A further examination of the linkage between exchange rate regimes and fiscal re-

[2]In the Latin America sample only the very early years of the Argentinean currency board arrangement are observed (1991-1994). As the time series of that sample starts from 1960, the currency board effect may be negligible when considering the performance of Argentina relative to other countries.

straints is done by Alberola and Molina (2002). In a theoretical model which analyzes the financing of government expenditure, the authors consider two instruments, namely *monetary* and *fiscal seigniorage*. Monetary seigniorage is defined as the process of money creation and fiscal seigniorage as the increase in public debt holding by the central bank. Based on an estimation using a broad IMF data set, it is shown that monetary seigniorage has no significant influence on the fiscal deficit, whereas fiscal seigniorage by its nature has. As sketched in their model, a standard fixed exchange-rate regime does not increase fiscal discipline as there is no prevention from creating fiscal seigniorage. However, under a currency board system claims of the government disappear (or are reduced significantly) from the balance sheet due to the strict features of the currency board, which are fixed by law (see chapter 2). Therefore, fiscal seigniorage is strongly decreased under a currency board arrangement or even completely impossible in an orthodox system.[3] They come to the conclusion that a currency board arrangement creates fiscal discipline and cement their results by an empirical investigation.

Grigonytė (2003) analyzes the impact of currency boards on fiscal discipline in ten central and eastern European countries. By estimating cross country regressions the author exhibits that the three currency board countries Estonia, Lithuania, and Bulgaria show a certain degree of fiscal discipline. The econometric studies suggest that a currency board arrangement decreases government expenditure and improves the public balance. The result can also easily be seen from the raw data, where the three countries show a level of debt relative to GDP clearly below the average level of the rest of the countries belonging to the European Union. Furthermore, Estonia and Lithuania attained fiscal surpluses in the recent years and were also far above the average of the sample. We will come back to this point again in the next section.

[3] The notion "orthodox currency board" describes a very strict interpretation of a currency board system, where the policy makers have no opportunity to make use of monetary policy instruments. This interpretation is mainly used for the theoretical analysis of currency board systems and was introduced in chapter 2.

Table 4.1: Modern Currency Board Systems

Country	Estonia	Lithuania	Bulgaria	Bosnia and Herzegovina	Hong-Kong	Argentina	ECCB)*
Time of introduction	1992	1994	1997	1997	1983	1991	1983
Establishment by law	Law on the Security for the Est. Kroon	Law on the Credibility of the Litas	Law on the Bulgarian National Bank	Law on the CBBiH	Exchange Fund Ordinance	Convertibility Law of 1991	The ECCB Agreement Act 1983
Motivation	Macroeconomic Stabilization	Macroeconomic Stabilization	Macroeconomic Stabilization	Dayton Peace Accord: Postwar Reconstruction	Restore Confidence	Macroeconomic Stabilization	Establish Confidence
Authority in Charge	Bank of Estonia	Bank of Lithuania	Bulgarian National Bank	Central Bank of BiH	Exchange Fund	Banco Central de la República Argentina	ECCB
Official Parity	EEK 15.6466 = EUR 1.00	LTL 3.4528 = EUR 1.00	BGN 1.9588 = EUR 1.00	BKM 1.9588 EUR 1.00	HKD 7.80 = USD 1.00	ARS 1.00 = USD 1.00	XCD 2.70 = USD 1.00
Former Parities (until)	EEK 8.00 = 1.00 DEM (01.01.1999)	LTL 4.00 = 1.00 USD (02.02.2002)	BGN 1000 = 1.00 DEM (01.01.1999)	BKM 1.00 = 1.00 DEM (01.01.1999)	–	–	–
Strategy	Euro Changeover 01.01.2008	Euro Changeover 01.01.2007	EU Accession	EU Accession	Political Independence (China)	–	Maintenance of Mon. Stability

Sources: Enoch and Gulde (1998), Ho (2002), Stukenbrock (2004) and the Eastern Caribbean Central Bank (www.eccb-centralbank.org).

)* The Eastern Caribbean Central Bank was established in October 1983. It is the Monetary Authority for a group of eight island economies – namely Anguilla, Antigua and Barbuda, Commonwealth of Dominica, Grenada, Montserrat, St Kitts and Nevis, St Lucia, and St Vincent and the Grenadines.

The Exchange Fund Ordinance in Hong-Kong is no formal currency board law, hence its contents are not typical for a modern currency board arrangement.

Note that the Argentinean Convertibility Plan was repealed during the crisis in February of 2002.

4.3 Experience from Recent and Present Currency Board Systems

Before attending to the theoretical analysis of a currency board arrangement, it is useful to ask how the currency board countries have fared so far. Therefore, we give a brief survey of present and recent currency board systems. We discuss the motivation of operating under a currency board and lay our focus, again, on the development of the fiscal deficit and the debt level of the single countries by also taking the development of further key indicators as inflation and economic growth into account. The background and some features of the particular systems are summarized in table 4.1, to which we will also refer in the following.[4]

4.3.1 Baltic Experience: Estonia and Lithuania

We begin our survey with the experience in the two Baltic currency board countries, namely Estonia and Lithuania. Both countries decided to introduce a currency board system early after the breakdown of the Soviet Union to escape from the instabilities that emerged within the ruble area. At first, Estonia established a currency board system in 1992 and was followed by Lithuania two years after. The success of both systems has been noteworthy: Inflation was stabilized quickly and came down from three digit levels at the beginning of political independence to a one digit level within a short period of time. Low inflation rates have been maintained without severe fluctuations since 1997. Over the last ten years, inflation rates have on average been clearly below the level of CPI inflation achieved by almost all other central and eastern European transition countries. The superior performance is also empirically confirmed by Gulde et al. (2000). The success of the systems in Estonia and Lithuania has also been shown in the proceedings on the road to the European Monetary Union. Both countries show stable economic growth and an outstanding development of the budget deficits and total government debts: Estonia is actually the top performer of all EU member-states by attaining a budget surplus over the last five years and showing the smallest total government debt relative to GDP of 4.9%. Lithuania also belongs to the top-performing countries with having realized small budget deficits over the recent years and showing a debt ratio to GDP of around 20%.

[4] A broad survey of modern currency board systems can be found in Stukkenbrock (2004).

Therefore, both countries seem to be prepared to an early adoption of the Euro.[5]

4.3.2 Bulgarian Currency Board

To regain credibility and to overcome the twin crisis – a combination of a debt and banking crisis – Bulgaria introduced a currency board in 1997 (Berlemann and Nenovsky, 2004). Immediately after the introduction, inflation was pinned down from hyperinflation to a stable level and reached a one digit rate within the first year. Bulgaria returned to solid growth rates with exception of the year 1999, when the Kosovo crisis put some pressure on the Bulgarian economy. On the way to the accession to the EU on 1 January 2007, Bulgaria showed a high degree of fiscal discipline with a fiscal surplus of currently 2.3% of GDP and a total debt level around the average level of the Central and Eastern European transition countries of about 30% of GDP. In this context it should be mentioned that that the fiscal surplus was favorably affected by earnings resulting from privatization, but this should not detract from the success of the currency board arrangement.

4.3.3 Bosnian Currency Board

The Dayton Peace Agreement enforced the creation of a currency board system in Bosnia in 1997 with the ulterior motive to gain credibility of monetary policy in a fragile economic and political environment and simultaneously not giving too much independence to local authorities (Ho, 2002). The fiscal deficit of Bosnia was steadily improving in the recent years and reached a surplus in the years 2003 and 2004. Total government debt was also reduced continuously to a sound level of 29.1% in 2005. Fiscal policy was accompanied by solid economic growth and low inflation rates during the last five years.[6]

[5] Originally, Estonia desired to introduce the Euro in 2007. However, as the projected inflation rate was slightly above the required reference value according to the Maastricht Convergence Criteria, the government delayed its strategy of the changeover to the Euro (Bank of Estonia, 2006). On May 16th 2006, the European Commission and the European Central Bank evaluated Lithuania for the state of convergence. Both institutions suggested to reject Lithuania's plan to adopt the Euro at the beginning of 2007 as the inflation criterion was slightly missed and is expected to be missed in the near future, too (European Central Bank, 2006; European Commission, 2006). This interpretation of the inflation criterion is judged critically among many economists. For a detailed discussion see for example Buiter and Sibert (2006) and Feuerstein and Grimm (2006a).

[6] The statistical data can be found on the webpage of the Bank of Bosnia: http://cbbh.ba/en/statistics.html, 8 November 2006.

4.3.4 Hong-Kong Currency Board

The motivation of the introduction of the Hong-Kong currency board was also to restore confidence: The reason for the precipitating of the meltdown of the flexible exchange rate regime at the end of the 1970s and the beginning of the 1980s was the confidence crisis, which was accruing from uncertainties surrounding Hong Kong's political transition. After several years of relatively high inflation rates of above 10%, Hong-Kong established its currency board in 1983 (Chiu, 2001). Although having been attacked several times by speculators, the exchange rate peg has never been changed and the currency board is still operating. The key economic indicators were stabilizing in 2005 characterized by low inflation rates (after a period of deflation), a solid growth rate of GDP, and a balanced fiscal budget in 2004 after two periods of fiscal deficits with a peak of around 4.8% of GDP. However, Hong-Kong shows an accumulated fiscal surplus of higher than 20% of GDP and thus fiscal discipline plays no major role at the time (International Monetary Fund, 2006).

4.3.5 Argentinean Currency Board

Notwithstanding the well-performing and the findings of sound fiscal policies in the countries considered so far, the failure of the Argentinean Convertibility Plan in 2002 has brought some scepticism concerning the actual benefits of hard peg regimes. In this context, De la Torre et al. (2005, p.184) claim that "the advantages of hard pegs have been greatly overstated". According to their opinion, the currency board did not provide nominal stability and did also not foster fiscal or monetary discipline. Therefore, we consider thoroughly what went wrong in Argentina after a brief summary of the first years under the Convertibility Plan.[7]

Out of similar reasons as in Bulgaria, after the failure of several stability programs the Argentinean government decided to adopt a currency board system to combat hyperinflation and to regain credibility of monetary policy. The decision seemed to be a great success during the period of 1991-1998: The macroeconomic environment was improved impressively and economic growth returned. Even the Tequila Crisis of Mexico in 1994 had no major impact on the Argentinean economy. However, fiscal policy being on a solid path at the beginning – also favorably influenced by revenues from privatizations – the situation became worse from 1994 on. The budget deficit deteriorated caused by a change

[7] For a detailed discussion see Mussa (2002).

in the social security system which led to an unexpected amount of additional expenditure and by an increase of debts in the provinces, which occupied a high degree of autonomy in their decision-making and budget planning. Furthermore, already in the first years of the currency board a financial dollarization persisted and even increased during that time. This has been "a key factor behind the ambivalence of investor confidence in the currency board" (De la Torre et al., 2005, p.196). The behavior of investors, of course, was also influenced by the omnipresent history of economic fragility in Argentina.

The bad times in Argentina began after 1998, when it slipped into a great recession accompanied by high unemployment levels, which were triggered by a sudden stop of foreign direct investments. The situation became worse, when the competitiveness of Argentina came under great pressure after Brazil's currency fell sharply over 50% towards the US-$ in 1999. At the same time the peso appreciated together with its anchor currency, the US-$, against the Euro, which contributed to a further loss of competitiveness resulting in a drastic jump up of the current account deficit. As the biggest trading partner of Argentina were Brazil and southern European countries, one has to criticize the choice of the US-$ as an anchor currency. This argument is strengthened by the fact that Argentina's economy is relatively closed. It, therefore, was far from meeting the conditions of an optimal currency area with the United States (Feldstein, 2002).

Due to the large fraction of net foreign debts, denominated primarily in US-$, an exit of the currency board with the intention to devalue did not seem to be an option of escape, as the real value of outstanding debt was supposed to rise consequently to an unsustainable amount. We take up this point in the theoretical part of this chapter. Argentina's situation became unfavourable and is circumscribed by the notion of the *currency-growth-debt trap* in De La Torre et al. (2005, p.196): "The currency was overvalued, growth was faltering, and the debt was hard to service". To maintain the fixed exchange rate, it was necessary to pursue a restrictive fiscal policy which aggravated the recession and reduced economic growth and led simultaneously to a painful deflation because of the existence of great labor market frictions (Aschinger, 2002). To regain competitiveness, a real devaluation, which is under a pegged exchange rate only attainable by falling prices, was necessary. This, however, was hardly possible, on the one hand, because of significant labor market inflexibilities as aforementioned, and, on the other hand, because of the necessary size of devaluation: Calvo et al. (2004) show in their examination that Argentina would have needed to depreciate its real exchange rate by 46 percent in order to reach a balanced current account.[8] However, a real depreciation was also not a possible strategy as it would have triggered an increase of the burden of real debts, also when assuming at the same time that the real interest rate would not change remarkably.

Consequently, the bad economic conditions led finally to an abandonment of the currency board system and the largest default in history at the beginning of 2002, despite the support packages granted by the IMF at the beginning of the crisis and the establishment of the *corralito*, which was an attempt to limit capital outflows by freezing domestic deposits.

The Argentinean crisis is widely discussed in literature. We refer only to a few of them in this survey. The role of the currency board is thereby judged in quite different ways. De la Torre et al. (2005) and Feldstein (2002) are sceptical about the benefits resulting from a hard peg regime. In contrast, Berlemann and Nenovsky (2004) conclude that in Bulgaria and Argentina the introduction of a currency board arrangement proved to be a successful strategy to overcome a deep financial crisis and to restore financial stability, but several mistakes in Argentina brought back economic instabilities.

We argue that a currency board system can contribute to macroeconomic stabilization. What went wrong in Argentina was a mixture of "political malpractice" and "bad luck". The relatively closed economy and the big trading partners being from Latin America and Europe suggest that the choice of the US-$ as an anchor currency was a mistake and contradicted the idea of participating in an optimal currency area. So, it seemed to be obvious that sooner or later strong asymmetric business cycles and economic shocks are likely to appear. And, indeed, by the real appreciation to the most important trading partners the Argentinean economy was driven into a recession. In addition, fiscal policy seemed to be on a solid path at the beginning. However, delayed and dubious economic reforms together with the occurence of corruption, the excessive debts caused by the independent provinces, and the existence of labor market inflexibilities put additional pressure on the Argentinean economy. We, therefore, state that the Argentinean case in fact has shown that a currency board system is not a guarantee of long-term stability per se, but the features of the Convertibility Law, which is the common denomination of the Argentinean currency board system, were in several points not compatible with the

[8]The great size of the depreciation stems from the fact that Argentina is a relatively closed economy as aforementioned, meaning that only a sharp decrease in relative prices to the main trading partners will generate enough gains in net exports to balance the current account.

economic environment.

4.3.6 Summary

To summarize, the findings in this survey about modern currency board systems supports the hypothesis that the introduction of a currency board leads to more fiscal discipline and to a sustainable debt growth. The case of Argentina seems to be an exemption. Therefore, we draw the conclusion that a currency board may be a useful instrument to (re)gain macroeconomic stability after a crisis or during a transition period. However, the repealing of the Convertibility Plan in Argentina in 2002 and the resulting default have shown that the features of the currency board system have to be implemented in a compatible way with the specific structure of the economy. Of course, an exit strategy to adopt the Euro like in Estonia, Lithuania and in the medium term probably in Bulgaria and Bosnia would have helped to gain credibility. But, the development in Hong-Kong has been favorable over a period of almost 23 years.

In the remainder of this chapter, we analyze how the introduction of a currency board arrangement affects the choice of government expenditure and the optimal amount of debt by using a theoretical model. Thereby, we examine the question whether a currency board can in fact increase fiscal discipline. Furthermore, we show which basic conditions guarantee high stability under a currency board and what circumstances lead to an abandonment of the system. We conclude our theoretical work by a comparison of a currency board and a standard fixed exchange-rate regime. Within the following sections, we frequently refer to the empirical findings which were elaborated and summarized here.

4.4 Model

In this section, we first enhance the model of chapter 3, and derive the basic model equations and the policy makers' target function. Thereby, we also show the trade-offs faced by the monetary and fiscal authorities. Second, we explain the sequence of the decision-making under a currency board system and a standard peg regime.

4.4.1 Model Equations

We consider a small open economy with a credibility problem caused by macroeconomic instabilities like e.g. a currency or debt crisis or by a lack of experience in policy-making during a transition period. The economy comprises the monetary authority, a government which decides upon fiscal policy, and the private sector. The monetary authority does not necessarily make its decisions independently, but may be influenced or overridden by the national government (Tornell and Velasco, 1998).

The output gap in period t is given by a modified Lucas-supply function of the form[9]

$$y_t = \gamma(\pi_t - \pi_t^e) + wg_t \qquad \gamma, w > 0\ . \tag{4.1}$$

The expression $\gamma(\pi_t - \pi_t^e)$ is a measure for the effect of surprise inflation on output: Workers demand nominal wages that are sufficiently high to cover expected average future price increases. As unexpectedly high inflation leads ex post to lower real wages, it increases employment and, thereby, output.

Additionally, output is driven by the fiscal policy variable, g_t, which equals total government expenditure minus tax revenues and is, therefore, very close to the definition of the *fiscal deficit*. For simplification, we will use the term "government expenditure" to describe g_t henceforth. The term wg_t reflects the impact of fiscal policy on output. We assume that government expenditure comprises, on the one hand, supply-side policy (e.g. produced output of state-owned companies or granting of subsidies), which enters the modified Lucas-supply function and, on the other hand, demand of goods from public authorities. We discuss this point more detailed, when introducing the inflation equation later in the section.[10]

Note that exactly speaking y_t denotes the deviation of output from its natural level (i.e. output gap), as log natural output is normalized to zero. For reasons of clarity, we refer to y_t by using the notion "output", henceforth.

The (real) stock of debts in period t is denoted by b_t and is given by

$$b_t = \frac{(1+r)(1+qe_t)}{(1+\pi_t)} b_{t-1} + g_t\ . \tag{4.2}$$

This means that the outstanding stock of debts b_{t-1} and the corresponding real interest payments, which are represented by the fraction $(1+r)(1+qe_2)/(1+\pi_t)$, plus the government spending deficit g_t have to be financed by the end of period stock of real debts

[9] Note that analogously to the previous chapter, we use a log-linearized model where the logarithmic natural output level equals zero.

[10] The assumption that fiscal policy can affect both the demand and the supply side is primarily used for mathematical purpose to avoid corner solutions. In our analysis, we focus on a demand-side oriented fiscal policy, as this seems to be the case which accords best with reality for developing and transition countries. Therefore, w should be of small size. From a theoretical aspect, however, the two possible directions of fiscal policy leave room for more flexibility for the application of our basic model.

b_t. The nominal interest rate r is taken as constant as we consider a small open economy with perfect capital mobility; q is the fraction of foreign debts on total debt, which is taken exogenously, and e_t denotes the percentage change of the nominal exchange-rate in price notation.

In the following, we define $\tilde{b}_{t-1}$ as the outstanding level of debt plus interest rate payments, i.e. $\tilde{b}_{t-1} := (1 + r)b_{t-1}$. Then, we can rewrite equation (4.2) as

$$b_t = \frac{(1 + qe_t)}{(1 + \pi_t)}\tilde{b}_{t-1} + g_t \ . \tag{4.3}$$

Multiplying the first summand by $1 = (1 - \pi_t)/(1 - \pi_t)$ yields

$$b_t = \frac{(1 + qe_t)(1 - \pi_t)}{1 - \pi_t^2}\tilde{b}_{t-1} + g_t \ . \tag{4.4}$$

Furthermore, as π_t is assumed to be a small number close to zero, we use the approximation $\pi_t^2 \approx 0$ and obtain

$$b_t = (1 - \pi_t + qe_t - q\pi_t e_t)\tilde{b}_{t-1} + g_t \ . \tag{4.5}$$

As $q\pi_t e_t$ is approximately zero (again following the same idea as above), we finally obtain the resource constraint for the government, which is given by

$$b_t = (1 + qe_t - \pi_t)\tilde{b}_{t-1} + g_t \ . \tag{4.6}$$

The resource constraint used here is similar to that in Sachs, Tornell and Velasco (1996) and Aloy, Moreno and Nancy (2003).[11] Now, it is obvious that a devaluation, i.e. an increasing e_t, increases (foreign) debts directly. In contrast, a higher inflation rate π_t leads to a decrease of the overall debt level. Government expenditure g_t and the stocks of debts b_t and b_{t-1} are measured as shares of GDP. This seems to be most in line with the definition of the natural output, which was normalized to one (log natural output was normalized to zero, respectively).

To motivate the inflation equation used in our later analysis, it is necessary to look more precisely on the idea of its derivation. It is assumed that in the considered economy there exist tradeable and non-tradeable goods.[12] Hence, inflation in period t is given

[11] Sachs et al. also include a seigniorage term $\mu(\pi_t - \pi^e_{t,t-1})$, which is neglected in our analysis.

[12] The degree of openness of an economy in this model is considered as the fraction of tradeable goods relative to all produced goods of this economy (and is thus mainly represented by κ and β, two parameters we will introduce on the next page). Note that this definition is not typical. Usually openness is defined as export volume relative to GDP or as the sum of exports and imports with respect to GDP. We already mentioned the importance of openness in chapter 2, and in this chapter when discussing the breakdown of the Argentinean currency board system in section 4.3.

by the weighted sum of inflation in the tradeable good sector, π_t^T, and inflation in the non-tradeable good sector, π_t^N. We can formulate the following equation:

$$\pi_t = \xi \pi_t^N + (1 - \xi)\pi_t^T \ , \quad 0 \leq \xi \leq 1 \ , \tag{4.7}$$

where ξ denotes the weight of non-tradeable goods relative to all consumption goods. We assume that in the tradeable good sector firms act under perfect competition, so that the stochastic purchasing power parity holds. Then, inflation in the tradeable goods sector equals

$$\pi_t^T = \pi_t^* + e_t + \varepsilon_t \ , \tag{4.8}$$

where π_t^* denotes foreign inflation, e_t the change of the exchange rate in period t, and ε_t is a random PPP-shock. We assume that the considered economy pegs its exchange rate to a stable anchor currency like for example the US-$, EUR or JPY to (re)gain credibility of its domestic currency. Hence, it is no major restriction to assume that the foreign inflation rate π^* is relatively low and can, thus, in the further analysis be considered as being approximately zero.

In the non-tradeable sector, a certain degree of price-inflexibility caused for example by price regulations in the good market sector and labor market frictions is supposed to occur. Inflation for non-tradeable goods is given by

$$\pi_t^N = \widetilde{\pi}_t + \iota g_t \ , \tag{4.9}$$

where $\widetilde{\pi}_t$ reflects the part of inflation which depends on the asymmetric shock ε_t, on exchange rate movements,[13] on wage-setting behavior of firms in the non-tradeable good sector and on the degree of price stickiness (which is closely linked to the wage-setting behavior).[14] The last term of (4.9) represents the effect of government expenditure used for demand of goods to create surprise inflation, where $\iota > 0$ is a weight factor. We assume

[13]We assume that a move of the nominal exchange rate changes the price of some input goods, which lead also to a certain adjustment of output prices in the non-tradeable good sector. However, the existence of price rigidities hampers a complete adjustment of prices and some inflation inertia – i.e. the need for further adjustment in the following periods – remains.

[14]We forbear from a formal exposition of $\widetilde{\pi}_t$ as a function of all the enumerated effects to keep the number of parameters and variables tractable. Note, that in this context the wage-setting argument is used for a more tangible motivation of some parameters, although the labor market is not explicitly introduced into the model.

to have a complete home-bias in government expenditure which means that public demand comprises only home-produced goods.

To summarize, fiscal policy works in two directions: A fraction of government expenditure is used to raise supply of goods, as mentioned when explaining equation (4.1) and the rest is used for demand of goods aiming on pushing output above its natural level by creating unanticipated inflation. The latter case is the reason for having a time-inconsistency problem of fiscal policy and monetary policy at the same time.

Referring to equation (4.7) and the idea briefly sketched above, overall inflation in period t can be formulated as

$$\pi_t = \kappa e_t + \beta g_t + \phi_t \ , \quad 0 \leq \kappa \leq 1 \text{ and } \beta > 0 \ . \tag{4.10}$$

The exact size of the parameters κ and β depends mainly on the extent of wage and price rigidities which are included in $\widetilde{\pi}_t$. If markets were completely flexible, κ would be equal to one and β equal to zero, i.e. the stochastic purchasing power parity would hold for both the tradeable and the non-tradeable good sector.[15]

Finally, ϕ_t is a random shock, which describes the current effect of the PPP-shock ε_t on overall inflation π_t. Analogously to the previous chapter, we assume that ϕ_t follows an AR(1)-process:[16]

$$\phi_t = \eta \phi_{t-1} + u_t \ , \quad \eta \in (0,1) \ , \tag{4.11}$$

where the shock innovations u_t are identically, independently distributed with zero mean and $\sigma_u^2 > 0$ for all t. We, additionally, assume in the sections 4.5 and 4.6.1 that there is no inherited shock from the first period, i.e. $\phi_1 = u_1$. We depart from this assumption in subsection 4.6.2 to establish a fair comparison of a currency board and a peg and allow for some shock persistence from period zero.

To motivate the inflation equation from the viewpoint of empirical evidence, we refer to Catão and Terrones (2003): They have shown by using a sample of 107 countries

[15] In principal, $\beta < 0$ would also be possible if fiscal policy is mainly characterized by granting production subsidies. As this seems to be highly implausible for developing countries and emerging market economies, on which we focus here, we strictly exclude $\beta < 0$.

[16] As explained already in chapter 3, ϕ_t represents an asymmetric shock that changes the equilibrium real exchange rate, which reflects e.g. asymmetric business-cycle movements, demand-side effects and exchange rate movements between the anchor currency and the currencies of third countries. $\phi_t > 0$ corresponds to the necessity of a real appreciation and $\phi_t < 0$ corresponds to the necessity of a real depreciation, respectively.

over the period from 1960 until 2001 that there is a strong positive association of fiscal deficit and inflation.[17] This result can be used as a further justification of the form of our inflation equation and especially for the influence of government expenditure g_t on inflation. Remember in this context that, by definition, g_t is closely linked to the fiscal deficit.

The policy makers' objective is to minimize a quadratic loss function which depends on present and future expected inflation, and output as well as on outstanding debt at the end of the world T. The intertemporal loss function is given by

$$\begin{aligned} \Lambda_t &= \sum_{s=t}^{T} \rho^{s-t} E_t \left[L_s(\pi_s, y_s) \right] + \frac{1}{2} \rho^{T-t} \theta_b E_t \left[b_T \right]^2 + \rho^{T-t} \delta c^i \qquad (4.12) \\ &= \sum_{s=t}^{T} \frac{1}{2} \rho^{s-t} E_t \left[(\theta_\pi \pi_s^2 + (y_s - k)^2) \right] + \frac{1}{2} \rho^{T-t} \theta_b E_t \left[b_T \right]^2 + \rho^{T-t} \delta c^i, \end{aligned}$$

where ρ is the government's intertemporal discount factor, $E(b_T)^2$ is the expected loss resulting from outstanding real debt in the final period T and $\theta_b > 0$ is the policy makers' relative weight on these debts.[18] Analogously, θ_π is the policy makers' relative weight of the inflation target. Political costs c^i arise, whenever a policy maker under a fixed exchange-rate system, i.e. for our case under a standard peg or a currency board, decides to realign its exchange rate peg or central rate, respectively. δ is a dummy variable, which equals one when leaving the peg or the currency board and equals zero otherwise. The superscript i of political costs denotes the type of the exchange-rate system to be withdrawn, i.e. a standard peg system or a currency board arrangement. The policy maker wants to push output above its natural level, which is characterized by the parameter k. Therefore, the parameter k together with the weight of the inflation target θ_π, can be considered as the main factors which determine the time inconsistency problem of the policy makers, like in chapter 3.

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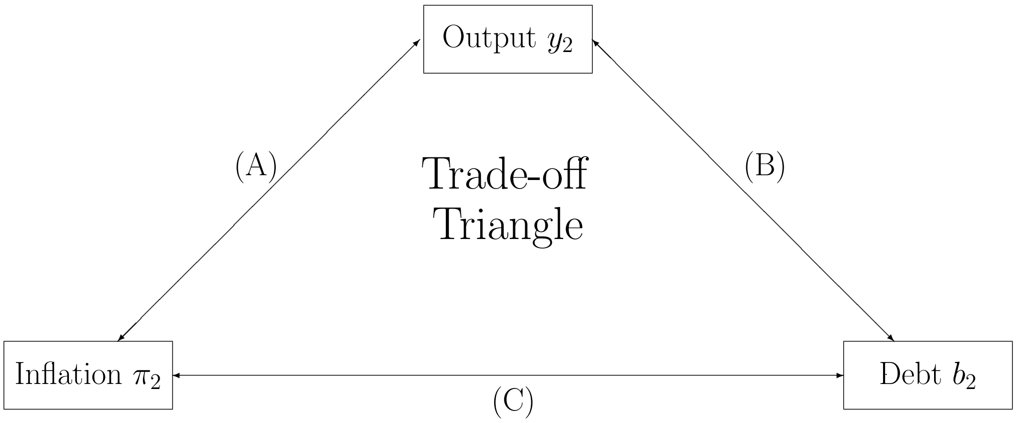
[17] The sample used in Catão and Terrones (2003) comprises advanced countries, emerging-market countries and other developing countries. They find that "fiscal deficits have been shown to matter not only during high hyperinflations but also under moderate inflation ranges [...]". Furthermore, the positive correlation between the fiscal deficit and inflation appeared significantly in all groups of countries and "surprisingly strong over a broad range of developing countries [...] ", (p.26). These are exactly the countries in the center of our analysis.

[18] By applying the debt term in the loss function, we guarantee that debt accumulation is not for free: This means that we avoid costless accumulation of debt, i.e. this assumption guarantees that the policy maker abstains from creating an unlimited amount of debts.

Figure 4.1: Debt-, Inflation- and Output Target



We restrict our further examination to a time-horizon of two periods, by focussing on the policy makers' decisions for the second period. Therefore, we rewrite (4.12) and consider the second-period loss under the assumption that the "world will end after that period". Then, L_2 is given by[19]

$$L_2 = \frac{1}{2}\left[(y_2 - k)^2 + \theta_\pi \pi_2^2 + \theta_b b_2^2\right] + \delta c^i \ . \tag{4.13}$$

To summarize, the government dislikes inflation and debts, however, it prefers more output to less, even beyond the natural output level (depending on k). The structure of the model implies that several trade-offs may occur, which are illustrated by the *Trade-off Triangle* in figure 4.1 and explained in the following.

A) *Inflation-output Trade-off:* On the one hand, the policy maker has an incentive to push output above its natural level by creating surprise inflation, on the other hand, inflation itself contributes directly to the policy loss.

B) *Output-debt Trade-off:* Fiscal policy has an incentive to push output to the desired level via its policy instrument g_2, at the same time fiscal policy (or to be more precisely the fiscal deficit) has to be financed by an increasing debt which accounts by itself for a policy loss.

C) *Debt-inflation Trade-off:* On the one hand, the policy maker has an incentive to lower its real debt level by a rise of the inflation rate, on the other hand, inflation itself contributes directly to the policy loss.

[19]Note that the intertemporal exposition of the loss function is needed to motivate the second-period debt term $\theta_b b_2^2$ in the loss function.

Summary of the Main Model Equations

To keep track of the main equations of the model to which we refer in the following sections, we illustrate them here again in the two-period notation:

- Policy Makers' Target Function:

$$L_2 = \frac{1}{2}\left[(y_2 - k)^2 + \theta_\pi \pi_2^2 + \theta_b b_2^2\right] + \delta c^i \ . \tag{4.13}$$

- Output Equation:

$$y_2 = \gamma(\pi_2 - \pi_2^e) + w g_2 \tag{4.1}$$

- Debt Equation:[20]

$$b_2 = (1 + q e_2 - \pi_2) b_1 + g_2 \tag{4.6}$$

- Inflation Equation:

$$\pi_2 = \kappa e_2 + \beta g_2 + \phi_2 \tag{4.10}$$

4.4.2 Timing of Political Decision-Making

The currency board system is characterized as in chapter 3. A key feature of the currency board is its establishment by law. Enoch and Gulde (1998) consider a sound legal basis as an essential issue when introducing a currency board, "because a currency board arrangement derives much of its credibility from the changes required in the central bank law concerning exchange rate adjustments", (p.42). The legal anchor of the currency board arrangement gives reason that an abandonment of the system is not possible over night.[21] As a currency board is established by law, we state that it can only be repealed, if this was announced one period in advance, i.e. before the private sector made its expectations on inflation. The sequence of decision-making of the private sector and the fiscal and monetary authority is depicted in figure 4.2.[22] In our model, the decision whether to

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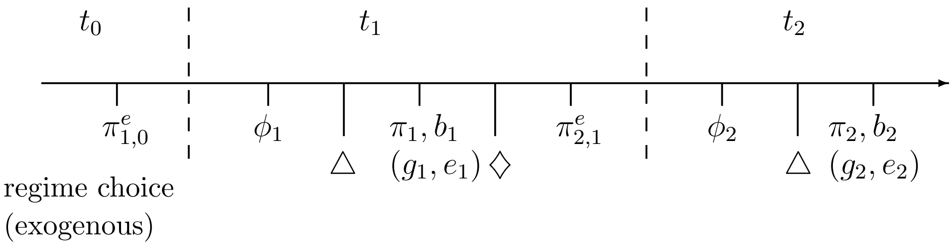
[20] To be correct, b_1 denotes the inherited debt level which also includes nominal interest rate payments, and it should, thus, be denoted by $\tilde{b}_1$ to be consistent with the definition above. In a two period-setting, in which "the world ends" after period 2, we can drop the tilde for simplicity; but, the reader should note that, henceforth, b_1 comprises the inherited debt level including nominal interest rates.

[21] For a reference to the legal base of currency boards in practice see table 4.1 again.

[22] As we, typically, consider countries with a credibility and stability problem, the central bank may not be acting independently as aforementioned. Therefore, we assume that the central bank and the government act simultaneously and can be considered as a "single authority". We further assume, that strategic behavior of fiscal and monetary authorities does not matter here.

Figure 4.2: Sequence of the Model



repeal the currency board in a certain period or not is announced one period in advance – here in the picture at time $\diamondsuit$ before the private sector forms second period expectations $\pi^e_{2,1}$.

In the case of a standard peg, the fixed exchange rate can be abandoned after observing the shock which would actually be possible in each period and is labeled in figure 4.2 by $\triangle$. As we, analogously to chapter 3, presume that a standard peg has some commitment value, too, we assume that the policy maker will leave the peg earliest after the realization of ϕ_2. This means that a standard peg system survives at least in the first period after its introduction.[23] The private sector has rational expectations about inflation $\pi^e_{2,1}$, which are formed before the future shock ϕ_2 is observed. For a better understanding of the sequence, we use the notation $\pi^e_{2,1}$ in figure 4.2 for the private's expected inflation of the second period, which was created in the first period. For reasons of brevity. we use in the calculations only the first indicator, which means $\pi^e_{2,1}$ is rewritten as π^e_2.

If the exchange rate is fixed, the monetary authority has to abstain from an active policy and surprise inflation can only be created by a rise in government expenditure g_t. If the exchange rate is flexible, inflation can be influenced by a change of the nominal exchange rate and by fiscal policy.[24]

[23]We do not model the optimal regime choice. In this model, we compare the losses occurring under a standard peg regime and a currency board system. Both systems are introduced in period 0.

[24]Note that in figure 4.2 the arguments in the brackets, g_t and e_t, are the policy variables of the fiscal and monetary authorities through which the targets π_t, y_t and b_t are determined.

4.5 Government Debts and Fiscal Discipline

4.5.1 Comparison of a Currency Board and a Standard Peg Regime

Subject of this section is to find out whether a standard fixed-exchange rate regime or a currency board system induces higher fiscal discipline and a more conservative amount of debts. Hence, we do not focus on the decision of maintaining or abandoning both types of fixed exchange-rate regimes. In fact, we aim at a ***comparison of the fiscal policy and the choice of the debt level under a currency board and a standard peg while both systems are maintained.***

To be more precisely: The idea is to elaborate a theoretical explanation for the findings of the empirical literature demonstrated in chapter 4.3, which says that currency board systems increase fiscal discipline. Therefore, we compare losses occurring under both exchange-rate regimes from an *interim perspective*.[25] This means that we make the assumption that the fixed exchange rate is maintained in the second period under both regimes (a standard peg and a currency board), but their survival *was not obvious ex ante* and, therefore, the private sector may have a positive devaluation expectation under a peg.

The optimization problem under both systems follows the same pattern. The systems differ only from the expected inflation rate of the private sector due to the different timing of political decision-making. We refer to this point again at the end of this subsection.

As the monetary authorities abstain from a devaluation, the resource constraint is simply given by

$$b_2 = b_1 - b_1\pi_2 + g_2 \ . \tag{4.14}$$

Note that we have no longer to distinguish between domestic and foreign debts, because exchange rate movements are excluded here: Neither a revaluation, which would reduce the net value of real foreign debts, nor a devaluation, which would increase the real foreign debt level, will take place. The policy maker optimizes the second period loss function

$$L_2 \;=\; \frac{1}{2}(y_2 - k)^2 + \frac{1}{2}\theta_\pi \pi_2^2 + \frac{1}{2}\theta_b b_2^2$$

[25] Note that it makes no sense to compare fiscal policy outcomes of both fixed exchange-rate regimes, if one or both systems have already been repealed. This problem belongs to another interesting aspect, i.e. the stability of both systems, and is analyzed in section 4.6.1.

with respect to the debt restriction (4.6), the output equation (4.1) and the inflation equation (4.10). The first-order conditions of the problem are given by

$$
\begin{aligned}
(y_2 - k) + \lambda_1 &= 0 && (4.15)\\
\theta_\pi \pi_2 - \gamma\lambda_1 + \lambda_2 + \lambda_3 b_1 &= 0 && (4.16)\\
\theta_b b_2 + \lambda_3 &= 0 && (4.17)\\
w\lambda_1 + \beta\lambda_2 + \lambda_3 &= 0\ , && (4.18)
\end{aligned}
$$

and by the three restrictions (4.1), (4.6) and (4.10). The optimization problem is solved in appendix B.1, where we determine the optimal values for b_2, π_2, y_2, and g_2 analytically.

For a better exposition, we define $A := w+\gamma\beta$ and $B := (1-\beta b_1)$ in appendix B.1. Due to the domain of the parameters, it is obvious that A is strictly positive. B is also assumed to be positive, as for developing countries and transition economies total government debt ratios, b_1, lie typically between 20% and 60% of GDP and as the parameter β is supposed to be sufficiently small. Note in this context that the Argentinean default on sovereign debts in 2002 happened to be at a level of public debts of around 60% of GDP. Applying the abbreviations A and B, we obtain the optimal debt value

$$b_2^* = \frac{(k + \gamma\pi_2^e + wb_1)AB}{\theta_b B^2 + A^2 + \theta_\pi \beta^2} - \frac{(\phi - \beta b_1)(\theta_\pi \beta + A(\gamma + wb_1))}{\theta_b B^2 + A^2 + \theta_\pi \beta^2}\ . \tag{4.19}$$

We also obtain the optimal values for π_2, y_2 and g_2 given by

$$\pi_2^* = \frac{\beta A(k + \gamma\pi_2^e) + \phi_2(\theta_b B + wA) - \theta_b \beta B b_1}{\theta_b B^2 + A^2 + \theta_\pi \beta^2}\ . \tag{4.20}$$

The optimal rate of inflation depends positively on the second period shock, ϕ_2, and it is pushed by the desired output level k above its natural level. On the contrary, a higher weight of the debts in the loss function, θ_b, may reduce inflation: In the considered case, the central bank cannot create inflation to reduce (the domestic part) of the real public debts, because the exchange rate is fixed. Therefore, a reduction of government expenditure, which actually lowers inflation, remains the only way how fiscal policy can reduce debts. A higher weight of inflation in the loss function, θ_π, makes inflation more costly and, hence, contributes to lower inflation.

Optimal output amounts to

$$y_2^* \;=\; \frac{kA^2 - \theta_b b_1 AB - \gamma\pi_2^e(\theta_b B^2 + \theta_\pi \beta^2) + \phi_2(\theta_b B(\gamma + wb_1) - \theta_\pi w\beta)}{\theta_b B^2 + A^2 + \theta_\pi \beta^2}\ , \tag{4.21}$$

where y_2 depends positively on the second period shock, ϕ_2, due to our assumption of a very small ω and a relatively small β. Output depends also positively on the desired output target k.

The optimal fiscal policy is given by

$$g_2^* = \frac{(k+\pi_2^e)A + \phi_2(\theta_b B - \theta_\pi \beta + A(w+\gamma+wb_1)) + b_1(A(\gamma\beta - 1) + w - \theta_b B)}{\theta_b B^2 + A^2 + \theta_\pi \beta^2} \;, \tag{4.22}$$

where a high desired output target k and a large γ, the parameter which measures the effect of surprise inflation on output, both lead to an incentive to create surprise inflation by raising demand and, hence, contribute to the policy makers' optimal choice of a higher fiscal deficit g_2. In contrast, an extremely high θ_b reduces government expenditure as the reduction of debts is of major interest.

In the following, we calculate the first derivatives of the policy variables with respect to π_2^e and show that the signs are univocally determined:

$$\frac{\partial b_2}{\partial \pi_2^e} = \frac{\gamma A B}{\theta_b B^2 + A^2 + \theta_\pi \beta^2} > 0 \tag{4.23}$$

$$\frac{\partial \pi_2}{\partial \pi_2^e} = \frac{\gamma \beta A}{\theta_b B^2 + A^2 + \theta_\pi \beta^2} > 0 \tag{4.24}$$

$$\frac{\partial g_2}{\partial \pi_2^e} = \frac{A}{\theta_b B^2 + A^2 + \theta_\pi \beta^2} > 0 \;. \tag{4.25}$$

As the expected inflation rate under a currency board is according to empirical evidence smaller compared to the expected inflation under a standard peg regime, the following result holds:[26]

Result 1. *The lower π_2^e the higher is the discipline of fiscal policy and the lower are the debts under the condition that the exchange rate is not changed.*
As a currency board is associated with a lower expected inflation compared to a standard peg system, a currency board leads to more fiscal discipline and to a lower debt level.

Furthermore, we can show that

$$\frac{\partial y_2}{\partial \pi_2^e} = -\frac{\gamma(\theta_b B^2 + \theta_\pi \beta^2)}{\theta_b B^2 + A^2 + \theta_\pi \beta^2} < 0 \;, \tag{4.26}$$

i.e. output depends negatively on the inflation expectations. This stands in line with the empirical findings of e.g. Ghosh et al. (2000) when analyzing the effects from

[26] Empirical evidence and a theoretical proof of $\pi_2^{e,CB} < \pi_2^{e,Peg}$ is added in the subsection 4.5.2. To be more accurate, the lower expected inflation rate under a currency board compared to a standard peg regime can be traced back to a lower expected devaluation under a currency board compared to a peg.

introducing a currency board arrangement on economic growth. Therefore, we draw the following conclusion:

Result 2. *The choice of a currency board can contribute to a higher growth of output due to the lower inflation expectations of the private sector.*

4.5.2 Inflation Expectations

In this subsection, we compare the expected inflation of the private sector under a currency board system and a standard fixed exchange-rate regime. At first, we derive the condition for which the expected inflation rate is lower under a currency board arrangement than under a standard peg by using a theoretical approach to justify the assumption made in the previous subsection. Subsequently, we refer to empirical work suggesting that a currency board system may contribute to higher credibility which is expressed by lower inflation expectations.

Theoretical Approach

To derive inflation expectations, we use a similar calculation as in section 4.5.1. The policy maker optimizes the following Lagrangean, where we introduce e_2, which denotes the change of the second period exchange rate:

$$\begin{aligned}\mathcal{L}_2 &= \frac{1}{2}(y_2-k)^2+\frac{1}{2}\theta_\pi\pi_2^2+\frac{1}{2}\theta_b b_2^2 \\ &\quad +\lambda_1(y_2-\gamma(\pi_2-\pi_2^e)-wg_2) \\ &\quad +\lambda_2(\pi_2-\phi_2-\kappa e_2-\beta g_2) \\ &\quad +\lambda_3(b_2-b_1(1+qe_2-\pi_2)-g_2)\ . \end{aligned} \tag{4.27}$$

At first, we treat e_2 like an additional parameter. The idea is to derive the expected inflation rate depending on an expected change of e_2, to show in which direction a devaluation drives the expected inflation π_2^e.[27]

The calculation follows the same pattern as in the previous subsection and is shifted to appendix B.3. By assuming rational expectations of the private sector, expected inflation

[27] Our analysis is limited to cases of a devaluation, as the need of an appreciation under a fixed exchange-rate regime is assumed to be less problematic as it usually does not indicate a typical crisis scenario and is thus less interesting for the purpose of our analysis.

π_2^e is determined by

$$\pi_2^e = \frac{\beta Ak + \eta\phi_1(\theta_b B + wA) - b_1\beta\theta_b B + e_2^e(w\kappa A + \theta_b B(\kappa - \beta b_1 q))}{\theta_\pi \beta^2 + wA + \theta_b B^2} . \tag{4.28}$$

Due to the announcement of maintaining or abandoning the currency board one period in advance, the expected inflation rate under a currency board is given by

$$\pi_2^e\Big|_{e_2^e=0} = \frac{\beta Ak + \eta\phi_1(\theta_b B + wA) - b_1\beta\theta_b B}{\theta_\pi \beta^2 + wA + \theta_b B^2} . \tag{4.29}$$

Under a standard peg regime, a sudden realignment after the observation of the second period shock, ϕ_2, is possible. How the private sector's expectation about second period inflation is influenced by an expected devaluation of e_2^e is given by

$$\frac{\partial \pi_2^e}{\partial e_2^e} = \frac{w\kappa A + \theta_b B(\kappa - \beta b_1 q)}{\theta_\pi \beta^2 + A^2 + \theta_b B^2} . \tag{4.30}$$

If foreign debts are not too high represented by a relatively small value of q and the economy is quite open (represented by κ approaching one), the sign of the terms inside the brackets is positive and an expected devaluation causes an increase of privates' inflation expectations.

Furthermore, we know from the common literature that by assuming a certain distribution of the new shock u_2, multiple equilibria may occur under a standard peg regime, as pointed out in chapter 3. As we do not focus on the exact value for the inflation expectations of both systems, but on a qualitative comparison, we do not calculate the equilibria explicitly. If we assume that an appreciation of the exchange rate is excluded as in the graphical explanation of the equilibrium exchange rate in Obstfeld (1996), i.e. $e_2 \geq 0$, we can avail ourself on the fact that the private sector is aware that with a certain probability – depending on the realization of the second period shock – the policy maker will make use of her escape clause and devalue the currency under a standard peg system. Therefore, we can state that the expected inflation rate is a (probability weighted) mixture of the expected inflation rate for the case of defending the peg, which equals that of a currency board system given by (4.29), and for the case of devaluing the exchange rate, given by equation (4.28). According to the sketched idea, the following result holds:

Result 3. *The private sector has a lower expected inflation under a currency board arrangement than under a standard peg system, if* $(w\kappa A + \theta_b B(\kappa - \beta b_1 q)) > 0$.
This condition is unambiguously fulfilled if the fraction of foreign debts relative to total

debts is not too large and the economy is relatively open, i.e. $\kappa > \beta b_1 q$.[28]

As a stable fixed exchange-rate system is typically accompanied by a relatively small ratio of foreign debts to total debts and a high openness towards the anchor currency, κ, it may be no major restriction to use result 3, which states that $\pi_2^{e,CB} < \pi_2^{e,Peg}$, in section 4.5.1.

However, the opposite may be true for a relatively closed economy characterized by a small value of the parameter κ and a large amount of public debts denominated in the foreign currency: If the government devalues its currency, the real value of outstanding debts increases. Therefore, a high level of foreign debts can have two different effects: It may prevent the government from a devaluation and thereby implies fiscal discipline or it may force the government to default on its debts (as it happened in Argentina in 2001 as highlighted in section 4.3).

Note that it is easy to understand the argumentation used here, when a revaluation of the exchange rate is excluded. However, when referring to the common literature on time inconsistency, a desired output level above the natural level creates an inflation bias, which makes a devaluation more likely than a revaluation. Therefore abstaining from the possibility of revaluation is not necessary for obtaining result 3, but makes it more easy to capture the line of arguments and avoids at the same time the distinction of further cases, which are necessary to examine when inherited shocks and debts from the past are considered.

Empirical Findings

The two common proceedings to estimate inflation expectations are to evaluate survey data or to use the difference of the nominal and real interest rates of non-indexed and indexed governments bonds as a proxy for expected inflation. Survey data can only be obtained for Bulgaria and government-bond yields (especially those of indexed bonds) are hardly available for any currency board country on a monthly basis, which would be necessary to obtain enough observations for a meaningful estimation. Therefore, we abstain from an own estimation and refer to two empirical papers, which suggest that expected inflation under a currency board is lower than under a standard fixed exchange-rate regime.

Ghosh et al. (2000) compare inflation rates for countries with currency boards and

[28] If we consider the extreme case that a standard peg is highly credible, i.e. the probability of a devaluation equals zero, the expected inflation is the same under both systems.

standard peg regimes and show that inflation under currency board regimes is lower than under standard fixed-exchange rate regimes. The findings can be used for claiming from an ex post view that also the inflation expectations should be lower when operating a currency board.

Carlson and Valev (2000) use survey data for Bulgaria to analyze whether the introduction of a currency board system lowers expectations of inflation. The survey was conducted a short time before the currency board was introduced and a follow-up survey was conducted 10 months later. The authors show that already in the first survey, the people had a lower expected inflation rate due to the near introduction of the currency board arrangement. In the follow-up survey, it became obvious that the introduction of a currency board did indeed lower inflation expectations as well as actual inflation rates.

4.6 Stability of a Currency Board System

We analyze the stability of currency board systems by focussing on two aspects. First, we try to figure out under which conditions a policy maker operating a currency board system announces the continuity or the abandonment of the currency board. We examine thoroughly how the change of some characteristic parameters like e.g. the policy makers' desired output deviation k, the volatility of the PPP-shock σ_u^2, the inherited shock from the first period $\eta\phi_1$, the inherited debt level b_1 or the fraction of foreign debts on total public debts q can influence the policy makers' decision. We use a numerical approach to obtain the results.

Second, we compare the stability of a currency board system with that of a standard fixed exchange-rate regime for a given set of parameters. Here, we use a similar concept of the *credibility of an exchange rate system* as in chapter 3, which is referred to the probability that the considered exchange rate regime survives the second period. Analogically to the proceedings of the first part, we examine how the variation of some characteristic parameters changes the credibility of a standard peg compared to that of a currency board system by using four numerical scenarios. We derive the conditions under which the currency board system induces a credibility advantage.

4.6.1 Decision-Making under a Currency Board

As the decision whether to repeal or to maintain a currency board regime is made before the "new shock" is realized, we have to compare the expected losses of the next period for both cases. We suppose that, whenever a monetary authority decides to repeal the currency board, the exchange rate will be adjusted optimally, which means that the new system can be characterized by a free floating exchange rate regime in our two-period framework.[29] The policy maker announces the continuity of the currency board system in the next period if the expected loss of the free float plus political costs c^{CB}, which arise when giving up the currency board, exceed the expected loss when maintaining the currency board. Therefore, the critical threshold where the policy maker is indifferent between both systems is determined by

$$E(L_2^{float}) + c^{CB} = E(L_2^{CB}) \ . \tag{4.31}$$

The solution is briefly sketched in appendix B.4. To gain insights into the solution, we use two numerical examples in the following for a quantitative exploration. Our analysis focusses on situations, in which the policy makers' main problem is whether to devalue or not. Therefore, we assume that besides the existence of the time-inconsistency problem, a negative shock hits the economy in the first period. We make this assumption, as we focus on examining situations characterized by the existence of a credibility problem, which usually do not occur when a currency is revaluated.

The numerical values of the parameters in a first scenario and a short explanation of each parameter are depicted in table 4.2. In this scenario, the expected second period loss under a currency board system is lower compared to a free float system. The exact numerical values for the expected losses in both cases are shown in table 4.3. Therefore, in this scenario the policy maker would announce the continuity of the currency board in period 1 — before the privates negotiate their wages (= build their expectation of the second period inflation rate) and before the PPP-shock of the second period hits the economy. However, we do not focus on the explicit values of the losses under this scenario. In fact, the aim is to show how the variation of a particular parameter value can influence the expected losses in both cases and, thus, the stability of the currency board system, while keeping the rest of the parameters fixed. We measure *the stability of the currency board* by the *distance of the expected loss function in the currency board and the free float case* in this section. The sensitivity analysis of the expected losses of scenario I is done in figure 4.3.[30] The pictures confirm the results found in chapter 3, where the credibility of a

[29]Note that in this subsection, when referring to the free float system, we discuss the case of announcing the abolishment of the currency board arrangement.

[30]Note that the vertical line in the single plots denotes the parameter set, which was used in Scenario I.

Table 4.2: Numerical Example, Scenario I

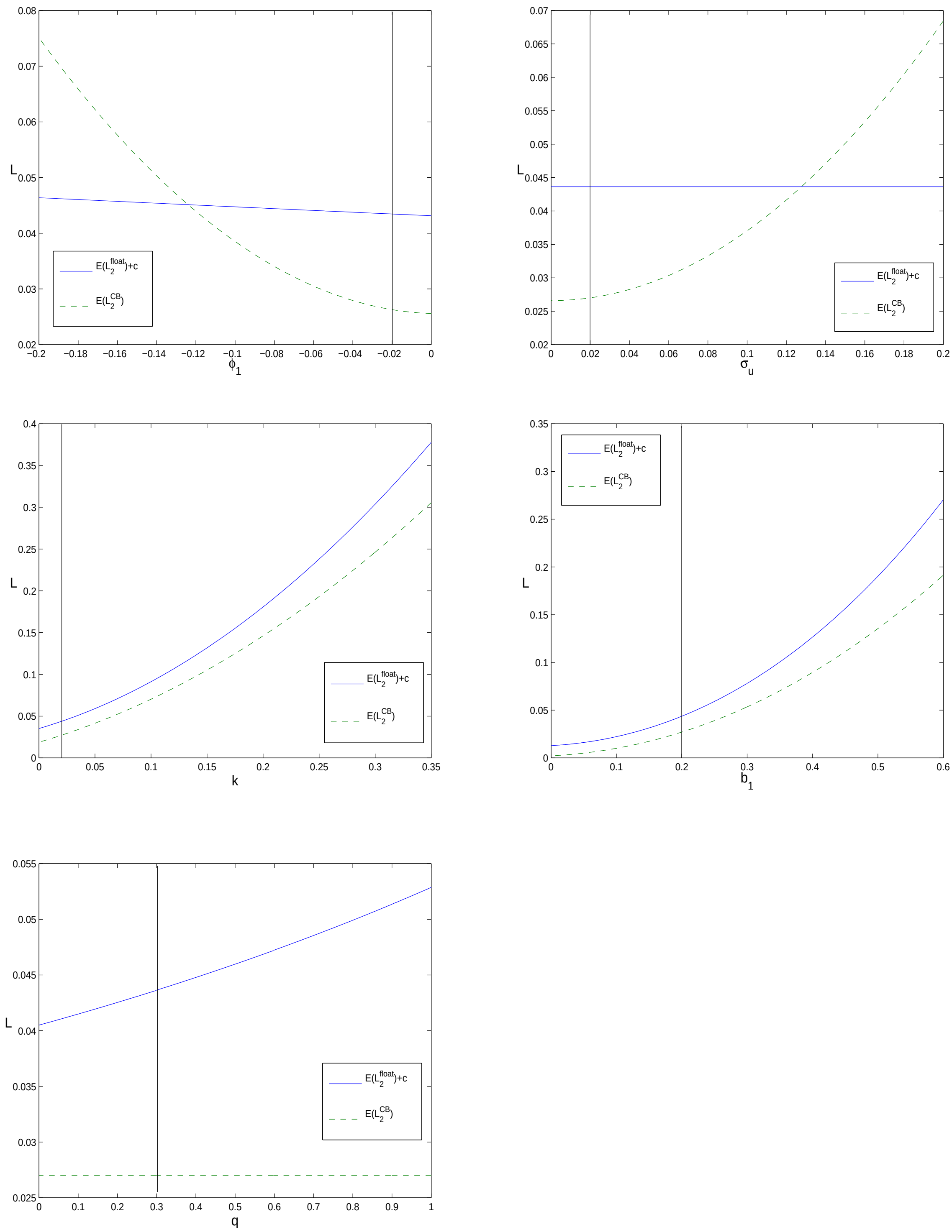
Parameter	Numerical Value	Explanation
k	0.02	Desired output deviation from the natural level
γ	1.00	Effect of surprise inflation on output
w	0.09	Effect of fiscal policy on output
		(only fiscal policy addressed upon the supply side)
b_1	0.20	Inherited stock of public debts
q	0.30	Ratio of foreign public debts to total public debts
κ	0.80	Effect of the change of the exchange rate on inflation
β	0.30	Effect of fiscal policy on inflation
		(only fiscal policy addressed upon the demand side)
ϕ_1	-0.03	PPP-shock of the first period
σ_u	0.02	Standard deviation of the PPP-shock
η	0.80	Shock persistence
θ_b	0.2	Policy makers' relative weight on debt target
θ_π	1.20	Policy makers' relative weight on inflation target
c^{CB}	0.012	Political costs

Table 4.3: Expected Losses in Scenario I

Expected loss under a currency board	Expected loss under a free float
$E(L_2^{CB}) = 0.0270$	$E(L_2^{Float}) + c^{CB} = 0.0436$

currency board is examined without taking the effects of fiscal policy and public debts into account: A high (negative) inherited shock from the first period ϕ_1 and a high volatility of the shock innovation in the second period σ_u^2 increases the expected second-period loss under a currency board relative to the expected loss under free float system. This result stems from the fact that an announcement to maintain the currency board system prevents the policy maker from offsetting the shock in the second period. Therefore, when the first period shock reaches some critical threshold the policy maker will announce to repeal the currency board in the second period to lower its expected policy loss. A high volatility of the "new shock" works in the same direction, as it increases the risk that the economy will be hit by a large shock in the second period to which the policy maker cannot react under a currency board system.

Figure 4.3: Sensitivity of Expected Losses (Scenario I)



A larger desired output level, k, which together with a relatively small θ_π reflects the main cause for the time inconsistency problem of fiscal and monetary policy (i.e. fiscal and monetary policy try to create surprise inflation to raise output), leads to a higher stability of the currency board system. The time inconsistency problem of monetary policy is solved under a currency board as the monetary authorities' decision is well-known at the point in time where the privates create their expectations of inflation (see also chapter 4.5). However, the time inconsistency problem of fiscal policy given by the desire of the policy maker to create surprise inflation by raising government demand persists.

The reason for an increasing level of k to improve the stability of a currency board system, anyway, is based on the quadratic loss function, i.e. a higher deviation from the target levels leads to a disproportionately high increase of the policy loss. In the case of repealing the currency board arrangement, the policy maker can mix two instruments to raise inflation, the exchange rate (e_2) and government expenditure (g_2), whereas when maintaining the currency board only fiscal policy (g_2) is available. An increase in g_2 primarily leads to higher debts and an increase of e_2 primarily leads to an increase of inflation. As both, debts and inflation enter the loss function quadratically, the costs of each instrument increase disproportionately high by more intensive use. So, in the case of repealing the currency board, a higher value of k leads to a disproportionately higher incentive to generate surprise inflation, compared to the case when the currency board is maintained. Hence, the time inconsistency problem is more severe in the case of abandoning the currency board, which is also reflected by a higher expected inflation of the private sector. Therefore, the result is still in line with chapter 3, where merely the time inconsistency problem of monetary policy existed.

The last two pictures of figure 4.3 suggest that a rise in the inherited debt level, b_1, and an increasing fraction of foreign debts on total debts, q, leads to a higher stability of the currency board system:

A rise in b_1 makes the *time inconsistence problem of monetary policy more severe*, as the monetary authority has now the incentive to create inflation for two reasons: (i) create surprise inflation to push output above the natural level, and (ii) create inflation to devalue the level of outstanding real government debts. As the currency board solves the time-inconsistency problem of monetary policy, a higher b_1 leads to a higher stability of the currency board.[31]

[31] Note that we solidly focus on the consequences in variations of b_1 in our model, where we do not incorporate the possibility of a default on government debts or binding credit market constraints, which

A higher value of q does not influence the loss if the currency board is maintained, i.e. a devaluation is excluded. However, a desired output level above the natural level and a negative first period shock (see the parameter values in example 1) as well as a high π_2^e triggers the policy maker to a relatively large devaluation when an exit of the currency board arrangement was announced before, and the exchange rate can be adjusted freely. The same devaluation leads in case of a larger q to a higher growth of real debts, where the growth rate itself depends on the size of foreign debts, meaning that a higher q contributes to higher policy loss and, hence, it makes a currency board more stable.

To abstain from sweeping something under the rug, we point out that when b_1 and k are varied the result may be contrary if the policy maker puts an extremely high weight on public debts in the loss function, which is given by an extraordinary high θ_b. However, as for developing countries such an extremely conservative behavior is not observable, our results of example 1 seem to be robust with respect to the most relevant political scenarios.

In the following, we apply our analysis once more to a "stressful" scenario for a policy maker operating a currency board system. Now, a larger negative PPP-shock inherited from the first period, a higher shock volatility, a greater weight of the inflation target in the loss function and zero exit costs are assumed in scenario II.[32] The exact parameter values used here are given by table 4.4. The choice of the parameter set enables to exhibit

Table 4.4: Numerical Example, Scenario II

Parameter	Numerical Value	Parameter	Numerical Value
k	0.02	ϕ_1	-0.06
γ	1.00	σ_u	0.04
w	0.09	η	0.80
b_1	0.30	θ_b	0.20
q	0.30	θ_π	2.00
κ	0.80	c^{CB}	0.00
β	0.30		

an example in which, contrary to our first scenario, the policy maker decides to announce

may both play a major role from a practical point of view.

[32]The value of b_1 was also slightly increased for a better graphical exposition, but the same qualitative results could be obtained for $b_1 = 0.2$, which was used in scenario I.

to exit the currency board system in the second period. We make the free float case (i.e. the incentive of abandoning the currency board) more attractive by choosing a higher policy weight of the inflation goal, a higher volatility of the new shock, a raising inherited first period shock, and exit costs of zero. The comparison of the losses of a free float and a currency board is given in table 4.5.

Table 4.5: Expected Losses in Scenario II

Expected loss under a currency board	Expected loss under a free float
$E(L_2^{CB}) = 0.0586$	$E(L_2^{Float}) = 0.0439$

Our main interest of the examination of example II is to find out, whether variations in the parameters $\phi_1, \sigma_u^2, k, b_1$ and q work in the same direction as in example 1. The results are depicted from figure 4.4.

From a qualitative view the results obtained in scenario II stand strongly in line with the results in scenario I. One more aspect, which is emphasized by the plots of this scenario, is noteworthy to mention: The pictures in the second row illustrate, that a currency board has an *intrinsic commitment value*, i.e. its stability does not solely depend on the existence of the political costs c^{CB}, as we find cases in which the continuity of the currency board seems advantageous even when the exit costs c^{CB} are equal to zero. This result holds due to the characteristics of the currency board arrangement that it is defined by law and, therefore, cannot be abolished on short notice.[33] This stands in contrast to the findings of Irwin (2004).

To complete this analysis and to strengthen one central result, we take up once more the variation of k to analyze, how a more severe (overall) time inconsistency problem contributes to the stability of the currency board when directly comparing scenarios I and II.

Figure 4.5 shows that despite a higher shock volatility, a negative inherited first period shock, and zero exit costs the loss of a currency board system is still increasing less intensely when k rises compared to the loss of a flexible exchange rate system and thus a currency board achieves a stability gain. Therefore, we emphasize once more that a

[33] The time inconsistency problem of monetary policy is completely solved, but, the time inconsistency problem of fiscal policy remains. As aforementioned, the overall time inconsistency problem is reduced as surprise inflation is more costly if fiscal policy remains the only instrument to increase inflation. This is the reason for the intrinsic credibility of a currency board arrangement.

Figure 4.4: Sensitivity of Expected Losses (Scenario II)

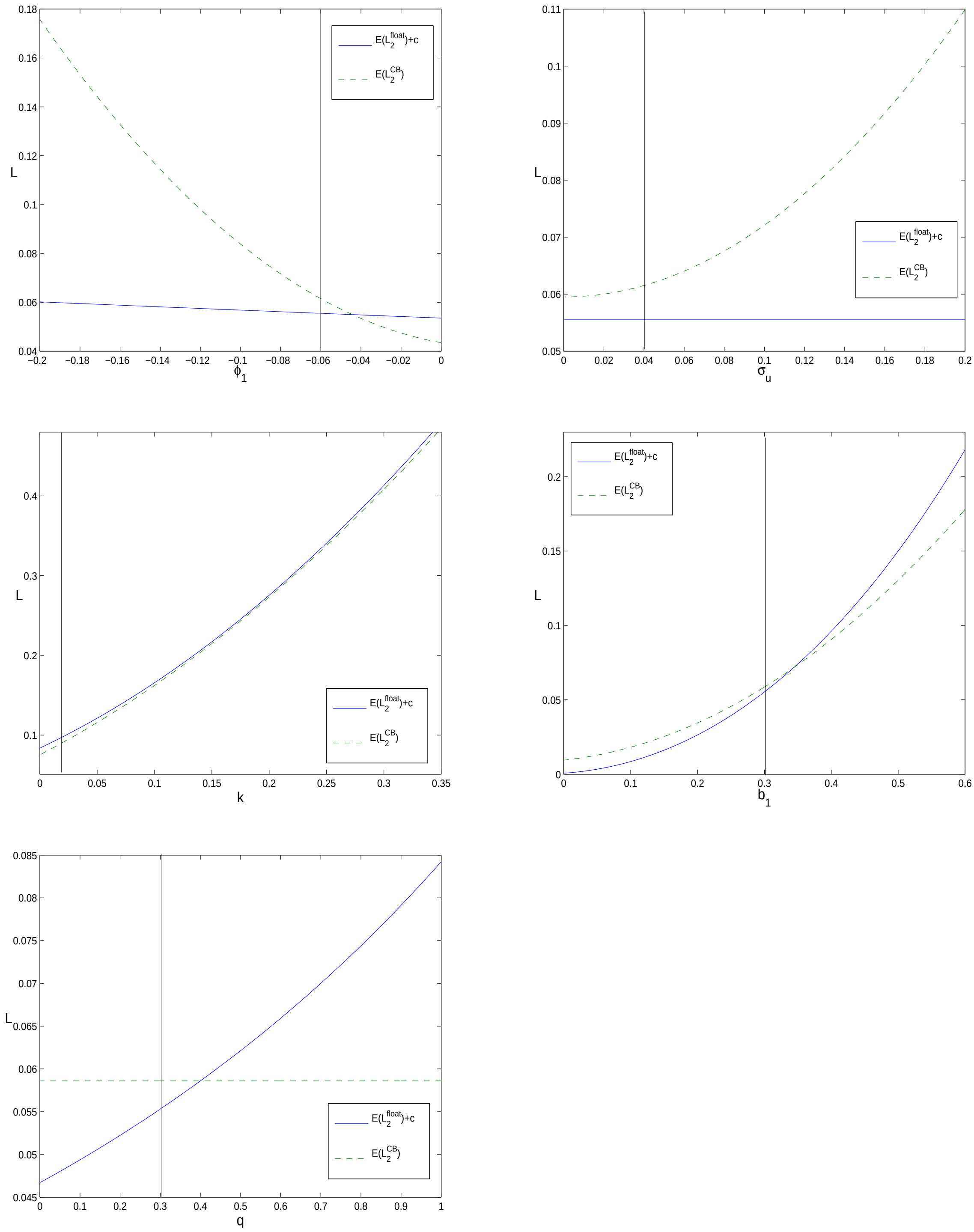
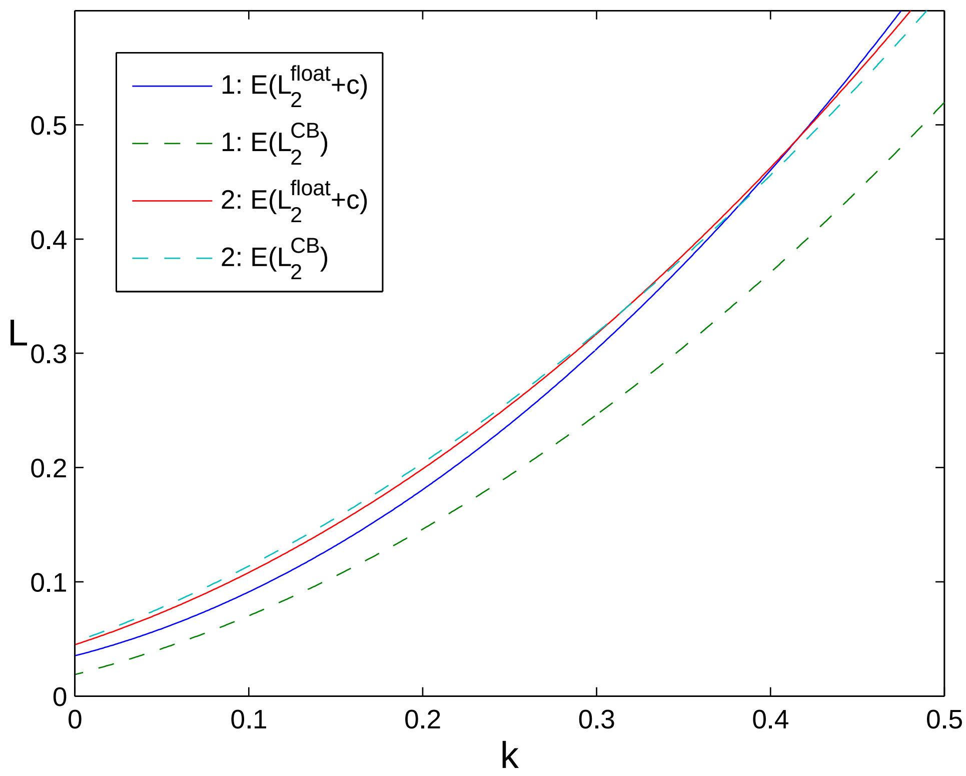


Figure 4.5: Time Inconsistency Problem: Comparison of Scenario I and II



currency board system can reduce the (overall) time inconsistency problem. However, for a given k, of course, the stability of a currency board decreases in scenario II compared to scenario I.

This subsection is concluded by a summary of the central results found in the two scenarios.

Numerical Result 4. *The stability of a currency board system – measured as the difference of the expected losses when deciding to maintain or to repeal the system – is relatively high in cases characterized by*

a) a relatively low volatility of the "new shock" u_2,

b) a relatively low inherited first period shock $\eta\phi_1$,

c) a relatively high fraction of foreign debts q,

d) a relatively high desired output level k,

e) and a relatively high inherited first period debt b_1.

4.6.2 Comparison of a Currency Board and a Standard Peg

The credibility of a (standard) fixed exchange-rate system and a currency board system is measured by the probability that the system survives the second period. The proceeding is motivated by the approach of Drazen and Masson (1994) and Feuerstein and Grimm (2006b). The comparison of both exchange rate systems is done by using the maintaining probabilities of the systems. The technical part of this subsection is shifted to appendix B.5, where the support of the uniformly distributed "new shock" is characterized and the explicit formulas for the probabilities of maintaining the currency board and defending the standard peg system are derived.

In this section we refer to four numerical examples to compare the currency board system and the standard peg regime. The comparison is done in a similar way as in the previous section: We use a given set of parameters and calculate the maintaining probability of a currency board and the probability of defending the peg regime. Then we analyze how the probabilities change when the value of some parameter is varied. In our analysis, we aim at finding the absolute credibility advantage (given if one exchange rate system has a higher maintaining probability than the other one) and at the change of the difference of the maintaining probabilities caused by the variation of a particular parameter. To derive the results from our numerical examples, which are supposed to hold "more generally", we lay a greater importance into the latter case. We begin our analysis with scenario I and scenario II from section 4.6.1 (see tables 4.2 and 4.4). As the probability of maintaining a standard peg regime depends strongly on the exit costs, we add exit costs of $c = 0.012$ in scenario II. We, furthermore, assume that the exit costs under both systems are of the same size. Due to the nature of these costs, however, the repealing of a currency board should go along with higher political costs, as was already pointed out in chapter 3. Hence, the credibility of currency board may be underestimated in several cases. Therefore, the arguments in favor of a currency board are actually even stronger than stated here.

As already mentioned in previous sections, multiple equilibria may occur under a standard peg regime. We abstain here from calculating these equilibria explicitly, but assume analogously to section 4.6.2 that the inflation expectation of the private sector is a mixture of the inflation expectation under a free float system and a currency board system (= ultimate fix):

$$\pi_2^{e,Peg} = \alpha\pi_2^{e,float} + (1-\alpha)\pi_2^{e,CB} \qquad \text{with} \quad \alpha \in [0,1] \;, \tag{4.32}$$

where $\pi_2^{e,CB}$ is the expected inflation under a currency board system when its continuity in the future period has already been announced. We discriminate between three cases of the private sector's inflation expectations under a peg system:

a) Full anticipated credibility: $\alpha = 0$, $\pi_2^{e,Peg} = \pi_2^{e,CB}$

b) Partial anticipated credibility: $0 < \alpha < 1$, $\pi_2^{e,Peg} = (1-\alpha)\pi_2^{e,CB} + \alpha\pi_2^{e,Float}$

c) Zero anticipated credibility: $\alpha = 1$, $\pi_2^{e,Peg} = \pi_2^{e,Float}$

Now, we can calculate the maintaining probabilities for the three peg cases and for the currency board arrangement in scenario I and II. In the following, α is set equal to 0.5 for the case of partial anticipated credibility of the privates. Table 4.6 shows that the currency

Table 4.6: Maintaining Probabilities for the Scenarios I and II

	Currency board	Peg ($\alpha = 0$)	Peg ($\alpha = 0.5$)	Peg ($\alpha = 1$)
Scenario I	1	0.9890	0.9311	0.7882
Scenario II	1	0.5773	0.5318	0.4577

board system is maintained with probability one in both scenarios, which means that the policy maker will always announce to keep on operating the currency board system. Technically speaking, the support of the shock ϕ_1 is a subset of the maintaining-interval of the currency board, which was derived in appendix B.5. The peg system is maintained with a slightly lower probability in scenario I, which is decreasing with an increasing level of α. Table 4.6 also shows that in the second scenario, the peg is maintained with a probability around 50% for all three cases of α. Therefore, the currency board has a credibility advantage compared to a standard peg in both scenarios.

Remark. *Prima facie the result seems unexpected: Although the peg system has one more degree of freedom than the currency board, i.e. a policy maker can decide whether to leave or maintain the peg after the shock realization, and although presuming that $\alpha = 0$, which means that the private sector anticipates full credibility of the peg, a currency board performs better than a standard peg. We, however, do not focus on a welfare comparison, but on a comparison of the credibility/stability of an exchange rate system. Therefore, we state that a policy maker under a currency board maintains the system with a probability of one for the given set of parameters. In contrast, the policy maker operating a standard peg system will abandon the peg and, hence, makes use of the escape clause when a large* *unfavorable shock occurs. For $\alpha = 0$ this can indeed be welfare-improving compared to the welfare achieved under a maintained currency board.*

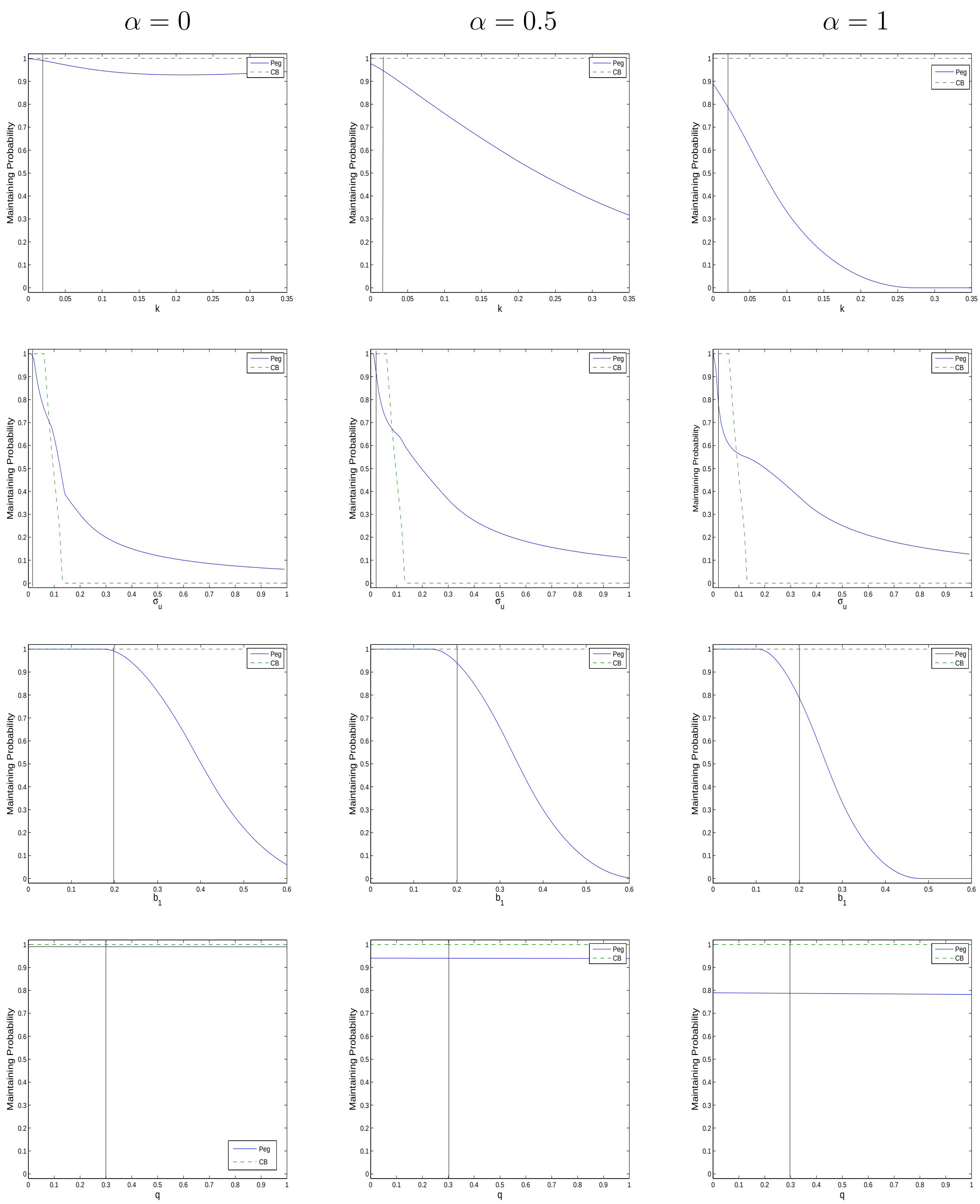
Not the result of scenario I itself, but, rather the explanation which parameters exactly drive the credibility of both exchange rate systems is of major interest in the following. Figure 4.6 shows, how the credibility of both systems react to variations of the characteristic parameters.

The main result, which can be drawn from figure 4.6 is that an increasing volatility of the new shock σ_u^2 decreases the maintaining probability of both systems, but increases the relative credibility of a standard peg regime. The result can be traced back on the timing of the policy makers' decision-making: A high volatility of the shock implies a high risk to bear a large future shock. Whereas under a currency board system a sudden exit in period 2 is not possible, the monetary authority operating a softer peg regime can do a realignment in period 2, surprisingly. Therefore, a currency board is announced to be abandoned with a higher probability in the first period, and the credibility of a standard peg relative to a currency board rises with an increasing σ_u^2.

Furthermore, figure 4.6 suggests that for high values of θ_b, i.e. the policy maker puts a high weight on debts in her loss function, a standard peg gains a relative credibility advantage. A high θ_b triggers the policy maker to create inflation to lower real debts, which is easily possible by a devaluation when the ratio of foreign debts to total debts, q, is small as in the considered case. Therefore, the exit clause in the second period under a standard peg may increase the relative credibility of the peg, meaning that if no favorable second period shock materializes, the policy maker will devalue the currency. This option is not possible under a currency board and, hence, may provide an incentive to the policy maker to repeal the currency board more quickly.

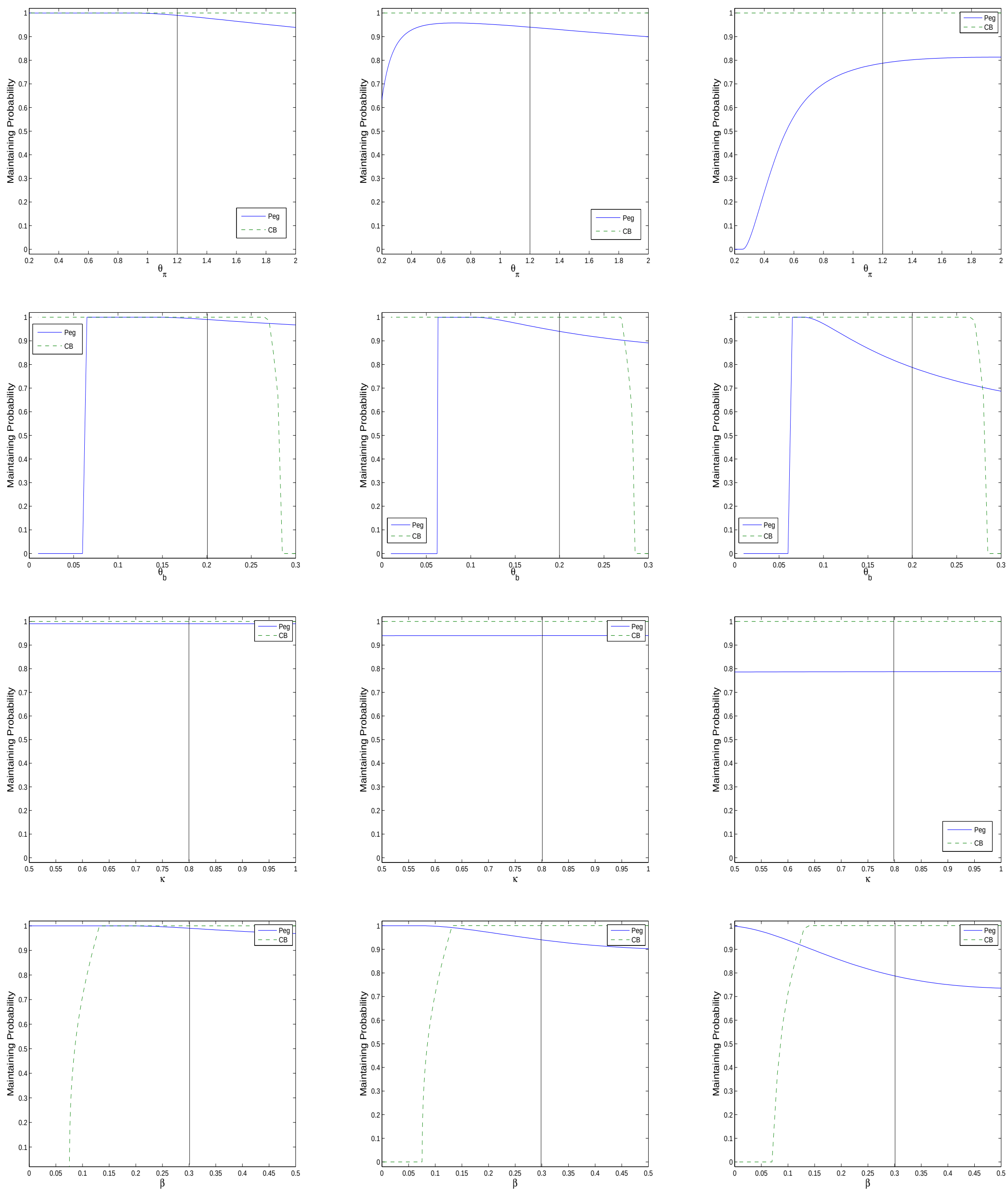
The pictures in the fourth row suggest that a rising β leads to a relative credibility advantage of a currency board. Note that β and κ are linked by the inflation equation (4.10) and, therefore, an increase of β is supposed to be accompanied by a falling κ. As we here vary only one parameter, we restricted the variation of β for a given κ on the interval $[0, 0.5]$. Hence, we restrict κ out of the same reason on the interval $[0.5, 1]$. By choosing these domains of κ and β, we assume that fiscal policy affects inflation, however, the main force driving inflation is still the central bank: Assume, for instance, that the government raises demand by one percent and the central bank devalues the currency by one percent at the same time, then the devaluation should have a greater effect on the inflation rate than fiscal policy has.

Figure 4.6: Sensitivity of the Maintaining Probabilities in Scenario I



Sensitivity of the Maintaining Probabilities in Scenario I (ad figure 4.6)

$\alpha = 0$ $\alpha = 0.5$ $\alpha = 1$



Unfortunately, for the rest of the parameters the probability of maintaining a currency board remains at 100% and, hence, we cannot draw further conclusions on the changes in the relative credibility of a currency board to a standard peg.

We mention here, for the sake of completeness, that the results from the graphs of scenario II stand almost in line with the results of scenario I. We abstain, therefore, from an explicit discussion.

To get further insights into the comparison of the credibility of both regimes, we use two stress scenarios. We, now, drop the assumption, which was made in section 4.4 and presume, instead, that a negative inherited shock hit the economy in period zero. Analogically to section 4.6.1, we want to examine the behavior of the maintaining probabilities of both systems, when devaluation pressure resulting from a negative inherited shock is present. The shock has to occur in period zero (and not in period one) to establish a fair setting for the comparison of the currency board and the peg system. Furthermore, we have chosen the parameter values with the intension to establish a scenario, where the policy makers' advantage under both fixed exchange-rate systems seems to be very limited, i.e. besides the inherited shock from period zero, we assume a relatively high shock volatility, a relatively high κ, and a relatively small β to appear. This makes monetary policy more important as a stabilization tool, as the effect of fiscal policy was strongly reduced by the parameter choice in the scenarios III and IV and, thus, weakens the credibility of both fixed exchange-rate regimes. The parameter values for both scenarios are exhibited in table 4.7. The maintaining probabilities in the two scenarios are shown in table 4.8. In scenario III the maintaining probability of a currency board system is below 100%, and in scenario IV a repealing of the currency board is announced for any first period shock, meaning that the interval in which the currency board is defended lies completely outside the support of the shock ϕ_1 or shrinks to zero (for technical details see appendix B.5).

Table 4.7: Numerical Examples: Stress Scenarios III and IV

Parameter	Stress scenario III	Stress scenario IV
k	0.02	0.02
γ	1.00	1.00
σ_u	0.06	0.10
w	0.09	0.09
η	0.80	0.80
ϕ_1	-0.04	-0.04
b_1	0.30	0.20
θ_b	0.20	0.20
θ_π	1.50	1.50
q	0.50	0.20
κ	0.80	0.90
β	0.30	0.20
c^{CB}	0.012	0.012
$\eta\phi_0$	-0.04	-0.04

Table 4.8: Maintaining Probabilities for the Stress Scenarios III and IV

	Currency board	Peg ($\alpha = 0$)	Peg ($\alpha = 0.5$)	Peg ($\alpha = 1$)
Scenario III	0.9367	0.4310	0.3913	0.3128
Scenario IV	0	0.5239	0.5256	0.4843

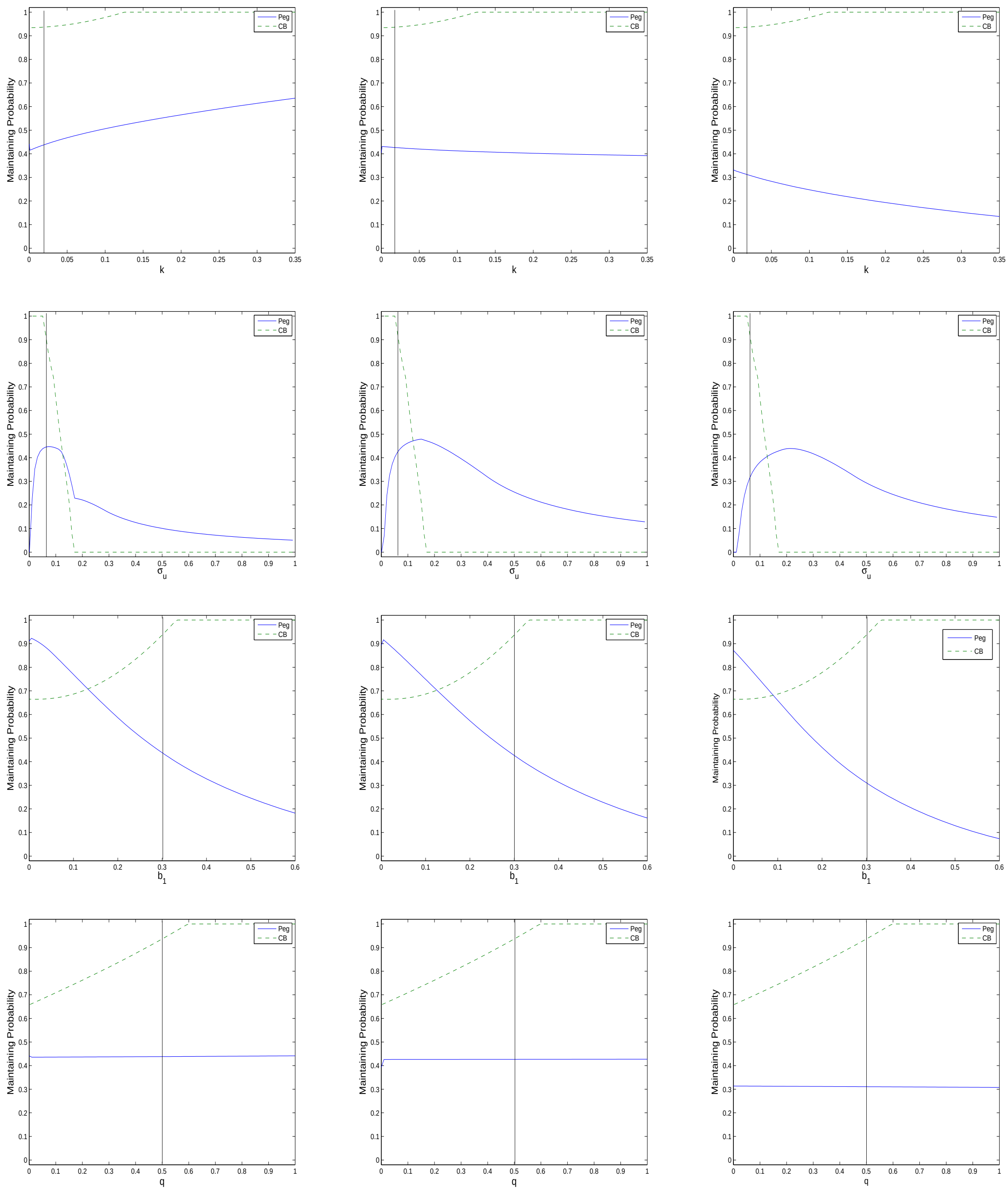
The examination of the sensitivity of the maintaining probabilities to a parameter change is done by a discussion of the figures 4.7 and 4.8, in the following.
In both stress scenarios, the three results found in scenario I and II are confirmed from a qualitative perspective:

- an increasing shock volatility σ_u leads again to a credibility gain of the standard peg, while the absolute credibility is decreasing in both systems,
- an increasing β leads also to a credibility advantage of the currency board,
- a greater value of θ_b contributes to a credibility gain of the standard peg regime.

Fortunately, we can derive further results from the two stress scenarios: We start with the parameter k, the desired output level of the fiscal and monetary policy authorities,

Figure 4.7: Sensitivity of the Maintaining Probabilities in Scenario III

$\alpha = 0$ $\alpha = 0.5$ $\alpha = 1$



Sensitivity of the Maintaining Probabilities in Scenario III (ad figure 4.7)

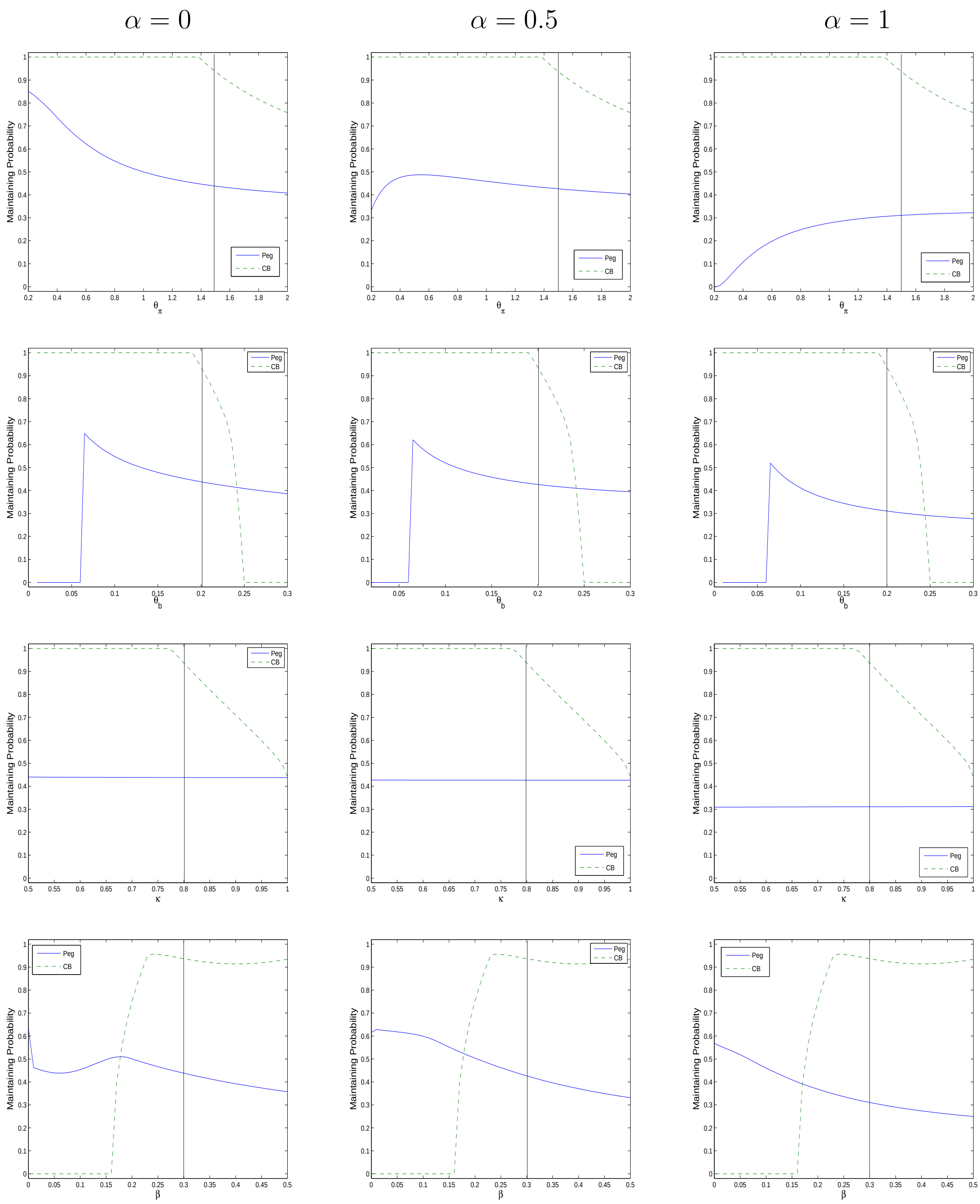
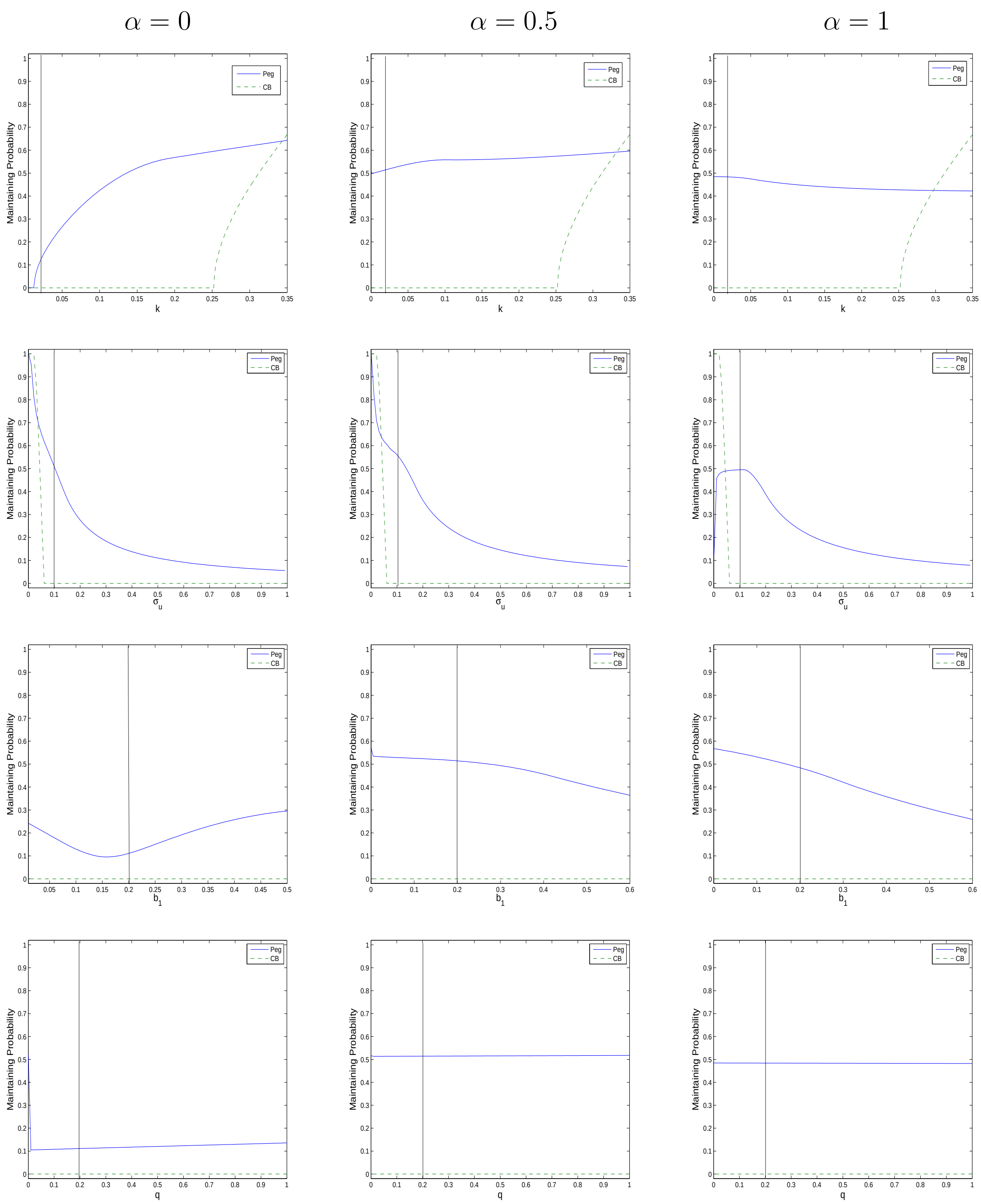
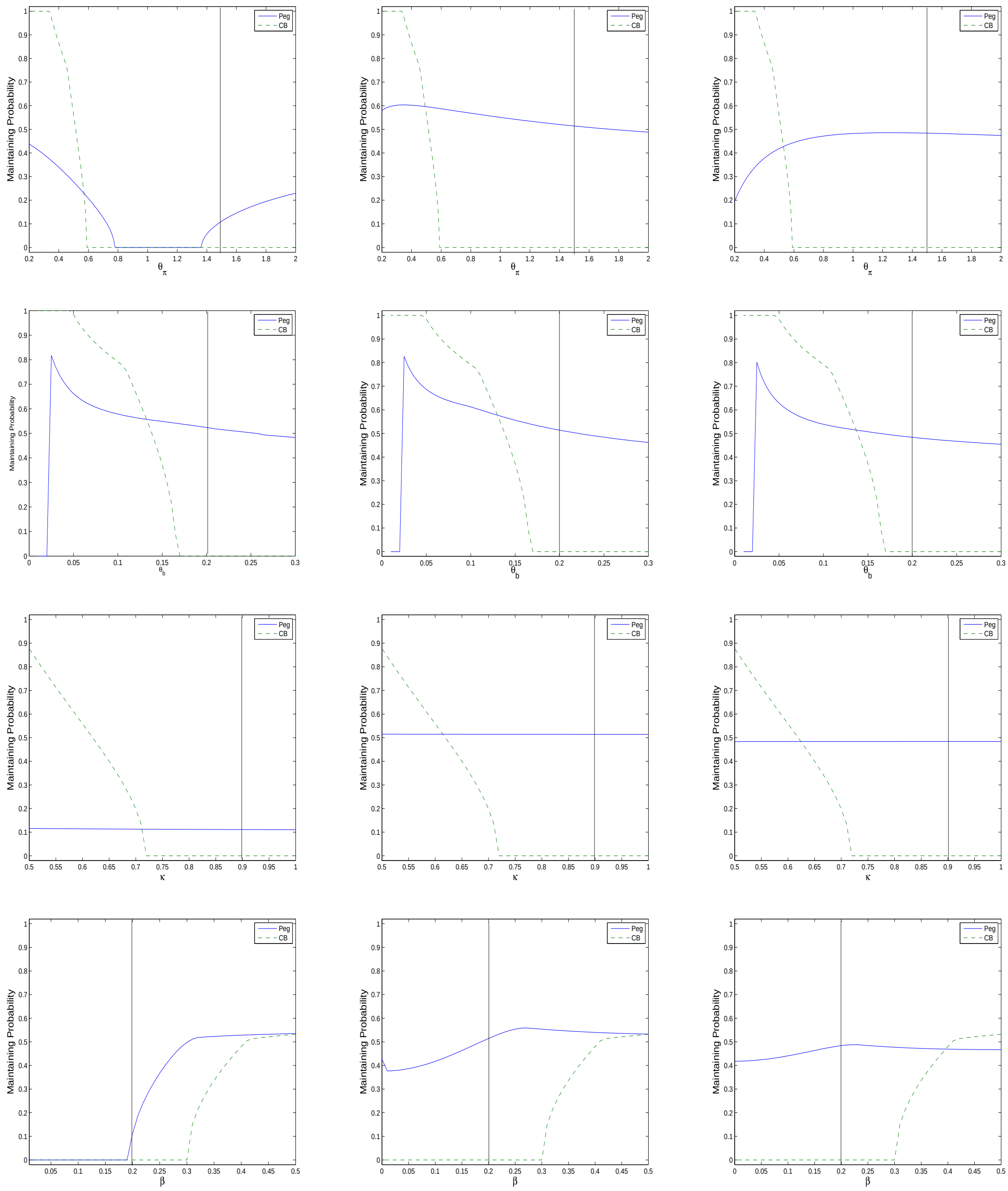


Figure 4.8: Sensitivity of the Maintaining Probabilities in Scenario IV



Sensitivity of the Maintaining Probabilities in Scenario IV (ad figure 4.8)

$\alpha = 0$ $\alpha = 0.5$ $\alpha = 1$



which determines (together with θ_π) the time inconsistency problem of monetary policy. While remembering the results from section 4.6.1, where we pointed out that the time inconsistency problem of monetary policy is completely solved and that at the same time the overall time inconsistency problem is reduced under a currency board system – which is not the case under a standard peg due to the existence of an escape clause in period 2 – one would suppose that a higher k leads to a relative credibility gain under a currency board system. The plots of scenario III and IV support that view, if α is greater than zero. This result comes as no surprise, because $\alpha = 0$ implies a full anticipated credibility of the private sector and in that case the time inconsistency problem of monetary policy is also solved under a peg by the definition of equation (4.32).

A decrease in θ_b leaves room for a more expansionary fiscal policy, which implies a more severe time inconsistency problem (of fiscal policy). As a currency board reduces the time inconsistency problem of *fiscal policy*, as mentioned before, which is not the case for a standard peg, the credibility of a currency board increases compared to a standard peg.

Scenario III further suggests that a higher inherited debt level from the first period, b_1, leads to a credibility gain of a currency board system. The explanation corresponds to that in the previous section: a higher b_1 amounts to a higher loss stemming from debts in the policy makers' loss function, if debts are not reduced in the second period. To reduce debts, the monetary authority has a higher incentive to create inflation, which at the same time aggravates the time inconsistency problem of *monetary policy*. As the time inconsistency problem of monetary policy is completely solved under a currency board system, but remains, at least partly, under a standard peg system, a higher b_1 suggests a relative credibility gain of a currency board system.

A higher fraction of foreign government debts, q, leads ceteris paribus also to a credibility gain of the currency board system. An increasing q makes the escape clause under the peg, i.e. the possibility to devalue the currency, less important, because only the fraction of government debts denominated in the home currency can be reduced. The fraction of foreign government debts may even increase in real terms.

To summarize the findings of this subsection, we can state that in the first two scenarios, which presumably accord best with reality, a currency board has (in frequent cases) an absolute credibility advantage compared to a standard peg. The argument is strengthened by the fact that we have assumed equal exit costs under both systems and have simulated the probability of maintaining the peg over the total range of possible shocks ϕ_1 to guarantee a fair comparison (see appendix B.5). This supports the empirical findings that currency board systems have proven to be stable exchange rate systems over several years, with only one abandonment (or breakdown) in Argentina, which was discussed in section 4.3.

The comparison of the credibility of a currency board and standard peg gives reason to formulate the following conclusion:

Numerical result 5. *The credibility of a currency board system and of a standard peg are measured by the probability of maintaining the system in the next period. Using this definition, the analysis of scenario I-IV states that a currency board gains a credibility advantage compared to a standard peg when*

a) the time inconsistency problem is severe (represented by a high value of k, and a small value of θ_π)

b) σ_u is low, which reduces the risk of bearing a high shock in the future period,

c) θ_b is relatively low and q relatively high, making a devaluation less necessary as a stabilization tool,

d) b_1 is large, which again leads to a more severe time inconsistency problem of monetary policy, and

e) β is relatively large and κ relatively small, again making monetary policy (=a devaluation) as a stabilization tool less important.

Result e) makes only sense from a static perspective or from a purely theoretical point of view. When discussing the stability of fixed exchange-rate regimes, a high degree of openness (κ approaching one) and no major frictions are generally seen as an essential prerequisite for a long-term stability, as shown in chapter 2. Otherwise, asymmetric shocks to the anchor currency or a strong accumulation of debts will make a repealing of the currency board or a realignment under the peg inevitable in the intermediate-term. To analyze such effects, a dynamic setting will be a necessary premise.

4.7 Conclusion

In this chapter we examined two major issues, the *fiscal sustainability* and the *stability* of currency board arrangements. In section 4.2, we summarized empirical and theoretical work about fixed exchange-rate systems and their impact on fiscal policy. The conclusions drawn from the individual papers are quite different. Some argue that choosing a hard peg facilitates fiscal soundness, others find a contrary interrelationship, but only a few of them divided the hard peg regimes into subgroups and, thereby, focussed especially on currency boards. Two empirical studies did this and both, FATAS and ROSE (2001) and GRYGONITE (2003), found that currency board countries actually tend to have a higher degree of fiscal discipline compared to other types of exchange rate systems. Before attending to a theoretical exploration of this aspect, we gave an overview of recent and present currency board systems in section 4.3. Thereby, it was shown that countries operating a currency board had not only performed well with respect to fiscal policy, but had also proven to have stable exchange-rate systems for several years.

In section 4.4, we introduced the basic model. The main feature of a currency board system is its anchorage by law. We state, therefore, that a currency board gains a high commitment value and cannot be abandoned surprisingly, but only if its abandonment was announced one period in advance. Thus, the time inconsistency problem of monetary policy, which means that a policy maker tries to increase the output level by creating surprise inflation through a devaluation, is solved. Though, the possibility that fiscal policy is used to create surprise inflation via an expansion of demand remains. And, therefore, the (overall) time inconsistency problem is not completely solved under a currency board either, but it is reduced. In contrast, the policy maker operating a standard peg regime can make use of her escape clause in every period.

We showed in section 4.5 that a currency board leads to more fiscal discipline than a peg system due to lower inflation expectations, meaning that the fiscal deficits and debts will be lower under a currency board system. The lower inflation expectations under a currency board are justified, on the one hand, by using a theoretical approach and, on the other hand, by referring to empiricism.

The examination of the stability of a currency board system was subject of section 4.6. First, we analyzed how the stability of a currency board changes when varying characteristic parameters by using numerical examples. The stability of a currency board is measured by the difference of the expected losses occurring when a currency board system is announced to be maintained in the next period and when it is announced to be abandoned. We showed that the stability of a currency board decreases, when a large (negative) future shock is likely to materialize due to a negative inherited shock and a high shock volatility. A currency board becomes more stable, if the time inconsistency problem is severe, as it is partially solved through the timing of the decision-making under a currency board. The stability of a currency board also increases by a higher ratio of foreign debts relative to total debts and by an increasing level of overall debt levels. A higher ratio of foreign debts makes a devaluation ceteris paribus more costly and reduces, thus, the advantage of repealing the currency board. From an empirical perspective, however, the result seems to be somewhat surprising. The reason for that may be found in not taking moral hazard aspects into account. A high level of (foreign) debts can create an incentive of the policy maker to default on its debts like for example 1999 in Ecuador and 2002 in Argentina, a behavior which is not covered in our model.

Besides that, we also showed that a currency board system has an intrinsic commitment value, which means that we find scenarios for which the currency board is maintained although no exit costs exist. This characteristic originates from the reduction of the time inconsistency problem, as aforementioned.

Second, we compare a currency board system and a standard peg system by introducing the concept of the "credibility of an exchange-rate system", which is defined as the probability that the current exchange rate system will still be in operation in the following period. The results, although including the effects of fiscal policy, stand almost in line with the results of chapter 3. We showed by using several numerical scenarios, that a standard peg gains a credibility advantage if the stabilization of future shocks is of paramount interest. This is the case if a high shock volatility exists. The result is traced back on the existence of an escape clause under a peg, meaning that a policy maker can realign its currency optimally after a large unfavorable shock hit the economy, which is not possible under a currency board. In contrast, if the time inconsistency problem is the dominant problem, a currency board gains a credibility advantage compared to a standard peg system. Furthermore, we found that a larger amount of government debts and a higher ratio of foreign debts increases the credibility of a currency board relative to a standard peg. Also, a higher impact of fiscal policy on inflation while at the same time the impact of a devaluation on inflation is decreasing makes the exchange rate policy less important as a stabilization tool and, hence, raises the credibility of a currency board system relatively to the credibility of a standard peg.

To get insights into the debt evolution and the stability of a currency board system over an intermediate time-horizon, we focus on an enhancement of the model to more periods in our future research. However, as the parametrization makes the model already woefully complicated in the two-period setting, further restrictive assumptions would be necessary.

An additional interesting modification of the model would be to incorporate nominal interest rates endogenously: If debts are accumulated, private creditors would increase the level of the rate of returns on government bonds to be willing to accept further debts due to the demand of a higher risk premia.[34] We suppose that this would also affect the stability and credibility of a currency board system. In this context a consideration of borrowing constraints would be meaningful, too.

[34]Of course, the argument only holds, if the *Ricardian Equivalence Theorem* does not apply here. This seems to be the case for emerging market economies.

Part II

Monetary and Fiscal Policies in a Monetary Union

Chapter 5

Introduction to Part II

The second part of this dissertation examines monetary and fiscal policy interactions in a currency area. It includes four *interdependent chapters* which are based on GRIMM and RIED (2006): In this chapter, we give first an overview of related literature and an outline of the structure and the main findings of part II. The second part of the introduction is referred to the EMU. To motivate our work from an empirical point of view, we show that inflation and growth differentials have been significant and persistent since the inception in 1999 and discuss related empirical studies.

We lay out the model from its foundations in microeconomics in chapter 6: In section 6.1 we consider a currency area consisting of two regions and derive the region-specific output and inflation equations. In addition, we derive a second-order approximation of the welfare criterion in section 6.2, and show that the fiscal policy makers' target function aim at stabilizing region-specific inflation and region-specific output, whereas the common central bank focusses on the whole currency area by minimizing a weighted-sum of the social losses occurring in both regions.

The examination of fiscal and monetary policy interactions is done in chapter 7. We highlight in section 7.1 the main building blocks of the "macroeconomic part" of our model and characterize the different scenarios of strategic behavior of fiscal and monetary policies. The evaluation of the different scenarios is done in section 7.2. We consider both, supply-side fiscal policy, which is in line with the fiscal policy used in our microfounded model, and demand-side policy. Analogously to the proceedings in the related literature, we run a numerical simulation to obtain the results. For the supply-side policy, we use a parametrization which is almost standard in this kind of literature. The parameters of the demand-side fiscal policy are chosen for a quantitative exploration. The results under both fiscal policies seem to be robust for parameter variations as shown in a sensitivity analysis. Chapter 8 concludes the second part of this dissertation.

5.1 Outline and Related Literature

A country participating in a currency union has to abstain from sovereign monetary policy. Union-wide monetary policy aims at stabilizing inflation and output in the whole currency area and cannot pay attention to every specific country in its decision-making.[1] Instead, national fiscal policies are typically concerned with their single country and not the union as a whole. This gives rise to a variety of possible strategic behaviors. National fiscal policies can help monetary policy to maximize union-wide welfare (Gali and Monacelli 2002, 2005a, and Benigno 2004), they can try to adjust the outcomes of monetary policy to maximize nationwide welfare (Dixit 2001, Uhlig 2002), or they can be used to maximize the probability of the current government staying in office after the next elections (Beetsma and Uhlig 1999). In our model, we allow for all three possibilities. We analyze monetary and fiscal policy interactions in a monetary union in various scenarios and indicate the scenarios that are preferable from a welfare perspective.

The literature on monetary and fiscal policy in a monetary union is vast, so we only refer to articles of special importance to our work:[2] Dixit and Lambertini (2003b) consider monetary-fiscal policy interactions in a monetary union. They assume that the participating regions and their policy goals are symmetric and in line with the common central bank's target. Accordingly, it is hardly surprising that ideal output and inflation levels can be achieved – even without coordination of the fiscal authorities and the common central bank, and without the need for monetary commitment.

Dixit (2001), Dixit and Lambertini (2003a) and Lambertini (2004, 2006a) check also the implications of this model for asymmetric policy goals in the case where monetary policy is conservative in the sense of Rogoff (1985). One of their major findings is that fiscal discretion destroys the positive effect of monetary commitment, and that fiscal cooperation typically leads to less efficient outcomes than independently acting fiscal policies.

Lombardo and Sutherland (2004) construct a symmetric, two-country model that features government spending in the utility function. They find that the last result can

[1] The central bank's target function will be derived in chapter 6.

[2] We refer the reader to De Grauwe (2003) for an overview of the field, as well as for references to less recent literature.

be overturned if the share of steady-state government spending in output is positive and supply shocks are not perfectly negatively correlated. Nonetheless, for plausible parameter values the welfare gains of fiscal cooperation are small.

Dixit and Lambertini (2001) allow for some heterogeneities by assuming that fiscal and monetary authorities may have conflicting output and inflation goals. They show that without commitment or leadership by either authority the ideal levels of output and inflation cannot be attained.

Chari and Kehoe (2004) take a closer look at the desirability of fiscal debt-constraints. They find that such constraints are undesirable if monetary commitment is possible, whereas the opposite holds if the central bank cannot commit to its policy. The latter is the result of a time-inconsistency problem of monetary policy, which leads to free-riding behavior by the fiscal authorities.

In the very recent literature, the topic of monetary and fiscal interactions has also been dealt with in dynamic stochastic general-equilibrium models. However, the emphasis in most of these papers is not so much on strategic behavior and game-theoretical scenarios. Gali and Monacelli (2005a) e.g. analyze optimal fiscal and monetary policies in a monetary union where all policy agents care about union-wide variables and Ferrero (2005) considers a two-region model and compares the optimal policies to simple policy rules where all policy agents care about union-wide variables. Canzoneri et al. (2005) study the interactions between monetary and fiscal policy in a monetary union and compare the results of their New Keynesian model with the data. They also assess the effects of regional asymmetries on welfare, but they assume that fiscal policy is described by exogenously given processes for government spending and distortionary taxes.[3] Lambertini (2006b) attempts to combine the game-theoretical approach of the static models with features of dynamic models. To do so, she assumes that fiscal authorities can commit to their policies. Also, she assumes that government spending is exogenously given. All these dynamic models have in common that fiscal policies are given exogenously or reduced to very simple rules.

In a series of papers, van Aarle et al. (2001) and (2002), Engwerda et al. (2002) and Garretsen et al. (2005) focus on macroeconomic policy interactions in national fiscal policies and the monetary policy of a common central bank by using a New Keynesian

[3]As alternative specifications they consider fiscal policy rules making movements in the budget deficit lead to reactions either in government spending or in tax rates. In our model, by contrast, the government budget is always balanced.

framework. Of these papers, van Aarle et al. (2002) is the one most closely related to our model. They compare the outcomes of different scenarios by distinguishing between non-cooperation, partial cooperation, and full cooperation between monetary and fiscal policies. They find that the stability of coalitions depends strongly on the policy makers' preferences. When the countries are very heterogeneous, non-cooperative behavior is the most likely outcome.

We consider a static two-country model with a single currency and one monetary policy conducted by a common central bank. Each country or region has its own fiscal authority that maximizes its objective function with the arguments output and inflation. The equations of the basic model and the loss functions are derived from microfoundation, by enhancing and modifying the Dixit and Lambertini (2003a, 2003b) approach. Our contribution here is to accurately model the possibility of various differences between two regions in a *heterogeneous monetary union.*

As an application of the theory, the countries participating in the Economic and Monetary Union (EMU) are far from being homogeneous. Both the differentials of output growth and inflation dispersion, have been significant and rather persistent as will be shown in the following section. The spread of the key macroeconomic indicators in the participating countries will presumably become even larger when the ten new EU member states adopt the Euro. Hence, it seems appropriate to incorporate those heterogeneities when analyzing the interactions of monetary and fiscal policies in a currency area like the Euro area.

We do this in two steps. First, we derive the inflation and output equation from microfoundation and state that terms of trade (i.e. inflation differentials) and a country-specific productivity shock, both affect the region-specific output levels. Second, we take the view that national fiscal policies are concerned with national output and inflation targets, whereas they are not directly concerned with output growth and price changes in other parts of the union, unless they decide to cooperate. As a simple illustration relating to the EMU, the Greek finance minister considers the current wage and house-price increases in Ireland not to be of major importance for his economy. Additionally, we assume that fiscal authorities have target rates for output and inflation that are higher than the welfare optimizing rates. Monetary policy is assumed to aim at the union-wide optimal rates in terms of welfare.

We analyze the fiscal policy makers' and central bank's losses in various scenarios. Policies can be conducted simultaneously in the Nash scenario, or sequentially in Stackelberg leadership scenarios for each policy. Alternatively, policies can be coordinated between some or all authorities. We investigate the implications for output, inflation, and various policy loss functions in a numerical analysis, and using a sensitivity analysis, we show that the ranking of the scenarios is relatively robust across different degrees of heterogeneity. We, furthermore, compare the outcomes for the different scenarios for two types of fiscal policies: supply-side and demand-side policy.

Our numerical results suggest that from the viewpoint of welfare maximization joint cooperation between all policy makers, and monetary leadership produce the smallest losses. The result holds for both cases, supply-side and demand-side fiscal policy. Furthermore, we find that the more asymmetric the regions, the larger the overall losses and the higher the relative gains from a first mover advantage of monetary policy and from joint cooperation.

5.2 Heterogeneities in the European Monetary Union

Much empirical research referring to the EMU focusses on the existing heterogeneities between its member countries. These heterogeneities may contribute to growth differences and to inflation dispersion among the member countries. The growth differentials are depicted in figure 5.1.[4] The figure shows real GDP growth rates of the countries that feature the most disparate movements: Output growth differentials have been rather persistent over the recent years. While the real GDP growth rate has been most of the time below the Euro area average level in Germany and Italy since the introduction of the Euro, it has been above average in countries like Luxembourg, Greece and Ireland.

The existing theoretical literature on monetary and fiscal policy in a monetary union is well aware of the differences in GDP growth, whereas inflation dispersion is discussed much less often. We, therefore, take a closer look at this aspect and recapitulate explanations to this given in the literature. Inflation differentials per se are a well-known observation, they appear in every "currency area", like for example among individual states of the U. S., or within regions of larger countries, e. g. between eastern and southern parts of Germany. We nonetheless should pay special attention to inflation dispersion in the European Monetary Union, because both the size of inflation differentials and the

[4] The data is obtained from the Eurostat webpage. The Euro area (EU-12) average level of output growth before the introduction of the Euro in 1999 and before the participation of Greece in the EMU in 2001 is artificially calculated by using the same country-weights that were used in 2006. GDP growth rates for 2006 are estimates.

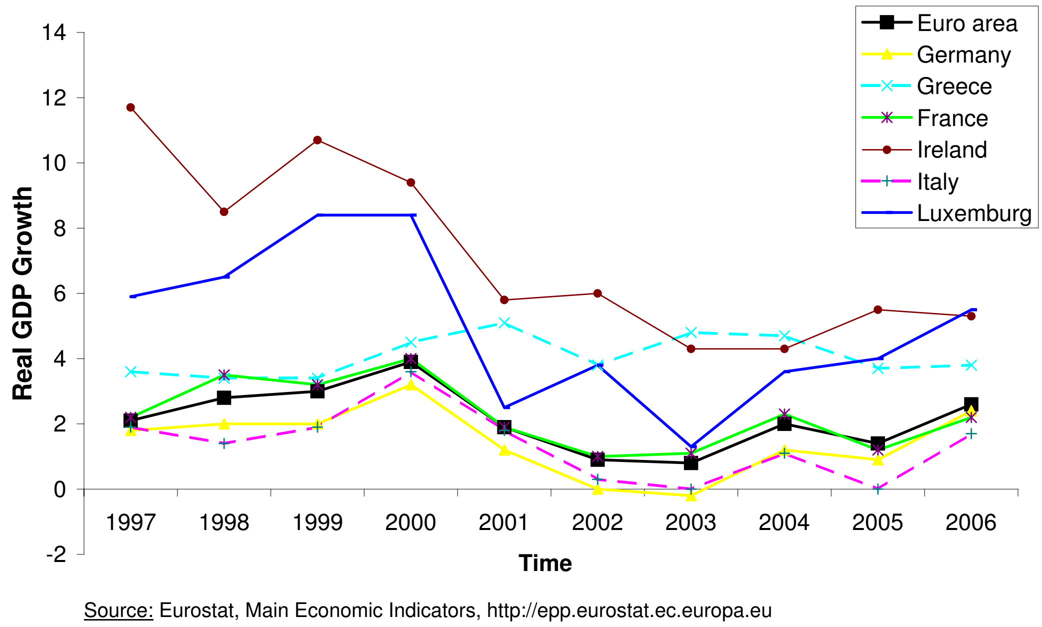


Figure 5.1: Real GDP Growth Differentials in the European Monetary Union

variability of inflation rates between member states of the EMU is higher than in the within-country cases described above, as is well documented in Honohan and Lane (2003). This may stem from (i) substantial structural differences, (ii) good-market frictions, (iii) labor-market inflexibilities and (iv) different exposures to extra-union trade among the European countries.[5] In the following, we want to look more thoroughly at the developments in the Euro-zone since its inception in 1999. After that, we give a brief survey of the literature on inflation differentials and discuss the implications for monetary and fiscal policies.

Between the participating countries of the European Monetary Union inflation differentials, interestingly, have risen again since the beginning of the Euro adoption, after a period of decline from the beginning of the nineties until 1998, as figure 5.2 documents for some selected countries.[6] Inflation dispersion has also been proven to be rather persis-

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[5]For the first three arguments, see e. g. chapters 4 and 8 in De Grauwe (2003). For the last argument, see Honohan and Lane (2003).

[6]Of course, the rise of inflation differentials after 1999 can be traced back on the fact that exchange-rate policy was no longer available as a stabilization instrument, but it was used before 1999 (within a +/-15% exchange-rate margin). However, the persistence of the inflation differentials which has been observed since 1999 until today is somewhat surprising.

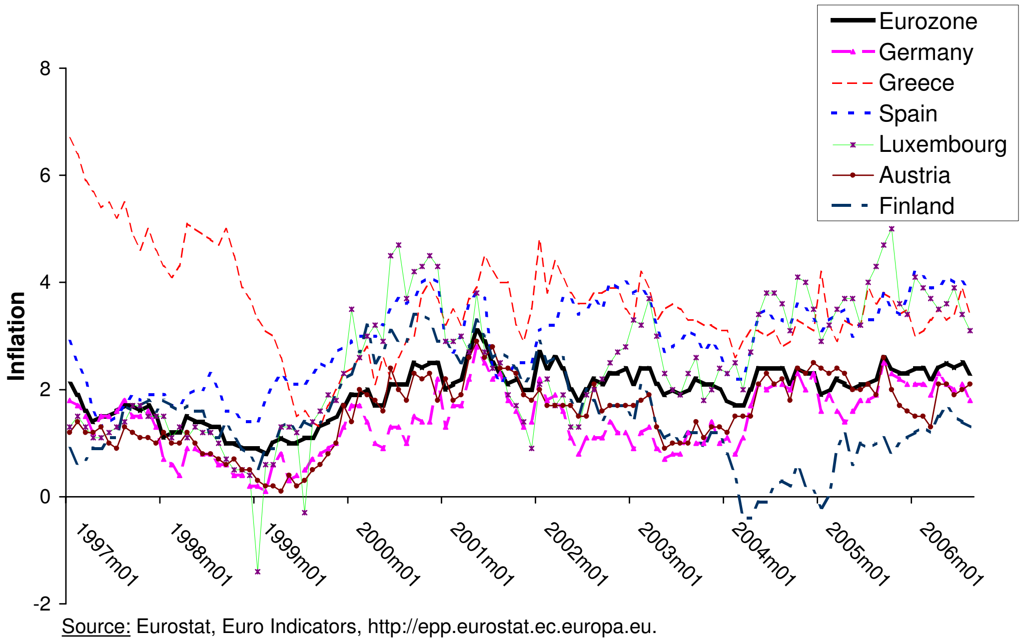

Figure 5.2: Inflation Differentials in the European Monetary Union

tent, which means that the group of countries with inflation rates above average inflation of the the Euro-zone and the group of countries with lower than average inflation rates have comprised the same countries during the last five years: Inflation has been highest in Greece, Spain and Luxembourg reaching levels between 3 and around 4 percent during the last three years and has been most of the time above the average-rate of the Euro-zone since 1999, as shown in figure 5.2. Also Portugal and Ireland showed inflation rates above the EMU average level during this period.[7] The countries at the lower end are Austria, Finland and Germany.

At first glance, the observation seems to be quite unexpected, as one would think that a high-inflation country tends to lose competitiveness relative to the other countries participating in the currency area as a consequence of the real appreciation. A real appreciation dampens demand for commodities in the high-inflation country and leads to a decline of inflation towards the average level of the currency area. The opposite would be true for the low inflation country. We call this the *competitiveness effect*.

[7]Ireland had the highest inflation rates of 5.3 percent in 2000 and well above 4 percent from 2000 until 2004, but converged to a level slightly above the EU average, recently. The reason for that is discussed in the following.

In the literature, there are several approaches to explain the existence and persistence of inflation dispersion. ANDRÉS et al. (2003) analyze the effects of heterogeneities in economic structures like the degree of competition, openness and the appearance of nominal inertias on inflation differentials in response to asymmetric shocks. They parameterize a two-country model of a monetary union, where the countries show a different economic structure in the features characterized above and a different degree of openness which is closely linked to the size of a home-biased behavior in consumption. By allowing moderate differences in nominal frictions and real rigidities, like e.g. habit formation and investment costs, they are able to "generate sizeable inflation differentials in a model calibrated to match the most salient long-run features of the average big EMU economies".[8]

ANGELONI and EHRMANN (2004) use a stylized empirical model of the European Monetary Union. They state that besides the above explained effect of a changing degree of *competitiveness* by real exchange rate movements, additionally a *dis-equilibrating* effect resulting from the differences in inflation rates can appear: a high inflation rate in one country relative to another leads according to the well-known FISHER-relation to a lower real interest rate compared to the other country, since all countries share the same nominal interest rate. Hence, the original inflation differentials can be strengthened through demand-side effects (via real investments), if the *dis-equilibrating effect dominates the competitiveness effect*, which was explained already.

HOFMANN and REMSPERGER (2005) mention further potential causes which contribute to inflation differentials. The first and, probably, the most popular one is the so called BALASSA-SAMUELSON effect. Caused by heterogeneities in the productivity, the country with a lower productivity will achieve productivity gains during the catch-up phase. These gains mainly accrue in the sector of tradeable goods due to the pressure stemming from international competition. Productivity gains lead, of course, to increasing nominal wages in the tradeable sector. When assuming full mobility of labor between the tradeable and non-tradeable sector, nominal wages in the non-tradeable sector also rise until the same wage level in both sectors is achieved. This again implies a rise of goods prices in the non-tradeable sector leading to inflation. HOFMANN and REMSPERGER compare several empirical studies and conclude that on average only a small part of inflation in the European Monetary Union can be explained by the BALASSA-SAMUELSON effect. MICHAELIS and MINICH (2004) share this view in their analysis.

Another aspect in HOFMANN and REMSPERGER as well as in MICHAELIS and MINICH

[8] ANDRÉS et al. (2003), p. 31

is the occurrence of temporary shocks. By differing consumption habits the composition of the Harmonized Index of Consumer Prices (HICP) varies over the countries and leads to different exposures to common price shocks (oil prices, raw materials etc.). A similar result is found by Honohan and Lane (2003). They claim that the Euro depreciation against the U. S.-$ at the beginning of the European Monetary Union had a different impact on the individual countries. Ireland, for instance, whose main trading partners are the United States and Great Britain, both outside the currency area, faced an increase of import prices resulting in a rise of output prises. The effect was strengthened by a higher demand from overseas. The reverse effect has been observed since the starting of the appreciation period of the Euro, and Ireland's inflation rate now converges to the Union's average level.

To summarize, there are several reasons for inflation differentials in the European Monetary Union, most of them of structural origin:

(i) Different cultures, languages, and habits, which lead to a restricted mobility of labor.

(ii) Different trading partners, which lead to an asymmetric ability to react to price shocks and to unexpected nominal exchange rate movements of the Euro against other currencies.

(iii) Different and uncoordinated national government expenditure and tax policies.

(iv) The existence of nominal rigidities, which lead to asymmetric effects of monetary policy on the different currency area countries.

These are only some explanations for the inflation dispersion, which is higher and more persistent than in the US-$ area and, probably, will become more relevant after the eastern enlargement of the European Monetary Union.

Possible consequences are that the existence of output and inflation differentials may lead to a different behavior of fiscal policies, which focus on national target variables. We take up this point in the microfoundation of our model and in our policy analysis. Another point of interest is, whether the common central bank should take heterogeneities into account, and should, thus, pay attention to country-specific targets, or whether it should stabilize union-wide variables.

Chapter 6

Microfoundation

6.1 A Microfounded Two-Country Model

We consider a general-equilibrium monetary model with monopolistic distortions and staggered prices. The model is closely related to Dixit and Lambertini (2003b) and refers to the seminal work of Blanchard and Kiyotaki (1987).[1] In the economy there exists an infinity of consumption goods over the unit interval. These are imperfect substitutes. Households derive utility from consumption and from holding real money balances and suffer from work. Each household produces a specific good and consumes a bundle of goods. We will, henceforth, refer to a representative household as a "producer-consumer". There are two regions, home H and foreign F, with the population on the segment $[0, n)$ belonging to the home region H and the remaining population belonging to the foreign region F, with $0 \leq n \leq 1$.[2]

6.1.1 The Problem of a Producer-Consumer

A producer-consumer j in region $i \in \{H, F\}$ derives utility

$$U_i^j = \left(\frac{C^j}{\gamma}\right)^{\gamma} \left(\frac{M_i^j/P^i}{1-\gamma}\right)^{1-\gamma} - \left(\frac{d_i}{\beta}\right)(Y_i^j)^{\beta}, \quad \gamma \in (0,1),\ d_i > 0,\ \beta \geq 1. \tag{6.1}$$

[1]For a detailed explanation of the basic model see appendix A in Dixit and Lambertini (2003b) and also Obstfeld and Rogoff (1996, chapter 10).

[2]This setting is taken from Benigno (2004). Other related models are Lombardo and Sutherland (2004), Ferrero (2005), and Gali and Monacelli (2005b).

The utility function depends on consumption, real money balances, and labor. The producer-consumer derives a positive utility from consumption of goods and from the stock of real money, while the parameter γ captures the elasticity of substitution between the two. Labor, which, for simplicity, is assumed to be a linear function of output and is therefore replaced by output itself, contributes negatively to the utility of agent j. Here, $1+\beta$ is the elasticity of the marginal disutility of labor. The stochastic variable d_i captures both the scaling of disutility of labor and the fluctuations in total factor productivity. Changes in this variable may be interpreted as changes in technology.[3] The total consumption of agent j – who for reasons of exposition is assumed to live in region H – is given by[4]

$$C^j \equiv \frac{(C_H^j)^{\nu^H}(C_F^j)^{1-\nu^H}}{(\nu^H)^{\nu^H}(1-\nu^H)^{1-\nu^H}}, \tag{6.2}$$

where ν^H is a preference shifter with $n \leq \nu^H \leq 1$ that allows for a home bias in consumption.[5] We assume that all regions exhibit the same home bias, i.e. we henceforth use $\nu \equiv \nu^H = \nu^F$.

Consumption of goods from each region is given by

$$C_H^j = \left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n c^j(h)^{\frac{\theta-1}{\theta}} dh\right]^{\frac{\theta}{\theta-1}}, \quad C_F^j = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\theta}} \int_n^1 c^j(f)^{\frac{\theta-1}{\theta}} df\right]^{\frac{\theta}{\theta-1}}, \tag{6.3}$$

where h is a generic good produced in region H, f a generic good produced in region F, and $\theta > 1$ the elasticity of substitution between different goods in the same region.[6] The elasticity of substitution of the home and foreign bundles of goods equals one. The corresponding consumer-price indices – with subscripts denoting the place of production

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[3]To appreciate this, assume a production function of $Y_i^j = A_i N_i^j$ with total factor productivity A_i and hours N_i^j. Then rewrite the second summand in the utility function as $\frac{d_i}{\beta}(N_i^j)^\beta$ with the help of the definition $d_i \equiv \delta_i A_i^{-\beta}$, where δ_i captures the disutility of labor. In the welfare derivation in section 6.2 below, we will define $d_i \equiv \delta_i \xi_i$, where for simplicity $\delta_i = 1$ and ξ_i is a stochastic variable capturing technological progress.

[4]For an agent j living in region F, total consumption is given by $C^j \equiv \frac{(C_F^j)^{\nu^F}(C_H^j)^{1-\nu^F}}{(\nu^F)^{\nu^F}(1-\nu^F)^{1-\nu^F}}$ for all $j \in [n, 1]$.

[5]To our knowledge, this model is the first two-region model of a monetary union that features the possibility of more than proportional demand for goods produced in the agent's home economy.

[6]The weights $(1/n)^{(1/\theta)}$ and $(1/(1-n))^{(1/\theta)}$ are a "normalization with the implication that an increase in the number of products does not affect marginal utility after optimization". See Blanchard and Kiyotaki (1987), p. 649.

and superscripts denoting variables specific to agent j or region i – are

$$P^H \equiv (P^H_H)^\nu (P^H_F)^{1-\nu} \qquad \text{and} \qquad P^F \equiv (P^F_F)^\nu (P^F_H)^{1-\nu}, \tag{6.4}$$

where

$$P^i_H \equiv \left[\frac{1}{n}\int_0^n p^i(h)^{1-\theta} dh\right]^{\frac{1}{1-\theta}} \quad \text{and} \quad P^i_F \equiv \left[\frac{1}{1-n}\int_n^1 p^i(f)^{1-\theta} df\right]^{\frac{1}{1-\theta}} \tag{6.5}$$

denote the market-price indices of goods consumed in region i and produced in region H and F, respectively. Note that the price index P^H is defined as the minimum expenditure necessary for purchasing goods leading to a consumption index C^j of size one[7] and the price indexes P^i_H and P^i_F are defined as the minimum expenditure required to purchase goods resulting in consumption indexes C^j_H and C^j_F, which equal one.

Although producers would have an incentive to set different prices across regions because of the home bias in consumption, we exclude this possibility by assuming that goods-market arbitrage leads to identical prices across borders such that $P^H_H = P^F_H = P_H$ and $P^H_F = P^F_F = P_F$.[8] With output produced by agent j in region i denoted by Y^j_i, the budget constraint for this agent is

$$\int_0^n p^i(h)c^j(h)dh + \int_n^1 p^i(f)c^j(f)df + M^j_i = p^i(j)Y^j_i(1-\tau_i) - P_iT_i + \bar{M}^j_i \equiv I^j_i. \tag{6.6}$$

The budget constraint guarantees that the sum of consumption expenditures plus money demand equals nominal net income I^j_i, which is the sum of sale revenues from the good produced and beginning-of-period money holdings minus net tax payments.

In each region, a government pursues its fiscal policy by making use of four instruments: a tax rate τ_i proportional to sales, real lump-sum taxes T_i, government consumption G^i, and wasteful government expenditures X^i. Government consumption of goods G^i is defined symmetrically to private consumption, as given in equation (6.3). Sale taxes could also be negative with the interpretation of subsidies. Also, lump-sum transfers $T_i < 0$ are

[7] The same argument also holds for region F.

[8] In our theoretical model, inflation differentials occur due to the home-bias effect, as the composition of the consumption bundles differ in both regions. This assumption is somewhat critical with reference to the Euro zone, where significant price differences for the same product exist in different countries (including tradeable goods).

possible. For the two regional government budget constraints we have

$$\begin{aligned}\int_0^n p^H(j)y(j)\tau_H dj + nP_H T_H &= \chi^H[\nu P^H G^H + (1-\nu)P^F G^F] + (1-\chi^H)X^H \\ &\equiv I_H^g \qquad (6.7)\\ \int_n^1 p^F(j)y(j)\tau_F dj + (1-n)P_F T_F &= \chi^F[\nu P^F G^F + (1-\nu)P^H G^H] + (1-\chi^F)X^F \\ &\equiv I_F^g. \qquad (6.8)\end{aligned}$$

Following Dixit and Lambertini (2003b) we assume that the government can spend its budget on government consumption G^i or it can be wasted, X^i, ruled by the weight $\chi^i \in [0,1]$.

Consumption maximization is done in two steps. First, suppose that C_H^j is a single good instead of an aggregate. Then, utility maximization of agent j in region H subject to the corresponding aggregated budget constraint implies the two first-order conditions

$$\lambda_{BC} = \left(\frac{C^j}{\gamma}\right)^{\gamma-1}\left(\frac{M_H^j/P^H}{1-\gamma}\right)^{1-\gamma}\nu\frac{C^j}{P_H^H C_H^j}, \qquad (6.9)$$

$$\lambda_{BC} = \left(\frac{C^j}{\gamma}\right)^{\gamma}\left(\frac{M_H^j/P^H}{1-\gamma}\right)^{-\gamma}\frac{1}{P^H}. \qquad (6.10)$$

Equalizing the two equations by replacing the Lagrange multiplier λ_{BC} and noting that $\frac{P^i C^j}{\gamma} = \frac{M_i^j}{1-\gamma} = I_i^j$ leads to[9]

$$C_H^j = \nu\left(\frac{P^H}{P_H^H}\right)C^j. \qquad (6.11)$$

Second, maximizing C_H^j with respect to two generic elements $c^j(h)$ and $c^j(h')$, subject to $\int_0^n P^i(h)c^j(h)dh = Z$, leads to

$$c^j(h) = \left(\frac{p^i(h)}{p^i(h')}\right)^{-\theta} c^j(h') \; . \qquad (6.12)$$

Then, replacing $c^j(h)$ in equation (6.3) by the right-hand side of the previous equation

[9] This is a result of the Cobb-Douglas structure of the utility function.

gives

$$\begin{aligned}
C_H^j &= \left[\left(\frac{1}{n}\right)^{\frac{1}{\theta}}\int_0^n \left(\left(\frac{p^i(h)}{p^i(h')}\right)^{-\theta} c^j(h')\right)^{\frac{\theta-1}{\theta}} dh\right]^{\frac{\theta}{\theta-1}} \\
&= \left[p^i(h')^{\theta-1}c^j(h')^{\frac{\theta-1}{\theta}}\left(\frac{1}{n}\right)^{\frac{1}{\theta}}\int_0^n p^i(h)^{1-\theta}dh\right]^{\frac{\theta}{\theta-1}} \\
&= \left[\left(\frac{1}{n}\right)^{\frac{1-\theta}{\theta}}(p^i(h'))^{\theta-1}(c^j(h'))^{\frac{\theta-1}{\theta}}\left[\frac{1}{n}\int_0^n (p^i(h))^{1-\theta}dh\right]\right]^{\frac{\theta}{\theta-1}} \\
&= n(p^i(h'))^{\theta}c^j(h')\underbrace{\left[\frac{1}{n}\int_0^n (p^i(h))^{1-\theta}dh\right]^{-\frac{\theta}{1-\theta}}}_{=(P_H^i)^{-\theta}} \\
&= p^i(h')^{\theta}c^j(h')n(P_H^i)^{-\theta},
\end{aligned}$$

which implies

$$c^j(h) = \frac{1}{n}\left(\frac{p^i(h)}{P_H^i}\right)^{-\theta} C_H^j. \tag{6.13}$$

Adding steps one and two plus the symmetric results for the foreign good (for ease of exposition agent j is still assumed to live in region H) results in[10]

$$c^j(h) = \frac{\nu}{n}\left(\frac{p^H(h)}{P_H^H}\right)^{-\theta}\frac{P^H}{P_H^H}C^j \quad \text{and} \quad c^j(f) = \frac{1-\nu}{1-n}\left(\frac{p^H(f)}{P_F^H}\right)^{-\theta}\frac{P^H}{P_F^H}C^j. \tag{6.14}$$

We assume that government spending is subject to the same home bias as private consumption expenditures. This assumption lies between the extreme positions of no home bias in government expenditures, as proposed by LOMBARDO and SUTHERLAND (2004), and complete home bias, as proposed by BEETSMA and JENSEN (2002), BENIGNO (2004) and GALI and MONACELLI (2005a).[11] The symmetric results for optimal expenditures of the home government are

$$g^H(h) = \frac{\nu}{n}\left(\frac{p^H(h)}{P_H^H}\right)^{-\theta}\frac{P^H}{P_H^H}G^H \quad \text{and} \quad g^H(f) = \frac{1-\nu}{1-n}\left(\frac{p^H(f)}{P_F^H}\right)^{-\theta}\frac{P^H}{P_F^H}G^H. \tag{6.15}$$

---

[10] An agent j of region F would demand $c^j(h) = \frac{1-\nu}{n}\left(\frac{p^H(h)}{P_H^F}\right)^{-\theta}\frac{P^F}{P_H^F}C^j$ and $c^j(f) = \frac{\nu}{1-n}\left(\frac{p^F(f)}{P_F^F}\right)^{-\theta}\frac{P^F}{P_F^F}C^j$.

[11] Our solution is in line with the comment by LEITH (2004) alluded to by LOMBARDO and SUTHERLAND (2004) in footnote 8. GALI and MONACELLI (2005a) cite "evidence on a strong home bias in government procurement" in their footnote 8.

6.1.2 Terms of Trade and Aggregate Demand

As set out before, the law of one price holds in the economy considered, i.e. $p^H(h) = p^F(h)$ and $p^H(f) = p^F(f)$. Nonetheless, agents appreciate consumption of domestically produced goods more. Hence, the (consumer) price index in the home region P^H includes a larger share of domestic goods than the (consumer) price index in the foreign region P^F. This implies non-trivial terms of trade. Defining the terms of trade as the price of imports relative to the price of exports,[12] from the viewpoint of economy i we have

$$S_i \equiv \frac{P^i_{-i}}{P^{-i}_i} = \frac{P_{-i}}{P_i}, \tag{6.16}$$

where "not i" is denoted by "$-i$". The latter equality holds as the rate of substitution between domestic goods is constant in both economies, so that the basket of domestically produced goods has the same composition in both economies, though not the same relative size. Therefore a change in the price index of domestically produced goods has the same impact on e.g. P^F_H and on P^H_H and we can drop the superscript. It is useful to relate the terms of trade to the consumer price indices P^H and P^F and to the price indices of goods produced in each region, P_H and P_F, using the definitions $P^H \equiv (P_H)^\nu (P_F)^{1-\nu}$ and $P^F \equiv (P_F)^\nu (P_H)^{1-\nu}$, which are generalizations of (6.4):

$$\frac{P^H}{P_H} = (S_H)^{1-\nu}, \quad \frac{P^H}{P_F} = \frac{1}{(S_H)^\nu}, \quad \frac{P^F}{P_H} = (S_H)^\nu \quad \text{and} \quad \frac{P^F}{P_F} = \frac{1}{(S_H)^{1-\nu}}. \tag{6.17}$$

In the case of an identical home bias in both regions, which we are assuming here, the ratios of the two measures of inflation are inversely related to each other:[13] $S_i = 1/S_{-i}$. Movements in the terms of trade imply movements in relative prices and, therefore, shift demand across the border. Using the terms of trade and the fact that $C^j = \frac{\gamma I^j_i}{P^i}$, we can rewrite the first-order condition of the producer-consumers with respect to their consump-

[12] This notation is the reciprocal of the usual definition, see e. g. OBSTFELD and ROGOFF (1996), p. 242. The notation is in line with the standard literature from the viewpoint of the foreign economy.

[13] See GALI and MONACELLI (2002) for a similar treatment in a small open economy setting.

tion of a single good and – in a similar manner – to their money holdings M_i^j as

$$
\begin{aligned}
c^j(h) &= \frac{\nu}{n}\left(\frac{p^H(h)}{P_H}\right)^{-\theta}\frac{\gamma I_H^j}{P_H}, && (6.18)\\
c^j(f) &= \frac{1-\nu}{1-n}\left(\frac{p^H(f)}{P_F}\right)^{-\theta}\frac{\gamma I_H^j}{P_F}, && (6.19)\\
c^j(h) &= \frac{1-\nu}{n}\left(\frac{p^H(h)}{P_H}\right)^{-\theta}\frac{\gamma I_F^j}{P_H}, && (6.20)\\
c^j(f) &= \frac{\nu}{1-n}\left(\frac{p^F(f)}{P_F}\right)^{-\theta}\frac{\gamma I_F^j}{P_F}, && (6.21)\\
M_i^j &= (1-\gamma)I_i^j. && (6.22)
\end{aligned}
$$

The first two equations determine a home resident's optimal choice of home and foreign goods, the next two equations determine the analog for a foreign resident, while the last equation shows the optimality condition with respect to money holdings.

Total nominal expenditure by consumers in region H is $I_H = \int_0^n I_H^j dj$, while in region F it is $I_F = \int_n^1 I_F^j dj$. The demand function for a good h is given by

$$
\begin{aligned}
Y^d(h) &= \int_0^1 c^j(h)dj + \chi^H g^H(h) + \chi^F g^F(h)\\
&= \left(\frac{p^H(h)}{P_H}\right)^{-\theta}\frac{1}{n}\\
&\quad \cdot\left[\gamma\frac{\nu I_H + (1-\nu)I_F}{P_H} + \nu\chi^H\frac{P^H}{P_H}G^H + (1-\nu)\chi^F\frac{P^F}{P_H}G^F\right]. \qquad (6.23)
\end{aligned}
$$

Similarly, the demand for a certain foreign good f is given by

$$
\begin{aligned}
Y^d(f) &= \int_0^1 c^j(f)dj + \chi^H g^H(f) + \chi^F g^F(f)\\
&= \left(\frac{p^F(f)}{P_F}\right)^{-\theta}\frac{1}{1-n}\\
&\quad \cdot\left[\gamma\frac{(1-\nu)I_H + \nu I_F}{P_F} + \nu\chi^F\frac{P^F}{P_F}G^F + (1-\nu)\chi^H\frac{P^H}{P_F}G^H\right]. \qquad (6.24)
\end{aligned}
$$

Again denoting "not i" by $-i$, we define a variable proportional to "wealth":

$$
W \equiv \gamma\frac{\nu I_i + (1-\nu)I_{-i}}{P_i} + \nu\chi^i\frac{P^i}{P_i}G^i + (1-\nu)\chi^{-i}\frac{P^{-i}}{P_i}G^{-i}. \qquad (6.25)
$$

At this point it is useful to note that this definition includes the terms of trade between domestic and foreign goods, as $I_i = \frac{P^iC}{\gamma}$ measures the nominal consumption expenditures using the level of the consumer price index (CPI), while the denominator involves the level of the producer price index (PPI) as a reference. Using the identities from (6.17), one can easily transform this notation into one that includes real expenditures and the terms of trade S:

$$W = \begin{cases} \nu(S_H)^{1-\nu}\left(\gamma\frac{I_H}{P^H} + \chi^H G^H\right) + (1-\nu)(S_H)^{\nu}\left(\gamma\frac{I_F}{P^F} + \chi^F G^F\right) & \text{if } i = H, \\ \nu(S_H)^{\nu-1}\left(\gamma\frac{I_F}{P^F} + \chi^F G^F\right) + (1-\nu)(S_H)^{-\nu}\left(\gamma\frac{I_H}{P^H} + \chi^H G^H\right) & \text{if } i = F. \end{cases}$$

To obtain a single equation for demand, we define the following weights:

$$w_i = \begin{cases} n & \text{if } i = H, \\ 1-n & \text{if } i = F. \end{cases}$$

Then, demand for a specific good j from region i amounts to

$$Y^d(j) = \left(\frac{p^i(j)}{P_i}\right)^{-\theta}\frac{W}{w_i}. \tag{6.26}$$

Analogously to BENIGNO (2004), the smaller a region is (i.e. the higher the degree of openness), the larger the terms of trade effect will be on regional output (included in the W term).[14]

6.1.3 Price Setting

When selling the product, each producer is a monopolist. The producer, therefore, decides upon the price of the product by maximizing the indirect utility function. The indirect utility function is obtained by plugging $C^j = \frac{\gamma I_i^j}{P^i}$ and $M_i^j = (1-\gamma)I_i^j$ into the utility function (6.1), replacing I_i^j by the right-hand side of the budget constraint, replacing the price ratio with the help of equation (6.26), and simplifying:

$$U_i^j = (1-\tau_i)\left(\frac{W}{w_i}\right)^{\frac{1}{\theta}}(Y_i^j)^{\frac{\theta-1}{\theta}} - T_i + \frac{\bar{M}_i^j}{P_i} - \left(\frac{d_i}{\beta}\right)(Y_i^j)^{\beta}. \tag{6.27}$$

[14] Note that our demand functions are more complicated than the ones in BENIGNO (2004) because of the preference parameter ν. This destroys the identity $P^H = P^F$ that holds in BENIGNO (2004) as long as $\nu^H \neq n$. If $\nu^H = \nu^F = n$ and $1-\nu^H = 1-\nu^F = 1-n$, the consumer price indices of both regions are identical, and the demand functions become as simple as in BENIGNO (2004).

The indirect utility function of agent j is maximized with respect to the price $p^i(j)$, noting that the output produced by agent j is equal to its demand, i.e. $Y_i^j = Y^d(j)$.[15]

$$\begin{aligned}
\frac{\partial U_i^j}{\partial p^i(j)} &= (1-\tau_i)\left(\frac{W}{w_i}\right)^{\frac{1}{\theta}}\left[\frac{\theta-1}{\theta}(Y_i^j)^{-\frac{1}{\theta}}(-\theta)\left(\frac{p^i(j)}{P_i}\right)^{-\theta-1}\left(\frac{W}{w_i}\right)\right] \\
&\quad -\frac{d_i}{\beta}\beta(Y_i^j)^{\beta-1}(-\theta)\left(\frac{p^i(j)}{P_i}\right)^{-\theta-1}\left(\frac{W}{w_i}\right) \\
&= (1-\tau_i)\left(\frac{W}{w_i}\right)^{\frac{\theta+1}{\theta}}(1-\theta)\left(\frac{p^i(j)}{P_i}\right)^{-\theta-1+1}\left(\frac{W}{w_i}\right)^{\frac{-1}{\theta}} \\
&\quad +d_i\theta\left(\frac{p^i(j)}{P_i}\right)^{-\theta(\beta-1+1)}\left(\frac{W}{w_i}\right)^{\beta-1+1} \\
&= (1-\tau_i)\frac{W}{w_i}(1-\theta)\left(\frac{p^i(j)}{P_i}\right)^{-\theta}+d_i\theta\left(\frac{W}{w_i}\right)^{\beta}\left(\frac{p^i(j)}{P_i}\right)^{-\theta\beta-1} \stackrel{!}{=} 0.
\end{aligned} \tag{6.28}$$

We obtain the optimal ratio of prices by

$$\begin{aligned}
\left(\frac{p^i(j)}{P_i}\right) &= \left(\frac{-d_i\theta\left(\frac{W}{w_i}\right)^{\beta-1}}{(1-\tau_i)(1-\theta)}\right)^{\frac{1}{-\theta+\theta\beta+1}} \\
&= \left(\frac{\theta d_i}{(\theta-1)(1-\tau_i)}\left(\frac{W}{w_i}\right)^{\beta-1}\right)^{\frac{1}{1+\theta(\beta-1)}}.
\end{aligned} \tag{6.29}$$

Furthermore, we assume that some prices are fixed in advance, comparable to a static version of the staggered price-setting introduced by CALVO (1983). A fraction Φ^i of producers cannot change their prices and thus have to charge the same prices as in the past, whereas a fraction $(1-\Phi^i)$ of producers is able to set their prices freely after the realization of the shocks in region i. The price level of goods from region H is a weighted sum of the average of pre-set prices $E[\bar{p}^H(h)]$ and the newly set prices $\tilde{p}^H(h)$, which due to symmetry are equal for all producers. Based on equation (6.5), we obtain

$$P_H^{1-\theta} = \Phi^H(E\bar{p}^H(h))^{1-\theta} + (1-\Phi^H)(\tilde{p}^H(h))^{1-\theta}. \tag{6.30}$$

For goods produced in region F the equivalent equation is

$$P_F^{1-\theta} = \Phi^F(E\bar{p}^F(f))^{1-\theta} + (1-\Phi^F)(\tilde{p}^F(f))^{1-\theta}. \tag{6.31}$$

[15] As the decision of a single individual has only marginal impact on terms of trade and the price indices, this effect is neglected in the optimization.

For convenience, the price ratio in region i may be defined as

$$\lambda_i \equiv \Phi^i \left(\frac{E\bar{p}^i(j)}{P_i} \right)^{1-\theta} + (1-\Phi^i) \left(\frac{\tilde{p}^i(j)}{P_i} \right)^{1-\theta} = 1. \tag{6.32}$$

In line with equation (6.4), the aggregate consumer price index in region i is given by

$$\begin{aligned} P^H &= \left[\Phi^H \left(E\bar{p}^H(h) \right)^{1-\theta} + (1-\Phi^H) \left(\tilde{p}^H(h) \right)^{1-\theta} \right]^{\frac{\nu}{1-\theta}} \\ &\quad \cdot \left[\Phi^F \left(E\bar{p}^F(f) \right)^{1-\theta} + (1-\Phi^F) \left(\tilde{p}^F(f) \right)^{1-\theta} \right]^{\frac{1-\nu}{1-\theta}} \end{aligned} \tag{6.33}$$

$$\begin{aligned} P^F &= \left[\Phi^F \left(E\bar{p}^F(f) \right)^{1-\theta} + (1-\Phi^F) \left(\tilde{p}^F(f) \right)^{1-\theta} \right]^{\frac{\nu}{1-\theta}} \\ &\quad \cdot \left[\Phi^H \left(E\bar{p}^H(h) \right)^{1-\theta} + (1-\Phi^H) \left(\tilde{p}^H(h) \right)^{1-\theta} \right]^{\frac{1-\nu}{1-\theta}}. \end{aligned} \tag{6.34}$$

This can be written in terms of the overall price level[16]

$$\begin{aligned} P &\equiv (P^H)^n (P^F)^{1-n} \\ &= \left[\Phi^H \left(E\bar{p}^H(h) \right)^{1-\theta} + (1-\Phi^H) \left(\tilde{p}^H(h) \right)^{1-\theta} \right]^{\frac{n\nu+(1-n)(1-\nu)}{1-\theta}} \\ &\quad \cdot \left[\Phi^F \left(E\bar{p}^F(f) \right)^{1-\theta} + (1-\Phi^F) \left(\tilde{p}^F(f) \right)^{1-\theta} \right]^{\frac{n(1-\nu)+(1-n)\nu}{1-\theta}}. \end{aligned} \tag{6.35}$$

6.1.4 Aggregate Output and Fiscal Policy

Aggregate output in each region is defined by the following equations:

$$Y_H \equiv \int_0^n \frac{p^H(h)Y(h)}{P_H} dh \quad \text{and} \quad Y_F \equiv \int_n^1 \frac{p^F(f)Y(f)}{P_F} df. \tag{6.36}$$

Using the demand functions (6.23) and (6.24) as well as the price index definitions (6.5), and denoting the lower and upper integral limits of each region i by lli and uli, respectively,[17] aggregate output produced in region i can be rewritten as

$$\begin{aligned} Y_i &= \int_{lli}^{uli} \frac{p^i(j)}{P_i} \left(\frac{p^i(j)}{P_i} \right)^{-\theta} \frac{W}{w_i} dj = \left[\int_{lli}^{uli} \left(\frac{p^i(j)}{P_i} \right)^{1-\theta} dj \right] \frac{W}{w_i} \\ &= \lambda_i \frac{W}{w_i} = \frac{W}{w_i}. \end{aligned} \tag{6.37}$$

[16]Note that the numerators of the exponents add up exactly to one.

[17]I.e., $lli = \begin{cases} 0 & \text{if } i = H, \\ n & \text{if } i = F, \end{cases}$ and $uli = \begin{cases} n & \text{if } i = H, \\ 1 & \text{if } i = F. \end{cases}$

Essentially, this implies that the goods' supply in region i is equal to its demand, which according to equation (6.25) originates from both regions. Total output is given as the geometric average of output in both regions:

$$Y \equiv Y_H^n Y_F^{1-n}. \tag{6.38}$$

We specify fiscal policy as follows: Each fiscal authority uses per-capita taxes T_i to subsidize production, i. e., $T_i > 0$, $\tau_i < 0$. We assume for the moment that there is no other government spending, i.e. $\chi_i = X^i = G^i = 0$. In this case, wealth W simplifies to

$$\begin{aligned}
W &= \gamma \frac{\nu I_i + (1-\nu) I_{-i}}{P_i} \\
&= \frac{\gamma}{P_i}\left[\nu \int_{lli}^{uli} I_i^j dj + (1-\nu)\int_{ll-i}^{ul-i} I_{-i}^j dj\right] \\
&= \frac{\gamma}{P_i}[\nu \int_{lli}^{uli} \left(p^i(j) Y_i^j (1-\tau_i) - P_i T_i + \bar{M}_i^j\right) dj \\
&\quad + (1-\nu)\int_{ll-i}^{ul-i} \left(p^{-i}(j) Y_{-i}^j (1-\tau_{-i}) - P_{-i} T_{-i} + \bar{M}_{-i}^j\right) dj] \\
&= \frac{\gamma}{P_i}\left[\nu\left(P_i Y_i (1-\tau_i) - P_i T_i + \bar{M}_i\right) + (1-\nu)\left(P_{-i} Y_{-i}(1-\tau_{-i}) - P_{-i} T_{-i} + \bar{M}_{-i}\right)\right] \\
&= \frac{\gamma}{P_i}\left[\nu \bar{M}_i + (1-\nu)\bar{M}_{-i} + \nu P_i \frac{W}{w_i} + (1-\nu) P_{-i}\frac{W}{w_{-i}}\right] \\
\Leftrightarrow W &= \gamma \frac{\bar{M}}{P} \frac{1}{1 - \gamma\frac{\nu}{w_i} - \gamma\frac{1-\nu}{w_{-i}} S_i},
\end{aligned} \tag{6.39}$$

where we assume identical beginning-of-period real money holdings for all agents $\frac{\bar{M}}{P} = \frac{\bar{M}_i^j}{P_i}$ and for all i, j.[18] This fiscal policy uses distortionary taxation to offset market distortion due to monopolistic competition. Therefore, this type of fiscal policy is closest to the theoretical optimum. Nonetheless, our framework allows for various other fiscal policies.[19]

[18]Without the assumption of internationally identical money holdings $\bar{M}/P$ has to be replaced by $[n\bar{M}_i + (1-n)\bar{M}_{-i}]/P_i$.

[19]Two alternative fiscal policies – with distortionary taxation that is either wasted or used for government spending – are analyzed in Dixit and Lambertini (2003b): In the first, $\tau_i > 0$, , $\chi_i = G^i = T_i = 0$ and $X^i > 0$. In the second, $\tau_i > 0$ (as long as $G^i > 0$), $\chi_i = 1$, $T_i = 0$. Analyzing the effects of these policies might be a useful topic for future research.

6.1.5 Log-Linear Equilibrium Fluctuations: Price Setting

We log-linearize the model as follows: First, note that a linear approximation of equation (6.4) around $P_i = P^i = P$ for all i results in

$$\pi^H = \nu\pi_H + (1-\nu)\pi_F \qquad \text{and} \qquad \pi^F = \nu\pi_F + (1-\nu)\pi_H, \tag{6.40}$$

where the inflation rates are defined as percentage deviations of the respective price level from its steady-state level,[20] i.e.

$$\pi^i \equiv \log(P^i) - \log(\bar{P}^i), \text{ given } \bar{P}^i \neq 0. \tag{6.41}$$

Then, equations (6.30) and (6.31) linearize[21] to

$$\pi_H = \Phi^H\bar{\pi}_H + (1-\Phi^H)\tilde{\pi}_H \qquad \text{and} \qquad \pi_F = \Phi^F\bar{\pi}_F + (1-\Phi^F)\tilde{\pi}_F. \tag{6.42}$$

Combining the results gives

$$\pi^H = \nu(\Phi^H\bar{\pi}_H + (1-\Phi^H)\tilde{\pi}_H) + (1-\nu)(\Phi^F\bar{\pi}_F + (1-\Phi^F)\tilde{\pi}_F) \tag{6.43}$$

$$\pi^F = \nu(\Phi^F\bar{\pi}_F + (1-\Phi^F)\tilde{\pi}_F) + (1-\nu)(\Phi^H\bar{\pi}_H + (1-\Phi^H)\tilde{\pi}_H). \tag{6.44}$$

Now, we turn to the optimal price a producer would set if he could choose the price freely. According to Dixit and Lambertini (2003a), we refer to the idea of Calvo-staggered pricing, which reflects a dynamic setting (for details see again Calvo, 1983). Analogously to the procedure proposed by Dixit and Lambertini, we introduce a discount factor η with $\eta < 1$ (which means that pseudo-future period utilities have a lower weight than present utility). We, first, assume that η equals unity to explain the "intuitional proceeding". In the case where prices are allowed to change, the optimal log price equals

$$\begin{aligned} \tilde{\pi}_H &= (1-\Phi^H)\pi_H^j + \Phi^H\bar{\pi}_H \\ \tilde{\pi}_F &= (1-\Phi^F)\pi_F^j + \Phi^F\bar{\pi}_F. \end{aligned}$$

where π_i^j is the log steady-state deviation of the price that would be optimal if prices could be adjusted freely. The log price set by producer j is a sum of the weighted optimal

[20] Under the assumption that $\bar{P}^i \equiv 1$, one can equivalently define $\pi^i \equiv \log(P^i)$.

[21] To appreciate this, compare the following procedure undertaken with a simplified, yet similar equation:
$P^b = \phi Q^b + (1-\phi)R^b \quad \Rightarrow \quad \bar{P}^b e^{b\pi} = \phi\bar{Q}^b e^{b\bar{\pi}} + (1-\phi)\bar{R}^b e^{b\tilde{\pi}}$, which is approximately equal to
$\bar{P}^b(1+b\pi) = \phi\bar{Q}^b(1+b\bar{\pi}) + (1-\phi)\bar{R}^b(1+b\tilde{\pi}) \quad \Rightarrow \quad b\pi = \phi\frac{\bar{Q}^b}{\bar{P}^b}b\bar{\pi} + (1-\phi)\frac{\bar{R}^b}{\bar{P}^b}b\tilde{\pi}$. As the fractions are equal to unity, this simplifies to $\pi = \phi\bar{\pi} + (1-\phi)\tilde{\pi}$.

price of producer j, if prices were fully flexible, and the weighted price that maximizes the expected indirect utility, if prices are to be fixed in future periods. The weights equal the probability of being able, $(1 - \Phi^i)$, or not being able, Φ^i, to change the price in the following period(s).

Now we come back to the discount factor $\eta < 1$: As already mentioned, the individuals place lower weight on future utilities. Therefore, the fact that the producer cannot change the price in future periods with a certain probability is expressed by a lower weight than the pure probability of future price setting (given by $\eta\Phi^i$) and a higher weight for the present period $(1 - \eta\Phi^i)$. Hence, we obtain

$$\tilde{\pi}_H = (1 - \Phi^H\eta)\pi_H^j + \Phi^H\eta\bar{\pi}_H, \quad (6.45)$$

$$\tilde{\pi}_F = (1 - \Phi^F\eta)\pi_F^j + \Phi^F\eta\bar{\pi}_F. \quad (6.46)$$

In the case of $\eta = 0$, this setting would be purely static: Here, the (deviation from the steady state of the) optimal price once an individual is allowed to change price $\tilde{\pi}_i$ is identical to the price that is optimal for the current period only, as there are no future periods to form expectations about.

Using equations (6.45) and (6.46) to replace the optimal prices in the consumer price indices (6.43) and (6.44) gives

$$\begin{aligned} \pi^H &= \nu\Phi^H[1 + (1 - \Phi^H)\eta]\bar{\pi}_H + \nu(1 - \Phi^H)(1 - \Phi^H\eta)\pi_H^j \\ &\quad + (1 - \nu)\Phi^F[1 + (1 - \Phi^F)\eta]\bar{\pi}_F + (1 - \nu)(1 - \Phi^F)(1 - \Phi^F\eta)\pi_F^j \quad (6.47) \\ \pi^F &= \nu\Phi^F[1 + (1 - \Phi^F)\eta]\bar{\pi}_F + \nu(1 - \Phi^F)(1 - \Phi^F\eta)\pi_F^j \\ &\quad + (1 - \nu)\Phi^F[1 + (1 - \Phi^F)\eta]\bar{\pi}_H + (1 - \nu)(1 - \Phi^F)(1 - \Phi^F\eta)\pi_H^j\ . \quad (6.48) \end{aligned}$$

The overall inflation rate can be calculated by using the previous equations together with equation (6.35):

$$\begin{aligned} \pi &= n\pi^H + (1 - n)\pi^F \quad (6.49) \\ &= [n\nu + (1 - n)(1 - \nu)]\pi_H + [n(1 - \nu) + (1 - n)\nu]\pi_F. \quad (6.50) \end{aligned}$$

Equation (6.49) states that union-wide inflation is the sum of the regional CPI inflation weighted by the size of each region. The second equation (6.50) links union-wide inflation to the PPI inflation rates in each region, where the influence of regional PPI inflation depends on both the size of the region and the preference of agents for goods from that region.

6.1.6 Inflation Determination

In general, a producer sets its price by maximizing the indirect utility function which results in equation (6.29) above. A log-linear approximation of this equation around the steady state, solved for the relative deviation of wealth from its steady state level, $\hat{W}$, is

$$\hat{W} = \frac{1+\theta(\beta-1)}{\beta-1}(\hat{p}^i(j) - \pi_i) - \frac{1}{\beta-1}\hat{d}_i - \frac{\bar{\tau}_i}{\beta-1}\hat{\tau}_i, \tag{6.51}$$

where $\pi_i \equiv \hat{P}_i$, and a "hat" above a variable denotes percentage deviations of the variable from its steady state.[22] To replace $\hat{W}$ in the last expression, we log-linearize the policy-dependent wealth equation.

For the fiscal policy considered here, we use equation (6.39), and obtain the result

$$\hat{W} = \frac{\gamma\bar{m}}{\omega}\hat{m} + \frac{\gamma(1-\nu)}{\omega w_{-i}}s_i, \tag{6.52}$$

where ω is given by $\omega \equiv 1-\gamma[\frac{\nu}{w_i} + \frac{1-\nu}{w_{-i}}]$ and $s_i \equiv \hat{S}_i = \pi_{-i} - \pi_i$. $\hat{m} = \widehat{M/P}$ is the change in the beginning-of-period real money holdings.

In the next step, equation (6.51) – with $\hat{W}$ replaced by the fiscal-policy-dependent equation – is evaluated at both $E[\hat{p}^i(j)] \equiv \bar{\pi}_i$, the (log deviation of the) price that maximizes the future indirect utility, and at $\hat{p}_i^j \equiv \pi_i^j$, the (log deviation of the) price that maximizes the current period indirect utility. Starting with the first case $\bar{\pi}_i$, we obtain

$$\begin{aligned}
\bar{\pi}_i &= E[\pi_i] + \frac{1}{1+\theta(\beta-1)}E[\hat{d}_i] + \frac{\bar{\tau}_i}{1+\theta(\beta-1)}E[\hat{\tau}_i] \\
&\quad + \frac{\beta-1}{1+\theta(\beta-1)}E\left[\frac{\gamma\bar{m}}{\omega}\hat{m} + \frac{\gamma(1-\nu)}{\omega w_{-i}}s_i\right] \\
&= E[\pi_i] + \frac{1}{1+\theta(\beta-1)}E[\hat{d}_i] + \frac{\bar{\tau}_i}{1+\theta(\beta-1)}E[\hat{\tau}_i] \\
&\quad + \frac{\beta-1}{1+\theta(\beta-1)}\frac{\gamma\bar{m}}{\omega}E[\hat{m}] + \frac{\beta-1}{1+\theta(\beta-1)}\frac{\gamma(1-\nu)}{\omega w_{-i}}E[s_i] \\
&= \frac{1}{1+\theta(\beta-1)}E[\hat{d}_i] + \frac{\beta-1}{1+\theta(\beta-1)}\frac{\gamma\bar{m}}{\omega}E[\hat{m}] + \frac{\bar{\tau}_i}{1+\theta(\beta-1)}E[\hat{\tau}_i] \\
&\quad + \frac{\beta-1}{1+\theta(\beta-1)}\frac{\gamma(1-\nu)}{\omega w_{-i}}E[s_i] + E[\pi_i] \\
&= \frac{1}{1+\theta(\beta-1)}E[\hat{d}_i] + \frac{\beta-1}{1+\theta(\beta-1)}\frac{\gamma\bar{m}}{\omega}E[\hat{m}] + \frac{\bar{\tau}_i}{1+\theta(\beta-1)}E[\hat{\tau}_i] \\
&\quad + \left(1 - \frac{\beta-1}{1+\theta(\beta-1)}\frac{\gamma(1-\nu)}{\omega w_{-i}}\right)E[\pi_i] + \frac{\beta-1}{1+\theta(\beta-1)}\frac{\gamma(1-\nu)}{\omega w_{-i}}E[\pi_{-i}] \\
&= \bar{\omega}_{0,i} + \omega_1 E[\hat{\tau}_i] + \omega_2 E[\hat{\tau}_{-i}] + \omega_3 E[\pi_i] + \omega_4 E[\pi_{-i}],
\end{aligned} \tag{6.53}$$

[22] For the approximation of the fiscal policy term note that $\widehat{(1-\tau_i)} = \frac{-\bar{\tau}_i}{1-\bar{\tau}_i}\hat{\tau}_i$.

where $\bar{\omega}_{0,i} \equiv \frac{1}{1+\theta(\beta-1)}E[\hat{d}_i] + \frac{\beta-1}{1+\theta(\beta-1)}\frac{\gamma\bar{m}}{\omega}E[\hat{m}]$, $\omega_1 \equiv \frac{\bar{\tau}_i}{1+\theta(\beta-1)}$, $\omega_2 \equiv 0$, $\omega_3 \equiv \left(1 - \frac{\beta-1}{1+\theta(\beta-1)}\frac{\gamma(1-\nu)}{\omega w_{-i}}\right)$ and $\omega_4 \equiv \frac{\beta-1}{1+\theta(\beta-1)}\frac{\gamma(1-\nu)}{\omega w_{-i}}$.[23] Note that via $s_i = \pi_{-i} - \pi_i$ the terms in s_i have been replaced by terms in π_i and π_{-i}. Accordingly, for the price that maximizes the current period indirect utility only, we obtain

$$\begin{aligned}
\pi_i^j &= \frac{1}{1+\theta(\beta-1)}\hat{d}_i + \frac{\beta-1}{1+\theta(\beta-1)}\frac{\gamma\bar{m}}{\omega}\hat{m} + \frac{\bar{\tau}_i}{1+\theta(\beta-1)}\hat{\tau}_i \\
&\quad + \left(1 - \frac{\beta-1}{1+\theta(\beta-1)}\frac{\gamma(1-\nu)}{\omega w_{-i}}\right)\pi_i + \frac{\beta-1}{1+\theta(\beta-1)}\frac{\gamma(1-\nu)}{\omega w_{-i}}\pi_{-i} \qquad (6.54) \\
&= \omega_{0,i} + \omega_1\hat{\tau}_i + \omega_2\hat{\tau}_{-i} + \omega_3\pi_i + \omega_4\pi_{-i}. \qquad (6.55)
\end{aligned}$$

Using equations (6.42), (6.45) and (6.46), we obtain an equation that expresses the regional producer inflation rate in terms of the log of the price that maximizes the future indirect utility and the price that maximizes the current period indirect utility only:

$$\pi_i = \rho^i\bar{\pi}_i + (1-\rho^i)\pi_i^j, \qquad \rho^i = \Phi^i[1 + (1-\Phi^i)\eta]. \qquad (6.56)$$

Henceforth, we will neglect the superscript i for the parameter ρ for reasons of clarity, because the results derived in the following have exactly the same structure for both regions.

We use (6.56) and combine the two log prices in equations (6.53) and (6.55):

$$\begin{aligned}
\pi_i &= \rho\left[\bar{\omega}_{0,i} + \omega_1 E[\hat{\tau}_i] + \omega_2 E[\hat{\tau}_{-i}] + \omega_3 E[\pi_i] + \omega_4 E[\pi_{-i}]\right] \\
&\quad + (1-\rho)\left[\omega_{0,i} + \omega_1\hat{\tau}_i + \omega_2\hat{\tau}_{-i} + \omega_3\pi_i + \omega_4\pi_{-i}\right]. \qquad (6.57)
\end{aligned}$$

For the other region, analog steps yield

$$\begin{aligned}
\pi_{-i} &= \rho\left[\bar{\omega}_{0,-i} + \omega_1 E[\hat{\tau}_{-i}] + \omega_2 E[\hat{\tau}_i] + \omega_3 E[\pi_{-i}] + \omega_4 E[\pi_i]\right] \\
&\quad + (1-\rho)\left[\omega_{0,-i} + \omega_1\hat{\tau}_{-i} + \omega_2\hat{\tau}_i + \omega_3\pi_{-i} + \omega_4\pi_i\right], \qquad (6.58)
\end{aligned}$$

where $\omega_{0,-i}$ differs only from $\omega_{0,i}$ by the stochastic disutility of labor variable $\hat{d}_{-i}$ instead of $\hat{d}_i$.

Combining (6.57) and (6.58) and solving this system of equations for the region-specific

[23] We add the term ω_2 to show that under alternative fiscal policies this spillover effect can be non-zero.

inflation rates, one obtains

$$
\begin{aligned}
\pi_i = \;& \Omega\rho\left[\bar{\omega}_{0,i} + \frac{(1-\rho)\omega_4}{1-(1-\rho)\omega_3}\bar{\omega}_{0,-i}\right] \\
&+\Omega\rho\left[\left(\bar{\omega}_1 + \frac{(1-\rho)\omega_4}{1-(1-\rho)\omega_3}\omega_2\right)E[\hat{\tau}_i] + \left(\omega_2 + \frac{(1-\rho)\omega_4}{1-(1-\rho)\omega_3}\omega_1\right)E[\hat{\tau}_{-i}]\right] \\
&+\Omega\rho\left[\left(\omega_3 + \frac{(1-\rho)\omega_4}{1-(1-\rho)\omega_3}\omega_4\right)E[\pi_i] + \left(\omega_4 + \frac{(1-\rho)\omega_4}{1-(1-\rho)\omega_3}\omega_3\right)E[\pi_{-i}]\right] \\
&+\Omega(1-\rho)\left[\omega_{0,i} + \frac{(1-\rho)\omega_4}{1-(1-\rho)\omega_3}\omega_{0,-i}\right] \\
&+\Omega(1-\rho)\left(\omega_1 + \frac{(1-\rho)\omega_4}{1-(1-\rho)\omega_3}\omega_2\right)\hat{\tau}_i \\
&+\Omega(1-\rho)\left(\omega_2 + \frac{(1-\rho)\omega_4}{1-(1-\rho)\omega_3}\omega_1\right)\hat{\tau}_{-i}
\end{aligned}
\tag{6.59}
$$

with $\Omega \equiv \frac{1-(1-\rho)\,\omega_3}{[1-(1-\rho)\,\omega_3]^2-[(1-\rho)\,\omega_4]^2}$. Written in a more compact way, this result will be used in the following sections:

$$
\pi_i = \mu_i + c^i\hat{\tau}_i + c^{-i}\hat{\tau}_{-i}, \quad i \in \{H, F\}. \tag{6.60}
$$

Referring to the supply-side fiscal policy introduced above, we assume that $\omega_2 = 0$ and obtain[24]

$$
\begin{aligned}
\mu_i \equiv \;& \Omega\rho\left[\bar{\omega}_{0,i} + \frac{(1-\rho)\omega_4}{1-(1-\rho)\omega_3}\bar{\omega}_{0,-i}\right] + \Omega\rho\left[\omega_1 E[\hat{\tau}_i] + \frac{(1-\rho)\omega_4}{1-(1-\rho)\omega_3}\omega_1 E[\hat{\tau}_{-i}]\right] \\
&+\Omega\rho\left[\left(\omega_3 + \frac{(1-\rho)\omega_4}{1-(1-\rho)\omega_3}\omega_4\right)E[\pi_i] + \left(\omega_4 + \frac{(1-\rho)\omega_4}{1-(1-\rho)\omega_3}\omega_3\right)E[\pi_{-i}]\right] \\
&+\Omega(1-\rho)\left[\omega_{0,i} + \frac{(1-\rho)\omega_4}{1-(1-\rho)\omega_3}\omega_{0,-i}\right] \;,
\end{aligned}
$$

which captures not only the terms dependent on monetary policy, but also the expectational terms and the stochastic terms, as shown by Dixit and Lambertini (2003a). The parameters

$$
c^i \equiv \Omega(1-\rho)\omega_1 \qquad \text{and} \qquad c^{-i} \equiv \Omega(1-\rho)\frac{(1-\rho)\omega_4}{1-(1-\rho)\omega_3}\omega_1
$$

denote the impact of domestic and foreign fiscal policy on inflation, respectively. Equation (6.60) states that regional PPI inflation can be explained as the outcome of influences from monetary policy and stochastic events, from fiscal policy in the same region, and from fiscal policy in the other region.

[24] Note that the calculations made so far hold more generally to facilitate enhancement of the micro-model with respect to other types of fiscal policies. Henceforth, we assume that $\omega_2 = 0$.

6.1.7 Output Determination

To obtain an equation for regional output y_i, we start with equation (6.37) and plug in equation (6.29):

$$
\begin{aligned}
Y_i &= \int_{lli}^{uli} \left(\frac{p^i(j)}{P_i}\right)^{1-\theta} dj \frac{W}{w_i} \\
&= \int_{lli}^{uli} \left(\frac{\theta \hat{d}_i}{(\theta-1)(1-\tau_i)} \left(\frac{W}{w_i}\right)^{\beta-1}\right)^{\frac{1-\theta}{1+\theta(\beta-1)}} \frac{W}{w_i} dj \\
&= \left(\frac{\theta \hat{d}_i}{(\theta-1)(1-\tau_i)} \left(\frac{W}{w_i}\right)^{\beta-1}\right)^{\frac{1-\theta}{1+\theta(\beta-1)}} \frac{W}{w_i}.
\end{aligned}
$$

Log-linearizing this equation and using the notation $y_i \equiv \hat{Y}_i$, we obtain

$$
y_i = \frac{1-\theta}{1+\theta(\beta-1)}\hat{d}_i + \frac{1-\theta}{1+\theta(\beta-1)}\bar{\tau}_i\hat{\tau}_i + \frac{(\beta-1)(1-\theta)}{1+\theta(\beta-1)}\hat{W} + \hat{W}. \tag{6.61}
$$

Now we follow the procedure in Dixit and Lambertini (2003b) and apply equation (6.51) in two ways: First, we replace the first $\hat{W}$ in (6.51) with i indices and the second $\hat{W}$ with $-i$ indices. We thus obtain

$$
\begin{aligned}
y_i &= \frac{1-\theta}{1+\theta(\beta-1)}\hat{d}_i + \frac{1-\theta}{1+\theta(\beta-1)}\bar{\tau}_i\hat{\tau}_i \\
&\quad + \frac{(\beta-1)(1-\theta)}{1+\theta(\beta-1)}\left[\frac{1+\theta(\beta-1)}{\beta-1}(\hat{p}^i(j)-\pi_i) - \frac{1}{\beta-1}\hat{d}_i - \frac{\bar{\tau}_i}{\beta-1}\hat{\tau}_i\right] \\
&\quad + \left[\frac{1+\theta(\beta-1)}{\beta-1}(\hat{p}^{-i}(j)-\pi_{-i}) - \frac{1}{\beta-1}\hat{d}_{-i} - \frac{\bar{\tau}_{-i}}{\beta-1}\hat{\tau}_{-i}\right].
\end{aligned} \tag{6.62}
$$

Second, we do the same thing the other way round, leading to

$$
\begin{aligned}
y_i &= \frac{1-\theta}{1+\theta(\beta-1)}\hat{d}_i + \frac{1-\theta}{1+\theta(\beta-1)}\bar{\tau}_i\hat{\tau}_i \\
&\quad + \frac{(\beta-1)(1-\theta)}{1+\theta(\beta-1)}\left[\frac{1+\theta(\beta-1)}{\beta-1}(\hat{p}^{-i}(j)-\pi_{-i}) - \frac{1}{\beta-1}\hat{d}_{-i} - \frac{\bar{\tau}_{-i}}{\beta-1}\hat{\tau}_{-i}\right] \\
&\quad + \left[\frac{1+\theta(\beta-1)}{\beta-1}(\hat{p}^{i}(j)-\pi_{i}) - \frac{1}{\beta-1}\hat{d}_{i} - \frac{\bar{\tau}_{i}}{\beta-1}\hat{\tau}_{i}\right].
\end{aligned} \tag{6.63}
$$

In the next step, we add up the two equations and divide by two. We evaluate $\hat{p}^i(j)$ in both regions for the flexible price firms, i. e. we replace $\hat{p}^i(j)$ by π_i^j, the price that maximizes current period indirect utility only. Replacing π_i^j with equation (6.56) and simplifying leads to

$$
\begin{aligned}
y_i = &\left(\frac{1-\theta}{2[1+\theta(\beta-1)]} - \frac{1}{2(\beta-1)}\right)\bar{\tau}_i\hat{\tau}_i - \left(\frac{1-\theta}{2[1+\theta(\beta-1)]} + \frac{1}{2(\beta-1)}\right)\bar{\tau}_{-i}\hat{\tau}_{-i} \\
&+ \frac{\beta\rho}{(\beta-1)(1-\rho)}(\pi_i - \bar{\pi}_i + \pi_{-i} - \bar{\pi}_{-i}) \\
&+ \left(\frac{1-\theta}{2[1+\theta(\beta-1)]} - \frac{1}{2(\beta-1)}\right)\hat{d}_i - \left(\frac{1-\theta}{2[1+\theta(\beta-1)]} + \frac{1}{2(\beta-1)}\right)d_{-i} \\
= &\left(\frac{1-\theta}{2[1+\theta(\beta-1)]} - \frac{1}{2(\beta-1)}\right)\bar{\tau}_i\hat{\tau}_i - \left(\frac{1-\theta}{2[1+\theta(\beta-1)]} + \frac{1}{2(\beta-1)}\right)\bar{\tau}_{-i}\hat{\tau}_{-i} \\
&+ \frac{2\beta\rho}{(\beta-1)(1-\rho)}(\pi_i - \bar{\pi}_i) + \frac{\beta\rho}{(\beta-1)(1-\rho)}(s_i - \bar{s}_i) \\
&+ \left(\frac{1-\theta}{2[1+\theta(\beta-1)]} - \frac{1}{2(\beta-1)}\right)\hat{d}_i - \left(\frac{1-\theta}{2[1+\theta(\beta-1)]} + \frac{1}{2(\beta-1)}\right)d_{-i},
\end{aligned}
\tag{6.64}
$$

The notation $\bar{s}_i = E[s_i]$ is used to denote region i's expected terms of trade. Given the steady state of $\bar{P}_i = \bar{P}^i = \bar{P}$ for all i, we have $\bar{s}_i \equiv 0$ so that we can drop this term. For ease of exposition we rewrite the last equation as follows:

$$
y_i = \bar{y}_i + a^i\hat{\tau}_i + a^{i,-i}\hat{\tau}_{-i} + b^i(\pi_i - \pi_i^e) + \kappa^i s_i + \phi_i, \tag{6.65}
$$

where $\bar{y}_i = 0$, $a^i \equiv \left(\frac{1-\theta}{2[1+\theta(\beta-1)]} - \frac{1}{2(\beta-1)}\right)\bar{\tau}_i$ captures the effect of the home country's fiscal policy instrument and $a^{i,-i} \equiv -\left(\frac{1-\theta}{2[1+\theta(\beta-1)]} + \frac{1}{2(\beta-1)}\right)\bar{\tau}_{-i}$ the effect of foreign fiscal policy on domestic output.[25] The effect of domestic surprise inflation on output is captured by $b^i \equiv \frac{2\beta\rho}{(\beta-1)(1-\rho)}$, with $\pi_i^e = \bar{\pi}_i = E[\pi_i]$, whereas the effect of a surprise change in the terms of trade, s_i, is measured by $\kappa^i \equiv \frac{\beta\rho}{(\beta-1)(1-\rho)}$. The variable ϕ_i replaces the effects of both productivity shocks, given by

$$
\phi_i = \left(\frac{1-\theta}{2[1+\theta(\beta-1)]} - \frac{1}{2(\beta-1)}\right)\hat{d}_i - \left(\frac{1-\theta}{2[1+\theta(\beta-1)]} + \frac{1}{2(\beta-1)}\right)\hat{d}_{-i}.
$$

Henceforth, ϕ_i is denoted as the "region-specific" output shock. In the policy analysis done in section 7.1, we will focus attention on equations (6.60) and (6.65), which summarize the microeconomic model.

[25]Note that the steady-state level of subsidies $\bar{\tau}_i$ is negative, as will be shown in section 7.2. Therefore, an expansionary fiscal policy is given if $\tau_i < \bar{\tau}_i$, i.e. if $\hat{\tau}_i = \frac{\tau_i - \bar{\tau}_i}{\bar{\tau}_i} > 0$. It is important to keep this in mind in order to follow the fiscal policy description in section 7.1.

6.2 A Utility Based Welfare Criterion

In this section, we derive a second-order approximation of welfare, based on the utility of a representative agent. The resulting welfare criterion will then be used to evaluate the various policies we consider in our model. The method of approximating welfare based on utility is well documented in the literature, beginning with Rotemberg and Woodford (1998), and Clarida, Gali and Gertler (2002) for a two-country model. In our model, we look at both regional and union-wide welfare and, thus, combine the closed economy model with that of an open economy. Our approach is based on the textbook treatment of Woodford (2003) and appendix B of Dixit and Lambertini (2003a).

One important insight shall become clear and is stressed in the upcoming sections: nationwide welfare depends on the national producer price index (PPI) inflation, as opposed to consumer price index (CPI) inflation. This means that a welfare maximizing policy authority should target the prices of goods produced within this region, and not those consumed in the region, which also include foreign goods' prices. This finding is in line with Gali and Monacelli (2002, 2005a), but in contrast to many papers on this topic, e. g. Dixit and Lambertini (2001, 2003b). From a theoretical point of view, we give the following explanation: In a microfounded *yeoman-farmer* model,[26] the agents' aversion of inflation variation is a result of the "disutility of labor"-part in the utility function. As a result of labor earnings that are assumed to be restricted to the home region, the home price level and, thereby, home inflation becomes important for home residents.[27] Hence, a concerned domestic policy authority should stabilize domestic inflation and domestic output in the short-run.[28]

In the following, we sketch the approximation method, derive the steady state around which we approximate, and derive the welfare criteria for one region and for the union as a whole.

[26]See Woodford (2003), pp. 144f.

[27]To apply this to the EMU, we state that mobility of labor appears still to be very low in the Euro-area due to cultural and structural barriers.

[28]Note that also fluctuations in foreign prices influences the consumption behavior and, therefore, affects also the utility of a household in region H. These prices, however, cannot be controlled by the fiscal policy maker in region H. So, the domestic fiscal authority focusses on a stabilization of home inflation and home output.

6.2.1 Approximation Method and Optimal Policy Instrument

In general, a variable $y \equiv \log(Y/\bar{Y})$ is approximated by a second-order Taylor series at the steady state $Y = \bar{Y}$:

$$\begin{aligned}\log(Y/\bar{Y}) &\approx \log(\bar{Y}/\bar{Y}) + \frac{1}{\bar{Y}}(Y-\bar{Y}) - \frac{1}{\bar{Y}^2}\frac{(Y-\bar{Y})^2}{2} \\ &\approx 0 + \frac{Y-\bar{Y}}{\bar{Y}} - \frac{1}{2}\left(\frac{Y-\bar{Y}}{\bar{Y}}\right)^2 \;, \end{aligned} \tag{6.66}$$

which can be transformed to $\frac{Y-\bar{Y}}{\bar{Y}} \approx y + \frac{1}{2}\left(\frac{Y-\bar{Y}}{\bar{Y}}\right)^2$. Then, notice that $y^2 = \left(\log(Y/\bar{Y})\right)^2$, approximated by a second-order Taylor series at the same point $Y = \bar{Y}$, looks as follows:

$$\begin{aligned}\left(\log\frac{Y}{\bar{Y}}\right)^2 &\approx \left(\log\frac{\bar{Y}}{\bar{Y}}\right)^2 + 2\left(\log\frac{\bar{Y}}{\bar{Y}}\right)\frac{1}{\bar{Y}}(Y-\bar{Y}) + 2\frac{1}{(\bar{Y})^2}\frac{(Y-\bar{Y})^2}{2} \\ &\approx 0 + 0 + \left(\frac{Y-\bar{Y}}{\bar{Y}}\right)^2 \;, \end{aligned} \tag{6.67}$$

which is exactly minus twice the last term in the first approximation. Therefore, up to second-order,

$$\frac{Y-\bar{Y}}{\bar{Y}} \approx y + \frac{1}{2}y^2 \;. \tag{6.68}$$

6.2.2 Steady State and Optimal Fiscal Policy

To understand the welfare implications of various policies, we evaluate the welfare costs of deviating from the optimal steady state. In doing so, we first evaluate the optimal steady state values, which would occur under flexible prices. Then, we calculate a second-order approximation of the households' utility function and simplify it to obtain a canonical representation of the welfare function.

The representative agent j living in region i derives utility from equation (6.1). Plugging in the first-order conditions and using the budget constraint yields the indirect utility function, as given in equation (6.27). Using equation (6.37), we can replace $\frac{W}{w_i}$ by $Y_i = Y_i^j$ for all j, where the latter equality only holds for flexible prices. Neglecting the utility from real balances, which is in line with what Rotemberg and Woodford (1999) call the *cashless limit* of the economy, we obtain

$$U_i^j = (1-\tau_i)Y_i^j - T_i - \frac{d_i}{\beta}(Y_i^j)^\beta. \tag{6.69}$$

Analogously to the previous section, we assume that fiscal policy uses only the lump-sum tax (or transfer) and the consumption tax (or subsidy) τ_i for its actions, while government spending (or waste) is set equal to zero. The budget constraint for the fiscal authority in region i is given by

$$T_i = -Y_i \tau_i \ . \tag{6.70}$$

As flexible prices imply identical pricing decisions for all producers, the left hand side of the first-order condition (6.29) becomes unity. Using again equation (6.37) and assuming a steady state in which all stochastic terms are equal to their expected value, i. e. $\bar{d}_i = E[d_i]$, the steady state *natural* level of output in the flexible price scenario is given by

$$\bar{Y}_i^{j,n} = \left(\frac{(\theta - 1)(1 - \bar{\tau}_i)}{\theta \bar{d}_i} \right)^{\frac{1}{\beta - 1}} . \tag{6.71}$$

We will approximate utility around this steady state given in equation (6.71), i. e. we linearize the utility function around a point in which prices are flexible and stochastic terms are equal to their expected values. Note, however, that we do not necessarily assume that fiscal policy is efficient. Thus, the overall distortion in the steady state output level is a result of both, market power and fiscal policy.[29] When taxes were set efficiently, they would eliminate the distortions stemming from monopolistic power. Taking the derivative of equation (6.69) with respect to τ_i, while taking equation (6.71) into account, gives the efficient tax rate

$$\tau_i^{\text{eff}} = \frac{1}{1 - \theta} \ , \quad \theta > 1, \tag{6.72}$$

which can be inserted into equation (6.71) to calculate the corresponding level of output in region i. We, then, obtain

$$Y_i^{j,\text{eff}} = \left(\frac{\theta - 1}{\theta d_i - 1} \right)^{\frac{1}{\beta - 1}} . \tag{6.73}$$

6.2.3 Derivation of a Regional Utility Based Welfare Criterion

A welfare maximizing fiscal policy in the home region optimizes the utility function of a representative agent j living in region H, which is given by equation (6.1). We aggregate all agents living in region H, which simplifies the consumption part due to the symmetry of this problem, while it leaves us with an integral in the labor/output-part, as staggered pricing implies a different behavior for different agents.

[29] See Woodford (2003), 293f. His parameter Φ_y equals our parameter κ used later in this section, where we keep with the notation in Dixit and Lambertini (2003a), Appendix B.

We obtain

$$\begin{aligned} U^H &= \gamma u(C^H, M_H/P^H) - \frac{1}{n}\int_0^n v_j(Y_H^j;\xi_H)dj \\ &= \gamma\left(\frac{C^H}{\gamma}\right)^{\gamma}\left(\frac{M_H/P^H}{1-\gamma}\right)^{1-\gamma} - \frac{1}{n}\int_0^n \left(\frac{d_H}{\beta}\right)(Y_H^j)^{\beta}dj. \end{aligned} \tag{6.74}$$

Note that we do not consider the fraction of utility that originates from real balances, as we focus on the cashless limit, following the seminal work of Rotemberg and Woodford (1998). Therefore, we only consider the fraction γ stemming from the $u(.)$-term.

The notation ξ is used to capture all stochastic disturbances to the model. More specifically, we assume that there exists a regional technology shock ξ_i that directly affects production of region i. As explained in the beginning of section 6.1, we abstain from modeling the production-side explicitly, but use instead a yeoman farmer model. The technology shock is, therefore, treated as being a part of the "disutility of labor"- term, which means that d_i is assumed to be a random variable, as defined at the beginning of the previous section.[30] We approximate around the flexible price steady state level of consumption of households in region H, which — for reasons of brevity only — is denoted by $\bar{C}$ from now on.[31]

Approximation of the $u(.)$-part of the Utility Function

We begin with the approximation of the $u(.)$-part in the utility function (6.74) around its steady state level under flexible prices and a given, constant fiscal policy by using a second-order Taylor series:

$$\begin{aligned} \tilde{u} &= \gamma\Big(\bar{u} + u_C\tilde{C} + u_\xi\tilde{\xi}_H + u_m\tilde{m} + \frac{1}{2}u_{CC}\tilde{C}^2 + \frac{1}{2}u_{\xi\xi}\tilde{\xi}_H^2 + \frac{1}{2}u_{mm}\tilde{m}^2 + \\ &\quad u_{Cm}\tilde{C}\tilde{m} + u_{C\xi}\tilde{C}\tilde{\xi}_H + u_{m\xi}\tilde{m}\tilde{\xi}_H\Big) + \mathcal{O}(||\xi_H||^3)\ , \end{aligned} \tag{6.75}$$

where a variable with a tilde (e.g. $\tilde{X}$) denotes the absolute deviation from the respective steady state level ($\bar{X}$), i. e., for home consumption we define $\tilde{C} \equiv C - \bar{C}$. The term $\mathcal{O}(||\xi_H||^3)$ denotes that the residual of the approximation is of third or higher order,

[30] Placing the regional productivity shock ξ_H after the semicolon in the disutility-part of equation (6.74) indicates direct dependency of the realization of the productivity shock.

[31] For a better exposition of the approximation of the utility function, we omit the superscripts H in the consumption terms. Note that the steady state level of consumption can also be rewritten in terms of domestic output Y_H, as it is done later in this section.

where the expression $||\xi_H||$ is a bound on the amplitude of the exogenous disturbances. The effects of third or higher order deviations of the various variables from their steady state levels will be neglected. A subscript on u or v denotes the first derivative of v or u with respect to the argument indicated by the subscript (u_C is for example the first derivative of u with respect to consumption C). Correspondingly, we use two subscripts after u or v to denote second derivatives. Furthermore, we use the notation m for domestic real money balance, i. e. $m \equiv M_H/P^H$.

Summarizing terms that are independent of policy by t.i.p., we can rewrite equation (6.75) as

$$\begin{aligned} \tilde{u} &= \gamma \left(\bar{u} + u_C \tilde{C} + u_m \tilde{m} + \frac{1}{2} u_{CC} \tilde{C}^2 + \frac{1}{2} u_{mm} \tilde{m}^2 + u_{Cm} \tilde{C} \tilde{m} + u_{C\xi} \tilde{C} \tilde{\xi}_H \right) \\ & \quad + \text{t.i.p.} + \mathcal{O}(||\xi_H||^3) \ . \end{aligned} \tag{6.76}$$

Combining the first-order conditions of the utility function with respect to consumption and money holdings leads to[32]

$$\begin{aligned} M_H &= (1-\gamma) I_H = \frac{1-\gamma}{\gamma} P^H C \\ \Leftrightarrow m &= \frac{1-\gamma}{\gamma} C \ . \end{aligned} \tag{6.77}$$

After log-linearization, we apply this equation for $\tilde{m}$ with $\tilde{m} = \frac{1-\gamma}{\gamma} \tilde{C}$.

The representative agent consumes goods of both regions, the home and the foreign region. Home consumption can be expressed by $C = \gamma k Y_H^{\nu} Y_F^{1-\nu}$, where $k \equiv (1-\nu)^{1-\nu} \nu^{\nu}$ and Y_i with $i = H, F$ denotes output of each region i.

The long-run steady state under flexible prices within the monetary union is given by

$$\bar{C} = \bar{Y}_H \ , \tag{6.78}$$

where C is overall consumption of region H. This means that in the long-run steady state domestic output (i. e. real income) equals the demand for commodities of the representative agents living in region H. This assumption holds, because government spending is assumed to be zero for the considered supply-side fiscal policy, which was introduced in section 6.1, and because international debt holdings are excluded in our examination.[33]

[32] For the derivation of this equation see section 6.1.

[33] Despite the one-period horizon, it is common when using this type of model to approximate around the long-term flexible price equilibrium. See e.g. Rotemberg and Woodford (1998).

Applying the Taylor expansion of second order as explained at the beginning of this section, we can substitute for

$$\tilde{C} = \bar{Y}_H \left(\hat{Y}_H + \frac{1}{2}\hat{Y}_H^2 + \mathcal{O}(||\xi_H||^3) \right) \ , \tag{6.79}$$

where we make use of the definition $\hat{Y}_H \equiv \log(Y_H/\bar{Y}_H)$.

In the steady state, the following relations (6.80) - (6.85) hold:[34]

$$\bar{m} = \frac{(1-\gamma)\bar{Y}_H}{\gamma}, \tag{6.80}$$

$$u_C = \gamma \bar{Y}_H^{\gamma-1} \frac{1}{\gamma^\gamma} \left(\frac{\bar{m}}{1-\gamma} \right)^{1-\gamma} = \left(\frac{\bar{Y}_H}{\gamma} \right)^{\gamma-1} \left(\frac{\bar{Y}_H}{\gamma} \right)^{1-\gamma} = 1, \tag{6.81}$$

$$\begin{aligned} u_m &= (1-\gamma) \left(\frac{\bar{Y}_H}{\gamma} \right)^{\gamma} \frac{\bar{m}^{-\gamma}}{(1-\gamma)^{1-\gamma}} = \left(\frac{\bar{Y}_H}{\gamma} \right)^{\gamma} \left(\frac{\bar{m}}{1-\gamma} \right)^{-\gamma} \\ &= \left(\frac{\bar{Y}_H}{\gamma} \right)^{\gamma} \left(\frac{\bar{Y}_H}{\gamma} \right)^{-\gamma} = 1, \end{aligned} \tag{6.82}$$

$$\begin{aligned} u_{CC} &= (\gamma-1) \frac{\bar{Y}_H^{\gamma-2}}{\gamma^{\gamma-1}} \left(\frac{\bar{m}}{1-\gamma} \right)^{1-\gamma} = \frac{\gamma-1}{\gamma} \left(\frac{\bar{Y}_H}{\gamma} \right)^{\gamma-2} \left(\frac{\bar{Y}_H}{\gamma} \right)^{1-\gamma} \\ &= -\frac{1-\gamma}{\bar{Y}_H}, \end{aligned} \tag{6.83}$$

$$\begin{aligned} u_{mm} &= -\gamma(1-\gamma) \left(\frac{\bar{Y}_H}{\gamma} \right)^{\gamma} \frac{\bar{m}^{-\gamma-1}}{(1-\gamma)^{1-\gamma}} = -\frac{\gamma}{1-\gamma} \left(\frac{\bar{m}}{1-\gamma} \right)^{-\gamma-1} \left(\frac{\bar{Y}_H}{\gamma} \right)^{\gamma} \\ &= -\frac{\gamma}{1-\gamma} \left(\frac{\bar{m}}{1-\gamma} \right)^{-\gamma-1} \left(\frac{\bar{m}}{1-\gamma} \right)^{\gamma} = -\frac{\gamma}{\bar{m}}, \end{aligned} \tag{6.84}$$

$$\begin{aligned} u_{Cm} &= \gamma(1-\gamma) \frac{\bar{Y}_H^{\gamma-1}}{\gamma^\gamma} \frac{\bar{m}^{-\gamma}}{(1-\gamma)^{1-\gamma}} = \left(\frac{\bar{Y}_H}{\gamma} \right)^{\gamma-1} \left(\frac{\bar{m}}{1-\gamma} \right)^{-\gamma} \\ &= \left(\frac{\bar{Y}_H}{\gamma} \right)^{\gamma-1} \left(\frac{\bar{Y}_H}{\gamma} \right)^{-\gamma} = \frac{\gamma}{\bar{Y}_H} \ . \end{aligned} \tag{6.85}$$

If we use equation (6.75) and insert m and the derivatives given above, we obtain equation

[34] To help the reader with the notation, we point out that e.g. u_C denotes the first derivative of u with respect C, u_{CC} denotes the second derivative of u with respect to C and u_{Cm} denotes the cross derivative with respect to C and m.

(6.86) after some mathematical manipulations:

$$\begin{aligned}
\tilde{u} &= \gamma\Bigg(\bar{u} + u_C\tilde{C} + u_m\tilde{m} + \frac{1}{2}u_{CC}\tilde{C}^2 + \frac{1}{2}u_{mm}\tilde{m}^2 + u_{C\xi}\tilde{C}\tilde{\xi}_H \\
&\quad + u_{Cm}\tilde{C}\tilde{m}\Bigg) + \text{t.i.p.} + \mathcal{O}(||\xi_H||^3) \\
&= \gamma\left(\bar{u} + u_C\left(\hat{Y}_H + \frac{1}{2}\hat{Y}_H\right)\bar{Y}_H + u_m\frac{1-\gamma}{\gamma}\left(\hat{Y}_H + \frac{1}{2}\hat{Y}_H\right)\bar{Y}_H\right. \\
&\quad + \frac{1}{2}u_{CC}\left(\hat{Y}_H + \frac{1}{2}\hat{Y}_H\right)^2\bar{Y}_H^2 + \frac{1}{2}\frac{(1-\gamma)^2}{\gamma^2}u_{mm}\left(\hat{Y}_H + \frac{1}{2}\hat{Y}_H\right)^2\bar{Y}_H^2 \\
&\quad \left. + u_{C\xi}\tilde{\xi}_H\left(\hat{Y}_H + \frac{1}{2}\hat{Y}_H\right)\bar{Y}_H + u_{Cm}\frac{1-\gamma}{\gamma}\left(\hat{Y}_H + \frac{1}{2}\hat{Y}_H\right)^2\bar{Y}_H^2\right) + \text{t.i.p.} + \mathcal{O}(||\xi||^3) \\
&= \gamma\bar{Y}_H\left(u_C\left(\hat{Y}_H + \frac{1}{2}\hat{Y}_H\right) + u_m\frac{1-\gamma}{\gamma}\left(\hat{Y}_H + \frac{1}{2}\hat{Y}_H\right) + \frac{1}{2}u_{CC}\hat{Y}_H^2\bar{Y}_H\right. \\
&\quad \left. + \frac{1}{2}u_{mm}\frac{(1-\gamma)^2}{\gamma^2}\hat{Y}_H^2\bar{Y}_H + u_{c\xi}\tilde{\xi}_H\hat{Y}_H + u_{Cm}\frac{1-\gamma}{\gamma}\hat{Y}_H^2\bar{Y}_H\right) + \text{t.i.p.} + \mathcal{O}(||\xi_H||^3) \\
&= \gamma u_C\bar{Y}_H\left(\hat{Y}_H\left(1 + \frac{u_m}{u_C}\frac{1-\gamma}{\gamma} + \frac{u_{C\xi}\tilde{\xi}_H}{u_C}\right)\right. \\
&\quad \left. + \frac{1}{2}\hat{Y}_H^2\left(1 + \frac{u_m}{u_C}\frac{1-\gamma}{\gamma} + \frac{u_{CC}}{u_C}\bar{Y}_H + \frac{u_{mm}}{u_C}\frac{(1-\gamma)^2}{\gamma^2}\bar{Y}_H + 2\frac{u_{Cm}}{u_C}\frac{1-\gamma}{\gamma}\bar{Y}_H\right)\right) \\
&\quad + \text{t.i.p.} + \mathcal{O}(||\xi_H||^3) \\
&= \gamma u_C\bar{Y}_H\left(\hat{Y}\left(1 + \frac{1-\gamma}{\gamma} + u_{C\xi}\tilde{\xi}_H\right)\right. \\
&\quad \left. + \frac{1}{2}\hat{Y}^2\left(1 + \frac{1-\gamma}{\gamma} - 1 + \gamma - 1 + \gamma + 2(1-\gamma)\right)\right) + \text{t.i.p.} + \mathcal{O}(||\xi_H||^3) \\
&= \gamma u_C\bar{Y}_H\left(\hat{Y}_H\left(1 + \frac{1-\gamma}{\gamma} + u_{c\xi}\tilde{\xi}_H\right)\right. \\
&\quad \left. + \frac{1}{2}\hat{Y}_H^2\left(1 + \frac{1-\gamma}{\gamma}\right)\right) \\
&= u_C\bar{Y}\left(\hat{Y}_H(\gamma + 1 - \gamma + \gamma u_{C\xi}\tilde{\xi}_H\right) + \frac{1}{2}\left(\hat{Y}_H^2\left(\gamma + 1 - \gamma\right)\right) + \text{t.i.p.} + \mathcal{O}(||\xi_H||^3) \\
&= u_C\bar{Y}_H\left(\hat{Y}_H\left(1 + \gamma u_{C\xi}\tilde{\xi}_H\right) + \frac{1}{2}\hat{Y}_H^2\right) + \text{t.i.p.} + \mathcal{O}(||\xi_H||^3).
\end{aligned}$$

We define $q_1 \equiv -\frac{\gamma u_{C\xi}\tilde{\xi}}{u_{CC}\bar{Y}_H}$ and obtain for the $u(.)$-part of the loss function the approximation

$$\tilde{u} = u_C\bar{Y}\left(\hat{Y}_H\left(1 + (1-\gamma)q_1\right) + \frac{1}{2}\hat{Y}_H^2\right) + \text{t.i.p.} + \mathcal{O}(||\xi_H||^3)\ , \tag{6.86}$$

where t.i.p. summarizes, again, the terms which cannot be influenced by policy.

Approximation of the $v(.)$-part of the Utility Function

Second order Taylor expansion of the disutility of labor part for a representative agent j (in a certain state of nature) leads to

$$\begin{aligned}\tilde{v}_j &= \tilde{Y}_H^j v_Y + \frac{1}{2} v_{YY} (\tilde{Y}_H^j)^2 + v_\xi \tilde{\xi}_H + \frac{1}{2} v_{\xi\xi} \tilde{\xi}_H^2 + v_{Y\xi} \tilde{Y}_H^j \tilde{\xi}_H + \mathcal{O}(||\xi_H||^3) \\ &= \tilde{Y}_H^j v_Y + \frac{1}{2} v_{YY} (\tilde{Y}_H^j)^2 + v_{Y\xi} \tilde{Y}_H^j \tilde{\xi}_H + \text{t.i.p.} + \mathcal{O}(||\xi_H||^3) \ . \end{aligned} \tag{6.87}$$

By using the Taylor-approximation

$$\tilde{Y}_H^j = \bar{Y}_H^j \left(\hat{Y}_H^j + \frac{1}{2} (\hat{Y}_H^j)^2 + \mathcal{O}(||\xi_H||^3) \right) \ , \tag{6.88}$$

we can rewrite equation (6.87) as

$$\begin{aligned}\tilde{v}_j &= \bar{Y}_H^j v_Y \left(\hat{Y}_H^j + \frac{1}{2} (\hat{Y}_H^j)^2 + \frac{1}{2} \frac{v_{YY}}{v_Y} \left(\hat{Y}_H^j + \frac{1}{2} (\hat{Y}_H^j)^2 \right)^2 \bar{Y}_H \right. \\ &\quad \left. + \frac{v_{Y\xi} \tilde{\xi}_H}{v_Y} \left(\hat{Y}_H^j + \frac{1}{2} (\hat{Y}_H^j)^2 \right) \right) + \text{t.i.p.} + \mathcal{O}(||\xi_H||^3) \\ &= \bar{Y}_H^j v_Y \left(\hat{Y}_H^j + \frac{1}{2} (\hat{Y}_H^j)^2 + \frac{1}{2} \frac{v_{YY}}{v_Y} \left(\hat{Y}_H^j \right)^2 + \frac{v_{Y\xi} \tilde{\xi}_H}{v_Y} \hat{Y}_H^j \right) + \text{t.i.p.} + \mathcal{O}(||\xi_H||^3) \\ &= \bar{Y}_H^j v_Y \left[\hat{Y}_H^j \left(1 + \frac{v_{Y\xi} \tilde{\xi}_H}{v_Y} \right) + \frac{(\hat{Y}_H^j)^2}{2} \left(1 + \frac{v_{YY} \bar{Y}_H^j}{v_Y} \right) \right] + \text{t.i.p.} + \mathcal{O}(||\xi_H||^3) \ . \end{aligned} \tag{6.89}$$

By maximizing the utility function (6.74) and combining the first-order conditions, we obtain

$$v_Y = u_C (1 - \kappa) \ , \tag{6.90}$$

where the marginal disutility of producing output v_Y is equal to

$$v_Y \equiv \frac{\partial v}{\partial Y_H^j} = d_H (Y_H^j)^{\beta - 1} \ . \tag{6.91}$$

Inserting the symmetric steady state output (under fully flexible prices) given by equation (6.71) results

$$v_Y = \bar{d}_H \left(\frac{(\theta - 1)(1 - \bar{\tau}_H)}{\theta \bar{d}_H} \right) = \frac{(1 - \bar{\tau}_H)(\theta - 1)}{\theta} \ . \tag{6.92}$$

Analogously to the proceedings in WOODFORD, we define a parameter κ as a size which "summarizes the overall distortions in the steady state output level as a result of both taxes and market power": [35]

$$\kappa \equiv 1 - \frac{(1-\bar{\tau}_H)(\theta-1)}{\theta}. \tag{6.93}$$

Equation (6.90) says that the marginal utility from consumption equals the marginal disutility from labor. Using this condition with the disutility part (6.89) of an average representative agent, we obtain

$$\begin{aligned}
\tilde{v} &= \frac{1}{n}\int_0^n v_j d_j \\
&= \bar{Y}_H u_C \left((1-\kappa)\left(1+\frac{v_{\xi Y}\tilde{\xi}_H}{v_Y}\right) E(\hat{Y}_H^j) + \frac{1}{2}E((\hat{Y}_H^j)^2)(1-\kappa)\left(1+\frac{v_{YY}\bar{Y}_H}{v_Y}\right)\right) \\
&\quad +\text{t.i.p.} + \mathcal{O}(||\xi_H||^3) \ .
\end{aligned} \tag{6.94}$$

Note that the expectations operator has to be used here, since agents are different from each other with respect to their pricing decisions. We calculate the second derivative of v with respect to the steady state output:

$$\begin{aligned}
v_{YY} &= (\beta-1)d_H(Y_H^j)^{\beta-2} = \frac{(\beta-1)d_H(Y_H^j)^{\beta-1}}{Y_H^j} \\
&= \frac{(\beta-1)(\theta-1)(1-\tau^H)}{\theta Y_H^j} = \frac{(\beta-1)(1-\kappa)}{Y_H^j} = \frac{(\beta-1)v_Y}{Y_H^j} \ .
\end{aligned} \tag{6.95}$$

Solving for v_Y yields

$$v_Y = \frac{Y_H^j}{\beta-1} v_{YY} \ . \tag{6.96}$$

Replacing v_Y in equation 6.94 yields

$$\begin{aligned}
\tilde{v} &= \bar{Y}_H u_C \left(\left[(1-\kappa) + (1-\kappa)(\beta-1)\frac{v_{YY}\tilde{\xi}_H}{v_{YY}\bar{Y}_H} \right] E(\hat{Y}_j^H) \right. \\
&\qquad \left. + E((\hat{Y}_H^j)^2)(1-\kappa)(1+\beta-1) \right) + \text{t.i.p.} + \mathcal{O}(||\xi_H||^3) \qquad (6.97) \\
&= \bar{Y}_H u_C \left(\left[1-\kappa-(\beta-1)\frac{v_{Y\xi}\tilde{\xi}_H}{v_{YY}\bar{Y}_H} \right] + \frac{\beta}{2}\left[(E\hat{Y}_H^j)^2 + \text{Var}Y_H^j \right] \right) \\
&\quad + \text{t.i.p.} + \mathcal{O}(||\xi_H||^3) \ . \qquad (6.98)
\end{aligned}$$

[35] See WOODFORD (2003), appendix E, p. 394.

To obtain equation (6.98) from (6.97), we refer to DIXIT and LAMBERTINI (2003a), who assume that "κ is small", meaning that it can be neglected if it enters (6.97) multiplicatively. This is possible as $v_{YY}\tilde{\xi}_H$ is on average significantly smaller than $v_{YY}\bar{Y}_H$. Furthermore, we use the definition of the variance and replace $E[(\hat{Y}_H^j)^2]$ by $(E[\hat{Y}_H^j])^2 + \text{Var}Y_H^j$.

A second-order Taylor approximation of the CES-aggregator Y_H of home goods leads to[36]

$$\hat{Y}_H = E\hat{Y}_H^j + \frac{1}{2}\left(1 - \frac{1}{\theta}\right)\text{Var}\hat{Y}_H^j + \mathcal{O}(||\xi_H||^3)\ . \tag{6.99}$$

Solving for $E\hat{Y}_H^j$, we have

$$E\hat{Y}_H^j = \hat{Y}_H - \frac{1}{2}\left(1 - \frac{1}{\theta}\right)\text{Var}\hat{Y}_H^j + \mathcal{O}(||\xi_H||^3), \tag{6.100}$$

and inserting into equation (6.98) yields

$$\begin{aligned}\tilde{v} \ &= \ \bar{Y}_H u_C\left([1 - \kappa + (\beta - 1)q_2]\,\hat{Y}_H + \frac{\beta}{2}\hat{Y}_H^2 + \frac{1}{2}\left[\beta - 1 + \frac{1}{\theta}\right]\text{Var}\hat{Y}_H^j\right) \\ &\quad + \text{t.i.p.} + \mathcal{O}(||\xi_H||^3)\ ,\end{aligned} \tag{6.101}$$

where the definition $q_2 \equiv -\frac{v_{Y\xi}\tilde{\xi}_H}{\bar{Y}_H v_{YY}}$ is used.

Derivation of the Welfare Function

Subtracting (6.101) from (6.86) yields social welfare

$$\begin{aligned}U^H \ &= \ u_C\bar{Y}_H\left(\hat{Y}\left(1 + (1-\gamma)q_1\right) + \frac{1}{2}\hat{Y}^2\right) \\ &\quad - u_C\bar{Y}_H\left([1 - \kappa + (\beta - 1)q_2]\,\hat{Y}_H + \frac{\beta}{2}\hat{Y}_H^2 + \frac{1}{2}\left[\beta - 1 + \frac{1}{\theta}\right]\text{Var}\hat{Y}_H^j\right) \\ &\quad + \text{t.i.p.} + \mathcal{O}(||\xi_H||^3) \\ &= -\frac{u_C\bar{Y}_H}{2}\left(\hat{Y}_H^2(\beta - 1) - 2\hat{Y}_H\left[q_1(1-\gamma) + q_2(\beta - 1) + \kappa\right]\right. \\ &\quad \left. + \frac{1 + \theta(\beta - 1)}{\theta}\text{Var}\hat{Y}_H^j\right) + \text{t.i.p.} + \mathcal{O}(||\xi_H||^3)\ .\end{aligned} \tag{6.102}$$

When loglinearizing the CES-aggregator over domestically produced differentiated goods, we obtain (see WOODFORD (2003), p. 396)

$$\hat{Y}_H^j = \log \bar{Y}_H - \theta(\log p^H(j) - \log P_H). \tag{6.103}$$

[36]See WOODFORD (2003) chapter 6 and appendix E, or ROTEMBERG and WOODFORD (1998).

The variance of $\log \hat{Y}_H^j$ is given by

$$\mathrm{Var}(\hat{Y}_H^j) = \mathrm{Var}(\log \bar{Y}_H - \theta(\log p^H(j) - \log P_H)) = \theta^2 \mathrm{Var}(\log p^H(j)). \tag{6.104}$$

The last equation incorporates a very important finding of our model: As the domestic agent j only works in the home-region to produce the domestic good Y_H^j, and as the production of this good matters for his utility (via the disutility part of labor in the utility function), he also cares only about the variability of this output. Hence, only the price level of domestically produced goods matters for the welfare of domestic agents. While this finding is well documented in the theoretical literature,[37] it is much less noted in applied work. Of course, this finding is overturned once one assumes high labor mobility, such that a representative agent of one region produces goods in both regions. So far, for the European Monetary Union a high degree of labor mobility seems not to be backed by the data.

To apply the variance of domestic prices to the welfare equation for obtaining the inflation target, we need first to refer to the staggered price setting: We assume that a certain fraction Φ^H of firms (=producer-consumers) is not able to adjust the prices in response to a shock, while a fraction $(1 - \Phi^H)$ can freely change their prices after a shock occurs.

Using the overall inflation rate in region H given by equation (6.42) as well as the "pseudo-intertemporal" equation (6.45) for the optimal inflation rate $\tilde{\pi}_H$ derived in the previous section, we can express the inflation rate as a combination of variables that have a single period interpretation:

$$\pi_H = \rho\bar{\pi}_H + (1-\rho)\pi_H^j, \qquad \rho = \Phi^H[1 + (1 - \Phi^H)\eta], \tag{6.105}$$

which is just a restatement of equation (6.56) for region H.[38] Again, $\bar{\pi}_H$ is the average inflation rate that arises when prices are set before the shocks occur, while π_H^j is the price that is optimal for the current period only, i. e. after uncertainty about the stochastic processes is resolved. With this single period representation we are able to apply the result of the first example in Woodford (2003, pp. 397f.) for our next steps. Firms that have to set prices before the shock materializes will set them identically according to the expected value of the optimal price for the period, i. e.

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[37]See Woodford (2003) or Gali and Monacelli (2002).

[38]This and the following steps are all loglinear approximations and are thus only accurate up to a residual of order $\mathcal{O}(||\xi_H||^2)$ or higher.

$$\log \bar{\pi}_H = E[\pi_H^j], \tag{6.106}$$

where E denotes the expectations operator. Subtracting the expectation of equation (6.105) from itself and noting that $E[\bar{\pi}_H] = \bar{\pi}_H$, we obtain

$$\begin{aligned} \pi_H - E[\pi_H] &= (1-\rho)(\pi_H^j - E[\pi_H^j]) \\ &= (1-\rho)(\pi_H^j - \bar{\pi}_H). \end{aligned} \tag{6.107}$$

We, now, combine equations (6.104) and (6.107). The relation between the variance of $\log p^H(j)$ and the inflation goal in the representative agent's utility function is given by

$$\begin{aligned} \operatorname{Var} \log p^H(j) &= \rho(1-\rho)(\pi_H^j - \bar{\pi}_H)^2 = \frac{\rho(1-\rho)}{(1-\rho)^2}(\pi_H - E[\pi_H])^2 \\ &= \frac{\rho}{1-\rho}(\pi_H - \bar{\pi}_H)^2 \ . \end{aligned} \tag{6.108}$$

Inserting relation (6.108) into (6.102) yields

$$\begin{aligned} U^H = & -\frac{\bar{Y} u_C}{2} \left(\hat{Y}_H^2(\beta-1) - 2\hat{Y}_H \left[q_1(1-\gamma) + q_2(\beta-1) + \kappa \right] \right. \\ & \left. + \frac{\rho\theta(1+\theta(\beta-1))}{1-\rho}(\pi_H - \bar{\pi}_H) \right) + \text{t.i.p.} + \mathcal{O}(||\xi_H||^3) \ . \end{aligned} \tag{6.109}$$

To obtain the output goal in the welfare function, we perform some mathematical manipulations:[39]

$$\begin{aligned} & \hat{Y}_H^2(\beta-1) - 2\hat{Y}_H \left[q_1(1-\gamma) + q_2(\beta-1) \right] - 2\kappa \hat{Y}_H \\ & = (\beta-1) \left(\hat{Y}_H^2 - 2\hat{Y}_H \frac{q_1(1-\gamma) + q_2(\beta-1)}{\beta-1} - \frac{2\hat{Y}_H}{\beta-1}\kappa \right) \ . \end{aligned} \tag{6.110}$$

The *natural* rate of output in region H, $\hat{Y}_H^n$, which materializes under flexible prices in the setting of monopolistic competition, is given by a loglinearization of equation (6.71). It can be expressed in terms of the region-specific variables q_1 and q_2:[40]

$$\hat{Y}_H^n = \frac{(1-\gamma)q_1 + (\beta-1)q_2}{\beta-1} \ . \tag{6.111}$$

[39]Note that q_1 and q_2, which were defined above, are region specific notations, as it contains first and second derivatives of a representative household's utility, who lives in region H.

[40]To see this, note that the loglinearized version of equation (6.71) reads $\hat{Y}_H^n = \frac{-1}{\beta-1}(\hat{d}_i + \frac{\bar{\tau}_i}{1-\bar{\tau}_i}\hat{\tau}_i)$. Rewriting the terms in the last parentheses in absolute deviations from steady state, we obtain $\hat{Y}_H^n = \frac{-1}{\beta-1}(\frac{d_i}{\bar{d}_i} - \frac{1-\tau_i}{1-\bar{\tau}_i}) = \frac{-1}{\beta-1}(Y_i/\bar{Y}_i)$. Given our assumptions, $\xi_H = d_H$, $u_{C,\xi} = 0$ and the other partial derivatives as given in the text, equation (6.111) is only a more general notation for the equation given in this footnote.

A detailed derivation of the natural rate of output for a similar utility function (but restricted to a single-country examination) can be found in WOODFORD (2003), chapter 3. Using this result and adding terms from the t.i.p.-part, we can rewrite (6.110) as

$$\begin{aligned}
&(\beta-1)\left(\hat{Y}_H^2 - 2\hat{Y}_H\hat{Y}_H^n + (\hat{Y}_H^n)^2 - 2\frac{\hat{Y}_H\kappa}{\beta-1} - 2\frac{\hat{Y}_H^n\kappa}{\beta-1} + \frac{\kappa^2}{(\beta-1)^2}\right) + 2\frac{\hat{Y}_H^n\kappa}{\beta-1} - \frac{\kappa^2}{(\beta-1)^2} \\
&= (\beta-1)\left((\hat{Y}_H - \hat{Y}_H^n)^2 - 2\frac{(\hat{Y}_H - \hat{Y}_H^n)\kappa}{\beta-1} + \frac{\kappa^2}{(\beta-1)^2}\right) + \text{t.i.p.} \\
&= (\beta-1)\left((\hat{Y}_H - \hat{Y}_H^n)^2 - 2(\hat{Y}_H - \hat{Y}_H^n)\bar{y}_H^{\text{diff}} + (\bar{y}_H^{\text{diff}})^2\right) + \text{t.i.p.} \qquad (6.112)
\end{aligned}$$

Here, we have used the notation $\bar{y}_H^{\text{diff}}$ for the log deviation of steady state level of output associated with zero inflation, as given by an aggregated version of equation (6.71), from the steady state of efficient output given by equation (6.73), evaluated at $d_i = \bar{d}_i$.[41] Formally,

$$\begin{aligned}
\bar{y}_H^{\text{diff}} &= \log\left(\frac{\bar{Y}_H}{\bar{Y}_H^{\text{eff}}}\right) = \log\left[\frac{\left(\frac{(\theta-1)(1-\bar{\tau}_H)}{\theta\bar{d}_H}\right)^{\frac{1}{\beta-1}}}{\left(\frac{\theta-1}{\theta\bar{d}_H-1}\right)^{\frac{1}{\beta-1}}}\right] = \frac{1}{\beta-1}\log\left(\frac{(\theta-1)(1-\bar{\tau}_H)}{\theta}\right) \\
&\approx \frac{1}{\beta-1}\left(\frac{(\theta-1)(1-\bar{\tau}_H)}{\theta} - 1\right) = -\frac{\kappa}{\beta-1}, \qquad (6.113)
\end{aligned}$$

where $\bar{d}_H = 1$, the approximation holds only for values of $(\theta-1)(1-\bar{\tau}_H)\theta^{-1}$ close to one, and the last equation uses the notation of equation (6.93). The variable $\bar{y}_H^{\text{diff}}$ summarizes the overall distortions in steady state output. Inserting (6.112) into equation (6.109) yields

$$\begin{aligned}
U^H &= -\frac{\bar{Y}u_C}{2}\left(\frac{\rho}{1-\rho}\theta\left[1+\theta(\beta-1)\right](\pi_H - \bar{\pi}_H)^2 + (\beta-1)(\hat{Y}_H - \hat{Y}_H^n - \bar{y}_H^{\text{diff}})^2\right) \\
&\quad + \text{t.i.p.} + \mathcal{O}(||\xi_H||^3)\ . \qquad (6.114)
\end{aligned}$$

Rearranging terms — and writing y_i instead of $\hat{Y}_i$ as well as $\bar{\bar{y}}_H$ instead of $\hat{Y}_H^n + \bar{y}_H^{\text{diff}}$ — results in the welfare criterion for the home region,

$$L_H = \frac{1}{2}\left((\pi_H - \bar{\pi}_H)^2 + \theta_H(y_H - \bar{\bar{y}}_H)^2\right) + \text{t.i.p.} + \mathcal{O}(||\xi_H||^3)\ , \qquad (6.115)$$

where θ_H is the relative weight of the output goal, which can be expressed by

$$\theta_H = \frac{(1-\rho)(\beta-1)}{\rho\theta[1+\theta(\beta-1)]}.$$

[41] Observe that in the current setting, the steady state of output under zero inflation coincides with the steady state of output under flexible prices, see WOODFORD (2001, p. 16) or WOODFORD (2003).

The optimization problem of social welfare in the foreign region follows the same principle. Therefore, we can state that the foreign welfare criterion equals

$$L_F = \frac{1}{2}\left((\pi_F - \bar{\pi}_F)^2 + \theta_F(y_F - \bar{\bar{y}}_F)^2\right) + \text{t.i.p.} + \mathcal{O}(||\xi_H||^3) \ , \tag{6.116}$$

where θ_F is the output goal of the foreign region. Remark that our notation in terms of percentage changes in output differs slightly from the one with an "output gap" typically used in the literature. The reason for this is twofold. First, we want to express the variables in our policy analysis throughout in terms of inflation and output. Second, we prefer the explicit notation of observables rather than using the empirically less precise concept of an "output gap".

We follow the assumption in WOODFORD (2003, chapter 6, section 3.1) that the inefficiency wedge κ is zero (or of order $\mathcal{O}(||\xi_H||^3)$, so that we can neglect it in the quadratic approximation of expected utility). Implications of this are that (a) the steady state of output is equal to the efficient steady state, (b) the percentage deviations of the natural rate of output are equal to those of the efficient rate (up to second order), and (c) the natural rate of output is (up to second order) equal to the efficient rate of output. Thus, $\bar{y}_H^{\text{diff}} = \hat{Y}_H^n = \bar{\bar{y}}_H = y_H = 0$, and the output argument in the welfare maximizing loss functions simplifies to just the percentage deviation of regional output from its steady state $y_H = 0$.

6.2.4 Derivation of a Union-wide Utility Based Welfare Criterion

One can imagine two forms of union-wide welfare: On the one hand, welfare in the whole union can be seen as the utility of a representative agent of the whole union, who produces union-wide output and cares about union-wide prices. On the other hand, union-wide welfare can be seen as the weighted average of regional welfare in both regions.

To derive a utility-based welfare criterion of the first type from our micro-approach, we abstract from differentials within the union and use union-wide variables: Union-wide inflation is defined as $\pi = n\pi_H + (1-n)\pi_F$, and analogously, union-wide output is defined as $y = ny_H + (1-n)y_F$. The maximization problem of a policy authority corresponds to that of the region-specific welfare criterion, derived in the previous subsection, after having replaced the region-specific variables by the corresponding union-wide variables. Hence, we can write down the union-wide welfare criterion as

$$L_{U,I} = \frac{1}{2}\left((\pi - \bar{\pi})^2 + \theta_M(y - \bar{\bar{y}})^2\right) + \text{t.i.p.} + \mathcal{O}(||\xi_H||^3) \ , \tag{6.117}$$

where $\bar{\bar{y}}_H = n\bar{\bar{y}}_H + (1-n)\bar{\bar{y}}_F$.

Alternatively, a policy maker can optimize a weighted sum of each single region's social loss function. As an example, Pappa (2004) uses a central bank loss function of this type when analyzing welfare-effects under a coordinated monetary policy. The idea of its derivation is that a policy maker tries to maximize the utility of a "union-wide representative household", who behaves with a fraction of n like an agent of region H and with a fraction of $(1-n)$ like an agent of region F. The welfare criterion then equals the sum of the region-specific welfare functions, weighted by the size of the corresponding region. It has the form

$$\begin{aligned} L_{U,II} &= \frac{1}{2}\left[n\left((\pi_H - \bar{\pi}_H)^2 + \theta_H(y_H - \bar{\bar{y}}_H)^2\right)\right. \\ &\quad \left. + (1-n)\left((\pi_F - \bar{\pi}_F)^2 + \theta_F(y_F - \bar{\bar{y}}_F)^2\right)\right] + \text{t.i.p.} + \mathcal{O}(||\xi_H||^3) \ . \quad (6.118) \end{aligned}$$

The two possibilities are not only important from the perspective of their microfoundation, but they are also important for economic policy analysis. In this context, it is not clear which loss function is most eligible for a common, union-wide monetary policy. Gros and Hefeker (2002) see a conflict in the fact that the European Central Bank is responsible for the average performance of the Euro-zone as a whole, whereas the European Union was created to serve the interests of its single member states. In their model they compare the welfare impact of a common monetary policy which minimizes a weighted average of national losses to a common monetary policy focussing on the minimization of the loss function treating Euro-area wide variables. One result is that the inflation bias might be higher under a central bank that cares about area-wide variables than under the one which cares about national variables. The main policy conclusion is that in most cases a common central bank that takes national variables into account outweighs a common central bank which optimizes a loss function considering Euro-area wide variables from a welfare-perspective, when remarkable heterogeneities exist.

In a further paper, Aksoy et al. (2002) analyze the consequences of economic and institutional asymmetries on the effectiveness of a common central bank's policy in a currency area. Using a model similar to Rudebusch and Svensson (1999), assuming that the policy maker optimizes a loss function depending on variability of output, inflation and interest rates, they pose the question "whether the national representatives in the ECB-Council take a union-wide perspective when deciding about monetary policy, or [...] give a high weight to national economic conditions when taking [...] decisions" (p. 444). From a more practical viewpoint – the decision-making problems that arise in a heterogeneous monetary union – they come to a different conclusion as Gros and Hefeker:

Welfare is higher when having an ECB-Board focussing on Euro-wide variables compared to a regime in which all members of the Governing Council take national targets into account. This is based on the fact that the individual decision makers tend to offset each other when asymmetries in shocks or transmission processes are significant.

We tested both possibilities to obtain the numerical results of section 7.2 and found that the differences of the results using both types of social loss functions were marginal, and did not change the qualitative order of the results. Therefore, we restrict our analysis to a use of the second type of the central bank's target function in the remainder of our analysis.

Chapter 7

Policy Analysis

7.1 Setup of the Policy Analysis

In this section, we analyze the strategic behavior of monetary and fiscal policies in a monetary union. The monetary union consists of two regions with independently acting fiscal policies and an independent common central bank exercising monetary policy.

We begin with the establishment of the setting used for the analysis of fiscal and monetary policy interactions. This summarizes the main model equations derived in the previous sections 6.1 and 6.2, extended by additional assumptions made in this section. We, furthermore, highlight the different scenarios in which we analyze the strategic behavior of monetary and fiscal policies. We distinguish between scenarios of simultaneous policy actions and those in which policy makers act sequentially.

7.1.1 Framework

As in the micro-model we consider two regions, previously denoted by home region H and foreign region F. From now on, we will use the notation region A and region B instead, to take a neutral point of view. We consider a region to be defined by a set of countries characterized by a high degree of homogeneity and exposed to similar shocks. Thus, fiscal policies within one such region can be considered to be coordinated, as each region has to optimize a similar problem. Alternatively, one region could capture one specific country of interest, while the other region refers to the "rest of the monetary union".

In the whole currency area, the population is given by a continuum of agents on the interval $[0, 1]$, with $[0, n]$ living in region A and $[n, 1]$ in region B. The fiscal authority in region i chooses a policy variable τ_i, with $i = A, B$, where τ_i is a shortcut to $\hat{\tau}_i$, the notation used in the previous chapter. Fiscal policy affects national output, y_i, and inflation, π_i, as well as union-wide output, y, and inflation, π.[1] Union-wide variables are given by the weighted sum of the region-specific levels, where the weights of the regions are given by n and $(1-n)$, respectively. In the following, we discuss the essential building blocks of our model:

Output Equation of Country i

Output in region i is derived in the micro-model in section 6.1 and explicitly given in equation (6.65). For convenience, we restate it here:

$$y_i = \bar{y}_i + a^i\tau_i + a^{i,j}\tau_j + b^i(\pi_i - \pi_i^e) + \kappa^i s_i + \phi_i \ , \tag{7.1}$$

where j denotes "not region i".

According to Kydland and Prescott (1979) and Barro and Gordon (1983), surprise inflation may generate an increase in the national output level. Workers demand nominal wages that are sufficiently high to cover expected average future price increases. When the inflation rate reaches an unexpectedly high level, i. e. $\pi > \pi^e$, it leads ex post to lower real wages and increases both employment and output. Therefore, b_i has a positive sign.

A higher τ_i corresponds to a more expansionary fiscal policy. It can be interpreted (i) as subsidies granted by the fiscal policies to reduce the frictions stemming from monopolistic power (an interpretation of τ_i which is in line with our microfoundation of section 6.1 and which is also typically used in New Keynesian Dynamic Stochastic General Equilibrium (DSGE) models), and (ii) as demand for public goods (which accords better with actual fiscal policies in the European Monetary Union).[2]

We consider both interpretations of fiscal policies in section 7.2. In both cases an expansionary fiscal policy is represented by an increase in τ_i. Furthermore, we derived that fiscal policies have positive spillovers onto the other region. For the baseline supply-side fiscal policy[3] as well as for the demand-side fiscal policy both a^i and a^{ij} have a positive

[1] More precisely, y_i denotes the percentage deviation of output from its steady state. Henceforth, we use "output" for reasons of brevity.

[2] While we have not incorporated this interpretation in our model, one can easily see from inspecting the fairly general budget constraint of the fiscal authorities given in equation (6.7) and (6.8) that this is a straightforward exercise.

[3] In New Keynesian DSGE models, fiscal policy is described by supply side policy, which aims at

sign.[4]

The term $\kappa^i s_i$ denotes the change in the current account, where $\kappa^i > 0$ and the terms of trade, s_i, from the perspective of region i, which is given by the log-linear approximation of equation (6.16):

$$s_i = (\pi_j - \pi_i) \ . \tag{7.2}$$

We know from empirical studies that the terms of trade effect also depends on the region's size. This means that a smaller region typically has a higher κ^i, implying that inflation differentials have a greater effect on output, something that is missing here.[5] A higher inflation rate in region j than in region i corresponds to a real depreciation of region i and thus increases its net exports. This shift of consumption from foreign goods (region j) to domestic goods (region i) increases domestic income.

Finally, a random shock ϕ_i enters the output equation, which is an i.i.d. shock with an expected value of zero and variance $\sigma^2_{\phi_i}$. In the microfounded model we have shown that this shock is the weighted sum of the deviations of the two regional (stationary) productivity processes from their respective steady states.[6]

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eliminating the distortions stemming from monopoly power. We, thus, denote supply-side policy as baseline case, henceforth.

[4]Note that a^i and a^{ij} can principally also be negative, e.g. for demand-side policy if an expansionary governmental policy has a great negative impact on private consumption or investment. We will, however, neglect such situations in our analysis.

[5]Note that we implicitly assume that the intensity of trade inside the currency area is high enough for effects from outside the union to be neglected. Another possibility for eliminating outside effects is to assume that all regions within the monetary union have similar trade relations with the rest of the world and are, thus, immaterial for our results. This is a critical assumption, as we pointed out in chapter 5.2 that empirically also trade relations with third regions matter, especially in the case of Ireland and Finland.

[6]Dixit and Lambertini (2003a) use a nonlinear shock vector of structural parameters, as laid out in the appendix of their paper. We depart from this, as we find it difficult to assume distributions of the single parameters and also the correlations between them. Instead, we make the standard RBC assumption that output is influenced by a normally distributed random variable capturing total factor productivity. This shock may be interpreted as comprising the effects of the shock vector in Dixit and Lambertini. Our shock effects output deviation as well as inflation directly, but also indirectly through the policy variables.

Inflation Equation of Country i

Inflation differences within the monetary union are caused by asymmetric shocks and country-specific fiscal policy actions. Thus, inflation in region i evolves according to

$$\pi^i = \mu + c^i \tau_i + c^{ij} \tau_j \ , \tag{7.3}$$

as derived in section 6.1 and stated in equation (6.60). Again, the superscript j denotes "not region i". The central bank influences a policy variable μ, where we assume that monetary policy has the same impact on inflation in both regions.[7] Analogously to Dixit and Lambertini (2003a), "μ stands for some actual policy variable such as the base money supply or a nominal interest rate, and determines a component of the price level," (p. 1525). Therefore, a higher μ implies a more expansionary monetary policy.

The parameter c^i refers to the influence of national fiscal policies on inflation, while c^{ij} measures the effect of foreign expenditure on region i's inflation rate, i. e. it captures the spillover effects stemming from fiscal policy.

Note that the parameters c^i and c^{ij} can have either sign. Dixit and Lambertini (2003a) indicate that the sign of the parameters may become negative when tax cuts and subsidies raise the supply of goods and are at the same time financed by income taxes, which lead to a crowding out of private demand. This is in line with the microfounded model of section 6.1. By contrast, a positive sign appears when fiscal policies are characterized by demand-side policies. This effect may be stronger if government expenditures are financed by distortionary production taxes reducing supply. c^i and c^{ij} have the same sign, but the absolute value of c^i is supposed to be higher than that of c^{ij}, i. e., direct effects from fiscal policies are stronger than the resulting spillovers to the other region.

Rational Expectations

The private sector has rational expectations about inflation, i. e. the following condition holds:

$$\pi_i^e = E(\pi_i). \tag{7.4}$$

[7] In this context Adão et al (2004) show that monetary policy cannot be used to offset idiosyncratic shocks within different countries belonging to a monetary union, as common monetary policy affects the monetary union as a whole.

Target Functions of Fiscal Authorities

Fiscal authorities minimize a quadratic loss function, that aims at national inflation and national output. The functional form of the loss function is identical to that of regional welfare, derived in the previous section and explicitly stated in equations (6.115) and (6.116):

$$L_{F^i} = \frac{1}{2}\left[(\pi_i - \pi_F^i)^2 + \theta_F^i (y_i - y_F^i)^2\right] \ . \tag{7.5}$$

Note that π_F^i is the fiscal policy's inflation target in region i, and y_F^i is the desired output level of the fiscal authority in region i. According to the utility-based welfare criterion derived in section 6.2, these reference values should be equal to zero for inflation and to the flexible price output plus the steady state deviation from the efficient steady state in the case of output.[8] If both fiscal authorities and the monetary authority agree on the targets, the first-best situation with the highest possible welfare can be obtained. This is demonstrated in Dixit and Lambertini (2003b) and corresponds to the joint cooperation case in our model, which will be introduced later.

However, EMU national governments and the ECB have often disagreed about the appropriate strategy for their policies. Therefore, we deviate from the microeconomic model by presuming that the fiscal targets deviate from the socially optimal level. More specifically, *for inflation and output we assume target levels that are both above the socially optimal levels*. This might be justified by the fiscal policy makers' desire to attain greater government size (Fatas and Rose, 2001) and/or their incentive to maximize reelection probability (Beetsma and Uhlig, 1999). To illustrate this, one can easily imagine that fiscal authorities are able to deceive their voters about the socially optimal targets, particularly during election campaigns. This would be espacially true of a monetary union, where fiscal policy communicates with the *domestic* society, while monetary policy is *centralized* and concerned with the whole society of the monetary union. Accordingly, it communicates with the private sector of the individual regions from a greater distance.

Furthermore, the inflation and output targets of fiscal policies in both regions may: Economically intuitive reasons for considering different inflation targets on the part of the agents may be given (i) by home-bias effects in the consumption of goods, (ii) by different elasticities of substitution in the representative agents' utility function across regions, or (iii) by different proportions of tradeable and non-tradeable goods in both regions.

[8] With the simplifying assumptions made below equation (6.116) the optimal target for output is also zero.

In our microeconomic model we have incorporated a home-bias effect in consumption and considered region-specific productivity shocks, which represent possible reasons for different fiscal targets in the two regions.

Target Function of the Common Central Bank

The common central bank is assumed to optimize the union-wide social welfare function derived in section 6.2. Our major justification for this assumption is the relatively high independence of central banks in industrialized countries. We choose the second specification in regional variables, stated in equation (6.118), which we state again here, using a notation with indices M to denote monetary policy:

$$\begin{aligned} L_M &= \frac{1}{2}\left[n\left((\pi_A - \pi_M^A)^2 + \theta_M^A (y_A - y_M^A)^2\right)\right. \\ &\quad \left. + (1-n)\left((\pi_B - \pi_M^B)^2 + \theta_M^B (y_B - y_M^B)^2\right)\right] \ . \end{aligned} \tag{7.6}$$

In the case of excessive fiscal targets, as motivated above, we can state that the central bank is relatively conservative in comparison to fiscal policies, given by $\pi_M^i < \pi_F^i$ and $y_M^i < y_F^i$ for all i. Our model differs in that respect from the approach of Dixit and Lambertini (2003b). They assume that fiscal policies act in a socially optimal manner and the central bank is too conservative, whereas we claim that the central bank maximizes union-wide welfare, and fiscal policies act in too expansionary a way.

7.1.2 Scenarios

The different weights on output stabilization and the different output and inflation targets of monetary and fiscal policies give rise to trade-offs among policy makers. Whereas the fiscal authorities attach greater importance to output stabilization (and to pushing output and inflation above their natural levels), the common central bank puts a relatively higher weight on the stabilization of inflation. These conflicting targets induce strategic behavior among the policy makers, which is examined in the following.

In our model, we restrict the analysis to scenarios under discretion. Discretionary policies accord best with reality, as no major central bank makes any kind of binding commitment on the way of conducting future monetary policy.[9] Fiscal policies in the

[9] See Clarida, Gali and Gertler (1999). As an example of the lack of a binding commitment, the European Central Bank, although officially announcing “close to, but below two percent” as its target for a harmonized index of consumer prices inflation, regularly failed to achieve this goal in the last five years, as figure 5.2 shows.

European Monetary Union seem also far away from committing themselves to certain rules. In this context, one can possibly consider the compliance of the stability and growth pact, which is not too strict a rule. However, it has been violated by several EMU-countries during unfavorable economic times without having been confronted to major sanctions until today. And one currently cannot imagine that national fiscal authorities would accept to commit to strict common rules. Beetsma and Jensen (2005) state that actual fiscal policies in the European Monetary Union are primarily conducted with the objective of serving national interests rather than union-wide-interests.

In this respect, we want to understand the nature of optimal monetary and fiscal policies when both, fiscal and monetary policy makers, move simultaneously and its nature, when one of them moves first and, thereby, takes leadership. Furthermore, we want to allow for coordinated as well as uncoordinated policy-making between all policy authorities. In the following, we give a brief overview of the scenarios that will be analyzed:

a) Monetary and fiscal policies take place simultaneously and are uncoordinated. Fiscal policies aim at national targets and monetary policy aims at a union-wide loss function.

b) Monetary and fiscal policy actions are conducted simultaneously and coordinated in the sense of minimizing a union-wide loss function. Thereby, the three policy makers mutually agree on the weights and targets on inflation and output.

c) Monetary and fiscal policy decisions are again made simultaneously. Monetary policy optimizes the union-wide target function given by equation (7.6). Fiscal policies of region A and B are coordinated in the sense of optimizing also a union-wide target function, but differ in their weights and targets from those of the common central bank.

d) Fiscal policies are uncoordinated and act as a first mover (Stackelberg leader). The common central bank is second mover (Stackelberg follower) and reacts optimally in response to the fiscal policy decision.

e) The common central bank acts first (Stackelberg leader). Fiscal policies respond optimally given the action of the common central bank (Stackelberg follower).

f) Fiscal policy makers cooperate and move first. Monetary policy reacts to the decision made by the national fiscal authorities.

g) Monetary policy is Stackelberg leader and fiscal policies, which are coordinated, are Stackelberg followers.

As all scenarios considered in cases a) – g) include only discretionary policies, the expectations of the private sector on inflation have already been made beforehand and are, thus, taken as given: by contracting wages in advance, the private sector acts as a Stackelberg leader against the policy authorities. Remember that the central bank is assumed to be more conservative than fiscal policies in all scenarios besides scenario b), where all policy authorities cooperate.

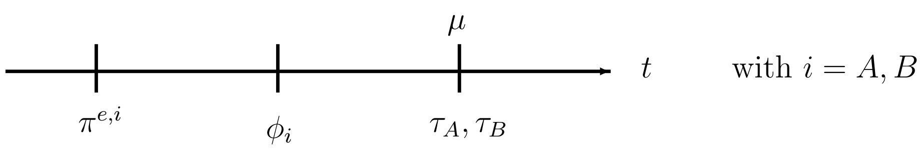
7.1.3 Strategic Behavior and Simultaneous Decision-Making

In this subsection we consider the scenario in which both fiscal authorities and the common central bank choose their optimal policies simultaneously. As the analytical results are dreadfully complicated, we restrict our policy analysis to a numerical examination undertaken in section 7.2. The explicit analytical solutions of the different scenarios, which are used in our MATLAB-simulations can be found in appendix C and the corresponding MATLAB simulation-files are attached in appendix D.

Nash Behavior

First, we consider the scenario of uncoordinated fiscal and monetary policies. The policy makers decide upon their optimal policies after having observed the realizations of the region-specific shocks. Thus, they take the households' expectations on inflation as given. For better understanding, the sequence is depicted in figure 7.1.

Figure 7.1: Time Structure for Simultaneous Decision-Making



The optimization problem for the policy authorities is briefly sketched in the following: Country A's fiscal policy maker optimizes the social loss function (7.5) with respect to τ_A, while taking the decision of the other region's fiscal policy, τ_B, and the policy choice of the common central bank, μ, as given.

Using equations (7.5) and (7.6) as well as the definitions of output and inflation, we obtain the following first-order conditions:

$$\frac{\partial L_F^i}{\partial \tau^i} = (\pi_i - \pi_F^i)c^i + \theta_{F^i}(y_i - y_F^i)(a^i + b^i c^i + \kappa^i(c^{ji} - c^i)) = 0, \tag{7.7}$$

with $i = A, B$,$j = A, B$ and $i \neq j$,

$$\frac{\partial L_M}{\partial \mu} = (\pi - n\pi_M^A - (1-n)\pi_M^B) + n\theta_M^A b^A(y_A - y_M^A) + (1-n)\theta_M^B b^B(y_B - y_M^B) = 0 \ , \tag{7.8}$$

with $\pi = n\pi_A + (1-n)\pi_B$. Solving the fiscal first-order condition of region A for π_A yields

$$\pi_A = \pi_F^A - \theta_F^A(y_A - y_F^A)\left(\frac{a^A + \kappa^A c^{BA}}{c^A} + b^A - \kappa^A\right) \ . \tag{7.9}$$

For most of the scenarios when considering demand-side fiscal policy, there exists a negative correlation between inflation and output, since the parameters a^A, b^A, c^A and c^{AB} are assumed to be positive in our policy analysis.[10] However, if the terms of trade effect is dominant, i. e. if κ^A is extremely large, inflation depends positively on output. Hofmann and Remsperger (2005) show that the inflation differentials in the EMU have been largely persistent since 1999. This implies – at least up to now – that terms of trade effects have tended to be rather small and, therefore, strengthen the view stated above. We will consider this issue in greater depth in the sensitivity analysis in section 7.2. The first-order condition for the fiscal policy of region B follows the same scheme.

Monetary policy optimizes the union-wide social loss function (7.6), taking the fiscal policy actions and the expectations of the private sector as given. Solving for π leads to

$$\pi = n\pi_M^A + (1-n)\pi_M^B - n\theta_M^A b^A(y_A - y_M^A) - (1-n)\theta_M^B b^B(y_B - y_M^B) \ . \tag{7.10}$$

As the parameters b^A and b^B are assumed to be strictly positive, the first-order condition of monetary policy reflects also a negative correlation between inflation and the national output levels. The complete analytical solution is given in appendix C.

Cooperation of Monetary and Fiscal Policies

According to many economists and politicians, coordination plays a crucial role. This is emphasized by the fact that regions and international organizations create institutions like the stability and growth pact and aim at further common targets like tax harmonizations,

[10]For fiscal policy aiming at reducing monopolistic distortions by granting subsidies, the opposite correlation is likely to hold as shown in section 7.2.

which are only a few examples of coordination instruments. In this subsection, we analyze the scenario of coordination under discretion characterized by an agreement of the political authorities on common policy goals, i. e. $\pi_F^A = \pi_F^B = \pi_M = \pi_{JC}$, $y_F^A = y_F^B = y_M = y_{JC}$ and $\theta_F^A = \theta_F^B = \theta_M = \theta_{JC}$, where the subscript JC denotes the "joint cooperation" scenario. The timing of political decision-making corresponds to the Nash scenario and is illustrated in figure 7.1. We assume here, that the policy makers share a combined loss function of the following kind:

$$\begin{aligned} L_{JC} &= n\frac{1}{2}[(\pi_A - \pi_{JC})^2 + \theta_{JC}(y_A - y_{JC})^2] \\ &\quad +(1-n)\frac{1}{2}[(\pi_B - \pi_{JC})^2 + \theta_{JC}(y_B - y_{JC})^2] \ . \end{aligned} \tag{7.11}$$

The minimizing problem follows the same pattern as in the Nash scenario, the only difference being that all authorities face the same loss function. We treat the joint cooperation case as if the policy makers were committed to the socially optimal targets, i.e. *we assume that all policy makers aim at attaining the social optimum in this scenario, and that the private sector is aware of that when forming its expectations about inflation.* We do not incorporate possible deviations from this strategy, though this could be an interesting enhancement of this model. Thus, the first-best optimum for the private agents is attainable under joint cooperation. Dixit and Lambertini (2003b) use the same assumption in their model. We return to this point in section 7.2. The analytical solutions are shifted to appendix D.

Independent Monetary Policy and Cooperation between Fiscal Policies

In situations of union-wide stagnation of economic growth and rising unemployment the fiscal authorities may put the common central bank under pressure to lower interest rates and to increase money growth. In a case where the common central bank maintains its focus on the primary objective, i. e. a high degree of price stability, Beetsma and Bovenberg (1998) argue that coordination of fiscal players, in the sense that each fiscal player internalizes the effects of a unilateral tax change on the other fiscal players, is profitable, at least for the fiscal policy makers. This may encourage the fiscal authorities to coordinate their tax decisions so as to induce the common central bank to set the inflation rate in the direction preferred among the fiscal authorities. However, this behavior may lead to an increase in inflation, taxes and expenditures at the same time and harms welfare.

In our model, the fiscal authorities optimize a similar loss function as in the joint cooperation scenario, but with target values of inflation and output above the socially optimal levels. The fiscal objective function of both regions is given by

$$\begin{aligned} L_{FC} &= n\frac{1}{2}\left[(\pi_A - \pi^A_{FC})^2 + \theta_{FC}(y_A - y^A_{FC})^2\right] \\ &\quad +(1-n)\frac{1}{2}\left[(\pi_B - \pi^B_{FC})^2 + \theta_{FC}(y_B - y^B_{FC})^2\right] \ , \end{aligned} \tag{7.12}$$

where the subscript FC denotes "fiscal cooperation". The results of Beetsma and Bovenberg (1998) for cooperation of fiscal policies are almost in line with our numerical results shown in section 7.2 for the case of a demand-side policy.

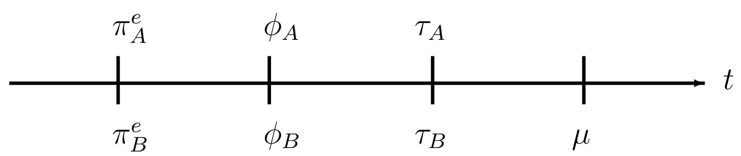
7.1.4 Strategic Behavior and Sequential Decision-Making

The policy choices made by monetary and fiscal authorities may possibly take place at different times due to certain pre-scheduled rules, bureaucracy, or special intrinsic features of the political institutions. Therefore, we focus here on interactions between fiscal and monetary policies when both authorities act sequentially. The analytical results are again shifted to the appendix C and D, and the evaluation of the different scenarios follows in section 7.2.

Stackelberg Leadership of Fiscal Policy

We begin with the scenario of fiscal leadership, i. e. fiscal policy makers have to decide on their policy actions before monetary policy has been implemented and after having observed the realization of the regional shocks ϕ_i. Accordingly, they take the household's inflation expectation as given. Beetsma and Bovenberg (1998) argue that fiscal leadership seems to be more likely when monetary policy can be implemented and adjusted more quickly than fiscal policy. This may be applicable when choices for taxes and subsidies are accompanied by bureaucratic and legislative processes that provide the fiscal authority with leadership over monetary policy. The sequence in that scenario is depicted in figure 7.2.

Figure 7.2: Time Structure for Sequential Decision-Making (Fisc. Leadership)

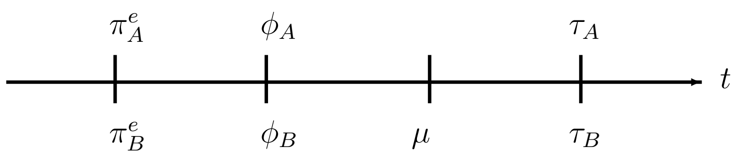


The solution of the game is obtained by backward induction. Solving the monetary policy's optimization problem at the second stage of the game leads to the optimal choice of μ while taking the fiscal policy variables τ_A and τ_B as given. The monetary reaction function is again given by (7.8). In the first stage, the fiscal policy maker of region i optimizes τ_i to react to the action taken by the policy maker of region j, τ_j, and subject to the monetary reaction function, which is derived from the second stage of the game. For the analytical solution see the appendix C and D.

Stackelberg Leadership of Monetary Policy

In contrast to the previous case, monetary policy attains Stackelberg leadership over fiscal policies if it only affects the economy with a lapse of time exceeding the legislative and bureaucratic time need for fiscal policy decision-making. The timing is shown in figure 7.3. The solution is similar to the former scenario of fiscal leadership. In the second stage,

Figure 7.3: Time Structure for Sequential Decision-Making (Mon. Leadership)



fiscal policy makers minimize the loss function (7.5) analogously to the Nash scenario shown above, given the other region's fiscal policy and the monetary policy variable μ. The common central bank chooses μ in the first stage, given the best responses of the fiscal policies τ_A and τ_B.

Fiscal Cooperation and Sequential Policy Actions

Analogously to the fiscal corporation scenario where the policy makers choose their optimal policies simultaneously, one can also assume coordination between national fiscal policies when the decision-making on monetary and fiscal policies takes place at different stages. The motivation for a coordinated fiscal policy in a sequential policy game corresponds to that of fiscal coordination in a simultaneous game. Accordingly, we also analyze scenarios (i) *fiscal cooperation when fiscal policy moves first* and (ii) *fiscal cooperation when monetary policy moves first.*

The time structure of scenario (i) corresponds to the one in figure 7.2, while the time structure of scenario (ii) corresponds to that in figure 7.3. The optimization problem under both scenarios follows the same pattern as in the corresponding sequential scenarios without coordination and are, therefore, omitted in this section.[11]

7.2 Results

In the following we derive numerical results for the seven scenarios of strategic behavior between monetary and fiscal authorities introduced in the previous section.

We, first, analyze our baseline case with a supply-side fiscal policy as described in the microfoundation in section 6.1 and show the calibration of the model. Second, we show the evaluation methods used for the ranking of the different scenarios. Third, we run simulations for the case of a homogeneous and a heterogeneous monetary union by using the structural parameters from the microfounded model of sections 6.1 and 6.2. In this case, fiscal policy aims at granting production subsidies and levy per-capita taxes to reduce the distortions caused by monopoly power. We use the results from the homogeneous monetary union as a reference case, and compare the rankings of different scenarios in the heterogeneous case. Fourth, we strengthen our results by using a sensitivity analysis regarding both the structural parameters and the policy targets. Fifth, we run the simulations once more, now analyzing the strategic interactions among fiscal and monetary policies when fiscal policy is described by demand-side policy. Thereby, we do not refer to the parameter values of our microfoundation, but choose instead the parameter values of the reduced-form model (macroeconomic equations of section 7.1) in a quantitative exploration. In this case, the parameters $a^i, a^{i,j}, c^i$ and c^{ij} have positive signs as already discussed in the previous section. I. e., expansionary fiscal policy raises output and inflation at the same time. Like in the supply-side case, we strengthen the results of our numerical settings by checking the sensitivity of our results to variations in the macroeconomic parameters. Sixth, we show that it is possible to nest the results of Dixit and Lambertini (2001, 2003a and 2003b) in our model.

[11] Please consult appendix C and D for details of the analytical solution.

7.2.1 Calibration

We calibrate the structural parameters of the model in accordance with the standard literature, as referred to in Dixit and Lambertini (2003a, appendix F). The elasticity of marginal disutility of labor is set at 0.45, a value proposed by Blanchard and Fischer (1989).[12] This implies that the disutility parameter β, which is one plus the inverse of the elasticity of marginal disutility of labor, has the value $\beta = 3.22$. The Calvo-stickiness parameters Φ^H and Φ^F are set at a moderate value of 0.5, implying an average price to be fixed for three periods. The elasticity of substitution between goods of the same region is set at $\theta = 11$, as in Dixit and Lambertini (2003a). Obstfeld and Rogoff (2001) discuss the literature that has found values between 1 and above 20. Note that the elasticity of substitution between goods of different regions is set to unity, as in Benigno (2004). In setting the steady state of the technology parameter as $\bar{d}_i = 1$ and the subjective discount factor as $\eta = 0.98$ we strictly follow Dixit and Lambertini (2003a). The steady-state value for the fiscal policy instrument is assumed to be set optimally, i. e. to offset monopolistic distortion. Via $\bar{\tau}_i = 1/(1-\theta)$ we obtain a subsidy rate of ten percent for both regions in the steady state.

We look here at two different cases. In the first case, both regions have the same size ($n = 1 - n = 0.5$) and are completely symmetric, with identical structural parameters, identical fiscal policies, and no home bias ($\nu^H = \nu^F = 0.5$). In the second case, region B accounts for only 30 percent of the union and displays more price rigidities. The latter assumption is based on the findings of Benigno and Lopez-Salido (2004). They estimate the price rigidity in five core EMU countries and identify substantial heterogeneities.[13]

In the second case we presume that there is also a considerable home bias in consumption in both regions, thus following Anderson and van Wincoop (2003).

Given the values stated above, we can calculate the various parameters a^i, b^i, c^i and κ^i in the model equations. Also, we can infer the values in the policy loss functions maximizing social welfare. In the symmetric case, these are target values for inflation and output of both zero, and a weight on output of $\theta_M^A = \theta_M^B = 0.00763$. In the asymmetric

[12]The authors discuss this parameter on page 341. Dixit and Lambertini (2003a) assume unit wage elasticity and thus less curvature.

[13]The average price duration varies between around four quarters in the Netherlands and Germany and up to 17 quarters in Spain, implying price rigidity parameters between 0.75 and 0.94. We will choose numbers between 0.5 and 0.58, following the more conservative estimates of Bils and Klenow (2004). For a closer look at European data, the reader is referred to Dhyne et al. (2005).

case, the output weight for region B rises to $\theta_M^B = 0.01046$, while all other socially optimal target values remain the same.

As stated earlier, we assume that the common central bank sticks to these values, while the fiscal policy authorities may deviate from them. There may be various reasons for such deviation: For example systematic mismeasurement by the fiscal authorities or the fiscal authorities maximizing a different objective function they are able to conceal from the households. This was substantiated in section 7.1. More particularly, we assume that the fiscal policy authorities put equal weight on output and inflation of unity. Furthermore, fiscal policies have higher target values for output $y_F^A = y_F^B = 0.015$ and inflation $\pi_F^A = \pi_F^B = 0.02$. In the asymmetric case, fiscal policy in region B even puts a weight of $\theta_F^B = 1.25$ on output, sets its output target at $y_F^B = 0.025$ and its inflation target at $\pi_F^B = 0.03$, which could be seen as the result of its self-perception as a high growth catch-up region. Table 7.1 summarizes this calibration. As in Dixit and Lambertini (2003a) the stochastic term is calibrated to match the variance of output around its steady state to be plus/minus six percent, as is the case for the U. S.

As set out in section 7.1, we assume that the private sector has rational expectations about inflation. In our analytical calculations we treat π_A^e and π_B^e as given (see appendix C). The inflation expectations of the private agents in both countries are determined in our model by iteration: In other words, we use an arbitrary starting value for the inflation expectations in both countries and repeat the optimization calculations until the inflation expectations differ from realized inflation by a value of less than 10^{-10} for both countries, while keeping the shock at its expected value of zero. This approach guarantees that $\pi_i^e = E(\pi_i)$ holds for $i = A, B$. After inflation expectations are determined, we simulate our model by averaging over 100,000 random draws of the stochastic processes.[14]

7.2.2 Evaluation Method

The main purpose of our numerical approach is to rank the different scenarios of strategic behavior displayed by monetary and fiscal policies by the losses they induce. We distinguish three approaches:

(i) Evaluation of the loss functions, referring to the policy exercised by the fiscal and monetary authorities. In each cooperation scenario, the corresponding loss function is a compromise between the cooperating authorities.

[14]The corresponding MATLAB-File can be found in appendix D.2.

Table 7.1: Calibration of the Baseline Model

Parameter	Value*	Alternative×	Explanation
Structural parameters			
n	0.50	0.70	Size of region A
ν	0.50	0.80†	Parameter capturing preference for home goods
β	3.22	3.22	One plus one over the elasticity of marginal disutility of labor
Φ	0.50	0.58	Fraction of firms that cannot adjust prices
θ	11.00	11.00	Elasticity of substitution between goods
d_i	1.00	1.00	Technology parameter
η	0.98	0.98	Subjective discount factor
$\bar{\tau}_i$	-0.10	-0.10	Steady state value of taxes
Loss functions			
θ_M^i	0.00736	0.01046	Central bank's weighting factor for output
π_M^i	0.00	0.00	Inflation target of the central bank
y_M^i	0.00	0.00	Output target of the central bank
θ_F^i	1.00	1.25	Fiscal policy's weighting factor for output
π_F^i	0.02	0.03	Inflation target of fiscal policy
y_F^i	0.015	0.025	Output target of fiscal policy

Remarks:

)* The term "Value" denotes the value chosen for both regions in the symmetric case and for region A in the asymmetric case.

)× "Alternative" denotes the value chosen for region B in the asymmetric case.

)† For the heterogeneous case the home-bias parameter is set in both regions equal to 0.8.

(ii) Evaluation of the region-specific loss functions. In each cooperation scenario, these are the region-specific loss functions the policy authorities would minimize if they were not cooperating. This approach allows us to infer whether cooperation scenarios are preferable for each participating policy authority.

(iii) Evaluation of social welfare. For each region, we calculate the welfare loss that arises due to deviations in output and inflation from the socially optimal values.

We show the losses involved in all three approaches in table 7.2 for the baseline model and in table 7.4 for demand-side fiscal policy. In our discussion we incorporate only the second and third approach. The reasoning behind this is as follows: In approach (i), the losses of the three policy authorities are based on the loss functions used in the optimization calculations. If the policy makers decide to cooperate, they usually compromise on targets that differ from their own *true preferences.* However, the "true losses" the policy makers face are still based on their specific preferences. Therefore, in approach (ii) we calculate the values of the policy makers' loss functions given by equations (7.5) and (7.6), irrespective of the loss function used for optimization in the relevant scenarios.[15] One should also take these losses into account when exploring whether joint cooperation among all policy makers or cooperation between fiscal policy makers can take place on a voluntary basis.

The region-specific social welfare losses of approach (iii) are given by[16]

$$\begin{aligned} L_A &= \frac{1}{2}((\pi_A - \pi_M^A)^2 + \theta_M^A(y_A - y_M^A)^2) \\ L_B &= \frac{1}{2}((\pi_B - \pi_M^B)^2 + \theta_M^B(y_B - y_M^B)^2). \end{aligned}$$

Additionally, we express the region-specific social losses in terms of an equivalent reduction in region-specific consumption units, following the example of LUCAS (2003): A scenario "performs best" when it shows the lowest reduction of consumption units compared to the consumption level in the social optimum. The calculation of the consumption-equivalent losses follows the approach of ADAM and BILLI (2005):

From equation (6.114) in section 6.2, we know that for region A

$$U^A = -\bar{Y}_A u_C L_A \tag{7.13}$$

[15] Note that by this definition the losses in case (ii) only differ from the losses in case (i) for the joint cooperation scenario and the scenarios of fiscal cooperation.

[16] Remember from section 7.1 that the central bank is assumed to optimize the union-wide social loss, which is a region-sized weighted sum of the social losses of region A and B.

holds. To derive a relation between a permanent reduction of consumption (given by δ_C^A percent), and the welfare loss, a second-order Taylor approximation of the utility loss is generated by

$$\begin{aligned} U^A &\approx \left(-\frac{u_C \bar{Y}_A \delta_C^A}{100} + u_{CC} \left(\frac{\bar{Y}_A \delta_C^A}{100} \right)^2 \right) \\ &= -u_C \bar{Y}_A \left(\frac{\delta_C^A}{100} - \frac{u_{CC} \bar{Y}_A}{u_C} \left(\frac{\delta_C^A}{100} \right)^2 \right) \\ &= -u_C \bar{Y}_A \left(\frac{\delta_C^A}{100} + \frac{(1-\gamma)\bar{Y}_A}{\bar{Y}_A} \left(\frac{\delta_C^A}{100} \right)^2 \right), \end{aligned} \tag{7.14}$$

where we have used the results of equations (6.81) and (6.83). Replacing $-\frac{U^A}{u_C \bar{Y}_A}$ by L_A yields

$$L_A = \frac{(\delta_C^A)^2}{100^2} + (1-\gamma)\frac{\delta_C^A}{100} \ . \tag{7.15}$$

To calculate the reduction of consumption equivalent to the social loss for region A, we solve for δ_C^A to obtain

$$\delta_C^A = 100 \frac{-1 + \sqrt{1 + 4(1-\gamma)L_A}}{2(1-\gamma)} \ . \tag{7.16}$$

The reduction of consumption equivalent to a certain welfare loss for region B can be obtained analogously. We use this transformation in the following subsections to make the welfare losses more tangible.

7.2.3 Monetary and Fiscal Policies in the Baseline Model

Now, we examine the results of the simulations for the supply-side fiscal policy described in section 6.1. The model calibration was explained in section 7.2.1 and is summarized in table 7.1. A summary of the results is given in table 7.2.

Table 7.2: Baseline Model: Analysis of Welfare and Policy Losses

	Symmetric case					*Asymmetric case*				
	Calculated Policy Losses			Equivalent Consumption Reduction, %		Calculated Policy Losses			Equivalent Consumption Reduction, %	
Policy	L_{FA}	L_{FB}	L_M	CR_A	CR_B	L_{FA}	L_{FB}	L_M	CR_A	CR_B
Nash	21.90936	21.90935	0.11895	0.012	0.012	23.82412	51.16257	0.36286	0.024	0.065
	(0.033)	(0.033)	(0.000)	(0.000)	(0.000)	(0.019)	(0.054)	(0.000)	(0.000)	(0.000)
Stackelberg, fiscal leadership	21.91073	21.91072	0.11582	0.012	0.012	23.80327	51.17020	0.35035	0.024	0.062
	(0.033)	(0.033)	(0.000)	(0.000)	(0.000)	(0.019)	(0.054)	(0.000)	(0.000)	(0.000)
Stackelberg, monetary leadership	23.63918	23.63917	0.01599	0.002	0.002	33.43925	85.52175	0.00407	0.000	0.001
	(0.032)	(0.032)	(0.000)	(0.000)	(0.000)	(0.016)	(0.055)	(0.000)	(0.000)	(0.000)
Cooperation	0.00001	0.00001	0.00001	0.000	0.000	0.00001	0.00001	0.00001	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
... region-specific policy losses	31.25024	31.25020	—	—	—	31.25001	101.25024	—	—	—
	(0.125)	(0.125)				(0.063)	(0.193)			
Fiscal cooperation, simultaneous	21.90926	21.90926	0.11848	0.012	0.012	32.02629	32.02629	0.36256	0.024	0.065
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.003)	(0.003)	(0.000)	(0.000)	(0.000)
... region-specific policy losses	21.90927	21.90926	—	—	—	23.82536	51.16177	—	—	—
	(0.033)	(0.033)				(0.019)	(0.054)			
Fiscal cooperation, fiscal leadership	21.64560	21.64560	0.11056	0.011	0.011	32.46600	32.46600	0.41334	0.022	0.087
	(0.010)	(0.010)	(0.000)	(0.000)	(0.000)	(0.042)	(0.042)	(0.001)	(0.000)	(0.000)
... region-specific policy losses	21.64561	21.64559	—	—	—	23.34691	53.74388	—	—	—
	(0.041)	(0.026)				(0.012)	(0.146)			
Fiscal cooperation, mon. leadership	31.24131	31.24131	0.00011	0.000	0.000	52.22477	52.22477	0.00012	0.000	0.000
	(0.377)	(0.377)	(0.000)	(0.000)	(0.000)	(0.434)	(0.434)	(0.000)	(0.000)	(0.000)
... region-specific policy losses	31.24143	31.24120	—	—	—	31.23737	101.19537	—	—	—
	(0.740)	(0.740)				(0.718)	(1.745)			

Remarks: L_{Fi} is fiscal loss in region i, L_M loss of the common central bank, all multiplied by 10^5. CR_i denotes welfare loss measured in terms of an equivalent permanent percent reduction in consumption in region i. The numbers in parentheses denote standard deviations.

Homogeneous Monetary Union

We begin with a comparison of the losses for the monetary and fiscal policy authorities in the symmetric case. The first columns of table 7.2 show that the fiscal authorities of both regions face the highest region-specific policy losses under cooperation and in the scenario where monetary policy moves first. The lowest fiscal losses occur when fiscal policies have the greatest influence, i. e. under the scenarios of fiscal cooperation when fiscal policies move first and under fiscal cooperation in the simultaneous scenario. The explanation is simple. Fiscal policies aim at higher inflation and higher output than the central bank, which targets socially optimal levels. Due to the low relative weight on output stabilization the central bank reacts strongly to offset inflation deviating from the socially optimum level. Fiscal policies, themselves, engage in a trade-off between inflation and output when fixing their own policy decisions. An expansionary fiscal policy pushes output above the socially optimal level by granting subsidies in order to lower production costs. Thus, it decreases inflation at the same time. Accordingly, output is higher than natural output and lower than the desired fiscal targets. Inflation is below the fiscal target levels and slightly below the social optimum. Note, however, that the central bank reacts strongly to the downward pressure of inflation with an expansionary monetary policy on account of the high weight on inflation in the target function.[17]

The loss in the Nash scenario is similar to that of the two scenarios where fiscal policies move first.

In the scenarios where monetary policy takes the lead (with or without coordination of fiscal policies), fiscal policies internalize the fact that the central bank cannot offset fiscal policy that is too expansionary. Therefore, fiscal policies are less expansionary, and output and inflation deviate from the fiscal targets to a higher degree than in the previously analyzed scenarios. This implies higher losses for the fiscal policy authorities. The highest losses occur when policy makers cooperate and agree on the socially optimal targets: On average the realized value for inflation is close to zero (but still dependent on stochastics) and output is at its lowest compared to the desired levels. It is, therefore, questionable whether overall cooperation aiming at socially optimal targets can be implemented in this setting.

[17]When fiscal policies are characterized by demand-side policy, an expansionary fiscal policy pushes both output and inflation into the desired direction. Therefore, fiscal policy is much more expansionary when fiscal policy raises demand, and the ranking of the losses differs from that under a supply-side fiscal policy.

Our assumption of a welfare maximizing monetary policy means that the rankings of the central bank losses correspond to the rankings of the union-wide social losses. The social losses, in turn, can be transformed into welfare equivalent consumption reductions relative to the social optimum. Accordingly, we consider only the consumption losses of the private agents in the following. We find that the ranking of the scenarios is quite different in comparison with the (fiscal) policy makers' losses (see again table 7.2). The first best can be attained in the cooperation scenario.[18] The consumption loss is also very low in both monetary leadership scenarios, i.e. when fiscal policies do not cooperate and when fiscal policies are coordinated. The highest social losses occur when fiscal policies are dominant in the sense of being Stackelberg leaders, and in the Nash scenario. In line with the explanation for the fiscal policy makers' losses, inflation and output levels are closest to the social optimum when monetary policy takes the lead (together, of course, with the joint cooperation case).

Heterogeneous Monetary Union

In our analysis of a heterogeneous monetary union we assume that the fiscal policy of region A follows the same strategy as in the homogeneous case, whereas the fiscal policy of region B targets higher levels of both inflation and output. Furthermore, we assume that region B is smaller than region A and is characterized by a slightly higher degree of price-stickiness. The exact parameter values for region A are again depicted in the second column of table 7.1, while the "alternative" parameter values for region B are summarized in the third column of this table. Results for the heterogeneous case are shown in columns seven to eleven of table 7.2.

Beginning with the losses for region A, we find that the values of the fiscal policy maker's losses are much higher for all scenarios in the heterogeneous case, except one: The cooperation scenario corresponds to the homogeneous case by definition, as all policy makers agree on the socially optimal targets. The ranking of the scenarios with respect to the region-specific fiscal policy makers' losses is similar to that in the homogeneous case: the highest losses occur when monetary policy has the greatest influence (monetary leadership scenarios), the smallest losses occur in the scenarios in which fiscal policies have the greatest influence (fiscal cooperation when fiscal policy takes leadership, fiscal cooperation and simultaneous decision-making, and fiscal leadership when monetary policy is uncoordinated), and in the Nash scenario. The fiscal policy maker again faces the

[18]The (monetary) policy loss is slightly larger than zero because of the shock in our simulation.

highest loss in the joint cooperation scenario. We observe almost the same ranking for region B, but the losses are higher compared to region A.

We find that the losses of the common central bank and, hence, also the consumption losses of the private agents show also a similar ranking as in the homogeneous monetary union: The lowest losses are attained when monetary policy moves first or when all policy makers agree on the socially optimal targets (=first best). The highest losses occur when fiscal policies moves first (uncoordinated and coordinated) and when fiscal policies are coordinated and monetary and fiscal policy decisions take place simultaneously. This result seems, at first glance, to be contrary to the findings of Lombardo and Sutherland (2004), who state that fiscal cooperation is welfare-improving. But a closer look reveals that our calibration of a unit elasticity of substitution between domestic and foreign goods also implies in Lombardo and Sutherland (2004), according to their Proposition 1, that fiscal cooperation is no longer welfare-improving.[19]

The welfare-equivalent consumption reductions under Nash, fiscal leadership, and the two fiscal cooperation scenarios with simultaneous actions or with fiscal leadership are about three times larger in the (smaller) region B. Also, the equivalent consumption reductions are relatively higher in the heterogeneous case compared to the homogeneous case, by about 50 percent for region A and a factor of above four for region B. This implies that a model of a homogeneous monetary union that does not properly take into account heterogeneities possibly underestimates the welfare effects of certain policies. This finding also suggests that homogeneity is a desirable feature of the currency area for all policy makers (fiscal and monetary authorities) and the private agents. We take up this point again in section 7.2.5 and consider the implications for the European Monetary Union.

7.2.4 Sensitivity Analysis for Supply-side Policy (Baseline-Case)

Are the results of the previous section robust to changes in the structural parameters of the model? To examine this, we vary the structural parameters within plausible ranges. In figure 7.4 we plot the parameter variations that show the highest sensitivity of results. The corresponding parameters are the elasticity of marginal disutility of labor, (emdl), price rigidity, ϕ_i, and the elasticity of substitution, θ. We plot their effects on fiscal policy makers' losses and social welfare, which is equivalent to the central bank loss for both the

[19] Note also that Lombardo and Sutherland (2004) features government consumption in the utility function. We will refer to that point again in section 7.2.5.

symmetric and the asymmetric case.[20]

Variation of the Elasticity of Marginal Disutility of Labor

We vary the elasticity of marginal disutility of labor (emdl) between zero and one, where the lower bound is given in Blanchard and Fischer (1989), while the upper bound is often used in New Keynesian models, see e. g. Gali and Monacelli (2005a). The effects of these variations on the policy losses in the three simultaneous scenarios are depicted in the first row of figure 7.4, while the second row shows the effects in the four sequential scenarios.

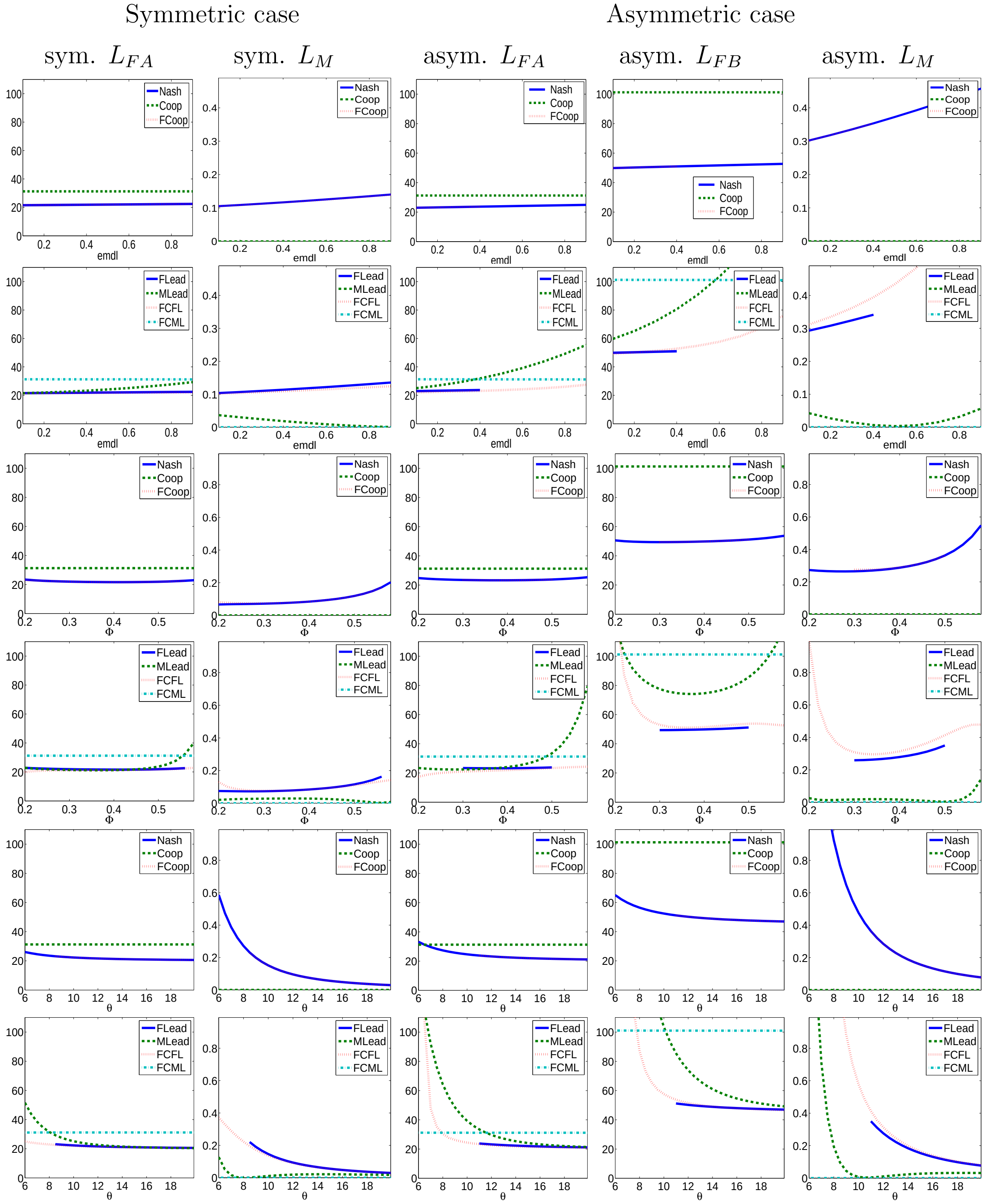
Increasing elasticity of marginal disutility of labor leads to higher central bank losses. This result is obvious as, given the other parameters, the same outcome is produced at higher cost, meaning that the same effort in the production of goods leads now to a higher reduction of utility than before.

Referring to the homogeneous case, we see that the rankings for both the fiscal authorities' losses and the central bank losses are stable: fiscal policies suffer from the smallest losses in the Nash scenario and if they obtain fiscal leadership, as in comparison with the other scenarios they are better able to pursue their inflation and output targets (above the socially optimal levels). The central bank's welfare function shows the smallest losses in the joint cooperation case (which determines the first best) and in the scenario where monetary policy takes leadership. In the latter scenario, the fiscal policies are restrained, as too expansionary a fiscal policy would lead to low inflation, which will not be corrected by the central bank afterwards. Therefore, monetary leadership has a disciplining effect on supply-side-oriented fiscal policies. The fact that joint cooperation leads to the first best from a welfare perspective comes as no surprise as all policy makers agree upon the socially optimal targets, as mentioned in the previous section.

In the heterogeneous case, the losses are higher for the fiscal policies of both regions, the one with the more conservative and the one with the more aggressive targets, and also for the central bank. However, the rankings seem to be robust with two exceptions: (i)

[20] In the figures we use the following abbreviations to save space: For the policy scenarios, Nash = Nash, Coop = cooperation, FCoop = fiscal cooperation, FLead = fiscal leadership, MLead = monetary leadership, FCFL = fiscal cooperation with fiscal leadership, FCML = fiscal cooperation with monetary leadership. The labels on the x-axis denote emdl = elasticity of marginal disutility of labor, Φ = Calvo parameter, i.e. the percentage of firms that cannot adjust their prices, and θ = elasticity of substitution between different goods produced in the same region.

Figure 7.4: Identical Parameter Variations in Region A and B (Supply-Side Policy)



When monetary policy moves first fiscal losses are strongly increasing for higher values of the elasticity of disutility of labor. (ii) The losses in the fiscal cooperation fiscal leadership cases “explode” to a value of 0.4, which may be an indication that there is no equilibrium in that case to which rational inflation expectations could converge. It would be interesting to take up this point in further research.

Variation of Price Rigidity

The third and fourth rows of figure 7.4 examine the effect of varying price rigidities on fiscal and monetary losses. The figure shows that the ranking of the scenarios is stable in the homogeneous and heterogeneous case for almost the whole parameter set, and it is in line with the results of table 7.2: Fiscal policies incur the smallest loss under fiscal leadership, whereas monetary policy suffers from the smallest losses when it takes leadership and, of course, under the joint cooperation scenario. Again, the fiscal cooperation fiscal leadership scenario leads to dramatically increasing losses for more rigid prices, a factor that calls for analysis in future research.[21]

Variation of the Elasticity of Substitution of Consumption Goods

In the fifth and sixth rows of figure 7.4 we consider the effect of changes from the elasticity of substitution of consumption goods, θ, on the losses over the range discussed by Obstfeld and Rogoff (2001). The figure confirms one intuitive result, i.e. that an increasing θ leads to smaller fiscal policy and welfare losses: higher substitutability between goods implies fewer distortions from monopoly power. There is again one interesting exception. For a relatively small value of θ below 10 the losses explode, which, again, may conceivably induce indeterminacy of equilibria.

Summary of the Findings

For all parameter variations over the ranges used in the standard literature (see model-calibration), we find that the rankings of the different scenarios illustrated by table 7.2 are relatively robust. The sensitivity analysis has also confirmed that the losses in a heteroge-

[21]The variations of the intertemporal discount factor η, which determines the importance of “pseudo-future” periods relative to the present period in the producer-consumers price-setting behavior, show almost the same results as those indicated for variations of the price rigidity parameter. We, therefore, abstain from depicting and discussing the figures for η.

neous monetary union tend to be higher. From the perspective of welfare maximization, joint cooperation and monetary leadership are the best-performing scenarios.

7.2.5 Alternative Fiscal Policy: Demand-side Policy

In this section, we consider an alternative interpretation of fiscal policy: Governments try to push output and inflation above their natural levels by raising demand for public goods to lower unemployment below its natural rate. This type of fiscal policy is not incorporated in our microfounded model. Nevertheless, we are interested in analyzing how the different scenarios describing the interplay between policy makers are ranked when using a parametrization for demand-side oriented fiscal policies.[22]

A homogeneous monetary union

First, we consider the case of a homogeneous monetary union. The parametrization we use here is depicted in table 7.3. We assume that an expansionary fiscal policy leads to an increase in output, i. e. $a^i > 0$ and $a^{ij} > 0$. In contrast to the supply-side policy, the coefficients c^i and c^{ij}, which denote the effect of fiscal policies on inflation, now have a positive sign: The additional demand of goods by fiscal policies causes an upward pressure on goods' prices.[23] The values of the remaining parameters are equal to those in the previous subsection or have, at least, the same sign, but differ in their absolute size. The results are depicted in the first columns of table 7.4. The ranking of the policy makers' losses shows some interesting changes compared to the supply-side case: The policy losses in both fiscal-leadership scenarios, i. e. in the uncoordinated and coordinated scenarios, are higher compared to the losses in the scenarios where monetary policy moves first. The result is the outcome of a monetary policy that fights inflation strongly when fiscal policy moves first: The monetary policy variable μ exhibits a relatively strong negative value in the fiscal leadership scenarios compared to the monetary leadership scenarios. We also find that joint cooperation of fiscal and monetary policies — characterized by agreeing on the socially optimal output and inflation targets — reduces the losses of fiscal

[22] A microfoundation of demand-side oriented fiscal policy would also be possible, if adding public goods to the representative household's utility function. The reader is referred to Beetsma and Jensen (2005) or Lombardo and Sutherland (2004) for related models with this feature.

[23] We assume that the demand-side effect of fiscal policies outweighs the crowding-out effect on private demand.

policy makers when underlying their true preferences on domestic inflation and domestic output.[24]

Table 7.3: Demand-side Policy: Parameter Values in the Symmetric Case

Parameter	Value	Explanation
Output Equation		
$\bar{y}^i$	0	Natural output level
a^i	0.8	Effect of fiscal policy on the same region's output
a^{ij}	0.1	Effect of fiscal policy on the other region's output
b^i	1.0	Effect of surprise inflation on the same region's output
κ^i	0.2	Terms of trade effect of an inter-regional difference in inflation rates
σ_{ϕ^i}	0.01	Variance of regional shocks to output
Inflation Equation		
c^i	0.5	Effect of fiscal policy on the same region's inflation
c^{ij}	0.15	Effect of fiscal policy on the other region's inflation
Loss Functions		
θ_M^i	0.5	Central bank's weighting factor for output
π_M^i	0.0	Inflation target of the central bank
y_M^i	0.0	Output target of the central bank
θ_F^i	1.3	Fiscal policy's weighting factor for output
π_F^i	0.02	Inflation target of fiscal policy
y_F^i	0.015	Output target of fiscal policy
n	0.5	GDP share of region A on union-wide GDP

Remarks: $i = A, B$, $j = A, B$ and $j \neq i$.

[24] At first glance, this suggests that the cooperation solution would be easy to implement. However, we possibly have a prisoner's dilemma: if the fiscal policy maker under cooperation deviates from the cooperation strategy and optimizes his own target function, he can reduce the loss by more as long as the other policy maker abstains from deviating. Cooperation of all policy makers would then only be attainable if deviations were associated with a large enough imminent sanction.

Table 7.4: Demand-side Policy: Analysis of Welfare and Policy Losses

	Symmetric case					*Asymmetric case*				
	Calculated Policy Losses			Equivalent Consumption Reduction, %		Calculated Policy Losses			Equivalent Consumption Reduction, %	
Policy	L_{FA}	L_{FB}	L_M	CR_A	CR_B	L_{FA}	L_{FB}	L_M	CR_A	CR_B
Nash	59.88023	59.88042	23.80139	2.380	2.380	96.03430	80.34667	38.87501	4.229	3.090
	(0.536)	(0.536)	(0.002)	(0.000)	(0.000)	(0.494)	(1.029)	(0.103)	(0.000)	(0.000)
Stackelberg, fiscal leadership	43.39911	43.39928	10.66685	1.067	1.067	87.68806	36.68796	25.13275	2.729	2.009
	(0.477)	(0.477)	(0.002)	(0.000)	(0.000)	(0.436)	(0.749)	(0.169)	(0.000)	(0.000)
Stackelberg, monetary leaders.	21.78963	21.78971	1.35417	0.135	0.135	33.64416	56.98092	1.32407	0.037	0.355
	(0.229)	(0.229)	(0.002)	(0.000)	(0.000)	(0.172)	(0.415)	(0.057)	(0.000)	(0.000)
Cooperation	0.00103	0.00103	0.00103	0.000	0.000	0.00109	0.00109	0.00109	0.000	0.000
	(0.001)	(0.001)	(0.001)	(0.000)	(0.000)	(0.002)	(0.002)	(0.002)	(0.000)	(0.000)
... region-specific policy losses	34.62639	34.62642	—	—	—	34.62549	107.50532	—	—	—
	(0.063)	(0.063)				(0.008)	(0.312)			
Fiscal coop., simultaneous	60.76041	60.76041	24.41184	2.441	2.441	90.60180	90.60180	38.94619	4.237	3.095
	(0.002)	(0.002)	(0.002)	(0.000)	(0.000)	(0.039)	(0.039)	(0.106)	(0.000)	(0.000)
... region-specific policy losses	60.76031	60.76051	—	—	—	98.34502	71.62484	—	—	—
	(0.539)	(0.539)				(0.461)	(0.998)			
Fiscal coop., fiscal leadership	93.63063	93.63063	13.33663	1.334	1.334	495.16384	495.16384	155.18772	13.72	19.71
	(0.285)	(0.285)	(0.085)	(0.000)	(0.000)	(0.359)	(0.359)	(0.202)	(0.000)	(0.001)
... region-specific policy losses	93.63037	93.63089	—	—	—	387.47316	781.26691	—	—	—
	(0.369)	(0.591)				(0.347)	(1.386)			
Fiscal coop., mon. leadership	19.83643	19.83643	7.58544	0.759	0.759	32.72155	32.72155	21.99331	1.716	3.326
	(0.018)	(0.018)	(0.068)	(0.000)	(0.000)	(0.129)	(0.129)	(0.128)	(0.000)	(0.000)
... region-specific policy losses	19.83638	19.83649	—	—	—	36.21712	23.29640	—	—	—
	(0.285)	(0.286)				(0.181)	(0.668)			

Remarks: L_{Fi} is fiscal loss in region i, L_M loss of the common central bank, all multiplied by 10^5. CR_i denotes welfare loss measured in terms of an equivalent permanent percent reduction in consumption in region i. The numbers in parentheses denote standard deviations.

A further interesting result is that the scenarios in which monetary policy moves first show the smallest losses *for all policy makers*, the governments and the common central bank.

The ranking of the scenarios according to the welfare equivalent consumption losses corresponds almost to that found for supply-side fiscal policy. The first best is obtained if all policy makers agree on the socially optimal targets.[25] Monetary leadership leads to a lower consumption reduction compared to the fiscal leadership and the Nash scenario. Under both monetary leadership scenarios monetary policy has a great influence which leads to relatively low welfare equivalent consumption reductions. The result holds due to the common central bank's target function, which is assumed to maximize social welfare.

A heterogeneous monetary union

In a heterogeneous monetary union conflicts do not only occur between the targets of monetary and fiscal policies, but also among fiscal authorities themselves.

Therefore, we analyze also for the case of demand-side fiscal policy the interactions of monetary and fiscal policies in a monetary union comprising two heterogeneous regions. To relate this topic to a practical situation, one could think of the European Monetary Union, where region A describes the richer northern and central European countries and region B consists of southern and in the near future eastern European countries. We choose a parametrization to analyze strategic behavior under heterogeneities which can be applied to the situation in European Monetary Union. For the richer countries in region A we use the same structural parameters as in the homogeneous case. Region B, which can be called "catch-up"-region, is characterized by relatively higher output and inflation goals and also by a higher relative-weight on the output goal compared to region A. Furthermore, the parameters a^i, b^i and c^i are assumed to be relatively higher in the catch-up region, meaning that the same fiscal policy has a stronger impact on inflation and output in region B. This may be caused by a higher price-stickiness in the catch-up region due to greater labor-market frictions and a higher ratio of administered prices in the goods market.[26] The exact parameter values are listed in table 7.5. The results are

[25] Note, again, that this scenario is equivalent to a joint commitment scenarios due to our modeling: we assume that the private sector has rational expectations on inflation and knows the true structure of the policy maker's target function and identifies the scenario perfectly. This is a critical assumption as the true preferences of the policy makers do not necessarily coincide with the targets under cooperation meaning that a deviation from the current strategy cannot be excluded, but this is not our main focus.

[26] See Benigno and Lopez-Salido (2004) and Dhyne et al. (2005) for an empirical assessment of

depicted in the last columns of table 7.4. Compared to the symmetric case, the policy authorities' losses are higher for region A in a heterogeneous monetary union, although the structural parameters remained the same. The reason is too expansionary a policy in region B to which the common central bank reacts to. This implies that monetary policy acts stronger for region A compared to the homogeneous case.

The overall ranking of the scenarios for region A is similar to that in the symmetric case. The scenarios in which fiscal policy is dominant (leadership and/or fiscal cooperation scenarios) and the Nash scenario exhibit the highest losses. Remember, in this context, that the joint targeted inflation and output levels are a country-size weighted average of the single countries' preferences when fiscal policies cooperate.

The lowest welfare losses occur if monetary policy moves first or if all policies cooperate. Again, the relatively small losses under monetary dominance are due to the fact that, when monetary policy moves first, fiscal policy makers know that the central bank will not react to too expansionary a fiscal policy. Therefore, fiscal policy itself is more restrained compared to the fiscal leadership scenarios.

For region B we have some changes in the ranking: the three scenarios in which fiscal policies are dominant (cooperation of fiscal policies in a simultaneous game, in a sequential game where fiscal policies move first, and in the scenario where fiscal policies move first but act independently) and the Nash scenario show the highest losses. The reasons for the high losses under the two fiscal cooperation scenarios are easy to explain: As has been discussed for region A, the common central bank reacts strongly to fiscal policies by aiming at lower inflation rates. At the same time the fiscal targets, themselves, which are a weighted average of the regions' targets, are more conservative than region B's desired levels. Both effects together amount to an even higher loss compared to the one of region A.

The lowest losses for the fiscal authority in region B occur under (i) monetary leadership and fiscal cooperation, and (ii) leadership of fiscal policies. In (ii), fiscal policy of region B can follow its own targets best; in (i), fiscal policy makers recognize, again, that they cannot exploit monetary policy to reach their own goals due to the time-sequence of the game and abstain from too expansionary a fiscal policy.

The ranking of the central bank losses correspond again to the rankings of the union-wide social losses by our assumption of a welfare maximizing monetary policy. We, therefore, consider the welfare equivalent consumption losses in the following. A comparison

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this assumption.

Table 7.5: Demand-side Policy: Parameter Values in the Asymmetric Case

Parameter	$i = A$	$i = B$	Explanation
Output Equation			
$\bar{y}^i$	0	0	Natural output level
a^i	0.8	1.0	Effect of fiscal policy on the same region's output
a^{ij}	0.1	0.1	Effect of fiscal policy on the other region's output
b^i	1.0	1.5	Effect of surprise inflation on the same region's output
κ^i	0.2	0.2	Terms of trade effect of an inter-regional difference in inflation rates
$\sigma^2_{\phi^i}$	0.01	0.058	Variance of regional shocks to output
Inflation Equation			
c^i	0.5	0.7	Effect of fiscal policy on the same region's inflation
c^{ij}	0.15	0.15	Effect of fiscal policy on the other region's inflation
Loss Functions			
θ^i_M	0.5	0.5	Central bank's weighting factor for output
π^i_M	0.0	0.0	Inflation target of the central bank
y^i_M	0.0	0.0	Output target of the central bank
θ^i_F	1.3	2.0	Fiscal policy's weighting factor for output
π^i_F	0.02	0.03	Inflation target of fiscal policy
y^i_F	0.015	0.025	Output target of fiscal policy
n	0.7		GDP share of region A on union-wide GDP

Remarks: $i = A, B$, $j = A, B$ and $j \neq i$.

of the social losses under the symmetric and asymmetric case shows that, of course, the region closer to the social optimum achieves generally higher losses when heterogeneities exist. This may underline the meaning of the convergence criteria, which have to be fulfilled by the new European member states before admitted to adopt the Euro.[27]

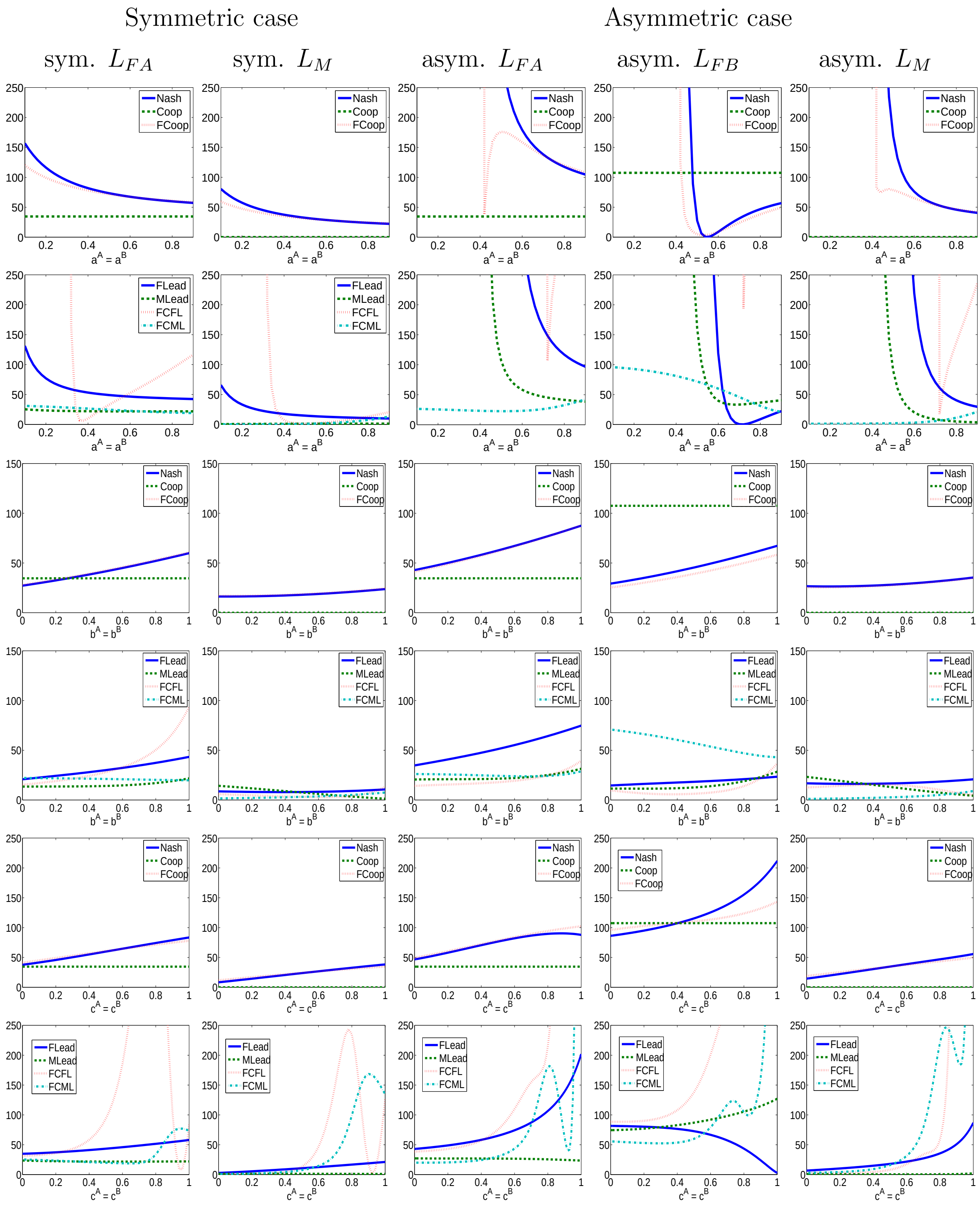
In our example, the targets of the fiscal policy maker in region A are closer to the social optimum, meaning that fiscal policy is more conservative in region A than in B. When comparing the welfare based consumption losses, we find for region A that the losses are higher than for region B in scenarios where fiscal policy is coordinated or/and has a first mover advantage, i. e. in scenarios where fiscal policies have the greatest influence. These are also the scenarios under which the private agents achieve the highest consumption losses in region B. The smallest consumption losses occur when monetary policy moves first, leading to equilibria of output and inflation closer to the social optimum, i.e. monetary policy gains a first mover-advantage in these scenarios.

7.2.6 Sensitivity Analysis for Demand-side Policy

Analogously to the procedure in section 7.2.4, we use a sensitivity analysis to examine whether the results derived in the previous section still hold when the structural parameters are varied. We begin with varying a^i, b^i and c^i for $i = A, B$ simultaneously in both regions. We consider how this affects the rankings of the different scenarios with respect to the fiscal policy makers' and central bank's loss functions. The result is shown in figure 7.5. After that, we examine how the rankings change, when a parameter is changed in one region while it is kept fixed for the other region. These results are depicted in the figures 7.6 and 7.7.

[27] We agree that a certain level of (real and nominal) convergence is a necessary requirement for the new EU member states to be able to abstain from an independent monetary policy as a stabilization tool, and to participate successfully in the European Monetary Union. However, the way in which the convergence criteria are used in practice is often debatable. E. g., an economic justification of the strict interpretation of the Maastricht Criteria in the assessment of Lithuania's state of preparation to adopt the Euro by stating that Lithuania has slightly missed the inflation criterion is questionable: the convergence criteria focus primarily on nominal convergence, but this may even hamper real convergence (see for a detailed discussion e.g. Feuerstein and Grimm, 2004 and Siebke et al., 2003).

Figure 7.5: Identical Parameter Variations in Region A and B (Demand-Side Policy)



Identical Variations of Parameters in Both Regions (Figure 7.5)

The first obvious result is that the losses in a heterogeneous monetary union are higher for almost all scenarios over the whole parameter ranges. A result that we also found for supply-side fiscal policy. The only exception is the cooperation case: Here we reach always the first best for the central bank and, thus, the private agents due to the agreement of all policy makers upon the socially optimal targets. However, if fiscal policies can, instead, enforce (partially) their own targets, the result may not hold any longer and cooperation can have worse outcomes than other scenarios exhibit.

Identical Variation of Both a^i

Beginning with a more detailed discussion of the different cases, we see that for an increasing a^i, which raises the influence of fiscal policies on output, the losses of the central bank and fiscal policy makers decrease in all scenarios except for fiscal cooperation in the symmetric case (=homogeneous currency area). We also find that a value of $a^i > 0.6$ leads to relatively small central bank losses in the scenarios where monetary policy has leadership, no matter whether fiscal policies cooperate or not. The opposite is true for the scenarios in which fiscal policies take leadership. The latter case stands in line with the findings of the simulation output discussed in the previous section, where we used $a^i = 0.8$.

In the asymmetric case (=heterogeneous currency area), monetary leadership contributes also to small losses when a^i is large for the fiscal policy maker of region A, which is the more conservative region: a large a^i facilitates an expansionary fiscal policy in region B to push inflation and output into the desired direction more easily. The central bank reacts with a strong contractionary fiscal policy. Therefore, the fiscal policy maker in country A suffers from too low an inflation.

For region B, the Nash scenario and the simultaneous fiscal cooperation scenario show the smallest losses around $a^i = 0.5$; fiscal leadership creates for $a^B > 0.6$ the smallest loss together with monetary leadership when fiscal policies cooperate.

The two scenarios of monetary leadership (with fiscal cooperation and with independently acting fiscal policies) lead to small welfare losses, as the central bank gains a first-mover advantage and is able to implement its policy mostly in line with its targets in these scenarios. Fiscal leadership, the Nash scenario, and fiscal cooperation when fiscal and monetary policies act simultaneously produce the highest central bank losses.

Identical Variation of Both b^i

We, here, consider the pictures in the third and fourth row of figure 7.5. An increasing b^i together with fiscal authorities' output targets above the socially optimal rate implies a more severe time inconsistency problem of fiscal policies as explained in section 7.1. Hence, an increasing b^i tends to produce higher policy losses in almost all cases.

In the symmetric case, both monetary leadership scenarios contribute to the smallest losses, whereas the Nash scenario, the fiscal cooperation scenario where fiscal policies move first and where fiscal and monetary policies act simultaneously lead to the highest losses for the fiscal policy makers. The realized losses are of a similar size. This supports the findings of the previous section.

Nearly the same ranking can be observed for the fiscal authority of region A in the heterogeneous case, with one exception: the fiscal leadership case creates higher losses. The reason is that the inflation and output targets of region B are above those of the relatively more conservative region A. The policy maker in A suffers, therefore, from a too expansionary a fiscal policy in B to which the central bank reacts with a restrictive monetary policy: for region A, output is above and inflation below the social optimum.

In contrast, for region B the cooperation scenario and the fiscal cooperation scenario (when all players act simultaneously) produce the highest policy losses. Under the uncoordinated fiscal leadership case the losses are relatively small, because the fiscal authority in region B can follow its own targets most aggressively.

From the viewpoint of the central bank, the monetary leadership scenario, and, interestingly, the fiscal cooperation and monetary leadership scenario lead to the best outcomes for $0 < b^i < 0.8$. This holds as fiscal cooperation dampens the extreme expansionary fiscal policy of country B as the policy maker of B internalizes now the target function of A. For a high value of b^i monetary leadership contributes again to the smallest central bank loss besides the joint cooperation scenario.

Identical Variation of Both c^i

The parameter c^i measures the direct impact of fiscal policies on inflation. The graphs in the fifth and sixth row of figure 7.5 show that a high value of c^i leads to a worse performance of all three fiscal cooperation cases (simultaneous movement, fiscal leadership and monetary leadership).

We find again that in the symmetric case the two monetary leadership scenarios produce relatively small losses for the fiscal authorities for values of $c^i < 0.5$. The highest losses occur under the Nash case, in the simultaneous fiscal cooperation case, and in the fiscal cooperation case where fiscal policies move first (with the exception of c^i close to 1). We have a similar ranking for the central bank losses as in the two former cases: Stackelberg leadership of monetary policy implies the lowest losses, again, besides the joint cooperation scenario.

For a heterogeneous monetary union, we observe similar rankings for the losses of the policy maker in region A and the common central bank. Region B, in contrast, faces small losses when fiscal policies move first and are uncoordinated.

Summary of the Findings For Identical Parameter Variations (Figure 7.5)

The sensitivity analysis has proven that the ranking of the different scenarios with respect to the fiscal policy makers' target function and the central bank's target function (= union-wide social loss function) given in table 7.4 is relatively stable for variations in a^i, b^i and c^i. The main implications, which we have found are summarized in the following:

- Joint cooperation, monetary leadership and monetary leadership with coordinated fiscal policies lead to the lowest central bank losses in a heterogeneous monetary union. This implies also that these three scenarios generate the smallest welfare losses for the private agents.

- The two monetary leadership scenarios contribute also to relatively small losses faced by the fiscal authorities in the case of a homogeneous monetary union.

- The two fiscal leadership scenarios (for coordinated and uncoordinated fiscal policies) and the Nash-scenario lead to relatively high losses for both, the fiscal authorities and the central bank, in a homogeneous monetary union.

- In a heterogeneous monetary union where region B is characterized by a more aggressive inflation and output goal compared to region A and the central bank, fiscal leadership produces in many scenarios the smallest losses for fiscal policy in region B, but high losses for the central bank and for fiscal policy in region A.

- For a low value of $a < 0.6$ and a high value of $c > 0.5$, the qualitative results become for several scenarios unstable. However, both $a > 0.6$ and $c < 0.5$ seem to accord best with reality.

Figure 7.6: Sensitivity Analysis for Demand-side Policy in the Symmetric Case

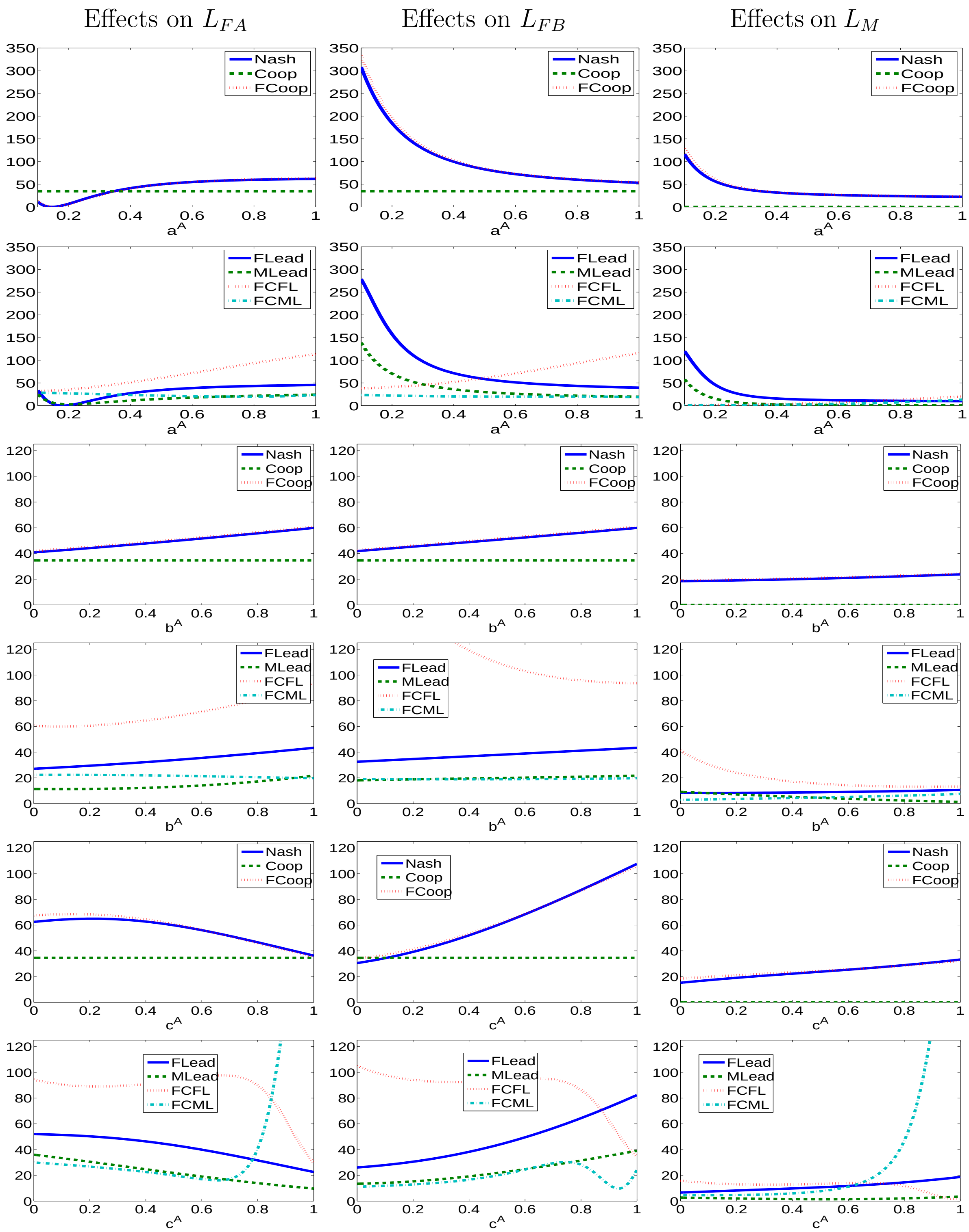
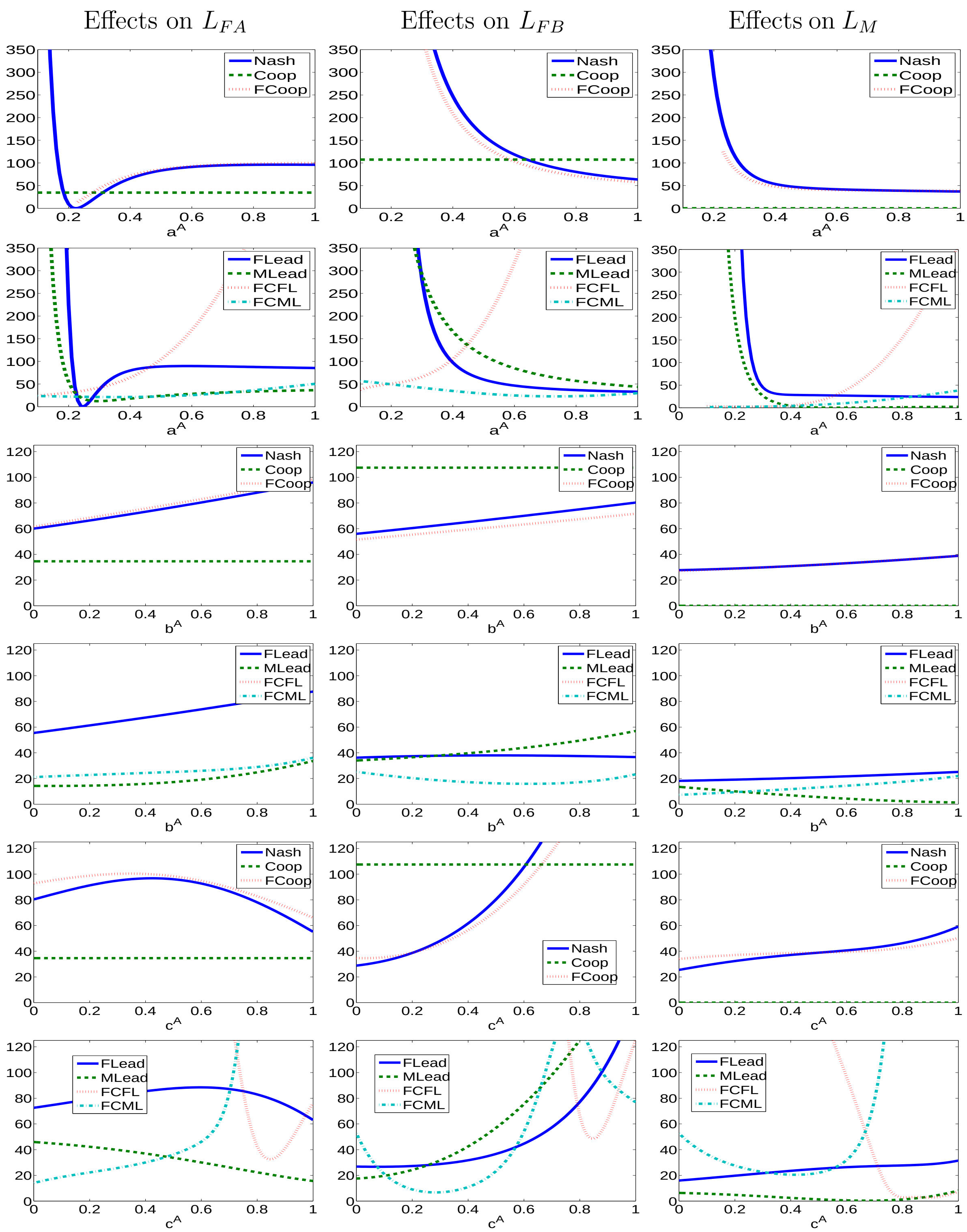


Figure 7.7: Sensitivity Analysis for Demand-side Policy in the Asymmetric Case



Individual Parameter Variations

Figures 7.6 and 7.7 depict the fiscal and monetary authorities' losses when the parameters a^A, b^A or c^A are varied in country A, while no changes occur in country B. The results are almost in line with the results of figure 7.5. For the homogeneous monetary union, the ranking of the results from table 7.4 are widely confirmed (see figure 7.6). For the heterogeneous monetary union, the findings of table 7.4 are confirmed for $a^A < 0.5$ and $c^A > 0.5$ by figure 7.7.

7.2.7 Nesting of the Results by Dixit and Lambertini

We are able to replicate the qualitative results of Dixit and Lambertini (2001, 2003a, 2003b) in our model, as explained in the following.

Dixit and Lambertini (2001) find that under Nash, $\pi^i_N < \pi_M < \pi^i_F$ and $y^i_N > y^i_F > y_M$, under Monetary Stackelberg $\pi^i_{STM} > \pi_M$ and under Fiscal Stackelberg $y^i_{STF} > y_M$. We obtain the same results if we set $\kappa^i = 0$ for all i and fulfill the conditions stated in equation (7) of that paper.

Dixit and Lambertini (2003b) find that if all authorities share the same output and inflation targets, those targets can be achieved without coordination and no matter how the timing of actions is or whether the output weights are equal or not. We obtain their results by setting $\kappa^i = c^{ij} = 0$ and equalizing the target values $\pi_M = \pi^A_F = \pi^B_F$ and $y_M = y^A_F = y^B_F$.

Dixit and Lambertini (2003a) find that $y_N < y_M \leq y_F$ and $\pi_N > \pi_F \geq \pi_M$. For the Stackelberg scenarios, no clear pattern emerges, as the stochastic terms can change either inequality sign. Furthermore, they simulate and explore the implications for welfare under different discretionary policy scenarios. We replicate their qualitative results by setting $n = 1$ and $\kappa^i = c^{i,j} = a^{ij} = 0$, $c^i < 0$, $\pi_M < \pi^A_F = \pi^B_F$ and $y_M = y^A_F = y^B_F$.

Chapter 8

Conclusion to Part II

In part II of this dissertation we have examined the interactions of fiscal and monetary policies in a monetary union. One main focus of our model was to derive a theoretical model that allows for capturing heterogeneities among the different countries participating in a monetary union, and for analyzing strategic interactions of fiscal and monetary authorities. We have surveyed the empirical findings and the literature on heterogeneities in the European Monetary Union in section 5.2. Thereby, we have shown that heterogeneities are a serious matter in the EMU, as inflation and output differentials have been and still are sizeable and highly persistent. It is obvious that the enlargement of the EMU will deepen the existing heterogeneities – at least during the first years of participation of the new EU member states, until a high degree of real convergence will be attained at some point in time.

Why do heterogeneities matter? The answer is quite simple. By adopting the Euro, the participating countries abstain from a monetary policy of their own and fiscal policy remains the only instrument for pursuing region-specific goals and stabilizing region-specific shocks. The common central bank has to implement a monetary policy that is most appropriate for the whole monetary union and it cannot respond to idiosyncratic shocks and country-specific political targets. This makes the role of fiscal policies more important and leaves room for strategic behavior in achieving national goals.

To examine these heterogeneities, we have enhanced and modified the model of Dixit and Lambertini (2003b). From the microfoundation we have established that terms of trade, i. e. inflation differentials, have an impact on regional output. In section 6.2 we derived a utility-based welfare criterion for each region as well as for the whole monetary union.

In our reduced-form framework, elaborated in section 7.1, we have introduced the different scenarios of strategic interactions between fiscal and monetary policies. In this context we have assumed that fiscal policies deviate from maximizing regional welfare and aiming, instead, at higher inflation and output levels compared to the union-wide central bank. By contrast, monetary policy is assumed to maximize union-wide welfare.

We have used simulations to evaluate the different scenarios of strategic behavior, and explored two types of fiscal policies: (i) a supply-side policy in line with the micro-model, where fiscal policies grant subsidies to increase output financed by per-head taxes, and (ii) demand-side policy, where fiscal policies try to push output by raising demand. We, further, have considered a heterogeneous monetary union with two different regions: a "conservative" region and a "catch-up" region. We have assumed that the desired inflation and output targets of the "conservative region" are relatively closer to the social optimum.

To evaluate the supply-side policy (i), we have used a calibration of our micro-model drawing upon the parameters from the standard economic literature. We have shown that the losses of fiscal policies are relatively small in the Nash scenario, in the fiscal leadership scenario (for both cooperation of fiscal policies and independently acting fiscal policies), and when fiscal policies cooperate and all policy makers move simultaneously. In these scenarios, fiscal policies achieve an output level closest to their preferred levels, whereas inflation is stabilized close to the socially optimal level by the common central bank.

The losses of monetary policy, which correspond to the welfare losses of the private agents, are lowest when monetary policy moves first. The first-best situation is attained when all policy makers agree upon the socially optimal levels. But as the central bank and fiscal policy makers consider different scenarios optimal, such an agreement appears to be unrealistic on a voluntary basis.

If fiscal authorities exercise a demand-side oriented policy (ii), which accords best with the policies in the EMU, fiscal leadership leads to relatively low losses for the catch-up region. In contrast, monetary leadership produces the lowest losses for the conservative region. If monetary policy moves first, fiscal cooperation would be preferable for the catch-up region, whereas the conservative region prefers to act independently.

From the viewpoint of monetary policy (and welfare), the smallest losses are again attained when monetary policy moves first and, of course, when monetary and fiscal policies agree on the socially optimal level.

In the EMU, fiscal policies appear primarily to track national interests. However, the analysis has shown that fiscal policies in a heterogeneous monetary union can contribute to high welfare losses under both types of fiscal policies. From a welfare perspective, monetary leadership or cooperation would then be a desirable scenario for both types of fiscal policy.

To summarize, if the authorities' preferences do not coincide, or are at least relatively far apart, worse outcomes are likely to occur. In such a case, designing the institutions so that monetary policy plays a lead role generates the smallest losses for the agents living in both regions, even with existing heterogeneities.

The European Central Bank aggressively pursues the price-stability goal, meaning that the inflation rate should not exceed 2%. Accordingly, it appears to act as a first mover, which is beneficial for welfare. At the same time, fiscal policies are restricted in their actions by the Stability and Growth Pact, which leaves less room for pursuing too excessive fiscal targets and implies a reduction of the trade-offs caused by strategic behavior. Recent experience, however, has shown that in bad times meeting the stability criteria may not be a very credible option for fiscal policies, especially, when the culprits judge their own sanctions, as has happened in the European Union. Therefore, reducing heterogeneities and bringing fiscal policies' targets closer to the socially optimal levels is an essential task in achieving a longer-term stability guarantee for the EMU.

Appendix A

Supplements to Chapter 3

In this appendix, we investigate the expected inflation rate in the case of a fixed exchange-rate system and derive sufficient conditions under which multiple equilibria can be excluded.

A.1 Calculation of the Expected Value of Inflation under a Peg

As mentioned in section 3.5.2, for certain parameter values the boundaries u_2^u or u_2^l may lie outside the support of the new shock u_2 which is uniformly distributed. In that case, we can replace the boundaries by $-A$ or A, respectively. To determine π_2^e, we thus define $\tilde{u}_2^l$ and $\tilde{u}_2^u$ as

$$\begin{aligned} \tilde{u}_2^l &= \min\{\max\{u_2^l, -A\}, A\} \\ \tilde{u}_2^u &= \min\{\max\{u_2^u, -A\}, A\} \ . \end{aligned} \tag{A.1}$$

For instance, $u_2^u > A$ implies $\tilde{u}_2^u = A$.

Using (A.1), we can rewrite (3.25) as

$$\begin{aligned} E_1(\pi_2) &= \frac{\tilde{u}_2^l + A}{2A} \cdot \Gamma(\pi_2^e, k) + \frac{A - \tilde{u}_2^u}{2A} \cdot \Gamma(\pi_2^e, k) + \left(\eta\phi_1 + \frac{\tilde{u}_2^u - \tilde{u}_2^l}{2A}\right) \frac{\tilde{u}_2^u - \tilde{u}_2^l}{2A} \\ &= \left(\eta\phi_1 + \frac{\tilde{u}_2^u - \tilde{u}_2^l}{2}\right) \frac{\tilde{u}_2^u - \tilde{u}_2^l}{2A} + \left(1 - \frac{\tilde{u}_2^u - \tilde{u}_2^l}{2A}\right) \Gamma(\pi_2^e, k) \ . \end{aligned} \tag{A.2}$$

$E_1(\pi_2)$ is a continuous function of π_2^e. The equilibrium condition is given by $\pi_2^e = E_1(\pi_2)$. To solve for the expected inflation π_2^e, we consider the following five different cases:

- **Case (i):** $u_2^u < -A$, i.e. the interval $[u_2^l, u_2^u]$ lies outside the support of u_2.

$$\begin{aligned} E_1(\pi_2|\tilde{u}_2^u = -A) &= \frac{\eta\phi_1 - 2A}{2} \cdot \frac{-A+A}{2A} + \frac{1-(-A+A)}{2A} \cdot \Gamma(\pi_2^e, k) \\ &= \Gamma(\pi_2^e, k) \end{aligned} \tag{A.3}$$

The solution equals the free-float equilibrium rate, as monetary authorities will always revalue.

- **Case (ii):** $u_2^u > -A$ and $u_2^l < -A$, i.e. $[u_2^l, u_2^u]$ lies partly in the support of u_2. In this case the expected value of π_2 is given by

$$\begin{aligned} E_1(\pi_2|u_2^u > -A \quad \wedge \quad u_2^l < -A) &= \left(\eta\phi_1 + \frac{u_2^u - A}{2}\right) \frac{u_2^u + A}{2A} + \left(1 - \frac{u_2^u + A}{2A}\right) \cdot \Gamma(\pi_2^e, k) \\ &= \Gamma(\pi_2^e, k) + \frac{u_2^u + A}{2A}\left[\eta\phi_1 + \frac{u_2^u - A}{2} - \Gamma(\pi_2^e, k)\right] \\ &= \Gamma(\pi_2^e, k) - \frac{1}{4A}\left(-\eta\phi_1 + \Gamma(\pi_2^e, k) + \sqrt{\frac{c}{\gamma^2+\theta}} + A\right) \\ &\quad \cdot \left[-\eta\phi_1 + \Gamma(\pi_2^e, k) - \sqrt{\frac{c}{\gamma^2+\theta}} + A\right] . \end{aligned} \tag{A.4}$$

- **Case (iii):** $-A \leq u_2^l < u_2^u \leq A$, i.e. $[u_2^l, u_2^u]$ is a subset of the support interval $[-A, A]$. This leads to

$$\begin{aligned} E_1(\pi_2|u_2^l > -A \quad \wedge \quad u_2^u < A) &= \left(\eta\phi_1 + \frac{u_2^u + u_2^l}{2}\right) \frac{u_2^u - u_2^l}{2A} + \left(1 - \frac{u_2^u - u_2^l}{2A}\right) \cdot \Gamma(\pi_2^e, k) \\ &= (\eta\phi_1 - \eta\phi_1 + \Gamma(\pi_2^e, k))\frac{1}{A}\sqrt{\frac{c}{\gamma^2+\theta}} + \left(1 - \frac{1}{A}\sqrt{\frac{c}{\gamma^2+\theta}}\right)\Gamma(\pi_2^e, k) \\ &= \Gamma(\pi_2^e, k) . \end{aligned} \tag{A.5}$$

- **Case (iv):** $u_2^l < A$ and $u_2^u > A$, i.e. the interval $[u_2^l, u_2^u]$ lies partly in $[-A, A]$.

$$\begin{aligned} E_1(\pi|u_2^l < A \quad \wedge \quad u_2^u > A) &= \frac{\left(\eta\phi_1 + \frac{1}{2}A + \frac{1}{2}u_2^l\right)(A - u_2^l)}{2A} \\ &\quad + \left(1 - \frac{A - u_2^l}{2A}\right) \cdot \Gamma(\pi_2^e, k) \\ &= \Gamma(\pi_2^e, k) + \frac{1}{4A}\left[A + \eta\phi_1 - \Gamma(\pi_2^e, k) + \sqrt{\frac{c}{\gamma^2+\theta}}\right] \\ &\quad \cdot \left(A + \eta\phi_1 - \Gamma(\pi_2^e, k) - \sqrt{\frac{c}{\gamma^2+\theta}}\right) . \end{aligned} \tag{A.6}$$

- **Case (v):** If $u_2^l > A$, i.e. the interval $[u_2^l, u_2^u]$ lies also outside of the support interval of u_2, the policy maker will always devalue.

Using $C := \sqrt{\frac{c}{\gamma^2+\theta}}$, we can rewrite the results for the different cases as

$$
\begin{aligned}
E_1(\pi_2|\text{case (i)}) &= \Gamma(\pi_2^e, k) \\
E_1(\pi_2|\text{case (ii)}) &= \Gamma(\pi_2^e, k) - \frac{1}{4A}\left(-\eta\phi_1 + \Gamma(\pi_2^e, k) + C + A\right)\left[-\eta\phi_1 + \Gamma(\pi_2^e, k) - C + A\right] \\
E_1(\pi_2|\text{case (iii)}) &= \Gamma(\pi_2^e, k) \\
E_1(\pi_2|\text{case (iv)}) &= \Gamma(\pi_2^e, k) + \frac{1}{4A}\left(-\eta\phi_1 + \Gamma(\pi_2^e, k) + C - A\right)\left[-\eta\phi_1 + \Gamma(\pi_2^e, k) - C - A\right] \\
E_1(\pi_2|\text{case (v)}) &= \Gamma(\pi_2^e, k) \; .
\end{aligned}
$$

A.2 Graph of $E_1(\pi_2)$ as a Function of π_2^e

The function $E_1(\pi_2)$ is defined by cases. As u_2^l and u_2^u depend positively on π_2^e (equations 3.23 and 3.24), the sequence of the respective intervals corresponds to the numbering of the cases (i) to (v).

The graph of $E_1(\pi_2)$ lies on the straight line $\Gamma(\pi_2^e, k)$ with the slope $\frac{\gamma^2}{\gamma^2+\theta} < 1$ in the three cases (i), (iii) and (iv). By investigating the second derivative of $E_1(\pi_2)$, it follows that the function is concave in case (ii) and convex in case (iv). Thus $E_1(\pi_2)$ is a straight line with two convexities – one above (case ii) and one below (case iv) the line as illustrated in figure A.1. The position of the line and the convexities depend on the parameters, in particular on k and $\eta\phi_1$.

A.3 Conditions for a Unique Equilibrium under a Peg

In the following section, we derive two conditions excluding the possibility of multiple equilibria under a peg.

A) From the description of the graph of $E_1(\pi_2)$ it is clear that the equilibrium is unique, if the function $E_1(\pi_2)$ cuts the bisecting line in the range of case (iii).

B) Moreover, we can exclude the existence of multiplicities if the slope of $E_1(\pi_2)$ does not exceed one in the two cases (ii) and (iv), too. We take up case (iv) for further examination, the argument in case (ii) is analogous.

The first derivative of $E_1(\pi_2|\text{case (iv)})$ is given by

$$\begin{aligned}\frac{dE_1(\pi_2|\text{case (iv)})}{d\pi_2^e} &= \frac{1}{2}\frac{\gamma^2(\gamma^2\pi_2^e + \gamma k + A\gamma^2 + A\theta - \eta\phi_1\theta - \eta\phi_1\gamma^2)}{A(\gamma^2+\theta)^2} \\ &= \frac{1}{2A}\frac{\gamma^2}{\gamma^2+\theta}\left[\Gamma(\pi_2^e, k) - \eta\phi_1 + A\right] \; . \qquad (A.7)\end{aligned}$$

The expression in equation (A.7) equals one if[1]

$$\pi_2^e = A + \eta\phi_1 + \frac{2A\theta^2 + 3\gamma^2 A\theta - \gamma^3 k + \gamma^2\eta\phi_1\theta}{\gamma^4} \; . \qquad (A.8)$$

The boundary between the ranges of the cases (iv) and (v) is given by

$$\pi_2^e = \left(A + \eta\phi_1 + \sqrt{\frac{c}{\gamma^2+\theta}}\right)\frac{\gamma^2+\theta}{\gamma^2} - \frac{k}{\gamma} \; . \qquad (A.9)$$

If that boundary lies left of the point where $\frac{dE_1(\pi_2)}{d\pi_2^e} = 1$, the slope of $E_1(\pi_2)$ is smaller than one also in the range of case (iv) because the function is convex in that interval. Thus, the following condition excludes multiplicities

$$\begin{aligned} & A + \eta\phi_1 + \frac{2A\theta^2 + 3\gamma^2 A\theta - \gamma^3 k + \gamma^2\eta\phi_1\theta}{\gamma^4} \\ & \qquad - \left[\left(A + \eta\phi_1 + \sqrt{\frac{c}{\gamma^2+\theta}}\right)\frac{\gamma^2+\theta}{\gamma^2} - \frac{k}{\gamma}\right] > 0 \\ \Leftrightarrow\;& \frac{1}{\gamma^4}\cdot\left(2A\theta^2 + 2\gamma^2 A\theta - \gamma^4\sqrt{\frac{c}{g^2+\theta}} - \gamma^2\theta\sqrt{\frac{c}{g^2+\theta}}\right) > 0 \\ \Leftrightarrow\;& 2A\theta(\gamma^2+\theta) - \gamma^2\sqrt{\frac{c}{\gamma^2+\theta}}(\gamma^2+\theta) > 0 \\ \Leftrightarrow\;& 2A\theta - \gamma^2\sqrt{\frac{c}{\gamma^2+\theta}} > 0 \; . \qquad (A.10)\end{aligned}$$

As by assumption $\sqrt{\frac{c}{\gamma^2+\theta}} \leq A$,

$$2A\theta - \gamma^2 A > 0 \qquad (A.11)$$

is sufficient for (A.10). Thus, multiplicities can be excluded, if $\theta > \frac{\gamma^2}{2}$.

Figure A.1 depicts $E(\pi_2)$ as a function of π_2^e for a set of parameters satisfying this condition (i.e. $\gamma = 0.7,\ k = 1,\ c^{pf} = 1.2,\ A = 1.5,\ \eta = 0.5,\ \phi_1 = \frac{-2}{5}A$ and $\theta = 0.5$). The unique equilibrium is denoted by E.

[1]Note, that we first determine the point where the slope equals one and check afterwards, if the point is in the range of case (iv).

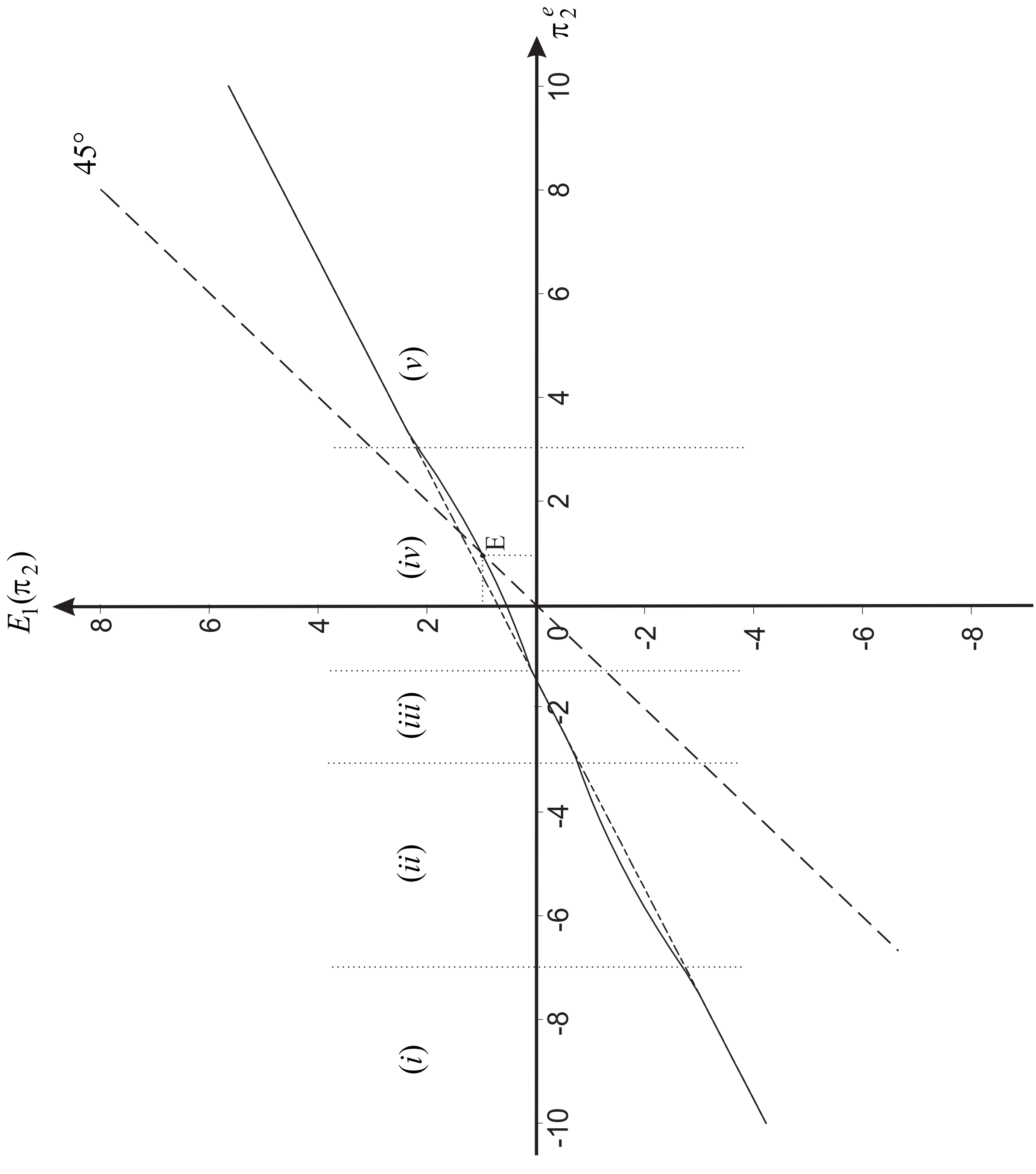


Figure A.1: Expected Value of Second Period Inflation under a Peg

Appendix B

Supplements to Chapter 4

B.1 Calculations for a Fixed Exchange Rate System

In the following part of the appendix, we determine optimal fiscal and monetary policy under a fixed exchange-rate system. At first, we make the assumption that expected inflation is treated as given. The results of the calculations are used in section 4.5.1 and in section 4.6. After that, we reformulate the loss function and determine the rational expectations equilibrium for the case of the exchange rate peg being defended with a probability of one.

B.1.1 Optimization with Exogenous π_2^e

The optimization problem of the policy maker is given by

$$\begin{aligned}\max_{y_2,\pi_2,b_2,g_2} \mathcal{L}_2 \quad = \quad & \frac{1}{2}(y_2 - k)^2 + \frac{1}{2}\theta_\pi \pi_2^2 + \frac{1}{2}\theta_b b_2^2 \\ & +\lambda_1(y_2 - \gamma(\pi_2 - \pi_2^e) - wg_2) \\ & +\lambda_2(\pi_2 - \phi_2 - \beta g_2) \\ & +\lambda_3(b_2 - b_1(1 - \pi_2) - g_2) \ . \qquad \text{(B.1)}\end{aligned}$$

The first-order conditions are

$$\frac{\partial \mathcal{L}_2}{\partial y_2} = (y_2 - k) + \lambda_1 = 0 \qquad \text{(B.2)}$$

$$\frac{\partial \mathcal{L}_2}{\partial \pi_2} = \theta_\pi \pi_2 - \gamma\lambda_1 + \lambda_2 + \lambda_3 b_1 = 0 \qquad \text{(B.3)}$$

$$\begin{aligned} \frac{\partial \mathcal{L}_2}{\partial b_2} &= \theta_b b_2 + \lambda_3 = 0 &\text{(B.4)} \\ \frac{\partial \mathcal{L}_2}{\partial g_2} &= w\lambda_1 + \beta\lambda_2 + \lambda_3 = 0 \ . &\text{(B.5)} \end{aligned}$$

Combining equations (B.2), (B.4) and (B.5) yields

$$\lambda_2 = -\frac{w\lambda_1}{\beta} - \frac{\lambda_3}{\beta} = \frac{\theta_b b_2}{\beta} + \frac{w}{\beta}(y_2 - k) \ . \tag{B.6}$$

By inserting (B.2), (B.4) and (B.6) into (B.3), we obtain

$$\begin{aligned} \theta\pi_2 &= k\frac{\gamma\beta + w}{\beta} - y_2\frac{\gamma\beta + w}{\beta} + \theta_b b_2 \frac{\beta b_1 - 1}{\beta} \\ \Leftrightarrow y_2 &= -\frac{\theta_\pi \beta}{\gamma\beta + w}\pi_2 + \theta_b b_2 \frac{\beta b_1 - 1}{\gamma\beta + w} + k \ . &\text{(B.7)} \end{aligned}$$

Restriction 3 (=debt equation) solved for g_2 yields

$$g_2 = b_2 + b_1\pi_2 - b_1. \tag{B.8}$$

Inserting into restriction 1 (=output equation) leads to

$$\begin{aligned} y_2 &= \gamma(\pi_2 - \pi_2^e) + wg_2 = \gamma\pi_2 - \gamma\pi_2^e + wb_2 + wb_1\pi_2 - wb_1 \\ &= \pi_2(\gamma + wb_1) + wb_2 - wb_1 - \gamma\pi_2^e. &\text{(B.9)} \end{aligned}$$

Combining (B.7) and (B.9) yields

$$\begin{aligned} \theta_\pi \pi_2 &= k\frac{\gamma\beta + w}{\beta} - \frac{\gamma\beta + w}{\beta}(\pi_2(\gamma + wb_1) + wb_2 - wb_1 - \gamma\pi_2^e) \\ &\quad + \theta_b b_2 \frac{\beta b_1 - 1}{\beta} \end{aligned}$$

$$\begin{aligned} \Leftrightarrow \pi_2\left(\frac{\theta_\pi\beta + (\gamma\beta + w)(\gamma + wb_1)}{\beta}\right) &= \frac{\gamma\beta + w}{\beta}(k + wb_1 + \gamma\pi_2^e) \\ &\quad + \frac{\theta_b(\beta b_1 - 1) - w(\gamma\beta + w)}{\beta} b_2 \ . &\text{(B.10)} \end{aligned}$$

Some further manipulations lead to

$$\pi_2 = -\frac{\theta_b(1 - \beta b_1) + w(\gamma\beta + w)}{(\gamma + wb_1)(\gamma\beta + w) + \theta_\pi\beta} b_2 + \frac{(k + \gamma\pi_2^e + wb_1)(\gamma\beta + w)}{(\gamma + wb_1)(\gamma\beta + w) + \theta_\pi\beta} \ . \tag{B.11}$$

Restriction 2 (=inflation equation) combined with restriction 3 (=debt equation) gives

$$\pi_2 = \frac{\beta}{1 - \beta b_1} b_2 + \frac{\phi_2 - \beta b_1}{1 - \beta b_1} \ . \tag{B.12}$$

Setting (B.11) equal to (B.12) leads to

$$\left[\frac{\beta}{1-\beta b_1} + \frac{\theta_b(1-\beta b_1) + w(\gamma\beta + w)}{(\gamma + wb_1)(\gamma\beta + w) + \theta_\pi\beta}\right] b_2 = \frac{(k + \gamma\pi_2^e + wb_1)(\gamma\beta + w)}{(\gamma + wb_1)(\gamma\beta + w) + \theta_\pi\beta} - \frac{\phi_2 - \beta b_1}{1-\beta b_1}$$
$$b_2 \frac{\beta[(\gamma + wb_1)(\gamma\beta + w) + \theta_\pi\beta] + \theta_b(1-\beta b_1)^2 + w(\gamma\beta + w)(1-\beta b_1)}{(1-\beta b_1)\left[(\gamma + wb_1)(\gamma\beta + w) + \theta_\pi\beta)\right]}$$
$$= \frac{(k + \gamma\pi_2^e + wb_1)(\gamma\beta + w)(1-\beta b_1) - (\phi_2 - \beta b_1)[(\gamma + wb_1)(\gamma\beta + w) + \theta_\pi\beta]}{(1-\beta b_1)\left[(\gamma + wb_1)(\gamma\beta + w) + \theta_\pi\beta)\right]} . \tag{B.13}$$

Solving for b_2 yields the optimum value of second period debts (for a given π_2^e) which equals

$$b_2^* = \frac{(k + \gamma\pi_2^e + wb_1)(\gamma\beta + w)(1-\beta b_1) - (\phi_2 - \beta b_1)\big((\gamma + wb_1)(\gamma\beta + w) + \theta_\pi\beta\big)}{(\gamma\beta + w)^2 + \theta_\pi\beta^2 + \theta_b(1-\beta b_1)^2} . \tag{B.14}$$

For a better exposition and to simplify further calculations, we define $A := w + \gamma\beta$ and $B := (1 - \beta b_1)$. Note that from the domain of parameters used here $A > 0$ and $B > 0$ is obvious. Therefore, we can rewrite b_2^* as

$$b_2^* = \frac{(k + \gamma\pi_2^e + wb_1)AB}{\theta_b B^2 + A^2 + \theta_\pi\beta^2} - \frac{(\phi_2 - \beta b_1)(\theta_\pi\beta + A(\gamma + wb_1))}{\theta_b B^2 + A^2 + \theta_\pi\beta^2} . \tag{B.15}$$

The optimal second period inflation rate and the optimal output y_2 are given by

$$\begin{aligned}
\pi_2^* &= \frac{\phi_2 - \beta b_1}{B} + \frac{\beta}{B} b_2^* \\
&= \frac{\beta A(k + \gamma\pi_2^e + wb_1)}{\theta_b B^2 + A^2 + \theta_\pi\beta^2} - \frac{\beta(\phi_2 - \beta b_1)(\theta_\pi\beta + A(\gamma + wb_1))}{B(\theta_b B^2 + A^2 + \theta_\pi\beta^2)} + \frac{\phi_2 - \beta b_1}{B} \\
&= \frac{\beta A(k + \gamma\pi_2^e + wb_1)}{\theta_b B^2 + A^2 + \theta_\pi\beta^2} + \frac{\phi_2 - \beta b_1}{B}\left[\frac{\theta_b B^2 + w^2 + w\beta(\gamma - wb_1 - \beta\gamma b_1)}{\theta_b B^2 + A^2 + \theta_\pi\beta^2}\right] \\
&= \frac{\beta A(k + \gamma\pi_2^e + wb_1)}{\theta_b B^2 + A^2 + \theta_\pi\beta^2} + \frac{(\phi_2 - \beta b_1)(\theta_b B + wA)}{\theta_b B^2 + A^2 + \theta_\pi\beta^2} \\
&= \frac{\beta A(k + \gamma\pi_2^e) + \phi_2(\theta_b B + wA) - \theta_b\beta B b_1}{\theta_b B^2 + A^2 + \theta_\pi\beta^2} ,
\end{aligned} \tag{B.16}$$

$$\begin{aligned}
y_2^* &= k - \theta_b b_2^* \frac{B}{A} - \theta_\pi \pi_2^* \frac{\beta}{A} \\
&= k - \frac{(k + \gamma\pi_2^e + wb_1)(\theta_b B^2 + \theta_\pi \beta^2)}{\theta_b B^2 + A^2 + \theta_\pi \beta^2} \\
&\quad - \frac{\phi_2 - \beta b_1}{A} \frac{\theta_\pi \beta(w^2 + w\beta(\gamma - wb_1 - \gamma\beta b_1)) - \theta_b B^2 (A(\gamma + wb_1))}{B(\theta_b B^2 + A^2 + \theta_\pi \beta^2)} \\
&= k - \frac{(k + \gamma\pi_2^e + wb_1)(\theta_b B^2 + \theta_\pi \beta^2)}{\theta_b B^2 + A^2 + \theta_\pi \beta^2} + \frac{(\phi_2 - \beta b_1)(\theta_b B(\gamma + wb_1) - \theta_\pi w\beta)}{\theta_b B^2 + A^2 + \theta_\pi \beta^2} \\
&= \frac{kA^2 - \theta_b b_1 AB - \gamma\pi_2^e(\theta_b B^2 + \theta_\pi \beta^2) + \phi_2(\theta_b B(\gamma + wb_1) - \theta_\pi w\beta)}{\theta_b B^2 + A^2 + \theta_\pi \beta^2} \ . \qquad \text{(B.17)}
\end{aligned}$$

To reach the optimum, the policy maker chooses a fiscal deficit of

$$\begin{aligned}
g_2^* &= b_2^* + b_1 \pi_2^* - b_1 \\
&= \frac{(k + \gamma\pi_2^e + wb_1)AB}{\theta_b B^2 + A^2 + \theta_\pi \beta^2} + \frac{\beta A(k + \gamma\pi_2^e)b_1 - \theta_b \beta B b_1^2}{\theta_b B^2 + A^2 + \theta_\pi \beta^2} \\
&\quad - \frac{(\phi_2 - \beta b_1)(\theta_\pi \beta + A(\gamma + wb_1))}{\theta_b B^2 + A^2 + \theta_\pi \beta^2} + \frac{\phi_2(\theta_b B + wA)}{\theta_b B^2 + A^2 + \theta_\pi \beta^2} - b_1 \qquad \text{(B.18)} \\
&= \frac{(k + \pi_2^e)A + \phi_2(\theta_b B - \theta_\pi \beta + A(w + \gamma + wb_1)) + b_1(A(\gamma\beta - 1) + w - \theta_b B)}{\theta_b B^2 + A^2 + \theta_\pi \beta^2} \ .
\end{aligned}$$

The derivatives of optimal second period output, inflation, debt, and of the policy instrument g_2 for π_2^e equals:

$$\begin{aligned}
\frac{\partial b_2}{\partial \pi_2^e} &= \frac{\gamma(\gamma\beta + w)(1 - \beta b_1)}{(\gamma\beta + w)^2 + \beta^2\theta_\pi + \theta_b(1 - \beta b_1)^2} > 0 \quad \text{for } \beta b_1 < 1 \qquad \text{(B.19)} \\
\frac{\partial \pi_2}{\partial \pi_2^e} &= \frac{\gamma\beta(\gamma\beta + w)}{(\gamma\beta + w)^2 + \beta^2\theta_\pi + \theta_b(1 - \beta b_1)^2} > 0 \qquad \text{(B.20)} \\
\frac{\partial g_2}{\partial \pi_2^e} &= \frac{\gamma\beta + w}{(\gamma\beta + w)^2 + \beta^2\theta_\pi + \theta_b(1 - \beta b_1)^2} > 0 \qquad \text{(B.21)} \\
\frac{\partial y_2}{\partial \pi_2^e} &= -\frac{\gamma(\theta_b B^2 + \theta_\pi \beta^2)}{(\gamma\beta + w)^2 + \beta^2\theta_\pi + \theta_b(1 - \beta b_1)^2} < 0 \ . \qquad \text{(B.22)}
\end{aligned}$$

B.1.2 Reformulation of the Loss Function as a Function of π_2

For a better handling in section 4.6, we rewrite the loss function by using the calculations of the first section of appendix B.1 (for a given π_2^e). From equation B.9, we have

$$y_2 = \pi_2(\gamma + wb_1) + wb_2 - wb_1 - \gamma\pi_2^e \ . \qquad \text{(B.23)}$$

Solving (B.12) for b_2 yields

$$b_2 = \frac{B}{\beta}\pi_2 - \frac{\phi_2 - \beta b_1}{\beta} = \frac{1 - \beta b_1}{\beta}\pi_2 - \frac{\phi_2}{\beta} + b_1 \ . \qquad \text{(B.24)}$$

Inserting (B.24) into (B.23) leads to

$$y_2 = \pi_2 \frac{w+\gamma\beta}{\beta} - \frac{w}{\beta}\phi_2 - \gamma\pi_2^e \ . \tag{B.25}$$

Plugging (B.24) and (B.25) into the loss function, we obtain

$$\begin{aligned} 2\cdot L_2^{CB}(\pi_2) &= \left(\pi_2 \frac{w+\gamma\beta}{\beta} - \frac{w}{\beta}\phi_2 - \gamma\pi_2^e - k\right)^2 + \theta_\pi \pi_2^2 \\ &+\theta_b\left(\frac{1-\beta b_1}{\beta}\pi_2 - \frac{\phi_2}{\beta} + b_1\right)^2 \ . \end{aligned} \tag{B.26}$$

B.1.3 Rational Expectations Equilibrium

We assume that the private sector has rational expectations on inflation. Then, the following condition holds:

$$\begin{aligned} \pi_2^e = E(\pi_2) &= \frac{\beta Ak + \beta A\gamma\pi_2^e + \eta\phi_1(\theta_b B + wA) - \theta_b\beta b_1 B}{\theta_b B^2 + A^2 + \theta_\pi\beta^2} \\ \Leftrightarrow \pi_2^e\left(\frac{\theta_b B^2 + A + \theta_\pi\beta^2 - \beta\gamma A}{\theta_b B^2 + A + \theta_\pi\beta^2}\right) &= \frac{\beta Ak + \eta\phi_1(\theta_b B + wA) - \theta_b\beta b_1 B}{\theta_b B^2 + A^2 + \theta_\pi\beta^2} \\ \Leftrightarrow \pi_2^e &= \frac{\beta Ak + \eta\phi_1(\theta_b B + wA) - \theta_b\beta b_1 B}{\theta_b B^2 + wA + \theta_\pi\beta^2} \ . \end{aligned} \tag{B.27}$$

Inserting (B.27) into equation (B.16) yields the rational expectation equilibrium value, given by[1]

$$\begin{aligned} \pi_2^{**} &= \frac{\beta Ak + \phi_2(\theta_b B + wA) - \theta_b\beta b_1 B}{\theta_b B^2 + A^2 + \theta_\pi\beta^2} \\ &+\frac{\beta\gamma A}{\theta_b B^2 + A^2 + \theta_\pi\beta^2}\cdot\frac{\beta Ak + \eta\phi_1(\theta_b B + wA) - \theta_b\beta B b_1}{\theta_b B^2 + wA + \theta_\pi\beta^2} \\ &= \frac{\beta Ak + \eta\phi_1(\theta_b B + wA) - \theta_b\beta b_1 B}{\theta_b B^2 + wA + \theta_\pi\beta^2} + u_2\frac{\theta_b B + wA}{\theta_b B^2 + A^2 + \theta_\pi\beta^2} \ . \end{aligned} \tag{B.28}$$

The equilibrium value for b_2 is obtained by plugging (B.28) into equation (B.24):

$$\begin{aligned} b_2^{**} &= \pi_2^{**}\frac{B}{\beta} - \frac{\phi_2}{\beta} + b_1 \\ &= \frac{B(\beta Ak + \eta\phi_1(\theta_b B + wA) - \theta_b b_1\beta B)}{\beta(\theta_b B^2 + wA + \theta_\pi\beta^2)} + b_1 - u_2\frac{A(\gamma + wb_1)}{\theta_b B^2 + A^2 + \theta_\pi\beta^2} \ , \end{aligned} \tag{B.29}$$

and y_2 can be obtained by plugging (B.28) into (B.25)

$$y_2^{**} = w\pi_2^{**} + \gamma u_2\frac{\theta_b B + wA}{\theta_b B^2 + A^2 + \theta_\pi\beta^2} - \frac{w}{\beta}\phi_2 \ . \tag{B.30}$$

[1] Rational expectations equilibrium values are denoted by the superscript "**".

The rational expectations equilibrium is used in the sections 4.5.2 and in section 4.6. Note that we, of course, can calculate the optimal size of government expenditure (=fiscal deficit) g_2 for the rational expectations equilibrium, too.

B.2 Calculation for a Flexible Exchange Rate System

We, here, derive optimal fiscal and monetary policy under flexible exchange rates. At the beginning of this section, we make again the assumption that expected inflation is exogenous. The results are used in section 4.5.1. Thereafter, we reformulate the loss function and determine the rational expectations equilibrium.

B.2.1 Optimization with an Exogenous π_2^e and an Exogenous Ratio of Foreign Debts q

Optimization problem of the policy maker:

$$\begin{aligned}\max_{y_2,\pi_2,b_2,g_2,e_2} \mathcal{L}_2 &= \frac{1}{2}(y_2-k)^2+\frac{1}{2}\theta_\pi\pi_2^2+\frac{1}{2}\theta_b b_2^2 \\ &+\lambda_1(y_2-\gamma(\pi_2-\pi_2^e)-wg_2) \\ &+\lambda_2(\pi_2-\phi_2-\kappa e_2-\beta g_2) \\ &+\lambda_3(b_2-b_1(1+qe_2-\pi_2)-g_2)\ , \end{aligned} \tag{B.31}$$

where e_2 and g_2 are the policy instruments to reach the output-, inflation- and debt-goal. The first-order conditions are

$$\frac{\partial\mathcal{L}_2}{\partial y_2} = (y_2-k)+\lambda_1 \Leftrightarrow \lambda_1 = k-y_2 = 0 \tag{B.32}$$

$$\frac{\partial\mathcal{L}_2}{\partial \pi_2} = \theta_\pi\pi_2-\gamma\lambda_1+\lambda_2+\lambda_3 b_1 = 0 \tag{B.33}$$

$$\frac{\partial\mathcal{L}_2}{\partial b_2} = \theta_b b_2+\lambda_3 = 0 \Leftrightarrow \lambda_3 = -\theta_b b_2 \tag{B.34}$$

$$\frac{\partial\mathcal{L}_2}{\partial g_2} = -w\lambda_1-\beta\lambda_2-\lambda_3 = 0 \tag{B.35}$$

$$\frac{\partial\mathcal{L}_2}{\partial e_2} = -\kappa\lambda_2-b_1 q\lambda_3 = 0\ . \tag{B.36}$$

Combining (B.36) and (B.34) yields

$$\lambda_2 = -\frac{b_1 q}{\kappa}\lambda_3 = \frac{q b_1\theta_b b_2}{\kappa}\ . \tag{B.37}$$

Plugging (B.37) and (B.32) into (B.35) leads to

$$\lambda_1 = -\frac{\beta}{w}\lambda_2 - \frac{\lambda_3}{w} = -\frac{qb_1\beta\theta_b b_2}{wk} + \frac{\theta_b b_2}{w} = \theta_b b_2 \frac{\kappa - q\beta b_1}{w\kappa} \ . \tag{B.38}$$

Using (B.34), (B.37), (B.38) and (B.33), we obtain

$$\begin{aligned}
\theta_\pi \pi_2 &= \gamma\lambda_1 - \lambda_2 - \lambda_3 b_1 \\
\Leftrightarrow \theta_\pi \pi_2 &= -\frac{qb_1}{\kappa}\theta_b b_2 + \theta_b b_2 b_1 + \frac{\gamma}{w\kappa}\theta_b b_2(\kappa - qb_1\beta) \\
\Leftrightarrow \theta_\pi \pi_2 &= \theta_b b_2 \left(b_1 - \frac{qb_1}{\kappa} + \frac{\gamma(\kappa - qb_1\beta)}{w\kappa} \right) \\
\Leftrightarrow \pi_2 &= \frac{\theta_b b_2}{\theta_\pi} \frac{\gamma(\kappa - q\beta b_1) - b_1 w(q - \kappa)}{w\kappa} \ .
\end{aligned} \tag{B.39}$$

The first-order condition with respect to λ_3 solved for g_2 equals

$$g_2 = b_2 - b_1(1 + qe_2 - \pi_2) = b_2 - b_1 q e_2 - b_1 + b_1\pi_2 \ . \tag{B.40}$$

The first-order condition with respect to λ_2 solved for e_2 yields

$$\begin{aligned}
\pi_2 &= \phi_2 + \kappa e_2 + \beta g_2 \\
\Leftrightarrow e_2 &= \frac{1}{\kappa}(\pi_2 - \phi_2 - \beta g_2).
\end{aligned} \tag{B.41}$$

Combining (B.40) and (B.41) leads to

$$g_2 = \frac{b_2\kappa}{\kappa - b_1 q\beta} + \frac{b_1(k - q)}{\kappa - b_1 q\beta}\pi_2 + \frac{b_1 q}{\kappa - b_1 q\beta}\phi_2 - \frac{b_1\kappa}{\kappa - b_1 q\beta} \ . \tag{B.42}$$

Using (B.32) and (B.38) yields

$$y_2 = k - \theta_b b_2 \frac{\kappa - q\beta b_1}{w\kappa} \ . \tag{B.43}$$

Inserting (B.41) into the output equation (first-order condition with respect to λ_1), we obtain

$$\begin{aligned}
y_2 &= \gamma(\pi_2 - \pi_2^e) + wg_2 = \gamma\pi_2 + wg_2 - \gamma\pi_2^e \\
&= \gamma\pi_2 + b_2\frac{w\kappa}{\kappa - q\beta b_1} + \pi_2\frac{wb_1(\kappa - q)}{\kappa - q\beta b_1} + \frac{qwb_1\phi_2}{\kappa - q\beta b_1} - \frac{wb_1\kappa}{\kappa - q\beta b_1} - \gamma\pi_2^e \\
&= \pi_2\frac{\gamma(\kappa - b_1 q\beta) + wb_1(\kappa - q)}{\kappa - b_1 q\beta} + \frac{w}{\kappa - b_1 q\beta}(\kappa b_2 + qb_1\phi_2 - b_1\kappa) - \gamma\pi_2^e \ .
\end{aligned} \tag{B.44}$$

Using (B.43) with (B.44) yields

$$\begin{aligned}
&\pi_2\frac{\gamma(\kappa - b_1 q\beta) + wb_1(\kappa - q) + w(\kappa b_2 + qb_1\phi_2 - b_1\kappa)}{\kappa - b_1 q\beta} - \gamma\pi_2^e = k - \theta_b b_2\frac{\kappa - q\beta b_1}{w\kappa} \\
&\pi_2 = \frac{(k + \gamma\pi_2^e)(\kappa - b_1 q\beta) - wb_1(q\phi_2 - \kappa)}{\gamma(\kappa - b_1 q\beta) + wb_1(\kappa - q)} - \frac{\theta_b(\kappa - b_1 q\beta)^2 + (w\kappa)^2}{w\kappa(\gamma(\kappa - b_1 q\beta) + wb_1(\kappa - q)}b_2 \ .
\end{aligned} \tag{B.45}$$

By combining (B.39) and (B.45), we obtain the optimal second period debt level b_2 as a function of inflation expectations of the private sector π_2^e by[2]

$$b_2^f = \frac{\theta_\pi w\kappa(\kappa - b_1 q\beta)(k + \gamma\pi_2^e) + \theta_\pi w^2 b_1 \kappa(\kappa - q\phi_2)}{\theta_b(\gamma(\kappa - b_1 q\beta) + b_1(\kappa - q))^2 + \theta_\pi\theta_b(\kappa - b_1 q\beta)^2 + \theta_\pi(w\kappa)^2} \ . \tag{B.46}$$

We define $E := \kappa - q\beta b_1$ and $F := wb_1(\kappa - q)$. E and F may both have either sign, mainly depending on the size of foreign debts q and the openness parameter κ. From an empirical perspective, however, a prerequisite for a stable exchange rate is a highly open economy, which guarantees the flexibility necessary to offset unfavorable shocks (this means κ is close to one); at the same time, a credibly stable exchange rate is characterized by the fact that creditors trust more in domestic debts, which contributes to a small q. If this holds for the considered economy, E and F are positive. Using the abbreviations, we can rewrite (B.46) as

$$b_2^f = \frac{E(k + \gamma\pi_2^e)\theta_\pi w\kappa + \theta_\pi w^2 b_1 \kappa(\kappa - q\phi_2)}{\theta_b(\gamma E + F)^2 + \theta_\pi\theta_b E^2 + \theta_\pi(w\kappa)^2} \ . \tag{B.47}$$

The optimal value for inflation is calculated from (B.39):

$$\begin{aligned}
\pi_2^f &= b_2^f \frac{\theta_b}{\theta_\pi} \frac{\gamma(\kappa - b_1 q\beta) + wb_1(\kappa - q)}{w\kappa} = b_2^f \frac{\theta_b}{\theta_\pi} \frac{\gamma E + F}{w\kappa} \\
&= \frac{\theta_b}{\theta_\pi} \cdot \frac{\gamma E + F}{w\kappa} \cdot \frac{E(k + \gamma\pi_2^e)\theta_\pi w\kappa + \theta_\pi w^2 b_1 \kappa(\kappa - q\phi_2)}{\theta_b(\gamma E + F)^2 + \theta_\pi\theta_b E^2 + \theta_\pi(w\kappa)^2} \\
&= \frac{\theta_b(\gamma E + F)\left[E(k + \gamma\pi_2^e) + wb_1(\kappa - q\phi_2)\right]}{\theta_b(\gamma E + F)^2 + \theta_\pi\theta_b E^2 + \theta_\pi(w\kappa)^2} \ ;
\end{aligned} \tag{B.48}$$

and optimal output is derived from (B.43) as

$$\begin{aligned}
y_2^f &= k - \theta_b b_2^f \frac{E}{w\kappa} \\
&= k - \frac{\theta_b E}{w\kappa} \cdot \frac{E(k + \gamma\pi_2^e)\theta_\pi w\kappa + \theta_\pi w^2 b_1 \kappa(\kappa - q\phi_2)}{\theta_b(\gamma E + F)^2 + \theta_\pi\theta_b E^2 + \theta_\pi(w\kappa)^2} \\
&= k - \frac{\theta_b E\left[(k + \gamma\pi_2^e)\theta_\pi E + \theta_\pi w b_1(\kappa - q\phi_2)\right]}{\theta_b(\gamma E + F)^2 + \theta_\pi\theta_b E^2 + \theta_\pi(w\kappa)^2} \ .
\end{aligned} \tag{B.49}$$

Combining the k-terms yields

$$k\left[\theta_b(\gamma E + F)^2 + \theta_\pi\theta_b E^2 + \theta_\pi(w\kappa)^2 - \theta_b\theta_\pi E^2\right] = k(\theta_b(\gamma E + F)^2 + \theta_\pi(w\kappa)^2) \ . \tag{B.50}$$

Inserting in (B.49) leads to

$$y_2^f = \frac{k(\theta_b(\gamma E + F)^2 + \theta_\pi(w\kappa)^2) - \theta_b\theta_\pi E^2\gamma\pi_2^e - \theta_b\theta_\pi E w b_1(\kappa - q\phi_2)}{\theta_b(\gamma E + F)^2 + \theta_\pi\theta_b E^2 + \theta_\pi(w\kappa)^2} \ . \tag{B.51}$$

[2]We denote the equilibrium values in the free float case by a superscript f, henceforth.

B.2.2 Reformulation of the Loss Function as a Function of π_2

To simplify the calculations in section 4.6 and appendix B.5, we rewrite the loss function of a flexible exchange rate system. As shocks are perfectly offset in this setting, output and debts can be expressed as a multiple of π_2^f. Therefore, the loss function has the simple form

$$\begin{aligned} 2\cdot L_2 &= (y_2-k)^2+\theta_\pi(\pi_2^f)^2+\theta_b b_2^2 \\ &= \left(-\frac{E\theta_\pi}{\gamma E+F}\pi_2^f\right)^2+\theta_\pi(\pi_2^f)^2+\left(\theta_b\theta\pi\frac{w\kappa}{\gamma E+F}\pi_2^f\right)^2 \\ &= \left(-\frac{(\kappa-q\beta b_1)\theta_\pi}{\gamma(\kappa-q\beta b_1)+wb_1(\kappa-q)}\pi_2\right)^2+\theta_\pi(\pi_2^f)^2 \\ &\quad+\left(\theta_b\theta_\pi\frac{w\kappa}{\gamma(\kappa-q\beta b_1)+wb_1(\kappa-q)}\pi_2^f\right)^2 \\ &= (\pi_2^f)^2\left(\theta_\pi+\theta_\pi^2\left[\frac{(\kappa-q\beta b_1)^2+(\theta_b w\kappa)^2}{(\gamma(\kappa-q\beta b_1)+wb_1(\kappa-q))^2}\right]\right)\ . \end{aligned} \tag{B.52}$$

B.2.3 Calculation of the Rational Expectations Equilibrium

Assuming rational expectations of inflation, we have

$$\pi_2^{e,f}=E(\pi_2^f)=\frac{\theta_b(\gamma E+F)\left[E(k+\gamma\pi_2^{e,f})+wb_1(\kappa-q\eta\phi_1)\right]}{\theta_b(\gamma E+F)^2+\theta_\pi\theta_b E^2+\theta_\pi(w\kappa)^2} \tag{B.53}$$

$$\begin{aligned} \Leftrightarrow\ &\pi_2^{e,f}\left(1-\frac{\theta_b(\gamma E+F)\gamma E}{\theta_b(\gamma E+F)^2+\theta_\pi\theta_b E^2+\theta_\pi(w\kappa)^2}\right) \\ =\ &\frac{\theta_b(\gamma E+F)\left[Ek+wb_1(\kappa-q\eta\phi_1)\right]}{\theta_b(\gamma E+F)^2+\theta_\pi\theta_b E^2+\theta_\pi(w\kappa)^2} \end{aligned} \tag{B.54}$$

$$\begin{aligned} \Leftrightarrow \pi_2^{e,f} &= \frac{\theta_b(\gamma E+F)\left[Ek+wb_1(\kappa-q\eta\phi_1)\right]}{\theta_b(\gamma E+F)^2+\theta_\pi\theta_b E^2+\theta_\pi(w\kappa)^2-\theta_b(\gamma E-F)E\gamma} \\ \Leftrightarrow \pi_2^{e,f} &= \frac{\theta_b(\gamma E+F)\left[Ek+wb_1(\kappa-q\eta\phi_1)\right]}{\theta_b F(F+\gamma E)+\theta_\pi(w\kappa)^2+\theta_\pi\theta_b E^2}\ . \end{aligned} \tag{B.55}$$

To check the correctness of the result, we plug (B.55) into (B.48):

$$\begin{aligned} \pi_2^{**f} &= \frac{\theta_b(\gamma E+F)\left[Ek+wb_1(\kappa-q\phi_2)\right]}{\theta_b(\gamma E+F)^2+\theta_\pi\theta_b E^2+\theta_\pi(w\kappa)^2}+\frac{\theta_b(\gamma E+F)\gamma E}{\theta_b(\gamma E+F)^2+\theta_\pi\theta_b E^2+\theta_\pi(w\kappa)^2} \\ &\quad\cdot\frac{\theta_b(\gamma E+F)\left[Ek+wb_1(\kappa-q\eta\phi_1)\right]}{\theta_b(\gamma E+F)^2+\theta_\pi\theta_b E^2+\theta_\pi(w\kappa)^2-\theta_b(\gamma E+F)E\gamma} \\ &= \frac{\theta_b(\gamma E+F)\left[Ek+wb_1(\kappa-q\eta\phi_1)\right]}{\theta_b F(F+\gamma E)+\theta_\pi(w\kappa)^2+\theta_\pi\theta_b E^2} \\ &\quad-\frac{\theta_b(\gamma E+F)wb_1 q u_2}{\theta_b(\gamma E+F)^2+\theta_\pi\theta_b E^2+\theta_\pi(w\kappa)^2}\ . \end{aligned} \tag{B.56}$$

B.3 Optimization of the Policy Maker, while Treating e_2 as a Parameter

This part of the appendix refers to section 4.5.2. We compare the privates' expectations of inflation under a currency board and a peg system. The optimization problem is similar to that in appendix B.2, but the change of the exchange rate e_2 is here treated like an additional parameter. The aim of this section is to show, how the equilibrium values and the inflation expectations change when e_2 is varied.

We have the following Lagrangean:

$$\begin{aligned}\mathcal{L}_2 \;=\;& \frac{1}{2}(y_2-k)^2+\frac{1}{2}\theta_\pi\pi_2^2+\frac{1}{2}\theta_b b_2^2\\ &+\lambda_1(y_2-\gamma(\pi_2-\pi_2^e)-wg_2)\\ &+\lambda_2(\pi_2-\phi_2-\kappa e_2-\beta g_2)\\ &+\lambda_3(b_2-b_1(1+qe_2-\pi_2)-g_2)\ .\end{aligned} \tag{B.57}$$

The first-order conditions are given by

$$\frac{\partial\mathcal{L}_2}{\partial y_2} = (y_2-k)+\lambda_1 \Leftrightarrow \lambda_1=k-y_2=0 \tag{B.58}$$

$$\frac{\partial\mathcal{L}_2}{\partial \pi_2} = \theta_\pi\pi_2-\gamma\lambda_1+\lambda_2+\lambda_3 b_1=0 \tag{B.59}$$

$$\frac{\partial\mathcal{L}_2}{\partial b_2} = \theta_b b_2+\lambda_3=0 \Leftrightarrow \lambda_3=-\theta_b b_2 \tag{B.60}$$

$$\frac{\partial\mathcal{L}_2}{\partial g_2} = -w\lambda_1-\beta\lambda_2-\lambda_3=0\ . \tag{B.61}$$

Solving equation (B.61) for λ_2 yields

$$\lambda_2 = -\frac{w}{\beta}\lambda_1-\frac{\lambda_3}{\beta}\ , \tag{B.62}$$

and by inserting (B.58) and (B.60) into (B.62) we have

$$\lambda_2 = \frac{w}{\beta}(y_2-k)+\frac{\theta_b b_2}{\beta}\ . \tag{B.63}$$

Plugging (B.58), (B.60) and (B.63) into (B.59) leads to

$$\begin{aligned}\theta_\pi\pi_2 &= \gamma\lambda_1-\lambda_2-b_1\lambda_3\\ &= \gamma(k-y_2)+\frac{w}{\beta}(k-y_2)-\frac{\theta_b b_2}{\beta}\theta_b b_1 b_2\\ &= (k-y_2)\frac{w+\gamma\beta}{\beta}-\theta_b b_2\frac{1-\beta b_1}{\beta}\\ &= (k-y_2)\frac{A}{\beta}-\theta_b b_2\frac{B}{\beta}\ ,\end{aligned} \tag{B.64}$$

where $A := (\gamma\beta + w) > 0$ and $B = 1 - \beta b_1 > 0$. Solving restriction 2 (=inflation equation) for g_2 yields

$$g_2 = \frac{1}{\beta}(\pi_2 - \phi_2 - \kappa e_2) \ . \tag{B.65}$$

Inserting this into restriction 1 (=output equation), we have

$$\begin{aligned} y_2 &= \gamma(\pi_2 - \pi_2^e) + w\frac{\pi_2 - \phi_2 - \kappa e_2}{\beta} \\ &= \frac{A}{\beta}\pi_2 - \gamma\pi_2^e - \frac{w}{\beta}\phi_2 - \frac{w\kappa}{\beta}e_2 \ . \end{aligned} \tag{B.66}$$

Combining (B.66) and (B.64) results in

$$\begin{aligned} \theta_\pi \pi_2 &= \frac{A}{\beta}\left(k - \frac{A}{\beta}\pi_2 + \gamma\pi_2^e + \frac{w}{\beta}\phi_2 + \frac{w\kappa}{\beta}e_2\right) - \theta_b b_2 \frac{B}{\beta} \\ \Leftrightarrow \pi_2 \frac{\theta_\pi \beta^2 + A^2}{\beta^2} &= \frac{A}{\beta}k + \frac{A\gamma}{\beta}\pi_2^e + \frac{Aw}{\beta^2}\phi_2 + \frac{w\kappa A}{\beta^2}e_2 - \theta_b b_2 \frac{B}{\beta} \\ \Leftrightarrow \pi_2 &= \frac{A\left[\beta k + \beta\gamma\pi_2^e + w\phi_2 + w\kappa e_2\right]}{\theta_\pi\beta^2 + A^2} - \theta_b b_2 \frac{\beta B}{\theta_\pi \beta^2 + A^2} \ . \end{aligned} \tag{B.67}$$

Solving restriction 3 (=debt equation) for g_2 yields

$$g_2 = b_2 - b_1(1 + qe_2 - \pi_2) \ . \tag{B.68}$$

Inserting (B.68) into restriction 2 leads to

$$\begin{aligned} \pi_2 &= \phi_2 + \kappa e_2 + \beta b_2 - \beta b_1 - \beta b_1 q e_2 + \beta b_1 \pi_2 \\ \Leftrightarrow \pi_2 &= \frac{\phi_2 - \beta b_1}{B} + e_2 \frac{\kappa - q\beta b_1}{B} + \frac{\beta}{B}b_2 \ . \end{aligned} \tag{B.69}$$

Combining (B.67) and (B.69) results in

$$\frac{A(\beta k + \beta\gamma\pi_2^e + w\phi_2 + w\kappa e_2)}{\theta_\pi\beta^2 + A^2} + \frac{\beta b_1 - \phi_2 - e_2(\kappa - q\beta b_1)}{B} = \frac{\beta}{B}b_2 + \frac{\theta_b \beta B}{\theta_\pi \beta^2 + A^2}b_2 \ . \tag{B.70}$$

Solving for b_2 yields

$$\begin{aligned} b_2 &= \frac{AB(k + \gamma\pi_2^e) - \phi_2(A(wb_1 + \gamma) + \theta_\pi\beta) + b_1(\theta_\pi\beta^2 + A^2)}{\theta_\pi\beta^2 + A^2 + \theta_b B^2} \\ &\quad + \frac{e_2(-\kappa A(wb_1 + \gamma) - \theta_\pi\beta(\kappa - q\beta b_1) + qb_1 A^2)}{\theta_\pi\beta^2 + A^2 + \theta_b B^2} \ . \end{aligned} \tag{B.71}$$

We obtain b_2 by using the above equation in B.69

$$\begin{aligned}
\pi_2 &= \frac{\phi_2 - \beta b_1 + e_2(\kappa - q\beta b_1)}{B} + \frac{\beta}{B} b_2 \\
&= \frac{[\phi_2 - \beta b_1 + e_2(\kappa - q\beta b_1)](\theta_\pi \beta^2 + A^2 + \theta_b B^2) + \beta AB(k + \gamma \pi_2^e)}{B(\theta_\pi \beta^2 + A^2 + \theta_b B^2)} \\
&\quad \frac{-\phi_2 \beta (A(wb_1 + \gamma) + \theta_\pi \beta) + e_2(-\kappa A\beta(wb_1 + \gamma) - \theta_\pi \beta^2(\kappa - q\beta b_1) + q\beta b_1 A^2)}{B(\theta_\pi \beta^2 + A^2 + \theta_b B^2)} \\
&= \frac{\beta AB(k + \gamma \pi_2^e) + \phi_2 B(\theta_b B + wA) - b_1 \beta \theta_b B^2 + e_2(w\kappa AB + \theta_b B^2(\kappa - \beta b_1 q))}{B(\theta_\pi \beta^2 + A^2 + \theta_b B^2)} \\
&= \frac{\beta A(k + \gamma \pi_2^e) + \phi_2(\theta_b B + wA) - b_1 \beta \theta_b B + e_2(w\kappa A + \theta_b B(\kappa - \beta b_1 q))}{\theta_\pi \beta^2 + A^2 + \theta_b B^2} \, . \qquad \text{(B.72)}
\end{aligned}$$

We can determine the expected inflation π_2^e (with rational expectations):

$$\begin{aligned}
\pi_2^e &= E(\pi_2) = \frac{\beta A(k + \gamma \pi_2^e) + \eta \phi_1(\theta_b B + wA) - b_1 \beta \theta_b B + e_2(w\kappa A + \theta_b B(\kappa - \beta b_1 q))}{\theta_\pi \beta^2 + A^2 + \theta_b B^2} \\
\Leftrightarrow &\ \pi_2^e \left(\frac{\theta_\pi \beta^2 + A^2 + \theta_b B^2 - \gamma \beta A}{\theta_\pi \beta^2 + A^2 + \theta_b B^2} \right) = \dots \\
&\dots \frac{\beta A k + \eta \phi_1(\theta_b B + wA) - b_1 \beta \theta_b B + e_2(w\kappa A + \theta_b B(\kappa - \beta b_1 q))}{\theta_\pi \beta^2 + A^2 + \theta_b B^2} \\
\Leftrightarrow &\ \pi_2^e = \frac{\beta A k + \eta \phi_1(\theta_b B + wA) - b_1 \beta \theta_b B + e_2(w\kappa A + \theta_b B(\kappa - \beta b_1 q))}{\theta_\pi \beta^2 + wA + \theta_b B^2} \, . \qquad \text{(B.73)}
\end{aligned}$$

<u>Assumption:</u> We neglect the possibility of a revaluation (cf Obstfeld, 1996).

<u>Solution:</u> If foreign debts are not to high, represented by a relatively low q (with q generally is part of the interval $\in (0, 1)$) and the economy is not very closed (represented by κ approaching one), a devaluation causes an increasing expectation of inflation under a float.[3]

<u>Conclusion:</u> As a standard peg system can surprisingly be abandoned after the realization of the shock (escape clause) for an optimal realignment (which corresponds to a free float in the two-period setting), the inflation expectation should be a mixture of the inflation expectations of both an ultimate fixed exchange-rate regime and a free float. Therefore, under the above derived cases, a currency board (ultimate fix) guarantees lower inflation expectations than a standard peg.

Note that in cases where the probability of a devaluation under a peg converges to zero, the inflation expectations in both systems become equal and $\pi_2^{e,CB} = \pi_2^{e,peg}$ holds.

[3] See also the first derivative of equation B.73, which is depicted in chapter 4.

B.4 Determination of the Threshold for Maintaining or Leaving the Currency Board

The rational expectation equilibrium values for y_2, π_2, and b_2 were derived in appendix part B.1 (currency board) and part B.2 (free float) and are used in the following.

The characteristics of a currency board that its abandonment must be announced in advance leads to the following maintaining condition:

$$E(L_2^f) + c^{CB} \geq E(L_2^{CB}) \ . \tag{B.74}$$

Note that the exit costs c^{CB} are given exogenously. The condition means that the policy maker compares the expected loss which would occur if she decides to maintain the currency board with the situation in which she decides to abolish the currency board and chooses the optimal exchange rate freely and optimally. The loss in the latter case, of course, corresponds to the expected loss under a free float. The comparison is done in the following.

We again make use of the following abbreviations, which were introduced in the appendices A and B:

$$A = w + \gamma\beta > 0; \quad B = 1 - \beta b_1 > 0; \quad F = wb_1(\kappa - q); \quad E = \kappa - q\beta b_1 \ .$$

Note that when referring to the domains of the particular parameters E and F the appearance of negative values cannot be excluded per se.

B.4.1 Determination of the Expected Loss under a Currency Board

Using equation (B.26), we calculate the loss under a currency board when inserting the rational expectations equilibrium values:

$$\begin{aligned} L_2^{cb}(\pi_2) &= \left(\pi_2 \frac{w + \gamma\beta}{\beta} - \frac{w}{\beta}\phi_2 - \gamma\pi_2^e - k\right)^2 + \theta_\pi \pi_2^2 \\ &\quad + \theta_b \left(\frac{1 - \beta b_1}{\beta}\pi_2 - \frac{\phi_2}{\beta} + b_1\right)^2 \ . \end{aligned} \tag{B.75}$$

We define $D := \theta_b B^2 + A^2 + \theta_\pi \beta^2$ and $\tilde{D} := \theta_b B^2 + wA + \theta_\pi \beta^2$. Both D and $\tilde{D}$ are obviously strictly positive. We use these abbreviations in the following and refer to appendix B.2, where

$$\gamma(\pi_2 - \pi_2^e) = u_2 \frac{\theta_b B + wA}{D} \ , \tag{B.76}$$

and

$$\begin{aligned}
\frac{w}{\beta}(\pi_2 - \phi_2) &= \frac{w}{\beta}(\pi_2 - \eta\phi_1 - u_2) \\
&= \frac{w}{\beta}\frac{\beta Ak + \eta\phi_1(\theta_b B + wA - \theta_b B^2 - wA - \theta_\pi\beta^2) - \theta_b\beta b_1 B}{\tilde{D}} \\
&\quad + u_2\frac{w}{\beta}\frac{\theta_b B + wA - \theta_b B^2 - A^2 - \theta_\pi\beta^2}{D} \\
&= w\frac{Ak + \eta\phi_1(\theta_b b_1 B - \theta_\pi\beta) - \theta_b b_1 B}{\tilde{D}} \\
&\quad + wu_2\frac{\theta_b b_1 B - \gamma A - \theta_\pi\beta}{D} \ . \qquad \text{(B.77)}
\end{aligned}$$

Combining both parts and adding "$-k$", we can rewrite the output goal in the loss function as

$$\begin{aligned}
(y_2 - k)^2 &= \left(w\frac{Ak + \eta\phi_1(\theta_b b_1 B - \theta_\pi\beta) - \theta_b b_1 B}{\tilde{D}} - k \right. \\
&\quad \left. + u_2\frac{wB\theta_b b_1 - \gamma wA - w\theta_\pi\beta + \theta_b B + wA}{D} \right)^2 \\
&= \left(k \underbrace{\left[\frac{wA - \tilde{D}}{\tilde{D}}\right]}_{=:Q_1} + \eta\phi_1 \underbrace{\left[\frac{\theta_b b_1 B - \theta_\pi\beta}{\tilde{D}}\right]}_{=:Q_2} \right. \\
&\quad \left. + u_2 \underbrace{\left[\frac{\theta_b B(wb_1 + 1) + wA(1-\gamma) - w\theta_\pi\beta}{D}\right]}_{=:Q_3} + \underbrace{\left[-\frac{\theta_b b_1 B}{\tilde{D}}\right]}_{=:Q_4} \right)^2 \ . \qquad \text{(B.78)}
\end{aligned}$$

The inflation goal $\theta_\pi\pi_2^2$ can be rewritten by using equation (B.16) as

$$\theta_\pi\pi_2^2 = \theta_\pi \left(k \underbrace{\left[\frac{\beta A}{\tilde{D}}\right]}_{=:Q_5} + \eta\phi_1 \underbrace{\left[\frac{\theta_b B + wA}{\tilde{D}}\right]}_{=:Q_6} + u_2 \underbrace{\left[\frac{\theta_b B + wA}{D}\right]}_{=:Q_7} + \underbrace{\left[\frac{-\theta_b b_1\beta B}{\tilde{D}}\right]}_{=:Q_8} \right)^2 \ . \qquad \text{(B.79)}$$

The third term of the loss function $\theta_b b^2$ can be transformed into

$$\begin{aligned}
\theta_b b_2^2 &= \theta_b \left[\frac{1}{\beta}(B\pi_2 - \eta\phi_1 - u_2) + b_1\right]^2 \\
&= \theta_b \left(\frac{\beta ABk + \eta\phi_1(\theta_b B^2 + wAB - \theta_b B^2 - wA - \theta_\pi\beta^2) - \theta_b\beta b_1 B^2}{\beta\tilde{D}} \right. \\
&\quad \left. + u_2\frac{\theta_b B^2 + wAB - \theta_b B^2 - A^2 - \theta_\pi\beta^2}{\beta D} + b_1 \right)^2
\end{aligned}$$

$$
\begin{aligned}
&= \theta_b \left(\frac{ABk + \eta\phi_1(wb_1A - \theta_\pi\beta) - \theta_b b_1 B^2}{\tilde{D}} + u_2 \frac{-A(wb_1+\gamma) - \theta_\pi\beta^2}{D} + b_1 \right)^2 \\
&= \theta_b \Bigg(k \underbrace{\left[\frac{AB}{\tilde{D}}\right]}_{=:Q_9} + \eta\phi_1 \underbrace{\left[\frac{wAb_1 - \theta_\pi\beta}{\tilde{D}}\right]}_{=:Q_{10}} + u_2 \underbrace{\left[\frac{-A(wb_1+\gamma) - \theta_\pi\beta}{D}\right]}_{=:Q_{11}} \\
&\quad + \underbrace{\left[b_1 - \frac{\theta_b b_1 B^2}{\tilde{D}}\right]}_{=:Q_{12}} \Bigg)^2 \ . \qquad \text{(B.80)}
\end{aligned}
$$

By using the abbreviations $Q_1, \ldots, Q_{12}$ in the loss function, we obtain

$$
\begin{aligned}
L_2^{cboard} &= k^2Q_1^2 + (\eta\phi)^2Q_2^2 + u_2^2Q_3^2 + Q_4^2 + 2k\eta\phi_1Q_1Q_2 + 2ku_2Q_1Q_3 + 2kQ_1Q_4 \\
&+2\eta\phi_1u_2Q_2Q_3 + 2\eta\phi_1Q_2Q_4 + 2u_2Q_3Q_4 \\
&+\theta_\pi(k^2Q_5^2 + (\eta\phi_1)^2Q_6^2 + u_2^2Q_7^2 + Q_8^2 + 2k\eta\phi_1Q_5Q_6 + 2ku_2Q_5Q_7 + 2kQ_5Q_8 \\
&+2\eta\phi_1u_2Q_6Q_7 + 2\eta\phi_1Q_6Q_8 + 2u_2Q_7Q_8) \\
&+\theta_b(k^2Q_9^2 + (\eta\phi_1)^2Q_{10}^2 + u_2^2Q_{11}^2 + Q_{12}^2 + 2k\eta\phi_1Q_9Q_{10} + 2ku_2Q_9Q_{11} + 2kQ_9Q_{12} \\
&+2\eta\phi_1u_2Q_{10}Q_{11} + 2\eta\phi_1Q_{10}Q_{12} + 2u_2Q_{11}Q_{12}) \ . \qquad \text{(B.81)}
\end{aligned}
$$

The expected value of L_2, by noticing that $E(u_2) = 0$ and $E(u_2^2) = \sigma_u^2$, equals

$$
\begin{aligned}
E(L_2^{cboard}) &= k^2Q_1^2 + (\eta\phi)^2Q_2^2 + \sigma_u^2Q_3^2 + Q_4^2 \\
&+2k\eta\phi_1Q_1Q_2 + 2kQ_1Q_4 + 2\eta\phi_1Q_2Q_4 \\
&+\theta_\pi(k^2Q_5^2 + (\eta\phi_1)^2Q_6^2 + \sigma_u^2Q_7^2 + Q_8^2 \\
&+2k\eta\phi_1Q_5Q_6 + 2kQ_5Q_8 + 2\eta\phi_1Q_6Q_8) \\
&+\theta_b(k^2Q_9^2 + (\eta\phi_1)^2Q_{10}^2 + \sigma_u^2Q_{11}^2 + Q_{12}^2 \\
&+2k\eta\phi_1Q_9Q_{10} + 2kQ_9Q_{12} + 2\eta\phi_1Q_{10}Q_{12}) \ . \qquad \text{(B.82)}
\end{aligned}
$$

B.4.2 Determination of the Expected Loss under a Free Float

Subject of this subsection is to determine the expected loss under a flexible exchange rate system. The second period loss was calculated in appendix B.2 and is given by equation (B.52):

$$
L_2 = (\pi_2^f)^2 \underbrace{\left(\theta_\pi + \theta_\pi^2 \left[\frac{(\kappa - q\beta b_1)^2 + (\theta_b w\kappa)^2}{(\gamma(\kappa - q\beta b_1) + wb_1(\kappa - q))^2} \right] \right)}_{=:R_0} = (\pi_2^f)^2 R_0 \ .
$$

Furthermore, we use the two definitions $T := \theta_b F(\gamma E + F) + \theta_\pi (w\kappa)^2 + \theta_\pi \theta_b E^2$ and $\tilde{T} := \theta_b(\gamma E + F)^2 + \theta_\pi (w\kappa)^2 + \theta_\pi \theta_b E^2$. Of course, $\tilde{T}$ is strictly positive. T is in most of all cases also strictly positive as the signs of E and F are closely linked by the parameter values of κ and q and have thus in most cases the same sign as discussed before.

Then, the rational expectations equilibrium of inflation, π_2^{**f} can be written as

$$
\begin{aligned}
(\pi_2^{**f})^2 &= \Bigg(k \underbrace{\left[\frac{\theta_b E(\gamma E + F)}{T}\right]}_{=:R_1} + \eta\phi_1 \underbrace{\left[\frac{-qwb_1}{T}\right]}_{=:R_2} \\
&\quad + u_2 \underbrace{\left[\frac{-\theta_b(\gamma E + F)wb_1 q}{\tilde{T}}\right]}_{=:R_3} + \underbrace{\left[\frac{wb_1\kappa}{T}\right]}_{=:R_4} \Bigg)^2 .
\end{aligned} \tag{B.83}
$$

The second period loss equals

$$
\begin{aligned}
L_2 &= R_0(kR_1 + \eta\phi_1 R_2 + u_2 R_3 + R_4)^2 \\
&= R_0(k^2 R_1^2 + (\eta\phi_1)^2 R_2^2 + u_2^2 R_3^2 + R_4^2 + 2k\eta\phi_1 R_1 R_2 + 2ku_2 R_1 R_3 \\
&\quad + 2kR_1 R_4 + 2\eta\phi_1 u_2 R_2 R_3 + 2\eta\phi_1 R_2 R_4 + 2u_2 R_3 R_4) .
\end{aligned} \tag{B.84}
$$

The expected value of L_2 is given by

$$
\begin{aligned}
E(L_2^{float}) &= R_0(k^2 R_1^2 + (\eta\phi_1)^2 R_2^2 + \sigma_u^2 R_3^2 + R_4^2 \\
&\quad + 2k\eta\phi_1 R_1 R_2 + 2kR_1 R_4 + 2\eta\phi_1 R_2 R_4) .
\end{aligned} \tag{B.85}
$$

B.4.3 Comparison of the Expected Losses

The difference of the expected losses under a currency board and a float is given by

$$
\begin{aligned}
D_2 &= E(L_2^{cboard}) - E(L_2^{float}) - c^{CB} \\
&= k^2 Q_1^2 + (\eta\phi)^2 Q_2^2 + \sigma_u^2 Q_3^2 + Q_4^2 + 2k\eta\phi_1 Q_1 Q_2 + 2kQ_1 Q_4 + 2\eta\phi_1 Q_2 Q_4 \\
&\quad + \theta_\pi(k^2 Q_5^2 + (\eta\phi_1)^2 Q_6^2 + \sigma_u^2 Q_7^2 + Q_8^2 + 2k\eta\phi_1 Q_5 Q_6 + 2kQ_5 Q_8 + 2\eta\phi_1 Q_6 Q_8) \\
&\quad + \theta_b(k^2 Q_9^2 + (\eta\phi_1)^2 Q_{10}^2 + \sigma_u^2 Q_{11}^2 + Q_{12}^2 + 2k\eta\phi_1 Q_9 Q_{10} + 2kQ_9 Q_{12} + 2\eta\phi_1 Q_{10} Q_{12}) \\
&\quad - R_0(k^2 R_1^2 + (\eta\phi_1)^2 R_2^2 + \sigma_u^2 R_3^2 + R_4^2 + 2k\eta\phi_1 R_1 R_2 + 2kR_1 R_4 + 2\eta\phi_1 R_2 R_4) - c^{CB} .
\end{aligned} \tag{B.86}
$$

As it is hardly possible to find an explicit solution, we do comparative statics to compare both systems in section 4.6.1 by using two numerical scenarios. The solution of D_2 is used for the MATLAB-simulations to obtain the numerical results and *is interpreted as a measure for the stability of a currency board system* in section 4.6.1.

B.5 Comparison of a Currency Board and a Standard Peg

Before going through the technical details of the comparison of a currency board and a standard peg, we introduce briefly the assumptions made in this section:

- To compare the standard peg regime and the currency board arrangement, we use the probability of maintaining each system. We, henceforth, interpret the probability of maintaining an exchange rate system as the credibility of that system (a similar approach can be found in Drazen and Masson, 1994 or Feuerstein and Grimm, 2006b).

- We have to calculate the maintaining probabilities by comparing the expected second period losses of a currency board / standard peg and the free float case from the view of period 0 after a shock materialized in period 0. In scenario I and II of section 4.6.2, we assume that there is no shock in period 0. Furthermore, we assume that the standard fixed exchange rate is defended in the first period (this means that the peg has an implicit commitment value in the first period after its introduction), but can be abandoned surprisingly in period 2. A currency board can only be abandoned in period 2, if this was announced in period one, before the private sector's wage bargaining (inflation expectations) takes place and before the second period shock occurs. Therefore, the maintaining interval of the currency board depends on ϕ_1 and the maintaining interval of the peg depends on ϕ_2. The preparations to establish a fair comparison of both systems despite the different timing of decision-making is done in the parts 2 and 3 of appendix B.5.

- When analyzing the standard peg system, multiplicities of π_2^e may occur if the private sector has rational expectations of inflation (see discussion in section 4.5). Therefore, in our calculations we treat π_2^e as an exogenous parameter. When comparing the credibility of a currency board arrangement with the credibility of a standard peg, we assume that inflation expectations under a standard peg are a mixture of expectations under a currency board (full credibility) and a free float system (zero credibility).

B.5.1 Second-period Loss Functions (for a given π_2^e)

In a standard peg regime, the policy maker decides whether to defend or to leave the exchange-rate peg after the realization of the second period shock. Therefore, a policy maker's decision is based on a comparison of the loss functions (B.26) and (B.52) for a given $\pi_2^{e,Peg}$ after the realization of the shock ϕ_2.

We begin with reformulating the loss function (B.26) by using the inflation equation (B.16), where π_2^e is taken as given. The first component of the loss function equals

$$\begin{aligned}(y_2-k)^2 &= \frac{1}{(\beta D)^2}\big((\beta Ak+\beta\gamma A\pi_2^e+\phi_2(\theta_b B+wA)-\theta_b\beta Bb_1)A \\ &\quad -\phi_2 w(\theta_b B^2+A^2+\theta_\pi\beta^2)-\gamma\pi_2^e(\theta_b\beta B^2+\beta A^2+\theta_\pi\beta^3) \\ &\quad -k(\theta_b\beta B^2+\beta A^2+\theta_\pi\beta^3)\big)^2 \\ &= \frac{1}{(\beta D)^2}\big(\beta(k(\theta_b B^2-\theta_\pi\beta^2)+\phi_2(\theta_b B(\gamma+wb_1)-w\theta_\pi\beta) \\ &\quad +\pi_2^e(-\gamma\theta_b B^2-\gamma\theta_\pi\beta^2)+(-\theta_b b_1 BA))\big)^2 \\ &= \frac{1}{D^2}\Bigg(k\underbrace{(\theta_b B^2-\theta_\pi\beta^2)}_{=:P_1})+\phi_2\underbrace{(\theta_b B(\gamma+wb_1)-w\theta_\pi\beta)}_{=:P_2} \\ &\quad +\pi_2^e\underbrace{(-\gamma\theta_b B^2-\gamma\theta_\pi\beta^2)}_{=:P_3}+\underbrace{(-\theta_b b_1 BA)}_{=:P_4}\Bigg)^2 . \end{aligned} \tag{B.87}$$

The second component of the loss function is rewritten as

$$\theta_\pi\pi_2^2=\frac{\theta_b}{D^2}\Bigg(k\underbrace{(\beta A)}_{=:P_5}+\phi_2\underbrace{(\theta_b B+wA)}_{=:P_6}+\pi_2^e\underbrace{(\gamma\beta A)}_{=:P_7}+\underbrace{(-\theta_b\beta Bb_1)}_{=:P_8}\Bigg)^2 . \tag{B.88}$$

The third component $\theta_b b_2^2$ equals

$$\begin{aligned}\theta_b b_2^2 &= \theta_b\left(\frac{B}{\beta}\pi_2-\frac{\phi_2}{\beta}+b_1\right)^2 \\ &= \theta_b\left(k\left[\frac{AB}{D}\right]+\phi_2\left[\frac{wAB-A^2-\theta_\pi\beta^2}{\beta D}\right]+\pi_2^e\left[\frac{\gamma AB}{D}\right]+\left[-\frac{\theta_b B^2 b_1}{D}+b_1\right]\right)^2 \\ &= \frac{\theta_b}{D^2}\Bigg(k\underbrace{[AB]}_{=:P_9}+\phi_2\underbrace{[-\gamma A-\theta_\pi\beta]}_{=:P_{10}}+\pi_2^e\underbrace{[\gamma AB]}_{=:P_{11}}+\underbrace{[b_1(A^2+\theta_\pi\beta^2)]}_{=:P_{12}}\Bigg)^2 . \end{aligned} \tag{B.89}$$

Using the definitions P_1 to P_{12} in the loss L_2^{Peg} simplifies to

$$\begin{aligned}L_2^{Peg} &= \frac{1}{D^2}\big[(kP_1+\phi_2P_2+\pi_2^eP_3+P_4)^2+\theta_\pi(kP_5+\phi_2P_6+\pi_2^eP_7+P_8)^2 \\ &\quad +\theta_b(kP_9+\phi_2P_{10}+\pi_2^eP_{11}+P_{12})^2\big] . \end{aligned} \tag{B.90}$$

Now, we rewrite the loss function (B.52) of the free float case by inserting the inflation equation (B.48) while, again, treating π_2^e as given:

$$\begin{aligned} L_2^f &= R_0 \frac{\theta_b(\gamma E + F)}{\tilde{T}^2} \left(k \underbrace{[E]}_{=:S_1} + \pi_2^e \underbrace{[\gamma E]}_{=:S_3} + \phi_2 \underbrace{[-qwb_1]}_{=:S_2} + \underbrace{[\kappa w b_1]}_{=:S_4} \right)^2 \\ &= R_0 \frac{\theta_b(\gamma E + F)}{\tilde{T}^2} (kS_1 + \phi_2 S_2 + \pi_2^e S_3 + S_4)^2 \ , \end{aligned} \tag{B.91}$$

where R_0 and $\tilde{T}$ were defined in the second section of appendix B.4.

B.5.2 Credibility of a Standard Peg

To derive the range of the random variable ϕ_2 characterizing the second period shock for which the exchange-rate peg is defended, we have to solve the following inequality for ϕ_2

$$L_2^{Peg} \ \leq \ L_2^f + c \ . \tag{B.92}$$

Using the expressions for the loss functions, just derived in the first part of appendix B.5, we obtain the condition where the peg is defended as

$$\begin{aligned} \frac{R_0\theta_b(\gamma E + F)}{\tilde{T}^2} & \left[k^2 S_1^2 + \phi_2^2 S_2^2 + (\pi_2^e)^2 S_3^2 + S_4^2 + 2k\phi_2 S_1 S_2 + 2k\pi_2^e S_1 S_3 + 2k S_1 S_4 \right. \\ & \left. +2\phi_2 \pi_2^e S_2 S_3 + 2\phi_2 S_2 S_4 + \pi_2^e S_3 S_4\right] + c^{Peg} \\ -\frac{1}{D^2} & \left[k^2 P_1^2 + \phi_2^2 P_2^2 + (\pi_2^e)^2 P_3^2 + P_4^2 + 2k\phi_2 P_1 P_2 + 2k\pi_2^e P_1 P_3 + 2k P_1 P_4 \right. \\ & +2\phi_2 \pi_2^e P_2 P_3 + 2\phi_2 P_2 P_4 + \pi_2^e P_3 P_4 \\ & \theta_\pi(k^2 P_5^2 + \phi_2^2 P_6^2 + (\pi_2^e)^2 P_7^2 + P_8^2 + 2k\phi_2 P_5 P_6 + 2k\pi_2^e P_5 P_7 + 2k P_5 P_8 \\ & +2\phi_2 \pi_2^e P_6 P_7 + 2\phi_2 P_6 P_8 + \pi_2^e P_7 P_8) \\ & \theta_b(k^2 P_9^2 + \phi_2^2 P_{10}^2 + (\pi_2^e)^2 P_{11}^2 + P_{12}^2 + 2k\phi_2 P_9 P_{10} + 2k\pi_2^e P_9 P_{11} + 2k P_9 P_{12} \\ & \left. +2\phi_2 \pi_2^e P_{10} P_{11} + 2\phi_2 P_{10} P_{12} + \pi_2^e P_{11} P_{12})\right] \ \geq \ 0 \ . \end{aligned} \tag{B.93}$$

We focus on deriving mathematical expressions for the boundaries of the "defending-interval" in case of a standard-peg regime, which can be determined if (B.93) holds with equality. The boundaries itself are functions which depend on expected inflation π_2^e and on the parameters $k, q, b_1, \eta\phi_1, \kappa, \beta, c^f, \theta_b, \theta_\pi$. We denote the set of parameters by $\mathcal{S}$ for reasons of clarity. Therefore, solving (B.93) for ϕ_2, we get the interval in which the standard peg is defended as

$$[\phi^{l,Peg}(\pi_2^e; \mathcal{S}), \phi^{u,Peg}(\pi_2^e; \mathcal{S})] \ . \tag{B.94}$$

Note that we set the probability of maintaining the standard peg equal to zero, if by solving the quadratic equation, which results from a comparison of the loss function for ϕ_2, the discriminant becomes negative. If we assume that the new shock u_2 is uniformly distributed with zero mean and standard deviation $\sigma_u > 0$, we obtain the boundaries of the interval of possible realizations of the second period shock ϕ_2 by[4]

$$\underline{\phi}_2 = \eta\phi_1 - \sigma_u\sqrt{3} \tag{B.95}$$
$$\overline{\phi}_2 = \eta\phi_1 + \sigma_u\sqrt{3}\ . \tag{B.96}$$

For the comparison of the standard peg and the currency board system, we define the credibility of both systems as the probability of maintaining the first-period exchange-rate systems also in the second period (see also FEUERSTEIN and GRIMM, 2006b). The probability of defending the exchange rate under a standard peg in period 2 for a given realization of ϕ_1 equals

$$Prob(\text{maintain Peg}|\phi_1) = \max\left[\frac{\min(\overline{\phi}_2, \phi_2^{u,Peg}(\pi_2^e;\mathcal{S})) - \max(\underline{\phi}_2, \phi_2^{l,Peg}(\pi_2^e;\mathcal{S}))}{\overline{\phi}_2 - \underline{\phi}_2}, 0\right]\ . \tag{B.97}$$

To avoid an unfair comparison due to the different timing of decision-making in the standard peg and the currency board system caused by a particular choice of ϕ_1, we calculate the probability of maintaining the peg as an average over all possible realizations of ϕ_1, given by

$$Prob(\text{maintain Peg}) = \int_{\underline{\phi}_1}^{\overline{\phi}_1} Prob\,(\text{maintain Peg}|\phi_1)\, d\phi_1\ . \tag{B.98}$$

Due to the assumption of a uniform distribution, we can approximate the probability by

$$Prob(\text{maintain Peg}) \approx \frac{1}{N}\sum_{n=0}^{N} Prob\left(\text{maintain Peg}\middle|\phi_1 = \frac{n}{N}\underline{\phi}_1 + \frac{N-n}{N}\overline{\phi}_1\right)\ . \tag{B.99}$$

In our numerical examples of section 4.6, we used $N = 50$ "drawings" from the distribution of ϕ_1.

[4]The expected value of a uniformly distributed random variable u is given by $\mu_u = (\overline{\phi} + \underline{\phi})/2$ and the variance is given by $\sigma_u^2 = (\overline{\phi} - \underline{\phi})^2/12$. Using both equations, we can compute the support of u explicitly.

B.5.3 Credibility of a Currency Board

To compare the standard peg system with that of a currency board, we have now to define how to measure the probability of maintaining the currency board. As decision-making under a currency board arrangement takes place in the first period, the maintaining-interval depends on the first period shock ϕ_1. Due to the assumption of a uniformly distributed shock, the support of ϕ_1 is given by

$$[\underline{\phi}_1, \overline{\phi}_1] = [-\sigma_u\sqrt{3}, \sigma_u\sqrt{3}].$$

In this context, remember the assumption already made in section 4.4 saying that $\phi_0 = 0$, which means that there is no inherited shock from period 0.

In the currency board case, multiple equilibria cannot occur due to the time structure assumed in the model. Therefore, we can insert the unique equilibrium value for π_2^e into the expected loss functions (see appendix B.4). To find the boundaries of the interval, in which the monetary authority announces the continuity of the currency board, we have to solve

$$E(L_2^{CB}) = E(L_2^f) + c^{CB}$$

for ϕ_1. We obtain two solutions, which determine the lower and upper boundaries of the maintaining-interval. They depend on the parameter set $k, q, b_1, \sigma_u, \kappa, \beta, c^f, \theta_b, \theta_\pi$, summarized by $\mathcal{R}$, henceforth. Analogically to the proceedings in the peg-case, we set the probability of maintaining the currency board equal to zero if the discriminant by solving the quadratic equation for ϕ_2 becomes negative. Then, the probability of announcing to maintain the peg can be calculated by

$$Prob(\text{maintain CB}) = \max\left[\frac{\min(\overline{\phi}_1, \phi_1^{u,CB}(\mathcal{R})) - \max(\underline{\phi}_1, \phi_1^{l,CB}(\mathcal{R}))}{\overline{\phi}_1 - \underline{\phi}_1}, 0\right]. \quad \text{(B.100)}$$

Appendix C

Supplements to Chapter 7

In this part of the appendix, we show the analytical solution of the fiscal and monetary authorities' optimization problem for two representative scenarios. The simulations in section 7.2 are based on these results: We show how to determine the equilibrium in the Nash scenario, where all policy makers act simultaneously without coordinating, and in the sequential game in which fiscal policies are Stackelberg leader but do not cooperate. We only sketch the solution of these two scenarios, because the other scenarios considered in our model follow the same pattern.
We, again, exhibit the main model equations, which were derived in the sections 6.1 and 6.2:

Output equation:

$$\begin{aligned} y_A &= \bar{y}_A + a^A \tau_A + a^{AB} \tau_B + b^A(\pi_A - \pi_A^e) + \kappa^A(\pi_B - \pi_A) && \text{(C.1)} \\ y_B &= \bar{y}_B + a^B \tau_B + a^{BA} \tau_A + b^B(\pi_B - \pi_B^e) + \kappa^B(\pi_A - \pi_B)\ . && \text{(C.2)} \\ & && \text{(C.3)} \end{aligned}$$

Inflation equation:

$$\begin{aligned} \pi_A &= \mu + c^A \tau_A + c^{AB} \tau_B && \text{(C.4)} \\ \pi_B &= \mu + c^B \tau_B + c^{BA} \tau_A\ . && \text{(C.5)} \end{aligned}$$

Basic loss functions:

$$
\begin{aligned}
L_M &= \frac{n}{2}[(\pi_A - \pi_M^A)^2 + \theta_M^A(y_A - y_M^A)^2] \\
&\quad + \frac{1-n}{2}[(\pi_B - \pi_M^B)^2 + \theta_M^B(y_B - y_M^B)^2] && \text{(C.6)} \\
L_F^A &= \frac{1}{2}\left[(\pi_A - \pi_F^A)^2 + \theta_F^A(y_A - y_F^A)^2\right] && \text{(C.7)} \\
L_F^B &= \frac{1}{2}\left[(\pi_B - \pi_F^B)^2 + \theta_F^B(y_B - y_F^B)^2\right] \; . && \text{(C.8)} \\
& && \text{(C.9)}
\end{aligned}
$$

C.1 A Simultaneous Scenario: Nash Behavior

In the Nash scenario every policy maker optimizes her own loss function with respect to her policy variable while taking the other policy maker's reaction into account. Using equations (C.1) to (C.5) as well as the definitions of total output and total inflation, we obtain the following first-order conditions:

$$
\begin{aligned}
\frac{\partial L_M}{\partial \mu} &= (\pi - \pi_M) + n\theta_M^A b^A(y_A - y_M) + (1-n)\theta_M^B b^B(y_B - y_M) = 0 && \text{(C.10)} \\
\frac{\partial L_F^A}{\partial \tau_A} &= (\pi_A - \pi_F^A)c^A + \theta_F^A(y_A - y_F^A)(a^A + b^A c^A - \kappa^A c^A) = 0 && \text{(C.11)} \\
\frac{\partial L_F^B}{\partial \tau_B} &= (\pi_B - \pi_F^B)c^B + \theta_F^B(y_B - y_F^B)(a^B + b^B c^B - \kappa^B c^B) = 0 \; . && \text{(C.12)}
\end{aligned}
$$

Inserting (C.1) to (C.5) into the first-order conditions results in three equations in the variables μ, τ^A and τ^B. In the order monetary policy, fiscal policy A, and fiscal policy B, the first-order conditions are rewritten as

$$
\begin{aligned}
0 &= \alpha_1\mu + \alpha_2\tau_A + \alpha_3\tau_B + \alpha_4 && \text{(C.13)} \\
0 &= \alpha_5\mu + \alpha_6\tau_A + \alpha_7\tau_B + \alpha_8 && \text{(C.14)} \\
0 &= \alpha_9\mu + \alpha_{10}\tau_A + \alpha_{11}\tau_B + \alpha_{12} \; , && \text{(C.15)}
\end{aligned}
$$

where the parameters α_i are given by

$$
\begin{aligned}
\alpha_1 &= 1 + n\theta_M^A (b^A)^2 + (1-n)\theta_M^B (b^B)^2 \\
\alpha_2 &= nc^A + (1-n)c^{BA} + n\theta_M^A b^A (a^A + b^A c^A + \kappa^A (c^{BA} - c^A)) \\
&\qquad + (1-n)\theta_M^B b^B (a^{BA} + b^B c^{BA} + \kappa^B (c^A - c^{BA})) \\
\alpha_3 &= nc^{AB} + (1-n)c^B + n\theta_M^A b^A (a^{AB} + b^A c^{AB} + \kappa^A (c^B - c^{AB})) \\
&\qquad + (1-n)\theta_M^B b^B (a^B + b^B c^B + \kappa^B (c^{AB} - c^B)) \\
\alpha_4 &= -\pi_M + n\theta_M^A b^A (\bar{y}_A - b^A \pi_A^e - y_M^A + \phi_A) \\
&\quad + (1-n)\theta_M^B b^B (\bar{y}_B - b^B \pi_B^e - y_M^B + \phi_B) \\
\alpha_5 &= c^A + \theta_F^A b^A [a^A + b^A c^A + \kappa^A (c^{BA} - c^A)] \\
\alpha_6 &= (c^A)^2 + \theta_F^A [a^A + b^A c^A + \kappa^A (c^{BA} - c^A)]^2 \\
\alpha_7 &= c^{AB} c^A + \theta_F^A [a^A + b^A c^A + \kappa^A (c^{BA} - c^A)][a^{AB} + b^A c^{AB} + \kappa^A (c^B - c^{AB})] \\
\alpha_8 &= -\pi_F^A c^A + \theta_F^A [a^A + b^A c^A + \kappa^A (c^{BA} - c^A)][\bar{y}_A - b^A \pi_A^e - y_F^A] \\
\alpha_9 &= c^B + \theta_F^B b^B [a^B + b^B c^B + \kappa^B (c^{AB} - c^B)] \\
\alpha_{10} &= c^{BA} c^B + \theta_F^B [a^B + b^B c^B + \kappa^B (c^{AB} - c^B)][a^{BA} + b^B c^{BA} + \kappa^B (c^A - c^{BA})] \\
\alpha_{11} &= (c^B)^2 + \theta_F^B [a^B + b^B c^B + \kappa^B (c^{AB} - c^B)]^2 \\
\alpha_{12} &= -\pi_F^B c^B + \theta_F^B [a^B + b^B c^B + \kappa^B (c^{AB} - c^B)][\bar{y}_B - b^B \pi_B^e - y_F^B] \; .
\end{aligned}
$$

The (three-)equation-system is then solved for μ, τ_A and τ_B. The according equilibrium values for y_i and π_i can be obtained by inserting the optimal values of the fiscal and monetary authorities' policy variable (C.1) to (C.5).[1]

C.2 A Sequential Scenario: Fiscal Leadership

When fiscal policies make their decisions before the common central bank chooses its policy, they set their τ_i before monetary policy chooses μ.

We obtain the leadership-equilibrium by backward induction. The central bank optimizes its policy given the fiscal policy choices τ_A and τ_B. $\frac{\partial L_M}{\partial \mu}$ equals equation (C.10), which is rearranged in equation (C.13).

Rearranging further, μ can be written as

$$\mu = -\frac{\alpha_2}{\alpha_1}\tau_A - \frac{\alpha_3}{\alpha_1}\tau_B - \frac{\alpha_4}{\alpha_1} \; , \tag{C.16}$$

[1] Note that the expected inflation rates are here taken as given. The rational expectations of inflation are determined in our simulations by iteration.

which is the monetary policy's reaction function depending on τ_A and τ_B.

Now, the fiscal first-order conditions can be determined, taking the monetary reaction function into account:

$$\frac{\partial L_F^A}{\partial \tau_A} = \beta_1(\pi_A - \pi_F^A) + \beta_2\theta_F^A(y_A - y_F^A) = 0 \tag{C.17}$$

$$\frac{\partial L_F^B}{\partial \tau_B} = \beta_3(\pi_B - \pi_F^B) + \beta_4\theta_F^B(y_B - y_F^B) = 0 \ , \tag{C.18}$$

where

$$\beta_1 = c^A - \frac{\alpha_2}{\alpha_1} \tag{C.19}$$

$$\beta_2 = a^A + b^A\left(c^A - \frac{\alpha_2}{\alpha_1}\right) + \kappa^A(c^{BA} - c^A) \tag{C.20}$$

$$\beta_3 = c^B - \frac{\alpha_3}{\alpha_1} \tag{C.21}$$

$$\beta_4 = a^B + b^B\left(c^B - \frac{\alpha_3}{\alpha_1}\right) + \kappa^B(c^{AB} - c^B) \ . \tag{C.22}$$

Inserting (C.1) to (C.5) and (C.16) into (C.17) and (C.18) results in

$$0 = \beta_5\tau_A + \beta_6\tau_B + \beta_7 \tag{C.23}$$

$$0 = \beta_8\tau_A + \beta_9\tau_B + \beta_{10} \ , \tag{C.24}$$

where

$$\begin{aligned}
\beta_5 &= \beta_1^2 + \beta_2^2\theta_F^A \\
\beta_6 &= \beta_1\left(c^{AB} - \frac{\alpha_3}{\alpha_1}\right) + \beta_2\theta_F^A\left[a^{AB} + b^A\left(c^{AB} - \frac{\alpha_3}{\alpha_1}\right) + \kappa^A\left(c^B - c^{AB}\right)\right] \\
\beta_7 &= \beta_1\left(-\pi_F^A - \frac{\alpha_4}{\alpha_1}\right) + \beta_2\theta_F^A\left[\bar{y}_A + b^A\left(-\pi_A^e - \frac{\alpha_4}{\alpha_1}\right) - y_F^A\right] \\
\beta_8 &= \beta_3\left(c^{BA} - \frac{\alpha_3}{\alpha_1}\right) + \beta_4\theta_F^B\left[a^{BA} + b^B\left(c^{BA} - \frac{\alpha_2}{\alpha_1}\right) + \kappa^B\left(c^A - c^{BA}\right)\right] \\
\beta_9 &= \beta_3^2 + \beta_4^2\theta_F^B \\
\beta_{10} &= \beta_3\left(-\pi_F^B - \frac{\alpha_4}{\alpha_1}\right) + \beta_4\theta_F^B\left[\bar{y}_B + b^B\left(-\pi_B^e - \frac{\alpha_4}{\alpha_1}\right) - y_F^B\right] \ .
\end{aligned}$$

This (two)-equation-system can be solved for τ^A and τ^B, where μ is replaced by the reaction function of monetary policy, which is given by equation (C.16).

Appendix D

MATLAB Codes used in Chapter 7

In this part of the appendix, we list the MATLAB-codes which are used for obtaining the simulation results in section 7.2. We begin with the calibration and the simulation file for supply-side policy, and abstain from showing the files for demand-side policy, because they have the same structure. After that we list the policy scenario files, in which the analytical solution of the particular scenarios are determined. The scenario files are loaded in the simulation files.

D.1 The Calibration Files: `grmp22.m`

There are four different files that are at the beginning of the MATLAB calculations: For the baseline model and for the supply-side model each one symmetric and one asymmetric case. As an example for these, we add the code for the asymmetric case of the baseline model.

```
% GR Main (Structural) Parameter file for the asymmetric case 2 and fiscal policy 2
% created Dec 2, 2006 by SR
disp('  '); close all; %clear all; %clc;
disp('Grimm Ried 2006:      Results for the asymmetric case 2 with fiscal
    policy 2 (subsidies) are calculated.      Please wait.');
disp('                Region B is assumed to stand for the new EU members,
    with smaller size, lower taxes and more rigid prices.');
% This file is used to evaluate the parameters of the Grimm and Ried (2006) paper.
% We give values to the structural parameters  and calculate the resulting
% parameters in the macro model.

% Baseline structural parameters to be calibrated:
```

```
if exist('n')~=1,       n       = 0.7; end; % Size of region H; between 0 and 1
if exist('nu')~=1,      nu      = 0.8; end; % Home bias parameter;
                                            % between 0.5 and 1
if exist('emdl')~=1,    emdl    = .45; end; % Elasticity of marginal disutility
                                            % of labor; between 0 and 0.45, perhaps
                                            % up to 1
if exist('Phi')~=1,     Phi     = 0.5; end; % Calvo stickiness parameter:
                                            % fraction of firms which cannot
                                            % readjust prices
if exist('Phiast')~=1,  Phiast  = 0.58; end;% We assume the same Calvo stickiness
                                            % for both regions, Phi = 0.01:.1:1
if exist('theta')~=1,   theta   = 11;  end; % elasticity of substitution
                                            % between goods;
                                            % between 1 and 25
if exist('dH')~=1,      dH      = 1;   end; % disutility of labor; includes
                                            % technology shock;
if exist('eta')~=1,     eta     = 0.98;end; % subjective discount factor;
                                            % between 0.5 and 1
if exist('tauHbar')~=1, tauHbar = 1/(1-theta); end; %Steady state value for
                                            % subsidies; set optimally to offset
                                            % the monopolistic dist.
                                            % for tauHbar =.01:.1:1
if exist('tauFbar')~=1, tauFbar = 1/(1-theta); end; %Steady state value for subsidies;
                                            % set optimally to offset the monopolistic
                                            % dist.

% Simple implications used here:
nuast   = nu;           % We assume the same home bias for both regions
betta   = 1+(1/emdl);   % disutility of labor, 2 implies unit wage elasticity
dF      = dH;           % disutility of labor; we assume identical values,
                        %except for the stochastic part of this
yHbar   = 0; yFbar = 0; % fixed by the microfounded model
gamma   = 0.8;          % share of consumption relative to real money balances
                        % in the utility

% Resulting parameters defined in the paper, together with their definitions
rho     = Phi*[1+(1-Phi)*eta];
rhoast  = Phiast*[1+(1-Phiast)*eta];
aH      = [[(1-theta)/[2*[1+theta*(betta-1)]]]-1/[2*(betta-1)]]*tauHbar;
aHF     = - [[(1-theta)/[2*[1+theta*(betta-1)]]]+1/[2*(betta-1)]]*tauFbar;
aF      = [[(1-theta)/[2*[1+theta*(betta-1)]]]-1/[2*(betta-1)]]*tauFbar;
aFH     = - [[(1-theta)/[2*[1+theta*(betta-1)]]]+1/[2*(betta-1)]]*tauHbar;
```

```
bH      = 2*betta*rho/[(betta-1)*(1-rho)];
bF      = 2*betta*rhoast/[(betta-1)*(1-rhoast)];
kappaH  = 0.5*bH;
kappaF  = 0.5*bF;
thetaH  = eta*Phi*(betta-1)/[[(1-eta*Phi)*theta*[1+theta*(betta-1)]]];
thetaF  = eta*Phiast*(betta-1)/[[(1-eta*Phiast)*theta*[1+theta*(betta-1)]]];
    omH     = 1 - gamma * [nu/n + (1-nu)/(1-n)];
% The following parameters depend on fiscal policy 2!
    omF     = 1 - gamma * [nu/(1-n) + (1-nu)/(n)];
    om1H    = tauHbar/[1+theta*(betta-1)];
    om1F    = tauFbar/[1+theta*(betta-1)];
    om2     = 0;
    om3H    = 1-[(betta-1)/[1+theta*(betta-1)]]*[gamma*(1-nu)/[omH*(1-n)]];
    om3F    = 1-[(betta-1)/[1+theta*(betta-1)]]*[gamma*(1-nu)/[omF*(n)]];
    om4H    = 1-om3H;
    om4F    = 1-om3F;
    OmH     = [1-(1-rho)*om3H]/[[1-(1-rho)*om3H]^2-[(1-rho)*om4H]^2];
    OmF     = [1-(1-rhoast)*om3F]/[[1-(1-rhoast)*om3F]^2-[(1-rhoast)*om4F]^2];
    cH      = OmH*(1-rho)*om1H;
    cHF     = OmH*(1-rho)*[(1-rho)*om4H*om1H]/[1-(1-rho)*om3H];
    cF      = OmF*(1-rhoast)*om1F;
    cFH     = OmF*(1-rhoast)*[(1-rhoast)*om4F*om1F]/[1-(1-rhoast)*om3F];

% Output:
disp('----------------------------------------------------------------------');
disp('Asymmetric Fiscal Policy 1 Case');
disp(sprintf('%s \t\t', 'a^H ', 'a^F ', 'a^HF', 'a^FH', 'b^H', 'b^F', 'kapa^H',...
    ... 'kapa^F', 'c^H', 'c^F', 'c^HF', 'c^FH', 'theta_H', 'theta_F'));
disp(sprintf('%3.5f \t', aH, aF, aHF, aFH, bH, bF, kappaH, kappaF, cH, cF, cHF,...
    ... cFH, thetaH, thetaF));
disp('----------------------------------------------------------------------');
% Transfer to the notation used in the macromodel and in the grmm
% (or, older, grmr) files:
aa = aH; ab = aF; aab = aHF; aba = aFH; ba = bH; bb = bF; ka = kappaH;
kb = kappaF; ca = cH; cb = cF; cab = cHF; cba = cFH;
thetama = thetaH; thetamb = thetaF; ybara = yHbar; ybarb = yFbar;

% Setting the remaining macro parameters:
% For the union:    For region A:       For region B:
% Target values for inflation and output for the different players and scenarios:
pim     = 0.00;     pima    = 0.00;     pimb    = 0.00;
ym      = 0.00;     yma     = 0.00;     ymb     = 0.00;
```

```
delta   = 0.00;     if exist('thetafa')~=1, thetafa = 1.00;   end;
                    if exist('thetafb')~=1, thetafb = 1.25;    end;
                    if exist('yfa')~=1,     yfa     = 0.015;  end;
                    if exist('yfb')~=1,     yfb     = 0.030;   end;
                    if exist('pifa')~=1,    pifa    = 0.02;   end;
                    if exist('pifb')~=1,    pifb    = 0.03;    end;
% Country-Specific Social Optimum in the Symmetric Case
% (identical with central bank's targets)
        piaso   = 0.0;      pibso   = 0.0;
        yaso    = 0;        ybso    = 0;
%The Cooperation Case:
                    thetajca= thetama;  thetajcb= thetamb;
pifjc   = pim;      %pifjca   = 0.03;    pifjcb  = 0.03;
yfjc    = ym;       %yfjca    = 0.04;    yfjcb   = 0.04;
%The Fiscal Cooperation Case:
        thetaafc= thetafa;  thetabfc= thetafb;
%The Stackelberg Fiscal Cooperation Case: inserting yaffc, piaffc, ybffc and pibffc
        piaffc  = pifa;     pibffc  = pifb;
        yaffc   = yfa;      ybffc   = yfb;

% Starting values for the rational inflation expectation algorithm:
 piean   = 0.0;      piebn   = 0.0;     %SV Nash
 pieastf = 0.0;      piebstf = 0.0;     %SV Stackelberg Fiscal Leader
 pieastm = 0.0;      piebstm = 0.0;     %SV Stackelberg Monetary Leader
 pieans  = 0.0;      piebns = 0.0;      %SV Non-Strategic Fiscal Policy
 pieajc  = 0.0;      piebjc = 0.0;      %SV Joint Collusion
 pieafc  = 0.0;      piebfc = 0.0;      %SV Fiscal Collusion
 pieafcstm = 0.0;    piebfcstm = 0.0;   %SV Fiscal Collusion Monetary Leadership
 pieafcstf = 0.0;    piebfcstf = 0.0;   %SV Fiscal Collusion Fiscal Leadership

% Calling the file grmm.m that calculates the results and gives the resulting tables:
grmm;   % Note that grmm now replaces grmr1
        % for the symmetric and grmr2 for the asymmetric case.
```

D.2 The Rational Expectations and Stochastic Simulations File: `grmm.m`

This file is used to evaluate the rational expectations for the inflation rates and to simulate the stochastics of the model.

```
% GR Main InterMediate File for all cases: This file is called by the grmp files.
% It calls all other files.

% Grimm Ried 2005: Main Program File that calls all the other files
% This is GRmainrational1 with correct calculation of inflation
% expectations.
% Loss is multiplied by 10^5 for a better overview.

mnvek=[];       xanvek=[];     xbnvek=[];     pianvek=[];    pibnvek=[];    pinvek =[];
yanvek =[];     ybnvek =[];    ynvek =[];     lfanvek=[];    lfbnvek =[];   lmnvek =[];
lnsoavek =[];  lnsobvek =[];  cnsoavek=[];   cnsobvek=[];   lfanpolvek =[];
lfbnpolvek =[];

mstmvek=[];     xastmvek=[];   xbstmvek=[];   piastmvek=[];  pibstmvek=[];  pistmvek =[];
yastmvek =[];  ybstmvek =[];  ystmvek =[];   lfastmvek =[]; lfbstmvek =[]; lmstmvek =[];
lstmsoavek =[];  lstmsobvek =[];  cstmsoavek=[];   cstmsobvek=[];   lfastmpolvek =[];
lfbstmpolvek =[];

mstfvek=[];     xastfvek=[];   xbstfvek=[];   piastfvek=[];  pibstfvek=[];  pistfvek =[];
yastfvek =[];  ybstfvek =[];  ystfvek = [];  lfastfvek =[]; lfbstfvek = [];lmstfvek = [];
lstfsoavek =[];  lstfsobvek =[];  cstfsoavek=[];   cstfsobvek=[];   lfastfpolvek =[];
lfbstfpolvek =[];

mjcvek=[];      xajcvek=[];    xbjcvek=[];    piajcvek=[];   pibjcvek=[];   pijcvek =[];
yajcvek =[];   ybjcvek =[];   yjcvek = [];   lfajcvek =[];  lfbjcvek = []; lmjcvek = [];
ljcsoavek =[];  ljcsobvek =[];  cjcsoavek=[];   cjcsobvek=[];   lfajcpolvek =[];
lfbjcpolvek =[];

mfcvek=[];      xafcvek=[];    xbfcvek=[];    piafcvek=[];   pibfcvek=[];   pifcvek =[];
yafcvek =[];   ybfcvek =[];   yfcvek = [];   lfafcvek =[];  lfbfcvek = []; lmfcvek = [];
lfcsoavek =[];  lfcsobvek =[];  cfcsoavek=[];   cfcsobvek=[];   lfafcpolvek =[];
lfbfcpolvek =[];

%Sequential Fiscal Collusion Cases
mfcstfvek=[];       xafcstfvek=[];      xbfcstfvek=[];     piafcstfvek=[];
```

```
pibfcstfvek=[];     pifcstfvek =[];     yafcstfvek =[];    ybfcstfvek =[];
yfcstfvek = [];     lfafcstfvek =[];    lfbfcstfvek = [];  lmfcstfvek = [];
lfcstfsoavek =[];   lfcstfsobvek =[];  cfcstfsoavek=[];    cfcstfsobvek=[];
lfafcstfpolvek =[]; lfbfcstfpolvek =[];

mfcstmvek=[];       xafcstmvek=[];      xbfcstmvek=[];      piafcstmvek=[];
pibfcstmvek=[];     pifcstmvek =[];     yafcstmvek =[];     ybfcstmvek =[];
yfcstmvek = [];     lfafcstmvek =[];    lfbfcstmvek = [];   lmfcstmvek = [];
lfcstmsoavek =[];   lfcstmsobvek =[];   cfcstmsoavek=[];    cfcstmsobvek=[];
lfafcstmpolvek =[]; lfbfcstmpolvek =[];

lcsoavek =[];  lcsobvek =[];  ccsoavek=[];   ccsobvek=[];   lfacpolvek =[];
lfbcpolvek =[];

phiavek = [];   phibvek = [];   zahl=-1;    % zahl starts with -1 as it counts only the
                                            % stochastic loops

% Averages:
b       = n * ba    + (1-n) * bb;
yf      = n * yfa   + (1-n) * yfb;
ybar    = n * ybara + (1-n) * ybarb;
pif     = n * pifa  + (1-n) * pifb;
pien    = n * piean + (1-n) * piebn;      %Nash Case !!!
% Shock starting values (If you do not want shocks, then set
% at the beginning of line 83.)
phia    = 0;  phib=0;

% Algorithm for rational inflation expectations:
expectationlooplength = 0;  expectationloopcriterion = 1;
expectationlooptolerance = 0.00000000001;
while   expectationloopcriterion > expectationlooptolerance
        expectationlooplength = expectationlooplength+1;
% Counter for the number of loops
% Calculating the unionwide averages:         % THE OLD FOR LOOP FOR EXPECTATIONS LOOKED
                                              % AS FOLLOWS: for j = 1:1:10; %for pie
        pif         = n * pifa      + (1-n) * pifb;
        pien        = n * piean     + (1-n) * piebn;
        piestm      = n * pieastm   + (1-n) * piebstm;
        piestf      = n * piebstf   + (1-n) * piebstf;
        piejc       = n * pieajc    + (1-n) * piebjc;
        piefc       = n * pieafc    + (1-n) * piebfc;
        piefcstm    = n * pieafcstm + (1-n) * piebfcstm; %Sequential Fiscal Collusion Cases
```

```
        piefcstf    = n * pieafcstf + (1-n) * piebfcstf; %Sequential Fiscal Collusion Cases
% Starting the actual calculations of the results:
        GRNasheval1;
        GRStackeval1;
        GRCollusioneval1;
        GRFiscalcollusioneval1;
        GRFCMonLeadershipEval1; %Sequential Fiscal Collusion Cases
        GRFCFiscLeadershipEval1;%Sequential Fiscal Collusion Cases
% Calculating the current expectation loop criterion
        expectationloopcriterion = (piean-pian)^2      +(piebn-pibn)^2         ...
        +(pieastf-piastf)^2    +(piebstf-pibstf)^2    +(pieastm-piastm)^2     ...
        +(piebstm-pibstm)^2    +(pieajc-piajc)^2      +(piebjc-pibjc)^2       ...
        +(pieafc-piafc)^2      +(piebfc-pibfc)^2      +(pieafcstm-piafcstm)^2 ...
        +(piebfcstm-pibfcstm)^2 +(pieafcstf-piafcstf)^2 +(piebfcstf-pibfcstf)^2;
% Fixing the inflation expectation in each round to the actual inflation
        piean       = pian;     piebn       = pibn;
        pieastf     = piastf;   piebstf     = pibstf;
        pieastm     = piastm;   piebstm     = pibstm;
        pieajc      = piajc;    piebjc      = pibjc;
        pieafc      = piafc;    piebfc      = pibfc;
        pieafcstm   = piafcstm; piebfcstm   = pibfcstm; %Sequential Fiscal Collusion Cases
        pieafcstf   = piafcstf; piebfcstf   = pibfcstf; %Sequential Fiscal Collusion Cases
end;
disp(sprintf('It took %u repetition(s) until the expectations converge to the true value.'...
    , expectationlooplength));

% Simulation of length k with stochastic shocks of size such that output gap variance
% is about plus minus 6 percent
for k = 1:1:100000;       %for shocks
    if  k==1
        phia = 0; phib = 0;
    else
         phia = (randn/10000)*6;  phib = (randn/10000)*6;
% Set a % at the beginning of this line if no random shocks wanted.
    end;
    phi=n*phia+(1-n)*phib;
        GRNasheval1;
        GRStackeval1;
        GRCollusioneval1;
        GRFiscalcollusioneval1;
        GRFCMonLeadershipEval1; %Sequential Fiscal Collusion Cases
        GRFCFiscLeadershipEval1;%Sequential Fiscal Collusion Cases
```

```
%       GRCommitmenteval1;

%Nash Case

mnvek=[mnvek ; mn];         xanvek=[xanvek ;xan];             xbnvek=[xbnvek ;xbn];
pianvek=[pianvek ; pian];   pibnvek=[pibnvek ; pibn];         pinvek =[ pinvek ; pin];
yanvek =[ yanvek ; yan];    ybnvek =[ ybnvek ; ybn];          ynvek =[ ynvek ;yn];
lfanvek =[lfanvek ;lfan];   lfbnvek =[ lfbnvek;lfbn];         lmnvek =[ lmnvek;lmn];
lnsoavek =[lnsoavek;lnsoa]; lnsobvek =[lnsobvek; lnsob];      cnsoavek=[cnsoavek;cnsoa];
cnsobvek=[cnsobvek;cnsob];  lfanpolvek =[lfanpolvek;lfanpol]; lfbnpolvek =[lfbnpolvek;lfbnpol];

%Stackelberg Monetary Leadership Case

mstmvek=[mstmvek ; mstm];           xastmvek=[xastmvek ;xastm];
xbstmvek=[xbstmvek ;xbstm];
piastmvek=[piastmvek ; piastm];     pibstmvek=[pibstmvek ; pibstm];
pistmvek =[ pistmvek ; pistm];
yastmvek =[ yastmvek ; yastm];      ybstmvek =[ ybstmvek ; ybstm];
ystmvek =[ ystmvek ;ystm];
lfastmvek =[lfastmvek ;lfastm];     lfbstmvek =[ lfbstmvek;lfbstm];
lmstmvek =[ lmstmvek;lmstm];
lstmsoavek =[lstmsoavek;lstmsoa];   lstmsobvek =[lstmsobvek; lstmsob];
cstmsoavek=[cstmsoavek;cstmsoa];
cstmsobvek=[cstmsobvek;cstmsob];    lfastmpolvek =[lfastmpolvek;lfastmpol];
lfbstmpolvek =[lfbstmpolvek;lfbstmpol];

%Stackelberg Fiscal Leadership Case

mstfvek=[mstfvek ; mstf];           xastfvek=[xastfvek ;xastf];
xbstfvek=[xbstfvek ;xbstf];
piastfvek=[piastfvek ; piastf];     pibstfvek=[pibstfvek ; pibstf];
pistfvek =[ pistfvek ; pistf];
yastfvek =[ yastfvek ; yastf];      ybstfvek =[ ybstfvek ; ybstf];
ystfvek =[ ystfvek ;ystf];
lfastfvek =[lfastfvek ;lfastf];     lfbstfvek =[ lfbstfvek;lfbstf];
lmstfvek =[ lmstfvek;lmstf];
lstfsoavek =[lstfsoavek;lstfsoa];   lstfsobvek =[lstfsobvek; lstfsob];
cstfsoavek=[cstfsoavek;cstfsoa];
cstfsobvek=[cstfsobvek;cstfsob];    lfastfpolvek =[lfastfpolvek;lfastfpol];
lfbstfpolvek =[lfbstfpolvek;lfbstfpol];

%Joint Collusion
```

```
mjcvek=[mjcvek ; mjc];              xajcvek=[xajcvek ;xajc];
xbjcvek=[xbjcvek ;xbjc];
piajcvek=[piajcvek ; piajc];        pibjcvek=[pibjcvek ; pibjc];
pijcvek =[ pijcvek ; pijc];
yajcvek =[ yajcvek ; yajc];         ybjcvek =[ ybjcvek ; ybjc];
yjcvek =[ yjcvek ;yjc];
lfajcvek =[lfajcvek ;lfajc];        lfbjcvek =[ lfbjcvek;lfbjc];
lmjcvek =[ lmjcvek;lmjc];
ljcsoavek =[ljcsoavek;ljcsoa];      ljcsobvek =[ljcsobvek; ljcsob];
cjcsoavek=[cjcsoavek;cjcsoa];
cjcsobvek=[cjcsobvek;cjcsob];       lfajcpolvek =[lfajcpolvek;lfajcpol];
lfbjcpolvek =[lfbjcpolvek;lfbjcpol];

%Simulataneous Fiscal Collusion

mfcvek=[mfcvek ; mfc];              xafcvek=[xafcvek ;xafc];
xbfcvek=[xbfcvek ;xbfc];
piafcvek=[piafcvek ; piafc];        pibfcvek=[pibfcvek ; pibfc];
pifcvek =[ pifcvek ; pifc];
yafcvek =[ yafcvek ; yafc];         ybfcvek =[ ybfcvek ; ybfc];
yfcvek =[ yfcvek ;yfc];
lfafcvek =[lfafcvek ;lfafc];        lfbfcvek =[ lfbfcvek;lfbfc];
lmfcvek =[ lmfcvek;lmfc];
lfcsoavek =[lfcsoavek;lfcsoa];      lfcsobvek =[lfcsobvek; lfcsob];
cfcsoavek=[cfcsoavek;cfcsoa];
cfcsobvek=[cfcsobvek;cfcsob];       lfafcpolvek =[lfafcpolvek;lfafcpol];
lfbfcpolvek =[lfbfcpolvek;lfbfcpol];

%Sequential Fiscal Collusion Cases

mfcstmvek=[mfcstmvek ; mfcstm];             xafcstmvek=[xafcstmvek ; xafcstm];
xbfcstmvek=[xbfcstmvek ;xbfcstm];
piafcstmvek=[piafcstmvek ; piafcstm];       pibfcstmvek=[pibfcstmvek ; pibfcstm];
pifcstmvek =[ pifcstmvek ; pifcstm];
yafcstmvek =[ yafcstmvek ; yafcstm];        ybfcstmvek =[ ybfcstmvek ; ybfcstm];
yfcstmvek =[ yfcstmvek ;yfcstm];
lfafcstmvek =[lfafcstmvek ;lfafcstm];       lfbfcstmvek =[ lfbfcstmvek;lfbfcstm];
lmfcstmvek =[ lmfcstmvek;lmfcstm];
lfcstmsoavek =[lfcstmsoavek;lfcstmsoa];     lfcstmsobvek =[lfcstmsobvek; lfcstmsob];
cfcstmsoavek=[cfcstmsoavek;cfcstmsoa];
cfcstmsobvek=[cfcstmsobvek;cfcstmsob];      lfafcstmpolvek =[lfafcstmpolvek;lfafcstmpol];
```

```
lfbfcstmpolvek =[lfbfcstmpolvek;lfbfcstmpol];

mfcstfvek=[mfcstfvek ; mfcstf];           xafcstfvek=[xafcstfvek ; xafcstf];
xbfcstfvek=[xbfcstfvek ;xbfcstf];
piafcstfvek=[piafcstfvek ; piafcstf];      pibfcstfvek=[pibfcstfvek ; pibfcstf];
pifcstfvek =[ pifcstfvek ; pifcstf];
yafcstfvek =[ yafcstfvek ; yafcstf];       ybfcstfvek =[ ybfcstfvek ; ybfcstf];
yfcstfvek =[ yfcstfvek ;yfcstf];
lfafcstfvek =[lfafcstfvek ;lfafcstf];      lfbfcstfvek =[ lfbfcstfvek;lfbfcstf];
lmfcstfvek =[ lmfcstfvek;lmfcstf];
lfcstfsoavek =[lfcstfsoavek;lfcstfsoa];    lfcstfsobvek =[lfcstfsobvek; lfcstfsob];
cfcstfsoavek=[cfcstfsoavek;cfcstfsoa];
cfcstfsobvek=[cfcstfsobvek;cfcstfsob];     lfafcstfpolvek =[lfafcstfpolvek;lfafcstfpol];
lfbfcstfpolvek =[lfbfcstfpolvek;lfbfcstfpol];


phiavek =[ phiavek ;phia];        phibvek =[ phibvek ;phib];
zahl=zahl+1; % For zahl, only stochastic simulations (k>0) count
end;

% Obtaining the expected values for all the results by using the nonstochastic
% first loop, k=0:

%Nash Case

mne=mnvek(1);          xane=xanvek(1);           xbne=xbnvek(1);
piane=pianvek(1);      pibne=pibnvek(1);         pine = pinvek(1);
yane = yanvek(1);      ybne = ybnvek(1);         yne = ynvek(1);
lfane =lfanvek(1);     lfbne = lfbnvek(1);       lmne = lmnvek(1);
lnsoae = lnsoavek(1);  lnsobe = lnsobvek(1);     cnsoae = cnsoavek(1);
cnsobe = cnsobvek(1);  lfanpole = lfanpolvek(1); lfbnpole = lfbnpolvek(1);

%Monetary Leadership

mstme=mstmvek(1);            xastme=xastmvek(1);            xbstme=xbstmvek(1);
piastme=piastmvek(1);        pibstme=pibstmvek(1);          pistme = pistmvek(1);
yastme = yastmvek(1);        ybstme = ybstmvek(1);          ystme = ystmvek(1);
lfastme =lfastmvek(1);       lfbstme = lfbstmvek(1);        lmstme = lmstmvek(1);
lstmsoae = lstmsoavek(1);    lstmsobe = lstmsobvek(1);      cstmsoae = cstmsoavek(1);
cstmsobe = cstmsobvek(1);    lfastmpole = lfastmpolvek(1);  lfbstmpole = lfbstmpolvek(1);

%Fiscal Leadership
```

```
mstfe=mstfvek(1);          xastfe=xastfvek(1);            xbstfe=xbstfvek(1);
piastfe=piastfvek(1);      pibstfe=pibstfvek(1);          pistfe = pistfvek(1);
yastfe = yastfvek(1);      ybstfe = ybstfvek(1);          ystfe = ystfvek(1);
lfastfe =lfastfvek(1);     lfbstfe = lfbstfvek(1);        lmstfe = lmstfvek(1);
lstfsoae = lstfsoavek(1);  lstfsobe = lstfsobvek(1);      cstfsoae = cstfsoavek(1);
cstfsobe = cstfsobvek(1);  lfastfpole = lfastfpolvek(1);  lfbstfpole = lfbstfpolvek(1);

%Joint Collusion

mjce=mjcvek(1);            xajce=xajcvek(1);              xbjce=xbjcvek(1);
piajce=piajcvek(1);        pibjce=pibjcvek(1);            pijce = pijcvek(1);
yajce = yajcvek(1);        ybjce = ybjcvek(1);            yjce = yjcvek(1);
lfajce =lfajcvek(1);       lfbjce = lfbjcvek(1);          lmjce = lmjcvek(1);
ljcsoae = ljcsoavek(1);    ljcsobe = ljcsobvek(1);        cjcsoae = cjcsoavek(1);
cjcsobe = cjcsobvek(1);    lfajcpole = lfajcpolvek(1);    lfbjcpole = lfbjcpolvek(1);

%Simultaneous Fiscal Collusion

mfce=mfcvek(1);            xafce=xafcvek(1);              xbfce=xbfcvek(1);
piafce=piafcvek(1);        pibfce=pibfcvek(1);            pifce = pifcvek(1);
yafce = yafcvek(1);        ybfce = ybfcvek(1);            yfce = yfcvek(1);
lfafce =lfafcvek(1);       lfbfce = lfbfcvek(1);          lmfce = lmfcvek(1);
lfcsoae = lfcsoavek(1);    lfcsobe = lfcsobvek(1);        cfcsoae = cfcsoavek(1);
cfcsobe = cfcsobvek(1);    lfafcpole = lfafcpolvek(1);    lfbfcpole = lfbfcpolvek(1);

%Sequential Fiscal Collusion Cases

mfcstme=mfcstmvek(1);          xafcstme=xafcstmvek(1);
xbfcstme=xbfcstmvek(1);
piafcstme=piafcstmvek(1);      pibfcstme=pibfcstmvek(1);
pifcstme = pifcstmvek(1);
yafcstme = yafcstmvek(1);      ybfcstme = ybfcstmvek(1);
yfcstme = yfcstmvek(1);
lfafcstme =lfafcstmvek(1);     lfbfcstme = lfbfcstmvek(1);
lmfcstme = lmfcstmvek(1);
lfcstmsoae = lfcstmsoavek(1);  lfcstmsobe = lfcstmsobvek(1);
cfcstmsoae = cfcstmsoavek(1);
cfcstmsobe = cfcstmsobvek(1);  lfafcstmpole = lfafcstmpolvek(1);
lfbfcstmpole = lfbfcstmpolvek(1);
```

```
mfcstfe=mfcstfvek(1);          xafcstfe=xafcstfvek(1);
xbfcstfe=xbfcstfvek(1);
piafcstfe=piafcstfvek(1);      pibfcstfe=pibfcstfvek(1);
pifcstfe = pifcstfvek(1);
yafcstfe = yafcstfvek(1);      ybfcstfe = ybfcstfvek(1);
yfcstfe = yfcstfvek(1);
lfafcstfe =lfafcstfvek(1);     lfbfcstfe = lfbfcstfvek(1);
lmfcstfe = lmfcstfvek(1);
lfcstfsoae = lfcstfsoavek(1);  lfcstfsobe = lfcstfsobvek(1);
cfcstfsoae = cfcstfsoavek(1);
cfcstfsobe = cfcstfsobvek(1);  lfafcstfpole = lfafcstfpolvek(1);
lfbfcstfpole = lfbfcstfpolvek(1);


% Obtaining the averages of the stochastic results:
mn=(1/zahl)*sum(mnvek(2:k));                   xan=(1/zahl)*sum(xanvek(2:k));
xbn=(1/zahl)*sum(xbnvek(2:k));
pian=(1/zahl)*sum(pianvek(2:k));               pibn=(1/zahl)*sum(pibnvek(2:k));
pin = (1/zahl)*sum(pinvek(2:k));
yan = (1/zahl)*sum(yanvek(2:k));               ybn = (1/zahl)*sum(ybnvek(2:k));
yn = (1/zahl)*sum(ynvek(2:k));
lfan = (1/zahl)*sum(lfanvek(2:k));             lfbn = (1/zahl)*sum(lfbnvek(2:k));
lmn = (1/zahl)*sum(lmnvek(2:k));
lnsoa = (1/zahl)*sum(lnsoavek(2:k));           lnsob = (1/zahl)*sum(lnsobvek(2:k));
cnsoa = (1/zahl)*sum(cnsoavek(2:k));
cnsob = (1/zahl)*sum(cnsobvek(2:k));           lfanpol = (1/zahl)*sum(lfanpolvek(2:k));
lfbnpol = (1/zahl)*sum(lfbnpolvek(2:k));

mstm=(1/zahl)*sum(mstmvek(2:k));               xastm=(1/zahl)*sum(xastmvek(2:k));
xbstm=(1/zahl)*sum(xbstmvek(2:k));
piastm=(1/zahl)*sum(piastmvek(2:k));           pibstm=(1/zahl)*sum(pibstmvek(2:k));
pistm = (1/zahl)*sum(pistmvek(2:k));
yastm = (1/zahl)*sum(yastmvek(2:k));           ybstm = (1/zahl)*sum(ybstmvek(2:k));
ystm = (1/zahl)*sum(ystmvek(2:k));
lfastm =(1/zahl)*sum(lfastmvek(2:k));          lfbstm = (1/zahl)*sum(lfbstmvek(2:k));
lmstm = (1/zahl)*sum(lmstmvek(2:k));
lstmsoa = (1/zahl)*sum(lstmsoavek(2:k));       lstmsob = (1/zahl)*sum(lstmsobvek(2:k));
cstmsoa = (1/zahl)*sum(cstmsoavek(2:k));
cstmsob = (1/zahl)*sum(cstmsobvek(2:k));       lfastmpol = (1/zahl)*sum(lfastmpolvek(2:k));
lfbstmpol = (1/zahl)*sum(lfbstmpolvek(2:k));

mstf=(1/zahl)*sum(mstfvek(2:k));               xastf=(1/zahl)*sum(xastfvek(2:k));
```

```
ystf = (1/zahl)*sum(ystfvek(2:k));
lfastf =(1/zahl)*sum(lfastfvek(2:k));        lfbstf = (1/zahl)*sum(lfbstfvek(2:k));
lmstf = (1/zahl)*sum(lmstfvek(2:k));
lstfsoa = (1/zahl)*sum(lstfsoavek(2:k));     lstfsob = (1/zahl)*sum(lstfsobvek(2:k));
cstfsoa = (1/zahl)*sum(cstfsoavek(2:k));
cstfsob = (1/zahl)*sum(cstfsobvek(2:k));     lfastfpol = (1/zahl)*sum(lfastfpolvek(2:k));
lfbstfpol = (1/zahl)*sum(lfbstfpolvek(2:k));

mjc=(1/zahl)*sum(mjcvek(2:k));               xajc=(1/zahl)*sum(xajcvek(2:k));
xbjc=(1/zahl)*sum(xbjcvek(2:k));
piajc=(1/zahl)*sum(piajcvek(2:k));           pibjc=(1/zahl)*sum(pibjcvek(2:k));
pijc = (1/zahl)*sum(pijcvek(2:k));
yajc = (1/zahl)*sum(yajcvek(2:k));           ybjc = (1/zahl)*sum(ybjcvek(2:k));
yjc = (1/zahl)*sum(yjcvek(2:k));
lfajc =(1/zahl)*sum(lfajcvek(2:k));          lfbjc = (1/zahl)*sum(lfbjcvek(2:k));
lmjc = (1/zahl)*sum(lmjcvek(2:k));
ljcsoa = (1/zahl)*sum(ljcsoavek(2:k));       ljcsob = (1/zahl)*sum(ljcsobvek(2:k));
cjcsoa = (1/zahl)*sum(cjcsoavek(2:k));
cjcsob = (1/zahl)*sum(cjcsobvek(2:k));       lfajcpol = (1/zahl)*sum(lfajcpolvek(2:k));
lfbjcpol = (1/zahl)*sum(lfbjcpolvek(2:k));

mfc=(1/zahl)*sum(mfcvek(2:k));               xafc=(1/zahl)*sum(xafcvek(2:k));
xbfc=(1/zahl)*sum(xbfcvek(2:k));
piafc=(1/zahl)*sum(piafcvek(2:k));           pibfc=(1/zahl)*sum(pibfcvek(2:k));
pifc = (1/zahl)*sum(pifcvek(2:k));
yafc = (1/zahl)*sum(yafcvek(2:k));           ybfc = (1/zahl)*sum(ybfcvek(2:k));
yfc = (1/zahl)*sum(yfcvek(2:k));
lfafc =(1/zahl)*sum(lfafcvek(2:k));          lfbfc = (1/zahl)*sum(lfbfcvek(2:k));
lmfc = (1/zahl)*sum(lmfcvek(2:k));
lfcsoa = (1/zahl)*sum(lfcsoavek(2:k));       lfcsob = (1/zahl)*sum(lfcsobvek(2:k));
cfcsoa = (1/zahl)*sum(cfcsoavek(2:k));
cfcsob = (1/zahl)*sum(cfcsobvek(2:k));       lfafcpol = (1/zahl)*sum(lfafcpolvek(2:k));
lfbfcpol = (1/zahl)*sum(lfbfcpolvek(2:k));

%Sequential Fiscal Collusion Cases

mfcstm=(1/zahl)*sum(mfcstmvek(2:k));         xafcstm=(1/zahl)*sum(xafcstmvek(2:k));
xbfcstm=(1/zahl)*sum(xbfcstmvek(2:k));
piafcstm=(1/zahl)*sum(piafcstmvek(2:k));     pibfcstm=(1/zahl)*sum(pibfcstmvek(2:k));
pifcstm = (1/zahl)*sum(pifcstmvek(2:k));
yafcstm = (1/zahl)*sum(yafcstmvek(2:k));     ybfcstm = (1/zahl)*sum(ybfcstmvek(2:k));
yfcstm = (1/zahl)*sum(yfcstmvek(2:k));
```

```
lfafcstm =(1/zahl)*sum(lfafcstmvek(2:k));     lfbfcstm = (1/zahl)*sum(lfbfcstmvek(2:k));
lmfcstm = (1/zahl)*sum(lmfcstmvek(2:k));
lfcstmsoa = (1/zahl)*sum(lfcstmsoavek(2:k));  lfcstmsob = (1/zahl)*sum(lfcstmsobvek(2:k));
cfcstmsoa = (1/zahl)*sum(cfcstmsoavek(2:k));
cfcstmsob = (1/zahl)*sum(cfcstmsobvek(2:k));
lfafcstmpol = (1/zahl)*sum(lfafcstmpolvek(2:k));
lfbfcstmpol = (1/zahl)*sum(lfbfcstmpolvek(2:k));

mfcstf=(1/zahl)*sum(mfcstfvek(2:k));          xafcstf=(1/zahl)*sum(xafcstfvek(2:k));
xbfcstf=(1/zahl)*sum(xbfcstfvek(2:k));
piafcstf=(1/zahl)*sum(piafcstfvek(2:k));      pibfcstf=(1/zahl)*sum(pibfcstfvek(2:k));
pifcstf = (1/zahl)*sum(pifcstfvek(2:k));
yafcstf = (1/zahl)*sum(yafcstfvek(2:k));      ybfcstf = (1/zahl)*sum(ybfcstfvek(2:k));
yfcstf = (1/zahl)*sum(yfcstfvek(2:k));
lfafcstf =(1/zahl)*sum(lfafcstfvek(2:k));     lfbfcstf = (1/zahl)*sum(lfbfcstfvek(2:k));
lmfcstf = (1/zahl)*sum(lmfcstfvek(2:k));
lfcstfsoa = (1/zahl)*sum(lfcstfsoavek(2:k));  lfcstfsob = (1/zahl)*sum(lfcstfsobvek(2:k));
cfcstfsoa = (1/zahl)*sum(cfcstfsoavek(2:k));
cfcstfsob = (1/zahl)*sum(cfcstfsobvek(2:k));
lfafcstfpol = (1/zahl)*sum(lfafcstfpolvek(2:k));
lfbfcstfpol = (1/zahl)*sum(lfbfcstfpolvek(2:k));

phia = (1/zahl)*sum(phiavek(2:k));            phib = (1/zahl)*sum(phibvek(2:k));

% Calculating the statistics: standard deviations, indicated by first letter s
smn=std(mnvek(2:k));                sxan=std(xanvek(2:k));
sxbn=std(xbnvek(2:k));
spian=std(pianvek(2:k));            spibn=std(pibnvek(2:k));
spin = std(pinvek(2:k));
syan = std(yanvek(2:k));            sybn = std(ybnvek(2:k));
syn = std(ynvek(2:k));
slfan =std(lfanvek(2:k));           slfbn = std(lfbnvek(2:k));
slmn = std(lmnvek(2:k));
slnsoa = std(lnsoavek(2:k));        slnsob = std(lnsobvek(2:k));
scnsoa = std(cnsoavek(2:k));
scnsob = std(cnsobvek(2:k));        slfanpol = std(lfanpolvek(2:k));
slfbnpol = std(lfbnpolvek(2:k));

smstm=std(mstmvek(2:k));            sxastm=std(xastmvek(2:k));
sxbstm=std(xbstmvek(2:k));
spiastm=std(piastmvek(2:k));        spibstm=std(pibstmvek(2:k));
spistm = std(pistmvek(2:k));
```

```
syastm = std(yastmvek(2:k));        sybstm = std(ybstmvek(2:k));
systm = std(ystmvek(2:k));
slfastm =std(lfastmvek(2:k));       slfbstm = std(lfbstmvek(2:k));
slmstm = std(lmstmvek(2:k));
slstmsoa = std(lstmsoavek(2:k));    slstmsob = std(lstmsobvek(2:k));
scstmsoa = std(cstmsoavek(2:k));
scstmsob = std(cstmsobvek(2:k));    slfastmpol = std(lfastmpolvek(2:k));
slfbstmpol = std(lfbstmpolvek(2:k));

smstf=std(mstfvek(2:k));            sxastf=std(xastfvek(2:k));
sxbstf=std(xbstfvek(2:k));
spiastf=std(piastfvek(2:k));        spibstf=std(pibstfvek(2:k));
spistf = std(pistfvek(2:k));
syastf = std(yastfvek(2:k));        sybstf = std(ybstfvek(2:k));
systf = std(ystfvek(2:k));
slfastf =std(lfastfvek(2:k));       slfbstf = std(lfbstfvek(2:k));
slmstf = std(lmstfvek(2:k));
slstfsoa = std(lstfsoavek(2:k));    slstfsob = std(lstfsobvek(2:k));
scstfsoa = std(cstfsoavek(2:k));
scstfsob = std(cstfsobvek(2:k));    slfastfpol = std(lfastfpolvek(2:k));
slfbstfpol = std(lfbstfpolvek(2:k));

smjc=std(mjcvek(2:k));                sxajc=std(xajcvek(2:k));
sxbjc=std(xbjcvek(2:k));
spiajc=std(piajcvek(2:k));            spibjc=std(pibjcvek(2:k));
spijc = std(pijcvek(2:k));
syajc = std(yajcvek(2:k));            sybjc = std(ybjcvek(2:k));
syjc = std(yjcvek(2:k));
slfajc =std(lfajcvek(2:k));           slfbjc = std(lfbjcvek(2:k));
slmjc = std(lmjcvek(2:k));
sljcsoa = std(ljcsoavek(2:k));        sljcsob = std(ljcsobvek(2:k));
scjcsoa = std(cjcsoavek(2:k));
scjcsob = std(cjcsobvek(2:k));        slfajcpol = std(lfajcpolvek(2:k));
slfbjcpol = std(lfbjcpolvek(2:k));

smfc=std(mfcvek(2:k));                sxafc=std(xafcvek(2:k));
sxbfc=std(xbfcvek(2:k));
spiafc=std(piafcvek(2:k));            spibfc=std(pibfcvek(2:k));
spifc = std(pifcvek(2:k));
syafc = std(yafcvek(2:k));            sybfc = std(ybfcvek(2:k));
syfc = std(yfcvek(2:k));
slfafc =std(lfafcvek(2:k));           slfbfc = std(lfbfcvek(2:k));
```

```
slmfc = std(lmfcvek(2:k));
slfcsoa = std(lfcsoavek(2:k));       slfcsob = std(lfcsobvek(2:k));
scfcsoa = std(cfcsoavek(2:k));
scfcsob = std(cfcsobvek(2:k));       slfafcpol = std(lfafcpolvek(2:k));
slfbfcpol = std(lfbfcpolvek(2:k));
%Sequential Fiscal Collusion Cases
smfcstm=std(mfcstmvek(2:k));         sxafcstm=std(xafcstmvek(2:k));
sxbfcstm=std(xbfcstmvek(2:k));
spiafcstm=std(piafcstmvek(2:k));     spibfcstm=std(pibfcstmvek(2:k));
spifcstm = std(pifcstmvek(2:k));
syafcstm = std(yafcstmvek(2:k));     sybfcstm = std(ybfcstmvek(2:k));
syfcstm = std(yfcstmvek(2:k));
slfafcstm =std(lfafcstmvek(2:k));    slfbfcstm = std(lfbfcstmvek(2:k));
slmfcstm = std(lmfcstmvek(2:k));
slfcstmsoa = std(lfcstmsoavek(2:k)); slfcstmsob = std(lfcstmsobvek(2:k));
scfcstmsoa = std(cfcstmsoavek(2:k));
scfcstmsob = std(cfcstmsobvek(2:k)); slfafcstmpol = std(lfafcstmpolvek(2:k));
slfbfcstmpol = std(lfbfcstmpolvek(2:k));

smfcstf=std(mfcstfvek(2:k));               sxafcstf=std(xafcstfvek(2:k));
sxbfcstf=std(xbfcstfvek(2:k));
spiafcstf=std(piafcstfvek(2:k));           spibfcstf=std(pibfcstfvek(2:k));
spifcstf = std(pifcstfvek(2:k));
syafcstf = std(yafcstfvek(2:k));           sybfcstf = std(ybfcstfvek(2:k));
syfcstf = std(yfcstfvek(2:k));
slfafcstf =std(lfafcstfvek(2:k));          slfbfcstf = std(lfbfcstfvek(2:k));
slmfcstf = std(lmfcstfvek(2:k));
slfcstfsoa = std(lfcstfsoavek(2:k));       slfcstfsob = std(lfcstfsobvek(2:k));
scfcstfsoa = std(cfcstfsoavek(2:k));
scfcstfsob = std(cfcstfsobvek(2:k));       slfafcstfpol = std(lfafcstfpolvek(2:k));
slfbfcstfpol = std(lfbfcstfpolvek(2:k));




%Beginning of the table of results

disp('-------- Grimm Ried (2006): Table of Results: Values, Standard Deviations
-----------------------------------------------------------');
disp(sprintf('%s \t\t', 'm  ', 'xa ', 'xb ', 'pia', 'pib', 'pi ', 'ya ', 'yb ', 'y  '));
disp('-----------------------------------------------------------------------------
```

```
-------------------------------------------------------');
disp('Nash Case ');
disp(sprintf('%3.5f \t', mn, xan, xbn, pian, pibn, pin, yan, ybn, yn));
disp(sprintf('%3.5f \t', smn, sxan, sxbn, spian, spibn, spin, syan, sybn, syn));
disp('-----------------------------------------------------------------------------
-------------------------------------------------------');
disp('Stackelberg Case 1: Fiscal Policies move first');
disp(sprintf('%3.5f \t', mstf, xastf, xbstf, piastf, pibstf, pistf, yastf, ybstf, ystf));
disp(sprintf('%3.5f \t', smstf, sxastf, sxbstf, spiastf, spibstf, spistf, syastf,...
    ...sybstf, systf));
disp('--------------------------------------------------------------------------------
----------------------------------------------------');
disp('Stackelberg Case 2: Monetary Policy moves first');
disp(sprintf('%3.5f \t', mstm, xastm, xbstm, piastm, pibstm, pistm, yastm, ybstm, ystm));
disp(sprintf('%3.5f \t', smstm, sxastm, sxbstm, spiastm, spibstm, spistm, syastm, sybstm,...
    ...systm));
disp('---------------------------------------------------------------------------------
---------------------------------------------------');
disp('Cooperation Case (with identical goals for all players)');
disp(sprintf('%3.5f \t', mjc, xajc, xbjc, piajc, pibjc, pijc, yajc, ybjc, yjc));
disp(sprintf('%3.5f \t', smjc, sxajc, sxbjc, spiajc, spibjc, spijc, syajc, sybjc, syjc));
disp('--------------------------------------------------------------------------------
----------------------------------------------------');
disp('Fiscal Cooperation Case (with identical goals for the fiscal players)');
disp(sprintf('%3.5f \t', mfc, xafc, xbfc, piafc, pibfc, pifc, yafc, ybfc, yfc));
disp(sprintf('%3.5f \t', smfc, sxafc, sxbfc, spiafc, spibfc, spifc, syafc, sybfc, syfc));
disp('--------------------------------------------------------------------------------
----------------------------------------------------');
disp('The Case of non-strategic fiscal policy');
disp(sprintf('%3.5f \t', mnsf, xansf, xbnsf, piansf, pibnsf, pinsf, yansf, ybnsf, ynsf));
disp('---------------------------------------------------------------------------------
---------------------------------------------------');
disp('Fiscal Cooperation 1: Fiscal Policies move first');
disp(sprintf('%3.5f \t', mfcstf, xafcstf, xbfcstf, piafcstf, pibfcstf, pifcstf, yafcstf,...
    ...ybfcstf, yfcstf));
disp(sprintf('%3.5f \t', smfcstf, sxafcstf, sxbfcstf, spiafcstf, spibfcstf, spifcstf,...
    ...syafcstf, sybfcstf, syfcstf));
disp('----------------------------------------------------------------------------------
--------------------------------------------------');
disp('Fiscal Cooperation 2: Monetary Policy moves first');
disp(sprintf('%3.5f \t', mfcstm, xafcstm, xbfcstm, piafcstm, pibfcstm, pifcstm, yafcstm,...
    ...ybfcstm, yfcstm));
```

```
disp(sprintf('%3.5f \t', smfcstm, sxafcstm, sxbfcstm, spiafcstm, spibfcstm, spifcstm,...
    ...syafcstm, sybfcstm, syfcstm));
disp('------------------------------------------------------------------------------------
---------------------------------------------------');
disp('  ');
disp('  ');
disp('  ');
disp('  ');
disp('  ');
disp('----------------------------------------------------------------------------------
----------------------------------------------------');
disp(sprintf('%s \t\t', 'L_FA ', 'L_FB ', 'L_M ', 'LSO_A', 'LSO_B', 'CSO_A ', 'CSO_B ',...
    'piea ', 'pieb '));
disp('----------------------------------------------------------------------------------
----------------------------------------------------');
disp('Nash Case ');
disp(sprintf('%3.5f \t', lfan, lfbn, lmn, lnsoa, lnsob, cnsoa, cnsob, piean, piebn));
disp(sprintf('%3.5f \t', slfan, slfbn, slmn, slnsoa, slnsob, scnsoa, scnsob));
disp('----------------------------------------------------------------------------------
----------------------------------------------------');
disp('Stackelberg Case 1: Fiscal Policies move first');
disp(sprintf('%3.5f \t', lfastf, lfbstf, lmstf, lstfsoa, lstfsob, cstfsoa, cstfsob,...
    ... pieastf, piebstf));
disp(sprintf('%3.5f \t', slfastf, slfbstf, slmstf, slstfsoa, slstfsob, scstfsoa,...
    ... scstfsob));
disp('----------------------------------------------------------------------------------
----------------------------------------------------');
disp('Stackelberg Case 2: Monetary Policy moves first');
disp(sprintf('%3.5f \t', lfastm, lfbstm, lmstm, lstmsoa, lstmsob, cstmsoa, cstmsob,...
    ... pieastm, piebstm));
disp(sprintf('%3.5f \t', slfastm, slfbstm, slmstm, slstmsoa, slstmsob, scstmsoa,...
    ... scstmsob));
disp('----------------------------------------------------------------------------------
----------------------------------------------------');
disp('Cooperation Case (with identical goals for all players)');
disp(sprintf('%3.5f \t', lfajc, lfbjc, lmjc, ljcsoa, ljcsob, cjcsoa, cjcsob, pieajc,...
    ... piebjc));
disp(sprintf('%3.5f \t', slfajc, slfbjc, slmjc, sljcsoa, sljcsob, scjcsoa, scjcsob));
disp('----------------------------------------------------------------------------------
----------------------------------------------------');
disp('Fiscal Cooperation Case (with identical goals for the fiscal players)');
disp(sprintf('%3.5f \t', lfafc, lfbfc, lmfc, lfcsoa, lfcsob, cfcsoa, cfcsob, pieafc,...
```

```
    piebfc));
disp(sprintf('%3.5f \t', slfafc, slfbfc, slmfc, slfcsoa, slfcsob, scfcsoa, scfcsob));
disp('-----------------------------------------------------------------------------------
---------------------------------------------------');
disp('The Case of non-strategic fiscal policy');
disp(sprintf('%3.5f \t', lfansf, lfbnsf, lmnsf, lnsfsoa, lnsfsob, cnsfsoa, cnsfsob));
disp('------------------------------------------------------------------------------------
--------------------------------------------------');
disp('Fiscal Cooperation 1: Fiscal Policies move first');
disp(sprintf('%3.5f \t', lfafcstf, lfbfcstf, lmfcstf, lfcstfsoa, lfcstfsob, cfcstfsoa,...
    ... cfcstfsob, pieafcstf, piebfcstf));
disp(sprintf('%3.5f \t', slfafcstf, slfbfcstf, slmfcstf, slfcstfsoa, slfcstfsob,...
    ... scfcstfsoa, scfcstfsob));
disp('-----------------------------------------------------------------------------------
---------------------------------------------------');
disp('Fiscal Cooperation 2: Monetary Policy moves first');
disp(sprintf('%3.5f \t', lfafcstm, lfbfcstm, lmfcstm, lfcstmsoa, lfcstmsob, cfcstmsoa,...
    ... cfcstmsob, pieafcstm, piebfcstm));
disp(sprintf('%3.5f \t', slfafcstm, slfbfcstm, slmfcstm, slfcstmsoa, slfcstmsob,...
    ... scfcstmsoa, scfcstmsob));
disp(sprintf('%3.5f \t', tlfafcstm, tlfbfcstm, tlmfcstm, tlfcstmsoa, tlfcstmsob,...
    ... tcfcstmsoa, tcfcstmsob));
disp('------------------------------------------------------------------------------------
--------------------------------------------------');
disp('  ');
disp('  ');
disp('  ');
disp('  ');
disp('  ');
disp('  ');
disp('  ');
disp('  ');
disp('------------------------------------------------------');
disp(sprintf('%s \t\t', 'L_FAPO', 'L_FBPO', 'phi_A ', 'phi_B '));
disp('------------------------------------------------------');
disp('Nash Case ');
disp(sprintf('%3.5f \t', lfanpol, lfbnpol, phia, phib));
disp(sprintf('%3.5f \t', slfanpol, slfbnpol, sphia, sphib));
disp(sprintf('%3.5f \t', tlfanpol, tlfbnpol, tphia, tphib));
disp('------------------------------------------------------');
disp('Stackelberg Case 1: Fiscal Policies move first');
disp(sprintf('%3.5f \t', lfastfpol, lfbstfpol, phia, phib));
```

```
disp(sprintf('%3.5f \t', slfastfpol, slfbstfpol, sphia, sphib));
disp(sprintf('%3.5f \t', tlfastfpol, tlfbstfpol, tphia, tphib));
disp('---------------------------------------------------');
disp('Stackelberg Case 2: Monetary Policy moves first');
disp(sprintf('%3.5f \t', lfastmpol, lfbstmpol, phia, phib));
disp(sprintf('%3.5f \t', slfastmpol, slfbstmpol, sphia, sphib));
disp(sprintf('%3.5f \t', tlfastmpol, tlfbstmpol, tphia, tphib));
disp('---------------------------------------------------');
disp('Cooperation Case (with identical goals for all players)');
disp(sprintf('%3.5f \t', lfajcpol, lfbjcpol, phia, phib));
disp(sprintf('%3.5f \t', slfajcpol, slfbjcpol, sphia, sphib));
disp(sprintf('%3.5f \t', tlfajcpol, tlfbjcpol, tphia, tphib));
disp('---------------------------------------------------');
disp('Fiscal Cooperation Case (with identical goals for the fiscal players)');
disp(sprintf('%3.5f \t', lfafcpol, lfbfcpol, phia, phib));
disp(sprintf('%3.5f \t', slfafcpol, slfbfcpol, sphia, sphib));
disp(sprintf('%3.5f \t', tlfafcpol, tlfbfcpol, tphia, tphib));
disp('---------------------------------------------------');
disp('The Case of non-strategic fiscal policy');
disp(sprintf('%3.5f \t', lfansfpol, lfbnsfpol, phia, phib));
%disp(sprintf('%3.5f \t', slfansfpol, slfbnsfpol, sphia, sphib));
%disp(sprintf('%3.5f \t', tlfansfpol, tlfbnsfpol, tphia, tphib));
disp('---------------------------------------------------');
disp('Fiscal Cooperation 1: Fiscal Policies move first');
disp(sprintf('%3.5f \t', lfafcstfpol, lfbfcstfpol, phia, phib));
disp(sprintf('%3.5f \t', slfafcstfpol, slfbfcstfpol, sphia, sphib));
disp(sprintf('%3.5f \t', tlfafcstfpol, tlfbfcstfpol, tphia, tphib));
disp('---------------------------------------------------');
disp('Fiscal Cooperation 2: Monetary Policy moves first');
disp(sprintf('%3.5f \t', lfafcstmpol, lfbfcstmpol, phia, phib));
disp(sprintf('%3.5f \t', slfafcstmpol, slfbfcstmpol, sphia, sphib));
disp(sprintf('%3.5f \t', tlfafcstmpol, tlfbfcstmpol, tphia, tphib));
disp('---------------------------------------------------');
```

D.3 Policy Scenario Files: `GRNash1.m` and `GRNasheval1.m`

These files calculate the results for Nash policies. First, the policy problem is solved. Then, the solutions for the policy problem are calculated in the file `GRNash1.m`, using MATLAB's symbolic math toolbox. Finally, the results for output, inflation and losses are calculated.

D.4 Policy Scenario File: `GRStackeval1.m`

D.5 Policy Scenario File: `GRCollusioneval1.m`

```
% Grimm Ried 2005 Jep
% Cooperation Solution for a two country setup with one monetary policy
% Sub-program, you should call GRMain1 to start the calculations

% Additional paramters to determine the Joint Loss Function
% (this is only helpful if we want to consider different thetajc for different
% countries later)
%thetajc     = thetam;   % This is only meaningful if A and B behave identically
%thetajca    = thetajc;
%thetajcb    = thetajc;
%pifjc       = pim;      % This is only meaningful if A and B behave identically
%pifjca      = pifjc;
%pifjcb      = pifjc;
%yfjc        = ym;       % This is only meaningful if A and B behave identically
%yfjca       = yfjc;
%yfjcb       = yfjc;

q1  = aa  + ba*ca   + ka*cba - ka*ca;
q2  = aba + bb*cba  + kb*ca  - kb*cba;
q3  = aab + ba*cab  + ka*cb  - ka*cab;
q4  = ab  + bb*cb   + kb*cab - kb*cb;
q5  = n * (ca          + thetajca * ba*q1)...
    ... + (1-n) * (cba         + thetajcb * bb*q2);
q6  = n * (ca^2        + thetajca * q1^2)...
    ... + (1-n) * (cba^2       + thetajcb * q2^2);
q7  = n * (cab*ca      + thetajca * q1*q3)
    ... + (1-n) * (cb*cba      + thetajcb * q2*q4);
q8  = n * (-pifjc*ca    + thetajca * (ybara-ba*pieajc - yfjc+phia)*q1)...
    ... + (1-n) * (-pifjc*cba   + thetajcb * (ybarb-bb*piebjc-yfjc+phib)*q2);
```

```
q9  = n * (cab         + thetajca * ba*q3)...
    ... + (1-n) * (cb          + thetajcb * bb*q4);
q10 = n * (ca*cab      + thetajca * q1*q3)...
    ... + (1-n) * (cba*cb      + thetajcb * q2*q4);
q11 = n * (cab^2       + thetajca * q3^2)...
    ... + (1-n) * (cb^2        + thetajcb * q4^2);
q12 = n * (-cab*pifjc   + thetajca * (ybara-ba*pieajc-yfjc+phia)*q3)...
    ... + (1-n) * (-cb*pifjc    + thetajcb * (ybarb-bb*piebjc-yfjc+phib)*q4);
q13 = n * (1           + thetajca * ba^2)...
    ... + (1-n) * (1           + thetajcb * bb^2);
q14 = n * (ca          + thetajca * ba * q1)...
    ... + (1-n) * (cba         + thetajcb * bb * q2);
q15 = n * (cab         + thetajca * ba * q3)...
    ... + (1-n) * (cb          + thetajcb * bb * q4);
q16 = n * (-pifjc      + thetajca * ba * (ybara - ba*pieajc - yfjc+phia))...
    ... + (1-n) * (-pifjc      + thetajcb * bb* (ybarb - bb*piebjc - yfjc+phib));

% Collusion solution: (mjc, xajc and xbjc are obtained
% by the symbolic calculation in GRJCom1.m)

mjc     = -(q10*q8*q15-q10*q7*q16+q6*q11*q16-q6*q12*q15+q7*q14*q12-q8*q14*q11)...
        .../(-q6*q15*q9-q14*q5*q11+q14*q7*q9+q15*q10*q5-q13*q10*q7+q6*q13*q11);
xajc    = (q5*q11*q16-q5*q12*q15-q7*q16*q9+q7*q13*q12+q8*q15*q9-q8*q13*q11)...
        .../(-q6*q15*q9-q14*q5*q11+q14*q7*q9+q15*q10*q5-q13*q10*q7+q6*q13*q11);
xbjc    = -(-q6*q16*q9+q14*q8*q9+q6*q13*q12+q16*q10*q5-q13*q10*q8-q14*q5*q12)/...
        ...(-q6*q15*q9-q14*q5*q11+q14*q7*q9+q15*q10*q5-q13*q10*q7+q6*q13*q11);



% The resulting inflation rates and output levels, obtained through eqs. (1) and (2):
piajc = mjc + ca*xajc + cab*xbjc;           pibjc = mjc + cb*xbjc + cba*xajc;
yajc  = ybara + aa*xajc + aab*xbjc + ba* (piajc-pieajc) + ka*(pibjc-piajc) + phia;
ybjc  = ybarb + ab*xbjc + aba*xajc + bb* (pibjc-piebjc) + kb*(piajc-pibjc) + phib;
pijc  = n * piajc  + (1-n) * pibjc;          yjc   = n * yajc   + (1-n) * ybjc;
lfajc = 10^5*0.5 * (n*( (piajc-pifjc)^2 + thetajca*(yajc-yfjc)^2 + 2*delta*abs(xajc))...
    ... +(1-n)*( (pibjc-pifjc)^2 + thetajca*(ybjc-yfjc)^2 + 2*delta*abs(xbjc)));
lfbjc = lfajc;%10^5*0.5 * ( (pibjc-pifjc)^2 + thetajca*(ybjc-yfjc)^2 + 2*delta*abs(xbjc));
lmjc  = 10^5*0.5 * ( n*((piajc-pifjc)^2 + thetajca*(yajc-yfjc)^2)...
    ... + (1-n)*((pibjc-pifjc)^2 + thetajcb*(ybjc-yfjc)^2));
%lmjc  = 0.5 * ( (pijc-pifjc)^2 + thetam*(yjc-yfjc)^2);

%Table of Results
```

```
%Social Optimal Country-Specific Losses
ljcsoa = 10^5*0.5 * ( (piajc-piaso)^2 + thetama*(yajc-yaso)^2);
ljcsob = 10^5*0.5 * ( (pibjc-pibso)^2 + thetamb*(ybjc-ybso)^2);

%Consumption Reduction equivalent to Social Welfare Loss
cjcsoa = 100*(-1+sqrt(1+4*(1-gamma)*(ljcsoa/10^5)))/(2*(1-gamma));
cjcsob = 100*(-1+sqrt(1+4*(1-gamma)*(ljcsob/10^5)))/(2*(1-gamma));

%Losses for policymakers (own preferences according to the uncoordinated Nash Case)
lfajcpol = 10^5*0.5 * ( (piajc-pifa)^2 + thetafa*(yajc-yfa)^2) ;
lfbjcpol = 10^5*0.5 * ( (pibjc-pifb)^2 + thetafb*(ybjc-yfb)^2) ;
```

D.6 Policy Scenario File: `GRFiscalcollusioneval1.m`

```
%used abbreviations
q1  = aa  + ba*ca   + ka*cba - ka*ca;
q2  = aba + bb*cba  + kb*ca  - kb*cba;
q3  = aab + ba*cab  + ka*cb  - ka*cab;
q4  = ab  + bb*cb   + kb*cab - kb*cb;

%Fiscal coefficients of country A
q5  = n * (ca          + thetaafc * ba*q1)...
    ... + (1-n) * (cba          + thetabfc * bb*q2);
q6  = n * (ca^2        + thetaafc * q1^2)...
    ... + (1-n) * (cba^2        + thetabfc * q2^2);
q7  = n * (cab*ca      + thetaafc * q1*q3)...
    ... + (1-n) * (cb*cba       + thetabfc * q2*q4);
q8  = n * (-piaffc*ca   + thetaafc * (ybara-ba*pieafc - yaffc+phia)*q1 + delta)...
    ... + (1-n) * (-pibffc*cba  + thetabfc * (ybarb-bb*piebfc-ybffc+phib)*q2);


%Fiscal coefficients of country B
q9  = n * (cab         + thetaafc * ba*q3)...
    ... + (1-n) * (cb           + thetabfc * bb*q4);
q10 = n * (ca*cab      + thetaafc * q1*q3)...
    ... + (1-n) * (cba*cb       + thetabfc * q2*q4);
q11 = n * (cab^2       + thetaafc * q3^2)...
    ... + (1-n) * (cb^2         + thetabfc * q4^2);
q12 = n * (-cab*piaffc  + thetaafc * (ybara-ba*pieafc-yaffc+phia)*q3)...
    ... + (1-n) * (-cb*pibffc   + thetabfc * (ybarb-bb*piebfc-ybffc+phib)*q4 + delta);
```

```
%Monetary Coefficients
q13  = 1+(n*thetama*ba^2+(1-n)*thetamb*bb^2);
q14  = n*ca  + (1-n)*cba + thetama*(ba  * n*q1     + bb*(1-n)*q2);
q15  = n*cab + (1-n)*cb  + thetamb*(ba  * n*q3     + bb*(1-n)*q4);
q16  = -pim              + (n*thetama*ba*(ybara-ba*piean -yma +phia)...
    ... +(1-n)*thetamb*bb*(ybarb-bb*piebn -ymb +phib));

% Fiscal Collusion solution: (mfc, xafc and xbfc are obtained by the symbolic
% calculation in Fiscalcollusion1.m)
mfc =(q6*q11*q16-q6*q12*q15+q7*q14*q12-q7*q16*q10+q8*q15*q10-q8*q14*q11)...
    ... /(-q5*q15*q10+q15*q9*q6+q13*q7*q10-q13*q6*q11-q14*q9*q7+q5*q14*q11);
xafc =(q5*q11*q16-q5*q12*q15-q9*q7*q16+q12*q13*q7-q11*q13*q8+q9*q8*q15)...
    ... /(q5*q15*q10-q15*q9*q6-q13*q7*q10+q13*q6*q11+q14*q9*q7-q5*q14*q11);
xbfc =-1/(q5*q15*q10-q15*q9*q6-q13*q7*q10+q13*q6*q11+q14*q9*q7-q5*q14*q11)...
    ...*(-q13*q8*q10-q5*q14*q12+q5*q16*q10-q16*q9*q6+q13*q6*q12+q14*q9*q8);

 % The resulting inflation rates and output levels, obtained through eqs. (1) and (2):
piafc = mfc + ca*xafc + cab*xbfc;           pibfc = mfc + cb*xbfc + cba*xafc;
yafc  = ybara + aa*xafc + aab*xbfc + ba* (piafc-pieafc) + ka*(pibfc-piafc) + phia;
ybfc  = ybarb + ab*xbfc + aba*xafc + bb* (pibfc-piebfc) + kb*(piafc-pibfc) + phib;
pifc  = n * piafc  + (1-n) * pibfc;          yfc   = n * yafc   + (1-n) * ybfc;
lfafc = 10^5*0.5 * ( n*((piafc-piaffc)^2 + thetaafc*(yafc-yaffc)^2...
    ... + 2* delta*abs(xafc))+(1-n)*((pibfc-pibffc)^2 + thetabfc*(ybfc-ybffc)^2...
    ... + 2* delta*abs(xbfc)));
lfbfc = lfafc;
lmfc  = 10^5*0.5 * ( n*((piafc-pima)^2 + thetama*(yafc-yma)^2)...
    ... + (1-n)*((pibfc-pimb)^2 + thetamb*(ybfc-ymb)^2));

%Social Optimal Country-Specific Losses
lfcsoa = 10^5*0.5 * ( (piafc-piaso)^2 + thetama*(yafc-yaso)^2);
lfcsob = 10^5*0.5 * ( (pibfc-pibso)^2 + thetamb*(ybfc-ybso)^2);

%Consumption Reduction equivalent to Social Welfare Loss
cfcsoa = 100*(-1+sqrt(1+4*(1-gamma)*(lfcsoa/10^5)))/(2*(1-gamma));
cfcsob = 100*(-1+sqrt(1+4*(1-gamma)*(lfcsob/10^5)))/(2*(1-gamma));

%Losses for policymakers (own preferences according to the uncoordinated Nash Case)
lfafcpol = 10^5*0.5 * ( (piafc-pifa)^2 + thetafa*(yafc-yfa)^2) ;
lfbfcpol = 10^5*0.5 * ( (pibfc-pifb)^2 + thetafb*(ybfc-yfb)^2) ;
```

D.7 Policy Scenario File: `GRFCFiscLeadershipEval1.m`

```
% Stackelberg Case: Fiscal Cooperation (1nd mover), Monetary Policy (follower)
% Calculations 23-08-2006

%2nd Stage of the game: Optimization of monetary policy

%used abbreviations
q1  = aa  + ba*ca   + ka*cba - ka*ca;
q2  = aba + bb*cba  + kb*ca  - kb*cba;
q3  = aab + ba*cab  + ka*cb  - ka*cab;
q4  = ab  + bb*cb   + kb*cab - kb*cb;

%Monetary coefficients of country A and B
q5  = n * (1    + thetama*ba^2)...
    ... + (1-n) * (1  + thetamb*bb^2);  %for m
q6  = n * (ca + thetama*ba*q1)...
    ... + (1-n) * (cba + thetamb*bb*q2); %for xa
q7  = n * (cab  + thetama*ba*q3)...
    ... + (1-n) * (cb  + thetamb*bb*q4); %for xb
q8  = n * (-pima  + thetama*ba*(ybara-ba*pieafcstf + phia-yma))...
    ... + (1-n) * (-pimb + thetamb *bb*(ybarb-bb*piebfcstf-ymb+phib)); %for parameters

% The reaction function of monetary policy to xa and xb is given by:
% mstfrf=-xa*q6/q5-xb*q7/q5-q8/q5;
% mstfrf means monetary policy reaction function
% under stackelberg leadership of fiscal policy

%1st stage of the game: optimization of fiscal policies with respect
% to the monetary reaction function:
%FOC of fiscal policy A

qq1=  n*(ca^2 -ca *q6/q5+q6^2/q5^2-q6/q5*ca  +thetaafc*(q1-ba*q6/q5)^2)...
    ... + (1-n)*(cba^2-cba*q6/q5+q6^2/q5^2-q6/q5*cba +thetabfc*(q2-bb*q6/q5)^2);
qq2=  n*((ca-q6/q5) *(cab-q7/q5) + thetaafc*(q1-ba*q6/q5)*(q3-ba*q7/q5))...
    ... + (1-n)*((cba-q6/q5)*(cb -q7/q5) + thetabfc*(q2-bb*q6/q5)*(q4-bb*q7/q5));
qq3=  n*((ca-q6/q5) *(-q8/q5-piaffc) + thetaafc*(q1-ba*q6/q5)...
    ... *(ybara-ba*q8/q5-ba*pieafcstf-yaffc+phia)+delta)...
    ... +(1-n)*((cba-q6/q5)*(-q8/q5-pibffc) + thetabfc*(q2-bb*q6/q5)...
    ... *(ybarb-bb*q8/q5-bb*piebfcstf-ybffc+phib));

%FOC of fiscal policy B
```

```
qq4= n*((cab-q7/q5)*(ca-q6/q5) + thetaafc*(q1-ba*q6/q5)*(q3-ba*q7/q5))...
    ... + (1-n)*((cb -q7/q5)*(cba-q6/q5)+ thetabfc*(q2-bb*q6/q5)*(q4-bb*q7/q5));
qq5= n*((cab-q7/q5)^2 + thetaafc*(q3-ba*q7/q5)^2)...
    ... + (1-n)*((cb-q7/q5)^2 + thetabfc*(q4-bb*q7/q5)^2);
qq6= n*((cab-q7/q5) *(-q8/q5-piaffc) + thetaafc*(q3-ba*q7/q5)...
    ... *(ybara-ba*q8/q5-bb*piebfcstf-yaffc+phia))...
    ... + (1-n)*((cb-q7/q5)*(-q8/q5-pibffc) + thetabfc*(q4-bb*q7/q5)...
    ... *(ybarb-bb*q8/q5-bb*piebfcstf-ybffc+phib) -delta);


%note, yaffc, ybffc, piaffc and pibffc are the targets of output and
%inflation of couzntry A and B under fiscal collusion


% Fiscal Collusion solution under fiscal leadership:
% Determination of xafcstf and xbfcstf from file GRFCstf.m
%

xafcstf = (qq2*qq6-qq3*qq5)/(qq1*qq5-qq4*qq2);

xbfcstf = -1/(qq1*qq5-qq4*qq2)*(qq1*qq6-qq4*qq3);

%From the monetary reaction function, we obtain

mfcstf=-xafcstf*q6/q5-xbfcstm*q7/q5-q8/q5;


% The resulting inflation rates and output levels, obtained through eqs. (1) and (2):
piafcstf = mfcstf + ca*xafcstf + cab*xbfcstf;
pibfcstf = mfcstf + cb*xbfcstf + cba*xafcstf;
yafcstf  = ybara + aa*xafcstf + aab*xbfcstf + ba* (piafcstf-pieafcstf)...
    ... + ka*(pibfcstf-piafcstf) + phia;
ybfcstf  = ybarb + ab*xbfcstf + aba*xafcstf + bb* (pibfcstf-piebfcstf)...
    ... + kb*(piafcstf-pibfcstf) + phib;
pifcstf  = n * piafcstf  + (1-n) * pibfcstf;
yfcstf   = n * yafcstf   + (1-n) * ybfcstf;
lfafcstf = 10^5*0.5 * ( n*((piafcstf-piaffc)^2 + thetaafc*(yafcstf-yaffc)^2...
    ... + 2* delta*xafcstf)...
    ... +(1-n)*((pibfcstf-pibffc)^2 + thetabfc*(ybfcstf-ybffc)^2 + 2* delta*xbfcstf));
lfbfcstf = lfafcstf;
lmfcstf  = 10^5*0.5 * ( n*((piafcstf-pima)^2 + thetama*(yafcstf-yma)^2)...
```

```
    ... + (1-n)*((pibfcstf-pimb)^2 + thetamb*(ybfcstf-ymb)^2));


%Table of Results
%Social Optimal Country-Specific Losses
lfcstfsoa = 10^5*0.5 * ( (piafcstf-piaso)^2 + thetama*(yafcstf-yaso)^2);
lfcstfsob = 10^5*0.5 * ( (pibfcstf-pibso)^2 + thetamb*(ybfcstf-ybso)^2);

%Consumption Reduction equivalent to Social Welfare Loss
cfcstfsoa = 100*(-1+sqrt(1+4*(1-gamma)*(lfcstfsoa/10^5)))/(2*(1-gamma));
cfcstfsob = 100*(-1+sqrt(1+4*(1-gamma)*(lfcstfsob/10^5)))/(2*(1-gamma));

%Losses for policymakers (own preferences according to the uncoordinated Nash Case)
lfafcstfpol = 10^5*0.5 * ( (piafcstf-pifa)^2 + thetafa*(yafcstf-yfa)^2 );
lfbfcstfpol = 10^5*0.5 * ( (pibfcstf-pifb)^2 + thetafb*(ybfcstf-yfb)^2 );
```

D.8 Policy Scenario File: `GRFCMonLeadershipEval1.m`

```
% Stackelberg Case: Fiscal Cooperation (2nd mover), Monetary Policy (leader)
% Calculations 23-08-2006
% Note: Calculations of q1 - q12 like in the simultaneous case of fiscal cooperation

%2nd Stage of the game

%used abbreviations
q1  = aa  + ba*ca   + ka*cba - ka*ca;
q2  = aba + bb*cba  + kb*ca  - kb*cba;
q3  = aab + ba*cab  + ka*cb  - ka*cab;
q4  = ab  + bb*cb   + kb*cab - kb*cb;

%Fiscal coefficients of country A
q5  = n * (ca^2         + thetaafc * q1^2)...
    ... + (1-n) * (cba^2        + thetabfc * q2^2);  %for xa
q6  = n * (ca*cab       + thetaafc * q1*q3)...
    ... + (1-n) * (cba*cb       + thetabfc * q2*q4); %for xb
q7  = n * (ca           + thetaafc * ba*q1)...
    ... + (1-n) * (cba          + thetabfc * bb*q2); %for m
q8  = n * (-piaffc*ca    + thetaafc * (ybara-ba*pieafcstm - yaffc+phia)*q1 + delta)...
    ... + (1-n) * (-pibffc*cba  + thetabfc * (ybarb-bb*piebfcstm-ybffc+phib)*q2);
                                                                  %for parameters
```

```
%Fiscal coefficients of country B
q9  = n * (ca*cab       + thetaafc * q3*q1)...
    ... + (1-n) * (cb*cba       + thetabfc * q2*q4); %for xa
q10 = n * (cab^2        + thetaafc * q3^2)...
    ... + (1-n) * (cb^2         + thetabfc * q4^2);  %for xb
q11 = n * (cab          + thetaafc * q3*ba)...
    ... + (1-n) * (cb           + thetabfc * q4*bb);                %for m
q12 = n * (-cab*piaffc  + thetaafc * (ybara-ba*pieafcstm-yaffc+phia)*q3)...
    ... + (1-n) * (-cb*pibffc + thetabfc * (ybarb-bb*piebfcstm-ybffc+phib)*q4 + delta);
    %for parameters

%note, yaffc, ybffc, piaffc and pibffc are the targets of output and inflation of
%country A and B under fiscal collusion

%Determination of the fiscal reaction functions given the monetary policy decision

%xarfml is the reaction function of fiscal policy A in the monetary leadership case
qq1=(q6*q11-q7*q10)/q10;
qq2=(q6*q12-q8*q10)/q10;
%xarfml = m*qq1 + qq2;

%xbrfml is the reaction function of fiscal policy B in the monetary leadership case
qq3=(q9*q7-q11*q5)/q5;
qq4=(q9*q8-q12*q5)/q5;
%xbrfml = m*qq3 + qq4;

%1st stage of the game: optimization of monetary policy with respect to the
%fiscal reaction functions
%FOC of monetary policy
%     n*((m+ca*(m*qq1 + qq2)+cab*(m*qq3 + qq4)-pima)*(1+ca*qq1+cab*qq3)
%       + thetama*(ybara+aa*(qq1*m+qq2)+aab*(qq3*m+qq4)+ba*(m+ca*(qq1*m+qq2)
%       +cab*(qq3*m+qq4)-pieafcstm)+ka*(cb*(qq3*m+qq4)+cba*(qq1+qq2*m)
%       -ca*(qq1*m+qq2)-cab*(qq3*m+qq4))+phia-yma)*(aa*qq1+aab*qq3
%       +ba*(1+ca*qq1+cab*qq3)+ka*(cb*qq3+cba*qq1-ca*qq1-cab*qq3)))
%       +(1-n)*((m+cb*(m*qq3 + qq4)+cba*(m*qq1 + qq2)-pimb)*(1+cb*qq3+cba*qq1) +
%       thetamb*(ybarb+ab*(qq3*m+qq4)+aba*(qq1*m+qq2)+bb*(m+cb*(qq3*m+qq4)
%       +cba*(qq1*m+qq2)-piebfcstm)+kb*(ca*(qq1*m+qq2)+cab*(qq3*m+qq4)-cb*(qq3*m+qq4)
%       -cba*(qq1*m+qq2))+phib-ybm)*(ab*qq3+aba*qq1+bb*(1+cb*qq3+cba*qq1)
%       +ka*(ca*qq1+cab*qq3-cb*qq3-cba*qq1)))=0;
```

```
% Fiscal Collusion solution under monetary leadership: (mfcstm, xafcstm and xbfcstm)
%Solution obtained from File GRFCstm

mfcstm = (-n*cab*qq4-thetamb*ybarb*bb+pimb*cba*qq1+thetamb*bb^2*piebfcstm...
    ... -n*ca*qq2-thetamb*phib*bb+pimb*cb*qq3+n*cb*qq4+n*cba*qq2-cba^2*qq2*qq1...
    ... -thetamb*ybarb*bb*cb*qq3-cb^2*qq4*qq3+thetamb*ymb*bb+n*thetamb*phib*ab*qq3...
    ... -n*thetama*phia*ba*cab*qq3+pimb-cb*qq4-cba*qq2+n*pima+n*pima*cab*qq3+...
    ... n*pima*ca*qq1-n*cab^2*qq4*qq3-n*ca^2*qq2*qq1+n*thetama*yma*ba-n*thetama*ybara*ba...
    ... -n*thetama*phia*ba+n*thetama*ba^2*pieafcstm-cb*qq4*cba*qq1-cba*qq2*cb*qq3...
    ... -thetamb*bb^2*cb*qq4-thetamb*ab*qq4*bb-thetamb*phib*ab*qq3-thetamb*bb^2*cba*qq2...
    ... +thetamb*ymb*aba*qq1-thetamb*phib*aba*qq1-thetamb*aba*qq2*bb+thetamb*ymb*ab*qq3...
    ... -thetamb*ab^2*qq4*qq3-thetamb*ybarb*ab*qq3-thetamb*ybarb*aba*qq1
    ... -thetamb*aba^2*qq2*qq1+n*thetamb*ybarb*bb-n*pimb*cba*qq1-n*thetamb*bb^2*piebfcstm
    ... +n*thetamb*phib*bb-n*pimb*cb*qq3+n*cba^2*qq2*qq1+n*cb^2*qq4*qq3
    ... -n*thetamb*ymb*bb+n*thetama*yma*ba*cab*qq3-n*ca*qq2*cab*qq3-n*cab*qq4*ca*qq1
    ... -n*thetama*ba^2*cab*qq4+n*thetama*yma*aa*qq1-n*thetama*ybara*aab*qq3...
xafcstm = mfcstm*qq1 + qq2;
xbfcstm = mfcstm*qq3 + qq4;


 % The resulting inflation rates and output levels, obtained through eqs. (1) and (2):
piafcstm = mfcstm + ca*xafcstm + cab*xbfcstm;
pibfcstm = mfcstm + cb*xbfcstm + cba*xafcstm;
yafcstm  = ybara + aa*xafcstm + aab*xbfcstm + ba* (piafcstm-pieafcstm)...
    ... + ka*(pibfcstm-piafcstm) + phia;
ybfcstm  = ybarb + ab*xbfcstm + aba*xafcstm + bb* (pibfcstm-piebfcstm)...
    ... + kb*(piafcstm-pibfcstm) + phib;
pifcstm  = n * piafcstm  + (1-n) * pibfcstm;
yfcstm   = n * yafcstm   + (1-n) * ybfcstm;
lfafcstm = 10^5*0.5 * ( n*((piafcstm-piaffc)^2 + thetaafc*(yafcstm-yaffc)^2...
    ... + 2* delta*xafcstm)
    ... +(1-n)*( (pibfcstm-pibffc)^2 + thetabfc*(ybfcstm-ybffc)^2...
    ... + 2* delta*xbfcstm));
lfbfcstm = 10^5*0.5 * ( n*((piafcstm-piaffc)^2 + thetaafc*(yafcstm-yaffc)^2...
    ... + 2* delta*xafcstm)+(1-n)*( (pibfcstm-pibffc)^2 + thetabfc*(ybfcstm-ybffc)^2...
    ... + 2* delta*xbfcstm));
lmfcstm  = 10^5*0.5 * ( n*((piafcstm-pima)^2   + thetama*(yafcstm-yma)^2)...
    ... +(1-n)*( (pibfcstm-pimb)^2   + thetamb* (ybfcstm-ymb)^2                    ));

%Social Optimal Country-Specific Losses
lfcstmsoa = 10^5*0.5 * ( (piafcstm-piaso)^2 + thetama*(yafcstm-yaso)^2);
lfcstmsob = 10^5*0.5 * ( (pibfcstm-pibso)^2 + thetamb*(ybfcstm-ybso)^2);
```

```
%Consumption Reduction equivalent to Social Welfare Loss
cfcstmsoa = 100*(-1+sqrt(1+4*(1-gamma)*(lfcstmsoa/10^5)))/(2*(1-gamma));
cfcstmsob = 100*(-1+sqrt(1+4*(1-gamma)*(lfcstmsob/10^5)))/(2*(1-gamma));

%Losses for policymakers (own preferences according to the uncoordinated Nash Case)
lfafcstmpol = 10^5*0.5 * ( (piafcstm-pifa)^2 + thetafa*(yafcstm-yfa)^2 );
lfbfcstmpol = 10^5*0.5 * ( (pibfcstm-pifb)^2 + thetafb*(ybfcstm-yfb)^2 );
```

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