

CHAPTER ONE

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**Definitions, Traditions,
and a General Framework
for Understanding
Complex Problem Solving**

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INTRODUCTION

Many of our daily activities involve problem solving of some sort. For example, we decide what to wear in the morning, which route to take to get to our office, which job-related duties to perform in which sequence once we arrive at our office, what to have for lunch, and so on. Of course, not all problem solving is alike. There are problems that can be solved with a few mental steps, and there are problems that require extensive thinking. There are problems that we have never encountered before, and there are problems we are familiar with. There are problems that have very clear goals, and there are problems where the goals are far from clear. Problems, then, can be distinguished on any number of meaningful dimensions, and the solution processes, the mental steps we engage in when solving a problem, may differ widely for different types of problems.

Given the multidimensionality of a problem, it may not come as a surprise to discover that different researchers, all claiming to study the phenomenon of problem solving, have on more than one occasion wholeheartedly disagreed with each other's conclusions. For example, many of those studying expert problem solving have maintained that experts typically use a forward-working (from the givens to the goal) approach to problem solving, whereas others have argued that experts work backward (from the goal to the givens). This apparent contradiction can be resolved if one considers the type of task that has been studied by the different researchers. It turns out that those claiming that experts prefer a forward-working approach have used problems that their experts were relatively familiar with, whereas those claiming the opposite tended to use tasks that were relatively novel for their experts (Smith, 1991). Thus, any general conclusion regarding expert problem solving, and indeed any conclusion regarding problem solving in general, can only be meaningful if we can all agree on what constitutes a problem and what constitutes problem solving.

In the first section of this introductory chapter, therefore, we present definitions of the terms *problem* and *problem solving* that have been offered in the past, and discuss why these definitions differ. In the second section, we discuss the main differences between the current North American and European mainstream approaches to studying problem solving, and argue that the differences between the two approaches are at least partially due to differences in how problem solving is defined. Finally, in the third section, we discuss how selecting a definition of problem solving constrains a theory of problem solving, and describe how a general theoretical framework for understanding problem solving that is based on the definition adopted within the European tradition, might look like.

DEFINITIONS OF PROBLEM SOLVING

"When I use a word," Humpty Dumpty said in a rather scornful tone, "it means just what I choose it to mean—neither more nor less." (Lewis Carroll, 1935, p. 119)

According to Webster's New Twentieth Century Dictionary (1983), a definition is "an explanation or statement of what a word or word phrase means or has meant." In the first section of this chapter, we present and compare various statements of the meaning of problem solving that have been offered in the past. We ask why we need an explicit definition of problem solving at all, and discuss why existing definitions differ. Finally, we present our thoughts on whether we can ever agree on a general definition of problem solving.

Explicit Definitions

Researchers in the area of problem solving have long been troubled by the absence of agreement on the exact meaning of many of the basic terms used (e.g., Smith, 1991). Among these basic terms are *expert*, *novice*, *heuristic*, *problem*, and even *problem solving* itself. Consider, for example, some of the better known definitions of problem solving that have been offered in the past:

- Problem solving is defined as any goal-directed sequence of cognitive operations. (Anderson, 1980, p. 257)
- . . . problem solving is defined here as a goal-directed sequence of cognitive and affective operations as well as behavioral responses for the purpose of adapting to internal or external demands or challenges. (Heppner & Krauskopf, 1987, p. 375)
- What you do, when you don't know what to do. (Wheatley, 1984, p. 1)

These definitions are examples of literally dozens and dozens of definitions that continue to be offered in the literature. Most of the definitions that one encounters in the literature differ primarily on three dimensions. First, they differ in terms of their *semantic content*, that is, in which actions and thoughts are classified as problem solving. To take two examples from the aforementioned, affectively coping with the loss of a close relative, for instance, would be considered problem solving by Heppner and Krauskopf, but would not be considered problem solving by Anderson. Second, the definitions differ in how fuzzy their *boundaries* are. The boundary of Anderson's definition

is clearly more precise, or less fuzzy, than the boundaries of the definitions presented by Heppner and Krauskopf or Wheatley. What exactly is meant, for instance, by Heppner and Krauskopf's "internal or external demands or challenges?" And what exactly does Wheatley's "what you do" include and exclude? And finally, the definitions differ in terms of their *category size*, that is, in how many events are classified as problem solving.

If we find it difficult to define *problem solving*, then perhaps we can at least agree on a definition of the more basic term *problem*. The following are some of the commonly cited definitions of *problem*:

- A problem exists when the goal that is sought is not directly attainable by the performance of a simple act available in the animal's repertoire; the solution calls for either a novel action or a new integration of available actions. (Thorndike, 1898, cited by Sheerer, 1963, p. 118)
- A problem occurs . . . if an organism follows a goal and does not know how to reach it. Whenever a given state cannot be converted into the desired state directly through the activation of obvious operations, thinking is needed to construct mediating actions. (Duncker, 1935, p. 1; translated by the authors)
- A question for which there is at the moment no answer is a problem. (Skinner, 1966, p. 225)
- A problem is a stimulus situation for which an organism does not have a ready response. (Davis, 1973, p. 12)
- A problem is a "stimulus situation for which an organism does not have a response," . . . a problem arises "when the individual cannot immediately and effectively respond to the situation." (Woods, Crow, Hoffman, & Wright, 1985, p. 1)
- A person is confronted with a problem when he wants something and does not know immediately what series of actions he can perform to get it. (Newell & Simon, 1972, p. 72)
- Whenever there is a gap between where you are now and where you want to be, and you don't know how to find a way to cross that gap, you have a problem. (Hayes, 1980, p. i)

As in the case of problem solving, it should be readily apparent that the one and only, universally accepted definition of what constitutes a problem does not exist. Most of the definitions in the literature appear to differ again on primarily three dimensions: (a) their semantic content, or more precisely, a focus on either the absence of a task-relevant *response* or the absence of a task-relevant *thought* that would lead to a solution for the task at hand, (b) the fuzziness of their boundaries, and (c) their category size, that is, in how many tasks are classified as problems.

For example, the definitions provided by Thorndike, Davis, and Woods et al., respectively, focus on the absence of an observable response, whereas most of the remaining definitions focus on the absence of a nonobservable thought or cognition. Thus, a known, yet complicated mathematical equation that requires hours to be solved and thus may not lead to an observable response for a long time, may be classified as a problem according to some definitions, but may not constitute a problem according to other definitions.

On the other hand, however, most, if not all, of the definitions just given do appear to share an important component, namely a focus on the *distance* between the task and the solver, rather than a focus on the nature of the task itself. That is, a problem is said to exist only if there is a gap between task and solver, or a barrier between the state given in the actual situation and the goal state in the head of the problem solver. A problem is not defined by task features, but rather by the interaction between task requirements and solver, that is, by the interaction between task characteristics and person characteristics. In general, gap definitions imply that the same task may constitute a problem for one solver, but not for another, whereas task definitions assume that a given task either constitutes, or does not constitute, a problem for all solvers.

Explicit Versus Implicit Definitions

The impression that all definitions of *problem* share a common aspect, namely a focus on the task-person interaction, quickly disappears, however, when we consider implicit, in addition to, explicit definitions of a problem. By an explicit definition, we mean a definition that is articulated in writing, or is at least stated verbally. Thus, all of the examples given previously constitute explicit definitions of problem and problem solving, respectively. By an implicit definition, in contrast, we mean an operational definition that one uses in one's research. Ideally, the implicit definition is accompanied by, and consistent with, an explicit definition. However, frequently the implicit definition is the only definition a researcher uses, and worse, on occasion, the implicit definition used may not be consistent with the explicit definition that one subscribes to.

By their very nature, implicit definitions are hidden and can be discovered only if one carefully examines the details of the research performed under the heading of problem solving. For instance, historically, much of what is generally considered problem-solving research has been concerned with subjects' performances on classroom tasks that are well-structured (e.g., "what does 15×16 equal?"). For these tasks, subjects typically do not immediately know the correct answer, although they know how to get the answers, that is, how to solve the task. Such tasks would not be considered

problems according to any of the explicit definitions previously discussed. However, by using these tasks in what is labeled as *problem-solving research*, one implicitly defines *problem* in terms of task characteristics.

Thus, explicitly, problems have been defined in terms of the interaction between task and individual; implicitly, however, problems frequently have been defined in terms of their task properties. If we accept the explicit definitions of *problem solving*, then research that has used exerciselike tasks cannot be relevant to our understanding of problem solving. The extensive literature on routine versus non-routine problem solving (Davis, 1985), for instance, would not belong to the area of problem solving. If we accept the implicit definitions, in contrast, then any theoretical treatment of problem solving that is based on an explicit definition cannot adequately cover our empirical findings (except in the unlikely case that both explicit and implicit definitions are identical in scope and content). For instance, much of what we know about the performance of novices on exercises and most of what we know about expertise would not accurately be included in problem-solving theory because most of the work to date has compared the performance of novices and experts on the solution of tasks that can only be considered exercises for the experts.

To summarize, definitions of *problem solving* and *problem*, both explicit and implicit, differ widely. Some definitions focus on the interaction between task characteristics and observable behavior, other definitions focus on the interaction between task characteristics and nonobservable cognitions, whereas still other definitions focus on task characteristics alone. In addition, definitions differ in their category size and in how precise, or how fuzzy, their boundaries are.

Why Do Definitions Differ?

Why is it that psychologists do not seem to be able to agree on definitions of *problem* and *problem solving*? After all, physicists appear to agree on the meaning of an atom, and mathematicians seem to have no quarrel with the meaning of an equation. But then, is it even necessary to subscribe to a common definition? Why not simply accept a variety of different definitions? Why not interpret a researcher's findings simply on the basis of the, explicit or implicit, definition provided by the researcher?

We believe that in order to meaningfully discuss these questions, it is helpful to distinguish between two aspects of a definition: (a) its purpose, and (b) its perceived usefulness. The primary purpose of a definition, we submit, is to facilitate accurate communication among all people who use the defined term. Whether or not a definition meets its purpose is directly related, for example, to how clearly the definition distinguishes between

aspects that are to be included and those that are to be excluded. In this regard, the better defined the boundaries of a term, that is, the less fuzzy the boundaries, the better a definition meets its purpose. For example, Anderson's (1980) definition of *problem solving* contains clearer boundaries than Wheatley's (1984) definition. Therefore, Anderson's definition is a better definition than Wheatley's if one applies the purpose criterion.

However, the primary reason for why one particular definition is favored over another one, we argue, is not that the one definition meets the purpose criterion and the other one does not. Rather, the reason for why one particular definition is favored has to do with what we call the *perceived usefulness of definitions*. By usefulness, at least in the context of theoretical and empirical psychological research, we mean the extent to which a definition allows meaningful generalizations to be drawn. An advantage on the purpose criterion does not automatically translate into an advantage on the usefulness criterion. Consider the following example. Assume *problem* is very narrowly defined such that it includes only addition problems (e.g., $2 + 1$). Further assume that we find empirically that second graders perform worse on these problems than third graders (hardly a surprise here). Note that although our narrow definition would score high on purpose because it very clearly specifies which tasks are considered problems and which ones are not, the definition would not allow us to generalize our findings to tasks other than simple addition problems. Thus, the usefulness of our definition would be severely limited. In general, the usefulness of a definition varies with the number of instances that are covered by the definition and over which meaningful generalizations are possible.

The Perceived Usefulness of a Definition

We argue that, again in the context of psychological research, meeting the purpose criterion is a prerequisite of a useful definition. Before a definition can be useful, one needs to be clear about what its boundaries are. Once the purpose criterion has been met, however, that is, once a definition has been formulated that has precise boundaries, what matters is the definition's usefulness. We further argue that the usefulness of a definition is not an objective property of a definition in the sense that a given definition is judged as more or less useful by everyone in the field. Rather, researchers differ in how useful they judge a definition. We believe that the perceived usefulness of definitions, on the one hand, is a consequence of a researcher's prior knowledge, beliefs, and theoretical goals, and, on the other hand, is primarily responsible for the variety of definitions we observe in most areas of psychology, including the area of problem solving.

The Role of Theoretical Goals

Hunt (1991) recently distinguished among three different goals that one may have in mind when studying problem solving, namely the extraction of scientific, engineering, or humanistic explanations. When the goal is to extract scientific explanations of problem solving, the focus is on the observation of empirical laws and the formulation of simple, but general principles from which the empirical laws can be deduced. Questions of interest for those interested in extracting scientific explanations of problem solving, are, for example, "Which cognitive processes are performed in which sequence to arrive at the solution of a problem," or "How are solutions to previous problems stored in our memory?"

When the goal of studying problem solving is to extract engineering explanations, in contrast, one is primarily interested in generating instructions that are useful for solving a particular set of problems. Thus, potentially interesting questions would be, "Does positive feedback facilitate problem solving?" or "How can we best structure the problem-solving context such that the probability of successful problem solving is optimized?" In general, scientific and engineering explanations differ in that scientific explanations search for general principles explaining the process of problem solving, whereas engineering explanations search for principles that increase the likelihood of success in problem solving. Although engineering principles might be derived from scientific principles, they need not be. In fact, we know a good deal about how to structure the environment such that success at problem solving is increased, without knowing why our methods work.

Humanistic explanations, finally, are personal interpretations of events such that the events make sense to us. Thus, humanistic explanations of the events described in a piece of art differ widely across interpreters, as do humanistic explanations of why it is that our paper was rejected by a prestigious journal, to use an example that is closer to home.

How is the perceived usefulness of a definition affected by theoretical goals? We argue that a definition is perceived as useful only if the definition is consistent with one's theoretical goals. If one's goal is to establish a scientific explanation, for example, then a definition of problem solving focusing on the cognitive operations by which a goal is accomplished (e.g., Anderson, 1980) is perceived as useful. If, in contrast, one's goal is to extract an engineering explanation, then any useful definition will probably need to focus on the interaction between problem solver and the environmental context that increases the likelihood of success. If one's goal is to generate a humanistic explanation, then a definition of problem solving may need to focus on the achievement of a subjectively meaningful interpretation of a situation to be perceived as useful.

In summary, what one wants to know about problem solving, that is the theoretical goals, determines to a certain extent whether or not one perceives a definition of problem solving as useful. Theoretical goals that one may have

are not limited to the ones mentioned previously, of course. For instance, if one is interested in a scientific explanation of problem solving that considers the neurophysiological substrates of problem solving, then a definition focusing on the cognitive processes would be rather meaningless. Instead, one would probably want to define problem solving in terms of the neurophysiological processes that occur while one engages in a certain type of behavior.

The Role of Beliefs

The perceived usefulness of a definition is affected by one's prior beliefs just as much as it is affected by one's theoretical goals. With prior beliefs, in this context, we mean primarily beliefs that determine how one goes about generating scientific, engineering, or humanistic explanations of problem solving. The effect of prior beliefs may be most evident in scientific explanations. Researchers differ, for instance, in the extent to which their theorizing considers the behavioral, cognitive, or neurophysiological level. If one holds the belief, for example, that theories should be stated primarily in terms of observable behavior, then a definition of problem solving in terms of cognitive operations would not be perceived as useful (see definitions by Davis, 1973; Skinner, 1966; Thorndike, 1898; Woods et al., 1985, previously mentioned). If, in contrast, one holds the belief that psychologically meaningful theories should be formulated at a neurophysiological level, then definitions in terms of both observable behavior and cognitive operations would be perceived as meaningless.

One may argue that our discussion of the effects of prior beliefs is obsolete, given our discussion of theoretical goals because different prior beliefs lead to different theoretical goals. That is, if one's prior belief is that humans can best be understood only at a behavioral level, for instance, then one's theoretical goal will be to formulate scientific explanations at a behavioral level. Thus, the effect of prior beliefs on accepting a definition as useful would be an indirect one, mediated, so to speak, by theoretical goals. Although this may indeed frequently be the case, we argue that it need not always be the case. Different prior beliefs may lead to the same theoretical goals, and the same prior belief may still lead to the formulation of different theoretical goals. For example, even though some researchers may disagree on the proper level at which theorizing should occur, they may agree on using the experimental method for generating scientific laws. Conversely, even though researchers may agree on what constitutes the proper level of theorizing, they may yet disagree on how to extract scientific knowledge.

The Role of Prior Knowledge

Definitions are not static concepts that, for a given set of prior beliefs and theoretical goals, remain unchanged over time. Rather, definitions are very dynamic concepts that change as new knowledge becomes available

To take a decidedly nonproblem-solving example, today's definitions of *music* would normally include what is labeled as rock and roll. But imagine a person who never heard this sort of music before; very likely, the definition of this person would label rock and roll as noise. Similarly, definitions of *art* would change dependent on a person's prior knowledge of Pablo Picasso's work. Prior knowledge thus places constraints on what is perceived as a meaningful definition.

This general principle has been rather obvious in the area of problem solving as well. Not too long ago, it was thought that any problem solving might be accomplished by a limited set of procedures. That is, it was argued that a limited number of general algorithms—sequences of mental steps that are guaranteed to arrive at a solution if a solution exists—might suffice to solve any problem of any kind (Groner, Groner, & Bischof, 1983). Groner et al., for instance, described how Raimundus Lullus, a Spanish philosopher in the 13th century, searched for a general algorithm that would produce every truth. Both Descartes (Adam & Tannery, 1908) and Leibniz (Gerhardt, 1880) also seemed to have believed that algorithms could be found that would solve at least all mathematical problems, perhaps even philosophical problems. Gödel (1931), eventually, put to rest the search for an universal algorithm by showing that certain mathematical problems could not be solved by any algorithm.

More recently, researchers have been searching for heuristic methods, rather than algorithms, that would be capable of solving problems of any kind (e.g., Duncker, 1935; Newell, Shaw, & Simon, 1958; Polya, 1945). Heuristics are general problem-solving procedures that do not guarantee that a solution is found if it exists. Heuristics merely increase the probability that a solution is found. Work on artificial intelligence, however, has more and more led to the conclusion that general heuristics are typically too general to be helpful for any but novel problems (e.g., Newell, 1983). Thus, the prevailing opinion at this time is that many problems cannot be solved by utilizing a small number of domain-general heuristics, but rather can only be solved by using domain-specific knowledge and procedures.

Can We Ever Agree on a Universal Definition of Problem Solving?

It follows from our discussion that different researchers will agree on a definition only if the definition is perceived as useful by all of them. Whether or not the definition is perceived as useful, in turn, is constrained by the prior beliefs, knowledge, and goals of the researchers. This implies that we can agree on a definition of problem solving only to the extent that we can agree on the constraints that are to be placed on the formulation of the definition.

Some constraints, we believe, are more readily acceptable to most researchers than are others. For instance, prior knowledge constraints are

relatively easy to accept because they are directly based on empirical research and are affected by opinion only to a limited degree. On the other hand, beliefs on how to best theorize about the human mind are probably tightly entrenched in researchers' thinking, and nearly impossible to change. Researchers' theoretical goals also differ, and might even change over time. Realistically, we cannot expect to find an universally accepted definition of problem solving. What we can expect to find, however, is a limited number of meaningful definitions that differ systematically in terms of the theoretical goals and beliefs that are held by their subscribers.

In summary, we distinguish between a definition's purpose and its perceived usefulness. The purpose of a definition is to facilitate accurate communication. A definition is perceived as useful if meaningful generalizations can be drawn across the cases, or instances, subsumed by the definition. We argue that generalizations are meaningful if they are consistent with what we (a) already know about the entity that is to be defined, and if they are consistent with, (b) our prior beliefs and, (c) our theoretical goals. We do not claim, of course, that these are the only variables affecting whether one perceives a definition as useful or not. Our general point is simply that different definitions exist because the perceived usefulness of a definition varies across people in a systematic way.

In the next section, we present definitions of problem solving that have been offered by the contributors to this volume, and discuss the theoretical goals that underlie these definitions. Our discussion serves three purposes. First, it serves as an illustration of our general point that different researchers adopt different definitions of problem solving because they differ in their theoretical goals and beliefs. Second, it allows us to clarify which topic the research described in this volume is exactly concerned with. Third, it allows us to compare the mainstream approach chosen by many European researchers with the mainstream approach chosen by many of their North American counterparts. To foreshadow our main point: We argue that the definitions adopted by many of the European and North American researchers differ, at least in emphasis, and that consequently the two areas of research are concerned with different phenomena that overlap only partially. The two approaches are thus best considered complementary.

DEFINITIONS OF PROBLEM SOLVING: THE EUROPEAN APPROACH

The Contributors' View

We asked all of the contributors to this volume to provide us with a short definition of *complex problem solving* (CPS) as they use the term in their chapter. Following are, in alphabetical order, their occasionally somewhat shortened and edited answers.

Beckmann & Gutbke. CPS represents a class of task demands the cognitive mastery of which calls for the recognition of causal relations among the variables of a system.

Berry. Deciding whether a task should be considered as being complex or not, seems to be a relative rather than an absolute issue. Some tasks seem to be complex when compared with many traditional experimental problem-solving tasks. In these cases, the large number of variables and their interconnectivity, the intransparency, the time lags, and the large number of goals to be met all contribute to task complexity.

Brehmer. I am concerned with peoples ability to handle tasks that are complex, dynamic (in the sense that their state changes both autonomously and as a consequence of the decision makers actions), and opaque (in the sense that the decision maker may not be able to directly see the task states or task structure).

Buchner. CPS is the successful interaction with task environments that are dynamic (i.e., change as a function of the user's interventions and/or as a function of time) and in which some, if not all, of the environment's regularities can only be revealed by successful exploration and integration of the information gained in that process.

Dörner. CPS concerns the behavior of people or groups of people in complex, dynamic, and intransparent situations where the exact structure and properties of the situations are relatively unknown. The complex problem solver permanently elaborates on her goals and constructs hypotheses about the (unknown) structure of the domain. He or she makes decisions and needs to control the results.

U. Funke. The main objective of applying paradigms of CPS research for personnel selection is to use more complex, meaningful, integrative, and realistic tasks requiring higher order thinking processes and skills. Personnel selection and training applications adopt research paradigms as a technology and lack a common definition of CPS.

Huber. CPS is the task of optimizing one or more target variables of a system by a series of decisions. The system consists of several variables, there are several alternative actions. Information about the system or system states is incomplete (e.g., not available or probabilistic) or delayed. A time component may be involved.

Kluwe. 1. Referring to the task environment, complex problems may be characterized by a large number of interrelated components (or variables); 2. Referring to the problem space, complex problems may be characterized

by a large number of different cognitive operations that are necessary for searching through the problem space (e.g., the number of steps of a program simulating solution search). 3. Complex problems, finally, may be decomposed into smaller subproblems.

Krems. A problem is called "complex" when goal-state and initial-state are clearly described, and when there is (a) no precise definition of the problem space (not complete) and/or (b) no precise definition of the operators available (what can be done). Both (a) and (b) depend on domain-specific features (e.g., the context, the number and connectivity of relevant variables) and on the level of expertise (amount of knowledge about the domain-specific features). In general, I use the term CPS as more or less equivalent to problem solving in semantically rich domains or in knowledge-rich tasks.

Although the definitions presented previously are all different from each other in some respects, they share some important characteristics. For example, all definitions describe problem solving as a cognitive, rather than behavioral or neurophysiological, activity. This should not surprise us, given that most researchers in this volume are interested in scientific or engineering explanations of problem solving. That is, they are interested in the mental steps underlying problem solving (e.g., Berry & Broadbent, this volume; Brehmer, this volume; Huber, this volume; J. Funke, this volume), in how problem-solving performance can be improved (e.g., Brehmer, this volume), or in how to best select people such that they can deal with known task demands (e.g., U. Funke, this volume). Furthermore, all researchers represented in this volume subscribe, at least implicitly, to the Information-Processing framework as the theoretical framework within which to formulate general psychological principles.

What may appear surprising, however, is the consistent lack of an emphasis on the interaction between problem and solver. That is, most definitions do not appear to be gap definitions; rather, most definitions define a problem in terms of the task specifications a solver faces. For example, Brehmer defines a problem as a task that is "complex, dynamic, . . . and opaque." Similarly, Kluwe states that a problem consists of a "large number of interrelated components." Berry refers to a relativistic point of view that, nevertheless, is restricted by task attributes. The emphasis on task features, rather than the task-solver interaction, is even more obvious when we consider implicit definitions of problem solving. For example, many of the contributors to this volume, and indeed most of those performing contemporary problem-solving research in Europe, use complex computerized scenarios as their problems. Because these problems are novel, subjects do not typically differ in terms of the background knowledge they bring to bear on the tasks. Thus, any

influence of prior knowledge is minimized, and the researchers' main interest is in how task characteristics affect problem solving.

The reason for the apparent emphasis in European research on task specifics, rather than on the task-solver interaction, is perhaps best understood historically. Therefore, a brief excursion into the recent history of problem-solving research is in order.

Historical Roots

Beginning with the early experimental work of the Gestaltists in Germany (e.g., Duncker, 1935), and continuing through the 1960s and early 1970s, research on problem solving was typically conducted with relatively simple, laboratory tasks (e.g., Duncker's X-ray problem; Ewert & Lambert's, 1932, disk problem, later known as Tower of Hanoi) that were novel to subjects (e.g., Mayer, 1992). Simple novel tasks were used for various reasons: They had clearly defined optimal solutions, they were solvable within a relatively short time frame, subjects' problem-solving steps could be traced, and so on. The underlying assumption was, of course, that simple tasks, such as the Tower of Hanoi, captured the main properties of real problems, and that the cognitive processes underlying subjects' solution attempts on simple problems were representative of the processes engaged in when solving "real" problems. Thus, simple problems were used for reasons of convenience, and generalizations to more complex problems were thought possible. Perhaps the best known and most impressive example of this line of research is the work by Newell and Simon (1972).

During the Gestaltists days and the 1960s and early 1970s, the prevailing definitions of problem solving at least implicitly contained three assumptions: (a) the theoretical goal was to understand the cognitive processes of a person solving a problem, (b) cognitive processes were guided by internal goals, and (c) perhaps most importantly, the cognitive processes were essentially the same for all kinds of problems. Problems were typically defined such that they represented situations for the person that could not be solved by the mere application of existing knowledge; thus, problems were typically domain-general, or knowledge-lean (e.g., VanLehn, 1989).

However, beginning in the 1970s, researchers became increasingly convinced that empirical findings and theoretical concepts derived from simple laboratory tasks were not generalizable to more complex, real-life problems. Even worse, it appeared that the processes underlying CPS in different domains were different from each other. These realizations have led to rather different responses in North America and Europe.

In North America, initiated by the work of Simon on learning by doing in semantically rich domains (e.g., Anzai & Simon, 1979; Bhaskar & Simon, 1977), researchers began to investigate problem solving separately in differ-

ent natural knowledge domains (e.g., physics, writing, chess playing) and gave up on their attempt to extract a global theory of problem solving (e.g., Sternberg & Frensch, 1991). Instead, these researchers frequently focus on the development of problem solving within a certain domain, that is on the development of expertise (e.g., Anderson, Boyle, & Reiser, 1985; Chase & Simon, 1973; Chi, Feltovich, & Glaser, 1981). Areas that have attracted rather intensive attention in North America include such diverse fields as reading (Stanovich & Cunningham, 1991), writing (Bryson, Bereiter, Scardamalia, & Joram, 1991), calculation (Sokol & McCloskey, 1991), political decision-making (Voss, Wolfe, Lawrence, & Engle, 1991), managerial problem-solving (Wagner, 1991), lawyers' reasoning (Amsel, Langer, & Loutzenhiser, 1991), mechanical problem-solving (Hegarty, 1991), problem solving in electronics (Lesgold & Lajoie, 1991), computer skills (Kay, 1991), and game playing (Frensch & Sternberg, 1991).

The European Situation

In Europe, two main approaches surfaced, one initiated by Broadbent (1977; see Berry & Broadbent, this volume) in Great Britain and the other one by Dörner (1975, 1985; see also Dörner, Drewes, & Reither, 1975; Dörner & Reither, 1978; see Dörner & Wearing, this volume) in Germany. The two approaches have in common an emphasis on relatively complex, semantically rich, computerized laboratory tasks that are constructed to be similar to real life problems. The approaches differ somewhat in their theoretical goals and methodology, however. The tradition initiated by Broadbent emphasizes the distinction between cognitive problem-solving processes that operate under awareness versus outside of awareness, and typically employs mathematically well-defined computerized systems. The tradition initiated by Dörner, on the other hand, is interested in the interplay of the cognitive, motivational, and social components of problem solving, and utilizes very complex computerized scenarios that contain up to 2,000 highly interconnected variables (e.g., Dörner, Kreuzig, Reither, & Stäudel's, 1983, Lohhausen project; Ringelband, Misiak, & Kluwe, 1990).

In summary, researchers' realization that problem-solving processes differ across knowledge domains and across levels of expertise and that, consequently, findings obtained in the laboratory cannot necessarily be generalized to problem-solving situations outside the laboratory, has during the past two decades, led to an emphasis on real world problem solving. This emphasis has been expressed quite differently in North America and Europe, however. Whereas North American research typically concentrates on studying problem solving in separate, natural knowledge domains, much of the European research focuses on novel, complex problems, and has been performed with computerized scenarios (see Funke, 1991, for an overview).

In essence, we argue that the current North American and European mainstream approaches to studying problem solving have adopted different definitions of problem solving. Much of the North American research compares the cognitive processes engaged in by experts and novices when faced with natural, knowledge-based tasks that constitute problems for novices and exercises for experts, and thus focuses **primarily** on learning. In contrast, much of the European work is conducted **with complex tasks** that are novel to all subjects. The emphasis here is on **the mental steps** underlying the solution process of novel and complex tasks.

An Integrated View

Following we offer our own definition of *CPS*, a definition that is firmly rooted in the European tradition and, in fact, incorporates many aspects of the definitions provided by the contributors to this volume (see previously discussed definitions). According to our definition,

CPS occurs to overcome barriers between a given state and a desired goal state by means of behavioral and/or cognitive, multistep activities. The given state, goal state, and barriers between given state and goal state are complex, change dynamically during problem solving, and are intransparent. The exact properties of the given state, goal state, and barriers are unknown to the solver at the outset. CPS implies the efficient interaction between a solver and the situational requirements of the task, and involves a solver's cognitive, emotional, personal, and social abilities and knowledge.

Notice the differences between our definition and the definitions that feature prominent in the North American tradition. Anderson (1980, see beginning of this chapter), for the North American approach for example, defined *problem solving* as "any goal-directed sequence of cognitive operations" (p. 257), regardless of whether the task is novel or familiar to the solver, regardless of whether or not the task is complex, and regardless of whether or not a single barrier or multiple barriers exist between given state and goal state. Our definition, in contrast, constrains potential problems by requiring that they be (a) novel tasks that subjects are unfamiliar with, (b) complex, (c) dynamically changing over time, and (d) intransparent. In order to solve these problems, a solver has to be able to anticipate what will happen over time, and has to consider side effects of potential actions.

Also, note that according to our definition CPS is not simply an extension of simple problem solving (SPS), that is, problem solving involving relatively simple laboratory problems. CPS and SPS are qualitatively different. For example, whereas in SPS typically a single barrier needs to be overcome, in CPS a large number of barriers exists. Because there are multiple barriers,

a single cognitive or behavioral activity may not be sufficient to reach the goal state. Rather, a well-planned, prioritized, set of cognitions and actions needs to be performed in order to get closer to the goal state.

In addition, note that in contrast to earlier, often implicit views, CPS is not viewed as deterministic in the sense that any problem-solving activity will always lead to the solution of a problem. Rather, CPS may lead to an approximate solution that may advance the solver but may not lead to actually solving the problem. For example, subjects performing the duties of the mayor of a computer-simulated town, may, even after some practice, still not be able to generate the best possible solution to a given problem. In fact, many, often computerized, tasks exist for which—due to the complex nonlinear relations among the task variables—the optimal solution is unknown. Of course, the absence of an optimal solution, while theoretically reasonable and even desirable, poses a problem to experimenters who want to determine the quality of subjects' performances, and to those who use microworlds for personnel selection purposes (see U. Funke, this volume).

Finally, because both the given state and goal state and also the barriers are intransparent in CPS, it is difficult for a solver to evaluate his or her progress toward problem solution. This makes it necessary for the solver to select and structure the interactions with the task such that information that is helpful for the evaluation of progress can be extracted.

Readers might want to keep our definition in mind when going through the following chapters. With a few exceptions (e.g., Krems, this volume), this definition or a somewhat modified version thereof, has been adopted by the contributors to this volume and by most of the European researchers in the area.

Summary

In summary, researchers adopt definitions that they perceive as useful, that is, that are consistent with their beliefs, knowledge, and theoretical goals. A definition can therefore be neither static nor commonly accepted. Rather, for any domain of research, a number of meaningful definitions coexist. In the area of problem-solving research, the current theoretical goals are quite different for North American and European researchers. The primary goal adopted by many North American researchers is to understand task performance and learning in natural knowledge domains. The primary goal adopted by many European researchers, in contrast, is to understand how people deal with complex, novel task situations. Because the theoretical goals differ for the two traditions, the definitions that have been adopted differ as well. Consequently, the two traditions are not concerned with the same phenomenon, and any comparison of research findings runs the risk of being meaningless.

A THEORETICAL FRAMEWORK FOR COMPLEX PROBLEM SOLVING

In the previous two sections, we argued that the choice of a definition is affected by a researcher's theoretical goals, beliefs, and prior knowledge, and that a definition, once selected, affects how a phenomenon is empirically approached. A definition not only affects how a subject is studied empirically, however; it also affects which theoretical explanations of a phenomenon are acceptable. At a general level, this point is rather obvious. After all, one needs to explain what one studies. In addition, however, there are many, much more subtle, interactions between a definition and a theory. For example, if problem solving is defined in terms of cognitive, rather than neurophysiological, biological, or behavioral, processes, then it makes little sense to construct a theory of problem solving at a neurophysiological, biological, or behavioral level. The theoretical level must thus match the level adopted in the definition. In general, just as the choice of a definition is affected by theoretical goals, beliefs, and prior knowledge, so is the choice of an acceptable theoretical framework.

In the following, we present our thoughts on how a general theoretical framework for understanding problem solving that is based on our definition of CPS might look like. Our framework is based on the assumptions that, (a) our theoretical goal is to understand the interplay among cognitive, motivational, personal, and social factors when complex, novel, dynamic, intransparent tasks are solved, and, (b) the interplay among the various components can best be understood within an Information-Processing model. The framework is constrained, of course, by what is known already about CPS as it is defined above. Following, we therefore present a brief, nonexhaustive list of the main empirical phenomena that have been demonstrated in recent years, thereby summarizing many of the findings presented in this volume.

Internal Subject Factors

Experience. CPS appears to vary with the amount of experience an individual has in the task domain at hand (e.g., Krems, this volume). Experience affects the likelihood of successful problem solving, but more importantly, it affects which strategies are employed. It influences, for instance, whether or not a person experiments with a task; that is, whether or not the person exhaustively tests the hypotheses about task relations and tries to falsify the assumptions.

Cognitive Variables. There is considerable evidence that cognitive variables, such as background knowledge, monitoring and evaluation strategies, and cognitive style affect CPS. There is even evidence indicating

that general intelligence, when measured appropriately, affects at least some aspects of CPS (e.g., Beckmann & Guthke, this volume). Also, it appears that at least under certain conditions, CPS performance and explicit task knowledge may be dissociable. That is, performance improvements can be found even in the absence of explicit knowledge about the task (e.g., Berry & Broadbent, this volume).

Noncognitive Variables. CPS appears to be enhanced by some noncognitive factors such as self-confidence, perseverance, motivation, and enjoyment. In general, both personality and social factors appear to influence CPS (e.g., Dörner & Wearing, this volume).

External Factors

Problem Structure. CPS appears to vary with the structure of the task including the semantics of the task, the complexity of the task, the transparency of the task, and so on (e.g., J. Funke, this volume).

Problem Context. The likelihood of successful CPS performance seems to vary with the semantic embeddedness of a task, that is, with whether or not the task is couched within a well-understood and familiar context (e.g., Huber, this volume).

Environmental Factors. Successful CPS performance is influenced by the environment within which a solver operates. This includes feedback and feedback delay, expectations, cooperation, peer pressure, and so on (e.g., Brehmer, this volume).

The Components of a Theory of CPS

These empirical findings have led us to construct a simple theoretical framework for understanding CPS that is depicted in Figure 1.1. The figure summarizes the basic components of our framework as well as the interrelations among the components. As can be seen, the framework contains three separate components: the problem solver, the task, and the environment.

Within the problem solver, we distinguish between static memory content and dynamic information processing. Memory is divided further into domain-general and domain-specific knowledge both of which affect CPS performance. Information processing includes the task strategies that are selected and the processes of task monitoring and progress evaluation. In addition, noncognitive problem-solver variables such as motivation and personality also factor into CPS performance.

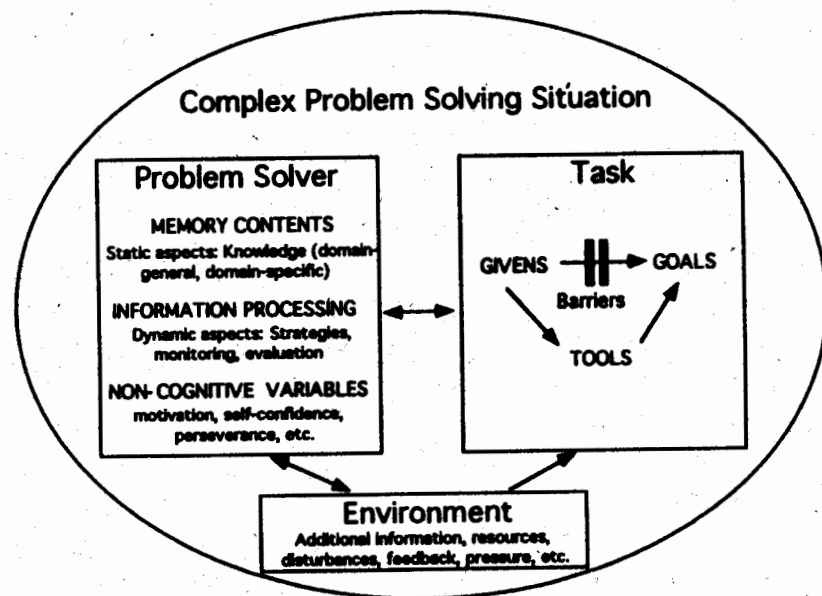


FIG. 1.1. CPS is viewed as the interaction between a problem solver and a task in the context of an environment. The figure shows only static aspects of the interaction. For additional information, see text.

The task itself is depicted in terms of the barriers that exist between a given state and a goal state (see our remarks on the gap definition). As explained previously, the barriers are assumed to be complex, dynamically changing, and intransparent; the transition from given to goal state is constrained by the problem solver's memory content and information processing, and by the tools that are available to the solver.

The environment includes the resources that are available for problem solving, as well as feedback, expectations, cooperation, peer pressure, disturbances, and so on. The environment affects both the problem solver and the task. It affects the problem solver by constraining the information processes that can be used and by influencing which knowledge is accessible. The environment affects the task by offering additional information, constraining which tools may be used, and so on. In addition, the environment can be changed actively by the problem solver but not by the task.

From this very simple view of CPS, it should become clear that two of the main questions that will need to be addressed by future research are as follows: Which components within the problem solver, task, and environment affect CPS in which way? How do the various components—the person, task, and environment—interact in affecting CPS performance? Clearly, much more research will need to be conducted before we can attempt to answer these questions.

SUMMARY

We argued that existing definitions of *problem solving* and *problem*, both explicit and implicit, differ widely. Some definitions focus on the interaction between task characteristics and observable behavior, other definitions focus on the interaction between task characteristics and nonobservable cognitions, whereas still other definitions focus on task characteristics alone. In addition, definitions differ in their category size and in how precise, or how fuzzy, their boundaries are.

We have made the distinction between a definition's purpose and its perceived usefulness. The purpose of a definition is to facilitate accurate communication. A definition is perceived as useful if meaningful generalizations can be drawn across the cases, or instances, subsumed by the definition. We have further argued that researchers adopt definitions that they perceive as useful, that is, that are consistent with their beliefs, knowledge, and theoretical goals. A definition can therefore be neither static nor commonly accepted. As an example, we stated that in the area of problem-solving research, the current theoretical goals are quite different for mainstream North American and European researchers. The primary goal adopted by many North American researchers is to understand task performance and learning in natural knowledge domains. The primary goal adopted by many European researchers, in contrast, is to understand how people deal with complex, novel task situations. Consequently, the definitions of problem solving adopted within the two approaches differ as well.

We have offered a definition of CPS that is consistent with the theoretical goal adopted by many of the European researchers, and, finally, have described our thoughts on how a theoretical framework of CPS that is based on this definition might look like.

In conclusion, we believe that research on CPS, despite its many shortcomings and despite the diverse approaches taken by North American and European researchers, carries the promise of a more realistic, real life approach to the psychology of action control within complex environments than has been offered in the past. This is a new and exciting area of research, and as the old saying goes, "Nothing is as promising as a beginning."

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