Seljuk Muqarnas along the Silk Road

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Abstract

The film *Seljuk Muqarnas along the Silk Road* gives an overview of muqarnas, stalactite vaults, in Seljuk style architecture (1038-1194). The muqarnas are located in portals and niches of caravansaraies, madrassas and mosques. Starting with the Sultan Han near Kayseri we follow the Silk Road westward till Konya and finally show the Arslanhane Camii in Ankara. Video recordings alternate with computer reconstructions and animations, explaining the assembly of a caravansaray and the composition of muqarnas. The timeline of the video is contained in this paper.
Muqarnas reconstructions follow mathematical rules and stylistic specialties. Possible combinations of elements in tiers can be expressed in their 2D plane projection by means of a directed graph. On the other hand, given a 2D plan we are able to determine the directed graphs which correspond to a muqarnas. After adjusting some free directions manually, the 3D models can be generated automatically.

1 Contents of the film

Starting near Kayseri we follow the Silk Road westward till Konya. On this route there are several caravansaraies, also called hans, of the Seljuk period (1038-1194). They lie within a day trip of a caravan apart from each other and gave shelter to the people and animals travelling along the Silk Road. Beginning with the Sultan Han near Kayseri (see fig. 1), we show that their architecture follow common rules. Using a computer reconstruction of Sarı Han, Avanos, the assembly of a han is explained in detail. These hans all have a courtyard with arcades and a covered section for accommodation especially during the cold period. High and windowless walls not only gave shelter from odd climate but also protected against robbery. A little mosque is often located as a stand-alone building in the middle of the courtyard or integrated in the entrance whereas a hammam can be found in the arcades. Video recordings show the well restored Sarı Han and Sultan Han, Aksaray and in contrast the ruins of Alay Han and Sadeddin Han. The portal to the covered section of Sultan Han, Kayseri (see fig. 2) is our first example of a computer reconstructed muqarnas. The second reconstruction shows the portal of the Çifte Medresse, Kayseri. A detailed description of the automated reconstruction is given by way of example of the portal of Alay Han near Aksaray followed by an example from Ağzıkara Han

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near Nevşehir. Muqarnas are an ornamentation located in portals and niches of important buildings like hans, madrassas and also mosques. A very fine example can be found in the Arslanhane Camii in Ankara, which is worth a side trip of the Silk Road.

1.1 Timeline

The timeline of the film *Seljuk Muqarnas along the Silk Road* gives a short overview of the different scenes. The video recordings were taken during a journey to Anatolia in April 2005. The reconstructions are done with the *Muqarnas Tool*, a software developed in the scope of our research. The reconstruction of Sarı Han is due to plans from [Erdmann 1961] and modeled with AutoCAD™ and Maya™.

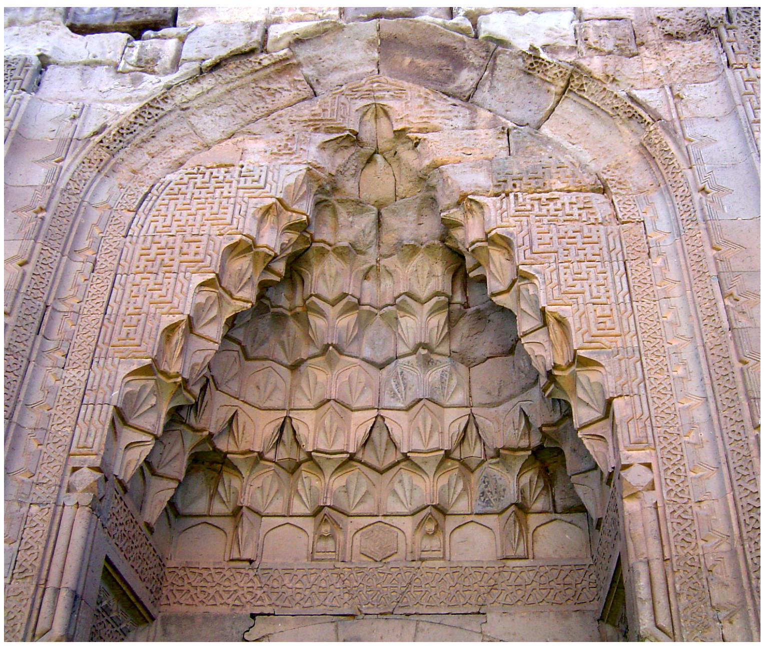

Figure 1. Portal to covered section at Sultan Han near Kayseri (1232 - 36).

Intro

0'00" IWR Visualization Group introduction sequence

0'27" Title

Seljuk muqarnas in Anatolia

0'43" **Map** showing part of the Silk Road

0'59" Sultan Han, Kayseri

2'48" Muqarnas reconstruction of Sultan Han near Kayseri

3'53" **Map** showing part of the Silk Road locating Kayseri

4’02” Çifte Medresse, Kayseri
4’21” Muqarnas reconstruction of Çifte Medresse
4’40” Hunat Camii, Kayseri
5’56” **Map** showing part of the Silk Road locating Sarı Han near Avanos
6’05” Sarı Han, Avanos
6’59” Reconstruction and animation of Sarı Han, explaining the assembly
9’25” **Map** showing part of the Silk Road locating Alay Han near Aksaray
9’34” Alay Han, Aksaray

Plan reconstruction and elements

10’44” Construction of muqarnas plan
11’41” Animations of muqarnas elements
12’29” Final reconstruction of muqarnas at Alay Han with the *Muqarnas Tool*

Seljuk muqarnas and computer reconstructions

13’12” **Map** showing part of the Silk Road locating Ağzıkara Han near Aksaray
13’21” Ağzıkara Han, Aksaray
13’46” Muqarnas reconstruction of Ağzıkara Han, Aksaray
14’44” **Map** showing part of the Silk Road locating Aksaray
14’53” Ulu Camii, Aksaray
15’40” Zinciriye Medresesi, Aksaray
16’17” **Map** showing part of the Silk Road locating Sultan Han near Aksaray
16’26” Sultan Han, Aksaray
17’46” Muqarnas reconstruction of Sultan Han, Aksaray
18’20” **Map** showing part of the Silk Road locating Sadeddin Han near Aksaray
18’29” Sadeddin Han, Aksaray
19’48” **Map** showing part of the Silk Road locating Konya
19’57” Serefeddin Camii, Konya
21’02” Karatay Madrasah, Konya
21’35” Selmiye Camii, Konya
22’15” **Map** showing part of the Silk Road locating Ankara
22’24” Inside of Arslanhane Camii (Ahi Serafettin Camii), Ankara
23’10” Portal outside of Arslanhane Camii, Ankara
23’33” Reconstruction and animation of the portal

Rolling title

24’16” The End
24’20” Rolling title with names of all contributors

1.2 Summary of animations and computer reconstructions

For a better search of the animation here is a listing of the starting times for the reconstructions in order of their appearance:

- 2'48" Muqarnas reconstruction of Sultan Han, Kayseri
- 4'21" Muqarnas reconstruction of Çifte Medresse, Kayseri
- 6'59" Reconstruction and animation of Sarı Han
- 10'44" Construction of a muqarnas plan, Alay Han, Aksaray
- 13'46" Muqarnas reconstruction of Ağzıkara Han, Aksaray
- 17'46" Muqarnas reconstruction of Sultan Han, Aksaray
- 23'33" Reconstruction and animation of the portal of Arslanhane Camii, Ankara

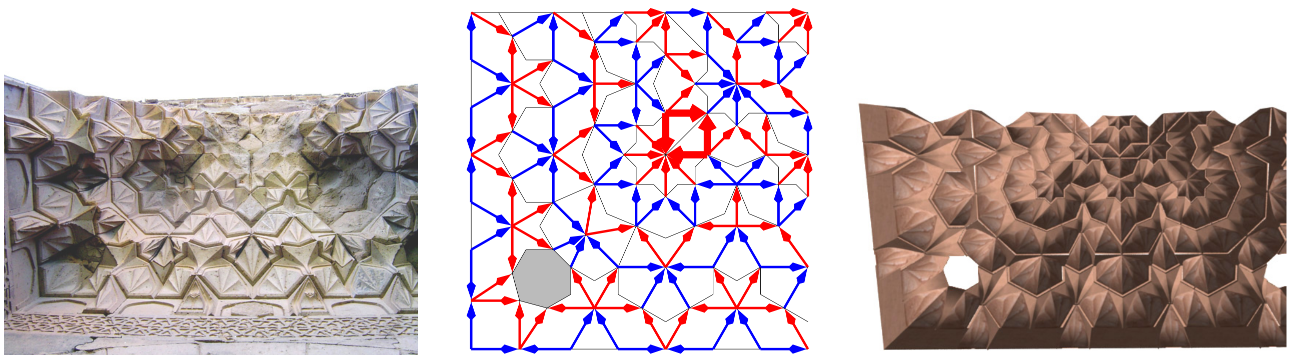

Figure 2. Portal to covered section at Sultan Han near Kayseri (1232-36), view from underneath.

2 Muqarnas: a geometrical ornamentation

Muqarnas, the Arabic word for stalactite vault, is a 3D architectural decoration composed of niche-like elements arranged in tiers. This ornamentation was developed around the tenth century and has a continuous tradition throughout various historical periods and regional highlights to the muqarnas designs still in use today [Yaghan 2001].

Most probably, muqarnas were designed as 2D projections on a ground plan. The first known example of a muqarnas design is a stucco plate found in the ruins of an Ilkhanid palace at Takht-i Sulayman (see [Harb 1978], [Dold, Harmsen 2005], [Harmsen 2006]). The geometric pattern incised on it shows the ground plan of a quarter muqarnas vault. This drawing comprises those elements described by the Timurid mathematician al-Kāshī (died 1429 A.D.). He gives the following definition [Dold 1992, Dold 1996]: *The muqarnas is a ceiling like a staircase with facets and a flat roof. Every facet intersects the adjacent one at either a right angle, or half a right angle, or their sum, or another combination* *of these two. The two facets can be thought of as standing on a plane parallel to the horizon. Above them is built either a flat surface, not parallel to the horizon, or two surfaces, either flat or curved, that constitute their roof. Both facets together with their roof are called one cell. Adjacent cells, which have their bases on one and the same surface parallel to the horizon, are called one tier.*

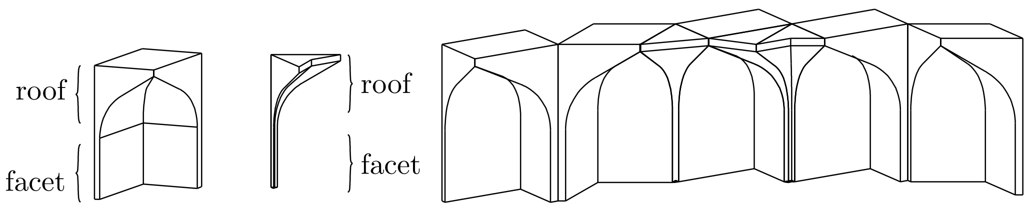

Figure 3. From left to right is shown a basic muqarnas cell, an intermediate element and how they can be combined in a tier.

A muqarnas consists of two kinds of elements of which one is a cell with expanded facets and the other one is a so-called intermediate element (fig. 3), connecting the roofs of two adjacent cells. As al-Kāshī explains in *Book IV* of his *Key of Arithmetic*, both kinds of elements stand on simple 2D geometrical figures. This means that the *plane projection* of an element, or the view from underneath, is one of the following: square; rhombus; half-rhombus; almond (deltoid) and its complement to a rhombus, a small biped; jug (a quarter octagon) and its complement to a square, a large biped; and barley-kernels (which only occur on the top tier of a muqarnas). Ilkhanid and Seljuk muqarnas differ in angles of the geometric elements. The older Seljuk style (1038-1194) shows a greater variety, it is less restricted in angles of the elements (see for example [Aslanapa 1971]). Secondly the facets have almost no height. In the film this is shown by shrinking the height of the facets in the Alay Han example down to zero. The texture of the reconstruction uses a scallop-like image, imitating the typical form of the roofs of Seljuk muqarnas elements (see fig. 8). The difference between algorithmic reconstruction and artistic freedom of Seljuk muqarnas is pointed out in detail in [Harmsen, Krömker 2006], following the chronology of their construction time.

3 Analyzing muqarnas structures

In order to convert a muqarnas plan into a 3D representation of the muqarnas, we need to know the position (in tiers), the orientation, and the type (cell or intermediate element) for each figure in the plan. To decide on its type, it is important to know which edges correspond to curved sides. If we know which sides are curved, we can decide on the central node and therefore the orientation of the figure. The direction of the curve in connection to its central node decodes the type of the element. Angle and diagonal can be calculated from the geometry of the plan. Having information about the curved sides then gives us the tier position of the element.

3.1 Decoding of muqarnas plans

For reconstructing the muqarnas from its 2D plan, we interpret the plan as a graph $P = P(N, E)$. That means we store the nodes N and edges E of the plan and not the polygons representing the elements. In general an edge corresponds to the sides of two elements, which are adjacent in the plane projection but do not necessarily lie on the same tier. In the left part of fig. 4 a cell and an intermediate element are drawn, with arcs pointing to the upper part of the curved side. In the plan P, the edges which are projections of curved sides appear as arcs and define a directed subgraph G of P (fig. 4, right part). If two arcs in G join in their endpoints these arcs define a cell, if they join in their initial points they may define an intermediate element. See also fig. 5 for examples of a plan P and its directed subgraph G.

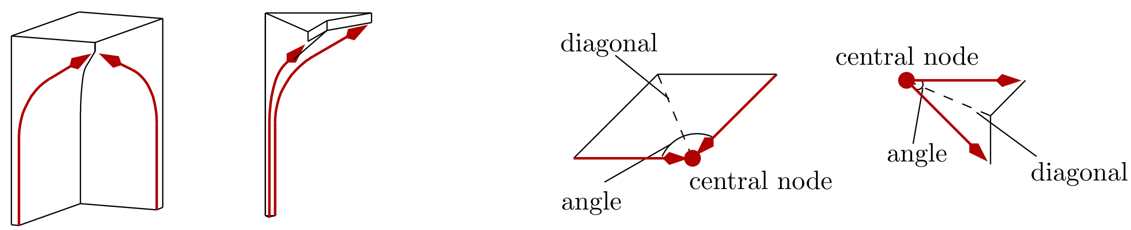

Figure 4. Cell and intermediate element with upward directions (left) and their 2D projection (right).

The height of the tiers is counted in integers. We remark that a node in the muqarnas plan can correspond to a vertex of several elements belonging to different tiers. We define the height $h(n)$ of a node $n \in N$ by the minimal height of the tier containing this vertex. Following the direction of an arc in the projection means going up to the next tier in the muqarnas. Hence the height difference between the initial point and the endpoint of an arc equals to one. The blue and red color of the arcs in the graphs alternate from tier to tier. In this way we can determine the heights for all nodes $n \in N$ belonging to G. Task is to find the directed graph from the plan and then reconstruct the virtual 3D muqarnas. All of our Seljuk examples are portals and we show only the left half of the symmetric plans. To determine the subgraph G from the plan P, we start with removing the edges which do not correspond to curved sides.

3.2 From 2D plans to directed subgraphs

The plan P (fig. 5, left) belongs to the portal of Arslanhane Camii in Ankara (fig. 10). In this plan we can recognize such edges not corresponding to curved sides. They divide a regular polygon of four edges, e.g. a parallelogram or a square, into two triangles or into two polygons of again four edges, e.g. a biped and a jug (see the plan in the middle of fig. 5). In 3D this division occurs whenever an intermediate element joins with a cell in the tier above. Therefore front sides of intermediate elements join to facets of cells and these dashed edges lie on the same plane. After removing these edges, we need to give a direction to each of the remaining edges (see right part of fig. 5). For a detailed explanation see [Harmsen 2006].

A muqarnas is oriented in upstairs direction. This gives us the following rules:

1. Arrows at the bottom boundary need to point inward.
2. Arrows at the center, need to point towards the center.

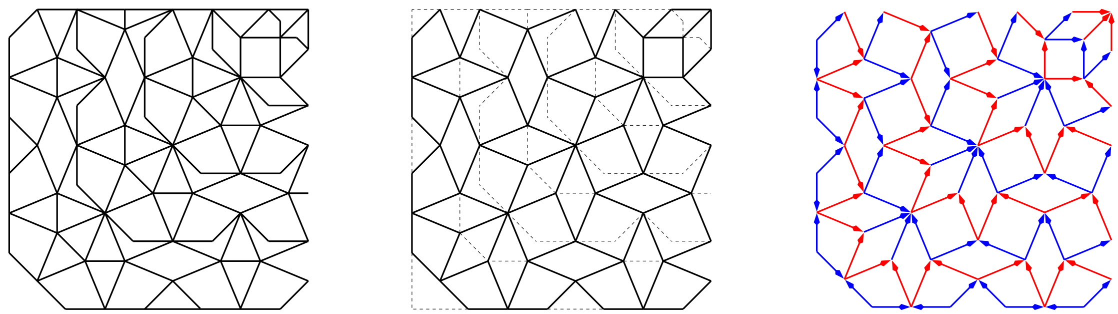

Figure 5. Left: The xfig input P. Middle: The automatically generated subgraph G with dotted edges which do not correspond to curved sides. Right: The directed subgraph G. (Example: Portal of Arslanhane Camii)

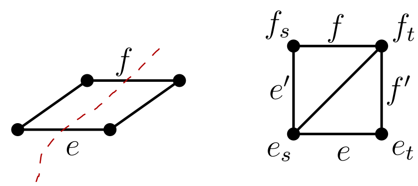

Figure 6. In the left part the edges e and f are opposite. The dashed line indicates the orbit to which they belong. Edges e and f in the right part are not opposite, because e_s joins to f_t.

By studying the possibilities of how the elements in Seljuk muqarnas can join, we conclude that edges $e, f \in G$ which are opposite in P need to have the same direction. By *opposite* we mean: If $e = \{e_s, e_t\}$ and $f = \{f_s, f_t\}$ are edges with initial point e_s resp. f_s and endpoint e_t resp. f_t such that there exist edges $e' = \{e_s, f_s\}$, $f' = \{e_t, f_t\}$ but no cross edge $\{e_s, f_t\}$ or $\{f_s, e_t\}$, then we say that e and f are opposite edges in P (fig. 6). We bind opposite edges together in one orbit and give the same direction to all edges of an orbit, which results in a decrease of the number of decisions to be made.
The decision on the direction of the remaining orbits can be done interactively (see section 4). Different directions of these orbits result in different muqarnas reconstructions. From the resulting set of mathematically possible muqarnas, and without a realization at hand, an art historian may choose which reconstruction is more likely in the context of the time period and region the muqarnas was built.

4 Automated reconstruction: From 2D plans to 3D ornaments

Our computer program combines the algorithm of plan analysis and muqarnas reconstruction in a complex tool, refered to as *Muqarnas Tool*. It comprises a plan editor and a viewer both based on *OpenGL* with the help of GLUT and GLUI libraries only. The plan editor is able to read, create and modify 2D plans based on the xfig file format (extension *.fig) using only basic parts of the xfig language. It generates regular xfig data. The viewer then can be used to visualize *.wrl data generated by the plan editor. Textures in either *.ppm raw data or JPEG data format can be used on the surface.

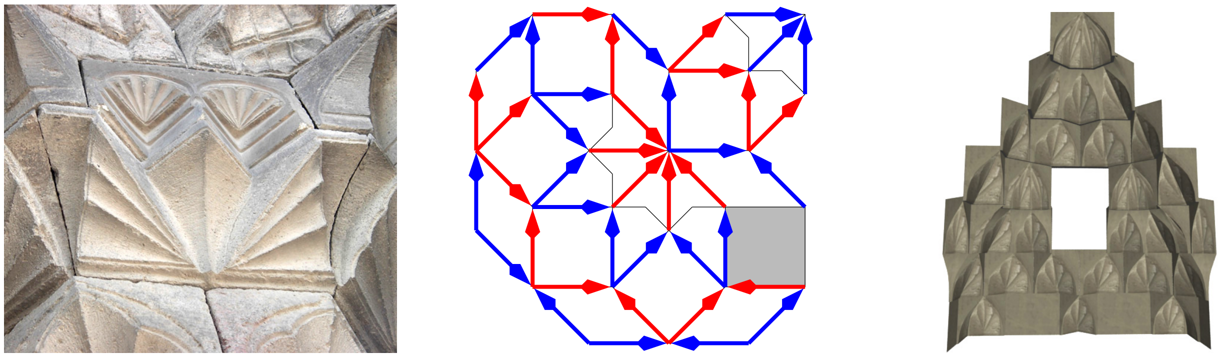

Figure 7. Entrance portal of the Çifte Medresse in Kayseri (1201). The picture on the left shows a central element involving two tiers. Half of this element corresponds to the gray colored square in the directed subgraph showing only half of the symmetric niche (middle picture). No arcs can be given to this element, and the automated reconstruction then fails at this location, showing a hole instead of an element.

4.1 Manual adjustment of the open directions

Creating new edges, moving the edges or just a node, and deleting can be done by simply selecting these edges or nodes by mouse click. The plan editor then calls the `PlanToGraph` program yielding a directed graph due to the rules defined in subsection 3.2. This must not be unique such that some of the edges will not become an arc. These missing directions can be set manually resulting in different muqarnas designs.

4.2 Reconstruction of muqarnas from the graph

The program `GraphToMuq` uses the information of the plan P and the graph G to generate VRML data representing the muqarnas. This program iterates over the nodes of the directed subgraph G and finds sets of arcs with same endpoints. If two arcs join in their endpoints, a cell is placed. The information of the plan P is used to find a possible fourth node of the cell. In the next step, the place left between arcs joining in their initial points is filled with intermediate elements.

4.3 Viewer output

The VRML data can be used in any VRML viewer and within the integrated viewer. This offers the opportunity of a quick judgment and possibly a revision of the decisions made so far. Besides, the output is suitable to be used on websites or with programs importing VRML format as a 3D geometric model. For rendering high-quality still images a `Renderman` output is generated and RIB-files can be saved for later use with different `shaders`.

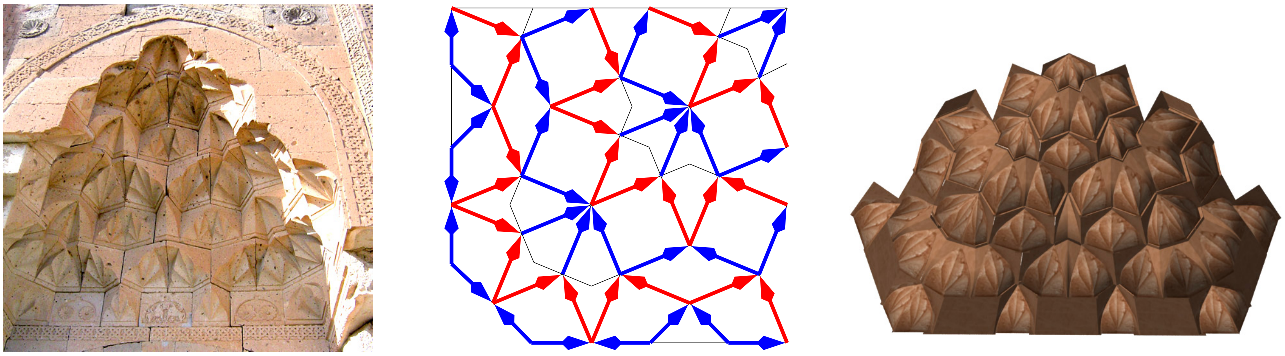

Figure 8. Portal of Alay Han near Aksaray (1180-1200).

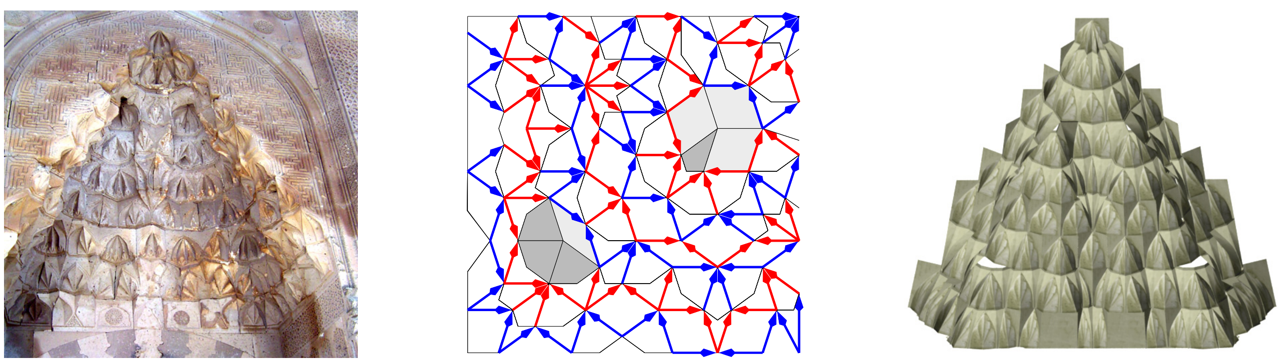

Figure 9. Entrance portal of Ağzıkara Han near Nevşehir (1242-43).

5 Reconstruction results

We have seen that a muqarnas design is a drawing which consists of polygons representing the plane projections of the muqarnas elements. The data used as test cases for Seljuk muqarnas are 2D projections of existing muqarnas ([Erdmann 1961], [Stierlin 2002]), thus providing comparisons for the results. As we already mentioned, Seljuk muqarnas have some specialties. They have scallop-like ornaments which we represent by textures, and the elements have almost no facets but consist of roofs only. They also show a greater variety of angles than e.g. the Ilkhanid muqarnas.

5.1 Selected examples of Seljuk portls

The portals of Alay Han (fig. 8) and of Arslanhane Camii (see figs. 5, 10) can be reconstructed automatically. In contrary the gray colored parts in the graphs of figs. 2, 7 and 9 show parts of the

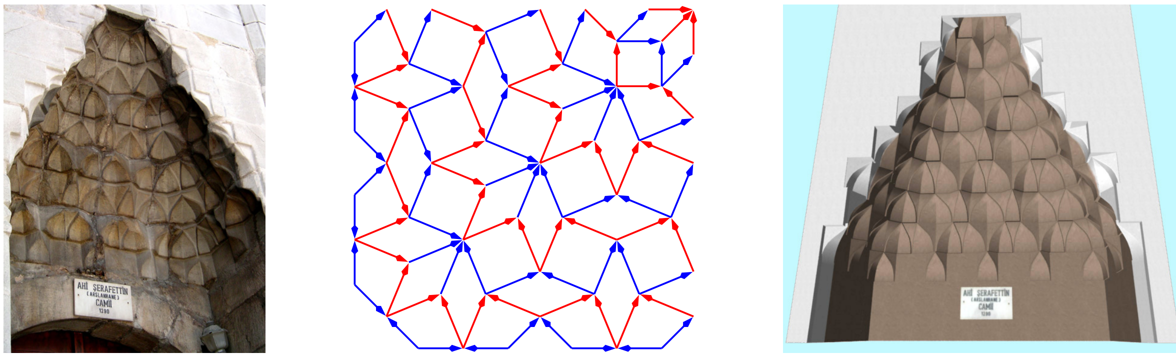

Figure 10. Portal of Arslanhane Camii in Ankara (1290) and its reconstruction (Renderman output).

muqarnas which cannot be filled with cells or intermediate elements. By applying our reconstruction algorithm gaps appear at these places in our virtually reconstructed muqarnas. This is because these polygons do not have two arcs joining in their endpoints (cell) or two arcs joining in their initial points (intermediate element). In the real muqarnas these parts are filled with ornamented stones. In the graph of the inner portal of Sultan Han near Kayseri (fig. 2) the gray part corresponds to a flat ornamented surface at the roof of the second tier. In the Çifte Medresse a stone involving two tiers is placed (see left picture in fig. 7). It has a (double) square as projection. For the edges of this element, the rules on constructions of muqarnas are contradictory, such that no arcs can be indicated here. The automated reconstruction then fails at this location, showing a hole instead of an element (see the reconstruction on the right in fig. 7). The problem is solved in the real muqarnas with an unusual element in this central part. In the entrance portal of the Ağzıkara Han (fig. 9) the gray areas are filled with a complex muqarnas-like structure. The lighter gray areas are influenced by the non-canonical parts.

5.2 Conclusions

Algorithmic reconstruction helps in understanding the rules and their stylistic specialties by which an architect or an artisan designs a muqarnas. Since these rules are not written but communicated from a master to his apprentice, a mathematical treatment can formalize the common rules and filter the artistic freedom. The algorithm is capable of calculating muqarnas reconstructions according to a given plane projection of a muqarnas. In this sense our tool can serve with analyzing styles and finding out about their differences.

Acknowledgment

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